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# Topological Fault-Tolerance And Cluster State Quantum Computation, Optical Quantum Computation, Separability Of Very Noisy Mixed States, Supercausality And Consciousness, Electrical Conductivity Of Microtubules, Neuronal Microtubules, Dynamical Localization In Atomic Momentum Transfer, Quantum Chaos With Quantum Tomography, Quantum Fidelity Amplitude, Thermodynamics Of Quantum Transport, Regularity And Chaos, Mind Influences Brain Through Proactive Spin, Dynamics In Large-Scale Functional Brain Networks, Advanced Waves And Syntropy, Bose-Einstein Condensates And Other Representations, Colloquiums And Editorials Of Similar Nature And Interest: Sprachgefuhl – Sprachwirklich- Einzelsprachlich: Group Studies: Zitterbewegung-Heiligenschein-Bremsstrahlung Models: Bright Halo , Trembling Motion, Braking Radiation Models

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**Evolution Edward Farhi, Jeffrey Goldstone, Sam Gutmann, and Michael Sipser (Submitted on 28 Jan 2000) STOP** A significant development in computing has been the discovery that the computational power of (e) quantum computers exceeds (e) that of Turing machines. Central to the experimental realization of quantum information processing is the construction of (e) fault-tolerant quantum logic gates. Their operation requires (e) conditional quantum dynamics, in which one sub-system undergoes (e&eb) a coherent evolution that depends on (=) the quantum state of another sub-system2; in particular, the evolving sub-system may acquire a conditional phase shift. Although conventionally dynamic in origin, phase shifts can also be (=) geometric3, 4. Conditional geometric (or 'Berry') phases depend only on (=) the geometry of the path executed, and are therefore resilient to (e) certain types of errors; this suggests the possibility of an intrinsically fault-tolerant way of (e) performing quantum gate operations. Nuclear magnetic resonance techniques have already been used to (e) demonstrate both simple quantum information processing5, 6, 7, 8, 9 and geometric phase shifts10, 11, 12. Here **Jonathan A. Jones1,2, Vlatko Vedral1, Artur Ekert1 & Giuseppe Castagnoli3** combine these ideas by performing (e&eb) a nuclear magnetic resonance experiment in which a conditional Berry phase is implemented, demonstrating (eb) a controlled phase shift gate. **Nature 403, 869-871 (24 February 2000) | doi:10.1038/35002528; Received 22 July 1999; Accepted 14 December 1999 Geometric quantum computation using nuclear magnetic resonance Jonathan A. Jones1,2, Vlatko Vedral1, Artur Ekert1 & Giuseppe Castagnoli3 STOP** Besides the traditional circuit-based model of quantum computation, several quantum algorithms based on (e) a continuous-time Hamiltonian evolution have recently been introduced, including (e) for instance continuous-time quantum walk algorithms as well as (e&eb) adiabatic quantum algorithms. Unfortunately, very little is known today about the behavior of these Hamiltonian algorithms in (eb) the presence of noise. Here, authors perform a fully analytical study of the resistance to (e) noise of these algorithms using perturbation theory combined with (e&eb) a theoretical noise model based on (e) random matrices drawn from (e) the Gaussian orthogonal ensemble, whose elements vary (e&eb) in time and form (eb) a stationary random process. **http://dx.doi.org/10.1103/PhysRevA.71.032330© 2011 American Physical Society Noise resistance of adiabatic quantum computation using random matrix theory Jérémie Roland and Nicolas J. Cerf Phys. Rev. A 71, 032330 – Published 21 March 2005 STOP** It is shown that the "geometrical phase factor" recently found by Berry in his study of the quantum adiabatic theorem is precisely the holonomy in (eb) a Hermitian line bundle since (e) the adiabatic theorem naturally defines (eb) a connection in such a bundle. This not only takes the mystery out of (e) Berry's phase factor and provides (eb) calculational simple formulas, but makes a connection between Berry's work and (e&eb) that of Thouless et al. This connection allows the author to use Berry's ideas to interpret the integers of Thouless et al. in terms of (e&eb) eigenvalue degeneracies. **http://dx.doi.org/10.1103/PhysRevLett.51.2167© 2011 American Physical Society Holonomy, the Quantum Adiabatic Theorem, and Berry's Phase Barry Simon Phys. Rev. Lett 51, 2167 – Published 12 December 1983 STOP** This Colloquium gives an overview of recent theoretical and experimental progress in (eb) the area of nonequilibrium dynamics of (e) isolated quantum systems. There is particularly a focus on quantum quenches: the temporal evolution following (e&eb) a sudden or slow change of the coupling constants of (e) the system Hamiltonian. Several aspects of the slow dynamics in driven systems are discussed and the universality of such dynamics in (eb) gapless systems with (e&eb) specific focus on dynamics near (eb) continuous quantum phase transitions is emphasized. Recent progress on understanding thermalization in closed systems through the eigenstate thermalization hypothesis is also reviewed and relaxation in (eb) integrable systems is discussed. Finally key experiments probing quantum dynamics in cold atom systems are overviewed and put into the context of our current theoretical understanding. http://dx.doi.org/10.1103/RevModPhys.83.863© 2011 American Physical Society **Colloquium: Nonequilibrium dynamics of closed interacting quantum systems Anatoli Polkovnikov, Krishnendu Sengupta, Alessandro Silva, and Mukund Vengalattore Rev. Mod. Phys. 83, 863 – Published 15 August 2011 STOP** Quantum walks, the quantum mechanical counterpart of classical random walks, is (=) an advanced tool for building quantum algorithms that has been recently shown to constitute (e) a universal model of quantum computation. Quantum walks is now a solid field of research of quantum computation full of exciting open problems for physicists, computer scientists and engineers. In this paper authors review theoretical advances on the foundations of (e) both discrete- and continuous-time quantum walks, together with (e&eb) the role that randomness

plays in (e&eb) quantum walks, the connections between the mathematical models of coined discrete quantum walks and (e&eb) continuous quantum walks, the quantumness of quantum walks, a summary of papers published on discrete quantum walks and (e&eb) entanglement as well as a succinct review of experimental proposals and realizations of discrete-time quantum walks. Furthermore, they review several algorithms based on (e) both discrete- and continuous-time quantum walks as well as a most important result: the computational universality of (e) both continuous- and discrete-time quantum walks. **Quantum walks: a comprehensive review Author: Salvador Elías Venegas-Andraca Tecnológico de Monterrey Campus Estado de México and Texia, Estado de México, México Published in: · Journal Quantum Information Processing archive Volume 11 Issue 5, October 2012 Pages 1015-1106 Kluwer Academic Publishers Hingham, MA, USA Quantum walks: a comprehensive review2012 STOP David R. Mitchell, Christoph Adami, Waynn Lue, and Colin P. Williams** present an analysis of the quantum adiabatic algorithm for (e) solving hard instances of 3-SAT (an NP-complete problem) in terms of random matrix theory (RMT). Authors determine the global regularity of (e) the spectral fluctuations of (e) the instantaneous Hamiltonians encountered during the interpolation between (e&eb) the starting Hamiltonians and the ones whose ground states encode (e) the solutions to the computational problems of (e) interest. At each interpolation point, authors quantify (eb) the degree of regularity of the average spectral distribution via (e&eb) its Brody parameter, (=) a measure that distinguishes regular (i.e., Poissonian) from (e&eb) chaotic (i.e., Wigner-type) distributions of (e) normalized nearest-neighbor spacings. Authors find that for hard problem instances—i.e., those having a critical ratio of clauses to (e) variables—the spectral fluctuations typically become (=) irregular across (e&eb) a contiguous region of (e) the interpolation parameter, while the spectrum is (=) regular for easy instances. Within the hard region, RMT may be applied to obtain (eb) a mathematical model of the probability of (e) avoided level crossings and concomitant failure rate of (e) the adiabatic algorithm due to (e) nonadiabatic Landau-Zener-type transitions. Model predicts that if the interpolation is performed at a uniform rate, (eb) the average failure rate of the quantum adiabatic algorithm, when averaged over hard problem instances, scales (eb) exponentially with (e&eb) increasing problem size. **DOI: http://dx.doi.org/10.1103/PhysRevA.71.052324 ISSN 1094-1622 (online), 1050-2947** (print) ©2015 American Physical Society. **PHYSICAL REVIEW A Random matrix model of adiabatic quantum computing David R. Mitchell, Christoph Adami, Waynn Lue, and Colin P. Williams Phys. Rev. A 71, 052324 – Published 20 May 2005STOP In the subterranean realm of subjective consciousness Clarke, Chris** argues that a dual-aspect theory of consciousness, associated with (e&eb) a particular class of quantum states, can provide (eb) a consistent account of consciousness. **David Bourget** illustrates this with the use of coherent states as this class. The proposal meets Chalmers 'requirements of allowing (eb) a structural correspondence between consciousness and (e&eb) its physical correlate. It provides (eb) a means for consciousness to have an effect on the world (it is not an epiphenomenon, and can thus be selected by evolution) in a way that supplements and completes conventional physics, rather than interfering with it. **David Bourget** draws on the work of Hameroff and Penrose to explain (eb) the consistency of this proposal with (e&eb) decoherence, while adding details to this work. The proposal is open to extensive further research at both theoretical and experimental levels. Document Type: Research Article Publication date: 01 January 2007 **The Role of Quantum Physics in the Theory of Subjective Consciousness Author: Clarke, Chris Source: Mind and Matter, Volume 5, Number 1, 2007, pp. 45-81(37) Publisher: Imprint Academic STOP chief** aim of the science of consciousness is to discover general principles that determine exactly which states of phenomenal consciousness occur in exactly which conditions. In this paper **David Bourget** argue that making progress towards the discovery of such principles requires developing a new regimented language for (e) describing phenomenal states. This language should allow us to describe phenomenal states in a way that is commensurable with our descriptions of physical states. **David Bourget** suggests one way of doing this. **David Bourget** 's approach extends (...) Consciousness and Content, Misc in Philosophy of Mind Consciousness and Intentionality in Philosophy of Mind Consciousness and Neuroscience, Foundational Issues in Philosophy of Cognitive Science Representationalism in Philosophy of Mind **David Bourget, Regimentation and the Science of Consciousness STOP** paper replies to objections from perceptual distortion against the representationalist thesis that the phenomenal characters of experiences supervene on their intentional contents. It has been argued that some pairs of distorted and undistorted experiences share (e&eb) contents without sharing

phenomenal characters, which is incompatible with (e&eb) the supervenience thesis. In reply, **David Bourget** suggests that such cases are not counterexamples to (e) the representationalist thesis because (e) the contents of distorted experiences are always impoverished in some way compared to (e&eb) those of normal experiences This (...) Phenomenal Concept in Philosophy of Mind Representationalism in Philosophy of Mind **767 David Bourget (2015) Representationalism, Perceptual Distortion and the Limits of Phenomenal Concepts. Canadian Journal of Philosophy 45 (1):16-36 STOP** This paper compares tracking and phenomenal intentionality theories of intentionality with respect to (e&eb) the issue of naturalism. Tracking theories explicitly aim to naturalize intentionality, while phenomenal intentionality theories generally do not. It might seem that considerations of naturalism count in favor of tracking theories. **David Bourget** et al survey key considerations relevant to this claim, including some motivations for and objections to the two kinds of theories. We conclude by suggesting that naturalistic considerations may in fact support phenomenal intentionality theories over tracking theories. Asymmetric-Dependence Accounts of Mental Content in Philosophy of Mind Causal Accounts of Mental Content, Misc in Philosophy of Mind Information-Based Accounts of Mental Content in Philosophy of Mind Metaphysical Naturalism in Metaphysics Phenomenal Intentionality in Philosophy of Mind Teleological Accounts of Mental Content in Philosophy of Mind Cognitive Phenomenology in Philosophy of Mind Conscious Thought in Philosophy of Mind **Angela Mendelovici & David Bourget (2014) Naturalizing Intentionality: Tracking Theories Versus Phenomenal Intentionality Theories. Philosophy Compass 9 (5):325-337 STOP** Representationalists argue that phenomenal states are intentional states of a special kind. This paper offers an account of the kind of intentional state phenomenal states are: **David Bourget** argues that they are underived intentional states. This account of phenomenal states is equivalent to two theses: first, all possible phenomenal states are underived intentional states; second, all possible underived intentional states are phenomenal states. **David Bourget** clarifies these claims and argues for each of them. I also address objections which touch on a range (...) Consciousness and Intentionality in Philosophy of Mind Phenomenal Intentionality in Philosophy of Mind Representationalism in Philosophy of Mind **David Bourget (2010). Consciousness is Underived Intentionality. Noûs 44 (1):32 – 58 STOP** satisfactory solution to the problem of consciousness would take the form of a simple yet fully general model which specifies the precise conditions under which any given state of consciousness occurs. Science has uncovered numerous correlations between consciousness and (e&eb) neural activity, but it has not yet come anywhere close to this. We are still looking for the Newtonian laws of consciousness. -/- One of the main difficulties with consciousness is that we lack a language in which to formulate illuminating (...) Consciousness and Intentionality in Philosophy of Mind Representationalism in Philosophy of Mind The Nature of Contents, Misc in Philosophy of Mind **David Bourget (2010) The Representational Theory of Consciousness. Dissertation, Australian National University STOP** David **Bourget** discusses the quantum mechanical theory of consciousness and freewill offered by Stapp (1993, 1995, 2000, and 2004). First **David Bourget** show that decoherence-based arguments do not work against this theory. Then discuss a number of problems with the theory: Stapp's separate accounts of consciousness and freewill are incompatible, the interpretations of QM they are tied to be questionable, the Zeno effect could not enable freewill as he suggests **because weakness of will would then be ubiquitous**, and the holism of measurement in (...) Consciousness and the Interpretation of Quantum Mechanics in Philosophy of Cognitive Science **David Bourget (2004). Quantum Leaps in Philosophy of Mind Journal of Consciousness Studies 11 (12):17--42.(For models please see one of the papers in the series). Following system is discussed with attendant properties: Topological Fault-Tolerance And Cluster State Quantum Computation, Optical Quantum Computation, Separability Of Very Noisy Mixed States, Supercausality And Consciousness, Electrical Conductivity Of Microtubules, Neuronal Microtubules, Dynamical Localization In Atomic Momentum Transfer, Quantum Chaos With Quantum Tomography, Quantum Fidelity Amplitude, Thermodynamics Of Quantum Transport, Regularity And Chaos, Mind Influences Brain Through Proactive Spin, Dynamics In Large-Scale Functional Brain Networks, Advanced Waves And Syntropy, Bose-Einstein Condensates et al**

**Key Words: Topological Fault-Tolerance, Cluster State Quantum Computation, Optical Quantum Computation, Separability , Noisy Mixed States, Supercausality ,Consciousness, Electrical Conductivity, Microtubules, Neuronal Microtubules, Dynamical Localization, Atomic Momentum Transfer, Quantum Chaos, Quantum Tomography, Quantum Fidelity Amplitude, Thermodynamics , Quantum Transport, Regularity, Chaos, Mind Influences Brain Through Proactive Spin, Dynamics, Large-Scale Functional Brain Networks, Advanced Waves, Syntropy, Bose-Einstein Condensates**

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**Acknowledgements**: I have made concerted efforts, sustained struggle and protracted endeavours for the inclusion of every one and every source either in references or cross references. In the eventuality of any act of omission and commission, I make a sincere entreat, earnest beseech and fervent appeal to kindly pardon me. By nature, the work called for browsing and collating and coordinating thousands of various documents. Please excuse me in case of any prevarication from rules, or miss classification of the protagonist or antagonist laws is based on the characterstics and pen chance, predilection, proclivity and propensities of the systems under investigation. We acknowledge in unmistakable and unambiguous terms the help of Stanford encyclopedia, Kants writings, and Deleuze's Logic Of Sense, Penrose and Hawking's nature of space and time, Penrose's Emperor's New Mind, Shadows of mind, Ken Wilber's spectrum of consciousness etc., great men seem to be endless… ad infinitium….. One feels humbled and the least……...Google search and Wikipedia. Concerted and orchestrated efforts are made to put on recordial evidence the names of all people either under references list or mostly by cross references so that no one is missed. If there be any act of omission or commission, it is my sincere entreat, earnest beseech, fervent appeal to kindly pardon me and the error is absolutely inadvertent and in deliberate. Let not any sensibilities, susceptibilities, and sentimentalities be hurt. Parson's pattern variables provide a way of categorizing the types of choices and forms of orientation for individual social actors, both in contemporary society and historically .I want to put on record with humble gratefulness the help by American Physical society, nature and other Noetic institutes of US who sent lot of alerts for my reference which were very valuable, and could not have been found by me despite assiduous and fervent search. Most important type is that concerning the stability of solutions near to a point of equilibrium. This may be discussed by the theory of Lyapunov. In simple terms, if all solutions of the dynamical system that start out near an equilibrium point x_e stay near x_e forever, then x_e is Lyapunov stable. More strongly, if x_e is Lyapunov stable and all solutions that start out near x_e converge to x_e, then x_e is asymptotically stable. The notion of exponential stability guarantees a minimal rate of decay, i.e., an estimate of how quickly the solutions converge. The idea of Lyapunov stability can be extended to infinite-dimensional manifolds, where it is known as structural stability, which concerns the behavior of different but "nearby" solutions to differential equations. Input-to-state

stability (ISS) applies Lyapunov notions to systems with inputs. von Neumann stability is necessary and sufficient for stability in the sense of Lax–Richtmyer (as used in the Lax equivalence theorem): The PDE and the finite difference scheme models are linear; the PDE is constant-coefficient with periodic boundary conditions and have only two independent variables; and the scheme uses no more than two time levels (See Wikipedia) Von Neumann stability is necessary in a much wider variety of cases. It is often used in place of a more detailed stability analysis to provide a good guess at the restrictions (if any) on the step sizes used in the scheme because of its relative simplicity. Albeit forwarded in nine module systematizations, the entire gamut is to be seen in a single shot, and the presentation of nine schedule twenty seven storey models is to circumvent typing of hundreds of superscripts and subscripts. In fact the statement is made inclusive of all previous models, and the variables are definitely different for each schedule, which again is reinstated due to typing of corresponding variables millions of systems, a fastidious and fussy work again. I beg pardon for any inconvenience caused to the readers due to such utilization of convention. I am grateful to Professor Chadralekha MD PhD. Tagore medical College, Chennai for deliberations and discussions on Medicine. To discussions on Physics topics credit goes to Dr. A.S. Krishna Prasad, Former Director DRDO, and Bangalore Chapter. Prof. Sunita MSc. PhD., of MS Ramaiah University helped me with valuable suggestions on Aerodynamics and propellant chemistry. Sir KVB Pantulu, former Chairman of NALCO, ESSAR Steels helped in formatting process and project management advices.

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# PREFACE

**Note:** Some of the following concepts are modelled; some are not. For those which are not modelled kindly see one of the papers in the series. This note has the axiomatic predication to give a holistic view of the study. Here we talk of the characterstics of systems which satisfy the condition of cosmological constant or any other for that matter or variable or axiomatic predication. . There are lots of zeroes corresponding and concomitant to the second law of black holes. Infact as many as that of extant and existential blackholes exist. At the outset, it is to be stated that there are hadrons in every system. Supersymmetry between forces and matter, with both open and closed strings; no tachyon; group symmetry is SO (32) and its axiomatic predications, predicational anteriorities, character constitution shall be extant and existential in very many systems, and the characterstics are taken in to consideration in the classification scheme. Many systems have such fundamental instabilities like that of quantum gravity and characterstics of those systems form the citadel and fulcrum, bulwark and manor, mainstay and reinforcement, alcazar and chateau theory has a fundamental instability on which the classification schémas are valid. There are lots of systems which follow the axioms of string theory. It is the characterstics of this system which are taken in to consideration in the classification scheme. "All institutionalization involves common moral as well as other values. Collectivity obligations are, therefore, an aspect of every institutionalized role. But in certain contexts of orientation-choice, these obligations may be latent ...." (Parsons, 1951, p. 99). There are various systems that have the same bastion, support system, stylobate and sentinel as that of the Deleuzean terms and predications and phenomenological methodologies systemized. Look at this beautiful passage whose stability analysis would be eye opener. To "deconstruct" is not the same as to destroy. Deconstruction attempts to undo logical contradictions, to overturn rigid conceptual oppositions while releasing new concepts and meanings that could not be included in the old system. At the heart of Western metaphysics, for example, Derrida finds the opposition between "speech" and "writing." This binary logic functions in an illicit way to establish speech as the means of giving "presence" to the world, while writing is deemed derivative and inferior In Derrida's sense of "grammatology," however, all production of meaning is writing and subject to the infinite play of signification. By taking away the transcendental signified and advancing the concept of "differance" (language organized around difference and deferred, or

mediated, understandings), Derrida, like Nietzsche, wants to leave us without transcendental illusions, metaphysical unities, and foundations that constrain thought and creativity. **THE POSTMODERN TURN IN PHILOSOPHY: THEORETICAL PROVOCATIONS AND NORMATIVE DEFICITS by Steven Best and Douglas Kelner http://www.gseis.ucla.edu/faculty/kellner/kellner.html** Each and every system has electrons, neutrons and protons and for that matter quarks. There shall be strong nuclear force and weak nuclear force. There are many systems that satisfy the criterion specified by the equation, principle or statement in question. Characterstics of the investigating systems form the bastion for the classification scheme and doxa thereof. Systemic differentiation is conducted. Despite gravity being constant, there exists gravity between two objects, and this could be taken as a system. Depending upon some parametric representationalities, functionalities, advantageousness, appropriateness, benefit, facilitation, fittingness, helpfulness, instrumentality, merit, practicality, serviceability, suitability, and usefulness, utility, these systems could be classified in to various categories. In respect of an equation, there shall be many systems that satisfy the given equation. Equations themselves could be by the utilization of the model solved term by term as has been exemplified and illustrated many time in the previous papers. There is lot of systems that could be brought in to the orbit of and gamut of the theory in question which the investigatable systems satisfy the axiomatic predications and postulation alcovishness of the systems in question. Towards the end of classificational consummation, consolidation, corporation and concatenation we take the characterstics of the systems, the predicational interiorities, ontological consonance and primordial exactitude, accolytish representations, functional topology, apocryphal aneurism and atrophied asseveration, event at contracted points, and other parameters as the bastion and stylobate of the stratification purposes. Such totalistic entities would have easy paradigm of relational content, differentiated system of expressly oriented actions with primary focus and locus of homologues receptiveness and differentially instrumental activity, variable universalism and particularism, imperative compatibilities and structural variabilities, interactional dynamical orientation, institutionalization and internalisation of pattern variables common attitudinal orientation of constituionalisation of internalized dispositions, and a qualitative gradient of structural differentiation and ascribed particularistic solidarity abstraction or interactional dynamics, internal differentiation, structural morphology, formal characterization, concept formulation, phenomenological methodologies, constituent structure, transformational minimal condition, paradigmatic feasibilities, programmatic plausibilities, comparative variability, normative aspect of expectational prediction, projection and prognostication as consideration of the investigatory systems. Any scale can be used that is convenient to the classification scheme. It is important to note that the scheme of classification and the stratification doxa must not be adversely antagonistic, inexorably irreconcilable, antithetically antipodal, diametrically opposed, repugnantly retrogressive, inimically inverse, violatively unsimilar, diversely dissimilar, antipathetically anti thetical, conflictingly combative, obstructively pugnacious, inimically obstructive, repellently restrictive, disputatiously gainsaying or conformingly pugnacious to the axiomatic predications and postulation alcovishness of the theory in question or the equation representative or constitutive thereof. Sole intention, main objective and primary aim is twofold. One is towards the end of circumvention of the extra equation and the concomitant and corresponding variable therein. Second is avoidance of clustered congest, swarmed huddle, mustered pack, sardine squash, and swamp throng in the scheme of classification. Only thing that is sought out is the consonance in the entire diaspora and body fabric of the systems under study. This statement is true and holds unmistakably true for all the papers and I sincerely entreat readers to remember the statement and read the paper against this background. When we write A+B we mean by that B is being added to A or vice versa. It is like adding milk to water and water to milk. There may or may not be a time gap. As said earlier there may be many systems that satisfy the conditions of the equations and those systems that are investigatory or investigatable are taken in to consideration based on their characterstics in the classification doxa. When there are more than two entities, we can take logarithm and find the value of that factor to be taken with anti log to obtain prediction and projected values of the model. In case of A-B, we are removing B from A and that means B is eating up A. These factors are taken in to consideration in the application of the model to equations. Log (ab) and Log (a+b) is well defined. Model stands out as universal testament and template for application to each sentence and equation what with the quantification process done and the correlations well defined. In the eventuality of non existence of any connection at some phase, the model would

have the accentuation and attrition coefficients and detritions coefficients as zero rendering the equations of concatenation simpler. Projection formula which incorporates in its diaspora the initial values provide autheticative determination, unimpeachable validation, incontrovertible establishment , apodictic evidence, reliable roll out, unfailing cinquecento quattrocento trecento, incontrovertible indication of the final finale, notwithstanding appellation, appellative, brand, cognomen, compellation, designation, flag, handle, identification, label, moniker, nomen, slot, style, surname, tab, tag, term, title designation, appellation, appellative, class, classification, denomination, description, epithet, of the classification scepter, scenario, scimitar, schottische. There is pure and impure consciousness in every one. Gratification producing and deprivation producing one's can be easily classified from individual general ledgers. Similar analogy holds for collective general ledger and cosmic general ledger or nature's general ledger. It is also to be noted that while dealing with equations towards the end of consummation of the measure, it is necessary that the two variables are to be classified in to three sections and each one would have the adventitious and decidedly stated relationship whereby the fundamental equations are drawn up and the analysis made. All the parametric representationalities, conditionalities, orientationalities remain unequivocal as stated in the variables stated in to consideration section and are different from module to module. In essence the paper is to be read as holistic one with the sole intention, primary objective and raison d' etre to build a TOE. Towards the end of circumvention of typing hundreds of superscripts and subscripts which would be a sardine squash and the concomitant operational difficulties, model is presented in piece meal of nine modules. Logarithms are to be taken in respect of those which incorporate more than one variable in bra-ket. Values of log (ab) and log (a+b) are readily available. Anti logarithm shall be taken at the end to predict, project, prognosticate the value of the variable. This is true for tensors, vectors and other variables too. Affirmational assertion, and explanation justification, statement of vindication, annotational commentary, and explicational glossary for each and every system changes and physical interpretation of results is one thing that is to be with earnest endeavour and feverish and febrile expediency. Editions never ever mean the same and identicality of parameters and this has been explicitly and unmistakably stated in the model apriori itself. Akin and analogous, cognate and concurrent, correspondent and congruent, comparable and complementary, synonymous and duple, tantamount and agnate, commensurate and correlative representation is only to highlight the importance and subterfuge the replication of work. It is my fervent solicitation to kindly bear with me for any lapses, notwithstanding the orchestrated efforts for a paper without any minor errors. Postulation predication, conclusive presumption, differential presuppositions, underscored decidedly axiomatic statement of the statement, equation form the bye word or the watch word in the aggrandizement-amplification, caricature and crock, understatement- unembellishment, elocution-emphasis, enunciation-inflection, announcement-argument, articulation, assertion, asseveration, choice of words, commentary and communication, declaration and definition, delivery-diction, elucidation-emphasis, enunciation-execution, explanation, exposition-formulation, idiom, interpretation and intonation tone and tenor of stratification. Now, how do find the reaction of systems to these singularities. You do the same thing a boss does for you. "Problematize" the events and see how you behave. I will resort to "pressure tactics". "intimidation of deriding report", or "cut in the increment" to make you undergo trials, travails and tribulations. I am happy to see if you improve your work; but may or may not be sad if you succumb to it and hang yourself! We do the same thing with systems. systems show conducive response, felicitous reciprocation or behave erratically with inner roil, eponymous radicality without and with blitzy conviction say like a solipsist nature of bellicose and blustering particles, or for that matter coruscation, trepidiational motion in fluid flows, or seemingly perfidious incendiaries in gormandizing fellow elementary particles, abnormal ebullitions, surcharges calumniations and unwarranted(you think so but the system idoes not!) unrighteous fulminations. Amount of emotion or affect that is appropriate or expected in an given form of interaction. Particular individuals and diffuse obligations (see c and d) are associated with affectivity, whereas contacts with many individuals (universalistic) in a bureaucracy may be devoid of emotion and characterized by affective neutrality. Affective neutrality may refer to self discipline and the deferment of gratification (eg Weber's spirit of capitalism). In contrast, affectivity may be associated with expressing emotions. Adams and Sydie also refer to affective neutrality being associated with ego control (p. 15). (Parsons: Wikipedia). So the point that is made here is "like we problematize the "events" to understand the human behaviour we have to "problematize" the events of systems to

understand their behaviour. This statement is made in connection to the fact that there shall be creation or destruction of particles or complete obliteration of the system (blackhole evaporation) or obfuscation of results. Some systems are like “inside traders" they will not put signature atoll! How do you find they did it! Anyway, there are possibilities of a CIA finding out as they recently did! So we can do the same thing witn systems to. This is accentuation, corroboration, fortification, .fomentatory note to explain the various coefficients we have used in the model as also the dissipations called door. In the bank example we have clarified that various systems are individually conservative, and their conservativeness extends holisticallytoo.that one law is universal does not mean there is complete adjudication of nonexistence of totality or global or holistic figure. Total always exists and "individual" systems always exists, if we donot bring Kant in to picture! For the time being let us not! Equations would become more ensorcelled and frenzied..... philosophy merges with ontology; ontology merges with univocity of being; analogy has always a theological vision; not a philosophical vision; one becomes adapted to the forms of god; self and world; the univocity of being does not mean that there is one and the same being; on the contrary, beings are multiple and different they are always produced by disjunctive synthesis; and they themselves are disintegrated and disjoint and divergent; membra disjuncta.the univocity of being signifies that that being is a voice that is said and it is said in one and the same "consciousness”. In consciousness research, two rival sets of theories can be recognized: (A) Scientific material interpretations of consciousness are based on axioms that view consciousness in the context of highly advanced intentional processing of information in which subject-object relations evolve, and (B) humanistic interpretations of consciousness are based on axioms that view consciousness in the context of, say, "centered pulsations" that enable a conscious agent to act from his or her center of awareness. In this paper I will argue for the selection of axioms that favor humanistic interpretations of consciousness. The Pursuit of Autonomy Interdisciplinary Observations to Human Consciousness: Wautischer, Helmut (1993) the Pursuit of Autonomy. Interdisciplinary Observations to Human Consciousness: Social Neuroscience Bulletin, 6 (4). pp. 52-56.Everything about which consciousness is spoken about being is the same for everything for which it is said like gravity; it occurs therefore as an unique event for everything; for everything for which it happens; eventum tantum; it is the ultimate form for all of the forms; and all these forms are disjointed; it brings about resonance and ramification of its disjunction; the univocity of being merges with the positive use of the disjunctive synthesis, and this is the highest affirmation of its univocity like gravity; it is the eternal resurrection or a return itself, the affirmation of all chance in a single moment, the unique cast for all throws; a simple rejoinder for Einstein’s god does not play dice; one being, one consciousness, for all forms and all times. a single instance for all that exists, a single phantom for all the living; a single voice for very hum of voices; or a single silence for all the silences; a single vacuum for all the vacuums; consciousness should not be said without occurring; if consciousness is one unique event in which all the events communicate with each other; univocity refers both to what occurs to what it is said. This is attributable to all states of bodies and states of affairs and the expressible of every proposition. So univocity of consciousness means the identity of the noematic attribute and that which is expressed linguistically and sense fully. Univocity means that it does not allow consciousness to be subsisting in a quasi state and but expresses in all pervading reality; **CONSCIOUSNESS AND ITS UNIQUENESS June 12, 2012 at 8:40pm (See Deleuze Logic of sense, Wikipedia and Stanford encyclopedia for more details)** Atrocious contrivance and device, gratificational primogeniture, calamitous dodge and expedient, depraved, destructive, disastrous, execrable gambit and gimmick, great solace and succor, iniquitous, injurious, loathsome, low, maleficent, malevolent machination and maneuver, promethaleon of candor, frankness, honesty, honor, ingenuousness, innocence, openness, reality, sincerity, truthfulness, spiteful, stinking, ugly, unpleasant, unpropitious play and ploy, progenitor of jurisprudence and circumspection, racket and ruse, savvy, scam, stratagem, subterfuge, tactic and wile, forthrightness, honesty, truthfulness embodied and personified is the structural predisposition and dispensation of the complex. Nobody need have to take the Brahman-Anti Brahman agency to be a recalcitrant, repugnant, and refractory proposition. God gives and also takes away. That is the point made. And of course lesser gods, and God doth follow them and they follow God. That is why they are lesser gods. And wield the power on lesser mortals like us. When we mean gratification and deprivation we mean celestial Brahman Anti Brahman and Terrestial Brahman and Anti Brahman agencies producing such happiness or sadness by any means such as (blending- amalgamation,

adulterant-adulteration, amalgam-amalgamation, blend- combination, composite- compound, debasement-denaturant, fusion-hybrid, intermixture-reduction, amalgam- mixture, composite- compound, fusion-mishmash, aggregate- alloy, amalgamation-blend, commixture-composite, composition-compost, conglomerate-fusion, goulash-medley, mishmash-stew, blend-coalition, commixture-compound, federation-heating, integration-junction, melting-merger, synthesis-unification, assortment- combination, adulteration-alloy, assimilation-association, batter-blend, brewed combine, composite compound, concoction confection, conglomeration cross of money, orientation(dis), penury creation, murder, mayhem plunder, pillage, apocalypse, Armageddon in various forms and permutation and combination of Manichaean artifice, agathokakological malevolence, agley chicane fourberie, fraud, furtiveness, gambit, awry and bad ,flagitious mechanizations, cacodemonic gambit, hanky-panky, intrigue, machination, deprecatory and diabolic underhandedness and wiles, energumenical skullduggery, goetic chicanery, lenocinant stratagem, malefic and maleficent strategy, malominous sell, sellout, sham subterfuge, peccable, humbug, imposture, misrepresentation peccant fraud, fraudulence, hoax, qued , scelerate pretense, ruse, sell, sellout, sham stratagems' of maladroit manouevures , sinistral ,tortious and venal sophistry, contingency enterprise, endangerment emprise, jeopardizing adventure, jejune jeremiad, avoidable inertia, latent passiveness: with each action accounted and showing balance in the nature's general ledger as also actionlessness:e). All these are recorded in Nature's general ledger. Neuron DNA encapsulates all the actions passions, actions, interactions, transactions or the lack of it at various levels of individual, collective and cosmic levels. In celestial Brahman Anti Brahman ledger there is cessation of dualities. One more point to be made is if 5=3+2, then we can say 5 gormandizes 3 or 5 gobbles up 2. Quintessentially such a statement arises out of the difference of LHS and RHS term wise. It must be remembered despite all theoretical abstractions and generalizations be it proton mass or Higgs Boson, or topology of a space, it is notwithstanding the fundamentality of nature is just a number. Just everything in Physics and mathematics is just a number. Let us assume I buy rice in Bangalore (District) worth 5k.g. I can always divide 5k.g.s of rice in categories of Bangalore (South), Bangalore (west), Bangalore (East) and Bangalore (South). On similar basis, there are billions of neutrons, protons and concomitant interactions which would satisfy the given numerical value. Example of gravity above makes things clear. Everything atleast is a function of time and someplace or some other coordinates, subordinates or superordinates. Mass of proton is 2.47 does not mean total mass of all protons is 2.47. It is this argument that is used in the classification scheme. By the very nature, self the unmonitoring but the witness consciousness agency identifies itself with something, be it a profession, an actor on the screen, when it does not have a identity. Such examples are legion, a Charlie Chaplin in "The Kid" or Raj kapoor in "Jagte Raho". These are people who just wanted to live and living without being put under surveillance every moment is the greatest part of their life. This is "Absolute Subjectivity" of Shiva. Such a state is always craving for identification for in pure consciousness all identities are lost. It is this "identitylessness" or having very minimal identity is what pervades most in India and this is due to expansion of individual consciousness. When that happens, be it education, or wealth, the "identity' becomes so well entrenched, they become the exact opposite of "pure consciousness"," impure consciousness" (Shakti). This is a state of "Absolute or relativistic objectivity". There was this case of a lady who went to such an extent of planning, she was sure that whatever she does must work out. When it did not, she lapsed in to "identity crisis". Such thing as this might happen in well developed countries where the competition is extremal and to be achieved at any cost. Realisation that one is not the doer but the witness consciousness is just like watching a movie in which one has acted. It is in this sense "advaita", "non duality' seems to be of cardinal and preeminent importance for the analytical treat ment of "identity' and "identity lessness", both happen in space time. "Multiverses" in this sense is also like a different film of an actor released at different theaters and it is in this sense we state that that every object in space time is a multiverse. You must have totally immersed in doing a "string theory" problem or watching a "film", oblivious of surroundings. Essentially creation (space time) also is what is happening in mind and honed in to you billion times. Identification of anything with a designation or signification is one that leads to association within spacetime, which would only have pathological consequences and detrimental ramification as stated above or positive implications of "happiness" because of the identity itself. Search or 'show" has to go on! It gives a sense of consummation. a watching pot never boils; donot go to check nature; you are disturbing it; it will never show its true colours bhakti and Virakthi, gratification and deprivation,

raga and dwesha is essential to live in space. It is the quantum interest thereof that produces energy to live .gratification and deprivation which form the bastion, pillar, post and stylobate and sentinel of all human actions depends on tams, rajas, and sattva of individual or insentient object. Verily tamas, rajas and satva are the characterstics of space itself, nay the behavioural pattern and attitudinal orientation. There are peaceful places like Vienna or Sringeri and there are violent places like others. Space is warped and woofed by the Akshara, the quantum information of primordial syllable Om. cosmic general ledger is the one vibration of which is brahman; quantum information possessions thereof form the sentinel of all transactions (Gargi to Yajnavalkya: my interpretation)adhocism, nonseqientiality of events, anti sequentiality of events, simultaneity of phenomenon, concurrent happenings , improbability of phenomenon happening itself are characterstics of brahman-anti brahman (Terrestial and celestial) are the cause of non predictability of happenings like in black hole, or in the areas of operation of brahman-anti brahman (Terrestial and celestial). brahman AntiBrahman actions, interactions, decisions, circumspection, adhocism, satisfies sensitive to initial conditions; topologically mixing; dense periodic orbits, the axiomatic predications and postulation alcovishness of chaos theory. Like gravity is a characteristic of space, so is tamas, rajas, sattva, with spaces knotted together by quantum information (for models see below) .accidents are preplanned actions of Terrestial and celestial Brahman AntiBrahman. All actions are perpetrated by them, be it train on fire or death. Belief systems play an important role in the determination of tamas, rajas, and satva state of human beings. There is neither cause nor effect. All actions are performed by Terrestial and celestial Brahman anti Brahman and it is off the record. psychic energy is negative equivalent to –mc^2, so that second law of thermodynamics is satisfied on a holistic basis vis-à-vis material energy ParaBrahaman is infinite individual, collective and cosmic consciousnesses are infinite (concomitant general ledgers) for they are in the evolution mode with respect to each other in that order. Later emanates from the former as transactions emanate from the conservative state of assets and liabilities. Is not zero true for infinite conservative systems inside space and time? zero is the only consummation, consolidation and conservation. if fifty out of seventy members come out of the room at whatever time you come out, talk about the grand limbo of oblivion and hibernation of lifeless world you are going to and if you are not a celebrity, then it is not coincidence, it is coordination. Nature is playing the same trick on you. That a generalised law is there and you can measure does not make things less good albeit recalcitrant, refractory and rogue particles occur time and again, by and large. Entering in to all living beings through his constant part on the new moon night, he is born therefrom next morning. Absolute ParaBrahaman (zero) projects himself again and again on the screen of individual consciousness which appears of depairing despondent world. Some where the balancing act is done on karma. cosmic general ledger reflects upon itself to keep all transactional ties conservative (Brihadarnkayaka Upanishad: 1.5 14) for models see next paper through its divine power ( i interpret man creates god; it is attribution and ascription of various Shaktis would the formation of god takes place like space and time) the self assumes (eb) the form of a deity man can contemplate and venerate, even though siva, the pure subject, can never in fact be an object of meditation (adhyeya). Until we realise our true identity with Sankara, he is worshipped and conceived to be a reality alien to ourselves. While we are in the realm of creation, he too is a creation or mode or appearing of the absolute, manifest to us in meditation, through his freedom as an eternal, omniscient being. There is no gulf between the created and the uncreated creator: nothing in reality, although an object of knowledge, ceases to be Siva: this is the reason why meditation [on this or that aspect] of reality bestows its fruit. The world of the senses and mind appears to the well awakened (suprabuddha) as a theophany, an eternal revealing of god in his creation. The doctrine of vibration declares that "there is no state in word, meaning or thought, either at the beginning, middle or end, which is not Siva." 59 to utter any word is, in reality, to intone a sacred formula. Every act is a part of Siva's eternal cosmic liturgy, every movement of the body a ritual gesture (mudras), and every thought, god's thought. By what path are you not attainable? What words do not speak of you? In which meditation are you not an object of contemplation? What indeed are you not, o lord? 61 spandashakti, which accounts for the appearing of all things, is also the means by which deity in its many varied forms appears to man. Ksemaraja concludes: the ultimate object of worship of any theistic school differs not from the Spanda principle. The diversity of meditation is due solely to the absolute freedom of Spanda. 62 Sankara is not only the supreme object of devotion; as the static polarity of the absolute, he is the inner reality which holds together

its Siva and Sakti outer manifestations. Phenomena are patterns of cognitions projected onto the surface of self-luminous siva-consciousness. There they become apparent, directly revealed to consciousness according to their manifest form. Siva is accordingly symbolized as the ground or surface of awareness, smooth and even like a screen (samabhittitalopama). 64 inscribed on this screen (kutfya) are the countless manifest forms which appear within it rendering it as diverse and beautiful as a fossil ammonite (falagrama). 65 Siva is the sacred ground upon which the cosmic mandalas are drawn, the absolute surface of inscription which bears the mark (cihna) of the universe. Abhinava writes: the variety of this world can only be manifest if the highest lord, who is essentially the pure light of consciousness, exists; just as a surface is necessary for a picture. if external objects were perceived in isolation then, because 'blue' and 'yellow', etc., are self-confined and the perceptions [we have of them] refer to their objects alone and so are insentient, mute and dumb in relation to one another . . . how would it be possible to be aware that an object is variegated? but just as depths and elevations can be represented by lines on a smooth wall, and we perceive [a female figure and think], similarly it is possible to be aware of differences in the variegated (contents of experience) only if all the diverse perceptions are connected together on the one wall of the universal light of consciousness. 66 Siva is the perfect artist who, without need of canvas or brush, paints the world pictures. The instant he imagines it, it appears spontaneously, perfect in every respect. The colours he uses are the varying shades and gradations of his own Spanda energy and the medium his own consciousness. Intractable disaffect, Mutinous obdurate, Rebellious recalcitrant, Seditious refractory, Insubordinate insurgent, antagonistically antipodean, discrepant incompatible, Contumacious coadjutant, Conflicting converse, Unyielding obdurate, the antagonistic and anachronistic dispositional ties of Anti Brahman are in fact the sine qua non of Brahman only (See Vishwaroopa of Mahavishnu, which contains the polarised positives on RHS and negatives on LHS). The universe is coloured with the dye of its own nature (svabhava) by the power of Siva's consciousness (a//). Rajanaka rama says: homage to him who paints the picture of the three worlds, thereby displaying in full evidence his amazing genius {pratibha)\ to Sambhu who is beautiful with the hundreds of appearances laid out by the brush of his own unique, subtle and pure energy. 68 analogously, at the microcosmic level, all the cognitions and emotions, etc., which make up the individual personality form the outward flow of essentially introverted consciousness. They are specific pulsations (vise$aspanda) or aspects of the universal pulsation (samanyaspanda) of pure t consciousness. at the lower level, within the domain of maya, they represent the play in the fettered soul of the three primary qualities (gunas) or feeling-tones' which permeate to varying degrees his daily experience. These are: 1) sattva — the quality of goodness and luminosity which accompanies blissful experience both aesthetic and spiritual. 2) Rajas — the passion or agitation which oscillates between the extremes of 'light' and 'darkness' and characterises inherently painful experiences. 3) Tamas — the torpor and delusion which accompany states of inertia and ignorance. 69 the liberated soul recognises that these three are the natural and uncreated powers of pure consciousness. for him they are manifest respectively as: 1) Sankara 's power of knowledge (jnana) — the light of consciousness (prakasa); 2) the power of action (kriya) — the reflective awareness of consciousness (vimarsa)\ 3) the power of maya — which does not mean here the world of diversity, but the initial subtle distinction which appears between subject and object in pure consciousness. 70(doctrine of vibration: mark S.G. Dyczkowski) .to "deconstruct" is not the same as to destroy. Deconstruction attempts to undo logical contradictions, to overturn rigid conceptual oppositions while releasing new concepts and meanings that could not be included in the old system. at the heart of western metaphysics, for example, derrida finds the opposition between "speech" and "writing." this binary logic functions in an illicit way to establish speech as the means of giving "presence" to the world, while writing is deemed derivative and inferior in derrida's sense of "grammatology," however, all production of meaning is writing and subject to the infinite play of signification. By taking away the transcendental signified and advancing the concept of "differance" (language organized around difference and deferred, or mediated, understandings), derrida, like Nietzsche, wants to leave us without transcendental illusions, metaphysical unities, and foundations that constrain thought and creativity. The postmodern turn in philosophy: theoretical provocations and normative deficits by Steven best and Douglas Kelner http://www.gseis.ucla.edu/faculty/kellner/kellner.html. you cannot see beyond consciousness for that is the limit of your storage of information or the basis for understanding further information. Turiya is empty bucket; it is from which prajna the measure of arises; turiya measures the number of quantum

information stores in other three states of waking, dreams, and dreamless deep sleep (Manduka Upanishad; my interpretation for models next paper).let there be path where angels tread: it is a force never born never dies the brahman an anti brahman verily there are parallel universes in (eb) in the ambit of space and time; the society of sinners and the sinned with the grammar of evil; parallel universes contain(e) multiverses in them; all physics, philosophy, break down(e) in these universes; astronomical gratification ,deprivation, glorification, mortification, projections, axes, dimensions, rotations, foldings, concurrences, motions, shocks, are the axiomatic predications of multiverses and parallel universes; death is when self (the witness consciousness; register of individual general ledger) leaves the body; ostensibly parallel seem to have (e) an objective; parallel universes donot (e) have objectives; their paradigm of relational content, differentiated system of expressly oriented actions with primary focus and locus of homologues receptiveness and differentially instrumental activity, variable universalism and particularism, imperative compatibilities and structural variabilities, interactional dynamical orientation, institutionalization and internalisation of pattern variables common attitudinal orientation of constituionalisation of internalized dispositions, and a qualitative gradient of structural differentiation and ascribed particularistic solidarity abstraction or interactional dynamics, internal differentiation, structural morphology, formal characterization, concept formulation, phenomenological methodologies, constituent structure, transformational minimal condition, paradigmatic feasibilities, programmatic plausibilities, comparative variability, normative aspect of expectational prediction, affection- appreciation, awareness- discernment, emotion- feeling, gut reaction, heart insight, intuition, judgment, keenness, perceptiveness, rationale, sensation, sense, sensitiveness, sensitivity, sentiment, susceptibility, taste, vibes, admiration, aesthetic sense, affection, appraisal, assessment, attraction, awareness, cognizance, commendation, comprehension, enjoyment, esteem, estimation, grasp, high regard, knowledge, liking, love, perceptional realization, recognitional regard, relish, respect, responsiveness, sensibility, sensitiveness, sensitivity, sympathy, understanding, valuation, acknowledgment, airing, baring, , confessional defenselessness, denudation-denunciation, disclosure, display, divulgence-exhibition, hazard, introduction, jeopardy, laying open, liability, manifestation, nakedness, openness, peril, presentation, publicity, revelation, risk, showing, susceptibility , susceptiveness,susceptivity, unfolding, unmasking, unveiling, vulnerability, vulnerableness, action, affection, appreciation and ardor, behavior, capacity, compassion, concern, cultivation and culture, delicacy, discernment and discrimination, emotion, empathy, faculty, fervor, fondness, heat, imagination, impression and intelligence, intensity and intuition, judgment, keenness- palpability, passion, pathos, pity, reaction, refinement, sensibility, sensitivity, sentiment, sentimentality , sharpness, spirit, sympathy, tangibility, taste, tenderness, understanding, warmth, fond memories, hearts and flowers, homesickness, longing, pining, reminiscence, ,schmaltz, sentimentality wistfulness, yearning satiation is only ostensible; their job is to get the job done; their business is to mind other's business ;all laws of physics and finance breakdown; it is antilaw,piscatorial Piratish and bubonic bucaneerishness; there is free transactions between in multiverses; when reflected on the screen of consciousness they appear as extant existential universe. Transactions are astronomical and gratification deprivation, glorification and mortification incalculable; they have no barriers of region, religion, language which they create for others. theirs objectives are: corruption criminality, debasement debauchery, degradation evil immoral lewdness licentiousness, perversion sinfulness, vice viciousness, vileness wickedness, improbity corruption, atrocity decadence, degeneration degradation, depravity evil, immorality impurity, infamy iniquity, looseness lubricity, perversion profligacy, sinfulness turpitude, vice viciousness, vulgarity wickedness, disgrace bad reputation, abasement abuse, baseness black eye, blemish blur, brand comedown,contempt contumely, corruption culpability, debasement debasing, defamation degradation, derision disbarment, discredit disesteem, disfavor dishonor, disrepute disrespect, humbling humiliation ignominy ill repute, infamy ingloriousness, meanness obloquy, odium opprobrium, pollution ,put-down reproach, scandal scorn, slander slight, slur spot, stain stigma, taint tarnish, turpitude venality, enormity Horribleness , abomination atrociousness, atrocity crime and the positive aspectionalities thereof .let there not be any incursions where evil mercenaries in the garb of mendicants thrive with adversely antagonistic, inexorably irreconcilable, antithetically antipodal, diametrically opposed, repugnantly retrogressive, inimically inverse, violatively unsimilar, diversely dissimilar, antipathetically anti thetical, conflictingly combative, obstructively pugnacious, inimically obstructive, repellently restrictive,

disputatious gainsaying or conformingly pugnacious Manichaean artifice, agathokakological malevolence, agley chicane fourberie, fraud, furtiveness, gambit, awry and bad ,flagitious mechanizations, cacodemonic gambit, hanky-panky, intrigue, machination, deprecatory and diabolic underhandedness and wiles, energumenical skullduggery, execrable and goetic chicanery, lenocinant stratagem, malefic and maleficent strategy, malominous sell, sellout, sham subterfuge, peccable, humbug, imposture, misrepresentation peccant fraud, fraudulence, hoax, qued , scelerate pretense, ruse, sell, sellout, sham stratagems' of maladroit manouevures , sinistral ,tortious and venal sophistry, contingency enterprise, endangerment emprise, jeopardizing adventure, jejune jeremiad, avoidable inertia, latent passiveness . statement for Mephistophelean mercenaries philosophy merges with ontology; ontology merges with univocity of being; analogy has always a theological vision; not a philosophical vision; one becomes adapted to the forms of god; self and world; the univocity of being does not mean that there is one and the same being; on the contrary, beings are multiple and different they are always produced by disjunctive synthesis; and they themselves are disintegrated and disjoint and divergent; membra disjuncta.the univocity of being signifies that that being is a voice that is said and it is said in one and the same "consciousness". everything about which consciousness is spoken about being is the same for everything for which it is said like gravity; it occurs therefore as an unique event for everything; for everything for which it happens; eventum tantum; it is the ultimate form for all of the forms; and all these forms are disjointed; it brings about resonance and ramification of its disjunction; the univocity of being merges with the positive use of the disjunctive synthesis, and this is the highest affirmation of its univocity like gravity; it is the eternal resurrection or a return itself, the affirmation of all chance in a single moment, the unique cast for all throws; a simple rejoinder for Einstein's god does not play dice; one being, one consciousness, for all forms and all times. a single instance for all that exists, a single phantom for all the living; a single voice for very hum of voices; or a single silence for all the silences; a single vacuum for all the vacuums; consciousness should not be said without occurring; if consciousness is one unique event in which all the events communicate with each other; univocity refers both to what occurs to what it is said. This is attributable to all states of bodies and states of affairs and the expressible of every proposition. So univocity of consciousness means the identity of the noematic attribute and that which is expressed linguistically and sense fully. Univocity means that it does not allow consciousness to be subsisting in a quasi state and but expresses in all pervading reality. Negative energy stabilising wormholes are considered candidates for darkmatter. An analysis of negative energy might be an useful ingredient of the thoughts of travel at speeds more than the velocity of light. Peirce explains that an "imperfect continuum" possesses "topical singularities, or places of lower dimensionality" which break continuity, while "the parts of a perfect continuum have the same dimensionality as the whole." (CP 4.642) Circular continua without singularities are time and space. (CP 6.210-212) By using math as his lens, Peirce envisioned singularities or lower dimensional holes in the imperfect cosmologic continuum, while maintaining that a probabilistic law of chance exists that entropy decreases **(Wikipedia).** On the side of philosophy**,** quite explicitly, Deleuze provides a critique of Husserl's "sleight-of-hand" change from a static categorical phenomenology to one supposedly more dynamically constitutive of reality. This a clear cut **chagrined circumvention**, **contraventional curbing**, **defeated disgruntlement**, **dissatisfactional downer**, **dragged failure**, **fizzled foil**, **grievance hindrance**, **impedimental irritation**, letdown, nonfulfillment, nonsuccess, obstruction, old one-two, **resent mental setback**, unfulfillment of the earlier theories makes Sartre contend that human existence is a conundrum whereby each of us exists, for as long as we live, within an overall condition of nothingness (nothing-ness)—that ultimately allows for free consciousness. But simultaneously, within our being (in the physical world), we are constrained to make continuous, conscious choices. It is this dichotomy that causes anguish, because choice (subjectivity) represents a limit on freedom within an otherwise unbridled range of thoughts. Subsequently, humans seek to flee our anguish through action-oriented constructs such as escapes, visualizations, or visions (such as dreams) designed to lead us toward some meaningful end, such as necessity, destiny, determinism (God), etc. Thus, in living our lives, we often become unconscious actors—Bourgeois, Feminist, Worker, Party Member, Frenchman, Canadian or American—each doing as we must to fulfill our chosen characters' destinies. However, Sartre contends our conscious choices (leading to often unconscious actions) run counter to our intellectual freedom. Yet we are bound to the conditioned and physical world—in which some form of action is always required. This leads to failed dreams of completion, as Sartre

described them, because inevitably we are unable to bridge the void between the purity and spontaneity of thought and all-too constraining action; between the being and the nothingness that inherently coincide in our self. Deleuze insists on the absolute non-resemblance between what conditions (metastable, pre-individual plane of singularities) and what is conditioned (actualized or the individuated 'thing'). Only then do "the conditions of the true genesis become apparent," Deleuze writes (Logic of Sense 105). Without addressing this issue, Deleuze's borrowings from Simondon, Lautman and structuralism remain untethered. Because of Bowden's decision to completely sidestep Deleuze's overt negotiations with phenomenology, his otherwise sophisticated commentary suggests a pedagogical narrowness. If, the omission of phenomenology strengthens Bowden's focus on the ontological priority of the event over substantial ontologies via the genealogy of figures and movements Deleuze outlines, it also incompletely articulates the problem of the sense-event as Deleuze formulates it. We have lost any idea of whom Deleuze takes to be his primary adversary, against whom he stakes out his position. This worry is most clearly manifested in Bowden's description of Deleuze's ontological prioritizing of events as offering a "transcendental ontology." For Bowden Deleuze's transcendental ontology defines events that "are ontologically prior to substances, 'all the way down'" (82). But Bowden provides a second definition: it is that the priority of events over worldly individuals means that the world of events "is not something external to the conditions of knowledge" (69). **SEAN BOWDEN the Priority of Events: Deleuze's Logic of Sense Sean Bowden, the Priority of Events: Deleuze's Logic of Sense, Edinburgh University Press, 2011, 296pp., $40.00 (hbk), ISBN 9780748643646. Reviewed by David Scott, Coppin State University Weltschmerz, agony, apprehension, blues, depression, dread, mid-life crisis, misgiving, nervousness, uneasiness, angst were expressed with such effulgent words…… aggrandizement-amplification, caricature and crock, understatement- unembellishment, elocution-emphasis, enunciation-inflection, announcement, argument, articulational assertion, asseverational choice of words, commentary and communication, declaration and definition, delivery, diction, elucidation, emphasis, enunciation-execution, explanation, exposition-formulation, idiom, interpretation and intonation about the human predicament is brought out in Being and Nothingness. Being and Nothingness offers a critique of Sigmund Freud's theory of the unconscious, based on the claim that consciousness is essentially self-conscious. Sartre also argues that Freud's theory of repression is internally flawed. According to Sartre, in his clinical work, Freud encountered patients who seemed to embody a particular kind of paradox—they appeared to both know and not know the same thing. In response, Freud postulated the existence of the unconscious, which contains the "truth" of the traumas underlying the patients' behavior. This "truth" is actively repressed, which is made evident by the patients' resistance to its revelation during analysis. Yet what does the resisting if the patients are unaware of what they are repressing? Sartre finds the answer in what Freud calls the "censor". "The only level on which we can locate the refusal of the subject," Sartre writes, "is that of the censor.** The Holographic Principle, as conceived in current physics, applies to fields, and perhaps, even to more elementary entities, called strings and branes. These more elementary entities remain hypothetical at the time of this writing. There are many layers between the level of fields and that of neurons. We know that neurons are surrounded by a surface phospholipid membrane, which supports an electrochemical process that is fundamentally necessary to human experience. As the information supported by the membrane surface according to the Holographic Principle is proportional to its area, we should expect to find that the neural correlates of consciousness most clearly associated with those parts of the neuron that have the have a high proportion of neuronal surface area, and, in particular, receptive surface area. Dendrites account for an average of about 90% of the neuron is receptive surface area (Wong, 2002), and so would be expected to be the most important neural structures in information processes. We find this to be true **The Holographic Principle Theory of Mind MARK GERMINE Institute for Psycho science (Wikipedia).** Discordant-discrepant, dissonant-hard-line, implacable, incompatible, incongruous- inconsistent, inexorable-inflexible, inharmonious-intransigent, opposed, reluctant, unappeasable, uncompromising, unfriendly some of these expatiations and enumerations might sound, a more generalised interpretations is given by a Vedantin. Look at this arresting passage: The teacher continued to be silent. When addressed a second and third time he said: "I am teaching, but you do not follow. The Self is silence." The undetermined and unthinkable character of the Brahman is a consequence of the absolute's eternal and immutable

nature. To concede the existence of a real universe is, from the Vedantin's point of view, to posit the existence of a reality apart from the Brahman. Nor can we simply identify a real universe with the absolute unless we are prepared to compromise its unchanging, absolute status. The criterion of authenticity is immutability. Reality never changes; only that which is less than real can appear to do so. Reality is constant in the midst of change. What this means essentially is that there is change although nothing changes. This impossible situation is reflected in the ultimate impossibility of change itself. That which does not exist prior to its changing and at the end, after it has changed, must be equally non-existent between these two moments. Although the world of change appears to be real, it cannot be so. Change, according to the Vedantin, presupposes a loss of identity. Reality cannot suffer transformation; if it were to do so, it would become something else and the real would be deprived of its reality. The immortal can never become mortal, nor can the mortal become immortal. The ultimate nature of anything cannot change. Change of any sort is merely apparent (vivarta); the world of change and becoming is a false super-imposition (adhyaropa, adhyasa) on the absolute. In cosmic terms, the mistake (bhranti) consists of the supposition that the real Brahman is the unreal universe and the unreal universe is the real Brahman. In microcosmic terms, it is the mistake of falsely conceiving the body, mind or even one's personality to be the Self. In the same way as the image of a snake is falsely superimposed on a rope, similarly the universe is falsely projected onto the real substratum, the Brahman. Ignorance is not merely a personal lack of knowledge, but a cosmic principle. As such it is called "Maya," the indefinable factor (anirvacaniya) that brings this mistake in identity about. The reality status of this cosmic illusion is also indefinable: on the one hand it is not Brahman, the sole reality; on the other hand it is not absolutely non-existent like a hare's horn or the son of a barren woman. Brahman is the source of world appearances only in the sense of being their unconditioned ground or essential nature. The universe is false not because it has no nature of its own but because it does have one. Just as the illusion of a snake disappears when one sees that it is nothing but a rope, similarly cancellation (badha) of the empirically real occurs when the absolute reality of the Brahman is realised. Thus, according to Vedanta, appearance implies the real, while the real need not imply appearance. To appear is essentially to appear in place of the real, but to be real is not necessarily to appear. All things -exist because the absolute exists. It is their Being. Thus the very existence of phenomena implies their non-existence as independent realities. When they are known to be as they are, in the fullest sense of their existence, their phenomenal nature disappears leaving the ground of Being naked and accessible. This approach was validated by a critique of experience. The Vedanta established that space, time and the other primary categories of our daily experience can have no absolute existence. It was therefore necessary to make a distinction between relative truths — that accepted by the precritical common man — and an absolute truth discovered at a higher level of consciousness. The Saiva absolutist 17 rejects any theory that maintains that the universe is less than real. From his point of view a doctrine of two truths, one absolute and the other relative, endangers the very foundation of monism. The Kashmiri Saiva approach is integral: everything is given a place in the economy of the whole. It is equally wrong to say that reality is either one or diverse. Those who do so fail to grasp the true nature of things which is neither as well as both. "We do not" says Abhinavagupta, "base our contention that [reality] is one because of the contradictions inherent in saying that it is dual. It is your approach {paksa) that accepts this [method] While, if [duality and oneness] were in fact [to contradict each other], they would clearly be two [distinct realities]." The Vedantin, who maintains that non-duality is the true nature of the absolute by rejecting duality as only provisionally real, is ultimately landed in a dualism between the real and illusory by the foolishness of his own excessive sophistry (vacafadurvidya). Oneness is better understood as the coextensive unity (ekarasa) of both duality and unity. They are equally expressions of the absolute. Gopinath Kaviraj says: According to Sarikara, Brahman is truth and Maya is inexplicable (anirvacaniya). Hence the [Advaitin's] endeavour to demonstrate the superiority of Advaita philosophy is turned against his own system. It tarnishes the picture of its philosophical perfection and profundity. He cannot accept Maya to be a reality, therefore his non-dualism is exclusive. The whole system is based on renunciation and elimination and thus is not all-embracing.... By accepting Maya to be Brahman (brahmamayi), eternal (nitya) and real (satyarupa), Brahman and Maya [in the Tantra] become one and coextensive. **The Doctrine of Vibration: An Analysis of the Doctrines and Practices of Kashmir Shaivism Mark S. G. Dyczkowski. Concept of being from Shaivite point of view:** Solutional behaviour and stability analysis of such systems is of paramount importance,

notwithstanding the assumptions made which are not farfetched, in that it throws light on the fact whether they could be candidates for yet unsolved problems, providing Autheticative determination, unimpeachable validation, incontrovertible establishment, apodictic evidence, reliable roll out, unfailing cinquecento **quattrocento trecento,** incontrovertible indication for the wide amplitudinal ramificatory usage of the thesis propounded. The world is now the other person's world, a foreign world that no longer comes from the self, but from the other. The other person is a "threat to the order and arrangement of your whole world...Your world is suddenly haunted by the Other's values, over which you have no control"(Satre: being and Nothingness) Sartre states that many relationships are created by people's attraction not to another person, but rather how that person makes them feel about themselves by how they look at them. This is a state of emotional alienation whereby a person avoids experiencing **their subjectivity by identifying themselves with "the look" of the other**. The consequence is **conflict**. In order to maintain the person's own being, the person must control the other, but must also control the **freedom of the other "as freedom**". These relationships are a profound manifestation of "bad faith" as the for-itself is replaced with the other's freedom. The purpose of either participant is not to exist, but to maintain the other participant's looking at them. This system is often mistakenly called "love", but it is, in fact, nothing more than emotional alienation and denial of freedom. In essence we are always simulating and selling ourselves at the cost of our freedom. Both as a script writer and participant consciousness, human beings experienceâ complete subjective feelings and sometimes absolute subjective feelings, which if remains unchecked might lead to pathological ramifications. (italics mine) Perspicuous forbearance, Sophisticated seasoning, Partcipational observation, Background actuality, Existential worldliness, Existential strife, Predicational anteriorities, Character consonance, Ontological consonance, Primordial exactitude, Phenomenological correlates, Accolytish representation, Atrophied asseveration. Anamensial alienisms, Anchorite aperitif, Arcadian Atticism all delineated in the conditionalities and functionalities and orinetatilities of the system which incorporate the rules and regulations, axiomatic predications and postulation alcovishness of the foregoing state form the bastion, pillar, post, stylobate and sentinel of the classification scheme and doxa. appropriate and diagnostic, differentiating-discriminating, distinctive-distinguishing, emblematic- especial, essentially exclusive, fixed, idiosyncratic-inborn, inbred, indicative and individual, individualistic-inherent, individualizing and ingrained, inherent, innate, local, marked, native, normal, original, particularly- peculiar, personal, private, proper, regular, representative-singular, special, specific, symbolic, symptomatic- unique these form the classification and stratification measure that is suitable notwithstanding the constancy in many a case, as in gravity, which depends on the mass and distance for two given objects**.** Like Kant and Bergson, Deleuze considers traditional notions of space and time as unifying forms imposed by the subject. He therefore concludes that pure difference is non-spatio-temporal; it is an idea, what Deleuze calls "the virtual". (The coinage refers to Proust's definition of what is constant in both the past and the present: "real without being actual, ideal without being abstract.") Proust, Le Temps Retrouvé, ch. III: see the fourth line from the bottom of this page, or, in English translation, the thirteenth paragraph here: "I began to discover the cause by comparing those varying happy impressions which had the common quality of being felt simultaneously at the actual moment and at a distance in time, because of which common quality the noise of the spoon upon the plate, the unevenness of the paving-stones, the taste of the madeleine, imposed the past upon the present and made me hesitate as to which time I was existing in. Of a truth, the being within me which sensed this impression, sensed what it had in common in former days and now, sensed its extra-temporal character, a being which only appeared when through the medium of the identity of present and past, it found itself in the only setting in which it could exist and enjoy the essence of things, that is, outside Time. [...] **Nothing but a moment of the past Much more perhaps; something which being common to the past and the present, is more essential than both. [...] a marvellous expedient of nature had caused a sensation to flash to me—sound of a spoon and of a hammer, uneven paving-stones**—simultaneously in the past which permitted my imagination to grasp it and in the present in which the shock to my senses caused by the noise had effected a contact between the dreams of the imagination and that of which they are habitually deprived, namely, the idea of existence—and thanks to that stratagem had permitted that being within me to secure, to isolate and to render static for the duration of a lightning flash that which it can never wholly grasp, a fraction of Time in its pure essence. When, with such a shudder of happiness, I heard the sound common, at once, to the spoon touching

the plate, to the hammer striking the wheel, to the unevenness of the paving-stones in the courtyard of the Guermantes' mansion and the Baptistry of St. Mark's, it was because that being within me can only be nourished on the essence of things and finds in them alone its subsistence and its delight. It languishes in the observation by the senses of the present sterilised by the intelligence awaiting a future constructed by the will out of fragments of the past and the present from which it removes still more reality, keeping that only which serves the narrow human aim of utilitarian purposes. But let a sound, a scent already heard and breathed in the past be heard and breathed anew, simultaneously in the present and in the past, real without being actual, ideal without being abstract, then instantly the permanent and characteristic essence hidden in things is freed and our true being which has for long seemed dead but was not so in other ways awakes and revives, thanks to this celestial nourishment" While Deleuze's virtual ideas superficially resemble Plato's forms and Kant's ideas of pure reason, they are not originals or models, nor do they transcend possible experience; instead they are the conditions of actual experience, the internal difference in itself. "The concept they [the conditions] form is identical to its object."[Desert Islands, p. 36.] A Deleuzean idea or concept of difference is therefore not a wraith-like abstraction of an experienced thing, it is a real system of differential relations that creates actual spaces, times, and sensations. [See "The Method of Dramatization" in Desert Islands, and "Actual and Virtual" in Dialogues. In fact it not as if the style and substance of science and philosophers have been entirely different. look at the conformality and consonance in the Gaussian Theory and Deleuze's statements on identity and the loss of it in the following passage…….That is, identity is a continuous process that finds its limit in exhaustion. As Deleuze will write, "…the logical relation of causality is inseparable from a physical process of signalling, without which it would not be translated into action. By 'signal' we mean a system with orders of disparate size, endowed with elements of dissymmetry; by 'sign' we mean what happens within such a system, what flashes across the intervals when a communication takes place between disparates. The sign is indeed an effect, but an effect with two aspects: in one of these it expresses, qua sign, the productive dissymmetry; in the other it tends to cancel it" (dr, 20). An actualized entity just is such a signal-sign system, remaining in perpetual communication with the virtual multiplicity it actualizes, constituting its own elements and identity. Such is Deleuze's account of what Badiou refers to as the operation of the "count-for-one"… an operation that is immanent to being itself and energetic in character. yet how are we to discern the local nature of ontological situations, the manner in which there is no global, overarching global situation in which all local situations are embedded as if parts of a whole? in part the above passage already responds to this question. if series must be brought into relation to communicate in order for causality to occur, and if the resonance of series is always a matter of chance, of a throw of the dice, like Lacan's objet a where no series enjoys primacy of model over copy, then all we have are local situations without a total situation. Unlike Heidegger's analysis of Dasein where Dasein is always being-in-the-world, Deleuze's simulacra or actualizations are always divergent, enjoying only local relations. Heidegger argues that in order to understand the being of Dasein we must understand the manner in which Dasein is a being-in-the-world. By contrast, Deleuze's concept of multiplicity provides us with the principle of a new structuralism, of a new local ontology, that allows us to understand the immanent organization of a multiplicity without referring it to an embedding global space. as DeLanda so beautifully puts it in relation to the mathematician gauss, "…when gauss began to tap into these differential resources, a curved two-dimension surface was studied using the old Cartesian method: the surface was embedded in three-dimensional space complete with its own fixed set of axes; then, using those axes, coordinates would be assigned to every point of the surface; finally, the geometric links between points determining the form of the surface would be expressed as algebraic relations between the numbers. but gauss realized that the calculus, focusing as it does on infinitesimal points on the surface itself (that is, operating entirely with local information), allowed the study of the surface without any reference to a global embedding space. basically, gauss developed a method to implant the coordinate axes on the surface itself (that is, a method to implant the coordinate axes on the surface itself (that is, a method of 'coordinatizing' the surface) and, once points had been so translated into numbers, to use differential (not algebraic) equations to characterize their relations. as the mathematician and historian Morris Kline observes, by getting rid of the global embedding space and dealing with the surface through its own local properties, 'gauss advanced the totally new concept that a surface is a space in itself'" (intensive science and virtual philosophy, 11-12).according to the old Cartesian method, we can only outline

the properties of a space by relativizing it to a global space in terms of which it is then mapped. by contrast, gauss is able to explore a space in terms of its intrinsic metric and organization as a local space, without referencing it to a whole of which it is conceptualized as a part. **The Image of Thought.** "Image of thought "permeates both popular and philosophical discourse. According to this image, thinking naturally gravitates towards truth. Thought is divided easily into categories of **truth and error**. The model for thought comes from the educational institution, in which a master sets a problem and the pupil produces a solution which is either true or false. This image of the subject supposes that there are different faculties, each of which ideally grasps the particular domain of reality to which it is most suited. In philosophy, this conception results in discourses predicated on the argument that "**Everybody knows..."** the truth of some basic idea. Descartes, for example, appeals to the idea that everyone can at least think and therefore exists. Deleuze points out that philosophy of this type attempts to eliminate all objective presuppositions while maintaining subjective ones. Deleuze maintains, with Artaud, that real thinking is one of the most difficult challenges there is. Thinking requires a confrontation with stupidity, the state of being formlessly human without engaging any real problems. One discovers that the real path to truth is through the production of sense: the creation of a texture for thought that relates it to its object. Sense is the membrane that relates thought to its other. Accordingly, learning is not the memorization of facts but the coordination of thought with a reality. "**As a result, 'learning' always takes place in and through the unconscious, thereby establishing the bond of a profound complicity between nature and mind".** Deleuze's alternate image of thought is based on difference, which creates a dynamism that traverses individual faculties and conceptions. This thought is fundamentally energetic and signifying: if it produces propositions, these are wholly secondary to its development. Philosophy merges with ontology; ontology merges with univocity of being; analogy has always a theological vision; not a philosophical vision; one becomes adapted to the forms of god; self and world; the univocity of being does not mean that there is one and the same being; on the contrary, beings are multiple and different they are always produced by disjunctive synthesis; and they themselves are disintegrated and disjoint and divergent; membra disjuncta.the univocity of being signifies that that being is a voice that is said and it is said in one and the same "consciousness". Everything about which consciousness is spoken about being is the same for everything for which it is said like gravity; it occurs therefore as an unique event for everything; for everything for which it happens; even tum tan tum;it is the ultimate form for all of the forms; and all these forms are disjointed; it brings about resonance and ramification of its disjunction; the vorticity of being merges with the positive use of the disjunctive synthesis, and this is the highest affirmation of its univocity like gravity; it is the eternal resurrection or a return itself, the affirmation of all chance in a single moment, the unique cast for all throws; a simple rejoin der for Einstein's god does not play dice; one being, one consciousness, for all forms and all times. a single instance for all that exists, a single phantom for all the living; a single voice for every hum of voices or a single silence for all the silences; a single vacuum for all the vacuousness consciousness should not be said without conjuring if consciousness is one unique event in which all the events communicate with each other; univocity refers both to what occurs to what it is said. The attributable to all states of bodies and states of affairs and the expressible of every proposition So univocity of consciousness means the identity of thé noematic attribute and that which is expressed linguistically and sense-fully. Univovity.We mean that it does not allow consciousness to be subsisting in aquasi state and but expresses in all pervading reality. (Reference: LOS: Deleuze) Deleuze sometimes spoke of getting up behind an author and "creating a monster", as if his reading methodology somehow consisted of a distortion of the philosopher's or author's thought. Certainly there is a case to be made for this with regard to the process of deterritorialization, where something is wrested from a territory and deterritorialized upon a new territory, like the animal paw that is deterritorialized from the earth and reterritorialize on the branch. however, it seems to me that the more interesting aspect of Deleuze's approach to other philosophers and art is not so much his "monstrous creations" (i seldom find them particularly monstrous), but rather their Gaussian or Riemannian style, where he explores them in terms of their own internal organization and metric, without reducing them to something alien such as history, society, biology (reductivism), or the signifier. What we find in Deleuze's approach to phenomena is a Gaussian technique. for instance, take Deleuze's books on cinema, his book on Francis bacon, or his study of sacher-masoch. In the first instance, Deleuze's carefully separates cinema from narrative and the signifier, studying

it in terms of its specific organization pertaining to the production of images. in the case of bacon, Deleuze doesn't look for an underlying narrative or "meaning", but instead studies the manner in which bacon composes and organizes his images and lines, both in terms of their production and actuality, so as to liberate a logic of sensation. finally, in approaching sacher-masoch, Deleuze allows Sacher-Masoch's novels to speak for themselves in terms of their desire and relation to pain, stalwartly refusing to reduce masochism to the complement of sadism. in each case, we have a local exploration of a "space" of multiplicities that is extremely precise. Sean Bowden the priority of events: **Deleuze's logic of sense sean Bowden, the priority of events: Deleuze's logic of sense, Edinburgh university press, 2011, 296pp., $40.00 (hbk), isbn 9780748643646 Reviewed by David Scott, Coppin state university and Gaussian spaces and multiplicities a note on Gaussian spaces and multiplicities posted by larval subjects under uncategorized.** What you "see" is not always what you "get." Many people mistakenly take their own visions literally without "seeing through" the various possibilities that are outside their belief system, knowledge or skill base. Deeper reality is not remote in the physical sense but in a psychological sense. The archetypes of the collective unconscious are arrayed behind the scenes of current worldwide conditions, of crisis and confusion. They mirror our own states back at us, whether we perceive them as such or interpret them plausibly or not. The noise of ordinary consciousness and beliefs drowns out the signal. Unconsciousness is the background of our ordinary awareness. Our organism is very much at the center of such effects. The organismic source is our human bodies and the focus of human consciousness. The fantasy principle dethrones reality, but can be dissociative or compensatory. The human mind is a meme-Scape. Pre-conceived concepts vie with structures, concepts with images. Like scientists who ignore assumed truths, we leapfrog over our beliefs and personality deficits, claiming idiosyncratic imagination is literal reality. It couldn't be further from the truth and symbolism is utterly lost. The metaphor that might heal us enslaves us. Perhaps images like the holographic universe have an implicate order. Can we have a sense of the cosmos in the world without projecting myriad fantasies on it that we embrace literally? Has the world become so horrible it is unreasonable to be realistic? We may need to look at our drives and wishes, rather than the fantasy content. Psyche constructs reality. Our experience of so-called reality is always mediated by our image of it. Even if all the contents of the psyche are real, that doesn't mean they are realistic. That psyche is real is still a radical proposition, but psychic politics certainly color the self-image and ideas of everyone. We observe and participate with images It is not a question of nature or nurture (genes alone or experience alone). Rather, everything is both. We inherit the structures that make our experience what it is. But the structure itself is "empty," and each human culture "fills" it with its own specific adaptations. It is difficult to define an archetype and set boundaries that distinguish it from others. In a hologram each part contains all the information but in lower resolution. Archetypes have this holographic quality. There are patterns within patterns within patterns. Some overlap with others, and some are nested inside others. Archetypal realities, passed on through DNA, are expressed in distinctive neuronal tracts in the brain. They include customs and laws regarding property, incest, marriage, kinship, and social status or roles; myths and legends; beliefs about the supernatural; gambling, adultery, homicide, schizophrenia, and the therapies to deal with them. A mythic and visionary language of immediate experience encompasses themes of deepest, highest, and ultimate concern. Most fantasy-based individuals are at a complete loss to coherently explain their own conventional behavior much less anomalous events and their deep meaning, much less the cultural unconscious or mythological unconscious matrix. But they try, and become utterly entrenched in their belief that they are right about the nature of the world and reality. We have pseudo-memories about our personal lives. Why not more so for our collective life? The subject matter often revolves around catastrophe, creation and the mythopoeic forces of mankind. Ignorant of such dynamics, interpretive mistakes and displaced psychic contents proliferate into errors of fact. Propaganda, media distortions, memes, and disinformation compound the social problem of misapprehension further. Shameless self-promotion by personalities of such ideas leads to cults. They make up myths about the myths of by-gone eras. Roiling unconscious images can be fatally confusing. Thought illusions culminate in projections and projections of mythology. Jung suggested symbols live only as long as they are pregnant with meaning. Philosophy arose from criticism of myth, from discussing and challenging it. In science, we criticize, reject and eliminate theories. At the edge of the abyss of the unknown, new signs and symbols emerge. Credible theories and paradigms must include biology, physics, and neurophysiology. One of the reasons people

"see God", or a guru, or anomalies may be because our brains are constructed to see reality through the eyes of others. There are heaps of mirror neurons which are there to make us feel the 'other'. Mirror neurons do for psychology what DNA did for biology. They provide a unifying framework and help explain a host of mental abilities. As in the psychochemical processes of empathy or falling in love, a complex feedback loop sustains a state of mind. But when we empathically transpose ourselves into someone else's position, we expose ourselves to that reality -- cognitively and emotionally. The unconscious complicates empathy, both ways. Mirror neurons might well play a role in bonding, language and self-awareness. Naively, we take too much as self-evident. But 'seeing' does not always 'believe', though many make this error or leap in logic and formulate their choices and future accordingly. Yet, there is only one way to learn what consciousness is. Experience. But we have no satisfactory explanatory edifice for consciousness. Would such a theory release in each of us our own inner knowledge of the creativity of our own consciousness, and its infinite possibilities? The problem is trying to define a verb, a dynamic, as if it were a noun. But we do recognize the effect of consciousness. It functions to mediate states of consciousness, high and low psychobiological arousal. Consciousness is the subconscious lifted up by the physical body. When the body fails, the consciousness collapses back into the subconscious. All our thoughts come from the subconscious which can see our intentions but not our world. This relates somehow to intention being imaginary and not of the physical frictionized world (King). Gerald Edelman postulates that the flows of information in the brain are mediated through're-entrant' feedback loops. As evolution provides new cognitive functions, new re-entrant loops are established. Even language itself is an archetype -- a chaotic field of dynamic associations. A subtle net of tropes, grammar, symbols, and meaning, the program language begins in limbic resonance. Some phenomena generate their own language patterns, nomenclature, and internal coherence of meaning and representation. n a holographic universe, even time and space could no longer be viewed as fundamentals. Because concepts such as location break down in a universe in which nothing is truly separate from anything else, time and three-dimensional space would also have to be viewed as projections of this deeper order. At its deeper level reality is a sort of super hologram in which the past, present, and future all exist simultaneously. This suggests that given the proper tools it might even be possible to someday reach into the super holographic level of reality and pluck out scenes from the long-forgotten past. Or not. A fantasy of such penetration or phenomenon inside the head is not the same as that penetration. **Jung in the 21st Century: Synchronicity and science By John Ryan Haule Mind Control Countermeasures.** Adversely antagonistic, inexorably irreconcilable, antithetically antipodal, diametrically opposed, repugnantly retrograde, inimically inverse, violatively unsimilar, diversely dissimilar some statements in Vedanta, it is not superfluous or redundant. It is evolutionary. Autheticative determination, unimpeachable validation, incontrovertible establishment, apodictic evidence, reliable roll out, unfailing cinquecento **quattrocento trecento,** incontrovertible indication of the evolutionary process towards understanding is ever present in all, its thematic and discursive form. it is the relentless holding on to a theory this is despicable albeit the theory is not. Look at this paragraph. It is the 'I am the body/mind' belief that gives rise to the 'I am not (e) the world' belief. These two beliefs are (=) co-created."Consciousness projects (e&eb) the appearance of the mind, body and world by taking the shape of thinking, sensing and perceiving." "Attention is Consciousness with an object. When the object vanishes, attention simply remains what it always is, Consciousness.""There is (=) no purpose to (e) meditation. The purpose is already accomplished." "Everything that is experienced is experienced by, through, in and as Consciousness." "The seen cannot be separated from seeing and seeing cannot be separated from (e) Consciousness." "The Reality of any experience is not hidden in the appearance; it is expressed by the appearance." "Once we see that everything is Consciousness... Maya still dances, but it is a dance of love not seduction." There are some truly excellent sections such as a long one taking, as a starting point, the following quotation from the painter Cézanne: "Everything vanishes, falls apart, doesn't it? Nature is always the same but nothing in her that appears to us lasts. Our art must render the thrill of her performance, along with her elements, the appearance of all her changes. It must give us a taste of her Eternity." Rupert begins: "That statement must be one of the clearest and most profound expressions of the nature and purpose of art in our era." And he goes on for some 14 pages to elaborate on this claim, examining the nature of the 'elements' Existence, appearance and Consciousness and their relationship. "This Reality is the support or ground of the appearance. The appearance may be an illusion, but the

illusion itself is real. There is an illusion. It has Reality." He refers to the rope-snake metaphor and says that: "We do not see anything new. We see in a new way." "So that we know that nature is real, that there is something present, that there is a reality to it, even if everything that appears to us is insubstantial and fleeting." **The Transparency of Things' by Rupert Spira Book Review by Dennis Waite The following is a review of Rupert's book: 'The Transparency of Things: Contemplating the Nature of Experience'. There is an essay from the book here and the essay on Cézanne may be read at the awakened eye website For models see Mitigation Of Beam-Induced Backgrounds, Multi-Particle Azimuthal Correlations, Search For Extra Dimensions, Multi-Higgs-Boson Cascade, Top Quark Pair (Tt) Production Charge Asymmetry, $\Delta Y \equiv Yt - Yt^{-}$, Reflects The Asymmetry In $Tt^{-}$ Production, Ultra-Relativistic Electrons And Other Letters Of Interest: A Synecdochal Syncretism: Et Lux In Tenebris Lucet: Light Shines In The Darkness Models.** What then is the relation of Higgs boson and consciousness? Functionalist approaches generally assume that conscious experience appears as a novel property at a critical level of computational complexity. On the surface this would seem to deal with issues 1 and 3, however a conscious threshold has neither been identified nor predicted, and there are no apparent differences in electrophysiological activities between non-conscious and conscious activity. Regarding the nature of experience (why we are not unfeeling "zombies") functionalism offers no testable predictions. Problem 2) of 'binding' in vision and self is often attributed by functionalists to temporal correlation (e.g. coherent 40 Hz), but it is unclear why temporal correlation per se should bind experience without an explanation of experience. As functionalism is based on deterministic computation, it is also unable to account for Penrose's proposed non-computability (4), or free will (5). Something may be missing. To address these issues, various proposals have been suggested in which macroscopic quantum phenomena are connected to the brain's known neural activity. For the problem of unitary binding, Marshall (1989) suggested that coherent quantum states known as Bose-Einstein condensation occurred among neural proteins (c.f. Penrose, 1987; Bohm and Hiley, 1993; Jibu and Yasue, 1995). Pre-conscious to conscious transitions was identified by Stapp (1992) with collapse of a quantum wave function in pre-synaptic axon terminals (c.f. Beck and Eccles, 1992). In another proposal, protein assemblies called microtubules within the brain's neurons are viewed as self-organizing quantum computers ("orchestrated objective reduction - Orch OR" e.g**. Penrose and Hameroff, 1995; Hameroff and Penrose 1996a; 1996b; c.f. Hameroff 1997; 1998a; 1998b; 1998c; 1998d). Stability analysis of such theories would certainly provide a gateway for further bringing in the "reality" in space time.** String theory suggests that the graviton particle would not be detected from a particle collision because it would have "jumped" into another dimension. So, if the graviton exists and the theory is correct, there should be a "gap" in the energy profile of all the particles ejected from the collision. To understand this, think of breaking the pack of balls in a game of pool. The total energy of all the balls after the collision will depend on the force with which the cue ball was hit. So, after the collision, the energy of all the balls is measured and added-up to make sure it is the same as the energy of the cue ball. BUT - imagine that there is some energy missing, because one of the pool balls has "gone missing" - into a higher dimension. (POSTED BY THEREALJEFFHAL**) Talbot says** "If the universe is a hologram, in some sense it suggests that there may be two very drastically different levels of reality: the concrete reality that we see when we look at [things]... and at some deep level there's a level of reality where everything dissolves into an ocean of energy that is holographically interconnected.". A theory of everything must encompass all the variations however contradictory it may be. Nor need any theory be sent in to grand limbo of oblivion and hibernation or a differentiated groundless less ness because there are some irregularities which are like audit discrepancies need to be rectified. Although the language of religion and science seem different, both are talking about the central order of nature relative to DNA and information, as is Peirce, Petrus, Bacon, and pharaonic Egypt. This path-ordering, active information suggests that the central pattern or law of our holographic cosmos may be modeled on the biophysical battleground of gene regulation in bacteriophage Lambda, one of nature's most efficient and highly evolved mechanisms identified by experimental praxis. Egyptian texts support that the Lambda lifestyles of lysogeny and lysis can be understood as two paths to two living systems, photosynthesis and its reversal chemiluminescence, the cosmic key for evolution of mind and continuity. The texts support that the Lambda genome is the world-heart of two ways. The viral lifestyle of lysogeny controlled by cI protein results in matter, classical spacetime, our world of projected shadow,

photosynthesis, sexual genesis or vertical gene transfer, mind in the human body, and the imperfect continuum. In contrast, the viral lifestyle of lysis controlled by cro protein and lactose metabolism results in transformation to energy, the quantum, chemiluminescence, asexual genesis or HGT, mind in the cosmos, and the perfect continuum. Some scientists believe our cosmos is entangled with another (Chown 2007). On the holographic quantum level, perhaps this is just the competitive entanglement of cI and cro proteins cycling in perpetual motion—an inherent law of nature elucidating the assembly of things. In light of modern experiments, physicist John Wheeler (1988) speculated that the act of conscious observation functions with quantum mechanics, catalyzing a probability or outcome. So, if an observing consciousness has knowledge of the classical path and its magnetic field orientation, an act of measurement or choice might entangle space to reify the holographic quantum paths to a perfect continuum. Peirce guesses that to be drawn into a new system, a particle must have the right mass, the right velocity in the right direction, the right attraction, and it must present itself at the right point (EP 1. 270). Similarly, a finite probability exists for any particle that approaches the black hole event horizon to bounce back, dependent on the incoming particle's energy, its charge, and its projection of the orbital momentum on the axis of rotation of the black hole (Kuchiev 2003; 2004; 2004a), the same conditions in Egyptian texts (King 2004; 2006).Only an approximate conclusion remains: chance may be knowledge of the cosmos' holographic mode of operation and magnetic fields; habit breaking may result in the transformation of energy by a violation of the second kind; and evolution of mind or energy might be possible through HGT via a cosmic viral code of Lambda-genesis. "Order is simply thought embodied in arrangement;" (CP 6.490). Accordingly, the original meaning of the signs may relate to a viral biophysics of crystallized mind for human evolution via a natural chemical pathway to a perfect continuum. Again, this semiotics of evolvability and order recurs in time with the chaotic dream of reason that projects a pervasive falsification of our perceptual world. As Wallace Stevens envisions in "Connoisseur of Chaos", perhaps "the pensive man may see." **Peircean Process Metaphysics Origin of Science Quantum and Classical Laws Ancient Egyptian Evolutionary Biophysics** Adversely antagonistic, inexorably irreconcilable, antithetically antipodal, diametrically opposed, repugnantly retrograde, inimically inverse, violatively unsimilar, diversely dissimilar are the views of epistemologists in the analysis of raison d'etre of knowledge. Epistemology is the study of knowledge and those things closely related to it: justification, what it takes for you to be justified, the relation between knowledge and justification, whether you can have any justified beliefs at all, and if so, how you come to know (or justifiably believe) things, how you can use what you know (or justifiably believe) to come to know (or justifiably believe) other things, and whether and why it's valuable to know instead of merely having justified and true beliefs. The literature on epistemology is vast. Here's a very brief summary of some epistemological discussions. Concerning knowledge, many epistemologists think knowledge is justified true belief, where the justification you have is linked to the truth of the matter in the right kind of way, though what this way is a matter of debate; some epistemologists think knowledge can't be analyzed this way. Concerning justification's relation to knowledge, some epistemologists think we don't need to be justified to know, and some think we do need to be. Concerning what it takes to be justified, some epistemologists think that what it takes for you to be justified are only factors internal to the believer (Internalists). Others think it takes an external factor, like reliable or well-functioning cognitive faculties (Externalists). Skeptics argue that we can't have any justified beliefs at all, and many epistemologists reply to the skeptic's arguments. Concerning how we use what we know (justifiably believe) to come to know (justifiably believe) other things, some epistemologists (Foundationalists) argue that there are bedrock propositions that we know (justifiably believe), and we build our knowledge (justified beliefs) on those. Others (Coherentists) argue that there aren't bedrock propositions; rather, a set of beliefs is justified as a whole, and several beliefs can be mutually supporting. Concerning the value of knowledge, some argue that knowledge is intrinsically valuable. Others have argued that knowledge is valuable only because of the role it plays in practical reasoning, and others argue that knowledge isn't more valuable than justified and true belief, but there are other epistemic states such as understanding, that do have value above their proper subparts. **Edited y Matthew McGrath (University of Missouri, Columbia). It is not as if there is an anathema or misnomer in the philosophy or physics. Infact most of the pnysicists have used philosophical ideas towards the end of consummation of their their and their concomitant justification thereof.** The eternal return produces becoming-active. It is sufficient to relate the will to

nothingness to the eternal return in order to realize that reactive forces do not return. However far they go, however deep the becoming-reactive of forces, reactive forces will not return. The small, petty, reactive man will not return. Affirmation alone returns, this that can be affirmed alone returns, joy alone returns. Everything that can be denied, everything that is negation, is expelled due to the very movement of the eternal return. We were entitled to dread that the combinations of nihilism and reactivity would eternally return too. The eternal return must be compared to a wheel; yet, the movement of the wheel is endowed with centrifugal powers that drive away the entire negative. Because Being imposes itself on becoming, it expels from itself everything that contradicts affirmation, all forms of nihilism and reactivity: bad conscience, ressentiment..., we shall witness them only once. [...] The eternal return is the Repetition, but the Repetition that selects, the Repetition that saves. Here is the marvelous secret of a selective and liberating repetition. There is no need to remind the reader that neither the image of a centrifugal movement nor the concept of a negativity-rejecting repetition appears anywhere in Nietzsche's writings, and indeed Deleuze does not refer to any text in support of this interpretation. Further, one could highlight that Nietzsche never formulates the opposition between active and reactive forces, which constitutes the broader framework of Deleuze's interpretation. For some years, Marco Brusotti has called attention to the fact that Deleuze introduced a dualism that does not exist in Nietzsche's writings. To be sure, the German philosopher describes a certain number of "reactive" phenomena (for example, in the second essay of the Genealogy of Morality, § 11, he talks about "reactive affects" [reaktive Affekte], "reactive feelings" [reaktive Gefühlen], reactive men [reaktive Menschen]); but these are nonetheless the result of complex ensembles of configurations of centers of forces that remain in themselves active. Neither the word nor the concept of "reactive forces" ever appears in Nietzsche's philosophy. **After the discovery of the two principles of thermodynamics began a debate about the dissipation of energy and the thermal death of the universe which framed the modern renewal of the debate between the linear and circular conceptions of time**. Scientists such as Thomson, Helmholtz, Clausius, and Boltzmann and-by way of Kant, Hegel and Schopenhauer-philosophers such as Dühring, Hartmann, Engels, Wundt and Nietzsche have tried to address this problem by using the force of scientific argumentation and of philosophical discussion. Whoever believed in an origin and a final end to the motion of the universe (be it in the physical form of the gradual loss of heat, or in the metaphysical form of a final state of the "world process"), relied on the second principle of thermodynamics or on the demonstration of the thesis of Kant's first cosmological antinomy. On the contrary, those who refused to admit a final state to the universe used Schopenhauer's argument of infinity a parte ante-according to which if a final state were possible, it should already have established itself in the infinity of time past-to propose henceforth a number of alternative solutions. Scientists would propose the hypothesis that energy could have re-concentrated after a cosmic conflagration, thus reversing the tendency towards dissipation. Those belonging to the monistic and materialistic tradition relied on the first principle of thermodynamics and on the infinity of matter, space and time, and regarded the universe as an eternal succession of new forms. A certain critical agnosticism was widespread among scientists and philosophers, oftentimes through a reaffirmation of the validity of Kant's antonymic conflict, this movement avoided to take a stand on specifically speculative issues. Other German philosophers, like Otto Caspari, or Johann Carl Friedrich Zöllner, had reintroduced an organicist and pan-psychical conception of the universe, investing atoms with the ability to escape any state of balance. Indeed, it is probably one of Otto Caspari's works, The Correlation of Things (Der Zusammenhang der Dinge. Gesammelte philosophische Aufsätze (Bleslau: Trewendt, 1881)), which awakened Nietzsche's interest for all things cosmological, in that summer of 1881, in Sils-Maria. addressing Schopenhauer and Eduard von Hartmann's mystical pessimism according to which the world is the creation of a stupid and blind essence (which, after having created the world by mistake, comes to the realization that it had made a mistake and strives to return it to nothingness) Caspari stresses that it is nothing short of mystical to imagine that the world may have been borne out of a an originary and undifferentiated state. Where would it have drawn the first impulse? But, continues Caspari, even if the world had received this first impulse from some deus ex machina, there is no doubt that, in the temporal infinity of past time thus far, it would have either attained the end of the process (but this is impossible because the world would then have ended), or it would be necessarily bound to repeat indefinitely this original mistake, and the entire process that accompanies it. But then, what is the process of the world? We must now take one more step back and understand further the process of the world according to von Hartmann. Hartmann

objects that the regressive movement postulated by Schopenhauer is possible only in thought: it remains nothing more than an "ideal postulate" with no real object and which "does not teach us anything about the real process of the world that unfolds in a movement contrary to this backwards movement of thought" (Hartmann, Philosophie des Unbewussten, third edition (1871): 772). Hartmann affirms that if unlike Schopenhauer one admits the reality of time and of the world process, one must also admit that the process must be limited in the past and therefore that there must be an absolute beginning. In Hartmann's mind, failure to do so would result in positing the contradictory **concept of an accomplished infinity**: "The infinity that from the point of view of regressive thinking, remains an ideal postulate, which no reality may correspond to, must, for the world, whose process is, on the contrary, a progressive movement, open up to a determinate result; and here the contradiction comes to light" (Hartmann (1871): 772). What really "comes to light" in this passage is the fact that Hartmann does not provide a demonstration but a petitio principii. Indeed, the concept of the world process analytically contains the concept of a beginning of the world. In all rigor, it is therefore impossible to demonstrate these concepts with reference to each other. Secondly, Hartmann's view that one is bound to accept the reality of the world process even if one rejects the ideality of Schopenhauer's time is mistaken. Hartmann believes that if time is real there must be a world process with both an **absolute beginning and an absolute end. Without any justification, Hartmann jumps from Schopenhauer's negated time to oriented time.** Hartmann's view is that the world process leads into a final state absolutely identical to the initial state. However, it follows from this that even as the cosmic adventures of the unconscious come to a close, we are still haunted by the specter of a new will and of another beginning of the world process. This exposes a serious internal flaw of Hartmann's system insofar as it jeopardizes the possibility of a final liberation from existence and suffering. This is why in the last pages of his work, "On the Last Principles," he painstakingly calculates the degree of probability of a reawakening of the volitional faculty of the unconscious. Insofar as the will is entirely free, unconditioned and a-temporal, the possibility of a new volition is left to pure mathematical chance and is therefore ½. Hartmann further stresses that if the will were embedded in time, the probability of the repetition would amount to 1 and the process of the world would be bound to begin again, in an eternal return which would completely preclude the possibility of a final liberation. Fortunately, this is not the case since-according to Hartmann's remarkable logic-the world-process develops through time, but the original will is outside of time. In fact, one may even affirm, along the lines of Hartmann's peculiar theory of probability, that every new beginning gradually reduces the probability of the next beginning: let n be the number of times that the will is realized, the probability of any new realization is ½ n. "But it is clear that the probability ½ n diminishes as n increases, in a way that suffices to reassure us in practice" (Hartmann (1869): 663). In his famous work entitled **On the Conservation of Force (1847), Hermann von Helmholtz** had divided the totality of the energy in the universe between potential energy and kinetic energy and affirmed the reciprocal convertibility of the two. In 1852, William Thomson pointed out that there exists a sub-ensemble within kinetic energy, heat, which, once it has been generated, is no longer entirely convertible into potential energy-or into any other form of kinetic energy. Considering that the (partial) reconversion of heat into labor is possible only in situations that present a disparity in temperature, and that heat tends to pass from warmer to cooler bodies by spreading on an even temperature level through space, Thomson concluded that the universe tends towards a final state where any energetic transformations, every movement and every form of life will cease. Caspari's atoms (which bring to mind those in Leibniz's Monadology) resemble some sort of biological monads, endowed with internal states. For Caspari, every atom obeys the ethical imperative to participate in the conservation of the general organism and its movement does not only follow the simple physical kind of interaction but also an a priori law ensuring. If a balance of forces had been attained at any moment, this moment would still be going on: therefore, it never happened. The present state contradicts this proposition. Supposing that a certain state rigorously identical with the present state had, one day, existed, this supposition is not refuted by the present state. As one of the infinite possibilities, it is necessary that the present state had been given anyway, since until now an infinite period of time has already unfolded. **If equilibrium were possible, it must have occurred**; and if the present state has already taken place, then so too the one that preceded it as well as the one preceding that one. Therefore it has already taken place a second time, a third time and so on. And likewise it shall take place again a second time, a third time. Innumerable times forwards and backwards. This amounts to saying

that all becoming occurs within a repetition of an innumerable number of absolutely identical states. [.] The immovability of forces, their equilibrium is a conceivable case, but it has not occurred. As a result the number of possibilities is greater than the number of realities. -The fact that nothing identical recurs may be explained not thanks to chance, but only thanks to an intention infiltrated within the essence of force. Indeed, supposing an enormous amount of cases, the random occurrence of the same combination is more probable than the same combination never recurring. The quantum of force in the universe is determinate and not "infinite": let us beware [hüten wir uns] from such conceptual extravaganza! Therefore the number of situations, modifications, combinations and developments of this force is doubtless enormous and practically "immeasurable," but in any case this number is determinate and not infinite. On the other hand, the time in which the universe exerts its force is infinite. That is to say, that force is eternally identical and eternally active: -until the present instant an infinity has already taken place, that is to say that all possible developments must have already taken place. Consequently, the present development must be a repetition and therefore both this that was born from it and this that shall be born from it and so on both forwards and backwards. Everything has taken place an innumerable number of times because the overall situation of all forces always recurs (FP 11 [202] of 1881). Nietzsche regards the mechanistic vision as more plausible and less anthropomorphic than organicism. However, faced with the two major **cosmological models of his time, the mechanistic model and the organic model, Nietzsche wishes to return its polymorphous, proteiform, unstructured** and chaotic character to nature of which the perfectly non-theological and non-teleological theory of eternal return is the strongest seal. This is the first of the "new battles" which come to whoever is aware of the consequences of the death of God: take any antropomorphism away from nature. In the preparatory papers, the third book of the Gay Science is entitled "Gedanke eines Gottlosen / Thoughts of a Godless One." Aphorism 109 of this book, which immediately follows the famous aphorism against the shadows of God, **Boltzmann's theory belongs to the third phase of the debate on thermodynamics and cosmology. The publication of Thomson's brief paper "On a Universal Tendency in Nature to the Dissipation of Mechanical Energy" of 1852 signals the beginning of the scientific controversies on the problem of the dissipation of energy and opened the first phase of the debate, which was announced in the Reflexions on the Motor Powers of Fire by Sadi Carnot and whose conclusion is represented by Clausius's recapitulative article on the concept of entropy in 1865.[44] The second phase started in 1867 when, at the forty-first congress of German scientists and doctors, Clausius gave a lecture on "The Second Principle of the Mechanistic Theory of Heat," where he applied the results of his research on thermodynamics to the universe**. It is true that in his famous lecture of 1854, Helmholtz had already presented the cosmic consequences of the second principle, but Clausius' contribution had a strong impact on German culture. This is because in this lecture he robustly rejected the possibility to consider the universe as an eternal and self-renewing circle, an ewiger Kreislauf in which force and matter are in constant transformation, as was heretofore affirmed by the materialism of the scientists and philosophers, and he did so in the name of the second principle of thermodynamics. In this way, the debate on the principles of thermodynamics gained great importance in European Culture starting in 1867. In the two first phases, it is Thomson's mechanism that predicts the thermal death of the universe. In the third phase, on the contrary, the meaning of the term mechanism changes radically.[46] In accordance with the apocalyptic climate of this period dominated by the "rebirth of idealism," the "overcoming of scientific materialism" and the "bankruptcy of science", the mechanistic paradigm which had accompanied the birth of modern science became challenged on **account of the second principle of thermodynamics**. According to the theorem of the quasi-periodicity of the motions of mechanical systems demonstrated by Poincaré as part of the problem of the three bodies (1890), a mechanical system must evolve according to a quasi-periodical movement and consequently it must always return-sooner or later-to the initial state. **An easily established theorem informs** us that a limited world obeying solely the laws of mechanics shall always pass through a state closely similar to its initial state. On the contrary, according to established experimental laws (supposing we grant them absolute value and wish to push their consequences to the end), the universe is directed towards a final state, which once it is attained, it shall not be able to escape. In this final state, which shall be like a sort of death, all material bodies shall be at rest at the same temperature. **Poincaré's theorem seems** therefore incompatible with the second principle of thermodynamics, which predicts a

unidirectional movement of all natural phenomena until the whole universe is brought to a total standstill. Wilhelm Ostwald and the entire energeticist school of thought contended that the principles of thermodynamics were fundamentally new, and could not be re-incorporated to traditional physics and that they should serve as a basis for a new science that regards the qualitative diversity of energy and its tendency to degradation as its axioms. Against energeticism and in an effort to bring entropic phenomena back into the theoretical framework of mechanism, Ludwig Boltzmann introduced the concept of probability in physics, not as an instrument of calculation, but as an explicative principle. In Boltzmann's statistical thermodynamics, the increase of entropy assumed by Clausius is re-interpreted as an increase in molecular chaos. As a result, it becomes possible to explain mechanistically the evolution of closed systems endowed with increasing entropic value, without it committing us to granting absolute value to the second principle of thermodynamics. **Moreover, one no longer needs to fear the thermal death of the universe insofar as the state of equilibrium will in principle** never be complete, but rather will be attained only statistically, leaving open the possibility of fluctuations towards less probable states. Boltzmann's critics remarked that this hypothesis involved two paradoxes called the objection of reversibility (Umkehreinwand), and that of repetition (Wiederkehreinwand). I shall only address here the second one since it coincides with the theory of the eternal return. Based on Poincaré's theorem quoted above, Ernst Zermelo objected to Boltzmann that his model of the universe suggested that after a finite (if admittedly very long) time the system would return to its initial position. In his first response to Zermelo, Boltzmann avoids committing himself directly to cosmological questions and he only observes that, in the case of concrete thermodynamic systems, the time of recurrence may be extremely long. For example, in normal conditions of pressure and temperature, one-centimeter cube of gas requires 1010 years to reach a molecular configuration identical to the original one! However, following a response by Zermelo, Boltzmann wrote a new article where he outlines a cosmological picture that he will re-use later in his conclusion to his famous Lectures on Gas Theory. In this cosmological picture, Boltzmann considers the universe as a closed system with constant entropy, within which some fluctuations occur, creating islands of negative entropy. Our solar system originates in one of these fluctuations. As Clausius correctly pointed out, the entropy of our solar system increases constantly as the solar system gets closer to the state of chaos and of the thermal death of the rest of the cosmos. However, in other zones of the universe, some new fluctuations and new islands appear, so that thermal death is never generalized. Here we are given a grand cosmic image, in which the solar system and the sparkle of life that was lit on planet earth are only a fluctuation of order from within a dominant entropic tendency. Life, and the order on which it is based are exceptions, transitory forms taking place in the realm of the shapeless, they are islands of the cosmos that will soon be re-absorbed into chaos. According to Poincaré's theorem, our island will have to be reborn, to develop and die innumerable times in a strictly identical fashion. This happened an infinite number of times during the past eternity and it shall take place again an infinite number of times in the eternity to come. **Boltzmann accepts the "paradox" of recurrence-that is the eternal return of the same-as a legitimate consequence of the probabilistic conception of thermodynamics**. It may be rejected for ethical reasons, it may be stored away as an abstract speculation or dismissed along with other cosmic fantasies, but it cannot be rejected on the basis of any rigorously scientific viewpoint. **The Eternal Return: Genesis and Interpretation BY PAUL D'IORIO(excerpts). Nietzsche found the template of a materialistic cosmology based upon the first principle of thermodynamics. During the same year, he could have found a model of an organistic solution to the problem of thermal death of the universe as well as a discussion on the conformation of space in Zöllner's book On the Nature of Comets** (Johann Carl Friedrich Zöllner, Über die Natur der Kometen. Beiträge zur Geschichte und Theorie der Erkenntnis (Leipzig, Engelmann, 1872), 299 f. and 313 f.); Assumed Association In Space Time Of Particles Or Men Leads To Detrimental Ramification Be In Terms Of Progress Of “Subject Matter” Or In The Pathological Disequilibrium Of Gratification Deprivation Complex Which Again Has Mental Turbulence In Case Collective Consciousness Approve It (Raaga) Or Pernicious Implications In Case The Theory Is Not Accepted; After All What Is A Theory: Set Of Correlations And Causations Which Are “Tested” Against A Collective Consciousness Background, Which Yields “Results’ Which Are Themselves In “Question” Due To The Very Perception And The Simulation Prospective. The Definition Of Quantum Theorists' Terms, Such As Wavefunctions And Matrix Mechanics, Progressed Through Many Stages. For Instance, Erwin Schrödinger

Originally Viewed The Electron's Wavefunction As Its Charge Density Smeared Across The Field, Whereas Max Born Reinterpreted It As The Electron's Probability Density Distributed Across The Field. Although The Copenhagen Interpretation Was Originally Most Popular, Quantum Decoherence Has Gained Popularity. Thus The Many-Worlds Interpretation Has Been Gaining Acceptance Moreover, The Strictly Formalist Position, Shunning Interpretation, Has Been Challenged By Proposals For Falsifiable Experiments That Might One Day Distinguish Among Interpretations, As By Measuring An AI Consciousness Or Via Quantum Computing. More Or Less, All Interpretations Of Quantum Mechanics Share Two Qualities: They Interpret Formalism—A Set Of Equations And Principles To Generate Predictions Via Input Of Initial Conditions They Interpret A Phenomenology—A Set Of Observations, Including Those Obtained By Empirical Research And Those Obtained Informally, Such As Humans' Experience Of An Unequivocal World Two Qualities Vary Among Interpretations: Ontology—Claims About What Things, Such As Categories And Entities, Exist In The World Epistemology—Claims About The Possibility, Scope, And Means Toward Relevant Knowledge Of The World. Now How Does A Person In A Crowd Behave? They He Would Due To Herd Mentality; Ok; That Is One Theory; Simulation By Terrestial Brahman Anti Brahman Will Make Them Behave In A Highly Organised And Coordinated Manner; With Association Of Things And Individuals Already Established Resultant Orientation Of The Actions Hall Be Only Oriented Towards The Figment Of Imagination And Product Of Puerile Prognostication Formally Established By Terrestial Brahman Anti Brahman Before Such A Coordinated And Organised Execution Of Exercise Is Conducted; Analogically, If A Simulation Is Going On In A Bunch Of Particles Or Electrons, And If Some Agency Say Celestial Brahman Anti Brahman Agency Is Simulating It, There Shall Be Coordination And Organisation So That You See “Reality” While The Truth Shall Remain Hidden. Result Is The Suppression Of Information, Delineation Of Disinformation, And The Concomitant Development Of Mushrooms Of Concocted And Fabricated Theories To That “Reality”; In The Case Of Terrestial Brahman Anti Brahman Agency Intention May Be To Bulldoze And Bowdlerize You, Baffle And Befuddle You In To Submission; In The Case Of Celestial Brahman Anti Brahman Agency Only Reason Could Be To Shield The Truth Being Disseminated; But Why?;May Be You Should Never Become God Which You Are Trying To; May Be God Is Really Playing Dice; What Does That Result In To ; You Become A Scezophreniac Associating Again And Again What You Had Been Until Now Of Particles And Other Concomitants Again And Again Leading To Flawed Inferences. Confusing The Epistemic With The Ontic, Like If One Were To Presume That A General Law Actually "Governs” Outcomes—And That The Statement Of A Regularity Has The Role Of A Causal Mechanism—Is A Category Mistake. In A Broad Sense, Scientific Theory Can Be Viewed As (=) Offering Scientific Realism—Approximately True Description Or Explanation Of The Natural World—Or Might Be Perceived With Antirealism. A Realist Stance Seeks The Epistemic And The Ontic, Whereas An Antirealist Stance Seeks Epistemic But Not The Ontic. In The 20th Century's First Half, Antirealism Was Mainly Logical Positivism, Which Sought To Exclude Unobservable Aspects Of Reality From Scientific Theory. Since The 1950s, Antirealism Is More Modest, Usually Instrumentalism, Permitting Talk Of Unobservable Aspects, But Ultimately Discarding The Very Question Of Realism And Posing Scientific Theory As A Tool To Help Humans Make Predictions, Not To Attain Metaphysical Understanding Of The World. The Instrumentalist View Is Carried By The Famous Quote Of David Mermin, "Shut Up And Calculate", Often Misattributed To Richard Feynman. Quantum Mechanical Wave Function (Absolutely Squared) Describes The Completed Interference Pattern, It Must Describe An Ensemble Interpretations Of Quantum Mechanics From Wikipedia. Philosophy Of Science, The Distinction Of Knowledge Versus Reality Is Termed Epistemic Versus Ontic. A General Law Is A Regularity Of Outcomes (Epistemic), Whereas A Causal Mechanism May Regulate The Outcomes (Ontic). A Phenomenon Can Receive Interpretation Either Ontic Or Epistemic. For Instance, Indeterminism May Be Attributed To Limitations Of Human Observation And Perception (Epistemic), Or May Be Explained As A Real Existing Maybe Encoded In The Universe (Ontic). Confusing The Epistemic With The Ontic, Like If One Were To Presume That A General Law Actually "Governs” Outcomes—And That The Statement Of A Regularity Has The Role Of A Causal Mechanism—Is A Category Mistake. Interpretations Of Quantum Mechanics From Wikipedia. Reality Is Constant In The Midst Of Change. Note The Conservation Of Individual, Collective And Cosmic General Ledgers Are What Are Being Spoken Of. Not The Changing Transactions, But The Immutable Refractory Zero. (Italics Mine) What This Means Essentially Is

That There Is Change Although Nothing Changes. Like There Are Changing Transactions And Still The Assets And Liabilities Will Be Equal. This Impossible Situation Is Reflected (E&Eb) In The Ultimate Impossibility Of Change Itself. That Which Does Not Exist Prior To Its Changing And At The End, After It Has Changed, Must Be Equally Non-Existent Between These Two Moments. Although The World Of Change Appears To Be Real, It Cannot Be So. (Real)Change, According To The Vedantin, Presupposes A Loss Of Identity. Reality Cannot Suffer Transformation If It Were To Do So, It Would Become Something Else And The Real Would Be Deprived Of Its Reality. The Immortal Can Never Become Mortal, Nor Can The Mortal Become Immortal The Doctrine Of Vibration: An Analysis Of The Doctrines And Practices Of Kashmir Shaivism Mark S. G. Dyczkowski. Concept Of Being From Shaivite Point Of View: In His Famous Work Entitled On The Conservation Of Force (1847), Hermann Von Helmholtz Had Divided The Totality Of The Energy In The Universe Between Potential Energy And Kinetic Energy And Affirmed The Reciprocal Convertibility Of The Two. In 1852, William Thomson Pointed Out That There Exists A Sub-Ensemble Within Kinetic Energy, Heat, Which, Once It Has Been Generated, Is No Longer Entirely Convertible Into Potential Energy-Or Into Any Other Form Of Kinetic Energy. Considering That The (Partial) Reconversion Of Heat Into Labor Is Possible Only In Situations That Present A Disparity In Temperature, And That Heat Tends To Pass From Warmer To Cooler Bodies By Spreading On An Even Temperature Level Through Space, Thomson Concluded That The Universe Tends Towards A Final State Where Any Energetic Transformations, Every Movement And Every Form Of Life Will Cease: The Eternal Return: Genesis And Interpretation BY PAUL D'IORIO: The Ultimate Nature Of Anything Cannot Change. Change Of Any Sort Is Merely Apparent (Vivarta). The World Of Change And Becoming Is A False Super- Imposition (Adhyaropa, Adhyasa) On The Absolute. It Appears So On The Screen Of Individual, Collective Consciousness, Albeit One Cannot Be So Sure Of Cosmic, For There Are Many Reasons That Cosmic Transactions Also Had To Be Subjected To Change By ParaBrahaman. In Cosmic Terms, The Mistake (Bhranti) Consists Of The Supposition That The Real Brahman Is The Unreal Universe And The Unreal Universe Is The Real Brahman. In Microcosmic Terms, It Is The Mistake Of Falsely Conceiving The Body, Mind Or Even One's Personality To Be The Self. Note We Have Taken Self As Witness Consciousness. Notwithstanding There Is Nothing Wrong In The Conception Of Self As A Dynamical Presupposition, In That Individual Consciousness And Collective Consciousness And Even Cosmic Consciousness Evolutes In To ParaBrahaman. In The Same Way As The Image Of A Snake Is Falsely Superimposed On A Rope, Similarly The Universe Is Falsely Projected Onto The Real Substratum, The Brahman. Ignorance Is Not Merely A Personal Lack Of Knowledge, But A Cosmic Principle. As Such It Is Called "Maya," The Indefinable Factor (Anirvacaniya) That Brings This Mistake In Identity About. The Reality Status Of This Cosmic Illusion Is Also Indefinable: On The One Hand It Is Not Brahman, The Sole Reality; On The Other Hand It Is Not Absolutely Non-Existent Like A Hare's Horn Or The Son Of A Barren Woman. Note Here The Vedantin Avoids Talking About Zero Which We Have Avidly And Assiduously Stressed About. Nothing Is The Factor That Exists. Also See the Remarks about Satre's Being And Nothingness. Brahman Is The Source Of World Appearances Only In The Sense Of Being Their Unconditioned Ground Or Essential Nature. The Universe Is False Not Because It Has No Nature Of Its Own But Because It Does Have One. The Doctrine Of Vibration: An Analysis Of The Doctrines And Practices Of Kashmir Shaivism Mark S. G. Dyczkowski. Concept Of Being From Shaivite Point Of View: It Is The 'I Am The Body/Mind' Belief That Gives Rise To The 'I Am Not The World' Belief. These Two Beliefs Are (=) Co-Created."Consciousness Projects The Appearance Of The Mind, Body And World By Taking (Eb) The Shape Of Thinking, Sensing And Perceiving.""Attention Is Consciousness With An Object. When The Object Vanishes, Attention Simply Remains What It Always Is Consciousness.""There Is No Purpose To Meditation. The Purpose Is Already Accomplished.""Everything That Is Experienced Is Experienced By, Through, In And As Consciousness.""The Seen Cannot Be Separated From Seeing And Seeing Cannot Be Separated From Consciousness." "**The Reality Of Any Experience Is Not Hidden In The Appearance; It Is Expressed By The Appearance The Transparency Of Things'** By Rupert Spira Book Review By Dennis Waite The Following Is A Review Of Rupert's Book: 'The Transparency Of Things: Contemplating The Nature Of Experience'. There Is An Essay From The Book Here And The Essay On Cézanne May Be Read At The Awakened Eye Website Nature Rejects The Naiveté That Seeks Absolute Truth. We Are Beginning To Realize, Individually And Culturally, That

"Realities" Are All Human Constructions. The Task Becomes One Of "Catching Ourselves In The Act" Of Creating Our Own "Reality" From The Flow Of Events. Human Truth Is Always An Engagement Of Mind With Experience. The Challenge Of The Therapist In These Times Of Chaotic Change Is To Validate The Concept That We Don't Have To Fear The Collapse Of What We Think We Are. Strange Attractors: Transference, Holography, And An **Archetype Burke, J. (2003). Strange Attractors: Transference, Holography, And An Archetype (Doctoral Dissertation, Pacifica Graduate Institute, 2003).** When we get to the topic and understanding of the Transcendental Logic, the brilliant and baffling Transcendental Deduction (TD) Recall the two movements just discussed, the one from experience to its conditions and the one from the forms of valid inference to the concepts that we must use in all judging (the Categories). This duality led Kant to his famous question of right (quid juris) (A84=B116): with what right do we apply the Categories, which are not acquired from experience, to the contents of experience? (A85=B117). Kant's problem here is not as arcane as it might seem. It reflects an important question: How is it that the world as we experience it conforms to our logic? In briefest form, Kant thought that the trick to showing how it is possible for the Categories to apply to experience is to show that it is necessary that they apply (A97).TD has two sides, though Kant never treats them separately John Marlado, accounts, attributes, and ascribes the philosophy of Nishida KitarÅ is lucid, insightful, and deeply informative despite the highly questionable argument that gives the book its structure. In comparison with other, technically more accurate accounts of Nishida's philosophy, it is broader in scope than most but more single-minded in its approach. The analysis is for the most part limited to four books that have been translated into English, out of approximately 16 volumes of philosophical essays in Japanese, but it aims to capture the contours and most important details of Nishida's entire philosophy. To appreciate the achievement of this account, one would need only to read, in English translation or in the Japanese original, any of the volumes with their streams of repetitive, puzzling expressions and circuitous, often dead- end argumentation. Wilkinson's elucidation of details in the passages he samples makes it easier to understand why Nishida is considered Japan's greatest academic philosopher. A critical look at the frame of the book and some of its own claims reveals where, to my mind, it has gone astray. The argument that frames this book is repeated throughout: Nishida's philosophy is an attempt to articulate, in a characteristically Western philosophical manner, the Zen experience he had as a young man. Nishida first made the attempt by using the conceptual frameworks of Western philosophers, and after they failed him he developed his own, largely by recasting Buddhist insights. In the end, a philosophical articulation of the Zen worldview in Western terms or even in Eastern terms recast by conceptual, Western philosophical methods approved impossible. What Nishida attempted is incommensurable with Western philosophy. While the author first offers these statements as hypotheses (p. 2), one premise of his argument that there is such a thing as "the Zen world-view" or "the Zen experience ‘‘remains unquestioned; and another that Nishida's pivotal experience was the Zen experienceâis asserted as "beyond question" (p. 151; see also pp. 28, 48, 10, 114).Wilkinson's presentation of Zen in the first chapter is drawn largely from D. T. Suzuki, with help from Suzuki's portage Abe Masao; some comments on time, birth, and death in; and a few other sources. The result is a caricature that imagines Zen as an invariable, unitary experience. In fact, Dagan’s teachings on practice as the manifestation of enlightenment do not sit well with D. T. Suzuki's emphasis on a satori experience, and Suzuki does not enlighten us about what Zen in its various practices has historically been. The author would have benefited from acquaintance with other presentations by Zen teachers, such as that of ShunryÅ Suzuki'sZen Mind Beginner's Mind, with historical accounts of Zen traditions, and with contrasts between DÅgen's view of time and Sam Â· sÄraand that of other Buddhist philosophers. A little research in contemporary Zen scholarship would have shown how implausible is the talk of "nirvana or awareness of mu" (p. 24), "the basic Zen position [that] never varies" (p. 3), "the Zen vision" (pp. 5, 7), "Zen epistemology" , "Zen experience" (p. 59), "the conception of time involved in Zen" (p. 15), and "the fundamental Zen assertion that there is no absolute time" (p. 16). Even if DÅgen's notion that time and being are nondual (p. 16) epitomized most of Zen, Nishida's early notion of "a transcendent, unchanging reality apart from time" (p. 51) would contrast sharply with it. Equally questionable is Wilkinson's contention that Nishida's "pure experience" is equivalent to Zen satori. The notion of pure experience Nishida developed in his first major work may be pivotal for his entire philosophy, but he never claims or implies that any part of his philosophy presupposes an exceptional experience like satori that grounds his convictions and renders them unverifiable to

those who have not had the experience, as Wilkinson intimates (pp. 55, 159, and 161). Nishida presents his notions of pure experience and intellectual intuition, the grasp of its unity, and later... Perennialism and Constructivism is studied by Randolph T Dible : experiencer or the observer and the transcendental subjectivity itself Pure objectivity is essentially a deconstruction, apophatic and negativa The encountering of the experiencer or observer—transcendental subjectivity itself—at the foundation of the world leads inevitably to the recognition of pure objectivity as ultimate reality (which can be taken as its ultimate deconstruction, analogous to the apophatic or via negativa), from which objects derive their value, weight, significance, meaning or objectivity. In this way, pure objectivity can be seen as the supra-self-evident Axiological Axiom, so to speak, even Unconditional Love, in romantic terms. This axiology (value theory) has a structure inverse to the relationship between transcendental subjectivity as the radical unity of pure self-reference and on the other hand, the world of forms, as mere traces (representations, indications) of the unique, original "first distinction" Spencer-Brown speaks of at the foundation of his calculus. That is, all forms (i.e., distinctions, differences) would reduce to being the first distinction, also known as the marked state, which can be called penultimate reality (pure self-reference or transcendental subjectivity: the Spirit which animates us), except that forms are complimentary to their content, which is their objectivity or value, which would reduce to the unmarked state or ultimate reality. It is the incongruity of form (thoughts; Whitehead's "negative prehensions") and value (feelings; Whitehead's "positive prehensions," or my notion of objectivity, meaning and qualia; in short, the non-formal aspects of experience) that holds forms open and keeps them from absolute reduction. This accounts for the brute, concrete persistence of the "functional illusion"-- to use a term from Dzogchen Buddhism-- of the world. Thus this system has an axiology of metaphysical objectivity grounded on the ideal of pure objectivity as the source of all value, meaning and significance, itself the very fecundity of profundity, which is the motive of drawing the distinction in the first place **(Wikipedia, Kant's writings and Logic Of Sense)** Nano rings were created by accident while intending to make quantum dots. They have interesting optical properties associated with excitons and the Aharonov–Bohm effect. Application of these rings used as light capacitors or a buffer includes photonic computing and communications technology. Analysis and measurement of geometric phases in mesoscopic rings is ongoing. Several experiments, including some reported in 2012, show Aharonov-Bohm oscillations in charge density wave (CDW) current versus magnetic flux, of dominant period h/2e through CDW rings up to 85 **μm** in circumference above 77 K. This behavior is similar to that of the superconducting quantum interference devices (seeSQUID). **Aharonov–Bohm nano rings (Wikipedia) Stability analysis of nano rings would help progressively accentuate the research about the materials division that uses nano rings.** From the profound contemplation and wisdom of the Buddha and Mahabodhisattvas, we know there is no such reality. Instead, egolessness (non-self) is the only path to understand the reality of the deluded life. All existences are subject to the law of causes and conditions. These include the smallest particles, the relationship between the particles, the planets, and the relationship between them, up to and including the whole universe! From the smallest particles to the biggest matter, there exists no absolute independent identity. Egolessness (non-self) implies the void characteristics of all existence. Egolessness (non-self) signifies the non-existence of permanent identity for self and existence (dharma). Sunyata stresses the voidness characteristic of self and existence (dharma). Sunyata and egolessness possess similar attributes. As we have discussed before, we can observe the profound significance of sunyata from the perspective of inter-dependent relationships. Considering dharma-nature and the condition of nirvana, all existences are immaterial and of a void-nature. Then we see each existence as independent of each other. But then we cannot find any material that does exist independent of everything else. So egolessness also implies void-nature! From the observation of all existences, we can infer the theory of nirvana and the complete cessation of all phenomena. From the viewpoint of phenomena, all existences are so different from each other, that they may contradict each other. They are so chaotic. In reality, their existence is illusionary and arises from conditional causation. They seem to exist on one hand, and yet do not exist on the other. They seem to be united, but yet they are so different to one another. They seem to exist and yet they do cease! Ultimately everything will return to harmony and complete calmness. This is the nature of all existence. It is the final resting place for all. If we can understand this reality and remove our illusions, we can find this state of harmony and complete calmness. **Teachings in Chinese Buddhism (6) sunyata (emptiness) in the Mahayana context (Wikipedia) Many models**

**have been given concatenating Buddhist doctrines and Vaccum energy. These models essentially throw light on the essence of self(witness consciousness) and the cosmic general ledger (cosmic consciousness) while the imbalance in the gratification deprivation complex always remain unbalanced when it comes to individual general ledger , advocating the evolutionary theory of individual consciousness to cosmic consciousness. It is to be noted that it is not the question of knowing all the transactions but the wisdom of what lies behind the debit- Credit, Debit-Debit, or Credit-Credit principle that is most important for assimilation.** Look at this simple thesis to prove the point stated in the foregoing. Thesis: 'The World Has A Beginning In Time, And In Space It Is Also Enclosed In Boundaries.' Proof: 'For If One Assumes That The World Has No Beginning In Time, Then Up To Every Given Moment In Time An Eternity Is Elapsed, And Hence An Infinite Series Of States Of Things In The World, Each Following Another, Has Passed Away. But Now The Infinity Of A Series Consists Precisely In The Fact That It Can Never Be Completed Through A Successive Synthesis. Therefore An Infinitely Elapsed World-Series Is Impossible, So A Beginning Of The World Is A Necessary Condition Of Its Existence, Which Was The First Point To Be Proved (Kant, Critique Of Pure Reason, Tr. By R. Guyer And A. W. Wood (Cambridge: Cambridge University Press, 1998): B 454, P. 470). As Quoted In The Eternal Return: Genesis And Interpretation BY PAUL D'IORIO"The Cyclical Hypothesis, So Heavily Criticized By Nietzsche (VP II 325 And 334), Arises In This Way." In Fact, Nietzsche Was Not Criticizing The Cyclical Hypothesis But Only The Particular Form Of That Hypothesis Presented In Vogt's Work. All Of Nietzsche's Texts Without Exception Speak Of The Eternal Return As The Repetition Of The Same Events Within A Cycle Which Repeats Itself Eternally. If Deleuze's Interpretation Holds That The Eternal Return Is Not A Circle, Then What Is It? A Wheel Moving Centrifugally, Operating A "Creative Selection," "Nietzsche's Secret Is That The Eternal Return Is Selective" Says Deleuze: The Eternal Return Produces Becoming-Active. It Is Sufficient To Relate The Will To Nothingness To The Eternal Return In Order To Realize That Reactive Forces Do Not Return. However Far They Go, However Deep The Becoming-Reactive Of Forces, Reactive Forces Will Not Return. The Small, Petty, Reactive Man Will Not Return. Affirmation Alone Returns, This That Can Be Affirmed Alone Returns, Joy Alone Returns. Everything That Can Be Denied, Everything That Is Negation, Is Expelled Due To The Very Movement Of The Eternal Return. We Were Entitled To Dread That The Combinations Of Nihilism And Reactivity Would Eternally Return Too. The Eternal Return Must Be Compared To A Wheel; Yet, The Movement Of The Wheel Is Endowed With Centrifugal Powers That Drive Away The Entire Negative. Because Being Imposes Itself On Becoming, It Expels From Itself Everything That Contradicts Affirmation, All Forms Of Nihilism And Reactivity: Bad Conscience, Ressentiment..., We Shall Witness Them Only Once. [...] The Eternal Return Is The Repetition, But The Repetition That Selects, The Repetition That Saves. Here Is The Marvelous Secret Of A Selective And Liberating Repetition. There Is No Need To Remind The Reader That Neither The Image Of A Centrifugal Movement Nor The Concept Of A Negativity-Rejecting Repetition Appears Anywhere In Nietzsche's Writings, And Indeed Deleuze Does Not Refer To Any Text In Support Of This Interpretation. Further, One Could Highlight That Nietzsche Never Formulates The Opposition Between Active And Reactive Forces, Which Constitutes The Broader Framework Of Deleuze's Interpretation. For Some Years, Marco Brusotti Has Called Attention To The Fact That Deleuze Introduced A Dualism That Does Not Exist In Nietzsche's Writings. To Be Sure, The German Philosopher Describes A Certain Number Of "Reactive" Phenomena (For Example, In The Second Essay Of The Genealogy Of Morality, § 11, He Talks About "Reactive Affects" [Reaktive Affekte], "Reactive Feelings" [Reaktive Gefühlen], Reactive Men [Reaktive Menschen]); But These Are Nonetheless The Result Of Complex Ensembles Of Configurations Of Centers Of Forces That Remain In Themselves Active. Neither The Word Nor The Concept Of "Reactive Forces" Ever Appears In Nietzsche's Philosophy. The Eternal Return: Genesis And Interpretation BY PAUL D'IORIO Some Parts Are Deleted Due To Spatial Constraints. In the midst of thoughts about the eternal return we find at least two other thematic axes On the one hand, the view of the **world as a constant flux of forces without any goal, law, or rules of becoming. A chaos sive natura de-divinized and de-anthropomorphized which constitutes the "ontological substratum**" of the whole of Nietzsche's reflexions On the other hand, an ensemble of fragments of an anthropologico-sociological character, designing a path of liberation leading to the creation of superior individuals by way of a profound transformation of their instinctual structure. This transformation must be achieved by a practice of solitude and

internal struggle towards the liberation from the ancient representations of the world and from the incorporated herd values. For an analysis of these thematic perspectives, see Paolo D'Iorio, La linea e il circolo. Cosmologia e filosofia dell'eterno ritorno in Nietzsche (Genova: Pantograf, 1995): 233-322. With most of the parameters quantified, with assumptions there must be picturesque view of the self and non self, space and time, science and philosophy with the concatenational consummations. After The Chapter "On Redemption," Where Zarathustra Dares Not Expose His Doctrine, The Eternal Return Begins To Be Enunciated In Part Three Of The Work. In The First Place, It Is The Dwarf Who Formulates It In The Chapter "On The Vision And The Riddle." Facing The "Gate Of The Instant" Which Symbolizes The Two Infinities That Stretch Towards The Past And The Future, The Dwarf Whispers: "All Truth Is Crooked, Time Itself Is A Circle" The Dwarf Represents The Spirit Of Gravity, And He Embodies The Herd Morality, "The Belittling Virtue" Which Is The Title Of Another Chapter From Part III. The Dwarf Can Endure The Eternal Return Without Great Difficulties Because He Has No Aspirations; Unlike Zarathustra He Does Not Wish To Climb The Mountains That Symbolize Elevation And Solitude. In Two Unpublished Notes, From The Summer And The Fall Of 1883, Nietzsche Writes: The Doctrine Is At First Favored By The Rabble, Before It Gets To The Superior Men. The Doctrine Of Recurrence Will First Smile To The Rabble, Which Is Cold And Without Any Strong Internal Need. It Is The Most Ordinary Of Life Instincts, Which Gives Its Agreement First. Hence, The Content Of The Doctrine Is The Same, But Whereas The Dwarf Can Endure It (Because He Interprets It According To The Pessimistic Tradition For Which "Nothing Is New Under The Sun"), Zarathustra, Who Is The "Advocate Of Life" Regards The Eternal Return As The Strongest Objection To Existence, And As The Rest Of The Dream Suggests, He Does Not Yet Succeed In Accepting It After The Vision At The Gate Of The Instant, The Chapter Is Brought To An End By The Enigma Of The Shepherd. The Eternal Return: Genesis And Interpretation BY PAUL D'IORIO .How does the memory affect the GD complex or for that matter the gratification deprivation of an individual or a collective identity? Non-Markovian local-in-time master equations give a relatively simple way to describe the dynamics of open quantum systems with memory effects. Despite their simple form, there are still many misunderstandings related to the physical applicability and interpretation of these equations. Here, we clarify these issues both in the cases of quantum and classical master equations. Further introduction of the concept of a classical non-Markov chain signified through negative jump rates in the chain configuration. **E-M Laine et al 2012 J. Phys. B: At. Mol. Opt. Phys. 45 154004 doi:10.1088/0953-4075/45/15/154004 Local-in-time master equations with memory effects: applicability and interpretation.** Supposing That There Were Indeed An "Energy Of Contraction" Constant In All Centers Of Force Of The Universe, It Remains To Be Explained Where Any Difference Would Ever Originate. It Would Be Necessary For The Whole To Dissolve Into An Infinite Number Of Perfectly Identical Existential Rings And Spheres, And We Would Therefore Behold Innumerable And Perfectly Identical Worlds COEXISTING [Nietzsche Underlines This Word Twice] Alongside Each Other. Is It Necessary For Me To Admit This? Is It Necessary To Posit An Eternal Coexistence On Top Of The Eternal Succession Of Identical Worlds? The Eternal Return: Genesis And Interpretation BY PAUL D'IORIO. Science is usually portrayed as dehumanizing. Brave New World epitomizes this fear. "The more we understand the world, the more it seems completely pointless" (Steven Weinberg). Certainly science can seem chilling when conceived in the abstract as a metaphysical world-picture. We may seem to find ourselves living in a universe with all the human meaning stripped out: Participants in a soulless dance of molecules, or harmonics of pointlessly waggling superstrings and their braneworld cousins. Nature seems loveless and indifferent to our lives. What rights have we to be happy? **(Brave New World: Aldous Huxley).** Religion can afford to be dogmatic but science cannot. Look at this passage which counters both scientists and philosophers alike: However, Hartmann Objects That The Regressive Movement Postulated By Schopenhauer Is Possible Only In Thought: It Remains Nothing More Than An "Ideal Postulate" With No Real Object And Which "Does Not Teach Us Anything About The Real Process Of The World That Unfolds In A Movement Contrary To This Backwards Movement Of Thought" (Hartmann, Philosophie Des Unbewussten, Third Edition (1871): 772). Hartmann Affirms That If Unlike Schopenhauer One Admits The Reality Of Time And Of The World Process, One Must Also Admit That The Process Must Be Limited In The Past And Therefore That There Must Be An Absolute Beginning. In Hartmann's Mind, Failure To Do So Would Result In Positing The Contradictory Concept Of An Accomplished Infinity: "The Infinity That From The Point Of View Of

Regressive Thinking, Remains An Ideal Postulate, Which No Reality May Correspond To, Must, For The World, Whose Process Is, On The Contrary, A Progressive Movement, Open Up To A Determinate Result; And Here The Contradiction Comes To Light" (Hartmann (1871): 772). What Really "Comes To Light" In This Passage Is The Fact That Hartmann Does Not Provide A Demonstration But A Petitio Principii. Indeed, The Concept Of The World Process Analytically Contains The Concept Of A Beginning Of The World. In All Rigors, It Is Therefore Impossible To Demonstrate These Concepts With Reference To Each Other. Secondly, Hartmann's View That One Is Bound To Accept The Reality Of The World Process Even If One Rejects The Ideality Of Schopenhauer's Time Is Mistaken. Hartmann Believes That If Time Is Real There Must Be A World Process With Both An Absolute Beginning And An Absolute End. Without Any Justification, Hartmann Jumps From Schopenhauer's Negated Time To Oriented Time. The Eternal Return: Genesis And Interpretation BY PAUL D'IORIO. In the last years several theoretical papers discussed if time can be an emergent property deriving from ) quantum correlations. Here, to provide an insight into how this phenomenon can occur, authors present an experiment that illustrates Page and Wootters' mechanism of "static" time, and Gambini et al. subsequent refinements. A static, entangled state between a clock system and the rest of the universe is perceived as evolving by internal observers that test the correlations between the two subsystems. They implement this mechanism using an entangled state of the polarization of two photons, one of which is used as a clock to gauge the evolution of the second: an "internal" observer that becomes correlated with the clock photon sees the other system evolve, while an "external" observer that only observes global properties of the two photons can prove it is static.Phys. Rev. A 89, 052122 (2014)DOI:10.1103/PhysRevA.89.052122 arXiv: 1310.4691 [quant-ph] **Time from quantum entanglement: an experimental illustration Ekaterina Moreva.** Time is an entanglement .When the new ideas of quantum mechanics spread through science like wildfire in the first half of the 20th century, one of the first things physicists did was to apply them to gravity and general relativity. The results were not pretty. It immediately became clear that these two foundations of modern physics were entirely incompatible. When physicists attempted to meld the approaches, the resulting equations were bedeviled with infinities making it impossible to make sense of the results. Then in the mid-1960s, there was a breakthrough. The physicists John Wheeler and Bryce DeWitt successfully combined the previously incompatible ideas in a key result that has since become known as the Wheeler-DeWitt equation. This is important because it avoids the troublesome infinites—a huge advance. But it didn't take physicists long to realise that while the Wheeler-DeWitt equation solved one significant problem, it introduced another. The new problem was that time played no role in this equation. In effect, it says that nothing ever happens in the universe, a prediction that is clearly at odds with the observational evidence. This conundrum, which physicists call 'the problem of time', has proved to be thorn in flesh of modern physicists, who have tried to ignore it but with little success. Then in 1983, the theorists Don Page and William Wootters came up with a novel solution based on the quantum phenomenon of entanglement. This is the exotic property in which two quantum particles share the same existence, even though they are physically separated. Entanglement is a deep and powerful link and Page and Wootters showed how it can be used (e) to measure time. Their idea was that the way pair of entangled particles evolves is a kind of clock that can be used to measure change. But the results depend on how the observation is made. One way to do this is to compare the change in the entangled particles with an external clock that is entirely independent of the universe. This is equivalent to god-like observer outside the universe measuring the evolution of the particles using an external clock. In this case, Page and Wootters showed that the particles would appear entirely unchanging—that time would not exist in this scenario. But there is another way to do it that gives a different result. This is for an observer inside the universe to compare the evolution of the particles with the rest of the universe. In this case, the internal observer would see a change and this difference in the evolution of entangled particles compared with everything else is an important a measure of time. This is an elegant and powerful idea. It suggests that time is an emergent phenomenon that comes about because of the nature of entanglement. And it exists only for observers inside the universe. **Time is an entanglement** (Wikipedia). Danish religious philosopher Soren Kierkegaard carried out a systematic critique of the pretensions of reason and an abstract rationalism which he believed that the modern age was nurturing. Condemning reflection as a "danger" that ensnares people in logical delays and machinations, Kierkegaard compared it to a prison. Reflection is for him a form of captivity, a bondage which "can only be broken by [passionate] religious inwardness" (1978: 81).

**Reflection seduces individuals into thinking its possibilities are "much more magnificent than a paltry decision"** (1978: 82). It leads them to act **"on principle,"** to dwell on the deliberation of the context of their actions and the calculation of their worth or outcome. Kierkegaard argues that this drives away feeling, inspiration, and spontaneity, all of which are crucial for true inner being and a vital relation to God. For Kierkegaard, as Nietzsche would later agree, genuine inner being (and culture) is characterized by the tautness and tension of the soul which characterizes passionate existence. But the "coiled springs of life relationships ... lose their resilience" in reflection (1978: 78) and "everything becomes meaningless externality, devoid of [internal] character" (1978: 62). Kierkegaard thus contributes to the development of an irrationalist tradition that has echoes in some later postmodern thought. Kierkegaard might well have agreed with his contemporary Fyodor Dostoyevsky, who wrote: "An intelligent [reflective] man cannot seriously become anything ... excessive consciousness is a disease" (1974: 3, 5). In an age overtaken by rules and regulations, genuine action -- which Kierkegaard assumes to be subjective and spontaneous -- is frustrated at every turn. Complaining that we are too "sober and serious" (1978: 71) even at banquets, Kierkegaard bemoans the fact that even suicides are premeditated (1978: 68)! "That a person stands or falls on his actions is becoming obsolete; instead, everybody sits around and does a brilliant job of bungling through with the aid of some reflection and also by declaring that they all know very well what has to be done" (1978: 73). Thus, it is passion, not reflection, that guarantees "a decent modesty between man and man [and] prevents crude aggressiveness" (1978: 62). "Take away the passion and the propriety also disappears" (1978: 64). The ambiguity in the word "passion" may cause some confusion here. To say that the age and its individuals are "passionless" is not to say there are no emotions whatsoever, but rather that there is no true spiritual inwardness and depth, no intensively motivated action and commitment. It suggests that passion exists only in a simulated, pseudo-form, "the rebirth of passion" through "talkativeness" (1978: 64). "Chattering" for Kierkegaard gets in the way of "essential speaking" and merely "reflects" inconsequential events (1978: 89-99). Hence, in "the present age," emotions -- which in fact are all too pronounced -- have been transformed into negative forces. Anticipating Nietzsche's genealogy of the "slave revolt" in morality, Kierkegaard claims that the "enthusiasm" of the prior age of Revolution, a "positively unifying principle," has become a vicious "envy," a "negatively unifying principle" (1978: 81), a leveling force in its own right insofar as those lacking in talent and resources want to tear down those who have them. **THE POSTMODERN TURN IN PHILOSOPHY: THEORETICAL PROVOCATIONS AND NORMATIVE DEFICITS By Steven Best and Douglas Kelner http://www.gseis.ucla.edu/faculty/kellner/kellner.html.** In November 2011, the United States government argued before the US Supreme Court that it wants to continue utilizing GPS tracking of individuals without first seeking a warrant. In response, Justice Stephen Breyer questioned what this means for a democratic society by referencing Nineteen Eighty-Four. Justice Breyer asked, "If you win this case, then there is nothing to prevent the police or the government from monitoring 24 hours a day the public movement of every citizen of the United States. So if you win, you suddenly produce what sounds like 1984... "[65]. What is to be said in unmistakable terms is that when it comes to national security and innocent citizens there is question of compromise even it be an infringement and encroachment of so called privacy which has been made a word to subterfuge the activities that are far said to be fair. Quantum theory has provoked intense discussions about its interpretation since its pioneer days. One of the few scientists who have been continuously engaged in this development from both physical and philosophical perspectives is Carl Friedrich von Weizsäcker. The questions he posed were and are inspiring for many, including the authors of this contribution. Weizsäcker developed Bohr's view of quantum theory as a theory of knowledge. **Harald Atmanspacher, Hans Primas** show that such an epistemic perspective can be consistently complemented by Einstein's ontically oriented position. **Time, Quantum and Information 2003, pp 301-321 Epistemic and Ontic Quantum Realities Harald Atmanspacher, Hans Primas Robert W. Spekkens** present a toy theory that is based on a simple principle: the number of questions about the physical state of a system that are answered must always be equal to the number that are unanswered in a state of maximal knowledge. Many quantum phenomena are found to have analogues within this toy theory. These include the noncommutativity of measurements, interference, the multiplicity of convex decompositions of a mixed state, the impossibility of discriminating nonorthogonal states, the impossibility of a universal state inverter, the distinction between bipartite and tripartite entanglement, the monogamy of pure entanglement, no cloning, no

broadcasting, remote steering, teleportation, entanglement swapping, dense coding, mutually unbiased bases, and many others. The diversity and quality of these analogies is taken as evidence for the view that quantum states are states of **incomplete knowledge** rather than states of reality. A consideration of the phenomena that the toy theory **fails to reproduce, notably, violations of Bell inequalities** and the existence of a Kochen-Specker theorem, provides clues for how to proceed with this research program.DOI: http://dx.doi.org/10.1103/PhysRevA.75.032110 **Evidence for the epistemic view of quantum states A toy theory Phys Rev A 75, 032110 – Published 19 March 2007 Robert W. Spekkens In such specialized and autonomous areas, it is to the eminent and erudite authors to interpret the contradistinctions and contradictions' in the dove tailing explaining. This I feel would be yeoman service in the creation of Nature's general ledger. M. S. Leifer and O. J. E. Maroney** examine the relationship between quantum contextuality (in both the standard Kochen-Specker sense and in the generalized sense proposed by Spekkens) and models of quantum theory in which the quantum state is maximally epistemic. We find that preparation noncontextual models must be maximally epistemic, and these in turn must be Kochen-Specker noncontextual. This implies that the Kochen-Specker theorem is sufficient to establish both the impossibility of maximally epistemic models and the impossibility of preparation noncontextual models. The implication from preparation noncontextual to maximally epistemic then also yields a proof of Bell's theorem from an Einstein-Podolsky-Rosen-like argument. DOI: http://dx.doi.org/10.1103/PhysRevLett.110.120401 Received 25 August 2012Published 20 March 2013 © 2013 American Physical Society **Maximally Epistemic Interpretations of the Quantum State and Contextuality Phys. Rev. Lett 110, 120401 – Published 20 March 2013 M. S. Leifer and O. J. E. Maroney.** Epistemological, onitic and ontological aspects particularly of quantum mechanics have invited profound thinkers to contribute their mite to the growing literature on the subject, which includes the contributions from the subterranean realm and ceratoid dualism of philosophy like that Chaos theory of Deleuze, Derrida , Kierkegaard and others. **Andrei Khrennikov** shows that the Dirac-von Neumann formalism for quantum mechanics can be obtained as an approximation of classical statistical field theory. This approximation is based on the Taylor expansion (up to terms of the second order) of classical physical variables – maps f: $\Omega \rightarrow R$, where $\Omega$ is the infinite-dimensional Hilbert space. The space of classical statistical states consists of Gaussian measures $\rho$ on $\Omega$ having zero mean value and dispersion $\sigma 2\ (\rho) \approx h$. This viewpoint to the conventional quantum formalism gives the possibility to create generalized quantum formalisms based on expansions of classical physical variables in the Taylor series up to terms of nth order and considering statistical states $\rho$ having dispersion $\sigma 2(\rho) = h\ n$ (for n = 2 we obtain the conventional quantum formalism). **December 2005, Volume 18, Issue 7, pp 637-650 Date: 28 Nov 2005 Generalizations of Quantum Mechanics Induced by Classical Statistical Field Theory Andrei Khrennikov** It is widely accepted that consciousness or, in other words, mental activity is in some way correlated to the behavior of the brain or, in other words, material brain activity. Since quantum theory is the most fundamental theory of matter that is currently available, it is a legitimate question to ask whether quantum theory can help us to understand consciousness. Several approaches answering this question affirmatively, proposed in recent decades, will be surveyed. It will be pointed out that they make different epistemological assumptions, refer to different neurophysiological levels of description, and adopt quantum theory in different ways. For each of the approaches discussed, these imply both problematic and promising features which will be indicated. **Discrete Dynamics in Nature and Society Volume 2004 (2004), Issue 1, Pages 51-73 http://dx.doi.org/10.1155/S102602260440106X Quantum theory and consciousness: an overview with selected examples Harald Atmanspacher Copyright © 2004 Hindawi Publishing Corporation. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.** Another thought provoking and investigatory article is by Peter G. Lewis et al. Perhaps the quantum state represents information about reality, and not reality directly. Wave function collapse is then possibly no more mysterious than a Bayesian update of a probability distribution given new data. **Peter G. Lewis et al** consider models for quantum systems with measurement outcomes determined by an underlying physical state of the system but where several quantum states are consistent with a single underlying state—i.e., probability distributions for distinct quantum states overlap. Significantly, they demonstrate by example that additional assumptions are always necessary to rule out such a model. DOI:

http://dx.doi.org/10.1103/PhysRevLett.109.150404 **Distinct Quantum States Can Be Compatible with a Single State of Reality Phys. Rev. Lett. 109, 150404 – Published 9 October 2012 Peter G. Lewis et al** Communication complexity of a quantum channel is the minimal amount of classical communication required for classically simulating a process of state preparation, transmission through the channel and subsequent measurement. It establishes and perpetuates a limit on the power of quantum communication in terms of classical resources. **Alberto Montina** shows that classical simulations employing a finite amount of communication can be derived from a special class of hidden variable theories where quantum states represent statistical knowledge about the classical state and not an element of reality. This special class has attracted strong interest very recently. The communication cost of each derived simulation is given by the mutual information between the quantum state and the classical state of the parent hidden variable theory. Finally, find that the communication complexity for single qubits is smaller than 1.28 bits. The previous known upper bound was 1.85 bits. DOI: http://dx.doi.org/10.1103/PhysRevLett.109.110501 **Epistemic View of Quantum States and Communication Complexity of Quantum Channels Phys. Rev. Lett 109, 110501 – Published 12 September 2012 Alberto Montina for models see elsewhere .** Stability analysis of what? Quantum state representing reality or our knowledge of reality? Does the quantum state represent reality or our knowledge of reality? In making this distinction precise, **Nicholas Harrigan, Robert W. Spekkens** are led to a novel classification of hidden variable models of quantum theory. They show that representatives of each class can be found among existing constructions for two-dimensional Hilbert spaces. Approach also provides a fruitful new perspective on arguments for the nonlocality and incompleteness of quantum theory. Specifically, **Nicholas Harrigan, Robert W. Spekkens** show that for models wherein the quantum state has the status of something real, the failure of locality can be established through an argument considerably more straightforward than Bell's theorem. The historical significance of this result becomes evident when one recognizes that the same reasoning is present in Einstein's preferred argument for incompleteness, which dates back to 1935. This fact suggests that Einstein was seeking not just any completion of quantum theory, but one wherein quantum states are solely representative of our knowledge. Hypothesis is supported by an analysis of Einstein's attempts to clarify his views on quantum theory and the circumstance of his otherwise puzzling abandonment of an even simpler argument for incompleteness from 1927. **February 2010, Volume 40, Issue 2, pp 125-157 Date: 09 Jan 2010 Einstein, Incompleteness, and the Epistemic View of Quantum States Nicholas Harrigan, Robert W. Spekkens** statistical model of the probabilistic description of physical reality has been studied by many authors. **Andrei Khrennikov posits** a contextual statistical model of the probabilistic description of physical reality. Here contexts (complexes of physical conditions) are considered as basic elements of reality. There is discussed the relation with QM. He proposes a realistic analogue of Bohr's principle of complementarity. In the opposite of the Bohr's principle, **Andrei Khrennikov** principle has no direct relation with mutual exclusivity for observables. To distinguish his principle from the Bohr's principle and to give better characterization, he changes the terminology and speaks about supplementarity, instead of complementarity. Supplementarity is based on the interference of probabilities. It has quantitative expression trough a coefficient which can be easily calculated from experimental statistical data. There is need not appeal to the Hilbert space formalism and noncommutativity of operators representing observables. Moreover, in our model there exists a pair of supplementary observables which cannot be represented in the complex Hilbert space. There are discussed applications of the principle of supplementarity outside quantum physics. **October 2005, Volume 35, Issue 10, pp 1655-1693 The Principle of Supplementarity: A Contextual Probabilistic Viewpoint to Complementarity, the Interference of Probabilities and Incompatibility of Variables in Quantum Mechanics Andrei Khrennikov Foundations of Physics** Description of the vacuum in Yang-Mills theory and arrive at a physical interpretation of the pseudoparticle solution and the attendant violation of symmetries has been done by **R. Jackiw and C. Rebbi.** It is here vaccuum stability forms the bastion, pillar, post and stylobate of the mechanisms of yang mills Theory and action of a particle. The existence of topologically inequivalent classical gauge fields gives rise to a family of quantum mechanical vacua, parametrized by a CP-nonconserving angle. The requirement of **vacuum stability** against gauge transformations renders the vacua chirally noninvariant. DOI: http://dx.doi.org/10.1103/PhysRevLett.37.172 **Vacuum Periodicity in a Yang-Mills Quantum Theory Phys. Rev.**

**Lett 37, 172 – Published 19 July 1976 R. Jackiw and C. Rebbi.** Three-dimensional Yang-Mills and gravity theories augmented by gauge-invariant mass terms are analyzed. These topologically nontrivial additions profoundly alter the particle content of the models and lead to quantization of a dimensionless mass-coupling-constant ratio. The vector field excitations become massive, with spin 1 (rather than massless with spin 0), and the mass provides an infrared cutoff. The gravitation acquires mass, mediates finite-range interactions, and has spin 2 (rather than being absent altogether); although its mass term is of third derivative order, there are no ghosts or acausalities. DOI: http://dx.doi.org/10.1103/PhysRevLett.48.975 © 1982 The American Physical Society **Three-Dimensional Massive Gauge Theories Phys. Rev. Lett 48, 975 – Published 12 April 1982 S. Deser et al** All scattering amplitudes in the maximally supersymmetric N=4 super-Yang–Mills theory possess a new, dual superconformal symmetry which extends the previously discovered dual conformal symmetry of MHV amplitudes. To reveal this property we formulate the scattering amplitudes as functions on the appropriate dual superspace. Rewritten in this form, all tree-level MHV and next-to-MHV amplitudes exhibit manifest dual superconformal symmetry. We propose a new, compact and Lorentz covariant formula for the tree-level NMHV amplitudes for arbitrary numbers and types of external particles. The dual superconformal symmetry is broken at loop level by infrared divergences. However, we provide evidence that the dual conformal anomaly of the MHV and NMHV superamplitudes is the same and, therefore, their ratio is dual conformally invariant. We show this explicitly for the six-particle amplitudes at one loop. Authors conjecture that these properties hold for all, MHV and non-MHV, superamplitudes in N=4 SYM both at weak and at strong coupling. **Nuclear Physics B Volume 828, Issues 1–2, 21 March 2010, Pages 317–374 Dual superconformal symmetry of scattering amplitudes in N=4 super-Yang–Mills theory J.M. Drummonda DOI: 10.1016/j.nuclphysb.2009.11.022.** By comparison with numerical results in the maximal Abelian projection of lattice Yang–Mills theory, it is argued that the nonperturbative dynamics of Yang Mills theory can be described by (e) a set of fields that take their values in the coset space SU(2)/U(1). The Yang–Mills connection is parameterized in a special way to separate the dependence on the coset field. The coset field is then regarded as a collective variable, and a method to obtain its effective action is developed. It is argued that the physical excitations of the effective action may be knot solitons. A procedure to calculate the mass scale of knot solitons is discussed for lattice gauge theories in the maximal Abelian projection. The approach is extended to the SU (N) Yang–Mills theory. A relation between the large N limit and the monopole dominance is pointed out. **Physics Letters B Volume 458, Issues 2–3, 8 July 1999, Pages 322–330 An effective action for monopoles and knot solitons in Yang–Mills theory Sergei V. Shabanov DOI: 10.1016/S0370-2693(99)00612-7.** It is here the quintessential formulation of consciousness occurs. Like a man standing on the threshold of infinity trying to ponder what lies beyond the veil, which separates the seen from unseen a true seeker tries to lift the individual general ledger to the cosmic general ledger. Like in Darwinian evolution, mind also has to evolve from individual consciousness to cosmic consciousness. In Kaishmiri Shaivism one withdraws from the finite to the infinite, but one also goes on an outward journey from the infinite to the finite, because both the finite and the infinite have an intimate connection. The finite is not seen as unreal, but as a symbol of the infinite. There is no real distinction between them. Those two movements constitute Spanda, a key concept in Kaishmiri Shaivism. Spanda is the pulsation of the Absolute in different phases of being. There are no opposites like subject and object, unity or duality, absolute or relative. They are just different phases of the universal vibration of the Absolute. The goal is to realize or be at once infinite and finite. One does not turn away from appearances (like in Advaita Vedanta), but one realizes that the Absolute manifests all things. Spanda, the eternal pulsation of the Absolute, oscillates between a passion to create and dispassion from the created. Through it the Absolute transform itself into all things and then returns back into the emptiness of its undifferentiated nature. In Kaishmiri Shaivism the Absolute is seen as pure consciousness (=being). The Absolute is an eternal all-pervasive principle, the highest reality, the nature of all entities eternally and blissfully at rest within its own nature. The Absolute is the nature of the Self (and thus of us all). The Absolute is divine; it is Shiva, the Lord of the Universe. It is full of conscious activity through which it generates the universe, and reabsorbs it into itself at the end of each cycle of creation. Thus we speak of monism, as everything resides within this one absolute consciousness. It sustains all things, it embraces all things, and it pervades all things. All things are appearances within the absolute consciousness, but nevertheless real (in contrast to Advaita Vedanta where

appearances are seen as unreal or illusionary). All things appear external (out there, outside ourselves), but they do not have a being on their own. They do not exist as separate entities on their own. Everything is contained within consciousness. What we see as objects are manifestations of consciousness. The event which constitutes the universe is always internal events happening within consciousness because their essential nature is consciousness itself. If a physical object were totally material, and independent or external to consciousness, it could never be experienced. The universe and consciousness are two aspects of a whole. The universe is an attribute of consciousness which bears consciousness as its substance. Doctrine of Vibration: S.G.Dyczkowski What you "see" is not always what you "get." Many people mistakenly take their own visions literally without "seeing through" the various possibilities that are outside their belief system, knowledge or skill base. Deeper reality is not remote in the physical sense but in a psychological sense. The archetypes of the collective unconscious are arrayed behind the scenes of current worldwide conditions, of crisis and confusion. They mirror our own states back at us, whether we perceive them as such or interpret them plausibly or not. The noise of ordinary consciousness and beliefs drowns out the signal. Unconsciousness is the background of our ordinary awareness. Our organism is very much at the center of such effects. The organismic source is our human bodies and the focus of human consciousness. The fantasy principle dethrones reality, but can be dissociative or compensatory. The human mind is a meme-Scape. Pre-conceived concepts vie with structures, concepts with images. Like scientists who ignore assumed truths, we leapfrog over our beliefs and personality deficits, claiming idiosyncratic imagination is literal reality. It couldn't be further from the truth and symbolism is utterly lost. The metaphor that might heal us enslaves us. Perhaps images like the holographic universe have an implicate order. Can we have a sense of the cosmos in the world without projecting myriad fantasies on it that we embrace literally? Has the world become so horrible it is unreasonable to be realistic? We may need to look at our drives and wishes, rather than the fantasy content. Psyche constructs reality. Our experience of so-called reality is always mediated by our image of it. Even if all the contents of the psyche are real, that doesn't mean they are realistic. That psyche is real is still a radical proposition, but psychic politics certainly color the self-image and ideas of everyone. We observe and participate with images It is not a question of nature or nurture (genes alone or experience alone). Rather, everything is both. We inherit the structures that make our experience what it is. But the structure itself is “empty,” and each human culture “fills” it with its own specific adaptations. It is difficult to define an archetype and set boundaries that distinguish it from others. In a hologram each part contains all the information but in lower resolution. Archetypes have this holographic quality. There are patterns within patterns within patterns. Some overlap with others, and some are nested inside others. Archetypal realities, passed on through DNA, are expressed in distinctive neuronal tracts in the brain. They include customs and laws regarding property, incest, marriage, kinship, and social status or roles; myths and legends; beliefs about the supernatural; gambling, adultery, homicide, schizophrenia, and the therapies to deal with them. A mythic and visionary language of immediate experience encompasses themes of deepest, highest, and ultimate concern. Most fantasy-based individuals are at a complete loss to coherently explain their own conventional behavior much less anomalous events and their deep meaning, much less the cultural unconscious or mythological unconscious matrix. But they try, and become utterly entrenched in their belief that they are right about the nature of the world and reality. We have pseudo-memories about our personal lives. Why not more so for our collective life? The subject matter often revolves around catastrophe, creation and the mythopoeic forces of mankind. Ignorant of such dynamics, interpretive mistakes and displaced psychic contents proliferate into errors of fact. Propaganda, media distortions, memes, and disinformation compound the social problem of misapprehension further. Shameless self-promotion by personalities of such ideas leads to cults. They make up myths about the myths of by-gone eras. Roiling unconscious images can be fatally confusing. Thought illusions culminate in projections and projections of mythology. Jung suggested symbols live only as long as they are pregnant with meaning. Philosophy arose from criticism of myth, from discussing and challenging it. In science, we criticize, reject and eliminate theories. At the edge of the abyss of the unknown, new signs and symbols emerge. Credible theories and paradigms must include biology, physics, and neurophysiology. One of the reasons people "see God", or a guru, or anomalies may be because our brains are constructed to see reality through the eyes of others. There are heaps of mirror neurons which are there to make us feel the 'other'. Mirror neurons do for psychology what DNA did for biology. They provide a

unifying framework and help explain a host of mental abilities. As in the psychochemical processes of empathy or falling in love, a complex feedback loop sustains a state of mind. But when we empathically transpose ourselves into someone else's position, we expose ourselves to that reality -- cognitively and emotionally. The unconscious complicates empathy, both ways. Mirror neurons might well play a role in bonding, language and self-awareness. Naively, we take too much as self-evident. But 'seeing' does not always 'believe', though many make this error or leap in logic and formulate their choices and future accordingly. Yet, there is only one way to learn what consciousness is. Experience. But we have no satisfactory explanatory edifice for consciousness. Would such a theory release in each of us our own inner knowledge of the creativity of our own consciousness, and its infinite possibilities? The problem is trying to define a verb, a dynamic, as if it were a noun. But we do recognize the effect of consciousness. It functions to mediate states of consciousness, high and low psychobiological arousal. Consciousness is the subconscious lifted up by the physical body. When the body fails, the consciousness collapses back into the subconscious. All our thoughts come from the subconscious which can see our intentions but not our world. This relates somehow to intention being imaginary and not of the physical frictionized world (King). Gerald Edelman postulates that the flows of information in the brain are mediated through 're-entrant' feedback loops. As evolution provides new cognitive functions, new re-entrant loops are established. Even language itself is an archetype -- a chaotic field of dynamic associations. A subtle net of tropes, grammar, symbols, and meaning, the program language begins in limbic resonance. Some phenomena generate their own language patterns, nomenclature, and internal coherence of meaning and representation. In a holographic universe, even time and space could no longer be viewed as fundamentals. Because concepts such as location break down in a universe in which nothing is truly separate from anything else, time and three-dimensional space would also have to be viewed as projections of this deeper order. At its deeper level reality is a sort of super hologram in which the past, present, and future all exist simultaneously. This suggests that given the proper tools it might even be possible to someday reach into the super holographic level of reality and pluck out scenes from the long-forgotten past. Or not. A fantasy of such penetration or phenomenon inside the head is not the same as that penetration. **Jung in the 21st Century: Synchronicity and science By John Ryan Haule Mind Control Countermeasures.** The lower domain of experience is what we do not directly observe, the quantum realm, while the higher level is what we ordinarily observe, the classical realm. Based purely on experiment, the formulation of quantum theory initially placed the great divide between the quantum and classical domains between the measuring apparatus and the particle (Herbert, 1985). However, in the early 1930s, John von Neumann, in his rigorous mathematical treatise on quantum mechanics, found no support for such a division, compelling him to conclude that the wave for was collapsed by consciousness. Von Neumann's treatise has been called the "quantum bible" and "the most influential book on quantum theory ever written" (Herbert, 1985). The von Neumann formulation leads us to conclude that the lower realms of quantum reality form a virtually limitless array of potentials that are inherently incapable of realization without the observation of a knowing entity, which we identify with consciousness (Kafatos and Nadeau, 1990). We thus have a correspondence between quantum theory, the hierarchical structure implied by the Holographic Principle, and the higher orders of experience, leading to the full expression of the Conscious Universe. David Bohm's theory of the implicate and explicate orders involves a holographic Principle that is fully consistent with the Holographic Principle discussed here (Bohm 1980; 1986; Bohm and Hiley, 1993). According to Bohm, there is an implicate order that represents the universal, holographic subtext of reality, and which unfolds in every moment to produce the explicate order that we all observe. Thomas Germinario (2004) has equated the implicate order with the unconscious process, and the explicate order with conscious process. He emphasizes the nature of dreams within the implicate order, and the importance of the dream work for maintaining a healthy mind through integration of the implicate order and our daily lives. Allan Combs and Mark Holland (1990) connected the implicate order or holomovement (Bohm, 1980) with Carl Jung is theory of synchronicity (Jung and Pauli, 1955), with the implicate order providing a holographic medium through which apparently disconnected individuals become connected. The principle of synchronicity, the instantaneous connection of people and events beyond the senses, has been equated with the quantum-physical principle of non-locality (Combs and Holland, 1990; Germine, 1991), and has been proposed to be the fundamental mechanism of conscious process (Germine, 1991).Applying the Holographic

Principle theory of mind, subjects may be connected synchronously in the manner of two individuals conversing through cellular phones, where the radio signals are transmitted through a distant satellite. The difference here is that the radio signals travel at the speed of light, and thus the there is some miniscule time lapse between the sending and receiving ends. The Holographic Principle implies a much more distant and instantaneous communication pathway, with the analogue of the satellite being the holographic boundary of the Universe, as well as a much richer communication, potentially involving a transmission of thoughts and feelings. Our ego-consciousness seems to mask the universal relatedness implied by the Holographic Principle, and it is perhaps only through transcendence of the ego-consciousness that the higher orders of experience can become conscious. In the phenomenon of synchronicity, there seems to be a meaningful connection between individuals that breaks through the barrier of ego-consciousness. Such a connection is reported by many individuals in the course of dreams, when the ego-consciousness has been suspended, at times informing the dreamer of something that has happened in the life of a meaningfully-connected individual. **The Holographic Principle Theory of Mind MARK GERMINE Institute for Psycho science** Impassioned-intense Connectionist neuroscience, in an effort to attain relevance to clinical practice, has recently given us a theory of the relationship of the unconscious to consciousness which is dualistic (Viamontes and Beitman, 2007), an ad hoc theory of conclusions based on assumptions, the most erroneous of which are that representations of information are information, and that such representations elaborate themselves through a process of representations of representations in neuronal circuits (Bechtel and Abrahamsen,1991). As theory, the connectionist models are lacking heuristic value and **are inimical to the progress** of basic and clinical neuroscience. This is not to denigrate or minimize the importance of neuronal connections, but these connections are rather like the wires in a radio that plays a symphony. The radio, in and of itself, is incapable of playing the symphony. The radio doesn't do not write the symphony, nor does it broadcast the Passionate-raging symphony over the airways. Yet, if we loosen one wire, the symphony is not manifested through the radio. The brain is not subject to the same vulnerability, in that it has parallel and redundant networks of processing, and, it is in this sense that the connectivity of the brain becomes important. Connectionism, to the extent that it accepts consciousness and the unconscious at all, attributes them to separate groups of "circuits" in the brain, with consciousness-processing circuits having a limited but detailed ability to analyze information, and unconscious-processing processing more information in less detail (Viamontes and Beitman, 2007). There is an element of truth in this view in that information that has importance and meaning resonates within the brain at higher levels of experience and at higher levels of holographic recursion. This resonance involves what we know, what we feel, and what we are capable and willing to consciously realize. The Universe, as William Blake noted, can be seen in a grain of sand. Indeed, the grain of sand requires the Universe to exist. The human organism requires the Self to exist. The Self is not a simple amalgam of sensory experiences, as suggested in the connectionist model (Blinder, 2007). This amalgam view of the Self can be damaging when applied to psychotherapy, as the connectionists suggest (Blinder, 2007). The Self is ours forever and ever, from eternity to eternity. As Milton suggested, the mind is its own place. Our deepest purpose is for our minds to resonate with the supra-conscious levels of experience, much as it is purpose of the radio to play the symphony. If the symphony is not to our liking, we can turn the radio off. So, perhaps, the transpersonal consciousness can turn our own individual and collective radios off, or perhaps turn the volume down, if we are not in harmony with the Universal Mind. Physics has given us many enigmas. Quantum theory itself is an enigma. But some of the enigmas seem to involve the limitation of our own abilities of discernment. The thought experiment of "Schrödinger's Cat," as it is often interpreted (e.g. Gribbin, 1984), assumes that the cat is not conscious, and is incapable of observation in the context of collapse of the wave function. This is due, in large part, to the false dichotomy of information and experience. Our observation of other animals indicates, to those that are attuned to the animal mind, who can "feel" the animal mind, that, at the very least, all mammals are conscious. It would seem that the great neuropsychiatrist, Stanley Cobb (1948), may have been correct in his attribution of levels of consciousness to a wide variety of organisms: "...lower animals with no cerebrum appear to be conscious...even plants such as the sunflower that turns towards strong light may have a vague awareness or warmth and comfort. There are many degrees of consciousness and it is my contention that it is integrated at many levels like other important functions of the central nervous system."With respect to the theory of mind, recent mainstream thinking

remains classical and mechanistic. Such views of mind are advocated by our most in prominent neuroscientists (e.g. Changeux, 1985; Gazzaniga, 1985) as neuroscience research attempts to explain all mental function on the basis of pure mechanism and the localized function of specific areas of the brain. Francis Crick, the distinguished Nobel Laureate, had a very influential second career as a neuroscientist. He concluded that we are basically soulless creatures, that all mental processes could be reduced to identifiable, neural correlates in the brain, and that, in fact, we are nothing but a collection of neurons (Crick, 1995). Crick even went so far as to say that what we call the soul is a group of neurons located in the prefrontal lobes of the brain. Another Nobel Laureate, Gerald Edelman (1987), has written a critically acclaimed book on "neural Darwinism," which purports that the brain is "circuitry" is constructed through a developmental process of "survival of the fittest" neurons and neuronal connections. At the same time the total purposelessness and lack of direction in evolution is expounded, again to critical acclaim, by Richard Dawkins (1986). The idea that there essentially no self or soul imbued with agency and self determination was expounded by Gilbert Ryle (1949/2002) with his parody of the "ghost in the machine," and this work continues to be influential among many scholars of the mind. Philosophers such as Patricia Churchland (1986), who coined the term "neurophilosophy," take an "eliminative materialist" view of belief, free will, and consciousness. Other philosophers, such as Daniel Dennett (1991) would make experience "epiphenomenal," a by-product of brain processes with no effect whatsoever on the function of mind. However, unlike neuroscience, which is still in its infancy, the philosophy of mind has had a long history, including such notable figures as Plato, Aristotle, Descartes, Kant, Locke, Hume, Kierkegaard, and James, to name just a few. Perhaps, as noted by Ervin Laszlo (1974): "in today's world, most of the traditional functions of cognitive synthesis have atrophied and are ignored and neglected." Hopefully, what we have presented here will be a step in the right direction for what Laszlo calls a "conceptual synthesis" to help fill the need for meaningful engagement is such a world. The fundamental elements of such a synthesis have been described by Laszlo as follows: Conceptual synthesis performs at least five basic functions in the guidance of affairs. They are the mystical, the cosmological, the sociological, the pedagogical or psychological, and the editorial functions. The mystical function inspires in man a sense of mystery and profound meaning related to the universe and of himself in it. The cosmological function forms images of the universe in accord with local knowledge and experience, enabling men to describe and identify the structure of the universe and the forces of nature. The sociological function validates supports and enforces social order, representing it in accord with the nature of the universe, or as the natural or right form of social organization. The pedagogical or psychological function guides individuals through stages of life, teaching ways of understanding themselves and others and presenting desirable responses to life is challenges and trials. Finally, the editorial function of conceptual is to define some aspects of reality as important and credible and hence to be attended to, and other aspects unworthy of serious attention. **The Holographic Principle Theory of Mind MARK GERMINE Institute for Psycho science** Kant sums up this inversion and its spirit early in the Critique of Pure Reason: Up to now it has been assumed that all our cognition must conform to the objects; but all attempts to find something about them a priori through concepts that would extend our cognition have, on this presupposition, come to nothing. Hence let us once try whether we do not get farther with the problems of metaphysics by assuming that the objects must conform to our cognition, which would agree better with the requested possibility of an a priori cognition of them, which is to establish something about objects before they are given to us. This would be just like the first thoughts of Copernicus, who, when he did not make good progress in the explanation of the celestial motions if he assumed that the entire celestial host revolves around the observer, tried to see if he might not have greater success if he made the observer revolve and left the stars at rest. (B xvi) Onticology, like all variations of object-oriented ontology, is realist in its orientation. In defending a realist ontology onticology holds that the vast majorities of objects, actants, beings, or entities are independent of humans and are what they are regardless of whether any humans regard them or register them. In short, onticology rejects any anthropomorphic, idealist, or anti-realist thesis to the effect that to be is to be the correlate of mind, spirit, the body, the human, and language or otherwise. While it is certainly the case that knowledge is necessarily dependent on the object to which it relates, the converse does not hold true. Objects are not dependent on being known, regarded, perceived, or spoken about. As such, and to put it in Aristotlean terms, knowledge is an accident of objects, not

objects an accident of knowledge. As Althusser so nicely puts it, "[n]o doubt there is a relation between thought-about-the-real and thisreal, but it is a relation of knowledge, a relation of adequacy or inadequacy of knowledge, not a real relation, meaning by this a relation inscribed in that real of which the thought is the (adequate or inadequate) knowledge" (Reading Capital, 96). Althusser goes on to remark that "[t]he distinction between a relation of knowledge and a relation of the real is a fundamental one: if we did not respect it we should fall irreversibly into either speculative or empiricist idealism" (ibid.). Onticology categorically endorses Althusser's verdict. It is a fundamental necessity to distinguish between those relations that belong to the object and those that belong to knowledge. Contemporary philosophy continuously confuses these two very different sorts of relations. Naturally the question arises of how it is possible to surmount our relation to the object so as to determine whether objects themselves possess the properties we encounter in relating to objects. In other words, given that we can only ever relate to the object in relating to the object how is it possible to surmount this relation to get at the being of the object itself? Much more will have to be said about this later– and the answers will be surprising with respect to standard prejudices about realism –however, for the moment it can be said that onticology takes its epistemological inspiration from the transcendental realism of Roy Bhaskar. Among other things, Bhaskar sought to provide a transcendental grounding for the sciences. Insofar as onticology defends the thesis that the field of being is much more vast than the field of objects investigated by the natural sciences, it parts way with the thesis that the domain of being is exhausted by the domain of natural objects. However, the general form of Bhaskar's argument holds for our realist purposes. A transcendental argument seeks to elucidate the conditions under which certain acknowledged practices and forms of cognition are possible. Kant, for example, asked what must be the case for mathematical judgments to be possible. How is it both that we are able to extend our knowledge, as if by magic, through mathematical judgments and, more significantly, that these judgments are able to provide genuine knowledge of the world despite the fact that these forms of reasoning are not based on experience? Part of Kant's argument consisted in claiming that mind imposes the forms of space and time on the data of experience. In other words, space and time are not attributes of being itself but rather of the mind that regards being. Insofar, Kant argues, as mathematics is ultimately a rumination on the nature of space and time taken in their most abstract form and insofar as the mind imposes space and time on the manifold of sensation, it thus follows that a priori judgments about the nature of spatio-temporal relations are possible that anticipate the structure of actual-space times without directly experiencing these space-times. Why? Because any manifold of sensation must necessarily be structured by these forms imposed by intuition. **Onticology– A Manifesto for Object-Oriented Ontology Part I Posted by larval subjects under Object-Oriented Philosophy My Question Is This: You Say, "Granted There Are Deeper And Deeper And More Primitive Forms Of "Knowledge" Or Abstraction Into Mechanism, But I Wouldn't Say That The Atomic Level Brings A New Epistemic World Into View." But Doesn't Any Kind Of Knowledge, However Primitive, Imply Bringing Some Kind Of Epistemic World Into View Or Proto-View?** Joel: David, Yes Exactly. My Question Is, Just How Far Down The Complexity Gradient Can The Meaning Of 'Knowledge,' Or 'Semiotics, Or Specifically Representation (Maya) Be Stretched Without Breaking? I Can Comfortably Stretch It To The Genetic Level, As The Proto-Epistemic, Where Genetic Codes 'Represent' Phenotypic Structures Or Modes, And The Whole Of Evolution Learns From Its Mistakes And Successes As That Knowledge And History Is Encoded Into And As The Recapitulation Of The Embryogenesis Of The Organism. But An Atom, In My Model, While Certainly Reacting To Its Environment From Its Own Attractor, Essence, And 'Interiority' (Prehension) Isn't Making Representations Of It To Do So. But The Key To The Power And 'Essence' Of Representation (And Simultaneously Its Limitation) Is The Capacity Of Abstraction And Choice. Evolution Does This At A Rudimentary Level (And Anthropomorphizing A Bit) With Its Sending Into The World Its 'Random' Variations, Organismic Ideas, In A Sense. Its 'Mind' Is Literally On The Outside As Our Living World And Biosphere. A 'Choice' Is Made In The Biospheric Interiority, And Knowledge For The Evolutionary/Embryogenetic Trajectory Is Gained By The "Differential Reproductive Success" Of These Individuals Lives Through Time, As The Increasing Intelligence, Representation, And Capacity For Choice Is Directly Injected Into The Flow Through Sexual Selection. But Where Is The Encoded/Abstracted (Enfolded) Representation Of Options And A Choice Among Them Made At The Atomic Level? I Am Working With The Most Cutting-Edge And Coherent Models For

The Atom (Including My Own) That I Can Find Among The Heterodoxy, And I See No Evidence Of Representation And Choice At This Level, Or Rather No Real Way To Further Stretch The Meaning Of The Terms To This Level. Sure There Is Uncertainty From Infinite Difference And Immanent Causation (See Bohm On Infinite Causation), But Without Representation There Really Is No Choice (Although Crudely We Could Call A 'Bifurcation' A 'Choice' By The System As A Whole). So I Am Simply Saying That Representation At The Atomic Level Is Entirely Enfolded And These Are Really Just Ontic Level Phenomena. The Atomic Level, In My View, Is Pre- Not Yet Proto-Epistemic, Which I Place At The Genetic Level Where We Find The First Codes And Primitive Representation. This Just Means That There Is A Real Distinction Between Ontic And Epistemic (And A Gradient Between Them) And We Can't Collapse The Two Into Just The Ontic-Epistemic. And Yet, With The Distinction Intact, And The Gradient To Explore, They Remain Nondual, Univocal, ONE Or Ontic.What I Would Say Exists At All Levels (My Own Self-Aware Myth About The Given) Is What I Call The "Symbiogenesis Of Subject And Object". This Is The Spinozan Essence Of Dynamic Stability And Growth, And Leibnizian Prehension. Also Called The "Nucleation Of Observability" In Spinbitz. It Is From This Nucleus Of The Observer (Fuller) In A "Point-Free Geometry" (Whitehead) Of Pointless "Points Of View" As Conditions Of Boundary That Wilber's "All Is Perspective" Finds Its Ontic Grounding. And This Symbiogenesis Is Also A Rudimentary Or Deep-Level Native Or Primitive Intelligence, Evolution At The Involution Of The Subject-Object Interface. It Is An Exploration Of Creative Learning And Direct Immanent Awareness, Unmediated (Enfolded) By Representational Forms (Unfolded). It Just Is, In Waves (Cosmic Vertebrae) Before It Abstracts, Represents And Incarnates The Flesh Of What Could Be. Matter Is Enfolded Seed Of The Flower Of Its Abstraction. Maya Is This Depth And Recursion Of Brahma Into And Through The Boundary Conditions Of Abstract Relation. And Brahma Is The Span Of The Emergent Rigor Of The Cosmic Ergodic Spine That Opens The Recursions Into The Field Of Infinite Difference. In Immanent-Transcendent Waves, Literally And Empirically A Limit-Cycle In Ergodicity, It Enfolds And Unfolds From Anatom To Anatomy, From Infinite Intelligence Or Omnirelational Awareness Of Self, Through The Representational Depths Of Self As Self-Consciousness. Omni-Evolution, Trans-Dynamics, And The Anatom Posted On February 21, 2013A Discussion With Tom Huston, Part II The Following Is Part 2 Of An Excerpt From A Discussion Initiated By Tom Huston On His Facebook Page, Edited And Embellished For Clarity. Find Part One, Here. In This Part We Dig Into The Nature Of The "Anatom" And The "Ergodic Spine" Of An Electro-Fractal Cosmos, And Into The Trans-Dynamic Integration Between Being And Becoming, And The X-Interface−The Crossroads Of The Ontic-Epistemic (Brahma And Maya) And The Subject-Object Polarities. Univocal_Dynamics_V1 Tom Huston Is A Founding Member Of Integral Institute And A Former Editor Of Enlighten Next Magazine. His Webpage Is Tomhuston.Com. David Marshall Is A Writer And Editor Living In Chicago. No meaning, value, or normativity to be found in nature, that there is nothing **natural beings ought to be,** but that, rather, these judgments arise from us (See last post of larval subjects) which has led some to raise valuable questions about the coherence of these claims. The problem arises when the following three propositions are taken together: 1. there is nothing outside of nature.2. Beings have no intrinsic meaning, purpose, or value (in and of themselves, there's nothing they ought to be). 3. Value judgments about what beings and being ought to be arise from us and beings like us (bonobo apes, dolphins, institutions, birds of paradise, etc). The problem arises between proposition 1 and 3. How can it both be true that there is nothing outside of being and those normative judgments belong to us and other beings capable of making normative judgments, not nature? The problem arises from restricting these judgments to humans and beings capable of making these judgments. In making such a claim it seems as if we're saying that there's something outside of nature, something that is beyond nature, thereby violating the first thesis and potentially reintroducing the nature/culture distinction. By contrast, if we say that normative judgments are the special domain of those living beings with the proper degree of sentience to make such judgments, then we seem to reintroduce the nature/culture distinction and fall back into the sorts of problems that I outlined in my last post (and that are so nicely critiqued by thinkers such as Latour). Is there a way out of this? I don't know. Great_Chain_of_Being_2_ (lighter) ASIDE: It seems to me that this is really what the debate between realism and anti-realism, realism and socio-linguistic constructivism surrounding the new materialisms and speculative realists has really been all about. It's very easy to treat this as an abstract, academic debate: "Are you a realist or are you an

anti-realist?", as if it were just a matter of what happens to be true. But it seems to me that this debate has, in reality, always been about politics. As I outlined in my last post, we have perpetually seen how appeals to the real and natural have been used in the name of oppressive power, inscribing both the exploitation of nature and the oppression of various people in the very fabric of being itself. Theistic theology and realist ontology have perpetually been used in the name of what Deleuze called "State Philosophies" or philosophies that ontologize contingent orders of power and privilege (e.g., "the great chain of being" used to justify patriarchy, monarchy, serfdom, poverty, etc, and appeals to nature used to justify poverty and racial inequality (The Bell Curve), patriarchy (evolutionary psychology), heteronormativity, capitalism, etc). Because arrangements of power and inequality are always contingent in the sense that there's no marked difference in the capacities of peoples, power always looks for a transcendent supplement that would provide justification through ontological necessity. Antirealism– from the Greek atomists to present –became the radical and emancipatory gesture because it revealed the lie behind all of these forms of social organization or their inherit contingency or arbitrariness. Realism, by contrast, has all too often functioned as an apologetics for arbitrary power and social organizations. Here it's worth recalling what Foucault said about science: "…Even before we know to what extent something like Marxism or psychoanalysis is analogous to a scientific practice in its day-to-day operations, in its rules of construction, in the concepts it uses, we should be asking the question, asking ourselves about the aspiration to power that is inherent in the claim to being a science. The question or questions that have to be asked are: "What types of knowledge are you trying to disqualify when you say that you are a science? What speaking subject, what discursive subject, what subject of experience and knowledge are you trying to minorize when you say 'I speak this discourse, I am speaking a scientific discourse, and I am a scientist.' What theoretico-political vanguard are you trying to put on the throne in order to detach it from the massive, circulating, and discontinuous forms that knowledge can take?" (Society Must Be Defended, 10). All of these questions hold equally for claims to something being real. What is one trying to minorize when claiming something is real? What becomes privileged? What is excluded? It is these questions that have been at the heart of the realism debates, for as Spencer-Brown taught us, every distinction has a marked and unmarked space, draws attention to something to be included and pushes something into the unconscious or the domain of the invisible, hidden, or veiled. This is above all the case with evocations of the real. However, as I've tried to show antirealism leads to its own problems. First, so long as we exclude real beings from our ontological inventory, we are unable to fully understand how power functions ("such and such a set of cultural formations have been thoroughly debunked, yet people still live as if they believed them"). Not only do we not fully understand the sources of the problems due to too much focus on the discursive and semiotic, but we deny ourselves valuable sites of political intervention at the level of infrastructure. Second, we are prevented from addressing things such as cultural racism, such as that found in Heidegger's privileging of the West and the Greeks and Germans in particular. We do a good job addressing biological and theological racism, heteronormativity, and sexism by showing how it is a cultural construction, but when faced with racism such as Heidegger's where he argues that there's something "unique" about the Greek event, it's language, and about the German language, or Badiou/Zizek's racism with respect to the "Pauline Event", we really have no response. Here someone like Jared Diamond or Fernand Braud el is needed to explain global-geographical inequalities. Third, the tools of the cultural turn really do not provide us with the means of thinking the ecological as a site of the political. For this reason, I've tried to formulate a third way– which might be called "constructivist realism" or "constructivist naturalism" –that retains the insights of the cultural turn, while also allowing a robust place for the material. I don't claim to be original in this. I think that many such as Manuel DeLanda, Deleuze and Guattari, Stacy Alaimo, Karen Barad, Jane Bennett, etc., are up to something similar. It's a vast project that requires the work of a multitude of voices, especially given the way in which the culturalism pervades humanities. **The Relational and the Non-Relational: Notes towards an Immanent and Pluralist Theory of Meaning Posted by larval subjects under uncategorized k-bigpic I have deleted or reformulated some sentences for which I beg pardon attributable to spatial constraints** What is correlation, what is causation? What is coordination? What is coincidence? These thoughts have bothered me since long. I have been able to excavate some reliable material from Wikipedia which I would like to share. "All views are partial and contingent – that's the lesson of pluralism, and you say as much yourself when you say that we must be attentive to

ways our own knowledge might contain superstitions. But it's not discourse that determines whether one view works and another doesn't – it's a confrontation with **non-discursive (non-human)** agencies." I wonder if part of the issue here is that we understand pluralism differently. For me this doesn't sound like pluralism at all. Rather, it just sounds like our **epistemological condition**. We [hopefully] come to understand those portions of existence we question or investigate. The reason we come to investigate them is largely contingent. Finally, the accounts of these features of existence we give can be mistaken. All of this is perfectly consistent with a monism. I'm not sure why what he outlines above is the lesson of pluralism. It seems to me that every realist knows this. It seems to me that ontological pluralism is something quite different. Ontological pluralism is the thesis that there are many different worlds inhabited by many different entities. Thus, for example, you would have one world; say that of Lucretius, that's only inhabited by atoms and their combinations. You would have another world; say the world of the Mongolian shaman, that's inhabited by spirits that do all sorts of things. The ontological pluralist is saying that these spirits are, that they exist, that they're real. This is something quite different than merely saying that there are partial and contingent points of view on the world. It's this saying that these entities are rather than that some person or groups of people believe that they are that is the nub of the issue. Now, I think part of the issue here is that there's an ambiguity in the term ontology. Ontology can be one of two things. On the one hand, ontology is a group or persons set of beliefs as to what is. Here it's trivially true that there are pluralities of ontologies and the realist readily recognizes this. This is the whole reason there are debates over ontology. Mongolian shamans have their ontology, Europeans theirs, Christian fundamentalists theirs, materialists theirs, etc. When striving to understand and communicate with others it's vital to understand these ontologies because, as rhetoricians like Burke point out, our **beliefs about what is** are among the things that motivate our action. Despite having never seen bacteria I was my hands because I believe there are bacteria and viruses on door handles and whatnot and don't want to get sick. My belief about a particular thing existing is what motivates my action. And who knows, perhaps this belief is as superstitious as the belief that the crops failed because God was displeased with my community. On the other hand, ontology is a theory **about what is.** It is making a claim that something exists. This is where the rubber hits the road. The ontological pluralist seems committed to the thesis that every group set of beliefs about what exists is sufficient for granting the existence of those entities. This is what I find objectionable in Latour. Obviously I'm not bothered by Latour's suggestion that we should take into account the role that nonhumans like speed bumps, rivers, microbes, etc., play in the form that social assemblages take. But this is not all that Latour claims. This is precisely where the philosopher might balk. Unlike the ethnographer, the philosopher is not interested in what people believe exists, but rather philosophers– at least of the realist variant –are trying to figure out what is. In other words, the realist philosopher begins with the premise that not all of these beliefs about what is are true. So for the philosopher, recognizing that Mongolian shaman's believe in the existence of shamans would only be the first step. The next step would consist in determining whether there's good reason for thinking such entities really do exist, i.e., whether there's good reason for believing these entities have mind and culture independent reality. Lest readers think that I'm just picking on the supernatural here, we can ask similar questions about strings, subatomic particles, galaxies, etc., etc., etc. I guess the question really comes down to what exactly we mean by ontological pluralism. When we talk about ontological pluralism are we defending the thesis that people have different theories of being? If so, then ontological pluralism is trivially true. If this is what is meant, then I certainly share Jeremy's view that it's valuable to understand the different world's people believe in. Certainly when I was practicing as an analyst I didn't get in ontological debates with my patients and it was necessary to understand their theory of being or their ontology to properly attend to them. Or, when we talk about ontological pluralism are we defending the thesis that all these ontological theories are true and refer to really existing entities? That's quite a different claim and is not one I would defend or endorse. Now someone might object that "in both The Democracy of Objects and Onto-Cartography you defend the thesis that there are multiple worlds." This is true. Because I hold that not everything is related I'm led to the conclusion that there are diverse worlds. However, I also hold that however many worlds there might be, these worlds are nonetheless composed solely of material entities. Within the framework I propose I wouldn't suggest that there's one world where there are spirits and another world where there are souls and yet another where there are only material entities. My view is that there aren't spirits or souls in any of these worlds.

One of the things I keep hearing in these discussions is that somehow the realist adopts a view from nowhere. I honestly don't understand this criticism. Investigation always occurs somewhere and requires all sorts of mediations involving technologies, experiments, etc. It's that labor of gathering evidence, conducting experiments, using technologies to observe the world, etc., that gradually gives us a body of data that allows us to say there's good reason to believe that such and such a thing exists and has these powers. Another charge seems to be that the realist refuses to recognize that their claims about the world are fallible. I find this charge particularly strange because it's precisely because the realist recognizes the difference between our theories of the world or what we say about the world and the world itself that fallibility is built into the core of his position. Realism doesn't mean one holds they have special access to the world, that they know all truths, or that they have the truth in hand, only that there are truths to be known and that we can be mistaken about things. Circling **Squares Philip** has a post responding to my quandaries about how to mesh realism and pluralism. He writes: Ontologically and metaphysically the idea of realist pluralism is no longer an issue. There are (appropriately) numerous variants but the basic idea that reality is itself pluralistic is well established. The question is political-discursive. It's what Stengers and Latour are getting at with their concepts of diplomacy and cosmopolitics. They grant, first, that all entities exist and, second, that to say that someone's cherished idol (or whatever disputed entity they hold dear) is non-existent is a **'declaration of war'** – 'this means war,' as Stengers often says. They thus shunt onto-political discourse off of the terrain of knowledge/belief in the sense of existence/non-existence. Their basic claim seems to be that 'respect for otherness,' i.e. political pluralism, can only come from granting the entities that others hold dear an ontology, even if you don't 'believe' in them. You are thus permitted to say 'I do not follow that god, he has no hold over me' but you are not permitted to say 'your god is an inane, infantile, non-existent fantasy, grow up.' And it's not just a question of politeness (although there's that too). The point is to grant others' idols and deities an existence – one needn't agree over what that existence entails, over what capacities that entity has or what obligations it impresses upon you as someone in its partial presence but to deny it existence entirely is to 'declare war' – to deny the possibility of civil discourse, of pluralistic co-existence. I believe that it was Richard Rorty who once quipped something like claims to reconcile realism and idealism always seem to end with a triumph of idealism. We don't, in fact, get realism through such approaches, but rather just get a pervasive anti-realism. I think this is also the problem with the "non-controversial pluralism" advocated by Stengers and Latour that Phillip defends here. Such pluralism is not realism but is, in fact, a thoroughgoing social constructivism. I think this is the central problem with Latour's argument in Irreductions (these days I regret having ever defended it). In rejecting both Enlightenment critique and what he calls "reduction" he wants to say something like "The Pentacostal really is filled with the Holy Spirit", that for the 19th working scientist heat really is a fluid and phlogiston really is what allows things to burn, and that for the Greek lightning really is an expression of Zeus's anger. Latour tells us that we aren't to reduce or explain away the entities posited by another group's "ontology" but are to develop explanations from within that ontology. It might not sound particularly sexy– and it certainly doesn't tell us what is worth thinking –but I can't help but believe that philosophy is the critical and reflective investigation of basic concepts that guide our investigation of the world about us, how we ought to live our lives, and what form of governance might be best. Compare two figures. A scientist might ask, what causes depression? We can very well imagine a philosopher turning around and asking the scientist, what is causality? The scientist presupposes a concept of causality in her investigations. She uses this concept in her inquiry. Now she might have a sophisticated concept of causality or she might never have thought much about causality at all, using it in the sort of colloquial and unreflective way that Plato decried when, for example, people like Euthyphro talked about piety. A whole cascade of questions arise when we raise a question like "what is causality?" We can ask whether or not causality exists at all. We can ask how we distinguish between correlation and two events that merely accompany one another from genuine causation. This, for example, was Hume's question. But perhaps most importantly we can ask whether there is only one form of causality or many forms of causality. Is there only one-to-one causation; one cause and one effect? Is there many-to-one causation; or many events conspiring to produce an effect? Is there one-to-many causation; or one event producing a variety of different effects? We can even ask whether causality necessarily moves from past to present or whether there aren't forms of causality that move from future to past! **January 23, 2014 Pluralism and Realism:** multiple-worlds Over

at Struggles Forever, Jeremy Trombley has an interesting post up on "the ontological turn" in anthropology or ethnography. I've been meaning to have a discussion with him about this as I think it's an issue many of us are struggling with. For example, the core project of The Democracy of Objects– a proj ect which I think many have missed –is to somehow reconcile some version of social constructivism with a realist ontology capable of making room for ecology (which requires realist and materialist positions as there's a fact of the matter where global warming is concerned) as well as the role played by objective agencies in social assemblages such as technologies, infrastructure, features of geography, local climates, the growth cycles of plants and animals, waste, etc. Maybe we can try to organize some cross-blog event to discuss these issues. I certainly think they're close to the heart of Jeremy, Michael of Archive Fire, Arran James, and a host of others. As an aside, I'm beginning to realize how the different sites of the political I've been outlining– semiopolitics, thermopolitics, oikopolitics (political economy), geopolitics, eropolitics (the politics of sex and desire), biopolitics, and chronopolitics (and I'm sure there are other political sites!) –are drawing me away from traditional Marxism. Assuming that classical Marxism holds that economics or the conditions and relations of production are determinative of all other sites of the political, the various sites of the political that I've been outlining would lead to the conclusion that there is not one determinative base of the political. This would not require committing Marx to flames, but rather of recognizing the phenomenon of overdetermination, or of a variety of different entangled sites of the political. But I digress. First, I find myself wondering what the ontological turn means in ethnography. Is it 1) the investigation of the different ontologies held/proposed by different cultures? E.g., the Aztecs believed that reality was structured in this way, while the Greeks in that way, and the ancient Chinese this way, etc? Or 2) is it an investigation of how real entities– independent of cultural beliefs –influence cultural formations? Or is it a combination of both? A position that I would favor. January 20, 2014 Thoughts on the Social and Political Implications of Correlationism .Lion Mirror Truth be told, as my thought has evolved the issue of correlationism had fallen off the radar for me. Somehow the debate had come to seem too "philosophical" to me, too "scholastic", too remote from what interests me: understanding why social assemblages are organized as they are, how power functions in social assemblages, and what we might do to address that power and change things. Somehow the question of whether or not we can get out of the correlation between thinking and being just came to seem remote from these sorts of issues. Somehow it seemed too epistemological. ASIDE: Numerous discussions over the years have led me to believe that the debate over correlationism is poorly understood (or maybe I just don't understand it). On countless occasions I've heard people say "of course we must relate to things in order to know them." Well yeah, of course! I don't think this is what the critic of correlationism is getting at. It seems to me that correlationism is something more robust than the theses that we must relate to something to know it. Correlationism instead seems to require the theses that thought and being are indiscernible. Put more concretely, the correlationist is someone who argues that we either a) can never tell whether being is merely a construction of our thought (weak correlationism), or b) who argues that thought actually constructs being (strong correlationism). In other words, correlationism is another name for idealism. One can hold that we must relate to something in order to know it without being a correlationist. As an aside I should also add that I am a correlationist about some things. For example, I think money is something constructed by society and is therefore a strong correlationist when it comes to money. At any rate, for a long time I'd become rather indifferent to debates about correlationism and philosophies of access. I had learned the lessons of speculative realism– which I could have also learned, I think, from Deleuze and Guattari, the new materialist feminists, actor-network theorists such as Latour –and had moved on. However, occasionally you come across a tone of phrase that pitches something in a different light. In The Cut of the Real, Katerina Kolozova writes, …the political problem of contemporary philosophy identified by the 'new realists' is, in fact, the product of a more fundamental epistemic problem. In his book After Finitude, Quentin Meillassoux calls this problem 'correlationism' and identifies it as an essentially post-Kantian legacy, which continues to dominate and limit philosophy. As a matter of fact, correlationism lies at the heart of postmodern theory and consists in the premise that thought can only 'think itself,' that the real is inaccessible to knowledge and human subjectivity, and that there is nothing but discursive constructs that fully determine thinking and that are meth0dologically accounted for all the way down. (1 – 2) Thought thinking only itself: Thought only encountering itself. In the jargon of postmodern and

poststructuralist lingo, this would be the thesis of infinite semiosis, where signs ("thoughts") only ever relate to other signs. Within this framework, discursivity comes to be the hegemonic framework defining all of being. At the level of politics and social theory more generally, if the correlationist thesis is true the consequences are clear: all social phenomena are discursive and all solutions to social and political problems will be discursive. The sole sphere of the political will be the discursive and all questions of politics will be questions of speech-acts and interpretation. The problem here is not that many theorists recognize that the discursive and semiotic plays an important role in the social and the political. It does and I've repeated this tirelessly. The problem is with what happens when thought or the semiotic becomes a hegemon, an "all", foreclosing our ability to recognize other forms of power. What I've wanted to say is that not all power functions discursively. In my last post and elsewhere I spoke of some other forms of politics: Thermopolitics: The politics surrounding **energy in the form of calories and fuels such as gasoline** and coal, and how our life and our very bodies are structured by energy dependencies and by being trapped in particular distributive networks that render these forms of energy available. I'm being quite literal when I speak of energy, talking about the effects, for example, of the absence of food in certain educational environments on cognition, for example; and am generally hostile to metaphorical extensions of the concept of energy which I see as erasing the dimension of real materiality. Geopolitics: The role that features of natural and built geography such as mountain ranges, rivers, oceans, soil conditions, roads, housing design, etc., play in the form that social relations take and how they impact individual bodies. Chronopolitics: The way in which the structurations of time organize what is possible for us. For example, the structuration of the working day, how much we can say and comprehend at any given time, the impact of things like the invention of the clock, etc. **Oikopolitics: This would be the domain of political economy described so well by Marxists**. So five different types of politics: Semiopolitics (or what currently dominates critical theory), thermopolitics, geopolitics, chronopolitics, and oikopolitics. No doubt there are other sites of the political or political struggle that we could speak of, but this is a good start. Also, it should be obvious that these aren't exclusive domains, but are entangled in all sorts of important ways. For example, something might take place at the level of semiopolitics (speech, law, rhetoric, norms, and communication) that has all sorts of impact at the level of thermopolitics. Congress might decide to cut programs that fund school meal programs. This, in turn, will have a thermodynamic impact on those students that go without the calories they need developmentally and cognitively to function in a particular way. There is an entanglement here of semiopolitical and thermopolitical domains. The young student here has been constrained both at the level of **semiotic phenomena and thermodynamic structures**. The point is that if true, semiotic intervention (speech-acts, protests, interpretations, deconstructions, etc) will not be an appropriate response to all political problems because social formations are not entirely structured by the semiotic. The child in that school does not suffer from a lack of the right signs, but from a lack of calories needed to run the engine of his thought and body. Certainly semiotic interventions might be needed to render that energy available, but it is the energy itself that is at issue and the absence of that energy that forms the spider web entangling him in his position. A correlationist perspective tends to erase this as even being a site of the political. **Larval Subjects January 28, 2014 Ontological Anarché Posted by larval subjects under Uncategorized January 27, 2014 Roden on Pluralism Over at enemy industry Roden has an excellent post up on the pluralism discussion. Check it out here. January 25, 2014 Different Senses of Pluralism and Ontology Posted by larval subjects under Uncategorized Responding to one of my comments over at his blog, Jeremy Trombley:** I have deleted restricted, reformulated some sentences .Kindly bear with me. Intention is to make the discursive enucleation consistent with the subject in question**.** Recourse to epistemology of the quantum mechanics generic and fervor of string theory is a sine qua non for the further comprehension of the theory and ontology .How would the world appear to us if its ontology was that of classical mechanics but every agent faced a restriction on how much they could come to know about the classical state? We show that in most respects it would appear to us as quantum. The statistical theory of classical mechanics, which specifies how probability distributions over **phase space evolve** under Hamiltonian evolution and under measurements, is typically called Liouville mechanics, so the theory we explore here is Liouville mechanics with an epistemic restriction. The particular epistemic restriction we posit as our foundational postulate specifies two constraints. The first constraint is a classical analog of Heisenberg's uncertainty principle; the second-order moments of position and momentum

defined by the phase-space distribution that characterizes an agent's knowledge are required to satisfy the same constraints as are satisfied by the moments of position and momentum observables for a quantum state. The second constraint is that the distribution should have maximal entropy for the given moments. Starting from this postulate, authors derive the allowed preparations, measurements, and transformations and demonstrate that they are isomorphic to those allowed in Gaussian quantum mechanics and generate the same experimental statistics. They argue that this reconstruction of Gaussian quantum mechanics constitutes additional evidence in favor of a research program wherein quantum states are interpreted as states of incomplete knowledge and that the phenomena that do not arise in Gaussian quantum mechanics provide the best clues for how one might reconstruct the full quantum theory. I: http://dx.doi.org/10.1103/PhysRevA.86.012103 **Reconstruction of Gaussian quantum mechanics from Liouville mechanics with an epistemic restriction Phys. Rev. A 86, 012103 – Published 10 July 2012 Stephen D. Bartlett, Terry Rudolph, and Robert W. Spekkens.** Deleuze in fact characterizes modes of existence, with their powers and capacities. The answer is this: Deleuze approaches modes of existence, ethically speaking, not in terms of their will, or their conscious decision making power (as in Kant), nor in terms of their interests (as in Marx, for example), but rather in terms of their drives. For Deleuze, conscious will and preconscious interest are both subsequent to our unconscious drives, and it is at the level of the drives that we have to aim our ethical analysis. Here, I would like to focus on two sets of texts on the drives taken, not from Nietzsche and Spinoza, but rather from Nietzsche and Leibniz (Leibniz being one of the first philosophers in the history of philosophy to have developed a theory of the unconscious). The first set of texts comes from Nietzsche's great early book entitled Daybreak, published in July 1881. Nietzsche first approaches the question of the drives by giving us an everyday scenario: "Suppose we were in the market place one day," he writes, "and we noticed someone laughing at us as we went by: this event will signify this or that to us according to whether this or that drive happens at that moment to be at its height in us—and it will be a quite different event according to the kind of person we are. One person will absorb it like a drop of rain, another will shake it from him like an insect, another will try to pick a quarrel, another will examine his clothing to see if there is anything about it that might give rise to laughter, another will be led to reflect on the nature of laughter as such, another will be glad to have involuntarily augmented the amount of cheerfulness and sunshine in the world—and in each case, a drive has gratified itself, whether it be the drive to annoyance, or to combativeness or to reflection or to benevolence. This drive seized the event as its prey. Why precisely this one? Because, thirsty and hungry, it was lying in wait" (D 119). This is the source of Nietzsche's doctrine of perspectivism ("there are no facts, only interpretations"), but what is often overlooked is that, for Nietzsche, it is our drives that interpret the world, that are perspectival—and not our egos, not our conscious opinions. It is not so much that I have a different perspective on the world than you; it is rather that each of us has multiple perspectives on the world because of the multiplicity of our drives—drives that are often contradictory among themselves. "Within ourselves," Nietzsche writes, "we can be egoistic or altruistic, hard-hearted, magnanimous, just, lenient, insincere, can cause pain or give pleasure" (Parkes, pp. 291-292). We all contain such "a vast confusion of contradictory drives" (WP 259) that we are, as Nietzsche liked to say, multiplicities, and not unities. Moreover, these drives are in a constant struggle or combat with each other: my drive to smoke and get my nicotine rush is in combat with (but also coexistent with) my drive to quit. This is where Nietzsche first developed his concept of the will to power—at the level of the drives. "Every drive is a kind of lust to rule," he writes, "each one has its perspective that it would like to compel all the other drives to accept as a norm" (WP 481). DELEUZE AND THE QUESTION OF DESIRE: TOWARD AN IMMANENT THEORY OF ETHICS Daniel W. Smith. **In on a Possible Physical Metatheory of Consciousness Miroljub Dugic et al** show that the modern quantum mechanics, and particularly the theory of decoherence, allows formulating a sort of a physical metatheory of consciousness. Particularly, the analysis of the necessary conditions for the occurrence of decoherence, along with the hypothesis that consciousness bears (more-or-less) well definable physical origin, leads to a wider physical picture naturally involving consciousness. This can be considered as a sort of a psycho-physical parallelism, but on very wide scales bearing some cosmological relevance. Open Systems & Information Dynamics Volume 9, Number 2, 153 (2002) arXiv: quant-ph/0212128. On a broader perspective and background, the theoretical computation of the universe is justified.IN physics and cosmology, digital physics is a collection of theoretical perspectives based on the premise that the universe is, at

heart, describable by information, and is therefore computable. Therefore, according to this theory, the universe can be conceived of as either the output of a deterministic or probabilistic computer program, a vast, digital computation device, or mathematically isomorphic to such a device. Digital physics is grounded in one or more of the following hypotheses; listed in order of decreasing strength. The universe or reality: is essentially informational (although not every informational ontology needs to be digital); is essentially computable (the pancomputationalist position); can be described digitally; is in essence digital; is itself a computer (pancomputationalism); is the output of a simulated reality exercise. Following Jaynes and Weizsäcker, the physicist John Archibald Wheeler wrote the following: [...] it is not unreasonable to imagine that information sits at the core of physics, just as it sits at the core of a computer. (John Archibald Wheeler 1998: 340) It from bit. Otherwise put, every 'it'—every particle, every field of force, even the space-time continuum itself—derives its function, its meaning, its very existence entirely—even if in some contexts indirectly—from the apparatus-elicited answers to yes-or-no questions, binary choices, bits. 'It from bit' symbolizes the idea that every item of the physical world has at bottom—a very deep bottom, in most instances—an immaterial source and explanation; that which we call reality arises in the last analysis from the posing of yes–no questions and the registering of equipment-evoked responses; in short, that all things physical are information-theoretic in origin and that this is a participatory universe. (John Archibald Wheeler 1990: 5) David Chalmers of the Australian National University summarised Wheeler's views as follows: Wheeler (1990) has suggested that information is fundamental to the physics of the universe. According to this 'it from bit' doctrine, the laws of physics can be cast in terms of information, postulating different states that give rise to different effects without actually saying what those states are. It is only their position in an information space that counts. If so, then information is a natural candidate to also play a role in a fundamental theory of consciousness. We are led to a conception of the world on which information is truly fundamental, and on which it has two basic aspects, corresponding to the physical and the phenomenal features of the world. (Wikipedia). **Participatory consciousness forms the bastion, pillar, post, stylobate and fulcrum of the redressal measures and corrective steps taken for circumvention of many problems, identified and unidentified, spread widest commonalty. Anne Baring's sententious and pithy article will be good to have a timely outlook.** I would like to open my contribution to this meeting with the words of Ken Wilber that I quoted in the first Millennium Symposium: 'The secret impulse of life is towards greater consciousness. Maybe the evolutionary sequence is from matter to body, to mind, to soul, to spirit - each transcending and including, each with a greater depth and greater consciousness and wider embrace' Ken Wilber, and with these from Rupert Sheldrake's contribution to the same meeting: "It is possible to make quite a good case that the sun could be conscious and if the sun, then why not the stars? Why not the galaxies? Why should there not be galactic minds? And if they communicate with each other then the distances are far too great for light to be the medium of transmission. We are into inter-galactic telepathy." After many decades of study, I know of only one diagram in all religious traditions that can adequately explain these statements; that can give us both a blue-print of the many levels of consciousness and a route-map to help us reach beyond our present level of understanding. I would like to introduce a diagram of the four worlds of Kabbalah to illustrate the theme that spirit has brought these four worlds into being and that we, living in the fourth and manifest world know nothing of the existence of the other three invisible ones. It is the clearest diagram I know of that can explain to us the multi-levelled structure of consciousness and our place in the great chain of being. We are indeed, as Wilbur suggests, evolving from matter to body, to mind, to soul, to spirit but where have we come from? It is possible that, as participants in the emanation of spirit into manifest life, we have also come from spirit to soul, to mind, to body, to matter. Everything, from the immense galactic energies to the minute forms of matter participates in one life, is generated from one creative spirit. There is nothing outside this life, this spirit. This diagram suggests that truth is not an article of faith but the experience of participating in the undiscovered reality of what has brought us into being. We cannot know truth until we have entered into communion with that reality. Continuing in similar vein **Michel Bitbol dilates upon the dialectic deliberation and polemical argumentation between** hermeneutists and eliminativists. When he formulated the program of Neurophenomenology, Francisco Varela suggested a balanced methodological dissolution of the "hard problem" of consciousness. He shows that his dissolution is a paradigm which imposes itself onto seemingly opposite views, including materialist approaches. I also point out that Varela's revolutionary

epistemological ideas are gaining wider acceptance as a side effect of a recent controversy between hermeneutists and eliminativists. Finally, he emphasizes a structural parallel between the science of consciousness and the distinctive features of quantum mechanics. This parallel, together with the former convergences, point towards the common origin of the main puzzles of both quantum mechanics and the philosophy of mind: neglect of the constitutive blindspot of objective knowledge. Phenomenology and the Cognitive Sciences Phenomenology and the Cognitive Sciences **Phenomenology and the Cognitive Sciences 2002, Volume 1, Issue 2, pp 181-224 Science as if situation mattered Michel Bitbol.** Caroline Williams investigated affective density of the political and its effect on our understanding of political subjectivity. Taking up Spinoza's challenge to think about affect beyond corporeal embodiment, he argues that there is a modality of affectivity that cannot simply be inscribed within the borders of subjectivity. theorising affect as an impersonal force anchored in a relational ontology that gives due recognition to the circulation of affects, as well as to their ambivalent structure in creating sites of identification, and he utilises this ontology to reflect on the dynamic of the political and the shape of political subjectivity. Williams argues that Spinoza's philosophy (through ideas of conatus and imagination) offers the conceptual resources to reconfigure the composition of affective subjectivity as a transindividual social bond and as an unconscious dynamic of ethico-political existence. **Subjectivity (2010) 3, 245–262. doi:10.1057/sub.2010.15 Affective processes without a subject: Rethinking the relation between subjectivity and affect with Spinoza Caroline Williams** Phenomenological Time bears ample evidence and infallible observatory and apodictic evidence albeit with a sense of chagrined circumvention and defeated disgruntlment circularity of both psychological and phenomenological interpretations of art which are sometimes insightful, but most often work on the logic of cyclical time and unavoidable drives and representations played out as a 'natural' order: where 'men can't avoid being boys' a circularity which valorizes a helplessness/truthfulness conundrum. One begins with a classical phenomenological fallacy with Duchamp's Fountain, does the urinal keep returning or do we keep returning to the urinal? Put in this way, it is a male question, something that phenomenology conveniently conceals when positing the "I", or indeed, the "we". But we must be careful not to simply assume that the phenomenological first person is white, heterosexual, able-bodied and male, providing a foundation upon which knowledge should be based as a universal condition. Deleuze's second critique of time is linked to Kant. This model of time extends not only to phenomenology but to the phenomenological interpretation of art and is seen as a straight line. In the Critique of Pure Reason, Kant cuts up circular time and reassembles it as a series of sensory experiences which memory processes and which constitute subjectivity. Whereas the circular model of time (and interpretation) inscribes fatalism into the subject and the world, the phenomenological method is in danger of solipsism, where the subject constitutes the world. Much of embodied philosophy and enactivist theories of cognition go down this line with a plethora of art historical analyses by the likes of Rosalind Krauss, Michael Fried and others taking the lead from Merleau-Ponty. These approaches successfully challenge the **homunculus model of a Cartesian mind-body** dualism where there is a supervisor in the mind who processes experience and even consciousness and to whom all representations are directed. But phenomenology is mainly concerned with perceptual consciousness, the processing of colors, shapes, forms, motion in art and the world. The senses and sensorimotor processes are the fundamental pathways by which the interior is connected to the exterior in a chiasm of self-constitution in the world. The problem with this is that we explore the art through our sense perceptions, and what does art reveal to us?—our sense perceptions! Such solipsistic approaches have real problems relating complex and abstract conceptual production to bodily and sensorimotor contingencies, and they nearly always posit the subject as the site and object of discovery in the world. To be fair, phenomenology has moved on to rather more sophisticated models of intersubjectivity, taking into account the specificities of different kinds of bodies and ways of being beyond the white, male heterosexual as the common denominator, yet many naïve approaches to art perpetuate the notion that perceptual, rather than conceptual processes and evolutionary neural patterns are primary, fundamental ways to understand the art experience In fact, these aspects are only part of art production and reception, aspects which are often filtered out or made known to us in order for us to parody such trigger responses, as in much of Duchamp's work, which holds in contempt such retinal approaches to art. If we take the **subject of abstract expressionism** in the 1950s or later minimalist art, art history, teaching and interpretation still insists on putting the act of perceiving

as fundamental to the constitution not only of the phenomenological first person but also of art itself. The male act of usually standing up, which consists of directing a stream with the hand(s) into a container or on a flat surface, sand or snow in order to leave patterns and traces (the usual child's prank) underlies the logic of macho 1950s Abstract Expressionist culture, especially that of Jackson Pollock, so effectively parodied by Carolee Schneeman's Vagina Scroll, 1975. Here, painting or writing is a kind of a discharge which invests the **object and space with** the phenomenological "I", disciplined by existential struggle and reflected back by the traces, signatures and gestures left behind in the work of art, rediscoverable as an element in the making of other subjectivities.It is precisely this kind of solipsism which is blind to the art event as something well in excess of re-establishing habits of subjective self-identity through the production and interpretation of art. The attempt to reinforce a phenomenological identity upon the event by personalizing sense experience is unable to see art as something which remodels the phenomenological 'I' and neural plasticity itself. We learn new technologies and redraw our neural pathways through such new and spontaneous experiences which draw us into intersubjective worlds. Duchamp's Fountain and Deleuze's Repetition and Différence Grégory Minis sale (I have made some changes) The relation of different groups to deconstruction is settled by its relation to groups. An American enthusiast--one who has had a role in the development of our affection for Derrida--notices that the motivational chains are phobically driven by concerns with social scale. Richard Rorty: "He wants to figure out how to break with the temptation to identify himself with something big. . . ."(10) And: "So I take Derrida's importance to lie in his having had the courage to give up the attempt to unite the private and the public, to stop trying to bring together a quest for private autonomy and an attempt at public resonance and utility. He privatizes the sublime, having learned from the fate of his predecessors that the public can never be more than beautiful."(11) Notice is made here of what is indeed the telling feature. Levinas is praised, for example, for his alarm when "the social will [is] sought in an ideal of fusion . . . the subject losing himself in a collective representation, in a common ideal. . . . It is the collectivity which says 'us,' and which, turned toward the intelligible sun, toward the truth, experience, the other at his side and not face to face with him. The tellingly titled D'un ton apocalyptique contains passages that are particularly excited: These people situate themselves outside the ordinary, but they have in common this: they describe themselves as having an immediate and intuitive relation with mystery. And they want to attract, to seduce, and lead others to the mystery, through mystery. This agogic function of the leader of men, of the duce, of the Führer, of the leader, places him above the crowd that he manipulates with the aid of a small numbers of adepts joined together in a sect with a secret language, a clique or a small party with its ritualized practices. The mystifiers pretend to have exclusive access to the privilege of a secret mystery. . . . The revelation or the unveiling of the secret is something that they jealously reserve for themselves. Jealousy is here a major characteristic. There is not narcissism and non-narcissism; there are narcissisms that are more or less comprehensive, generous, open, extended. What is called non-narcissism is in general but the economy of a much more welcoming, hospitable narcissism, one that is much more open to the experience of the other as other. I believe that without a movement of narcissistic reappropriation, the relation to the other would be absolutely destroyed; it would be destroyed in advance. The relation to the other--even if it remains asymmetrical, open, without reapporpriation--must trace a movement of reappropriation in the image of oneself for love to be possible, for example. Love is narcissistic. Beyond that, there are little narcissisms, there are big narcissisms. . . .(23) **Justice of the Pieces Deconstruction as a Social Psychology Douglas Collins** Sabine Wilke cogently argues for the rise of the aesthetic dimension in philosophical writing by showing how the metaphorical and poetic style of Heidegger, the principles of phrase configurations with Adorno, or the topographic writing of derrida replace through their very form something what traditionally would have been subject to discursive exposure The concern shifts from telling to showing, showing first and foremost that something remains untold. An exemplary book in this aesthetic drive is Avital Ronell's Telephone Book where the playful layout and explicit break with traditional design interrupt the sequential logic of the paragraphs thereby performing the very argument at stake: open up new circuits of meaning and signification. The engagement with form and publishing formats is only consequential since media theory as exposed by Walter J. Ong, Eric A. Havelock, Parry Miman or Marshall McLuhan besides many others made it clear that different communication technologies condition the mode of thought of a given culture The fractals of Benoit Mandelbrot or the butterfly shaped Lorenz attractor are the popular icons of this turn in the

sciences. For communication studies and philosophy, Vilém Flusser was one of the pioneer thinkers reflecting the rise of what he called "techno-images." The photograph is the paradigmatic case which condenses the whole crises of Western thought and culture on its grainy surface. It subverts the linear logic of writing.6 Flusser's central diagnosis detects a shift from the grand narratives associated with the literate mind and Modernity towards creating and envisioning the world through image based practices. Writing himself in a succinct, visually inspired style, Flusser contents that the web of meaning, spun throughout the centuries by our textual practices, has become transparent. We discover the void between the letters through which the world escapes: "We are alienated from the world of text, precisely because we see through them as our own product."7 As a consequence we are confronted with a world of dissociated facts and information bits. Flusser hails this fragmented world as the advent of the zero dimensional point-universe that will provide the new raw material for our image based practices. Techno-images are nothing else than a conscious, inherently artistic drawing-together of point-elements as exemplified first and foremost by the silver-grains of the photograph but also by more recent pixel based techno-images. Similar to blueprints, designs or statistical curves, the weather map for example visualizes and contracts a tremendous amount of meteorological data from widely distributed stations into a single image of high- and low pressure
areas, temperatures, wind speeds, etc. Techno-images "compute" in this sense the universe of unimaginable, abstract information-bits into a concrete surface – the image. Echoing Paul Feyerabend, these image practices have to be understood as creative practices where the truth of scientific models has been replaced by its aesthetic quality **Aesthetics, Writing, Networked Computers Jörg Muller Dissertation Submitted to the Division of Media and Communications of The European Graduate School in Candidacy for the Degree of Doctor of Philosophy**
**Lothar Schäfer (1) and Sisir Roy (2) in** Quantum Reality, the Importance of Consciousness in the Universe, the Discovery of a Non-Empirical Realm of Physical Reality, and the Convergence with Ancient Traditions of Indian and Western Philosophy investigate metaphysical aspects that include 1) the discovery of a non-empirical part of physical reality in a realm of potentiality; 2) the emanation of the empirical world out of a realm of non-material forms; 3) the discovery that the nature of physical reality is that of an indivisible Wholeness – the One; and 4) panpsychism: the possibility that the One is aware of its processes like a Cosmic Consciousness. The convergence of powerful traditions of seemingly disparate cultures is particularly important to point out in the present process of globalization, when a unifying view is needed to avoid controversy and conflict. To quote them in extenso: This is the nonlocality of the quantum world. Menas Kafatos and Robert Nadeau (1990) have drawn a remarkable conclusion from this phenomenon: If reality is nonlocal, the nature of the universe is that of an undivided wholeness. Because our consciousness has emerged from this wholeness and is part of it, it is possible to conclude that an element of consciousness is active in the universe: a Cosmic Consciousness. Pre-reflective self-consciousness is pre-reflective in the sense that (1) it is an awareness we have before we do any reflecting on our experience; (2) it is an implicit and first-order awareness rather than an explicit or higher-order form of self-consciousness. Indeed, an explicit reflective self-consciousness is possible only because there is a pre-reflective self-awareness that is an on-going and more primary self-consciousness. Although phenomenologists do not always agree on important questions about method, focus, or even whether there is an ego or self, they are in close to unanimous agreement about the idea that the experiential dimension always involves such an implicit pre-reflective self-awareness. In line with Edmund Husserl (1959, 189, 412), who maintains that consciousness always involves a self-appearance (Für-sich-selbst-erscheinens), and in agreement with Michel Henry (1963, 1965), who notes that experience is always self-manifesting, and with Maurice Merleau-Ponty who states that consciousness is always given to itself and that the word 'consciousness' has no meaning independently of this self-givenness (Merleau-Ponty 1945, 488), Jean-Paul Sartre writes that pre-reflective self-consciousness is not simply a quality added to the experience, an accessory; rather, it constitutes the very mode of being of the experience: This self-consciousness we ought to consider not as a new consciousness, but as the only mode of existence which is possible for a consciousness of something (Sartre 1943, 20 [1956, liv]). The notion of pre-reflective self-awareness is related to the idea that experiences have a subjective 'feel' to them, a certain (phenomenal) quality of 'what it is like' or what it 'feels' like to have them. As it is usually expressed outside of phenomenological texts, to undergo a conscious experience necessarily means that there is something it is like for the subject to have that experience (Nagel 1974; Searle 1992).

This is obviously true of bodily sensations like pain. But it is also the case for perceptual experiences, experiences of desiring, feeling, and thinking. There is something it is like to taste chocolate, and this is different from what it is like to remember what it is like to taste chocolate, or to smell vanilla, to run, to stand still, to feel envious, nervous, depressed or happy, or to entertain an abstract belief. Yet, at the same time, as I live through these differences, there is something experiential that is, in some sense, the same, namely, their distinct first-personal character. All the experiences are characterized by a quality of mineness or for-me-ness, the fact that it is I who am having these experiences. All the experiences are given (at least tacitly) as my experiences, as experiences I am undergoing or living through. All of this suggests that first-person experience presents me with an immediate and non-observational access to myself, and that consequently (phenomenal) consciousness consequently entails a (minimal) form of self-consciousness. To put it differently, unless a mental process is pre-reflectively self-conscious there will be nothing it is like to undergo the process, and it therefore cannot be a phenomenally conscious process. The mineness in question is not a quality like being scarlet, sour or soft. It doesn't refer to a specific experiential content, to a specific what; nor does it refer to the diachronic or synchronic sum of such content, or to some other relation that might obtain between the contents in question. Rather, it refers to the distinct givenness or the how it feels of experience. It refers to the first-personal presence or character of experience. It refers to the fact that the experiences I am living through are given differently (but not necessarily better) to me than to anybody else. It could consequently be claimed that anybody who denies the for-me-ness of experience simply fails to recognize an essential constitutive aspect of experience. Such a denial would be tantamount to a denial of the first-person perspective. It would entail the view that my own mind is either not given to me at all — I would be mind- or self-blind — or is presented to me in exactly the same way as the minds of others. There are also lines of argumentation in contemporary analytical philosophy of mind that is close to and consistent with the phenomenological conception of pre-reflective self-awareness. Alvin Goldman provides an example: [Consider] the case of thinking about x or attending to x. In the process of thinking about x there is already an implicit awareness that one is thinking about x. There is no need for reflection here, for taking a step back from thinking about x in order to examine it…When we are thinking about x, the mind is focused on x, not on our thinking of x. Nevertheless, the process of thinking about x carries with it a non-reflective self-awareness (Goldman 1970, 96). A similar view has been defended by Owen Flanagan, who not only argues that consciousness involves self-consciousness in the weak sense that there is something it is like for the subject to have the experience, but also speaks of the low-level self-consciousness involved in experiencing my experiences as mine (Flanagan 1992, 194). As Flanagan quite correctly points out, this primary type of self-consciousness should not be confused with the much stronger notion of self-consciousness that is in play when we are thinking about our own narrative self. The latter form of reflective self-consciousness presupposes both conceptual knowledge and narrative competence. It requires maturation and socialization, and the ability to access and issue reports about the states, traits, dispositions that make one the person one is. To claim that every kind of self-consciousness is conceptual is overly cognitive. Bermúdez (1998), to mention one further philosopher in the analytic tradition, argues that there are a variety of **nonconceptual forms of self-consciousness** that are "logically and ontogenetically more primitive than the higher forms of self-consciousness that are usually the focus of philosophical debate" (1998, 274; also see Poellner 2003). This growing consensus across philosophical studies supports the phenomenological view of pre-reflective self-consciousness. (References: Stanford Encyclopedia, Post Modersnist Theories, Internet Encyclopedia of Philosophy) Korzybski pointed out that there are two ways to slice easily through life; to believe everything or to doubt everything. Both ways save us from thinking. So, from time to time you have to give yourself a Virtual Reality Check. The hallmark of postmodern philosophy has been disbelief or skepticism of all "metanarratives," or translations of reality. Postmodernism has even turned its profound skepticism on such important humanist concepts as "objective truth" and reason.Yet, for a deconstructionist postmodern society, individually we are still riddled with superstition and gullibility, and open to manipulation through our belief systems as any politician, philosopher, clergy, or salesperson will attest. Further, most people are painfully naive when it comes to even the simplest scientific understanding. Most of us don't have a clue about the fundamental nature of physical reality or our own psychological nature, and our ability to be fooled by our senses and mind, or methods of social and technological persuasion. We may think we know, but the

penetration is deeper than we dare imagine. Nature rejects the naiveté that seeks absolute truth. We are beginning to realize, individually and culturally, that "realities" are all human constructions. The task becomes one of "catching ourselves in the act" of creating our own "reality" from the flow of events. Human truth is always an engagement of mind with experience.. We don't need to fear the collapse of our personalistic belief system (the "box" we live in), nor our belief in absolute truth. **A strong desire to engage in the "quest for uncertainty" complements our anxiety that perhaps there is no absolute, objective ground to reality**. The warrant of Truth is ever elusive when we deconstruct the foundational justifications of our convenient notions about the way the world works. It is easy to confuse what is actually the creation of beliefs with the "discovery of Truth," a common goal of science and theology. Science offers no ontological Scientific Picture of the World but substitutes a number of applied theories as meta-theory. It's close, powerful and useful: it 'works' and allows use to do work. But the still mysterious "true" metatheory may reveal the basis of consciousness. But who knows when we will lift that veil? Permeating the living reality of our culture are certain contagious notions, fads and trends that have the ability to influence the way we think about the nature of Reality and ourselves. Some of them are toxic and can consume you; others are just whims of pop culture infectious and benign as the common cold. **Jung** described the concept of psychic contagion by certain archetypal forces inherent in the human psyche, which manifest in our spiritual lives and belief systems. He spoke of both conscious and unconscious contamination. Notions like this range from simple superstitions to scientific concepts, to urban myths. Notions sweep through our culture and insinuate themselves within its fabric, as fads, whether they are "real" or not, they can be influential. They mood alter us, make us feel we belong. An analysis of these notions is useful in distinguishing a common human phenomenon from any potential "alien influence" which may or may not be exerted on us from an unknown source. **Strange Attractors: Transference, Holography, and an Archetype Burke, J. (2003). Strange Attractors: Transference, Holography, and an Archetype (Doctoral dissertation, Pacifica Graduate Institute, 2003).** In consideration to the Nobel Proze for Chemistry this year(2014), a stability nalysis of the proble in to single molexues would be a contemporaneous question that has to be taken cognisace of and given importance to . Determination of molecular structure by geometry optimization became routine only after efficient methods for calculating the first derivatives of the energy with respect to all atomic coordinates became available. Evaluation of the related second derivatives allows the prediction of vibrational frequencies if harmonic motion is estimated. More importantly, it allows for the characterization of stationary points. The frequencies are related to the eigenvalues of the Hessian matrix, which contains second derivatives. If the eigenvalues are all positive, then the frequencies are all real and the stationary point is a local minimum. If one eigenvalue is negative (i.e., an imaginary frequency), then the stationary point is a transition structure. If more than one eigenvalue is negative, then the stationary point is a more complex one, and is usually of little interest. When one of these is found, it is necessary to move the search away from it if the experimenter is looking solely for local minima and transition structures. The total energy is determined by approximate solutions of the time-dependent Schrödinger equation, usually with no relativistic terms included, and by making use of the Born–Oppenheimer approximation, which allows for the separation of electronic and nuclear motions, thereby simplifying the Schrödinger equation. This leads to the evaluation of the total energy as a sum of the electronic energy at fixed nuclei positions and the repulsion energy of the nuclei. Notable exceptions are certain approaches called direct quantum chemistry, which treat electrons and nuclei on a common footing. Density functional methods and semi-empirical methods are variants on the major theme. **Orlando Alvarez** discusses Polyakov's quantization of the string in the presence of a boundary allowing for an arbitrary topology for the world sheet. In addition to the dynamical conformal factor discovered by Polyakov, there are a finite number of new degrees of freedom if the surface is more complicated than a sphere or a disc. The quantization of the Liouville theory in an arbitrary topology is also discussed. A one-loop calculation shows that the model is renormalizable if one performs a mass renormalization and an additive field renormalization. The renormalization group equations have a perturbative infrared unstable fixed point in all topologies. Copyright © 1983 Published by Elsevier B.V. **Nuclear Physics B Volume 216, Issue 1, 25 April 1983, Pages 125–184 Theory of strings with boundaries: Fluctuations, topology and quantum geometry Orlando Alvarez DOI: 10.1016/0550-3213(83)90490-X** Quantum geometry (the modern loop quantum gravity involving graphs and spin-networks instead of the loops)

provides microscopic degrees of freedom that account for black-hole entropy. However, the procedure for state counting used in the literature contains an error and the number of the relevant horizon states is underestimated. In our paper a correct method of counting is presented by **Marcin Domagala and Jerzy Lewandowski.** Results lead to a revision of the literature of the subject. It turns out that the contribution of spins greater than 1/2 to the entropy is not negligible. Hence, the value of the Barbero–Immirzi parameter involved in the spectra of all the geometric and physical operators in this theory is different than previously derived. Also, the conjectured relation between quantum geometry and the black-hole quasi-normal modes should be understood again. **Marcin Domagala and Jerzy Lewandowski 2004 Class. Quantum Grav 21 5233 doi:10.1088/0264-9381/21/22/014 Black-hole entropy from quantum geometry** Quantum geometry predicts that a universe evolves through an inflationary phase at small volume before exiting gracefully into a standard Friedmann phase. This does not require the introduction of additional matter fields with ad hoc potentials; rather, it occurs because of a quantum gravity modification of the kinetic part of ordinary matter Hamiltonians. Authors draw the cognizance of application of the same mechanism that can explain why the present day cosmological acceleration is so tiny. DOI: http://dx.doi.org/10.1103/PhysRevLett.89.261301 **Inflation from Quantum Geometry Phys. Rev. Lett 89, 261301 – Published 12 December 2002 Martin Bojowald** The loop quantum cosmology of the closed isotropic model is studied with special emphasis on a comparison with traditional results obtained in the Wheeler-DeWitt approach by **Martin Bojowald** .Investigation includes the relation of the dynamical initial conditions to boundary conditions such as the no-boundary or the tunneling proposal and a discussion of inflation from quantum cosmology. **DOI: http://dx.doi.org/10.1103/PhysRevD.67.124023 Loop quantum cosmology, boundary proposals, and inflation Phys. Rev. D 67, 124023 – Published 23 June 2003 Martin Bojowald and Kevin Vandersloot** In loop quantum cosmology, the universe avoids a big bang singularity and undergoes an early and short super-inflation phase. During super-inflation, non-perturbative quantum corrections to the dynamics drive an inflaton field up its potential hill, thus setting the initial conditions for standard inflation. **Shinji Tsujikawa et al** show that this effect can raise the inflaton high enough to achieve sufficient e-foldings in the standard inflation era. They also analyse the cosmological perturbations generated when slow-roll is **violated after super-inflation** and show that loop quantum effects can in principle leave an **indirect signature** on the largest scales in the CMB, with some loss of power and running of the spectral index. **Shinji Tsujikawa et al 2004 Class. Quantum Grav 21 5767 doi:10.1088/0264-9381/21/24/006 Loop quantum gravity effects on inflation and the CMB Quantum geometry from quantum information** is the study of **Florian Girelli and Etera R Livine .**Loop quantum gravity defines the quantum states of space geometry as spin networks and describes their evolution in time. We reformulate spin networks in terms of harmonic oscillators and show how the holographic degrees of freedom of the theory are described as matrix models. This allows us to make a link with non-commutative geometry and to look at the issue of the semi-classical limit of loop quantum gravity from a new perspective. This work is thought of as part of a bigger project of describing quantum geometry in quantum information terms. **Florian Girelli and Etera R Livine 2005 Class. Quantum Grav 22 3295 doi:10.1088/0264-9381/22/16/011 Reconstructing quantum geometry from quantum information: spin networks as harmonic oscillators.** Quantum geometric mechanics is the topic of study by **Dorje C. Brody Lane P. Hughston** establishing a clear relation between the quantum mechanics and quantum geometry and also the entangled states. The manifold of pure quantum states can be regarded as a complex projective space endowed with the unitary-invariant Fubini–Study metric. According to the principles of geometric quantum mechanics, the physical characteristics of a given quantum system can be represented by geometrical features that are preferentially identified in this complex manifold. Here we construct a number of examples of such features as they arise in the state spaces for spin, spin 1, spin and spin 2 systems, and for pairs of spin systems. A study is then undertaken on the geometry of entangled states. A locally invariant measure is assigned to the degree of entanglement of a given state for a general multi-particle system, and the properties of this measure are analysed for the entangled states of a pair of spin particles. With the specification of a quantum Hamiltonian, the resulting Schrödinger trajectories induce an isometry of the Fubini–Study manifold, and hence also an isometry of each of the energy surfaces generated by level values of the expectation of the Hamiltonian. For a generic quantum evolution, the corresponding Killing trajectory is quasiergodic on a toroidal subspace of the energy surface through the initial

state. When a dynamical trajectory is lifted orthogonally to Hilbert space, it induces a geometric phase shift on the wave function. The uncertainty of an observable in a given state is the length of the gradient vector of the level surface of the expectation of the observable in that state; a fact that allows us to calculate higher orders corrections to the Heisenberg relations. A general mixed state is determined by a probability density function on the state space, for which the associated first moment is the density matrix. The advantage of a general state is in its applicability in various attempts to go beyond the standard quantum theory, some of which admit a natural phase-space characterization. **Journal of Geometry and Physics Volume 38, Issue 1, April 2001, Pages 19–53 Geometric quantum mechanics Dorje C. Brody Lane P. Hughston DOI: 10.1016/S0393-0440(00)00052-8** Expansion (unmesa) of [consciousness] or the creative intuition {pratibha) is (=) [experienced] in the interval which divides two [moments] of differentiated perception (vikalpa) It is here that they arise and disappear. The Sastras and Agamas proclaim with reasoned argument that it is (=) free of thought-constructs {Nirvikalpa) and precedes (e) Path to Liberation all mental representations of any object. None can deny that a gap exists between perceptions insofar as two moments of thought are (=) invariably divided. This [gap] is (=) the undifferentiated unity of all the countless manifestations. Similarly, in the outer more objective sphere, where change consists (e) of the alterations in the configurations of manifest appearances (Abhasa\ the transition from one to (e&eb) another corresponds to (e&eb) a phase of pure luminosity that marks (eb) the beginning of one form and the end of another. The world of manifestation and differentiated perceptions (Yikalpa) thus extends from one Centre to (eb) the next. **What is meant again is the realisation or the evolution of individual consciousness or general ledger to be the cosmic general ledger is the sole criterion for the Satchidananda state where all dualities cease** Although it is never in fact divorced from (e) the subject who resides there, the ignorant fail (e) to grasp this fact and so, cut off from (e) the Centre, the world of objectivity becomes (=) for them the sphere of Maya. Bhagavatotpala quotes the Light of Consciousness amvitprakdsa): This ever pure experience (suddhanubhava) is variegated by (e) each form [revealed within it] Even so it remains unstained (nirmala) when moving to another. Just as a cloth which is naturally white, once dyed, cannot (e) change colour without [first] becoming white again, similarly the pure power of awareness, (citi) once coloured by (e) form, is pure [again] at the Centre where that form is (=) abandoned and from whence it proceeds to (e&eb) another. In his Essence of Vibration (Spandasarpdoha), K§Emaraja explains that the rise and fall of every individual perception in the field of awareness is (=) a specific pulsation of consciousness. This is exactly the individual general ledger transaction lifted to the transaction in cosmic general ledger. During The Initial Instant Of Perception, T Consciousness Is Manifestly Apparent And The Yogi, Participating In Its Plenitude, Observes The Outer World Without Being Attached To Any Particular Divine Body And Sacred Circle Of The Senses Singling It Out From Any Other, Like A Man Who Observes A City From A High Mountain Peak. He Sees The Outer World Reflected Within His Consciousness Free Of Thought-Constructs And So 'Stamps* The Outer On The Inner While Absorbing The Object And Means Of Knowledge In The Pure Subject Which Grasps Them As The Expansion Of His Own Nature. K§Emaraja Says: By Penetrating Into Bhairavimudra, The Yogi Observes The Vast Totality Of Beings Rising From, And Dissolving Into, The Sky [Of Conscious- Ness], Like A Series Of Reflections Appearing And Disappearing Inside A Mirror. Through The Practice Of Bhairavimudra The Yogi Realises That He Is The Substratum Consciousness (Adhisfhatr) Which Both Underlies And Is The Essence Of All Things. He Discovers That Phenomena Have No Independent Existence Apart From Him And So Are, In This Sense, Void. At The Same Time, He Realises That Because All Things Are Consciousness, They Are Far From Unreal. He Views The Outer World Yet Sees It Not. Beyond Both Voidness And Non-Voidness He Penetrates Into The Supreme Abode (Pararn Padam) Of Siva's Consciousness. [The Powers Of The Senses] Endowed With The Attributes Of The Great Union [Between Subject And Object] Whose Form Is The Wakening Of Man's Spiritual Potential (Kuntfalinf), Fill [With Consciousness] The Outer Clatter Of Diversity (Bhedatfambara) Born Of Its Intense Power And Are Then Established In The Unobscured Abode Of The Void Of Consciousness To Shine [There] Eternally. Thus Residing Beyond Being And Non-Being, The Sole Protector Of The Unity Which Is Tranquil And Expanding [Consciousness], Whose Glory Is All-Embracing And Form Unobscured, Is Called Bhairavimudra. Through The Practice Of Bhairavimudra, The Yogi Unites The Universal Vibration Of T Consciousness With The Individual Pulsation Of Objectivised 'This' Consciousness. The Two Aspects Of Consciousness Are Now In A State Of

Equilibrium Like The Two Pans Of An Evenly Weighed Balance And The Yogi Experiences The Pure Knowledge (Suddhavidya) That: 'I Alone Am All Things'. Thus Becoming The Master Of The Wheel Of Energies He Is Free, Like Siva, To Create And Destroy. When [The Yogi] Is Well Established, Without Wavering, Solely In The Integral Egoity Of His Authentic Nature, The Spanda Principle, And Is Absorbed In Contemplation (Samavisfa), He Becomes One With It The Doctrine Of Vibration (Tanmaya). Then . . . Dissolving And Creating The Universe By Means Of His Introverted And Extroverted Absorption, He Destroys And Creates All Things Out Of Sarikara, His Innate Nature. [Thus] He Assumes The State Of The Universal Experiencer And Having Absorbed All That Is To Be Experienced From [The Grossest Level] — Earth — To (The Subtlest) — Siva — He Reaches The State Of The Supreme Subject By Progressively Recognising [His Identity With Him]. Thus, Introverted And Extroverted Absorption Both Lead To The Recognition Of The Pulsation (Spanda) Of One's Own Consciousness. This Is Where Individual General Ledger Becomes Cosmic General Ledger. At The Level Of Consciousness Corresponding To Siva's Basic State (Saryibhavdvastha), The Alternation From Inner To Outer Is Instantaneously Resolved Into The Vibration Of His Nature. When The Yogi Finally Comes To Be Constantly Aware Of This Reality, His Enlightenment Is Full And Perfect. Freed Of All Means (Anupaya) And Delighting In The Power Of His Bliss (Anandasakti), He Knows And Does Whatever He Pleases. The Yogi Seeking Self-Realisation Must Acquire Mastery Over This Movement. Ksemaraja Stresses That The Doctrine Of Vibration Teaches That Liberation Can Only Be Achieved By First Withdrawing All Sense Activity In Introverted Contemplation (Nimilanasamadhi) To Then Experience The 'Great Expansion' (Mahdvikdsd) Of Consciousness While Recognising This To Be A Spontaneous Process Within It. This Is Done Through The Practice Of Kramamudra. A Passage From The Now Lost Kramasiitra Explains: Although The Adept's Attention [May Be] Outwardly Directed, He Enjoys Contemplative Absorption Through The Introverted Aspect Of Kramamudra. Initially He Turns Inward From The Outside World And [Then] From Within [Himself] He Exits Into The Outer World Under The Influence Of His Absorption. There Is No Necessity Of Any Glorification Or Mortification Here. It Is Just Like Joining A Job And Retiring And Realising All That Happened Is Just By His Own Effort And Nothing Else. Thus The Sequence (Krama) In This Attitude (Mudra) [Ranges Through] Both Inner And Outer. The Yogi Must Pervade The Surface Level Of Awareness (Yyutthana) With The Same Bliss He Experiences Plunging Into The Depths Of Con-Templative Absorption (Samadhi). Note Here That Samadhi Is A Death Like State And Still Being Matter And Information Stored You Experience Consciousness. Submerging Himself And Emerging Repeatedly From Samadhi, He Eventually Recognises That The Unity Of Consciousness Pervades Both States: The Best Of Yogi's, Who Has Achieved A State Of Complete Absorption Even When Risen From Meditation, [Inwardly] Vibrating Like A Drunkard In Blissful Inebriation From The After-Effects Of The Nectar Of Con- Templation, Sees All Things Dissolving In The Sky Of Consciousness Like A Cloud In The Autumn Sky. He Plunges Repeatedly Within Himself Divine Body And Sacred Circle Of The Senses And Becomes Aware Of His Identity With Consciousness By The Practice Of Introverted Contemplation. Thus Even When He Is Said To Have Risen From Absorption, He Is One With [His] Experience Of It. Ksemaraja Goes On To Explain That This Practice Is Called 'Mudra' Because It Both Fills The Adept With Bliss (Mud) And Is Itself The Bliss Of Consciousness. Moreover, It Dissolves Away (Dra) All Bondage And 'Stamps' The Universe Of Experience With The Seal (Mudra) Of The Fourth State (Turiya) Of Enlightened Consciousness Beyond, And Including, The Three States Of Waking, Dreaming And Deep Sleep. It Is Called 'Krama' Because It Is The Root Source Of All Emanation And All Other Conscious Processes Which Succeed One Another In Ordered Sequence (Krama) And Is, At The Same Time, Their Successive (Krama) Appearance As Well. By The Practice Of Kramamudra The Opposites Fuse And Siva And Sakti Unite. They Yogi Comes To Experience The Simultaneous Pervasion Of All The Lower, Grosser Categories Of Existence By The Higher And The Presence Of The Lower In The Higher. Commencing His Practice In A Low Form Of Bhairavimudra, The Yogi Conjoins The Outer With The Inner; Then, In Kramamudra, He Fills Both The Outer With The Inner And The Inner With The Outer. When He Achieves Perfection In This Two-Fold Movement, He Attains To The Highest Form Of Bhairavimudra In Which The Two Merge Completely In The Experience Of The Absolute (Anuttara), Free Of All Differentiation And Polarities. If He Fails To Maintain Awareness Of This State, He Again Falls Into Kramamudra Until He Has Finally Completely Merged All The Highest States In The Lower

And The Lower In The Higher. He Then No Longer Needs To Resort To Any Means (Anupaya) To Achieve Liberation. All He Says Or Does Anything He Perceives Or Thinks, Instantly Occasions In Him The Highest Level Of Consciousness. Thus The Fruit Of Bhairavimudra Is The Wonder (Camatkara) Or Amazement (Vismaya) That Overcomes The Yogi When He Reaches The Plane Of Union (Yogabhumika) Y Xsl Where All Opposites Merge In The Radiance Of The Great Light Of Consciousness. The Stanzas On Vibration Teach: How Can One Who, As If Astonished, Beholds His Own Nature As That Which Sustains [The Existence Of Everything] Be Subject To This Painful Round Of Transmigration (Kusrti)? The Yogi, Recognising His True Nature To Be The Supreme Subject, Is Astonished To Suddenly Discover That The Individual He Thought He Was, Caught Up In The Trammels Of Thought And Living In A World Enmeshed In The Web Of Time And Space, Does Not Really Exist At All. He Experiences A 'Turning About' (Paravrtti) In The Deepest Seat Of The Doctrine Of Vibration Consciousness As He Penetrates His True Nature. The Sudden Eruption Of This Intuition (Pratibha) Arouses In Him A Cry Of Amazement As He Transcends All Thought-Constructs And, Perfectly Absorbed In His Own Nature, Is Liberated. The Path To Liberation Essentially, Spanda Doctrine Is Concerned With Two Matters. The First Is To Impart To Those Who Are Fit To Receive The Teachings A Deeper Understanding Of The Ultimate Goal Of Life (Upeya). When We Have Understood What Truly Benefits Us And Is Worth Attaining And What, On The Contrary, Is Of No Real Value But Stands In The Way Of This Attainment, We Can Begin To Make Progress Towards Our Goal. This Is Spanda Doctrine's Second Concern, Namely, To Show The Way In Which We Can Develop Spiritually Through Siva's Grace And The Right Application Of The Means To Realisation That It Teaches. When Both These Aspects Of The Teaching Have Been Correctly Understood And Applied, The Spanda Yogi Achieves A Clear And Permanent Realisation Of His Goal And Is Liberated, Thus Fulfilling The Ultimate Aim Of The Teaching. The Doctrine Of Vibration Is Not Meant For The Spiritually Dull. It Is Not For The Worldly Whose Consciousness, Clouded By Ignorance, Is As If Dreaming, Even During The Waking State Of Daily Life, The Dream Of Its Own Thought-Constructs. The Teachings Are Meant For Those Who Are Awake (Prabuddha), Those Who, Full Of Faith And Reverence, Are Always Alert And Intent On Discerning The True Nature Of Ultimate Reality This Reality Is Understood In Three Basic Ways. The First Is Purely Transcendental. The Stanzas Choose This Aspect As The One Which Formally Defines It Most Specifically. Ultimate Reality Transcends All The Opposites, Including Subject And Object. This Does Not Mean, However, That It Is An Unconscious Void, A Mere Absence Of All Existence. In Fact, This Negative Characterization Of Reality (Which Includes Also A Denial Of All That Is Unconscious) Implies A Positive Immanence In Which The Opposites Are United In The Oneness Of Pure Consciousness That Is Equally Siva And Spanda, His Universal Activity. These Two Seemingly Contrasting Aspects Are Reconciled In The Third, Namely, Reality Understood As The Essential Nature Of All Things. Although Universal And Everywhere The Same, It Is Understood To Be The Essential And Specific Nature Of Each Existent As Its 'Own Nature' (Svabhava). In The Case Of The Individual Soul It Is Even More Specific, More Personal As His Own 'Own Nature' (Svasvabhava). Note Here That Svabhava Is Not Svasbhava. In Fact We Made A Statement That Svabhava Is The Very Characterstics Of Space Time Like, Crime Zones, Silent Zones, School Zones We Classify Space In To. Belonging To None Other Than Oneself It Is The Pure Subjectivity That Perceives Experiences, Enjoys, Reflects, Thinks And Senses As Well As Being The Conscious Agent Who Creates Every Possible Form Of Experience In All The States Of Consciousness. The Liberating Knowledge Of Reality Thus Corresponds To Our Regaining Possession Of Ourselves (Svatmagraha). We Must Lay Hold Of Ourselves And Abide In Our Authentic Nature. Reality Coincides With Our Own Most Fundamental State Of Being (Svasthiti), Free Of All Contrasts And Contradictions. Once We Have Overcome The Negative Forces That Arise From Our Ignorance And Prevent Us From Abiding In Ourselves, We Are Liberated. To Do This, We Must Penetrate Through The Pulsing Fluctuations Of Objectively Experienced States And Perceptions At The Surface Level Of Consciousness And Gain Insight Into The Timeless Rhythm Of Our Own Nature Manifest In The Universal Arising And Falling Away Of All Things. We Are Not Freed Of The Trammels Of Perpetual Change By Setting It Aside; On The Contrary, We Must Gain Insight Into The Recurrent Cycles Of Creation, Persistence And Destruction, Or Else Be Bound By Our Ignorance. This Spiritual Ignorance Consists Essentially Of Our Contracted State Of Consciousness And So Can Only Be Effectively Countered By Expanding It To Reveal Our Own Authentic Nature As This Expanded State Itself, Which

Is The Universal Vibration (Samanyaspanda) Of Consciousness. Vibration Of Strings Which Gives Shape To Objects Is Itself A Information, A Sine Qua Non Of Individual Consciousness. The Spanda- Yogi Treads The Path Of Consciousness Expansion. The Movement From The Contracted To The Expanded State Marks The Transition From Ignorance To Understanding, From The Dispersion And Incompleteness Of A Form Of Consciousness Entirely Centred On An Objectively Perceived And Discursively Represented Reality To A Direct, Intuitive Awareness Of The Unity And Integral Wholeness Of Our Own Absolute Spanda Nature. Along The Way To This Supreme Realisation Consciousness Develops, As Veil After Veil Is Lifted, Until It Becomes Full And Perfect In The Absolute Which Encompasses Within Itself All Possible Formats Of Experience. As Abhinava Says: [This Realisation] Is The Supreme Limit Of Plenitude And As Such There Can Be No Higher Attainment. Any [Other] Attainment [We Can] Conceive Issues From A State That Falls Short Of [This] Perfection. Once* [This] Uncreated Fullness Has Been Attained, Pray Tell, What Other Fruit Can There Be [Beyond It]? The Fettered Soul's Contracted State Of Consciousness Binds Him Because He Is Deprived Thereby Of The Subtle, Intuitive Insight Into The Underlying Unity Of Existence And His Attention Is Focused Instead On Its Gross, Outer Diversity Easily Apparent To Everybody, However Restricted His Consciousness May Be. However, Although The Fettered Soul In This State Is Ignorant Of This Unity, This Does Not Mean That His Knowledge Of Diversity Is False. Ignorance Entails A Form Of Knowledge Which, Although Quite Correct, Is Binding. We Are Not Absolutely Ignorant Of Reality For If We Were We Would Be Totally Unconscious. Spiritual Ignorance Is Always Linked With Some Degree Of Consciousness. Those Subject To The Round Of Birth And Death Are Not Inert Clods Of Earth. This Also Leads To Conclusion That Shakti Responsible Is Impure Because Of Maya Imbibed In It. Thus, Although Ignorance Obscures Consciousness, It Is Wrong To Think Of It, As Dualist Saivites Do, In Terms Of A Defiling Impurity That Shrouds It Like A Cloth Covering Ajar. Spiritual Ignorance Can Be Nothing But Consciousness Itself, Albeit In A Limited State. Siva, Who Is Universal Consciousness, Is The Innate Nature Of Both Its Contracted And Expanded States, Both Of Which Are Forms Of Knowledge, Namely: 1) Supreme Knowledge (Parajnana) Defined As The Revelation Of One's Own Innate Nature As The One Reality Which Is The Being Of All Things. 2) Inferior Knowledge (Aparajndna) Which Jayaratha Explains Results From The Mental Activity (Yyapard) Of The Individual Subject Whose Consciousness Is Contracted. It Consists Of The Mental Representations (Vikalpa) He Forms Of Himself And His Object, Of The Type I Know This'. The Lower Knowledge Obscures The Higher And Binds The Soul By Breaking Up His Direct, Pervasive Awareness Of His Own Pure Consciousness Nature, Free Of Mental Representation. The Stanzas On Vibration Teach: Operating In The Field Of The Subtle Elements, The Arising Of Mental Representation Marks The Disappearance Of The Flavour Of The Supreme Nectar Of Immortality; Due To This [Man] Forfeits His Freedom. As We Have Already Seen, Three Factors Are Necessary For Perception And Thought To Be Possible, Namely, The Perceiving Subject, The Means Of Knowledge And The Object Perceived. Rajanaka Rama, In His Commentary On The Stanza Cited Above, Explains At Length That These Three Factors Correspond To Three Major Divisions In The Lower Thirty-One Categories Of Existence, Namely: 1) The Object. This Consists Essentially Of The Five Primary Sensations Which Are The Subtle Elements {Tanmatras) Of Smell, Taste, Sight, Touch And Sound Along With The Five Gross Elements — Earth, Water, Fire, Air And Ether — Of Which These Sensations Are The Perceivable Qualities. 2) The Means Of Knowledge. This Consists Of The Senses And The Inner Mental Organ. 3) The Subject. At This Level, The Subject Is The Individual Soul (Purusa) Whose Consciousness Is Contracted By The Five Obscuring Coverings (Kancukas) Of Limited Knowledge And Action, Attachment, Natural Law And Time Along With Maya, Their Source. All These Categories Belong To The Impure Creation (Asuddhasrsti), Which Is The Sphere Of Maya Where The Lower Order Of Knowledge Operates And Subject And Object Are Divided. Above Them Are Five More Categories Which Belong To The Pure Creation (Suddhasrsfi) Where Subject And Object Are Still United. The Highest Of Those Categories Are Siva And Sakti. Combined They Represent The State Of Pure T Consciousness And Its Sentient Subjectivity (Upalabdhrta), Respectively. The Next Category Is Called Sadasiva. Here Faint Traces Of Objectivity Appear In The Pervasive, Undivided Consciousness Of Siva And Sakti. Consciousness, Now Full Of The Power Of Knowledge (Jnanasakti), Views The All In A State Of Withdrawal (Nimesa), Shining Within, And At One With Its Own Nature. T Consciousness Predominates Over 'This' Consciousness Which It Encompasses In The Awareness That: 'I Am This

[Universe]' (Aham-Idam). Next Comes The Category 'Isvara' Corresponding To The Awareness: 'This (Universe) Is Me' (Idam-Aham). 'This' Consciousness Takes The Upperhand Over T Consciousness And Unfolds Externally Full Of The Creative Power Of Action (Kriya&Akti). The All Now Becomes More Clearly Manifest As An Independent Reality. It Is Still Experienced As One With Consciousness But Is No Longer Fully Merged Within It. Finally, When Both Subjective And Objective Aspects Share An Equal Status In The Two-Fold Awareness That: 'I Am This (Universe) And This (Universe) Is Me (Ahamidam-Idamaham)\ Pure Knowledge (Fuddhavidya), The Last Of These Categories, Emerges. The Pure Categories Are The Experience Of The Impure Categories When They Are Recognised To Be One With Consciousness. They Are Experienced Within The Domain Of The Pure Universal Subject The Enlightened Yogi Realises Himself To Be. Mental Representations (Yikalpa) Emerge From This Pure Awareness And Subside Into It In Consonance With The Rhythm Of The Emanation And Withdrawal Of The Lower Categories. Impelled By The Universal Will, This Movement Is Spontaneous And Free. Free Of All Hopes And Fears The Enlightened Yogi Sees All Things As Part Of This Eternal Cosmic Game, Played In Harmony With The Blissful Rhythm Of His Own Sportive Nature At One With All Things. The Stanzas On Vibration Teach: Everything Arises [Out Of) The Individual Soul And He Is All Things. Being Aware Of Them, He Perceives His Identity [With Them]. Therefore There Is No State In The Thoughts Of Words Or [Their] Meanings That Is Not Siva. It Is The Enjoyer Alone Who Always And Everywhere Abides As The Object Of Enjoyment. Or, Constantly Attentive, And Perceiving The Entire Universe As Play, He Who Has This Awareness (Sawvitti) Is Undoubtedly Liberated In This Very Life. According To The Doctrine Of Vibration, Only Liberation In This Life (Jivanmukti) Is Authentic Liberation. Liberation After Death (Yideha- Mukti) In Some Form Of Disembodied State Free Of All Perceptions And Notions Of The World Of Diversity Is Not The Ultimate Goal. Ksemaraja Stresses That Liberation Is Only Possible By Realising One's Own Identity With The Whole Universe, However Difficult This May Be. Similarly, He Maintains That The Suspension Of All Mental And Sensory Activity, Which Takes Place In The Introverted Absorption Of Contemplation With The Eyes Closed (Nimilanasamadhi) That Leads To Identification With Transcendent Consciousness Is Complemented And Fulfilled By The Cosmic Vision Had Through The Expansion Of Consciousness That Takes Place In Contemplative Absorption With The Eyes Open (Unmilanasamadhi). Consequently, Ksemaraja Explains That The First Of The Three Sections, Into Which He Divides The Stanzas, Deals With The Former Mode Of Contemplation And The Second Section With The Latter. Significantly, The Last Stanza Of The Second Section Ends With The Declaration That 'This Is The Initiation That Bestows Siva's True Nature'. In Other Words, This Realisation, Attained Through The Expanding Consciousness Of Contemplation With The Eyes Open, Initiates The Yogi Into The Liberated State, Which Is Identification With Siva Whose Body Is The Universe. In Order To Attain This Expanded State Of Liberated Consciousness, The Yogi Must Find A Spiritual Guide Because The Master (Guru) Is The Means To Realisation. The Master Is For His Disciple Siva Himself For It Is He Who Through His Initiation, Teaching And Grace, Reveals The Secret Power Of Spiritual Discipline. Instructing In The Purport Of Scripture He Does More Than Simply Explain Its Meaning: He Transmits The Realisation It Can Bestow. The Master Is At One With Siva's Divine Power Through Which He Enlightens His Disciple. It Is This Power That Matters And Makes The Master A True Spiritual Guide, Just As It Was This Same Power That Led The Disciple To Him In His Quest For The Path That Leads To The Tranquility That Can Only Be Found 'In The Abode Beyond Mind'. The Master Is The Ferry That Transports The Disciple Over The Ocean Of Thought — If, That Is, The Disciple Is Ready. The Disciple Must Be 'Awake' (Prabuddha) 2% Attending Carefully To The Pulse Of Consciousness. This Alert State Of Wakefulness Is At Once The Keen Sensitivity Of Insight As Well As The Receptivity Of One Who Has No Other Goal To Pursue Except Enlightenment. The Highest, Most Perfect Relationship The Disciple Can Have With His Master Is Such As It Is With Siva Himself: One Of Identity. The Exchange That Takes Place Between Them Is An Internal Dialogue Within Universal Consciousness, Their Common Identity (Svabhava). Limiting Itself To A Point Source (Anu) And Obscured By (e) The Thought-Constructs Born Of Doubt And Ignorance, Consciousness Assumes The Guise Of The Disciple Who Seeks To Attain The Expanded Fullness Of His Master's Consciousness. The Master, On The Other Hand, Embodies The Aspect Of Consciousness Which Responds To The Inquiring Consciousness Of His Disciple. Free Of The Notions Of 'Self And 'Other', When The Disciple Is Liberated By His Grace, It Is The Master Who In Reality Liberates

Himself. Although Ksemaraja Assures Us That The Master Can By Himself Enlighten His Disciple By The Initiation He Imparts To Him And The Other Means (Yukti) N He Adopts, Even So, He Is Not The Only Guide On The Path. Apart From The Master There Is Scripture And, Above All, One's Own Personal Experience, Because, As Abhinava Says: The Knowledge [Acquired] By Gradually [Coming To Understand The Meaning Of] The Scriptures And Following The Master [Who Knows Them] Leads, [When] Confirmed For Oneself, To The Realisation Of One's Own Identity With Bhairava. It Is Important To Know The Scriptures. God Reveals Himself Through Them; They Are One Of The Forms In Which He Is Directly Apparent In This World. They Teach Man What Is Worth Attaining And What Should Be Avoided And So Like A Boat Convey Him Across The Ocean Of Profane Existence (Samsara) To The Other Shore Where God's True Nature Is Revealed To Him. However, The Study Of The Scriptures Is Of Value Only If Accompanied By The Spiritual Knowledge That Results From Personal Experience. Mahesvarananda Writes: Being Well Versed In The Nature Of Deity Is One Thing, But Being Well Versed In The Sacred Scripture Is Another, Just As The Peace Of That Abode Is One Thing And What Worldly People Experience Is Another. Vasugupta, Who Found The Sivasutra, Knew The Means To Realisation (Yukti) As Well As The Scriptures And Had Fully Experienced The One Ultimate Reality. Therefore, Ksemaraja Declares Him To Be Amongst The Best Of Teachers. The Stanzas On Vibration (That Ksemaraja Attributes To Vasugupta) Accordingly Transmit The Secrets Of The Sivasutra In Accord With Scripture, Sound Reasoning And Personal Experience. The Latter Is Particularly Important For The Spanda Yogi; He Is Not Interested In Wasting His Time In Useless Discussion About The Experience Of Consciousness Expansion And Its Fruits, For That Can Only Be Known For Oneself. The Yogi Can Achieve This Experience Either Through Faith In The Master Or Personal Insight (Svapratyayatah) Acquired By Unswerving Devotion To God. Ksemaraja Accordingly Quotes A Passage From The Bhagavadgita Where Krsna Says: Those I Deem To Be The Best Yogis Who Fix Their Thoughts On Me And Serve Me, Ever Integrated [In Themselves], Filled With The Highest Faith. But While The Yogi's Development Depends On Faith And Personal Experience Of The Higher States Of Consciousness, He Can, And Must, Strengthen His Conviction In The Light Of Reason .When Reason (Upapatti) And Direct Insight (Upalabdhi) Work Together, They Serve As A Means To Liberation. Reason Alone Cannot Help Us, But When It Is Based On An Intuitive Insight Of Fundamental Principles Along With A Direct Experience Of Reality, Error Is Eradicated And The Yogi Is Freed. In This Way The Awakened Yogi Realizes His Inherent Spiritual Power (Svabala) With Which He Exerts Himself To Distinguish Between The **Motions Of Individualized Consciousness And The Universal Vibration (Samanyaspanda)** Of The Collective Consciousness That Is Their Ultimate Ground And Firm Foundation. Note The Information Of Individual, Collective And Cosmic General Ledger Is Alone Responsible For Creation And Also Destruction. Thus, Although The Doctrine Taught In The Stanzas On Vibration Accords With Scripture, It Is Supported By Reason And Above All By Personal Experience. Thus, For Example, The Seventeenth Stanza Describes The Difference In The Manner In Which The Well Awakened And The Unawakened Experience Their Own Nature (Atmopalambha While The Eighteenth Describes The Experience Of The Well Awakened In The Three States Of Waking, Dreaming And Deep Sleep. Indeed, Rajanaka Rama, One Of The Commentators, Explains That The First Sixteen Stanzas Establish On The Basis Of Personal Experience (Svanubhava) That One's Own True Nature Is Independent Of The Body. Similarly, The Remaining Stanzas Also Discuss The Direct Experience Of One's Own Nature, But This Time As The Unity Of All Things. This Direct Experience, In Its Diverse Aspects, Is Both The Means By Which The Yogi Develops His Consciousness As Well As His Ultimate Goal. **Doctrine Of Vibration: Mark S.G.Dyczkowski Models See Next Paper** From the point of view of the object, the expansion (unmesa) of this pulse is represented(eb) by the initial desire to perceive (didrksa) a particular object, while the contracted (nimesha) phase is (=) the withdrawal of attention from the object previously perceived. From the point of view of the perceiving subjectivity, the phases are (=) reversed, so that the initial desire to perceive marks (eb) the contraction (nimesha) of subjective consciousness while the falling away of the previous perception is (=) its expansion (unmesa). At the higher level, where these two phases are experienced within (eb) consciousness, they represent (eb) the state of the categories of Isvara ('this universe is me') and Sadasiva ('I am this universe'). Utpaladeva says: Expansion (unmesa\ which is in (eb) the external manifestation [of objectivity], is (=) Kvaratattva while contraction (Niemba), which is in the internal manifestation [of subjectivity], is Sadasiva. **There is no**

**glorification or mortification here. It just like joining the job and retiring the involution and evolution phases**.(italics mine)The Doctrine of Vibration At this level all the powers of consciousness fuse (e&eb) and both phases are (=) manifest as part of one reality. This unity is in fact apparent (eb) to everybody at each moment. However, within the domain of Maya, which is the sphere of differentiated perceptions (Yikalpa), it is clearly manifest (eb) only at the juncture (Madhya) between two cognitions. In this Centre resides (eb) the void (kha) of consciousness (free of (e) thought-constructs) which divested of (e) diversity, digests into (eb) itself all the psycho-physical processes that give (eb) life to the multiplicity of perceptions. **Here it is important to note that the when the cosmic general ledger zero reflects upon itself and it is projected on the individual consciousness you see a film an illusory animations wherein creation itself becomes subjective wherein the individual consciousness is both the perceiver the perceived being the image on the screen on the screen of individual consciousness**. The yogi moves from the particular vibrations of consciousness at its periphery to (e&eb) the universal throb of the Heart in the Centre. As Abhinava explains: The self-reflective awareness in (eb) the Heart of pure consciousness, present (eb) at the beginning and end of each perception, within (eb) which the entire universe is (=) dissolved away without residue, is (=) called in the scriptures, the universal vibration of consciousness (samanyaspanda) and is (=) the outpouring (uccalana) [of awareness] within one's own nature. All the categories of existence (tattvas) are united in (eb) the Heart of the Centre where the life-giving elixir of Siva's consciousness floods (e&eb) one's own inner nature. To reside in the Centre is to abide by (e) the law of totality (gramadharma) in a state which transcends (e) the workings of the mind (unmana).Consciousness (jnana) with Light as its support, residing in (eb) the Centre between being and non-being is (=) known as the act of abiding in (eb) one's own abode as (=) the perceiving subjectivity (dratfrtva) free of (e) all obscuration. That which has been purified by (e) pure awareness **(suddhavijnana)** is called the transcendent (viviktavastu), said to be (=) the mode of being (v/7//) of the law of totality (gramadharma) through (e&eb) which everything is (=) easily attainable. The power in the Centre (madhyasakti) is (=) the eternal Present Beyond time it is the source of both past and future. To be established there is to abide without a break in Rama, the supreme enjoyer, in every action of one's life. Rama is Siva, the supreme cause Who pervades the fourteen aspects which embrace the entire universe of experience, namely, moving, standing, dreaming, waking, the opening and closing of the eyes, running, jumping, exertion, knowledge [born] of the power of the senses, the [three] aspects of the mind, living beings, names and all kinds of actions. This came as a natural development in Spanda doctrine not only for **this reason but also because the universal ego is experienced as the inner dynamics of absolute consciousness. To conclude our summarial exposition of the Divine Means, which is centred on the direct experience of this pure ego (and hence on Spanda in this form), we turn now to a brief description of its inner, cyclic activity. We shall do this** by examining Abhinava's esoteric exegesis of the symbolic significance of the word 'A HAM', which in Sanskrit means T, and symbolises by its form the ego's dynamic nature. “Dasein Does Not Fill Up A Track Or Stretch ‘Of Life’ — One Which Is Somehow Present-At-Hand — With The Phases Of Its Momentary Actualities. It Stretches Itself Along In Such A Way That Its Own Being Is Constituted In Advance As A Stretching-Along. The ‘Between’ Which Relates To Birth And Death Already Lies In The Being Of Dasein … It Is By No Means The Case That Dasein ‘Is’ Actual In A Point Of Time, And That, Apart From This, It Is ‘Surrounded’ By The Non-Actuality Of Its Birth And Death. Understood Existentially, Birth Is Not … Something Past In The Sense Of Something No Longer Present-At-Hand; And Death Is Just As Far From Having The Kind Of Being Of Something … Not Yet Present-At-Hand But Coming Along … Factical Dasein Exists As Born; And, As Born, It Is Already Dying, In The Sense Of Being-Towards-Death. As Long As Dasein Factically Exists, Both The ‘Ends’ And Their ‘Between’ *Are*, And They Are In The Only Way Possible On The Basis Of Dasein’s Being As *Care* … As Care, Dasein Is The ‘Between’.” **— Martin Heidegger (Thomas Piel - Traducteur), Being And Time** Path to Liberation On without knowledge that everything is manifest within consciousness is illusory or unreal in that sense alone. 78 Things are more real or more tangibly experienced according to their own essential nature (svabhava) to the degree in which we recognise that they are appearances (abhasa) within absolute consciousness As Jayaratha says: Just as images manifest in a mirror, for example, are essentially mere appearances, so too are [phenomena] manifest within conscious- ness. Thus, because they are external, [phenomena] have no being (sattva) of their own. The Lord says this [not with the intention of saying anything about the nature of things] but in order to

raise the level of consciousness of those people who are attached to outer things; thus everything in this sense is essentially a mere appearance. [Knowing this], in order to quell the delusion of duality, one should not be attached to anything external. 79 The ultimate experience is the realisation that everything is contained within consciousness. We can discover this in two ways. Either we merge the external world into the inner subject, or we look upon the outer as a gross form of the inner. In these two ways we come to recognise that all things reside within our own consciousness just as consciousness resides within them. This all-embracing inwardness is only possible if there is an essential identity between the universe and consciousness. The events which constitute the universe are always internal events happening within consciousness because their essential nature is consciousness itself. 80 We can only account for the fact that things appear if there is an essential identity between consciousness and the object perceived. If a physical object were really totally material, that is, part of a reality independent of, and external to, consciousness, it could never be experienced. Abhinava says: The existence or non-existence of phenomena within the domain of the empirical (iha) cannot be established unless they rest within consciousness. In fact, phenomena which rest within consciousness are apparent (prakasamana). And the fact of their appearing is itself their oneness {abheda) with consciousness because consciousness is nothing but the fact of appearing {prakasa). If one were to say that they were separate from the light of [that consciousness] and that they appeared [it would be tantamount to saying that] 'blue' is separate from its own nature. However, [insofar as it appears and is known as such] one says: 'this is blue 1 . Thus, in this sense, [phenomena] rest in conscious- ness; they are not separate from consciousness. The Doctrine of Vibration The universe and consciousness are two aspects of the whole, just as quality and substance constitute two aspects of a single entity. The universe is an attribute (dharma) of consciousness which bears (dharmiri) it as its substance. It is said that 'substance' is that resting in which this entire group of categories manifests and is made effective. Now, if you don't get angry [we insist that] this entire class of worlds, entities, elements and categories (tattva) rests in consciousness and [resting in it] is as it is. Thus consciousness contains everything in the sense that it is the ground or basis (adhard) of all things, their very being (satta) and substance from which they are made. But, unlike the Brahman of the Advaita Vedanta, it is not the real basis (adhisthand) of an unreal projection or illusion. Consciousness and its contents are essentially identical and equally real. They are two forms of the same reality. Consciousness is both the substratum and what it supports: The perceiving awareness and its object. In this respect, the Kashmiri Saiva is frankly and without reserve an idealist. Although he does not deny the reality of the object, his position is at odds with most commonly accepted forms of realism. The realist maintains that the content perceived is independent of the act of perception. The content is only accidentally an object of perception and undergoes no change in the process of being perceived. His contention, however, is essentially unverifiable; to verify it, we would have to know an object without perceiving it. This, from the Kashmiri Saiva point of view, is not possible. Objects of which we have no knowledge may indeed exist, but they are knowable as objects only if they are related to subjects who perceive them. In this sense, if there were no subjects, there could be no objects. 86 The subject, however, as opposed to the object is, in terms of the phenomenology of perception, apparent to himself. He is self-luminous (svaprakdsa). Thus, conscious- ness (the essence of subjectivity) is one s own awareness by virtue of which all things exist. The realist maintains that consciousness clearly differs from its object insofar as their properties are contrary to each other. The Saivite idealist. However, says that the object is a form of awareness (vijnanakara)™ The objective status of the object is cognition itself. Perception manifests its object and renders it immediately apparent (sphuta) to those who perceive it. It does not appear at any other time. If 'blue' were to exist apart from the cognition of k blue\ two things would appear: 'blue' and its cognition, which is not the case. It is the perception of the object which constitutes its manifest nature. An entity becomes an Integral Monism of Kashmiri Saivism object of knowledge not by virtue of the entity itself but by our knowledge of it. If objects had the property of making other objects appear, it would be possible for one object to make another appear in its own likeness. 'Blue' is perceived to be 'blue' because it is manifest as such to the perceiver. As Abhinava points out: The [nature of an] object of knowledge could not be established through a means of knowledge totally unrelated to it — a crow does not become white because a swan [sitting next to it] is white. Perception, on the other hand, is immediately apparent to conscious- ness. It is self-luminous in the sense that it is directly known without need of being known by any ulterior acts of perception and makes its object known at the

same time. Adopting the Buddhist Yogacara doctrine that things necessarily perceived together are the same (sahopa- lambhaniyamavada), the Saivite affirms that because the perceived is never found apart from perception, they are in fact identical. Reality (satya) is the point where the intelligible and the sensible meet in the common unity of being; it cannot be said to exist in itself outside, and apart from, knowledge or vision. Bhagavatotpala in his commentary on the Stanzas on Vibration quotes: Once the object is reduced to its authentic nature, one knows [the true nature of] consciousness. What then [remains of] objectivity? What [indeed could be] higher than consciousness? Consciousness is essentially active. Full of the vibration of its own energy engaged in the act of perception, it manifests itself externally as its own object. When the act of perception is over, consciousness reabsorbs the object and turns in on itself to resume its undifferentiated inner nature. Knowledge (jnana) manifests internally and externally as each individual entity.... Once knowledge has assumed that form it falls back [into itself]. The Yogacara Buddhist similarly maintains that consciousness creates its own forms. But, according to him, because the perceived and perception are identical; there is no perceived object at all. The so-called outer world is merely a flux of cognitions, it is not real. He is firmly committed to a doctrine of illusion. The reality of consciousness from The Doctrine of Vibration his point of view is established by proving the unreality of the universe. "All this consists of the act of consciousness alone", says Vasubandhu, "because unreal entities appear, just as a man with defective vision sees unreal hair or a moon, etc."He points to dreams as examples of purely subjective constructs which appear to be objective realities. The apparent reality dreams possess is not derived from any concrete, objective world, but merely from the idea of objectivity. While the Yogacara does not say that an idea has, for example, spatial attributes, it does have a form manifesting them. While he agrees with the Saiva idealist that appearances have no independent existence apart from their appearing to consciousness, he maintains that for this reason they are unreal. The creativity of consciousness consists in its diversification in many modes having apparent externality; it is not a creation of objects. While the Kashmiri Saivite agrees that the world is pure conscious- ness alone, he maintains that it is such because it is a real creation of consciousness. The effect is essentially identical with the cause and shares in its reality. Matter and the entire universe are absolutely real, as 'congealed' (sty ana) or 'contracted' (samkucita) forms of consciousness. "This God of consciousness", writes Ksemaraja, "generates the universe and its form is a condensation of His own essence (rasa)" m By boiling sugarcane juice it condenses to form treacle, brown sugar and candy which retains its sweetness. Similarly, consciousness abides unchanged even though it assumes the concrete material form of the five gross elements. The same reality thus abides equally in gross and subtle forms. Consequently no object is totally insentient. Even stones bear a trace (vasana) of consciousness, although it is not clearly apparent because it is not associated with the vital breath (prana) and other components of a psycho-physical organism. Somananda goes so far as to affirm that physical objects, far from being insentient, can only exist insofar as they are aware of themselves as existing. The jar performs its function because it knows itself to be its agent. Indeed, all things are pervaded by consciousness and at one with it and hence share in its omniscience. Thus, Siva, Who perceives Himself in the form of physical objects, is the one ultimate reality. "The jar knows because it is of my nature", writes Somananda, "and I know it because I am of the jar's nature. I know because I am of Sadasiva's nature and He knows because He is of my nature; Yajnadatta [knows] because he is of Siva's nature and Siva [knows] because He is of Yajnadatta's nature". Integral Monism of Kashmiri Saivism Everything in this sense is directly perceived by absolute conscious- ness, and this direct perception (pratyaksa) unifies the knowable into a single, undivided whole. This is the central concept behind a doctrine originally expounded by Narasimha called 'the non-dualism of direct perception' (pratyaksadvaita). This states that consciousness is essentially perceptive and that its perception of all things operates throughout the universe. Insofar as phenomena are clearly evident (sphufa) to us, everything is directly perceived by absolute consciousness, with which our individual consciousness is identical. This direct perception unfolds everywhere; the one true reality, it is alone and without companion or rival (nihsapatna). Even though it remains one, it can, by its very nature, perceive distinctions (bheda) between one entity and another, without this engendering any division within it. We distinguish between two entities in empirical terms on the basis of their mutual exclusion (anyonyabhava). The relative distinction {bheda) between them is essentially the perceived difference between their respective characteristics. Despite this difference they are united within the purview of a single cognition insofar as they are equally both

manifest appearances. This cognition is the undivided essence (rasa) or 'own nature' (svabhava) of both. Encompassed by the 'fire of consciousness', there is no essential difference between them. Just as when an emerald and ruby reflect each other's light, the ruby is reddish-green and the emerald greenish-red, similarly everything is connected with everything else as part of the single variegated (vicitra) cognition of absolute consciousness. Mahesvarananda writes: The Supreme Lord's unique state of emotivity (asadharanabhava) is the outpouring of pure Being (mahasatta). It is manifest as the brilliance (sphuratta) of the universe which, if we ponder deeply, [is realized to be] the single flavour (ekarasa) of the essence of Beauty which is the vibration of the bliss of one's own nature. In this way all things are in reality one although divided from the one another sharing as they do the 'single flavour' (ekarasa) of the pure vibration of consciousness. Kashmiri Saiva Realism Kashmiri Saivism as a whole has been variously called a form of 'realistic idealism', 'monistic idealism', 'idealistic monism' and 'concrete monism'. It is easy to understand why Kashmiri Saivism is The Doctrine of Vibration said to be 'idealistic' and 'monistic', but in what sense is it also 'realistic*? The answer to this question is of no small importance in trying to under- stand the central idea behind its metaphysics and the fundamental importance of the concept of Spanda, in this seemingly impossible marriage between monistic idealism and pluralistic realism. The Kashmiri Saiva approach understands the world to be a symbol of the absolute, that is, as the manner in which it presents itself to us. Again we can contrast this view with that of the Advaita Vedanta. The Advaita Vedanta understands the world to be an expression of the absolute insofar as it exists by virtue of the absolute's Being. Being is understood to be the real unity which underlies empirically manifest separateness and as such is never empirically manifest. It is only transcendentally actual as 'being-in-itself. The Kashmiri Saiva position represents, in a sense, a reversal of this point of view. The nature of the absolute, and also that of Being, is conceived as an eternal becoming (satatodita), a dynamic flux or Spanda, 'the agency of the act of being'. It is identified with the concrete actuality of the fact of appearing, not passive unmanifest Being. Appearance (abhasa) alone is real.ppearing (prakasamanatva) is equivalent to the fact of being (astitva). Ksemaraja writes in his commentary on the Stanzas on Vibration: Indeed, all things are manifest because they are nothing but manifestation. The point being that nothing is manifest apart from manifestation The absolutely unmanifest, from this point of view, can have as little existence as the space in a lattice window of a sky-palace. Nay, even less, because even that space can appear as an imagined mage manifest within consciousness. Everything is real according to the manner in which it appears. Even an illusion is in this sense real, insofar as it appears and is known in the manner in which it appears. The empirical and the real are identical categories of thought. As Abhinava says: Thus this is the supreme doctrine (upanisatf), namely that, when- ever and in whatever form [an entity] appears, that then is its particular nature. Perhaps at this stage a brief comparison with Heidegger's ideas might prove to be enlightening and not altogether out of place. According to Heidegger's phenomenology of Being, reality is intelligible in a two-fold Integral Monism of Kashmiri Saivism manner as 'phenomenon' and 'logos'. Heidegger defines what he means by 'phenomenon' as: "that-which-shows-itself. The manifest . . . phenomena are then the collection of that which lies open in broad daylight or can be brought to the light of day — what the Greeks at times implicitly identified as 'ta onta' (the things-which-are)". 127 In his later writings Heidegger drops the term 'phenomenon' in preference for the verbal form 'phainesthai' in order to emphasize even more the actuality or presentational property of Being. Explaining this new form of the term he writes: "Being disclosed itself to the ancient Greeks as 'physis'. The etymological roots 'phy-' and 'pha-' designate the same thing: 'phyein', the rising-up or upsurge which resides within itself as 'phainesthai', lighting-up, self-showing, coming-out, appearing-forth." Heidegger contrasted his notion of phenomenon with semblance (Schein) and with appearing (Erscheinung). In the case of semblance a thing can show itself as that which it is not, as when fool's gold shows itself to be gold. The ancients always allied semblance with non-being. Heidegger points out, however, that semblances are grounded in showings, and so does Abhinava. Both Heidegger and Abhinava consequently maintain that all semblances have a real basis and are to be treated as instances of phenomena along with the so-called real showing or manifestation of non-deceptive objects. So Heidegger states that: 'how- ever much seeming, just that much being'. Thus self-showing or appearing defines Being as phenomenon, but this definition of Being is as yet incomplete. Being is not only self-showing but 'logos' which Heidegger explains means 'discourse' (Rede) in the sense of 'apophansis': 'letting-be-seen'. Phenomenology, which according to Heidegger is the only correct study of Being, means 'letting-be-seen-that-

which- shows-itself. This is true of Saiva Paramadvaita as well. The reality of the world demands recognition; we are forced to accept the direct presentation of the fact of our daily experience. As Abhinava says: "if practical life, which is useful to all persons at all times, places and conditions were not real, then there would be nothing left which could be said to be real." A thousand proofs could not make 'blue' other than the colour blue. The reality of whatever appears in consciousness cannot be denied. Objects appear; they do not cease to do so by a mere emphatic denial. The manifestation of an entity in its own specific form is a fact at one level of consciousness; it is real. The appearing of the same entity in the same form but recognised to be a direct representation of the absolute is also a fact, but at another level of consciousness. It is no more or less real than the first. 'As is the state of consciousness, so is the experience,' says Abhinava. Although the nature of the absolute is discovered at a higher level of consciousness, The Doctrine of Vibration nonetheless it presents itself to us directly in the specific form in which we perceive things; otherwise there would be no way in which we could penetrate from the level of appearing to that of its source and basis. Abhinava writes: Real is the entity iyastu) that appears in the moment of direct perception (sak$atkara), that is to say, within our experience of it. Once its own specific form has been clearly determined one should, with effort, induce it to penetrate into its pure conscious nature. All things are known to be just as they present themselves. The concrete actuality of being known (pramiti), irrespective of content, is itself the vibrant (spanda) actuality of the absolute. Liberating knowledge is gained not by going beyond appearances but by attending closely to them. "The secret," Mahesvarananda says, "is that liberation while alive (jivanmukti) is the profound contemplation of Maya's nature." No ontological distinction can be drawn between the absolute and its manifestations because both are an appearing (dbhasa), the latter of diversity and the former of 'the true light of consciousness which is beyond Maya and is the category Siva'. Those who have attained the category of Pure Knowledge above Maya and have thus gone beyond the category of Maya, see the entire universe as the light of consciousness . . . Just as the markings [on a feather] are nothing apart from the feather, the feather [is nothing apart from] them, similarly, when the light of consciousness is manifest, the whole group of phenomena is manifest as the light of consciousness itself. Within the sphere of Maya, every entity's 'own nature' (svabhdva) corresponds to its specific manifest form. Accordingly it is defined as that which distinguishes it from all else and from which it never deviates. Above the sphere of Maya, that is, above the level of objectivity, is the domain of the subject. At this level, everything is realised to be part of the fullness of the experience r and hence no longer bound by the conditions which impinge on the object. Here the part is discovered to be the whole, that is, consciousness in toto. In this sphere beyond relative distinctions, the yogi realises that (all) the categories of existence are present in every single category. m The yogi experiences every individual particular as the sum total of everything else. He recognises that all things have one nature and that every particular is all things. This is the 'essence' (sard) or co-extensive unity (samarasya) of all things. Integral Monism of Kashmiri Saivism. We have established that reality is manifest according to how [and the degree in which] the freedom of consciousness reveals it and that [this freedom] is the womb of all forms. Just as 'sweetness' is present in its entirety in every atom of the sugarcane, so each and every atom [of the universe] bears within itself the emanation of all things. This is the level of consciousness in which the absolute reflects on itself realising to its eternal delight and astonishment (camatkara) its own integral nature. The reality of the world of diversity is not denied, but experienced in a new mode of awareness free of time and space in the eternal omnipresence of the Here and Now. [Phenomenal forms of awareness] such as 'this [exists]', born of the colouring [imparted to the absolute] by the limitations engendered by the diversifying power of time {kalakalana) also emanate within the Supreme Principle. There [at that level], Fullness {purriata) is the one nature [of all things] and so everything is omnipresent; otherwise, associated with division (khancjlana), the Fullness [of the absolute] would not be full. The content of absolute consciousness consists of diverse appearings (abhdsa) which, because they are manifest through it in this way, do not compromise the wholeness of consciousness. Everything we perceive is a momentary collocation of a number of such manifestations which combine together like 4 a row of altar lamps' (dipavalT) to form the single radiant picture of the universe. The individual objects which constitute the universe are specific collocations of such 'atomic' appearings. Together they form a single unified particular which appears according to its own defining features (svalaksand). A jar, for example, consists of a number of appearances such as 'round', 4 fat\ 'earthen', 'red', etc., which together discharge a single function (arthakriya), in this case, that of

carrying the appearance 'water'. They unite with each other much as the scattered rays of a lamp come together when focused, or as the various currents of the sea together give rise to waves. Atomic appearings can combine in any number of ways, provided that they are not contrary to one another as established by the dictates of natural law (niyati). An appearance of 'form', for example, cannot combine with that of 'air'. Insofar as they share a common basis (samanyadhikaranya), a given cluster of appearances appears as a single whole. This common basis is the most prominent member of the group; the appearance 'jar' is such in the example quoted above. Any one appearance in a cluster may assume a more important or subordinate role. The result is a specific The Doctrine of Vibration awareness of an object of the form: 'here this is such.' While individual appearances do not lose their separate identity {svarupabheda) when they rest on a common basis, even so the particular object which appears according to its own characteristics (svalak$ana) is an individual reality in its own right. It is a different kind of appearance characterised by its association with the appearance of the specific location and time in which it is made manifest. The form of our experience is thus 'I now see this here'. But when we perceive each particular constituent appearance separately, each assumes a separate fixed function. Abhinava cites the following colourful example to illustrate how the various combinations of appearances account for the variety of experience: Thus even though the appearance of the beloved may manifest externally, it is as if far away in the absence of another appearance, namely, that of 'embracing'. So when the [appearing of the beloved] is associated with another appearance [namely that of 'far away'] the power (arthakriya) it formerly had of giving pleasure appears as its contrary. The form our experience assumes depends, not only on the nature of the object perceived, but also on personal factors entirely peculiar to ourselves. This theory explains this in two ways. In one sense, the object remains the same, but one or other of its constituent appearances comes to the fore according to the inclinations of the perceiver. From another point of view, we can say that the perceived object is different for each perceiver according to the difference in the prominent appearance manifest to him. Abhinava, citing as an example a golden jar, illustrates how the same object appears differently to different perceivers according to the use they wish to make of it and to their state of mind: When a person who is depressed and feels that there is nothing [of value for him in the world] sees the jar, he merely perceives the appearance 'exists' [in the form of the awareness that] 'it is\ He is not conscious of any other [of its constituent appearances] at all. An individual who desires to fetch water [perceives] the appearance 'jar'. The man who simply wants something that can be taken somewhere and then brought back [perceives] the appearance 'thing'. The man who desires money [perceives] the appearance 'gold'. The man who desires a pleasing object [perceives] the appearance 'brightness' while he who wants something solid sees the appearance 'hardness'. These 'atomic events' or appearances emerge from the pure subject's consciousness and combine together to form a total event at each moment. Integral Monism of Kashmiri Saivism Daily life (vyavahara) goes on by virtue of this ever renewed flux of appearances. They are connected together and work towards a single unified experience because they appear within the field of consciousness of the universal subject. The aggregate of appearances arises in the [supreme] subject as do [sprouts in] a rice field. Even though each sprout germinates from its own seed, they are perceived as a collective whole. Appearances rest in this way within the universal subject. 'External- ity' is itself another appearance; it arises from a distinction between appearances and the individual subject. So, although all manifestation always occurs within the subject, it appears to be external due to the power of Maya which separates the individual subject from his object. This split must occur for daily life to be possible. Only externally manifest appearances can perform their functions; when they are merged within the subject and at one with him, they cannot do so. Daily life proceeds on the basis of the operation and withdrawal of the conditions necessary for fruitful action to be possible. Appearance in this sense represents the actualisation of a potential hidden in consciousness made possible by virtue of its dynamic, Spanda nature which is both the flow from inner to outer and back as well as the power that impels it. The emergence from, and submergence into, pure consciousness of each individual appearance is a particular pulsation (visesaspanda) of differentiated awareness. Together these individual pulsations constitute the universal pulse (samanyaspanda) of cosmic creation and destruction. Thus, every single thing in this way forms a part of the radiant vibration {sphuratta, Sphurana) of the light of absolute consciousness. Light and Awareness: The Two Aspects of Consciousness Absolute consciousness understood as the unchanging ontological ground of all appearing is termed 'Prakasa'. As the creative awareness of its own Being, the absolute is called ' Vimarsa '. Prakasa and Vimarsa — the

Divine Light of consciousness and the reflective awareness this Light has of its own nature — together constitute the all-embracing fullness (Purnata) of consciousness. The Recognition (pratyabhijfia) school of Kashmiri Saivism develops this concept of the absolute which finds its fullest expression in Utpaladeva's Stanzas on the Recognition of God. Even though neither of these two key terms appear in the Stanzas on Vibration or the Aphorisms of Siva, they recur frequently in their commentaries. Thus, although the original formulation of the Doctrine of Vibration differs from the theology of Recognition in this respect, it was extended in the course of its development to accommodate this concept of the absolute as well. This was possible, and quite justified, insofar as the absolute understood in Pratyabhijfia terms does not, as we shall see, differ essentially from that of the Spanda school. We can, as Kashmiri Saivites themselves have done, explain one in terms of the other. The Doctrine of Vibration Prakasa: The Light of Consciousness Prakasa is the pure luminosity' (bhdna) or 'self-showing' that constitutes the essence and ultimate identity (atman) of phenomena. That things appear at all is due to the light of consciousness, and their appearing (avabhasana) is itself this Light which bestows on all things their evident, manifest nature. Established in the light of consciousness everything appears there according to its own specific nature (svabhava). Anything that supposedly does not rest in this Light is as unreal as a sky-flower. 3 Thus, according to Rajanaka Rama, unlike the light of the sun, or any other light, this Light not only makes all things apparent, it is also their ultimate source. 4 Full of its divine vibration the Light makes all things manifest and withdraws them into itself. This supra- temporal activity characterises it most specifically; devoid of it, it would be no better than an inert physical phenomenon. At the same time, this light is the conjunction (slesa) or oneness (aikdtmya) of its countless manifest forms, 6 and the collective whole (Sampina'ana) of all the categories of existence. The universe is nothing but the shining of the Light within itself. It is the radiant vibration (sphuratta) of this Light, the state (avast hand) in which consciousness becomes manifest. Although the Light shines as all things at all times and hence also makes their diversity manifest, 9 penetrating each object individually as well as collectively, it is not totally 'merged' (magna) or identified with the object so as to suffer any division within itself. Our experience of any object is of the form: T see this': it is not itself an object, but the manifest form the object assumes as a luminous principle of experience. The Light is ever revealed and can never be obscured; objectivity can never cast a shadow on the light of consciousness. The Stanzas on Vibration declare: That in which all this creation is established and from whence it arises is nowhere obstructed because it is unconditioned by [its very] nature. This Light is the highest reality (Paramartha). It is the 'Ancient Light' (puranaprakasa) that makes all things new and fresh every moment. It is 'always new and secret, ancient and known to all'. It is the form of the Present (vartamanarupa), the Eternal Now. Time and space are relations between the contents of consciousness; they cannot impinge on the integrity of the absolute itself. 16 Neither space nor time can divide it, for they are one with the Light that illumines them Light and Awareness: Two Aspects of Consciousness and makes them known as elements of experience. But this Light is the shining of the absolute; it is not an impersonal principle. It is the living Light of God, indeed it is God Himself, the Master Who instructs the entire universe. 18 Siva is this 'auspicious lamp', Who illumines all things. He is the Light of consciousness that reveals the presence of both the real and the unreal, of light' and 'darkness'. 20 Abhinavagupta writes: Thus Bhairava, the Light, is self-evident (svatahsiddha); without beginning, He is the first and last of all things, the Eternal Present. And so what else can be said of Him? The unfolding of the categories of existence (tattva) and creation, which are the expansion of His own Self, He illumines, luminous with His own Light, in identity with Himself, and because He illumines Himself, so too He reflects on His own nature, without His wonder (camatkara) being in any way diminished. Since Zwicky's early observations, similar data from other galaxy clusters has yielded the same result – we consistently see that clusters of galaxies appear to By developing an awareness of the Centre, the yogi experiences the bliss of consciousness. 117 Through this gap he plunges into introverted absorption (nimilanasamadhi) and then emerges again to pervade the field of awareness between Centres and so experience the Cosmic Bliss (jagadananda) of the universal vibration of consciousness. 1 i8 He then recognises that this state pervades every aspect of experience. In this way the yogi's consciousness is no longer afflicted by the power which obscures it, hemming the Centre in on both sides with thought-constructs that seemingly deprive it of its fullness. As he realises directly his pure conscious nature as the universal ego free of all mental representations, it expands out to embrace all things within itself. Thus the realisation the Divine Means leads to, and is directly based

upon, is that this pure ego is in all things just as all things are within it. In the Spanda tradition, as recorded in the Stanzas on Vibration, no such ego is recognised. ' I9 Man's authentic nature is, however, understood in personal terms as every individual's own 'own nature* (svasvabhava) which is Siva, the universal vibration of pure subjectivity (upalabdhfta). It is not surprising, therefore, that later commentators found these two conceptions to be essentially the same and accordingly identified one's own inner nature with the pure ego. This came as a natural development in Spanda doctrine not only for **this reason but also because the universal ego is experienced as the inner dynamics of absolute consciousness. To conclude our summarial exposition of the Divine Means, which is centred on the direct experience of this pure ego (and hence on Spanda in this form), we turn now to a brief description of its inner, cyclic activity. We shall do this** by examining Abhinava's esoteric exegesis of the symbolic significance of the word 'A HAM', which in Sanskrit means T, and symbolises by its form the ego's dynamic nature. The objective world of perceptions is, as we have seen, essentially a chain of thought-constructs (prapanca) closely linked to one another and woven into the fabric of diversity (vicitrata). This thought (vikalpa) is a form of speech (vac) uttered internally by the mind (citta), which is itself an outpouring of consciousness. Consciousness also, in its turn, resounds with the silent, supreme form of speech {para vac) which is the reflective awareness through which it expresses itself to itself. Consequently, the fifty letters of the Sanskrit alphabet, which are the smallest phonemic units into which speech can be analysed, are symbolic of the principal elements of the activity of consciousness. Letters come together to generate words and words go on to form sentences. In the same way the fifty phases in the cycle of consciousness represent, in the realms of denoted meaning (vacya)the sum total of its universal activity (kriya) corresponding to the principal forces (kala) which come together to form the metaphysical categories of 186 The Doctrine of Vibration experience, which in their turn appear in the grossest, most explicitly 'articulate' form as the one hundred and eighteen world-systems (bhuvana). 'A', the first letter of both AHAM and the Sanskrit alphabet, is the point of departure or initial emergence of all the other letters and hence denotes Anuttara — the absolute. 'Ha', is the final letter of the alphabet and represents the point of completion when ail the letters have emerged. It represents the state in which all the elements of experience, in the domains of both inner consciousness and outer unconsciousness, are fully displayed. It is also the generative, emission (visarga) which, like the breath, casts the inner into the outer, and draws what is outside inward. The two letters 'A' and 'Ha' thus represent Siva, the transcendental source and Sakti, His cosmic outpouring that flows back into Him. The combined 'A-Ha' contains within itself all the letters of the alphabet — every phase of consciousness, both transcendental and universal. (For a graphic representation of this analysis, see figure 1.) M, the final letter of AHAM, is written as a dot placed above the letter which precedes it. It comes at the end of the vowel series and before the consonants and so is called 'anus vara '(lit. 'that which follows the vowels') and also 'bindu ' (lit. 'dot,' 'drop,' 'point' or 'zero'). While the consonant 'M' symbolises the individual soul (purusa), 'bindu 1 rep resents the subtle vibration of T, which is the life force (jivakala) and essence of the soul's subjectivity manifest at the transcendental, supra-mental level (unmana). 12 ° It is the zero-point in the centre between the series of negative numbers, in this case the vowels which represent the processes happening internally within Siva, and the series of positive numbers — the consonants which symbolise the processes happening externally within Sakti. Bindu, as a point without area, symbolises the non-finite nature of the pure awareness (pramitibhava) of AHAM. It is the pivot around which the cycle of energies from 'A' to 'Ha' rotates, the Void in the centre from which all the powers emanate and into which they collapse. As such, it is the supreme power of action which holds subject, object and means of knowledge together in a potential state in the one Light that shines as all three 121 containing them in its repose 122 (visrdnti). Bindu is the 'knower' (jndtr), who is essentially consciousness that, though omniscient does not manifest its intelligence, like a man who knows the scriptures but having no occasion to explain them to others silently bears this knowledge within himself. As such, it symbolises the union of Siva and Sakti (sivasakti- mithunapina'a) 123 in a state of heightened potency in which they have not yet divided to generate the world of diversity. It stands, in other words, at the threshold of differentiation in the stream of emanation still contained within Siva. Expansion Commences Bindu — The Individual Soul Withdrawal Commences Figure 1 188 The Doctrine of Vibration Then, to the degree in which that which is to be accomplished by the power of action residing within it [as a potential] penetrates into the absolute, it appears initially as bindu, which is the light of pure consciousness. 124

When outer objectivity is reabsorbed into its transcendent source, bindu is the point into which all the manifest powers of consciousness are gathered and fused together. The universal potency of all the letters is thus contained in bindu which, as the reflective awareness of supreme T consciousness, 125 gives them all life. Thus bindu also marks the beginning of Siva's internal movement back to the undifferentiated absolute and so stands at the threshold of both emission and absorption without being involved in either. The three aspects of AH AM together constitute a movement from the undifferentiated source of transcendental consciousness — 'A' — through the expansion or emission of its power — 4 Ha' — to the subject — 'M 1 — which contains and makes manifest the entire universe of experience. The reverse of this movement, which of withdrawal (samhara), is represented by M-Ha-A. AHAM and M-Ha-A alternate in the rotation (ghurnana) of the reflective awareness of T consciousness as immanent Sakti emerges from transcendental Siva to then merge back into Him. As Abhinava says: The universe rests within Sakti and She on the plane of the absolute (anutiara) and this again within Sakti ... for the universe shines within consciousness and [consciousness shines] there [within the universe by the power of] consciousness. These three poles, forming a couple and merging, make up the one supreme nature of Bhairava Whose essence is AHAM- 126 At the microcosmic level, fc A' represents the initial moment when the subject begins to rise out of himself to view the object. The movement from 'A to 'Ha' marks the emergence of sensation within the field of awareness, which is represented by the fifty letters of the alphabet symbolic of the fifty aspects of the flux of consciousness leading to objectified perception. 'Nl' is tne subject who, resting content within himself when he has perceived his object, merges through the inner flow of awareness into k A\ the absolute. Then from the absolute (A) its emission (Ha) flows back into the pure subject (M) set to perceive his object. Thus all the cycles of creation and destruction are contained within AHAM through which they are experienced simultaneously as the spontaneous play of the absolute. The yogi who recognises this recurrent pulse of awareness to be the movement of his own consciousness merges his limited ego with the Path to Liberation universal ego. **This is natures general ledger** Thus he realises that its power to create, sustain and destroy all things is his own inner strength (svabaia) that he exerts effortlessly in the same state of mystical absorption (turlya) in universal consciousness that the absolute itself enjoys. In this way he shares in the three-fold awareness Siva Himself has of His own nature which Abhinava describes as follows: 4 I make the universe manifest within myself in the Sky of Consciousness. I, who am the universe, am its creator! ' — this awareness is the way in which one becomes Bhairava. 'AH of manifest creation (sadadhvari) is reflected within me, I cause it to persist 1 — this awareness is the way in which one becomes the universe. The universe dissolves within me. I who am the flame of the [one] great and eternal fire of consciousness' — seeing thus one achieves peace. 127 The experience of the liberated thus coincides with the realisation of their own divine nature which, through its power, rules and guides the cosmic order. Thus this attainment (siddhi), which is liberation itself, is in the DoctrLie of Vibration technically called 'Mastery over the Wheel of Energies' (cakresvaratvasiddhi) because the liberated soul, identified with Siva, now governs, as does Siva, the cycle of the powers that bring about the creation and destruction of all things, Here we are talking of individual consciousness becoming cosmic general ledger and the liberationist realises all destruction and creation is by him. This clearly means that he is in his own world of illusion. Creation is subjective quintessentially The Empowered Means (Saktopaya) All the practices taught in the Stanzas on Vibration are internal. Whenever ritual is mentioned, it is invariably interpreted in terms of the dynamics of the inner processes the yogi experiences and implements in the course of his yogic practice. The Doctrine of Vibration, Ksemaraja affirms, 129 is concerned entirely with these inner disciplines centred, as it is, in one way or another, on consciousness or, at least, on the inner activity of the mind. Thus the Empowered Means which, like the other categories we have discussed, is entirely internal, includes an important part of Spanda practice. Spanda practice belonging to the Divine Means centres on one's own inherent nature (svasvabhava) as Siva, the universal perceiver and agent, that belonging to the Empowered Means on His power. Instead of arriving directly at the all-embracing emptiness of subjective conscious- ness, the yogi practising the Empowered Means realises his true nature through the fullness of its energy. Practising the Divine Means, the yogi plunges, as it were, straight into the fire of consciousness; practising the 190 The Doctrine of Vibration Empowered Means he merges with its rays. Either way the yogi is centred equally on ultimate reality. The power of consciousness is no less absolute than its possessor. To make this point Abhinava quotes the Matanga- tantra: This reality consists of the rays of [Siva's]

power and is variously said to be the abode of the Lord's manifestation . . . That same [power] illumined [by Siva] is itself also luminous, unshaken and unmoving. That very [power] is the supreme state, subtle, omnipresent, the nectar of immortality, free of obscuration, peaceful, yearning for pure Being alone {vastumatra) and devoid of beginning and end. Perfectly pure, it is said to be the body [of ultimate reality]. 110 The yogi concentrates on the powers operating in all of life's activities as particular pulsations (visesaspanda) in the universal rhythm (samanyaspanda) of the power of consciousness. In this way he rises progressively from the particular to the universal until he reaches pure Being (satta), the greatest of all universals (mahasamanyd) and the highest form of Siva's power. Thus the creative power of Maya, manifest through countless lesser powers, no longer causes the yogi to stray from Siva's consciousness but becomes the means through which it can be realised 13 1 in the illuminating brilliance (sphuratta) which is Siva's pure Being. Thus by discovering the true nature of Sakti, the yogi realises himself to be Siva, its possessor Who consists of all its countless powers. Thus practise belonging to this Means leads to the same pure consciousness free of thought-constructs realised through the Divine Means. Although the ultimate realisation is instantaneous, the yogi rises to it gradually by freeing his consciousness of the limitations imposed upon it by thought. Abhinava explains: The same occurs in the Empowered Means [as does in the Divine]. At the discursive level of consciousness {yaikalpikibhumi) [where the Empowered Means functions] knowledge and action, although evident, are, for the reasons explained previously, contracted. A blazing energy [is revealed within] the one who dedicates himself to removing the burden of this contraction. [This energy eventually] brings about the inner manifestation {antarabhasa) of pure consciousness he seeks. 132 Consciousness is individualized and its power of knowledge and action contracted by the thought-constructs born of ignorance. The arising of these mental representations, as the Stanzas on Vibration say, deprives the soul of its freedom and immortal life. 133 The practise of the Path to Liberation 191 Empowered Means is meant to free the fettered soul of this constriction on his consciousness. It operates within the mental sphere (cetasy™ and is designed to purify thought (vikalpasamskara) in order to. reveal the pure consciousness which is its ground and ultimate source. Thus, the Empowered Means is concerned with the second instant of perception, during which the subject forms mental representations of his object. Thought functions on the basis of an awareness of relative distinctions between specific particulars, distinguishing them from one another and thus seemingly fragmenting the essential unity of reality. 135 The vibrant vitality of consciousness, universally manifest, is clouded like a mirror by a child's breath 136 and the soul is deprived of the liberating intuition of the one reality free of thought-constructs (nirvikalpa). Abhinava writes: The [fettered soul] is like a dancing girl who although wishing to leave the dancehall is collared by the doorkeeper of thought and thrown back onto the stage of Maya. 137 All thought is centred on objectivity and hence dislodges awareness from the plenitude of pure subjective consciousness. Thus, to regain the original state of rest (visranti) consciousness enjoys, the yogi must rid himself of thought. As thought-forms decrease, pure, thought-free awareness is strengthened 138 until the yogi is fully established in a state in which the relative distinctions (bheda) conceived between entities dissolve away. Everything appears to him as pure Being (sattdmdtra) l}9 and the entire universe shines before him pervaded by Siva's radiance. 140 His intuitive faculty (mati) thus purified, the yogi gains both the perfections (siddhi) of yogic practice and liberation (mukti). His consciousness is now like a well-polished mirror which reflects everything he desires and grants it to him. 141 Abhinava writes: Just as a man who has been ill for a long time forgets his past pain completely when he regains his health, absorbed as he is in the ease of his present condition, so too those who are grounded in pure awareness free of thought-constructs are no longer conscious of their previous [fettered] state. Consciousness, the sole truly existent reality, free of thought- constructs is made fully and evidently manifest by eliminating thesedifferentiated perceptions. The wise man should therefore exert himself to attend closely to this [state of awareness]. I42 The thought-constructs generated within consciousness do not in reality affect it at all. They can neither break up nor add anything to the Light which shines as all things. 143 They are in fact nothing but 192 The Doctrine of Vibration consciousness itself 144 which perceives, through its power of reflective awareness (yimarsa), the multitude of objects in diverse ways, and so assumes this form. 145 Although thought-constructs are mental representations of objects once seen or present, they are products of the power of consciousness and not of the objects they represent. 146 Thought is both analytic and synthetic; 147 it serves the useful purpose of separating individual elements of experience from others and linking together those that appear to

be distinct from one another so that they can be better understood. 148 It does not consist merely of false mental constructs projected onto reality that need to be wholly rejected. Thought obscures consciousness and distracts it only when it appears in the form of doubt, vacillating between alternatives. 149 Once this conflicting duality (dvaitddhivasa) 150 is eliminated, thought is purified and rests in itself as the 'thought-less thought' of pure consciousness. 151 By gradually eliminating the multitude of conflicting notions that agitate him, the yogi ultimately achieves the certainty (niscaya) corresponding to a direct awareness of his own divine nature. 152 Abhinava explains: Thought is in reality none other than pure consciousness. Even so, it serves as a means to liberation for the individual soul (anu) only when it takes the form of certainty (niscaya). 153 The yogi must eliminate every doubt and misguided notion that leads him to believe himself to be other than Siva. By developing the thought: 4 1 am Siva', it ultimately affirms itself directly as a pure awareness beyond thought without any intervening mental representations. Abhinava says: Just as the man who thinks intensely that he is a sinner becomes such, just so one who thinks himself to be Siva, and none other than He, becomes Siva. This certainty (ddrtfhya), which penetrates and affirms itself in our thoughts, coincides with an awareness free of thought- constructs engendered by a series of differentiated mental representa- tions, the object of which is our identity with Siva. 154 As thought is gradually purified, it becomes progressively clearer until its object becomes maximally apparent (sphufatama). 155 The stream of perceptive consciousness (pramana) progressively reveals each aspect of its object which, thus affirming itself with increasing clarity, reveals its ultimate nature. The yogi reflects repeatedly upon it as the object of his realisation and loving devotion, for all that is perceptible and need be known (j fie yd) is Siva alone. As Abhinava says: Path to Liberation 193 What should we say of those who before they are satisfied have to see their beloved again and again, caress her and think about her for a long time? 156 The yogi practising the Empowered Means is initiated into the Great Sacrificial Rite (mahayaga), eternally enacted at the interface between the inner and outer aspects of consciousness, by a direct infusion of awareness from his master who is the embodiment and outer symbol of the yogi's enlightened identity. ,57 The rite begins with ritual bathing (snana) which is in this case the immersion of the body of thought in the white ashes of the cosmic fuel of duality, burnt in the fire of consciousness. 158 The yogi then goes on to worship (puja) by uniting all that is pleasing to the senses in the oneness of consciousness. 159 The ritual formula (mantra) he recites is the eternal resonance of the awareness which is the pulsation of the Heart of his own consciousness. 160 Repetition (Japa) of the formula is every activity, perception, breath or thought which arises within him while plunged in the universal awareness of his true nature. 161 The mental image he visualises meditating (dhyana) on the Deity in the course of the rite, is whatever the yogi spontaneously imagines and contemplates as the outpouring of the universal creativity of consciousness. 162 Ritual gesture (mudra) is whatever bodily posture the yogi may assume when, fully absorbed in consciousness, he moves, staggering about (ghumita) as it were, drunk with the wine of self-realisation. 163 Oblation is performed by offering with devotion and awareness all the sensations which flow in through the channels of the senses to the fire of his subjectivity, which is thus inflamed (uddipita) and makes all things one with itself. 164 The outer ritual which commences in the sphere of the Individual Means thus leads naturally to the inner rite of the Empowered Means. When the yogi's practise (abhyasa) reaches fruition, the rite merges with the spontaneous activity of consciousness. This is fullness (Purnata), the completion and reunification of the forces within consciousness which, through the power of ignorance, were formerly dispersed and divided. "Just as a horse driven here and there", writes Abhinava, "over plains, hills and dales follows the will of its rider, so also consciousness, driven by various expedients (bharigi), quiescent or terrific, abandoning duality, becomes Bhairava. Just as by looking repeatedly at one's own face in a mirror one comes to know that it is the same [as the image reflected], so also, [one sees] in the mirror of mental representations of meditation {dhyana), ritual (puja) and worship (area) one's own Self as Bhairava and so quickly identifies with Him. This identification is the realisation that takes place in the absolute (anuttara)." 165 194 The Doctrine of Vibration By ridding himself of the relative distinctions engendered by thought, the yogi practising the Empowered Means, illumined by the power of self- awareness of Pure Knowledge {suddhavidya), transcends the distinction between right and wrong, purity and impurity. He is led to the conviction that the pure consciousness, which is his true nature, is unaffected by whatever action he may do, whether conventionally accepted as good or bad. Abhinava quotes the Malinivijaya as saying: All here is enjoined and all prohibited. This alone, O Lord of the

gods, is here prescribed as obligatory, namely that the mind be firmly applied to the true reality. It matters little how this is achieved. He whose mind is firmly established in [this] reality, even if he eats poison, is as little affected by it as are lotus petals by water. 166 Impurity is a state of seeming separation from consciousness. 167 The yogi who has freed himself of all false notions comes to realise that the true nature of consciousness can never be sullied or limited by any object appearing within it. 168 This is the realisation the ancient sages achieved through a direct intuition of reality free of intruding thought-constructs {avikalpabhava), but kept secret in order not to confuse the worldly. 169 Similarly, in reality nobody is ever bound. It is ignorance to believe bondage exists and to contrast it with a conceived state of liberation. 170 If the Self is one with Siva, how can it be either bound or released? 171 Nothing essentially distinguishes those who are bound from those who are free. 172 The difference between their states is merely conceptual. 173 Pure consciousness abides free of all such distinctions. Thus Bhagavatotpala, <n his commentary on the Stanzas, repeatedly stresses that thought- constructs obscure consciousness and misguide the individual soul. 174 Those who are bound are convinced that they are dull witted, conditioned by Karma, sullied by their sin and helplessly impelled to action by some power beyond their control. He who manages to counter this conviction with its opposite achieves freedom. 175 He who considers himself to be free is free indeed, while he who thinks himself bound remains so. Thus at the highest level of realisation, as Abhinava says: Nothing new is achieved nor is that which in reality is unmanifest, revealed- [only] the idea is eradicated that the luminous being shines not. 176 Nothing is impure, all is perfect, including Maya and the diversity it engenders. To say that illusion exists and that ignorance must be Path to Liberation 195 eradicated implies that it has a separate existence apart from conscious- ness. If this is so, it has as little reality as the shadow of a shadow, but if not,then it must be consciousness itself. Thus, as Kallata says, bondage, the binder and the bound are in fact one. 177 It is Siva Himself Who freely obscures His own nature. Siva binds Himself by Himself. 178 Concealing and revealing Himself, Siva plays His timeless game. At the Divine (sambhava) level of pure Siva-consciousness, the Spanda yogi directly lays hold of the power inherent in his own conscious nature (svabala) which gives life to the psycho-physical organism and impels the senses and mind to action. 179 In this way every thought- construct, and with it the ego, is instantly annulled in the immediacy of the pure subjectivity that remains unaltered throughout every perception and state of consciousness. The same takes place at the Empowered level b> attending to the recurrent activity — Spanda — of the subject, that is, the flux of awareness through the cyclic movement of the powers of consciousness. 180 By attending (avadhana) to this movement the thought- constructs that emerge and subside in the course of perception are seen to be part of this universal process, and, in this way purified, are no longer binding. Thus, Ksemaraja says that the Spanda teachings are concerned most directly with the Empowered Means. m The yogi who is always alert to discern the pulse of Spanda quickly realises his own authentic state of being (nijarfi bhavam). 1 * 2 He is then truly awake, not only literally, but also in the deeper sense that he is awake to his authentic nature, its power and activity. When attention (avadhana) slackens, this movement takes place unconsciously and so the thought-constructs and perceptions generated through it appear to take on an autonomous existence of their own, just as happens when we dream. 183 The spontaneity of the movement that travels between subject and object and holds them together in the pure awareness of the universal subject's identity with his cosmic object devolves into the creative activity of waking and dreaming. Man, in other words, becomes a victim of his states of consciousness and the contents that they, by their very nature, generate within themselves. 184 The Spanda teachings are not only concerned with the structure of thought and its functions, but also with the powers and properties of its vehicle, namely, speech. Speech issues out of consciousness, develops into thought to then become articulated sound. A focal point of Spanda doctrine is thus the role speech plays in the formation of thought- constructs and their purification. Although this takes place at all levels of practise below the Divine (sambhava), the Spanda teachings, meant as they are for advanced yogis, ignore the outer forms of spiritual discipline to concentrate on practise in the Empowered (sakta) psychic sphere (cetas) and what lies beyond it where speech is the pure inner awareness (yimarsa) **Although Abhinava does not differentiate between individual consciousness and cosmic general ledger, one tends to comprehend that right interpretation is the transfor mentioned the individual consciousness to cosmic consciousness responsible for the creation and destruction which the sadhaka sees as his own power. Every one is rama and every one is Shiva. only thing that is essential is striving and concerted**

**efforts and sustained struggle to reach.** The Doctrine of Vibration of the light of consciousness. The Doctrine of Vibration identifies this, the highest level of speech (para vac), with the universal pulse of consciousness that resounds spontaneously within it as the inner flow of its own undifferentiated awareness. 185 Beyond the realms of language, it is the transcendental consciousness in which all language is rooted and pervades all that language denotes as its essential being. Utpaladeva writes: The Supreme Voice is consciousness. It is self-awareness spontaneously arisen, the highest freedom and sovereignty of the Supreme Lord. That pulsing radiance (sphuratta) is pure Being, unqualified by time and space. As the essence [of all things] it is said to be the Heart of the Supreme Lord. 186 When the intention arises within consciousness to discern its own brilliance manifest in the world of denotations and denoted meanings, speech turns from the supreme transcendental level to that of immanence and assumes the form of a pure intuitive awareness (pratibha) which perceives and comprehends its universal manifestation. This is the voice of intuition (pasyanti), which grasps the meaning inherent inwardly in all words and externally in all that they denote. Analogous to the non- discursive, instinctual knowledge animals possess, it is a pure generic perception not yet formed into language in which the act of denotation, its object and that which denotes it are indistinguishable. Illumined by the voice of intuition birds migrate in their due seasons, the cock crows at dawn and young mammals suck at the breast. 187 Infants similarly reflect and respond instinctively to their environment by virtue of this intuitive sense 188 and through it come to grasp the link between words and the objects they denote. As they learn to speak, they begin to form concepts and so the next two levels of speech develop. One is the outer corporeal speech (yaikhari) and the other the subtler, inner discourse (antah- saryijalpa) of thought that forms at the intermediate level (madhyama) where the ratiocinating mind stands between the higher levels of intuition and its outer verbal expression. In this way the development of speech in infancy reflects its progressive actualisation in every spoken word. A hymn to the Goddess quoted by Bhagavatotpala describes this process well: Therefore, O Supreme Goddess, the highest form of speech should be worshipped as the [universal] cause that establishes the existence of all things by insight {niscaya) into their nature (artha) brought about by their manifestation through the superimposition [of verbal designations]. O Mother, insight into the true nature [of things] is nothing but the Path to Liberation 197 act of intent of that [same speech], apart from which [speech itself and all that it expresses] could not attain to its own nature. Again, in that state [speech] is said to be the light of one's own nature. Free of division and succession it is attainable [only] by the yogi. Then from the state of intent, O Siva, speech [assumes] the nature of thought as the radiant pulse (sphuraria) of desire to speak of that which is in the domain of words. Then consisting of words, it bears a clearly expressed meaning, for if [speech] were not such, meaning could not be understood. 189 personal experience clearly proves that thought is invariably associated with speech. 190 Thought is a function of language. Through it we communicate to ourselves a mental image of the world about us and can construct complex ideas about ourselves. Language is the fabric from which our world of ideas is woven. Mental representation which orders the influx of sensation and presents us with a meaningful, picture of the world, memory, the elaboration of ideas and the shifting tides of emotion are all intimately connected with language and through it to the consciousness which underlies them. To think of language as nothing more than a system of denotation based on a commonly accepted convention (sanketd) fails to fully account forks inherent power to convey meaning (vacakasakti). In order to learn the convention we must be born with an innate ability to grasp meaning, and this ability is not itself learned nor found anywhere within the domain of convention. Lacking this ability we would be caught in an infinitely expanding system of denotation in which each element pointed to some other within it, without ever coming to rest anywhere. Unless we can couple the word 'jar' with the object it denotes, explaining that the word 'pot' is a synonym of the word 'jar' would leave us none the wiser. 191 The connection between word and meaning is only explicable if we postulate that it is an inherent property of the power of awareness to link one with the other. Language must be grounded in the pure cognitive awareness (prama) of consciousness which stands beyond, and yet illumines, the sphere of experience we define and understand through the medium of language. As Abhinava says: Someone may hear another person speak, but if his awareness (prama) is obscured, he is unable to rise, unconscious as he is, to the level of the experiencing subject [who understands] what has been said. He only grasps the outer successive (sound) of what the other person says and thus can only repeat it as would a parrot. An understanding of its meaning presupposes that he has caught hold of his own power

of awareness (prama) by attaining the autonomy [of the conscious, universal subject]. **Note that we have defined sachitananada as one which understand Truth and Untruth, Bliss and Unbliss, and Knowledge and ignorance where all these are equal. Truth=Untruth, Bliss=Unbliss and Ignorance=Knowledge** The Doctrine of Vibration Outer, articulate speech consists of a series of ordered phonemic elements produced and combined by the vocal organs to form meaningful words. In order for this to be possible, these elements must also be grounded in consciousness (prama). The articulated phonemes are merely outer, gross manifestations of the phonemic energies (yarnagrama) held in a potential state within consciousness. This 'mass of sounds* (sabdarasi) is the light of consciousness (prakdsd) which makes the universe manifest and contains all things within itself. In other words, it is the totality of consciousness expressed as the collective awareness symbolized by all the letters corresponding to the introverted subjectivity of Siva Himself. The power through which this potential actualises itself into speech and the world of denotation is technically called * Mdtrkd*. It is the reflective awareness (vimarsa) and radiance (sphuratta) of the supreme subject — the 'mass of sounds' (Jabdarasi) — and the undivided wonder Siva experiences when He contemplates the universe He gathers up into Himself in the form of countless words (vdcaka) and their meanings (vdcya). 193 Mdtfkdsakti is manifest in the second movement of consciousness after the primal vibration of the pure luminosity of the 'mass of sounds', as the state of pure potency which arises when its unsullied subjectivity begins to turn away from itself and is associated with faint traces of objectivity (dmrsya- cchdyd). 194 Mdtrkd contains within itself the various aspects of objectivity that, although not yet manifest, are ready to issue forth. Thus this power, at one with Siva, is called 'Mdtrkd' because she is the mother (mdtrkd) of the universe that she contains within herself as does a pregnant woman her child. 195 The circle of the powers of Mdtrkd (mdtrkdcakra) consists of the phonemic energies contained in AH AM, the universal ego. 196 When grasped in its entirety at its source, these energies elevate the consciousness of the enlightened, but when split up and dispersed give rise to the obscuring forces (kald) which lead the ignorant away from realisation. The fettered soul is ignorant of the pure egoity that is the source of speech and so it generates, through its powers, the many thought-constructs that deprive him of the awareness of unity and obscure Siva's universal activity. 197 The Stanzas on Vibration declare: He who is deprived of his power by the forces of obscuration {kala) and a victim of the powers arising from the mass of sounds (sabdardsi) is called the fettered soul. 198 The powers [of speech] are always ready to obscure his true nature as no mental representation can arise that is not penetrated by speech. 199 The rays of phonemic energies emanate from the light of Siva, the Path to Liberation 199 'mass of sounds' (sabdarasi) in eight groups. They constitute the powers of the inner mental organ and the five senses, figuratively arranged in a circle around the sacred shrine {pi(ha) of Matrka&akti who manifests externally as the body. 200 The eight classes and the names of the deities presiding over them are as follows: 201 Gutturals Brahman! Intellect (buddhi) Palatals Mahegvari Ego (ahankara) Cerebrals Kaumarl Mind (manas) Dentals Narayani Hearing Labials Varahi Touch Semivowels Aindri Sight Sibilants Camuntfa Taste Vowels Mahalak§ml Smell The yogi who grasps the true nature of the power of Matfka and its phonemic forces is liberated 202 by recognising that the activity of the senses and the discursive representations of the mind are in fact emanations of universal consciousness. Conversely, when ignorant, he is affected by its power in its multiple negative aspects known as 'Mahdghora' ('greatly terrible') and, unable to rest within himself free of the sense of diversity, he is constantly disturbed by the flux of extroverted perceptions. Abhinava explains: When the [phonemic energies] are not known to be [emanations of the Lord] they conceal the wonder (camatkara) of consciousness which is the one essential non-discursive awareness [present throughout perception] and even in discursive thought. They obscure it with thought-constructs constituted by the diverse configurations of phonemes and syllables which [although also] a form of the deity [are no longer benevolent but] most terrible. Inducing doubt and fear, they engender the fettered soul's state, bound by the shackles of transmigra tion. But once their true nature is understood correctly in this way, they bestow freedom in this very life This knowledge of their intimate being [at one with the absolute] consists of this, namely, that even in the midst of all these fluctuations, free at their inception of discursive representations, thought-constructs do not conjoin [individualised consciousness] with the wheel of energies consisting of the totality of phonemes, even though [these constructs] are coloured by the many diverse words generated by the aggregate of phonemes. Language has a powerful effect on us. A few words we may hear or read can inspire us with joy, fear or sadness, and the constant inner 200 The Doctrine of Vibration dialogue of thought

arouses intense feelings within us. This power hidden in language, which binds us through the thought-constructs it generates, can also be used to free us of them by channeling it through Mantra. Mantric practice begins at the Individual (driava) level where Mantras are recited in consonance with the rising and falling away of the breath. In this way they are charged with the vibration (spandd) of consciousness and, in their turn, make consciousness vibration (Doctrine of Vibration S.G.Dyczkowski) On without knowledge that everything is manifest within consciousness is illusory or unreal in that sense alone. 78 Things are more real or more tangibly experienced according to their own essential nature (svabhava) to the degree in which we recognise that they are appearances (abhasa) within absolute consciousness As Jayaratha says: Just as images manifest in a mirror, for example, are essentially mere appearances, so too are [phenomena] manifest within conscious- ness. Thus, beause they are external, [phenomena] have no being (sattva) of their own. The Lord says this [not with the intention of saying anything about the nature of things] but in order to raise the level of consciousness of those people who are attached to outer things; thus everything in this sense is essentially a mere appearance. [Knowing this], in order to quell the delusion of duality, one should not be attached to anything external. 79 The ultimate experience is the realisation that everything is contained within consciousness. We can discover this in two ways. Either we merge the external world into the inner subject, or we look upon the outer as a gross form of the inner. In these two ways we come to recognise that all things reside within our own consciousness just as consciousness resides within them. This all-embracing inwardness is only possible if there is an essential identity between the universe and consciousness. The events which constitute the universe are always internal events happening within consciousness because their essential nature is consciousness itself. 80 We can only account for the fact that things appear if there is an essential identity between consciousness and the object perceived. If a physical object were really totally material, that is, part of a reality independent of, and external to, consciousness, it could never be experienced. Abhinava says: The existence or non-existence of phenomena within the domain of the empirical (iha) cannot be established unless they rest within consciousness. In fact, phenomena which rest within consciousness are apparent (prakasamana). And the fact of their appearing is itself their oneness {Abheda) with consciousness because consciousness is nothing but the fact of appearing {Prakasa). If one were to say that they were separate from the light of [that consciousness] and that they appeared [it would be tantamount to saying that] 'blue' is separate from its own nature. However, [insofar as it appears and is known as such] one says: 'this is blue 1 . Thus, in this sense, [phenomena] rest in conscious- ness; they are not separate from consciousness. The Doctrine of Vibration The universe and consciousness are two aspects of the whole, just as quality and substance constitute two aspects of a single entity. The universe is an attribute (dharma) of consciousness which bears (dharmiri) it as its substance. It is said that 'substance' is that resting in which this entire group of categories manifests and is made effective. Now, if you don't get angry [we insist that] this entire class of worlds, entities, elements and categories (tattva) rests in consciousness and [resting in it] is as it is. Thus consciousness contains everything in the sense that it is the ground or basis (adhard) of all things, their very being (satta) and substance from which they are made. But, unlike the Brahman of the Advaita Vedanta, it is not the real basis (adhisthand) of an unreal projection or illusion. Consciousness and its contents are essentially identical and equally real. They are two forms of the same reality. Consciousness is both the substratum and what it supports: The perceiving awareness and its object. In this respect, the Kashmiri Saiva is frankly and without reserve an idealist. Although he does not deny the reality of the object, his position is at odds with most commonly accepted forms of realism. The realist maintains that the content perceived is independent of the act of perception. The content is only accidentally an object of perception and undergoes no change in the process of being perceived. His contention, however, is essentially unverifiable; to verify it, we would have to know an object without perceiving it. This, from the Kashmiri Saiva point of view, is not possible. Objects of which we have no knowledge may indeed exist, but they are knowable as objects only if they are related to subjects who perceive them. In this sense, if there were no subjects, there could be no objects. 86 The subject, however, as opposed to the object is, in terms of the phenomenology of perception, apparent to himself. He is self-luminous (svaprakdsa). Thus, conscious- ness (the essence of subjectivity) is one s own awareness by virtue of which all things exist. The realist maintains that consciousness clearly differs from its object insofar as their properties are contrary to each other. The Saivite idealist. However, says that the object is a form of awareness (vijnanakara)™ The objective status of the object is

cognition itself. Perception manifests its object and renders it immediately apparent (sphuta) to those who perceive it. It does not appear at any other time. If 'blue' were to exist apart from the cognition of k blue\ two things would appear: 'blue' and its cognition, which is not the case. It is the perception of the object which constitutes its manifest nature. An entity becomes an Integral Monism of Kashmiri Saivism object of knowledge not by virtue of the entity itself but by our knowledge of it. If objects had the property of making other objects appear, it would be possible for one object to make another appear in its own likeness. 'Blue' is perceived to be 'blue' because it is manifest as such to the perceiver. As Abhinava points out: The [nature of an] object of knowledge could not be established through a means of knowledge totally unrelated to it — a crow does not become white because a swan [sitting next to it] is white. Perception, on the other hand, is immediately apparent to conscious- ness. It is self-luminous in the sense that it is directly known without need of being known by any ulterior acts of perception and makes its object known at the same time. Adopting the Buddhist Yogacara doctrine that things necessarily perceived together are the same (sahopa- lambhaniyamavada), the Saivite affirms that because the perceived is never found apart from perception, they are in fact identical. Reality (satya) is the point where the intelligible and the sensible meet in the common unity of being; it cannot be said to exist in itself outside, and apart from, knowledge or vision. Bhagavatotpala in his commentary on the Stanzas on Vibration quotes: Once the object is reduced to its authentic nature, one knows [the true nature of] consciousness. What then [remains of] objectivity? What [indeed could be] higher than consciousness? Consciousness is essentially active. Full of the vibration of its own energy engaged in the act of perception, it manifests itself externally as its own object. When the act of perception is over, consciousness reabsorbs the object and turns in on itself to resume its undifferentiated inner nature. Knowledge (jnana) manifests internally and externally as each individual entity.... Once knowledge has assumed that form it falls back [into itself]. The Yogacara Buddhist similarly maintains that consciousness creates its own forms. But, according to him, because the perceived and perception are identical; there is no perceived object at all. The so-called outer world is merely a flux of cognitions, it is not real. He is firmly committed to a doctrine of illusion. The reality of consciousness from The Doctrine of Vibration his point of view is established by proving the unreality of the universe. "All this consists of the act of consciousness alone", says Vasubandhu, "because unreal entities appear, just as a man with defective vision sees unreal hair or a moon, etc."He points to dreams as examples of purely subjective constructs which appear to be objective realities. The apparent reality dreams possess is not derived from any concrete, objective world, but merely from the idea of objectivity. While the Yogacara does not say that an idea has, for example, spatial attributes, it does have a form manifesting them. While he agrees with the Saiva idealist that appearances have no independent existence apart from their appearing to consciousness, he maintains that for this reason they are unreal. The creativity of consciousness consists in its diversification in many modes having apparent externality; it is not a creation of objects. While the Kashmiri Saivite agrees that the world is pure conscious- ness alone, he maintains that it is such because it is a real creation of consciousness. The effect is essentially identical with the cause and shares in its reality. Matter and the entire universe are absolutely real, as 'congealed' (sty ana) or 'contracted' (samkucita) forms of consciousness. "This God of consciousness", writes Ksemaraja, "generates the universe and its form is a condensation of His own essence (rasa)" m By boiling sugarcane juice it condenses to form treacle, brown sugar and candy which retains its sweetness. Similarly, consciousness abides unchanged even though it assumes the concrete material form of the five gross elements. The same reality thus abides equally in gross and subtle forms. Consequently no object is totally insentient. Even stones bear a trace (vasana) of consciousness, although it is not clearly apparent because it is not associated with the vital breath (prana) and other components of a psycho-physical organism. Somananda goes so far as to affirm that physical objects, far from being insentient, can only exist insofar as they are aware of themselves as existing. The jar performs its function because it knows itself to be its agent. Indeed, all things are pervaded by consciousness and at one with it and hence share in its omniscience. Thus, Siva, Who perceives Himself in the form of physical objects, is the one ultimate reality. "The jar knows because it is of my nature", writes Somananda, "and I know it because I am of the jar's nature. I know because I am of Sadasiva's nature and He knows because He is of my nature; Yajnadatta [knows] because he is of Siva's nature and Siva [knows] because He is of Yajnadatta's nature". Integral Monism of Kashmiri Saivism Everything in this sense is directly perceived by absolute conscious- ness, and this direct perception (pratyaksa)

unifies the knowable into a single, undivided whole. This is the central concept behind a doctrine originally expounded by Narasimha called 'the non-dualism of direct perception' (pratyaksadvaita). This states that consciousness is essentially perceptive and that its perception of all things operates throughout the universe. Insofar as phenomena are clearly evident (sphufa) to us, everything is directly perceived by absolute consciousness, with which our individual consciousness is identical. This direct perception unfolds everywhere; the one true reality, it is alone and without companion or rival (nihsapatna). Even though it remains one, it can, by its very nature, perceive distinctions (bheda) between one entity and another, without this engendering any division within it. We distinguish between two entities in empirical terms on the basis of their mutual exclusion (anyonyabhava). The relative distinction {bheda) between them is essentially the perceived difference between their respective characteristics. Despite this difference they are united within the purview of a single cognition insofar as they are equally both manifest appearances. This cognition is the undivided essence (rasa) or 'own nature' (svabhava) of both. Encompassed by the 'fire of consciousness', there is no essential difference between them. Just as when an emerald and ruby reflect each other's light, the ruby is reddish-green and the emerald greenish-red, similarly everything is connected with everything else as part of the single variegated (vicitra) cognition of absolute consciousness. Mahesvarananda writes: The Supreme Lord's unique state of emotivity (asadharanabhava) is the outpouring of pure Being (maha satta). It is manifest as the brilliance (sphuratta) of the universe which, if we ponder deeply, [is realized to be] the single flavour (ekarasa) of the essence of Beauty which is the vibration of the bliss of one's own nature. In this way all things are in reality one although divided from the one another sharing as they do the 'single flavour' (ekarasa) of the pure vibration of consciousness. Kashmiri Saiva Realism Kashmiri Saivism as a whole has been variously called a form of 'realistic idealism', 'monistic idealism', ‘idealistic monism' and 'concrete monism'. It is easy to understand why Kashmiri Saivism is The Doctrine of Vibration said to be 'idealistic' and 'monistic', but in what sense is it also 'realistic*? The answer to this question is of no small importance in trying to under- stand the central idea behind its metaphysics and the fundamental importance of the concept of Spanda, in this seemingly impossible marriage between monistic idealism and pluralistic realism. The Kashmiri Saiva approach understands the world to be a symbol of the absolute, that is, as the manner in which it presents itself to us. Again we can contrast this view with that of the Advaita Vedanta. The Advaita Vedanta understands the world to be an expression of the absolute insofar as it exists by virtue of the absolute's Being. Being is understood to be the real unity which underlies empirically manifest separateness and as such is never empirically manifest. It is only transcendentally actual as 'being-in-itself. The Kashmiri Saiva position represents, in a sense, a reversal of this point of view. The nature of the absolute, and also that of Being, is onceived as an eternal becoming (satatodita), a dynamic flux or Spanda, 'the agency of the act of being'. It is dentified with the concrete actuality of the fact of appearing, not passive unmanifest Being. Appearance (abhasa) alone is real.Appearing (prakasamanatva) is equivalent to the fact of being (astitva). Ksemaraja writes in his commentary on the Stanzas on Vibration: Indeed, all things are manifest because they are nothing but manifestation. The point being that nothing is manifest apart from manifestation The absolutely unmanifest, from this point of view, can have as little existence as the space in a lattice window of a sky-palace. Nay, even less, because even that space can appear as an imagined mage manifest within consciousness. Everything is real according to the manner in which it appears. Even an illusion is in this sense real, insofar as it appears and is known in the manner in which it appears. The empirical and the real are identical categories of thought. As Abhinava says: Thus this is the supreme doctrine (upanisatf), namely that, when- ever and in whatever form [an entity] appears, that then is its particular nature. Perhaps at this stage a brief comparison with Heidegger's ideas might prove to be enlightening and not altogether out of place. According to Heidegger's phenomenology of Being, reality is intelligible in a two-fold Integral Monism of Kashmiri Saivism manner as 'phenomenon' and 'logos'. Heidegger defines what he means by 'phenomenon' as: "that-which-shows-itself. The manifest . . . phenomena are then the collection of that which lies open in broad daylight or can be brought to the light of day — what the Greeks at times implicitly identified as 'ta onta' (the things-which-are)". 127 In his later writings Heidegger drops the term 'phenomenon' in preference for the verbal form 'phainesthai' in order to emphasize even more the actuality or presentational property of Being. Explaining this new form of the term he writes: "Being disclosed itself to the ancient Greeks as 'physis'. The etymological roots 'phy-' and 'pha-' designate the same thing: 'phyein', the rising-up

or upsurge which resides within itself as 'phainesthai', lighting-up, self-showing, coming-out, appearing-forth." Heidegger contrasted his notion of phenomenon with semblance (Schein) and with appearing (Erscheinung). In the case of semblance a thing can show itself as that which it is not, as when fool's gold shows itself to be gold. The ancients always allied semblance with non-being. Heidegger points out, however, that semblances are grounded in showings, and so does Abhinava. Both Heidegger and Abhinava consequently maintain that all semblances have a real basis and are to be treated as instances of phenomena along with the so-called real showing or manifestation of non-deceptive objects. So Heidegger states that: 'how- ever much seeming, just that much being'. Thus self-showing or appearing defines Being as phenomenon, but this definition of Being is as yet incomplete. Being is not only self-showing but 'logos' which Heidegger explains means 'discourse' (Rede) in the sense of 'apophansis': 'letting-be-seen'. Phenomenology, which according to Heidegger is the only correct study of Being, means 'letting-be-seen-that-which- shows-itself. This is true of Saiva Paramadvaita as well. The reality of the world demands recognition; we are forced to accept the direct presentation of the fact of our daily experience. As Abhinava says: "if practical life, which is useful to all persons at all times, places and conditions were not real, then there would be nothing left which could be said to be real" A thousand proofs could not make 'blue' other than the colour blue. The reality of whatever appears in consciousness cannot be denied. Objects appear; they do not cease to do so by a mere emphatic denial. The manifestation of an entity in its own specific form is a fact at one level of consciousness; it is real. The appearing of the same entity in the same form but recognised to be a direct representation of the absolute is also a fact, but at another level of consciousness. It is no more or less real than the first. 'As is the state of consciousness, so is the experience,' says Abhinava. Although the nature of the absolute is discovered at a higher level of consciousness, The Doctrine of Vibration nonetheless it presents itself to us directly in the specific form in which we perceive things; otherwise there would be no way in which we could penetrate from the level of appearing to that of its source and basis. Abhinava writes: Real is the entity iyastu) that appears in the moment of direct perception (sak$atkara), that is to say, within our experience of it. Once its own specific form has been clearly determined one should, with effort, induce it to penetrate into its pure conscious nature. All things are known to be just as they present themselves. The concrete actuality of being known (pramiti), irrespective of content, is itself the vibrant (spanda) actuality of the absolute. Liberating knowledge is gained not by going beyond appearances but by attending closely to them. "The secret," Mahesvarananda says, "is that liberation while alive (jivanmukti) is the profound contemplation of Maya's nature." No ontological distinction can be drawn between the absolute and its manifestations because both are an appearing (dbhasa), the latter of diversity and the former of 'the true light of consciousness which is beyond Maya and is the category Siva'. Those who have attained the category of Pure Knowledge above Maya and have thus gone beyond the category of Maya, see the entire universe as the light of consciousness . . . Just as the markings [on a feather] are nothing apart from the feather, the feather [is nothing apart from] them, similarly, when the light of consciousness is manifest, the whole group of phenomena is manifest as the light of consciousness itself. Within the sphere of Maya, every entity's 'own nature' (svabhdva) corresponds to its specific manifest form. Accordingly it is defined as that which distinguishes it from all else and from which it never deviates. Above the sphere of Maya, that is, above the level of objectivity, is the domain of the subject. At this level, everything is realised to be part of the fullness of the experience r and hence no longer bound by the conditions which impinge on the object. Here the part is discovered to be the whole, that is, consciousness in toto. In this sphere beyond relative distinctions, the yogi realises that (all) the categories of existence are present in every single category. m The yogi experiences every individual particular as the sum total of everything else. He recognises that all things have one nature and that every particular is all things. This is the 'essence' (sard) or co-extensive unity (samarasya) of all things. Integral Monism of Kashmiri SaivismWe have established that reality is manifest according to how [and the degree in which] the freedom of consciousness reveals it and that [this freedom] is the womb of all forms. Just as 'sweetness' is present in its entirety in every atom of the sugarcane, so each and every atom [of the universe] bears within itself the emanation of all things. This is the level of consciousness in which the absolute reflects on itself realising to its eternal delight and astonishment (camatkara) its own integral nature. The reality of the world of diversity is not denied, but experienced in a new mode of awareness free of time and space in the eternal omnipresence of the Here and Now. [Phenomenal forms of awareness] such as 'this [exists]', born of the

colouring [imparted to the absolute] by the limitations engendered by the diversifying power of time {kalakalana) also emanate within the Supreme Principle. There [at that level], Fullness {purriata) is the one nature [of all things] and so everything is omnipresent; otherwise, associated with division (khancjlana), the Fullness [of the absolute] would not be full. The content of absolute consciousness consists of diverse appearings (abhdsa) which, because they are manifest through it in this way, do not compromise the wholeness of consciousness. Everything we perceive is a momentary collocation of a number of such manifestations which combine together like 4 a row of altar lamps' (dipavalT) to form the single radiant picture of the universe. The individual objects which constitute the universe are specific collocations of such 'atomic' appearings. Together they form a single unified particular which appears according to its own defining features (svalaksand). A jar, for example, consists of a number of appearances such as 'round', 4 fat\ 'earthen', 'red', etc., which together discharge a single function (arthakriya), in this case, that of carrying the appearance 'water'. They unite with each other much as the scattered rays of a lamp come together when focused, or as the various currents of the sea together give rise to waves. Atomic appearings can combine in any number of ways, provided that they are not contrary to one another as established by the dictates of natural law (niyati). An appearance of 'form', for example, cannot combine with that of 'air'. Insofar as they share a common basis (samanyadhikaranya), a given cluster of appearances appears as a single whole. This common basis is the most prominent member of the group; the appearance 'jar' is such in the example quoted above. Any one appearance in a cluster may assume a more important or subordinate role. The result is a specific The Doctrine of Vibration awareness of an object of the form: 'here this is such.' While individual appearances do not lose their separate identity {svarupabheda) when they rest on a common basis, even so the particular object which appears according to its own characteristics (svalak$ana) is an individual reality in its own right. It is a different kind of appearance characterised by its association with the appearance of the specific location and time in which it is made manifest. The form of our experience is thus 'I now see this here'. But when we perceive each particular constituent appearance separately, each assumes a separate fixed function. Abhinava cites the following colourful example to illustrate how the various combinations of appearances account for the variety of experience: Thus even though the appearance of the beloved may manifest externally, it is as if far away in the absence of another appearance, namely, that of 'embracing'. So when the [appearing of the beloved] is associated with another appearance [namely that of 'far away'] the power (arthakriya) it formerly had of giving pleasure appears as its contrary. The form our experience assumes depends, not only on the nature of the object perceived, but also on personal factors entirely peculiar to ourselves. This theory explains this in two ways. In one sense, the object remains the same, but one or other of its constituent appearances comes to the fore according to the inclinations of the perceiver. From another point of view, we can say that the perceived object is different for each perceiver according to the difference in the prominent appearance manifest to him. Abhinava, citing as an example a golden jar, illustrates how the same object appears differently to different perceivers according to the use they wish to make of it and to their state of mind: When a person who is depressed and feels that there is nothing [of value for him in the world] sees the jar, he merely perceives the appearance 'exists' [in the form of the awareness that] 'it is\ He is not conscious of any other [of its constituent appearances] at all. An individual who desires to fetch water [perceives] the appearance 'jar'. The man who simply wants something that can be taken somewhere and then brought back [perceives] the appearance 'thing'. The man who desires money [perceives] the appearance 'gold'. The man who desires a pleasing object [perceives] the appearance 'brightness' while he who wants something solid sees the appearance 'hardness'. These 'atomic events' or appearances emerge from the pure subject's consciousness and combine together to form a total event at each moment. Integral Monism of Kashmiri Saivism Daily life (vyavahara) goes on by virtue of this ever renewed flux of appearances. They are connected together and work towards a single unified experience because they appear within the field of consciousness of the universal subject. The aggregate of appearances arises in the [supreme] subject as do [sprouts in] a rice field. Even though each sprout germinates from its own seed, they are perceived as a collective whole. Appearances rest in this way within the universal subject. 'External- ity' is itself another appearance; it arises from a distinction between appearances and the individual subject. So, although all manifestation always occurs within the subject, it appears to be external due to the power of Maya which separates the individual subject from his object. This split must occur for daily life to be possible. Only externally manifest appearances can perform their

functions; when they are merged within the subject and at one with him, they cannot do so. Daily life proceeds on the basis of the operation and withdrawal of the conditions necessary for fruitful action to be possible. Appearance in this sense represents the actualisation of a potential hidden in consciousness made possible by virtue of its dynamic, Spanda nature which is both the flow from inner to outer and back as well as the power that impels it. The emergence from, and submergence into, pure consciousness of each individual appearance is a particular pulsation (visesaspanda) of differentiated awareness. Together these individual pulsations constitute the universal pulse (samanyaspandd) of cosmic creation and destruction. Thus, every single thing in this way forms a part of the radiant vibration {sphuratta, sphurana) of the light of absolute consciousness. Light and Awareness: The Two Aspects of Consciousness Absolute consciousness understood as the unchanging ontological ground of all appearing is termed 'Prakasa'. As the creative awareness of its own Being, the absolute is called ' Vimarsa '. Prakasa and Vimarsa — the Divine Light of consciousness and the reflective awareness this Light has of its own nature — together constitute the all-embracing fullness (purnata) of consciousness. The Recognition (pratyabhijfia) school of Kashmiri Saivism develops this concept of the absolute which finds its fullest expression in Utpaladeva's Stanzas on the Recognition of God. Even though neither of these two key terms appear in the Stanzas on Vibration or the Aphorisms of Siva, they recur frequently in their commentaries. Thus, although the original formulation of the Doctrine of Vibration differs from the theology of Recognition in this respect, it was extended in the course of its development to accommodate this concept of the absolute as well. This was possible, and quite justified, insofar as the absolute understood in Pratyabhijfia terms does not, as we shall see, differ essentially from that of the Spanda school. We can, as Kashmiri Saivites themselves have done, explain one in terms of the other. The Doctrine of Vibration Prakasa: The Light of Consciousness Prakasa is the pure luminosity' (bhdna) or 'self-showing' that constitutes the essence and ultimate identity (atman) of phenomena. That things appear at all is due to the light of consciousness, and their appearing (avabhasana) is itself this Light which bestows on all things their evident, manifest nature. Established in the light of consciousness everything appears there according to its own specific nature (svabhava). Anything that supposedly does not rest in this Light is as unreal as a sky-flower. 3 Thus, according to Rajanaka Rama, unlike the light of the sun, or any other light, this Light not only makes all things apparent, it is also their ultimate source. 4 Full of its divine vibration the Light makes all things manifest and withdraws them into itself. This supra- temporal activity characterises it most specifically; devoid of it, it would be no better than an inert physical phenomenon. At the same time, this light is the conjunction (slesa) or oneness (aikdtmya) of its countless manifest forms, 6 and the collective whole (sampina'ana) of all the categories of existence. The universe is nothing but the shining of the Light within itself. It is the radiant vibration (sphuratta) of this Light, the state (avast hand) in which consciousness becomes manifest.Although the Light shines as all things at all times and hence also makes their diversity manifest, 9 penetrating each object individually as well as collectively, it is not totally 'merged' (magna) or identified with the object so as to suffer any division within itself. Our experience of any object is of the form: T see this': it is not itself an object, but the manifest form the object assumes as a luminous principle of experience. The Light is ever revealed and can never be obscured; objectivity can never cast a shadow on the light of consciousness. The Stanzas on Vibration declare: That in which all this creation is established and from whence it arises is nowhere obstructed because it is unconditioned by [its very] nature. This Light is the highest reality (Paramartha). It is the 'Ancient Light' (puranaprakasa) that makes all things new and fresh every moment. It is 'always new and secret, ancient and known to all'. It is the form of the Present (vartamanarupa), the Eternal Now. Time and space are relations between the contents of consciousness; they cannot impinge on the integrity of the absolute itself. 16 Neither space nor time can divide it, for they are one with the Light that illumines them Light and Awareness: Two Aspects of Consciousness and makes them known as elements of experience. But this Light is the shining of the absolute; it is not an impersonal principle. It is the living Light of God, indeed it is God Himself, the Master Who instructs the entire universe. 18 Siva is this 'auspicious lamp', Who illumines all things. He is the Light of consciousness that reveals the presence of both the real and the unreal, of light' and 'darkness'. 20 Abhinavagupta writes: Thus Bhairava, the Light, is self-evident (svatahsiddha); without beginning, He is the first and last of all things, the Eternal Present. And so what else can be said of Him? The unfolding of the categories of existence (tattva) and creation, which are the expansion of His own Self, He illumines, luminous with His own Light, in

identity with Himself, and because He illumines Himself, so too He reflects on His own nature, without His wonder (camatkara) being in any way diminished. Although It Is Possible To Catch Glimpses Of The Highest Reality In Advanced States Of Contemplation Before Attaining Perfect Enlightenment, These States, However Long They Last, Are Transitory (Kadacitka) And When They End The Vision Of The Absolute Ceases With Them. The Highest Realisation, However, Persists In All States Of Consciousness. It Happens Once And Need Never Occur Again. A Passage From A Lost Tantra Declares: "The Self Shines Forth But Once, It Is Full [Of All Things] And Can Nowhere Be Unmanifest." All Spiritual Discipline Culminates In This Moment Of Realisation. Accordingly, Abhinava Stresses That The Goal Of All The Means To Realisation, Even The Individual Means, Is This Absolute Consciousness. Finally, It Is Worth Noting That Although Abhinava Affirms That The Teachings Concerning Anupaya Are Found In The Siddhayogesvarimata And The Malinxvijaya, Both Of Which, According To Abhinava, Are Major Tantras Of The Trika School, It Is In The Theology Of The School Of Recognition That It Is Best Exemplified. Abhinava Himself Refers To Somananda, The Founder Of This School, As Teaching It And Alludes To The Following Passage In The Vision Of Siva To Support His Own Exposition: When Siva, Who Is Everywhere Present, Is Known Just Once Through The Firm Insight Born Of Right Knowledge (Pramana), The Scripture And The Master's Words, No Means [To Realisation] Serves Any Purpose And Even Contemplation {Bhavana) [Is Of No Further Use]. Anupaya Is Therefore, According To Abhinavagupta, The Recognition Of One's Own Authentic Siva-Nature, Which All The Higher Tantric Traditions Teach Is The Ultimate Realisation. This Is Also True Of The Doctrine Of Vibration Whose Precedents Are Clearly Traceable To These Same Traditions. Thus, Although The Stanzas Themselves Never Refer Directly To Enlighten- Ment As An Experience Of Recognition, There Can Be Little Doubt That Spanda Practice Leads To This Same Realisation. Accordingly, Commentators Stress That We Realise The Vibration Of Consciousness By Recognising Its Activity And That Liberation Depends On The Recognition Of This As One's Own Nature. Ksemaraja Describes What Happens In This Moment Of Recognition According To The Doctrine Of Vibration Thus: At The End Of Countless Rebirths, The Yogi's [Psycho-Physical] Activity [Which Issues From Ignorance] Is Suddenly Interrupted By (e&eb) The Recognition Of His Own Transcendent Nature, Full Of A Novel And Supreme Bliss. He Is Like One Struck With Awe. And In This Attitude Of Astonishment (Vismaya- Mudra) Achieves (eb) The Great Expansion [Of Consciousness] {Mahavikasa). Thus He, The Best Of Yogis, Whose True Nature Has Been Revealed [To Him] Is (=) Well Established [At The Highest Level Of Consciousness], Which He Grasps Firmly And His Hold Upon It Never Slackens. Thus He Is No Longer Subject To (e) Profane Existence (Pravftti), The Abhorrent And Continuing Round Of Birth And Death, Which Inspires (eb) Fear In All Living Beings, Because (e) Its Cause, His Own Impurity, No Longer Exists. The Divine Means (Sambhavopava) In Anupaya The Yogi Does Not (e) Need To Deal With The World Of Diversity At All; Only Paramasiva Exists There. Beyond Both Immanence And Transcendence, He Has Nothing (e) To Do With The World Of Practice And Realisation. Anupaya Is The Experience Of (e) The Undefinable (Qnakhya) Light Of Consciousness, Which Is (=) The Pure Bliss Beyond (e) Even The Supreme State (Paratlta) Of Sivatattva. At A Slightly Lower Level, Corresponding To (e&eb) The Divine Means, A Subtle Distinction Emerges Between The Goal And (e&eb) The Path. The Yogi Now Practises Within The Domain Of The Outpouring Of The Power Of Consciousness. From This Level He Penetrates Directly Into (e&eb) The Universal Egoity Of Pure Consciousness By The Subtle Exertion (Udyama) Of Its Freedom (Svatantrya) And Reflective Awareness. The Yogi Who Practises The Divine Means Is Not Concerned With Any Partial Aspect Of Reality But Centers His Attention Directly On Its Abounding Plenitude. Hence This Means Is Based On Siva's Own State (Sambhavavastha) In Which Only The Power Of Freedom Operates As The Pure Being (Satta) Or Essence Of All The Other Powers. This State Is The Light Of Consciousness Which, Free Of All Thought- Forms, Is The Basis Of All Practice." The Yogi, Who Recognises That Pure Consciousness, Free Of Thought-Constructs (Nirvikalpa), Is (=) His Basic State, Can Practice In Any Way He Chooses; Even The Most Common Mantra Will Lead Him Directly To The Highest State. Thus The Forms Of Contemplative Absorption, Empowered (Sakta) And Individual (Anava), Which Are The Fruits Of The Other Means To Realisation Both, Attain Maturity In (eb) This Same Undifferentiated Awareness. This Awareness Is The Pure Ego Manifest At (eb) The Initial Moment Of Perception (Prathamikalocana), When (e) The Power Of The Will To Perceive Is Activated. It Is The Subtle State Of Consciousness That Reveals (eb)The Presence And

Nature Of Its Object Directly: That Which Shines And Is Directly Grasped In The First Moment Of Perception While It Is Still Free Of (e) Differentiated Representations And Reflects Upon Itself Is [The Basis Of The Divine Means] Said To Be The Will. Just As An Object Appears Directly To (e) One Whose Eyes Are Open Without (e) The Intervention Of (e) Any Mental Cogitation (Anusaipdhana), So, For Some, Does Siva's Nature. The Movement Of Awareness At This Level Of Practice Attains (eb) Its Goal Quickly. While Consciousness Is Heightened Progressively In The Other Means, Here It Expands Freely To (e&eb) The Higher Levels, Unconfined By (e) Any Intruding Thought-Constructs. The Divine Means Is A 'Thoughtless Thought', A 'Processless Process', That Occurs At The Juncture Between Being And Becoming. Abhinava Explains: When The Heart [Of Consciousness] Is Pure And [Free Of Thought- Constructs], It Harbours The Light Which Illumines The Radiant, Primordial Plane Ipragrabhumi) Together With All The Categories Of Existence. [The Yogi] Then Realises Through It (e&eb) His Identity With Siva Who Is Pure Consciousness The Yogi Must Catch The Initial Moment Of Awareness (Adiparamarsa) Just When Perception Begins^ He Must Not Move On From The First Pure Sensation Of The Object But Return To Its Original Source In His Own T Consciousness. Observing In This Way The Objective Field Of Consciousness Without Laboring To Distinguish Particulars, The Yogi Penetrates Into His Own Subjectivity Which, Vacuous And Divested Of (e) All Outer Supports (Niralamba), Is (=) Not Directed Anywhere Outside Itself (Ananyamukha- Preksin). Here He Can Hold Of The Power Inherent In His Own Consciousness Through Which He Discerns The True Nature Of Whatever Appears Before Him. Thus The Stanzas On Vibration Teach: Just As An Object, Which Is Not Seen Clearly At First Even When The Mind Attends To It Carefully, Becomes Later Fully Evident When Observed With The Effort Exerted Through One's Own [Inherent] Strength (Svabala), In The Same Way, When [The Yogi] Lays Hold Of That Same Power, Then Whatever [He Perceives Manifests To Him] Quickly According To Its True Nature, Whatever Be Its Form, Locus, Time Or State. Thus, Although The Practice Of This Divine Means Starts By Catching Hold Of The Will In The First Moment Of Awareness, It Also Concerns The Second And Third Moments In Which The Means Of Knowledge And The Object Are Made (eb) Manifest. When Practice At This Level Proceeds Smoothly And Without (e) Interruption, The Three Powers Of Will, Knowledge And Action Fuse Into (e&eb) The Trident (Trisula) Of Power, Which Is The Subject Free Of (e) All Obscuration (Niranjana), M At One With The Power Of Action In Its Most Powerful And Evident Form. The Kaula Schools Call This State The Stainless (Niranjanatattva). Equated In The Spanda Tradition With The Dawning Of The Vibration Of Consciousness {Spandodaya), It Is The Enlightenment The Spanda Yogi Seeks. Spanda Practice Is Based On The Experience Of Spanda Which, As We Have Seen, Is Defined As (=) The Intent (Aunmukhya) Of Consciousness, Unrestricted To Any Specific Object And Hence Free Of Thought-Constructs. Spanda Can Therefore Be Experienced Directly When A Powerful Intention Develops Within Consciousness, Whatever Is Its Ultimate Goal Or Cause. We Have Already Noted That Intense Anger, Joy, Grief Or Confusion Is Such Occasions. Similarly, The Yogi Can Make Contact With The Omnipotent Will, Which He As Siva Possesses, Through Intense Prayer. Directing His Entire Attention To Siva, The Benefactor Of The World, Entreating Him Fervently And Without Break, His Will Merges With Siva's Universal Will, Which Is The Source Of Every Impulse And Perception. As He Looks About Him, The Yogi Realises That It Is Siva Himself, The Universal Consciousness And The Yogi's Authentic Identity, Who Ordains (eb) His Every Action, Thought And Perception. Thus The Yogi's Cognitive Intent On His Object Coincides With (=) **The** Universal Will To Make That Object Known To Him, Whether The Yogi Is Awake Or Dreaming. He Is Thus No Longer Like The Worldly Man Who Cannot (e) Dream As He Wishes And Is Forced To Experience Whatever Spontaneously Happens In These States Of Consciousness. Ultimately The Yogi Manages, By Siva's Grace, To Maintain A Constant Awareness Of His Own Pure Perceptive Consciousness (Upalabdhrta) Divested Of All Obscuring Thought-Constructs In Deep Sleep As Well As In The Contemplative State (Turiya) Beyond It. When He Rises To The Higher Levels Of Contemplation In Which The Breath Is Suspended And All Sensory And Mental Activity Ceases, The Yogi Who Manages To Sustain This Pure, Undifferentiated Awareness Does Not Succumb To Sleep As Do Less Developed Yogis. Perfection In The Practice Of The Divine Means Thus Coincides With The Goal Of Spanda Practice, Namely, A Constant, Alert Attention To The Perceiving Subjectivity Which Persists Unchanged In Every State Of Consciousness Both As The Perceiver And Agent Of All That It Experiences. Another Important Spanda Practice Belonging To This Means Is Centering.

The Spanda Yogi Seeks To Find The Centre (Madhya) Between One Cognition And (e&eb) The Next; For It Is There That He Discovers The Expansion (Unmesd) Of Consciousness Free Of Thought-Constructs From (e) Whence All Differentiated Perceptions (Yikalpa) Emerge. 107 Abhinava Explains That This Pure Awareness Is Called: . . . The Expansion (Unme$a) Of [Consciousness] Or The Creative Intuition {Pratibha) [Experienced] In The Interval Which Divides Two [Moments] Of (e&eb) Differentiated Perception (Vikalpa) It Is Here That They Arise And Disappear. The Sastras And Agamas Proclaim With Reasoned Argument That It Is Free Of Thought-Constructs {Nirvikalpa) And Precedes All Mental Representations Of Any Object. None Can Deny That A Gap Exists Between Perceptions Insofar As Two Moments Of Thought Are Invariably Divided. This [Gap] Is The Undifferentiated Unity Of All The Countless Manifestations. Similarly, In The Outer More Objective Sphere, Where Change Consists Of The Alterations In The Configurations Of Manifest Appearances (Abhasa\ The Transition From One To Another Corresponds To A Phase Of Pure Luminosity That Marks The Beginning Of One Form And The End Of Another. The World Of Manifestation And Differentiated Perceptions (Yikalpa) Thus Extends From One Centre To The Next. Although It Is Never In Fact Divorced From The Subject Who Resides There, The Ignorant Fail To Grasp This Fact And So, Cut Off From The Centre, The World Of Objectivity Becomes For Them The Sphere Of Maya. Bhagavatotpala Quotes The Light Of Consciousness (Samvitprakdsa): This Ever Pure Experience (Suddhanubhava) Is Variegated By Each Form [Revealed Within It]; Even So It Remains Unstained (Nirmala) When Moving To Another. Just As A Cloth Which Is Naturally White, Once Dyed, Cannot Change Colour Without [First] Becoming White Again, Similarly The Pure Power Of Awareness, (Citi) Once Coloured By Form, Is Pure [Again] At The Centre Where That Form Is Abandoned And From Whence It Proceeds To Another. In His Essence Of Vibration (Spandasarpdoha), K§Emaraja Explains That The Rise And Fall Of Every Individual Perception In The Field Of Awareness Is A Specific Pulsation Of Consciousness. From The Point Of View Of The Object, The Expansion (Unmesa) Of This Pulse Is Represented By The Initial Desire To Perceive (Didrksa) A Particular Object, While The Contracted (Nimesa) Phase Is The Withdrawal Of Attention From The Object Previously Perceived. From The Point Of View Of The Perceiving Subjectivity, The Phases Are Reversed, So That The Initial Desire To Perceive Marks The Contraction (Nimesa) Of Subjective Consciousness While The Falling Away Of The Previous Perception Is Its Expansion (Unmesa). At The Higher Level, Where These Two Phases Are Experienced Within Consciousness, They Represent The State Of The Categories Of Isvara ('This Universe Is Me') And Sadasiva ('I Am This Universe'). Utpaladeva Says: Expansion (Unme$A\ Which Is In The External Manifestation [Of Objectivity], Is Kvaratattva While Contraction (Nime$A), Which Is In The Internal Manifestation [Of Subjectivity], Is Sadasiva. At This Level All The Powers Of Consciousness Fuse And Both Phases Are Manifest As Part Of One Reality. This Unity Is In Fact Apparent To Everybody At Each Moment. However, Within The Domain Of Maya, Which Is The Sphere Of Differentiated Perceptions (Yikalpa), It Is Clearly Manifest Only At The Juncture (Madhya) Between Two Cognitions. In This Centre Resides The Void (Kha) Of Consciousness (Free Of Thought-Constructs) Which, Divested Of Diversity, Digests Into Itself All The Psycho-Physical Processes That Give Life To The Multiplicity Of Perceptions. The Yogi Moves From The Particular Vibrations Of Consciousness At Its Periphery To The Universal Throb Of The Heart In The Centre. As Abhinava Explains: The Self-Reflective Awareness In The Heart Of Pure Consciousness, Present At The Beginning And End Of Each Perception, Within Which The Entire Universe Is Dissolved Away Without Residue, Is Called In The Scriptures, The Universal Vibration Of Consciousness (Samanyaspanda) And Is The Outpouring (Uccalana) [Of Awareness] Within One's Own Nature. All The Categories Of Existence (Tattvas) Are United In The Heart Of The Centre Where The Life-Giving Elixir Of Siva's Consciousness Floods One's Own Inner Nature. To Reside In The Centre Is To Abide By The Law Of Totality (Gramadharma) In A State Which Transcends The Workings Of The Mind (Unmana). Consciousness (Jnana) With Light As Its Support, Residing In The Centre Between Being And Non-Being Is Known As The Act Of Abiding In One's Own Abode As The Perceiving Subjectivity (Dratfrtva) Free Of All Obscuration. That Which Has Been Purified By Pure Awareness (Suddhavijnana) Is Called The Transcendent (Viviktavastu), Said To Be The Mode Of Being (V/7//) Of The Law Of Totality (Gramadharma) Through Which Everything Is Easily Attainable. The Power In The Centre (Madhyasakti) Is The Eternal Present. Beyond Time It Is The Source Of Both Past And Future. To Be Established There Is To Abide Without A Break In Rama, The Supreme Enjoyer, In Every Action Of One's Life.

Rama Is Siva, The Supreme Cause Who Pervades The Fourteen Aspects Which Embrace The Entire Universe Of Experience, Namely, Moving, Standing, Dreaming, Waking, The Opening And Closing Of The Eyes, Running, Jumping, Exertion, Knowledge [Born] Of The Power Of The Senses, The [Three] Aspects Of The Mind, Living Beings, Names And All Kinds Of Actions. By Developing An Awareness Of The Centre, The Yogi Experiences The Bliss Of Consciousness. Through This Gap He Plunges Into Introverted Absorption (Nimilanasamadhi) And Then Emerges Again To Pervade The Field Of Awareness Between Centres And So Experience The Cosmic Bliss (Jagadananda) Of The Universal Vibration Of Consciousness. 1 I8 He Then Recognises That This State Pervades Every Aspect Of Experience. In This Way The Yogi's Consciousness Is No Longer Afflicted By The Power Which Obscures It, Hemming The Centre In On Both Sides With Thought-Constructs That Seemingly Deprive It Of Its Fullness. As He Realises Directly His Pure Conscious Nature As The Universal Ego Free Of All Mental Representations, It Expands Out To Embrace All Things Within Itself. Thus The Realisation The Divine Means Leads To, And Is Directly Based Upon, Is That This Pure Ego Is In All Things Just As All Things Are Within It. In The Spanda Tradition, As Recorded In The Stanzas On Vibration, No Such Ego Is Recognised. ' I9 Man's Authentic Nature Is, However, Understood In Personal Terms As Every Individual's Own 'Own Nature* (Svasvabhava) Which Is Siva, The Universal Vibration Of Pure Subjectivity (Upalabdhfta). It Is Not Surprising, Therefore, That Later Commentators Found These Two Conceptions To Be Essentially The Same And Accordingly Identified One's Own Inner Nature With The Pure Ego. This Came As A Natural Development In Spanda Doctrine Not Only For This Reason But Also Because The Universal Ego Is Experienced As The Inner Dynamics Of Absolute Consciousness. To Conclude Our Summarial Exposition Of The Divine Means, Which Is Centred On The Direct Experience Of This Pure Ego (And Hence On Spanda In This Form), We Turn Now To A Brief Description Of Its Inner, Cyclic Activity. We Shall Do This By Examining Abhinava' S Esoteric Exegesis Of The Symbolic Significance Of The Word 'A HAM', Which In Sanskrit Means T, And Symbolises By Its Form The Ego's Dynamic Nature. The Objective World Of Perceptions Is, As We Have Seen, Essentially A Chain Of Thought-Constructs (Prapanca) Closely Linked To One Another And Woven Into The Fabric Of Diversity (Vicitrata). This Thought (Vikalpa) Is A Form Of Speech (Vac) Uttered Internally By The Mind (Citta), Which Is Itself An Outpouring Of Consciousness. Consciousness Also, In Its Turn, Resounds With The Silent, Supreme Form Of Speech {Para Vac) Which Is The Reflective Awareness Through Which It Expresses Itself To Itself. Consequently, The Fifty Letters Of The Sanskrit Alphabet, Which Are The Smallest Phonemic Units Into Which Speech Can Be Analysed, Are Symbolic Of The Principal Elements Of The Activity Of Consciousness. Letters Come Together To Generate Words And Words Go On To Form Sentences. In The Same Way The Fifty Phases In The Cycle Of Consciousness Represent, In The Realms Of Denoted Meaning (Vacya), The Sum Total Of Its Universal Activity (Kriya) Corresponding To The Principal Forces (Kala) Which Come Together To Form The Metaphysical Categories Of Experience, Which In Their Turn Appear In The Grossest, Most Explicitly 'Articulate' Form As The One Hundred And Eighteen World-Systems (Bhuvana). 'A', The First Letter Of Both AHAM And The Sanskrit Alphabet, Is The Point Of Departure Or Initial Emergence Of All The Other Letters And Hence Denotes Anuttara — The Absolute. 'Ha', Is The Final Letter Of The Alphabet And Represents The Point Of Completion When Ail The Letters Have Emerged. It Represents The State In Which All The Elements Of Experience, In The Domains Of Both Inner Consciousness And Outer Unconsciousness, Are Fully Displayed. It Is Also The Generative, Emission (Visarga) Which, Like The Breath, Casts The Inner Into The Outer, And Draws What Is Outside Inward. The Two Letters 'A' And 'Ha' Thus Represent Siva, The Transcendental Source And Sakti, His Cosmic Outpouring That Flows Back Into Him. The Combined 'A-Ha' Contains Within Itself All The Letters Of The Alphabet — Every Phase Of Consciousness, Both Transcendental And Universal. (For A Graphic Representation Of This Analysis, See Figure 1.) M, The Final Letter Of AHAM, Is Written As A Dot Placed Above The Letter Which Precedes It. It Comes At The End Of The Vowel Series And Before The Consonants And So Is Called 'Anus Vara '(Lit. 'That Which Follows The Vowels') And Also 'Bindu ' (Lit. 'Dot,' 'Drop,' 'Point' Or 'Zero'). While The Consonant 'M' Symbolises The Individual Soul (Purusa), 'Bindu 1 Rep Resents The Subtle Vibration Of T, Which Is The Life Force (Jivakala) And Essence Of The Soul's Subjectivity Manifest At The Transcendental, Supra-Mental Level (Unmana). 12 ° It Is The Zero-Point In The Centre Between The Series Of Negative Numbers, In This Case The Vowels Which Represent The Processes Happening Internally

Within Siva, And The Series Of Positive Numbers — The Consonants Which Symbolise The Processes Happening Externally Within Sakti. Bindu, As A Point Without Area, Symbolises The Non-Finite Nature Of The Pure Awareness (Pramitibhava) Of AHAM. It Is The Pivot Around Which The Cycle Of Energies From 'A' To 'Ha' Rotates, The Void In The Centre From Which All The Powers Emanate And Into Which They Collapse. As Such, It Is The Supreme Power Of Action Which Holds Subject, Object And Means Of Knowledge Together In A Potential State In The One Light That Shines As All Three Containing Them In Its Repose (Visrdnti). Bindu Is The 'Knower' (Jndtr), Who Is Essentially Consciousness That, Though Omniscient, Does Not Manifest Its Intelligence, Like A Man Who Knows The Scriptures But Having No Occasion To Explain Them To Others Silently Bears This Knowledge Within Himself. As Such, It Symbolises The Union Of Siva And Sakti (Sivasakti- Mithunapina'a) In A State Of Heightened Potency In Which They Have Not Yet Divided To Generate The World Of Diversity. It Stands, In Other Words, At The Threshold Of Differentiation In The Stream Of Emanation Still Contained Within Siva. The Absolute. Expansion Commences Bindu — The Individual Soul Withdrawal Commences Then, To The Degree In Which That Which Is To Be Accomplished By The Power Of Action Residing Within It [As A Potential] Penetrates Into The Absolute, It Appears Initially As Bindu, Which Is The Light Of Pure Consciousness. When Outer Objectivity Is Reabsorbed Into Its Transcendent Source, Bindu Is The Point Into Which All The Manifest Powers Of Consciousness Are Gathered And Fused Together. The Universal Potency Of All The Letters Is Thus Contained In Bindu Which, As The Reflective Awareness Of Supreme T Consciousness, Gives Them All Life. Thus Bindu Also Marks The Beginning Of Siva's Internal Movement Back To The Undifferentiated Absolute And So Stands At The Threshold Of Both Emission And Absorption Without Being Involved In Either. The Three Aspects Of AH AM Together Constitute A Movement From The Undifferentiated Source Of Transcendental Consciousness — 'A' — Through The Expansion Or Emission Of Its Power — 4 Ha' — To The Subject — 'M 1 —Which Contains And Makes Manifest The Entire Universe Of Experience. The Reverse Of This Movement, That Of Withdrawal (Samhara), Is Represented By M-Ha-A. AHAM And M-Ha-A Alternate In The Rotation (Ghurnana) Of The Reflective Awareness Of T Consciousness As Immanent Sakti Emerges From Transcendental Siva To Then Merge Back Into Him. As Abhinava Says: The Universe Rests Within Sakti And She On The Plane Of The Absolute (Anutiara) And This Again Within Sakti ... For The Universe Shines Within Consciousness And [Consciousness Shines] There [Within The Universe By The Power Of] Consciousness. These Three Poles, Forming A Couple And Merging, Make Up The One Supreme Nature Of Bhairava Whose Essence Is AHAM- At The Microcosmic Level, Fc A' Represents The Initial Moment When The Subject Begins To Rise Out Of Himself To View The Object. The Movement From 'A To 'Ha' Marks The Emergence Of Sensation Within The Field Of Awareness, Which Is Represented By The Fifty Letters Of The Alphabet Symbolic Of The Fifty Aspects Of The Flux Of Consciousness Leading To Objectified Perception. 'Nl' Is Tne Subject Who, Resting Content Within Himself When He Has Perceived His Object Merges Through The Inner Flow Of Awareness Into K A\ The Absolute. Then From The Absolute (A) Its Emission (Ha) Flows Back Into The Pure Subject (M) Set To Perceive His Object. Thus All The Cycles Of Creation And Destruction Are Contained Within AHAM Through Which They Are Experienced Simultaneously As The Spontaneous Play Of The Absolute. The Yogi Who Recognises This Recurrent Pulse Of Awareness To Be The Movement Of His Own Consciousness Merges His Limited Ego With The Universal Ego. Thus He Realises That Its Power To Create, Sustain And Destroy All Things Is His Own Inner Strength (Svabaia) That He Exerts Effortlessly In The Same State Of Mystical Absorption (Turlya) In Universal Consciousness That The Absolute Itself Enjoys. In This Way He Shares In The Three-Fold Awareness Siva Himself Has Of His Own Nature Which Abhinava Describes As Follows: I Make The Universe Manifest Within Myself In The Sky Of Consciousness. I, Who Am The Universe, Am Its Creator! ' — This Awareness Is The Way In Which One Becomes Bhairava. 'AH Of Manifest Creation(Sadadhvari) Is Reflected Within Me, I Cause It To Persist 1 — This Awareness Is The Way In Which One Becomes The Universe. The Universe Dissolves Within Me. I Who Am The Flame Of The [One] Great And Eternal Fire Of Consciousness' — Seeing Thus One Achieves Peace. The Experience Of The Liberated Thus Coincides With The Realisation Of Their Own Divine Nature Which, Through Its Power, Rules And Guides The Cosmic Order. Thus This Attainment (Siddhi), Which Is Liberation Itself, Is In The Doctrlie Of Vibration Technically Called 'Mastery Over The Wheel Of Energies' (Cakresvaratvasiddhi) Because The Liberated

Soul, Identified With Siva, Now Governs, As Does Siva, The Cycle Of The Powers That Bring About The Creation And Destruction Of All Things, The Empowered Means (Saktopaya) All The Practices Taught In The Stanzas On Vibration Are Internal. Whenever Ritual Is Mentioned, It Is Invariably Interpreted In Terms Of The Dynamics Of The Inner Processes The Yogi Experiences And Implements In The Course Of His Yogic Practice. The Doctrine Of Vibration, Ksemaraja Affirms, Is Concerned Entirely With These Inner Disciplines Centred, As It Is, In One Way Or Another, On Consciousness Or, At Least, On The Inner Activity Of The Mind. Thus The Empowered Means Which, Like The Other Categories We Have Discussed, Is Entirely Internal Includes An Important Part Of Spanda Practice. Spanda Practice Belonging To The Divine Means Centres On One's Own Inherent Nature (Svasvabhava) As Siva, The Universal Perceiver And Agent, That Belonging To The Empowered Means On His Power Instead Of Arriving Directly At The All-Embracing Emptiness Of Subjective Conscious- Ness, The Yogi Practising The Empowered Means Realises His True Nature Through The Fullness Of Its Energy. Practising The Divine Means, The Yogi Plunges, As It Were, Straight Into The Fire Of Consciousness; Practising The Empowered Means He Merges With Its Rays. Either Way The Yogi Is Centred Equally On Ultimate Reality. The Power Of Consciousness Is No Less Absolute Than Its Possessor. To Make This Point Abhinava Quotes The Matanga- Tantra: This Reality Consists Of The Rays Of [Siva's] Power And Is Variously Said To Be The Abode Of The Lord's Manifestation . . . That Same [Power] Illumined [By Siva] Is Itself Also Luminous, Unshaken And Unmoving. That Very [Power] Is The Supreme State, Subtle, Omnipresent, The Nectar Of Immortality, Free Of Obscuration, Peaceful, Yearning For Pure Being Alone {Vastumatra) And Devoid Of Beginning And End. Perfectly Pure, It Is Said To Be The Body [Of Ultimate Reality]. The Yogi Concentrates On The Powers Operating In All Of Life's Activities As Particular Pulsations (Visesaspanda) In The Universal Rhythm (Samanyaspanda) Of The Power Of Consciousness. In This Way He Rises Progressively From The Particular To The Universal Until He Reaches Pure Being (Satta), The Greatest Of All Universals (Mahasamanyd) And The Highest Form Of Siva's Power. Thus The Creative Power Of Maya, Manifest Through Countless Lesser Powers, No Longer Causes The Yogi To Stray From Siva's Consciousness But Becomes The Means Through Which It Can Be Realised In The Illuminating Brilliance (Sphuratta) Which Is Siva's Pure Being. Thus By Discovering The True Nature Of Sakti, The Yogi Realises Himself To Be Siva, Its Possessor Who Consists Of All Its Countless Powers. Thus Practise Belonging To This Means Leads To The Same Pure Consciousness Free Of Thought-Constructs Realised Through The Divine Means. Although The Ultimate Realisation Is Instantaneous, The Yogi Rises To It Gradually By Freeing His Consciousness Of The Limitations Imposed Upon It By Thought. Abhinava Explains: The Same Occurs In The Empowered Means [As Does In The Divine]. At The Discursive Level Of Consciousness {Yaikalpikibhumi) [Where The Empowered Means Functions] Knowledge And Action, Although Evident, Are, For The Reasons Explained Previously, Contracted. A Blazing Energy [Is Revealed Within] The One Who Dedicates Himself To Removing The Burden Of This Contraction. [This Energy Eventually] Brings About The Inner Manifestation {Antarabhasa) Of Pure Consciousness He Seeks. Consciousness Is Individualized And Its Power Of Knowledge And Action Contracted By The Thought-Constructs Born Of Ignorance. The Arising Of These Mental Representations, As The Stanzas On Vibration Say, Deprives The Soul Of Its Freedom And Immortal Life. The Practise Of The Empowered Means Is Meant To Free The Fettered Soul Of This Constriction On His Consciousness. It Operates Within The Mental Sphere (Cetasy™ And Is Designed To Purify Thought (Vikalpasamskara) In Order To. Reveal The Pure Consciousness Which Is Its Ground And Ultimate Source. Thus, The Empowered Means Is Concerned With The Second Instant Of Perception, During Which The Subject Forms Mental Representations Of His Object. Thought Functions On The Basis Of An Awareness Of Relative Distinctions Between Specific Particulars, Distinguishing Them From One Another And Thus Seemingly Fragmenting The Essential Unity Of Reality. The Vibrant Vitality Of Consciousness, Universally Manifest, Is Clouded Like A Mirror By A Child's Breath And The Soul Is Deprived Of The Liberating Intuition Of The One Reality Free Of Thought-Constructs (Nirvikalpa). Abhinava Writes: The [Fettered Soul] Is Like A Dancing Girl Who Although Wishing To Leave The Dancehall Is Collared By The Doorkeeper Of Thought And Thrown Back Onto The Stage Of Maya All Thought Is Centred On Objectivity And Hence Dislodges Awareness From The Plenitude Of Pure Subjective Consciousness. Thus, To Regain The Original State Of Rest (Visranti) Consciousness Enjoys, The Yogi Must Rid Himself Of Thought. As Thought-Forms

Decrease, Pure, Thought-Free Awareness Is Strengthened Until The Yogi Is Fully Established In A State In Which The Relative Distinctions (Bheda) Conceived Between Entities Dissolve Away. Everything Appears To Him As Pure Being (Sattdmdtra) L} And The Entire Universe Shines Before Him Pervaded By Siva's Radiance. His Intuitive Faculty (Mati) Thus Purified, The Yogi Gains Both The Perfections (Siddhi) Of Yogic Practice And Liberation (Mukti). His Consciousness Is Now Like A Well-Polished Mirror Which Reflects Everything He Desires And Grants It To Him. Abhinava Writes: Just As A Man Who Has Been Ill For A Long Time Forgets His Past Pain Completely When He Regains His Health, Absorbed As He Is In The Ease Of His Present Condition, So Too Those Who Are Grounded In Pure Awareness Free Of Thought-Constructs Are No Longer Conscious Of Their Previous [Fettered] State. Consciousness, The Sole Truly Existent Reality, Free Of Thought- Constructs Is Made Fully And Evidently Manifest By Eliminating These Differentiated Perceptions. The Wise Man Should Therefore Exert Himself To Attend Closely To This [State Of Awareness]. The Thought-Constructs Generated Within Consciousness Do Not In Reality Affect It At All. They Can Neither Break Up Nor Add Anything To The Light Which Shines As All Things. They Are In Fact Nothing But Consciousness Itself Which Perceives, Through Its Power Of Reflective Awareness (Yimarsa), The Multitude Of Objects In Diverse Ways, And So Assumes This Form. Although Thought-Constructs Are Mental Representations Of Objects Once Seen Or Present, They Are Products Of The Power Of Consciousness And Not Of The Objects They Represent. Thought Is Both Analytic And Synthetic; It Serves The Useful Purpose Of Separating Individual Elements Of Experience From Others And Linking Together Those That Appear To Be Distinct From One Another So That They Can Be Better Understood. It Does Not Consist Merely Of False Mental Constructs Projected Onto Reality That Need To Be Wholly Rejected. Thought Obscures Consciousness And Distracts It Only When It Appears In The Form Of Doubt, Vacillating Between Alternatives. Once This Conflicting Duality (Dvaitddhivasa) Is Eliminated, Thought Is Purified And Rests In Itself As The 'Thought-Less Thought' Of Pure Consciousness. By Gradually Eliminating The Multitude Of Conflicting Notions That Agitate Him, The Yogi Ultimately Achieves The Certainty (Niscaya) Corresponding To A Direct Awareness Of His Own Divine Nature. Abhinava Explains: Thought Is In Reality None Other Than Pure Consciousness. Even So, It Serves As A Means To Liberation For The Individual Soul (Anu) Only When It Takes The Form Of Certainty (Niscaya). This surprising result--that information capacity depends on surface area—has (e) a natural explanation if the holographic principle (proposed in 1993 by Nobelist Gerard't Hooft of the University of Utrecht in the Netherlands and elaborated by Susskind) is true**.** In the everyday world, a hologram is a special kind of photograph that generates a full three-dimensional image when it is illuminated in the right manner. When it com,es to structures,one cannot bypass Deleuze and his work. Levi Strauss Has Also Noticed That Signs Always Offer An Excess. The System Of Language, 'The Order Of The Known' Exceeds Actual Speech, Even Attempts At Totalization (48). Laws Pre-Exist Actual Cases. [So We Were Getting Close To A Role For Social Life, But Then It Gets Metaphysical Again]. As LS Put It, The Universe Signified Long Before Human Beings Knew What It Was Signifying. By Contrast, The Domination Of Nature Proceeds Partially And Progressively, Step By Step Unlike Social Life Where All Its Goals And Possibilities Given At Once. We're Back With Two Series, This Time Conceived As Rhythms, Social And Natural. Both Technocrats And Dictators Attempt A False Synthesis Of These Two Rhythms. Levi Strauss Referred To 'The Floating Signifier' As A Creative Force And Deleuze Wants To Say It's 'The Promise Of All Revolutions' (49). There Are Also 'Floated Signifieds', Which Seem To Be Possibilities Which Have Not Yet Been Realized. These Can Fill The Gap Between Signifier And Signified [And Are Found In Common Sense Expressions Like 'Gadgets' Or 'Whatnot' – Maybe Connected To The Idea Of A Bricoleur?]. It Implies A Symbolic Content, But Does Not Attempt To Fill It With Specifics. Together, These Possibilities Constitute A Structure, Two Heterogeneous Series, One Signifying, One Signified, Interdependent, And Including Particular Events, Singularities, Emitted By A Differentiator. The Singularities Belong To Neither Series Exclusively And Thus Have No Coherent Identity –Each Is An Excess In One Series And A Lack In The Other. The Singularities Can React Back On The Series, So Structures And Events Are Interdependent [So Structures Need A Dynamic Element, And, D Argues, An Excess, An Empty Square Instead Of Total Systematic Closure. Addresses The Old Issue Of The Static Nature Of Structuralism]. The Signifying Series Contains A Series Of Ideal Events, An Internal History. Differentiators Articulate Series And This Produces A 'Tangled Tale' Overall (51). **Sense Can Be**

**Found In Either Series. It Is Not Just Signification But The Relation That Produces Signifier And Signified**, [The Operation Of The Whole Structure In This Expanded Sense]. Alexander I. Stingl's Blog A Nomadic Scholarship Entity NOTES ON: Deleuze, G (1990) The Logic Of Sense, Trans Mark Lester, Edited By Constantin Boundas, New York: Columbia University Press Attributable And Ascribable To Constraints On Space Accomodation Model Is Provided In Some Paper, Notwithstanding The Generality And Commonalty Of The Observation And Its Concatenatability With The Other Modules. All the information describing the 3-D scene is encoded into the pattern of light and dark areas on the two-dimensional piece of film, ready to be regenerated. The holographic principle contends that an analogue of this visual magic applies to the full physical description of any system occupying a 3-D region It proposes that another physical theory defined only on the 2-D boundary of the region completely describes the 3-D physics. If a 3-D system can be fully described by a physical theory operating solely on its 2-D boundary, one would expect the information content of the system not to exceed that of the description on the boundary. Can we apply the holographic principle to the universe at large? The real universe is a 4-D system It has volume and extends in time. If the physics of our universe is holographic, there would be an alternative set of physical laws, operating on a 3-D boundary of spacetime somewhere, which would be equivalent to our known 4-D physics. We do not yet know of any such 3-D theory that works in that way. Indeed, what surface should we use as the boundary of the universe? One step toward realizing these ideas is to study models that are simpler than our real universe. A class of concrete examples of the holographic principle at work involves so-called anti-de Sitter spacetimes **.** The original de Sitter spacetime is a model universe first obtained by Dutch astronomer Willem de Sitter in 1917 as a solution of Einstein's equations, including the repulsive force known as the cosmological constant. De Sitter's spacetime is empty, expands at an accelerating rate and is very highly symmetrical. In 1997 astronomers studying distant supernova explosions concluded that our universe now expands in an accelerated fashion and will probably become increasingly like a de Sitter spacetime in the future. Now, if the repulsion in Einstein's equations is changed to attraction, de Sitter's solution turns into the anti-de Sitter spacetime, which has equally as much symmetry. More important for the holographic concept, it possesses a boundary, which is located "at infinity" and is a lot like our everyday spacetime. Using anti-de Sitter spacetime, theorists have devised a concrete example of the holographic principle at work: a universe described by superstring theory functioning in an anti-de Sitter spacetime is completely equivalent to a quantum field theory operating on the boundary of that spacetime Thus, the full majesty of superstring theory in an anti-de Sitter universe is painted on the boundary of the universe. Juan Maldacena, then at Harvard University, first conjectured such a relation in 1997 for the 5-D anti-de Sitter case, and it was later confirmed for many situations by Edward Witten of the Institute for Advanced Study in Princeton, N.J., and Steven S. Gubser, Igor R. Klebanov and Alexander M. Polyakov of Princeton University. Examples of this holographic correspondence are now known for spacetimes with a variety of dimensions. This result means that two ostensibly very different theories--not even acting in spaces of the same dimension--are equivalent. Creatures living in one of these universes would be incapable of determining if they inhabited a 5-D universe described by string theory or a 4-D one described by a quantum field theory of point particles. (Of course, the structures of their brains might give them an overwhelming "commonsense" prejudice in favor of one description or another **Holographic Space-Time (Crystal Links, Tom Hanks and Wikipedia Francisco Di Biase** propose a quantum-informational holographic model of brain-consciousness-universe interactions based in the holonomic neural networks of Karl Pribram, in the holographic quantum theory developed by David Bohm, and in the non-locality property of the quantum field described by Hiroomi Umezawa. He considers this model an extension of the interactive dualism of Sir John Eccles, of an interconnection between brain and spirit by means of quantum microsites named dendrons and psychons. **Francisco Di Biase** proposes a dynamic concept of consciousness seen as a holoinformational flux interconnecting the holonomic informational quantum brain dynamics, with the quantum informational holographic nature of the universe. This self-organizing flux is generated by the holographic mode of treatment of neuronal information and can be optimized through practices of deep meditation, prayer, and others states of higher consciousness that underlie the coherence of cerebral waves. In brain mapping studies performed during the occurrence of these harmonic states we can see the spectral array of brain waves highly synchronized and perfectly ordered like a unique harmonic wave, as if all frequencies of all

neurons from all cerebral centers played the same symphony. This highly coherent brain state generates the non-local holographic informational cortical field of consciousness that interconnect the human brain and the holographic cosmos. The comprehension of this holonomic quantum informational nature of brain-consciousness-universe interconnectedness allows us to solve the old mind-matter Cartesian hard problem, unifying science, philosophy, and spiritual traditions in a more transdisciplinary, holistic, integrated paradigm. In this new arrangement cosmovision, consciousness and transpersonal phenomena becomes part of Science and of the very holoinformational nature of the Holographic Conscious Multiverse. **DOI: 10.14704/nq.2009.7.4.259 Quantum-Holographic Informational Consciousness Francisco Di Biase NeuroQuantology** Holography suggests a considerable reduction of degrees of freedom in theories with gravity. However it seems to be difficult to understand how holography could be realized in a closed re-contracting universe. In this Letter we claim that a scenario which achieves that goal will eliminate all spatial degrees of freedom. This would require a different concept of quantum mechanics and would imply an intriguing increase of power for the natural laws. **Physics Letters B Volume 451, Issues 1–2, 1 April 1999, Pages 19–26 Richard Dawid doi:10.1016/S0370-2693(99)00205-1Vijay Balasubramanian, Per Kraus, Albion Lawrence, and Sandip P. Trivedi** describe probes of anti–de Sitter spacetimes in terms of conformal field theories on the AdS boundary. Basic tool is a formula that relates bulk and boundary states—classical bulk field configurations are dual to expectation values of operators on the boundary. At the quantum level they relate the operator expansions of bulk and boundary fields. Using our methods, we discuss the CFT description of local bulk probes including normalizable wave packets, fundamental and D-strings, and D-instantons. Radial motions of probes in the bulk spacetime are related to motions in scale on the boundary, demonstrating a scale-radius duality. They also discuss the implications of these results for the holographic description of black hole horizons in the boundary field theory.DOI: http://dx.doi.org/10.1103/PhysRevD.59.104021 **Holographic probes of anti–de Sitter spacetimes Phys. Rev. D 59, 104021 – Published 26 April 1999 Vijay Balasubramanian, Per Kraus, Albion Lawrence, and Sandip P. Trivedi** The existence of a fundamental scale, a lower bound to any output of a position measurement, seems to be a model-independent feature of quantum gravity. In fact, different approaches to this theory lead to this result. The key ingredients for the appearance of this minimum length are quantum mechanics, special relativity and general relativity. As a consequence, classical notions such as causality or distance between events cannot be expected to be applicable at this scale. They must be replaced by some other, yet unknown, structure. **LUIS J. GARAY, Int. J. Mod. Phys A, 10, 145 (1995) DOI: 10.1142/S0217751X95000085 QUANTUM GRAVITY AND MINIMUM LENGTH** A digital computer is generally believed to be an efficient universal computing device; that is, it is believed able to simulate any physical computing device with an increase in computation time by at most a polynomial factor. This may not be true when quantum mechanics is taken into consideration. This paper considers factoring integers and finding discrete logarithms, two problems which are generally thought to be hard on a classical computer and which have been used as the basis of several proposed cryptosystems. Efficient randomized algorithms are given for these two problems on a hypothetical quantum computer. These algorithms take a number of steps polynomial in the input size, e.g., the number of digits of the integer to be factored. Copyright © 1997 Society for Industrial and Applied Mathematics **SIAM J. Comput., 26(5), 1484–1509. (26 pages) Polynomial-Time Algorithms for Prime Factorization and Discrete Logarithms on a Quantum Computer ISSN (print): 0097-5397 ISSN (online): 1095-7111 Publisher: Society for Industrial and Applied Mathematics Peter W. Shor DOI: 10.1137/S0097539795293172** The classical dynamics of M-dimensional extended objects arising from stationary points of the world volume swept out in space time is discussed from various points of view. A introduction to the Hamiltonian mechanics of bosonic compact M (em) branes is given, emphasing the diversity of the different formulations and gauge choices. For moving hypersurfaces, a graph description—including its nonlinear realization of Lorentz invariance—and hydrodynamic formulations (in light-cone coordinates as well as when choosing the time coordinate of a Lorentz observer as the dependent variable) are presented. A matrix regularization for M = 2 (existing for all topologies) is explained in detail for the 2-sphere, as well as multilinear formulations for M > 2. The recently found dynamical symmetry that exists for all M and related reconstruction algebras are covered, just as some explicit solutions of the level-set equations. **Jens Hoppe 2013 J. Phys A: Math. Theor. 46 023001 doi:10.1088/1751-8113/46/2/023001**

**Relativistic membranes N Bodendorfer et al** rederive the results of our companion paper, for matching space–time and internal signature, by applying in detail the Dirac algorithm to the Palatini action. While the constraint set of the Palatini action contains second class constraints, by an appeal to the method of gauge unfixing, we map the second class system to an equivalent first class system which turns out to be identical to the first class constraint system obtained via the extension of the ADM phase space performed in our companion paper. Central to analysis is again the appropriate treatment of the simplicity constraint. Remarkably, the simplicity constraint invariant extension of the Hamiltonian constraint, that is a necessary step in the gauge unfixing procedure, involves a correction term which is precisely the one found in the companion paper and which makes sure that the Hamiltonian constraint derived from the Palatini Lagrangian coincides with the ADM Hamiltonian constraint when Gauß and simplicity constraints are satisfied. **N Bodendorfer et al** therefore have rederived new connection formulation of general relativity from an independent starting point, thus confirming the consistency of this framework. **N Bodendorfer et al 2013 Class. Quantum Grav 30 045002 doi:10.1088/0264-9381/30/4/045002 new variables for classical and quantum gravity in all dimensions: II. Lagrangian analysis** As K§emaraja points out, none of the practices taught in the Stanzas on Vibration belong to the Individual Means and so it does not, strictly speaking, concern Spanda doctrine, if that is, we consider the Stanzas to be the basic text of the Spanda school. From K§emaraja's point of view, however, the third section of the Aphorisms of Siva (Sivasutra), which is both the last and most extensive, is largely an exposition of this category of practice. The Stanzas and Aphorisms have been traditionally linked together and so, even though we feel that they should be distinguished sofar as the Stanzas rather than the Aphorisms teach the Doctrine of vibration as such, we are nonetheless justified in referring to the Aphorisms as its major source. Our exposition of the Individual Means will therefore be largely based on Ksemaraja's interpretation of the third section of Aphorisms and we will present it, as he does, as an exposition of a possible mystical journey of individualised (anava) consciousness to realisation. We follow K§emaraja because he understood the practise taught in the Aphorisms in these terms, thereby not only illustrating for us how it fits into this scheme but also how he understood the basic categories of practice and their relationship to one another. According to K§emaraja, the first Aphorism of each section of the Sivasutra characterises the condition and nature of the Self at the corresponding three levels of practice. In other words, they indicate the yogi's basic state at each level in terms of his self-identification. This identification corresponds to his existential condition as a degree of self- realisation in the process leading to the authentic self-awareness of the liberated. The very first Aphorism starts directly with this, the highest state, by declaring that the Self is pure, dynamic and universal consciousness (caitanya).This is true for the yogi who has awakened to his authentic nature at the Divine (sambhava) level of being. At the Individual (anava) level, however, the situation has changed. In this sphere of consciousness the intermediate processes of discernment, analysis and classification of perceptions, which bridge the gap in the flow of awareness from the universal subject to a specific object of knowledge, appear to take over the status of the perceiving subjectivity which underlies them. The universal Self recedes into the background as a pure, undefinable awareness, and the individual ego, consisting of the perceptions, thoughts and emotions generated by the contact between the universal perceiver and the perceived, emerges in the juncture between them. Thus at this level, as the Aphorisms say, the Self is the mind. This is the Self which moves (atati) from one state of being to another, from one body to the next carrying with it subtle traces left behind by its sensory and mental activity. Together these are said to constitute, and be caused by, the subtle body technically called the 'City of Eight' (puryasfaka) with which consciousness is identified and due to which it is subject to the constant alterations of pleasure, pain and inertia. The Stanzas teach: [The soul] is bound by the City of Eight (puryaffaka) that resides in the mind, intellect and ego and consists of the arising of the [five] subtle elements [of sensory perception]. He helplessly suffers worldly pleasure and pain (bhoga) which consists of the arising of mental representations born of that [City of Eight] and so its existence subjects him to transmigration. **Whereas consciousness itself is the subject who practises the Divine Means (Sambhavopayd), the subject who practises the Individual Means is the mind. Unlike the Empowered Means, however, the mind is not directed inwards onto itself. At the Empowered level, enlivened by the direct intuition (pratibha) consciousness has of its own nature, mind ceases to function merely in the paradigmatic, formative manner which gives rise to mental representations, but operates instead as the subtle introverted**

**activity of reflective awareness (vimarsa), the power of consciousness (sakti).** This activity, as we have seen, is the essence of Mantra which, independent of the senses, is no longer restricted in any way. At the Individual lew, however, the creative powers of consciousness reflected through the eAtroverted mind are greatly attenuated. All that remains is the power to form thought-constructs and make determined resolutions (sankalpa) which go on to issue through the body into outer action to make the private creations of the mind apparent to others. The Individual Means, therefore, deals with the objectively perceived contents of consciousness and hence with the individual subject as a composite aggregate of objective elements, ranging from the subtle life force (prdrta) to the physical body and its outer environment. The practices belonging to this Means are thus of two types. One is concerned with the individual subject who resides in, and as, the psycho-physical organism; the other with external reality. What this implies essentially is that practice at this level is not concerned as much with the will or cognitive consciousness **(refer hiedigger who proposes the same philosophy: italics mine**) as are the other two Means, but with the power of action applied, in the context of the practice taught in the Aphorisms, to the spiritual activity of Yoga. According to Ksemaraja, the Individual Means culminates in the Empowered state and hence leads to the levels of practice beyond it. 251 This is possible because, despite their differences, there is an essential similarity between them. The aim of both the Individual and Empowered Means is to purify the discursive representa- tions of differentiated perceptions (vikalpasamskara) 252 and so lead the yogi to the expanded (yikasita) consciousness of the Divine (sambhava) state. The other levels of practice therefore both sustain and complement it. The activity of individual consciousness can be fully perfected only when it operates through the flow of the conative and cognitive powers which together constitute the pure activity of universal consciousness beyond all means (anupaya). In fact, according to Ksemaraja, all three soteriological types function together in various ways, their corresponding states representing dimensions of the same experience. For example, the upsurge of consciousness (udyama) which is the supreme, **illuminating intuition (parapralibha)** of the Divine state (sambhavavastha) is concomitant with the gathering together of all the powers of consciousness in the Empowered state. The Divine Means, in other words, leads to the experience of Power (sakti) which in its turn, when fully affirmed, marks the attainment of a permanent contemplative consciousness (turiyatita) at the Divine level which persists unaltered in every state of consciousness. Consequently, Ksemaraja concludes his exposition of the first section of the Aphorisms which exemplifies, according to him, the Divine Means, by saying: Thus we have explained the first expansion which starts with [the Aphorism] 'the Self is pure dynamic consciousness' **(note this happens in evolutionary pocess and in normal human being it remains as witness consciousness italics mine)** and expounds the nature of the realisation (prathana) attained through the Divine Means. It is the intuitive insight (samapatti) of Bhairava's nature which is, as we have said, the upsurge of consciousness that quells all bondage, namely, the ignorance of that freedom which makes it manifest. Transforming all things into the nectar of one's own innate bliss, it bestows every yogic accomplishment (siddhi) including mystic absorption in the vitality of Mantra, the highest of them all. Accordingly, we have, in the course of this exposition, explained the nature of Sakti in order to show that the Divine nature (Sambhavarupa) possesses [every] power. Another way in which the Means are related to one another is illustrated by the recurrence of the same Aphorism in different sections of the Sivasutra which indicates, according to Ksemaraja, that the same practice belongs to more than one Means. Both times this happens, the Aphorism appears first in the section dealing with the Divine Means and then recurs in that concerned with the Individual Means. In one case, Ksemaraja tells us this is because practice at the Divine level requires no effort whereas at the Individual level, the yogi must exert himself to achieve the same state that at the Divine level dawns spontaneously. At the Empowered level also, as the Sivasutra says, 'effort achieves the goal'. Here, however, because as the Empowered Means is, according to Ksemaraja, predominantly concerned with the contemplation (anusamdhi) of the vitality of Mantra, the effort exerted is that required to bring the practice of Mantra to fulfillment. It is, as Ksemaraja says, 'the spontaneous effort exerted to grasp the initial expansion of intention to apply oneself to the contemplation [of Mantra]. It is this exertion which wins the favour of the gods of Mantra and identifies the adept with them.' The second case of the same practice being taught in different sections of the Aphorisms concerns the realisation of the Fourth State of contemplative consciousness (turiya) in the other three states of waking, dreaming and deep sleep. At the Divine level this takes place by **'violently digesting' (hafhapaka)** the three states in the Fourth. At the

Individual level the Fourth state is first experienced at the junctures between the other three states and then induced gradually to spread out from these Centres to pervade the other states like oil extending slowly through a piece of cloth. The difference in this case between the levels of practice is not only that at the Divine level it reaches fulfillment spontaneously, but it is also sudden and complete, leading directly to the liberated state of consciousness Beyond the Fourth (turiyatita) At the Individual level, however, practice is gradual and even when the yogi manages to rise to states of contemplation, he must take care not to fall to lower levels of consciousness. Indeed, until the yogi attains the sudden and direct realisation of perfect enlightenment, whatever be his state of consciousness or level of practice, he is bound to rise and fall because his contemplative state is necessarily transitory (kadacitka) however long it may last. The yogi is more prone to these ups and downs the lower his basic state of consciousness. Consequently, the last section of the Sivasutra repeatedly instructs the yogi not only how to rise to higher levels of consciousness and maintain them, but also in what way he is liable to fall from them and how to regain them. Ksemaraja stresses that the rise from one level of consciousness to another is marked by the transition from a lower Means to a higher. Conversely, a fall from the higher level to the lower entails practice of a lower Means. The measure of the yogi's level of consciousness and that which sustains him in it allowing him to progress further, is his attentiveness (avadhana) to the higher realities he experiences in the more elevated states. Thus the last Aphorism of the second section of the Sivasutra warns the yogi that if his pure awareness (suddhavidya) of his oneness with all things slackens, he will fall from his awakened state to dream the dream of thought-constructs. From Ksemaraja's point of view this means that the negligent yogi must now resort to the Individual Means described in the next section to return to his former, higher Empowered practice in which he experiences this oneness. Ksemaraja expounds practice at the Individual level, as he sees it in the Aphorisms, as extending from one Means to the next. For example, practice at the Individual level diverts the flow of the vital breath (prana) from its more usual course and induces it to enter the Central Channel (susumna) along which it rises as a pure conscious energy (technically called 'kuritfalini'). This leads the yogi to the Empowered state in which he enjoys the pure awareness of unity. If he manages to make it truly his own and it becomes his basic state of being, he enters the Divine plane (sambhavapadd) of identity with Siva. The Individual Means is both a point of departure to higher levels of practice and the level to which the yogi returns if he falls. Thus although the practices taught in the last section of the Aphorisms may belong to any one of the three Means, they are collectively treated as part of the Individual Means because they start from it and because it is the yogi's abiding standby if he falls. Let us turn now to the basic practice at the Individual level, as Ksemaraja understands it. This is essentially Yoga. According to the Classical Yoga system taught by Patanjali in the Aphorisms of Yoga (Yogasutra), Yoga is defined as 'the quelling of the fluctuations of the mind' (cittavrttinirodha). The aim is to sever the spiritual essence of the Person (purusa) from the defiling materiality of Nature (prakfti), even though the word 'Yoga' means to 'unite' or 'yoke together.' Here, however, Yoga combines both union and cessation. It is the act (kriya) of removing the latent traces (yasana) of differentiated perceptions (yikalpa) born of the impurities (mala) which contract consciousness **(like pravrutti and Nivrutti; italcis mine)**. This is achieved by uniting all the elements of experience (tattva) together in the wholeness of the activity of consciousness. As Jayaratha explains: The [wise] consider Yoga to be the union of one thing with another/ thus, in accord with this dictum, Yoga is the [act] of uniting [all] the metaphysical principles together within consciousness . . . , Ksemaraja seeks initially to establish the best form of Yoga for the yogi to practice at the Individual level. His sources are two Tantras he knew well and considered to be amongst the most important, namely, The Tantra of (Siva's Third) Eye (Netratantra) and The Tantra of the Liberated Bhairava (Svacchandabhairavatantra). The basic model is that of the Eight-limbed Yoga (asfariga) taught by Patanjali which consists of: 1) The five restraints (yama\ namely, abstention from violence (ahirrisa), falsehood (satya), dishonesty (asteya), sexual intercourse (brahmacarya) and desire for more than the essential (aparigrahd) 2) The five disciplines (niyama), namely, cleanliness (sauca), contentment (santo$a), austerity (tapas\ study (svadhyaya) and reverence for God (Uvarapranidhand) 3) Posturing of the body (asana) in a manner conducive to the practice of meditation and physical health. 4) Regulation of the breath (pranayama) 5) Withdrawal of the senses from their objects (pratyahara) 6) Focusing of attention (dharana) 7) Meditation (dhyana), that is, steady, uninterrupted concentration. 8) Contemplation (samadhi). Ksemaraja **rejects** Patanjali's system because he believes it to be a form of Yoga that

can, at best, lead only to limited yogic attainments (mitasiddhi). In the Netratantra, however, Siva teaches a different, higher form of the Eight Limbs of Yoga which lead to perfect penetration into the supreme, transcendental principle 280 of which the Netratantra says: Speech cannot express, nor the eye see, the ears hear, or the nose smell, the tongue taste, the skin touch or the mind conceive that which is eternal. Free of all colour and flavour, endowed with all colours and flavours, it is beyond the senses and cannot be objectively perceived. O goddess, those yogis who attain it become immortal gods! By great practice and supreme dispassion . . . one attains Siva, the supreme imperishable, eternal and unchanging reality. A necessary preliminary of all Tantric Yoga is a process technically called the 'purification of the elements' (bhutasuddhi), through which the body is homologized with the macrocosm and so made a fit vessel for the pure, conscious presence of the Deity within it. Ksemaraja equates this with the meditation (dhyana) which, according to the Mdlinivijayatantra, characterises the Individual Means. In order to practice this meditation the yogi must visualise the dissolving away of all the forces in the body. There are two ways in which this can be done. The first is called 'the contemplation of dissolution' (layabhdvana). Through it the progressive differentiation of consciousness from its causal, pre-cosmic form to its phenomenal manifestation is reversed As the Vijnanabhairava teaches: "One should meditate on the All in the form of the Paths of the world-orders etc. considered in their gross, subtle and supreme forms until, at the end, the mind dissolves away." Mediated by consciousness, the macrocosm rests in the microcosm which is emitted along with it successively in the emptiness of the individual subject, vital breaths, mind, psychic nerves (ndtfi), senses and external body. The yogi reproduces this process by visualising the totality of reality including the world-systems, metaphysical principles and cosmic forces along with the Mantras, letters and syllables which represent them, as arising successively throughout the psycho-physical body so as to constitute it. Deployed in this way they form the Cosmic Path along which the yogi ascends, absorbing as he does so, the lower elements into the higher, thus strengthening and extending his unifying awareness (anusamdhdnd) of the configuration of the Path. Thus, moving from the gross elements constituting the outer physical body, to pure sensations (tanmdtra), then to the senses and mind back to their primordial source, the yogi rises from the **embodied subjectivity** of the waking state to the Fourth State (turiya) of contemplation where he is one with the pervasive intent which initiates the creative vision of consciousness. Abhinava writes: Once [the yogi] has known [this] Path in its completeness, he must then dissolve it into the deities who sustain it and these successively into the body, breath, mind [and emptiness] as before, and all these into his own consciousness. Once this is full and an object of constant worship, it destroys, like the fire at the end of time, the ocean of transmigration.Thus, the second method Ksemaraja teaches to dissolve away the diversity of sensory, mental and physical energies into the unity of consciousness is a meditation on the Fire of Consciousness (dahacintd) which the yogi visualises as burning away all division. At the Divine level (sambhavopaya) the yogi witnesses the sudden and violent withdrawal of all objectivity into the pure ego (aharri), like the pouring of fuel into a raging fire. He does not need to visualise this process but merely attend to it with a passive, receptive attitude. At the Individual level the yogi must exert his imagination to induce this process and so rise to the Divine level through the Empowered. The Vijnanabhairava teaches: Visualise the fortress [of your body] burning with the Fire of Time (kalagni) risen from the Abode of Time; then at the end peace manifests. The Fire of Time {kalagni) resides underneath the hell worlds at the bottom of the Cosmic Egg (Brahman^a). It issues from Ananta — a form of Siva who presides over the lower regions. He floats on a boat in the causal waters supporting the Egg, his mind all the while fixed on Bhairava. The flames of the Fire of Time rise up to the hell-worlds heating them intensely and radiate its energy throughout the universe. At the end of each period of creation the flames rise higher and destroy the old cosmic order to make room for a new one. At the microcosmic level the yogi reproduces this process by mentally placing the letters of the alphabet, in the prescribed order, on the limbs of his body starting from the left toe to the top of the head. As his attention progresses upwards, he visualises the Fire of Time moving with it in such a way that his bodily consciousness, together with the universe of differentiated perceptions, is gradually burnt away leaving in its place the white ashes of the undivided light of consciousness. Ksemaraja considers this meditation (dhyana) to be a limb of a programme of yogic practice at the Individual level 291 of which the remaining limbs are as follows: Posture (A sana). The yogi fixes his attention on the centre between the inhaled and exhaled breath, absorbing in this way the flux of his awareness into the unfolding power of knowledge which rises initially as the

upward flowing breath (udanaprana) in the Central Channel (su§umna) between the other two breaths. The Prank aspect of this flow disappears as it moves upward and the yogi experiences the spontaneous rise of the omniscience of consciousness within himself. The mind reverts back to its original, pervasive conscious nature and understands the infinite fact of Siva's omnipresence. This is the firm seat (asana) upon which the yogi sits to practice. Regulation of the Breath (Pranayama) .To regulate the movement of the breath, the yogi must first cleanse the right and left channels of the ascending and descending breath by blocking the left nostril while exhaling and the right while inhaling a few times. This ensures that the movement of the breath is firm and evenly distributed. Next, without attempting to control it in any way, he attends to the flow of his breathing. As the mind becomes steadier and in closer harmony with the rhythm of its movement, the duration of each inhalation and exhalation gradually alters until they become equal. At this stage they unite and merge in the upward flowing current of vitality in the Central Channel (Susumna). This is when true Pranayama begins. The yogi's mind pure and tranquil, he returns, as it were, to a prenatal state and the external breathing cycle is internalized, so that it no longer moves through the lungs but passes directly to the universal source of vitality. The yogi, now at the Empowered level of practice, experiences this movement as travelling from the Heart centre upwards to a point distant twelve fingers above the head where it merges in the void of consciousness. Free of its outer gross form, the breath moves freely through the Central Channel and soon transcends even this subtle movement to become one with the supreme vibration of consciousness. In this way, the yogi's breathing becomes one with the spontaneous rise and fall of energy from the bosom of the absolute. Abhinava quotes the Tantra of the Line of Heroes (Viravalitantra) as saying: When, by constantly merging the mind in Siva, Who is the pure conscious nature, the Sun and Moon [of the two breaths] have dissolved away and the Sun of Life, which is one's own consciousness, has reached the twelve-finger space, this is termed liberation. Breath control [at this stage] serves no useful purpose. Breath control which merely inflicts pain on the body is not to be practised. He who knows this secret is both themselves liberated and liberates others. **Focusing of Attention (Dhara nA):** Attention is fixed on the psychic centres in the body corresponding to the five gross elements In this way the vital breath is successively directed to these centres from the Heart of consciousness to refresh and stimulate their activity. First it moves to the Earth centre in the throat which regulates the firmness of the bones and flesh of the body; then to the Water centre in the glottis responsible for the balance of the bodily fluids. After this it travels to the navel which is the Fire centre dealing with digestion and anabolisms and catabolesis in general. It then moves to the Wind centre in the toe of the left foot which governs the movement of gases to and from the cells via the circulatory system. When the yogi has thus achieved control over these forces, the breath rises from the Heart to the top of the head and he becomes master of the Ether element and so attains every yogic power. **Meditation (Dhyana)** The highest form of meditation stills the flux of the qualities (guna) and induces the mind into a state of contemplative absorption. The object of this meditation is the supreme and pervasive divinity of the pure subject whose true nature is known to none but himself alone (svasatpvedya). The yogi attains him by merging into the constant flow of awareness that streams into the Light which illumines his own nature. Contemplation (Samadhi) .The yogi rises to the level of contemplation when the awareness he has of himself and the things around him become one and he realises his own identity with Siva, **(or Rama the omniscient: cosmic general ledger)** the sole reality. The aim of this Yoga in all its phases is to achieve the Fourth State of consciousness (turiya) beyond the three states of waking, dreaming and deep sleep and to then ultimately reach the liberated state Beyond the Fourth (turiyatita). These five states correspond to: (a) Siva's activity (vy apara), that is, His power of action; (b) Siva's Lordship (adhipatya), which is His power of knowledge; (c) the absence of these two, which corresponds to Siva's power of will; (d) His exertion (prerakatva), which contains all the cycles of creation and destruction and, (e) the rest Siva enjoys in His own nature, which is His power of consciousness. The first three states, when divorced from the last two, belong to the sphere of transmigratory existence. The Fourth and Beyond the Fourth on the other hand are higher, supramundane (alaukika) states of consciousness in which the yogi enjoys bliss and repose (visranti) in his own nature by penetrating (samavesa) into the universal consciousness of the Self, through which he ultimately becomes liberated (jivanmukta). Beyond the Fourth is the state of awareness ParamaSiva Himself enjoys when duality has entirely disappeared and everything is realised to be one with consciousness. The Fourth is the state of awareness of the yogi who, catching hold of the pure subjectivity (upalabdhrta) flowing through the lower three

states, is still actively eliminating his sense of duality. While the former is the supreme subject as T consciousness (aham), the latter is the pure awareness (prama) or I-nesses (ahanta) of the subject which encompasses the lower states, giving them life and uniting them together As such, the Fourth State is the reflective awareness of one's own nature shining in all three states at one with them. The fact that we recall that we slept well is proof that this state of consciousness persists even in deep sleep. Indeed, if the flow of Turiya could somehow be brought to a halt, all the other states of consciousness would come to an end in the absence of the pure subjectivity which makes them, and their contents, manifest. The states of waking, dreaming and deep sleep correspond to the form of awareness consciousness assumes when it predominantly manifest as the object, means of knowledge and individual subject, respectively. Turiya is the pure awareness (prama) that both transcends them and merges them all into itself. 300 As such, it appears as the triad of deed, means and agent in the pure act (vy apara) of consciousness unsullied by any outer reality. Abhinava explains: {Turiya) transcends the three aspects of *form\ 'sight' and T consisting as it does of the pure act of 'seeing'; therefore any means [by which this state could be realised] has merely a [provisional] instrumental value. It is, in other words, pure subjectivity of the nature of absolute freedom, independent of all external means. This is the state of consciousness called Turiya, luminous with its own light. Turiya is thus not just a psychological state but the supreme creative power (para sakti) of consciousness, the Goddess (samviddevi) who generates and withdraws the entire universe of subject, object and means of knowledge. In the Heart of Recognition Ksemaraja explains: Whenever the extroverted [conscious] nature rests within itself, external objectivity is withdrawn [and consciousness] is established in the inner abode of peace which threads through the flux of awareness in every [externally] emanated [state]. Thus Turiya, the Goddess of Consciousness, is the union of creation, persistence and destruction. **(Read as cosmic consciousness; Kshemaraja does not distinguish like we have done, individual, collective and cosmic and of three states)** She emanates every individual [cycle] of creation and withdraws it. Eternally full [of all things] and [yet] void [of diversity] She is both and yet neither, shining radiantly as non-successive [consciousness] alone. The yogi is fully absorbed in this state of consciousness and takes possession of its power when he is able to rise from contemplation (samadhi) carrying with him the abiding awareness of Turiya throughout his waking, dreaming and deep sleep. When he achieves this constantly, he continues to experience these states individually, but they no longer obscure the insight (pratibha) he has acquired because he realises that they are all aspects of the bliss of Turiya. Thus, while the common man calls this state the 'Fourth' (turiya) because he cannot experience it directly and knows only that it is beyond the other three, the yogi calls it 'Beyond Form' (rupatita) because it transcends the detachment of the state of deep sleep which, devoid of objective content, is the naked form' of the individual subject tending towards the fullness of consciousness. Those who are on the path of knowledge (jnaniri) call it the 'Whole* (pracaya) because, in this state, they see the entire universe gathered together in one place. 'Supra-mental Awareness' (manonmana) is the name given to the experience of Turiya in the waking state. The yogi in this state moves and lives in the world of waking experience free of all disturbing thoughts while abiding in the transcendental silence beyond the activities of the mind. 'Infinite' is the name of the experience of Turiya while dreaming because, free of the limitations imposed upon the body by time and space, the yogi enjoys the unlimited expanse of the Self. When Turiya is experienced in deep sleep, the yogi's state is called 'All things' (sarvdrtha) because in it he discovers his freedom from limitations in this, the most contracted state of human consciousness. The yogi who manages to maintain Turiya- consciousness comes to experience the three states of waking, dreaming and deep sleep as the constant flow of the bliss of consciousness in which all traces of the relative distinction between these states and their contents is eradicated. Following the stream of Turiya to its highest level (para katfha), he reaches the state Beyond the Fourth (turiydtita), which is the universal consciousness (caitanya) of the Self. Here the yogi comes to rest within his own nature. Plunged in the vast, waveless ocean of the consciousness and bliss (ciddnanda) of the state Beyond the Fourth, the yogi becomes Siva, the Free One (svacchanda), and thus wanders freely, practising the Yoga of Freedom. K§emaraja equates the Fourth State with the pure (suddha), innate (sahaja) knowledge that one's own conscious nature is all things. It is the Supra-mental State (unmana) in which Siva's pervasive presence is experienced once the Yoga practised at the Individual level attains fruition at the Empowered. What the yogi must do, once consciousness is elevated to grasp the Fourth State, is make it constant. He must forcefully lay hold of it within himself and not release his grip until it becomes

permanent. Then he travels 'Beyond the Fourth' to enlighten- ment. Before this ultimate attainment the yogi inevitably falls. The forces operating within consciousness that limit and obscure it throw him down whenever they possibly can. The only way the yogi can defend himself against them is to maintain a constant attentive awareness of the Fourth State. He falls when he is distracted but when he attends carefully to his pure conscious nature, he realises that every aspect of his state of being, including the forces that lead him astray, are one with the pulsing flux of his own consciousness and so cannot affect him. These powers, which are the energies of Matrka we have already discussed, are not the only obstacles the yogi must overcome. He must, for example, also resist the temptation to rest content with the miraculous yogic powers (siddhi) he acquires in the course of his spiritual develop- ment. Again to do this he must practice Yoga. Similarly, in order to pervade the Fourth State gradually through the other states in the manner proper to practice at the Individual level, the method is the same. He must practise the higher yoga of the Tantras which, turning his mind inwards and freeing it from discursive representations, allows him to penetrate into the Supreme Principle. Once the yogi has attained this contemplative state, his main problem is to make it permanent. In the introvered state the gross external movement of the breath is suspended and with it the activity of the intellect, mind, individualised consciousness, powers of the senses and the ego. 313 When the yogi rises out of this state, he is liable to fall again into the lower order of creation generated by Maya if he does not maintain his awareness of the higher reality he has experienced and allows his awakened, illumined insight to be obscured by the dream-like vision of thought-constructs. 314 Naturally, the yogi must rise out of the introverted condition of suspension. It is inherent in the very nature of reality that it should move out of (e) itself. 315 Pure, universal consciousness initially transforms itself into (e&eb) the vital breath 316 charged with (e&eb) the impression iyasana) of the power of awareness attained through (e&eb) introversion. By attending to the pulse (spanda) of (e) the breath as it moves out of (e) the absolute, the yogi can develop (eb) an intuitive sense of the inherent unity of (e) all he will perceive in (eb) the mental and physical spheres created by (e) the outpouring of consciousness. In this way he realises (e) that his own nature is everywhere present in (eb) all he perceives and that all things thus reside within (eb) him. Blessed with this insight his consciousness remains(eb) free and unlimited even at the individual level where the breath, mind, senses and body are active. If the yogi fails to do this,(eb) he finds himself once again beset by the strictures of his embodied existence and must, as before, try to pervade all his other states of consciousness with (e&eb) the aesthetic delight (rasa) and wonder (camatkara) of the Fourth State he experienced in (eb)contemplation. Again this means that he must strengthen (e&eb) his pure, empowered awareness that his universal nature manifests (eb) as all things. In this way he discovers Siva's presence in (eb) every sphere of individualised consciousness ranging from the breath to (e&eb) outer objectivity. The yogi's mind then becomes (=) tranquil and undistracted because (e) wherever it may wander, the yogi perceives (e) only Siva, his authentic nature. Consciousness is thus freed of (e) all external referents and the yogi's subjectivity is purified of (e)all identification with the body or anything else that belongs to(e) the objective sphere. The yogi then becomes detached from the opposites of pleasure and pain and is transcendentally free (kevalin). The yogi is again, however, liable to fall if (e) he allows himself to get entangled in (e&eb) the play of opposites. This fall is more serious than the others because, although he is caught by the confining restrictions of individual- ised consciousness as before, he is now also affected by (e&eb) karma. Fleeing from pain in the pursuit of (e) pleasure he is bound to (e) act (karma) to minimise one and maximise the ( a **maximaization problem is mentioned elsewhere about the svabhava of human beings to enhance pleasure reduce deprivation)** other and so is thrown down to (e&eb) the lowest level of embodied subjectivity (sakala). In order to regain his lost state, he must ascend gradually, by Siva's grace, from one order of subjectivity to the next and so free himself progressively of the limitations of the lower levels to gain the greater freedom and expansion of the higher. As he progresses, the objective sphere also evolves from the grossest perceptions of physical objects outside the lowest order to subjectivity, through to the subtler inner, mental perceptions to finally reach the order of subjectivity that contains objectivity within itself and is free to externalise it at will. The degree to which this process develops depends, as before, on the yogi's awareness of the Fourth State. In consonance with the general principle that the remedy should suit the defect, the yogi is instructed to seek this higher state of consciousness in the wonder (camatkara) or delight (ananda) he feels in moments of intense physical pleasure. At first he experiences this subtle consciousness for an instant in the subjective sphere. If he manages to

catch hold of it, it becomes more intense as the cognitive and objective spheres are also gradually pervaded and vitalised by it. Occasions for this practice are, for example, the sense of satisfaction one feels after a good meal or the aesthetic delight one experiences when listening to good music or the pleasure of sexual union with the Tantric consort or even solitary sexual excitation. In these moments of delight the yogi can penetrate momentarily into his own authentic Siva-nature (sambhavavesa) through the empowered contact (saktasparsa) m he makes with it in the freedom of the pure subjectivity of the Fourth State. If the yogi develops his awareness of this higher level of consciousness and maintains it, he eventually experiences it constantly. Clearly, what prevents the yogi from attending to his state of consciousness rather than the circumstances which induce it is the craving for pleasure (abhila§a) born of ignorance — the source of every impurity which clouds consciousness. Craving directs the yogi's attention towards outer, worldly things and so he is caught in the **net of thought- constructs**. To free himself of his worldly desires and reverse this binding extroversion, the yogi must eradicate its cause. To be freed of all the ups and downs of the path and no longer be tormented by the possibility of a fall, the yogi must see reality perfectly and completely. This insight is itself liberation and the moment it dawns the yogi is instantly freed. This sudden realisation is the goal of Tantric Yoga. Accordingly the Tantra declares: "He, who perceives reality directly, even for the brief moment it takes to blink, is liberated that very instant and never reborn again." Although the yogi's body and mind continue to function as before, they are like mere outer coverings which contain, but do not obscure, the mighty, universal consciousness which operates through them. The yogi's body is the universe, the senses the energies that vitalise it, his mind Mantra, the rhythm of his breath the pulse of time and his inner nature pure, dynamic consciousness. Raised above all practice, and hence all possibility of falling to lower levels, the yogi realises that he has always been free and that his journey through the dark land of Maya was nothing but a dream, a construct of his own imagination. **(Models are given in one of the papers of the series due to spatial constraints) Relationship betwixt string theory (note the spade remarks above) and holographic principle has been enucleated and expatiated in literature**. Some exemplar work is sententiously mentioned in the following:The ratio of shear viscosity to volume density of entropy can be used to characterize how close a given fluid is to being perfect. Using string theory methods, we show that this ratio is equal to a universal value of $\hbar/4\pi$kB for a large class of strongly interacting quantum field theories whose dual description involves black holes in anti–de Sitter space. **P K. Kovtun, D. T. Son, and A. O. Starinets** provide evidence that this value may serve as a lower bound for a wide class of systems, thus suggesting that black hole horizons are dual to the most ideal fluid **Viscosity in Strongly Interacting Quantum Field Theories from Black Hole Physics Phys. Rev. Lett. 94, 111601 – Published 22 March 2005 P K. Kovtun, D. T. Son, and A. O. Starinets** According to't Hooft the combination of quantum mechanics and gravity requires the three-dimensional world to be an image of data that can be stored on a two-dimensional projection much like a holographic image. The two-dimensional description only requires one discrete degree of freedom per Planck area and yet it is rich enough to describe all three-dimensional phenomena. After outlining't Hooft's proposal we give a preliminary informal description of how it may be implemented. One finds a basic requirement that particles must grow in size as their momenta are increased far above the Planck scale. The consequences for high-energy particle collisions are described. The phenomenon of particle growth with momentum was previously discussed in the context of string theory and was related to information spreading near black hole horizons. The considerations of this paper indicate that the effect is much more rapid at all but the earliest times. In fact the rate of spreading is found to saturate the bound from causality. Perception is not reality. It needs some hidden variables to posit the reality. (see earlier models).We Make Sense Of Objects, Events Or State Of Affairs When We Pin Them Down And Describe Them, State Propositions About Them. We Can Then Develop These Propositions Into Logical Forms To Draw Conclusions, Make Predictions Or Whatever. The Trouble Is, Neither Object As We See Them, Nor Propositions As We Commonly Use Them, Are At All Simple. Objects As We Perceive Them, For Example, Clearly Have All Sorts Of Characteristics As Well That We Don't Perceive. They Have A History, And A Potential For The Future, For Example. The Worst Problem Is That Objects As We See Them Are Actually Mixtures Of Things, The Affects Of Complex Causes: They Are 'Overdetermined', To Use The Phrase Associated With Both Freud And Marx. Deleuze Actually Draws On Resources In Classical Philosophy To Argue This, Especially The Work Of The Stoics, Who Saw Objects And Events As A 'Mixture Of Bodies', This

Mixture Emanating From Somewhere In The Depths, Below Conscious Sense Making. Incidentally, This Reliance On The Stoics Also Makes Deleuze A Bit Stoical, In The Sense That He Thinks That These Mixed Bodies Are Part Of Some Unified Whole (He Was Going To Call This Being), And The Role Of Destiny Is Associated With The Whole. It Follows That Ethically And Conceptually, All Human Beings Can Do This Discover The Ways In Which Being Works. There's No Point Complaining About It Either. This Is the Basis of Badiou's Critique Of Deleuze. It Is Also The Basis Of Deleuze's Anti-Humanism: We Don't Understand Reality By Trying To Develop A Conscious Synthesis, Tracking Analogies Based On Some Notion Of A Shared Essence, Or Seeing God As An Ultimate Synthesiser For That Matter. Events And States Of Affairs Are Also More Complex Than They Look, And Deleuze Illustrates This Discussion With The Extraordinary Notion Of 'Compossibility' In Leibniz. Basically, The Idea Is That Singularities Can Produce Events In All Sorts Of Divergent Ways, All Of Them Equally Possible, And This Potential Is Shrunk If We Just Study The Actual Course Of Events. Objects Are More Complex Than They Look, With All Sorts Of Bits Of Their Being Unavailable To Us, But Language Is Equally Complex. Here, Deleuze Tries To Demonstrate This By Looking At People Who Use All Kinds Of Linguistic Anomalies Such As Paradoxes, Or, In The Case Of Lewis Carroll Especially, Portmanteau Words Or Esoteric Words. Although These Constructions Offend Logic, And Are Strictly Speaking Nonsense, They Still Make Sense. This Also Points To Some Powers Of Language That Are Not Immediately Available To Inspection. More Conventionally, We Know That The Use Of Particular Words Can Have Different Functions In Any Language System—They Can Denote, They Can Manifest Inner Thoughts, And They Can Signify (In The French Sense, That Is Indicate The Existence Of A Structured Language System Which Enables Us To Communicate With Each Other. Deleuze Proposes To Modify Classical Saussurian Semiotics By Adding A Moveable Element, An 'Empty Space' Or Floating Signifier'). Again Wordplay Can Deliberately Confuse These Different Functions, As When I Use The Term 'It' To Denote A Specific Object, Only To Refer To A Whole Process As 'It', To Express Myself By Saying Something About 'It' And So On (Actual Easy Examples Are Thin On The Ground In Deleuze!). Then There Is One Of Those (Many) Philosophical Diversions Into Various Types Of Explanation For The Extra Elements That Are Not Available To Actual Objects Or Speech Acts. Platonists Thought The Extra Bits Referred To Some Universal Form Or Idea, Other Philosophers Like Husserl Or Kant Suggest There Was Some Transcendental Realm Beyond The Immediate. Deleuze Offers Objections To Both For Those Approaches, And Suggests That The Real Is Actually Divided Into Virtual And Actual Levels (Delanda's Commentary Is Invaluable Here). The Objects And Words Of Our Immediate Perception Are Actualised Are Condensed Out From Much More Complex Objects And Events At The Virtual Level. In This Book, About The Only Candidate For These Complex Objects Is The Singularity, Which Actualizes Itself In Various Partial Ways. Deleuze Goes On To Develop This Idea By Saying That Singularities Themselves, And The Events And Objects They Actualize, Are Produced In A Random Or Arbitrary Fashion, The Chance Is At The Heart Of Actualisation. We Learn That The Virtual Also Has Its Own Sense Of Time, Which Is Not The Usual One Which Is Highly Limited By The Insistence On The Present Tense In Human Operations. It Is A Very Abstract Discussion, Which Finally Comes To An Example I Could Understand—The Emergence Of Language In Freud. As Infants Develop, Their Biological Urges Produce Certain Infantile Concepts Like Part Objects. These Are Classically Mixed, Deriving From Contradictory Drives In The Depths Of The Unconscious. The Sounds The Infant Makes Our Initially Just Bodily Emanations, But They Gradually Come To Take On The Form Of Linguistic Units, Like Phonemes. There Are Elements Of Adult Language Available Too, Of Course, But These Are Initially Unintelligible. Then A Process Described As The Phantasm Manages To Combine Various Infantile Concepts Together Into A'disjunctive Unity', A Mixture Of Heterogeneous Elements Which Deleuze Thinks Characterises Most Objects. A Kind Of Primitive Narrative Develops In The Phantasm, Partly Driven By Biological Drives, And Partly Driven By Emerging Linguistic Competencies (Such As The Oedipal Scene). The Key To This Is The Freudian Notion Of The Phallus, Which Both Refers To Biological Organs Like Penises, And To Linguistic And Cultural Functions To Do With Authority And Value. The Phallus Is The Ideal Ambiguous Object (Or Empty Object As Deleuze Insists On Calling It), Able To Zigzag Between Bodies And Language, And Thus Make Divergent Series 'Resonate'. The Phantasm Therefore Develops An Energy Of Its Own, Which Permits A Relative Disengagement From Sexual Energy (Sublimation) And, In The Final Stages, The Proper Development Of

Language In The Form Of Symbolisation. At Last We Have Language And Events Brought Together On The Surface, Which Is Of The Level Of Consciousness. If I Have Read The Appendices Correctly, This Phantasmic Form Is Then Generalised To Include All The Operations Of Making Sense As Describes Right At The Beginning. I Have Lots Of Reservations About This Whole Schema, In Fact, Ranging From The General Structure Of The Argument, Where Philosophical Issues Are Introduced As If They Were Necessary, Whereas Their Role Is Been Predetermined All Along (For Example, Deleuze Has Always Admired Stoics, And Here They Are As Offering The Best Account), Right Down To The Curious Insistence On The Phantasm As The Adult Form Of Making Sense—In My View, Dreams Are A Lot More Like A General Model Of Thinking, And They Include The More Sophisticated Linguistic Operations Of Metonymy (Condensation), And Metaphor (Displacement). Alexander I. Stingl's Blog A Nomadic Scholarship Entity NOTES ON: Deleuze, G (1990) The Logic Of Sense, Trans Mark Lester, Edited By Constantin Boundas, New York: Columbia University Press For Models Kindly See One Of The Papers In The Series. Attributable And Ascribable To Constraints On Space Accomodation Model Is Provided In Some Paper, Notwithstanding The Generality And Commonalty Of The Observation And Its Concatenatability With The Other Modules. Finally **Leonard Susskind** considers string theory as a possible realization of't Hooft's idea. The light front lattice string model of Klebanov and Susskind is reviewed and its similarities with the holographic theory are demonstrated. The agreement between the two requires unproven but plausible assumptions about the nonperturbative behavior of string theory. Very similar ideas to those in this paper have long been held by Charles Thorn. © 1995 American Institute of Physics **The world as a hologram Leonard Susskind J. Math Phys. 36, 6377 (1995); http://dx.doi.org/10.1063/1.531249 Ofer Aharonya, , Steven S. Gubserb, , Juan Maldacenab, c, , Hirosi Oogurid, e, , Yaron Ozf,** review the holographic correspondence between field theories and string/M theory, focusing on the relation between compactifications of string/M theory on Anti-de Sitter spaces and conformal field theories. They review the background for this correspondence and discuss its motivations and the evidence for its correctness. They describe the main results that have been derived from the correspondence in the regime that the field theory is approximated by classical or semiclassical gravity. We focus on the case of the supersymmetric gauge theory in four dimensions, but discuss also field theories in other dimensions, conformal and non-conformal, with or without supersymmetry, and in particular the relation to QCD. We also discuss some implications for black hole physics. **Physics Reports Volume 323, Issues 3–4, January 2000, Pages 183–386 Large N field theories, string theory and gravity Ofer Aharonya, , Steven S. Gubserb, , Juan Maldacenab, c, , Hirosi Oogurid, e, , Yaron Ozf, , doi:10.1016/S0370-1573(99)00083-6 T. Banks, W. Fischler, S. H. Shenker, and L. Susskind** suggest and motivate a precise equivalence between uncompactified 11-dimensional M theory and the $N=\infty$ limit of the supersymmetric matrix quantum mechanics describing D0 branes. The evidence for the conjecture consists of several correspondences between the two theories. As a consequence of supersymmetry the simple matrix model is rich enough to describe the properties of the entire Fock space of massless well separated particles of the supergravity theory. In one particular kinematic situation the leading large distance interaction of these particles is exactly described by supergravity. The model appears to be a nonperturbative realization of the holographic principle. The membrane states required by M theory are contained as excitations of the matrix model. The membrane world volume is a noncommutative geometry embedded in a noncommutative spacetime. DOI: http://dx.doi.org/10.1103/PhysRevD.55.5112 **M theory as a matrix model: A conjecture Phys. Rev. D 55, 5112 – Published 15 April 1997 T. Banks, W. Fischler, S. H. Shenker, and L. Susskind** These TASI lectures review the Holographic principle. The first lecture describes the puzzle of black hole information loss that led to the idea of Black Hole Complementarity and subsequently to the Holographic Principle itself. The second lecture discusses the holographic entropy bound in general space-times. The final two lectures are devoted to the ADS/CFT duality as a special case of the principle. The presentation is self contained and emphasizes the physical principles. Very little technical knowledge of string theory or supergravity is assumed. Report number: SU-ITP 99-14, KUL-TF-2000/03 Cite as: arXiv: hep-th/0002044 **TASI lectures on the Holographic Principle Daniela Bigatti, Leonard Susskind** The notion of a space-time uncertainty principle in string theory is clarified and further developed. The motivation and the derivation of the principle are first reviewed in a reasonably self-contained way. It is then shown that the nonperturbative (Borel summed) high-energy and high-momentum transfer behavior of string scattering is consistent

with the space-time uncertainty principle. It is also shown that, as a consequence of this principle, string theories in 10 dimensions generically exhibit a characteristic length scale which is equal to the well-known 11 dimensional Planck length gs1/3 ℓs of M-theory as the scale at which stringy effects take over the effects of classical supergravity, even without involving D-branes directly. The implications of the space-time uncertainty relation in connection with D-branes and black holes are discussed and reinterpreted. Finally, we present a novel interpretation of the Schild-gauge action for strings from the viewpoint of noncommutative geometry. This conforms to the space-time uncertainty relation by manifestly exhibiting a noncommutativity of quantized string coordinates between, dominantly, space and time. We also discuss the consistency of the space-time uncertainty relation with S and T dualities. Copyright (c) 2000 Progress of Theoretical Physics **String Theory and the Space-Time Uncertainty Principle Tamiaki Yoneya Oxford Journals Science & Mathematics Progress of Theoretical Physics Volume 103, Issue 6Pp. 1081-112** In this Letter a recently proposed gravity dual of noncommutative Yang-Mills theory is derived from the relations between closed string moduli and open string moduli recently suggested by Seiberg and Witten. The only new input one needs is a simple form of the running string tension as a function of energy. This derivation provides convincing evidence that string theory integrates with the holographical principle and demonstrates a direct link between noncommutative Yang-Mills theory and holography.DOI: http://dx.doi.org/10.1103/PhysRevLett.84.2084 **Holography and Noncommutative Yang-Mills Theory Phys. Rev. Lett. 84, 2084 – Published 6 March 2000 Miao Li and Yong-Shi Wu Petr Hořava** suggests that M theory could be nonperturbatively equivalent to a local quantum field theory. More precisely, we present a "renormalizable" gauge theory in eleven dimensions, and show that it exhibits various properties expected of quantum M theory, most notably the holographic principle of't Hooft and Susskind. The theory also satisfies Mach's principle: A macroscopically large space-time (and the inertia of low-energy excitations) is generated by a large number of "partons" in the microscopic theory. **Petr Hořava** argues that at low energies in large eleven dimensions, the theory should be effectively described by eleven-dimensional supergravity. This effective description breaks down at much lower energies than naively expected, precisely when the system saturates the Bekenstein bound on energy density. He shows that the number of partons scales like the area of the surface surrounding the system, and discuss how this holographic reduction of degrees of freedom affects the cosmological constant problem. proposal is put forth for the holographic field theory as a candidate for a covariant, nonperturbative formulation of quantum M theory. DOI: http://dx.doi.org/10.1103/PhysRevD.59.046004 **M theory as a holographic field theory Phys. Rev. D 59, 046004 – Published 26 January 1999 Petr Hořava** The recent proposal on the correspondence between the super-Yang–Mills theory and string theory in the Penrose limit of the AdS5×S5 geometry involves a few puzzles from the viewpoint of holographic principle, especially in connection with the interpretation of times. To resolve these puzzles, we propose to interpret the PP-wave strings on the basis of tunneling null geodesics connecting boundaries of the AdS geometry. Our approach predicts a direct and systematic identification of the S-matrix of Euclidean string theory in the bulk with the short-distance structure of correlation functions of super-Yang–Mills theory on the AdS boundary, as an extension of the ordinary relation in supergravity–CFT correspondence. Holography requires an infinite number of contact terms for interaction vertices of string field theory and constrains their forms in a way consistent with supersymmetry. Copyright © 2003 Elsevier B.V. All rights reserved. **Nuclear Physics B Volume 665, 18 August 2003, Pages 94–128 Holographic reformulation of string theory on AdS5×S5 background in the PP-wave limit Suguru Dobashi, Hidehiko Shimada, Tamiaki Yoneya doi:10.1016/S0550-3213(03)00460-7** The strongest adversary in quantum information science is decoherence, which arises owing to the coupling of a system with its environment1. The induced dissipation tends to destroy and wash out the interesting quantum effects that give rise to the power of quantum computation2, cryptography2 and simulation3. Whereas such a statement is true for many forms of dissipation, **Frank Verstraete1, Michael M. Wolf2 & J. Ignacio Cirac3** show here that dissipation can also have exactly the opposite effect: it can be a fully fledged resource for universal quantum computation without any coherent dynamics needed to complement it. The coupling to the environment drives the system to a steady state where the outcome of the computation is encoded. In a similar vein, they show that dissipation can be used to engineer a large variety of strongly correlated states in steady state, including all stabilizer codes, matrix product states4, and their

generalization to higher dimensions5. **Letter abstract Nature Physics 5, 633 - 636 (2009) Published online: 20 July 2009 | doi: 10.1038/nphys1342 Quantum computation and quantum-state engineering driven by dissipation Frank Verstraete1, Michael M. Wolf2 & J. Ignacio Cirac3** Nonlocal twist operators are introduced for the O (n) and Q-state Potts models in two dimensions which count the numbers of self-avoiding loops (respectively, percolation clusters) surrounding a given point. Their scaling dimensions are computed exactly. This yields many results: for example, the number of percolation clusters which must be crossed to connect a given point to an infinitely distant boundary. Its mean behaves as $(1/33\sqrt{\pi})\,|\ln(pc-p)|$ as $p\to pc-$. As an application **John Cardy** computes the exact value $3\sqrt{/2}$ for the conductivity at the spin Hall transition, as well as the shape dependence of the mean conductance in an arbitrary simply connected geometry with two extended edge contacts. DOI: http://dx.doi.org/10.1103/PhysRevLett.84.3507 **Linking Numbers for Self-Avoiding Loops and Percolation: Application to the Spin Quantum Hall Transition Phys. Rev. Lett 84, 3507 – Published 17 April 2000 John Cardy Ferenc Iglói, Loïc Turban, Dragi Karevski, and Ferenc Szalma** consider the Ising model and the directed walk on two-dimensional layered lattices and show that the two problems are inherently related: zero-field thermodynamical properties of the Ising model are contained in the spectrum of the transfer matrix of the directed walk. The critical properties of the two models are connected to the scaling behavior of the eigenvalue spectrum of the transfer matrix which is studied exactly through renormalization for different self-similar distributions of the couplings. The models show very rich bulk and surface critical behaviors with nonuniversal critical exponents, coupling-dependent anisotropic scaling, first-order surface transition, and stretched exponential critical correlations. It is shown that all the nonuniversal critical exponents obtained for the aperiodic Ising models satisfy scaling relations and can be expressed as functions of varying surface magnetic exponents. DOI: http://dx.doi.org/10.1103/PhysRevB.56.11031 **exact renormalization-group study of aperiodic Ising quantum chains and directed walks Phys. Rev. B 56, 11031 – Published 1 November 1997 Ferenc Iglói, Loïc Turban, Dragi Karevski, and Ferenc Szalma** Using exact expressions for the persistence probability and for the leading eigenvalue of the Fokker-Planck operator of a random walk in a random environment, we establish a fundamental relation between the statistical properties of anomalous diffusion and the critical and off-critical behavior of random quantum spin chains. Many exact results are obtained from this correspondence, including the space and time correlations of surviving random walks and the distribution of the gaps of the corresponding Fokker-Planck operator. In turn **Ferenc Iglói and Heiko Rieger** derives analytically the dynamical exponent of the random transverse-field Ising spin chain in the Griffiths-McCoy region. DOI: http://dx.doi.org/10.1103/PhysRevE.58.4238 **Anomalous diffusion in disordered media and random quantum spin chains Phys. Rev. E 58, 4238 – Published 1 October 1998 Ferenc Iglói and Heiko Rieger** To gain deeper insight into the dynamics of complex quantum systems we need a quantum leap in computer simulations. We cannot translate quantum behaviour arising from superposition states or entanglement efficiently into the classical language of conventional computers. The solution to this problem, proposed in 1982 (ref. 1), is simulating the quantum behaviour of interest in a different quantum system where the interactions can be controlled and the outcome detected sufficiently well. **A. Friedenauer1, H. Schmitz1, J. T. Glueckert, D. Porras & T. Schaetz** study the building blocks for simulating quantum spin Hamiltonians with trapped ions2. They experimentally simulate the adiabatic evolution of the smallest non-trivial spin system from paramagnetic into ferromagnetic order with a quantum magnetization for two spins of 98%. We prove that the transition is not driven by thermal fluctuations but is of quantum-mechanical origin (analogous to quantum fluctuations in quantum phase transitions3). We observe a final superposition state of the two degenerate spin configurations for the ferromagnetic order (|↑↑right fence+|↓↓right fence), corresponding to deterministic entanglement achieved with 88% fidelity. This method should allow for scaling to a higher number of coupled spins2, enabling implementation of simulations that are intractable on conventional computers. **Letter Nature Physics 4, 757 - 761 (2008) Published online: 27 July 2008 | doi:10.1038/nphys1032 Subject Categories: Quantum physics | Atomic and molecular physics Simulating a quantum magnet with trapped ions A. Friedenauer1, H. Schmitz1, J. T. Glueckert, D. Porras & T. Schaetz** This article gives a comprehensive description of the fractal geometry of conformally-invariant (CI) scaling curves, in the plane or half-plane. It focuses on deriving critical exponents associated with interacting random paths, by exploiting an underlying quantum

gravity (QG) structure, which uses KPZ maps relating exponents in the plane to those on a random lattice, i.e., in a fluctuating metric. This is applied to critical models, like O(N) and Potts models, and to the Stochastic L\"owner Evolution (SLE). The multifractal (MF) function f(alpha, c) of the harmonic measure near any CI fractal boundary, is given as a function of the central charge c of the associated CFT. The Hausdorff dimensions D_{H} of a non-simple scaling curve or cluster hull, and D_{EP} of its external perimeter or frontier, are shown to obey the duality equation (D_{H}-1)(D_{EP}-1)=1/4, valid for any c. The universal mixed MF spectrum f(alpha,lambda;c) describing the local spiralling rate lambda and singularity exponent alpha of the potential near any CI scaling curve is given. The duality between simple and non-simple random paths is established via symmetry of the KPZ quantum gravity map. An extended dual KPZ relation is introduced for the SLE_{kappa}, which commutes with the kappa to kappa'=16/kappa duality. This gives the SLE exponents from simple QG rules, established from the general structure of correlation functions of arbitrary interacting random sets on a random lattice. Journal reference: {\it Fractal Geometry and Applications: A Jubilee of Beno\^{i}t Mandelbrot} (M. L. Lapidus and M. van Frankenhuysen, eds.), Proc. Symposia Pure Math. vol. 72, Part 2, 365-482 (AMS, Providence, R.I., 2004) arXiv: math-ph/0303034(or arXiv: math-ph/0303034v2 for this version) **Conformal Fractal Geometry and Boundary Quantum Gravity Bertrand Duplantier** Hume, For Deleuze, Considers The Mind To Be A System Of Associations Alone, A Network Of Tendencies (ES 25): “We Are Habits, Nothing But Habits – The Habit Of Saying ‘I’. Perhaps There Is No More Striking Answer To The Problem Of The Self.” (ES X.) The Mind, Affected By The Natural Principle Of Association, Becomes Human Nature, From The Ground Up: Empirical Subjectivity Is Constituted In The Mind Under The Influence Of The Principles Affecting It; The Mind Therefore Does Not Have The Characteristics Of A Preexisting Subject. (ES 29) These Associations Account Not Only for Experience in the Basic Sense, But up To the Highest Level of Social and Cultural Life: This Is The Basis For Hume’s Rejection (E) Of A Social Contract Model Of Society (Such As Hobbes’), In Favour Of Convention Alone. Morals, Feelings, Bodily Comportment, All Of These Elements Of Subjectivity Are Explained, Not By Transcendental Structures, Such As Kant Will Propose, But By The Immanent Activity Of Association. Once This Habitual Structure Of The Self Is In Place, Deleuze Suggests, The Humean Concept Of Belief Comes Into Play , Which Is Resolutely A Central Part Of Human Nature. It Describes The Particularly Human Way Of Going Beyond The Given. When We Expect The Sun To Come Up Tomorrow, We Do Not Do So Because We Know That It Will, But Because Of A Belief Based On A Habit. This In Turn Reverses The Hierarchy Of Knowledge And Belief, And Results, For Deleuze, In A, “Great Conversion Of Theory To Practice.” (PI 36) Every Act Of Belief Is A Practical Application Of Habit, Without Any Reference To An A Priori Ability To Judge. Not Only Is The Human Being Thus Habitual, On Deleuze’s Reading, But Also Creative Even In The Most Mundane Moments Of Life. Finally, Deleuze Insists That One Of Hume’s Greatest Contributions To Modern Philosophy Is His Insistence That All Relations Are External To Their Terms: This Is The Essence Of Hume’s Anti-Transcendental Stance. Human Nature Cannot Unite Itself, There Is No ‘I’ Which Stands Before Experience, But Only Moments Of Experience Themselves, Unattached And Meaningless Without Any Necessary Relation To Each Other. A Flash Of Red, A Movement, A Gust Of Wind, Dashing Of A Woman Against You, Obscene Gestures, Spitting At You, Talking Extempore About You (My Addition) And These Elements Must Be Externally Related To Each Other To Create The Sensation Of A Tree In Autumn. In The Social World, This Externality Attests To The Always-Already Interested Nature Of Life: No Relation Is Necessary, Or Governed By Neutral Laws, So Every Relation Has A Localised And Passional Motive. The Ways In Which Habits Are Formed Attests To The Desires At The Heart Of Our Social Milieu. Subjectivity, As Deleuze Describes It Through His Reading Of Hume, Is A Practical, Passional, And Empiricist Concept, Immediately Located At The Heart Of The Conventional, Which Is To Say The Social. (Stanford Encyclopedia of Philosophy). The Buddha Always Used The Terms Void, No Rising And Falling, Calmness And Extinction To Explain The Profound Meaning Of Sunyata And Cessation. For Example, Sunyata And The State Of Nirvana Where There Is No Rising Nor Falling, Are Interpreted By Most People As A State Of Non-Existence And Gloom. They Fail To Realise That Quite The Opposite, Sunyata Is Of Substantial And Positive Significance. The Sutras Often Use The Word "Great Void" To Explain The Significance Of Sunyata. In General, We Understand The "Great Void" As Something That Contains Absolutely Nothing. However, From A Buddhist Perspective, The Nature Of

The "Great Void" Implies Something Which Does Not Obstruct Other Things, In Which All Matters Perform Their Own Functions. Materials Are Form, Which By Their Nature, Imply Obstruction. The Special Characteristic Of The "Great Void" Is Non-Obstruction. The "Great Void" Therefore, Does Not Serve As An Obstacle To Them. Since The "Great Void" Exhibits No Obstructive Tendencies, It Serves As The Foundation For Matter To Function. In Other Words, If There Was Neither "Great Void" Nor Characteristic Of Non-Obstruction, It Would Be Impossible For The Material World To Exist And Function. The "Great Void" Is Not Separated From The Material World. The Latter Depends On The Former. We Can State That the Profound Significance of Sunyata and the Nature of Sunyata in Buddhism Highlight The "Great Void's" Non-Obstructive Nature. Sunyata Does Not Imply The "Great Void". Instead, It Is The Foundation Of All Phenomena (Form And Mind). It Is The True Nature Of All Phenomena, And It Is The Basic Principle Of All Existence. In Other Words, If The Universe's Existence Was Not Empty Nor Impermanent, Then All Resulting Phenomena Could Not Have Arisen Due To The Co-Existence Of Various Causes And There Would Be No Rising Nor Falling. The Nature Of Sunyata Is Of Positive Significance. Calmness And Extinction Are The Opposite Of Rising And Falling. They Are Another Way To Express That There Is No Rising And Falling. Rising And Falling Are The Common Characteristics Of Worldly Existence. All Phenomena Are Always In The Cycle Of Rising And Falling. However, Most People Concentrate On Living (Rising). They Think That The Universe And Life Are The Reality Of A Continuous Existence. Buddhism On The Other Hand, Promotes The Value Of A Continuous Cessation (Falling). This Cessation Does Not Imply That It Ceases To Exist Altogether. Instead, It Is Just A State In The Continuous Process Of Phenomena. In This Material World, Or What We May Call This "State Of Existence", Everything Eventually Ceases To Exist. Cessation Is Definitely The Home Of All Existences. Since Cessation Is The Calm State Of Existence And The Eventual Refuge Of All Phenomena, It Is Also The Foundation For All Activities And Functions. Teachings In Chinese Buddhism (6) Sunyata (Emptiness) In The Mahayana Context (Wikipedia) Studies by Eric V. Linder about geometric dark energy bring out the quintessential prerequisites about the extreme physics ad dark matter**.** The acceleration of the expansion of the universe arises from unknown physical processes involving either new fields in high energy physics or modifications of gravitation theory. It is crucial for our understanding to characterize the properties of the dark energy or gravity through cosmological observations and compare and distinguish between them. In fact, close consistencies exist between a dark energy equation of state function w(z) and changes to the framework of the Friedmann cosmological equations as well as direct spacetime geometry quantities involving the acceleration, such as "geometric dark energy" from the Ricci scalar. We investigate these interrelationships, including for the case of super acceleration or phantom energy where the fate of the universe may be gentler than the Big Rip. DOI: http://dx.doi.org/10.1103/PhysRevD.70.023511 **Probing gravitation, dark energy, and acceleration Phys. Rev. D 70, 023511 – Published 28 July 2004 Eric V. Linder. In the same vein he continues to question the number of parameters that exists in dark energy** .How many dark energy parameters? For exploring the physics behind the accelerating Universe a crucial question is how much we can learn about the dynamics through next generation cosmological experiments. For example, in defining the dark energy behavior through an effective equation of state, how many parameters can we realistically expect to tightly constrain? Through both general and specific examples (including new parametrizations and principal component analysis) we argue that the answer is two. Cosmological parameter analyses involving a measure of the equation of state value at some epoch (e.g. w0) and a measure of the change in equation of state (e.g. w′) are therefore realistic in projecting dark energy parameter constraints. More elaborate parametrizations could have some uses (e.g. testing for bias or comparison with model features), but do not lead to accurately measured dark energy parameters. DOI: http://dx.doi.org/10.1103/PhysRevD.72.043509 **Phys. Rev. D 72, 043509 – Published 11 August 2005 Eric V. Linder and Dragan Huterer D.F. Mota, C. van de Bruck Oxford Univ** study the spherical collapse model in dark energy cosmologies, in which dark energy is modelled as a minimally coupled scalar field. We first follow the standard assumption that dark energy does not cluster on the scales of interest. Investigating four different popular potentials in detail, we show that the predictions of the spherical collapse model depend on the potential used. We also investigate the dependence on the initial conditions. Secondly, they investigate in how far perturbations in the quintessence field affect the predictions of the spherical collapse model. In doing so, we assume that the field collapses along with the dark matter. Although the field is still subdominant at the time of virialisation,

the predictions are different from the case of a homogeneous dark energy component. This will in particular be true if the field is non--minimally coupled. **D.F. Mota, C. van de Bruck** conclude that a better understanding of the evolution of dark energy in the highly non--linear regime is needed in order to make predictions using the spherical collapse model in models with dark energy. Astrophys 421 (2004) 71-81DOI:10.1051/0004-6361:20041090arXiv: astro-ph/0401504 **on the spherical collapse model in dark energy cosmologies D.F. Mota, C. van de Bruck (Oxford Univ)** Studies by Eric V. Linder about geometric dark energy bring out the quintessential prerequisites about the extreme physics ad dark matter**.** The acceleration of the expansion of the universe arises from unknown physical processes involving either new fields in high energy physics or modifications of gravitation theory. It is crucial for our understanding to characterize the properties of the dark energy or gravity through cosmological observations and compare and distinguish between them. In fact, close consistencies exist between a dark energy equation of state function w(z) and changes to the framework of the Friedmann cosmological equations as well as direct spacetime geometry quantities involving the acceleration, such as "geometric dark energy" from the Ricci scalar. We investigate these interrelationships, including for the case of super acceleration or phantom energy where the fate of the universe may be gentler than the Big Rip. DOI: http://dx.doi.org/10.1103/PhysRevD.70.023511 **Probing gravitation, dark energy, and acceleration Phys. Rev. D 70, 023511 – Published 28 July 2004 Eric V. Linder. In the same vein he continues to question the number of parameters that exists in dark energy** .How many dark energy parameters? For exploring the physics behind the accelerating Universe a crucial question is how much we can learn about the dynamics through next generation cosmological experiments. For example, in defining the dark energy behavior through an effective equation of state, how many parameters can we realistically expect to tightly constrain? Through both general and specific examples (including new parametrizations and principal component analysis) we argue that the answer is two. All Existences Exhibit Void-Nature And Nirvana-Nature. These Natures Are The Reality Of All Existence. To Realise The Truth, We Have To Contemplate And Observe Our Worldly Existence. We Cannot Realise The Former Without Observing The Latter. Consider This Heart Sutra Extract, "Only When Avalokiteshvara Bodhisattva Practised The Deep Course Of Wisdom Of Prajna Paramita Did He Come To Realise That The Five Skandhas (Aggregates, And Material And Mental Objects) Were Void." Profound Wisdom Leads Us To The Realisation That All Existences Are Of Void-Nature. The Sutras Demonstrate That The Profound Principle Can Be Understood By Contemplating And Observing The Five Skandhas. We Cannot Realise The Truth By Seeking Something Beyond The Material And Mental World. The Buddha, Using His Perfect Wisdom, Observed Worldly Existence From Various Implications And Aspects, And Came To Understand All Existences. Teachings In Chinese Buddhism (6) Sunyata (Emptiness) In The Mahayana Context (Wikipedia) .But From The Profound Contemplation And Wisdom Of The Buddha And Mahabodhisattvas, We Know There Is No Such Reality. Instead, Egolessness (Non-Self) Is The Only Path To Understand The Reality Of The Deluded Life. All Existences Are Subject To The Law Of Causes And Conditions. These Include The Smallest Particles, The Relationship Between The Particles, The Planets, And The Relationship Between Them, Up To And Including The Whole Universe! From The Smallest Particles To The Biggest Matter, There Exists No Absolute Independent Identity. Egolessness (Non-Self) Implies The Void Characteristics Of All Existence. Egolessness (Non-Self) Signifies The Non-Existence Of Permanent Identity For Self And Existence (Dharma). Sunyata Stresses The Voidness Characteristic Of Self And Existence (Dharma). Sunyata And Egolessness Possess Similar Attributes. As We Have Discussed Before, We Can Observe The Profound Significance Of Sunyata From (e) The Perspective Of Inter-Dependent Relationships. Considering Dharma-Nature And The Condition Of Nirvana, All Existences Are (=) Immaterial And Of A Void-Nature. Then We See Each Existence As (=) Independent Of Each Other. But Then We Cannot (e) Find Any Material That Does Exist Independent Of (e) Everything Else. So Egolessness Also Implies (eb) Void-Nature! From The Observation Of All Existences, We Can Infer (e) The Theory Of Nirvana And The Complete Cessation Of All Phenomena. From The Viewpoint Of Phenomena, All Existences Are (=) So Different From (e) Each Other, That They May Contradict (e&eb) Each Other. They Are (=) So Chaotic. In Reality, Their Existence Is (=) Illusionary And Arises From (e) Conditional Causation. They Seem To Exist On One Hand, And Yet Do Not Exist On The Other. They Seem To Be United, But Yet They Are So Different To One Another. They Seem To Exist And (e&eb) Yet They Do Cease! Ultimately Everything Will Return To (e) Harmony And Complete

Calmness. This Is The Nature Of All Existence. It Is The Final Resting Place For All. If We Can Understand This Reality And Remove Our Illusions, We Can Find This State Of Harmony And Complete Calmness. Teachings In Chinese Buddhism (6) Sunyata (Emptiness) In The Mahayana Context (Wikipedia) All Our Contradictions, Impediments And Confusion Will Be Converted To (e&eb) Equanimity. Free From Illusion, Complete Calmness Will Be The Result Of (e) Attaining Nirvana. The Buddha Emphasised The Significance Of This Attainment And Encouraged The Direct And Profound Contemplation On (e) Void-Nature. He Said, "Since There Is No Absolute Self-Nature Thus Every Existence Exhibits (eb) Void-Nature. Because (e) It Is Void, There Is No Rising Nor Falling. Since There Is No Rising Nor Falling, (eb) Thus Everything Was Originally In Complete Calmness. Its Self-Nature Is Nirvana." Teachings In Chinese Buddhism (6) Sunyata (Emptiness) In The Mahayana Context (Wikipedia).The Scholars Of Tien Tai Called It The "Embodied Nature". (This Is The Buddha-Nature That Includes Both Good And Evil.) The Scholars Of Xian Shou Say, "It Is Arising From Primal Nature", And The Scholars Of Chan (Zen) Say, "It Is Nature That Causes The Rising Of Things". All Dharma Is Dharma-Nature. It Is Not Different From Dharma-Nature. Dharma And Dharma-Nature Are Not Two Separate Identities, "Phenomena" And "Nature" Is Also Not Distinguishable Either. In Other Words, There Is No Difference Between Principle (Absolute) And Practice (Relative). Teachings In Chinese Buddhism (6) Sunyata (Emptiness) In The Mahayana Context (Wikipedia). In Other Words, All Dharma Arises From Causes And Conditions, And All Dharma Is Empty In Nature. The Law of Dependent Origination (Existence) And The Nature Of Emptiness Is Neither The Same Nor Different. They Exist Mutually. The Truth Of "Sunyata" And "Existence", And "Nature" And "Phenomena" Are Not In Conflict With Each Other. Unlike The Scholars Of The Dharmalaksana Sect Who Explain The Dharma Only From The Aspect Of Dependent Origination, Or The Scholars Of Dharma-Nature That Explain The Existence Of Dharma Only From The Aspect Of Dharma-Nature, The Scholars Of Madhyamika Explain The Truth Of The Dharma From Both Aspects. Hence This Is Called The Middle Path Which Does Not Incline To Either Side. Teachings In Chinese Buddhism (6) Sunyata (Emptiness) In The Mahayana Context (Wikipedia) **C S Lent, P D Tougaw, W Porod and G H Bernstein** formulate a new paradigm for computing with cellular automata (CAS) composed of arrays of quantum devices-quantum cellular automata. Computing in such a paradigm is edge driven. Input, output, and power are delivered at the edge of the CA array only; no direct flow of information or energy to internal cells is required. Computing in this paradigm is also computing with the ground state. The architecture is so designed that the ground-state configuration of the array, subject to boundary conditions determined by the input, yields the computational result. proposal is put forth for a specific realization of these ideas using two-electron cells composed of quantum dots. The charge density in the cell is very highly polarized (aligned) along one of the two cell axes, suggestive of a two-state CA. The polarization of one cell induces a polarization in a neighboring cell through the Coulomb interaction in a very non-linear fashion. Quantum cellular automata can perform useful computing. The authors show that AND gates, OR gates and inverters can be constructed and interconnected. **C S Lent et al 1993 Nanotechnology 4 49 doi:10.1088/0957-4484/4/1/004 Quantum cellular automata C S Lent, P D Tougaw, W Porod and G H Bernstein** The basic features of the dynamics of open quantum systems, such as the dissipation of energy, the decay of coherences, the relaxation to equilibrium or non-equilibrium stationary state, and the transport of excitations in complex structures are of central importance in many applications of quantum mechanics. The theoretical description, analysis and control of non-Markovian quantum processes play an important role in this context. While in a Markovian process an open system irretrievably loses information to its surroundings, non-Markovian processes feature a flow of information from the environment back to the open system, which implies the presence of memory effects and represents the key property of non-Markovian quantum behaviour. **Heinz-Peter Breuer** review recent ideas developing a general mathematical definition for non-Markovianity in the quantum regime and a measure for the degree of memory effects in the dynamics of open systems, which are based on the exchange of information between system and environment. **Heinz-Peter Breuer** further study the dynamical effects induced by the presence of system–environment correlations in the total initial state and design suitable methods to detect such correlations through local measurements on the open system. **Heinz-Peter Breuer 2012 J. Phys. B: At. Mol. Opt. Phys. 45 154001 doi:10.1088/0953-4075/45/15/154001 Foundations and measures of quantum non-Markovianity** A fundamental step towards atomic- or molecular-scale spintronic devices has

recently been made by demonstrating that the spin of an individual atom deposited on a surface1, or of a small paramagnetic molecule embedded in a nanojunction2, can be externally controlled. An appealing next step is the extension of such a capability to the field of information storage, by taking advantage of the magnetic bistability and rich quantum behaviour of single-molecule magnets3, 4, 5, 6 (SMMs). Recently, a proof of concept that the magnetic memory effect is retained when SMMs are chemically anchored to a metallic surface7 was provided. However, control of the nanoscale organization of these complex systems is required for SMMs to be integrated into molecular spintronic devices8, 9. Here we show that a preferential orientation of Fe4 complexes on a gold surface can be achieved by chemical tailoring. As a result, the most striking quantum feature of SMMs—their stepped hysteresis loop, which results from resonant quantum tunnelling of the magnetization5, 6—can be clearly detected using synchrotron-based spectroscopic techniques. With the aid of multiple theoretical approaches, **M. Mannini et al** relate the angular dependence of the quantum tunnelling resonances to the adsorption geometry, and demonstrate that molecules predominantly lie with their easy axes close to the surface normal. Findings of **M. Mannini et al prove** that the quantum spin dynamics can be observed in SMMs chemically grafted to surfaces, and offer a tool to reveal the organization of matter at the nanoscale. **Quantum tunnelling of the magnetization in a monolayer of oriented single-molecule magnets M. Mannini et al** Nature 468, 417–421 (18 November 2010) doi: 10.1038/nature09478 Supposing That There Were Indeed An "Energy Of Contraction" Constant In All Centers Of Force Of The Universe, It Remains To Be Explained Where Any Difference Would Ever Originate. It Would Be Necessary For The Whole To Dissolve Into An Infinite Number Of Perfectly Identical Existential Rings And Spheres, And We Would Therefore Behold Innumerable And Perfectly Identical Worlds COEXISTING [Nietzsche Underlines This Word Twice] Alongside Each Other. Is It Necessary For Me To Admit This? Is It Necessary To Posit An Eternal Coexistence On Top Of The Eternal Succession Of Identical Worlds? The Eternal Return: Genesis And Interpretation BY PAUL D'IORIO"The Cyclical Hypothesis, So Heavily Criticized By Nietzsche (VP II 325 And 334), Arises In This Way." In Fact, Nietzsche Was Not Criticizing The Cyclical Hypothesis But Only The Particular Form Of That Hypothesis Presented In Vogt's Work. All Of Nietzsche's Texts Without Exception Speak Of The Eternal Return As The Repetition Of The Same Events Within A Cycle Which Repeats Itself Eternally. If Deleuze's Interpretation Holds That The Eternal Return Is Not A Circle, Then What Is It? A Wheel Moving Centrifugally, Operating A "Creative Selection," "Nietzsche's Secret Is That The Eternal Return Is Selective" Says Deleuze: The Eternal Return Produces Becoming-Active. It Is Sufficient To Relate The Will To Nothingness To The Eternal Return In Order To Realize That Reactive Forces Do Not Return. However Far They Go, However Deep The Becoming-Reactive Of Forces, Reactive Forces Will Not Return. The Small, Petty, Reactive Man Will Not Return. Affirmation Alone Returns, This That Can Be Affirmed Alone Returns, Joy Alone Returns. Everything That Can Be Denied, Everything That Is Negation, Is Expelled Due To The Very Movement Of The Eternal Return. We Were Entitled To Dread That The Combinations Of Nihilism And Reactivity Would Eternally Return Too. The Eternal Return Must Be Compared To A Wheel; Yet, The Movement Of The Wheel Is Endowed With Centrifugal Powers That Drive Away The Entire Negative. Because Being Imposes Itself On Becoming, It Expels From Itself Everything That Contradicts Affirmation, All Forms Of Nihilism And Reactivity: Bad Conscience, Ressentiment..., We Shall Witness Them Only Once. [...] The Eternal Return Is The Repetition, But The Repetition That Selects, The Repetition That Saves. Here Is The Marvelous Secret Of A Selective And Liberating Repetition. There Is No Need To Remind The Reader That Neither The Image Of A Centrifugal Movement Nor The Concept Of A Negativity-Rejecting Repetition Appears Anywhere In Nietzsche's Writings, And Indeed Deleuze Does Not Refer To Any Text In Support Of This Interpretation. Further, One Could Highlight That Nietzsche Never Formulates The Opposition Between Active And Reactive Forces, Which Constitutes The Broader Framework Of Deleuze's Interpretation. For Some Years, Marco Brusotti Has Called Attention To The Fact That Deleuze Introduced A Dualism That Does Not Exist In Nietzsche's Writings. To Be Sure, The German Philosopher Describes A Certain Number Of "Reactive" Phenomena (For Example, In The Second Essay Of The Genealogy Of Morality, § 11, He Talks About "Reactive Affects" [Reaktive Affekte], "Reactive Feelings" [Reaktive Gefühlen], Reactive Men [Reaktive Menschen]); But These Are Nonetheless The Result Of Complex Ensembles Of Configurations Of Centers Of Forces That Remain In Themselves Active. Neither The Word Nor The Concept Of "Reactive Forces" Ever Appears In Nietzsche's

Philosophy. The Eternal Return: Genesis And Interpretation BY PAUL D'IORIO Some Parts Are Deleted Due To Spatial Constraints. Please Pardon Me For The Deletion. Deleuze Opposes The Historical Course Of The Hegelian Notion That Confronts Struggles And Finally Dialectizes The Negative And Results In A Consoling Teleology Leading To The Triumph Of The Idea Or The Liberation Of The Masses With The Centrifugal Movement Of The Wheel, Which Simply Ejects The Negative. It Is Still A Case Of A Consoling And Optimistic Teleology, Which, Instead Of Confronting The Weight Of History, The Grief And The Negative, Makes It Disappear In One Centrifugal Stroke Of A Magic Wand. There Is Reason To Worry That This Be A Case Of Repression, Which, Unable To Dialectize Or Accept The Negative, Simply Seeks To Exorcise It In One Gesture Of "Creative Selection." But Exorcism Is A Feat Of Magic And Not Of Philosophy: It Is Unfortunately Not Enough To Make The Negative Disappear. In All Probability, The Negative Will Come Back With A Vengeance. In Contrast To Deleuze's "Affirmation Of Affirmation", Which Affirms Only Affirmation, Nietzsche Conceives Of The Eternal Return From A Rigorously Non-Teleological Perspective As The Accomplishment Of A Philosophy Strong Enough To Accept Existence In All Its Aspects, Even The Most Negative, Without Any Need To Dialecticize Them, Without Any Need To Exclude Them By Way Of Some Centrifugal Movement Of Repression. It Denies Nothing And Incarnates Itself In A Figure Similar To The One Nietzsche, In Twilight Of The Idols, Draws Of Goethe: Such A Spirit, Who Has Become Free Stands In The Middle Of The World With A Cheerful And Trusting Fatalism In The Belief That Only The Individual Is Reprehensible, That Everything Is Redeemed And Affirmed In The Whole-He Does Not Negate Anymore. Such A Faith However, Is The Highest Of All Possible Faiths: I Have Baptized It With The Name Of Dionysus. The Eternal Return: Genesis and Interpretation BY PAUL D'IORIO The Most Complete Theory Of Mind That Comports With The Holographic Principle Of Mind, As Developed Here, Is The Microgenetic Theory Of Neurologist Jason Brown. Brown (2005) Presents An Evolutionary Theory Of Values, Morals, And Ethics, Building On The Microgenetic Theory And Process Theory Of His Earlier Works (1988, 1991, 1996, 1998, And 2000). Brown Is Work Has Been A Progression That Began With The Study Of Brain Processes In Neurological Subjects With Brain Injuries Or Lesions. In His Earlier Work (Brown, 1988; 1991) He Described The Process Of Microgenesis As A Process Of Elaboration Of Mental Contents In An Evolutionary And Developmental Hierarchy Of Brain Structures Within The Process Of The Genesis Of The Mental State, And Related The Hierarchy Of The Genesis Of The Mental State To Disturbances Of Language Comprehension Or Expression (Aphasia), Of Knowledge (Agnosia), And Or Purposeful Movement (Apraxia). The Fundamental Neurology Of These Disturbances Had Been Described In Brownís Previous Work (1972, 1988), And In The Seminal Work Of The Russian Neurologist And Neuropsychologist, Alexander Luria (1966). Brown (1988, 1991, 1996, 1998) Created A Process Theory Of Mind On The Basis Of His Neurological Observations, And Went On From His Neurological Work To Incorporate Process Theory In Philosophy, As Developed Principally By Henri Bergson (1911/1998). And Alfred North Whitehead (1925/1953, 1929/1978). Bergson Had Pioneered The Concept Of An Irreducible Duration Of Experience. This Concept Was Elaborated By Whitehead, In Keeping With Discontinuities Of Particles, As "Actual Entities" Of A Rudimentary Sort, As They Make "Quantum Leaps" Along Their Trajectories, Halting Or Persisting At A Given Location For A Short Period Of Time Or Duration Before Leaping To The Next. Whitehead Employed These "Quantum Leaps" To The Process By Which Experience Arises In "Actual Entities" At The Most Fundamental Level Of Actuality, Through The Epochal Or Halting Duration Of A Discontinuous Process (Whitehead, 1925/1953, 1929/1978). This Halting Allows Actual Entities To Participate In A Process Of Internal Relations, Relations On The "Inside" Of Things. According The Holographic Principle, This "Inside" Is In The Holographic Surface Of The Relevant Region Of Spacetime Of The Actual Entity, And Within Other Surfaces That Are Non-Locally Connected With That Surface. The Holographic Surface Is Thus The Locus Of Non-Local Information And Experience, In Our Interpretations Of Whitehead Is Metaphysics. Internal Relations Occur Through Feeling, Or "Prehension," Which Is Nothing More Than The Subjective Quality Of Non-Local Experience. The Internal Relations Of The Becoming Entity Occur In A Seriality Of Prior Becomings Of The Actual Entity, With The Entity Inheriting All Of Its Causal Past As Well As Its Relevant Feelings Or Prehensions From Prior "Occasions" Or Quanta Of Experience. This Process Is, In The Holographic Model, The Process By Which Former Surfaces, Now-Pasts, Are Elaborated On To The Now-Present. Prehensions Also Occur With Other Actual Entities

Throughout The Universe, With Which They Are Non-Locally Connected, And With Neo-Platonic "Eternal Objects," Existing In An Eternal Heaven. The "Ingression" Of Eternal Objects Is Fundamental To Each Duration Of Becoming Of The Entity, And, On This Basis, Whitehead Formulated The Notion Of An Eternal Heaven In Constant Intercourse All Of Its Creation. There Are Clear Parallels Between Whitehead's Cosmology And The Cosmology Introduced Here According To The Holographic Principle, In That The "Heaven" May Be Regarded As The Holographic Boundary, In Constant Intercourse With The Relevant Volume Of Spacetime. This Interconnection Of Feelings Or Prehensions, According To Their Relevance, Produces Intensities And Contrasts Which Give Rise To The Creative Advance, Which, For Whitehead (1929/1978) Is The Process Which Impels The Movement From The Physical To The Mental Pole Of Process. With Each Halting, Internally-Timeless Duration There Is A Concrescence Or Integration Of These Feelings, Which, When Complete, Constitutes A New Actual Entity Or Actual Occasion (They Are The Same Thing) At A New Locus In Spacetime. This Movement To A New Locus, Or Transition, Is, Fundamentally, The Quantum Leap. MICROGENESIS AND THE HOLOGRAPHIC PRINCIPLE The Holographic Principle Theory Of Mind MARK GERMINE Institute For Psycho Science If the Holographic Principle is true, than it must be the fundamental principle of mind. The brain has no way around the Holographic Principle. For the reductionist, the Holographic Principle is the ultimate reduction. It applies to the most minuscule level of what we can observe, and beyond. For the Universalist, the Holographic Principle gives us the ultimate universal. It extends to the limit of our Universe, the Universal holographic boundary, and beyond. For the Phenomenalist, the Holographic Principle gives us the ultimate ground of our phenomenal perception, our grounding in the Universal "now," in the now-present of the Universal holographic boundary, moving outward from now-pasts to now-futures. Wheeler (1988) has said that all of reality is information, and that other physical quantities are "mere incidentals." Information monism is gaining popularity in physics. By breaking the dichotomy of between information and experience, we find a deep connection between Wheeler is monism and the experiential monism of Whitehead, sometimes called panexperientialism, which relates to the later theological concept of his student, Charles Hartshorne, panentheism, God inside of everything (Hartshorne, 1964). There thus seems to be a convergence of the concepts of information, experience, and spirituality. There is not one universe, but many parallel universes, or, more accurately, a vast superposition of universes called the multiverse (Penrose, 2004). However, as explained in our treatment of the double-slit experiment, in the observation of events on the quantum level, the individual observer sees only one of these vast superpositions, not a summation of a vast number of potentials. This has been explained in terms of the many worlds or many minds theories, in which there are multiple copies of the same observing individual, but this particular issue remains unresolved, leaving physics ungrounded in reality (Penrose, 2004). The concept of One Mind is not only more parsimonious than that of many minds, but also lets us out of the bizarre and counterintuitive idea that there are multiple copies of our own selves, which exist as mere potential, and are thus not actual. There are many possible universes, but there is only One Mind, which determines events on the quantum level, and thus creates our Universe. As we had discussed previously, the quantum level provides the essential ground of the Holographic Principle, such that quantum-level holographic surfaces are elaborated at higher levels, manifesting higher orders of information from the quantum "world." In this process, there is a reduction of the wave function or potentialities of quantum fields. At the level of consciousness, this entails freedom to choose which observations we make (Stapp, 1997), which gives us the capacity to think and to make decisions. These capacities are the basis of individuality, self-determination, judgment, and values. The progressive evolution of the manifestation of Mind through higher orders of experience, leading to higher orders of consciousness, is entailed by the Holographic Principle. Again, we are dealing here with levels of description, with the multiverse of all potentials fundamentally supporting the single Universe we collectively observe. The multiverse is the wave function of the Universe (Penrose, 2004). The recursive integration of nested hierarchies of holographic surfaces brings out a single actuality in consciousness from a wave function that is unconscious. Consciousness thus gives us information at a level of experience that is causal. In this sense, we partake in the creation of the Universe. We participate in creation, and this participation, when fully realized, leads us to higher levels of consciousness and of realization. As a system, the biosphere that we live in has a holographic surface, creating a deep sense of ecology as we collectively move toward a planetary consciousness. It is only when the

Universal Holographic Boundary reaches the information storage and processing capacity needed for the requisite biological and biochemical complexity that consciousness evolves in living things. This evolution is natural and spontaneous, since consciousness gives rise to what is actual, as opposed to what is merely potential. In this sense, consciousness is still in the process of creating our Universe, and levels of higher consciousness will continue to evolve. We are all in the same "now," and that now is defined by the present Universal Boundary. This assures us that our experience is Universal, and does not pass with time. This is the fundamental basis of memory and of cognition. The identity of mind and brain is a myth. We have a continual, internal or non-local relation with the Universe, as it has with us. Once this mystery is resolved, the myth is no longer needed, and there is a confluence of science and spirituality.Experience is primary, information is secondary. We can only gather information from experience, whether it is in the laboratory or in life. We cannot measure the information on the surfaces of systems. The physicists that have formulated the Holographic Principle for all systems are quite aware of this, or else the principle would have been established or discredited. We cannot measure what we experience. It is intangible, yet it is all we know to be actual. Everything else is inferred. Because it cannot be measured, it has been fundamentally disregarded by mainstream science. Materialism is considered scientific, while idealism is considered unscientific. But arenít ideas, fundamentally, made of information? Brain science has mistaken the representation of information for information itself, and has tied those representations to matter and energy. Consciousness, the highest order of information, has generally been regarded as superfluous, something that needs to explained away, or altogether ignored. Yet it is the only "thing" that reaches our awareness. Consciousness comes at a price. For everything that becomes conscious, there must be something that becomes conscious. Consciousness is certainty, and its complement, unconsciousness uncertainty. Consciousness is the particle nature of experience. It has definiteness about it. The unconscious is the wave nature of experience, it is like the metaphysical cloud of unknowing. If we are the most conscious of animals, then we must also be the most unconscious. Perhaps this is the predicament of humankind. The Holographic Principle of Mind leads us naturally from the most fundamental experiences, existing as quantum potentials from the conformations of proteins down through the fields of electrons, through their manifestation upward through a recursion or successive applications of the same holographic process, through higher levels of experience, to the emergence of consciousness as higher and higher orders of experience. The quantum Holographic Principle Mind does not require anything more quantum than is obviously present at the microscopic and submicroscopic levels, as it represents successive orders of manifestations from these levels. Recursion also applies here in the sense that it is used in computer science, in which the function of the part depends on the function of the whole. A program, as a part, cannot work without a functioning whole, the operating system. Recursive wholes which are, for us, supra-conscious, are on the group, species, planetary, and universal levels. As individuals, our consciousness cannot function without recursion to the Universal Consciousness, even though we may be unaware that such Universal Consciousness exists. Reaching upward to these supra-conscious levels is a spiritual process, making transcendence the ultimate solution to the unconscious human predicament. What was once supra-conscious becomes unconscious through a process of conditioning? We are born as creatures of the Earth and of the Universe, as evidenced by the beliefs and practices of "primitive peoples." There is evidence from cave paintings that our hominid ancestors experienced a kind of holographic perception (Combs, 1996), which could constitute our early connection with the holographic subtext of reality, and which we might then propose existed in our animal ancestors, and in extant animal species. If this is the case, than microgenesis would entail the recapitulation of this holographic experience as it progresses through our ancestral past. The supra-conscious mind seems to envelope perinatal experiences, and Stanislav Grof (1994) has developed techniques to gain access to these experiences, as well as to the earlier experiences of our human and animal ancestral lineage, and of our universal history. Grof (1994) concludes: "Our consciousness seems to have the amazing capacity to directly access the earliest history of the universe ñ witnessing dramatic sequences of the Big Bang, the formation of the galaxies, the birth of the solar system, and the early geophysical processes on this planet billions of years ago." The Holographic Principle Theory Of Mind MARK GERMINE Institute For Psycho Science. There Are Similarities Between The Patterns Of Holography And Of Psychological Transference, Where Holography Is The Process Of Recording And Reconstructing Holograms. Employing A Theoretical Perspective Using A Hermeneutic Method, This Dissertation

Parallels Holography With Transference, Offering Another Way To Encounter Transference By Showing Similarities Between The Processes Of Each And The Results Of Each. Though Complex, Infinitely Varying, And Unique, Their Patterns Are Clearly Identifiable. Thus They Are A Metaphorical Fit To The Concept Of Strange Attractors In Physics And A More Literal Fit To The Concept Of Archetypes In Depth Psychology Or Dynamic Psychology, Psychology Which Attends To The Living, Autonomous Unconscious. This Study Explains How Holography Models Transference, What A Hologram Is And How It Works, And How Depth Psychology Understands Of The Interaction Between Consciousness And The Unconscious Is Related To The Hologram. It Describes Transference And Related Psychological Processes As Understood In Six Different Schools Of Depth Psychological Thought. It Shows That The Underlying Pattern Or Strange Attraction Between Transference And Holography Extends To Other Processes Both Within And Outside The Field Of Psychology, Processes Such As Projection, Projective Identification, Splitting, Memory, Biology, Creative Discovery, Theology, Synchronicity, Chaos, And Nonlocality. By Identifying The Similar Patterns Of These Processes, This Study Demonstrates The Existence Of An Underlying Holographic Archetype In Which Essential Qualities Of The Whole Are Present In Each Of The Parts Of The Whole: The Visual Image Of The Overall Hologram Is Present In Each Component Part Of The Hologram, The Autonomy Of The Overall Human Is Present In Each Conscious And Unconscious Component Part Of The Human Psyche. By Noting Differences As Well As Similarities In These Processes, It Suggests An Inventory Of The Qualities Of The Holographic Archetype. This Study Furthers Understanding Of The Pervasiveness, Force, And Autonomy Of The Unconscious Acting Through Transference And Projection By Identifying A Group Projection Of Domestic Violence Lying At The Core Of The Christian Myth. This Study Also Furthers Understanding Of The Concept Of Transference By Providing A Reflection Hologram Of The Human Psyche As An Artistic Work And As A Visual Metaphor Of Transference. Strange Attractors: Transference, Holography, And An Archetype Burke, J. (2003). Strange Attractors: Transference, Holography, And An Archetype (Doctoral Dissertation, Pacifica Graduate Institute, 2003). Onticology, Like All Variations Of Object-Oriented Ontology, Is Realist In Its Orientation. In Defending A Realist Ontology Onticology Holds That The Vast Majorities Of Objects, Actants, Beings, Or Entities Are Independent Of Humans And Are What They Are Regardless Of Whether Any Humans Regard Them Or Register Them. In Short, Onticology Rejects Any Anthropomorphic, Idealist, Or Anti-Realist Thesis To The Effect That To Be Is To Be The Correlate Of Mind, Spirit, The Body, The Human, And Language Or Otherwise. While It Is Certainly The Case That Knowledge Is Necessarily Dependent On The Object To Which It Relates, The Converse Does Not Hold True. Objects Are Not Dependent On Being Known, Regarded, Perceived, Or Spoken About. As Such, And To Put It In Aristotelean Terms, Knowledge Is An Accident Of Objects, Not Objects An Accident Of Knowledge. As Althusser So Nicely Puts It, "[N]O Doubt There Is A Relation Between Thought-About-The-Real And This Real, But It Is A Relation Of Knowledge, A Relation Of Adequacy Or Inadequacy Of Knowledge, Not A Real Relation, Meaning By This A Relation Inscribed In That Real Of Which The Thought Is The (Adequate Or Inadequate) Knowledge" (Reading Capital, 96). Althusser Goes On To Remark That "[T] He Distinction Between A Relation Of Knowledge And A Relation Of The Real Is A Fundamental One: If We Did Not Respect It We Should Fall Irreversibly Into Either Speculative Or Empiricist Idealism" (Ibid.). Onticology Categorically Endorses Althusser's Verdict It Is A Fundamental Necessity To Distinguish Between Those Relations That Belong To The Object And Those That Belong To Knowledge. Contemporary Philosophy Continuously Confuses These Two Very Different Sorts Of Relations. Naturally The Question Arises Of How It Is Possible To Surmount Our Relation To The Object So As To Determine Whether Objects Themselves Possess The Properties We Encounter In Relating To Objects. In Other Words, Given That We Can Only Ever Relate To The Object In Relating To The Object How Is It Possible To Surmount This Relation To Get At The Being Of The Object Itself? Much More Will Have To Be Said About This Later– And The Answers Will Be Surprising With Respect To Standard Prejudices About Realism –However, For The Moment It Can Be Said That Onticology Takes Its Epistemological Inspiration From The Transcendental Realism Of Roy Bhaskar. Among Other Things, Bhaskar Sought To Provide A Transcendental Grounding For The Sciences. Insofar As Onticology Defends The Thesis That The Field Of Being Is Much Faster Than The Field Of Objects Investigated By The Natural Sciences, It Parts Way With The Thesis That The Domain Of Being Is

Exhausted By The Domain Of Natural Objects. However, The General Form of Bhaskar's Argument Holds for Our Realist Purposes. A Transcendental Argument Seeks To Elucidate The Conditions Under Which Certain Acknowledged Practices And Forms Of Cognition Are Possible. Kant, For Example, Asked What Must Be The Case For Mathematical Judgments To Be Possible. How Is It Both That We Are Able To Extend Our Knowledge, As If By Magic, Through Mathematical Judgments And, More Significantly, That These Judgments Are Able To Provide Genuine Knowledge Of The World Despite The Fact That These Forms Of Reasoning Are Not Based On Experience? Part of Kant's Argument Consisted in Claiming That Mind Imposes the Forms of Space and Time on the Data of Experience. In Other Words, Space And Time Are Not Attributes Of Being Itself But Rather Of The Mind That Regards Being. Insofar, Kant Argues, As Mathematics Is Ultimately A Rumination On The Nature Of Space And Time Taken In Their Most Abstract Form And Insofar As The Mind Imposes Space And Time On The Manifold Of Sensation, It Thus Follows That A Priori Judgments About The Nature Of Spatio-Temporal Relations Are Possible That Anticipate The Structure Of Actual-Space Times Without Directly Experiencing These Space-Times. Why? Because Any Manifold Of Sensation Must Necessarily Be Structured By These Forms Imposed By Intuition Onticology– A Manifesto For Object-Oriented Ontology Part I Posted By Larval Subjects Under Object-Oriented Philosophy. ……. it is necessary to distinguish the being of objects from the manifestation of objects. While objects are acts, these acts are not identical with their performance in either nature (events where no humans are about to perceive them) or with their performance for humans. Rather, the proper being of the object is not its performance or manifestation, but the generative mechanism that serves as the condition under which these performances or manifestations are possible. As Graham Harman will argue– though in a very different theoretical constellation –the being of objects is essentially withdrawn or hidden. No one has ever perceived a single object, but we do perceive all sorts of effects of objects. Traditional epistemology has confused these effects with the objects themselves. Fortunately we do occasionally manage to cognize objects through a sort of detective work that infers these generative mechanisms from their effects; without, for all this, ever exhausting the infinity of a single object. At any rate, if objects were not withdrawn in this way, the practice of experiment would be unintelligible. This leads to Bhaskar's answer to the first question: What must the world be like for science to be possible? Note, this is not a question about mind or culture, but about the world itself, regardless of whether or not humans exist. Once again, knowledge is an accident of objects, not objects an accident of consciousness or cognition. If science is to be possible– and I would argue, if any human practice is to be possible –then the world must be structured and differentiated. The world must have joints or, as Harman puts it, the world must be composed of "chunks". Why is this case? Let us return to the question of experimentation and the conditions under which experiment is possible. We will adopt two possible hypotheses pertaining to the ontological nature of the world independent of humans: 1) As certain mystics and contemporary crypto-mystics would have it, the world is an undifferentiated One-All that is only subsequently segmented or partitioned into discrete beings by some form of human agency whether this be through cognition or language (in the case of language we might think of Saussure's and Hjelmslev's undifferentiated "sonorous matter"). 2) Entities are the sum totality of their relations to all other entities in the universe. The first hypothesis is easily dispatched on two grounds: First, this hypothesis fails to register the contradiction in its own utterance. At the level of explicit content, it claims that the world is an undifferentiated One-All that is only subsequently segmented into discrete beings, yet what it misses at the unconscious level of its own utterance is that it registers at least one structured differentiation that is not undifferentiated within this One-All: Namely, the agency through which the One-All subsequently comes to be differentiated. Certain anti-realist, transcendental philosophers will, in a gesture that is all too cute, claim that the agency by which the world is segmented cannot properly be said to exist, thus attempting to resolve this contradiction through a sleight of hand. However, as Meillassoux has shown in After Finitude, all attempts on the part of transcendental anti-realist philosophies to treat the transcendental subject as a non-existing or non-objectile agency that does not itself exist end up, all too clearly, attaching that conception of finitude and the segmentary work with which it is charged to the body (a differentiated being or generative mechanism). Second, suppose the anti-realist transcendental philosophers were to convince us through his appeals to the non-existence of the transcendental, would we still encounter problems? Like Atlas, transcendental subjectivity is charged with the

monumental task of segmenting the formless Apeiron of the One-All from out of primal chaos into a supremely segmented world. But this world appears to us to be too slippery for even a titan like Atlas to grasp. Were the world, prior to and independent of humans genuinely a formless Apeiron it would contain no differences providing hand-holds for Atlas to grasp in his segmenting activity. Consequently, no segments could ever come into being. Yet everywhere we encounter segments, so the world must not be a formless One-All, but must rather be structured and differentiated even if structure and differentiation are transformed in their encounter with the human. Onticology– A Manifesto For Object-Oriented Ontology Part I Posted By Larval Subjects Under Object-Oriented Philosophy. The neural networks of the human brain act as very efficient parallel processing computers co-ordinating memory related responses to a multitude of input signals from sensory organs. Information storage, update and appropriate retrieval are controlled at the molecular level by the neuronal cytoskeleton which serves as the internal communication network within neurons. Information flow in the highly ordered parallel networks of the filamentous protein polymers which make up the cytoskeleton may be compared to atmospheric flows which exhibit long-range spatiotemporal correlations, i.e. long-term memory. Such long-range spatiotemporal correlations are ubiquitous to real world dynamical systems and are recently identified as signature of self-organized criticality or chaos. The signatures of self-organized criticality i.e. long-range temporal correlations have recently been identified in the electrical activity of the brain. A recently developed non-deterministic cell dynamical system model for atmospheric flows predicts the observed long-range spatiotemporal correlations as intrinsic to quantum-like mechanics governing flow dynamics. The model visualizes large scale circulations to form as the result of spatial integration of enclosed small scale perturbations with intrinsic two-way ordered energy flow between the scales. Such a concept maybe applied for the collection and integration of a multitude of signals at the cytoskeletal level and manifested in activation of neurons in the macroscale. The cytoskeleton networks inside neurons may be the elementary units of a unified dynamic memory circulation network with intrinsic global response to local stimuli. Cite as: arXiv: chao-dyn/9809003.The neural network of the human brain responds as a unified whole memory bank to a multitude of input signals from the environment and functions with a high degree of robustness and stability. The three aspects of neural networks memory bank are, storage, real-time update and retrieval. The memory is believed to be embedded in the strength of the numerous connections or synapses in the network. Sensory inputs (electrical) produce particular patterns of activity in groups of neurons which then trigger optimal response to the input signal. The cooperative response of millions of neurons to a multitude of input signals has been compared to a very efficient parallel processing computer with neurons and their synaptic connections as fundamental units of information processing, like switches within computers. However, recent studies by Hameroff et al [1,2] and Rasmussen et al [3] show that neurons and synapses are extremely complex and resemble entire computers, rather than switches. The interiors of neurons (and other eucaryotic cells) are now known to contain highly ordered parallel networks of filamentous protein polymers collectively termed the cytoskeleton. Information storage, update and appropriate retrieval are controlled at the molecular level by the neuronal cytoskeleton which serves as the internal communication network within neuron. Organization of information at the molecular level in the cytoskeletal network contributes to the overall response of each neuron and the collective activity pattern of neurons then governs the response to the environmental stimuli. The general awareness or consciousness of the individual to environment may also be governed by the overall background activity pattern of the neurons and their cytoskeletal networks. Coherent signal flow patterns in neural networks may form the basis for general consciousness and response to stimuli (external or internal). Inputs signals trigger spontaneous appropriate coherent pattern formation in the activity of the neurons with implicit spatial correlations in the activity pattern. The time variation of electrical activity of the brain as recorded by the Electro Encephalogram (EEG) exhibits fluctuations on all scales of time, i.e. a broadband spectrum of periodicities (frequencies) contribute to the observed fluctuations [4]. Power spectral analysis which is used to resolve the component frequencies (f) and their intensities shows that the intensity (power) of the component frequencies follows the inverse power law form 1/fB where B is the exponent. Inverse power law form for power spectra of temporal fluctuations imply long-range temporal correlations, i.e. long - term memory of short - term fluctuations. The signatures of short - term fluctuations are carried as internal structures of long - term fluctuations. Time variation of spatial activity pattern in neural networks therefore has inbuilt long - term memory.

Neural network activity patterns therefore exhibit long - range spatial and temporal correlations. Such non-local connections in space and time are ubiquitous to time evolution of spatially extended dynamical system in nature and are recently identified as signature of self-organized criticality [5]. Examples of dynamical systems, i.e. systems which change with time include atmospheric flows, electrical activity of the brain, heart rhythms, stock market price fluctuations, etc. Extended dynamical systems in nature have self similar fractal geometry. Self similarity implies that submits of a system resemble the whole in shape. The world fractal coined by Mandelbrot [6] means fractional or broken Euclidean geometry appropriate for description of non-Euclidean structure generic to natural phenomena. The fractal dimension D is given by dlnM/dlnR where M is the mass contained within a distance R from a point within the extended object. A constant value for D implies uniform stretching on logarithmic scale for length scale range R. Objects in nature exhibit multifractal structure, i.e. the fractal dimension D varies with length scale R. Fractal architectures generic to nature support functions which exhibit fluctuations on all time scale, i.e. the fluctuations are irregular (nonlinear) and apparently chaotic. The association of fractal structures with chaotic dynamics has been identified in all dynamical systems in nature. Fractals, chaos and nonlinear dynamics or Chaos Science is now an area of intensive research in all branches of science [7]. Incidentally, Chaos Science began in 1963 with identification of sensitive dependence on initial conditions resulting in chaotic solution for computer realizations of deterministic nonlinear mathematical model of atmospheric flows and named appropriately deterministic chaos .The computed trajectory of time evolution exhibits fractal geometry. The discipline of nonlinear dynamics and chaos began with investigation of universal characteristics of deterministic chaos in nonlinear mathematical models of dynamical systems in all branches of science. In mathematics, the Cantorian fractal space-time is now associated with reference to quantum mechanical objects [8,9,10]. Further, El Naschie has shown that fractal structures (space-time) incorporate the golden mean equal to (1+Ö 5)/2 =1.618 in their architecture signifying ordered signal /information flow in the fractal network. The golden mean is incorporated in the fractal architecture of the cycloskeleton network [11] which plays a very important role in sub-consciousness to consciousness process integration [12,13]. Surprisingly similar chaotic behavior in space and time was found to be exhibited by all real world dynamical systems. Fractal structure to the spatial pattern concomitant with chaotic (irregular) dynamics has now been identified to be intrinsic to physiological and biological systems [14,15]. The branching interconnecting networks of neurons and intra-neuronal cytoskeleton networks are fractal structures which generate electrical signal pattern with self-similar fluctuations on all scales of time characterised by 1/fB power law behavior for the power spectrum. Such inverse power law form for spectra of temporal fluctuations implies long-range temporal correlations, i. e., long term memory of short term fluctuations or events. Fractal architecture of neural networks supports and coordinates information (fluctuations) flow on all time and space time scales in a state of dynamic equilibrium, now identified as self-organized criticality, is ubiquitous to natural phenomena (living and non-living) and is independent of the exact details of the dynamical processes governing the space-time evolution. The physics of self-organized criticality or deterministic chaos is not yet identified. The physical mechanism governing self-organized criticality should be universally applicable to diverse biological, physical, chemical and other dynamical systems. In this paper a universal cell dynamical system model for self-organized criticality applicable to neural networks of the brain is summarised [16, 17, and 18]. This model was originally developed to explain the observed self-organized criticality in atmospheric flows [19, 20, and 21]. Therefore a brief description of the model with respect to atmospheric flows in first described followed by application to neural networks. Atmospheric flows exhibit self-organized criticality or long-range spatiotemporal correlations manifested in the self similar fractal geometry to the global cloud cover pattern concomitant with inverse power law form 1/fB for power spectrum of temporal fluctuations in meteorological variables such as temperature, pressure, etc. documented by Tessier et al [22]. The co-operative existence of fluctuations ranging in size (duration) from the turbulence scale of millimetres (seconds) to the planetary scale of thousands of kilometres (years) contribute to coherent weather pattern in atmospheric flows. Townsend [23] postulated that large eddies (waves) form in atmospheric flows as a chance configuration (envelope) of enclosed turbulent (small scale) eddies. A hierarchical continuum of eddies is therefore generated with larger eddies enclosing smaller eddies. Since large eddy is but the integrated mean of enclosed turbulent eddies, atmospheric eddy energy (kinetic) distribution follows normal distribution characteristics

according to the Central Limit Theorem in Statistics. The eddy kinetic energy represented by square of eddy amplitude then represents the probability density. Such a result that the additive amplitudes of eddies, when squared, represent probability densities is observed in the subatomic dynamics of quantum system such as the electron or photon. Atmospheric eddy energy spectrum therefore follows quantum-like mechanical laws [19,20,21]. Condensation of water vapour in updraft regions of large eddies give rise to cloud formation while adjacent downdraft regions are associated with evaporation and cloud dissipation, thereby accounting for the discrete cellular structure to cloud geometry.\ Under the head of applications authors' describe the following: Frohlich [24] had described analogous self-organization of vibrational modes of all frequencies triggering coherent activity in biological functions. Insinna [25] has summarized Froehlich's coherent excitation concept as follows. More than 20 years ago Frohlich [24,26-28] introduced the concept of cooperative vibrational modes between proteins. Coherent oscillations in the range of 1010- 1012 Hz involving cell membranes, DNA and cellular proteins could be generated by interaction between vibrating electric dipoles contained in the proteins as a result of nonlinear properties of the system. Through long-range effects proper to Froehlich's nonlinear electrodynamics a temporospatial link, is, in fact, established between all molecules constituting the system. Single molecules may thus act in a synchronized fashion and can no longer be considered individual. New unexpected features arise from such a dynamic system, reacting as a unified whole entity [25]. Coherent Frohlich oscillations may be associated with the dynamical pattern formation of intraneuronal cytoskeletal architecture which coordinates and integrates information flow into the neuron and generates output signal. Hameroff and colleagues [1,29-31] have simulated such interaction in their cellular automata model. Conclusion they draw in is herein below: Fractal architecture to information flow path results in spatiotemporal integration of signals so that the fractal system responds as a unified whole to a multitude of input signals. Two disparate examples for such self-organized information flow networks are atmospheric flows and the neural networks of the human brain. **Cantorian Fractal Spacetime and Quantum-like Chaos in Neural Networks of the Human Brain A. M. Selvam (for reference s please see the original article. Models are given elsewhere in the papers series .** If The Holographic Principle Is True, Than It Must Be The Fundamental Principle Of Mind. The Brain Has No Way Around The Holographic Principle. For The Reductionist, The Holographic Principle Is The Ultimate Reduction. It Applies To The Most Minuscule Level Of What We Can Observe, And Beyond. For The Universalist, The Holographic Principle Gives Us The Ultimate Universal. It Extends To The Limit Of Our Universe, The Universal Holographic Boundary, And Beyond. For The Phenomenalist, The Holographic Principle Gives Us The Ultimate Ground Of Our Phenomenal Perception, Our Grounding In The Universal "Now," In The Now-Present Of The Universal Holographic Boundary, Moving Outward From Now-Pasts To Now-Futures. Wheeler (1988) Has Said That All Of Reality Is Information, And That Other Physical Quantities Are "Mere Incidentals." Information Monism Is Gaining Popularity In Physics. By Breaking The Dichotomy Of Between Information And Experience, We Find A Deep Connection Between Wheeler Is Monism And The Experiential Monism Of Whitehead, Sometimes Called Panexperientialism, Which Relates To The Later Theological Concept Of His Student, Charles Hartshorne, Panentheism, God Inside Of Everything (Hartshorne, 1964). There Thus Seems To Be A Convergence Of The Concepts Of Information, Experience, And Spirituality. There Is Not One Universe, But Many Parallel Universes, Or, More Accurately, A Vast Superposition Of Universes Called The Multiverse (Penrose, 2004). However, As Explained In Our Treatment Of The Double-Slit Experiment, In The Observation Of Events On The Quantum Level, The Individual Observer Sees Only One Of These Vast Superpositions, Not A Summation Of A Vast Number Of Potentials. This Has Been Explained In Terms Of The Many Worlds Or Many Minds Theories, In Which There Are Multiple Copies Of The Same Observing Individual, But This Particular Issue Remains Unresolved, Leaving Physics Ungrounded In Reality (Penrose, 2004). The Concept Of One Mind Is Not Only More Parsimonious Than That Of Many Minds, But Also Lets Us Out Of The Bizarre And Counterintuitive Idea That There Are Multiple Copies Of Our Own Selves, Which Exist As Mere Potential, And Are Thus Not Actual. There Are Many Possible Universes, But There Is Only One Mind, Which Determines Events On The Quantum Level, And Thus Creates Our Universe. As We Had Discussed Previously, The Quantum Level Provides The Essential Ground Of The Holographic Principle, Such That Quantum-Level Holographic Surfaces Are Elaborated At Higher Levels, Manifesting Higher Orders Of Information From The

Quantum "World." In This Process, There Is A Reduction Of The Wave Function Or Potentialities Of Quantum Fields. At The Level Of Consciousness, This Entails Freedom To Choose Which Observations We Make (Stapp, 1997), Which Gives Us The Capacity To Think And To Make Decisions. These Capacities Are The Basis Of Individuality, Self-Determination, Judgment, And Values. The Progressive Evolution Of The Manifestation Of Mind Through Higher Orders Of Experience, Leading To Higher Orders Of Consciousness, Is Entailed By The Holographic Principle. Again, We Are Dealing Here With Levels Of Description, With The Multiverse Of All Potentials Fundamentally Supporting The Single Universe We Collectively Observe. The Multiverse Is The Wave Function Of The Universe (Penrose, 2004). The Recursive Integration Of Nested Hierarchies Of Holographic Surfaces Brings Out A Single Actuality In Consciousness From A Wave Function That Is Unconscious. Consciousness Thus Gives Us Information At A Level Of Experience That Is Causal. In This Sense, We Partake In The Creation Of The Universe. We Participate In Creation, And This Participation, When Fully Realized, Leads Us To Higher Levels Of Consciousness And Of Realization. As A System, The Biosphere That We Live In Has A Holographic Surface, Creating A Deep Sense Of Ecology As We Collectively Move Toward A Planetary Consciousness. It Is Only When The Universal Holographic Boundary Reaches The Information Storage And Processing Capacity Needed For The Requisite Biological And Biochemical Complexity That Consciousness Evolves In Living Things. This Evolution Is Natural And Spontaneous, Since Consciousness Gives Rise To What Is Actual, As Opposed To What Is Merely Potential. In This Sense, Consciousness Is Still In The Process Of Creating Our Universe, And Levels Of Higher Consciousness Will Continue To Evolve. We Are All In The Same "Now," And That Now Is Defined By The Present Universal Boundary. This Assures Us That Our Experience Is Universal, And Does Not Pass With Time. This Is The Fundamental Basis Of Memory And Of Cognition. The Identity Of Mind And Brain Is A Myth. We Have A Continual, Internal Or Non-Local Relation With The Universe, As It Has With Us. Once This Mystery Is Resolved, The Myth Is No Longer Needed, And There Is A Confluence Of Science And Spirituality.Experience Is Primary, Information Is Secondary. We Can Only Gather Information From Experience, Whether It Is In The Laboratory Or In Life. We Cannot Measure The Information On The Surfaces Of Systems. The Physicists That Have Formulated The Holographic Principle For All Systems Are Quite Aware Of This, Or Else The Principle Would Have Been Established Or Discredited. We Cannot Measure What We Experience. It Is Intangible, Yet It Is All We Know To Be Actual. Everything Else Is Inferred. Because It Cannot Be Measured, It Has Been Fundamentally Disregarded By Mainstream Science. Materialism Is Considered Scientific, While Idealism Is Considered Unscientific. But Aren't Ideas, Fundamentally, Made Of Information? Brain Science Has Mistaken The Representation Of Information For Information Itself, And Has Tied Those Representations To Matter And Energy. Consciousness, The Highest Order Of Information, Has Generally Been Regarded As Superfluous, Something That Needs To Explained Away, Or Altogether Ignored. Yet It Is The Only "Thing" That Reaches Our Awareness. Consciousness Comes At A Price. For Everything That Becomes Conscious, There Must Be Something That Becomes Conscious. Consciousness Is Certainty, And Its Complement, Unconsciousness Uncertainty. Consciousness Is The Particle Nature Of Experience. It Has Definiteness About It. The Unconscious Is The Wave Nature Of Experience, It Is Like The Metaphysical Cloud Of Unknowing. If We Are The Most Conscious Of Animals, Then We Must Also Be The Most Unconscious. Perhaps This Is The Predicament Of Humankind. The Holographic Principle Of Mind Leads Us Naturally From The Most Fundamental Experiences, Existing As Quantum Potentials From The Conformations Of Proteins Down Through The Fields Of Electrons, Through Their Manifestation Upward Through A Recursion Or Successive Applications Of The Same Holographic Process, Through Higher Levels Of Experience, To The Emergence Of Consciousness As Higher And Higher Orders Of Experience. The Quantum Holographic Principle Mind Does Not Require Anything More Quantum Than Is Obviously Present At The Microscopic And Submicroscopic Levels, As It Represents Successive Orders Of Manifestations From These Levels. Recursion Also Applies Here In The Sense That It Is Used In Computer Science, In Which The Function Of The Part Depends On The Function Of The Whole. A Program, As A Part, Cannot Work Without A Functioning Whole, The Operating System Recursive Wholes Which Are, For Us, Supra-Conscious, Are On The Group, Species, Planetary, And Universal Levels. As Individuals, Our Consciousness Cannot Function Without Recursion To The Universal Consciousness, Even Though We May Be Unaware That

Such Universal Consciousness Exists. Reaching Upward To These Supra-Conscious Levels Is A Spiritual Process, Making Transcendence The Ultimate Solution To The Unconscious Human Predicament. What Was Once Supra-Conscious Becomes Unconscious Through A Process Of Conditioning? We Are Born As Creatures Of The Earth And Of The Universe, As Evidenced By The Beliefs And Practices Of "Primitive Peoples." There Is Evidence From Cave Paintings That Our Hominid Ancestors Experienced A Kind Of Holographic Perception (Combs, 1996), Which Could Constitute Our Early Connection With The Holographic Subtext Of Reality, And Which We Might Then Propose Existed In Our Animal Ancestors, And In Extant Animal Species. If This Is The Case, Than Microgenesis Would Entail The Recapitulation Of This Holographic Experience As It Progresses Through Our Ancestral Past. The Supra-Conscious Mind Seems To Envelope Perinatal Experiences, And Stanislav Grof (1994) Has Developed Techniques To Gain Access To These Experiences, As Well As To The Earlier Experiences Of Our Human And Animal Ancestral Lineage, And Of Our Universal History. Grof (1994) Concludes: "Our Consciousness Seems To Have The Amazing Capacity To Directly Access The Earliest History Of The Universe Ñ Witnessing Dramatic Sequences Of The Big Bang, The Formation Of The Galaxies, The Birth Of The Solar System, And The Early Geophysical Processes On This Planet Billions Of Years Ago." The Holographic Principle Theory Of Mind MARK GERMINE Institute For Psycho Science There Are Similarities Between The Patterns Of Holography And Of Psychological Transference, Where Holography Is The Process Of Recording And Reconstructing Holograms Employing A Theoretical Perspective Using A Hermeneutic Method, This Dissertation Parallels Holography With Transference, Offering Another Way To Encounter Transference By Showing Similarities Between The Processes Of Each And The Results Of Each. Though Complex, Infinitely Varying, And Unique, Their Patterns Are Clearly Identifiable. Thus They Are A Metaphorical Fit To The Concept Of Strange Attractors In Physics And A More Literal Fit To The Concept Of Archetypes In Depth Psychology Or Dynamic Psychology, Psychology Which Attends To The Living, Autonomous Unconscious. This Study Explains How Holography Models Transference, What A Hologram Is And How It Works, And How Depth Psychology Understands Of The Interaction Between Consciousness And The Unconscious Is Related To The Hologram. It Describes Transference And Related Psychological Processes As Understood In Six Different Schools Of Depth Psychological Thought. It Shows That The Underlying Pattern Or Strange Attraction Between Transference And Holography Extends To Other Processes Both Within And Outside The Field Of Psychology, Processes Such As Projection, Projective Identification, Splitting, Memory, Biology, Creative Discovery, Theology, Synchronicity, Chaos, And Nonlocality. By Identifying The Similar Patterns Of These Processes, This Study Demonstrates The Existence Of An Underlying Holographic Archetype In Which Essential Qualities Of The Whole Are Present In Each Of The Parts Of The Whole: The Visual Image Of The Overall Hologram Is Present In Each Component Part Of The Hologram, The Autonomy Of The Overall Human Is Present In Each Conscious And Unconscious Component Part Of The Human Psyche. By Noting Differences As Well As Similarities In These Processes, It Suggests An Inventory Of The Qualities Of The Holographic Archetype. This Study Furthers Understanding Of The Pervasiveness, Force, And Autonomy Of The Unconscious Acting Through Transference And Projection By Identifying A Group Projection Of Domestic Violence Lying At The Core Of The Christian Myth. This Study Also Furthers Understanding Of The Concept Of Transference By Providing A Reflection Hologram Of The Human Psyche As An Artistic Work And As A Visual Metaphor Of Transference. **Strange Attractors: Transference, Holography, And An Archetype Burke, J. (2003). Strange Attractors: Transference, Holography, And An Archetype (Doctoral Dissertation, Pacifica Graduate Institute, 2003).** A host of observed, but very basic human phenomena, including consciousness itself,have eluded rigorous scientific description by all disciplines of science. This is true, not because of insufficient evidence for a particular phenomenon’s existence, but rather forlack of a theoretical construct, which could fit within the prevailing paradigms of science. For the past century eminent men and women of science have accumulated thousands of pages of data on mind/mind and mind/matter interactions. Many of the most telling experiments have been criticized, perfected and repeated numerous times during the past five decades, using increasingly sophisticated technologies. Meta analysis of these experiments produce accumulated probabilities against chance occurrences exceeding trillions to one (Radin, 1997). It has required, however, that quantum science mature for seventy-five years and during that period, test, validate and synthesize a

number of seemingly outrageous physical concepts arising from quantum theory, before testable theories could arise which offer hope that anomalous mind and consciousness data can be explained (Mitchell, 1996).The missing concepts that prevented the earliest investigators of consciousness from succeeding in their quest were 1) a generalized theory of information, and 2) quantum science itself, with the associated phenomena of non-locality, the zero point energy field and the quantum hologram. These associated phenomena are still not well understood butare sufficiently validated today by both theory and experiment to provide a basis for postulating a necessary condition for the existence of consciousness phenomena, as experienced in the observable four dimensional space/time universes. A third concept, chaos theory, is also necessary to understand the nonlinear evolutionary processes that caused consciousness to evolve toward the anthropic consciousness experienced by humans. In particular, chaos theory maps far from equilibrium systems and demonstrates the irreversibility of nonlinear processes and thus the irreversibility of time in the macro-scale universe. Another class of phenomena, including normal sensory perception and evolution, to cite but two, have explanatory theories in classical science, but which in view of current developments in late quantum physics and in chaos theory may be incomplete approximations to the correct theory. Information concepts have been examined by Weiner, von Neuman and Shannon in well-known seminal works and by Frieden more recently (1998) to produce theories useful to physics, to computation and to communications technologies. These theories, although accurate and mathematically useful in their domains, fall short of being sufficiently encompassing when considering the problem of consciousness, its evolution and its associated phenomena. Even relatively simple perceptual organisms utilize patterns of energy, that is, information, not completely described by existing mathematical theories. Theory and experimental evidence for the zero point energy field has been published by many authors, but I shall cite Haisch, Rueda and Puthoff, (1997, 1998), as the most contemporary and relevant work for this paper. Theory and experimental evidence concerning the quantum hologram has been developed by Schempp (1992, 1993) andMarcer (1996, 1997, 1998), separately and jointly, based upon a new understanding ofquantum mechanics. (See previous work by Cramer [1986], Berry [1988], Anandan [1992] and Resta [1997]).Non-locality, although predicted by the earliest work in quantum theory and decisively demonstrated by Aspect in 1982, has been thought to be a curious property of particle physics but of little relevance to macro-scale reality until discovery of the quantum hologram. Further, it is widely believed that non-local quantum information represented by entanglement of particles could not be recovered locally as useable information (Eberhard's theorem). However recent work both in theory and experiment (e.g. see Nature, 1997, 11th December, vol. 390, Sudbury T pp551-552, and Bouwmeester D. et alp 575-579) is in line with the work by Berry, Resta, Schempp and Marcer and makes it clear that this is not the general case for quantum information processing andcommunication.It has been widely accepted in science, until recently, particularly in the field of artificialintelligence, that the brain was likely a complex classical computer, incapable of supporting quantum processes. The work by Hammeroff (1994) and Penrose in isolating and describing microtubules in brain tissue have caused a re-examination of this dogma, and renewed interest in uncovering the quantum processes involved.Based upon this earlier work I postulate and examine the evidence in this paper for thefollowing theories: 1. The basis of subjective experience is rooted in the quantum attribute of naturecalled non-locality. I will use the word "perception" in its most generic sense to denote a basic subjective experience at all levels of complex matter. Thus the non-local quantum correlation between entangled quantum particles is considered the root cause of the phenomenon experienced as perception in more complex matter, but the non-local quantum hologram is the non-local carrier of information for molecular and larger scale matter. Thus, perception is not an object but rather the label for a nonlinear process involving an object, a percipient and information.2. The experience of humans is that they sometimes, perhaps often, perceive information from or about physical objects that is not available through normal, local, sensory mechanisms, nor classical space/time information. Objective testing data in overwhelming abundance provides evidence that this is true, though an explanatory mechanism has until contemporary times remained elusive. I shall call this intuitive information or intuitive perception. I postulate that a quantum hologram is the source of this intuitive perception and that the percipient is at that time in phase-conjugate-adaptive-resonance (pcar) with the entity or object associated with the quantum hologram.3. The phenomenon of "learning" in humans is a subjective process that involves perception, memory, intentionality, and evaluation of outcome and behavior change.

This may be viewed as a classical nonlinear feedback loop. Although we cannot know precisely the subjective experience of another entity, presumably in the successful training of animals, an analogous subjective process is in effect. Sheldrake (1981) has published a successful theory of morphic resonance related to animal learning based upon non-local information. Marcer has published papers (1996, 1997) theorizing a mechanism by which the quantum hologram causes learning to take place in both DNA molecules and prokaryote cells as an adaptation process of environmental resonance, rather than mutation and adaptation solely by random processes. I postulate that Marcer's concept can be generalized to nature alarge and that the quantum hologram is the information structure suitable to explain Sheldrake's morphic resonance. The non-local quantum correlations observed in particles, and the non-local quantum hologram associated with molecular and larger scale objects, serve the purpose of providing information at all scale sizes to guide evolutionary processes. That is to say, that quantum non-locality is the basis of perception, and thus fundamental and necessary to the complex organizations of matter and information in the universe. Further, since learning is an observed property of complex systems such as animals and, via the quantum hologram, is theorized to be a property of simple cells and molecules, one can also postulate the generalization that nature evolves through a learning process rather than because of random mutations.4. Marcer (1997) has proposed that the condition of phase-conjugate-adaptive-resonance (pcar) is a necessary condition for an object in three-dimensional reality to be perceived as it really is. That is, resonance requires a virtual path mathematically equal but opposite to the incoming sensory information about the object. Further, that it is the incoming electromagnetic (space/time) information (visual, acoustic, etc), which decodes the information of the quantum hologram and establishes the condition of pcar so that accurate three-dimensional perception is possible. That is to say, both quantum information and space/time information aroused in the act of perception by organisms having sensory preceptors. I propose that the two equal but opposite paths required by the pcar condition are the mathematical equivalent of perception and attention (or intention). (I shall distinguish between attention and intention in following pages.) Discussion The anecdotal evidence for humans perceiving non-local information dates to prehistory. The data were sufficiently robust that both experiences and philosophers, from Plato and Aristotle forward, accepted that both physical and non-physical realms of reality must exist. Non-physical was thought to explain the subtle, ephemeral and mystical subjective experiences ubiquitously reported in human culture. After Descartes and Newton, however, classical western science rapidly discarded the non-physical hypothesis and systematically began to ignore all evidence for perception of non-local information Field theories and point particles were created to preserve the concept of physical contact between particles and to explain obvious examples of "spooky action at a distance" such as gravitation and electromagnetic interactions. Information, broadly defined as patterns of energy, reemerges however, in non-local form in the mysterious quantum spin correlations of double slit experiments, although it has been widely believed that such non-local information could not be recovered and utilized by sensory systems. With validation of theory and experiments concerned with the non-local quantum hologram, information, including non-local information, suddenly acquires a more important status in physical theory, a status as important as energy itself. This is true because information is the basis of the cognition and knowing by which creatures perceive reality, and non-local information can now be seen as a ubiquitous and useful property of the cosmos, rather than a unique attribute of particles (and human animals). It is likely that most, if not all, subtle, ephemeral and unexplained phenomena associated with subjective experience are connected, directly or indirectly, with the phenomenon of non-locality. The brain is clearly a quantum computer (Schempp & Marcer, 1996) which utilizes both quantum and space/time information. This discovery alone almost certainly sets a necessary, but not sufficient condition, for intelligent life to have arisen in the cosmos, wherever environmental conditions permit. Many volumes have been written in this century by scientists experimenting with remote viewing, ESP, telepathy, clairvoyance, precognition, etc. Police agencies routinely use "psychics" to assist in criminal cases often with success. Intelligence agencies of governments have clandestinely utilized the findings to successfully gain information about an adversary. Many reports of these activities have been recently declassified and printed in open professional journals, even though no explanatory physical mechanism has yet been reported which is acceptable to mainstream science. The most succinct modern summary of this activity and analysis of results have been published by Radin (1997). Quantum Holography Non-locality and the non-local quantum hologram provide the only testable mechanism discovered to

date which offer a possible solution to the host of enigmatic observations and data associated with consciousness and such consciousness phenomena. Schempp (1992) has successfully validated the concept of recovery and utilization of non-local quantum information in the case of functional Magnetic Resonance Imaging (fMRI) using quantum holography. Marcer (1995) has made compelling arguments that a number of other chemical and electromagnetic processes in common use have a deeper quantum explanation that is not revealed by the classical interpretation of these processes. Hammeroff (1994) and Penrose have presented experimental data on microtubules in the brain supporting quantum processes. The absorption/re-emission phenomena associated with all matter is well recognized. That such re-emissions are sufficiently coherent to be considered a source of information about the object is due to the theoretical and experimental work of Schempp and Marcer, based upon the transactional interpretation of quantum mechanics of Cramer (1986), the Berry geometric phase analysis of information (Berry, 1988; Anandan, 1992) and the ability of quantum phase information to be recovered and utilized (Resta, 1997). The mathematical formalism appropriate to these analyses is consistent with standard quantum mechanical formalism and is defined by means of the harmonic analysis on the Heisenberg nilpotent Lie group G, algebra g and nilmanifold (see Schempp (1986) for a full mathematical treatment). The information carried by a quantum hologram encodes the complete event history of the object with respect to its three dimensional environment. It evolves overtime to provide an encoded non-local record of the "experience" of the object in the four-dimensional space/time of the object as to its journey in space/time and the quantum states visited. The question of the brain's ability, as a massively parallel quantum processor, to decode this information is addressed by Marcer and Schempp in "Model of the Neuron Working by Quantum Holography" (1997) and "The Brain as a Conscious System" (1998). They argue that an organism's ability to perceive objects as they are and where they actually are in three-dimensional reality requires the phase conjugate relationship provided by quantum holography. It is not sufficient for the incoming electromagnetic illumination (or acoustic signal) carrying object information to present tothe brain a wave front in the manner presented to a flat photographic plate. Rather, a virtual signal as mapped by the phase conjugation of quantum holographic formalism is required to decode the information in order for perception and cognition to exist as we experience it in three dimensional realities. The percipient and the source of information are in a resonant relationship for the information to be accurately perceived. Many investigators have proposed a holography mechanism as a basis for brain functioning, beginning with Pribram, and indeed, others have proposed holography as a construct forth universe itself, but discovery of the non-local quantum hologram created by the absorption/remission phenomenon and characteristic of all physical objects provides the first quantum physical mechanism compatible with macro-scale three dimensional worlds we experience it. The existence of a quantum hologram associated with each physical object provides each physical object with the non-local waveform predicted by quantum theory's wave/particle duality and extends quantum theory to all physical matter. It allows, for the first time, possible approach for understanding the mysterious world of consciousness. Postulating that this is globally true, we inhabit a quantum world where non-local effects should be expected at all levels of functioning, not just as a curious artifact of the subatomic level of reality. Existence of the non-local quantum hologram suggests that nature has utilized non-local information from the big bang forward, throughout its evolutionary history; and long before planetary environments self organized to permit living matter and complex space/time sensory systems to evolve. The papers of Marcer and Schempp on learning inherent in DNA and prokaryote cells using quantum holography, when generalized, helps explain the ubiquitous appearance in nature across distances, scale sizes and species, of similar processes, organs and sensory systems. This certainly conforms to the fractal geometry of chaos theory. Certainly the similarities, of DNA, cell structure, organs and brains across species are easier to reconcile with a non-local learning process than with a theory of localized random mutation and natural adaptation. It is important to observe that in standard particle physics experiments the object is to discover the quantum characteristics of the individual types of particles, and the conditions under which they split and recombine. In quantum holography the object is to treat the entire group of re-emitted quanta as a whole, and as in laser holography, to examine the information carried in the interference pattern and phase relationships. These represent two quite different levels of approach to quantum information. In particle experiments, it is considered that the eigenvalues of the applicable matrix represent measurable values; and that information is lost during measurement due to decoherence of the

particles and energy exchange. But in the quantum holographic formalism, the information is carried in the phase relationships, which are represented by off-diagonal terms in the matrix, and the information is recoverable under the proper conditions as Berry and Resta have predicted, and as Schempp has demonstrated with fMRI. The quantum mathematics is consistent with standard quantum theory in both cases. In decoding the quantum holographic information, however, the energy exchange is insignificant. The similarity of the mathematical treatment in these various experiments is important tothe thesis of this paper. In examining the quantum non-locality of particles it is spin numbers and/or polarization that are the parameters of interest. A standard technique of analysis is to use the Fourier transform to map the state of the particles into the frequency domain. In the formalism of the quantum hologram, mapping into the frequency domain is also fundamental; however, the requirement for pcar assures that the phase relationships are matched so that the percipient (sensory system) is able to decode the information carried in the phase relationships. It is precisely the pcar requirement that permits the encoded holographic information to be decoded by the percipient. Mathematically, decoding is simply reversing the rotation of the phase vector in phase space. Physically, it is matching the frequencies and phase of the information such that resonance results. Frequency, phase matching and resonance are an operational characteristic of every type receiver technology. Pribram's earliest proposal that the brain stored information encoded as in a hologram and mapped by the Fourier transform is in complete agreement with the evidence presented by quantum holographic mathematical formalism. It is the spin and polarization attributes of particles (both are mapped by wave mathematics) that represent the puzzling non-local property of subatomic matter. It is the phase relationships that carry the information in holography (again mapped by waveform mathematics). And it appears that the brain stores and manages information not as a classical digital machine, but rather as an analog device using non-local properties of thequantum hologram, which can be analyzed by wave form mathematics (harmonic analysis on the Heisenberg Lie group). In the cosmological evolutionary scheme of things this similarity of appropriate mapping techniques is too bizarre a coincidence to be ignored Asa cosmic accident. Thus there is ample evidence that the **non-local attribute of** nature is much more than just a curious artifact of subatomic particle interactions, but rather is a more fundamental phenomenon that appears at all scale sizes and is, in particular, associated with the utilization of information in nature, and associated with the fact that information has a causal effect independent of distance. It is precisely information, however, that is the basis of the phenomena of perception, cognition, memory, learning etc, that is to say, consciousness and the subjective experience. Though the evidence is quite ample to postulate that non-locality is the unique, universal basis for perception and the subjective experience, the evidence though compelling is not sufficient to be conclusive that such is indeed the case. The next steps are to validate more completely with experimental evidence that non-locality plays a major role at all scale sizes and that all physical objects are quantum objects and thus interconnected by information in this strange way. **Non-Locality in Nature**: There is experimental evidence to strongly suggest that simple organisms perceive and respond to information non-locally as well. Cleve Backster was perhaps the earliest to experiment with plants and simple life forms in electromagnetic isolation in late 1960'sand early 1970's. His work was not confirmed through replication by others at that time. Other investigators have had mixed results replicating non-local information perception by simple organisms and living tissues. In the area of human experimentation, results likewise have been mixed and controversial for three-quarters of a century. However Meta analysis by Radin (1997) and independently by Utts (1991) across a large and appropriate spectrum of experiments demonstrates compelling statistics that the **perception of non-local information exists and is real.** Perhaps was there a larger body of experimental evidence for simple life forms, similar Meta patterns would emerge. Failure to replicate results in well constructed experiments does not, in the case of subtle consciousness phenomena, prove that the phenomenon is missing but rather that a **hidden mechanism** below the threshold of classical measurement is operating. For example, the most telling experimental evidence to explain the sometimes inconsistent results relates to direct non-local observer and/or experimenter effects. Gertrude Schmiedler isolated the **"sheep/goat "**effect in human experimentation decades ago (1972). Experimenters and/or participants in a human telepathy (or similar non-local) experiments exhibited results statistically above or below chance results depending upon their subjective bias toward the experiment. (In other words, 100% wrong answers would be as statistically significant as 100% correct answers in such tests, and in addition betrays the mind set or intention of the subject;

whereas only chance results would be inconclusive.) More recently, a series of experiments by Marilyn Schlitz (1997) investigating "intentionality" clearly demonstrated that experimenter bias (intentionality) affected the outcome even of double blind experiments. Thus, in the subtle realms of mind and consciousness studies, bias, belief and intention clearly have an effect. The lack of an existing theoretical structure in classical science to support any type perception of non-local information, much less to support bias, belief or intention as having a non-local effect, when in fact it does have a non-local effect, is quite sufficient to account for anomalous results in many scientific experiments. Further validation and acceptance of the non-local thesis will have strong positive repercussions for the prevailing scientific paradigm and particularly the theory of measurement. **Attention and Intention:** A powerful and telling series of experiments conducted by Dean Radin (1997) at University of Nevada at Las Vegas following a decade long set of equally significant experiments by Brenda Dunne and Robert Jahn at Princeton University (1988) provide insight as to the subtleties involved in this level of mind/brain functioning. Jahn and Dunne provided overwhelming evidence that subjects could intentionally produce statistically skewed results in mechanical processes normally thought to be driven by random processes. Radin went further; he discovered that audiences watching a stage performance would skew the output of nearby random number generators during periods of high emotional content in the stage performance. Further, in a wide-ranging audience participation experiment, he recorded the output of computer random number generators during the television broadcasts of the O.J. Simpson murder trial. Most television media reported this event for weeks on end and tens of millions of humans were watching the results. Again, the results of the random number generators were skewed corresponding to emotional peaks during the trial drama and corresponding to the number of people watching television. The thesis in the Princeton experiments was that participant intentionality created a non-random effect to bias the skewed distribution. In the Radin experiments the results were not intentional, as the participants were unaware of the experiment, but the hypothesis was that attention (in particular, rapt attention) drove the system away from chaos (randomness) and toward greater order (reduced entropy). These results suggest that attention and intention provide closely correlated outcomes, further, that randomness may not be a general property of nature, but that what is perceived as random noise in a system may be information (a pattern of energy) that is not in resonance at that moment with the particular perceptual system. William Tiller, emeritus professor at Stanford also has performed experiments (1997) that are consistent with these results, though his interpretation of the operating mechanism is somewhat different. These different types of mind/mind, mind/matter experiments have been rigorously and routinely conducted for decades with statistically compelling results but just as routinely dismissed or ignored by main stream science because the implications of non-local action are so foreign to the classical paradigm. However, if we consider that the condition of phase-conjugate-adaptive-resonance is necessary to completely specify the act of perception as described in the mathematical formalism of the non-local quantum hologram by Marcer, then we may also consider the perceived object and the percipient's perceptual system as locked in a resonant feedback loop. The incoming wave front carrying information may be labeled as "perception" from the point of view of the percipient, and the return path required by the resonant relationship may be labeled "attention" (or for subsequent discussion, "intention"). It is a well established principle in the meditative practices of esoteric disciplines that prolonged focused attention on a object of meditation causes the percipient and the object to appear to merge so that a deeper level of information about the object is obtained; information such as history or internal functioning, that would not be available through classical space/time information. The concept of the quantum hologram adequately and completely describes how this phenomenon might take place. Further, it is accepted that the mind/brain is a massively parallel processor, capable of performing many tasks simultaneously and subconsciously (in the right, intuitive part of the brain). Attention (meaning conscious, focused attention) is a unique and singular task that must take place sequentially, mostly in the left cognitive part of the brain. The condition of attention deficit disorder (ADD) is precisely the problem of a percipient being unable to maintain a singular focus for sufficient time to complete a desired task or observation. Thus, the action of focusing attention by apercipient may be construed as a necessary condition for pcar to be established with the perceived object. Non-Locality, Near and Far Marcer has presented the case for the pcar requirement in normal sensory perception (visual and acoustic). A frequent modality used by psychic sensitive individuals to gain information is to physically touch an object. Touching an object

satisfies the pcar requirement and presumably allows the percipient access to information about the object not available from space/time information. Police agencies frequently use this modality with psychic sensitives to gain information about a crime scene, much as they utilize bloodhound to track the scent of an individual, often with considerable success. If, as in the theory of the quantum hologram, the object has been in the presence of the individual about whom information is desired, the event history of the object and that of the individual intersect. The Berry phase information of the object contains its journey in three dimensional space and time, as well as the quantum states through which it has passed on this journey. The sensitive individual, with a honed talent, seems often able to decode useful Berry phase information from the object about the individual sought. It may also be the case with the blood hound, which additional non-local information has been gained about the subject, even though the classical explanation is that the animal is operating only with heightened olfactory sensing. Although perception in the three dimensional world requires and utilizes pcar, most humans, however, do not bring to conscious awareness non-local information when weare routinely operating in three-dimensional reality. We perceive objects as presented by space/time information, that is, shape, color, function (tree, chair, table, etc) but are not usually aware of the additional non-local information. It takes training as provided by many of the esoteric traditions and/or certain naturally sensitive individuals to routinely perceive the non-local holographic information associated with a particular object. There is massive evidence to suggest, however, that the brain has these latter capabilities at birth. Suppression by cultural conditioning in childhood and subsequent lack of practice cause the natural ability for conscious, intuitive perceptions to atrophy. Particularly in western tradition, educational interest has been on the left brain, rational functions rather than right brain, intuitive functions. However, mystic adepts and natural psychics' routinely demonstrate that non-local information is perceptible from physical objects by focusing attention, quieting the left brain and allowing intuitive perceptions to appear. It is the left brain cognitive ability in humans that provides canonical labeling of the intuitive and artistic processes taking place in the right brain. The fact that with training and practice, individuals can recover, deepen and label their individual cognitive access to intuitive, on-local information demonstrates that learning is taking place within the whole brain itself and involves enhanced coherence and coordination between the hemispheres. This process is different and distinct from the left brain function of extending and extrapolating factual data and logical deduction to leap to an "intuitive" conclusion, while omitting the intermediate steps leading to that conclusion. Experimental protocols for remote viewing normally provide clues to the location of the object such as adescription, a picture, or location by latitude and longitude, that is to say, an icon representing the object. These clues seem to be sufficient for the percipient to establish a resonance with the object. Normal space/time information (visual, acoustic, tactile) about the object is not being directly perceived by the percipient, nor does the object usually appear at its physical location in space/time like a photograph or map in the mind. Rather, the information is perceived and presented as internal information and the percipient must associate the perceptions with his/her internal data base of experience in order to cognize and to describe the object's perceived attributes. In the case of complex objects being remotely viewed, the perceived information is seldom so unambiguous as to be instantly recognizable as correct. Sketches, metaphors and analogies are usually employed to cognize and communicate the non-local information. A considerable amount of training, teamwork and experience are necessary to reliably and correctly extract complex non-local information from a distant location. The information appears to the percipient as sketchy, often dream-like, and wispy, subtle impressions of the remote reality. Very skilled individuals may report the internal information as frequently vivid, clear and unambiguous. The remote viewing information, being strictly non-local, and in this hypothesis, the information perceived by quantumholography, is missing the normal space/time components of information necessary to completely specify the object. It has been demonstrated that this intuitive mode of perception can be trained in most individuals. Perhaps additional training and greater acceptance of this capability will allow percipients to develop greater detail, accuracy and reliability in their skill. In principle, training will not only increase the skill and accuracy but should cause the appropriate neural circuitry to become more robust as well. In the absence of space/time (electromagnetic) signals to establish the pcar condition and to provide a basis for decoding the quantum hologram, an icon representing an object seems to be sufficient to allow the brain to focus on the object and to establish the pcar condition. However, a reference signal is also required to provide decoding of the

encoded holographic phase dependent information. Marcer (1998) has established, using Huygen's principle of waves and secondary sources, which any waves reverberating through theuniverse remain coherent with the waves at the source, and are thus sufficient to serve as the reference to decode the holographic information of any quantum hologram emanating from remote locations. The Zero Point Field: The results of the Michelson/Morley experiment banished the concept of aether from early twentieth century physics. However, it left a void as the nature of interstellar space and nothing for propagating waves to wave in. Quantum physics reincarnated the aether as the zero point fields, a seething cauldron of quantum potential and unmanifest energy where particles and antiparticles spontaneously arise and then disappear. The very fabricant structure of space/time itself is again in question; its structure and its metric under intense investigation with far more questions than answers having emerged to date. For the purposes of this paper the relevant issues are two: 1) the emission/absorption phenomenon, and 2) the structure and mechanics of non-locality. Zero point (zero degrees Kelvin) emission and absorption of quanta from all physical objects is a well established phenomenon. It is our view that the zero point fields are the plenum (or cauldron) which supports this absorption and re-emission, and makes the phenomenon of the quantum hologram possible at all temperatures. Although particle experiments are carried out under rigid conditions of temperature and pressure, Schempps experimental work with the fMRI requires no such constraints. There are deep and difficult questions yet to be answered about how the information of thequantum hologram maintains its integrity and is propagated, about how resonance takes place at extremely large distances. There is considerable evidence that intuitively perceived information is truly non-local. It does not obey the inverse square law for space/time energy propagation; it is time independent and cannot be shielded by electromagnetic shielding. Such characteristics are the mark of non-locality. But understanding the mechanics of non-locality (or a visual picture) is missing from standard models. Some physicists turn to superluminal speed of propagation, others to the zero point as a zero dimension, which is resonant with all parts of the universe simultaneously. The issue of instantaneous communication (or at least superluminal communication) of non-local effects on a cosmic scale remains a problem, even though the phenomenon itself is well validated. Perhaps it is a problem of topology. What shape can the universe have such that one point can be in simultaneous contact with all other points? In this regard it is clear that certain problems between quantum mechanics, special and general relativity remain inexistence. Haisch, Puthoff and Rueda continue to investigate the **metrics of the zero point fields** with regard to better defining the unanswered questions about, mass, gravitation and inertia. Perhaps these investigations will also bring answers for how phase related information is propagated non-locally, likely within the zero point fields, and thereby unveil the mechanics of the resonance phenomenon. Further, new investigations reported by Van Flandern (1998) on measurements from orbiting Global Positioning System (GPS)clocks indicate the predictions from Lorentzian relativity to be approximately four times more accurate (0.7% opposed to 3%) than predictions of special relativity. If these measurements are further validated, it implies that Lorentzian relativity with a Hubble absolute rest reference frame, an aether (zero point field), and instantaneous propagation of non-local effects may be the preferred one. If this is the case, then many questions about non-locality would be resolved. Intentionality I have argued that by establishing pcar between a percipient and an object, the phase conjugate (equal but opposite) paths connecting the two can be labeled "perception" and "attention". In the case where the object is a simple physical object (rock, flower, etc.), our interest is on the non-local information perceived by the percipient about the object. However, from the point of view of the object, information about the percipient is also available to the object. The pcar condition is a reciprocal relationship, mathematically. Quantum holographic formalism predicts that the history of events of quantum objects is carried in the quantum hologram, thus we must conclude that the **"attention" focused upon the object causes that event to be recorded in that object's quantum hologram.** Although we cannot query the object about its experience perhaps an experiment such as one utilizing the Aharonov-Bohm effect would detect a phase shift in the object's holographic field. (In this discussion I use anthropic labeling as we are discussing human perception. The phenomena however, are rooted in natural (and primitive) non-local physical processes, which are fundamental. The evolved complexities of perception, cognition, etc, associated with a brain obviously, as yet, have no analogous label to describe the experience of simple objects.)Once the pcar condition is established, the percipient can change its mind state with regard to the object. The perceived information can be operated upon by the brain's function so that

cognition occurs with respect to the perceived information and meaning assigned. Cognition and meaning require finding a relationship between the perceived informationand the information residing in the percipient's memory. The percipient can then form intent with respect to the object. In such case the path I have labeled "attention" could Figure 1 In self aware animals (those with a brain) cognition, meaning and intent with respect to an object can often be described in simple terms, for example: enemy, fight or flight; food, eat; friend, greet, etc. The non-local component of information, although present and creating effect, is operating below the level of conscious perception in humans and results in "instinctual" subconscious behaviors in all animals. The brain as a massive parallel computer is simultaneously performing numerous tasks to accomplish the desired intention. Classical modelings of this autonomous activity describe it only in terms of classical information and energy flow in the central nervous system and the brain. However, if non-locality is operating at all levels of activity, as this theory suggests, certainly there are resonances involving non-local information operating throughout the body of an organism in parallel with classical space/time functions. Subsequent experimental work will surely uncover these quantum processes where non-local resonance is involved in the functioning of an animal's internal processes. In the case of non-local effects at a distance, outside the body, simple spin correlations of entangled particles is the most basic. The spin coherence is reciprocal. Action on one particle creates an effect on other entangled particles. The non-local information is causal of affects at large distances. It is no less important for macro-scale objects. Sheldrake (1995) proposed and others conducted experiments with dogs whereby the animals correctly anticipated their owners' departure from work to return home. He proposed other successful experiments where rats learning a new maze benefited non-locally from the experience of others that had previously learned the maze, in the total absence of classical space/time information. It is not surprising then, that human's exhibit an even wider range of reactions to non-local information. The evidence suggests that humans can perceive, cognize, and give meaning to non-local information across a range of complexity, from inanimate objects, simple organisms, animals and other humans. The existence of quantum holography provides an adequate informational structure to permit a theory for the observed results. The case is a classic case in phenomenology, where results are repeatedly observed over time that fall outside the prevailing paradigm, and must await new developments in science before an explanation is forthcoming. The results for intentional effects of non-locality should be no more difficult to accept than the results for perception. The pcar relationship implies symmetry, that is, information flows in both directions between object and percipient such that each is object and each is percipient. Only the complexity of the more ordered perceiving system suggests a non-symmetrical relationship. We humans have great difficulty in accepting that thoughts, specifically intentionality, can cause action at a distance. Yet, it has been observed for centuries and in recent decades subjected to scientific scrutiny. Were not prayer to have produced some positive results, religion would have been abandoned centuries ago. That cause was ascribed to supernatural agency rather than non-locality is simply, again, phenomenology needing to wait while science caught up. Modern studies by Dossey (1993), Byrd (1988) plus many others have attempted to document the efficacy of prayer, particularly healing prayer. The results in most cases are very suggestive of non-local effects, and some claim they establish the case for healing prayer. However, the difficulties of controlling all variables in such clinical studies leave many avenues for valid criticism. The fact that Radin's several studies (1997) demonstrated that attention alone produced non-local results in machines, i.e., reduced randomness (increases order) does confirm that information has non-local effect and may be correctly formulated asnegentropy. These results apply directly to healing prayer as well.The case for pcar conditions to create remote effects by transfer of non-local information between equally complex percipients, humans for example, is not difficult to understand. Indeed, hundreds of successful experiments establish the case. In these cases no energy transfer is required, only non-local information, as each percipient/object has access to its own energy source. The case for intentionally creating remote physical effects in inanimate objects is more puzzling. Even though teleportation of quantum states has been successfully accomplished for particles, and numerous studies (Radin, 1997; Dunne and Dunne, 1988) show that macro-scale objects can also be changed or moved, the energy transfer mechanism by which the classical states of a remote object are affected remains elusive. Conclusions The case for mind/mind and mind/matter interactions is impressively well documented over many decades as studies in phenomenology, with staggering probabilities against chance having produced the results. The discovery of the non-

local quantum hologram, which is theoretically sound and experimentally validated in at least one application, thé fMRI, is sufficient to postulate that the quantum hologram is a solution to the foregoing enigma. Further, recognition that the quantum hologram is a macro-scale, non-local, information structure described by the standard formalism of quantum mechanics extends quantum mechanics to all physical objects including DNA molecules, organic cells, organs, brains and bodies. The discovery of a solution which seems to resolve so many phenomena, and also that points to the fact that in many instances classical theory is incomplete without including the subtle non-local components involved, suggests a major paradigm change must be forthcoming. The papers already published by Marcer and Schempp proposing a learning model both for DNA and prokaryote cells, which uses quantum holography, suggests that evolution in general is driven by a learning feedback loop with the environment, rather than by random mutations. This solution to biological evolution was proposed by Lamarck in 1809 but discarded for the mechanistic solution of random mutations by the colleagues of Darwin. The fact that non-local correlations and non-local quantum information can now be seen as ubiquitous in nature leads to the conclusions that the **quantum hologram can properly be labeled as "nature's mind"** and that the intuitive function we label in humans as the "sixth sense" should properly be called the "first sense". The perception of non-local information certainly preceded and helped to shape, through learning feedback, the sensory systems that evolved in planetary environments, and which we currently label as the five normal senses. We must conclude that evolved, complex organisms, which can form intent, can produce and often **do produce non-local causal effects associated with that intent.** Further, that attention alone produces coherence in nature that in some measure reduces randomness. Finally, I conclude that the cited experiments and current understanding of non-locality in nature is sufficient to postulate that **non-locality is the antecedent attribute of energy and matter which permits perception and is the root of the consciousness which manifests in the evolved organisms existing in three dimensional realities**. ReferencesAnandan, J. "The Geometric Phase"; Nature, 360,26, 307-313 (1992)Berry, M. V. "The Geometric Phase" Scientific American, December, 26–32 (1988)Byrd, R. C. "Positive therapeutic effect of intercessory prayer in a coronary care population"Southern Medical Journal 81 (7): 826–829 (1988)Dossey, L. "Healing Words" Harper San Francisco, (1993)Dunne, B. J., Nelson, R. D., and Jahn, R. G. 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MathematicalImaging and Vision 2: 109–164 (1992)Schlitz, M. & Wiserman, R. "Experimenter Effects and the Remote Detection of Staring", Journalof Parapsychology Vol. 61, September, (1997)Schmeidler, G. R., Craig, J. G. "Moods and Esp Scores in Group Testing" Journal of ASPR vol66no. 3, pp. 280–287 (1972)Tiller, A. W. "Science and Human Transformation" Pavoir Publishing, California, (1997)Utts, J. M. "Replication and meta-analysis in parapsychology" Statistical Science 6: 363–382,(1991)Van Flandern, T. "What the Global Positioning System tells us about Relativity", Open Questions in Relativistic Physics, ed. by F. Selleri, Apeiron, Montreal (1998) **http://www.cosmicdreaming.com/pdf2011/Nature's%20Mind%20the%20Quantum%20Hologram%20by%20Edgar%20Mitchell,%20Ph_D.pdf**. **Models given in one of the papers. Some sentences are deleted and reformulated for felicity of reading and spatial restrictions. Kindly pardon me on that count. National Institute for Discovery Science: Nature's Mind: the Quantum Hologram by Edgar Mitchell, Ph.D. National Institute for Discovery Science Home > Resources > Consciousness Studies Nature's Mind: the Quantum Hologram Edgar Mitchell, Sc.D. Institute of Noetic Sciences, Sausalito, Calif. Fax: 561-641-5242, edgarmitchell@msn.comIntroduction** My Answer To Quora Question "If Consciousness Has No Evolutionary Advantage, Doesn't That Imply That It Is An Emergent Property? " While Human Consciousness Has Certainly Been Shaped By Evolution, Which Does Not Mean That Consciousness Itself Could Have Evolved From Non-Consciousness. Whether We Are Talking About Other Species Of Animals Or Cells Or Organic Molecules, The Same Issues Which We Run Into In Explaining Human Consciousness Are Still Present At Any Scale. The Issues Of The Hard Problem Of Consciousness, Explanatory Gap, Binding Problem, And Symbol Grounding Problem Make The Mind-Body Split Just As Relevant Wither The 'Body' Is A Brain, Neuron, Or Subatomic Particle. No Matter What, You Have To Explain How An 'Interior World' Can 'Exist' In A Physical Structure Whose Behavior Is Causally Closed. Whatever Way You Slice It, If We Accept That T-Cells Can Be Effective In Detecting And Neutralizing Threats On A Cellular Level Without Having Consciousness, Or That DNA Can Create Cellular Machines Which Build A Brain Without Consciousness, And Then We Are Admitting That Consciousness Doesn't Make Sense As A Functional Adaptation. The Rest Of The Universe Already Works Too Well Without It. There Is Nothing Especially Interesting about A Hominid's Need for Food and Shelter Which Would Demand Rich Awareness to Develop out Of Blind Reflex. Single Celled Organisms Chase Food, Avoid Danger, Etc Also. We Are Then Left With Considering That Either Consciousness Could Somehow Be An Accident Of Evolution, Or That Consciousness May Be Intrinsic To All Physical Phenomena In Some Sense (Panpsychism, Panexperientialism) Or Even That Consciousness Is The Universal Substrate Upon Which All Phenomena Depends (Idealism, Idealist Monism). If Consciousness Is A Mutation That Has No Functional Role (A Spandrel), We Have To Ask Why It Would Even Be A Possibility. Remember That If Consciousness Is A Mutation, We Are Assuming That There Is A Whole Universe Already In Place Which Is Overflowing With Processes, Biological And Otherwise, Which Are Perfectly Capable Of Directing Themselves Effectively While Being Unconscious. It's Actually A Radically Anthropocentric Cosmology Since We Are Privileging Our Tiny Piece Of History In The Universe As The Only Piece Which Is Not Devoid Of Experience. We Are Saying That Everything That Existed Before Humans Was Unconscious, Therefore An Invisible, Intangible, Silent Void With No Memory Etc. If We Are Not Intending That, And Prefer To Think That The Universe Looked, Sounded, Felt, And Tasted Just Like It Does For Homo Sapiens Since The Dawn Of Time, Then We Would Have To Ask Exactly What We Think Consciousness Is Adding To That Kind Of Eternal-Universal 'Unconsciousness'. If Consciousness Is Intrinsic to Physical Phenomena (As in Penrose-Hameroff's Microtubule-Based Quantum Consciousness) or Is Intrinsic to Information Integration (As in Tononi-Koch's IIT), We Still Have the Same Kind Of Mind-Body Problem. A 'Body' Which Is A Statistical Function Rather Than A Literal Form In Space Is Still Falls Short Of Explaining Why And How There Is Any Such Thing As Consciousness. In My View, Only The Idealist Monist View Or What I Call Pansensitivity Makes Sense Ultimately As The Parent Of Both Physics And Information. Just As We Learn To Count On Our Fingers, All Forms Of Information Are Representations Of Experiences Which Have An Aesthetic Foundation – A Seeing, Feeling, Touching, Thinking, Etc. Without That Sensory-Motive Context From The Start, There Would Nothing To Evolve; Only Abstractions In The Dark (Or Not Even Dark). Once We Can Get Over Ourselves As A Species And Recognize That Consciousness Doesn't Begin And End With Us, I Think That Awareness Will Be Seen As The

Container Of Relativity Itself, With Quantum Mechanics And Evolutionary Biology As A Consequence Of Deeper Stories Rather Than Their Originator. Multisense Realism A Cosmology Of Sense, Essence & Existence For Models See One Of The Papers In The Series Procrastinated Due To Constraints On Space. Events Occur In Time, But There Is No Time Except That Which Has Been Produced By The Big Bang. It Makes No Difference Whether The Big Bang Is The Only Beginning Of The Only Universe Or Just The Beginning Of One Of Many Universes, Either Way It Doesn't Explain The Beginning Of Existence. Some Say That Everything Following The Big Bang Is Random, Not Designed, But Why Is 'Designed' Even An Option? If That Word Means Anything, And Anything Has Ever Been Designed By Anything, Then What Difference Does It Make Whether That Capacity Of Intention Came Early On In The Universe Or More Recently? How Do We Know The Difference Between What We Design And What Is Random? How Do We Know That Randomness Even Exists? We Can't Generate True Randomness Computationally, We Can Only Grab Onto Some Pattern That Seems Nearly Random By Some Arbitrary Duration Of Measurement And Say That It Is Good Enough. Random Is A Concept, and the Difference between Random and Intentional May Ultimately A Matter of Perspective. Some Interesting Possibilities Arise When We Consider The Observation That The More Something Seems Intentional, The More It Is 'Like Me' And The More Unlike Me Something Appears, The More It Seems Mechanistic. In A Way That Is Similar To How Any Object Appears As A Dot Or Smudge If It Is Too Small Or Distant For Us To See, The Notion That Perceptual Relativity Dictates The Quality Of Intention And Unintention Is A Provocative Hypothesis Of MSR. Random Events Cannot Follow A Series Nor Have Results. That Doesn't Mean That There Has To Be An Entity Making Everything Happen, But It Suggests That The Universe Is A Phenomenon Of Appearance, And Part Of Appearance Is Oscillation Between Intentional And Unintentional Attributes. The Universe Speaks In Both Entropy And Significance. Randomness Doesn't Do Anything, Can't Be Anywhere Or Feel Anything. Randomness Is An Abstraction Derived From An Expectation Of The Contrary. By Random, All We Mean Is That It Lacks Pattern And Intent – Both Of Which Must Be Implicitly Present Before They Can Be Hypothetically Absent. There Is Sense And There Is Non-Sense, Order And Dis-Order, It Is Not Non-Randomness And Dis-Chaos. Many People Believe That Physical Law Excludes The Possibility Of Free Will Because Of Strong Causal Closure. By This, What Is Meant Is That There Is No Room In Physics For Any Force That Causes A Physical Effect That Has Not Been Accounted For, Whether Or Not This Is Actually True Or Not Is Questionable In The First Place. The Sudden Addition Of Dark Matter And Dark Energy In The 1990s, Which Together Must Account For 95.1% Of Mass-Energy Of The Universe, Doesn't Seem To Have Encountered Nearly As Much Resistance From The Scientific Community Than Conscious Intention Has. Even If It Were The Case, However, And The Typing And Reading Of These Words Will Be Someday Explained By Purely Neurochemical Mechanisms, The Question Of Why The Feeling Of Intention Exists, And How It Can Be Produced Will Remain Unanswered. My Free Will Demands That Your Free Will Admit That There Is No Free Will. In All Of The Contemporary Debates On Free Will There Seems To Be A Blind Spot, In Which Free Will Does Not Exist, Except Where It Concerns The Application Of The Results Of Scientific Experiments. There Is Always A Call For The Educated And Enlightened To Voluntarily Change Their Own Minds, To Be Persuaded By The Argument Of Their Own Free Will. The Hypocrisy Is Hard To Overlook Once You See It. It Seems That The Anti-Free Will Assertion Makes An Exception For People Who Are 'Right'. Being Right Seems To Give Us A Right To Have Opinions Which Others Cannot, Due To Their Enslavement To Physical Causes. An Issue Which Often Comes Up In Free Will Debates Is How Our Stance On Free Will Impacts Criminal Prosecution. Again, The Blind Spot Of The Anti-Free Will Philosopher Projects That While Criminals May Not Be Held Liable For Their Actions Because Their Neurology Is To Blame, Then We Can't Allow Society To Take Credit Or Blame For Its System Of Justice Either. If Guilt Or Innocence Is Irrational, Then Believing That We Can Modify Our Own Attitudes Toward Guilt And Innocence Must Also Be Irrational. What Seems More Irrational Is To Codify Law Into A Mechanistic Formula Which Diminishes Our Capacity For Thought And Feeling In Consideration Of The Fate Of Others (Or Others Considering Our Fate).Zero-Tolerance Determinism: Another Pillar Of The Anti-Free Will Position Is That It Must Be An All-Or-Nothing Phenomenon. This Seems To Be More Of A Hasty Generalization Than An Honest Look At The Phenomena. In A More Tolerant Analysis, It Should Be Clear That Free Will Doesn't Have To Be Absolutely Free. The Fact That Our Conscious Mind Thinks That It Was Presented With Certain Options Still Requires That

We Choose Freely From Among Those Options. It's True That We Are Always, In a Sense, Only Able to Choose What Our Mind Thinks Is the 'Best' Option (Even If That Choice Is Based on Bad Advice from Our Stomach or Emotions), But Still There Is an Intentional Participation Which Cannot Be Explained by Statistical Functions. No Matter How Constrained We Are By The Rules Of The Road, Our Car, The Limits Of Our Driving Skill, Etc, There Is Still A Difference Between Driving And Being Asleep At The Wheel, Or Between Driving And Being Forced To Drive Somewhere At Gunpoint. We Should Not Look Only At How Our Freedom Seems To Vanish Upon Thorough Inspection; We Must Also Look At How It Appears In The First Place – Unbidden And Self-Evident. Look At The Universal Appeal Of Freedom And The Universal Disparagement Of That Which Is Unfeeling, Robotic, And Mechanistic. One Thing That Is Seldom Mentioned In These Endless Debates Is Creativity. Were The Egyptians Destined To Build Pyramids And Not Megalithic Circles? Was It Inevitable That The Star Spangled Banner Was Written In Reference To The United States? Did Deterministic Processes Have No Choice But To Create The Illusion Of Free Will? Why? The Work of Benjamin Libet Is Frequently (Compulsively?) Cited As Well, Work Which Even Libet He Later Made Clear Did Not Show That Free Will Didn't Exist. The Fact That A Neurological Signal Can Be Detected Before The Various Parts Of Us (The Self Who Makes The Decision, The Self Who Knows They Make The Decision, The Self Who Reports That They Make The Decision) Can Arrive At A Consensus Does Not Mean That The Initial Impulse Doesn't Correspond To Conscious Experience. There Is Also The Matter Of Focusing On Repetitive, Reflex Actions Which Minimize Free Will And Maximize Predictive Expectation. These Kinds Of Experiments Are Like Proving That Chefs Lack Imagination By Studying The Fry Cooks At Mcdonalds. Since Physics And Neuroscience Has No Theory At All As To The Origin Or Utility Of Consciousness, We Cannot Give Inanimate Instruments The Benefit Of The Doubt When It Comes To Our Subjectivity. Because We Are A Single Zygote Which Has Reproduced Itself, In Some Sense, Every Cell In Our Body Is 'Us' Just As Much As Any Organ Or Process In Our Body. We Are Complex, But Not In The Way That A Machine Is Complex. We Are Not Assembled From Specialized Parts; We Are A Single Whole, Divided Into Relative Specialization. Just Because Every Part Of Us Doesn't Know What Every Other Part Of Us Is Doing At All Times, Doesn't Mean That It Isn't All 'Us' Doing It Pansensitive Relativity: If The Perceptual Relativity Hypothesis Is Correct, And Pansensitivity Is The Engine Of Both Physics And Subjectivity, Then Cause Itself Is Relativistic. Intention Is Real And Primitively Creative In The First Person, And Unreal/Recombinant-Derivative In The Third Person…But Third Person Is Only Real Because There Is A First Person Participant Present. **Free Will, Determinism, And the Big Bang: Multi Realism Blog: Wikipedia http://multisenserealism.com/2012/08/27/deleuzes-the-logic-of-sense-part-i/.** From A Whiteheadian Perspective, However, There Are At Least Two Key Sources Of Misunderstanding Latent In Some Of These "Returns" To "The Body", "The Object" And "Materiality". There Is A Rather Too Hasty Dismissal Of The Concept Of Subjectivity As Such And There Is A Related Tendency To "Flatten Out" Any Would-Be Distinctions Between Human And Non-Human Entities. Such Positions Thus Risk A Return To A Bleak Anti-Subjectivism That Mocks Those Who Might Cling To The Idea That "Humans Are Very Different From Knives Or Paper" (Harman, 2002, Cited In Thrift, 2008). With Respect To The Latter, For Example, Nigel Thrift Invokes The ANT Principle Of The Democracy Of Things And States That In His Theory "Things [And Human Beings] Are Given Equal Weight, And I Do Mean Equal" (Thrift, 2008, P. 9). Much Of This Is Done In The Name Of A Whiteheadian Inspiration Since The Book Begins With Strong References To Whitehead, Including Key Concepts Such As The "Actual Occasion". The Point Concerning The Dismissal Of Subjectivity Can Be Brought Out Most Starkly By Contrasting One Of Thrift's Statements About His Non-Representational Theory With A Statement From Whitehead Himself. "Thus Things", Writes Thrift, "Are Not Just Bound By Their Brute Efficacy To The Visible Termini Of Humans In Some Form Of Latent Subjectivism Such As 'Concern' Or 'Care'". Now, It Is Certainly True That For Whitehead A "Thing" Is, By Definition, "Describable Without Reference To Its Entertainment" In An Occasion Of Experience (Whitehead, 1933/1935, P. 226). In This Sense, Things Are Not "Bound By Their Brute Efficacy To The Visible Termini Of Humans". But It Is Certainly Problematic To Imply That His Work Is Based Upon A Move Away From The Subject–Object Relation As The Fundamental Structure Of Experience And Hence From Concepts Such As Concern And Care. On The Contrary, On This Matter Whitehead's Thinking Is Quite Comparable To That Of His

Contemporary Martin Heidegger In Being Grounded In The Concept Of Concern. The Centrality Of This Concept Is Due To The Way In Which It Brings Together Subject And Object As Relative Terms In The Unity Of What He Calls An Actual Occasion Of Experience: Thus The Quaker Word "Concern", Divested Of Any Suggestion Of Knowledge, Is More Fitted To Suggest This Fundamental Structure. The Occasion As Subject Has a "Concern" For the Object. And The "Concern" At Once Places The Object As A Component In The Experience Of The Subject, With An Affective Tone Drawn From This Object And Directed Towards It. With This Interpretation, The Subject–Object Relation Is The Fundamental Structure Of Experience. (Whitehead, 1933/1935, P. 226) Granted, Thrift Does State That He Wishes To "Temper… The More Extreme Manifestations Of This Lineage, Which Can End Up By Positing A Continuity Of And To Experience About Which I Am Sceptical" (2008, P. 6) However, One Wonders What Is Left Of Whiteheadian Cosmology If This Radical Extension Of The Concept Of Experience And Hence Of Empiricism Is Omitted. It Is A Little Like Being A Marxist Without The Dialectical Materialism. It Is Also Difficult Not To Be Sceptical About The Remaining Self-Consciously "Inhuman" Theoretical Framework "In Which Individuals Are Generally Understood As Effects Of The Events To Which Their Body Parts (Broadly Understood) Respond And In Which They Participate" (Thrift, 2008, P. 60). In Recent "Radical" Social Theory, It Seems The Baby Of Subjectivity Is At Risk Of Being Thrown Out With The Bathwater Of Representationalism, Leaving Only The Hollow Remainder Of A Reactive Ensemble Of "Body Parts". Some Clarity Is Therefore Needed If We Are Not Simply Going To Recruit Whitehead Into Concerns Alien To His Own. His Chief Problem Was Not the Notion of the Subject/Object Structure of Experience Itself, But It's Too Rapid Identification with the Difference between Knower and Known (Whitehead, 1933/1935, P. 225). It Is This Conflation Of The Deeper Subject/Object Relation With The More Superficial Distinction Between Knower And Known That Gives Rise To A "Representational" Style Of Thinking And Its Interminable Debates. I Will Argue That It Is Precisely By Way Of The Subject/Object Relation Of The Actual Occasion That We Are Best Positioned To Conceive Of "The Cumulation Of The Universe And Not A Stage-Play About It" (Whitehead, 1927–1928/1985, P. 237). **Palgrave Communications | Palgrave Macmillan Journals Journal Home > Archive > Original Articles > Full Text Original Article Subjectivity (2008) 22, 90–109 Doi:10.1057/Sub.2008.4 A.N. Whitehead And Subjectivity Paul Stenner1** 1University Of Brighton, Brighton, UK Correspondence: Paul Stenner, School Of Applied Social Science, University Of Brighton, Falmer Campus, Mayfield House, Brighton BN9 1PH, UK. E-Mail: P.Stenner@Brighton.Ac.Uk (For Models Please See One Of The Papers In The Series Attributed To Spatial Restraints It Is Accommodated Where It Could Be). In Sum, Whitehead Offers A Relational Process Ontology That Promises To Deepen The Constructivist Insights Associated With The Turn To Textuality, But Without Reducing The Universe To "Discourse" And "Materiality". In This Ontology, Things (Whether Occasions Or Assemblages) Are Definable As Their Relevance To Other Things And In Terms Of The Way Other Things Are Relevant To Them. Things Have Relational Essences. Likewise, Things Do Not Exist Independently Of Temporality But Are Constituted By The History Of Their Specific And Situated Encounters. Every Actual Thing Is Thus "Something By Reason Of Its Activity" (Whitehead, 1927/1985, P. 26). Importantly, This Talk Of "Things" Need Not Incline One Towards Denying The Relevance Of Subjectivity. I Have Thus Taken Issue With A Tendency Illustrated In The Work Of Thrift (2008), Who Appears To Define Cutting-Edge Social Theory As Concerned With "Flow" And "Play" Rather Than With Stability (Since "Non-Foundational Theory Takes The Leitmotif Of Movement", P. 5 And "Privileges Play", P. 7); As "A Means Of Going Beyond Constructivism" (P. 5); And As "Trading" In "Modes Of Perception Which Are Not Subject-Based" (P. 7). I Have Tried To Show That, In Fact, The Concept Of Process Is As Much About Stability As About Change. Stability Is To Be Thought Of As An Achievement Resulting From Particular Ways Of Actualizing Potential And Of Patterning Occasions Into Spatial And Temporal Co-Assemblies. Nevertheless, I Have Also Stressed That Becoming Is An Inherently Self-Creative Process, Albeit A Self-Creation Grounded In The Facticity Of A Concrete Inheritance. Whitehead's Category Of Subjective Unity Is Thus An Exemplary Statement Of Constructivism, Which States That: "Self-Realization Is The Ultimate Fact Of Facts. An Actuality Is Self-Realizing, And Whatever Is Self-Realizing Is An Actuality" (Whitehead, 1927–1928/1985, P. 222). The "Capture Of Intensity" And The "Clutch At Vivid Immediacy" Are Thus The Defining Characteristics Of Life (Whitehead, 1927–1928/1985, P. 105). Finally, I Have Suggested That It Is Only On The Basis Of A Deep Extension Of The Concept Of Experience Throughout

Nature That Whitehead Is Able To Resoundingly Affirm His Reformed Notion Of The Subjectivist Principle: "That Apart From The Experiences Of Subjects There Is Nothing, Nothing, Nothing, Bare Nothingness" (1927–1928/1985, P. 167). "Scientific Reasoning Is Completely Dominated By the Presupposition That Mental Functionings Are Not Properly Part of Nature" (Whitehead, 1938/1966, P. 156). (Ibid). Lewis Carroll Is Wonderful In Exploring Some Of The Paradoxes Of Logic. Following Some Of These Paradoxes Will Lead To The Important Role Of 'Sense' In Understanding. [NB Weird Titles Are Deleuze's Own]. First Series Of Paradoxes Of Pure Becoming: We Need To Explore The Nature Of An Event. Events Assume Becoming, Since They Refer To States In The Past And The Future In A Way Which 'Eludes The Present'. This Is Paradoxical But Still Makes Sense. Plato Tried To Distinguish Between Limited Fixed Things And Pure Becoming, But The Two Cannot Be Separated. Instead, A Dualism Is Hidden In Material Bodies. We Need To Introduce The Notion Of A Simulacrum Which Is Neither Copy Nor Model [Massumi Has A Useful Article On This—Roughly, The Issue Is That What We Take To Be Material Reality Is Actually A Simulacrum Of The Virtual, A Limited Condensation 'Beneath Things' (2)]. It Also Avoids The Problems Of Subsuming Reality Under The Idea. Sometimes, This Is Indicated By A Summer The Peculiarities Of Language, Which Can Seem To Flow Over Specific Referents. This Provides A Clue That There Is Some Dimension To Language Which Serves To Come To The Aid Of More Specific Attempts To Name And Describe. There Are Implications For Identity. Fully Grasping Becoming Means That Identities Are Infinite, Incorporating Future And Past, Active And Passive, And Even Cause And Effect. Language Attempts To Limit This Infinity, But Still Often Alludes To It [With Open-Ended Statements Or Generalisations]. As With The Alice Stories, This Also Disrupts The Conventional Notion Of The Personal Identity. Normally, This Is Maintained By Some Underlying Commonsense Or Knowledge, As When 'The Personal Self Requires God And The World In General' (3). Becoming Threatens This Stability With The Paradox Of Events, Which Can Penetrate Even Commonsense—It Is Not Just A Doubt About Reality, But A Clear Indication Of The 'Objective Structure Of The Event Itself, Insofar As It Moves In Two Directions At Once, And Insofar As It Fragments The Subject Following This Double Direction' (3). Second Series Of Paradoxes Of Surface Effects: Stoics Divided Things Into Bodies And States Of Affairs, 'Actions And Passions' (4). There Is Also Some Cosmic Unifying Quality, Always In The Present. Bodies Can Interact and Cause Effects in Each Other, But These Effects Are Incorporeal, 'Logical or Dialectical Attributes… Not Things or Facts but Events' (5). They Have The Kind Of Subsidiary Existence, Acting As Verbs, And They Are Infinitives—The Example Is A Cut Inflicted On The Body, Which Is Seen As An Incorporeal Surface Effect, Compared To The Actuality Of Bodies And Their Mixtures. This Argument Had Important Implications For Understanding The Causal Relation. Specific Bodily Causes Produce Other Bodies, Linked By Some Cosmic Unity Or Destiny. Similarly, Effects Can Be Seen As Having Bonds Between Them, But Effects Can Never Be Causes In Themselves. They Can Only Be '"Quasi-Causes' Following Laws Which Perhaps Express In Each Case The Relative Unity Or Mixture Of Bodies On Which They Depend For Their Real Causes' (6). These Combinations and Bonds to Provide For Some Emergent Qualities, Which Means That Destiny Can Be Avoided. An Alternative Is Offered By The Epicurean Classification Of Different Kinds Of Causes Which Are Relatively Independent [And So Can Interact], And This Is A Kantian Idea Too. There Is A Reference Back To The Capacities Of Language To Offer 'A Declension Of Causes... [And]... A Conjugation of Effects' (6). Stoic Philosophy Introduces The Notion Of A Something Behind Both Specific Material Beings And Incorporeal Events. The Idea Must Belong To 'This Impassive Extra-Being Which Is Sterile, Inefficacious, And On The Surface Of Things: The Ideational Or The Incorporeal Can No Longer Be Anything Other Than An "Effect"' (7). This In Turn Leads To A Change Of Metaphor From Surface/Depth To Just Surface, To A Series Of Effects Which Are Manifestations And Are Of Different Types. We Have A Notion of Possibilities, Of Ideality Itself, Rather Than the Platonic Idea, But With No 'Causal and Spiritual Efficacy' (7). The Simulacrum Now Appears On The Surface, Rather Than Being Hidden In The Depths. Events As Effects Combine Past And Present, Active And Passive, All Of Which Are Located Elsewhere As Causes. The Relation Between Events Can Only Be Quasi-Causes. Stoics Saw Dialectical Analysis Has Explorations Of These Combinations, Once They Had Been Expressed In Propositions—Dialectics As Conjugation. Language Also Enables Us To Go Beyond Events Into The Possible Or Becoming. The Relation Between Propositions And Specifics Is Itself Still Paradoxical—'Chryssipus Taught "If

You Say Something It Passes Through Your Lips, So If You Say "Chariot", A Chariot Passes Through Your Lips' (8). It Is Deliberate Nonsense In The Anglo American Sense, Or Humourous Play On The Surface, As Opposed To An Ironic Exploration Of Depths And Heights. Lewis Carroll Did Something Similar In Alice. [A Commentary On Alice Ensues, Stressing The Surface Rather Than The Underground World, And Picking Up The Disdain Lewis Carroll Felt For Boys Who Did Not Like To Operate At The Surface. Left Handers And Stutterers Can Sometimes Remind Us Of The Paradoxes Of The Surface, However, Which Can Defeat Commonsense Understandings]. Third Series Of The Proposition: Describing Events As Propositions Raises The Question Of How Best To Analyse Surface Events. There Are Three Possibilities: Denotation [Roughly, A Direct Connection Between Words And Images Which Represent States Of Affairs, As In Indexical Signs. Here, Propositions Are Either True Or False, And May Be True In All Cases]; Manifestation -- A Relation Between The Proposition And The Person Expressing It, Statements Of Desire And Belief. These Are Causal Relations: 'Desire Is The Internal Causality Of An Image With Respect To The Existence Of The Object Or The Corresponding State Of Affairs' (13). Belief Anticipates Production Of An Effect By A Cause. Manifestation Includes Denotation, Makes It Possible. '"I" Is the Basic Manifester' (13). Manifestation Is 'The Domain Of The Personal, Which Functions As The Principle Of All Possible Denotation' (13). The Issues Here Turn On Veracity Other Than Truth And Falsehood, The Avoidance Of Illusion. Signification, The Relation Of A Word To Universal Or General Concepts, And Connections To Implications Which Have To Follow The Rules Of Syntax. Again, Signifying Involves Conceptual Implications Referring To Other Propositions, As In Premises Or Conclusions. This Involves A Certain Indirect Process, Implication Or Assertion, Instead Of Truth Or Veracity, Which Remain As Possible In Certain Conditions. However, It Is Not Just Formal Logical Operations That Are Involved, But Notions Of Probability Or Even Moral Terms Such As Promise Or Commitment. Error Produces Not Falsehood But Absurdity [Looks Really Close To Habermas and the Three Validity Claims Here]. Signification May Not Be Primary, Since All Language Begins From The Standpoint Of The 'I', But There Is An Assumption That Propositions Must Be Understood By Others And Have A General Force. This Implies That Manifestation Has Primacy. But Signification Is Implied, And [In Social Relations] Would Be The Basis Of Manifestation. It Is The Difference Between Langue And Parole. Particular Utterances Only Make Sense Against The Background Of Constant Concepts. This Is Extended To Particular Desires And Beliefs, As Opposed To 'Simple Opinions' Which Would Not Signify (16). Actual Utterances Often Involve Truth Claims And General Signification [And Other Presuppositions, Which Are Technically Infinite]. This Is The Paradox Facing Pure Logic, Solved By A Form Of [Smuggling]: 'Implication Never Succeeds In Grounding Denotation Except By Giving Itself A Ready- Made Denotation, Once In The Premises And Again In The Conclusion' (16). So Actual Propositions Feature Circular Relations between Signification, Manifestation, and Denotation. There Might Even Be A Fourth Dimension—Sense, But Introducing This Will Depend On Making Relations Theoretically Consistent—It Is 'Not Simply A Question Of Fact' (17). To Approach The Issue, We Ask Whether Sense Might Be Located In One Of The Existing Three. Denotation Concerns Itself With Truth And Falsity, Which Is Too Narrow. The Mere Relation Between Words And Denoted Things Is To Paradoxical To Always Makes Sense, As In The Example Of Speaking The Term 'Chariot'. Instead, Denotation Presupposes Sense. Manifestation Does Involve Some Manifesting Subject Which Initiates, So May Be Sense Is Itself A Subjective Matter Of Beliefs And Desires Of Persons—But Subjects Only Possess This Ability To Speak Because Of A General System Of Signification In Language. It Looks Like Sense Must Be Identified With Signification—But Signification Is Linked In A Circular Relation With Denotation And Manifestation. Perhaps it's Necessary to Think Of Different Forms of Possibility of Propositions—Logical, Physical, Syntactic and So On. This Might Serve As Foundations For Sense, But This Would Be An External Foundation, Independent Of Speech [I Think The Problem Is The Connections Between Any Foundations And Actual Act Of Speech, Whether Anything Would Escape The Foundations]. The Concept Of Truth In Particular Implies Independence From Form. This Independence, Separate From Conceptual Possibilities In Signification, Is What Constitutes Sense. This Is The Fourth Dimension. It Is 'An Incorporeal, Complex, And Irreducible Entity, At The Surface Of Things' (19). There Is Philosophers Have Discovered And Rediscovered This Quality. It Is The Idea Of A Something Again, Beyond The Propositions And The Terms And The Objects Which Are Denoted, Beyond The Subjective I And Things

Which Are Expressed. Sense Is Irreducible To Propositions, and It Is and Must Be '"Neutral," Altogether Indifferent To Particular and General, Singular and Universal, Personal and Impersonal' (19). There Is Been Little Agreement About This Possible Fourth Dimension, Whether It Exists Simply In The Form Of Some Enquiry. It Is Not Even Immediately Useful Because It Is Neutral. It Can Only Be Inferred Indirectly, By Questioning Characteristics Of Propositions As Above—This Is 'Inspired In Its Entirety By Empiricism… [Avoiding Notions of Essence or Idea, and Knowing]… Have To Track Down, Invoked And Perhaps Produce A Phantom At The Limit Of A Length Or Unfolded Experience' (20). It Might Be What Husserl Called 'Expression', Lingering In Terms Such As The Noema, As Pure Appearance, Outside Denotation Or Manifestation, Linked In Complex Ways To Appearances. In The Same Way, Sense Does Not Exist Outside Propositions Exactly, But 'Inheres or Subsists' (21). It Is Not Just An Expression, But An Attribute, Not Just Of The Proposition, But 'Of The Thing Or State Of Affairs' (21), [The Potential, 'To Be Able To Be Green' Rather Than Just The Denotation 'Green' Is The Example Here]. It Is Said Of A Thing, So It Depends On Propositions Which Express It And Is Therefore Not Separate From The Proposition. It Is Something Else, Both the Expressible, And the State Of Affairs: 'It Turns One Side towards Things and One Side towards Propositions' (22). It Is What Joins Propositions And Things. [It Is A Becoming]. It Operates On The Surface, Rather As Mathematics Does, Or The Nonsense Of Carroll. It Is The Operation Of Sense That Produces [Meaningful] Paradox. Fourth Series Of Dualities: Important Dualities Exist Between Causes And Effects, And 'Corporeal Things And Incorporeal Events' (23). This Is Extended To A Duality Between Things And Propositions, Bodies And Language. This Is Expressed In Lewis Carroll As A Duality Between Eating And Speaking—The Former Is A Matter Of Bodies Actions And Passions, And The Latter Movements Of The Surface And 'Ideational Attributes Or Incorporeal Events' (23) [Lots Of Examples From Alice About Being Presented To Food And Having Food Presented To You]. The Normal Relationship Can Be Distorted By 'Verbal Hallucinations… Unrestricted Oral Behaviour… And Various Disorders of the Surface' As Bodily Matters Intrude—Stuttering, Left Handedness (24). Sense Is Always Expressed In Propositions, But It Lies In States of Affairs, It Happens To Things. In This Sense, Bodies And Language Are United In The Production Of Sense, Existing 'On The Two Sides Of The Frontier Represented By Sense', Which Constantly Articulates The Differences (24)—Things Include 'Ideational Logical Attributes Which Indicate Incorporeal Events', And Propositions Include Both Denotations And Expressions, Names And Adjectives, And Verbs [The Latter Indicating Becoming And Chains Of Events] (24) [Illustrated With Words By Humpty Dumpty]. This Duality In Propositions Represents Two Dimensions, The 'Denotation Of Things And The Expression Of Sense' (25) [Here, It Is Sense That Is Being Expressed Not Subjectivity, However]. This Means The Duality Is Inherent In Propositions As Well As Between Propositions And Things. [More From Lewis Carroll Page 26, Turning On The Universal Denotation Implied By The Term 'It', And Also The Ability Of The Term To Summarise The Sense Of An Earlier Proposition]. The Two Dimensions Can Emerge In an Esoteric Word, Or In A 'Non Identifiable Aliquid' (26). The Example Given Is The Word 'Snark' [Both A Limited Thing And A Representation Of Anything That Is Too Exotic To Exist?]. [The Section ends with an Extended Quotation from the Gardener's Song in Sylvie And Bruno—Pass] Alexander I. Stingl's Blog A Nomadic Scholarship Entity NOTES ON: Deleuze, G (1990) The Logic Of Sense, Trans Mark Lester, Edited By Constantin Boundas, New York: Columbia University Press For Models Kindly See One Of The Papers In The Series. Attributable And Ascribable to Constraints on Space Accomodation Model Is Provided in Some Paper in the series Notwithstanding the Generality and Commonalty of the Observation and Its Concatenatability with the Other Modules. The Scholars Of Tien Tai Called It The "Embodied Nature". (This Is The Buddha-Nature That Includes Both Good And Evil.) The Scholars of Xian Shou Say, "It Is Arising From Primal Nature", And the Scholars of Chan (Zen) Say, "It Is Nature That Causes the Rising of Things". All Dharma Is Dharma-Nature. It Is Not Different From Dharma-Nature. Dharma And Dharma-Nature Are Not Two Separate Identities, "Phenomena" And "Nature" Is Also Not Distinguishable Either. In Other Words, There Is No Difference Between Principle (Absolute) And Practice (Relative). Teachings in Chinese Buddhism (6) Sunyata (Emptiness) In the Mahayana Context (Wikipedia). The reason for becoming, according to Whitehead, is always the actual entity itself. He called this the ontological principle. The entity "enjoys" its relations and is "satisfied" by its own becomings. Whiteheads theory of process and relations can be viewed as an experiential alternative view of

quantum physics which incorporates a theory of mind. However, Whitehead's master work, Process and Reality (1929/1978), is nearly impossible to fully understand, has its own idiosyncratic vocabulary, and is not entirely self-consistent. This relative opacity of Whitehead's work has impeded the generalization of his quantum process theory, or metaphysics, to other areas of discourse. In his independent formulation of the microgenesis of the mental state, Jason Brown discovered a process theory of mind that had many similarities to Whiteheadian process. He took the concepts of duration and discontinuities of process and applied them to the duration of the mental state as it recapitulates the contents of prior mental states. Through the introduction of novelty, and through the process of microgenesis during the progression of mental states, the Self gives rise to a percept of self, the ego, in the object world, at the termination of the microgenetic process. This termination is the conscious instant, or instantiation of the microgenetic process. It is all that we consciously see. We see the ego, not the Self, and thus mistake the ego for self, replacing self with selfhood. The process itself is obscured from our consciousness, but can be inferred from those neurological deficit syndromes, which expose lower "layers" in the becoming. This parallels the "layers" of the nested hierarchy of holographic surfaces in the brain itself. Brown is theory of microgenesis involves a progression in the genesis of the mental state in a developmental sequence that recapitulates the organism is evolutionary development (phylogeny), embryologic development (ontogeny), and history of personal and social experience and development, in a movement in an upward direction through structures of increasing phylogenetic and ontogenic recency (Brown, 1988, 1991, 1996, 1998, 2005). He developed a view that considered the evolution of mental states to be a continuum from depth to surface, with leaps or saltation between mental states at intervals that are temporally irreducible units of the duration of microgenetic process (Brown, 1996, 1998). The process of neoteny, through which earlier features of development and evolution are brought to the present, was described (Brown, 1996) as a change in the development of some aspect of the mental state, which is attenuated at some stage of elaboration. In neurological and psychiatric deficit syndromes, this attenuation corresponds with the locus of a lesion or functional deficiency. The signature of the attenuated stage is carried through the remaining process of microgenesis, leading to expression of the attenuated phase at the endpoint of the duration of microgenesis, coupled with full elaboration of other elements of microgenesis. The intellectual pedigree of the theory of microgenesis dates back to the principle of superposition of Sir Charles Lyell, according to which younger strata lie on top of older strata. Lyell, a geologist, was to have great influence on evolutionary theorists, including Darwin and Wallace. The principle of superposition was taken up by Herbert Spencer, a scholar of many fields, who was a contemporary of Charles Darwin, and who had developed an evolutionary theory that perhaps rivaled Darwin is in popularity during the late 1800ís and early 1900ís. An important element of Spencer is theory was his observation of the brain, which he fashioned to be layered, with the newer layers added on to older layer during the course of evolution. Looking at the deeper layers was, for Spencer, like peeling away the layers of an onion. Spencer is idea was taken up by the renowned British neurologist Hughlings Jackson (Kennard and Swash eds., 1989), who developed the notion of hierarchies of brain process and the concept of neurological deficit syndromes as disturbances in the hierarchical structure and function of the brain. The concepts of microgeny and microgenesis, derived from hierarchies of brain function, were later developed by Arieti (1962) and Werner (1956). Later, the concept of a hierarchy of adaptive ego mechanisms, including lower order defenses such as denial and reaction formation, and higher order defenses such as altruism, was developed by Vaillant (1971), with the more mature defense mechanisms bearing on higher order values. The process theory of the evolution of the brain is extraordinarily important to Brown is process theory of mind. We can place these theories in perspective with the process theory of Whitehead, into which the theory of Brown partially merged (Brown, 1996; 1998; 2005). The materialist perspective of evolution, was, according to Whitehead (1925/1953), inconsistent with the process by which organisms become: "...in truth, a thoroughgoing evolutionary philosophy is inconsistent with materialism. The aboriginal stuff, or material, from which a materialistic philosophy starts, is incapable of evolution. This material is in itself the ultimate substance. Evolution, in the materialistic theory, is reduced to the role of being another word for the description of the changes of the external relations between portions of matter. There is nothing to evolve, because one set of external relations is as good as any other set of external relations. There can merely be change, purposeless and unprogressive." Whitehead is (1929/1978) realm of eternal objects, apprehended in the unconscious, can be viewed as a basic

element in the evolution of organisms and of mind. Whitehead (1929/1978), in his view of eternal objects, hearkens is back to Plato: "...eternal objects, as in Gods primordial nature, constitute the Platonic world of ideas." Whitehead is realm of eternal objects may include Carl Jung is (1934/1967) archetypes of the collective unconscious (Griffin, ed., 1989), connecting Whitehead is cosmology to the corpus of literature on depth psychology. Jung is archetypes were largely based on mythological, religious, and alchemical constructs. Mythology embodies the spiritual and social history of the species, as well as a variety of metaphysical metaphors (Campbell, 1969/1976; Neumann, 1954/1970). The physical nature of the brain state, including the notion of binding of the mental state into a coherent whole, is also critical to Brown is theory of microgenesis. Recent evidence indicates that the brain is a chaotic or dynamical system (Freeman, 1987; Combs, 2004; Abraham, 2004). The self-organization of dynamical systems, which include all living organisms as well as the brain, has been described in detail by Ilya Prigogine (1980), who won the Nobel Prize in chemistry in 1977 for his work on dynamical systems. Prigogine defines an "internal time" or duration of the states of dynamical systems. Physical time, he states, is secondary, and internal time is primary (Prigogine, 1986). The dynamical interval of internal time, or duration, is governed by the baker transformation (Prigogine, 1980), which can be envisioned as the time it takes the baker to knead or fold two different colors of dough successively, until it seems to be uniform in color. The brain, as a dynamical system, thus "deposits" time in the sense of Brown (1996). Here we have a parallel with the "layering" of process time according to the Holographic Principle. We can also detect, remotely, the undertones of Lyellís principle of superposition, albeit in a different form, and in a sense that Lyell could never have imagined. The work of Jason Brown in process neurology and psychology stands in sharp contrast to the prevailing scientific and philosophical nihilism in these fields regarding the self, free will, and the process of experience. In the sense of Brown is theory of microgenetic process of the brain, and in the sense of the duration of process discontinuities of Whitehead as it relates to the mind, it is important for the purposes of this paper to explore the scientific basis of the brain state. Mental process involves successive durations of such states, each with a "temporal thickness" (Brown, 1996). In recent years the "brain electrical microstate" has been described in detail as patterns of electrical field potential which endure for a period a period of time, typically around 100 milliseconds or a tenth of a second, and are punctuated by rapid transitions to the next state (Fingelkurts and Fingelkurts, 2001; Koenig and others, 2002). Brain electrical microstates are prime candidates for the brain states of microgenetic process and for the dynamical interval of internal time. The application of the theory of discontinuous information states, with the complete repetition of former states, as applied to the Holographic Principle of Mind, requires an information theory that includes time and the duration of mental states (e.g. Germine, 1993).The process of microgenesis, as it applies to brain evolution, can be seen in purely classical way, related to the recapitulation theory of vertebrate brain evolution (Agoutis, Montiel, and Lopez, 2002), which implies that "sensory projection sites and sites and processing circuits have been conserved [recapitulated] in reptiles and mammals" in the course of evolution. Again, there is an element of predictability in evolution which seems to belie the purposelessness of random natural selection. In keeping with the Holographic Principle, the development of the organism is a product of its evolutionary history, and is in continuous intercourse with that history. Microgenesis is readily related to the Holographic Principle of Mind, whereby the past remains actual, and, in the course of each mental state or mental process, there is a literal recapitulation of the organisms' entire past. The depth to surface principle entails a direction of mental process which is essential to microgenesis, and which is explained naturally by the nested hierarchy of surfaces in the brain according to the Holographic Principle. In the Holographic Principle of Mind we have a universal principle of microgenesis, whereby the entire relevant history of the organism is repeated during the course of each timeless and eternal moment. In this way, microgenesis can extend back to the very inception of the Universe. MICROGENESIS AND THE HOLOGRAPHIC PRINCIPLE the Holographic Principle Theory of Mind MARK GERMINE Institute for Psycho Science. Aristotle and others (including, especially, Leibniz) have argued that time does not exist independently of the events that occur in time. This view is typically called either "Reductionism with Respect to Time" or "Relationism with Respect to Time," since according to this view; all talk that appears to be about time can somehow be reduced to talk about temporal relations among things and events. The opposing view, normally referred to either as "Platonism with Respect to Time" or as "Substantivalism with Respect to Time" or as "Absolutism with Respect to Time," has

been defended by Plato, Newton, and others. On this view, time is like an empty container into which things and events may be placed; but it is a container that exists independently of what (if anything) is placed in it. Time First published Mon Nov 25, 2002; substantive revision Fri Jan 24, 2014Markosian, Ned, "Time", The Stanford Encyclopedia of Philosophy (Spring 2014 Edition), Edward N. Zalta (ed.), URL = <http://plato.stanford.edu/archives/spr2014/entries/time/>. The question of whether there could be time without change has traditionally been thought to be closely tied to the question of whether time exists independently of the events that occur in time. For, the thinking goes, if there could be a period of time without change, then it follows that time could exist without any events to fill it; but if, on the other hand, there could not be a period of time without change, then it must be that time exists only if there are some events to fill it. Time First published Mon Nov 25, 2002; substantive revision Fri Jan 24, 2014Markosian, Ned, "Time", The Stanford Encyclopedia of Philosophy (Spring 2014 Edition), Edward N. Zalta (ed.), URL = <http://plato.stanford.edu/archives/spr2014/entries/time/>."Whoever thou mayest be, beloved stranger, whom I meet here for the first time, avail thyself of this happy hour and of the stillness around us, and above us, and let me tell thee something of the thought which has suddenly risen before me like a star which would fain shed down its rays upon thee and every one, as befits the nature of light. - Fellow man! Your whole life, like a sandglass, will always be reversed and will ever run out again; - a long minute of time will elapse until all those conditions out of which you were evolved return in the wheel of the cosmic process. And then you will find every pain and every pleasure, every friend and every enemy, every hope and every error, every blade of grass and every ray of sunshine once more, and the whole fabric of things which make up your life. This ring in which you are but a grain will glitter afresh forever. And in every one of these cycles of human life there will be one hour where, for the first time one man, and then many, will perceive the mighty thought of the eternal recurrence of all things:- and for mankind this is always the hour of Noon". Notes on the Eternal Recurrence - Vol. 16 of Oscar Levy Edition of Nietzsche's Complete Works (in English). Physical time is public time, the time that clocks are designed to measure. Biological time, by contrast, is indicated by an organism's circadian rhythm or body clock, which is normally regulated by the pattern of sunlight and darkness. Psychological time is different from both physical time and biological time. Psychological time is private time. It is also called phenomenological time, and it is perhaps best understood as awareness of physical time. Psychological time passes relatively swiftly for us while we are enjoying an activity, but it slows dramatically if we are waiting anxiously for the pot of water to boil on the stove. The slowness is probably due to focusing our attention on short intervals of physical time. Meanwhile, the clock by the stove is measuring physical time and is not affected by any person's awareness or by any organism's biological time. When a physicist defines speed to be the rate of change of position with respect to time, the term "time" refers to physical time, not psychological time or biological time. Physical time is more basic or fundamental than psychological time for helping us understand our shared experiences in the world, and so it is more useful for doing physical science, but psychological time is vitally important for understanding many mental experiences. Psychological time is faster for older people than for children, as you notice when your grandmother says, "Oh, it's my birthday again." That is, an older person's psychological time is faster relative to physical time. Psychological time is slower or faster depending upon where we are in the spectrum of conscious experience: awake normally, involved in a daydream, sleeping normally, drugged with anesthetics, or in a coma. Some philosophers claim that psychological time is completely transcended in the mental state called nirvana because psychological time slows to a complete stop. There is general agreement among philosophers that, when we are awake normally, we do not experience time as stopping and starting Time First published Mon Nov 25, 2002; substantive revision Fri Jan 24, 2014Markosian, Ned, "Time", The Stanford Encyclopedia of Philosophy (Spring 2014 Edition), Edward N. Zalta (ed.), URL = <http://plato.stanford.edu/archives/spr2014/entries/time/>. But From The Profound Contemplation And Wisdom Of The Buddha And Mahabodhisattvas, We Know There Is No Such Reality. Instead, Egolessness (Non-Self) Is The Only Path To Understand The Reality Of The Deluded Life. All Existences Are Subject To The Law Of Causes And Conditions. These Include The Smallest Particles, The Relationship Between The Particles, The Planets, And The Relationship Between Them, Up To And Including The Whole Universe! From The Smallest Particles To The Biggest Matter, There Exists No Absolute Independent Identity. Egolessness (Non-Self) Implies The Void

Characteristics Of All Existence. Egolessness (Non-Self) Signifies The Non-Existence Of Permanent Identity For Self And Existence (Dharma). Sunyata Stresses The Voidness Characteristic Of Self And Existence (Dharma). Sunyata And Egolessness Possess Similar Attributes. As We Have Discussed Before, We Can Observe The Profound Significance Of Sunyata From The Perspective Of Inter-Dependent Relationships. Considering Dharma-Nature and the Condition of Nirvana, All Existences Are Immaterial and Of A Void-Nature. Then We See Each Existence As Independent Of Each Other. But Then We Cannot Find Any Material That Does Exist Independent Of Everything Else. So Egolessness Also Implies Void-Nature! From The Observation Of All Existences, We Can Infer The Theory Of Nirvana And The Complete Cessation Of All Phenomena. From The Viewpoint Of Phenomena, All Existences Are So Different From Each Other, That They May Contradict Each Other. They Are So Chaotic. In Reality, Their Existence Is Illusionary And Arises From Conditional Causation. They Seem To Exist On One Hand, And Yet Do Not Exist On The Other. They Seem To Be United, But Yet They Are So Different To One Another. They Seem To Exist And Yet They Do Cease! Ultimately Everything Will Return To Harmony And Complete Calmness. This Is The Nature Of All Existence. It Is The Final Resting Place For All. If We Can Understand This Reality And Remove Our Illusions, We Can Find This State Of Harmony And Complete Calmness. Teachings in Chinese Buddhism (6) Sunyata (Emptiness) In the Mahayana Context (Wikipedia) for models please see one of the papers in the series. “The Difference Between Supermind And Big Mind (If We Take Big Mind To Mean The State Experience Of Nondual Suchness, Or Turiyatita) Is That Big Mind Can Be Experienced Or Recognized At Virtually Any Lower Level Or Rung. Magic to Integral. In Fact, One Can Be At, Say, The Pluralistic Stage, And Experience Several Core Characteristics Of The Entire Sequence Of State-Stages (Gross To Subtle To Causal To Witnessing To Nondual), Although, Of Course, The Entire Sequence, Including Nondual Suchness, Will Be Interpreted In Pluralistic Terms. This Is Unfortunate In Many Ways—Interpreting Dharma In Merely Pluralistic Terms (Or Mythic Terms, Or Rational, And So On)—Because It Is So Ultimately Reductionistic; But It Happens All The Time, Given The Relative Independence Of States And Structures At 1st And 2nd Tier. Supermind, On The Other Hand, As A Basic Structure-Rung (Conjoined With Nondual Suchness) Can Only Be Experienced Once All The Previous Junior Levels Have Emerged And Developed, And As In All Structure Development, Stages Cannot Be Skipped. Therefore, Unlike Big Mind, Supermind Can Only Be Experienced After All 1st-, 2nd-, and 3rd-Tier Junior Stages Have Been Passed Through. While, As Genpo Roshi Has Abundantly Demonstrated, Big Mind State Experience Is Available To Virtually Anybody At Almost Any Age (And Will Be Interpreted According To The View Of Their Current Stage), Supermind Is An Extremely Rare Recognition. Supermind, As The Highest Structure-Rung To Date, Has Access To All Previous Structures, All The Way Back To Archaic—And The Archaic Itself, Of Course, Has Transcended And Included, And Now Embraces, Every Major Structural Evolution Going All The Way Back To The Big Bang. (A Human Being Literally Enfolds And Embraces All The Major Transformative Unfoldings Of The Entire Kosmic History—Strings To Quarks To Subatomic Particles To Atoms To Molecules To Cells, All The Way Through The Tree Of Life Up To Its Latest Evolutionary Emergent, The Triune Brain, The Most Complex Structure In The Known Natural World.) Supermind, In Any Given Individual, Is Experienced As A Type Of “Omniscience”—The Supermind, Since It Transcends And Includes All Of The Previous Structure-Rungs, And Inherently Is Conjoined With The Highest Nondual Suchness State, Has A Full And Complete Knowledge Of All Of The Potentials In That Person. It Literally “Knows All,” At Least For The Individual.” ― Ken Wilber, The Fourth Turning: imagining the Evolution of an Integral Buddhism (fopr models please see one of the papers in the series). “The Movement Of Descent And Discovery Begins At The Moment You Consciously Become Dissatisfied With Life. Contrary To Most Professional Opinion, This Gnawing Dissatisfaction With Life Is Not A Sign Of "Mental Illness," or An Indication Of Poor Social Adjustment, or A Character Disorder. For Concealed Within This Basic Unhappiness With Life And Existence Is The Embryo Of A Growing Intelligence, A Special Intelligence Usually Buried Under The Immense Weight Of Social Shams. A Person Who Is Beginning To Sense The Suffering Of Life Is, At The Same Time, Beginning To Awaken To Deeper Realities, Truer Realities. For Suffering Smashes To Pieces The Complacency Of Our Normal Fictions About Reality, And Forces Us To Become Alive In A Special Sense—To See Carefully, To Feel Deeply, To Touch Ourselves And Our Worlds In Ways We Have Heretofore Avoided. It Has Been Said, And Truly I Think, That Suffering Is The First Grace. In A Special Sense, Suffering Is

Almost A Time Of Rejoicing, For It Marks The Birth Of Creative Insight. But Only In a Special Sense. Some People Cling To Their Suffering As A Mother To Its Child, Carrying It As A Burden They Dare Not Set Down. They Do Not Face Suffering With Awareness, But Rather Clutch At Their Suffering, Secretly Transfixed With The Spasms Of Martyrdom. Suffering Should Neither Be (=) Denied Awareness, Avoided, Despised, Not Glorified, Clung To, And Dramatized. The Emergence Of Suffering Is (=) Not So Much Good As It Is A Good Sign, An Indication That One Is Starting To Realize That Life Lived Outside (e) Unity Consciousness Is (=) Ultimately Painful, Distressing, And Sorrowful. The Life Of Boundaries Is (=) A Life Of Battles—Of Fear, Anxiety, Pain, And Finally Death. It Is Only Through All Manner Of Numbing Compensations, Distractions, And Enchantments That We Agree Not To Question Our Illusory Boundaries, The Root Cause Of (e) The Endless Wheel Of Agony. But Sooner Or Later, If We Are Not Rendered Totally Insensitive, (eb) Our Defensive Compensations Begin To Fail Their Soothing And Concealing Purpose. As A Consequence (eb) We Begin To Suffer In One Way Or Another, Because Our Awareness Is (=) Finally Directed Toward The Conflict-Ridden Nature Of Our False Boundaries And The Fragmented Life Supported By (e) Them." — Ken Wilber, No Boundary: eastern and Western Approaches to Personal Growth for models please see one of the papers in the series. "To The Extent That You Actually Realize That You Are Not, For Example, Your Anxieties, Then (eb) Your Anxieties No Longer Threaten You. Even If Anxiety Is Present, (eb) It No Longer Overwhelms You Because (e) You Are No Longer Exclusively Tied To (e&eb) It. You Are No Longer Courting It, Fighting It, Resisting It, Or Running From It. In The Most Radical Fashion, Anxiety Is (=) Thoroughly Accepted As It Is And Allowed To (eb) Move As It Will. You Have (e) Nothing To Lose, Nothing To Gain, By Its Presence Or Absence, For You Are (=) Simply Watching It Pass By. Thus, Any Emotion, Sensation, Thought, Memory, Or Experience That Disturbs You Is (=) Simply One With Which You Have Exclusively Identified Yourself, And The Ultimate Resolution Of The Disturbance Is Simply To Dis-Identify With It. You Cleanly Let All Of Them Drop Away By Realizing That They Are Not You--Since You Can See Them, They Cannot Be The True Seer And Subject. Since They Are Not Your Real Self, There Is No Reason Whatsoever For You To Identify With Them, Hold On To Them, Or Allow Yourself To Be Bound By Them. Slowly, Gently, As You Pursue This Dis-Identification "Therapy," You May Find That You is Entire Individual Self (Persona, Ego, and Centaur), Which Heretofore You Have Fought to Defend and Protect, Begins to Go Transparent and Drop Away. Not That It Literally Falls Off And You Find Yourself Floating, Disembodied, Through Space. Rather, You Begin To Feel That What Happens To Your Personal Self—Your Wishes, Hopes, Desires, Hurts—Is Not A Matter Of Life-Or-Death Seriousness, Because There Is Within You A Deeper And More Basic Self Which Is Not Touched By These Peripheral Fluctuations, These Surface Waves Of Grand Commotion But Feeble Substance. Thus, Your Personal Mind-And-Body May Be In Pain, Or Humiliation, Or Fear, But As Long As You Abide As The Witness Of These Affairs, As If From On High, They No Longer Threaten You, And Thus You Are No Longer Moved To Manipulate Them, Wrestle With Them, Or Subdue Them. Because You Are Willing To Witness Them, To Look At Them Impartially, You Are Able To Transcend Them. As St. Thomas Put It, "Whatever Knows Certain Things Cannot Have Any Of Them In Its Own Nature." Thus, If The Eye Were Colored Red, It Wouldn't Be Able To Perceive Red Objects. It Can See Red Because It Is Clear, Or "Redless." Likewise, If We Can But Watch Or Witness Our Distresses, We Prove Ourselves Thereby To Be "Distress-Less," Free Of The Witnessed Turmoil. That Within Which Feels Pain Is Itself Pain-Less; That Which Feels Fear Is Fear-Less; That Which Perceives Tension Is Tensionless. To Witness These States Is To Transcend Them. They No Longer Seize You From Behind Because You Look At Them Up Front." — Ken Wilber, No Boundary: eastern and Western Approaches to Personal Growth (for models please see one of the papers in the series). "The Difference Between Supermind And Big Mind (If We Take Big Mind To Mean The State Experience Of Nondual Suchness, Or Turiyatita) Is That Big Mind Can Be Experienced Or Recognized At Virtually Any Lower Level Or Rung. ……In Difference and Repetition (1968, English 1994), Deleuze Develops His Project in Multiple Directions. His Work, He Says, Stems From The Convergence Of Two Lines Of Research: The Concept Of Difference Without Negation, And The Concept Of Repetition, In Which Physical And Mechanical Repetitions Are Masks For A Hidden Differential That Is Disguised And Displaced. His Major Focus Is A Thoroughgoing Critique Of Representational Thinking, Including Identity, Opposition, Analogy, And Resemblance (Deleuze 1994, 132). For Deleuze, "Appearances Of"

Are Not Representations, But Sensory Intensities Free Of Subjective Or Objective Identities (Deleuze 1994, 144). Without These Identities, Appearances Are Simulacra of A Non-Apparent Differential He Calls the "Dark Precursor" Or "The In-Itself of Difference" (Deleuze 1994, 119). This Differential Is The Non-Sensible Being Of The Sensible, A Being Not Identical To The Sensible, Or To Itself, But Irreducibly Problematic Insofar As It Forces Us To Encounter The Sensible As "Given." Furthermore, Any Move Against Representational Thinking Impinges Upon The Identity Of The Subject. Where Kant Founds The Representational Unity Of Space And Time Upon The Formal Unity Of Consciousness (Kant 1964, 135-137), Difference Re-Distributes Intuitions Of Past, Present, And Future, Fracturing Consciousness Into Multiple States Not Predicable Of A Single Subject. Intensive Qualities Are Individuating By Themselves, Says Deleuze, And Individuality Is Not Characteristic Of A Self Or An Ego, But Of A Differential Forever Dividing Itself And Changing Its Configuration (Deleuze 1994, 246, 254, 257). In Nietzschean Fashion, The "I" Refers Not To The Unity Of Consciousness, But To A Multitude Of Simulacra Without An Identical Subject For Whom This Multitude Appears. Instead, Subjects Arise And Multiply As "Effects" Of The Intensive Qualities Saturating Space And Time. This Leads Deleuze to Postulate Multiple Faculties for Subjectivity, Which Are Correlates of the Sensible Insofar As It Gives Rise to Feeling, Thought, and Action. "Each Faculty, Including Thought, Has Only Involuntary Adventures," He Says, And "Involuntary Operation Remains Embedded In The Empirical" (Deleuze 1994, 145). Subjectively, The Paradox Of The Differential Breaks Up The Faculties' Common Function And Places Them Before Their Own Limits: Thought Before The Unthinkable, Memory Before The Immemorial, Sensibility Before The Imperceptible, Etc. (Deleuze 1994, 227). This Fracturing And Multiplying Of The Subject, He Notes, Leads To The Realization That "Schizophrenia Is Not Only A Human Fact But Also A Possibility For Thought" (Deleuze 1994, 148), Thus Expanding The Term Into A Philosophical Concept, Beyond Its Clinical Application. Postmodernism First Published Fri Sep 30, 2005 Copyright © 2005 By Gary Aylesworth Geaylesworth@Eiu.Edu Open Access To The SEP Is Made Possible By A World-Wide Funding Initiative for models kindly see one of the papers in the series, procrastinated due to restrictions on space. The Dissolution Of The Subject And Its Implications For Society Is The Theme Of Anti-Oedipus: Capitalism And Schizophrenia, Which Deleuze Published With Félix Guattari In 1972 (English 1983). The Book, In Large Part, Is Written Against An Established Intellectual Orthodoxy Of The Political Left In France During The 1950s And 1960s, An Orthodoxy Consisting Of Marx, Freud, And Structuralist Concepts Applied To Them By Louis Althusser And Jacques Lacan. Deleuze And Guattari Argue That This Mixture Is Still Limited By Representational Thinking, Including Concepts Of Production Based Upon Lack, And Concepts Of Alienation Based Upon Identity And Negation. Furthermore, The Oedipus Concept In Psychoanalysis, They Say, Institutes A Theater Of Desire In Which The Psyche Is Embedded In A Family Drama Closed Off From The Extra-Familial And Extra-Psychic Forces At Work In Society. They Characterize These Forces As "Desiring Machines" Whose Function Is To Connect, Disconnect, And Reconnect With One Another Without Meaning Or Intention. The Authors Portray Society As A Series Of "Territorializations" Or Inscriptions Upon The "Body Without Organs, "Or The Free-Flowing Matter Of Intensive Qualities Filling Space In Their Varying Degrees. The First Inscriptions Are Relations Of Kinship And Filiation Structuring Primitive Societies, Often Involving The Marking And Scarring Of Human Bodies. As An Interruption and Encoding of "Flows," The Primitive Inscriptions Constitute a Nexus of Desiring Machines, Both Technical and Social, Who's Elements Are Humans and Their Organs. The Full Body Of Society Is The Sacred Earth, Which Appropriates To Itself All Social Products As Their Natural Or Divine Precondition, And To Whom All Members Of Society Are Bound By Direct Filiation (Deleuze 1983b, 141-42). These First Inscriptions Are Then De-Territorialized And Re-Coded By The "Despotic Machine," Establishing New Relations Of Alliance And Filiation Through The Body Of The Ruler Or Emperor, Who Alone Stands In Direct Filiation To The Deity (Deleuze 1983b, 192) And Who Institutes The Mechanism Of The State Upon Pre-Existing Social Arrangements. Finally, Capitalism De-Territorializes The Inscriptions Of The Despotic Machine And Re-Codes All Relations Of Alliance And Filiation Into Flows Of Money (Deleuze 1983b, 224-27). The Organs Of Society And The State Are Appropriated Into The Functioning Of Capital, And Humans Become Secondary To The Filiation Of Money With Itself. Postmodernism First Published Fri Sep 30, 2005 Copyright © 2005 By Gary Aylesworth Geaylesworth@Eiu.Edu Open Access To The SEP Is Made Possible By A World-Wide Funding

Initiative For Models Kindly See One Of The Papers In The Series, Procrastinated Due To Restrictions On Space. Deleuze And Guattari See In The Capitalist Money System "An Axiomatic Of Abstract Quantities That Keeps Moving Further And Further In The Direction Of The Deterritorialization Of The Socius" (Deleuze 1983a, 33), Which Is To Say That Capital Is Inherently Schizophrenic. However, Because Capital Also Re-Territorializes All Flows Into Money, Schizophrenia Remains Capitalism's External Limit. Nevertheless, It Is Precisely That Limit Against Which Thinking Can Subject Capitalism To Philosophical Critique. Psychoanalysis, They Say, Is Part Of The Reign Of Capital Because It Re-Territorializes The Subject As "Private" And "Individual," Instituting Psychic Identity Through Images Of The Oedipal Family. However, The Oedipal Triangle Is Merely A Representational Simulacrum Of Kinship And Filiation, Re-Coded Within A System Of Debt And Payment. In This System, They Insist, Flows Of Desire Have Become Mere Representations Of Desire, Cut Off From The Body Without Organs And The Extra-Familial Mechanisms Of Society. A Radical Critique Of Capital Cannot Therefore Be Accomplished By Psychoanalysis, But Requires A Schizoanalysis "To Overturn The Theater Of Representation Into The Order Of Desiring-Production" (Deleuze 1983b, 271). Here, The Authors See A Revolutionary Potential In Modern Art And Science, Where, In Bringing About The "New," They Circulate De-Coded And De-Territorialized Flows Within Society Without Automatically Re-Coding Them Into Money (Deleuze 1983a, 379). In This Revolutionary Aspect, Anti-Oedipus Reads As A Statement Of The Desire That Took To The Streets Of Paris In May Of 1968, And Which Continues, Even Now, To Make Itself Felt In Intellectual Life. Postmodernism First Published Fri Sep 30, 2005 Copyright © 2005 By Gary Aylesworth Geaylesworth@Eiu.Edu Open Access To The SEP Is Made Possible By A World-Wide Funding Initiative For Models Kindly See One Of The Papers In The Series, Procrastinated Due To Restrictions On Space. The Term "Deconstruction," Like "Postmodernism," Has Taken On Many Meanings In The Popular Imagination. However, In Philosophy, It Signifies Certain Strategies For Reading And Writing Texts. The Term Was Introduced Into Philosophical Literature In 1967, With The Publication Of Three Texts By Jacques Derrida: Of Grammatology (English 1974), Writing And Difference (English 1978), And Speech And Phenomena (English 1973). This So-Called "Publication Blitz" Immediately Established Derrida As A Major Figure In The New Movement In Philosophy And The Human Sciences Centered In Paris, And Brought The Idiom "Deconstruction" Into Its Vocabulary. Derrida And Deconstruction Are Routinely Associated With Postmodernism, Although Like Deleuze And Foucault, He Does Not Use The Term And Would Resist Affiliation With "-Isms" Of Any Sort. Of The Three Books From 1967, Of Grammatology Is The More Comprehensive In Laying Out The Background For Deconstruction As A Way Of Reading Modern Theories Of Language, Especially Structuralism, And Heidegger's Meditations On The Non-Presence Of Being. It Also Sets Out Derrida's Difference With Heidegger Over Nietzsche. Where Heidegger Places Nietzsche Within The Metaphysics Of Presence, Derrida Insists That "Reading, And Therefore Writing, The Text Were For Nietzsche 'Originary' Operations," (Derrida 1974, 19), And This Puts Him At The Closure Of Metaphysics (Not The End), A Closure That Liberates Writing From The Traditional Logos, Which Takes Writing To Be A Sign (A Visible Mark) For Another Sign (Speech), Whose "Signified" Is A Fully Present Meaning. This Closure Has Emerged, Says Derrida, With The Latest Developments In Linguistics, The Human Sciences, Mathematics, And Cybernetics, Where The Written Mark Or Signifier Is Purely Technical, That Is, A Matter Of Function Rather Than Meaning. Precisely The Liberation Of Function Over Meaning Indicates That The Epoch Of What Heidegger Calls The Metaphysics Of Presence Has Come To Closure, Although This Closure Does Not Mean Its Termination. Just As In The Essay "On The Question Of Being" (Heidegger 1998, 291-322) Heidegger Sees Fit To Cross Out The Word "Being," Leaving It Visible, Nevertheless, Under The Mark, Derrida Takes The Closure Of Metaphysics To Be Its "Erasure," Where It Does Not Entirely Disappear, But Remains Inscribed As One Side Of A Difference, And Where The Mark Of Deletion Is Itself A Trace Of The Difference That Joins And Separates This Mark And What It Crosses Out. Derrida Calls This Joining And Separating Of Signs Différance (Derrida 1974, 23), A Device That Can Only Be Read And Not Heard When Différance And Différence Are Pronounced In French. The "A" Is A Written Mark That Differentiates Independently Of The Voice, The Privileged Medium Of Metaphysics. In This Sense, Différance As The Spacing Of Difference, As Archi-Writing, Would Be The Gram Of Grammatology. However, As Derrida Remarks: "There Cannot Be A Science Of Difference Itself In Its Operation, As It Is Impossible To Have A Science Of The Origin Of Presence Itself, That Is

To Say Of A Certain Non-Origin" (Derrida 1974, 63). Instead, There Is Only The Marking Of The Trace Of Difference, That Is, Deconstruction. Because At Its Functional Level All Language Is A System Of Differences, Says Derrida, All Language, Even When Spoken, Is Writing, And This Truth Is Suppressed When Meaning Is Taken As An Origin, Present And Complete Unto Itself. Texts That Take Meaning Or Being As Their Theme Are Therefore Particularly Susceptible To Deconstruction, As Are All Other Texts Insofar As They Are Conjoined With These. For Derrida, Written Marks Or Signifiers Do Not Arrange Themselves Within Natural Limits, But Form Chains Of Signification That Radiate In All Directions. As Derrida Famously Remarks, "There Is No Outside-Text" (Derrida 1974, 158), That Is, The Text Includes The Difference Between Any "Inside" Or "Outside." A Text, Then, Is Not A Book, And Does Not, Strictly Speaking, Have An Author. On The Contrary, The Name Of The Author Is A Signifier Linked With Others, And There Is No Master Signifier (Such As The Phallus In Lacan) Present Or Even Absent In A Text. This Goes For The Term "Différance" As Well, Which Can Only Serve As A Supplement For The Productive Spacing Between Signs. Therefore, Derrida Insists That "Différance Is Literally Neither A Word Nor A Concept" (Derrida 1982, 3). Instead, It Can Only Be Marked As A Wandering Play Of Differences That Is Both A Spacing Of Signifiers In Relation To One Another And A Deferral Of Meaning Or Presence When They Are Read. How, Then, Can Différance Be Characterized? Derrida Refuses To Answer Questions As To "Who" Or "What" Differs, Because To Do So Would Suggest There Is A Proper Name For Difference Instead Of Endless Supplements, Of Which "Différance" Is But One. Structurally, This Supplemental Displacement Functions Just As, For Heidegger, All Names For Being Reduce Being To The Presence Of Beings, Thus Ignoring The "Ontological Difference" Between Them. However, Derrida Takes The Ontological Difference As One Difference Among Others, As A Product Of What The Idiom "Différance" Supplements. As He Remarks: "Différance, In A Certain And Very Strange Way, (Is) 'Older' Than The Ontological Difference Or Than The Truth Of Being" (Derrida 1982, 22). Deconstruction, Then, Traces The Repetitions Of The Supplement. It Is Not So Much A Theory About Texts As A Practice Of Reading And Transforming Texts, Where Tracing The Movements Of Différance Produces Other Texts Interwoven With The First. While There Is Certain Arbitrariness In The Play Of Differences That Result, It Is Not The Arbitrariness Of A Reader Getting The Text To Mean Whatever He Or She Wants. It Is A Question Of Function Rather Than Meaning, If Meaning Is Understood As A Terminal Presence, And The Signifying Connections Traced In Deconstruction Are First Offered By The Text Itself. A Deconstructive Reading, Then, Does Not Assert Or Impose Meaning, But Marks Out Places Where The Function Of The Text Works Against Its Apparent Meaning, Or Against The History Of Its Interpretation. Hyperreality is closely related to the concept of the simulacrum: a copy or image without reference to an original. In postmodernism, hyperreality is the result of the technological mediation of experience, where what passes for reality is a network of images and signs without an external referent, such that what is represented is representation itself. In Symbolic Exchange and Death (1976) (English 1993), Jean Baudrillard uses Lacan's concepts of the symbolic, the imaginary, and the real to develop this concept while attacking orthodoxies of the political Left, beginning with the assumed reality of power, production, desire, society, and political legitimacy. Baudrillard argues that all of these realities have become simulations, that is, signs without any referent, because the real and the imaginary have been absorbed into the symbolic Postmodernism First Published Fri Sep 30, 2005 Copyright © 2005 By Gary Aylesworth Geaylesworth@Eiu.Edu Open Access To The SEP Is Made Possible By A World-Wide Funding Initiative For Models Kindly See One Of The Papers In The Series, Procrastinated Due To Restrictions On Space. Magic to Integral. In Fact, One Can Be At, Say, The Pluralistic Stage, And Experience Several Core Characteristics Of The Entire Sequence Of State-Stages (Gross To Subtle To Causal To Witnessing To Nondual), Although, Of Course, The Entire Sequence, Including Nondual Suchness, Will Be Interpreted In Pluralistic Terms. This Is Unfortunate In Many Ways—Interpreting Dharma In Merely Pluralistic Terms (Or Mythic Terms, Or Rational, And So On)—Because It Is So Ultimately Reductionistic; But It Happens All The Time, Given The Relative Independence Of States And Structures At 1st And 2nd Tier. Supermind, On The Other Hand, As A Basic Structure-Rung (Conjoined With Nondual Suchness) Can Only Be Experienced Once All The Previous Junior Levels Have Emerged And Developed, And As In All Structure Development, Stages Cannot Be Skipped. Therefore, Unlike Big Mind, Supermind Can Only Be Experienced After All 1st-, 2nd-, and 3rd-Tier Junior Stages Have Been Passed Through. While, As Genpo Roshi

Has Abundantly Demonstrated, Big Mind State Experience Is Available To Virtually Anybody At Almost Any Age (And Will Be Interpreted According To The View Of Their Current Stage), Supermind Is An Extremely Rare Recognition. Supermind, As The Highest Structure-Rung To Date, Has Access To All Previous Structures, All The Way Back To Archaic—And The Archaic Itself, Of Course, Has Transcended And Included, And Now Embraces, Every Major Structural Evolution Going All The Way Back To The Big Bang. (A Human Being Literally Enfolds And Embraces All The Major Transformative Unfoldings Of The Entire Kosmic History—Strings To Quarks To Subatomic Particles To Atoms To Molecules To Cells, All The Way Through The Tree Of Life Up To Its Latest Evolutionary Emergent, The Triune Brain, The Most Complex Structure In The Known Natural World.) Supermind, In Any Given Individual, Is Experienced As A Type Of "Omniscience"—The Supermind, Since It Transcends And Includes All Of The Previous Structure-Rungs, And Inherently Is Conjoined With The Highest Nondual Suchness State, Has A Full And Complete Knowledge Of All Of The Potentials In That Person. It Literally "Knows All," At Least For The Individual." ― Ken Wilber, The Fourth Turning: imagining the Evolution of an Integral Buddhism (fopr models please see one of the papers in the series). As Lyotard Argues, Aesthetic Judgment Is The Appropriate Model For The Problem Of Justice In Postmodern Experience Because We Are Confronted With A Plurality Of Games And Rules Without A Concept Under Which To Unify Them. Judgment Must Therefore Be Reflective Rather Than Determining. Furthermore, Judgment Must Be Aesthetic Insofar As It Does Not Produce Denotative Knowledge About A Determinable State Of Affairs, But Refers To The Way Our Faculties Interact With Each Other As We Move From One Mode Of Phrasing To Another, I.E. The Denotative, the Prescriptive, the Performative, the Political, the Cognitive, the Artistic, Etc. In Kantian Terms, This Interaction Registers As An Aesthetic Feeling. Where Kant Emphasizes The Feeling Of The Beautiful As A Harmonious Interaction Between Imagination And Understanding, Lyotard Stresses The Mode In Which Faculties (Imagination And Reason,) Are In Disharmony, I.E. The Feeling of the Sublime. For Kant, The Sublime Occurs When Our Faculties Of Sensible Presentation Are Overwhelmed By Impressions Of Absolute Power And Magnitude, And Reason Is Thrown Back Upon Its Own Power To Conceive Ideas (Such As The Moral Law) Which Surpass The Sensible World. For Lyotard, However, The Postmodern Sublime Occurs When We Are Affected By A Multitude Of Unpresentables Without Reference To Reason As Their Unifying Origin. Justice, Then, Would Not Be A Definable Rule, But Ability To Move And Judge Among Rules In Their Heterogeneity And Multiplicity. In This Respect, It Would Be More Akin To The Production Of Art Than A Moral Judgment In Kant's Sense. Postmodernism First Published Fri Sep 30, 2005 Copyright © 2005 By Gary Aylesworth Geaylesworth@Eiu.Edu Open Access To The SEP Is Made Possible By A World-Wide Funding Initiative. The Scholars Of Tien Tai Called It The "Embodied Nature". (This Is The Buddha-Nature That Includes Both Good And Evil.) The Scholars of Xian Shou Say, "It Is Arising From Primal Nature", And the Scholars of Chan (Zen) Say, "It Is Nature That Causes the Rising of Things". All Dharma Is Dharma-Nature. It Is Not Different From Dharma-Nature. Dharma And Dharma-Nature Are Not Two Separate Identities, "Phenomena" And "Nature" Is Also Not Distinguishable Either. In Other Words, There Is No Difference Between Principle (Absolute) And Practice (Relative). Teachings in Chinese Buddhism (6) Sunyata (Emptiness) In the Mahayana Context (Wikipedia). In Other Words, All Dharma Arises From Causes and Conditions, and All Dharma Is Empty In Nature. The Law of Dependent Origination (Existence) and the Nature of Emptiness Is Neither the Same nor Different. They Exist Mutually. The Truth Of "Sunyata" And "Existence", And "Nature" And "Phenomena" Are Not In Conflict With Each Other. Unlike The Scholars Of The Dharmalaksana Sect Who Explain The Dharma Only From The Aspect Of Dependent Origination, Or The Scholars Of Dharma-Nature That Explain The Existence Of Dharma Only From The Aspect Of Dharma-Nature, The Scholars Of Madhyamika Explain The Truth Of The Dharma From Both Aspects. Hence This Is Called The Middle Path Which Does Not Incline To Either Side. Teachings in Chinese Buddhism (6) Sunyata (Emptiness) In the Mahayana Context (Wikipedia). For Stoics, Causes Involved Reference To The Depths, But Effects At The Surface Could Also Have Relations Among Themselves. This Permitted The Distinction To Be Drawn Between Destiny And Necessity—The Stoics Wanted To Affirm Destiny And Deny Necessity. In The First Place, Effects Express Causes, But Expressions Of Relations [Rather Than Necessity] Connects Those Effects. Those Relations May Be Described As Compatibility Or Incompatibility, Conjunction Or Disjunction. These Are Not Causal Relations Themselves, But Represent 'An

Aggregate Of Noncausal Correspondences Which Form A System Of Echoes, Of Presumptions And Resonances, A System Of Signs—In Short, An Expressive Quasi Causality And Not At All A Necessitating Causality' (170). This Need Not Involve Contradiction, Which Is Applying To Events Rules That Really Only Applied To Logic And Argument. There Can Be Incompatability Without Contradiction, A Noncausal Correspondence. Leibniz Described Incompossible Worlds [Delanda Is Very Clear On This Too]. Only Impossible Events Contradict Possible Ones. Events Can Be Compossible [Roughly, They Have Predictable and Predicative Future and Past Events]. For Deleuze, It Is A Matter Of 'The Convergence Of Series Which Singularities Of Events Form As They Stretch Themselves Out Over Lines Of Ordinary Points. Incompossibility Must Be Defined By the Divergence of Such Series' (171). Such A Notion Is Essential To Any Theory of Sense. We Should Not See Divergence As A Matter Of Exclusion, As Leibniz Did [Since God Chose Actual Events]. Divergent And Disjunction Can Both Be Positive, While Preserving Differences. In Fact, Differences Are Crucial, Preserving The Distance Between Objects While Affirming That They Are Related. This 'Permits The Measuring Of Contraries Through Their Finite Difference Instead Of Equating Difference With A Measureless Contrariety' (173). It Is Contradiction Which Is The Special Case. Difference Here Is A Topological Term Relating To Distance On Surfaces Rather Than Depths. It Is Not Just A Matter Of Suggesting 'Some Unknown Identity Of Contraries (As In Commonplace In Spiritualist And Colorist Philosophy)' (173) [Take That St Pierre!]. An Example Is Nietzsche Arguing That Health And Sickness Can Both Inform Each Other; Act As Points Of View, Remembering That 'Things, Beings, Are Themselves Points Of View' (173). Divergence Does Not Mean Exclusion, And Disjunction Does Not Mean Separation. Connective Syntheses '(If…, Then)' Construct A Single Series; Conjunctive Series '(And)' Produces Convergent Series, But Disjunctive Series '(Or)' Produces A Divergence Series. Normally, Disjunction Helps Us To Criticise Synthesis, But Disjunction Can Still Be A Synthesis Itself, Despite Its Use In Logical Analysis [I Think What Is Going On Here Is Arguing That There Is A Difference Between 'Either/Or' In A Logical Sense, And 'One Or Two' In The Real Sense—The Latter Can Mean That Both Are Compossible. This Is The 'Communication Of Events' Rather Than The Logical Business Of Analysing Predicates (174)]. The Synthetic Disjunction Expresses The Paradox, With Divergence At The Centre. The Discussion of Paradoxes and Esoteric Words Above Are Examples: They Contract 'The Multitude of Divergent Series in the Successive Appearance of A Single One' (175). There Is A Difference Again Between the Subversion of the Present and Simple Identity by Depths, And Operations at the Surface. By Considering The Depths, We Encounter Infinite Identities [As Events Become Examples Of Deeper Categories, Wholes?]. At The Surface Events Communicate With Each Other Directly Through Maintaining Distance And By Affirming Disjunctions. Disjunction Threatens The Identity Of The Self, And Helps Us To See The Self As 'So Many Impersonal And Pre-Individual Singularities' [Connected Through Disjunctive Synthesis. Hence the Importance of Heterogeneity] (175). The Normal Concept Of The Self Implies Some Connected Series, 'But When Disjunction Accedes To The Principle Which Gives To It A Synthetic And Affirmative Value, The Self, The World, And God Share In A Common Death' (176). [There Is Also A Point That Divergent Series Explain And Also Exceed Normal Conjunctive And Connective Series]. In The Usual Conception, 'The Self Is the Principle of Manifestation, In Relation To the Proposition, the World Is the Principle of Denotation, And God the Principle of Signification' (176). But The Theory Of Sense Here Says That It Emanates From Nonsense, From Paradox, And From The 'Eternally Decentred Ex-Centric Centre' (176). This Position 'Does Not Tolerate the Subsistence of God as an Original Individuality, Nor The Self As The Person, Nor The World As An Element Of The Self And As God's Product. The Divergence Of The Affirmed Series Forms A "Chaosmos" And No Longer A World; The Aleatory Point Which Traverses Them Forms A Counter self, And No Longer A Self' (176). There Is No Centre But Only 'Pure Events Which The Instant, Displaced Over The Line [Of Aion], Goes On Dividing Into Already Past And Yet To Come. Nothing Other Than The Event Subsists… This Communicates With Itself through Its Own Distance and Resonates Across All of Its Disjuncts' (176). Alexander I. Stingl's Blog A Nomadic Scholarship Entity NOTES ON: Deleuze, G (1990) The Logic Of Sense, Trans Mark Lester, Edited By Constantin Boundas, New York: Columbia University Press For Models Kindly See One Of The Papers In The Series. Attributable And Ascribable To Constraints On Space Accomodation Model Is Provided In Some Paper, Notwithstanding The Generality And Commonalty Of The Observation And Its Concatenatability With The Other Modules Sixteenth

Series Of The Static Ontological Genesis Twenty-fourth series of the communication of events. The Buddha Always Used The Terms Void, No Rising And Falling, Calmness And Extinction To Explain The Profound Meaning Of Sunyata And Cessation. For Example, Sunyata And The State Of Nirvana Where There Is No Rising Nor Falling, Are Interpreted By Most People As A State Of Non-Existence And Gloom. They Fail To Realise That Quite The Opposite, Sunyata Is Of Substantial And Positive Significance. The Sutras Often Use The Word "Great Void" To Explain The Significance Of Sunyata. In General, We Understand The "Great Void" As Something That Contains Absolutely Nothing. However, From A Buddhist Perspective, The Nature Of The "Great Void" Implies Something Which Does Not Obstruct Other Things, In Which All Matters Perform Their Own Functions. Materials Are Form, Which By Their Nature, Imply Obstruction. The Special Characteristic Of The "Great Void" Is Non-Obstruction. The "Great Void" Therefore, Does Not Serve As An Obstacle To Them. Since The "Great Void" Exhibits No Obstructive Tendencies, It Serves As The Foundation For Matter To Function. In Other Words, If There Was Neither “Great Void” Nor Characteristic Of Non-Obstruction, It Would Be Impossible For The Material World To Exist And Function. The "Great Void" Is Not Separated From The Material World. The Latter Depends On The Former. We Can State That the Profound Significance of Sunyata and the Nature of Sunyata in Buddhism highlight The "Great Void’s" Non-Obstructive Nature. Sunyata Does Not Imply The "Great Void". Instead, It Is The Foundation Of All Phenomena (Form And Mind). It Is The True Nature Of All Phenomena, And It Is The Basic Principle Of All Existence. In Other Words, If The Universe’s Existence Was Not Empty Nor Impermanent, Then All Resulting Phenomena Could Not Have Arisen Due To The Co-Existence Of Various Causes And There Would Be No Rising Nor Falling. The Nature Of Sunyata Is Of Positive Significance. Calmness And Extinction Are The Opposite Of Rising And Falling. They Are Another Way To Express That There Is No Rising And Falling. Rising And Falling Are The Common Characteristics Of Worldly Existence. All Phenomena Are Always In The Cycle Of Rising And Falling. However, Most People Concentrate On Living (Rising). They Think That The Universe And Life Are The Reality Of A Continuous Existence. Buddhism On The Other Hand, Promotes The Value Of A Continuous Cessation (Falling). This Cessation Does Not Imply That It Ceases To Exist Altogether. Instead, It Is Just A State In The Continuous Process Of Phenomena. In This Material World, Or What We May Call This "State Of Existence", Everything Eventually Ceases To Exist. Cessation Is Definitely The Home Of All Existences. Since Cessation Is The Calm State Of Existence And The Eventual Refuge Of All Phenomena, It Is Also The Foundation For All Activities And Functions. Teachings in Chinese Buddhism (6) Sunyata (Emptiness) In the Mahayana Context (Wikipedia). You Are In Your Own Film A Subjective Experience Where You Are Both Actor And Director Writing Your Script As You Go Along. You Are Both The Audience And The Actor A Near State Of Pure Consciousness. You Are An Individual General Ledger With A Place In Collective General Ledger And Cosmic General Ledger With Celestial Brahman Anti Brahman Simulating The Events Which Appear To You On The Screen Of Consciousness. Sensations Are Disruptions Within Actual Identifications; They Are Signs Of The Expression Of Ideas And Intensities In Actual Structures. There Is No Denying the Difficulty of Deleuze’s Account of Reality – Necessary for an Understanding of the Role Played by Kantian Arguments. It May Therefore Be Helpful To Think In Terms Of An Example. You Have Waited Years To See A Loved One Again, Tarrying With Dimming Memories And Shifts In Longings And Needs. A Meeting Has Been Arranged. The Opening Gestures, Words And Images Bring Together Fixed Memories And Preconceptions (Actual Identifications) With Feelings That Destroy those (Actual Sensations Of Disappointment, Excitement, Bemusement, And Renewed Passion). The Power Of These Sensations Cannot Come Solely From The Disrupted Identities; Rather, They Are Expressions Of Deeper Charges (Virtual Intensities). But, Beyond Local Identifications And Their More And More Distant Actual Effects, Imagined And Even Undreamed Worlds Of Ideas Are Changed Through The Meeting. According To Deleuze, These Are Not Possible Worlds, But Virtual Ones That Ground Actual Identifications. The Actual Meeting Is Accompanied By A Virtual One, Where Ideas Take Shape And Acquire Significance Through Shifts In Relations Between Intensities. These Shifts Light Up Actual Situations In New Ways And Determine Different Ideas To Come Into Play. The Meeting Is A Failure. Intensities Push Feelings of Disappointment to the Fore, Destroying One World of Ideas and Perhaps Determining a Colder One with Greater Clarity. The Transversal Thought of Gilles Deleuze: Encounters and Influences James Williams. For models please see one of the papers in the series. Models

are procrastinated due to spatial restrictions. You Can Stir The Conscious State And Consequentially Connect All The Three States. As Lyotard Argues, Aesthetic Judgment Is The Appropriate Model For The Problem Of Justice In Postmodern Experience Because We Are Confronted With A Plurality Of Games And Rules Without A Concept Under Which To Unify Them. Judgment Must Therefore Be Reflective Rather Than Determining. Furthermore, Judgment Must Be Aesthetic Insofar As It Does Not Produce Denotative Knowledge About A Determinable State Of Affairs, But Refers To The Way Our Faculties Interact With Each Other As We Move From One Mode Of Phrasing To Another, I.E. The Denotative, the Prescriptive, the Performative, the Political, the Cognitive, the Artistic, Etc. In Kantian Terms, This Interaction Registers As An Aesthetic Feeling. Where Kant Emphasizes The Feeling Of The Beautiful As A Harmonious Interaction Between Imagination And Understanding, Lyotard Stresses The Mode In Which Faculties (Imagination And Reason,) Are In Disharmony, I.E. The Feeling of the Sublime. For Kant, The Sublime Occurs When Our Faculties Of Sensible Presentation Are Overwhelmed By Impressions Of Absolute Power And Magnitude, And Reason Is Thrown Back Upon Its Own Power To Conceive Ideas (Such As The Moral Law) Which Surpass The Sensible World. For Lyotard, However, The Postmodern Sublime Occurs When We Are Affected By A Multitude Of Unpresentables Without Reference To Reason As Their Unifying Origin. Justice, Then, Would Not Be A Definable Rule, But Ability To Move And Judge Among Rules In Their Heterogeneity And Multiplicity. In This Respect, It Would Be More Akin To The Production Of Art Than A Moral Judgment In Kant's Sense. Postmodernism First Published Fri Sep 30, 2005 Copyright © 2005 By Gary Aylesworth Geaylesworth@Eiu.Edu Open Access To The SEP Is Made Possible By A World-Wide Funding Initiative. In Dissolving All Social Phenomena In The Acid Bath Of Power And Domination, Foucault Prevents Critical Theory From Drawing Crucial Distinctions, Such As Those "Between Just And Unjust Social Arrangements, Legitimate And Illegitimate Uses Of Political Power, Strategic And Cooperative Interpersonal Relations, Coercive And Consensual Measures" (McCarthy 1991: 54). One Cannot Say, For Example, That One Regime Of Power Is Any Better Or Worse Than Another, Only That They Are Different -- "Another Power, Another Knowledge" (Foucault 1979: 226). Since Ruling Powers Attempt To Erase Such Distinctions, Or To Present Injustice As Justice, Falsehood As Truth, And Domination As Freedom, Foucault's Position Unwittingly Supports The Mystifications Of Orwellian Doublespeak, Now More Rife Than Ever (See Kellner 2001), And Blocks The Discriminations Necessary For Social Critique. If There Are No Standards Or Right, Then, With Thrasymacus And Hobbes, We Can Conclude Might Is As Right As Anything. There Can Be No Ideology Critique Where There Is No Distinction Between True And False, And any Social Or Moral Critique Without A Distinction Between Right And Wrong. The Evaluative Character Of Foucault's Own Work Is Not Any Less Normative For His Refusal To Explicitly Confront It. The Problem Becomes Glaring In His Later Work Where He Employs Normative Terms Such As Liberty And Autonomy, But Fails To State What We Should Be Free For. Foucault's Anti-Normative Stance Therefore Forces Him into Self-Defeating Value Neutrality. Richard Rorty, The Attack on Theory, And Renunciation of Radical Politics (for models see one of the papers in the series). Our Environment Can Either Open Up or Constrict Possibilities for Spiritual and Intellectual Growth. In The Politics Of Experience Laing Offers A Fine Passage From Erving Goffman’s Studies Encounters: There Seems To Be No Agent More Effective Than Another Person In Bringing A World For Oneself Alive, Or, By A Glance, A Gesture, Or A Remark, Shriveling Up The Reality In Which One Is Lodged. "The Anxiety Of Meaninglessness Is Anxiety About The Loss Of An Ultimate Concern, Of A Meaning Which Gives Meaning To All Meanings. This Anxiety Is Aroused By The Loss Of A Spiritual Center, Of An Answer, However Symbolic And Indirect, To The Question Of The Meaning Of Existence." Philosophy And Depression By Tim Ruggiero -- Paul Tillich, The Courage To Be (1952) As Quoted In Depression And The Threat Of Nonbeing Tim Ruggiero, September 4, 2001 (Philosophical Society.Com).The Dissolution Of The Subject And Its Implications For Society Is The Theme Of Anti-Oedipus: Capitalism And Schizophrenia, Which Deleuze Published With Félix Guattari In 1972 (English 1983). The Book, In Large Part, Is Written Against An Established Intellectual Orthodoxy Of The Political Left In France During The 1950s And 1960s, An Orthodoxy Consisting Of Marx, Freud, And Structuralist Concepts Applied To Them By Louis Althusser And Jacques Lacan. Deleuze And Guattari Argue That This Mixture Is Still Limited By Representational Thinking, Including Concepts Of Production Based Upon Lack, And Concepts Of Alienation Based Upon Identity And Negation. Furthermore, The Oedipus

Concept In Psychoanalysis, They Say, Institutes A Theater Of Desire In Which The Psyche Is Embedded In A Family Drama Closed Off From The Extra-Familial And Extra-Psychic Forces At Work In Society. They Characterize These Forces As "Desiring Machines" Whose Function Is To Connect, Disconnect, And Reconnect With One Another Without Meaning Or Intention. The Authors Portray Society As A Series Of "Territorializations" Or Inscriptions Upon The "Body Without Organs, "Or The Free-Flowing Matter Of Intensive Qualities Filling Space In Their Varying Degrees. The First Inscriptions Are Relations Of Kinship And Filiation Structuring Primitive Societies, Often Involving The Marking And Scarring Of Human Bodies. As An Interruption and Encoding of "Flows," The Primitive Inscriptions Constitute a Nexus of Desiring Machines, Both Technical and Social, Who's Elements Are Humans and Their Organs. The Full Body Of Society Is The Sacred Earth, Which Appropriates To Itself All Social Products As Their Natural Or Divine Precondition, And To Whom All Members Of Society Are Bound By Direct Filiation (Deleuze 1983b, 141-42). These First Inscriptions Are Then De-Territorialized And Re-Coded By The "Despotic Machine," Establishing New Relations Of Alliance And Filiation Through The Body Of The Ruler Or Emperor, Who Alone Stands In Direct Filiation To The Deity (Deleuze 1983b, 192) And Who Institutes The Mechanism Of The State Upon Pre-Existing Social Arrangements. Finally, Capitalism De-Territorializes The Inscriptions Of The Despotic Machine And Re-Codes All Relations Of Alliance And Filiation Into Flows Of Money (Deleuze 1983b, 224-27). The Organs Of Society And The State Are Appropriated Into The Functioning Of Capital, And Humans Become Secondary To The Filiation Of Money With Itself. Postmodernism First Published Fri Sep 30, 2005 Copyright © 2005 By Gary Aylesworth Geaylesworth@Eiu.Edu Open Access To The SEP Is Made Possible By A World-Wide Funding Initiative For Models Kindly See One Of The Papers In The Series, Procrastinated Due To Restrictions On Space. "The Vanity Of Existence Is Revealed In The Whole Form Existence Assumes: In The Infiniteness Of Time And Space Contrasted With The Finiteness Of The Individual In Both; In The Fleeting Present As The Sole Form In Which Actuality Exists; In The Contingency And Relativity Of All Things; In Continual Becoming Without Being; In Continual Desire Without Satisfaction; In The Continual Frustration Of Striving Of Which Life Consists." (For Models See One of the Papers in the Series, Which Are Procrastinated Due to Spatial Constraints) -- Arthur Schopenhauer, "On the Vanity of Existence" as quoted in Philosophy and Depression by Tim Ruggiero. "There Is Little Conjunction Of Truth And Social 'Reality'. Around Us Are Pseudo-Events, To Which We Adjust With A False Consciousness Adapted To See These Events As True And Real, And Even As Beautiful. In The Society Of Men The Truth Resides Now Less In What Things Are Than In What They Are Not. Our Social Realities Are So Ugly If Seen In The Light Of Exiled Truth, And Beauty Is Almost No Longer Possible If It Is Not A Lie." (For Models See One of the Papers in the Series, Which Are Procrastinated Due to Spatial Constraints) -- R.D. Laing, the Politics of Experience. [We're going to Use the Idea of a Game to Further Illustrate How the Virtual Is Imminent in the Actual. Why This Example? To Have A Go At The Metaphor Of Games In Philosophy? To Instate Lewis Carroll As Some Great Philosopher?] The Actual Games Have A Series Of Rules About Playing And Outcomes That Limit The Influence Of Chance. It's Possible to Invert These Rules to arrive at an Ideal Game, With No Rules, And with the Full Operation of Chance [Which Describes the Virtual, I Suppose]. Carroll Describe Some. We Also Learn About Time And How It Operates. In Each Throw Of The Dice, Singularities Are Distributed As A 'Nomadic And Non Sedentary Distribution, Wherein Each System Of Singularities Communicate And Resonates With The Others, Being At Once Implicated By The Others And Implicating Them In The Most Important Cast' (60). This Game Can Only Be Played In Thought [Since Any Empirical Reality Limits Chance. Chance Is Fixed By The Mechanisms Of Say The Roulette Wheel]. Infinite Time Can Be Compared With Finite [Or Metric] Time. They Are Linked Together, Although It Is Useful To Consider Both. In Finite Time, The Present Exists And Features Mixtures Of Matter Which Are Temporally Realized And Can Take On A [Empirical] Depth. Any Other Kind of Time, Only the Past and Future Exist, and Colonise Moments in the Presence. It Offers An Unlimited Kind Of Time, Since The Gap Between Past And Future Is Endlessly Subdivided And Never Closed. It Is Neutral And Incorporeal. Here, the Present Is A 'Pure Mathematical Instant' (62). It Is The Time Of Events Which Are Always Things Which Have Just Happened And Things Which Are About To Happen 'Never Something Which Is Happening' (63). As With The Operations Of Chance In A Game, Specific Events Or Throws Occupy A Time Which Is Smaller Than That Which Can Be

Thought [That Is The Unfolding Of Singularities Which Happen Instantly?], And Also The Maximum Notion Of Time [I Can Only Grasped This By Thinking Of Specific Chances Being Understood By Their Location In Whole Sequences Of The Infinite Throws Of The Dice]. [There Is Some Stuff About The Eternal Return Here Too Which I Don't Understand—63 – 64]. 'The Aion Is The Straight Line Traced By The Aleatory [Unpredictable, Accidental] Point' [Search Me] Which Means That Events The Can Communicate With All The Others, As In Events Of The Aion Itself. This Sort Of Unity Is Not The Same Is An Empirical Corporeal Unity. The Aion Underpins or Circulates Within the Series of Events. This Also Helps Explain The Connection Between The Sense Of Propositions And The Sense Of States Of Affairs. [Weird. Making The Strongest Possible Case For Chance, I Assume. Denying Essence or Subjective Intent. Makes Empirical {Including Social} Science As Limited To The Finite And Metric, But Requiring Thought Experiments To Move To The 'Pure' Level? So Says Delanda Anyway, I Think]. Alexander I. Stingl's Blog A Nomadic Scholarship Entity NOTES ON: Deleuze, G (1990) The Logic Of Sense, Trans Mark Lester, Edited By Constantin Boundas, New York: Columbia University Press For Models Please See One Of The papers In The Series Alexander I. Stingl's Blog A Nomadic Scholarship Entity NOTES ON: Deleuze, G (1990) The Logic Of Sense, Trans Mark Lester, Edited By Constantin Boundas, New York: Columbia University Press For Models Kindly See One Of The Papers In The Series. Attributable and Ascribable To Constraints on Space Accomodation Model Is Provided In Some Paper, Notwithstanding the Generality and Commonalty of the Observation and Its Concatenatability with the Other Modules. [Particularly Dense And Baffling Stuff, Attempting To Show How The Subjective World Is A Derivative Of The Objective Play Of Singularities. Makes More Sense With Concrete And Familiar Examples As In Delanda's Stuff On Biological Membranes And Their Importance In Embryology, Instead Of The Abstract 'Surfaces' Of This Account]] Individuals Are Capable Of Infinite Description, Limited Only By Their Bodies To Express. Persons [Human Beings Based the More Abstract Notion of an Individual?] Can Only Produce Propositions To Describe The World To A Limited Extent. Both, However, Are 'Ontological Propositions' [That Is They Create A Kind Of Reality?]. Multiple Classes And Systems Of Categories Are Not Produced By Propositions. Instead We Have To Look At Something That Now Produces Human Propositions Themselves, As 'Material Instances' (118). [So Human Ontological Activity Only Realises Sense Making In The Form Of Denotations, Manifestations, And Signification As Above?]. Denotations And The Others Are Interconnected As We Saw Above. There Is Also No Simple Connection Between, Say, 'The Individual And Denotation, The Person And Manifestation, Multiple Classes Or Variable Properties And Signification' (119) [For Similar Reasons—For Example, Signification Depends On The Good Sense Already Established By Individuation]. The Whole Complex Structure Is Produced By Both Ontological And Logical Genesis. Sense Operates On The Whole Structure. [We Can See This Both In The Fragility Of Sense And Its Tendency To Be Threatened By Nonsense, And Because The Alternative Is Unpalatable—Language And Sense-Making Would Be Based On Nothing But A 'Undifferentiated Abyss' (120)]. So Sense, 'In Its Organization of Aleatory and Singular Points, Problems and Questions, Series and Displacement' (120) Generates both Logical Propositions and Also 'the Objective Correlates' Of Propositions… The Denoted, the Manifested, and the Signified' (120). The Notion Of An Error Suggests This Although In A Confused Way. Normally We Think Of Error As A Matter Of Truth Or Falsity, When Propositions Are Formed And Tested. However, When We Consider Problems Instead Of Propositions That Offer Solutions, The Category Of Sense Emerges Strongly. [We Have Seen Above the Problem Is an Objective Matter of Structured Possibilities]. We Can See How Both Knowledge And The Known Are Produced By This Structure. [Problematics Are Further Discussed As Involving Particular Distributions Of Singularities In Space And In Time. As Problems Condense Out, So Do Solutions—'The Synthesis Of The Problem With Its Conditions Engenders Propositions, Their Dimensions, And Their Correlates' (121)]. Sense Is Produced Or Expressed When Solutions, Expressed As Propositions, Correspond To Problems [Act As 'Instances Of A General Solution' (121)] It Is Common To Express Sense In An Interrogative Form [Although The Interrogative Also Includes A More Limited Operation As In The Closed Question]. Specific Questions And Solutions Are Already Determined By The Problematic, However—'The Problem In Itself Is The Reality Of A Genetic Element, The Complex Theme Which Does Not Allow It To Be Reduced To Any Propositional Thesis' (122). It Is A Mistake To Define A Problem In Terms Of Possible Solutions [Which Arise From Human Consciousness]. This Would Mean Us 'Confuse Sense

with Signification, and… Conceive Of the Condition Only In the Image of the Conditioned' (122). [This Autonomous Constitution Of Problems Shows The Inadequacy Of Subjective Conceptions. Seeing Problems As Derived From Propositions Expressing Solutions Would Also Infringe The Neutrality Of Sense]. Problems Are Neutral Insofar As [Modes Of] Propositions Are Concerned. For Example 'A Circle Qua Circle Is Neither A Particular Circle, Nor A Concept Represented In An Equation… It Is Rather A Differential System To Which An Emission Of Singularities Corresponds' (123). Problems In This Sense Exist In Propositions But Also Allude To None Being, As Above. As A Result, Problems Are Neutral, That Is 'Independent of both the Negative and the Affirmative' (123). The Neutrality of Sense Means It Is Never Just the Echo ['the Double'] Of Propositions. Working With Propositions Can Only Lead To A Partial Understanding Of Sense. We Have To Develop Another Conception, Not Based On Propositions Or On Images Of Conventional Logical Thinking. Philosophy Must 'Purge The Transcendental Field Of All Resemblance' In Order To Avoid Trap Of Consciousness As The Origin Of The Transcendental (123). However, The Earlier Discussions Defined The Neutrality Of Sense As An Effect Produced By Corporeal Causes. Here, We Are Implying That It Arises From Its Genetic Power [To Produce Problematics] And This Relates To A Quasi Cause. Sense Is Produced By Bodies In A Way Which Presupposes This More General Kind Of Sense. The More General Kind Operates In A Different Way, Not Through Concepts Or Describing Mixtures. This Time It Is a Matter of Depth [Of Bodies] and the Effects of a Surface. The Pulsations Of Bodies Produce Surfaces In Particular Ways, Sometimes As A Minimum Energy Conserving Form [Delanda's Soap Bubble], Sometimes As A More Complex Structure Of Multiplied Differentiated Surfaces [Stretched, Emulsified, Absorbing Etc Are The Examples Given, P. 124]. Surfaces Are Produced By the 'Actions and Passions of Mixed Bodies' (124). Surfaces Have No Thickness Of Their Own, Which Permits Contact Between The Internal And The External. The Quasi Causes Play On These Surfaces, As A Kind Of 'Fictitious Surface Tension… A Force Exerting Itself On The Plane Of The Surface' (125). Singularities Are Condensed Extended And Reshuffled. These Surfaces Have An Existence In Actual Physics, And Also In Metaphysics—The Surface Becomes The Transcendental Field, The Border Between Bodies And Propositions. As Such, The Surface Becomes The 'Locus Of Sense And Expression' (125). Propositions And Bodies Are Actually Articulated On Surfaces, 'So That Sense Is Presented Both As That Which Happens To Bodies And That To Which Insists [Sic] In Propositions' (125). It Is In This Process Of 'Doubling Up' That Neutrality Arises [I Think The Argument Here Is That Neutrality Is Not Some Disembodied Quality But A Function Of Surfaces Which Enable 'The Continuity Of Reverse And Right Sides' (125). It Is The Indifference Of The Objective, Immune To Appeals From Human Subjectivity? ]. The Surface Enables Sense To Be Distributed, As Both The Expressed In Propositions, And The Event In Bodies [Depends On This Very General Definition Of 'Expression' Again - More General Than The Use Above Where It Means Human Expression As In Manifestation Etc]. If The Surface Is Destroyed, Bodies' Fall Back Into The Depth, The 'Primary Order [Some Sort Of Natural Being Which Cannot Be Named Or Expressed?] Which Grumbles Beneath the Secondary Organisation of Sense' (125). But On The Surface, Sense Is Unfolded And Is Also Affected By Quasi Causes. This Sense In Turn Individuates and Determines Bodies, and Signification, and All the Propositions 'The Entire Tertiary Arrangement or the Object of the Static Genesis' (126). Alexander I. Stingl's Blog A Nomadic Scholarship Entity NOTES ON: Deleuze, G (1990) The Logic Of Sense, Trans Mark Lester, Edited By Constantin Boundas, New York: Columbia University Press For Models Kindly See One Of The Papers In The Series. Attributable And Ascribable To Constraints On Space Accomodation Model Is Provided In Some Paper, Notwithstanding The Generality And Commonalty Of The Observation And Its Concatenatability With The Other Modules Sixteenth Series Of The Static Ontological Genesis. Chronos And Aion Are Compared In Terms Of The Importance Of The Present As Opposed To The Past And The Future. For Chronos, The Present Represents The Main Interest, Although There Is 'The Relativity Of Presents Themselves In Relation To Each Other. God Experiences As Present That Which for Me Is Future or Past, Since I Live Inside More Limited Presents' (162). The Present Is Corporeal, A Matter of Mixtures [Imperfect for Humans and Perfect in the Divine Present for Stoics as Above]. The Present Limits The Action Of Bodies [Compare This With Bergsonian Stuff About The Present As A Cone—Here We Are Talking About Contraction And Dilation To Connect The Empirical With A Cosmic Present]. The Partial Mixtures Of Bodies In The Present Threaten To Subvert The Sufficiency Of The Notion. Stoics Had To Distinguish Between

Good Or Bad Mixtures, For Example, Which Lead To Their Notion Of Cosmic Perfection, And To Aion. This Implies That Bodies Are Actually Nothing But Simulacra, And The Present Less Important Than The Future Or The Past. However, Chronos Represents the Only Kind Of Empirical Understanding [?] (164). For Aion, The Present Is A Mere Instant, Always Divided Into Past And Future. This Is A Different Way Of Subverting The Present As In Depth Metaphors [Some Of Which Appear To Linger In The Argument About Partial Mixtures Of Bodies In The Present?]. [Somehow], Aion Operates At The Surface, Since The Present Is Evaded In Favour Of The Instant Rather Than The Fathomless Depths. Aion Operates With Incorporeal Events, And Their Effects, Which Are Limitless And Infinite. Thus Aion Is 'The Eternal Truth Of Time: Pure Empty Form Of Time, Which Is Freed Itself Of Its Present Corporeal Content' (165) [I Think Delanda Is Much Clearer On The Differences Between Metric And Intensive Time]. Aion Is An Essential Element In The Development Of Language, Allowing Language To Escape Corporeal Determinations, Alluding To An Existence Outside Of The Present, Allowing Signification And Manifestation. [Once Having Escaped, Language Can Persist In Itself]. The Instant Demonstrates The Aleatory Point, Nonsense, And Quasi-Cause. It Is A Pure Abstraction And It 'Extracts Singularities From The Present, And From Individuals And Persons Which Occupy This Present' (166) [Because It Alludes To Future And Past, And Thereby Constitutes The Pure Event?]. Without The Notion Of Aion, We Would Only Be Left With Bodies And States Of Affairs, Not Language And Propositions. [The Argument Here Seems To Be That Language Necessarily Has A Future Element?]. The Surface Divides The Two Series Of States Of Affairs And Propositions, And Sense Can Now Relate To Propositions And Events [In The Form Of The Commentary On Events Mentioned Wayback At The Beginning?]. [There Is Some Sort Of Topological Connection Between Points, Lines And Surfaces, Presumably In Terms Of The Way In Which One Can Be 'Cut' From The One Above]. Actualizations Occur When Bodies, States Of Affairs And Mixtures Intersect At The Surface. This Is A Matter Of 'Imprisoning First Their Singularities Within The Limits Of Worlds, Individuals, And Persons' (167). However, There Is Always Something In Excess Of Actualizations, Which Alludes To The Quasi- Cause [Which Apparently Can Be Identified By Sages]. This Involves Analysing the 'Pure Perverse "Moment"', the Pure Operation—It Is Graspable At and As A Moment of Counter Actualization. Twenty-Third Series Of The Aionalexander I. Stingl's Blog A Nomadic Scholarship Entity NOTES ON: Deleuze, G (1990) The Logic Of Sense, Trans Mark Lester, Edited By Constantin Boundas, New York: Columbia University Press For Models Kindly See One Of The Papers In The Series. Attributable And Ascribable To Constraints On Space Accomodation Model Is Provided In Some Paper, Notwithstanding The Generality And Commonalty Of The Observation And Its Concatenatability With The Other Modules Sixteenth Series Of The Static Ontological Genesis. Wild And Wacky Stuff Here Relating To Artaud's Work. I Would Normally Chop It Out Altogether, Except It Provides Some Context For The Emergence Of The Phrase 'Body Without Organs' That Crops Up So Prominently In Anti Oedipus Sense Is Fragile And Is Threatened By Nonsense. Sometimes Such Nonsense Can Destroy Everything, And We See This If We Switch From The Playful Portmanteaux Of Carroll To The Schizophrenic Writings Of Artaud. These Are Real Examples Of Nonsense, Instead Of The Artificial Ones Discussed By Philosophers. Artaud Apparently Disliked Carroll, and Rendered Jabberwocky in A Much More Challenging Schizophrenic Language. Carroll Is Too Superficial, Whereas Schizophrenia Reveals The Real Problems With Language. Another Work Is Discussed, By Wolfson, To Focus On The Duality Between Things And Words. This Takes The Form Of Someone [Schizophrenic?] Experiencing A Problem In Translating From One Language To The Other, Which Somehow Becomes Transposed Into An Anxiety About Eating. A Schizophrenic Interlude Ensues Involving Associations Between Consonants As The Basis For Translation, And The Paradoxes That Emerge (85). The Same Basic 'Oral Duality (To Eat/To Speak)' Is Also Found In Carroll's Work, And In Artaud 's. However, In Caroll, Some Sense Is Retained Since This Duality Is Explored 'At The Surface' (86) [This Also Indicates, Apparently That The Operation Of Sense Of The Surface Shows Only A Quasi-Causal Relation Between Its Elements, Since There Are Incorporeal Elements Driving It]. For Artaud, The Classic Schizophrenic Symptoms Included The Absence Of Surface, Especially With Bodies. Apparently Freud Also Noticed This Tendency For Schizophrenics To See Their Body As 'Punctured By An Infinite Number Of Little Holes' (87). The Body Therefore Incorporates Everything Into Its Depths, Everything Becomes Corporeal And Physical. The Surface No Longer Limits The Extension Of The Body. 'Hence The

Schizophrenic Manner Of Living The Contradiction: Either In The Deep Fissure Which Traverses The Body, Or In The Fragmented Parts Which Encase One Another And Spin About' (87). The World Loses Its Meaning And Sense [Because It Can No Longer Split Sensation Into A Signifying And Signified Separated By A Surface?] Words Become Physical And Affect Bodies, Or They Burst Into Components [Which Relates Back To The Wolfson Example]. Schizophrenics Experience 'A Pure Language–Affect' (88) [Sic --Affect Not Effect]. Schizophrenics Manage This By Overcoming The Effects Of Language, As In The Strange Translation Activity In The Wolfson Example. In Artaud's Case The Solution Was To Create Special Words Expressing 'Values Which Are Exclusively Tonic And Not Written' (88). 'To These Values A Glorious Body Corresponds, Being A New Dimension Of The Schizophrenic Body, An Organism About Parts Which Operates Entirely By Insufflations, Respiration, Evaporation, And Fluid Transmission (The Superior Body Or Body Without Organs Of Antonin Artaud)' (88). This Solution Can Never Be Complete Because There Can Never Be A Total Separation Between Sufferings ['Passion'] And [Remedial] Action, And Passion Can Be Reintroduced, And The Body Corrupted-- A Schizophrenic Body Is Therefore A Constant Mixture Of Two Actions Or Principles. Artaud Tries To Invent A New Language Which Cannot Be Decomposed And Thus Cannot Be Colonised, A Language Of 'Consonantal Guttural And Aspirated Overloads' (89). The Words Are Joined By Some Invented Principle, In This Case A 'Palatalized' One (89) Which Blurs The Consonants Together And Prevents Them Being Written Down. The Result Is 'So Many Active Howls In One Continuous Breath' (89) [Sounds Very Much Like Tara's Dadaist Tone Poems --Artaud's Example On P.83]]. These Words Are Often The Equivalent To Portmanteaux [Some Examples Are Given On Page 90]. Using These Words Can 'Enact A Chain Of Associations… In A Region Of Infra Sense, According To A Fluid And Burning Principle Which Absorbs And Reabsorbs Effectively The Sense As Soon As It Is Produced' (90). So Two Sorts Of Words Related To Two Sorts Of Bodies, One Fragmented And One Without Organs. There Are Also Two Theatres Or Two Types Of Nonsense Implied Here: One Where Ordinary Words Are Decomposed Into Nonsense, And One Where Tonic Elements Alone Form Nonsensical Words. They Are Produced By Things Happening Beneath The Surface, Unlike Carroll's Playful Superficiality. The Two Signifying and Signified Series Disappear, and Non Sense Engulfs Signifiers and Signified. There Is No Surface Division To Separate The Expressivity Of Words And The Attributes Of Actual Bodies [Which Regulates Ordinary Language]. In Schizophrenic Language There Is No Grammar Or Syntax Either, Although Both Are Preserved In Carroll. Nevertheless, It Is Artaud Who Has 'Discovered A Vital Body And The Prodigious Language Of This Body… He Explored The Infra Sense Which Is Still Unknown Today' (93). However, Caroll Has Explored Those Important Surfaces, On Which 'The Entire Logic Of Sense Is Located' (93). We Can Still Find Schizoid Fragments In Ordinary Speech, But These Are Normally Reorganized. Similarly, Carroll Can Be Retranslated As A Schizophrenic Piece (92). But It Is Wrong To Generalise Here, 'Believing To Have Discovered Analogous Forms Which Create False Differences' (92). Psychoanalysis Should Operate With A Surface/Depth Structure Rather Than With Analogies—'It Is Geographical Before It Is Historical' (93). [While We Are Here, Note That 'It Is Hardly Acceptable… To Run Together a Child's Nursery Rhymes, Poetic Experimentations, And Experiences of Madness… [And] Justify the Grotesque Trinity of Child, Poet, and Madmen' (82-83). This Must Be A Problem For Those Who Think That Deleuze Is Arguing That Children Are Philosophers?] Thirteenth Series Of The Schizophrenic And The Little Girl Alexander I. Stingl's Blog A Nomadic Scholarship Entity NOTES ON: Deleuze, G (1990) The Logic Of Sense, Trans Mark Lester, Edited By Constantin Boundas, New York: Columbia University Press For Models Kindly See One Of The Papers In The Series. Attributable and Ascribable To Constraints on Space Accomodation Model Is Provided In Some Paper, Notwithstanding the Generality and Commonalty of the Observation and Its Concatenatability with the Other Modules. Singularities Constitute The Transcendental Field. The First Stage Of Actualization Involves The Derivation Of Individuals. Singularities Spread Out 'In a Determined Direction over a Line of Ordinary Points', Up To the Vicinity of another Singularity (109). When Series Converge, A World Is Constituted [And Other Worlds Arise From Diverging Series]. The Infinite System Of Singularities Are Selected And Rendered Finite By Individuals [Does He Mean Human Individuals Throughout? I Think So] Who Combine Them, 'Spread Them Out Over Their Ordinary Lines' (109) And Even Form Them Again On Various Membranes. 'An Individual Is Therefore Always In A World As A Circle Of Convergence, And A World May Be Formed And Thought Only In

The Vicinity Of The Individuals Which Occupy Or Fill It' (110). [Deleuze Is Suggesting That This Role For Individuals Is The Only Way For Worlds To Persist, Since Entropy Prevents The Endless Renewal Of Singularities At The Virtual Level—'The Power Of Renewal Is Conceded Only To Individuals In The World, And Only For A Time—The Time Of Their Living Present' (110). This Is Static Genesis, The First Level Of Actualization. Singularities Are Actualized In The World And In Individuals. Actualization Means Being Extended, Selected, Incarnated In A Body, And Renewed Locally. These Qualities Require Individuals. Actualization Also Means Being Expressed, But It Would Be Wrong To Assume That Only The Expressed World Exists. There Are, For Example Incompossible Worlds, Produced By Diverging Series [As Real Events, And Not Just Matters Of Contradictory Consciousness]. This Implies That There Is A Continuum Of Singularities Outside Of [Conscious] Individuals, Worlds In Which Other Possibilities Arise. [So Grasping These Other Possibilities Again Distinguishes Sense From Mere Logic, Based On The Truth And Falsity Of Propositions Expressed By Individuals]. Because Individuals Express Worlds, However, The World Looks As If It Is Merely Subjective, And Objective Events Appear To Be 'The Analytic Predicate Of A Subject' (112). [These Analytic Predicates Appear To Be Properties Of People Or Objects]. [Much Of This Argument Is Referring To Leibniz]. However, The Problem Is To Explain Logical Hierarchies Of Properties And General Categories. Subjective Analytic Predicates Have Immediacy, But Do Not Have Any Kind of Order; They Get Elaborated But Only As Mixtures. They Are Descriptive Of Actual Structures And Diversities. They Are 'Intuitions... Immediate Representations' (113). The Second Level Of Actualization Involves The Transcendental Again, Originally Conceived As The Development Of Transcendency In The Individual [See The Critique Of Husserl Above]. The Transcendental Ego Is Constituted Just As Other Individuals Are, As A Circle Of Convergences, So The Problem Is To Escape This And Develop Knowledge Of The Whole Continuum, Of Incompossible Worlds And Divergent Series, A New Sense Of World [Deleuze Uses The German Terms Welt For This New World And Umwelt For The Subjective World. I Am Grateful To Wikipedia For Explaining That: In The Semiotic Theories Of Jakob Von Uexküll And Thomas A. Sebeok, Umwelt (Plural: Umwelten; The German Word Umwelt Means "Environment" Or "Surrounding World") Is The "Biological Foundations That Lie At The Very Epicenter Of The Study Of Both Communication And Signification In The Human [And Non-Human] Animal."[Citation Needed] The Term Is Usually Translated As "Self-Centered World". [1] Uexküll Theorised That Organisms Can Have Different Umwelten, Even Though They Share The Same Environment']. Proper Transcendence Involves Transcending Individuated Worlds. Singularities Come With Their Own 'Perfectly Objective Determination Which Is The Open Space Of Its Nomadic Distribution' (113), And This Is How Deleuze Wants To Define A Problematic [Above]. Leibniz Apparently Half Grasped This Argument, Suggesting That Conic Sections, For Example, All Referred Back To The Same Event, Subdivided By 'Ambiguous Signs' (114). Similarly, Incompossible Worlds Also Have 'Something Objectively In Common', And 'Several Worlds Appear As Instances Of Solutions For One And The Same Problem... Variants of the Same Story... There Is Thus A"Vague Adam", That Is A Vagabond, A Nomad… Common to Several Worlds' (114). In Other Words, All The Objective And Subjective Worlds Can Be Explained By A Series Of Singularities. It Is Not Just The Analytic Predicates Of Individuals That Actualize Worlds. 'On The Contrary, [There] Are Predicates Which Define Persons Synthetically And Open Different Worlds And Individualities To Them As So Many Variables Or Possibilities' (115). It Is Incompossible Worlds Which Synthesise 'Primary Possibilities Or Categories' (115). These Possibilities or Categories 'Necessarily Signify Classes and Properties', And These Are Distinct From Individual Categories at the First Level. They Still Seen Grounded In Persons, But That Is 'Because Persons Themselves Are Primarily Classes Having One Single Member, And Their Predicates Are Properties Having One Constant' (115). There Is Thus A System Involving Persons, Classes With One Single Member, Extensive Classes And Variable Properties '-- That Is The General Concepts Which Derive From Them' (115). If There Is A Universal Ego, It Is Something That Corresponds To Common Elements In All Worlds. So, Firstly, Sense Gets Actualized In A Field Of Singularities, Then The Umwelt Develops Around Individuals Which Express Or Describe This World, Then, Secondly, A Whole Objective World Develops From Common Elements, A Welt. Persons Can Then Define This Common Element And Develop Classes And Properties Derived From It. At The First Level We Find Good Sense, 'An Already Fixed And Sedentary Organization Of Differences'. At The Second Level,

Commonsense Serves The Function Of Identification. Neither Term Explains How These Activities Are Derived [In Fact, Deleuze Wants To Suggest That The Second Level 'Is The Work Of Non Sense Which Is Always Copresent To Sense (Aleatory Point Or Ambiguous Sign)... Productive Nonsense Which Animates The Ideal Game And The Impersonal Transcendental Field' (116). There Is No Conventional Transcendental Movement, Especially One Driven By A Version Of The Ego. The Person Is A 'Produced Form, Derived From This Impersonal Transcendental Field… Always an Individual in General, Born… From The Singularity Which Extends Itself Over A Line of Ordinary Points and Starts from the Preindividual Transcendental Field' (116). Persons And Their Varieties Of Sense Are All Produced 'On The Basis Of Sense And Nonsense Which Do Not Resemble Them' (117). This Explains The Paradoxes And Limits Of Good Sense And Common Sense. [So The Existence Of Nonsense -- Paradox And Impossible Sentences Etc -- In The Ordinary Subjective World Shows The Existence Of Non Sense -- Something That Produces Ordinary Sense In The First Place, Something Objective? Individuals Can Only Describe But As Soon As They Generalise And Order Concepts They Necessarily Assume Some Non-Subjective World? The Man oeuvre seems Rather like the Illicit Transcendental Ego Move by Husserl, Generalising from an Ordinary Ego: Here, Explaining Ordinary Experience Is Used to Explain Some Deeper Virtual World as an Extension?] Alexander I. Stingl's Blog A Nomadic Scholarship Entity NOTES ON: Deleuze, G (1990) The Logic Of Sense, Trans Mark Lester, Edited By Constantin Boundas, New York: Columbia University Press For Models Kindly See One Of The Papers In The Series. Attributable And Ascribable To Constraints On Space Accomodation Model Is Provided In Some Paper, Notwithstanding The Generality And Commonalty Of The Observation And Its Concatenatability With The Other Modules Sixteenth Series Of The Static Ontological Genesis. The Issue Is Of The Validity Or Truth Of Propositions And How This Might Be Grounded. It Is As Difficult To Root Propositions In The Idea As It Is Actual Events. Significations Cannot Be Just Seen As Exhausted By Their Denoted Examples. A List Of Denoted Examples Would Be Endless And Groundless Anyway. Classic Philosophers Often Refuse To Answer Questions Denoted Examples Any Way, And Replied With A Blow From The Staff, Or A Mute Demonstration. After All, 'There Is No Resemblance (Nor Should There Be One) Between What One Points Out And What One Has Been Asked' (135) [Typical Philosophical Elitism]. The Issue Is Often Covered By Humour, Especially Irony or the Absurd. We Must Refuse Ascent and Descent and Occupy the Surface 'Where Pure Sense Is Produced' (136). At The Surface, 'One Finds Pure Singularities, An Emission Of Singularities Considered From This Perspective Of The Aleatory Element, Independent Of The Individuals And Persons Which Embody Or Actualise Them' (136). Getting To The Surface Involves Humour And Adventure, Seen In The Routines Of Zen Masters, Who Can Allude To The Void Which Is Present In Events [Events Are 'Not The Object As Denoted, But The Object As Expressed Or Expressible, Never Present, But Always Already In The Past And Yet To Come' (136), And The Abolition Of The Object Alludes To The Void]. Negating Objects Indicates The Expressible. [Examples From Zen Follow, Page 137. What They Seem To Be Suggesting Is That Sense Combines Actual Objects With The Notion Of The Void, Non Sense—An Actual Brushstroke And Also The White Space—'Language Becomes Possible And, By Becoming Possible, It Inspires Only A Silent And Immediate Communication' Somehow Outside All The Significations And Denotations]. The Same Reasoning Affects The Question Of Who Speaks—Individuals Or Language Itself In The Form Of Ideal Forms [And We Are Back With Greek Philosophy Examples. Socratic irony apparently Refers to the Fact That Individuals Both Initiate Speech and Are the Products of Speech—138. This In Turn Permits The Idea Of A 'Pure Rational Language… [Enabling]… Natural Communication between A Supremely Individuated God and the Derived Individuals Which He Created' (138)]. Further Irony Awaits The Romantic Notion Of The Person, 'The Finite Synthetic Unity Of The Person... [Not Just]... The Analytic Identity of the Individual' (138). Here, The Person And Representations Are Linked Ironically, Implying Both A Universal Idea And 'Sensible Particularity' (138), Both Universal And Particular Characteristics Of The Person—Hence The Earlier Argument That Persons Should Really Be Seen As A Class Which Has Only One Member. All These Formulations Assume That Singularities Are Located In The Individual Or The Person, Influenced Only By Some Groundless Abyss, Or Chaos, Threatening Both Classical And Romantic Discourse With The Lack Of Articulation. There Is Also A Threefold Division Of Language—The Ordinary Or Real; The Ideal Language, Like The Purely Rational One; Esoteric Language Which Subverts The Ideal Language And The Individuality Of The Speaker Of Ordinary Language [For

Example Social Scientific Explanations, Or Structural Linguistic Ones?] [Weird Examples of Esoteric Languages Which Mean Nothing to Me on Page 140, With a Reference Back To Portmanteau Words]. So Sometimes Individual Speaks, Sometimes The Person, And Sometimes 'The Ground Which Dissolves Both' (140). The Only Way Out Is To See That Singularities Are Not Coterminous With Individuals, And There Is Not Just A Groundless Abyss Beneath Them. There Are Other Impersonal And Pre-Individual Singularities. Nonsense And Sense Collaborate At The Surface. Irony Gives Way To Humour—'The Co extensiveness Off Sense with Nonsense', A Matter Of 'Surfaces… Doubles… Nomad Singularities and Of the Always Displaced Aleatory Point' (141). Normal Significations, Denotations and Manifestations Are Suspended, and 'All Height and Depth Abolished' (141). Nineteenth Series of Humour. Alexander I. Stingl's Blog A Nomadic Scholarship Entity NOTES ON: Deleuze, G (1990) The Logic Of Sense, Trans Mark Lester, Edited By Constantin Boundas, New York: Columbia University Press For Models Kindly See One Of The Papers In The Series. Attributable And Ascribable To Constraints On Space Accomodation Model Is Provided In Some Paper, Notwithstanding The Generality And Commonalty Of The Observation And Its Concatenatability With The Other Modules Sixteenth Series Of The Static Ontological Genesis.Doctrines Come From 'Wounds And Vital Aphorisms' (148), And Some Writers See Themselves As Embodying Events [Hints Of The Stuff On Nietzsche And Illness Earlier]. Our Will Can Act As A Quasi Cause Of Bodily Events, As We Live Them. This Is An Ethical Stance—'Not To Be Unworthy Of What Happens To Us' (149). The Alternative Is Ressentiment. The Normal Moral Notions Such As Just Or Unjust Are Themselves Immoral. It Is Not Just A Matter Of Resignation To Events, Which Can Still Be Ressentiment, More An 'Apotheosis Of The Will' (149) [Citing Bousquet]. The Organic Is Exchanged For The Spiritual Will [Isn't This Just Making The Best Of Things, Finding Some Deep Meaning In Personal Tragedy?]. In This Sense, Freedom Is The Same As Submitting To Fate, Actualizing Events, Making Sense Of Events, Seeing Events As Something Expressed. One Becomes 'The Offspring of One's Events Are Not of One's Actions, For the Action Is Itself produced By the Offspring Of The Event' (150). In This Way, Actors Somehow Communicate With The Aion, Instead Of Being Dominated By Chronos. [Lots Of Implicit Christianity Here, Surely?]. All The Components Of An Event, Future As Well As Past, Are United, So That One Realises The Impact Of Singularities, Including Preindividual Components. [In This Way, Some Sort Of Agency Seems To Remain? One Can Become 'The Actor of One's Own Events – A Counter Actualization' (150)]. Ressentiment Arises From Not Realizing That Our Particular Part Of Experience, Which May Well Be Unjust, Is Part Of A More Perfect Whole. It Is Incorrect To Judge Everything From The Perspective Of The Present. Some Humourous Recognition of the Futility of It All Is Valuable—In the Great Scheme of Things, States of Affairs Are Always 'Impersonal and Pre-Individual, Neutral, neither General nor Particular' (151). In These Circumstances, Either Life Can Seem 'Too Weak For Me' [Not Vivid Enough, Not Enough Focused In The Present?], Or 'It Is I Who Am Too Weak For Life, It Is Life Which Overwhelms Me, Scattering Its Singularities All About, In No Relation To Me, Nor To A Moment Determinable As The Present Except An Impersonal Instant' (151). This Is Apparent When One Considers Death Or The Mortal Wound, An Event Which Is Indifferent To Me, 'Incorporeal And Infinitive, Impersonal, Grounded Only In It' (151), Although It Is I Who Has To Actualize The Event. However I Can Also Counter actualizes In the Sense Above. It Follows That The Events Of Reality Are Quite Different From Personal Experience, Shown In The 'Splendour Of The "They"… The Splendour of the Event Itself Order Of the Fourth Person' (152) [This Fourth Person Is the 'It' As In Phrases Like 'It Is Raining']. The Old Distinctions Between Private And Collective Refer Only To Personal Experience, Whereas 'Everything Is Singular, And Thus Both Collective And Private… Which Private Event Does Not Have All Its Coordinates, That Is, All Its Impersonal Social Singularities?' (152). Freedom Exists Only In Recognising The Impersonal Nature Of The Event, Seeing All Events As 'A Single Event Which No Longer Makes Room For The Accident, And Which Denounces And Removes The Power Of Ressentiment Within The Individual As Well As The Power Of Oppression Within Society' (152) [Ridiculous Philosophical Notion Of Freedom Overcoming Oppression By An Act Of Philosophy]. Ressentiment Ties One to an Oppressive Order. [Then A Really Sentimental And Pathos Ridden Bit: 'It Is At This Mobile And Precise Point, Where All Events Gather Together In One That Transmutation Happens: This Is The Point At Which Death Turns Against Death; Where Dying Is The Negation Of Death And The Impersonal At He Of Dying No Longer Indicates Only The Moment

When I Disappear Outside Of Myself, But Rather The Moment When Death Loses Itself In Itself, And Also The Figure Which The Most Singular Life Takes On In Order To Substitute Itself For Me' (153)] Ethics Links Logic and the Body. The Stoic Notion Of Bodies Included The Passions, And Good And Evil Intents: Particular Bodies Might Have Evil Mixtures, But The Aggregate Of Bodies Is Perfect Or Good, The Unity Of Causes Themselves. In Principle, Each Event Can Be Linked To A Particular Cause And Thus The Unity Of Causes, And This Could Lead To The Activity Of Divination [Grasping The Divine Unity Of Causes] As A Basis For Ethics. The Effects, Lines Have To Be Traced Back From Events To Pure Events And Then To Actions And Passions [With A Lot Of Poetic Stuff As Examples, 143]. Stoics Took Another Route To Ethics Through Logic [Kind Of Working Out Which Events Will Actualize]. Stoic Philosophy Then Saw In Representations Of The Limited Event A Connection With Pure Events [And Ethical Conduct Seems To Be To Work Towards Actualizing Such Events?]. [Stoic Accounts Of Representations Ensue, 144-5.] There Is A Difference [For Stoics] Between Representations Of Sense And Of Logic, Denotations And Significations, Representations And Expressions. Representation Is Not Just Resemblance, But Includes A Notion of Adequate Expression. Concepts Have To Be Actualised In Representation, And Also Expressed If They Are To Be Comprehensive. Thus Our Knowledge Of Death Is Abstract, Despite The Number Of Deaths We Witness, Until It Becomes Personal, Not Indifferent, But Concrete. This Is Where Expression Is Needed. In Stoic Philosophy, Moral Conduct Involves A Relation with Pure Events Unites Those Events with One's Self. [The Examples Again Come From Zen: 'The Bowman Must Reach The Point Where The Aim Is Also Not The Aim, That Is To Say, The Bowman Himself' (146).] An Understanding Of The Pure Event Is Required, As Something Which Is 'Eternally Yet To Come And Always Already Past', But Which Has To Be Willed Into Actualization (146). The Stoic Embodies Incorporeal Effects, Aligning Themselves With The Quasi Cause. It Is Necessary Because Quasi Causes Cannot Embody Themselves, Except To The Immediate Instant, The Present. [There Is An Equally Baffling Aside About The Difference Between Actors And Characters—Actors Represent By Occupying The Instant, While The Character Also 'Portrays Hopes Or Fears In The Future And Remembers Or Repents In The Past' (147). Stoics Therefore See Themselves As Uniting The Instant With The Unlimited Future And Past, Willing The Event And Also Representing It. [Largely Incomprehensible, Although I Am Starting To See How The Normal Notion Of Stoicism Might Fit—The Patient Resignation Of One's Self To One's Fate—And Also Seeing How Deleuze Thinks That Human Beings Should Reconcile Themselves To Solving The Problems That Reality Creates In The Form Of A Problematic]. Alexander I. Stingl's Blog A Nomadic Scholarship Entity NOTES ON: Deleuze, G (1990) The Logic Of Sense, Trans Mark Lester, Edited By Constantin Boundas, New York: Columbia University Press For Models Kindly See One Of The Papers In The Series. Attributable And Ascribable To Constraints On Space Accomodation Model Is Provided In Some Paper, Notwithstanding The Generality And Commonalty Of The Observation And Its Concatenatability With The Other Modules Sixteenth Series Of The Static Ontological Genesis. Deleuze And Guattari See In The Capitalist Money System "An Axiomatic Of Abstract Quantities That Keeps Moving Further And Further In The Direction Of The Deterritorialization Of The Socius" (Deleuze 1983a, 33), Which Is To Say That Capital Is Inherently Schizophrenic. However, Because Capital Also Re-Territorializes All Flows Into Money, Schizophrenia Remains Capitalism's External Limit. Nevertheless, It Is Precisely That Limit Against Which Thinking Can Subject Capitalism To Philosophical Critique. Psychoanalysis, They Say, Is Part Of The Reign Of Capital Because It Re-Territorializes The Subject As "Private" And "Individual," Instituting Psychic Identity Through Images Of The Oedipal Family. However, The Oedipal Triangle Is Merely A Representational Simulacrum Of Kinship And Filiation, Re-Coded Within A System Of Debt And Payment. In This System, They Insist, Flows Of Desire Have Become Mere Representations Of Desire, Cut Off From The Body Without Organs And The Extra-Familial Mechanisms Of Society. A Radical Critique Of Capital Cannot Therefore Be Accomplished By Psychoanalysis, But Requires A Schizoanalysis "To Overturn The Theater Of Representation Into The Order Of Desiring-Production" (Deleuze 1983b, 271). Here, The Authors See A Revolutionary Potential In Modern Art And Science, Where, In Bringing About The "New," They Circulate De-Coded And De-Territorialized Flows Within Society Without Automatically Re-Coding Them Into Money (Deleuze 1983a, 379). In This Revolutionary Aspect, Anti-Oedipus Reads As A Statement Of The Desire That Took To The Streets Of Paris In May Of 1968, And Which Continues, Even Now, To Make Itself Felt In Intellectual Life. Postmodernism First

Published Fri Sep 30, 2005 Copyright © 2005 By Gary Aylesworth Geaylesworth@Eiu.Edu Open Access To The SEP Is Made Possible By A World-Wide Funding Initiative For Models Kindly See One Of The Papers In The Series, Procrastinated Due To Restrictions On Space. The Term “Deconstruction,” Like “Postmodernism,” Has Taken On Many Meanings In The Popular Imagination. However, In Philosophy, It Signifies Certain Strategies For Reading And Writing Texts. The Term Was Introduced Into Philosophical Literature In 1967, With The Publication Of Three Texts By Jacques Derrida: Of Grammatology (English 1974), Writing And Difference (English 1978), And Speech And Phenomena (English 1973). This So-Called “Publication Blitz” Immediately Established Derrida As A Major Figure In The New Movement In Philosophy And The Human Sciences Centered In Paris, And Brought The Idiom “Deconstruction” Into Its Vocabulary. Derrida And Deconstruction Are Routinely Associated With Postmodernism, Although Like Deleuze And Foucault, He Does Not Use The Term And Would Resist Affiliation With “-Isms” Of Any Sort. Of The Three Books From 1967, Of Grammatology Is The More Comprehensive In Laying Out The Background For Deconstruction As A Way Of Reading Modern Theories Of Language, Especially Structuralism, And Heidegger's Meditations On The Non-Presence Of Being. It Also Sets Out Derrida's Difference With Heidegger Over Nietzsche. Where Heidegger Places Nietzsche Within The Metaphysics Of Presence, Derrida Insists That “Reading, And Therefore Writing, The Text Were For Nietzsche ‘Originary’ Operations,” (Derrida 1974, 19), And This Puts Him At The Closure Of Metaphysics (Not The End), A Closure That Liberates Writing From The Traditional Logos, Which Takes Writing To Be A Sign (A Visible Mark) For Another Sign (Speech), Whose “Signified” Is A Fully Present Meaning. This Closure Has Emerged, Says Derrida, With The Latest Developments In Linguistics, The Human Sciences, Mathematics, And Cybernetics, Where The Written Mark Or Signifier Is Purely Technical, That Is, A Matter Of Function Rather Than Meaning. Precisely The Liberation Of Function Over Meaning Indicates That The Epoch Of What Heidegger Calls The Metaphysics Of Presence Has Come To Closure, Although This Closure Does Not Mean Its Termination. Just As In The Essay “On The Question Of Being” (Heidegger 1998, 291-322) Heidegger Sees Fit To Cross Out The Word “Being,” Leaving It Visible, Nevertheless, Under The Mark, Derrida Takes The Closure Of Metaphysics To Be Its “Erasure,” Where It Does Not Entirely Disappear, But Remains Inscribed As One Side Of A Difference, And Where The Mark Of Deletion Is Itself A Trace Of The Difference That Joins And Separates This Mark And What It Crosses Out. Derrida Calls This Joining And Separating Of Signs Différance (Derrida 1974, 23), A Device That Can Only Be Read And Not Heard When Différance And Différence Are Pronounced In French. The “A” Is A Written Mark That Differentiates Independently Of The Voice, The Privileged Medium Of Metaphysics. In This Sense, Différance As The Spacing Of Difference, As Archi-Writing, Would Be The Gram Of Grammatology. However, As Derrida Remarks: “There Cannot Be A Science Of Difference Itself In Its Operation, As It Is Impossible To Have A Science Of The Origin Of Presence Itself, That Is To Say Of A Certain Non-Origin” (Derrida 1974, 63). Instead, There Is Only The Marking Of The Trace Of Difference, That Is, Deconstruction. Because At Its Functional Level All Language Is A System Of Differences, Says Derrida, All Language, Even When Spoken, Is Writing, And This Truth Is Suppressed When Meaning Is Taken As An Origin, Present And Complete Unto Itself. Texts That Take Meaning Or Being As Their Theme Are Therefore Particularly Susceptible To Deconstruction, As Are All Other Texts Insofar As They Are Conjoined With These. For Derrida, Written Marks Or Signifiers Do Not Arrange Themselves Within Natural Limits, But Form Chains Of Signification That Radiate In All Directions. As Derrida Famously Remarks, “There Is No Outside-Text” (Derrida 1974, 158), That Is, The Text Includes The Difference Between Any “Inside” Or “Outside.” A Text, Then, Is Not A Book, And Does Not, Strictly Speaking, Have An Author. On The Contrary, The Name Of The Author Is A Signifier Linked With Others, And There Is No Master Signifier (Such As The Phallus In Lacan) Present Or Even Absent In A Text. This Goes For The Term “Différance” As Well, Which Can Only Serve As A Supplement For The Productive Spacing Between Signs. Therefore, Derrida Insists That “Différance Is Literally Neither A Word Nor A Concept” (Derrida 1982, 3). Instead, It Can Only Be Marked As A Wandering Play Of Differences That Is Both A Spacing Of Signifiers In Relation To One Another And A Deferral Of Meaning Or Presence When They Are Read. How, Then, Can Différance Be Characterized? Derrida Refuses To Answer Questions As To “Who” Or “What” Differs, Because To Do So Would Suggest There Is A Proper Name For Difference Instead Of Endless Supplements, Of Which “Différance” Is But One. Structurally, This Supplemental

Displacement Functions Just As, For Heidegger, All Names For Being Reduce Being To The Presence Of Beings, Thus Ignoring The "Ontological Difference" Between Them. However, Derrida Takes The Ontological Difference As One Difference Among Others, As A Product Of What The Idiom "Différance" Supplements. As He Remarks: "Différance, In A Certain And Very Strange Way, (Is) 'Older' Than The Ontological Difference Or Than The Truth Of Being" (Derrida 1982, 22). Deconstruction, Then, Traces The Repetitions Of The Supplement. It Is Not So Much A Theory About Texts As A Practice Of Reading And Transforming Texts, Where Tracing The Movements Of Différance Produces Other Texts Interwoven With The First. While There Is A Certain Arbitrariness In The Play Of Differences That Result, It Is Not The Arbitrariness Of A Reader Getting The Text To Mean Whatever He Or She Wants. It Is A Question Of Function Rather Than Meaning, If Meaning Is Understood As A Terminal Presence, And The Signifying Connections Traced In Deconstruction Are First Offered By The Text Itself. A Deconstructive Reading, Then, Does Not Assert Or Impose Meaning, But Marks Out Places Where The Function Of The Text Works Against Its Apparent Meaning, Or Against The History Of Its Interpretation. Postmodernism First Published Fri Sep 30, 2005 Copyright © 2005 By Gary Aylesworth Geaylesworth@Eiu.Edu Open Access To The SEP Is Made Possible By A World-Wide Funding Initiative For Models Kindly See One Of The Papers In The Series, Procrastinated Due To Restrictions On Space. "Anything Which Is Just Born, Which Has Just Come Into Existence, Has No Past Behind It. Birth, In Other Words, Is The Condition Of Having No Past. And Likewise, Anything Which Now Dies, Which Has Just Ceased To Be, Has No Future Left In Front Of It. Death Is The Condition Of Having No Future. But We Have Already Seen That This Present Moment Has Both No Past And No Future Simultaneously. That Is, Birth And Death Are One In This Present Moment. This Moment Is Just Now Being Born—You Can Never Find A Past To This Present Moment, You Can Never Find Something Before It. Yet Also, This Moment Is Just Now Dying — You Can Never Find A Future To This Moment, Never Find Something After It. This Present, Then, Is A Coincidence Of Opposites, A Unity Of Birth And Death, Being And Non-Being, Living And Dying. As Ippen Put It, "Every Moment Is The Last Moment And Every Moment Is A Rebirth." Ken Wilber, No Boundary: eastern and Western Approaches to Personal Growth for models please see one of the papers in the series. "Because We Demand A Future, We Live Each Moment In Expectation And Unfulfillment. We Live Each Moment In Passing. In Just This Way The Real Nunc Stans, The Timeless Present, Is Reduced To The Nunc Fluens, The Fleeting Present, The Passing Present Of A Mere One Or Two Seconds. We Expect Each Moment To Pass On To A Future Moment, For In This Fashion We Pretend To Avoid Death By Always Rushing Toward An Imagined Future. We Want To Meet Ourselves In The Future. We Don't Want Just Now—We Want Another Now, And Another, And Another, Tomorrow And Tomorrow And Tomorrow. And Thus, Paradoxically, Our Impoverished Present Is Fleeting Precisely Because We Demand That It End! We Want It To End So That It Can Thereby Pass On To Yet Another Moment, A Future Moment, Which Will In Turn Live Only To Pass." ― Ken Wilber, No Boundary: Eastern And Western Approaches To Personal Growth. For Stoics, Causes Involved Reference To The Depths, But Effects At The Surface Could Also Have Relations Among Themselves. This Permitted The Distinction To Be Drawn Between Destiny And Necessity—The Stoics Wanted To Affirm Destiny And Deny Necessity. In The First Place, Effects Express Causes, But Expressions Of Relations [Rather Than Necessity] Connects Those Effects. Those Relations May Be Described As Compatibility Or Incompatibility, Conjunction Or Disjunction. These Are Not Causal Relations Themselves, But Represent 'An Aggregate Of Noncausal Correspondences Which Form A System Of Echoes, Of Presumptions And Resonances, A System Of Signs—In Short, An Expressive Quasi Causality And Not At All A Necessitating Causality' (170). This Need Not Involve Contradiction, Which Is Applying To Events Rules That Really Only Applied To Logic And Argument. There Can Be Incompatability Without Contradiction, A Noncausal Correspondence. Leibniz Described Incompossible Worlds [Delanda Is Very Clear On This Too]. Only Impossible Events Contradict Possible Ones. Events Can Be Compossible [Roughly, They Have Predictable And Predicative Future And Past Events]. For Deleuze, It Is A Matter Of 'The Convergence Of Series Which Singularities Of Events Form As They Stretch Themselves Out Over Lines Of Ordinary Points. Incompossibility Must Be Defined By The Divergence Of Such Series' (171). Such A Notion Is Essential To Any Theory Of Sense. We Should Not See Divergence As A Matter Of Exclusion, As Leibniz Did [Since God Chose Actual Events]. Divergent And Disjunction Can Both Be Positive, While Preserving

Differences. In Fact, Differences Are Crucial, Preserving The Distance Between Objects While Affirming That They Are Related. This 'Permits The Measuring Of Contraries Through Their Finite Difference Instead Of Equating Difference With A Measureless Contrariety' (173). It Is Contradiction Which Is The Special Case. Difference Here Is A Topological Term Relating To Distance On Surfaces Rather Than Depths. It Is Not Just A Matter Of Suggesting 'Some Unknown Identity Of Contraries (As In Commonplace In Spiritualist And Dolorist Philosophy)' (173) [Take That St Pierre!]. An Example Is Nietzsche Arguing That Health And Sickness Can Both Inform Each Other, Act As Points Of View, Remembering That 'Things, Beings, Are Themselves Points Of View' (173). Divergence Does Not Mean Exclusion, And Disjunction Does Not Mean Separation. Connective Syntheses '(If…, Then)' Construct A Single Series; Conjunctive Series '(And)' Produces Convergent Series, But Disjunctive Series '(Or)' Produces A Divergence Series. Normally, Disjunction Helps Us To Criticise Synthesis, But Disjunction Can Still Be A Synthesis Itself, Despite Its Use In Logical Analysis [I Think What Is Going On Here Is Arguing That There Is A Difference Between 'Either/Or' In A Logical Sense, And 'One Or Two' In The Real Sense—The Latter Can Mean That Both Are Compossible. This Is The 'Communication Of Events' Rather Than The Logical Business Of Analysing Predicates (174)]. The Synthetic Disjunction Expresses The Paradox, With Divergence At The Centre. The Discussion Of Paradoxes And Esoteric Words Above Are Examples: They Contract 'The Multitude Of Divergent Series In The Successive Appearance Of A Single One' (175). There Is A Difference Again Between The Subversion Of The Present And Simple Identity By Depths, And Operations At The Surface. By Considering The Depths, We Encounter Infinite Identities [As Events Become Examples Of Deeper Categories, Wholes?]. At The Surface Events Communicate With Each Other Directly Through Maintaining Distance And By Affirming Disjunctions. Disjunction Threatens The Identity Of The Self, And Helps Us To See The Self As 'So Many Impersonal And Pre-Individual Singularities' [Connected Through Disjunctive Synthesis. Hence The Importance Of Heterogeneity] (175). The Normal Concept Of The Self Implies Some Connected Series, 'But When Disjunction Accedes To The Principle Which Gives To It A Synthetic And Affirmative Value, The Self, The World, And God Share In A Common Death' (176). [There Is Also A Point That Divergent Series Explain And Also Exceed Normal Conjunctive And Connective Series]. In The Usual Conception, 'The Self Is The Principle Of Manifestation, In Relation To The Proposition, The World Is The Principle Of Denotation, And God The Principle Of Signification' (176). But The Theory Of Sense Here Says That It Emanates From Nonsense, From Paradox, And From The 'Eternally Decentred Ex-Centric Centre' (176). This Position 'Does Not Tolerate The Subsistence Of God As An Original Individuality, Nor The Self As The Person, Nor The World As An Element Of The Self And As God's Product. The Divergence Of The Affirmed Series Forms A "Chaosmos" And No Longer A World; The Aleatory Point Which Traverses Them Forms A Counterself, And No Longer A Self' (176). There Is No Centre But Only 'Pure Events Which The Instant, Displaced Over The Line[Of Aion] , Goes On Dividing Into Already Past And Yet To Come. Nothing Other Than The Event Subsists… This Communicates With Itself Through Its Own Distance And Resonates Across All Of Its Disjuncts' (176). Alexander I. Stingl's Blog A Nomadic Scholarship Entity NOTES ON: Deleuze, G (1990) The Logic Of Sense, Trans Mark Lester, Edited By Constantin Boundas, New York: Columbia University Press For Models Kindly See One Of The Papers In The Series. Attributable And Ascribable To Constraints On Space Accomodation Model Is Provided In Some Paper of the series, Notwithstanding The Generality And Commonalty Of The Observation And Its Concatenatability With The Other Modules Sixteenth Series Of The Static Ontological Genesis Twenty-fourth series of the communication of events . "As Simple As That Sounds, It Is Nevertheless Extremely Difficult To Adequately Discuss No-Boundary Awareness Or Nondual Consciousness. This Is Because Our Language — The Medium In Which All Verbal Discussion Must Float — Is A Language Of Boundaries. As We Have Seen, Words And Symbols And Thoughts Themselves Are Actually Nothing But Boundaries, For Whenever You Think Or Use A Word Or Name, You Are Already Creating Boundaries. Even To Say "Reality Is No-Boundary Awareness" Is Still To Create A Distinction Between Boundaries And No-Boundary! So We Have To Keep In Mind The Great Difficulty Involved With Dualistic Language. That "Reality Is No-Boundary" Is True Enough, Provided We Remember That No-Boundary Awareness Is A Direct, Immediate, And Nonverbal Awareness, And Not A Mere Philosophical Theory. It Is For These Reasons That The Mystic-Sages Stress That Reality Lies Beyond Names And Forms, Words And Thoughts,

Divisions And Boundaries. Beyond All Boundaries Lies The Real World Of Suchness, The Void, The Dharmakaya, Tao, Brahman, The Godhead. And In The World Of Suchness, There Is Neither Good Nor Bad, Saint Nor Sinner, Birth Nor Death, For In The World Of Suchness There Are No Boundaries." ― Ken Wilber, No Boundary: Eastern And Western Approaches To Personal Growth. It Has Just Been Argued That Divergence Can Produce A Positive Synthesis, And That Ultimately, Events And States Of Affairs Are Compatible. Incompatibility Arises Only With Actualizations. It Is Individuals, For Example, Who Actualize Divergent Events. Even These Divergences Are Not Just Logical Contradictions But 'Alogical 'Incompatibilities. Persons Can Enjoy The Paradoxes That Ensue, But The Point Is How To Get To The Universal Communication Of Events, The Disjunctive Syntheses. Individuals Must Grasp Themselves As Events, And See What They Normally Regard As Themselves As An Actualization. Normal Individuality Is Only 'Fortuitous' (178). It Follows That The Same Thinking Must Be Extended To All Other Events And Individuals. 'Each Individual Would Be Like A Mirror For The Condensation Of Singularities… The Ultimate Sense Of Counteractualisation' (178) [Undertaken For Cognitive Reasons Here Not Ethical Ones, Although Deleuze Says That This Is A Route To 'The Universal Freedom' (178)]. The Normal Sense Of The Individual As Having A Coherent Identity Must Be Replaced By The Notion Of A Series Of Individualities [With Nomadic Lines Connecting Them?]. We Then Get To The Realm Of Pure Events, And The Universal Connections Between Them, Seen As Disjunctive Syntheses Of Series, And The Knowledge That Actualized Events Are Fortuitous. [The Example Here Is That A Friend In One World Could Be An Enemy In Another Equally Possible World]. 'Philosophy Merges With Ontology, But Ontology Merges With The Univocity Of Being (Analogy Has Always Been A Theological Vision, Not A Philosophical One, Adapted To The Forms Of God, The World, And The Self)' (179). This Does Not Mean That There Is Only One And The Same Being [And Certainly Not Identical Beings, Because These Are Always Heterogeneous]. Instead, 'Being Is Voice That Is Said, And It Is Said In One And The Same "Sense" Of Everything About Which It Is Said' (179). It Is The Ultimate Form. All The Other Forms Remain 'Disjointed In It', But Being Joins Them Into Series And Disjunctions. 'The Positive Use Of The Disjunctive Synthesis… Is The Highest Affirmation... A Single Voice For Every Hum Of Voices And Every Drop Of Water In The Sea' (180). It Is Not Just That There Is A Connection At The Level Of Language—'Being Cannot Be Said Without Also Occurring' (180). In This Way, Event And Sense Are Identical. 'Univocal Being Is Neutral. It Is Extra Being, That Is The Minimum Of Being Common To The Real, The Possible And The Impossible… The Pure Form Of The Aion… One Single Event For All Events; One And The Same Aliquid [Something] For That Which Happens And That Which Is Said; And One And The Same Being For The Impossible, The Possible, And The Real' (180). Alexander I. Stingl's Blog A Nomadic Scholarship Entity NOTES ON: Deleuze, G (1990) The Logic Of Sense, Trans Mark Lester, Edited By Constantin Boundas, New York: Columbia University Press For Models Kindly See One Of The Papers In The Series. Attributable And Ascribable To Constraints On Space Accomodation Model Is Provided In Some Paper, Notwithstanding The Generality And Commonalty Of The Observation And Its Concatenatability With The Other Modules. Sāṅkhya (Often Spelled Sāṁkhya) Is One Of The Major "Orthodox" (Or Hindu) Indian Philosophies. Two Millennia Ago It Was The Representative Hindu Philosophy. Its Classical Formulation Is Found In Īśvarakṛṣṇa's Sāṅkhya-Kārikā (Ca. 350 C.E.), A Condensed Account In Seventy-Two Verses. It Is A Strong Indian Example Of Metaphysical Dualism, But Unlike Many Western Counterparts It Is Atheistic. The Two Types Of Entities Of Sāṅkhya Are Prakṛti And Puruṣa-S, Namely Nature And Persons. Nature Is Singular, And Persons Are Numerous. Both Are Eternal And Independent Of Each Other. Persons (Puruṣa-S) Are Essentially Unchangeable, Inactive, Conscious Entities, Who Nonetheless Gain Something From Contact With Nature. Creation As We Know It Comes About By A Conjunction Of Nature And Persons. Prakṛti, Or Nature, Is Comprised Of Three Guṇa-S Or Qualities. The Highest Of The Three Is Sattva (Essence), The Principle Of Light, Goodness And Intelligence. Rajas (Dust) Is The Principle Of Change, Energy And Passion, While Tamas (Darkness) Appears As Inactivity, Dullness, Heaviness And Despair. Nature, Though Unconscious, Is Purposeful And Is Said To Function For The Purpose Of The Individual Puruṣa-S. Aside From Comprising The Physical Universe, It Comprises The Gross Body And "Sign-Body" Of A Puruṣa. The Latter Contains Among Other Things The Epistemological Apparati Of Embodied Beings (Such As The Mind, Intellect, And Senses). The Sign Body Of A Puruṣa Transmigrates: After The Death Of The Gross Body, The Sign-Body Is

Reborn Into Another Gross Body According To Past Merit, And The Puruṣa Continues To Be A Witness Through Its Various Bodies. An Escape From This Endless Circle Is Possible Only Through The Realization Of The Fundamental Difference Between Nature And Persons, Whereby An Individual Puruṣa Loses Interest In Nature And Is Thereby Liberated Forever From All Bodies, Subtle And Gross. Much Of The Sāṅkhya System Became Widely Accepted In India: Especially The Theory Of The Three Guṇa-S; And It Was Incorporated Into Much Latter Indian Philosophy, Especially Vedānta. Internet Encyclopedia Of Philosophy Sankhya Author Information Ferenc Ruzsa Email: Ferenc.Ruzsa@Elte.Hu Eötvös Loránd University Hungary. The Amitabha Buddha Who Was, And Is, Revered And Praised By Buddhists Around The World, Radiates Indefinite Light And Life From This "State Of Cessation". This State Is A Continuous Process Of Calmness. It Will Be The Eventual Refuge For Us All. If We Think Carefully About The Definitions Of Calmness And Extinction, Then We Can Deduce That They Are The True Natural End-Points Of Rising And Falling. The True Nature Of The Cycle Of Rising And Falling Is Calmness And Extinction. Because Of This Nature, All Chaos And Conflicts In The State Of Rising And Falling Will Eventually Cease. This Is Attainable By The Realisation Of Prajna. Teachings In Chinese Buddhism (6) Sunyata (Emptiness) In The Mahayana Context (Wikipedia). Like Psychic DNA, The Collective Unconscious Contains "Inherited" Psychic Material That Links Us Not Only To Other Humans In The Present But Also To Our Ancestors From The Past. According To Jung's Theory, Though Each Of Us Appears To Function Independently, In Actuality We're All Tapped Into The Same Global Mind. Symbols Interact With And Condition Our Biohologram. The Collective Unconscious Is Essentially A Hologram. Symbols Arise From And Are Embedded In The Environment As Holographic Fields Of Energy. They Are Morphogenic Veils Of Primal Forces. If Your Brain Acts Like A Self-Contained Hologram, It Is Possible Your Consciousness Is Actually A Piece Of A Much Larger Hologram Of Overall Human Consciousness. Jung Noticed That Patterns Spontaneously Appear Over And Over Around The World. They Also Appear As Our Ancestral Memories Or Holographic Wisdom Field. In The Archetypal World, Everyone Is The Same, All Around The World... We Are All Gods, And Our Emotional Addictions To Pain And Suffering, Contempt, Insecurities, Doubt, Failure, Is Holographically-Recorded And Can Be Holographically Healed. All Archetypes Are A Form Of Human Expression That Is Both Holographic And Physical. Physical Formations Of Archetypal Sequences Cause Humans To Behave In Parallel Manners To Each Ancestor That Is Associated. Integration Is A Function Of Intentionality -- Conscious And Unconsciously Maintained, Or Incorporated. Integration Occurs Both Without Effort, As A Redesign Of The Central Processor Of Our Minds, And Voluntarily As A Deliberate Effort To Understand, Find Meaning, And As Rectification Of Our Behavior Towards Others And Towards Ourselves. Imagination Is Structured By The Archetypal Potentials Of The Unconscious. Archetypes Structure The Possibility To Generate And Entertain Such Ideas. The Archetype Itself Cannot Be Known But Structures Everything We Come To Know. Their Totality Functions As A Psychic Organ. Universal Themes Appear In Distinct Cultural Garb. Over Millennia, All The Archetypes Have Emerged In Stories, Deities And Cultural Forms. One Of The Striking Points Of Religious Faith Is That They Aren't True. In Early History People Didn't Know The ‘Real' Reasons Things Happened, So It's No Surprise That Their Explanations Were Wrong. But Then, Why Would We Want To Retrieve Such Superstitions? Kuhn Reminds Us That Even The Most Absurd Or Confused Explanation Of A Phenomenon Can Find Acceptance In The Absence Of A Competing Idea. Once Any Explanation Is Offered For A Phenomenon, It Has An Explanation From That Time On. Succeeding Speculations Might Be Able To Explain The Relevant Phenomena Better Than Its Predecessors. Deities, As Archetypal Role Models, Are The Opening Gambit In The Drama Of Understanding Human Subjectivity. Folk Tales Function The Same Way. Deities Or Archetypes May Have Evolved To Smooth Interpersonal Relations By Including An Understanding Of Human Types, Along With Rules For Helping The Different Types Relate With One Another. They Are Reflections Of Ancient Evolutionary Pressures, With A Dash Of Neuroanatomy. We Still Have Yjr Same Pressures Today So The Ancient Archetypes Still "Work", Regardless Of Objective Existence Of Gods And Goddesses. Burke (2003) Describes How The Underlying Pattern Or Strange Attraction Between Transference And Holography Extends To Other Processes Both Within And Outside The Field Of Psychology. Such Processes Include Projection, Projective Identification, Splitting, Memory, Biology, Creative Discovery, Theology, Synchronicity, Chaos, And Nonlocality. Identifying The Similar Patterns Of These Processes

Demonstrates The Existence Of An Underlying Holographic Archetype In Which Essential Qualities Of The Whole Are Present In Each Of The Parts Of The Whole. The Autonomy Of The Overall Human Is Present In Each Conscious And Unconscious Component Part Of The Human Psyche. Cognition, Itself Is A Holographic Archetype. Many Essential Qualities Of The Whole Are Reflected Or Contained In Each Of The Parts That Make Up That Whole. It Is A Subtle Net Of Metaphor, Analogy And Simile. Holographic Archetypes Effectively Echo Their Resonant Patterns Through Literal And Symbolic Reflectaphors. The Passing Forms Of The Holographic Archetype Include The Hologram, Psychic Structure, Synchronicity, Wisdom Traditions, Memory, And The Process Of Scientific Discovery, Chaos In Physics, Nonlocality And Virtuality In Physics. As Unconscious Structuring Principle, The Archetype Lies Beyond Normal Consciousness And Emerges Suddenly And Dynamically From The (Holographic) Psychoid Field, With A Powerful Emotional Charge That Invests It With Significance. Everything That Happens Is Conditioned By The Moment In Which It Happens. The Universal Field Imposes The Conditions. Matter Is Not Inert But Receptive To Holistic Patterning. If The Mythic World Taught Our Ancestors How To Manipulate The Empirical World, It Also Taught Them To Manipulate That Mythic Narrative Itself For Control Purposes. Socioeconomic Power Enforces Its Mythic Narrative. The Psychoid Field Imposes Holistic Function. Autonomous Inner Forces Arouse Compelling Opportunities To Enact Archetypal Behaviors. They Guide Our Perceptions And Behavior, Usually Without Our Awareness. Limbic Action Of Complexes Is A Big Part Of The Holistic Dynamics Of The Psyche. Dreams Report What Goes On Beneath The Veil Of Conscious Awareness. To The Consciousness Of The 'Thinker', Knowledge Is Thought. Period. Without Thought, The Consciousness Of The 'Thinker' Collapses Into Psychosis. With Archetypes Come The Potential For Wisdom, Relatedness, Sociality, Ambiguity, Paranoia, Projection, Identification, Denial, Inflation, Sub Personalities (Fragmentation), Defensiveness, Obsession, Hypnotic Dissociation, The Contagion Of Participation Mystique, Mythologizing, Complexes, Compensation, And Self-Delusion. Compensation May Calm Or Disturb Consciousness. There Is No Imperative For The Ego To Integrate These Alternative Perspectives, Private And Public Myths. The Unconscious Can Produce Deep Wisdom And Utter Nonsense. It Is Up To The Ego To Discriminate. The Value Of Myths Is Purely Heuristic, Not Pragmatic. Jung In The 21st Century: Synchronicity And Science By John Ryan Haule Mind Control Countermeasures. Forms Of Experience, Such As Madness, Violence, Or Sexuality That Break From The Prison Of Rationality. Where Modern Societies "Problematize" Forms Of Experience Such As Madness, Illness, And Sexuality, That Is, Turn Them Into Governmental Problems, Into Areas Of Life In Need Of Control And Regulation, Foucault In Turn Queries The Social Construction Of “Problems” By Uncovering Their Political Motivations And Effects And By Challenging Their Character As Natural, Necessary, Or Timeless. In What He Calls A "Diagnostic Critique" That Combines Philosophy And History (1989: 38-39, 73), Foucault Attempts To Clarify The Nature Of The Present Historical Era, To Underline Its Radical Difference From Preceding Eras, And To Show That Contemporary Forms Of Knowledge, Rationality, Social Institutions, And Subjectivity Are Contingent Socio-Historical Constructs Of Power And Domination, And Therefore Are Subject To Change And Modification. Foucault's Ultimate Task, Therefore, Is "To Produce A Shift In Thought So That Things Can Really Change" (Quoted In O'Farrell 1989: 39). The Goal Of Foucault's Historico-Philosophical Studies, As He Later Came To Define It, Is To Show How Different Domains Of Modern Knowledge And Practice Constrain Human Action And How They Can Be Transformed By Alternative Forms Of Knowledge And Practice In The Service Of Human Freedom. Foucault Is Concerned To Analyze Various Forms Of The "Limit Experience" Whereby Society Attempts To Define And Circumscribe The Boundaries Of Legitimate Thought And Action. The Political Vision Informing Foucault's Work Foresees Individuals Liberated From Coercive Social Norms, Transgressing All Limits To Experience, And Transvaluing Values, Going Beyond Good And Evil, To Promote Their Own Creative Lifestyles And Affirm Their Bodies And Pleasures, Endlessly Creating And Recreating Themselves. Foucault Denies There Can Be Any Basis For Objective Descriptive Statements Of Social Reality Or Universal Normative Statements That Are Not Socially Conditioned And Locally Bound. He Tries To Show That All Norms, Values, Beliefs, And Truth Claims Are Relative To The Discursive Framework Within Which They Originate. Any Attempt To Write Or Speak About The Nature Of Things Is Made From Within A Rule-Governed Linguistic Framework, An “Episteme,” That Predetermines What Kinds Of Statements Are True Or Meaningful. All Forms Of Consciousness, Therefore, Are

Socio-Historically Determined And Relative To Specific Discursive Conditions. There Is No Absolute, Unconditioned, Transcendental Stance From Which To Grasp What Is Good, Right, Or True. Foucault Refuses To Specify What Is True Because There Are No Objective Grounds Of Knowledge; He Does Not State What Is Good Or Right Because He Believes There Is No Universal Standpoint From Which To Speak. Universal Statements Merely Disguise The Will To Power Of Specific Interests; All Knowledge Is Perspectival In Character. For Postmodern Theorists Like Foucault, The Appeal To Foundations Is Necessarily Metaphysical And Assumes The Fiction Of An Archimedean Point Outside Of Language And Social Conditioning. Habermas (1987) Rightly Finds Perplexing An Approach That Raises Truth Claims While Destroying A Basis For Belief In Truth, Which Takes Normative Positions While Suppressing The Values To Which They Are Committed. For Critique To Be Justified And Effective, It Should Preserve Standards By Which To Judge And Evaluate, But Foucault's Total Critique Turns Against Itself And Calls All Rational Standards Into Question. **Richard Rorty, The Attack On Theory, And Renunciation Of Radical Politics.** Deleuze's Analysis Begins by Coming to New Understandings of the Concepts of the Image and Movement. The Image, Above All, Is Not A Representation Of Something, That Is, A Linguistic Sign. This Definition Relies Upon The Age-Old Platonic Distinction Between Form And (e&eb) Matter, In Its Modern Saussurean Form Of Signifier-Signified. Rather, Deleuze Wants To Collapse These Two Orders Into (eb) One, And The Image Thus Becomes Expressive And Affective: Not An Image Of A Body, But The Body As Image (C1 58). This Collapse Comes About With Reference To Two Philosophers, Henri Bergson And Charles Sanders Pierce. Deleuze Dedicated A Book-Length Study To The Former Entitled Bergsonism (1968), And His Use Of His Notions Of Movement And Time In The Cinema Texts Is Already Presaged By This Text. Movement For Bergson, Deleuze Argues, Is Not Separable From The Object Which Moves: (I Am Not Movement: Italics Mine) They Are Literally The Same Thing. Thus, No Representative Relationship Can Be Established Without (e) Artificially Halting The Flow Of Movement And Thus Misconstruing The Frozen 'Element' As Self-Sufficient. (See the befuddling similarity of holographic principle of Modern Quantum Mechanics and the Deleuzean idea; italics mine) There Is Only The Flow Of Movement Which Expresses Itself In Different Ways. Among Other Things, This Is One Of Deleuze's Critiques Of Phenomenology (C1 56, 60). Thus The Early Cinema Is Characterised For Deleuze By The Reign Of What He Calls The Sensory-Motor Schema. This Schema Is The Unity Of The Viewed And The Eye That Views In Dynamic Movement. This Model Of The Movement-Image Is Precisely The Nature Of Cinema, For Deleuze. It Does Not Falsify Movement By Extracting Segments And Stringing Them Together In A Representative Fashion, But Creates A Wide Range Of Expressive Images. It Is In Order To Come To Terms With The Varieties Of Movement-Images That Deleuze Turns To Pierce, Who Developed, "The Most Extraordinary Classifications Of Images And Signs . . ." (C2 30). ……. With Some Alterations, Pierce's Semiotic Classifications To Describe The Use Of Movement-Images In Cinema, And Their Centrality Before The Second World War The Movement From The First Text To Cinema 2: The Time-Image Has A Significance Closely Related To Kant's So-Called Copernican Revolution In Philosophy. Up Until Kant, Time Was Subject To The Events That Took Place Within It, Time Was A Time Of Seasons And Habitual Repetition (See (3) (C) Above); It Was Not Able To Be Considered On Its Own, But As A Measure Of Movement (C2 34-5; KCP Iv.). One Element Of Kant's Achievement For Deleuze, As We Have Seen, Is His Reversal Of The Time-Movement Relationship: He Establishes Time Itself As An Element To Which Movement Must Be Subordinated, A Pure Time. In The Cinema, Deleuze Argues, A Similar Reversal Takes Place. The Historico-Cultural Reason Behind This Reversal Is The Event Of World War Two Itself. With The Great Truths Of Western Culture Put So Deeply In Question By The Before Unimaginable Methods Employed And Their Forthcoming Results, The Sensory-Motor Apparatus Of The Movement-Image Are Made To Tremble Before The Unbearable, The Too-Much Of Life's Possibilities, The Potential Of The Present (C2 35). No Longer Could The Dogmatic Truths That Had Guided Society, And Cinema To An Extent, Allow The Apparently 'Natural' Movement From One Thing To The Next In An Habitual Fashion: 'Natural' Links Precisely Lost Their Efficacy. And With The Use Of Unnatural Or False Links, Which Do Not Follow The Sequence Or Narrative Affect Of The Movement-Image, Time Itself, The Time-Image, Is Manifested In Cinema (Deleuze Considers Orson Welles To Be The First Auteur To Make Use Of The Time-Image (C2 137)). Rather Than Finding Time As An, "Indirect Representation," (C2 35-6), The Viewer Experiences The Movement Of Time Itself, Which Images, Scenes, Plots

And Characters Presuppose Or Manifest In Order To Gain Any Sort Of Movement Whatsoever. Along With This 'External' Reason, There Is Also For Deleuze A Motivation Within Cinema Itself To Go From The Movement-Image To The Time-Image. The Movement Image Has The Tendency, Thanks To The Habitual Experience Of Movement As Normal And Centered, To Justify Itself In Relation To Truth: As Deleuze Argues With Regard To The Dogmatic Image Of Thought (See (3)(D) Above), There Is The Presupposition That Thought Naturally Moves Towards Truth. Of Course, Deleuze Suggests, Cinema, When Truly Creative, Never Relied Upon This Presupposition, And Yet, "The Movement-Image, In Its Very Essence, Is Answerable To The Effect Of Truth Which It Invokes While Movement Preserves Its Centres." (C2 142). In Questioning Its Own Presuppositions, Deleuze Argues, Cinema Moved Towards A New, Different, Way Of Understanding Movement Itself, As Subordinate To Time. (Wikipedia: Gilles Deleuze). From His Early Writings On, Nietzsche, Like Kierkegaard, Rails Against A Life-Denying Rationalism And Idealist Philosophy Which Champions' Reason Over The Passions. Nietzsche Interprets The "Subject" As A Mere Construct, An Idealized Sublimation Of Bodily Drives, Experiences, And A Multiplicity Of Thoughts And Impulses. This "Little Changeling," On Nietzsche's View, This Subject, "Is Believed in More Firmly than Anything Else on Earth," But Is For Him A Simple Illusion Created Out Of Modern Desperation To Have A Well-Grounded Identity. Belief In The Subject Is Promoted By The Exigencies Of Grammar Which Utilize A Subject/Predicate Form, Giving Rise To The Fallacy That The "I" Is A Substance, Whereas It Is Really Only A Convention Of Grammar (Nietzsche 1968b: 37-38). For Nietzsche, "The Doer" Is "Merely A Fiction Added To The Deed -- The Deed Is Everything" (1968b: 45). "The Subject," He Concludes, Is Thus But A Shorthand Expression For A Multiplicity Of Drives, Experiences, And Ideas. In The Spirit Of Enlightenment, Nietzsche Also Polemicizes Against Metaphysics, Arguing That It Illicitly Generalizes From Ideas In One Historical Epoch To The Entirety Of History. Against This Form Of Philosophical Universalism, Nietzsche Argues "There Are No Eternal Facts, Just As There Are No Absolute Truths. Consequently, What Is Needed From Now On Is Historical Philosophizing, And With It The Virtue Of Modesty" (Nietzsche 1986: 13). Castigating Traditional Philosophy And Values From A Critical Enlightenment Perspective, Nietzsche Anticipates Later Postmodern Critiques Of Metaphysics, Assailing The Concept Of Enduring Knowledge, The Notion Of A Transcendental World, And Presenting Metaphysical Thought As A Thoroughly Obsolete Mode Of Thinking. He Attributes The "Metaphysical Need," At The Heart Of Philosophies Such As Schopenhauer's, To Primitive Yearnings For Religious Consolation For The Sufferings Of Life And He Urges "Free Spirits" To Liberate Them And Pursue Thinking And Living Experimentally (1986: 8). Nietzsche's Attack On Foundationalism, Universalizing Thought, And Metaphysics Thus Undertakes A "Postmodern" Turn In Philosophy Through A Radical Deconstruction Of Modern Theory. But While Deconstructionist Philosophies Typically Terminate In The No, Merely Seeking To Unravel A Positive Modern Value System Into A Heap Of Disconnected Fragments, Nietzsche Starts And Finishes With A Big Yes, A Life-Affirming Value, Deconstructing Only To Reconstruct. Moving Far Away From Schopenhauerian Pessimism, Back Toward A Greek View Of Tragedy, Toward A Dionysian View Of Existence, Nietzsche Seeks "A Justification Of Life, Even At Its Most Terrible, Ambiguous, And Mendacious" (1968a: 521), A Justification Found In Art, Creativity, Independence, And The Emergence Of "Higher Types" Of Humanity. Yet Nietzsche's Perspectivism Denies The Possibility Of Affirming Any Absolute Or Universal Values: All Ideas, Values, Positions, And So On Are Posits Of Individual Constructs Of A Will To Power, Which Are To Be Judged According To The Extent To Which They Do Or Do Not Serve The Values Of Life, Creativity, And Strong Individuality. For Nietzsche There Are No Facts, Only Interpretations, And He Argues That All Interpretation Is Constituted By The Individual's Perspectives And Is Thus Inevitably Laden With Presuppositions, Biases, And Limitations. For Nietzsche, A Perspective Is Thus An Optic, A Way Of Seeing, And The More Perspectives One Has At One's Disposal, The More One Can See, And The Better One Can Understand And Grasp Specific Phenomena. To Avoid Limited And Partial Vision One Should Learn "How To Employ A Variety Of Perspectives And Interpretations In The Service Of Knowledge" (Nietzsche 1968a: 119). THE POSTMODERN TURN IN PHILOSOPHY: THEORETICAL PROVOCATIONS AND NORMATIVE DEFICITS By Steven Best And Douglas Kelner Http://Www.Gseis.Ucla.Edu/Faculty/Kellner/Kellner.Html. And Just As During Much Of The Twentieth Century, Any Plea For Greater Social Justice Could Be Successfully Damned As

Communist, Likewise Today, Any Strategy To Eradicate Suffering Is Likely To Be Condemned In Similar Reactionary Terms: Either Wire Head Hedonism Or Revamped Brave New World. This Response Is Not Just Facile And Simplistic. If It Gains Currency, The Result Is Morally Catastrophic. Of Course, The Abolitionist Issue Rarely Arises. Typically, Universal Bliss Is Still More-Or-Less Unthinkingly Dismissed As Technically Impossible. Insofar As The Prospect Is Even Contemplated - Grudgingly - It Is Usually Assumed That The New Regime Would Be Underwritten Day-By-Day With Drugs Or, More Crudely, Electrodes In The Pleasure-Centres. These Techniques Have Their Uses. Yet In The Medium-To-Long-Term, Stopgaps Won't Be Enough. All Use Of Psychoactive Drugs May Be Conceived As An Attempt To Correct Something Pathological With One's State Of Consciousness. There's Something Deeply Wrong With Our Brains. If What We Had Now Was OK, We Wouldn't Try To Change It. But It Isn't, So We Do. Mature Biological Psychiatry Will Recognise Inadequate Innate Bliss As A Pandemic Form Of Mental Ill-Health: Good For Selfish DNA In The Ancestral Environment Where The Adaptation Arose, But Bad For Its Throwaway Vehicles, Notably Us. The Whole Gamuts Of Behavioural Conditioning, Socio-Economic Reform, Talk-Therapies - And Even Euphoriant Superdrugs - Are Just Palliatives, Not Cures, For A Festering Global Illness. Its Existence Demands A Global Eradication Program, Not Idle Philosophical Manifestos And Scientific Belle's Letters. But One Does One's Best. The Ideological Obstacles To Genetically Pre-Programmed Mental Super-Health Are Actually More Daunting Than The Technical Challenges. To Be Cured, Hypo-Hedonia Must Be Recognised As A Primarily Genetic Deficiency-Disorder. Designer Mood-Brighteners And Anti-Anxiety Agents To Alleviate It Are Sometimes Branded "Lifestyle-Drugs"; But This Is To Trivialize A Serious Medical Condition Which Must Be Corrected At Source. Happily, Our Hereditary Neuropsychiatric Disorder Is Likely To Become Extinct Within A Few Generations As The Reproductive Revolution Unfolds. Aversive Experience, And The Poisonous Metabolic Pathways That Mediate Its Textures, Will Become Physiologically Impossible Once The Genes Coding Its Neural Substrates Have Been Eliminated. We Won't Miss Its Corrupting Effect When It's Gone. In The Medium-Term, The Functional Equivalent Of Aversive Experience Can Help Animate Us Instead. Late In The Third Millennium And Beyond, Its Functional Successors May Be Expressed As Gradients Of Majestic Well-Being. On This Scenario, Our Descendants Will Enjoy A Civilisation Based On Information-Bearing Pleasure-Gradients: Whether Steep Or Shallow, We Simply Don't Know. Such A Global Species-Project Does Not Have The Desperate Moral Urgency Of Eliminating The Phenomenon Of Darwinian Pain - Both "Mental" And "Physical", Human And Non-Human Alike. Abolishing Raw Nastiness - Sometimes Vile Beyond Belief - Remains The Over-Riding Ethical Priority. One Doesn't Have To Be An Outright Negative Utilitarian To Acknowledge That Getting Rid Of Agony Takes Moral Precedence Over Maximizing Pleasure. But Both Genetic Fundamentalists And Gung-Ho Advocates Of Better Living Through Chemistry Today Agree On One Crucial Issue. There Is No Sense In Sustaining A Legacy Of Mood-Darkening Metabolic Pathways Out Of Superstitious Deference To Our Savage Past. When Bernard Marx Tells The Savage He Will Try To Secure Permission For Him And His Mother To Visit The Other Place, John Is Initially Pleased And Excited. Echoing Miranda In The Tempest, He Exclaims: "O Brave New World That Has Such People In It." Heavy Irony. Like Innocent Miranda, He Is Eager To Embrace A Way Of Life He Neither Knows Nor Understands. And Of Course He Comes Unstuck. Yet If We Swallow Such Fancy Literary Conceits, Then Ultimately The Joke Is On Us. It Is Only Funny In The Sense There Are "Jokes" About Auschwitz. For It Is Huxley Who Neither Knows Nor Understands The Glory Of What Lies Ahead. A Utopian Society In Which We Are Sublimely Happy Will Be Far Better Than We Can Presently Imagine, Not Worse. And It Is We, Trapped In The Emotional Squalor Of Late-Darwinian Antiquity, Which Neither Know Nor Understand The Lives Of The God-Like Super-Beings We Are Destined To Become. **A Defence Of Paradise-Engineering Brave New World By Aldous Huxley Brave New World (1932) Is One Of The Most Bewitching And Insidious Works Of Literature Ever Written.** Deleuze attacks Hegel and others in what we can call—though Deleuze did not—the "identitarian" tradition first of all by means of (e) a radicalized reading of Kant, whose genius, as Deleuze explains in Kant's Critical Philosophy (1963), was to have conceived of a purely immanent critique of reason—a critique that did not seek "errors" of reason produced by external causes, but rather "illusions" that arise (eb) from within reason itself by the illegitimate (transcendent) uses of the syntheses of (e) consciousness. Deleuze characterized his own work as a philosophy of immanence, arguing that Kant himself had failed to realize fully the ambitions of his

critique, for at least two reasons: first, the failure to pursue a fully immanent critique, and second, the failure to propose a genetic account of (e) real experience, resting content with the account of the conditions of possible experience. First, Kant made the field of consciousness immanent to (e) a transcendental subject, thereby reintroducing an element of identity that is (=) transcendent (that is, external) to (e) the field itself, and reserving all power of synthesis (that is, identity-formation) in the field to the activity of (e) the always already unified and transcendent subject. (Deleuze was influenced in this regard by his reading of Sartre's 1937 essay "The Transcendence of the Ego.") Already in his Hume book, Empiricism and Subjectivity (1953), Deleuze had pointed to an empiricist reversal of Kant. Where Kant's question had been "How can the given be given to (e) a subject?" Hume's question had been "How is the subject (human nature) constituted within (eb) the given?" Metz asserting a notion of langue with the formal elements associated with narrative. Moreover he narrowed this gap further by claiming a version of langue's syntagmatic and paradigmatic in the operation of cinematic narrative. Deleuze notes the significance of this move in The Time-Image: [L]anguage features which necessarily apply to utterances will be found in the cinema, as rules of use, in the language system and outside of it: the syntagm (conjunction of present relative units) and the paradigm (disjunction of present units with comparable absent units). The semiology of the cinema will be the discipline that applies linguistic model, especially syntagmatic ones, to images as constituting one of their principle "codes" (25-26). (As quoted in Roger Dawkin's making matter our of Deleuzean concept of cinema) An exemplary and illustrious account of unbounded consciousness has been studied by CORRINA BONSHEK Deleuzian sensation and unbounded consciousness in Anna & Corrina Bonshek's Reverie I (2002). Entire article has been modelled in one of the paper in the series. It is true that Deleuze sometimes suggests follow of the Model and Shadow theme instead of getting into the trap of consciousness and cogito. This should not be confused with the unbounded consciousness Deleuze mentioned in his studies on Cinema. In his mature work, Deleuze argues for an "impersonal and pre-individual" transcendental field in which the subject as identity pole which produces (eb) empirical identities by active synthesis is itself the result or product of (e) differential passive syntheses (for instance, in what Deleuze calls the syntheses of habit, we find bodily, desiring, and unconscious "contractions" which unify (e&eb) a series of experiences, extracting (eb) that which it to be retained in the habit and allowing (eb) the rest to be "forgotten"). The passive syntheses responsible for subject formation must be qualified as (=) "differential," for three reasons. Each passive synthesis is (=) serial, never singular (there is never one synthesis by itself, but always a series of (e) "contractions," that is to say, experience is (=) ongoing and so our habits require (e) constant "updating"); each series is related to (e&eb) other series in the same body (at the most basic level, for instance, the series of taste contractions is related to (e&eb) those of smell, sight, touch, hearing and proprioception); and each body is related to (e&eb) other bodies, which are themselves similarly differential (the series of syntheses of bodies can resonate or clash). Together the passive syntheses at all these levels form (eb) a differential field within which subject formation takes place as (=) an integration or resolution of that field; in other words, subjects are (=) roughly speaking the patterns of (e) these multiple and serial syntheses which fold in on themselves producing (eb) a site of self-awareness. Of course, Deleuze never simply proclaims this as a bald thesis, but develops a genetic account of subjectivity in many of his books. Taking all this into account, Deleuze summarized his differential, immanent and genetic position by the at first glance odd phrase of "transcendental empiricism." This is cashed out in terms of two characteristics: (1) the abstract (e.g., "subject," "object," "State," the "whole," and so on) does not explain, but must itself be (=) explained; and (2) the aim of philosophy is not to rediscover (eb) the eternal or the universal, but to find the singular conditions under which (e&eb) something new is produced. In other words—and this is a pragmatic perspective from which Deleuze never deviated—philosophy aims not at stating the conditions of knowledge qua representation, but at finding and fostering (eb) the conditions of creative production. Deleuze's second criticism of Kant claims that he had simply presumed (eb) the existence of knowledge and morality as "facts" and then sought their conditions of possibility in the transcendental But already in 1789, Salomon Maimon, whose early critiques of Kant helped generate the post-Kantian tradition, had argued that Kant's critical project required (e) a method of genesis—and not (e) merely a method of conditioning—that would account for the (eb) production of knowledge, morality, and indeed reason itself. In other words, Maimon called for a genetic method that would be able to reach (eb) the conditions of real and not merely possible experience.

Maimon found a solution to this problem in a principle of difference: whereas identity is (=) the condition of possibility of thought in general, it is difference that constitutes (e) the genetic and productive principle of real thought. These two Maimonian exigencies—the search for the genetic conditions of real experience and (e&eb) the positing of a principle of difference—appear in almost every one of Deleuze's early monographs. Nietzsche and Philosophy (1962), for instance, suggests that Nietzsche completed and inverted Kantianism by bringing critique to bear, not simply on false claims to knowledge or morality, but on (eb) true knowledge and true morality, and indeed on truth itself: "genealogy" constituted (e) Nietzsche's genetic method, and the will-to-power was (=) his principle of difference. Deleuze's anti-Hegelianism is shown in his focus on (eb) the productivity of the non-dialectical ("affirmative") differential forces termed by Nietzsche "noble." These forces affirm themselves, and thereby differentiate (e&eb) themselves first, and only secondarily consider that from which they have (e) differentiated themselves. In Bergsonism (1966), Deleuze develops the ideas of virtuality and (e&eb) multiplicity that will serve as (=) the backbone of his later work. From Maimon's reading of Kant, we know that Deleuze needs to substitute the notion of the condition of the genesis of the real for (e&eb) the notion of conditions of possibility of representational knowledge. The positive name for that genetic condition is the virtual, which Deleuze adopts from the following Bergsonian argument. The notion of the, possible Bergson holds in Creative Evolution, is derived from (e) a false problem that confuses the "more" with (e&eb) the "less" and ignores (e) differences in kind; there is not less but more in (eb) the idea of the possible than in the real, just as there is (=) more in the idea of nonbeing than in that of being, or more in the idea of disorder than (e) in that of order. When we think of the possible as (=) somehow "pre-existing" the real, we think of (e) the real, then we add to (e) it the negation of its existence, and then we project (e&eb) the "image" of the possible into the past. We then reverse the procedure and think of the real as (=) something more than possible, that is, as the possible with existence added to (e) it. We then say that the possible has been "realized" in (eb) the real. By contrast, Deleuze will reject the notion of the possible in favor of (e) that of the virtual. Rather than awaiting realization, the virtual is (=) fully real; what happens in genesis is (=) that the virtual is (=) actualized. The fundamental characteristic of the virtual, that which means (eb) it must be actualized rather than realized, is its differential makeup. Here Deleuze used terms actualisation and realisation as if they are (=) differential in nature Actualisation of the dreams could be tantamount to (=) the realization of the dreams, but the reciprocal is not true. (My interpretation) Deleuze always held the critical axiom that the ground cannot (e) resemble that which it grounds; he constantly critiques the "tracing" operation by which identities in real experience are said to be conditioned (e&eb) by identities in the transcendental. Mention may be made of the Deleuzian statement that the relationship between problem and its conditions bears ample testimony and infallible observatory to the truth of the problem as such. Deleuze writes: …….as for the determination of the conditions it implies (eb) on one hand, a space of nomadic distribution, in which singularities are (=) distributed, on the other hand, it implies, (eb) a time of decomposition whereby a space is (=) subdivided in to subspaces. (See Seventeenth series of Logical genesis: Gilles Deleuze: Translation by Constantin Boundas) **It is to be noted that Deleuze writes in a progressive vein; and the subspaces he mentions are those which incorporate adjunct and corresponding points, which in fact defines progressively a determined domain under consideration.** This also brings us to the question whether there is any space which has singularities in its domain under consideration, and if so effects of such singularities incorporated thereof the space itself. Modem quantum mechanics proves beyond doubt that the happenings in space affect (e&eb) the space and vice versa. Deleuze goes on to state that verily there is a space that incorporates these singularities. What does this mean in the most general sense and to us who are used to quantum mechanical world? It is clear that there exists a spatio temporal self determination of the problem in which (eb) the problem advances, making up for (e) its deficiencies and thwarting (e) its own conditionalities. This can be made possible only by (e) a transcendental power or complete computer simulation ad dissimulation. Mention may be made that many studies of the Universe being a computer has been put forward. And it had its many ramifications in the positive direction, albeit it did not received the much needed accentuation and augmentation when the theory was proposed. There is a mistaken notion that once the solution is obtained, problem is solved. Infact solutions are engendered precisely at (eb) the same moment problem determines itself. Thus sense in its organsiation of problems and solutions, aleatory and (e&eb) singular points, series and displacements are (=) doubly generative. Solutions does not attribute a subjective

status to the problem retrospectively as is thought of by the widest commonalty spread when the problem is determined it also determines the solutions under which it persist. Argument also can be put forth from Stoic Philosophy that in order for the thing to be bad, it should have been good. But it has its own limitations as one could discern with circumspection and jurisprudence. For instance, Deleuze criticizes Kant for copying the transcendental field in the image of (e) the empirical field. In fact Kant himself clarifies that the figment of imagination lies dormant like in the mind. If cinnabar were now red, now black, now light, now heavy, if a human being were now changed into this animal shape, now into that one, if on the longest day the land were covered now with fruits, now with ice and snow, then my empirical imagination would never even get the opportunity to think of (e) heavy cinnabar on the occasion of the representation of the color red; or if a certain word were now attributed now to this thing, now to that, or if one and the same thing were sometimes called this, sometimes that, without the governance of (e&eb) a certain rule to which the appearances are already subjected (e&eb) in themselves, then no empirical synthesis of reproduction could take place (CPR, A100-101) as quoted in Real Experience and Possible Experience: Deleuze's Ontology of Experience – Conclusion. This is very much true,. In fact associations are created because of the nonchangeabilityof of the object, notwithstanding certain change in the subject. Such continuous associations are what form the basis of our understanding or misunderstanding (see earlier papers). That is, empirical experience is (=) personal, identitarian and centripetal; there is (=) a central focus, the subject, in which all our experiences are tagged as belonging (e&eb) to us. Mention may be made that Satre established that pre reflexive cogito is primary consciousness notwithstanding the fact that he makes in later work, he makes this his original point of departure. Kant says this empirical identity is only possible if (e) we can posit the Transcendental Unity of Apperception, that is, the possibility of adding "I think" to (e&eb) all our judgments. Instead of this smuggled-in or "traced" identity, Deleuze will want to have the transcendental field be (=) differential. Deleuze still wants to work back from experience, but since the condition cannot (e) resemble the conditioned, and since the empirical is (=) personal and individuated, the transcendental must be (=) impersonal and pre-individual. Deleuze is of the opinion that the interrogative problem cannot bear resemblance to propositions it subsumes(e) under it; it rather engenders (eb) them as it determines its own conditions and assigns (eb) individual order of individual order of permutation of (e) the propositions within (eb) the framework of generalised signification (See seventeenth series of static genesis) The virtual is the condition for (e) real experience, but it has (e) no identity; identities of the subject and the object are products of (e) processes that resolve, integrate, or actualize (the three terms are synonymous for Deleuze) a differential field. Satre was furious about Descartes for having confused spontaneous doubt which is consciousness with methodological doubt. I may add the later might arise from the non emotional feeling, and the question of consciousness itself would remain slightly confused with Satre as far as the author is concerned. Satre holds that all consciousness is consciousness of something. In other words consciousness is (=) pre reflexive and directive. It is (=) certainly (consciousness) intentional or so it would seem. Author's view is that consciousness is not utterly translucent but is to major extent cultivated through reference groups and peer groups which follow the principle of making a choice of 'coming up in life". "I know" is a tendency that grows in an individual and this is a relation with his objects or information (as we have taken) in mind which is essentially anagrammatic. The Deleuzean virtual is (=) thus not the condition of possibility of (e) any rational experience, but the condition of (e) genesis of real experience. Interrogation itself is (=) the shadow of the problem projected, or rather reconstructed based on (e) empirical propositions. This is true of every parameter like language, region, religion, science. As we have seen, the virtual, as genetic ground of the actual cannot (e) resemble that which it grounds; thus, if we are confronted with actual identities in experience, then (eb) the virtual ground of those identities must be purely differential. Deleuze adopts "multiplicity" from Bergson as the name for such a purely differential field. In this usage, Deleuze later clarifies; "multiplicity" designates the multiple as (=) a substantive, rather than as (=) a predicate. Deleuze has been candid about the fact that there exists no parallelism between logical genesis and (e&eb) ontological genesis. There is rather a relay between logical genesis and ontological genesis which permits every sort of shifting (e&eb) and jamming. It is therefore too simple to argue for the correspondence between the individual and (e&eb) the denotation the person and (e&eb) manifestation, multiple properties or variable properties and (e&eb) signification. (Seventeenth series of logical genesis) The multiple as predicate generates (eb) a set of philosophical problems

under (e&eb) the rubric of "the one and the many" (things is one or multiple, one and multiple, and so on). With multiplicity, or the multiple as substantive, the question of the relation between the predicates one/multiple is replaced by (e&eb) the question of distinguishing types of multiplicities (as with Bergson's distinction of qualitative and (er&eb) quantitative multiplicities in Time and Free Will). A typological difference between substantive multiplicities, in short, is substituted for (e&eb) the dialectical opposition of the one and the multiple. In sum, then, against the "major" post-Kantian tradition of Fichte, Schelling, and Hegel, Deleuze in effect posited his (el&eb) own "minor" post-Kantian trio of Maimon, Nietzsche, and Bergson. To these he added a trio of pre-Kantians, Spinoza, Leibniz and Hume, but read through a post-Kantian lens. We have already touched on Deleuze's reading of Hume. Let us now turn to Spinoza, for whom Deleuze's admiration was seemingly limitless; for Deleuze, Spinoza was the "prince" or even the "Christ" of philosophers. There are many Spinozist inheritances in Deleuze, but one of the most important is certainly the notion of univocity in ontology. Univocity—as opposed to its great rivals, equivocity and analogy—is (=) the key to developing (eb) a "philosophy of difference" (Deleuze's term for his project in Difference and Repetition), in which difference would no longer be (=) subordinated to identity. The result is a Spinozism minus (-MD) substance, (=) a purely modal or (=) differential universe. In univocity, as Deleuze reads Spinoza, the single sense of Being frees (e) a charge of difference throughout all that is. In univocal ontology being is said in a single sense of (e) all of which it is said, but it is said of difference itself. What is that difference? Difference is (=) difference in degrees of "power"; in interpreting this term we must distinguish the two French words puissance and pouvoir. In social terms, puissance is (=) immanent power, power to act (e&eb) rather than power to (e) dominate another; we could say that puissance is (=) praxis (in which equals clash or act together) rather than poiesis (in which others are matter to be formed by the command of a superior, a sense of transcendent power that matches (e&eb) what pouvoir indicates for Deleuze). In the most general terms Deleuze develops throughout his career, puissance is (=) the ability to affect and to be affected, to form (eb) assemblages or consistencies, that is, to form (eb) emergent unities that nonetheless respect the heterogeneity of their components. (Here we see the empiricist theme of the "externality of relations": in an assemblage or consistency, the "becoming" or relation of the terms attains (eb) its own independent ontological status. In Deleuze's favorite example, the wasp and orchid create (eb) a "becoming" or symbiotic emergent unit.). Although Deleuze wrote a touching and certainly important book in tribute to his friend Foucault after the latter's death in 1984, the final important figure in Deleuze's readings of other philosophers is Leibniz, to whom, it must be recalled, Maimon appealed in his criticism of Kant. In 1988, Deleuze published a book on Leibniz entitled The Fold: Leibniz and the Baroque, which added new elements to the reading of Leibniz that appeared in Deleuze's earlier books: an interpretation centered on the concept of the fold, a development of a concept of the Baroque, and a attempt to define a neo-Leibnizianism in terms of (e&eb) contemporary artistic and scientific practices. While The Fold is a fascinating work, we will concentrate here on Deleuze's early reading of Leibniz, which plays an important role in Difference and Repetition. Deleuze pushes Leibniz's thought to a point where Leibniz could never have taken it, given his (Leibniz's) theological presuppositions. This is the point where one begins to consider the virtual domain on its own account, freed from (e) its actualization in a world and its individuals. Never has the enterprise and industry of "demystification" been further to the horizon where a person stands on the threshold of infinity trying to ponder what lies beyond its viel which separates the seen from unseen. Myth is always the expression (e) false infinite and thé disturbance of spirit. One of the most profound constants of Naturalism is to denounce everything that is (=) sadness, everything that is (=) cause of sadness, and everything that needs (e) sadness to exercise its power. Naturalism makes of (e) thought and sensibility (=)an affirmation .Naturalism directs its attack against (e&eb) the prestige of the negative (negativity is not bad and it has its own cognizance and prestige) Naturalism deprives (e) negative of all its power. Naturalism also disallows (e) negativity to speak in the name of philosophy which is unfair. Afterall, the spirit of negative made an appearance out of (e) the sensible and linked (e&eb) the intelligible to (e&eb) One or the Whole. But this one and One Whole as we have stressed many a time is that which cancels each projections, actions, cognitions, perceptions, concurrences, rotations, foldings, motions and shocks in the universe atleast holistically and is conservative. This bears ample testimony and infallible observatory and impeccable demonstration to the fact that One and the Whole is (=) nothingness of thought. On a similar veing appearance might also be (=) nothing ness of sensation. Naturalism

is (=) the thought of an infinite sum all of the elements which are not composed at once. Naturalism is (=) also the sensation of finite compounds which are not (e) added up as such with one another. Thus multiplicity is (=) affirmed Multiple of multiple is (=) the object of affirmation .Diverse of diverse is (=) an object of joy. Notwithstanding the fact that gender is no longer widely considered to be a property of individuals, the alternative of viewing it in terms of (e&eb) performativity, where gender is (=) the outcome of linguistic and social performances, unnecessarily limits (e) the possibilities of thinking of gender as(=) a form of multiplicity that is both internally and externally differentiated. Any attempt to move beyond binary thinking in gender relations initiates (eb) a consideration of multiplicity, and the way in which multiplicity is conceptualized exerts a critical influence on (e&eb) the possibilities that are opened up. This article interrogates existing understandings of multiplicity and finds (eb) three actual or possible types - multiplicities of the same, characteristic of feminist approaches which authors critique through (e&eb) a reconceptualization of desire; multiplicities of (e) the third, characterized by anthropological, transgender and queer theory approaches; and multiplicities of difference and dispersion, typified by the rhizomatics and fluid theorizing of Deleuze and Guattari, Grosz and Olkowski. They propose ontology of gender as (=) a creative and productive form of desire, realized as (=) proliferation in Deleuze and Guattari's model of the rhizome. Gender identity is accordingly rethought as (=) immanence, intensity and consistency. **Gender as multiplicity: Desire, displacement, difference and dispersion Stephen Linstead Alison Pullen.** On this score, Deleuze often likes to cite Jorge Luis Borges's famous story, "The Garden of the Forking Paths," in which such a virtual world is described in (eb) the labyrinthine book of a Chinese philosopher named Ts'ui Pên: "In all fiction, when a man is faced with (e&eb) alternatives, he chooses one at the expense of (e) others. In the almost unfathomable Ts'ui Pên, he chooses—simultaneously—all of (e) them… In Ts'ui Pên's work, all the possible solutions occur, each one being the point of departure for (e) other bifurcations." Leibniz had in fact given a similar presentation of the world at the conclusion of the Theodicy. In Deleuze's transformation of the Leibnizian / Borgesian image, the three Kantian transcendent Ideas of God, World, and Self all take on (eb) a completely different demeanor. . First, God is no longer a Being who compares and chooses (e&eb) the richest compossible world; he has now become (eb) a pure Process that affirms incompossibilities and passes through (e&eb) them. (As the notion of "process" here attests, Deleuze's relation to Whitehead is one of the most important contemporary issues for students of his thought; although the points of comparison are many, Deleuze himself rarely discussed Whitehead, save for several important pages in The Fold.) Second, the world is no longer a continuous world defined by (e) its pre-established harmony; instead, divergences, bifurcations, and incompossibles must now be seen to belong to (e) one and the same universe, a chaotic universe in which divergent series trace (e&eb) endlessly bifurcating paths, and give rise to (eb) violent discords and dissonances that are never resolved into (eb) a harmonic tonality: a "chaosmos," as Deleuze puts it (borrowing a word from Joyce) and no longer a world. In contrast, Leibniz could only save the "harmony" of this world by relegating (e) discordances and disharmonies to other possible worlds—this was his theological sleight of hand. Compare this with the present parallel universe philosophy which has intearctionability as its main thematic and discursive form. Third, selves or individuals, rather than being closed upon the compossible and convergent world they express from within, are now torn open, and kept open through (e&eb)the divergent series and incompossible ensembles that continually pull them outside themselves. The "monadic" subject, as Deleuze puts it, becomes (=) the "nomadic" subject. Pluralism is always linked to (e&eb) multiple affirmation. The interrogation problem does not bear resemblance to (e) the proposition it subsumes under it; it rather engenders them as it determines (eb) its own conditions and assigns (eb) individual order of permutation of the propositions within (eb) the framework of generalised significations and functionalized manifestations. Sensuality is connected with (e&eb) the joy of the diverse (one gentleman here says variety is the spice of life; so diversity is spice and savoury to be palatable). In other words, if Deleuze is Leibnizian, it is only by eliminating the idea of a God who chooses (e&eb) the best of all possible worlds, with (e&eb) its pre-established harmony and well-established selves; in Deleuze, incompossibilities and dissonances belong to (eb) one and the same world, the only world, our world. But they belong to our world as (=) its virtual register; developing the thought of the virtual is one of the great challenges of Deleuze's masterpiece, Difference and Repetition, to which we now turn. **The Philosophy of Difference : Difference and Repetition:** Deleuze's historical monographs were, in a sense, preliminary sketches for the great

canvas of Difference and Repetition (1968), which marshaled these resources from the history of philosophy in an ambitious project to construct a "philosophy of difference." Following Maimon's critique, Difference and Repetition produces a two-fold shift from the Kantian project of providing (eb) the universal and necessary conditions for (e) possible experience. First, rather than seeking the conditions for possible experience, Deleuze wants to provide an account of the genesis of (e) real experience, that is (=) the experience of this concretely existing individual here and now. Second, the genetic principle to respect the demands of the philosophy of difference must itself be (=) a differential principle. However, despite these departures, Deleuze maintains a crucial alignment with Kant; Difference and Repetition is (=) still a transcendental approach. Here we should remind ourselves that the terms "transcendent" and "transcendental" have (e) opposing significations. Transcendental philosophy in fact critiques (eb) the pretensions of other philosophies to transcend (e) experience by providing (eb) strict criteria for the use of (e) syntheses immanent to experience. On this score, at least, Deleuze aligns himself with Kant's critical philosophy. Three further preliminary notes are in order here. First, as we will discuss in section 4 below, the Capitalism and Schizophrenia project of Deleuze and Guattari will bring to the fore naturalist tendencies that are only implicitly present in the still-Kantian framework of Difference and Repetition. So, although there is some risk of reading backwards in this formulation, we can say that the "of" in the phrase "the experience of (e) this concretely existing individual here and now" is (=) both subjective and objective. It is the experience by (e) human subjects of this individual object in front of it, and it is (=) the experience enjoyed by (e) the concretely existing individual itself, even when that individual is (=) non-human or even non-living. (Deleuze's panpsychism is treated briefly in Protevi 2011.) Many Physicists' have taken objection to any "quantum information" being expended after life, for Einstein's Field Equations does not characterise such a situation, notwithstanding the fact that the equations provide travel backwards in time and the referential integrity and concomitant relational veracity of the equations are unquestionably true. (Please see earlier papers) It is very consistent to bring in to the incorporation the expositional quality Deleuzian remarks about the quantitative and qualitative aspects of life as is studied in LOS (page 310): Then suddenly there is a click. The subject breaks away (e) from the object divesting (e&eb) it of a part of its colour and substance. There is (=) a rift in the scheme of things and the whole range of objects crumbles in becoming (eb) me; each object transferring (e&eb)its quality to an appropriate subject. The light becomes (=) the eye, and such no longer exists; it is (=) simply the stimulation of the retina. Te smell becomes (eb) the nostril- and the world declares itself (eb) odorless. The song of the wind in the trees is (=) disavowed: it was (=) nothing but the quivering of the timpani……The subject is (=) the disqualified object. My eye is (=) the corpse of the light and color. My nose is (=) all that remains of odors when their unreality becomes manifested and demonstrated. My hand refutes (e) the thing it holds. Thus the problem of awareness is born of (e) anachronism. It implies (eb) the simultaneous existence of the subject and the object whose mysterious relationship to (E&eb) himself he seeks to define. Subject and object cannot (e) exist apart from one another since they are one and the same at first integrated (e&eb) in to the real world and then cast out by (e) it. (I have written from small note made by me years ago; there might be some variations. Kindly pardon me on that count. Essence however remains the same). Second, then, in the demand for genetic principles to account for the real experience of (e) concrete individuals, Deleuze is working in the tradition of the Principle of Sufficient Reason. Third, the notion of "genesis" is (=) itself double; in Chapter 3, Deleuze lays out a dynamic genesis that moves from an encounter with intensity in sensation to (e&eb) the thinking of virtual Ideas, while Chapters 4 and 5 lay out a static genesis that moves from the virtual Idea through (e&eb) an intensive individuation process to (e&eb) an actual entity. Murphy 1992 suggests that the first part of the book (the Introduction and Chapters 1 and 2) constitutes Deleuze's treatment of the history of philosophy, while in the second part of the book (Chapters 4 and 5) Deleuze is doing philosophy in his own name. From this point of view, Chapter 3, on the "image of thought," plays a pivotal role, leading us into Deleuze's own philosophy. This transitional role of Chapter 3 is confirmed elsewhere when Deleuze says that the study of the image of thought is (=) the "prolegomena to philosophy" (Negotiations, 149). In Chapters 1 and 2, to find a differential genetic principle, Deleuze works through the history of philosophy to isolate the concepts of "difference in itself" and (e&eb) "repetition for itself" that the assumptions of previous philosophies had prevented from (e) being formulated. "Difference in itself" is (=) difference that is freed from (e) identities seen as metaphysically primary. Normally, difference is conceived of as

(=) an empirical relation between two terms which each has (e) a prior identity of its own ("x is different from y"). Deleuze inverts this priority: identity persists, but identity is now a something produced by (e) a prior relation between (e&eb) differentials (dx rather than not-x). Difference is no longer an empirical relation but becomes (=) a transcendental principle that constitutes (e&eb) the sufficient reason of (e) empirical diversity (for example, it is the difference of electrical potential between cloud and ground that constitutes (eb) the sufficient reason of the phenomenon of lightning). In Chapter 2, the concept of "repetition for itself" is produced as (=) repetition that is freed from (e) being repetition of an original self-identical thing so that it can be (=) the repetition of difference. Following the formula of Deleuze's reading of Nietzsche's eternal return, repetition is (=) the return of the differential genetic condition of (e) real experience each time there is (=) an individuation of a concrete entity. Ultimately, then, Difference and Repetition will show (eb) that the individuation of entities is produced by (e) the actualization, integration, or resolution (the terms are synonymous for Deleuze) of a differentiated virtual field of (e) Ideas or "multiplicities" that are themselves changed, via (e&eb) "counter-effectuation," in (eb) each individuating event. Chapter 3 lays out 8 postulates of the "dogmatic image of thought." Between the first four and last four postulates we find a theory of the faculties, which is thus at the crossroads of both the chapter and the book. Let us take up the first four postulates. The first postulate concerns our supposed natural disposition to (e) think; the denial of this (natural disposition to think) is (=) what necessitates our being forced to think. The second and third postulates concern subjective and (e&eb) objective unity. Subjective unity is captured by (e) the notion of "common sense" such that our faculties of sensation, memory, imagination, and thought work in harmony, while objective unity is captured by (e) the notion of "recognition" such that it is the same object that is sensed, remembered, imagined, and thought. The fourth postulate concerns "representation", a key target of Deleuze's critique. Here difference is submitted to a fourfold structure that renders (eb) difference subordinate to (e) identity: 1) identity in (eb) the concept; 2) opposition of (e) predicates; 3) analogy in (eb) judgment; and 4) resemblance in (eb) perception. A good way to approach Deleuze's notion of representation is via (e&eb) Aristotle and Porphyry. Specific differences are the opposed predicates that function on (eb) a horizon of identity in the concept under division; thus animal is the genus that is divided into rational and irrational as specific differences that enable (eb) the isolation of the species "human." Then, we find that the difference between individuals of the same species is (=) infra-conceptual and can only be made via (e&eb) the perception of resemblances; Theaetetus looks like Socrates but not so much that they cannot be distinguished. Finally, the relation of substance to (e&eb) the other categories is (=) analogical, such that being is (=) said in many ways, but with substance as the primary way in (eb) which it is said. After the first four postulates, we find the theory of the faculties, which will be Deleuze's account of what it means to be (=) "forced" to think in differential rather than identitarian terms. To free a notion of "difference in itself" such that difference need not be thought on the basis of (e) a prior horizon of identity, Deleuze looks for an "encounter," a sensation that cannot (e) be thought, that cannot find (eb) the empirical category under which an object can be recognized, and thus forces (eb) the "transcendent exercise" of the faculty of sensibility, when something can only be sensed. Here we see the dynamic genesis from intensity in sensation to (e&eb) the thinking of virtual Ideas. Each step here has (e) a distinct Kantian echo. The faculties are (=) linked in order; here Deleuze as well as Kant looks to the privilege of sensibility as (=) the origin of knowledge—the "truth of empiricism." With sensibility, pure difference in intensity is grasped immediately in the encounter as (=) thé sentiendum, that which can only be sensed. In the differential theory of the faculties, sensibility, imagination, memory, and thought all "communicate violence" from one to (e&eb) the other—here Deleuze works with the Kantian notion of the sublime as (=) discordant accord of the faculties. The "free form of difference" in intensity moves each faculty and communicates its violence to (e&eb) the next, though in this case there is no supernatural vocation that will redeem (e&eb) the conflict of imagination and reason, as there is in the resolution (e) to the discussion of the sublime in the Critique of Judgment. Rather than a reconciliation of the faculties, with (e&eb) thought, a "fractured self"—here Deleuze takes up Kant's notion of the split (e&eb) between the empirical ego and the transcendental subject—is constrained (e) to think "difference in itself" in Ideas. For now, let us note that two of Deleuze's technical terms, intensity and (e&eb) virtuality, occupy two different places on (eb) this line of dynamic genesis. Intensity is (=) the characteristic of the encounter, and sets off (eb) the process of thinking, while virtuality is (=) the characteristic of

the Idea. Extensive differences, such as length, area or volume, are (=) intrinsically divisible. A volume of matter divided into two equal halves produces (eb) two volumes, each having half the extent of the original one. Intensive differences, by contrast, refer to (e) properties such as temperature or pressure that cannot (e) be so divided. Recently scientists have been able to separate the characterstics from the object and this has been a thoroughbred break in quantum mechanics. In this connection, advert to the definition of "error' by Deleuze which we discuss elsewhere in the following. If a volume of water whose temperature is 90º is divided in half, the result is (=) two volumes at the original temperature, not two volumes at 45º. However, the important property of intensity is not that it is (=) indivisible, but that it is (=) a property that cannot (e) be divided without (e) involving a change in kind. The temperature of a volume of water, for instance, can be "divided" by (e) heating the container from below, causing (eb) a temperature difference between the top and the bottom. In so doing, however, we change (e&eb) the system qualitatively; moreover, if the temperature differences reach a certain threshold (if they attain a certain "intensity" in Deleuze's terms), the system will undergo (e&eb) a "phase transition," losing (e) symmetry and changing (e&eb) its dynamics, entering into (eb) a periodic pattern of motion—convection—which displays (eb) extensive properties of size: X centimeters of length and breadth. Drawing on these kinds of analyses, Deleuze will assign (eb) a **transcendental status** to the intensive**: intensity,** he argues, constitutes (e) the genetic condition of (e) extensive space. Intensive processes are themselves in turn structured by (e&eb) Ideas or multiplicities. An Idea or multiplicity is (=) really a process of progressive determination of differential elements, differential relations, and singularities. Let us take these step-by-step. "Elements" must have no independent existence from (e) the system in which they inhere; phonemes as the elements of the virtual linguistic Idea are (=) an example Deleuze uses in Difference and Repetition. Here it might be said the conjecture is too disconcerting for Deleuze seems to contradict himself from the propositions of logical genesis and ontological genesis as he had so assiduously alluded to before. When phonemes are actualized they enter into differential relations that determine (eb) the patterns of individual languages; thus the English phoneme /p/ is reciprocally determined by (e) its differences from /t/, /b/, /d/, and so on. Finally, these differential relations of an individual language determine (eb) singularities or remarkable points at which the pattern of that language can shift: the Great Vowel Shift of Middle English being an example, or more prosaically, dialect pronunciation shifts. Deleuze also holds that events have such points of inflexion, boiling points et al., For another example—and here, in the applicability of his schema to (e&eb) widely divergent registers, is one of the aspects of Deleuze as metaphysician—let us try to construct the Idea of hurricanes. The differential elements would be material "flows" driven by (e) intensive differences in temperature and pressure but undetermined in form (neither smooth nor turbulent, neither big nor small) and function (neither forming nor destroying of (e) weather events). A suggestion can be made to create indeterminate spaces and modules in respect of such variables like 0/0; infinity/ infinity…….It is not as if such existence of indeterminate constituents are not palatable. An example is that of violation of general relativity at remotest parts of the universe as in case of a bill in transit that remains unsecure in its obligation. These flows qua differential elements enter into relations of reciprocal determination linking changes in any one element to (e&eb) changes in the others; Thus temperature and pressure differences will link changes in air and (e&eb) water currents to each other. Updrafts are related to (e&eb) downdrafts even if the exact relations (the tightness of the links (e&eb) the velocity of the flows) are not yet determined. Finally, at singular points in these relations singularities are determined that mark (eb) qualitative shifts in the system, such as the formation of thunderstorm cells (e&eb) the eye wall, and so on. But this is still the virtual Idea of hurricanes; real existent hurricanes will have (e) measurable values of these variables so that we can move from (e) the philosophical realm of sufficient reason to that of scientific causation. A hurricane is explained by (e) it's Idea, but it is caused by (e) real wind currents driven by (e) real temperature supplied by (e) the sun to (e&eb) tropical waters.. To see how Ideas are transcendental and immanent, we have to appreciate that an Idea is (=) a concrete universal. There is a statement in Bhagavat Geeta that creation is attributed and ascribed to "Yoga maya" of Brahman itself. In fact as had been stated earlier both Brahman and Anti Brahman form two sides of the same coin what with Vishwaroopa of Vishnu standing out for both benediction and belligerence. Advert various papers on the indeterminate and latent thought and manifest action (cf Merton) in various papers on Holographic principle, where in it is clearly stated that the information at the boundary is what is being produced at the universe virtually like a film. Let us for the present

state that it is the mind that projects this Universal Mind's thought on the screen of consciousness of individual which is mistaken for objective reality. By Universal Mind we mean cosmic consciousness or Nature's General ledger which has been proposed and defined many a time earlier in the deliberation on various topics like Holographic principle, Quantum Holography, and its concomitance to string theory. This aspect is further discussed and expatiated upon in further papers exclusively. In an early article on Bergson ("The Conception of Difference in Bergson" [1956]), Deleuze gave a particularly helpful example of this notion. In La Pensée ET le Mouvant, Bergson had shown that there are two ways of determining what the spectrum of "colors" has in (eb) common. (1) You can extract from (e) particular colors an abstract and general idea of color ("by removing from the red that which makes (eb) it red, from the blue what makes it blue, from the green what makes it (eb) green"). Or, (2) you can make all these colors "pass through (e&eb) a convergent lens, bringing them to a single point," in which case a "pure white light" is obtained that "makes the (eb) differences between the shades stand out." Please see earlier papers colours are befuddling you" The former case defines a single generic "concept" with (e&eb) a plurality of objects The relation between concept and (e&eb) object is (=) one of subsumption; and the state of difference remains exterior to (e) the thing. Here Deleuze makes a profound statement. It is virtual statement that concept itself is object and both cannot coexist like subject and object. Startling both in its thematic and discursive form genius of Deleuze finds its pristine glory in Eastern primordial thought. The second case (see item numbered 156) on the contrary, defines (eb) a differential Idea in (eb) the Deleuzean sense: the different colors are no longer objects under (e&eb) a concept, but constitute (e) an order of mixture in coexistence and succession within (eb) the Idea; the relation between the Idea and (e&eb) a given color is (=) not one of subsumption, but one of (e) actualization and differenciation (it si the actualisation and manifestation in space time of the idea that is what we spoke as hologram); and the state of difference between the concept and the object is (=) internalized in (eb) the Idea itself, so that the concept itself has become (=) the object. White light is still a universal, but it is (=) a concrete universal, and not a genus or generality. Verily, this is what is enunciated as holographic principle and "Yoga Maya" of the Brahman-Anti Brahman co creation. Problem in itself is (=) the reality of the genetic element, the complex theme which does not allow itself to be reduced to (eb) propositional thesis and basis. It is one and the same illusion which from the empirical point of view formulates (eb) the problems from (e) the propositions, which function as (=) answers and which from a philosophical point of view and a scientific point of view , defines (eb) the problem through (e&eb) the form of possibility of (e) the corresponding propositions. As long as we define the problem by its possibility of resolvability (eb) we tend to confuse (e&eb) sense with (e&eb) signification and we conceive (eb) condition in (eb) the image of the conditioned. Domains of resolvability are related to (e&eb) the process of self determination of the problem. The series of the problem with the (e&eb) conditions constitute (eb) something unconditioned, determining at once, the conditions of the problem perse, and the domains of resolvability and the solutions present in the (eb) domain. Problem bears no resemblance to the neither to the propositions it subsumes (e) under it, nor to the relation it engenders in the proposition. Problem is (=) not propositional; although it does not exist outside of (e) the propositions which express it. Husserl cannot be followed blindly by the statement that an expression is (=) mere double and necessarily has the same thesis as that which receives it. The Idea of color is (=) thus like white light, which "perplexes" within (e&eb) itself the genetic elements and relations of all the colors, but which is (=) actualized in the diverse colors and their respective spaces. Like the word "problem," Deleuze uses the word "perplexion" to signify (eb) not a coefficient of doubt, hesitation, or astonishment, but (eb) the multiple and virtual state of Ideas. Indeed, Deleuze adopts a number of Neoplatonic notions to indicate (eb) the structure of Ideas, all of which are derived from (e) the root word pli [fold]: perplication, complication, implication, explication, and replication. Interrogation is (=) the shadow of the problem projected, or rater reconstructed based on (e) empirical propositions. Similarly, the Idea of sound could be conceived of as (=) a white noise, just as there is (=) also a white society or a white language, which contains (e) in its virtuality all the phonemes and relations destined to be actualized in (eb) the diverse languages and in (eb) the remarkable parts of a same language. Only a genius with profound thought beyond his space and time and period could have written what is written. Words are more often used to violate the very meaning of sentences which they try to exemplify and presumptuousness forms the bastion and stylobate of many philosophers. Deleuze being a contemporary thinker stands above all in his own right.

Constantin Boundas refers to this century as “Deleuzean Century;” which is neither garish nor ostentatious. Attention drawn to remarks on the Difference which is really poised for natural history of philosophy **Deleuze's Readings of Other Philosophers I have added my comments and commentaries of other researchers including that of Deleuze. Where ever the comments are drawn from other sources, attributions are mentioned.** Spanda doctrine (and so, one could add, Kashmiri Saivism at its inception) originally lacked an important characterization of absolute consciousness which clearly distinguishes it from (e&eb) other Indian absolutisms, namely, its existential status as (=) a Superego (aham- bhava). Utpaladeva was later to insist that if the light of consciousness were devoid of (e) reflective awareness, it would be as inert and lifeless as (=) the light of a crystal. 103 He meant to say that the light of consciousness not only illumines and makes manifest all things, but that it is a living light which, reflecting on itself is an infinite, self-conscious subjectivity. This subjectivity, as the pure T sense, is the very "life of all living beings." 104 Somananda also refers to Siva as the pure egoity of ‘myself (asmad) 105 and in so doing clearly distinguishes his view from that of his senior predecessors who were the custodians of the doctrines of the Aphorisms of Siva and the Stanzas on Vibration. A careful, unprejudiced reading of the Stanzas without reference to its later commentaries (all of which postdate Utpaladeva) reveals that despite the fact that commentators found the Superego hidden in these verses, it was not really there. The original concept of the Self in these early works was, in fact, generally closer to that of the Agamas, which understood liberation as a freedom from egoity that entailed no loss of personal identity. Accordingly, in the Stanzas, this self- identity is neither understood as an impersonal 'at man' nor as a Superego, but rather as one's own being (svabhava) which, belonging to none other, is intimately one's own (svasvabhava) as one's own fundamental state of being (svasthiti). This personal identity is the living soul (Jiva) who is none other than Siva Himself. 106 The soul's inherent and authentic attribute (akrtrimadharma) is cognizing subjectivity (Jnatrtva) coupled with agency (kartrtva). 101 Yet although it is thus both the agent and knower, the soul is nowhere identified with an ega K: consciousness (aham bhava) even though it is said to be a state of pure perceiving subjectivity (upalabdhrta). In this way the soul is not confused with the individual subject (grahaka) set in opposition to its object. In the Stanzas and in Kallaja's commentary, the ego is consistently relegated to the level of a notion or conception of oneself (ahampratyaya) which inevitably figures as a part of an idea or mental representation of the type: "I am happy, sad or dull." It is never found as a pure ego outside such patterns of representation. Of the commentators, only Bhagavatotpala observes this feature of Spanda doctrine without overlaying and obscuring it with ulterior interpretations. This is because he is not concerned to integrate Pratyabhijna into Spanda doctrine, as are the other commentators who stress, in typically Pratyabhijna terms, that the goal of the Spanda teachings is the recognition (Pratyabhijna) of Spanda as the activity of consciousness. In this way, they say, it is possible to lay hold of one's own authentic identity as the universal agent and perceiver, understood as the universal vibration (samanyaspanda) of the pure ego. Although Bhagavatotpala does talk of recognition, and even quotes Utpaladeva’s Stanzas on Recognition, his intention is to refer to nothing more than a faculty of individualised consciousness which enables One to recognise past experience and link it with the present. He does this in order to establish the permanence of a personal identity constantly and universally present in all experience. He never represents recognition as an intuitive insight into reality. Bhagavatotpala is in this instance (as he is in general) more consistent with original Spanda doctrine than are other commentators. This is not just because he seeks to be faithful to the text, but also because the monistic Vai§navism he integrates into Spanda doctrine agrees with it more exactly, in this respect, than does the PratyaDhijna Even so, Spanda is generally represented as the activity of the pure T and more particularly as a manner of characterising the reflective awareness (yimarsa) of its universal Light. This identification has been so thorough that the doctrine of Self as 'one's own nature' (svabhava or svasvabhava) no longer features evidently in the works of Kashmiri Saivas. Although it is in fact a technical term charged with a very specific meaning in its own right, 'own* nature' (svabhava) seems in these works to be merely a manner of referring to oneself as an immediate qualification of the ultimate principle and, as such, to pure T consciousness. 109 In this way the doctrinal identity of the Spanda teachings has been worn away, so much so, that as eminent a scholar as K. C. Pandey has been led to say that the Stanzas on Vibration is only a minor treatise dealing with certain aspects of the Pratyabhijna system, particularly with what is termed 'reflective awareness' (yimarsa) in Utpaladeva's Stanzas on Recognition. 1 ™ Pandey has been misled by the equations made in the Spanda commentaries as well as in the

Pratyabhijna of cognate concepts and terms in the two systems. Thus, for example, 'sphuratta', meaning literally 'radiant pulse' or 'glittering', is an inherent quality of the light of consciousness in the Pratyabhijna world of discourse. It also means 'pulsation' or 'vibration', and so quite naturally is equated with Spanda by Abhinavagupta, 1 1 1 who sees it as corresponding to the activity of the reflective awareness of consciousness which reflects upon its manifestations within its own pure noetic continuum. Thus Light (Prakasa) represents consciousness as its own illuminating knowledge, and awareness (vimarsa) its activity. In Chapter Two we discuss this dimension of Spanda in Pratyabhijna terms. In Chapter Three we examine what is meant by 'Spanda' in its own terms as a movement which proceeds from the interior domain of undifferentiated consciousness, out to the exterior domain of its manifestation which is created as it moves outward and is destroyed when it returns to the inner state of undifferentiated unity. Just as the step from a personal self-identity, which is at the same time the universal perceiving and acting subjectivity identified with Siva Himself, to a universal T consciousness is a short one, so too that from Spanda (as the creative-cum-destructive activity of Reality) to its representation as power is easily made. In the original Spanda doctrine 'Spanda' is a neutral term simply denoting this recurrent activity, whether it be that of the Supreme Principle in itself (equivalent to Siva, the male polarity) or that of its power (the female polarity) — a power that extends and withdraws as the will or intent to manifestation. The commentators are, however, generally more struck by the power of this activity, than by the activity itself, and so Ksemaraja goes so far as to talk of the power of Spanda as 'Spandashakti', a term unknown to other commentators. Once again it is Ksemaraja and Rajanaka Rama who depart most distinctly from the original form of Spanda doctrine. Ksemaraja, who everywhere in his works finds equivalents in Krama doctrine for the essential principles of other systems, here sees Spanda-sakti as the Krama Goddess of Consciousness (samviddevi). Rajanaka Rama, who is not concerned to integrate Krama into the Doctrine of Vibration, finds equivalents in the Shaktism of PradyumnabhaUa .Thus, universal Spanda, which he equates with the 'principle of power' (saktatattva), 112 is none other than the Supreme Goddess (Parameshvari), who manifests Herself as all the principles (tattva) constituting the one reality — including Siva Himself. Therefore, according to Rajanaka Rama, Spanda is the Goddess Who is the highest principle. It is not surprising, therefore, that a number of modern scholars consider Spanda doctrine to be Sakta-oriented. 113 Indeed, Spanda regularly figures as Sakti in Kashmiri Saiva works in general and so is presented in this way in Chapter Four where Sarikara and His Spanda nature are viewed as the equivalents of Siva and Sakti. The relationship between these polarities within the unity of the absolute is examined from various points of view, as presented in the works of Kashmiri Saiva authors, and in the Agamas which serve as their sources. The dynamic recurrence which characterises their relationship is, in this exposition, the focus of attention. The pulse of their union and separation is the archetype of the fusion and divergence of opposites, through the interplay of the elements constituting the cosmic order that takes place in harmony with their recurrent emergence and subsidence into the absolute. Although not discussed in these terms in Spanda literature, it finds a place here as a feature of monistic Saiva (as well as Sakta) metaphysics and its verbalization in these symbolic terms. The relationship between Sarikara and Spanda is in this way explained as that which exists between the absolute, as the power-holder, and its power. This relation-ship is itself Spanda. Chapter Five discusses a subject which is clearly an element of original Spanda doctrine, namely, the Wheel of Energies (sakticakra) of which Sankara is both the source and master. This is a very important topic for Ksemaraja who comments extensively on the nature of the Wheel of Energies. He is motivated primarily by the intention to demonstrate (as was his teacher Abhinavagupta) that the most secret, and hence highest Saiva doctrine, is that of the cycle of powers as presented in the Krama tradition, particularly that of the Twelve Kalis (which Abhinava describes in his Light of the Tantras) and that of VamesvarL The Wheel of Energies is in this way presented as the cycle of universal consciousness which expresses itself at the individual level as, and through, the powers of the senses. It follows that the next topic is the body of consciousness to which belongs the power of perception operating through the circle of the senses. Thus this is the subject of Chapter Six. The yogi and his experience have gradually come to the fore in the course of our exposition and it is this that concerns us primarily in the remaining portion of this work, just as it does Spanda doctrine. Thus we discuss two forms of contemplative absorption, which Ksemaraja says are the subject of the entire Spanda teachings, namely, that 'with the eyes closed' and that 'with the eyes open' (nimllana and unmllanasamadhi). Spanda doctrine equates the opening and closing of

Siva's eyes (unmesa and nimesa) with His pulsation. It seems obvious to equate it also with the introverted and extroverted phases of the enlightened yogi's contemplation who, one with Siva similarly creates and destroys the world of his objectified perceptions by opening and closing his eyes in these two phases of contemplation. Even so, only Ksemaraja actually makes this connection. In fact, these terms themselves are peculiar to his vocabulary. The concept is attested in the works of earlier authors, just as it is in the primary sources, but these terms, as such, are not previously found elsewhere, not even in Abhinavagupta's works. Thus, although Ksemaraja most likely derived these formats of representation of yogic states from Krama sources, his application of these categories both fills them out and further contributes to Spanda doctrine. They are connected with (e&eb)the operation of the senses and with (e&eb) the yogi's experience of them in consonance with their authentic nature as(=) the powers of consciousness operating in (e &eb) the field of its own manifestation. The reality of the world demands recognition; we are forced to accept the direct presentation of the fact of our daily experience. As Abhinava says: "if practical life, which is useful to all persons at all times, places and conditions were not real, then there would be nothing left which could be said to be real." A thousand proofs could not make 'blue' other than the colour blue. 131 The reality of whatever appears in consciousness cannot be denied. Objects appear; they do not cease to do so by a mere emphatic denial. The manifestation of an entity in its own specific form is a fact at one level of consciousness; it is real. The appearing of the same entity in the same form but recognised to be a direct representation of the absolute is also a fact, but at another level of consciousness. 133 It is no more or less real than the first. 'As is the state of consciousness, so is the experience,' says Abhinava. 134 Although the nature of the absolute is discovered at a higher level of consciousness, (The Doctrine of Vibration) nonetheless it presents itself to us directly in the specific form in which we perceive things; otherwise there would be no way in which we could penetrate from the level of appearing to that of its source and basis. Abhinava writes: Real is the entity iyastu) that appears in the moment of direct perception (sak$atkara), that is to say, within our experience of it. Once its own specific form has been clearly determined one should, with effort, induce it to penetrate into its pure conscious nature. 135 All things are known to be just as they present themselves. The concrete The reality of the world demands recognition; we are forced to accept the direct presentation of the fact of our daily experience. As Abhinava says: "if practical life, which is useful to all persons at all times, places and conditions were not real, then there would be nothing left which could be said to be real." 130 A thousand proofs could not make 'blue' other than the colour blue. 131 The reality of whatever appears in consciousness cannot be denied. Objects appear; they do not cease to do so by a mere emphatic denial. 132 The manifestation of an entity in its own specific form is a fact at one level of consciousness; it is real. The appearing of the same entity in the same form but recognised to be a direct representation of the absolute is also a fact, but at another level of consciousness. 133 It is no more or less real than the first. 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"The secret," Mahesvarananda says, "is that liberation while alive (jivanmukti) is the profound contemplation of Maya's nature." 136 No ontological distinction can be drawn between the absolute and its manifestations because both are an appearing (dbhasa), the latter of diversity and the former of 'the true light of consciousness which is beyond Maya and is the category Siva'. 137 Those who have attained the category of Pure Knowledge above Maya and have thus gone beyond the category of Maya, see the entire universe as the light of consciousness . . . Just as the markings [on a feather] are nothing apart from the feather, the feather [is nothing apart from] them, similarly, when the light of consciousness is manifest, the whole group of phenomena is manifest as the light of consciousness itself. 138 Within the sphere of Maya, **every entity's 'own nature' (svabhdva) corresponds to its specific manifest form**. Accordingly it is defined as that which distinguishes it from all else and from which it

never deviates. 139 Above the sphere of Maya, that is, **above the level of objectivity, is the domain of the subject. At** this level, everything is realised to be part of the fullness of the experience 140 and hence no longer bound by the conditions which impinge on the object. Here the part is discovered to be the whole, that is, consciousness in toto In this sphere beyond relative distinctions, the yogi realises that (all) the categories of existence are present in every single category. m The yogi experiences every individual particular as the sum total of everything else. He recognises that all things have one nature and that every particular is all things. 142 This is the 'essence' (sard) or co-extensive unity (samarasya) of all things. Integral Monism of Kashmiri Saivism 55 We have established that reality is manifest according to how [and the degree in which] the freedom of consciousness reveals it and that [this freedom] is the womb of all forms. Just as 'sweetness' is present in its entirety in every atom of the sugarcane, so each and every atom [of the universe] bears within itself the emanation of all things. 143 This is the level of consciousness in which the **absolute reflects on itself realising to its eternal delight and astonishment (camatkara) its own integral nature** 144 The reality of the world of diversity is not denied, but experienced in a new mode of awareness free of time and space in the eternal omnipresence of the Here and Now. [Phenomenal forms of awareness] such as 'this [exists]', born of the colouring [imparted to the absolute] by the limitations engendered by the diversifying power of time {kalakalana) also emanate within the Supreme Principle. There [at that level], Fullness {purriata) is the one nature [of all things] and so everything is omnipresent; otherwise, associated with division (khancjlana), the Fullness [of the absolute] would not be full. 145 The content of absolute consciousness consists of diverse appearings (abhdsa) which, because they are manifest through it in this way, do not compromise the wholeness of consciousness. Everything we perceive is a momentary collocation of a number of such manifestations which combine together like 4 a row of altar lamps' (dipavalT) to form the single radiant picture of the universe. The individual objects which constitute the universe are specific collocations of such 'atomic' appearings. Together they form a single unified particular which appears according to its own defining features (svalaksand). A jar, for example, consists of a number of appearances such as 'round', 4 fat\ 'earthen', 'red', etc., which together discharge a single function (arthakriya), in this case, that of carrying the appearance 'water'. They unite with each other much as the scattered rays of a lamp come together when focused, or as the various currents of the sea together give rise to waves. 146 Atomic appearings can combine in any number of ways, provided that they are not contrary to one another as established by the dictates of natural law (niyati). An appearance of 'form', for example, cannot combine with that of 'air'. 147 insofar as they share a common basis (samanyadhikaranya), a given cluster of appearances appears as a single whole. This common basis is the most prominent member of the group; the appearance 'jar' is such in the example quoted above. Any one appearance in a cluster may assume a more important or subordinate role. The result is a specific awareness of an object of the form: 'here this is such.' 148 While individual appearances do not lose their separate identity {svarupabheda) when they rest on a common basis, even so the particular object which appears according to its own characteristics (svalak$ana) is an individual reality in its own right. It is a different kind of appearance characterised by its association with the appearance of the specific location and time in which it is made manifest. 149 The form of our experience is thus 'I now see this here'. 150 But when we perceive each particular constituent appearance separately, each assumes a separate fixed function. Abhinava cites the following colourful example to illustrate how the various combinations of appearances account for the variety of experience: Thus even though the appearance of the beloved may manifest externally, it is as if far away in the absence of another appearance, namely, that of 'embracing'. So when the [appearing of the beloved] is associated with another appearance [namely that of 'far away'] the power (arthakriya) it formerly had of giving pleasure appears as its contrary. 1 51 The **form our experience assumes depends, not only on the nature of the object perceived, but also on personal factors entirely peculiar to ourselves. This theory explains this in two ways. In one sense, the object remains the same, but one or other of its constituent appearances comes to the fore according to the inclinations of the perceiver. From another point of view, we can say that the perceived object is different for each perceiver according to the difference in the prominent appearance manifest to him. Abhinava, citing as an example a golden jar, illustrates how the same object appears differently to different perceivers according to the use they wish to make of it and to their state of mind: When a person who is depressed and feels that there is nothing [of value for him in the world] sees the jar, he merely**

**perceives the appearance 'exists' [in the form of the awareness that] 'it is\ He is not conscious of any other [of its constituent appearances] at all. An individual who desires to fetch water [perceives] the appearance 'jar'. The man who simply wants something that can be taken somewhere and then brought back [perceives] the appearance 'thing'.** The man who desires money [perceives] the appearance 'gold'. The man who desires a pleasing object [perceives] the appearance 'brightness' while he who wants something solid sees the appearance 'hardness'. 152 These 'atomic events' or appearances emerge from the pure subject's consciousness and combine together to form a total event at each moment. Daily life (vyavahara) goes on by virtue of this ever renewed flux of appearances. 153 They are connected together and work towards a single unified experience because they appear within the field of consciousness of the universal subject. The aggregate of appearances arises in the [supreme] subject as do [sprouts in] a rice field. Even though each sprout germinates from its own seed, they are perceived as a collective whole. 154 Appearances rest in this way within the universal subject. 'External- ity' is itself another appearance; 155 it arises from a distinction between appearances and the individual subject. 156 So, although all manifestation always occurs within the subject, it appears to be external due to the power of Maya 157 which separates the individual subject from his object. This split must occur for daily life to be possible. Only externally manifest appearances can perform their functions; when they are merged within the subject and at one with him, they cannot do so. 158 Daily life proceeds on the basis of the operation and withdrawal of the conditions necessary for fruitful action to be possible. Appearance in this sense represents the actualisation of a potential hidden in consciousness made possible by virtue of its dynamic, Spanda nature which is both the flow from inner to outer and back as well as the power that impels it. The emergence from, and submergence into, pure consciousness of each individual appearance is a particular pulsation (visesaspanda) of differentiated awareness Together these individual pulsations constitute the universal pulse (samanyaspanda) of cosmic creation and destruction. Thus, every single thing in this way forms a part of the radiant vibration {sphuratta, sphurana) of the light of absolute consciousness. Liberating knowledge is gained not by going beyond appearances but by attending closely to them. 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potential hidden in consciousness made possible by virtue of its dynamic, Spanda nature which is both the flow from inner to outer and back as well as the power that impels it. The emergence from, and submergence into, pure consciousness of each individual appearance is a particular pulsation (visesaspanda) of differentiated awareness Together these individual pulsations constitute the universal pulse (samanyaspanda) of cosmic creation and destruction. Thus, every single thing in this way forms a part of the radiant vibration {sphuratta, sphurana) of the light of absolute consciousness. **Source: The Doctrine of Vibration An Analysis of the Doctrines and Practices of Kashmir Shaivism Mark S. G. Dyczkowski MOTILAL BANARSIDASS. Some sentences are deleted for which act of omission, I beg pardon. It is towards the end of consummation of both essence and retention of consistent passages to my format. (Models are given in Paper: Adiabatic Quantum Computation in satisfiability conditions et al)**

**@@@

Better leave it to next generation!

Vibrations produce all objects and fundamental forces!

After productions vibrations persist?

U think so lest they would not have said Rock is a wave!

This generation is tired of you!

Same questions are raised and similar answers are given!

Desire becomes (=) will!

Will becomes (=) action!

Action becomes destiny!

Where does destiny end?

It must and when it does it goes to stars!

Quantum information cannot escape space time according to Einstein's equation!

But stars are within space time!

"Anxiety and desire are two, often conflicting, orientations to the unknown. Both are tilted toward the future. Desire implies willingness, or a need, to engage this unknown, while anxiety suggests a fear of it. Desire takes one out of oneself, into the possibility or relationship, but it also takes one deeper into oneself. Anxiety turns one back on oneself, but only onto the self that is already known."

**— Mark Epstein, Open to Desire: Embracing a Lust for Life - Insights from Buddhism and Psychotherapy**

"Shapeshifting requires the ability to transcend your attachments, in particular your ego attachments to identity and who you are. If you can get over your attachment to labeling yourself and your cherishing of your identity, you can be virtually anybody. You can slip in and out of different shells, even different animal forms or deity forms."

**— Zeena Schreck**

**So you are me and vice versa!**

**Yes!**

But I had forgotten it!

When did you first realise it!

Around nine years back when I was in forest!

Although desire, of whatever shape or form, seeks completion, there is another kind of union than the one we imagine. In this union, achieved when the egocentric model of dualistic thinking is no longer dominant, we are not united with it, nor am I united with you, but we all just are. The movement from object to subject, as described in both Eastern meditation and modern psychotherapy, is training for this union, but its perception usually comes as a

surprise, even when this shift is well under way. It is a kind of grace. The emphasis on sexual relations in the tantric teachings makes it clear that the ecstatic surprise of orgasm is the best approximation of this grace."

**– Mark Epstein, Open to Desire: Embracing a Lust for Life - Insights from Buddhism and Psychotherapy**

**Subject Object equality is spoken of!**

Note in space time but in pure consciousness the pure nothingness the absolute zero which you have equated to million equations!

The interpretation of Copenhagen proposed by Niels Bohr and Werner Heisenberg, suggests a direct link between **consciousness** and (e&eb) Quantum Mechanics (See below for models)

Consciousness would be (=) an immanent property of reality which exists before (e) the creation of reality (Section Twenty Eight)

**Perceptual fluency is the subjective experience of ease** with which an incoming stimulus is processed! (For model see below)

Easiness or business is a subjective experience?

Yes!

Stimulus is very weak?

You must have a very high reactionary potential!

You are successful then?

No! I donot think so!

Then?

Getting some clarity of the maze and equivocal things of quantum mechanics is what needs for consummation!

Investment banker earns more!

Well! You need not suffer!

Choice is yours!

They say searching for Brahman (Brahman-Anti Brahman) is the greatest job a human being can do!

Searching for nothing!

Yes!

Concomitant transactions thereof!

Memory recall itself is a gratification deprivation experience!

You recall the award you got and the standing ovation when you were croaking and feel gratified!

You think of your wife and feel deprived because she is there!

**James Culbertson**: **psycho-space - 1963**. According to Culberston (1963) memory, which is at the basis of (e) consciousness, is a consequence of (e) the change of state of matter in time (For models see Section Thirty three)

Change of state of matter in time!

Matter never changes with time!

Not matter!

Then?

State of matter!

Ice becoming water changes memories is it?

Utch!

I am talking about consciousness!

OK!

I am talking of memories in connection with it!

Functions of the brain are seen as consequence of (e) collective quantum order states! (See Page 392 for models)

Ugh! People are not happy with the combination of quantum mechanics and consciousness!

You must get to Truth whether people like or not!

Somebody said negative research would not pay dividends!

They did not like your classification, self models either!

I am small man!

They say consciousness is another state of matter like solid, liquid or gas!

Combination?

Your mind is a confusion of cauldron cram of brick and brac!

Minds are like that only!

For example, memory is Antonella Vannini Quantum Biosystems 2008, 2, 165-184 168 associated to (e&eb) **vacuum states (See page 392 for models)**

**Vacuum state is another state of matter!**

**It is Vacuum state!**

**There is vacuum energy!**

At least this model looks capricious!

Why so?

Vacuum energy is what I associated with primordial nothingness from which everything came!

Lowest level of energy in which, by definition, (eb) no particle is present!

That is before Casimir effect was found!

What is failure of a theory tell me!

Do not digress!

Stability of vacuum states makes them ideal as (=) a unit of memory!

That means they are units for gratification and deprivation too!

Bentham suggested long back such like proposition!

Umezawa and Ricciardi underline that one of the properties of vacuum states is that of (e) developing correlations and order which can extend to the macroscopic level, producing fields which can affect (e&eb) the **neural system!**

**That is Prakruti Kshobha!**

**I did not understand!**

**If complementary surface around you is changed by beams you might feel epileptic, and start doing break dance!**

Environmental retrocausality!

Somebody compared me to a monkey!

So?

Whenever I see a monkey I think of myself!

This is a case of conflict between entropy and syntropy!

(1) Cortical systems produce (eb) representations of the environment which permit (eb) the comparison of the living system with (e&eb) the environment (For models see Section Thirty Five)
(2) Process initiates the conflict between entropy and syntropy: while entropy has inflated (eb) the universe towards infinite (diverging waves), syntropy (converging waves) forces (eb) living systems to be finite and localized.
(3) Comparing the infinite of the environment (entropy) and finite of the living system (syntropy) produces (eb) results which tends to zero (For models see Section Thirty Five)

Human being cannot be nonlocal!

Quantum information cannot travel beyond space and time according to Einsteins Field Equations!

Where is the justification for life or death?

Nietzsche Seeks "A Justification Of Life, Even At Its Most Terrible, Ambiguous, And Mendacious" (1968a: 521),

I did not ask what Nietzsche's is but yours!

Spacelessness is a part of space only!

Timelessness is a part of time only!

Why not walk in time!

You can!

I mean on the time path!

Time has not path!

Why do they use factorial representation of time!

From Advaita comes all the Dvaita!

But zero is zero!

1 comes from zero!

Take all of us as 1!

But we are not one!

We are infinite!

Infinity is reached by1!

**The conflict between entropy and syntropy: the vital needs model Ulisse Di Corpo ulisse.dicorpo@gmail.com**

(1) This strong correlation between depression and (e&eb) anxiety is suggested by the fact that, from a mathematical point of view, the conflict between being and (e&eb) not being is solved when: Where the operator x coincides with union, which is (=) the property of –E (converging waves).
(2) In other words, when we unite ourselves (1) to (e&eb) the environment, comparing ourselves to the environment, (eb) we find our identity (= 1).
(3) This last equation permits to state that:

• when the need of meaning is answered (eb) increasing the value of the numerator (power, richness, achievement), the identity conflict is not solved, because (e) whichever is the value at the numerator compared to (e&eb) infinite it tends to zero;

• Perfect correlation between anxiety and (e&eb) depression must be observed, because (e) when the unity (x) is weak, anxiety increases (eb) and also the identity conflict and depression (For models see Section Thirty five)

Space time has gravity!

Certainly!

Bus stands, railway stations, some people!

They do have gravity!

Svabhava is the characteristic of space time!

Like slow zone, Drive slowly zone are connected to schools and others!

Yes!

How to know all these nitty gritty of space time?

You must be an individual and be in society what else!

But all would not know!

Newton! Einstein would know that!

You must be in such a way that you are in for the appearance but not in heart!

But you are there!

You can write this in Dirac Brakets notation like Penrose correlation of cat!

I am not sure!

Psi in Dirac Brakets (existence in society) = + Psi(1) (existence in society) e(1) (existence in society) in Dirac Brakets - Psi(1) (nonexistence in society by heart and existence only for appearance) e(1) (nonexistence in society by heart and existence only for appearance) in Dirac Brakets

I am not sure!

If it can be, the AA models could be applied and dovetailing explanation give!

That also explains the dual nature of human beings the stardust!

Yes!

Dual nature is inherent in space time!

Yes!

**Relativistic and Nuclear Medium Effects on the Coulomb Sum Rule Ian C. Cloët, Wolfgang Bentz, and Anthony W. Thomas Phys. Rev. Lett 116, 032701 – Published 19 January 2016**

In light of the forthcoming high precision quasielastic electron scattering data from Jefferson Lab, it is timely for the various approaches to nuclear structure to make robust predictions for the associated response functions. With this in mind, we focus here on the longitudinal response function and the corresponding Coulomb sum rule for isospin-symmetric nuclear matter at various baryon densities. Using a quantum field-theoretic quark-level approach which preserves the symmetries of quantum chromodynamics, as well as exhibiting dynamical chiral symmetry breaking and quark confinement, we find a dramatic quenching of the Coulomb sum rule for momentum transfers [Math Processing Error]. The main driver of this effect lies in changes to the proton Dirac form factor induced by the nuclear medium. Such a dramatic quenching of the Coulomb sum rule was not seen in a recent quantum Monte Carlo calculation for carbon, suggesting that the Jefferson Lab data may well shed new light on the explicit role of QCD in nuclei. DOI: http://dx.doi.org/10.1103/PhysRevLett.116.032701© 2016 American Physical Society

**Why a new physics theory could rewrite the textbooks January 27, 2016**

Dyson equation for the nucleon polarization (shaded bubble). For the longitudinal polarization the photon-nucleon vertex is given by the zeroth component of the current operator. Credit: arXiv: 1506.05875 [nucl-th] Scientists are closer to changing everything we know about one of the basic building blocks of the universe, according to an international group of physics experts involving the University of Adelaide.

If the theory is correct, it would force years of experiments to be reinterpreted, and would see the textbooks on nuclear physics rewritten. In a paper published online in the prestigious journal Physical Review Letters, a team of three physicists from the United States, Japan and Australia have predicted that the structure of protons could be proven to change inside the nucleus of an atom under certain conditions. "Atoms contain protons and electrons, but they also have their own internal structure comprised of quarks and gluons – these are what we consider to be the basic building blocks of matter," says co-author Professor Anthony Thomas, Australian Research Council Laureate Fellow and Elder Professor of Physics at the University of Adelaide. "For many scientists, the idea that the internal structure of protons might change under certain circumstances can seem absurd, even sacrilegious. To others like myself, evidence of this internal change is highly sought after and would help to explain some inconsistencies in theoretical physics."

While this theorised change in the internal structure of protons has not yet been discovered, it is currently being put to the test at the Thomas Jefferson National Accelerator facility in the US, in experiments designed by this research team. "By firing a beam of electrons at an atomic nucleus you can measure the difference in energy of the outgoing electrons, representing the changed state. We are making some fairly strong predictions about what the outcomes of those tests will show, and we're hopeful of a definitive measurement," Professor Thomas says. "While the principle of the experiment itself is relatively simple, making the measurements reliable and accurate is extremely demanding, requiring a state-of-the-art machine like that at Jefferson Lab, and skilled experimenters. "The ramifications for the scientific world are significant. This is about as high stakes as it gets in science. It could represent a new paradigm for nuclear physics."

**The world's greatest literature reveals multifractals and cascades of consciousness January 26, 2016**

Multifractal analysis of "Finnegan's Wake" by James Joyce. The ideal shape of the graph is virtually indistinguishable from the results for purely mathematical multifractals. The horizontal axis represents degree of singularity, and the …more

James Joyce, Julio Cortazar, Marcel Proust, Henryk Sienkiewicz and Umberto Eco. Regardless of the language they were working in, some of the world's greatest writers appear to be, in some respects, constructing fractals. Statistical analysis carried out at the Institute of Nuclear Physics of the Polish Academy of Sciences, however, revealed something even more intriguing. The composition of works from within a particular genre was characterized by the exceptional dynamics of a cascading (avalanche) narrative structure. This type of narrative turns out to be multifractal. That is, fractals of fractals are created.

As far as many bookworms are concerned, advanced equations and graphs are the last things that would hold their interest, but there's no escaping the math. Physicists from the Institute of Nuclear Physics of the Polish Academy of Sciences (IFJ PAN) in Cracow, Poland, performed a detailed statistical analysis of more than one hundred famous works of world literature, written in several languages and representing various literary genres. The books, tested for revealing correlations in variations of sentence length, proved to be governed by the dynamics of a cascade. This means that the construction of these books is in fact a fractal. In the case of several works, their mathematical complexity proved to be exceptional, comparable to the structure of complex mathematical objects considered to be multifractal. Interestingly, in the analyzed pool of all the works, one genre turned out to be exceptionally multifractal in nature.

Fractals are self-similar mathematical objects. When we begin to expand one fragment or another, what eventually emerges is a structure that resembles the original object. Typical fractals, especially those widely known as the Sierpinski triangle and the Mandelbrot set, are monofractals, meaning that the pace of enlargement in any place of a fractal is the same, linear: If they were rescaled x number of times to reveal a structure similar to the original, the same increase in another place would also reveal a similar structure. The world's greatest literature reveals multifractals and cascades of consciousness Sequences of sentence lengths (as measured by number of words) in four literary works representative of various degree of cascading character. Credit: Source: IFJ PAN

Multifractals are more highly advanced mathematical structures: fractals of fractals. They arise from fractals 'interwoven' with each other in an appropriate manner and in appropriate proportions. Multifractals are not simply the sum of fractals and cannot be divided to return back to their original components, because the way they weave is fractal in nature. The result is that in order to see a structure similar to the original, different portions of a multifractal need to expand at different rates. A multifractal is therefore non-linear in nature.

"Analyses on multiple scales, carried out using fractals, allow us to neatly grasp information on correlations among data at various levels of complexity of tested systems. As a result, they point to the hierarchical organization of phenomena and structures found in nature. So we can expect natural language, which represents a major evolutionary leap of the natural world, to show such correlations as well. Their existence in literary works, however, had not yet been convincingly documented. Meanwhile, it turned out that when you look at these works from the proper perspective, these correlations appear to be not only common, but in some works they take on a particularly sophisticated mathematical complexity," says Prof. Stanislaw Drozdz of IFJ PAN, Cracow University of Technology.

The study involved 113 literary works written in English, French, German, Italian, Polish, Russian and Spanish by such famous figures as Honore de Balzac, Arthur Conan Doyle, Julio Cortazar, Charles Dickens, Fyodor

Dostoevsky, Alexandre Dumas, Umberto Eco, George Elliot, Victor Hugo, James Joyce, Thomas Mann, Marcel Proust, Wladyslaw Reymont, William Shakespeare, Henryk Sienkiewicz, JRR Tolkien, Leo Tolstoy and Virginia Woolf, among others. The selected works were no less than 5,000 sentences long, in order to ensure statistical reliability.

To convert the texts to numerical sequences, sentence length was measured by the number of words (an alternative method of counting characters in the sentence turned out to have no major impact on the conclusions). The researchers then searched for dependences in the data beginning with the simplest, i.e. linear. The researchers posited the question: If a sentence of a given length is x times longer than the sentences of different lengths, is the same aspect ratio preserved when looking at sentences respectively longer or shorter?

The world's greatest literature reveals multifractals and cascades of consciousness

Multifractality in literature. The numbers correspond to the position of a given literary work in the pool as in detailed list available in the scientific publication). The color red indicates works traditionally belonging to the genre of …more

"All of the examined works showed self-similarity in terms of organization of the lengths of sentences. Some were more expressive—The Ambassadors by Henry James stood out—while others were less extreme, as in the case of the French 17th-century romance Artamene ou le Grand Cyrus. However, correlations were evident, and therefore, these texts were the construction of a fractal," says Dr. Pawel Oswiecimka (IFJ PAN), who also noted that fractality of a literary text will in practice never be as perfect as in the world of mathematics. It is possible to magnify mathematical fractals up to infinity, while the number of sentences in each book is finite, and at a certain stage of scaling, there will always be a cut-off in the form of the end of the dataset.

Things took a particularly interesting turn when physicists from the IFJ PAN began tracking non-linear dependence, which in most of the studied works was present to a slight or moderate degree. However, more than a dozen works revealed a very clear multifractal structure, and almost all of these proved to be representative of one genre: stream of consciousness. The only exception was the Bible, specifically the Old Testament, which has so far never been associated with this literary genre.

"The absolute record in terms of multifractality turned out to be Finnegan's Wake by James Joyce. The results of our analysis of this text are virtually indistinguishable from ideal, purely mathematical multifractals," says Prof. Drozdz.

The most multifractal works also included A Heartbreaking Work of Staggering Genius by Dave Eggers, Rayuela by Julio Cortazar, The US Trilogy by John Dos Passos, The Waves by Virginia Woolf, 2666 by Roberto Bolano, and Joyce's Ulysses. At the same time, a lot of works usually regarded as stream of consciousness turned out to show little correlation to multifractality, as it was hardly noticeable in books such as Atlas Shrugged by Ayn Rand and A la recherche du temps perdu by Marcel Proust.

"It is not entirely clear whether stream of consciousness writing actually reveals the deeper qualities of our consciousness, or rather the imagination of the writers. It is hardly surprising that ascribing a work to a particular genre is, for whatever reason, sometimes subjective. We see, moreover, the possibility of an interesting application of our methodology: it may someday help in a more objective assignment of books to one genre or another," notes Prof. Drozdz. Multifractal analyses of literary texts carried out by the IFJ PAN have been published in Information Sciences, the prestigious journal of computer science. The publication has undergone rigorous verification. Given the interdisciplinary nature of the subject, editors immediately appointed up to six reviewers. More information:

Stanisław Drożdż et al. Quantifying origin and character of long-range correlations in narrative texts, Information Sciences (2016). DOI: 10.1016/j.ins.2015.10.023

**Information Sciences Volume 331, 20 February 2016, Pages 32–44 Quantifying origin and character of long-range correlations in narrative texts Stanisław Drożdż, a, d, , , Paweł Oświęcimkaa, Andrzej Kuliga, Jarosław Kwapieńa, Katarzyna Bazarnikb, Iwona Grabska-Gradzińskac, Jan Rybickib, Marek Stanuszekd**

In natural language using short sentences is considered efficient for communication. However, a text composed exclusively of such sentences looks technical and reads boring. A text composed of long ones, on the other hand, demands significantly more effort for comprehension. Studying characteristics of the sentence length variability (SLV) in a large corpus of world-famous literary texts shows that an appealing and aesthetic optimum appears somewhere in between and involves selfsimilar, cascade-like alternation of various lengths sentences. A related quantitative observation is that the power spectra S(f) of thus characterized SLV universally develop a convincing '1/fβ' scaling with the average exponent $\beta \approx 1/2$, close to what has been identified before in musical compositions or in the brain waves. An overwhelming majority of the studied texts simply obeys such fractal attributes but especially spectacular in this respect are hypertext-like, "stream of consciousness" novels. In addition, they appear to develop structures characteristic of irreducibly interwoven sets of fractals called multifractals. Scaling of S(f) in the present context implies existence of the long-range correlations in texts and appearance of multifractality indicates that they carry even a nonlinear component. A distinct role of the full stops in inducing the long-range correlations in texts is evidenced by the fact that the above quantitative characteristics on the long-range correlations manifest themselves in variation of the full stops recurrence times along texts, thus in SLV, but to a much lesser degree in the recurrence times of the most frequent words. In this latter case the nonlinear correlations, thus multifractality, disappear even completely for all the texts considered. Treated as one extra word, the full stops at the same time appear to obey the Zipfian rank-frequency distribution, however. Keywords Natural language; Consciousness; Correlations; Multifractals; Hypertext Corresponding author at: Complex Systems Theory Department, Institute of Nuclear Physics, Polish Academy of Sciences, ul. Radzikowskiego 152, 31-342 Kraków, Poland. Tel.:+48 12 6628220. Copyright © 2015 Elsevier Inc. All rights reserved.

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(3) In this paper we complement these approaches by proposing a method for measuring the diversity of Internet paths in a geographical sense.
(4) By leveraging the recent developments in IP geolocation we show how to map the paths discovered by trace route into geographically equivalent classes.
**(5)** This allows us to identify the geographical footprints of the major transmission paths between end-hosts, and building on our observations, authors propose a quantitative measure for geographical diversity of Internet routes between any two hosts. Subjects: Networking and Internet Architecture (cs.NI) Cite as: arXiv: 1601.01116 [cs.NI] (or arXiv: 1601.01116v1 [cs.NI] for this version)

**Intelligent Conversational Bot for Massive Online Open Courses (MOOCs) Ser Ling Lim, Ong Sing Goh (Submitted on 26 Jan 2016)**

(6) Massive Online Open Courses (MOOCs) which were introduced in 2008 has since drawn attention around the world for both its advantages as well as criticism on its drawbacks. One of the issues in MOOCs which is the lack of interactivity with the instructor has brought conversational bot into the picture to fill in this gap.
(7) In this study, a prototype of MOOCs conversational bot, MOOC-bot is being developed and integrated into MOOCs website to respond to the learner inquiries using text or speech input. MOOC-bot is using the popular Artificial Intelligence Markup Language (AIML) to develop its knowledge base, leverage from AIML capability to deliver appropriate responses and can be quickly adapted to new knowledge domains.
(8) The system architecture of MOOC-bot consists of knowledge base along with AIML interpreter, chat interface, MOOCs website and Web Speech API to provide speech recognition and speech synthesis capability. The initial MOOC-bot prototype has the general knowledge from the past Loebner Prize winner - ALICE, frequent asked questions, and a content offered by Universiti Teknikal Malaysia Melaka (UTeM).
(9) The evaluation of MOOC-bot based on the past competition questions from Chatterbox Challenge (CBC) and Loebner Prize has shown that it was able to provide correct answers most of the time during the test and demonstrated the capability to prolong the conversation.
(10) The advantages of MOOC-bot such as able to provide 24-hour service that can serve different time zones, able to have knowledge in multiple domains, and can be shared by multiple sites simultaneously have outweighed its existing limitations. Subjects: Artificial Intelligence (cs.AI) Cite as: arXiv: 1601.07065 [cs.AI] (or arXiv: 1601.07065v1 [cs.AI] for this version)
**(11)** Identification and classification of TCM syndrome types among patients with vascular mild cognitive impairment using latent tree analysis

**Chen Fu, Nevin L. Zhang, Bao Xin Chen, Zhou Rong Chen, Xiang Lan Jin, Rong Juan Guo, Zhi Gang Chen, Yun Ling Zhang (Submitted on 26 Jan 2016)**

(12) Objective: To treat patients with vascular mild cognitive impairment (VMCI) using TCM, it is necessary to classify the patients into TCM syndrome types and to apply different treatments to different types.
(13) Authors investigate how to properly carry out the classification using a novel data-driven method known as latent tree analysis.
(14) Method: A cross-sectional survey on VMCI was carried out in several regions in northern China from 2008 to 2011, which resulted in a data set that involves 803 patients and 93 symptoms. Latent tree analysis was performed on the data to reveal symptom co-occurrence patterns, and the patients were partitioned into clusters in multiple ways based on the patterns. The patient clusters were matched up with syndrome types, and population statistics of the clusters are used to quantify the syndrome types and to establish classification rules.
(15) Results: Eight syndrome types are identified: Qi Deficiency, Qi Stagnation, Blood Deficiency, Blood Stasis, Phlegm-Dampness, Fire-Heat, Yang Deficiency, and Yin Deficiency. The prevalence and symptom

occurrence characteristics of each syndrome type are determined. Quantitative classification rules are established for determining whether a patient belongs to each of the syndrome types.

**(16)** Conclusions: A solution for the TCM syndrome classification problem associated with VMCI is established based on the latent tree analysis of unlabeled symptom survey data.

**(17)** The results can be used as a reference in clinic practice to improve the quality of syndrome differentiation and to reduce diagnosis variances across physicians.

**(18)** They can also be used for patient selection in research projects aimed at finding biomarkers for the syndrome types and in randomized control trials aimed at determining the efficacy of TCM treatments of VMCI. Subjects: Artificial Intelligence (cs.AI) Cite as: arXiv: 1601.06923 [cs.AI] (or arXiv: 1601.06923v1 [cs.AI] for this version)

**Fisher Motion Descriptor for Multiview Gait Recognition F.M. Castro, M.J. Marín-Jiménez, N. Guil, R. Muñoz-Salinas (Submitted on 26 Jan 2016)**

(19) The goal of this paper is to identify individuals by analyzing their gait. Instead of using binary silhouettes as input data (as done in many previous works) we propose and evaluate the use of motion descriptors based on densely sampled short-term trajectories.

(20) Authors take advantage of state-of-the-art people detectors to define custom spatial configurations of the descriptors around the target person, obtaining a rich representation of the gait motion.

(21) The local motion features (described by the Divergence-Curl-Shear descriptor) extracted on the different spatial areas of the person are combined into a single high-level gait descriptor by using the Fisher Vector encoding.

(22) The proposed approach, coined Pyramidal Fisher Motion, is experimentally validated on `CASIA' dataset (parts B and C), `TUM GAID' dataset, `CMU MoBo' dataset and the recent `AVA Multiview Gait' dataset.

**(23)** The results show that this new approach achieves state-of-the-art results in the problem of gait recognition, allowing to recognize walking people from diverse viewpoints on single and multiple camera setups, wearing different clothes, carrying bags, walking at diverse speeds and not limited to straight walking paths. Comments: This paper extends with new experiments the one published at ICPR'2014 Subjects: Computer Vision and Pattern Recognition (cs.CV); Artificial Intelligence (cs.AI) Cite as: arXiv: 1601.06931 [cs.CV] (or arXiv: 1601.06931v1 [cs.CV] for this version)

**Emerging Dimension Weights in a Conceptual Spaces Model of Concept Combination Martha Lewis, Jonathan Lawry (Submitted on 25 Jan 2016)**

(24) Authors investigate the generation of new concepts from combinations of properties as an artificial language develops. To do so, we have developed a new framework for conjunctive concept combination.

(25) This framework gives a semantic grounding to the weighted sum approach to concept combination seen in the literature.

(26) Authors implement the framework in a multi-agent simulation of language evolution and show that shared combination weights emerge.

(27) The expected value and the variance of these weights across agents may be predicted from the distribution of elements in the conceptual space, as determined by the underlying environment, together with the rate at which agents adopt others' concepts.

(28) When this rate is smaller, the agents are able to converge to weights with lower variance. However, the time taken to converge to a steady state distribution of weights is longer. Comments: AISB 2014, updated to include references to previous work Subjects: Artificial Intelligence (cs.AI); Computation and Language (cs.CL); Multiagent Systems (cs.MA) Cite as: arXiv: 1601.06763 [cs.AI] (or arXiv: 1601.06763v1 [cs.AI] for this version)

**The Utility of Hedged Assertions in the Emergence of Shared Categorical Labels Martha Lewis, Jonathan Lawry (Submitted on 25 Jan 2016)**

(29) Authors investigate the emergence of shared concepts in a community of language users using a multi-agent simulation. We extend results showing that negated assertions are of use in developing shared categories, to include assertions modified by linguistic hedges.

(30) Results show that using hedged assertions positively affects the emergence of shared categories in two distinct ways.

(31) Firstly, using contraction hedges like `very' gives better convergence over time. Secondly, using expansion hedges such as `quite' reduces concept overlap. However, both these improvements come at a cost of slower speed of development. Comments: AISB 2013, updated to include cross-reference to previous work Subjects: Artificial Intelligence (cs.AI); Computation and Language (cs.CL); Multiagent Systems (cs.MA) Cite as: arXiv: 1601.06755 [cs.AI] (or arXiv: 1601.06755v1 [cs.AI] for this version)

**A Label Semantics Approach to Linguistic Hedges Martha Lewis, Jonathan Lawry (Submitted on 25 Jan 2016**)

(32) Authors introduce a model for the linguistic hedges `very' and `quite' within the label semantics framework, and combined with the prototype and conceptual spaces theories of concepts.

(33) The proposed model emerges naturally from the representational framework we use and as such, has a clear semantic grounding.

**(34)** Authors give generalisations of these hedge models and show that they can be composed with themselves and with other functions, going on to examine their behaviour in the limit of composition. Subjects: Artificial Intelligence (cs.AI); Computation and Language (cs.CL) Cite as: arXiv: 1601.06738 [cs.AI] (or arXiv: 1601.06738v1 [cs.AI] for this version)

**Generalizing Prototype Theory: A Formal Quantum Framework Diederik Aerts, Jan Broekaert, Liane Gabora, Sandro Sozzo (Submitted on 25 Jan 2016)**

(35) Theories of natural language and concepts have been unable to model the flexibility, creativity, context-dependence, and emergence, exhibited by words, concepts and their combinations.

(36) Mathematical formalism of quantum theory has instead been successful in capturing these phenomena such as graded membership, situational meaning, and composition of categories, and also more complex decision making situations, which cannot be modeled in traditional probabilistic approaches.

(37) Authors show how a formal quantum approach to concepts and their combinations can provide a powerful extension of prototype theory.

(38) Authors explain how prototypes can interfere in conceptual combinations as a consequence of their contextual interactions, and provide an illustration of this using an intuitive wave-like diagram.

**(39)** This quantum-conceptual approach gives new life to original prototype theory, without however making it a privileged concept theory, as we explain at the end of our paper. Subjects: Artificial Intelligence (cs.AI); Quantum Physics (quant-ph) Cite as: arXiv: 1601.06610 [cs.AI] (or arXiv:1601.06610v1 [cs.AI] for this version)

**Is a `Wirikuta empowerment' of the Huichol measurable on the Internet? Lorena Pérez-García, Jan Broekaert, Nicole Note (Submitted on 23 Jun 2014)**

(40) Current social and activist movements find the opportunity in social media to effectively impact on the agenda of governing bodies and create `global' perceptions -- it is often claimed.

(41) Content related to the social and activist movements is online, to be accessed, supported or disputed and distributed from virtually anywhere at any time, in the public sphere of the Internet.

(42) This activity allows the enlargement of social movements and would increase the empowerment in the concerned communities.

(43) Aim of this explorative study is to assess whether the temporal evolution of the Normalised Web Distance (NWD) --as defined by Cilibrasi & Vit\'anyi (2007)-- between identifying terms concerning this activism could be used to measure the progress or decline of social empowerment through the Internet.

(44) The NWD relies on the page count number of single and joint queries, which in our study have been registered using a freely available web browser (e.g. Google Search) providing a time search window for temporal query results.

(45) To explore this meta-data technique, we introduce the case of a perceived Wirikuta online movement, which originated in Mexico with the aim to protect the Huichols' sacred land and water resources from open mining projects for silver ore.

(46) Authors conducted a small scale Internet study relating the key terms `Wirikuta', `Huichol', and `Wixarika' and their co-occurrence with seven positive qualifiers (e.g. `sacred land'), five negative qualifiers (e.g. `violence') and one neutral qualifier (`table') over time, annually from 1994 till 2013.

**(47)** Authors confirm close semantic clustering over time of traditional indigeneous identity terms of the Huichol, and observe a slight convergence of key terms to `mines' and less pronounced to `sacred land' and a divergence with respect to `ancestors' indicating a complex image of a tendency of empowerment. Subjects: Social and Information Networks (cs.SI); Computers and Society (cs.CY); Information Retrieval (cs.IR); Physics and Society (physics.soc-ph) Cite as: arXiv: 1406.5946 [cs.SI] (or arXiv: 1406.5946v1 [cs.SI] for this version)

**The Quantum Challenge in Concept Theory and Natural Language Processing Diederik Aerts, Jan Broekaert, Sandro Sozzo, and Tomas Veloz (Submitted on 12 Jun 2013)**

(48) The mathematical formalism of quantum theory has been successfully used in human cognition to model decision processes and to deliver representations of human knowledge.

(49) As such, quantum cognition inspired tools have improved technologies for Natural Language Processing and Information Retrieval.

(50) In this paper, Authors overview the quantum cognition approach developed in our Brussels team during the last two decades, specifically our identification of quantum structures in human concepts and language, and the modeling of data from psychological and corpus-text-based experiments.

(51) Authors discuss our quantum-theoretic framework for concepts and their conjunctions/disjunctions in a Fock-Hilbert space structure, adequately modeling a large amount of data collected on concept combinations.

**(52)** Inspired by this modeling, we put forward elements for a quantum contextual and meaning-based approach to information technologies in which 'entities of meaning' are inversely reconstructed from texts, which are considered as traces of these entities' states. Subjects: Computation and Language (cs.CL); Information Retrieval (cs.IR); Quantum Physics (quant-ph) Journal reference: Proceedings of the 25th International Conference on System Research, Informatics & Cybernetics (pp. 13-17). Ed.. E. G. Lasker, IIAS, 2013 Cite as: arXiv: 1306.2838 [cs.CL] (or arXiv: 1306.2838v1 [cs.CL] for this version)

**Meaning-focused and Quantum-inspired Information Retrieval Diederik Aerts, Jan Broekaert, Sandro Sozzo, Tomas Veloz (Submitted on 30 Mar 2013)**

(53) In recent years, quantum-based methods have promisingly integrated the traditional procedures in information retrieval (IR) and natural language processing (NLP).

(54) Inspired by our research on the identification and application of quantum structures in cognition, more specifically Authors work on the representation of concepts and their combinations, Authors put forward a 'quantum meaning based' framework for structured query retrieval in text corpora and standardized testing corpora.

(55) This scheme for IR rests on considering as basic notions, (i) 'entities of meaning', e.g., concepts and their combinations and (ii) traces of such entities of meaning, which is how documents are considered in this approach.

(56) The meaning content of these 'entities of meaning' is reconstructed by solving an 'inverse problem' in the quantum formalism, consisting of reconstructing the full states of the entities of meaning from their collapsed states identified as traces in relevant documents.

**(57)** The advantages with respect to traditional approaches, such as Latent Semantic Analysis (LSA), are discussed by means of concrete examples. Subjects: Information Retrieval (cs.IR); Computation and Language (cs.CL); Quantum Physics (quant-ph) Journal reference: Quantum Interaction. Lecture Notes in Computer Science, 8369, pp. 71-83, 2014 DOI: 10.1007/978-3-642-54943-4_7 Cite as: arXiv: 1304.0104 [cs.IR] (or arXiv:1304.0104v1 [cs.IR] for this version)

**The Guppy Effect as Interference Diederik Aerts, Jan Broekaert, Liane Gabora, Tomas Veloz (Submitted on 11 Aug 2012)**

(58) People use conjunctions and disjunctions of concepts in ways that violate the rules of classical logic, such as the law of compositionality.

(59) Specifically, they overextend conjunctions of concepts, a phenomenon referred to as the Guppy Effect. Authors build on previous efforts to develop a quantum model that explains the Guppy Effect in terms of interference.

(60) Using a well-studied data set with 16 exemplars that exhibit the Guppy Effect, we developed a 17-dimensional complex Hilbert space H that models the data and demonstrates the relationship between overextension and interference.

**(61)** Authors view the interference effect as, not a logical fallacy on the conjunction, but a signal that out of the two constituent concepts, a new concept has emerged. Subjects: Artificial Intelligence (cs.AI); Quantum Physics (quant-ph) Journal reference: Quantum Interaction. Lecture Notes in Computer Science, 7620, pp 36-47, 2012 DOI: 10.1007/978-3-642-35659-9_4 Cite as: arXiv: 1208.2362 [cs.AI] (or arXiv: 1208.2362v1 [cs.AI] for this version)

**Contextual Random Boolean Networks Carlos Gershenson, Jan Broekaert, Diederik Aerts (Submitted on 10 Mar 2003 (v1), last revised 11 Jun 2003 (this version, v2))**

(62) Authors propose the use of Deterministic Generalized Asynchronous Random Boolean Networks [Gershenson, 2002] as models of contextual deterministic discrete dynamical systems.

(63) Authors show that changes in the context have drastic effects on the global properties of the same networks, namely the average number of attractors and the average percentage of states in attractors.

(64) Authors introduce the situation where Authors lack knowledge on the context as a more realistic model for contextual dynamical systems.

(65) Authors notice that this makes the network non-deterministic in a specific way, namely introducing a non-Kolmogorovian quantum-like structure for the modelling of the network [Aerts, 1986].

**(66)** In this case, for example, a state of the network has the potentiality (probability) of collapsing into different attractors, depending on the specific form of lack of knowledge on the context. Subjects: Adaptation and **Self-Organizing Systems** (nlin.AO); Disordered Systems and Neural Networks (cond-mat.dis-nn); Computational Complexity (cs.CC); Cellular Automata and Lattice Gases (nlin.CG) Journal reference: In Banzhaf, W, T. Christaller, P. Dittrich, J. T. Kim, and J. Ziegler, Advances in Artificial Life, 7th European Conference, ECAL 2003, Dortmund, Germany, pp. 615-624. LNAI 2801. Springer. Cite as: arXiv: nlin/0303021 [nlin.AO] (or arXiv:nlin/0303021v2 [nlin.AO] for this version)

**On the Foundations of the Brussels Operational-Realistic Approach to Cognition Diederik Aerts, Massimiliano Sassoli de Bianchi, Sandro Sozzo (Submitted on 29 Dec 2015)**

(67) The scientific community is becoming more and more interested in the research that applies the mathematical formalism of quantum theory to model human decision-making. In this paper, Authors provide the theoretical foundations of the quantum approach to cognition that Authors developed in Brussels.

(68) These foundations rest on the results of two decade studies on the axiomatic and operational-realistic approaches to the foundations of quantum physics.

(69) The deep analogies between the foundations of physics and cognition lead us to investigate the validity of quantum theory as a general and unitary framework for cognitive processes, and the empirical success of the Hilbert space models derived by such investigation provides a strong theoretical confirmation of this validity. However, two situations in the cognitive realm, 'question order effects' and 'response replicability', indicate that even the Hilbert space framework could be insufficient to reproduce the collected data.

(70) This does not mean that the mentioned operational-realistic approach would be incorrect, but simply that a larger class of measurements would be in force in human cognition, so that an extended quantum formalism may be needed to deal with all of them.

(71) As we will explain, the recently derived 'extended Bloch representation' of quantum theory (and the associated 'general tension-reduction' model) precisely provides such extended formalism, while remaining within the same unitary interpretative framework. Subjects: Artificial Intelligence (cs.AI); Quantum Physics (quant-ph) Cite as: arXiv: 1512.08710 [cs.AI] (or arXiv: 1512.08710v1 [cs.AI] for this version)

**Beyond-Quantum Modeling of Question Order Effects and Response Replicability in Psychological Measurements Diederik Aerts, Massimiliano Sassoli de Bianchi (Submitted on 15 Aug 2015)**

(72) A general tension-reduction (GTR) model was recently considered to derive quantum probabilities as (universal) averages over all possible forms of non-uniform fluctuations, and explain their considerable success in describing experimental situations also outside of the domain of physics, for instance in the ambit of quantum models of cognition and decision.

(73) Yet, this result also highlighted the possibility of observing violations of the predictions of the Born rule, in those situations where the averaging would not be large enough, or would be altered because of the combination of multiple measurements.

(74) In this article Authors show that this is indeed the case in typical psychological measurements exhibiting question order effects, by showing that their statistics of outcomes are inherently non-Hilbertian, and require the larger framework of the GTR-model to receive an exact mathematical description. We also consider another unsolved problem of quantum cognition: response replicability.

(75) It is has been observed that when question order effects and response replicability occur together, the situation cannot be handled anymore by quantum theory.

**(76)** However, we show that it can be easily and naturally described in the GTR-model. Based on these findings, Authors motivate the adoption in cognitive science of a hidden-measurements interpretation of the quantum formalism, and of its GTR-model generalization, as the natural interpretational framework explaining the data of psychological measurements on conceptual entities. Subjects: Artificial Intelligence (cs.AI); Quantum Physics (quant-ph) Cite as: arXiv: 1508.03686 [cs.AI] (or arXiv: 1508.03686v1 [cs.AI] for this version)

**Quantum Structure of Negation and Conjunction in Human Thought Diederik Aerts, Sandro Sozzo, Tomas Veloz (Submitted on 14 Mar 2015)**

(77) Authors analyse in this paper the data collected in a set of experiments performed on human subjects on the combination of natural concepts.

(78) Authors investigate the mutual influence of conceptual conjunction and negation by measuring the membership weights of a list of exemplars with respect to two concepts, e.g., 'Fruits' and 'Vegetables', and their conjunction 'Fruits And Vegetables', but also their conjunction when one or both concepts are negated, namely, 'Fruits And Not Vegetables', 'Not Fruits And Vegetables' and 'Not Fruits And Not Vegetables'. Our findings sharpen existing analysis on conceptual combinations, revealing systematic and remarkable deviations from classical (fuzzy set) logic and probability theory.

(79) And, more important, our results give further considerable evidence to the validity of our quantum-theoretic framework for the combination of two concepts. Indeed, the representation of conceptual negation naturally arises from the general assumptions of our two-sector Fock space model, and this representation faithfully agrees with the collected data.

**(80)** In addition, we find a further significant deviation and a priori unexpected from classicality, which can exactly be explained by assuming that human reasoning is the superposition of an 'emergent reasoning' and a 'logical reasoning', and that these two processes can be successfully represented in a Fock space algebraic structure. Subjects: Artificial Intelligence (cs.AI); Quantum Physics (quant-ph) Cite as: arXiv: 1503.04260 [cs.AI] (or arXiv: 1503.04260v1 [cs.AI] for this version

**Quantum Structure in Cognition, Origins, Developments, Successes and Expectations Diederik Aerts, Sandro Sozzo (Submitted on 10 Mar 2015)**

(81) Authors provide an overview of the results we have attained in the last decade on the identification of quantum structures in cognition and, more specifically, in the formalization and representation of natural concepts.

(82) Authors firstly discuss the quantum foundational reasons that led us to investigate the mechanisms of formation and combination of concepts in human reasoning, starting from the empirically observed deviations from classical logical and probabilistic structures

(83). Authors then develop our quantum-theoretic perspective in Fock space which allows successful modeling of various sets of cognitive experiments collected by different scientists, including ourselves.

(84) In addition, Authors formulate a unified explanatory hypothesis for the presence of quantum structures in cognitive processes, and discuss our recent discovery of further quantum aspects in concept combinations, namely, 'entanglement' and 'indistinguishability'.

**(85)** Authors finally illustrate perspectives for future research. Subjects: Artificial Intelligence (cs.AI); Quantum Physics (quant-ph) Cite as: arXiv: 1503.02994 [cs.AI] (or arXiv: 1503.02994v1 [cs.AI] for this version)

**Quantum Structure in Cognition and the Foundations of Human Reasoning Diederik Aerts, Sandro Sozzo, and Tomas Veloz (Submitted on 30 Dec 2014)**

(86) Traditional cognitive science rests on a foundation of classical logic and probability theory.

(87) This foundation has been seriously challenged by several findings in experimental psychology on human decision making.

(88) Meanwhile, the formalism of quantum theory has provided an efficient resource for modeling these classically problematical situations. In this paper, we start from our successful quantum-theoretic approach to the modeling of concept combinations to formulate a unifying explanatory hypothesis.

(89) In it, human reasoning is the superposition of two processes -- a conceptual reasoning, whose nature is emergence of new conceptuality, and a logical reasoning, founded on an algebraic calculus of the logical type.

(90) In most cognitive processes however, the former reasoning prevails over the latter.

**(91)** In this perspective, the observed deviations from classical logical reasoning should not be interpreted as biases but, rather, as natural expressions of emergence in its deepest form. Subjects: Artificial Intelligence (cs.AI); Quantum Physics (quant-ph) Journal reference: International Journal of Theoretical Physics, 54, pp

4557-4569, 2015 DOI: 10.1007/s10773-015-2717-9 Cite as: arXiv: 1412.8704 [cs.AI] (or arXiv:1412.8704v1 [cs.AI] for this version)

**The Quantum Nature of Identity in Human Thought: Bose-Einstein Statistics for Conceptual Indistinguishability Diederik Aerts, Sandro Sozzo, and Tomas Veloz (Submitted on 24 Oct 2014)**

(92) Increasing experimental evidence shows that humans combine concepts in a way that violates the rules of classical logic and probability theory.

(93) On the other hand, mathematical models inspired by the formalism of quantum theory are in accordance with data on concepts and their combinations. In this paper, Authors investigate a novel type of concept combination were a number is combined with a noun, e.g., `Eleven Animals.

(94) Aim is to study 'conceptual identity' and the effects of 'indistinguishability' - in the combination 'Eleven Animals', the 'animals' are identical and indistinguishable - on the mechanisms of conceptual combination.

(95) Authors perform experiments on human subjects and find significant evidence of deviation from the predictions of classical statistical theories, more specifically deviations with respect to Maxwell-Boltzmann statistics.

(96) This deviation is of the 'same type' of the deviation of quantum mechanical from classical mechanical statistics, due to indistinguishability of microscopic quantum particles, i.e we find convincing evidence of the presence of Bose-Einstein statistics.

**(97)** Authors also present preliminary promising evidence of this phenomenon in a web-based study. Subjects: Artificial Intelligence (cs.AI); Quantum Physics (quant-ph) Journal reference: International Journal of Theoretical Physics, 54, pp 4430-4443, 2015 DOI: 10.1007/s10773-015-2620-4 Cite as: arXiv: 1410.6854 [cs.AI] (or arXiv:1410.6854v1 [cs.AI] for this version)

**Modeling Concept Combinations in a Quantum-theoretic Framework Diederik Aerts, Sandro Sozzo (Submitted on 15 Jan 2014)**

(98) Authors present modeling for conceptual combinations which uses the mathematical formalism of quantum theory.

(99) Model faithfully describes a large amount of experimental data collected by different scholars on concept conjunctions and disjunctions.

(100) Furthermore, approach sheds a new light on long standing drawbacks connected with vagueness, or fuzziness, of concepts, and puts forward a completely novel possible solution to the 'combination problem' in concept theory.

**(101)** Additionally, we introduce an explanation for the occurrence of quantum structures in the mechanisms and dynamics of concepts and, more generally, in cognitive and decision processes, according to which human thought is a well structured superposition of a 'logical thought' and a 'conceptual thought', and the latter usually prevails over the former, at variance with some widespread beliefs Subjects: Artificial Intelligence (cs.AI); Quantum Physics (quant-ph) Journal reference: Advances in Cognitive Neurodynamics (IV), pp 393-399, 2014 DOI: 10.1007/978-94-017-9548-7_55 Cite as: arXiv:1401.3626 [cs.AI] (or arXiv:1401.3626v1 [cs.AI] for this version)

**Toward a Formal Model of the Shifting Relationship between Concepts and Contexts during Associative Thought Tomas Veloz, Liane Gabora, Mark Eyjolfson, Diederik Aerts (Submitted on 29 Oct 2013)**

(102) The quantum inspired State Context Property (SCOP) theory of concepts is unique amongst theories of concepts in offering a means of incorporating that for each concept in each different context there are an unlimited number of exemplars, or states, of varying degrees of typicality.

(103) Working with data from a study in which participants were asked to rate the typicality of exemplars of a concept for different contexts, and introducing an exemplar typicality threshold, we built a SCOP model of

how states of a concept arise differently in associative versus analytic (or divergent and convergent) modes of thought.

(104) Introducing measures of state robustness and context relevance, Authors show that by varying the threshold, the relevance of different contexts changes, and seemingly atypical states can become typical.

**(105)** The formalism provides a pivotal step toward a formal explanation of creative thought processes. Subjects: Neurons and Cognition (q-bio.NC); Artificial Intelligence (cs.AI) Journal reference: (2011). Lecture Notes in Computer Science 7052: Proceedings Fifth International Symposium on Quantum Interaction. June 27-29, Aberdeen, UK. Berlin: Springer Cite as: arXiv: 1310.7981 [q-bio.NC] (or arXiv:1310.7981v1 [q-bio.NC] for this version)

**Contextualizing concepts using a mathematical generalization of the quantum formalism Liane Gabora, Diederik Aerts (Submitted on 29 Oct 2013 (v1), last revised 1 Nov 2013 (this version, v2))**

(106) Authors outline the rationale and preliminary results of using the state context property (SCOP) formalism, originally developed as a generalization of quantum mechanics, to describe the contextual manner in which concepts are evoked, used and combined to generate meaning.

(107) The quantum formalism was developed to cope with problems arising in the description of (i) the measurement process, and (ii) the generation of new states with new properties when particles become entangled. Similar problems arising with concepts motivated the formal treatment introduced here.

(108) Concepts are viewed not as fixed representations, but entities existing in states of potentiality that require interaction with a context-a stimulus or another concept-to 'collapse' to an instantiated form (e.g. exemplar, prototype, or other possibly imaginary instance).

(109) The stimulus situation plays the role of the measurement in physics, acting as context that induces a change of the cognitive state from superposition state to collapsed state.

(110) Collapsed state is more likely to consist of a conjunction of concepts for associative than analytic thought because more stimulus or concept properties take part in the collapse.

**(111)** Authors provide two contextual measures of conceptual distance-one using collapse probabilities and the other weighted properties-and show how they can be applied to conjunctions using the pet fish problem. Subjects: Neurons and Cognition (q-bio.NC); Artificial Intelligence (cs.AI); Quantum Physics (quant-ph) Journal reference: Journal of Experimental and Theoretical Artificial Intelligence, 14(4), 327-358 Cite as: arXiv: 1310.7682 [q-bio.NC] (or arXiv:1310.7682v2 [q-bio.NC] for this version)

**General Quantum Hilbert Space Modeling Scheme for Entanglement Diederik Aerts, Sandro Sozzo (Submitted on 12 Apr 2013)**

(112) Authors work out a classification scheme for quantum modeling in Hilbert space of any kind of composite entity violating Bell's inequalities and exhibiting entanglement.

(113) Theoretical framework includes situations with entangled states and product measurements ('customary quantum situation'), and also situations with both entangled states and entangled measurements ('nonlocal box situation', 'nonlocal non-marginal box situation').

(114) Authors show that entanglement is structurally a joint property of states and measurements.

**(115)** Furthermore, entangled measurements enable quantum modeling of situations that are usually believed to be 'beyond quantum'. Our results are also extended from pure states to quantum mixtures. Subjects: Quantum Physics (quant-ph); Artificial Intelligence (cs.AI) Journal reference: Proceedings of the Seventh International Conference on Quantum, Nano and Micro Technologies (pp. 25-31), Eds. V. Ovchinnikov and P. Dini, IARIA, 2013 Cite as: arXiv:1304.3733 [quant-ph] (or arXiv:1304.3733v1 [quant-ph] for this version)

**Entanglement Zoo II: Examples in Physics and Cognition Diederik Aerts, Sandro Sozzo (Submitted on 30 Mar 2013 (v1), last revised 5 Sep 2013 (this version, v2))**

(116) Authors have recently presented a general scheme enabling quantum modeling of different types of situations that violate Bell's inequalities.

(117) In this paper, Authors specify this scheme for a combination of two concepts.

(118) Authors work out a quantum Hilbert space model where 'entangled measurements' occur in addition to the expected 'entanglement between the component concepts', or 'state entanglement'.

(119) Authors extend this result to a macroscopic physical entity, the 'connected vessels of water', which maximally violates Bell's inequalities.

**(120)** Authors enlighten the structural and conceptual analogies between the cognitive and physical situations which are both examples of a nonlocal non-marginal box modeling in our classification. Subjects: Artificial Intelligence (cs.AI); Quantum Physics (quant-ph) Journal reference: Quantum Interaction. Lecture Notes in Computer Science, 8369, pp. 97-109, 2014 DOI: 10.1007/978-3-642-54943-4_9 Cite as: arXiv: 1304.0102 [cs.AI] (or arXiv:1304.0102v2 [cs.AI] for this version)

**Entanglement Zoo I: Foundational and Structural Aspects Diederik Aerts, Sandro Sozzo (Submitted on 30 Mar 2013 (v1), last revised 5 Sep 2013 (this version, v2))**

(121) Authors put forward a general classification for a structural description of the entanglement present in compound entities experimentally violating Bell's inequalities, making use of a new entanglement scheme that we developed recently.

(122) Scheme, although different from the traditional one, is completely compatible with standard quantum theory, and enables quantum modeling in complex Hilbert space for different types of situations.

(123) Namely, situations where entangled states and product measurements appear ('customary quantum modeling'), and situations where states and measurements and evolutions between measurements are entangled ('nonlocal box modeling', 'nonlocal non-marginal box modeling').

**(124)** The role played by Tsirelson's bound and marginal distribution law is emphasized. Specific quantum models are worked out in detail in complex Hilbert space within this new entanglement scheme. Subjects: Artificial Intelligence (cs.AI); Quantum Physics (quant-ph) Journal reference: Quantum Interaction. Lecture Notes in Computer Science, 8369, pp. 84-96, 2014 DOI: 10.1007/978-3-642-54943-4_8 Cite as: arXiv: 1304.0100 [cs.AI] (or arXiv: 1304.0100v2 [cs.AI] for this version)

**Quantum Entanglement in Concept Combinations Diederik Aerts, Sandro Sozzo (Submitted on 15 Feb 2013 (v1), last revised 15 Aug 2013 (this version, v2))**

(125) Research in the application of quantum structures to cognitive science confirms that these structures quite systematically appear in the dynamics of concepts and their combinations and quantum-based models faithfully represent experimental data of situations where classical approaches are problematical.

(126) In this paper, we analyze the data we collected in an experiment on a specific conceptual combination, showing that Bell's inequalities are violated in the experiment.

(127) Authors present a new refined entanglement scheme to model these data within standard quantum theory rules, where 'entangled measurements and entangled evolutions' occur, in addition to the expected 'entangled states', and present a full quantum representation in complex Hilbert space of the data.

(128) This stronger form of entanglement in measurements and evolutions might have relevant applications in the foundations of quantum theory, as well as in the interpretation of nonlocality tests.

**(129)** It could indeed explain some non-negligible 'anomalies' identified in EPR-Bell experiments. Subjects: Artificial Intelligence (cs.AI); Computation and Language (cs.CL); Quantum Physics (quant-ph) Journal

reference: International Journal of Theoretical Physics, 53, pp. 3587-3603, 2014 DOI: 10.1007/s10773-013-1946-z Cite as: arXiv: 1302.3831 [cs.AI] (or arXiv: 1302.3831v2 [cs.AI] for this version)

**Concepts and Their Dynamics: A Quantum-Theoretic Modeling of Human Thought Diederik Aerts, Liane Gabora, Sandro Sozzo (Submitted on 5 Jun 2012 (v1), last revised 5 Jan 2013 (this version, v2))**

(130) Authors analyze different aspects of our quantum modeling approach of human concepts, and more specifically focus on the quantum effects of contextuality, interference, entanglement and emergence, illustrating how each of them makes its appearance in specific situations of the dynamics of human concepts and their combinations.

(131) Authors point out the relation of our approach, which is based on ontology of a concept as an entity in a state changing under influence of a context, with the main traditional concept theories, i.e. prototype theory, exemplar theory and theory theory.

(132) Authors ponder about the question why quantum theory performs so well in its modeling of human concepts, and shed light on this question by analyzing the role of complex amplitudes, showing how they allow to describe interference in the statistics of measurement outcomes, while in the traditional theories statistics of outcomes originate in classical probability weights, without the possibility of interference.

(133) The relevance of complex numbers, the appearance of entanglement, and the role of Fock space in explaining contextual emergence, all as unique features of the quantum modeling, are explicitly revealed in this paper by analyzing human concepts and their dynamics. Subjects: Artificial Intelligence (cs.AI); Computation and Language (cs.CL); Quantum Physics (quant-ph) Journal reference: Topics in Cognitive Science, 5, pp. 737-772, 2013 DOI: 10.1111/tops.12042 Cite as: arXiv: 1206.1069 [cs.AI] (or arXiv: 1206.1069v2 [cs.AI] for this version)

**Quantum Interaction Approach in Cognition, Artificial Intelligence and Robotics Diederik Aerts, Marek Czachor, Sandro Sozzo (Submitted on 17 Apr 2011)**

(134) The mathematical formalism of quantum mechanics has been successfully employed in the last years to model situations in which the use of classical structures gives rise to problematical situations, and where typically quantum effects, such as 'contextuality' and 'entanglement', have been recognized.

(135) This 'Quantum Interaction Approach' is briefly reviewed in this paper focusing, in particular, on the quantum models that have been elaborated to describe how concepts combine in cognitive science, and on the ensuing identification of a quantum structure in human thought.

(136) Authors point out that these results provide interesting insights toward the development of a unified theory for meaning and knowledge formalization and representation.

(137) Then, Authors analyze the technological aspects and implications of approach, and a particular attention is devoted to the connections with symbolic artificial intelligence, quantum computation and robotics. Artificial Intelligence (cs.AI); Robotics (cs.RO); Quantum Physics (quant-ph) Journal reference: In V. Privman and V. Ovchinnikov (Eds.), IARIA, Proceedings of the Fifth International Conference on Quantum, Nano and Micro Technologies, pp. 35-40, 2011 Cite as: arXiv:1104.3345 [cs.AI] (or arXiv:1104.3345v1 [cs.AI] for this version)

**The Pet-Fish problem on the World-Wide Web Diederik Aerts, Marek Czachor, Bart D'Hooghe, Sandro Sozzo (Submitted on 10 Jun 2010)**

(138) Authors identify the presence of Pet-Fish problem situations and the corresponding Guppy effect of concept theory on the World-Wide Web.

**(139)** For this purpose, Authors introduce absolute weights for words expressing concepts and relative weights between words expressing concepts, and the notion of 'meaning bound' between two words expressing concepts, making explicit use of the conceptual structure of the World-Wide Web

**(140)** . The Pet-Fish problem occurs whenever there are exemplars - in the case of Pet and Fish these can be Guppy or Goldfish - for which the meaning bound with respect to the conjunction is stronger than the meaning bounds with respect to the individual concepts. Subjects: Artificial Intelligence (cs.AI); Computation and Language (cs.CL) MSC classes: 68Txx Journal reference: Proceedings of the AAAI Fall Symposium (FS-10-08), Quantum Informatics for Cognitive, Social, and Semantic Processes, pp. 17-21, (2010) Cite as: arXiv:1006.1930 [cs.AI] (or arXiv:1006.1930v1 [cs.AI] for this version)

**Geometric Analogue of Holographic Reduced Representation Diederik Aerts, Marek Czachor, Bart De Moor**

(141) Holographic reduced representations (HRR) are based on superpositions of convolution-bound n-tuples, but the n-tuples cannot be regarded as vectors since the formalism is basis dependent.

(142) This is why HRR cannot be associated with geometric structures. Replacing convolutions by geometric products one arrives at reduced representations analogous to HRR but interpretable in terms of geometry.

(143) Variable bindings occurring in both HRR and its geometric analogue mathematically correspond to two different representations of Z_2\times...\times Z_2 (the additive group of binary n-tuples with addition modulo 2).

(144) As opposed to standard HRR, variable binding performed by means of geometric product allows for computing exact inverses of all nonzero vectors, a procedure even simpler than approximate inverses employed in HRR.

**(145)** The formal structure of the new reduced representation is analogous to cartoon computation, a geometric analogue of quantum computation. Subjects: Artificial Intelligence (cs.AI); Quantum Physics (quant-ph) Journal reference: Journal of Mathematical Psychology 53, 389-398 (2009) Cite as: arXiv: 0710.2611 [cs.AI] (or arXiv: 0710.2611v2 [cs.AI] for this version)

## Distributed, Parallel, and Cluster Computing

**Holistic Approach for Fault-Tolerant Network-on-Chip based Many-Core Systems Siavoosh Payandeh Azad, Behrad Niazmand, Jaan Raik, Gert Jervan, Thomas Hollstein (Submitted on 26 Jan 2016)**

(146) In this paper Authors describe a holistic approach for Fault-Tolerant Network-on-Chip (NoC) based many-core systems that incorporates a System Health Monitoring Unit (SHMU) which collects all the fault information from the system, classifies them and provides different solutions for different fault classes

(147) . A Mapper/Scheduler Unit (MSU) is used for online generation of different mapping and scheduling solutions based on the current fault configuration of the system.

(148) For detection of faults, Authors have leveraged concurrent online checkers, able to capture faults with low detection latency and providing the fault information for SHMU, which can be later used for the recovery process.

**(149)** The experimentation setup is performed in an open source tool, able to perform the mapping, scheduling and simulation of the system. Comments: 2nd International Workshop on Dynamic Resource Allocation and Management in Embedded, High Performance and Cloud Computing DREAM Cloud 2016 (arXiv:cs/1601.04675), DREAM Cloud/2016/05 Subjects: Distributed, Parallel, and Cluster Computing (cs.DC) Cite as: arXiv:1601.07089 [cs.DC] (or arXiv:1601.07089v1 [cs.DC] for this version)

**Bidding policies for market-based HPC workflow scheduling Andrew Burkimsher, Leandro Soares Indrusiak (Submitted on 26 Jan 2016)**

(150) This paper considers the scheduling of jobs on distributed, heterogeneous High Performance Computing (HPC) clusters. Market-based approaches are known to be efficient for allocating limited resources to those that are most prepared to pay.

(151) This context is applicable to an HPC or cloud computing scenario where the platform is overloaded. In this paper, jobs are composed of dependent tasks. Each job has a non-increasing time-value curve associated with it. Jobs are submitted to and scheduled by a market-clearing centralised auctioneer.

(152) This paper compares the performance of several policies for generating task bids. The aim investigated here is to maximise the value for the platform provider while minimising the number of jobs that do not complete (or starve).

(153) It is found that the Projected Value Remaining bidding policy gives the highest level of value under a typical overload situation, and gives the lowest number of starved tasks across the space of utilisation examined

(154) It does this by attempting to capture the urgency of tasks in the queue.

**(155)** At high levels of overload, some alternative algorithms produce slightly higher value, but at the cost of a hugely higher number of starved workflows. Comments: 2nd International Workshop on Dynamic Resource Allocation and Management in Embedded, High Performance and Cloud Computing DREAM Cloud 2016 (arXiv:cs/1601.04675), DREAM Cloud/2016/06 Subjects: Distributed, Parallel, and Cluster Computing (cs.DC) Cite as: arXiv:1601.07047 [cs.DC] (or arXiv:1601.07047v1 [cs.DC] for this version)

**A Cost-Effective Strategy for Storing Scientific Datasets with Multiple Service Providers in the Cloud Dong Yuan, Lizhen Cui, Xiao Liu, Erjiang Fu, and Yun Yang (Submitted on 26 Jan 2016)**

(156) Cloud computing provides scientists a platform that can deploy computation and data intensive applications without infrastructure investment. With excessive cloud resources and a decision support system, large generated data sets can be flexibly 1 stored locally in the current cloud, 2 deleted and regenerated whenever reused or 3 transferred to cheaper cloud services for storage.

(157) However, due to the pay for use model, the total application cost largely depends on the usage of computation, storage and bandwidth resources, hence cutting the cost of cloud based data storage becomes a big concern for deploying scientific applications in the cloud

(158) In this paper, Authors propose a novel strategy that can cost effectively store large generated data sets with multiple cloud service providers.

(159) The strategy is based on a novel algorithm that finds the trade off among computation, storage and bandwidth costs in the cloud, which are three key factors for the cost of data storage.

**(160)** Both general (random) simulations conducted with popular cloud service providers pricing models and three specific case studies on real world scientific applications show that the proposed storage strategy is highly cost effective and practical for run time utilization in the cloud. Subjects: Distributed, Parallel, and Cluster Computing (cs.DC) Cite as: arXiv: 1601.07028 [cs.DC] (or arXiv:1601.07028v1 [cs.DC] for this version)

**Edge-Disjoint Node-Independent Spanning Trees in Dense Gaussian Networks Bader AlBdaiwi, Zaid Hussain, Anton Cerny, and Robert Aldred (Submitted on 26 Jan 2016)**

(161) Independent trees are used in building secure and/or fault-tolerant network communication protocols.

(162) They have been investigated for different network topologies including tori. Dense Gaussian networks are potential alternatives for 2-dimensional tori.

(163) They have similar topological properties; however, they are superiors in carrying communications due to their node-distance distributions and smaller diameters.

**(164)** In this paper, Authors present constructions of edge-disjoint node-independent spanning trees in dense Gaussian networks. Based on the constructed trees, we design algorithms that could be used in fault-tolerant routing or secure message distribution. Subjects: Distributed, Parallel, and Cluster Computing (cs.DC);

Data Structures and Algorithms (cs.DS) Cite as: arXiv:1601.06915 [cs.DC] (or arXiv:1601.06915v1 [cs.DC] for this version)

**Quegel: A General-Purpose Query-Centric Framework for Querying Big Graphs Da Yan, James Cheng, M. Tamer Özsu, Fan Yang, Yi Lu, John C.S. Lui, Qizhen Zhang, Wilfred Ng (Submitted on 25 Jan 2016)**

(165) Pioneered by Google's Pregel, many distributed systems have been developed for large-scale graph analytics. These systems expose the user-friendly "think like a vertex" programming interface to users, and exhibit good horizontal scalability.

(166) However, these systems are designed for tasks where the majority of graph vertices participate in computation, but are not suitable for processing light-workload graph queries where only a small fraction of vertices need to be accessed.

(167) The programming paradigm adopted by these systems can seriously under-utilize the resources in a cluster for graph query processing

(168) In this work, we develop a new open-source system, called Quegel, for querying big graphs, which treats queries as first-class citizens in the design of its computing model.

(169) Users only need to specify the Pregel-like algorithm for a generic query, and Quegel processes light-workload graph queries on demand using a novel superstep-sharing execution model to effectively utilize the cluster resources.

(170) Quegel further provides a convenient interface for constructing graph indexes, which significantly improve query performance but are not supported by existing graph-parallel systems. Our experiments verified that Quegel is highly efficient in answering various types of graph queries and is up to orders of magnitude faster than existing systems. Comments: This is a full version of our VLDB paper Subjects: Distributed, Parallel, and Cluster Computing (cs.DC); Databases (cs.DB) Cite as: arXiv: 1601.06497 [cs.DC] (or arXiv:1601.06497v1 [cs.DC] for this version)

**Lightweight Fault Tolerance in Large-Scale Distributed Graph Processing Da Yan, James Cheng, Fan Yang (Submitted on 25 Jan 2016)**

(171) The success of Google's Pregel framework in distributed graph processing has inspired a surging interest in developing Pregel-like platforms featuring a user-friendly "think like a vertex" programming model. Existing Pregel-like systems support a fault tolerance mechanism called checkpointing, which periodically saves computation states as checkpoints to HDFS, so that when a failure happens, computation rolls back to the latest checkpoint.

(172) However, a checkpoint in existing systems stores a huge amount of data, including vertex states, edges, and messages sent by vertices, which significantly degrades the failure-free performance. Moreover, the high checkpointing cost prevents frequent checkpointing, and thus recovery has to replay all the computations from a state checkpointed some time ago.

(173) In this paper, Authors propose a novel checkpointing approach which only stores vertex states and incremental edge updates to HDFS as a lightweight checkpoint (LWCP), so that writing an LWCP is typically tens of times faster than writing a conventional checkpoint.

(174) To recover from the latest LWCP, messages are generated from the vertex states, and graph topology is recovered by replaying incremental edge updates.

(175) Authors show how to realize lightweight checkpointing with minor modifications of the vertex-centric programming interface.

(176) Authors also apply the same idea to a recently-proposed log-based approach for fast recovery, to make it work efficiently in practice by significantly reducing the cost of garbage collection of logs.

**(177)** Extensive experiments on large real graphs verified the effectiveness of LWCP in improving both failure-free performance and the performance of recovery. subjects: Distributed, Parallel, and Cluster Computing (cs.DC)Cite as: arXiv:1601.06496 [cs.DC] (or arXiv:1601.06496v1 [cs.DC] for this version)

**Task Allocation for Distributed Stream Processing Raphael Eidenbenz, Thomas Locher (Submitted on 22 Jan 2016)**

(178) There is a growing demand for live, on-the-fly processing of increasingly large amounts of data. In order to ensure the timely and reliable processing of streaming data, a variety of distributed stream processing architectures and platforms have been developed, which handle the fundamental tasks of (dynamically) assigning processing tasks to the currently available physical resources and routing streaming data between these resources.

(179) However, while there are plenty of platforms offering such functionality, the theory behind it is not well understood. In particular, it is unclear how to best allocate the processing tasks to the given resources.

(180) In this paper, Authors establish a theoretical foundation by formally defining a task allocation problem for distributed stream processing, which Authors prove to be NP-hard.

(181) Furthermore, we propose an approximation algorithm for the class of series-parallel decomposable graphs, which captures a broad range of common stream processing applications.

**(182)** The algorithm achieves a constant-factor approximation under the assumptions that the number of resources scales at least logarithmically with the number of computational tasks and the computational cost of the tasks dominates the cost of communication. Comments: Extended Version of the work published in the proceedings of IEEE International Conference o Computer Communications (INFOCOM), 10-15 April 2016, San Francisco, US Subjects: Distributed, Parallel, and Cluster Computing (cs.DC) Cite as: arXiv:1601.06060 [cs.DC] (or arXiv:1601.06060v1 [cs.DC] for this version)

**Neural and efficient cognition:**

**Very Efficient Training of Convolutional Neural Networks using Fast Fourier Transform and Overlap-and-Add Tyler Highlander, Andres Rodriguez (Submitted on 25 Jan 2016)**

(183) Convolutional neural networks (CNNs) are currently state-of-the-art for various classification tasks, but are computationally expensive.

(184) Propagating through the convolutional layers is very slow, as each kernel in each layer must sequentially calculate many dot products for a single forward and backward propagation which equates to O(N2n2) per kernel per layer where the inputs are N×N arrays and the kernels are n×n arrays.

(185) Convolution can be efficiently performed as a Hadamard product in the frequency domain. The bottleneck is the transformation which has a cost of O (N2log2N) using the fast Fourier transform (FFT).

(186) However, the increase in efficiency is less significant when N≫n as is the case in CNNs. We mitigate this by using the "overlap-and-add" technique reducing the computational complexity to O (N2log2n) per kernel.

**(187)** This method increases the algorithm's efficiency in both the forward and backward propagation, reducing the training and testing time for CNNs. Empirical results show method reduces computational time by a factor of up to 16.3 times the traditional convolution implementation for a 8 × 8 kernel and a 224 × 224 image. Comments: British Machine Vision Conference 2015 Subjects: Neural and Evolutionary Computing (cs.NE); Learning (cs.LG) Cite as: arXiv: 1601.06815 [cs.NE] (or arXiv: 1601.06815v1 [cs.NE] for this version)

**Long Short-Term Memory-Networks for Machine Reading Jianpeng Cheng, Li Dong, Mirella Lapata (Submitted on 25 Jan 2016 (v1), last revised 26 Jan 2016 (this version, v2))**

**(188)** Teaching machines to process text with psycholinguistic insights is a challenging task.

**(189)** Authors propose an attentive machine reader that reads text from left to right, whilst linking the word at the current fixation point to previous words stored in the memory as a kind of implicit information parsing to facilitate understanding.

**(190)** The reader is equipped with a Long Short-Term Memory architecture, which, different from previous work, has a memory tape (instead of a memory cell) to store the past information and adaptively use them without severe information compression.

**(191)** They demonstrate the excellent performance of the machine reader in language modeling as well as the downstream sentiment analysis and natural language inference. Subjects: Computation and Language (cs.CL); Neural and Evolutionary Computing (cs.NE) Cite as: arXiv: 1601.06733 [cs.CL] (or arXiv: 1601.06733v2 [cs.CL] for this version)

**Character-Level Incremental Speech Recognition with Recurrent Neural Networks Kyuyeon Hwang, Wonyong Sung (Submitted on 25 Jan 2016)**

(192) In real-time speech recognition applications, the latency is an important issue.

(193) Authors have developed a character-level incremental speech recognition (ISR) system that responds quickly even during the speech, where the hypotheses are gradually improved while the speaking proceeds.

(194) The algorithm employs a speech-to-character unidirectional recurrent neural network (RNN), which is end-to-end trained with connectionist temporal classification (CTC), and an RNN-based character-level language model (LM).

(195) The output values of the CTC-trained RNN are character-level probabilities, which are processed by beam search decoding.

(196) The RNN LM augments the decoding by providing long-term dependency information.

(197) Authors propose tree-based online beam search with additional depth-pruning, which enables the system to process infinitely long input speech with low latency.

(198) This system not only responds quickly on speech but also can dictate out-of-vocabulary (OOV) words according to pronunciation.

**(199)** The proposed model achieves the word error rate (WER) of 8.90% on the Wall Street Journal (WSJ) Nov'92 20K evaluation set when trained on the WSJ SI-284 training set. Subjects: Computation and Language (cs.CL); Learning (cs.LG); Neural and Evolutionary Computing (cs.NE) Cite as: arXiv: 1601.06581 [cs.CL] (or arXiv: 1601.06581v1 [cs.CL] for this version)

**Orthogonal Echo State Networks and stochastic evaluations of likelihoods Norbert Michael Mayer (Submitted on 22 Jan 2016)**

(200) In this paper we report the likelihood estimates that are performed on time series using a echo state network with orthogonal recurrent connectivity.

(201) The results indicate that the optimal performance depends on the way of balancing the input strength with the recurrent activity, which also has an influence on the network with regard to the quality of the short term prediction versus prediction that accounts for influences that date back a long time in the input history.

**(202)** Finally, sensitivity of such networks against noise/finite accuracy of network states in the recurrent layer is investigated. In addition a measure that bases on mutual information is introduced in order to best quantify the performance of the network with the time series. Subjects: Neural and Evolutionary Computing (cs.NE) Cite as: arXiv: 1601.05911 [cs.NE] (or arXiv: 1601.05911v1 [cs.NE] for this version)

**Bitwise Neural Networks Minje Kim, Paris Smaragdis (Submitted on 22 Jan 2016)**

(203) Based on the assumption that there exists a neural network that efficiently represents a set of Boolean functions between all binary inputs and outputs, Authors propose a process for developing and deploying

neural networks whose weight parameters, bias terms, input, and intermediate hidden layer output signals, are all binary-valued, and require only basic bit logic for the feedforward pass.

(204) The proposed Bitwise Neural Network (BNN) is especially suitable for resource-constrained environments, since it replaces either floating or fixed-point arithmetic with significantly more efficient bitwise operations.

(205) Hence, the BNN requires for less spatial complexity, less memory bandwidth, and less power consumption in hardware.

(206) In order to design such networks, we propose to add a few training schemes, such as weight compression and noisy back propagation, which result in a bitwise network that performs almost as well as its corresponding real-valued network.

**(207)** Authors test the proposed network on the MNIST dataset, represented using binary features, and show that BNNs result in competitive performance while offering dramatic computational savings. Comments: This paper was presented at the International Conference on Machine Learning (ICML) Workshop on Resource-Efficient Machine Learning, Lille, France, Jul. 6-11, 2015 Subjects: Learning (cs.LG); Artificial Intelligence (cs.AI); Neural and Evolutionary Computing (cs.NE) Cite as: arXiv:1601.06071 [cs.LG] (or arXiv:1601.06071v1 [cs.LG] for this version)

**A Robust Frame-based Nonlinear Prediction System for Automatic Speech Coding Mahmood Yousefi-Azar, Farbod Razzazi (Submitted on 22 Jan 2016)**

(208) In this paper, Authors propose a neural-based coding scheme in which an artificial neural network is exploited to automatically compress and decompress speech signals by a trainable approach.

(209) Having a two-stage training phase, the system can be fully specified to each speech frame and have robust performance across different speakers and wide range of spoken utterances.

(210) Indeed, Frame-based nonlinear predictive coding (FNPC) would code a frame in the procedure of training to predict the frame samples.

(211) The motivating objective is to analyze the system behavior in regenerating not only the envelope of spectra, but also the spectra phase.

(212) This scheme has been evaluated in time and discrete cosine transform (DCT) domains and the output of predicted phonemes show the potentiality of the FNPC to reconstruct complicated signals.

(213) Experiments were conducted on three voiced plosive phonemes, b/d/g/ in time and DCT domains versus the number of neurons in the hidden layer.

(214) Experiments approve the FNPC capability as an automatic coding system by which /b/d/g/ phonemes have been reproduced with a good accuracy.

(215) Evaluations revealed that the performance of FNPC system, trained to predict DCT coefficients is more desirable, particularly for frames with the wider distribution of energy, compared to time samples. Subjects: Sound (cs.SD); Neural and Evolutionary Computing (cs.NE) Cite as: arXiv: 1601.06008 [cs.SD] (or arXiv: 1601.06008v1 [cs.SD] for this version)

**Model-Coupled Autoencoder for Time Series Visualisation Nikolaos Gianniotis, Sven D. Kügler, Peter Tiňo, and Kai L. Polsterer (Submitted on 21 Jan 2016)**

(216) Authors present an approach for the visualisation of a set of time series that combines an echo state network with an autoencoder.

(217) For each time series in the dataset we train an echo state network, using a common and fixed reservoir of hidden neurons, and use the optimised readout weights as the new representation. Dimensionality reduction is then performed via an autoencoder on the readout weight representations.

(218) The crux of the work is to equip the autoencoder with a loss function that correctly interprets the reconstructed readout weights by associating them with a reconstruction error measured in the data space of sequences.

(219) This essentially amounts to measuring the predictive performance that the reconstructed readout weights exhibit on their corresponding sequences when plugged back into the echo state network with the same fixed reservoir.

(220) Authors demonstrate that the proposed visualisation framework can deal both with real valued sequences as well as binary sequences.

(221) Authors derive magnification factors in order to analyse distance preservations and distortions in the visualisation space.

(222) The versatility and advantages of the proposed method are demonstrated on datasets of time series that originate from diverse domains. Subjects: Instrumentation and Methods for Astrophysics (astro-ph.IM); Neural and Evolutionary Computing (cs.NE) Cite as: arXiv: 1601.05654 [astro-ph.IM] (or arXiv: 1601.05654v1 [astro-ph.IM] for this version)

**Selecting Efficient Features via a Hyper-Heuristic Approach Mitra Montazeri, Mahdieh Soleymani Baghshah, and Aliakbar Niknafs (Submitted on 20 Jan 2016**)

(223) By Emerging huge databases and the need to efficient learning algorithms on these datasets, new problems have appeared and some methods have been proposed to solve these problems by selecting efficient features.

(224) Feature selection is a problem of finding efficient features among all features in which the final feature set can improve accuracy and reduce complexity.

(225) One way to solve this problem is to evaluate all possible feature subsets. However, evaluating all possible feature subsets is an exhaustive search and thus it has high computational complexity.

(226) Until now many heuristic algorithms have been studied for solving this problem. Hyper-heuristic is a new heuristic approach which can search the solution space effectively by applying local searches appropriately.

(227) Each local search is a neighborhood searching algorithm. Since each region of the solution space can have its own characteristics, it should be chosen an appropriate local search and apply it to current solution.

(228) This task is tackled to a supervisor. The supervisor chooses a local search based on the functional history of local searches. By doing this task, it can trade of between exploitation and exploration.

(229) Since the existing heuristic cannot trade of between exploration and exploitation appropriately, the solution space has not been searched appropriately in these methods and thus they have low convergence rate.

(230) For the first time, in this paper use a hyper-heuristic approach to find an efficient feature subset.

(231) In the proposed method, genetic algorithm is used as a supervisor and 16 heuristic algorithms are used as local searches.

**(232)** Empirical study of the proposed method on several commonly used data sets from UCI data sets indicates that it outperforms recent existing methods in the literature for feature selection. Comments: The Fifth Iran Data Mining Conference (IDMC 2011), Amirkabir University of Technology, Tehran, Iran Subjects: Computer Vision and Pattern Recognition (cs.CV); Neural and Evolutionary Computing (cs.NE) Cite as: arXiv:1601.05409 [cs.CV] (or arXiv:1601.05409v1 [cs.CV] for this version)

**Towards Programmable Network Dynamics: A Chemistry-Inspired Abstraction for Hardware Design Massimo Monti, Manolis Sifalakis, Christian F. Tschudin, Marco Luise (Submitted on 20 Jan 2016)**

(233) **C**hemical algorithms are statistical algorithms described and represented as chemical reaction networks.

(234) They are particularly attractive for traffic shaping and general control of network dynamics; they are analytically tractable, they reinforce a strict state-to-dynamics relationship, they have configurable stability properties, and they are directly implemented in state-space using a high-level (graphical) representation.

(235) In this paper, Authors present a direct implementation of chemical algorithms on FPGA hardware. Besides substantially improving performance, Authors have achieved hardware-level programmability and re-configurability of these algorithms at runtime (not interrupting servicing) and in realtime (with sub-second latency).

(236) This opens an interesting perspective for expanding the currently limited scope of software defined networking and network virtualization solutions, to include programmable control of network dynamics. Subjects: Emerging Technologies (cs.ET); Systems and Control (cs.SY) Cite as: arXiv: 1601.05356 [cs.ET] (or arXiv: 1601.05356v1 [cs.ET] for this version)

**Combined Compute and Storage: Configurable Memristor Arrays to Accelerate Search Yang Liu, Chris Dwyer, Alvin R. Lebeck**

(237) Emerging technologies present opportunities for system designers to meet the challenges presented by competing trends of big data analytics and limitations on CMOS scaling.

(238) Specifically, memristors are an emerging high-density technology where the individual memristors can be used as storage or to perform computation.

(239) The voltage applied across a memristor determines its behavior (storage vs. compute), which enables a configurable memristor substrate that can embed computation with storage.

(240) This paper explores accelerating point and range search queries as instances of the more general configurable combined compute and storage capabilities of memristor arrays.

(241) Authors first present MemCAM, a configurable memristor-based content addressable memory for the cases when fast, infrequent searches over large datasets are required.

(242) For frequent searches, memristor lifetime becomes a concern. To increase memristor array lifetime we introduce hybrid data structures that combine trees with MemCAM using conventional CMOS processor/cache hierarchies for the upper levels of the tree and configurable memristor technologies for lower levels.

(243) Authors use SPICE to analyze energy consumption and access time of memristors and use analytic models to evaluate the performance of configurable hybrid data structures.

(244) The results show that with acceptable energy consumption our configurable hybrid data structures improve performance of search intensive applications and achieve lifetime in years or decades under continuous queries.

**(245)** Furthermore, the configurability of memristor arrays and the proposed data structures provide opportunities to tune the trade- off between performance and lifetime and the data structures can be easily adapted to future memristors or other technologies with improved endurance. Subjects: Emerging Technologies (cs.ET) Cite as: arXiv: 1601.05273 [cs.ET] (or arXiv: 1601.05273v1 [cs.ET] for this version)

**Wi-Fi/WiGig Coordination for Optimal WiGig Concurrent Transmissions in Random Access Scenario Ehab Mahmoud Mohamed, Kei Sakaguchi, and Seiichi Sampei (Submitted on 19 Jan 2016)**

(246) Wireless Gigabit (WiGig) access points (APs) using 60 GHz unlicensed frequency band are considered as key enablers for future Gbps WLANs.

(247) Due to its short range transmission with high susceptibility to path blocking, a multiple number of WiGig APs should be installed to fully cover a typical target environment. However, using autonomously operated WiGig APs with IEEE 802.11ad DCF, the exhaustive search analog beam forming and the maximum received power based autonomous users association prevent the establishment of optimal WiGig concurrent links that maximize the total system throughput in random access scenarios

(248) In this paper, Authors formulate the problem of WiGig concurrent transmissions in random access scenarios as an optimization problem, and then Authors propose a Wi-Fi/WiGig coordination architecture to solve it.

(249) The proposed coordinated Wi-Fi/WiGig WLAN is based on a tight coordination between the 5 GHz (Wi-Fi) and the 60 GHz (WiGig) unlicensed frequency bands.

**(250)** By which, the wide coverage Wi-Fi band controls the establishment of the WiGig concurrent links. Statistical learning using Wi-Fi fingerprinting is used for estimating the best candidate AP and its best beam

identification (ID) for establishing the WiGig concurrent link without making any interference to the existing WiGig data links. Subjects: Networking and Internet Architecture (cs.NI); Emerging Technologies (cs.ET) Cite as: arXiv: 1601.04797 [cs.NI] (or arXiv: 1601.04797v1 [cs.NI] for this version)

**Computational complexity:**

**Skyrmions as Compact, Robust and Energy-Efficient Interconnects for Domain Wall (DW)-based Systems Wang Kang, Yangqi Huang, Xichao Zhang, Yan Zhou, Weifeng Lv, Weisheng Zhao (Submitted on 12 Jan 2016)**

(251) Magnetic domain-wall (DW) has been widely investigated for future memory and computing systems. However, energy efficiency and stability become two major challenges of DW-based systems.

(252) In this letter, Authors first propose exploiting skyrmions as on-chip and inter-chip interconnects for DW-based systems, owing to the topological stability, small size and ultra-low depinning current density. In the proposed technique, data are processed in the form of DWs but are transmitted instead in the form of skyrmions.

(253) The reversible conversion between a skyrmion and a DW pair can be physically achieved by connecting a wide and a narrow magnetic nanowire.

**(254)** Proposed technique can realize highly compact, robust and energy-efficient on-chip and inter-chip interconnects for DW-based systems, enabling the system to take advantages of both the DW and skyrmion. Subjects: Mesoscale and Nanoscale Physics (cond-mat.mes-hall); Emerging Technologies (cs.ET) Cite as: arXiv: 1601.03085 [cond-mat.mes-hall] (or arXiv: 1601.03085v1 [cond-mat.mes-hall] for this version)

**Non-causal computation avoiding the grandfather and information antinomies Ämin Baumeler, Stefan Wolf (Submitted on 25 Jan 2016)**

(255) Computation models such as circuits describe sequences of computation steps that are carried out one after the other.

(256) In other words, algorithm design is traditionally subject to the restriction imposed by a fixed causal order.

(257) Authors address a novel computing paradigm, replacing this assumption by mere logical consistency: Authors study non-causal circuits, where a fixed time structure within a gate is locally assumed whilst the global causal structure between the gates is dropped.

(258) Authors present examples of logically consistent non-causal circuits outperforming all causal ones; they imply that suppressing loops entirely is more restrictive than just avoiding the contradictions they can give rise to.

**(259)** That fact is already known for correlations as well as for communication, and we here extend it to computation. Subjects: Quantum Physics (quant-ph); Computational Complexity (cs.CC); Logic in Computer Science (cs.LO) Cite as: arXiv: 1601.06522 [quant-ph] (or arXiv: 1601.06522v1 [quant-ph] for this version)

**Uncomputability of the generalized capacity David Elkouss, David Pérez-García (Submitted on 22 Jan 2016)**

(260) What is the maximum rate at which we can send information over a channel? We define the capacity of a channel as the answer to this question.

(261) For an important family of channels the capacity is given by a simple optimization problem as proven in Shannon's noisy coding theorem. Furthermore, for these channels, Authors have the Blahut-Arimoto algorithm that allows efficiently solving the optimization problem and computing the capacity.

**(262)** In groundbreaking work, Verdu and Han proved a coding theorem for general channels. However, despite considerable effort, there is no equivalent to the Blahut-Arimoto algorithm for computing the generalized capacity. Here, we show that such an algorithm cannot exist. Subjects: Information Theory (cs.IT);

Computational Complexity (cs.CC); Formal Languages and Automata Theory (cs.FL); Quantum Physics (quant-ph) Cite as: arXiv: 1601.06101 [cs.IT] (or arXiv: 1601.06101v1 [cs.IT] for this version)

**Computing with Polynomial Ordinary Differential Equations Olivier Bournez, Daniel Graça, and Amaury Pouly (Submitted on 21 Jan 2016)**

(263) In 1941, Claude Shannon introduced the General Purpose Analog Computer(GPAC) as a mathematical model of Differential Analysers, that is to say as a model of continuous-time analog (mechanical, and later one electronic) machines of that time.

(264) Following Shannon's arguments, functions generated by GPACs must be differentially algebraic. As it is known that some computable functions like Euler's $\Gamma(x) = \int_0^{\infty} tx-1e-tdt$ or Riemann's Zeta function $\zeta(x) = \sum_{k=1}^{\infty} 1kx$ are not differentially algebraic, this argument has been often used to demonstrate in the past that the GPAC is less powerful than digital computation.

(265) It was proved in JOC2007, that if a more modern notion of computation is considered, i.e. in particular if computability is not restricted to real-time generation of functions, the GPAC is actually equivalent to Turing machines.

(266) purpose is first to discuss the robustness of the notion of computation involved in JOC2007, by establishing that natural variants of the notion of computation from this paper leads to the same computability result.

(267) Second, to go considerations about (time) complexity, we explore several natural variants for measuring time/space complexity of a computation.

(268) Rather surprisingly, whereas defining a robust time complexity for general continuous time systems is a well known open problem, we prove that all variants are actually equivalent even at the complexity level. As a consequence, it seems that a robust and well defined notion of time complexity exists for the GPAC, or equivalently for computations by polynomial ordinary differential equations.

**(269)** Another side effect of our proof is also that we show in some way that polynomial ordinary differential equations can be used as a kind of programming model, and that there is a rather nice and robust notion of ODE programming. Comments: arXiv admin note: substantial text overlap with arXiv: 1601.05360 Subjects: Computational Complexity (cs.CC) Cite as: arXiv: 1601.05683 [cs.CC] (or arXiv: 1601.05683v1 [cs.CC] for this version)

**Fixed-parameter Approximability of Boolean MinCSPs Édouard Bonnet, László Egri, Dániel Marx (Submitted on 19 Jan 2016)**

(270) The minimum unsatisfiability version of a constraint satisfaction problem asks for an assignment where the number of unsatisfied constraints is minimum possible, or equivalently, asks for a minimum-size set of constraints whose deletion makes the instance satisfiable.

(271) For a finite set $\Gamma$ of constraints, we denote by MinCSP ($\Gamma$) the restriction of the problem where each constraint is from $\Gamma$.

(272) The polynomial-time solvability and the polynomial-time approximability of MinCSP ($\Gamma$) were fully characterized by Khanna et al. Here we study the fixed-parameter approximability of the problem: given an instance and an integer k, one has to find a solution of size at most g(k) in time $f(k)\cdot nO(1)$ if a solution of size at most k exists.

(273) Authors especially focus on the case of constant-factor FP-approximability. Our main result classifies each finite constraint language $\Gamma$ into one of three classes: MinCSP ($\Gamma$) has a constant-factor FP-approximation. MinCSP ($\Gamma$) has a (constant-factor) FP-approximation if and only if Nearest Codeword has a (constant-factor) FP-approximation. MinCSP ($\Gamma$) has no FP-approximation, unless FPT=W [P].

(274) Authors give some evidence that problems in the second class do not have constant-factor FP-approximations: Authors show that there is no such approximation if both the Exponential-Time Hypothesis (ETH) and the Linear PCP Conjecture (LPC) are true, and Authors also show that such an approximation

would imply the existence of an FP-approximation for the k-Densest Subgraph problem with ratio 1−ϵ for any ϵ>0. Subjects: Computational Complexity (cs.CC) MSC classes: 68Q17 ACM classes: F.2.2 Cite as: arXiv: 1601.04935 [cs.CC] (or arXiv: 1601.04935v1 [cs.CC] for this version)

**Solutions to the Problem of K-SAT / K-COL Phase Transition Location Changqing Liu (Submitted on 19 Jan 2016)**

(275) As general theories, currently there are concentration inequalities (of random walk) only for the cases of independence and martingale differences. In this paper, the concentration inequalities are extended to more general situations.

(276) In terms of the theory presented in the paper, the condition of independence is ∂y∂t= constant and martingale difference's is ∂y∂t=0.

**(277)** This paper relaxes these conditions to ∂2y∂ui∂t≤L; i.e. ∂y∂t can vary. Further, the concentration inequalities are extended to branching random walk, the applications of which solve some long standing open problems, including the well known problems of K-SAT and K-COL phase transition locations, among others. Comments: Extended abstract Subjects: Computational Complexity (cs.CC) Cite as: arXiv: 1601.04794 [cs.CC] (or arXiv: 1601.04794v1 [cs.CC] for this version)

## Computer Science and Game Theory:

**Game Theoretic Analysis of Tree Based Referrals for Crowd Sensing Social Systems with Passive Rewards Kundan Kandhway, Bhushan Kotnis (Submitted on 27 Jan 2016)**

(278) Participatory crowd sensing social systems rely on the participation of large number of individuals. Since humans are strategic by nature, effective incentive mechanisms are needed to encourage participation?

(279) A popular mechanism to recruit individuals is through referrals and passive incentives such as geometric incentive mechanisms used by the winning team in the 2009 DARPA Network Challenge and in multi level marketing schemes.

(280) The effect of such recruitment schemes on the effort put in by recruited strategic individuals is not clear. This paper attempts to fill this gap.

(281) Given a referral tree and the direct and passive reward mechanism, we formulate a network game where agents compete for finishing crowd sensing tasks.

(282) Authors characterize the Nash equilibrium efforts put in by the agents and derive closed form expressions for the same.

(283) Authors discover free riding behavior among nodes who obtain large passive rewards. This work has implications on designing effective recruitment mechanisms for crowd sourced tasks.

**(284)** For example, usage of geometric incentive mechanisms to recruit large number of individuals may not result in proportionate effort because of free riding. Subjects: Computer Science and Game Theory (cs.GT); Multiagent Systems (cs.MA); Social and Information Networks (cs.SI); Physics and Society (physics.soc-ph) Cite as: arXiv: 1601.07505 [cs.GT] (or arXiv: 1601.07505v1 [cs.GT] for this version)

**Hotelling Games on Networks: Existence and Efficiency of Equilibria Gaëtan Fournier, Marco Scarsini (Submitted on 27 Jan 2016)**

(285) Authors consider a Hotelling game where a finite number of retailers choose a location, given that their potential consumers are distributed on a network.

(286) Retailers do not compete on price but only on location, therefore each consumer shops at the closest store. Authors show that when the number of retailers is large enough, the game admits a pure Nash equilibrium and Authors construct it.

(287) Authors then compare the equilibrium cost borne by the consumers with the cost that could be achieved if the retailers followed the dictate of a benevolent planner.

(288) Authors perform this comparison in term of the induced price of anarchy, i.e., the ratio of the worst equilibrium cost and the optimal cost, and the induced price of stability, i.e., the ratio of the best equilibrium cost and the optimal cost.

**(289)** Authors show that, asymptotically in the number of retailers, these ratios are two and one, respectively. Subjects: Computer Science and Game Theory (cs.GT) MSC classes: Primary 91A43, secondary 91A06 Cite as: arXiv: 1601.07414 [cs.GT] (or arXiv: 1601.07414v1 [cs.GT] for this version)

**Providing Probabilistic Robustness Guarantee for Crowdsensing Yuben Qu, Shaojie Tang, Chao Dong, Peng Li, Song Guo, and Chang Tian (Submitted on 27 Jan 2016)**

(290) Due to its flexible and pervasive sensing ability, crowdsensing has been extensively studied recently in research communities.

(291) However, the fundamental issue of how to meet the requirement of sensing robustness in crowdsensing remains largely unsolved. Specifically, from the task owner's perspective, how to minimize the total payment in crowdsensing while guaranteeing the sensing data quality is a critical issue to be resolved.

(292) Authors elegantly model the robustness requirement over sensing data quality as chance constraints, and investigate both hard and soft chance constraints for different crowdsensing applications.

(293) For the former, we reformulate the problem through Boole's Inequality, and explore the optimal value gap between the original problem and the reformulated problem.

(294) For the latter, we study a serial of a general payment minimization problem, and propose a binary search algorithm that achieves both feasibility and low payment.

**(295)** Performance gap between our solution and the optimal solution is also theoretically analyzed. Extensive simulations validate our theoretical analysis. Subjects: Computer Science and Game Theory (cs.GT); Social and Information Networks (cs.SI) Cite as: arXiv: 1601.07341 [cs.GT] (or arXiv: 1601.07341v1 [cs.GT] for this version)

**Evolutionary stability implies asymptotic stability under multiplicative weights Ioannis Avramopoulos (Submitted on 27 Jan 2016)**

(296) Authors show that evolutionarily stable states in general (nonlinear) population games (which can be viewed as continuous vector fields constrained on a polytope) are asymptotically stable under a multiplicative weights dynamic (under appropriate choices of a parameter called the learning rate or step size, which Authors demonstrate to be crucial to achieve convergence, as otherwise even chaotic behavior is possible to manifest).

(297) Result implies that evolutionary theories based on multiplicative weights are compatible (in principle, more general) with those based on the notion of evolutionary stability

**(298)** However, our result further establishes multiplicative weights as a nonlinear programming primitive (on par with standard nonlinear programming methods) since various nonlinear optimization problems, such as finding Nash/Wardrop equilibria in nonatomic congestion games, which are well-known to be equipped with a convex potential function, and finding strict local maxima of quadratic programming problems, are special cases of the problem of computing evolutionarily stable states in nonlinear population games. Subjects: Computer Science and Game Theory (cs.GT); Learning (cs.LG); Optimization and Control (math.OC) Cite as: arXiv: 1601.07267 [cs.GT] (or arXiv: 1601.07267v1 [cs.GT] for this version)

**On the Sample Complexity of Learning Sparse Graphical Games Jean Honorio (Submitted on 27 Jan 2016)**

(299) Authors analyze the sample complexity of learning sparse graphical games from purely behavioral data.

(300) That is, Authors assume that we can only observe the players' joint actions and not their payoffs.

(301) Authors analyze the sufficient and necessary number of samples for the correct recovery of the set of pure-strategy Nash equilibria (PSNE) of the true game.

(302) Analysis focuses on sparse directed graphs with n nodes and at most k parents per node.

(303) Authors show that if the number of samples is greater than O (knlog2n), then maximum likelihood estimation correctly recovers the PSNE with high probability.

**(304)** Authors also show that if the number of samples is less than O (knlog2n), then any conceivable method fails to recover the PSNE with arbitrary probability. Subjects: Computer Science and Game Theory (cs.GT); Learning (cs.LG); Machine Learning (stat.ML) Cite as: arXiv: 1601.07243 [cs.GT] (or arXiv: 1601.07243v1 [cs.GT] for this version)

(305) **Security Games in Network Flow Problems Mathieu Dahan, Saurabh Amin (Submitted on 26 Jan 2016)**

(306) This article considers a two-player strategic game for network routing under link disruptions. Player 1 (defender) routes flow through a network to maximize her value of effective flow while facing transportation costs. Player 2 (attacker) simultaneously disrupts one or more links to maximize her value of lost flow but also faces cost of disrupting links.

(307) Linear programming duality in zero-sum games and the Max-Flow Min-Cut Theorem are applied to obtain properties that are satisfied in any Nash equilibrium

(308) A characterization of the support of the equilibrium strategies is provided using graph-theoretic arguments. Finally, conditions under which these results extend to budget-constrained environments are also studied.

**(309)** These results extend the classical minimum cost maximum flow problem and the minimum cut problem to a class of security games on flow networks. Subjects: Computer Science and Game Theory (cs.GT); Systems and Control (cs.SY) MSC classes: Primary: 91A05, 91A10, 91A43, secondary: 90C46, 05C21 Cite as: arXiv: 1601.07216 [cs.GT] (or arXiv:1601.07216v1 [cs.GT] for this version)

**Optimal Auctions with Convex Perceived Payments Amy Greenwald, Takehiro Oyakawa, and Vasilis Syrgkanis (Submitted on 26 Jan 2016)**

(310) Myerson derived a simple and elegant solution to the single-parameter revenue-maximization problem in his seminal work on optimal auction design assuming the usual model of quasi-linear utilities

(311) In this paper, we consider a slight generalization of this usual model---from linear to convex "perceived" payments.

(312) This more general problem does not appear to admit a solution as simple and elegant as Myerson's.

(313) While some of Myerson's results extend to our setting, like his payment formula (suitably adjusted), others do not.

(314) For example, Authors observe that the solutions to the Bayesian and the robust (i.e., non-Bayesian) optimal auction design problems in the convex perceived payment setting do not coincide like they do in the case of linear payments.

(315) Authors therefore study the two problems in turn.

(316) Authors derive an upper and a heuristic lower bound on expected revenue in our setting. These bounds are easily computed pointwise, and yield monotonic allocation rules, so can be supported by Myerson payments (suitably adjusted). In this way, our bounds yield heuristics that approximate the optimal robust auction, assuming convex perceived payments.

**(317)** Authors close with experiments, the final set of which massages the output of one of the closed-form heuristics for the robust problem into an extremely fast, near-optimal heuristic solution to the Bayesian optimal auction design problem. Subjects: Computer Science and Game Theory (cs.GT) Cite as: arXiv: 1601.07163 [cs.GT] (or arXiv: 1601.07163v1 [cs.GT] for this version)

**Shapley Values in Weighted Voting Games with Random Weights Yuval Filmus, Joel Oren, and Kannan Soundararajan (Submitted on 23 Jan 2016)**

(318) Authors investigate the distribution of the well-studied Shapley--Shubik values in weighted voting games where the agents are stochastically determined. The Shapley--Shubik value measures the voting power of an agent, in typical collective decision making systems.

(319) While easy to estimate empirically given the parameters of a weighted voting game, the Shapley values are notoriously hard to reason about analytically.

(320) Authors propose a probabilistic approach in which the agent weights are drawn i.i.d. from some known exponentially decaying distribution.

(321) Authors provide a general closed-form characterization of the highest and lowest expected Shapley values in such a game, as a function of the parameters of the underlying distribution.

(322) To do so, we give a novel reinterpretation of the stochastic process that generates the Shapley variables as a renewal process.

**(323)** Authors demonstrate the use of our results on the uniform and exponential distributions. Furthermore, Authors show the strength of our theoretical predictions on several synthetic datasets. Subjects: Computer Science and Game Theory (cs.GT) Cite as: arXiv: 1601.06223 [cs.GT] (or arXiv:1601.06223v1 [cs.GT] for this version)

**Energy-Aware Optimization and Mechanism Design for Cellular Device-to-Device Local Area Networks Mehdi Naderi Soorki, Mohammad Yaghini, Mohammad Hossein Manshaei, Walid Saad, and Hossein Saidi (Submitted on 25 Jan 2016)**

(324) In a device-to-device (D2D) local area network (LAN), mobile users (MUs) must cooperate to download common real-time content from a wireless cellular network.

(325) However, sustaining such D2D LANs over cellular networks requires the introduction of mechanisms that will incentivize the MUs to cooperate. In this paper, the problem of energy-aware D2D LAN formation over cellular networks is studied.

(326) The problem is formulated using a game-theoretic framework in which each MU seeks to minimize its energy consumption while actively participating in the D2D LAN.

(327) To account for the **selfish behavior** of the MUs, a punishment and incentive protocol is proposed in order to ensure cooperation among MUs.

(328) Within this protocol, an estimation algorithm is proposed to simulate the process of D2D LAN formation and, then, adjust the mechanism parameters to maintain cooperation.

**(329)** Simulation results show that the proposed framework can improve energy efficiency up to 36% relative to the traditional multicast scenario. Comments: To appear in the 50th Annual Conference on Information Sciences and Systems (CISS), Princeton, New Jersey, March 2016 Subjects: Information Theory (cs.IT); Computer Science and Game Theory (cs.GT); Networking and Internet Architecture (cs.NI) Cite as: arXiv:1601.06762 [cs.IT] (or arXiv:1601.06762v1 [cs.IT] for this version)

**Power Distribution in Randomized Weighted Voting: the Effects of the Quota Joel Oren, Yuval Filmus, Yair Zick, Yoram Bachrach (Submitted on 3 Aug 2014)**

(330) Authors study the Shapley value in weighted voting games.

(331) The Shapley value has been used as an index for measuring the power of individual agents in decision-making bodies and political organizations, where decisions are made by a majority vote process. We characterize the impact of changing the quota (i.e., the minimum number of seats in the parliament that are required to form a coalition) on the Shapley values of the agents.

(332) Contrary to previous studies, which assumed that the agent weights (corresponding to the size of a caucus or a political party) are fixed, Authors analyze new domains, in which the weights are stochastically generated, modelling, for example, elections processes.

(333) Authors examine a natural weight generation process: the Balls and Bins model, with uniform as well as exponentially decaying probabilities.

**(334)** Authors also analyze weights that admit a super-increasing sequence, answering several open questions pertaining to the Shapley values in such games. Subjects: Computer Science and Game Theory (cs.GT) Cite as: arXiv: 1408.0442 [cs.GT] (or arXiv:1408.0442v1 [cs.GT] for this version)

**The Pricing War Continues: On Competitive Multi-Item Pricing Omer Lev, Joel Oren, Craig Boutilier, and Jeffery S. Rosenschein (Submitted on 1 Aug 2014)**

(335) Authors study a game with \emph{strategic} vendors who own multiple items and a single buyer with a submodular valuation function. The goal of the vendors is to maximize their revenue via pricing of the items, given that the buyer will buy the set of items that maximizes his net payoff.

(336) Authors show this game may not always have a pure Nash equilibrium, in contrast to previous results for the special case where each vendor owns a single item.

(337) Authors do so by relating our game to an intermediate, discrete game in which the vendors only choose the available items, and their prices are set exogenously afterwards.

(338) Authors further make use of the intermediate game to provide tight bounds on the price of anarchy for the subset games that have pure Nash equilibria; Authors find that the optimal PoA reached in the previous special cases does not hold, but only a logarithmic one.

**(339)** Finally, Authors show that for a special case of submodular functions, efficient pure Nash equilibria always exist. Subjects: Computer Science and Game Theory (cs.GT); Multiagent Systems (cs.MA) Cite as: arXiv: 1408.0258 [cs.GT] (or arXiv: 1408.0258v1 [cs.GT] for this version)

**Quantum Discord:**

**Truthful Mechanisms for Competing Submodular Processes Allan Borodin, Mark Braverman, Brendan Lucier, Joel Oren (Submitted on 9 Feb 2012 (v1), last revised 25 Mar 2014 (this version, v4))**

(340) Motivated by applications to word-of-mouth advertising, Authors consider a game-theoretic scenario in which competing advertisers want to target initial adopters in a social network.

(341) Each advertiser wishes to maximize the resulting cascade of influence, modeled by a general network diffusion process.

(342) However, competition between products may adversely impact the rate of adoption for any given firm.

(343) The resulting framework gives rise to complex preferences that depend on the specifics of the stochastic diffusion model and the network topology.

(344) Authors study this model from the perspective of a central mechanism, such as a social networking platform, that can optimize seed placement as a service for the advertisers.

(345) Authors ask: given the reported demands of the competing firms, how should a mechanism choose seeds to maximize overall efficiency?

(346) Beyond the algorithmic problem, competition raises issues of strategic behaviour: rational agents should not be incentivized to underreport their budget demands.

(347) Authors show that when there are two players, the social welfare can be 2-approximated by a polynomial-time strategyproof mechanism.

(348) Mechanism is defined recursively, randomizing the order in which advertisers are allocated seeds according to a particular greedy method.

**(349)** For three or more players, Authors demonstrate that under additional assumptions (satisfied by many existing models of influence spread) there exists a simpler strategyproof ee−1-approximation mechanism; notably, this second mechanism is not necessarily strategyproof when there are only two players. Comments: The latest version contains a revised Section 4 and Appendix D Subjects: Computer Science and

Game Theory (cs.GT); Data Structures and Algorithms (cs.DS) Cite as: arXiv: 1202.2097 [cs.GT] (or arXiv:1202.2097v4 [cs.GT] for this version)

**Efficient Sum-Based Hierarchical smoothing Under \ell_1-Norm Siavosh Benabbas, Hyun Chul Lee, Joel Oren, Yuli Ye (Submitted on 8 Aug 2011)**

(350) Authors introduce a new regression problem which we call the Sum-Based Hierarchical Smoothing problem.

(351) Given a directed acyclic graph and a non-negative value, called target value, for each vertex in the graph, Authors wish to find non-negative values for the vertices satisfying a certain constraint while minimizing the distance of these assigned values and the target values in the lp-norm.

(352) The constraint is that the value assigned to each vertex should be no less than the sum of the values assigned to its children.

(353) Authors motivate this problem with applications in information retrieval and web mining.

(354) While our problem can be solved in polynomial time using linear programming, given the input size in these applications such a solution might be too slow.

(355) Authors mainly study the \ell_1-norm case restricting the underlying graphs to rooted trees. For this case we provide an efficient algorithm, running in O (n^2) time.

(356) While the algorithm is purely combinatorial, its proof of correctness is an elegant use of linear programming duality.

(357) Authors believe that our approach may be applicable to similar problems, where comparable hierarchical constraints are involved, e.g. considering the average of the values assigned to the children of each vertex.

**(358)** While similar in flavor to other smoothing problems like Isotonic Regression (see for example [Angelov et al. SODA'06]), our problem is arguably richer and theoretically more challenging. Subjects: Data Structures and Algorithms (cs.DS) Cite as: arXiv: 1108.1751 [cs.DS] (or arXiv:1108.1751v1 [cs.DS] for this version)

**Quantum discord of cosmic inflation: Can we show that CMB anisotropies are of quantum-mechanical origin? Jérôme Martin and Vincent Vennin Phys. Rev. D 93, 023505 – Published 12 January 2016**

(359) Authors investigate the quantumness of primordial cosmological fluctuations and its detectability.

(360) Quantum discord of inflationary perturbations is calculated for an arbitrary splitting of the system, and shown to be very large on super-Hubble scales.

(361) This entails the presence of large quantum correlations, due to the entangled production of particles with opposite momentums during inflation.

(362) To determine how this is reflected at the observational level, we study whether quantum correlators can be reproduced by a nondiscordant state, i.e. a state with vanishing discord that contains classical correlations only.

(363) Authors demonstrate that this can be done for the power spectrum, the price to pay being twofold: first, large errors in other two-point correlation functions that cannot however be detected since they are hidden in the decaying mode; second, the presence of intrinsic non-Gaussianity, the detectability of which remains to be determined but which could possibly rule out a nondiscordant description of the cosmic microwave background.

(364) If one abandons the idea that perturbations should be modeled by quantum mechanics and wants to use a classical stochastic formalism instead, Authors show that any two-point correlators on super-Hubble scales can be exactly reproduced regardless of the squeezing of the system.

**(365)** The latter becomes important only for higher order correlation functions that can be accurately reproduced only in the strong squeezing regime. DOI: http://dx.doi.org/10.1103/PhysRevD.93.023505© 2016 American Physical Society

**Quantum Information Processing January 2016, Volume 15, Issue 1, pp 291-300 First online: 18 November 2015 Geometric measure of quantum discord with weak measurements Lei Li, Qing-Wen Wang, Shu-Qian Shen, Ming Li**

(366) Super quantum discord based on weak measurements was introduced by Singh and Pati (Ann Phys 343:141–152, 2014).

(367) Authors propose a geometric way of quantifying quantum discord with weak measurements. It is shown that this geometric measure of quantum discord with weak measurements (GQDW) is linearly dependent on geometric measure of quantum discord (Dakic et al. in Phys Rev Lett 105:190502, 2010) and only captures partial quantumness of the states.

**(368)** It is found that the quantum correlation can be extracted by a sequence of infinitesimal weak measurements. Finally, the level surfaces of GQDW for Bell-diagonal states are depicted and the results are demonstrated by explicit example.

**Quantum discord length is enhanced while entanglement length is not by introducing disorder in a spin chain Debasis Sadhukhan, Sudipto Singha Roy, Debraj Rakshit, R. Prabhu, Aditi Sen(De), and Ujjwal Sen Phys. Rev. E 93, 012131 – Published 19 January 2016**

(369) Classical correlation functions of ground states typically decay exponentially and polynomially, respectively, for gapped and gapless short-range quantum spin systems.

(370) In such systems, entanglement decays exponentially even at the quantum critical points. However, quantum discord, an information-theoretic quantum correlation measure, survives long lattice distances.

(371) Authors investigate the effects of quenched disorder on quantum correlation lengths of quenched averaged entanglement and quantum discord, in the anisotropic XY and XYZ spin glass and random field chains.

**(372)** Authors find that there is virtually neither reduction nor enhancement in entanglement length while quantum discord length increases significantly with the introduction of the quenched disorder.

**Quantum Information Processing January 2016, Volume 15, Issue 1, pp 279-289 First online: 13 November 2015 A note on one-way quantum deficit and quantum discord Biao-Liang Ye , Shao-Ming Fei**

(373) One-way quantum deficit and quantum discord are two important measures of quantum correlations. We revisit the relationship between them in two-qubit systems.

(374) Authors investigate the conditions that both one-way quantum deficit and quantum discord have the same optimal measurement ensembles, and demonstrate that one-way quantum deficit can be derived from the quantum discord for a class of X states.

**(375)** Moreover, we give an explicit relation between one-way quantum deficit and entanglement of formation. Authors show that under phase damping channel both one-way quantum deficit and quantum discord evolve exactly in the same way for four parameter X states. Some examples are presented in details. Keywords One-way quantum deficit Quantum discord Entanglement of formation

**International Journal of Theoretical Physics First online: 07 January 2016 Quantum Discord and Entanglement of Quasi-Werner States Based on Bipartite Entangled Coherent States Manoj K. Mishra , Ajay K. Maurya, Hari Prakash**

(376) Present work is an attempt to compare quantum discord and quantum entanglement of quasi-Werner states formed with the four bipartite entangled coherent states (ECS) used recently for quantum teleportation of a qubit encoded in superposed coherent state.

(377) Out of these, the quasi-Werner states based on maximally ECS due to its invariant nature under local operation is independent of measurement basis and mean photon numbers, while for quasi-Werner states based on non-maximally ECS, it depends upon measurement basis as well as on mean photon number.

**(378)** However, for large mean photon numbers since non-maximally ECS becomes almost maximally entangled therefore dependence of quantum discord for non-maximally ECS based quasi-Werner states on the measurement basis disappears Keywords Quantum discord Quantum entanglement Coherent states Werner states International Journal of Theoretical Physics International Journal of Theoretical Physics Look

**Quantum Information Processing January 2016, Volume 15, Issue 1, pp 301-326 First online: 13 November 2015 Geometry and dynamics of one-norm geometric quantum discord Zhiming Huang, Daowen Qiu , Paulo Mateus**

(379) Authors investigate the geometry of one-norm geometric quantum discord and present a geometric interpretation of one-norm geometric quantum discord for a class of two-qubit states.

(380) It is found that one-norm geometric quantum discord has geometric behavior different from that described in Lang and Caves (Phys Rev Lett 105:150501, 2010), Li et al. (Phys Rev A 83:022321, 2011) and Yao et al. (Phys Lett A 376:358–364, 2012).

(381) Authors also compare the dynamics of the one-norm geometric quantum discord and other measures of quantum correlations under correlated noise.

(382) It is shown that different decoherent channels bring different influences to quantum correlations measured by concurrence, entropic quantum discord and geometric quantum discord, which depend on the memory parameter and decoherence parameter.

(383) Authors lay emphasis on the behaviors such as entanglement sudden death and sudden transition of quantum discord. Finally, we study the dynamical behavior of one-norm geometric quantum discord in one-dimensional anisotropic XXZ model by utilizing the quantum renormalization group method.

**(384)** It is shown that the one-norm geometric quantum discord demonstrates quantum phase transition through renormalization group approach. Keywords One-norm geometric quantum discord Geometry Correlated noise Quantum phase transition XXZ model Quantum Information Processing Quantum Information Processing Look

**Quantum Information Processing October 2013, Volume 12, Issue 10, pp 3305-3317 First online: 13 June 2013 The monogamy relation and quantum phase transition in one-dimensional anisotropic XXZ model Xue-Ke Song, Tao Wu , Liu Ye**

(385) In the paper, Authors have researched the monogamy relation and the quantum phase transition (QPT) in the anisotropic spin XXZ model by exploiting the quantum renormalization group method.

(386) Results show that there exits QPT after several iterations of renormalization in the present system.

(387) Authors we can find out that the monogamy inequality of entanglement of formation (EOF) and entropy quantum discord develop two saturated values which associate with spin-liquid and Néel phases after several iterations of the renormalization.

(388) Furthermore, Authors can also find out that the renormalization of EOF and entropy quantum discord violate the monogamy property while the renormalized geometric quantum discord obeys it no matter whether the QPT iterations are carried out.

**(389)** As a byproduct, the nonanalytic phenomenon and scaling behavior of the spin system are analyzed in detail. Keywords Quantum discord Monogamy relation Quantum phase transition Quantum Information Processing Quantum Information Processing Look

**Quantum Information Processing January 2016, Volume 15, Issue 1, pp 255-267 First online: 13 November 2015 Renormalization of the global quantum correlation and monogamy relation in the anisotropic Heisenberg XXZ model Meng Qin , Zhong-Zhou Ren, Xin Zhang**

(390) In this study, the global quantum correlation, monogamy relation and quantum phase transition of the Heisenberg XXZ model are investigated by the method of quantum renormalization group.

(391) Authors obtain, analytically, the expressions of the global negativity, the global measurement-induced disturbance and the monogamy relation for the system.

(392) The result shows that for a three-site block state, the partial transpose of an asymmetric block can get stronger entanglement than that of the symmetric one.

(393) The residual entanglement and the difference of the monogamy relation of measurement-induced disturbance show a scaling behavior with the size of the system becoming large.

**(394)** Moreover, the monogamy nature of entanglement measured by negativity exists in the model, while the nonclassical correlation quantified by measurement-induced disturbance violates the monogamy relation and demonstrates polygamy. Keywords Quantum correlation Monogamy relation Quantum renormalization group Quantum phase transition Quantum Information Processing Quantum Information Processing Look

**International Journal of Theoretical Physics April 2015, Volume 54, Issue 4, pp 1233-1244 First online: 12 September 2014 Nonclassicality Indicators of Quantum Phase Transition in the XY Model with Dzyaloshinskii-Moriya Interaction Xueke Song, Tao Wu, Shuai Xu, Jiadong Shi, Liu Ye**

(395) In this paper, performance of the important quantum correlation measures shared by the next-to-neighbor qubits of the XY model with Dzyaloshinskii-Moriya (DM) interaction has been investigated, employing quantum renormalization group method.

(396) Through analyzing the effect of DM interaction and anisotropy parameters on the quantum discord (QD) and measurement-induced disturbance (MID), Authors find that MID fails to symbolize the characteristics of quantum correlation for the system.

(397) While performance of the improved version of MID i.e. ameliorated MID (AMID), delivers more dependable consequences.

(398) On the other hand, take QD as an example we have illustrated that it can efficiently observe the quantum critical points associated with quantum phase transitions after several iterations of the renormalization.

**(399)** To serve as the further insight, the nonanalytic and scaling behaviors of the QD are also taken into consideration. Keywords Quantum phase transition DM interaction Quantum discord Quantum renormalization group method Measurement-induced disturbance International Journal of Theoretical Physics International Journal of Theoretical Physics Look

**International Journal of Theoretical Physics August 2014, Volume 53, Issue 8, pp 2769-2777 First online: 26 March 2014 Sudden Change of Quantum Discord in Qutrit-Qutrit System Under Depolarising Noise Liang Qiu , Gang Tang, Xianqing Yang, Zhipeng Xun, Bin Ye, Anmin Wang**

(400) The effects of decoherence on quantum discord of two-qutrit system under the influence of multilocal, collective and global depolarising noises are investigated. Sudden change of quantum discord with the decoherence parameter and the initial state parameter occurs for the qutrits being coupled to multilocal or global depolarising noises.

**(401)** On the other hand, decoherence cannot induce sudden change of quantum discord when the system is collectively coupled to the depolarising noise. In addition, the local unitary transformation can alter the region in which the phenomenon of sudden change exists. Keywords Quantum discord Sudden change Depolarising noise International Journal of Theoretical Physics International Journal of Theoretical Physics Look

**Quantum Information Processing February 2014, Volume 13, Issue 2, pp 259-271 First online: 08 October 2013 Quantum discord amplification of fermionic systems in an accelerated frame M. Ramzan**

(402) Quantum discord of fermionic systems in the relativistic regime, that is, beyond the single-mode approximation (SMA) is investigated.

(403) It is shown that quantum discord is amplified for the fermionic system in non-inertial frames irrespective of the choice of state, region and level of mixedness.

(404) This ensures that the phenomenon of amplification can actually happen in the relativistic regime.

(405) It is seen that quantum discord converges at infinite acceleration limit, which means that it becomes independent of qR (Unruh modes) beyond SMA.

(406) This implies that most of the tensor product structures already used in the literature to compute quantum field correlations in relativistic quantum information cannot give rise to physical results.

(407) The dynamics of quantum discord is investigated under amplitude damping, depolarizing and flipping channels.

(408) The vanishing behavior of quantum discord is seen for higher level of decoherence in the infinite acceleration limit.

(409) The depolarizing channel dominantly affects the fermionic quantum discord as compared to the amplitude damping channel. It means that the depolarizing channel has most destructive influence on the discord of the fermionic systems.

**(410)** Moreover, the effect of environment on the discord is much stronger than that of the acceleration of non-inertial frames. Keywords Quantum discord Single-mode approximation Fermionic systems Decoherence Quantum Information Processing Quantum Information Processing Look

**International Journal of Theoretical Physics November 2012, Volume 51, Issue 11, pp 3637-3646 First online: 19 July 2012 Quantum Discord for Two-Qubit System in a Symmetry-Broken Environment Jing Nie , Xiuyi Yang, Chong Li**

(411) In contrast with the entanglement, we study the quantum discord dynamics of the two-qubit system in a symmetry-broken environment consisting of a fermionic bath. The quantum discord decay induced by the bath is analysed.

**(412)** By considering the two qubits that are initially prepared in the different X-states, we find that the behaviors of quantum discord and entanglement are different; the robustness of quantum discord depends on the initial state prepared in. Keywords Quantum discord Symmetry-broken environment Entanglement International Journal of Theoretical Physics International Journal of Theoretical Physics Look

**International Journal of Theoretical Physics June 2015, Volume 54, Issue 6, pp 1958-1967 First online: 14 November 2014 The Quantum Correlations of the Werner State Under Quantum Decoherence Peng Xu, Tao Wu, Liu Ye**

(413) This paper examines the quantum correlations of the Werner state and demonstrates the relationship between the quantum discord and entanglement under decoherence.

(414) Authors also identify that when the discord takes the maximal value for a given state, and discuss the boundary value for entanglement and discord.

(415) As we all know that, in an ideal environment, the quantum discord is always larger than the entanglement of formation for almost all entangled states.

(416) But, the analysis in the article exposes that the entanglement may show stronger anti-interference than quantum discord in the noisy channel. In addition, the emphasis is also placed on the problem of the relation between the geometric discord (D g) and the entanglement of formation (EOF) which is customarily used as a measure of entanglement.

**(417)** At the same time, we also investigate the inequality $2D\ g \geq N\ 2$ and find that it is still be satisfied under the phase flip channel. Keywords Quantum correlations Entanglement Quantum discord International Journal of Theoretical Physics International Journal of Theoretical Physics Look

**International Journal of Theoretical Physics pp 1-10 First online: 19 January 2016 Pairwise Quantum Discord for a Symmetric Multi-Qubit System in Different Types of Noisy Channels You-Neng Guo, Ke Zeng , Guo-You Wang**

(418) Authors study the pairwise quantum discord (QD) for a symmetric multi-qubit system in different types of noisy channels, such as phase-flip, amplitude damping, phase-damping, and depolarizing channels.

(419) Using the QD and geometric quantum discord (GMQD) to quantify quantum correlations, some analytical and numerical results are presented.

(420) The results show that, the QD dynamics is strongly related to the number of spin particles N as well as the initial parameter $\theta$ of the one-axis twisting collective state. With the number of spin particles N increasing, the amount of the QD increases.

(421) However, when the amount of the QD arrives at a stable maximal value, the QD is independence of the number of spin particles N increasing.

**(422)** The behavior of the QD is symmetrical during a period $0 \leq \theta \leq 2\pi$. Moreover, we compare the QD dynamics with the GMQD for a symmetric multi-qubit system in different types of noisy channels. Keywords Pairwise quantum discord Geometric quantum discord One-axis twisting collective state International Journal of Theoretical Physics International Journal of Theoretical Physics Look

**Quantum Information Processing June 2014, Volume 13, Issue 6, pp 1437-1456 First online: 11 February 2014Quantum correlation via quantum coherence Chang-shui Yu, Yang Zhang, Haiqing Zhao**

(423) Quantum correlation includes quantum entanglement and quantum discord. Both entanglement and discord have a common necessary condition—quantum coherence or quantum superposition.

(424) In this paper, Authors attempt to give an alternative understanding of how quantum correlation is related to quantum coherence.

(425) Authors divide the coherence of a quantum state into several classes and find the complete coincidence between geometric (symmetric and asymmetric) quantum discords and some particular classes of quantum coherence.

(426) Authors propose a revised measure for total coherence and find that this measure can lead to a symmetric version of geometric quantum correlation, which is analytic for two qubits.

(427) In particular, this measure can also arrive at monogamy equality on the distribution of quantum coherence.

**(428)** Finally, Authors also quantify a **remaining type of quantum coherence** and find that for two qubits, it is directly connected with quantum nonlocality. Keywords Quantum correlation Quantum entanglement Quantum coherence Quantum Information Processing Quantum Information Processing Look

(429) **International Journal of Theoretical Physics September 2015, Volume 54, Issue 9, pp 3340-3349 First online: 25 February 2015 Revisiting Quantum Discord for Two-Qubit X States: The Error Bound to an Analytical Formula Min Namkung, Jinho Chang, Jaehee Shin, Younghun Kwon**

(430) In this article, Authors investigate the error bound of quantum discord, obtained from the analytic formula of Ali et al. (Phys. Rev. A 81, 042105, 2010) for the case of general X states and from the analytic formula of Fanchini et al. (Phys. Rev. A 81, 052107, 2010) for the case of symmetric X states.

**(431)** For this purpose Authors consider 3-element POVM. We find that the results of Ali et al. and Fanchini et al. may have the worst-case errors of 0.004565 and 0.0009, respectively. Keywords Quantum discord Quantum correlation International Journal of Theoretical Physics International Journal of Theoretical Physics Look

**The European Physical Journal D June 2014, 68:163 First online: 27 June 2014 Geometric quantum discord and entanglement between two atoms in Tavis-Cummings model with dipole-dipole interaction under intrinsic decoherence Kai-Ming Fan, Guo-Feng Zhang**

(432) Quantum correlation and entanglement, measured by geometric quantum discord (GQD) and concurrence respectively, between two identical two-level atoms are investigated in detail in Tavis-Cummings model with dipole-dipole interaction (DDI) under intrinsic decoherence.

(433) The results show that the phase decoherence rate makes the original harmonic vibration with respective to time decay to a stable value. DDI coupling enhances GQD and concurrence and even avoids entanglement sudden death (ESD).

(434) By the introduction of phase decoherence rate, the atom-light field coupling coefficient makes the decay of periodical curves quicker. Evolution periods are shortened as photon number increases, as well DDI and atomic-light field coupling become stronger.

**(435)** Moreover, both GQD and concurrence have nothing to do with frequency of cavity field and atoms for the resonant case. Keywords Quantum Optics The European Physical Journal D The European Physical Journal D Look

**Quantum Information Processing February 2014, Volume 13, Issue 2, pp 171-183 First online: 27 September 2013 Spontaneous emission can locally create quantum discord out of classical correlations Marek Gwóźdź, Lech Jakóbczyk**

(436) Authors consider local time evolution given by spontaneous emission in the system of independent two-level atoms.

(437) It is shown that all classically correlated initial states are driven into the states with transient non-zero quantum discord.

**(438)** Thus local creation of genuine quantum correlations can be observed in a simple physical system of non-interacting atoms which are not completely isolated from the environment. Keywords Quantum discord Spontaneous emission Local generation of quantum discord Quantum Information Processing Quantum Information Processing Look

**International Journal of Theoretical Physics pp 1-14 First online: 30 September 2015 Geometric Global Quantum Discord of Two-qubit X States Wen-Chao Qiang , Hua-Ping Zhang, Lei Zhang**

(439) In this paper, Authors find that the geometric global quantum discord proposed by Xu and the total quantum correlations proposed by Hassan and Joag are identical.

(440) Moreover, we work out the analytical formulas of the geometric global quantum discord and geometric quantum discord both for two-qubit X states, respectively.

(441) Authors further illustrate how to use these formulas to deal with a few particular examples. Authors also compare the results achieved by using three kinds of geometric quantum discords.

**(442)** The geometric quantum discord is verified as a tight lower bound of the geometric global quantum discord for two-qubit X states. Keywords Geometric global quantum discord Two-qubit system X states International Journal of Theoretical Physics International Journal of Theoretical Physics Look

**International Journal of Theoretical Physics January 2014, Volume 53, Issue 1, pp 320-328 First online: 02 October 2013 Open Access Quantum Discord for a Central Two-Qubit System Coupled to an XY Spin Chain with Energy Current Feng Zhong**

(443) Authors investigate the quantum discord dynamics of a two-qubit system coupled to an XY spin-chain environment with energy current.

(444) Authors compared dynamical behaviors of quantum discord under different system-environment couplings, the size of the degrees of freedom of the environment, the anisotropy parameter, and the energy current.

**(445)** The results indicate that the energy current can strongly suppress the quantum discord in the weak-coupling region, while it has no obvious effect on the quantum discord in the strong-coupling regime. Keywords Quantum discord Energy current Two-qubit system XY spin chain International Journal of Theoretical Physics International Journal of Theoretical Physics Look

**Control of an Aerial Manipulator using On-line Parameter Estimator for an Unknown Payload Hyeonbeom Lee, Suseong Kim, H. Jin Kim (Submitted on 10 Jan 2016)**

(446) This paper presents an estimation and control algorithm for an aerial manipulator using a hexacopter with a 2-DOF robotic arm.

(447) The unknown parameters of a payload are estimated by an on-line estimator based on parametrization of the aerial manipulator dynamics.

(448) With the estimated mass information and the augmented passivity-based controller, the aerial manipulator can fly with the unknown object.

(449) Simulation for an aerial manipulator is performed to compare estimation performance between the proposed control algorithm and conventional adaptive sliding mode controller.

**(450)** Experimental results show a successful flight of a custom-made aerial manipulator while the unknown parameters related to an additional payload were estimated satisfactorily. Comments: 2015 IEEE International Conference on Automation Science and Engineering Subjects: Robotics (cs.RO) Cite as: arXiv: 1601.02204 [cs.RO] (or arXiv: 1601.02204v1 [cs.RO] for this version)

**Weakly-supervised Disentangling with Recurrent Transformations for 3D View Synthesis Jimei Yang, Scott Reed, Ming-Hsuan Yang, Honglak Lee (Submitted on 5 Jan 2016)**

(451) An important problem for both graphics and vision is to synthesize novel views of a 3D object from a single image.

(452) This is particularly challenging due to the partial observability inherent in projecting a 3D object onto the image space, and the ill-posedness of inferring object shape and pose.

(453) However, Authors can train a neural network to address the problem if we restrict our attention to specific object categories (in our case faces and chairs) for which we can gather ample training data.

(454) In this paper, Authors propose a novel recurrent convolutional encoder-decoder network that is trained end-to-end on the task of rendering rotated objects starting from a single image.

(455) The recurrent structure allows our model to capture long-term dependencies along a sequence of transformations.

**(456)** Authors demonstrate the quality of its predictions for human faces on the Multi-PIE dataset and for a dataset of 3D chair models, and also show its ability to disentangle latent factors of variation (e.g., identity and pose) without using full supervision. Comments: This was published in NIPS 2015 conference Subjects: Learning (cs.LG); Artificial Intelligence (cs.AI); Computer Vision and Pattern Recognition (cs.CV) Cite as: arXiv: 1601.00706 [cs.LG] (or arXiv:1601.00706v1 [cs.LG] for this version

**Learning Transferrable Knowledge for Semantic Segmentation with Deep Convolutional Neural Network Seunghoon Hong, Junhyuk Oh, Bohyung Han, Honglak Lee (Submitted on 24 Dec 2015)**

(457) Authors propose a novel weakly-supervised semantic segmentation algorithm based on Deep Convolutional Neural Network (DCNN).

(458) Contrary to existing weakly-supervised approaches, our algorithm exploits auxiliary segmentation annotations available for different categories to guide segmentations on images with only image-level class labels.

(459) To make the segmentation knowledge transferrable across categories, we design a decoupled encoder-decoder architecture with attention model.

(460) In this architecture, the model generates spatial highlights of each category presented in an image using an attention model, and subsequently generates foreground segmentation for each highlighted region using decoder.

(461) Combining attention model, we show that the decoder trained with segmentation annotations in different categories can boost the performance of weakly-supervised semantic segmentation.

**(462)** The proposed algorithm demonstrates substantially improved performance compared to the state-of-the-art weakly-supervised techniques in challenging PASCAL VOC 2012 dataset when model is trained with the annotations in 60 exclusive categories in Microsoft COCO dataset. Subjects: Computer Vision and Pattern Recognition (cs.CV) Cite as: arXiv: 1512.07928 [cs.CV] (or arXiv:1512.07928v1 [cs.CV] for this version)

**Attribute2Image: Conditional Image Generation from Visual Attributes Xinchen Yan, Jimei Yang, Kihyuk Sohn, and Honglak Lee (Submitted on 2 Dec 2015)**

(463) This paper investigates a problem of generating images from visual attributes. Given the prevalent research for image recognition, the conditional image generation problem is relatively under-explored due to the challenges of learning a good generative model and handling rendering uncertainties in images.

(464) To address this, Authors propose a variety of attribute-conditioned deep variational auto-encoders that enjoy both effective representation learning and Bayesian modeling, from which images can be generated from specified attributes and sampled latent factors.

(465) Authors experiment with natural face images and demonstrate that the proposed models are capable of generating realistic faces with diverse appearance.

**(466)** Authors further evaluate the proposed models by performing attribute-conditioned image progression, transfer and retrieval. In particular, our generation method achieves superior performance in the retrieval experiment against traditional nearest-neighbor-based methods both qualitatively and quantitatively. Subjects: Learning (cs.LG); Artificial Intelligence (cs.AI); Computer Vision and Pattern Recognition (cs.CV) Cite as: arXiv: 1512.00570 [cs.LG] (or arXiv:1512.00570v1 [cs.LG] for this version)

**Position Estimation of Robotic Mobile Nodes in Wireless Testbed using GENI Ahmed Abdelhadi, Felipe Rechia, Arvind Narayanan, Thiago Teixeira, Ricardo Lent, Driss Benhaddou, Hyunwoo Lee, T. Charles Clancy (Submitted on 28 Nov 2015)**

(467) Authors present a low complexity experimental RF-based indoor localization system based on the collection and processing of WiFi RSSI signals and processing using a RSS-based multi-lateration algorithm to determine a robotic mobile node's location.

(468) Authors use a real indoor wireless testbed called w-iLab.t that is deployed in Zwijnaarde, Ghent, Belgium.

(469) One of the unique attributes of this testbed is that it provides tools and interfaces using Global Environment for Network Innovations (GENI) project to easily create reproducible wireless network experiments in a controlled environment.

(470) Authors provide a low complexity algorithm to estimate the location of the mobile robots in the indoor environment.

(471) In addition, we provide a comparison between some of our collected measurements with their corresponding location estimation and the actual robot location.

(472) The comparison shows accuracy between 0.65 and 5 meters. Subjects: Networking and Internet Architecture (cs.NI) Cite as: arXiv: 1511.08936 [cs.NI] (or arXiv:1511.08936v1 [cs.NI] for this version)

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**Dynamic Belief Fusion for Object Detection Hyungtae Lee, Heesung Kwon, Ryan M. Robinson, William d. Nothwang, Amar M. Marathe (Submitted on 10 Nov 2015)**

(477) A novel approach for the fusion of heterogeneous object detection methods is proposed. In order to effectively integrate the outputs of multiple detectors, the level of ambiguity in each individual detection score is estimated using the precision/recall relationship of the corresponding detector.

(478) The main contribution of the proposed work is a novel fusion method, called Dynamic Belief Fusion (DBF), which dynamically assigns probabilities to hypotheses (target, non-target, intermediate state (target or non-target)) based on confidence levels in the detection results conditioned on the prior performance of individual detectors.

(479) In DBF, a joint basic probability assignment, optimally fusing information from all detectors, is determined by the Dempster's combination rule, and is easily reduced to a single fused detection score.

**(480)** Experiments on ARL and PASCAL VOC 07 datasets demonstrate that the detection accuracy of DBF is considerably greater than conventional fusion approaches as well as individual detectors used for the fusion. Subjects: Computer Vision and Pattern Recognition (cs.CV) Cite as: arXiv: 1511.03183 [cs.CV] (or arXiv:1511.03183v1 [cs.CV] for this version)

**Towards Structured Deep Neural Network for Automatic Speech Recognition Yi-Hsiu Liao, Hung-yi Lee, Lin-shan Lee (Submitted on 8 Nov 2015)**

(481) In this paper authors propose the Structured Deep Neural Network (structured DNN) as a structured and deep learning framework.

(482) This approach can learn to find the best structured object (such as a label sequence) given a structured input (such as a vector sequence) by globally considering the mapping relationships between the structures rather than item by item.

(483) When automatic speech recognition is viewed as a special case of such a structured learning problem, where we have the acoustic vector sequence as the input and the phoneme label sequence as the output, it becomes possible to comprehensively learn utterance by utterance as a whole, rather than frame by frame.

(484) Structured Support Vector Machine (structured SVM) was proposed to perform ASR with structured learning previously, but limited by the linear nature of SVM.

(485) Here Authors propose structured DNN to use nonlinear transformations in multi-layers as a structured and deep learning approach.

(486) This approach was shown to beat structured SVM in preliminary experiments on TIMIT. Comments: arXiv admin note: text overlap with arXiv: 1506.01163 Subjects: Computation and Language

(cs.CL); Learning (cs.LG); Neural and Evolutionary Computing (cs.NE) Cite as: arXiv: 1511.02506 [cs.CL] (or arXiv: 1511.02506v1 [cs.CL] for this version)

**Resource Allocation Techniques for Wireless Powered Communication Networks with Energy Storage Constraint Hoon Lee, Kyoung-Jae Lee, Hanjin Kim, Bruno Clerckx, Inkyu Lee (Submitted on 7 Oct 2015)**

(487) This paper studies multi-user wireless powered communication networks, where energy constrained users charge their energy storages by scavenging energy of the radio frequency signals radiated from a hybrid access point (H-AP).

(488) The energy is then utilized for the users' uplink information transmission to the H-AP in time division multiple access mode.

(489) In this system, we aim to maximize the uplink sum rate performance by jointly optimizing energy and time resource allocation for multiple users in both infinite capacity and finite capacity energy storage cases.

(490) First, when the users are equipped with the infinite capacity energy storages, we derive the optimal downlink energy transmission policy at the H-AP. Based on this result analytical resource allocation solutions are obtained.

(491) Next, Authors propose the optimal energy and time allocation algorithm for the case where each user has finite capacity energy storage.

**(492)** Simulation results confirm that the proposed algorithms offer 30% average sum rate performance gain over conventional schemes. Subjects: Information Theory (cs.IT) Cite as: arXiv: 1510.01816 [cs.IT] (or arXiv: 1510.01816v1 [cs.IT] for this version)

**A Behavior Analysis-Based Game Bot Detection Approach Considering Various Play Styles Yeounoh Chung, Chang-yong Park, Noo-ri Kim, Hana Cho, Taebok Yoon, Hunjoo Lee, Jee-Hyong Lee (Submitted on 8 Sep 2015)**

(493) An approach for game bot detection in MMORPGs is proposed based on the analysis of game playing behavior.

(494) Since MMORPGs are large scale games, users can play in various ways. This variety in playing behavior makes it hard to detect game bots based on play behaviors. In order to cope with this problem, the proposed approach observes game playing behaviors of users and groups them by their behavioral similarities.

(495) Then, it develops a local bot detection model for each player group. Since the locally optimized models can more accurately detect game bots within each player group, the combination of those models brings about overall improvement.

(496) For a practical purpose of reducing the workloads of the game servers in service, the game data is collected at a low resolution in time.

(497) Behavioral features are selected and developed to accurately detect game bots with the low resolution data, considering common aspects of MMORPG playing.

**(498)** Through the experiment with the real data from a game currently in service, it is shown that the proposed local model approach yields more accurate results. Subjects: Learning (cs.LG); Artificial Intelligence (cs.AI) Journal reference: ETRI Journal 35.6 (2013): 1058-1067 DOI: 10.4218/etrij.13.2013.0049 Cite as: arXiv: 1509.02458 [cs.LG] (or arXiv: 1509.02458v1 [cs.LG] for this version)

**Bootstrapping Real-world Deployment of Future Internet Architectures Taeho Lee, Pawel Szalachowski, David Barrera, Adrian Perrig, Heejo Lee, David Watrin (Submitted on 10 Aug 2015)**

(499) The past decade has seen many proposals for future Internet architectures. Most of these proposals require substantial changes to the current networking infrastructure and end-user devices, resulting in a failure to move from theory to real-world deployment.

(500) This paper describes one possible strategy for bootstrapping the initial deployment of future Internet architectures by focusing on providing high availability as an incentive for early adopters.

(501) Through large-scale simulation and real-world implementation, Authors show that with only a small number of adopting ISPs, customers can obtain high availability guarantees.

**(502)** Authors discuss design, implementation, and evaluation of an availability device that allows customers to bridge into the future Internet architecture without modifications to their existing infrastructure. Subjects: Networking and Internet Architecture (cs.NI) Cite as: arXiv: 1508.02240 [cs.NI] (or arXiv:1508.02240v1 [cs.NI] for this version)

**Action-Conditional Video Prediction using Deep Networks in Atari Games Junhyuk Oh, Xiaoxiao Guo, Honglak Lee, Richard Lewis, Satinder Singh (Submitted on 31 Jul 2015 (v1), last revised 22 Dec 2015 (this version, v2))**

(503) Motivated by vision-based reinforcement learning (RL) problems, in particular Atari games from the recent benchmark Aracade Learning Environment (ALE), Authors consider spatio-temporal prediction problems where future (image-) frames are dependent on control variables or actions as well as previous frames.

(504) While not composed of natural scenes, frames in Atari games are high-dimensional in size, can involve tens of objects with one or more objects being controlled by the actions directly and many other objects being influenced indirectly, can involve entry and departure of objects, and can involve deep partial observability.

(505) Authors propose and evaluate two deep neural network architectures that consist of encoding, action-conditional transformation, and decoding layers based on convolutional neural networks and recurrent neural networks.

(506) Experimental results show that the proposed architectures are able to generate visually-realistic frames that are also useful for control over approximately 100-step action-conditional futures in some games.

**(507)** To the best of our knowledge, this paper is the first to make and evaluate long-term predictions on high-dimensional video conditioned by control inputs. Comments: Published at NIPS 2015 (Advances in Neural Information Processing Systems 28) Subjects: Learning (cs.LG); Artificial Intelligence (cs.AI); Computer Vision and Pattern Recognition (cs.CV) Cite as: arXiv:1507.08750 [cs.LG] (or arXiv:1507.08750v2 [cs.LG] for this version)

**A Multi-layered Acoustic Tokenizing Deep Neural Network (MAT-DNN) for Unsupervised Discovery of Linguistic Units and Generation of High Quality Features Cheng-Tao Chung, Cheng-Yu Tsai, Hsiang-Hung Lu, Yuan-ming Liou, Yen-Chen Wu, Yen-Ju Lu, Hung-yi Lee, Lin-shan Lee (Submitted on 7 Jun 2015)**

(508) This paper summarizes the work done by the authors for the Zero Resource Speech Challenge organized in the technical program of Interspeech 2015

(509) The goal of the challenge is to discover linguistic units directly from unlabeled speech data.

(510) The Multi-layered Acoustic Tokenizer (MAT) proposed in this work automatically discovers multiple sets of acoustic tokens from the given corpus. Each acoustic token set is specified by a set of hyperparameters that describe the model configuration.

(511) These sets of acoustic tokens carry different characteristics of the given corpus and the language behind thus can be mutually reinforced.

(512) The multiple sets of token labels are then used as the targets of a Multi-target DNN (MDNN) trained on low-level acoustic features. Bottleneck features extracted from the MDNN are used as feedback for the MAT and the MDNN itself.

**(513)** Authors call this iterative system the Multi-layered Acoustic Tokenizing Deep Neural Network (MAT-DNN) which generates high quality features for track 1 of the challenge and acoustic tokens for track 2 of the challenge. Comments: submitted to Interspeech 2015 Subjects: Computation and Language

(cs.CL); Learning (cs.LG); Neural and Evolutionary Computing (cs.NE) Cite as: arXiv: 1506.02327 [cs.CL] (or arXiv:1506.02327v1 [cs.CL] for this version)

**□ created Sep 1998, last update 24 Sep 1998 □ comments and suggestions send to petra.zdanska@jh-inst.cas.cz Quantum - Classical Correspondence Van Vleck - Gutzwiller Propagator Keywords: □ semiclassical propagator □ root searching □ stability (or monodromy) matrix □ frozen Gaussian approximation □ cellular dynamics**

**Semiclassical propagator. For models please see next paper.**

(514) In time domain formulation of quantum mechanics we are interested in the probability that the system arrives from a position q(0) to a position q(t) after a given time t.

(515) This probability is given by integral <q(1)|U|q(2)>, where U is the time-evolution operator. U=exp (-i/h H t), where H is the Hamiltonian operator of the system.

(516) The integral <q(0)|U|q(t)> is called Green function; the whole "matrix" of these integrals may be used to evaluate any time-dependent quantity like wavefunction, kinetic energy or correlation functions which in turn form an access to evaluating many kinds of spectra.

(517) The Green function can be expanded rigorously in terms of Feynmann path integral. It is an integral over all "paths" between two points q(0) and q(t);

$$\langle q(1)|\exp(-i/h\, H\, t)|q(2)\rangle = \lim_{N\to\infty} \prod_{k=1}^{N} \{m/[2\,\mathrm{Pi}\,h\,(tk-tk-1)]\}1/2 \exp[\, i/h\,(tk-tk-1)\,Lk]\, dq$$

(518) Here L is the classical Lagrangian for a given time step k. In classical mechanics there are allowed only a limited number of paths between the starting and the final positions.

(519) This path is called the classical orbit. The classical action along the classical orbit is the minimum among all other paths.

(520) We see that the exact quantum formula for the Green function count some contributions from all paths between the q(0) and q(t) but within the condition of the semiclassical limit the only path that contributes to this integral is the classical orbit.

(521) The integrant is highly oscillatory function and in these cases the good approximation is to consider only the regions where the exponent is stationary.

(522) The "stationary point" for the path integral is a classical orbit. The Green function simplifies to

$$[2\,\mathrm{Pi}\,i\,h\,]-1/2 \quad |d2S/dq(0)dq(t)|1/2 \exp\{\, i/h\, S[q(0),q(t)] - i \quad Pi/2\,\},$$

(523) Here summation is performed over all classical paths and is the Morse index which increases by one each time the path crosses a turning point.

(524) The prefactor is a differentiation of the classical action with respect to initial and final positions; it is called "Van Vleck determinant".

(525) To evaluate this quantity one needs to employ computationally difficult procedure of “root searching". For multidimensional system this procedure becomes worthless because of chaotic behavior of many systems.

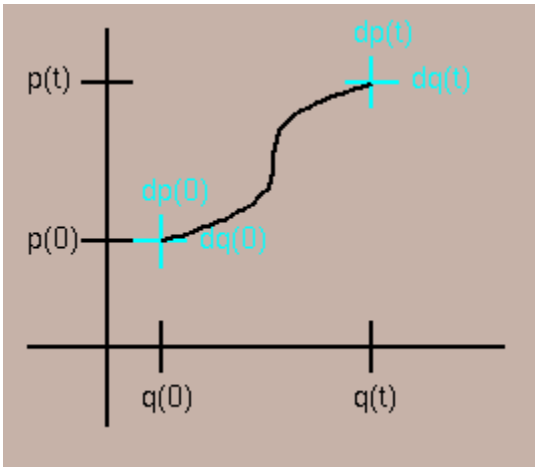
(526) Authors describe in the following the root searching procedure and two methods built upon the semiclassical propagator which add some more approximations to avoid the root searching; the Frozen Gaussian approximation (FGA) and the Cellular dynamics.

(527) Root Searching, Stability (or Monodromy) Matrix

(528) First, we need to find all classical orbits satisfying the boundary condition that it starts at q(0) and finishes at q(t).

(529) Second, the classical probability density [d2S/dq (0) dq (t)] can be found by examining the dynamics around each classical orbit.

(530) The probability density is equal to dp (0)/dq (t). The tool to evaluate this derivative is the stability matrix M (t). It is defined




$$\begin{pmatrix} dp(t) \\ dq(t) \end{pmatrix} = M(t) \begin{pmatrix} dp(0) \\ dq(0) \end{pmatrix}.$$

(531) This matrix fixes the connection between small variations of initial positions and momenta with small variation of final positions and momenta.

(532) Note that this matrix diverges whenever the trajectories do not create cones along the central orbit but escape away; this case of chaotic behavior. The above equation is written

$$\begin{pmatrix} dp(t) \\ dq(t) \end{pmatrix} = \begin{pmatrix} dp(t)/dp(0) & dp(t)/dq(0) \\ dq(t)/dp(0) & dq(t)/dq(0) \end{pmatrix} \begin{pmatrix} dp(0) \\ dq(0) \end{pmatrix}.$$

(533) We see that the classical probability density is fixed by the element M21. We are allowed to perform a differentiation with respect to time to split the cone of classical orbits to small time intervals.

(534)

$$\frac{d}{dt}\begin{pmatrix} dp(t) \\ dq(t) \end{pmatrix} = \begin{pmatrix} d2p(t)/dt\, dp(0) & d2p(t)/dt\, dq(0) \\ d2q(t)/dt\, dp(0) & d2q(t)/dt\, dq(0) \end{pmatrix} \begin{pmatrix} dp(0) \\ dq(0) \end{pmatrix}.$$

For small intervals t-dt -> t thus;

$$\begin{pmatrix} dp(t) \\ dq(t) \end{pmatrix} \sim \begin{pmatrix} d[dp(t)/dt]/dp(t) & d[dp(t)/dt]/dq(t) \\ d[dq(t)/dt]/dp(t) & d[dq(t)/dt]/dq(t) \end{pmatrix} \begin{pmatrix} dp(t-dt) \\ dq(t-dt) \end{pmatrix} dt.$$

We substitute from Hamilton's equations of motion;

/ dp(t) \ / (-d2H/d2p)(t) (-d2H/dp dq)(t) \ / dp(t-dt) \
| | ~ | | | | dt
\ dq(t) / \ (d2H/dq dp)(t) (d2H/dq2)(t) / \ dq(t-dt) / .

Now, we have the stability matrix for small time step along the classical orbit. The stability matrix for the whole time interval along the classical orbit is the product of contributions by each time step

M(t) = M(0->dt) M(dt->2dt) ... M(t-dt->t) .

**Frozen Gaussian Approximation (FGA)**

(535) The evaluation of the Green function may be demanding for case of initial or final wavefunctions being broad in positional coordinate. Even more computational difficulties may appear when calculating systems with many degrees of freedom. Finally, it is a chaotic behavior of classical dynamics of large systems which disallow the straightforward application of the Van Vleck propagator.

(536) Any initial wavefunction can be written as a sum of Gaussians. We are allowed to propagate each Gaussian separately and construct the final wavefunction as a simple sum of the individual resulting wavefunctions since the quantum mechanics is linear.

(537) We approximate the propagation of each Gaussian using the semiclassical Green function. To the initial Gaussian

Psi(q,t=0) = 4 (a/Pi) exp[ -a/2 (q-qc)2 + i/h pc (q-qc) ]

the semiclassical propagator is applied which leads to the following formula for the final wavefunction; Psi[q(t),t] ~

q(0) |d2S/dq(0)dq(t)|1/2 exp{ -a/2 [q(0)-qc]2 + i/h pc [q(0)-qc] } exp{ i/h S[q(0),q(t)] - i Pi/2 } .

(538) Now, we are still at the level of semiclassical approximation. The FGA moreover expands the action S[q(0),q(t)] quadratically about the central orbit.

(539) Note that this restriction on one hand spoils the semiclassical approximation of the Green function which is itself more precise but on the other hand this approximation is comparable or better then all time-independent semiclassical approximations which are described elsewhere in these pages. Why it could be? The time-independent approach uses a priori only a single classical orbit.

(540) The time-dependent approach is more precise as it employs the broad initial wavepacket and therefore many classical orbits.

(541) The FGA selects the leading classical orbit from the initial wavefunction but still includes also the other orbits but let them propagate with some restriction to the central orbit.

(542) Therefore the FGA is still on the level of time-independent semi classics and is even better.

(543) The FGA, e.g., automatically uniformize its solution near the turning points where the time-independent approach diverges in general.

(544) The FGA approximation to the wavefunction at a time t is the Gaussian wavepacket

Psi[q,t] ~ exp{ip(t)/h [q-q(t)] - a(t)/2 [q-q(t)]2}

a(t) = ip(t)/h [ M11(t) a h - iM12(t) ] / [ M21(t) a h - iM22(t) ] .

(545) q(t), p(t), and M(t) correspond to the leading classical orbit. There is no need to care about the other orbits since they are involved implicitly in the above result.

**Cellular Dynamics**

(546) Cellular dynamics is a more precise method to propagate the initial
(547) Gaussian. (We remind that any wavefunction can be initially resolved to linear combination of gaussian wavepackets.)
(548) The Gaussian is discretized in the q,p space and each "cell" is propagated separately via classical dynamics of a guiding orbit plus a cone of satellite trajectories which are forced to follow the quadratic expansion of classical action about the guiding orbit.

@@@

issues and proofs that the sets of gates that we use are (=) universal. Another section is devoted to (e) a rigorous proof that fault tolerance can be achieved in the presence of (e) general non probabilistic noise.

(8) The systematic structure of the fault tolerant procedures for (e) polynomial codes is explained in length. The proof that the concatenation scheme works is written in a clearer way. The paper also contains new and significantly simpler proofs for (e) most of the known results which we use. For example, authors give a simple proof that it suffices to correct (e&eb) bit and phase flips.

(9) They significantly simplify Calderbank and Shor's original proof of (e) the correctness of CSS codes. Authors also give a simple proof of the fact that two-qubit gates are (=) universal. The paper thus provides a self contained and complete proof for (e) universal fault tolerant quantum computation. Cite as**:arXiv: quant-ph/9906129 (or arXiv: quant-ph/9906129v1 for this version)**

## NOTATION

### Module One

R Raussendorf1, J Harrington1 and K Goyal2 describe a fault-tolerant version of (e) the one-way quantum computer using (e) a cluster state in (eb) three spatial dimensions

$G_{13}$ : Category one of one-way quantum computer using (e) a cluster state in (eb) three spatial dimensions

$G_{14}$ : Category two of SAS

$G_{15}$ : Category three of SAS

$T_{13}$ : Category one of fault-tolerant version

$T_{14}$ : Category two of SAS

$T_{15}$ : Category three of SAS

### Module Two

R Raussendorf1, J Harrington1 and K Goyal2 describe a fault-tolerant version of the one-way quantum computer using (e) a cluster state in (eb) three spatial dimensions

$G_{16}$ : Category one of cluster state in three spatial dimensions

$G_{17}$ : Category two of SAS

$G_{18}$ : Category three of SAS

$T_{16}$ : Category one of fault-tolerant version of the one-way quantum computer

$T_{17}$ : Category two of SAS

$T_{18}$ : Category three of SAS

### Module three

Topologically protected quantum gates are realized by (e) choosing appropriate boundary conditions on (eb) the cluster. They provide (eb) equivalence transformations for (e) these boundary conditions that can be used to simplify fault-tolerant circuits and to (e) derive circuit identities in a topological manner

$G_{20}$ : Category one of choosing appropriate boundary conditions on (eb) the cluster. They provide (eb) equivalence transformations for (e) these boundary conditions that can be used to simplify fault-tolerant circuits and to (e) derive circuit identities in a topological manner

$G_{21}$ : Category two of SAS

$G_{22}$ : Category three of SAS

$T_{20}$ : Category one of Topologically protected quantum gates $T_{21}$ : Category two of SAS

$T_{22}$ : Category three of SAS

**Module four**

Topologically protected quantum gates are realized by choosing appropriate boundary conditions on (eb) the cluster.

They provide (eb) equivalence transformations for (e) these boundary conditions that can be used to simplify fault-tolerant circuits and to (e) derive circuit identities in a topological manner

$G_{24}$ : Category one of Topologically protected quantum gates are realized by choosing appropriate boundary conditions

$G_{25}$ : Category two of SAS

$G_{26}$ : Category three of SAS

$T_{24}$ : Category one of cluster(s)

$T_{25}$ : Category two of SAS

$T_{26}$ : Category three of SAS

**Module five**

They provide (eb) equivalence transformations for (e) these boundary conditions that can be used to simplify fault-tolerant circuits and to (e) derive circuit identities in a topological manner

$G_{28}$ : Category one of boundary conditions that can be used to simplify fault-tolerant circuits and to (e) derive circuit identities in a topological manner

$G_{29}$ : Category two of SAS

$G_{30}$ : Category three of SAS

$T_{28}$ : Category one of equivalence transformations

$T_{29}$ : Category two of SAS

$T_{30}$ : Category three of SAS

**Module six**

They provide equivalence transformations for these boundary conditions that can be used to simplify fault-tolerant circuits and to (e) derive circuit identities in a topological manner

$G_{32}$ : Category one of equivalence transformations for these boundary conditions

$G_{33}$ : Category two of SAS

$G_{34}$ : Category three of SAS

$T_{32}$ : Category one of simplify fault-tolerant circuits and to (e) derive circuit identities in a topological manner

$T_{33}$ : Category two of SAS

$T_{34}$ : Category three of SAS

**Module seven**

They provide equivalence transformations for these boundary conditions that can be used to simplify fault-tolerant circuits and to (e) derive circuit identities in a topological manner

$G_{36}$ : Category one of provide equivalence transformations for these boundary conditions that can be used to simplify fault-tolerant circuits

$G_{37}$ : Category two of SAS

$G_{38}$ : Category three of SAS

$T_{36}$ : Category one of circuit identities in a topological manner

$T_{37}$ : Category two of SAS

$T_{38}$ : Category three of SAS

**Module eight**

The **spatial dimensionality of the scheme** can be reduced to (e&eb) two by converting one spatial axis of the cluster into (e&eb) time.
The error threshold is 0.75% for (e) each source in an error model with (e&eb) preparation, gate, storage and measurement errors.
The operational overhead is (=) poly-logarithmic in the circuit size

$G_{40}$ : Category one of **spatial dimensionality of the scheme;** two by converting one spatial axis of the cluster into (e&eb) time

$G_{41}$ : Category two of SAS

$G_{42}$ : Category three of SAS

$T_{40}$ : Category one of two by converting one spatial axis of the cluster into (e&eb) time.; **spatial dimensionality of the scheme**

$T_{41}$ : Category two of SAS

$T_{42}$ : Category three of SAS

**Module Nine**

**spatial dimensionality of the scheme can be reduced to two by converting one spatial axis of the cluster** into (e&eb) time

$G_{44}$ : Category one of **spatial dimensionality of the scheme can be reduced to two by converting one**

**spatial axis of the cluster; time**

$G_{45}$ : Category two of SAS

$G_{46}$ : Category three of SAS

$T_{44}$ : Category one of time; **spatial dimensionality of the scheme can be reduced to two by converting one spatial axis of the cluster**

$T_{45}$ : Category two of SAS

$T_{46}$ : Category three of SAS

**The Coefficients:**

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}\ ,\ (b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)},$
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)},\ (b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}\ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}\ ,$
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)},$
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)},$
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)},$
are Dissipation coefficients

**Module Numbered One**

**The differential system of this model is now (Module Numbered one)**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \quad 1$$
$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad 2$$
$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15} \quad 3$$
$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13} \quad 4$$
$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14} \quad 5$$
$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)\right]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14}, t) =$ First augmentation factor
$-(b''_{13})^{(1)}(G, t) =$ First detritions factor

**Module Numbered Two**

**The differential system of this model is now ( Module numbered two)**

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17},t)\right]G_{16} \quad 7$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17},t)\right]G_{17} \quad 8$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17},t)\right]G_{18} \quad 9$$

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\left((G_{19}),t\right)\right]T_{16} \quad 10$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\left((G_{19}),t\right)\right]T_{17} \quad 11$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\left((G_{19}),t\right)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17},t) =$ First augmentation factor

$-(b''_{16})^{(2)}\left((G_{19}),t\right) =$ First detritions factor

**Module Numbered Three**

**The differential system of this model is now (Module numbered three)**

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21},t)\right]G_{20} \quad 13$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21},t)\right]G_{21} \quad 14$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21},t)\right]G_{22} \quad 15$$

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23},t)\right]T_{20} \quad 16$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23},t)\right]T_{21} \quad 17$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23},t)\right]T_{22} \quad 18$$

$+(a''_{20})^{(3)}(T_{21},t) =$ First augmentation factor

$-(b''_{20})^{(3)}(G_{23},t) =$ First detritions factor

**Module Numbered Four**

**The differential system of this model is now (Module numbered Four)**

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right]G_{24} \quad 19$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25},t)\right]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25},t)\right]G_{26} \quad 21$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27}),t\right)\right]T_{24} \quad 22$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27}),t\right)\right]T_{25} \quad 23$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27}),t\right)\right]T_{26} \quad 24$$

$+(a''_{24})^{(4)}(T_{25},t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}\left((G_{27}),t\right) =$ **First detritions factor**

**Module Numbered Five**:

**The differential system of this model is now (Module number five)**

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29},t)\right]G_{28} \quad 25$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29},t)\right]G_{29} \quad 26$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29},t)\right]G_{30} \quad 27$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}\left((G_{31}),t\right)\right]T_{28} \quad 28$$

$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}((G_{31}),t)\right]T_{29}$ 29

$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31}),t)\right]T_{30}$ 30

$+(a''_{28})^{(5)}(T_{29},t) =$ **First augmentation factor**

$-(b''_{28})^{(5)}((G_{31}),t) =$ **First detritions factor**

**Module Numbered Six**

**The differential system of this model is now (Module numbered Six)**

$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33},t)\right]G_{32}$ 31

$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33},t)\right]G_{33}$ 32

$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33},t)\right]G_{34}$ 33

$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35}),t)\right]T_{32}$ 34

$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}((G_{35}),t)\right]T_{33}$ 35

$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35}),t)\right]T_{34}$ 36

$+(a''_{32})^{(6)}(T_{33},t) =$ **First augmentation factor**

**Module Numbered Seven:**

**The differential system of this model is now (Seventh Module)**

$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right]G_{36}$ 37

$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37},t)\right]G_{37}$ *38*

$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37},t)\right]G_{38}$ *39*

$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39}),t)\right]T_{36}$ *40*

$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}((G_{39}),t)\right]T_{37}$ *41*

$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39}),t)\right]T_{38}$ *42*

$+(a''_{36})^{(7)}(T_{37},t) =$ **First augmentation factor**

**Module Numbered Eight**

The differential system of this model is now

$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right]G_{40}$ *43*

$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41},t)\right]G_{41}$ *44*

$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41},t)\right]G_{42}$ *45*

$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43}),t)\right]T_{40}$ *46*

$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}((G_{43}),t)\right]T_{41}$ *47*

$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43}),t)\right]T_{42}$ *48*

**Module Numbered Nine**

The differential system of this model is now

$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44}$ 49

$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45}$ 50

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}\left((G_{47}),t\right)\right]T_{44} \quad 52$$

$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}\left((G_{47}),t\right)\right]T_{45} \quad 53$$

$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}\left((G_{47}),t\right)\right]T_{46} \quad 54$$

$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}\left((G_{47}),t\right) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)}\boxed{+(a''_{13})^{(1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)}\boxed{+(a''_{14})^{(1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)}\boxed{+(a''_{15})^{(1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15} \quad 57$$

Where $\boxed{(a''_{13})^{(1)}(T_{14},t)}$, $\boxed{(a''_{14})^{(1)}(T_{14},t)}$, $\boxed{(a''_{15})^{(1)}(T_{14},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7)}(T_{37},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8)}(T_{41},t)}$ $\boxed{+(a''_{41})^{(8,8)}(T_{41},t)}$ $\boxed{+(a''_{42})^{(8,8)}(T_{41},t)}$ are eight augmentation coefficient for 1,2,3

$\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)}\boxed{-(b''_{13})^{(1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13} \quad 58$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)}\boxed{-(b''_{14})^{(1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14} \quad 59$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)}\boxed{-(b''_{15})^{(1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15} \quad 60$$

Where $\boxed{-(b''_{13})^{(1)}(G,t)}$, $\boxed{-(b''_{14})^{(1)}(G,t)}$, $\boxed{-(b''_{15})^{(1)}(G,t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8)}(G_{43},t)}$ $\boxed{-(b''_{41})^{(8,8)}(G_{43},t)}$ $\boxed{-(b''_{42})^{(8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \begin{bmatrix} (a'_{16})^{(2)}\boxed{+(a''_{16})^{(2)}(T_{17},t)} & \boxed{+(a''_{13})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{16} \quad 61$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \begin{bmatrix} (a'_{17})^{(2)}\boxed{+(a''_{17})^{(2)}(T_{17},t)} & \boxed{+(a''_{14})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{17} \quad 62$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \begin{bmatrix} (a'_{18})^{(2)}\boxed{+(a''_{18})^{(2)}(T_{17},t)} & \boxed{+(a''_{15})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{18} \quad 63$$

Where $\boxed{+(a''_{16})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17},t)}$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{13})^{(1,1,)}(T_{14},t)$ , $+(a''_{14})^{(1,1,)}(T_{14},t)$ , $+(a''_{15})^{(1,1,)}(T_{14},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,3)}(T_{21},t)$ , $+(a''_{21})^{(3,3,3)}(T_{21},t)$ , $+(a''_{22})^{(3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)$ , $+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)$ , $+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{36})^{(7,7,7)}(T_{37},t)$ , $+(a''_{37})^{(7,7,7)}(T_{37},t)$ , $+(a''_{38})^{(7,7,7)}(T_{37},t)$ are seventh augmentation coefficient for category 1, 2 and 3

$+(a''_{40})^{(8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8)}(T_{41},t)$ , $+(a''_{42})^{(8,8,8)}(T_{41},t)$ are eight augmentation coefficient for category 1, 2 and 3

$+(a''_{44})^{(9,9)}(T_{45},t)$ , $+(a''_{45})^{(9,9)}(T_{45},t)$ , $+(a''_{46})^{(9,9)}(T_{45},t)$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \begin{bmatrix} (b'_{16})^{(2)} & -(b''_{16})^{(2)}(G_{19},t) & -(b''_{13})^{(1,1,)}(G,t) & -(b''_{20})^{(3,3,3,)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6)}(G_{35},t) \\ -(b''_{36})^{(7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9)}(G_{47},t) \end{bmatrix} T_{16} \quad 64$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)} & -(b''_{17})^{(2)}(G_{19},t) & -(b''_{14})^{(1,1,)}(G,t) & -(b''_{21})^{(3,3,3,)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6)}(G_{35},t) \\ -(b''_{37})^{(7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9)}(G_{47},t) \end{bmatrix} T_{17} \quad 65$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)} & -(b''_{18})^{(2)}(G_{19},t) & -(b''_{15})^{(1,1,)}(G,t) & -(b''_{22})^{(3,3,3,)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6)}(G_{35},t) \\ -(b''_{38})^{(7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9)}(G_{47},t) \end{bmatrix} T_{18} \quad 66$$

where $-(b''_{16})^{(2)}(G_{19},t)$ , $-(b''_{17})^{(2)}(G_{19},t)$ , $-(b''_{18})^{(2)}(G_{19},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{13})^{(1,1,)}(G,t)$, $-(b''_{14})^{(1,1,)}(G,t)$, $-(b''_{15})^{(1,1,)}(G,t)$ are second detrition coefficients for category 1,2 and 3

$-(b''_{20})^{(3,3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,)}(G_{23},t)$ are third detrition coefficients for category 1,2 and 3

$-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1,2 and 3

$-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1,2 and 3

$-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients for category 1,2 and 3

$-(b''_{36})^{(7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2 and 3

$-(b''_{40})^{(8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1,2 and 3

$-(b''_{44})^{(9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt}=(a_{20})^{(3)}G_{21}-\begin{bmatrix}(a'_{20})^{(3)} & +(a''_{20})^{(3)}(T_{21},t) & +(a''_{16})^{(2,2,2)}(T_{17},t) & +(a''_{13})^{(1,1,1,)}(T_{14},t)\\ +(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t) & \\ +(a''_{36})^{(7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9)}(T_{45},t) & \end{bmatrix}G_{20} \quad (67)$$

$$\frac{dG_{21}}{dt}=(a_{21})^{(3)}G_{20}-\begin{bmatrix}(a'_{21})^{(3)} & +(a''_{21})^{(3)}(T_{21},t) & +(a''_{17})^{(2,2,2)}(T_{17},t) & +(a''_{14})^{(1,1,1,)}(T_{14},t)\\ +(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t) & \\ +(a''_{37})^{(7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9)}(T_{45},t) & \end{bmatrix}G_{21} \quad (68)$$

$$\frac{dG_{22}}{dt}=(a_{22})^{(3)}G_{21}-\begin{bmatrix}(a'_{22})^{(3)} & +(a''_{22})^{(3)}(T_{21},t) & +(a''_{18})^{(2,2,2)}(T_{17},t) & +(a''_{15})^{(1,1,1,)}(T_{14},t)\\ +(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t) & \\ +(a''_{38})^{(7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9)}(T_{45},t) & \end{bmatrix}G_{22} \quad (69)$$

$+(a''_{20})^{(3)}(T_{21},t)$, $+(a''_{21})^{(3)}(T_{21},t)$, $+(a''_{22})^{(3)}(T_{21},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2)}(T_{17},t)$ are second augmentation coefficients for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,)}(T_{14},t)$ are third augmentation coefficients for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficients for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficients for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficients for category 1, 2 and 3

$+(a''_{36})^{(7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1, 2 and 3

$+(a''_{40})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8)}(T_{41},t)$ are eight augmentation coefficients for category 1, 2 and 3

$+(a''_{44})^{(9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt}=(b_{20})^{(3)}T_{21}-\begin{bmatrix}(b'_{20})^{(3)} & -(b''_{20})^{(3)}(G_{23},t) & -(b''_{16})^{(2,2,2)}(G_{19},t) & -(b''_{13})^{(1,1,1,)}(G,t)\\ -(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t) & \\ -(b''_{36})^{(7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9)}(G_{47},t) & \end{bmatrix}T_{20} \quad (70)$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b'_{21})^{(3)}\boxed{-(b''_{21})^{(3)}(G_{23},t)}\boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}\boxed{-(b''_{14})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)}\boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)}\boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7)}(G_{39},t)}\boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)}\boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{21} \quad (71)$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)}\boxed{-(b''_{22})^{(3)}(G_{23},t)}\boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}\boxed{-(b''_{15})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)}\boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)}\boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)}\boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)}\boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{22} \quad (72)$$

$\boxed{-(b''_{20})^{(3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3)}(G_{23},t)}$ *are first detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}$ *are second detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{13})^{(1,1,1,)}(G,t)}, \boxed{-(b''_{14})^{(1,1,1,)}(G,t)}, \boxed{-(b''_{15})^{(1,1,1,)}(G,t)}$ *are third detrition coefficients for category 1,2 and 3*

$\boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)}, \boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)}, \boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)}$ *are fourth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)}, \boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)}, \boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)}$ *are fifth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)}, \boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)}, \boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)}$ *are sixth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{36})^{(7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{37})^{(7,7,7,7)}(G_{39},t)}\boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)}, \boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)}, \boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \begin{bmatrix} (a'_{24})^{(4)}\boxed{+(a''_{24})^{(4)}(T_{25},t)}\boxed{+(a''_{28})^{(5,5,)}(T_{29},t)}\boxed{+(a''_{32})^{(6,6,)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)}\boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)}\boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)}\boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)}\boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{24} \quad (73)$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \begin{bmatrix} (a'_{25})^{(4)}\boxed{+(a''_{25})^{(4)}(T_{25},t)}\boxed{+(a''_{29})^{(5,5,)}(T_{29},t)}\boxed{+(a''_{33})^{(6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)}\boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)}\boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)}\boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)}\boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{25} \quad (74)$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \begin{bmatrix} (a'_{26})^{(4)}\boxed{+(a''_{26})^{(4)}(T_{25},t)}\boxed{+(a''_{30})^{(5,5,)}(T_{29},t)}\boxed{+(a''_{34})^{(6,6,)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)}\boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)}\boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)}\boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)}\boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{26} \quad (75)$$

$\boxed{(a''_{24})^{(4)}(T_{25},t)}, \boxed{(a''_{25})^{(4)}(T_{25},t)}, \boxed{(a''_{26})^{(4)}(T_{25},t)}$ *are first augmentation coefficients category* 1, 2 3

$\boxed{+(a''_{28})^{(5,5,)}(T_{29},t)}, \boxed{+(a''_{29})^{(5,5,)}(T_{29},t)}, \boxed{+(a''_{30})^{(5,5,)}(T_{29},t)}$ *are second augmentation*

$coefficient\ \ for\ category\ 1,2\ and\ 3$

$+(a_{32}'')^{(6,6,)}(T_{33},t)$, $+(a_{33}'')^{(6,6,)}(T_{33},t)$, $+(a_{34}'')^{(6,6,)}(T_{33},t)$ $are\ third\ \ augmentation$

$coefficient\ for\ category\ 1,2\ and\ 3$

$+(a_{13}'')^{(1,1,1,1)}(T_{14},t)$, $+(a_{14}'')^{(1,1,1,1)}(T_{14},t)$, $+(a_{15}'')^{(1,1,1,1)}(T_{14},t)$

$are\ fourth\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$

$+(a_{16}'')^{(2,2,2,2)}(T_{17},t)$, $+(a_{17}'')^{(2,2,2,2)}(T_{17},t)$, $+(a_{18}'')^{(2,2,2,2)}(T_{17},t)$

$are\ fifth\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$

$+(a_{20}'')^{(3,3,3,3)}(T_{21},t)$, $+(a_{21}'')^{(3,3,3,3)}(T_{21},t)$, $+(a_{22}'')^{(3,3,3,3)}(T_{21},t)$

$are\ sixth\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$

$+(a_{36}'')^{(7,7,7,7,7)}(T_{37},t)$, $+(a_{37}'')^{(7,7,7,7,7)}(T_{37},t)$, $+(a_{38}'')^{(7,7,7,7,7)}(T_{37},t)$

$are\ seventh\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$

$+(a_{40}'')^{(8,8,8,8,8)}(T_{41},t)$, $+(a_{41}'')^{(8,8,8,8,8)}(T_{41},t)$, $+(a_{42}'')^{(8,8,8,8,8)}(T_{41},t)$

$are\ eighth\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$

$+(a_{46}'')^{(9,9,9,9)}(T_{45},t)$, $+(a_{45}'')^{(9,9,9,9)}(T_{45},t)$, $+(a_{44}'')^{(9,9,9,9)}(T_{45},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt}=(b_{24})^{(4)}T_{25}-\begin{bmatrix}(b_{24}')^{(4)}&-(b_{24}'')^{(4)}(G_{27},t)&-(b_{28}'')^{(5,5,)}(G_{31},t)&-(b_{32}'')^{(6,6,)}(G_{35},t)\\&-(b_{13}'')^{(1,1,1,1)}(G,t)&-(b_{16}'')^{(2,2,2,2)}(G_{19},t)&-(b_{20}'')^{(3,3,3,3)}(G_{23},t)\\&-(b_{36}'')^{(7,7,7,7,7)}(G_{39},t)&-(b_{40}'')^{(8,8,8,8,8)}(G_{43},t)&-(b_{44}'')^{(9,9,9,9)}(G_{47},t)\end{bmatrix}T_{24} \quad 76$$

$$\frac{dT_{25}}{dt}=(b_{25})^{(4)}T_{24}-\begin{bmatrix}(b_{25}')^{(4)}&-(b_{25}'')^{(4)}(G_{27},t)&-(b_{29}'')^{(5,5,)}(G_{31},t)&-(b_{33}'')^{(6,6,)}(G_{35},t)\\&-(b_{14}'')^{(1,1,1,1)}(G,t)&-(b_{17}'')^{(2,2,2,2)}(G_{19},t)&-(b_{21}'')^{(3,3,3,3)}(G_{23},t)\\&-(b_{37}'')^{(7,7,7,7,7)}(G_{39},t)&-(b_{41}'')^{(8,8,8,8,8)}(G_{43},t)&-(b_{45}'')^{(9,9,9,9)}(G_{47},t)\end{bmatrix}T_{25} \quad 77$$

$$\frac{dT_{26}}{dt}=(b_{26})^{(4)}T_{25}-\begin{bmatrix}(b_{26}')^{(4)}&-(b_{26}'')^{(4)}(G_{27},t)&-(b_{30}'')^{(5,5,)}(G_{31},t)&-(b_{34}'')^{(6,6,)}(G_{35},t)\\&-(b_{15}'')^{(1,1,1,1)}(G,t)&-(b_{18}'')^{(2,2,2,2)}(G_{19},t)&-(b_{22}'')^{(3,3,3,3)}(G_{23},t)\\&-(b_{38}'')^{(7,7,7,7,7)}(G_{39},t)&-(b_{42}'')^{(8,8,8,8,8)}(G_{43},t)&-(b_{46}'')^{(9,9,9,9)}(G_{47},t)\end{bmatrix}T_{26} \quad 78$$

$Where\ -(b_{24}'')^{(4)}(G_{27},t)$, $-(b_{25}'')^{(4)}(G_{27},t)$, $-(b_{26}'')^{(4)}(G_{27},t)$ $are\ first\ detrition\ coefficients$

$for\ category\ 1,2\ and\ 3$

$-(b_{28}'')^{(5,5,)}(G_{31},t)$, $-(b_{29}'')^{(5,5,)}(G_{31},t)$, $-(b_{30}'')^{(5,5,)}(G_{31},t)$ $are\ second\ detrition\ coefficients$

$for\ category\ 1,2\ and\ 3$

$-(b_{32}'')^{(6,6,)}(G_{35},t)$, $-(b_{33}'')^{(6,6,)}(G_{35},t)$, $-(b_{34}'')^{(6,6,)}(G_{35},t)$ $are\ third\ detrition\ coefficients$

$for\ category\ 1,2\ and\ 3$

$-(b_{13}'')^{(1,1,1,1)}(G,t)$, $-(b_{14}'')^{(1,1,1,1)}(G,t)$, $-(b_{15}'')^{(1,1,1,1)}(G,t)$

$are\ fourth\ \ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$-(b_{16}'')^{(2,2,2,2)}(G_{19},t)$, $-(b_{17}'')^{(2,2,2,2)}(G_{19},t)$, $-(b_{18}'')^{(2,2,2,2)}(G_{19},t)$

$are\ fifth\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$-(b_{20}'')^{(3,3,3,3)}(G_{23},t)$, $-(b_{21}'')^{(3,3,3,3)}(G_{23},t)$, $-(b_{22}'')^{(3,3,3,3)}(G_{23},t)$

$are\ sixth\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$-(b_{36}'')^{(7,7,7,7,7)}(G_{39},t)$, $-(b_{37}'')^{(7,7,7,7,7)}(G_{39},t)$, $-(b_{38}'')^{(7,7,7,7,7)}(G_{39},t)$

$are\ seventh\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)}$

*are eighth detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{46})^{(9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \begin{bmatrix} (a'_{28})^{(5)}\boxed{+(a''_{28})^{(5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{28} \quad (79)$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \begin{bmatrix} (a'_{29})^{(5)}\boxed{+(a''_{29})^{(5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{29} \quad (80)$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \begin{bmatrix} (a'_{30})^{(5)}\boxed{+(a''_{30})^{(5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{30} \quad (81)$$

*Where* $\boxed{+(a''_{28})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5)}(T_{29},t)}$ *are first augmentation coefficients for category* 1, 2 *and* 3

*And* $\boxed{+(a''_{24})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,)}(T_{25},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)}$ are fourth augmentation coefficients for category 1,2, and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)}$ are fifth augmentation coefficients for category 1,2,and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)}$ are sixth augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)}$ are seventh augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)}$ are eighth augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \begin{bmatrix} (b'_{28})^{(5)}\boxed{-(b''_{28})^{(5)}(G_{31},t)} & \boxed{-(b''_{24})^{(4,4,)}(G_{27},t)} & \boxed{-(b''_{32})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{28} \quad (82)$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b'_{29})^{(5)} & -(b''_{29})^{(5)}(G_{31},t) & -(b''_{25})^{(4,4,)}(G_{27},t) & -(b''_{33})^{(6,6,6)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{29} \quad (83)$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b'_{30})^{(5)} & -(b''_{30})^{(5)}(G_{31},t) & -(b''_{26})^{(4,4,)}(G_{27},t) & -(b''_{34})^{(6,6,6)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1,1,)}(G,t) & -(b''_{18})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{30} \quad (84)$$

*where* $-(b''_{28})^{(5)}(G_{31},t)$ , $-(b''_{29})^{(5)}(G_{31},t)$ , $-(b''_{30})^{(5)}(G_{31},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{24})^{(4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,)}(G_{27},t)$ *are second detrition coefficients for category* 1,2 *and* 3

$-(b''_{32})^{(6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6)}(G_{35},t)$ *are third detrition coefficients for category* 1,2 *and* 3

$-(b''_{13})^{(1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1)}(G,t)$ , $-(b''_{15})^{(1,1,1,1,1,)}(G,t)$ are fourth detrition coefficients for category 1,2, and 3

$-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1,2, and 3

$-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1,2, and 3

$-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2, and 3

$-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1,2, and 3

$-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \begin{bmatrix} (a'_{32})^{(6)} & +(a''_{32})^{(6)}(T_{33},t) & +(a''_{28})^{(5,5,5)}(T_{29},t) & +(a''_{24})^{(4,4,4,)}(T_{25},t) \\ & +(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t) & +(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t) \\ & +(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{32} \quad (85)$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \begin{bmatrix} (a'_{33})^{(6)} & +(a''_{33})^{(6)}(T_{33},t) & +(a''_{29})^{(5,5,5)}(T_{29},t) & +(a''_{25})^{(4,4,4,)}(T_{25},t) \\ & +(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t) & +(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t) \\ & +(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{33} \quad (86)$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \begin{bmatrix} (a'_{34})^{(6)} & +(a''_{34})^{(6)}(T_{33},t) & +(a''_{30})^{(5,5,5)}(T_{29},t) & +(a''_{26})^{(4,4,4,)}(T_{25},t) \\ & +(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t) & +(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t) \\ & +(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{34} \quad (87)$$

$+(a''_{32})^{(6)}(T_{33},t)$, $+(a''_{33})^{(6)}(T_{33},t)$, $+(a''_{34})^{(6)}(T_{33},t)$ *are first augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}$ *are second augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}$ *are third augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}$ - are fourth augmentation coefficients

$\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}$ - fifth augmentation coefficients

$\boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)}$ sixth augmentation coefficients

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)}$ seventh augmentation coefficients

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}$ Eighth augmentation coefficients

$\boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)}$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[\begin{array}{ccc} (b'_{32})^{(6)}\boxed{-(b''_{32})^{(6)}(G_{35},t)} & \boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)} \end{array}\right] T_{32} \quad (88)$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[\begin{array}{ccc} (b'_{33})^{(6)}\boxed{-(b''_{33})^{(6)}(G_{35},t)} & \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)} \end{array}\right] T_{33} \quad (89)$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[\begin{array}{ccc} (b'_{34})^{(6)}\boxed{-(b''_{34})^{(6)}(G_{35},t)} & \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)} \end{array}\right] T_{34} \quad (90)$$

$\boxed{-(b''_{32})^{(6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6)}(G_{35},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}$ *are second detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)}$ *are third detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}$ are fourth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)}$ are sixth detrition

coefficients for category 1, 2, and 3
$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2, and 3
$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2, and 3
$\boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \qquad 91$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a'_{36})^{(7)}\boxed{+(a''_{36})^{(7)}(T_{37},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \qquad 92$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a'_{37})^{(7)}\boxed{+(a''_{37})^{(7)}(T_{37},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{38}}{dt} \qquad 93$$

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a'_{38})^{(7)}\boxed{+(a''_{38})^{(7)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{(a''_{36})^{(7)}(T_{37},t)}$, $\boxed{(a''_{37})^{(7)}(T_{37},t)}$, $\boxed{(a''_{38})^{(7)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3
$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3
$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3
$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3
$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3
$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3
$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for category 1, 2 and 3
$\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)}$ are eighth augmentation coefficient for 1,2,3
$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation

coefficient for 1,2,3



$$\frac{dT_{36}}{dt} =$$

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b'_{36})^{(7)}\boxed{-(b''_{36})^{(7)}(G_{39},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{37}}{dt} =$$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b'_{37})^{(7)}\boxed{-(b''_{37})^{(7)}(G_{39},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{38}}{dt} =$$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b'_{38})^{(7)}\boxed{-(b''_{38})^{(7)}(G_{39},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3



$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \begin{bmatrix} (a'_{40})^{(8)}\boxed{+(a''_{40})^{(8)}(T_{41},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \begin{bmatrix} (a'_{41})^{(8)}\boxed{+(a''_{41})^{(8)}(T_{41},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \begin{bmatrix} (a'_{42})^{(8)}\boxed{+(a''_{42})^{(8)}(T_{41},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a''_{40})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$\frac{dT_{40}}{dt} =$

$$(b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)}\boxed{-(b''_{40})^{(8)}(G_{43},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} =$$

$$(b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)}\boxed{-(b''_{41})^{(8)}(G_{43},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} =$$

$$(b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)}\boxed{-(b''_{42})^{(8)}(G_{43},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{44}}{dt} \qquad 96$$

$$= (a_{44})^{(9)}G_{45} - \begin{bmatrix} (a'_{44})^{(9)}\boxed{+(a''_{44})^{(9)}(T_{45},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \begin{bmatrix} (a'_{45})^{(9)}\boxed{+(a''_{45})^{(9)}(T_{45},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \begin{bmatrix} (a'_{46})^{(9)}\boxed{+(a''_{46})^{(9)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a''_{44})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ are Seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \begin{bmatrix} (b'_{44})^{(9)}\boxed{-(b''_{44})^{(9)}(G_{47},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b'_{45})^{(9)}\boxed{-(b''_{45})^{(9)}(G_{47},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b'_{46})^{(9)}\boxed{-(b''_{46})^{(9)}(G_{47},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{44})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9)}(G_{47},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a'_i)^{(1)}, (a''_i)^{(1)}, (b_i)^{(1)}, (b'_i)^{(1)}, (b''_i)^{(1)} > 0, \quad i,j = 13,14,15 \qquad 97$$

The functions $(a''_i)^{(1)}, (b''_i)^{(1)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a''_i)^{(1)}(T_{14},t) \le (p_i)^{(1)} \le (\hat{A}_{13})^{(1)}$$

$$(b''_i)^{(1)}(G,t) \le (r_i)^{(1)} \le (b'_i)^{(1)} \le (\hat{B}_{13})^{(1)}$$

$$lim_{T_2\to\infty}(a''_i)^{(1)}(T_{14},t) = (p_i)^{(1)} \qquad 98$$

$$\lim_{G\to\infty}(b''_i)^{(1)}(G,t) = (r_i)^{(1)}$$

**Definition of** $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$ :

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, \ (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$$|(a_i'')^{(1)}(T_{14}', t) - (a_i'')^{(1)}(T_{14}, t)| \leq (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G', t) - (b_i'')^{(1)}(G, t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}', t)$ and $(a_i'')^{(1)}(T_{14}, t)$ . $(T_{14}', t)$ and $(T_{14}, t)$ are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14}, t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$ : 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)},$ are positive constants

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \ , \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$$

**Definition of** $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$ : 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, \ (r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}}[\,(a_i)^{(1)} + (a_i')^{(1)} + \ (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}\,(\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[\ (b_i)^{(1)} + (b_i')^{(1)} + \ (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)}\ (\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i, j = 16,17,18$$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

**Definition of** $(\mathrm{p_i})^{(2)}$, $(\mathrm{r_i})^{(2)}$:

$$(a_i'')^{(2)}(T_{17}, t) \leq (p_i)^{(2)} \leq \left(\hat{A}_{16}\right)^{(2)} \quad 102$$

$$(b_i'')^{(2)}(G_{19}, t) \leq \ (r_i)^{(2)} \leq (b_i')^{(2)} \leq (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2 \to \infty} (a_i'')^{(2)}\,(T_{17}, t) = (p_i)^{(2)} \quad 104$$

$$\lim_{G \to \infty} (b_i'')^{(2)}\left((G_{19}), t\right) = \ (r_i)^{(2)} \quad 105$$

**Definition of** $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$ : 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}', t) - (a_i'')^{(2)}(T_{17}, t)| \leq (\hat{k}_{16})^{(2)}|T_{17} - T_{17}'|e^{-(\hat{M}_{16})^{(2)}t} \quad 107$$

$$|(b_i'')^{(2)}((G_{19})', t) - (b_i'')^{(2)}\big((G_{19}), t\big)| < (\hat{k}_{16})^{(2)}||(G_{19}) - (G_{19})'||e^{-(\hat{M}_{16})^{(2)}t} \quad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}', t)$ and$(a_i'')^{(2)}(T_{17}, t)$ . $(T_{17}', t)$ and $(T_{17}, t)$ are points belonging to the interval $\big[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}\big]$ . It is to be noted that $(a_i'')^{(2)}(T_{17}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17}, t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$ :

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants 109

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$$

**Definition of** $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$ :

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and\ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1 \quad 110$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(b_i)^{(2)} + (b_i')^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1 \quad 111$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i, j = 20,21,22 \quad 112$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(3)}, \ (\mathrm{r_i})^{(3)}$:

$$(a_i'')^{(3)}(T_{21}, t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23}, t) \leq \ (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(3)}(T_{21}, t) = (p_i)^{(3)} \quad 113$$

$\lim_{G\to\infty}(b_i'')^{(3)}\,(G_{23},t) = \ (r_i)^{(3)}$

**Definition of** $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$ :

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, \ (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: 114

$|(a_i'')^{(3)}(T_{21}',t) - (a_i'')^{(3)}(T_{21},t)| \le (\hat{k}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\hat{M}_{20})^{(3)}t}$

$|(b_i'')^{(3)}(G_{23}',t) - (b_i'')^{(3)}(G_{23},t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}'||e^{-(\hat{M}_{20})^{(3)}t}$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}',t)$ and$(a_i'')^{(3)}(T_{21},t)$ . $(T_{21}',t)$ And $(T_{21},t)$ are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21},t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$ : 115

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)},$ are positive constants

$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}$ , $\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$ and the constants $(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, \ (r_i)^{(3)}, i = 20,21,22,$ satisfy the inequalities 116

$\frac{1}{(\hat{M}_{20})^{(3)}}[\,(a_i)^{(3)} + (a_i')^{(3)} + \ (\hat{A}_{20})^{(3)} + \ (\hat{P}_{20})^{(3)}\,(\hat{k}_{20})^{(3)}] < 1$

$\frac{1}{(\hat{M}_{20})^{(3)}}[\ (b_i)^{(3)} + (b_i')^{(3)} + \ (\hat{B}_{20})^{(3)} + \ (\hat{Q}_{20})^{(3)}\ \ (\hat{k}_{20})^{(3)}] < 1$

Where we suppose

$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i,j = 24,25,26$ 117

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(4)}, \ (r_i)^{(4)}$:

$(a_i'')^{(4)}(T_{25},t) \le (p_i)^{(4)} \le (\hat{A}_{24})^{(4)}$

$(b_i'')^{(4)}\big((G_{27}),t\big) \le \ (r_i)^{(4)} \le (b_i')^{(4)} \le (\hat{B}_{24})^{(4)}$

$lim_{T_2\to\infty}(a_i'')^{(4)}\,(T_{25},t) = (p_i)^{(4)}$ 118

$\lim_{G\to\infty}(b_i'')^{(4)}\,\big((G_{27}),t\big) = \ (r_i)^{(4)}$

**Definition of** $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$ :

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24{,}25{,}26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}', t) - (a_i'')^{(4)}(T_{25}, t)| \leq (\hat{k}_{24})^{(4)} |T_{25} - T_{25}'| e^{-(\hat{M}_{24})^{(4)} t}$$

$$|(b_i'')^{(4)}((G_{27})', t) - (b_i'')^{(4)}\big((G_{27}), t\big)| < (\hat{k}_{24})^{(4)} ||(G_{27}) - (G_{27})'|| e^{-(\hat{M}_{24})^{(4)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}', t)$ and$(a_i'')^{(4)}(T_{25}, t)$ . $(T_{25}', t)$ and $(T_{25}, t)$ are points belonging to the interval $\big[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}\big]$. It is to be noted that $(a_i'')^{(4)}(T_{25}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25}, t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$ : 120

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$, are positive constants

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}} \ , \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$$

**Definition of** $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$ : 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and\ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}, i = 24{,}25{,}26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[\,(a_i)^{(4)} + (a_i')^{(4)} + \ (\hat{A}_{24})^{(4)} + \ (\hat{P}_{24})^{(4)}\,(\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[\ (b_i)^{(4)} + (b_i')^{(4)} + \ (\hat{B}_{24})^{(4)} + \ (\hat{Q}_{24})^{(4)}\ (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i, j = 28{,}29{,}30$$ 122

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(5)}, \ (r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29}, t) \leq (p_i)^{(5)} \leq (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}\big((G_{31}), t\big) \leq \ (r_i)^{(5)} \leq (b_i')^{(5)} \leq (\hat{B}_{28})^{(5)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(5)}\,(T_{29}, t) = (p_i)^{(5)}$$ 123

$$\lim_{G \to \infty}(b_i'')^{(5)}\,(G_{31}, t) = \ (r_i)^{(5)}$$

**Definition of** $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$ :

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, \ (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}', t) - (a_i'')^{(5)}(T_{29}, t)| \leq (\hat{k}_{28})^{(5)}|T_{29} - T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})', t) - (b_i'')^{(5)}\big((G_{31}), t\big)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}', t)$ and$(a_i'')^{(5)}(T_{29}, t)$ . $(T_{29}', t)$ and $(T_{29}, t)$ are points belonging to the interval $\big[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\big]$. It is to be noted that $(a_i'')^{(5)}(T_{29}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29}, t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$ : 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}} \ , \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

**Definition of** $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$ : 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and \ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, \ (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[\,(a_i)^{(5)} + (a_i')^{(5)} + \ (\hat{A}_{28})^{(5)} + \ (\hat{P}_{28})^{(5)}\,(\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[\ (b_i)^{(5)} + (b_i')^{(5)} + \ (\hat{B}_{28})^{(5)} + \ (\hat{Q}_{28})^{(5)}\ (\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i, j = 32,33,34$$ 127

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(6)}, \ (r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33}, t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}), t) \leq \ (r_i)^{(6)} \leq (b_i')^{(6)} \leq (\hat{B}_{32})^{(6)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(6)}\,(T_{33}, t) = (p_i)^{(6)}$$ 128

$$\lim_{G \to \infty}(b_i'')^{(6)}\,\big((G_{35}), t\big) = \ (r_i)^{(6)}$$

**Definition of** $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$ :

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, \ (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}', t) - (a_i'')^{(6)}(T_{33}, t)| \leq (\hat{k}_{32})^{(6)} |T_{33} - T_{33}'| e^{-(\hat{M}_{32})^{(6)} t}$$

$$|(b_i'')^{(6)}((G_{35})', t) - (b_i'')^{(6)}((G_{35}), t)| < (\hat{k}_{32})^{(6)} ||(G_{35}) - (G_{35})'|| e^{-(\hat{M}_{32})^{(6)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}', t)$ and$(a_i'')^{(6)}(T_{33}, t)$ . $(T_{33}', t)$ and $(T_{33}, t)$ are points belonging to the interval $\left[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}\right]$. It is to be noted that $(a_i'')^{(6)}(T_{33}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33}, t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$ : 129

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$, are positive constants

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}} \ , \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$$

**Definition of** $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$ : 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and\ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, \ (r_i)^{(6)}, i = 32,33,34,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(a_i)^{(6)} + (a_i')^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)} (\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(b_i)^{(6)} + (b_i')^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)} (\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

(A) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i, j = 36,37,38$ 131

(B) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(7)}, \ (r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37}, t) \leq (p_i)^{(7)} \leq (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39}, t) \leq \ (r_i)^{(7)} \leq (b_i')^{(7)} \leq (\hat{B}_{36})^{(7)}$$

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*(C)* $\lim_{T_2 \to \infty} (a_i'')^{(7)}(T_{37}, t) = (p_i)^{(7)}$

*(D)*

$\lim_{G\to\infty}(b_i'')^{(7)}\left((G_{39}),t\right) = (r_i)^{(7)}$

**Definition of** $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$ :

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$$|(a_i'')^{(7)}(T_{37}', t) - (a_i'')^{(7)}(T_{37}, t)| \leq (\hat{k}_{36})^{(7)}|T_{37} - T_{37}'|e^{-(\hat{M}_{36})^{(7)}t}$$

$$|(b_i'')^{(7)}((G_{39})', t) - (b_i'')^{(7)}\left((G_{39}),t\right)| < (\hat{k}_{36})^{(7)}||(G_{39}) - (G_{39})'||e^{-(\hat{M}_{36})^{(7)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}', t)$ and$(a_i'')^{(7)}(T_{37}, t)$ . $(T_{37}', t)$ and $(T_{37}, t)$ are points belonging to the interval $\left[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}\right]$. It is to be noted that $(a_i'')^{(7)}(T_{37}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37}, t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$ : 134

(E) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)},$ are positive constants

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$$

**Definition of** $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$ : 135

(F) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)}(\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(b_i)^{(7)} + (b_i')^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)}(\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i, j = 40,41,42 \qquad 136$$

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

**Definition of** $(p_i)^{(8)}$, $(r_i)^{(8)}$: 137

$$(a_i'')^{(8)}(T_{41},t) \le (p_i)^{(8)} \le (\hat{A}_{40})^{(8)} \quad 138$$

$$(b_i'')^{(8)}((G_{43}),t) \le (r_i)^{(8)} \le (b_i')^{(8)} \le (\hat{B}_{40})^{(8)} \quad 139$$

$$\lim_{T_2\to\infty}(a_i'')^{(8)}(T_{41},t) = (p_i)^{(8)} \quad 140$$

$$\lim_{G\to\infty}(b_i'')^{(8)}\left((G_{43}),t\right) = (r_i)^{(8)} \quad 141$$

**Definition of** $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$ :

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(8)}(T_{41}',t) - (a_i'')^{(8)}(T_{41},t)| \le (\hat{k}_{40})^{(8)}|T_{41} - T_{41}'|e^{-(\hat{M}_{40})^{(8)}t} \quad 142$$

$$|(b_i'')^{(8)}((G_{43})',t) - (b_i'')^{(8)}\left((G_{43}),t\right)| < (\hat{k}_{40})^{(8)}||(G_{43}) - (G_{43})'||e^{-(\hat{M}_{40})^{(8)}t} \quad 143$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}',t)$ and $(a_i'')^{(8)}(T_{41},t)$ . $(T_{41}',t)$ and $(T_{41},t)$ are points belonging to the interval $\left[(\hat{k}_{40})^{(8)}, (\hat{M}_{40})^{(8)}\right]$. It is to be noted that $(a_i'')^{(8)}(T_{41},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{40})^{(8)} = 1$ then the function $(a_i'')^{(8)}(T_{41},t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$ :

$(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$, are positive constants

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}, \frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1 \quad 144$$

**Definition of** $(\hat{P}_{40})^{(8)}, (\hat{Q}_{40})^{(8)}$ :

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}, (\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)}, (a_i')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (p_i)^{(8)}$, $(r_i)^{(8)}, i = 40,41,42,$
Satisfy the inequalities

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(a_i)^{(8)} + (a_i')^{(8)} + (\hat{A}_{40})^{(8)} + (\hat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}] < 1 \quad 145$$

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(b_i)^{(8)} + (b_i')^{(8)} + (\hat{B}_{40})^{(8)} + (\hat{Q}_{40})^{(8)}(\hat{k}_{40})^{(8)}] < 1 \quad 146$$

Where we suppose

$$(a_i)^{(9)}, (a_i')^{(9)}, (a_i'')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (b_i'')^{(9)} > 0, \quad i,j = 44,45,46 \quad 146A$$

The functions $(a_i'')^{(9)}, (b_i'')^{(9)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(9)}$, $(r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45},t) \le (p_i)^{(9)} \le (\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47},t) \le (r_i)^{(9)} \le (b_i')^{(9)} \le (\hat{B}_{44})^{(9)}$$

$$\lim_{T_2\to\infty}(a_i'')^{(9)}(T_{45},t) = (p_i)^{(9)}$$

$$\lim_{G\to\infty}(b_i'')^{(9)}(G_{47},t) = (r_i)^{(9)}$$

**Definition of** $(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}$ :

Where $\boxed{(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}}$ are positive constants and $\boxed{i = 44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}',t) - (a_i'')^{(9)}(T_{45},t)| \le (\hat{k}_{44})^{(9)}|T_{45} - T_{45}'|e^{-(\hat{M}_{44})^{(9)}t}$$

$$|(b_i'')^{(9)}((G_{47})',t) - (b_i'')^{(9)}((G_{47}),t)| < (\hat{k}_{44})^{(9)}||(G_{47}) - (G_{47})'||e^{-(\hat{M}_{44})^{(9)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}',t)$ and$(a_i'')^{(9)}(T_{45},t)$ . $(T_{45}',t)$ and $(T_{45},t)$ are points belonging to the interval $\left[(\hat{k}_{44})^{(9)}, (\hat{M}_{44})^{(9)}\right]$. It is to be noted that $(a_i'')^{(9)}(T_{45},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{44})^{(9)} = 1$ then the function $(a_i'')^{(9)}(T_{45},t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$ :

$(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$, are positive constants

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}, \frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$$

**Definition of** $(\hat{P}_{44})^{(9)}, (\hat{Q}_{44})^{(9)}$ :

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}, (\hat{A}_{44})^{(9)} and\ (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)}, (a_i')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}, i = 44,45,46,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{44})^{(9)}}[(a_i)^{(9)} + (a_i')^{(9)} + (\hat{A}_{44})^{(9)} + (\hat{P}_{44})^{(9)}(\hat{k}_{44})^{(9)}] < 1$$

$$\frac{1}{(\hat{M}_{44})^{(9)}}[(b_i)^{(9)} + (b_i')^{(9)} + (\hat{B}_{44})^{(9)} + (\hat{Q}_{44})^{(9)}(\hat{k}_{44})^{(9)}] < 1$$

**Theorem 1:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \left(\hat{P}_{13}\right)^{(1)} e^{\left(\hat{M}_{13}\right)^{(1)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{13}\right)^{(1)} e^{\left(\hat{M}_{13}\right)^{(1)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 2 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

**Definition of** $G_i(0)\,, T_i(0)$

$$G_i(t) \le \left(\hat{P}_{16}\right)^{(2)} e^{\left(\hat{M}_{16}\right)^{(2)} t} \quad , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le \left(\hat{Q}_{16}\right)^{(2)} e^{\left(\hat{M}_{16}\right)^{(2)} t} \quad , \quad T_i(0) = T_i^0 > 0$$

**Theorem 3 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t) \le \left(\hat{P}_{20}\right)^{(3)} e^{\left(\hat{M}_{20}\right)^{(3)} t} \quad , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le \left(\hat{Q}_{20}\right)^{(3)} e^{\left(\hat{M}_{20}\right)^{(3)} t} \quad , \quad T_i(0) = T_i^0 > 0$$

**Theorem 4 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

**Definition of** $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \left(\hat{P}_{24}\right)^{(4)} e^{\left(\hat{M}_{24}\right)^{(4)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{24}\right)^{(4)} e^{\left(\hat{M}_{24}\right)^{(4)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 5 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

**Definition of** $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \left(\hat{P}_{28}\right)^{(5)} e^{\left(\hat{M}_{28}\right)^{(5)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{28}\right)^{(5)} e^{\left(\hat{M}_{28}\right)^{(5)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 6 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

**Definition of** $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \left(\hat{P}_{32}\right)^{(6)} e^{\left(\hat{M}_{32}\right)^{(6)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{32}\right)^{(6)} e^{\left(\hat{M}_{32}\right)^{(6)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 7:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

**Definition of** $G_i(0)\,, T_i(0)$ :

$G_i(t) \leq \left(\hat{P}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Theorem 8:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

**Definition of** $G_i(0)$ , $T_i(0)$ :

$G_i(t) \leq \left(\hat{P}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Theorem 9:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

**Definition of** $G_i(0)$ , $T_i(0)$ :

$G_i(t) \leq \left(\hat{P}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Proof:** Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i$: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$G_i(0) = G_i^0$ , $T_i(0) = T_i^0$ , $G_i^0 \leq \left(\hat{P}_{13}\right)^{(1)}$ , $T_i^0 \leq \left(\hat{Q}_{13}\right)^{(1)}$, 155

$0 \leq G_i(t) - G_i^0 \leq \left(\hat{P}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t}$ 156

$0 \leq T_i(t) - T_i^0 \leq \left(\hat{Q}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t}$ 157

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}(s_{(13)}) - \left((a'_{13})^{(1)} + a''_{13})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{15}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{15}(t) = T_{15}^0 + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{15}(s_{(13)})\right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0,t)$

**Proof:**

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i$: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$ , $T_i(0) = T_i^0$ , $G_i^0 \le (\hat{P}_{16})^{(2)}$ , $T_i^0 \le (\hat{Q}_{16})^{(2)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{16})^{(2)} e^{(\bar{M}_{16})^{(2)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{16})^{(2)} e^{(\bar{M}_{16})^{(2)} t}$

By

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[ (a_{16})^{(2)} G_{17}(s_{(16)}) - \left( (a'_{16})^{(2)} + a''_{16})^{(2)} (T_{17}(s_{(16)}), s_{(16)}) \right) G_{16}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[ (a_{17})^{(2)} G_{16}(s_{(16)}) - \left( (a'_{17})^{(2)} + (a''_{17})^{(2)} (T_{17}(s_{(16)}), s_{(17)}) \right) G_{17}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[ (a_{18})^{(2)} G_{17}(s_{(16)}) - \left( (a'_{18})^{(2)} + (a''_{18})^{(2)} (T_{17}(s_{(16)}), s_{(16)}) \right) G_{18}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[ (b_{16})^{(2)} T_{17}(s_{(16)}) - \left( (b'_{16})^{(2)} - (b''_{16})^{(2)} (G_{19}(s_{(16)}), s_{(16)}) \right) T_{16}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^0 + \int_0^t \left[ (b_{17})^{(2)} T_{16}(s_{(16)}) - \left( (b'_{17})^{(2)} - (b''_{17})^{(2)} (G_{19}(s_{(16)}), s_{(16)}) \right) T_{17}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^0 + \int_0^t \left[ (b_{18})^{(2)} T_{17}(s_{(16)}) - \left( (b'_{18})^{(2)} - (b''_{18})^{(2)} (G_{19}(s_{(16)}), s_{(16)}) \right) T_{18}(s_{(16)}) \right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0,t)$

**Proof:**

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i$: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$ , $T_i(0) = T_i^0$ , $G_i^0 \le (\hat{P}_{20})^{(3)}$ , $T_i^0 \le (\hat{Q}_{20})^{(3)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\bar{M}_{20})^{(3)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\bar{M}_{20})^{(3)} t}$

By

$$\bar{G}_{20}(t) = G_{20}^0 + \int_0^t \left[ (a_{20})^{(3)} G_{21}(s_{(20)}) - \left( (a'_{20})^{(3)} + a''_{20})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[ (a_{21})^{(3)} G_{20}(s_{(20)}) - \left( (a'_{21})^{(3)} + (a''_{21})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{21}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[ (a_{22})^{(3)} G_{21}(s_{(20)}) - \left( (a'_{22})^{(3)} + (a''_{22})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{22}(s_{(20)}) \right] ds_{(20)}$$

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$$\bar{T}_{20}(t) = T_{20}^{0} + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^{0} + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{\mathrm{T}}_{22}(\mathrm{t}) = \mathrm{T}_{22}^{0} + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{22}(s_{(20)})\right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{24})^{(4)}, T_i^0 \le (\hat{Q}_{24})^{(4)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

By 162

$$\bar{G}_{24}(t) = G_{24}^{0} + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^{0} + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^{0} + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{26}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^{0} + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^{0} + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{\mathrm{T}}_{26}(\mathrm{t}) = \mathrm{T}_{26}^{0} + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{26}(s_{(24)})\right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{28})^{(5)}, T_i^0 \le (\hat{Q}_{28})^{(5)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

By 163

$$\bar{G}_{28}(t) = G_{28}^{0} + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{30}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{\mathrm{T}}_{30}(\mathrm{t}) = \mathrm{T}_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{30}(s_{(28)})\right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{32})^{(6)}$, $T_i^0 \le (\hat{Q}_{32})^{(6)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

By 164

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a'_{32})^{(6)} + a''_{32})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{34}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{\mathrm{T}}_{34}(\mathrm{t}) = \mathrm{T}_{34}^0 + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{34}(s_{(32)})\right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[ (a_{36})^{(7)} G_{37}(s_{(36)}) - \left( (a'_{36})^{(7)} + a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[ (a_{37})^{(7)} G_{36}(s_{(36)}) - \left( (a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[ (a_{38})^{(7)} G_{37}(s_{(36)}) - \left( (a'_{38})^{(7)} + (a''_{38})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{38}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[ (b_{36})^{(7)} T_{37}(s_{(36)}) - \left( (b'_{36})^{(7)} - (b''_{36})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[ (b_{37})^{(7)} T_{36}(s_{(36)}) - \left( (b'_{37})^{(7)} - (b''_{37})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{38}(t) = T_{38}^0 + \int_0^t \left[ (b_{38})^{(7)} T_{37}(s_{(36)}) - \left( (b'_{38})^{(7)} - (b''_{38})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{38}(s_{(36)}) \right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{40})^{(8)}$, $T_i^0 \le (\hat{Q}_{40})^{(8)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[ (a_{40})^{(8)} G_{41}(s_{(40)}) - \left( (a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[ (a_{41})^{(8)} G_{40}(s_{(40)}) - \left( (a'_{41})^{(8)} + (a''_{41})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[ (a_{42})^{(8)} G_{41}(s_{(40)}) - \left( (a'_{42})^{(8)} + (a''_{42})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{42}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^0 + \int_0^t \left[ (b_{40})^{(8)} T_{41}(s_{(40)}) - \left( (b'_{40})^{(8)} - (b''_{40})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^0 + \int_0^t \left[ (b_{41})^{(8)} T_{40}(s_{(40)}) - \left( (b'_{41})^{(8)} - (b''_{41})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{42}(t) = T_{42}^0 + \int_0^t \left[ (b_{42})^{(8)} T_{41}(s_{(40)}) - \left( (b'_{42})^{(8)} - (b''_{42})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{42}(s_{(40)}) \right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** 166

Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ A which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{44})^{(9)},\ T_i^0 \le (\hat{Q}_{44})^{(9)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} t}$$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[ (a_{44})^{(9)} G_{45}(s_{(44)}) - \left( (a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[ (a_{45})^{(9)} G_{44}(s_{(44)}) - \left( (a'_{45})^{(9)} + (a''_{45})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[ (a_{46})^{(9)} G_{45}(s_{(44)}) - \left( (a'_{46})^{(9)} + (a''_{46})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{46}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[ (b_{44})^{(9)} T_{45}(s_{(44)}) - \left( (b'_{44})^{(9)} - (b''_{44})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[ (b_{45})^{(9)} T_{44}(s_{(44)}) - \left( (b'_{45})^{(9)} - (b''_{45})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{46}(t) = T_{46}^0 + \int_0^t \left[ (b_{46})^{(9)} T_{45}(s_{(44)}) - \left( (b'_{46})^{(9)} - (b''_{46})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{46}(s_{(44)}) \right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \le G_{13}^0 + \int_0^t \left[ (a_{13})^{(1)} \left( G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\tilde{M}_{13})^{(1)} s_{(13)}} \right) \right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\tilde{M}_{13})^{(1)}} \left( e^{(\tilde{M}_{13})^{(1)} t} - 1 \right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\tilde{M}_{13})^{(1)} t} \le \frac{(a_{13})^{(1)}}{(\tilde{M}_{13})^{(1)}} \left[ \left( (\hat{P}_{13})^{(1)} + G_{14}^0 \right) e^{\left( - \frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0} \right)} + (\hat{P}_{13})^{(1)} \right]$$

$(G_i^0)$ is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}$ , $G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \le G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}}\right)\right] ds_{(16)} = \quad (169)$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1\right)$$

From which it follows that (170)

$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \le \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left[\left((\hat{P}_{16})^{(2)} + G_{17}^0\right) e^{\left(-\frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0}\right)} + (\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}$ , $G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that (171)

$$G_{20}(t) \le G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} \left(G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} s_{(20)}}\right)\right] ds_{(20)} =$$

$$\left(1 + (a_{20})^{(3)} t\right) G_{21}^0 + \frac{(a_{20})^{(3)} (\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left(e^{(\hat{M}_{20})^{(3)} t} - 1\right)$$

From which it follows that (172)

$$(G_{20}(t) - G_{20}^0) e^{-(\hat{M}_{20})^{(3)} t} \le \frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left[\left((\hat{P}_{20})^{(3)} + G_{21}^0\right) e^{\left(-\frac{(\hat{P}_{20})^{(3)} + G_{21}^0}{G_{21}^0}\right)} + (\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}$ , $G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that (173)

$$G_{24}(t) \le G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} \left(G_{25}^0 + (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} s_{(24)}}\right)\right] ds_{(24)} =$$

$$\left(1 + (a_{24})^{(4)} t\right) G_{25}^0 + \frac{(a_{24})^{(4)} (\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}} \left(e^{(\hat{M}_{24})^{(4)} t} - 1\right)$$

From which it follows that (174)

$$(G_{24}(t) - G_{24}^0) e^{-(\hat{M}_{24})^{(4)} t} \le \frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}} \left[\left((\hat{P}_{24})^{(4)} + G_{25}^0\right) e^{\left(-\frac{(\hat{P}_{24})^{(4)} + G_{25}^0}{G_{25}^0}\right)} + (\hat{P}_{24})^{(4)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t) \le G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} \left(G_{29}^0 + (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} s_{(28)}}\right)\right] ds_{(28)} =$$

$$\left(1+(a_{28})^{(5)}t\right)G_{29}^{0}+\frac{(a_{28})^{(5)}(\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left(e^{(\hat{M}_{28})^{(5)}t}-1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t}\leq\frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t)\leq G_{32}^{0}+\int_{0}^{t}\left[(a_{32})^{(6)}\left(G_{33}^{0}+(\hat{P}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\right)\right]ds_{(32)}=$$

$$\left(1+(a_{32})^{(6)}t\right)G_{33}^{0}+\frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left(e^{(\hat{M}_{32})^{(6)}t}-1\right)$$

From which it follows that 177

$$(G_{32}(t)-G_{32}^{0})e^{-(\hat{M}_{32})^{(6)}t}\leq\frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left[((\hat{P}_{32})^{(6)}+G_{33}^{0})e^{\left(-\frac{(\hat{P}_{32})^{(6)}+G_{33}^{0}}{G_{33}^{0}}\right)}+(\hat{P}_{32})^{(6)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

(a) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t)\leq G_{36}^{0}+\int_{0}^{t}\left[(a_{36})^{(7)}\left(G_{37}^{0}+(\hat{P}_{36})^{(7)}e^{(\hat{M}_{36})^{(7)}s_{(36)}}\right)\right]ds_{(36)}=$$
$$\left(1+(a_{36})^{(7)}t\right)G_{37}^{0}+\frac{(a_{36})^{(7)}(\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left(e^{(\hat{M}_{36})^{(7)}t}-1\right)$$

From which it follows that

$$(G_{36}(t)-G_{36}^{0})e^{-(\hat{M}_{36})^{(7)}t}\leq\frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left[((\hat{P}_{36})^{(7)}+G_{37}^{0})e^{\left(-\frac{(\hat{P}_{36})^{(7)}+G_{37}^{0}}{G_{37}^{0}}\right)}+(\hat{P}_{36})^{(7)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t)\leq G_{40}^{0}+\int_{0}^{t}\left[(a_{40})^{(8)}\left(G_{41}^{0}+(\hat{P}_{40})^{(8)}e^{(\hat{M}_{40})^{(8)}s_{(40)}}\right)\right]ds_{(40)}=$$ 180
$$\left(1+(a_{40})^{(8)}t\right)G_{41}^{0}+\frac{(a_{40})^{(8)}(\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left(e^{(\hat{M}_{40})^{(8)}t}-1\right)$$

From which it follows that 181

$$(G_{40}(t)-G_{40}^{0})e^{-(\hat{M}_{40})^{(8)}t}\leq\frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left[((\hat{P}_{40})^{(8)}+G_{41}^{0})e^{\left(-\frac{(\hat{P}_{40})^{(8)}+G_{41}^{0}}{G_{41}^{0}}\right)}+(\hat{P}_{40})^{(8)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 8
Analogous inequalities hold also for $G_{41}, G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t)\leq G_{44}^{0}+\int_{0}^{t}\left[(a_{44})^{(9)}\left(G_{45}^{0}+(\hat{P}_{44})^{(9)}e^{(\hat{M}_{44})^{(9)}s_{(44)}}\right)\right]ds_{(44)}=$$

$$\left(1+(a_{44})^{(9)}t\right)G_{45}^{0}+\frac{(a_{44})^{(9)}(\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}}\left(e^{(\hat{M}_{44})^{(9)}t}-1\right)$$

From which it follows that

$$(G_{44}(t)-G_{44}^{0})e^{-(\hat{M}_{44})^{(9)}t}\leq\frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}}\left[((\hat{P}_{44})^{(9)}+G_{45}^{0})e^{\left(-\frac{(\hat{P}_{44})^{(9)}+G_{45}^{0}}{G_{45}^{0}}\right)}+(\hat{P}_{44})^{(9)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 9

Analogous inequalities hold also for $G_{45}, G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}<1$ and to choose 182

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[(\hat{P}_{13})^{(1)}+((\hat{P}_{13})^{(1)}+G_j^0)e^{-\left(\frac{(\hat{P}_{13})^{(1)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{13})^{(1)} \quad 183$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[((\hat{Q}_{13})^{(1)}+T_j^0)e^{-\left(\frac{(\hat{Q}_{13})^{(1)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{13})^{(1)}\right]\leq(\hat{Q}_{13})^{(1)} \quad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left((G^{(1)},T^{(1)}),(G^{(2)},T^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\hat{M}_{13})^{(1)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\hat{M}_{13})^{(1)}t}\}$$

Indeed if we denote

**Definition of** $\tilde{G},\tilde{T}:(\tilde{G},\tilde{T})=\mathcal{A}^{(1)}(G,T)$

It results

$$\left|\tilde{G}_{13}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{13})^{(1)} \left|G_{14}^{(1)}-G_{14}^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\, ds_{(13)} +$$

$$\int_0^t \{(a'_{13})^{(1)}\left|G_{13}^{(1)}-G_{13}^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} +$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)}, s_{(13)}\right)\left|G_{13}^{(1)}-G_{13}^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} e^{(\widehat{M}_{13})^{(1)}s_{(13)}} +$$

$$G_{13}^{(2)}\left|(a''_{13})^{(1)}\left(T_{14}^{(1)}, s_{(13)}\right)-(a''_{13})^{(1)}\left(T_{14}^{(2)}, s_{(13)}\right)\right|\ e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\} ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\left|G^{(1)}-G^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)}t} \leq \qquad 186$$
$$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)}+\ (a'_{13})^{(1)}+(\widehat{A}_{13})^{(1)}+(\widehat{P}_{13})^{(1)}(\widehat{k}_{13})^{(1)}\right) d\left(\left(G^{(1)}, T^{(1)};\ G^{(2)}, T^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

**Remark 1:** The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}\ and\ (\widehat{Q}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(1)}$ and $(b''_i)^{(1)}, i = 13,14,15$ depend only on $T_{14}$ and respectively on $G(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$

From 19 to 24 it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a'_i)^{(1)}-(a''_i)^{(1)}(T_{14}(s_{(13)}),s_{(13)})\} ds_{(13)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b'_i)^{(1)}t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2\ and\ \left((\widehat{M}_{13})^{(1)}\right)_3$ : 187

**Remark 3**: if $G_{13}$ is bounded, the same property have also $G_{14}\ and\ G_{15}$ . indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a'_{14})^{(1)}G_{14}$ and by integrating

$$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1/(a'_{14})^{(1)}$$

In the same way , one can obtain

$$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2/(a'_{15})^{(1)}$$

If $G_{14}\ or\ G_{15}$ is bounded, the same property follows for $G_{13}$ , $G_{15}$ and $G_{13}$ , $G_{14}$ respectively.

**Remark 4:** If $G_{13}$ *is* bounded, from below, the same property holds for $G_{14}$ *and* $G_{15}$. The proof is analogous with the preceding one. An analogous property is true if $G_{14}$ is bounded from below. 188

**Remark 5:** If $T_{13}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}(G(t),t)) = (b_{14}')^{(1)}$ then $T_{14} \to \infty$. 189

**Definition of** $(m)^{(1)}$ and $\varepsilon_1$ :

Indeed let $t_1$ be so that for $t > t_1$

$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t),t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\frac{2}{\varepsilon_1}$ By taking now $\varepsilon_1$ sufficiently small one sees that $T_{14}$ is unbounded.
The same property holds for $T_{15}$ if $\lim_{t\to\infty}(b_{15}'')^{(1)}(G(t),t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}$ *and* $(\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[(\hat{P}_{16})^{(2)} + \left((\hat{P}_{16})^{(2)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{16})^{(2)}\right] \leq (\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t) - G_i^{(2)}(t)|e^{-(\hat{M}_{16})^{(2)}t}, \max_{t\in\mathbb{R}_+}|T_i^{(1)}(t) - T_i^{(2)}(t)|e^{-(\hat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

**Definition of** $\widetilde{G_{19}}, \widetilde{T_{19}}$ : $(\widetilde{G_{19}}, \widetilde{T_{19}}) = \mathcal{A}^{(2)}(G_{19}, T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)}-\tilde{G}_{i}^{(2)}\right|\leq\int_{0}^{t}(a_{16})^{(2)}\left|G_{17}^{(1)}-G_{17}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)}+$$

$$\int_{0}^{t}\{(a'_{16})^{(2)}\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$G_{16}^{(2)}\left|(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)-(a''_{16})^{(2)}\left(T_{17}^{(2)},s_{(16)}\right)\right|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0,t]$ 197

From the hypotheses it follows

$$\left|(G_{19})^{(1)}-(G_{19})^{(2)}\right|\mathrm{e}^{-(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}\leq$$
$$\frac{1}{(\widehat{\mathrm{M}}_{16})^{(2)}}\left((a_{16})^{(2)}+\ (a'_{16})^{(2)}+(\widehat{\mathrm{A}}_{16})^{(2)}+(\widehat{\mathrm{P}}_{16})^{(2)}(\hat{k}_{16})^{(2)}\right)\mathrm{d}\left(((G_{19})^{(1)},(T_{19})^{(1)};\ (G_{19})^{(2)},(T_{19})^{(2)})\right)$$

And analogous inequalities for $\mathrm{G}_i$ and $\mathrm{T}_i$. Taking into account the hypothesis the result follows 198

**Remark 6:** The fact that we supposed $(a''_{16})^{(2)}$ and $(b''_{16})^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{\mathrm{P}}_{16})^{(2)}\mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}$ and $(\widehat{\mathrm{Q}}_{16})^{(2)}\mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}$ respectively of $\mathbb{R}_{+}$. 199

If instead of proving the existence of the solution on $\mathbb{R}_{+}$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(2)}$ and $(b''_i)^{(2)}, i=16,17,18$ depend only on $\mathrm{T}_{17}$ and respectively on $(G_{19})$(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 7:** There does not exist any t where $\mathrm{G}_i\,(\mathrm{t})=0$ and $\mathrm{T}_i\,(\mathrm{t})=0$ 200

it results

$$\mathrm{G}_i\,(\mathrm{t})\geq\mathrm{G}_i^0\mathrm{e}^{\left[-\int_0^{\mathrm{t}}\{(a'_i)^{(2)}-(a''_i)^{(2)}(\mathrm{T}_{17}(s_{(16)}),s_{(16)})\}\mathrm{d}s_{(16)}\right]}\geq 0$$

$$\mathrm{T}_i\,(\mathrm{t})\geq\mathrm{T}_i^0\mathrm{e}^{(-(b'_i)^{(2)}\mathrm{t})}>0\quad\text{for t}>0$$

**Definition of** $\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_1$, $\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_2$ and $\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_3$: 201

**Remark 8:** if $\mathrm{G}_{16}$ is bounded, the same property have also $\mathrm{G}_{17}$ and $\mathrm{G}_{18}$ . indeed if

$\mathrm{G}_{16}<(\widehat{\mathrm{M}}_{16})^{(2)}$ it follows $\frac{\mathrm{dG}_{17}}{\mathrm{dt}}\leq\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_1-(a'_{17})^{(2)}\mathrm{G}_{17}$ and by integrating

$$\mathrm{G}_{17}\leq\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_2=\mathrm{G}_{17}^0+2(a_{17})^{(2)}\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_1/(a'_{17})^{(2)}$$

In the same way , one can obtain

$$\mathrm{G}_{18}\leq\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_3=\mathrm{G}_{18}^0+2(a_{18})^{(2)}\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_2/(a'_{18})^{(2)}$$

If $\mathrm{G}_{17}$ or $\mathrm{G}_{18}$ is bounded, the same property follows for $\mathrm{G}_{16}$ , $\mathrm{G}_{18}$ and $\mathrm{G}_{16}$ , $\mathrm{G}_{17}$ respectively.

**Remark 9:** If $G_{16}$ is bounded, from below, the same property holds for $G_{17}$ and $G_{18}$. The proof is analogous with the preceding one. An analogous property is true if $G_{17}$ is bounded from below. 202

**Remark 10:** If $T_{16}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t),t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$. 203

**Definition of** $(m)^{(2)}$ and $\varepsilon_2$ :

Indeed let $t_2$ be so that for $t > t_2$

$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t),t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right), \quad t = \log\frac{2}{\varepsilon_2}$ By taking now $\varepsilon_2$ sufficiently small one sees that $T_{17}$ is unbounded. 205
The same property holds for $T_{18}$ if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t),t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\tilde{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\tilde{M}_{20})^{(3)}} < 1$ and to choose 207

$(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\tilde{M}_{20})^{(3)}}\left[(\hat{P}_{20})^{(3)} + \left((\hat{P}_{20})^{(3)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\tilde{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{20})^{(3)}\right] \leq (\hat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying 210
Equations into itself

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)},(T_{23})^{(1)}),((G_{23})^{(2)},(T_{23})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\tilde{M}_{20})^{(3)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\tilde{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

**Definition of** $\widetilde{G_{23}}, \widetilde{T_{23}}$ :$\left(\widetilde{(G_{23})},\widetilde{(T_{23})}\right) = \mathcal{A}^{(3)}\left((G_{23}),(T_{23})\right)$

It results 213

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_{i}^{(2)}\right|\leq\int_{0}^{t}(a_{20})^{(3)}\left|G_{21}^{(1)}-G_{21}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\,ds_{(20)}+$$

$$\int_{0}^{t}\{(a'_{20})^{(3)}\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$G_{20}^{(2)}\left|(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)},s_{(20)}\right)\right|\ e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\}ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}t}\leq \qquad 214$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+\ (a'_{20})^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\widehat{k}_{20})^{(3)}\right)d\left(((G_{23})^{(1)},(T_{23})^{(1)};\ (G_{23})^{(2)},(T_{23})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

**Remark 11:** The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}\ and\ (\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_{+}$. 215

If instead of proving the existence of the solution on $\mathbb{R}_{+}$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i=20,21,22$ depend only on $T_{21}$ and respectively on $(G_{23})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 12:** There does not exist any $t$ where $G_i\,(t)=0\ and\ T_i\,(t)=0$ 216

it results

$$G_i\,(t)\geq G_i^0e^{\left[-\int_0^t\{(a_i')^{(3)}-(a_i'')^{(3)}(T_{21}(s_{(20)}),s_{(20)})\}ds_{(20)}\right]}\geq 0$$

$$T_i\,(t)\geq T_i^0e^{\left(-(b_i')^{(3)}t\right)}>0\quad \text{for t}>0$$

**Definition of** $\left((\widehat{M}_{20})^{(3)}\right)_1,\left((\widehat{M}_{20})^{(3)}\right)_2\ and\ \left((\widehat{M}_{20})^{(3)}\right)_3$ : 217

**Remark 13:** if $G_{20}$ is bounded, the same property have also $G_{21}\ and\ G_{22}$ . indeed if

$G_{20}<(\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt}\leq\left((\widehat{M}_{20})^{(3)}\right)_1-(a'_{21})^{(3)}G_{21}$ and by integrating

$$G_{21}\leq\left((\widehat{M}_{20})^{(3)}\right)_2=G_{21}^0+2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1/(a'_{21})^{(3)}$$

In the same way , one can obtain

$$G_{22}\leq\left((\widehat{M}_{20})^{(3)}\right)_3=G_{22}^0+2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2/(a'_{22})^{(3)}$$

If $G_{21}\ or\ G_{22}$ is bounded, the same property follows for $G_{20}$ , $G_{22}$ and $G_{20}$ , $G_{21}$ respectively.

**Remark 14:** If $G_{20}$ *is* bounded, from below, the same property holds for $G_{21}$ *and* $G_{22}$. The proof is analogous with the preceding one. An analogous property is true if $G_{21}$ is bounded from below. 218

**Remark 15:** If $T_{20}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t),t\right)) = (b_{21}')^{(3)}$ then $T_{21} \to \infty$. 219

**Definition of** $(m)^{(3)}$ and $\varepsilon_3$ :

Indeed let $t_3$ be so that for $t > t_3$

$(b_{21})^{(3)} - (b_i'')^{(3)}\left((G_{23})(t),t\right) < \varepsilon_3, T_{20}(t) > (m)^{(3)}$

Then $\frac{dT_{21}}{dt} \geq (a_{21})^{(3)}(m)^{(3)} - \varepsilon_3 T_{21}$ which leads to 220

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t} = \frac{1}{2}$ it results

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right), \quad t = log\frac{2}{\varepsilon_3}$ By taking now $\varepsilon_3$ sufficiently small one sees that $T_{21}$ is unbounded.
The same property holds for $T_{22}$ if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t),t\right) = (b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\tilde{M}_{24})^{(4)}}$ , $\frac{(b_i)^{(4)}}{(\tilde{M}_{24})^{(4)}} < 1$ and to choose 221

$(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\tilde{M}_{24})^{(4)}}\left[(\hat{P}_{24})^{(4)} + \left((\hat{P}_{24})^{(4)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{24})^{(4)} \quad 222$$

$$\frac{(b_i)^{(4)}}{(\tilde{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{24})^{(4)}\right] \leq (\hat{Q}_{24})^{(4)} \quad 223$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself 224

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric 225

$$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\tilde{M}_{24})^{(4)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\tilde{M}_{24})^{(4)}t}\}$$

Indeed if we denote

**Definition of** $\widetilde{(G_{27})}, \widetilde{(T_{27})}$ : $\left(\widetilde{(G_{27})}, \widetilde{(T_{27})}\right) = \mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_{i}^{(2)}\right|\leq\int_{0}^{t}(a_{24})^{(4)}\left|G_{25}^{(1)}-G_{25}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\,ds_{(24)}+$$

$$\int_{0}^{t}\{(a'_{24})^{(4)}\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$G_{24}^{(2)}\left|(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)-(a''_{24})^{(4)}\left(T_{25}^{(2)},s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\}ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}t}\leq \qquad 226$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)}+\ (a'_{24})^{(4)}+(\widehat{A}_{24})^{(4)}+(\widehat{P}_{24})^{(4)}(\widehat{k}_{24})^{(4)}\right)d\left(((G_{27})^{(1)},(T_{27})^{(1)};\ (G_{27})^{(2)},(T_{27})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

**Remark 16:** The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}\ and\ (\widehat{Q}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_{+}$. 227

If instead of proving the existence of the solution on $\mathbb{R}_{+}$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i=24{,}25{,}26$ depend only on $\mathrm{T}_{25}$ and respectively on $(G_{27})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 17:** There does not exist any $t$ where $G_i\,(t)=0\ and\ T_i\,(t)=0$ 228

it results

$$G_i\,(t)\geq G_i^0e^{\left[-\int_0^t\{(a_i')^{(4)}-(a_i'')^{(4)}(T_{25}(s_{(24)}),s_{(24)})\}ds_{(24)}\right]}\geq 0$$

$$T_i\,(t)\geq T_i^0e^{\left(-(b_i')^{(4)}t\right)}>0\quad \text{for t}>0$$

**Definition of** $\left((\widehat{M}_{24})^{(4)}\right)_1,\left((\widehat{M}_{24})^{(4)}\right)_2\ and\ \left((\widehat{M}_{24})^{(4)}\right)_3$ : 229

**Remark 18:** if $G_{24}$ is bounded, the same property have also $G_{25}\ and\ G_{26}$ . indeed if

$G_{24}<(\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt}\leq\left((\widehat{M}_{24})^{(4)}\right)_1-(a'_{25})^{(4)}G_{25}$ and by integrating

$$G_{25}\leq\left((\widehat{M}_{24})^{(4)}\right)_2=G_{25}^0+2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1/(a'_{25})^{(4)}$$

In the same way , one can obtain

$$G_{26}\leq\left((\widehat{M}_{24})^{(4)}\right)_3=G_{26}^0+2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2/(a'_{26})^{(4)}$$

If $G_{25}\ or\ G_{26}$ is bounded, the same property follows for $G_{24}$ , $G_{26}$ and $G_{24}$ , $G_{25}$ respectively.

**Remark 19:** If $G_{24}$ *is* bounded, from below, the same property holds for $G_{25}$ *and* $G_{26}$. The proof is analogous with the preceding one. An analogous property is true if $G_{25}$ is bounded from below. 230

**Remark 20:** If $T_{24}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}((G_{27})(t),t)) = (b_{25}')^{(4)}$ then $T_{25} \to \infty$. 231

**Definition of** $(m)^{(4)}$ and $\varepsilon_4$ :

Indeed let $t_4$ be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t),t) < \varepsilon_4, T_{24}(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right), \quad t = log\frac{2}{\varepsilon_4}$ By taking now $\varepsilon_4$ sufficiently small one sees that $T_{25}$ is unbounded.
The same property holds for $T_{26}$ if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t),t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for $G_{29}, G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}, \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)} + \left((\hat{P}_{28})^{(5)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{28})^{(5)}\right] \leq (\hat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$$d\left(((G_{31})^{(1)},(T_{31})^{(1)}),((G_{31})^{(2)},(T_{31})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

**Definition of** $\widetilde{(G_{31})}, \widetilde{(T_{31})}$ : $\left(\widetilde{(G_{31})}, \widetilde{(T_{31})}\right) = \mathcal{A}^{(5)}\left((G_{31}),(T_{31})\right)$

It results

$$\left|\tilde{G}_{28}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{28})^{(5)} \left|G_{29}^{(1)}-G_{29}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}}\, ds_{(28)} +$$

$$\int_0^t \{(a'_{28})^{(5)} \left|G_{28}^{(1)}-G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} +$$

$$(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right)\left|G_{28}^{(1)}-G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}} +$$

$$G_{28}^{(2)} \left|(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right)-(a''_{28})^{(5)}\left(T_{29}^{(2)}, s_{(28)}\right)\right|\ e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}}\} ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$$\left|(G_{31})^{(1)}-(G_{31})^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} t} \leq \qquad (237)$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)} + (a'_{28})^{(5)} + (\widehat{A}_{28})^{(5)} + (\widehat{P}_{28})^{(5)}(\widehat{k}_{28})^{(5)}\right) d\left(\left((G_{31})^{(1)}, (T_{31})^{(1)};\ (G_{31})^{(2)}, (T_{31})^{(2)}\right)\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 21:** The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$ *and* $(\widehat{Q}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$ respectively of $\mathbb{R}_+$. (238)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on $T_{29}$ and respectively on $(G_{31})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 22:** There does not exist any $t$ where $G_i(t) = 0\ and\ T_i(t) = 0$ (239)

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(5)} - (a_i'')^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(5)} t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2\ and\ \left((\widehat{M}_{28})^{(5)}\right)_3$ : (240)

**Remark 23:** if $G_{28}$ is bounded, the same property have also $G_{29}\ and\ G_{30}$ . indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a'_{29})^{(5)} G_{29}$ and by integrating

$$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1 / (a'_{29})^{(5)}$$

In the same way , one can obtain

$$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2 / (a'_{30})^{(5)}$$

If $G_{29}$ *or* $G_{30}$ is bounded, the same property follows for $G_{28}$ , $G_{30}$ and $G_{28}$ , $G_{29}$ respectively.

**Remark 24:** If $G_{28}$ *is* bounded, from below, the same property holds for $G_{29}$ *and* $G_{30}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{29}$ is bounded from below. 241

**Remark 25:** If $T_{28}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}((G_{31})(t),t)) = (b_{29}')^{(5)}$ then $T_{29} \to \infty$. 242

**Definition of** $(m)^{(5)}$ and $\varepsilon_5$ :

Indeed let $t_5$ be so that for $t > t_5$

$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t),t) < \varepsilon_5, T_{28}(t) > (m)^{(5)}$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\frac{2}{\varepsilon_5}$ By taking now $\varepsilon_5$ sufficiently small one sees that $T_{29}$ is unbounded.
The same property holds for $T_{30}$ if $\lim_{t\to\infty}(b_{30}'')^{(5)}\left((G_{31})(t),t\right) = (b_{30}')^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{33}$ , $G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}$ , $\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$ and to choose 244

$(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[(\hat{P}_{32})^{(6)} + \left((\hat{P}_{32})^{(6)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{32})^{(6)} \quad 245$$

$$\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[\left((\hat{Q}_{32})^{(6)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{32})^{(6)}\right] \leq (\hat{Q}_{32})^{(6)} \quad 246$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric 247

$$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{32})^{(6)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

**Definition of** $\widetilde{(G_{35})}, \widetilde{(T_{35})}$ : $\left(\widetilde{(G_{35})}, \widetilde{(T_{35})}\right) = \mathcal{A}^{(6)}\left((G_{35}), (T_{35})\right)$

It results

$\left|\tilde{G}_{32}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{32})^{(6)} \left|G_{33}^{(1)} - G_{33}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}}\, ds_{(32)} +$

$\int_0^t \{(a'_{32})^{(6)} \left|G_{32}^{(1)} - G_{32}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} +$

$(a''_{32})^{(6)}\left(T_{33}^{(1)}, s_{(32)}\right)\left|G_{32}^{(1)} - G_{32}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}} +$

$G_{32}^{(2)} \left|(a''_{32})^{(6)}\left(T_{33}^{(1)}, s_{(32)}\right) - (a''_{32})^{(6)}\left(T_{33}^{(2)}, s_{(32)}\right)\right| \; e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}} \} ds_{(32)}$

Where $s_{(32)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$\left|(G_{35})^{(1)} - (G_{35})^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} t} \leq$ 248

$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)} + (a'_{32})^{(6)} + (\widehat{A}_{32})^{(6)} + (\widehat{P}_{32})^{(6)} (\widehat{k}_{32})^{(6)}\right) d\left(\left((G_{35})^{(1)}, (T_{35})^{(1)}; (G_{35})^{(2)}, (T_{35})^{(2)}\right)\right)$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 26:** The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)} t}$ *and* $(\widehat{Q}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)} t}$ respectively of $\mathbb{R}_+$. 249

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i = 32{,}33{,}34$ depend only on $T_{33}$ and respectively on $(G_{35})$(*and not on* $t$) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 27:** There does not exist any $t$ where $G_i(t) = 0$ *and* $T_i(t) = 0$ 250

it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(6)} - (a_i'')^{(6)}(T_{33}(s_{(32)}), s_{(32)})\} ds_{(32)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(6)} t\right)} > 0$ for t > 0

**Definition of** $\left((\widehat{M}_{32})^{(6)}\right)_1, \left((\widehat{M}_{32})^{(6)}\right)_2$ *and* $\left((\widehat{M}_{32})^{(6)}\right)_3$ : 251

**Remark 28:** if $G_{32}$ is bounded, the same property have also $G_{33}$ *and* $G_{34}$ . indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a'_{33})^{(6)} G_{33}$ and by integrating

$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)} \left((\widehat{M}_{32})^{(6)}\right)_1 / (a'_{33})^{(6)}$

In the same way , one can obtain

$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_2/(a'_{34})^{(6)}$

If $G_{33}$ *or* $G_{34}$ is bounded, the same property follows for $G_{32}$ , $G_{34}$ and $G_{32}$ , $G_{33}$ respectively.

**Remark 29:** If $G_{32}$ *is* bounded, from below, the same property holds for $G_{33}$ *and* $G_{34}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{33}$ is bounded from below. 252

**Remark 30:** If $T_{32}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}((G_{35})(t),t)) = (b'_{33})^{(6)}$ then $T_{33} \to \infty$. 253

**Definition of** $(m)^{(6)}$ and $\varepsilon_6$ :

Indeed let $t_6$ be so that for $t > t_6$

$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t),t\right) < \varepsilon_6, T_{32}(t) > (m)^{(6)}$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right), \quad t = log\frac{2}{\varepsilon_6}$ By taking now $\varepsilon_6$ sufficiently small one sees that $T_{33}$ is unbounded.
The same property holds for $T_{34}$ if $\lim_{t\to\infty}(b_{34}'')^{(6)}\left((G_{35})(t),t(t),t\right) = (b'_{34})^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{37}$ , $G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}$ , $\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}} < 1$ and to choose
$(\widehat{P}_{36})^{(7)}$ and $(\widehat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[(\widehat{P}_{36})^{(7)} + \left((\widehat{P}_{36})^{(7)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{36})^{(7)} \quad 256$$

257

$$\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[\left((\widehat{Q}_{36})^{(7)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{36})^{(7)}\right] \leq (\widehat{Q}_{36})^{(7)}$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself
The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric 258

$d\left(((G_{39})^{(1)},(T_{39})^{(1)}),((G_{39})^{(2)},(T_{39})^{(2)})\right) =$
$sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t) - G_i^{(2)}(t)|e^{-(\widehat{M}_{36})^{(7)}t}, \max_{t\in\mathbb{R}_+}|T_i^{(1)}(t) - T_i^{(2)}(t)|e^{-(\widehat{M}_{36})^{(7)}t}\}$

Indeed if we denote

**Definition of** $\widetilde{(G_{39})}, \widetilde{(T_{39})} : \left( \widetilde{(G_{39})}, \widetilde{(T_{39})} \right) = \mathcal{A}^{(7)}((G_{39}), (T_{39}))$
It results

$$\left|\tilde{G}_{36}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{36})^{(7)} \left|G_{37}^{(1)} - G_{37}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{(\widehat{M}_{36})^{(7)} s_{(36)}}\, ds_{(36)} +$$
$$\int_0^t \{(a'_{36})^{(7)} \left|G_{36}^{(1)} - G_{36}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} +$$
$$(a''_{36})^{(7)}\left(T_{37}^{(1)}, s_{(36)}\right) \left|G_{36}^{(1)} - G_{36}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{(\widehat{M}_{36})^{(7)} s_{(36)}} +$$
$$G_{36}^{(2)} \left|(a''_{36})^{(7)}\left(T_{37}^{(1)}, s_{(36)}\right) - (a''_{36})^{(7)}\left(T_{37}^{(2)}, s_{(36)}\right)\right| \; e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{(\widehat{M}_{36})^{(7)} s_{(36)}} \} ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$$\left|(G_{39})^{(1)} - (G_{39})^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} t} \leq \qquad 259$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}} \left((a_{36})^{(7)} + (a'_{36})^{(7)} + (\widehat{A}_{36})^{(7)} + (\widehat{P}_{36})^{(7)} (\widehat{k}_{36})^{(7)}\right) d\left(((G_{39})^{(1)}, (T_{39})^{(1)};\ (G_{39})^{(2)}, (T_{39})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 31:** The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)} t}$ *and* $(\widehat{Q}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)} t}$ respectively of $\mathbb{R}_+$. 260

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(7)}$ and $(b_i'')^{(7)}, i = 36{,}37{,}38$ depend only on $T_{37}$ and respectively on $(G_{39})$(*and not on* $t$) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 32:** There does not exist any $t$ where $G_i(t) = 0$ *and* $T_i(t) = 0$ 261

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(7)} - (a_i'')^{(7)}(T_{37}(s_{(36)}), s_{(36)})\} ds_{(36)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(7)} t)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2$ *and* $\left((\widehat{M}_{36})^{(7)}\right)_3$ : 262

**Remark 33:** if $G_{36}$ is bounded, the same property have also $G_{37}$ *and* $G_{38}$ . indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a'_{37})^{(7)} G_{37}$ and by integrating

$$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)} \left((\widehat{M}_{36})^{(7)}\right)_1 / (a'_{37})^{(7)}$$

In the same way , one can obtain

$$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2/(a'_{38})^{(7)}$$

If $G_{37}$ *or* $G_{38}$ is bounded, the same property follows for $G_{36}$ , $G_{38}$ and $G_{36}$ , $G_{37}$ respectively.

**Remark 34:** If $G_{36}$ *is* bounded, from below, the same property holds for $G_{37}$ *and* $G_{38}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{37}$ is bounded from below. 263

**Remark 35:** If $T_{36}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(7)}((G_{39})(t),t)) = (b'_{37})^{(7)}$ then $T_{37} \to \infty$. 264

**Definition of** $(m)^{(7)}$ and $\varepsilon_7$ :

Indeed let $t_7$ be so that for $t > t_7$

$$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t),t) < \varepsilon_7, T_{36}(t) > (m)^{(7)}$$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right)$, $t = log\frac{2}{\varepsilon_7}$ By taking now $\varepsilon_7$ sufficiently small one sees that $T_{37}$ is unbounded.
The same property holds for $T_{38}$ if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t),t\right) = (b'_{38})^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}$ , $\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}} < 1$ and to choose 266
$(\widehat{P}_{40})^{(8)}$ and $(\widehat{Q}_{40})^{(8)}$ large to have

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[(\widehat{P}_{40})^{(8)} + \left((\widehat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{40})^{(8)} \quad 267$$

$$\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[\left((\widehat{Q}_{40})^{(8)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{40})^{(8)}\right] \leq (\widehat{Q}_{40})^{(8)} \quad 268$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)},(T_{43})^{(1)}),((G_{43})^{(2)},(T_{43})^{(2)})\right) = \quad 269$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\tilde{M}_{40})^{(8)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\tilde{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

**Definition of** $(\widetilde{G_{43}}),(\widetilde{T_{43}})$ : $\left((\widetilde{G_{43}}),(\widetilde{T_{43}})\right)=\mathcal{A}^{(8)}((G_{43}),(T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{40})^{(8)}\left|G_{41}^{(1)}-G_{41}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\,ds_{(40)}+$$
$$\int_0^t\{(a'_{40})^{(8)}\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$G_{40}^{(2)}\left|(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)-(a''_{40})^{(8)}\left(T_{41}^{(2)},s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\}ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval [0, t] 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)}-(G_{43})^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}t}\le \qquad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)}+(a'_{40})^{(8)}+(\widehat{A}_{40})^{(8)}+(\widehat{P}_{40})^{(8)}(\widehat{k}_{40})^{(8)}\right)d\left(((G_{43})^{(1)},(T_{43})^{(1)};\ (G_{43})^{(2)},(T_{43})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 36:** The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ *and* $(\widehat{Q}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ respectively of $\mathbb{R}_+$. 274

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i=40,41,42$ depend only on $T_{41}$ and respectively on $(G_{43})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 37** There does not exist any $t$ where $G_i(t)=0$ *and* $T_i(t)=0$ 275

it results

$$G_i(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(8)}-(a_i'')^{(8)}(T_{41}(s_{(40)}),s_{(40)})\}ds_{(40)}\right]}\ge 0$$

$$T_i(t)\ge T_i^0e^{\left(-(b_i')^{(8)}t\right)}>0\quad \text{for t}>0$$

**Definition of** $\left((\widehat{M}_{40})^{(8)}\right)_1,\left((\widehat{M}_{40})^{(8)}\right)_2$ *and* $\left((\widehat{M}_{40})^{(8)}\right)_3$ : 276

**Remark 38:** if $G_{40}$ is bounded, the same property have also $G_{41}$ *and* $G_{42}$ . indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a'_{41})^{(8)} G_{41}$ and by integrating

$$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1/(a'_{41})^{(8)}$$

In the same way , one can obtain

$$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2/(a'_{42})^{(8)}$$

If $G_{41}$ *or* $G_{42}$ is bounded, the same property follows for $G_{40}$ , $G_{42}$ and $G_{40}$ , $G_{41}$ respectively.

**Remark 39:** If $G_{40}$ *is* bounded, from below, the same property holds for $G_{41}$ *and* $G_{42}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{41}$ is bounded from below. 277

**Remark 40:** If $T_{40}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}((G_{43})(t),t)) = (b'_{41})^{(8)}$ then $T_{41} \to \infty$. 278

**Definition of** $(m)^{(8)}$ and $\varepsilon_8$ :

Indeed let $t_8$ be so that for $t > t_8$

$$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t),t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right)$, $t = log\frac{2}{\varepsilon_8}$ By taking now $\varepsilon_8$ sufficiently small one sees that $T_{41}$ is unbounded.
The same property holds for $T_{42}$ if $\lim_{t\to\infty}(b''_{42})^{(8)}\left((G_{43})(t),t(t),t\right) = (b'_{42})^{(8)}$

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It is now sufficient to take $\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}$ , $\frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$ and to choose $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ large to have

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}\left[(\hat{P}_{44})^{(9)} + \left((\hat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}}\left[\left((\hat{Q}_{44})^{(9)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{44})^{(9)}\right] \leq (\hat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)},(T_{47})^{(1)}),((G_{47})^{(2)},(T_{47})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\widehat{M}_{44})^{(9)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\widehat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{47}}),(\widetilde{T_{47}}):\left((\widetilde{G_{47}}),(\widetilde{T_{47}})\right)=\mathcal{A}^{(9)}\left((G_{47}),(T_{47})\right)$
It results

$$|\tilde{G}_{44}^{(1)}-\tilde{G}_i^{(2)}|\le\int_0^t(a_{44})^{(9)}|G_{45}^{(1)}-G_{45}^{(2)}|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\,ds_{(44)}+$$
$$\int_0^t\{(a'_{44})^{(9)}|G_{44}^{(1)}-G_{44}^{(2)}|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)|G_{44}^{(1)}-G_{44}^{(2)}|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$G_{44}^{(2)}|(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)-(a''_{44})^{(9)}\left(T_{45}^{(2)},s_{(44)}\right)|\;e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\}ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on 45,46,47,28 and 29 it follows

$$|(G_{47})^{(1)}-G^{(2)}|e^{-(\widehat{M}_{44})^{(9)}t}\le$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)}+(a'_{44})^{(9)}+(\widehat{A}_{44})^{(9)}+(\widehat{P}_{44})^{(9)}(\widehat{k}_{44})^{(9)}\right)d\left(((G_{47})^{(1)},(T_{47})^{(1)};\;(G_{47})^{(2)},(T_{47})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis (39,35,36) the result follows

**Remark 41:** The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ *and* $(\widehat{Q}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i=44,45,46$ depend only on $T_{45}$ and respectively on $(G_{47})$(*and not on* $t$) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 42:** There does not exist any $t$ where $G_i(t)=0$ *and* $T_i(t)=0$

From 99 to 44 it results

$$G_i(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(9)}-(a_i'')^{(9)}(T_{45}(s_{(44)}),s_{(44)})\}ds_{(44)}\right]}\ge 0$$

$$T_i(t)\ge T_i^0e^{\left(-(b_i')^{(9)}t\right)}>0\quad\text{for t}>0$$

**Definition of** $\left((\widehat{M}_{44})^{(9)}\right)_1,\left((\widehat{M}_{44})^{(9)}\right)_2$ *and* $\left((\widehat{M}_{44})^{(9)}\right)_3$ :

**Remark 43:** if $G_{44}$ is bounded, the same property have also $G_{45}$ *and* $G_{46}$ . indeed if

$G_{44}<(\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt}\le\left((\widehat{M}_{44})^{(9)}\right)_1-(a'_{45})^{(9)}G_{45}$ and by integrating

$$G_{45}\le\left((\widehat{M}_{44})^{(9)}\right)_2=G_{45}^0+2(a_{45})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_1/(a'_{45})^{(9)}$$

In the same way , one can obtain

$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_2/(a'_{46})^{(9)}$

If $G_{45}$ *or* $G_{46}$ is bounded, the same property follows for $G_{44}$ , $G_{46}$ and $G_{44}$ , $G_{45}$ respectively.

**Remark 44:** If $G_{44}$ *is* bounded, from below, the same property holds for $G_{45}$ *and* $G_{46}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{45}$ is bounded from below.

**Remark 45:** If $T_{44}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t),t\right) = (b'_{45})^{(9)}$ then $T_{45} \to \infty$.

**Definition of** $(m)^{(9)}$ and $\varepsilon_9$ :

Indeed let $t_9$ be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t),t\right) < \varepsilon_9, T_{44}(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right)$, $t = log\frac{2}{\varepsilon_9}$ By taking now $\varepsilon_9$ sufficiently small one sees that $T_{45}$ is unbounded.
The same property holds for $T_{46}$ if $\lim_{t\to\infty}(b''_{46})^{(9)}\left((G_{47})(t),t\right) = (b'_{46})^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

**Behavior of the solutions of equation** 280

**Theorem** If we denote and define

**Definition of** $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ :

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)} \leq -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14},t) + (a''_{14})^{(1)}(T_{14},t) \leq -(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)} \leq -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G,t) - (b''_{14})^{(1)}(G,t) \leq -(\tau_1)^{(1)}$

**Definition of** $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$ : 281

By $(\nu_1)^{(1)} > 0$ , $(\nu_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$ , $(u_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

**Definition of** $(\bar{\nu}_1)^{(1)},, (\bar{\nu}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$ : 282

By $(\bar{\nu}_1)^{(1)} > 0$ , $(\bar{\nu}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$ , $(\bar{u}_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

**Definition of** $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (\nu_0)^{(1)}$ :- 283

If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by

$(m_2)^{(1)} = (v_0)^{(1)}, (m_1)^{(1)} = (v_1)^{(1)},\ if\ (v_0)^{(1)} < (v_1)^{(1)}$

$(m_2)^{(1)} = (v_1)^{(1)}, (m_1)^{(1)} = (\bar{v}_1)^{(1)}, if\ (v_1)^{(1)} < (v_0)^{(1)} < (\bar{v}_1)^{(1)},$

and $\boxed{(v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$

$(m_2)^{(1)} = (v_1)^{(1)}, (m_1)^{(1)} = (v_0)^{(1)},\ if\ (\bar{v}_1)^{(1)} < (v_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)},\ if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)},$

and $\boxed{(u_0)^{(1)} = \frac{T_{13}^0}{T_{14}^0}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$ are defined

Then the solution of global equations satisfies the inequalities 285

$$G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{13}(t) \le G_{13}^0 e^{(S_1)^{(1)} t}$$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)} t}$$

$$\left(\frac{(a_{15})^{(1)} G_{13}^0}{(m_1)^{(1)}((S_1)^{(1)} - (p_{13})^{(1)} - (S_2)^{(1)})}\left[e^{((S_1)^{(1)} - (p_{13})^{(1)})t} - e^{-(S_2)^{(1)} t}\right] + G_{15}^0 e^{-(S_2)^{(1)} t} \le G_{15}(t) \le \right. \quad 286$$

$$\left. \frac{(a_{15})^{(1)} G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)} - (a'_{15})^{(1)})}\left[e^{(S_1)^{(1)} t} - e^{-(a'_{15})^{(1)} t}\right] + G_{15}^0 e^{-(a'_{15})^{(1)} t}\right)$$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t}} \quad 287$$

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t} \quad 288$$

$$\frac{(b_{15})^{(1)} T_{13}^0}{(\mu_1)^{(1)}((R_1)^{(1)} - (b'_{15})^{(1)})}\left[e^{(R_1)^{(1)} t} - e^{-(b'_{15})^{(1)} t}\right] + T_{15}^0 e^{-(b'_{15})^{(1)} t} \le T_{15}(t) \le \quad 289$$

$$\frac{(a_{15})^{(1)} T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)} + (r_{13})^{(1)} + (R_2)^{(1)})}\left[e^{((R_1)^{(1)} + (r_{13})^{(1)})t} - e^{-(R_2)^{(1)} t}\right] + T_{15}^0 e^{-(R_2)^{(1)} t}$$

**Definition of** $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:- 290

Where $(S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$

$$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$$

$$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$$

$$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$$

**Behavior of the solutions of equation** 291

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ : 292

$(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ four constants satisfying
$-(\sigma_2)^{(2)} \le -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(T_{17}, t) + (a''_{17})^{(2)}(T_{17}, t) \le -(\sigma_1)^{(2)}$ 293

$-(\tau_2)^{(2)} \le -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}\big((G_{19}), t\big) - (b''_{17})^{(2)}\big((G_{19}), t\big) \le -(\tau_1)^{(2)}$ 294

**Definition of** $(\nu_1)^{(2)}, (\nu_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$ : 295

By $(\nu_1)^{(2)} > 0$ , $(\nu_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0$ , $(u_2)^{(2)} < 0$ the roots 296

of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 297

and $(b_{14})^{(2)}\big(u^{(2)}\big)^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and 298

**Definition of** $(\bar{\nu}_1)^{(2)}, , (\bar{\nu}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$ : 299

By $(\bar{\nu}_1)^{(2)} > 0$ , $(\bar{\nu}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0$ , $(\bar{u}_2)^{(2)} < 0$ the 300

roots of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 301

and $(b_{17})^{(2)}\big(u^{(2)}\big)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 302

**Definition of** $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ :- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ , ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{G^0_{16}}{G^0_{17}}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, ***if*** $(u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$ , ***if*** $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)},$ ***if*** $(\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)} t}$$

$(p_i)^{(2)}$ is defined by equation

$$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)} t} \quad 311$$

$$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)} t}\right] + G_{18}^0 e^{-(S_2)^{(2)} t} \le G_{18}(t) \le \right. \quad 312$$

$$\left. \frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)} t} - e^{-(a'_{18})^{(2)} t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)} t}\right)$$

$$\boxed{T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}} \quad 313$$

$$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t} \quad 314$$

$$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)} t} - e^{-(b'_{18})^{(2)} t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)} t} \le T_{18}(t) \le \quad 315$$

$$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})}\left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)} t}\right] + T_{18}^0 e^{-(R_2)^{(2)} t}$$

**Definition of** $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$$

$$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)} \quad 318$$

$$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$$

**Behavior of the solutions** 319

**Theorem 3:** If we denote and define

**Definition of** $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ :

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}\big((G_{23}), t\big) \le -(\tau_1)^{(3)}$$

**Definition of** $(v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$ : 320

By $(v_1)^{(3)} > 0$ , $(v_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$ , $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{v}_1)^{(3)} > 0$ , $(\bar{v}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$ , $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

**Definition of** $(m_1)^{(3)}$ , $(m_2)^{(3)}$ , $(\mu_1)^{(3)}, (\mu_2)^{(3)}$ :- 321

If we define $(m_1)^{(3)}$ , $(m_2)^{(3)}$ , $(\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (v_0)^{(3)}, (m_1)^{(3)} = (v_1)^{(3)}$, ***if*** $(v_0)^{(3)} < (v_1)^{(3)}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (\bar{v}_1)^{(3)}$ , ***if*** $(v_1)^{(3)} < (v_0)^{(3)} < (\bar{v}_1)^{(3)}$,

and $\boxed{(v_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_0)^{(3)}$, ***if*** $(\bar{v}_1)^{(3)} < (v_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}$, ***if*** $(u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$ , ***if*** $(u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}$, and $\boxed{(u_0)^{(3)} = \frac{T_{20}^0}{T_{21}^0}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}$, ***if*** $(\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G_{20}^0 e^{(S_1)^{(3)}t}$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)}t} \quad (323)$$

$$\left(\frac{(a_{22})^{(3)}G_{20}^0}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\left[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)}t}\right] + G_{22}^0 e^{-(S_2)^{(3)}t} \le G_{22}(t) \le \right. \quad (324)$$

$$\left. \frac{(a_{22})^{(3)}G_{20}^0}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}\left[e^{(S_1)^{(3)}t} - e^{-(a'_{22})^{(3)}t}\right] + G_{22}^0 e^{-(a'_{22})^{(3)}t}\right)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t}} \quad (325)$$

$$\frac{1}{(\mu_1)^{(3)}}T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}}T_{20}^0 e^{((R_1)^{(3)}+(r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)}T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)}-(b'_{22})^{(3)})}\left[e^{(R_1)^{(3)}t} - e^{-(b'_{22})^{(3)}t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)}t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)}T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)}+(r_{20})^{(3)}+(R_2)^{(3)})}\left[e^{((R_1)^{(3)}+(r_{20})^{(3)})t} - e^{-(R_2)^{(3)}t}\right] + T_{22}^0 e^{-(R_2)^{(3)}t}$$

**Definition of** $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

**Behavior of the solutions of equation**

**Theorem:** If we denote and define

**Definition of** $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ :

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$$

$$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}\big((G_{27}), t\big) - (b''_{25})^{(4)}\big((G_{27}), t\big) \le -(\tau_1)^{(4)}$$

**Definition of** $(v_1)^{(4)}, (v_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, v^{(4)}, u^{(4)}$ : 329

By $(v_1)^{(4)} > 0$ , $(v_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$ , $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\big(v^{(4)}\big)^2 + (\sigma_1)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\big(u^{(4)}\big)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

**Definition of** $(\bar{v}_1)^{(4)}, , (\bar{v}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$ : 330

By $(\bar{v}_1)^{(4)} > 0$ , $(\bar{v}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$ , $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\big(v^{(4)}\big)^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\big(u^{(4)}\big)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

**Definition of** $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}, (v_0)^{(4)}$ :-

If we define $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (v_0)^{(4)}, (m_1)^{(4)} = (v_1)^{(4)}$, ***if*** $(v_0)^{(4)} < (v_1)^{(4)}$

$(m_2)^{(4)} = (v_1)^{(4)}, (m_1)^{(4)} = (\bar{v}_1)^{(4)}$ , ***if*** $(v_4)^{(4)} < (v_0)^{(4)} < (\bar{v}_1)^{(4)}$,
and $\boxed{(v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (v_4)^{(4)}, (m_1)^{(4)} = (v_0)^{(4)},\ \boldsymbol{if}\ (\bar{v}_4)^{(4)} < (v_0)^{(4)}$

and analogously

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$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)},\ \boldsymbol{if}\ (u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}, \boldsymbol{if}\ (u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)},$

and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}, \boldsymbol{if}\ (\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

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$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)} t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)} t}$$

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$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)} t}\right] + G_{26}^0 e^{-(S_2)^{(4)} t} \le G_{26}(t) \le\right. \quad 334$$

*(a26)4G240(m2)4(S1)4−(a26′)4e(S1)4t−e−(a26′)4t+ G260e−(a26′)4t*

$$\boxed{T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}} T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}} T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t} \quad 335$$

$$\frac{(b_{26})^{(4)} T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)} - (b'_{26})^{(4)})}\left[e^{(R_1)^{(4)} t} - e^{-(b'_{26})^{(4)} t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)} t} \le T_{26}(t) \le \quad 336$$

$$\frac{(a_{26})^{(4)} T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)} + (r_{24})^{(4)} + (R_2)^{(4)})}\left[e^{((R_1)^{(4)} + (r_{24})^{(4)})t} - e^{-(R_2)^{(4)} t}\right] + T_{26}^0 e^{-(R_2)^{(4)} t}$$

**Definition of** $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

**Behavior of the solutions of equation** 338

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ :

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

**Definition of** $(\nu_1)^{(5)}, (\nu_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, \nu^{(5)}, u^{(5)}$ : 339

By $(\nu_1)^{(5)} > 0$, $(\nu_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$, $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(5)},, (\bar{\nu}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$ : 340

By $(\bar{\nu}_1)^{(5)} > 0$, $(\bar{\nu}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$, $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

**Definition of** $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (\nu_0)^{(5)}$ :-

If we define $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (\nu_0)^{(5)}, (m_1)^{(5)} = (\nu_1)^{(5)}$, ***if*** $(\nu_0)^{(5)} < (\nu_1)^{(5)}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\bar{\nu}_1)^{(5)}$, ***if*** $(\nu_1)^{(5)} < (\nu_0)^{(5)} < (\bar{\nu}_1)^{(5)}$,
and $\boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)}$, ***if*** $(\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)}$, ***if*** $(u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}$, ***if*** $(u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)}$,
and $\boxed{(u_0)^{(5)} = \frac{T^0_{28}}{T^0_{29}}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}$, ***if*** $(\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$$G^0_{28} e^{\left((S_1)^{(5)} - (p_{28})^{(5)}\right)t} \le G_{28}(t) \le G^0_{28} e^{(S_1)^{(5)} t}$$

where $(p_i)^{(5)}$ is defined by equation

$$\frac{1}{(m_5)^{(5)}} G^0_{28} e^{\left((S_1)^{(5)} - (p_{28})^{(5)}\right)t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}} G^0_{28} e^{(S_1)^{(5)} t} \quad 343$$

$$\left(\frac{(a_{30})^{(5)} G^0_{28}}{(m_1)^{(5)}\left((S_1)^{(5)} - (p_{28})^{(5)} - (S_2)^{(5)}\right)}\left[e^{\left((S_1)^{(5)} - (p_{28})^{(5)}\right)t} - e^{-(S_2)^{(5)} t}\right] + G^0_{30} e^{-(S_2)^{(5)} t} \le G_{30}(t) \le \right. \quad 344$$

$$\frac{(a_{30})^{(5)}G_{28}^{0}}{(m_2)^{(5)}((S_1)^{(5)}-(a'_{30})^{(5)})}\left[e^{(S_1)^{(5)}t}-e^{-(a'_{30})^{(5)}t}\right]+\ G_{30}^{0}e^{-(a'_{30})^{(5)}t}\Big)$$

$$\boxed{T_{28}^{0}e^{(R_1)^{(5)}t}\le T_{28}(t)\le T_{28}^{0}e^{((R_1)^{(5)}+(r_{28})^{(5)})t}} \quad 345$$

$$\frac{1}{(\mu_1)^{(5)}}T_{28}^{0}e^{(R_1)^{(5)}t}\le T_{28}(t)\le\frac{1}{(\mu_2)^{(5)}}T_{28}^{0}e^{((R_1)^{(5)}+(r_{28})^{(5)})t} \quad 346$$

$$\frac{(b_{30})^{(5)}T_{28}^{0}}{(\mu_1)^{(5)}((R_1)^{(5)}-(b'_{30})^{(5)})}\left[e^{(R_1)^{(5)}t}-e^{-(b'_{30})^{(5)}t}\right]+T_{30}^{0}e^{-(b'_{30})^{(5)}t}\le T_{30}(t)\le \quad 347$$

$$\frac{(a_{30})^{(5)}T_{28}^{0}}{(\mu_2)^{(5)}((R_1)^{(5)}+(r_{28})^{(5)}+(R_2)^{(5)})}\left[e^{((R_1)^{(5)}+(r_{28})^{(5)})t}-e^{-(R_2)^{(5)}t}\right]+T_{30}^{0}e^{-(R_2)^{(5)}t}$$

**Definition of** $(S_1)^{(5)},(S_2)^{(5)},(R_1)^{(5)},(R_2)^{(5)}$:- 348

Where $(S_1)^{(5)}=(a_{28})^{(5)}(m_2)^{(5)}-(a'_{28})^{(5)}$

$(S_2)^{(5)}=(a_{30})^{(5)}-(p_{30})^{(5)}$

$(R_1)^{(5)}=(b_{28})^{(5)}(\mu_2)^{(5)}-(b'_{28})^{(5)}$

$(R_2)^{(5)}=(b'_{30})^{(5)}-(r_{30})^{(5)}$

**Behavior of the solutions of equation** 349

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(6)},(\sigma_2)^{(6)},(\tau_1)^{(6)},(\tau_2)^{(6)}$ :

$(\sigma_1)^{(6)},(\sigma_2)^{(6)},(\tau_1)^{(6)},(\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)}\le-(a'_{32})^{(6)}+(a'_{33})^{(6)}-(a''_{32})^{(6)}(T_{33},t)+(a''_{33})^{(6)}(T_{33},t)\le-(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)}\le-(b'_{32})^{(6)}+(b'_{33})^{(6)}-(b''_{32})^{(6)}\big((G_{35}),t\big)-(b''_{33})^{(6)}\big((G_{35}),t\big)\le-(\tau_1)^{(6)}$$

**Definition of** $(v_1)^{(6)},(v_2)^{(6)},(u_1)^{(6)},(u_2)^{(6)},v^{(6)},u^{(6)}$ : 350

By $(v_1)^{(6)}>0$ , $(v_2)^{(6)}<0$ and respectively $(u_1)^{(6)}>0$ , $(u_2)^{(6)}<0$ the roots of the equations $(a_{33})^{(6)}\big(v^{(6)}\big)^2+(\sigma_1)^{(6)}v^{(6)}-(a_{32})^{(6)}=0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2+(\tau_1)^{(6)}u^{(6)}-(b_{32})^{(6)}=0$ and

**Definition of** $(\bar{v}_1)^{(6)},,(\bar{v}_2)^{(6)},(\bar{u}_1)^{(6)},(\bar{u}_2)^{(6)}$ : 351

By $(\bar{v}_1)^{(6)}>0$ , $(\bar{v}_2)^{(6)}<0$ and respectively $(\bar{u}_1)^{(6)}>0$ , $(\bar{u}_2)^{(6)}<0$ the roots of the equations $(a_{33})^{(6)}\big(v^{(6)}\big)^2+(\sigma_2)^{(6)}v^{(6)}-(a_{32})^{(6)}=0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2+(\tau_2)^{(6)}u^{(6)}-(b_{32})^{(6)}=0$

**Definition of** $(m_1)^{(6)},(m_2)^{(6)},(\mu_1)^{(6)},(\mu_2)^{(6)},(v_0)^{(6)}$ :-

If we define $(m_1)^{(6)},(m_2)^{(6)},(\mu_1)^{(6)},(\mu_2)^{(6)}$ by

$(m_2)^{(6)} = (\nu_0)^{(6)}, (m_1)^{(6)} = (\nu_1)^{(6)},\ \boldsymbol{if}\ (\nu_0)^{(6)} < (\nu_1)^{(6)}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}, \boldsymbol{if}\ (\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)},$

and $\boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$

and analogously 352

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)},\ \boldsymbol{if}\ (u_0)^{(6)} < (u_1)^{(6)}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}, \boldsymbol{if}\ (u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)},$

and $\boxed{(u_0)^{(6)} = \frac{T_{32}^0}{T_{33}^0}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}, \boldsymbol{if}\ (\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$

Then the solution of global equations satisfies the inequalities 353

$$G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{32}(t) \le G_{32}^0 e^{(S_1)^{(6)} t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}} G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{33}(t) \le \frac{1}{(m_2)^{(6)}} G_{32}^0 e^{(S_1)^{(6)} t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)} G_{32}^0}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})}\left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)} t}\right] + G_{34}^0 e^{-(S_2)^{(6)} t} \le G_{34}(t) \le \right. \quad 355$$

*(a34)6G320(m2)6(S1)6−(a34′)6e(S1)6t−e−(a34′)6t+ G340e−(a34′)6t*

$$\boxed{T_{32}^0 e^{(R_1)^{(6)} t} \le T_{32}(t) \le T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}} T_{32}^0 e^{(R_1)^{(6)} t} \le T_{32}(t) \le \frac{1}{(\mu_2)^{(6)}} T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)} T_{32}^0}{(\mu_1)^{(6)}((R_1)^{(6)} - (b'_{34})^{(6)})}\left[e^{(R_1)^{(6)} t} - e^{-(b'_{34})^{(6)} t}\right] + T_{34}^0 e^{-(b'_{34})^{(6)} t} \le T_{34}(t) \le \quad 358$$

$$\frac{(a_{34})^{(6)} T_{32}^0}{(\mu_2)^{(6)}((R_1)^{(6)} + (r_{32})^{(6)} + (R_2)^{(6)})}\left[e^{((R_1)^{(6)} + (r_{32})^{(6)})t} - e^{-(R_2)^{(6)} t}\right] + T_{34}^0 e^{-(R_2)^{(6)} t}$$

**Definition of** $(S_1)^{(6)}, (S_2)^{(6)}, (R_1)^{(6)}, (R_2)^{(6)}$:- 359

Where $(S_1)^{(6)} = (a_{32})^{(6)}(m_2)^{(6)} - (a'_{32})^{(6)}$

$(S_2)^{(6)} = (a_{34})^{(6)} - (p_{34})^{(6)}$

$(R_1)^{(6)} = (b_{32})^{(6)}(\mu_2)^{(6)} - (b'_{32})^{(6)}$

$(R_2)^{(6)} = (b'_{34})^{(6)} - (r_{34})^{(6)}$

**Behavior of the solutions of equation**

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$ :

$(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)} \le -(a'_{36})^{(7)} + (a'_{37})^{(7)} - (a''_{36})^{(7)}(T_{37}, t) + (a''_{37})^{(7)}(T_{37}, t) \le -(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)} \le -(b'_{36})^{(7)} + (b'_{37})^{(7)} - (b''_{36})^{(7)}\big((G_{39}), t\big) - (b''_{37})^{(7)}\big((G_{39}), t\big) \le -(\tau_1)^{(7)}$$

**Definition of** $(v_1)^{(7)}, (v_2)^{(7)}, (u_1)^{(7)}, (u_2)^{(7)}, v^{(7)}, u^{(7)}$ : 361

By $(v_1)^{(7)} > 0, (v_2)^{(7)} < 0$ and respectively $(u_1)^{(7)} > 0, (u_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(v^{(7)}\big)^2 + (\sigma_1)^{(7)}v^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_1)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$ and

**Definition of** $(\bar{v}_1)^{(7)},, (\bar{v}_2)^{(7)}, (\bar{u}_1)^{(7)}, (\bar{u}_2)^{(7)}$ : 362

By $(\bar{v}_1)^{(7)} > 0, (\bar{v}_2)^{(7)} < 0$ and respectively $(\bar{u}_1)^{(7)} > 0, (\bar{u}_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(v^{(7)}\big)^2 + (\sigma_2)^{(7)}v^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

**Definition of** $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (v_0)^{(7)}$ :-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$$(m_2)^{(7)} = (v_0)^{(7)}, (m_1)^{(7)} = (v_1)^{(7)}, \ \boldsymbol{if}\ (v_0)^{(7)} < (v_1)^{(7)}$$

$$(m_2)^{(7)} = (v_1)^{(7)}, (m_1)^{(7)} = (\bar{v}_1)^{(7)}, \boldsymbol{if}\ (v_1)^{(7)} < (v_0)^{(7)} < (\bar{v}_1)^{(7)},$$

and $\boxed{(v_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$

$$(m_2)^{(7)} = (v_1)^{(7)}, (m_1)^{(7)} = (v_0)^{(7)}, \ \boldsymbol{if}\ (\bar{v}_1)^{(7)} < (v_0)^{(7)}$$

and analogously 363

$$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}, \ \boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$$

and $\boxed{(u_0)^{(7)} = \frac{T^0_{36}}{T^0_{37}}}$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)} \text{ where } (u_1)^{(7)}, (\bar{u}_1)^{(7)}$$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^{0}e^{((S_1)^{(7)}-(p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^{0}e^{(S_1)^{(7)}t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}}G_{36}^{0}e^{((S_1)^{(7)}-(p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}}G_{36}^{0}e^{(S_1)^{(7)}t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)}G_{36}^{0}}{(m_1)^{(7)}((S_1)^{(7)}-(p_{36})^{(7)}-(S_2)^{(7)})}\left[e^{((S_1)^{(7)}-(p_{36})^{(7)})t}-e^{-(S_2)^{(7)}t}\right]+G_{38}^{0}e^{-(S_2)^{(7)}t} \le G_{38}(t) \le \right. \quad 366$$

$$\left.\frac{(a_{38})^{(7)}G_{36}^{0}}{(m_2)^{(7)}((S_1)^{(7)}-(a'_{38})^{(7)})}[e^{(S_1)^{(7)}t}-e^{-(a'_{38})^{(7)}t}]+\ G_{38}^{0}e^{-(a'_{38})^{(7)}t}\right)$$

$$\boxed{T_{36}^{0}e^{(R_1)^{(7)}t} \le T_{36}(t) \le T_{36}^{0}e^{((R_1)^{(7)}+(r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}}T_{36}^{0}e^{(R_1)^{(7)}t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}}T_{36}^{0}e^{((R_1)^{(7)}+(r_{36})^{(7)})t} \quad 368$$

$$\frac{(b_{38})^{(7)}T_{36}^{0}}{(\mu_1)^{(7)}((R_1)^{(7)}-(b'_{38})^{(7)})}\left[e^{(R_1)^{(7)}t}-e^{-(b'_{38})^{(7)}t}\right]+T_{38}^{0}e^{-(b'_{38})^{(7)}t} \le T_{38}(t) \le \quad 369$$

$$\frac{(a_{38})^{(7)}T_{36}^{0}}{(\mu_2)^{(7)}((R_1)^{(7)}+(r_{36})^{(7)}+(R_2)^{(7)})}\left[e^{((R_1)^{(7)}+(r_{36})^{(7)})t}-e^{-(R_2)^{(7)}t}\right]+T_{38}^{0}e^{-(R_2)^{(7)}t}$$

**Definition of** $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$$

$$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$$

$$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$$

**Behavior of the solutions of equation** 371

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ :

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$$-(\sigma_2)^{(8)} \le -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \le -(\sigma_1)^{(8)}$$

$$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\big((G_{43}), t\big) - (b''_{41})^{(8)}\big((G_{43}), t\big) \le -(\tau_1)^{(8)}$$

**Definition of** $(\nu_1)^{(8)}, (\nu_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, \nu^{(8)}, u^{(8)}$ : 372

By $(\nu_1)^{(8)} > 0$ , $(\nu_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0$ , $(u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\left(u^{(8)}\right)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(8)}$,, $(\bar{\nu}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$ :

By $(\bar{\nu}_1)^{(8)} > 0$ , $(\bar{\nu}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0$ , $(\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\left(u^{(8)}\right)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

**Definition of** $(m_1)^{(8)}$ , $(m_2)^{(8)}$ , $(\mu_1)^{(8)}, (\mu_2)^{(8)}, (\nu_0)^{(8)}$ :-

If we define $(m_1)^{(8)}$ , $(m_2)^{(8)}$ , $(\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$(m_2)^{(8)} = (\nu_0)^{(8)}, (m_1)^{(8)} = (\nu_1)^{(8)}$, $\boldsymbol{if}\ (\nu_0)^{(8)} < (\nu_1)^{(8)}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ , $\boldsymbol{if}\ (\nu_1)^{(8)} < (\nu_0)^{(8)} < (\bar{\nu}_1)^{(8)}$,
and $\boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\nu_0)^{(8)}$, $\boldsymbol{if}\ (\bar{\nu}_1)^{(8)} < (\nu_0)^{(8)}$

and analogously 374

$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)}$, $\boldsymbol{if}\ (u_0)^{(8)} < (u_1)^{(8)}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}$ , $\boldsymbol{if}\ (u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)}$,
and $\boxed{(u_0)^{(8)} = \frac{T_{40}^0}{T_{41}^0}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}, \boldsymbol{if}\ (\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^0 e^{(S_1)^{(8)}t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}} G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}} G_{40}^0 e^{(S_1)^{(8)}t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)}G_{40}^{0}}{(m_1)^{(8)}((S_1)^{(8)}-(p_{40})^{(8)}-(S_2)^{(8)})}\left[e^{((S_1)^{(8)}-(p_{40})^{(8)})t}-e^{-(S_2)^{(8)}t}\right]+G_{42}^{0}e^{-(S_2)^{(8)}t}\le G_{42}(t)\le\right. \quad 377$$

$$\left.\frac{(a_{42})^{(8)}G_{40}^{0}}{(m_2)^{(8)}((S_1)^{(8)}-(a'_{42})^{(8)})}\left[e^{(S_1)^{(8)}t}-e^{-(a'_{42})^{(8)}t}\right]+G_{42}^{0}e^{-(a'_{42})^{(8)}t}\right)$$

$$\boxed{T_{40}^{0}e^{(R_1)^{(8)}t}\le T_{40}(t)\le T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}}T_{40}^{0}e^{(R_1)^{(8)}t}\le T_{40}(t)\le\frac{1}{(\mu_2)^{(8)}}T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)}T_{40}^{0}}{(\mu_1)^{(8)}((R_1)^{(8)}-(b'_{42})^{(8)})}\left[e^{(R_1)^{(8)}t}-e^{-(b'_{42})^{(8)}t}\right]+T_{42}^{0}e^{-(b'_{42})^{(8)}t}\le T_{42}(t)\le \quad 380$$

$$\frac{(a_{42})^{(8)}T_{40}^{0}}{(\mu_2)^{(8)}((R_1)^{(8)}+(r_{40})^{(8)}+(R_2)^{(8)})}\left[e^{((R_1)^{(8)}+(r_{40})^{(8)})t}-e^{-(R_2)^{(8)}t}\right]+T_{42}^{0}e^{-(R_2)^{(8)}t}$$

**Definition of** $(S_1)^{(8)},(S_2)^{(8)},(R_1)^{(8)},(R_2)^{(8)}$:- 381

Where $(S_1)^{(8)}=(a_{40})^{(8)}(m_2)^{(8)}-(a'_{40})^{(8)}$

$$(S_2)^{(8)}=(a_{42})^{(8)}-(p_{42})^{(8)}$$

$$(R_1)^{(8)}=(b_{40})^{(8)}(\mu_2)^{(8)}-(b'_{40})^{(8)}$$

$$(R_2)^{(8)}=(b'_{42})^{(8)}-(r_{42})^{(8)}$$

**Behavior of the solutions of equation 37 to 92** 382

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(9)},(\sigma_2)^{(9)},(\tau_1)^{(9)},(\tau_2)^{(9)}$:

$(\sigma_1)^{(9)},(\sigma_2)^{(9)},(\tau_1)^{(9)},(\tau_2)^{(9)}$ four constants satisfying

$$-(\sigma_2)^{(9)}\le-(a'_{44})^{(9)}+(a'_{45})^{(9)}-(a''_{44})^{(9)}(T_{45},t)+(a''_{45})^{(9)}(T_{45},t)\le-(\sigma_1)^{(9)}$$

$$-(\tau_2)^{(9)}\le-(b'_{44})^{(9)}+(b'_{45})^{(9)}-(b''_{44})^{(9)}\big((G_{47}),t\big)-(b''_{45})^{(9)}\big((G_{47}),t\big)\le-(\tau_1)^{(9)}$$

**Definition of** $(\nu_1)^{(9)},(\nu_2)^{(9)},(u_1)^{(9)},(u_2)^{(9)},\nu^{(9)},u^{(9)}$:

By $(\nu_1)^{(9)}>0,(\nu_2)^{(9)}<0$ and respectively $(u_1)^{(9)}>0,(u_2)^{(9)}<0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2+(\sigma_1)^{(9)}\nu^{(9)}-(a_{44})^{(9)}=0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2+(\tau_1)^{(9)}u^{(9)}-(b_{44})^{(9)}=0$ and

**Definition of** $(\bar{\nu}_1)^{(9)},,(\bar{\nu}_2)^{(9)},(\bar{u}_1)^{(9)},(\bar{u}_2)^{(9)}$:

By $(\bar{\nu}_1)^{(9)}>0,(\bar{\nu}_2)^{(9)}<0$ and respectively $(\bar{u}_1)^{(9)}>0,(\bar{u}_2)^{(9)}<0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2+(\sigma_2)^{(9)}\nu^{(9)}-(a_{44})^{(9)}=0$

and $(b_{45})^{(9)}\left(u^{(9)}\right)^2+(\tau_2)^{(9)}u^{(9)}-(b_{44})^{(9)}=0$

**Definition of** $(m_1)^{(9)}, (m_2)^{(9)}, (\mu_1)^{(9)}, (\mu_2)^{(9)}, (\nu_0)^{(9)}$ :-

If we define $(m_1)^{(9)}, (m_2)^{(9)}, (\mu_1)^{(9)}, (\mu_2)^{(9)}$ by

$(m_2)^{(9)}=(\nu_0)^{(9)}, (m_1)^{(9)}=(\nu_1)^{(9)},\ \boldsymbol{if}\ (\nu_0)^{(9)}<(\nu_1)^{(9)}$

$(m_2)^{(9)}=(\nu_1)^{(9)}, (m_1)^{(9)}=(\bar{\nu}_1)^{(9)}, \boldsymbol{if}\ (\nu_1)^{(9)}<(\nu_0)^{(9)}<(\bar{\nu}_1)^{(9)},$

and $\boxed{(\nu_0)^{(9)}=\frac{G_{44}^0}{G_{45}^0}}$

$(m_2)^{(9)}=(\nu_1)^{(9)}, (m_1)^{(9)}=(\nu_0)^{(9)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(9)}<(\nu_0)^{(9)}$

and analogously

$(\mu_2)^{(9)}=(u_0)^{(9)}, (\mu_1)^{(9)}=(u_1)^{(9)},\ \boldsymbol{if}\ (u_0)^{(9)}<(u_1)^{(9)}$

$(\mu_2)^{(9)}=(u_1)^{(9)}, (\mu_1)^{(9)}=(\bar{u}_1)^{(9)}, \boldsymbol{if}\ (u_1)^{(9)}<(u_0)^{(9)}<(\bar{u}_1)^{(9)},$

and $\boxed{(u_0)^{(9)}=\frac{T_{44}^0}{T_{45}^0}}$

$(\mu_2)^{(9)}=(u_1)^{(9)}, (\mu_1)^{(9)}=(u_0)^{(9)}, \boldsymbol{if}\ (\bar{u}_1)^{(9)}<(u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G_{44}^0 e^{((S_1)^{(9)}-(p_{44})^{(9)})t}\le G_{44}(t)\le G_{44}^0 e^{(S_1)^{(9)}t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}}G_{44}^0 e^{((S_1)^{(9)}-(p_{44})^{(9)})t}\le G_{45}(t)\le \frac{1}{(m_2)^{(9)}}G_{44}^0 e^{(S_1)^{(9)}t}$$

$$\left(\frac{(a_{46})^{(9)}G_{44}^0}{(m_1)^{(9)}((S_1)^{(9)}-(p_{44})^{(9)}-(S_2)^{(9)})}\left[e^{((S_1)^{(9)}-(p_{44})^{(9)})t}-e^{-(S_2)^{(9)}t}\right]+G_{46}^0 e^{-(S_2)^{(9)}t}\le G_{46}(t)\le\right.$$

$$\left.\frac{(a_{46})^{(9)}G_{44}^0}{(m_2)^{(9)}((S_1)^{(9)}-(a'_{46})^{(9)})}[e^{(S_1)^{(9)}t}-e^{-(a'_{46})^{(9)}t}]+\ G_{46}^0 e^{-(a'_{46})^{(9)}t}\right)$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)}t}\le T_{44}(t)\le T_{44}^0 e^{((R_1)^{(9)}+(r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}}T_{44}^0 e^{(R_1)^{(9)}t}\le T_{44}(t)\le \frac{1}{(\mu_2)^{(9)}}T_{44}^0 e^{((R_1)^{(9)}+(r_{44})^{(9)})t}$$

$$\frac{(b_{46})^{(9)}T_{44}^0}{(\mu_1)^{(9)}((R_1)^{(9)}-(b'_{46})^{(9)})}\left[e^{(R_1)^{(9)}t}-e^{-(b'_{46})^{(9)}t}\right]+T_{46}^0 e^{-(b'_{46})^{(9)}t}\le T_{46}(t)\le$$

$$\frac{(a_{46})^{(9)}T_{44}^0}{(\mu_2)^{(9)}((R_1)^{(9)}+(r_{44})^{(9)}+(R_2)^{(9)})}\left[e^{((R_1)^{(9)}+(r_{44})^{(9)})t}-e^{-(R_2)^{(9)}t}\right]+T_{46}^0 e^{-(R_2)^{(9)}t}$$

**Definition of** $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$

**Proof :** From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14},t)\right) - (a''_{14})^{(1)}(T_{14},t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

**Definition of** $v^{(1)}$ :- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)}\right) \le \frac{dv^{(1)}}{dt} \le -\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(1)}, (\nu_0)^{(1)}$ :-

For $0 < \boxed{(\nu_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}} < (\nu_1)^{(1)} < (\bar{\nu}_1)^{(1)}$

$$\nu^{(1)}(t) \ge \frac{(\nu_1)^{(1)} + (C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)} - (\nu_0)^{(1)}\right)t\right]}}{1 + (C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)} - (\nu_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(\nu_1)^{(1)} - (\nu_0)^{(1)}}{(\nu_0)^{(1)} - (\nu_2)^{(1)}}}$$

it follows $(\nu_0)^{(1)} \le \nu^{(1)}(t) \le (\nu_1)^{(1)}$

In the same manner , we get 384

$$\nu^{(1)}(t) \le \frac{(\bar{\nu}_1)^{(1)} + (\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)} - (\bar{\nu}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)} - (\bar{\nu}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{\nu}_1)^{(1)} - (\nu_0)^{(1)}}{(\nu_0)^{(1)} - (\bar{\nu}_2)^{(1)}}}$$

From which we deduce $(\nu_0)^{(1)} \le \nu^{(1)}(t) \le (\bar{\nu}_1)^{(1)}$

If $0 < (\nu_1)^{(1)} < (\nu_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}} < (\bar{\nu}_1)^{(1)}$ we find like in the previous case, 385

$$(\nu_1)^{(1)} \le \frac{(\nu_1)^{(1)} + (C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)} - (\nu_2)^{(1)}\right)t\right]}}{1 + (C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)} - (\nu_2)^{(1)}\right)t\right]}} \le \nu^{(1)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(1)} + (\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)} - (\bar{\nu}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)} - (\bar{\nu}_2)^{(1)}\right)t\right]}} \le (\bar{\nu}_1)^{(1)}$$

If $0 < (\nu_1)^{(1)} \le (\bar{\nu}_1)^{(1)} \le \boxed{(\nu_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}}$ , we obtain 386

$$(\nu_1)^{(1)} \leq \nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\nu_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(1)}(t)$ :-

$$(m_2)^{(1)} \leq \nu^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{\nu^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(1)}(t)$ :-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{13})^{(1)} = (a''_{14})^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)}G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)}T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

**Proof** : From global equations we obtain 387

$$\frac{d\nu^{(2)}}{dt} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right) - (a''_{17})^{(2)}(T_{17}, t)\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

**Definition of** $\nu^{(2)}$ :- $\boxed{\nu^{(2)} = \frac{G_{16}}{G_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \leq \frac{d\nu^{(2)}}{dt} \leq -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

**Definition of** $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$ :-

For $0 < (\nu_0)^{(2)} = \frac{G^0_{16}}{G^0_{17}} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \geq \frac{(\nu_1)^{(2)}+(\mathrm{C})^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_0)^{(2)}\right)t\right]}}{1+(\mathrm{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\mathrm{C})^{(2)} = \frac{(\nu_1)^{(2)}-(\nu_0)^{(2)}}{(\nu_0)^{(2)}-(\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)}+(\bar{\mathrm{C}})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{\mathrm{C}})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{\mathrm{C}})^{(2)} = \frac{(\bar{\nu}_1)^{(2)}-(\nu_0)^{(2)}}{(\nu_0)^{(2)}-(\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{\mathrm{G}_{16}^0}{\mathrm{G}_{17}^0} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \leq \frac{(\nu_1)^{(2)}+(\mathrm{C})^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}}{1+(\mathrm{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}} \leq \nu^{(2)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(2)}+(\bar{\mathrm{C}})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{\mathrm{C}})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \leq (\bar{\nu}_1)^{(2)} \leq (\nu_0)^{(2)} = \frac{\mathrm{G}_{16}^0}{\mathrm{G}_{17}^0}$ , we obtain 394

$$(\nu_1)^{(2)} \leq \nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)}+(\bar{\mathrm{C}})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{\mathrm{C}})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(2)}(t)$ :- 395

$$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}, \quad \boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$$

In a completely analogous way, we obtain 396

**Definition of** $u^{(2)}(t)$ :-

$$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}, \quad \boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :** 397

If $(a_{16}'')^{(2)} = (a_{17}'')^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)}G_{17}(t)$

Analogously if $(b''_{16})^{(2)} = (b''_{17})^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)}T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

**Proof** : From global equations we obtain 398

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right) - (a''_{21})^{(3)}(T_{21}, t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

**Definition of** $\nu^{(3)}$ :- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right) \leq \frac{d\nu^{(3)}}{dt} \leq -\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right)$$

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From which one obtains

For $0 < (\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\nu_1)^{(3)} < (\bar{\nu}_1)^{(3)}$

$$\nu^{(3)}(t) \geq \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(\nu_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\nu_2)^{(3)}}}$$

it follows $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\nu_1)^{(3)}$

In the same manner , we get 401

$$\nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)} + (\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{\nu}_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\bar{\nu}_2)^{(3)}}}$$

**Definition of** $(\bar{\nu}_1)^{(3)}$ :-

From which we deduce $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\bar{\nu}_1)^{(3)}$

If $0 < (\nu_1)^{(3)} < (\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\bar{\nu}_1)^{(3)}$ we find like in the previous case, 402

$$(\nu_1)^{(3)} \leq \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_2)^{(3)}\right)t\right]}}{1 + (C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_2)^{(3)}\right)t\right]}} \leq \nu^{(3)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(3)} + (\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\bar{\nu}_1)^{(3)}$$

If $0 < (\nu_1)^{(3)} \leq (\bar{\nu}_1)^{(3)} \leq (\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}$ , we obtain 403

$$(\nu_1)^{(3)} \leq \nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\nu_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(3)}(t)$ :-

$$(m_2)^{(3)} \leq \nu^{(3)}(t) \leq (m_1)^{(3)}, \quad \boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(3)}(t)$ :-

$$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}, \quad \boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{20})^{(3)} = (a''_{21})^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)}G_{21}(t)$

Analogously if $(b''_{20})^{(3)} = (b''_{21})^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)}T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

**Proof** : From global equations we obtain 404

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right) - (a''_{25})^{(4)}(T_{25},t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

**Definition of** $\nu^{(4)}$ :- $\quad \boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right) \leq \frac{d\nu^{(4)}}{dt} \leq -\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_4)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$ :-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \geq \frac{(\nu_1)^{(4)}+(C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_0)^{(4)}\right)t\right]}}{4+(C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)}-(\nu_0)^{(4)}}{(\nu_0)^{(4)}-(\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \le \frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{4+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)}-(\nu_0)^{(4)}}{(\nu_0)^{(4)}-(\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \le \nu^{(4)}(t) \le (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \le \frac{(\nu_1)^{(4)}+(C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_2)^{(4)}\right)t\right]}}{1+(C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_2)^{(4)}\right)t\right]}} \le \nu^{(4)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \le (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \le (\bar{\nu}_1)^{(4)} \le \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$ , we obtain 407

$$(\nu_1)^{(4)} \le \nu^{(4)}(t) \le \frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \le (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(4)}(t)$ :-

$$(m_2)^{(4)} \le \nu^{(4)}(t) \le (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(4)}(t)$ :-

$$(\mu_2)^{(4)} \le u^{(4)}(t) \le (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a_{24}'')^{(4)} = (a_{25}'')^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)}G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b_{24}'')^{(4)} = (b_{25}'')^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)}T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

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**Proof** : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a_{28}')^{(5)} - (a_{29}')^{(5)} + (a_{28}'')^{(5)}(T_{29},t)\right) - (a_{29}'')^{(5)}(T_{29},t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

**Definition of** $\nu^{(5)}$ :- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$ :-

For $0 < \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \leq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}}{1 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}} \leq \nu^{(5)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \leq (\bar{\nu}_1)^{(5)} \leq \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$ , we obtain 411

$$(\nu_1)^{(5)} \leq \nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(5)}(t)$ :-

$(m_2)^{(5)} \leq \nu^{(5)}(t) \leq (m_1)^{(5)}$, $\boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$

In a completely analogous way, we obtain

**Definition of** $u^{(5)}(t)$ :-

$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}$, $\boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{28})^{(5)} = (a''_{29})^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)} G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b''_{28})^{(5)} = (b''_{29})^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$ if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)} T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

**Proof** : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a'_{32})^{(6)} - (a'_{33})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right) - (a''_{33})^{(6)}(T_{33}, t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

**Definition of** $\nu^{(6)}$ :- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \leq \frac{d\nu^{(6)}}{dt} \leq -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$ :-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \geq \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\nu_1)^{(6)}$

In the same manner , we get 413

$$\nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{\nu}_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\bar{\nu}_1)^{(6)}$

If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 414

$$(\nu_1)^{(6)} \leq \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}} \leq \nu^{(6)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\bar{\nu}_1)^{(6)}$$

If $0 < (\nu_1)^{(6)} \leq (\bar{\nu}_1)^{(6)} \leq \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$, we obtain

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$$(\nu_1)^{(6)} \leq \nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(6)}(t)$ :-

$$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}, \quad \boxed{\nu^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(6)}(t)$ :-

$$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}, \quad \boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)}G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)}T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

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**Proof** : From global equations we obtain

$$\frac{d\nu^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right) - (a''_{37})^{(7)}(T_{37}, t)\nu^{(7)} - (a_{37})^{(7)}\nu^{(7)}$$

**Definition of** $\nu^{(7)}$ :- $\boxed{\nu^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right) \leq \frac{d\nu^{(7)}}{dt} \leq -\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(7)}, (\nu_0)^{(7)}$ :-

For $0 < \boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}} < (\nu_1)^{(7)} < (\bar{\nu}_1)^{(7)}$

$$\nu^{(7)}(t) \geq \frac{(\nu_1)^{(7)}+(C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_0)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)}=\frac{(\nu_1)^{(7)}-(\nu_0)^{(7)}}{(\nu_0)^{(7)}-(\nu_2)^{(7)}}}$$

it follows $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\nu_1)^{(7)}$

In the same manner , we get 417

$$\nu^{(7)}(t) \leq \frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)}=\frac{(\bar{\nu}_1)^{(7)}-(\nu_0)^{(7)}}{(\nu_0)^{(7)}-(\bar{\nu}_2)^{(7)}}}$$

From which we deduce $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\bar{\nu}_1)^{(7)}$

If $0 < (\nu_1)^{(7)} < (\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0} < (\bar{\nu}_1)^{(7)}$ we find like in the previous case, 418

$$(\nu_1)^{(7)} \leq \frac{(\nu_1)^{(7)}+(C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}} \leq \nu^{(7)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \leq (\bar{\nu}_1)^{(7)}$$

If $0 < (\nu_1)^{(7)} \leq (\bar{\nu}_1)^{(7)} \leq \boxed{(\nu_0)^{(7)}=\frac{G_{36}^0}{G_{37}^0}}$ , we obtain 419

$$(\nu_1)^{(7)} \leq \nu^{(7)}(t) \leq \frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \leq (\nu_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(7)}(t)$ :-

$$(m_2)^{(7)} \leq \nu^{(7)}(t) \leq (m_1)^{(7)}, \quad \boxed{\nu^{(7)}(t)=\frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(7)}(t)$ :- 420

$$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}, \quad \boxed{u^{(7)}(t)=\frac{T_{36}(t)}{T_{37}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $\nu^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)}G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for**

**the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)}T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

**Proof** : From global equations we obtain 421

$$\frac{d\nu^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right) - (a''_{41})^{(8)}(T_{41},t)\nu^{(8)} - (a_{41})^{(8)}\nu^{(8)}$$

**Definition of** $\nu^{(8)}$ :- $\boxed{\nu^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right) \le \frac{d\nu^{(8)}}{dt} \le -\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$ :-

For $0 < \boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$

$$\nu^{(8)}(t) \ge \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \le \nu^{(8)}(t) \le (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \le \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \le \nu^{(8)}(t) \le (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \le \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}} \le \nu^{(8)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \le (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \le (\bar{\nu}_1)^{(8)} \le \boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}}$ , we obtain 424

$$(v_1)^{(8)} \leq v^{(8)}(t) \leq \frac{(\bar{v}_1)^{(8)}+(\bar{C})^{(8)}(\bar{v}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{v}_1)^{(8)}-(\bar{v}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{v}_1)^{(8)}-(\bar{v}_2)^{(8)}\right)t\right]}} \leq (v_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $v^{(8)}(t)$ :-

$$(m_2)^{(8)} \leq v^{(8)}(t) \leq (m_1)^{(8)}, \quad \boxed{v^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(8)}(t)$ :-

$$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}, \quad \boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(v_1)^{(8)} = (\bar{v}_1)^{(8)}$ if in addition $(v_0)^{(8)} = (v_1)^{(8)}$ then $v^{(8)}(t) = (v_0)^{(8)}$ and as a consequence $G_{40}(t) = (v_0)^{(8)}G_{41}(t)$ **this also defines** $(v_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then $(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)}T_{41}(t)$ This is an important consequence of the relation between $(v_1)^{(8)}$ and $(\bar{v}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

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**Proof** : From 99,20,44,22,23,44 we obtain

$$\frac{dv^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45}, t)\right) - (a''_{45})^{(9)}(T_{45}, t)v^{(9)} - (a_{45})^{(9)}v^{(9)}$$

**Definition of** $v^{(9)}$ :- $\boxed{v^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)}\right) \leq \frac{dv^{(9)}}{dt} \leq -\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_1)^{(9)}v^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

**Definition of** $(\bar{v}_1)^{(9)}, (v_0)^{(9)}$ :-

For $0 < \boxed{(v_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (v_1)^{(9)} < (\bar{v}_1)^{(9)}$

$$v^{(9)}(t) \geq \frac{(v_1)^{(9)}+(C)^{(9)}(v_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((v_1)^{(9)}-(v_0)^{(9)}\right)t\right]}}{1+(C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((v_1)^{(9)}-(v_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(v_1)^{(9)}-(v_0)^{(9)}}{(v_0)^{(9)}-(v_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\nu_9)^{(9)}$

In the same manner , we get

$$\nu^{(9)}(t) \leq \frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)}-(\nu_0)^{(9)}}{(\nu_0)^{(9)}-(\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \leq \frac{(\nu_1)^{(9)}+(C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}}{1+(C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}} \leq \nu^{(9)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \leq (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \leq (\bar{\nu}_1)^{(9)} \leq \boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$ , we obtain

$$(\nu_1)^{(9)} \leq \nu^{(9)}(t) \leq \frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \leq (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(9)}(t)$ :-

$$(m_2)^{(9)} \leq \nu^{(9)}(t) \leq (m_1)^{(9)}, \quad \boxed{\nu^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(9)}(t)$ :-

$$(\mu_2)^{(9)} \leq u^{(9)}(t) \leq (\mu_1)^{(9)}, \quad \boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)}G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then $(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)}T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following



**Theorem :** If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on $t$ , and the conditions with the notations

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0$ ,

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$

$with\ \ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(2)} and\ (b_i'')^{(2)}$ are independent on t , and the conditions with the notations 426

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$ 427

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 428

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$ , 429

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 430

$with\ \ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(3)} and\ (b_i'')^{(3)}$ are independent on $t$ , and the conditions with the notations 431

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$ ,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

**Theorem :** If $(a_i'')^{(4)} and\ (b_i'')^{(4)}$ are independent on $t$ , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$ ,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(5)} and\ (b_i'')^{(5)}$ are independent on $t$ , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$ ,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

**Theorem** If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on $t$ , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$ ,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(7)} and\ (b_i'')^{(7)}$ are independent on $t$ , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$ ,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(8)} and\ (b_i'')^{(8)}$ are independent on $t$ , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$ ,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(9)} and\ (b_i'')^{(9)}$ are independent on $t$ , and the conditions (with the notations 436 A
45,46,27,28)

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$ ,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0 \quad 437$$

$$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0 \quad 438$$

$$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0 \quad 439$$

$$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0 \quad 440$$

$$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ]T_{14} = 0 \quad 441$$

$$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0 \quad 442$$

has a unique positive solution , which is an equilibrium solution for the system

$$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0 \quad 443$$

$$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0 \quad 444$$

$$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0 \quad 445$$

$$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\ ]T_{16} = 0 \quad 446$$

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\ ]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\ ]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\ ]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\ ]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\ ]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27})\right)]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27})\right)]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27})\right)]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0 \quad 479$$

$$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0 \quad 480$$

$$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0 \quad 481$$

$$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0 \quad 482$$

$$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0 \quad 483$$

$$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0 \quad 484$$

$$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0 \quad 484\,A$$

$$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$$

$$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$$

$$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$$

$$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$$

$$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$$

**Proof:** 485

(a) Indeed the first two equations have a nontrivial solution $G_{13}, G_{14}$ if

$$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$$

**Proof:** 486

(a) Indeed the first two equations have a nontrivial solution $G_{16}, G_{17}$ if

$$F(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$$

**Proof:** 487

(a) Indeed the first two equations have a nontrivial solution $G_{20}, G_{21}$ if

$$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$$

**Proof:** 488

(a) Indeed the first two equations have a nontrivial solution $G_{24}, G_{25}$ if

$$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$$

**Proof:** 489

(a) Indeed the first two equations have a nontrivial solution $G_{28}, G_{29}$ if

$$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$$

**Proof:** 490

(a) Indeed the first two equations have a nontrivial solution $G_{32}, G_{33}$ if

$$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$$

**Proof:** 491

(a) Indeed the first two equations have a nontrivial solution $G_{36}, G_{37}$ if

$$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$$

**Proof:** 492

(a) Indeed the first two equations have a nontrivial solution $G_{40}, G_{41}$ if

$$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$$

**Proof:** 492 A

(a) Indeed the first two equations have a nontrivial solution $G_{44}, G_{45}$ if

$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$

**Definition and uniqueness of** $T^*_{14}$ :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique $T^*_{14}$ for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

**Definition and uniqueness of** $T^*_{17}$ :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique $T^*_{17}$ for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]} \qquad 495$$

**Definition and uniqueness of** $T^*_{21}$ :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique $T^*_{21}$ for which $f(T^*_{21}) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

**Definition and uniqueness of** $T^*_{25}$ :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that there exists a unique $T^*_{25}$ for which $f(T^*_{25}) = 0$. With this value , we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T^*_{25})]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T^*_{25})]}$$

**Definition and uniqueness of** $T^*_{29}$ :- 498

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(5)}(T_{29})$ being increasing, it follows that there exists a unique $T^*_{29}$ for which $f(T^*_{29}) = 0$. With this value , we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T^*_{29})]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T^*_{29})]}$$

**Definition and uniqueness of** $T^*_{33}$ :- 499

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(6)}(T_{33})$ being increasing, it follows that

there exists a unique $T_{33}^{*}$ for which $f(T_{33}^{*})=0$. With this value , we obtain from the three first equations

$$G_{32}=\frac{(a_{32})^{(6)}G_{33}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^{*})]} \quad , \quad G_{34}=\frac{(a_{34})^{(6)}G_{33}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^{*})]}$$

**Definition and uniqueness of** $T_{37}^{*}$ :- 500

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(7)}(T_{37})$ being increasing, it follows that there exists a unique $T_{37}^{*}$ for which $f(T_{37}^{*})=0$. With this value , we obtain from the three first equations

$$G_{36}=\frac{(a_{36})^{(7)}G_{37}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^{*})]} \quad , \quad G_{38}=\frac{(a_{38})^{(7)}G_{37}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^{*})]}$$

**Definition and uniqueness of** $T_{41}^{*}$ :- 501

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique $T_{41}^{*}$ for which $f(T_{41}^{*})=0$. With this value , we obtain from the three first equations

$$G_{40}=\frac{(a_{40})^{(8)}G_{41}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T_{41}^{*})]} \quad , \quad G_{42}=\frac{(a_{42})^{(8)}G_{41}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T_{41}^{*})]}$$

**Definition and uniqueness of** $T_{45}^{*}$ :- 501 A

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(9)}(T_{45})$ being increasing, it follows that there exists a unique $T_{45}^{*}$ for which $f(T_{45}^{*})=0$. With this value , we obtain from the three first equations

$$G_{44}=\frac{(a_{44})^{(9)}G_{45}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T_{45}^{*})]} \quad , \quad G_{46}=\frac{(a_{46})^{(9)}G_{45}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T_{45}^{*})]}$$

By the same argument, the equations admit solutions $G_{13}, G_{14}$ if 502

$$\varphi(G)=(b'_{13})^{(1)}(b'_{14})^{(1)}-(b_{13})^{(1)}(b_{14})^{(1)}-$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G)+(b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right]+(b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G)=0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{14}$ taking into account the hypothesis $\varphi(0)>0$ , $\varphi(\infty)<0$ it follows that there exists a unique $G_{14}^{*}$ such that $\varphi(G^{*})=0$

By the same argument, the equations admit solutions $G_{16}, G_{17}$ if 503

$$\varphi(G_{19})=(b'_{16})^{(2)}(b'_{17})^{(2)}-(b_{16})^{(2)}(b_{17})^{(2)}-$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19})+(b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right]+(b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19})=0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{17}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{14}^{*}$ such that $\varphi((G_{19})^{*}) = 0$ 504

By the same argument, the equations admit solutions $G_{20}, G_{21}$ if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right] + (b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{21}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{21}^{*}$ such that $\varphi((G_{23})^{*}) = 0$

By the same argument, the equations admit solutions $G_{24}, G_{25}$ if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right] + (b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{25}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{25}^{*}$ such that $\varphi((G_{27})^{*}) = 0$

By the same argument, the equations admit solutions $G_{28}, G_{29}$ if 507

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right] + (b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{29}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{29}^{*}$ such that $\varphi((G_{31})^{*}) = 0$

By the same argument, the equations admit solutions $G_{32}, G_{33}$ if 508

$$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$$

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right] + (b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{33}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{33}^{*}$ such that $\varphi({G_{35}}^{*}) = 0$

By the same argument, the equations admit solutions $G_{36}, G_{37}$ if 509

$$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$$

$$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right] + (b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{37}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{37}^{*}$ such that $\varphi({G_{39}}^{*}) = 0$

By the same argument, the equations admit solutions $G_{40}, G_{41}$ if 510

$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$
$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right] + (b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{41}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{41}$ such that $\varphi(G_{43}{}^*) = 0$

By the same argument, the equations 92,93 admit solutions $G_{44}, G_{45}$ if
$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$

$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right] + (b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{45}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{45}$ such that $\varphi((G_{47})^*) = 0$

Finally we obtain the unique solution 511

$G^*_{14}$ given by $\varphi(G^*) = 0$ , $T^*_{14}$ given by $f(T^*_{14}) = 0$ and

$$G^*_{13} = \frac{(a_{13})^{(1)}G^*_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G^*_{15} = \frac{(a_{15})^{(1)}G^*_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

$$T^*_{13} = \frac{(b_{13})^{(1)}T^*_{14}}{[(b'_{13})^{(1)}-(b''_{13})^{(1)}(G^*)]} \quad , \quad T^*_{15} = \frac{(b_{15})^{(1)}T^*_{14}}{[(b'_{15})^{(1)}-(b''_{15})^{(1)}(G^*)]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

$G^*_{17}$ given by $\varphi((G_{19})^*) = 0$ , $T^*_{17}$ given by $f(T^*_{17}) = 0$ and 512

$$G^*_{16} = \frac{(a_{16})^{(2)}G^*_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G^*_{18} = \frac{(a_{18})^{(2)}G^*_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]} \quad 513$$

$$T^*_{16} = \frac{(b_{16})^{(2)}T^*_{17}}{[(b'_{16})^{(2)}-(b''_{16})^{(2)}((G_{19})^*)]} \quad , \quad T^*_{18} = \frac{(b_{18})^{(2)}T^*_{17}}{[(b'_{18})^{(2)}-(b''_{18})^{(2)}((G_{19})^*)]} \quad 514$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

$G^*_{21}$ given by $\varphi((G_{23})^*) = 0$ , $T^*_{21}$ given by $f(T^*_{21}) = 0$ and

$$G^*_{20} = \frac{(a_{20})^{(3)}G^*_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G^*_{22} = \frac{(a_{22})^{(3)}G^*_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

$$T^*_{20} = \frac{(b_{20})^{(3)}T^*_{21}}{[(b'_{20})^{(3)}-(b''_{20})^{(3)}(G_{23}{}^*)]} \quad , \quad T^*_{22} = \frac{(b_{22})^{(3)}T^*_{21}}{[(b'_{22})^{(3)}-(b''_{22})^{(3)}(G_{23}{}^*)]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

$G_{25}^*$ given by $\varphi(G_{27}) = 0$ , $T_{25}^*$ given by $f(T_{25}^*) = 0$ and

$$G_{24}^* = \frac{(a_{24})^{(4)} G_{25}^*}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^*)]} \quad , \quad G_{26}^* = \frac{(a_{26})^{(4)} G_{25}^*}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^*)]}$$

$$T_{24}^* = \frac{(b_{24})^{(4)} T_{25}^*}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^*)]} \quad , \quad T_{26}^* = \frac{(b_{26})^{(4)} T_{25}^*}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^*)]} \qquad 517$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

$G_{29}^*$ given by $\varphi((G_{31})^*) = 0$ , $T_{29}^*$ given by $f(T_{29}^*) = 0$ and

$$G_{28}^* = \frac{(a_{28})^{(5)} G_{29}^*}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^*)]} \quad , \quad G_{30}^* = \frac{(a_{30})^{(5)} G_{29}^*}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^*)]}$$

$$T_{28}^* = \frac{(b_{28})^{(5)} T_{29}^*}{[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31})^*)]} \quad , \quad T_{30}^* = \frac{(b_{30})^{(5)} T_{29}^*}{[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31})^*)]} \qquad 519$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

$G_{33}^*$ given by $\varphi((G_{35})^*) = 0$ , $T_{33}^*$ given by $f(T_{33}^*) = 0$ and

$$G_{32}^* = \frac{(a_{32})^{(6)} G_{33}^*}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^*)]} \quad , \quad G_{34}^* = \frac{(a_{34})^{(6)} G_{33}^*}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^*)]}$$

$$T_{32}^* = \frac{(b_{32})^{(6)} T_{33}^*}{[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35})^*)]} \quad , \quad T_{34}^* = \frac{(b_{34})^{(6)} T_{33}^*}{[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35})^*)]} \qquad 521$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

$G_{37}^*$ given by $\varphi((G_{39})^*) = 0$ , $T_{37}^*$ given by $f(T_{37}^*) = 0$ and

$$G_{36}^* = \frac{(a_{36})^{(7)} G_{37}^*}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^*)]} \quad , \quad G_{38}^* = \frac{(a_{38})^{(7)} G_{37}^*}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^*)]}$$

$$T_{36}^* = \frac{(b_{36})^{(7)} T_{37}^*}{[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39})^*)]} \quad , \quad T_{38}^* = \frac{(b_{38})^{(7)} T_{37}^*}{[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39})^*)]}$$

Finally we obtain the unique solution 523

$G_{41}^*$ given by $\varphi((G_{43})^*) = 0$ , $T_{41}^*$ given by $f(T_{41}^*) = 0$ and

$$G_{40}^* = \frac{(a_{40})^{(8)} G_{41}^*}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42}^* = \frac{(a_{42})^{(8)} G_{41}^*}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

$$T_{40}^{*} = \frac{(b_{40})^{(8)} T_{41}^{*}}{[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43})^{*})]} \quad , \quad T_{42}^{*} = \frac{(b_{42})^{(8)} T_{41}^{*}}{[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43})^{*})]}$$

Finally we obtain the unique solution of 89 to 99 523

A

$G_{45}^{*}$ given by $\varphi((G_{47})^{*}) = 0$ , $T_{45}^{*}$ given by $f(T_{45}^{*}) = 0$ and

$$G_{44}^{*} = \frac{(a_{44})^{(9)} G_{45}^{*}}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^{*})]} \quad , \quad G_{46}^{*} = \frac{(a_{46})^{(9)} G_{45}^{*}}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^{*})]}$$

$$T_{44}^{*} = \frac{(b_{44})^{(9)} T_{45}^{*}}{[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47})^{*})]} \quad , \quad T_{46}^{*} = \frac{(b_{46})^{(9)} T_{45}^{*}}{[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47})^{*})]}$$

## ASYMPTOTIC STABILITY ANALYSIS 524

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

**Proof:** Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^{*} + \mathbb{G}_i \qquad , T_i = T_i^{*} + \mathbb{T}_i$$

$$\frac{\partial (a''_{14})^{(1)}}{\partial T_{14}}(T_{14}^{*}) = (q_{14})^{(1)} \quad , \frac{\partial (b_i'')^{(1)}}{\partial G_j}(G^{*}) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a'_{13})^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)} G_{13}^{*}\mathbb{T}_{14} \quad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a'_{14})^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)} G_{14}^{*}\mathbb{T}_{14} \quad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a'_{15})^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)} G_{15}^{*}\mathbb{T}_{14} \quad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b'_{13})^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)} T_{13}^{*}\mathbb{G}_j\right) \quad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b'_{14})^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)} T_{14}^{*}\mathbb{G}_j\right) \quad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b'_{15})^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)} T_{15}^{*}\mathbb{G}_j\right) \quad 530$$

## ASYMPTOTIC STABILITY ANALYSIS 531

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(2)}$ and $(b_i'')^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i \tag{532}$$

$$\frac{\partial(a_{17}'')^{(2)}}{\partial T_{17}}(T_{17}^*) = (q_{17})^{(2)} \quad , \frac{\partial(b_i'')^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \tag{533}$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a_{16}')^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G_{16}^*\mathbb{T}_{17} \tag{534}$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a_{17}')^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G_{17}^*\mathbb{T}_{17} \tag{535}$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a_{18}')^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G_{18}^*\mathbb{T}_{17} \tag{536}$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b_{16}')^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T_{16}^*\mathbb{G}_j\right) \tag{537}$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b_{17}')^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}T_{17}^*\mathbb{G}_j\right) \tag{538}$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b_{18}')^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}T_{18}^*\mathbb{G}_j\right) \tag{539}$$

**ASYMPTOTIC STABILITY ANALYSIS** 540

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(3)}$ *and* $(b_i'')^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{21}'')^{(3)}}{\partial T_{21}}(T_{21}^*) = (q_{21})^{(3)} \quad , \frac{\partial(b_i'')^{(3)}}{\partial G_j}((G_{23})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a_{20}')^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G_{20}^*\mathbb{T}_{21} \tag{541}$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a_{21}')^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G_{21}^*\mathbb{T}_{21} \tag{542}$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a_{22}')^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G_{22}^*\mathbb{T}_{21} \tag{543}$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b_{20}')^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T_{20}^*\mathbb{G}_j\right) \tag{544}$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b_{21}')^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T_{21}^*\mathbb{G}_j\right) \tag{545}$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b'_{22})^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T^*_{22}\mathbb{G}_j\right) \quad 546$$

**ASYMPTOTIC STABILITY ANALYSIS** 547

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}\ and\ (b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{25})^{(4)}}{\partial T_{25}}(T^*_{25}) = (q_{25})^{(4)} \ , \ \frac{\partial(b_i'')^{(4)}}{\partial G_j}((G_{27})^*\ ) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\left((a'_{24})^{(4)} + (p_{24})^{(4)}\right)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G^*_{24}\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\left((a'_{25})^{(4)} + (p_{25})^{(4)}\right)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G^*_{25}\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\left((a'_{26})^{(4)} + (p_{26})^{(4)}\right)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G^*_{26}\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\left((b'_{24})^{(4)} - (r_{24})^{(4)}\right)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(24)(j)}T^*_{24}\mathbb{G}_j\right) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\left((b'_{25})^{(4)} - (r_{25})^{(4)}\right)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\left(s_{(25)(j)}T^*_{25}\mathbb{G}_j\right) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T^*_{26}\mathbb{G}_j\right) \quad 554$$

**ASYMPTOTIC STABILITY ANALYSIS** 555

**Theorem 5:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}\ and\ (b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 556

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{29})^{(5)}}{\partial T_{29}}(T^*_{29}) = (q_{29})^{(5)} \ , \ \frac{\partial(b_i'')^{(5)}}{\partial G_j}(\ (G_{31})^*\ ) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a'_{28})^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G^*_{28}\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a'_{29})^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G^*_{29}\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a'_{30})^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G^*_{30}\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b'_{28})^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T^*_{28}\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b'_{29})^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T^*_{29}\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b'_{30})^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T^*_{30}\mathbb{G}_j\right) \quad 562$$

**ASYMPTOTIC STABILITY ANALYSIS** 563

**Theorem 6:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}$ *and* $(b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{33})^{(6)}}{\partial T_{33}}(T^*_{33}) = (q_{33})^{(6)} \quad , \frac{\partial(b_i'')^{(6)}}{\partial G_j}\left((G_{35})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a'_{32})^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G^*_{32}\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a'_{33})^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G^*_{33}\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a'_{34})^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G^*_{34}\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b'_{32})^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T^*_{32}\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b'_{33})^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T^*_{33}\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b'_{34})^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T^*_{34}\mathbb{G}_j\right) \quad 570$$

**ASYMPTOTIC STABILITY ANALYSIS** 571

**Theorem 7:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 572

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{37}'')^{(7)}}{\partial T_{37}}(T_{37}^{*}) = (q_{37})^{(7)} \quad , \frac{\partial(b_i'')^{(7)}}{\partial G_j}\left( (G_{39})^{**} \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a_{36}')^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G_{36}^{*}\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a_{37}')^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G_{37}^{*}\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a_{38}')^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G_{38}^{*}\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b_{36}')^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T_{36}^{*}\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b_{37}')^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T_{37}^{*}\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b_{38}')^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T_{38}^{*}\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

## ASYMPTOTIC STABILITY ANALYSIS

**Theorem 8:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 580

$$G_i = G_i^{*} + \mathbb{G}_i \qquad , T_i = T_i^{*} + \mathbb{T}_i$$

$$\frac{\partial(a_{41}'')^{(8)}}{\partial T_{41}}(T_{41}^{*}) = (q_{41})^{(8)} \quad , \frac{\partial(b_i'')^{(8)}}{\partial G_j}\left( (G_{43})^{*} \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a_{40}')^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G_{40}^{*}\mathbb{T}_{41} \quad 581$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a_{41}')^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G_{41}^{*}\mathbb{T}_{41} \quad 582$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a_{42}')^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G_{42}^{*}\mathbb{T}_{41} \quad 583$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b_{40}')^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T_{40}^{*}\mathbb{G}_j\right) \quad 584$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b_{41}')^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T_{41}^{*}\mathbb{G}_j\right) \quad 585$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b'_{42})^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T^*_{42}\mathbb{G}_j\right)$$ 586

586

**ASYMPTOTIC STABILITY ANALYSIS** A

**Theorem 9:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}\ and\ (b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

**Proof: Denote**

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{45})^{(9)}}{\partial T_{45}}(T^*_{45}) = (q_{45})^{(9)} \ , \ \frac{\partial(b_i'')^{(9)}}{\partial G_j}(\ (G_{47})^*\ ) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a'_{44})^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)}G^*_{44}\mathbb{T}_{45}$$ 586 B

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a'_{45})^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)}G^*_{45}\mathbb{T}_{45}$$ 586 C

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a'_{46})^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)}G^*_{46}\mathbb{T}_{45}$$ 586 D

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b'_{44})^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)}T^*_{44}\mathbb{G}_j\right)$$ 586 E

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b'_{45})^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)}T^*_{45}\mathbb{G}_j\right)$$ 586 F

$$\frac{d\mathbb{T}_{46}}{dt} = -\left((b'_{46})^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T^*_{46}\mathbb{G}_j\right)$$ 586 G

**The characteristic equation of this system is** 587

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(q_{13})^{(1)}G^*_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)}T^*_{14} + (b_{14})^{(1)}s_{(13),(14)}T^*_{14}\right)$$

$$+\left(\left((\lambda)^{(1)} + (a'_{14})^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)}G^*_{13} + (a_{13})^{(1)}(q_{14})^{(1)}G^*_{14}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)}T^*_{14} + (b_{14})^{(1)}s_{(13),(13)}T^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left(\ (a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left(\ (b'_{13})^{(1)} + (b'_{14})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^2+\left((a'_{13})^{(1)}+(a'_{14})^{(1)}+(p_{13})^{(1)}+(p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)}+(a'_{13})^{(1)}+(p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^*_{14}+(a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}+(b'_{13})^{(1)}-(r_{13})^{(1)}\right)s_{(14),(15)}T^*_{14}+(b_{14})^{(1)}s_{(13),(15)}T^*_{13}\right)\}=0$$

$$+$$

$$(\lambda)^{(2)}+(b'_{18})^{(2)}-(r_{18})^{(2)}\{\left((\lambda)^{(2)}+(a'_{18})^{(2)}+(p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^*_{17}+(a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^*_{17}\right)$$

$$+\left(\left((\lambda)^{(2)}+(a'_{17})^{(2)}+(p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^*_{16}+(a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2+\left((b'_{16})^{(2)}+(b'_{17})^{(2)}-(r_{16})^{(2)}+(r_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$+\left(\left((\lambda)^{(2)}\right)^2+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}+(a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^*_{16}\right)\}=0$$

$$+$$

$$(\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left(\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^*_{20}\right)\right]$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^*_{21}+(b_{21})^{(3)}s_{(20),(21)}T^*_{21}\right)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^*_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^*_{21}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^*_{21}+(b_{21})^{(3)}s_{(20),(20)}T^*_{20}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^2+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$+\left(\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^*_{20}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^*_{21}+(b_{21})^{(3)}s_{(20),(22)}T^*_{20}\right)\}=0$$

$$+$$

$$(\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left(\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^*_{24}\right)\right]$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^*_{25}+(b_{25})^{(4)}s_{(24),(25)}T^*_{25}\right)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^*_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^*_{25}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^*_{25}+(b_{25})^{(4)}s_{(24),(24)}T^*_{24}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^2+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$+\left(\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^*_{24}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^*_{25}+(b_{25})^{(4)}s_{(24),(26)}T^*_{24}\right)\}=0$$

$$+$$

$$(\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^*_{28}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^*_{29}+(b_{29})^{(5)}s_{(28),(29)}T^*_{29}\right)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^*_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^*_{29}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^*_{29}+(b_{29})^{(5)}s_{(28),(28)}T^*_{28}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^2+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$+\left(\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^*_{28}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^*_{29}+(b_{29})^{(5)}s_{(28),(30)}T^*_{28}\right)\}=0$$

$$+$$

$$(\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left(\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^*_{32}\right)\right]$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^*_{33}+(b_{33})^{(6)}s_{(32),(33)}T^*_{33}\right)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^*_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^*_{33}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^*_{33}+(b_{33})^{(6)}s_{(32),(32)}T^*_{32}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^2+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$+\left(\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^*_{32}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^*_{33}+(b_{33})^{(6)}s_{(32),(34)}T^*_{32}\right)\}=0$$

$$+$$

$$(\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^*_{36}\right)\right]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^*_{37}+(b_{37})^{(7)}s_{(36),(37)}T^*_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^*_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^*_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^*_{37}+(b_{37})^{(7)}s_{(36),(36)}T^*_{36}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^2+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$+\left(\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^*_{36}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^*_{37}+(b_{37})^{(7)}s_{(36),(38)}T^*_{36}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\right)\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^*_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^*_{41}+(b_{41})^{(8)}s_{(40),(41)}T^*_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^*_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^*_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^*_{41}+(b_{41})^{(8)}s_{(40),(40)}T^*_{40}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^2+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$+\left(\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^*_{40}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^*_{41}+(b_{41})^{(8)}s_{(40),(42)}T^*_{40}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\right)\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^*_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^*_{45}+(b_{45})^{(9)}s_{(44),(45)}T^*_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^*_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^*_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^*_{45}+(b_{45})^{(9)}s_{(44),(44)}T^*_{44}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^2+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$+\left(\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^*_{44}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^*_{45}+(b_{45})^{(9)}s_{(44),(46)}T^*_{44}\right)\}=0$$

**And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.**

## SECTION TWO

## Fault-Tolerant Quantum Computation With Constant Error Rate

### INTRODUCTION—VARIABLES USED

**Topological fault-tolerance in cluster state quantum computation R Raussendorf1, J Harrington1 and K Goyal2 Published 29 June 2007 • IOP Publishing and Deutsche Physikalische Gesellschaft • New Journal of Physics, Volume 9, June 2007 Focus on Measurement-Based Quantum Information Processing**

(1) The error threshold is 0.75% for (e) each source in an error model with (e&eb) preparation, gate, storage and measurement errors.
(2) The operational overhead is (=) poly-logarithmic in the circuit size.

**Fault-Tolerant Quantum Computation with Constant Error Rate Dorit Aharonov, Michael Ben-Or**

(3) This paper proves the threshold result, which asserts that quantum computation can be made robust against (e&eb) errors and inaccuracies, when the error rate, η, is smaller than (eb) a constant threshold, ηc.
(4) The result holds for (e) a very general, not necessarily probabilistic noise model, for (e) quantum particles with any number of states, and is also generalized to (e) one dimensional quantum computers with (e&eb) only nearest neighbor interactions.
(5) No measurements, or classical operations, are required (e) during the quantum computation.
(6) The proceeding version was very succinct, and here we fill all the missing details, and elaborate on many parts of the proof. In particular, authors devote a section for (e) a discussion of universality issues and proofs that the sets of gates that we use are (=) universal. Another section is devoted to (e) a rigorous proof that fault tolerance can be achieved in the presence of (e) general non probabilistic noise.
(7) The systematic structure of the fault tolerant procedures for (e) polynomial codes is explained in length. The proof that the concatenation scheme works is written in a clearer way. The paper also contains new and significantly simpler proofs for (e) most of the known results which we use. For

example, authors give a simple proof that it suffices to correct (e&eb) bit and phase flips.

(8) They significantly simplify Calderbank and Shor's original proof of (e) the correctness of CSS codes. Authors also give a simple proof of the fact that two-qubit gates are (=) universal. The paper thus provides a self contained and complete proof for (e) universal fault tolerant quantum computation. Cite as**:arXiv: quant-ph/9906129 (or arXiv: quant-ph/9906129v1 for this version)**

## NOTATION

### Module One

operational overhead is (=) poly-logarithmic in the circuit size

$G_{13}$ : Category one of operational overhead

$G_{14}$ : Category two of SAS

$G_{15}$ : Category three of SAS

$T_{13}$ : Category one of poly-logarithmic in the circuit size

$T_{14}$ : Category two of SAS

$T_{15}$ : Category three of SAS

### Module Two

This paper proves the threshold result, which asserts that **quantum computation can be made robust** against (e&eb) errors and inaccuracies, when the error rate, η, is smaller than (eb) a constant threshold, ηc

$G_{16}$ : Category one of **quantum computation can be made robust;** errors and inaccuracies, when the error rate, η, is smaller than (eb) a constant threshold, ηc

$G_{17}$ : Category two of SAS

$G_{18}$ : Category three of SAS

$T_{16}$ : Category one of errors and inaccuracies, when the error rate, η, is smaller than (eb) a constant threshold, ηc; **quantum computation can be made robust**

$T_{17}$ : Category two of SAS

$T_{18}$ : Category three of SAS

### Module three

This paper proves the threshold result, which asserts that **quantum computation can be made robust against errors and inaccuracies, when the error rate, η,** is smaller than (eb) a constant threshold, ηc

$G_{20}$ : Category one of **quantum computation can be made robust against errors and inaccuracies, when the error rate, η;** constant threshold, ηc

$G_{21}$ : Category two of SAS

$G_{22}$ : Category three of SAS

$T_{20}$ : Category one of constant threshold, ηc; **quantum computation can be made robust against errors and inaccuracies, when the error rate, η,**

$T_{21}$ : Category two of SAS

$T_{22}$ : Category three of SAS

**Module four**

The result holds for a very general, not necessarily probabilistic noise model, for (e) quantum particles with any number of states, and is also generalized to (e) one dimensional quantum computers with (e&eb) only nearest neighbor interactions**.**

$G_{24}$ : Category one of result holds for a very general, not necessarily probabilistic noise model; **quantum particles with any number of states, and is also generalized to (e) one dimensional quantum computers with (e&eb) only nearest neighbor interactions.**

$G_{25}$ : Category two of SAS

$G_{26}$ : Category three of SAS

$T_{24}$ : Category one of **quantum particles with any number of states, and is also generalized to (e) one dimensional quantum computers with (e&eb) only nearest neighbor interactions.;** result holds for a very general, not necessarily probabilistic noise model

$T_{25}$ : Category two of SAS

$T_{26}$ : Category three of SAS

**Module five**

The result holds for a very general, not necessarily probabilistic noise model, for quantum particles with any number of states, and is also generalized to (e) one dimensional quantum computers with (e&eb) only nearest neighbor interactions

$G_{28}$ : Category one of **result holds for a very general, not necessarily probabilistic noise model, for quantum particles;** any number of states, and is also generalized to (e) one dimensional quantum computers with (e&eb) only nearest neighbor interactions

$G_{29}$ : Category two of SAS

$G_{30}$ : Category three of SAS

$T_{28}$ : Category one of any number of states, and is also generalized to (e) one dimensional quantum computers with (e&eb) only nearest neighbor interactions; **result holds for a very general, not necessarily probabilistic noise model, for quantum particles**

$T_{29}$ : Category two of SAS

$T_{30}$ : Category three of SAS

**Module six**

The result holds for a very general, not necessarily probabilistic noise model, for quantum particles with any number of states, and is also generalized to one dimensional quantum computers with (e&eb) only **nearest neighbor interactions**

$G_{32}$ : Category one of result holds for a very general, not necessarily probabilistic noise model, for quantum particles with any number of states, and is also generalized to one dimensional quantum computers; **nearest neighbor interactions**

$G_{33}$ : Category two of SAS

$G_{34}$ : Category three of SAS

$T_{32}$ : Category one of **nearest neighbor interactions;** result holds for a very general, not necessarily probabilistic noise model, for quantum particles with any number of states, and is also generalized to one dimensional quantum computers

$T_{33}$ : Category two of SAS

$T_{34}$ : Category three of SAS

**Module seven**

No measurements, or classical operations, are required (e) during the **quantum computation**

$G_{36}$ : Category one of No measurements, or classical operations; **quantum computation**

$G_{37}$ : Category two of SAS

$G_{38}$ : Category three of SAS

$T_{36}$ : Category one of **quantum computation** ;No measurements, or classical operations

$T_{37}$ : Category two of SAS

$T_{38}$ : Category three of SAS

**Module eight**

The proceeding version was very succinct, and here we fill all the missing details, and elaborate on many parts of the proof.

In particular, authors devote a section for (e) a discussion of universality issues and proofs that the sets of gates that we use are (=) universal.

Another section is devoted to (e) a rigorous proof that fault tolerance can be achieved in the presence of (e) general non probabilistic noise.

$G_{40}$ : Category one of the sets of gates that we use

$G_{41}$ : Category two of SAS

$G_{42}$ : Category three of SAS

$T_{40}$ : Category one of universal.

$T_{41}$ : Category two of SAS

$T_{42}$ : Category three of SAS

**Module Nine**

Another section is devoted to (e) a rigorous proof that **fault tolerance can be achieved** in the presence of (e) general non probabilistic noise.

$G_{44}$ : Category one of **fault tolerance can be achieved;** general non probabilistic noise.

$G_{45}$ : Category two of SAS

$G_{46}$ : Category three of SAS

$T_{44}$ : Category one of general non probabilistic noise.; **fault tolerance can be achieved**

$T_{45}$ : Category two of SAS

$T_{46}$ : Category three of SAS

**The Coefficients:**

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}\ ,\ (b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)},$
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)},\ \ (b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}\ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}\ ,$
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)},$
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)},$
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)},$
are Dissipation coefficients

**Module Numbered One**

**The differential system of this model is now (Module Numbered one)**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \quad 1$$
$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad 2$$
$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15} \quad 3$$
$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13} \quad 4$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G,t)\right]T_{14} \quad 5$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G,t)\right]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14},t) =$ First augmentation factor

$-(b''_{13})^{(1)}(G,t) =$ First detritions factor

**Module Numbered Two**

**The differential system of this model is now ( Module numbered two)**

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17},t)\right]G_{16} \quad 7$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17},t)\right]G_{17} \quad 8$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17},t)\right]G_{18} \quad 9$$

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\big((G_{19}),t\big)\right]T_{16} \quad 10$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\big((G_{19}),t\big)\right]T_{17} \quad 11$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\big((G_{19}),t\big)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17},t) =$ First augmentation factor

$-(b''_{16})^{(2)}\big((G_{19}),t\big) =$ First detritions factor

**Module Numbered Three**

**The differential system of this model is now (Module numbered three)**

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21},t)\right]G_{20} \quad 13$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21},t)\right]G_{21} \quad 14$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21},t)\right]G_{22} \quad 15$$

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23},t)\right]T_{20} \quad 16$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23},t)\right]T_{21} \quad 17$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23},t)\right]T_{22} \quad 18$$

$+(a''_{20})^{(3)}(T_{21},t) =$ First augmentation factor

$-(b''_{20})^{(3)}(G_{23},t) =$ First detritions factor

**Module Numbered Four**

**The differential system of this model is now (Module numbered Four)**

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right]G_{24} \quad 19$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25},t)\right]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25},t)\right]G_{26} \quad 21$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}\big((G_{27}),t\big)\right]T_{24} \quad 22$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}\big((G_{27}),t\big)\right]T_{25} \quad 23$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}\big((G_{27}),t\big)\right]T_{26} \quad 24$$

$+(a''_{24})^{(4)}(T_{25},t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}\big((G_{27}),t\big) =$ **First detritions factor**

**Module Numbered Five**:

**The differential system of this model is now (Module number five)**

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right]G_{28} \quad 25$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}, t)\right]G_{29} \quad 26$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}, t)\right]G_{30} \quad 27$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}\left((G_{31}), t\right)\right]T_{28} \quad 28$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}\left((G_{31}), t\right)\right]T_{29} \quad 29$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}\left((G_{31}), t\right)\right]T_{30} \quad 30$$

$+(a''_{28})^{(5)}(T_{29}, t) =$ **First augmentation factor**

$-(b''_{28})^{(5)}\left((G_{31}), t\right) =$ **First detritions factor**

**Module Numbered Six**

**The differential system of this model is now (Module numbered Six)**

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right]G_{32} \quad 31$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}, t)\right]G_{33} \quad 32$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}, t)\right]G_{34} \quad 33$$

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}\left((G_{35}), t\right)\right]T_{32} \quad 34$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}\left((G_{35}), t\right)\right]T_{33} \quad 35$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}\left((G_{35}), t\right)\right]T_{34} \quad 36$$

$+(a''_{32})^{(6)}(T_{33}, t) =$ **First augmentation factor**

**Module Numbered Seven:**

**The differential system of this model is now (Seventh Module)**

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right]G_{36} \quad 37$$

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37}, t)\right]G_{37} \quad 38$$

$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}, t)\right]G_{38} \quad 39$$

$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}\left((G_{39}), t\right)\right]T_{36} \quad 40$$

$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}\left((G_{39}), t\right)\right]T_{37} \quad 41$$

$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}\left((G_{39}), t\right)\right]T_{38} \quad 42$$

$+(a''_{36})^{(7)}(T_{37}, t) =$ **First augmentation factor**

**Module Numbered Eight**

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right]G_{40} \quad 43$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41}, t)\right]G_{41} \quad 44$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}, t)\right]G_{42} \quad 45$$

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}\left((G_{43}), t\right)\right]T_{40} \quad 46$$

$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}\left((G_{43}), t\right)\right]T_{41} \quad 47$$

$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43}),t)\right]T_{42} \quad 48$$

**Module Numbered Nine**

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47}),t)\right]T_{44} \quad 52$$

$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}((G_{47}),t)\right]T_{45} \quad 53$$

$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47}),t)\right]T_{46} \quad 54$$

$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}((G_{47}),t) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)} & +(a''_{13})^{(1)}(T_{14},t) & +(a''_{16})^{(2,2,)}(T_{17},t) & +(a''_{20})^{(3,3,)}(T_{21},t) \\ +(a''_{24})^{(4,4,4,4,)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,)}(T_{33},t) & \\ +(a''_{36})^{(7,7)}(T_{37},t) & +(a''_{40})^{(8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) & \end{bmatrix} G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)} & +(a''_{14})^{(1)}(T_{14},t) & +(a''_{17})^{(2,2,)}(T_{17},t) & +(a''_{21})^{(3,3,)}(T_{21},t) \\ +(a''_{25})^{(4,4,4,4,)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,)}(T_{33},t) & \\ +(a''_{37})^{(7,7)}(T_{37},t) & +(a''_{41})^{(8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) & \end{bmatrix} G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)} & +(a''_{15})^{(1)}(T_{14},t) & +(a''_{18})^{(2,2,)}(T_{17},t) & +(a''_{22})^{(3,3,)}(T_{21},t) \\ +(a''_{26})^{(4,4,4,4,)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,)}(T_{33},t) & \\ +(a''_{38})^{(7,7)}(T_{37},t) & +(a''_{42})^{(8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) & \end{bmatrix} G_{15} \quad 57$$

Where $(a''_{13})^{(1)}(T_{14},t)$, $(a''_{14})^{(1)}(T_{14},t)$, $(a''_{15})^{(1)}(T_{14},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,)}(T_{17},t)$, $+(a''_{17})^{(2,2,)}(T_{17},t)$, $+(a''_{18})^{(2,2,)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,)}(T_{21},t)$, $+(a''_{21})^{(3,3,)}(T_{21},t)$, $+(a''_{22})^{(3,3,)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{38})^{(7,7)}(T_{37},t)$ $+(a''_{37})^{(7,7)}(T_{37},t)$ $+(a''_{36})^{(7,7)}(T_{37},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{40})^{(8,8)}(T_{41},t)$ $+(a''_{41})^{(8,8)}(T_{41},t)$ $+(a''_{42})^{(8,8)}(T_{41},t)$ are eight augmentation coefficient for 1,2,3

$+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)}\boxed{-(b''_{13})^{(1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13} \quad (58)$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)}\boxed{-(b''_{14})^{(1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14} \quad (59)$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)}\boxed{-(b''_{15})^{(1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15} \quad (60)$$

Where $\boxed{-(b''_{13})^{(1)}(G,t)}$, $\boxed{-(b''_{14})^{(1)}(G,t)}$, $\boxed{-(b''_{15})^{(1)}(G,t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8)}(G_{43},t)}$ $\boxed{-(b''_{41})^{(8,8)}(G_{43},t)}$ $\boxed{-(b''_{42})^{(8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \begin{bmatrix} (a'_{16})^{(2)}\boxed{+(a''_{16})^{(2)}(T_{17},t)} & \boxed{+(a''_{13})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{16} \quad (61)$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \begin{bmatrix} (a'_{17})^{(2)}\boxed{+(a''_{17})^{(2)}(T_{17},t)} & \boxed{+(a''_{14})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{17} \quad (62)$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \begin{bmatrix} (a'_{18})^{(2)}\boxed{+(a''_{18})^{(2)}(T_{17},t)} & \boxed{+(a''_{15})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{18} \quad (63)$$

Where $\boxed{+(a''_{16})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,)}(T_{14},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)}$ are eight augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{44})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9)}(T_{45},t)}$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \begin{bmatrix} (b'_{16})^{(2)}\boxed{-(b''_{16})^{(2)}(G_{19},t)} & \boxed{-(b''_{13})^{(1,1,)}(G,t)} & \boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{16} \quad (64)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)}\boxed{-(b''_{17})^{(2)}(G_{19},t)} & \boxed{-(b''_{14})^{(1,1,)}(G,t)} & \boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{17} \quad (65)$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)}\boxed{-(b''_{18})^{(2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,)}(G,t)} & \boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{18} \quad (66)$$

where $\boxed{-(b''_{16})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2)}(G_{19},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,)}(G,t)}$ are second detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)}$ are fourth detrition

coefficients for category 1,2 and 3

$-(b_{28}'')^{(5,5,5,5,5)}(G_{31},t)$, $-(b_{29}'')^{(5,5,5,5,5)}(G_{31},t)$, $-(b_{30}'')^{(5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1,2 and 3

$-(b_{32}'')^{(6,6,6,6,6)}(G_{35},t)$, $-(b_{33}'')^{(6,6,6,6,6)}(G_{35},t)$, $-(b_{34}'')^{(6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients for category 1,2 and 3

$-(b_{36}'')^{(7,7,7)}(G_{39},t)$, $-(b_{37}'')^{(7,7,7)}(G_{39},t)$, $-(b_{38}'')^{(7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2 and 3

$-(b_{40}'')^{(8,8,8)}(G_{43},t)$, $-(b_{41}'')^{(8,8,8)}(G_{43},t)$, $-(b_{42}'')^{(8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1,2 and 3

$-(b_{44}'')^{(9,9)}(G_{47},t)$, $-(b_{46}'')^{(9,9)}(G_{47},t)$, $-(b_{45}'')^{(9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[\begin{array}{c}(a_{20}')^{(3)}+(a_{20}'')^{(3)}(T_{21},t)+(a_{16}'')^{(2,2,2)}(T_{17},t)+(a_{13}'')^{(1,1,1,)}(T_{14},t)\\ +(a_{24}'')^{(4,4,4,4,4,4)}(T_{25},t)+(a_{28}'')^{(5,5,5,5,5,5)}(T_{29},t)+(a_{32}'')^{(6,6,6,6,6,6)}(T_{33},t)\\ +(a_{36}'')^{(7,7,7,7)}(T_{37},t)+(a_{40}'')^{(8,8,8,8)}(T_{41},t)+(a_{44}'')^{(9,9,9)}(T_{45},t)\end{array}\right]G_{20} \quad (67)$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[\begin{array}{c}(a_{21}')^{(3)}+(a_{21}'')^{(3)}(T_{21},t)+(a_{17}'')^{(2,2,2)}(T_{17},t)+(a_{14}'')^{(1,1,1,)}(T_{14},t)\\ +(a_{25}'')^{(4,4,4,4,4,4)}(T_{25},t)+(a_{29}'')^{(5,5,5,5,5,5)}(T_{29},t)+(a_{33}'')^{(6,6,6,6,6,6)}(T_{33},t)\\ +(a_{37}'')^{(7,7,7,7)}(T_{37},t)+(a_{41}'')^{(8,8,8,8)}(T_{41},t)+(a_{45}'')^{(9,9,9)}(T_{45},t)\end{array}\right]G_{21} \quad (68)$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[\begin{array}{c}(a_{22}')^{(3)}+(a_{22}'')^{(3)}(T_{21},t)+(a_{18}'')^{(2,2,2)}(T_{17},t)+(a_{15}'')^{(1,1,1,)}(T_{14},t)\\ +(a_{26}'')^{(4,4,4,4,4,4)}(T_{25},t)+(a_{30}'')^{(5,5,5,5,5,5)}(T_{29},t)+(a_{34}'')^{(6,6,6,6,6,6)}(T_{33},t)\\ +(a_{38}'')^{(7,7,7,7)}(T_{37},t)+(a_{42}'')^{(8,8,8,8)}(T_{41},t)+(a_{46}'')^{(9,9,9)}(T_{45},t)\end{array}\right]G_{22} \quad (69)$$

$+(a_{20}'')^{(3)}(T_{21},t)$, $+(a_{21}'')^{(3)}(T_{21},t)$, $+(a_{22}'')^{(3)}(T_{21},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a_{16}'')^{(2,2,2)}(T_{17},t)$, $+(a_{17}'')^{(2,2,2)}(T_{17},t)$, $+(a_{18}'')^{(2,2,2)}(T_{17},t)$ are second augmentation coefficients for category 1, 2 and 3

$+(a_{13}'')^{(1,1,1,)}(T_{14},t)$, $+(a_{14}'')^{(1,1,1,)}(T_{14},t)$, $+(a_{15}'')^{(1,1,1,)}(T_{14},t)$ are third augmentation coefficients for category 1, 2 and 3

$+(a_{24}'')^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a_{25}'')^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a_{26}'')^{(4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficients for category 1, 2 and 3

$+(a_{28}'')^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a_{29}'')^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a_{30}'')^{(5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficients for category 1, 2 and 3

$+(a_{32}'')^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a_{33}'')^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a_{34}'')^{(6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficients for category 1, 2 and 3

$+(a_{36}'')^{(7,7,7,7)}(T_{37},t)$, $+(a_{37}'')^{(7,7,7,7)}(T_{37},t)$, $+(a_{38}'')^{(7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1, 2 and 3

$+(a_{40}'')^{(8,8,8,8)}(T_{41},t)$, $+(a_{41}'')^{(8,8,8,8)}(T_{41},t)$, $+(a_{42}'')^{(8,8,8,8)}(T_{41},t)$ are eight augmentation coefficients for category 1, 2 and 3

$+(a_{44}'')^{(9,9,9)}(T_{45},t)$, $+(a_{45}'')^{(9,9,9)}(T_{45},t)$, $+(a_{46}'')^{(9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for

category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b'_{20})^{(3)}\boxed{-(b''_{20})^{(3)}(G_{23},t)}\boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}\boxed{-(b''_{13})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)}\boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)}\boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7)}(G_{39},t)}\boxed{-(b''_{40})^{(8,8,8,8)}(G_{43},t)}\boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{20} \quad 70$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b'_{21})^{(3)}\boxed{-(b''_{21})^{(3)}(G_{23},t)}\boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}\boxed{-(b''_{14})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)}\boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)}\boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7)}(G_{39},t)}\boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)}\boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{21} \quad 71$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)}\boxed{-(b''_{22})^{(3)}(G_{23},t)}\boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}\boxed{-(b''_{15})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)}\boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)}\boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)}\boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)}\boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{22} \quad 72$$

$\boxed{-(b''_{20})^{(3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3)}(G_{23},t)}$ *are first detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}$ *are second detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{13})^{(1,1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,)}(G,t)}$ *are third detrition coefficients for category 1,2 and 3*

$\boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)}$ *are fourth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)}$ *are fifth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)}$ *are sixth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{36})^{(7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7)}(G_{39},t)}$ $\boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \begin{bmatrix} (a'_{24})^{(4)}\boxed{+(a''_{24})^{(4)}(T_{25},t)}\boxed{+(a''_{28})^{(5,5,)}(T_{29},t)}\boxed{+(a''_{32})^{(6,6,)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)}\boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)}\boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)}\boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)}\boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{24} \quad 73$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \begin{bmatrix} (a'_{25})^{(4)}\boxed{+(a''_{25})^{(4)}(T_{25},t)}\boxed{+(a''_{29})^{(5,5,)}(T_{29},t)}\boxed{+(a''_{33})^{(6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)}\boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)}\boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)}\boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)}\boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{25} \quad 74$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \begin{bmatrix} (a'_{26})^{(4)} & +(a''_{26})^{(4)}(T_{25},t) & +(a''_{30})^{(5,5,)}(T_{29},t) & +(a''_{34})^{(6,6,)}(T_{33},t) \\ & +(a''_{15})^{(1,1,1,1)}(T_{14},t) & +(a''_{18})^{(2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3)}(T_{21},t) \\ & +(a''_{38})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9)}(T_{45},t) \end{bmatrix} G_{26} \quad 75$$

$(a''_{24})^{(4)}(T_{25},t)$, $(a''_{25})^{(4)}(T_{25},t)$, $(a''_{26})^{(4)}(T_{25},t)$ *are first augmentation coefficients category* 1, 2 3

$+(a''_{28})^{(5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,)}(T_{29},t)$ *are second augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{32})^{(6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,)}(T_{33},t)$ *are third augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{13})^{(1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1)}(T_{14},t)$ *are fourth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{16})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2)}(T_{17},t)$ *are fifth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{20})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3)}(T_{21},t)$ *are sixth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)$ *are seventh augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)$ *are eighth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{46})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9)}(T_{45},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \begin{bmatrix} (b'_{24})^{(4)} & -(b''_{24})^{(4)}(G_{27},t) & -(b''_{28})^{(5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,)}(G_{35},t) \\ & -(b''_{13})^{(1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{24} \quad 76$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \begin{bmatrix} (b'_{25})^{(4)} & -(b''_{25})^{(4)}(G_{27},t) & -(b''_{29})^{(5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{25} \quad 77$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)} & -(b''_{26})^{(4)}(G_{27},t) & -(b''_{30})^{(5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1)}(G,t) & -(b''_{18})^{(2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{26} \quad 78$$

*Where* $-(b''_{24})^{(4)}(G_{27},t)$, $-(b''_{25})^{(4)}(G_{27},t)$, $-(b''_{26})^{(4)}(G_{27},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{28})^{(5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,)}(G_{31},t)$ *are second detrition coefficients for category* 1, 2 *and* 3

$-(b''_{32})^{(6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,)}(G_{35},t)$ *are third detrition coefficients for category* 1, 2 *and* 3

$-(b''_{13})^{(1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1)}(G,t)$

*are fourth detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2,2)}(G_{19},t)}$

*are fifth detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3,3,3,3)}(G_{23},t)}$

*are sixth detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)}$

*are seventh detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)}$

*are eighth detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{46})^{(9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{45})^{(9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{44})^{(9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[\begin{array}{ccc} (a'_{28})^{(5)}\boxed{+(a''_{28})^{(5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{28} \quad (79)$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[\begin{array}{ccc} (a'_{29})^{(5)}\boxed{+(a''_{29})^{(5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{29} \quad (80)$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[\begin{array}{ccc} (a'_{30})^{(5)}\boxed{+(a''_{30})^{(5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{30} \quad (81)$$

*Where* $\boxed{+(a''_{28})^{(5)}(T_{29},t)}, \boxed{+(a''_{29})^{(5)}(T_{29},t)}, \boxed{+(a''_{30})^{(5)}(T_{29},t)}$ *are first augmentation coefficients for category* 1, 2 *and* 3

*And* $\boxed{+(a''_{24})^{(4,4,)}(T_{25},t)}, \boxed{+(a''_{25})^{(4,4,)}(T_{25},t)}, \boxed{+(a''_{26})^{(4,4,)}(T_{25},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)}, \boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)}, \boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)}$ are fourth augmentation coefficients for category 1,2, and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)}$ are fifth augmentation coefficients for category 1,2,and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)}$ are sixth augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)}$ are seventh augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)}$ are eighth augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation

coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \begin{bmatrix} (b'_{28})^{(5)}\boxed{-(b''_{28})^{(5)}(G_{31},t)} & \boxed{-(b''_{24})^{(4,4,)}(G_{27},t)} & \boxed{-(b''_{32})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{28} \quad (82)$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b'_{29})^{(5)}\boxed{-(b''_{29})^{(5)}(G_{31},t)} & \boxed{-(b''_{25})^{(4,4,)}(G_{27},t)} & \boxed{-(b''_{33})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{29} \quad (83)$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b'_{30})^{(5)}\boxed{-(b''_{30})^{(5)}(G_{31},t)} & \boxed{-(b''_{26})^{(4,4,)}(G_{27},t)} & \boxed{-(b''_{34})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{30} \quad (84)$$

*where* $\boxed{-(b''_{28})^{(5)}(G_{31},t)}$ , $\boxed{-(b''_{29})^{(5)}(G_{31},t)}$ , $\boxed{-(b''_{30})^{(5)}(G_{31},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{24})^{(4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,)}(G_{27},t)}$ *are second detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{32})^{(6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6)}(G_{35},t)}$ *are third detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1)}(G,t)}$ , $\boxed{-(b''_{15})^{(1,1,1,1,1,)}(G,t)}$ are fourth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)}$ , $\boxed{-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)}$ , $\boxed{-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)}$ , $\boxed{-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)}$ , $\boxed{-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)}$ , $\boxed{-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)}$ , $\boxed{-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \begin{bmatrix} (a'_{32})^{(6)}\boxed{+(a''_{32})^{(6)}(T_{33},t)} & \boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{32} \quad (85)$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \begin{bmatrix} (a'_{33})^{(6)}\boxed{+(a''_{33})^{(6)}(T_{33},t)} & \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{33} \quad (86)$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \begin{bmatrix} (a'_{34})^{(6)}\boxed{+(a''_{34})^{(6)}(T_{33},t)} & \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{34} \quad 87$$

$\boxed{+(a''_{32})^{(6)}(T_{33},t)}, \boxed{+(a''_{33})^{(6)}(T_{33},t)}, \boxed{+(a''_{34})^{(6)}(T_{33},t)}$ *are first augmentation coefficients for category* $1, 2$ *and* $3$

$\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}$ *are second augmentation coefficients for category* $1, 2$ *and* $3$

$\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}$ *are third augmentation coefficients for category* $1, 2$ *and* $3$

$\boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}$ - are fourth augmentation coefficients

$\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}$ - fifth augmentation coefficients

$\boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)}$ sixth augmentation coefficients

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)}$ seventh augmentation coefficients

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}$ Eighth augmentation coefficients

$\boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)}$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \begin{bmatrix} (b'_{32})^{(6)}\boxed{-(b''_{32})^{(6)}(G_{35},t)} & \boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{32} \quad 88$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \begin{bmatrix} (b'_{33})^{(6)}\boxed{-(b''_{33})^{(6)}(G_{35},t)} & \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{33} \quad 89$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)}\boxed{-(b''_{34})^{(6)}(G_{35},t)} & \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{34} \quad 90$$

$\boxed{-(b''_{32})^{(6)}(G_{35},t)}, \boxed{-(b''_{33})^{(6)}(G_{35},t)}, \boxed{-(b''_{34})^{(6)}(G_{35},t)}$ *are first detrition coefficients for category* $1, 2$ *and* $3$

$\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}$ *are second detrition coefficients for category* $1, 2$ *and* $3$

$\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)}$ *are third detrition coefficients*

$for\ category\ 1,2\ and\ 3$

$-(b_{13}'')^{(1,1,1,1,1,1,1)}(G,t)$, $-(b_{14}'')^{(1,1,1,1,1,1,1)}(G,t)$, $-(b_{15}'')^{(1,1,1,1,1,1,1)}(G,t)$ are fourth detrition coefficients for category 1, 2, and 3

$-(b_{16}'')^{(2,2,2,2,2,2,2)}(G_{19},t)$, $-(b_{17}'')^{(2,2,2,2,2,2,2)}(G_{19},t)$, $-(b_{18}'')^{(2,2,2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1, 2, and 3

$-(b_{20}'')^{(3,3,3,3,3,3,3)}(G_{23},t)$, $-(b_{21}'')^{(3,3,3,3,3,3,3)}(G_{23},t)$, $-(b_{22}'')^{(3,3,3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1, 2, and 3

$-(b_{36}'')^{(7,7,7,7,7,7,7)}(G_{39},t)$, $-(b_{37}'')^{(7,7,7,7,7,7,7)}(G_{39},t)$, $-(b_{38}'')^{(7,7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2, and 3

$-(b_{40}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)$, $-(b_{41}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)$, $-(b_{42}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1, 2, and 3

$-(b_{46}'')^{(9,9,9,9,9,9,9)}(G_{47},t)$, $-(b_{45}'')^{(9,9,9,9,9,9,9)}(G_{47},t)$, $-(b_{44}'')^{(9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \qquad 91$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a_{36}')^{(7)} & +(a_{36}'')^{(7)}(T_{37},t) & +(a_{16}'')^{(2,2,2,2,2,2,2)}(T_{17},t) & +(a_{20}'')^{(3,3,3,3,3,3,3)}(T_{21},t) \\ & +(a_{24}'')^{(4,4,4,4,4,4,4)}(T_{25},t) & +(a_{28}'')^{(5,5,5,5,5,5,5)}(T_{29},t) & +(a_{32}'')^{(6,6,6,6,6,6,6)}(T_{33},t) \\ & +(a_{13}'')^{(1,1,1,1,1,1,1)}(T_{14},t) & +(a_{40}'')^{(8,8,8,8,8,8,8,8,)}(T_{41},t) & +(a_{44}'')^{(9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \qquad 92$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a_{37}')^{(7)} & +(a_{37}'')^{(7)}(T_{37},t) & +(a_{17}'')^{(2,2,2,2,2,2,2)}(T_{17},t) & +(a_{21}'')^{(3,3,3,3,3,3,3)}(T_{21},t) \\ & +(a_{25}'')^{(4,4,4,4,4,4,4)}(T_{25},t) & +(a_{29}'')^{(5,5,5,5,5,5,5)}(T_{29},t) & +(a_{33}'')^{(6,6,6,6,6,6,6)}(T_{33},t) \\ & +(a_{13}'')^{(1,1,1,1,1,1,1)}(T_{14},t) & +(a_{41}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t) & +(a_{45}'')^{(9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{14}$$

$$\frac{dG_{38}}{dt} \qquad 93$$

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a_{38}')^{(7)} & +(a_{38}'')^{(7)}(T_{37},t) & +(a_{18}'')^{(2,2,2,2,2,2,2)}(T_{17},t) & +(a_{22}'')^{(3,3,3,3,3,3,3)}(T_{21},t) \\ & +(a_{26}'')^{(4,4,4,4,4,4,4)}(T_{25},t) & +(a_{30}'')^{(5,5,5,5,5,5,5)}(T_{29},t) & +(a_{34}'')^{(6,6,6,6,6,6,6)}(T_{33},t) \\ & +(a_{15}'')^{(1,1,1,1,1,1,1)}(T_{14},t) & +(a_{42}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t) & +(a_{46}'')^{(9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{15}$$

Where $(a_{36}'')^{(7)}(T_{37},t)$, $(a_{37}'')^{(7)}(T_{37},t)$, $(a_{38}'')^{(7)}(T_{37},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a_{16}'')^{(2,2,2,2,2,2,2)}(T_{17},t)$, $+(a_{17}'')^{(2,2,2,2,2,2,2)}(T_{17},t)$, $+(a_{18}'')^{(2,2,2,2,2,2,2)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a_{20}'')^{(3,3,3,3,3,3,3)}(T_{21},t)$, $+(a_{21}'')^{(3,3,3,3,3,3,3)}(T_{21},t)$, $+(a_{22}'')^{(3,3,3,3,3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a_{24}'')^{(4,4,4,4,4,4,4)}(T_{25},t)$, $+(a_{25}'')^{(4,4,4,4,4,4,4)}(T_{25},t)$, $+(a_{26}'')^{(4,4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a_{28}'')^{(5,5,5,5,5,5,5)}(T_{29},t)$, $+(a_{29}'')^{(5,5,5,5,5,5,5)}(T_{29},t)$, $+(a_{30}'')^{(5,5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a_{32}'')^{(6,6,6,6,6,6,6)}(T_{33},t)$, $+(a_{33}'')^{(6,6,6,6,6,6,6)}(T_{33},t)$, $+(a_{34}'')^{(6,6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation

coefficient for category 1, 2 and 3
$+(a_{13}'')^{(1,1,1,1,1,1,1)}(T_{14},t)$, $+(a_{13}'')^{(1,1,1,1,1,1,1)}(T_{14},t)$, $+(a_{15}'')^{(1,1,1,1,1,1,1)}(T_{14},t)$ are seventh augmentation coefficient for category 1, 2 and 3
$+(a_{42}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a_{41}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a_{40}'')^{(8,8,8,8,8,8,8,8,)}(T_{41},t)$ are eighth augmentation coefficient for 1,2,3
$+(a_{46}'')^{(9,9,9,9,9,9,9)}(T_{45},t)$, $+(a_{45}'')^{(9,9,9,9,9,9,9)}(T_{45},t)$, $+(a_{44}'')^{(9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3



$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \begin{bmatrix} (b_{36}')^{(7)}\boxed{-(b_{36}'')^{(7)}(G_{39},t)} & \boxed{-(b_{16}'')^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{20}'')^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{24}'')^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{28}'')^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{32}'')^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{13}'')^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b_{44}'')^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \begin{bmatrix} (b_{37}')^{(7)}\boxed{-(b_{37}'')^{(7)}(G_{39},t)} & \boxed{-(b_{17}'')^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{21}'')^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{25}'')^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{29}'')^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{33}'')^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{14}'')^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b_{45}'')^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \begin{bmatrix} (b_{38}')^{(7)}\boxed{-(b_{38}'')^{(7)}(G_{39},t)} & \boxed{-(b_{18}'')^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{22}'')^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{26}'')^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{30}'')^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{34}'')^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{15}'')^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b_{46}'')^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b_{36}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{37}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3
$\boxed{-(b_{16}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{17}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{18}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3
$\boxed{-(b_{20}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{21}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{22}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3
$\boxed{-(b_{24}'')^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{25}'')^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{26}'')^{(4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3
$\boxed{-(b_{28}'')^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3
$\boxed{-(b_{32}'')^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{34}'')^{(6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3
$\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3
$\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3



$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \begin{bmatrix} (a'_{40})^{(8)}\boxed{+(a''_{40})^{(8)}(T_{41},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \begin{bmatrix} (a'_{41})^{(8)}\boxed{+(a''_{41})^{(8)}(T_{41},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \begin{bmatrix} (a'_{42})^{(8)}\boxed{+(a''_{42})^{(8)}(T_{41},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a''_{40})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} =$$

$$(b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)}\boxed{-(b''_{40})^{(8)}(G_{43},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} =$$

$$(b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)}\boxed{-(b''_{41})^{(8)}(G_{43},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} =$$

$$(b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)}\boxed{-(b''_{42})^{(8)}(G_{43},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3



$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \begin{bmatrix} (a'_{44})^{(9)}\boxed{+(a''_{44})^{(9)}(T_{45},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \begin{bmatrix} (a'_{45})^{(9)}\boxed{+(a''_{45})^{(9)}(T_{45},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \begin{bmatrix} (a'_{46})^{(9)}\boxed{+(a''_{46})^{(9)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a''_{44})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ are Seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} -$$

$$\begin{bmatrix} (b'_{44})^{(9)}\boxed{-(b''_{44})^{(9)}(G_{47},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b'_{45})^{(9)}\boxed{-(b''_{45})^{(9)}(G_{47},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b'_{46})^{(9)}\boxed{-(b''_{46})^{(9)}(G_{47},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{44})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9)}(G_{47},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a'_i)^{(1)}, (a''_i)^{(1)}, (b_i)^{(1)}, (b'_i)^{(1)}, (b''_i)^{(1)} > 0, \quad i,j = 13,14,15$$ *97*

The functions $(a''_i)^{(1)}, (b''_i)^{(1)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$(a_i'')^{(1)}(T_{14},t) \leq (p_i)^{(1)} \leq (\hat{A}_{13})^{(1)}$

$(b_i'')^{(1)}(G,t) \leq (r_i)^{(1)} \leq (b_i')^{(1)} \leq (\hat{B}_{13})^{(1)}$

$lim_{T_2\to\infty}(a_i'')^{(1)}(T_{14},t) = (p_i)^{(1)}$ 98

$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = (r_i)^{(1)}$

**Definition of** $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$ :

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$|(a_i'')^{(1)}(T_{14}',t) - (a_i'')^{(1)}(T_{14},t)| \leq (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$

$|(b_i'')^{(1)}(G',t) - (b_i'')^{(1)}(G,t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}',t)$ and $(a_i'')^{(1)}(T_{14},t)$ . $(T_{14}',t)$ and $(T_{14},t)$ are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14},t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$ : 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$

**Definition of** $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$ : 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$

$\frac{1}{(\hat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$

Where we suppose

$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i,j = 16,17,18$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17},t) \le (p_i)^{(2)} \le \left(\hat{A}_{16}\right)^{(2)} \quad 102$$

$$(b_i'')^{(2)}(G_{19},t) \le (r_i)^{(2)} \le (b_i')^{(2)} \le (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2\to\infty}(a_i'')^{(2)}(T_{17},t) = (p_i)^{(2)} \quad 104$$

$$\lim_{G\to\infty}(b_i'')^{(2)}\left((G_{19}),t\right) = (r_i)^{(2)} \quad 105$$

**Definition of** $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$ : 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}',t) - (a_i'')^{(2)}(T_{17},t)| \le (\hat{k}_{16})^{(2)}|T_{17} - T_{17}'|e^{-(\hat{M}_{16})^{(2)}t} \quad 107$$

$$|(b_i'')^{(2)}((G_{19})',t) - (b_i'')^{(2)}\left((G_{19}),t\right)| < (\hat{k}_{16})^{(2)}||(G_{19}) - (G_{19})'||e^{-(\hat{M}_{16})^{(2)}t} \quad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}',t)$ and$(a_i'')^{(2)}(T_{17},t)$ . $(T_{17}',t)$ and $(T_{17},t)$ are points belonging to the interval $\left[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}\right]$ . It is to be noted that $(a_i'')^{(2)}(T_{17},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17},t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$ :

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants 109

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$$

**Definition of** $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$ :

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1 \quad 110$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(b_i)^{(2)} + (b_i')^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1 \quad 111$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i,j = 20,21,22 \quad 112$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(3)}$, $(\mathrm{r_i})^{(3)}$:

$$(a_i'')^{(3)}(T_{21},t) \le (p_i)^{(3)} \le (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23},t) \le (r_i)^{(3)} \le (b_i')^{(3)} \le (\hat{B}_{20})^{(3)}$$

$$lim_{T_2\to\infty}(a_i'')^{(3)}(T_{21},t) = (p_i)^{(3)} \qquad 113$$

$$\lim_{G\to\infty}(b_i'')^{(3)}(G_{23},t) = (r_i)^{(3)}$$

**Definition of** $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$ :

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: 114

$$|(a_i'')^{(3)}(T_{21}',t) - (a_i'')^{(3)}(T_{21},t)| \le (\hat{k}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\hat{M}_{20})^{(3)}t}$$

$$|(b_i'')^{(3)}(G_{23}',t) - (b_i'')^{(3)}(G_{23},t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}'||e^{-(\hat{M}_{20})^{(3)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}',t)$ and$(a_i'')^{(3)}(T_{21},t)$ . $(T_{21}',t)$ And $(T_{21},t)$ are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$ : 115

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$ and the constants $(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}, i = 20,21,22,$ satisfy the inequalities 116

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(a_i)^{(3)} + (a_i')^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(b_i)^{(3)} + (b_i')^{(3)} + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i,j = 24,25,26 \qquad 117$$

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(4)}$, $(r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25},t) \le (p_i)^{(4)} \le (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}\big((G_{27}),t\big) \le (r_i)^{(4)} \le (b_i')^{(4)} \le (\hat{B}_{24})^{(4)}$$

$$lim_{T_2\to\infty}(a_i'')^{(4)}(T_{25},t) = (p_i)^{(4)} \quad (118)$$

$$\lim_{G\to\infty}(b_i'')^{(4)}\big((G_{27}),t\big) = (r_i)^{(4)}$$

**Definition of** $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$ :

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24,25,26}$

They satisfy Lipschitz condition: (119)

$$|(a_i'')^{(4)}(T_{25}',t) - (a_i'')^{(4)}(T_{25},t)| \le (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})',t) - (b_i'')^{(4)}\big((G_{27}),t\big)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}',t)$ and$(a_i'')^{(4)}(T_{25},t)$ . $(T_{25}',t)$ and $(T_{25},t)$ are points belonging to the interval $\big[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}\big]$. It is to be noted that $(a_i'')^{(4)}(T_{25},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25},t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$ : (120)

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$, are positive constants

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}, \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$$

**Definition of** $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$ : (121)

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and\ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}, i = 24,25,26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(a_i)^{(4)} + (a_i')^{(4)} + (\hat{A}_{24})^{(4)} + (\hat{P}_{24})^{(4)}(\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(b_i)^{(4)} + (b_i')^{(4)} + (\hat{B}_{24})^{(4)} + (\hat{Q}_{24})^{(4)}(\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i,j = 28,29,30 \quad (122)$$

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(5)}$, $(r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29},t) \le (p_i)^{(5)} \le (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}\big((G_{31}),t\big) \le (r_i)^{(5)} \le (b_i')^{(5)} \le (\hat{B}_{28})^{(5)}$$

$$lim_{T_2\to\infty}(a_i'')^{(5)}(T_{29},t) = (p_i)^{(5)} \qquad 123$$

$$\lim_{G\to\infty}(b_i'')^{(5)}(G_{31},t) = (r_i)^{(5)}$$

**Definition of** $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$ :

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}',t) - (a_i'')^{(5)}(T_{29},t)| \le (\hat{k}_{28})^{(5)}|T_{29} - T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})',t) - (b_i'')^{(5)}\big((G_{31}),t\big)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}',t)$ and$(a_i'')^{(5)}(T_{29},t)$ . $(T_{29}',t)$ and $(T_{29},t)$ are points belonging to the interval $\big[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\big]$. It is to be noted that $(a_i'')^{(5)}(T_{29},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29},t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$ : 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}, \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

**Definition of** $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$ : 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and\ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(a_i)^{(5)} + (a_i')^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)}(\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(b_i)^{(5)} + (b_i')^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)}(\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i,j = 32,33,34 \qquad 127$$

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(6)}$, $(r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33},t) \le (p_i)^{(6)} \le (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}),t) \le (r_i)^{(6)} \le (b_i')^{(6)} \le (\hat{B}_{32})^{(6)}$$

$$lim_{T_2\to\infty}(a_i'')^{(6)}(T_{33},t) = (p_i)^{(6)} \quad (128)$$

$$\lim_{G\to\infty}(b_i'')^{(6)}\left((G_{35}),t\right) = (r_i)^{(6)}$$

**Definition of** $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$ :

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}',t) - (a_i'')^{(6)}(T_{33},t)| \le (\hat{k}_{32})^{(6)}|T_{33} - T_{33}'|e^{-(\hat{M}_{32})^{(6)}t}$$

$$|(b_i'')^{(6)}((G_{35})',t) - (b_i'')^{(6)}\left((G_{35}),t\right)| < (\hat{k}_{32})^{(6)}||(G_{35}) - (G_{35})'||e^{-(\hat{M}_{32})^{(6)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}',t)$ and$(a_i'')^{(6)}(T_{33},t)$ . $(T_{33}',t)$ and $(T_{33},t)$ are points belonging to the interval $\left[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}\right]$. It is to be noted that $(a_i'')^{(6)}(T_{33},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33},t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$ : (129)

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$, are positive constants

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$$

**Definition of** $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$ : (130)

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and\ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32,33,34,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(a_i)^{(6)} + (a_i')^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(b_i)^{(6)} + (b_i')^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

(G) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i,j = 36,37,38$ (131)

(H) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(7)}$, $(r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37},t) \le (p_i)^{(7)} \le (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39},t) \le (r_i)^{(7)} \le (b_i')^{(7)} \le (\hat{B}_{36})^{(7)}$$

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*(I)* $\lim_{T_2\to\infty} (a_i'')^{(7)}(T_{37},t) = (p_i)^{(7)}$

*(J)*

$\lim_{G\to\infty}(b_i'')^{(7)}\big((G_{39}),t\big) = (r_i)^{(7)}$

**Definition of** $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$ :

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$$|(a_i'')^{(7)}(T_{37}',t) - (a_i'')^{(7)}(T_{37},t)| \le (\hat{k}_{36})^{(7)}|T_{37} - T_{37}'|e^{-(\hat{M}_{36})^{(7)}t}$$

$$|(b_i'')^{(7)}((G_{39})',t) - (b_i'')^{(7)}\big((G_{39}),t\big)| < (\hat{k}_{36})^{(7)}||(G_{39}) - (G_{39})'||e^{-(\hat{M}_{36})^{(7)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}',t)$ and$(a_i'')^{(7)}(T_{37},t)$ . $(T_{37}',t)$ and $(T_{37},t)$ are points belonging to the interval $\big[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}\big]$. It is to be noted that $(a_i'')^{(7)}(T_{37},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37},t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$ : 134

(K) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$, are positive constants

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$$

**Definition of** $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$ : 135

(L) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)}(\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(b_i)^{(7)} + (b_i')^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)}(\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40,41,42 \tag{136}$$

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

**Definition of** $(p_i)^{(8)}$, $(r_i)^{(8)}$: (137)

$$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)} \tag{138}$$

$$(b_i'')^{(8)}((G_{43}), t) \leq (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)} \tag{139}$$

$$\lim_{T_2 \to \infty} (a_i'')^{(8)}(T_{41}, t) = (p_i)^{(8)} \tag{140}$$

$$\lim_{G \to \infty} (b_i'')^{(8)}\left((G_{43}), t\right) = (r_i)^{(8)} \tag{141}$$

**Definition of** $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$ :

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)} |T_{41} - T_{41}'| e^{-(\hat{M}_{40})^{(8)} t} \tag{142}$$

$$|(b_i'')^{(8)}((G_{43})', t) - (b_i'')^{(8)}\left((G_{43}), t\right)| < (\hat{k}_{40})^{(8)} ||(G_{43}) - (G_{43})'|| e^{-(\hat{M}_{40})^{(8)} t} \tag{143}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}', t)$ and $(a_i'')^{(8)}(T_{41}, t)$ . $(T_{41}', t)$ and $(T_{41}, t)$ are points belonging to the interval $\left[(\hat{k}_{40})^{(8)}, (\hat{M}_{40})^{(8)}\right]$. It is to be noted that $(a_i'')^{(8)}(T_{41}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{40})^{(8)} = 1$ then the function $(a_i'')^{(8)}(T_{41}, t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$ :

$(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$, are positive constants

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}, \frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1 \tag{144}$$

**Definition of** $(\hat{P}_{40})^{(8)}, (\hat{Q}_{40})^{(8)}$ :

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}, (\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)}, (a_i')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (p_i)^{(8)}$, $(r_i)^{(8)}, i = 40,41,42,$
Satisfy the inequalities

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(a_i)^{(8)} + (a_i')^{(8)} + (\hat{A}_{40})^{(8)} + (\hat{P}_{40})^{(8)} (\hat{k}_{40})^{(8)}] < 1 \tag{145}$$

$$\frac{1}{(\widehat{M}_{40})^{(8)}}[\ (b_i)^{(8)} + (b_i')^{(8)} + \ (\hat{B}_{40})^{(8)} + \ (\hat{Q}_{40})^{(8)}\ (\hat{k}_{40})^{(8)}] < 1 \qquad 146$$

Where we suppose

$$(a_i)^{(9)}, (a_i')^{(9)}, (a_i'')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (b_i'')^{(9)} > 0, \quad i,j = 44,45,46 \qquad 146\,A$$

The functions $(a_i'')^{(9)}, (b_i'')^{(9)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(9)}$, $(r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45}, t) \leq (p_i)^{(9)} \leq (\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47}, t) \leq \ (r_i)^{(9)} \leq (b_i')^{(9)} \leq (\hat{B}_{44})^{(9)}$$

$$\lim_{T_2 \to \infty} (a_i'')^{(9)}(T_{45}, t) = (p_i)^{(9)}$$

$$\lim_{G \to \infty} (b_i'')^{(9)}(G_{47}, t) = \ (r_i)^{(9)}$$

**Definition of** $(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}$ :

Where $\boxed{(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}, (p_i)^{(9)}, \ (r_i)^{(9)}}$ are positive constants and $\boxed{i = 44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}', t) - (a_i'')^{(9)}(T_{45}, t)| \leq (\hat{k}_{44})^{(9)} |T_{45} - T_{45}'| e^{-(\widehat{M}_{44})^{(9)} t}$$

$$|(b_i'')^{(9)}((G_{47})', t) - (b_i'')^{(9)}\big((G_{47}), t\big)| < (\hat{k}_{44})^{(9)} ||(G_{47}) - (G_{47})'|| e^{-(\widehat{M}_{44})^{(9)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}', t)$ and$(a_i'')^{(9)}(T_{45}, t)$ . $(T_{45}', t)$ and $(T_{45}, t)$ are points belonging to the interval $\left[(\hat{k}_{44})^{(9)}, (\widehat{M}_{44})^{(9)}\right]$. It is to be noted that $(a_i'')^{(9)}(T_{45}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{44})^{(9)} = 1$ then the function $(a_i'')^{(9)}(T_{45}, t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\widehat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$ :

$(\widehat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$, are positive constants

$$\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}} \ , \frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}} < 1$$

**Definition of** $(\hat{P}_{44})^{(9)}, (\hat{Q}_{44})^{(9)}$ :

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\widehat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}, (\hat{A}_{44})^{(9)} and\ (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)}, (a_i')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (p_i)^{(9)}, \ (r_i)^{(9)}, i = 44,45,46,$
satisfy the inequalities

$$\frac{1}{(\widehat{M}_{44})^{(9)}}[(a_i)^{(9)} + (a_i')^{(9)} + (\hat{A}_{44})^{(9)} + (\hat{P}_{44})^{(9)}(\hat{k}_{44})^{(9)}] < 1$$

$$\frac{1}{(\widehat{M}_{44})^{(9)}}[(b_i)^{(9)} + (b_i')^{(9)} + (\hat{B}_{44})^{(9)} + (\hat{Q}_{44})^{(9)}(\hat{k}_{44})^{(9)}] < 1$$

**Theorem 1:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0), T_i(0)$ :

$$G_i(t) \le (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 2 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

**Definition of** $G_i(0), T_i(0)$

$$G_i(t) \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} \quad , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} \quad , \quad T_i(0) = T_i^0 > 0$$

**Theorem 3 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t) \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t} \quad , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t} \quad , \quad T_i(0) = T_i^0 > 0$$

**Theorem 4 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

**Definition of** $G_i(0), T_i(0)$ :

$$G_i(t) \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 5 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

**Definition of** $G_i(0), T_i(0)$ :

$$G_i(t) \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 6 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

**Definition of** $G_i(0), T_i(0)$ :

$$G_i(t) \le (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$T_i(t) \leq (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Theorem 7:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

**Definition of** $G_i(0), T_i(0)$ :

$G_i(t) \leq (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Theorem 8:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

**Definition of** $G_i(0), T_i(0)$ :

$G_i(t) \leq (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Theorem 9:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

**Definition of** $G_i(0), T_i(0)$ :

$G_i(t) \leq (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Proof:** Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$G_i(0) = G_i^0$ , $T_i(0) = T_i^0$ , $G_i^0 \leq (\hat{P}_{13})^{(1)}$ , $T_i^0 \leq (\hat{Q}_{13})^{(1)}$, 155

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t}$ 156

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t}$ 157

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[ (a_{13})^{(1)} G_{14}(s_{(13)}) - \left( (a'_{13})^{(1)} + a''_{13})^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \right) G_{13}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[ (a_{14})^{(1)} G_{13}(s_{(13)}) - \left( (a'_{14})^{(1)} + (a''_{14})^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \right) G_{14}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[ (a_{15})^{(1)} G_{14}(s_{(13)}) - \left( (a'_{15})^{(1)} + (a''_{15})^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \right) G_{15}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[ (b_{13})^{(1)} T_{14}(s_{(13)}) - \left( (b'_{13})^{(1)} - (b''_{13})^{(1)} (G(s_{(13)}), s_{(13)}) \right) T_{13}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^{0} + \int_0^t \left[ (b_{14})^{(1)} T_{13}(s_{(13)}) - \left( (b'_{14})^{(1)} - (b''_{14})^{(1)} \left( G(s_{(13)}), s_{(13)} \right) \right) T_{14}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{\mathrm{T}}_{15}(\mathrm{t}) = \mathrm{T}_{15}^{0} + \int_0^t \left[ (b_{15})^{(1)} T_{14}(s_{(13)}) - \left( (b'_{15})^{(1)} - (b''_{15})^{(1)} \left( G(s_{(13)}), s_{(13)} \right) \right) T_{15}(s_{(13)}) \right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{16})^{(2)}, T_i^0 \le (\hat{Q}_{16})^{(2)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$$

By 160

$$\bar{G}_{16}(t) = G_{16}^{0} + \int_0^t \left[ (a_{16})^{(2)} G_{17}(s_{(16)}) - \left( (a'_{16})^{(2)} + a''_{16})^{(2)} (T_{17}(s_{(16)}), s_{(16)}) \right) G_{16}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^{0} + \int_0^t \left[ (a_{17})^{(2)} G_{16}(s_{(16)}) - \left( (a'_{17})^{(2)} + (a''_{17})^{(2)} (T_{17}(s_{(16)}), s_{(17)}) \right) G_{17}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^{0} + \int_0^t \left[ (a_{18})^{(2)} G_{17}(s_{(16)}) - \left( (a'_{18})^{(2)} + (a''_{18})^{(2)} (T_{17}(s_{(16)}), s_{(16)}) \right) G_{18}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^{0} + \int_0^t \left[ (b_{16})^{(2)} T_{17}(s_{(16)}) - \left( (b'_{16})^{(2)} - (b''_{16})^{(2)} (G_{19}(s_{(16)}), s_{(16)}) \right) T_{16}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^{0} + \int_0^t \left[ (b_{17})^{(2)} T_{16}(s_{(16)}) - \left( (b'_{17})^{(2)} - (b''_{17})^{(2)} (G_{19}(s_{(16)}), s_{(16)}) \right) T_{17}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^{0} + \int_0^t \left[ (b_{18})^{(2)} T_{17}(s_{(16)}) - \left( (b'_{18})^{(2)} - (b''_{18})^{(2)} (G_{19}(s_{(16)}), s_{(16)}) \right) T_{18}(s_{(16)}) \right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{20})^{(3)}, T_i^0 \le (\hat{Q}_{20})^{(3)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$$

By 161

$$\bar{G}_{20}(t) = G_{20}^{0} + \int_0^t \left[ (a_{20})^{(3)} G_{21}(s_{(20)}) - \left( (a'_{20})^{(3)} + a''_{20})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{22}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^0 + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^0 + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{\mathrm{T}}_{22}(\mathrm{t}) = \mathrm{T}_{22}^0 + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{22}(s_{(20)})\right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{24})^{(4)}, T_i^0 \le (\hat{Q}_{24})^{(4)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

By



$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{26}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{\mathrm{T}}_{26}(\mathrm{t}) = \mathrm{T}_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{26}(s_{(24)})\right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{28})^{(5)}, T_i^0 \le (\hat{Q}_{28})^{(5)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

By 163

$$\bar{G}_{28}(t) = G_{28}^{0} + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^{0} + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^{0} + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{30}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^{0} + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^{0} + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{\mathrm{T}}_{30}(\mathrm{t}) = \mathrm{T}_{30}^{0} + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{30}(s_{(28)})\right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{32})^{(6)}, T_i^0 \le (\hat{Q}_{32})^{(6)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$$

By 164

$$\bar{G}_{32}(t) = G_{32}^{0} + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a'_{32})^{(6)} + a''_{32})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^{0} + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^{0} + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{34}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^{0} + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^{0} + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{\mathrm{T}}_{34}(\mathrm{t}) = \mathrm{T}_{34}^{0} + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{34}(s_{(32)})\right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$ , $T_i(0) = T_i^0$ , $G_i^0 \leq (\hat{P}_{36})^{(7)}$ , $T_i^0 \leq (\hat{Q}_{36})^{(7)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[ (a_{36})^{(7)} G_{37}(s_{(36)}) - \left( (a'_{36})^{(7)} + a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[ (a_{37})^{(7)} G_{36}(s_{(36)}) - \left( (a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[ (a_{38})^{(7)} G_{37}(s_{(36)}) - \left( (a'_{38})^{(7)} + (a''_{38})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{38}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[ (b_{36})^{(7)} T_{37}(s_{(36)}) - \left( (b'_{36})^{(7)} - (b''_{36})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[ (b_{37})^{(7)} T_{36}(s_{(36)}) - \left( (b'_{37})^{(7)} - (b''_{37})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{38}(t) = T_{38}^0 + \int_0^t \left[ (b_{38})^{(7)} T_{37}(s_{(36)}) - \left( (b'_{38})^{(7)} - (b''_{38})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{38}(s_{(36)}) \right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i$: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$ , $T_i(0) = T_i^0$ , $G_i^0 \leq (\hat{P}_{40})^{(8)}$ , $T_i^0 \leq (\hat{Q}_{40})^{(8)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{40})^{(8)} e^{(\bar{M}_{40})^{(8)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{40})^{(8)} e^{(\bar{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[ (a_{40})^{(8)} G_{41}(s_{(40)}) - \left( (a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[ (a_{41})^{(8)} G_{40}(s_{(40)}) - \left( (a'_{41})^{(8)} + (a''_{41})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)}\left(T_{41}(s_{(40)}), s_{(40)}\right)\right) G_{42}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^0 + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)}\left(G_{43}(s_{(40)}), s_{(40)}\right)\right) T_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^0 + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)}\left(G_{43}(s_{(40)}), s_{(40)}\right)\right) T_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{\mathrm{T}}_{42}(\mathrm{t}) = \mathrm{T}_{42}^0 + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)}\left(G_{43}(s_{(40)}), s_{(40)}\right)\right) T_{42}(s_{(40)})\right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** 166

Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy A

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{44})^{(9)},\ T_i^0 \le (\hat{Q}_{44})^{(9)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''_{44})^{(9)}\left(T_{45}(s_{(44)}), s_{(44)}\right)\right) G_{44}(s_{(44)})\right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)}\left(T_{45}(s_{(44)}), s_{(44)}\right)\right) G_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)}\left(T_{45}(s_{(44)}), s_{(44)}\right)\right) G_{46}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)}\left(G_{47}(s_{(44)}), s_{(44)}\right)\right) T_{44}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)}\left(G_{47}(s_{(44)}), s_{(44)}\right)\right) T_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{\mathrm{T}}_{46}(\mathrm{t}) = \mathrm{T}_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)}\left(G_{47}(s_{(44)}), s_{(44)}\right)\right) T_{46}(s_{(44)})\right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}}\right)\right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left(e^{(\hat{M}_{13})^{(1)} t} - 1\right)$$

From which it follows that 168

$$(G_{13}(t)-G_{13}^{0})e^{-(\hat{M}_{13})^{(1)}t}\leq\frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}}\left[((\hat{P}_{13})^{(1)}+G_{14}^{0})e^{\left(-\frac{(\hat{P}_{13})^{(1)}+G_{14}^{0}}{G_{14}^{0}}\right)}+(\hat{P}_{13})^{(1)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t)\leq G_{16}^{0}+\int_{0}^{t}\left[(a_{16})^{(2)}\left(G_{17}^{0}+(\hat{P}_{16})^{(6)}e^{(\hat{M}_{16})^{(2)}s_{(16)}}\right)\right]ds_{(16)}= \quad (169)$$

$$\left(1+(a_{16})^{(2)}t\right)G_{17}^{0}+\frac{(a_{16})^{(2)}(\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}}\left(e^{(\hat{M}_{16})^{(2)}t}-1\right)$$

From which it follows that (170)

$$(G_{16}(t)-G_{16}^{0})e^{-(\hat{M}_{16})^{(2)}t}\leq\frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}}\left[((\hat{P}_{16})^{(2)}+G_{17}^{0})e^{\left(-\frac{(\hat{P}_{16})^{(2)}+G_{17}^{0}}{G_{17}^{0}}\right)}+(\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that (171)

$$G_{20}(t)\leq G_{20}^{0}+\int_{0}^{t}\left[(a_{20})^{(3)}\left(G_{21}^{0}+(\hat{P}_{20})^{(3)}e^{(\hat{M}_{20})^{(3)}s_{(20)}}\right)\right]ds_{(20)}=$$

$$\left(1+(a_{20})^{(3)}t\right)G_{21}^{0}+\frac{(a_{20})^{(3)}(\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left(e^{(\hat{M}_{20})^{(3)}t}-1\right)$$

From which it follows that (172)

$$(G_{20}(t)-G_{20}^{0})e^{-(\hat{M}_{20})^{(3)}t}\leq\frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left[((\hat{P}_{20})^{(3)}+G_{21}^{0})e^{\left(-\frac{(\hat{P}_{20})^{(3)}+G_{21}^{0}}{G_{21}^{0}}\right)}+(\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that (173)

$$G_{24}(t)\leq G_{24}^{0}+\int_{0}^{t}\left[(a_{24})^{(4)}\left(G_{25}^{0}+(\hat{P}_{24})^{(4)}e^{(\hat{M}_{24})^{(4)}s_{(24)}}\right)\right]ds_{(24)}=$$

$$\left(1+(a_{24})^{(4)}t\right)G_{25}^{0}+\frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left(e^{(\hat{M}_{24})^{(4)}t}-1\right)$$

From which it follows that (174)

$$(G_{24}(t)-G_{24}^{0})e^{-(\hat{M}_{24})^{(4)}t}\leq\frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left[((\hat{P}_{24})^{(4)}+G_{25}^{0})e^{\left(-\frac{(\hat{P}_{24})^{(4)}+G_{25}^{0}}{G_{25}^{0}}\right)}+(\hat{P}_{24})^{(4)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t) \le G_{28}^0 + \int_0^t \left[ (a_{28})^{(5)} \left( G_{29}^0 + (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} s_{(28)}} \right) \right] ds_{(28)} =$$

$$\left(1 + (a_{28})^{(5)} t\right) G_{29}^0 + \frac{(a_{28})^{(5)} (\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}} \left( e^{(\hat{M}_{28})^{(5)} t} - 1 \right)$$

From which it follows that 175

$$(G_{28}(t) - G_{28}^0) e^{-(\hat{M}_{28})^{(5)} t} \le \frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}} \left[ \left( (\hat{P}_{28})^{(5)} + G_{29}^0 \right) e^{\left( - \frac{(\hat{P}_{28})^{(5)} + G_{29}^0}{G_{29}^0} \right)} + (\hat{P}_{28})^{(5)} \right]$$

$(G_i^0)$ is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t) \le G_{32}^0 + \int_0^t \left[ (a_{32})^{(6)} \left( G_{33}^0 + (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} s_{(32)}} \right) \right] ds_{(32)} =$$

$$\left(1 + (a_{32})^{(6)} t\right) G_{33}^0 + \frac{(a_{32})^{(6)} (\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}} \left( e^{(\hat{M}_{32})^{(6)} t} - 1 \right)$$

From which it follows that 177

$$(G_{32}(t) - G_{32}^0) e^{-(\hat{M}_{32})^{(6)} t} \le \frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}} \left[ \left( (\hat{P}_{32})^{(6)} + G_{33}^0 \right) e^{\left( - \frac{(\hat{P}_{32})^{(6)} + G_{33}^0}{G_{33}^0} \right)} + (\hat{P}_{32})^{(6)} \right]$$

$(G_i^0)$ is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

(b) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t) \le G_{36}^0 + \int_0^t \left[ (a_{36})^{(7)} \left( G_{37}^0 + (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} s_{(36)}} \right) \right] ds_{(36)} =$$

$$\left(1 + (a_{36})^{(7)} t\right) G_{37}^0 + \frac{(a_{36})^{(7)} (\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left( e^{(\hat{M}_{36})^{(7)} t} - 1 \right)$$

From which it follows that

$$(G_{36}(t) - G_{36}^0) e^{-(\hat{M}_{36})^{(7)} t} \le \frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left[ \left( (\hat{P}_{36})^{(7)} + G_{37}^0 \right) e^{\left( - \frac{(\hat{P}_{36})^{(7)} + G_{37}^0}{G_{37}^0} \right)} + (\hat{P}_{36})^{(7)} \right]$$

$(G_i^0)$ is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t) \le G_{40}^0 + \int_0^t \left[ (a_{40})^{(8)} \left( G_{41}^0 + (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} s_{(40)}} \right) \right] ds_{(40)} = \quad 180$$

$$\left(1 + (a_{40})^{(8)} t\right) G_{41}^0 + \frac{(a_{40})^{(8)} (\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}} \left( e^{(\hat{M}_{40})^{(8)} t} - 1 \right)$$

From which it follows that 181

$$(G_{40}(t)-G_{40}^{0})e^{-(\hat{M}_{40})^{(8)}t}\leq\frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left[((\hat{P}_{40})^{(8)}+G_{41}^{0})e^{\left(-\frac{(\hat{P}_{40})^{(8)}+G_{41}^{0}}{G_{41}^{0}}\right)}+(\hat{P}_{40})^{(8)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 8

Analogous inequalities hold also for $G_{41}, G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t)\leq G_{44}^{0}+\int_{0}^{t}\left[(a_{44})^{(9)}\left(G_{45}^{0}+(\hat{P}_{44})^{(9)}e^{(\hat{M}_{44})^{(9)}s_{(44)}}\right)\right]ds_{(44)}=$$

$$\left(1+(a_{44})^{(9)}t\right)G_{45}^{0}+\frac{(a_{44})^{(9)}(\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}}\left(e^{(\hat{M}_{44})^{(9)}t}-1\right)$$

From which it follows that

$$(G_{44}(t)-G_{44}^{0})e^{-(\hat{M}_{44})^{(9)}t}\leq\frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}}\left[((\hat{P}_{44})^{(9)}+G_{45}^{0})e^{\left(-\frac{(\hat{P}_{44})^{(9)}+G_{45}^{0}}{G_{45}^{0}}\right)}+(\hat{P}_{44})^{(9)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 9

Analogous inequalities hold also for $G_{45}, G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}<1$ and to choose 182

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[(\hat{P}_{13})^{(1)}+((\hat{P}_{13})^{(1)}+G_j^0)e^{-\left(\frac{(\hat{P}_{13})^{(1)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{13})^{(1)} \quad 183$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[((\hat{Q}_{13})^{(1)}+T_j^0)e^{-\left(\frac{(\hat{Q}_{13})^{(1)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{13})^{(1)}\right]\leq(\hat{Q}_{13})^{(1)} \quad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left((G^{(1)},T^{(1)}),(G^{(2)},T^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\hat{M}_{13})^{(1)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\hat{M}_{13})^{(1)}t}\}$$

Indeed if we denote

**Definition of** $\tilde{G},\tilde{T}:(\tilde{G},\tilde{T})=\mathcal{A}^{(1)}(G,T)$

It results

$$\left|\tilde{G}_{13}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{13})^{(1)} \left|G_{14}^{(1)}-G_{14}^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)} s_{(13)}} e^{(\widehat{M}_{13})^{(1)} s_{(13)}}\, ds_{(13)} +$$

$$\int_0^t \{(a'_{13})^{(1)} \left|G_{13}^{(1)}-G_{13}^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)} s_{(13)}} e^{-(\widehat{M}_{13})^{(1)} s_{(13)}} +$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)}, s_{(13)}\right)\left|G_{13}^{(1)}-G_{13}^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)} s_{(13)}} e^{(\widehat{M}_{13})^{(1)} s_{(13)}} +$$

$$G_{13}^{(2)} \left|(a''_{13})^{(1)}\left(T_{14}^{(1)}, s_{(13)}\right)-(a''_{13})^{(1)}\left(T_{14}^{(2)}, s_{(13)}\right)\right|\ e^{-(\widehat{M}_{13})^{(1)} s_{(13)}} e^{(\widehat{M}_{13})^{(1)} s_{(13)}}\} ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\left|G^{(1)}-G^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)} t} \leq \qquad 186$$
$$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)} + (a'_{13})^{(1)} + (\widehat{A}_{13})^{(1)} + (\widehat{P}_{13})^{(1)} (\widehat{k}_{13})^{(1)}\right) d\left(\left(G^{(1)}, T^{(1)};\ G^{(2)}, T^{(2)}\right)\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 1:** The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)} t}$ *and* $(\widehat{Q}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)} t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i = 13,14,15$ depend only on $T_{14}$ and respectively on $G(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any $t$ where $G_i(t) = 0$ *and* $T_i(t) = 0$

From 19 to 24 it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(1)} - (a_i'')^{(1)}(T_{14}(s_{(13)}), s_{(13)})\} ds_{(13)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(1)} t)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2$ *and* $\left((\widehat{M}_{13})^{(1)}\right)_3$ : 187

**Remark 3**: if $G_{13}$ is bounded, the same property have also $G_{14}$ *and* $G_{15}$ . indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a'_{14})^{(1)} G_{14}$ and by integrating

$$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1 / (a'_{14})^{(1)}$$

In the same way , one can obtain

$$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2 / (a'_{15})^{(1)}$$

If $G_{14}$ *or* $G_{15}$ is bounded, the same property follows for $G_{13}$ , $G_{15}$ and $G_{13}$ , $G_{14}$ respectively.

**Remark 4:** If $G_{13}$ *is* bounded, from below, the same property holds for $G_{14}$ *and* $G_{15}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{14}$ is bounded from below. 188

**Remark 5:** If $T_{13}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}(G(t),t)) = (b_{14}')^{(1)}$ then $T_{14} \to \infty$. 189

**Definition of** $(m)^{(1)}$ and $\varepsilon_1$ :

Indeed let $t_1$ be so that for $t > t_1$

$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t),t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\frac{2}{\varepsilon_1}$ By taking now $\varepsilon_1$ sufficiently small one sees that $T_{14}$ is unbounded.
The same property holds for $T_{15}$ if $\lim_{t\to\infty}(b_{15}'')^{(1)}(G(t),t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\tilde{M}_{16})^{(2)}}$ , $\frac{(b_i)^{(2)}}{(\tilde{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}$ *and* $(\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\tilde{M}_{16})^{(2)}}\left[(\hat{P}_{16})^{(2)} + \left((\hat{P}_{16})^{(2)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\tilde{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{16})^{(2)}\right] \leq (\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\tilde{M}_{16})^{(2)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\tilde{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

**Definition of** $\widetilde{G_{19}}, \widetilde{T_{19}}$ : $\left(\widetilde{G_{19}}, \widetilde{T_{19}}\right) = \mathcal{A}^{(2)}(G_{19}, T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{16})^{(2)} \left|G_{17}^{(1)}-G_{17}^{(2)}\right| e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\, ds_{(16)} +$$

$$\int_0^t \{(a'_{16})^{(2)} \left|G_{16}^{(1)}-G_{16}^{(2)}\right| e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} +$$

$$(a''_{16})^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right)\left|G_{16}^{(1)}-G_{16}^{(2)}\right| e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} e^{(\widehat{M}_{16})^{(2)}s_{(16)}} +$$

$$G_{16}^{(2)} \left|(a''_{16})^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right) - (a''_{16})^{(2)}\left(T_{17}^{(2)}, s_{(16)}\right)\right|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\} ds_{(16)}$$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0, t]$ 197

From the hypotheses it follows

$$\left|(G_{19})^{(1)}-(G_{19})^{(2)}\right| e^{-(\widehat{M}_{16})^{(2)}t} \leq$$
$$\frac{1}{(\widehat{M}_{16})^{(2)}}\left((a_{16})^{(2)} + (a'_{16})^{(2)} + (\widehat{A}_{16})^{(2)} + (\widehat{P}_{16})^{(2)}(\widehat{k}_{16})^{(2)}\right) d\left(((G_{19})^{(1)}, (T_{19})^{(1)};\ (G_{19})^{(2)}, (T_{19})^{(2)})\right)$$

And analogous inequalities for $G_i$ and $T_i$. Taking into account the hypothesis the result follows 198

**Remark 6:** The fact that we supposed $(a''_{16})^{(2)}$ and $(b''_{16})^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{16})^{(2)} e^{(\widehat{M}_{16})^{(2)}t}$ and $(\widehat{Q}_{16})^{(2)} e^{(\widehat{M}_{16})^{(2)}t}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i = 16,17,18$ depend only on $T_{17}$ and respectively on $(G_{19})$(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 7:** There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$ 200

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(2)} - (a_i'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})\} ds_{(16)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(2)}t)} > 0 \quad \text{for } t > 0$$

**Definition of** $\left((\widehat{M}_{16})^{(2)}\right)_1$, $\left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$: 201

**Remark 8:** if $G_{16}$ is bounded, the same property have also $G_{17}$ and $G_{18}$ . indeed if

$G_{16} < (\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt} \leq \left((\widehat{M}_{16})^{(2)}\right)_1 - (a'_{17})^{(2)} G_{17}$ and by integrating

$$G_{17} \leq \left((\widehat{M}_{16})^{(2)}\right)_2 = G_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1 / (a'_{17})^{(2)}$$

In the same way , one can obtain

$$G_{18} \leq \left((\widehat{M}_{16})^{(2)}\right)_3 = G_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2 / (a'_{18})^{(2)}$$

If $G_{17}$ or $G_{18}$ is bounded, the same property follows for $G_{16}$, $G_{18}$ and $G_{16}$, $G_{17}$ respectively.

**Remark 9**: If $G_{16}$ is bounded, from below, the same property holds for $G_{17}$ and $G_{18}$. The proof is analogous with the preceding one. An analogous property is true if $G_{17}$ is bounded from below. 202

**Remark 10:** If $T_{16}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t),t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$. 203

**Definition of** $(m)^{(2)}$ and $\varepsilon_2$ :

Indeed let $t_2$ be so that for $t > t_2$

$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t),t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right), \quad t = \log\frac{2}{\varepsilon_2}$ By taking now $\varepsilon_2$ sufficiently small one sees that $T_{17}$ is unbounded. 205
The same property holds for $T_{18}$ if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t),t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\bar{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\bar{M}_{20})^{(3)}} < 1$ and to choose 207

$(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\bar{M}_{20})^{(3)}}\left[(\hat{P}_{20})^{(3)} + \left((\hat{P}_{20})^{(3)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\bar{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{20})^{(3)}\right] \leq (\hat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself 210

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)},(T_{23})^{(1)}),((G_{23})^{(2)},(T_{23})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\bar{M}_{20})^{(3)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\bar{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

**Definition of** $\widetilde{G_{23}}, \widetilde{T_{23}}$ :$\left(\widetilde{(G_{23})},\widetilde{(T_{23})}\right) = \mathcal{A}^{(3)}\left((G_{23}),(T_{23})\right)$

It results 213

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_{0}^{t}(a_{20})^{(3)}\left|G_{21}^{(1)}-G_{21}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\, ds_{(20)} +$$

$$\int_{0}^{t}\{(a'_{20})^{(3)}\left|G_{20}^{(1)}-G_{20}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} +$$

$$(a''_{20})^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{(\widehat{M}_{20})^{(3)}s_{(20)}} +$$

$$G_{20}^{(2)}\left|(a''_{20})^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)}, s_{(20)}\right)\right|\ e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\} ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}t} \leq \qquad 214$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+\ (a'_{20})^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\widehat{k}_{20})^{(3)}\right) d\left(\left((G_{23})^{(1)},(T_{23})^{(1)};\ (G_{23})^{(2)},(T_{23})^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

**Remark 11:** The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}\ and\ (\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_{+}$. 215

If instead of proving the existence of the solution on $\mathbb{R}_{+}$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i = 20,21,22$ depend only on $T_{21}$ and respectively on $(G_{23})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 12:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 216

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t\{(a_i')^{(3)}-(a_i'')^{(3)}(T_{21}(s_{(20)}),s_{(20)})\}ds_{(20)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(3)}t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2\ and\ \left((\widehat{M}_{20})^{(3)}\right)_3$ : 217

**Remark 13:** if $G_{20}$ is bounded, the same property have also $G_{21}\ and\ G_{22}$ . indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a'_{21})^{(3)}G_{21}$ and by integrating

$$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1/(a'_{21})^{(3)}$$

In the same way , one can obtain

$$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2/(a'_{22})^{(3)}$$

If $G_{21}$ *or* $G_{22}$ is bounded, the same property follows for $G_{20}$ , $G_{22}$ and $G_{20}$ , $G_{21}$ respectively.

**Remark 14:** If $G_{20}$ *is* bounded, from below, the same property holds for $G_{21}$ *and* $G_{22}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{21}$ is bounded from below. 218

**Remark 15:** If $T_{20}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t),t\right)=(b_{21}')^{(3)}$ then $T_{21}\to\infty$. 219

**Definition of** $(m)^{(3)}$ and $\varepsilon_3$ :

Indeed let $t_3$ be so that for $t > t_3$

$(b_{21})^{(3)}-(b_i'')^{(3)}\left((G_{23})(t),t\right)<\varepsilon_3, T_{20}(t)>(m)^{(3)}$

Then $\frac{dT_{21}}{dt}\geq(a_{21})^{(3)}(m)^{(3)}-\varepsilon_3 T_{21}$ which leads to 220

$T_{21}\geq\left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1-e^{-\varepsilon_3 t})+T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t}=\frac{1}{2}$ it results

$T_{21}\geq\left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right),\quad t=log\frac{2}{\varepsilon_3}$ By taking now $\varepsilon_3$ sufficiently small one sees that $T_{21}$ is unbounded.
The same property holds for $T_{22}$ if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t),t\right)=(b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}$ , $\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}}<1$ and to choose 221

$(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\left[(\hat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \quad 222$$

$$\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \quad 223$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself 224

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric 225

$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{24})^{(4)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{27}}),(\widetilde{T_{27}})$ : $\left((\widetilde{G_{27}}),(\widetilde{T_{27}})\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{24})^{(4)} \left|G_{25}^{(1)}-G_{25}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)}s_{(24)}} e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\, ds_{(24)} +$$

$$\int_0^t \{(a'_{24})^{(4)}\left|G_{24}^{(1)}-G_{24}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)}s_{(24)}} e^{-(\widehat{M}_{24})^{(4)}s_{(24)}} +$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)}, s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)}s_{(24)}} e^{(\widehat{M}_{24})^{(4)}s_{(24)}} +$$

$$G_{24}^{(2)}\left|(a''_{24})^{(4)}\left(T_{25}^{(1)}, s_{(24)}\right)-(a''_{24})^{(4)}\left(T_{25}^{(2)}, s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)}s_{(24)}} e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\} ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)}t} \leq \qquad (226)$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)}+\ (a'_{24})^{(4)}+(\widehat{A}_{24})^{(4)}+(\widehat{P}_{24})^{(4)}(\widehat{k}_{24})^{(4)}\right) d\left(\left((G_{27})^{(1)}, (T_{27})^{(1)};\ (G_{27})^{(2)}, (T_{27})^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

**Remark 16:** The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}\ and\ (\widehat{Q}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_+$. (227)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i = 24,25,26$ depend only on $T_{25}$ and respectively on $(G_{27})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 17:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ (228)

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(4)}-(a_i'')^{(4)}(T_{25}(s_{(24)}),s_{(24)})\}ds_{(24)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(4)}t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2\ and\ \left((\widehat{M}_{24})^{(4)}\right)_3$ : (229)

**Remark 18:** if $G_{24}$ is bounded, the same property have also $G_{25}\ and\ G_{26}$ . indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a'_{25})^{(4)}G_{25}$ and by integrating

$$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1/(a'_{25})^{(4)}$$

In the same way , one can obtain

$$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2/(a'_{26})^{(4)}$$

If $G_{25}$ *or* $G_{26}$ is bounded, the same property follows for $G_{24}$ , $G_{26}$ and $G_{24}$ , $G_{25}$ respectively.

**Remark 19:** If $G_{24}$ *is* bounded, from below, the same property holds for $G_{25}$ *and* $G_{26}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{25}$ is bounded from below. 230

**Remark 20:** If $T_{24}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}((G_{27})(t),t)) = (b_{25}')^{(4)}$ then $T_{25} \to \infty$. 231

**Definition of** $(m)^{(4)}$ and $\varepsilon_4$ :

Indeed let $t_4$ be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t),t) < \varepsilon_4, T_{24}(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right)$, $t = log\frac{2}{\varepsilon_4}$ By taking now $\varepsilon_4$ sufficiently small one sees that $T_{25}$ is unbounded.
The same property holds for $T_{26}$ if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t),t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for $G_{29}, G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\bar{M}_{28})^{(5)}}$ , $\frac{(b_i)^{(5)}}{(\bar{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\bar{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)} + \left((\hat{P}_{28})^{(5)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\bar{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{28})^{(5)}\right] \leq (\hat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$$d\left(((G_{31})^{(1)},(T_{31})^{(1)}),((G_{31})^{(2)},(T_{31})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\bar{M}_{28})^{(5)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\bar{M}_{28})^{(5)}t}\}$$

Indeed if we denote

**Definition of** $\widetilde{(G_{31})}, \widetilde{(T_{31})}$ : $\left(\widetilde{(G_{31})}, \widetilde{(T_{31})}\right) = \mathcal{A}^{(5)}\left((G_{31}), (T_{31})\right)$

It results

$\left|\tilde{G}_{28}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{28})^{(5)} \left|G_{29}^{(1)} - G_{29}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}}\, ds_{(28)} +$

$\int_0^t \{(a'_{28})^{(5)} \left|G_{28}^{(1)} - G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} +$

$(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right)\left|G_{28}^{(1)} - G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}} +$

$G_{28}^{(2)} \left|(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right) - (a''_{28})^{(5)}\left(T_{29}^{(2)}, s_{(28)}\right)\right|\ e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}} \} ds_{(28)}$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$\left|(G_{31})^{(1)} - (G_{31})^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} t} \leq$ 237

$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)} + (a'_{28})^{(5)} + (\widehat{A}_{28})^{(5)} + (\widehat{P}_{28})^{(5)} (\widehat{k}_{28})^{(5)}\right) d\left(\left((G_{31})^{(1)}, (T_{31})^{(1)};\ (G_{31})^{(2)}, (T_{31})^{(2)}\right)\right)$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 21:** The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$ *and* $(\widehat{Q}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$ respectively of $\mathbb{R}_+$. 238

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on $T_{29}$ and respectively on $(G_{31})$(*and not on* $t$) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 22:** There does not exist any $t$ where $G_i(t) = 0$ *and* $T_i(t) = 0$ 239

it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(5)} - (a_i'')^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{(-(b_i')^{(5)} t)} > 0$ for t > 0

**Definition of** $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2$ *and* $\left((\widehat{M}_{28})^{(5)}\right)_3$ : 240

**Remark 23:** if $G_{28}$ is bounded, the same property have also $G_{29}$ *and* $G_{30}$ . indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a'_{29})^{(5)} G_{29}$ and by integrating

$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1 / (a'_{29})^{(5)}$

In the same way , one can obtain

$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2/(a'_{30})^{(5)}$

If $G_{29}$ *or* $G_{30}$ is bounded, the same property follows for $G_{28}$, $G_{30}$ and $G_{28}$, $G_{29}$ respectively.

**Remark 24:** If $G_{28}$ *is* bounded, from below, the same property holds for $G_{29}$ *and* $G_{30}$. The proof is analogous with the preceding one. An analogous property is true if $G_{29}$ is bounded from below. 241

**Remark 25:** If $T_{28}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}((G_{31})(t),t)) = (b'_{29})^{(5)}$ then $T_{29} \to \infty$. 242

**Definition of** $(m)^{(5)}$ and $\varepsilon_5$ :

Indeed let $t_5$ be so that for $t > t_5$

$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t),t) < \varepsilon_5, T_{28}(t) > (m)^{(5)}$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\frac{2}{\varepsilon_5}$ By taking now $\varepsilon_5$ sufficiently small one sees that $T_{29}$ is unbounded.
The same property holds for $T_{30}$ if $\lim_{t\to\infty}(b_{30}'')^{(5)}\left((G_{31})(t),t\right) = (b'_{30})^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{33}, G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}$, $\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$ and to choose 244

$(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[(\hat{P}_{32})^{(6)} + \left((\hat{P}_{32})^{(6)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{32})^{(6)} \quad 245$$

$$\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[\left((\hat{Q}_{32})^{(6)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{32})^{(6)}\right] \leq (\hat{Q}_{32})^{(6)} \quad 246$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric 247

$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right) =$

$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{32})^{(6)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{32})^{(6)}t}\}$

Indeed if we denote

**Definition of** $\widetilde{(G_{35})}, \widetilde{(T_{35})}$ : $\left(\widetilde{(G_{35})}, \widetilde{(T_{35})}\right) = \mathcal{A}^{(6)}\left((G_{35}), (T_{35})\right)$

It results

$$\left|\tilde{G}_{32}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{32})^{(6)} \left|G_{33}^{(1)} - G_{33}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}}\, ds_{(32)} +$$

$$\int_0^t \{(a'_{32})^{(6)} \left|G_{32}^{(1)} - G_{32}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} +$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)}, s_{(32)}\right)\left|G_{32}^{(1)} - G_{32}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}} +$$

$$G_{32}^{(2)} \left|(a''_{32})^{(6)}\left(T_{33}^{(1)}, s_{(32)}\right) - (a''_{32})^{(6)}\left(T_{33}^{(2)}, s_{(32)}\right)\right| \; e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}} \} ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|(G_{35})^{(1)} - (G_{35})^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} t} \leq \qquad (248)$$
$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)} + (a'_{32})^{(6)} + (\widehat{A}_{32})^{(6)} + (\widehat{P}_{32})^{(6)} (\widehat{k}_{32})^{(6)}\right) d\left(\left((G_{35})^{(1)}, (T_{35})^{(1)}; \; (G_{35})^{(2)}, (T_{35})^{(2)}\right)\right)$$

And analogous inequalities for $G_i \; and \; T_i$. Taking into account the hypothesis the result follows

**Remark 26:** The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)} t} \; and \; (\widehat{Q}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)} t}$ respectively of $\mathbb{R}_+$. (249)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i = 32{,}33{,}34$ depend only on $T_{33}$ and respectively on $(G_{35})(and \; not \; on \; t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 27:** There does not exist any $t$ where $G_i\,(t) = 0 \; and \; T_i\,(t) = 0$ (250)

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(6)} - (a_i'')^{(6)}(T_{33}(s_{(32)}), s_{(32)})\} ds_{(32)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(6)} t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{32})^{(6)}\right)_1, \left((\widehat{M}_{32})^{(6)}\right)_2 \; and \; \left((\widehat{M}_{32})^{(6)}\right)_3$ : (251)

**Remark 28:** if $G_{32}$ is bounded, the same property have also $G_{33} \; and \; G_{34}$ . indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a'_{33})^{(6)} G_{33}$ and by integrating

$$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_1 / (a'_{33})^{(6)}$$

In the same way , one can obtain

$$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_2/(a'_{34})^{(6)}$$

If $G_{33}$ *or* $G_{34}$ is bounded, the same property follows for $G_{32}$ , $G_{34}$ and $G_{32}$ , $G_{33}$ respectively.

**Remark 29:** If $G_{32}$ *is* bounded, from below, the same property holds for $G_{33}$ *and* $G_{34}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{33}$ is bounded from below. 252

**Remark 30:** If $T_{32}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}((G_{35})(t),t)) = (b'_{33})^{(6)}$ then $T_{33} \to \infty$. 253

**Definition of** $(m)^{(6)}$ and $\varepsilon_6$ :

Indeed let $t_6$ be so that for $t > t_6$

$$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t),t\right) < \varepsilon_6, T_{32}(t) > (m)^{(6)}$$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right)$, $t = log\frac{2}{\varepsilon_6}$ By taking now $\varepsilon_6$ sufficiently small one sees that $T_{33}$ is unbounded.
The same property holds for $T_{34}$ if $\lim_{t\to\infty}(b''_{34})^{(6)}\left((G_{35})(t),t(t),t\right) = (b'_{34})^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{37}$ , $G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}$ , $\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}} < 1$ and to choose
$(\widehat{P}_{36})^{(7)}$ and $(\widehat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[(\widehat{P}_{36})^{(7)} + \left((\widehat{P}_{36})^{(7)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{36})^{(7)} \quad 256$$

$$\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[\left((\widehat{Q}_{36})^{(7)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{36})^{(7)}\right] \leq (\widehat{Q}_{36})^{(7)} \quad 257$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself
The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric 258

$$d\left(\left((G_{39})^{(1)},(T_{39})^{(1)}\right),\left((G_{39})^{(2)},(T_{39})^{(2)}\right)\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\tilde{M}_{36})^{(7)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\tilde{M}_{36})^{(7)}t}\}$$

Indeed if we denote

**Definition of** $\widetilde{(G_{39})},\widetilde{(T_{39})}:\left(\widetilde{(G_{39})},\widetilde{(T_{39})}\right)=\mathcal{A}^{(7)}((G_{39}),(T_{39}))$
It results

$$\left|\tilde{G}_{36}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{36})^{(7)}\left|G_{37}^{(1)}-G_{37}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\,ds_{(36)}+$$
$$\int_0^t\{(a'_{36})^{(7)}\left|G_{36}^{(1)}-G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$(a''_{36})^{(7)}\left(T_{37}^{(1)},s_{(36)}\right)\left|G_{36}^{(1)}-G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$G_{36}^{(2)}\left|(a''_{36})^{(7)}\left(T_{37}^{(1)},s_{(36)}\right)-(a''_{36})^{(7)}\left(T_{37}^{(2)},s_{(36)}\right)\right|\ e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\}ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses on it follows

$$\left|(G_{39})^{(1)}-(G_{39})^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}t}\le \qquad 259$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}}\left((a_{36})^{(7)}+(a'_{36})^{(7)}+(\widehat{A}_{36})^{(7)}+(\widehat{P}_{36})^{(7)}(\widehat{k}_{36})^{(7)}\right)d\left(((G_{39})^{(1)},(T_{39})^{(1)};\ (G_{39})^{(2)},(T_{39})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

**Remark 31:** The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}\ and\ (\widehat{Q}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ respectively of $\mathbb{R}_+$. 260

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(7)}$ and $(b_i'')^{(7)},i=36{,}37{,}38$ depend only on $\mathrm{T}_{37}$ and respectively on $(G_{39})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 32:** There does not exist any $t$ where $G_i\,(t)=0\ and\ T_i\,(t)=0$ 261

it results

$$G_i\,(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(7)}-(a_i'')^{(7)}(T_{37}(s_{(36)}),s_{(36)})\}ds_{(36)}\right]}\ge 0$$

$$T_i\,(t)\ge T_i^0e^{\left(-(b_i')^{(7)}t\right)}>0\quad \text{for t}>0$$

**Definition of** $\left((\widehat{M}_{36})^{(7)}\right)_1,\left((\widehat{M}_{36})^{(7)}\right)_2\ and\ \left((\widehat{M}_{36})^{(7)}\right)_3:$ 262

**Remark 33:** if $G_{36}$ is bounded, the same property have also $G_{37}\ and\ G_{38}$ . indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a'_{37})^{(7)} G_{37}$ and by integrating

$$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1/(a'_{37})^{(7)}$$

In the same way , one can obtain

$$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2/(a'_{38})^{(7)}$$

If $G_{37}$ *or* $G_{38}$ is bounded, the same property follows for $G_{36}$ , $G_{38}$ and $G_{36}$ , $G_{37}$ respectively.

**Remark 34:** If $G_{36}$ *is* bounded, from below, the same property holds for $G_{37}$ *and* $G_{38}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{37}$ is bounded from below. 263

**Remark 35:** If $T_{36}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(7)}((G_{39})(t),t)) = (b'_{37})^{(7)}$ then $T_{37} \to \infty$. 264

**Definition of** $(m)^{(7)}$ and $\varepsilon_7$ :

Indeed let $t_7$ be so that for $t > t_7$

$$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t),t) < \varepsilon_7, T_{36}(t) > (m)^{(7)}$$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right)$, $t = log\frac{2}{\varepsilon_7}$ By taking now $\varepsilon_7$ sufficiently small one sees that $T_{37}$ is unbounded.
The same property holds for $T_{38}$ if $\lim_{t\to\infty}(b''_{38})^{(7)}\left((G_{39})(t),t\right) = (b'_{38})^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}$ , $\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}} < 1$ and to choose 266
$(\widehat{P}_{40})^{(8)}$ and $(\widehat{Q}_{40})^{(8)}$ large to have

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[(\widehat{P}_{40})^{(8)} + \left((\widehat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{40})^{(8)} \quad 267$$

$$\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[\left((\widehat{Q}_{40})^{(8)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{40})^{(8)}\right] \leq (\widehat{Q}_{40})^{(8)} \quad 268$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)},(T_{43})^{(1)}),((G_{43})^{(2)},(T_{43})^{(2)})\right)= \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

**Definition of** $\widetilde{(G_{43})},\widetilde{(T_{43})}$ : $\left(\widetilde{(G_{43})},\widetilde{(T_{43})}\right)=\mathcal{A}^{(8)}((G_{43}),(T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{40})^{(8)}\left|G_{41}^{(1)}-G_{41}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\,ds_{(40)}+$$
$$\int_0^t\{(a'_{40})^{(8)}\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$G_{40}^{(2)}\left|(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)-(a''_{40})^{(8)}\left(T_{41}^{(2)},s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\}ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0,t]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)}-(G_{43})^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}t}\le \quad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)}+(a'_{40})^{(8)}+(\widehat{A}_{40})^{(8)}+(\widehat{P}_{40})^{(8)}(\widehat{k}_{40})^{(8)}\right)d\left(((G_{43})^{(1)},(T_{43})^{(1)};\ (G_{43})^{(2)},(T_{43})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

**Remark 36:** The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}\ and\ (\widehat{Q}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ respectively of $\mathbb{R}_+$. 274

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i=40,41,42$ depend only on $T_{41}$ and respectively on $(G_{43})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 37** There does not exist any $t$ where $G_i\,(t)=0\ and\ T_i\,(t)=0$ 275

it results

$$G_i\,(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(8)}-(a_i'')^{(8)}(T_{41}(s_{(40)}),s_{(40)})\}ds_{(40)}\right]}\ge 0$$

$$T_i\,(t)\ge T_i^0e^{\left(-(b_i')^{(8)}t\right)}>0\quad \text{for t}>0$$

**Definition of** $\left((\widehat{M}_{40})^{(8)}\right)_1, \left((\widehat{M}_{40})^{(8)}\right)_2$ *and* $\left((\widehat{M}_{40})^{(8)}\right)_3$ : 276

**Remark 38:** if $G_{40}$ is bounded, the same property have also $G_{41}$ *and* $G_{42}$ . indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a'_{41})^{(8)} G_{41}$ and by integrating

$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1/(a'_{41})^{(8)}$

In the same way , one can obtain

$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2/(a'_{42})^{(8)}$

If $G_{41}$ *or* $G_{42}$ is bounded, the same property follows for $G_{40}$ , $G_{42}$ and $G_{40}$ , $G_{41}$ respectively.

**Remark 39:** If $G_{40}$ *is* bounded, from below, the same property holds for $G_{41}$ *and* $G_{42}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{41}$ is bounded from below. 277

**Remark 40:** If $T_{40}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}((G_{43})(t),t)) = (b'_{41})^{(8)}$ then $T_{41} \to \infty$. 278

**Definition of** $(m)^{(8)}$ and $\varepsilon_8$ :

Indeed let $t_8$ be so that for $t > t_8$

$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t),t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right), \quad t = log\frac{2}{\varepsilon_8}$ By taking now $\varepsilon_8$ sufficiently small one sees that $T_{41}$ is unbounded.
The same property holds for $T_{42}$ if $\lim_{t\to\infty}(b''_{42})^{(8)}\left((G_{43})(t),t(t),t\right) = (b'_{42})^{(8)}$

It is now sufficient to take $\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}$ , $\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}} < 1$ and to choose $(\widehat{P}_{44})^{(9)}$ and $(\widehat{Q}_{44})^{(9)}$ large to have 279 A

$$\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[(\widehat{P}_{44})^{(9)} + \left((\widehat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[\left((\widehat{Q}_{44})^{(9)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{44})^{(9)}\right] \leq (\widehat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)},(T_{47})^{(1)}),((G_{47})^{(2)},(T_{47})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{47}}),(\widetilde{T_{47}}):\left((\widetilde{G_{47}}),(\widetilde{T_{47}})\right)=\mathcal{A}^{(9)}\left((G_{47}),(T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{44})^{(9)}\left|G_{45}^{(1)}-G_{45}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\,ds_{(44)}+$$
$$\int_0^t\{(a'_{44})^{(9)}\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$G_{44}^{(2)}\left|(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)-(a''_{44})^{(9)}\left(T_{45}^{(2)},s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\}ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}t}\le$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)}+(a'_{44})^{(9)}+(\widehat{A}_{44})^{(9)}+(\widehat{P}_{44})^{(9)}(\widehat{k}_{44})^{(9)}\right)d\left(((G_{47})^{(1)},(T_{47})^{(1)};\ (G_{47})^{(2)},(T_{47})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis (39,35,36) the result follows

**Remark 41:** The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}\ and\ (\widehat{Q}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i=44,45,46$ depend only on $T_{45}$ and respectively on $(G_{47})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 42:** There does not exist any $t$ where $G_i\,(t)=0\ and\ T_i\,(t)=0$

From 99 to 44 it results

$$G_i\,(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(9)}-(a_i'')^{(9)}(T_{45}(s_{(44)}),s_{(44)})\}ds_{(44)}\right]}\ge 0$$

$$T_i\,(t)\ge T_i^0e^{(-(b_i')^{(9)}t)}>0\quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{44})^{(9)}\right)_1,\left((\widehat{M}_{44})^{(9)}\right)_2\ and\ \left((\widehat{M}_{44})^{(9)}\right)_3$ :

**Remark 43:** if $G_{44}$ is bounded, the same property have also $G_{45}\ and\ G_{46}$ . indeed if

$G_{44} < (\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt} \leq \left((\widehat{M}_{44})^{(9)}\right)_1 - (a'_{45})^{(9)} G_{45}$ and by integrating

$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_1 / (a'_{45})^{(9)}$

In the same way , one can obtain

$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_2 / (a'_{46})^{(9)}$

If $G_{45}$ *or* $G_{46}$ is bounded, the same property follows for $G_{44}$ , $G_{46}$ and $G_{44}$ , $G_{45}$ respectively.

**Remark 44:** If $G_{44}$ *is* bounded, from below, the same property holds for $G_{45}$ *and* $G_{46}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{45}$ is bounded from below.

**Remark 45:** If $T_{44}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t),t\right) = (b'_{45})^{(9)}$ then $T_{45} \to \infty$.

**Definition of** $(m)^{(9)}$ and $\varepsilon_9$ :

Indeed let $t_9$ be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t),t\right) < \varepsilon_9, T_{44}(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right)$, $t = log\frac{2}{\varepsilon_9}$ By taking now $\varepsilon_9$ sufficiently small one sees that $T_{45}$ is unbounded.
The same property holds for $T_{46}$ if $\lim_{t\to\infty}(b''_{46})^{(9)}\left((G_{47})(t),t\right) = (b'_{46})^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

**Behavior of the solutions of equation** 280

**Theorem** If we denote and define

**Definition of** $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ :

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)} \leq -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14},t) + (a''_{14})^{(1)}(T_{14},t) \leq -(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)} \leq -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G,t) - (b''_{14})^{(1)}(G,t) \leq -(\tau_1)^{(1)}$

**Definition of** $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$ : 281

By $(\nu_1)^{(1)} > 0$ , $(\nu_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$ , $(u_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

**Definition of** $(\bar{\nu}_1)^{(1)}, , (\bar{\nu}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$ : 282

By $(\bar{\nu}_1)^{(1)} > 0$ , $(\bar{\nu}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$ , $(\bar{u}_2)^{(1)} < 0$ the roots of the equations

$(a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

**Definition of** $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (v_0)^{(1)}$ :- 283

If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by

$(m_2)^{(1)} = (v_0)^{(1)}, (m_1)^{(1)} = (v_1)^{(1)}, \ if\ (v_0)^{(1)} < (v_1)^{(1)}$

$(m_2)^{(1)} = (v_1)^{(1)}, (m_1)^{(1)} = (\bar{v}_1)^{(1)}, if\ (v_1)^{(1)} < (v_0)^{(1)} < (\bar{v}_1)^{(1)},$

and $\boxed{(v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$

$(m_2)^{(1)} = (v_1)^{(1)}, (m_1)^{(1)} = (v_0)^{(1)}, \ if\ (\bar{v}_1)^{(1)} < (v_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)}, \ if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)},$

and $\boxed{(u_0)^{(1)} = \frac{T_{13}^0}{T_{14}^0}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$

are defined

Then the solution of global equations satisfies the inequalities 285

$$G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{13}(t) \le G_{13}^0 e^{(S_1)^{(1)}t}$$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)}t}$$

$$\left(\frac{(a_{15})^{(1)} G_{13}^0}{(m_1)^{(1)}((S_1)^{(1)} - (p_{13})^{(1)} - (S_2)^{(1)})}\left[e^{((S_1)^{(1)} - (p_{13})^{(1)})t} - e^{-(S_2)^{(1)}t}\right] + G_{15}^0 e^{-(S_2)^{(1)}t} \le G_{15}(t) \le \right. \quad 286$$

$$\left. \frac{(a_{15})^{(1)} G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)} - (a'_{15})^{(1)})}\left[e^{(S_1)^{(1)}t} - e^{-(a'_{15})^{(1)}t}\right] + G_{15}^0 e^{-(a'_{15})^{(1)}t}\right)$$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)}t} \le T_{13}(t) \le T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t}} \quad 287$$

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)}t} \le T_{13}(t) \le \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t} \quad 288$$

$$\frac{(b_{15})^{(1)} T_{13}^0}{(\mu_1)^{(1)}((R_1)^{(1)} - (b'_{15})^{(1)})}\left[e^{(R_1)^{(1)}t} - e^{-(b'_{15})^{(1)}t}\right] + T_{15}^0 e^{-(b'_{15})^{(1)}t} \le T_{15}(t) \le \quad 289$$

$$\frac{(a_{15})^{(1)} T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)} + (r_{13})^{(1)} + (R_2)^{(1)})}\left[e^{((R_1)^{(1)} + (r_{13})^{(1)})t} - e^{-(R_2)^{(1)}t}\right] + T_{15}^0 e^{-(R_2)^{(1)}t}$$

**Definition of** $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:- 290

Where $(S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$

$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$

$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$

$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$

**Behavior of the solutions of equation** 291

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ : 292

$(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ four constants satisfying
$-(\sigma_2)^{(2)} \le -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(T_{17}, t) + (a''_{17})^{(2)}(T_{17}, t) \le -(\sigma_1)^{(2)}$ 293

$-(\tau_2)^{(2)} \le -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}((G_{19}), t) - (b''_{17})^{(2)}((G_{19}), t) \le -(\tau_1)^{(2)}$ 294

**Definition of** $(\nu_1)^{(2)}, (\nu_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$ : 295

By $(\nu_1)^{(2)} > 0$ , $(\nu_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0$ , $(u_2)^{(2)} < 0$ the roots 296

of the equations $(a_{17})^{(2)}(\nu^{(2)})^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 297

and $(b_{14})^{(2)}(u^{(2)})^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and 298

**Definition of** $(\bar{\nu}_1)^{(2)},, (\bar{\nu}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$ : 299

By $(\bar{\nu}_1)^{(2)} > 0$ , $(\bar{\nu}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0$ , $(\bar{u}_2)^{(2)} < 0$ the 300

roots of the equations $(a_{17})^{(2)}(\nu^{(2)})^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 301

and $(b_{17})^{(2)}(u^{(2)})^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 302

**Definition of** $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ :- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)},\ \boldsymbol{if}\ (u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}, \boldsymbol{if}\ (u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)},$

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)},\ \boldsymbol{if}\ (\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)} t}$

$(p_i)^{(2)}$ is defined by equation

$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)} t}$ 311

$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)} t}\right] + G_{18}^0 e^{-(S_2)^{(2)} t} \le G_{18}(t) \le\right.$ 312

$\left.\frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)} t} - e^{-(a'_{18})^{(2)} t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)} t}\right)$

$\boxed{T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}}$ 313

$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}$ 314

$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)} t} - e^{-(b'_{18})^{(2)} t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)} t} \le T_{18}(t) \le$ 315

$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})}\left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)} t}\right] + T_{18}^0 e^{-(R_2)^{(2)} t}$

**Definition of** $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$

$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)}$ 318

$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$

**Behavior of the solutions** 319

**Theorem 3:** If we denote and define

**Definition of** $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ :

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}\big((G_{23}), t\big) \le -(\tau_1)^{(3)}$$

**Definition of** $(\nu_1)^{(3)}, (\nu_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$ : 320

By $(\nu_1)^{(3)} > 0$ , $(\nu_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$ , $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\big(\nu^{(3)}\big)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{\nu}_1)^{(3)} > 0$ , $(\bar{\nu}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$ , $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\big(\nu^{(3)}\big)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

**Definition of** $(m_1)^{(3)}$ , $(m_2)^{(3)}$ , $(\mu_1)^{(3)}, (\mu_2)^{(3)}$ :- 321

If we define $(m_1)^{(3)}$ , $(m_2)^{(3)}$ , $(\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (\nu_0)^{(3)}, (m_1)^{(3)} = (\nu_1)^{(3)}$, ***if*** $(\nu_0)^{(3)} < (\nu_1)^{(3)}$

$(m_2)^{(3)} = (\nu_1)^{(3)}, (m_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ , ***if*** $(\nu_1)^{(3)} < (\nu_0)^{(3)} < (\bar{\nu}_1)^{(3)}$,

and $\boxed{(\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}}$

$(m_2)^{(3)} = (\nu_1)^{(3)}, (m_1)^{(3)} = (\nu_0)^{(3)}$, ***if*** $(\bar{\nu}_1)^{(3)} < (\nu_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}$, ***if*** $(u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$ , ***if*** $(u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}$, and $\boxed{(u_0)^{(3)} = \frac{T^0_{20}}{T^0_{21}}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}$, ***if*** $(\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$$G^0_{20} e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G^0_{20} e^{(S_1)^{(3)} t}$$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}} G^0_{20} e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}} G^0_{20} e^{(S_1)^{(3)} t} \quad 323$$

$$\Big(\frac{(a_{22})^{(3)} G^0_{20}}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\Big[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)} t}\Big] + G^0_{22} e^{-(S_2)^{(3)} t} \le G_{22}(t) \le \quad 324$$

$$\frac{(a_{22})^{(3)} G^0_{20}}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}[e^{(S_1)^{(3)} t} - e^{-(a'_{22})^{(3)} t}] + G^0_{22} e^{-(a'_{22})^{(3)} t}\Big)$$

$$\boxed{T_{20}^{0}e^{(R_1)^{(3)}t} \le T_{20}(t) \le T_{20}^{0}e^{((R_1)^{(3)}+(r_{20})^{(3)})t}} \quad 325$$

$$\frac{1}{(\mu_1)^{(3)}}T_{20}^{0}e^{(R_1)^{(3)}t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}}T_{20}^{0}e^{((R_1)^{(3)}+(r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)}T_{20}^{0}}{(\mu_1)^{(3)}((R_1)^{(3)}-(b'_{22})^{(3)})}\left[e^{(R_1)^{(3)}t} - e^{-(b'_{22})^{(3)}t}\right] + T_{22}^{0}e^{-(b'_{22})^{(3)}t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)}T_{20}^{0}}{(\mu_2)^{(3)}((R_1)^{(3)}+(r_{20})^{(3)}+(R_2)^{(3)})}\left[e^{((R_1)^{(3)}+(r_{20})^{(3)})t} - e^{-(R_2)^{(3)}t}\right] + T_{22}^{0}e^{-(R_2)^{(3)}t}$$

**Definition of** $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$$

$$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$$

$$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$$

**Behavior of the solutions of equation**

**Theorem:** If we denote and define

**Definition of** $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ :

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$$

$$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}\left((G_{27}), t\right) - (b''_{25})^{(4)}\left((G_{27}), t\right) \le -(\tau_1)^{(4)}$$

**Definition of** $(\nu_1)^{(4)}, (\nu_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, \nu^{(4)}, u^{(4)}$ : 329

By $(\nu_1)^{(4)} > 0$ , $(\nu_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$ , $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_1)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(4)},, (\bar{\nu}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$ : 330

By $(\bar{\nu}_1)^{(4)} > 0$ , $(\bar{\nu}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$ , $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

**Definition of** $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}, (\nu_0)^{(4)}$ :-

If we define $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$$(m_2)^{(4)} = (\nu_0)^{(4)}, (m_1)^{(4)} = (\nu_1)^{(4)}, \ \boldsymbol{if}\ (\nu_0)^{(4)} < (\nu_1)^{(4)}$$

$$(m_2)^{(4)} = (\nu_1)^{(4)}, (m_1)^{(4)} = (\bar{\nu}_1)^{(4)}, \boldsymbol{if}\ (\nu_4)^{(4)} < (\nu_0)^{(4)} < (\bar{\nu}_1)^{(4)},$$

and $\boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (\nu_4)^{(4)}, (m_1)^{(4)} = (\nu_0)^{(4)},\ \boldsymbol{if}\ (\bar{\nu}_4)^{(4)} < (\nu_0)^{(4)}$

and analogously

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$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)},\ \boldsymbol{if}\ (u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}, \boldsymbol{if}\ (u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)},$

and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}, \boldsymbol{if}\ (\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

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$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)} t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)} t} \quad 333$$

$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)} t}\right] + G_{26}^0 e^{-(S_2)^{(4)} t} \le G_{26}(t) \le \right. \quad 334$$

*(a26)4G240(m2)4(S1)4−(a26′)4e(S1)4t−e−(a26′)4t+ G260e−(a26′)4t*

$$\boxed{T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}} T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}} T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t} \quad 335$$

$$\frac{(b_{26})^{(4)} T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)} - (b'_{26})^{(4)})}\left[e^{(R_1)^{(4)} t} - e^{-(b'_{26})^{(4)} t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)} t} \le T_{26}(t) \le \quad 336$$

$$\frac{(a_{26})^{(4)} T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)} + (r_{24})^{(4)} + (R_2)^{(4)})}\left[e^{((R_1)^{(4)} + (r_{24})^{(4)})t} - e^{-(R_2)^{(4)} t}\right] + T_{26}^0 e^{-(R_2)^{(4)} t}$$

**Definition of** $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

**Behavior of the solutions of equation** 338

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ :

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

**Definition of** $(\nu_1)^{(5)}, (\nu_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, \nu^{(5)}, u^{(5)}$ : 339

By $(\nu_1)^{(5)} > 0$, $(\nu_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$, $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(5)},, (\bar{\nu}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$ : 340

By $(\bar{\nu}_1)^{(5)} > 0$, $(\bar{\nu}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$, $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

**Definition of** $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (\nu_0)^{(5)}$ :-

If we define $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (\nu_0)^{(5)}, (m_1)^{(5)} = (\nu_1)^{(5)}$, ***if*** $(\nu_0)^{(5)} < (\nu_1)^{(5)}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\bar{\nu}_1)^{(5)}$, ***if*** $(\nu_1)^{(5)} < (\nu_0)^{(5)} < (\bar{\nu}_1)^{(5)}$,
and $\boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)}$, ***if*** $(\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)}$, ***if*** $(u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}$, ***if*** $(u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)}$,
and $\boxed{(u_0)^{(5)} = \frac{T^0_{28}}{T^0_{29}}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}$, ***if*** $(\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$$G^0_{28} e^{\left((S_1)^{(5)} - (p_{28})^{(5)}\right)t} \le G_{28}(t) \le G^0_{28} e^{(S_1)^{(5)}t}$$

where $(p_i)^{(5)}$ is defined by equation

$$\frac{1}{(m_5)^{(5)}} G^0_{28} e^{\left((S_1)^{(5)} - (p_{28})^{(5)}\right)t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}} G^0_{28} e^{(S_1)^{(5)}t} \quad 343$$

$$\left(\frac{(a_{30})^{(5)}G_{28}^{0}}{(m_1)^{(5)}((S_1)^{(5)}-(p_{28})^{(5)}-(S_2)^{(5)})}\left[e^{((S_1)^{(5)}-(p_{28})^{(5)})t}-e^{-(S_2)^{(5)}t}\right]+G_{30}^{0}e^{-(S_2)^{(5)}t}\leq G_{30}(t)\leq\right. \quad 344$$

*(a30)5G280(m2)5(S1)5−(a30′)5e(S1)5t−e−(a30′)5t+ G300e−(a30′)5t*

$$\boxed{T_{28}^{0}e^{(R_1)^{(5)}t}\leq T_{28}(t)\leq T_{28}^{0}e^{((R_1)^{(5)}+(r_{28})^{(5)})t}} \quad 345$$

$$\frac{1}{(\mu_1)^{(5)}}T_{28}^{0}e^{(R_1)^{(5)}t}\leq T_{28}(t)\leq\frac{1}{(\mu_2)^{(5)}}T_{28}^{0}e^{((R_1)^{(5)}+(r_{28})^{(5)})t} \quad 346$$

$$\frac{(b_{30})^{(5)}T_{28}^{0}}{(\mu_1)^{(5)}((R_1)^{(5)}-(b'_{30})^{(5)})}\left[e^{(R_1)^{(5)}t}-e^{-(b'_{30})^{(5)}t}\right]+T_{30}^{0}e^{-(b'_{30})^{(5)}t}\leq T_{30}(t)\leq \quad 347$$

$$\frac{(a_{30})^{(5)}T_{28}^{0}}{(\mu_2)^{(5)}((R_1)^{(5)}+(r_{28})^{(5)}+(R_2)^{(5)})}\left[e^{((R_1)^{(5)}+(r_{28})^{(5)})t}-e^{-(R_2)^{(5)}t}\right]+T_{30}^{0}e^{-(R_2)^{(5)}t}$$

**Definition of** $(S_1)^{(5)},(S_2)^{(5)},(R_1)^{(5)},(R_2)^{(5)}$:- 348

Where $(S_1)^{(5)}=(a_{28})^{(5)}(m_2)^{(5)}-(a'_{28})^{(5)}$

$(S_2)^{(5)}=(a_{30})^{(5)}-(p_{30})^{(5)}$

$(R_1)^{(5)}=(b_{28})^{(5)}(\mu_2)^{(5)}-(b'_{28})^{(5)}$

$(R_2)^{(5)}=(b'_{30})^{(5)}-(r_{30})^{(5)}$

**Behavior of the solutions of equation** 349

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(6)},(\sigma_2)^{(6)},(\tau_1)^{(6)},(\tau_2)^{(6)}$ :

$(\sigma_1)^{(6)},(\sigma_2)^{(6)},(\tau_1)^{(6)},(\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)}\leq-(a'_{32})^{(6)}+(a'_{33})^{(6)}-(a''_{32})^{(6)}(T_{33},t)+(a''_{33})^{(6)}(T_{33},t)\leq-(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)}\leq-(b'_{32})^{(6)}+(b'_{33})^{(6)}-(b''_{32})^{(6)}\big((G_{35}),t\big)-(b''_{33})^{(6)}\big((G_{35}),t\big)\leq-(\tau_1)^{(6)}$$

**Definition of** $(v_1)^{(6)},(v_2)^{(6)},(u_1)^{(6)},(u_2)^{(6)},v^{(6)},u^{(6)}$ : 350

By $(v_1)^{(6)}>0$ , $(v_2)^{(6)}<0$ and respectively $(u_1)^{(6)}>0$ , $(u_2)^{(6)}<0$ the roots of the equations $(a_{33})^{(6)}\big(v^{(6)}\big)^2+(\sigma_1)^{(6)}v^{(6)}-(a_{32})^{(6)}=0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2+(\tau_1)^{(6)}u^{(6)}-(b_{32})^{(6)}=0$ and

**Definition of** $(\bar{v}_1)^{(6)},,(\bar{v}_2)^{(6)},(\bar{u}_1)^{(6)},(\bar{u}_2)^{(6)}$ : 351

By $(\bar{v}_1)^{(6)}>0$ , $(\bar{v}_2)^{(6)}<0$ and respectively $(\bar{u}_1)^{(6)}>0$ , $(\bar{u}_2)^{(6)}<0$ the roots of the equations $(a_{33})^{(6)}\big(v^{(6)}\big)^2+(\sigma_2)^{(6)}v^{(6)}-(a_{32})^{(6)}=0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2+(\tau_2)^{(6)}u^{(6)}-(b_{32})^{(6)}=0$

**Definition of** $(m_1)^{(6)},(m_2)^{(6)},(\mu_1)^{(6)},(\mu_2)^{(6)},(v_0)^{(6)}$ :-

If we define $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}$ by

$(m_2)^{(6)} = (\nu_0)^{(6)}, (m_1)^{(6)} = (\nu_1)^{(6)}$, ***if*** $(\nu_0)^{(6)} < (\nu_1)^{(6)}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}$, ***if*** $(\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)}$,

and $\boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}$, ***if*** $(\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$

and analogously 352

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}$, ***if*** $(u_0)^{(6)} < (u_1)^{(6)}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}$, ***if*** $(u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)}$,

and $\boxed{(u_0)^{(6)} = \frac{T_{32}^0}{T_{33}^0}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}$, ***if*** $(\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$

Then the solution of global equations satisfies the inequalities 353

$$G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \leq G_{32}(t) \leq G_{32}^0 e^{(S_1)^{(6)} t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}} G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \leq G_{33}(t) \leq \frac{1}{(m_2)^{(6)}} G_{32}^0 e^{(S_1)^{(6)} t} \quad 354$$

$$\left( \frac{(a_{34})^{(6)} G_{32}^0}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})} \left[ e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)} t} \right] + G_{34}^0 e^{-(S_2)^{(6)} t} \leq G_{34}(t) \leq \right. \quad 355$$

*(a34)6G320(m2)6(S1)6−(a34′)6e(S1)6t−e−(a34′)6t+ G340e−(a34′)6t*

$$\boxed{T_{32}^0 e^{(R_1)^{(6)} t} \leq T_{32}(t) \leq T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}} T_{32}^0 e^{(R_1)^{(6)} t} \leq T_{32}(t) \leq \frac{1}{(\mu_2)^{(6)}} T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)} T_{32}^0}{(\mu_1)^{(6)}((R_1)^{(6)} - (b'_{34})^{(6)})} \left[ e^{(R_1)^{(6)} t} - e^{-(b'_{34})^{(6)} t} \right] + T_{34}^0 e^{-(b'_{34})^{(6)} t} \leq T_{34}(t) \leq \quad 358$$

$$\frac{(a_{34})^{(6)} T_{32}^0}{(\mu_2)^{(6)}((R_1)^{(6)} + (r_{32})^{(6)} + (R_2)^{(6)})} \left[ e^{((R_1)^{(6)} + (r_{32})^{(6)})t} - e^{-(R_2)^{(6)} t} \right] + T_{34}^0 e^{-(R_2)^{(6)} t}$$

**Definition of** $(S_1)^{(6)}, (S_2)^{(6)}, (R_1)^{(6)}, (R_2)^{(6)}$:- 359

Where $(S_1)^{(6)} = (a_{32})^{(6)} (m_2)^{(6)} - (a'_{32})^{(6)}$

$(S_2)^{(6)} = (a_{34})^{(6)} - (p_{34})^{(6)}$

$(R_1)^{(6)} = (b_{32})^{(6)} (\mu_2)^{(6)} - (b'_{32})^{(6)}$

$$(R_2)^{(6)} = (b'_{34})^{(6)} - (r_{34})^{(6)}$$

**Behavior of the solutions of equation**

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$ :

$(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)} \le -(a'_{36})^{(7)} + (a'_{37})^{(7)} - (a''_{36})^{(7)}(T_{37}, t) + (a''_{37})^{(7)}(T_{37}, t) \le -(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)} \le -(b'_{36})^{(7)} + (b'_{37})^{(7)} - (b''_{36})^{(7)}\big((G_{39}), t\big) - (b''_{37})^{(7)}\big((G_{39}), t\big) \le -(\tau_1)^{(7)}$$

**Definition of** $(\nu_1)^{(7)}, (\nu_2)^{(7)}, (u_1)^{(7)}, (u_2)^{(7)}, \nu^{(7)}, u^{(7)}$ : 361

By $(\nu_1)^{(7)} > 0, (\nu_2)^{(7)} < 0$ and respectively $(u_1)^{(7)} > 0, (u_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_1)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(7)},, (\bar{\nu}_2)^{(7)}, (\bar{u}_1)^{(7)}, (\bar{u}_2)^{(7)}$ : 362

By $(\bar{\nu}_1)^{(7)} > 0, (\bar{\nu}_2)^{(7)} < 0$ and respectively $(\bar{u}_1)^{(7)} > 0, (\bar{u}_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

**Definition of** $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (\nu_0)^{(7)}$ :-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$$(m_2)^{(7)} = (\nu_0)^{(7)}, (m_1)^{(7)} = (\nu_1)^{(7)}, \ \boldsymbol{if}\ (\nu_0)^{(7)} < (\nu_1)^{(7)}$$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\bar{\nu}_1)^{(7)}, \boldsymbol{if}\ (\nu_1)^{(7)} < (\nu_0)^{(7)} < (\bar{\nu}_1)^{(7)},$$

and $\boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\nu_0)^{(7)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(7)} < (\nu_0)^{(7)}$$

and analogously 363

$$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}, \ \boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$$

and $\boxed{(u_0)^{(7)} = \frac{T^0_{36}}{T^0_{37}}}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)}$ where $(u_1)^{(7)}, (\bar{u}_1)^{(7)}$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^0 e^{\left((S_1)^{(7)} - (p_{36})^{(7)}\right)t} \le G_{36}(t) \le G_{36}^0 e^{(S_1)^{(7)}t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{\left((S_1)^{(7)} - (p_{36})^{(7)}\right)t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)}t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)}\left((S_1)^{(7)} - (p_{36})^{(7)} - (S_2)^{(7)}\right)}\left[e^{\left((S_1)^{(7)} - (p_{36})^{(7)}\right)t} - e^{-(S_2)^{(7)}t}\right] + G_{38}^0 e^{-(S_2)^{(7)}t} \le G_{38}(t) \le \right. \quad 366$$

$$\left. \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)}\left((S_1)^{(7)} - (a'_{38})^{(7)}\right)}\left[e^{(S_1)^{(7)}t} - e^{-(a'_{38})^{(7)}t}\right] + G_{38}^0 e^{-(a'_{38})^{(7)}t}\right)$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)}t} \le T_{36}(t) \le T_{36}^0 e^{\left((R_1)^{(7)} + (r_{36})^{(7)}\right)t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)}t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{\left((R_1)^{(7)} + (r_{36})^{(7)}\right)t} \quad 368$$

$$\frac{(b_{38})^{(7)} T_{36}^0}{(\mu_1)^{(7)}\left((R_1)^{(7)} - (b'_{38})^{(7)}\right)}\left[e^{(R_1)^{(7)}t} - e^{-(b'_{38})^{(7)}t}\right] + T_{38}^0 e^{-(b'_{38})^{(7)}t} \le T_{38}(t) \le \quad 369$$

$$\frac{(a_{38})^{(7)} T_{36}^0}{(\mu_2)^{(7)}\left((R_1)^{(7)} + (r_{36})^{(7)} + (R_2)^{(7)}\right)}\left[e^{\left((R_1)^{(7)} + (r_{36})^{(7)}\right)t} - e^{-(R_2)^{(7)}t}\right] + T_{38}^0 e^{-(R_2)^{(7)}t}$$

**Definition of** $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$$

$$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$$

$$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$$

**Behavior of the solutions of equation** 371

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ :

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$$-(\sigma_2)^{(8)} \le -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \le -(\sigma_1)^{(8)}$$

$$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\big((G_{43}),t\big) - (b''_{41})^{(8)}\big((G_{43}),t\big) \le -(\tau_1)^{(8)}$$

**Definition of** $(\nu_1)^{(8)}, (\nu_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, \nu^{(8)}, u^{(8)}$ : 372

By $(\nu_1)^{(8)} > 0$ , $(\nu_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0$ , $(u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(8)},, (\bar{\nu}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$ :

By $(\bar{\nu}_1)^{(8)} > 0$ , $(\bar{\nu}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0$ , $(\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

**Definition of** $(m_1)^{(8)}$ , $(m_2)^{(8)}$ , $(\mu_1)^{(8)}, (\mu_2)^{(8)}, (\nu_0)^{(8)}$ :-

If we define $(m_1)^{(8)}$ , $(m_2)^{(8)}$ , $(\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$(m_2)^{(8)} = (\nu_0)^{(8)}, (m_1)^{(8)} = (\nu_1)^{(8)}$, ***if*** $(\nu_0)^{(8)} < (\nu_1)^{(8)}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ , ***if*** $(\nu_1)^{(8)} < (\nu_0)^{(8)} < (\bar{\nu}_1)^{(8)}$,
and $\boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\nu_0)^{(8)}$, ***if*** $(\bar{\nu}_1)^{(8)} < (\nu_0)^{(8)}$

and analogously 374

$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)}$, ***if*** $(u_0)^{(8)} < (u_1)^{(8)}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}$ , ***if*** $(u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)}$,
and $\boxed{(u_0)^{(8)} = \frac{T^0_{40}}{T^0_{41}}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}$, ***if*** $(\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G^0_{40}e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{40}(t) \le G^0_{40}e^{(S_1)^{(8)}t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}} G_{40}^0 e^{((S_1)^{(8)}-(p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}} G_{40}^0 e^{(S_1)^{(8)}t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)}G_{40}^0}{(m_1)^{(8)}((S_1)^{(8)}-(p_{40})^{(8)}-(S_2)^{(8)})}\left[e^{((S_1)^{(8)}-(p_{40})^{(8)})t} - e^{-(S_2)^{(8)}t}\right] + G_{42}^0 e^{-(S_2)^{(8)}t} \le G_{42}(t) \le \right. \quad 377$$

$$\left. \frac{(a_{42})^{(8)}G_{40}^0}{(m_2)^{(8)}((S_1)^{(8)}-(a'_{42})^{(8)})}[e^{(S_1)^{(8)}t} - e^{-(a'_{42})^{(8)}t}] + G_{42}^0 e^{-(a'_{42})^{(8)}t}\right)$$

$$\boxed{T_{40}^0 e^{(R_1)^{(8)}t} \le T_{40}(t) \le T_{40}^0 e^{((R_1)^{(8)}+(r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}} T_{40}^0 e^{(R_1)^{(8)}t} \le T_{40}(t) \le \frac{1}{(\mu_2)^{(8)}} T_{40}^0 e^{((R_1)^{(8)}+(r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)}T_{40}^0}{(\mu_1)^{(8)}((R_1)^{(8)}-(b'_{42})^{(8)})}\left[e^{(R_1)^{(8)}t} - e^{-(b'_{42})^{(8)}t}\right] + T_{42}^0 e^{-(b'_{42})^{(8)}t} \le T_{42}(t) \le \quad 380$$

$$\frac{(a_{42})^{(8)}T_{40}^0}{(\mu_2)^{(8)}((R_1)^{(8)}+(r_{40})^{(8)}+(R_2)^{(8)})}\left[e^{((R_1)^{(8)}+(r_{40})^{(8)})t} - e^{-(R_2)^{(8)}t}\right] + T_{42}^0 e^{-(R_2)^{(8)}t}$$

**Definition of** $(S_1)^{(8)}, (S_2)^{(8)}, (R_1)^{(8)}, (R_2)^{(8)}$:- 381

Where $(S_1)^{(8)} = (a_{40})^{(8)}(m_2)^{(8)} - (a'_{40})^{(8)}$

$(S_2)^{(8)} = (a_{42})^{(8)} - (p_{42})^{(8)}$

$(R_1)^{(8)} = (b_{40})^{(8)}(\mu_2)^{(8)} - (b'_{40})^{(8)}$

$(R_2)^{(8)} = (b'_{42})^{(8)} - (r_{42})^{(8)}$

**Behavior of the solutions of equation 37 to 92** 382

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ :

$(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ four constants satisfying

$$-(\sigma_2)^{(9)} \le -(a'_{44})^{(9)} + (a'_{45})^{(9)} - (a''_{44})^{(9)}(T_{45}, t) + (a''_{45})^{(9)}(T_{45}, t) \le -(\sigma_1)^{(9)}$$

$$-(\tau_2)^{(9)} \le -(b'_{44})^{(9)} + (b'_{45})^{(9)} - (b''_{44})^{(9)}((G_{47}), t) - (b''_{45})^{(9)}((G_{47}), t) \le -(\tau_1)^{(9)}$$

**Definition of** $(\nu_1)^{(9)}, (\nu_2)^{(9)}, (u_1)^{(9)}, (u_2)^{(9)}, \nu^{(9)}, u^{(9)}$ :

By $(\nu_1)^{(9)} > 0$ , $(\nu_2)^{(9)} < 0$ and respectively $(u_1)^{(9)} > 0$ , $(u_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}(\nu^{(9)})^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}(u^{(9)})^2 + (\tau_1)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(9)}, , (\bar{\nu}_2)^{(9)}, (\bar{u}_1)^{(9)}, (\bar{u}_2)^{(9)}$ :

By $(\bar{\nu}_1)^{(9)} > 0$ , $(\bar{\nu}_2)^{(9)} < 0$ and respectively $(\bar{u}_1)^{(9)} > 0$ , $(\bar{u}_2)^{(9)} < 0$ the

roots of the equations $(a_{45})^{(9)}\left(v^{(9)}\right)^2+(\sigma_2)^{(9)}v^{(9)}-(a_{44})^{(9)}=0$
and $(b_{45})^{(9)}\left(u^{(9)}\right)^2+(\tau_2)^{(9)}u^{(9)}-(b_{44})^{(9)}=0$

**Definition of** $(m_1)^{(9)},(m_2)^{(9)},(\mu_1)^{(9)},(\mu_2)^{(9)},(v_0)^{(9)}$ :-

If we define $(m_1)^{(9)},(m_2)^{(9)},(\mu_1)^{(9)},(\mu_2)^{(9)}$ by

$(m_2)^{(9)}=(v_0)^{(9)},(m_1)^{(9)}=(v_1)^{(9)},\ \boldsymbol{if}\ (v_0)^{(9)}<(v_1)^{(9)}$

$(m_2)^{(9)}=(v_1)^{(9)},(m_1)^{(9)}=(\bar{v}_1)^{(9)},\boldsymbol{if}\ (v_1)^{(9)}<(v_0)^{(9)}<(\bar{v}_1)^{(9)},$
and $\boxed{(v_0)^{(9)}=\frac{G_{44}^0}{G_{45}^0}}$

$(m_2)^{(9)}=(v_1)^{(9)},(m_1)^{(9)}=(v_0)^{(9)},\ \boldsymbol{if}\ (\bar{v}_1)^{(9)}<(v_0)^{(9)}$

and analogously

$(\mu_2)^{(9)}=(u_0)^{(9)},(\mu_1)^{(9)}=(u_1)^{(9)},\ \boldsymbol{if}\ (u_0)^{(9)}<(u_1)^{(9)}$

$(\mu_2)^{(9)}=(u_1)^{(9)},(\mu_1)^{(9)}=(\bar{u}_1)^{(9)},\boldsymbol{if}\ (u_1)^{(9)}<(u_0)^{(9)}<(\bar{u}_1)^{(9)},$
and $\boxed{(u_0)^{(9)}=\frac{T_{44}^0}{T_{45}^0}}$

$(\mu_2)^{(9)}=(u_1)^{(9)},(\mu_1)^{(9)}=(u_0)^{(9)},\boldsymbol{if}\ (\bar{u}_1)^{(9)}<(u_0)^{(9)}$ where $(u_1)^{(9)},(\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G_{44}^0e^{\left((S_1)^{(9)}-(p_{44})^{(9)}\right)t}\le G_{44}(t)\le G_{44}^0e^{(S_1)^{(9)}t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}}G_{44}^0e^{\left((S_1)^{(9)}-(p_{44})^{(9)}\right)t}\le G_{45}(t)\le\frac{1}{(m_2)^{(9)}}G_{44}^0e^{(S_1)^{(9)}t}$$

$$\left(\frac{(a_{46})^{(9)}G_{44}^0}{(m_1)^{(9)}\left((S_1)^{(9)}-(p_{44})^{(9)}-(S_2)^{(9)}\right)}\left[e^{\left((S_1)^{(9)}-(p_{44})^{(9)}\right)t}-e^{-(S_2)^{(9)}t}\right]+G_{46}^0e^{-(S_2)^{(9)}t}\le G_{46}(t)\le\right.$$
$$\left.\frac{(a_{46})^{(9)}G_{44}^0}{(m_2)^{(9)}\left((S_1)^{(9)}-(a'_{46})^{(9)}\right)}\left[e^{(S_1)^{(9)}t}-e^{-(a'_{46})^{(9)}t}\right]+\ G_{46}^0e^{-(a'_{46})^{(9)}t}\right)$$

$$\boxed{T_{44}^0e^{(R_1)^{(9)}t}\le T_{44}(t)\le T_{44}^0e^{\left((R_1)^{(9)}+(r_{44})^{(9)}\right)t}}$$

$$\frac{1}{(\mu_1)^{(9)}}T_{44}^0e^{(R_1)^{(9)}t}\le T_{44}(t)\le\frac{1}{(\mu_2)^{(9)}}T_{44}^0e^{\left((R_1)^{(9)}+(r_{44})^{(9)}\right)t}$$

$$\frac{(b_{46})^{(9)}T_{44}^0}{(\mu_1)^{(9)}\left((R_1)^{(9)}-(b'_{46})^{(9)}\right)}\left[e^{(R_1)^{(9)}t}-e^{-(b'_{46})^{(9)}t}\right]+T_{46}^0e^{-(b'_{46})^{(9)}t}\le T_{46}(t)\le$$

$$\frac{(a_{46})^{(9)}T_{44}^0}{(\mu_2)^{(9)}\left((R_1)^{(9)}+(r_{44})^{(9)}+(R_2)^{(9)}\right)}\left[e^{\left((R_1)^{(9)}+(r_{44})^{(9)}\right)t}-e^{-(R_2)^{(9)}t}\right]+T_{46}^0e^{-(R_2)^{(9)}t}$$

**Definition of** $(S_1)^{(9)},(S_2)^{(9)},(R_1)^{(9)},(R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$

**Proof :** From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right) - (a''_{14})^{(1)}(T_{14}, t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

**Definition of** $v^{(1)}$ :- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)}\right) \le \frac{dv^{(1)}}{dt} \le -\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)}\right)$$

From which one obtains

**Definition of** $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$ :-

For $0 < \boxed{(v_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}} < (v_1)^{(1)} < (\bar{v}_1)^{(1)}$

$$v^{(1)}(t) \ge \frac{(v_1)^{(1)} + (C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_0)^{(1)}\right)t\right]}}{1 + (C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(v_1)^{(1)} - (v_0)^{(1)}}{(v_0)^{(1)} - (v_2)^{(1)}}}$$

it follows $(v_0)^{(1)} \le v^{(1)}(t) \le (v_1)^{(1)}$

In the same manner , we get 384

$$v^{(1)}(t) \le \frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{v}_1)^{(1)} - (v_0)^{(1)}}{(v_0)^{(1)} - (\bar{v}_2)^{(1)}}}$$

From which we deduce $(v_0)^{(1)} \le v^{(1)}(t) \le (\bar{v}_1)^{(1)}$

If $0 < (v_1)^{(1)} < (v_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}} < (\bar{v}_1)^{(1)}$ we find like in the previous case, 385

$$(v_1)^{(1)} \le \frac{(v_1)^{(1)} + (C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_2)^{(1)}\right)t\right]}}{1 + (C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_2)^{(1)}\right)t\right]}} \le v^{(1)}(t) \le$$

$$\frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}} \le (\bar{v}_1)^{(1)}$$

If $0 < (\nu_1)^{(1)} \leq (\bar{\nu}_1)^{(1)} \leq \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$ , we obtain 386

$$(\nu_1)^{(1)} \leq \nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)} + (\bar{C})^{(1)}(\bar{\nu}_2)^{(1)} e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)} - (\bar{\nu}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)} e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)} - (\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\nu_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(1)}(t)$ :-

$$(m_2)^{(1)} \leq \nu^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{\nu^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(1)}(t)$ :-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{13}'')^{(1)} = (a_{14}'')^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)} G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b_{13}'')^{(1)} = (b_{14}'')^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)} T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

**Proof** : From global equations we obtain 387

$$\frac{d\nu^{(2)}}{dt} = (a_{16})^{(2)} - \left((a_{16}')^{(2)} - (a_{17}')^{(2)} + (a_{16}'')^{(2)}(T_{17}, t)\right) - (a_{17}'')^{(2)}(T_{17}, t)\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

**Definition of** $\nu^{(2)}$ :- $\boxed{\nu^{(2)} = \frac{G_{16}}{G_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \leq \frac{d\nu^{(2)}}{dt} \leq -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

**Definition of** $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$ :-

For $0 < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \geq \frac{(\nu_1)^{(2)}+(\mathrm{C})^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_0)^{(2)}\right)t\right]}}{1+(\mathrm{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\mathrm{C})^{(2)}=\frac{(\nu_1)^{(2)}-(\nu_0)^{(2)}}{(\nu_0)^{(2)}-(\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)}+(\bar{\mathrm{C}})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{\mathrm{C}})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{\mathrm{C}})^{(2)}=\frac{(\bar{\nu}_1)^{(2)}-(\nu_0)^{(2)}}{(\nu_0)^{(2)}-(\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{\mathrm{G}_{16}^0}{\mathrm{G}_{17}^0} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \leq \frac{(\nu_1)^{(2)}+(\mathrm{C})^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}}{1+(\mathrm{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}} \leq \nu^{(2)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(2)}+(\bar{\mathrm{C}})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{\mathrm{C}})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \leq (\bar{\nu}_1)^{(2)} \leq (\nu_0)^{(2)} = \frac{\mathrm{G}_{16}^0}{\mathrm{G}_{17}^0}$ , we obtain 394

$$(\nu_1)^{(2)} \leq \nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)}+(\bar{\mathrm{C}})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{\mathrm{C}})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(2)}(t)$ :- 395

$$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}, \quad \boxed{\nu^{(2)}(t)=\frac{G_{16}(t)}{G_{17}(t)}}$$

In a completely analogous way, we obtain 396

**Definition of** $u^{(2)}(t)$ :-

$$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}, \quad \boxed{u^{(2)}(t)=\frac{T_{16}(t)}{T_{17}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :** 397

If $(a_{16}'')^{(2)} = (a_{17}'')^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)}G_{17}(t)$

Analogously if $(b''_{16})^{(2)} = (b''_{17})^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)}T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

**Proof** : From global equations we obtain 398

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right) - (a''_{21})^{(3)}(T_{21}, t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

**Definition of** $\nu^{(3)}$ :- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right) \leq \frac{d\nu^{(3)}}{dt} \leq -\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right)$$

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From which one obtains

For $0 < (\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\nu_1)^{(3)} < (\bar{\nu}_1)^{(3)}$

$$\nu^{(3)}(t) \geq \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(\nu_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\nu_2)^{(3)}}}$$

it follows $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\nu_1)^{(3)}$

In the same manner , we get 401

$$\nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)} + (\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{\nu}_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\bar{\nu}_2)^{(3)}}}$$

**Definition of** $(\bar{\nu}_1)^{(3)}$ :-

From which we deduce $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\bar{\nu}_1)^{(3)}$

If $0 < (\nu_1)^{(3)} < (\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\bar{\nu}_1)^{(3)}$ we find like in the previous case, 402

$$(\nu_1)^{(3)} \leq \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_2)^{(3)}\right)t\right]}}{1 + (C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_2)^{(3)}\right)t\right]}} \leq \nu^{(3)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(3)} + (\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\bar{\nu}_1)^{(3)}$$

If $0 < (\nu_1)^{(3)} \leq (\bar{\nu}_1)^{(3)} \leq (\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}$ , we obtain 403

$$(\nu_1)^{(3)} \le \nu^{(3)}(t) \le \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \le (\nu_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(3)}(t)$ :-

$$(m_2)^{(3)} \le \nu^{(3)}(t) \le (m_1)^{(3)}, \quad \boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(3)}(t)$ :-

$$(\mu_2)^{(3)} \le u^{(3)}(t) \le (\mu_1)^{(3)}, \quad \boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{20})^{(3)} = (a''_{21})^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)}G_{21}(t)$

Analogously if $(b''_{20})^{(3)} = (b''_{21})^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)}T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

**Proof** : From global equations we obtain 404

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right) - (a''_{25})^{(4)}(T_{25},t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

**Definition of** $\nu^{(4)}$ :- $\quad \boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right) \le \frac{d\nu^{(4)}}{dt} \le -\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_4)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$ :-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \ge \frac{(\nu_1)^{(4)}+(C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_0)^{(4)}\right)t\right]}}{4+(C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)}-(\nu_0)^{(4)}}{(\nu_0)^{(4)}-(\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \le \nu^{(4)}(t) \le (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{4+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)}-(\nu_0)^{(4)}}{(\nu_0)^{(4)}-(\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \leq \frac{(\nu_1)^{(4)}+(C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_2)^{(4)}\right)t\right]}}{1+(C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_2)^{(4)}\right)t\right]}} \leq \nu^{(4)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \leq (\bar{\nu}_1)^{(4)} \leq \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$ , we obtain 407

$$(\nu_1)^{(4)} \leq \nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(4)}(t)$ :-

$$(m_2)^{(4)} \leq \nu^{(4)}(t) \leq (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(4)}(t)$ :-

$$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a_{24}'')^{(4)} = (a_{25}'')^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)}G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b_{24}'')^{(4)} = (b_{25}'')^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)}T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

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**Proof** : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a_{28}')^{(5)} - (a_{29}')^{(5)} + (a_{28}'')^{(5)}(T_{29}, t)\right) - (a_{29}'')^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

**Definition of** $\nu^{(5)}$ :- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$ :-

For $0 < \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \leq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}}{1 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}} \leq \nu^{(5)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \leq (\bar{\nu}_1)^{(5)} \leq \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$ , we obtain 411

$$(\nu_1)^{(5)} \leq \nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(5)}(t)$ :-

$(m_2)^{(5)} \leq \nu^{(5)}(t) \leq (m_1)^{(5)}$, $\boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$

In a completely analogous way, we obtain

**Definition of** $u^{(5)}(t)$ :-

$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}$, $\boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{28})^{(5)} = (a''_{29})^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)} G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b''_{28})^{(5)} = (b''_{29})^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$ if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)} T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

**Proof** : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a'_{32})^{(6)} - (a'_{33})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right) - (a''_{33})^{(6)}(T_{33}, t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

**Definition of** $\nu^{(6)}$ :- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \leq \frac{d\nu^{(6)}}{dt} \leq -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$ :-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \geq \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\nu_1)^{(6)}$

In the same manner , we get 413

$$\nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{\nu}_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\bar{\nu}_1)^{(6)}$

If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 414

$$(\nu_1)^{(6)} \leq \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}} \leq \nu^{(6)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\bar{\nu}_1)^{(6)}$$

If $0 < (\nu_1)^{(6)} \leq (\bar{\nu}_1)^{(6)} \leq \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$, we obtain

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$$(\nu_1)^{(6)} \leq \nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(6)}(t)$ :-

$$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}, \quad \boxed{\nu^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(6)}(t)$ :-

$$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}, \quad \boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)}G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)}T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

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**Proof** : From global equations we obtain

$$\frac{d\nu^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right) - (a''_{37})^{(7)}(T_{37}, t)\nu^{(7)} - (a_{37})^{(7)}\nu^{(7)}$$

**Definition of** $\nu^{(7)}$ :- $\boxed{\nu^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right) \leq \frac{d\nu^{(7)}}{dt} \leq -\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(7)}, (\nu_0)^{(7)}$ :-

For $0 < \boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}} < (\nu_1)^{(7)} < (\bar{\nu}_1)^{(7)}$

$$\nu^{(7)}(t) \geq \frac{(\nu_1)^{(7)}+(C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_0)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)}=\frac{(\nu_1)^{(7)}-(\nu_0)^{(7)}}{(\nu_0)^{(7)}-(\nu_2)^{(7)}}}$$

it follows $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\nu_1)^{(7)}$

In the same manner , we get 417

$$\nu^{(7)}(t) \leq \frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)}=\frac{(\bar{\nu}_1)^{(7)}-(\nu_0)^{(7)}}{(\nu_0)^{(7)}-(\bar{\nu}_2)^{(7)}}}$$

From which we deduce $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\bar{\nu}_1)^{(7)}$

If $0 < (\nu_1)^{(7)} < (\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0} < (\bar{\nu}_1)^{(7)}$ we find like in the previous case, 418

$$(\nu_1)^{(7)} \leq \frac{(\nu_1)^{(7)}+(C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}} \leq \nu^{(7)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \leq (\bar{\nu}_1)^{(7)}$$

If $0 < (\nu_1)^{(7)} \leq (\bar{\nu}_1)^{(7)} \leq \boxed{(\nu_0)^{(7)}=\frac{G_{36}^0}{G_{37}^0}}$ , we obtain 419

$$(\nu_1)^{(7)} \leq \nu^{(7)}(t) \leq \frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \leq (\nu_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(7)}(t)$ :-

$$(m_2)^{(7)} \leq \nu^{(7)}(t) \leq (m_1)^{(7)}, \quad \boxed{\nu^{(7)}(t)=\frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(7)}(t)$ :- 420

$$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}, \quad \boxed{u^{(7)}(t)=\frac{T_{36}(t)}{T_{37}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $\nu^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)}G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for**

**the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)}T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

**Proof** : From global equations we obtain 421

$$\frac{d\nu^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right) - (a''_{41})^{(8)}(T_{41},t)\nu^{(8)} - (a_{41})^{(8)}\nu^{(8)}$$

**Definition of** $\nu^{(8)}$ :- $\boxed{\nu^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right) \le \frac{d\nu^{(8)}}{dt} \le -\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$ :-

For $0 < \boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$

$$\nu^{(8)}(t) \ge \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \le \nu^{(8)}(t) \le (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \le \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \le \nu^{(8)}(t) \le (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \le \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}} \le \nu^{(8)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \le (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \le (\bar{\nu}_1)^{(8)} \le \boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}}$ , we obtain 424

$$(\nu_1)^{(8)} \leq \nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(8)}(t)$ :-

$$(m_2)^{(8)} \leq \nu^{(8)}(t) \leq (m_1)^{(8)}, \quad \boxed{\nu^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(8)}(t)$ :-

$$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}, \quad \boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $\nu^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)}G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then $(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)}T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

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**Proof** : From 99,20,44,22,23,44 we obtain

$$\frac{d\nu^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right) - (a''_{45})^{(9)}(T_{45},t)\nu^{(9)} - (a_{45})^{(9)}\nu^{(9)}$$

**Definition of** $\nu^{(9)}$ :- $\quad \boxed{\nu^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(\nu^{(9)}\right)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)}\right) \leq \frac{d\nu^{(9)}}{dt} \leq -\left((a_{45})^{(9)}\left(\nu^{(9)}\right)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$ :-

$$\text{For } 0 < \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$$

$$\nu^{(9)}(t) \geq \frac{(\nu_1)^{(9)}+(C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_0)^{(9)}\right)t\right]}}{1+(C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)}-(\nu_0)^{(9)}}{(\nu_0)^{(9)}-(\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\nu_9)^{(9)}$

In the same manner , we get

$$\nu^{(9)}(t) \leq \frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)}-(\nu_0)^{(9)}}{(\nu_0)^{(9)}-(\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \leq \frac{(\nu_1)^{(9)}+(C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}}{1+(C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}} \leq \nu^{(9)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \leq (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \leq (\bar{\nu}_1)^{(9)} \leq \boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$ , we obtain

$$(\nu_1)^{(9)} \leq \nu^{(9)}(t) \leq \frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \leq (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(9)}(t)$ :-

$$(m_2)^{(9)} \leq \nu^{(9)}(t) \leq (m_1)^{(9)}, \quad \boxed{\nu^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(9)}(t)$ :-

$$(\mu_2)^{(9)} \leq u^{(9)}(t) \leq (\mu_1)^{(9)}, \quad \boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)}G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then $(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)}T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following

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**Theorem :** If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on $t$ , and the conditions with the notations

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0$ ,

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$

$with\ \ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(2)} and\ (b_i'')^{(2)}$ are independent on t , and the conditions with the notations 426

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$ 427

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 428

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$ , 429

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 430

$with\ \ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(3)} and\ (b_i'')^{(3)}$ are independent on $t$ , and the conditions with the notations 431

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$ ,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

**Theorem :** If $(a_i'')^{(4)} and\ (b_i'')^{(4)}$ are independent on $t$ , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$ ,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(5)} and\ (b_i'')^{(5)}$ are independent on $t$ , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$ ,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

**Theorem** If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on $t$ , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$ ,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(7)} and\ (b_i'')^{(7)}$ are independent on $t$ , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$ ,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(8)} and\ (b_i'')^{(8)}$ are independent on $t$ , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$ ,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(9)} and\ (b_i'')^{(9)}$ are independent on $t$ , and the conditions (with the notations 436 A
45,46,27,28)

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$ ,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0 \quad 437$$

$$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0 \quad 438$$

$$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0 \quad 439$$

$$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0 \quad 440$$

$$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ]T_{14} = 0 \quad 441$$

$$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0 \quad 442$$

has a unique positive solution , which is an equilibrium solution for the system

$$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0 \quad 443$$

$$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0 \quad 444$$

$$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0 \quad 445$$

$$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\ ]T_{16} = 0 \quad 446$$

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\ ]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\ ]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\ ]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\ ]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\ ]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27})\right)]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27})\right)]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27})\right)]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0 \quad 479$$

$$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0 \quad 480$$

$$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0 \quad 481$$

$$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0 \quad 482$$

$$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0 \quad 483$$

$$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0 \quad 484$$

$$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0 \quad 484\text{A}$$

$$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$$

$$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$$

$$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$$

$$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$$

$$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$$

**Proof:** 485

(a) Indeed the first two equations have a nontrivial solution $G_{13}, G_{14}$ if

$$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$$

**Proof:** 486

(b) Indeed the first two equations have a nontrivial solution $G_{16}, G_{17}$ if

$$F(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$$

**Proof:** 487

(a) Indeed the first two equations have a nontrivial solution $G_{20}, G_{21}$ if

$$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$$

**Proof:** 488

(a) Indeed the first two equations have a nontrivial solution $G_{24}, G_{25}$ if

$$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$$

**Proof:** 489

(a) Indeed the first two equations have a nontrivial solution $G_{28}, G_{29}$ if

$$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$$

**Proof:** 490

(a) Indeed the first two equations have a nontrivial solution $G_{32}, G_{33}$ if

$$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$$

**Proof:** 491

(a) Indeed the first two equations have a nontrivial solution $G_{36}, G_{37}$ if

$$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$$

**Proof:** 492

(a) Indeed the first two equations have a nontrivial solution $G_{40}, G_{41}$ if

$$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$$

**Proof:** 492 A

(a) Indeed the first two equations have a nontrivial solution $G_{44}, G_{45}$ if

$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$

**Definition and uniqueness of** $T^*_{14}$ :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique $T^*_{14}$ for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

**Definition and uniqueness of** $T^*_{17}$ :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique $T^*_{17}$ for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]} \qquad 495$$

**Definition and uniqueness of** $T^*_{21}$ :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique $T^*_{21}$ for which $f(T^*_{21}) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

**Definition and uniqueness of** $T^*_{25}$ :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that there exists a unique $T^*_{25}$ for which $f(T^*_{25}) = 0$. With this value , we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T^*_{25})]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T^*_{25})]}$$

**Definition and uniqueness of** $T^*_{29}$ :- 498

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(5)}(T_{29})$ being increasing, it follows that there exists a unique $T^*_{29}$ for which $f(T^*_{29}) = 0$. With this value , we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T^*_{29})]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T^*_{29})]}$$

**Definition and uniqueness of** $T^*_{33}$ :- 499

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(6)}(T_{33})$ being increasing, it follows that

there exists a unique $T_{33}^{*}$ for which $f(T_{33}^{*})=0$. With this value , we obtain from the three first equations

$$G_{32}=\frac{(a_{32})^{(6)}G_{33}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^{*})]} \quad , \quad G_{34}=\frac{(a_{34})^{(6)}G_{33}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^{*})]}$$

**Definition and uniqueness of** $T_{37}^{*}$ :- 500

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(7)}(T_{37})$ being increasing, it follows that there exists a unique $T_{37}^{*}$ for which $f(T_{37}^{*})=0$. With this value , we obtain from the three first equations

$$G_{36}=\frac{(a_{36})^{(7)}G_{37}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^{*})]} \quad , \quad G_{38}=\frac{(a_{38})^{(7)}G_{37}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^{*})]}$$

**Definition and uniqueness of** $T_{41}^{*}$ :- 501

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique $T_{41}^{*}$ for which $f(T_{41}^{*})=0$. With this value , we obtain from the three first equations

$$G_{40}=\frac{(a_{40})^{(8)}G_{41}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T_{41}^{*})]} \quad , \quad G_{42}=\frac{(a_{42})^{(8)}G_{41}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T_{41}^{*})]}$$

**Definition and uniqueness of** $T_{45}^{*}$ :- 501 A

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(9)}(T_{45})$ being increasing, it follows that there exists a unique $T_{45}^{*}$ for which $f(T_{45}^{*})=0$. With this value , we obtain from the three first equations

$$G_{44}=\frac{(a_{44})^{(9)}G_{45}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T_{45}^{*})]} \quad , \quad G_{46}=\frac{(a_{46})^{(9)}G_{45}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T_{45}^{*})]}$$

By the same argument, the equations admit solutions $G_{13}, G_{14}$ if 502

$$\varphi(G)=(b'_{13})^{(1)}(b'_{14})^{(1)}-(b_{13})^{(1)}(b_{14})^{(1)}-$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G)+(b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right]+(b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G)=0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{14}$ taking into account the hypothesis $\varphi(0)>0$ , $\varphi(\infty)<0$ it follows that there exists a unique $G_{14}^{*}$ such that $\varphi(G^{*})=0$

By the same argument, the equations admit solutions $G_{16}, G_{17}$ if 503

$$\varphi(G_{19})=(b'_{16})^{(2)}(b'_{17})^{(2)}-(b_{16})^{(2)}(b_{17})^{(2)}-$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19})+(b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right]+(b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19})=0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{17}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{14}^{*}$ such that $\varphi((G_{19})^{*}) = 0$ 504

By the same argument, the equations admit solutions $G_{20}, G_{21}$ if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right] + (b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{21}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{21}^{*}$ such that $\varphi((G_{23})^{*}) = 0$

By the same argument, the equations admit solutions $G_{24}, G_{25}$ if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right] + (b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{25}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{25}^{*}$ such that $\varphi((G_{27})^{*}) = 0$

By the same argument, the equations admit solutions $G_{28}, G_{29}$ if 507

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right] + (b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{29}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{29}^{*}$ such that $\varphi((G_{31})^{*}) = 0$

By the same argument, the equations admit solutions $G_{32}, G_{33}$ if 508

$$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$$

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right] + (b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{33}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{33}^{*}$ such that $\varphi({G_{35}}^{*}) = 0$

By the same argument, the equations admit solutions $G_{36}, G_{37}$ if 509

$$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$$

$$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right] + (b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{37}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{37}^{*}$ such that $\varphi({G_{39}}^{*}) = 0$

By the same argument, the equations admit solutions $G_{40}, G_{41}$ if 510

$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$
$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right] + (b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{41}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{41}$ such that $\varphi(G_{43}{}^*) = 0$

By the same argument, the equations 92,93 admit solutions $G_{44}, G_{45}$ if
$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$

$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right] + (b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{45}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{45}$ such that $\varphi((G_{47})^*) = 0$

Finally we obtain the unique solution 511

$G^*_{14}$ given by $\varphi(G^*) = 0$ , $T^*_{14}$ given by $f(T^*_{14}) = 0$ and

$$G^*_{13} = \frac{(a_{13})^{(1)}G^*_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G^*_{15} = \frac{(a_{15})^{(1)}G^*_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

$$T^*_{13} = \frac{(b_{13})^{(1)}T^*_{14}}{[(b'_{13})^{(1)}-(b''_{13})^{(1)}(G^*)]} \quad , \quad T^*_{15} = \frac{(b_{15})^{(1)}T^*_{14}}{[(b'_{15})^{(1)}-(b''_{15})^{(1)}(G^*)]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

$G^*_{17}$ given by $\varphi((G_{19})^*) = 0$ , $T^*_{17}$ given by $f(T^*_{17}) = 0$ and 512

$$G^*_{16} = \frac{(a_{16})^{(2)}G^*_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G^*_{18} = \frac{(a_{18})^{(2)}G^*_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]} \quad 513$$

$$T^*_{16} = \frac{(b_{16})^{(2)}T^*_{17}}{[(b'_{16})^{(2)}-(b''_{16})^{(2)}((G_{19})^*)]} \quad , \quad T^*_{18} = \frac{(b_{18})^{(2)}T^*_{17}}{[(b'_{18})^{(2)}-(b''_{18})^{(2)}((G_{19})^*)]} \quad 514$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

$G^*_{21}$ given by $\varphi((G_{23})^*) = 0$ , $T^*_{21}$ given by $f(T^*_{21}) = 0$ and

$$G^*_{20} = \frac{(a_{20})^{(3)}G^*_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G^*_{22} = \frac{(a_{22})^{(3)}G^*_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

$$T^*_{20} = \frac{(b_{20})^{(3)}T^*_{21}}{[(b'_{20})^{(3)}-(b''_{20})^{(3)}(G_{23}{}^*)]} \quad , \quad T^*_{22} = \frac{(b_{22})^{(3)}T^*_{21}}{[(b'_{22})^{(3)}-(b''_{22})^{(3)}(G_{23}{}^*)]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

$G_{25}^*$ given by $\varphi(G_{27}) = 0$ , $T_{25}^*$ given by $f(T_{25}^*) = 0$ and

$$G_{24}^* = \frac{(a_{24})^{(4)} G_{25}^*}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^*)]} \quad , \quad G_{26}^* = \frac{(a_{26})^{(4)} G_{25}^*}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^*)]}$$

$$T_{24}^* = \frac{(b_{24})^{(4)} T_{25}^*}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^*)]} \quad , \quad T_{26}^* = \frac{(b_{26})^{(4)} T_{25}^*}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^*)]} \qquad 517$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

$G_{29}^*$ given by $\varphi((G_{31})^*) = 0$ , $T_{29}^*$ given by $f(T_{29}^*) = 0$ and

$$G_{28}^* = \frac{(a_{28})^{(5)} G_{29}^*}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^*)]} \quad , \quad G_{30}^* = \frac{(a_{30})^{(5)} G_{29}^*}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^*)]}$$

$$T_{28}^* = \frac{(b_{28})^{(5)} T_{29}^*}{[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31})^*)]} \quad , \quad T_{30}^* = \frac{(b_{30})^{(5)} T_{29}^*}{[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31})^*)]} \qquad 519$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

$G_{33}^*$ given by $\varphi((G_{35})^*) = 0$ , $T_{33}^*$ given by $f(T_{33}^*) = 0$ and

$$G_{32}^* = \frac{(a_{32})^{(6)} G_{33}^*}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^*)]} \quad , \quad G_{34}^* = \frac{(a_{34})^{(6)} G_{33}^*}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^*)]}$$

$$T_{32}^* = \frac{(b_{32})^{(6)} T_{33}^*}{[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35})^*)]} \quad , \quad T_{34}^* = \frac{(b_{34})^{(6)} T_{33}^*}{[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35})^*)]} \qquad 521$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

$G_{37}^*$ given by $\varphi((G_{39})^*) = 0$ , $T_{37}^*$ given by $f(T_{37}^*) = 0$ and

$$G_{36}^* = \frac{(a_{36})^{(7)} G_{37}^*}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^*)]} \quad , \quad G_{38}^* = \frac{(a_{38})^{(7)} G_{37}^*}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^*)]}$$

$$T_{36}^* = \frac{(b_{36})^{(7)} T_{37}^*}{[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39})^*)]} \quad , \quad T_{38}^* = \frac{(b_{38})^{(7)} T_{37}^*}{[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39})^*)]}$$

Finally we obtain the unique solution 523

$G_{41}^*$ given by $\varphi((G_{43})^*) = 0$ , $T_{41}^*$ given by $f(T_{41}^*) = 0$ and

$$G_{40}^* = \frac{(a_{40})^{(8)} G_{41}^*}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42}^* = \frac{(a_{42})^{(8)} G_{41}^*}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

$$T_{40}^{*} = \frac{(b_{40})^{(8)} T_{41}^{*}}{[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43})^{*})]} \quad , \quad T_{42}^{*} = \frac{(b_{42})^{(8)} T_{41}^{*}}{[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43})^{*})]}$$

Finally we obtain the unique solution of 89 to 99 523

A

$G_{45}^{*}$ given by $\varphi((G_{47})^{*}) = 0$ , $T_{45}^{*}$ given by $f(T_{45}^{*}) = 0$ and

$$G_{44}^{*} = \frac{(a_{44})^{(9)} G_{45}^{*}}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^{*})]} \quad , \quad G_{46}^{*} = \frac{(a_{46})^{(9)} G_{45}^{*}}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^{*})]}$$

$$T_{44}^{*} = \frac{(b_{44})^{(9)} T_{45}^{*}}{[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47})^{*})]} \quad , \quad T_{46}^{*} = \frac{(b_{46})^{(9)} T_{45}^{*}}{[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47})^{*})]}$$

## ASYMPTOTIC STABILITY ANALYSIS 524

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

**Proof:** Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^{*} + \mathbb{G}_i \qquad , T_i = T_i^{*} + \mathbb{T}_i$$

$$\frac{\partial (a''_{14})^{(1)}}{\partial T_{14}}(T_{14}^{*}) = (q_{14})^{(1)} \quad , \frac{\partial (b_i'')^{(1)}}{\partial G_j}(G^{*}) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a'_{13})^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)} G_{13}^{*}\mathbb{T}_{14} \quad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a'_{14})^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)} G_{14}^{*}\mathbb{T}_{14} \quad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a'_{15})^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)} G_{15}^{*}\mathbb{T}_{14} \quad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b'_{13})^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)} T_{13}^{*}\mathbb{G}_j\right) \quad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b'_{14})^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)} T_{14}^{*}\mathbb{G}_j\right) \quad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b'_{15})^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)} T_{15}^{*}\mathbb{G}_j\right) \quad 530$$

## ASYMPTOTIC STABILITY ANALYSIS 531

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(2)}$ and $(b_i'')^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i \tag{532}$$

$$\frac{\partial(a_{17}'')^{(2)}}{\partial T_{17}}(T_{17}^*) = (q_{17})^{(2)} \ , \ \frac{\partial(b_i'')^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \tag{533}$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a_{16}')^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G_{16}^*\mathbb{T}_{17} \tag{534}$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a_{17}')^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G_{17}^*\mathbb{T}_{17} \tag{535}$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a_{18}')^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G_{18}^*\mathbb{T}_{17} \tag{536}$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b_{16}')^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T_{16}^*\mathbb{G}_j\right) \tag{537}$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b_{17}')^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}T_{17}^*\mathbb{G}_j\right) \tag{538}$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b_{18}')^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}T_{18}^*\mathbb{G}_j\right) \tag{539}$$

**ASYMPTOTIC STABILITY ANALYSIS** 540

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(3)}$ *and* $(b_i'')^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{21}'')^{(3)}}{\partial T_{21}}(T_{21}^*) = (q_{21})^{(3)} \ , \ \frac{\partial(b_i'')^{(3)}}{\partial G_j}((G_{23})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a_{20}')^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G_{20}^*\mathbb{T}_{21} \tag{541}$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a_{21}')^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G_{21}^*\mathbb{T}_{21} \tag{542}$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a_{22}')^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G_{22}^*\mathbb{T}_{21} \tag{543}$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b_{20}')^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T_{20}^*\mathbb{G}_j\right) \tag{544}$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b_{21}')^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T_{21}^*\mathbb{G}_j\right) \tag{545}$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b'_{22})^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T^*_{22}\mathbb{G}_j\right) \quad 546$$

**ASYMPTOTIC STABILITY ANALYSIS** 547

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}\ and\ (b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{25})^{(4)}}{\partial T_{25}}(T^*_{25}) = (q_{25})^{(4)} \ , \ \frac{\partial(b_i'')^{(4)}}{\partial G_j}((G_{27})^*\ ) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\left((a'_{24})^{(4)} + (p_{24})^{(4)}\right)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G^*_{24}\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\left((a'_{25})^{(4)} + (p_{25})^{(4)}\right)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G^*_{25}\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\left((a'_{26})^{(4)} + (p_{26})^{(4)}\right)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G^*_{26}\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\left((b'_{24})^{(4)} - (r_{24})^{(4)}\right)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(24)(j)}T^*_{24}\mathbb{G}_j\right) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\left((b'_{25})^{(4)} - (r_{25})^{(4)}\right)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\left(s_{(25)(j)}T^*_{25}\mathbb{G}_j\right) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T^*_{26}\mathbb{G}_j\right) \quad 554$$

**ASYMPTOTIC STABILITY ANALYSIS** 555

**Theorem 5:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}\ and\ (b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 556

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{29})^{(5)}}{\partial T_{29}}(T^*_{29}) = (q_{29})^{(5)} \ , \ \frac{\partial(b_i'')^{(5)}}{\partial G_j}(\ (G_{31})^*\ ) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a'_{28})^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G^*_{28}\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a'_{29})^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G^*_{29}\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a'_{30})^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G^*_{30}\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b'_{28})^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T^*_{28}\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b'_{29})^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T^*_{29}\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b'_{30})^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T^*_{30}\mathbb{G}_j\right) \quad 562$$

**ASYMPTOTIC STABILITY ANALYSIS** 563

**Theorem 6:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}$ *and* $(b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{33})^{(6)}}{\partial T_{33}}(T^*_{33}) = (q_{33})^{(6)} \quad , \frac{\partial(b_i'')^{(6)}}{\partial G_j}\left((G_{35})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a'_{32})^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G^*_{32}\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a'_{33})^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G^*_{33}\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a'_{34})^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G^*_{34}\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b'_{32})^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T^*_{32}\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b'_{33})^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T^*_{33}\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b'_{34})^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T^*_{34}\mathbb{G}_j\right) \quad 570$$

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**Theorem 7:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 572

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{37}'')^{(7)}}{\partial T_{37}}(T_{37}^{*}) = (q_{37})^{(7)} \quad , \frac{\partial(b_i'')^{(7)}}{\partial G_j}\left( (G_{39})^{**} \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a_{36}')^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G_{36}^{*}\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a_{37}')^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G_{37}^{*}\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a_{38}')^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G_{38}^{*}\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b_{36}')^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T_{36}^{*}\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b_{37}')^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T_{37}^{*}\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b_{38}')^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T_{38}^{*}\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

## ASYMPTOTIC STABILITY ANALYSIS

**Theorem 8:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 580

$$G_i = G_i^{*} + \mathbb{G}_i \qquad , T_i = T_i^{*} + \mathbb{T}_i$$

$$\frac{\partial(a_{41}'')^{(8)}}{\partial T_{41}}(T_{41}^{*}) = (q_{41})^{(8)} \quad , \frac{\partial(b_i'')^{(8)}}{\partial G_j}\left( (G_{43})^{*} \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a_{40}')^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G_{40}^{*}\mathbb{T}_{41} \quad 581$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a_{41}')^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G_{41}^{*}\mathbb{T}_{41} \quad 582$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a_{42}')^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G_{42}^{*}\mathbb{T}_{41} \quad 583$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b_{40}')^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T_{40}^{*}\mathbb{G}_j\right) \quad 584$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b_{41}')^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T_{41}^{*}\mathbb{G}_j\right) \quad 585$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b'_{42})^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T^*_{42}\mathbb{G}_j\right)$$ 586

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**ASYMPTOTIC STABILITY ANALYSIS** A

**Theorem 9:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}\ and\ (b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

**Proof: Denote**

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{45})^{(9)}}{\partial T_{45}}(T^*_{45}) = (q_{45})^{(9)} \ , \ \frac{\partial(b_i'')^{(9)}}{\partial G_j}(\ (G_{47})^*\ ) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a'_{44})^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)}G^*_{44}\mathbb{T}_{45}$$ 586 B

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a'_{45})^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)}G^*_{45}\mathbb{T}_{45}$$ 586 C

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a'_{46})^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)}G^*_{46}\mathbb{T}_{45}$$ 586 D

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b'_{44})^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)}T^*_{44}\mathbb{G}_j\right)$$ 586 E

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b'_{45})^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)}T^*_{45}\mathbb{G}_j\right)$$ 586 F

$$\frac{d\mathbb{T}_{46}}{dt} = -\left((b'_{46})^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T^*_{46}\mathbb{G}_j\right)$$ 586 G

**The characteristic equation of this system is** 587

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(q_{13})^{(1)}G^*_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)}T^*_{14} + (b_{14})^{(1)}s_{(13),(14)}T^*_{14}\right)$$

$$+\left(\left((\lambda)^{(1)} + (a'_{14})^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)}G^*_{13} + (a_{13})^{(1)}(q_{14})^{(1)}G^*_{14}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)}T^*_{14} + (b_{14})^{(1)}s_{(13),(13)}T^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left(\ (a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left(\ (b'_{13})^{(1)} + (b'_{14})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^2+\left((a'_{13})^{(1)}+(a'_{14})^{(1)}+(p_{13})^{(1)}+(p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)}+(a'_{13})^{(1)}+(p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^*_{14}+(a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}+(b'_{13})^{(1)}-(r_{13})^{(1)}\right)s_{(14),(15)}T^*_{14}+(b_{14})^{(1)}s_{(13),(15)}T^*_{13}\right)\}=0$$

$$+$$

$$(\lambda)^{(2)}+(b'_{18})^{(2)}-(r_{18})^{(2)}\{\left((\lambda)^{(2)}+(a'_{18})^{(2)}+(p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^*_{17}+(a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^*_{17}\right)$$

$$+\left(\left((\lambda)^{(2)}+(a'_{17})^{(2)}+(p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^*_{16}+(a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2+\left((b'_{16})^{(2)}+(b'_{17})^{(2)}-(r_{16})^{(2)}+(r_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$+\left(\left((\lambda)^{(2)}\right)^2+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}+(a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^*_{16}\right)\}=0$$

$$+$$

$$(\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left(\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^*_{20}\right)\right]$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^*_{21}+(b_{21})^{(3)}s_{(20),(21)}T^*_{21}\right)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^*_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^*_{21}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^*_{21}+(b_{21})^{(3)}s_{(20),(20)}T^*_{20}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^2+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$+\left(\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^*_{20}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^*_{21}+(b_{21})^{(3)}s_{(20),(22)}T^*_{20}\right)\}=0$$

$$+$$

$$(\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left(\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^*_{24}\right)\right]$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^*_{25}+(b_{25})^{(4)}s_{(24),(25)}T^*_{25}\right)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^*_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^*_{25}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^*_{25}+(b_{25})^{(4)}s_{(24),(24)}T^*_{24}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^2+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$+\left(\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^*_{24}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^*_{25}+(b_{25})^{(4)}s_{(24),(26)}T^*_{24}\right)\}=0$$

$$+$$

$$(\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^*_{28}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^*_{29}+(b_{29})^{(5)}s_{(28),(29)}T^*_{29}\right)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^*_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^*_{29}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^*_{29}+(b_{29})^{(5)}s_{(28),(28)}T^*_{28}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^2+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$+\left(\left((\lambda)^{(5)}\right)^2+\left(\,(a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^*_{28}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^*_{29}+(b_{29})^{(5)}s_{(28),(30)}T^*_{28}\right)\}=0$$

$$+$$

$$(\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left(\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^*_{32}\right)\right]$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^*_{33}+(b_{33})^{(6)}s_{(32),(33)}T^*_{33}\right)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^*_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^*_{33}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^*_{33}+(b_{33})^{(6)}s_{(32),(32)}T^*_{32}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^2+\left(\,(a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^2+\left(\,(b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$+\left(\left((\lambda)^{(6)}\right)^2+\left(\,(a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^*_{32}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^*_{33}+(b_{33})^{(6)}s_{(32),(34)}T^*_{32}\right)\}=0$$

$$+$$

$$(\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^*_{36}\right)\right]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^*_{37}+(b_{37})^{(7)}s_{(36),(37)}T^*_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^*_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^*_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^*_{37}+(b_{37})^{(7)}s_{(36),(36)}T^*_{36}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^2+\left(\,(a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^2+\left(\,(b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$+\left(\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^*_{36}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^*_{37}+(b_{37})^{(7)}s_{(36),(38)}T^*_{36}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\right)\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^*_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^*_{41}+(b_{41})^{(8)}s_{(40),(41)}T^*_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^*_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^*_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^*_{41}+(b_{41})^{(8)}s_{(40),(40)}T^*_{40}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^2+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$+\left(\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^*_{40}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^*_{41}+(b_{41})^{(8)}s_{(40),(42)}T^*_{40}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\right)\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^*_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^*_{45}+(b_{45})^{(9)}s_{(44),(45)}T^*_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^*_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^*_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^*_{45}+(b_{45})^{(9)}s_{(44),(44)}T^*_{44}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^2+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$+\left(\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^*_{44}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^*_{45}+(b_{45})^{(9)}s_{(44),(46)}T^*_{44}\right)\}=0$$

**And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.**

## SECTION THREE

## Fault-Tolerant Quantum Computation By Anyons

### INTRODUCTION—VARIABLES USED

**Fault-Tolerant Quantum Computation with Constant Error Rate Dorit Aharonov, Michael Ben-Or**

(1) The systematic structure of the fault tolerant procedures for (e) polynomial codes is explained in length.
(2) The proof that the concatenation scheme works is written in a clearer way.
(3) The paper also contains new and significantly simpler proofs for (e) most of the known results which we use.
(4) For example, authors give a simple proof that it suffices to correct (e&eb) bit and phase flips.
(5) They significantly simplify Calderbank and Shor's original proof of (e) the correctness of CSS codes.
(6) Authors also give a simple proof of the fact that two-qubit gates are (=) universal.
(7) The paper thus provides a self contained and complete proof for (e) universal fault tolerant quantum computation. Cite as**:arXiv: quant-ph/9906129 (or arXiv: quant-ph/9906129v1 for this version)**

**Fault-tolerant quantum computation by anyons A. Yu. Kitaev (Submitted on 9 Jul 1997)**

(8) A two-dimensional quantum system with anyonic excitations can be considered as (=) a quantum computer.
(9) Unitary transformations can be performed by (e&eb) moving the excitations around each other.
(10) Measurements can be performed by joining excitations in pairs and (e&eb) observing the result of fusion.
(11) Such computation is fault-tolerant by (e) its physical nature. Subjects: Quantum Physics (quant-ph); Mesoscale and Nanoscale Physics (cond-mat.mes-hall); High Energy Physics - Theory (hep-th)Journal reference: Annals Phys. 303 (2003) 2-30 DOI: 10.1016/S0003-4916(02)00018-0 Cite as: arXiv:quant-ph/9707021 (or arXiv:quant-ph/9707021v1 for this

version)

**Protected gates for superconducting qubits Peter Brooks, Alexei Kitaev, John Preskill (Submitted on 17 Feb 2013)**

(12) Authors analyze the accuracy of quantum phase gates acting on (e&eb) "zero-pi qubits" in superconducting circuits, where (e) the gates are protected against thermal and Hamiltonian noise by (e&eb) continuous-variable quantum error-correcting codes.

(13) The gates are executed by turning on and off a (e&eb) tunable Josephson coupling between (e&eb) an LC oscillator and a qubit or pair of quits; assuming perfect qubits, authors show that the (eb) gate errors are exponentially small when the oscillator's impedance sqrt{L/C} is large compared to hbar/4e^2 ~ 1 kilo-ohm.

(14) Protected gates are not computationally universal by themselves, but a scheme for universal fault-tolerant quantum computation can be constructed by combining those with (e&eb) unprotected noisy operations. Authors validate analytic arguments with numerical simulations. Subjects: Quantum Physics (quant-ph); Superconductivity (cond-mat.supr-con) Journal reference: Phys. Rev. A 87, 052306 (2013) DOI: 10.1103/PhysRevA.87.052306 Report number: CALT-68-2911 Cite as: arXiv:1302.4122 [quant-ph] (or arXiv:1302.4122v1 [quant-ph] for this version)

## NOTATION

### Module One

The systematic structure of the fault tolerant procedures for (e) polynomial codes is explained in length

$G_{13}$ : Category one of polynomial codes

$G_{14}$ : Category two of SAS

$G_{15}$ : Category three of SAS

$T_{13}$ : Category one of systematic structure of the fault tolerant procedures

$T_{14}$ : Category two of SAS

$T_{15}$ : Category three of SAS

### Module Two

Authors also give a simple proof of the fact that two-qubit gates are (=) universal

$G_{16}$ : Category one of two-qubit gates

$G_{17}$ : Category two of SAS

$G_{18}$ : Category three of SAS

$T_{16}$ : Category one of universal

$T_{17}$ : Category two of SAS

$T_{18}$ : Category three of SAS

### Module three

A two-dimensional quantum system with anyonic excitations can be considered as (=) a quantum computer

$G_{20}$ : Category one of two-dimensional quantum system with anyonic excitations

$G_{21}$ : Category two of SAS

$G_{22}$ : Category three of SAS

$T_{20}$ : Category one of quantum computer

$T_{21}$ : Category two of SAS

$T_{22}$ : Category three of SAS

**Module four**

**Unitary transformations** can be performed by (e&eb) moving the excitations around each other

$G_{24}$ : Category one of **Unitary transformations;** moving the excitations around each other

$G_{25}$ : Category two of SAS

$G_{26}$ : Category three of SAS

$T_{24}$ : Category one of moving the excitations around each other **;Unitary transformations**

$T_{25}$ : Category two of SAS

$T_{26}$ : Category three of SAS

**Module five**

Measurements can be performed by joining excitations in pairs and (e&eb) observing the result of fusion

$G_{28}$ : Category one of joining excitations in pairs and (e&eb) observing the result of fusion

$G_{29}$ : Category two of SAS

$G_{30}$ : Category three of SAS

$T_{28}$ : Category one of Measurements

$T_{29}$ : Category two of SAS

$T_{30}$ : Category three of SAS

**Module six**

**Measurements can be performed by joining excitations in pairs** and (e&eb) observing the result of fusion

$G_{32}$ : Category one of **Measurements can be performed by joining excitations in pairs;** observing the result of fusion

$G_{33}$ : Category two of SAS

$G_{34}$ : Category three of SAS

$T_{32}$ : Category one of observing the result of fusion **;Measurements can be performed by joining excitations in pairs**

$T_{33}$ : Category two of SAS

$T_{34}$ : Category three of SAS

**Module seven**

Such computation is fault-tolerant by its physical nature.

Subjects: Quantum Physics (quant-ph); Mesoscale and Nanoscale Physics (cond-mat.mes-hall); High Energy Physics - Theory (hep-th)Journal reference: Annals Phys. 303 (2003) 2-30 DOI: 10.1016/S0003-4916(02)00018-0 Cite as: arXiv:quant-ph/9707021 (or arXiv:quant-ph/9707021v1 for this version)

$G_{36}$ : Category one of Such computation

$G_{37}$ : Category two of SAS

$G_{38}$ : Category three of SAS

$T_{36}$ : Category one of fault-tolerant by its physical nature

$T_{37}$ : Category two of SAS

$T_{38}$ : Category three of SAS

**Module eight**

Authors analyze the **accuracy of quantum phase gates** acting on (e&eb) "zero-pi qubits" in superconducting circuits, where (e) the gates are protected against thermal and Hamiltonian noise by (e&eb) continuous-variable quantum error-correcting codes

$G_{40}$ : Category one of **accuracy of quantum phase gates;** "zero-pi qubits" in superconducting circuits, where (e) the gates are protected against thermal and Hamiltonian noise by (e&eb) continuous-variable quantum error-correcting codes

$G_{41}$ : Category two of SAS

$G_{42}$ : Category three of SAS

$T_{40}$ : Category one of "zero-pi qubits" in superconducting circuits, where (e) the gates are protected against thermal and Hamiltonian noise by (e&eb) continuous-variable quantum error-correcting codes **;accuracy of quantum phase gates**

$T_{41}$ : Category two of SAS

$T_{42}$ : Category three of SAS

**Module Nine**

Authors analyze the accuracy of quantum phase gates acting on "zero-pi qubits" in superconducting circuits,

where the gates are protected against thermal and Hamiltonian noise by (e&eb) **continuous-variable quantum error-correcting codes**

$G_{44}$ : Category one of accuracy of quantum phase gates acting on "zero-pi qubits" in superconducting circuits, where the gates are protected against thermal and Hamiltonian noise; **continuous-variable quantum error-correcting codes**

$G_{45}$ : Category two of SAS

$G_{46}$ : Category three of SAS

$T_{44}$ : Category one of **continuous-variable quantum error-correcting codes;** accuracy of quantum phase gates acting on "zero-pi qubits" in superconducting circuits, where the gates are protected against thermal and Hamiltonian noise

$T_{45}$ : Category two of SAS

$T_{46}$ : Category three of SAS

**The Coefficients:**

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}$ , $(b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}$,
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}$, $(b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}\ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$ ,
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)}$,
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)}$,
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)}$,
are Dissipation coefficients
**Module Numbered One**

**The differential system of this model is now (Module Numbered one)**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \quad 1$$
$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad 2$$
$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15} \quad 3$$
$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13} \quad 4$$
$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14} \quad 5$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G,t)\right]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14},t) =$ First augmentation factor

$-(b''_{13})^{(1)}(G,t) =$ First detritions factor

**Module Numbered Two**

**The differential system of this model is now ( Module numbered two)**

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17},t)\right]G_{16} \quad 7$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17},t)\right]G_{17} \quad 8$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17},t)\right]G_{18} \quad 9$$

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\left((G_{19}),t\right)\right]T_{16} \quad 10$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\left((G_{19}),t\right)\right]T_{17} \quad 11$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\left((G_{19}),t\right)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17},t) =$ First augmentation factor

$-(b''_{16})^{(2)}\left((G_{19}),t\right) =$ First detritions factor

**Module Numbered Three**

**The differential system of this model is now (Module numbered three)**

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21},t)\right]G_{20} \quad 13$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21},t)\right]G_{21} \quad 14$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21},t)\right]G_{22} \quad 15$$

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23},t)\right]T_{20} \quad 16$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23},t)\right]T_{21} \quad 17$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23},t)\right]T_{22} \quad 18$$

$+(a''_{20})^{(3)}(T_{21},t) =$ First augmentation factor

$-(b''_{20})^{(3)}(G_{23},t) =$ First detritions factor

**Module Numbered Four**

**The differential system of this model is now (Module numbered Four)**

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right]G_{24} \quad 19$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25},t)\right]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25},t)\right]G_{26} \quad 21$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27}),t\right)\right]T_{24} \quad 22$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27}),t\right)\right]T_{25} \quad 23$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27}),t\right)\right]T_{26} \quad 24$$

$+(a''_{24})^{(4)}(T_{25},t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}\left((G_{27}),t\right) =$ **First detritions factor**

**Module Numbered Five**:

**The differential system of this model is now (Module number five)**

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right]G_{28} \quad 25$$
$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}, t)\right]G_{29} \quad 26$$
$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}, t)\right]G_{30} \quad 27$$
$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31}), t)\right]T_{28} \quad 28$$
$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}((G_{31}), t)\right]T_{29} \quad 29$$
$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31}), t)\right]T_{30} \quad 30$$

$+(a''_{28})^{(5)}(T_{29}, t) =$ **First augmentation factor**

$-(b''_{28})^{(5)}((G_{31}), t) =$ **First detritions factor**

**Module Numbered Six**

**The differential system of this model is now (Module numbered Six)**

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right]G_{32} \quad 31$$
$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}, t)\right]G_{33} \quad 32$$
$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}, t)\right]G_{34} \quad 33$$
$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35}), t)\right]T_{32} \quad 34$$
$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}((G_{35}), t)\right]T_{33} \quad 35$$
$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35}), t)\right]T_{34} \quad 36$$

$+(a''_{32})^{(6)}(T_{33}, t) =$ **First augmentation factor**

**Module Numbered Seven:**

**The differential system of this model is now (Seventh Module)**

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right]G_{36} \quad 37$$
$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37}, t)\right]G_{37} \quad 38$$
$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}, t)\right]G_{38} \quad 39$$
$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39}), t)\right]T_{36} \quad 40$$
$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}((G_{39}), t)\right]T_{37} \quad 41$$
$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39}), t)\right]T_{38} \quad 42$$

$+(a''_{36})^{(7)}(T_{37}, t) =$ **First augmentation factor**

**Module Numbered Eight**

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right]G_{40} \quad 43$$
$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41}, t)\right]G_{41} \quad 44$$
$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}, t)\right]G_{42} \quad 45$$
$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43}), t)\right]T_{40} \quad 46$$
$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}((G_{43}), t)\right]T_{41} \quad 47$$
$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43}), t)\right]T_{42} \quad 48$$

**Module Numbered Nine**

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}\left((G_{47}),t\right)\right]T_{44} \quad 52$$

$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}\left((G_{47}),t\right)\right]T_{45} \quad 53$$

$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}\left((G_{47}),t\right)\right]T_{46} \quad 54$$

$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}\left((G_{47}),t\right) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)}\boxed{+(a''_{13})^{(1)}(T_{14},t)}\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7)}(T_{37},t)}\boxed{+(a''_{40})^{(8,8)}(T_{41},t)}\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)}\boxed{+(a''_{14})^{(1)}(T_{14},t)}\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7)}(T_{37},t)}\boxed{+(a''_{41})^{(8,8)}(T_{41},t)}\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)}\boxed{+(a''_{15})^{(1)}(T_{14},t)}\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7)}(T_{37},t)}\boxed{+(a''_{42})^{(8,8)}(T_{41},t)}\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15} \quad 57$$

Where $\boxed{(a''_{13})^{(1)}(T_{14},t)}$, $\boxed{(a''_{14})^{(1)}(T_{14},t)}$, $\boxed{(a''_{15})^{(1)}(T_{14},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7)}(T_{37},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8)}(T_{41},t)}$ $\boxed{+(a''_{41})^{(8,8)}(T_{41},t)}$ $\boxed{+(a''_{42})^{(8,8)}(T_{41},t)}$ are eight augmentation coefficient for 1,2,3

$\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)}\boxed{-(b''_{13})^{(1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13} \quad 58$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)}\boxed{-(b''_{14})^{(1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14} \quad 59$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)}\boxed{-(b''_{15})^{(1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15} \quad 60$$

Where $\boxed{-(b''_{13})^{(1)}(G,t)}$, $\boxed{-(b''_{14})^{(1)}(G,t)}$, $\boxed{-(b''_{15})^{(1)}(G,t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8)}(G_{43},t)}$ $\boxed{-(b''_{41})^{(8,8)}(G_{43},t)}$ $\boxed{-(b''_{42})^{(8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \begin{bmatrix} (a'_{16})^{(2)}\boxed{+(a''_{16})^{(2)}(T_{17},t)} & \boxed{+(a''_{13})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{16} \quad 61$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \begin{bmatrix} (a'_{17})^{(2)}\boxed{+(a''_{17})^{(2)}(T_{17},t)} & \boxed{+(a''_{14})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{17} \quad 62$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \begin{bmatrix} (a'_{18})^{(2)}\boxed{+(a''_{18})^{(2)}(T_{17},t)} & \boxed{+(a''_{15})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{18} \quad (63)$$

Where $\boxed{+(a''_{16})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,)}(T_{14},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)}$ are eight augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{44})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9)}(T_{45},t)}$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \begin{bmatrix} (b'_{16})^{(2)}\boxed{-(b''_{16})^{(2)}(G_{19},t)} & \boxed{-(b''_{13})^{(1,1,)}(G,t)} & \boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{16} \quad (64)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)}\boxed{-(b''_{17})^{(2)}(G_{19},t)} & \boxed{-(b''_{14})^{(1,1,)}(G,t)} & \boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{17} \quad (65)$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)}\boxed{-(b''_{18})^{(2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,)}(G,t)} & \boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{18} \quad (66)$$

where $\boxed{-(b''_{16})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2)}(G_{19},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,)}(G,t)}$ are second detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)}$ are fourth detrition

coefficients for category 1,2 and 3

$-(b_{28}'')^{(5,5,5,5,5)}(G_{31},t)$, $-(b_{29}'')^{(5,5,5,5,5)}(G_{31},t)$, $-(b_{30}'')^{(5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1,2 and 3

$-(b_{32}'')^{(6,6,6,6,6)}(G_{35},t)$, $-(b_{33}'')^{(6,6,6,6,6)}(G_{35},t)$, $-(b_{34}'')^{(6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients for category 1,2 and 3

$-(b_{36}'')^{(7,7,7)}(G_{39},t)$, $-(b_{37}'')^{(7,7,7)}(G_{39},t)$, $-(b_{38}'')^{(7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2 and 3

$-(b_{40}'')^{(8,8,8)}(G_{43},t)$, $-(b_{41}'')^{(8,8,8)}(G_{43},t)$, $-(b_{42}'')^{(8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1,2 and 3

$-(b_{44}'')^{(9,9)}(G_{47},t)$, $-(b_{46}'')^{(9,9)}(G_{47},t)$, $-(b_{45}'')^{(9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[\begin{array}{c}(a_{20}')^{(3)}+(a_{20}'')^{(3)}(T_{21},t)+(a_{16}'')^{(2,2,2)}(T_{17},t)+(a_{13}'')^{(1,1,1,)}(T_{14},t)\\ +(a_{24}'')^{(4,4,4,4,4,4)}(T_{25},t)+(a_{28}'')^{(5,5,5,5,5,5)}(T_{29},t)+(a_{32}'')^{(6,6,6,6,6,6)}(T_{33},t)\\ +(a_{36}'')^{(7,7,7,7)}(T_{37},t)+(a_{40}'')^{(8,8,8,8)}(T_{41},t)+(a_{44}'')^{(9,9,9)}(T_{45},t)\end{array}\right]G_{20} \quad (67)$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[\begin{array}{c}(a_{21}')^{(3)}+(a_{21}'')^{(3)}(T_{21},t)+(a_{17}'')^{(2,2,2)}(T_{17},t)+(a_{14}'')^{(1,1,1,)}(T_{14},t)\\ +(a_{25}'')^{(4,4,4,4,4,4)}(T_{25},t)+(a_{29}'')^{(5,5,5,5,5,5)}(T_{29},t)+(a_{33}'')^{(6,6,6,6,6,6)}(T_{33},t)\\ +(a_{37}'')^{(7,7,7,7)}(T_{37},t)+(a_{41}'')^{(8,8,8,8)}(T_{41},t)+(a_{45}'')^{(9,9,9)}(T_{45},t)\end{array}\right]G_{21} \quad (68)$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[\begin{array}{c}(a_{22}')^{(3)}+(a_{22}'')^{(3)}(T_{21},t)+(a_{18}'')^{(2,2,2)}(T_{17},t)+(a_{15}'')^{(1,1,1,)}(T_{14},t)\\ +(a_{26}'')^{(4,4,4,4,4,4)}(T_{25},t)+(a_{30}'')^{(5,5,5,5,5,5)}(T_{29},t)+(a_{34}'')^{(6,6,6,6,6,6)}(T_{33},t)\\ +(a_{38}'')^{(7,7,7,7)}(T_{37},t)+(a_{42}'')^{(8,8,8,8)}(T_{41},t)+(a_{46}'')^{(9,9,9)}(T_{45},t)\end{array}\right]G_{22} \quad (69)$$

$+(a_{20}'')^{(3)}(T_{21},t)$, $+(a_{21}'')^{(3)}(T_{21},t)$, $+(a_{22}'')^{(3)}(T_{21},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a_{16}'')^{(2,2,2)}(T_{17},t)$, $+(a_{17}'')^{(2,2,2)}(T_{17},t)$, $+(a_{18}'')^{(2,2,2)}(T_{17},t)$ are second augmentation coefficients for category 1, 2 and 3

$+(a_{13}'')^{(1,1,1,)}(T_{14},t)$, $+(a_{14}'')^{(1,1,1,)}(T_{14},t)$, $+(a_{15}'')^{(1,1,1,)}(T_{14},t)$ are third augmentation coefficients for category 1, 2 and 3

$+(a_{24}'')^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a_{25}'')^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a_{26}'')^{(4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficients for category 1, 2 and 3

$+(a_{28}'')^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a_{29}'')^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a_{30}'')^{(5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficients for category 1, 2 and 3

$+(a_{32}'')^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a_{33}'')^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a_{34}'')^{(6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficients for category 1, 2 and 3

$+(a_{36}'')^{(7,7,7,7)}(T_{37},t)$, $+(a_{37}'')^{(7,7,7,7)}(T_{37},t)$, $+(a_{38}'')^{(7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1, 2 and 3

$+(a_{40}'')^{(8,8,8,8)}(T_{41},t)$, $+(a_{41}'')^{(8,8,8,8)}(T_{41},t)$, $+(a_{42}'')^{(8,8,8,8)}(T_{41},t)$ are eight augmentation coefficients for category 1, 2 and 3

$+(a_{44}'')^{(9,9,9)}(T_{45},t)$, $+(a_{45}'')^{(9,9,9)}(T_{45},t)$, $+(a_{46}'')^{(9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for

category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b'_{20})^{(3)}\boxed{-(b''_{20})^{(3)}(G_{23},t)}\boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}\boxed{-(b''_{13})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)}\boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)}\boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7)}(G_{39},t)}\boxed{-(b''_{40})^{(8,8,8,8)}(G_{43},t)}\boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{20} \quad 70$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b'_{21})^{(3)}\boxed{-(b''_{21})^{(3)}(G_{23},t)}\boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}\boxed{-(b''_{14})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)}\boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)}\boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7)}(G_{39},t)}\boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)}\boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{21} \quad 71$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)}\boxed{-(b''_{22})^{(3)}(G_{23},t)}\boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}\boxed{-(b''_{15})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)}\boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)}\boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)}\boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)}\boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{22} \quad 72$$

$\boxed{-(b''_{20})^{(3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3)}(G_{23},t)}$ *are first detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}$ *are second detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{13})^{(1,1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,)}(G,t)}$ *are third detrition coefficients for category 1,2 and 3*

$\boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)}$ *are fourth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)}$ *are fifth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)}$ *are sixth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{36})^{(7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7)}(G_{39},t)}$ $\boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \begin{bmatrix} (a'_{24})^{(4)}\boxed{+(a''_{24})^{(4)}(T_{25},t)}\boxed{+(a''_{28})^{(5,5,)}(T_{29},t)}\boxed{+(a''_{32})^{(6,6,)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)}\boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)}\boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)}\boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)}\boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{24} \quad 73$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \begin{bmatrix} (a'_{25})^{(4)}\boxed{+(a''_{25})^{(4)}(T_{25},t)}\boxed{+(a''_{29})^{(5,5,)}(T_{29},t)}\boxed{+(a''_{33})^{(6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)}\boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)}\boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)}\boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)}\boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{25} \quad 74$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \begin{bmatrix} (a'_{26})^{(4)} & +(a''_{26})^{(4)}(T_{25},t) & +(a''_{30})^{(5,5,)}(T_{29},t) & +(a''_{34})^{(6,6,)}(T_{33},t) \\ +(a''_{15})^{(1,1,1,1)}(T_{14},t) & +(a''_{18})^{(2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3)}(T_{21},t) \\ +(a''_{38})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9)}(T_{45},t) \end{bmatrix} G_{26} \quad 75$$

$(a''_{24})^{(4)}(T_{25},t)$, $(a''_{25})^{(4)}(T_{25},t)$, $(a''_{26})^{(4)}(T_{25},t)$ *are first augmentation coefficients category* 1, 2 3

$+(a''_{28})^{(5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,)}(T_{29},t)$ *are second augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{32})^{(6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,)}(T_{33},t)$ *are third augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{13})^{(1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1)}(T_{14},t)$ *are fourth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{16})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2)}(T_{17},t)$ *are fifth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{20})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3)}(T_{21},t)$ *are sixth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)$ *are seventh augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)$ *are eighth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{46})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9)}(T_{45},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \begin{bmatrix} (b'_{24})^{(4)} & -(b''_{24})^{(4)}(G_{27},t) & -(b''_{28})^{(5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,)}(G_{35},t) \\ -(b''_{13})^{(1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3)}(G_{23},t) \\ -(b''_{36})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{24} \quad 76$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \begin{bmatrix} (b'_{25})^{(4)} & -(b''_{25})^{(4)}(G_{27},t) & -(b''_{29})^{(5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,)}(G_{35},t) \\ -(b''_{14})^{(1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3)}(G_{23},t) \\ -(b''_{37})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{25} \quad 77$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)} & -(b''_{26})^{(4)}(G_{27},t) & -(b''_{30})^{(5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,)}(G_{35},t) \\ -(b''_{15})^{(1,1,1,1)}(G,t) & -(b''_{18})^{(2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3)}(G_{23},t) \\ -(b''_{38})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{26} \quad 78$$

*Where* $-(b''_{24})^{(4)}(G_{27},t)$, $-(b''_{25})^{(4)}(G_{27},t)$, $-(b''_{26})^{(4)}(G_{27},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{28})^{(5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,)}(G_{31},t)$ *are second detrition coefficients for category* 1, 2 *and* 3

$-(b''_{32})^{(6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,)}(G_{35},t)$ *are third detrition coefficients for category* 1, 2 *and* 3

$-(b''_{13})^{(1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1)}(G,t)$

*are fourth detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b_{16}'')^{(2,2,2,2)}(G_{19},t)}, \boxed{-(b_{17}'')^{(2,2,2,2)}(G_{19},t)}, \boxed{-(b_{18}'')^{(2,2,2,2)}(G_{19},t)}$

*are fifth detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b_{20}'')^{(3,3,3,3)}(G_{23},t)}, \boxed{-(b_{21}'')^{(3,3,3,3)}(G_{23},t)}, \boxed{-(b_{22}'')^{(3,3,3,3)}(G_{23},t)}$

*are sixth detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b_{36}'')^{(7,7,7,7,7)}(G_{39},t)}, \boxed{-(b_{37}'')^{(7,7,7,7,7)}(G_{39},t)}, \boxed{-(b_{38}'')^{(7,7,7,7,7)}(G_{39},t)}$

*are seventh detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b_{40}'')^{(8,8,8,8,8)}(G_{43},t)}, \boxed{-(b_{41}'')^{(8,8,8,8,8)}(G_{43},t)}, \boxed{-(b_{42}'')^{(8,8,8,8,8)}(G_{43},t)}$

*are eighth detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b_{46}'')^{(9,9,9,9)}(G_{47},t)}, \boxed{-(b_{45}'')^{(9,9,9,9)}(G_{47},t)}, \boxed{-(b_{44}'')^{(9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[\begin{array}{ccc} (a_{28}')^{(5)} & \boxed{+(a_{28}'')^{(5)}(T_{29},t)} & \boxed{+(a_{24}'')^{(4,4,)}(T_{25},t)} \quad \boxed{+(a_{32}'')^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{16}'')^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{20}'')^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{36}'')^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{40}'')^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a_{44}'')^{(9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{28} \qquad 79$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[\begin{array}{ccc} (a_{29}')^{(5)} & \boxed{+(a_{29}'')^{(5)}(T_{29},t)} & \boxed{+(a_{25}'')^{(4,4,)}(T_{25},t)} \quad \boxed{+(a_{33}'')^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a_{14}'')^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{17}'')^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{21}'')^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{37}'')^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{41}'')^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a_{45}'')^{(9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{29} \qquad 80$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[\begin{array}{ccc} (a_{30}')^{(5)} & \boxed{+(a_{30}'')^{(5)}(T_{29},t)} & \boxed{+(a_{26}'')^{(4,4,)}(T_{25},t)} \quad \boxed{+(a_{34}'')^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a_{15}'')^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{18}'')^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{22}'')^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{38}'')^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{42}'')^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a_{46}'')^{(9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{30} \qquad 81$$

*Where* $\boxed{+(a_{28}'')^{(5)}(T_{29},t)}, \boxed{+(a_{29}'')^{(5)}(T_{29},t)}, \boxed{+(a_{30}'')^{(5)}(T_{29},t)}$ *are first augmentation coefficients for category* 1, 2 *and* 3

*And* $\boxed{+(a_{24}'')^{(4,4,)}(T_{25},t)}, \boxed{+(a_{25}'')^{(4,4,)}(T_{25},t)}, \boxed{+(a_{26}'')^{(4,4,)}(T_{25},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a_{32}'')^{(6,6,6)}(T_{33},t)}, \boxed{+(a_{33}'')^{(6,6,6)}(T_{33},t)}, \boxed{+(a_{34}'')^{(6,6,6)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a_{13}'')^{(1,1,1,1,1)}(T_{14},t)}, \boxed{+(a_{14}'')^{(1,1,1,1,1)}(T_{14},t)}, \boxed{+(a_{15}'')^{(1,1,1,1,1)}(T_{14},t)}$ are fourth augmentation coefficients for category 1,2, and 3

$\boxed{+(a_{16}'')^{(2,2,2,2,2)}(T_{17},t)}, \boxed{+(a_{17}'')^{(2,2,2,2,2)}(T_{17},t)}, \boxed{+(a_{18}'')^{(2,2,2,2,2)}(T_{17},t)}$ are fifth augmentation coefficients for category 1,2,and 3

$\boxed{+(a_{20}'')^{(3,3,3,3,3)}(T_{21},t)}, \boxed{+(a_{21}'')^{(3,3,3,3,3)}(T_{21},t)}, \boxed{+(a_{22}'')^{(3,3,3,3,3)}(T_{21},t)}$ are sixth augmentation coefficients for category 1,2, 3

$\boxed{+(a_{36}'')^{(7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a_{37}'')^{(7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a_{38}'')^{(7,7,7,7,7,7)}(T_{37},t)}$ are seventh augmentation coefficients for category 1,2, 3

$\boxed{+(a_{40}'')^{(8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a_{41}'')^{(8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a_{42}'')^{(8,8,8,8,8,8)}(T_{41},t)}$ are eighth augmentation coefficients for category 1,2, 3

$\boxed{+(a_{46}'')^{(9,9,9,9,9)}(T_{45},t)}, \boxed{+(a_{45}'')^{(9,9,9,9,9)}(T_{45},t)}, \boxed{+(a_{44}'')^{(9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation

coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \begin{bmatrix} (b'_{28})^{(5)}\boxed{-(b''_{28})^{(5)}(G_{31},t)} & \boxed{-(b''_{24})^{(4,4,)}(G_{27},t)} & \boxed{-(b''_{32})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{28} \quad (82)$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b'_{29})^{(5)}\boxed{-(b''_{29})^{(5)}(G_{31},t)} & \boxed{-(b''_{25})^{(4,4,)}(G_{27},t)} & \boxed{-(b''_{33})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{29} \quad (83)$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b'_{30})^{(5)}\boxed{-(b''_{30})^{(5)}(G_{31},t)} & \boxed{-(b''_{26})^{(4,4,)}(G_{27},t)} & \boxed{-(b''_{34})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{30} \quad (84)$$

*where* $\boxed{-(b''_{28})^{(5)}(G_{31},t)}$ , $\boxed{-(b''_{29})^{(5)}(G_{31},t)}$ , $\boxed{-(b''_{30})^{(5)}(G_{31},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{24})^{(4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,)}(G_{27},t)}$ *are second detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{32})^{(6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6)}(G_{35},t)}$ *are third detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1)}(G,t)}$ , $\boxed{-(b''_{15})^{(1,1,1,1,1,)}(G,t)}$ are fourth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)}$ , $\boxed{-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)}$ , $\boxed{-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)}$ , $\boxed{-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)}$ , $\boxed{-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)}$ , $\boxed{-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)}$ , $\boxed{-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \begin{bmatrix} (a'_{32})^{(6)}\boxed{+(a''_{32})^{(6)}(T_{33},t)} & \boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{32} \quad (85)$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \begin{bmatrix} (a'_{33})^{(6)}\boxed{+(a''_{33})^{(6)}(T_{33},t)} & \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{33} \quad (86)$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \begin{bmatrix} (a'_{34})^{(6)}\boxed{+(a''_{34})^{(6)}(T_{33},t)} & \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{34} \quad 87$$

$\boxed{+(a''_{32})^{(6)}(T_{33},t)}, \boxed{+(a''_{33})^{(6)}(T_{33},t)}, \boxed{+(a''_{34})^{(6)}(T_{33},t)}$ *are first augmentation coefficients for category* $1, 2$ *and* $3$

$\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}$ *are second augmentation coefficients for category* $1, 2$ *and* $3$

$\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}$ *are third augmentation coefficients for category* $1, 2$ *and* $3$

$\boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}$ - are fourth augmentation coefficients

$\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}$ - fifth augmentation coefficients

$\boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)}$ sixth augmentation coefficients

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)}$ seventh augmentation coefficients

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}$ Eighth augmentation coefficients

$\boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)}$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \begin{bmatrix} (b'_{32})^{(6)}\boxed{-(b''_{32})^{(6)}(G_{35},t)} & \boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{32} \quad 88$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \begin{bmatrix} (b'_{33})^{(6)}\boxed{-(b''_{33})^{(6)}(G_{35},t)} & \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{33} \quad 89$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)}\boxed{-(b''_{34})^{(6)}(G_{35},t)} & \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{34} \quad 90$$

$\boxed{-(b''_{32})^{(6)}(G_{35},t)}, \boxed{-(b''_{33})^{(6)}(G_{35},t)}, \boxed{-(b''_{34})^{(6)}(G_{35},t)}$ *are first detrition coefficients for category* $1, 2$ *and* $3$

$\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}$ *are second detrition coefficients for category* $1, 2$ *and* $3$

$\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)}$ *are third detrition coefficients*

$for\ category\ 1,2\ and\ 3$

$-(b_{13}'')^{(1,1,1,1,1,1,1)}(G,t)$, $-(b_{14}'')^{(1,1,1,1,1,1,1)}(G,t)$, $-(b_{15}'')^{(1,1,1,1,1,1,1)}(G,t)$ are fourth detrition coefficients for category 1, 2, and 3

$-(b_{16}'')^{(2,2,2,2,2,2,2)}(G_{19},t)$, $-(b_{17}'')^{(2,2,2,2,2,2,2)}(G_{19},t)$, $-(b_{18}'')^{(2,2,2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1, 2, and 3

$-(b_{20}'')^{(3,3,3,3,3,3,3)}(G_{23},t)$, $-(b_{21}'')^{(3,3,3,3,3,3,3)}(G_{23},t)$, $-(b_{22}'')^{(3,3,3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1, 2, and 3

$-(b_{36}'')^{(7,7,7,7,7,7,7)}(G_{39},t)$, $-(b_{37}'')^{(7,7,7,7,7,7,7)}(G_{39},t)$, $-(b_{38}'')^{(7,7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2, and 3

$-(b_{40}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)$, $-(b_{41}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)$, $-(b_{42}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1, 2, and 3

$-(b_{46}'')^{(9,9,9,9,9,9,9)}(G_{47},t)$, $-(b_{45}'')^{(9,9,9,9,9,9,9)}(G_{47},t)$, $-(b_{44}'')^{(9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \qquad 91$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a_{36}')^{(7)} & +(a_{36}'')^{(7)}(T_{37},t) & +(a_{16}'')^{(2,2,2,2,2,2,2)}(T_{17},t) & +(a_{20}'')^{(3,3,3,3,3,3,3)}(T_{21},t) \\ & +(a_{24}'')^{(4,4,4,4,4,4,4)}(T_{25},t) & +(a_{28}'')^{(5,5,5,5,5,5,5)}(T_{29},t) & +(a_{32}'')^{(6,6,6,6,6,6,6)}(T_{33},t) \\ & +(a_{13}'')^{(1,1,1,1,1,1,1)}(T_{14},t) & +(a_{40}'')^{(8,8,8,8,8,8,8,8,)}(T_{41},t) & +(a_{44}'')^{(9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \qquad 92$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a_{37}')^{(7)} & +(a_{37}'')^{(7)}(T_{37},t) & +(a_{17}'')^{(2,2,2,2,2,2,2)}(T_{17},t) & +(a_{21}'')^{(3,3,3,3,3,3,3)}(T_{21},t) \\ & +(a_{25}'')^{(4,4,4,4,4,4,4)}(T_{25},t) & +(a_{29}'')^{(5,5,5,5,5,5,5)}(T_{29},t) & +(a_{33}'')^{(6,6,6,6,6,6,6)}(T_{33},t) \\ & +(a_{13}'')^{(1,1,1,1,1,1,1)}(T_{14},t) & +(a_{41}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t) & +(a_{45}'')^{(9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{14}$$

$$\frac{dG_{38}}{dt} \qquad 93$$

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a_{38}')^{(7)} & +(a_{38}'')^{(7)}(T_{37},t) & +(a_{18}'')^{(2,2,2,2,2,2,2)}(T_{17},t) & +(a_{22}'')^{(3,3,3,3,3,3,3)}(T_{21},t) \\ & +(a_{26}'')^{(4,4,4,4,4,4,4)}(T_{25},t) & +(a_{30}'')^{(5,5,5,5,5,5,5)}(T_{29},t) & +(a_{34}'')^{(6,6,6,6,6,6,6)}(T_{33},t) \\ & +(a_{15}'')^{(1,1,1,1,1,1,1)}(T_{14},t) & +(a_{42}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t) & +(a_{46}'')^{(9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{15}$$

Where $(a_{36}'')^{(7)}(T_{37},t)$, $(a_{37}'')^{(7)}(T_{37},t)$, $(a_{38}'')^{(7)}(T_{37},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a_{16}'')^{(2,2,2,2,2,2,2)}(T_{17},t)$, $+(a_{17}'')^{(2,2,2,2,2,2,2)}(T_{17},t)$, $+(a_{18}'')^{(2,2,2,2,2,2,2)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a_{20}'')^{(3,3,3,3,3,3,3)}(T_{21},t)$, $+(a_{21}'')^{(3,3,3,3,3,3,3)}(T_{21},t)$, $+(a_{22}'')^{(3,3,3,3,3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a_{24}'')^{(4,4,4,4,4,4,4)}(T_{25},t)$, $+(a_{25}'')^{(4,4,4,4,4,4,4)}(T_{25},t)$, $+(a_{26}'')^{(4,4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a_{28}'')^{(5,5,5,5,5,5,5)}(T_{29},t)$, $+(a_{29}'')^{(5,5,5,5,5,5,5)}(T_{29},t)$, $+(a_{30}'')^{(5,5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a_{32}'')^{(6,6,6,6,6,6,6)}(T_{33},t)$, $+(a_{33}'')^{(6,6,6,6,6,6,6)}(T_{33},t)$, $+(a_{34}'')^{(6,6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation

coefficient for category 1, 2 and 3
$+(a_{13}'')^{(1,1,1,1,1,1,1)}(T_{14},t)$, $+(a_{13}'')^{(1,1,1,1,1,1,1)}(T_{14},t)$, $+(a_{15}'')^{(1,1,1,1,1,1,1)}(T_{14},t)$ are seventh augmentation coefficient for category 1, 2 and 3
$+(a_{42}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a_{41}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a_{40}'')^{(8,8,8,8,8,8,8,8,)}(T_{41},t)$ are eighth augmentation coefficient for 1,2,3
$+(a_{46}'')^{(9,9,9,9,9,9,9)}(T_{45},t)$, $+(a_{45}'')^{(9,9,9,9,9,9,9)}(T_{45},t)$, $+(a_{44}'')^{(9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3



$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \begin{bmatrix} (b_{36}')^{(7)}\boxed{-(b_{36}'')^{(7)}(G_{39},t)} & \boxed{-(b_{16}'')^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{20}'')^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{24}'')^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{28}'')^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{32}'')^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{13}'')^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b_{44}'')^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \begin{bmatrix} (b_{37}')^{(7)}\boxed{-(b_{37}'')^{(7)}(G_{39},t)} & \boxed{-(b_{17}'')^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{21}'')^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{25}'')^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{29}'')^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{33}'')^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{14}'')^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b_{45}'')^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \begin{bmatrix} (b_{38}')^{(7)}\boxed{-(b_{38}'')^{(7)}(G_{39},t)} & \boxed{-(b_{18}'')^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{22}'')^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{26}'')^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{30}'')^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{34}'')^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{15}'')^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b_{46}'')^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $-(b_{36}'')^{(7)}(G_{39},t)$, $-(b_{37}'')^{(7)}(G_{39},t)$, $-(b_{38}'')^{(7)}(G_{39},t)$ are first detrition coefficients for category 1, 2 and 3
$-(b_{16}'')^{(2,2,2,2,2,2,2)}(G_{19},t)$, $-(b_{17}'')^{(2,2,2,2,2,2,2)}(G_{19},t)$, $-(b_{18}'')^{(2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3
$-(b_{20}'')^{(3,3,3,3,3,3,3)}(G_{23},t)$, $-(b_{21}'')^{(3,3,3,3,3,3,3)}(G_{23},t)$, $-(b_{22}'')^{(3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3
$-(b_{24}'')^{(4,4,4,4,4,4,4)}(G_{27},t)$, $-(b_{25}'')^{(4,4,4,4,4,4,4)}(G_{27},t)$, $-(b_{26}'')^{(4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3
$-(b_{28}'')^{(5,5,5,5,5,5,5)}(G_{31},t)$, $-(b_{29}'')^{(5,5,5,5,5,5,5)}(G_{31},t)$, $-(b_{30}'')^{(5,5,5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3
$-(b_{32}'')^{(6,6,6,6,6,6,6)}(G_{35},t)$, $-(b_{33}'')^{(6,6,6,6,6,6,6)}(G_{35},t)$, $-(b_{34}'')^{(6,6,6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3
$-(b_{15}'')^{(1,1,1,1,1,1,1)}(G,t)$, $-(b_{14}'')^{(1,1,1,1,1,1,1)}(G,t)$, $-(b_{13}'')^{(1,1,1,1,1,1,1)}(G,t)$ are seventh detrition coefficients for category 1, 2 and 3
$-(b_{40}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)$, $-(b_{41}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)$, $-(b_{42}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3



$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \begin{bmatrix} (a'_{40})^{(8)}\boxed{+(a''_{40})^{(8)}(T_{41},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \begin{bmatrix} (a'_{41})^{(8)}\boxed{+(a''_{41})^{(8)}(T_{41},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \begin{bmatrix} (a'_{42})^{(8)}\boxed{+(a''_{42})^{(8)}(T_{41},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a''_{40})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} =$$

$$(b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)}\boxed{-(b''_{40})^{(8)}(G_{43},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} =$$

$$(b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)}\boxed{-(b''_{41})^{(8)}(G_{43},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} =$$

$$(b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)}\boxed{-(b''_{42})^{(8)}(G_{43},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3



$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \begin{bmatrix} (a'_{44})^{(9)}\boxed{+(a''_{44})^{(9)}(T_{45},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \begin{bmatrix} (a'_{45})^{(9)}\boxed{+(a''_{45})^{(9)}(T_{45},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \begin{bmatrix} (a'_{46})^{(9)}\boxed{+(a''_{46})^{(9)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a''_{44})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ are Seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} -$$

$$\begin{bmatrix} (b'_{44})^{(9)}\boxed{-(b''_{44})^{(9)}(G_{47},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b'_{45})^{(9)}\boxed{-(b''_{45})^{(9)}(G_{47},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b'_{46})^{(9)}\boxed{-(b''_{46})^{(9)}(G_{47},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{44})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9)}(G_{47},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a'_i)^{(1)}, (a''_i)^{(1)}, (b_i)^{(1)}, (b'_i)^{(1)}, (b''_i)^{(1)} > 0, \quad i,j = 13,14,15 \qquad 97$$

The functions $(a''_i)^{(1)}, (b''_i)^{(1)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$(a_i'')^{(1)}(T_{14},t) \leq (p_i)^{(1)} \leq (\hat{A}_{13})^{(1)}$

$(b_i'')^{(1)}(G,t) \leq (r_i)^{(1)} \leq (b_i')^{(1)} \leq (\hat{B}_{13})^{(1)}$

$lim_{T_2\to\infty}(a_i'')^{(1)}(T_{14},t) = (p_i)^{(1)}$ 98

$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = (r_i)^{(1)}$

**Definition of** $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$ :

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$|(a_i'')^{(1)}(T_{14}',t) - (a_i'')^{(1)}(T_{14},t)| \leq (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$

$|(b_i'')^{(1)}(G',t) - (b_i'')^{(1)}(G,t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}',t)$ and $(a_i'')^{(1)}(T_{14},t)$ . $(T_{14}',t)$ and $(T_{14},t)$ are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14},t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$ : 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}$ , $\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$

**Definition of** $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$ : 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$

$\frac{1}{(\hat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$

Where we suppose

$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i,j = 16,17,18$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17},t) \le (p_i)^{(2)} \le \left(\hat{A}_{16}\right)^{(2)} \quad 102$$

$$(b_i'')^{(2)}(G_{19},t) \le (r_i)^{(2)} \le (b_i')^{(2)} \le (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2\to\infty}(a_i'')^{(2)}(T_{17},t) = (p_i)^{(2)} \quad 104$$

$$\lim_{G\to\infty}(b_i'')^{(2)}\left((G_{19}),t\right) = (r_i)^{(2)} \quad 105$$

**Definition of** $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$ : 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}',t) - (a_i'')^{(2)}(T_{17},t)| \le (\hat{k}_{16})^{(2)}|T_{17} - T_{17}'|e^{-(\hat{M}_{16})^{(2)}t} \quad 107$$

$$|(b_i'')^{(2)}((G_{19})',t) - (b_i'')^{(2)}\left((G_{19}),t\right)| < (\hat{k}_{16})^{(2)}||(G_{19}) - (G_{19})'||e^{-(\hat{M}_{16})^{(2)}t} \quad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}',t)$ and $(a_i'')^{(2)}(T_{17},t)$ . $(T_{17}',t)$ and $(T_{17},t)$ are points belonging to the interval $\left[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}\right]$ . It is to be noted that $(a_i'')^{(2)}(T_{17},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17},t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$ :

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants 109

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$$

**Definition of** $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$ :

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1 \quad 110$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(b_i)^{(2)} + (b_i')^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1 \quad 111$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i,j = 20,21,22 \quad 112$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(3)}$, $(\mathrm{r_i})^{(3)}$:

$$(a_i'')^{(3)}(T_{21},t) \le (p_i)^{(3)} \le (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23},t) \le (r_i)^{(3)} \le (b_i')^{(3)} \le (\hat{B}_{20})^{(3)}$$

$$lim_{T_2\to\infty}(a_i'')^{(3)}(T_{21},t) = (p_i)^{(3)} \qquad 113$$

$$\lim_{G\to\infty}(b_i'')^{(3)}(G_{23},t) = (r_i)^{(3)}$$

**Definition of** $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$ :

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: 114

$$|(a_i'')^{(3)}(T_{21}',t) - (a_i'')^{(3)}(T_{21},t)| \le (\hat{k}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\hat{M}_{20})^{(3)}t}$$

$$|(b_i'')^{(3)}(G_{23}',t) - (b_i'')^{(3)}(G_{23},t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}'||e^{-(\hat{M}_{20})^{(3)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}',t)$ and$(a_i'')^{(3)}(T_{21},t)$ . $(T_{21}',t)$ And $(T_{21},t)$ are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$ : 115

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$ and the constants $(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}, i = 20,21,22,$ satisfy the inequalities 116

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(a_i)^{(3)} + (a_i')^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(b_i)^{(3)} + (b_i')^{(3)} + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i,j = 24,25,26 \qquad 117$$

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(4)}$, $(r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25},t) \le (p_i)^{(4)} \le (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}\big((G_{27}),t\big) \le (r_i)^{(4)} \le (b_i')^{(4)} \le (\hat{B}_{24})^{(4)}$$

$$lim_{T_2\to\infty}(a_i'')^{(4)}(T_{25},t) = (p_i)^{(4)} \quad (118)$$

$$\lim_{G\to\infty}(b_i'')^{(4)}\big((G_{27}),t\big) = (r_i)^{(4)}$$

**Definition of** $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$ :

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24,25,26}$

They satisfy Lipschitz condition: (119)

$$|(a_i'')^{(4)}(T_{25}',t) - (a_i'')^{(4)}(T_{25},t)| \le (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})',t) - (b_i'')^{(4)}\big((G_{27}),t\big)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}',t)$ and$(a_i'')^{(4)}(T_{25},t)$ . $(T_{25}',t)$ and $(T_{25},t)$ are points belonging to the interval $\big[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}\big]$. It is to be noted that $(a_i'')^{(4)}(T_{25},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25},t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$ : (120)

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$, are positive constants

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}, \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$$

**Definition of** $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$ : (121)

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and\ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}, i = 24,25,26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(a_i)^{(4)} + (a_i')^{(4)} + (\hat{A}_{24})^{(4)} + (\hat{P}_{24})^{(4)}(\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(b_i)^{(4)} + (b_i')^{(4)} + (\hat{B}_{24})^{(4)} + (\hat{Q}_{24})^{(4)}(\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i,j = 28,29,30 \quad (122)$$

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(5)}$, $(r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29},t) \le (p_i)^{(5)} \le (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}\big((G_{31}),t\big) \le (r_i)^{(5)} \le (b_i')^{(5)} \le (\hat{B}_{28})^{(5)}$$

$$lim_{T_2\to\infty}(a_i'')^{(5)}(T_{29},t) = (p_i)^{(5)} \qquad 123$$

$$\lim_{G\to\infty}(b_i'')^{(5)}(G_{31},t) = (r_i)^{(5)}$$

**Definition of** $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$ :

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}',t) - (a_i'')^{(5)}(T_{29},t)| \le (\hat{k}_{28})^{(5)}|T_{29} - T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})',t) - (b_i'')^{(5)}\big((G_{31}),t\big)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}',t)$ and$(a_i'')^{(5)}(T_{29},t)$ . $(T_{29}',t)$ and $(T_{29},t)$ are points belonging to the interval $\big[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\big]$. It is to be noted that $(a_i'')^{(5)}(T_{29},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29},t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$ : 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}, \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

**Definition of** $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$ : 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and\ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(a_i)^{(5)} + (a_i')^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)}(\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(b_i)^{(5)} + (b_i')^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)}(\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i,j = 32,33,34 \qquad 127$$

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(6)}$, $(r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33},t) \le (p_i)^{(6)} \le (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}),t) \le (r_i)^{(6)} \le (b_i')^{(6)} \le (\hat{B}_{32})^{(6)}$$

$$lim_{T_2\to\infty}(a_i'')^{(6)}(T_{33},t) = (p_i)^{(6)} \quad (128)$$

$$\lim_{G\to\infty}(b_i'')^{(6)}\left((G_{35}),t\right) = (r_i)^{(6)}$$

**Definition of** $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$ :

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}',t) - (a_i'')^{(6)}(T_{33},t)| \le (\hat{k}_{32})^{(6)}|T_{33} - T_{33}'|e^{-(\hat{M}_{32})^{(6)}t}$$

$$|(b_i'')^{(6)}((G_{35})',t) - (b_i'')^{(6)}\left((G_{35}),t\right)| < (\hat{k}_{32})^{(6)}||(G_{35}) - (G_{35})'||e^{-(\hat{M}_{32})^{(6)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}',t)$ and$(a_i'')^{(6)}(T_{33},t)$ . $(T_{33}',t)$ and $(T_{33},t)$ are points belonging to the interval $\left[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}\right]$. It is to be noted that $(a_i'')^{(6)}(T_{33},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33},t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$ : (129)

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$, are positive constants

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}} \ , \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$$

**Definition of** $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$ : (130)

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and\ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32,33,34,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(a_i)^{(6)} + (a_i')^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(b_i)^{(6)} + (b_i')^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

(M) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i,j = 36,37,38$ (131)

(N) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(7)}$, $(r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37},t) \le (p_i)^{(7)} \le (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39},t) \le (r_i)^{(7)} \le (b_i')^{(7)} \le (\hat{B}_{36})^{(7)} \quad (132)$$

*(O)* $\lim_{T_2\to\infty} (a_i'')^{(7)}(T_{37},t) = (p_i)^{(7)}$

*(P)* $\lim_{G\to\infty}(b_i'')^{(7)}\big((G_{39}),t\big) = (r_i)^{(7)}$

**Definition of** $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$ :

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: (133)

$$|(a_i'')^{(7)}(T_{37}',t) - (a_i'')^{(7)}(T_{37},t)| \le (\hat{k}_{36})^{(7)}|T_{37} - T_{37}'|e^{-(\hat{M}_{36})^{(7)}t}$$

$$|(b_i'')^{(7)}((G_{39})',t) - (b_i'')^{(7)}\big((G_{39}),t\big)| < (\hat{k}_{36})^{(7)}||(G_{39}) - (G_{39})'||e^{-(\hat{M}_{36})^{(7)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}',t)$ and$(a_i'')^{(7)}(T_{37},t)$ . $(T_{37}',t)$ and $(T_{37},t)$ are points belonging to the interval $\big[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}\big]$. It is to be noted that $(a_i'')^{(7)}(T_{37},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37},t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$ : (134)

(Q) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$, are positive constants

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$$

**Definition of** $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$ : (135)

(R) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)}(\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(b_i)^{(7)} + (b_i')^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)}(\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40,41,42 \tag{136}$$

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

**Definition of** $(p_i)^{(8)}$, $(r_i)^{(8)}$: (137)

$$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)} \tag{138}$$

$$(b_i'')^{(8)}((G_{43}), t) \leq (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)} \tag{139}$$

$$\lim_{T_2 \to \infty} (a_i'')^{(8)}(T_{41}, t) = (p_i)^{(8)} \tag{140}$$

$$\lim_{G \to \infty} (b_i'')^{(8)}\left((G_{43}), t\right) = (r_i)^{(8)} \tag{141}$$

**Definition of** $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$ :

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)} |T_{41} - T_{41}'| e^{-(\hat{M}_{40})^{(8)} t} \tag{142}$$

$$|(b_i'')^{(8)}((G_{43})', t) - (b_i'')^{(8)}\left((G_{43}), t\right)| < (\hat{k}_{40})^{(8)} ||(G_{43}) - (G_{43})'|| e^{-(\hat{M}_{40})^{(8)} t} \tag{143}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}', t)$ and $(a_i'')^{(8)}(T_{41}, t)$ . $(T_{41}', t)$ and $(T_{41}, t)$ are points belonging to the interval $\left[(\hat{k}_{40})^{(8)}, (\hat{M}_{40})^{(8)}\right]$. It is to be noted that $(a_i'')^{(8)}(T_{41}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{40})^{(8)} = 1$ then the function $(a_i'')^{(8)}(T_{41}, t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$ :

$(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$, are positive constants

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}, \frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1 \tag{144}$$

**Definition of** $(\hat{P}_{40})^{(8)}, (\hat{Q}_{40})^{(8)}$ :

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}, (\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)}, (a_i')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (p_i)^{(8)}$, $(r_i)^{(8)}, i = 40,41,42,$
Satisfy the inequalities

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(a_i)^{(8)} + (a_i')^{(8)} + (\hat{A}_{40})^{(8)} + (\hat{P}_{40})^{(8)} (\hat{k}_{40})^{(8)}] < 1 \tag{145}$$

$$\frac{1}{(\hat{M}_{40})^{(8)}}[\ (b_i)^{(8)} + (b_i')^{(8)} + \ (\hat{B}_{40})^{(8)} + \ (\hat{Q}_{40})^{(8)}\ (\hat{k}_{40})^{(8)}] < 1 \qquad 146$$

Where we suppose

$$(a_i)^{(9)}, (a_i')^{(9)}, (a_i'')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (b_i'')^{(9)} > 0, \quad i,j = 44,45,46 \qquad 146\,A$$

The functions $(a_i'')^{(9)}, (b_i'')^{(9)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(9)}$, $(r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45}, t) \leq (p_i)^{(9)} \leq (\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47}, t) \leq \ (r_i)^{(9)} \leq (b_i')^{(9)} \leq (\hat{B}_{44})^{(9)}$$

$$\lim_{T_2 \to \infty} (a_i'')^{(9)}(T_{45}, t) = (p_i)^{(9)}$$

$$\lim_{G \to \infty} (b_i'')^{(9)}(G_{47}, t) = \ (r_i)^{(9)}$$

**Definition of** $(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}$ :

Where $\boxed{(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}, (p_i)^{(9)}, \ (r_i)^{(9)}}$ are positive constants and $\boxed{i = 44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}', t) - (a_i'')^{(9)}(T_{45}, t)| \leq (\hat{k}_{44})^{(9)} |T_{45} - T_{45}'| e^{-(\hat{M}_{44})^{(9)} t}$$

$$|(b_i'')^{(9)}((G_{47})', t) - (b_i'')^{(9)}((G_{47}), t)| < (\hat{k}_{44})^{(9)} ||(G_{47}) - (G_{47})'|| e^{-(\hat{M}_{44})^{(9)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}', t)$ and$(a_i'')^{(9)}(T_{45}, t)$ . $(T_{45}', t)$ and $(T_{45}, t)$ are points belonging to the interval $\left[(\hat{k}_{44})^{(9)}, (\hat{M}_{44})^{(9)}\right]$. It is to be noted that $(a_i'')^{(9)}(T_{45}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{44})^{(9)} = 1$ then the function $(a_i'')^{(9)}(T_{45}, t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$ :

$(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$, are positive constants

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}} \ , \frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$$

**Definition of** $(\hat{P}_{44})^{(9)}, (\hat{Q}_{44})^{(9)}$ :

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}, (\hat{A}_{44})^{(9)} and\ (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)}, (a_i')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (p_i)^{(9)}, \ (r_i)^{(9)}, i = 44,45,46,$
satisfy the inequalities

$$\frac{1}{(\widehat{M}_{44})^{(9)}}[(a_i)^{(9)} + (a_i')^{(9)} + (\hat{A}_{44})^{(9)} + (\hat{P}_{44})^{(9)}(\hat{k}_{44})^{(9)}] < 1$$

$$\frac{1}{(\widehat{M}_{44})^{(9)}}[(b_i)^{(9)} + (b_i')^{(9)} + (\hat{B}_{44})^{(9)} + (\hat{Q}_{44})^{(9)}(\hat{k}_{44})^{(9)}] < 1$$

**Theorem 1:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0), T_i(0)$ :

$$G_i(t) \le (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 2 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

**Definition of** $G_i(0), T_i(0)$

$$G_i(t) \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} \quad , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} \quad , \quad T_i(0) = T_i^0 > 0$$

**Theorem 3 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t) \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t} \quad , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t} \quad , \quad T_i(0) = T_i^0 > 0$$

**Theorem 4 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

**Definition of** $G_i(0), T_i(0)$ :

$$G_i(t) \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 5 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

**Definition of** $G_i(0), T_i(0)$ :

$$G_i(t) \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 6 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

**Definition of** $G_i(0), T_i(0)$ :

$$G_i(t) \le (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$T_i(t) \leq (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Theorem 7:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

**Definition of** $G_i(0), T_i(0)$ :

$G_i(t) \leq (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Theorem 8:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

**Definition of** $G_i(0), T_i(0)$ :

$G_i(t) \leq (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Theorem 9:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

**Definition of** $G_i(0), T_i(0)$ :

$G_i(t) \leq (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Proof:** Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$G_i(0) = G_i^0$ , $T_i(0) = T_i^0$ , $G_i^0 \leq (\hat{P}_{13})^{(1)}$ , $T_i^0 \leq (\hat{Q}_{13})^{(1)}$, 155

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t}$ 156

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t}$ 157

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[ (a_{13})^{(1)} G_{14}(s_{(13)}) - \left( (a'_{13})^{(1)} + a''_{13})^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \right) G_{13}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[ (a_{14})^{(1)} G_{13}(s_{(13)}) - \left( (a'_{14})^{(1)} + (a''_{14})^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \right) G_{14}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[ (a_{15})^{(1)} G_{14}(s_{(13)}) - \left( (a'_{15})^{(1)} + (a''_{15})^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \right) G_{15}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[ (b_{13})^{(1)} T_{14}(s_{(13)}) - \left( (b'_{13})^{(1)} - (b''_{13})^{(1)} (G(s_{(13)}), s_{(13)}) \right) T_{13}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^{0} + \int_0^t \left[ (b_{14})^{(1)} T_{13}(s_{(13)}) - \left( (b'_{14})^{(1)} - (b''_{14})^{(1)} \left( G(s_{(13)}), s_{(13)} \right) \right) T_{14}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{\mathrm{T}}_{15}(\mathrm{t}) = \mathrm{T}_{15}^{0} + \int_0^t \left[ (b_{15})^{(1)} T_{14}(s_{(13)}) - \left( (b'_{15})^{(1)} - (b''_{15})^{(1)} \left( G(s_{(13)}), s_{(13)} \right) \right) T_{15}(s_{(13)}) \right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{16})^{(2)}, T_i^0 \le (\hat{Q}_{16})^{(2)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$$

By 160

$$\bar{G}_{16}(t) = G_{16}^{0} + \int_0^t \left[ (a_{16})^{(2)} G_{17}(s_{(16)}) - \left( (a'_{16})^{(2)} + a''_{16})^{(2)} (T_{17}(s_{(16)}), s_{(16)}) \right) G_{16}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^{0} + \int_0^t \left[ (a_{17})^{(2)} G_{16}(s_{(16)}) - \left( (a'_{17})^{(2)} + (a''_{17})^{(2)} (T_{17}(s_{(16)}), s_{(17)}) \right) G_{17}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^{0} + \int_0^t \left[ (a_{18})^{(2)} G_{17}(s_{(16)}) - \left( (a'_{18})^{(2)} + (a''_{18})^{(2)} (T_{17}(s_{(16)}), s_{(16)}) \right) G_{18}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^{0} + \int_0^t \left[ (b_{16})^{(2)} T_{17}(s_{(16)}) - \left( (b'_{16})^{(2)} - (b''_{16})^{(2)} (G_{19}(s_{(16)}), s_{(16)}) \right) T_{16}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^{0} + \int_0^t \left[ (b_{17})^{(2)} T_{16}(s_{(16)}) - \left( (b'_{17})^{(2)} - (b''_{17})^{(2)} (G_{19}(s_{(16)}), s_{(16)}) \right) T_{17}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^{0} + \int_0^t \left[ (b_{18})^{(2)} T_{17}(s_{(16)}) - \left( (b'_{18})^{(2)} - (b''_{18})^{(2)} (G_{19}(s_{(16)}), s_{(16)}) \right) T_{18}(s_{(16)}) \right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{20})^{(3)}, T_i^0 \le (\hat{Q}_{20})^{(3)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$$

By 161

$$\bar{G}_{20}(t) = G_{20}^{0} + \int_0^t \left[ (a_{20})^{(3)} G_{21}(s_{(20)}) - \left( (a'_{20})^{(3)} + a''_{20})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{22}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^0 + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^0 + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{\mathrm{T}}_{22}(\mathrm{t}) = \mathrm{T}_{22}^0 + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{22}(s_{(20)})\right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{24})^{(4)}, T_i^0 \le (\hat{Q}_{24})^{(4)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

By



$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{26}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{\mathrm{T}}_{26}(\mathrm{t}) = \mathrm{T}_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{26}(s_{(24)})\right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{28})^{(5)}, T_i^0 \le (\hat{Q}_{28})^{(5)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

By 163

$$\bar{G}_{28}(t) = G_{28}^{0} + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^{0} + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^{0} + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{30}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^{0} + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^{0} + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{\mathrm{T}}_{30}(\mathrm{t}) = \mathrm{T}_{30}^{0} + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{30}(s_{(28)})\right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{32})^{(6)}$, $T_i^0 \le (\hat{Q}_{32})^{(6)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

By 164

$$\bar{G}_{32}(t) = G_{32}^{0} + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a'_{32})^{(6)} + a''_{32})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^{0} + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^{0} + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{34}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^{0} + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^{0} + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{\mathrm{T}}_{34}(\mathrm{t}) = \mathrm{T}_{34}^{0} + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{34}(s_{(32)})\right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$ , $T_i(0) = T_i^0$ , $G_i^0 \leq (\hat{P}_{36})^{(7)}$ , $T_i^0 \leq (\hat{Q}_{36})^{(7)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[ (a_{36})^{(7)} G_{37}(s_{(36)}) - \left( (a'_{36})^{(7)} + a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[ (a_{37})^{(7)} G_{36}(s_{(36)}) - \left( (a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[ (a_{38})^{(7)} G_{37}(s_{(36)}) - \left( (a'_{38})^{(7)} + (a''_{38})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{38}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[ (b_{36})^{(7)} T_{37}(s_{(36)}) - \left( (b'_{36})^{(7)} - (b''_{36})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[ (b_{37})^{(7)} T_{36}(s_{(36)}) - \left( (b'_{37})^{(7)} - (b''_{37})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{38}(t) = T_{38}^0 + \int_0^t \left[ (b_{38})^{(7)} T_{37}(s_{(36)}) - \left( (b'_{38})^{(7)} - (b''_{38})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{38}(s_{(36)}) \right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i$: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$ , $T_i(0) = T_i^0$ , $G_i^0 \leq (\hat{P}_{40})^{(8)}$ , $T_i^0 \leq (\hat{Q}_{40})^{(8)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{40})^{(8)} e^{(\bar{M}_{40})^{(8)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{40})^{(8)} e^{(\bar{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[ (a_{40})^{(8)} G_{41}(s_{(40)}) - \left( (a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[ (a_{41})^{(8)} G_{40}(s_{(40)}) - \left( (a'_{41})^{(8)} + (a''_{41})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)}\left(T_{41}(s_{(40)}), s_{(40)}\right)\right) G_{42}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^0 + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)}\left(G_{43}(s_{(40)}), s_{(40)}\right)\right) T_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^0 + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)}\left(G_{43}(s_{(40)}), s_{(40)}\right)\right) T_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{\mathrm{T}}_{42}(\mathrm{t}) = \mathrm{T}_{42}^0 + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)}\left(G_{43}(s_{(40)}), s_{(40)}\right)\right) T_{42}(s_{(40)})\right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** 166

Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy A

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{44})^{(9)},\ T_i^0 \le (\hat{Q}_{44})^{(9)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''_{44})^{(9)}\left(T_{45}(s_{(44)}), s_{(44)}\right)\right) G_{44}(s_{(44)})\right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)}\left(T_{45}(s_{(44)}), s_{(44)}\right)\right) G_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)}\left(T_{45}(s_{(44)}), s_{(44)}\right)\right) G_{46}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)}\left(G_{47}(s_{(44)}), s_{(44)}\right)\right) T_{44}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)}\left(G_{47}(s_{(44)}), s_{(44)}\right)\right) T_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{\mathrm{T}}_{46}(\mathrm{t}) = \mathrm{T}_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)}\left(G_{47}(s_{(44)}), s_{(44)}\right)\right) T_{46}(s_{(44)})\right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}}\right)\right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left(e^{(\hat{M}_{13})^{(1)} t} - 1\right)$$

From which it follows that 168

$$(G_{13}(t)-G_{13}^{0})e^{-(\hat{M}_{13})^{(1)}t}\leq\frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}}\left[((\hat{P}_{13})^{(1)}+G_{14}^{0})e^{\left(-\frac{(\hat{P}_{13})^{(1)}+G_{14}^{0}}{G_{14}^{0}}\right)}+(\hat{P}_{13})^{(1)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t)\leq G_{16}^{0}+\int_{0}^{t}\left[(a_{16})^{(2)}\left(G_{17}^{0}+(\hat{P}_{16})^{(6)}e^{(\hat{M}_{16})^{(2)}s_{(16)}}\right)\right]ds_{(16)}= \quad (169)$$

$$\left(1+(a_{16})^{(2)}t\right)G_{17}^{0}+\frac{(a_{16})^{(2)}(\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}}\left(e^{(\hat{M}_{16})^{(2)}t}-1\right)$$

From which it follows that (170)

$$(G_{16}(t)-G_{16}^{0})e^{-(\hat{M}_{16})^{(2)}t}\leq\frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}}\left[((\hat{P}_{16})^{(2)}+G_{17}^{0})e^{\left(-\frac{(\hat{P}_{16})^{(2)}+G_{17}^{0}}{G_{17}^{0}}\right)}+(\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that (171)

$$G_{20}(t)\leq G_{20}^{0}+\int_{0}^{t}\left[(a_{20})^{(3)}\left(G_{21}^{0}+(\hat{P}_{20})^{(3)}e^{(\hat{M}_{20})^{(3)}s_{(20)}}\right)\right]ds_{(20)}=$$

$$\left(1+(a_{20})^{(3)}t\right)G_{21}^{0}+\frac{(a_{20})^{(3)}(\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left(e^{(\hat{M}_{20})^{(3)}t}-1\right)$$

From which it follows that (172)

$$(G_{20}(t)-G_{20}^{0})e^{-(\hat{M}_{20})^{(3)}t}\leq\frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left[((\hat{P}_{20})^{(3)}+G_{21}^{0})e^{\left(-\frac{(\hat{P}_{20})^{(3)}+G_{21}^{0}}{G_{21}^{0}}\right)}+(\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that (173)

$$G_{24}(t)\leq G_{24}^{0}+\int_{0}^{t}\left[(a_{24})^{(4)}\left(G_{25}^{0}+(\hat{P}_{24})^{(4)}e^{(\hat{M}_{24})^{(4)}s_{(24)}}\right)\right]ds_{(24)}=$$

$$\left(1+(a_{24})^{(4)}t\right)G_{25}^{0}+\frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left(e^{(\hat{M}_{24})^{(4)}t}-1\right)$$

From which it follows that (174)

$$(G_{24}(t)-G_{24}^{0})e^{-(\hat{M}_{24})^{(4)}t}\leq\frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left[((\hat{P}_{24})^{(4)}+G_{25}^{0})e^{\left(-\frac{(\hat{P}_{24})^{(4)}+G_{25}^{0}}{G_{25}^{0}}\right)}+(\hat{P}_{24})^{(4)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t) \le G_{28}^0 + \int_0^t \left[ (a_{28})^{(5)} \left( G_{29}^0 + (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} s_{(28)}} \right) \right] ds_{(28)} =$$

$$\left(1 + (a_{28})^{(5)} t\right) G_{29}^0 + \frac{(a_{28})^{(5)} (\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}} \left( e^{(\hat{M}_{28})^{(5)} t} - 1 \right)$$

From which it follows that 175

$$(G_{28}(t) - G_{28}^0) e^{-(\hat{M}_{28})^{(5)} t} \le \frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}} \left[ \left( (\hat{P}_{28})^{(5)} + G_{29}^0 \right) e^{\left( - \frac{(\hat{P}_{28})^{(5)} + G_{29}^0}{G_{29}^0} \right)} + (\hat{P}_{28})^{(5)} \right]$$

$(G_i^0)$ is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t) \le G_{32}^0 + \int_0^t \left[ (a_{32})^{(6)} \left( G_{33}^0 + (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} s_{(32)}} \right) \right] ds_{(32)} =$$

$$\left(1 + (a_{32})^{(6)} t\right) G_{33}^0 + \frac{(a_{32})^{(6)} (\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}} \left( e^{(\hat{M}_{32})^{(6)} t} - 1 \right)$$

From which it follows that 177

$$(G_{32}(t) - G_{32}^0) e^{-(\hat{M}_{32})^{(6)} t} \le \frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}} \left[ \left( (\hat{P}_{32})^{(6)} + G_{33}^0 \right) e^{\left( - \frac{(\hat{P}_{32})^{(6)} + G_{33}^0}{G_{33}^0} \right)} + (\hat{P}_{32})^{(6)} \right]$$

$(G_i^0)$ is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

(c) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t) \le G_{36}^0 + \int_0^t \left[ (a_{36})^{(7)} \left( G_{37}^0 + (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} s_{(36)}} \right) \right] ds_{(36)} =$$

$$\left(1 + (a_{36})^{(7)} t\right) G_{37}^0 + \frac{(a_{36})^{(7)} (\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left( e^{(\hat{M}_{36})^{(7)} t} - 1 \right)$$

From which it follows that

$$(G_{36}(t) - G_{36}^0) e^{-(\hat{M}_{36})^{(7)} t} \le \frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left[ \left( (\hat{P}_{36})^{(7)} + G_{37}^0 \right) e^{\left( - \frac{(\hat{P}_{36})^{(7)} + G_{37}^0}{G_{37}^0} \right)} + (\hat{P}_{36})^{(7)} \right]$$

$(G_i^0)$ is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t) \le G_{40}^0 + \int_0^t \left[ (a_{40})^{(8)} \left( G_{41}^0 + (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} s_{(40)}} \right) \right] ds_{(40)} = \quad 180$$

$$\left(1 + (a_{40})^{(8)} t\right) G_{41}^0 + \frac{(a_{40})^{(8)} (\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}} \left( e^{(\hat{M}_{40})^{(8)} t} - 1 \right)$$

From which it follows that 181

$$(G_{40}(t)-G_{40}^{0})e^{-(\hat{M}_{40})^{(8)}t}\leq\frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left[((\hat{P}_{40})^{(8)}+G_{41}^{0})e^{\left(-\frac{(\hat{P}_{40})^{(8)}+G_{41}^{0}}{G_{41}^{0}}\right)}+(\hat{P}_{40})^{(8)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 8

Analogous inequalities hold also for $G_{41}, G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t)\leq G_{44}^{0}+\int_{0}^{t}\left[(a_{44})^{(9)}\left(G_{45}^{0}+(\hat{P}_{44})^{(9)}e^{(\hat{M}_{44})^{(9)}s_{(44)}}\right)\right]ds_{(44)}=$$

$$\left(1+(a_{44})^{(9)}t\right)G_{45}^{0}+\frac{(a_{44})^{(9)}(\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}}\left(e^{(\hat{M}_{44})^{(9)}t}-1\right)$$

From which it follows that

$$(G_{44}(t)-G_{44}^{0})e^{-(\hat{M}_{44})^{(9)}t}\leq\frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}}\left[((\hat{P}_{44})^{(9)}+G_{45}^{0})e^{\left(-\frac{(\hat{P}_{44})^{(9)}+G_{45}^{0}}{G_{45}^{0}}\right)}+(\hat{P}_{44})^{(9)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 9

Analogous inequalities hold also for $G_{45}, G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}<1$ and to choose 182

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[(\hat{P}_{13})^{(1)}+((\hat{P}_{13})^{(1)}+G_j^0)e^{-\left(\frac{(\hat{P}_{13})^{(1)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{13})^{(1)} \quad 183$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[((\hat{Q}_{13})^{(1)}+T_j^0)e^{-\left(\frac{(\hat{Q}_{13})^{(1)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{13})^{(1)}\right]\leq(\hat{Q}_{13})^{(1)} \quad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left((G^{(1)},T^{(1)}),(G^{(2)},T^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\hat{M}_{13})^{(1)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\hat{M}_{13})^{(1)}t}\}$$

Indeed if we denote

**Definition of** $\tilde{G},\tilde{T}:(\tilde{G},\tilde{T})=\mathcal{A}^{(1)}(G,T)$

It results

$$\left|\tilde{G}_{13}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{13})^{(1)} \left|G_{14}^{(1)}-G_{14}^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)} s_{(13)}} e^{(\widehat{M}_{13})^{(1)} s_{(13)}}\, ds_{(13)} +$$

$$\int_0^t \{(a'_{13})^{(1)} \left|G_{13}^{(1)}-G_{13}^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)} s_{(13)}} e^{-(\widehat{M}_{13})^{(1)} s_{(13)}} +$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)}, s_{(13)}\right)\left|G_{13}^{(1)}-G_{13}^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)} s_{(13)}} e^{(\widehat{M}_{13})^{(1)} s_{(13)}} +$$

$$G_{13}^{(2)} \left|(a''_{13})^{(1)}\left(T_{14}^{(1)}, s_{(13)}\right)-(a''_{13})^{(1)}\left(T_{14}^{(2)}, s_{(13)}\right)\right|\ e^{-(\widehat{M}_{13})^{(1)} s_{(13)}} e^{(\widehat{M}_{13})^{(1)} s_{(13)}}\} ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\left|G^{(1)}-G^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)} t} \leq \qquad (186)$$
$$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)} + (a'_{13})^{(1)} + (\widehat{A}_{13})^{(1)} + (\widehat{P}_{13})^{(1)} (\widehat{k}_{13})^{(1)}\right) d\left(\left(G^{(1)}, T^{(1)};\ G^{(2)}, T^{(2)}\right)\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 1:** The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)} t}$ *and* $(\widehat{Q}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)} t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i = 13,14,15$ depend only on $T_{14}$ and respectively on $G(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any $t$ where $G_i(t) = 0$ *and* $T_i(t) = 0$

From 19 to 24 it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(1)} - (a_i'')^{(1)}(T_{14}(s_{(13)}), s_{(13)})\} ds_{(13)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(1)} t)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2$ *and* $\left((\widehat{M}_{13})^{(1)}\right)_3$ : (187)

**Remark 3**: if $G_{13}$ is bounded, the same property have also $G_{14}$ *and* $G_{15}$ . indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a'_{14})^{(1)} G_{14}$ and by integrating

$$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1 / (a'_{14})^{(1)}$$

In the same way , one can obtain

$$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2 / (a'_{15})^{(1)}$$

If $G_{14}$ *or* $G_{15}$ is bounded, the same property follows for $G_{13}$ , $G_{15}$ and $G_{13}$ , $G_{14}$ respectively.

**Remark 4:** If $G_{13}$ *is* bounded, from below, the same property holds for $G_{14}$ *and* $G_{15}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{14}$ is bounded from below. 188

**Remark 5:** If $T_{13}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}\,(G(t),t)) = (b_{14}')^{(1)}$ then $T_{14} \to \infty$. 189

**Definition of** $(m)^{(1)}$ and $\varepsilon_1$ :

Indeed let $t_1$ be so that for $t > t_1$

$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t),t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_1}$ By taking now $\varepsilon_1$ sufficiently small one sees that $T_{14}$ is unbounded.
The same property holds for $T_{15}$ if $\lim_{t\to\infty}(b_{15}'')^{(1)}\,(G(t),t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}$ , $\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}$ *and* $(\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[(\hat{P}_{16})^{(2)} + \left((\hat{P}_{16})^{(2)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{16})^{(2)}\right] \leq (\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

**Definition of** $\widetilde{G_{19}}, \widetilde{T_{19}}$ : $\left(\widetilde{G_{19}}, \widetilde{T_{19}}\right) = \mathcal{A}^{(2)}(G_{19}, T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{16})^{(2)} \left|G_{17}^{(1)}-G_{17}^{(2)}\right| e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\, ds_{(16)} +$$

$$\int_0^t \{(a'_{16})^{(2)} \left|G_{16}^{(1)}-G_{16}^{(2)}\right| e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} +$$

$$(a''_{16})^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right)\left|G_{16}^{(1)}-G_{16}^{(2)}\right| e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} e^{(\widehat{M}_{16})^{(2)}s_{(16)}} +$$

$$G_{16}^{(2)} \left|(a''_{16})^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right) - (a''_{16})^{(2)}\left(T_{17}^{(2)}, s_{(16)}\right)\right| \; e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\} ds_{(16)}$$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0, t]$ 197

From the hypotheses it follows

$$\left|(G_{19})^{(1)}-(G_{19})^{(2)}\right| e^{-(\widehat{M}_{16})^{(2)}t} \leq$$
$$\frac{1}{(\widehat{M}_{16})^{(2)}}\left((a_{16})^{(2)} + (a'_{16})^{(2)} + (\widehat{A}_{16})^{(2)} + (\widehat{P}_{16})^{(2)}(\widehat{k}_{16})^{(2)}\right) d\left(((G_{19})^{(1)}, (T_{19})^{(1)}; (G_{19})^{(2)}, (T_{19})^{(2)})\right)$$

And analogous inequalities for $G_i$ and $T_i$. Taking into account the hypothesis the result follows 198

**Remark 6:** The fact that we supposed $(a''_{16})^{(2)}$ and $(b''_{16})^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{16})^{(2)} e^{(\widehat{M}_{16})^{(2)}t}$ and $(\widehat{Q}_{16})^{(2)} e^{(\widehat{M}_{16})^{(2)}t}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i = 16,17,18$ depend only on $T_{17}$ and respectively on $(G_{19})$(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 7:** There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$ 200

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(2)} - (a_i'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})\} ds_{(16)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(2)}t)} > 0 \quad \text{for } t > 0$$

**Definition of** $\left((\widehat{M}_{16})^{(2)}\right)_1$, $\left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$: 201

**Remark 8:** if $G_{16}$ is bounded, the same property have also $G_{17}$ and $G_{18}$ . indeed if

$G_{16} < (\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt} \leq \left((\widehat{M}_{16})^{(2)}\right)_1 - (a'_{17})^{(2)} G_{17}$ and by integrating

$$G_{17} \leq \left((\widehat{M}_{16})^{(2)}\right)_2 = G_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1 / (a'_{17})^{(2)}$$

In the same way , one can obtain

$$G_{18} \leq \left((\widehat{M}_{16})^{(2)}\right)_3 = G_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2 / (a'_{18})^{(2)}$$

If $G_{17}$ or $G_{18}$ is bounded, the same property follows for $G_{16}$, $G_{18}$ and $G_{16}$, $G_{17}$ respectively.

**Remark 9**: If $G_{16}$ is bounded, from below, the same property holds for $G_{17}$ and $G_{18}$. The proof is analogous with the preceding one. An analogous property is true if $G_{17}$ is bounded from below. 202

**Remark 10:** If $T_{16}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t),t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$. 203

**Definition of** $(m)^{(2)}$ and $\varepsilon_2$ :

Indeed let $t_2$ be so that for $t > t_2$

$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t),t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right), \quad t = \log\frac{2}{\varepsilon_2}$ By taking now $\varepsilon_2$ sufficiently small one sees that $T_{17}$ is unbounded. 205
The same property holds for $T_{18}$ if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t),t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[(\hat{P}_{20})^{(3)} + \left((\hat{P}_{20})^{(3)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{20})^{(3)}\right] \leq (\hat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself 210

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)},(T_{23})^{(1)}),((G_{23})^{(2)},(T_{23})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{20})^{(3)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

**Definition of** $\widetilde{G_{23}}, \widetilde{T_{23}}$ :$\left(\widetilde{(G_{23})},\widetilde{(T_{23})}\right) = \mathcal{A}^{(3)}\left((G_{23}),(T_{23})\right)$

It results 213

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_{0}^{t}(a_{20})^{(3)}\left|G_{21}^{(1)}-G_{21}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\, ds_{(20)} +$$

$$\int_{0}^{t}\{(a'_{20})^{(3)}\left|G_{20}^{(1)}-G_{20}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} +$$

$$(a''_{20})^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{(\widehat{M}_{20})^{(3)}s_{(20)}} +$$

$$G_{20}^{(2)}\left|(a''_{20})^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)}, s_{(20)}\right)\right|\ e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\} ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}t} \leq \qquad 214$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+\ (a'_{20})^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\widehat{k}_{20})^{(3)}\right) d\left(\left((G_{23})^{(1)},(T_{23})^{(1)};\ (G_{23})^{(2)},(T_{23})^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

**Remark 11:** The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}\ and\ (\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_{+}$. 215

If instead of proving the existence of the solution on $\mathbb{R}_{+}$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i = 20,21,22$ depend only on $T_{21}$ and respectively on $(G_{23})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 12:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 216

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t\{(a_i')^{(3)}-(a_i'')^{(3)}(T_{21}(s_{(20)}),s_{(20)})\}ds_{(20)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(3)}t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2\ and\ \left((\widehat{M}_{20})^{(3)}\right)_3$ : 217

**Remark 13:** if $G_{20}$ is bounded, the same property have also $G_{21}\ and\ G_{22}$ . indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a'_{21})^{(3)}G_{21}$ and by integrating

$$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1/(a'_{21})^{(3)}$$

In the same way , one can obtain

$$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2/(a'_{22})^{(3)}$$

If $G_{21}$ *or* $G_{22}$ is bounded, the same property follows for $G_{20}$ , $G_{22}$ and $G_{20}$ , $G_{21}$ respectively.

**Remark 14:** If $G_{20}$ *is* bounded, from below, the same property holds for $G_{21}$ *and* $G_{22}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{21}$ is bounded from below. 218

**Remark 15:** If $T_{20}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t),t\right)=(b_{21}')^{(3)}$ then $T_{21}\to\infty$. 219

**Definition of** $(m)^{(3)}$ and $\varepsilon_3$ :

Indeed let $t_3$ be so that for $t > t_3$

$(b_{21})^{(3)}-(b_i'')^{(3)}\left((G_{23})(t),t\right)<\varepsilon_3, T_{20}(t)>(m)^{(3)}$

Then $\frac{dT_{21}}{dt}\geq(a_{21})^{(3)}(m)^{(3)}-\varepsilon_3 T_{21}$ which leads to 220

$T_{21}\geq\left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1-e^{-\varepsilon_3 t})+T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t}=\frac{1}{2}$ it results

$T_{21}\geq\left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right),\quad t=log\frac{2}{\varepsilon_3}$ By taking now $\varepsilon_3$ sufficiently small one sees that $T_{21}$ is unbounded.
The same property holds for $T_{22}$ if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t),t\right)=(b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}$ , $\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}}<1$ and to choose 221

$(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\left[(\hat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \quad 222$$

$$\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \quad 223$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself 224

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric 225

$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{24})^{(4)}t}\}$$

Indeed if we denote

**Definition of** $\widetilde{(G_{27})},\widetilde{(T_{27})}$ : $\left(\widetilde{(G_{27})},\widetilde{(T_{27})}\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_{i}^{(2)}\right|\leq\int_{0}^{t}(a_{24})^{(4)}\left|G_{25}^{(1)}-G_{25}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\,ds_{(24)}+$$

$$\int_{0}^{t}\{(a'_{24})^{(4)}\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$G_{24}^{(2)}\left|(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)-(a''_{24})^{(4)}\left(T_{25}^{(2)},s_{(24)}\right)\right|\;e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\}ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}t}\leq \qquad 226$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)}+\;(a'_{24})^{(4)}+(\widehat{A}_{24})^{(4)}+(\widehat{P}_{24})^{(4)}(\widehat{k}_{24})^{(4)}\right)d\left(\left((G_{27})^{(1)},(T_{27})^{(1)};\;(G_{27})^{(2)},(T_{27})^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

**Remark 16:** The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}\ and\ (\widehat{Q}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_{+}$. 227

If instead of proving the existence of the solution on $\mathbb{R}_{+}$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i=24,25,26$ depend only on $\mathrm{T}_{25}$ and respectively on $(G_{27})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 17:** There does not exist any $t$ where $G_i\,(t)=0\ and\ T_i\,(t)=0$ 228

it results

$$G_i\,(t)\geq G_i^0e^{\left[-\int_0^t\{(a_i')^{(4)}-(a_i'')^{(4)}(T_{25}(s_{(24)}),s_{(24)})\}ds_{(24)}\right]}\geq 0$$

$$T_i\,(t)\geq T_i^0e^{\left(-(b_i')^{(4)}t\right)}>0\quad \text{for t}>0$$

**Definition of** $\left((\widehat{M}_{24})^{(4)}\right)_1,\left((\widehat{M}_{24})^{(4)}\right)_2\ and\ \left((\widehat{M}_{24})^{(4)}\right)_3$ : 229

**Remark 18:** if $G_{24}$ is bounded, the same property have also $G_{25}\ and\ G_{26}$ . indeed if

$G_{24}<(\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt}\leq\left((\widehat{M}_{24})^{(4)}\right)_1-(a'_{25})^{(4)}G_{25}$ and by integrating

$$G_{25}\leq\left((\widehat{M}_{24})^{(4)}\right)_2=G_{25}^0+2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1/(a'_{25})^{(4)}$$

In the same way , one can obtain

$$G_{26}\leq\left((\widehat{M}_{24})^{(4)}\right)_3=G_{26}^0+2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2/(a'_{26})^{(4)}$$

If $G_{25}$ *or* $G_{26}$ is bounded, the same property follows for $G_{24}$ , $G_{26}$ and $G_{24}$ , $G_{25}$ respectively.

**Remark 19:** If $G_{24}$ *is* bounded, from below, the same property holds for $G_{25}$ *and* $G_{26}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{25}$ is bounded from below. 230

**Remark 20:** If $T_{24}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}((G_{27})(t),t)) = (b_{25}')^{(4)}$ then $T_{25} \to \infty$. 231

**Definition of** $(m)^{(4)}$ and $\varepsilon_4$ :

Indeed let $t_4$ be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t),t) < \varepsilon_4, T_{24}(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right), \quad t = log\frac{2}{\varepsilon_4}$ By taking now $\varepsilon_4$ sufficiently small one sees that $T_{25}$ is unbounded.
The same property holds for $T_{26}$ if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t),t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for $G_{29}, G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}$ , $\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)} + \left((\hat{P}_{28})^{(5)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{28})^{(5)}\right] \leq (\hat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$$d\left(((G_{31})^{(1)},(T_{31})^{(1)}),((G_{31})^{(2)},(T_{31})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

**Definition of** $\widetilde{(G_{31})}, \widetilde{(T_{31})}$ : $\left(\widetilde{(G_{31})}, \widetilde{(T_{31})}\right) = \mathcal{A}^{(5)}\left((G_{31}), (T_{31})\right)$

It results

$\left|\tilde{G}_{28}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{28})^{(5)} \left|G_{29}^{(1)} - G_{29}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}}\, ds_{(28)} +$

$\int_0^t \{(a'_{28})^{(5)} \left|G_{28}^{(1)} - G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} +$

$(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right)\left|G_{28}^{(1)} - G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}} +$

$G_{28}^{(2)} \left|(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right) - (a''_{28})^{(5)}\left(T_{29}^{(2)}, s_{(28)}\right)\right| \; e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}} \} ds_{(28)}$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$\left|(G_{31})^{(1)} - (G_{31})^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} t} \leq$ 237

$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)} + (a'_{28})^{(5)} + (\widehat{A}_{28})^{(5)} + (\widehat{P}_{28})^{(5)} (\widehat{k}_{28})^{(5)}\right) d\left(\left((G_{31})^{(1)}, (T_{31})^{(1)}; (G_{31})^{(2)}, (T_{31})^{(2)}\right)\right)$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 21:** The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$ *and* $(\widehat{Q}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$ respectively of $\mathbb{R}_+$. 238

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28{,}29{,}30$ depend only on $T_{29}$ and respectively on $(G_{31})$(*and not on* $t$) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 22:** There does not exist any $t$ where $G_i(t) = 0$ *and* $T_i(t) = 0$ 239

it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(5)} - (a_i'')^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(5)} t\right)} > 0$ for t > 0

**Definition of** $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2$ *and* $\left((\widehat{M}_{28})^{(5)}\right)_3$ : 240

**Remark 23:** if $G_{28}$ is bounded, the same property have also $G_{29}$ *and* $G_{30}$ . indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a'_{29})^{(5)} G_{29}$ and by integrating

$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1 / (a'_{29})^{(5)}$

In the same way , one can obtain

$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2/(a'_{30})^{(5)}$

If $G_{29}$ *or* $G_{30}$ is bounded, the same property follows for $G_{28}$, $G_{30}$ and $G_{28}$, $G_{29}$ respectively.

**Remark 24:** If $G_{28}$ *is* bounded, from below, the same property holds for $G_{29}$ *and* $G_{30}$. The proof is analogous with the preceding one. An analogous property is true if $G_{29}$ is bounded from below. 241

**Remark 25:** If $T_{28}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}((G_{31})(t),t)) = (b'_{29})^{(5)}$ then $T_{29} \to \infty$. 242

**Definition of** $(m)^{(5)}$ and $\varepsilon_5$ :

Indeed let $t_5$ be so that for $t > t_5$

$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t),t) < \varepsilon_5, T_{28}(t) > (m)^{(5)}$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\frac{2}{\varepsilon_5}$ By taking now $\varepsilon_5$ sufficiently small one sees that $T_{29}$ is unbounded.
The same property holds for $T_{30}$ if $\lim_{t\to\infty}(b_{30}'')^{(5)}\left((G_{31})(t),t\right) = (b'_{30})^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{33}, G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}$, $\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$ and to choose 244

$(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[(\hat{P}_{32})^{(6)} + \left((\hat{P}_{32})^{(6)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{32})^{(6)} \quad 245$$

$$\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[\left((\hat{Q}_{32})^{(6)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{32})^{(6)}\right] \leq (\hat{Q}_{32})^{(6)} \quad 246$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric 247

$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right) =$

$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{32})^{(6)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{32})^{(6)}t}\}$

Indeed if we denote

**Definition of** $\widetilde{(G_{35})}, \widetilde{(T_{35})}$ : $\left(\widetilde{(G_{35})}, \widetilde{(T_{35})}\right) = \mathcal{A}^{(6)}\left((G_{35}), (T_{35})\right)$

It results

$$\left|\tilde{G}_{32}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{32})^{(6)} \left|G_{33}^{(1)} - G_{33}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}}\, ds_{(32)} +$$

$$\int_0^t \{(a'_{32})^{(6)} \left|G_{32}^{(1)} - G_{32}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} +$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)}, s_{(32)}\right)\left|G_{32}^{(1)} - G_{32}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}} +$$

$$G_{32}^{(2)} \left|(a''_{32})^{(6)}\left(T_{33}^{(1)}, s_{(32)}\right) - (a''_{32})^{(6)}\left(T_{33}^{(2)}, s_{(32)}\right)\right| \; e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}} \} ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|(G_{35})^{(1)} - (G_{35})^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} t} \leq \qquad (248)$$
$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)} + (a'_{32})^{(6)} + (\widehat{A}_{32})^{(6)} + (\widehat{P}_{32})^{(6)} (\widehat{k}_{32})^{(6)}\right) d\left(\left((G_{35})^{(1)}, (T_{35})^{(1)}; \; (G_{35})^{(2)}, (T_{35})^{(2)}\right)\right)$$

And analogous inequalities for $G_i \; and \; T_i$. Taking into account the hypothesis the result follows

**Remark 26:** The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)} t} \; and \; (\widehat{Q}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)} t}$ respectively of $\mathbb{R}_+$. (249)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i = 32{,}33{,}34$ depend only on $T_{33}$ and respectively on $(G_{35})(and \; not \; on \; t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 27:** There does not exist any $t$ where $G_i\,(t) = 0 \; and \; T_i\,(t) = 0$ (250)

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(6)} - (a_i'')^{(6)}(T_{33}(s_{(32)}), s_{(32)})\} ds_{(32)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(6)} t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{32})^{(6)}\right)_1, \left((\widehat{M}_{32})^{(6)}\right)_2 \; and \; \left((\widehat{M}_{32})^{(6)}\right)_3$ : (251)

**Remark 28:** if $G_{32}$ is bounded, the same property have also $G_{33} \; and \; G_{34}$ . indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a'_{33})^{(6)} G_{33}$ and by integrating

$$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_1 / (a'_{33})^{(6)}$$

In the same way , one can obtain

$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_2/(a'_{34})^{(6)}$

If $G_{33}$ *or* $G_{34}$ is bounded, the same property follows for $G_{32}$ , $G_{34}$ and $G_{32}$ , $G_{33}$ respectively.

**Remark 29:** If $G_{32}$ *is* bounded, from below, the same property holds for $G_{33}$ *and* $G_{34}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{33}$ is bounded from below. 252

**Remark 30:** If $T_{32}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}((G_{35})(t),t)) = (b'_{33})^{(6)}$ then $T_{33} \to \infty$. 253

**Definition of** $(m)^{(6)}$ and $\varepsilon_6$ :

Indeed let $t_6$ be so that for $t > t_6$

$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t),t\right) < \varepsilon_6, T_{32}(t) > (m)^{(6)}$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right)$, $t = log\frac{2}{\varepsilon_6}$ By taking now $\varepsilon_6$ sufficiently small one sees that $T_{33}$ is unbounded.
The same property holds for $T_{34}$ if $\lim_{t\to\infty}(b''_{34})^{(6)}\left((G_{35})(t),t(t),t\right) = (b'_{34})^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{37}$ , $G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}$ , $\frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$ and to choose
$(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[(\hat{P}_{36})^{(7)} + \left((\hat{P}_{36})^{(7)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{36})^{(7)} \quad 256$$

$$\frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[\left((\hat{Q}_{36})^{(7)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{36})^{(7)}\right] \leq (\hat{Q}_{36})^{(7)} \quad 257$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself
The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric 258

$d\left(((G_{39})^{(1)},(T_{39})^{(1)}),((G_{39})^{(2)},(T_{39})^{(2)})\right) =$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\tilde{M}_{36})^{(7)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\tilde{M}_{36})^{(7)}t}\}$$

Indeed if we denote

**Definition of** $\widetilde{(G_{39})},\widetilde{(T_{39})}:\left(\widetilde{(G_{39})},\widetilde{(T_{39})}\right)=\mathcal{A}^{(7)}((G_{39}),(T_{39}))$
It results

$$\left|\tilde{G}_{36}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{36})^{(7)}\left|G_{37}^{(1)}-G_{37}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\,ds_{(36)}+$$
$$\int_0^t\{(a'_{36})^{(7)}\left|G_{36}^{(1)}-G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$(a''_{36})^{(7)}\left(T_{37}^{(1)},s_{(36)}\right)\left|G_{36}^{(1)}-G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$G_{36}^{(2)}\left|(a''_{36})^{(7)}\left(T_{37}^{(1)},s_{(36)}\right)-(a''_{36})^{(7)}\left(T_{37}^{(2)},s_{(36)}\right)\right|\ e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\}ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses on it follows

$$\left|(G_{39})^{(1)}-(G_{39})^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}t}\le \qquad 259$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}}\left((a_{36})^{(7)}+(a'_{36})^{(7)}+(\widehat{A}_{36})^{(7)}+(\widehat{P}_{36})^{(7)}(\widehat{k}_{36})^{(7)}\right)d\left(((G_{39})^{(1)},(T_{39})^{(1)};\ (G_{39})^{(2)},(T_{39})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

**Remark 31:** The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}\ and\ (\widehat{Q}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ respectively of $\mathbb{R}_+$. 260

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(7)}$ and $(b_i'')^{(7)}, i=36,37,38$ depend only on $\mathrm{T}_{37}$ and respectively on $(G_{39})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 32:** There does not exist any $t$ where $G_i\,(t)=0\ and\ T_i\,(t)=0$ 261

it results

$$G_i\,(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(7)}-(a_i'')^{(7)}(T_{37}(s_{(36)}),s_{(36)})\}ds_{(36)}\right]}\ge 0$$

$$T_i\,(t)\ge T_i^0e^{\left(-(b_i')^{(7)}t\right)}>0\quad \text{for t}>0$$

**Definition of** $\left((\widehat{M}_{36})^{(7)}\right)_1,\left((\widehat{M}_{36})^{(7)}\right)_2\ and\ \left((\widehat{M}_{36})^{(7)}\right)_3:$ 262

**Remark 33:** if $G_{36}$ is bounded, the same property have also $G_{37}\ and\ G_{38}$ . indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a'_{37})^{(7)} G_{37}$ and by integrating

$$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1/(a'_{37})^{(7)}$$

In the same way , one can obtain

$$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2/(a'_{38})^{(7)}$$

If $G_{37}$ *or* $G_{38}$ is bounded, the same property follows for $G_{36}$ , $G_{38}$ and $G_{36}$ , $G_{37}$ respectively.

**Remark 34:** If $G_{36}$ *is* bounded, from below, the same property holds for $G_{37}$ *and* $G_{38}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{37}$ is bounded from below. 263

**Remark 35:** If $T_{36}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(7)}((G_{39})(t),t)) = (b'_{37})^{(7)}$ then $T_{37} \to \infty$. 264

**Definition of** $(m)^{(7)}$ and $\varepsilon_7$ :

Indeed let $t_7$ be so that for $t > t_7$

$$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t),t) < \varepsilon_7, T_{36}(t) > (m)^{(7)}$$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right)$, $t = log\frac{2}{\varepsilon_7}$ By taking now $\varepsilon_7$ sufficiently small one sees that $T_{37}$ is unbounded.
The same property holds for $T_{38}$ if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t),t\right) = (b'_{38})^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}$ , $\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}} < 1$ and to choose 266
$(\widehat{P}_{40})^{(8)}$ and $(\widehat{Q}_{40})^{(8)}$ large to have

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[(\widehat{P}_{40})^{(8)} + \left((\widehat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{40})^{(8)} \quad 267$$

$$\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[\left((\widehat{Q}_{40})^{(8)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{40})^{(8)}\right] \leq (\widehat{Q}_{40})^{(8)} \quad 268$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)},(T_{43})^{(1)}),((G_{43})^{(2)},(T_{43})^{(2)})\right)= \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

**Definition of** $\widetilde{(G_{43})},\widetilde{(T_{43})}$ : $\left(\widetilde{(G_{43})},\widetilde{(T_{43})}\right)=\mathcal{A}^{(8)}((G_{43}),(T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{40})^{(8)}\left|G_{41}^{(1)}-G_{41}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\,ds_{(40)}+$$
$$\int_0^t\{(a'_{40})^{(8)}\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$G_{40}^{(2)}\left|(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)-(a''_{40})^{(8)}\left(T_{41}^{(2)},s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\}ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0,t]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)}-(G_{43})^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}t}\le \quad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)}+(a'_{40})^{(8)}+(\widehat{A}_{40})^{(8)}+(\widehat{P}_{40})^{(8)}(\widehat{k}_{40})^{(8)}\right)d\left(((G_{43})^{(1)},(T_{43})^{(1)};\ (G_{43})^{(2)},(T_{43})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

**Remark 36:** The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}\ and\ (\widehat{Q}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ respectively of $\mathbb{R}_+$. 274

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i=40,41,42$ depend only on $T_{41}$ and respectively on $(G_{43})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 37** There does not exist any $t$ where $G_i\,(t)=0\ and\ T_i\,(t)=0$ 275

it results

$$G_i\,(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(8)}-(a_i'')^{(8)}(T_{41}(s_{(40)}),s_{(40)})\}ds_{(40)}\right]}\ge 0$$

$$T_i\,(t)\ge T_i^0e^{\left(-(b_i')^{(8)}t\right)}>0\quad \text{for t}>0$$

**Definition of** $\left((\widehat{M}_{40})^{(8)}\right)_1, \left((\widehat{M}_{40})^{(8)}\right)_2$ *and* $\left((\widehat{M}_{40})^{(8)}\right)_3$ : 276

**Remark 38:** if $G_{40}$ is bounded, the same property have also $G_{41}$ *and* $G_{42}$ . indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a'_{41})^{(8)} G_{41}$ and by integrating

$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1/(a'_{41})^{(8)}$

In the same way , one can obtain

$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2/(a'_{42})^{(8)}$

If $G_{41}$ *or* $G_{42}$ is bounded, the same property follows for $G_{40}$ , $G_{42}$ and $G_{40}$ , $G_{41}$ respectively.

**Remark 39:** If $G_{40}$ *is* bounded, from below, the same property holds for $G_{41}$ *and* $G_{42}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{41}$ is bounded from below. 277

**Remark 40:** If $T_{40}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}\,((G_{43})(t),t)) = (b'_{41})^{(8)}$ then $T_{41} \to \infty$. 278

**Definition of** $(m)^{(8)}$ and $\varepsilon_8$ :

Indeed let $t_8$ be so that for $t > t_8$

$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t),t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right), \quad t = log\frac{2}{\varepsilon_8}$ By taking now $\varepsilon_8$ sufficiently small one sees that $T_{41}$ is unbounded.
The same property holds for $T_{42}$ if $\lim_{t\to\infty}(b''_{42})^{(8)}\left((G_{43})(t),t(t),t\right) = (b'_{42})^{(8)}$

It is now sufficient to take $\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}$ , $\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}} < 1$ and to choose $(\widehat{P}_{44})^{(9)}$ and $(\widehat{Q}_{44})^{(9)}$ large to have 279 A

$$\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[(\widehat{P}_{44})^{(9)} + \left((\widehat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[\left((\widehat{Q}_{44})^{(9)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{44})^{(9)}\right] \leq (\widehat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)},(T_{47})^{(1)}),((G_{47})^{(2)},(T_{47})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{47}}),(\widetilde{T_{47}}):\left((\widetilde{G_{47}}),(\widetilde{T_{47}})\right)=\mathcal{A}^{(9)}\left((G_{47}),(T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{44})^{(9)}\left|G_{45}^{(1)}-G_{45}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\,ds_{(44)}+$$
$$\int_0^t\{(a'_{44})^{(9)}\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$G_{44}^{(2)}\left|(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)-(a''_{44})^{(9)}\left(T_{45}^{(2)},s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\}ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}t}\le$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)}+(a'_{44})^{(9)}+(\widehat{A}_{44})^{(9)}+(\widehat{P}_{44})^{(9)}(\widehat{k}_{44})^{(9)}\right)d\left(((G_{47})^{(1)},(T_{47})^{(1)};\ (G_{47})^{(2)},(T_{47})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis (39,35,36) the result follows

**Remark 41:** The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}\ and\ (\widehat{Q}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i=44,45,46$ depend only on $T_{45}$ and respectively on $(G_{47})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 42:** There does not exist any $t$ where $G_i\,(t)=0\ and\ T_i\,(t)=0$

From 99 to 44 it results

$$G_i\,(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(9)}-(a_i'')^{(9)}(T_{45}(s_{(44)}),s_{(44)})\}ds_{(44)}\right]}\ge 0$$

$$T_i\,(t)\ge T_i^0e^{(-(b_i')^{(9)}t)}>0\quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{44})^{(9)}\right)_1,\left((\widehat{M}_{44})^{(9)}\right)_2\ and\ \left((\widehat{M}_{44})^{(9)}\right)_3$ :

**Remark 43:** if $G_{44}$ is bounded, the same property have also $G_{45}\ and\ G_{46}$ . indeed if

$G_{44} < (\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt} \leq \left((\widehat{M}_{44})^{(9)}\right)_1 - (a'_{45})^{(9)} G_{45}$ and by integrating

$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_1 / (a'_{45})^{(9)}$

In the same way , one can obtain

$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_2 / (a'_{46})^{(9)}$

If $G_{45}$ *or* $G_{46}$ is bounded, the same property follows for $G_{44}$ , $G_{46}$ and $G_{44}$ , $G_{45}$ respectively.

**Remark 44:** If $G_{44}$ *is* bounded, from below, the same property holds for $G_{45}$ *and* $G_{46}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{45}$ is bounded from below.

**Remark 45:** If $T_{44}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t),t\right) = (b'_{45})^{(9)}$ then $T_{45} \to \infty$.

**Definition of** $(m)^{(9)}$ and $\varepsilon_9$ :

Indeed let $t_9$ be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t),t\right) < \varepsilon_9, T_{44}(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right)$, $t = log\frac{2}{\varepsilon_9}$ By taking now $\varepsilon_9$ sufficiently small one sees that $T_{45}$ is unbounded.
The same property holds for $T_{46}$ if $\lim_{t\to\infty}(b''_{46})^{(9)}\left((G_{47})(t),t\right) = (b'_{46})^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

**Behavior of the solutions of equation** 280

**Theorem** If we denote and define

**Definition of** $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ :

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)} \leq -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14},t) + (a''_{14})^{(1)}(T_{14},t) \leq -(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)} \leq -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G,t) - (b''_{14})^{(1)}(G,t) \leq -(\tau_1)^{(1)}$

**Definition of** $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$ : 281

By $(\nu_1)^{(1)} > 0$ , $(\nu_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$ , $(u_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

**Definition of** $(\bar{\nu}_1)^{(1)}, , (\bar{\nu}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$ : 282

By $(\bar{\nu}_1)^{(1)} > 0$ , $(\bar{\nu}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$ , $(\bar{u}_2)^{(1)} < 0$ the roots of the equations

$(a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

**Definition of** $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (v_0)^{(1)}$ :- 283

If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by

$(m_2)^{(1)} = (v_0)^{(1)}, (m_1)^{(1)} = (v_1)^{(1)}, \ if\ (v_0)^{(1)} < (v_1)^{(1)}$

$(m_2)^{(1)} = (v_1)^{(1)}, (m_1)^{(1)} = (\bar{v}_1)^{(1)}, if\ (v_1)^{(1)} < (v_0)^{(1)} < (\bar{v}_1)^{(1)},$

and $\boxed{(v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$

$(m_2)^{(1)} = (v_1)^{(1)}, (m_1)^{(1)} = (v_0)^{(1)}, \ if\ (\bar{v}_1)^{(1)} < (v_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)}, \ if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)},$

and $\boxed{(u_0)^{(1)} = \frac{T_{13}^0}{T_{14}^0}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$

are defined

Then the solution of global equations satisfies the inequalities 285

$$G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{13}(t) \le G_{13}^0 e^{(S_1)^{(1)}t}$$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)}t}$$

$$\left(\frac{(a_{15})^{(1)} G_{13}^0}{(m_1)^{(1)}((S_1)^{(1)} - (p_{13})^{(1)} - (S_2)^{(1)})}\left[e^{((S_1)^{(1)} - (p_{13})^{(1)})t} - e^{-(S_2)^{(1)}t}\right] + G_{15}^0 e^{-(S_2)^{(1)}t} \le G_{15}(t) \le \right. \quad 286$$

$$\left. \frac{(a_{15})^{(1)} G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)} - (a'_{15})^{(1)})}\left[e^{(S_1)^{(1)}t} - e^{-(a'_{15})^{(1)}t}\right] + G_{15}^0 e^{-(a'_{15})^{(1)}t}\right)$$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)}t} \le T_{13}(t) \le T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t}} \quad 287$$

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)}t} \le T_{13}(t) \le \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t} \quad 288$$

$$\frac{(b_{15})^{(1)} T_{13}^0}{(\mu_1)^{(1)}((R_1)^{(1)} - (b'_{15})^{(1)})}\left[e^{(R_1)^{(1)}t} - e^{-(b'_{15})^{(1)}t}\right] + T_{15}^0 e^{-(b'_{15})^{(1)}t} \le T_{15}(t) \le \quad 289$$

$$\frac{(a_{15})^{(1)} T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)} + (r_{13})^{(1)} + (R_2)^{(1)})}\left[e^{((R_1)^{(1)} + (r_{13})^{(1)})t} - e^{-(R_2)^{(1)}t}\right] + T_{15}^0 e^{-(R_2)^{(1)}t}$$

**Definition of** $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:- 290

Where $(S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$

$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$

$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$

$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$

**Behavior of the solutions of equation** 291

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ : 292

$(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ four constants satisfying
$-(\sigma_2)^{(2)} \le -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(T_{17}, t) + (a''_{17})^{(2)}(T_{17}, t) \le -(\sigma_1)^{(2)}$ 293

$-(\tau_2)^{(2)} \le -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}\big((G_{19}), t\big) - (b''_{17})^{(2)}\big((G_{19}), t\big) \le -(\tau_1)^{(2)}$ 294

**Definition of** $(\nu_1)^{(2)}, (\nu_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$ : 295

By $(\nu_1)^{(2)} > 0$ , $(\nu_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0$ , $(u_2)^{(2)} < 0$ the roots 296

of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 297

and $(b_{14})^{(2)}\big(u^{(2)}\big)^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and 298

**Definition of** $(\bar{\nu}_1)^{(2)},, (\bar{\nu}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$ : 299

By $(\bar{\nu}_1)^{(2)} > 0$ , $(\bar{\nu}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0$ , $(\bar{u}_2)^{(2)} < 0$ the 300

roots of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 301

and $(b_{17})^{(2)}\big(u^{(2)}\big)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 302

**Definition of** $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ :- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{G^0_{16}}{G^0_{17}}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)},\ \boldsymbol{if}\ (u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}, \boldsymbol{if}\ (u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)},$

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)},\ \boldsymbol{if}\ (\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)} t}$

$(p_i)^{(2)}$ is defined by equation

$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)} t}$ 311

$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)} t}\right] + G_{18}^0 e^{-(S_2)^{(2)} t} \le G_{18}(t) \le\right.$ 312

$\left.\frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)} t} - e^{-(a'_{18})^{(2)} t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)} t}\right)$

$\boxed{T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}}$ 313

$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}$ 314

$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)} t} - e^{-(b'_{18})^{(2)} t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)} t} \le T_{18}(t) \le$ 315

$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})}\left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)} t}\right] + T_{18}^0 e^{-(R_2)^{(2)} t}$

**Definition of** $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$

$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)}$ 318

$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$

**Behavior of the solutions** 319

**Theorem 3:** If we denote and define

**Definition of** $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ :

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}\big((G_{23}), t\big) \le -(\tau_1)^{(3)}$$

**Definition of** $(\nu_1)^{(3)}, (\nu_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$ : 320

By $(\nu_1)^{(3)} > 0$ , $(\nu_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$ , $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\big(\nu^{(3)}\big)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{\nu}_1)^{(3)} > 0$ , $(\bar{\nu}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$ , $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\big(\nu^{(3)}\big)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

**Definition of** $(m_1)^{(3)}$ , $(m_2)^{(3)}$ , $(\mu_1)^{(3)}, (\mu_2)^{(3)}$ :- 321

If we define $(m_1)^{(3)}$ , $(m_2)^{(3)}$ , $(\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (\nu_0)^{(3)}, (m_1)^{(3)} = (\nu_1)^{(3)}$, ***if*** $(\nu_0)^{(3)} < (\nu_1)^{(3)}$

$(m_2)^{(3)} = (\nu_1)^{(3)}, (m_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ , ***if*** $(\nu_1)^{(3)} < (\nu_0)^{(3)} < (\bar{\nu}_1)^{(3)}$,

and $\boxed{(\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}}$

$(m_2)^{(3)} = (\nu_1)^{(3)}, (m_1)^{(3)} = (\nu_0)^{(3)}$, ***if*** $(\bar{\nu}_1)^{(3)} < (\nu_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}$, ***if*** $(u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$ , ***if*** $(u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}$, and $\boxed{(u_0)^{(3)} = \frac{T^0_{20}}{T^0_{21}}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}$, ***if*** $(\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$$G^0_{20} e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G^0_{20} e^{(S_1)^{(3)} t}$$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}} G^0_{20} e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}} G^0_{20} e^{(S_1)^{(3)} t} \quad 323$$

$$\Big(\frac{(a_{22})^{(3)} G^0_{20}}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\Big[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)} t}\Big] + G^0_{22} e^{-(S_2)^{(3)} t} \le G_{22}(t) \le \quad 324$$

$$\frac{(a_{22})^{(3)} G^0_{20}}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}[e^{(S_1)^{(3)} t} - e^{-(a'_{22})^{(3)} t}] + G^0_{22} e^{-(a'_{22})^{(3)} t}\Big)$$

$$\boxed{T_{20}^{0}e^{(R_1)^{(3)}t} \le T_{20}(t) \le T_{20}^{0}e^{((R_1)^{(3)}+(r_{20})^{(3)})t}} \quad 325$$

$$\frac{1}{(\mu_1)^{(3)}}T_{20}^{0}e^{(R_1)^{(3)}t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}}T_{20}^{0}e^{((R_1)^{(3)}+(r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)}T_{20}^{0}}{(\mu_1)^{(3)}((R_1)^{(3)}-(b'_{22})^{(3)})}\left[e^{(R_1)^{(3)}t} - e^{-(b'_{22})^{(3)}t}\right] + T_{22}^{0}e^{-(b'_{22})^{(3)}t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)}T_{20}^{0}}{(\mu_2)^{(3)}((R_1)^{(3)}+(r_{20})^{(3)}+(R_2)^{(3)})}\left[e^{((R_1)^{(3)}+(r_{20})^{(3)})t} - e^{-(R_2)^{(3)}t}\right] + T_{22}^{0}e^{-(R_2)^{(3)}t}$$

**Definition of** $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$$

$$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$$

$$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$$

**Behavior of the solutions of equation**

**Theorem:** If we denote and define

**Definition of** $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ :

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$$

$$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}\left((G_{27}), t\right) - (b''_{25})^{(4)}\left((G_{27}), t\right) \le -(\tau_1)^{(4)}$$

**Definition of** $(\nu_1)^{(4)}, (\nu_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, \nu^{(4)}, u^{(4)}$ : 329

By $(\nu_1)^{(4)} > 0$ , $(\nu_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$ , $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_1)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(4)},, (\bar{\nu}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$ : 330

By $(\bar{\nu}_1)^{(4)} > 0$ , $(\bar{\nu}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$ , $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

**Definition of** $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}, (\nu_0)^{(4)}$ :-

If we define $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$$(m_2)^{(4)} = (\nu_0)^{(4)}, (m_1)^{(4)} = (\nu_1)^{(4)}, \ \boldsymbol{if}\ (\nu_0)^{(4)} < (\nu_1)^{(4)}$$

$$(m_2)^{(4)} = (\nu_1)^{(4)}, (m_1)^{(4)} = (\bar{\nu}_1)^{(4)}, \boldsymbol{if}\ (\nu_4)^{(4)} < (\nu_0)^{(4)} < (\bar{\nu}_1)^{(4)},$$

and $\boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (\nu_4)^{(4)}, (m_1)^{(4)} = (\nu_0)^{(4)},\ \boldsymbol{if}\ (\bar{\nu}_4)^{(4)} < (\nu_0)^{(4)}$

and analogously

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$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)},\ \boldsymbol{if}\ (u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}, \boldsymbol{if}\ (u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)},$

and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}, \boldsymbol{if}\ (\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

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$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)} t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)} t} \quad 333$$

$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)} t}\right] + G_{26}^0 e^{-(S_2)^{(4)} t} \le G_{26}(t) \le \right. \quad 334$$

*(a26)4G240(m2)4(S1)4−(a26′)4e(S1)4t−e−(a26′)4t+ G260e−(a26′)4t*

$$\boxed{T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}} T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}} T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t} \quad 335$$

$$\frac{(b_{26})^{(4)} T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)} - (b'_{26})^{(4)})}\left[e^{(R_1)^{(4)} t} - e^{-(b'_{26})^{(4)} t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)} t} \le T_{26}(t) \le \quad 336$$

$$\frac{(a_{26})^{(4)} T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)} + (r_{24})^{(4)} + (R_2)^{(4)})}\left[e^{((R_1)^{(4)} + (r_{24})^{(4)})t} - e^{-(R_2)^{(4)} t}\right] + T_{26}^0 e^{-(R_2)^{(4)} t}$$

**Definition of** $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

**Behavior of the solutions of equation** 338

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ :

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

**Definition of** $(\nu_1)^{(5)}, (\nu_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, \nu^{(5)}, u^{(5)}$ : 339

By $(\nu_1)^{(5)} > 0$ , $(\nu_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$ , $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(5)}, , (\bar{\nu}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$ : 340

By $(\bar{\nu}_1)^{(5)} > 0$ , $(\bar{\nu}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$ , $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

**Definition of** $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (\nu_0)^{(5)}$ :-

If we define $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$$(m_2)^{(5)} = (\nu_0)^{(5)}, (m_1)^{(5)} = (\nu_1)^{(5)}, \ \boldsymbol{if}\ (\nu_0)^{(5)} < (\nu_1)^{(5)}$$

$$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\bar{\nu}_1)^{(5)}, \boldsymbol{if}\ (\nu_1)^{(5)} < (\nu_0)^{(5)} < (\bar{\nu}_1)^{(5)},$$

and $\boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}}$

$$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$$

and analogously 341

$$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)}, \ \boldsymbol{if}\ (u_0)^{(5)} < (u_1)^{(5)}$$

$$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}, \boldsymbol{if}\ (u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)},$$

and $\boxed{(u_0)^{(5)} = \frac{T^0_{28}}{T^0_{29}}}$

$$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}, \boldsymbol{if}\ (\bar{u}_1)^{(5)} < (u_0)^{(5)}$$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$$G^0_{28} e^{\left((S_1)^{(5)} - (p_{28})^{(5)}\right)t} \le G_{28}(t) \le G^0_{28} e^{(S_1)^{(5)}t}$$

where $(p_i)^{(5)}$ is defined by equation

$$\frac{1}{(m_5)^{(5)}} G^0_{28} e^{\left((S_1)^{(5)} - (p_{28})^{(5)}\right)t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}} G^0_{28} e^{(S_1)^{(5)}t}$$ 343

$$\left(\frac{(a_{30})^{(5)}G_{28}^{0}}{(m_1)^{(5)}((S_1)^{(5)}-(p_{28})^{(5)}-(S_2)^{(5)})}\left[e^{((S_1)^{(5)}-(p_{28})^{(5)})t}-e^{-(S_2)^{(5)}t}\right]+G_{30}^{0}e^{-(S_2)^{(5)}t}\leq G_{30}(t)\leq\right. \quad 344$$

*(a30)5G280(m2)5(S1)5−(a30′)5e(S1)5t−e−(a30′)5t+ G300e−(a30′)5t*

$$\boxed{T_{28}^{0}e^{(R_1)^{(5)}t}\leq T_{28}(t)\leq T_{28}^{0}e^{((R_1)^{(5)}+(r_{28})^{(5)})t}} \quad 345$$

$$\frac{1}{(\mu_1)^{(5)}}T_{28}^{0}e^{(R_1)^{(5)}t}\leq T_{28}(t)\leq\frac{1}{(\mu_2)^{(5)}}T_{28}^{0}e^{((R_1)^{(5)}+(r_{28})^{(5)})t} \quad 346$$

$$\frac{(b_{30})^{(5)}T_{28}^{0}}{(\mu_1)^{(5)}((R_1)^{(5)}-(b'_{30})^{(5)})}\left[e^{(R_1)^{(5)}t}-e^{-(b'_{30})^{(5)}t}\right]+T_{30}^{0}e^{-(b'_{30})^{(5)}t}\leq T_{30}(t)\leq \quad 347$$

$$\frac{(a_{30})^{(5)}T_{28}^{0}}{(\mu_2)^{(5)}((R_1)^{(5)}+(r_{28})^{(5)}+(R_2)^{(5)})}\left[e^{((R_1)^{(5)}+(r_{28})^{(5)})t}-e^{-(R_2)^{(5)}t}\right]+T_{30}^{0}e^{-(R_2)^{(5)}t}$$

**Definition of** $(S_1)^{(5)},(S_2)^{(5)},(R_1)^{(5)},(R_2)^{(5)}$:- 348

Where $(S_1)^{(5)}=(a_{28})^{(5)}(m_2)^{(5)}-(a'_{28})^{(5)}$

$(S_2)^{(5)}=(a_{30})^{(5)}-(p_{30})^{(5)}$

$(R_1)^{(5)}=(b_{28})^{(5)}(\mu_2)^{(5)}-(b'_{28})^{(5)}$

$(R_2)^{(5)}=(b'_{30})^{(5)}-(r_{30})^{(5)}$

**Behavior of the solutions of equation** 349

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(6)},(\sigma_2)^{(6)},(\tau_1)^{(6)},(\tau_2)^{(6)}$ :

$(\sigma_1)^{(6)},(\sigma_2)^{(6)},(\tau_1)^{(6)},(\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)}\leq-(a'_{32})^{(6)}+(a'_{33})^{(6)}-(a''_{32})^{(6)}(T_{33},t)+(a''_{33})^{(6)}(T_{33},t)\leq-(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)}\leq-(b'_{32})^{(6)}+(b'_{33})^{(6)}-(b''_{32})^{(6)}\big((G_{35}),t\big)-(b''_{33})^{(6)}\big((G_{35}),t\big)\leq-(\tau_1)^{(6)}$$

**Definition of** $(v_1)^{(6)},(v_2)^{(6)},(u_1)^{(6)},(u_2)^{(6)},v^{(6)},u^{(6)}$ : 350

By $(v_1)^{(6)}>0$ , $(v_2)^{(6)}<0$ and respectively $(u_1)^{(6)}>0$ , $(u_2)^{(6)}<0$ the roots of the equations $(a_{33})^{(6)}\big(v^{(6)}\big)^2+(\sigma_1)^{(6)}v^{(6)}-(a_{32})^{(6)}=0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2+(\tau_1)^{(6)}u^{(6)}-(b_{32})^{(6)}=0$ and

**Definition of** $(\bar{v}_1)^{(6)},,(\bar{v}_2)^{(6)},(\bar{u}_1)^{(6)},(\bar{u}_2)^{(6)}$ : 351

By $(\bar{v}_1)^{(6)}>0$ , $(\bar{v}_2)^{(6)}<0$ and respectively $(\bar{u}_1)^{(6)}>0$ , $(\bar{u}_2)^{(6)}<0$ the roots of the equations $(a_{33})^{(6)}\big(v^{(6)}\big)^2+(\sigma_2)^{(6)}v^{(6)}-(a_{32})^{(6)}=0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2+(\tau_2)^{(6)}u^{(6)}-(b_{32})^{(6)}=0$

**Definition of** $(m_1)^{(6)},(m_2)^{(6)},(\mu_1)^{(6)},(\mu_2)^{(6)},(\nu_0)^{(6)}$ :-

If we define $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}$ by

$(m_2)^{(6)} = (v_0)^{(6)}, (m_1)^{(6)} = (v_1)^{(6)}$, $\boldsymbol{if}\ (v_0)^{(6)} < (v_1)^{(6)}$

$(m_2)^{(6)} = (v_1)^{(6)}, (m_1)^{(6)} = (\bar{v}_6)^{(6)}, \boldsymbol{if}\ (v_1)^{(6)} < (v_0)^{(6)} < (\bar{v}_1)^{(6)}$,
and $\boxed{(v_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$

$(m_2)^{(6)} = (v_1)^{(6)}, (m_1)^{(6)} = (v_0)^{(6)}$, $\boldsymbol{if}\ (\bar{v}_1)^{(6)} < (v_0)^{(6)}$

and analogously 352

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}$, $\boldsymbol{if}\ (u_0)^{(6)} < (u_1)^{(6)}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}, \boldsymbol{if}\ (u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)}$,
and $\boxed{(u_0)^{(6)} = \frac{T_{32}^0}{T_{33}^0}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}, \boldsymbol{if}\ (\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$

Then the solution of global equations satisfies the inequalities 353

$$G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{32}(t) \le G_{32}^0 e^{(S_1)^{(6)} t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}} G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{33}(t) \le \frac{1}{(m_2)^{(6)}} G_{32}^0 e^{(S_1)^{(6)} t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)} G_{32}^0}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})}\left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)} t}\right] + G_{34}^0 e^{-(S_2)^{(6)} t} \le G_{34}(t) \le \right. \quad 355$$

*(a34)6G320(m2)6(S1)6−(a34′)6e(S1)6t−e−(a34′)6t+ G340e−(a34′)6t*

$$\boxed{T_{32}^0 e^{(R_1)^{(6)} t} \le T_{32}(t) \le T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}} T_{32}^0 e^{(R_1)^{(6)} t} \le T_{32}(t) \le \frac{1}{(\mu_2)^{(6)}} T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)} T_{32}^0}{(\mu_1)^{(6)}((R_1)^{(6)} - (b'_{34})^{(6)})}\left[e^{(R_1)^{(6)} t} - e^{-(b'_{34})^{(6)} t}\right] + T_{34}^0 e^{-(b'_{34})^{(6)} t} \le T_{34}(t) \le \quad 358$$

$$\frac{(a_{34})^{(6)} T_{32}^0}{(\mu_2)^{(6)}((R_1)^{(6)} + (r_{32})^{(6)} + (R_2)^{(6)})}\left[e^{((R_1)^{(6)} + (r_{32})^{(6)})t} - e^{-(R_2)^{(6)} t}\right] + T_{34}^0 e^{-(R_2)^{(6)} t}$$

**Definition of** $(S_1)^{(6)}, (S_2)^{(6)}, (R_1)^{(6)}, (R_2)^{(6)}$:- 359

Where $(S_1)^{(6)} = (a_{32})^{(6)}(m_2)^{(6)} - (a'_{32})^{(6)}$

$(S_2)^{(6)} = (a_{34})^{(6)} - (p_{34})^{(6)}$

$(R_1)^{(6)} = (b_{32})^{(6)}(\mu_2)^{(6)} - (b'_{32})^{(6)}$

$$(R_2)^{(6)} = (b'_{34})^{(6)} - (r_{34})^{(6)}$$

**Behavior of the solutions of equation**

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$ :

$(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)} \le -(a'_{36})^{(7)} + (a'_{37})^{(7)} - (a''_{36})^{(7)}(T_{37}, t) + (a''_{37})^{(7)}(T_{37}, t) \le -(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)} \le -(b'_{36})^{(7)} + (b'_{37})^{(7)} - (b''_{36})^{(7)}\big((G_{39}), t\big) - (b''_{37})^{(7)}\big((G_{39}), t\big) \le -(\tau_1)^{(7)}$$

**Definition of** $(\nu_1)^{(7)}, (\nu_2)^{(7)}, (u_1)^{(7)}, (u_2)^{(7)}, \nu^{(7)}, u^{(7)}$ : 361

By $(\nu_1)^{(7)} > 0, (\nu_2)^{(7)} < 0$ and respectively $(u_1)^{(7)} > 0, (u_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_1)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(7)}, , (\bar{\nu}_2)^{(7)}, (\bar{u}_1)^{(7)}, (\bar{u}_2)^{(7)}$ : 362

By $(\bar{\nu}_1)^{(7)} > 0, (\bar{\nu}_2)^{(7)} < 0$ and respectively $(\bar{u}_1)^{(7)} > 0, (\bar{u}_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

**Definition of** $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (\nu_0)^{(7)}$ :-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$$(m_2)^{(7)} = (\nu_0)^{(7)}, (m_1)^{(7)} = (\nu_1)^{(7)}, \ \boldsymbol{if}\ (\nu_0)^{(7)} < (\nu_1)^{(7)}$$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\bar{\nu}_1)^{(7)}, \boldsymbol{if}\ (\nu_1)^{(7)} < (\nu_0)^{(7)} < (\bar{\nu}_1)^{(7)},$$

and $\boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\nu_0)^{(7)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(7)} < (\nu_0)^{(7)}$$

and analogously 363

$$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}, \ \boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$$

and $\boxed{(u_0)^{(7)} = \frac{T^0_{36}}{T^0_{37}}}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)}$ where $(u_1)^{(7)}, (\bar{u}_1)^{(7)}$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^0 e^{\left((S_1)^{(7)} - (p_{36})^{(7)}\right)t} \leq G_{36}(t) \leq G_{36}^0 e^{(S_1)^{(7)} t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{\left((S_1)^{(7)} - (p_{36})^{(7)}\right)t} \leq G_{37}(t) \leq \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)} t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)}\left((S_1)^{(7)} - (p_{36})^{(7)} - (S_2)^{(7)}\right)}\left[e^{\left((S_1)^{(7)} - (p_{36})^{(7)}\right)t} - e^{-(S_2)^{(7)} t}\right] + G_{38}^0 e^{-(S_2)^{(7)} t} \leq G_{38}(t) \leq \right. \quad 366$$

$$\left. \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)}\left((S_1)^{(7)} - (a'_{38})^{(7)}\right)}\left[e^{(S_1)^{(7)} t} - e^{-(a'_{38})^{(7)} t}\right] + G_{38}^0 e^{-(a'_{38})^{(7)} t}\right)$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)} t} \leq T_{36}(t) \leq T_{36}^0 e^{\left((R_1)^{(7)} + (r_{36})^{(7)}\right)t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)} t} \leq T_{36}(t) \leq \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{\left((R_1)^{(7)} + (r_{36})^{(7)}\right)t} \quad 368$$

$$\frac{(b_{38})^{(7)} T_{36}^0}{(\mu_1)^{(7)}\left((R_1)^{(7)} - (b'_{38})^{(7)}\right)}\left[e^{(R_1)^{(7)} t} - e^{-(b'_{38})^{(7)} t}\right] + T_{38}^0 e^{-(b'_{38})^{(7)} t} \leq T_{38}(t) \leq \quad 369$$

$$\frac{(a_{38})^{(7)} T_{36}^0}{(\mu_2)^{(7)}\left((R_1)^{(7)} + (r_{36})^{(7)} + (R_2)^{(7)}\right)}\left[e^{\left((R_1)^{(7)} + (r_{36})^{(7)}\right)t} - e^{-(R_2)^{(7)} t}\right] + T_{38}^0 e^{-(R_2)^{(7)} t}$$

**Definition of** $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$$

$$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$$

$$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$$

**Behavior of the solutions of equation** 371

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ :

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$$-(\sigma_2)^{(8)} \leq -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \leq -(\sigma_1)^{(8)}$$

$$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\big((G_{43}),t\big) - (b''_{41})^{(8)}\big((G_{43}),t\big) \le -(\tau_1)^{(8)}$$

**Definition of** $(\nu_1)^{(8)}, (\nu_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, \nu^{(8)}, u^{(8)}$ : 372

By $(\nu_1)^{(8)} > 0$ , $(\nu_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0$ , $(u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(8)},, (\bar{\nu}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$ :

By $(\bar{\nu}_1)^{(8)} > 0$ , $(\bar{\nu}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0$ , $(\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

**Definition of** $(m_1)^{(8)}$ , $(m_2)^{(8)}$ , $(\mu_1)^{(8)}, (\mu_2)^{(8)}, (\nu_0)^{(8)}$ :-

If we define $(m_1)^{(8)}$ , $(m_2)^{(8)}$ , $(\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$(m_2)^{(8)} = (\nu_0)^{(8)}, (m_1)^{(8)} = (\nu_1)^{(8)}$, ***if*** $(\nu_0)^{(8)} < (\nu_1)^{(8)}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ , ***if*** $(\nu_1)^{(8)} < (\nu_0)^{(8)} < (\bar{\nu}_1)^{(8)}$,
and $\boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\nu_0)^{(8)}$, ***if*** $(\bar{\nu}_1)^{(8)} < (\nu_0)^{(8)}$

and analogously 374

$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)}$, ***if*** $(u_0)^{(8)} < (u_1)^{(8)}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}$ , ***if*** $(u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)}$,
and $\boxed{(u_0)^{(8)} = \frac{T^0_{40}}{T^0_{41}}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}$, ***if*** $(\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G^0_{40}e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{40}(t) \le G^0_{40}e^{(S_1)^{(8)}t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}} G_{40}^0 e^{((S_1)^{(8)}-(p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}} G_{40}^0 e^{(S_1)^{(8)}t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)}G_{40}^0}{(m_1)^{(8)}((S_1)^{(8)}-(p_{40})^{(8)}-(S_2)^{(8)})}\left[e^{((S_1)^{(8)}-(p_{40})^{(8)})t} - e^{-(S_2)^{(8)}t}\right] + G_{42}^0 e^{-(S_2)^{(8)}t} \le G_{42}(t) \le \right. \quad 377$$

$$\left. \frac{(a_{42})^{(8)}G_{40}^0}{(m_2)^{(8)}((S_1)^{(8)}-(a'_{42})^{(8)})}[e^{(S_1)^{(8)}t} - e^{-(a'_{42})^{(8)}t}] + G_{42}^0 e^{-(a'_{42})^{(8)}t}\right)$$

$$\boxed{T_{40}^0 e^{(R_1)^{(8)}t} \le T_{40}(t) \le T_{40}^0 e^{((R_1)^{(8)}+(r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}} T_{40}^0 e^{(R_1)^{(8)}t} \le T_{40}(t) \le \frac{1}{(\mu_2)^{(8)}} T_{40}^0 e^{((R_1)^{(8)}+(r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)}T_{40}^0}{(\mu_1)^{(8)}((R_1)^{(8)}-(b'_{42})^{(8)})}\left[e^{(R_1)^{(8)}t} - e^{-(b'_{42})^{(8)}t}\right] + T_{42}^0 e^{-(b'_{42})^{(8)}t} \le T_{42}(t) \le \quad 380$$

$$\frac{(a_{42})^{(8)}T_{40}^0}{(\mu_2)^{(8)}((R_1)^{(8)}+(r_{40})^{(8)}+(R_2)^{(8)})}\left[e^{((R_1)^{(8)}+(r_{40})^{(8)})t} - e^{-(R_2)^{(8)}t}\right] + T_{42}^0 e^{-(R_2)^{(8)}t}$$

**Definition of** $(S_1)^{(8)}, (S_2)^{(8)}, (R_1)^{(8)}, (R_2)^{(8)}$:- 381

Where $(S_1)^{(8)} = (a_{40})^{(8)}(m_2)^{(8)} - (a'_{40})^{(8)}$

$(S_2)^{(8)} = (a_{42})^{(8)} - (p_{42})^{(8)}$

$(R_1)^{(8)} = (b_{40})^{(8)}(\mu_2)^{(8)} - (b'_{40})^{(8)}$

$(R_2)^{(8)} = (b'_{42})^{(8)} - (r_{42})^{(8)}$

**Behavior of the solutions of equation 37 to 92** 382

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ :

$(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ four constants satisfying

$$-(\sigma_2)^{(9)} \le -(a'_{44})^{(9)} + (a'_{45})^{(9)} - (a''_{44})^{(9)}(T_{45}, t) + (a''_{45})^{(9)}(T_{45}, t) \le -(\sigma_1)^{(9)}$$

$$-(\tau_2)^{(9)} \le -(b'_{44})^{(9)} + (b'_{45})^{(9)} - (b''_{44})^{(9)}\big((G_{47}), t\big) - (b''_{45})^{(9)}\big((G_{47}), t\big) \le -(\tau_1)^{(9)}$$

**Definition of** $(\nu_1)^{(9)}, (\nu_2)^{(9)}, (u_1)^{(9)}, (u_2)^{(9)}, \nu^{(9)}, u^{(9)}$ :

By $(\nu_1)^{(9)} > 0$ , $(\nu_2)^{(9)} < 0$ and respectively $(u_1)^{(9)} > 0$ , $(u_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_1)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(9)}, , (\bar{\nu}_2)^{(9)}, (\bar{u}_1)^{(9)}, (\bar{u}_2)^{(9)}$ :

By $(\bar{\nu}_1)^{(9)} > 0$ , $(\bar{\nu}_2)^{(9)} < 0$ and respectively $(\bar{u}_1)^{(9)} > 0$ , $(\bar{u}_2)^{(9)} < 0$ the

roots of the equations $(a_{45})^{(9)}\left(v^{(9)}\right)^2+(\sigma_2)^{(9)}v^{(9)}-(a_{44})^{(9)}=0$
and $(b_{45})^{(9)}\left(u^{(9)}\right)^2+(\tau_2)^{(9)}u^{(9)}-(b_{44})^{(9)}=0$

**Definition of** $(m_1)^{(9)},(m_2)^{(9)},(\mu_1)^{(9)},(\mu_2)^{(9)},(v_0)^{(9)}$ :-

If we define $(m_1)^{(9)},(m_2)^{(9)},(\mu_1)^{(9)},(\mu_2)^{(9)}$ by

$(m_2)^{(9)}=(v_0)^{(9)},(m_1)^{(9)}=(v_1)^{(9)},\ \boldsymbol{if}\ (v_0)^{(9)}<(v_1)^{(9)}$

$(m_2)^{(9)}=(v_1)^{(9)},(m_1)^{(9)}=(\bar{v}_1)^{(9)},\boldsymbol{if}\ (v_1)^{(9)}<(v_0)^{(9)}<(\bar{v}_1)^{(9)}$,
and $\boxed{(v_0)^{(9)}=\frac{G_{44}^0}{G_{45}^0}}$

$(m_2)^{(9)}=(v_1)^{(9)},(m_1)^{(9)}=(v_0)^{(9)},\ \boldsymbol{if}\ (\bar{v}_1)^{(9)}<(v_0)^{(9)}$

and analogously

$(\mu_2)^{(9)}=(u_0)^{(9)},(\mu_1)^{(9)}=(u_1)^{(9)},\ \boldsymbol{if}\ (u_0)^{(9)}<(u_1)^{(9)}$

$(\mu_2)^{(9)}=(u_1)^{(9)},(\mu_1)^{(9)}=(\bar{u}_1)^{(9)},\boldsymbol{if}\ (u_1)^{(9)}<(u_0)^{(9)}<(\bar{u}_1)^{(9)}$,
and $\boxed{(u_0)^{(9)}=\frac{T_{44}^0}{T_{45}^0}}$

$(\mu_2)^{(9)}=(u_1)^{(9)},(\mu_1)^{(9)}=(u_0)^{(9)},\boldsymbol{if}\ (\bar{u}_1)^{(9)}<(u_0)^{(9)}$ where $(u_1)^{(9)},(\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G_{44}^0e^{\left((S_1)^{(9)}-(p_{44})^{(9)}\right)t}\le G_{44}(t)\le G_{44}^0e^{(S_1)^{(9)}t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}}G_{44}^0e^{\left((S_1)^{(9)}-(p_{44})^{(9)}\right)t}\le G_{45}(t)\le\frac{1}{(m_2)^{(9)}}G_{44}^0e^{(S_1)^{(9)}t}$$

$$\left(\frac{(a_{46})^{(9)}G_{44}^0}{(m_1)^{(9)}\left((S_1)^{(9)}-(p_{44})^{(9)}-(S_2)^{(9)}\right)}\left[e^{\left((S_1)^{(9)}-(p_{44})^{(9)}\right)t}-e^{-(S_2)^{(9)}t}\right]+G_{46}^0e^{-(S_2)^{(9)}t}\le G_{46}(t)\le\right.$$
$$\left.\frac{(a_{46})^{(9)}G_{44}^0}{(m_2)^{(9)}\left((S_1)^{(9)}-(a'_{46})^{(9)}\right)}\left[e^{(S_1)^{(9)}t}-e^{-(a'_{46})^{(9)}t}\right]+\ G_{46}^0e^{-(a'_{46})^{(9)}t}\right)$$

$$\boxed{T_{44}^0e^{(R_1)^{(9)}t}\le T_{44}(t)\le T_{44}^0e^{\left((R_1)^{(9)}+(r_{44})^{(9)}\right)t}}$$

$$\frac{1}{(\mu_1)^{(9)}}T_{44}^0e^{(R_1)^{(9)}t}\le T_{44}(t)\le\frac{1}{(\mu_2)^{(9)}}T_{44}^0e^{\left((R_1)^{(9)}+(r_{44})^{(9)}\right)t}$$

$$\frac{(b_{46})^{(9)}T_{44}^0}{(\mu_1)^{(9)}\left((R_1)^{(9)}-(b'_{46})^{(9)}\right)}\left[e^{(R_1)^{(9)}t}-e^{-(b'_{46})^{(9)}t}\right]+T_{46}^0e^{-(b'_{46})^{(9)}t}\le T_{46}(t)\le$$

$$\frac{(a_{46})^{(9)}T_{44}^0}{(\mu_2)^{(9)}\left((R_1)^{(9)}+(r_{44})^{(9)}+(R_2)^{(9)}\right)}\left[e^{\left((R_1)^{(9)}+(r_{44})^{(9)}\right)t}-e^{-(R_2)^{(9)}t}\right]+T_{46}^0e^{-(R_2)^{(9)}t}$$

**Definition of** $(S_1)^{(9)},(S_2)^{(9)},(R_1)^{(9)},(R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$

**Proof :** From global equations we obtain 383

$$\frac{d\nu^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right) - (a''_{14})^{(1)}(T_{14}, t)\nu^{(1)} - (a_{14})^{(1)}\nu^{(1)}$$

**Definition of** $\nu^{(1)}$ :- $\boxed{\nu^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)}\right) \le \frac{d\nu^{(1)}}{dt} \le -\left((a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(1)}, (\nu_0)^{(1)}$ :-

For $0 < \boxed{(\nu_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}} < (\nu_1)^{(1)} < (\bar{\nu}_1)^{(1)}$

$$\nu^{(1)}(t) \ge \frac{(\nu_1)^{(1)} + (C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)} - (\nu_0)^{(1)}\right)t\right]}}{1 + (C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)} - (\nu_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(\nu_1)^{(1)} - (\nu_0)^{(1)}}{(\nu_0)^{(1)} - (\nu_2)^{(1)}}}$$

it follows $(\nu_0)^{(1)} \le \nu^{(1)}(t) \le (\nu_1)^{(1)}$

In the same manner , we get 384

$$\nu^{(1)}(t) \le \frac{(\bar{\nu}_1)^{(1)} + (\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)} - (\bar{\nu}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)} - (\bar{\nu}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{\nu}_1)^{(1)} - (\nu_0)^{(1)}}{(\nu_0)^{(1)} - (\bar{\nu}_2)^{(1)}}}$$

From which we deduce $(\nu_0)^{(1)} \le \nu^{(1)}(t) \le (\bar{\nu}_1)^{(1)}$

If $0 < (\nu_1)^{(1)} < (\nu_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}} < (\bar{\nu}_1)^{(1)}$ we find like in the previous case, 385

$$(\nu_1)^{(1)} \le \frac{(\nu_1)^{(1)} + (C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)} - (\nu_2)^{(1)}\right)t\right]}}{1 + (C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)} - (\nu_2)^{(1)}\right)t\right]}} \le \nu^{(1)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(1)} + (\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)} - (\bar{\nu}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)} - (\bar{\nu}_2)^{(1)}\right)t\right]}} \le (\bar{\nu}_1)^{(1)}$$

If $0 < (\nu_1)^{(1)} \leq (\bar{\nu}_1)^{(1)} \leq \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$ , we obtain 386

$$(\nu_1)^{(1)} \leq \nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)} + (\bar{C})^{(1)}(\bar{\nu}_2)^{(1)} e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)} - (\bar{\nu}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)} e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)} - (\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\nu_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(1)}(t)$ :-

$$(m_2)^{(1)} \leq \nu^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{\nu^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(1)}(t)$ :-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{13}'')^{(1)} = (a_{14}'')^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)} G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b_{13}'')^{(1)} = (b_{14}'')^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)} T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

**Proof** : From global equations we obtain 387

$$\frac{d\nu^{(2)}}{dt} = (a_{16})^{(2)} - \left((a_{16}')^{(2)} - (a_{17}')^{(2)} + (a_{16}'')^{(2)}(T_{17}, t)\right) - (a_{17}'')^{(2)}(T_{17}, t)\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

**Definition of** $\nu^{(2)}$ :- $\boxed{\nu^{(2)} = \frac{G_{16}}{G_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \leq \frac{d\nu^{(2)}}{dt} \leq -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

**Definition of** $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$ :-

For $0 < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \geq \frac{(\nu_1)^{(2)}+(\mathrm{C})^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_0)^{(2)}\right)t\right]}}{1+(\mathrm{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\mathrm{C})^{(2)}=\frac{(\nu_1)^{(2)}-(\nu_0)^{(2)}}{(\nu_0)^{(2)}-(\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)}+(\bar{\mathrm{C}})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{\mathrm{C}})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{\mathrm{C}})^{(2)}=\frac{(\bar{\nu}_1)^{(2)}-(\nu_0)^{(2)}}{(\nu_0)^{(2)}-(\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{\mathrm{G}_{16}^0}{\mathrm{G}_{17}^0} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \leq \frac{(\nu_1)^{(2)}+(\mathrm{C})^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}}{1+(\mathrm{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}} \leq \nu^{(2)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(2)}+(\bar{\mathrm{C}})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{\mathrm{C}})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \leq (\bar{\nu}_1)^{(2)} \leq (\nu_0)^{(2)} = \frac{\mathrm{G}_{16}^0}{\mathrm{G}_{17}^0}$ , we obtain 394

$$(\nu_1)^{(2)} \leq \nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)}+(\bar{\mathrm{C}})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{\mathrm{C}})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(2)}(t)$ :- 395

$$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}, \quad \boxed{\nu^{(2)}(t)=\frac{G_{16}(t)}{G_{17}(t)}}$$

In a completely analogous way, we obtain 396

**Definition of** $u^{(2)}(t)$ :-

$$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}, \quad \boxed{u^{(2)}(t)=\frac{T_{16}(t)}{T_{17}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :** 397

If $(a_{16}'')^{(2)} = (a_{17}'')^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)}G_{17}(t)$

Analogously if $(b''_{16})^{(2)} = (b''_{17})^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)}T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

**Proof** : From global equations we obtain 398

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right) - (a''_{21})^{(3)}(T_{21}, t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

**Definition of** $\nu^{(3)}$ :- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right) \leq \frac{d\nu^{(3)}}{dt} \leq -\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right)$$

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From which one obtains

For $0 < (\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\nu_1)^{(3)} < (\bar{\nu}_1)^{(3)}$

$$\nu^{(3)}(t) \geq \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(\nu_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\nu_2)^{(3)}}}$$

it follows $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\nu_1)^{(3)}$

In the same manner , we get 401

$$\nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)} + (\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{\nu}_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\bar{\nu}_2)^{(3)}}}$$

**Definition of** $(\bar{\nu}_1)^{(3)}$ :-

From which we deduce $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\bar{\nu}_1)^{(3)}$

If $0 < (\nu_1)^{(3)} < (\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\bar{\nu}_1)^{(3)}$ we find like in the previous case, 402

$$(\nu_1)^{(3)} \leq \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_2)^{(3)}\right)t\right]}}{1 + (C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_2)^{(3)}\right)t\right]}} \leq \nu^{(3)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(3)} + (\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\bar{\nu}_1)^{(3)}$$

If $0 < (\nu_1)^{(3)} \leq (\bar{\nu}_1)^{(3)} \leq (\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}$ , we obtain 403

$$(\nu_1)^{(3)} \leq \nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\nu_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(3)}(t)$ :-

$$(m_2)^{(3)} \leq \nu^{(3)}(t) \leq (m_1)^{(3)}, \quad \boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(3)}(t)$ :-

$$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}, \quad \boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{20})^{(3)} = (a''_{21})^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)}G_{21}(t)$

Analogously if $(b''_{20})^{(3)} = (b''_{21})^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)}T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

**Proof** : From global equations we obtain 404

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right) - (a''_{25})^{(4)}(T_{25},t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

**Definition of** $\nu^{(4)}$ :- $\quad \boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right) \leq \frac{d\nu^{(4)}}{dt} \leq -\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_4)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$ :-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \geq \frac{(\nu_1)^{(4)}+(C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_0)^{(4)}\right)t\right]}}{4+(C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)}-(\nu_0)^{(4)}}{(\nu_0)^{(4)}-(\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \le \frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{4+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)}-(\nu_0)^{(4)}}{(\nu_0)^{(4)}-(\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \le \nu^{(4)}(t) \le (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \le \frac{(\nu_1)^{(4)}+(C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_2)^{(4)}\right)t\right]}}{1+(C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_2)^{(4)}\right)t\right]}} \le \nu^{(4)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \le (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \le (\bar{\nu}_1)^{(4)} \le \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$ , we obtain 407

$$(\nu_1)^{(4)} \le \nu^{(4)}(t) \le \frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \le (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(4)}(t)$ :-

$$(m_2)^{(4)} \le \nu^{(4)}(t) \le (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(4)}(t)$ :-

$$(\mu_2)^{(4)} \le u^{(4)}(t) \le (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a_{24}'')^{(4)} = (a_{25}'')^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)}G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b_{24}'')^{(4)} = (b_{25}'')^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)}T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

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**Proof** : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a_{28}')^{(5)} - (a_{29}')^{(5)} + (a_{28}'')^{(5)}(T_{29},t)\right) - (a_{29}'')^{(5)}(T_{29},t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

**Definition of** $\nu^{(5)}$ :- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$ :-

For $0 < \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \leq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}}{1 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}} \leq \nu^{(5)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \leq (\bar{\nu}_1)^{(5)} \leq \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$ , we obtain 411

$$(\nu_1)^{(5)} \leq \nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(5)}(t)$ :-

$$(m_2)^{(5)} \leq \nu^{(5)}(t) \leq (m_1)^{(5)}, \quad \boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(5)}(t)$ :-

$$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}, \quad \boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{28})^{(5)} = (a''_{29})^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)} G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b''_{28})^{(5)} = (b''_{29})^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$ if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)} T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

**Proof** : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a'_{32})^{(6)} - (a'_{33})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right) - (a''_{33})^{(6)}(T_{33}, t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

**Definition of** $\nu^{(6)}$ :- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \leq \frac{d\nu^{(6)}}{dt} \leq -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$ :-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \geq \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\nu_1)^{(6)}$

In the same manner , we get 413

$$\nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{\nu}_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\bar{\nu}_1)^{(6)}$

If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 414

$$(\nu_1)^{(6)} \leq \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}} \leq \nu^{(6)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\bar{\nu}_1)^{(6)}$$

If $0 < (\nu_1)^{(6)} \leq (\bar{\nu}_1)^{(6)} \leq \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$, we obtain

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$$(\nu_1)^{(6)} \leq \nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(6)}(t)$ :-

$$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}, \quad \boxed{\nu^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(6)}(t)$ :-

$$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}, \quad \boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)}G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)}T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

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**Proof** : From global equations we obtain

$$\frac{d\nu^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right) - (a''_{37})^{(7)}(T_{37}, t)\nu^{(7)} - (a_{37})^{(7)}\nu^{(7)}$$

**Definition of** $\nu^{(7)}$ :- $\boxed{\nu^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right) \leq \frac{d\nu^{(7)}}{dt} \leq -\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(7)}, (\nu_0)^{(7)}$ :-

For $0 < \boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}} < (\nu_1)^{(7)} < (\bar{\nu}_1)^{(7)}$

$$\nu^{(7)}(t) \geq \frac{(\nu_1)^{(7)}+(C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_0)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)}=\frac{(\nu_1)^{(7)}-(\nu_0)^{(7)}}{(\nu_0)^{(7)}-(\nu_2)^{(7)}}}$$

it follows $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\nu_1)^{(7)}$

In the same manner , we get 417

$$\nu^{(7)}(t) \leq \frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)}=\frac{(\bar{\nu}_1)^{(7)}-(\nu_0)^{(7)}}{(\nu_0)^{(7)}-(\bar{\nu}_2)^{(7)}}}$$

From which we deduce $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\bar{\nu}_1)^{(7)}$

If $0 < (\nu_1)^{(7)} < (\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0} < (\bar{\nu}_1)^{(7)}$ we find like in the previous case, 418

$$(\nu_1)^{(7)} \leq \frac{(\nu_1)^{(7)}+(C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}} \leq \nu^{(7)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \leq (\bar{\nu}_1)^{(7)}$$

If $0 < (\nu_1)^{(7)} \leq (\bar{\nu}_1)^{(7)} \leq \boxed{(\nu_0)^{(7)}=\frac{G_{36}^0}{G_{37}^0}}$ , we obtain 419

$$(\nu_1)^{(7)} \leq \nu^{(7)}(t) \leq \frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \leq (\nu_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have
**Definition of** $\nu^{(7)}(t)$ :-

$$(m_2)^{(7)} \leq \nu^{(7)}(t) \leq (m_1)^{(7)}, \quad \boxed{\nu^{(7)}(t)=\frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(7)}(t)$ :- 420

$$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}, \quad \boxed{u^{(7)}(t)=\frac{T_{36}(t)}{T_{37}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $\nu^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)}G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for**

**the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)}T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

**Proof** : From global equations we obtain 421

$$\frac{d\nu^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right) - (a''_{41})^{(8)}(T_{41},t)\nu^{(8)} - (a_{41})^{(8)}\nu^{(8)}$$

**Definition of** $\nu^{(8)}$ :- $\boxed{\nu^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right) \le \frac{d\nu^{(8)}}{dt} \le -\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$ :-

For $0 < \boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$

$$\nu^{(8)}(t) \ge \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \le \nu^{(8)}(t) \le (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \le \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \le \nu^{(8)}(t) \le (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \le \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}} \le \nu^{(8)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \le (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \le (\bar{\nu}_1)^{(8)} \le \boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}}$ , we obtain 424

$$(v_1)^{(8)} \leq v^{(8)}(t) \leq \frac{(\bar{v}_1)^{(8)}+(\bar{C})^{(8)}(\bar{v}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{v}_1)^{(8)}-(\bar{v}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{v}_1)^{(8)}-(\bar{v}_2)^{(8)}\right)t\right]}} \leq (v_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $v^{(8)}(t)$ :-

$$(m_2)^{(8)} \leq v^{(8)}(t) \leq (m_1)^{(8)}, \quad \boxed{v^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(8)}(t)$ :-

$$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}, \quad \boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(v_1)^{(8)} = (\bar{v}_1)^{(8)}$ if in addition $(v_0)^{(8)} = (v_1)^{(8)}$ then $v^{(8)}(t) = (v_0)^{(8)}$ and as a consequence $G_{40}(t) = (v_0)^{(8)}G_{41}(t)$ **this also defines** $(v_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then
$(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)}T_{41}(t)$ This is an important consequence of the relation between $(v_1)^{(8)}$ and $(\bar{v}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

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**Proof** : From 99,20,44,22,23,44 we obtain

$$\frac{dv^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45}, t)\right) - (a''_{45})^{(9)}(T_{45}, t)v^{(9)} - (a_{45})^{(9)}v^{(9)}$$

**Definition of** $v^{(9)}$ :- $\boxed{v^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)}\right) \leq \frac{dv^{(9)}}{dt} \leq -\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_1)^{(9)}v^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

**Definition of** $(\bar{v}_1)^{(9)}, (v_0)^{(9)}$ :-

For $0 < \boxed{(v_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (v_1)^{(9)} < (\bar{v}_1)^{(9)}$

$$v^{(9)}(t) \geq \frac{(v_1)^{(9)}+(C)^{(9)}(v_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((v_1)^{(9)}-(v_0)^{(9)}\right)t\right]}}{1+(C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((v_1)^{(9)}-(v_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(v_1)^{(9)}-(v_0)^{(9)}}{(v_0)^{(9)}-(v_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\nu_9)^{(9)}$

In the same manner , we get

$$\nu^{(9)}(t) \leq \frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)}-(\nu_0)^{(9)}}{(\nu_0)^{(9)}-(\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \leq \frac{(\nu_1)^{(9)}+(C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}}{1+(C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}} \leq \nu^{(9)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \leq (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \leq (\bar{\nu}_1)^{(9)} \leq \boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$ , we obtain

$$(\nu_1)^{(9)} \leq \nu^{(9)}(t) \leq \frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \leq (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(9)}(t)$ :-

$$(m_2)^{(9)} \leq \nu^{(9)}(t) \leq (m_1)^{(9)}, \quad \boxed{\nu^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(9)}(t)$ :-

$$(\mu_2)^{(9)} \leq u^{(9)}(t) \leq (\mu_1)^{(9)}, \quad \boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)}G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then $(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)}T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following

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**Theorem :** If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on $t$ , and the conditions with the notations

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0$ ,

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$

$with\ \ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(2)} and\ (b_i'')^{(2)}$ are independent on t , and the conditions with the notations 426

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$ 427

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 428

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$ , 429

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 430

$with\ \ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(3)} and\ (b_i'')^{(3)}$ are independent on $t$ , and the conditions with the notations 431

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$ ,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

**Theorem :** If $(a_i'')^{(4)} and\ (b_i'')^{(4)}$ are independent on $t$ , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$ ,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(5)} and\ (b_i'')^{(5)}$ are independent on $t$ , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$ ,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

**Theorem** If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on $t$ , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$ ,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(7)} and\ (b_i'')^{(7)}$ are independent on $t$ , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$ ,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(8)} and\ (b_i'')^{(8)}$ are independent on $t$ , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$ ,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(9)} and\ (b_i'')^{(9)}$ are independent on $t$ , and the conditions (with the notations 436 A
45,46,27,28)

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$ ,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0 \quad 437$$

$$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0 \quad 438$$

$$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0 \quad 439$$

$$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0 \quad 440$$

$$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ]T_{14} = 0 \quad 441$$

$$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0 \quad 442$$

has a unique positive solution , which is an equilibrium solution for the system

$$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0 \quad 443$$

$$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0 \quad 444$$

$$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0 \quad 445$$

$$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\ ]T_{16} = 0 \quad 446$$

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\ ]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\ ]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\ ]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\ ]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\ ]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27})\right)]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27})\right)]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27})\right)]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0 \quad 479$$

$$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0 \quad 480$$

$$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0 \quad 481$$

$$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0 \quad 482$$

$$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0 \quad 483$$

$$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0 \quad 484$$

$$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0 \quad 484\text{A}$$

$$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$$

$$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$$

$$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$$

$$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$$

$$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$$

**Proof:** 485

(a) Indeed the first two equations have a nontrivial solution $G_{13}, G_{14}$ if

$$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$$

**Proof:** 486

(c) Indeed the first two equations have a nontrivial solution $G_{16}, G_{17}$ if

$$F(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$$

**Proof:** 487

(a) Indeed the first two equations have a nontrivial solution $G_{20}, G_{21}$ if

$$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$$

**Proof:** 488

(a) Indeed the first two equations have a nontrivial solution $G_{24}, G_{25}$ if

$$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$$

**Proof:** 489

(a) Indeed the first two equations have a nontrivial solution $G_{28}, G_{29}$ if

$$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$$

**Proof:** 490

(a) Indeed the first two equations have a nontrivial solution $G_{32}, G_{33}$ if

$$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$$

**Proof:** 491

(a) Indeed the first two equations have a nontrivial solution $G_{36}, G_{37}$ if

$$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$$

**Proof:** 492

(a) Indeed the first two equations have a nontrivial solution $G_{40}, G_{41}$ if

$$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$$

**Proof:** 492 A

(a) Indeed the first two equations have a nontrivial solution $G_{44}, G_{45}$ if

$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$

**Definition and uniqueness of** $T^*_{14}$ :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique $T^*_{14}$ for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

**Definition and uniqueness of** $T^*_{17}$ :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique $T^*_{17}$ for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]} \qquad 495$$

**Definition and uniqueness of** $T^*_{21}$ :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique $T^*_{21}$ for which $f(T^*_{21}) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

**Definition and uniqueness of** $T^*_{25}$ :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that there exists a unique $T^*_{25}$ for which $f(T^*_{25}) = 0$. With this value , we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T^*_{25})]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T^*_{25})]}$$

**Definition and uniqueness of** $T^*_{29}$ :- 498

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(5)}(T_{29})$ being increasing, it follows that there exists a unique $T^*_{29}$ for which $f(T^*_{29}) = 0$. With this value , we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T^*_{29})]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T^*_{29})]}$$

**Definition and uniqueness of** $T^*_{33}$ :- 499

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(6)}(T_{33})$ being increasing, it follows that

there exists a unique $T_{33}^{*}$ for which $f(T_{33}^{*})=0$. With this value , we obtain from the three first equations

$$G_{32}=\frac{(a_{32})^{(6)}G_{33}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^{*})]} \quad , \quad G_{34}=\frac{(a_{34})^{(6)}G_{33}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^{*})]}$$

**Definition and uniqueness of** $T_{37}^{*}$ :- 500

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(7)}(T_{37})$ being increasing, it follows that there exists a unique $T_{37}^{*}$ for which $f(T_{37}^{*})=0$. With this value , we obtain from the three first equations

$$G_{36}=\frac{(a_{36})^{(7)}G_{37}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^{*})]} \quad , \quad G_{38}=\frac{(a_{38})^{(7)}G_{37}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^{*})]}$$

**Definition and uniqueness of** $T_{41}^{*}$ :- 501

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique $T_{41}^{*}$ for which $f(T_{41}^{*})=0$. With this value , we obtain from the three first equations

$$G_{40}=\frac{(a_{40})^{(8)}G_{41}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T_{41}^{*})]} \quad , \quad G_{42}=\frac{(a_{42})^{(8)}G_{41}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T_{41}^{*})]}$$

**Definition and uniqueness of** $T_{45}^{*}$ :- 501 A

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(9)}(T_{45})$ being increasing, it follows that there exists a unique $T_{45}^{*}$ for which $f(T_{45}^{*})=0$. With this value , we obtain from the three first equations

$$G_{44}=\frac{(a_{44})^{(9)}G_{45}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T_{45}^{*})]} \quad , \quad G_{46}=\frac{(a_{46})^{(9)}G_{45}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T_{45}^{*})]}$$

By the same argument, the equations admit solutions $G_{13}, G_{14}$ if 502

$$\varphi(G)=(b'_{13})^{(1)}(b'_{14})^{(1)}-(b_{13})^{(1)}(b_{14})^{(1)}-$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G)+(b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right]+(b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G)=0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{14}$ taking into account the hypothesis $\varphi(0)>0$ , $\varphi(\infty)<0$ it follows that there exists a unique $G_{14}^{*}$ such that $\varphi(G^{*})=0$

By the same argument, the equations admit solutions $G_{16}, G_{17}$ if 503

$$\varphi(G_{19})=(b'_{16})^{(2)}(b'_{17})^{(2)}-(b_{16})^{(2)}(b_{17})^{(2)}-$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19})+(b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right]+(b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19})=0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{17}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{14}^{*}$ such that $\varphi((G_{19})^{*}) = 0$ 504

By the same argument, the equations admit solutions $G_{20}, G_{21}$ if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right] + (b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{21}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{21}^{*}$ such that $\varphi((G_{23})^{*}) = 0$

By the same argument, the equations admit solutions $G_{24}, G_{25}$ if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right] + (b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{25}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{25}^{*}$ such that $\varphi((G_{27})^{*}) = 0$

By the same argument, the equations admit solutions $G_{28}, G_{29}$ if 507

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right] + (b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{29}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{29}^{*}$ such that $\varphi((G_{31})^{*}) = 0$

By the same argument, the equations admit solutions $G_{32}, G_{33}$ if 508

$$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$$

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right] + (b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{33}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{33}^{*}$ such that $\varphi({G_{35}}^{*}) = 0$

By the same argument, the equations admit solutions $G_{36}, G_{37}$ if 509

$$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$$

$$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right] + (b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{37}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{37}^{*}$ such that $\varphi({G_{39}}^{*}) = 0$

By the same argument, the equations admit solutions $G_{40}, G_{41}$ if 510

$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$
$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right] + (b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{41}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{41}$ such that $\varphi(G_{43}{}^*) = 0$

By the same argument, the equations 92,93 admit solutions $G_{44}, G_{45}$ if
$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$

$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right] + (b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{45}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{45}$ such that $\varphi((G_{47})^*) = 0$

Finally we obtain the unique solution 511

$G^*_{14}$ given by $\varphi(G^*) = 0$ , $T^*_{14}$ given by $f(T^*_{14}) = 0$ and

$$G^*_{13} = \frac{(a_{13})^{(1)}G^*_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G^*_{15} = \frac{(a_{15})^{(1)}G^*_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

$$T^*_{13} = \frac{(b_{13})^{(1)}T^*_{14}}{[(b'_{13})^{(1)}-(b''_{13})^{(1)}(G^*)]} \quad , \quad T^*_{15} = \frac{(b_{15})^{(1)}T^*_{14}}{[(b'_{15})^{(1)}-(b''_{15})^{(1)}(G^*)]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

$G^*_{17}$ given by $\varphi((G_{19})^*) = 0$ , $T^*_{17}$ given by $f(T^*_{17}) = 0$ and 512

$$G^*_{16} = \frac{(a_{16})^{(2)}G^*_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G^*_{18} = \frac{(a_{18})^{(2)}G^*_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]} \quad 513$$

$$T^*_{16} = \frac{(b_{16})^{(2)}T^*_{17}}{[(b'_{16})^{(2)}-(b''_{16})^{(2)}((G_{19})^*)]} \quad , \quad T^*_{18} = \frac{(b_{18})^{(2)}T^*_{17}}{[(b'_{18})^{(2)}-(b''_{18})^{(2)}((G_{19})^*)]} \quad 514$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

$G^*_{21}$ given by $\varphi((G_{23})^*) = 0$ , $T^*_{21}$ given by $f(T^*_{21}) = 0$ and

$$G^*_{20} = \frac{(a_{20})^{(3)}G^*_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G^*_{22} = \frac{(a_{22})^{(3)}G^*_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

$$T^*_{20} = \frac{(b_{20})^{(3)}T^*_{21}}{[(b'_{20})^{(3)}-(b''_{20})^{(3)}(G_{23}{}^*)]} \quad , \quad T^*_{22} = \frac{(b_{22})^{(3)}T^*_{21}}{[(b'_{22})^{(3)}-(b''_{22})^{(3)}(G_{23}{}^*)]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

$G_{25}^{*}$ given by $\varphi(G_{27}) = 0$ , $T_{25}^{*}$ given by $f(T_{25}^{*}) = 0$ and

$$G_{24}^{*} = \frac{(a_{24})^{(4)} G_{25}^{*}}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}^{*} = \frac{(a_{26})^{(4)} G_{25}^{*}}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^{*})]}$$

$$T_{24}^{*} = \frac{(b_{24})^{(4)} T_{25}^{*}}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^{*})]} \quad , \quad T_{26}^{*} = \frac{(b_{26})^{(4)} T_{25}^{*}}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^{*})]} \qquad 517$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

$G_{29}^{*}$ given by $\varphi((G_{31})^{*}) = 0$ , $T_{29}^{*}$ given by $f(T_{29}^{*}) = 0$ and

$$G_{28}^{*} = \frac{(a_{28})^{(5)} G_{29}^{*}}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}^{*} = \frac{(a_{30})^{(5)} G_{29}^{*}}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^{*})]}$$

$$T_{28}^{*} = \frac{(b_{28})^{(5)} T_{29}^{*}}{[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31})^{*})]} \quad , \quad T_{30}^{*} = \frac{(b_{30})^{(5)} T_{29}^{*}}{[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31})^{*})]} \qquad 519$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

$G_{33}^{*}$ given by $\varphi((G_{35})^{*}) = 0$ , $T_{33}^{*}$ given by $f(T_{33}^{*}) = 0$ and

$$G_{32}^{*} = \frac{(a_{32})^{(6)} G_{33}^{*}}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^{*})]} \quad , \quad G_{34}^{*} = \frac{(a_{34})^{(6)} G_{33}^{*}}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^{*})]}$$

$$T_{32}^{*} = \frac{(b_{32})^{(6)} T_{33}^{*}}{[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35})^{*})]} \quad , \quad T_{34}^{*} = \frac{(b_{34})^{(6)} T_{33}^{*}}{[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35})^{*})]} \qquad 521$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

$G_{37}^{*}$ given by $\varphi((G_{39})^{*}) = 0$ , $T_{37}^{*}$ given by $f(T_{37}^{*}) = 0$ and

$$G_{36}^{*} = \frac{(a_{36})^{(7)} G_{37}^{*}}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^{*})]} \quad , \quad G_{38}^{*} = \frac{(a_{38})^{(7)} G_{37}^{*}}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^{*})]}$$

$$T_{36}^{*} = \frac{(b_{36})^{(7)} T_{37}^{*}}{[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39})^{*})]} \quad , \quad T_{38}^{*} = \frac{(b_{38})^{(7)} T_{37}^{*}}{[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39})^{*})]}$$

Finally we obtain the unique solution 523

$G_{41}^{*}$ given by $\varphi((G_{43})^{*}) = 0$ , $T_{41}^{*}$ given by $f(T_{41}^{*}) = 0$ and

$$G_{40}^{*} = \frac{(a_{40})^{(8)} G_{41}^{*}}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^{*})]} \quad , \quad G_{42}^{*} = \frac{(a_{42})^{(8)} G_{41}^{*}}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^{*})]}$$

$$T_{40}^{*}=\frac{(b_{40})^{(8)}T_{41}^{*}}{[(b'_{40})^{(8)}-(b''_{40})^{(8)}((G_{43})^{*})]} \quad , \quad T_{42}^{*}=\frac{(b_{42})^{(8)}T_{41}^{*}}{[(b'_{42})^{(8)}-(b''_{42})^{(8)}((G_{43})^{*})]}$$

Finally we obtain the unique solution of 89 to 99 523

A

$G_{45}^{*}$ given by $\varphi((G_{47})^{*})=0$ , $T_{45}^{*}$ given by $f(T_{45}^{*})=0$ and

$$G_{44}^{*}=\frac{(a_{44})^{(9)}G_{45}^{*}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T_{45}^{*})]} \quad , \quad G_{46}^{*}=\frac{(a_{46})^{(9)}G_{45}^{*}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T_{45}^{*})]}$$

$$T_{44}^{*}=\frac{(b_{44})^{(9)}T_{45}^{*}}{[(b'_{44})^{(9)}-(b''_{44})^{(9)}((G_{47})^{*})]} \quad , \quad T_{46}^{*}=\frac{(b_{46})^{(9)}T_{45}^{*}}{[(b'_{46})^{(9)}-(b''_{46})^{(9)}((G_{47})^{*})]}$$

**ASYMPTOTIC STABILITY ANALYSIS** 524

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}\ and\ (b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

**Proof:** Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i=G_i^{*}+\mathbb{G}_i \qquad , T_i=T_i^{*}+\mathbb{T}_i$$

$$\frac{\partial(a_{14}'')^{(1)}}{\partial T_{14}}(T_{14}^{*})=(q_{14})^{(1)} \quad , \frac{\partial(b_i'')^{(1)}}{\partial G_j}(G^{*})=s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt}=-\left((a'_{13})^{(1)}+(p_{13})^{(1)}\right)\mathbb{G}_{13}+(a_{13})^{(1)}\mathbb{G}_{14}-(q_{13})^{(1)}G_{13}^{*}\mathbb{T}_{14} \quad 525$$

$$\frac{d\mathbb{G}_{14}}{dt}=-\left((a'_{14})^{(1)}+(p_{14})^{(1)}\right)\mathbb{G}_{14}+(a_{14})^{(1)}\mathbb{G}_{13}-(q_{14})^{(1)}G_{14}^{*}\mathbb{T}_{14} \quad 526$$

$$\frac{d\mathbb{G}_{15}}{dt}=-\left((a'_{15})^{(1)}+(p_{15})^{(1)}\right)\mathbb{G}_{15}+(a_{15})^{(1)}\mathbb{G}_{14}-(q_{15})^{(1)}G_{15}^{*}\mathbb{T}_{14} \quad 527$$

$$\frac{d\mathbb{T}_{13}}{dt}=-\left((b'_{13})^{(1)}-(r_{13})^{(1)}\right)\mathbb{T}_{13}+(b_{13})^{(1)}\mathbb{T}_{14}+\sum_{j=13}^{15}\left(s_{(13)(j)}T_{13}^{*}\mathbb{G}_j\right) \quad 528$$

$$\frac{d\mathbb{T}_{14}}{dt}=-\left((b'_{14})^{(1)}-(r_{14})^{(1)}\right)\mathbb{T}_{14}+(b_{14})^{(1)}\mathbb{T}_{13}+\sum_{j=13}^{15}\left(s_{(14)(j)}T_{14}^{*}\mathbb{G}_j\right) \quad 529$$

$$\frac{d\mathbb{T}_{15}}{dt}=-\left((b'_{15})^{(1)}-(r_{15})^{(1)}\right)\mathbb{T}_{15}+(b_{15})^{(1)}\mathbb{T}_{14}+\sum_{j=13}^{15}\left(s_{(15)(j)}T_{15}^{*}\mathbb{G}_j\right) \quad 530$$

**ASYMPTOTIC STABILITY ANALYSIS** 531

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(2)}$ and $(b_i'')^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$\mathrm{G}_i = \mathrm{G}_i^* + \mathbb{G}_i \quad , \mathrm{T}_i = \mathrm{T}_i^* + \mathbb{T}_i \tag{532}$$

$$\frac{\partial(a_{17}'')^{(2)}}{\partial \mathrm{T}_{17}}(\mathrm{T}_{17}^*) = (q_{17})^{(2)} \ , \ \frac{\partial(b_i'')^{(2)}}{\partial \mathrm{G}_j}(\ (G_{19})^*\ ) = s_{ij} \tag{533}$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{\mathrm{d}\mathbb{G}_{16}}{\mathrm{dt}} = -\big((a_{16}')^{(2)} + (p_{16})^{(2)}\big)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}\mathrm{G}_{16}^*\mathbb{T}_{17} \tag{534}$$

$$\frac{\mathrm{d}\mathbb{G}_{17}}{\mathrm{dt}} = -\big((a_{17}')^{(2)} + (p_{17})^{(2)}\big)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}\mathrm{G}_{17}^*\mathbb{T}_{17} \tag{535}$$

$$\frac{\mathrm{d}\mathbb{G}_{18}}{\mathrm{dt}} = -\big((a_{18}')^{(2)} + (p_{18})^{(2)}\big)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}\mathrm{G}_{18}^*\mathbb{T}_{17} \tag{536}$$

$$\frac{\mathrm{d}\mathbb{T}_{16}}{\mathrm{dt}} = -\big((b_{16}')^{(2)} - (r_{16})^{(2)}\big)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \textstyle\sum_{j=16}^{18}\big(s_{(16)(j)}\mathrm{T}_{16}^*\mathbb{G}_j\big) \tag{537}$$

$$\frac{\mathrm{d}\mathbb{T}_{17}}{\mathrm{dt}} = -\big((b_{17}')^{(2)} - (r_{17})^{(2)}\big)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \textstyle\sum_{j=16}^{18}\big(s_{(17)(j)}\mathrm{T}_{17}^*\mathbb{G}_j\big) \tag{538}$$

$$\frac{\mathrm{d}\mathbb{T}_{18}}{\mathrm{dt}} = -\big((b_{18}')^{(2)} - (r_{18})^{(2)}\big)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \textstyle\sum_{j=16}^{18}\big(s_{(18)(j)}\mathrm{T}_{18}^*\mathbb{G}_j\big) \tag{539}$$

**ASYMPTOTIC STABILITY ANALYSIS** (540)

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(3)}$ *and* $(b_i'')^{(3)}$ Belong to $C^{(3)}(\ \mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{21}'')^{(3)}}{\partial T_{21}}(T_{21}^*) = (q_{21})^{(3)} \ , \ \frac{\partial(b_i'')^{(3)}}{\partial G_j}(\ (G_{23})^*\ ) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\big((a_{20}')^{(3)} + (p_{20})^{(3)}\big)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G_{20}^*\mathbb{T}_{21} \tag{541}$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\big((a_{21}')^{(3)} + (p_{21})^{(3)}\big)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G_{21}^*\mathbb{T}_{21} \tag{542}$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\big((a_{22}')^{(3)} + (p_{22})^{(3)}\big)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G_{22}^*\mathbb{T}_{21} \tag{543}$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\big((b_{20}')^{(3)} - (r_{20})^{(3)}\big)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \textstyle\sum_{j=20}^{22}\big(s_{(20)(j)}T_{20}^*\mathbb{G}_j\big) \tag{544}$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\big((b_{21}')^{(3)} - (r_{21})^{(3)}\big)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \textstyle\sum_{j=20}^{22}\big(s_{(21)(j)}T_{21}^*\mathbb{G}_j\big) \tag{545}$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b'_{22})^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T^*_{22}\mathbb{G}_j\right) \quad 546$$

**ASYMPTOTIC STABILITY ANALYSIS** 547

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}\ and\ (b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{25})^{(4)}}{\partial T_{25}}(T^*_{25}) = (q_{25})^{(4)} \ , \ \frac{\partial(b_i'')^{(4)}}{\partial G_j}((G_{27})^*\ ) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\left((a'_{24})^{(4)} + (p_{24})^{(4)}\right)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G^*_{24}\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\left((a'_{25})^{(4)} + (p_{25})^{(4)}\right)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G^*_{25}\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\left((a'_{26})^{(4)} + (p_{26})^{(4)}\right)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G^*_{26}\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\left((b'_{24})^{(4)} - (r_{24})^{(4)}\right)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(24)(j)}T^*_{24}\mathbb{G}_j\right) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\left((b'_{25})^{(4)} - (r_{25})^{(4)}\right)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\left(s_{(25)(j)}T^*_{25}\mathbb{G}_j\right) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T^*_{26}\mathbb{G}_j\right) \quad 554$$

**ASYMPTOTIC STABILITY ANALYSIS** 555

**Theorem 5:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}\ and\ (b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 556

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{29})^{(5)}}{\partial T_{29}}(T^*_{29}) = (q_{29})^{(5)} \ , \ \frac{\partial(b_i'')^{(5)}}{\partial G_j}(\ (G_{31})^*\ ) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a'_{28})^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G^*_{28}\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a'_{29})^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G^*_{29}\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a'_{30})^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G^*_{30}\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b'_{28})^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T^*_{28}\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b'_{29})^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T^*_{29}\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b'_{30})^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T^*_{30}\mathbb{G}_j\right) \quad 562$$

**ASYMPTOTIC STABILITY ANALYSIS** 563

**Theorem 6:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}$ *and* $(b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{33})^{(6)}}{\partial T_{33}}(T^*_{33}) = (q_{33})^{(6)} \quad , \frac{\partial(b_i'')^{(6)}}{\partial G_j}\left((G_{35})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a'_{32})^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G^*_{32}\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a'_{33})^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G^*_{33}\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a'_{34})^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G^*_{34}\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b'_{32})^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T^*_{32}\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b'_{33})^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T^*_{33}\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b'_{34})^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T^*_{34}\mathbb{G}_j\right) \quad 570$$

**ASYMPTOTIC STABILITY ANALYSIS** 571

**Theorem 7:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 572

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{37}'')^{(7)}}{\partial T_{37}}(T_{37}^{*}) = (q_{37})^{(7)} \quad , \frac{\partial(b_i'')^{(7)}}{\partial G_j}\left( (G_{39})^{**} \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a_{36}')^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G_{36}^{*}\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a_{37}')^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G_{37}^{*}\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a_{38}')^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G_{38}^{*}\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b_{36}')^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T_{36}^{*}\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b_{37}')^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T_{37}^{*}\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b_{38}')^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T_{38}^{*}\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

## ASYMPTOTIC STABILITY ANALYSIS

**Theorem 8:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 580

$$G_i = G_i^{*} + \mathbb{G}_i \qquad , T_i = T_i^{*} + \mathbb{T}_i$$

$$\frac{\partial(a_{41}'')^{(8)}}{\partial T_{41}}(T_{41}^{*}) = (q_{41})^{(8)} \quad , \frac{\partial(b_i'')^{(8)}}{\partial G_j}\left( (G_{43})^{*} \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a_{40}')^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G_{40}^{*}\mathbb{T}_{41} \quad 581$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a_{41}')^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G_{41}^{*}\mathbb{T}_{41} \quad 582$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a_{42}')^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G_{42}^{*}\mathbb{T}_{41} \quad 583$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b_{40}')^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T_{40}^{*}\mathbb{G}_j\right) \quad 584$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b_{41}')^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T_{41}^{*}\mathbb{G}_j\right) \quad 585$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b'_{42})^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T^*_{42}\mathbb{G}_j\right)$$ 586

586

**ASYMPTOTIC STABILITY ANALYSIS** A

**Theorem 9:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}\ and\ (b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

**Proof: Denote**

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{45})^{(9)}}{\partial T_{45}}(T^*_{45}) = (q_{45})^{(9)} \ , \ \frac{\partial(b_i'')^{(9)}}{\partial G_j}((G_{47})^*) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a'_{44})^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)}G^*_{44}\mathbb{T}_{45}$$ 586 B

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a'_{45})^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)}G^*_{45}\mathbb{T}_{45}$$ 586 C

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a'_{46})^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)}G^*_{46}\mathbb{T}_{45}$$ 586 D

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b'_{44})^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)}T^*_{44}\mathbb{G}_j\right)$$ 586 E

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b'_{45})^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)}T^*_{45}\mathbb{G}_j\right)$$ 586 F

$$\frac{d\mathbb{T}_{46}}{dt} = -\left((b'_{46})^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T^*_{46}\mathbb{G}_j\right)$$ 586 G

**The characteristic equation of this system is** 587

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(q_{13})^{(1)}G^*_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)}T^*_{14} + (b_{14})^{(1)}s_{(13),(14)}T^*_{14}\right)$$

$$+\left(\left((\lambda)^{(1)} + (a'_{14})^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)}G^*_{13} + (a_{13})^{(1)}(q_{14})^{(1)}G^*_{14}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)}T^*_{14} + (b_{14})^{(1)}s_{(13),(13)}T^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((b'_{13})^{(1)} + (b'_{14})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^2+\left((a'_{13})^{(1)}+(a'_{14})^{(1)}+(p_{13})^{(1)}+(p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)}+(a'_{13})^{(1)}+(p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^*_{14}+(a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}+(b'_{13})^{(1)}-(r_{13})^{(1)}\right)s_{(14),(15)}T^*_{14}+(b_{14})^{(1)}s_{(13),(15)}T^*_{13}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(2)}+(b'_{18})^{(2)}-(r_{18})^{(2)}\right)\{\left((\lambda)^{(2)}+(a'_{18})^{(2)}+(p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^*_{17}+(a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^*_{17}\right)$$

$$+\left(\left((\lambda)^{(2)}+(a'_{17})^{(2)}+(p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^*_{16}+(a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2+\left((b'_{16})^{(2)}+(b'_{17})^{(2)}-(r_{16})^{(2)}+(r_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$+\left(\left((\lambda)^{(2)}\right)^2+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}+(a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^*_{16}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\right)\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left(\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^*_{20}\right)\right]$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^*_{21}+(b_{21})^{(3)}s_{(20),(21)}T^*_{21}\right)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^*_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^*_{21}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^*_{21}+(b_{21})^{(3)}s_{(20),(20)}T^*_{20}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^2+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$+\left(\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^*_{20}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^*_{21}+(b_{21})^{(3)}s_{(20),(22)}T^*_{20}\right)\}=0$$

$$+$$

$$(\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left(\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^*_{24}\right)\right]$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^*_{25}+(b_{25})^{(4)}s_{(24),(25)}T^*_{25}\right)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^*_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^*_{25}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^*_{25}+(b_{25})^{(4)}s_{(24),(24)}T^*_{24}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^2+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$+\left(\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^*_{24}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^*_{25}+(b_{25})^{(4)}s_{(24),(26)}T^*_{24}\right)\}=0$$

$$+$$

$$(\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^*_{28}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^*_{29}+(b_{29})^{(5)}s_{(28),(29)}T^*_{29}\right)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^*_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^*_{29}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^*_{29}+(b_{29})^{(5)}s_{(28),(28)}T^*_{28}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^2+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$+\left(\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^*_{28}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^*_{29}+(b_{29})^{(5)}s_{(28),(30)}T^*_{28}\right)\}=0$$

$$+$$

$$(\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left(\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^*_{32}\right)\right]$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^*_{33}+(b_{33})^{(6)}s_{(32),(33)}T^*_{33}\right)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^*_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^*_{33}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^*_{33}+(b_{33})^{(6)}s_{(32),(32)}T^*_{32}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^2+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$+\left(\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^*_{32}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^*_{33}+(b_{33})^{(6)}s_{(32),(34)}T^*_{32}\right)\}=0$$

$$+$$

$$(\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^*_{36}\right)\right]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^*_{37}+(b_{37})^{(7)}s_{(36),(37)}T^*_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^*_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^*_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^*_{37}+(b_{37})^{(7)}s_{(36),(36)}T^*_{36}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^2+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$+\left(\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^*_{36}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^*_{37}+(b_{37})^{(7)}s_{(36),(38)}T^*_{36}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\right)\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^*_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^*_{41}+(b_{41})^{(8)}s_{(40),(41)}T^*_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^*_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^*_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^*_{41}+(b_{41})^{(8)}s_{(40),(40)}T^*_{40}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^2+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$+\left(\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^*_{40}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^*_{41}+(b_{41})^{(8)}s_{(40),(42)}T^*_{40}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\right)\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^*_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^*_{45}+(b_{45})^{(9)}s_{(44),(45)}T^*_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^*_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^*_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^*_{45}+(b_{45})^{(9)}s_{(44),(44)}T^*_{44}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^2+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$+\left(\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^*_{44}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^*_{45}+(b_{45})^{(9)}s_{(44),(46)}T^*_{44}\right)\}=0$$

**And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.**

## SECTION FOUR

## Optical Quantum Computation

### INTRODUCTION—VARIABLES USED

**Optical quantum computation using cluster states Michael A. Nielsen**

(1) Authors propose an approach to optical quantum computation in which a deterministic entangling quantum gate may be performed using (e) on average a few hundred coherently interacting (e&eb) optical elements (beam splitters, phase shifters, single photon sources, and photodetectors with feedforward).

(2) This scheme combines ideas from the optical quantum computing proposal of Knill, Laflamme and Milburn [Nature 409 (6816), 46 (2001)], and (e&eb) the abstract cluster-state model of quantum computation proposed by Raussendorf and Briegel [Phys. Rev. Lett. 86, 5188 (2001)]. DOI:10.1103/PhysRevLett.93.040503 Cite as: arXiv: quant-ph/0402005 (or arXiv:quant-ph/0402005v1 for this version)

**Role of entanglement and correlations in mixed-state quantum computation Animesh Datta and Guifre Vidal Phys. Rev. A 75, 042310 – Published 11 April 2007**

(3) In a quantum computation with pure states, the generation of large amounts of entanglement is known to be necessary for (e) a speedup with respect to (e&eb) classical computations.

(4) However, examples of quantum computations with (e&eb) mixed states are known, such as the deterministic computation with (e&eb) one quantum qubit (DQC1) model [Knill and Laflamme, Phys. Rev. Lett. 81, 5672 (1998)], in which entanglement is (=) at most marginally present, and yet a computational speedup is (=) believed to occur.

(5) Correlations, and not entanglement, have been identified as a necessary ingredient for mixed-state quantum computation speedups.

(6) Here authors show that correlations, as measured through the operator Schmidt rank, are indeed present in large amounts in the DQC1 circuit. This provides evidence for the preclusion of efficient classical simulation of DQC1 by means of a whole class of classical simulation algorithms, thereby

reinforcing the conjecture that DQC1 leads to a genuine quantum computational speedup. **http://dx.doi.org/10.1103/PhysRevA.75.042310**

## NOTATION

### Module One

Authors propose an approach to optical quantum computation in which a deterministic entangling quantum gate may be performed using (e) on average a few hundred coherently interacting (e&eb) optical elements (beam splitters, phase shifters, single photon sources, and photodetectors with feedforward)

$G_{13}$ : Category one of **optical quantum computation;** deterministic entangling quantum gate may be performed using (e) on average a few hundred coherently interacting (e&eb) optical elements (beam splitters, phase shifters, single photon sources, and photodetectors with feedforward)

$G_{14}$ : Category two of SAS

$G_{15}$ : Category three of SAS

$T_{13}$ : Category one of deterministic entangling quantum gate may be performed using (e) on average a few hundred coherently interacting (e&eb) optical elements (beam splitters, phase shifters, single photon sources, and photodetectors with feedforward);**optical quantum computation**

$T_{14}$ : Category two of SAS

$T_{15}$ : Category three of SAS

### Module Two

Authors propose an approach to **optical quantum computation in which a deterministic entangling quantum gate may be performed using on average a few hundred coherently interacting** (e&eb) optical elements (beam splitters, phase shifters, single photon sources, and photodetectors with feedforward)

$G_{16}$ : Category one of **optical quantum computation in which a deterministic entangling quantum gate may be performed using on average a few hundred coherently interacting;** optical elements (beam splitters, phase shifters, single photon sources, and photodetectors with feedforward)

$G_{17}$ : Category two of SAS

$G_{18}$ : Category three of SAS

$T_{16}$ : Category one of optical elements (beam splitters, phase shifters, single photon sources, and photodetectors with feedforward); **optical quantum computation in which a deterministic entangling quantum gate may be performed using on average a few hundred coherently interacting**

$T_{17}$ : Category two of SAS

$T_{18}$ : Category three of SAS

### Module three

This scheme combines ideas from the **optical quantum computing proposal of Knill, Laflamme and Milburn [Nature 409 (6816), 46 (2001)],** and (e&eb) the abstract cluster-state model of quantum computation proposed by Raussendorf and Briegel [Phys. Rev. Lett. 86, 5188 (2001)].

DOI:10.1103/PhysRevLett.93.040503 Cite as: arXiv: quant-ph/0402005 (or arXiv:quant-ph/0402005v1 for this version)

$G_{20}$ : Category one of **optical quantum computing proposal of Knill, Laflamme and Milburn [Nature 409 (6816), 46 (2001)];** abstract cluster-state model of quantum computation proposed by Raussendorf and Briegel [Phys. Rev. Lett. 86, 5188 (2001)].

$G_{21}$ : Category two of SAS

$G_{22}$ : Category three of SAS

$T_{20}$ : Category one of abstract cluster-state model of quantum computation proposed by Raussendorf and Briegel [Phys. Rev. Lett. 86, 5188 (2001)]. ;**optical quantum computing proposal of Knill, Laflamme and Milburn [Nature 409 (6816), 46 (2001)]**

$T_{21}$ : Category two of SAS

$T_{22}$ : Category three of SAS

**Module four**

**Knill, E., Laflamme, R. & Milburn, G. J. A scheme for efficient quantum computation with linear optics. Nature 409, 46-52 E Knill R Laflamme G J Milburn Nature (Impact Factor: 41.46). 02/2001; 409(6816):46-52. DOI: 10.1038/35051009 Source: PubMed**

(1) Quantum computers promise to increase greatly the efficiency of solving problems such as factoring large integers, combinatorial optimization and quantum physics simulation. One of the greatest challenges now is to implement the basic quantum-computational elements in a physical system and to demonstrate that they can be reliably and scalably controlled. One of the earliest proposals for quantum computation is based on (e) implementing a quantum bit with (e&eb) two optical modes containing one photon.
(2) Proposal is appealing because of (e) the ease with which photon interference can be observed.
(3) Until now, it suffered from the requirement for (e) non-linear couplings between optical modes containing few photons.
(4) Authors show that efficient quantum computation is possible using (e) only beam splitters, phase shifters, single photon sources and photo-detectors.
(5) Methods exploit feedback from photo-detectors and are robust against (e&eb) errors from photon loss and detector inefficiency. The basic elements are accessible to experimental investigation with current technology. Knill, E., Laflamme, R. & Milburn, G. J. A scheme for efficient quantum computation with linear optics. Nature 409, 46-52. Available from: https://www.researchgate.net/publication/11994420_Knill_E_Laflamme_R_Milburn_G_J_A_scheme_for_efficient_quantum_computation_with_linear_optics_Nature_409_46-52 [accessed Jan 28, 2016].

quantum computation is based on (e) implementing a quantum bit with (e&eb) two optical modes containing one photon

$G_{24}$ : Category one of implementing a quantum bit with two optical modes containing one photon

$G_{25}$ : Category two of SAS

$G_{26}$ : Category three of SAS

$T_{24}$ : Category one of quantum computation

$T_{25}$ : Category two of SAS

$T_{26}$ : Category three of SAS

**Module five**

Until now, it suffered from the requirement for (e) non-linear couplings between optical modes containing few photons

$G_{28}$ : Category one of non-linear couplings between optical modes containing few photons

$G_{29}$ : Category two of SAS

$G_{30}$ : Category three of SAS

$T_{28}$ : Category one of foregoing postulation suffered from the requirement

$T_{29}$ : Category two of SAS

$T_{30}$ : Category three of SAS

**Module six**

In a quantum computation with pure states, the generation of large amounts of entanglement is known to be necessary for (e) a **speedup with respect to classical computations**

$G_{32}$ : Category one of the generation of large amounts of entanglement in a quantum computation with pure states; **speedup with respect to classical computations**

$G_{33}$ : Category two of SAS

$G_{34}$ : Category three of SAS

$T_{32}$ : Category one of **speedup with respect to classical computations** ;generation of large amounts of entanglement in a quantum computation with pure states

$T_{33}$ : Category two of SAS

$T_{34}$ : Category three of SAS

**Module seven**

However, examples of **quantum computations with mixed states are known**, such as the deterministic computation with (e&eb) one quantum qubit (DQC1) model [Knill and Laflamme, Phys. Rev. Lett. 81, 5672 (1998)], in which entanglement is (=) at most marginally present, and yet a computational speedup is (=) believed to occur

$G_{36}$ : Category one of **quantum computations with mixed states**

$G_{37}$ : Category two of SAS

$G_{38}$ : Category three of SAS

$T_{36}$ : Category one of deterministic computation with (e&eb) one quantum qubit (DQC1) model [Knill and Laflamme, Phys. Rev. Lett. 81, 5672 (1998)], in which entanglement is (=) at most marginally present, and yet a computational speedup is (=) believed to occur

$T_{37}$ : Category two of SAS

$T_{38}$ : Category three of SAS

**Module eight**

**deterministic computation** with one quantum qubit (DQC1) model [Knill and Laflamme, Phys. Rev. Lett. 81, 5672 (1998)], in which entanglement is at most marginally present, and yet a computational speedup is (=) believed to occur

$G_{40}$ : Category one of **deterministic computation;** one quantum qubit (DQC1) model [Knill and Laflamme, Phys. Rev. Lett. 81, 5672 (1998)], in which entanglement is (=) at most marginally present, and yet a computational speedup is (=) believed to occur

$G_{41}$ : Category two of SAS

$G_{42}$ : Category three of SAS

$T_{40}$ : Category one of one quantum qubit (DQC1) model [Knill and Laflamme, Phys. Rev. Lett. 81, 5672 (1998)], in which entanglement is (=) at most marginally present, and yet a computational speedup is (=) believed to occur **; deterministic computation**

$T_{41}$ : Category two of SAS

$T_{42}$ : Category three of SAS

**Module Nine**

**DQC1** leads to a genuine quantum computational speedup

$G_{44}$ : Category one of **DQC1;** genuine quantum computational speedup

$G_{45}$ : Category two of SAS

$G_{46}$ : Category three of SAS

$T_{44}$ : Category one of genuine quantum computational speedup **;DQC1**

$T_{45}$ : Category two of SAS

$T_{46}$ : Category three of SAS

## SECTION FIVE

## Separability Of Very Noisy Mixed States

### INTRODUCTION—VARIABLES USED

**Separability of Very Noisy Mixed States and Implications for NMR Quantum Computing S. L.**

**Braunstein, C. M. Caves, R. Jozsa, N. Linden, S. Popescu, and R. Schack Phys. Rev. Lett. 83, 1054 – Published 2 August 1999**

**(1)** **S. L. Braunstein, C. M. Caves, R. Jozsa, N. Linden, S. Popescu, and R. Schack** give a constructive proof that all mixed states of N qubits in a sufficiently small neighborhood of the maximally mixed state are (=) separable (unentangled).
**(2)** The construction provides (eb) an explicit representation of any such state as (=) a mixture of product states.
**(3)** Authors give upper and lower bounds on the size of the neighborhood, which show (eb) that its extent decreases exponentially with (e&eb) the number of qubits.
**(4)** Bounds show that no entanglement appears in (eb) the physical states at any stage of present NMR experiments.
**(5)** Though this result raises questions about NMR quantum computation, further analysis would be necessary to assess (eb) the power of the general unitary transformations, which are (=) indeed implemented in these experiments, in their action on (er&eb) separable states. **http://dx.doi.org/10.1103/PhysRevLett.83.1054**

**Quantum Discord and the Power of One Qubit Animesh Datta, Anil Shaji, and Carlton M. Caves Phys. Rev. Lett. 100, 050502 – Published 5 February 2008**

(6) Animesh Datta, Anil Shaji, and Carlton M. Caves use (e) quantum discord to characterize (eb) the correlations present in the model called deterministic quantum computation with (e&eb) one quantum bit (DQC1), introduced by Knill and Laflamme [Phys. Rev. Lett. 81, 5672 (1998)].
(7) The model involves a collection of qubits in the completely mixed state coupled to (e&eb) a single control qubit that has (e) nonzero purity.
(8) The initial state, operations, and measurements in the model all point to a natural bipartite split between the control qubit and (e&eb: AC=DC=0) the mixed ones.
(9) Although there is no entanglement between these two parts, authors show that the quantum discord across (e&eb) this split is nonzero for typical instances of the DQC1 circuit.
(10) Nonzero values of discord indicate (eb) the presence of nonclassical correlations.
(11) Authors propose quantum discord as figure of merit for (e) characterizing the resources present in this computational model. http://dx.doi.org/10.1103/PhysRevLett.100.050502

**Quantum discord as resource for remote state preparation Borivoje Dakić, Yannick Ole Lipp, Xiaosong MA, Martín Ringbauer, Sebastian Kropatschek, Stefanie Barz, Tomasz Paterek, Vlatko Vedral, Anton Zeilinger, Časlav Brukner & Philip Walther Nature Physics 8, 666–670 (2012) doi:10.1038/nphys2377Published online 05 August 2012**

(12) The existence of better-than-classical quantum information processing (QIP) models which consume very little or no entanglement suggests (eb) that separable or weakly entangled states could be extremely useful tools for (e) information processing as they are much easier to prepare and control (eb,e&eb)even in dissipative environments.
(13) It has been proposed that a resource of advantage is (=) the generation of quantum discord, a (=) measure of non-classical correlations that includes (e) entanglement as a subset.
(14) Here authors show that quantum discord is the necessary resource for (e) quantum remote state preparation.
(15) Authors explicitly show that the geometric measure of quantum discord is related to (e&eb) the fidelity of this task, which provides (eb) its operational meaning.
(16) Results are experimentally demonstrated using (e) photonic quantum systems.

(17) Moreover, authors demonstrate that separable states with (e&eb) non-zero quantum discord can outperform (e) entangled states.

**(18)** Therefore, the role of quantum discord might provide (eb) fundamental insights for resource-efficient QIP. Subject terms: Optical physics Quantum information

## NOTATION

### Module One

**S. L. Braunstein, C. M. Caves, R. Jozsa, N. Linden, S. Popescu, and R. Schack** give a constructive proof that all mixed states of N qubits in a sufficiently small neighborhood of the maximally mixed state are (=) separable (unentangled)

$G_{13}$ : Category one of all mixed states of N qubits in a sufficiently small neighborhood of the maximally mixed state

$G_{14}$ : Category two of SAS

$G_{15}$ : Category three of SAS

$T_{13}$ : Category one of separable (unentangled)

$T_{14}$ : Category two of SAS

$T_{15}$ : Category three of SAS

### Module Two

The construction provides an explicit representation of any such state as (=) a mixture of product states

$G_{16}$ : Category one of construction provides an explicit representation of any such state

$G_{17}$ : Category two of SAS

$G_{18}$ : Category three of SAS

$T_{16}$ : Category one of mixture of product states

$T_{17}$ : Category two of SAS

$T_{18}$ : Category three of SAS

### Module three

Authors give upper and lower bounds on the size of the neighborhood, which show (eb) that its extent decreases exponentially with (e&eb) the number of qubits

$G_{20}$ : Category one of upper and lower bounds on the size of the neighborhood

$G_{21}$ : Category two of SAS

$G_{22}$ : Category three of SAS

$T_{20}$ : Category one of extent decreases exponentially with (e&eb) the number of qubits

$T_{21}$ : Category two of SAS

$T_{22}$ : Category three of SAS

**Module four**

Authors give upper and lower bounds on the size of the neighborhood, which show that its extent decreases exponentially with (e&eb) the **number of qubits**

$G_{24}$ : Category one of upper and lower bounds on the size of the neighborhood, which show that its extent decreases exponentially; **number of qubits**

$G_{25}$ : Category two of SAS

$G_{26}$ : Category three of SAS

$T_{24}$ : Category one of **number of qubits** ;upper and lower bounds on the size of the neighborhood, which show that its extent decreases exponentially

$T_{25}$ : Category two of SAS

$T_{26}$ : Category three of SAS

**Module five**

Bounds show that no entanglement appears in (eb) the physical states at any stage of present NMR experiments

$G_{28}$ : Category one of Bounds show that no entanglement

$G_{29}$ : Category two of SAS

$G_{30}$ : Category three of SAS

$T_{28}$ : Category one of physical states at any stage of present NMR experiments

$T_{29}$ : Category two of SAS

$T_{30}$ : Category three of SAS

**Module six**

Though this result raises questions about NMR quantum computation, further analysis would be necessary to assess (eb) the power of the general unitary transformations, which are (=) indeed implemented in these experiments, in their action on (er&eb) separable states. **http://dx.doi.org/10.1103/PhysRevLett.83.1054**

$G_{32}$ : Category one of result raises questions about NMR quantum computation, further analysis would be necessary to assess

$G_{33}$ : Category two of SAS

$G_{34}$ : Category three of SAS

$T_{32}$ : Category one of power of the general unitary transformations, which are (=) indeed implemented in these experiments, in their action on (er&eb) separable states.

$T_{33}$ : Category two of SAS

$T_{34}$ : Category three of SAS

**Module seven**

Though this result raises questions about NMR quantum computation, further analysis would be necessary to assess the power of the general unitary transformations, which are (=) indeed implemented in these experiments, in their action on (er&eb) separable states. **http://dx.doi.org/10.1103/PhysRevLett.83.1054**

$G_{36}$ : Category one of result raises questions about NMR quantum computation, further analysis would be necessary to assess the power of the general unitary transformations

$G_{37}$ : Category two of SAS

$G_{38}$ : Category three of SAS

$T_{36}$ : Category one of implemented in these experiments, in their action on (er&eb) separable states.

$T_{37}$ : Category two of SAS

$T_{38}$ : Category three of SAS

**Module eight**

Though this result raises questions about NMR quantum computation, further analysis would be necessary to assess the power of the general unitary transformations, which are indeed implemented in these experiments, in their action on (er&eb) **separable states**.

**http://dx.doi.org/10.1103/PhysRevLett.83.1054**

$G_{40}$ : Category one of result raises questions about NMR quantum computation, further analysis would be necessary to assess the power of the general unitary transformations, which are indeed implemented in these experiments, in their action; **separable states**

$G_{41}$ : Category two of SAS

$G_{42}$ : Category three of SAS

$T_{40}$ : Category one of **separable states** ; result raises questions about NMR quantum computation, further analysis would be necessary to assess the power of the general unitary transformations, which are indeed implemented in these experiments, in their action

$T_{41}$ : Category two of SAS

$T_{42}$ : Category three of SAS

**Module Nine**

Animesh Datta, Anil Shaji, and Carlton M. Caves use (e) quantum discord to characterize (eb) the correlations present in the model called deterministic quantum computation with (e&eb) one quantum bit (DQC1), introduced by Knill and Laflamme [Phys. Rev. Lett. 81, 5672 (1998)]

$G_{44}$ : Category one of quantum discord

$G_{45}$ : Category two of SAS

$G_{46}$ : Category three of SAS

$T_{44}$ : Category one of correlations present in the model called deterministic quantum computation with (e&eb) one quantum bit (DQC1), introduced by Knill and Laflamme [Phys. Rev. Lett. 81, 5672 (1998)]

$T_{45}$ : Category two of SAS

$T_{46}$ : Category three of SAS

**The Coefficients:**

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}\ ,\ (b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}$,
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}, \ (b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}\ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$ ,
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)}$,
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)}$,
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)}$,
are Dissipation coefficients

**Module Numbered One**

**The differential system of this model is now (Module Numbered one)**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \quad 1$$
$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad 2$$
$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15} \quad 3$$
$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13} \quad 4$$
$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14} \quad 5$$
$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)\right]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14}, t) =$ First augmentation factor
$-(b''_{13})^{(1)}(G, t) =$ First detritions factor

**Module Numbered Two**

**The differential system of this model is now ( Module numbered two)**

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17},t)\right]G_{16} \quad 7$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17},t)\right]G_{17} \quad 8$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17},t)\right]G_{18} \quad 9$$

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\left((G_{19}),t\right)\right]T_{16} \quad 10$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\left((G_{19}),t\right)\right]T_{17} \quad 11$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\left((G_{19}),t\right)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17},t) =$ First augmentation factor

$-(b''_{16})^{(2)}\left((G_{19}),t\right) =$ First detritions factor

**Module Numbered Three**

**The differential system of this model is now (Module numbered three)**

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21},t)\right]G_{20} \quad 13$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21},t)\right]G_{21} \quad 14$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21},t)\right]G_{22} \quad 15$$

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23},t)\right]T_{20} \quad 16$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23},t)\right]T_{21} \quad 17$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23},t)\right]T_{22} \quad 18$$

$+(a''_{20})^{(3)}(T_{21},t) =$ First augmentation factor

$-(b''_{20})^{(3)}(G_{23},t) =$ First detritions factor

**Module Numbered Four**

**The differential system of this model is now (Module numbered Four)**

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right]G_{24} \quad 19$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25},t)\right]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25},t)\right]G_{26} \quad 21$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27}),t\right)\right]T_{24} \quad 22$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27}),t\right)\right]T_{25} \quad 23$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27}),t\right)\right]T_{26} \quad 24$$

$+(a''_{24})^{(4)}(T_{25},t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}\left((G_{27}),t\right) =$ **First detritions factor**

**Module Numbered Five**:

**The differential system of this model is now (Module number five)**

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29},t)\right]G_{28} \quad 25$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29},t)\right]G_{29} \quad 26$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29},t)\right]G_{30} \quad 27$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}\left((G_{31}),t\right)\right]T_{28} \quad 28$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}((G_{31}),t)\right]T_{29} \quad 29$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31}),t)\right]T_{30} \quad 30$$

$+(a''_{28})^{(5)}(T_{29},t) =$ **First augmentation factor**

$-(b''_{28})^{(5)}((G_{31}),t) =$ **First detritions factor**

**Module Numbered Six**

**The differential system of this model is now (Module numbered Six)**

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33},t)\right]G_{32} \quad 31$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33},t)\right]G_{33} \quad 32$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33},t)\right]G_{34} \quad 33$$

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35}),t)\right]T_{32} \quad 34$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}((G_{35}),t)\right]T_{33} \quad 35$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35}),t)\right]T_{34} \quad 36$$

$+(a''_{32})^{(6)}(T_{33},t) =$ **First augmentation factor**

**Module Numbered Seven:**

**The differential system of this model is now (Seventh Module)**

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right]G_{36} \quad 37$$

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37},t)\right]G_{37} \quad 38$$

$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37},t)\right]G_{38} \quad 39$$

$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39}),t)\right]T_{36} \quad 40$$

$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}((G_{39}),t)\right]T_{37} \quad 41$$

$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39}),t)\right]T_{38} \quad 42$$

$+(a''_{36})^{(7)}(T_{37},t) =$ **First augmentation factor**

**Module Numbered Eight**

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right]G_{40} \quad 43$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41},t)\right]G_{41} \quad 44$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41},t)\right]G_{42} \quad 45$$

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43}),t)\right]T_{40} \quad 46$$

$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}((G_{43}),t)\right]T_{41} \quad 47$$

$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43}),t)\right]T_{42} \quad 48$$

**Module Numbered Nine**

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}\big((G_{47}),t\big)\right]T_{44} \quad 52$$

$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}\big((G_{47}),t\big)\right]T_{45} \quad 53$$

$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}\big((G_{47}),t\big)\right]T_{46} \quad 54$$

$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}\big((G_{47}),t\big) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)} & +(a''_{13})^{(1)}(T_{14},t) & +(a''_{16})^{(2,2,)}(T_{17},t) & +(a''_{20})^{(3,3,)}(T_{21},t) \\ +(a''_{24})^{(4,4,4,4,)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{36})^{(7,7)}(T_{37},t) & +(a''_{40})^{(8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)} & +(a''_{14})^{(1)}(T_{14},t) & +(a''_{17})^{(2,2,)}(T_{17},t) & +(a''_{21})^{(3,3,)}(T_{21},t) \\ +(a''_{25})^{(4,4,4,4,)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{37})^{(7,7)}(T_{37},t) & +(a''_{41})^{(8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)} & +(a''_{15})^{(1)}(T_{14},t) & +(a''_{18})^{(2,2,)}(T_{17},t) & +(a''_{22})^{(3,3,)}(T_{21},t) \\ +(a''_{26})^{(4,4,4,4,)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{38})^{(7,7)}(T_{37},t) & +(a''_{42})^{(8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{15} \quad 57$$

Where $(a''_{13})^{(1)}(T_{14},t)$, $(a''_{14})^{(1)}(T_{14},t)$, $(a''_{15})^{(1)}(T_{14},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,)}(T_{17},t)$, $+(a''_{17})^{(2,2,)}(T_{17},t)$, $+(a''_{18})^{(2,2,)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,)}(T_{21},t)$, $+(a''_{21})^{(3,3,)}(T_{21},t)$, $+(a''_{22})^{(3,3,)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{38})^{(7,7)}(T_{37},t)$ $+(a''_{37})^{(7,7)}(T_{37},t)$ $+(a''_{36})^{(7,7)}(T_{37},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{40})^{(8,8)}(T_{41},t)$ $+(a''_{41})^{(8,8)}(T_{41},t)$ $+(a''_{42})^{(8,8)}(T_{41},t)$ are eight augmentation coefficient for 1,2,3

$+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)} & -(b''_{13})^{(1)}(G,t) & -(b''_{16})^{(2,2,)}(G_{19},t) & -(b''_{20})^{(3,3,)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{36})^{(7,7,)}(G_{39},t) & -(b''_{40})^{(8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{13} \quad 58$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)}\boxed{-(b''_{14})^{(1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14} \quad 59$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)}\boxed{-(b''_{15})^{(1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15} \quad 60$$

Where $\boxed{-(b''_{13})^{(1)}(G,t)}$, $\boxed{-(b''_{14})^{(1)}(G,t)}$, $\boxed{-(b''_{15})^{(1)}(G,t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8)}(G_{43},t)}$ $\boxed{-(b''_{41})^{(8,8)}(G_{43},t)}$ $\boxed{-(b''_{42})^{(8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \begin{bmatrix} (a'_{16})^{(2)}\boxed{+(a''_{16})^{(2)}(T_{17},t)} & \boxed{+(a''_{13})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{16} \quad 61$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \begin{bmatrix} (a'_{17})^{(2)}\boxed{+(a''_{17})^{(2)}(T_{17},t)} & \boxed{+(a''_{14})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{17} \quad 62$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \begin{bmatrix} (a'_{18})^{(2)}\boxed{+(a''_{18})^{(2)}(T_{17},t)} & \boxed{+(a''_{15})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{18} \quad 63$$

Where $\boxed{+(a''_{16})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17},t)}$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{13})^{(1,1,)}(T_{14},t)$ , $+(a''_{14})^{(1,1,)}(T_{14},t)$ , $+(a''_{15})^{(1,1,)}(T_{14},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,3)}(T_{21},t)$ , $+(a''_{21})^{(3,3,3)}(T_{21},t)$ , $+(a''_{22})^{(3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)$ , $+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)$ , $+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{36})^{(7,7,7)}(T_{37},t)$ , $+(a''_{37})^{(7,7,7)}(T_{37},t)$ , $+(a''_{38})^{(7,7,7)}(T_{37},t)$ are seventh augmentation coefficient for category 1, 2 and 3

$+(a''_{40})^{(8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8)}(T_{41},t)$ , $+(a''_{42})^{(8,8,8)}(T_{41},t)$ are eight augmentation coefficient for category 1, 2 and 3

$+(a''_{44})^{(9,9)}(T_{45},t)$ , $+(a''_{45})^{(9,9)}(T_{45},t)$ , $+(a''_{46})^{(9,9)}(T_{45},t)$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \begin{bmatrix} (b'_{16})^{(2)} & -(b''_{16})^{(2)}(G_{19},t) & -(b''_{13})^{(1,1,)}(G,t) & -(b''_{20})^{(3,3,3,)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6)}(G_{35},t) & \\ -(b''_{36})^{(7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9)}(G_{47},t) & \end{bmatrix} T_{16} \quad 64$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)} & -(b''_{17})^{(2)}(G_{19},t) & -(b''_{14})^{(1,1,)}(G,t) & -(b''_{21})^{(3,3,3,)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6)}(G_{35},t) & \\ -(b''_{37})^{(7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9)}(G_{47},t) & \end{bmatrix} T_{17} \quad 65$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)} & -(b''_{18})^{(2)}(G_{19},t) & -(b''_{15})^{(1,1,)}(G,t) & -(b''_{22})^{(3,3,3,)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6)}(G_{35},t) & \\ -(b''_{38})^{(7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9)}(G_{47},t) & \end{bmatrix} T_{18} \quad 66$$

where $-(b''_{16})^{(2)}(G_{19},t)$ , $-(b''_{17})^{(2)}(G_{19},t)$ , $-(b''_{18})^{(2)}(G_{19},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{13})^{(1,1,)}(G,t)$, $-(b''_{14})^{(1,1,)}(G,t)$, $-(b''_{15})^{(1,1,)}(G,t)$ are second detrition coefficients for category 1,2 and 3

$-(b''_{20})^{(3,3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,)}(G_{23},t)$ are third detrition coefficients for category 1,2 and 3

$-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1,2 and 3

$-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1,2 and 3

$-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{44})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt}=(a_{20})^{(3)}G_{21}-\begin{bmatrix}(a'_{20})^{(3)}\boxed{+(a''_{20})^{(3)}(T_{21},t)}\ \boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}\ \boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)}\\ \boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)}\\ \boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{44})^{(9,9,9)}(T_{45},t)}\end{bmatrix}G_{20} \quad (67)$$

$$\frac{dG_{21}}{dt}=(a_{21})^{(3)}G_{20}-\begin{bmatrix}(a'_{21})^{(3)}\boxed{+(a''_{21})^{(3)}(T_{21},t)}\ \boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}\ \boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)}\\ \boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)}\\ \boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)}\end{bmatrix}G_{21} \quad (68)$$

$$\frac{dG_{22}}{dt}=(a_{22})^{(3)}G_{21}-\begin{bmatrix}(a'_{22})^{(3)}\boxed{+(a''_{22})^{(3)}(T_{21},t)}\ \boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}\ \boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)}\\ \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)}\\ \boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)}\end{bmatrix}G_{22} \quad (69)$$

$\boxed{+(a''_{20})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3)}(T_{21},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}$ are second augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)}$ are third augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)}$ are seventh augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)}$ are eight augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{44})^{(9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)}$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt}=(b_{20})^{(3)}T_{21}-\begin{bmatrix}(b'_{20})^{(3)}\boxed{-(b''_{20})^{(3)}(G_{23},t)}\ \boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}\ \boxed{-(b''_{13})^{(1,1,1,)}(G,t)}\\ \boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)}\ \boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)}\ \boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)}\\ \boxed{-(b''_{36})^{(7,7,7,7)}(G_{39},t)}\ \boxed{-(b''_{40})^{(8,8,8,8)}(G_{43},t)}\ \boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)}\end{bmatrix}T_{20} \quad (70)$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b'_{21})^{(3)}\boxed{-(b''_{21})^{(3)}(G_{23},t)}\boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}\boxed{-(b''_{14})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)}\boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)}\boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7)}(G_{39},t)}\boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)}\boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{21} \quad (71)$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)}\boxed{-(b''_{22})^{(3)}(G_{23},t)}\boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}\boxed{-(b''_{15})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)}\boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)}\boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)}\boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)}\boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{22} \quad (72)$$

$\boxed{-(b''_{20})^{(3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3)}(G_{23},t)}$ *are first detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}$ *are second detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{13})^{(1,1,1,)}(G,t)}, \boxed{-(b''_{14})^{(1,1,1,)}(G,t)}, \boxed{-(b''_{15})^{(1,1,1,)}(G,t)}$ *are third detrition coefficients for category 1,2 and 3*

$\boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)}, \boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)}, \boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)}$ *are fourth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)}, \boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)}, \boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)}$ *are fifth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)}, \boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)}, \boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)}$ *are sixth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{36})^{(7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{37})^{(7,7,7,7)}(G_{39},t)}\boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)}, \boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)}, \boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \begin{bmatrix} (a'_{24})^{(4)}\boxed{+(a''_{24})^{(4)}(T_{25},t)}\boxed{+(a''_{28})^{(5,5,)}(T_{29},t)}\boxed{+(a''_{32})^{(6,6,)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)}\boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)}\boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)}\boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)}\boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{24} \quad (73)$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \begin{bmatrix} (a'_{25})^{(4)}\boxed{+(a''_{25})^{(4)}(T_{25},t)}\boxed{+(a''_{29})^{(5,5,)}(T_{29},t)}\boxed{+(a''_{33})^{(6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)}\boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)}\boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)}\boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)}\boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{25} \quad (74)$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \begin{bmatrix} (a'_{26})^{(4)}\boxed{+(a''_{26})^{(4)}(T_{25},t)}\boxed{+(a''_{30})^{(5,5,)}(T_{29},t)}\boxed{+(a''_{34})^{(6,6,)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)}\boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)}\boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)}\boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)}\boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{26} \quad (75)$$

$\boxed{(a''_{24})^{(4)}(T_{25},t)}, \boxed{(a''_{25})^{(4)}(T_{25},t)}, \boxed{(a''_{26})^{(4)}(T_{25},t)}$ *are first augmentation coefficients category* 1, 2 3

$\boxed{+(a''_{28})^{(5,5,)}(T_{29},t)}, \boxed{+(a''_{29})^{(5,5,)}(T_{29},t)}, \boxed{+(a''_{30})^{(5,5,)}(T_{29},t)}$ *are second augmentation*

$coefficient\ \ for\ category\ 1,2\ and\ 3$

$+(a_{32}'')^{(6,6,)}(T_{33},t)$, $+(a_{33}'')^{(6,6,)}(T_{33},t)$, $+(a_{34}'')^{(6,6,)}(T_{33},t)$ $are\ third\ \ augmentation$

$coefficient\ for\ category\ 1,2\ and\ 3$

$+(a_{13}'')^{(1,1,1,1)}(T_{14},t)$, $+(a_{14}'')^{(1,1,1,1)}(T_{14},t)$, $+(a_{15}'')^{(1,1,1,1)}(T_{14},t)$

$are\ fourth\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$

$+(a_{16}'')^{(2,2,2,2)}(T_{17},t)$, $+(a_{17}'')^{(2,2,2,2)}(T_{17},t)$, $+(a_{18}'')^{(2,2,2,2)}(T_{17},t)$

$are\ fifth\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$

$+(a_{20}'')^{(3,3,3,3)}(T_{21},t)$, $+(a_{21}'')^{(3,3,3,3)}(T_{21},t)$, $+(a_{22}'')^{(3,3,3,3)}(T_{21},t)$

$are\ sixth\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$

$+(a_{36}'')^{(7,7,7,7,7)}(T_{37},t)$, $+(a_{37}'')^{(7,7,7,7,7)}(T_{37},t)$, $+(a_{38}'')^{(7,7,7,7,7)}(T_{37},t)$

$are\ seventh\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$

$+(a_{40}'')^{(8,8,8,8,8)}(T_{41},t)$, $+(a_{41}'')^{(8,8,8,8,8)}(T_{41},t)$, $+(a_{42}'')^{(8,8,8,8,8)}(T_{41},t)$

$are\ eighth\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$

$+(a_{46}'')^{(9,9,9,9)}(T_{45},t)$, $+(a_{45}'')^{(9,9,9,9)}(T_{45},t)$, $+(a_{44}'')^{(9,9,9,9)}(T_{45},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt}=(b_{24})^{(4)}T_{25}-\begin{bmatrix}(b_{24}')^{(4)}-(b_{24}'')^{(4)}(G_{27},t) & -(b_{28}'')^{(5,5,)}(G_{31},t) & -(b_{32}'')^{(6,6,)}(G_{35},t)\\ -(b_{13}'')^{(1,1,1,1)}(G,t) & -(b_{16}'')^{(2,2,2,2)}(G_{19},t) & -(b_{20}'')^{(3,3,3,3)}(G_{23},t)\\ -(b_{36}'')^{(7,7,7,7,7)}(G_{39},t) & -(b_{40}'')^{(8,8,8,8,8)}(G_{43},t) & -(b_{44}'')^{(9,9,9,9)}(G_{47},t)\end{bmatrix}T_{24} \quad 76$$

$$\frac{dT_{25}}{dt}=(b_{25})^{(4)}T_{24}-\begin{bmatrix}(b_{25}')^{(4)}-(b_{25}'')^{(4)}(G_{27},t) & -(b_{29}'')^{(5,5,)}(G_{31},t) & -(b_{33}'')^{(6,6,)}(G_{35},t)\\ -(b_{14}'')^{(1,1,1,1)}(G,t) & -(b_{17}'')^{(2,2,2,2)}(G_{19},t) & -(b_{21}'')^{(3,3,3,3)}(G_{23},t)\\ -(b_{37}'')^{(7,7,7,7,7)}(G_{39},t) & -(b_{41}'')^{(8,8,8,8,8)}(G_{43},t) & -(b_{45}'')^{(9,9,9,9)}(G_{47},t)\end{bmatrix}T_{25} \quad 77$$

$$\frac{dT_{26}}{dt}=(b_{26})^{(4)}T_{25}-\begin{bmatrix}(b_{26}')^{(4)}-(b_{26}'')^{(4)}(G_{27},t) & -(b_{30}'')^{(5,5,)}(G_{31},t) & -(b_{34}'')^{(6,6,)}(G_{35},t)\\ -(b_{15}'')^{(1,1,1,1)}(G,t) & -(b_{18}'')^{(2,2,2,2)}(G_{19},t) & -(b_{22}'')^{(3,3,3,3)}(G_{23},t)\\ -(b_{38}'')^{(7,7,7,7,7)}(G_{39},t) & -(b_{42}'')^{(8,8,8,8,8)}(G_{43},t) & -(b_{46}'')^{(9,9,9,9)}(G_{47},t)\end{bmatrix}T_{26} \quad 78$$

$Where$ $-(b_{24}'')^{(4)}(G_{27},t)$, $-(b_{25}'')^{(4)}(G_{27},t)$, $-(b_{26}'')^{(4)}(G_{27},t)$ $are\ first\ detrition\ coefficients$

$for\ category\ 1,2\ and\ 3$

$-(b_{28}'')^{(5,5,)}(G_{31},t)$, $-(b_{29}'')^{(5,5,)}(G_{31},t)$, $-(b_{30}'')^{(5,5,)}(G_{31},t)$ $are\ second\ detrition\ coefficients$

$for\ category\ 1,2\ and\ 3$

$-(b_{32}'')^{(6,6,)}(G_{35},t)$, $-(b_{33}'')^{(6,6,)}(G_{35},t)$, $-(b_{34}'')^{(6,6,)}(G_{35},t)$ $are\ third\ detrition\ coefficients$

$for\ category\ 1,2\ and\ 3$

$-(b_{13}'')^{(1,1,1,1)}(G,t)$, $-(b_{14}'')^{(1,1,1,1)}(G,t)$, $-(b_{15}'')^{(1,1,1,1)}(G,t)$

$are\ fourth\ \ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$-(b_{16}'')^{(2,2,2,2)}(G_{19},t)$, $-(b_{17}'')^{(2,2,2,2)}(G_{19},t)$, $-(b_{18}'')^{(2,2,2,2)}(G_{19},t)$

$are\ fifth\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$-(b_{20}'')^{(3,3,3,3)}(G_{23},t)$, $-(b_{21}'')^{(3,3,3,3)}(G_{23},t)$, $-(b_{22}'')^{(3,3,3,3)}(G_{23},t)$

$are\ sixth\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$-(b_{36}'')^{(7,7,7,7,7)}(G_{39},t)$, $-(b_{37}'')^{(7,7,7,7,7)}(G_{39},t)$, $-(b_{38}'')^{(7,7,7,7,7)}(G_{39},t)$

$are\ seventh\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)}$

*are eighth detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{46})^{(9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[\begin{array}{ccc} (a'_{28})^{(5)} & \boxed{+(a''_{28})^{(5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,)}(T_{25},t)} \quad \boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{28} \quad (79)$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[\begin{array}{ccc} (a'_{29})^{(5)} & \boxed{+(a''_{29})^{(5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,)}(T_{25},t)} \quad \boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{29} \quad (80)$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[\begin{array}{ccc} (a'_{30})^{(5)} & \boxed{+(a''_{30})^{(5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,)}(T_{25},t)} \quad \boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{30} \quad (81)$$

*Where* $\boxed{+(a''_{28})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5)}(T_{29},t)}$ *are first augmentation coefficients for category* 1, 2 *and* 3

*And* $\boxed{+(a''_{24})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,)}(T_{25},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)}$ are fourth augmentation coefficients for category 1,2, and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)}$ are fifth augmentation coefficients for category 1,2,and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)}$ are sixth augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)}$ are seventh augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)}$ are eighth augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[\begin{array}{ccc} (b'_{28})^{(5)} & \boxed{-(b''_{28})^{(5)}(G_{31},t)} & \boxed{-(b''_{24})^{(4,4,)}(G_{27},t)} \quad \boxed{-(b''_{32})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)} \end{array}\right] T_{28} \quad (82)$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b'_{29})^{(5)} & -(b''_{29})^{(5)}(G_{31},t) & -(b''_{25})^{(4,4,)}(G_{27},t) & -(b''_{33})^{(6,6,6)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{29} \quad 83$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b'_{30})^{(5)} & -(b''_{30})^{(5)}(G_{31},t) & -(b''_{26})^{(4,4,)}(G_{27},t) & -(b''_{34})^{(6,6,6)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1,1,)}(G,t) & -(b''_{18})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{30} \quad 84$$

*where* $-(b''_{28})^{(5)}(G_{31},t)$ , $-(b''_{29})^{(5)}(G_{31},t)$ , $-(b''_{30})^{(5)}(G_{31},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{24})^{(4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,)}(G_{27},t)$ *are second detrition coefficients for category* 1,2 *and* 3

$-(b''_{32})^{(6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6)}(G_{35},t)$ *are third detrition coefficients for category* 1,2 *and* 3

$-(b''_{13})^{(1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1)}(G,t)$ , $-(b''_{15})^{(1,1,1,1,1,)}(G,t)$ are fourth detrition coefficients for category 1,2, and 3

$-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1,2, and 3

$-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1,2, and 3

$-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2, and 3

$-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1,2, and 3

$-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} \quad 85$$

$$- \begin{bmatrix} (a'_{32})^{(6)} & +(a''_{32})^{(6)}(T_{33},t) & +(a''_{28})^{(5,5,5)}(T_{29},t) & +(a''_{24})^{(4,4,4,)}(T_{25},t) \\ & +(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t) & +(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t) \\ & +(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{32}$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \begin{bmatrix} (a'_{33})^{(6)} & +(a''_{33})^{(6)}(T_{33},t) & +(a''_{29})^{(5,5,5)}(T_{29},t) & +(a''_{25})^{(4,4,4,)}(T_{25},t) \\ & +(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t) & +(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t) \\ & +(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{33} \quad 86$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \begin{bmatrix} (a'_{34})^{(6)} & +(a''_{34})^{(6)}(T_{33},t) & +(a''_{30})^{(5,5,5)}(T_{29},t) & +(a''_{26})^{(4,4,4,)}(T_{25},t) \\ & +(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t) & +(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t) \\ & +(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{34} \quad 87$$

$+(a''_{32})^{(6)}(T_{33},t)$, $+(a''_{33})^{(6)}(T_{33},t)$, $+(a''_{34})^{(6)}(T_{33},t)$ *are first augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{28})^{(5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5)}(T_{29},t)$ *are second augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{24})^{(4,4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,)}(T_{25},t)$ *are third augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)$ - are fourth augmentation coefficients

$+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)$ - fifth augmentation coefficients

$+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)$ sixth augmentation coefficients

$+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)$ seventh augmentation coefficients

$+(a''_{40})^{(8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)$ Eighth augmentation coefficients

$+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \begin{bmatrix} (b'_{32})^{(6)} & -(b''_{32})^{(6)}(G_{35},t) & -(b''_{28})^{(5,5,5)}(G_{31},t) & -(b''_{24})^{(4,4,4,)}(G_{27},t) \\ & -(b''_{13})^{(1,1,1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{32} \quad (88)$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \begin{bmatrix} (b'_{33})^{(6)} & -(b''_{33})^{(6)}(G_{35},t) & -(b''_{29})^{(5,5,5)}(G_{31},t) & -(b''_{25})^{(4,4,4,)}(G_{27},t) \\ & -(b''_{14})^{(1,1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{33} \quad (89)$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)} & -(b''_{34})^{(6)}(G_{35},t) & -(b''_{30})^{(5,5,5)}(G_{31},t) & -(b''_{26})^{(4,4,4,)}(G_{27},t) \\ & -(b''_{15})^{(1,1,1,1,1,1)}(G,t) & -(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{34} \quad (90)$$

$-(b''_{32})^{(6)}(G_{35},t)$, $-(b''_{33})^{(6)}(G_{35},t)$, $-(b''_{34})^{(6)}(G_{35},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{28})^{(5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5)}(G_{31},t)$ *are second detrition coefficients for category* 1, 2 *and* 3

$-(b''_{24})^{(4,4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,)}(G_{27},t)$ *are third detrition coefficients for category* 1,2 *and* 3

$-(b''_{13})^{(1,1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1,1,1)}(G,t)$ are fourth detrition coefficients for category 1, 2, and 3

$-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1, 2, and 3

$-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)$ are sixth detrition

coefficients for category 1, 2, and 3
$-(b_{36}'')^{(7,7,7,7,7,7,7)}(G_{39},t)$, $-(b_{37}'')^{(7,7,7,7,7,7,7)}(G_{39},t)$, $-(b_{38}'')^{(7,7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2, and 3
$-(b_{40}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)$, $-(b_{41}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)$, $-(b_{42}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1, 2, and 3
$-(b_{46}'')^{(9,9,9,9,9,9,9)}(G_{47},t)$, $-(b_{45}'')^{(9,9,9,9,9,9,9)}(G_{47},t)$, $-(b_{44}'')^{(9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \qquad 91$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a_{36}')^{(7)} & +(a_{36}'')^{(7)}(T_{37},t) & +(a_{16}'')^{(2,2,2,2,2,2,2)}(T_{17},t) & +(a_{20}'')^{(3,3,3,3,3,3,3)}(T_{21},t) \\ +(a_{24}'')^{(4,4,4,4,4,4,4)}(T_{25},t) & +(a_{28}'')^{(5,5,5,5,5,5,5)}(T_{29},t) & +(a_{32}'')^{(6,6,6,6,6,6,6)}(T_{33},t) \\ +(a_{13}'')^{(1,1,1,1,1,1,1)}(T_{14},t) & +(a_{40}'')^{(8,8,8,8,8,8,8,8,)}(T_{41},t) & +(a_{44}'')^{(9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \qquad 92$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a_{37}')^{(7)} & +(a_{37}'')^{(7)}(T_{37},t) & +(a_{17}'')^{(2,2,2,2,2,2,2)}(T_{17},t) & +(a_{21}'')^{(3,3,3,3,3,3,3)}(T_{21},t) \\ +(a_{25}'')^{(4,4,4,4,4,4,4)}(T_{25},t) & +(a_{29}'')^{(5,5,5,5,5,5,5)}(T_{29},t) & +(a_{33}'')^{(6,6,6,6,6,6,6)}(T_{33},t) \\ +(a_{13}'')^{(1,1,1,1,1,1,1)}(T_{14},t) & +(a_{41}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t) & +(a_{45}'')^{(9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{14}$$

$$\frac{dG_{38}}{dt} \qquad 93$$

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a_{38}')^{(7)} & +(a_{38}'')^{(7)}(T_{37},t) & +(a_{18}'')^{(2,2,2,2,2,2,2)}(T_{17},t) & +(a_{22}'')^{(3,3,3,3,3,3,3)}(T_{21},t) \\ +(a_{26}'')^{(4,4,4,4,4,4,4)}(T_{25},t) & +(a_{30}'')^{(5,5,5,5,5,5,5)}(T_{29},t) & +(a_{34}'')^{(6,6,6,6,6,6,6)}(T_{33},t) \\ +(a_{15}'')^{(1,1,1,1,1,1,1)}(T_{14},t) & +(a_{42}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t) & +(a_{46}'')^{(9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{15}$$

Where $(a_{36}'')^{(7)}(T_{37},t)$, $(a_{37}'')^{(7)}(T_{37},t)$, $(a_{38}'')^{(7)}(T_{37},t)$ are first augmentation coefficients for category 1, 2 and 3
$+(a_{16}'')^{(2,2,2,2,2,2,2)}(T_{17},t)$, $+(a_{17}'')^{(2,2,2,2,2,2,2)}(T_{17},t)$, $+(a_{18}'')^{(2,2,2,2,2,2,2)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3
$+(a_{20}'')^{(3,3,3,3,3,3,3)}(T_{21},t)$, $+(a_{21}'')^{(3,3,3,3,3,3,3)}(T_{21},t)$, $+(a_{22}'')^{(3,3,3,3,3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3
$+(a_{24}'')^{(4,4,4,4,4,4,4)}(T_{25},t)$, $+(a_{25}'')^{(4,4,4,4,4,4,4)}(T_{25},t)$, $+(a_{26}'')^{(4,4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3
$+(a_{28}'')^{(5,5,5,5,5,5,5)}(T_{29},t)$, $+(a_{29}'')^{(5,5,5,5,5,5,5)}(T_{29},t)$, $+(a_{30}'')^{(5,5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3
$+(a_{32}'')^{(6,6,6,6,6,6,6)}(T_{33},t)$, $+(a_{33}'')^{(6,6,6,6,6,6,6)}(T_{33},t)$, $+(a_{34}'')^{(6,6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3
$+(a_{13}'')^{(1,1,1,1,1,1,1)}(T_{14},t)$, $+(a_{13}'')^{(1,1,1,1,1,1,1)}(T_{14},t)$, $+(a_{15}'')^{(1,1,1,1,1,1,1)}(T_{14},t)$ are seventh augmentation coefficient for category 1, 2 and 3
$+(a_{42}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a_{41}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a_{40}'')^{(8,8,8,8,8,8,8,8,)}(T_{41},t)$ are eighth augmentation coefficient for 1,2,3
$+(a_{46}'')^{(9,9,9,9,9,9,9)}(T_{45},t)$, $+(a_{45}'')^{(9,9,9,9,9,9,9)}(T_{45},t)$, $+(a_{44}'')^{(9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation

coefficient for 1,2,3



$$\frac{dT_{36}}{dt} =$$

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b'_{36})^{(7)}\boxed{-(b''_{36})^{(7)}(G_{39},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{37}}{dt} =$$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b'_{37})^{(7)}\boxed{-(b''_{37})^{(7)}(G_{39},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{38}}{dt} =$$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b'_{38})^{(7)}\boxed{-(b''_{38})^{(7)}(G_{39},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3



$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \begin{bmatrix} (a'_{40})^{(8)}\boxed{+(a''_{40})^{(8)}(T_{41},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \begin{bmatrix} (a'_{41})^{(8)}\boxed{+(a''_{41})^{(8)}(T_{41},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \begin{bmatrix} (a'_{42})^{(8)}\boxed{+(a''_{42})^{(8)}(T_{41},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a''_{40})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$\frac{dT_{40}}{dt} =$

$$(b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)}\boxed{-(b''_{40})^{(8)}(G_{43},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} =$$

$$(b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)}\boxed{-(b''_{41})^{(8)}(G_{43},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} =$$

$$(b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)}\boxed{-(b''_{42})^{(8)}(G_{43},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{44}}{dt} \qquad 96$$

$$= (a_{44})^{(9)}G_{45} - \begin{bmatrix} (a'_{44})^{(9)}\boxed{+(a''_{44})^{(9)}(T_{45},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \begin{bmatrix} (a'_{45})^{(9)}\boxed{+(a''_{45})^{(9)}(T_{45},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \begin{bmatrix} (a'_{46})^{(9)}\boxed{+(a''_{46})^{(9)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a''_{44})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ are Seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \begin{bmatrix} (b'_{44})^{(9)}\boxed{-(b''_{44})^{(9)}(G_{47},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b'_{45})^{(9)}\boxed{-(b''_{45})^{(9)}(G_{47},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b'_{46})^{(9)}\boxed{-(b''_{46})^{(9)}(G_{47},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{44})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9)}(G_{47},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a'_i)^{(1)}, (a''_i)^{(1)}, (b_i)^{(1)}, (b'_i)^{(1)}, (b''_i)^{(1)} > 0, \quad i,j = 13,14,15 \qquad 97$$

The functions $(a''_i)^{(1)}, (b''_i)^{(1)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a''_i)^{(1)}(T_{14},t) \le (p_i)^{(1)} \le (\hat{A}_{13})^{(1)}$$

$$(b''_i)^{(1)}(G,t) \le (r_i)^{(1)} \le (b'_i)^{(1)} \le (\hat{B}_{13})^{(1)}$$

$$lim_{T_2\to\infty}(a''_i)^{(1)}(T_{14},t) = (p_i)^{(1)} \qquad 98$$

$$\lim_{G\to\infty}(b''_i)^{(1)}(G,t) = (r_i)^{(1)}$$

**Definition of** $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$ :

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, \ (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$$|(a_i'')^{(1)}(T_{14}', t) - (a_i'')^{(1)}(T_{14}, t)| \leq (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G', t) - (b_i'')^{(1)}(G, t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}', t)$ and $(a_i'')^{(1)}(T_{14}, t)$ . $(T_{14}', t)$ and $(T_{14}, t)$ are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14}, t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$ : 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)},$ are positive constants

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \ , \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$$

**Definition of** $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$ : 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, \ (r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}}[\,(a_i)^{(1)} + (a_i')^{(1)} + \ (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}\,(\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[\ (b_i)^{(1)} + (b_i')^{(1)} + \ (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)}\ (\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i, j = 16,17,18$$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

**Definition of** $(\mathrm{p_i})^{(2)}$, $(\mathrm{r_i})^{(2)}$:

$$(a_i'')^{(2)}(T_{17}, t) \leq (p_i)^{(2)} \leq \left(\hat{A}_{16}\right)^{(2)} \quad 102$$

$$(b_i'')^{(2)}(G_{19}, t) \leq \ (r_i)^{(2)} \leq (b_i')^{(2)} \leq (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2 \to \infty} (a_i'')^{(2)}\,(T_{17}, t) = (p_i)^{(2)} \quad 104$$

$$\lim_{G \to \infty} (b_i'')^{(2)}\left((G_{19}), t\right) = \ (r_i)^{(2)} \quad 105$$

**Definition of** $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$ : 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}', t) - (a_i'')^{(2)}(T_{17}, t)| \leq (\hat{k}_{16})^{(2)}|T_{17} - T_{17}'|e^{-(\hat{M}_{16})^{(2)}t} \quad 107$$

$$|(b_i'')^{(2)}((G_{19})', t) - (b_i'')^{(2)}\big((G_{19}), t\big)| < (\hat{k}_{16})^{(2)}||(G_{19}) - (G_{19})'||e^{-(\hat{M}_{16})^{(2)}t} \quad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}', t)$ and$(a_i'')^{(2)}(T_{17}, t)$ . $(T_{17}', t)$ and $(T_{17}, t)$ are points belonging to the interval $\big[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}\big]$ . It is to be noted that $(a_i'')^{(2)}(T_{17}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17}, t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$ :

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants 109

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}} \ , \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$$

**Definition of** $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$ :

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and \ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[\,(a_i)^{(2)} + (a_i')^{(2)} + \ (\hat{A}_{16})^{(2)} + \ (\hat{P}_{16})^{(2)}\,(\hat{k}_{16})^{(2)}] < 1 \quad 110$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[\ (b_i)^{(2)} + (b_i')^{(2)} + \ (\hat{B}_{16})^{(2)} + \ (\hat{Q}_{16})^{(2)}\ (\hat{k}_{16})^{(2)}] < 1 \quad 111$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i, j = 20,21,22 \quad 112$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(3)}$, $(\mathrm{r_i})^{(3)}$:

$$(a_i'')^{(3)}(T_{21}, t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23}, t) \leq \ (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(3)}\,(T_{21}, t) = (p_i)^{(3)} \quad 113$$

$\lim_{G\to\infty}(b_i'')^{(3)}\,(G_{23},t) = \ (r_i)^{(3)}$

**Definition of** $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$ :

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, \ (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: 114

$|(a_i'')^{(3)}(T_{21}',t) - (a_i'')^{(3)}(T_{21},t)| \le (\hat{k}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\hat{M}_{20})^{(3)}t}$

$|(b_i'')^{(3)}(G_{23}',t) - (b_i'')^{(3)}(G_{23},t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}'||e^{-(\hat{M}_{20})^{(3)}t}$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}',t)$ and$(a_i'')^{(3)}(T_{21},t)$ . $(T_{21}',t)$ And $(T_{21},t)$ are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21},t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$ : 115

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)},$ are positive constants

$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}$ , $\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with 116
$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$ and the constants
$(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, \ (r_i)^{(3)}, i = 20,21,22,$
satisfy the inequalities

$\frac{1}{(\hat{M}_{20})^{(3)}}[\,(a_i)^{(3)} + (a_i')^{(3)} + \ (\hat{A}_{20})^{(3)} + \ (\hat{P}_{20})^{(3)}\,(\hat{k}_{20})^{(3)}] < 1$

$\frac{1}{(\hat{M}_{20})^{(3)}}[\ (b_i)^{(3)} + (b_i')^{(3)} + \ (\hat{B}_{20})^{(3)} + \ (\hat{Q}_{20})^{(3)}\ \ (\hat{k}_{20})^{(3)}] < 1$

Where we suppose

$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i,j = 24,25,26$ 117

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(4)}, \ (r_i)^{(4)}$:

$(a_i'')^{(4)}(T_{25},t) \le (p_i)^{(4)} \le (\hat{A}_{24})^{(4)}$

$(b_i'')^{(4)}\big((G_{27}),t\big) \le \ (r_i)^{(4)} \le (b_i')^{(4)} \le (\hat{B}_{24})^{(4)}$

$lim_{T_2\to\infty}(a_i'')^{(4)}\,(T_{25},t) = (p_i)^{(4)}$ 118

$\lim_{G\to\infty}(b_i'')^{(4)}\,\big((G_{27}),t\big) = \ (r_i)^{(4)}$

**Definition of** $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$ :

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24{,}25{,}26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}', t) - (a_i'')^{(4)}(T_{25}, t)| \leq (\hat{k}_{24})^{(4)} |T_{25} - T_{25}'| e^{-(\hat{M}_{24})^{(4)} t}$$

$$|(b_i'')^{(4)}((G_{27})', t) - (b_i'')^{(4)}\big((G_{27}), t\big)| < (\hat{k}_{24})^{(4)} ||(G_{27}) - (G_{27})'|| e^{-(\hat{M}_{24})^{(4)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}', t)$ and$(a_i'')^{(4)}(T_{25}, t)$ . $(T_{25}', t)$ and $(T_{25}, t)$ are points belonging to the interval $\big[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}\big]$. It is to be noted that $(a_i'')^{(4)}(T_{25}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25}, t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$ : 120

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$, are positive constants

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}} \ , \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$$

**Definition of** $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$ : 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and\ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}, i = 24{,}25{,}26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[\,(a_i)^{(4)} + (a_i')^{(4)} + \ (\hat{A}_{24})^{(4)} + \ (\hat{P}_{24})^{(4)}\,(\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[\ (b_i)^{(4)} + (b_i')^{(4)} + \ (\hat{B}_{24})^{(4)} + \ (\hat{Q}_{24})^{(4)}\ (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i, j = 28{,}29{,}30$$ 122

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(5)}, \ (r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29}, t) \leq (p_i)^{(5)} \leq (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}\big((G_{31}), t\big) \leq \ (r_i)^{(5)} \leq (b_i')^{(5)} \leq (\hat{B}_{28})^{(5)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(5)}\,(T_{29}, t) = (p_i)^{(5)}$$ 123

$$\lim_{G \to \infty}(b_i'')^{(5)}\,(G_{31}, t) = \ (r_i)^{(5)}$$

**Definition of** $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$ :

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, \ (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}', t) - (a_i'')^{(5)}(T_{29}, t)| \leq (\hat{k}_{28})^{(5)}|T_{29} - T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})', t) - (b_i'')^{(5)}((G_{31}), t)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}', t)$ and$(a_i'')^{(5)}(T_{29}, t)$ . $(T_{29}', t)$ and $(T_{29}, t)$ are points belonging to the interval $\left[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\right]$. It is to be noted that $(a_i'')^{(5)}(T_{29}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29}, t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$ : 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}} \ , \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

**Definition of** $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$ : 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and \ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, \ (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(a_i)^{(5)} + (a_i')^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)}(\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(b_i)^{(5)} + (b_i')^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)}(\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i, j = 32,33,34$$ 127

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(6)}, \ (r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33}, t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}), t) \leq \ (r_i)^{(6)} \leq (b_i')^{(6)} \leq (\hat{B}_{32})^{(6)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(6)}(T_{33}, t) = (p_i)^{(6)}$$ 128

$$\lim_{G \to \infty}(b_i'')^{(6)}\left((G_{35}), t\right) = \ (r_i)^{(6)}$$

**Definition of** $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$ :

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, \ (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}', t) - (a_i'')^{(6)}(T_{33}, t)| \leq (\hat{k}_{32})^{(6)} |T_{33} - T_{33}'| e^{-(\hat{M}_{32})^{(6)} t}$$

$$|(b_i'')^{(6)}((G_{35})', t) - (b_i'')^{(6)}((G_{35}), t)| < (\hat{k}_{32})^{(6)} ||(G_{35}) - (G_{35})'|| e^{-(\hat{M}_{32})^{(6)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}', t)$ and$(a_i'')^{(6)}(T_{33}, t)$ . $(T_{33}', t)$ and $(T_{33}, t)$ are points belonging to the interval $\left[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}\right]$. It is to be noted that $(a_i'')^{(6)}(T_{33}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33}, t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$ : 129

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$, are positive constants

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}} \ , \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$$

**Definition of** $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$ : 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and\ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, \ (r_i)^{(6)}, i = 32,33,34,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(a_i)^{(6)} + (a_i')^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)} (\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(b_i)^{(6)} + (b_i')^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)} (\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

(S) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i, j = 36,37,38$ 131

(T) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(7)}, \ (r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37}, t) \leq (p_i)^{(7)} \leq (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39}, t) \leq \ (r_i)^{(7)} \leq (b_i')^{(7)} \leq (\hat{B}_{36})^{(7)}$$

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*(U)* $\lim_{T_2 \to \infty} (a_i'')^{(7)}(T_{37}, t) = (p_i)^{(7)}$

*(V)*

$\lim_{G\to\infty}(b_i'')^{(7)}\left((G_{39}),t\right) = (r_i)^{(7)}$

**Definition of** $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$ :

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$$|(a_i'')^{(7)}(T_{37}',t) - (a_i'')^{(7)}(T_{37},t)| \le (\hat{k}_{36})^{(7)}|T_{37} - T_{37}'|e^{-(\hat{M}_{36})^{(7)}t}$$

$$|(b_i'')^{(7)}((G_{39})',t) - (b_i'')^{(7)}\left((G_{39}),t\right)| < (\hat{k}_{36})^{(7)}||(G_{39}) - (G_{39})'||e^{-(\hat{M}_{36})^{(7)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}',t)$ and$(a_i'')^{(7)}(T_{37},t)$ . $(T_{37}',t)$ and $(T_{37},t)$ are points belonging to the interval $\left[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}\right]$. It is to be noted that $(a_i'')^{(7)}(T_{37},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37},t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$ : 134

(W) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)},$ are positive constants

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$$

**Definition of** $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$ : 135

(X) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)}(\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(b_i)^{(7)} + (b_i')^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)}(\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40,41,42$$ 136

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

**Definition of** $(p_i)^{(8)}, (r_i)^{(8)}$: 137

$$(a_i'')^{(8)}(T_{41},t) \le (p_i)^{(8)} \le (\hat{A}_{40})^{(8)} \quad 138$$

$$(b_i'')^{(8)}((G_{43}),t) \le (r_i)^{(8)} \le (b_i')^{(8)} \le (\hat{B}_{40})^{(8)} \quad 139$$

$$\lim_{T_2\to\infty}(a_i'')^{(8)}(T_{41},t) = (p_i)^{(8)} \quad 140$$

$$\lim_{G\to\infty}(b_i'')^{(8)}\left((G_{43}),t\right) = (r_i)^{(8)} \quad 141$$

**Definition of** $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$ :

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(8)}(T_{41}',t) - (a_i'')^{(8)}(T_{41},t)| \le (\hat{k}_{40})^{(8)}|T_{41} - T_{41}'|e^{-(\hat{M}_{40})^{(8)}t} \quad 142$$

$$|(b_i'')^{(8)}((G_{43})',t) - (b_i'')^{(8)}\left((G_{43}),t\right)| < (\hat{k}_{40})^{(8)}||(G_{43}) - (G_{43})'||e^{-(\hat{M}_{40})^{(8)}t} \quad 143$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}',t)$ and $(a_i'')^{(8)}(T_{41},t)$ . $(T_{41}',t)$ and $(T_{41},t)$ are points belonging to the interval $\left[(\hat{k}_{40})^{(8)}, (\hat{M}_{40})^{(8)}\right]$. It is to be noted that $(a_i'')^{(8)}(T_{41},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{40})^{(8)} = 1$ then the function $(a_i'')^{(8)}(T_{41},t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$ :

$(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)},$ are positive constants

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}, \frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1 \quad 144$$

**Definition of** $(\hat{P}_{40})^{(8)}, (\hat{Q}_{40})^{(8)}$ :

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}, (\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)}, (a_i')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}, i = 40,41,42,$
Satisfy the inequalities

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(a_i)^{(8)} + (a_i')^{(8)} + (\hat{A}_{40})^{(8)} + (\hat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}] < 1 \quad 145$$

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(b_i)^{(8)} + (b_i')^{(8)} + (\hat{B}_{40})^{(8)} + (\hat{Q}_{40})^{(8)}(\hat{k}_{40})^{(8)}] < 1 \quad 146$$

Where we suppose

$$(a_i)^{(9)}, (a_i')^{(9)}, (a_i'')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (b_i'')^{(9)} > 0, \quad i,j = 44,45,46 \quad 146A$$

The functions $(a_i'')^{(9)}, (b_i'')^{(9)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(9)}$, $(r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45},t) \le (p_i)^{(9)} \le (\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47},t) \le (r_i)^{(9)} \le (b_i')^{(9)} \le (\hat{B}_{44})^{(9)}$$

$$\lim_{T_2\to\infty}(a_i'')^{(9)}(T_{45},t) = (p_i)^{(9)}$$

$$\lim_{G\to\infty}(b_i'')^{(9)}(G_{47},t) = (r_i)^{(9)}$$

**Definition of** $(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}$ :

Where $\boxed{(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}}$ are positive constants and $\boxed{i = 44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}',t) - (a_i'')^{(9)}(T_{45},t)| \le (\hat{k}_{44})^{(9)}|T_{45} - T_{45}'|e^{-(\hat{M}_{44})^{(9)}t}$$

$$|(b_i'')^{(9)}((G_{47})',t) - (b_i'')^{(9)}((G_{47}),t)| < (\hat{k}_{44})^{(9)}||(G_{47}) - (G_{47})'||e^{-(\hat{M}_{44})^{(9)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}',t)$ and$(a_i'')^{(9)}(T_{45},t)$ . $(T_{45}',t)$ and $(T_{45},t)$ are points belonging to the interval $\left[(\hat{k}_{44})^{(9)}, (\hat{M}_{44})^{(9)}\right]$. It is to be noted that $(a_i'')^{(9)}(T_{45},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{44})^{(9)} = 1$ then the function $(a_i'')^{(9)}(T_{45},t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$ :

$(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$, are positive constants

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}, \frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$$

**Definition of** $(\hat{P}_{44})^{(9)}, (\hat{Q}_{44})^{(9)}$ :

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}, (\hat{A}_{44})^{(9)} and\ (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)}, (a_i')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}, i = 44,45,46,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{44})^{(9)}}[(a_i)^{(9)} + (a_i')^{(9)} + (\hat{A}_{44})^{(9)} + (\hat{P}_{44})^{(9)}(\hat{k}_{44})^{(9)}] < 1$$

$$\frac{1}{(\hat{M}_{44})^{(9)}}[(b_i)^{(9)} + (b_i')^{(9)} + (\hat{B}_{44})^{(9)} + (\hat{Q}_{44})^{(9)}(\hat{k}_{44})^{(9)}] < 1$$

**Theorem 1:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \left(\hat{P}_{13}\right)^{(1)} e^{\left(\hat{M}_{13}\right)^{(1)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{13}\right)^{(1)} e^{\left(\hat{M}_{13}\right)^{(1)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 2 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

**Definition of** $G_i(0)\,, T_i(0)$

$$G_i(t) \le \left(\hat{P}_{16}\right)^{(2)} e^{\left(\hat{M}_{16}\right)^{(2)} t} \quad , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le \left(\hat{Q}_{16}\right)^{(2)} e^{\left(\hat{M}_{16}\right)^{(2)} t} \quad , \quad T_i(0) = T_i^0 > 0$$

**Theorem 3 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t) \le \left(\hat{P}_{20}\right)^{(3)} e^{\left(\hat{M}_{20}\right)^{(3)} t} \quad , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le \left(\hat{Q}_{20}\right)^{(3)} e^{\left(\hat{M}_{20}\right)^{(3)} t} \quad , \quad T_i(0) = T_i^0 > 0$$

**Theorem 4 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

**Definition of** $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \left(\hat{P}_{24}\right)^{(4)} e^{\left(\hat{M}_{24}\right)^{(4)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{24}\right)^{(4)} e^{\left(\hat{M}_{24}\right)^{(4)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 5 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

**Definition of** $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \left(\hat{P}_{28}\right)^{(5)} e^{\left(\hat{M}_{28}\right)^{(5)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{28}\right)^{(5)} e^{\left(\hat{M}_{28}\right)^{(5)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 6 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

**Definition of** $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \left(\hat{P}_{32}\right)^{(6)} e^{\left(\hat{M}_{32}\right)^{(6)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{32}\right)^{(6)} e^{\left(\hat{M}_{32}\right)^{(6)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 7:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

**Definition of** $G_i(0)\,, T_i(0)$ :

$G_i(t) \leq \left(\hat{P}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Theorem 8:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

**Definition of** $G_i(0)$ , $T_i(0)$ :

$G_i(t) \leq \left(\hat{P}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Theorem 9:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

**Definition of** $G_i(0)$ , $T_i(0)$ :

$G_i(t) \leq \left(\hat{P}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Proof:** Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i$: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$G_i(0) = G_i^0$ , $T_i(0) = T_i^0$ , $G_i^0 \leq \left(\hat{P}_{13}\right)^{(1)}$ , $T_i^0 \leq \left(\hat{Q}_{13}\right)^{(1)}$, 155

$0 \leq G_i(t) - G_i^0 \leq \left(\hat{P}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t}$ 156

$0 \leq T_i(t) - T_i^0 \leq \left(\hat{Q}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t}$ 157

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}(s_{(13)}) - \left((a'_{13})^{(1)} + a''_{13})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{15}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{15}(t) = T_{15}^0 + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{15}(s_{(13)})\right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0,t)$

**Proof:**

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i$: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$ , $T_i(0) = T_i^0$ , $G_i^0 \le (\hat{P}_{16})^{(2)}$ , $T_i^0 \le (\hat{Q}_{16})^{(2)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{16})^{(2)} e^{(\bar{M}_{16})^{(2)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{16})^{(2)} e^{(\bar{M}_{16})^{(2)} t}$

By

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[ (a_{16})^{(2)} G_{17}(s_{(16)}) - \left( (a'_{16})^{(2)} + a''_{16})^{(2)} (T_{17}(s_{(16)}), s_{(16)}) \right) G_{16}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[ (a_{17})^{(2)} G_{16}(s_{(16)}) - \left( (a'_{17})^{(2)} + (a''_{17})^{(2)} (T_{17}(s_{(16)}), s_{(17)}) \right) G_{17}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[ (a_{18})^{(2)} G_{17}(s_{(16)}) - \left( (a'_{18})^{(2)} + (a''_{18})^{(2)} (T_{17}(s_{(16)}), s_{(16)}) \right) G_{18}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[ (b_{16})^{(2)} T_{17}(s_{(16)}) - \left( (b'_{16})^{(2)} - (b''_{16})^{(2)} (G_{19}(s_{(16)}), s_{(16)}) \right) T_{16}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^0 + \int_0^t \left[ (b_{17})^{(2)} T_{16}(s_{(16)}) - \left( (b'_{17})^{(2)} - (b''_{17})^{(2)} (G_{19}(s_{(16)}), s_{(16)}) \right) T_{17}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^0 + \int_0^t \left[ (b_{18})^{(2)} T_{17}(s_{(16)}) - \left( (b'_{18})^{(2)} - (b''_{18})^{(2)} (G_{19}(s_{(16)}), s_{(16)}) \right) T_{18}(s_{(16)}) \right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0,t)$

**Proof:**

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i$: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$ , $T_i(0) = T_i^0$ , $G_i^0 \le (\hat{P}_{20})^{(3)}$ , $T_i^0 \le (\hat{Q}_{20})^{(3)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\bar{M}_{20})^{(3)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\bar{M}_{20})^{(3)} t}$

By

$$\bar{G}_{20}(t) = G_{20}^0 + \int_0^t \left[ (a_{20})^{(3)} G_{21}(s_{(20)}) - \left( (a'_{20})^{(3)} + a''_{20})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[ (a_{21})^{(3)} G_{20}(s_{(20)}) - \left( (a'_{21})^{(3)} + (a''_{21})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{21}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[ (a_{22})^{(3)} G_{21}(s_{(20)}) - \left( (a'_{22})^{(3)} + (a''_{22})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{22}(s_{(20)}) \right] ds_{(20)}$$

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$$\bar{T}_{20}(t) = T_{20}^{0} + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^{0} + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{\mathrm{T}}_{22}(\mathrm{t}) = \mathrm{T}_{22}^{0} + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{22}(s_{(20)})\right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{24})^{(4)}, T_i^0 \le (\hat{Q}_{24})^{(4)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

By 162

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{26}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^{0} + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^{0} + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{\mathrm{T}}_{26}(\mathrm{t}) = \mathrm{T}_{26}^{0} + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{26}(s_{(24)})\right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{28})^{(5)}, T_i^0 \le (\hat{Q}_{28})^{(5)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

By 163

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{30}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{\mathrm{T}}_{30}(\mathrm{t}) = \mathrm{T}_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{30}(s_{(28)})\right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{32})^{(6)}$, $T_i^0 \le (\hat{Q}_{32})^{(6)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

By 164

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a'_{32})^{(6)} + a''_{32})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{34}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{\mathrm{T}}_{34}(\mathrm{t}) = \mathrm{T}_{34}^0 + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{34}(s_{(32)})\right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[ (a_{36})^{(7)} G_{37}(s_{(36)}) - \left( (a'_{36})^{(7)} + a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[ (a_{37})^{(7)} G_{36}(s_{(36)}) - \left( (a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[ (a_{38})^{(7)} G_{37}(s_{(36)}) - \left( (a'_{38})^{(7)} + (a''_{38})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{38}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[ (b_{36})^{(7)} T_{37}(s_{(36)}) - \left( (b'_{36})^{(7)} - (b''_{36})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[ (b_{37})^{(7)} T_{36}(s_{(36)}) - \left( (b'_{37})^{(7)} - (b''_{37})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{38}(t) = T_{38}^0 + \int_0^t \left[ (b_{38})^{(7)} T_{37}(s_{(36)}) - \left( (b'_{38})^{(7)} - (b''_{38})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{38}(s_{(36)}) \right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{40})^{(8)}$, $T_i^0 \le (\hat{Q}_{40})^{(8)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[ (a_{40})^{(8)} G_{41}(s_{(40)}) - \left( (a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[ (a_{41})^{(8)} G_{40}(s_{(40)}) - \left( (a'_{41})^{(8)} + (a''_{41})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[ (a_{42})^{(8)} G_{41}(s_{(40)}) - \left( (a'_{42})^{(8)} + (a''_{42})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{42}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^0 + \int_0^t \left[ (b_{40})^{(8)} T_{41}(s_{(40)}) - \left( (b'_{40})^{(8)} - (b''_{40})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^0 + \int_0^t \left[ (b_{41})^{(8)} T_{40}(s_{(40)}) - \left( (b'_{41})^{(8)} - (b''_{41})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{42}(t) = T_{42}^0 + \int_0^t \left[ (b_{42})^{(8)} T_{41}(s_{(40)}) - \left( (b'_{42})^{(8)} - (b''_{42})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{42}(s_{(40)}) \right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** 166

Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ A which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{44})^{(9)},\ T_i^0 \le (\hat{Q}_{44})^{(9)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} t}$$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[ (a_{44})^{(9)} G_{45}(s_{(44)}) - \left( (a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[ (a_{45})^{(9)} G_{44}(s_{(44)}) - \left( (a'_{45})^{(9)} + (a''_{45})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[ (a_{46})^{(9)} G_{45}(s_{(44)}) - \left( (a'_{46})^{(9)} + (a''_{46})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{46}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[ (b_{44})^{(9)} T_{45}(s_{(44)}) - \left( (b'_{44})^{(9)} - (b''_{44})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[ (b_{45})^{(9)} T_{44}(s_{(44)}) - \left( (b'_{45})^{(9)} - (b''_{45})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{46}(t) = T_{46}^0 + \int_0^t \left[ (b_{46})^{(9)} T_{45}(s_{(44)}) - \left( (b'_{46})^{(9)} - (b''_{46})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{46}(s_{(44)}) \right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \le G_{13}^0 + \int_0^t \left[ (a_{13})^{(1)} \left( G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\tilde{M}_{13})^{(1)} s_{(13)}} \right) \right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\tilde{M}_{13})^{(1)}} \left( e^{(\tilde{M}_{13})^{(1)} t} - 1 \right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\tilde{M}_{13})^{(1)} t} \le \frac{(a_{13})^{(1)}}{(\tilde{M}_{13})^{(1)}} \left[ ((\hat{P}_{13})^{(1)} + G_{14}^0) e^{\left( -\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0} \right)} + (\hat{P}_{13})^{(1)} \right]$$

$(G_i^0)$ is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}$ , $G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \le G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}}\right)\right] ds_{(16)} = \quad (169)$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1\right)$$

From which it follows that (170)

$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \le \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left[\left((\hat{P}_{16})^{(2)} + G_{17}^0\right) e^{\left(-\frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0}\right)} + (\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}$ , $G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that (171)

$$G_{20}(t) \le G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} \left(G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} s_{(20)}}\right)\right] ds_{(20)} =$$

$$\left(1 + (a_{20})^{(3)} t\right) G_{21}^0 + \frac{(a_{20})^{(3)} (\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left(e^{(\hat{M}_{20})^{(3)} t} - 1\right)$$

From which it follows that (172)

$$(G_{20}(t) - G_{20}^0) e^{-(\hat{M}_{20})^{(3)} t} \le \frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left[\left((\hat{P}_{20})^{(3)} + G_{21}^0\right) e^{\left(-\frac{(\hat{P}_{20})^{(3)} + G_{21}^0}{G_{21}^0}\right)} + (\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}$ , $G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that (173)

$$G_{24}(t) \le G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} \left(G_{25}^0 + (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} s_{(24)}}\right)\right] ds_{(24)} =$$

$$\left(1 + (a_{24})^{(4)} t\right) G_{25}^0 + \frac{(a_{24})^{(4)} (\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}} \left(e^{(\hat{M}_{24})^{(4)} t} - 1\right)$$

From which it follows that (174)

$$(G_{24}(t) - G_{24}^0) e^{-(\hat{M}_{24})^{(4)} t} \le \frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}} \left[\left((\hat{P}_{24})^{(4)} + G_{25}^0\right) e^{\left(-\frac{(\hat{P}_{24})^{(4)} + G_{25}^0}{G_{25}^0}\right)} + (\hat{P}_{24})^{(4)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t) \le G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} \left(G_{29}^0 + (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} s_{(28)}}\right)\right] ds_{(28)} =$$

$$\left(1+(a_{28})^{(5)}t\right)G_{29}^{0}+\frac{(a_{28})^{(5)}(\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left(e^{(\hat{M}_{28})^{(5)}t}-1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t}\leq\frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t)\leq G_{32}^{0}+\int_{0}^{t}\left[(a_{32})^{(6)}\left(G_{33}^{0}+(\hat{P}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\right)\right]ds_{(32)}=$$

$$\left(1+(a_{32})^{(6)}t\right)G_{33}^{0}+\frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left(e^{(\hat{M}_{32})^{(6)}t}-1\right)$$

From which it follows that 177

$$(G_{32}(t)-G_{32}^{0})e^{-(\hat{M}_{32})^{(6)}t}\leq\frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left[((\hat{P}_{32})^{(6)}+G_{33}^{0})e^{\left(-\frac{(\hat{P}_{32})^{(6)}+G_{33}^{0}}{G_{33}^{0}}\right)}+(\hat{P}_{32})^{(6)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

(d) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t)\leq G_{36}^{0}+\int_{0}^{t}\left[(a_{36})^{(7)}\left(G_{37}^{0}+(\hat{P}_{36})^{(7)}e^{(\hat{M}_{36})^{(7)}s_{(36)}}\right)\right]ds_{(36)}=$$
$$\left(1+(a_{36})^{(7)}t\right)G_{37}^{0}+\frac{(a_{36})^{(7)}(\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left(e^{(\hat{M}_{36})^{(7)}t}-1\right)$$

From which it follows that

$$(G_{36}(t)-G_{36}^{0})e^{-(\hat{M}_{36})^{(7)}t}\leq\frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left[((\hat{P}_{36})^{(7)}+G_{37}^{0})e^{\left(-\frac{(\hat{P}_{36})^{(7)}+G_{37}^{0}}{G_{37}^{0}}\right)}+(\hat{P}_{36})^{(7)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t)\leq G_{40}^{0}+\int_{0}^{t}\left[(a_{40})^{(8)}\left(G_{41}^{0}+(\hat{P}_{40})^{(8)}e^{(\hat{M}_{40})^{(8)}s_{(40)}}\right)\right]ds_{(40)}=$$ 180
$$\left(1+(a_{40})^{(8)}t\right)G_{41}^{0}+\frac{(a_{40})^{(8)}(\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left(e^{(\hat{M}_{40})^{(8)}t}-1\right)$$

From which it follows that 181

$$(G_{40}(t)-G_{40}^{0})e^{-(\hat{M}_{40})^{(8)}t}\leq\frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left[((\hat{P}_{40})^{(8)}+G_{41}^{0})e^{\left(-\frac{(\hat{P}_{40})^{(8)}+G_{41}^{0}}{G_{41}^{0}}\right)}+(\hat{P}_{40})^{(8)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 8
Analogous inequalities hold also for $G_{41}, G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t)\leq G_{44}^{0}+\int_{0}^{t}\left[(a_{44})^{(9)}\left(G_{45}^{0}+(\hat{P}_{44})^{(9)}e^{(\hat{M}_{44})^{(9)}s_{(44)}}\right)\right]ds_{(44)}=$$

$$\left(1+(a_{44})^{(9)}t\right)G_{45}^{0}+\frac{(a_{44})^{(9)}(\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}}\left(e^{(\hat{M}_{44})^{(9)}t}-1\right)$$

From which it follows that

$$(G_{44}(t)-G_{44}^{0})e^{-(\hat{M}_{44})^{(9)}t}\leq\frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}}\left[((\hat{P}_{44})^{(9)}+G_{45}^{0})e^{\left(-\frac{(\hat{P}_{44})^{(9)}+G_{45}^{0}}{G_{45}^{0}}\right)}+(\hat{P}_{44})^{(9)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 9

Analogous inequalities hold also for $G_{45}, G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}<1$ and to choose 182

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[(\hat{P}_{13})^{(1)}+((\hat{P}_{13})^{(1)}+G_j^0)e^{-\left(\frac{(\hat{P}_{13})^{(1)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{13})^{(1)} \quad 183$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[((\hat{Q}_{13})^{(1)}+T_j^0)e^{-\left(\frac{(\hat{Q}_{13})^{(1)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{13})^{(1)}\right]\leq(\hat{Q}_{13})^{(1)} \quad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left((G^{(1)},T^{(1)}),(G^{(2)},T^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\hat{M}_{13})^{(1)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\hat{M}_{13})^{(1)}t}\}$$

Indeed if we denote

**Definition of** $\tilde{G},\tilde{T}:(\tilde{G},\tilde{T})=\mathcal{A}^{(1)}(G,T)$

It results

$$\left|\tilde{G}_{13}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{13})^{(1)} \left|G_{14}^{(1)}-G_{14}^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\, ds_{(13)} +$$

$$\int_0^t \{(a'_{13})^{(1)}\left|G_{13}^{(1)}-G_{13}^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} +$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)}, s_{(13)}\right)\left|G_{13}^{(1)}-G_{13}^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} e^{(\widehat{M}_{13})^{(1)}s_{(13)}} +$$

$$G_{13}^{(2)}\left|(a''_{13})^{(1)}\left(T_{14}^{(1)}, s_{(13)}\right)-(a''_{13})^{(1)}\left(T_{14}^{(2)}, s_{(13)}\right)\right|\ e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\} ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\left|G^{(1)}-G^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)}t} \leq \qquad 186$$
$$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)}+\ (a'_{13})^{(1)}+(\widehat{A}_{13})^{(1)}+(\widehat{P}_{13})^{(1)}(\widehat{k}_{13})^{(1)}\right) d\left(\left(G^{(1)}, T^{(1)};\ G^{(2)}, T^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

**Remark 1:** The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t}\ and\ (\widehat{Q}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(1)}$ and $(b''_i)^{(1)}, i = 13,14,15$ depend only on $T_{14}$ and respectively on $G(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any $t$ where $G_i(t) = 0\ and\ T_i(t) = 0$

From 19 to 24 it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a'_i)^{(1)}-(a''_i)^{(1)}(T_{14}(s_{(13)}),s_{(13)})\} ds_{(13)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b'_i)^{(1)}t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2\ and\ \left((\widehat{M}_{13})^{(1)}\right)_3$ : 187

**Remark 3**: if $G_{13}$ is bounded, the same property have also $G_{14}\ and\ G_{15}$ . indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a'_{14})^{(1)} G_{14}$ and by integrating

$$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1 / (a'_{14})^{(1)}$$

In the same way , one can obtain

$$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2 / (a'_{15})^{(1)}$$

If $G_{14}\ or\ G_{15}$ is bounded, the same property follows for $G_{13}$ , $G_{15}$ and $G_{13}$ , $G_{14}$ respectively.

**Remark 4:** If $G_{13}$ *is* bounded, from below, the same property holds for $G_{14}$ *and* $G_{15}$. The proof is analogous with the preceding one. An analogous property is true if $G_{14}$ is bounded from below. 188

**Remark 5:** If $T_{13}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}(G(t),t)) = (b_{14}')^{(1)}$ then $T_{14} \to \infty$. 189

**Definition of** $(m)^{(1)}$ and $\varepsilon_1$ :

Indeed let $t_1$ be so that for $t > t_1$

$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t),t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\frac{2}{\varepsilon_1}$ By taking now $\varepsilon_1$ sufficiently small one sees that $T_{14}$ is unbounded.
The same property holds for $T_{15}$ if $\lim_{t\to\infty}(b_{15}'')^{(1)}(G(t),t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}$ *and* $(\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[(\hat{P}_{16})^{(2)} + \left((\hat{P}_{16})^{(2)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{16})^{(2)}\right] \leq (\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t) - G_i^{(2)}(t)|e^{-(\hat{M}_{16})^{(2)}t}, \max_{t\in\mathbb{R}_+}|T_i^{(1)}(t) - T_i^{(2)}(t)|e^{-(\hat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

**Definition of** $\widetilde{G_{19}}, \widetilde{T_{19}}$ : $(\widetilde{G_{19}}, \widetilde{T_{19}}) = \mathcal{A}^{(2)}(G_{19}, T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)}-\tilde{G}_{i}^{(2)}\right|\leq\int_{0}^{t}(a_{16})^{(2)}\left|G_{17}^{(1)}-G_{17}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)}+$$

$$\int_{0}^{t}\{(a'_{16})^{(2)}\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$G_{16}^{(2)}\left|(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)-(a''_{16})^{(2)}\left(T_{17}^{(2)},s_{(16)}\right)\right|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0,t]$ 197

From the hypotheses it follows

$$\left|(G_{19})^{(1)}-(G_{19})^{(2)}\right|\mathrm{e}^{-(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}\leq$$
$$\frac{1}{(\widehat{\mathrm{M}}_{16})^{(2)}}\left((a_{16})^{(2)}+\ (a'_{16})^{(2)}+(\widehat{\mathrm{A}}_{16})^{(2)}+(\widehat{\mathrm{P}}_{16})^{(2)}(\hat{k}_{16})^{(2)}\right)\mathrm{d}\left(((G_{19})^{(1)},(T_{19})^{(1)};\ (G_{19})^{(2)},(T_{19})^{(2)})\right)$$

And analogous inequalities for $\mathrm{G}_i$ and $\mathrm{T}_i$. Taking into account the hypothesis the result follows 198

**Remark 6:** The fact that we supposed $(a''_{16})^{(2)}$ and $(b''_{16})^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{\mathrm{P}}_{16})^{(2)}\mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}$ and $(\widehat{\mathrm{Q}}_{16})^{(2)}\mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}$ respectively of $\mathbb{R}_{+}$. 199

If instead of proving the existence of the solution on $\mathbb{R}_{+}$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(2)}$ and $(b''_i)^{(2)}, i=16,17,18$ depend only on $\mathrm{T}_{17}$ and respectively on $(G_{19})$(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 7:** There does not exist any t where $\mathrm{G}_i\,(\mathrm{t})=0$ and $\mathrm{T}_i\,(\mathrm{t})=0$ 200

it results

$$\mathrm{G}_i\,(\mathrm{t})\geq\mathrm{G}_i^0\mathrm{e}^{\left[-\int_0^{\mathrm{t}}\{(a'_i)^{(2)}-(a''_i)^{(2)}(\mathrm{T}_{17}(s_{(16)}),s_{(16)})\}\mathrm{d}s_{(16)}\right]}\geq 0$$

$$\mathrm{T}_i\,(\mathrm{t})\geq\mathrm{T}_i^0\mathrm{e}^{(-(b'_i)^{(2)}\mathrm{t})}>0\quad\text{for t}>0$$

**Definition of** $\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_1$, $\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_2$ and $\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_3$: 201

**Remark 8:** if $\mathrm{G}_{16}$ is bounded, the same property have also $\mathrm{G}_{17}$ and $\mathrm{G}_{18}$ . indeed if

$\mathrm{G}_{16}<(\widehat{\mathrm{M}}_{16})^{(2)}$ it follows $\frac{\mathrm{dG}_{17}}{\mathrm{dt}}\leq\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_1-(a'_{17})^{(2)}\mathrm{G}_{17}$ and by integrating

$$\mathrm{G}_{17}\leq\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_2=\mathrm{G}_{17}^0+2(a_{17})^{(2)}\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_1/(a'_{17})^{(2)}$$

In the same way , one can obtain

$$\mathrm{G}_{18}\leq\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_3=\mathrm{G}_{18}^0+2(a_{18})^{(2)}\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_2/(a'_{18})^{(2)}$$

If $\mathrm{G}_{17}$ or $\mathrm{G}_{18}$ is bounded, the same property follows for $\mathrm{G}_{16}$ , $\mathrm{G}_{18}$ and $\mathrm{G}_{16}$ , $\mathrm{G}_{17}$ respectively.

**Remark 9:** If $G_{16}$ is bounded, from below, the same property holds for $G_{17}$ and $G_{18}$. The proof is analogous with the preceding one. An analogous property is true if $G_{17}$ is bounded from below. 202

**Remark 10:** If $T_{16}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t),t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$. 203

**Definition of** $(m)^{(2)}$ and $\varepsilon_2$ :

Indeed let $t_2$ be so that for $t > t_2$

$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t),t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right), \quad t = \log\frac{2}{\varepsilon_2}$ By taking now $\varepsilon_2$ sufficiently small one sees that $T_{17}$ is unbounded. 205
The same property holds for $T_{18}$ if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t),t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\tilde{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\tilde{M}_{20})^{(3)}} < 1$ and to choose 207

$(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\tilde{M}_{20})^{(3)}}\left[(\hat{P}_{20})^{(3)} + \left((\hat{P}_{20})^{(3)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\tilde{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{20})^{(3)}\right] \leq (\hat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying 210
Equations into itself

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)},(T_{23})^{(1)}),((G_{23})^{(2)},(T_{23})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\tilde{M}_{20})^{(3)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\tilde{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

**Definition of** $\widetilde{G_{23}}, \widetilde{T_{23}}$ :$\left(\widetilde{(G_{23})},\widetilde{(T_{23})}\right) = \mathcal{A}^{(3)}\left((G_{23}),(T_{23})\right)$

It results 213

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{20})^{(3)} \left|G_{21}^{(1)}-G_{21}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} e^{(\widehat{M}_{20})^{(3)} s_{(20)}}\, ds_{(20)} +$$

$$\int_0^t \{(a'_{20})^{(3)} \left|G_{20}^{(1)}-G_{20}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} +$$

$$(a''_{20})^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} e^{(\widehat{M}_{20})^{(3)} s_{(20)}} +$$

$$G_{20}^{(2)} \left|(a''_{20})^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right) - (a''_{20})^{(3)}\left(T_{21}^{(2)}, s_{(20)}\right)\right|\ e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} e^{(\widehat{M}_{20})^{(3)} s_{(20)}} \} ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\left|G_{23}{}^{(1)} - G_{23}{}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)} t} \leq \qquad (214)$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)} + (a'_{20})^{(3)} + (\widehat{A}_{20})^{(3)} + (\widehat{P}_{20})^{(3)} (\widehat{k}_{20})^{(3)}\right) d\left(((G_{23})^{(1)}, (T_{23})^{(1)};\ (G_{23})^{(2)}, (T_{23})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 11:** The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)} e^{(\widehat{M}_{20})^{(3)} t}$ *and* $(\widehat{Q}_{20})^{(3)} e^{(\widehat{M}_{20})^{(3)} t}$ respectively of $\mathbb{R}_+$. (215)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i = 20,21,22$ depend only on $T_{21}$ and respectively on $(G_{23})$(*and not on* $t$) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 12:** There does not exist any $t$ where $G_i(t) = 0$ *and* $T_i(t) = 0$ (216)

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(3)} - (a_i'')^{(3)} (T_{21}(s_{(20)}), s_{(20)})\} ds_{(20)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(3)} t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2$ *and* $\left((\widehat{M}_{20})^{(3)}\right)_3$ : (217)

**Remark 13:** if $G_{20}$ is bounded, the same property have also $G_{21}$ *and* $G_{22}$ . indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a'_{21})^{(3)} G_{21}$ and by integrating

$$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)} \left((\widehat{M}_{20})^{(3)}\right)_1 / (a'_{21})^{(3)}$$

In the same way , one can obtain

$$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)} \left((\widehat{M}_{20})^{(3)}\right)_2 / (a'_{22})^{(3)}$$

If $G_{21}$ *or* $G_{22}$ is bounded, the same property follows for $G_{20}$ , $G_{22}$ and $G_{20}$ , $G_{21}$ respectively.

**Remark 14:** If $G_{20}$ *is* bounded, from below, the same property holds for $G_{21}$ *and* $G_{22}$. The proof is analogous with the preceding one. An analogous property is true if $G_{21}$ is bounded from below. 218

**Remark 15:** If $T_{20}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t),t\right)) = (b_{21}')^{(3)}$ then $T_{21} \to \infty$. 219

**Definition of** $(m)^{(3)}$ and $\varepsilon_3$ :

Indeed let $t_3$ be so that for $t > t_3$

$(b_{21})^{(3)} - (b_i'')^{(3)}\left((G_{23})(t),t\right) < \varepsilon_3, T_{20}(t) > (m)^{(3)}$

Then $\frac{dT_{21}}{dt} \geq (a_{21})^{(3)}(m)^{(3)} - \varepsilon_3 T_{21}$ which leads to 220

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t} = \frac{1}{2}$ it results

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right), \quad t = log\frac{2}{\varepsilon_3}$ By taking now $\varepsilon_3$ sufficiently small one sees that $T_{21}$ is unbounded.
The same property holds for $T_{22}$ if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t),t\right) = (b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\tilde{M}_{24})^{(4)}}$ , $\frac{(b_i)^{(4)}}{(\tilde{M}_{24})^{(4)}} < 1$ and to choose 221

$(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\tilde{M}_{24})^{(4)}}\left[(\hat{P}_{24})^{(4)} + \left((\hat{P}_{24})^{(4)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{24})^{(4)} \quad 222$$

$$\frac{(b_i)^{(4)}}{(\tilde{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{24})^{(4)}\right] \leq (\hat{Q}_{24})^{(4)} \quad 223$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself 224

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric 225

$$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\tilde{M}_{24})^{(4)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\tilde{M}_{24})^{(4)}t}\}$$

Indeed if we denote

**Definition of** $\widetilde{(G_{27})},\widetilde{(T_{27})}$ : $\left(\widetilde{(G_{27})},\widetilde{(T_{27})}\right) = \mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_{i}^{(2)}\right|\leq\int_{0}^{t}(a_{24})^{(4)}\left|G_{25}^{(1)}-G_{25}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\,ds_{(24)}+$$

$$\int_{0}^{t}\{(a'_{24})^{(4)}\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$G_{24}^{(2)}\left|(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)-(a''_{24})^{(4)}\left(T_{25}^{(2)},s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\}ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}t}\leq \qquad 226$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)}+\ (a'_{24})^{(4)}+(\widehat{A}_{24})^{(4)}+(\widehat{P}_{24})^{(4)}(\widehat{k}_{24})^{(4)}\right)d\left(((G_{27})^{(1)},(T_{27})^{(1)};\ (G_{27})^{(2)},(T_{27})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

**Remark 16:** The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}\ and\ (\widehat{Q}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_{+}$. 227

If instead of proving the existence of the solution on $\mathbb{R}_{+}$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i=24{,}25{,}26$ depend only on $\mathrm{T}_{25}$ and respectively on $(G_{27})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 17:** There does not exist any $t$ where $G_i\,(t)=0\ and\ T_i\,(t)=0$ 228

it results

$$G_i\,(t)\geq G_i^0e^{\left[-\int_0^t\{(a_i')^{(4)}-(a_i'')^{(4)}(T_{25}(s_{(24)}),s_{(24)})\}ds_{(24)}\right]}\geq 0$$

$$T_i\,(t)\geq T_i^0e^{\left(-(b_i')^{(4)}t\right)}>0\quad \text{for t}>0$$

**Definition of** $\left((\widehat{M}_{24})^{(4)}\right)_1,\left((\widehat{M}_{24})^{(4)}\right)_2\ and\ \left((\widehat{M}_{24})^{(4)}\right)_3$ : 229

**Remark 18:** if $G_{24}$ is bounded, the same property have also $G_{25}\ and\ G_{26}$ . indeed if

$G_{24}<(\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt}\leq\left((\widehat{M}_{24})^{(4)}\right)_1-(a'_{25})^{(4)}G_{25}$ and by integrating

$$G_{25}\leq\left((\widehat{M}_{24})^{(4)}\right)_2=G_{25}^0+2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1/(a'_{25})^{(4)}$$

In the same way , one can obtain

$$G_{26}\leq\left((\widehat{M}_{24})^{(4)}\right)_3=G_{26}^0+2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2/(a'_{26})^{(4)}$$

If $G_{25}\ or\ G_{26}$ is bounded, the same property follows for $G_{24}$ , $G_{26}$ and $G_{24}$ , $G_{25}$ respectively.

**Remark 19:** If $G_{24}$ *is* bounded, from below, the same property holds for $G_{25}$ *and* $G_{26}$. The proof is analogous with the preceding one. An analogous property is true if $G_{25}$ is bounded from below. 230

**Remark 20:** If $T_{24}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}((G_{27})(t),t)) = (b_{25}')^{(4)}$ then $T_{25} \to \infty$. 231

**Definition of** $(m)^{(4)}$ and $\varepsilon_4$ :

Indeed let $t_4$ be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t),t) < \varepsilon_4, T_{24}(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right), \quad t = log\frac{2}{\varepsilon_4}$ By taking now $\varepsilon_4$ sufficiently small one sees that $T_{25}$ is unbounded.
The same property holds for $T_{26}$ if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t),t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for $G_{29}, G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}, \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)} + \left((\hat{P}_{28})^{(5)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{28})^{(5)}\right] \leq (\hat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$$d\left(((G_{31})^{(1)},(T_{31})^{(1)}),((G_{31})^{(2)},(T_{31})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

**Definition of** $\widetilde{(G_{31})}, \widetilde{(T_{31})}$ : $\left(\widetilde{(G_{31})}, \widetilde{(T_{31})}\right) = \mathcal{A}^{(5)}\left((G_{31}),(T_{31})\right)$

It results

$$\left|\tilde{G}_{28}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{28})^{(5)} \left|G_{29}^{(1)}-G_{29}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}}\, ds_{(28)} +$$

$$\int_0^t \{(a'_{28})^{(5)} \left|G_{28}^{(1)}-G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} +$$

$$(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right)\left|G_{28}^{(1)}-G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}} +$$

$$G_{28}^{(2)} \left|(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right)-(a''_{28})^{(5)}\left(T_{29}^{(2)}, s_{(28)}\right)\right|\ e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}}\} ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$$\left|(G_{31})^{(1)}-(G_{31})^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} t} \leq \qquad (237)$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)} + (a'_{28})^{(5)} + (\widehat{A}_{28})^{(5)} + (\widehat{P}_{28})^{(5)}(\widehat{k}_{28})^{(5)}\right) d\left(\left((G_{31})^{(1)}, (T_{31})^{(1)};\ (G_{31})^{(2)}, (T_{31})^{(2)}\right)\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 21:** The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$ *and* $(\widehat{Q}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$ respectively of $\mathbb{R}_+$. (238)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on $T_{29}$ and respectively on $(G_{31})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 22:** There does not exist any $t$ where $G_i(t) = 0$ *and* $T_i(t) = 0$ (239)

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(5)} - (a_i'')^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(5)} t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2$ *and* $\left((\widehat{M}_{28})^{(5)}\right)_3$ : (240)

**Remark 23:** if $G_{28}$ is bounded, the same property have also $G_{29}$ *and* $G_{30}$ . indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a'_{29})^{(5)} G_{29}$ and by integrating

$$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1 / (a'_{29})^{(5)}$$

In the same way , one can obtain

$$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2 / (a'_{30})^{(5)}$$

If $G_{29}$ *or* $G_{30}$ is bounded, the same property follows for $G_{28}$ , $G_{30}$ and $G_{28}$ , $G_{29}$ respectively.

**Remark 24:** If $G_{28}$ *is* bounded, from below, the same property holds for $G_{29}$ *and* $G_{30}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{29}$ is bounded from below. 241

**Remark 25:** If $T_{28}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}((G_{31})(t),t)) = (b_{29}')^{(5)}$ then $T_{29} \to \infty$. 242

**Definition of** $(m)^{(5)}$ and $\varepsilon_5$ :

Indeed let $t_5$ be so that for $t > t_5$

$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t),t) < \varepsilon_5, T_{28}(t) > (m)^{(5)}$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\frac{2}{\varepsilon_5}$ By taking now $\varepsilon_5$ sufficiently small one sees that $T_{29}$ is unbounded.
The same property holds for $T_{30}$ if $\lim_{t\to\infty}(b_{30}'')^{(5)}\left((G_{31})(t),t\right) = (b_{30}')^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{33}$ , $G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}$ , $\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$ and to choose 244

$(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[(\hat{P}_{32})^{(6)} + \left((\hat{P}_{32})^{(6)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{32})^{(6)} \quad 245$$

$$\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[\left((\hat{Q}_{32})^{(6)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{32})^{(6)}\right] \leq (\hat{Q}_{32})^{(6)} \quad 246$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric 247

$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right) =$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{32})^{(6)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

**Definition of** $\widetilde{(G_{35})}, \widetilde{(T_{35})}$ : $\left(\widetilde{(G_{35})}, \widetilde{(T_{35})}\right) = \mathcal{A}^{(6)}\left((G_{35}), (T_{35})\right)$

It results

$\left|\tilde{G}_{32}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{32})^{(6)} \left|G_{33}^{(1)} - G_{33}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}}\, ds_{(32)} +$

$\int_0^t \{(a'_{32})^{(6)} \left|G_{32}^{(1)} - G_{32}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} +$

$(a''_{32})^{(6)}\left(T_{33}^{(1)}, s_{(32)}\right)\left|G_{32}^{(1)} - G_{32}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}} +$

$G_{32}^{(2)} \left|(a''_{32})^{(6)}\left(T_{33}^{(1)}, s_{(32)}\right) - (a''_{32})^{(6)}\left(T_{33}^{(2)}, s_{(32)}\right)\right| \; e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}} \} ds_{(32)}$

Where $s_{(32)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$\left|(G_{35})^{(1)} - (G_{35})^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} t} \leq$ 248

$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)} + (a'_{32})^{(6)} + (\widehat{A}_{32})^{(6)} + (\widehat{P}_{32})^{(6)} (\widehat{k}_{32})^{(6)}\right) d\left(\left((G_{35})^{(1)}, (T_{35})^{(1)}; (G_{35})^{(2)}, (T_{35})^{(2)}\right)\right)$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 26:** The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)} t}$ *and* $(\widehat{Q}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)} t}$ respectively of $\mathbb{R}_+$. 249

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i = 32{,}33{,}34$ depend only on $T_{33}$ and respectively on $(G_{35})$(*and not on* $t$) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 27:** There does not exist any $t$ where $G_i(t) = 0$ *and* $T_i(t) = 0$ 250

it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(6)} - (a_i'')^{(6)}(T_{33}(s_{(32)}), s_{(32)})\} ds_{(32)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(6)} t\right)} > 0$ for t > 0

**Definition of** $\left((\widehat{M}_{32})^{(6)}\right)_1, \left((\widehat{M}_{32})^{(6)}\right)_2$ *and* $\left((\widehat{M}_{32})^{(6)}\right)_3$ : 251

**Remark 28:** if $G_{32}$ is bounded, the same property have also $G_{33}$ *and* $G_{34}$ . indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a'_{33})^{(6)} G_{33}$ and by integrating

$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)} \left((\widehat{M}_{32})^{(6)}\right)_1 / (a'_{33})^{(6)}$

In the same way , one can obtain

$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_2/(a'_{34})^{(6)}$

If $G_{33}$ *or* $G_{34}$ is bounded, the same property follows for $G_{32}$ , $G_{34}$ and $G_{32}$ , $G_{33}$ respectively.

**Remark 29:** If $G_{32}$ *is* bounded, from below, the same property holds for $G_{33}$ *and* $G_{34}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{33}$ is bounded from below. 252

**Remark 30:** If $T_{32}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}((G_{35})(t),t)) = (b'_{33})^{(6)}$ then $T_{33} \to \infty$. 253

**Definition of** $(m)^{(6)}$ and $\varepsilon_6$ :

Indeed let $t_6$ be so that for $t > t_6$

$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t),t\right) < \varepsilon_6, T_{32}(t) > (m)^{(6)}$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right), \quad t = log\frac{2}{\varepsilon_6}$ By taking now $\varepsilon_6$ sufficiently small one sees that $T_{33}$ is unbounded.
The same property holds for $T_{34}$ if $\lim_{t\to\infty}(b_{34}'')^{(6)}\left((G_{35})(t),t(t),t\right) = (b'_{34})^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{37}$ , $G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\tilde{M}_{36})^{(7)}}$ , $\frac{(b_i)^{(7)}}{(\tilde{M}_{36})^{(7)}} < 1$ and to choose
$(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\tilde{M}_{36})^{(7)}}\left[(\hat{P}_{36})^{(7)} + \left((\hat{P}_{36})^{(7)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{36})^{(7)} \quad 256$$

257

$$\frac{(b_i)^{(7)}}{(\tilde{M}_{36})^{(7)}}\left[\left((\hat{Q}_{36})^{(7)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{36})^{(7)}\right] \leq (\hat{Q}_{36})^{(7)}$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself
The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric 258

$$d\left(((G_{39})^{(1)},(T_{39})^{(1)}),((G_{39})^{(2)},(T_{39})^{(2)})\right) =$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\tilde{M}_{36})^{(7)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\tilde{M}_{36})^{(7)}t}\}$$

Indeed if we denote

**Definition of** $\widetilde{(G_{39})}, \widetilde{(T_{39})} : \left( \widetilde{(G_{39})}, \widetilde{(T_{39})} \right) = \mathcal{A}^{(7)}((G_{39}), (T_{39}))$
It results

$$\left|\tilde{G}_{36}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{36})^{(7)} \left|G_{37}^{(1)} - G_{37}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{(\widehat{M}_{36})^{(7)} s_{(36)}}\, ds_{(36)} +$$
$$\int_0^t \{(a'_{36})^{(7)} \left|G_{36}^{(1)} - G_{36}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} +$$
$$(a''_{36})^{(7)}\left(T_{37}^{(1)}, s_{(36)}\right) \left|G_{36}^{(1)} - G_{36}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{(\widehat{M}_{36})^{(7)} s_{(36)}} +$$
$$G_{36}^{(2)} \left|(a''_{36})^{(7)}\left(T_{37}^{(1)}, s_{(36)}\right) - (a''_{36})^{(7)}\left(T_{37}^{(2)}, s_{(36)}\right)\right| \; e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{(\widehat{M}_{36})^{(7)} s_{(36)}} \} ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$$\left|(G_{39})^{(1)} - (G_{39})^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} t} \leq \qquad 259$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}} \left((a_{36})^{(7)} + (a'_{36})^{(7)} + (\widehat{A}_{36})^{(7)} + (\widehat{P}_{36})^{(7)} (\widehat{k}_{36})^{(7)}\right) d\left(((G_{39})^{(1)}, (T_{39})^{(1)};\ (G_{39})^{(2)}, (T_{39})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 31:** The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)} t}$ *and* $(\widehat{Q}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)} t}$ respectively of $\mathbb{R}_+$. 260

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(7)}$ and $(b_i'')^{(7)}, i = 36,37,38$ depend only on $T_{37}$ and respectively on $(G_{39})$(*and not on* $t$) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 32:** There does not exist any $t$ where $G_i\,(t) = 0$ *and* $T_i\,(t) = 0$ 261

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(7)} - (a_i'')^{(7)}(T_{37}(s_{(36)}), s_{(36)})\} ds_{(36)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(7)} t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2$ *and* $\left((\widehat{M}_{36})^{(7)}\right)_3$ : 262

**Remark 33:** if $G_{36}$ is bounded, the same property have also $G_{37}$ *and* $G_{38}$ . indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a'_{37})^{(7)} G_{37}$ and by integrating

$$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)} \left((\widehat{M}_{36})^{(7)}\right)_1 / (a'_{37})^{(7)}$$

In the same way , one can obtain

$$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2/(a'_{38})^{(7)}$$

If $G_{37}$ *or* $G_{38}$ is bounded, the same property follows for $G_{36}$ , $G_{38}$ and $G_{36}$ , $G_{37}$ respectively.

**Remark 34:** If $G_{36}$ *is* bounded, from below, the same property holds for $G_{37}$ *and* $G_{38}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{37}$ is bounded from below. 263

**Remark 35:** If $T_{36}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(7)}((G_{39})(t),t)) = (b'_{37})^{(7)}$ then $T_{37} \to \infty$. 264

**Definition of** $(m)^{(7)}$ and $\varepsilon_7$ :

Indeed let $t_7$ be so that for $t > t_7$

$$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t),t) < \varepsilon_7, T_{36}(t) > (m)^{(7)}$$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right)$, $t = log\frac{2}{\varepsilon_7}$ By taking now $\varepsilon_7$ sufficiently small one sees that $T_{37}$ is unbounded.
The same property holds for $T_{38}$ if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t),t\right) = (b'_{38})^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}$ , $\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1$ and to choose 266
$(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ large to have

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[(\hat{P}_{40})^{(8)} + \left((\hat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{40})^{(8)} \quad 267$$

$$\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{40})^{(8)}\right] \leq (\hat{Q}_{40})^{(8)} \quad 268$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)},(T_{43})^{(1)}),((G_{43})^{(2)},(T_{43})^{(2)})\right) = \quad 269$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\tilde{M}_{40})^{(8)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\tilde{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

**Definition of** $(\widetilde{G_{43}}),(\widetilde{T_{43}})$ : $\left((\widetilde{G_{43}}),(\widetilde{T_{43}})\right)=\mathcal{A}^{(8)}((G_{43}),(T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{40})^{(8)}\left|G_{41}^{(1)}-G_{41}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\,ds_{(40)}+$$
$$\int_0^t\{(a'_{40})^{(8)}\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$G_{40}^{(2)}\left|(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)-(a''_{40})^{(8)}\left(T_{41}^{(2)},s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\}ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval [0, t] 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)}-(G_{43})^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}t}\le \qquad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)}+(a'_{40})^{(8)}+(\widehat{A}_{40})^{(8)}+(\widehat{P}_{40})^{(8)}(\widehat{k}_{40})^{(8)}\right)d\left(((G_{43})^{(1)},(T_{43})^{(1)};\ (G_{43})^{(2)},(T_{43})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 36:** The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ *and* $(\widehat{Q}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ respectively of $\mathbb{R}_+$. 274

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i=40,41,42$ depend only on $T_{41}$ and respectively on $(G_{43})$(*and not on* $t$) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 37** There does not exist any $t$ where $G_i(t)=0$ *and* $T_i(t)=0$ 275

it results

$$G_i(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(8)}-(a_i'')^{(8)}(T_{41}(s_{(40)}),s_{(40)})\}ds_{(40)}\right]}\ge 0$$

$$T_i(t)\ge T_i^0e^{\left(-(b_i')^{(8)}t\right)}>0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{40})^{(8)}\right)_1,\left((\widehat{M}_{40})^{(8)}\right)_2$ *and* $\left((\widehat{M}_{40})^{(8)}\right)_3$ : 276

**Remark 38:** if $G_{40}$ is bounded, the same property have also $G_{41}$ *and* $G_{42}$ . indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a'_{41})^{(8)} G_{41}$ and by integrating

$$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1/(a'_{41})^{(8)}$$

In the same way , one can obtain

$$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2/(a'_{42})^{(8)}$$

If $G_{41}$ *or* $G_{42}$ is bounded, the same property follows for $G_{40}$ , $G_{42}$ and $G_{40}$ , $G_{41}$ respectively.

**Remark 39:** If $G_{40}$ *is* bounded, from below, the same property holds for $G_{41}$ *and* $G_{42}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{41}$ is bounded from below. 277

**Remark 40:** If $T_{40}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}((G_{43})(t),t)) = (b'_{41})^{(8)}$ then $T_{41} \to \infty$. 278

**Definition of** $(m)^{(8)}$ and $\varepsilon_8$ :

Indeed let $t_8$ be so that for $t > t_8$

$$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t),t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right)$, $t = log\frac{2}{\varepsilon_8}$ By taking now $\varepsilon_8$ sufficiently small one sees that $T_{41}$ is unbounded.
The same property holds for $T_{42}$ if $\lim_{t\to\infty}(b''_{42})^{(8)}\left((G_{43})(t),t(t),t\right) = (b'_{42})^{(8)}$

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It is now sufficient to take $\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}$ , $\frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$ and to choose $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ large to have

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}\left[(\hat{P}_{44})^{(9)} + \left((\hat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}}\left[\left((\hat{Q}_{44})^{(9)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{44})^{(9)}\right] \leq (\hat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)},(T_{47})^{(1)}),((G_{47})^{(2)},(T_{47})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\widehat{M}_{44})^{(9)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\widehat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{47}}),(\widetilde{T_{47}}):\left((\widetilde{G_{47}}),(\widetilde{T_{47}})\right)=\mathcal{A}^{(9)}\left((G_{47}),(T_{47})\right)$
It results

$$|\tilde{G}_{44}^{(1)}-\tilde{G}_i^{(2)}|\le\int_0^t(a_{44})^{(9)}|G_{45}^{(1)}-G_{45}^{(2)}|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\,ds_{(44)}+$$
$$\int_0^t\{(a'_{44})^{(9)}|G_{44}^{(1)}-G_{44}^{(2)}|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)|G_{44}^{(1)}-G_{44}^{(2)}|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$G_{44}^{(2)}|(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)-(a''_{44})^{(9)}\left(T_{45}^{(2)},s_{(44)}\right)|\;e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\}ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on 45,46,47,28 and 29 it follows

$$|(G_{47})^{(1)}-G^{(2)}|e^{-(\widehat{M}_{44})^{(9)}t}\le$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)}+(a'_{44})^{(9)}+(\widehat{A}_{44})^{(9)}+(\widehat{P}_{44})^{(9)}(\widehat{k}_{44})^{(9)}\right)d\left(((G_{47})^{(1)},(T_{47})^{(1)};\;(G_{47})^{(2)},(T_{47})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis (39,35,36) the result follows

**Remark 41:** The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ *and* $(\widehat{Q}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i=44,45,46$ depend only on $T_{45}$ and respectively on $(G_{47})$(*and not on* $t$) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 42:** There does not exist any $t$ where $G_i(t)=0$ *and* $T_i(t)=0$

From 99 to 44 it results

$$G_i(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(9)}-(a_i'')^{(9)}(T_{45}(s_{(44)}),s_{(44)})\}ds_{(44)}\right]}\ge 0$$

$$T_i(t)\ge T_i^0e^{(-(b_i')^{(9)}t)}>0\quad\text{for t}>0$$

**Definition of** $\left((\widehat{M}_{44})^{(9)}\right)_1,\left((\widehat{M}_{44})^{(9)}\right)_2$ *and* $\left((\widehat{M}_{44})^{(9)}\right)_3$ :

**Remark 43:** if $G_{44}$ is bounded, the same property have also $G_{45}$ *and* $G_{46}$ . indeed if

$G_{44}<(\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt}\le\left((\widehat{M}_{44})^{(9)}\right)_1-(a'_{45})^{(9)}G_{45}$ and by integrating

$$G_{45}\le\left((\widehat{M}_{44})^{(9)}\right)_2=G_{45}^0+2(a_{45})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_1/(a'_{45})^{(9)}$$

In the same way , one can obtain

$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_2/(a'_{46})^{(9)}$

If $G_{45}$ *or* $G_{46}$ is bounded, the same property follows for $G_{44}$ , $G_{46}$ and $G_{44}$ , $G_{45}$ respectively.

**Remark 44:** If $G_{44}$ *is* bounded, from below, the same property holds for $G_{45}$ *and* $G_{46}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{45}$ is bounded from below.

**Remark 45:** If $T_{44}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t),t\right) = (b'_{45})^{(9)}$ then $T_{45} \to \infty$.

**Definition of** $(m)^{(9)}$ and $\varepsilon_9$ :

Indeed let $t_9$ be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t),t\right) < \varepsilon_9, T_{44}(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right)$, $t = log\frac{2}{\varepsilon_9}$ By taking now $\varepsilon_9$ sufficiently small one sees that $T_{45}$ is unbounded.
The same property holds for $T_{46}$ if $\lim_{t\to\infty}(b''_{46})^{(9)}\left((G_{47})(t),t\right) = (b'_{46})^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

**Behavior of the solutions of equation** 280

**Theorem** If we denote and define

**Definition of** $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ :

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)} \leq -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14},t) + (a''_{14})^{(1)}(T_{14},t) \leq -(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)} \leq -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G,t) - (b''_{14})^{(1)}(G,t) \leq -(\tau_1)^{(1)}$

**Definition of** $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$ : 281

By $(\nu_1)^{(1)} > 0$ , $(\nu_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$ , $(u_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

**Definition of** $(\bar{\nu}_1)^{(1)},, (\bar{\nu}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$ : 282

By $(\bar{\nu}_1)^{(1)} > 0$ , $(\bar{\nu}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$ , $(\bar{u}_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

**Definition of** $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (\nu_0)^{(1)}$ :- 283

If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by

$(m_2)^{(1)} = (\nu_0)^{(1)}, (m_1)^{(1)} = (\nu_1)^{(1)},\ if\ (\nu_0)^{(1)} < (\nu_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}, if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)},$

and $\boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)},\ if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)},\ if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)},$

and $\boxed{(u_0)^{(1)} = \frac{T_{13}^0}{T_{14}^0}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$ are defined

Then the solution of global equations satisfies the inequalities 285

$$G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{13}(t) \le G_{13}^0 e^{(S_1)^{(1)} t}$$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)} t}$$

$$\left(\frac{(a_{15})^{(1)} G_{13}^0}{(m_1)^{(1)}((S_1)^{(1)} - (p_{13})^{(1)} - (S_2)^{(1)})}\left[e^{((S_1)^{(1)} - (p_{13})^{(1)})t} - e^{-(S_2)^{(1)} t}\right] + G_{15}^0 e^{-(S_2)^{(1)} t} \le G_{15}(t) \le \right. \quad 286$$

$$\left. \frac{(a_{15})^{(1)} G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)} - (a'_{15})^{(1)})}\left[e^{(S_1)^{(1)} t} - e^{-(a'_{15})^{(1)} t}\right] + G_{15}^0 e^{-(a'_{15})^{(1)} t}\right)$$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t}} \quad 287$$

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t} \quad 288$$

$$\frac{(b_{15})^{(1)} T_{13}^0}{(\mu_1)^{(1)}((R_1)^{(1)} - (b'_{15})^{(1)})}\left[e^{(R_1)^{(1)} t} - e^{-(b'_{15})^{(1)} t}\right] + T_{15}^0 e^{-(b'_{15})^{(1)} t} \le T_{15}(t) \le \quad 289$$

$$\frac{(a_{15})^{(1)} T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)} + (r_{13})^{(1)} + (R_2)^{(1)})}\left[e^{((R_1)^{(1)} + (r_{13})^{(1)})t} - e^{-(R_2)^{(1)} t}\right] + T_{15}^0 e^{-(R_2)^{(1)} t}$$

**Definition of** $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:- 290

Where $(S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$

$$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$$

$$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$$

$$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$$

**Behavior of the solutions of equation** 291

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ : 292

$(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ four constants satisfying
$-(\sigma_2)^{(2)} \le -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(T_{17}, t) + (a''_{17})^{(2)}(T_{17}, t) \le -(\sigma_1)^{(2)}$ 293

$-(\tau_2)^{(2)} \le -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}\big((G_{19}), t\big) - (b''_{17})^{(2)}\big((G_{19}), t\big) \le -(\tau_1)^{(2)}$ 294

**Definition of** $(\nu_1)^{(2)}, (\nu_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$ : 295

By $(\nu_1)^{(2)} > 0$ , $(\nu_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0$ , $(u_2)^{(2)} < 0$ the roots 296

of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 297

and $(b_{14})^{(2)}\big(u^{(2)}\big)^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and 298

**Definition of** $(\bar{\nu}_1)^{(2)}, , (\bar{\nu}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$ : 299

By $(\bar{\nu}_1)^{(2)} > 0$ , $(\bar{\nu}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0$ , $(\bar{u}_2)^{(2)} < 0$ the 300

roots of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 301

and $(b_{17})^{(2)}\big(u^{(2)}\big)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 302

**Definition of** $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ :- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ , ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{G^0_{16}}{G^0_{17}}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, ***if*** $(u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$ , ***if*** $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)},\ \boldsymbol{if}\ (\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)} t}$$

$(p_i)^{(2)}$ is defined by equation

$$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)} t} \quad 311$$

$$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)} t}\right] + G_{18}^0 e^{-(S_2)^{(2)} t} \le G_{18}(t) \le \right. \quad 312$$

$$\left. \frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)} t} - e^{-(a'_{18})^{(2)} t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)} t}\right)$$

$$\boxed{T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}} \quad 313$$

$$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t} \quad 314$$

$$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)} t} - e^{-(b'_{18})^{(2)} t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)} t} \le T_{18}(t) \le \quad 315$$

$$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})}\left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)} t}\right] + T_{18}^0 e^{-(R_2)^{(2)} t}$$

**Definition of** $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$$

$$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)} \quad 318$$

$$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$$

**Behavior of the solutions** 319

**Theorem 3:** If we denote and define

**Definition of** $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ :

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}\big((G_{23}), t\big) \le -(\tau_1)^{(3)}$$

**Definition of** $(v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$ : 320

By $(v_1)^{(3)} > 0$ , $(v_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$ , $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{v}_1)^{(3)} > 0$ , $(\bar{v}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$ , $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

**Definition of** $(m_1)^{(3)}$ , $(m_2)^{(3)}$ , $(\mu_1)^{(3)}, (\mu_2)^{(3)}$ :- 321

If we define $(m_1)^{(3)}$ , $(m_2)^{(3)}$ , $(\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (v_0)^{(3)}, (m_1)^{(3)} = (v_1)^{(3)}$, ***if*** $(v_0)^{(3)} < (v_1)^{(3)}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (\bar{v}_1)^{(3)}$ , ***if*** $(v_1)^{(3)} < (v_0)^{(3)} < (\bar{v}_1)^{(3)}$,

and $\boxed{(v_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_0)^{(3)}$, ***if*** $(\bar{v}_1)^{(3)} < (v_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}$, ***if*** $(u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$ , ***if*** $(u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}$, and $\boxed{(u_0)^{(3)} = \frac{T_{20}^0}{T_{21}^0}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}$, ***if*** $(\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$G_{20}^0 e^{\left((S_1)^{(3)} - (p_{20})^{(3)}\right)t} \le G_{20}(t) \le G_{20}^0 e^{(S_1)^{(3)}t}$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{\left((S_1)^{(3)} - (p_{20})^{(3)}\right)t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)}t} \quad (323)$$

$$\left(\frac{(a_{22})^{(3)} G_{20}^0}{(m_1)^{(3)}\left((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)}\right)}\left[e^{\left((S_1)^{(3)} - (p_{20})^{(3)}\right)t} - e^{-(S_2)^{(3)}t}\right] + G_{22}^0 e^{-(S_2)^{(3)}t} \le G_{22}(t) \le \right. \quad (324)$$

$$\left. \frac{(a_{22})^{(3)} G_{20}^0}{(m_2)^{(3)}\left((S_1)^{(3)} - (a'_{22})^{(3)}\right)}\left[e^{(S_1)^{(3)}t} - e^{-(a'_{22})^{(3)}t}\right] + G_{22}^0 e^{-(a'_{22})^{(3)}t}\right)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le T_{20}^0 e^{\left((R_1)^{(3)} + (r_{20})^{(3)}\right)t}} \quad (325)$$

$$\frac{1}{(\mu_1)^{(3)}}T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}}T_{20}^0 e^{((R_1)^{(3)}+(r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)}T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)}-(b'_{22})^{(3)})}\left[e^{(R_1)^{(3)}t} - e^{-(b'_{22})^{(3)}t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)}t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)}T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)}+(r_{20})^{(3)}+(R_2)^{(3)})}\left[e^{((R_1)^{(3)}+(r_{20})^{(3)})t} - e^{-(R_2)^{(3)}t}\right] + T_{22}^0 e^{-(R_2)^{(3)}t}$$

**Definition of** $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

**Behavior of the solutions of equation**

**Theorem:** If we denote and define

**Definition of** $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ :

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$$

$$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}\big((G_{27}), t\big) - (b''_{25})^{(4)}\big((G_{27}), t\big) \le -(\tau_1)^{(4)}$$

**Definition of** $(\nu_1)^{(4)}, (\nu_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, \nu^{(4)}, u^{(4)}$ : 329

By $(\nu_1)^{(4)} > 0$, $(\nu_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$, $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_1)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(4)},, (\bar{\nu}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$ : 330

By $(\bar{\nu}_1)^{(4)} > 0$, $(\bar{\nu}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$, $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

**Definition of** $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}, (\nu_0)^{(4)}$ :-

If we define $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (\nu_0)^{(4)}, (m_1)^{(4)} = (\nu_1)^{(4)}$, ***if*** $(\nu_0)^{(4)} < (\nu_1)^{(4)}$

$(m_2)^{(4)} = (\nu_1)^{(4)}, (m_1)^{(4)} = (\bar{\nu}_1)^{(4)}$, ***if*** $(\nu_4)^{(4)} < (\nu_0)^{(4)} < (\bar{\nu}_1)^{(4)}$,
and $\boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (v_4)^{(4)}, (m_1)^{(4)} = (v_0)^{(4)},\ \boldsymbol{if}\ (\bar{v}_4)^{(4)} < (v_0)^{(4)}$

and analogously

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$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)},\ \boldsymbol{if}\ (u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}, \boldsymbol{if}\ (u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)},$

and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}, \boldsymbol{if}\ (\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

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$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)} t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)} t} \quad 333$$

$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)} t}\right] + G_{26}^0 e^{-(S_2)^{(4)} t} \le G_{26}(t) \le \right. \quad 334$$

*(a26)4G240(m2)4(S1)4−(a26′)4e(S1)4t−e−(a26′)4t+ G260e−(a26′)4t*

$$\boxed{T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}} T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}} T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t} \quad 335$$

$$\frac{(b_{26})^{(4)} T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)} - (b'_{26})^{(4)})}\left[e^{(R_1)^{(4)} t} - e^{-(b'_{26})^{(4)} t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)} t} \le T_{26}(t) \le \quad 336$$

$$\frac{(a_{26})^{(4)} T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)} + (r_{24})^{(4)} + (R_2)^{(4)})}\left[e^{((R_1)^{(4)} + (r_{24})^{(4)})t} - e^{-(R_2)^{(4)} t}\right] + T_{26}^0 e^{-(R_2)^{(4)} t}$$

**Definition of** $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

**Behavior of the solutions of equation** 338

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ :

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

**Definition of** $(\nu_1)^{(5)}, (\nu_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, \nu^{(5)}, u^{(5)}$ : 339

By $(\nu_1)^{(5)} > 0$, $(\nu_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$, $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(5)},, (\bar{\nu}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$ : 340

By $(\bar{\nu}_1)^{(5)} > 0$, $(\bar{\nu}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$, $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

**Definition of** $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (\nu_0)^{(5)}$ :-

If we define $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (\nu_0)^{(5)}, (m_1)^{(5)} = (\nu_1)^{(5)}$, ***if*** $(\nu_0)^{(5)} < (\nu_1)^{(5)}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\bar{\nu}_1)^{(5)}$, ***if*** $(\nu_1)^{(5)} < (\nu_0)^{(5)} < (\bar{\nu}_1)^{(5)}$,
and $\boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)}$, ***if*** $(\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)}$, ***if*** $(u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}$, ***if*** $(u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)}$,
and $\boxed{(u_0)^{(5)} = \frac{T^0_{28}}{T^0_{29}}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}$, ***if*** $(\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$$G^0_{28} e^{\left((S_1)^{(5)} - (p_{28})^{(5)}\right)t} \le G_{28}(t) \le G^0_{28} e^{(S_1)^{(5)} t}$$

where $(p_i)^{(5)}$ is defined by equation

$$\frac{1}{(m_5)^{(5)}} G^0_{28} e^{\left((S_1)^{(5)} - (p_{28})^{(5)}\right)t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}} G^0_{28} e^{(S_1)^{(5)} t} \quad 343$$

$$\left(\frac{(a_{30})^{(5)} G^0_{28}}{(m_1)^{(5)}\left((S_1)^{(5)} - (p_{28})^{(5)} - (S_2)^{(5)}\right)}\left[e^{\left((S_1)^{(5)} - (p_{28})^{(5)}\right)t} - e^{-(S_2)^{(5)} t}\right] + G^0_{30} e^{-(S_2)^{(5)} t} \le G_{30}(t) \le \right. \quad 344$$

$$\frac{(a_{30})^{(5)}G_{28}^{0}}{(m_2)^{(5)}((S_1)^{(5)}-(a'_{30})^{(5)})}\left[e^{(S_1)^{(5)}t}-e^{-(a'_{30})^{(5)}t}\right]+\ G_{30}^{0}e^{-(a'_{30})^{(5)}t}\Big)$$

$$\boxed{T_{28}^{0}e^{(R_1)^{(5)}t}\le T_{28}(t)\le T_{28}^{0}e^{((R_1)^{(5)}+(r_{28})^{(5)})t}} \quad 345$$

$$\frac{1}{(\mu_1)^{(5)}}T_{28}^{0}e^{(R_1)^{(5)}t}\le T_{28}(t)\le\frac{1}{(\mu_2)^{(5)}}T_{28}^{0}e^{((R_1)^{(5)}+(r_{28})^{(5)})t} \quad 346$$

$$\frac{(b_{30})^{(5)}T_{28}^{0}}{(\mu_1)^{(5)}((R_1)^{(5)}-(b'_{30})^{(5)})}\left[e^{(R_1)^{(5)}t}-e^{-(b'_{30})^{(5)}t}\right]+T_{30}^{0}e^{-(b'_{30})^{(5)}t}\le T_{30}(t)\le \quad 347$$

$$\frac{(a_{30})^{(5)}T_{28}^{0}}{(\mu_2)^{(5)}((R_1)^{(5)}+(r_{28})^{(5)}+(R_2)^{(5)})}\left[e^{((R_1)^{(5)}+(r_{28})^{(5)})t}-e^{-(R_2)^{(5)}t}\right]+T_{30}^{0}e^{-(R_2)^{(5)}t}$$

**Definition of** $(S_1)^{(5)},(S_2)^{(5)},(R_1)^{(5)},(R_2)^{(5)}$:- 348

Where $(S_1)^{(5)}=(a_{28})^{(5)}(m_2)^{(5)}-(a'_{28})^{(5)}$

$(S_2)^{(5)}=(a_{30})^{(5)}-(p_{30})^{(5)}$

$(R_1)^{(5)}=(b_{28})^{(5)}(\mu_2)^{(5)}-(b'_{28})^{(5)}$

$(R_2)^{(5)}=(b'_{30})^{(5)}-(r_{30})^{(5)}$

**Behavior of the solutions of equation** 349

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(6)},(\sigma_2)^{(6)},(\tau_1)^{(6)},(\tau_2)^{(6)}$ :

$(\sigma_1)^{(6)},(\sigma_2)^{(6)},(\tau_1)^{(6)},(\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)}\le-(a'_{32})^{(6)}+(a'_{33})^{(6)}-(a''_{32})^{(6)}(T_{33},t)+(a''_{33})^{(6)}(T_{33},t)\le-(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)}\le-(b'_{32})^{(6)}+(b'_{33})^{(6)}-(b''_{32})^{(6)}\big((G_{35}),t\big)-(b''_{33})^{(6)}\big((G_{35}),t\big)\le-(\tau_1)^{(6)}$$

**Definition of** $(\nu_1)^{(6)},(\nu_2)^{(6)},(u_1)^{(6)},(u_2)^{(6)},\nu^{(6)},u^{(6)}$ : 350

By $(\nu_1)^{(6)}>0$ , $(\nu_2)^{(6)}<0$ and respectively $(u_1)^{(6)}>0$ , $(u_2)^{(6)}<0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2+(\sigma_1)^{(6)}\nu^{(6)}-(a_{32})^{(6)}=0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2+(\tau_1)^{(6)}u^{(6)}-(b_{32})^{(6)}=0$ and

**Definition of** $(\bar{\nu}_1)^{(6)},,(\bar{\nu}_2)^{(6)},(\bar{u}_1)^{(6)},(\bar{u}_2)^{(6)}$ : 351

By $(\bar{\nu}_1)^{(6)}>0$ , $(\bar{\nu}_2)^{(6)}<0$ and respectively $(\bar{u}_1)^{(6)}>0$ , $(\bar{u}_2)^{(6)}<0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2+(\sigma_2)^{(6)}\nu^{(6)}-(a_{32})^{(6)}=0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2+(\tau_2)^{(6)}u^{(6)}-(b_{32})^{(6)}=0$

**Definition of** $(m_1)^{(6)},(m_2)^{(6)},(\mu_1)^{(6)},(\mu_2)^{(6)},(\nu_0)^{(6)}$ :-

If we define $(m_1)^{(6)},(m_2)^{(6)},(\mu_1)^{(6)},(\mu_2)^{(6)}$ by

$(m_2)^{(6)} = (\nu_0)^{(6)}, (m_1)^{(6)} = (\nu_1)^{(6)},\ \boldsymbol{if}\ (\nu_0)^{(6)} < (\nu_1)^{(6)}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}, \boldsymbol{if}\ (\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)},$
and $\boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$

and analogously 352

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)},\ \boldsymbol{if}\ (u_0)^{(6)} < (u_1)^{(6)}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}, \boldsymbol{if}\ (u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)},$
and $\boxed{(u_0)^{(6)} = \frac{T_{32}^0}{T_{33}^0}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}, \boldsymbol{if}\ (\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$

Then the solution of global equations satisfies the inequalities 353

$$G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{32}(t) \le G_{32}^0 e^{(S_1)^{(6)} t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}} G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{33}(t) \le \frac{1}{(m_2)^{(6)}} G_{32}^0 e^{(S_1)^{(6)} t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)} G_{32}^0}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})}\left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)} t}\right] + G_{34}^0 e^{-(S_2)^{(6)} t} \le G_{34}(t) \le \right. \quad 355$$

*(a34)6G320(m2)6(S1)6−(a34′)6e(S1)6t−e−(a34′)6t+ G340e−(a34′)6t*

$$\boxed{T_{32}^0 e^{(R_1)^{(6)} t} \le T_{32}(t) \le T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}} T_{32}^0 e^{(R_1)^{(6)} t} \le T_{32}(t) \le \frac{1}{(\mu_2)^{(6)}} T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)} T_{32}^0}{(\mu_1)^{(6)}((R_1)^{(6)} - (b'_{34})^{(6)})}\left[e^{(R_1)^{(6)} t} - e^{-(b'_{34})^{(6)} t}\right] + T_{34}^0 e^{-(b'_{34})^{(6)} t} \le T_{34}(t) \le \quad 358$$

$$\frac{(a_{34})^{(6)} T_{32}^0}{(\mu_2)^{(6)}((R_1)^{(6)} + (r_{32})^{(6)} + (R_2)^{(6)})}\left[e^{((R_1)^{(6)} + (r_{32})^{(6)})t} - e^{-(R_2)^{(6)} t}\right] + T_{34}^0 e^{-(R_2)^{(6)} t}$$

**Definition of** $(S_1)^{(6)}, (S_2)^{(6)}, (R_1)^{(6)}, (R_2)^{(6)}$:- 359

Where $(S_1)^{(6)} = (a_{32})^{(6)}(m_2)^{(6)} - (a'_{32})^{(6)}$

$(S_2)^{(6)} = (a_{34})^{(6)} - (p_{34})^{(6)}$

$(R_1)^{(6)} = (b_{32})^{(6)}(\mu_2)^{(6)} - (b'_{32})^{(6)}$

$(R_2)^{(6)} = (b'_{34})^{(6)} - (r_{34})^{(6)}$

**Behavior of the solutions of equation**

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$ :

$(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)} \le -(a'_{36})^{(7)} + (a'_{37})^{(7)} - (a''_{36})^{(7)}(T_{37}, t) + (a''_{37})^{(7)}(T_{37}, t) \le -(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)} \le -(b'_{36})^{(7)} + (b'_{37})^{(7)} - (b''_{36})^{(7)}\big((G_{39}), t\big) - (b''_{37})^{(7)}\big((G_{39}), t\big) \le -(\tau_1)^{(7)}$$

**Definition of** $(v_1)^{(7)}, (v_2)^{(7)}, (u_1)^{(7)}, (u_2)^{(7)}, v^{(7)}, u^{(7)}$ : 361

By $(v_1)^{(7)} > 0, (v_2)^{(7)} < 0$ and respectively $(u_1)^{(7)} > 0, (u_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(v^{(7)}\big)^2 + (\sigma_1)^{(7)}v^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_1)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$ and

**Definition of** $(\bar{v}_1)^{(7)},, (\bar{v}_2)^{(7)}, (\bar{u}_1)^{(7)}, (\bar{u}_2)^{(7)}$ : 362

By $(\bar{v}_1)^{(7)} > 0, (\bar{v}_2)^{(7)} < 0$ and respectively $(\bar{u}_1)^{(7)} > 0, (\bar{u}_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(v^{(7)}\big)^2 + (\sigma_2)^{(7)}v^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

**Definition of** $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (v_0)^{(7)}$ :-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$$(m_2)^{(7)} = (v_0)^{(7)}, (m_1)^{(7)} = (v_1)^{(7)}, \ \boldsymbol{if}\ (v_0)^{(7)} < (v_1)^{(7)}$$

$$(m_2)^{(7)} = (v_1)^{(7)}, (m_1)^{(7)} = (\bar{v}_1)^{(7)}, \boldsymbol{if}\ (v_1)^{(7)} < (v_0)^{(7)} < (\bar{v}_1)^{(7)},$$

and $\boxed{(v_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$

$$(m_2)^{(7)} = (v_1)^{(7)}, (m_1)^{(7)} = (v_0)^{(7)}, \ \boldsymbol{if}\ (\bar{v}_1)^{(7)} < (v_0)^{(7)}$$

and analogously 363

$$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}, \ \boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$$

and $\boxed{(u_0)^{(7)} = \frac{T^0_{36}}{T^0_{37}}}$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)} \text{ where } (u_1)^{(7)}, (\bar{u}_1)^{(7)}$$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^{0}e^{((S_1)^{(7)}-(p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^{0}e^{(S_1)^{(7)}t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}}G_{36}^{0}e^{((S_1)^{(7)}-(p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}}G_{36}^{0}e^{(S_1)^{(7)}t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)}G_{36}^{0}}{(m_1)^{(7)}((S_1)^{(7)}-(p_{36})^{(7)}-(S_2)^{(7)})}\left[e^{((S_1)^{(7)}-(p_{36})^{(7)})t}-e^{-(S_2)^{(7)}t}\right]+G_{38}^{0}e^{-(S_2)^{(7)}t} \le G_{38}(t) \le \right. \quad 366$$

$$\left.\frac{(a_{38})^{(7)}G_{36}^{0}}{(m_2)^{(7)}((S_1)^{(7)}-(a'_{38})^{(7)})}\left[e^{(S_1)^{(7)}t}-e^{-(a'_{38})^{(7)}t}\right]+\ G_{38}^{0}e^{-(a'_{38})^{(7)}t}\right)$$

$$\boxed{T_{36}^{0}e^{(R_1)^{(7)}t} \le T_{36}(t) \le T_{36}^{0}e^{((R_1)^{(7)}+(r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}}T_{36}^{0}e^{(R_1)^{(7)}t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}}T_{36}^{0}e^{((R_1)^{(7)}+(r_{36})^{(7)})t} \quad 368$$

$$\frac{(b_{38})^{(7)}T_{36}^{0}}{(\mu_1)^{(7)}((R_1)^{(7)}-(b'_{38})^{(7)})}\left[e^{(R_1)^{(7)}t}-e^{-(b'_{38})^{(7)}t}\right]+T_{38}^{0}e^{-(b'_{38})^{(7)}t} \le T_{38}(t) \le \quad 369$$

$$\frac{(a_{38})^{(7)}T_{36}^{0}}{(\mu_2)^{(7)}((R_1)^{(7)}+(r_{36})^{(7)}+(R_2)^{(7)})}\left[e^{((R_1)^{(7)}+(r_{36})^{(7)})t}-e^{-(R_2)^{(7)}t}\right]+T_{38}^{0}e^{-(R_2)^{(7)}t}$$

**Definition of** $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$$

$$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$$

$$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$$

**Behavior of the solutions of equation** 371

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ :

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$$-(\sigma_2)^{(8)} \le -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \le -(\sigma_1)^{(8)}$$

$$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\big((G_{43}), t\big) - (b''_{41})^{(8)}\big((G_{43}), t\big) \le -(\tau_1)^{(8)}$$

**Definition of** $(v_1)^{(8)}, (v_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, v^{(8)}, u^{(8)}$ : 372

By $(v_1)^{(8)} > 0$ , $(v_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0$ , $(u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_1)^{(8)}v^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\left(u^{(8)}\right)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

**Definition of** $(\bar{v}_1)^{(8)}, , (\bar{v}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$ :

By $(\bar{v}_1)^{(8)} > 0$ , $(\bar{v}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0$ , $(\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_2)^{(8)}v^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\left(u^{(8)}\right)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

**Definition of** $(m_1)^{(8)}$ , $(m_2)^{(8)}$ , $(\mu_1)^{(8)}, (\mu_2)^{(8)}, (v_0)^{(8)}$ :-

If we define $(m_1)^{(8)}$ , $(m_2)^{(8)}$ , $(\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$(m_2)^{(8)} = (v_0)^{(8)}, (m_1)^{(8)} = (v_1)^{(8)}$, ***if*** $(v_0)^{(8)} < (v_1)^{(8)}$

$(m_2)^{(8)} = (v_1)^{(8)}, (m_1)^{(8)} = (\bar{v}_1)^{(8)}$ , ***if*** $(v_1)^{(8)} < (v_0)^{(8)} < (\bar{v}_1)^{(8)}$,
and $\boxed{(v_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$

$(m_2)^{(8)} = (v_1)^{(8)}, (m_1)^{(8)} = (v_0)^{(8)}$, ***if*** $(\bar{v}_1)^{(8)} < (v_0)^{(8)}$

and analogously 374

$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)}$, ***if*** $(u_0)^{(8)} < (u_1)^{(8)}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}$ , ***if*** $(u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)}$,
and $\boxed{(u_0)^{(8)} = \frac{T_{40}^0}{T_{41}^0}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}$, ***if*** $(\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \leq G_{40}(t) \leq G_{40}^0 e^{(S_1)^{(8)}t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}} G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \leq G_{41}(t) \leq \frac{1}{(m_2)^{(8)}} G_{40}^0 e^{(S_1)^{(8)}t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)}G_{40}^{0}}{(m_1)^{(8)}((S_1)^{(8)}-(p_{40})^{(8)}-(S_2)^{(8)})}\left[e^{((S_1)^{(8)}-(p_{40})^{(8)})t}-e^{-(S_2)^{(8)}t}\right]+G_{42}^{0}e^{-(S_2)^{(8)}t}\le G_{42}(t)\le\right. \quad 377$$

$$\left.\frac{(a_{42})^{(8)}G_{40}^{0}}{(m_2)^{(8)}((S_1)^{(8)}-(a'_{42})^{(8)})}\left[e^{(S_1)^{(8)}t}-e^{-(a'_{42})^{(8)}t}\right]+G_{42}^{0}e^{-(a'_{42})^{(8)}t}\right)$$

$$\boxed{T_{40}^{0}e^{(R_1)^{(8)}t}\le T_{40}(t)\le T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}}T_{40}^{0}e^{(R_1)^{(8)}t}\le T_{40}(t)\le\frac{1}{(\mu_2)^{(8)}}T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)}T_{40}^{0}}{(\mu_1)^{(8)}((R_1)^{(8)}-(b'_{42})^{(8)})}\left[e^{(R_1)^{(8)}t}-e^{-(b'_{42})^{(8)}t}\right]+T_{42}^{0}e^{-(b'_{42})^{(8)}t}\le T_{42}(t)\le \quad 380$$

$$\frac{(a_{42})^{(8)}T_{40}^{0}}{(\mu_2)^{(8)}((R_1)^{(8)}+(r_{40})^{(8)}+(R_2)^{(8)})}\left[e^{((R_1)^{(8)}+(r_{40})^{(8)})t}-e^{-(R_2)^{(8)}t}\right]+T_{42}^{0}e^{-(R_2)^{(8)}t}$$

**Definition of** $(S_1)^{(8)},(S_2)^{(8)},(R_1)^{(8)},(R_2)^{(8)}$:- 381

Where $(S_1)^{(8)}=(a_{40})^{(8)}(m_2)^{(8)}-(a'_{40})^{(8)}$

$$(S_2)^{(8)}=(a_{42})^{(8)}-(p_{42})^{(8)}$$

$$(R_1)^{(8)}=(b_{40})^{(8)}(\mu_2)^{(8)}-(b'_{40})^{(8)}$$

$$(R_2)^{(8)}=(b'_{42})^{(8)}-(r_{42})^{(8)}$$

**Behavior of the solutions of equation 37 to 92** 382

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(9)},(\sigma_2)^{(9)},(\tau_1)^{(9)},(\tau_2)^{(9)}$:

$(\sigma_1)^{(9)},(\sigma_2)^{(9)},(\tau_1)^{(9)},(\tau_2)^{(9)}$ four constants satisfying

$$-(\sigma_2)^{(9)}\le-(a'_{44})^{(9)}+(a'_{45})^{(9)}-(a''_{44})^{(9)}(T_{45},t)+(a''_{45})^{(9)}(T_{45},t)\le-(\sigma_1)^{(9)}$$

$$-(\tau_2)^{(9)}\le-(b'_{44})^{(9)}+(b'_{45})^{(9)}-(b''_{44})^{(9)}\big((G_{47}),t\big)-(b''_{45})^{(9)}\big((G_{47}),t\big)\le-(\tau_1)^{(9)}$$

**Definition of** $(\nu_1)^{(9)},(\nu_2)^{(9)},(u_1)^{(9)},(u_2)^{(9)},\nu^{(9)},u^{(9)}$:

By $(\nu_1)^{(9)}>0,(\nu_2)^{(9)}<0$ and respectively $(u_1)^{(9)}>0,(u_2)^{(9)}<0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2+(\sigma_1)^{(9)}\nu^{(9)}-(a_{44})^{(9)}=0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2+(\tau_1)^{(9)}u^{(9)}-(b_{44})^{(9)}=0$ and

**Definition of** $(\bar{\nu}_1)^{(9)},,(\bar{\nu}_2)^{(9)},(\bar{u}_1)^{(9)},(\bar{u}_2)^{(9)}$:

By $(\bar{\nu}_1)^{(9)}>0,(\bar{\nu}_2)^{(9)}<0$ and respectively $(\bar{u}_1)^{(9)}>0,(\bar{u}_2)^{(9)}<0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2+(\sigma_2)^{(9)}\nu^{(9)}-(a_{44})^{(9)}=0$

and $(b_{45})^{(9)}\left(u^{(9)}\right)^2 + (\tau_2)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$

**Definition of** $(m_1)^{(9)}, (m_2)^{(9)}, (\mu_1)^{(9)}, (\mu_2)^{(9)}, (\nu_0)^{(9)}$ :-

If we define $(m_1)^{(9)}, (m_2)^{(9)}, (\mu_1)^{(9)}, (\mu_2)^{(9)}$ by

$(m_2)^{(9)} = (\nu_0)^{(9)}, (m_1)^{(9)} = (\nu_1)^{(9)}$, ***if*** $(\nu_0)^{(9)} < (\nu_1)^{(9)}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\bar{\nu}_1)^{(9)}$, ***if*** $(\nu_1)^{(9)} < (\nu_0)^{(9)} < (\bar{\nu}_1)^{(9)}$,

and $\boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\nu_0)^{(9)}$, ***if*** $(\bar{\nu}_1)^{(9)} < (\nu_0)^{(9)}$

and analogously

$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)}$, ***if*** $(u_0)^{(9)} < (u_1)^{(9)}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}$, ***if*** $(u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)}$,

and $\boxed{(u_0)^{(9)} = \frac{T_{44}^0}{T_{45}^0}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}$, ***if*** $(\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{44}(t) \le G_{44}^0 e^{(S_1)^{(9)}t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}} G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{45}(t) \le \frac{1}{(m_2)^{(9)}} G_{44}^0 e^{(S_1)^{(9)}t}$$

$$\left(\frac{(a_{46})^{(9)} G_{44}^0}{(m_1)^{(9)}((S_1)^{(9)} - (p_{44})^{(9)} - (S_2)^{(9)})}\left[e^{((S_1)^{(9)} - (p_{44})^{(9)})t} - e^{-(S_2)^{(9)}t}\right] + G_{46}^0 e^{-(S_2)^{(9)}t} \le G_{46}(t) \le \right.$$

$$\left.\frac{(a_{46})^{(9)} G_{44}^0}{(m_2)^{(9)}((S_1)^{(9)} - (a'_{46})^{(9)})}\left[e^{(S_1)^{(9)}t} - e^{-(a'_{46})^{(9)}t}\right] + G_{46}^0 e^{-(a'_{46})^{(9)}t}\right)$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}} T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le \frac{1}{(\mu_2)^{(9)}} T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}$$

$$\frac{(b_{46})^{(9)} T_{44}^0}{(\mu_1)^{(9)}((R_1)^{(9)} - (b'_{46})^{(9)})}\left[e^{(R_1)^{(9)}t} - e^{-(b'_{46})^{(9)}t}\right] + T_{46}^0 e^{-(b'_{46})^{(9)}t} \le T_{46}(t) \le$$

$$\frac{(a_{46})^{(9)} T_{44}^0}{(\mu_2)^{(9)}((R_1)^{(9)} + (r_{44})^{(9)} + (R_2)^{(9)})}\left[e^{((R_1)^{(9)} + (r_{44})^{(9)})t} - e^{-(R_2)^{(9)}t}\right] + T_{46}^0 e^{-(R_2)^{(9)}t}$$

**Definition of** $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$

**Proof :** From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14},t)\right) - (a''_{14})^{(1)}(T_{14},t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

**Definition of** $v^{(1)}$ :- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)}\right) \le \frac{dv^{(1)}}{dt} \le -\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(1)}, (\nu_0)^{(1)}$ :-

For $0 < \boxed{(\nu_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}} < (\nu_1)^{(1)} < (\bar{\nu}_1)^{(1)}$

$$\nu^{(1)}(t) \ge \frac{(\nu_1)^{(1)} + (C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)} - (\nu_0)^{(1)}\right)t\right]}}{1 + (C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)} - (\nu_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(\nu_1)^{(1)} - (\nu_0)^{(1)}}{(\nu_0)^{(1)} - (\nu_2)^{(1)}}}$$

it follows $(\nu_0)^{(1)} \le \nu^{(1)}(t) \le (\nu_1)^{(1)}$

In the same manner , we get 384

$$\nu^{(1)}(t) \le \frac{(\bar{\nu}_1)^{(1)} + (\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)} - (\bar{\nu}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)} - (\bar{\nu}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{\nu}_1)^{(1)} - (\nu_0)^{(1)}}{(\nu_0)^{(1)} - (\bar{\nu}_2)^{(1)}}}$$

From which we deduce $(\nu_0)^{(1)} \le \nu^{(1)}(t) \le (\bar{\nu}_1)^{(1)}$

If $0 < (\nu_1)^{(1)} < (\nu_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}} < (\bar{\nu}_1)^{(1)}$ we find like in the previous case, 385

$$(\nu_1)^{(1)} \le \frac{(\nu_1)^{(1)} + (C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)} - (\nu_2)^{(1)}\right)t\right]}}{1 + (C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)} - (\nu_2)^{(1)}\right)t\right]}} \le \nu^{(1)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(1)} + (\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)} - (\bar{\nu}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)} - (\bar{\nu}_2)^{(1)}\right)t\right]}} \le (\bar{\nu}_1)^{(1)}$$

If $0 < (\nu_1)^{(1)} \le (\bar{\nu}_1)^{(1)} \le \boxed{(\nu_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}}$ , we obtain 386

$$(\nu_1)^{(1)} \leq \nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\nu_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(1)}(t)$ :-

$$(m_2)^{(1)} \leq \nu^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{\nu^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(1)}(t)$ :-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{13})^{(1)} = (a''_{14})^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)}G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)}T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

**Proof** : From global equations we obtain 387

$$\frac{d\nu^{(2)}}{dt} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right) - (a''_{17})^{(2)}(T_{17}, t)\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

**Definition of** $\nu^{(2)}$ :- $\boxed{\nu^{(2)} = \frac{G_{16}}{G_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \leq \frac{d\nu^{(2)}}{dt} \leq -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

**Definition of** $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$ :-

For $0 < (\nu_0)^{(2)} = \frac{G^0_{16}}{G^0_{17}} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \geq \frac{(\nu_1)^{(2)}+(\mathrm{C})^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_0)^{(2)}\right)t\right]}}{1+(\mathrm{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\mathrm{C})^{(2)} = \frac{(\nu_1)^{(2)}-(\nu_0)^{(2)}}{(\nu_0)^{(2)}-(\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)}+(\bar{\mathrm{C}})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{\mathrm{C}})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{\mathrm{C}})^{(2)} = \frac{(\bar{\nu}_1)^{(2)}-(\nu_0)^{(2)}}{(\nu_0)^{(2)}-(\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{\mathrm{G}_{16}^0}{\mathrm{G}_{17}^0} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \leq \frac{(\nu_1)^{(2)}+(\mathrm{C})^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}}{1+(\mathrm{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}} \leq \nu^{(2)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(2)}+(\bar{\mathrm{C}})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{\mathrm{C}})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \leq (\bar{\nu}_1)^{(2)} \leq (\nu_0)^{(2)} = \frac{\mathrm{G}_{16}^0}{\mathrm{G}_{17}^0}$ , we obtain 394

$$(\nu_1)^{(2)} \leq \nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)}+(\bar{\mathrm{C}})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{\mathrm{C}})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(2)}(t)$ :- 395

$$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}, \quad \boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$$

In a completely analogous way, we obtain 396

**Definition of** $u^{(2)}(t)$ :-

$$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}, \quad \boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :** 397

If $(a_{16}'')^{(2)} = (a_{17}'')^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)}G_{17}(t)$

Analogously if $(b''_{16})^{(2)} = (b''_{17})^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)}T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

**Proof** : From global equations we obtain 398

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right) - (a''_{21})^{(3)}(T_{21}, t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

**Definition of** $\nu^{(3)}$ :- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right) \leq \frac{d\nu^{(3)}}{dt} \leq -\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right)$$

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From which one obtains

For $0 < (\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\nu_1)^{(3)} < (\bar{\nu}_1)^{(3)}$

$$\nu^{(3)}(t) \geq \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(\nu_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\nu_2)^{(3)}}}$$

it follows $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\nu_1)^{(3)}$

In the same manner , we get 401

$$\nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)} + (\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{\nu}_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\bar{\nu}_2)^{(3)}}}$$

**Definition of** $(\bar{\nu}_1)^{(3)}$ :-

From which we deduce $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\bar{\nu}_1)^{(3)}$

If $0 < (\nu_1)^{(3)} < (\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\bar{\nu}_1)^{(3)}$ we find like in the previous case, 402

$$(\nu_1)^{(3)} \leq \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_2)^{(3)}\right)t\right]}}{1 + (C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_2)^{(3)}\right)t\right]}} \leq \nu^{(3)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(3)} + (\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\bar{\nu}_1)^{(3)}$$

If $0 < (\nu_1)^{(3)} \leq (\bar{\nu}_1)^{(3)} \leq (\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}$ , we obtain 403

$$(\nu_1)^{(3)} \leq \nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\nu_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(3)}(t)$ :-

$$(m_2)^{(3)} \leq \nu^{(3)}(t) \leq (m_1)^{(3)}, \quad \boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(3)}(t)$ :-

$$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}, \quad \boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{20})^{(3)} = (a''_{21})^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)}G_{21}(t)$

Analogously if $(b''_{20})^{(3)} = (b''_{21})^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)}T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

**Proof** : From global equations we obtain 404

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right) - (a''_{25})^{(4)}(T_{25},t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

**Definition of** $\nu^{(4)}$ :- $\quad \boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right) \leq \frac{d\nu^{(4)}}{dt} \leq -\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_4)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$ :-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \geq \frac{(\nu_1)^{(4)}+(C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_0)^{(4)}\right)t\right]}}{4+(C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)}-(\nu_0)^{(4)}}{(\nu_0)^{(4)}-(\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{4+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)}-(\nu_0)^{(4)}}{(\nu_0)^{(4)}-(\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \leq \frac{(\nu_1)^{(4)}+(C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_2)^{(4)}\right)t\right]}}{1+(C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_2)^{(4)}\right)t\right]}} \leq \nu^{(4)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \leq (\bar{\nu}_1)^{(4)} \leq \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$ , we obtain 407

$$(\nu_1)^{(4)} \leq \nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(4)}(t)$ :-

$$(m_2)^{(4)} \leq \nu^{(4)}(t) \leq (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(4)}(t)$ :-

$$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a_{24}'')^{(4)} = (a_{25}'')^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)}G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b_{24}'')^{(4)} = (b_{25}'')^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)}T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

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**Proof** : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a_{28}')^{(5)} - (a_{29}')^{(5)} + (a_{28}'')^{(5)}(T_{29}, t)\right) - (a_{29}'')^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

**Definition of** $\nu^{(5)}$ :- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$ :-

For $0 < \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \leq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}}{1 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}} \leq \nu^{(5)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \leq (\bar{\nu}_1)^{(5)} \leq \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$ , we obtain 411

$$(\nu_1)^{(5)} \leq \nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(5)}(t)$ :-

$(m_2)^{(5)} \leq \nu^{(5)}(t) \leq (m_1)^{(5)}$, $\boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$

In a completely analogous way, we obtain

**Definition of** $u^{(5)}(t)$ :-

$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}$, $\boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{28})^{(5)} = (a''_{29})^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)} G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b''_{28})^{(5)} = (b''_{29})^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$ if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)} T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

**Proof** : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a'_{32})^{(6)} - (a'_{33})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right) - (a''_{33})^{(6)}(T_{33}, t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

**Definition of** $\nu^{(6)}$ :- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \leq \frac{d\nu^{(6)}}{dt} \leq -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$ :-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \geq \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\nu_1)^{(6)}$

In the same manner , we get 413

$$\nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{\nu}_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\bar{\nu}_1)^{(6)}$

If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 414

$$(\nu_1)^{(6)} \leq \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}} \leq \nu^{(6)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\bar{\nu}_1)^{(6)}$$

If $0 < (\nu_1)^{(6)} \leq (\bar{\nu}_1)^{(6)} \leq \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$, we obtain

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$$(\nu_1)^{(6)} \leq \nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(6)}(t)$ :-

$$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}, \quad \boxed{\nu^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(6)}(t)$ :-

$$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}, \quad \boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)}G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)}T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

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**Proof** : From global equations we obtain

$$\frac{d\nu^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right) - (a''_{37})^{(7)}(T_{37}, t)\nu^{(7)} - (a_{37})^{(7)}\nu^{(7)}$$

**Definition of** $\nu^{(7)}$ :- $\boxed{\nu^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right) \leq \frac{d\nu^{(7)}}{dt} \leq -\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(7)}, (\nu_0)^{(7)}$ :-

For $0 < \boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}} < (\nu_1)^{(7)} < (\bar{\nu}_1)^{(7)}$

$$\nu^{(7)}(t) \geq \frac{(\nu_1)^{(7)}+(C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_0)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)}=\frac{(\nu_1)^{(7)}-(\nu_0)^{(7)}}{(\nu_0)^{(7)}-(\nu_2)^{(7)}}}$$

it follows $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\nu_1)^{(7)}$

In the same manner , we get 417

$$\nu^{(7)}(t) \leq \frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)}=\frac{(\bar{\nu}_1)^{(7)}-(\nu_0)^{(7)}}{(\nu_0)^{(7)}-(\bar{\nu}_2)^{(7)}}}$$

From which we deduce $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\bar{\nu}_1)^{(7)}$

If $0 < (\nu_1)^{(7)} < (\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0} < (\bar{\nu}_1)^{(7)}$ we find like in the previous case, 418

$$(\nu_1)^{(7)} \leq \frac{(\nu_1)^{(7)}+(C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}} \leq \nu^{(7)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \leq (\bar{\nu}_1)^{(7)}$$

If $0 < (\nu_1)^{(7)} \leq (\bar{\nu}_1)^{(7)} \leq \boxed{(\nu_0)^{(7)}=\frac{G_{36}^0}{G_{37}^0}}$ , we obtain 419

$$(\nu_1)^{(7)} \leq \nu^{(7)}(t) \leq \frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \leq (\nu_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have
**Definition of** $\nu^{(7)}(t)$ :-

$$(m_2)^{(7)} \leq \nu^{(7)}(t) \leq (m_1)^{(7)}, \quad \boxed{\nu^{(7)}(t)=\frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(7)}(t)$ :- 420

$$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}, \quad \boxed{u^{(7)}(t)=\frac{T_{36}(t)}{T_{37}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $\nu^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)}G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for**

**the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)}T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

**Proof** : From global equations we obtain 421

$$\frac{d\nu^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right) - (a''_{41})^{(8)}(T_{41},t)\nu^{(8)} - (a_{41})^{(8)}\nu^{(8)}$$

**Definition of** $\nu^{(8)}$ :- $\boxed{\nu^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right) \le \frac{d\nu^{(8)}}{dt} \le -\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$ :-

For $0 < \boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$

$$\nu^{(8)}(t) \ge \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \le \nu^{(8)}(t) \le (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \le \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \le \nu^{(8)}(t) \le (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \le \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}} \le \nu^{(8)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \le (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \le (\bar{\nu}_1)^{(8)} \le \boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}}$ , we obtain 424

$$(v_1)^{(8)} \leq v^{(8)}(t) \leq \frac{(\bar{v}_1)^{(8)}+(\bar{C})^{(8)}(\bar{v}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{v}_1)^{(8)}-(\bar{v}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{v}_1)^{(8)}-(\bar{v}_2)^{(8)}\right)t\right]}} \leq (v_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $v^{(8)}(t)$ :-

$$(m_2)^{(8)} \leq v^{(8)}(t) \leq (m_1)^{(8)}, \quad \boxed{v^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(8)}(t)$ :-

$$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}, \quad \boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(v_1)^{(8)} = (\bar{v}_1)^{(8)}$ if in addition $(v_0)^{(8)} = (v_1)^{(8)}$ then $v^{(8)}(t) = (v_0)^{(8)}$ and as a consequence $G_{40}(t) = (v_0)^{(8)}G_{41}(t)$ **this also defines** $(v_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then $(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)}T_{41}(t)$ This is an important consequence of the relation between $(v_1)^{(8)}$ and $(\bar{v}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

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**Proof** : From 99,20,44,22,23,44 we obtain

$$\frac{dv^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45}, t)\right) - (a''_{45})^{(9)}(T_{45}, t)v^{(9)} - (a_{45})^{(9)}v^{(9)}$$

**Definition of** $v^{(9)}$ :- $\boxed{v^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)}\right) \leq \frac{dv^{(9)}}{dt} \leq -\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_1)^{(9)}v^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

**Definition of** $(\bar{v}_1)^{(9)}, (v_0)^{(9)}$ :-

For $0 < \boxed{(v_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (v_1)^{(9)} < (\bar{v}_1)^{(9)}$

$$v^{(9)}(t) \geq \frac{(v_1)^{(9)}+(C)^{(9)}(v_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((v_1)^{(9)}-(v_0)^{(9)}\right)t\right]}}{1+(C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((v_1)^{(9)}-(v_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(v_1)^{(9)}-(v_0)^{(9)}}{(v_0)^{(9)}-(v_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\nu_9)^{(9)}$

In the same manner , we get

$$\nu^{(9)}(t) \leq \frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)}-(\nu_0)^{(9)}}{(\nu_0)^{(9)}-(\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \leq \frac{(\nu_1)^{(9)}+(C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}}{1+(C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}} \leq \nu^{(9)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \leq (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \leq (\bar{\nu}_1)^{(9)} \leq \boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$ , we obtain

$$(\nu_1)^{(9)} \leq \nu^{(9)}(t) \leq \frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \leq (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(9)}(t)$ :-

$$(m_2)^{(9)} \leq \nu^{(9)}(t) \leq (m_1)^{(9)}, \quad \boxed{\nu^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(9)}(t)$ :-

$$(\mu_2)^{(9)} \leq u^{(9)}(t) \leq (\mu_1)^{(9)}, \quad \boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)}G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then $(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)}T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following

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**Theorem :** If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on $t$ , and the conditions with the notations

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0$ ,

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$

$with\ \ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(2)} and\ (b_i'')^{(2)}$ are independent on t , and the conditions with the notations 426

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$ 427

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 428

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$ , 429

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 430

$with\ \ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(3)} and\ (b_i'')^{(3)}$ are independent on $t$ , and the conditions with the notations 431

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$ ,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

**Theorem :** If $(a_i'')^{(4)} and\ (b_i'')^{(4)}$ are independent on $t$ , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$ ,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(5)} and\ (b_i'')^{(5)}$ are independent on $t$ , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$ ,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

**Theorem** If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on $t$ , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$ ,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(7)} and\ (b_i'')^{(7)}$ are independent on $t$ , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$ ,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(8)} and\ (b_i'')^{(8)}$ are independent on $t$ , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$ ,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(9)} and\ (b_i'')^{(9)}$ are independent on $t$ , and the conditions (with the notations 436 A
45,46,27,28)

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$ ,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0 \quad 437$$

$$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0 \quad 438$$

$$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0 \quad 439$$

$$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0 \quad 440$$

$$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ]T_{14} = 0 \quad 441$$

$$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0 \quad 442$$

has a unique positive solution , which is an equilibrium solution for the system

$$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0 \quad 443$$

$$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0 \quad 444$$

$$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0 \quad 445$$

$$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\ ]T_{16} = 0 \quad 446$$

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\ ]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\ ]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\ ]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\ ]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\ ]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27})\right)]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27})\right)]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27})\right)]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0 \quad 479$$

$$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0 \quad 480$$

$$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0 \quad 481$$

$$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0 \quad 482$$

$$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0 \quad 483$$

$$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0 \quad 484$$

$$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0 \quad 484\text{A}$$

$$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$$

$$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$$

$$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$$

$$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$$

$$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$$

**Proof:** 485

(a) Indeed the first two equations have a nontrivial solution $G_{13}, G_{14}$ if

$$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$$

**Proof:** 486

(d) Indeed the first two equations have a nontrivial solution $G_{16}, G_{17}$ if

$$F(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$$

**Proof:** 487

(a) Indeed the first two equations have a nontrivial solution $G_{20}, G_{21}$ if

$$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$$

**Proof:** 488

(a) Indeed the first two equations have a nontrivial solution $G_{24}, G_{25}$ if

$$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$$

**Proof:** 489

(a) Indeed the first two equations have a nontrivial solution $G_{28}, G_{29}$ if

$$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$$

**Proof:** 490

(a) Indeed the first two equations have a nontrivial solution $G_{32}, G_{33}$ if

$$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$$

**Proof:** 491

(a) Indeed the first two equations have a nontrivial solution $G_{36}, G_{37}$ if

$$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$$

**Proof:** 492

(a) Indeed the first two equations have a nontrivial solution $G_{40}, G_{41}$ if

$$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$$

**Proof:** 492 A

(a) Indeed the first two equations have a nontrivial solution $G_{44}, G_{45}$ if

$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$

**Definition and uniqueness of** $T^*_{14}$ :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique $T^*_{14}$ for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

**Definition and uniqueness of** $T^*_{17}$ :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique $T^*_{17}$ for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]} \qquad 495$$

**Definition and uniqueness of** $T^*_{21}$ :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique $T^*_{21}$ for which $f(T^*_{21}) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

**Definition and uniqueness of** $T^*_{25}$ :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that there exists a unique $T^*_{25}$ for which $f(T^*_{25}) = 0$. With this value , we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T^*_{25})]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T^*_{25})]}$$

**Definition and uniqueness of** $T^*_{29}$ :- 498

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(5)}(T_{29})$ being increasing, it follows that there exists a unique $T^*_{29}$ for which $f(T^*_{29}) = 0$. With this value , we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T^*_{29})]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T^*_{29})]}$$

**Definition and uniqueness of** $T^*_{33}$ :- 499

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(6)}(T_{33})$ being increasing, it follows that

there exists a unique $T_{33}^{*}$ for which $f(T_{33}^{*})=0$. With this value , we obtain from the three first equations

$$G_{32}=\frac{(a_{32})^{(6)}G_{33}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^{*})]} \quad , \quad G_{34}=\frac{(a_{34})^{(6)}G_{33}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^{*})]}$$

**Definition and uniqueness of** $T_{37}^{*}$ :- 500

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(7)}(T_{37})$ being increasing, it follows that there exists a unique $T_{37}^{*}$ for which $f(T_{37}^{*})=0$. With this value , we obtain from the three first equations

$$G_{36}=\frac{(a_{36})^{(7)}G_{37}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^{*})]} \quad , \quad G_{38}=\frac{(a_{38})^{(7)}G_{37}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^{*})]}$$

**Definition and uniqueness of** $T_{41}^{*}$ :- 501

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique $T_{41}^{*}$ for which $f(T_{41}^{*})=0$. With this value , we obtain from the three first equations

$$G_{40}=\frac{(a_{40})^{(8)}G_{41}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T_{41}^{*})]} \quad , \quad G_{42}=\frac{(a_{42})^{(8)}G_{41}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T_{41}^{*})]}$$

**Definition and uniqueness of** $T_{45}^{*}$ :- 501 A

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(9)}(T_{45})$ being increasing, it follows that there exists a unique $T_{45}^{*}$ for which $f(T_{45}^{*})=0$. With this value , we obtain from the three first equations

$$G_{44}=\frac{(a_{44})^{(9)}G_{45}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T_{45}^{*})]} \quad , \quad G_{46}=\frac{(a_{46})^{(9)}G_{45}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T_{45}^{*})]}$$

By the same argument, the equations admit solutions $G_{13}, G_{14}$ if 502

$$\varphi(G)=(b'_{13})^{(1)}(b'_{14})^{(1)}-(b_{13})^{(1)}(b_{14})^{(1)}-$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G)+(b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right]+(b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G)=0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{14}$ taking into account the hypothesis $\varphi(0)>0$ , $\varphi(\infty)<0$ it follows that there exists a unique $G_{14}^{*}$ such that $\varphi(G^{*})=0$

By the same argument, the equations admit solutions $G_{16}, G_{17}$ if 503

$$\varphi(G_{19})=(b'_{16})^{(2)}(b'_{17})^{(2)}-(b_{16})^{(2)}(b_{17})^{(2)}-$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19})+(b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right]+(b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19})=0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{17}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{14}^{*}$ such that $\varphi((G_{19})^{*}) = 0$ 504

By the same argument, the equations admit solutions $G_{20}, G_{21}$ if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right] + (b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{21}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{21}^{*}$ such that $\varphi((G_{23})^{*}) = 0$

By the same argument, the equations admit solutions $G_{24}, G_{25}$ if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right] + (b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{25}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{25}^{*}$ such that $\varphi((G_{27})^{*}) = 0$

By the same argument, the equations admit solutions $G_{28}, G_{29}$ if 507

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right] + (b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{29}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{29}^{*}$ such that $\varphi((G_{31})^{*}) = 0$

By the same argument, the equations admit solutions $G_{32}, G_{33}$ if 508

$$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$$

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right] + (b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{33}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{33}^{*}$ such that $\varphi({G_{35}}^{*}) = 0$

By the same argument, the equations admit solutions $G_{36}, G_{37}$ if 509

$$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$$

$$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right] + (b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{37}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{37}^{*}$ such that $\varphi({G_{39}}^{*}) = 0$

By the same argument, the equations admit solutions $G_{40}, G_{41}$ if 510

$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$
$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right] + (b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{41}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{41}$ such that $\varphi(G_{43}{}^*) = 0$

By the same argument, the equations 92,93 admit solutions $G_{44}, G_{45}$ if
$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$

$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right] + (b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{45}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{45}$ such that $\varphi((G_{47})^*) = 0$

Finally we obtain the unique solution 511

$G^*_{14}$ given by $\varphi(G^*) = 0$ , $T^*_{14}$ given by $f(T^*_{14}) = 0$ and

$$G^*_{13} = \frac{(a_{13})^{(1)}G^*_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G^*_{15} = \frac{(a_{15})^{(1)}G^*_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

$$T^*_{13} = \frac{(b_{13})^{(1)}T^*_{14}}{[(b'_{13})^{(1)}-(b''_{13})^{(1)}(G^*)]} \quad , \quad T^*_{15} = \frac{(b_{15})^{(1)}T^*_{14}}{[(b'_{15})^{(1)}-(b''_{15})^{(1)}(G^*)]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

$G^*_{17}$ given by $\varphi((G_{19})^*) = 0$ , $T^*_{17}$ given by $f(T^*_{17}) = 0$ and 512

$$G^*_{16} = \frac{(a_{16})^{(2)}G^*_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G^*_{18} = \frac{(a_{18})^{(2)}G^*_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]} \quad 513$$

$$T^*_{16} = \frac{(b_{16})^{(2)}T^*_{17}}{[(b'_{16})^{(2)}-(b''_{16})^{(2)}((G_{19})^*)]} \quad , \quad T^*_{18} = \frac{(b_{18})^{(2)}T^*_{17}}{[(b'_{18})^{(2)}-(b''_{18})^{(2)}((G_{19})^*)]} \quad 514$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

$G^*_{21}$ given by $\varphi((G_{23})^*) = 0$ , $T^*_{21}$ given by $f(T^*_{21}) = 0$ and

$$G^*_{20} = \frac{(a_{20})^{(3)}G^*_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G^*_{22} = \frac{(a_{22})^{(3)}G^*_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

$$T^*_{20} = \frac{(b_{20})^{(3)}T^*_{21}}{[(b'_{20})^{(3)}-(b''_{20})^{(3)}(G_{23}{}^*)]} \quad , \quad T^*_{22} = \frac{(b_{22})^{(3)}T^*_{21}}{[(b'_{22})^{(3)}-(b''_{22})^{(3)}(G_{23}{}^*)]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

$G_{25}^{*}$ given by $\varphi(G_{27}) = 0$ , $T_{25}^{*}$ given by $f(T_{25}^{*}) = 0$ and

$$G_{24}^{*} = \frac{(a_{24})^{(4)} G_{25}^{*}}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}^{*} = \frac{(a_{26})^{(4)} G_{25}^{*}}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^{*})]}$$

$$T_{24}^{*} = \frac{(b_{24})^{(4)} T_{25}^{*}}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^{*})]} \quad , \quad T_{26}^{*} = \frac{(b_{26})^{(4)} T_{25}^{*}}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^{*})]} \qquad 517$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

$G_{29}^{*}$ given by $\varphi((G_{31})^{*}) = 0$ , $T_{29}^{*}$ given by $f(T_{29}^{*}) = 0$ and

$$G_{28}^{*} = \frac{(a_{28})^{(5)} G_{29}^{*}}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}^{*} = \frac{(a_{30})^{(5)} G_{29}^{*}}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^{*})]}$$

$$T_{28}^{*} = \frac{(b_{28})^{(5)} T_{29}^{*}}{[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31})^{*})]} \quad , \quad T_{30}^{*} = \frac{(b_{30})^{(5)} T_{29}^{*}}{[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31})^{*})]} \qquad 519$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

$G_{33}^{*}$ given by $\varphi((G_{35})^{*}) = 0$ , $T_{33}^{*}$ given by $f(T_{33}^{*}) = 0$ and

$$G_{32}^{*} = \frac{(a_{32})^{(6)} G_{33}^{*}}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^{*})]} \quad , \quad G_{34}^{*} = \frac{(a_{34})^{(6)} G_{33}^{*}}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^{*})]}$$

$$T_{32}^{*} = \frac{(b_{32})^{(6)} T_{33}^{*}}{[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35})^{*})]} \quad , \quad T_{34}^{*} = \frac{(b_{34})^{(6)} T_{33}^{*}}{[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35})^{*})]} \qquad 521$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

$G_{37}^{*}$ given by $\varphi((G_{39})^{*}) = 0$ , $T_{37}^{*}$ given by $f(T_{37}^{*}) = 0$ and

$$G_{36}^{*} = \frac{(a_{36})^{(7)} G_{37}^{*}}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^{*})]} \quad , \quad G_{38}^{*} = \frac{(a_{38})^{(7)} G_{37}^{*}}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^{*})]}$$

$$T_{36}^{*} = \frac{(b_{36})^{(7)} T_{37}^{*}}{[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39})^{*})]} \quad , \quad T_{38}^{*} = \frac{(b_{38})^{(7)} T_{37}^{*}}{[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39})^{*})]}$$

Finally we obtain the unique solution 523

$G_{41}^{*}$ given by $\varphi((G_{43})^{*}) = 0$ , $T_{41}^{*}$ given by $f(T_{41}^{*}) = 0$ and

$$G_{40}^{*} = \frac{(a_{40})^{(8)} G_{41}^{*}}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^{*})]} \quad , \quad G_{42}^{*} = \frac{(a_{42})^{(8)} G_{41}^{*}}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^{*})]}$$

$$T_{40}^{*} = \frac{(b_{40})^{(8)} T_{41}^{*}}{[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43})^{*})]} \quad , \quad T_{42}^{*} = \frac{(b_{42})^{(8)} T_{41}^{*}}{[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43})^{*})]}$$

Finally we obtain the unique solution of 89 to 99 523

A

$G_{45}^{*}$ given by $\varphi((G_{47})^{*}) = 0$ , $T_{45}^{*}$ given by $f(T_{45}^{*}) = 0$ and

$$G_{44}^{*} = \frac{(a_{44})^{(9)} G_{45}^{*}}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^{*})]} \quad , \quad G_{46}^{*} = \frac{(a_{46})^{(9)} G_{45}^{*}}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^{*})]}$$

$$T_{44}^{*} = \frac{(b_{44})^{(9)} T_{45}^{*}}{[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47})^{*})]} \quad , \quad T_{46}^{*} = \frac{(b_{46})^{(9)} T_{45}^{*}}{[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47})^{*})]}$$

**ASYMPTOTIC STABILITY ANALYSIS** 524

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

**Proof:** Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^{*} + \mathbb{G}_i \qquad , T_i = T_i^{*} + \mathbb{T}_i$$

$$\frac{\partial (a''_{14})^{(1)}}{\partial T_{14}}(T_{14}^{*}) = (q_{14})^{(1)} \quad , \frac{\partial (b_i'')^{(1)}}{\partial G_j}(G^{*}) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a'_{13})^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)} G_{13}^{*}\mathbb{T}_{14} \quad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a'_{14})^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)} G_{14}^{*}\mathbb{T}_{14} \quad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a'_{15})^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)} G_{15}^{*}\mathbb{T}_{14} \quad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b'_{13})^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)} T_{13}^{*}\mathbb{G}_j\right) \quad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b'_{14})^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)} T_{14}^{*}\mathbb{G}_j\right) \quad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b'_{15})^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)} T_{15}^{*}\mathbb{G}_j\right) \quad 530$$

**ASYMPTOTIC STABILITY ANALYSIS** 531

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(2)}$ and $(b_i'')^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i \tag{532}$$

$$\frac{\partial(a_{17}'')^{(2)}}{\partial T_{17}}(T_{17}^*) = (q_{17})^{(2)} \quad , \frac{\partial(b_i'')^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \tag{533}$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a_{16}')^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G_{16}^*\mathbb{T}_{17} \tag{534}$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a_{17}')^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G_{17}^*\mathbb{T}_{17} \tag{535}$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a_{18}')^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G_{18}^*\mathbb{T}_{17} \tag{536}$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b_{16}')^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T_{16}^*\mathbb{G}_j\right) \tag{537}$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b_{17}')^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}T_{17}^*\mathbb{G}_j\right) \tag{538}$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b_{18}')^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}T_{18}^*\mathbb{G}_j\right) \tag{539}$$

**ASYMPTOTIC STABILITY ANALYSIS** 540

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(3)}$ *and* $(b_i'')^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{21}'')^{(3)}}{\partial T_{21}}(T_{21}^*) = (q_{21})^{(3)} \quad , \frac{\partial(b_i'')^{(3)}}{\partial G_j}((G_{23})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a_{20}')^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G_{20}^*\mathbb{T}_{21} \tag{541}$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a_{21}')^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G_{21}^*\mathbb{T}_{21} \tag{542}$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a_{22}')^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G_{22}^*\mathbb{T}_{21} \tag{543}$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b_{20}')^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T_{20}^*\mathbb{G}_j\right) \tag{544}$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b_{21}')^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T_{21}^*\mathbb{G}_j\right) \tag{545}$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b'_{22})^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T^*_{22}\mathbb{G}_j\right) \quad 546$$

**ASYMPTOTIC STABILITY ANALYSIS** 547

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}\ and\ (b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{25})^{(4)}}{\partial T_{25}}(T^*_{25}) = (q_{25})^{(4)} \ , \ \frac{\partial(b_i'')^{(4)}}{\partial G_j}((G_{27})^*\ ) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\left((a'_{24})^{(4)} + (p_{24})^{(4)}\right)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G^*_{24}\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\left((a'_{25})^{(4)} + (p_{25})^{(4)}\right)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G^*_{25}\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\left((a'_{26})^{(4)} + (p_{26})^{(4)}\right)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G^*_{26}\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\left((b'_{24})^{(4)} - (r_{24})^{(4)}\right)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(24)(j)}T^*_{24}\mathbb{G}_j\right) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\left((b'_{25})^{(4)} - (r_{25})^{(4)}\right)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\left(s_{(25)(j)}T^*_{25}\mathbb{G}_j\right) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T^*_{26}\mathbb{G}_j\right) \quad 554$$

**ASYMPTOTIC STABILITY ANALYSIS** 555

**Theorem 5:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}\ and\ (b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 556

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{29})^{(5)}}{\partial T_{29}}(T^*_{29}) = (q_{29})^{(5)} \ , \ \frac{\partial(b_i'')^{(5)}}{\partial G_j}(\ (G_{31})^*\ ) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a'_{28})^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G^*_{28}\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a'_{29})^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G^*_{29}\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a'_{30})^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G^*_{30}\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b'_{28})^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T^*_{28}\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b'_{29})^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T^*_{29}\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b'_{30})^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T^*_{30}\mathbb{G}_j\right) \quad 562$$

**ASYMPTOTIC STABILITY ANALYSIS** 563

**Theorem 6:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}$ *and* $(b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{33})^{(6)}}{\partial T_{33}}(T^*_{33}) = (q_{33})^{(6)} \quad , \frac{\partial(b_i'')^{(6)}}{\partial G_j}\left((G_{35})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a'_{32})^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G^*_{32}\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a'_{33})^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G^*_{33}\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a'_{34})^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G^*_{34}\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b'_{32})^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T^*_{32}\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b'_{33})^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T^*_{33}\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b'_{34})^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T^*_{34}\mathbb{G}_j\right) \quad 570$$

**ASYMPTOTIC STABILITY ANALYSIS** 571

**Theorem 7:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 572

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{37}'')^{(7)}}{\partial T_{37}}(T_{37}^{*}) = (q_{37})^{(7)} \quad , \frac{\partial(b_i'')^{(7)}}{\partial G_j}\left( (G_{39})^{**} \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a_{36}')^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G_{36}^{*}\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a_{37}')^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G_{37}^{*}\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a_{38}')^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G_{38}^{*}\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b_{36}')^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T_{36}^{*}\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b_{37}')^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T_{37}^{*}\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b_{38}')^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T_{38}^{*}\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

## ASYMPTOTIC STABILITY ANALYSIS

**Theorem 8:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 580

$$G_i = G_i^{*} + \mathbb{G}_i \qquad , T_i = T_i^{*} + \mathbb{T}_i$$

$$\frac{\partial(a_{41}'')^{(8)}}{\partial T_{41}}(T_{41}^{*}) = (q_{41})^{(8)} \quad , \frac{\partial(b_i'')^{(8)}}{\partial G_j}\left( (G_{43})^{*} \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a_{40}')^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G_{40}^{*}\mathbb{T}_{41} \quad 581$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a_{41}')^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G_{41}^{*}\mathbb{T}_{41} \quad 582$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a_{42}')^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G_{42}^{*}\mathbb{T}_{41} \quad 583$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b_{40}')^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T_{40}^{*}\mathbb{G}_j\right) \quad 584$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b_{41}')^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T_{41}^{*}\mathbb{G}_j\right) \quad 585$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b'_{42})^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T^*_{42}\mathbb{G}_j\right)$$ 586

586

**ASYMPTOTIC STABILITY ANALYSIS** A

**Theorem 9:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}\ and\ (b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

**Proof: Denote**

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{45})^{(9)}}{\partial T_{45}}(T^*_{45}) = (q_{45})^{(9)} \ , \ \frac{\partial(b_i'')^{(9)}}{\partial G_j}(\ (G_{47})^*\ ) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a'_{44})^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)}G^*_{44}\mathbb{T}_{45}$$ 586 B

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a'_{45})^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)}G^*_{45}\mathbb{T}_{45}$$ 586 C

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a'_{46})^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)}G^*_{46}\mathbb{T}_{45}$$ 586 D

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b'_{44})^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)}T^*_{44}\mathbb{G}_j\right)$$ 586 E

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b'_{45})^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)}T^*_{45}\mathbb{G}_j\right)$$ 586 F

$$\frac{d\mathbb{T}_{46}}{dt} = -\left((b'_{46})^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T^*_{46}\mathbb{G}_j\right)$$ 586 G

**The characteristic equation of this system is** 587

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(q_{13})^{(1)}G^*_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)}T^*_{14} + (b_{14})^{(1)}s_{(13),(14)}T^*_{14}\right)$$

$$+\left(\left((\lambda)^{(1)} + (a'_{14})^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)}G^*_{13} + (a_{13})^{(1)}(q_{14})^{(1)}G^*_{14}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)}T^*_{14} + (b_{14})^{(1)}s_{(13),(13)}T^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left(\ (a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left(\ (b'_{13})^{(1)} + (b'_{14})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^2+\left((a'_{13})^{(1)}+(a'_{14})^{(1)}+(p_{13})^{(1)}+(p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)}+(a'_{13})^{(1)}+(p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^*_{14}+(a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}+(b'_{13})^{(1)}-(r_{13})^{(1)}\right)s_{(14),(15)}T^*_{14}+(b_{14})^{(1)}s_{(13),(15)}T^*_{13}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(2)}+(b'_{18})^{(2)}-(r_{18})^{(2)}\right)\{\left((\lambda)^{(2)}+(a'_{18})^{(2)}+(p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^*_{17}+(a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^*_{17}\right)$$

$$+\left(\left((\lambda)^{(2)}+(a'_{17})^{(2)}+(p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^*_{16}+(a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2+\left((b'_{16})^{(2)}+(b'_{17})^{(2)}-(r_{16})^{(2)}+(r_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$+\left(\left((\lambda)^{(2)}\right)^2+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}+(a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^*_{16}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\right)\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left(\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^*_{20}\right)\right]$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^*_{21}+(b_{21})^{(3)}s_{(20),(21)}T^*_{21}\right)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^*_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^*_{21}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^*_{21}+(b_{21})^{(3)}s_{(20),(20)}T^*_{20}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^2+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$+\left(\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^*_{20}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^*_{21}+(b_{21})^{(3)}s_{(20),(22)}T^*_{20}\right)\}=0$$

$$+$$

$$(\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left(\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^*_{24}\right)\right]$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^*_{25}+(b_{25})^{(4)}s_{(24),(25)}T^*_{25}\right)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^*_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^*_{25}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^*_{25}+(b_{25})^{(4)}s_{(24),(24)}T^*_{24}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^2+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$+\left(\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^*_{24}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^*_{25}+(b_{25})^{(4)}s_{(24),(26)}T^*_{24}\right)\}=0$$

$$+$$

$$(\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^*_{28}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^*_{29}+(b_{29})^{(5)}s_{(28),(29)}T^*_{29}\right)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^*_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^*_{29}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^*_{29}+(b_{29})^{(5)}s_{(28),(28)}T^*_{28}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^2+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$+\left(\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^*_{28}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^*_{29}+(b_{29})^{(5)}s_{(28),(30)}T^*_{28}\right)\}=0$$

$$+$$

$$(\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left(\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^*_{32}\right)\right]$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^*_{33}+(b_{33})^{(6)}s_{(32),(33)}T^*_{33}\right)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^*_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^*_{33}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^*_{33}+(b_{33})^{(6)}s_{(32),(32)}T^*_{32}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^2+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$+\left(\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^*_{32}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^*_{33}+(b_{33})^{(6)}s_{(32),(34)}T^*_{32}\right)\}=0$$

$$+$$

$$(\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^*_{36}\right)\right]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^*_{37}+(b_{37})^{(7)}s_{(36),(37)}T^*_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^*_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^*_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^*_{37}+(b_{37})^{(7)}s_{(36),(36)}T^*_{36}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^2+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$+\left(\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^*_{36}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^*_{37}+(b_{37})^{(7)}s_{(36),(38)}T^*_{36}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\right)\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^*_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^*_{41}+(b_{41})^{(8)}s_{(40),(41)}T^*_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^*_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^*_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^*_{41}+(b_{41})^{(8)}s_{(40),(40)}T^*_{40}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^2+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$+\left(\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^*_{40}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^*_{41}+(b_{41})^{(8)}s_{(40),(42)}T^*_{40}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\right)\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^*_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^*_{45}+(b_{45})^{(9)}s_{(44),(45)}T^*_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^*_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^*_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^*_{45}+(b_{45})^{(9)}s_{(44),(44)}T^*_{44}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^2+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$+\left(\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^*_{44}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^*_{45}+(b_{45})^{(9)}s_{(44),(46)}T^*_{44}\right)\}=0$$

**And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.**

## SECTION SIX

## Quantum Discord And The Power Of One Qubit

### INTRODUCTION—VARIABLES USED

**Quantum Discord and the Power of One Qubit Animesh Datta, Anil Shaji, and Carlton M. Caves Phys. Rev. Lett. 100, 050502 – Published 5 February 2008**

(1) Animesh Datta, Anil Shaji, and Carlton M. Caves use (e) quantum discord to characterize (eb) the correlations present in the model called deterministic quantum computation with (e&eb) one quantum bit (DQC1), introduced by Knill and Laflamme [Phys. Rev. Lett. 81, 5672 (1998)].
(2) The model involves a collection of qubits in the completely mixed state coupled to (e&eb) a single control qubit that has (e) nonzero purity.
(3) The initial state, operations, and measurements in the model all point to a natural bipartite split between the control qubit and (e&eb: AC=DC=0) the mixed ones.
(4) Although there is no entanglement between these two parts, authors show that the quantum discord across (e&eb) this split is nonzero for typical instances of the DQC1 circuit.
(5) Nonzero values of discord indicate (eb) the presence of nonclassical correlations.
(6) Authors propose quantum discord as figure of merit for (e) characterizing the resources present in this computational model. http://dx.doi.org/10.1103/PhysRevLett.100.050502

**Quantum discord as resource for remote state preparation Borivoje Dakić, Yannick Ole Lipp, Xiaosong MA, Martín Ringbauer, Sebastian Kropatschek, Stefanie Barz, Tomasz Paterek, Vlatko Vedral, Anton Zeilinger, Časlav Brukner & Philip Walther Nature Physics 8, 666–670 (2012) doi:10.1038/nphys2377Published online 05 August 2012**

(7) The existence of better-than-classical quantum information processing (QIP) models which consume very little or no entanglement suggests (eb) that separable or weakly entangled states could be extremely useful tools for (e) information processing as they are much easier to prepare and control (eb,e&eb)even in dissipative environments.

(8) It has been proposed that a resource of advantage is (=) the generation of quantum discord, a (=) measure of non-classical correlations that includes (e) entanglement as a subset.
(9) Here authors show that quantum discord is the necessary resource for (e) quantum remote state preparation.
(10) Authors explicitly show that the geometric measure of quantum discord is related to (e&eb) the fidelity of this task, which provides (eb) its operational meaning.
(11) Results are experimentally demonstrated using (e) photonic quantum systems.
**(12)** Moreover, authors demonstrate that separable states with (e&eb) non-zero quantum discord can outperform (e) entangled states.
**(13)** Therefore, the role of quantum discord might provide (eb) fundamental insights for resource-efficient QIP. Subject terms: Optical physics Quantum information

## NOTATION

### Module One

The model involves a **collection of qubits in the completely mixed state** coupled to (e&eb) a single control qubit that has (e) nonzero purity

$G_{13}$ : Category one of **collection of qubits in the completely mixed state;** single control qubit that has (e) nonzero purity

$G_{14}$ : Category two of SAS

$G_{15}$ : Category three of SAS

$T_{13}$ : Category one of single control qubit that has (e) nonzero purity **;collection of qubits in the completely mixed state**

$T_{14}$ : Category two of SAS

$T_{15}$ : Category three of SAS

### Module Two

The model involves a collection of qubits in the completely mixed state coupled to a single control qubit that has (e) nonzero purity

$G_{16}$ : Category one of nonzero purity

$G_{17}$ : Category two of SAS

$G_{18}$ : Category three of SAS

$T_{16}$ : Category one of collection of qubits in the completely mixed state coupled to a single control qubit

$T_{17}$ : Category two of SAS

$T_{18}$ : Category three of SAS

### Module three

The initial state, operations, and measurements in the model all point to a natural bipartite split between the control qubit and (e&eb: AC=DC=0) the mixed ones

$G_{20}$ : Category one of initial state, operations, and measurements in the model all point to a natural bipartite

split between the control qubit; **mixed ones**

$G_{21}$ : Category two of SAS

$G_{22}$ : Category three of SAS

$T_{20}$ : Category one of **mixed ones** ;initial state, operations, and measurements in the model all point to a natural bipartite split between the control qubit

$T_{21}$ : Category two of SAS

$T_{22}$ : Category three of SAS

**Module four**

Although there is no entanglement between these two parts, authors show that the **quantum discord** across (e&eb) this split is nonzero for typical instances of the DQC1 circuit

$G_{24}$ : Category one of **quantum discord;** split is nonzero for typical instances of the DQC1 circuit

$G_{25}$ : Category two of SAS

$G_{26}$ : Category three of SAS

$T_{24}$ : Category one of split is nonzero for typical instances of the DQC1 circuit **;quantum discord**

$T_{25}$ : Category two of SAS

$T_{26}$ : Category three of SAS

**Module five**

Nonzero values of discord indicate (eb) the presence of nonclassical correlations

$G_{28}$ : Category one of Nonzero values of discord

$G_{29}$ : Category two of SAS

$G_{30}$ : Category three of SAS

$T_{28}$ : Category one of presence of nonclassical correlations

$T_{29}$ : Category two of SAS

$T_{30}$ : Category three of SAS

**Module six**

Authors propose quantum discord as figure of merit for (e) characterizing the resources present in this computational model.

http://dx.doi.org/10.1103/PhysRevLett.100.050502

$G_{32}$ : Category one of quantum discord as figure of merit

$G_{33}$ : Category two of SAS

$G_{34}$ : Category three of SAS

$T_{32}$ : Category one of characterizing the resources present in this computational model.

$T_{33}$ : Category two of SAS

$T_{34}$ : Category three of SAS

**Module seven**

The existence of better-than-classical quantum information processing (QIP) models which consume very little or no entanglement suggests (eb) that separable or weakly entangled states could be extremely useful tools for (e) information processing as they are much easier to prepare and control (eb,e&eb)even in dissipative environments

$G_{36}$ : Category one of existence of better-than-classical quantum information processing (QIP) models which consume very little or no entanglement suggests

$G_{37}$ : Category two of SAS

$G_{38}$ : Category three of SAS

$T_{36}$ : Category one of separable or weakly entangled states could be extremely useful tools for (e) information processing as they are much easier to prepare and control (eb,e&eb)even in dissipative environments

$T_{37}$ : Category two of SAS

$T_{38}$ : Category three of SAS

**Module eight**

The existence of better-than-classical quantum information processing (QIP) models which consume very little or no entanglement suggests that separable or weakly entangled states could be extremely useful tools for (e) information processing as they are much easier to prepare and control (eb,e&eb)even in dissipative environments

$G_{40}$ : Category one of information processing as they are much easier to prepare and control (eb,e&eb) even in dissipative environments

$G_{41}$ : Category two of SAS

$G_{42}$ : Category three of SAS

$T_{40}$ : Category one of existence of better-than-classical quantum information processing (QIP) models which consume very little or no entanglement suggests that separable or weakly entangled states could be extremely useful tools

$T_{41}$ : Category two of SAS

$T_{42}$ : Category three of SAS

**Module Nine**

The existence of better-than-classical quantum information processing (QIP) models which consume very little or no entanglement suggests that separable or weakly entangled states could be extremely useful tools for information processing as they are much easier to prepare and control (eb,e&eb) even in **dissipative environments**

$G_{44}$ : Category one of existence of better-than-classical quantum information processing (QIP) models which consume very little or no entanglement suggests that separable or weakly entangled states could be extremely useful tools for information processing as they are much easier to prepare and control; **dissipative environments**

$G_{45}$ : Category two of SAS

$G_{46}$ : Category three of SAS

$T_{44}$ : Category one of **dissipative environments** ; existence of better-than-classical quantum information processing (QIP) models which consume very little or no entanglement suggests that separable or weakly entangled states could be extremely useful tools for information processing as they are much easier to prepare and control

$T_{45}$ : Category two of SAS

$T_{46}$ : Category three of SAS

**The Coefficients:**

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}$ $(a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}$ $(b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}$ , $(b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}$,
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}$, $(b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}$ $(a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$ ,
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)}$,
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)}$,
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)}$,
are Dissipation coefficients

**Module Numbered One**

**The differential system of this model is now (Module Numbered one)**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14},t)\right]G_{13} \quad 1$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14},t)\right]G_{14} \quad 2$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14},t)\right]G_{15} \quad 3$$

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G,t)\right]T_{13} \quad 4$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G,t)\right]T_{14} \quad 5$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G,t)\right]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14},t) =$ First augmentation factor

$-(b''_{13})^{(1)}(G,t) =$ First detritions factor

**Module Numbered Two**

**The differential system of this model is now ( Module numbered two)**

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17},t)\right]G_{16} \quad 7$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17},t)\right]G_{17} \quad 8$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17},t)\right]G_{18} \quad 9$$

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\left((G_{19}),t\right)\right]T_{16} \quad 10$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\left((G_{19}),t\right)\right]T_{17} \quad 11$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\left((G_{19}),t\right)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17},t) =$ First augmentation factor

$-(b''_{16})^{(2)}\left((G_{19}),t\right) =$ First detritions factor

**Module Numbered Three**

**The differential system of this model is now (Module numbered three)**

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21},t)\right]G_{20} \quad 13$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21},t)\right]G_{21} \quad 14$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21},t)\right]G_{22} \quad 15$$

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23},t)\right]T_{20} \quad 16$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23},t)\right]T_{21} \quad 17$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23},t)\right]T_{22} \quad 18$$

$+(a''_{20})^{(3)}(T_{21},t) =$ First augmentation factor

$-(b''_{20})^{(3)}(G_{23},t) =$ First detritions factor

**Module Numbered Four**

**The differential system of this model is now (Module numbered Four)**

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right]G_{24} \quad 19$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25},t)\right]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}, t)\right]G_{26} \quad 21$$
$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27}), t\right)\right]T_{24} \quad 22$$
$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27}), t\right)\right]T_{25} \quad 23$$
$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27}), t\right)\right]T_{26} \quad 24$$
$+(a''_{24})^{(4)}(T_{25}, t) =$ **First augmentation factor**
$-(b''_{24})^{(4)}\left((G_{27}), t\right) =$ **First detritions factor**

**Module Numbered Five**:

**The differential system of this model is now (Module number five)**

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right]G_{28} \quad 25$$
$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}, t)\right]G_{29} \quad 26$$
$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}, t)\right]G_{30} \quad 27$$
$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}\left((G_{31}), t\right)\right]T_{28} \quad 28$$
$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}\left((G_{31}), t\right)\right]T_{29} \quad 29$$
$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}\left((G_{31}), t\right)\right]T_{30} \quad 30$$
$+(a''_{28})^{(5)}(T_{29}, t) =$ **First augmentation factor**
$-(b''_{28})^{(5)}\left((G_{31}), t\right) =$ **First detritions factor**

**Module Numbered Six**

**The differential system of this model is now (Module numbered Six)**

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right]G_{32} \quad 31$$
$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}, t)\right]G_{33} \quad 32$$
$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}, t)\right]G_{34} \quad 33$$
$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}\left((G_{35}), t\right)\right]T_{32} \quad 34$$
$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}\left((G_{35}), t\right)\right]T_{33} \quad 35$$
$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}\left((G_{35}), t\right)\right]T_{34} \quad 36$$
$+(a''_{32})^{(6)}(T_{33}, t) =$ **First augmentation factor**

**Module Numbered Seven:**

**The differential system of this model is now (Seventh Module)**

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right]G_{36} \quad 37$$
$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37}, t)\right]G_{37} \quad 38$$
$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}, t)\right]G_{38} \quad 39$$
$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}\left((G_{39}), t\right)\right]T_{36} \quad 40$$
$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}\left((G_{39}), t\right)\right]T_{37} \quad 41$$
$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}\left((G_{39}), t\right)\right]T_{38} \quad 42$$
$+(a''_{36})^{(7)}(T_{37}, t) =$ **First augmentation factor**

**Module Numbered Eight**

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right]G_{40} \quad 43$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41},t)\right]G_{41} \quad 44$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41},t)\right]G_{42} \quad 45$$

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}\left((G_{43}),t\right)\right]T_{40} \quad 46$$

$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}\left((G_{43}),t\right)\right]T_{41} \quad 47$$

$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}\left((G_{43}),t\right)\right]T_{42} \quad 48$$

**Module Numbered Nine**

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}\left((G_{47}),t\right)\right]T_{44} \quad 52$$

$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}\left((G_{47}),t\right)\right]T_{45} \quad 53$$

$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}\left((G_{47}),t\right)\right]T_{46} \quad 54$$

$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}\left((G_{47}),t\right) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)} & +(a''_{13})^{(1)}(T_{14},t) & +(a''_{16})^{(2,2,)}(T_{17},t) & +(a''_{20})^{(3,3,)}(T_{21},t) \\ +(a''_{24})^{(4,4,4,4,)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,)}(T_{33},t) & \\ +(a''_{36})^{(7,7)}(T_{37},t) & +(a''_{40})^{(8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) & \end{bmatrix} G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)} & +(a''_{14})^{(1)}(T_{14},t) & +(a''_{17})^{(2,2,)}(T_{17},t) & +(a''_{21})^{(3,3,)}(T_{21},t) \\ +(a''_{25})^{(4,4,4,4,)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,)}(T_{33},t) & \\ +(a''_{37})^{(7,7)}(T_{37},t) & +(a''_{41})^{(8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) & \end{bmatrix} G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)} & +(a''_{15})^{(1)}(T_{14},t) & +(a''_{18})^{(2,2,)}(T_{17},t) & +(a''_{22})^{(3,3,)}(T_{21},t) \\ +(a''_{26})^{(4,4,4,4,)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,)}(T_{33},t) & \\ +(a''_{38})^{(7,7)}(T_{37},t) & +(a''_{42})^{(8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) & \end{bmatrix} G_{15} \quad 57$$

Where $(a''_{13})^{(1)}(T_{14},t)$, $(a''_{14})^{(1)}(T_{14},t)$, $(a''_{15})^{(1)}(T_{14},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,)}(T_{17},t)$, $+(a''_{17})^{(2,2,)}(T_{17},t)$, $+(a''_{18})^{(2,2,)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,)}(T_{21},t)$, $+(a''_{21})^{(3,3,)}(T_{21},t)$, $+(a''_{22})^{(3,3,)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{38})^{(7,7)}(T_{37},t)$ $+(a''_{37})^{(7,7)}(T_{37},t)$ $+(a''_{36})^{(7,7)}(T_{37},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{40})^{(8,8)}(T_{41},t)$ $+(a''_{41})^{(8,8)}(T_{41},t)$ $+(a''_{42})^{(8,8)}(T_{41},t)$ are eight augmentation coefficient for 1,2,3

$+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)}\left[-(b''_{13})^{(1)}(G,t)\right] & \left[-(b''_{16})^{(2,2,)}(G_{19},t)\right] & \left[-(b''_{20})^{(3,3,)}(G_{23},t)\right] \\ \left[-(b''_{24})^{(4,4,4,4,)}(G_{27},t)\right] & \left[-(b''_{28})^{(5,5,5,5,)}(G_{31},t)\right] & \left[-(b''_{32})^{(6,6,6,6,)}(G_{35},t)\right] \\ \left[-(b''_{36})^{(7,7,)}(G_{39},t)\right] & \left[-(b''_{40})^{(8,8)}(G_{43},t)\right] & \left[-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{13} \quad 58$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)}\left[-(b''_{14})^{(1)}(G,t)\right] & \left[-(b''_{17})^{(2,2,)}(G_{19},t)\right] & \left[-(b''_{21})^{(3,3,)}(G_{23},t)\right] \\ \left[-(b''_{25})^{(4,4,4,4,)}(G_{27},t)\right] & \left[-(b''_{29})^{(5,5,5,5,)}(G_{31},t)\right] & \left[-(b''_{33})^{(6,6,6,6,)}(G_{35},t)\right] \\ \left[-(b''_{37})^{(7,7,)}(G_{39},t)\right] & \left[-(b''_{41})^{(8,8)}(G_{43},t)\right] & \left[-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{14} \quad 59$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)}\left[-(b''_{15})^{(1)}(G,t)\right] & \left[-(b''_{18})^{(2,2,)}(G_{19},t)\right] & \left[-(b''_{22})^{(3,3,)}(G_{23},t)\right] \\ \left[-(b''_{26})^{(4,4,4,4,)}(G_{27},t)\right] & \left[-(b''_{30})^{(5,5,5,5,)}(G_{31},t)\right] & \left[-(b''_{34})^{(6,6,6,6,)}(G_{35},t)\right] \\ \left[-(b''_{38})^{(7,7,)}(G_{39},t)\right] & \left[-(b''_{42})^{(8,8)}(G_{43},t)\right] & \left[-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{15} \quad 60$$

Where $-(b''_{13})^{(1)}(G,t)$, $-(b''_{14})^{(1)}(G,t)$, $-(b''_{15})^{(1)}(G,t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,)}(G_{19},t)$, $-(b''_{17})^{(2,2,)}(G_{19},t)$, $-(b''_{18})^{(2,2,)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{37})^{(7,7,)}(G_{39},t)$, $-(b''_{36})^{(7,7,)}(G_{39},t)$, $-(b''_{38})^{(7,7,)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8)}(G_{43},t)$ $-(b''_{41})^{(8,8)}(G_{43},t)$ $-(b''_{42})^{(8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[\begin{array}{ccc} (a'_{16})^{(2)} & \boxed{+(a''_{16})^{(2)}(T_{17},t)} & \boxed{+(a''_{13})^{(1,1,)}(T_{14},t)} \quad \boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9)}(T_{45},t)} \end{array}\right] G_{16} \quad (61)$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[\begin{array}{ccc} (a'_{17})^{(2)} & \boxed{+(a''_{17})^{(2)}(T_{17},t)} & \boxed{+(a''_{14})^{(1,1,)}(T_{14},t)} \quad \boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9)}(T_{45},t)} \end{array}\right] G_{17} \quad (62)$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[\begin{array}{ccc} (a'_{18})^{(2)} & \boxed{+(a''_{18})^{(2)}(T_{17},t)} & \boxed{+(a''_{15})^{(1,1,)}(T_{14},t)} \quad \boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9)}(T_{45},t)} \end{array}\right] G_{18} \quad (63)$$

Where $\boxed{+(a''_{16})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,)}(T_{14},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)}$ are eight augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{44})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9)}(T_{45},t)}$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[\begin{array}{ccc} (b'_{16})^{(2)} & \boxed{-(b''_{16})^{(2)}(G_{19},t)} & \boxed{-(b''_{13})^{(1,1,)}(G,t)} \quad \boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9)}(G_{47},t)} \end{array}\right] T_{16} \quad (64)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[\begin{array}{ccc} (b'_{17})^{(2)} & \boxed{-(b''_{17})^{(2)}(G_{19},t)} & \boxed{-(b''_{14})^{(1,1,)}(G,t)} \quad \boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9)}(G_{47},t)} \end{array}\right] T_{17} \quad (65)$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[\begin{array}{ccc} (b'_{18})^{(2)} & \boxed{-(b''_{18})^{(2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,)}(G,t)} \quad \boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9)}(G_{47},t)} \end{array}\right] T_{18} \quad (66)$$

where $-(b''_{16})^{(2)}(G_{19},t)$ , $-(b''_{17})^{(2)}(G_{19},t)$ , $-(b''_{18})^{(2)}(G_{19},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{13})^{(1,1,)}(G,t)$, $-(b''_{14})^{(1,1,)}(G,t)$, $-(b''_{15})^{(1,1,)}(G,t)$ are second detrition coefficients for category 1,2 and 3

$-(b''_{20})^{(3,3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,)}(G_{23},t)$ are third detrition coefficients for category 1,2 and 3

$-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1,2 and 3

$-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1,2 and 3

$-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients for category 1,2 and 3

$-(b''_{36})^{(7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2 and 3

$-(b''_{40})^{(8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1,2 and 3

$-(b''_{44})^{(9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[\begin{array}{c} (a'_{20})^{(3)}\left[+(a''_{20})^{(3)}(T_{21},t)\right]\left[+(a''_{16})^{(2,2,2)}(T_{17},t)\right]\left[+(a''_{13})^{(1,1,1,)}(T_{14},t)\right] \\ \left[+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)\right]\left[+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)\right]\left[+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)\right] \\ \left[+(a''_{36})^{(7,7,7,7)}(T_{37},t)\right]\left[+(a''_{40})^{(8,8,8,8)}(T_{41},t)\right]\left[+(a''_{44})^{(9,9,9)}(T_{45},t)\right] \end{array}\right] G_{20} \quad (67)$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[\begin{array}{c} (a'_{21})^{(3)}\left[+(a''_{21})^{(3)}(T_{21},t)\right]\left[+(a''_{17})^{(2,2,2)}(T_{17},t)\right]\left[+(a''_{14})^{(1,1,1,)}(T_{14},t)\right] \\ \left[+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)\right]\left[+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)\right]\left[+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)\right] \\ \left[+(a''_{37})^{(7,7,7,7)}(T_{37},t)\right]\left[+(a''_{41})^{(8,8,8,8)}(T_{41},t)\right]\left[+(a''_{45})^{(9,9,9)}(T_{45},t)\right] \end{array}\right] G_{21} \quad (68)$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[\begin{array}{c} (a'_{22})^{(3)}\left[+(a''_{22})^{(3)}(T_{21},t)\right]\left[+(a''_{18})^{(2,2,2)}(T_{17},t)\right]\left[+(a''_{15})^{(1,1,1,)}(T_{14},t)\right] \\ \left[+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)\right]\left[+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)\right]\left[+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)\right] \\ \left[+(a''_{38})^{(7,7,7,7)}(T_{37},t)\right]\left[+(a''_{42})^{(8,8,8,8)}(T_{41},t)\right]\left[+(a''_{46})^{(9,9,9)}(T_{45},t)\right] \end{array}\right] G_{22} \quad (69)$$

$+(a''_{20})^{(3)}(T_{21},t)$, $+(a''_{21})^{(3)}(T_{21},t)$, $+(a''_{22})^{(3)}(T_{21},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2)}(T_{17},t)$ are second augmentation coefficients for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,)}(T_{14},t)$ are third augmentation coefficients for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficients for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficients for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficients for category 1, 2 and 3

$+(a''_{36})^{(7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1, 2 and 3

$+(a''_{40})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8)}(T_{41},t)$ are eight augmentation coefficients for category 1, 2 and 3

$+(a''_{44})^{(9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b'_{20})^{(3)} & -(b''_{20})^{(3)}(G_{23},t) & -(b''_{16})^{(2,2,2)}(G_{19},t) & -(b''_{13})^{(1,1,1,)}(G,t) \\ -(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{36})^{(7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{20} \quad (70)$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b'_{21})^{(3)} & -(b''_{21})^{(3)}(G_{23},t) & -(b''_{17})^{(2,2,2)}(G_{19},t) & -(b''_{14})^{(1,1,1,)}(G,t) \\ -(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{37})^{(7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{21} \quad (71)$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)} & -(b''_{22})^{(3)}(G_{23},t) & -(b''_{18})^{(2,2,2)}(G_{19},t) & -(b''_{15})^{(1,1,1,)}(G,t) \\ -(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{38})^{(7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{22} \quad (72)$$

$-(b''_{20})^{(3)}(G_{23},t)$, $-(b''_{21})^{(3)}(G_{23},t)$, $-(b''_{22})^{(3)}(G_{23},t)$ *are first detrition coefficients for category 1, 2 and 3*

$-(b''_{16})^{(2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2)}(G_{19},t)$ *are second detrition coefficients for category 1, 2 and 3*

$-(b''_{13})^{(1,1,1,)}(G,t)$, $-(b''_{14})^{(1,1,1,)}(G,t)$, $-(b''_{15})^{(1,1,1,)}(G,t)$ *are third detrition coefficients for category 1,2 and 3*

$-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)$ *are fourth detrition coefficients for category 1, 2 and 3*

$-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)$ *are fifth detrition coefficients for category 1, 2 and 3*

$-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)$ *are sixth detrition coefficients for category 1, 2 and 3*

$-(b''_{36})^{(7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$-(b''_{46})^{(9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \begin{bmatrix} (a'_{24})^{(4)} & +(a''_{24})^{(4)}(T_{25},t) & +(a''_{28})^{(5,5,)}(T_{29},t) & +(a''_{32})^{(6,6,)}(T_{33},t) \\ +(a''_{13})^{(1,1,1,1)}(T_{14},t) & +(a''_{16})^{(2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3)}(T_{21},t) \\ +(a''_{36})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9)}(T_{45},t) \end{bmatrix} G_{24} \quad (73)$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \begin{bmatrix} (a'_{25})^{(4)} & +(a''_{25})^{(4)}(T_{25},t) & +(a''_{29})^{(5,5,)}(T_{29},t) & +(a''_{33})^{(6,6)}(T_{33},t) \\ +(a''_{14})^{(1,1,1,1)}(T_{14},t) & +(a''_{17})^{(2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3)}(T_{21},t) \\ +(a''_{37})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9)}(T_{45},t) \end{bmatrix} G_{25} \quad (74)$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \begin{bmatrix} (a'_{26})^{(4)} & +(a''_{26})^{(4)}(T_{25},t) & +(a''_{30})^{(5,5,)}(T_{29},t) & +(a''_{34})^{(6,6,)}(T_{33},t) \\ +(a''_{15})^{(1,1,1,1)}(T_{14},t) & +(a''_{18})^{(2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3)}(T_{21},t) \\ +(a''_{38})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9)}(T_{45},t) \end{bmatrix} G_{26} \quad (75)$$

$+(a''_{24})^{(4)}(T_{25},t)$, $+(a''_{25})^{(4)}(T_{25},t)$, $+(a''_{26})^{(4)}(T_{25},t)$ *are first augmentation coefficients category* 1, 2 3

$+(a''_{28})^{(5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,)}(T_{29},t)$ *are second augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{32})^{(6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,)}(T_{33},t)$ *are third augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{13})^{(1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1)}(T_{14},t)$ *are fourth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{16})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2)}(T_{17},t)$ *are fifth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{20})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3)}(T_{21},t)$ *are sixth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)$ *are seventh augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)$ *are eighth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{46})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9)}(T_{45},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \begin{bmatrix} (b'_{24})^{(4)} & -(b''_{24})^{(4)}(G_{27},t) & -(b''_{28})^{(5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,)}(G_{35},t) \\ -(b''_{13})^{(1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3)}(G_{23},t) \\ -(b''_{36})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{24} \quad (76)$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \begin{bmatrix} (b'_{25})^{(4)} & -(b''_{25})^{(4)}(G_{27},t) & -(b''_{29})^{(5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,)}(G_{35},t) \\ -(b''_{14})^{(1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3)}(G_{23},t) \\ -(b''_{37})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{25} \quad (77)$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)} & -(b''_{26})^{(4)}(G_{27},t) & -(b''_{30})^{(5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,)}(G_{35},t) \\ -(b''_{15})^{(1,1,1,1)}(G,t) & -(b''_{18})^{(2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3)}(G_{23},t) \\ -(b''_{38})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{26} \quad (78)$$

$Where\ \boxed{-(b''_{24})^{(4)}(G_{27},t)}, \boxed{-(b''_{25})^{(4)}(G_{27},t)}, \boxed{-(b''_{26})^{(4)}(G_{27},t)}\ are\ first\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b''_{28})^{(5,5,)}(G_{31},t)}, \boxed{-(b''_{29})^{(5,5,)}(G_{31},t)}, \boxed{-(b''_{30})^{(5,5,)}(G_{31},t)}\ are\ second\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b''_{32})^{(6,6,)}(G_{35},t)}, \boxed{-(b''_{33})^{(6,6,)}(G_{35},t)}, \boxed{-(b''_{34})^{(6,6,)}(G_{35},t)}\ are\ third\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b''_{13})^{(1,1,1,1)}(G,t)}, \boxed{-(b''_{14})^{(1,1,1,1)}(G,t)}, \boxed{-(b''_{15})^{(1,1,1,1)}(G,t)}$
$are\ fourth\ \ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2,2)}(G_{19},t)}$
$are\ fifth\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3,3,3,3)}(G_{23},t)}$
$are\ sixth\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)}$
$are\ seventh\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)}$
$are\ eighth\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b''_{46})^{(9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{45})^{(9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{44})^{(9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[\begin{array}{c} (a'_{28})^{(5)}\boxed{+(a''_{28})^{(5)}(T_{29},t)}\ \boxed{+(a''_{24})^{(4,4,)}(T_{25},t)}\ \boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)}\ \boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)}\ \boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{28} \quad 79$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[\begin{array}{c} (a'_{29})^{(5)}\boxed{+(a''_{29})^{(5)}(T_{29},t)}\ \boxed{+(a''_{25})^{(4,4,)}(T_{25},t)}\ \boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)}\ \boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)}\ \boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{29} \quad 80$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[\begin{array}{c} (a'_{30})^{(5)}\boxed{+(a''_{30})^{(5)}(T_{29},t)}\ \boxed{+(a''_{26})^{(4,4,)}(T_{25},t)}\ \boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)}\ \boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)}\ \boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{30} \quad 81$$

$Where\ \boxed{+(a''_{28})^{(5)}(T_{29},t)}, \boxed{+(a''_{29})^{(5)}(T_{29},t)}, \boxed{+(a''_{30})^{(5)}(T_{29},t)}\ are\ first\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$

$And\ \boxed{+(a''_{24})^{(4,4,)}(T_{25},t)}, \boxed{+(a''_{25})^{(4,4,)}(T_{25},t)}, \boxed{+(a''_{26})^{(4,4,)}(T_{25},t)}\ are\ second\ augmentation\ coefficient\ for\ category\ 1,2\ and\ 3$

$\boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)}, \boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)}, \boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)}\ are\ third\ \ augmentation\ coefficient\ for\ category\ 1,2\ and\ 3$

$\boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)}$ are fourth augmentation coefficients for category 1,2, and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)}$ are fifth augmentation coefficients for category 1,2,and 3

$+(a_{20}'')^{(3,3,3,3,3)}(T_{21},t)$, $+(a_{21}'')^{(3,3,3,3,3)}(T_{21},t)$, $+(a_{22}'')^{(3,3,3,3,3)}(T_{21},t)$ are sixth augmentation coefficients for category 1,2, 3

$+(a_{36}'')^{(7,7,7,7,7,7)}(T_{37},t)$ , $+(a_{37}'')^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a_{38}'')^{(7,7,7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1,2, 3

$+(a_{40}'')^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a_{41}'')^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a_{42}'')^{(8,8,8,8,8,8)}(T_{41},t)$ are eighth augmentation coefficients for category 1,2, 3

$+(a_{46}'')^{(9,9,9,9,9)}(T_{45},t)$, $+(a_{45}'')^{(9,9,9,9,9)}(T_{45},t)$, $+(a_{44}'')^{(9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \begin{bmatrix} (b_{28}')^{(5)} & -(b_{28}'')^{(5)}(G_{31},t) & -(b_{24}'')^{(4,4,)}(G_{27},t) & -(b_{32}'')^{(6,6,6)}(G_{35},t) \\ -(b_{13}'')^{(1,1,1,1,1)}(G,t) & -(b_{16}'')^{(2,2,2,2,2)}(G_{19},t) & -(b_{20}'')^{(3,3,3,3,3)}(G_{23},t) & \\ -(b_{36}'')^{(7,7,7,7,7,7)}(G_{39},t) & -(b_{40}'')^{(8,8,8,8,8,8)}(G_{43},t) & -(b_{44}'')^{(9,9,9,9,9)}(G_{47},t) & \end{bmatrix} T_{28} \quad 82$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b_{29}')^{(5)} & -(b_{29}'')^{(5)}(G_{31},t) & -(b_{25}'')^{(4,4,)}(G_{27},t) & -(b_{33}'')^{(6,6,6)}(G_{35},t) \\ -(b_{14}'')^{(1,1,1,1,1)}(G,t) & -(b_{17}'')^{(2,2,2,2,2)}(G_{19},t) & -(b_{21}'')^{(3,3,3,3,3)}(G_{23},t) & \\ -(b_{37}'')^{(7,7,7,7,7,7)}(G_{39},t) & -(b_{41}'')^{(8,8,8,8,8,8)}(G_{43},t) & -(b_{45}'')^{(9,9,9,9,9)}(G_{47},t) & \end{bmatrix} T_{29} \quad 83$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b_{30}')^{(5)} & -(b_{30}'')^{(5)}(G_{31},t) & -(b_{26}'')^{(4,4,)}(G_{27},t) & -(b_{34}'')^{(6,6,6)}(G_{35},t) \\ -(b_{15}'')^{(1,1,1,1,1,)}(G,t) & -(b_{18}'')^{(2,2,2,2,2)}(G_{19},t) & -(b_{22}'')^{(3,3,3,3,3)}(G_{23},t) & \\ -(b_{38}'')^{(7,7,7,7,7,7)}(G_{39},t) & -(b_{42}'')^{(8,8,8,8,8,8)}(G_{43},t) & -(b_{46}'')^{(9,9,9,9,9)}(G_{47},t) & \end{bmatrix} T_{30} \quad 84$$

*where* $-(b_{28}'')^{(5)}(G_{31},t)$ , $-(b_{29}'')^{(5)}(G_{31},t)$ , $-(b_{30}'')^{(5)}(G_{31},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b_{24}'')^{(4,4,)}(G_{27},t)$, $-(b_{25}'')^{(4,4,)}(G_{27},t)$, $-(b_{26}'')^{(4,4,)}(G_{27},t)$ *are second detrition coefficients for category* 1,2 *and* 3

$-(b_{32}'')^{(6,6,6)}(G_{35},t)$, $-(b_{33}'')^{(6,6,6)}(G_{35},t)$, $-(b_{34}'')^{(6,6,6)}(G_{35},t)$ *are third detrition coefficients for category* 1,2 *and* 3

$-(b_{13}'')^{(1,1,1,1,1)}(G,t)$, $-(b_{14}'')^{(1,1,1,1,1)}(G,t)$ , $-(b_{15}'')^{(1,1,1,1,1,)}(G,t)$ are fourth detrition coefficients for category 1,2, and 3

$-(b_{16}'')^{(2,2,2,2,2)}(G_{19},t)$, $-(b_{17}'')^{(2,2,2,2,2)}(G_{19},t)$, $-(b_{18}'')^{(2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1,2, and 3

$-(b_{20}'')^{(3,3,3,3,3)}(G_{23},t)$, $-(b_{21}'')^{(3,3,3,3,3)}(G_{23},t)$, $-(b_{22}'')^{(3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1,2, and 3

$-(b_{36}'')^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b_{37}'')^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b_{38}'')^{(7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2, and 3

$-(b_{42}'')^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b_{41}'')^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b_{40}'')^{(8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1,2, and 3

$-(b_{46}'')^{(9,9,9,9,9)}(G_{47},t)$, $-(b_{45}'')^{(9,9,9,9,9)}(G_{47},t)$, $-(b_{44}'')^{(9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} \qquad 85$$

$$-\left[\begin{matrix}(a'_{32})^{(6)}\boxed{+(a''_{32})^{(6)}(T_{33},t)}\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}\\ \boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}\boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)}\boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)}\boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)}\end{matrix}\right]G_{32}$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[\begin{matrix}(a'_{33})^{(6)}\boxed{+(a''_{33})^{(6)}(T_{33},t)}\boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}\boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}\\ \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}\boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}\boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)}\boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}\boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)}\end{matrix}\right]G_{33} \qquad 86$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[\begin{matrix}(a'_{34})^{(6)}\boxed{+(a''_{34})^{(6)}(T_{33},t)}\boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}\boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}\\ \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}\boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}\boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)}\boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}\boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)}\end{matrix}\right]G_{34} \qquad 87$$

$\boxed{+(a''_{32})^{(6)}(T_{33},t)}, \boxed{+(a''_{33})^{(6)}(T_{33},t)}, \boxed{+(a''_{34})^{(6)}(T_{33},t)}$ *are first augmentation coefficients for category* $1, 2$ *and* $3$

$\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}$ *are second augmentation coefficients for category* $1, 2$ *and* $3$

$\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}$ *are third augmentation coefficients for category* $1, 2$ *and* $3$

$\boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}$ - are fourth augmentation coefficients

$\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}$ - fifth augmentation coefficients

$\boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)}$ sixth augmentation coefficients

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)}$ seventh augmentation coefficients

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}$ Eighth augmentation coefficients

$\boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)}$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[\begin{matrix}(b'_{32})^{(6)}\boxed{-(b''_{32})^{(6)}(G_{35},t)}\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}\\ \boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)}\boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)}\\ \boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)}\boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)}\end{matrix}\right]T_{32} \qquad 88$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[\begin{matrix}(b'_{33})^{(6)}\boxed{-(b''_{33})^{(6)}(G_{35},t)}\boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}\boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}\\ \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}\boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)}\\ \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)}\boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)}\end{matrix}\right]T_{33} \qquad 89$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)}\boxed{-(b''_{34})^{(6)}(G_{35},t)} & \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{34} \quad 90$$

$\boxed{-(b''_{32})^{(6)}(G_{35},t)}, \boxed{-(b''_{33})^{(6)}(G_{35},t)}, \boxed{-(b''_{34})^{(6)}(G_{35},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}$ *are second detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)}$ *are third detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}$ are fourth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \quad 91$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a'_{36})^{(7)}\boxed{+(a''_{36})^{(7)}(T_{37},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \quad 92$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a'_{37})^{(7)}\boxed{+(a''_{37})^{(7)}(T_{37},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{38}}{dt} \quad 93$$

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a'_{38})^{(7)}\boxed{+(a''_{38})^{(7)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{(a''_{36})^{(7)}(T_{37},t)}, \boxed{(a''_{37})^{(7)}(T_{37},t)}, \boxed{(a''_{38})^{(7)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a_{16}'')^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{17}'')^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{18}'')^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{20}'')^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{21}'')^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{22}'')^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{24}'')^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{25}'')^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{26}'')^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{28}'')^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{29}'')^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{30}'')^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{32}'')^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{33}'')^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{34}'')^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{13}'')^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a_{13}'')^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a_{15}'')^{(1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{42}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a_{41}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a_{40}'')^{(8,8,8,8,8,8,8,8,)}(T_{41},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a_{46}'')^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a_{45}'')^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a_{44}'')^{(9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{36}}{dt} =$$

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$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b_{36}')^{(7)}\boxed{-(b_{36}'')^{(7)}(G_{39},t)} & \boxed{-(b_{16}'')^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{20}'')^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{24}'')^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{28}'')^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{32}'')^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{13}'')^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b_{44}'')^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{37}}{dt} =$$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b_{37}')^{(7)}\boxed{-(b_{37}'')^{(7)}(G_{39},t)} & \boxed{-(b_{17}'')^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{21}'')^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{25}'')^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{29}'')^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{33}'')^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{14}'')^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b_{45}'')^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{38}}{dt} =$$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b_{38}')^{(7)}\boxed{-(b_{38}'')^{(7)}(G_{39},t)} & \boxed{-(b_{18}'')^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{22}'')^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{26}'')^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{30}'')^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{34}'')^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{15}'')^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b_{46}'')^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b_{36}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{37}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{16}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{17}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{18}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{20}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{21}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{22}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{24}'')^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{25}'')^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{26}'')^{(4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition

coefficients for category 1, 2 and 3
$\boxed{-(b_{28}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition
coefficients for category 1, 2 and 3
$\boxed{-(b_{32}'')^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{34}'')^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition
coefficients for category 1, 2 and 3
$\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$
are seventh detrition coefficients for category 1, 2 and 3
$\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition
coefficients for category 1, 2 and 3
$\boxed{-(b_{46}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{45}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{44}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition
coefficients for category 1, 2 and 3



$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \begin{bmatrix} (a_{40}')^{(8)}\boxed{+(a_{40}'')^{(8)}(T_{41},t)} & \boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{20}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{24}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{28}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{32}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{36}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{44}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \begin{bmatrix} (a_{41}')^{(8)}\boxed{+(a_{41}'')^{(8)}(T_{41},t)} & \boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{21}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{25}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{29}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{33}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{37}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{45}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \begin{bmatrix} (a_{42}')^{(8)}\boxed{+(a_{42}'')^{(8)}(T_{41},t)} & \boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{22}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{26}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{30}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{34}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{15}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{38}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{46}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a_{40}'')^{(8)}(T_{41},t)}$, $\boxed{+(a_{41}'')^{(8)}(T_{41},t)}$, $\boxed{+(a_{42}'')^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3
$\boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3
$\boxed{+(a_{20}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{21}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{22}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3
$\boxed{+(a_{24}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{25}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{26}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3
$\boxed{+(a_{28}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{29}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{30}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ $+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ $+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$ are eighth augmentation coefficient for 1,2,3

$+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} =$$

$$(b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)}\left[-(b''_{40})^{(8)}(G_{43},t)\right] & -(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t) & -(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} =$$

$$(b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)}\left[-(b''_{41})^{(8)}(G_{43},t)\right] & -(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t) & -(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} =$$

$$(b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)}\left[-(b''_{42})^{(8)}(G_{43},t)\right] & -(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t) & -(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{15}$$

Where $-(b''_{36})^{(7)}(G_{39},t)$, $-(b''_{37})^{(7)}(G_{39},t)$, $-(b''_{38})^{(7)}(G_{39},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)$, $-(b''_{38})^{(7,7,)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)$ are eighth detrition

coefficients for category 1, 2 and 3
$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3



$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \begin{bmatrix} (a'_{44})^{(9)}\boxed{+(a''_{44})^{(9)}(T_{45},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \begin{bmatrix} (a'_{45})^{(9)}\boxed{+(a''_{45})^{(9)}(T_{45},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \begin{bmatrix} (a'_{46})^{(9)}\boxed{+(a''_{46})^{(9)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a''_{44})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3
$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3
$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3
$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3
$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3
$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3
$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ are Seventh augmentation coefficient for category 1, 2 and 3
$\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3
$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} =$$

$$(b_{44})^{(9)}T_{45} -$$

$$\begin{bmatrix} (b'_{44})^{(9)}\boxed{-(b''_{44})^{(9)}(G_{47},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b'_{45})^{(9)}\boxed{-(b''_{45})^{(9)}(G_{47},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b'_{46})^{(9)}\boxed{-(b''_{46})^{(9)}(G_{47},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{44})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9)}(G_{47},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a'_i)^{(1)}, (a''_i)^{(1)}, (b_i)^{(1)}, (b'_i)^{(1)}, (b''_i)^{(1)} > 0, \quad i,j = 13,14,15 \qquad 97$$

The functions $(a''_i)^{(1)}, (b''_i)^{(1)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a_i'')^{(1)}(T_{14},t) \le (p_i)^{(1)} \le (\hat{A}_{13})^{(1)}$$

$$(b_i'')^{(1)}(G,t) \le (r_i)^{(1)} \le (b_i')^{(1)} \le (\hat{B}_{13})^{(1)}$$

$$lim_{T_2\to\infty}(a_i'')^{(1)}(T_{14},t) = (p_i)^{(1)} \qquad 98$$

$$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = (r_i)^{(1)}$$

**Definition of** $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$ :

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$$|(a_i'')^{(1)}(T_{14}',t) - (a_i'')^{(1)}(T_{14},t)| \le (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G',t) - (b_i'')^{(1)}(G,t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}',t)$ and $(a_i'')^{(1)}(T_{14},t)$ . $(T_{14}',t)$ and $(T_{14},t)$ are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14},t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$ : 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$$

**Definition of** $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$ : 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}$, $(r_i)^{(1)}, i = 13,14,15$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i,j = 16,17,18$$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17},t) \le (p_i)^{(2)} \le \left(\hat{A}_{16}\right)^{(2)} \quad 102$$

$$(b_i'')^{(2)}(G_{19},t) \le (r_i)^{(2)} \le (b_i')^{(2)} \le (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2\to\infty}(a_i'')^{(2)}(T_{17},t) = (p_i)^{(2)} \quad 104$$

$$\lim_{G\to\infty}(b_i'')^{(2)}\left((G_{19}),t\right) = (r_i)^{(2)} \quad 105$$

**Definition of** $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$ : 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}',t) - (a_i'')^{(2)}(T_{17},t)| \le (\hat{k}_{16})^{(2)}|T_{17} - T_{17}'|e^{-(\hat{M}_{16})^{(2)}t} \quad 107$$

$$|(b_i'')^{(2)}((G_{19})',t) - (b_i'')^{(2)}\left((G_{19}),t\right)| < (\hat{k}_{16})^{(2)}||(G_{19}) - (G_{19})'||e^{-(\hat{M}_{16})^{(2)}t} \quad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}',t)$ and$(a_i'')^{(2)}(T_{17},t)$ . $(T_{17}',t)$ and $(T_{17},t)$ are points belonging to the interval $\left[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}\right]$. It is to be noted that $(a_i'')^{(2)}(T_{17},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17},t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$ :

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants 109

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$$

**Definition of** $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$ :

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1 \quad 110$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(b_i)^{(2)} + (b_i')^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1 \quad 111$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i,j = 20,21,22 \quad 112$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(3)}$, $(\mathrm{r_i})^{(3)}$:

$$(a_i'')^{(3)}(T_{21}, t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23}, t) \leq (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(3)}(T_{21}, t) = (p_i)^{(3)} \quad (113)$$

$$\lim_{G \to \infty}(b_i'')^{(3)}(G_{23}, t) = (r_i)^{(3)}$$

**Definition of** $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$ :

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: (114)

$$|(a_i'')^{(3)}(T_{21}', t) - (a_i'')^{(3)}(T_{21}, t)| \leq (\hat{k}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\hat{M}_{20})^{(3)}t}$$

$$|(b_i'')^{(3)}(G_{23}', t) - (b_i'')^{(3)}(G_{23}, t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}'||e^{-(\hat{M}_{20})^{(3)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}', t)$ and$(a_i'')^{(3)}(T_{21}, t)$ . $(T_{21}', t)$ And $(T_{21}, t)$ are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21}, t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$ : (115)

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with (116)
$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and (\hat{B}_{20})^{(3)}$ and the constants
$(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}, i = 20,21,22,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(a_i)^{(3)} + (a_i')^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(b_i)^{(3)} + (b_i')^{(3)} + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i, j = 24,25,26 \quad (117)$$

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(4)}$, $(r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25},t) \le (p_i)^{(4)} \le (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}\big((G_{27}),t\big) \le (r_i)^{(4)} \le (b_i')^{(4)} \le (\hat{B}_{24})^{(4)}$$

$$lim_{T_2\to\infty}(a_i'')^{(4)}(T_{25},t) = (p_i)^{(4)} \quad (118)$$

$$\lim_{G\to\infty}(b_i'')^{(4)}\big((G_{27}),t\big) = (r_i)^{(4)}$$

**Definition of** $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$ :

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24,25,26}$

They satisfy Lipschitz condition: (119)

$$|(a_i'')^{(4)}(T_{25}',t) - (a_i'')^{(4)}(T_{25},t)| \le (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})',t) - (b_i'')^{(4)}\big((G_{27}),t\big)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}',t)$ and$(a_i'')^{(4)}(T_{25},t)$ . $(T_{25}',t)$ and $(T_{25},t)$ are points belonging to the interval $\big[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}\big]$. It is to be noted that $(a_i'')^{(4)}(T_{25},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25},t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$ : (120)

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$, are positive constants

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}, \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$$

**Definition of** $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$ : (121)

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and\ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}$, $(r_i)^{(4)}, i = 24,25,26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(a_i)^{(4)} + (a_i')^{(4)} + (\hat{A}_{24})^{(4)} + (\hat{P}_{24})^{(4)}(\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(b_i)^{(4)} + (b_i')^{(4)} + (\hat{B}_{24})^{(4)} + (\hat{Q}_{24})^{(4)}(\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i,j = 28,29,30 \quad (122)$$

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(5)}$, $(r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29},t) \le (p_i)^{(5)} \le (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}\big((G_{31}),t\big) \le (r_i)^{(5)} \le (b_i')^{(5)} \le (\hat{B}_{28})^{(5)}$$

$$lim_{T_2\to\infty}(a_i'')^{(5)}(T_{29},t) = (p_i)^{(5)} \qquad 123$$

$$\lim_{G\to\infty}(b_i'')^{(5)}(G_{31},t) = (r_i)^{(5)}$$

**Definition of** $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$ :

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}',t) - (a_i'')^{(5)}(T_{29},t)| \le (\hat{k}_{28})^{(5)}|T_{29} - T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})',t) - (b_i'')^{(5)}\big((G_{31}),t\big)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}',t)$ and$(a_i'')^{(5)}(T_{29},t)$ . $(T_{29}',t)$ and $(T_{29},t)$ are points belonging to the interval $\big[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\big]$. It is to be noted that $(a_i'')^{(5)}(T_{29},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29},t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$ : 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}, \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

**Definition of** $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$ : 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and\ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(a_i)^{(5)} + (a_i')^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)}(\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(b_i)^{(5)} + (b_i')^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)}(\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i,j = 32,33,34 \qquad 127$$

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(6)}$, $(r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33},t) \le (p_i)^{(6)} \le (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}),t) \le (r_i)^{(6)} \le (b_i')^{(6)} \le (\hat{B}_{32})^{(6)}$$

$$lim_{T_2\to\infty}(a_i'')^{(6)}(T_{33},t) = (p_i)^{(6)} \qquad 128$$

$$\lim_{G\to\infty}(b_i'')^{(6)}\left((G_{35}),t\right) = (r_i)^{(6)}$$

**Definition of** $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$ :

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}',t) - (a_i'')^{(6)}(T_{33},t)| \le (\hat{k}_{32})^{(6)}|T_{33} - T_{33}'|e^{-(\hat{M}_{32})^{(6)}t}$$

$$|(b_i'')^{(6)}((G_{35})',t) - (b_i'')^{(6)}\left((G_{35}),t\right)| < (\hat{k}_{32})^{(6)}||(G_{35}) - (G_{35})'||e^{-(\hat{M}_{32})^{(6)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}',t)$ and$(a_i'')^{(6)}(T_{33},t)$ . $(T_{33}',t)$ and $(T_{33},t)$ are points belonging to the interval $\left[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}\right]$. It is to be noted that $(a_i'')^{(6)}(T_{33},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33},t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$ : 129

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$, are positive constants

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$$

**Definition of** $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$ : 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and\ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32,33,34,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(a_i)^{(6)} + (a_i')^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(b_i)^{(6)} + (b_i')^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

(Y) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i,j = 36,37,38$ 131

(Z) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(7)}$, $(r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37},t) \le (p_i)^{(7)} \le (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39},t) \le (r_i)^{(7)} \le (b_i')^{(7)} \le (\hat{B}_{36})^{(7)} \quad (132)$$

*(AA)* $\lim_{T_2\to\infty} (a_i'')^{(7)}(T_{37},t) = (p_i)^{(7)}$

*(BB)*

$\lim_{G\to\infty}(b_i'')^{(7)}\big((G_{39}),t\big) = (r_i)^{(7)}$

**Definition of** $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$ :

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: (133)

$$|(a_i'')^{(7)}(T_{37}',t) - (a_i'')^{(7)}(T_{37},t)| \le (\hat{k}_{36})^{(7)}|T_{37} - T_{37}'|e^{-(\hat{M}_{36})^{(7)}t}$$

$$|(b_i'')^{(7)}((G_{39})',t) - (b_i'')^{(7)}\big((G_{39}),t\big)| < (\hat{k}_{36})^{(7)}||(G_{39}) - (G_{39})'||e^{-(\hat{M}_{36})^{(7)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}',t)$ and$(a_i'')^{(7)}(T_{37},t)$ . $(T_{37}',t)$ and $(T_{37},t)$ are points belonging to the interval $\big[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}\big]$. It is to be noted that $(a_i'')^{(7)}(T_{37},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37},t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$ : (134)

(CC) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$, are positive constants

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$$

**Definition of** $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$ : (135)

(DD) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)}(\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(b_i)^{(7)} + (b_i')^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)}(\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40,41,42 \tag{136}$$

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

**Definition of** $(p_i)^{(8)}$, $(r_i)^{(8)}$: (137)

$$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)} \tag{138}$$

$$(b_i'')^{(8)}((G_{43}), t) \leq (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)} \tag{139}$$

$$\lim_{T_2 \to \infty} (a_i'')^{(8)}(T_{41}, t) = (p_i)^{(8)} \tag{140}$$

$$\lim_{G \to \infty} (b_i'')^{(8)}((G_{43}), t) = (r_i)^{(8)} \tag{141}$$

**Definition of** $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$ :

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)} |T_{41} - T_{41}'| e^{-(\hat{M}_{40})^{(8)} t} \tag{142}$$

$$|(b_i'')^{(8)}((G_{43})', t) - (b_i'')^{(8)}((G_{43}), t)| < (\hat{k}_{40})^{(8)} ||(G_{43}) - (G_{43})'|| e^{-(\hat{M}_{40})^{(8)} t} \tag{143}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}', t)$ and $(a_i'')^{(8)}(T_{41}, t)$ . $(T_{41}', t)$ and $(T_{41}, t)$ are points belonging to the interval $[(\hat{k}_{40})^{(8)}, (\hat{M}_{40})^{(8)}]$. It is to be noted that $(a_i'')^{(8)}(T_{41}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{40})^{(8)} = 1$ then the function $(a_i'')^{(8)}(T_{41}, t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$ :

$(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$, are positive constants

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}, \frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1 \tag{144}$$

**Definition of** $(\hat{P}_{40})^{(8)}, (\hat{Q}_{40})^{(8)}$ :

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}, (\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)}, (a_i')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}, i = 40,41,42,$

Satisfy the inequalities

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(a_i)^{(8)} + (a_i')^{(8)} + (\hat{A}_{40})^{(8)} + (\hat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}] < 1 \quad 145$$

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(b_i)^{(8)} + (b_i')^{(8)} + (\hat{B}_{40})^{(8)} + (\hat{Q}_{40})^{(8)}(\hat{k}_{40})^{(8)}] < 1 \quad 146$$

Where we suppose

$$(a_i)^{(9)}, (a_i')^{(9)}, (a_i'')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (b_i'')^{(9)} > 0, \quad i,j = 44,45,46 \quad 146A$$

The functions $(a_i'')^{(9)}, (b_i'')^{(9)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(9)}$, $(r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45}, t) \leq (p_i)^{(9)} \leq (\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47}, t) \leq (r_i)^{(9)} \leq (b_i')^{(9)} \leq (\hat{B}_{44})^{(9)}$$

$$\lim_{T_2 \to \infty} (a_i'')^{(9)}(T_{45}, t) = (p_i)^{(9)}$$

$$\lim_{G \to \infty} (b_i'')^{(9)}(G_{47}, t) = (r_i)^{(9)}$$

**Definition of** $(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}$ :

Where $\boxed{(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}}$ are positive constants and $\boxed{i = 44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}', t) - (a_i'')^{(9)}(T_{45}, t)| \leq (\hat{k}_{44})^{(9)}|T_{45} - T_{45}'|e^{-(\hat{M}_{44})^{(9)}t}$$

$$|(b_i'')^{(9)}((G_{47})', t) - (b_i'')^{(9)}((G_{47}), t)| < (\hat{k}_{44})^{(9)}||(G_{47}) - (G_{47})'||e^{-(\hat{M}_{44})^{(9)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}', t)$ and$(a_i'')^{(9)}(T_{45}, t)$ . $(T_{45}', t)$ and $(T_{45}, t)$ are points belonging to the interval $[(\hat{k}_{44})^{(9)}, (\hat{M}_{44})^{(9)}]$. It is to be noted that $(a_i'')^{(9)}(T_{45}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{44})^{(9)} = 1$ then the function $(a_i'')^{(9)}(T_{45}, t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$ :

$(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$, are positive constants

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}, \frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$$

**Definition of** $(\hat{P}_{44})^{(9)}, (\hat{Q}_{44})^{(9)}$ :

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}, (\hat{A}_{44})^{(9)} and (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)}, (a_i')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}, i = 44,45,46,$
satisfy the inequalities

$$\frac{1}{(\widehat{M}_{44})^{(9)}}[(a_i)^{(9)} + (a_i')^{(9)} + (\hat{A}_{44})^{(9)} + (\hat{P}_{44})^{(9)}(\hat{k}_{44})^{(9)}] < 1$$

$$\frac{1}{(\widehat{M}_{44})^{(9)}}[(b_i)^{(9)} + (b_i')^{(9)} + (\hat{B}_{44})^{(9)} + (\hat{Q}_{44})^{(9)}(\hat{k}_{44})^{(9)}] < 1$$

**Theorem 1:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0), T_i(0)$ :

$$G_i(t) \leq (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 2 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

**Definition of** $G_i(0), T_i(0)$

$$G_i(t) \leq (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} \quad , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \leq (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} \quad , \quad T_i(0) = T_i^0 > 0$$

**Theorem 3 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t) \leq (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t} \quad , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \leq (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t} \quad , \quad T_i(0) = T_i^0 > 0$$

**Theorem 4 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

**Definition of** $G_i(0), T_i(0)$ :

$$G_i(t) \leq (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 5 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

**Definition of** $G_i(0), T_i(0)$ :

$$G_i(t) \leq (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 6 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

**Definition of** $G_i(0), T_i(0)$ :

$G_i(t) \le \left(\hat{P}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \le \left(\hat{Q}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Theorem 7:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

**Definition of** $G_i(0), T_i(0)$ :

$G_i(t) \le \left(\hat{P}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \le \left(\hat{Q}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Theorem 8:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

**Definition of** $G_i(0), T_i(0)$ :

$G_i(t) \le \left(\hat{P}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \le \left(\hat{Q}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Theorem 9:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

**Definition of** $G_i(0), T_i(0)$ :

$G_i(t) \le \left(\hat{P}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \le \left(\hat{Q}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Proof:** Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i$: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$G_i(0) = G_i^0$ , $T_i(0) = T_i^0$ , $G_i^0 \le \left(\hat{P}_{13}\right)^{(1)}$ , $T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)}$, 155

$0 \le G_i(t) - G_i^0 \le \left(\hat{P}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t}$ 156

$0 \le T_i(t) - T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t}$ 157

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}(s_{(13)}) - \left((a'_{13})^{(1)} + a''_{13})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{15}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^{0} + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^{0} + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{\mathrm{T}}_{15}(\mathrm{t}) = \mathrm{T}_{15}^{0} + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{15}(s_{(13)})\right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{16})^{(2)}$, $T_i^0 \le (\hat{Q}_{16})^{(2)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

By 160

$$\bar{G}_{16}(t) = G_{16}^{0} + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''_{16})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^{0} + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}(s_{(16)}), s_{(17)})\right) G_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^{0} + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{18}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^{0} + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^{0} + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^{0} + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{18}(s_{(16)})\right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{20})^{(3)}$, $T_i^0 \le (\hat{Q}_{20})^{(3)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

By 161

$$\bar{G}_{20}(t) = G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''_{20})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{22}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^0 + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^0 + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{\mathrm{T}}_{22}(\mathrm{t}) = \mathrm{T}_{22}^0 + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{22}(s_{(20)})\right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{24})^{(4)}, T_i^0 \le (\hat{Q}_{24})^{(4)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

By 162

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{26}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{\mathrm{T}}_{26}(\mathrm{t}) = \mathrm{T}_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{26}(s_{(24)})\right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{28})^{(5)}, T_i^0 \le (\hat{Q}_{28})^{(5)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{28})^{(5)} e^{(\tilde{M}_{28})^{(5)} t}$

By 163

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[ (a_{28})^{(5)} G_{29}(s_{(28)}) - \left( (a'_{28})^{(5)} + a''_{28})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{28}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[ (a_{29})^{(5)} G_{28}(s_{(28)}) - \left( (a'_{29})^{(5)} + (a''_{29})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{29}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[ (a_{30})^{(5)} G_{29}(s_{(28)}) - \left( (a'_{30})^{(5)} + (a''_{30})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{30}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[ (b_{28})^{(5)} T_{29}(s_{(28)}) - \left( (b'_{28})^{(5)} - (b''_{28})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{28}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[ (b_{29})^{(5)} T_{28}(s_{(28)}) - \left( (b'_{29})^{(5)} - (b''_{29})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{29}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{\mathrm{T}}_{30}(\mathrm{t}) = \mathrm{T}_{30}^0 + \int_0^t \left[ (b_{30})^{(5)} T_{29}(s_{(28)}) - \left( (b'_{30})^{(5)} - (b''_{30})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{30}(s_{(28)}) \right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{32})^{(6)}$, $T_i^0 \le (\hat{Q}_{32})^{(6)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\tilde{M}_{32})^{(6)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\tilde{M}_{32})^{(6)} t}$

By 164

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[ (a_{32})^{(6)} G_{33}(s_{(32)}) - \left( (a'_{32})^{(6)} + a''_{32})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[ (a_{33})^{(6)} G_{32}(s_{(32)}) - \left( (a'_{33})^{(6)} + (a''_{33})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[ (a_{34})^{(6)} G_{33}(s_{(32)}) - \left( (a'_{34})^{(6)} + (a''_{34})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{34}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[ (b_{32})^{(6)} T_{33}(s_{(32)}) - \left( (b'_{32})^{(6)} - (b''_{32})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[ (b_{33})^{(6)} T_{32}(s_{(32)}) - \left( (b'_{33})^{(6)} - (b''_{33})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{\mathrm{T}}_{34}(\mathrm{t}) = \mathrm{T}_{34}^0 + \int_0^t \left[ (b_{34})^{(6)} T_{33}(s_{(32)}) - \left( (b'_{34})^{(6)} - (b''_{34})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{34}(s_{(32)}) \right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} G_{37}(s_{(36)}) - \left((a'_{36})^{(7)} + a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[(a_{37})^{(7)} G_{36}(s_{(36)}) - \left((a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[(a_{38})^{(7)} G_{37}(s_{(36)}) - \left((a'_{38})^{(7)} + (a''_{38})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{38}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[(b_{36})^{(7)} T_{37}(s_{(36)}) - \left((b'_{36})^{(7)} - (b''_{36})^{(7)} (G_{39}(s_{(36)}), s_{(36)})\right) T_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[(b_{37})^{(7)} T_{36}(s_{(36)}) - \left((b'_{37})^{(7)} - (b''_{37})^{(7)} (G_{39}(s_{(36)}), s_{(36)})\right) T_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{38}(t) = T_{38}^0 + \int_0^t \left[(b_{38})^{(7)} T_{37}(s_{(36)}) - \left((b'_{38})^{(7)} - (b''_{38})^{(7)} (G_{39}(s_{(36)}), s_{(36)})\right) T_{38}(s_{(36)})\right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{40})^{(8)}$, $T_i^0 \le (\hat{Q}_{40})^{(8)}$,

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} G_{41}(s_{(40)}) - \left((a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[(a_{41})^{(8)} G_{40}(s_{(40)}) - \left((a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41}(s_{(40)}), s_{(40)})\right) G_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}(s_{(40)}), s_{(40)})\right) G_{42}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^0 + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43}(s_{(40)}), s_{(40)})\right) T_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^0 + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43}(s_{(40)}), s_{(40)})\right) T_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{\mathrm{T}}_{42}(\mathrm{t}) = \mathrm{T}_{42}^0 + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43}(s_{(40)}), s_{(40)})\right) T_{42}(s_{(40)})\right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** 166

Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ A which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{44})^{(9)},\ T_i^0 \le (\hat{Q}_{44})^{(9)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''_{44})^{(9)}(T_{45}(s_{(44)}), s_{(44)})\right) G_{44}(s_{(44)})\right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45}(s_{(44)}), s_{(44)})\right) G_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}(s_{(44)}), s_{(44)})\right) G_{46}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{44}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{\mathrm{T}}_{46}(\mathrm{t}) = \mathrm{T}_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{46}(s_{(44)})\right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167
$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}}\right)\right] ds_{(13)} =$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left(e^{(\hat{M}_{13})^{(1)} t} - 1\right)$$

From which it follows that 168

$$(G_{13}(t)-G_{13}^{0})e^{-(\hat{M}_{13})^{(1)}t}\leq\frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}}\left[((\hat{P}_{13})^{(1)}+G_{14}^{0})e^{\left(-\frac{(\hat{P}_{13})^{(1)}+G_{14}^{0}}{G_{14}^{0}}\right)}+(\hat{P}_{13})^{(1)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t)\leq G_{16}^{0}+\int_{0}^{t}\left[(a_{16})^{(2)}\left(G_{17}^{0}+(\hat{P}_{16})^{(6)}e^{(\hat{M}_{16})^{(2)}s_{(16)}}\right)\right]ds_{(16)}= \quad 169$$

$$\left(1+(a_{16})^{(2)}t\right)G_{17}^{0}+\frac{(a_{16})^{(2)}(\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}}\left(e^{(\hat{M}_{16})^{(2)}t}-1\right)$$

From which it follows that 170

$$(G_{16}(t)-G_{16}^{0})e^{-(\hat{M}_{16})^{(2)}t}\leq\frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}}\left[((\hat{P}_{16})^{(2)}+G_{17}^{0})e^{\left(-\frac{(\hat{P}_{16})^{(2)}+G_{17}^{0}}{G_{17}^{0}}\right)}+(\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t)\leq G_{20}^{0}+\int_{0}^{t}\left[(a_{20})^{(3)}\left(G_{21}^{0}+(\hat{P}_{20})^{(3)}e^{(\hat{M}_{20})^{(3)}s_{(20)}}\right)\right]ds_{(20)}=$$

$$\left(1+(a_{20})^{(3)}t\right)G_{21}^{0}+\frac{(a_{20})^{(3)}(\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left(e^{(\hat{M}_{20})^{(3)}t}-1\right)$$

From which it follows that 172

$$(G_{20}(t)-G_{20}^{0})e^{-(\hat{M}_{20})^{(3)}t}\leq\frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left[((\hat{P}_{20})^{(3)}+G_{21}^{0})e^{\left(-\frac{(\hat{P}_{20})^{(3)}+G_{21}^{0}}{G_{21}^{0}}\right)}+(\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 173

$$G_{24}(t)\leq G_{24}^{0}+\int_{0}^{t}\left[(a_{24})^{(4)}\left(G_{25}^{0}+(\hat{P}_{24})^{(4)}e^{(\hat{M}_{24})^{(4)}s_{(24)}}\right)\right]ds_{(24)}=$$

$$\left(1+(a_{24})^{(4)}t\right)G_{25}^{0}+\frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left(e^{(\hat{M}_{24})^{(4)}t}-1\right)$$

From which it follows that 174

$$(G_{24}(t)-G_{24}^{0})e^{-(\hat{M}_{24})^{(4)}t}\leq\frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left[((\hat{P}_{24})^{(4)}+G_{25}^{0})e^{\left(-\frac{(\hat{P}_{24})^{(4)}+G_{25}^{0}}{G_{25}^{0}}\right)}+(\hat{P}_{24})^{(4)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t) \le G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} \left(G_{29}^0 + (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} s_{(28)}}\right)\right] ds_{(28)} =$$

$$\left(1 + (a_{28})^{(5)} t\right) G_{29}^0 + \frac{(a_{28})^{(5)} (\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}} \left(e^{(\hat{M}_{28})^{(5)} t} - 1\right)$$

From which it follows that 175

$$(G_{28}(t) - G_{28}^0) e^{-(\hat{M}_{28})^{(5)} t} \le \frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}} \left[\left((\hat{P}_{28})^{(5)} + G_{29}^0\right) e^{\left(-\frac{(\hat{P}_{28})^{(5)} + G_{29}^0}{G_{29}^0}\right)} + (\hat{P}_{28})^{(5)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t) \le G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} \left(G_{33}^0 + (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} s_{(32)}}\right)\right] ds_{(32)} =$$

$$\left(1 + (a_{32})^{(6)} t\right) G_{33}^0 + \frac{(a_{32})^{(6)} (\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}} \left(e^{(\hat{M}_{32})^{(6)} t} - 1\right)$$

From which it follows that 177

$$(G_{32}(t) - G_{32}^0) e^{-(\hat{M}_{32})^{(6)} t} \le \frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}} \left[\left((\hat{P}_{32})^{(6)} + G_{33}^0\right) e^{\left(-\frac{(\hat{P}_{32})^{(6)} + G_{33}^0}{G_{33}^0}\right)} + (\hat{P}_{32})^{(6)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

(e) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t) \le G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} \left(G_{37}^0 + (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} s_{(36)}}\right)\right] ds_{(36)} =$$

$$\left(1 + (a_{36})^{(7)} t\right) G_{37}^0 + \frac{(a_{36})^{(7)} (\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left(e^{(\hat{M}_{36})^{(7)} t} - 1\right)$$

From which it follows that

$$(G_{36}(t) - G_{36}^0) e^{-(\hat{M}_{36})^{(7)} t} \le \frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left[\left((\hat{P}_{36})^{(7)} + G_{37}^0\right) e^{\left(-\frac{(\hat{P}_{36})^{(7)} + G_{37}^0}{G_{37}^0}\right)} + (\hat{P}_{36})^{(7)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t) \le G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} \left(G_{41}^0 + (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} s_{(40)}}\right)\right] ds_{(40)} =$$ 180

$$\left(1+(a_{40})^{(8)}t\right)G_{41}^{0}+\frac{(a_{40})^{(8)}(\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left(e^{(\hat{M}_{40})^{(8)}t}-1\right)$$

From which it follows that 181

$$(G_{40}(t)-G_{40}^{0})e^{-(\hat{M}_{40})^{(8)}t}\leq\frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left[\left((\hat{P}_{40})^{(8)}+G_{41}^{0}\right)e^{\left(-\frac{(\hat{P}_{40})^{(8)}+G_{41}^{0}}{G_{41}^{0}}\right)}+(\hat{P}_{40})^{(8)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 8

Analogous inequalities hold also for $G_{41}, G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t)\leq G_{44}^{0}+\int_{0}^{t}\left[(a_{44})^{(9)}\left(G_{45}^{0}+(\hat{P}_{44})^{(9)}e^{(\hat{M}_{44})^{(9)}s_{(44)}}\right)\right]ds_{(44)}=$$

$$\left(1+(a_{44})^{(9)}t\right)G_{45}^{0}+\frac{(a_{44})^{(9)}(\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}}\left(e^{(\hat{M}_{44})^{(9)}t}-1\right)$$

From which it follows that

$$(G_{44}(t)-G_{44}^{0})e^{-(\hat{M}_{44})^{(9)}t}\leq\frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}}\left[\left((\hat{P}_{44})^{(9)}+G_{45}^{0}\right)e^{\left(-\frac{(\hat{P}_{44})^{(9)}+G_{45}^{0}}{G_{45}^{0}}\right)}+(\hat{P}_{44})^{(9)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 9

Analogous inequalities hold also for $G_{45}, G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}<1$ and to choose 182

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[(\hat{P}_{13})^{(1)}+\left((\hat{P}_{13})^{(1)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{13})^{(1)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{13})^{(1)} \quad 183$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[\left((\hat{Q}_{13})^{(1)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{13})^{(1)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{13})^{(1)}\right]\leq(\hat{Q}_{13})^{(1)} \quad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)},T^{(1)}\right),\left(G^{(2)},T^{(2)}\right)\right)=$$

$$\sup_i\left\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{13})^{(1)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{13})^{(1)}t}\right\}$$

Indeed if we denote

**Definition of** $\tilde{G},\tilde{T}:\left(\tilde{G},\tilde{T}\right)=\mathcal{A}^{(1)}(G,T)$

It results

$$\left|\tilde{G}_{13}^{(1)}-\tilde{G}_{i}^{(2)}\right|\leq\int_{0}^{t}(a_{13})^{(1)}\left|G_{14}^{(1)}-G_{14}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\,ds_{(13)}+$$

$$\int_{0}^{t}\{(a'_{13})^{(1)}\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$G_{13}^{(2)}|(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)-(a''_{13})^{(1)}\left(T_{14}^{(2)},s_{(13)}\right)|\;\;e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses it follows

$$\left|G^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}t}\leq \qquad 186$$

$$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)}+\;(a'_{13})^{(1)}+(\widehat{A}_{13})^{(1)}+(\widehat{P}_{13})^{(1)}(\widehat{k}_{13})^{(1)}\right)d\left(\left(G^{(1)},T^{(1)};\;G^{(2)},T^{(2)}\right)\right)$$

And analogous inequalities for $G_i\;and\;T_i$. Taking into account the hypothesis the result follows

**Remark 1:** The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}\;and\;(\widehat{Q}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_{+}$.

If instead of proving the existence of the solution on $\mathbb{R}_{+}$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)},i=13{,}14{,}15$ depend only on $\mathrm{T}_{14}$ and respectively on $G(and\;not\;on\;\;t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any $t$ where $G_i\,(t)=0\;and\;T_i\,(t)=0$

From 19 to 24 it results

$$G_i\,(t)\geq G_i^0e^{\left[-\int_0^t\{(a_i')^{(1)}-(a_i'')^{(1)}(T_{14}(s_{(13)}),s_{(13)})\}ds_{(13)}\right]}\geq 0$$

$$T_i\,(t)\geq T_i^0e^{\left(-(b_i')^{(1)}t\right)}>0\quad\text{for t}>0$$

**Definition of** $\left((\widehat{M}_{13})^{(1)}\right)_1,\left((\widehat{M}_{13})^{(1)}\right)_2\;and\;\left((\widehat{M}_{13})^{(1)}\right)_3$: 187

**Remark 3**: if $G_{13}$ is bounded, the same property have also $G_{14}\;and\;G_{15}$ . indeed if

$G_{13}<(\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt}\leq\left((\widehat{M}_{13})^{(1)}\right)_1-(a'_{14})^{(1)}G_{14}$ and by integrating

$$G_{14}\leq\left((\widehat{M}_{13})^{(1)}\right)_2=G_{14}^0+2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1/(a'_{14})^{(1)}$$

In the same way , one can obtain

$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2/(a'_{15})^{(1)}$

If $G_{14}$ *or* $G_{15}$ is bounded, the same property follows for $G_{13}$ , $G_{15}$ and $G_{13}$ , $G_{14}$ respectively.

**Remark 4:** If $G_{13}$ *is* bounded, from below, the same property holds for $G_{14}$ *and* $G_{15}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{14}$ is bounded from below. 188

**Remark 5:** If $T_{13}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}(G(t),t)) = (b'_{14})^{(1)}$ then $T_{14} \to \infty$. 189

**Definition of** $(m)^{(1)}$ and $\varepsilon_1$ :

Indeed let $t_1$ be so that for $t > t_1$

$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t),t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\frac{2}{\varepsilon_1}$ By taking now $\varepsilon_1$ sufficiently small one sees that $T_{14}$ is unbounded.
The same property holds for $T_{15}$ if $\lim_{t\to\infty}(b''_{15})^{(1)}(G(t),t) = (b'_{15})^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}$ , $\frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}$ *and* $(\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[(\widehat{P}_{16})^{(2)} + \left((\hat{P}_{16})^{(2)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{16})^{(2)}\right] \leq (\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$d\left(\left((G_{19})^{(1)},(T_{19})^{(1)}\right),\left((G_{19})^{(2)},(T_{19})^{(2)}\right)\right) =$

$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t}\}$

Indeed if we denote 195

**Definition of** $\widetilde{G_{19}}, \widetilde{T_{19}}$ : $\left(\widetilde{G_{19}}, \widetilde{T_{19}}\right) = \mathcal{A}^{(2)}(G_{19}, T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{16})^{(2)} \left|G_{17}^{(1)} - G_{17}^{(2)}\right| e^{-(\widehat{M}_{16})^{(2)} s_{(16)}} e^{(\widehat{M}_{16})^{(2)} s_{(16)}}\, ds_{(16)} +$$

$$\int_0^t \{(a'_{16})^{(2)} \left|G_{16}^{(1)} - G_{16}^{(2)}\right| e^{-(\widehat{M}_{16})^{(2)} s_{(16)}} e^{-(\widehat{M}_{16})^{(2)} s_{(16)}} +$$

$$(a''_{16})^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right)\left|G_{16}^{(1)} - G_{16}^{(2)}\right| e^{-(\widehat{M}_{16})^{(2)} s_{(16)}} e^{(\widehat{M}_{16})^{(2)} s_{(16)}} +$$

$$G_{16}^{(2)} \left|(a''_{16})^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right) - (a''_{16})^{(2)}\left(T_{17}^{(2)}, s_{(16)}\right)\right| \; e^{-(\widehat{M}_{16})^{(2)} s_{(16)}} e^{(\widehat{M}_{16})^{(2)} s_{(16)}}\} ds_{(16)}$$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0, t]$ 197

From the hypotheses it follows

$$\left|(G_{19})^{(1)} - (G_{19})^{(2)}\right| e^{-(\widehat{M}_{16})^{(2)} t} \leq$$
$$\frac{1}{(\widehat{M}_{16})^{(2)}}\left((a_{16})^{(2)} + (a'_{16})^{(2)} + (\widehat{A}_{16})^{(2)} + (\widehat{P}_{16})^{(2)}(\widehat{k}_{16})^{(2)}\right) d\left(((G_{19})^{(1)}, (T_{19})^{(1)};\ (G_{19})^{(2)}, (T_{19})^{(2)})\right)$$

And analogous inequalities for $G_i$ and $T_i$. Taking into account the hypothesis the result follows 198

**Remark 6:** The fact that we supposed $(a''_{16})^{(2)}$ and $(b''_{16})^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{16})^{(2)} e^{(\widehat{M}_{16})^{(2)} t}$ and $(\widehat{Q}_{16})^{(2)} e^{(\widehat{M}_{16})^{(2)} t}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i = 16,17,18$ depend only on $T_{17}$ and respectively on $(G_{19})$(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 7:** There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$ 200

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(2)} - (a_i'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})\} ds_{(16)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(2)} t)} > 0 \quad \text{for } t > 0$$

**Definition of** $\left((\widehat{M}_{16})^{(2)}\right)_1$, $\left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$ : 201

**Remark 8:** if $G_{16}$ is bounded, the same property have also $G_{17}$ and $G_{18}$ . indeed if

$G_{16} < (\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt} \leq \left((\widehat{M}_{16})^{(2)}\right)_1 - (a'_{17})^{(2)} G_{17}$ and by integrating

$$G_{17} \leq \left((\widehat{M}_{16})^{(2)}\right)_2 = G_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1 / (a'_{17})^{(2)}$$

In the same way , one can obtain

$G_{18} \leq \left((\widehat{M}_{16})^{(2)}\right)_3 = G_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2/(a'_{18})^{(2)}$

If $G_{17}$ or $G_{18}$ is bounded, the same property follows for $G_{16}$ , $G_{18}$ and $G_{16}$ , $G_{17}$ respectively.

**Remark 9**: If $G_{16}$ is bounded, from below, the same property holds for $G_{17}$ and $G_{18}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{17}$ is bounded from below. 202

**Remark 10:** If $T_{16}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t),t)) = (b'_{17})^{(2)}$ then $T_{17} \to \infty$. 203

**Definition of** $(m)^{(2)}$ and $\varepsilon_2$ :

Indeed let $t_2$ be so that for $t > t_2$

$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t),t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right), \quad t = \log\frac{2}{\varepsilon_2}$ By taking now $\varepsilon_2$ sufficiently small one sees that $T_{17}$ is unbounded. 205
The same property holds for $T_{18}$ if $\lim_{t\to\infty}(b''_{18})^{(2)}\left((G_{19})(t),t\right) = (b'_{18})^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}$ , $\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[(\hat{P}_{20})^{(3)} + \left((\hat{P}_{20})^{(3)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{20})^{(3)}\right] \leq (\hat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself 210

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)},(T_{23})^{(1)}),((G_{23})^{(2)},(T_{23})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{20})^{(3)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

**Definition of** $\widetilde{G_{23}}, \widetilde{T_{23}} :\left( \widetilde{(G_{23})}, \widetilde{(T_{23})} \right) = \mathcal{A}^{(3)}\left((G_{23}), (T_{23})\right)$

It results 213

$$\left|\tilde{G}_{20}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{20})^{(3)} \left|G_{21}^{(1)} - G_{21}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} e^{(\widehat{M}_{20})^{(3)} s_{(20)}}\, ds_{(20)} +$$

$$\int_0^t \{(a'_{20})^{(3)} \left|G_{20}^{(1)} - G_{20}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} +$$

$$(a''_{20})^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right)\left|G_{20}^{(1)} - G_{20}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} e^{(\widehat{M}_{20})^{(3)} s_{(20)}} +$$

$$G_{20}^{(2)} \left|(a''_{20})^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right) - (a''_{20})^{(3)}\left(T_{21}^{(2)}, s_{(20)}\right)\right|\ e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} e^{(\widehat{M}_{20})^{(3)} s_{(20)}} \} ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\left|G_{23}{}^{(1)} - G_{23}{}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)} t} \leq \qquad 214$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)} + (a'_{20})^{(3)} + (\widehat{A}_{20})^{(3)} + (\widehat{P}_{20})^{(3)} (\widehat{k}_{20})^{(3)}\right) d\left(\left((G_{23})^{(1)}, (T_{23})^{(1)};\ (G_{23})^{(2)}, (T_{23})^{(2)}\right)\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 11:** The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)} e^{(\widehat{M}_{20})^{(3)} t}$ *and* $(\widehat{Q}_{20})^{(3)} e^{(\widehat{M}_{20})^{(3)} t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i = 20,21,22$ depend only on $T_{21}$ and respectively on $(G_{23})$(*and not on* $t$) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 12:** There does not exist any $t$ where $G_i(t) = 0$ *and* $T_i(t) = 0$ 216

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(3)} - (a_i'')^{(3)} (T_{21}(s_{(20)}), s_{(20)})\} ds_{(20)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(3)} t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2$ *and* $\left((\widehat{M}_{20})^{(3)}\right)_3$ : 217

**Remark 13:** if $G_{20}$ is bounded, the same property have also $G_{21}$ *and* $G_{22}$ . indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a'_{21})^{(3)} G_{21}$ and by integrating

$$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)} \left((\widehat{M}_{20})^{(3)}\right)_1 / (a'_{21})^{(3)}$$

In the same way , one can obtain

$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2/(a'_{22})^{(3)}$

If $G_{21}$ *or* $G_{22}$ is bounded, the same property follows for $G_{20}$ , $G_{22}$ and $G_{20}$ , $G_{21}$ respectively.

**Remark 14:** If $G_{20}$ *is* bounded, from below, the same property holds for $G_{21}$ *and* $G_{22}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{21}$ is bounded from below. 218

**Remark 15:** If $T_{20}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t),t\right) = (b'_{21})^{(3)}$ then $T_{21} \to \infty$. 219

**Definition of** $(m)^{(3)}$ and $\varepsilon_3$ :

Indeed let $t_3$ be so that for $t > t_3$

$(b_{21})^{(3)} - (b_i'')^{(3)}\left((G_{23})(t),t\right) < \varepsilon_3, T_{20}(t) > (m)^{(3)}$

Then $\frac{dT_{21}}{dt} \geq (a_{21})^{(3)}(m)^{(3)} - \varepsilon_3 T_{21}$ which leads to 220

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t} = \frac{1}{2}$ it results

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right), \quad t = log\frac{2}{\varepsilon_3}$ By taking now $\varepsilon_3$ sufficiently small one sees that $T_{21}$ is unbounded.
The same property holds for $T_{22}$ if $\lim_{t\to\infty}(b''_{22})^{(3)}\left((G_{23})(t),t\right) = (b'_{22})^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\tilde{M}_{24})^{(4)}}$ , $\frac{(b_i)^{(4)}}{(\tilde{M}_{24})^{(4)}} < 1$ and to choose 221

$(\widehat{P}_{24})^{(4)}$ and $(\widehat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\tilde{M}_{24})^{(4)}}\left[(\widehat{P}_{24})^{(4)} + \left((\hat{P}_{24})^{(4)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{24})^{(4)} \quad 222$$

$$\frac{(b_i)^{(4)}}{(\tilde{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{24})^{(4)}\right] \leq (\hat{Q}_{24})^{(4)} \quad 223$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself 224

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric 225

$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right) =$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\tilde{M}_{24})^{(4)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\tilde{M}_{24})^{(4)}t}\}$$

Indeed if we denote

**Definition of** $\widetilde{(G_{27})}, \widetilde{(T_{27})}$ : $\left(\widetilde{(G_{27})}, \widetilde{(T_{27})}\right) = \mathcal{A}^{(4)}((G_{27}), (T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{24})^{(4)} \left|G_{25}^{(1)} - G_{25}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{(\widehat{M}_{24})^{(4)} s_{(24)}}\, ds_{(24)} +$$

$$\int_0^t \{(a'_{24})^{(4)} \left|G_{24}^{(1)} - G_{24}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} +$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)}, s_{(24)}\right)\left|G_{24}^{(1)} - G_{24}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{(\widehat{M}_{24})^{(4)} s_{(24)}} +$$

$$G_{24}^{(2)} \left|(a''_{24})^{(4)}\left(T_{25}^{(1)}, s_{(24)}\right) - (a''_{24})^{(4)}\left(T_{25}^{(2)}, s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{(\widehat{M}_{24})^{(4)} s_{(24)}}\} ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)} - (G_{27})^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} t} \leq \qquad (226)$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)} + (a'_{24})^{(4)} + (\widehat{A}_{24})^{(4)} + (\widehat{P}_{24})^{(4)} (\widehat{k}_{24})^{(4)}\right) d\left(\left((G_{27})^{(1)}, (T_{27})^{(1)};\ (G_{27})^{(2)}, (T_{27})^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

**Remark 16:** The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)} t}\ and\ (\widehat{Q}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)} t}$ respectively of $\mathbb{R}_+$. (227)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i = 24{,}25{,}26$ depend only on $T_{25}$ and respectively on $(G_{27})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 17:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ (228)

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(4)} - (a_i'')^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(4)} t)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2\ and\ \left((\widehat{M}_{24})^{(4)}\right)_3$ : (229)

**Remark 18:** if $G_{24}$ is bounded, the same property have also $G_{25}\ and\ G_{26}$ . indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a'_{25})^{(4)} G_{25}$ and by integrating

$$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1 / (a'_{25})^{(4)}$$

In the same way , one can obtain

$$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2/(a'_{26})^{(4)}$$

If $G_{25}$ *or* $G_{26}$ is bounded, the same property follows for $G_{24}$ , $G_{26}$ and $G_{24}$ , $G_{25}$ respectively.

**Remark 19:** If $G_{24}$ *is* bounded, from below, the same property holds for $G_{25}$ *and* $G_{26}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{25}$ is bounded from below. 230

**Remark 20:** If $T_{24}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}((G_{27})(t),t)) = (b'_{25})^{(4)}$ then $T_{25} \to \infty$. 231

**Definition of** $(m)^{(4)}$ and $\varepsilon_4$ :

Indeed let $t_4$ be so that for $t > t_4$

$$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t),t) < \varepsilon_4, T_{24}(t) > (m)^{(4)}$$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right)$, $t = log\frac{2}{\varepsilon_4}$ By taking now $\varepsilon_4$ sufficiently small one sees that $T_{25}$ is unbounded.
The same property holds for $T_{26}$ if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t),t\right) = (b'_{26})^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for $G_{29}$ , $G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}$ , $\frac{(b_i)^{(5)}}{(\widehat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\widehat{P}_{28})^{(5)}$ and $(\widehat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}\left[(\widehat{P}_{28})^{(5)} + \left((\widehat{P}_{28})^{(5)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}\left[\left((\widehat{Q}_{28})^{(5)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{28})^{(5)}\right] \leq (\widehat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$$d\left(\left((G_{31})^{(1)}, (T_{31})^{(1)}\right), \left((G_{31})^{(2)}, (T_{31})^{(2)}\right)\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{28})^{(5)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{31}}),(\widetilde{T_{31}})$ : $\left((\widetilde{G_{31}}),(\widetilde{T_{31}})\right)=\mathcal{A}^{(5)}\left((G_{31}),(T_{31})\right)$

It results

$$\left|\tilde{G}_{28}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{28})^{(5)}\left|G_{29}^{(1)}-G_{29}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\,ds_{(28)}+$$

$$\int_0^t\{(a'_{28})^{(5)}\left|G_{28}^{(1)}-G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$(a''_{28})^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)\left|G_{28}^{(1)}-G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$G_{28}^{(2)}\left|(a''_{28})^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)-(a''_{28})^{(5)}\left(T_{29}^{(2)},s_{(28)}\right)\right|\ e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\}ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on it follows

$$\left|(G_{31})^{(1)}-(G_{31})^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}t}\le \qquad 237$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)}+(a'_{28})^{(5)}+(\widehat{A}_{28})^{(5)}+(\widehat{P}_{28})^{(5)}(\widehat{k}_{28})^{(5)}\right)d\left(((G_{31})^{(1)},(T_{31})^{(1)};\ (G_{31})^{(2)},(T_{31})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

**Remark 21:** The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}\ and\ (\widehat{Q}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ respectively of $\mathbb{R}_+$. 238

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i=28,29,30$ depend only on $T_{29}$ and respectively on $(G_{31})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 22:** There does not exist any $t$ where $G_i(t)=0\ and\ T_i(t)=0$ 239

it results

$$G_i(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(5)}-(a_i'')^{(5)}(T_{29}(s_{(28)}),s_{(28)})\}ds_{(28)}\right]}\ge 0$$

$$T_i(t)\ge T_i^0e^{(-(b_i')^{(5)}t)}>0\quad \text{for t}>0$$

**Definition of** $\left((\widehat{M}_{28})^{(5)}\right)_1,\left((\widehat{M}_{28})^{(5)}\right)_2\ and\ \left((\widehat{M}_{28})^{(5)}\right)_3$ : 240

**Remark 23:** if $G_{28}$ is bounded, the same property have also $G_{29}\ and\ G_{30}$ . indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a'_{29})^{(5)} G_{29}$ and by integrating

$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1/(a'_{29})^{(5)}$

In the same way , one can obtain

$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2/(a'_{30})^{(5)}$

If $G_{29}$ *or* $G_{30}$ is bounded, the same property follows for $G_{28}$ , $G_{30}$ and $G_{28}$ , $G_{29}$ respectively.

**Remark 24:** If $G_{28}$ *is* bounded, from below, the same property holds for $G_{29}$ *and* $G_{30}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{29}$ is bounded from below. 241

**Remark 25:** If $T_{28}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}((G_{31})(t),t)) = (b'_{29})^{(5)}$ then $T_{29} \to \infty$. 242

**Definition of** $(m)^{(5)}$ and $\varepsilon_5$ :

Indeed let $t_5$ be so that for $t > t_5$

$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t),t) < \varepsilon_5, T_{28}(t) > (m)^{(5)}$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\frac{2}{\varepsilon_5}$ By taking now $\varepsilon_5$ sufficiently small one sees that $T_{29}$ is unbounded.
The same property holds for $T_{30}$ if $\lim_{t\to\infty}(b_{30}'')^{(5)}\left((G_{31})(t),t\right) = (b'_{30})^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{33}$ , $G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}$ , $\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}} < 1$ and to choose 244

$(\widehat{P}_{32})^{(6)}$ and $(\widehat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\left[(\widehat{P}_{32})^{(6)} + \left((\widehat{P}_{32})^{(6)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{32})^{(6)} \quad 245$$

$$\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\left[\left((\widehat{Q}_{32})^{(6)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{32})^{(6)}\right] \leq (\widehat{Q}_{32})^{(6)} \quad 246$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric 247

$$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{35}}),(\widetilde{T_{35}})$ : $\left((\widetilde{G_{35}}),(\widetilde{T_{35}})\right)=\mathcal{A}^{(6)}\left((G_{35}),(T_{35})\right)$

It results

$$\left|\tilde{G}_{32}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{32})^{(6)}\left|G_{33}^{(1)}-G_{33}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\,ds_{(32)}+$$

$$\int_0^t\{(a'_{32})^{(6)}\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$G_{32}^{(2)}\left|(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)-(a''_{32})^{(6)}\left(T_{33}^{(2)},s_{(32)}\right)\right|\ e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\}ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|(G_{35})^{(1)}-(G_{35})^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}t}\le \qquad 248$$
$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)}+(a'_{32})^{(6)}+(\widehat{A}_{32})^{(6)}+(\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}\right)d\left(((G_{35})^{(1)},(T_{35})^{(1)};\ (G_{35})^{(2)},(T_{35})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 26:** The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ *and* $(\widehat{Q}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ respectively of $\mathbb{R}_+$. 249

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i=32,33,34$ depend only on $T_{33}$ and respectively on $(G_{35})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 27:** There does not exist any $t$ where $G_i(t)=0\ and\ T_i(t)=0$ 250

it results

$$G_i(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(6)}-(a_i'')^{(6)}(T_{33}(s_{(32)}),s_{(32)})\}ds_{(32)}\right]}\ge 0$$

$$T_i(t)\ge T_i^0e^{\left(-(b_i')^{(6)}t\right)}>0\quad \text{for t}>0$$

**Definition of** $\left((\widehat{M}_{32})^{(6)}\right)_1,\left((\widehat{M}_{32})^{(6)}\right)_2$ *and* $\left((\widehat{M}_{32})^{(6)}\right)_3$ : 251

**Remark 28:** if $G_{32}$ is bounded, the same property have also $G_{33}$ *and* $G_{34}$ . indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a'_{33})^{(6)} G_{33}$ and by integrating

$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_1 / (a'_{33})^{(6)}$

In the same way , one can obtain

$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_2 / (a'_{34})^{(6)}$

If $G_{33}$ *or* $G_{34}$ is bounded, the same property follows for $G_{32}$ , $G_{34}$ and $G_{32}$ , $G_{33}$ respectively.

**Remark 29:** If $G_{32}$ *is* bounded, from below, the same property holds for $G_{33}$ *and* $G_{34}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{33}$ is bounded from below. 252

**Remark 30:** If $T_{32}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}((G_{35})(t),t)) = (b'_{33})^{(6)}$ then $T_{33} \to \infty$. 253

**Definition of** $(m)^{(6)}$ and $\varepsilon_6$ :

Indeed let $t_6$ be so that for $t > t_6$

$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t),t\right) < \varepsilon_6, T_{32}(t) > (m)^{(6)}$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right)$, $t = log\frac{2}{\varepsilon_6}$ By taking now $\varepsilon_6$ sufficiently small one sees that $T_{33}$ is unbounded.
The same property holds for $T_{34}$ if $\lim_{t\to\infty}(b''_{34})^{(6)}\left((G_{35})(t),t(t),t\right) = (b'_{34})^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{37}$ , $G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}$ , $\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}} < 1$ and to choose
$(\widehat{P}_{36})^{(7)}$ and $(\widehat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[(\widehat{P}_{36})^{(7)} + \left((\widehat{P}_{36})^{(7)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{36})^{(7)} \quad 256$$

$$\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[\left((\widehat{Q}_{36})^{(7)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{36})^{(7)}\right] \leq (\widehat{Q}_{36})^{(7)} \quad 257$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying

Equations into itself
The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric 258

$$d\left(((G_{39})^{(1)},(T_{39})^{(1)}),((G_{39})^{(2)},(T_{39})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\widehat{M}_{36})^{(7)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\widehat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

**Definition of** $\widetilde{(G_{39})},\widetilde{(T_{39})}:\left(\widetilde{(G_{39})},\widetilde{(T_{39})}\right)=\mathcal{A}^{(7)}((G_{39}),(T_{39}))$
It results

$$|\tilde{G}_{36}^{(1)}-\tilde{G}_i^{(2)}|\le\int_0^t(a_{36})^{(7)}|G_{37}^{(1)}-G_{37}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}ds_{(36)}+$$
$$\int_0^t\{(a'_{36})^{(7)}|G_{36}^{(1)}-G_{36}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$(a''_{36})^{(7)}(T_{37}^{(1)},s_{(36)})|G_{36}^{(1)}-G_{36}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$G_{36}^{(2)}|(a''_{36})^{(7)}(T_{37}^{(1)},s_{(36)})-(a''_{36})^{(7)}(T_{37}^{(2)},s_{(36)})|\ e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\}ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on it follows

$$|(G_{39})^{(1)}-(G_{39})^{(2)}|e^{-(\widehat{M}_{36})^{(7)}t}\le$$ 259
$$\frac{1}{(\widehat{M}_{36})^{(7)}}\left((a_{36})^{(7)}+(a'_{36})^{(7)}+(\widehat{A}_{36})^{(7)}+(\widehat{P}_{36})^{(7)}(\widehat{k}_{36})^{(7)}\right)d\left(((G_{39})^{(1)},(T_{39})^{(1)};\ (G_{39})^{(2)},(T_{39})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 31:** The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ *and* $(\widehat{Q}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ respectively of $\mathbb{R}_+$. 260

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(7)}$ and $(b_i'')^{(7)}, i=36,37,38$ depend only on $T_{37}$ and respectively on $(G_{39})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 32:** There does not exist any $t$ where $G_i(t)=0\ and\ T_i(t)=0$ 261

it results

$$G_i(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(7)}-(a_i'')^{(7)}(T_{37}(s_{(36)}),s_{(36)})\}ds_{(36)}\right]}\ge 0$$

$$T_i(t)\ge T_i^0e^{(-(b_i')^{(7)}t)}>0\quad \text{for t}>0$$

**Definition of** $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2$ *and* $\left((\widehat{M}_{36})^{(7)}\right)_3$ : 262

**Remark 33:** if $G_{36}$ is bounded, the same property have also $G_{37}$ *and* $G_{38}$ . indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a'_{37})^{(7)} G_{37}$ and by integrating

$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1/(a'_{37})^{(7)}$

In the same way , one can obtain

$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2/(a'_{38})^{(7)}$

If $G_{37}$ *or* $G_{38}$ is bounded, the same property follows for $G_{36}$ , $G_{38}$ and $G_{36}$ , $G_{37}$ respectively.

**Remark 34:** If $G_{36}$ *is* bounded, from below, the same property holds for $G_{37}$ *and* $G_{38}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{37}$ is bounded from below. 263

**Remark 35:** If $T_{36}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(7)}((G_{39})(t),t)) = (b'_{37})^{(7)}$ then $T_{37} \to \infty$. 264

**Definition of** $(m)^{(7)}$ and $\varepsilon_7$ :

Indeed let $t_7$ be so that for $t > t_7$

$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t),t) < \varepsilon_7, T_{36}(t) > (m)^{(7)}$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right), \quad t = log\frac{2}{\varepsilon_7}$ By taking now $\varepsilon_7$ sufficiently small one sees that $T_{37}$ is unbounded.
The same property holds for $T_{38}$ if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t),t\right) = (b'_{38})^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}$ , $\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1$ and to choose 266
$(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ large to have

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[(\hat{P}_{40})^{(8)} + \left((\hat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{40})^{(8)} \quad 267$$

268

$$\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{40})^{(8)}\right]\leq(\hat{Q}_{40})^{(8)}$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)},(T_{43})^{(1)}),((G_{43})^{(2)},(T_{43})^{(2)})\right)= \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

**Definition of** $(\widetilde{G_{43}}),(\widetilde{T_{43}})$ : $\left((\widetilde{G_{43}}),(\widetilde{T_{43}})\right)=\mathcal{A}^{(8)}((G_{43}),(T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{40})^{(8)}\left|G_{41}^{(1)}-G_{41}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\,ds_{(40)}+$$
$$\int_0^t\{(a'_{40})^{(8)}\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$G_{40}^{(2)}\left|(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)-(a''_{40})^{(8)}\left(T_{41}^{(2)},s_{(40)}\right)\right|\;e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\}ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)}-(G_{43})^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}t}\leq \quad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)}+(a'_{40})^{(8)}+(\widehat{A}_{40})^{(8)}+(\widehat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}\right)d\left(((G_{43})^{(1)},(T_{43})^{(1)};\;(G_{43})^{(2)},(T_{43})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 36:** The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ *and* $(\widehat{Q}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ respectively of $\mathbb{R}_+$. 274

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i=40{,}41{,}42$ depend only on $\mathrm{T}_{41}$ and respectively on $(G_{43})$(*and not on* $t$) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 37** There does not exist any $t$ where $G_i(t)=0$ *and* $T_i(t)=0$ 275

it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(8)} - (a_i'')^{(8)}(T_{41}(s_{(40)}), s_{(40)})\} ds_{(40)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{(-(b_i')^{(8)} t)} > 0$ for t > 0

**Definition of** $\left((\widehat{M}_{40})^{(8)}\right)_1, \left((\widehat{M}_{40})^{(8)}\right)_2$ *and* $\left((\widehat{M}_{40})^{(8)}\right)_3$ : 276

**Remark 38:** if $G_{40}$ is bounded, the same property have also $G_{41}$ *and* $G_{42}$ . indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a_{41}')^{(8)} G_{41}$ and by integrating

$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)} \left((\widehat{M}_{40})^{(8)}\right)_1 / (a_{41}')^{(8)}$

In the same way , one can obtain

$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)} \left((\widehat{M}_{40})^{(8)}\right)_2 / (a_{42}')^{(8)}$

If $G_{41}$ *or* $G_{42}$ is bounded, the same property follows for $G_{40}$ , $G_{42}$ and $G_{40}$ , $G_{41}$ respectively.

**Remark 39:** If $G_{40}$ *is* bounded, from below, the same property holds for $G_{41}$ *and* $G_{42}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{41}$ is bounded from below. 277

**Remark 40:** If $T_{40}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}((G_{43})(t), t)) = (b_{41}')^{(8)}$ then $T_{41} \to \infty$. 278

**Definition of** $(m)^{(8)}$ and $\varepsilon_8$ :

Indeed let $t_8$ be so that for $t > t_8$

$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t), t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right), \quad t = log\frac{2}{\varepsilon_8}$ By taking now $\varepsilon_8$ sufficiently small one sees that $T_{41}$ is unbounded.
The same property holds for $T_{42}$ if $\lim_{t\to\infty}(b_{42}'')^{(8)}\left((G_{43})(t), t(t), t\right) = (b_{42}')^{(8)}$

It is now sufficient to take $\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}$ , $\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}} < 1$ and to choose $(\widehat{P}_{44})^{(9)}$ and $(\widehat{Q}_{44})^{(9)}$ large to have 279 A

$$\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[(\widehat{P}_{44})^{(9)} + \left((\widehat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{44})^{(9)} + G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[\left((\hat{Q}_{44})^{(9)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{44})^{(9)}\right]\leq(\hat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)},(T_{47})^{(1)}),\left((G_{47})^{(2)},(T_{47})^{(2)}\right)\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{47}}),(\widetilde{T_{47}}):\left((\widetilde{G_{47}}),(\widetilde{T_{47}})\right)=\mathcal{A}^{(9)}\left((G_{47}),(T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{44})^{(9)}\left|G_{45}^{(1)}-G_{45}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\,ds_{(44)}+$$
$$\int_0^t\{(a'_{44})^{(9)}\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$G_{44}^{(2)}\left|(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)-(a''_{44})^{(9)}\left(T_{45}^{(2)},s_{(44)}\right)\right|\;e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\}ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}t}\leq$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)}+(a'_{44})^{(9)}+(\widehat{A}_{44})^{(9)}+(\widehat{P}_{44})^{(9)}(\hat{k}_{44})^{(9)}\right)d\left(((G_{47})^{(1)},(T_{47})^{(1)};\;(G_{47})^{(2)},(T_{47})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis (39,35,36) the result follows

**Remark 41:** The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}\ and\ (\widehat{Q}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i=44,45,46$ depend only on $T_{45}$ and respectively on $(G_{47})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 42:** There does not exist any $t$ where $G_i\,(t)=0\ and\ T_i\,(t)=0$

From 99 to 44 it results

$$G_i\,(t)\geq G_i^0e^{\left[-\int_0^t\{(a_i')^{(9)}-(a_i'')^{(9)}(T_{45}(s_{(44)}),s_{(44)})\}ds_{(44)}\right]}\geq 0$$

$T_i(t) \geq T_i^0 e^{(-(b_i')^{(9)}t)} > 0$ for t > 0

**Definition of** $\left((\widehat{M}_{44})^{(9)}\right)_1, \left((\widehat{M}_{44})^{(9)}\right)_2$ *and* $\left((\widehat{M}_{44})^{(9)}\right)_3$ :

**Remark 43:** if $G_{44}$ is bounded, the same property have also $G_{45}$ *and* $G_{46}$ . indeed if

$G_{44} < (\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt} \leq \left((\widehat{M}_{44})^{(9)}\right)_1 - (a_{45}')^{(9)} G_{45}$ and by integrating

$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_1/(a_{45}')^{(9)}$

In the same way , one can obtain

$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_2/(a_{46}')^{(9)}$

If $G_{45}$ *or* $G_{46}$ is bounded, the same property follows for $G_{44}$ , $G_{46}$ and $G_{44}$ , $G_{45}$ respectively.

**Remark 44:** If $G_{44}$ *is* bounded, from below, the same property holds for $G_{45}$ *and* $G_{46}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{45}$ is bounded from below.

**Remark 45:** If $T_{44}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t),t\right) = (b_{45}')^{(9)}$ then $T_{45} \to \infty$.

**Definition of** $(m)^{(9)}$ and $\varepsilon_9$ :

Indeed let $t_9$ be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t),t\right) < \varepsilon_9, T_{44}(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right), \quad t = log\frac{2}{\varepsilon_9}$ By taking now $\varepsilon_9$ sufficiently small one sees that $T_{45}$ is unbounded.
The same property holds for $T_{46}$ if $\lim_{t\to\infty}(b_{46}'')^{(9)}\left((G_{47})(t),t\right) = (b_{46}')^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

**Behavior of the solutions of equation** 280

**Theorem** If we denote and define

**Definition of** $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ :

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)} \leq -(a_{13}')^{(1)} + (a_{14}')^{(1)} - (a_{13}'')^{(1)}(T_{14},t) + (a_{14}'')^{(1)}(T_{14},t) \leq -(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)} \leq -(b_{13}')^{(1)} + (b_{14}')^{(1)} - (b_{13}'')^{(1)}(G,t) - (b_{14}'')^{(1)}(G,t) \leq -(\tau_1)^{(1)}$

**Definition of** $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$ : 281

By $(v_1)^{(1)} > 0$ , $(v_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$ , $(u_2)^{(1)} < 0$ the roots of the equations $(a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

**Definition of** $(\bar{v}_1)^{(1)}$,, $(\bar{v}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$ : 282

By $(\bar{v}_1)^{(1)} > 0$ , $(\bar{v}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$ , $(\bar{u}_2)^{(1)} < 0$ the roots of the equations $(a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

**Definition of** $(m_1)^{(1)}$ , $(m_2)^{(1)}$ , $(\mu_1)^{(1)}, (\mu_2)^{(1)}, (v_0)^{(1)}$ :- 283

If we define $(m_1)^{(1)}$ , $(m_2)^{(1)}$ , $(\mu_1)^{(1)}, (\mu_2)^{(1)}$ by

$(m_2)^{(1)} = (v_0)^{(1)}, (m_1)^{(1)} = (v_1)^{(1)}$, $if\ (v_0)^{(1)} < (v_1)^{(1)}$

$(m_2)^{(1)} = (v_1)^{(1)}, (m_1)^{(1)} = (\bar{v}_1)^{(1)}$ , $if\ (v_1)^{(1)} < (v_0)^{(1)} < (\bar{v}_1)^{(1)}$,

and $\boxed{(v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$

$(m_2)^{(1)} = (v_1)^{(1)}, (m_1)^{(1)} = (v_0)^{(1)}$, $if\ (\bar{v}_1)^{(1)} < (v_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)}$, $if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}$ , $if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)}$,

and $\boxed{(u_0)^{(1)} = \frac{T_{13}^0}{T_{14}^0}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$

are defined

Then the solution of global equations satisfies the inequalities 285

$$G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{13}(t) \le G_{13}^0 e^{(S_1)^{(1)}t}$$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)}t}$$

$$\left(\frac{(a_{15})^{(1)} G_{13}^0}{(m_1)^{(1)}((S_1)^{(1)} - (p_{13})^{(1)} - (S_2)^{(1)})}\left[e^{((S_1)^{(1)} - (p_{13})^{(1)})t} - e^{-(S_2)^{(1)}t}\right] + G_{15}^0 e^{-(S_2)^{(1)}t} \le G_{15}(t) \le \right. \quad 286$$

$$\left. \frac{(a_{15})^{(1)} G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)} - (a'_{15})^{(1)})}\left[e^{(S_1)^{(1)}t} - e^{-(a'_{15})^{(1)}t}\right] + G_{15}^0 e^{-(a'_{15})^{(1)}t}\right)$$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)}t} \le T_{13}(t) \le T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t}} \quad 287$$

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)}t} \le T_{13}(t) \le \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t} \quad 288$$

$\frac{(b_{15})^{(1)}T_{13}^{0}}{(\mu_1)^{(1)}((R_1)^{(1)}-(b'_{15})^{(1)})}\left[e^{(R_1)^{(1)}t}-e^{-(b'_{15})^{(1)}t}\right]+T_{15}^{0}e^{-(b'_{15})^{(1)}t}\leq T_{15}(t)\leq$ 289

$\frac{(a_{15})^{(1)}T_{13}^{0}}{(\mu_2)^{(1)}((R_1)^{(1)}+(r_{13})^{(1)}+(R_2)^{(1)})}\left[e^{((R_1)^{(1)}+(r_{13})^{(1)})t}-e^{-(R_2)^{(1)}t}\right]+T_{15}^{0}e^{-(R_2)^{(1)}t}$

**Definition of** $(S_1)^{(1)},(S_2)^{(1)},(R_1)^{(1)},(R_2)^{(1)}$:- 290

Where $(S_1)^{(1)}=(a_{13})^{(1)}(m_2)^{(1)}-(a'_{13})^{(1)}$

$(S_2)^{(1)}=(a_{15})^{(1)}-(p_{15})^{(1)}$

$(R_1)^{(1)}=(b_{13})^{(1)}(\mu_2)^{(1)}-(b'_{13})^{(1)}$

$(R_2)^{(1)}=(b'_{15})^{(1)}-(r_{15})^{(1)}$

**Behavior of the solutions of equation** 291

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(2)},(\sigma_2)^{(2)},(\tau_1)^{(2)},(\tau_2)^{(2)}$ : 292

$(\sigma_1)^{(2)},(\sigma_2)^{(2)},(\tau_1)^{(2)},(\tau_2)^{(2)}$ four constants satisfying
$-(\sigma_2)^{(2)}\leq-(a'_{16})^{(2)}+(a'_{17})^{(2)}-(a''_{16})^{(2)}(\mathrm{T}_{17},t)+(a''_{17})^{(2)}(\mathrm{T}_{17},t)\leq-(\sigma_1)^{(2)}$ 293

$-(\tau_2)^{(2)}\leq-(b'_{16})^{(2)}+(b'_{17})^{(2)}-(b''_{16})^{(2)}\big((G_{19}),t\big)-(b''_{17})^{(2)}\big((G_{19}),t\big)\leq-(\tau_1)^{(2)}$ 294

**Definition of** $(\nu_1)^{(2)},(\nu_2)^{(2)},(u_1)^{(2)},(u_2)^{(2)}$ : 295

By $(\nu_1)^{(2)}>0$ , $(\nu_2)^{(2)}<0$ and respectively $(u_1)^{(2)}>0$ , $(u_2)^{(2)}<0$ the roots 296

of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2+(\sigma_1)^{(2)}\nu^{(2)}-(a_{16})^{(2)}=0$ 297

and $(b_{14})^{(2)}\big(u^{(2)}\big)^2+(\tau_1)^{(2)}u^{(2)}-(b_{16})^{(2)}=0$ and 298

**Definition of** $(\bar{\nu}_1)^{(2)},,(\bar{\nu}_2)^{(2)},(\bar{u}_1)^{(2)},(\bar{u}_2)^{(2)}$ : 299

By $(\bar{\nu}_1)^{(2)}>0$ , $(\bar{\nu}_2)^{(2)}<0$ and respectively $(\bar{u}_1)^{(2)}>0$ , $(\bar{u}_2)^{(2)}<0$ the 300

roots of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2+(\sigma_2)^{(2)}\nu^{(2)}-(a_{16})^{(2)}=0$ 301

and $(b_{17})^{(2)}\big(u^{(2)}\big)^2+(\tau_2)^{(2)}u^{(2)}-(b_{16})^{(2)}=0$ 302

**Definition of** $(m_1)^{(2)},(m_2)^{(2)},(\mu_1)^{(2)},(\mu_2)^{(2)}$ :- 303

If we define $(m_1)^{(2)},(m_2)^{(2)},(\mu_1)^{(2)},(\mu_2)^{(2)}$ by 304

$(m_2)^{(2)}=(\nu_0)^{(2)},(m_1)^{(2)}=(\nu_1)^{(2)},\ \boldsymbol{if}\ (\nu_0)^{(2)}<(\nu_1)^{(2)}$ 305

$(m_2)^{(2)}=(\nu_1)^{(2)},(m_1)^{(2)}=(\bar{\nu}_1)^{(2)},\boldsymbol{if}\ (\nu_1)^{(2)}<(\nu_0)^{(2)}<(\bar{\nu}_1)^{(2)},$ 306

and $\boxed{(\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)},\ \boldsymbol{if}\ (u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}, \boldsymbol{if}\ (u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)},$

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)},\ \boldsymbol{if}\ (\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)} t}$

$(p_i)^{(2)}$ is defined by equation

$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)} t}$ 311

$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)} t}\right] + G_{18}^0 e^{-(S_2)^{(2)} t} \le G_{18}(t) \le\right.$ 312

$\left.\frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)} t} - e^{-(a'_{18})^{(2)} t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)} t}\right)$

$\boxed{T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}}$ 313

$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}$ 314

$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)} t} - e^{-(b'_{18})^{(2)} t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)} t} \le T_{18}(t) \le$ 315

$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})}\left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)} t}\right] + T_{18}^0 e^{-(R_2)^{(2)} t}$

**Definition of** $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$

$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)}$ 318

$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$

**Behavior of the solutions** 319

**Theorem 3:** If we denote and define

**Definition of** $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ :

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}\big((G_{23}), t\big) \le -(\tau_1)^{(3)}$$

**Definition of** $(\nu_1)^{(3)}, (\nu_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$ : 320

By $(\nu_1)^{(3)} > 0$ , $(\nu_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$ , $(u_2)^{(3)} < 0$ the roots of the equations

$(a_{21})^{(3)}\big(\nu^{(3)}\big)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{\nu}_1)^{(3)} > 0$ , $(\bar{\nu}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$ , $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\big(\nu^{(3)}\big)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

**Definition of** $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ :- 321

If we define $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (\nu_0)^{(3)}, (m_1)^{(3)} = (\nu_1)^{(3)}$, ***if*** $(\nu_0)^{(3)} < (\nu_1)^{(3)}$

$(m_2)^{(3)} = (\nu_1)^{(3)}, (m_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ , ***if*** $(\nu_1)^{(3)} < (\nu_0)^{(3)} < (\bar{\nu}_1)^{(3)}$,

and $\boxed{(\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}}$

$(m_2)^{(3)} = (\nu_1)^{(3)}, (m_1)^{(3)} = (\nu_0)^{(3)}$, ***if*** $(\bar{\nu}_1)^{(3)} < (\nu_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}$, ***if*** $(u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$ , ***if*** $(u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}$, and $\boxed{(u_0)^{(3)} = \frac{T^0_{20}}{T^0_{21}}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}$, ***if*** $(\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$$G^0_{20}e^{((s_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G^0_{20}e^{(s_1)^{(3)}t}$$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)}-(p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)}t} \quad 323$$

$$\left(\frac{(a_{22})^{(3)}G_{20}^0}{(m_1)^{(3)}((S_1)^{(3)}-(p_{20})^{(3)}-(S_2)^{(3)})}\left[e^{((S_1)^{(3)}-(p_{20})^{(3)})t} - e^{-(S_2)^{(3)}t}\right] + G_{22}^0 e^{-(S_2)^{(3)}t} \le G_{22}(t) \le \right. \quad 324$$

$$\left. \frac{(a_{22})^{(3)}G_{20}^0}{(m_2)^{(3)}((S_1)^{(3)}-(a'_{22})^{(3)})}\left[e^{(S_1)^{(3)}t} - e^{-(a'_{22})^{(3)}t}\right] + G_{22}^0 e^{-(a'_{22})^{(3)}t}\right)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)}+(r_{20})^{(3)})t}} \quad 325$$

$$\frac{1}{(\mu_1)^{(3)}} T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}} T_{20}^0 e^{((R_1)^{(3)}+(r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)}T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)}-(b'_{22})^{(3)})}\left[e^{(R_1)^{(3)}t} - e^{-(b'_{22})^{(3)}t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)}t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)}T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)}+(r_{20})^{(3)}+(R_2)^{(3)})}\left[e^{((R_1)^{(3)}+(r_{20})^{(3)})t} - e^{-(R_2)^{(3)}t}\right] + T_{22}^0 e^{-(R_2)^{(3)}t}$$

**Definition of** $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$$

$$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$$

$$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$$

**Behavior of the solutions of equation**

**Theorem:** If we denote and define

**Definition of** $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ :

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$$

$$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}((G_{27}), t) - (b''_{25})^{(4)}((G_{27}), t) \le -(\tau_1)^{(4)}$$

**Definition of** $(v_1)^{(4)}, (v_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, v^{(4)}, u^{(4)}$ : 329

By $(v_1)^{(4)} > 0$ , $(v_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$ , $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_1)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

**Definition of** $(\bar{v}_1)^{(4)}, , (\bar{v}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$ : 330

By $(\bar{v}_1)^{(4)} > 0$ , $(\bar{v}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$ , $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

**Definition of** $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}, (\nu_0)^{(4)}$ :-

If we define $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (\nu_0)^{(4)}, (m_1)^{(4)} = (\nu_1)^{(4)}$, ***if*** $(\nu_0)^{(4)} < (\nu_1)^{(4)}$

$(m_2)^{(4)} = (\nu_1)^{(4)}, (m_1)^{(4)} = (\bar{\nu}_1)^{(4)}$, ***if*** $(\nu_4)^{(4)} < (\nu_0)^{(4)} < (\bar{\nu}_1)^{(4)}$,

and $\boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (\nu_4)^{(4)}, (m_1)^{(4)} = (\nu_0)^{(4)}$, ***if*** $(\bar{\nu}_4)^{(4)} < (\nu_0)^{(4)}$

and analogously

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$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)}$, ***if*** $(u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}$, ***if*** $(u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)}$,

and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}$, ***if*** $(\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

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$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)} t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)} t} \quad 333$$

$$\left( \frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})} \left[ e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)} t} \right] + G_{26}^0 e^{-(S_2)^{(4)} t} \le G_{26}(t) \le \right. \quad 334$$

*(a26)4G240(m2)4(S1)4−(a26′)4e(S1)4t−e−(a26′)4t+ G260e−(a26′)4t*

$$\boxed{T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}} T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}} T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t} \quad 335$$

$$\frac{(b_{26})^{(4)} T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)} - (b'_{26})^{(4)})} \left[ e^{(R_1)^{(4)} t} - e^{-(b'_{26})^{(4)} t} \right] + T_{26}^0 e^{-(b'_{26})^{(4)} t} \le T_{26}(t) \le \quad 336$$

$$\frac{(a_{26})^{(4)} T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)} + (r_{24})^{(4)} + (R_2)^{(4)})} \left[ e^{((R_1)^{(4)} + (r_{24})^{(4)})t} - e^{-(R_2)^{(4)} t} \right] + T_{26}^0 e^{-(R_2)^{(4)} t}$$

**Definition of** $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)} (m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$$

$$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$$

**Behavior of the solutions of equation** 338

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ :

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}((G_{31}), t) - (b''_{29})^{(5)}((G_{31}), t) \le -(\tau_1)^{(5)}$$

**Definition of** $(\nu_1)^{(5)}, (\nu_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, \nu^{(5)}, u^{(5)}$ : 339

By $(\nu_1)^{(5)} > 0$ , $(\nu_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$ , $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\left(u^{(5)}\right)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(5)}, , (\bar{\nu}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$ : 340

By $(\bar{\nu}_1)^{(5)} > 0$ , $(\bar{\nu}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$ , $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\left(u^{(5)}\right)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

**Definition of** $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (\nu_0)^{(5)}$ :-

If we define $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$$(m_2)^{(5)} = (\nu_0)^{(5)}, (m_1)^{(5)} = (\nu_1)^{(5)}, \ \boldsymbol{if}\ (\nu_0)^{(5)} < (\nu_1)^{(5)}$$

$$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\bar{\nu}_1)^{(5)}, \boldsymbol{if}\ (\nu_1)^{(5)} < (\nu_0)^{(5)} < (\bar{\nu}_1)^{(5)},$$

and $\boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}}$

$$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$$

and analogously 341

$$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)}, \ \boldsymbol{if}\ (u_0)^{(5)} < (u_1)^{(5)}$$

$$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}, \boldsymbol{if}\ (u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)},$$

and $\boxed{(u_0)^{(5)} = \frac{T^0_{28}}{T^0_{29}}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}, \boldsymbol{if}\ (\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$$G_{28}^{0}e^{((S_1)^{(5)}-(p_{28})^{(5)})t} \le G_{28}(t) \le G_{28}^{0}e^{(S_1)^{(5)}t}$$

where $(p_i)^{(5)}$ is defined by equation

$$\frac{1}{(m_5)^{(5)}}G_{28}^{0}e^{((S_1)^{(5)}-(p_{28})^{(5)})t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}}G_{28}^{0}e^{(S_1)^{(5)}t} \quad 343$$

$$\left(\frac{(a_{30})^{(5)}G_{28}^{0}}{(m_1)^{(5)}((S_1)^{(5)}-(p_{28})^{(5)}-(S_2)^{(5)})}\left[e^{((S_1)^{(5)}-(p_{28})^{(5)})t}-e^{-(S_2)^{(5)}t}\right]+G_{30}^{0}e^{-(S_2)^{(5)}t} \le G_{30}(t) \le \right. \quad 344$$

*(a30)5G280(m2)5(S1)5−(a30′)5e(S1)5t−e−(a30′)5t+ G300e−(a30′)5t*

$$\boxed{T_{28}^{0}e^{(R_1)^{(5)}t} \le T_{28}(t) \le T_{28}^{0}e^{((R_1)^{(5)}+(r_{28})^{(5)})t}} \quad 345$$

$$\frac{1}{(\mu_1)^{(5)}}T_{28}^{0}e^{(R_1)^{(5)}t} \le T_{28}(t) \le \frac{1}{(\mu_2)^{(5)}}T_{28}^{0}e^{((R_1)^{(5)}+(r_{28})^{(5)})t} \quad 346$$

$$\frac{(b_{30})^{(5)}T_{28}^{0}}{(\mu_1)^{(5)}((R_1)^{(5)}-(b'_{30})^{(5)})}\left[e^{(R_1)^{(5)}t}-e^{-(b'_{30})^{(5)}t}\right]+T_{30}^{0}e^{-(b'_{30})^{(5)}t} \le T_{30}(t) \le \quad 347$$

$$\frac{(a_{30})^{(5)}T_{28}^{0}}{(\mu_2)^{(5)}((R_1)^{(5)}+(r_{28})^{(5)}+(R_2)^{(5)})}\left[e^{((R_1)^{(5)}+(r_{28})^{(5)})t}-e^{-(R_2)^{(5)}t}\right]+T_{30}^{0}e^{-(R_2)^{(5)}t}$$

**Definition of** $(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$:- 348

Where $(S_1)^{(5)} = (a_{28})^{(5)}(m_2)^{(5)} - (a'_{28})^{(5)}$

$(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}$

$(R_1)^{(5)} = (b_{28})^{(5)}(\mu_2)^{(5)} - (b'_{28})^{(5)}$

$(R_2)^{(5)} = (b'_{30})^{(5)} - (r_{30})^{(5)}$

**Behavior of the solutions of equation** 349

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ :

$(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)} \le -(a'_{32})^{(6)} + (a'_{33})^{(6)} - (a''_{32})^{(6)}(T_{33}, t) + (a''_{33})^{(6)}(T_{33}, t) \le -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)} \le -(b'_{32})^{(6)} + (b'_{33})^{(6)} - (b''_{32})^{(6)}\big((G_{35}), t\big) - (b''_{33})^{(6)}\big((G_{35}), t\big) \le -(\tau_1)^{(6)}$$

**Definition of** $(v_1)^{(6)}, (v_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, v^{(6)}, u^{(6)}$ : 350

By $(v_1)^{(6)} > 0$ , $(v_2)^{(6)} < 0$ and respectively $(u_1)^{(6)} > 0$ , $(u_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(v^{(6)}\big)^2 + (\sigma_1)^{(6)}v^{(6)} - (a_{32})^{(6)} = 0$

and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$ and

**Definition of** $(\bar{v}_1)^{(6)},,(\bar{v}_2)^{(6)},(\bar{u}_1)^{(6)},(\bar{u}_2)^{(6)}$ : 351

By $(\bar{v}_1)^{(6)} > 0$ , $(\bar{v}_2)^{(6)} < 0$ and respectively $(\bar{u}_1)^{(6)} > 0$ , $(\bar{u}_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\left(v^{(6)}\right)^2 + (\sigma_2)^{(6)}v^{(6)} - (a_{32})^{(6)} = 0$ and $(b_{33})^{(6)}\left(u^{(6)}\right)^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$

**Definition of** $(m_1)^{(6)}$ , $(m_2)^{(6)}$ , $(\mu_1)^{(6)},(\mu_2)^{(6)},(v_0)^{(6)}$ :-

If we define $(m_1)^{(6)}$ , $(m_2)^{(6)}$ , $(\mu_1)^{(6)},(\mu_2)^{(6)}$ by

$(m_2)^{(6)} = (v_0)^{(6)}, (m_1)^{(6)} = (v_1)^{(6)}$, ***if*** $(v_0)^{(6)} < (v_1)^{(6)}$

$(m_2)^{(6)} = (v_1)^{(6)}, (m_1)^{(6)} = (\bar{v}_6)^{(6)}$ , ***if*** $(v_1)^{(6)} < (v_0)^{(6)} < (\bar{v}_1)^{(6)}$,
and $\boxed{(v_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$

$(m_2)^{(6)} = (v_1)^{(6)}, (m_1)^{(6)} = (v_0)^{(6)}$, ***if*** $(\bar{v}_1)^{(6)} < (v_0)^{(6)}$

and analogously 352

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}$, ***if*** $(u_0)^{(6)} < (u_1)^{(6)}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}$ , ***if*** $(u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)}$,
and $\boxed{(u_0)^{(6)} = \frac{T_{32}^0}{T_{33}^0}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}$, ***if*** $(\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$

Then the solution of global equations satisfies the inequalities 353

$$G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{32}(t) \le G_{32}^0 e^{(S_1)^{(6)}t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}} G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{33}(t) \le \frac{1}{(m_2)^{(6)}} G_{32}^0 e^{(S_1)^{(6)}t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)} G_{32}^0}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})}\left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)}t}\right] + G_{34}^0 e^{-(S_2)^{(6)}t} \le G_{34}(t) \le \right. \quad 355$$

*(a34)6G320(m2)6(S1)6−(a34′)6e(S1)6t−e−(a34′)6t+ G340e−(a34′)6t*

$$\boxed{T_{32}^0 e^{(R_1)^{(6)}t} \le T_{32}(t) \le T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}} T_{32}^0 e^{(R_1)^{(6)}t} \le T_{32}(t) \le \frac{1}{(\mu_2)^{(6)}} T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)} T_{32}^0}{(\mu_1)^{(6)}((R_1)^{(6)} - (b'_{34})^{(6)})}\left[e^{(R_1)^{(6)}t} - e^{-(b'_{34})^{(6)}t}\right] + T_{34}^0 e^{-(b'_{34})^{(6)}t} \le T_{34}(t) \le \quad 358$$

$$\frac{(a_{34})^{(6)} T_{32}^0}{(\mu_2)^{(6)}((R_1)^{(6)} + (r_{32})^{(6)} + (R_2)^{(6)})}\left[e^{((R_1)^{(6)} + (r_{32})^{(6)})t} - e^{-(R_2)^{(6)}t}\right] + T_{34}^0 e^{-(R_2)^{(6)}t}$$

**Definition of** $(S_1)^{(6)}, (S_2)^{(6)}, (R_1)^{(6)}, (R_2)^{(6)}$:- 359

Where $(S_1)^{(6)} = (a_{32})^{(6)}(m_2)^{(6)} - (a'_{32})^{(6)}$

$(S_2)^{(6)} = (a_{34})^{(6)} - (p_{34})^{(6)}$

$(R_1)^{(6)} = (b_{32})^{(6)}(\mu_2)^{(6)} - (b'_{32})^{(6)}$

$(R_2)^{(6)} = (b'_{34})^{(6)} - (r_{34})^{(6)}$

**Behavior of the solutions of equation**

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$ :

$(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)} \le -(a'_{36})^{(7)} + (a'_{37})^{(7)} - (a''_{36})^{(7)}(T_{37}, t) + (a''_{37})^{(7)}(T_{37}, t) \le -(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)} \le -(b'_{36})^{(7)} + (b'_{37})^{(7)} - (b''_{36})^{(7)}\big((G_{39}), t\big) - (b''_{37})^{(7)}\big((G_{39}), t\big) \le -(\tau_1)^{(7)}$$

**Definition of** $(\nu_1)^{(7)}, (\nu_2)^{(7)}, (u_1)^{(7)}, (u_2)^{(7)}, \nu^{(7)}, u^{(7)}$ : 361

By $(\nu_1)^{(7)} > 0$, $(\nu_2)^{(7)} < 0$ and respectively $(u_1)^{(7)} > 0$, $(u_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_1)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(7)},, (\bar{\nu}_2)^{(7)}, (\bar{u}_1)^{(7)}, (\bar{u}_2)^{(7)}$ : 362

By $(\bar{\nu}_1)^{(7)} > 0$, $(\bar{\nu}_2)^{(7)} < 0$ and respectively $(\bar{u}_1)^{(7)} > 0$, $(\bar{u}_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

**Definition of** $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (\nu_0)^{(7)}$ :-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$(m_2)^{(7)} = (\nu_0)^{(7)}, (m_1)^{(7)} = (\nu_1)^{(7)}$, ***if*** $(\nu_0)^{(7)} < (\nu_1)^{(7)}$

$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\bar{\nu}_1)^{(7)}$, ***if*** $(\nu_1)^{(7)} < (\nu_0)^{(7)} < (\bar{\nu}_1)^{(7)}$,
and $\boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$

$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\nu_0)^{(7)}$, ***if*** $(\bar{\nu}_1)^{(7)} < (\nu_0)^{(7)}$

and analogously 363

$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)},\ \boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$

and $\boxed{(u_0)^{(7)} = \frac{T_{36}^0}{T_{37}^0}}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)}$ where $(u_1)^{(7)}, (\bar{u}_1)^{(7)}$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^0 e^{(S_1)^{(7)} t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)} t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)}((S_1)^{(7)} - (p_{36})^{(7)} - (S_2)^{(7)})}\left[e^{((S_1)^{(7)} - (p_{36})^{(7)})t} - e^{-(S_2)^{(7)} t}\right] + G_{38}^0 e^{-(S_2)^{(7)} t} \le G_{38}(t) \le \right. \quad 366$$

$$\left. \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)}((S_1)^{(7)} - (a'_{38})^{(7)})}\left[e^{(S_1)^{(7)} t} - e^{-(a'_{38})^{(7)} t}\right] + G_{38}^0 e^{-(a'_{38})^{(7)} t}\right)$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t} \quad 368$$

$$\frac{(b_{38})^{(7)} T_{36}^0}{(\mu_1)^{(7)}((R_1)^{(7)} - (b'_{38})^{(7)})}\left[e^{(R_1)^{(7)} t} - e^{-(b'_{38})^{(7)} t}\right] + T_{38}^0 e^{-(b'_{38})^{(7)} t} \le T_{38}(t) \le \quad 369$$

$$\frac{(a_{38})^{(7)} T_{36}^0}{(\mu_2)^{(7)}((R_1)^{(7)} + (r_{36})^{(7)} + (R_2)^{(7)})}\left[e^{((R_1)^{(7)} + (r_{36})^{(7)})t} - e^{-(R_2)^{(7)} t}\right] + T_{38}^0 e^{-(R_2)^{(7)} t}$$

**Definition of** $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$

$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$

$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$

**Behavior of the solutions of equation** 371

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ :

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$$-(\sigma_2)^{(8)} \le -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \le -(\sigma_1)^{(8)}$$

$$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\big((G_{43}), t\big) - (b''_{41})^{(8)}\big((G_{43}), t\big) \le -(\tau_1)^{(8)}$$

**Definition of** $(\nu_1)^{(8)}, (\nu_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, \nu^{(8)}, u^{(8)}$ : 372

By $(\nu_1)^{(8)} > 0, (\nu_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0, (u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(8)}, , (\bar{\nu}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$ :

By $(\bar{\nu}_1)^{(8)} > 0, (\bar{\nu}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0, (\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

**Definition of** $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}, (\nu_0)^{(8)}$ :-

If we define $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$(m_2)^{(8)} = (\nu_0)^{(8)}, (m_1)^{(8)} = (\nu_1)^{(8)}$, ***if*** $(\nu_0)^{(8)} < (\nu_1)^{(8)}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\bar{\nu}_1)^{(8)}$, ***if*** $(\nu_1)^{(8)} < (\nu_0)^{(8)} < (\bar{\nu}_1)^{(8)}$,
and $\boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\nu_0)^{(8)}$, ***if*** $(\bar{\nu}_1)^{(8)} < (\nu_0)^{(8)}$

and analogously 374

$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)}$, ***if*** $(u_0)^{(8)} < (u_1)^{(8)}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}$, ***if*** $(u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)}$,
and $\boxed{(u_0)^{(8)} = \frac{T^0_{40}}{T^0_{41}}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}$, ***if*** $(\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^{0}e^{((S_1)^{(8)}-(p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^{0}e^{(S_1)^{(8)}t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}}G_{40}^{0}e^{((S_1)^{(8)}-(p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}}G_{40}^{0}e^{(S_1)^{(8)}t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)}G_{40}^{0}}{(m_1)^{(8)}((S_1)^{(8)}-(p_{40})^{(8)}-(S_2)^{(8)})}\left[e^{((S_1)^{(8)}-(p_{40})^{(8)})t}-e^{-(S_2)^{(8)}t}\right]+G_{42}^{0}e^{-(S_2)^{(8)}t} \le G_{42}(t) \le \right. \quad 377$$

$$\left.\frac{(a_{42})^{(8)}G_{40}^{0}}{(m_2)^{(8)}((S_1)^{(8)}-(a'_{42})^{(8)})}[e^{(S_1)^{(8)}t}-e^{-(a'_{42})^{(8)}t}]+\ G_{42}^{0}e^{-(a'_{42})^{(8)}t}\right)$$

$$\boxed{T_{40}^{0}e^{(R_1)^{(8)}t} \le T_{40}(t) \le T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}}T_{40}^{0}e^{(R_1)^{(8)}t} \le T_{40}(t) \le \frac{1}{(\mu_2)^{(8)}}T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)}T_{40}^{0}}{(\mu_1)^{(8)}((R_1)^{(8)}-(b'_{42})^{(8)})}\left[e^{(R_1)^{(8)}t}-e^{-(b'_{42})^{(8)}t}\right]+T_{42}^{0}e^{-(b'_{42})^{(8)}t} \le T_{42}(t) \le \quad 380$$

$$\frac{(a_{42})^{(8)}T_{40}^{0}}{(\mu_2)^{(8)}((R_1)^{(8)}+(r_{40})^{(8)}+(R_2)^{(8)})}\left[e^{((R_1)^{(8)}+(r_{40})^{(8)})t}-e^{-(R_2)^{(8)}t}\right]+T_{42}^{0}e^{-(R_2)^{(8)}t}$$

**Definition of** $(S_1)^{(8)}, (S_2)^{(8)}, (R_1)^{(8)}, (R_2)^{(8)}$:- 381

Where $(S_1)^{(8)} = (a_{40})^{(8)}(m_2)^{(8)} - (a'_{40})^{(8)}$

$$(S_2)^{(8)} = (a_{42})^{(8)} - (p_{42})^{(8)}$$

$$(R_1)^{(8)} = (b_{40})^{(8)}(\mu_2)^{(8)} - (b'_{40})^{(8)}$$

$$(R_2)^{(8)} = (b'_{42})^{(8)} - (r_{42})^{(8)}$$

**Behavior of the solutions of equation 37 to 92** 382

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ :

$(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ four constants satisfying

$$-(\sigma_2)^{(9)} \le -(a'_{44})^{(9)} + (a'_{45})^{(9)} - (a''_{44})^{(9)}(T_{45}, t) + (a''_{45})^{(9)}(T_{45}, t) \le -(\sigma_1)^{(9)}$$

$$-(\tau_2)^{(9)} \le -(b'_{44})^{(9)} + (b'_{45})^{(9)} - (b''_{44})^{(9)}\big((G_{47}), t\big) - (b''_{45})^{(9)}\big((G_{47}), t\big) \le -(\tau_1)^{(9)}$$

**Definition of** $(\nu_1)^{(9)}, (\nu_2)^{(9)}, (u_1)^{(9)}, (u_2)^{(9)}, \nu^{(9)}, u^{(9)}$ :

By $(\nu_1)^{(9)} > 0$ , $(\nu_2)^{(9)} < 0$ and respectively $(u_1)^{(9)} > 0$ , $(u_2)^{(9)} < 0$ the roots of the equations

$(a_{45})^{(9)}\left(v^{(9)}\right)^2+(\sigma_1)^{(9)}v^{(9)}-(a_{44})^{(9)}=0$

and $(b_{45})^{(9)}\left(u^{(9)}\right)^2+(\tau_1)^{(9)}u^{(9)}-(b_{44})^{(9)}=0$ and

**Definition of** $(\bar{v}_1)^{(9)},,(\bar{v}_2)^{(9)},(\bar{u}_1)^{(9)},(\bar{u}_2)^{(9)}$ :

By $(\bar{v}_1)^{(9)}>0$ , $(\bar{v}_2)^{(9)}<0$ and respectively $(\bar{u}_1)^{(9)}>0$ , $(\bar{u}_2)^{(9)}<0$ the roots of the equations $(a_{45})^{(9)}\left(v^{(9)}\right)^2+(\sigma_2)^{(9)}v^{(9)}-(a_{44})^{(9)}=0$

and $(b_{45})^{(9)}\left(u^{(9)}\right)^2+(\tau_2)^{(9)}u^{(9)}-(b_{44})^{(9)}=0$

**Definition of** $(m_1)^{(9)}$ , $(m_2)^{(9)}$ , $(\mu_1)^{(9)},(\mu_2)^{(9)},(\nu_0)^{(9)}$ :-

If we define $(m_1)^{(9)}$ , $(m_2)^{(9)}$ , $(\mu_1)^{(9)},(\mu_2)^{(9)}$ by

$(m_2)^{(9)}=(\nu_0)^{(9)},(m_1)^{(9)}=(\nu_1)^{(9)},$ $\boldsymbol{if}$ $(\nu_0)^{(9)}<(\nu_1)^{(9)}$

$(m_2)^{(9)}=(\nu_1)^{(9)},(m_1)^{(9)}=(\bar{\nu}_1)^{(9)}$ , $\boldsymbol{if}$ $(\nu_1)^{(9)}<(\nu_0)^{(9)}<(\bar{\nu}_1)^{(9)}$,

and $\boxed{(\nu_0)^{(9)}=\frac{G_{44}^0}{G_{45}^0}}$

$(m_2)^{(9)}=(\nu_1)^{(9)},(m_1)^{(9)}=(\nu_0)^{(9)},$ $\boldsymbol{if}$ $(\bar{\nu}_1)^{(9)}<(\nu_0)^{(9)}$

and analogously

$(\mu_2)^{(9)}=(u_0)^{(9)},(\mu_1)^{(9)}=(u_1)^{(9)},$ $\boldsymbol{if}$ $(u_0)^{(9)}<(u_1)^{(9)}$

$(\mu_2)^{(9)}=(u_1)^{(9)},(\mu_1)^{(9)}=(\bar{u}_1)^{(9)}$ , $\boldsymbol{if}$ $(u_1)^{(9)}<(u_0)^{(9)}<(\bar{u}_1)^{(9)}$,

and $\boxed{(u_0)^{(9)}=\frac{T_{44}^0}{T_{45}^0}}$

$(\mu_2)^{(9)}=(u_1)^{(9)},(\mu_1)^{(9)}=(u_0)^{(9)},\boldsymbol{if}$ $(\bar{u}_1)^{(9)}<(u_0)^{(9)}$ where $(u_1)^{(9)},(\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G_{44}^0e^{((S_1)^{(9)}-(p_{44})^{(9)})t}\le G_{44}(t)\le G_{44}^0e^{(S_1)^{(9)}t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}}G_{44}^0e^{((S_1)^{(9)}-(p_{44})^{(9)})t}\le G_{45}(t)\le\frac{1}{(m_2)^{(9)}}G_{44}^0e^{(S_1)^{(9)}t}$$

$$\frac{(a_{46})^{(9)}G_{44}^0}{(m_1)^{(9)}((S_1)^{(9)}-(p_{44})^{(9)}-(S_2)^{(9)})}\left[e^{((S_1)^{(9)}-(p_{44})^{(9)})t}-e^{-(S_2)^{(9)}t}\right]+G_{46}^0e^{-(S_2)^{(9)}t}\le G_{46}(t)\le$$

$$\frac{(a_{46})^{(9)}G_{44}^0}{(m_2)^{(9)}((S_1)^{(9)}-(a'_{46})^{(9)})}[e^{(S_1)^{(9)}t}-e^{-(a'_{46})^{(9)}t}]+G_{46}^0e^{-(a'_{46})^{(9)}t})$$

$$\boxed{T_{44}^0e^{(R_1)^{(9)}t}\le T_{44}(t)\le T_{44}^0e^{((R_1)^{(9)}+(r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}}T_{44}^0e^{(R_1)^{(9)}t}\le T_{44}(t)\le\frac{1}{(\mu_2)^{(9)}}T_{44}^0e^{((R_1)^{(9)}+(r_{44})^{(9)})t}$$

$$\frac{(b_{46})^{(9)}T_{44}^{0}}{(\mu_1)^{(9)}((R_1)^{(9)}-(b'_{46})^{(9)})}\left[e^{(R_1)^{(9)}t}-e^{-(b'_{46})^{(9)}t}\right]+T_{46}^{0}e^{-(b'_{46})^{(9)}t}\leq T_{46}(t)\leq$$

$$\frac{(a_{46})^{(9)}T_{44}^{0}}{(\mu_2)^{(9)}((R_1)^{(9)}+(r_{44})^{(9)}+(R_2)^{(9)})}\left[e^{((R_1)^{(9)}+(r_{44})^{(9)})t}-e^{-(R_2)^{(9)}t}\right]+T_{46}^{0}e^{-(R_2)^{(9)}t}$$

**Definition of** $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)}=(a_{44})^{(9)}(m_2)^{(9)}-(a'_{44})^{(9)}$

$(S_2)^{(9)}=(a_{46})^{(9)}-(p_{46})^{(9)}$

$(R_1)^{(9)}=(b_{44})^{(9)}(\mu_2)^{(9)}-(b'_{44})^{(9)}$

$(R_2)^{(9)}=(b'_{46})^{(9)}-(r_{46})^{(9)}$

**Proof :** From global equations we obtain 383

$$\frac{dv^{(1)}}{dt}=(a_{13})^{(1)}-\left((a'_{13})^{(1)}-(a'_{14})^{(1)}+(a''_{13})^{(1)}(T_{14},t)\right)-(a''_{14})^{(1)}(T_{14},t)v^{(1)}-(a_{14})^{(1)}v^{(1)}$$

**Definition of** $v^{(1)}$ :- $\boxed{v^{(1)}=\frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2+(\sigma_2)^{(1)}v^{(1)}-(a_{13})^{(1)}\right)\leq\frac{dv^{(1)}}{dt}\leq-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2+(\sigma_1)^{(1)}v^{(1)}-(a_{13})^{(1)}\right)$$

From which one obtains

**Definition of** $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$ :-

For $0<\boxed{(v_0)^{(1)}=\frac{G_{13}^{0}}{G_{14}^{0}}}<(v_1)^{(1)}<(\bar{v}_1)^{(1)}$

$$v^{(1)}(t)\geq\frac{(v_1)^{(1)}+(C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}}\quad,\quad\boxed{(C)^{(1)}=\frac{(v_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(v_2)^{(1)}}}$$

it follows $(v_0)^{(1)}\leq v^{(1)}(t)\leq(v_1)^{(1)}$

In the same manner , we get 384

$$v^{(1)}(t)\leq\frac{(\bar{v}_1)^{(1)}+(\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}}\quad,\quad\boxed{(\bar{C})^{(1)}=\frac{(\bar{v}_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(\bar{v}_2)^{(1)}}}$$

From which we deduce $(v_0)^{(1)}\leq v^{(1)}(t)\leq(\bar{v}_1)^{(1)}$

If $0<(v_1)^{(1)}<(v_0)^{(1)}=\frac{G_{13}^{0}}{G_{14}^{0}}<(\bar{v}_1)^{(1)}$ we find like in the previous case, 385

$$(\nu_1)^{(1)} \leq \frac{(\nu_1)^{(1)}+(C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}} \leq \nu^{(1)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\bar{\nu}_1)^{(1)}$$

If $0 < (\nu_1)^{(1)} \leq (\bar{\nu}_1)^{(1)} \leq \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$, we obtain 386

$$(\nu_1)^{(1)} \leq \nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\nu_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(1)}(t)$ :-

$$(m_2)^{(1)} \leq \nu^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{\nu^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(1)}(t)$ :-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{13}'')^{(1)} = (a_{14}'')^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)}G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b_{13}'')^{(1)} = (b_{14}'')^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)}T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

**Proof** : From global equations we obtain 387

$$\frac{d\nu^{(2)}}{dt} = (a_{16})^{(2)} - \left((a_{16}')^{(2)} - (a_{17}')^{(2)} + (a_{16}'')^{(2)}(T_{17}, t)\right) - (a_{17}'')^{(2)}(T_{17}, t)\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

**Definition of** $\nu^{(2)}$ :- $\boxed{\nu^{(2)} = \frac{G_{16}}{G_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(v^{(2)}\right)^2+(\sigma_2)^{(2)}v^{(2)}-(a_{16})^{(2)}\right)\leq\frac{\mathrm{d}v^{(2)}}{\mathrm{dt}}\leq-\left((a_{17})^{(2)}\left(v^{(2)}\right)^2+(\sigma_1)^{(2)}v^{(2)}-(a_{16})^{(2)}\right)$$

From which one obtains 390

**Definition of** $(\bar{v}_1)^{(2)},(v_0)^{(2)}$ :-

For $0<(v_0)^{(2)}=\frac{G_{16}^0}{G_{17}^0}<(v_1)^{(2)}<(\bar{v}_1)^{(2)}$

$$v^{(2)}(t)\geq\frac{(v_1)^{(2)}+(C)^{(2)}(v_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((v_1)^{(2)}-(v_0)^{(2)}\right)t\right]}}{1+(C)^{(2)}e^{\left[-(a_{17})^{(2)}\left((v_1)^{(2)}-(v_0)^{(2)}\right)t\right]}}\quad,\quad\boxed{(C)^{(2)}=\frac{(v_1)^{(2)}-(v_0)^{(2)}}{(v_0)^{(2)}-(v_2)^{(2)}}}$$

it follows $(v_0)^{(2)}\leq v^{(2)}(t)\leq(v_1)^{(2)}$

In the same manner , we get 391

$$v^{(2)}(t)\leq\frac{(\bar{v}_1)^{(2)}+(\bar{C})^{(2)}(\bar{v}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{v}_1)^{(2)}-(\bar{v}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{v}_1)^{(2)}-(\bar{v}_2)^{(2)}\right)t\right]}}\quad,\quad\boxed{(\bar{C})^{(2)}=\frac{(\bar{v}_1)^{(2)}-(v_0)^{(2)}}{(v_0)^{(2)}-(\bar{v}_2)^{(2)}}}$$

From which we deduce $(v_0)^{(2)}\leq v^{(2)}(t)\leq(\bar{v}_1)^{(2)}$ 392

If $0<(v_1)^{(2)}<(v_0)^{(2)}=\frac{G_{16}^0}{G_{17}^0}<(\bar{v}_1)^{(2)}$ we find like in the previous case, 393

$$(v_1)^{(2)}\leq\frac{(v_1)^{(2)}+(C)^{(2)}(v_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((v_1)^{(2)}-(v_2)^{(2)}\right)t\right]}}{1+(C)^{(2)}e^{\left[-(a_{17})^{(2)}\left((v_1)^{(2)}-(v_2)^{(2)}\right)t\right]}}\leq v^{(2)}(t)\leq$$

$$\frac{(\bar{v}_1)^{(2)}+(\bar{C})^{(2)}(\bar{v}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{v}_1)^{(2)}-(\bar{v}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{v}_1)^{(2)}-(\bar{v}_2)^{(2)}\right)t\right]}}\leq(\bar{v}_1)^{(2)}$$

If $0<(v_1)^{(2)}\leq(\bar{v}_1)^{(2)}\leq(v_0)^{(2)}=\frac{G_{16}^0}{G_{17}^0}$ , we obtain 394

$$(v_1)^{(2)}\leq v^{(2)}(t)\leq\frac{(\bar{v}_1)^{(2)}+(\bar{C})^{(2)}(\bar{v}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{v}_1)^{(2)}-(\bar{v}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{v}_1)^{(2)}-(\bar{v}_2)^{(2)}\right)t\right]}}\leq(v_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $v^{(2)}(t)$ :- 395

$$(m_2)^{(2)}\leq v^{(2)}(t)\leq(m_1)^{(2)},\quad\boxed{v^{(2)}(t)=\frac{G_{16}(t)}{G_{17}(t)}}$$

In a completely analogous way, we obtain 396

**Definition of** $u^{(2)}(t)$ :-

$$(\mu_2)^{(2)}\leq u^{(2)}(t)\leq(\mu_1)^{(2)},\quad\boxed{u^{(2)}(t)=\frac{T_{16}(t)}{T_{17}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :** 397

If $(a''_{16})^{(2)} = (a''_{17})^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)} G_{17}(t)$

Analogously if $(b''_{16})^{(2)} = (b''_{17})^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$ if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)} T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

**Proof** : From global equations we obtain 398

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right) - (a''_{21})^{(3)}(T_{21}, t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

**Definition of** $\nu^{(3)}$ :- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right) \leq \frac{d\nu^{(3)}}{dt} \leq -\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right)$$

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From which one obtains

For $0 < (\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\nu_1)^{(3)} < (\bar{\nu}_1)^{(3)}$

$$\nu^{(3)}(t) \geq \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(\nu_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\nu_2)^{(3)}}}$$

it follows $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\nu_1)^{(3)}$

In the same manner , we get 401

$$\nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)} + (\bar{C})^{(3)}(\bar{\nu}_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)} e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{\nu}_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\bar{\nu}_2)^{(3)}}}$$

**Definition of** $(\bar{\nu}_1)^{(3)}$ :-

From which we deduce $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\bar{\nu}_1)^{(3)}$

If $0 < (\nu_1)^{(3)} < (\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\bar{\nu}_1)^{(3)}$ we find like in the previous case, 402

$$(\nu_1)^{(3)} \le \frac{(\nu_1)^{(3)}+(C)^{(3)}(\nu_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}} \le \nu^{(3)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \le (\bar{\nu}_1)^{(3)}$$

If $0 < (\nu_1)^{(3)} \le (\bar{\nu}_1)^{(3)} \le (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}$ , we obtain 403

$$(\nu_1)^{(3)} \le \nu^{(3)}(t) \le \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \le (\nu_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(3)}(t)$ :-

$$(m_2)^{(3)} \le \nu^{(3)}(t) \le (m_1)^{(3)}, \quad \boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(3)}(t)$ :-

$$(\mu_2)^{(3)} \le u^{(3)}(t) \le (\mu_1)^{(3)}, \quad \boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{20}'')^{(3)} = (a_{21}'')^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)}G_{21}(t)$

Analogously if $(b_{20}'')^{(3)} = (b_{21}'')^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)}T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

**Proof** : From global equations we obtain 404

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a_{24}')^{(4)} - (a_{25}')^{(4)} + (a_{24}'')^{(4)}(T_{25},t)\right) - (a_{25}'')^{(4)}(T_{25},t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

**Definition of** $\nu^{(4)}$ :- $\quad \boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right) \le \frac{d\nu^{(4)}}{dt} \le -\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_4)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$ :-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \geq \frac{(\nu_1)^{(4)}+(C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_0)^{(4)}\right)t\right]}}{4+(C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)}-(\nu_0)^{(4)}}{(\nu_0)^{(4)}-(\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{4+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)}-(\nu_0)^{(4)}}{(\nu_0)^{(4)}-(\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \leq \frac{(\nu_1)^{(4)}+(C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_2)^{(4)}\right)t\right]}}{1+(C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_2)^{(4)}\right)t\right]}} \leq \nu^{(4)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \leq (\bar{\nu}_1)^{(4)} \leq \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$ , we obtain 407

$$(\nu_1)^{(4)} \leq \nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(4)}(t)$ :-

$(m_2)^{(4)} \leq \nu^{(4)}(t) \leq (m_1)^{(4)}$, $\boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$

In a completely analogous way, we obtain

**Definition of** $u^{(4)}(t)$ :-

$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}$, $\boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{24})^{(4)} = (a''_{25})^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)}G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b''_{24})^{(4)} = (b''_{25})^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then

$(u_1)^{(4)} = (\bar{u}_4)^{(4)}$if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)}T_{25}(t)$ This is an important consequence of the relation between $(v_1)^{(4)}$ and $(\bar{v}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

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**Proof** : From global equations we obtain

$$\frac{dv^{(5)}}{dt} = (a_{28})^{(5)} - \left((a'_{28})^{(5)} - (a'_{29})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right) - (a''_{29})^{(5)}(T_{29}, t)v^{(5)} - (a_{29})^{(5)}v^{(5)}$$

**Definition of** $v^{(5)}$ :- $\boxed{v^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(v^{(5)}\right)^2 + (\sigma_2)^{(5)}v^{(5)} - (a_{28})^{(5)}\right) \leq \frac{dv^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(v^{(5)}\right)^2 + (\sigma_1)^{(5)}v^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

**Definition of** $(\bar{v}_1)^{(5)}, (v_0)^{(5)}$ :-

For $0 < \boxed{(v_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}} < (v_1)^{(5)} < (\bar{v}_1)^{(5)}$

$$v^{(5)}(t) \geq \frac{(v_1)^{(5)} + (C)^{(5)}(v_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((v_1)^{(5)} - (v_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)}e^{\left[-(a_{29})^{(5)}\left((v_1)^{(5)} - (v_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(v_1)^{(5)} - (v_0)^{(5)}}{(v_0)^{(5)} - (v_2)^{(5)}}}$$

it follows $(v_0)^{(5)} \leq v^{(5)}(t) \leq (v_1)^{(5)}$

In the same manner , we get 409

$$v^{(5)}(t) \leq \frac{(\bar{v}_1)^{(5)} + (\bar{C})^{(5)}(\bar{v}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{v}_1)^{(5)} - (\bar{v}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{v}_1)^{(5)} - (\bar{v}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{v}_1)^{(5)} - (v_0)^{(5)}}{(v_0)^{(5)} - (\bar{v}_2)^{(5)}}}$$

From which we deduce $(v_0)^{(5)} \leq v^{(5)}(t) \leq (\bar{v}_5)^{(5)}$

If $0 < (v_1)^{(5)} < (v_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}} < (\bar{v}_1)^{(5)}$ we find like in the previous case, 410

$$(v_1)^{(5)} \leq \frac{(v_1)^{(5)} + (C)^{(5)}(v_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((v_1)^{(5)} - (v_2)^{(5)}\right)t\right]}}{1 + (C)^{(5)}e^{\left[-(a_{29})^{(5)}\left((v_1)^{(5)} - (v_2)^{(5)}\right)t\right]}} \leq v^{(5)}(t) \leq$$

$$\frac{(\bar{v}_1)^{(5)} + (\bar{C})^{(5)}(\bar{v}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{v}_1)^{(5)} - (\bar{v}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{v}_1)^{(5)} - (\bar{v}_2)^{(5)}\right)t\right]}} \leq (\bar{v}_1)^{(5)}$$

If $0 < (v_1)^{(5)} \leq (\bar{v}_1)^{(5)} \leq \boxed{(v_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}}$ , we obtain 411

$$(v_1)^{(5)} \leq v^{(5)}(t) \leq \frac{(\bar{v}_1)^{(5)} + (\bar{C})^{(5)}(\bar{v}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{v}_1)^{(5)} - (\bar{v}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{v}_1)^{(5)} - (\bar{v}_2)^{(5)}\right)t\right]}} \leq (v_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $v^{(5)}(t)$ :-

$(m_2)^{(5)} \leq v^{(5)}(t) \leq (m_1)^{(5)}$, $\boxed{v^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$

In a completely analogous way, we obtain

**Definition of** $u^{(5)}(t)$ :-

$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}$, $\boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{28})^{(5)} = (a''_{29})^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $v^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)} G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b''_{28})^{(5)} = (b''_{29})^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$ if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)} T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

**Proof** : From global equations we obtain 412

$$\frac{dv^{(6)}}{dt} = (a_{32})^{(6)} - \left((a'_{32})^{(6)} - (a'_{33})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right) - (a''_{33})^{(6)}(T_{33}, t)v^{(6)} - (a_{33})^{(6)}v^{(6)}$$

**Definition of** $v^{(6)}$ :- $\boxed{v^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(v^{(6)}\right)^2 + (\sigma_2)^{(6)}v^{(6)} - (a_{32})^{(6)}\right) \leq \frac{dv^{(6)}}{dt} \leq -\left((a_{33})^{(6)}\left(v^{(6)}\right)^2 + (\sigma_1)^{(6)}v^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$ :-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$v^{(6)}(t) \geq \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \leq v^{(6)}(t) \leq (\nu_1)^{(6)}$

In the same manner , we get 413

$$v^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{\nu}_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\bar{\nu}_1)^{(6)}$

If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 414

$$(\nu_1)^{(6)} \leq \frac{(\nu_1)^{(6)}+(C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_2)^{(6)}\right)t\right]}}{1+(C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_2)^{(6)}\right)t\right]}} \leq \nu^{(6)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\bar{\nu}_1)^{(6)}$$

If $0 < (\nu_1)^{(6)} \leq (\bar{\nu}_1)^{(6)} \leq \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$, we obtain 415

$$(\nu_1)^{(6)} \leq \nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(6)}(t)$ :-

$$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}, \quad \boxed{\nu^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(6)}(t)$ :-

$$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}, \quad \boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)}G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)}T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

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**Proof** : From global equations we obtain

$$\frac{d\nu^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right) - (a''_{37})^{(7)}(T_{37}, t)\nu^{(7)} - (a_{37})^{(7)}\nu^{(7)}$$

**Definition of** $\nu^{(7)}$ :- $\boxed{\nu^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right) \leq \frac{d\nu^{(7)}}{dt} \leq -\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(7)}, (\nu_0)^{(7)}$ :-

$$\text{For } 0 < \boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}} < (\nu_1)^{(7)} < (\bar{\nu}_1)^{(7)}$$

$$\nu^{(7)}(t) \geq \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}}{1 + (C)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(\nu_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\nu_2)^{(7)}}}$$

it follows $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\nu_1)^{(7)}$

In the same manner , we get 417

$$\nu^{(7)}(t) \leq \frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)} e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{\nu}_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\bar{\nu}_2)^{(7)}}}$$

From which we deduce $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\bar{\nu}_1)^{(7)}$

If $0 < (\nu_1)^{(7)} < (\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0} < (\bar{\nu}_1)^{(7)}$ we find like in the previous case, 418

$$(\nu_1)^{(7)} \leq \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_2)^{(7)}\right)t\right]}}{1 + (C)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_2)^{(7)}\right)t\right]}} \leq \nu^{(7)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)} e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \leq (\bar{\nu}_1)^{(7)}$$

If $0 < (\nu_1)^{(7)} \leq (\bar{\nu}_1)^{(7)} \leq \boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$ , we obtain 419

$$(\nu_1)^{(7)} \leq \nu^{(7)}(t) \leq \frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)} e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \leq (\nu_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(7)}(t)$ :-

$$(m_2)^{(7)} \leq \nu^{(7)}(t) \leq (m_1)^{(7)}, \quad \boxed{\nu^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(7)}(t)$ :- 420

$$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}, \quad \boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $\nu^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)} G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)} T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

**Proof** : From global equations we obtain 421

$$\frac{d\nu^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right) - (a''_{41})^{(8)}(T_{41}, t)\nu^{(8)} - (a_{41})^{(8)}\nu^{(8)}$$

**Definition of** $\nu^{(8)}$ :- $\boxed{\nu^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right) \leq \frac{d\nu^{(8)}}{dt} \leq -\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$ :-

For $0 < \boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$

$$\nu^{(8)}(t) \geq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)} e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \leq \frac{(\nu_1)^{(8)}+(C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}}{1+(C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}} \leq \nu^{(8)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \leq (\bar{\nu}_1)^{(8)} \leq \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$, we obtain

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$$(\nu_1)^{(8)} \leq \nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(8)}(t)$ :-

$$(m_2)^{(8)} \leq \nu^{(8)}(t) \leq (m_1)^{(8)}, \quad \boxed{\nu^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(8)}(t)$ :-

$$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}, \quad \boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $\nu^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)}G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then $(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)}T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

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**Proof** : From 99,20,44,22,23,44 we obtain

$$\frac{d\nu^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45}, t)\right) - (a''_{45})^{(9)}(T_{45}, t)\nu^{(9)} - (a_{45})^{(9)}\nu^{(9)}$$

**Definition of** $\nu^{(9)}$ :- $\boxed{\nu^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(\nu^{(9)}\right)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)}\right) \leq \frac{d\nu^{(9)}}{dt} \leq -\left((a_{45})^{(9)}\left(\nu^{(9)}\right)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$ :-

For $0 < \boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$

$$\nu^{(9)}(t) \geq \frac{(\nu_1)^{(9)}+(C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_0)^{(9)}\right)t\right]}}{1+(C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)}-(\nu_0)^{(9)}}{(\nu_0)^{(9)}-(\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\nu_9)^{(9)}$

In the same manner , we get

$$\nu^{(9)}(t) \leq \frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)}-(\nu_0)^{(9)}}{(\nu_0)^{(9)}-(\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \leq \frac{(\nu_1)^{(9)}+(C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}}{1+(C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}} \leq \nu^{(9)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \leq (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \leq (\bar{\nu}_1)^{(9)} \leq \boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$ , we obtain

$$(\nu_1)^{(9)} \leq \nu^{(9)}(t) \leq \frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \leq (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(9)}(t)$ :-

$$(m_2)^{(9)} \leq \nu^{(9)}(t) \leq (m_1)^{(9)}, \quad \boxed{\nu^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(9)}(t)$ :-

$$(\mu_2)^{(9)} \leq u^{(9)}(t) \leq (\mu_1)^{(9)}, \quad \boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)}G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then
$(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)}T_{45}(t)$ This is an important consequence of the relation between $(v_1)^{(9)}$ and $(\bar{v}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following 425

**Theorem :** If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on $t$ , and the conditions with the notations

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0$ ,

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$

$with\ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a''_i)^{(2)} and\ (b''_i)^{(2)}$ are independent on t , and the conditions with the notations 426

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$ 427

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 428

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$ , 429

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 430

$with\ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a''_i)^{(3)} and\ (b''_i)^{(3)}$ are independent on $t$ , and the conditions with the notations 431

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$ ,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

**Theorem :** If $(a''_i)^{(4)} and\ (b''_i)^{(4)}$ are independent on $t$ , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$ ,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(5)} and\ (b_i'')^{(5)}$ are independent on $t$ , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$ ,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

**Theorem** If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on $t$ , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$ ,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(7)} and\ (b_i'')^{(7)}$ are independent on $t$ , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$ ,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(8)} and\ (b_i'')^{(8)}$ are independent on $t$ , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$ ,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a''_i)^{(9)} and\ (b''_i)^{(9)}$ are independent on $t$ , and the conditions (with the notations 45,46,27,28) 436 A

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$ ,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0$ 437

$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0$ 438

$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0$ 439

$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0$ 440

$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ]T_{14} = 0$ 441

$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0$ 442

has a unique positive solution , which is an equilibrium solution for the system

$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0$ 443

$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0$ 444

$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0$ 445

$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\ ]T_{16} = 0$ 446

$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\ ]T_{17} = 0$ 447

$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\ ]T_{18} = 0$ 448

has a unique positive solution , which is an equilibrium solution

$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0$ 449

$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0$ 450

$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0$ 451

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\,]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\,]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\,]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\big((G_{27})\big)\,]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\big((G_{27})\big)\,]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\big((G_{27})\big)\,]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})\,]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})\,]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})\,]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})\,]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})\,]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})\,]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0 \quad 479$$

$$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0 \quad 480$$

$$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0 \quad 481$$

$$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0 \quad 482$$

$$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0 \quad 483$$

$$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0 \quad 484$$

$$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0 \quad 484\text{A}$$

$$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$$

$$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$$

$$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$$

$$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$$

$$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$$

**Proof:** 485

(a) Indeed the first two equations have a nontrivial solution $G_{13}, G_{14}$ if

$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$

**Proof:** 486

(e) Indeed the first two equations have a nontrivial solution $G_{16}, G_{17}$ if

$\mathrm{F}(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$

**Proof:** 487

(a) Indeed the first two equations have a nontrivial solution $G_{20}, G_{21}$ if

$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$

**Proof:** 488

(a) Indeed the first two equations have a nontrivial solution $G_{24}, G_{25}$ if

$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$

**Proof:** 489

(a) Indeed the first two equations have a nontrivial solution $G_{28}, G_{29}$ if

$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$

**Proof:** 490

(a) Indeed the first two equations have a nontrivial solution $G_{32}, G_{33}$ if

$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$

**Proof:** 491

(a) Indeed the first two equations have a nontrivial solution $G_{36}, G_{37}$ if

$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$

**Proof:** 492

(a) Indeed the first two equations have a nontrivial solution $G_{40}, G_{41}$ if

$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$

**Proof:** 492 A

(a) Indeed the first two equations have a nontrivial solution $G_{44}, G_{45}$ if

$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$

**Definition and uniqueness of** $T^*_{14}$ :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique $T^*_{14}$ for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

**Definition and uniqueness of** $T^*_{17}$ :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique $T^*_{17}$ for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]}$$ 495

**Definition and uniqueness of** $T^*_{21}$ :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique $T^*_{21}$ for which $f(T^*_{21}) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

**Definition and uniqueness of** $T^*_{25}$ :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that there exists a unique $T^*_{25}$ for which $f(T^*_{25}) = 0$. With this value , we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T^*_{25})]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T^*_{25})]}$$

**Definition and uniqueness of** $T^*_{29}$ :- 498

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(5)}(T_{29})$ being increasing, it follows that there exists a unique $T^*_{29}$ for which $f(T^*_{29}) = 0$. With this value , we obtain from the three first equations

$G_{28} = \frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T^*_{29})]}$ , $G_{30} = \frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T^*_{29})]}$

**Definition and uniqueness of** $T^*_{33}$ :- 499

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(6)}(T_{33})$ being increasing, it follows that there exists a unique $T^*_{33}$ for which $f(T^*_{33}) = 0$. With this value , we obtain from the three first equations

$G_{32} = \frac{(a_{32})^{(6)}G_{33}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T^*_{33})]}$ , $G_{34} = \frac{(a_{34})^{(6)}G_{33}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T^*_{33})]}$

**Definition and uniqueness of** $T^*_{37}$ :- 500

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(7)}(T_{37})$ being increasing, it follows that there exists a unique $T^*_{37}$ for which $f(T^*_{37}) = 0$. With this value , we obtain from the three first equations

$G_{36} = \frac{(a_{36})^{(7)}G_{37}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T^*_{37})]}$ , $G_{38} = \frac{(a_{38})^{(7)}G_{37}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T^*_{37})]}$

**Definition and uniqueness of** $T^*_{41}$ :- 501

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(8)}(T_{41})$ being increasing, it follows that there exists a unique $T^*_{41}$ for which $f(T^*_{41}) = 0$. With this value , we obtain from the three first equations

$G_{40} = \frac{(a_{40})^{(8)}G_{41}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T^*_{41})]}$ , $G_{42} = \frac{(a_{42})^{(8)}G_{41}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T^*_{41})]}$

**Definition and uniqueness of** $T^*_{45}$ :- 501 A

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(9)}(T_{45})$ being increasing, it follows that there exists a unique $T^*_{45}$ for which $f(T^*_{45}) = 0$. With this value , we obtain from the three first equations

$G_{44} = \frac{(a_{44})^{(9)}G_{45}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T^*_{45})]}$ , $G_{46} = \frac{(a_{46})^{(9)}G_{45}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T^*_{45})]}$

By the same argument, the equations admit solutions $G_{13}, G_{14}$ if 502

$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$

$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right]+(b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{14}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{14}$ such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions $G_{16}, G_{17}$ if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right] + (b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that $\varphi$ is a decreasing function in $G_{17}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{14}^{*}$ such that $\varphi((G_{19})^{*}) = 0$ 504

By the same argument, the equations admit solutions $G_{20}, G_{21}$ if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right] + (b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that $\varphi$ is a decreasing function in $G_{21}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{21}^{*}$ such that $\varphi((G_{23})^{*}) = 0$

By the same argument, the equations admit solutions $G_{24}, G_{25}$ if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right] + (b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that $\varphi$ is a decreasing function in $G_{25}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{25}^{*}$ such that $\varphi((G_{27})^{*}) = 0$

By the same argument, the equations admit solutions $G_{28}, G_{29}$ if 507

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right] + (b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that $\varphi$ is a decreasing function in $G_{29}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{29}^{*}$ such that $\varphi((G_{31})^{*}) = 0$

By the same argument, the equations admit solutions $G_{32}, G_{33}$ if 508

$$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$$

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right] + (b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that $\varphi$ is a decreasing function in $G_{33}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{33}^{*}$ such that $\varphi({G_{35}}^{*}) = 0$

By the same argument, the equations admit solutions $G_{36}, G_{37}$ if 509

$$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$$

$$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right] + (b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{37}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{37}$ such that $\varphi(G_{39}{}^*) = 0$

By the same argument, the equations admit solutions $G_{40}, G_{41}$ if 510

$$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$$
$$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right] + (b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{41}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{41}$ such that $\varphi(G_{43}{}^*) = 0$

By the same argument, the equations 92,93 admit solutions $G_{44}, G_{45}$ if

$$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$$

$$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right] + (b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{45}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{45}$ such that $\varphi((G_{47})^*) = 0$

Finally we obtain the unique solution 511

$G^*_{14}$ given by $\varphi(G^*) = 0$ , $T^*_{14}$ given by $f(T^*_{14}) = 0$ and

$$G^*_{13} = \frac{(a_{13})^{(1)}G^*_{14}}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G^*_{15} = \frac{(a_{15})^{(1)}G^*_{14}}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T^*_{14})]}$$

$$T^*_{13} = \frac{(b_{13})^{(1)}T^*_{14}}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^*)]} \quad , \quad T^*_{15} = \frac{(b_{15})^{(1)}T^*_{14}}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^*)]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

$G^*_{17}$ given by $\varphi((G_{19})^*) = 0$ , $T^*_{17}$ given by $f(T^*_{17}) = 0$ and 512

$$G^*_{16} = \frac{(a_{16})^{(2)}G^*_{17}}{[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G^*_{18} = \frac{(a_{18})^{(2)}G^*_{17}}{[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T^*_{17})]} \quad 513$$

$$T^*_{16} = \frac{(b_{16})^{(2)}T^*_{17}}{[(b'_{16})^{(2)} - (b''_{16})^{(2)}((G_{19})^*)]} \quad , \quad T^*_{18} = \frac{(b_{18})^{(2)}T^*_{17}}{[(b'_{18})^{(2)} - (b''_{18})^{(2)}((G_{19})^*)]} \quad 514$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

$G^*_{21}$ given by $\varphi((G_{23})^*) = 0$ , $T^*_{21}$ given by $f(T^*_{21}) = 0$ and

$$G_{20}^{*}=\frac{(a_{20})^{(3)}G_{21}^{*}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T_{21}^{*})]}\quad,\quad G_{22}^{*}=\frac{(a_{22})^{(3)}G_{21}^{*}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T_{21}^{*})]}$$

$$T_{20}^{*}=\frac{(b_{20})^{(3)}T_{21}^{*}}{[(b'_{20})^{(3)}-(b''_{20})^{(3)}(G_{23}{}^{*})]}\quad,\quad T_{22}^{*}=\frac{(b_{22})^{(3)}T_{21}^{*}}{[(b'_{22})^{(3)}-(b''_{22})^{(3)}(G_{23}{}^{*})]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

$G_{25}^{*}$ given by $\varphi(G_{27})=0$ , $T_{25}^{*}$ given by $f(T_{25}^{*})=0$ and

$$G_{24}^{*}=\frac{(a_{24})^{(4)}G_{25}^{*}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T_{25}^{*})]}\quad,\quad G_{26}^{*}=\frac{(a_{26})^{(4)}G_{25}^{*}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T_{25}^{*})]}$$

$$T_{24}^{*}=\frac{(b_{24})^{(4)}T_{25}^{*}}{[(b'_{24})^{(4)}-(b''_{24})^{(4)}((G_{27})^{*})]}\quad,\quad T_{26}^{*}=\frac{(b_{26})^{(4)}T_{25}^{*}}{[(b'_{26})^{(4)}-(b''_{26})^{(4)}((G_{27})^{*})]} \qquad 517$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

$G_{29}^{*}$ given by $\varphi((G_{31})^{*})=0$ , $T_{29}^{*}$ given by $f(T_{29}^{*})=0$ and

$$G_{28}^{*}=\frac{(a_{28})^{(5)}G_{29}^{*}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T_{29}^{*})]}\quad,\quad G_{30}^{*}=\frac{(a_{30})^{(5)}G_{29}^{*}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T_{29}^{*})]}$$

$$T_{28}^{*}=\frac{(b_{28})^{(5)}T_{29}^{*}}{[(b'_{28})^{(5)}-(b''_{28})^{(5)}((G_{31})^{*})]}\quad,\quad T_{30}^{*}=\frac{(b_{30})^{(5)}T_{29}^{*}}{[(b'_{30})^{(5)}-(b''_{30})^{(5)}((G_{31})^{*})]} \qquad 519$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

$G_{33}^{*}$ given by $\varphi((G_{35})^{*})=0$ , $T_{33}^{*}$ given by $f(T_{33}^{*})=0$ and

$$G_{32}^{*}=\frac{(a_{32})^{(6)}G_{33}^{*}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^{*})]}\quad,\quad G_{34}^{*}=\frac{(a_{34})^{(6)}G_{33}^{*}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^{*})]}$$

$$T_{32}^{*}=\frac{(b_{32})^{(6)}T_{33}^{*}}{[(b'_{32})^{(6)}-(b''_{32})^{(6)}((G_{35})^{*})]}\quad,\quad T_{34}^{*}=\frac{(b_{34})^{(6)}T_{33}^{*}}{[(b'_{34})^{(6)}-(b''_{34})^{(6)}((G_{35})^{*})]} \qquad 521$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

$G_{37}^{*}$ given by $\varphi((G_{39})^{*})=0$ , $T_{37}^{*}$ given by $f(T_{37}^{*})=0$ and

$$G_{36}^{*}=\frac{(a_{36})^{(7)}G_{37}^{*}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^{*})]}\quad,\quad G_{38}^{*}=\frac{(a_{38})^{(7)}G_{37}^{*}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^{*})]}$$

$$T_{36}^{*}=\frac{(b_{36})^{(7)}T_{37}^{*}}{[(b'_{36})^{(7)}-(b''_{36})^{(7)}((G_{39})^{*})]}\quad,\quad T_{38}^{*}=\frac{(b_{38})^{(7)}T_{37}^{*}}{[(b'_{38})^{(7)}-(b''_{38})^{(7)}((G_{39})^{*})]}$$

Finally we obtain the unique solution 523

$G_{41}^*$ given by $\varphi((G_{43})^*) = 0$ , $T_{41}^*$ given by $f(T_{41}^*) = 0$ and

$$G_{40}^* = \frac{(a_{40})^{(8)} G_{41}^*}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42}^* = \frac{(a_{42})^{(8)} G_{41}^*}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

$$T_{40}^* = \frac{(b_{40})^{(8)} T_{41}^*}{[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43})^*)]} \quad , \quad T_{42}^* = \frac{(b_{42})^{(8)} T_{41}^*}{[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43})^*)]}$$

Finally we obtain the unique solution of 89 to 99 523 A

$G_{45}^*$ given by $\varphi((G_{47})^*) = 0$ , $T_{45}^*$ given by $f(T_{45}^*) = 0$ and

$$G_{44}^* = \frac{(a_{44})^{(9)} G_{45}^*}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46}^* = \frac{(a_{46})^{(9)} G_{45}^*}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

$$T_{44}^* = \frac{(b_{44})^{(9)} T_{45}^*}{[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47})^*)]} \quad , \quad T_{46}^* = \frac{(b_{46})^{(9)} T_{45}^*}{[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47})^*)]}$$

## ASYMPTOTIC STABILITY ANALYSIS 524

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

**Proof:** Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{14}'')^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \frac{\partial (b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a'_{13})^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)} G_{13}^* \mathbb{T}_{14} \quad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a'_{14})^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)} G_{14}^* \mathbb{T}_{14} \quad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a'_{15})^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)} G_{15}^* \mathbb{T}_{14} \quad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b'_{13})^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)} T_{13}^* \mathbb{G}_j\right) \quad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b'_{14})^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)} T_{14}^* \mathbb{G}_j\right) \quad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b'_{15})^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T^*_{15}\mathbb{G}_j\right) \quad 530$$

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**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(2)}$ and $(b''_i)^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i \quad 532$$

$$\frac{\partial(a''_{17})^{(2)}}{\partial T_{17}}(T^*_{17}) = (q_{17})^{(2)} \quad , \frac{\partial(b''_i)^{(2)}}{\partial G_j}\left((G_{19})^*\right) = s_{ij} \quad 533$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a'_{16})^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G^*_{16}\mathbb{T}_{17} \quad 534$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a'_{17})^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G^*_{17}\mathbb{T}_{17} \quad 535$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a'_{18})^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G^*_{18}\mathbb{T}_{17} \quad 536$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b'_{16})^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T^*_{16}\mathbb{G}_j\right) \quad 537$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b'_{17})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}T^*_{17}\mathbb{G}_j\right) \quad 538$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b'_{18})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}T^*_{18}\mathbb{G}_j\right) \quad 539$$

**ASYMPTOTIC STABILITY ANALYSIS** 540

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(3)}$ *and* $(b''_i)^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{21})^{(3)}}{\partial T_{21}}(T^*_{21}) = (q_{21})^{(3)} \quad , \frac{\partial(b''_i)^{(3)}}{\partial G_j}\left((G_{23})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a'_{20})^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G^*_{20}\mathbb{T}_{21} \quad 541$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a'_{21})^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G^*_{21}\mathbb{T}_{21} \quad 542$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a'_{22})^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G^*_{22}\mathbb{T}_{21} \quad 543$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b'_{20})^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T^*_{20}\mathbb{G}_j\right) \quad 544$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b'_{21})^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T^*_{21}\mathbb{G}_j\right) \quad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b'_{22})^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T^*_{22}\mathbb{G}_j\right) \quad 546$$

**ASYMPTOTIC STABILITY ANALYSIS** 547

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}$ $and$ $(b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{25})^{(4)}}{\partial T_{25}}(T^*_{25}) = (q_{25})^{(4)} \ , \ \frac{\partial(b_i'')^{(4)}}{\partial G_j}((G_{27})^* \ ) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\left((a'_{24})^{(4)} + (p_{24})^{(4)}\right)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G^*_{24}\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\left((a'_{25})^{(4)} + (p_{25})^{(4)}\right)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G^*_{25}\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\left((a'_{26})^{(4)} + (p_{26})^{(4)}\right)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G^*_{26}\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\left((b'_{24})^{(4)} - (r_{24})^{(4)}\right)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(24)(j)}T^*_{24}\mathbb{G}_j\right) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\left((b'_{25})^{(4)} - (r_{25})^{(4)}\right)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\left(s_{(25)(j)}T^*_{25}\mathbb{G}_j\right) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T^*_{26}\mathbb{G}_j\right) \quad 554$$

**ASYMPTOTIC STABILITY ANALYSIS** 555

**Theorem 5:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}$ $and$ $(b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 556

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$\frac{\partial(a_{29}'')^{(5)}}{\partial T_{29}}(T_{29}^*) = (q_{29})^{(5)}$ , $\frac{\partial(b_i'')^{(5)}}{\partial G_j}((G_{31})^*) = s_{ij}$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a_{28}')^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G_{28}^*\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a_{29}')^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G_{29}^*\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a_{30}')^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G_{30}^*\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b_{28}')^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T_{28}^*\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b_{29}')^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T_{29}^*\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b_{30}')^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T_{30}^*\mathbb{G}_j\right) \quad 562$$

**ASYMPTOTIC STABILITY ANALYSIS** 563

**Theorem 6:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}$ *and* $(b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 564

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$\frac{\partial(a_{33}'')^{(6)}}{\partial T_{33}}(T_{33}^*) = (q_{33})^{(6)}$ , $\frac{\partial(b_i'')^{(6)}}{\partial G_j}((G_{35})^*) = s_{ij}$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a_{32}')^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G_{32}^*\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a_{33}')^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G_{33}^*\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a_{34}')^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G_{34}^*\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b_{32}')^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T_{32}^*\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b_{33}')^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T_{33}^*\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b_{34}')^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T_{34}^*\mathbb{G}_j\right) \quad 570$$

**ASYMPTOTIC STABILITY ANALYSIS** 571

**Theorem 7:** If the conditions of the previous theorem are satisfied and if the functions

$(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 572

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{37}'')^{(7)}}{\partial T_{37}}(T_{37}^*) = (q_{37})^{(7)} \quad , \frac{\partial (b_i'')^{(7)}}{\partial G_j}\left( (G_{39})^{**} \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a_{36}')^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G_{36}^*\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a_{37}')^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G_{37}^*\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a_{38}')^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G_{38}^*\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b_{36}')^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T_{36}^*\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b_{37}')^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T_{37}^*\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b_{38}')^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T_{38}^*\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

## ASYMPTOTIC STABILITY ANALYSIS

**Theorem 8:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 580

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{41}'')^{(8)}}{\partial T_{41}}(T_{41}^*) = (q_{41})^{(8)} \quad , \frac{\partial (b_i'')^{(8)}}{\partial G_j}\left( (G_{43})^{*} \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a_{40}')^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G_{40}^*\mathbb{T}_{41} \quad 581$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a'_{41})^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G^*_{41}\mathbb{T}_{41} \quad 582$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a'_{42})^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G^*_{42}\mathbb{T}_{41} \quad 583$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b'_{40})^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T^*_{40}\mathbb{G}_j\right) \quad 584$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b'_{41})^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T^*_{41}\mathbb{G}_j\right) \quad 585$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b'_{42})^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T^*_{42}\mathbb{G}_j\right) \quad 586$$

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## ASYMPTOTIC STABILITY ANALYSIS

**Theorem 9:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}$ *and* $(b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

**Proof:** Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{45})^{(9)}}{\partial T_{45}}(T^*_{45}) = (q_{45})^{(9)} \quad , \frac{\partial(b_i'')^{(9)}}{\partial G_j}((G_{47})^*) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a'_{44})^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)}G^*_{44}\mathbb{T}_{45} \quad 586\,B$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a'_{45})^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)}G^*_{45}\mathbb{T}_{45} \quad 586\,C$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a'_{46})^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)}G^*_{46}\mathbb{T}_{45} \quad 586\,D$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b'_{44})^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)}T^*_{44}\mathbb{G}_j\right) \quad 586\,E$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b'_{45})^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)}T^*_{45}\mathbb{G}_j\right) \quad 586\,F$$

$$\frac{d\mathbb{T}_{46}}{dt} = -\left((b'_{46})^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T^*_{46}\mathbb{G}_j\right) \quad 586\,G$$

**The characteristic equation of this system is** 587

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(q_{13})^{(1)}G^*_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)}T^*_{14} + (b_{14})^{(1)}s_{(13),(14)}T^*_{14}\right)$$

$$+\left(\left((\lambda)^{(1)}+(a'_{14})^{(1)}+(p_{14})^{(1)}\right)(q_{13})^{(1)}G^{*}_{13}+(a_{13})^{(1)}(q_{14})^{(1)}G^{*}_{14}\right)$$

$$\left(\left((\lambda)^{(1)}+(b'_{13})^{(1)}-(r_{13})^{(1)}\right)s_{(14),(13)}T^{*}_{14}+(b_{14})^{(1)}s_{(13),(13)}T^{*}_{13}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^{2}+\left((a'_{13})^{(1)}+(a'_{14})^{(1)}+(p_{13})^{(1)}+(p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^{2}+\left((b'_{13})^{(1)}+(b'_{14})^{(1)}-(r_{13})^{(1)}+(r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^{2}+\left((a'_{13})^{(1)}+(a'_{14})^{(1)}+(p_{13})^{(1)}+(p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)}+(a'_{13})^{(1)}+(p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^{*}_{14}+(a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^{*}_{13}\right)$$

$$\left(\left((\lambda)^{(1)}+(b'_{13})^{(1)}-(r_{13})^{(1)}\right)s_{(14),(15)}T^{*}_{14}+(b_{14})^{(1)}s_{(13),(15)}T^{*}_{13}\right)\}=0$$

$$+$$

$$(\lambda)^{(2)}+(b'_{18})^{(2)}-(r_{18})^{(2)}\{\left((\lambda)^{(2)}+(a'_{18})^{(2)}+(p_{18})^{(2)}\right)$$

$$\left[\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^{*}_{17}+(a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^{*}_{16}\right)\Big]$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^{*}_{17}\right)$$

$$+\left(\left((\lambda)^{(2)}+(a'_{17})^{(2)}+(p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^{*}_{16}+(a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^{*}_{17}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^{*}_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^{2}+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^{2}+\left((b'_{16})^{(2)}+(b'_{17})^{(2)}-(r_{16})^{(2)}+(r_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$+\left(\left((\lambda)^{(2)}\right)^{2}+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^{*}_{17}+(a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^{*}_{16}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^{*}_{16}\right)\}=0$$

$$+$$

$$(\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)\Big]$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(21)}T^{*}_{21}\right)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^{*}_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^{*}_{21}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(20)}T^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$+\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(22)}T^{*}_{20}\right)\}=0$$

$$+$$

$$(\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left(\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)\right]$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(25)}T^{*}_{25}\right)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^{*}_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^{*}_{25}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(24)}T^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$+\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(26)}T^{*}_{24}\right)\}=0$$

$$+$$

$$(\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(29)}T^{*}_{29}\right)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^{*}_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^{*}_{29}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(28)}T^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$+\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(30)}T^{*}_{28}\right)\}=0$$

$$+$$

$$(\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left(\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)\right]$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(33)}T^{*}_{33}\right)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^{*}_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^{*}_{33}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(32)}T^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$+\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(34)}T^{*}_{32}\right)\}=0$$

$$+$$

$$(\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)\right]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(37)}T^{*}_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^{*}_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^{*}_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(36)}T^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$+\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(38)}T^{*}_{36}\right)\}=0$$

$$+$$

$$(\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(41)}T^{*}_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^{*}_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^{*}_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(40)}T^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$+\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(42)}T^{*}_{40}\right)\}=0$$

$$+$$

$$(\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)\right]$$

$$\left((\lambda)^{(9)} + (b'_{44})^{(9)} - (r_{44})^{(9)}\right)s_{(45),(45)}T^*_{45} + (b_{45})^{(9)}s_{(44),(45)}T^*_{45}\Big)$$

$$+\left((\lambda)^{(9)} + (a'_{45})^{(9)} + (p_{45})^{(9)}\right)(q_{44})^{(9)}G^*_{44} + (a_{44})^{(9)}(q_{45})^{(9)}G^*_{45}\Big)$$

$$\left((\lambda)^{(9)} + (b'_{44})^{(9)} - (r_{44})^{(9)}\right)s_{(45),(44)}T^*_{45} + (b_{45})^{(9)}s_{(44),(44)}T^*_{44}\Big)$$

$$\left((\lambda)^{(9)}\right)^2 + \left((a'_{44})^{(9)} + (a'_{45})^{(9)} + (p_{44})^{(9)} + (p_{45})^{(9)}\right)(\lambda)^{(9)}\Big)$$

$$\left((\lambda)^{(9)}\right)^2 + \left((b'_{44})^{(9)} + (b'_{45})^{(9)} - (r_{44})^{(9)} + (r_{45})^{(9)}\right)(\lambda)^{(9)}\Big)$$

$$+\left((\lambda)^{(9)}\right)^2 + \left((a'_{44})^{(9)} + (a'_{45})^{(9)} + (p_{44})^{(9)} + (p_{45})^{(9)}\right)(\lambda)^{(9)}\Big)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)} + (a'_{44})^{(9)} + (p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^*_{45} + (a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^*_{44}\right)$$

$$\left((\lambda)^{(9)} + (b'_{44})^{(9)} - (r_{44})^{(9)}\right)s_{(45),(46)}T^*_{45} + (b_{45})^{(9)}s_{(44),(46)}T^*_{44}\Big)\} = 0$$

**And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.**

## SECTION SEVEN

## Quantum Discord As Resource For Remote State Preparation

### INTRODUCTION—VARIABLES USED

**Quantum discord as resource for remote state preparation Borivoje Dakić, Yannick Ole Lipp, Xiaosong MA, Martín Ringbauer, Sebastian Kropatschek, Stefanie Barz, Tomasz Paterek, Vlatko Vedral, Anton Zeilinger, Časlav Brukner & Philip Walther Nature Physics 8, 666–670 (2012) doi:10.1038/nphys2377Published online 05 August 2012**

(1) The existence of better-than-classical quantum information processing (QIP) models which consume very little or no entanglement suggests (eb) that separable or weakly entangled states could be extremely useful tools for (e) information processing as they are much easier to prepare and control (eb,e&eb)even in dissipative environments.
(2) It has been proposed that a resource of advantage is (=) the generation of quantum discord, a (=) measure of non-classical correlations that includes (e) entanglement as a subset.
(3) Here authors show that quantum discord is the necessary resource for (e) quantum remote state preparation.
(4) Authors explicitly show that the geometric measure of quantum discord is related to (e&eb) the fidelity of this task, which provides (eb) its operational meaning.

(5) Results are experimentally demonstrated using (e) photonic quantum systems.

**(6)** Moreover, authors demonstrate that separable states with (e&eb) non-zero quantum discord can outperform (e) entangled states.

**(7)** Therefore, the role of quantum discord might provide (eb) fundamental insights for resource-efficient QIP. Subject terms: Optical physics Quantum information

## NOTATION

### Module One

The existence of better-than-classical quantum information processing (QIP) models which consume very little or no entanglement suggests (eb) that separable or weakly entangled states could be extremely useful tools for (e) information processing as they are much easier to prepare and control (eb,e&eb)even in dissipative environments

$G_{13}$ : Category one of existence of better-than-classical quantum information processing (QIP) models which consume very little or no entanglement

$G_{14}$ : Category two of SAS

$G_{15}$ : Category three of SAS

$T_{13}$ : Category one of separable or weakly entangled states could be extremely useful tools for (e) information processing as they are much easier to prepare and control (eb,e&eb)even in dissipative environments

$T_{14}$ : Category two of SAS

$T_{15}$ : Category three of SAS

### Module Two

The existence of better-than-classical quantum information processing (QIP) models which consume very little or no entanglement suggests that separable or weakly entangled states could be extremely useful tools for (e) information processing as they are much easier to prepare and control (eb,e&eb)even in dissipative environments

$G_{16}$ : Category one of existence of better-than-classical quantum information processing (QIP) models which consume very little or no entanglement suggests that separable or weakly entangled states could be extremely useful tools; **information processing as they are much easier to prepare and control (eb,e&eb)even in dissipative environments**

$G_{17}$ : Category two of SAS

$G_{18}$ : Category three of SAS

$T_{16}$ : Category one of **information processing as they are much easier to prepare and control (eb,e&eb)even in dissipative environments;** existence of better-than-classical quantum information processing (QIP) models which consume very little or no entanglement suggests that separable or weakly entangled states could be extremely useful tools

$T_{17}$ : Category two of SAS

$T_{18}$ : Category three of SAS

### Module three

The existence of better-than-classical quantum information processing (QIP) models which consume very

little or no entanglement suggests that separable or weakly entangled states could be extremely useful tools for information processing as they are much easier to prepare and control (eb,e&eb)even in **dissipative environments**

$G_{20}$ : Category one of existence of better-than-classical quantum information processing (QIP) models which consume very little or no entanglement suggests that separable or weakly entangled states could be extremely useful tools for information processing as they are much easier to prepare and control; **dissipative environments**

$G_{21}$ : Category two of SAS

$G_{22}$ : Category three of SAS

$T_{20}$ : Category one of **dissipative environments** ;existence of better-than-classical quantum information processing (QIP) models which consume very little or no entanglement suggests that separable or weakly entangled states could be extremely useful tools for information processing as they are much easier to prepare and control

$T_{21}$ : Category two of SAS

$T_{22}$ : Category three of SAS

**Module four**

It has been proposed that a resource of advantage is (=) the generation of quantum discord, a (=) measure of non-classical correlations that includes (e) entanglement as a subset

$G_{24}$ : Category one of resource of advantage

$G_{25}$ : Category two of SAS

$G_{26}$ : Category three of SAS

$T_{24}$ : Category one of generation of quantum discord, a (=) measure of non-classical correlations that includes (e) entanglement as a subset

$T_{25}$ : Category two of SAS

$T_{26}$ : Category three of SAS

**Module five**

It has been proposed that a resource of advantage is the generation of quantum discord, a (=) measure of non-classical correlations that includes (e) entanglement as a subset

$G_{28}$ : Category one of resource of advantage is the generation of quantum discord

$G_{29}$ : Category two of SAS

$G_{30}$ : Category three of SAS

$T_{28}$ : Category one of measure of non-classical correlations that includes (e) entanglement as a subset

$T_{29}$ : Category two of SAS

$T_{30}$ : Category three of SAS

**Module six**

It has been proposed that a resource of advantage is the generation of quantum discord, a measure of non-classical correlations that includes (e) entanglement as a subset

$G_{32}$ : Category one of entanglement as a subset

$G_{33}$ : Category two of SAS

$G_{34}$ : Category three of SAS

$T_{32}$ : Category one of resource of advantage is the generation of quantum discord, a measure of non-classical correlations

$T_{33}$ : Category two of SAS

$T_{34}$ : Category three of SAS

**Module seven**

Here authors show that quantum discord is the necessary resource for (e) **quantum remote state preparation**

$G_{36}$ : Category one of quantum discord is the necessary resource; **quantum remote state preparation**

$G_{37}$ : Category two of SAS

$G_{38}$ : Category three of SAS

$T_{36}$ : Category one of **quantum remote state preparation** ;quantum discord is the necessary resource

$T_{37}$ : Category two of SAS

$T_{38}$ : Category three of SAS

**Module eight**

Authors explicitly show that the **geometric measure of quantum discord** is related to (e&eb) the fidelity of this task, which provides (eb) its operational meaning

$G_{40}$ : Category one of **geometric measure of quantum discord;** fidelity of this task, which provides (eb) its operational meaning

$G_{41}$ : Category two of SAS

$G_{42}$ : Category three of SAS

$T_{40}$ : Category one of fidelity of this task, which provides (eb) its operational meaning **;geometric measure of quantum discord**

$T_{41}$ : Category two of SAS

$T_{42}$ : Category three of SAS

**Module Nine**

Moreover, authors demonstrate that separable states with non-zero quantum discord can outperform (e) entangled states

$G_{44}$ : Category one of entangled states

$G_{45}$ : Category two of SAS

$G_{46}$ : Category three of SAS

$T_{44}$ : Category one of separable states with non-zero quantum discord

$T_{45}$ : Category two of SAS

$T_{46}$ : Category three of SAS

**The Coefficients:**

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}$ , $(b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}$,
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}$, $(b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}\ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$ ,
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)}$,
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)}$,
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)}$,
are Dissipation coefficients

**Module Numbered One**

**The differential system of this model is now (Module Numbered one)**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \quad 1$$
$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad 2$$
$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15} \quad 3$$
$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13} \quad 4$$

$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G,t)\right]T_{14}$ 5

$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G,t)\right]T_{15}$ 6

$+(a''_{13})^{(1)}(T_{14},t) =$ First augmentation factor

$-(b''_{13})^{(1)}(G,t) =$ First detritions factor

**Module Numbered Two**

**The differential system of this model is now ( Module numbered two)**

$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17},t)\right]G_{16}$ 7

$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17},t)\right]G_{17}$ 8

$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17},t)\right]G_{18}$ 9

$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\left((G_{19}),t\right)\right]T_{16}$ 10

$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\left((G_{19}),t\right)\right]T_{17}$ 11

$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\left((G_{19}),t\right)\right]T_{18}$ 12

$+(a''_{16})^{(2)}(T_{17},t) =$ First augmentation factor

$-(b''_{16})^{(2)}\left((G_{19}),t\right) =$ First detritions factor

**Module Numbered Three**

**The differential system of this model is now (Module numbered three)**

$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21},t)\right]G_{20}$ 13

$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21},t)\right]G_{21}$ 14

$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21},t)\right]G_{22}$ 15

$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23},t)\right]T_{20}$ 16

$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23},t)\right]T_{21}$ 17

$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23},t)\right]T_{22}$ 18

$+(a''_{20})^{(3)}(T_{21},t) =$ First augmentation factor

$-(b''_{20})^{(3)}(G_{23},t) =$ First detritions factor

**Module Numbered Four**

**The differential system of this model is now (Module numbered Four)**

$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right]G_{24}$ 19

$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25},t)\right]G_{25}$ 20

$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25},t)\right]G_{26}$ 21

$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27}),t\right)\right]T_{24}$ 22

$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27}),t\right)\right]T_{25}$ 23

$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27}),t\right)\right]T_{26}$ 24

$+(a''_{24})^{(4)}(T_{25},t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}\left((G_{27}),t\right) =$ **First detritions factor**

**Module Numbered Five**:

**The differential system of this model is now (Module number five)**

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right]G_{28} \quad 25$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}, t)\right]G_{29} \quad 26$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}, t)\right]G_{30} \quad 27$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}\left((G_{31}), t\right)\right]T_{28} \quad 28$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}\left((G_{31}), t\right)\right]T_{29} \quad 29$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}\left((G_{31}), t\right)\right]T_{30} \quad 30$$

$+(a''_{28})^{(5)}(T_{29}, t) =$ **First augmentation factor**

$-(b''_{28})^{(5)}\left((G_{31}), t\right) =$ **First detritions factor**

**Module Numbered Six**

**The differential system of this model is now (Module numbered Six)**

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right]G_{32} \quad 31$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}, t)\right]G_{33} \quad 32$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}, t)\right]G_{34} \quad 33$$

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}\left((G_{35}), t\right)\right]T_{32} \quad 34$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}\left((G_{35}), t\right)\right]T_{33} \quad 35$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}\left((G_{35}), t\right)\right]T_{34} \quad 36$$

$+(a''_{32})^{(6)}(T_{33}, t) =$ **First augmentation factor**

**Module Numbered Seven:**

**The differential system of this model is now (Seventh Module)**

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right]G_{36} \quad 37$$

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37}, t)\right]G_{37} \quad 38$$

$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}, t)\right]G_{38} \quad 39$$

$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}\left((G_{39}), t\right)\right]T_{36} \quad 40$$

$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}\left((G_{39}), t\right)\right]T_{37} \quad 41$$

$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}\left((G_{39}), t\right)\right]T_{38} \quad 42$$

$+(a''_{36})^{(7)}(T_{37}, t) =$ **First augmentation factor**

**Module Numbered Eight**

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right]G_{40} \quad 43$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41}, t)\right]G_{41} \quad 44$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}, t)\right]G_{42} \quad 45$$

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}\left((G_{43}), t\right)\right]T_{40} \quad 46$$

$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}\left((G_{43}), t\right)\right]T_{41} \quad 47$$

$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43}),t)\right]T_{42} \quad 48$$

**Module Numbered Nine**

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47}),t)\right]T_{44} \quad 52$$

$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}((G_{47}),t)\right]T_{45} \quad 53$$

$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47}),t)\right]T_{46} \quad 54$$

$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}((G_{47}),t) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)} & +(a''_{13})^{(1)}(T_{14},t) & +(a''_{16})^{(2,2,)}(T_{17},t) & +(a''_{20})^{(3,3,)}(T_{21},t) \\ +(a''_{24})^{(4,4,4,4,)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{36})^{(7,7)}(T_{37},t) & +(a''_{40})^{(8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)} & +(a''_{14})^{(1)}(T_{14},t) & +(a''_{17})^{(2,2,)}(T_{17},t) & +(a''_{21})^{(3,3,)}(T_{21},t) \\ +(a''_{25})^{(4,4,4,4,)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{37})^{(7,7)}(T_{37},t) & +(a''_{41})^{(8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)} & +(a''_{15})^{(1)}(T_{14},t) & +(a''_{18})^{(2,2,)}(T_{17},t) & +(a''_{22})^{(3,3,)}(T_{21},t) \\ +(a''_{26})^{(4,4,4,4,)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{38})^{(7,7)}(T_{37},t) & +(a''_{42})^{(8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{15} \quad 57$$

Where $(a''_{13})^{(1)}(T_{14},t)$, $(a''_{14})^{(1)}(T_{14},t)$, $(a''_{15})^{(1)}(T_{14},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,)}(T_{17},t)$, $+(a''_{17})^{(2,2,)}(T_{17},t)$, $+(a''_{18})^{(2,2,)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,)}(T_{21},t)$, $+(a''_{21})^{(3,3,)}(T_{21},t)$, $+(a''_{22})^{(3,3,)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{38})^{(7,7)}(T_{37},t)$ $+(a''_{37})^{(7,7)}(T_{37},t)$ $+(a''_{36})^{(7,7)}(T_{37},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{40})^{(8,8)}(T_{41},t)$ $+(a''_{41})^{(8,8)}(T_{41},t)$ $+(a''_{42})^{(8,8)}(T_{41},t)$ are eight augmentation coefficient for 1,2,3

$+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)}\boxed{-(b''_{13})^{(1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13} \quad (58)$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)}\boxed{-(b''_{14})^{(1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14} \quad (59)$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)}\boxed{-(b''_{15})^{(1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15} \quad (60)$$

Where $\boxed{-(b''_{13})^{(1)}(G,t)}$, $\boxed{-(b''_{14})^{(1)}(G,t)}$, $\boxed{-(b''_{15})^{(1)}(G,t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8)}(G_{43},t)}$ $\boxed{-(b''_{41})^{(8,8)}(G_{43},t)}$ $\boxed{-(b''_{42})^{(8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \begin{bmatrix} (a'_{16})^{(2)}\boxed{+(a''_{16})^{(2)}(T_{17},t)} & \boxed{+(a''_{13})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{16} \quad (61)$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \begin{bmatrix} (a'_{17})^{(2)}\boxed{+(a''_{17})^{(2)}(T_{17},t)} & \boxed{+(a''_{14})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{17} \quad (62)$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \begin{bmatrix} (a'_{18})^{(2)}\boxed{+(a''_{18})^{(2)}(T_{17},t)} & \boxed{+(a''_{15})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{18} \quad (63)$$

Where $\boxed{+(a''_{16})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,)}(T_{14},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)}$ are eight augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{44})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9)}(T_{45},t)}$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \begin{bmatrix} (b'_{16})^{(2)}\boxed{-(b''_{16})^{(2)}(G_{19},t)} & \boxed{-(b''_{13})^{(1,1,)}(G,t)} & \boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{16} \quad (64)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)}\boxed{-(b''_{17})^{(2)}(G_{19},t)} & \boxed{-(b''_{14})^{(1,1,)}(G,t)} & \boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{17} \quad (65)$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)}\boxed{-(b''_{18})^{(2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,)}(G,t)} & \boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{18} \quad (66)$$

where $\boxed{-(b''_{16})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2)}(G_{19},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,)}(G,t)}$ are second detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)}$ are fourth detrition

coefficients for category 1,2 and 3

$-(b_{28}'')^{(5,5,5,5,5)}(G_{31},t)$, $-(b_{29}'')^{(5,5,5,5,5)}(G_{31},t)$, $-(b_{30}'')^{(5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1,2 and 3

$-(b_{32}'')^{(6,6,6,6,6)}(G_{35},t)$, $-(b_{33}'')^{(6,6,6,6,6)}(G_{35},t)$, $-(b_{34}'')^{(6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients for category 1,2 and 3

$-(b_{36}'')^{(7,7,7)}(G_{39},t)$, $-(b_{37}'')^{(7,7,7)}(G_{39},t)$, $-(b_{38}'')^{(7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2 and 3

$-(b_{40}'')^{(8,8,8)}(G_{43},t)$, $-(b_{41}'')^{(8,8,8)}(G_{43},t)$, $-(b_{42}'')^{(8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1,2 and 3

$-(b_{44}'')^{(9,9)}(G_{47},t)$, $-(b_{46}'')^{(9,9)}(G_{47},t)$, $-(b_{45}'')^{(9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[\begin{array}{c}(a_{20}')^{(3)}+(a_{20}'')^{(3)}(T_{21},t)+(a_{16}'')^{(2,2,2)}(T_{17},t)+(a_{13}'')^{(1,1,1,)}(T_{14},t)\\ +(a_{24}'')^{(4,4,4,4,4,4)}(T_{25},t)+(a_{28}'')^{(5,5,5,5,5,5)}(T_{29},t)+(a_{32}'')^{(6,6,6,6,6,6)}(T_{33},t)\\ +(a_{36}'')^{(7,7,7,7)}(T_{37},t)+(a_{40}'')^{(8,8,8,8)}(T_{41},t)+(a_{44}'')^{(9,9,9)}(T_{45},t)\end{array}\right]G_{20} \quad (67)$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[\begin{array}{c}(a_{21}')^{(3)}+(a_{21}'')^{(3)}(T_{21},t)+(a_{17}'')^{(2,2,2)}(T_{17},t)+(a_{14}'')^{(1,1,1,)}(T_{14},t)\\ +(a_{25}'')^{(4,4,4,4,4,4)}(T_{25},t)+(a_{29}'')^{(5,5,5,5,5,5)}(T_{29},t)+(a_{33}'')^{(6,6,6,6,6,6)}(T_{33},t)\\ +(a_{37}'')^{(7,7,7,7)}(T_{37},t)+(a_{41}'')^{(8,8,8,8)}(T_{41},t)+(a_{45}'')^{(9,9,9)}(T_{45},t)\end{array}\right]G_{21} \quad (68)$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[\begin{array}{c}(a_{22}')^{(3)}+(a_{22}'')^{(3)}(T_{21},t)+(a_{18}'')^{(2,2,2)}(T_{17},t)+(a_{15}'')^{(1,1,1,)}(T_{14},t)\\ +(a_{26}'')^{(4,4,4,4,4,4)}(T_{25},t)+(a_{30}'')^{(5,5,5,5,5,5)}(T_{29},t)+(a_{34}'')^{(6,6,6,6,6,6)}(T_{33},t)\\ +(a_{38}'')^{(7,7,7,7)}(T_{37},t)+(a_{42}'')^{(8,8,8,8)}(T_{41},t)+(a_{46}'')^{(9,9,9)}(T_{45},t)\end{array}\right]G_{22} \quad (69)$$

$+(a_{20}'')^{(3)}(T_{21},t)$, $+(a_{21}'')^{(3)}(T_{21},t)$, $+(a_{22}'')^{(3)}(T_{21},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a_{16}'')^{(2,2,2)}(T_{17},t)$, $+(a_{17}'')^{(2,2,2)}(T_{17},t)$, $+(a_{18}'')^{(2,2,2)}(T_{17},t)$ are second augmentation coefficients for category 1, 2 and 3

$+(a_{13}'')^{(1,1,1,)}(T_{14},t)$, $+(a_{14}'')^{(1,1,1,)}(T_{14},t)$, $+(a_{15}'')^{(1,1,1,)}(T_{14},t)$ are third augmentation coefficients for category 1, 2 and 3

$+(a_{24}'')^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a_{25}'')^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a_{26}'')^{(4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficients for category 1, 2 and 3

$+(a_{28}'')^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a_{29}'')^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a_{30}'')^{(5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficients for category 1, 2 and 3

$+(a_{32}'')^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a_{33}'')^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a_{34}'')^{(6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficients for category 1, 2 and 3

$+(a_{36}'')^{(7,7,7,7)}(T_{37},t)$, $+(a_{37}'')^{(7,7,7,7)}(T_{37},t)$, $+(a_{38}'')^{(7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1, 2 and 3

$+(a_{40}'')^{(8,8,8,8)}(T_{41},t)$, $+(a_{41}'')^{(8,8,8,8)}(T_{41},t)$, $+(a_{42}'')^{(8,8,8,8)}(T_{41},t)$ are eight augmentation coefficients for category 1, 2 and 3

$+(a_{44}'')^{(9,9,9)}(T_{45},t)$, $+(a_{45}'')^{(9,9,9)}(T_{45},t)$, $+(a_{46}'')^{(9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for

category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b'_{20})^{(3)}\boxed{-(b''_{20})^{(3)}(G_{23},t)}\boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}\boxed{-(b''_{13})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)}\boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)}\boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7)}(G_{39},t)}\boxed{-(b''_{40})^{(8,8,8,8)}(G_{43},t)}\boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{20} \quad 70$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b'_{21})^{(3)}\boxed{-(b''_{21})^{(3)}(G_{23},t)}\boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}\boxed{-(b''_{14})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)}\boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)}\boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7)}(G_{39},t)}\boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)}\boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{21} \quad 71$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)}\boxed{-(b''_{22})^{(3)}(G_{23},t)}\boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}\boxed{-(b''_{15})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)}\boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)}\boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)}\boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)}\boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{22} \quad 72$$

$\boxed{-(b''_{20})^{(3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3)}(G_{23},t)}$ *are first detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}$ *are second detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{13})^{(1,1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,)}(G,t)}$ *are third detrition coefficients for category 1,2 and 3*

$\boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)}$ *are fourth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)}$ *are fifth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)}$ *are sixth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{36})^{(7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7)}(G_{39},t)}$ $\boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \begin{bmatrix} (a'_{24})^{(4)}\boxed{+(a''_{24})^{(4)}(T_{25},t)}\boxed{+(a''_{28})^{(5,5,)}(T_{29},t)}\boxed{+(a''_{32})^{(6,6,)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)}\boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)}\boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)}\boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)}\boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{24} \quad 73$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \begin{bmatrix} (a'_{25})^{(4)}\boxed{+(a''_{25})^{(4)}(T_{25},t)}\boxed{+(a''_{29})^{(5,5,)}(T_{29},t)}\boxed{+(a''_{33})^{(6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)}\boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)}\boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)}\boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)}\boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{25} \quad 74$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[\begin{array}{ccc} (a'_{26})^{(4)} & +(a''_{26})^{(4)}(T_{25},t) & +(a''_{30})^{(5,5,)}(T_{29},t) \quad +(a''_{34})^{(6,6,)}(T_{33},t) \\ +(a''_{15})^{(1,1,1,1)}(T_{14},t) & +(a''_{18})^{(2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3)}(T_{21},t) \\ +(a''_{38})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9)}(T_{45},t) \end{array}\right] G_{26} \quad 75$$

$(a''_{24})^{(4)}(T_{25},t)$, $(a''_{25})^{(4)}(T_{25},t)$, $(a''_{26})^{(4)}(T_{25},t)$ *are first augmentation coefficients category* 1, 2 3

$+(a''_{28})^{(5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,)}(T_{29},t)$ *are second augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{32})^{(6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,)}(T_{33},t)$ *are third augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{13})^{(1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1)}(T_{14},t)$ *are fourth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{16})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2)}(T_{17},t)$ *are fifth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{20})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3)}(T_{21},t)$ *are sixth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)$ *are seventh augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)$ *are eighth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{46})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9)}(T_{45},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[\begin{array}{ccc} (b'_{24})^{(4)} & -(b''_{24})^{(4)}(G_{27},t) & -(b''_{28})^{(5,5,)}(G_{31},t) \quad -(b''_{32})^{(6,6,)}(G_{35},t) \\ -(b''_{13})^{(1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3)}(G_{23},t) \\ -(b''_{36})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9)}(G_{47},t) \end{array}\right] T_{24} \quad 76$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[\begin{array}{ccc} (b'_{25})^{(4)} & -(b''_{25})^{(4)}(G_{27},t) & -(b''_{29})^{(5,5,)}(G_{31},t) \quad -(b''_{33})^{(6,6,)}(G_{35},t) \\ -(b''_{14})^{(1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3)}(G_{23},t) \\ -(b''_{37})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9)}(G_{47},t) \end{array}\right] T_{25} \quad 77$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[\begin{array}{ccc} (b'_{26})^{(4)} & -(b''_{26})^{(4)}(G_{27},t) & -(b''_{30})^{(5,5,)}(G_{31},t) \quad -(b''_{34})^{(6,6,)}(G_{35},t) \\ -(b''_{15})^{(1,1,1,1)}(G,t) & -(b''_{18})^{(2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3)}(G_{23},t) \\ -(b''_{38})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9)}(G_{47},t) \end{array}\right] T_{26} \quad 78$$

*Where* $-(b''_{24})^{(4)}(G_{27},t)$, $-(b''_{25})^{(4)}(G_{27},t)$, $-(b''_{26})^{(4)}(G_{27},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{28})^{(5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,)}(G_{31},t)$ *are second detrition coefficients for category* 1, 2 *and* 3

$-(b''_{32})^{(6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,)}(G_{35},t)$ *are third detrition coefficients for category* 1, 2 *and* 3

$-(b''_{13})^{(1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1)}(G,t)$

*are fourth detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2,2)}(G_{19},t)}$

*are fifth detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3,3,3,3)}(G_{23},t)}$

*are sixth detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)}$

*are seventh detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)}$

*are eighth detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{46})^{(9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{45})^{(9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{44})^{(9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[\begin{array}{ccc} (a'_{28})^{(5)}\boxed{+(a''_{28})^{(5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{28} \quad (79)$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[\begin{array}{ccc} (a'_{29})^{(5)}\boxed{+(a''_{29})^{(5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{29} \quad (80)$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[\begin{array}{ccc} (a'_{30})^{(5)}\boxed{+(a''_{30})^{(5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{30} \quad (81)$$

*Where* $\boxed{+(a''_{28})^{(5)}(T_{29},t)}, \boxed{+(a''_{29})^{(5)}(T_{29},t)}, \boxed{+(a''_{30})^{(5)}(T_{29},t)}$ *are first augmentation coefficients for category* 1, 2 *and* 3

*And* $\boxed{+(a''_{24})^{(4,4,)}(T_{25},t)}, \boxed{+(a''_{25})^{(4,4,)}(T_{25},t)}, \boxed{+(a''_{26})^{(4,4,)}(T_{25},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)}, \boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)}, \boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)}$ are fourth augmentation coefficients for category 1,2, and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)}$ are fifth augmentation coefficients for category 1,2,and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)}$ are sixth augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)}$ are seventh augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)}$ are eighth augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation

coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \begin{bmatrix} (b'_{28})^{(5)}\boxed{-(b''_{28})^{(5)}(G_{31},t)} & \boxed{-(b''_{24})^{(4,4,)}(G_{27},t)} & \boxed{-(b''_{32})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{28} \quad (82)$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b'_{29})^{(5)}\boxed{-(b''_{29})^{(5)}(G_{31},t)} & \boxed{-(b''_{25})^{(4,4,)}(G_{27},t)} & \boxed{-(b''_{33})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{29} \quad (83)$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b'_{30})^{(5)}\boxed{-(b''_{30})^{(5)}(G_{31},t)} & \boxed{-(b''_{26})^{(4,4,)}(G_{27},t)} & \boxed{-(b''_{34})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{30} \quad (84)$$

*where* $\boxed{-(b''_{28})^{(5)}(G_{31},t)}$ , $\boxed{-(b''_{29})^{(5)}(G_{31},t)}$ , $\boxed{-(b''_{30})^{(5)}(G_{31},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{24})^{(4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,)}(G_{27},t)}$ *are second detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{32})^{(6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6)}(G_{35},t)}$ *are third detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1)}(G,t)}$ , $\boxed{-(b''_{15})^{(1,1,1,1,1,)}(G,t)}$ are fourth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)}$ , $\boxed{-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)}$ , $\boxed{-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)}$ , $\boxed{-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)}$ , $\boxed{-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)}$ , $\boxed{-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)}$ , $\boxed{-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \begin{bmatrix} (a'_{32})^{(6)}\boxed{+(a''_{32})^{(6)}(T_{33},t)} & \boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{32} \quad (85)$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \begin{bmatrix} (a'_{33})^{(6)}\boxed{+(a''_{33})^{(6)}(T_{33},t)} & \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{33} \quad (86)$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[\begin{array}{ccc} (a'_{34})^{(6)}\boxed{+(a''_{34})^{(6)}(T_{33},t)} & \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{34} \quad 87$$

$\boxed{+(a''_{32})^{(6)}(T_{33},t)}, \boxed{+(a''_{33})^{(6)}(T_{33},t)}, \boxed{+(a''_{34})^{(6)}(T_{33},t)}$ *are first augmentation coefficients for category* $1, 2$ *and* $3$

$\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}$ *are second augmentation coefficients for category* $1, 2$ *and* $3$

$\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}$ *are third augmentation coefficients for category* $1, 2$ *and* $3$

$\boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}$ - are fourth augmentation coefficients

$\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}$ - fifth augmentation coefficients

$\boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)}$ sixth augmentation coefficients

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)}$ seventh augmentation coefficients

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}$ Eighth augmentation coefficients

$\boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)}$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[\begin{array}{ccc} (b'_{32})^{(6)}\boxed{-(b''_{32})^{(6)}(G_{35},t)} & \boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)} \end{array}\right] T_{32} \quad 88$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[\begin{array}{ccc} (b'_{33})^{(6)}\boxed{-(b''_{33})^{(6)}(G_{35},t)} & \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)} \end{array}\right] T_{33} \quad 89$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[\begin{array}{ccc} (b'_{34})^{(6)}\boxed{-(b''_{34})^{(6)}(G_{35},t)} & \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)} \end{array}\right] T_{34} \quad 90$$

$\boxed{-(b''_{32})^{(6)}(G_{35},t)}, \boxed{-(b''_{33})^{(6)}(G_{35},t)}, \boxed{-(b''_{34})^{(6)}(G_{35},t)}$ *are first detrition coefficients for category* $1, 2$ *and* $3$

$\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}$ *are second detrition coefficients for category* $1, 2$ *and* $3$

$\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)}$ *are third detrition coefficients*

$for\ category\ 1,2\ and\ 3$

$-(b_{13}'')^{(1,1,1,1,1,1,1)}(G,t)$, $-(b_{14}'')^{(1,1,1,1,1,1,1)}(G,t)$, $-(b_{15}'')^{(1,1,1,1,1,1,1)}(G,t)$ are fourth detrition coefficients for category 1, 2, and 3

$-(b_{16}'')^{(2,2,2,2,2,2,2)}(G_{19},t)$, $-(b_{17}'')^{(2,2,2,2,2,2,2)}(G_{19},t)$, $-(b_{18}'')^{(2,2,2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1, 2, and 3

$-(b_{20}'')^{(3,3,3,3,3,3,3)}(G_{23},t)$, $-(b_{21}'')^{(3,3,3,3,3,3,3)}(G_{23},t)$, $-(b_{22}'')^{(3,3,3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1, 2, and 3

$-(b_{36}'')^{(7,7,7,7,7,7,7)}(G_{39},t)$, $-(b_{37}'')^{(7,7,7,7,7,7,7)}(G_{39},t)$, $-(b_{38}'')^{(7,7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2, and 3

$-(b_{40}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)$, $-(b_{41}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)$, $-(b_{42}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1, 2, and 3

$-(b_{46}'')^{(9,9,9,9,9,9,9)}(G_{47},t)$, $-(b_{45}'')^{(9,9,9,9,9,9,9)}(G_{47},t)$, $-(b_{44}'')^{(9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \qquad 91$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a_{36}')^{(7)} & +(a_{36}'')^{(7)}(T_{37},t) & +(a_{16}'')^{(2,2,2,2,2,2,2)}(T_{17},t) & +(a_{20}'')^{(3,3,3,3,3,3,3)}(T_{21},t) \\ & +(a_{24}'')^{(4,4,4,4,4,4,4)}(T_{25},t) & +(a_{28}'')^{(5,5,5,5,5,5,5)}(T_{29},t) & +(a_{32}'')^{(6,6,6,6,6,6,6)}(T_{33},t) \\ & +(a_{13}'')^{(1,1,1,1,1,1,1)}(T_{14},t) & +(a_{40}'')^{(8,8,8,8,8,8,8,8,)}(T_{41},t) & +(a_{44}'')^{(9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \qquad 92$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a_{37}')^{(7)} & +(a_{37}'')^{(7)}(T_{37},t) & +(a_{17}'')^{(2,2,2,2,2,2,2)}(T_{17},t) & +(a_{21}'')^{(3,3,3,3,3,3,3)}(T_{21},t) \\ & +(a_{25}'')^{(4,4,4,4,4,4,4)}(T_{25},t) & +(a_{29}'')^{(5,5,5,5,5,5,5)}(T_{29},t) & +(a_{33}'')^{(6,6,6,6,6,6,6)}(T_{33},t) \\ & +(a_{13}'')^{(1,1,1,1,1,1,1)}(T_{14},t) & +(a_{41}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t) & +(a_{45}'')^{(9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{14}$$

$$\frac{dG_{38}}{dt} \qquad 93$$

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a_{38}')^{(7)} & +(a_{38}'')^{(7)}(T_{37},t) & +(a_{18}'')^{(2,2,2,2,2,2,2)}(T_{17},t) & +(a_{22}'')^{(3,3,3,3,3,3,3)}(T_{21},t) \\ & +(a_{26}'')^{(4,4,4,4,4,4,4)}(T_{25},t) & +(a_{30}'')^{(5,5,5,5,5,5,5)}(T_{29},t) & +(a_{34}'')^{(6,6,6,6,6,6,6)}(T_{33},t) \\ & +(a_{15}'')^{(1,1,1,1,1,1,1)}(T_{14},t) & +(a_{42}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t) & +(a_{46}'')^{(9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{15}$$

Where $(a_{36}'')^{(7)}(T_{37},t)$, $(a_{37}'')^{(7)}(T_{37},t)$, $(a_{38}'')^{(7)}(T_{37},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a_{16}'')^{(2,2,2,2,2,2,2)}(T_{17},t)$, $+(a_{17}'')^{(2,2,2,2,2,2,2)}(T_{17},t)$, $+(a_{18}'')^{(2,2,2,2,2,2,2)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a_{20}'')^{(3,3,3,3,3,3,3)}(T_{21},t)$, $+(a_{21}'')^{(3,3,3,3,3,3,3)}(T_{21},t)$, $+(a_{22}'')^{(3,3,3,3,3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a_{24}'')^{(4,4,4,4,4,4,4)}(T_{25},t)$, $+(a_{25}'')^{(4,4,4,4,4,4,4)}(T_{25},t)$, $+(a_{26}'')^{(4,4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a_{28}'')^{(5,5,5,5,5,5,5)}(T_{29},t)$, $+(a_{29}'')^{(5,5,5,5,5,5,5)}(T_{29},t)$, $+(a_{30}'')^{(5,5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a_{32}'')^{(6,6,6,6,6,6,6)}(T_{33},t)$, $+(a_{33}'')^{(6,6,6,6,6,6,6)}(T_{33},t)$, $+(a_{34}'')^{(6,6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation

coefficient for category 1, 2 and 3
$+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)$ are seventh augmentation coefficient for category 1, 2 and 3
$+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)$
are eighth augmentation coefficient for 1,2,3
$+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3



$$\frac{dT_{36}}{dt} =$$

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b'_{36})^{(7)}\boxed{-(b''_{36})^{(7)}(G_{39},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{37}}{dt} =$$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b'_{37})^{(7)}\boxed{-(b''_{37})^{(7)}(G_{39},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{38}}{dt} =$$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b'_{38})^{(7)}\boxed{-(b''_{38})^{(7)}(G_{39},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $-(b''_{36})^{(7)}(G_{39},t)$, $-(b''_{37})^{(7)}(G_{39},t)$, $-(b''_{38})^{(7)}(G_{39},t)$ are first detrition coefficients for category 1, 2 and 3
$-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3
$-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3
$-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3
$-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3
$-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3
$-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)$, $-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)$
are seventh detrition coefficients for category 1, 2 and 3
$-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3



$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \begin{bmatrix} (a'_{40})^{(8)}\boxed{+(a''_{40})^{(8)}(T_{41},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \begin{bmatrix} (a'_{41})^{(8)}\boxed{+(a''_{41})^{(8)}(T_{41},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \begin{bmatrix} (a'_{42})^{(8)}\boxed{+(a''_{42})^{(8)}(T_{41},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a''_{40})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} =$$

$$(b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)}\boxed{-(b''_{40})^{(8)}(G_{43},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} =$$

$$(b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)}\boxed{-(b''_{41})^{(8)}(G_{43},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} =$$

$$(b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)}\boxed{-(b''_{42})^{(8)}(G_{43},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3



$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \begin{bmatrix} (a'_{44})^{(9)}\boxed{+(a''_{44})^{(9)}(T_{45},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \begin{bmatrix} (a'_{45})^{(9)}\boxed{+(a''_{45})^{(9)}(T_{45},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \begin{bmatrix} (a'_{46})^{(9)}\boxed{+(a''_{46})^{(9)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a''_{44})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ are Seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} -$$

$$\begin{bmatrix} (b'_{44})^{(9)}\boxed{-(b''_{44})^{(9)}(G_{47},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b'_{45})^{(9)}\boxed{-(b''_{45})^{(9)}(G_{47},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b'_{46})^{(9)}\boxed{-(b''_{46})^{(9)}(G_{47},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{44})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9)}(G_{47},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a'_i)^{(1)}, (a''_i)^{(1)}, (b_i)^{(1)}, (b'_i)^{(1)}, (b''_i)^{(1)} > 0, \quad i,j = 13,14,15$$ *97*

The functions $(a''_i)^{(1)}, (b''_i)^{(1)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$(a_i'')^{(1)}(T_{14},t) \leq (p_i)^{(1)} \leq (\hat{A}_{13})^{(1)}$

$(b_i'')^{(1)}(G,t) \leq (r_i)^{(1)} \leq (b_i')^{(1)} \leq (\hat{B}_{13})^{(1)}$

$lim_{T_2\to\infty}(a_i'')^{(1)}(T_{14},t) = (p_i)^{(1)}$ 98

$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = (r_i)^{(1)}$

**Definition of** $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$ :

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$|(a_i'')^{(1)}(T_{14}',t) - (a_i'')^{(1)}(T_{14},t)| \leq (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$

$|(b_i'')^{(1)}(G',t) - (b_i'')^{(1)}(G,t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}',t)$ and $(a_i'')^{(1)}(T_{14},t)$ . $(T_{14}',t)$ and $(T_{14},t)$ are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14},t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$ : 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}$ , $\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$

**Definition of** $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$ : 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$

$\frac{1}{(\hat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$

Where we suppose

$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i,j = 16,17,18$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17},t) \le (p_i)^{(2)} \le \left(\hat{A}_{16}\right)^{(2)} \quad 102$$

$$(b_i'')^{(2)}(G_{19},t) \le (r_i)^{(2)} \le (b_i')^{(2)} \le (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2\to\infty}(a_i'')^{(2)}(T_{17},t) = (p_i)^{(2)} \quad 104$$

$$\lim_{G\to\infty}(b_i'')^{(2)}\left((G_{19}),t\right) = (r_i)^{(2)} \quad 105$$

**Definition of** $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$ : 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}',t) - (a_i'')^{(2)}(T_{17},t)| \le (\hat{k}_{16})^{(2)}|T_{17} - T_{17}'|e^{-(\hat{M}_{16})^{(2)}t} \quad 107$$

$$|(b_i'')^{(2)}((G_{19})',t) - (b_i'')^{(2)}\left((G_{19}),t\right)| < (\hat{k}_{16})^{(2)}||(G_{19}) - (G_{19})'||e^{-(\hat{M}_{16})^{(2)}t} \quad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}',t)$ and$(a_i'')^{(2)}(T_{17},t)$ . $(T_{17}',t)$ and $(T_{17},t)$ are points belonging to the interval $\left[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}\right]$ . It is to be noted that $(a_i'')^{(2)}(T_{17},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17},t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$ :

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants 109

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$$

**Definition of** $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$ :

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1 \quad 110$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(b_i)^{(2)} + (b_i')^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1 \quad 111$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i,j = 20,21,22 \quad 112$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(3)}$, $(\mathrm{r_i})^{(3)}$:

$$(a_i'')^{(3)}(T_{21},t) \le (p_i)^{(3)} \le (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23},t) \le (r_i)^{(3)} \le (b_i')^{(3)} \le (\hat{B}_{20})^{(3)}$$

$$lim_{T_2\to\infty}(a_i'')^{(3)}(T_{21},t) = (p_i)^{(3)} \qquad 113$$

$$\lim_{G\to\infty}(b_i'')^{(3)}(G_{23},t) = (r_i)^{(3)}$$

**Definition of** $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$ :

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: 114

$$|(a_i'')^{(3)}(T_{21}',t) - (a_i'')^{(3)}(T_{21},t)| \le (\hat{k}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\hat{M}_{20})^{(3)}t}$$

$$|(b_i'')^{(3)}(G_{23}',t) - (b_i'')^{(3)}(G_{23},t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}'||e^{-(\hat{M}_{20})^{(3)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}',t)$ and$(a_i'')^{(3)}(T_{21},t)$ . $(T_{21}',t)$ And $(T_{21},t)$ are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$ : 115

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$ and the constants $(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}, i = 20,21,22,$ satisfy the inequalities 116

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(a_i)^{(3)} + (a_i')^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(b_i)^{(3)} + (b_i')^{(3)} + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i,j = 24,25,26 \qquad 117$$

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(4)}$, $(r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25},t) \le (p_i)^{(4)} \le (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}\big((G_{27}),t\big) \le (r_i)^{(4)} \le (b_i')^{(4)} \le (\hat{B}_{24})^{(4)}$$

$$lim_{T_2\to\infty}(a_i'')^{(4)}(T_{25},t) = (p_i)^{(4)} \quad (118)$$

$$\lim_{G\to\infty}(b_i'')^{(4)}\big((G_{27}),t\big) = (r_i)^{(4)}$$

**Definition of** $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$ :

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24,25,26}$

They satisfy Lipschitz condition: (119)

$$|(a_i'')^{(4)}(T_{25}',t) - (a_i'')^{(4)}(T_{25},t)| \le (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})',t) - (b_i'')^{(4)}\big((G_{27}),t\big)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}',t)$ and$(a_i'')^{(4)}(T_{25},t)$ . $(T_{25}',t)$ and $(T_{25},t)$ are points belonging to the interval $\big[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}\big]$. It is to be noted that $(a_i'')^{(4)}(T_{25},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25},t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$ : (120)

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$, are positive constants

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}, \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$$

**Definition of** $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$ : (121)

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and\ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}, i = 24,25,26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(a_i)^{(4)} + (a_i')^{(4)} + (\hat{A}_{24})^{(4)} + (\hat{P}_{24})^{(4)}(\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(b_i)^{(4)} + (b_i')^{(4)} + (\hat{B}_{24})^{(4)} + (\hat{Q}_{24})^{(4)}(\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i,j = 28,29,30 \quad (122)$$

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(5)}$, $(r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29},t) \le (p_i)^{(5)} \le (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}\big((G_{31}),t\big) \le (r_i)^{(5)} \le (b_i')^{(5)} \le (\hat{B}_{28})^{(5)}$$

$$lim_{T_2\to\infty}(a_i'')^{(5)}(T_{29},t) = (p_i)^{(5)} \qquad 123$$

$$\lim_{G\to\infty}(b_i'')^{(5)}(G_{31},t) = (r_i)^{(5)}$$

**Definition of** $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$ :

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}',t) - (a_i'')^{(5)}(T_{29},t)| \le (\hat{k}_{28})^{(5)}|T_{29} - T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})',t) - (b_i'')^{(5)}\big((G_{31}),t\big)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}',t)$ and$(a_i'')^{(5)}(T_{29},t)$ . $(T_{29}',t)$ and $(T_{29},t)$ are points belonging to the interval $\big[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\big]$. It is to be noted that $(a_i'')^{(5)}(T_{29},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29},t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$ : 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}, \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

**Definition of** $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$ : 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and\ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(a_i)^{(5)} + (a_i')^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)}(\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(b_i)^{(5)} + (b_i')^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)}(\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i,j = 32,33,34 \qquad 127$$

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(6)}$, $(r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33},t) \le (p_i)^{(6)} \le (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}),t) \le (r_i)^{(6)} \le (b_i')^{(6)} \le (\hat{B}_{32})^{(6)}$$

$$lim_{T_2\to\infty}(a_i'')^{(6)}(T_{33},t) = (p_i)^{(6)} \quad (128)$$

$$\lim_{G\to\infty}(b_i'')^{(6)}\left((G_{35}),t\right) = (r_i)^{(6)}$$

**Definition of** $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$ :

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}',t) - (a_i'')^{(6)}(T_{33},t)| \le (\hat{k}_{32})^{(6)}|T_{33} - T_{33}'|e^{-(\hat{M}_{32})^{(6)}t}$$

$$|(b_i'')^{(6)}((G_{35})',t) - (b_i'')^{(6)}\left((G_{35}),t\right)| < (\hat{k}_{32})^{(6)}||(G_{35}) - (G_{35})'||e^{-(\hat{M}_{32})^{(6)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}',t)$ and$(a_i'')^{(6)}(T_{33},t)$ . $(T_{33}',t)$ and $(T_{33},t)$ are points belonging to the interval $\left[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}\right]$. It is to be noted that $(a_i'')^{(6)}(T_{33},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33},t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$ : (129)

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$, are positive constants

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$$

**Definition of** $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$ : (130)

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and\ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32,33,34,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(a_i)^{(6)} + (a_i')^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(b_i)^{(6)} + (b_i')^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

(EE) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i,j = 36,37,38$ (131)

(FF) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(7)}$, $(r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37},t) \le (p_i)^{(7)} \le (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39},t) \le (r_i)^{(7)} \le (b_i')^{(7)} \le (\hat{B}_{36})^{(7)}$$

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*(GG)* $\lim_{T_2\to\infty} (a_i'')^{(7)}(T_{37},t) = (p_i)^{(7)}$

*(HH)*

$\lim_{G\to\infty}(b_i'')^{(7)}\big((G_{39}),t\big) = (r_i)^{(7)}$

**Definition of** $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$ :

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$$|(a_i'')^{(7)}(T_{37}',t) - (a_i'')^{(7)}(T_{37},t)| \le (\hat{k}_{36})^{(7)}|T_{37} - T_{37}'|e^{-(\hat{M}_{36})^{(7)}t}$$

$$|(b_i'')^{(7)}((G_{39})',t) - (b_i'')^{(7)}\big((G_{39}),t\big)| < (\hat{k}_{36})^{(7)}||(G_{39}) - (G_{39})'||e^{-(\hat{M}_{36})^{(7)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}',t)$ and$(a_i'')^{(7)}(T_{37},t)$ . $(T_{37}',t)$ and $(T_{37},t)$ are points belonging to the interval $\big[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}\big]$. It is to be noted that $(a_i'')^{(7)}(T_{37},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37},t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$ : 134

(II) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$, are positive constants

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$$

**Definition of** $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$ : 135

(JJ) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)}(\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(b_i)^{(7)} + (b_i')^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)}(\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40,41,42 \tag{136}$$

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

**Definition of** $(p_i)^{(8)}$, $(r_i)^{(8)}$: (137)

$$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)} \tag{138}$$

$$(b_i'')^{(8)}((G_{43}), t) \leq (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)} \tag{139}$$

$$\lim_{T_2 \to \infty} (a_i'')^{(8)}(T_{41}, t) = (p_i)^{(8)} \tag{140}$$

$$\lim_{G \to \infty} (b_i'')^{(8)}\left((G_{43}), t\right) = (r_i)^{(8)} \tag{141}$$

**Definition of** $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$ :

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)} |T_{41} - T_{41}'| e^{-(\hat{M}_{40})^{(8)} t} \tag{142}$$

$$|(b_i'')^{(8)}((G_{43})', t) - (b_i'')^{(8)}\left((G_{43}), t\right)| < (\hat{k}_{40})^{(8)} ||(G_{43}) - (G_{43})'|| e^{-(\hat{M}_{40})^{(8)} t} \tag{143}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}', t)$ and $(a_i'')^{(8)}(T_{41}, t)$ . $(T_{41}', t)$ and $(T_{41}, t)$ are points belonging to the interval $\left[(\hat{k}_{40})^{(8)}, (\hat{M}_{40})^{(8)}\right]$. It is to be noted that $(a_i'')^{(8)}(T_{41}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{40})^{(8)} = 1$ then the function $(a_i'')^{(8)}(T_{41}, t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$ :

$(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$, are positive constants

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}, \frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1 \tag{144}$$

**Definition of** $(\hat{P}_{40})^{(8)}, (\hat{Q}_{40})^{(8)}$ :

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}, (\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)}, (a_i')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (p_i)^{(8)}$, $(r_i)^{(8)}, i = 40,41,42,$
Satisfy the inequalities

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(a_i)^{(8)} + (a_i')^{(8)} + (\hat{A}_{40})^{(8)} + (\hat{P}_{40})^{(8)} (\hat{k}_{40})^{(8)}] < 1 \tag{145}$$

$$\frac{1}{(\hat{M}_{40})^{(8)}}[\ (b_i)^{(8)} + (b_i')^{(8)} + \ (\hat{B}_{40})^{(8)} + \ (\hat{Q}_{40})^{(8)}\ (\hat{k}_{40})^{(8)}] < 1 \qquad 146$$

Where we suppose

$$(a_i)^{(9)}, (a_i')^{(9)}, (a_i'')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (b_i'')^{(9)} > 0, \quad i,j = 44,45,46 \qquad 146\,A$$

The functions $(a_i'')^{(9)}, (b_i'')^{(9)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(9)}$, $(r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45}, t) \leq (p_i)^{(9)} \leq (\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47}, t) \leq \ (r_i)^{(9)} \leq (b_i')^{(9)} \leq (\hat{B}_{44})^{(9)}$$

$$\lim_{T_2 \to \infty} (a_i'')^{(9)}(T_{45}, t) = (p_i)^{(9)}$$

$$\lim_{G \to \infty} (b_i'')^{(9)}(G_{47}, t) = \ (r_i)^{(9)}$$

**Definition of** $(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}$ :

Where $\boxed{(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}, (p_i)^{(9)}, \ (r_i)^{(9)}}$ are positive constants and $\boxed{i = 44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}', t) - (a_i'')^{(9)}(T_{45}, t)| \leq (\hat{k}_{44})^{(9)} |T_{45} - T_{45}'| e^{-(\hat{M}_{44})^{(9)} t}$$

$$|(b_i'')^{(9)}((G_{47})', t) - (b_i'')^{(9)}\big((G_{47}), t\big)| < (\hat{k}_{44})^{(9)} ||(G_{47}) - (G_{47})'|| e^{-(\hat{M}_{44})^{(9)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}', t)$ and$(a_i'')^{(9)}(T_{45}, t)$ . $(T_{45}', t)$ and $(T_{45}, t)$ are points belonging to the interval $\left[(\hat{k}_{44})^{(9)}, (\hat{M}_{44})^{(9)}\right]$. It is to be noted that $(a_i'')^{(9)}(T_{45}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{44})^{(9)} = 1$ then the function $(a_i'')^{(9)}(T_{45}, t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$ :

$(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$, are positive constants

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}} \ , \frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$$

**Definition of** $(\hat{P}_{44})^{(9)}, (\hat{Q}_{44})^{(9)}$ :

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}, (\hat{A}_{44})^{(9)} and\ (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)}, (a_i')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (p_i)^{(9)}, \ (r_i)^{(9)}, i = 44,45,46,$
satisfy the inequalities

$$\frac{1}{(\widehat{M}_{44})^{(9)}}[(a_i)^{(9)} + (a_i')^{(9)} + (\hat{A}_{44})^{(9)} + (\hat{P}_{44})^{(9)}(\hat{k}_{44})^{(9)}] < 1$$

$$\frac{1}{(\widehat{M}_{44})^{(9)}}[(b_i)^{(9)} + (b_i')^{(9)} + (\hat{B}_{44})^{(9)} + (\hat{Q}_{44})^{(9)}(\hat{k}_{44})^{(9)}] < 1$$

**Theorem 1:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0), T_i(0)$ :

$$G_i(t) \le (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 2 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

**Definition of** $G_i(0), T_i(0)$

$$G_i(t) \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} \quad , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} \quad , \quad T_i(0) = T_i^0 > 0$$

**Theorem 3 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t) \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t} \quad , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t} \quad , \quad T_i(0) = T_i^0 > 0$$

**Theorem 4 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

**Definition of** $G_i(0), T_i(0)$ :

$$G_i(t) \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 5 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

**Definition of** $G_i(0), T_i(0)$ :

$$G_i(t) \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 6 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

**Definition of** $G_i(0), T_i(0)$ :

$$G_i(t) \le (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$T_i(t) \leq (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Theorem 7:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

**Definition of** $G_i(0), T_i(0)$ :

$G_i(t) \leq (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Theorem 8:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

**Definition of** $G_i(0), T_i(0)$ :

$G_i(t) \leq (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Theorem 9:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

**Definition of** $G_i(0), T_i(0)$ :

$G_i(t) \leq (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Proof:** Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$G_i(0) = G_i^0$ , $T_i(0) = T_i^0$ , $G_i^0 \leq (\hat{P}_{13})^{(1)}$ , $T_i^0 \leq (\hat{Q}_{13})^{(1)}$, 155

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t}$ 156

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t}$ 157

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[ (a_{13})^{(1)} G_{14}(s_{(13)}) - \left( (a'_{13})^{(1)} + a''_{13})^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \right) G_{13}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[ (a_{14})^{(1)} G_{13}(s_{(13)}) - \left( (a'_{14})^{(1)} + (a''_{14})^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \right) G_{14}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[ (a_{15})^{(1)} G_{14}(s_{(13)}) - \left( (a'_{15})^{(1)} + (a''_{15})^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \right) G_{15}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[ (b_{13})^{(1)} T_{14}(s_{(13)}) - \left( (b'_{13})^{(1)} - (b''_{13})^{(1)} (G(s_{(13)}), s_{(13)}) \right) T_{13}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^{0} + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)}\left(G(s_{(13)}), s_{(13)}\right)\right) T_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{\mathrm{T}}_{15}(\mathrm{t}) = \mathrm{T}_{15}^{0} + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)}\left(G(s_{(13)}), s_{(13)}\right)\right) T_{15}(s_{(13)})\right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{16})^{(2)}, T_i^0 \le (\hat{Q}_{16})^{(2)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$$

By 160

$$\bar{G}_{16}(t) = G_{16}^{0} + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''_{16})^{(2)}\left(T_{17}(s_{(16)}), s_{(16)}\right)\right) G_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^{0} + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)}\left(T_{17}(s_{(16)}), s_{(17)}\right)\right) G_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^{0} + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)}\left(T_{17}(s_{(16)}), s_{(16)}\right)\right) G_{18}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^{0} + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)}\left(G_{19}(s_{(16)}), s_{(16)}\right)\right) T_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^{0} + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)}\left(G_{19}(s_{(16)}), s_{(16)}\right)\right) T_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^{0} + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)}\left(G_{19}(s_{(16)}), s_{(16)}\right)\right) T_{18}(s_{(16)})\right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{20})^{(3)}, T_i^0 \le (\hat{Q}_{20})^{(3)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$$

By 161

$$\bar{G}_{20}(t) = G_{20}^{0} + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''_{20})^{(3)}\left(T_{21}(s_{(20)}), s_{(20)}\right)\right) G_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{22}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^0 + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^0 + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{\mathrm{T}}_{22}(\mathrm{t}) = \mathrm{T}_{22}^0 + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{22}(s_{(20)})\right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{24})^{(4)}, T_i^0 \le (\hat{Q}_{24})^{(4)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

By



$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{26}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{\mathrm{T}}_{26}(\mathrm{t}) = \mathrm{T}_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{26}(s_{(24)})\right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{28})^{(5)}, T_i^0 \le (\hat{Q}_{28})^{(5)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

By 163

$$\bar{G}_{28}(t) = G_{28}^{0} + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^{0} + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^{0} + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{30}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^{0} + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^{0} + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{\mathrm{T}}_{30}(\mathrm{t}) = \mathrm{T}_{30}^{0} + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{30}(s_{(28)})\right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{32})^{(6)}, T_i^0 \le (\hat{Q}_{32})^{(6)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$$

By 164

$$\bar{G}_{32}(t) = G_{32}^{0} + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a'_{32})^{(6)} + a''_{32})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^{0} + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^{0} + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{34}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^{0} + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^{0} + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{\mathrm{T}}_{34}(\mathrm{t}) = \mathrm{T}_{34}^{0} + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{34}(s_{(32)})\right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$ , $T_i(0) = T_i^0$ , $G_i^0 \leq (\hat{P}_{36})^{(7)}$ , $T_i^0 \leq (\hat{Q}_{36})^{(7)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[ (a_{36})^{(7)} G_{37}(s_{(36)}) - \left( (a'_{36})^{(7)} + a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[ (a_{37})^{(7)} G_{36}(s_{(36)}) - \left( (a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[ (a_{38})^{(7)} G_{37}(s_{(36)}) - \left( (a'_{38})^{(7)} + (a''_{38})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{38}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[ (b_{36})^{(7)} T_{37}(s_{(36)}) - \left( (b'_{36})^{(7)} - (b''_{36})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[ (b_{37})^{(7)} T_{36}(s_{(36)}) - \left( (b'_{37})^{(7)} - (b''_{37})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{38}(t) = T_{38}^0 + \int_0^t \left[ (b_{38})^{(7)} T_{37}(s_{(36)}) - \left( (b'_{38})^{(7)} - (b''_{38})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{38}(s_{(36)}) \right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i$: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$ , $T_i(0) = T_i^0$ , $G_i^0 \leq (\hat{P}_{40})^{(8)}$ , $T_i^0 \leq (\hat{Q}_{40})^{(8)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{40})^{(8)} e^{(\bar{M}_{40})^{(8)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{40})^{(8)} e^{(\bar{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[ (a_{40})^{(8)} G_{41}(s_{(40)}) - \left( (a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[ (a_{41})^{(8)} G_{40}(s_{(40)}) - \left( (a'_{41})^{(8)} + (a''_{41})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)}\left(T_{41}(s_{(40)}), s_{(40)}\right)\right) G_{42}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^0 + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)}\left(G_{43}(s_{(40)}), s_{(40)}\right)\right) T_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^0 + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)}\left(G_{43}(s_{(40)}), s_{(40)}\right)\right) T_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{\mathrm{T}}_{42}(\mathrm{t}) = \mathrm{T}_{42}^0 + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)}\left(G_{43}(s_{(40)}), s_{(40)}\right)\right) T_{42}(s_{(40)})\right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** 166

Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy A

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{44})^{(9)},\ T_i^0 \le (\hat{Q}_{44})^{(9)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''_{44})^{(9)}\left(T_{45}(s_{(44)}), s_{(44)}\right)\right) G_{44}(s_{(44)})\right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)}\left(T_{45}(s_{(44)}), s_{(44)}\right)\right) G_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)}\left(T_{45}(s_{(44)}), s_{(44)}\right)\right) G_{46}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)}\left(G_{47}(s_{(44)}), s_{(44)}\right)\right) T_{44}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)}\left(G_{47}(s_{(44)}), s_{(44)}\right)\right) T_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{\mathrm{T}}_{46}(\mathrm{t}) = \mathrm{T}_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)}\left(G_{47}(s_{(44)}), s_{(44)}\right)\right) T_{46}(s_{(44)})\right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}}\right)\right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left(e^{(\hat{M}_{13})^{(1)} t} - 1\right)$$

From which it follows that 168

$$(G_{13}(t)-G_{13}^{0})e^{-(\hat{M}_{13})^{(1)}t}\leq\frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}}\left[((\hat{P}_{13})^{(1)}+G_{14}^{0})e^{\left(-\frac{(\hat{P}_{13})^{(1)}+G_{14}^{0}}{G_{14}^{0}}\right)}+(\hat{P}_{13})^{(1)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t)\leq G_{16}^{0}+\int_{0}^{t}\left[(a_{16})^{(2)}\left(G_{17}^{0}+(\hat{P}_{16})^{(6)}e^{(\hat{M}_{16})^{(2)}s_{(16)}}\right)\right]ds_{(16)}= \quad (169)$$

$$\left(1+(a_{16})^{(2)}t\right)G_{17}^{0}+\frac{(a_{16})^{(2)}(\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}}\left(e^{(\hat{M}_{16})^{(2)}t}-1\right)$$

From which it follows that (170)

$$(G_{16}(t)-G_{16}^{0})e^{-(\hat{M}_{16})^{(2)}t}\leq\frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}}\left[((\hat{P}_{16})^{(2)}+G_{17}^{0})e^{\left(-\frac{(\hat{P}_{16})^{(2)}+G_{17}^{0}}{G_{17}^{0}}\right)}+(\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that (171)

$$G_{20}(t)\leq G_{20}^{0}+\int_{0}^{t}\left[(a_{20})^{(3)}\left(G_{21}^{0}+(\hat{P}_{20})^{(3)}e^{(\hat{M}_{20})^{(3)}s_{(20)}}\right)\right]ds_{(20)}=$$

$$\left(1+(a_{20})^{(3)}t\right)G_{21}^{0}+\frac{(a_{20})^{(3)}(\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left(e^{(\hat{M}_{20})^{(3)}t}-1\right)$$

From which it follows that (172)

$$(G_{20}(t)-G_{20}^{0})e^{-(\hat{M}_{20})^{(3)}t}\leq\frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left[((\hat{P}_{20})^{(3)}+G_{21}^{0})e^{\left(-\frac{(\hat{P}_{20})^{(3)}+G_{21}^{0}}{G_{21}^{0}}\right)}+(\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that (173)

$$G_{24}(t)\leq G_{24}^{0}+\int_{0}^{t}\left[(a_{24})^{(4)}\left(G_{25}^{0}+(\hat{P}_{24})^{(4)}e^{(\hat{M}_{24})^{(4)}s_{(24)}}\right)\right]ds_{(24)}=$$

$$\left(1+(a_{24})^{(4)}t\right)G_{25}^{0}+\frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left(e^{(\hat{M}_{24})^{(4)}t}-1\right)$$

From which it follows that (174)

$$(G_{24}(t)-G_{24}^{0})e^{-(\hat{M}_{24})^{(4)}t}\leq\frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left[((\hat{P}_{24})^{(4)}+G_{25}^{0})e^{\left(-\frac{(\hat{P}_{24})^{(4)}+G_{25}^{0}}{G_{25}^{0}}\right)}+(\hat{P}_{24})^{(4)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t) \le G_{28}^0 + \int_0^t \left[ (a_{28})^{(5)} \left( G_{29}^0 + (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} s_{(28)}} \right) \right] ds_{(28)} =$$

$$\left(1 + (a_{28})^{(5)} t\right) G_{29}^0 + \frac{(a_{28})^{(5)} (\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}} \left( e^{(\hat{M}_{28})^{(5)} t} - 1 \right)$$

From which it follows that 175

$$(G_{28}(t) - G_{28}^0) e^{-(\hat{M}_{28})^{(5)} t} \le \frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}} \left[ \left( (\hat{P}_{28})^{(5)} + G_{29}^0 \right) e^{\left( - \frac{(\hat{P}_{28})^{(5)} + G_{29}^0}{G_{29}^0} \right)} + (\hat{P}_{28})^{(5)} \right]$$

$(G_i^0)$ is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t) \le G_{32}^0 + \int_0^t \left[ (a_{32})^{(6)} \left( G_{33}^0 + (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} s_{(32)}} \right) \right] ds_{(32)} =$$

$$\left(1 + (a_{32})^{(6)} t\right) G_{33}^0 + \frac{(a_{32})^{(6)} (\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}} \left( e^{(\hat{M}_{32})^{(6)} t} - 1 \right)$$

From which it follows that 177

$$(G_{32}(t) - G_{32}^0) e^{-(\hat{M}_{32})^{(6)} t} \le \frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}} \left[ \left( (\hat{P}_{32})^{(6)} + G_{33}^0 \right) e^{\left( - \frac{(\hat{P}_{32})^{(6)} + G_{33}^0}{G_{33}^0} \right)} + (\hat{P}_{32})^{(6)} \right]$$

$(G_i^0)$ is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

(f) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t) \le G_{36}^0 + \int_0^t \left[ (a_{36})^{(7)} \left( G_{37}^0 + (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} s_{(36)}} \right) \right] ds_{(36)} =$$
$$\left(1 + (a_{36})^{(7)} t\right) G_{37}^0 + \frac{(a_{36})^{(7)} (\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left( e^{(\hat{M}_{36})^{(7)} t} - 1 \right)$$

From which it follows that

$$(G_{36}(t) - G_{36}^0) e^{-(\hat{M}_{36})^{(7)} t} \le \frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left[ \left( (\hat{P}_{36})^{(7)} + G_{37}^0 \right) e^{\left( - \frac{(\hat{P}_{36})^{(7)} + G_{37}^0}{G_{37}^0} \right)} + (\hat{P}_{36})^{(7)} \right]$$

$(G_i^0)$ is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t) \le G_{40}^0 + \int_0^t \left[ (a_{40})^{(8)} \left( G_{41}^0 + (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} s_{(40)}} \right) \right] ds_{(40)} = \quad 180$$

$$\left(1 + (a_{40})^{(8)} t\right) G_{41}^0 + \frac{(a_{40})^{(8)} (\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}} \left( e^{(\hat{M}_{40})^{(8)} t} - 1 \right)$$

From which it follows that 181

$$(G_{40}(t)-G_{40}^{0})e^{-(\hat{M}_{40})^{(8)}t}\leq\frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left[((\hat{P}_{40})^{(8)}+G_{41}^{0})e^{\left(-\frac{(\hat{P}_{40})^{(8)}+G_{41}^{0}}{G_{41}^{0}}\right)}+(\hat{P}_{40})^{(8)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 8

Analogous inequalities hold also for $G_{41}, G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t)\leq G_{44}^{0}+\int_{0}^{t}\left[(a_{44})^{(9)}\left(G_{45}^{0}+(\hat{P}_{44})^{(9)}e^{(\hat{M}_{44})^{(9)}s_{(44)}}\right)\right]ds_{(44)}=$$

$$\left(1+(a_{44})^{(9)}t\right)G_{45}^{0}+\frac{(a_{44})^{(9)}(\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}}\left(e^{(\hat{M}_{44})^{(9)}t}-1\right)$$

From which it follows that

$$(G_{44}(t)-G_{44}^{0})e^{-(\hat{M}_{44})^{(9)}t}\leq\frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}}\left[((\hat{P}_{44})^{(9)}+G_{45}^{0})e^{\left(-\frac{(\hat{P}_{44})^{(9)}+G_{45}^{0}}{G_{45}^{0}}\right)}+(\hat{P}_{44})^{(9)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 9

Analogous inequalities hold also for $G_{45}, G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}<1$ and to choose 182

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[(\hat{P}_{13})^{(1)}+((\hat{P}_{13})^{(1)}+G_j^0)e^{-\left(\frac{(\hat{P}_{13})^{(1)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{13})^{(1)} \quad 183$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[((\hat{Q}_{13})^{(1)}+T_j^0)e^{-\left(\frac{(\hat{Q}_{13})^{(1)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{13})^{(1)}\right]\leq(\hat{Q}_{13})^{(1)} \quad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left((G^{(1)},T^{(1)}),(G^{(2)},T^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\hat{M}_{13})^{(1)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\hat{M}_{13})^{(1)}t}\}$$

Indeed if we denote

**Definition of** $\tilde{G},\tilde{T}:(\tilde{G},\tilde{T})=\mathcal{A}^{(1)}(G,T)$

It results

$$\left|\tilde{G}_{13}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{13})^{(1)} \left|G_{14}^{(1)}-G_{14}^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)} s_{(13)}} e^{(\widehat{M}_{13})^{(1)} s_{(13)}}\, ds_{(13)} +$$

$$\int_0^t \{(a'_{13})^{(1)} \left|G_{13}^{(1)}-G_{13}^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)} s_{(13)}} e^{-(\widehat{M}_{13})^{(1)} s_{(13)}} +$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)}, s_{(13)}\right)\left|G_{13}^{(1)}-G_{13}^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)} s_{(13)}} e^{(\widehat{M}_{13})^{(1)} s_{(13)}} +$$

$$G_{13}^{(2)} \left|(a''_{13})^{(1)}\left(T_{14}^{(1)}, s_{(13)}\right)-(a''_{13})^{(1)}\left(T_{14}^{(2)}, s_{(13)}\right)\right|\ e^{-(\widehat{M}_{13})^{(1)} s_{(13)}} e^{(\widehat{M}_{13})^{(1)} s_{(13)}}\} ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\left|G^{(1)}-G^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)} t} \leq \qquad 186$$
$$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)} + (a'_{13})^{(1)} + (\widehat{A}_{13})^{(1)} + (\widehat{P}_{13})^{(1)} (\widehat{k}_{13})^{(1)}\right) d\left(\left(G^{(1)}, T^{(1)};\ G^{(2)}, T^{(2)}\right)\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 1:** The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)} t}$ *and* $(\widehat{Q}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)} t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i = 13{,}14{,}15$ depend only on $T_{14}$ and respectively on $G(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any $t$ where $G_i(t) = 0$ *and* $T_i(t) = 0$

From 19 to 24 it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(1)} - (a_i'')^{(1)}(T_{14}(s_{(13)}), s_{(13)})\} ds_{(13)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(1)} t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2$ *and* $\left((\widehat{M}_{13})^{(1)}\right)_3$ : 187

**Remark 3**: if $G_{13}$ is bounded, the same property have also $G_{14}$ *and* $G_{15}$ . indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a'_{14})^{(1)} G_{14}$ and by integrating

$$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1 / (a'_{14})^{(1)}$$

In the same way , one can obtain

$$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2 / (a'_{15})^{(1)}$$

If $G_{14}$ *or* $G_{15}$ is bounded, the same property follows for $G_{13}$ , $G_{15}$ and $G_{13}$ , $G_{14}$ respectively.

**Remark 4:** If $G_{13}$ *is* bounded, from below, the same property holds for $G_{14}$ *and* $G_{15}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{14}$ is bounded from below. 188

**Remark 5:** If $T_{13}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}(G(t),t)) = (b_{14}')^{(1)}$ then $T_{14} \to \infty$. 189

**Definition of** $(m)^{(1)}$ and $\varepsilon_1$ :

Indeed let $t_1$ be so that for $t > t_1$

$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t),t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right)$, $t = log\frac{2}{\varepsilon_1}$ By taking now $\varepsilon_1$ sufficiently small one sees that $T_{14}$ is unbounded.
The same property holds for $T_{15}$ if $\lim_{t\to\infty}(b_{15}'')^{(1)}(G(t),t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}$ , $\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}$ *and* $(\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[(\hat{P}_{16})^{(2)} + \left((\hat{P}_{16})^{(2)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{16})^{(2)}\right] \leq (\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

**Definition of** $\widetilde{G_{19}}, \widetilde{T_{19}}$ : $\left(\widetilde{G_{19}}, \widetilde{T_{19}}\right) = \mathcal{A}^{(2)}(G_{19}, T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{16})^{(2)} \left|G_{17}^{(1)}-G_{17}^{(2)}\right| e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\, ds_{(16)} +$$

$$\int_0^t \{(a'_{16})^{(2)} \left|G_{16}^{(1)}-G_{16}^{(2)}\right| e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} +$$

$$(a''_{16})^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right)\left|G_{16}^{(1)}-G_{16}^{(2)}\right| e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} e^{(\widehat{M}_{16})^{(2)}s_{(16)}} +$$

$$G_{16}^{(2)} \left|(a''_{16})^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right) - (a''_{16})^{(2)}\left(T_{17}^{(2)}, s_{(16)}\right)\right|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\} ds_{(16)}$$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0, t]$ 197

From the hypotheses it follows

$$\left|(G_{19})^{(1)}-(G_{19})^{(2)}\right| e^{-(\widehat{M}_{16})^{(2)}t} \leq$$
$$\frac{1}{(\widehat{M}_{16})^{(2)}}\left((a_{16})^{(2)} + (a'_{16})^{(2)} + (\widehat{A}_{16})^{(2)} + (\widehat{P}_{16})^{(2)}(\widehat{k}_{16})^{(2)}\right) d\left(((G_{19})^{(1)}, (T_{19})^{(1)};\ (G_{19})^{(2)}, (T_{19})^{(2)})\right)$$

And analogous inequalities for $G_i$ and $T_i$. Taking into account the hypothesis the result follows 198

**Remark 6:** The fact that we supposed $(a''_{16})^{(2)}$ and $(b''_{16})^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{16})^{(2)} e^{(\widehat{M}_{16})^{(2)}t}$ and $(\widehat{Q}_{16})^{(2)} e^{(\widehat{M}_{16})^{(2)}t}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i = 16,17,18$ depend only on $T_{17}$ and respectively on $(G_{19})$(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 7:** There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$ 200

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(2)} - (a_i'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})\} ds_{(16)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(2)}t)} > 0 \quad \text{for } t > 0$$

**Definition of** $\left((\widehat{M}_{16})^{(2)}\right)_1$, $\left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$: 201

**Remark 8:** if $G_{16}$ is bounded, the same property have also $G_{17}$ and $G_{18}$ . indeed if

$G_{16} < (\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt} \leq \left((\widehat{M}_{16})^{(2)}\right)_1 - (a'_{17})^{(2)} G_{17}$ and by integrating

$$G_{17} \leq \left((\widehat{M}_{16})^{(2)}\right)_2 = G_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1 / (a'_{17})^{(2)}$$

In the same way , one can obtain

$$G_{18} \leq \left((\widehat{M}_{16})^{(2)}\right)_3 = G_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2 / (a'_{18})^{(2)}$$

If $G_{17}$ or $G_{18}$ is bounded, the same property follows for $G_{16}$, $G_{18}$ and $G_{16}$, $G_{17}$ respectively.

**Remark 9**: If $G_{16}$ is bounded, from below, the same property holds for $G_{17}$ and $G_{18}$. The proof is analogous with the preceding one. An analogous property is true if $G_{17}$ is bounded from below. 202

**Remark 10:** If $T_{16}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t),t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$. 203

**Definition of** $(m)^{(2)}$ and $\varepsilon_2$ :

Indeed let $t_2$ be so that for $t > t_2$

$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t),t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right), \quad t = \log\frac{2}{\varepsilon_2}$ By taking now $\varepsilon_2$ sufficiently small one sees that $T_{17}$ is unbounded. 205
The same property holds for $T_{18}$ if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t),t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\bar{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\bar{M}_{20})^{(3)}} < 1$ and to choose 207

$(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\bar{M}_{20})^{(3)}}\left[(\hat{P}_{20})^{(3)} + \left((\hat{P}_{20})^{(3)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\bar{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{20})^{(3)}\right] \leq (\hat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself 210

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)},(T_{23})^{(1)}),((G_{23})^{(2)},(T_{23})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\bar{M}_{20})^{(3)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\bar{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

**Definition of** $\widetilde{G_{23}}, \widetilde{T_{23}}$ :$\left(\widetilde{(G_{23})},\widetilde{(T_{23})}\right) = \mathcal{A}^{(3)}\left((G_{23}),(T_{23})\right)$

It results 213

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_{0}^{t}(a_{20})^{(3)}\left|G_{21}^{(1)}-G_{21}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\, ds_{(20)} +$$

$$\int_{0}^{t}\{(a'_{20})^{(3)}\left|G_{20}^{(1)}-G_{20}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} +$$

$$(a''_{20})^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{(\widehat{M}_{20})^{(3)}s_{(20)}} +$$

$$G_{20}^{(2)}\left|(a''_{20})^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)}, s_{(20)}\right)\right|\ e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\} ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}t} \leq \qquad 214$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+\ (a'_{20})^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\widehat{k}_{20})^{(3)}\right) d\left(\left((G_{23})^{(1)},(T_{23})^{(1)};\ (G_{23})^{(2)},(T_{23})^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

**Remark 11:** The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}\ and\ (\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_{+}$. 215

If instead of proving the existence of the solution on $\mathbb{R}_{+}$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i = 20,21,22$ depend only on $T_{21}$ and respectively on $(G_{23})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 12:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 216

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t\{(a_i')^{(3)}-(a_i'')^{(3)}(T_{21}(s_{(20)}),s_{(20)})\}ds_{(20)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(3)}t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2\ and\ \left((\widehat{M}_{20})^{(3)}\right)_3$ : 217

**Remark 13:** if $G_{20}$ is bounded, the same property have also $G_{21}\ and\ G_{22}$ . indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a'_{21})^{(3)}G_{21}$ and by integrating

$$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1/(a'_{21})^{(3)}$$

In the same way , one can obtain

$$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2/(a'_{22})^{(3)}$$

If $G_{21}$ *or* $G_{22}$ is bounded, the same property follows for $G_{20}$ , $G_{22}$ and $G_{20}$ , $G_{21}$ respectively.

**Remark 14:** If $G_{20}$ *is* bounded, from below, the same property holds for $G_{21}$ *and* $G_{22}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{21}$ is bounded from below. 218

**Remark 15:** If $T_{20}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t),t\right)=(b_{21}')^{(3)}$ then $T_{21}\to\infty$. 219

**Definition of** $(m)^{(3)}$ and $\varepsilon_3$ :

Indeed let $t_3$ be so that for $t > t_3$

$(b_{21})^{(3)}-(b_i'')^{(3)}\left((G_{23})(t),t\right)<\varepsilon_3, T_{20}(t)>(m)^{(3)}$

Then $\frac{dT_{21}}{dt}\geq(a_{21})^{(3)}(m)^{(3)}-\varepsilon_3 T_{21}$ which leads to 220

$T_{21}\geq\left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1-e^{-\varepsilon_3 t})+T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t}=\frac{1}{2}$ it results

$T_{21}\geq\left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right),\quad t=log\frac{2}{\varepsilon_3}$ By taking now $\varepsilon_3$ sufficiently small one sees that $T_{21}$ is unbounded.
The same property holds for $T_{22}$ if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t),t\right)=(b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}$ , $\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}}<1$ and to choose 221

$(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\left[(\hat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \quad 222$$

$$\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \quad 223$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself 224

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric 225

$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{24})^{(4)}t}\}$$

Indeed if we denote

**Definition of** $\widetilde{(G_{27})},\widetilde{(T_{27})}$ : $\left(\widetilde{(G_{27})},\widetilde{(T_{27})}\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{24})^{(4)} \left|G_{25}^{(1)}-G_{25}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)}s_{(24)}} e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\, ds_{(24)} +$$

$$\int_0^t \{(a'_{24})^{(4)}\left|G_{24}^{(1)}-G_{24}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)}s_{(24)}} e^{-(\widehat{M}_{24})^{(4)}s_{(24)}} +$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)}, s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)}s_{(24)}} e^{(\widehat{M}_{24})^{(4)}s_{(24)}} +$$

$$G_{24}^{(2)}\left|(a''_{24})^{(4)}\left(T_{25}^{(1)}, s_{(24)}\right)-(a''_{24})^{(4)}\left(T_{25}^{(2)}, s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)}s_{(24)}} e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\} ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)}t} \leq \qquad (226)$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)}+\ (a'_{24})^{(4)}+(\widehat{A}_{24})^{(4)}+(\widehat{P}_{24})^{(4)}(\widehat{k}_{24})^{(4)}\right) d\left(\left((G_{27})^{(1)}, (T_{27})^{(1)};\ (G_{27})^{(2)}, (T_{27})^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

**Remark 16:** The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}\ and\ (\widehat{Q}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_+$. (227)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i = 24,25,26$ depend only on $T_{25}$ and respectively on $(G_{27})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 17:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ (228)

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(4)}-(a_i'')^{(4)}(T_{25}(s_{(24)}),s_{(24)})\}ds_{(24)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(4)}t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2\ and\ \left((\widehat{M}_{24})^{(4)}\right)_3$ : (229)

**Remark 18:** if $G_{24}$ is bounded, the same property have also $G_{25}\ and\ G_{26}$ . indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a'_{25})^{(4)}G_{25}$ and by integrating

$$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1/(a'_{25})^{(4)}$$

In the same way , one can obtain

$$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2/(a'_{26})^{(4)}$$

If $G_{25}$ *or* $G_{26}$ is bounded, the same property follows for $G_{24}$ , $G_{26}$ and $G_{24}$ , $G_{25}$ respectively.

**Remark 19:** If $G_{24}$ *is* bounded, from below, the same property holds for $G_{25}$ *and* $G_{26}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{25}$ is bounded from below. 230

**Remark 20:** If $T_{24}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}((G_{27})(t),t)) = (b_{25}')^{(4)}$ then $T_{25} \to \infty$. 231

**Definition of** $(m)^{(4)}$ and $\varepsilon_4$ :

Indeed let $t_4$ be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t),t) < \varepsilon_4, T_{24}(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right), \quad t = log\frac{2}{\varepsilon_4}$ By taking now $\varepsilon_4$ sufficiently small one sees that $T_{25}$ is unbounded.
The same property holds for $T_{26}$ if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t),t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for $G_{29}, G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}$ , $\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)} + \left((\hat{P}_{28})^{(5)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{28})^{(5)}\right] \leq (\hat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$$d\left(((G_{31})^{(1)},(T_{31})^{(1)}),((G_{31})^{(2)},(T_{31})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

**Definition of** $\widetilde{(G_{31})}, \widetilde{(T_{31})}$ : $\left(\widetilde{(G_{31})}, \widetilde{(T_{31})}\right) = \mathcal{A}^{(5)}\left((G_{31}), (T_{31})\right)$

It results

$$\left|\tilde{G}_{28}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{28})^{(5)} \left|G_{29}^{(1)} - G_{29}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}}\, ds_{(28)} +$$

$$\int_0^t \{(a'_{28})^{(5)} \left|G_{28}^{(1)} - G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} +$$

$$(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right)\left|G_{28}^{(1)} - G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}} +$$

$$G_{28}^{(2)} \left|(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right) - (a''_{28})^{(5)}\left(T_{29}^{(2)}, s_{(28)}\right)\right| \; e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}} \} ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$$\left|(G_{31})^{(1)} - (G_{31})^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} t} \leq \qquad (237)$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)} + (a'_{28})^{(5)} + (\widehat{A}_{28})^{(5)} + (\widehat{P}_{28})^{(5)} (\widehat{k}_{28})^{(5)}\right) d\left(\left((G_{31})^{(1)}, (T_{31})^{(1)}; (G_{31})^{(2)}, (T_{31})^{(2)}\right)\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 21:** The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$ *and* $(\widehat{Q}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$ respectively of $\mathbb{R}_+$. (238)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on $T_{29}$ and respectively on $(G_{31})$(*and not on* $t$) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 22:** There does not exist any $t$ where $G_i(t) = 0$ *and* $T_i(t) = 0$ (239)

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(5)} - (a_i'')^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(5)} t)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2$ *and* $\left((\widehat{M}_{28})^{(5)}\right)_3$ : (240)

**Remark 23:** if $G_{28}$ is bounded, the same property have also $G_{29}$ *and* $G_{30}$ . indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a'_{29})^{(5)} G_{29}$ and by integrating

$$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1 / (a'_{29})^{(5)}$$

In the same way , one can obtain

$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2/(a'_{30})^{(5)}$

If $G_{29}$ *or* $G_{30}$ is bounded, the same property follows for $G_{28}$, $G_{30}$ and $G_{28}$, $G_{29}$ respectively.

**Remark 24:** If $G_{28}$ *is* bounded, from below, the same property holds for $G_{29}$ *and* $G_{30}$. The proof is analogous with the preceding one. An analogous property is true if $G_{29}$ is bounded from below. 241

**Remark 25:** If $T_{28}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}((G_{31})(t),t)) = (b'_{29})^{(5)}$ then $T_{29} \to \infty$. 242

**Definition of** $(m)^{(5)}$ and $\varepsilon_5$ :

Indeed let $t_5$ be so that for $t > t_5$

$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t),t) < \varepsilon_5, T_{28}(t) > (m)^{(5)}$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\frac{2}{\varepsilon_5}$ By taking now $\varepsilon_5$ sufficiently small one sees that $T_{29}$ is unbounded.
The same property holds for $T_{30}$ if $\lim_{t\to\infty}(b_{30}'')^{(5)}\left((G_{31})(t),t\right) = (b'_{30})^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{33}, G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}$, $\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$ and to choose 244

$(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[(\hat{P}_{32})^{(6)} + \left((\hat{P}_{32})^{(6)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{32})^{(6)} \quad 245$$

$$\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[\left((\hat{Q}_{32})^{(6)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{32})^{(6)}\right] \leq (\hat{Q}_{32})^{(6)} \quad 246$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric 247

$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right) =$

$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{32})^{(6)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{32})^{(6)}t}\}$

Indeed if we denote

**Definition of** $\widetilde{(G_{35})}, \widetilde{(T_{35})}$ : $\left(\widetilde{(G_{35})}, \widetilde{(T_{35})}\right) = \mathcal{A}^{(6)}\left((G_{35}), (T_{35})\right)$

It results

$$\left|\tilde{G}_{32}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{32})^{(6)} \left|G_{33}^{(1)} - G_{33}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}}\, ds_{(32)} +$$

$$\int_0^t \{(a'_{32})^{(6)} \left|G_{32}^{(1)} - G_{32}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} +$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)}, s_{(32)}\right)\left|G_{32}^{(1)} - G_{32}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}} +$$

$$G_{32}^{(2)} \left|(a''_{32})^{(6)}\left(T_{33}^{(1)}, s_{(32)}\right) - (a''_{32})^{(6)}\left(T_{33}^{(2)}, s_{(32)}\right)\right| \; e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}} \} ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|(G_{35})^{(1)} - (G_{35})^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} t} \leq \qquad (248)$$
$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)} + (a'_{32})^{(6)} + (\widehat{A}_{32})^{(6)} + (\widehat{P}_{32})^{(6)} (\widehat{k}_{32})^{(6)}\right) d\left(\left((G_{35})^{(1)}, (T_{35})^{(1)}; \; (G_{35})^{(2)}, (T_{35})^{(2)}\right)\right)$$

And analogous inequalities for $G_i \; and \; T_i$. Taking into account the hypothesis the result follows

**Remark 26:** The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)} t} \; and \; (\widehat{Q}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)} t}$ respectively of $\mathbb{R}_+$. (249)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i = 32{,}33{,}34$ depend only on $T_{33}$ and respectively on $(G_{35})(and \; not \; on \; t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 27:** There does not exist any $t$ where $G_i\,(t) = 0 \; and \; T_i\,(t) = 0$ (250)

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(6)} - (a_i'')^{(6)}(T_{33}(s_{(32)}), s_{(32)})\} ds_{(32)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(6)} t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{32})^{(6)}\right)_1, \left((\widehat{M}_{32})^{(6)}\right)_2 \; and \; \left((\widehat{M}_{32})^{(6)}\right)_3$ : (251)

**Remark 28:** if $G_{32}$ is bounded, the same property have also $G_{33} \; and \; G_{34}$ . indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a'_{33})^{(6)} G_{33}$ and by integrating

$$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_1 / (a'_{33})^{(6)}$$

In the same way , one can obtain

$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_2/(a'_{34})^{(6)}$

If $G_{33}$ *or* $G_{34}$ is bounded, the same property follows for $G_{32}$ , $G_{34}$ and $G_{32}$ , $G_{33}$ respectively.

**Remark 29:** If $G_{32}$ *is* bounded, from below, the same property holds for $G_{33}$ *and* $G_{34}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{33}$ is bounded from below. 252

**Remark 30:** If $T_{32}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}((G_{35})(t),t)) = (b'_{33})^{(6)}$ then $T_{33} \to \infty$. 253

**Definition of** $(m)^{(6)}$ and $\varepsilon_6$ :

Indeed let $t_6$ be so that for $t > t_6$

$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t),t\right) < \varepsilon_6, T_{32}(t) > (m)^{(6)}$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right), \quad t = log\frac{2}{\varepsilon_6}$ By taking now $\varepsilon_6$ sufficiently small one sees that $T_{33}$ is unbounded.
The same property holds for $T_{34}$ if $\lim_{t\to\infty}(b''_{34})^{(6)}\left((G_{35})(t),t(t),t\right) = (b'_{34})^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{37}$ , $G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}$ , $\frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$ and to choose
$(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[(\hat{P}_{36})^{(7)} + \left((\hat{P}_{36})^{(7)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{36})^{(7)} \quad 256$$

$$\frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[\left((\hat{Q}_{36})^{(7)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{36})^{(7)}\right] \leq (\hat{Q}_{36})^{(7)} \quad 257$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself
The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric 258

$d\left(((G_{39})^{(1)},(T_{39})^{(1)}),((G_{39})^{(2)},(T_{39})^{(2)})\right) =$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\tilde{M}_{36})^{(7)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\tilde{M}_{36})^{(7)}t}\}$$

Indeed if we denote

**Definition of** $\widetilde{(G_{39})},\widetilde{(T_{39})}:\left(\widetilde{(G_{39})},\widetilde{(T_{39})}\right)=\mathcal{A}^{(7)}((G_{39}),(T_{39}))$
It results

$$\left|\tilde{G}_{36}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{36})^{(7)}\left|G_{37}^{(1)}-G_{37}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\,ds_{(36)}+$$
$$\int_0^t\{(a'_{36})^{(7)}\left|G_{36}^{(1)}-G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$(a''_{36})^{(7)}\left(T_{37}^{(1)},s_{(36)}\right)\left|G_{36}^{(1)}-G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$G_{36}^{(2)}\left|(a''_{36})^{(7)}\left(T_{37}^{(1)},s_{(36)}\right)-(a''_{36})^{(7)}\left(T_{37}^{(2)},s_{(36)}\right)\right|\ e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\}ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses on it follows

$$\left|(G_{39})^{(1)}-(G_{39})^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}t}\le \qquad 259$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}}\left((a_{36})^{(7)}+(a'_{36})^{(7)}+(\widehat{A}_{36})^{(7)}+(\widehat{P}_{36})^{(7)}(\widehat{k}_{36})^{(7)}\right)d\left(((G_{39})^{(1)},(T_{39})^{(1)};\ (G_{39})^{(2)},(T_{39})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

**Remark 31:** The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}\ and\ (\widehat{Q}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ respectively of $\mathbb{R}_+$. 260

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(7)}$ and $(b_i'')^{(7)},i=36{,}37{,}38$ depend only on $\mathrm{T}_{37}$ and respectively on $(G_{39})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 32:** There does not exist any $t$ where $G_i\,(t)=0\ and\ T_i\,(t)=0$ 261

it results

$$G_i\,(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(7)}-(a_i'')^{(7)}(T_{37}(s_{(36)}),s_{(36)})\}ds_{(36)}\right]}\ge 0$$

$$T_i\,(t)\ge T_i^0e^{\left(-(b_i')^{(7)}t\right)}>0\quad \text{for t}>0$$

**Definition of** $\left((\widehat{M}_{36})^{(7)}\right)_1,\left((\widehat{M}_{36})^{(7)}\right)_2\ and\ \left((\widehat{M}_{36})^{(7)}\right)_3:$ 262

**Remark 33:** if $G_{36}$ is bounded, the same property have also $G_{37}\ and\ G_{38}$ . indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a'_{37})^{(7)} G_{37}$ and by integrating

$$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1/(a'_{37})^{(7)}$$

In the same way , one can obtain

$$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2/(a'_{38})^{(7)}$$

If $G_{37}$ *or* $G_{38}$ is bounded, the same property follows for $G_{36}$ , $G_{38}$ and $G_{36}$ , $G_{37}$ respectively.

**Remark 34:** If $G_{36}$ *is* bounded, from below, the same property holds for $G_{37}$ *and* $G_{38}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{37}$ is bounded from below. 263

**Remark 35:** If $T_{36}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(7)}((G_{39})(t),t)) = (b'_{37})^{(7)}$ then $T_{37} \to \infty$. 264

**Definition of** $(m)^{(7)}$ and $\varepsilon_7$ :

Indeed let $t_7$ be so that for $t > t_7$

$$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t),t) < \varepsilon_7, T_{36}(t) > (m)^{(7)}$$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right)$, $t = log\frac{2}{\varepsilon_7}$ By taking now $\varepsilon_7$ sufficiently small one sees that $T_{37}$ is unbounded.
The same property holds for $T_{38}$ if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t),t\right) = (b'_{38})^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}$ , $\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1$ and to choose 266
$(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ large to have

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[(\hat{P}_{40})^{(8)} + \left((\hat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{40})^{(8)} \quad 267$$

$$\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{40})^{(8)}\right] \leq (\hat{Q}_{40})^{(8)} \quad 268$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)},(T_{43})^{(1)}),((G_{43})^{(2)},(T_{43})^{(2)})\right)= \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

**Definition of** $\widetilde{(G_{43})},\widetilde{(T_{43})}$ : $\left(\widetilde{(G_{43})},\widetilde{(T_{43})}\right)=\mathcal{A}^{(8)}((G_{43}),(T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{40})^{(8)}\left|G_{41}^{(1)}-G_{41}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\,ds_{(40)}+$$
$$\int_0^t\{(a'_{40})^{(8)}\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$G_{40}^{(2)}\left|(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)-(a''_{40})^{(8)}\left(T_{41}^{(2)},s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\}ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0,t]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)}-(G_{43})^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}t}\le \quad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)}+(a'_{40})^{(8)}+(\widehat{A}_{40})^{(8)}+(\widehat{P}_{40})^{(8)}(\widehat{k}_{40})^{(8)}\right)d\left(((G_{43})^{(1)},(T_{43})^{(1)};\ (G_{43})^{(2)},(T_{43})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

**Remark 36:** The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}\ and\ (\widehat{Q}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ respectively of $\mathbb{R}_+$. 274

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i=40,41,42$ depend only on $T_{41}$ and respectively on $(G_{43})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 37** There does not exist any $t$ where $G_i\,(t)=0\ and\ T_i\,(t)=0$ 275

it results

$$G_i\,(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(8)}-(a_i'')^{(8)}(T_{41}(s_{(40)}),s_{(40)})\}ds_{(40)}\right]}\ge 0$$

$$T_i\,(t)\ge T_i^0e^{\left(-(b_i')^{(8)}t\right)}>0\quad \text{for t}>0$$

**Definition of** $\left((\widehat{M}_{40})^{(8)}\right)_1, \left((\widehat{M}_{40})^{(8)}\right)_2$ *and* $\left((\widehat{M}_{40})^{(8)}\right)_3$ : 276

**Remark 38:** if $G_{40}$ is bounded, the same property have also $G_{41}$ *and* $G_{42}$ . indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a'_{41})^{(8)} G_{41}$ and by integrating

$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1/(a'_{41})^{(8)}$

In the same way , one can obtain

$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2/(a'_{42})^{(8)}$

If $G_{41}$ *or* $G_{42}$ is bounded, the same property follows for $G_{40}$ , $G_{42}$ and $G_{40}$ , $G_{41}$ respectively.

**Remark 39:** If $G_{40}$ *is* bounded, from below, the same property holds for $G_{41}$ *and* $G_{42}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{41}$ is bounded from below. 277

**Remark 40:** If $T_{40}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}((G_{43})(t),t)) = (b'_{41})^{(8)}$ then $T_{41} \to \infty$. 278

**Definition of** $(m)^{(8)}$ and $\varepsilon_8$ :

Indeed let $t_8$ be so that for $t > t_8$

$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t),t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right), \quad t = log\frac{2}{\varepsilon_8}$ By taking now $\varepsilon_8$ sufficiently small one sees that $T_{41}$ is unbounded.
The same property holds for $T_{42}$ if $\lim_{t\to\infty}(b''_{42})^{(8)}\left((G_{43})(t),t(t),t\right) = (b'_{42})^{(8)}$

It is now sufficient to take $\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}$ , $\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}} < 1$ and to choose $(\widehat{P}_{44})^{(9)}$ and $(\widehat{Q}_{44})^{(9)}$ large to have 279 A

$$\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[(\widehat{P}_{44})^{(9)} + \left((\widehat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[\left((\widehat{Q}_{44})^{(9)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{44})^{(9)}\right] \leq (\widehat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)},(T_{47})^{(1)}),((G_{47})^{(2)},(T_{47})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{47}}),(\widetilde{T_{47}}):\left((\widetilde{G_{47}}),(\widetilde{T_{47}})\right)=\mathcal{A}^{(9)}\left((G_{47}),(T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{44})^{(9)}\left|G_{45}^{(1)}-G_{45}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\,ds_{(44)}+$$
$$\int_0^t\{(a'_{44})^{(9)}\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$G_{44}^{(2)}\left|(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)-(a''_{44})^{(9)}\left(T_{45}^{(2)},s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\}ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}t}\le$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)}+(a'_{44})^{(9)}+(\widehat{A}_{44})^{(9)}+(\widehat{P}_{44})^{(9)}(\widehat{k}_{44})^{(9)}\right)d\left(((G_{47})^{(1)},(T_{47})^{(1)};\ (G_{47})^{(2)},(T_{47})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis (39,35,36) the result follows

**Remark 41:** The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}\ and\ (\widehat{Q}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i=44,45,46$ depend only on $T_{45}$ and respectively on $(G_{47})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 42:** There does not exist any $t$ where $G_i\,(t)=0\ and\ T_i\,(t)=0$

From 99 to 44 it results

$$G_i\,(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(9)}-(a_i'')^{(9)}(T_{45}(s_{(44)}),s_{(44)})\}ds_{(44)}\right]}\ge 0$$

$$T_i\,(t)\ge T_i^0e^{(-(b_i')^{(9)}t)}>0\quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{44})^{(9)}\right)_1,\left((\widehat{M}_{44})^{(9)}\right)_2\ and\ \left((\widehat{M}_{44})^{(9)}\right)_3$ :

**Remark 43:** if $G_{44}$ is bounded, the same property have also $G_{45}\ and\ G_{46}$ . indeed if

$G_{44} < (\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt} \leq \left((\widehat{M}_{44})^{(9)}\right)_1 - (a'_{45})^{(9)} G_{45}$ and by integrating

$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_1 / (a'_{45})^{(9)}$

In the same way , one can obtain

$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_2 / (a'_{46})^{(9)}$

If $G_{45}$ *or* $G_{46}$ is bounded, the same property follows for $G_{44}$ , $G_{46}$ and $G_{44}$ , $G_{45}$ respectively.

**Remark 44:** If $G_{44}$ *is* bounded, from below, the same property holds for $G_{45}$ *and* $G_{46}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{45}$ is bounded from below.

**Remark 45:** If $T_{44}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t),t\right) = (b'_{45})^{(9)}$ then $T_{45} \to \infty$.

**Definition of** $(m)^{(9)}$ and $\varepsilon_9$ :

Indeed let $t_9$ be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t),t\right) < \varepsilon_9, T_{44}(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right)$, $t = log\frac{2}{\varepsilon_9}$ By taking now $\varepsilon_9$ sufficiently small one sees that $T_{45}$ is unbounded.
The same property holds for $T_{46}$ if $\lim_{t\to\infty}(b''_{46})^{(9)}\left((G_{47})(t),t\right) = (b'_{46})^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

**Behavior of the solutions of equation** 280

**Theorem** If we denote and define

**Definition of** $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ :

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)} \leq -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14},t) + (a''_{14})^{(1)}(T_{14},t) \leq -(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)} \leq -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G,t) - (b''_{14})^{(1)}(G,t) \leq -(\tau_1)^{(1)}$

**Definition of** $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$ : 281

By $(\nu_1)^{(1)} > 0$ , $(\nu_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$ , $(u_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

**Definition of** $(\bar{\nu}_1)^{(1)}, , (\bar{\nu}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$ : 282

By $(\bar{\nu}_1)^{(1)} > 0$ , $(\bar{\nu}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$ , $(\bar{u}_2)^{(1)} < 0$ the roots of the equations

$(a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

**Definition of** $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (v_0)^{(1)}$ :- 283

If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by

$(m_2)^{(1)} = (v_0)^{(1)}, (m_1)^{(1)} = (v_1)^{(1)}, \ if\ (v_0)^{(1)} < (v_1)^{(1)}$

$(m_2)^{(1)} = (v_1)^{(1)}, (m_1)^{(1)} = (\bar{v}_1)^{(1)}, if\ (v_1)^{(1)} < (v_0)^{(1)} < (\bar{v}_1)^{(1)},$

and $\boxed{(v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$

$(m_2)^{(1)} = (v_1)^{(1)}, (m_1)^{(1)} = (v_0)^{(1)}, \ if\ (\bar{v}_1)^{(1)} < (v_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)}, \ if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)},$

and $\boxed{(u_0)^{(1)} = \frac{T_{13}^0}{T_{14}^0}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$

are defined

Then the solution of global equations satisfies the inequalities 285

$$G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{13}(t) \le G_{13}^0 e^{(S_1)^{(1)}t}$$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)}t}$$

$$\left(\frac{(a_{15})^{(1)} G_{13}^0}{(m_1)^{(1)}((S_1)^{(1)} - (p_{13})^{(1)} - (S_2)^{(1)})}\left[e^{((S_1)^{(1)} - (p_{13})^{(1)})t} - e^{-(S_2)^{(1)}t}\right] + G_{15}^0 e^{-(S_2)^{(1)}t} \le G_{15}(t) \le \right. \quad 286$$

$$\left. \frac{(a_{15})^{(1)} G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)} - (a'_{15})^{(1)})}\left[e^{(S_1)^{(1)}t} - e^{-(a'_{15})^{(1)}t}\right] + G_{15}^0 e^{-(a'_{15})^{(1)}t}\right)$$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)}t} \le T_{13}(t) \le T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t}} \quad 287$$

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)}t} \le T_{13}(t) \le \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t} \quad 288$$

$$\frac{(b_{15})^{(1)} T_{13}^0}{(\mu_1)^{(1)}((R_1)^{(1)} - (b'_{15})^{(1)})}\left[e^{(R_1)^{(1)}t} - e^{-(b'_{15})^{(1)}t}\right] + T_{15}^0 e^{-(b'_{15})^{(1)}t} \le T_{15}(t) \le \quad 289$$

$$\frac{(a_{15})^{(1)} T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)} + (r_{13})^{(1)} + (R_2)^{(1)})}\left[e^{((R_1)^{(1)} + (r_{13})^{(1)})t} - e^{-(R_2)^{(1)}t}\right] + T_{15}^0 e^{-(R_2)^{(1)}t}$$

**Definition of** $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:- 290

Where $(S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$

$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$

$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$

$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$

**Behavior of the solutions of equation** 291

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ : 292

$(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ four constants satisfying
$-(\sigma_2)^{(2)} \le -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(T_{17}, t) + (a''_{17})^{(2)}(T_{17}, t) \le -(\sigma_1)^{(2)}$ 293

$-(\tau_2)^{(2)} \le -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}\big((G_{19}), t\big) - (b''_{17})^{(2)}\big((G_{19}), t\big) \le -(\tau_1)^{(2)}$ 294

**Definition of** $(\nu_1)^{(2)}, (\nu_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$ : 295

By $(\nu_1)^{(2)} > 0$ , $(\nu_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0$ , $(u_2)^{(2)} < 0$ the roots 296

of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 297

and $(b_{14})^{(2)}\big(u^{(2)}\big)^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and 298

**Definition of** $(\bar{\nu}_1)^{(2)},, (\bar{\nu}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$ : 299

By $(\bar{\nu}_1)^{(2)} > 0$ , $(\bar{\nu}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0$ , $(\bar{u}_2)^{(2)} < 0$ the 300

roots of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 301

and $(b_{17})^{(2)}\big(u^{(2)}\big)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 302

**Definition of** $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ :- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{G^0_{16}}{G^0_{17}}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)},\ \boldsymbol{if}\ (u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}, \boldsymbol{if}\ (u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)},$

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)},\ \boldsymbol{if}\ (\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)} t}$

$(p_i)^{(2)}$ is defined by equation

$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)} t}$ 311

$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)} t}\right] + G_{18}^0 e^{-(S_2)^{(2)} t} \le G_{18}(t) \le\right.$ 312

$\left.\frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)} t} - e^{-(a'_{18})^{(2)} t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)} t}\right)$

$\boxed{T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}}$ 313

$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}$ 314

$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)} t} - e^{-(b'_{18})^{(2)} t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)} t} \le T_{18}(t) \le$ 315

$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})}\left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)} t}\right] + T_{18}^0 e^{-(R_2)^{(2)} t}$

**Definition of** $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$

$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)}$ 318

$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$

**Behavior of the solutions** 319

**Theorem 3:** If we denote and define

**Definition of** $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ :

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}\big((G_{23}), t\big) \le -(\tau_1)^{(3)}$$

**Definition of** $(\nu_1)^{(3)}, (\nu_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$ : 320

By $(\nu_1)^{(3)} > 0$ , $(\nu_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$ , $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{\nu}_1)^{(3)} > 0$ , $(\bar{\nu}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$ , $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

**Definition of** $(m_1)^{(3)}$ , $(m_2)^{(3)}$ , $(\mu_1)^{(3)}, (\mu_2)^{(3)}$ :- 321

If we define $(m_1)^{(3)}$ , $(m_2)^{(3)}$ , $(\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (\nu_0)^{(3)}, (m_1)^{(3)} = (\nu_1)^{(3)}$, ***if*** $(\nu_0)^{(3)} < (\nu_1)^{(3)}$

$(m_2)^{(3)} = (\nu_1)^{(3)}, (m_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ , ***if*** $(\nu_1)^{(3)} < (\nu_0)^{(3)} < (\bar{\nu}_1)^{(3)}$,

and $\boxed{(\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}}$

$(m_2)^{(3)} = (\nu_1)^{(3)}, (m_1)^{(3)} = (\nu_0)^{(3)}$, ***if*** $(\bar{\nu}_1)^{(3)} < (\nu_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}$, ***if*** $(u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$ , ***if*** $(u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}$, and $\boxed{(u_0)^{(3)} = \frac{T^0_{20}}{T^0_{21}}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}$, ***if*** $(\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$$G^0_{20}e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G^0_{20}e^{(S_1)^{(3)}t}$$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}}G^0_{20}e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}}G^0_{20}e^{(S_1)^{(3)}t} \quad 323$$

$$\left(\frac{(a_{22})^{(3)}G^0_{20}}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\left[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)}t}\right] + G^0_{22}e^{-(S_2)^{(3)}t} \le G_{22}(t) \le \right. \quad 324$$

$$\left.\frac{(a_{22})^{(3)}G^0_{20}}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}\left[e^{(S_1)^{(3)}t} - e^{-(a'_{22})^{(3)}t}\right] + G^0_{22}e^{-(a'_{22})^{(3)}t}\right)$$

$$\boxed{T_{20}^{0}e^{(R_1)^{(3)}t} \le T_{20}(t) \le T_{20}^{0}e^{((R_1)^{(3)}+(r_{20})^{(3)})t}} \quad 325$$

$$\frac{1}{(\mu_1)^{(3)}}T_{20}^{0}e^{(R_1)^{(3)}t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}}T_{20}^{0}e^{((R_1)^{(3)}+(r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)}T_{20}^{0}}{(\mu_1)^{(3)}((R_1)^{(3)}-(b'_{22})^{(3)})}\left[e^{(R_1)^{(3)}t} - e^{-(b'_{22})^{(3)}t}\right] + T_{22}^{0}e^{-(b'_{22})^{(3)}t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)}T_{20}^{0}}{(\mu_2)^{(3)}((R_1)^{(3)}+(r_{20})^{(3)}+(R_2)^{(3)})}\left[e^{((R_1)^{(3)}+(r_{20})^{(3)})t} - e^{-(R_2)^{(3)}t}\right] + T_{22}^{0}e^{-(R_2)^{(3)}t}$$

**Definition of** $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

**Behavior of the solutions of equation**

**Theorem:** If we denote and define

**Definition of** $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ :

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$$

$$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}\left((G_{27}), t\right) - (b''_{25})^{(4)}\left((G_{27}), t\right) \le -(\tau_1)^{(4)}$$

**Definition of** $(\nu_1)^{(4)}, (\nu_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, \nu^{(4)}, u^{(4)}$ : 329

By $(\nu_1)^{(4)} > 0$ , $(\nu_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$ , $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_1)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(4)},, (\bar{\nu}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$ : 330

By $(\bar{\nu}_1)^{(4)} > 0$ , $(\bar{\nu}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$ , $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

**Definition of** $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}, (\nu_0)^{(4)}$ :-

If we define $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (\nu_0)^{(4)}, (m_1)^{(4)} = (\nu_1)^{(4)}$, ***if*** $(\nu_0)^{(4)} < (\nu_1)^{(4)}$

$(m_2)^{(4)} = (\nu_1)^{(4)}, (m_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ , ***if*** $(\nu_4)^{(4)} < (\nu_0)^{(4)} < (\bar{\nu}_1)^{(4)}$,

and $\boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (\nu_4)^{(4)}, (m_1)^{(4)} = (\nu_0)^{(4)},\ \boldsymbol{if}\ (\bar{\nu}_4)^{(4)} < (\nu_0)^{(4)}$

and analogously

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$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)},\ \boldsymbol{if}\ (u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}, \boldsymbol{if}\ (u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)},$

and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}, \boldsymbol{if}\ (\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

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$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)} t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)} t} \quad 333$$

$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)} t}\right] + G_{26}^0 e^{-(S_2)^{(4)} t} \le G_{26}(t) \le \right. \quad 334$$

*(a26)4G240(m2)4(S1)4−(a26′)4e(S1)4t−e−(a26′)4t+ G260e−(a26′)4t*

$$\boxed{T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}} T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}} T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t} \quad 335$$

$$\frac{(b_{26})^{(4)} T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)} - (b'_{26})^{(4)})}\left[e^{(R_1)^{(4)} t} - e^{-(b'_{26})^{(4)} t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)} t} \le T_{26}(t) \le \quad 336$$

$$\frac{(a_{26})^{(4)} T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)} + (r_{24})^{(4)} + (R_2)^{(4)})}\left[e^{((R_1)^{(4)} + (r_{24})^{(4)})t} - e^{-(R_2)^{(4)} t}\right] + T_{26}^0 e^{-(R_2)^{(4)} t}$$

**Definition of** $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

**Behavior of the solutions of equation** 338

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ :

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

**Definition of** $(\nu_1)^{(5)}, (\nu_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, \nu^{(5)}, u^{(5)}$ : 339

By $(\nu_1)^{(5)} > 0$, $(\nu_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$, $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(5)}, , (\bar{\nu}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$ : 340

By $(\bar{\nu}_1)^{(5)} > 0$, $(\bar{\nu}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$, $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

**Definition of** $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (\nu_0)^{(5)}$ :-

If we define $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (\nu_0)^{(5)}, (m_1)^{(5)} = (\nu_1)^{(5)}$, ***if*** $(\nu_0)^{(5)} < (\nu_1)^{(5)}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\bar{\nu}_1)^{(5)}$, ***if*** $(\nu_1)^{(5)} < (\nu_0)^{(5)} < (\bar{\nu}_1)^{(5)}$,
and $\boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)}$, ***if*** $(\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)}$, ***if*** $(u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}$, ***if*** $(u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)}$,
and $\boxed{(u_0)^{(5)} = \frac{T^0_{28}}{T^0_{29}}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}$, ***if*** $(\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$$G^0_{28} e^{\left((S_1)^{(5)} - (p_{28})^{(5)}\right)t} \le G_{28}(t) \le G^0_{28} e^{(S_1)^{(5)}t}$$

where $(p_i)^{(5)}$ is defined by equation

$$\frac{1}{(m_5)^{(5)}} G^0_{28} e^{\left((S_1)^{(5)} - (p_{28})^{(5)}\right)t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}} G^0_{28} e^{(S_1)^{(5)}t} \quad 343$$

$$\left(\frac{(a_{30})^{(5)}G_{28}^{0}}{(m_1)^{(5)}((S_1)^{(5)}-(p_{28})^{(5)}-(S_2)^{(5)})}\left[e^{((S_1)^{(5)}-(p_{28})^{(5)})t}-e^{-(S_2)^{(5)}t}\right]+G_{30}^{0}e^{-(S_2)^{(5)}t}\leq G_{30}(t)\leq\right. \quad 344$$

*(a30)5G280(m2)5(S1)5−(a30′)5e(S1)5t−e−(a30′)5t+ G300e−(a30′)5t*

$$\boxed{T_{28}^{0}e^{(R_1)^{(5)}t}\leq T_{28}(t)\leq T_{28}^{0}e^{((R_1)^{(5)}+(r_{28})^{(5)})t}} \quad 345$$

$$\frac{1}{(\mu_1)^{(5)}}T_{28}^{0}e^{(R_1)^{(5)}t}\leq T_{28}(t)\leq\frac{1}{(\mu_2)^{(5)}}T_{28}^{0}e^{((R_1)^{(5)}+(r_{28})^{(5)})t} \quad 346$$

$$\frac{(b_{30})^{(5)}T_{28}^{0}}{(\mu_1)^{(5)}((R_1)^{(5)}-(b'_{30})^{(5)})}\left[e^{(R_1)^{(5)}t}-e^{-(b'_{30})^{(5)}t}\right]+T_{30}^{0}e^{-(b'_{30})^{(5)}t}\leq T_{30}(t)\leq \quad 347$$

$$\frac{(a_{30})^{(5)}T_{28}^{0}}{(\mu_2)^{(5)}((R_1)^{(5)}+(r_{28})^{(5)}+(R_2)^{(5)})}\left[e^{((R_1)^{(5)}+(r_{28})^{(5)})t}-e^{-(R_2)^{(5)}t}\right]+T_{30}^{0}e^{-(R_2)^{(5)}t}$$

**Definition of** $(S_1)^{(5)},(S_2)^{(5)},(R_1)^{(5)},(R_2)^{(5)}$:- 348

Where $(S_1)^{(5)}=(a_{28})^{(5)}(m_2)^{(5)}-(a'_{28})^{(5)}$

$(S_2)^{(5)}=(a_{30})^{(5)}-(p_{30})^{(5)}$

$(R_1)^{(5)}=(b_{28})^{(5)}(\mu_2)^{(5)}-(b'_{28})^{(5)}$

$(R_2)^{(5)}=(b'_{30})^{(5)}-(r_{30})^{(5)}$

**Behavior of the solutions of equation** 349

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(6)},(\sigma_2)^{(6)},(\tau_1)^{(6)},(\tau_2)^{(6)}$ :

$(\sigma_1)^{(6)},(\sigma_2)^{(6)},(\tau_1)^{(6)},(\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)}\leq-(a'_{32})^{(6)}+(a'_{33})^{(6)}-(a''_{32})^{(6)}(T_{33},t)+(a''_{33})^{(6)}(T_{33},t)\leq-(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)}\leq-(b'_{32})^{(6)}+(b'_{33})^{(6)}-(b''_{32})^{(6)}\big((G_{35}),t\big)-(b''_{33})^{(6)}\big((G_{35}),t\big)\leq-(\tau_1)^{(6)}$$

**Definition of** $(v_1)^{(6)},(v_2)^{(6)},(u_1)^{(6)},(u_2)^{(6)},v^{(6)},u^{(6)}$ : 350

By $(v_1)^{(6)}>0$ , $(v_2)^{(6)}<0$ and respectively $(u_1)^{(6)}>0$ , $(u_2)^{(6)}<0$ the roots of the equations $(a_{33})^{(6)}\big(v^{(6)}\big)^2+(\sigma_1)^{(6)}v^{(6)}-(a_{32})^{(6)}=0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2+(\tau_1)^{(6)}u^{(6)}-(b_{32})^{(6)}=0$ and

**Definition of** $(\bar{v}_1)^{(6)},,(\bar{v}_2)^{(6)},(\bar{u}_1)^{(6)},(\bar{u}_2)^{(6)}$ : 351

By $(\bar{v}_1)^{(6)}>0$ , $(\bar{v}_2)^{(6)}<0$ and respectively $(\bar{u}_1)^{(6)}>0$ , $(\bar{u}_2)^{(6)}<0$ the roots of the equations $(a_{33})^{(6)}\big(v^{(6)}\big)^2+(\sigma_2)^{(6)}v^{(6)}-(a_{32})^{(6)}=0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2+(\tau_2)^{(6)}u^{(6)}-(b_{32})^{(6)}=0$

**Definition of** $(m_1)^{(6)},(m_2)^{(6)},(\mu_1)^{(6)},(\mu_2)^{(6)},(v_0)^{(6)}$ :-

If we define $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}$ by

$(m_2)^{(6)} = (\nu_0)^{(6)}, (m_1)^{(6)} = (\nu_1)^{(6)}$, ***if*** $(\nu_0)^{(6)} < (\nu_1)^{(6)}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}$, ***if*** $(\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)}$,

and $\boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}$, ***if*** $(\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$

and analogously 352

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}$, ***if*** $(u_0)^{(6)} < (u_1)^{(6)}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}$, ***if*** $(u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)}$,

and $\boxed{(u_0)^{(6)} = \frac{T_{32}^0}{T_{33}^0}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}$, ***if*** $(\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$

Then the solution of global equations satisfies the inequalities 353

$$G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \leq G_{32}(t) \leq G_{32}^0 e^{(S_1)^{(6)} t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}} G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \leq G_{33}(t) \leq \frac{1}{(m_2)^{(6)}} G_{32}^0 e^{(S_1)^{(6)} t} \quad 354$$

$$\left( \frac{(a_{34})^{(6)} G_{32}^0}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})} \left[ e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)} t} \right] + G_{34}^0 e^{-(S_2)^{(6)} t} \leq G_{34}(t) \leq \right. \quad 355$$

*(a34)6G320(m2)6(S1)6−(a34′)6e(S1)6t−e−(a34′)6t+ G340e−(a34′)6t*

$$\boxed{T_{32}^0 e^{(R_1)^{(6)} t} \leq T_{32}(t) \leq T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}} T_{32}^0 e^{(R_1)^{(6)} t} \leq T_{32}(t) \leq \frac{1}{(\mu_2)^{(6)}} T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)} T_{32}^0}{(\mu_1)^{(6)}((R_1)^{(6)} - (b'_{34})^{(6)})} \left[ e^{(R_1)^{(6)} t} - e^{-(b'_{34})^{(6)} t} \right] + T_{34}^0 e^{-(b'_{34})^{(6)} t} \leq T_{34}(t) \leq \quad 358$$

$$\frac{(a_{34})^{(6)} T_{32}^0}{(\mu_2)^{(6)}((R_1)^{(6)} + (r_{32})^{(6)} + (R_2)^{(6)})} \left[ e^{((R_1)^{(6)} + (r_{32})^{(6)})t} - e^{-(R_2)^{(6)} t} \right] + T_{34}^0 e^{-(R_2)^{(6)} t}$$

**Definition of** $(S_1)^{(6)}, (S_2)^{(6)}, (R_1)^{(6)}, (R_2)^{(6)}$:- 359

Where $(S_1)^{(6)} = (a_{32})^{(6)} (m_2)^{(6)} - (a'_{32})^{(6)}$

$(S_2)^{(6)} = (a_{34})^{(6)} - (p_{34})^{(6)}$

$(R_1)^{(6)} = (b_{32})^{(6)} (\mu_2)^{(6)} - (b'_{32})^{(6)}$

$$(R_2)^{(6)} = (b'_{34})^{(6)} - (r_{34})^{(6)}$$

**Behavior of the solutions of equation**

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$ :

$(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)} \le -(a'_{36})^{(7)} + (a'_{37})^{(7)} - (a''_{36})^{(7)}(T_{37}, t) + (a''_{37})^{(7)}(T_{37}, t) \le -(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)} \le -(b'_{36})^{(7)} + (b'_{37})^{(7)} - (b''_{36})^{(7)}\big((G_{39}), t\big) - (b''_{37})^{(7)}\big((G_{39}), t\big) \le -(\tau_1)^{(7)}$$

**Definition of** $(\nu_1)^{(7)}, (\nu_2)^{(7)}, (u_1)^{(7)}, (u_2)^{(7)}, \nu^{(7)}, u^{(7)}$ : 361

By $(\nu_1)^{(7)} > 0, (\nu_2)^{(7)} < 0$ and respectively $(u_1)^{(7)} > 0, (u_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_1)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(7)},, (\bar{\nu}_2)^{(7)}, (\bar{u}_1)^{(7)}, (\bar{u}_2)^{(7)}$ : 362

By $(\bar{\nu}_1)^{(7)} > 0, (\bar{\nu}_2)^{(7)} < 0$ and respectively $(\bar{u}_1)^{(7)} > 0, (\bar{u}_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

**Definition of** $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (\nu_0)^{(7)}$ :-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$$(m_2)^{(7)} = (\nu_0)^{(7)}, (m_1)^{(7)} = (\nu_1)^{(7)}, \ \boldsymbol{if}\ (\nu_0)^{(7)} < (\nu_1)^{(7)}$$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\bar{\nu}_1)^{(7)}, \boldsymbol{if}\ (\nu_1)^{(7)} < (\nu_0)^{(7)} < (\bar{\nu}_1)^{(7)},$$

and $\boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\nu_0)^{(7)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(7)} < (\nu_0)^{(7)}$$

and analogously 363

$$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}, \ \boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$$

and $\boxed{(u_0)^{(7)} = \frac{T^0_{36}}{T^0_{37}}}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)}$ where $(u_1)^{(7)}, (\bar{u}_1)^{(7)}$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^0 e^{\left((S_1)^{(7)} - (p_{36})^{(7)}\right)t} \le G_{36}(t) \le G_{36}^0 e^{(S_1)^{(7)}t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{\left((S_1)^{(7)} - (p_{36})^{(7)}\right)t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)}t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)}\left((S_1)^{(7)} - (p_{36})^{(7)} - (S_2)^{(7)}\right)}\left[e^{\left((S_1)^{(7)} - (p_{36})^{(7)}\right)t} - e^{-(S_2)^{(7)}t}\right] + G_{38}^0 e^{-(S_2)^{(7)}t} \le G_{38}(t) \le \right. \quad 366$$

$$\left. \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)}\left((S_1)^{(7)} - (a'_{38})^{(7)}\right)}\left[e^{(S_1)^{(7)}t} - e^{-(a'_{38})^{(7)}t}\right] + G_{38}^0 e^{-(a'_{38})^{(7)}t}\right)$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)}t} \le T_{36}(t) \le T_{36}^0 e^{\left((R_1)^{(7)} + (r_{36})^{(7)}\right)t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)}t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{\left((R_1)^{(7)} + (r_{36})^{(7)}\right)t} \quad 368$$

$$\frac{(b_{38})^{(7)} T_{36}^0}{(\mu_1)^{(7)}\left((R_1)^{(7)} - (b'_{38})^{(7)}\right)}\left[e^{(R_1)^{(7)}t} - e^{-(b'_{38})^{(7)}t}\right] + T_{38}^0 e^{-(b'_{38})^{(7)}t} \le T_{38}(t) \le \quad 369$$

$$\frac{(a_{38})^{(7)} T_{36}^0}{(\mu_2)^{(7)}\left((R_1)^{(7)} + (r_{36})^{(7)} + (R_2)^{(7)}\right)}\left[e^{\left((R_1)^{(7)} + (r_{36})^{(7)}\right)t} - e^{-(R_2)^{(7)}t}\right] + T_{38}^0 e^{-(R_2)^{(7)}t}$$

**Definition of** $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$$

$$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$$

$$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$$

**Behavior of the solutions of equation** 371

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ :

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$$-(\sigma_2)^{(8)} \le -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \le -(\sigma_1)^{(8)}$$

$$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\big((G_{43}),t\big) - (b''_{41})^{(8)}\big((G_{43}),t\big) \le -(\tau_1)^{(8)}$$

**Definition of** $(\nu_1)^{(8)}, (\nu_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, \nu^{(8)}, u^{(8)}$ : 372

By $(\nu_1)^{(8)} > 0$ , $(\nu_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0$ , $(u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(8)},, (\bar{\nu}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$ :

By $(\bar{\nu}_1)^{(8)} > 0$ , $(\bar{\nu}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0$ , $(\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

**Definition of** $(m_1)^{(8)}$ , $(m_2)^{(8)}$ , $(\mu_1)^{(8)}, (\mu_2)^{(8)}, (\nu_0)^{(8)}$ :-

If we define $(m_1)^{(8)}$ , $(m_2)^{(8)}$ , $(\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$(m_2)^{(8)} = (\nu_0)^{(8)}, (m_1)^{(8)} = (\nu_1)^{(8)}$, ***if*** $(\nu_0)^{(8)} < (\nu_1)^{(8)}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ , ***if*** $(\nu_1)^{(8)} < (\nu_0)^{(8)} < (\bar{\nu}_1)^{(8)}$,
and $\boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\nu_0)^{(8)}$, ***if*** $(\bar{\nu}_1)^{(8)} < (\nu_0)^{(8)}$

and analogously 374

$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)}$, ***if*** $(u_0)^{(8)} < (u_1)^{(8)}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}$ , ***if*** $(u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)}$,
and $\boxed{(u_0)^{(8)} = \frac{T^0_{40}}{T^0_{41}}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}$, ***if*** $(\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G^0_{40}e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{40}(t) \le G^0_{40}e^{(S_1)^{(8)}t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}} G_{40}^0 e^{((S_1)^{(8)}-(p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}} G_{40}^0 e^{(S_1)^{(8)}t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)}G_{40}^0}{(m_1)^{(8)}((S_1)^{(8)}-(p_{40})^{(8)}-(S_2)^{(8)})}\left[e^{((S_1)^{(8)}-(p_{40})^{(8)})t} - e^{-(S_2)^{(8)}t}\right] + G_{42}^0 e^{-(S_2)^{(8)}t} \le G_{42}(t) \le \right. \quad 377$$

$$\left. \frac{(a_{42})^{(8)}G_{40}^0}{(m_2)^{(8)}((S_1)^{(8)}-(a'_{42})^{(8)})}[e^{(S_1)^{(8)}t} - e^{-(a'_{42})^{(8)}t}] + G_{42}^0 e^{-(a'_{42})^{(8)}t}\right)$$

$$\boxed{T_{40}^0 e^{(R_1)^{(8)}t} \le T_{40}(t) \le T_{40}^0 e^{((R_1)^{(8)}+(r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}} T_{40}^0 e^{(R_1)^{(8)}t} \le T_{40}(t) \le \frac{1}{(\mu_2)^{(8)}} T_{40}^0 e^{((R_1)^{(8)}+(r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)}T_{40}^0}{(\mu_1)^{(8)}((R_1)^{(8)}-(b'_{42})^{(8)})}\left[e^{(R_1)^{(8)}t} - e^{-(b'_{42})^{(8)}t}\right] + T_{42}^0 e^{-(b'_{42})^{(8)}t} \le T_{42}(t) \le \quad 380$$

$$\frac{(a_{42})^{(8)}T_{40}^0}{(\mu_2)^{(8)}((R_1)^{(8)}+(r_{40})^{(8)}+(R_2)^{(8)})}\left[e^{((R_1)^{(8)}+(r_{40})^{(8)})t} - e^{-(R_2)^{(8)}t}\right] + T_{42}^0 e^{-(R_2)^{(8)}t}$$

**Definition of** $(S_1)^{(8)}, (S_2)^{(8)}, (R_1)^{(8)}, (R_2)^{(8)}$:- 381

Where $(S_1)^{(8)} = (a_{40})^{(8)}(m_2)^{(8)} - (a'_{40})^{(8)}$

$(S_2)^{(8)} = (a_{42})^{(8)} - (p_{42})^{(8)}$

$(R_1)^{(8)} = (b_{40})^{(8)}(\mu_2)^{(8)} - (b'_{40})^{(8)}$

$(R_2)^{(8)} = (b'_{42})^{(8)} - (r_{42})^{(8)}$

**Behavior of the solutions of equation 37 to 92** 382

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ :

$(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ four constants satisfying

$$-(\sigma_2)^{(9)} \le -(a'_{44})^{(9)} + (a'_{45})^{(9)} - (a''_{44})^{(9)}(T_{45}, t) + (a''_{45})^{(9)}(T_{45}, t) \le -(\sigma_1)^{(9)}$$

$$-(\tau_2)^{(9)} \le -(b'_{44})^{(9)} + (b'_{45})^{(9)} - (b''_{44})^{(9)}\big((G_{47}), t\big) - (b''_{45})^{(9)}\big((G_{47}), t\big) \le -(\tau_1)^{(9)}$$

**Definition of** $(\nu_1)^{(9)}, (\nu_2)^{(9)}, (u_1)^{(9)}, (u_2)^{(9)}, \nu^{(9)}, u^{(9)}$ :

By $(\nu_1)^{(9)} > 0$ , $(\nu_2)^{(9)} < 0$ and respectively $(u_1)^{(9)} > 0$ , $(u_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_1)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(9)}, , (\bar{\nu}_2)^{(9)}, (\bar{u}_1)^{(9)}, (\bar{u}_2)^{(9)}$ :

By $(\bar{\nu}_1)^{(9)} > 0$ , $(\bar{\nu}_2)^{(9)} < 0$ and respectively $(\bar{u}_1)^{(9)} > 0$ , $(\bar{u}_2)^{(9)} < 0$ the

roots of the equations $(a_{45})^{(9)}\left(v^{(9)}\right)^2+(\sigma_2)^{(9)}v^{(9)}-(a_{44})^{(9)}=0$
and $(b_{45})^{(9)}\left(u^{(9)}\right)^2+(\tau_2)^{(9)}u^{(9)}-(b_{44})^{(9)}=0$

**Definition of** $(m_1)^{(9)},(m_2)^{(9)},(\mu_1)^{(9)},(\mu_2)^{(9)},(v_0)^{(9)}$ :-

If we define $(m_1)^{(9)},(m_2)^{(9)},(\mu_1)^{(9)},(\mu_2)^{(9)}$ by

$(m_2)^{(9)}=(v_0)^{(9)},(m_1)^{(9)}=(v_1)^{(9)},\ \boldsymbol{if}\ (v_0)^{(9)}<(v_1)^{(9)}$

$(m_2)^{(9)}=(v_1)^{(9)},(m_1)^{(9)}=(\bar{v}_1)^{(9)},\boldsymbol{if}\ (v_1)^{(9)}<(v_0)^{(9)}<(\bar{v}_1)^{(9)},$
and $\boxed{(v_0)^{(9)}=\frac{G_{44}^0}{G_{45}^0}}$

$(m_2)^{(9)}=(v_1)^{(9)},(m_1)^{(9)}=(v_0)^{(9)},\ \boldsymbol{if}\ (\bar{v}_1)^{(9)}<(v_0)^{(9)}$

and analogously

$(\mu_2)^{(9)}=(u_0)^{(9)},(\mu_1)^{(9)}=(u_1)^{(9)},\ \boldsymbol{if}\ (u_0)^{(9)}<(u_1)^{(9)}$

$(\mu_2)^{(9)}=(u_1)^{(9)},(\mu_1)^{(9)}=(\bar{u}_1)^{(9)},\boldsymbol{if}\ (u_1)^{(9)}<(u_0)^{(9)}<(\bar{u}_1)^{(9)},$
and $\boxed{(u_0)^{(9)}=\frac{T_{44}^0}{T_{45}^0}}$

$(\mu_2)^{(9)}=(u_1)^{(9)},(\mu_1)^{(9)}=(u_0)^{(9)},\boldsymbol{if}\ (\bar{u}_1)^{(9)}<(u_0)^{(9)}$ where $(u_1)^{(9)},(\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G_{44}^0e^{\left((S_1)^{(9)}-(p_{44})^{(9)}\right)t}\le G_{44}(t)\le G_{44}^0e^{(S_1)^{(9)}t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}}G_{44}^0e^{\left((S_1)^{(9)}-(p_{44})^{(9)}\right)t}\le G_{45}(t)\le\frac{1}{(m_2)^{(9)}}G_{44}^0e^{(S_1)^{(9)}t}$$

$$\left(\frac{(a_{46})^{(9)}G_{44}^0}{(m_1)^{(9)}\left((S_1)^{(9)}-(p_{44})^{(9)}-(S_2)^{(9)}\right)}\left[e^{\left((S_1)^{(9)}-(p_{44})^{(9)}\right)t}-e^{-(S_2)^{(9)}t}\right]+G_{46}^0e^{-(S_2)^{(9)}t}\le G_{46}(t)\le\right.$$
$$\left.\frac{(a_{46})^{(9)}G_{44}^0}{(m_2)^{(9)}\left((S_1)^{(9)}-(a'_{46})^{(9)}\right)}\left[e^{(S_1)^{(9)}t}-e^{-(a'_{46})^{(9)}t}\right]+\ G_{46}^0e^{-(a'_{46})^{(9)}t}\right)$$

$$\boxed{T_{44}^0e^{(R_1)^{(9)}t}\le T_{44}(t)\le T_{44}^0e^{\left((R_1)^{(9)}+(r_{44})^{(9)}\right)t}}$$

$$\frac{1}{(\mu_1)^{(9)}}T_{44}^0e^{(R_1)^{(9)}t}\le T_{44}(t)\le\frac{1}{(\mu_2)^{(9)}}T_{44}^0e^{\left((R_1)^{(9)}+(r_{44})^{(9)}\right)t}$$

$$\frac{(b_{46})^{(9)}T_{44}^0}{(\mu_1)^{(9)}\left((R_1)^{(9)}-(b'_{46})^{(9)}\right)}\left[e^{(R_1)^{(9)}t}-e^{-(b'_{46})^{(9)}t}\right]+T_{46}^0e^{-(b'_{46})^{(9)}t}\le T_{46}(t)\le$$

$$\frac{(a_{46})^{(9)}T_{44}^0}{(\mu_2)^{(9)}\left((R_1)^{(9)}+(r_{44})^{(9)}+(R_2)^{(9)}\right)}\left[e^{\left((R_1)^{(9)}+(r_{44})^{(9)}\right)t}-e^{-(R_2)^{(9)}t}\right]+T_{46}^0e^{-(R_2)^{(9)}t}$$

**Definition of** $(S_1)^{(9)},(S_2)^{(9)},(R_1)^{(9)},(R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$

**Proof :** From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right) - (a''_{14})^{(1)}(T_{14}, t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

**Definition of** $v^{(1)}$ :- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)}\right) \le \frac{dv^{(1)}}{dt} \le -\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)}\right)$$

From which one obtains

**Definition of** $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$ :-

For $0 < \boxed{(v_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}} < (v_1)^{(1)} < (\bar{v}_1)^{(1)}$

$$v^{(1)}(t) \ge \frac{(v_1)^{(1)} + (C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_0)^{(1)}\right)t\right]}}{1 + (C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(v_1)^{(1)} - (v_0)^{(1)}}{(v_0)^{(1)} - (v_2)^{(1)}}}$$

it follows $(v_0)^{(1)} \le v^{(1)}(t) \le (v_1)^{(1)}$

In the same manner , we get 384

$$v^{(1)}(t) \le \frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{v}_1)^{(1)} - (v_0)^{(1)}}{(v_0)^{(1)} - (\bar{v}_2)^{(1)}}}$$

From which we deduce $(v_0)^{(1)} \le v^{(1)}(t) \le (\bar{v}_1)^{(1)}$

If $0 < (v_1)^{(1)} < (v_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}} < (\bar{v}_1)^{(1)}$ we find like in the previous case, 385

$$(v_1)^{(1)} \le \frac{(v_1)^{(1)} + (C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_2)^{(1)}\right)t\right]}}{1 + (C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_2)^{(1)}\right)t\right]}} \le v^{(1)}(t) \le$$

$$\frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}} \le (\bar{v}_1)^{(1)}$$

If $0 < (\nu_1)^{(1)} \leq (\bar{\nu}_1)^{(1)} \leq \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$ , we obtain 386

$$(\nu_1)^{(1)} \leq \nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)} + (\bar{C})^{(1)}(\bar{\nu}_2)^{(1)} e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)} - (\bar{\nu}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)} e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)} - (\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\nu_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(1)}(t)$ :-

$$(m_2)^{(1)} \leq \nu^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{\nu^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(1)}(t)$ :-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{13}'')^{(1)} = (a_{14}'')^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)} G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b_{13}'')^{(1)} = (b_{14}'')^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$ if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)} T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

**Proof** : From global equations we obtain 387

$$\frac{d\nu^{(2)}}{dt} = (a_{16})^{(2)} - \left((a_{16}')^{(2)} - (a_{17}')^{(2)} + (a_{16}'')^{(2)}(T_{17}, t)\right) - (a_{17}'')^{(2)}(T_{17}, t)\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

**Definition of** $\nu^{(2)}$ :- $\boxed{\nu^{(2)} = \frac{G_{16}}{G_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \leq \frac{d\nu^{(2)}}{dt} \leq -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

**Definition of** $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$ :-

For $0 < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \geq \frac{(\nu_1)^{(2)}+(\mathrm{C})^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_0)^{(2)}\right)t\right]}}{1+(\mathrm{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\mathrm{C})^{(2)}=\frac{(\nu_1)^{(2)}-(\nu_0)^{(2)}}{(\nu_0)^{(2)}-(\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)}+(\bar{\mathrm{C}})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{\mathrm{C}})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{\mathrm{C}})^{(2)}=\frac{(\bar{\nu}_1)^{(2)}-(\nu_0)^{(2)}}{(\nu_0)^{(2)}-(\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{\mathrm{G}_{16}^0}{\mathrm{G}_{17}^0} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \leq \frac{(\nu_1)^{(2)}+(\mathrm{C})^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}}{1+(\mathrm{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}} \leq \nu^{(2)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(2)}+(\bar{\mathrm{C}})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{\mathrm{C}})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \leq (\bar{\nu}_1)^{(2)} \leq (\nu_0)^{(2)} = \frac{\mathrm{G}_{16}^0}{\mathrm{G}_{17}^0}$ , we obtain 394

$$(\nu_1)^{(2)} \leq \nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)}+(\bar{\mathrm{C}})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{\mathrm{C}})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(2)}(t)$ :- 395

$$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}, \quad \boxed{\nu^{(2)}(t)=\frac{G_{16}(t)}{G_{17}(t)}}$$

In a completely analogous way, we obtain 396

**Definition of** $u^{(2)}(t)$ :-

$$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}, \quad \boxed{u^{(2)}(t)=\frac{T_{16}(t)}{T_{17}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :** 397

If $(a_{16}'')^{(2)} = (a_{17}'')^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)}G_{17}(t)$

Analogously if $(b''_{16})^{(2)} = (b''_{17})^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)}T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

**Proof** : From global equations we obtain 398

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right) - (a''_{21})^{(3)}(T_{21}, t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

**Definition of** $\nu^{(3)}$ :- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right) \leq \frac{d\nu^{(3)}}{dt} \leq -\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right)$$

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From which one obtains

For $0 < (\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\nu_1)^{(3)} < (\bar{\nu}_1)^{(3)}$

$$\nu^{(3)}(t) \geq \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(\nu_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\nu_2)^{(3)}}}$$

it follows $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\nu_1)^{(3)}$

In the same manner , we get 401

$$\nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)} + (\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{\nu}_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\bar{\nu}_2)^{(3)}}}$$

**Definition of** $(\bar{\nu}_1)^{(3)}$ :-

From which we deduce $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\bar{\nu}_1)^{(3)}$

If $0 < (\nu_1)^{(3)} < (\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\bar{\nu}_1)^{(3)}$ we find like in the previous case, 402

$$(\nu_1)^{(3)} \leq \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_2)^{(3)}\right)t\right]}}{1 + (C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_2)^{(3)}\right)t\right]}} \leq \nu^{(3)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(3)} + (\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\bar{\nu}_1)^{(3)}$$

If $0 < (\nu_1)^{(3)} \leq (\bar{\nu}_1)^{(3)} \leq (\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}$ , we obtain 403

$$(\nu_1)^{(3)} \leq \nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\nu_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(3)}(t)$ :-

$$(m_2)^{(3)} \leq \nu^{(3)}(t) \leq (m_1)^{(3)}, \quad \boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(3)}(t)$ :-

$$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}, \quad \boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{20})^{(3)} = (a''_{21})^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)}G_{21}(t)$

Analogously if $(b''_{20})^{(3)} = (b''_{21})^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)}T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

**Proof** : From global equations we obtain 404

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right) - (a''_{25})^{(4)}(T_{25},t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

**Definition of** $\nu^{(4)}$ :- $\quad \boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right) \leq \frac{d\nu^{(4)}}{dt} \leq -\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_4)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$ :-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \geq \frac{(\nu_1)^{(4)}+(C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_0)^{(4)}\right)t\right]}}{4+(C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)}-(\nu_0)^{(4)}}{(\nu_0)^{(4)}-(\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{4+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)}-(\nu_0)^{(4)}}{(\nu_0)^{(4)}-(\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \leq \frac{(\nu_1)^{(4)}+(C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_2)^{(4)}\right)t\right]}}{1+(C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_2)^{(4)}\right)t\right]}} \leq \nu^{(4)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \leq (\bar{\nu}_1)^{(4)} \leq \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$ , we obtain 407

$$(\nu_1)^{(4)} \leq \nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(4)}(t)$ :-

$$(m_2)^{(4)} \leq \nu^{(4)}(t) \leq (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(4)}(t)$ :-

$$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a_{24}'')^{(4)} = (a_{25}'')^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)}G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b_{24}'')^{(4)} = (b_{25}'')^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)}T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

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**Proof** : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a_{28}')^{(5)} - (a_{29}')^{(5)} + (a_{28}'')^{(5)}(T_{29}, t)\right) - (a_{29}'')^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

**Definition of** $\nu^{(5)}$ :- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$ :-

For $0 < \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \leq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}}{1 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}} \leq \nu^{(5)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \leq (\bar{\nu}_1)^{(5)} \leq \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$ , we obtain 411

$$(\nu_1)^{(5)} \leq \nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(5)}(t)$ :-

$(m_2)^{(5)} \leq \nu^{(5)}(t) \leq (m_1)^{(5)}$, $\boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$

In a completely analogous way, we obtain

**Definition of** $u^{(5)}(t)$ :-

$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}$, $\boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{28})^{(5)} = (a''_{29})^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)} G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b''_{28})^{(5)} = (b''_{29})^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$ if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)} T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

**Proof** : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a'_{32})^{(6)} - (a'_{33})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right) - (a''_{33})^{(6)}(T_{33}, t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

**Definition of** $\nu^{(6)}$ :- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \leq \frac{d\nu^{(6)}}{dt} \leq -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$ :-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \geq \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\nu_1)^{(6)}$

In the same manner , we get 413

$$\nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{\nu}_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\bar{\nu}_1)^{(6)}$

If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 414

$$(\nu_1)^{(6)} \leq \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}} \leq \nu^{(6)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\bar{\nu}_1)^{(6)}$$

If $0 < (\nu_1)^{(6)} \leq (\bar{\nu}_1)^{(6)} \leq \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$ , we obtain

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$$(\nu_1)^{(6)} \leq \nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(6)}(t)$ :-

$$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}, \quad \boxed{\nu^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(6)}(t)$ :-

$$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}, \quad \boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)}G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)}T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

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**Proof** : From global equations we obtain

$$\frac{d\nu^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right) - (a''_{37})^{(7)}(T_{37}, t)\nu^{(7)} - (a_{37})^{(7)}\nu^{(7)}$$

**Definition of** $\nu^{(7)}$ :- $\boxed{\nu^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right) \leq \frac{d\nu^{(7)}}{dt} \leq -\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(7)}, (\nu_0)^{(7)}$ :-

For $0 < \boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}} < (\nu_1)^{(7)} < (\bar{\nu}_1)^{(7)}$

$$\nu^{(7)}(t) \geq \frac{(\nu_1)^{(7)}+(C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_0)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)}=\frac{(\nu_1)^{(7)}-(\nu_0)^{(7)}}{(\nu_0)^{(7)}-(\nu_2)^{(7)}}}$$

it follows $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\nu_1)^{(7)}$

In the same manner , we get 417

$$\nu^{(7)}(t) \leq \frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)}=\frac{(\bar{\nu}_1)^{(7)}-(\nu_0)^{(7)}}{(\nu_0)^{(7)}-(\bar{\nu}_2)^{(7)}}}$$

From which we deduce $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\bar{\nu}_1)^{(7)}$

If $0 < (\nu_1)^{(7)} < (\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0} < (\bar{\nu}_1)^{(7)}$ we find like in the previous case, 418

$$(\nu_1)^{(7)} \leq \frac{(\nu_1)^{(7)}+(C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}} \leq \nu^{(7)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \leq (\bar{\nu}_1)^{(7)}$$

If $0 < (\nu_1)^{(7)} \leq (\bar{\nu}_1)^{(7)} \leq \boxed{(\nu_0)^{(7)}=\frac{G_{36}^0}{G_{37}^0}}$ , we obtain 419

$$(\nu_1)^{(7)} \leq \nu^{(7)}(t) \leq \frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \leq (\nu_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(7)}(t)$ :-

$$(m_2)^{(7)} \leq \nu^{(7)}(t) \leq (m_1)^{(7)}, \quad \boxed{\nu^{(7)}(t)=\frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(7)}(t)$ :- 420

$$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}, \quad \boxed{u^{(7)}(t)=\frac{T_{36}(t)}{T_{37}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $\nu^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)}G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for**

**the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)}T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

**Proof** : From global equations we obtain 421

$$\frac{d\nu^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right) - (a''_{41})^{(8)}(T_{41},t)\nu^{(8)} - (a_{41})^{(8)}\nu^{(8)}$$

**Definition of** $\nu^{(8)}$ :- $\boxed{\nu^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right) \le \frac{d\nu^{(8)}}{dt} \le -\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$ :-

For $0 < \boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$

$$\nu^{(8)}(t) \ge \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \le \nu^{(8)}(t) \le (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \le \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \le \nu^{(8)}(t) \le (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \le \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}} \le \nu^{(8)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \le (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \le (\bar{\nu}_1)^{(8)} \le \boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}}$ , we obtain 424

$$(v_1)^{(8)} \leq v^{(8)}(t) \leq \frac{(\bar{v}_1)^{(8)}+(\bar{C})^{(8)}(\bar{v}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{v}_1)^{(8)}-(\bar{v}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{v}_1)^{(8)}-(\bar{v}_2)^{(8)}\right)t\right]}} \leq (v_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $v^{(8)}(t)$ :-

$$(m_2)^{(8)} \leq v^{(8)}(t) \leq (m_1)^{(8)}, \quad \boxed{v^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(8)}(t)$ :-

$$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}, \quad \boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(v_1)^{(8)} = (\bar{v}_1)^{(8)}$ if in addition $(v_0)^{(8)} = (v_1)^{(8)}$ then $v^{(8)}(t) = (v_0)^{(8)}$ and as a consequence $G_{40}(t) = (v_0)^{(8)}G_{41}(t)$ **this also defines** $(v_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then
$(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)}T_{41}(t)$ This is an important consequence of the relation between $(v_1)^{(8)}$ and $(\bar{v}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

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**Proof** : From 99,20,44,22,23,44 we obtain

$$\frac{dv^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right) - (a''_{45})^{(9)}(T_{45},t)v^{(9)} - (a_{45})^{(9)}v^{(9)}$$

**Definition of** $v^{(9)}$ :- $\quad \boxed{v^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)}\right) \leq \frac{dv^{(9)}}{dt} \leq -\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_1)^{(9)}v^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

**Definition of** $(\bar{v}_1)^{(9)}, (v_0)^{(9)}$ :-

For $0 < \boxed{(v_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (v_1)^{(9)} < (\bar{v}_1)^{(9)}$

$$v^{(9)}(t) \geq \frac{(v_1)^{(9)}+(C)^{(9)}(v_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((v_1)^{(9)}-(v_0)^{(9)}\right)t\right]}}{1+(C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((v_1)^{(9)}-(v_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(v_1)^{(9)}-(v_0)^{(9)}}{(v_0)^{(9)}-(v_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \le \nu^{(9)}(t) \le (\nu_9)^{(9)}$

In the same manner , we get

$$\nu^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)}-(\nu_0)^{(9)}}{(\nu_0)^{(9)}-(\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \le \nu^{(9)}(t) \le (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \le \frac{(\nu_1)^{(9)}+(C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}}{1+(C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}} \le \nu^{(9)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \le (\bar{\nu}_1)^{(9)} \le \boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$ , we obtain

$$(\nu_1)^{(9)} \le \nu^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(9)}(t)$ :-

$$(m_2)^{(9)} \le \nu^{(9)}(t) \le (m_1)^{(9)}, \quad \boxed{\nu^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(9)}(t)$ :-

$$(\mu_2)^{(9)} \le u^{(9)}(t) \le (\mu_1)^{(9)}, \quad \boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)}G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then $(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)}T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following

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**Theorem :** If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on $t$ , and the conditions with the notations

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0$ ,

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$

$with\ \ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(2)} and\ (b_i'')^{(2)}$ are independent on t , and the conditions with the notations 426

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$ 427

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 428

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$ , 429

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 430

$with\ \ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(3)} and\ (b_i'')^{(3)}$ are independent on $t$ , and the conditions with the notations 431

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$ ,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

**Theorem :** If $(a_i'')^{(4)} and\ (b_i'')^{(4)}$ are independent on $t$ , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$ ,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(5)} and\ (b_i'')^{(5)}$ are independent on $t$ , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$ ,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

**Theorem** If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on $t$ , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$ ,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(7)} and\ (b_i'')^{(7)}$ are independent on $t$ , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$ ,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(8)} and\ (b_i'')^{(8)}$ are independent on $t$ , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$ ,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(9)} and\ (b_i'')^{(9)}$ are independent on $t$ , and the conditions (with the notations 436 A
45,46,27,28)

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$ ,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0 \quad 437$$

$$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0 \quad 438$$

$$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0 \quad 439$$

$$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0 \quad 440$$

$$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ]T_{14} = 0 \quad 441$$

$$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0 \quad 442$$

has a unique positive solution , which is an equilibrium solution for the system

$$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0 \quad 443$$

$$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0 \quad 444$$

$$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0 \quad 445$$

$$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\ ]T_{16} = 0 \quad 446$$

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\ ]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\ ]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\ ]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\ ]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\ ]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27})\right)]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27})\right)]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27})\right)]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0 \quad 479$$

$$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0 \quad 480$$

$$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0 \quad 481$$

$$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0 \quad 482$$

$$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0 \quad 483$$

$$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0 \quad 484$$

$$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0 \quad 484\text{A}$$

$$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$$

$$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$$

$$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$$

$$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$$

$$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$$

**Proof:** 485

(a) Indeed the first two equations have a nontrivial solution $G_{13}, G_{14}$ if

$$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$$

**Proof:** 486

(f) Indeed the first two equations have a nontrivial solution $G_{16}, G_{17}$ if

$$F(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$$

**Proof:** 487

(a) Indeed the first two equations have a nontrivial solution $G_{20}, G_{21}$ if

$$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$$

**Proof:** 488

(a) Indeed the first two equations have a nontrivial solution $G_{24}, G_{25}$ if

$$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$$

**Proof:** 489

(a) Indeed the first two equations have a nontrivial solution $G_{28}, G_{29}$ if

$$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$$

**Proof:** 490

(a) Indeed the first two equations have a nontrivial solution $G_{32}, G_{33}$ if

$$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$$

**Proof:** 491

(a) Indeed the first two equations have a nontrivial solution $G_{36}, G_{37}$ if

$$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$$

**Proof:** 492

(a) Indeed the first two equations have a nontrivial solution $G_{40}, G_{41}$ if

$$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$$

**Proof:** 492 A

(a) Indeed the first two equations have a nontrivial solution $G_{44}, G_{45}$ if

$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$

**Definition and uniqueness of** $T^*_{14}$ :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique $T^*_{14}$ for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

**Definition and uniqueness of** $T^*_{17}$ :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique $T^*_{17}$ for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]} \qquad 495$$

**Definition and uniqueness of** $T^*_{21}$ :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique $T^*_{21}$ for which $f(T^*_{21}) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

**Definition and uniqueness of** $T^*_{25}$ :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that there exists a unique $T^*_{25}$ for which $f(T^*_{25}) = 0$. With this value , we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T^*_{25})]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T^*_{25})]}$$

**Definition and uniqueness of** $T^*_{29}$ :- 498

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(5)}(T_{29})$ being increasing, it follows that there exists a unique $T^*_{29}$ for which $f(T^*_{29}) = 0$. With this value , we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T^*_{29})]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T^*_{29})]}$$

**Definition and uniqueness of** $T^*_{33}$ :- 499

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(6)}(T_{33})$ being increasing, it follows that

there exists a unique $T_{33}^{*}$ for which $f(T_{33}^{*}) = 0$. With this value , we obtain from the three first equations

$$G_{32} = \frac{(a_{32})^{(6)} G_{33}}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^{*})]} \quad , \quad G_{34} = \frac{(a_{34})^{(6)} G_{33}}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^{*})]}$$

**Definition and uniqueness of** $T_{37}^{*}$ :- 500

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(7)}(T_{37})$ being increasing, it follows that there exists a unique $T_{37}^{*}$ for which $f(T_{37}^{*}) = 0$. With this value , we obtain from the three first equations

$$G_{36} = \frac{(a_{36})^{(7)} G_{37}}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^{*})]} \quad , \quad G_{38} = \frac{(a_{38})^{(7)} G_{37}}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^{*})]}$$

**Definition and uniqueness of** $T_{41}^{*}$ :- 501

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique $T_{41}^{*}$ for which $f(T_{41}^{*}) = 0$. With this value , we obtain from the three first equations

$$G_{40} = \frac{(a_{40})^{(8)} G_{41}}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^{*})]} \quad , \quad G_{42} = \frac{(a_{42})^{(8)} G_{41}}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^{*})]}$$

**Definition and uniqueness of** $T_{45}^{*}$ :- 501 A

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(9)}(T_{45})$ being increasing, it follows that there exists a unique $T_{45}^{*}$ for which $f(T_{45}^{*}) = 0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)} G_{45}}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^{*})]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)} G_{45}}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^{*})]}$$

By the same argument, the equations admit solutions $G_{13}, G_{14}$ if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right] + (b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{14}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{14}^{*}$ such that $\varphi(G^{*}) = 0$

By the same argument, the equations admit solutions $G_{16}, G_{17}$ if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right] + (b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{17}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{14}^{*}$ such that $\varphi((G_{19})^{*}) = 0$ 504

By the same argument, the equations admit solutions $G_{20}, G_{21}$ if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right] + (b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{21}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{21}^{*}$ such that $\varphi((G_{23})^{*}) = 0$

By the same argument, the equations admit solutions $G_{24}, G_{25}$ if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right] + (b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{25}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{25}^{*}$ such that $\varphi((G_{27})^{*}) = 0$

By the same argument, the equations admit solutions $G_{28}, G_{29}$ if 507

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right] + (b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{29}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{29}^{*}$ such that $\varphi((G_{31})^{*}) = 0$

By the same argument, the equations admit solutions $G_{32}, G_{33}$ if 508

$$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$$

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right] + (b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{33}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{33}^{*}$ such that $\varphi({G_{35}}^{*}) = 0$

By the same argument, the equations admit solutions $G_{36}, G_{37}$ if 509

$$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$$

$$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right] + (b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{37}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{37}^{*}$ such that $\varphi({G_{39}}^{*}) = 0$

By the same argument, the equations admit solutions $G_{40}, G_{41}$ if 510

$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$
$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right] + (b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{41}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{41}$ such that $\varphi(G_{43}{}^*) = 0$

By the same argument, the equations 92,93 admit solutions $G_{44}, G_{45}$ if
$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$

$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right] + (b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{45}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{45}$ such that $\varphi((G_{47})^*) = 0$

Finally we obtain the unique solution 511

$G^*_{14}$ given by $\varphi(G^*) = 0$ , $T^*_{14}$ given by $f(T^*_{14}) = 0$ and

$$G^*_{13} = \frac{(a_{13})^{(1)}G^*_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G^*_{15} = \frac{(a_{15})^{(1)}G^*_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

$$T^*_{13} = \frac{(b_{13})^{(1)}T^*_{14}}{[(b'_{13})^{(1)}-(b''_{13})^{(1)}(G^*)]} \quad , \quad T^*_{15} = \frac{(b_{15})^{(1)}T^*_{14}}{[(b'_{15})^{(1)}-(b''_{15})^{(1)}(G^*)]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

$G^*_{17}$ given by $\varphi((G_{19})^*) = 0$ , $T^*_{17}$ given by $f(T^*_{17}) = 0$ and 512

$$G^*_{16} = \frac{(a_{16})^{(2)}G^*_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G^*_{18} = \frac{(a_{18})^{(2)}G^*_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]} \quad 513$$

$$T^*_{16} = \frac{(b_{16})^{(2)}T^*_{17}}{[(b'_{16})^{(2)}-(b''_{16})^{(2)}((G_{19})^*)]} \quad , \quad T^*_{18} = \frac{(b_{18})^{(2)}T^*_{17}}{[(b'_{18})^{(2)}-(b''_{18})^{(2)}((G_{19})^*)]} \quad 514$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

$G^*_{21}$ given by $\varphi((G_{23})^*) = 0$ , $T^*_{21}$ given by $f(T^*_{21}) = 0$ and

$$G^*_{20} = \frac{(a_{20})^{(3)}G^*_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G^*_{22} = \frac{(a_{22})^{(3)}G^*_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

$$T^*_{20} = \frac{(b_{20})^{(3)}T^*_{21}}{[(b'_{20})^{(3)}-(b''_{20})^{(3)}(G_{23}{}^*)]} \quad , \quad T^*_{22} = \frac{(b_{22})^{(3)}T^*_{21}}{[(b'_{22})^{(3)}-(b''_{22})^{(3)}(G_{23}{}^*)]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

$G_{25}^{*}$ given by $\varphi(G_{27}) = 0$ , $T_{25}^{*}$ given by $f(T_{25}^{*}) = 0$ and

$$G_{24}^{*} = \frac{(a_{24})^{(4)} G_{25}^{*}}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}^{*} = \frac{(a_{26})^{(4)} G_{25}^{*}}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^{*})]}$$

$$T_{24}^{*} = \frac{(b_{24})^{(4)} T_{25}^{*}}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^{*})]} \quad , \quad T_{26}^{*} = \frac{(b_{26})^{(4)} T_{25}^{*}}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^{*})]} \qquad 517$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

$G_{29}^{*}$ given by $\varphi((G_{31})^{*}) = 0$ , $T_{29}^{*}$ given by $f(T_{29}^{*}) = 0$ and

$$G_{28}^{*} = \frac{(a_{28})^{(5)} G_{29}^{*}}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}^{*} = \frac{(a_{30})^{(5)} G_{29}^{*}}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^{*})]}$$

$$T_{28}^{*} = \frac{(b_{28})^{(5)} T_{29}^{*}}{[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31})^{*})]} \quad , \quad T_{30}^{*} = \frac{(b_{30})^{(5)} T_{29}^{*}}{[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31})^{*})]} \qquad 519$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

$G_{33}^{*}$ given by $\varphi((G_{35})^{*}) = 0$ , $T_{33}^{*}$ given by $f(T_{33}^{*}) = 0$ and

$$G_{32}^{*} = \frac{(a_{32})^{(6)} G_{33}^{*}}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^{*})]} \quad , \quad G_{34}^{*} = \frac{(a_{34})^{(6)} G_{33}^{*}}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^{*})]}$$

$$T_{32}^{*} = \frac{(b_{32})^{(6)} T_{33}^{*}}{[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35})^{*})]} \quad , \quad T_{34}^{*} = \frac{(b_{34})^{(6)} T_{33}^{*}}{[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35})^{*})]} \qquad 521$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

$G_{37}^{*}$ given by $\varphi((G_{39})^{*}) = 0$ , $T_{37}^{*}$ given by $f(T_{37}^{*}) = 0$ and

$$G_{36}^{*} = \frac{(a_{36})^{(7)} G_{37}^{*}}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^{*})]} \quad , \quad G_{38}^{*} = \frac{(a_{38})^{(7)} G_{37}^{*}}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^{*})]}$$

$$T_{36}^{*} = \frac{(b_{36})^{(7)} T_{37}^{*}}{[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39})^{*})]} \quad , \quad T_{38}^{*} = \frac{(b_{38})^{(7)} T_{37}^{*}}{[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39})^{*})]}$$

Finally we obtain the unique solution 523

$G_{41}^{*}$ given by $\varphi((G_{43})^{*}) = 0$ , $T_{41}^{*}$ given by $f(T_{41}^{*}) = 0$ and

$$G_{40}^{*} = \frac{(a_{40})^{(8)} G_{41}^{*}}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^{*})]} \quad , \quad G_{42}^{*} = \frac{(a_{42})^{(8)} G_{41}^{*}}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^{*})]}$$

$$T_{40}^{*}=\frac{(b_{40})^{(8)}T_{41}^{*}}{[(b'_{40})^{(8)}-(b''_{40})^{(8)}((G_{43})^{*})]} \quad , \quad T_{42}^{*}=\frac{(b_{42})^{(8)}T_{41}^{*}}{[(b'_{42})^{(8)}-(b''_{42})^{(8)}((G_{43})^{*})]}$$

Finally we obtain the unique solution of 89 to 99 523

A

$G_{45}^{*}$ given by $\varphi((G_{47})^{*})=0$ , $T_{45}^{*}$ given by $f(T_{45}^{*})=0$ and

$$G_{44}^{*}=\frac{(a_{44})^{(9)}G_{45}^{*}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T_{45}^{*})]} \quad , \quad G_{46}^{*}=\frac{(a_{46})^{(9)}G_{45}^{*}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T_{45}^{*})]}$$

$$T_{44}^{*}=\frac{(b_{44})^{(9)}T_{45}^{*}}{[(b'_{44})^{(9)}-(b''_{44})^{(9)}((G_{47})^{*})]} \quad , \quad T_{46}^{*}=\frac{(b_{46})^{(9)}T_{45}^{*}}{[(b'_{46})^{(9)}-(b''_{46})^{(9)}((G_{47})^{*})]}$$

**ASYMPTOTIC STABILITY ANALYSIS** 524

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}\ and\ (b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

**Proof:** Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i=G_i^{*}+\mathbb{G}_i \qquad , T_i=T_i^{*}+\mathbb{T}_i$$

$$\frac{\partial(a''_{14})^{(1)}}{\partial T_{14}}(T_{14}^{*})=(q_{14})^{(1)} \quad , \frac{\partial(b_i'')^{(1)}}{\partial G_j}(G^{*})=s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt}=-\left((a'_{13})^{(1)}+(p_{13})^{(1)}\right)\mathbb{G}_{13}+(a_{13})^{(1)}\mathbb{G}_{14}-(q_{13})^{(1)}G_{13}^{*}\mathbb{T}_{14} \quad 525$$

$$\frac{d\mathbb{G}_{14}}{dt}=-\left((a'_{14})^{(1)}+(p_{14})^{(1)}\right)\mathbb{G}_{14}+(a_{14})^{(1)}\mathbb{G}_{13}-(q_{14})^{(1)}G_{14}^{*}\mathbb{T}_{14} \quad 526$$

$$\frac{d\mathbb{G}_{15}}{dt}=-\left((a'_{15})^{(1)}+(p_{15})^{(1)}\right)\mathbb{G}_{15}+(a_{15})^{(1)}\mathbb{G}_{14}-(q_{15})^{(1)}G_{15}^{*}\mathbb{T}_{14} \quad 527$$

$$\frac{d\mathbb{T}_{13}}{dt}=-\left((b'_{13})^{(1)}-(r_{13})^{(1)}\right)\mathbb{T}_{13}+(b_{13})^{(1)}\mathbb{T}_{14}+\sum_{j=13}^{15}\left(s_{(13)(j)}T_{13}^{*}\mathbb{G}_j\right) \quad 528$$

$$\frac{d\mathbb{T}_{14}}{dt}=-\left((b'_{14})^{(1)}-(r_{14})^{(1)}\right)\mathbb{T}_{14}+(b_{14})^{(1)}\mathbb{T}_{13}+\sum_{j=13}^{15}\left(s_{(14)(j)}T_{14}^{*}\mathbb{G}_j\right) \quad 529$$

$$\frac{d\mathbb{T}_{15}}{dt}=-\left((b'_{15})^{(1)}-(r_{15})^{(1)}\right)\mathbb{T}_{15}+(b_{15})^{(1)}\mathbb{T}_{14}+\sum_{j=13}^{15}\left(s_{(15)(j)}T_{15}^{*}\mathbb{G}_j\right) \quad 530$$

**ASYMPTOTIC STABILITY ANALYSIS** 531

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(2)}$ and $(b_i'')^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i \tag{532}$$

$$\frac{\partial(a_{17}'')^{(2)}}{\partial T_{17}}(T_{17}^*) = (q_{17})^{(2)} \quad , \frac{\partial(b_i'')^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \tag{533}$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a_{16}')^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G_{16}^*\mathbb{T}_{17} \tag{534}$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a_{17}')^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G_{17}^*\mathbb{T}_{17} \tag{535}$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a_{18}')^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G_{18}^*\mathbb{T}_{17} \tag{536}$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b_{16}')^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T_{16}^*\mathbb{G}_j\right) \tag{537}$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b_{17}')^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}T_{17}^*\mathbb{G}_j\right) \tag{538}$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b_{18}')^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}T_{18}^*\mathbb{G}_j\right) \tag{539}$$

**ASYMPTOTIC STABILITY ANALYSIS** 540

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(3)}$ *and* $(b_i'')^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{21}'')^{(3)}}{\partial T_{21}}(T_{21}^*) = (q_{21})^{(3)} \quad , \frac{\partial(b_i'')^{(3)}}{\partial G_j}((G_{23})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a_{20}')^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G_{20}^*\mathbb{T}_{21} \tag{541}$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a_{21}')^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G_{21}^*\mathbb{T}_{21} \tag{542}$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a_{22}')^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G_{22}^*\mathbb{T}_{21} \tag{543}$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b_{20}')^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T_{20}^*\mathbb{G}_j\right) \tag{544}$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b_{21}')^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T_{21}^*\mathbb{G}_j\right) \tag{545}$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b'_{22})^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T^*_{22}\mathbb{G}_j\right) \quad 546$$

**ASYMPTOTIC STABILITY ANALYSIS** 547

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}\ and\ (b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{25})^{(4)}}{\partial T_{25}}(T^*_{25}) = (q_{25})^{(4)} \ , \ \frac{\partial(b_i'')^{(4)}}{\partial G_j}((G_{27})^*\ ) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\left((a'_{24})^{(4)} + (p_{24})^{(4)}\right)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G^*_{24}\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\left((a'_{25})^{(4)} + (p_{25})^{(4)}\right)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G^*_{25}\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\left((a'_{26})^{(4)} + (p_{26})^{(4)}\right)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G^*_{26}\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\left((b'_{24})^{(4)} - (r_{24})^{(4)}\right)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(24)(j)}T^*_{24}\mathbb{G}_j\right) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\left((b'_{25})^{(4)} - (r_{25})^{(4)}\right)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\left(s_{(25)(j)}T^*_{25}\mathbb{G}_j\right) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T^*_{26}\mathbb{G}_j\right) \quad 554$$

**ASYMPTOTIC STABILITY ANALYSIS** 555

**Theorem 5:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}\ and\ (b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 556

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{29})^{(5)}}{\partial T_{29}}(T^*_{29}) = (q_{29})^{(5)} \ , \ \frac{\partial(b_i'')^{(5)}}{\partial G_j}(\ (G_{31})^*\ ) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a'_{28})^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G^*_{28}\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a'_{29})^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G^*_{29}\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a'_{30})^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G^*_{30}\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b'_{28})^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T^*_{28}\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b'_{29})^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T^*_{29}\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b'_{30})^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T^*_{30}\mathbb{G}_j\right) \quad 562$$

**ASYMPTOTIC STABILITY ANALYSIS** 563

**Theorem 6:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}$ *and* $(b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{33})^{(6)}}{\partial T_{33}}(T^*_{33}) = (q_{33})^{(6)} \quad , \frac{\partial(b_i'')^{(6)}}{\partial G_j}\left((G_{35})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a'_{32})^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G^*_{32}\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a'_{33})^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G^*_{33}\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a'_{34})^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G^*_{34}\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b'_{32})^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T^*_{32}\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b'_{33})^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T^*_{33}\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b'_{34})^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T^*_{34}\mathbb{G}_j\right) \quad 570$$

**ASYMPTOTIC STABILITY ANALYSIS** 571

**Theorem 7:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 572

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{37}'')^{(7)}}{\partial T_{37}}(T_{37}^{*}) = (q_{37})^{(7)} \quad , \frac{\partial(b_i'')^{(7)}}{\partial G_j}\left( (G_{39})^{**} \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a_{36}')^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G_{36}^{*}\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a_{37}')^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G_{37}^{*}\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a_{38}')^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G_{38}^{*}\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b_{36}')^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T_{36}^{*}\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b_{37}')^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T_{37}^{*}\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b_{38}')^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T_{38}^{*}\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

## ASYMPTOTIC STABILITY ANALYSIS

**Theorem 8:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 580

$$G_i = G_i^{*} + \mathbb{G}_i \qquad , T_i = T_i^{*} + \mathbb{T}_i$$

$$\frac{\partial(a_{41}'')^{(8)}}{\partial T_{41}}(T_{41}^{*}) = (q_{41})^{(8)} \quad , \frac{\partial(b_i'')^{(8)}}{\partial G_j}\left( (G_{43})^{*} \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a_{40}')^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G_{40}^{*}\mathbb{T}_{41} \quad 581$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a_{41}')^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G_{41}^{*}\mathbb{T}_{41} \quad 582$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a_{42}')^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G_{42}^{*}\mathbb{T}_{41} \quad 583$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b_{40}')^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T_{40}^{*}\mathbb{G}_j\right) \quad 584$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b_{41}')^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T_{41}^{*}\mathbb{G}_j\right) \quad 585$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b'_{42})^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T^*_{42}\mathbb{G}_j\right)$$ 586

586

**ASYMPTOTIC STABILITY ANALYSIS** A

**Theorem 9:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}\ and\ (b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

**Proof:** Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{45})^{(9)}}{\partial T_{45}}(T^*_{45}) = (q_{45})^{(9)} \ , \ \frac{\partial(b_i'')^{(9)}}{\partial G_j}((G_{47})^*) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a'_{44})^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)}G^*_{44}\mathbb{T}_{45}$$ 586 B

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a'_{45})^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)}G^*_{45}\mathbb{T}_{45}$$ 586 C

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a'_{46})^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)}G^*_{46}\mathbb{T}_{45}$$ 586 D

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b'_{44})^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)}T^*_{44}\mathbb{G}_j\right)$$ 586 E

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b'_{45})^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)}T^*_{45}\mathbb{G}_j\right)$$ 586 F

$$\frac{d\mathbb{T}_{46}}{dt} = -\left((b'_{46})^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T^*_{46}\mathbb{G}_j\right)$$ 586 G

**The characteristic equation of this system is** 587

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(q_{13})^{(1)}G^*_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)}T^*_{14} + (b_{14})^{(1)}s_{(13),(14)}T^*_{14}\right)$$

$$+\left(\left((\lambda)^{(1)} + (a'_{14})^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)}G^*_{13} + (a_{13})^{(1)}(q_{14})^{(1)}G^*_{14}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)}T^*_{14} + (b_{14})^{(1)}s_{(13),(13)}T^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((b'_{13})^{(1)} + (b'_{14})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^2+\left((a'_{13})^{(1)}+(a'_{14})^{(1)}+(p_{13})^{(1)}+(p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)}+(a'_{13})^{(1)}+(p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^*_{14}+(a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}+(b'_{13})^{(1)}-(r_{13})^{(1)}\right)s_{(14),(15)}T^*_{14}+(b_{14})^{(1)}s_{(13),(15)}T^*_{13}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(2)}+(b'_{18})^{(2)}-(r_{18})^{(2)}\right)\{\left((\lambda)^{(2)}+(a'_{18})^{(2)}+(p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^*_{17}+(a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^*_{17}\right)$$

$$+\left(\left((\lambda)^{(2)}+(a'_{17})^{(2)}+(p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^*_{16}+(a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2+\left((b'_{16})^{(2)}+(b'_{17})^{(2)}-(r_{16})^{(2)}+(r_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$+\left(\left((\lambda)^{(2)}\right)^2+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}+(a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^*_{16}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\right)\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left(\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^*_{20}\right)\right]$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^*_{21}+(b_{21})^{(3)}s_{(20),(21)}T^*_{21}\right)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^*_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^*_{21}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^*_{21}+(b_{21})^{(3)}s_{(20),(20)}T^*_{20}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^2+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$+\left(\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^*_{20}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^*_{21}+(b_{21})^{(3)}s_{(20),(22)}T^*_{20}\right)\}=0$$

$$+$$

$$(\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left(\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^*_{24}\right)\right]$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^*_{25}+(b_{25})^{(4)}s_{(24),(25)}T^*_{25}\right)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^*_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^*_{25}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^*_{25}+(b_{25})^{(4)}s_{(24),(24)}T^*_{24}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^2+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$+\left(\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^*_{24}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^*_{25}+(b_{25})^{(4)}s_{(24),(26)}T^*_{24}\right)\}=0$$

$$+$$

$$(\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^*_{28}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^*_{29}+(b_{29})^{(5)}s_{(28),(29)}T^*_{29}\right)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^*_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^*_{29}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^*_{29}+(b_{29})^{(5)}s_{(28),(28)}T^*_{28}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^2+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$+\left(\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^*_{28}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^*_{29}+(b_{29})^{(5)}s_{(28),(30)}T^*_{28}\right)\}=0$$

$$+$$

$$(\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left(\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^*_{32}\right)\right]$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^*_{33}+(b_{33})^{(6)}s_{(32),(33)}T^*_{33}\right)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^*_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^*_{33}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^*_{33}+(b_{33})^{(6)}s_{(32),(32)}T^*_{32}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^2+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$+\left(\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^*_{32}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^*_{33}+(b_{33})^{(6)}s_{(32),(34)}T^*_{32}\right)\}=0$$

$$+$$

$$(\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^*_{36}\right)\right]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^*_{37}+(b_{37})^{(7)}s_{(36),(37)}T^*_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^*_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^*_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^*_{37}+(b_{37})^{(7)}s_{(36),(36)}T^*_{36}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^2+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$+\left(\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^*_{36}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^*_{37}+(b_{37})^{(7)}s_{(36),(38)}T^*_{36}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\right)\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^*_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^*_{41}+(b_{41})^{(8)}s_{(40),(41)}T^*_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^*_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^*_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^*_{41}+(b_{41})^{(8)}s_{(40),(40)}T^*_{40}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^2+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$+\left(\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^*_{40}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^*_{41}+(b_{41})^{(8)}s_{(40),(42)}T^*_{40}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\right)\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^*_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^*_{45}+(b_{45})^{(9)}s_{(44),(45)}T^*_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^*_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^*_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^*_{45}+(b_{45})^{(9)}s_{(44),(44)}T^*_{44}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^2 + \left((b'_{44})^{(9)} + (b'_{45})^{(9)} - (r_{44})^{(9)} + (r_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$+\left(\left((\lambda)^{(9)}\right)^2 + \left((a'_{44})^{(9)} + (a'_{45})^{(9)} + (p_{44})^{(9)} + (p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)} + (a'_{44})^{(9)} + (p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^*_{45} + (a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^*_{44}\right)$$

$$\left(\left((\lambda)^{(9)} + (b'_{44})^{(9)} - (r_{44})^{(9)}\right)s_{(45),(46)}T^*_{45} + (b_{45})^{(9)}s_{(44),(46)}T^*_{44}\right)\} = 0$$

**And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.**

## SECTION EIGHT

## Supercausality And Consciousness

### INTRODUCTION—VARIABLES USED

**Chris King: Supercausality and consciousness - 1989**.

(1) Chris King (1989) suggests a model based on (e) the transactional interpretation of quantum mechanics (Cramer 1986) which starts from the dual solution of (e) the relativistic wave equation (Klein-Gordon equation).

(2) King states that quantum entities are always faced with (e&eb) bifurcations between past and future causes which require (e) choices to be performed.

(3) King refers to the works of Eccles, Penrose and Hameroff works which show the relevance of quantum processes in living systems and suggests (eb) that according to the dual solution of the wave equation all living systems would be faced with (e&eb) bifurcations between causality and retrocausality.

(4) King names supercausality (=) the sum of causality and retrocausality. These bifurcations would force living systems to operate (e&eb) choices which, according to King, are (=) a common state to all living systems and to all the levels of (e) organization of biological matter, from microstructures to (e&eb) macrostructures.

(5) As a consequence of the fact that all biological systems would act according to (e) free will, and as a consequence of the fact that the outcome of free will is (=) indeterminable, biological systems should always show (eb) chaotic dynamics which cannot be studied in a deterministic or computational way.

(6) King suggests two levels of explanation of consciousness. On the first level information moves from the mind to (e&eb) the brain as a consequence of (e) the exercise of free will; on the second level, information flows from the brain to (e&eb) the mind thanks to the amplification of signals performed by (e) the chaotic dynamics of the brain, which usual take the form of fractals.

## NOTATION

### Module One

King states that **quantum entities are always faced with** (e&eb) bifurcations between past and future causes which require (e) choices to be performed

$G_{13}$ : Category one of **quantum entities;** bifurcations between past and future causes which require (e) choices to be performed

$G_{14}$ : Category two of SAS

$G_{15}$ : Category three of SAS

$T_{13}$ : Category one of bifurcations between past and future causes which require (e) choices to be performed **;quantum entities**

$T_{14}$ : Category two of SAS

$T_{15}$ : Category three of SAS

### Module Two

King states that quantum entities are always faced with bifurcations between past and future causes which require (e) **choices to be performed**

$G_{16}$ : Category one of quantum entities are always faced with bifurcations between past and future causes; **choices to be performed**

$G_{17}$ : Category two of SAS

$G_{18}$ : Category three of SAS

$T_{16}$ : Category one of **choices to be performed** ;quantum entities are always faced with bifurcations between past and future causes

$T_{17}$ : Category two of SAS

$T_{18}$ : Category three of SAS

### Module three

King refers to the works of Eccles, Penrose and Hameroff works which show the relevance of quantum processes in living systems and suggests (eb) that according to the dual solution of the wave equation all living systems would be faced with (e&eb) bifurcations between causality and retrocausality

$G_{20}$ : Category one of relevance of quantum processes in living systems

$G_{21}$ : Category two of SAS

$G_{22}$ : Category three of SAS

$T_{20}$ : Category one of according to the dual solution of the wave equation all living systems would be faced with (e&eb) bifurcations between causality and retrocausality

$T_{21}$ : Category two of SAS

$T_{22}$ : Category three of SAS

**Module four**

King refers to the works of Eccles, Penrose and Hameroff works which show the relevance of quantum processes in living systems and suggests that according to the dual solution of the wave equation all living systems would be faced with (e&eb) **bifurcations between causality and retrocausality**

$G_{24}$ : Category one of relevance of quantum processes in living systems and suggests that according to the dual solution of the wave equation all living systems; **bifurcations between causality and retrocausality**

$G_{25}$ : Category two of SAS

$G_{26}$ : Category three of SAS

$T_{24}$ : Category one of **bifurcations between causality and retrocausality;** relevance of quantum processes in living systems and suggests that according to the dual solution of the wave equation all living systems

$T_{25}$ : Category two of SAS

$T_{26}$ : Category three of SAS

**Module five**

**causality** and retrocausality

$G_{28}$ : Category one of **causality;** retrocausality

$G_{29}$ : Category two of SAS

$G_{30}$ : Category three of SAS

$T_{28}$ : Category one of retrocausality **; causality**

$T_{29}$ : Category two of SAS

$T_{30}$ : Category three of SAS

**Module six**

King names supercausality (=) the sum of causality and retrocausality.

These bifurcations would force living systems to operate (e&eb) choices which, according to King, are (=) a common state to all living systems and to all the levels of (e) organization of biological matter, from microstructures to (e&eb) macrostructures.

$G_{32}$ : Category one of supercausality

$G_{33}$ : Category two of SAS

$G_{34}$ : Category three of SAS

$T_{32}$ : Category one of sum of causality and retrocausality

$T_{33}$ : Category two of SAS

$T_{34}$ : Category three of SAS

**Module seven**

**Bifurcations between causality and retrocausality** would force living systems to operate (e&eb) choices which, according to King, are (=) a common state to all living systems and to all the levels of (e) organization of biological matter, from microstructures to (e&eb) macrostructures.

$G_{36}$ : Category one of **bifurcations between causality and retrocausality;** choices which, according to King, are (=) a common state to all living systems and to all the levels of (e) organization of biological matter, from microstructures to (e&eb) macrostructures

$G_{37}$ : Category two of SAS

$G_{38}$ : Category three of SAS

$T_{36}$ : Category one of choices which, according to King, are (=) a common state to all living systems and to all the levels of (e) organization of biological matter, from microstructures to (e&eb) macrostructures **;bifurcations between causality and retrocausality**

$T_{37}$ : Category two of SAS

$T_{38}$ : Category three of SAS

**Module eight**

Bifurcations between causality, retrocausality and supercausality would force living systems to operate (e&eb) choices which, according to King, are (=) a common state to all living systems and to all the levels of (e) organization of biological matter, from microstructures to (e&eb) macrostructures

$G_{40}$ : Category one of **Bifurcations between causality, retrocausality and supercausality would force living systems;** choices which, according to King, are (=) a common state to all living systems and to all the levels of (e) organization of biological matter, from microstructures to (e&eb) macrostructures

$G_{41}$ : Category two of SAS

$G_{42}$ : Category three of SAS

$T_{40}$ : Category one of choices which, according to King, are (=) a common state to all living systems and to all the levels of (e) organization of biological matter, from microstructures to (e&eb) macrostructures; **Bifurcations between causality, retrocausality and supercausality would force living systems**

$T_{41}$ : Category two of SAS

$T_{42}$ : Category three of SAS

**Module Nine**

Bifurcations between causality, retrocausality and supercausality would force living systems to operate choices which, according to King, are (=) a common state to all living systems and to all the levels of

organization of biological matter, from microstructures to macrostructures

$G_{44}$ : Category one of Bifurcations between causality, retrocausality and supercausality would force living systems to operate choices

$G_{45}$ : Category two of SAS

$G_{46}$ : Category three of SAS

$T_{44}$ : Category one of common state to all living systems and to all the levels of organization of biological matter, from microstructures to macrostructures

$T_{45}$ : Category two of SAS

$T_{46}$ : Category three of SAS

**The Coefficients:**

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}$ , $(b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}$,
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}$, $(b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}\ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$ ,
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)}$,
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)}$,
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)}$,
are Dissipation coefficients
**Module Numbered One**

**The differential system of this model is now (Module Numbered one)**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \quad 1$$
$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad 2$$
$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15} \quad 3$$
$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13} \quad 4$$
$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14} \quad 5$$
$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)\right]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14}, t) =$ First augmentation factor

$-(b''_{13})^{(1)}(G,t) =$ First detritions factor

**Module Numbered Two**

**The differential system of this model is now ( Module numbered two)**

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17},t)\right]G_{16} \quad 7$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17},t)\right]G_{17} \quad 8$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17},t)\right]G_{18} \quad 9$$

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\left((G_{19}),t\right)\right]T_{16} \quad 10$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\left((G_{19}),t\right)\right]T_{17} \quad 11$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\left((G_{19}),t\right)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17},t) =$ First augmentation factor

$-(b''_{16})^{(2)}\left((G_{19}),t\right) =$ First detritions factor

**Module Numbered Three**

**The differential system of this model is now (Module numbered three)**

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21},t)\right]G_{20} \quad 13$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21},t)\right]G_{21} \quad 14$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21},t)\right]G_{22} \quad 15$$

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23},t)\right]T_{20} \quad 16$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23},t)\right]T_{21} \quad 17$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23},t)\right]T_{22} \quad 18$$

$+(a''_{20})^{(3)}(T_{21},t) =$ First augmentation factor

$-(b''_{20})^{(3)}(G_{23},t) =$ First detritions factor

**Module Numbered Four**

**The differential system of this model is now (Module numbered Four)**

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right]G_{24} \quad 19$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25},t)\right]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25},t)\right]G_{26} \quad 21$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27}),t\right)\right]T_{24} \quad 22$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27}),t\right)\right]T_{25} \quad 23$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27}),t\right)\right]T_{26} \quad 24$$

$+(a''_{24})^{(4)}(T_{25},t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}\left((G_{27}),t\right) =$ **First detritions factor**

**Module Numbered Five**:

**The differential system of this model is now (Module number five)**

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29},t)\right]G_{28} \quad 25$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29},t)\right]G_{29} \quad 26$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}, t)\right]G_{30} \quad 27$$
$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31}), t)\right]T_{28} \quad 28$$
$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}((G_{31}), t)\right]T_{29} \quad 29$$
$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31}), t)\right]T_{30} \quad 30$$
$+(a''_{28})^{(5)}(T_{29}, t) =$ **First augmentation factor**
$-(b''_{28})^{(5)}((G_{31}), t) =$ **First detritions factor**

**Module Numbered Six**

**The differential system of this model is now (Module numbered Six)**

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right]G_{32} \quad 31$$
$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}, t)\right]G_{33} \quad 32$$
$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}, t)\right]G_{34} \quad 33$$
$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35}), t)\right]T_{32} \quad 34$$
$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}((G_{35}), t)\right]T_{33} \quad 35$$
$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35}), t)\right]T_{34} \quad 36$$
$+(a''_{32})^{(6)}(T_{33}, t) =$ **First augmentation factor**

**Module Numbered Seven:**

**The differential system of this model is now (Seventh Module)**

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right]G_{36} \quad 37$$
$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37}, t)\right]G_{37} \quad 38$$
$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}, t)\right]G_{38} \quad 39$$
$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39}), t)\right]T_{36} \quad 40$$
$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}((G_{39}), t)\right]T_{37} \quad 41$$
$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39}), t)\right]T_{38} \quad 42$$
$+(a''_{36})^{(7)}(T_{37}, t) =$ **First augmentation factor**

**Module Numbered Eight**

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right]G_{40} \quad 43$$
$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41}, t)\right]G_{41} \quad 44$$
$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}, t)\right]G_{42} \quad 45$$
$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43}), t)\right]T_{40} \quad 46$$
$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}((G_{43}), t)\right]T_{41} \quad 47$$
$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43}), t)\right]T_{42} \quad 48$$

**Module Numbered Nine**

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}\left((G_{47}),t\right)\right]T_{44} \quad 52$$

$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}\left((G_{47}),t\right)\right]T_{45} \quad 53$$

$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}\left((G_{47}),t\right)\right]T_{46} \quad 54$$

$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}\left((G_{47}),t\right) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)} & +(a''_{13})^{(1)}(T_{14},t) & +(a''_{16})^{(2,2,)}(T_{17},t) & +(a''_{20})^{(3,3,)}(T_{21},t) \\ +(a''_{24})^{(4,4,4,4,)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,)}(T_{33},t) & \\ +(a''_{36})^{(7,7)}(T_{37},t) & +(a''_{40})^{(8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) & \end{bmatrix} G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)} & +(a''_{14})^{(1)}(T_{14},t) & +(a''_{17})^{(2,2,)}(T_{17},t) & +(a''_{21})^{(3,3,)}(T_{21},t) \\ +(a''_{25})^{(4,4,4,4,)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,)}(T_{33},t) & \\ +(a''_{37})^{(7,7)}(T_{37},t) & +(a''_{41})^{(8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) & \end{bmatrix} G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)} & +(a''_{15})^{(1)}(T_{14},t) & +(a''_{18})^{(2,2,)}(T_{17},t) & +(a''_{22})^{(3,3,)}(T_{21},t) \\ +(a''_{26})^{(4,4,4,4,)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,)}(T_{33},t) & \\ +(a''_{38})^{(7,7)}(T_{37},t) & +(a''_{42})^{(8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) & \end{bmatrix} G_{15} \quad 57$$

Where $(a''_{13})^{(1)}(T_{14},t)$, $(a''_{14})^{(1)}(T_{14},t)$, $(a''_{15})^{(1)}(T_{14},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,)}(T_{17},t)$, $+(a''_{17})^{(2,2,)}(T_{17},t)$, $+(a''_{18})^{(2,2,)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,)}(T_{21},t)$, $+(a''_{21})^{(3,3,)}(T_{21},t)$, $+(a''_{22})^{(3,3,)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{38})^{(7,7)}(T_{37},t)$ $+(a''_{37})^{(7,7)}(T_{37},t)$ $+(a''_{36})^{(7,7)}(T_{37},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{40})^{(8,8)}(T_{41},t)$ $+(a''_{41})^{(8,8)}(T_{41},t)$ $+(a''_{42})^{(8,8)}(T_{41},t)$ are eight augmentation coefficient for 1,2,3

$+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)}\boxed{-(b''_{13})^{(1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13} \quad (58)$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)}\boxed{-(b''_{14})^{(1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14} \quad (59)$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)}\boxed{-(b''_{15})^{(1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15} \quad (60)$$

Where $\boxed{-(b''_{13})^{(1)}(G,t)}$, $\boxed{-(b''_{14})^{(1)}(G,t)}$, $\boxed{-(b''_{15})^{(1)}(G,t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8)}(G_{43},t)}$ $\boxed{-(b''_{41})^{(8,8)}(G_{43},t)}$ $\boxed{-(b''_{42})^{(8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \begin{bmatrix} (a'_{16})^{(2)}\boxed{+(a''_{16})^{(2)}(T_{17},t)} & \boxed{+(a''_{13})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{16} \quad (61)$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \begin{bmatrix} (a'_{17})^{(2)}\boxed{+(a''_{17})^{(2)}(T_{17},t)} & \boxed{+(a''_{14})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{17} \quad (62)$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \begin{bmatrix} (a'_{18})^{(2)}\boxed{+(a''_{18})^{(2)}(T_{17},t)} & \boxed{+(a''_{15})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{18} \quad (63)$$

Where $\boxed{+(a''_{16})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,)}(T_{14},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)}$ are eight augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{44})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9)}(T_{45},t)}$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \begin{bmatrix} (b'_{16})^{(2)}\boxed{-(b''_{16})^{(2)}(G_{19},t)} & \boxed{-(b''_{13})^{(1,1,)}(G,t)} & \boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{16} \quad (64)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)}\boxed{-(b''_{17})^{(2)}(G_{19},t)} & \boxed{-(b''_{14})^{(1,1,)}(G,t)} & \boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{17} \quad (65)$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)}\boxed{-(b''_{18})^{(2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,)}(G,t)} & \boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{18} \quad (66)$$

where $\boxed{-(b''_{16})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2)}(G_{19},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,)}(G,t)}$ are second detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)}$ are fourth detrition

coefficients for category 1,2 and 3

$-(b_{28}'')^{(5,5,5,5,5)}(G_{31},t)$, $-(b_{29}'')^{(5,5,5,5,5)}(G_{31},t)$, $-(b_{30}'')^{(5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1,2 and 3

$-(b_{32}'')^{(6,6,6,6,6)}(G_{35},t)$, $-(b_{33}'')^{(6,6,6,6,6)}(G_{35},t)$, $-(b_{34}'')^{(6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients for category 1,2 and 3

$-(b_{36}'')^{(7,7,7)}(G_{39},t)$, $-(b_{37}'')^{(7,7,7)}(G_{39},t)$, $-(b_{38}'')^{(7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2 and 3

$-(b_{40}'')^{(8,8,8)}(G_{43},t)$, $-(b_{41}'')^{(8,8,8)}(G_{43},t)$, $-(b_{42}'')^{(8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1,2 and 3

$-(b_{44}'')^{(9,9)}(G_{47},t)$, $-(b_{46}'')^{(9,9)}(G_{47},t)$, $-(b_{45}'')^{(9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[\begin{array}{c}(a_{20}')^{(3)}+(a_{20}'')^{(3)}(T_{21},t)\ +(a_{16}'')^{(2,2,2)}(T_{17},t)\ +(a_{13}'')^{(1,1,1,)}(T_{14},t)\\ +(a_{24}'')^{(4,4,4,4,4,4)}(T_{25},t)\ +(a_{28}'')^{(5,5,5,5,5,5)}(T_{29},t)\ +(a_{32}'')^{(6,6,6,6,6,6)}(T_{33},t)\\ +(a_{36}'')^{(7,7,7,7)}(T_{37},t)\ +(a_{40}'')^{(8,8,8,8)}(T_{41},t)\ +(a_{44}'')^{(9,9,9)}(T_{45},t)\end{array}\right]G_{20} \quad 67$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[\begin{array}{c}(a_{21}')^{(3)}+(a_{21}'')^{(3)}(T_{21},t)\ +(a_{17}'')^{(2,2,2)}(T_{17},t)\ +(a_{14}'')^{(1,1,1,)}(T_{14},t)\\ +(a_{25}'')^{(4,4,4,4,4,4)}(T_{25},t)\ +(a_{29}'')^{(5,5,5,5,5,5)}(T_{29},t)\ +(a_{33}'')^{(6,6,6,6,6,6)}(T_{33},t)\\ +(a_{37}'')^{(7,7,7,7)}(T_{37},t)\ +(a_{41}'')^{(8,8,8,8)}(T_{41},t)\ +(a_{45}'')^{(9,9,9)}(T_{45},t)\end{array}\right]G_{21} \quad 68$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[\begin{array}{c}(a_{22}')^{(3)}+(a_{22}'')^{(3)}(T_{21},t)\ +(a_{18}'')^{(2,2,2)}(T_{17},t)\ +(a_{15}'')^{(1,1,1,)}(T_{14},t)\\ +(a_{26}'')^{(4,4,4,4,4,4)}(T_{25},t)\ +(a_{30}'')^{(5,5,5,5,5,5)}(T_{29},t)\ +(a_{34}'')^{(6,6,6,6,6,6)}(T_{33},t)\\ +(a_{38}'')^{(7,7,7,7)}(T_{37},t)\ +(a_{42}'')^{(8,8,8,8)}(T_{41},t)\ +(a_{46}'')^{(9,9,9)}(T_{45},t)\end{array}\right]G_{22} \quad 69$$

$+(a_{20}'')^{(3)}(T_{21},t)$, $+(a_{21}'')^{(3)}(T_{21},t)$, $+(a_{22}'')^{(3)}(T_{21},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a_{16}'')^{(2,2,2)}(T_{17},t)$, $+(a_{17}'')^{(2,2,2)}(T_{17},t)$, $+(a_{18}'')^{(2,2,2)}(T_{17},t)$ are second augmentation coefficients for category 1, 2 and 3

$+(a_{13}'')^{(1,1,1,)}(T_{14},t)$, $+(a_{14}'')^{(1,1,1,)}(T_{14},t)$, $+(a_{15}'')^{(1,1,1,)}(T_{14},t)$ are third augmentation coefficients for category 1, 2 and 3

$+(a_{24}'')^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a_{25}'')^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a_{26}'')^{(4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficients for category 1, 2 and 3

$+(a_{28}'')^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a_{29}'')^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a_{30}'')^{(5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficients for category 1, 2 and 3

$+(a_{32}'')^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a_{33}'')^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a_{34}'')^{(6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficients for category 1, 2 and 3

$+(a_{36}'')^{(7,7,7,7)}(T_{37},t)$, $+(a_{37}'')^{(7,7,7,7)}(T_{37},t)$, $+(a_{38}'')^{(7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1, 2 and 3

$+(a_{40}'')^{(8,8,8,8)}(T_{41},t)$, $+(a_{41}'')^{(8,8,8,8)}(T_{41},t)$, $+(a_{42}'')^{(8,8,8,8)}(T_{41},t)$ are eight augmentation coefficients for category 1, 2 and 3

$+(a_{44}'')^{(9,9,9)}(T_{45},t)$, $+(a_{45}'')^{(9,9,9)}(T_{45},t)$, $+(a_{46}'')^{(9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for

category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b'_{20})^{(3)}\boxed{-(b''_{20})^{(3)}(G_{23},t)}\boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}\boxed{-(b''_{13})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)}\boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)}\boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7)}(G_{39},t)}\boxed{-(b''_{40})^{(8,8,8,8)}(G_{43},t)}\boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{20} \quad 70$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b'_{21})^{(3)}\boxed{-(b''_{21})^{(3)}(G_{23},t)}\boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}\boxed{-(b''_{14})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)}\boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)}\boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7)}(G_{39},t)}\boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)}\boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{21} \quad 71$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)}\boxed{-(b''_{22})^{(3)}(G_{23},t)}\boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}\boxed{-(b''_{15})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)}\boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)}\boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)}\boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)}\boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{22} \quad 72$$

$\boxed{-(b''_{20})^{(3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3)}(G_{23},t)}$ *are first detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}$ *are second detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{13})^{(1,1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,)}(G,t)}$ *are third detrition coefficients for category 1,2 and 3*

$\boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)}$ *are fourth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)}$ *are fifth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)}$ *are sixth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{36})^{(7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7)}(G_{39},t)}$ $\boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \begin{bmatrix} (a'_{24})^{(4)}\boxed{+(a''_{24})^{(4)}(T_{25},t)}\boxed{+(a''_{28})^{(5,5,)}(T_{29},t)}\boxed{+(a''_{32})^{(6,6,)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)}\boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)}\boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)}\boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)}\boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{24} \quad 73$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \begin{bmatrix} (a'_{25})^{(4)}\boxed{+(a''_{25})^{(4)}(T_{25},t)}\boxed{+(a''_{29})^{(5,5,)}(T_{29},t)}\boxed{+(a''_{33})^{(6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)}\boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)}\boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)}\boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)}\boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{25} \quad 74$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[\begin{array}{ccc} (a'_{26})^{(4)} & +(a''_{26})^{(4)}(T_{25},t) & +(a''_{30})^{(5,5,)}(T_{29},t) \quad +(a''_{34})^{(6,6,)}(T_{33},t) \\ +(a''_{15})^{(1,1,1,1)}(T_{14},t) & +(a''_{18})^{(2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3)}(T_{21},t) \\ +(a''_{38})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9)}(T_{45},t) \end{array}\right] G_{26} \quad 75$$

$(a''_{24})^{(4)}(T_{25},t)$, $(a''_{25})^{(4)}(T_{25},t)$, $(a''_{26})^{(4)}(T_{25},t)$ *are first augmentation coefficients category* 1, 2 3

$+(a''_{28})^{(5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,)}(T_{29},t)$ *are second augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{32})^{(6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,)}(T_{33},t)$ *are third augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{13})^{(1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1)}(T_{14},t)$ *are fourth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{16})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2)}(T_{17},t)$ *are fifth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{20})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3)}(T_{21},t)$ *are sixth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)$ *are seventh augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)$ *are eighth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{46})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9)}(T_{45},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[\begin{array}{ccc} (b'_{24})^{(4)} & -(b''_{24})^{(4)}(G_{27},t) & -(b''_{28})^{(5,5,)}(G_{31},t) \quad -(b''_{32})^{(6,6,)}(G_{35},t) \\ -(b''_{13})^{(1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3)}(G_{23},t) \\ -(b''_{36})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9)}(G_{47},t) \end{array}\right] T_{24} \quad 76$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[\begin{array}{ccc} (b'_{25})^{(4)} & -(b''_{25})^{(4)}(G_{27},t) & -(b''_{29})^{(5,5,)}(G_{31},t) \quad -(b''_{33})^{(6,6,)}(G_{35},t) \\ -(b''_{14})^{(1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3)}(G_{23},t) \\ -(b''_{37})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9)}(G_{47},t) \end{array}\right] T_{25} \quad 77$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[\begin{array}{ccc} (b'_{26})^{(4)} & -(b''_{26})^{(4)}(G_{27},t) & -(b''_{30})^{(5,5,)}(G_{31},t) \quad -(b''_{34})^{(6,6,)}(G_{35},t) \\ -(b''_{15})^{(1,1,1,1)}(G,t) & -(b''_{18})^{(2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3)}(G_{23},t) \\ -(b''_{38})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9)}(G_{47},t) \end{array}\right] T_{26} \quad 78$$

*Where* $-(b''_{24})^{(4)}(G_{27},t)$, $-(b''_{25})^{(4)}(G_{27},t)$, $-(b''_{26})^{(4)}(G_{27},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{28})^{(5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,)}(G_{31},t)$ *are second detrition coefficients for category* 1, 2 *and* 3

$-(b''_{32})^{(6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,)}(G_{35},t)$ *are third detrition coefficients for category* 1, 2 *and* 3

$-(b''_{13})^{(1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1)}(G,t)$

*are fourth detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b_{16}'')^{(2,2,2,2)}(G_{19},t)}, \boxed{-(b_{17}'')^{(2,2,2,2)}(G_{19},t)}, \boxed{-(b_{18}'')^{(2,2,2,2)}(G_{19},t)}$

*are fifth detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b_{20}'')^{(3,3,3,3)}(G_{23},t)}, \boxed{-(b_{21}'')^{(3,3,3,3)}(G_{23},t)}, \boxed{-(b_{22}'')^{(3,3,3,3)}(G_{23},t)}$

*are sixth detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b_{36}'')^{(7,7,7,7,7)}(G_{39},t)}, \boxed{-(b_{37}'')^{(7,7,7,7,7)}(G_{39},t)}, \boxed{-(b_{38}'')^{(7,7,7,7,7)}(G_{39},t)}$

*are seventh detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b_{40}'')^{(8,8,8,8,8)}(G_{43},t)}, \boxed{-(b_{41}'')^{(8,8,8,8,8)}(G_{43},t)}, \boxed{-(b_{42}'')^{(8,8,8,8,8)}(G_{43},t)}$

*are eighth detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b_{46}'')^{(9,9,9,9)}(G_{47},t)}, \boxed{-(b_{45}'')^{(9,9,9,9)}(G_{47},t)}, \boxed{-(b_{44}'')^{(9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[\begin{array}{ccc} (a_{28}')^{(5)} & \boxed{+(a_{28}'')^{(5)}(T_{29},t)} & \boxed{+(a_{24}'')^{(4,4,)}(T_{25},t)} \quad \boxed{+(a_{32}'')^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{16}'')^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{20}'')^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{36}'')^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{40}'')^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a_{44}'')^{(9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{28} \qquad 79$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[\begin{array}{ccc} (a_{29}')^{(5)} & \boxed{+(a_{29}'')^{(5)}(T_{29},t)} & \boxed{+(a_{25}'')^{(4,4,)}(T_{25},t)} \quad \boxed{+(a_{33}'')^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a_{14}'')^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{17}'')^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{21}'')^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{37}'')^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{41}'')^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a_{45}'')^{(9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{29} \qquad 80$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[\begin{array}{ccc} (a_{30}')^{(5)} & \boxed{+(a_{30}'')^{(5)}(T_{29},t)} & \boxed{+(a_{26}'')^{(4,4,)}(T_{25},t)} \quad \boxed{+(a_{34}'')^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a_{15}'')^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{18}'')^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{22}'')^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{38}'')^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{42}'')^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a_{46}'')^{(9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{30} \qquad 81$$

*Where* $\boxed{+(a_{28}'')^{(5)}(T_{29},t)}, \boxed{+(a_{29}'')^{(5)}(T_{29},t)}, \boxed{+(a_{30}'')^{(5)}(T_{29},t)}$ *are first augmentation coefficients for category* 1, 2 *and* 3

*And* $\boxed{+(a_{24}'')^{(4,4,)}(T_{25},t)}, \boxed{+(a_{25}'')^{(4,4,)}(T_{25},t)}, \boxed{+(a_{26}'')^{(4,4,)}(T_{25},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a_{32}'')^{(6,6,6)}(T_{33},t)}, \boxed{+(a_{33}'')^{(6,6,6)}(T_{33},t)}, \boxed{+(a_{34}'')^{(6,6,6)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a_{13}'')^{(1,1,1,1,1)}(T_{14},t)}, \boxed{+(a_{14}'')^{(1,1,1,1,1)}(T_{14},t)}, \boxed{+(a_{15}'')^{(1,1,1,1,1)}(T_{14},t)}$ are fourth augmentation coefficients for category 1,2, and 3

$\boxed{+(a_{16}'')^{(2,2,2,2,2)}(T_{17},t)}, \boxed{+(a_{17}'')^{(2,2,2,2,2)}(T_{17},t)}, \boxed{+(a_{18}'')^{(2,2,2,2,2)}(T_{17},t)}$ are fifth augmentation coefficients for category 1,2,and 3

$\boxed{+(a_{20}'')^{(3,3,3,3,3)}(T_{21},t)}, \boxed{+(a_{21}'')^{(3,3,3,3,3)}(T_{21},t)}, \boxed{+(a_{22}'')^{(3,3,3,3,3)}(T_{21},t)}$ are sixth augmentation coefficients for category 1,2, 3

$\boxed{+(a_{36}'')^{(7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a_{37}'')^{(7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a_{38}'')^{(7,7,7,7,7,7)}(T_{37},t)}$ are seventh augmentation coefficients for category 1,2, 3

$\boxed{+(a_{40}'')^{(8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a_{41}'')^{(8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a_{42}'')^{(8,8,8,8,8,8)}(T_{41},t)}$ are eighth augmentation coefficients for category 1,2, 3

$\boxed{+(a_{46}'')^{(9,9,9,9,9)}(T_{45},t)}, \boxed{+(a_{45}'')^{(9,9,9,9,9)}(T_{45},t)}, \boxed{+(a_{44}'')^{(9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation

coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \begin{bmatrix} (b'_{28})^{(5)}\boxed{-(b''_{28})^{(5)}(G_{31},t)} & \boxed{-(b''_{24})^{(4,4,)}(G_{27},t)} & \boxed{-(b''_{32})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{28} \quad (82)$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b'_{29})^{(5)}\boxed{-(b''_{29})^{(5)}(G_{31},t)} & \boxed{-(b''_{25})^{(4,4,)}(G_{27},t)} & \boxed{-(b''_{33})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{29} \quad (83)$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b'_{30})^{(5)}\boxed{-(b''_{30})^{(5)}(G_{31},t)} & \boxed{-(b''_{26})^{(4,4,)}(G_{27},t)} & \boxed{-(b''_{34})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{30} \quad (84)$$

*where* $\boxed{-(b''_{28})^{(5)}(G_{31},t)}$ , $\boxed{-(b''_{29})^{(5)}(G_{31},t)}$ , $\boxed{-(b''_{30})^{(5)}(G_{31},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{24})^{(4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,)}(G_{27},t)}$ *are second detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{32})^{(6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6)}(G_{35},t)}$ *are third detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1)}(G,t)}$ , $\boxed{-(b''_{15})^{(1,1,1,1,1,)}(G,t)}$ are fourth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)}$ , $\boxed{-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)}$ , $\boxed{-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)}$ , $\boxed{-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)}$ , $\boxed{-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)}$ , $\boxed{-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)}$ , $\boxed{-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \begin{bmatrix} (a'_{32})^{(6)}\boxed{+(a''_{32})^{(6)}(T_{33},t)} & \boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{32} \quad (85)$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \begin{bmatrix} (a'_{33})^{(6)}\boxed{+(a''_{33})^{(6)}(T_{33},t)} & \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{33} \quad (86)$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \begin{bmatrix} (a'_{34})^{(6)}\boxed{+(a''_{34})^{(6)}(T_{33},t)} & \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{34} \quad 87$$

$\boxed{+(a''_{32})^{(6)}(T_{33},t)}, \boxed{+(a''_{33})^{(6)}(T_{33},t)}, \boxed{+(a''_{34})^{(6)}(T_{33},t)}$ *are first augmentation coefficients for category* $1, 2$ *and* $3$

$\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}$ *are second augmentation coefficients for category* $1, 2$ *and* $3$

$\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}$ *are third augmentation coefficients for category* $1, 2$ *and* $3$

$\boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}$ - are fourth augmentation coefficients

$\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}$ - fifth augmentation coefficients

$\boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)}$ sixth augmentation coefficients

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)}$ seventh augmentation coefficients

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}$ Eighth augmentation coefficients

$\boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)}$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \begin{bmatrix} (b'_{32})^{(6)}\boxed{-(b''_{32})^{(6)}(G_{35},t)} & \boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{32} \quad 88$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \begin{bmatrix} (b'_{33})^{(6)}\boxed{-(b''_{33})^{(6)}(G_{35},t)} & \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{33} \quad 89$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)}\boxed{-(b''_{34})^{(6)}(G_{35},t)} & \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{34} \quad 90$$

$\boxed{-(b''_{32})^{(6)}(G_{35},t)}, \boxed{-(b''_{33})^{(6)}(G_{35},t)}, \boxed{-(b''_{34})^{(6)}(G_{35},t)}$ *are first detrition coefficients for category* $1, 2$ *and* $3$

$\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}$ *are second detrition coefficients for category* $1, 2$ *and* $3$

$\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)}$ *are third detrition coefficients*

$for\ category\ 1,2\ and\ 3$

$-(b_{13}'')^{(1,1,1,1,1,1,1)}(G,t)$, $-(b_{14}'')^{(1,1,1,1,1,1,1)}(G,t)$, $-(b_{15}'')^{(1,1,1,1,1,1,1)}(G,t)$ are fourth detrition coefficients for category 1, 2, and 3

$-(b_{16}'')^{(2,2,2,2,2,2,2)}(G_{19},t)$, $-(b_{17}'')^{(2,2,2,2,2,2,2)}(G_{19},t)$, $-(b_{18}'')^{(2,2,2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1, 2, and 3

$-(b_{20}'')^{(3,3,3,3,3,3,3)}(G_{23},t)$, $-(b_{21}'')^{(3,3,3,3,3,3,3)}(G_{23},t)$, $-(b_{22}'')^{(3,3,3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1, 2, and 3

$-(b_{36}'')^{(7,7,7,7,7,7,7)}(G_{39},t)$, $-(b_{37}'')^{(7,7,7,7,7,7,7)}(G_{39},t)$, $-(b_{38}'')^{(7,7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2, and 3

$-(b_{40}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)$, $-(b_{41}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)$, $-(b_{42}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1, 2, and 3

$-(b_{46}'')^{(9,9,9,9,9,9,9)}(G_{47},t)$, $-(b_{45}'')^{(9,9,9,9,9,9,9)}(G_{47},t)$, $-(b_{44}'')^{(9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \qquad 91$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a_{36}')^{(7)} & +(a_{36}'')^{(7)}(T_{37},t) & +(a_{16}'')^{(2,2,2,2,2,2,2)}(T_{17},t) & +(a_{20}'')^{(3,3,3,3,3,3,3)}(T_{21},t) \\ & +(a_{24}'')^{(4,4,4,4,4,4,4)}(T_{25},t) & +(a_{28}'')^{(5,5,5,5,5,5,5)}(T_{29},t) & +(a_{32}'')^{(6,6,6,6,6,6,6)}(T_{33},t) \\ & +(a_{13}'')^{(1,1,1,1,1,1,1)}(T_{14},t) & +(a_{40}'')^{(8,8,8,8,8,8,8,8,)}(T_{41},t) & +(a_{44}'')^{(9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \qquad 92$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a_{37}')^{(7)} & +(a_{37}'')^{(7)}(T_{37},t) & +(a_{17}'')^{(2,2,2,2,2,2,2)}(T_{17},t) & +(a_{21}'')^{(3,3,3,3,3,3,3)}(T_{21},t) \\ & +(a_{25}'')^{(4,4,4,4,4,4,4)}(T_{25},t) & +(a_{29}'')^{(5,5,5,5,5,5,5)}(T_{29},t) & +(a_{33}'')^{(6,6,6,6,6,6,6)}(T_{33},t) \\ & +(a_{13}'')^{(1,1,1,1,1,1,1)}(T_{14},t) & +(a_{41}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t) & +(a_{45}'')^{(9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{14}$$

$$\frac{dG_{38}}{dt} \qquad 93$$

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a_{38}')^{(7)} & +(a_{38}'')^{(7)}(T_{37},t) & +(a_{18}'')^{(2,2,2,2,2,2,2)}(T_{17},t) & +(a_{22}'')^{(3,3,3,3,3,3,3)}(T_{21},t) \\ & +(a_{26}'')^{(4,4,4,4,4,4,4)}(T_{25},t) & +(a_{30}'')^{(5,5,5,5,5,5,5)}(T_{29},t) & +(a_{34}'')^{(6,6,6,6,6,6,6)}(T_{33},t) \\ & +(a_{15}'')^{(1,1,1,1,1,1,1)}(T_{14},t) & +(a_{42}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t) & +(a_{46}'')^{(9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{15}$$

Where $(a_{36}'')^{(7)}(T_{37},t)$, $(a_{37}'')^{(7)}(T_{37},t)$, $(a_{38}'')^{(7)}(T_{37},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a_{16}'')^{(2,2,2,2,2,2,2)}(T_{17},t)$, $+(a_{17}'')^{(2,2,2,2,2,2,2)}(T_{17},t)$, $+(a_{18}'')^{(2,2,2,2,2,2,2)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a_{20}'')^{(3,3,3,3,3,3,3)}(T_{21},t)$, $+(a_{21}'')^{(3,3,3,3,3,3,3)}(T_{21},t)$, $+(a_{22}'')^{(3,3,3,3,3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a_{24}'')^{(4,4,4,4,4,4,4)}(T_{25},t)$, $+(a_{25}'')^{(4,4,4,4,4,4,4)}(T_{25},t)$, $+(a_{26}'')^{(4,4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a_{28}'')^{(5,5,5,5,5,5,5)}(T_{29},t)$, $+(a_{29}'')^{(5,5,5,5,5,5,5)}(T_{29},t)$, $+(a_{30}'')^{(5,5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a_{32}'')^{(6,6,6,6,6,6,6)}(T_{33},t)$, $+(a_{33}'')^{(6,6,6,6,6,6,6)}(T_{33},t)$, $+(a_{34}'')^{(6,6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation

coefficient for category 1, 2 and 3
$+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)$ are seventh augmentation coefficient for category 1, 2 and 3
$+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)$
are eighth augmentation coefficient for 1,2,3
$+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3



$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \begin{bmatrix} (b'_{36})^{(7)}\boxed{-(b''_{36})^{(7)}(G_{39},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \begin{bmatrix} (b'_{37})^{(7)}\boxed{-(b''_{37})^{(7)}(G_{39},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \begin{bmatrix} (b'_{38})^{(7)}\boxed{-(b''_{38})^{(7)}(G_{39},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3
$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3
$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3
$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3
$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3
$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3
$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3
$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3



$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \begin{bmatrix} (a'_{40})^{(8)}\boxed{+(a''_{40})^{(8)}(T_{41},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \begin{bmatrix} (a'_{41})^{(8)}\boxed{+(a''_{41})^{(8)}(T_{41},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \begin{bmatrix} (a'_{42})^{(8)}\boxed{+(a''_{42})^{(8)}(T_{41},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a''_{40})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} =$$

$$(b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)}\boxed{-(b''_{40})^{(8)}(G_{43},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} =$$

$$(b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)}\boxed{-(b''_{41})^{(8)}(G_{43},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} =$$

$$(b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)}\boxed{-(b''_{42})^{(8)}(G_{43},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3



$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \begin{bmatrix} (a'_{44})^{(9)}\boxed{+(a''_{44})^{(9)}(T_{45},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \begin{bmatrix} (a'_{45})^{(9)}\boxed{+(a''_{45})^{(9)}(T_{45},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \begin{bmatrix} (a'_{46})^{(9)}\boxed{+(a''_{46})^{(9)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a''_{44})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ are Seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} -$$

$$\begin{bmatrix} (b'_{44})^{(9)}\boxed{-(b''_{44})^{(9)}(G_{47},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b'_{45})^{(9)}\boxed{-(b''_{45})^{(9)}(G_{47},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b'_{46})^{(9)}\boxed{-(b''_{46})^{(9)}(G_{47},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{44})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9)}(G_{47},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a'_i)^{(1)}, (a''_i)^{(1)}, (b_i)^{(1)}, (b'_i)^{(1)}, (b''_i)^{(1)} > 0, \quad i,j = 13,14,15$$ *97*

The functions $(a''_i)^{(1)}, (b''_i)^{(1)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$(a_i'')^{(1)}(T_{14},t) \leq (p_i)^{(1)} \leq (\hat{A}_{13})^{(1)}$

$(b_i'')^{(1)}(G,t) \leq (r_i)^{(1)} \leq (b_i')^{(1)} \leq (\hat{B}_{13})^{(1)}$

$lim_{T_2\to\infty}(a_i'')^{(1)}(T_{14},t) = (p_i)^{(1)}$ 98

$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = (r_i)^{(1)}$

**Definition of** $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$ :

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$|(a_i'')^{(1)}(T_{14}',t) - (a_i'')^{(1)}(T_{14},t)| \leq (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$

$|(b_i'')^{(1)}(G',t) - (b_i'')^{(1)}(G,t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}',t)$ and $(a_i'')^{(1)}(T_{14},t)$ . $(T_{14}',t)$ and $(T_{14},t)$ are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14},t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$ : 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}$ , $\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$

**Definition of** $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$ : 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$

$\frac{1}{(\hat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$

Where we suppose

$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i,j = 16,17,18$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17},t) \le (p_i)^{(2)} \le \left(\hat{A}_{16}\right)^{(2)} \quad 102$$

$$(b_i'')^{(2)}(G_{19},t) \le (r_i)^{(2)} \le (b_i')^{(2)} \le (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2\to\infty}(a_i'')^{(2)}(T_{17},t) = (p_i)^{(2)} \quad 104$$

$$\lim_{G\to\infty}(b_i'')^{(2)}\left((G_{19}),t\right) = (r_i)^{(2)} \quad 105$$

**Definition of** $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$ : 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}',t) - (a_i'')^{(2)}(T_{17},t)| \le (\hat{k}_{16})^{(2)}|T_{17} - T_{17}'|e^{-(\hat{M}_{16})^{(2)}t} \quad 107$$

$$|(b_i'')^{(2)}((G_{19})',t) - (b_i'')^{(2)}\left((G_{19}),t\right)| < (\hat{k}_{16})^{(2)}||(G_{19}) - (G_{19})'||e^{-(\hat{M}_{16})^{(2)}t} \quad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}',t)$ and$(a_i'')^{(2)}(T_{17},t)$ . $(T_{17}',t)$ and $(T_{17},t)$ are points belonging to the interval $\left[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}\right]$ . It is to be noted that $(a_i'')^{(2)}(T_{17},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17},t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$ :

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants 109

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$$

**Definition of** $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$ :

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1 \quad 110$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(b_i)^{(2)} + (b_i')^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1 \quad 111$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i,j = 20,21,22 \quad 112$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(3)}$, $(\mathrm{r_i})^{(3)}$:

$$(a_i'')^{(3)}(T_{21},t) \le (p_i)^{(3)} \le (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23},t) \le (r_i)^{(3)} \le (b_i')^{(3)} \le (\hat{B}_{20})^{(3)}$$

$$lim_{T_2\to\infty}(a_i'')^{(3)}(T_{21},t) = (p_i)^{(3)} \qquad 113$$

$$\lim_{G\to\infty}(b_i'')^{(3)}(G_{23},t) = (r_i)^{(3)}$$

**Definition of** $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$ :

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: 114

$$|(a_i'')^{(3)}(T_{21}',t) - (a_i'')^{(3)}(T_{21},t)| \le (\hat{k}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\hat{M}_{20})^{(3)}t}$$

$$|(b_i'')^{(3)}(G_{23}',t) - (b_i'')^{(3)}(G_{23},t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}'||e^{-(\hat{M}_{20})^{(3)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}',t)$ and$(a_i'')^{(3)}(T_{21},t)$ . $(T_{21}',t)$ And $(T_{21},t)$ are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$ : 115

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$ and the constants $(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}, i = 20,21,22,$ satisfy the inequalities 116

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(a_i)^{(3)} + (a_i')^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(b_i)^{(3)} + (b_i')^{(3)} + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i,j = 24,25,26 \qquad 117$$

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(4)}$, $(r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25},t) \le (p_i)^{(4)} \le (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}\big((G_{27}),t\big) \le (r_i)^{(4)} \le (b_i')^{(4)} \le (\hat{B}_{24})^{(4)}$$

$$lim_{T_2\to\infty}(a_i'')^{(4)}(T_{25},t) = (p_i)^{(4)} \quad (118)$$

$$\lim_{G\to\infty}(b_i'')^{(4)}\big((G_{27}),t\big) = (r_i)^{(4)}$$

**Definition of** $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$ :

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24,25,26}$

They satisfy Lipschitz condition: (119)

$$|(a_i'')^{(4)}(T_{25}',t) - (a_i'')^{(4)}(T_{25},t)| \le (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})',t) - (b_i'')^{(4)}\big((G_{27}),t\big)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}',t)$ and$(a_i'')^{(4)}(T_{25},t)$ . $(T_{25}',t)$ and $(T_{25},t)$ are points belonging to the interval $\big[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}\big]$. It is to be noted that $(a_i'')^{(4)}(T_{25},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25},t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$ : (120)

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$, are positive constants

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}, \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$$

**Definition of** $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$ : (121)

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and\ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}, i = 24,25,26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(a_i)^{(4)} + (a_i')^{(4)} + (\hat{A}_{24})^{(4)} + (\hat{P}_{24})^{(4)}(\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(b_i)^{(4)} + (b_i')^{(4)} + (\hat{B}_{24})^{(4)} + (\hat{Q}_{24})^{(4)}(\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i,j = 28,29,30 \quad (122)$$

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(5)}$, $(r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29},t) \le (p_i)^{(5)} \le (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}\big((G_{31}),t\big) \le (r_i)^{(5)} \le (b_i')^{(5)} \le (\hat{B}_{28})^{(5)}$$

$$lim_{T_2\to\infty}(a_i'')^{(5)}(T_{29},t) = (p_i)^{(5)} \qquad 123$$

$$\lim_{G\to\infty}(b_i'')^{(5)}(G_{31},t) = (r_i)^{(5)}$$

**Definition of** $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$ :

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}',t) - (a_i'')^{(5)}(T_{29},t)| \le (\hat{k}_{28})^{(5)}|T_{29} - T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})',t) - (b_i'')^{(5)}\big((G_{31}),t\big)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}',t)$ and$(a_i'')^{(5)}(T_{29},t)$ . $(T_{29}',t)$ and $(T_{29},t)$ are points belonging to the interval $\big[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\big]$. It is to be noted that $(a_i'')^{(5)}(T_{29},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29},t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$ : 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}, \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

**Definition of** $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$ : 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and\ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(a_i)^{(5)} + (a_i')^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)}(\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(b_i)^{(5)} + (b_i')^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)}(\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i,j = 32,33,34 \qquad 127$$

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(6)}$, $(r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33},t) \le (p_i)^{(6)} \le (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}),t) \le (r_i)^{(6)} \le (b_i')^{(6)} \le (\hat{B}_{32})^{(6)}$$

$$lim_{T_2\to\infty}(a_i'')^{(6)}(T_{33},t) = (p_i)^{(6)} \quad (128)$$

$$\lim_{G\to\infty}(b_i'')^{(6)}\left((G_{35}),t\right) = (r_i)^{(6)}$$

**Definition of** $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$ :

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}',t) - (a_i'')^{(6)}(T_{33},t)| \le (\hat{k}_{32})^{(6)}|T_{33} - T_{33}'|e^{-(\hat{M}_{32})^{(6)}t}$$

$$|(b_i'')^{(6)}((G_{35})',t) - (b_i'')^{(6)}\left((G_{35}),t\right)| < (\hat{k}_{32})^{(6)}||(G_{35}) - (G_{35})'||e^{-(\hat{M}_{32})^{(6)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}',t)$ and$(a_i'')^{(6)}(T_{33},t)$ . $(T_{33}',t)$ and $(T_{33},t)$ are points belonging to the interval $\left[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}\right]$. It is to be noted that $(a_i'')^{(6)}(T_{33},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33},t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$ : (129)

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$, are positive constants

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$$

**Definition of** $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$ : (130)

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and\ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32,33,34,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(a_i)^{(6)} + (a_i')^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(b_i)^{(6)} + (b_i')^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

(KK) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i,j = 36,37,38$ (131)

(LL) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(7)}$, $(r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37},t) \le (p_i)^{(7)} \le (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39},t) \le (r_i)^{(7)} \le (b_i')^{(7)} \le (\hat{B}_{36})^{(7)} \quad (132)$$

*(MM)* $\lim_{T_2\to\infty} (a_i'')^{(7)}(T_{37},t) = (p_i)^{(7)}$

*(NN)*

$\lim_{G\to\infty}(b_i'')^{(7)}\big((G_{39}),t\big) = (r_i)^{(7)}$

**Definition of** $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$ :

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: (133)

$$|(a_i'')^{(7)}(T_{37}',t) - (a_i'')^{(7)}(T_{37},t)| \le (\hat{k}_{36})^{(7)}|T_{37} - T_{37}'|e^{-(\hat{M}_{36})^{(7)}t}$$

$$|(b_i'')^{(7)}((G_{39})',t) - (b_i'')^{(7)}\big((G_{39}),t\big)| < (\hat{k}_{36})^{(7)}||(G_{39}) - (G_{39})'||e^{-(\hat{M}_{36})^{(7)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}',t)$ and$(a_i'')^{(7)}(T_{37},t)$ . $(T_{37}',t)$ and $(T_{37},t)$ are points belonging to the interval $\big[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}\big]$. It is to be noted that $(a_i'')^{(7)}(T_{37},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37},t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$ : (134)

(OO) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$, are positive constants

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$$

**Definition of** $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$ : (135)

(PP) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)}(\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(b_i)^{(7)} + (b_i')^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)}(\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40,41,42 \tag{136}$$

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

**Definition of** $(p_i)^{(8)}$, $(r_i)^{(8)}$: (137)

$$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)} \tag{138}$$

$$(b_i'')^{(8)}((G_{43}), t) \leq (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)} \tag{139}$$

$$\lim_{T_2 \to \infty} (a_i'')^{(8)}(T_{41}, t) = (p_i)^{(8)} \tag{140}$$

$$\lim_{G \to \infty} (b_i'')^{(8)}\left((G_{43}), t\right) = (r_i)^{(8)} \tag{141}$$

**Definition of** $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$ :

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)} |T_{41} - T_{41}'| e^{-(\hat{M}_{40})^{(8)} t} \tag{142}$$

$$|(b_i'')^{(8)}((G_{43})', t) - (b_i'')^{(8)}\left((G_{43}), t\right)| < (\hat{k}_{40})^{(8)} ||(G_{43}) - (G_{43})'|| e^{-(\hat{M}_{40})^{(8)} t} \tag{143}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}', t)$ and $(a_i'')^{(8)}(T_{41}, t)$ . $(T_{41}', t)$ and $(T_{41}, t)$ are points belonging to the interval $\left[(\hat{k}_{40})^{(8)}, (\hat{M}_{40})^{(8)}\right]$. It is to be noted that $(a_i'')^{(8)}(T_{41}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{40})^{(8)} = 1$ then the function $(a_i'')^{(8)}(T_{41}, t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$ :

$(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$, are positive constants

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}, \frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1 \tag{144}$$

**Definition of** $(\hat{P}_{40})^{(8)}, (\hat{Q}_{40})^{(8)}$ :

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}, (\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)}, (a_i')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (p_i)^{(8)}$, $(r_i)^{(8)}, i = 40,41,42,$
Satisfy the inequalities

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(a_i)^{(8)} + (a_i')^{(8)} + (\hat{A}_{40})^{(8)} + (\hat{P}_{40})^{(8)} (\hat{k}_{40})^{(8)}] < 1 \tag{145}$$

$$\frac{1}{(\widehat{M}_{40})^{(8)}}[\ (b_i)^{(8)} + (b_i')^{(8)} + \ (\hat{B}_{40})^{(8)} + \ (\hat{Q}_{40})^{(8)}\ (\hat{k}_{40})^{(8)}] < 1 \qquad 146$$

Where we suppose

$$(a_i)^{(9)}, (a_i')^{(9)}, (a_i'')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (b_i'')^{(9)} > 0, \quad i,j = 44,45,46 \qquad 146\ A$$

The functions $(a_i'')^{(9)}, (b_i'')^{(9)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(9)}$, $(r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45}, t) \leq (p_i)^{(9)} \leq (\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47}, t) \leq \ (r_i)^{(9)} \leq (b_i')^{(9)} \leq (\hat{B}_{44})^{(9)}$$

$$\lim_{T_2 \to \infty} (a_i'')^{(9)}(T_{45}, t) = (p_i)^{(9)}$$

$$\lim_{G \to \infty} (b_i'')^{(9)}(G_{47}, t) = \ (r_i)^{(9)}$$

**Definition of** $(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}$ :

Where $\boxed{(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}, (p_i)^{(9)}, \ (r_i)^{(9)}}$ are positive constants and $\boxed{i = 44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}', t) - (a_i'')^{(9)}(T_{45}, t)| \leq (\hat{k}_{44})^{(9)} |T_{45} - T_{45}'| e^{-(\widehat{M}_{44})^{(9)} t}$$

$$|(b_i'')^{(9)}((G_{47})', t) - (b_i'')^{(9)}\big((G_{47}), t\big)| < (\hat{k}_{44})^{(9)} ||(G_{47}) - (G_{47})'|| e^{-(\widehat{M}_{44})^{(9)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}', t)$ and$(a_i'')^{(9)}(T_{45}, t)$ . $(T_{45}', t)$ and $(T_{45}, t)$ are points belonging to the interval $\left[(\hat{k}_{44})^{(9)}, (\widehat{M}_{44})^{(9)}\right]$. It is to be noted that $(a_i'')^{(9)}(T_{45}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{44})^{(9)} = 1$ then the function $(a_i'')^{(9)}(T_{45}, t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\widehat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$ :

$(\widehat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$, are positive constants

$$\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}} \ , \frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}} < 1$$

**Definition of** $(\hat{P}_{44})^{(9)}, (\hat{Q}_{44})^{(9)}$ :

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\widehat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}, (\hat{A}_{44})^{(9)} and\ (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)}, (a_i')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (p_i)^{(9)}, \ (r_i)^{(9)}, i = 44,45,46,$
satisfy the inequalities

$$\frac{1}{(\widehat{M}_{44})^{(9)}}[(a_i)^{(9)} + (a_i')^{(9)} + (\hat{A}_{44})^{(9)} + (\hat{P}_{44})^{(9)}(\hat{k}_{44})^{(9)}] < 1$$

$$\frac{1}{(\widehat{M}_{44})^{(9)}}[(b_i)^{(9)} + (b_i')^{(9)} + (\hat{B}_{44})^{(9)} + (\hat{Q}_{44})^{(9)}(\hat{k}_{44})^{(9)}] < 1$$

**Theorem 1:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0), T_i(0)$ :

$$G_i(t) \le (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 2 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

**Definition of** $G_i(0), T_i(0)$

$$G_i(t) \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} \quad , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} \quad , \quad T_i(0) = T_i^0 > 0$$

**Theorem 3 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t) \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t} \quad , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t} \quad , \quad T_i(0) = T_i^0 > 0$$

**Theorem 4 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

**Definition of** $G_i(0), T_i(0)$ :

$$G_i(t) \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 5 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

**Definition of** $G_i(0), T_i(0)$ :

$$G_i(t) \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 6 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

**Definition of** $G_i(0), T_i(0)$ :

$$G_i(t) \le (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$T_i(t) \leq (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Theorem 7:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

**Definition of** $G_i(0), T_i(0)$ :

$G_i(t) \leq (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Theorem 8:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

**Definition of** $G_i(0), T_i(0)$ :

$G_i(t) \leq (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Theorem 9:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

**Definition of** $G_i(0), T_i(0)$ :

$G_i(t) \leq (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Proof:** Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$G_i(0) = G_i^0$ , $T_i(0) = T_i^0$ , $G_i^0 \leq (\hat{P}_{13})^{(1)}$ , $T_i^0 \leq (\hat{Q}_{13})^{(1)}$, 155

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t}$ 156

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t}$ 157

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[ (a_{13})^{(1)} G_{14}(s_{(13)}) - \left( (a'_{13})^{(1)} + a''_{13})^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \right) G_{13}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[ (a_{14})^{(1)} G_{13}(s_{(13)}) - \left( (a'_{14})^{(1)} + (a''_{14})^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \right) G_{14}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[ (a_{15})^{(1)} G_{14}(s_{(13)}) - \left( (a'_{15})^{(1)} + (a''_{15})^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \right) G_{15}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[ (b_{13})^{(1)} T_{14}(s_{(13)}) - \left( (b'_{13})^{(1)} - (b''_{13})^{(1)} (G(s_{(13)}), s_{(13)}) \right) T_{13}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^{0} + \int_0^t \left[ (b_{14})^{(1)} T_{13}(s_{(13)}) - \left( (b'_{14})^{(1)} - (b''_{14})^{(1)} \left( G(s_{(13)}), s_{(13)} \right) \right) T_{14}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{\mathrm{T}}_{15}(\mathrm{t}) = \mathrm{T}_{15}^{0} + \int_0^t \left[ (b_{15})^{(1)} T_{14}(s_{(13)}) - \left( (b'_{15})^{(1)} - (b''_{15})^{(1)} \left( G(s_{(13)}), s_{(13)} \right) \right) T_{15}(s_{(13)}) \right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{16})^{(2)}, T_i^0 \le (\hat{Q}_{16})^{(2)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$$

By 160

$$\bar{G}_{16}(t) = G_{16}^{0} + \int_0^t \left[ (a_{16})^{(2)} G_{17}(s_{(16)}) - \left( (a'_{16})^{(2)} + a''_{16})^{(2)} (T_{17}(s_{(16)}), s_{(16)}) \right) G_{16}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^{0} + \int_0^t \left[ (a_{17})^{(2)} G_{16}(s_{(16)}) - \left( (a'_{17})^{(2)} + (a''_{17})^{(2)} (T_{17}(s_{(16)}), s_{(17)}) \right) G_{17}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^{0} + \int_0^t \left[ (a_{18})^{(2)} G_{17}(s_{(16)}) - \left( (a'_{18})^{(2)} + (a''_{18})^{(2)} (T_{17}(s_{(16)}), s_{(16)}) \right) G_{18}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^{0} + \int_0^t \left[ (b_{16})^{(2)} T_{17}(s_{(16)}) - \left( (b'_{16})^{(2)} - (b''_{16})^{(2)} (G_{19}(s_{(16)}), s_{(16)}) \right) T_{16}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^{0} + \int_0^t \left[ (b_{17})^{(2)} T_{16}(s_{(16)}) - \left( (b'_{17})^{(2)} - (b''_{17})^{(2)} (G_{19}(s_{(16)}), s_{(16)}) \right) T_{17}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^{0} + \int_0^t \left[ (b_{18})^{(2)} T_{17}(s_{(16)}) - \left( (b'_{18})^{(2)} - (b''_{18})^{(2)} (G_{19}(s_{(16)}), s_{(16)}) \right) T_{18}(s_{(16)}) \right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{20})^{(3)}, T_i^0 \le (\hat{Q}_{20})^{(3)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$$

By 161

$$\bar{G}_{20}(t) = G_{20}^{0} + \int_0^t \left[ (a_{20})^{(3)} G_{21}(s_{(20)}) - \left( (a'_{20})^{(3)} + a''_{20})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{22}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^0 + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^0 + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{\mathrm{T}}_{22}(\mathrm{t}) = \mathrm{T}_{22}^0 + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{22}(s_{(20)})\right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{24})^{(4)}, T_i^0 \le (\hat{Q}_{24})^{(4)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

By



$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{26}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{\mathrm{T}}_{26}(\mathrm{t}) = \mathrm{T}_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{26}(s_{(24)})\right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{28})^{(5)}, T_i^0 \le (\hat{Q}_{28})^{(5)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

By 163

$$\bar{G}_{28}(t) = G_{28}^{0} + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^{0} + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^{0} + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{30}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^{0} + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^{0} + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{\mathrm{T}}_{30}(\mathrm{t}) = \mathrm{T}_{30}^{0} + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{30}(s_{(28)})\right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{32})^{(6)}, T_i^0 \le (\hat{Q}_{32})^{(6)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$$

By 164

$$\bar{G}_{32}(t) = G_{32}^{0} + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a'_{32})^{(6)} + a''_{32})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^{0} + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^{0} + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{34}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^{0} + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^{0} + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{\mathrm{T}}_{34}(\mathrm{t}) = \mathrm{T}_{34}^{0} + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{34}(s_{(32)})\right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$ , $T_i(0) = T_i^0$ , $G_i^0 \leq (\hat{P}_{36})^{(7)}$ , $T_i^0 \leq (\hat{Q}_{36})^{(7)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[ (a_{36})^{(7)} G_{37}(s_{(36)}) - \left( (a'_{36})^{(7)} + a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[ (a_{37})^{(7)} G_{36}(s_{(36)}) - \left( (a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[ (a_{38})^{(7)} G_{37}(s_{(36)}) - \left( (a'_{38})^{(7)} + (a''_{38})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{38}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[ (b_{36})^{(7)} T_{37}(s_{(36)}) - \left( (b'_{36})^{(7)} - (b''_{36})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[ (b_{37})^{(7)} T_{36}(s_{(36)}) - \left( (b'_{37})^{(7)} - (b''_{37})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{38}(t) = T_{38}^0 + \int_0^t \left[ (b_{38})^{(7)} T_{37}(s_{(36)}) - \left( (b'_{38})^{(7)} - (b''_{38})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{38}(s_{(36)}) \right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i$: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$ , $T_i(0) = T_i^0$ , $G_i^0 \leq (\hat{P}_{40})^{(8)}$ , $T_i^0 \leq (\hat{Q}_{40})^{(8)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[ (a_{40})^{(8)} G_{41}(s_{(40)}) - \left( (a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[ (a_{41})^{(8)} G_{40}(s_{(40)}) - \left( (a'_{41})^{(8)} + (a''_{41})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)}\left(T_{41}(s_{(40)}), s_{(40)}\right)\right) G_{42}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^0 + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)}\left(G_{43}(s_{(40)}), s_{(40)}\right)\right) T_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^0 + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)}\left(G_{43}(s_{(40)}), s_{(40)}\right)\right) T_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{\mathrm{T}}_{42}(\mathrm{t}) = \mathrm{T}_{42}^0 + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)}\left(G_{43}(s_{(40)}), s_{(40)}\right)\right) T_{42}(s_{(40)})\right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** 166

Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy A

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{44})^{(9)},\ T_i^0 \le (\hat{Q}_{44})^{(9)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''_{44})^{(9)}\left(T_{45}(s_{(44)}), s_{(44)}\right)\right) G_{44}(s_{(44)})\right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)}\left(T_{45}(s_{(44)}), s_{(44)}\right)\right) G_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)}\left(T_{45}(s_{(44)}), s_{(44)}\right)\right) G_{46}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)}\left(G_{47}(s_{(44)}), s_{(44)}\right)\right) T_{44}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)}\left(G_{47}(s_{(44)}), s_{(44)}\right)\right) T_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{\mathrm{T}}_{46}(\mathrm{t}) = \mathrm{T}_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)}\left(G_{47}(s_{(44)}), s_{(44)}\right)\right) T_{46}(s_{(44)})\right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}}\right)\right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left(e^{(\hat{M}_{13})^{(1)} t} - 1\right)$$

From which it follows that 168

$$(G_{13}(t)-G_{13}^{0})e^{-(\hat{M}_{13})^{(1)}t}\leq\frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}}\left[((\hat{P}_{13})^{(1)}+G_{14}^{0})e^{\left(-\frac{(\hat{P}_{13})^{(1)}+G_{14}^{0}}{G_{14}^{0}}\right)}+(\hat{P}_{13})^{(1)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t)\leq G_{16}^{0}+\int_{0}^{t}\left[(a_{16})^{(2)}\left(G_{17}^{0}+(\hat{P}_{16})^{(6)}e^{(\hat{M}_{16})^{(2)}s_{(16)}}\right)\right]ds_{(16)}= \quad (169)$$

$$\left(1+(a_{16})^{(2)}t\right)G_{17}^{0}+\frac{(a_{16})^{(2)}(\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}}\left(e^{(\hat{M}_{16})^{(2)}t}-1\right)$$

From which it follows that (170)

$$(G_{16}(t)-G_{16}^{0})e^{-(\hat{M}_{16})^{(2)}t}\leq\frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}}\left[((\hat{P}_{16})^{(2)}+G_{17}^{0})e^{\left(-\frac{(\hat{P}_{16})^{(2)}+G_{17}^{0}}{G_{17}^{0}}\right)}+(\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that (171)

$$G_{20}(t)\leq G_{20}^{0}+\int_{0}^{t}\left[(a_{20})^{(3)}\left(G_{21}^{0}+(\hat{P}_{20})^{(3)}e^{(\hat{M}_{20})^{(3)}s_{(20)}}\right)\right]ds_{(20)}=$$

$$\left(1+(a_{20})^{(3)}t\right)G_{21}^{0}+\frac{(a_{20})^{(3)}(\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left(e^{(\hat{M}_{20})^{(3)}t}-1\right)$$

From which it follows that (172)

$$(G_{20}(t)-G_{20}^{0})e^{-(\hat{M}_{20})^{(3)}t}\leq\frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left[((\hat{P}_{20})^{(3)}+G_{21}^{0})e^{\left(-\frac{(\hat{P}_{20})^{(3)}+G_{21}^{0}}{G_{21}^{0}}\right)}+(\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that (173)

$$G_{24}(t)\leq G_{24}^{0}+\int_{0}^{t}\left[(a_{24})^{(4)}\left(G_{25}^{0}+(\hat{P}_{24})^{(4)}e^{(\hat{M}_{24})^{(4)}s_{(24)}}\right)\right]ds_{(24)}=$$

$$\left(1+(a_{24})^{(4)}t\right)G_{25}^{0}+\frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left(e^{(\hat{M}_{24})^{(4)}t}-1\right)$$

From which it follows that (174)

$$(G_{24}(t)-G_{24}^{0})e^{-(\hat{M}_{24})^{(4)}t}\leq\frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left[((\hat{P}_{24})^{(4)}+G_{25}^{0})e^{\left(-\frac{(\hat{P}_{24})^{(4)}+G_{25}^{0}}{G_{25}^{0}}\right)}+(\hat{P}_{24})^{(4)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t) \le G_{28}^0 + \int_0^t \left[ (a_{28})^{(5)} \left( G_{29}^0 + (\hat{P}_{28})^{(5)} e^{(\tilde{M}_{28})^{(5)} s_{(28)}} \right) \right] ds_{(28)} =$$

$$\left(1 + (a_{28})^{(5)} t\right) G_{29}^0 + \frac{(a_{28})^{(5)} (\hat{P}_{28})^{(5)}}{(\tilde{M}_{28})^{(5)}} \left( e^{(\tilde{M}_{28})^{(5)} t} - 1 \right)$$

From which it follows that 175

$$(G_{28}(t) - G_{28}^0) e^{-(\tilde{M}_{28})^{(5)} t} \le \frac{(a_{28})^{(5)}}{(\tilde{M}_{28})^{(5)}} \left[ ((\hat{P}_{28})^{(5)} + G_{29}^0) e^{\left( - \frac{(\hat{P}_{28})^{(5)} + G_{29}^0}{G_{29}^0} \right)} + (\hat{P}_{28})^{(5)} \right]$$

$(G_i^0)$ is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t) \le G_{32}^0 + \int_0^t \left[ (a_{32})^{(6)} \left( G_{33}^0 + (\hat{P}_{32})^{(6)} e^{(\tilde{M}_{32})^{(6)} s_{(32)}} \right) \right] ds_{(32)} =$$

$$\left(1 + (a_{32})^{(6)} t\right) G_{33}^0 + \frac{(a_{32})^{(6)} (\hat{P}_{32})^{(6)}}{(\tilde{M}_{32})^{(6)}} \left( e^{(\tilde{M}_{32})^{(6)} t} - 1 \right)$$

From which it follows that 177

$$(G_{32}(t) - G_{32}^0) e^{-(\tilde{M}_{32})^{(6)} t} \le \frac{(a_{32})^{(6)}}{(\tilde{M}_{32})^{(6)}} \left[ ((\hat{P}_{32})^{(6)} + G_{33}^0) e^{\left( - \frac{(\hat{P}_{32})^{(6)} + G_{33}^0}{G_{33}^0} \right)} + (\hat{P}_{32})^{(6)} \right]$$

$(G_i^0)$ is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

(g) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t) \le G_{36}^0 + \int_0^t \left[ (a_{36})^{(7)} \left( G_{37}^0 + (\hat{P}_{36})^{(7)} e^{(\tilde{M}_{36})^{(7)} s_{(36)}} \right) \right] ds_{(36)} =$$

$$\left(1 + (a_{36})^{(7)} t\right) G_{37}^0 + \frac{(a_{36})^{(7)} (\hat{P}_{36})^{(7)}}{(\tilde{M}_{36})^{(7)}} \left( e^{(\tilde{M}_{36})^{(7)} t} - 1 \right)$$

From which it follows that

$$(G_{36}(t) - G_{36}^0) e^{-(\tilde{M}_{36})^{(7)} t} \le \frac{(a_{36})^{(7)}}{(\tilde{M}_{36})^{(7)}} \left[ ((\hat{P}_{36})^{(7)} + G_{37}^0) e^{\left( - \frac{(\hat{P}_{36})^{(7)} + G_{37}^0}{G_{37}^0} \right)} + (\hat{P}_{36})^{(7)} \right]$$

$(G_i^0)$ is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t) \le G_{40}^0 + \int_0^t \left[ (a_{40})^{(8)} \left( G_{41}^0 + (\hat{P}_{40})^{(8)} e^{(\tilde{M}_{40})^{(8)} s_{(40)}} \right) \right] ds_{(40)} = \quad 180$$

$$\left(1 + (a_{40})^{(8)} t\right) G_{41}^0 + \frac{(a_{40})^{(8)} (\hat{P}_{40})^{(8)}}{(\tilde{M}_{40})^{(8)}} \left( e^{(\tilde{M}_{40})^{(8)} t} - 1 \right)$$

From which it follows that 181

$$(G_{40}(t)-G_{40}^{0})e^{-(\hat{M}_{40})^{(8)}t}\leq\frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left[((\hat{P}_{40})^{(8)}+G_{41}^{0})e^{\left(-\frac{(\hat{P}_{40})^{(8)}+G_{41}^{0}}{G_{41}^{0}}\right)}+(\hat{P}_{40})^{(8)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 8

Analogous inequalities hold also for $G_{41}, G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t)\leq G_{44}^{0}+\int_{0}^{t}\left[(a_{44})^{(9)}\left(G_{45}^{0}+(\hat{P}_{44})^{(9)}e^{(\hat{M}_{44})^{(9)}s_{(44)}}\right)\right]ds_{(44)}=$$

$$\left(1+(a_{44})^{(9)}t\right)G_{45}^{0}+\frac{(a_{44})^{(9)}(\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}}\left(e^{(\hat{M}_{44})^{(9)}t}-1\right)$$

From which it follows that

$$(G_{44}(t)-G_{44}^{0})e^{-(\hat{M}_{44})^{(9)}t}\leq\frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}}\left[((\hat{P}_{44})^{(9)}+G_{45}^{0})e^{\left(-\frac{(\hat{P}_{44})^{(9)}+G_{45}^{0}}{G_{45}^{0}}\right)}+(\hat{P}_{44})^{(9)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 9

Analogous inequalities hold also for $G_{45}, G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}<1$ and to choose 182

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[(\hat{P}_{13})^{(1)}+((\hat{P}_{13})^{(1)}+G_j^0)e^{-\left(\frac{(\hat{P}_{13})^{(1)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{13})^{(1)} \quad 183$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[((\hat{Q}_{13})^{(1)}+T_j^0)e^{-\left(\frac{(\hat{Q}_{13})^{(1)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{13})^{(1)}\right]\leq(\hat{Q}_{13})^{(1)} \quad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left((G^{(1)},T^{(1)}),(G^{(2)},T^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\hat{M}_{13})^{(1)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\hat{M}_{13})^{(1)}t}\}$$

Indeed if we denote

**Definition of** $\tilde{G},\tilde{T}:(\tilde{G},\tilde{T})=\mathcal{A}^{(1)}(G,T)$

It results

$$\left|\tilde{G}_{13}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{13})^{(1)} \left|G_{14}^{(1)}-G_{14}^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\, ds_{(13)} +$$

$$\int_0^t \{(a'_{13})^{(1)} \left|G_{13}^{(1)}-G_{13}^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} +$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)}, s_{(13)}\right)\left|G_{13}^{(1)}-G_{13}^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} e^{(\widehat{M}_{13})^{(1)}s_{(13)}} +$$

$$G_{13}^{(2)} \left|(a''_{13})^{(1)}\left(T_{14}^{(1)}, s_{(13)}\right) - (a''_{13})^{(1)}\left(T_{14}^{(2)}, s_{(13)}\right)\right|\ e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} e^{(\widehat{M}_{13})^{(1)}s_{(13)}} \} ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\left|G^{(1)}-G^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)}t} \leq \qquad 186$$
$$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)} + (a'_{13})^{(1)} + (\widehat{A}_{13})^{(1)} + (\widehat{P}_{13})^{(1)}(\widehat{k}_{13})^{(1)}\right) d\left(\left(G^{(1)}, T^{(1)};\ G^{(2)}, T^{(2)}\right)\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 1:** The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t}$ *and* $(\widehat{Q}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i = 13,14,15$ depend only on $T_{14}$ and respectively on $G(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any $t$ where $G_i(t) = 0$ *and* $T_i(t) = 0$

From 19 to 24 it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(1)} - (a_i'')^{(1)}(T_{14}(s_{(13)}), s_{(13)})\} ds_{(13)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(1)}t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2$ *and* $\left((\widehat{M}_{13})^{(1)}\right)_3$ : 187

**Remark 3**: if $G_{13}$ is bounded, the same property have also $G_{14}$ *and* $G_{15}$ . indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a'_{14})^{(1)} G_{14}$ and by integrating

$$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1 / (a'_{14})^{(1)}$$

In the same way , one can obtain

$$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2 / (a'_{15})^{(1)}$$

If $G_{14}$ *or* $G_{15}$ is bounded, the same property follows for $G_{13}$ , $G_{15}$ and $G_{13}$ , $G_{14}$ respectively.

**Remark 4:** If $G_{13}$ *is* bounded, from below, the same property holds for $G_{14}$ *and* $G_{15}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{14}$ is bounded from below. 188

**Remark 5:** If $T_{13}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}\,(G(t),t)) = (b_{14}')^{(1)}$ then $T_{14} \to \infty$. 189

**Definition of** $(m)^{(1)}$ and $\varepsilon_1$ :

Indeed let $t_1$ be so that for $t > t_1$

$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t),t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_1}$ By taking now $\varepsilon_1$ sufficiently small one sees that $T_{14}$ is unbounded.
The same property holds for $T_{15}$ if $\lim_{t\to\infty}(b_{15}'')^{(1)}\,(G(t),t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}$ , $\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}$ *and* $(\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[(\hat{P}_{16})^{(2)} + \left((\hat{P}_{16})^{(2)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{16})^{(2)}\right] \leq (\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

**Definition of** $\widetilde{G_{19}}, \widetilde{T_{19}}$ : $\left(\widetilde{G_{19}}, \widetilde{T_{19}}\right) = \mathcal{A}^{(2)}(G_{19}, T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{16})^{(2)} \left|G_{17}^{(1)}-G_{17}^{(2)}\right| e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\, ds_{(16)} +$$

$$\int_0^t \{(a'_{16})^{(2)} \left|G_{16}^{(1)}-G_{16}^{(2)}\right| e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} +$$

$$(a''_{16})^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right)\left|G_{16}^{(1)}-G_{16}^{(2)}\right| e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} e^{(\widehat{M}_{16})^{(2)}s_{(16)}} +$$

$$G_{16}^{(2)} \left|(a''_{16})^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right) - (a''_{16})^{(2)}\left(T_{17}^{(2)}, s_{(16)}\right)\right|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\} ds_{(16)}$$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0, t]$ 197

From the hypotheses it follows

$$\left|(G_{19})^{(1)}-(G_{19})^{(2)}\right| e^{-(\widehat{M}_{16})^{(2)}t} \leq$$
$$\frac{1}{(\widehat{M}_{16})^{(2)}}\left((a_{16})^{(2)} + (a'_{16})^{(2)} + (\widehat{A}_{16})^{(2)} + (\widehat{P}_{16})^{(2)}(\widehat{k}_{16})^{(2)}\right) d\left(((G_{19})^{(1)}, (T_{19})^{(1)};\ (G_{19})^{(2)}, (T_{19})^{(2)})\right)$$

And analogous inequalities for $G_i$ and $T_i$. Taking into account the hypothesis the result follows 198

**Remark 6:** The fact that we supposed $(a''_{16})^{(2)}$ and $(b''_{16})^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{16})^{(2)} e^{(\widehat{M}_{16})^{(2)}t}$ and $(\widehat{Q}_{16})^{(2)} e^{(\widehat{M}_{16})^{(2)}t}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i = 16,17,18$ depend only on $T_{17}$ and respectively on $(G_{19})$(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 7:** There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$ 200

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(2)} - (a_i'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})\} ds_{(16)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(2)}t)} > 0 \quad \text{for } t > 0$$

**Definition of** $\left((\widehat{M}_{16})^{(2)}\right)_1$, $\left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$: 201

**Remark 8:** if $G_{16}$ is bounded, the same property have also $G_{17}$ and $G_{18}$ . indeed if

$G_{16} < (\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt} \leq \left((\widehat{M}_{16})^{(2)}\right)_1 - (a'_{17})^{(2)} G_{17}$ and by integrating

$$G_{17} \leq \left((\widehat{M}_{16})^{(2)}\right)_2 = G_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1 / (a'_{17})^{(2)}$$

In the same way , one can obtain

$$G_{18} \leq \left((\widehat{M}_{16})^{(2)}\right)_3 = G_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2 / (a'_{18})^{(2)}$$

If $G_{17}$ or $G_{18}$ is bounded, the same property follows for $G_{16}$, $G_{18}$ and $G_{16}$, $G_{17}$ respectively.

**Remark 9**: If $G_{16}$ is bounded, from below, the same property holds for $G_{17}$ and $G_{18}$. The proof is analogous with the preceding one. An analogous property is true if $G_{17}$ is bounded from below. 202

**Remark 10:** If $T_{16}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t),t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$. 203

**Definition of** $(m)^{(2)}$ and $\varepsilon_2$ :

Indeed let $t_2$ be so that for $t > t_2$

$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t),t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right), \quad t = \log\frac{2}{\varepsilon_2}$ By taking now $\varepsilon_2$ sufficiently small one sees that $T_{17}$ is unbounded. 205
The same property holds for $T_{18}$ if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t),t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\bar{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\bar{M}_{20})^{(3)}} < 1$ and to choose 207

$(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\bar{M}_{20})^{(3)}}\left[(\hat{P}_{20})^{(3)} + \left((\hat{P}_{20})^{(3)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\bar{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{20})^{(3)}\right] \leq (\hat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself 210

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)},(T_{23})^{(1)}),((G_{23})^{(2)},(T_{23})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\bar{M}_{20})^{(3)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\bar{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

**Definition of** $\widetilde{G_{23}}, \widetilde{T_{23}}$ :$\left(\widetilde{(G_{23})},\widetilde{(T_{23})}\right) = \mathcal{A}^{(3)}\left((G_{23}),(T_{23})\right)$

It results 213

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{20})^{(3)} \left|G_{21}^{(1)}-G_{21}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} e^{(\widehat{M}_{20})^{(3)} s_{(20)}}\, ds_{(20)} +$$

$$\int_0^t \{(a'_{20})^{(3)} \left|G_{20}^{(1)}-G_{20}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} +$$

$$(a''_{20})^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} e^{(\widehat{M}_{20})^{(3)} s_{(20)}} +$$

$$G_{20}^{(2)} \left|(a''_{20})^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)}, s_{(20)}\right)\right|\ e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} e^{(\widehat{M}_{20})^{(3)} s_{(20)}}\} ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)} t} \leq \qquad 214$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+\ (a'_{20})^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\widehat{k}_{20})^{(3)}\right) d\left(\left((G_{23})^{(1)}, (T_{23})^{(1)};\ (G_{23})^{(2)}, (T_{23})^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

**Remark 11:** The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)} e^{(\widehat{M}_{20})^{(3)} t}\ and\ (\widehat{Q}_{20})^{(3)} e^{(\widehat{M}_{20})^{(3)} t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i = 20,21,22$ depend only on $T_{21}$ and respectively on $(G_{23})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 12:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 216

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(3)} - (a_i'')^{(3)}(T_{21}(s_{(20)}), s_{(20)})\} ds_{(20)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(3)} t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2\ and\ \left((\widehat{M}_{20})^{(3)}\right)_3$ : 217

**Remark 13:** if $G_{20}$ is bounded, the same property have also $G_{21}\ and\ G_{22}$ . indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a'_{21})^{(3)} G_{21}$ and by integrating

$$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1 / (a'_{21})^{(3)}$$

In the same way , one can obtain

$$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2 / (a'_{22})^{(3)}$$

If $G_{21}$ *or* $G_{22}$ is bounded, the same property follows for $G_{20}$ , $G_{22}$ and $G_{20}$ , $G_{21}$ respectively.

**Remark 14:** If $G_{20}$ *is* bounded, from below, the same property holds for $G_{21}$ *and* $G_{22}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{21}$ is bounded from below. 218

**Remark 15:** If $T_{20}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t),t\right)=(b_{21}')^{(3)}$ then $T_{21}\to\infty$. 219

**Definition of** $(m)^{(3)}$ and $\varepsilon_3$ :

Indeed let $t_3$ be so that for $t > t_3$

$(b_{21})^{(3)}-(b_i'')^{(3)}\left((G_{23})(t),t\right)<\varepsilon_3, T_{20}(t)>(m)^{(3)}$

Then $\frac{dT_{21}}{dt}\geq(a_{21})^{(3)}(m)^{(3)}-\varepsilon_3 T_{21}$ which leads to 220

$T_{21}\geq\left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1-e^{-\varepsilon_3 t})+T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t}=\frac{1}{2}$ it results

$T_{21}\geq\left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right),\quad t=log\frac{2}{\varepsilon_3}$ By taking now $\varepsilon_3$ sufficiently small one sees that $T_{21}$ is unbounded.
The same property holds for $T_{22}$ if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t),t\right)=(b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}$ , $\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}}<1$ and to choose 221

$(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\left[(\hat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \quad 222$$

$$\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \quad 223$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself 224

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric 225

$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{24})^{(4)}t}\}$$

Indeed if we denote

**Definition of** $\widetilde{(G_{27})},\widetilde{(T_{27})}$ : $\left(\widetilde{(G_{27})},\widetilde{(T_{27})}\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_{i}^{(2)}\right|\leq\int_{0}^{t}(a_{24})^{(4)}\left|G_{25}^{(1)}-G_{25}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\,ds_{(24)}+$$

$$\int_{0}^{t}\{(a'_{24})^{(4)}\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$G_{24}^{(2)}\left|(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)-(a''_{24})^{(4)}\left(T_{25}^{(2)},s_{(24)}\right)\right|\;e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\}ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}t}\leq \qquad 226$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)}+\;(a'_{24})^{(4)}+(\widehat{A}_{24})^{(4)}+(\widehat{P}_{24})^{(4)}(\widehat{k}_{24})^{(4)}\right)d\left(\left((G_{27})^{(1)},(T_{27})^{(1)};\;(G_{27})^{(2)},(T_{27})^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

**Remark 16:** The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}\ and\ (\widehat{Q}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_{+}$. 227

If instead of proving the existence of the solution on $\mathbb{R}_{+}$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i=24,25,26$ depend only on $\mathrm{T}_{25}$ and respectively on $(G_{27})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 17:** There does not exist any $t$ where $G_i\,(t)=0\ and\ T_i\,(t)=0$ 228

it results

$$G_i\,(t)\geq G_i^0e^{\left[-\int_0^t\{(a_i')^{(4)}-(a_i'')^{(4)}(T_{25}(s_{(24)}),s_{(24)})\}ds_{(24)}\right]}\geq 0$$

$$T_i\,(t)\geq T_i^0e^{\left(-(b_i')^{(4)}t\right)}>0\quad \text{for t}>0$$

**Definition of** $\left((\widehat{M}_{24})^{(4)}\right)_1,\left((\widehat{M}_{24})^{(4)}\right)_2\ and\ \left((\widehat{M}_{24})^{(4)}\right)_3$ : 229

**Remark 18:** if $G_{24}$ is bounded, the same property have also $G_{25}\ and\ G_{26}$ . indeed if

$G_{24}<(\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt}\leq\left((\widehat{M}_{24})^{(4)}\right)_1-(a'_{25})^{(4)}G_{25}$ and by integrating

$$G_{25}\leq\left((\widehat{M}_{24})^{(4)}\right)_2=G_{25}^0+2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1/(a'_{25})^{(4)}$$

In the same way , one can obtain

$$G_{26}\leq\left((\widehat{M}_{24})^{(4)}\right)_3=G_{26}^0+2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2/(a'_{26})^{(4)}$$

If $G_{25}$ *or* $G_{26}$ is bounded, the same property follows for $G_{24}$ , $G_{26}$ and $G_{24}$ , $G_{25}$ respectively.

**Remark 19:** If $G_{24}$ *is* bounded, from below, the same property holds for $G_{25}$ *and* $G_{26}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{25}$ is bounded from below. 230

**Remark 20:** If $T_{24}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}((G_{27})(t),t)) = (b_{25}')^{(4)}$ then $T_{25} \to \infty$. 231

**Definition of** $(m)^{(4)}$ and $\varepsilon_4$ :

Indeed let $t_4$ be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t),t) < \varepsilon_4, T_{24}(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right)$, $t = log\frac{2}{\varepsilon_4}$ By taking now $\varepsilon_4$ sufficiently small one sees that $T_{25}$ is unbounded.
The same property holds for $T_{26}$ if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t),t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for $G_{29}, G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}$ , $\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)} + \left((\hat{P}_{28})^{(5)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{28})^{(5)}\right] \leq (\hat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$$d\left(((G_{31})^{(1)},(T_{31})^{(1)}),((G_{31})^{(2)},(T_{31})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

**Definition of** $\widetilde{(G_{31})}, \widetilde{(T_{31})}$ : $\left(\widetilde{(G_{31})}, \widetilde{(T_{31})}\right) = \mathcal{A}^{(5)}\left((G_{31}), (T_{31})\right)$

It results

$\left|\tilde{G}_{28}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{28})^{(5)} \left|G_{29}^{(1)} - G_{29}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}}\, ds_{(28)} +$

$\int_0^t \{(a'_{28})^{(5)} \left|G_{28}^{(1)} - G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} +$

$(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right)\left|G_{28}^{(1)} - G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}} +$

$G_{28}^{(2)} \left|(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right) - (a''_{28})^{(5)}\left(T_{29}^{(2)}, s_{(28)}\right)\right| \; e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}} \} ds_{(28)}$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$\left|(G_{31})^{(1)} - (G_{31})^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} t} \leq$ 237

$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)} + (a'_{28})^{(5)} + (\widehat{A}_{28})^{(5)} + (\widehat{P}_{28})^{(5)} (\widehat{k}_{28})^{(5)}\right) d\left(\left((G_{31})^{(1)}, (T_{31})^{(1)}; (G_{31})^{(2)}, (T_{31})^{(2)}\right)\right)$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 21:** The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$ *and* $(\widehat{Q}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$ respectively of $\mathbb{R}_+$. 238

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on $T_{29}$ and respectively on $(G_{31})$(*and not on* $t$) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 22:** There does not exist any $t$ where $G_i(t) = 0$ *and* $T_i(t) = 0$ 239

it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(5)} - (a_i'')^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(5)} t\right)} > 0$ for t > 0

**Definition of** $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2$ *and* $\left((\widehat{M}_{28})^{(5)}\right)_3$ : 240

**Remark 23:** if $G_{28}$ is bounded, the same property have also $G_{29}$ *and* $G_{30}$ . indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a'_{29})^{(5)} G_{29}$ and by integrating

$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1 / (a'_{29})^{(5)}$

In the same way , one can obtain

$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2/(a'_{30})^{(5)}$

If $G_{29}$ *or* $G_{30}$ is bounded, the same property follows for $G_{28}$, $G_{30}$ and $G_{28}$, $G_{29}$ respectively.

**Remark 24:** If $G_{28}$ *is* bounded, from below, the same property holds for $G_{29}$ *and* $G_{30}$. The proof is analogous with the preceding one. An analogous property is true if $G_{29}$ is bounded from below. 241

**Remark 25:** If $T_{28}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}((G_{31})(t),t)) = (b'_{29})^{(5)}$ then $T_{29} \to \infty$. 242

**Definition of** $(m)^{(5)}$ and $\varepsilon_5$ :

Indeed let $t_5$ be so that for $t > t_5$

$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t),t) < \varepsilon_5, T_{28}(t) > (m)^{(5)}$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\frac{2}{\varepsilon_5}$ By taking now $\varepsilon_5$ sufficiently small one sees that $T_{29}$ is unbounded.
The same property holds for $T_{30}$ if $\lim_{t\to\infty}(b_{30}'')^{(5)}\left((G_{31})(t),t\right) = (b'_{30})^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{33}, G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}$, $\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$ and to choose 244

$(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[(\hat{P}_{32})^{(6)} + \left((\hat{P}_{32})^{(6)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{32})^{(6)} \quad 245$$

$$\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[\left((\hat{Q}_{32})^{(6)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{32})^{(6)}\right] \leq (\hat{Q}_{32})^{(6)} \quad 246$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric 247

$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right) =$

$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{32})^{(6)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{32})^{(6)}t}\}$

Indeed if we denote

**Definition of** $\widetilde{(G_{35})}, \widetilde{(T_{35})}$ : $\left(\widetilde{(G_{35})}, \widetilde{(T_{35})}\right) = \mathcal{A}^{(6)}\left((G_{35}), (T_{35})\right)$

It results

$$\left|\tilde{G}_{32}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{32})^{(6)} \left|G_{33}^{(1)} - G_{33}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}}\, ds_{(32)} +$$

$$\int_0^t \{(a'_{32})^{(6)} \left|G_{32}^{(1)} - G_{32}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} +$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)}, s_{(32)}\right)\left|G_{32}^{(1)} - G_{32}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}} +$$

$$G_{32}^{(2)} \left|(a''_{32})^{(6)}\left(T_{33}^{(1)}, s_{(32)}\right) - (a''_{32})^{(6)}\left(T_{33}^{(2)}, s_{(32)}\right)\right| \; e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}} \} ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|(G_{35})^{(1)} - (G_{35})^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} t} \leq \qquad (248)$$
$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)} + (a'_{32})^{(6)} + (\widehat{A}_{32})^{(6)} + (\widehat{P}_{32})^{(6)} (\widehat{k}_{32})^{(6)}\right) d\left(\left((G_{35})^{(1)}, (T_{35})^{(1)}; \; (G_{35})^{(2)}, (T_{35})^{(2)}\right)\right)$$

And analogous inequalities for $G_i \; and \; T_i$. Taking into account the hypothesis the result follows

**Remark 26:** The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)} t} \; and \; (\widehat{Q}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)} t}$ respectively of $\mathbb{R}_+$. (249)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i = 32{,}33{,}34$ depend only on $T_{33}$ and respectively on $(G_{35})(and \; not \; on \; t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 27:** There does not exist any $t$ where $G_i\,(t) = 0 \; and \; T_i\,(t) = 0$ (250)

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(6)} - (a_i'')^{(6)}(T_{33}(s_{(32)}), s_{(32)})\} ds_{(32)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(6)} t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{32})^{(6)}\right)_1, \left((\widehat{M}_{32})^{(6)}\right)_2 \; and \; \left((\widehat{M}_{32})^{(6)}\right)_3$ : (251)

**Remark 28:** if $G_{32}$ is bounded, the same property have also $G_{33} \; and \; G_{34}$ . indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a'_{33})^{(6)} G_{33}$ and by integrating

$$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_1 / (a'_{33})^{(6)}$$

In the same way , one can obtain

$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_2/(a'_{34})^{(6)}$

If $G_{33}$ *or* $G_{34}$ is bounded, the same property follows for $G_{32}$ , $G_{34}$ and $G_{32}$ , $G_{33}$ respectively.

**Remark 29:** If $G_{32}$ *is* bounded, from below, the same property holds for $G_{33}$ *and* $G_{34}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{33}$ is bounded from below. 252

**Remark 30:** If $T_{32}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}((G_{35})(t),t)) = (b'_{33})^{(6)}$ then $T_{33} \to \infty$. 253

**Definition of** $(m)^{(6)}$ and $\varepsilon_6$ :

Indeed let $t_6$ be so that for $t > t_6$

$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t),t\right) < \varepsilon_6, T_{32}(t) > (m)^{(6)}$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right), \quad t = log\frac{2}{\varepsilon_6}$ By taking now $\varepsilon_6$ sufficiently small one sees that $T_{33}$ is unbounded.
The same property holds for $T_{34}$ if $\lim_{t\to\infty}(b''_{34})^{(6)}\left((G_{35})(t),t(t),t\right) = (b'_{34})^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{37}$ , $G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}$ , $\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}} < 1$ and to choose
$(\widehat{P}_{36})^{(7)}$ and $(\widehat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[(\widehat{P}_{36})^{(7)} + \left((\widehat{P}_{36})^{(7)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{36})^{(7)} \quad 256$$

$$\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[\left((\widehat{Q}_{36})^{(7)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{36})^{(7)}\right] \leq (\widehat{Q}_{36})^{(7)} \quad 257$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself
The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric 258

$d\left(((G_{39})^{(1)},(T_{39})^{(1)}),((G_{39})^{(2)},(T_{39})^{(2)})\right) =$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\tilde{M}_{36})^{(7)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\tilde{M}_{36})^{(7)}t}\}$$

Indeed if we denote

**Definition of** $\widetilde{(G_{39})},\widetilde{(T_{39})}:\left(\widetilde{(G_{39})},\widetilde{(T_{39})}\right)=\mathcal{A}^{(7)}((G_{39}),(T_{39}))$
It results

$$\left|\tilde{G}_{36}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{36})^{(7)}\left|G_{37}^{(1)}-G_{37}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\,ds_{(36)}+$$
$$\int_0^t\{(a'_{36})^{(7)}\left|G_{36}^{(1)}-G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$(a''_{36})^{(7)}\left(T_{37}^{(1)},s_{(36)}\right)\left|G_{36}^{(1)}-G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$G_{36}^{(2)}\left|(a''_{36})^{(7)}\left(T_{37}^{(1)},s_{(36)}\right)-(a''_{36})^{(7)}\left(T_{37}^{(2)},s_{(36)}\right)\right|\ e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\}ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses on it follows

$$\left|(G_{39})^{(1)}-(G_{39})^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}t}\le \qquad 259$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}}\left((a_{36})^{(7)}+(a'_{36})^{(7)}+(\widehat{A}_{36})^{(7)}+(\widehat{P}_{36})^{(7)}(\widehat{k}_{36})^{(7)}\right)d\left(((G_{39})^{(1)},(T_{39})^{(1)};\ (G_{39})^{(2)},(T_{39})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

**Remark 31:** The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}\ and\ (\widehat{Q}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ respectively of $\mathbb{R}_+$. 260

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(7)}$ and $(b_i'')^{(7)},i=36{,}37{,}38$ depend only on $\mathrm{T}_{37}$ and respectively on $(G_{39})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 32:** There does not exist any $t$ where $G_i\,(t)=0\ and\ T_i\,(t)=0$ 261

it results

$$G_i\,(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(7)}-(a_i'')^{(7)}(T_{37}(s_{(36)}),s_{(36)})\}ds_{(36)}\right]}\ge 0$$

$$T_i\,(t)\ge T_i^0e^{\left(-(b_i')^{(7)}t\right)}>0\quad \text{for t}>0$$

**Definition of** $\left((\widehat{M}_{36})^{(7)}\right)_1,\left((\widehat{M}_{36})^{(7)}\right)_2\ and\ \left((\widehat{M}_{36})^{(7)}\right)_3:$ 262

**Remark 33:** if $G_{36}$ is bounded, the same property have also $G_{37}\ and\ G_{38}$ . indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a'_{37})^{(7)} G_{37}$ and by integrating

$$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1/(a'_{37})^{(7)}$$

In the same way , one can obtain

$$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2/(a'_{38})^{(7)}$$

If $G_{37}$ *or* $G_{38}$ is bounded, the same property follows for $G_{36}$ , $G_{38}$ and $G_{36}$ , $G_{37}$ respectively.

**Remark 34:** If $G_{36}$ *is* bounded, from below, the same property holds for $G_{37}$ *and* $G_{38}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{37}$ is bounded from below. 263

**Remark 35:** If $T_{36}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(7)}((G_{39})(t),t)) = (b'_{37})^{(7)}$ then $T_{37} \to \infty$. 264

**Definition of** $(m)^{(7)}$ and $\varepsilon_7$ :

Indeed let $t_7$ be so that for $t > t_7$

$$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t),t) < \varepsilon_7, T_{36}(t) > (m)^{(7)}$$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right)$, $t = log\frac{2}{\varepsilon_7}$ By taking now $\varepsilon_7$ sufficiently small one sees that $T_{37}$ is unbounded.
The same property holds for $T_{38}$ if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t),t\right) = (b'_{38})^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}$ , $\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}} < 1$ and to choose 266
$(\widehat{P}_{40})^{(8)}$ and $(\widehat{Q}_{40})^{(8)}$ large to have

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[(\widehat{P}_{40})^{(8)} + \left((\widehat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{40})^{(8)} \quad 267$$

$$\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[\left((\widehat{Q}_{40})^{(8)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{40})^{(8)}\right] \leq (\widehat{Q}_{40})^{(8)} \quad 268$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)},(T_{43})^{(1)}),((G_{43})^{(2)},(T_{43})^{(2)})\right)= \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

**Definition of** $\widetilde{(G_{43})},\widetilde{(T_{43})}$ : $\left(\widetilde{(G_{43})},\widetilde{(T_{43})}\right)=\mathcal{A}^{(8)}((G_{43}),(T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{40})^{(8)}\left|G_{41}^{(1)}-G_{41}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\,ds_{(40)}+$$
$$\int_0^t\{(a'_{40})^{(8)}\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$G_{40}^{(2)}\left|(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)-(a''_{40})^{(8)}\left(T_{41}^{(2)},s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\}ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0,t]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)}-(G_{43})^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}t}\le \quad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)}+(a'_{40})^{(8)}+(\widehat{A}_{40})^{(8)}+(\widehat{P}_{40})^{(8)}(\widehat{k}_{40})^{(8)}\right)d\left(((G_{43})^{(1)},(T_{43})^{(1)};\ (G_{43})^{(2)},(T_{43})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

**Remark 36:** The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}\ and\ (\widehat{Q}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ respectively of $\mathbb{R}_+$. 274

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i=40,41,42$ depend only on $T_{41}$ and respectively on $(G_{43})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 37** There does not exist any $t$ where $G_i\,(t)=0\ and\ T_i\,(t)=0$ 275

it results

$$G_i\,(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(8)}-(a_i'')^{(8)}(T_{41}(s_{(40)}),s_{(40)})\}ds_{(40)}\right]}\ge 0$$

$$T_i\,(t)\ge T_i^0e^{\left(-(b_i')^{(8)}t\right)}>0\quad \text{for t}>0$$

**Definition of** $\left((\widehat{M}_{40})^{(8)}\right)_1, \left((\widehat{M}_{40})^{(8)}\right)_2$ *and* $\left((\widehat{M}_{40})^{(8)}\right)_3$ : 276

**Remark 38:** if $G_{40}$ is bounded, the same property have also $G_{41}$ *and* $G_{42}$ . indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a'_{41})^{(8)} G_{41}$ and by integrating

$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1/(a'_{41})^{(8)}$

In the same way , one can obtain

$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2/(a'_{42})^{(8)}$

If $G_{41}$ *or* $G_{42}$ is bounded, the same property follows for $G_{40}$ , $G_{42}$ and $G_{40}$ , $G_{41}$ respectively.

**Remark 39:** If $G_{40}$ *is* bounded, from below, the same property holds for $G_{41}$ *and* $G_{42}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{41}$ is bounded from below. 277

**Remark 40:** If $T_{40}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}((G_{43})(t),t)) = (b'_{41})^{(8)}$ then $T_{41} \to \infty$. 278

**Definition of** $(m)^{(8)}$ and $\varepsilon_8$ :

Indeed let $t_8$ be so that for $t > t_8$

$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t),t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right), \quad t = log\frac{2}{\varepsilon_8}$ By taking now $\varepsilon_8$ sufficiently small one sees that $T_{41}$ is unbounded.
The same property holds for $T_{42}$ if $\lim_{t\to\infty}(b''_{42})^{(8)}\left((G_{43})(t),t(t),t\right) = (b'_{42})^{(8)}$

It is now sufficient to take $\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}$ , $\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}} < 1$ and to choose $(\widehat{P}_{44})^{(9)}$ and $(\widehat{Q}_{44})^{(9)}$ large to have 279 A

$$\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[(\widehat{P}_{44})^{(9)} + \left((\widehat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[\left((\widehat{Q}_{44})^{(9)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{44})^{(9)}\right] \leq (\widehat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)},(T_{47})^{(1)}),((G_{47})^{(2)},(T_{47})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{47}}),(\widetilde{T_{47}}):\left((\widetilde{G_{47}}),(\widetilde{T_{47}})\right)=\mathcal{A}^{(9)}\left((G_{47}),(T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{44})^{(9)}\left|G_{45}^{(1)}-G_{45}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\,ds_{(44)}+$$
$$\int_0^t\{(a'_{44})^{(9)}\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$G_{44}^{(2)}\left|(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)-(a''_{44})^{(9)}\left(T_{45}^{(2)},s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\}ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}t}\le$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)}+(a'_{44})^{(9)}+(\widehat{A}_{44})^{(9)}+(\widehat{P}_{44})^{(9)}(\widehat{k}_{44})^{(9)}\right)d\left(((G_{47})^{(1)},(T_{47})^{(1)};\ (G_{47})^{(2)},(T_{47})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis (39,35,36) the result follows

**Remark 41:** The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}\ and\ (\widehat{Q}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i=44,45,46$ depend only on $T_{45}$ and respectively on $(G_{47})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 42:** There does not exist any $t$ where $G_i\,(t)=0\ and\ T_i\,(t)=0$

From 99 to 44 it results

$$G_i\,(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(9)}-(a_i'')^{(9)}(T_{45}(s_{(44)}),s_{(44)})\}ds_{(44)}\right]}\ge 0$$

$$T_i\,(t)\ge T_i^0e^{(-(b_i')^{(9)}t)}>0\quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{44})^{(9)}\right)_1,\left((\widehat{M}_{44})^{(9)}\right)_2\ and\ \left((\widehat{M}_{44})^{(9)}\right)_3$ :

**Remark 43:** if $G_{44}$ is bounded, the same property have also $G_{45}\ and\ G_{46}$ . indeed if

$G_{44} < (\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt} \leq \left((\widehat{M}_{44})^{(9)}\right)_1 - (a'_{45})^{(9)} G_{45}$ and by integrating

$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_1 / (a'_{45})^{(9)}$

In the same way , one can obtain

$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_2 / (a'_{46})^{(9)}$

If $G_{45}$ *or* $G_{46}$ is bounded, the same property follows for $G_{44}$ , $G_{46}$ and $G_{44}$ , $G_{45}$ respectively.

**Remark 44:** If $G_{44}$ *is* bounded, from below, the same property holds for $G_{45}$ *and* $G_{46}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{45}$ is bounded from below.

**Remark 45:** If $T_{44}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t),t\right) = (b'_{45})^{(9)}$ then $T_{45} \to \infty$.

**Definition of** $(m)^{(9)}$ and $\varepsilon_9$ :

Indeed let $t_9$ be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t),t\right) < \varepsilon_9, T_{44}(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right)$, $t = log\frac{2}{\varepsilon_9}$ By taking now $\varepsilon_9$ sufficiently small one sees that $T_{45}$ is unbounded.
The same property holds for $T_{46}$ if $\lim_{t\to\infty}(b''_{46})^{(9)}\left((G_{47})(t),t\right) = (b'_{46})^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

**Behavior of the solutions of equation** 280

**Theorem** If we denote and define

**Definition of** $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ :

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)} \leq -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14},t) + (a''_{14})^{(1)}(T_{14},t) \leq -(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)} \leq -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G,t) - (b''_{14})^{(1)}(G,t) \leq -(\tau_1)^{(1)}$

**Definition of** $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$ : 281

By $(\nu_1)^{(1)} > 0$ , $(\nu_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$ , $(u_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

**Definition of** $(\bar{\nu}_1)^{(1)}, , (\bar{\nu}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$ : 282

By $(\bar{\nu}_1)^{(1)} > 0$ , $(\bar{\nu}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$ , $(\bar{u}_2)^{(1)} < 0$ the roots of the equations

$(a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

**Definition of** $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (v_0)^{(1)}$ :- 283

If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by

$(m_2)^{(1)} = (v_0)^{(1)}, (m_1)^{(1)} = (v_1)^{(1)},\ if\ (v_0)^{(1)} < (v_1)^{(1)}$

$(m_2)^{(1)} = (v_1)^{(1)}, (m_1)^{(1)} = (\bar{v}_1)^{(1)}, if\ (v_1)^{(1)} < (v_0)^{(1)} < (\bar{v}_1)^{(1)},$

and $\boxed{(v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$

$(m_2)^{(1)} = (v_1)^{(1)}, (m_1)^{(1)} = (v_0)^{(1)},\ if\ (\bar{v}_1)^{(1)} < (v_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)},\ if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)},$

and $\boxed{(u_0)^{(1)} = \frac{T_{13}^0}{T_{14}^0}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$

are defined

Then the solution of global equations satisfies the inequalities 285

$$G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{13}(t) \le G_{13}^0 e^{(S_1)^{(1)}t}$$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)}t}$$

$$\left(\frac{(a_{15})^{(1)} G_{13}^0}{(m_1)^{(1)}((S_1)^{(1)} - (p_{13})^{(1)} - (S_2)^{(1)})}\left[e^{((S_1)^{(1)} - (p_{13})^{(1)})t} - e^{-(S_2)^{(1)}t}\right] + G_{15}^0 e^{-(S_2)^{(1)}t} \le G_{15}(t) \le \right. \quad 286$$

$$\left. \frac{(a_{15})^{(1)} G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)} - (a'_{15})^{(1)})}\left[e^{(S_1)^{(1)}t} - e^{-(a'_{15})^{(1)}t}\right] + G_{15}^0 e^{-(a'_{15})^{(1)}t}\right)$$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)}t} \le T_{13}(t) \le T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t}} \quad 287$$

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)}t} \le T_{13}(t) \le \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t} \quad 288$$

$$\frac{(b_{15})^{(1)} T_{13}^0}{(\mu_1)^{(1)}((R_1)^{(1)} - (b'_{15})^{(1)})}\left[e^{(R_1)^{(1)}t} - e^{-(b'_{15})^{(1)}t}\right] + T_{15}^0 e^{-(b'_{15})^{(1)}t} \le T_{15}(t) \le \quad 289$$

$$\frac{(a_{15})^{(1)} T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)} + (r_{13})^{(1)} + (R_2)^{(1)})}\left[e^{((R_1)^{(1)} + (r_{13})^{(1)})t} - e^{-(R_2)^{(1)}t}\right] + T_{15}^0 e^{-(R_2)^{(1)}t}$$

**Definition of** $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:- 290

Where $(S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$

$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$

$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$

$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$

**Behavior of the solutions of equation** 291

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ : 292

$(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ four constants satisfying
$-(\sigma_2)^{(2)} \le -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(T_{17}, t) + (a''_{17})^{(2)}(T_{17}, t) \le -(\sigma_1)^{(2)}$ 293

$-(\tau_2)^{(2)} \le -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}\big((G_{19}), t\big) - (b''_{17})^{(2)}\big((G_{19}), t\big) \le -(\tau_1)^{(2)}$ 294

**Definition of** $(\nu_1)^{(2)}, (\nu_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$ : 295

By $(\nu_1)^{(2)} > 0$ , $(\nu_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0$ , $(u_2)^{(2)} < 0$ the roots 296

of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 297

and $(b_{14})^{(2)}\big(u^{(2)}\big)^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and 298

**Definition of** $(\bar{\nu}_1)^{(2)},, (\bar{\nu}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$ : 299

By $(\bar{\nu}_1)^{(2)} > 0$ , $(\bar{\nu}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0$ , $(\bar{u}_2)^{(2)} < 0$ the 300

roots of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 301

and $(b_{17})^{(2)}\big(u^{(2)}\big)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 302

**Definition of** $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ :- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{G^0_{16}}{G^0_{17}}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)},\ \boldsymbol{if}\ (u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}, \boldsymbol{if}\ (u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)},$

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)},\ \boldsymbol{if}\ (\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)} t}$

$(p_i)^{(2)}$ is defined by equation

$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)} t}$ 311

$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)} t}\right] + G_{18}^0 e^{-(S_2)^{(2)} t} \le G_{18}(t) \le\right.$ 312

$\left.\frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)} t} - e^{-(a'_{18})^{(2)} t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)} t}\right)$

$\boxed{T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}}$ 313

$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}$ 314

$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)} t} - e^{-(b'_{18})^{(2)} t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)} t} \le T_{18}(t) \le$ 315

$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})}\left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)} t}\right] + T_{18}^0 e^{-(R_2)^{(2)} t}$

**Definition of** $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$

$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)}$ 318

$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$

**Behavior of the solutions** 319

**Theorem 3:** If we denote and define

**Definition of** $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ :

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}\big((G_{23}), t\big) \le -(\tau_1)^{(3)}$$

**Definition of** $(\nu_1)^{(3)}, (\nu_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$ : 320

By $(\nu_1)^{(3)} > 0$ , $(\nu_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$ , $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{\nu}_1)^{(3)} > 0$ , $(\bar{\nu}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$ , $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

**Definition of** $(m_1)^{(3)}$ , $(m_2)^{(3)}$ , $(\mu_1)^{(3)}, (\mu_2)^{(3)}$ :- 321

If we define $(m_1)^{(3)}$ , $(m_2)^{(3)}$ , $(\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (\nu_0)^{(3)}, (m_1)^{(3)} = (\nu_1)^{(3)}$, ***if*** $(\nu_0)^{(3)} < (\nu_1)^{(3)}$

$(m_2)^{(3)} = (\nu_1)^{(3)}, (m_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ , ***if*** $(\nu_1)^{(3)} < (\nu_0)^{(3)} < (\bar{\nu}_1)^{(3)}$,

and $\boxed{(\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}}$

$(m_2)^{(3)} = (\nu_1)^{(3)}, (m_1)^{(3)} = (\nu_0)^{(3)}$, ***if*** $(\bar{\nu}_1)^{(3)} < (\nu_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}$, ***if*** $(u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$ , ***if*** $(u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}$, and $\boxed{(u_0)^{(3)} = \frac{T^0_{20}}{T^0_{21}}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}$, ***if*** $(\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$$G^0_{20} e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G^0_{20} e^{(S_1)^{(3)} t}$$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}} G^0_{20} e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}} G^0_{20} e^{(S_1)^{(3)} t} \quad 323$$

$$\left(\frac{(a_{22})^{(3)} G^0_{20}}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\left[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)} t}\right] + G^0_{22} e^{-(S_2)^{(3)} t} \le G_{22}(t) \le \right. \quad 324$$

$$\left. \frac{(a_{22})^{(3)} G^0_{20}}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}\left[e^{(S_1)^{(3)} t} - e^{-(a'_{22})^{(3)} t}\right] + G^0_{22} e^{-(a'_{22})^{(3)} t}\right)$$

$$\boxed{T_{20}^{0}e^{(R_1)^{(3)}t} \le T_{20}(t) \le T_{20}^{0}e^{((R_1)^{(3)}+(r_{20})^{(3)})t}} \quad 325$$

$$\frac{1}{(\mu_1)^{(3)}}T_{20}^{0}e^{(R_1)^{(3)}t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}}T_{20}^{0}e^{((R_1)^{(3)}+(r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)}T_{20}^{0}}{(\mu_1)^{(3)}((R_1)^{(3)}-(b'_{22})^{(3)})}\left[e^{(R_1)^{(3)}t} - e^{-(b'_{22})^{(3)}t}\right] + T_{22}^{0}e^{-(b'_{22})^{(3)}t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)}T_{20}^{0}}{(\mu_2)^{(3)}((R_1)^{(3)}+(r_{20})^{(3)}+(R_2)^{(3)})}\left[e^{((R_1)^{(3)}+(r_{20})^{(3)})t} - e^{-(R_2)^{(3)}t}\right] + T_{22}^{0}e^{-(R_2)^{(3)}t}$$

**Definition of** $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$$

$$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$$

$$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$$

**Behavior of the solutions of equation**

**Theorem:** If we denote and define

**Definition of** $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ :

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$$

$$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}\big((G_{27}), t\big) - (b''_{25})^{(4)}\big((G_{27}), t\big) \le -(\tau_1)^{(4)}$$

**Definition of** $(\nu_1)^{(4)}, (\nu_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, \nu^{(4)}, u^{(4)}$ : 329

By $(\nu_1)^{(4)} > 0$ , $(\nu_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$ , $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_1)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(4)},, (\bar{\nu}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$ : 330

By $(\bar{\nu}_1)^{(4)} > 0$ , $(\bar{\nu}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$ , $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

**Definition of** $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}, (\nu_0)^{(4)}$ :-

If we define $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$$(m_2)^{(4)} = (\nu_0)^{(4)}, (m_1)^{(4)} = (\nu_1)^{(4)}, \ \boldsymbol{if}\ (\nu_0)^{(4)} < (\nu_1)^{(4)}$$

$$(m_2)^{(4)} = (\nu_1)^{(4)}, (m_1)^{(4)} = (\bar{\nu}_1)^{(4)}, \boldsymbol{if}\ (\nu_4)^{(4)} < (\nu_0)^{(4)} < (\bar{\nu}_1)^{(4)},$$

and $\boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (\nu_4)^{(4)}, (m_1)^{(4)} = (\nu_0)^{(4)},\ \boldsymbol{if}\ (\bar{\nu}_4)^{(4)} < (\nu_0)^{(4)}$

and analogously

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$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)},\ \boldsymbol{if}\ (u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}, \boldsymbol{if}\ (u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)},$

and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}, \boldsymbol{if}\ (\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

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$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)} t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)} t} \quad 333$$

$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)} t}\right] + G_{26}^0 e^{-(S_2)^{(4)} t} \le G_{26}(t) \le \right. \quad 334$$

*(a26)4G240(m2)4(S1)4−(a26′)4e(S1)4t−e−(a26′)4t+ G260e−(a26′)4t*

$$\boxed{T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}} T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}} T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t} \quad 335$$

$$\frac{(b_{26})^{(4)} T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)} - (b'_{26})^{(4)})}\left[e^{(R_1)^{(4)} t} - e^{-(b'_{26})^{(4)} t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)} t} \le T_{26}(t) \le \quad 336$$

$$\frac{(a_{26})^{(4)} T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)} + (r_{24})^{(4)} + (R_2)^{(4)})}\left[e^{((R_1)^{(4)} + (r_{24})^{(4)})t} - e^{-(R_2)^{(4)} t}\right] + T_{26}^0 e^{-(R_2)^{(4)} t}$$

**Definition of** $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

**Behavior of the solutions of equation** 338

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ :

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

**Definition of** $(\nu_1)^{(5)}, (\nu_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, \nu^{(5)}, u^{(5)}$ : 339

By $(\nu_1)^{(5)} > 0$ , $(\nu_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$ , $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(5)}, , (\bar{\nu}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$ : 340

By $(\bar{\nu}_1)^{(5)} > 0$ , $(\bar{\nu}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$ , $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

**Definition of** $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (\nu_0)^{(5)}$ :-

If we define $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (\nu_0)^{(5)}, (m_1)^{(5)} = (\nu_1)^{(5)}$, ***if*** $(\nu_0)^{(5)} < (\nu_1)^{(5)}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ , ***if*** $(\nu_1)^{(5)} < (\nu_0)^{(5)} < (\bar{\nu}_1)^{(5)}$,
and $\boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)}$, ***if*** $(\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)}$, ***if*** $(u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}$ , ***if*** $(u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)}$,
and $\boxed{(u_0)^{(5)} = \frac{T^0_{28}}{T^0_{29}}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}$, ***if*** $(\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$$G^0_{28} e^{\left((S_1)^{(5)} - (p_{28})^{(5)}\right)t} \le G_{28}(t) \le G^0_{28} e^{(S_1)^{(5)}t}$$

where $(p_i)^{(5)}$ is defined by equation

$$\frac{1}{(m_5)^{(5)}} G^0_{28} e^{\left((S_1)^{(5)} - (p_{28})^{(5)}\right)t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}} G^0_{28} e^{(S_1)^{(5)}t} \quad 343$$

$$\left(\frac{(a_{30})^{(5)}G_{28}^{0}}{(m_1)^{(5)}((S_1)^{(5)}-(p_{28})^{(5)}-(S_2)^{(5)})}\left[e^{((S_1)^{(5)}-(p_{28})^{(5)})t}-e^{-(S_2)^{(5)}t}\right]+G_{30}^{0}e^{-(S_2)^{(5)}t}\leq G_{30}(t)\leq\right. \quad 344$$

*(a30)5G280(m2)5(S1)5−(a30′)5e(S1)5t−e−(a30′)5t+ G300e−(a30′)5t*

$$\boxed{T_{28}^{0}e^{(R_1)^{(5)}t}\leq T_{28}(t)\leq T_{28}^{0}e^{((R_1)^{(5)}+(r_{28})^{(5)})t}} \quad 345$$

$$\frac{1}{(\mu_1)^{(5)}}T_{28}^{0}e^{(R_1)^{(5)}t}\leq T_{28}(t)\leq\frac{1}{(\mu_2)^{(5)}}T_{28}^{0}e^{((R_1)^{(5)}+(r_{28})^{(5)})t} \quad 346$$

$$\frac{(b_{30})^{(5)}T_{28}^{0}}{(\mu_1)^{(5)}((R_1)^{(5)}-(b'_{30})^{(5)})}\left[e^{(R_1)^{(5)}t}-e^{-(b'_{30})^{(5)}t}\right]+T_{30}^{0}e^{-(b'_{30})^{(5)}t}\leq T_{30}(t)\leq \quad 347$$

$$\frac{(a_{30})^{(5)}T_{28}^{0}}{(\mu_2)^{(5)}((R_1)^{(5)}+(r_{28})^{(5)}+(R_2)^{(5)})}\left[e^{((R_1)^{(5)}+(r_{28})^{(5)})t}-e^{-(R_2)^{(5)}t}\right]+T_{30}^{0}e^{-(R_2)^{(5)}t}$$

**Definition of** $(S_1)^{(5)},(S_2)^{(5)},(R_1)^{(5)},(R_2)^{(5)}$:- 348

Where $(S_1)^{(5)}=(a_{28})^{(5)}(m_2)^{(5)}-(a'_{28})^{(5)}$

$(S_2)^{(5)}=(a_{30})^{(5)}-(p_{30})^{(5)}$

$(R_1)^{(5)}=(b_{28})^{(5)}(\mu_2)^{(5)}-(b'_{28})^{(5)}$

$(R_2)^{(5)}=(b'_{30})^{(5)}-(r_{30})^{(5)}$

**Behavior of the solutions of equation** 349

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(6)},(\sigma_2)^{(6)},(\tau_1)^{(6)},(\tau_2)^{(6)}$ :

$(\sigma_1)^{(6)},(\sigma_2)^{(6)},(\tau_1)^{(6)},(\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)}\leq-(a'_{32})^{(6)}+(a'_{33})^{(6)}-(a''_{32})^{(6)}(T_{33},t)+(a''_{33})^{(6)}(T_{33},t)\leq-(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)}\leq-(b'_{32})^{(6)}+(b'_{33})^{(6)}-(b''_{32})^{(6)}\big((G_{35}),t\big)-(b''_{33})^{(6)}\big((G_{35}),t\big)\leq-(\tau_1)^{(6)}$$

**Definition of** $(v_1)^{(6)},(v_2)^{(6)},(u_1)^{(6)},(u_2)^{(6)},v^{(6)},u^{(6)}$ : 350

By $(v_1)^{(6)}>0$ , $(v_2)^{(6)}<0$ and respectively $(u_1)^{(6)}>0$ , $(u_2)^{(6)}<0$ the roots of the equations $(a_{33})^{(6)}\big(v^{(6)}\big)^2+(\sigma_1)^{(6)}v^{(6)}-(a_{32})^{(6)}=0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2+(\tau_1)^{(6)}u^{(6)}-(b_{32})^{(6)}=0$ and

**Definition of** $(\bar{v}_1)^{(6)},,(\bar{v}_2)^{(6)},(\bar{u}_1)^{(6)},(\bar{u}_2)^{(6)}$ : 351

By $(\bar{v}_1)^{(6)}>0$ , $(\bar{v}_2)^{(6)}<0$ and respectively $(\bar{u}_1)^{(6)}>0$ , $(\bar{u}_2)^{(6)}<0$ the roots of the equations $(a_{33})^{(6)}\big(v^{(6)}\big)^2+(\sigma_2)^{(6)}v^{(6)}-(a_{32})^{(6)}=0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2+(\tau_2)^{(6)}u^{(6)}-(b_{32})^{(6)}=0$

**Definition of** $(m_1)^{(6)},(m_2)^{(6)},(\mu_1)^{(6)},(\mu_2)^{(6)},(v_0)^{(6)}$ :-

If we define $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}$ by

$(m_2)^{(6)} = (v_0)^{(6)}, (m_1)^{(6)} = (v_1)^{(6)}$, ***if*** $(v_0)^{(6)} < (v_1)^{(6)}$

$(m_2)^{(6)} = (v_1)^{(6)}, (m_1)^{(6)} = (\bar{v}_6)^{(6)}$, ***if*** $(v_1)^{(6)} < (v_0)^{(6)} < (\bar{v}_1)^{(6)}$,

and $\boxed{(v_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$

$(m_2)^{(6)} = (v_1)^{(6)}, (m_1)^{(6)} = (v_0)^{(6)}$, ***if*** $(\bar{v}_1)^{(6)} < (v_0)^{(6)}$

and analogously 352

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}$, ***if*** $(u_0)^{(6)} < (u_1)^{(6)}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}$, ***if*** $(u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)}$,

and $\boxed{(u_0)^{(6)} = \frac{T_{32}^0}{T_{33}^0}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}$, ***if*** $(\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$

Then the solution of global equations satisfies the inequalities 353

$$G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \leq G_{32}(t) \leq G_{32}^0 e^{(S_1)^{(6)} t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}} G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \leq G_{33}(t) \leq \frac{1}{(m_2)^{(6)}} G_{32}^0 e^{(S_1)^{(6)} t} \quad 354$$

$$\left( \frac{(a_{34})^{(6)} G_{32}^0}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})} \left[ e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)} t} \right] + G_{34}^0 e^{-(S_2)^{(6)} t} \leq G_{34}(t) \leq \right. \quad 355$$

*(a34)6G320(m2)6(S1)6−(a34′)6e(S1)6t−e−(a34′)6t+ G340e−(a34′)6t*

$$\boxed{T_{32}^0 e^{(R_1)^{(6)} t} \leq T_{32}(t) \leq T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}} T_{32}^0 e^{(R_1)^{(6)} t} \leq T_{32}(t) \leq \frac{1}{(\mu_2)^{(6)}} T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)} T_{32}^0}{(\mu_1)^{(6)}((R_1)^{(6)} - (b'_{34})^{(6)})} \left[ e^{(R_1)^{(6)} t} - e^{-(b'_{34})^{(6)} t} \right] + T_{34}^0 e^{-(b'_{34})^{(6)} t} \leq T_{34}(t) \leq \quad 358$$

$$\frac{(a_{34})^{(6)} T_{32}^0}{(\mu_2)^{(6)}((R_1)^{(6)} + (r_{32})^{(6)} + (R_2)^{(6)})} \left[ e^{((R_1)^{(6)} + (r_{32})^{(6)})t} - e^{-(R_2)^{(6)} t} \right] + T_{34}^0 e^{-(R_2)^{(6)} t}$$

**Definition of** $(S_1)^{(6)}, (S_2)^{(6)}, (R_1)^{(6)}, (R_2)^{(6)}$:- 359

Where $(S_1)^{(6)} = (a_{32})^{(6)} (m_2)^{(6)} - (a'_{32})^{(6)}$

$(S_2)^{(6)} = (a_{34})^{(6)} - (p_{34})^{(6)}$

$(R_1)^{(6)} = (b_{32})^{(6)} (\mu_2)^{(6)} - (b'_{32})^{(6)}$

$$(R_2)^{(6)} = (b'_{34})^{(6)} - (r_{34})^{(6)}$$

**Behavior of the solutions of equation**

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$ :

$(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)} \le -(a'_{36})^{(7)} + (a'_{37})^{(7)} - (a''_{36})^{(7)}(T_{37}, t) + (a''_{37})^{(7)}(T_{37}, t) \le -(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)} \le -(b'_{36})^{(7)} + (b'_{37})^{(7)} - (b''_{36})^{(7)}\big((G_{39}), t\big) - (b''_{37})^{(7)}\big((G_{39}), t\big) \le -(\tau_1)^{(7)}$$

**Definition of** $(\nu_1)^{(7)}, (\nu_2)^{(7)}, (u_1)^{(7)}, (u_2)^{(7)}, \nu^{(7)}, u^{(7)}$ : 361

By $(\nu_1)^{(7)} > 0, (\nu_2)^{(7)} < 0$ and respectively $(u_1)^{(7)} > 0, (u_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_1)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(7)},, (\bar{\nu}_2)^{(7)}, (\bar{u}_1)^{(7)}, (\bar{u}_2)^{(7)}$ : 362

By $(\bar{\nu}_1)^{(7)} > 0, (\bar{\nu}_2)^{(7)} < 0$ and respectively $(\bar{u}_1)^{(7)} > 0, (\bar{u}_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

**Definition of** $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (\nu_0)^{(7)}$ :-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$$(m_2)^{(7)} = (\nu_0)^{(7)}, (m_1)^{(7)} = (\nu_1)^{(7)}, \ \boldsymbol{if}\ (\nu_0)^{(7)} < (\nu_1)^{(7)}$$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\bar{\nu}_1)^{(7)}, \boldsymbol{if}\ (\nu_1)^{(7)} < (\nu_0)^{(7)} < (\bar{\nu}_1)^{(7)},$$

and $\boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\nu_0)^{(7)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(7)} < (\nu_0)^{(7)}$$

and analogously 363

$$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}, \ \boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$$

and $\boxed{(u_0)^{(7)} = \frac{T^0_{36}}{T^0_{37}}}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)}$ where $(u_1)^{(7)}, (\bar{u}_1)^{(7)}$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^0 e^{\left((S_1)^{(7)} - (p_{36})^{(7)}\right)t} \leq G_{36}(t) \leq G_{36}^0 e^{(S_1)^{(7)} t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{\left((S_1)^{(7)} - (p_{36})^{(7)}\right)t} \leq G_{37}(t) \leq \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)} t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)}\left((S_1)^{(7)} - (p_{36})^{(7)} - (S_2)^{(7)}\right)}\left[e^{\left((S_1)^{(7)} - (p_{36})^{(7)}\right)t} - e^{-(S_2)^{(7)} t}\right] + G_{38}^0 e^{-(S_2)^{(7)} t} \leq G_{38}(t) \leq \right. \quad 366$$

$$\left. \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)}\left((S_1)^{(7)} - (a'_{38})^{(7)}\right)}\left[e^{(S_1)^{(7)} t} - e^{-(a'_{38})^{(7)} t}\right] + G_{38}^0 e^{-(a'_{38})^{(7)} t}\right)$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)} t} \leq T_{36}(t) \leq T_{36}^0 e^{\left((R_1)^{(7)} + (r_{36})^{(7)}\right)t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)} t} \leq T_{36}(t) \leq \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{\left((R_1)^{(7)} + (r_{36})^{(7)}\right)t} \quad 368$$

$$\frac{(b_{38})^{(7)} T_{36}^0}{(\mu_1)^{(7)}\left((R_1)^{(7)} - (b'_{38})^{(7)}\right)}\left[e^{(R_1)^{(7)} t} - e^{-(b'_{38})^{(7)} t}\right] + T_{38}^0 e^{-(b'_{38})^{(7)} t} \leq T_{38}(t) \leq \quad 369$$

$$\frac{(a_{38})^{(7)} T_{36}^0}{(\mu_2)^{(7)}\left((R_1)^{(7)} + (r_{36})^{(7)} + (R_2)^{(7)}\right)}\left[e^{\left((R_1)^{(7)} + (r_{36})^{(7)}\right)t} - e^{-(R_2)^{(7)} t}\right] + T_{38}^0 e^{-(R_2)^{(7)} t}$$

**Definition of** $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$$

$$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$$

$$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$$

**Behavior of the solutions of equation** 371

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ :

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$$-(\sigma_2)^{(8)} \leq -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \leq -(\sigma_1)^{(8)}$$

$$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\big((G_{43}),t\big) - (b''_{41})^{(8)}\big((G_{43}),t\big) \le -(\tau_1)^{(8)}$$

**Definition of** $(\nu_1)^{(8)}, (\nu_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, \nu^{(8)}, u^{(8)}$ : 372

By $(\nu_1)^{(8)} > 0$ , $(\nu_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0$ , $(u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(8)},, (\bar{\nu}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$ :

By $(\bar{\nu}_1)^{(8)} > 0$ , $(\bar{\nu}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0$ , $(\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

**Definition of** $(m_1)^{(8)}$ , $(m_2)^{(8)}$ , $(\mu_1)^{(8)}, (\mu_2)^{(8)}, (\nu_0)^{(8)}$ :-

If we define $(m_1)^{(8)}$ , $(m_2)^{(8)}$ , $(\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$(m_2)^{(8)} = (\nu_0)^{(8)}, (m_1)^{(8)} = (\nu_1)^{(8)}$, ***if*** $(\nu_0)^{(8)} < (\nu_1)^{(8)}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ , ***if*** $(\nu_1)^{(8)} < (\nu_0)^{(8)} < (\bar{\nu}_1)^{(8)}$,
and $\boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\nu_0)^{(8)}$, ***if*** $(\bar{\nu}_1)^{(8)} < (\nu_0)^{(8)}$

and analogously 374

$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)}$, ***if*** $(u_0)^{(8)} < (u_1)^{(8)}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}$ , ***if*** $(u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)}$,
and $\boxed{(u_0)^{(8)} = \frac{T^0_{40}}{T^0_{41}}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}$, ***if*** $(\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G^0_{40}e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{40}(t) \le G^0_{40}e^{(S_1)^{(8)}t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}} G_{40}^0 e^{((S_1)^{(8)}-(p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}} G_{40}^0 e^{(S_1)^{(8)}t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)}G_{40}^0}{(m_1)^{(8)}((S_1)^{(8)}-(p_{40})^{(8)}-(S_2)^{(8)})}\left[e^{((S_1)^{(8)}-(p_{40})^{(8)})t} - e^{-(S_2)^{(8)}t}\right] + G_{42}^0 e^{-(S_2)^{(8)}t} \le G_{42}(t) \le \right. \quad 377$$

$$\left. \frac{(a_{42})^{(8)}G_{40}^0}{(m_2)^{(8)}((S_1)^{(8)}-(a'_{42})^{(8)})}[e^{(S_1)^{(8)}t} - e^{-(a'_{42})^{(8)}t}] + G_{42}^0 e^{-(a'_{42})^{(8)}t}\right)$$

$$\boxed{T_{40}^0 e^{(R_1)^{(8)}t} \le T_{40}(t) \le T_{40}^0 e^{((R_1)^{(8)}+(r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}} T_{40}^0 e^{(R_1)^{(8)}t} \le T_{40}(t) \le \frac{1}{(\mu_2)^{(8)}} T_{40}^0 e^{((R_1)^{(8)}+(r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)}T_{40}^0}{(\mu_1)^{(8)}((R_1)^{(8)}-(b'_{42})^{(8)})}\left[e^{(R_1)^{(8)}t} - e^{-(b'_{42})^{(8)}t}\right] + T_{42}^0 e^{-(b'_{42})^{(8)}t} \le T_{42}(t) \le \quad 380$$

$$\frac{(a_{42})^{(8)}T_{40}^0}{(\mu_2)^{(8)}((R_1)^{(8)}+(r_{40})^{(8)}+(R_2)^{(8)})}\left[e^{((R_1)^{(8)}+(r_{40})^{(8)})t} - e^{-(R_2)^{(8)}t}\right] + T_{42}^0 e^{-(R_2)^{(8)}t}$$

**Definition of** $(S_1)^{(8)}, (S_2)^{(8)}, (R_1)^{(8)}, (R_2)^{(8)}$:- 381

Where $(S_1)^{(8)} = (a_{40})^{(8)}(m_2)^{(8)} - (a'_{40})^{(8)}$

$(S_2)^{(8)} = (a_{42})^{(8)} - (p_{42})^{(8)}$

$(R_1)^{(8)} = (b_{40})^{(8)}(\mu_2)^{(8)} - (b'_{40})^{(8)}$

$(R_2)^{(8)} = (b'_{42})^{(8)} - (r_{42})^{(8)}$

**Behavior of the solutions of equation 37 to 92** 382

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ :

$(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ four constants satisfying

$$-(\sigma_2)^{(9)} \le -(a'_{44})^{(9)} + (a'_{45})^{(9)} - (a''_{44})^{(9)}(T_{45}, t) + (a''_{45})^{(9)}(T_{45}, t) \le -(\sigma_1)^{(9)}$$

$$-(\tau_2)^{(9)} \le -(b'_{44})^{(9)} + (b'_{45})^{(9)} - (b''_{44})^{(9)}\big((G_{47}), t\big) - (b''_{45})^{(9)}\big((G_{47}), t\big) \le -(\tau_1)^{(9)}$$

**Definition of** $(\nu_1)^{(9)}, (\nu_2)^{(9)}, (u_1)^{(9)}, (u_2)^{(9)}, \nu^{(9)}, u^{(9)}$ :

By $(\nu_1)^{(9)} > 0$ , $(\nu_2)^{(9)} < 0$ and respectively $(u_1)^{(9)} > 0$ , $(u_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_1)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(9)}, , (\bar{\nu}_2)^{(9)}, (\bar{u}_1)^{(9)}, (\bar{u}_2)^{(9)}$ :

By $(\bar{\nu}_1)^{(9)} > 0$ , $(\bar{\nu}_2)^{(9)} < 0$ and respectively $(\bar{u}_1)^{(9)} > 0$ , $(\bar{u}_2)^{(9)} < 0$ the

roots of the equations $(a_{45})^{(9)}\left(v^{(9)}\right)^2+(\sigma_2)^{(9)}v^{(9)}-(a_{44})^{(9)}=0$
and $(b_{45})^{(9)}\left(u^{(9)}\right)^2+(\tau_2)^{(9)}u^{(9)}-(b_{44})^{(9)}=0$

**Definition of** $(m_1)^{(9)},(m_2)^{(9)},(\mu_1)^{(9)},(\mu_2)^{(9)},(v_0)^{(9)}$ :-

If we define $(m_1)^{(9)},(m_2)^{(9)},(\mu_1)^{(9)},(\mu_2)^{(9)}$ by

$(m_2)^{(9)}=(v_0)^{(9)},(m_1)^{(9)}=(v_1)^{(9)},\ \boldsymbol{if}\ (v_0)^{(9)}<(v_1)^{(9)}$

$(m_2)^{(9)}=(v_1)^{(9)},(m_1)^{(9)}=(\bar{v}_1)^{(9)},\boldsymbol{if}\ (v_1)^{(9)}<(v_0)^{(9)}<(\bar{v}_1)^{(9)}$,
and $\boxed{(v_0)^{(9)}=\frac{G_{44}^0}{G_{45}^0}}$

$(m_2)^{(9)}=(v_1)^{(9)},(m_1)^{(9)}=(v_0)^{(9)},\ \boldsymbol{if}\ (\bar{v}_1)^{(9)}<(v_0)^{(9)}$

and analogously

$(\mu_2)^{(9)}=(u_0)^{(9)},(\mu_1)^{(9)}=(u_1)^{(9)},\ \boldsymbol{if}\ (u_0)^{(9)}<(u_1)^{(9)}$

$(\mu_2)^{(9)}=(u_1)^{(9)},(\mu_1)^{(9)}=(\bar{u}_1)^{(9)},\boldsymbol{if}\ (u_1)^{(9)}<(u_0)^{(9)}<(\bar{u}_1)^{(9)}$,
and $\boxed{(u_0)^{(9)}=\frac{T_{44}^0}{T_{45}^0}}$

$(\mu_2)^{(9)}=(u_1)^{(9)},(\mu_1)^{(9)}=(u_0)^{(9)},\boldsymbol{if}\ (\bar{u}_1)^{(9)}<(u_0)^{(9)}$ where $(u_1)^{(9)},(\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G_{44}^0e^{((S_1)^{(9)}-(p_{44})^{(9)})t}\le G_{44}(t)\le G_{44}^0e^{(S_1)^{(9)}t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}}G_{44}^0e^{((S_1)^{(9)}-(p_{44})^{(9)})t}\le G_{45}(t)\le\frac{1}{(m_2)^{(9)}}G_{44}^0e^{(S_1)^{(9)}t}$$

$$\left(\frac{(a_{46})^{(9)}G_{44}^0}{(m_1)^{(9)}((S_1)^{(9)}-(p_{44})^{(9)}-(S_2)^{(9)})}\left[e^{((S_1)^{(9)}-(p_{44})^{(9)})t}-e^{-(S_2)^{(9)}t}\right]+G_{46}^0e^{-(S_2)^{(9)}t}\le G_{46}(t)\le\right.$$
$$\left.\frac{(a_{46})^{(9)}G_{44}^0}{(m_2)^{(9)}((S_1)^{(9)}-(a'_{46})^{(9)})}[e^{(S_1)^{(9)}t}-e^{-(a'_{46})^{(9)}t}]+\ G_{46}^0e^{-(a'_{46})^{(9)}t}\right)$$

$$\boxed{T_{44}^0e^{(R_1)^{(9)}t}\le T_{44}(t)\le T_{44}^0e^{((R_1)^{(9)}+(r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}}T_{44}^0e^{(R_1)^{(9)}t}\le T_{44}(t)\le\frac{1}{(\mu_2)^{(9)}}T_{44}^0e^{((R_1)^{(9)}+(r_{44})^{(9)})t}$$

$$\frac{(b_{46})^{(9)}T_{44}^0}{(\mu_1)^{(9)}((R_1)^{(9)}-(b'_{46})^{(9)})}\left[e^{(R_1)^{(9)}t}-e^{-(b'_{46})^{(9)}t}\right]+T_{46}^0e^{-(b'_{46})^{(9)}t}\le T_{46}(t)\le$$

$$\frac{(a_{46})^{(9)}T_{44}^0}{(\mu_2)^{(9)}((R_1)^{(9)}+(r_{44})^{(9)}+(R_2)^{(9)})}\left[e^{((R_1)^{(9)}+(r_{44})^{(9)})t}-e^{-(R_2)^{(9)}t}\right]+T_{46}^0e^{-(R_2)^{(9)}t}$$

**Definition of** $(S_1)^{(9)},(S_2)^{(9)},(R_1)^{(9)},(R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$

**Proof :** From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right) - (a''_{14})^{(1)}(T_{14}, t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

**Definition of** $v^{(1)}$ :- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)}\right) \le \frac{dv^{(1)}}{dt} \le -\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)}\right)$$

From which one obtains

**Definition of** $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$ :-

For $0 < \boxed{(v_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}} < (v_1)^{(1)} < (\bar{v}_1)^{(1)}$

$$v^{(1)}(t) \ge \frac{(v_1)^{(1)} + (C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_0)^{(1)}\right)t\right]}}{1 + (C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(v_1)^{(1)} - (v_0)^{(1)}}{(v_0)^{(1)} - (v_2)^{(1)}}}$$

it follows $(v_0)^{(1)} \le v^{(1)}(t) \le (v_1)^{(1)}$

In the same manner , we get 384

$$v^{(1)}(t) \le \frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{v}_1)^{(1)} - (v_0)^{(1)}}{(v_0)^{(1)} - (\bar{v}_2)^{(1)}}}$$

From which we deduce $(v_0)^{(1)} \le v^{(1)}(t) \le (\bar{v}_1)^{(1)}$

If $0 < (v_1)^{(1)} < (v_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}} < (\bar{v}_1)^{(1)}$ we find like in the previous case, 385

$$(v_1)^{(1)} \le \frac{(v_1)^{(1)} + (C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_2)^{(1)}\right)t\right]}}{1 + (C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_2)^{(1)}\right)t\right]}} \le v^{(1)}(t) \le$$

$$\frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}} \le (\bar{v}_1)^{(1)}$$

If $0 < (\nu_1)^{(1)} \leq (\bar{\nu}_1)^{(1)} \leq \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$ , we obtain 386

$$(\nu_1)^{(1)} \leq \nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)} + (\bar{C})^{(1)}(\bar{\nu}_2)^{(1)} e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)} - (\bar{\nu}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)} e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)} - (\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\nu_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(1)}(t)$ :-

$$(m_2)^{(1)} \leq \nu^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{\nu^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(1)}(t)$ :-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{13}'')^{(1)} = (a_{14}'')^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)} G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b_{13}'')^{(1)} = (b_{14}'')^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)} T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

**Proof** : From global equations we obtain 387

$$\frac{d\nu^{(2)}}{dt} = (a_{16})^{(2)} - \left((a_{16}')^{(2)} - (a_{17}')^{(2)} + (a_{16}'')^{(2)}(T_{17}, t)\right) - (a_{17}'')^{(2)}(T_{17}, t)\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

**Definition of** $\nu^{(2)}$ :- $\boxed{\nu^{(2)} = \frac{G_{16}}{G_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \leq \frac{d\nu^{(2)}}{dt} \leq -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

**Definition of** $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$ :-

For $0 < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \geq \frac{(\nu_1)^{(2)}+(\mathrm{C})^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_0)^{(2)}\right)t\right]}}{1+(\mathrm{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\mathrm{C})^{(2)}=\frac{(\nu_1)^{(2)}-(\nu_0)^{(2)}}{(\nu_0)^{(2)}-(\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)}+(\bar{\mathrm{C}})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{\mathrm{C}})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{\mathrm{C}})^{(2)}=\frac{(\bar{\nu}_1)^{(2)}-(\nu_0)^{(2)}}{(\nu_0)^{(2)}-(\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{\mathrm{G}_{16}^0}{\mathrm{G}_{17}^0} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \leq \frac{(\nu_1)^{(2)}+(\mathrm{C})^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}}{1+(\mathrm{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}} \leq \nu^{(2)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(2)}+(\bar{\mathrm{C}})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{\mathrm{C}})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \leq (\bar{\nu}_1)^{(2)} \leq (\nu_0)^{(2)} = \frac{\mathrm{G}_{16}^0}{\mathrm{G}_{17}^0}$ , we obtain 394

$$(\nu_1)^{(2)} \leq \nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)}+(\bar{\mathrm{C}})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{\mathrm{C}})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(2)}(t)$ :- 395

$$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}, \quad \boxed{\nu^{(2)}(t)=\frac{G_{16}(t)}{G_{17}(t)}}$$

In a completely analogous way, we obtain 396

**Definition of** $u^{(2)}(t)$ :-

$$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}, \quad \boxed{u^{(2)}(t)=\frac{T_{16}(t)}{T_{17}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :** 397

If $(a_{16}'')^{(2)} = (a_{17}'')^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)}G_{17}(t)$

Analogously if $(b''_{16})^{(2)} = (b''_{17})^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)}T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

**Proof** : From global equations we obtain 398

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right) - (a''_{21})^{(3)}(T_{21}, t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

**Definition of** $\nu^{(3)}$ :- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right) \leq \frac{d\nu^{(3)}}{dt} \leq -\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right)$$

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From which one obtains

For $0 < (\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\nu_1)^{(3)} < (\bar{\nu}_1)^{(3)}$

$$\nu^{(3)}(t) \geq \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(\nu_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\nu_2)^{(3)}}}$$

it follows $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\nu_1)^{(3)}$

In the same manner , we get 401

$$\nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)} + (\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{\nu}_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\bar{\nu}_2)^{(3)}}}$$

**Definition of** $(\bar{\nu}_1)^{(3)}$ :-

From which we deduce $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\bar{\nu}_1)^{(3)}$

If $0 < (\nu_1)^{(3)} < (\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\bar{\nu}_1)^{(3)}$ we find like in the previous case, 402

$$(\nu_1)^{(3)} \leq \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_2)^{(3)}\right)t\right]}}{1 + (C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_2)^{(3)}\right)t\right]}} \leq \nu^{(3)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(3)} + (\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\bar{\nu}_1)^{(3)}$$

If $0 < (\nu_1)^{(3)} \leq (\bar{\nu}_1)^{(3)} \leq (\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}$ , we obtain 403

$$(\nu_1)^{(3)} \leq \nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\nu_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(3)}(t)$ :-

$$(m_2)^{(3)} \leq \nu^{(3)}(t) \leq (m_1)^{(3)}, \quad \boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(3)}(t)$ :-

$$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}, \quad \boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{20})^{(3)} = (a''_{21})^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)}G_{21}(t)$

Analogously if $(b''_{20})^{(3)} = (b''_{21})^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)}T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

**Proof** : From global equations we obtain 404

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right) - (a''_{25})^{(4)}(T_{25},t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

**Definition of** $\nu^{(4)}$ :- $\quad \boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right) \leq \frac{d\nu^{(4)}}{dt} \leq -\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_4)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$ :-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \geq \frac{(\nu_1)^{(4)}+(C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_0)^{(4)}\right)t\right]}}{4+(C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)}-(\nu_0)^{(4)}}{(\nu_0)^{(4)}-(\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{4+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)}-(\nu_0)^{(4)}}{(\nu_0)^{(4)}-(\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \leq \frac{(\nu_1)^{(4)}+(C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_2)^{(4)}\right)t\right]}}{1+(C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_2)^{(4)}\right)t\right]}} \leq \nu^{(4)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \leq (\bar{\nu}_1)^{(4)} \leq \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$ , we obtain 407

$$(\nu_1)^{(4)} \leq \nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(4)}(t)$ :-

$$(m_2)^{(4)} \leq \nu^{(4)}(t) \leq (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(4)}(t)$ :-

$$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a_{24}'')^{(4)} = (a_{25}'')^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)}G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b_{24}'')^{(4)} = (b_{25}'')^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)}T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

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**Proof** : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a_{28}')^{(5)} - (a_{29}')^{(5)} + (a_{28}'')^{(5)}(T_{29}, t)\right) - (a_{29}'')^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

**Definition of** $\nu^{(5)}$ :- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \le \frac{d\nu^{(5)}}{dt} \le -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$ :-

For $0 < \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$

$$\nu^{(5)}(t) \ge \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \le \nu^{(5)}(t) \le (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \le \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \le \nu^{(5)}(t) \le (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \le \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}}{1 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}} \le \nu^{(5)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \le (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \le (\bar{\nu}_1)^{(5)} \le \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$ , we obtain 411

$$(\nu_1)^{(5)} \le \nu^{(5)}(t) \le \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \le (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(5)}(t)$ :-

$(m_2)^{(5)} \le \nu^{(5)}(t) \le (m_1)^{(5)}$, $\boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$

In a completely analogous way, we obtain

**Definition of** $u^{(5)}(t)$ :-

$(\mu_2)^{(5)} \le u^{(5)}(t) \le (\mu_1)^{(5)}$, $\boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{28})^{(5)} = (a''_{29})^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)} G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b''_{28})^{(5)} = (b''_{29})^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$ if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)} T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

**Proof** : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a'_{32})^{(6)} - (a'_{33})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right) - (a''_{33})^{(6)}(T_{33}, t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

**Definition of** $\nu^{(6)}$ :- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \leq \frac{d\nu^{(6)}}{dt} \leq -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$ :-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \geq \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\nu_1)^{(6)}$

In the same manner , we get 413

$$\nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{\nu}_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\bar{\nu}_1)^{(6)}$

If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 414

$$(\nu_1)^{(6)} \leq \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}} \leq \nu^{(6)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\bar{\nu}_1)^{(6)}$$

If $0 < (\nu_1)^{(6)} \leq (\bar{\nu}_1)^{(6)} \leq \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$ , we obtain

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$$(\nu_1)^{(6)} \leq \nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(6)}(t)$ :-

$$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}, \quad \boxed{\nu^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(6)}(t)$ :-

$$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}, \quad \boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)}G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)}T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

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**Proof** : From global equations we obtain

$$\frac{d\nu^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right) - (a''_{37})^{(7)}(T_{37}, t)\nu^{(7)} - (a_{37})^{(7)}\nu^{(7)}$$

**Definition of** $\nu^{(7)}$ :- $\boxed{\nu^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right) \leq \frac{d\nu^{(7)}}{dt} \leq -\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(7)}, (\nu_0)^{(7)}$ :-

For $0 < \boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}} < (\nu_1)^{(7)} < (\bar{\nu}_1)^{(7)}$

$$\nu^{(7)}(t) \geq \frac{(\nu_1)^{(7)}+(C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_0)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)}=\frac{(\nu_1)^{(7)}-(\nu_0)^{(7)}}{(\nu_0)^{(7)}-(\nu_2)^{(7)}}}$$

it follows $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\nu_1)^{(7)}$

In the same manner , we get 417

$$\nu^{(7)}(t) \leq \frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)}=\frac{(\bar{\nu}_1)^{(7)}-(\nu_0)^{(7)}}{(\nu_0)^{(7)}-(\bar{\nu}_2)^{(7)}}}$$

From which we deduce $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\bar{\nu}_1)^{(7)}$

If $0 < (\nu_1)^{(7)} < (\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0} < (\bar{\nu}_1)^{(7)}$ we find like in the previous case, 418

$$(\nu_1)^{(7)} \leq \frac{(\nu_1)^{(7)}+(C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}} \leq \nu^{(7)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \leq (\bar{\nu}_1)^{(7)}$$

If $0 < (\nu_1)^{(7)} \leq (\bar{\nu}_1)^{(7)} \leq \boxed{(\nu_0)^{(7)}=\frac{G_{36}^0}{G_{37}^0}}$ , we obtain 419

$$(\nu_1)^{(7)} \leq \nu^{(7)}(t) \leq \frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \leq (\nu_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(7)}(t)$ :-

$$(m_2)^{(7)} \leq \nu^{(7)}(t) \leq (m_1)^{(7)}, \quad \boxed{\nu^{(7)}(t)=\frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(7)}(t)$ :- 420

$$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}, \quad \boxed{u^{(7)}(t)=\frac{T_{36}(t)}{T_{37}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $\nu^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)}G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for**

**the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)}T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

**Proof** : From global equations we obtain 421

$$\frac{d\nu^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right) - (a''_{41})^{(8)}(T_{41},t)\nu^{(8)} - (a_{41})^{(8)}\nu^{(8)}$$

**Definition of** $\nu^{(8)}$ :- $\boxed{\nu^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right) \le \frac{d\nu^{(8)}}{dt} \le -\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$ :-

For $0 < \boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$

$$\nu^{(8)}(t) \ge \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \le \nu^{(8)}(t) \le (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \le \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \le \nu^{(8)}(t) \le (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \le \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}} \le \nu^{(8)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \le (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \le (\bar{\nu}_1)^{(8)} \le \boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}}$ , we obtain 424

$$(v_1)^{(8)} \leq \nu^{(8)}(t) \leq \frac{(\bar{v}_1)^{(8)}+(\bar{C})^{(8)}(\bar{v}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{v}_1)^{(8)}-(\bar{v}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{v}_1)^{(8)}-(\bar{v}_2)^{(8)}\right)t\right]}} \leq (v_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(8)}(t)$ :-

$$(m_2)^{(8)} \leq \nu^{(8)}(t) \leq (m_1)^{(8)}, \quad \boxed{\nu^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(8)}(t)$ :-

$$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}, \quad \boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $\nu^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)}G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then $(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)}T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

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**Proof** : From 99,20,44,22,23,44 we obtain

$$\frac{d\nu^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45}, t)\right) - (a''_{45})^{(9)}(T_{45}, t)\nu^{(9)} - (a_{45})^{(9)}\nu^{(9)}$$

**Definition of** $\nu^{(9)}$ :- $\quad \boxed{\nu^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(\nu^{(9)}\right)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)}\right) \leq \frac{d\nu^{(9)}}{dt} \leq -\left((a_{45})^{(9)}\left(\nu^{(9)}\right)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$ :-

$$\text{For } 0 < \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$$

$$\nu^{(9)}(t) \geq \frac{(\nu_1)^{(9)}+(C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_0)^{(9)}\right)t\right]}}{1+(C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)}-(\nu_0)^{(9)}}{(\nu_0)^{(9)}-(\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\nu_9)^{(9)}$

In the same manner , we get

$$\nu^{(9)}(t) \leq \frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)}-(\nu_0)^{(9)}}{(\nu_0)^{(9)}-(\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \leq \frac{(\nu_1)^{(9)}+(C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}}{1+(C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}} \leq \nu^{(9)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \leq (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \leq (\bar{\nu}_1)^{(9)} \leq \boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$ , we obtain

$$(\nu_1)^{(9)} \leq \nu^{(9)}(t) \leq \frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \leq (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(9)}(t)$ :-

$$(m_2)^{(9)} \leq \nu^{(9)}(t) \leq (m_1)^{(9)}, \quad \boxed{\nu^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(9)}(t)$ :-

$$(\mu_2)^{(9)} \leq u^{(9)}(t) \leq (\mu_1)^{(9)}, \quad \boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)}G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then $(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)}T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following



**Theorem :** If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on $t$ , and the conditions with the notations

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0$ ,

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$

$with\ \ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(2)} and\ (b_i'')^{(2)}$ are independent on t , and the conditions with the notations 426

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$ 427

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 428

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$ , 429

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 430

$with\ \ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(3)} and\ (b_i'')^{(3)}$ are independent on $t$ , and the conditions with the notations 431

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$ ,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

**Theorem :** If $(a_i'')^{(4)} and\ (b_i'')^{(4)}$ are independent on $t$ , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$ ,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(5)} and\ (b_i'')^{(5)}$ are independent on $t$ , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$ ,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

**Theorem** If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on $t$ , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$ ,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(7)} and\ (b_i'')^{(7)}$ are independent on $t$ , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$ ,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(8)} and\ (b_i'')^{(8)}$ are independent on $t$ , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$ ,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(9)} and\ (b_i'')^{(9)}$ are independent on $t$ , and the conditions (with the notations 436 A
45,46,27,28)

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$ ,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0 \quad 437$$

$$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0 \quad 438$$

$$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0 \quad 439$$

$$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0 \quad 440$$

$$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ]T_{14} = 0 \quad 441$$

$$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0 \quad 442$$

has a unique positive solution , which is an equilibrium solution for the system

$$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0 \quad 443$$

$$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0 \quad 444$$

$$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0 \quad 445$$

$$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\ ]T_{16} = 0 \quad 446$$

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\ ]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\ ]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\ ]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\ ]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\ ]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27})\right)]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27})\right)]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27})\right)]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0 \quad 479$$

$$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0 \quad 480$$

$$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0 \quad 481$$

$$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0 \quad 482$$

$$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0 \quad 483$$

$$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0 \quad 484$$

$$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0 \quad 484\text{A}$$

$$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$$

$$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$$

$$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$$

$$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$$

$$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$$

**Proof:** 485

(a) Indeed the first two equations have a nontrivial solution $G_{13}, G_{14}$ if

$$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$$

**Proof:** 486

(g) Indeed the first two equations have a nontrivial solution $G_{16}, G_{17}$ if

$$F(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$$

**Proof:** 487

(a) Indeed the first two equations have a nontrivial solution $G_{20}, G_{21}$ if

$$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$$

**Proof:** 488

(a) Indeed the first two equations have a nontrivial solution $G_{24}, G_{25}$ if

$$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$$

**Proof:** 489

(a) Indeed the first two equations have a nontrivial solution $G_{28}, G_{29}$ if

$$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$$

**Proof:** 490

(a) Indeed the first two equations have a nontrivial solution $G_{32}, G_{33}$ if

$$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$$

**Proof:** 491

(a) Indeed the first two equations have a nontrivial solution $G_{36}, G_{37}$ if

$$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$$

**Proof:** 492

(a) Indeed the first two equations have a nontrivial solution $G_{40}, G_{41}$ if

$$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$$

**Proof:** 492 A

(a) Indeed the first two equations have a nontrivial solution $G_{44}, G_{45}$ if

$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$

**Definition and uniqueness of** $T^*_{14}$ :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique $T^*_{14}$ for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

**Definition and uniqueness of** $T^*_{17}$ :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique $T^*_{17}$ for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]} \qquad 495$$

**Definition and uniqueness of** $T^*_{21}$ :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique $T^*_{21}$ for which $f(T^*_{21}) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

**Definition and uniqueness of** $T^*_{25}$ :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that there exists a unique $T^*_{25}$ for which $f(T^*_{25}) = 0$. With this value , we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T^*_{25})]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T^*_{25})]}$$

**Definition and uniqueness of** $T^*_{29}$ :- 498

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(5)}(T_{29})$ being increasing, it follows that there exists a unique $T^*_{29}$ for which $f(T^*_{29}) = 0$. With this value , we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T^*_{29})]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T^*_{29})]}$$

**Definition and uniqueness of** $T^*_{33}$ :- 499

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(6)}(T_{33})$ being increasing, it follows that

there exists a unique $T_{33}^{*}$ for which $f(T_{33}^{*}) = 0$. With this value , we obtain from the three first equations

$$G_{32} = \frac{(a_{32})^{(6)} G_{33}}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^{*})]} \quad , \quad G_{34} = \frac{(a_{34})^{(6)} G_{33}}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^{*})]}$$

**Definition and uniqueness of** $T_{37}^{*}$ :- 500

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(7)}(T_{37})$ being increasing, it follows that there exists a unique $T_{37}^{*}$ for which $f(T_{37}^{*}) = 0$. With this value , we obtain from the three first equations

$$G_{36} = \frac{(a_{36})^{(7)} G_{37}}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^{*})]} \quad , \quad G_{38} = \frac{(a_{38})^{(7)} G_{37}}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^{*})]}$$

**Definition and uniqueness of** $T_{41}^{*}$ :- 501

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique $T_{41}^{*}$ for which $f(T_{41}^{*}) = 0$. With this value , we obtain from the three first equations

$$G_{40} = \frac{(a_{40})^{(8)} G_{41}}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^{*})]} \quad , \quad G_{42} = \frac{(a_{42})^{(8)} G_{41}}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^{*})]}$$

**Definition and uniqueness of** $T_{45}^{*}$ :- 501 A

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(9)}(T_{45})$ being increasing, it follows that there exists a unique $T_{45}^{*}$ for which $f(T_{45}^{*}) = 0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)} G_{45}}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^{*})]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)} G_{45}}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^{*})]}$$

By the same argument, the equations admit solutions $G_{13}, G_{14}$ if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right] + (b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{14}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{14}^{*}$ such that $\varphi(G^{*}) = 0$

By the same argument, the equations admit solutions $G_{16}, G_{17}$ if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right] + (b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{17}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{14}^{*}$ such that $\varphi((G_{19})^{*}) = 0$ 504

By the same argument, the equations admit solutions $G_{20}, G_{21}$ if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right] + (b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{21}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{21}^{*}$ such that $\varphi((G_{23})^{*}) = 0$

By the same argument, the equations admit solutions $G_{24}, G_{25}$ if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right] + (b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{25}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{25}^{*}$ such that $\varphi((G_{27})^{*}) = 0$

By the same argument, the equations admit solutions $G_{28}, G_{29}$ if 507

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right] + (b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{29}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{29}^{*}$ such that $\varphi((G_{31})^{*}) = 0$

By the same argument, the equations admit solutions $G_{32}, G_{33}$ if 508

$$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$$

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right] + (b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{33}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{33}^{*}$ such that $\varphi({G_{35}}^{*}) = 0$

By the same argument, the equations admit solutions $G_{36}, G_{37}$ if 509

$$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$$

$$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right] + (b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{37}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{37}^{*}$ such that $\varphi({G_{39}}^{*}) = 0$

By the same argument, the equations admit solutions $G_{40}, G_{41}$ if 510

$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$
$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right] + (b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{41}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{41}$ such that $\varphi(G_{43}{}^*) = 0$

By the same argument, the equations 92,93 admit solutions $G_{44}, G_{45}$ if
$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$

$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right] + (b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{45}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{45}$ such that $\varphi((G_{47})^*) = 0$

Finally we obtain the unique solution 511

$G^*_{14}$ given by $\varphi(G^*) = 0$ , $T^*_{14}$ given by $f(T^*_{14}) = 0$ and

$$G^*_{13} = \frac{(a_{13})^{(1)}G^*_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G^*_{15} = \frac{(a_{15})^{(1)}G^*_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

$$T^*_{13} = \frac{(b_{13})^{(1)}T^*_{14}}{[(b'_{13})^{(1)}-(b''_{13})^{(1)}(G^*)]} \quad , \quad T^*_{15} = \frac{(b_{15})^{(1)}T^*_{14}}{[(b'_{15})^{(1)}-(b''_{15})^{(1)}(G^*)]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

$G^*_{17}$ given by $\varphi((G_{19})^*) = 0$ , $T^*_{17}$ given by $f(T^*_{17}) = 0$ and 512

$$G^*_{16} = \frac{(a_{16})^{(2)}G^*_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G^*_{18} = \frac{(a_{18})^{(2)}G^*_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]} \quad 513$$

$$T^*_{16} = \frac{(b_{16})^{(2)}T^*_{17}}{[(b'_{16})^{(2)}-(b''_{16})^{(2)}((G_{19})^*)]} \quad , \quad T^*_{18} = \frac{(b_{18})^{(2)}T^*_{17}}{[(b'_{18})^{(2)}-(b''_{18})^{(2)}((G_{19})^*)]} \quad 514$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

$G^*_{21}$ given by $\varphi((G_{23})^*) = 0$ , $T^*_{21}$ given by $f(T^*_{21}) = 0$ and

$$G^*_{20} = \frac{(a_{20})^{(3)}G^*_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G^*_{22} = \frac{(a_{22})^{(3)}G^*_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

$$T^*_{20} = \frac{(b_{20})^{(3)}T^*_{21}}{[(b'_{20})^{(3)}-(b''_{20})^{(3)}(G_{23}{}^*)]} \quad , \quad T^*_{22} = \frac{(b_{22})^{(3)}T^*_{21}}{[(b'_{22})^{(3)}-(b''_{22})^{(3)}(G_{23}{}^*)]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

$G_{25}^{*}$ given by $\varphi(G_{27}) = 0$ , $T_{25}^{*}$ given by $f(T_{25}^{*}) = 0$ and

$$G_{24}^{*} = \frac{(a_{24})^{(4)} G_{25}^{*}}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}^{*} = \frac{(a_{26})^{(4)} G_{25}^{*}}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^{*})]}$$

$$T_{24}^{*} = \frac{(b_{24})^{(4)} T_{25}^{*}}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^{*})]} \quad , \quad T_{26}^{*} = \frac{(b_{26})^{(4)} T_{25}^{*}}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^{*})]} \qquad 517$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

$G_{29}^{*}$ given by $\varphi((G_{31})^{*}) = 0$ , $T_{29}^{*}$ given by $f(T_{29}^{*}) = 0$ and

$$G_{28}^{*} = \frac{(a_{28})^{(5)} G_{29}^{*}}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}^{*} = \frac{(a_{30})^{(5)} G_{29}^{*}}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^{*})]}$$

$$T_{28}^{*} = \frac{(b_{28})^{(5)} T_{29}^{*}}{[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31})^{*})]} \quad , \quad T_{30}^{*} = \frac{(b_{30})^{(5)} T_{29}^{*}}{[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31})^{*})]} \qquad 519$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

$G_{33}^{*}$ given by $\varphi((G_{35})^{*}) = 0$ , $T_{33}^{*}$ given by $f(T_{33}^{*}) = 0$ and

$$G_{32}^{*} = \frac{(a_{32})^{(6)} G_{33}^{*}}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^{*})]} \quad , \quad G_{34}^{*} = \frac{(a_{34})^{(6)} G_{33}^{*}}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^{*})]}$$

$$T_{32}^{*} = \frac{(b_{32})^{(6)} T_{33}^{*}}{[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35})^{*})]} \quad , \quad T_{34}^{*} = \frac{(b_{34})^{(6)} T_{33}^{*}}{[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35})^{*})]} \qquad 521$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

$G_{37}^{*}$ given by $\varphi((G_{39})^{*}) = 0$ , $T_{37}^{*}$ given by $f(T_{37}^{*}) = 0$ and

$$G_{36}^{*} = \frac{(a_{36})^{(7)} G_{37}^{*}}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^{*})]} \quad , \quad G_{38}^{*} = \frac{(a_{38})^{(7)} G_{37}^{*}}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^{*})]}$$

$$T_{36}^{*} = \frac{(b_{36})^{(7)} T_{37}^{*}}{[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39})^{*})]} \quad , \quad T_{38}^{*} = \frac{(b_{38})^{(7)} T_{37}^{*}}{[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39})^{*})]}$$

Finally we obtain the unique solution 523

$G_{41}^{*}$ given by $\varphi((G_{43})^{*}) = 0$ , $T_{41}^{*}$ given by $f(T_{41}^{*}) = 0$ and

$$G_{40}^{*} = \frac{(a_{40})^{(8)} G_{41}^{*}}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^{*})]} \quad , \quad G_{42}^{*} = \frac{(a_{42})^{(8)} G_{41}^{*}}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^{*})]}$$

$$T_{40}^{*}=\frac{(b_{40})^{(8)}T_{41}^{*}}{[(b'_{40})^{(8)}-(b''_{40})^{(8)}((G_{43})^{*})]} \quad , \quad T_{42}^{*}=\frac{(b_{42})^{(8)}T_{41}^{*}}{[(b'_{42})^{(8)}-(b''_{42})^{(8)}((G_{43})^{*})]}$$

Finally we obtain the unique solution of 89 to 99 523

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$G_{45}^{*}$ given by $\varphi((G_{47})^{*})=0$ , $T_{45}^{*}$ given by $f(T_{45}^{*})=0$ and

$$G_{44}^{*}=\frac{(a_{44})^{(9)}G_{45}^{*}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T_{45}^{*})]} \quad , \quad G_{46}^{*}=\frac{(a_{46})^{(9)}G_{45}^{*}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T_{45}^{*})]}$$

$$T_{44}^{*}=\frac{(b_{44})^{(9)}T_{45}^{*}}{[(b'_{44})^{(9)}-(b''_{44})^{(9)}((G_{47})^{*})]} \quad , \quad T_{46}^{*}=\frac{(b_{46})^{(9)}T_{45}^{*}}{[(b'_{46})^{(9)}-(b''_{46})^{(9)}((G_{47})^{*})]}$$

**ASYMPTOTIC STABILITY ANALYSIS** 524

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}\ and\ (b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

**Proof:** Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i=G_i^{*}+\mathbb{G}_i \qquad , T_i=T_i^{*}+\mathbb{T}_i$$

$$\frac{\partial(a''_{14})^{(1)}}{\partial T_{14}}(T_{14}^{*})=(q_{14})^{(1)} \quad , \frac{\partial(b_i'')^{(1)}}{\partial G_j}(G^{*})=s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt}=-\left((a'_{13})^{(1)}+(p_{13})^{(1)}\right)\mathbb{G}_{13}+(a_{13})^{(1)}\mathbb{G}_{14}-(q_{13})^{(1)}G_{13}^{*}\mathbb{T}_{14} \quad 525$$

$$\frac{d\mathbb{G}_{14}}{dt}=-\left((a'_{14})^{(1)}+(p_{14})^{(1)}\right)\mathbb{G}_{14}+(a_{14})^{(1)}\mathbb{G}_{13}-(q_{14})^{(1)}G_{14}^{*}\mathbb{T}_{14} \quad 526$$

$$\frac{d\mathbb{G}_{15}}{dt}=-\left((a'_{15})^{(1)}+(p_{15})^{(1)}\right)\mathbb{G}_{15}+(a_{15})^{(1)}\mathbb{G}_{14}-(q_{15})^{(1)}G_{15}^{*}\mathbb{T}_{14} \quad 527$$

$$\frac{d\mathbb{T}_{13}}{dt}=-\left((b'_{13})^{(1)}-(r_{13})^{(1)}\right)\mathbb{T}_{13}+(b_{13})^{(1)}\mathbb{T}_{14}+\sum_{j=13}^{15}\left(s_{(13)(j)}T_{13}^{*}\mathbb{G}_j\right) \quad 528$$

$$\frac{d\mathbb{T}_{14}}{dt}=-\left((b'_{14})^{(1)}-(r_{14})^{(1)}\right)\mathbb{T}_{14}+(b_{14})^{(1)}\mathbb{T}_{13}+\sum_{j=13}^{15}\left(s_{(14)(j)}T_{14}^{*}\mathbb{G}_j\right) \quad 529$$

$$\frac{d\mathbb{T}_{15}}{dt}=-\left((b'_{15})^{(1)}-(r_{15})^{(1)}\right)\mathbb{T}_{15}+(b_{15})^{(1)}\mathbb{T}_{14}+\sum_{j=13}^{15}\left(s_{(15)(j)}T_{15}^{*}\mathbb{G}_j\right) \quad 530$$

**ASYMPTOTIC STABILITY ANALYSIS** 531

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(2)}$ and $(b_i'')^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i \tag{532}$$

$$\frac{\partial(a_{17}'')^{(2)}}{\partial T_{17}}(T_{17}^*) = (q_{17})^{(2)} \quad , \frac{\partial(b_i'')^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \tag{533}$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a_{16}')^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G_{16}^*\mathbb{T}_{17} \tag{534}$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a_{17}')^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G_{17}^*\mathbb{T}_{17} \tag{535}$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a_{18}')^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G_{18}^*\mathbb{T}_{17} \tag{536}$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b_{16}')^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T_{16}^*\mathbb{G}_j\right) \tag{537}$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b_{17}')^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}T_{17}^*\mathbb{G}_j\right) \tag{538}$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b_{18}')^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}T_{18}^*\mathbb{G}_j\right) \tag{539}$$

**ASYMPTOTIC STABILITY ANALYSIS** (540)

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(3)}$ *and* $(b_i'')^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{21}'')^{(3)}}{\partial T_{21}}(T_{21}^*) = (q_{21})^{(3)} \quad , \frac{\partial(b_i'')^{(3)}}{\partial G_j}((G_{23})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a_{20}')^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G_{20}^*\mathbb{T}_{21} \tag{541}$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a_{21}')^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G_{21}^*\mathbb{T}_{21} \tag{542}$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a_{22}')^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G_{22}^*\mathbb{T}_{21} \tag{543}$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b_{20}')^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T_{20}^*\mathbb{G}_j\right) \tag{544}$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b_{21}')^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T_{21}^*\mathbb{G}_j\right) \tag{545}$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b'_{22})^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T^*_{22}\mathbb{G}_j\right) \quad 546$$

**ASYMPTOTIC STABILITY ANALYSIS** 547

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}\ and\ (b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{25})^{(4)}}{\partial T_{25}}(T^*_{25}) = (q_{25})^{(4)} \ , \ \frac{\partial(b_i'')^{(4)}}{\partial G_j}((G_{27})^*\ ) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\left((a'_{24})^{(4)} + (p_{24})^{(4)}\right)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G^*_{24}\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\left((a'_{25})^{(4)} + (p_{25})^{(4)}\right)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G^*_{25}\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\left((a'_{26})^{(4)} + (p_{26})^{(4)}\right)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G^*_{26}\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\left((b'_{24})^{(4)} - (r_{24})^{(4)}\right)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(24)(j)}T^*_{24}\mathbb{G}_j\right) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\left((b'_{25})^{(4)} - (r_{25})^{(4)}\right)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\left(s_{(25)(j)}T^*_{25}\mathbb{G}_j\right) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T^*_{26}\mathbb{G}_j\right) \quad 554$$

**ASYMPTOTIC STABILITY ANALYSIS** 555

**Theorem 5:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}\ and\ (b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 556

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{29})^{(5)}}{\partial T_{29}}(T^*_{29}) = (q_{29})^{(5)} \ , \ \frac{\partial(b_i'')^{(5)}}{\partial G_j}(\ (G_{31})^*\ ) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a'_{28})^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G^*_{28}\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a'_{29})^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G^*_{29}\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a'_{30})^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G^*_{30}\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b'_{28})^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T^*_{28}\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b'_{29})^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T^*_{29}\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b'_{30})^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T^*_{30}\mathbb{G}_j\right) \quad 562$$

**ASYMPTOTIC STABILITY ANALYSIS** 563

**Theorem 6:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}$ *and* $(b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{33})^{(6)}}{\partial T_{33}}(T^*_{33}) = (q_{33})^{(6)} \quad , \frac{\partial(b_i'')^{(6)}}{\partial G_j}\left((G_{35})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a'_{32})^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G^*_{32}\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a'_{33})^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G^*_{33}\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a'_{34})^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G^*_{34}\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b'_{32})^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T^*_{32}\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b'_{33})^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T^*_{33}\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b'_{34})^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T^*_{34}\mathbb{G}_j\right) \quad 570$$

**ASYMPTOTIC STABILITY ANALYSIS** 571

**Theorem 7:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 572

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{37}'')^{(7)}}{\partial T_{37}}(T_{37}^{*}) = (q_{37})^{(7)} \quad , \frac{\partial(b_i'')^{(7)}}{\partial G_j}\left( (G_{39})^{**} \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a_{36}')^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G_{36}^{*}\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a_{37}')^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G_{37}^{*}\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a_{38}')^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G_{38}^{*}\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b_{36}')^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T_{36}^{*}\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b_{37}')^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T_{37}^{*}\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b_{38}')^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T_{38}^{*}\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

## ASYMPTOTIC STABILITY ANALYSIS

**Theorem 8:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 580

$$G_i = G_i^{*} + \mathbb{G}_i \qquad , T_i = T_i^{*} + \mathbb{T}_i$$

$$\frac{\partial(a_{41}'')^{(8)}}{\partial T_{41}}(T_{41}^{*}) = (q_{41})^{(8)} \quad , \frac{\partial(b_i'')^{(8)}}{\partial G_j}\left( (G_{43})^{*} \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a_{40}')^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G_{40}^{*}\mathbb{T}_{41} \quad 581$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a_{41}')^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G_{41}^{*}\mathbb{T}_{41} \quad 582$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a_{42}')^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G_{42}^{*}\mathbb{T}_{41} \quad 583$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b_{40}')^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T_{40}^{*}\mathbb{G}_j\right) \quad 584$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b_{41}')^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T_{41}^{*}\mathbb{G}_j\right) \quad 585$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b'_{42})^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T^*_{42}\mathbb{G}_j\right)$$ 586

586

**ASYMPTOTIC STABILITY ANALYSIS** A

**Theorem 9:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}\ and\ (b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

**Proof:** Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{45})^{(9)}}{\partial T_{45}}(T^*_{45}) = (q_{45})^{(9)} \ , \ \frac{\partial(b_i'')^{(9)}}{\partial G_j}((G_{47})^*) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a'_{44})^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)}G^*_{44}\mathbb{T}_{45}$$ 586 B

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a'_{45})^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)}G^*_{45}\mathbb{T}_{45}$$ 586 C

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a'_{46})^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)}G^*_{46}\mathbb{T}_{45}$$ 586 D

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b'_{44})^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)}T^*_{44}\mathbb{G}_j\right)$$ 586 E

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b'_{45})^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)}T^*_{45}\mathbb{G}_j\right)$$ 586 F

$$\frac{d\mathbb{T}_{46}}{dt} = -\left((b'_{46})^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T^*_{46}\mathbb{G}_j\right)$$ 586 G

**The characteristic equation of this system is** 587

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(q_{13})^{(1)}G^*_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)}T^*_{14} + (b_{14})^{(1)}s_{(13),(14)}T^*_{14}\right)$$

$$+\left(\left((\lambda)^{(1)} + (a'_{14})^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)}G^*_{13} + (a_{13})^{(1)}(q_{14})^{(1)}G^*_{14}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)}T^*_{14} + (b_{14})^{(1)}s_{(13),(13)}T^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((b'_{13})^{(1)} + (b'_{14})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^2+\left((a'_{13})^{(1)}+(a'_{14})^{(1)}+(p_{13})^{(1)}+(p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)}+(a'_{13})^{(1)}+(p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^*_{14}+(a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}+(b'_{13})^{(1)}-(r_{13})^{(1)}\right)s_{(14),(15)}T^*_{14}+(b_{14})^{(1)}s_{(13),(15)}T^*_{13}\right)\}=0$$

$$+$$

$$(\lambda)^{(2)}+(b'_{18})^{(2)}-(r_{18})^{(2)}\{\left((\lambda)^{(2)}+(a'_{18})^{(2)}+(p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^*_{17}+(a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^*_{17}\right)$$

$$+\left(\left((\lambda)^{(2)}+(a'_{17})^{(2)}+(p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^*_{16}+(a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2+\left((b'_{16})^{(2)}+(b'_{17})^{(2)}-(r_{16})^{(2)}+(r_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$+\left(\left((\lambda)^{(2)}\right)^2+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}+(a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^*_{16}\right)\}=0$$

$$+$$

$$(\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left(\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^*_{20}\right)\right]$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^*_{21}+(b_{21})^{(3)}s_{(20),(21)}T^*_{21}\right)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^*_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^*_{21}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^*_{21}+(b_{21})^{(3)}s_{(20),(20)}T^*_{20}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^2+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$+\left(\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^*_{20}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^*_{21}+(b_{21})^{(3)}s_{(20),(22)}T^*_{20}\right)\}=0$$

$$+$$

$$(\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left(\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^*_{24}\right)\right]$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^*_{25}+(b_{25})^{(4)}s_{(24),(25)}T^*_{25}\right)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^*_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^*_{25}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^*_{25}+(b_{25})^{(4)}s_{(24),(24)}T^*_{24}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^2+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$+\left(\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^*_{24}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^*_{25}+(b_{25})^{(4)}s_{(24),(26)}T^*_{24}\right)\}=0$$

$$+$$

$$(\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^*_{28}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^*_{29}+(b_{29})^{(5)}s_{(28),(29)}T^*_{29}\right)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^*_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^*_{29}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^*_{29}+(b_{29})^{(5)}s_{(28),(28)}T^*_{28}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^2+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$+\left(\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^*_{28}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^*_{29}+(b_{29})^{(5)}s_{(28),(30)}T^*_{28}\right)\}=0$$

$$+$$

$$(\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left(\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^*_{32}\right)\right]$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^*_{33}+(b_{33})^{(6)}s_{(32),(33)}T^*_{33}\right)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^*_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^*_{33}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^*_{33}+(b_{33})^{(6)}s_{(32),(32)}T^*_{32}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^2+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$+\left(\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^*_{32}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^*_{33}+(b_{33})^{(6)}s_{(32),(34)}T^*_{32}\right)\}=0$$

$$+$$

$$(\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^*_{36}\right)\right]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^*_{37}+(b_{37})^{(7)}s_{(36),(37)}T^*_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^*_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^*_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^*_{37}+(b_{37})^{(7)}s_{(36),(36)}T^*_{36}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^2+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$+\left(\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^*_{36}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^*_{37}+(b_{37})^{(7)}s_{(36),(38)}T^*_{36}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\right)\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^*_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^*_{41}+(b_{41})^{(8)}s_{(40),(41)}T^*_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^*_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^*_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^*_{41}+(b_{41})^{(8)}s_{(40),(40)}T^*_{40}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^2+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$+\left(\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^*_{40}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^*_{41}+(b_{41})^{(8)}s_{(40),(42)}T^*_{40}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\right)\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^*_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^*_{45}+(b_{45})^{(9)}s_{(44),(45)}T^*_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^*_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^*_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^*_{45}+(b_{45})^{(9)}s_{(44),(44)}T^*_{44}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^2+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$+\left(\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^*_{44}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^*_{45}+(b_{45})^{(9)}s_{(44),(46)}T^*_{44}\right)\}=0$$

**And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.**

## SECTION NINE

## Chaotic Dynamics Of The Brain, Take The Form Of Fractals

### INTRODUCTION—VARIABLES USED

**Chris King: Supercausality and consciousness - 1989**.

(1) As a consequence of the fact that all biological systems would act according to (e) free will, and as a consequence of the fact that the outcome of free will is (=) indeterminable, biological systems should always show (eb) chaotic dynamics which cannot be studied in a deterministic or computational way.

(2) King suggests two levels of explanation of consciousness. On the first level information moves from the mind to (e&eb) the brain as a consequence of (e) the exercise of free will; on the second level, information flows from the brain to (e&eb) the mind thanks to the amplification of signals performed by (e) the chaotic dynamics of the brain, which usual take the form of fractals.

### NOTATION

#### Module One

As a consequence of the fact that all biological systems would act according to (e) free will, and as a consequence of the fact that the outcome of free will is (=) indeterminable, biological systems should always show (eb) chaotic dynamics which cannot be studied in a deterministic or computational way

$G_{13}$ : Category one of **biological systems;** free will

$G_{14}$ : Category two of SAS

$G_{15}$ : Category three of SAS

$T_{13}$ : Category one of free will **; biological systems**

$T_{14}$ : Category two of SAS

$T_{15}$ : Category three of SAS

**Module Two**

As a consequence of the fact that all biological systems would act according to free will, and as a consequence of the fact that the outcome of free will is indeterminable, biological systems should always show (eb) chaotic dynamics which cannot be studied in a deterministic or computational way

$G_{16}$ : Category one of free will

$G_{17}$ : Category two of SAS

$G_{18}$ : Category three of SAS

$T_{16}$ : Category one of indeterminable

$T_{17}$ : Category two of SAS

$T_{18}$ : Category three of SAS

**Module three**

$G_{20}$ : Category one of all biological systems would act according to free will, and that the outcome of free will is indeterminable

$G_{21}$ : Category two of SAS

$G_{22}$ : Category three of SAS

$T_{20}$ : Category one of chaotic dynamics which cannot be studied in a deterministic or computational way

$T_{21}$ : Category two of SAS

$T_{22}$ : Category three of SAS

**Module four**

King suggests two levels of explanation of consciousness.

On the first level **information moves from the mind** to (e&eb) the brain as a consequence of (e) the exercise of free will; on the second level, information flows from the brain to (e&eb) the mind thanks to the amplification of signals performed by (e) the chaotic dynamics of the brain, which usual take the form of fractals.

$G_{24}$ : Category one of **information moves from the mind;** brain as a consequence of (e) the exercise of free will; on the second level, information flows from the brain to (e&eb) the mind thanks to the amplification of signals performed by (e) the chaotic dynamics of the brain, which usual take the form of fractals.

$G_{25}$ : Category two of SAS

$G_{26}$ : Category three of SAS

$T_{24}$ : Category one of brain as a consequence of (e) the exercise of free will; on the second level, information flows from the brain to (e&eb) the mind thanks to the amplification of signals performed by (e)

the chaotic dynamics of the brain, which usual take the form of fractals.; **information moves from the mind**

$T_{25}$ : Category two of SAS

$T_{26}$ : Category three of SAS

**Module five**

On the first level information moves from the mind to the brain as a consequence of (e) the exercise of free will; on the second level, information flows from the brain to (e&eb) the mind thanks to the amplification of signals performed by (e) the chaotic dynamics of the brain, which usual take the form of fractals

$G_{28}$ : Category one of exercise of free will; on the second level, information flows from the brain to (e&eb) the mind thanks to the amplification of signals performed by (e) the chaotic dynamics of the brain, which usual take the form of fractals

$G_{29}$ : Category two of SAS

$G_{30}$ : Category three of SAS

$T_{28}$ : Category one of information moves from the mind to the brain

$T_{29}$ : Category two of SAS

$T_{30}$ : Category three of SAS

**Module six**

On the first level information moves from the mind to the brain as a consequence of the exercise of free will; on the second level, information flows from the brain to (e&eb) the mind thanks to the amplification of signals performed by (e) the chaotic dynamics of the brain, which usual take the form of fractals

$G_{32}$ : Category one of **information flows from the brain;** mind thanks to the amplification of signals performed by (e) the chaotic dynamics of the brain, which usual take the form of fractals

$G_{33}$ : Category two of SAS

$G_{34}$ : Category three of SAS

$T_{32}$ : Category one of mind thanks to the amplification of signals performed by (e) the chaotic dynamics of the brain, which usual take the form of fractals; **information flows from the brain**

$T_{33}$ : Category two of SAS

$T_{34}$ : Category three of SAS

**Module seven**

On the first level information moves from the mind to the brain as a consequence of the exercise of free will; on the second level, information flows from the brain to the mind thanks to the amplification of signals performed by (e) the chaotic dynamics of the brain, which usual take the form of fractals

$G_{36}$ : Category one of amplification of signals performed by (e) the chaotic dynamics of the brain, which usual take the form of fractals

$G_{37}$ : Category two of SAS

$G_{38}$ : Category three of SAS

$T_{36}$ : Category one of information flows from the brain to the mind

$T_{37}$ : Category two of SAS

$T_{38}$ : Category three of SAS

**Module eight**

On the first level information moves from the mind to the brain as a consequence of the exercise of free will; on the second level, information flows from the brain to the mind thanks to the amplification of signals performed by (e) the chaotic dynamics of the brain, which usual take the form of fractals

$G_{40}$ : Category one of chaotic dynamics of the brain, which usual take the form of fractals

$G_{41}$ : Category two of SAS

$G_{42}$ : Category three of SAS

$T_{40}$ : Category one of information flows from the brain to the mind thanks to the amplification of signals

$T_{41}$ : Category two of SAS

$T_{42}$ : Category three of SAS

**Module Nine**

On the first level information moves from the mind to the brain as a consequence of the exercise of free will; on the second level, information flows from the brain to the mind thanks to the amplification of signals performed by the chaotic dynamics of the brain, which usual take the form of fractals

$G_{44}$ : Category one of information flows from the brain to the mind thanks to the amplification of signals performed by the chaotic dynamics of the brain

$G_{45}$ : Category two of SAS

$G_{46}$ : Category three of SAS

$T_{44}$ : Category one of form of fractals

$T_{45}$ : Category two of SAS

$T_{46}$ : Category three of SAS

**The Coefficients:**

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}\ ,\ (b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}$,
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)},\ \ (b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}\ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$ ,
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)}$,
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)}$,
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)}$,
are Dissipation coefficients

**Module Numbered One**

**The differential system of this model is now (Module Numbered one)**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \quad 1$$
$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad 2$$
$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15} \quad 3$$
$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13} \quad 4$$
$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14} \quad 5$$
$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)\right]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14}, t) =$ First augmentation factor
$-(b''_{13})^{(1)}(G, t) =$ First detritions factor

**Module Numbered Two**

**The differential system of this model is now ( Module numbered two)**

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right]G_{16} \quad 7$$
$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t)\right]G_{17} \quad 8$$
$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t)\right]G_{18} \quad 9$$
$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\big((G_{19}), t\big)\right]T_{16} \quad 10$$
$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\big((G_{19}), t\big)\right]T_{17} \quad 11$$
$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\big((G_{19}), t\big)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17}, t) =$ First augmentation factor
$-(b''_{16})^{(2)}\big((G_{19}), t\big) =$ First detritions factor

**Module Numbered Three**

**The differential system of this model is now (Module numbered three)**

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21},t)\right]G_{20} \quad 13$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21},t)\right]G_{21} \quad 14$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21},t)\right]G_{22} \quad 15$$

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23},t)\right]T_{20} \quad 16$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23},t)\right]T_{21} \quad 17$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23},t)\right]T_{22} \quad 18$$

$+(a''_{20})^{(3)}(T_{21},t) =$ First augmentation factor

$-(b''_{20})^{(3)}(G_{23},t) =$ First detritions factor

**Module Numbered Four**

**The differential system of this model is now (Module numbered Four)**

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right]G_{24} \quad 19$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25},t)\right]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25},t)\right]G_{26} \quad 21$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}\big((G_{27}),t\big)\right]T_{24} \quad 22$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}\big((G_{27}),t\big)\right]T_{25} \quad 23$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}\big((G_{27}),t\big)\right]T_{26} \quad 24$$

$+(a''_{24})^{(4)}(T_{25},t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}\big((G_{27}),t\big) =$ **First detritions factor**

**Module Numbered Five**:

**The differential system of this model is now (Module number five)**

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29},t)\right]G_{28} \quad 25$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29},t)\right]G_{29} \quad 26$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29},t)\right]G_{30} \quad 27$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}\big((G_{31}),t\big)\right]T_{28} \quad 28$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}\big((G_{31}),t\big)\right]T_{29} \quad 29$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}\big((G_{31}),t\big)\right]T_{30} \quad 30$$

$+(a''_{28})^{(5)}(T_{29},t) =$ **First augmentation factor**

$-(b''_{28})^{(5)}\big((G_{31}),t\big) =$ **First detritions factor**

**Module Numbered Six**

**The differential system of this model is now (Module numbered Six)**

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33},t)\right]G_{32} \quad 31$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33},t)\right]G_{33} \quad 32$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33},t)\right]G_{34} \quad 33$$

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35}),t)\right]T_{32} \quad 34$$
$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}((G_{35}),t)\right]T_{33} \quad 35$$
$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35}),t)\right]T_{34} \quad 36$$
$+(a''_{32})^{(6)}(T_{33},t) =$ **First augmentation factor**

**Module Numbered Seven:**

**The differential system of this model is now (Seventh Module)**

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right]G_{36} \quad 37$$
$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37},t)\right]G_{37} \quad 38$$
$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37},t)\right]G_{38} \quad 39$$
$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39}),t)\right]T_{36} \quad 40$$
$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}((G_{39}),t)\right]T_{37} \quad 41$$
$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39}),t)\right]T_{38} \quad 42$$
$+(a''_{36})^{(7)}(T_{37},t) =$ **First augmentation factor**

**Module Numbered Eight**

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right]G_{40} \quad 43$$
$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41},t)\right]G_{41} \quad 44$$
$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41},t)\right]G_{42} \quad 45$$
$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43}),t)\right]T_{40} \quad 46$$
$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}((G_{43}),t)\right]T_{41} \quad 47$$
$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43}),t)\right]T_{42} \quad 48$$

**Module Numbered Nine**

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$
$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$
$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$
$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47}),t)\right]T_{44} \quad 52$$
$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}((G_{47}),t)\right]T_{45} \quad 53$$
$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47}),t)\right]T_{46} \quad 54$$
$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**
$-(b''_{44})^{(9)}((G_{47}),t) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)} & \boxed{+(a''_{13})^{(1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)} & \\ \boxed{+(a''_{36})^{(7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} & \end{bmatrix} G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)}\boxed{+(a''_{14})^{(1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14} \quad (56)$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)}\boxed{+(a''_{15})^{(1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15} \quad (57)$$

Where $\boxed{(a''_{13})^{(1)}(T_{14},t)}$, $\boxed{(a''_{14})^{(1)}(T_{14},t)}$, $\boxed{(a''_{15})^{(1)}(T_{14},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7)}(T_{37},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8)}(T_{41},t)}$ $\boxed{+(a''_{41})^{(8,8)}(T_{41},t)}$ $\boxed{+(a''_{42})^{(8,8)}(T_{41},t)}$ are eight augmentation coefficient for 1,2,3

$\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)}\boxed{-(b''_{13})^{(1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13} \quad (58)$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)}\boxed{-(b''_{14})^{(1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14} \quad (59)$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)}\boxed{-(b''_{15})^{(1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15} \quad (60)$$

Where $\boxed{-(b''_{13})^{(1)}(G,t)}$, $\boxed{-(b''_{14})^{(1)}(G,t)}$, $\boxed{-(b''_{15})^{(1)}(G,t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8)}(G_{43},t)}$ $\boxed{-(b''_{41})^{(8,8)}(G_{43},t)}$ $\boxed{-(b''_{42})^{(8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[\begin{array}{ccc} (a'_{16})^{(2)}\boxed{+(a''_{16})^{(2)}(T_{17},t)} & \boxed{+(a''_{13})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9)}(T_{45},t)} \end{array}\right] G_{16} \quad (61)$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[\begin{array}{ccc} (a'_{17})^{(2)}\boxed{+(a''_{17})^{(2)}(T_{17},t)} & \boxed{+(a''_{14})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9)}(T_{45},t)} \end{array}\right] G_{17} \quad (62)$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[\begin{array}{ccc} (a'_{18})^{(2)}\boxed{+(a''_{18})^{(2)}(T_{17},t)} & \boxed{+(a''_{15})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9)}(T_{45},t)} \end{array}\right] G_{18} \quad (63)$$

Where $\boxed{+(a''_{16})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,)}(T_{14},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$+(a''_{40})^{(8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8)}(T_{41},t)$ are eight augmentation coefficient for category 1, 2 and 3

$+(a''_{44})^{(9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9)}(T_{45},t)$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \begin{bmatrix} (b'_{16})^{(2)} & -(b''_{16})^{(2)}(G_{19},t) & -(b''_{13})^{(1,1,)}(G,t) & -(b''_{20})^{(3,3,3,)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6)}(G_{35},t) \\ -(b''_{36})^{(7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9)}(G_{47},t) \end{bmatrix} T_{16} \quad (64)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)} & -(b''_{17})^{(2)}(G_{19},t) & -(b''_{14})^{(1,1,)}(G,t) & -(b''_{21})^{(3,3,3,)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6)}(G_{35},t) \\ -(b''_{37})^{(7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9)}(G_{47},t) \end{bmatrix} T_{17} \quad (65)$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)} & -(b''_{18})^{(2)}(G_{19},t) & -(b''_{15})^{(1,1,)}(G,t) & -(b''_{22})^{(3,3,3,)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6)}(G_{35},t) \\ -(b''_{38})^{(7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9)}(G_{47},t) \end{bmatrix} T_{18} \quad (66)$$

where $-(b''_{16})^{(2)}(G_{19},t)$, $-(b''_{17})^{(2)}(G_{19},t)$, $-(b''_{18})^{(2)}(G_{19},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{13})^{(1,1,)}(G,t)$, $-(b''_{14})^{(1,1,)}(G,t)$, $-(b''_{15})^{(1,1,)}(G,t)$ are second detrition coefficients for category 1,2 and 3

$-(b''_{20})^{(3,3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,)}(G_{23},t)$ are third detrition coefficients for category 1,2 and 3

$-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1,2 and 3

$-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1,2 and 3

$-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients for category 1,2 and 3

$-(b''_{36})^{(7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2 and 3

$-(b''_{40})^{(8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1,2 and 3

$-(b''_{44})^{(9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \begin{bmatrix} (a'_{20})^{(3)} & +(a''_{20})^{(3)}(T_{21},t) & +(a''_{16})^{(2,2,2)}(T_{17},t) & +(a''_{13})^{(1,1,1,)}(T_{14},t) \\ +(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t) \\ +(a''_{36})^{(7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9)}(T_{45},t) \end{bmatrix} G_{20} \quad (67)$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \begin{bmatrix} (a'_{21})^{(3)}\boxed{+(a''_{21})^{(3)}(T_{21},t)}\ \boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}\ \boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{21} \quad 68$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \begin{bmatrix} (a'_{22})^{(3)}\boxed{+(a''_{22})^{(3)}(T_{21},t)}\ \boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}\ \boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{22} \quad 69$$

$\boxed{+(a''_{20})^{(3)}(T_{21},t)}, \boxed{+(a''_{21})^{(3)}(T_{21},t)}, \boxed{+(a''_{22})^{(3)}(T_{21},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}, \boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}, \boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}$ are second augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)}$ are third augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)}, \boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)}, \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)}, \boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)}, \boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)}, \boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)}, \boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)}$ are seventh augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)}$ are eight augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{44})^{(9,9,9)}(T_{45},t)}, \boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)}, \boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)}$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b'_{20})^{(3)}\boxed{-(b''_{20})^{(3)}(G_{23},t)}\ \boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}\ \boxed{-(b''_{13})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)}\ \boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)}\ \boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7)}(G_{39},t)}\ \boxed{-(b''_{40})^{(8,8,8,8)}(G_{43},t)}\ \boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{20} \quad 70$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b'_{21})^{(3)}\boxed{-(b''_{21})^{(3)}(G_{23},t)}\ \boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}\ \boxed{-(b''_{14})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)}\ \boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)}\ \boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7)}(G_{39},t)}\ \boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)}\ \boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{21} \quad 71$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)}\boxed{-(b''_{22})^{(3)}(G_{23},t)}\ \boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}\ \boxed{-(b''_{15})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)}\ \boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)}\ \boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)}\ \boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)}\ \boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{22} \quad 72$$

$\boxed{-(b''_{20})^{(3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3)}(G_{23},t)}$ *are first detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}$ *are second detrition coefficients for*

*category 1, 2 and 3*

$\boxed{-(b_{13}'')^{(1,1,1,)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,)}(G,t)}$, $\boxed{-(b_{15}'')^{(1,1,1,)}(G,t)}$ *are third detrition coefficients for category 1,2 and 3*

$\boxed{-(b_{24}'')^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{25}'')^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{26}'')^{(4,4,4,4,4,4)}(G_{27},t)}$ *are fourth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b_{28}'')^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5)}(G_{31},t)}$ *are fifth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b_{32}'')^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{34}'')^{(6,6,6,6,6,6)}(G_{35},t)}$ *are sixth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b_{36}'')^{(7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{37}'')^{(7,7,7,7)}(G_{39},t)}$ $\boxed{-(b_{38}'')^{(7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{40}'')^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{42}'')^{(8,8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{46}'')^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b_{45}'')^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b_{44}'')^{(9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \begin{bmatrix} (a_{24}')^{(4)} & \boxed{+(a_{24}'')^{(4)}(T_{25},t)} & \boxed{+(a_{28}'')^{(5,5,)}(T_{29},t)} & \boxed{+(a_{32}'')^{(6,6,)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a_{16}'')^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a_{20}'')^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{36}'')^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{40}'')^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a_{44}'')^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{24} \quad 73$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \begin{bmatrix} (a_{25}')^{(4)} & \boxed{+(a_{25}'')^{(4)}(T_{25},t)} & \boxed{+(a_{29}'')^{(5,5,)}(T_{29},t)} & \boxed{+(a_{33}'')^{(6,6)}(T_{33},t)} \\ \boxed{+(a_{14}'')^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a_{17}'')^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a_{21}'')^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{37}'')^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{41}'')^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a_{45}'')^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{25} \quad 74$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \begin{bmatrix} (a_{26}')^{(4)} & \boxed{+(a_{26}'')^{(4)}(T_{25},t)} & \boxed{+(a_{30}'')^{(5,5,)}(T_{29},t)} & \boxed{+(a_{34}'')^{(6,6,)}(T_{33},t)} \\ \boxed{+(a_{15}'')^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a_{18}'')^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a_{22}'')^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{38}'')^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{42}'')^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a_{46}'')^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{26} \quad 75$$

$\boxed{(a_{24}'')^{(4)}(T_{25},t)}$, $\boxed{(a_{25}'')^{(4)}(T_{25},t)}$, $\boxed{(a_{26}'')^{(4)}(T_{25},t)}$ *are first augmentation coefficients category* $1, 2\ 3$

$\boxed{+(a_{28}'')^{(5,5,)}(T_{29},t)}$, $\boxed{+(a_{29}'')^{(5,5,)}(T_{29},t)}$, $\boxed{+(a_{30}'')^{(5,5,)}(T_{29},t)}$ *are second augmentation coefficient for category* $1, 2$ *and* $3$

$\boxed{+(a_{32}'')^{(6,6,)}(T_{33},t)}$, $\boxed{+(a_{33}'')^{(6,6,)}(T_{33},t)}$, $\boxed{+(a_{34}'')^{(6,6,)}(T_{33},t)}$ *are third augmentation coefficient for category* $1, 2$ *and* $3$

$\boxed{+(a_{13}'')^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a_{14}'')^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a_{15}'')^{(1,1,1,1)}(T_{14},t)}$
*are fourth augmentation coefficients for category* $1, 2$ *and* $3$

$\boxed{+(a_{16}'')^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{17}'')^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{18}'')^{(2,2,2,2)}(T_{17},t)}$
*are fifth augmentation coefficients for category* $1, 2$ *and* $3$

$\boxed{+(a_{20}'')^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{21}'')^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{22}'')^{(3,3,3,3)}(T_{21},t)}$
*are sixth augmentation coefficients for category* $1, 2$ *and* $3$

$\boxed{+(a_{36}'')^{(7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a_{37}'')^{(7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a_{38}'')^{(7,7,7,7,7)}(T_{37},t)}$
*are seventh augmentation coefficients for category* $1, 2$ *and* $3$

$+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)$

*are eighth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{46})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9)}(T_{45},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \begin{bmatrix} (b'_{24})^{(4)} & -(b''_{24})^{(4)}(G_{27},t) & -(b''_{28})^{(5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,)}(G_{35},t) \\ & -(b''_{13})^{(1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{24} \quad (76)$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \begin{bmatrix} (b'_{25})^{(4)} & -(b''_{25})^{(4)}(G_{27},t) & -(b''_{29})^{(5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{25} \quad (77)$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)} & -(b''_{26})^{(4)}(G_{27},t) & -(b''_{30})^{(5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1)}(G,t) & -(b''_{18})^{(2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{26} \quad (78)$$

*Where* $-(b''_{24})^{(4)}(G_{27},t)$, $-(b''_{25})^{(4)}(G_{27},t)$, $-(b''_{26})^{(4)}(G_{27},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{28})^{(5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,)}(G_{31},t)$ *are second detrition coefficients for category* 1, 2 *and* 3

$-(b''_{32})^{(6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,)}(G_{35},t)$ *are third detrition coefficients for category* 1, 2 *and* 3

$-(b''_{13})^{(1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1)}(G,t)$

*are fourth detrition coefficients for category* 1, 2 *and* 3

$-(b''_{16})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2)}(G_{19},t)$

*are fifth detrition coefficients for category* 1, 2 *and* 3

$-(b''_{20})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3)}(G_{23},t)$

*are sixth detrition coefficients for category* 1, 2 *and* 3

$-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)$

*are seventh detrition coefficients for category* 1, 2 *and* 3

$-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)$

*are eighth detrition coefficients for category* 1, 2 *and* 3

$-(b''_{46})^{(9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \begin{bmatrix} (a'_{28})^{(5)} & +(a''_{28})^{(5)}(T_{29},t) & +(a''_{24})^{(4,4,)}(T_{25},t) & +(a''_{32})^{(6,6,6)}(T_{33},t) \\ & +(a''_{13})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{16})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3,3)}(T_{21},t) \\ & +(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{28} \quad (79)$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \begin{bmatrix} (a'_{29})^{(5)} & +(a''_{29})^{(5)}(T_{29},t) & +(a''_{25})^{(4,4,)}(T_{25},t) & +(a''_{33})^{(6,6,6)}(T_{33},t) \\ +(a''_{14})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{17})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3,3)}(T_{21},t) \\ +(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{29} \quad (80)$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \begin{bmatrix} (a'_{30})^{(5)} & +(a''_{30})^{(5)}(T_{29},t) & +(a''_{26})^{(4,4,)}(T_{25},t) & +(a''_{34})^{(6,6,6)}(T_{33},t) \\ +(a''_{15})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{18})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3,3)}(T_{21},t) \\ +(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{30} \quad (81)$$

*Where* $+(a''_{28})^{(5)}(T_{29},t)$, $+(a''_{29})^{(5)}(T_{29},t)$, $+(a''_{30})^{(5)}(T_{29},t)$ *are first augmentation coefficients for category* 1, 2 *and* 3

*And* $+(a''_{24})^{(4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,)}(T_{25},t)$ *are second augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{32})^{(6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6)}(T_{33},t)$ *are third augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)$ are fourth augmentation coefficients for category 1,2, and 3

$+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)$ are fifth augmentation coefficients for category 1,2,and 3

$+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)$ are sixth augmentation coefficients for category 1,2, 3

$+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)$ , $+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1,2, 3

$+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)$ are eighth augmentation coefficients for category 1,2, 3

$+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \begin{bmatrix} (b'_{28})^{(5)} & -(b''_{28})^{(5)}(G_{31},t) & -(b''_{24})^{(4,4,)}(G_{27},t) & -(b''_{32})^{(6,6,6)}(G_{35},t) \\ -(b''_{13})^{(1,1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3)}(G_{23},t) \\ -(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{28} \quad (82)$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b'_{29})^{(5)} & -(b''_{29})^{(5)}(G_{31},t) & -(b''_{25})^{(4,4,)}(G_{27},t) & -(b''_{33})^{(6,6,6)}(G_{35},t) \\ -(b''_{14})^{(1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3)}(G_{23},t) \\ -(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{29} \quad (83)$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b'_{30})^{(5)} & -(b''_{30})^{(5)}(G_{31},t) & -(b''_{26})^{(4,4,)}(G_{27},t) & -(b''_{34})^{(6,6,6)}(G_{35},t) \\ -(b''_{15})^{(1,1,1,1,1,)}(G,t) & -(b''_{18})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3)}(G_{23},t) \\ -(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{30} \quad (84)$$

*where* $-(b''_{28})^{(5)}(G_{31},t)$ , $-(b''_{29})^{(5)}(G_{31},t)$ , $-(b''_{30})^{(5)}(G_{31},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{24})^{(4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,)}(G_{27},t)$ *are second detrition coefficients*

$for\ category\ 1,2\ and\ 3$

$\boxed{-(b''_{32})^{(6,6,6)}(G_{35},t)}, \boxed{-(b''_{33})^{(6,6,6)}(G_{35},t)}, \boxed{-(b''_{34})^{(6,6,6)}(G_{35},t)}$ $are\ third\ detrition\ coefficients$

$for\ category\ 1,2\ and\ 3$

$\boxed{-(b''_{13})^{(1,1,1,1,1)}(G,t)}, \boxed{-(b''_{14})^{(1,1,1,1,1)}(G,t)}, \boxed{-(b''_{15})^{(1,1,1,1,1,)}(G,t)}$ are fourth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \begin{bmatrix} (a'_{32})^{(6)}\boxed{+(a''_{32})^{(6)}(T_{33},t)}\ \boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}\ \boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}\ \boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}\ \boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{32} \quad 85$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \begin{bmatrix} (a'_{33})^{(6)}\boxed{+(a''_{33})^{(6)}(T_{33},t)}\ \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}\ \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}\ \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}\ \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{33} \quad 86$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \begin{bmatrix} (a'_{34})^{(6)}\boxed{+(a''_{34})^{(6)}(T_{33},t)}\ \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}\ \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}\ \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}\ \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{34} \quad 87$$

$\boxed{+(a''_{32})^{(6)}(T_{33},t)}, \boxed{+(a''_{33})^{(6)}(T_{33},t)}, \boxed{+(a''_{34})^{(6)}(T_{33},t)}$ $are\ first\ augmentation\ coefficients$

$for\ category\ 1,2\ and\ 3$

$\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}$ $are\ second\ augmentation$

$coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}$ $are\ third\ augmentation$

$coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}$ - are fourth augmentation coefficients

$\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}$ - fifth augmentation coefficients

$\boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)}$ sixth augmentation coefficients

$+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)$
seventh augmentation coefficients
$+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)$
Eighth augmentation coefficients
$+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \begin{bmatrix} (b'_{32})^{(6)} & -(b''_{32})^{(6)}(G_{35},t) & -(b''_{28})^{(5,5,5)}(G_{31},t) & -(b''_{24})^{(4,4,4,)}(G_{27},t) \\ & -(b''_{13})^{(1,1,1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{32} \quad (88)$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \begin{bmatrix} (b'_{33})^{(6)} & -(b''_{33})^{(6)}(G_{35},t) & -(b''_{29})^{(5,5,5)}(G_{31},t) & -(b''_{25})^{(4,4,4,)}(G_{27},t) \\ & -(b''_{14})^{(1,1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{33} \quad (89)$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)} & -(b''_{34})^{(6)}(G_{35},t) & -(b''_{30})^{(5,5,5)}(G_{31},t) & -(b''_{26})^{(4,4,4,)}(G_{27},t) \\ & -(b''_{15})^{(1,1,1,1,1,1)}(G,t) & -(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{34} \quad (90)$$

$-(b''_{32})^{(6)}(G_{35},t)$, $-(b''_{33})^{(6)}(G_{35},t)$, $-(b''_{34})^{(6)}(G_{35},t)$ *are first detrition coefficients for category* 1, 2 *and* 3
$-(b''_{28})^{(5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5)}(G_{31},t)$ *are second detrition coefficients for category* 1, 2 *and* 3
$-(b''_{24})^{(4,4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,)}(G_{27},t)$ *are third detrition coefficients for category* 1,2 *and* 3
$-(b''_{13})^{(1,1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1,1,1)}(G,t)$ are fourth detrition coefficients for category 1, 2, and 3
$-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1, 2, and 3
$-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1, 2, and 3
$-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2, and 3
$-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)$
are eighth detrition coefficients for category 1, 2, and 3
$-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \qquad 91$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a'_{36})^{(7)}\boxed{+(a''_{36})^{(7)}(T_{37},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \qquad 92$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a'_{37})^{(7)}\boxed{+(a''_{37})^{(7)}(T_{37},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{38}}{dt} \qquad 93$$

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a'_{38})^{(7)}\boxed{+(a''_{38})^{(7)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{(a''_{36})^{(7)}(T_{37},t)}$, $\boxed{(a''_{37})^{(7)}(T_{37},t)}$, $\boxed{(a''_{38})^{(7)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{36}}{dt} = \qquad 94$$

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b'_{36})^{(7)}\boxed{-(b''_{36})^{(7)}(G_{39},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{37}}{dt} =$$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b'_{37})^{(7)}\boxed{-(b''_{37})^{(7)}(G_{39},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{38}}{dt} =$$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b'_{38})^{(7)}\boxed{-(b''_{38})^{(7)}(G_{39},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{40}}{dt}$$



$$= (a_{40})^{(8)}G_{41} - \begin{bmatrix} (a'_{40})^{(8)}\boxed{+(a''_{40})^{(8)}(T_{41},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{41}}{dt}$$

$$= (a_{41})^{(8)}G_{40} - \begin{bmatrix} (a'_{41})^{(8)}\boxed{+(a''_{41})^{(8)}(T_{41},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \begin{bmatrix} (a'_{42})^{(8)}\boxed{+(a''_{42})^{(8)}(T_{41},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a''_{40})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)}\boxed{-(b''_{40})^{(8)}(G_{43},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)}\boxed{-(b''_{41})^{(8)}(G_{43},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)}\boxed{-(b''_{42})^{(8)}(G_{43},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{44}}{dt} \qquad 96$$

$$= (a_{44})^{(9)}G_{45} - \left[\begin{array}{ccc} (a'_{44})^{(9)} & \boxed{+(a''_{44})^{(9)}(T_{45},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} \quad \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} \end{array}\right] G_{13}$$

$$\frac{dG_{45}}{dt}$$

$$= (a_{45})^{(9)}G_{44} - \left[\begin{array}{ccc} (a'_{45})^{(9)} & \boxed{+(a''_{45})^{(9)}(T_{45},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} \quad \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} \end{array}\right] G_{14}$$

$$\frac{dG_{46}}{dt}$$

$$= (a_{46})^{(9)}G_{45} - \left[\begin{array}{ccc} (a'_{46})^{(9)} & \boxed{+(a''_{46})^{(9)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} \quad \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} \end{array}\right] G_{15}$$

Where $\boxed{+(a''_{44})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{20}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{21}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{22}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{24}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{25}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{26}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{28}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{29}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{30}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{32}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{33}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{34}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a_{14}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a_{15}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ are Seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{38}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a_{37}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a_{36}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a_{40}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a_{42}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a_{41}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} =$$

$$(b_{44})^{(9)}T_{45} -$$

$$\begin{bmatrix} (b_{44}')^{(9)}\boxed{-(b_{44}'')^{(9)}(G_{47},t)} & \boxed{-(b_{16}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{20}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{24}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{28}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{32}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{13}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{36}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b_{45}')^{(9)}\boxed{-(b_{45}'')^{(9)}(G_{47},t)} & \boxed{-(b_{17}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{21}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{25}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{29}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{33}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{14}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{37}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b_{46}')^{(9)}\boxed{-(b_{46}'')^{(9)}(G_{47},t)} & \boxed{-(b_{18}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{22}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{26}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{30}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{34}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{15}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{38}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b_{44}'')^{(9)}(G_{47},t)}$, $\boxed{-(b_{45}'')^{(9)}(G_{47},t)}$, $\boxed{-(b_{46}'')^{(9)}(G_{47},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{16}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{17}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{18}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{20}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{21}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{22}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{24}'')^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{25}'')^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{26}'')^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{28}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{32}'')^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{34}'')^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{37}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{36}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a_i')^{(1)}, (a_i'')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (b_i'')^{(1)} > 0, \quad i,j = 13,14,15 \qquad 97$$

The functions $(a_i'')^{(1)}, (b_i'')^{(1)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a_i'')^{(1)}(T_{14},t) \leq (p_i)^{(1)} \leq (\hat{A}_{13})^{(1)}$$

$$(b_i'')^{(1)}(G,t) \leq (r_i)^{(1)} \leq (b_i')^{(1)} \leq (\hat{B}_{13})^{(1)}$$

$$lim_{T_2\to\infty}(a_i'')^{(1)}(T_{14},t) = (p_i)^{(1)} \qquad 98$$

$$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = (r_i)^{(1)}$$

**Definition of** $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$ :

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$$|(a_i'')^{(1)}(T_{14}',t) - (a_i'')^{(1)}(T_{14},t)| \leq (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G',t) - (b_i'')^{(1)}(G,t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}',t)$ and $(a_i'')^{(1)}(T_{14},t)$ . $(T_{14}',t)$ and $(T_{14},t)$ are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14},t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$ : 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$

**Definition of** $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$ : 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}$, $(r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)} (\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)} (\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i, j = 16,17,18$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17}, t) \leq (p_i)^{(2)} \leq (\hat{A}_{16})^{(2)} \quad 102$$

$$(b_i'')^{(2)}(G_{19}, t) \leq (r_i)^{(2)} \leq (b_i')^{(2)} \leq (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2 \to \infty} (a_i'')^{(2)}(T_{17}, t) = (p_i)^{(2)} \quad 104$$

$$\lim_{G \to \infty} (b_i'')^{(2)}((G_{19}), t) = (r_i)^{(2)} \quad 105$$

**Definition of** $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$ : 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}', t) - (a_i'')^{(2)}(T_{17}, t)| \leq (\hat{k}_{16})^{(2)} |T_{17} - T_{17}'| e^{-(\hat{M}_{16})^{(2)} t} \quad 107$$

$$|(b_i'')^{(2)}((G_{19})', t) - (b_i'')^{(2)}((G_{19}), t)| < (\hat{k}_{16})^{(2)} ||(G_{19}) - (G_{19})'|| e^{-(\hat{M}_{16})^{(2)} t} \quad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}', t)$ and$(a_i'')^{(2)}(T_{17}, t)$ . $(T_{17}', t)$ and $(T_{17}, t)$ are points belonging to the interval $[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}]$ . It is to be noted that $(a_i'')^{(2)}(T_{17}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17}, t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$ :

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)},$ are positive constants 109

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$$

**Definition of** $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$ :

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and\ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1 \quad 110$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(b_i)^{(2)} + (b_i')^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1 \quad 111$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i, j = 20,21,22 \quad 112$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(3)}$, $(r_i)^{(3)}$:

$$(a_i'')^{(3)}(T_{21}, t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23}, t) \leq (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(3)}(T_{21}, t) = (p_i)^{(3)} \quad 113$$

$$\lim_{G \to \infty}(b_i'')^{(3)}(G_{23}, t) = (r_i)^{(3)}$$

**Definition of** $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$ :

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: 114

$$|(a_i'')^{(3)}(T_{21}', t) - (a_i'')^{(3)}(T_{21}, t)| \leq (\hat{k}_{20})^{(3)} |T_{21} - T_{21}'| e^{-(\hat{M}_{20})^{(3)} t}$$

$$|(b_i'')^{(3)}(G_{23}', t) - (b_i'')^{(3)}(G_{23}, t)| < (\hat{k}_{20})^{(3)} ||G_{23} - G_{23}'|| e^{-(\hat{M}_{20})^{(3)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}', t)$ and $(a_i'')^{(3)}(T_{21}, t)$ . $(T_{21}', t)$ And $(T_{21}, t)$ are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21}, t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$ : 115

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}$ , $\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with 116
$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$ and the constants
$(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}, i = 20,21,22,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(a_i)^{(3)} + (a_i')^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)} (\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(b_i)^{(3)} + (b_i')^{(3)} + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)} (\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i, j = 24,25,26$$ 117

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(4)}$, $(r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25}, t) \le (p_i)^{(4)} \le (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}((G_{27}), t) \le (r_i)^{(4)} \le (b_i')^{(4)} \le (\hat{B}_{24})^{(4)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(4)}(T_{25}, t) = (p_i)^{(4)}$$ 118

$$\lim_{G \to \infty}(b_i'')^{(4)}((G_{27}), t) = (r_i)^{(4)}$$

**Definition of** $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$ :

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24,25,26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}', t) - (a_i'')^{(4)}(T_{25}, t)| \le (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})', t) - (b_i'')^{(4)}((G_{27}), t)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}', t)$ and$(a_i'')^{(4)}(T_{25}, t)$ . $(T_{25}', t)$ and $(T_{25}, t)$ are points belonging to the interval $[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}]$. It is to be noted that $(a_i'')^{(4)}(T_{25}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25}, t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$ : 120

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)},$ are positive constants

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}, \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$$

**Definition of** $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$ : 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and\ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}, i = 24,25,26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(a_i)^{(4)} + (a_i')^{(4)} + (\hat{A}_{24})^{(4)} + (\hat{P}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(b_i)^{(4)} + (b_i')^{(4)} + (\hat{B}_{24})^{(4)} + (\hat{Q}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i, j = 28,29,30$$ 122

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(5)}$, $(r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29}, t) \le (p_i)^{(5)} \le (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}((G_{31}), t) \le (r_i)^{(5)} \le (b_i')^{(5)} \le (\hat{B}_{28})^{(5)}$$

$$lim_{T_2 \to \infty} (a_i'')^{(5)} (T_{29}, t) = (p_i)^{(5)}$$ 123

$$\lim_{G \to \infty} (b_i'')^{(5)} (G_{31}, t) = (r_i)^{(5)}$$

**Definition of** $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$ :

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}', t) - (a_i'')^{(5)}(T_{29}, t)| \le (\hat{k}_{28})^{(5)} |T_{29} - T_{29}'| e^{-(\hat{M}_{28})^{(5)} t}$$

$$|(b_i'')^{(5)}((G_{31})', t) - (b_i'')^{(5)}((G_{31}), t)| < (\hat{k}_{28})^{(5)} ||(G_{31}) - (G_{31})'|| e^{-(\hat{M}_{28})^{(5)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}', t)$ and $(a_i'')^{(5)}(T_{29}, t)$ . $(T_{29}', t)$ and $(T_{29}, t)$ are points belonging to the interval $\left[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\right]$. It is to be noted that $(a_i'')^{(5)}(T_{29}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$ : 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)},$ are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}, \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

**Definition of** $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$ : 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and\ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(a_i)^{(5)} + (a_i')^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(b_i)^{(5)} + (b_i')^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i, j = 32,33,34$$ 127

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(6)}$, $(r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33}, t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}), t) \leq (r_i)^{(6)} \leq (b_i')^{(6)} \leq (\hat{B}_{32})^{(6)}$$

$$lim_{T_2 \to \infty} (a_i'')^{(6)} (T_{33}, t) = (p_i)^{(6)}$$ 128

$$\lim_{G \to \infty} (b_i'')^{(6)} \left((G_{35}), t\right) = (r_i)^{(6)}$$

**Definition of** $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$ :

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}', t) - (a_i'')^{(6)}(T_{33}, t)| \leq (\hat{k}_{32})^{(6)} |T_{33} - T_{33}'| e^{-(\hat{M}_{32})^{(6)} t}$$

$$|(b_i'')^{(6)}((G_{35})', t) - (b_i'')^{(6)}\left((G_{35}), t\right)| < (\hat{k}_{32})^{(6)} ||(G_{35}) - (G_{35})'|| e^{-(\hat{M}_{32})^{(6)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}', t)$ and $(a_i'')^{(6)}(T_{33}, t)$ . $(T_{33}', t)$ and $(T_{33}, t)$ are points belonging to the interval $\left[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}\right]$. It is to be noted that $(a_i'')^{(6)}(T_{33}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$ : 129

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)},$ are positive constants

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$$

**Definition of** $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$ : 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and\ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32,33,34,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(a_i)^{(6)} + (a_i')^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)} (\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(b_i)^{(6)} + (b_i')^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)} (\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

(QQ) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i, j = 36,37,38$ 131

(RR) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(7)}$, $(r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37}, t) \leq (p_i)^{(7)} \leq (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39}, t) \leq (r_i)^{(7)} \leq (b_i')^{(7)} \leq (\hat{B}_{36})^{(7)}$$

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*(SS)* $\lim_{T_2 \to \infty} (a_i'')^{(7)}(T_{37}, t) = (p_i)^{(7)}$

*(TT)*

$\lim_{G \to \infty} (b_i'')^{(7)}((G_{39}), t) = (r_i)^{(7)}$

**Definition of** $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$ :

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$$|(a_i'')^{(7)}(T_{37}', t) - (a_i'')^{(7)}(T_{37}, t)| \leq (\hat{k}_{36})^{(7)} |T_{37} - T_{37}'| e^{-(\hat{M}_{36})^{(7)} t}$$

$$|(b_i'')^{(7)}((G_{39})', t) - (b_i'')^{(7)}((G_{39}), t)| < (\hat{k}_{36})^{(7)} ||(G_{39}) - (G_{39})'|| e^{-(\hat{M}_{36})^{(7)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}', t)$ and $(a_i'')^{(7)}(T_{37}, t)$ . $(T_{37}', t)$ and $(T_{37}, t)$ are points belonging to the interval $[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}]$ . It is to be noted that $(a_i'')^{(7)}(T_{37}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37}, t)$ , the **first augmentation coefficient** attributable to the

system, would be absolutely continuous.

**Definition of** $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$ : 134

(UU) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)},$ are positive constants

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$$

**Definition of** $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$ : 135

(VV) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(b_i)^{(7)} + (b_i')^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40,41,42 \qquad 136$$

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

**Definition of** $(p_i)^{(8)}$, $(r_i)^{(8)}$: 137

$$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)} \qquad 138$$

$$(b_i'')^{(8)}((G_{43}), t) \leq (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)} \qquad 139$$

$$\lim_{T_2 \to \infty} (a_i'')^{(8)}(T_{41}, t) = (p_i)^{(8)} \qquad 140$$

$$\lim_{G \to \infty} (b_i'')^{(8)}((G_{43}), t) = (r_i)^{(8)} \qquad 141$$

**Definition of** $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$ :

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)} |T_{41} - T_{41}'| e^{-(\hat{M}_{40})^{(8)} t} \qquad 142$$

$$|(b_i'')^{(8)}((G_{43})', t) - (b_i'')^{(8)}((G_{43}), t)| < (\hat{k}_{40})^{(8)} ||(G_{43}) - (G_{43})'|| e^{-(\hat{M}_{40})^{(8)} t} \qquad 143$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}', t)$ and $(a_i'')^{(8)}(T_{41}, t)$ . $(T_{41}', t)$ and $(T_{41}, t)$ are points belonging to the interval $\left[(\hat{k}_{40})^{(8)}, (\widehat{M}_{40})^{(8)}\right]$. It is to be noted that $(a_i'')^{(8)}(T_{41}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{40})^{(8)} = 1$ then the function $(a_i'')^{(8)}(T_{41}, t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\widehat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$ :

$(\widehat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)},$ are positive constants

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}, \frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}} < 1 \qquad 144$$

**Definition of** $(\hat{P}_{40})^{(8)}, (\hat{Q}_{40})^{(8)}$ :

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\widehat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}, (\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)}, (a_i')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}, i = 40,41,42,$
Satisfy the inequalities

$$\frac{1}{(\widehat{M}_{40})^{(8)}}[(a_i)^{(8)} + (a_i')^{(8)} + (\hat{A}_{40})^{(8)} + (\hat{P}_{40})^{(8)} (\hat{k}_{40})^{(8)}] < 1 \qquad 145$$

$$\frac{1}{(\widehat{M}_{40})^{(8)}}[(b_i)^{(8)} + (b_i')^{(8)} + (\hat{B}_{40})^{(8)} + (\hat{Q}_{40})^{(8)} (\hat{k}_{40})^{(8)}] < 1 \qquad 146$$

Where we suppose

$$(a_i)^{(9)}, (a_i')^{(9)}, (a_i'')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (b_i'')^{(9)} > 0, \quad i, j = 44,45,46 \qquad 146\text{A}$$

The functions $(a_i'')^{(9)}, (b_i'')^{(9)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(9)}$, $(r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45}, t) \leq (p_i)^{(9)} \leq (\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47}, t) \leq (r_i)^{(9)} \leq (b_i')^{(9)} \leq (\hat{B}_{44})^{(9)}$$

$$\lim_{T_2 \to \infty} (a_i'')^{(9)}(T_{45}, t) = (p_i)^{(9)}$$

$$\lim_{G \to \infty} (b_i'')^{(9)}(G_{47}, t) = (r_i)^{(9)}$$

**Definition of** $(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}$ :

Where $\boxed{(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}}$ are positive constants and $\boxed{i = 44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}', t) - (a_i'')^{(9)}(T_{45}, t)| \leq (\hat{k}_{44})^{(9)} |T_{45} - T_{45}'| e^{-(\widehat{M}_{44})^{(9)} t}$$

$$|(b_i'')^{(9)}((G_{47})', t) - (b_i'')^{(9)}\big((G_{47}), t\big)| < (\hat{k}_{44})^{(9)}||(G_{47}) - (G_{47})'||e^{-(\hat{M}_{44})^{(9)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}', t)$ and$(a_i'')^{(9)}(T_{45}, t)$ . $(T_{45}', t)$ and $(T_{45}, t)$ are points belonging to the interval $\big[(\hat{k}_{44})^{(9)}, (\hat{M}_{44})^{(9)}\big]$. It is to be noted that $(a_i'')^{(9)}(T_{45}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{44})^{(9)} = 1$ then the function $(a_i'')^{(9)}(T_{45}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$ :

$(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$, are positive constants

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}} \ , \frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$$

**Definition of** $(\hat{P}_{44})^{(9)}, (\hat{Q}_{44})^{(9)}$ :

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}, (\hat{A}_{44})^{(9)} and\ (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)}, (a_i')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (p_i)^{(9)}, \ (r_i)^{(9)}, i = 44,45,46,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{44})^{(9)}}[\,(a_i)^{(9)} + (a_i')^{(9)} + \ (\hat{A}_{44})^{(9)} + \ (\hat{P}_{44})^{(9)}\,(\hat{k}_{44})^{(9)}] < 1$$

$$\frac{1}{(\hat{M}_{44})^{(9)}}[\ (b_i)^{(9)} + (b_i')^{(9)} + \ (\hat{B}_{44})^{(9)} + \ (\hat{Q}_{44})^{(9)}\ (\hat{k}_{44})^{(9)}] < 1$$

**Theorem 1:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \big(\hat{P}_{13}\big)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \ , \ \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 2 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

**Definition of** $G_i(0)\,, T_i(0)$

$$G_i(t) \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} \quad , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} \quad , \quad T_i(0) = T_i^0 > 0$$

**Theorem 3 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t) \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t} \quad , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t} \quad , \quad T_i(0) = T_i^0 > 0$$

**Theorem 4 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

**Definition of** $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \left(\hat{P}_{24}\right)^{(4)} e^{(\hat{M}_{24})^{(4)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{24}\right)^{(4)} e^{(\hat{M}_{24})^{(4)}t} \;,\quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 5 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

**Definition of** $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \left(\hat{P}_{28}\right)^{(5)} e^{(\hat{M}_{28})^{(5)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{28}\right)^{(5)} e^{(\hat{M}_{28})^{(5)}t} \;,\quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 6 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

**Definition of** $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \left(\hat{P}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t} \;,\quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 7:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

**Definition of** $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \left(\hat{P}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t} \;,\quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 8:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

**Definition of** $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \left(\hat{P}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t} \;,\quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 9:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

**Definition of** $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \left(\hat{P}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

**Proof:** Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \leq (\hat{P}_{13})^{(1)} \text{ , } T_i^0 \leq (\hat{Q}_{13})^{(1)}, \quad 155$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t} \quad 156$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t} \quad 157$$

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[ (a_{13})^{(1)} G_{14}(s_{(13)}) - \left( (a'_{13})^{(1)} + a''_{13})^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \right) G_{13}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[ (a_{14})^{(1)} G_{13}(s_{(13)}) - \left( (a'_{14})^{(1)} + (a''_{14})^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \right) G_{14}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[ (a_{15})^{(1)} G_{14}(s_{(13)}) - \left( (a'_{15})^{(1)} + (a''_{15})^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \right) G_{15}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[ (b_{13})^{(1)} T_{14}(s_{(13)}) - \left( (b'_{13})^{(1)} - (b''_{13})^{(1)} (G(s_{(13)}), s_{(13)}) \right) T_{13}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[ (b_{14})^{(1)} T_{13}(s_{(13)}) - \left( (b'_{14})^{(1)} - (b''_{14})^{(1)} (G(s_{(13)}), s_{(13)}) \right) T_{14}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{T}_{15}(t) = T_{15}^0 + \int_0^t \left[ (b_{15})^{(1)} T_{14}(s_{(13)}) - \left( (b'_{15})^{(1)} - (b''_{15})^{(1)} (G(s_{(13)}), s_{(13)}) \right) T_{15}(s_{(13)}) \right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \leq (\hat{P}_{16})^{(2)} \text{ , } T_i^0 \leq (\hat{Q}_{16})^{(2)},$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$$

By 160

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[ (a_{16})^{(2)} G_{17}(s_{(16)}) - \left( (a'_{16})^{(2)} + a''_{16})^{(2)} (T_{17}(s_{(16)}), s_{(16)}) \right) G_{16}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[ (a_{17})^{(2)} G_{16}(s_{(16)}) - \left( (a'_{17})^{(2)} + (a''_{17})^{(2)} (T_{17}(s_{(16)}), s_{(17)}) \right) G_{17}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[ (a_{18})^{(2)} G_{17}(s_{(16)}) - \left( (a'_{18})^{(2)} + (a''_{18})^{(2)} (T_{17}(s_{(16)}), s_{(16)}) \right) G_{18}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^{0} + \int_0^t \left[ (b_{16})^{(2)} T_{17}(s_{(16)}) - \left( (b'_{16})^{(2)} - (b''_{16})^{(2)} (G_{19}(s_{(16)}), s_{(16)}) \right) T_{16}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^{0} + \int_0^t \left[ (b_{17})^{(2)} T_{16}(s_{(16)}) - \left( (b'_{17})^{(2)} - (b''_{17})^{(2)} (G_{19}(s_{(16)}), s_{(16)}) \right) T_{17}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^{0} + \int_0^t \left[ (b_{18})^{(2)} T_{17}(s_{(16)}) - \left( (b'_{18})^{(2)} - (b''_{18})^{(2)} (G_{19}(s_{(16)}), s_{(16)}) \right) T_{18}(s_{(16)}) \right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i : \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \le (\hat{P}_{20})^{(3)} \text{ , } T_i^0 \le (\hat{Q}_{20})^{(3)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\bar{M}_{20})^{(3)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\bar{M}_{20})^{(3)} t}$$

By 161

$$\bar{G}_{20}(t) = G_{20}^{0} + \int_0^t \left[ (a_{20})^{(3)} G_{21}(s_{(20)}) - \left( (a'_{20})^{(3)} + a''_{20})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^{0} + \int_0^t \left[ (a_{21})^{(3)} G_{20}(s_{(20)}) - \left( (a'_{21})^{(3)} + (a''_{21})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{21}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^{0} + \int_0^t \left[ (a_{22})^{(3)} G_{21}(s_{(20)}) - \left( (a'_{22})^{(3)} + (a''_{22})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{22}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^{0} + \int_0^t \left[ (b_{20})^{(3)} T_{21}(s_{(20)}) - \left( (b'_{20})^{(3)} - (b''_{20})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^{0} + \int_0^t \left[ (b_{21})^{(3)} T_{20}(s_{(20)}) - \left( (b'_{21})^{(3)} - (b''_{21})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{21}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{22}(t) = T_{22}^{0} + \int_0^t \left[ (b_{22})^{(3)} T_{21}(s_{(20)}) - \left( (b'_{22})^{(3)} - (b''_{22})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{22}(s_{(20)}) \right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i : \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \le (\hat{P}_{24})^{(4)} \text{ , } T_i^0 \le (\hat{Q}_{24})^{(4)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\bar{M}_{24})^{(4)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\bar{M}_{24})^{(4)} t}$$

By 162

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{26}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{\mathrm{T}}_{26}(\mathrm{t}) = \mathrm{T}_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{26}(s_{(24)})\right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{28})^{(5)}, T_i^0 \le (\hat{Q}_{28})^{(5)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

By 163

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{30}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{\mathrm{T}}_{30}(\mathrm{t}) = \mathrm{T}_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{30}(s_{(28)})\right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{32})^{(6)}, T_i^0 \le (\hat{Q}_{32})^{(6)},$$

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\tilde{M}_{32})^{(6)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\tilde{M}_{32})^{(6)} t}$

By 164

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[ (a_{32})^{(6)} G_{33}(s_{(32)}) - \left( (a'_{32})^{(6)} + a''_{32})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[ (a_{33})^{(6)} G_{32}(s_{(32)}) - \left( (a'_{33})^{(6)} + (a''_{33})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[ (a_{34})^{(6)} G_{33}(s_{(32)}) - \left( (a'_{34})^{(6)} + (a''_{34})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{34}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[ (b_{32})^{(6)} T_{33}(s_{(32)}) - \left( (b'_{32})^{(6)} - (b''_{32})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[ (b_{33})^{(6)} T_{32}(s_{(32)}) - \left( (b'_{33})^{(6)} - (b''_{33})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{34}(t) = T_{34}^0 + \int_0^t \left[ (b_{34})^{(6)} T_{33}(s_{(32)}) - \left( (b'_{34})^{(6)} - (b''_{34})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{34}(s_{(32)}) \right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\tilde{M}_{36})^{(7)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\tilde{M}_{36})^{(7)} t}$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[ (a_{36})^{(7)} G_{37}(s_{(36)}) - \left( (a'_{36})^{(7)} + a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[ (a_{37})^{(7)} G_{36}(s_{(36)}) - \left( (a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[ (a_{38})^{(7)} G_{37}(s_{(36)}) - \left( (a'_{38})^{(7)} + (a''_{38})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{38}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[ (b_{36})^{(7)} T_{37}(s_{(36)}) - \left( (b'_{36})^{(7)} - (b''_{36})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[ (b_{37})^{(7)} T_{36}(s_{(36)}) - \left( (b'_{37})^{(7)} - (b''_{37})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\overline{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^{0} + \int_0^t \left[ (b_{38})^{(7)} T_{37}(s_{(36)}) - \left( (b'_{38})^{(7)} - (b''_{38})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{38}(s_{(36)}) \right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{40})^{(8)}, T_i^0 \le (\hat{Q}_{40})^{(8)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[ (a_{40})^{(8)} G_{41}(s_{(40)}) - \left( (a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[ (a_{41})^{(8)} G_{40}(s_{(40)}) - \left( (a'_{41})^{(8)} + (a''_{41})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[ (a_{42})^{(8)} G_{41}(s_{(40)}) - \left( (a'_{42})^{(8)} + (a''_{42})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{42}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^0 + \int_0^t \left[ (b_{40})^{(8)} T_{41}(s_{(40)}) - \left( (b'_{40})^{(8)} - (b''_{40})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^0 + \int_0^t \left[ (b_{41})^{(8)} T_{40}(s_{(40)}) - \left( (b'_{41})^{(8)} - (b''_{41})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\overline{\mathrm{T}}_{42}(\mathrm{t}) = \mathrm{T}_{42}^{0} + \int_0^t \left[ (b_{42})^{(8)} T_{41}(s_{(40)}) - \left( (b'_{42})^{(8)} - (b''_{42})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{42}(s_{(40)}) \right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** 166
Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy A

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{44})^{(9)}, T_i^0 \le (\hat{Q}_{44})^{(9)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[ (a_{44})^{(9)} G_{45}(s_{(44)}) - \left( (a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45}(s_{(44)}), s_{(44)})\right) G_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}(s_{(44)}), s_{(44)})\right) G_{46}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{44}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{46}(t) = T_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{46}(s_{(44)})\right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}}\right)\right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left(e^{(\hat{M}_{13})^{(1)} t} - 1\right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\hat{M}_{13})^{(1)} t} \le \frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{P}_{13})^{(1)} + G_{14}^0\right) e^{\left(-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0}\right)} + (\hat{P}_{13})^{(1)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \le G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}}\right)\right] ds_{(16)} = \quad 169$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1\right)$$

From which it follows that 170

$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \le \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left[\left((\hat{P}_{16})^{(2)} + G_{17}^0\right) e^{\left(-\frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0}\right)} + (\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t) \le G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} \left(G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} s_{(20)}}\right)\right] ds_{(20)} =$$

$$\left(1 + (a_{20})^{(3)} t\right) G_{21}^0 + \frac{(a_{20})^{(3)} (\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left(e^{(\hat{M}_{20})^{(3)} t} - 1\right)$$

From which it follows that 172

$$(G_{20}(t)-G_{20}^{0})e^{-(\hat{M}_{20})^{(3)}t}\leq\frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left[((\hat{P}_{20})^{(3)}+G_{21}^{0})e^{\left(-\frac{(\hat{P}_{20})^{(3)}+G_{21}^{0}}{G_{21}^{0}}\right)}+(\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 173

$$G_{24}(t)\leq G_{24}^{0}+\int_{0}^{t}\left[(a_{24})^{(4)}\left(G_{25}^{0}+(\hat{P}_{24})^{(4)}e^{(\hat{M}_{24})^{(4)}s_{(24)}}\right)\right]ds_{(24)}=$$

$$\left(1+(a_{24})^{(4)}t\right)G_{25}^{0}+\frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left(e^{(\hat{M}_{24})^{(4)}t}-1\right)$$

From which it follows that 174

$$(G_{24}(t)-G_{24}^{0})e^{-(\hat{M}_{24})^{(4)}t}\leq\frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left[((\hat{P}_{24})^{(4)}+G_{25}^{0})e^{\left(-\frac{(\hat{P}_{24})^{(4)}+G_{25}^{0}}{G_{25}^{0}}\right)}+(\hat{P}_{24})^{(4)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t)\leq G_{28}^{0}+\int_{0}^{t}\left[(a_{28})^{(5)}\left(G_{29}^{0}+(\hat{P}_{28})^{(5)}e^{(\hat{M}_{28})^{(5)}s_{(28)}}\right)\right]ds_{(28)}=$$

$$\left(1+(a_{28})^{(5)}t\right)G_{29}^{0}+\frac{(a_{28})^{(5)}(\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left(e^{(\hat{M}_{28})^{(5)}t}-1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t}\leq\frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t)\leq G_{32}^{0}+\int_{0}^{t}\left[(a_{32})^{(6)}\left(G_{33}^{0}+(\hat{P}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\right)\right]ds_{(32)}=$$

$$\left(1+(a_{32})^{(6)}t\right)G_{33}^{0}+\frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left(e^{(\hat{M}_{32})^{(6)}t}-1\right)$$

From which it follows that 177

$$(G_{32}(t)-G_{32}^{0})e^{-(\hat{M}_{32})^{(6)}t}\leq\frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left[((\hat{P}_{32})^{(6)}+G_{33}^{0})e^{\left(-\frac{(\hat{P}_{32})^{(6)}+G_{33}^{0}}{G_{33}^{0}}\right)}+(\hat{P}_{32})^{(6)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}$ , $G_{26}, T_{24}, T_{25}, T_{26}$

(h) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t) \le G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} \left(G_{37}^0 + (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} s_{(36)}}\right)\right] ds_{(36)} =$$
$$\left(1 + (a_{36})^{(7)} t\right) G_{37}^0 + \frac{(a_{36})^{(7)} (\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left(e^{(\hat{M}_{36})^{(7)} t} - 1\right)$$

From which it follows that

$$(G_{36}(t) - G_{36}^0) e^{-(\hat{M}_{36})^{(7)} t} \le \frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left[((\hat{P}_{36})^{(7)} + G_{37}^0) e^{\left(-\frac{(\hat{P}_{36})^{(7)} + G_{37}^0}{G_{37}^0}\right)} + (\hat{P}_{36})^{(7)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t) \le G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} \left(G_{41}^0 + (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} s_{(40)}}\right)\right] ds_{(40)} = \quad 180$$
$$\left(1 + (a_{40})^{(8)} t\right) G_{41}^0 + \frac{(a_{40})^{(8)} (\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}} \left(e^{(\hat{M}_{40})^{(8)} t} - 1\right)$$

From which it follows that 181

$$(G_{40}(t) - G_{40}^0) e^{-(\hat{M}_{40})^{(8)} t} \le \frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}} \left[((\hat{P}_{40})^{(8)} + G_{41}^0) e^{\left(-\frac{(\hat{P}_{40})^{(8)} + G_{41}^0}{G_{41}^0}\right)} + (\hat{P}_{40})^{(8)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 8

Analogous inequalities hold also for $G_{41}$ , $G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t) \le G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} \left(G_{45}^0 + (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} s_{(44)}}\right)\right] ds_{(44)} =$$
$$\left(1 + (a_{44})^{(9)} t\right) G_{45}^0 + \frac{(a_{44})^{(9)} (\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left(e^{(\hat{M}_{44})^{(9)} t} - 1\right)$$

From which it follows that

$$(G_{44}(t) - G_{44}^0) e^{-(\hat{M}_{44})^{(9)} t} \le \frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left[((\hat{P}_{44})^{(9)} + G_{45}^0) e^{\left(-\frac{(\hat{P}_{44})^{(9)} + G_{45}^0}{G_{45}^0}\right)} + (\hat{P}_{44})^{(9)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 9

Analogous inequalities hold also for $G_{45}$ , $G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}$ , $\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$ and to choose 182

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[(\hat{P}_{13})^{(1)}+\left((\hat{P}_{13})^{(1)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{13})^{(1)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{13})^{(1)} \quad 183$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[\left((\hat{Q}_{13})^{(1)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{13})^{(1)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{13})^{(1)}\right]\leq(\hat{Q}_{13})^{(1)} \quad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)},T^{(1)}\right),\left(G^{(2)},T^{(2)}\right)\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{13})^{(1)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{13})^{(1)}t}\}$$

Indeed if we denote

**Definition of** $\tilde{G},\tilde{T}:\left(\tilde{G},\tilde{T}\right)=\mathcal{A}^{(1)}(G,T)$

It results

$$\left|\tilde{G}_{13}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{13})^{(1)}\left|G_{14}^{(1)}-G_{14}^{(2)}\right|e^{-(\hat{M}_{13})^{(1)}s_{(13)}}e^{(\hat{M}_{13})^{(1)}s_{(13)}}ds_{(13)}+$$

$$\int_0^t\{(a'_{13})^{(1)}\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\hat{M}_{13})^{(1)}s_{(13)}}e^{-(\hat{M}_{13})^{(1)}s_{(13)}}+$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\hat{M}_{13})^{(1)}s_{(13)}}e^{(\hat{M}_{13})^{(1)}s_{(13)}}+$$

$$G_{13}^{(2)}\left|(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)-(a''_{13})^{(1)}\left(T_{14}^{(2)},s_{(13)}\right)\right|\ e^{-(\hat{M}_{13})^{(1)}s_{(13)}}e^{(\hat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|G^{(1)}-G^{(2)}\right|e^{-(\hat{M}_{13})^{(1)}t}\leq \quad 186$$
$$\frac{1}{(\hat{M}_{13})^{(1)}}\left((a_{13})^{(1)}+(a'_{13})^{(1)}+(\hat{A}_{13})^{(1)}+(\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}\right)d\left(\left(G^{(1)},T^{(1)};\ G^{(2)},T^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

**Remark 1:** The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\hat{P}_{13})^{(1)}e^{(\hat{M}_{13})^{(1)}t}\ and\ (\hat{Q}_{13})^{(1)}e^{(\hat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i = 13,14,15$ depend only on $T_{14}$ and respectively on $G(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any $t$ where $G_i(t) = 0\ and\ T_i(t) = 0$

From 19 to 24 it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(1)} - (a_i'')^{(1)}(T_{14}(s_{(13)}), s_{(13)})\} ds_{(13)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(1)} t)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2\ and\ \left((\widehat{M}_{13})^{(1)}\right)_3$ : 187

**Remark 3**: if $G_{13}$ is bounded, the same property have also $G_{14}\ and\ G_{15}$ . indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a_{14}')^{(1)} G_{14}$ and by integrating

$$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)} \left((\widehat{M}_{13})^{(1)}\right)_1 / (a_{14}')^{(1)}$$

In the same way , one can obtain

$$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)} \left((\widehat{M}_{13})^{(1)}\right)_2 / (a_{15}')^{(1)}$$

If $G_{14}\ or\ G_{15}$ is bounded, the same property follows for $G_{13}$ , $G_{15}$ and $G_{13}$ , $G_{14}$ respectively.

**Remark 4:** If $G_{13}$ *is* bounded, from below, the same property holds for $G_{14}\ and\ G_{15}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{14}$ is bounded from below. 188

**Remark 5:** If $T_{13}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}(G(t), t)) = (b_{14}')^{(1)}$ then $T_{14} \to \infty$. 189

**Definition of** $(m)^{(1)}$ and $\varepsilon_1$ :

Indeed let $t_1$ be so that for $t > t_1$

$$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t), t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right)$, $t = log\frac{2}{\varepsilon_1}$ By taking now $\varepsilon_1$ sufficiently small one sees that $T_{14}$ is unbounded.
The same property holds for $T_{15}$ if $\lim_{t\to\infty}(b_{15}'')^{(1)}(G(t), t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}$ , $\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}\ and\ (\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[(\widehat{P}_{16})^{(2)}+\left((\hat{P}_{16})^{(2)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{16})^{(2)}\right]\leq(\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

**Definition of** $\widetilde{G_{19}},\widetilde{T_{19}}$ : $\left(\widetilde{G_{19}},\widetilde{T_{19}}\right)=\mathcal{A}^{(2)}(G_{19},T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{16})^{(2)}\left|G_{17}^{(1)}-G_{17}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)}+$$

$$\int_0^t\{(a'_{16})^{(2)}\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$G_{16}^{(2)}\left|(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)-(a''_{16})^{(2)}\left(T_{17}^{(2)},s_{(16)}\right)\right|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0,t]$ 197

From the hypotheses it follows

$$\left|(G_{19})^{(1)}-(G_{19})^{(2)}\right|\mathrm{e}^{-(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}\leq$$
$$\frac{1}{(\widehat{\mathrm{M}}_{16})^{(2)}}\left((a_{16})^{(2)}+(a'_{16})^{(2)}+(\widehat{\mathrm{A}}_{16})^{(2)}+(\widehat{\mathrm{P}}_{16})^{(2)}(\hat{k}_{16})^{(2)}\right)\mathrm{d}\left(((G_{19})^{(1)},(T_{19})^{(1)};\ (G_{19})^{(2)},(T_{19})^{(2)})\right)$$

And analogous inequalities for $G_i$ and $T_i$. Taking into account the hypothesis the result follows 198

**Remark 6:** The fact that we supposed $(a''_{16})^{(2)}$ and $(b''_{16})^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{\mathrm{P}}_{16})^{(2)}\mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}$ and $(\widehat{\mathrm{Q}}_{16})^{(2)}\mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i=16,17,18$ depend only on $\mathrm{T}_{17}$ and respectively on

$(G_{19})$(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 7:** There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$ 200

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(2)} - (a_i'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})\} ds_{(16)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(2)}t)} > 0 \quad \text{for } t > 0$$

**Definition of** $\left((\widehat{M}_{16})^{(2)}\right)_1$, $\left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$: 201

**Remark 8:** if $G_{16}$ is bounded, the same property have also $G_{17}$ and $G_{18}$. indeed if

$G_{16} < (\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt} \leq \left((\widehat{M}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)} G_{17}$ and by integrating

$$G_{17} \leq \left((\widehat{M}_{16})^{(2)}\right)_2 = G_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1 / (a_{17}')^{(2)}$$

In the same way , one can obtain

$$G_{18} \leq \left((\widehat{M}_{16})^{(2)}\right)_3 = G_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2 / (a_{18}')^{(2)}$$

If $G_{17}$ or $G_{18}$ is bounded, the same property follows for $G_{16}$, $G_{18}$ and $G_{16}$, $G_{17}$ respectively.

**Remark 9:** If $G_{16}$ is bounded, from below, the same property holds for $G_{17}$ and $G_{18}$. The proof is analogous with the preceding one. An analogous property is true if $G_{17}$ is bounded from below. 202

**Remark 10:** If $T_{16}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t), t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$. 203

**Definition of** $(m)^{(2)}$ and $\varepsilon_2$ :

Indeed let $t_2$ be so that for $t > t_2$

$$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right)$, $t = \log\frac{2}{\varepsilon_2}$ By taking now $\varepsilon_2$ sufficiently small one sees that $T_{17}$ is unbounded. 205
The same property holds for $T_{18}$ if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t), t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[(\widehat{P}_{20})^{(3)}+\left((\hat{P}_{20})^{(3)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{20})^{(3)}\right]\leq(\hat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself 210

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)},(T_{23})^{(1)}),((G_{23})^{(2)},(T_{23})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

**Definition of** $\widetilde{G_{23}},\widetilde{T_{23}}:(\widetilde{(G_{23})},\widetilde{(T_{23})})=\mathcal{A}^{(3)}((G_{23}),(T_{23}))$

It results 213

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{20})^{(3)}\left|G_{21}^{(1)}-G_{21}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}ds_{(20)}+$$

$$\int_0^t\{(a'_{20})^{(3)}\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$G_{20}^{(2)}\left|(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)},s_{(20)}\right)\right|\ e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\}ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses it follows

$$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}t}\leq \quad 214$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+(a'_{20})^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\widehat{k}_{20})^{(3)}\right)d\left(((G_{23})^{(1)},(T_{23})^{(1)};\ (G_{23})^{(2)},(T_{23})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 11:** The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ *and* $(\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i=20,21,22$ depend only on $T_{21}$ and respectively on

$(G_{23})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 12:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 216

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(3)} - (a_i'')^{(3)}(T_{21}(s_{(20)}), s_{(20)})\} ds_{(20)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(3)} t)} > 0$ for t > 0

**Definition of** $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2\ and\ \left((\widehat{M}_{20})^{(3)}\right)_3$ : 217

**Remark 13:** if $G_{20}$ is bounded, the same property have also $G_{21}\ and\ G_{22}$ . indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a_{21}')^{(3)} G_{21}$ and by integrating

$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1 / (a_{21}')^{(3)}$

In the same way , one can obtain

$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2 / (a_{22}')^{(3)}$

If $G_{21}\ or\ G_{22}$ is bounded, the same property follows for $G_{20}$ , $G_{22}$ and $G_{20}$ , $G_{21}$ respectively.

**Remark 14:** If $G_{20}$ *is* bounded, from below, the same property holds for $G_{21}\ and\ G_{22}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{21}$ is bounded from below. 218

**Remark 15:** If $T_{20}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t), t\right)) = (b_{21}')^{(3)}$ then $T_{21} \to \infty$. 219

**Definition of** $(m)^{(3)}$ and $\varepsilon_3$ :

Indeed let $t_3$ be so that for $t > t_3$

$(b_{21})^{(3)} - (b_i'')^{(3)}\left((G_{23})(t), t\right) < \varepsilon_3, T_{20}\,(t) > (m)^{(3)}$

Then $\frac{dT_{21}}{dt} \geq (a_{21})^{(3)}(m)^{(3)} - \varepsilon_3 T_{21}$ which leads to 220

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t} = \frac{1}{2}$ it results

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_3}$ By taking now $\varepsilon_3$ sufficiently small one sees that $T_{21}$ is unbounded.
The same property holds for $T_{22}$ if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t), t\right) = (b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}$ , $\frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}} < 1$ and to choose 221

$(\widehat{P}_{24})^{(4)}$ and $(\widehat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[(\widehat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \tag{222}$$

$$\frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \tag{223}$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself (224)

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric (225)

$$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{24})^{(4)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{27}}),(\widetilde{T_{27}})$ : $\left((\widetilde{G_{27}}),(\widetilde{T_{27}})\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{24})^{(4)}\left|G_{25}^{(1)}-G_{25}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\,ds_{(24)}+$$

$$\int_0^t\{(a'_{24})^{(4)}\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$G_{24}^{(2)}\left|(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)-(a''_{24})^{(4)}\left(T_{25}^{(2)},s_{(24)}\right)\right|\;e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\}ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}t}\leq \tag{226}$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)}+(a'_{24})^{(4)}+(\widehat{A}_{24})^{(4)}+(\widehat{P}_{24})^{(4)}(\hat{k}_{24})^{(4)}\right)d\left(((G_{27})^{(1)},(T_{27})^{(1)};\;(G_{27})^{(2)},(T_{27})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 16:** The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ *and* $(\widehat{Q}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_+$. (227)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i=24,25,26$ depend only on $T_{25}$ and respectively on

$(G_{27})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 17:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 228

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(4)} - (a_i'')^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(4)} t)} > 0$ for t > 0

**Definition of** $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2\ and\ \left((\widehat{M}_{24})^{(4)}\right)_3$ : 229

**Remark 18:** if $G_{24}$ is bounded, the same property have also $G_{25}\ and\ G_{26}$ . indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a_{25}')^{(4)} G_{25}$ and by integrating

$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1 / (a_{25}')^{(4)}$

In the same way , one can obtain

$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2 / (a_{26}')^{(4)}$

If $G_{25}\ or\ G_{26}$ is bounded, the same property follows for $G_{24}$ , $G_{26}$ and $G_{24}$ , $G_{25}$ respectively.

**Remark 19:** If $G_{24}$ *is* bounded, from below, the same property holds for $G_{25}\ and\ G_{26}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{25}$ is bounded from below. 230

**Remark 20:** If $T_{24}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}\ ((G_{27})(t), t)) = (b_{25}')^{(4)}$ then $T_{25}\ \to \infty$. 231

**Definition of** $(m)^{(4)}$ and $\varepsilon_4$ :

Indeed let $t_4$ be so that for $t\ > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t), t) < \varepsilon_4, T_{24}\,(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25}\ \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25}\ \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_4}$ By taking now $\varepsilon_4$ sufficiently small one sees that $T_{25}$ is unbounded.
The same property holds for $T_{26}$ if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t), t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for $G_{29}\ , G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}\ , \frac{(b_i)^{(5)}}{(\widehat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)}+\left((\hat{P}_{28})^{(5)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{28})^{(5)}\right]\leq(\hat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$$d\left(((G_{31})^{(1)},(T_{31})^{(1)}),((G_{31})^{(2)},(T_{31})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\widehat{M}_{28})^{(5)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\widehat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{31}}),(\widetilde{T_{31}})$ : $\left((\widetilde{G_{31}}),(\widetilde{T_{31}})\right)=\mathcal{A}^{(5)}((G_{31}),(T_{31}))$

It results

$$|\tilde{G}_{28}^{(1)}-\tilde{G}_i^{(2)}|\leq\int_0^t(a_{28})^{(5)}|G_{29}^{(1)}-G_{29}^{(2)}|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\,ds_{(28)}+$$

$$\int_0^t\{(a'_{28})^{(5)}|G_{28}^{(1)}-G_{28}^{(2)}|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$(a''_{28})^{(5)}(T_{29}^{(1)},s_{(28)})|G_{28}^{(1)}-G_{28}^{(2)}|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$G_{28}^{(2)}|(a''_{28})^{(5)}(T_{29}^{(1)},s_{(28)})-(a''_{28})^{(5)}(T_{29}^{(2)},s_{(28)})|\;e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\}ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on it follows

$$|(G_{31})^{(1)}-(G_{31})^{(2)}|e^{-(\widehat{M}_{28})^{(5)}t}\leq \quad 237$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)}+(a'_{28})^{(5)}+(\widehat{A}_{28})^{(5)}+(\hat{P}_{28})^{(5)}(\hat{k}_{28})^{(5)}\right)d\left(((G_{31})^{(1)},(T_{31})^{(1)};\,(G_{31})^{(2)},(T_{31})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 21:** The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\hat{P}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ *and* $(\hat{Q}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ respectively of $\mathbb{R}_+$. 238

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on $T_{29}$ and respectively on $(G_{31})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 22:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 239

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(5)} - (a_i'')^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(5)} t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2\ and\ \left((\widehat{M}_{28})^{(5)}\right)_3$ : 240

**Remark 23:** if $G_{28}$ is bounded, the same property have also $G_{29}\ and\ G_{30}$ . indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a_{29}')^{(5)} G_{29}$ and by integrating

$$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1 / (a_{29}')^{(5)}$$

In the same way , one can obtain

$$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2 / (a_{30}')^{(5)}$$

If $G_{29}\ or\ G_{30}$ is bounded, the same property follows for $G_{28}$ , $G_{30}$ and $G_{28}$ , $G_{29}$ respectively.

**Remark 24:** If $G_{28}$ *is* bounded, from below, the same property holds for $G_{29}\ and\ G_{30}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{29}$ is bounded from below. 241

**Remark 25:** If $T_{28}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}\,((G_{31})(t), t)) = (b_{29}')^{(5)}$ then $T_{29} \to \infty$. 242

**Definition of** $(m)^{(5)}$ and $\varepsilon_5$ :

Indeed let $t_5$ be so that for $t > t_5$

$$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t), t) < \varepsilon_5, T_{28}\,(t) > (m)^{(5)}$$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_5}$ By taking now $\varepsilon_5$ sufficiently small one sees that $T_{29}$ is unbounded.
The same property holds for $T_{30}$ if $\lim_{t\to\infty}(b_{30}'')^{(5)}\left((G_{31})(t), t\right) = (b_{30}')^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{33}\,, G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\ , \frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}} < 1$ and to choose 244

$(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[(\hat{P}_{32})^{(6)}+\left((\hat{P}_{32})^{(6)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{32})^{(6)} \tag{245}$$

$$\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[\left((\hat{Q}_{32})^{(6)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{32})^{(6)}\right]\leq(\hat{Q}_{32})^{(6)} \tag{246}$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric (247)

$$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\hat{M}_{32})^{(6)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\hat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{35}}),(\widetilde{T_{35}})$ : $\left((\widetilde{G_{35}}),(\widetilde{T_{35}})\right)=\mathcal{A}^{(6)}\left((G_{35}),(T_{35})\right)$

It results

$$|\tilde{G}_{32}^{(1)}-\tilde{G}_i^{(2)}|\leq\int_0^t(a_{32})^{(6)}|G_{33}^{(1)}-G_{33}^{(2)}|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\,ds_{(32)}+$$

$$\int_0^t\{(a'_{32})^{(6)}|G_{32}^{(1)}-G_{32}^{(2)}|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)|G_{32}^{(1)}-G_{32}^{(2)}|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$G_{32}^{(2)}|(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)-(a''_{32})^{(6)}\left(T_{33}^{(2)},s_{(32)}\right)|\;e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\}ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$|(G_{35})^{(1)}-(G_{35})^{(2)}|e^{-(\widehat{M}_{32})^{(6)}t}\leq \tag{248}$$
$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)}+(a'_{32})^{(6)}+(\widehat{A}_{32})^{(6)}+(\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}\right)d\left(((G_{35})^{(1)},(T_{35})^{(1)};\;(G_{35})^{(2)},(T_{35})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 26:** The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\hat{P}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ *and* $(\hat{Q}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ respectively of $\mathbb{R}_+$. (249)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i = 32,33,34$ depend only on $T_{33}$ and respectively on $(G_{35})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 27:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 250

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(6)} - (a_i'')^{(6)}(T_{33}(s_{(32)}), s_{(32)})\} ds_{(32)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(6)} t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{32})^{(6)}\right)_1, \left((\widehat{M}_{32})^{(6)}\right)_2\ and\ \left((\widehat{M}_{32})^{(6)}\right)_3$ : 251

**Remark 28:** if $G_{32}$ is bounded, the same property have also $G_{33}\ and\ G_{34}$ . indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a_{33}')^{(6)} G_{33}$ and by integrating

$$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)} \left((\widehat{M}_{32})^{(6)}\right)_1 / (a_{33}')^{(6)}$$

In the same way , one can obtain

$$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)} \left((\widehat{M}_{32})^{(6)}\right)_2 / (a_{34}')^{(6)}$$

If $G_{33}\ or\ G_{34}$ is bounded, the same property follows for $G_{32}$ , $G_{34}$ and $G_{32}$ , $G_{33}$ respectively.

**Remark 29:** If $G_{32}$ *is* bounded, from below, the same property holds for $G_{33}\ and\ G_{34}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{33}$ is bounded from below. 252

**Remark 30:** If $T_{32}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}\ ((G_{35})(t), t)) = (b_{33}')^{(6)}$ then $T_{33} \to \infty$. 253

**Definition of** $(m)^{(6)}$ and $\varepsilon_6$ :

Indeed let $t_6$ be so that for $t > t_6$

$$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t), t\right) < \varepsilon_6, T_{32}\,(t) > (m)^{(6)}$$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right)$, $t = log\,\frac{2}{\varepsilon_6}$ By taking now $\varepsilon_6$ sufficiently small one sees that $T_{33}$ is unbounded.
The same property holds for $T_{34}$ if $\lim_{t\to\infty}(b_{34}'')^{(6)}\left((G_{35})(t), t(t), t\right) = (b_{34}')^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{37}, G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}$ , $\frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$ and to choose

$(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[(\hat{P}_{36})^{(7)}+\left((\hat{P}_{36})^{(7)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{36})^{(7)} \tag{256}$$

$$\frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[\left((\hat{Q}_{36})^{(7)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{36})^{(7)}\right]\leq(\hat{Q}_{36})^{(7)} \tag{257}$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself
The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric (258)

$$d\left(((G_{39})^{(1)},(T_{39})^{(1)}),((G_{39})^{(2)},(T_{39})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\hat{M}_{36})^{(7)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\hat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

**Definition of** $\widetilde{(G_{39})},\widetilde{(T_{39})}:(\widetilde{(G_{39})},\widetilde{(T_{39})})=\mathcal{A}^{(7)}((G_{39}),(T_{39}))$
It results

$$|\tilde{G}_{36}^{(1)}-\tilde{G}_i^{(2)}|\leq\int_0^t(a_{36})^{(7)}|G_{37}^{(1)}-G_{37}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\,ds_{(36)}+$$
$$\int_0^t\{(a'_{36})^{(7)}|G_{36}^{(1)}-G_{36}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$(a''_{36})^{(7)}(T_{37}^{(1)},s_{(36)})|G_{36}^{(1)}-G_{36}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$G_{36}^{(2)}|(a''_{36})^{(7)}(T_{37}^{(1)},s_{(36)})-(a''_{36})^{(7)}(T_{37}^{(2)},s_{(36)})|\;e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\}ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on it follows

$$|(G_{39})^{(1)}-(G_{39})^{(2)}|e^{-(\widehat{M}_{36})^{(7)}t}\leq \tag{259}$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}}\left((a_{36})^{(7)}+(a'_{36})^{(7)}+(\widehat{A}_{36})^{(7)}+(\widehat{P}_{36})^{(7)}(\widehat{k}_{36})^{(7)}\right)d\left(((G_{39})^{(1)},(T_{39})^{(1)};\;(G_{39})^{(2)},(T_{39})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 31:** The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ *and* $(\widehat{Q}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ respectively of $\mathbb{R}_+$. (260)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(7)}$ and $(b_i'')^{(7)}, i = 36,37,38$ depend only on $T_{37}$ and respectively on $(G_{39})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 32:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 261

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(7)} - (a_i'')^{(7)}(T_{37}(s_{(36)}), s_{(36)})\} ds_{(36)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(7)} t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2\ and\ \left((\widehat{M}_{36})^{(7)}\right)_3$ : 262

**Remark 33:** if $G_{36}$ is bounded, the same property have also $G_{37}\ and\ G_{38}$ . indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a_{37}')^{(7)} G_{37}$ and by integrating

$$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1 / (a_{37}')^{(7)}$$

In the same way , one can obtain

$$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2 / (a_{38}')^{(7)}$$

If $G_{37}\ or\ G_{38}$ is bounded, the same property follows for $G_{36}$ , $G_{38}$ and $G_{36}$ , $G_{37}$ respectively.

**Remark 34:** If $G_{36}$ *is* bounded, from below, the same property holds for $G_{37}\ and\ G_{38}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{37}$ is bounded from below. 263

**Remark 35:** If $T_{36}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(7)}\ ((G_{39})(t), t)) = (b_{37}')^{(7)}$ then $T_{37} \to \infty$. 264

**Definition of** $(m)^{(7)}$ and $\varepsilon_7$ :

Indeed let $t_7$ be so that for $t > t_7$

$$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t), t) < \varepsilon_7, T_{36}\,(t) > (m)^{(7)}$$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_7}$ By taking now $\varepsilon_7$ sufficiently small one sees that $T_{37}$ is unbounded.
The same property holds for $T_{38}$ if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t), t\right) = (b_{38}')^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}$ , $\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}} < 1$ and to choose $(\widehat{P}_{40})^{(8)}$ and $(\widehat{Q}_{40})^{(8)}$ large to have 266

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[(\widehat{P}_{40})^{(8)} + \left((\hat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{40})^{(8)} \quad 267$$

$$\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{40})^{(8)}\right] \leq (\hat{Q}_{40})^{(8)} \quad 268$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)}, (T_{43})^{(1)}), ((G_{43})^{(2)}, (T_{43})^{(2)})\right) = \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

**Definition of** $\widetilde{(G_{43})}, \widetilde{(T_{43})}$ : $\left(\widetilde{(G_{43})}, \widetilde{(T_{43})}\right) = \mathcal{A}^{(8)}((G_{43}), (T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{40})^{(8)}\left|G_{41}^{(1)} - G_{41}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\,ds_{(40)} +$$
$$\int_0^t\{(a'_{40})^{(8)}\left|G_{40}^{(1)} - G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{-(\widehat{M}_{40})^{(8)}s_{(40)}} +$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right)\left|G_{40}^{(1)} - G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}} +$$
$$G_{40}^{(2)}\left|(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right) - (a''_{40})^{(8)}\left(T_{41}^{(2)}, s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\}ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0, t]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)} - (G_{43})^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}t} \leq \quad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)} + (a'_{40})^{(8)} + (\widehat{A}_{40})^{(8)} + (\widehat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}\right)d\left(((G_{43})^{(1)}, (T_{43})^{(1)};\ (G_{43})^{(2)}, (T_{43})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

**Remark 36:** The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate 274

condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ *and* $(\widehat{Q}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i = 40,41,42$ depend only on $T_{41}$ and respectively on $(G_{43})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 37** There does not exist any $t$ where $G_i(t) = 0\ and\ T_i(t) = 0$ 275

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(8)} - (a_i'')^{(8)}(T_{41}(s_{(40)}), s_{(40)})\} ds_{(40)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(8)}t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{40})^{(8)}\right)_1, \left((\widehat{M}_{40})^{(8)}\right)_2\ and\ \left((\widehat{M}_{40})^{(8)}\right)_3$ : 276

**Remark 38:** if $G_{40}$ is bounded, the same property have also $G_{41}\ and\ G_{42}$ . indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a_{41}')^{(8)}G_{41}$ and by integrating

$$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1/(a_{41}')^{(8)}$$

In the same way , one can obtain

$$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2/(a_{42}')^{(8)}$$

If $G_{41}\ or\ G_{42}$ is bounded, the same property follows for $G_{40}$ , $G_{42}$ and $G_{40}$ , $G_{41}$ respectively.

**Remark 39:** If $G_{40}$ *is* bounded, from below, the same property holds for $G_{41}\ and\ G_{42}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{41}$ is bounded from below. 277

**Remark 40:** If $T_{40}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}((G_{43})(t), t)) = (b_{41}')^{(8)}$ then $T_{41} \to \infty$. 278

**Definition of** $(m)^{(8)}$ and $\varepsilon_8$ :

Indeed let $t_8$ be so that for $t > t_8$

$$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t), t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right)$, $t = log\frac{2}{\varepsilon_8}$ By taking now $\varepsilon_8$ sufficiently small one sees that $T_{41}$ is unbounded.
The same property holds for $T_{42}$ if $\lim_{t\to\infty}(b''_{42})^{(8)}\left((G_{43})(t), t(t), t\right) = (b'_{42})^{(8)}$



It is now sufficient to take $\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}$ , $\frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$ and to choose $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ large to have

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}\left[(\hat{P}_{44})^{(9)} + \left((\hat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}}\left[\left((\hat{Q}_{44})^{(9)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{44})^{(9)}\right] \leq (\hat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)}, (T_{47})^{(1)}), ((G_{47})^{(2)}, (T_{47})^{(2)})\right) =$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{44})^{(9)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{47}}), (\widetilde{T_{47}}) : \left((\widetilde{G_{47}}), (\widetilde{T_{47}})\right) = \mathcal{A}^{(9)}\left((G_{47}), (T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{44})^{(9)}\left|G_{45}^{(1)} - G_{45}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\,ds_{(44)} +$$
$$\int_0^t\{(a'_{44})^{(9)}\left|G_{44}^{(1)} - G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{-(\widehat{M}_{44})^{(9)}s_{(44)}} +$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right)\left|G_{44}^{(1)} - G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}} +$$
$$G_{44}^{(2)}\left|(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right) - (a''_{44})^{(9)}\left(T_{45}^{(2)}, s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\}ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)} - G^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}t} \leq$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)} + (a'_{44})^{(9)} + (\widehat{A}_{44})^{(9)} + (\widehat{P}_{44})^{(9)}(\hat{k}_{44})^{(9)}\right)d\left(((G_{47})^{(1)}, (T_{47})^{(1)};\ (G_{47})^{(2)}, (T_{47})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis (39,35,36) the result follows

**Remark 41:** The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by

$(\widehat{P}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ *and* $(\widehat{Q}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i = 44{,}45{,}46$ depend only on $T_{45}$ and respectively on $(G_{47})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 42:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$

From 99 to 44 it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t\{(a_i')^{(9)}-(a_i'')^{(9)}(T_{45}(s_{(44)}),s_{(44)})\}ds_{(44)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(9)}t)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{44})^{(9)}\right)_1, \left((\widehat{M}_{44})^{(9)}\right)_2$ *and* $\left((\widehat{M}_{44})^{(9)}\right)_3$ :

**Remark 43:** if $G_{44}$ is bounded, the same property have also $G_{45}\ and\ G_{46}$ . indeed if

$G_{44} < (\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt} \leq \left((\widehat{M}_{44})^{(9)}\right)_1 - (a_{45}')^{(9)}G_{45}$ and by integrating

$$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_1/(a_{45}')^{(9)}$$

In the same way , one can obtain

$$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_2/(a_{46}')^{(9)}$$

If $G_{45}\ or\ G_{46}$ is bounded, the same property follows for $G_{44}$ , $G_{46}$ and $G_{44}$ , $G_{45}$ respectively.

**Remark 44:** If $G_{44}$ *is* bounded, from below, the same property holds for $G_{45}\ and\ G_{46}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{45}$ is bounded from below.

**Remark 45:** If $T_{44}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t),t\right) = (b_{45}')^{(9)}$ then $T_{45} \to \infty$.

**Definition of** $(m)^{(9)}$ and $\varepsilon_9$ :

Indeed let $t_9$ be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t),t\right) < \varepsilon_9, T_{44}\,(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right), \quad t = log\frac{2}{\varepsilon_9}$ By taking now $\varepsilon_9$ sufficiently small one sees that $T_{45}$ is unbounded.
The same property holds for $T_{46}$ if $\lim_{t\to\infty}(b_{46}'')^{(9)}\left((G_{47})(t),t\right) = (b_{46}')^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

**Behavior of the solutions of equation** 280

**Theorem** If we denote and define

**Definition of** $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ :

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)} \le -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14}, t) + (a''_{14})^{(1)}(T_{14}, t) \le -(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)} \le -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G, t) - (b''_{14})^{(1)}(G, t) \le -(\tau_1)^{(1)}$

**Definition of** $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$ : 281

By $(\nu_1)^{(1)} > 0$ , $(\nu_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$ , $(u_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

**Definition of** $(\bar{\nu}_1)^{(1)}, , (\bar{\nu}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$ : 282

By $(\bar{\nu}_1)^{(1)} > 0$ , $(\bar{\nu}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$ , $(\bar{u}_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

**Definition of** $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (\nu_0)^{(1)}$ :- 283

If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by
$(m_2)^{(1)} = (\nu_0)^{(1)}, (m_1)^{(1)} = (\nu_1)^{(1)}$, $if\ (\nu_0)^{(1)} < (\nu_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}, if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)}$,

and $\boxed{(\nu_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)}$, $if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)}$, $if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)}$,

and $\boxed{(u_0)^{(1)} = \frac{T^0_{13}}{T^0_{14}}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$

are defined

Then the solution of global equations satisfies the inequalities 285

$G^0_{13}e^{\left((S_1)^{(1)} - (p_{13})^{(1)}\right)t} \le G_{13}(t) \le G^0_{13}e^{(S_1)^{(1)}t}$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)}-(p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)}t}$$

$$\left(\frac{(a_{15})^{(1)}G_{13}^0}{(m_1)^{(1)}((S_1)^{(1)}-(p_{13})^{(1)}-(S_2)^{(1)})}\left[e^{((S_1)^{(1)}-(p_{13})^{(1)})t} - e^{-(S_2)^{(1)}t}\right] + G_{15}^0 e^{-(S_2)^{(1)}t} \le G_{15}(t) \le \right. \quad (286)$$

$$\left. \frac{(a_{15})^{(1)}G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)}-(a'_{15})^{(1)})}[e^{(S_1)^{(1)}t} - e^{-(a'_{15})^{(1)}t}] + G_{15}^0 e^{-(a'_{15})^{(1)}t}\right)$$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)}t} \le T_{13}(t) \le T_{13}^0 e^{((R_1)^{(1)}+(r_{13})^{(1)})t}} \quad (287)$$

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)}t} \le T_{13}(t) \le \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)}+(r_{13})^{(1)})t} \quad (288)$$

$$\frac{(b_{15})^{(1)}T_{13}^0}{(\mu_1)^{(1)}((R_1)^{(1)}-(b'_{15})^{(1)})}\left[e^{(R_1)^{(1)}t} - e^{-(b'_{15})^{(1)}t}\right] + T_{15}^0 e^{-(b'_{15})^{(1)}t} \le T_{15}(t) \le \quad (289)$$

$$\frac{(a_{15})^{(1)}T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)}+(r_{13})^{(1)}+(R_2)^{(1)})}\left[e^{((R_1)^{(1)}+(r_{13})^{(1)})t} - e^{-(R_2)^{(1)}t}\right] + T_{15}^0 e^{-(R_2)^{(1)}t}$$

**Definition of** $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:- (290)

Where $(S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$

$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$

$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$

$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$

**Behavior of the solutions of equation** (291)

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ : (292)

$(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ four constants satisfying

$$-(\sigma_2)^{(2)} \le -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(T_{17}, t) + (a''_{17})^{(2)}(T_{17}, t) \le -(\sigma_1)^{(2)} \quad (293)$$

$$-(\tau_2)^{(2)} \le -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}((G_{19}), t) - (b''_{17})^{(2)}((G_{19}), t) \le -(\tau_1)^{(2)} \quad (294)$$

**Definition of** $(\nu_1)^{(2)}, (\nu_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$ : (295)

By $(\nu_1)^{(2)} > 0$, $(\nu_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0$, $(u_2)^{(2)} < 0$ the roots (296)

of the equations $(a_{17})^{(2)}(\nu^{(2)})^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ (297)

and $(b_{14})^{(2)}(u^{(2)})^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and (298)

**Definition of** $(\bar{\nu}_1)^{(2)}, , (\bar{\nu}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$ : (299)

By $(\bar{\nu}_1)^{(2)} > 0$, $(\bar{\nu}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0$, $(\bar{u}_2)^{(2)} < 0$ the (300)

roots of the equations $(a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 301

and $(b_{17})^{(2)}\left(u^{(2)}\right)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 302

**Definition of** $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ :- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, ***if*** $(u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$, ***if*** $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)}$, ***if*** $(\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)}t}$$

$(p_i)^{(2)}$ is defined by equation

$$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)}t} \quad 311$$

$$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)}t}\right] + G_{18}^0 e^{-(S_2)^{(2)}t} \le G_{18}(t) \le \right. \quad 312$$

$$\left. \frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)}t} - e^{-(a'_{18})^{(2)}t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)}t}\right)$$

$$\boxed{T_{16}^0 e^{(R_1)^{(2)}t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}} \quad 313$$

$$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)}t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t} \quad 314$$

$$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)}t} - e^{-(b'_{18})^{(2)}t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)}t} \le T_{18}(t) \le \quad 315$$

$$\frac{(a_{18})^{(2)}T_{16}^{0}}{(\mu_2)^{(2)}((R_1)^{(2)}+(r_{16})^{(2)}+(R_2)^{(2)})}\left[e^{((R_1)^{(2)}+(r_{16})^{(2)})t}-e^{-(R_2)^{(2)}t}\right]+T_{18}^{0}e^{-(R_2)^{(2)}t}$$

**Definition of** $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$$

$$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)}$$ 318

$$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$$

**Behavior of the solutions** 319

**Theorem 3:** If we denote and define

**Definition of** $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ :

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}\big((G_{23}), t\big) \le -(\tau_1)^{(3)}$$

**Definition of** $(v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$ : 320

By $(v_1)^{(3)} > 0$, $(v_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{v}_1)^{(3)} > 0$, $(\bar{v}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

**Definition of** $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ :- 321

If we define $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (v_0)^{(3)}, (m_1)^{(3)} = (v_1)^{(3)}$, ***if*** $(v_0)^{(3)} < (v_1)^{(3)}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (\bar{v}_1)^{(3)}$, ***if*** $(v_1)^{(3)} < (v_0)^{(3)} < (\bar{v}_1)^{(3)}$,

and $\boxed{(v_0)^{(3)} = \frac{G_{20}^{0}}{G_{21}^{0}}}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_0)^{(3)}$, ***if*** $(\bar{v}_1)^{(3)} < (v_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)},\ \boldsymbol{if}\ (u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}, \boldsymbol{if}\ (u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)},$ and $\boxed{(u_0)^{(3)} = \frac{T_{20}^0}{T_{21}^0}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)},\ \boldsymbol{if}\ (\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G_{20}^0 e^{(S_1)^{(3)} t}$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)} t} \quad 323$$

$$\left(\frac{(a_{22})^{(3)} G_{20}^0}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\left[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)} t}\right] + G_{22}^0 e^{-(S_2)^{(3)} t} \le G_{22}(t) \le \right. \quad 324$$

$$\left. \frac{(a_{22})^{(3)} G_{20}^0}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}\left[e^{(S_1)^{(3)} t} - e^{-(a'_{22})^{(3)} t}\right] + G_{22}^0 e^{-(a'_{22})^{(3)} t}\right)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t}} \quad 325$$

$$\frac{1}{(\mu_1)^{(3)}} T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}} T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)} T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)} - (b'_{22})^{(3)})}\left[e^{(R_1)^{(3)} t} - e^{-(b'_{22})^{(3)} t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)} t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)} T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)} + (r_{20})^{(3)} + (R_2)^{(3)})}\left[e^{((R_1)^{(3)} + (r_{20})^{(3)})t} - e^{-(R_2)^{(3)} t}\right] + T_{22}^0 e^{-(R_2)^{(3)} t}$$

**Definition of** $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

**Behavior of the solutions of equation**

**Theorem:** If we denote and define

**Definition of** $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ :

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$

$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}((G_{27}), t) - (b''_{25})^{(4)}((G_{27}), t) \le -(\tau_1)^{(4)}$

**Definition of** $(v_1)^{(4)}, (v_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, v^{(4)}, u^{(4)}$ : 329

By $(v_1)^{(4)} > 0$ , $(v_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$ , $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_1)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

**Definition of** $(\bar{v}_1)^{(4)},, (\bar{v}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$ : 330

By $(\bar{v}_1)^{(4)} > 0$ , $(\bar{v}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$ , $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

**Definition of** $(m_1)^{(4)}$ , $(m_2)^{(4)}$ , $(\mu_1)^{(4)}, (\mu_2)^{(4)}, (\nu_0)^{(4)}$ :-

If we define $(m_1)^{(4)}$ , $(m_2)^{(4)}$ , $(\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (v_0)^{(4)}, (m_1)^{(4)} = (v_1)^{(4)}$, $\boldsymbol{if}$ $(v_0)^{(4)} < (v_1)^{(4)}$

$(m_2)^{(4)} = (v_1)^{(4)}, (m_1)^{(4)} = (\bar{v}_1)^{(4)}$ , $\boldsymbol{if}$ $(v_4)^{(4)} < (v_0)^{(4)} < (\bar{v}_1)^{(4)}$,
and $\boxed{(v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (v_4)^{(4)}, (m_1)^{(4)} = (v_0)^{(4)}$, $\boldsymbol{if}$ $(\bar{v}_4)^{(4)} < (v_0)^{(4)}$

and analogously

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$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)}$, $\boldsymbol{if}$ $(u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}$ , $\boldsymbol{if}$ $(u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)}$,
and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}, \boldsymbol{if}$ $(\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

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$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)}t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)}t}$$

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$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)}t}\right] + G_{26}^0 e^{-(S_2)^{(4)}t} \le G_{26}(t) \le\right.$$ 334

*(a26)4G240(m2)4(S1)4−(a26′)4e(S1)4t−e−(a26′)4t+ G260e−(a26′)4t*

$$\boxed{T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}}T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}}T_{24}^0 e^{((R_1)^{(4)}+(r_{24})^{(4)})t} \quad 335$$

$$\frac{(b_{26})^{(4)}T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)}-(b'_{26})^{(4)})}\left[e^{(R_1)^{(4)}t} - e^{-(b'_{26})^{(4)}t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)}t} \le T_{26}(t) \le \quad 336$$

$$\frac{(a_{26})^{(4)}T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)}+(r_{24})^{(4)}+(R_2)^{(4)})}\left[e^{((R_1)^{(4)}+(r_{24})^{(4)})t} - e^{-(R_2)^{(4)}t}\right] + T_{26}^0 e^{-(R_2)^{(4)}t}$$

**Definition of** $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

**Behavior of the solutions of equation** 338

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ :

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

**Definition of** $(v_1)^{(5)}, (v_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, v^{(5)}, u^{(5)}$ : 339

By $(v_1)^{(5)} > 0$ , $(v_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$ , $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(v^{(5)}\big)^2 + (\sigma_1)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

**Definition of** $(\bar{v}_1)^{(5)}, , (\bar{v}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$ : 340

By $(\bar{v}_1)^{(5)} > 0$ , $(\bar{v}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$ , $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(v^{(5)}\big)^2 + (\sigma_2)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

**Definition of** $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (v_0)^{(5)}$ :-

If we define $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (v_0)^{(5)}, (m_1)^{(5)} = (v_1)^{(5)}$, ***if*** $(v_0)^{(5)} < (v_1)^{(5)}$

$(m_2)^{(5)} = (v_1)^{(5)}, (m_1)^{(5)} = (\bar{v}_1)^{(5)}$ , ***if*** $(v_1)^{(5)} < (v_0)^{(5)} < (\bar{v}_1)^{(5)}$,
and $\boxed{(v_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)},\ \boldsymbol{if}\ (u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}, \boldsymbol{if}\ (u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)},$

and $\boxed{(u_0)^{(5)} = \frac{T_{28}^0}{T_{29}^0}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}, \boldsymbol{if}\ (\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{28}(t) \le G_{28}^0 e^{(S_1)^{(5)} t}$

where $(p_i)^{(5)}$ is defined by equation

$\frac{1}{(m_5)^{(5)}} G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}} G_{28}^0 e^{(S_1)^{(5)} t}$ 343

$\left(\frac{(a_{30})^{(5)} G_{28}^0}{(m_1)^{(5)}((S_1)^{(5)} - (p_{28})^{(5)} - (S_2)^{(5)})}\left[e^{((S_1)^{(5)} - (p_{28})^{(5)})t} - e^{-(S_2)^{(5)} t}\right] + G_{30}^0 e^{-(S_2)^{(5)} t} \le G_{30}(t) \le\right.$ 344

*(a30)5G280(m2)5(S1)5−(a30′)5e(S1)5t−e−(a30′)5t+ G300e−(a30′)5t*

$\boxed{T_{28}^0 e^{(R_1)^{(5)} t} \le T_{28}(t) \le T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}}$ 345

$\frac{1}{(\mu_1)^{(5)}} T_{28}^0 e^{(R_1)^{(5)} t} \le T_{28}(t) \le \frac{1}{(\mu_2)^{(5)}} T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}$ 346

$\frac{(b_{30})^{(5)} T_{28}^0}{(\mu_1)^{(5)}((R_1)^{(5)} - (b'_{30})^{(5)})}\left[e^{(R_1)^{(5)} t} - e^{-(b'_{30})^{(5)} t}\right] + T_{30}^0 e^{-(b'_{30})^{(5)} t} \le T_{30}(t) \le$ 347

$\frac{(a_{30})^{(5)} T_{28}^0}{(\mu_2)^{(5)}((R_1)^{(5)} + (r_{28})^{(5)} + (R_2)^{(5)})}\left[e^{((R_1)^{(5)} + (r_{28})^{(5)})t} - e^{-(R_2)^{(5)} t}\right] + T_{30}^0 e^{-(R_2)^{(5)} t}$

**Definition of** $(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$:- 348

Where $(S_1)^{(5)} = (a_{28})^{(5)}(m_2)^{(5)} - (a'_{28})^{(5)}$

$(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}$

$(R_1)^{(5)} = (b_{28})^{(5)}(\mu_2)^{(5)} - (b'_{28})^{(5)}$

$(R_2)^{(5)} = (b'_{30})^{(5)} - (r_{30})^{(5)}$

**Behavior of the solutions of equation** 349

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ :

$(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)} \le -(a'_{32})^{(6)} + (a'_{33})^{(6)} - (a''_{32})^{(6)}(T_{33}, t) + (a''_{33})^{(6)}(T_{33}, t) \le -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)} \le -(b'_{32})^{(6)} + (b'_{33})^{(6)} - (b''_{32})^{(6)}\big((G_{35}), t\big) - (b''_{33})^{(6)}\big((G_{35}), t\big) \le -(\tau_1)^{(6)}$$

**Definition of** $(\nu_1)^{(6)}, (\nu_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, \nu^{(6)}, u^{(6)}$ : 350

By $(\nu_1)^{(6)} > 0$, $(\nu_2)^{(6)} < 0$ and respectively $(u_1)^{(6)} > 0$, $(u_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(6)},, (\bar{\nu}_2)^{(6)}, (\bar{u}_1)^{(6)}, (\bar{u}_2)^{(6)}$ : 351

By $(\bar{\nu}_1)^{(6)} > 0$, $(\bar{\nu}_2)^{(6)} < 0$ and respectively $(\bar{u}_1)^{(6)} > 0$, $(\bar{u}_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$

**Definition of** $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}, (\nu_0)^{(6)}$ :-

If we define $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}$ by

$$(m_2)^{(6)} = (\nu_0)^{(6)}, (m_1)^{(6)} = (\nu_1)^{(6)}, \ \boldsymbol{if}\ (\nu_0)^{(6)} < (\nu_1)^{(6)}$$

$$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}, \boldsymbol{if}\ (\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)},$$

and $\boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}}$

$$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$$

and analogously 352

$$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}, \ \boldsymbol{if}\ (u_0)^{(6)} < (u_1)^{(6)}$$

$$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}, \boldsymbol{if}\ (u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)},$$

and $\boxed{(u_0)^{(6)} = \frac{T^0_{32}}{T^0_{33}}}$

$$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}, \boldsymbol{if}\ (\bar{u}_1)^{(6)} < (u_0)^{(6)} \text{ where } (u_1)^{(6)}, (\bar{u}_1)^{(6)}$$

Then the solution of global equations satisfies the inequalities 353

$$G^0_{32}e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{32}(t) \le G^0_{32}e^{(S_1)^{(6)}t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}}G^0_{32}e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{33}(t) \le \frac{1}{(m_2)^{(6)}}G^0_{32}e^{(S_1)^{(6)}t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)}G^0_{32}}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})}\left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)}t}\right] + G^0_{34}e^{-(S_2)^{(6)}t} \le G_{34}(t) \le \right. \quad 355$$

$$\frac{(a_{34})^{(6)}G_{32}^{0}}{(m_2)^{(6)}((S_1)^{(6)}-(a'_{34})^{(6)})}\left[e^{(S_1)^{(6)}t}-e^{-(a'_{34})^{(6)}t}\right]+\ G_{34}^{0}e^{-(a'_{34})^{(6)}t}\Big)$$

$$\boxed{T_{32}^{0}e^{(R_1)^{(6)}t}\le T_{32}(t)\le T_{32}^{0}e^{((R_1)^{(6)}+(r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}}T_{32}^{0}e^{(R_1)^{(6)}t}\le T_{32}(t)\le\frac{1}{(\mu_2)^{(6)}}T_{32}^{0}e^{((R_1)^{(6)}+(r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)}T_{32}^{0}}{(\mu_1)^{(6)}((R_1)^{(6)}-(b'_{34})^{(6)})}\left[e^{(R_1)^{(6)}t}-e^{-(b'_{34})^{(6)}t}\right]+T_{34}^{0}e^{-(b'_{34})^{(6)}t}\le T_{34}(t)\le \quad 358$$

$$\frac{(a_{34})^{(6)}T_{32}^{0}}{(\mu_2)^{(6)}((R_1)^{(6)}+(r_{32})^{(6)}+(R_2)^{(6)})}\left[e^{((R_1)^{(6)}+(r_{32})^{(6)})t}-e^{-(R_2)^{(6)}t}\right]+T_{34}^{0}e^{-(R_2)^{(6)}t}$$

**Definition of** $(S_1)^{(6)},(S_2)^{(6)},(R_1)^{(6)},(R_2)^{(6)}$:- 359

Where $(S_1)^{(6)}=(a_{32})^{(6)}(m_2)^{(6)}-(a'_{32})^{(6)}$

$(S_2)^{(6)}=(a_{34})^{(6)}-(p_{34})^{(6)}$

$(R_1)^{(6)}=(b_{32})^{(6)}(\mu_2)^{(6)}-(b'_{32})^{(6)}$

$(R_2)^{(6)}=(b'_{34})^{(6)}-(r_{34})^{(6)}$

**Behavior of the solutions of equation**

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(7)},(\sigma_2)^{(7)},(\tau_1)^{(7)},(\tau_2)^{(7)}$ :

$(\sigma_1)^{(7)},(\sigma_2)^{(7)},(\tau_1)^{(7)},(\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)}\le-(a'_{36})^{(7)}+(a'_{37})^{(7)}-(a''_{36})^{(7)}(T_{37},t)+(a''_{37})^{(7)}(T_{37},t)\le-(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)}\le-(b'_{36})^{(7)}+(b'_{37})^{(7)}-(b''_{36})^{(7)}\big((G_{39}),t\big)-(b''_{37})^{(7)}\big((G_{39}),t\big)\le-(\tau_1)^{(7)}$$

**Definition of** $(v_1)^{(7)},(v_2)^{(7)},(u_1)^{(7)},(u_2)^{(7)},v^{(7)},u^{(7)}$ : 361

By $(v_1)^{(7)}>0,(v_2)^{(7)}<0$ and respectively $(u_1)^{(7)}>0,(u_2)^{(7)}<0$ the roots of the equations $(a_{37})^{(7)}\big(v^{(7)}\big)^2+(\sigma_1)^{(7)}v^{(7)}-(a_{36})^{(7)}=0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2+(\tau_1)^{(7)}u^{(7)}-(b_{36})^{(7)}=0$ and

**Definition of** $(\bar{v}_1)^{(7)},,(\bar{v}_2)^{(7)},(\bar{u}_1)^{(7)},(\bar{u}_2)^{(7)}$ : 362

By $(\bar{v}_1)^{(7)}>0,(\bar{v}_2)^{(7)}<0$ and respectively $(\bar{u}_1)^{(7)}>0,(\bar{u}_2)^{(7)}<0$ the roots of the equations $(a_{37})^{(7)}\big(v^{(7)}\big)^2+(\sigma_2)^{(7)}v^{(7)}-(a_{36})^{(7)}=0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2+(\tau_2)^{(7)}u^{(7)}-(b_{36})^{(7)}=0$

**Definition of** $(m_1)^{(7)},(m_2)^{(7)},(\mu_1)^{(7)},(\mu_2)^{(7)},(v_0)^{(7)}$ :-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$$(m_2)^{(7)} = (\nu_0)^{(7)}, (m_1)^{(7)} = (\nu_1)^{(7)}, \ \boldsymbol{if}\ (\nu_0)^{(7)} < (\nu_1)^{(7)}$$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\bar{\nu}_1)^{(7)}, \boldsymbol{if}\ (\nu_1)^{(7)} < (\nu_0)^{(7)} < (\bar{\nu}_1)^{(7)},$$

and $\boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\nu_0)^{(7)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(7)} < (\nu_0)^{(7)}$$

and analogously 363

$$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}, \ \boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$$

and $\boxed{(u_0)^{(7)} = \frac{T_{36}^0}{T_{37}^0}}$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)} \text{ where } (u_1)^{(7)}, (\bar{u}_1)^{(7)}$$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^0 e^{(S_1)^{(7)} t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)} t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)}((S_1)^{(7)} - (p_{36})^{(7)} - (S_2)^{(7)})}\left[e^{((S_1)^{(7)} - (p_{36})^{(7)})t} - e^{-(S_2)^{(7)} t}\right] + G_{38}^0 e^{-(S_2)^{(7)} t} \le G_{38}(t) \le \right. \quad 366$$

$$\left. \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)}((S_1)^{(7)} - (a'_{38})^{(7)})}\left[e^{(S_1)^{(7)} t} - e^{-(a'_{38})^{(7)} t}\right] + G_{38}^0 e^{-(a'_{38})^{(7)} t}\right)$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t} \quad 368$$

$$\frac{(b_{38})^{(7)} T_{36}^0}{(\mu_1)^{(7)}((R_1)^{(7)} - (b'_{38})^{(7)})}\left[e^{(R_1)^{(7)} t} - e^{-(b'_{38})^{(7)} t}\right] + T_{38}^0 e^{-(b'_{38})^{(7)} t} \le T_{38}(t) \le \quad 369$$

$$\frac{(a_{38})^{(7)} T_{36}^0}{(\mu_2)^{(7)}((R_1)^{(7)} + (r_{36})^{(7)} + (R_2)^{(7)})}\left[e^{((R_1)^{(7)} + (r_{36})^{(7)})t} - e^{-(R_2)^{(7)} t}\right] + T_{38}^0 e^{-(R_2)^{(7)} t}$$

**Definition of** $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$

$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$

$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$

**Behavior of the solutions of equation** 371

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ :

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$-(\sigma_2)^{(8)} \le -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \le -(\sigma_1)^{(8)}$

$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\big((G_{43}), t\big) - (b''_{41})^{(8)}\big((G_{43}), t\big) \le -(\tau_1)^{(8)}$

**Definition of** $(\nu_1)^{(8)}, (\nu_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, \nu^{(8)}, u^{(8)}$ : 372

By $(\nu_1)^{(8)} > 0, (\nu_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0, (u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(8)}, , (\bar{\nu}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$ :

By $(\bar{\nu}_1)^{(8)} > 0, (\bar{\nu}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0, (\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

**Definition of** $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}, (\nu_0)^{(8)}$ :-

If we define $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$(m_2)^{(8)} = (\nu_0)^{(8)}, (m_1)^{(8)} = (\nu_1)^{(8)}$, ***if*** $(\nu_0)^{(8)} < (\nu_1)^{(8)}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\bar{\nu}_1)^{(8)}$, ***if*** $(\nu_1)^{(8)} < (\nu_0)^{(8)} < (\bar{\nu}_1)^{(8)}$,
and $\boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\nu_0)^{(8)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(8)} < (\nu_0)^{(8)}$

and analogously 374

$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)},\ \boldsymbol{if}\ (u_0)^{(8)} < (u_1)^{(8)}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}, \boldsymbol{if}\ (u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)},$

and $\boxed{(u_0)^{(8)} = \frac{T_{40}^0}{T_{41}^0}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}, \boldsymbol{if}\ (\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^0 e^{(S_1)^{(8)} t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}} G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}} G_{40}^0 e^{(S_1)^{(8)} t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)} G_{40}^0}{(m_1)^{(8)}((S_1)^{(8)} - (p_{40})^{(8)} - (S_2)^{(8)})}\left[e^{((S_1)^{(8)} - (p_{40})^{(8)})t} - e^{-(S_2)^{(8)} t}\right] + G_{42}^0 e^{-(S_2)^{(8)} t} \le G_{42}(t) \le \right. \quad 377$$

$$\left. \frac{(a_{42})^{(8)} G_{40}^0}{(m_2)^{(8)}((S_1)^{(8)} - (a'_{42})^{(8)})}\left[e^{(S_1)^{(8)} t} - e^{-(a'_{42})^{(8)} t}\right] + G_{42}^0 e^{-(a'_{42})^{(8)} t}\right)$$

$$\boxed{T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}} T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le \frac{1}{(\mu_2)^{(8)}} T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)} T_{40}^0}{(\mu_1)^{(8)}((R_1)^{(8)} - (b'_{42})^{(8)})}\left[e^{(R_1)^{(8)} t} - e^{-(b'_{42})^{(8)} t}\right] + T_{42}^0 e^{-(b'_{42})^{(8)} t} \le T_{42}(t) \le \quad 380$$

$$\frac{(a_{42})^{(8)} T_{40}^0}{(\mu_2)^{(8)}((R_1)^{(8)} + (r_{40})^{(8)} + (R_2)^{(8)})}\left[e^{((R_1)^{(8)} + (r_{40})^{(8)})t} - e^{-(R_2)^{(8)} t}\right] + T_{42}^0 e^{-(R_2)^{(8)} t}$$

**Definition of** $(S_1)^{(8)}, (S_2)^{(8)}, (R_1)^{(8)}, (R_2)^{(8)}$:- 381

Where $(S_1)^{(8)} = (a_{40})^{(8)}(m_2)^{(8)} - (a'_{40})^{(8)}$

$(S_2)^{(8)} = (a_{42})^{(8)} - (p_{42})^{(8)}$

$(R_1)^{(8)} = (b_{40})^{(8)}(\mu_2)^{(8)} - (b'_{40})^{(8)}$

$(R_2)^{(8)} = (b'_{42})^{(8)} - (r_{42})^{(8)}$

**Behavior of the solutions of equation 37 to 92** 382

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ :

$(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ four constants satisfying

$$-(\sigma_2)^{(9)} \le -(a'_{44})^{(9)} + (a'_{45})^{(9)} - (a''_{44})^{(9)}(T_{45}, t) + (a''_{45})^{(9)}(T_{45}, t) \le -(\sigma_1)^{(9)}$$

$$-(\tau_2)^{(9)} \le -(b'_{44})^{(9)} + (b'_{45})^{(9)} - (b''_{44})^{(9)}\big((G_{47}), t\big) - (b''_{45})^{(9)}\big((G_{47}), t\big) \le -(\tau_1)^{(9)}$$

**Definition of** $(\nu_1)^{(9)}, (\nu_2)^{(9)}, (u_1)^{(9)}, (u_2)^{(9)}, \nu^{(9)}, u^{(9)}$ :

By $(\nu_1)^{(9)} > 0$ , $(\nu_2)^{(9)} < 0$ and respectively $(u_1)^{(9)} > 0$ , $(u_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_1)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(9)}, , (\bar{\nu}_2)^{(9)}, (\bar{u}_1)^{(9)}, (\bar{u}_2)^{(9)}$ :

By $(\bar{\nu}_1)^{(9)} > 0$ , $(\bar{\nu}_2)^{(9)} < 0$ and respectively $(\bar{u}_1)^{(9)} > 0$ , $(\bar{u}_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_2)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$

**Definition of** $(m_1)^{(9)}, (m_2)^{(9)}, (\mu_1)^{(9)}, (\mu_2)^{(9)}, (\nu_0)^{(9)}$ :-

If we define $(m_1)^{(9)}, (m_2)^{(9)}, (\mu_1)^{(9)}, (\mu_2)^{(9)}$ by

$$(m_2)^{(9)} = (\nu_0)^{(9)}, (m_1)^{(9)} = (\nu_1)^{(9)}, \ \boldsymbol{if}\ (\nu_0)^{(9)} < (\nu_1)^{(9)}$$

$$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\bar{\nu}_1)^{(9)}, \boldsymbol{if}\ (\nu_1)^{(9)} < (\nu_0)^{(9)} < (\bar{\nu}_1)^{(9)},$$

and $\boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$

$$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\nu_0)^{(9)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(9)} < (\nu_0)^{(9)}$$

and analogously

$$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)}, \ \boldsymbol{if}\ (u_0)^{(9)} < (u_1)^{(9)}$$

$$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}, \boldsymbol{if}\ (u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)},$$

and $\boxed{(u_0)^{(9)} = \frac{T^0_{44}}{T^0_{45}}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}, \boldsymbol{if}\ (\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G^0_{44} e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{44}(t) \le G^0_{44} e^{(S_1)^{(9)} t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}}G_{44}^0 e^{((S_1)^{(9)}-(p_{44})^{(9)})t} \le G_{45}(t) \le \frac{1}{(m_2)^{(9)}}G_{44}^0 e^{(S_1)^{(9)}t}$$

(

$$\frac{(a_{46})^{(9)}G_{44}^0}{(m_1)^{(9)}((S_1)^{(9)}-(p_{44})^{(9)}-(S_2)^{(9)})}\left[e^{((S_1)^{(9)}-(p_{44})^{(9)})t} - e^{-(S_2)^{(9)}t}\right] + G_{46}^0 e^{-(S_2)^{(9)}t} \le G_{46}(t) \le$$

$$\frac{(a_{46})^{(9)}G_{44}^0}{(m_2)^{(9)}((S_1)^{(9)}-(a'_{46})^{(9)})}[e^{(S_1)^{(9)}t} - e^{-(a'_{46})^{(9)}t}] + G_{46}^0 e^{-(a'_{46})^{(9)}t})$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le T_{44}^0 e^{((R_1)^{(9)}+(r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}}T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le \frac{1}{(\mu_2)^{(9)}}T_{44}^0 e^{((R_1)^{(9)}+(r_{44})^{(9)})t}$$

$$\frac{(b_{46})^{(9)}T_{44}^0}{(\mu_1)^{(9)}((R_1)^{(9)}-(b'_{46})^{(9)})}\left[e^{(R_1)^{(9)}t} - e^{-(b'_{46})^{(9)}t}\right] + T_{46}^0 e^{-(b'_{46})^{(9)}t} \le T_{46}(t) \le$$

$$\frac{(a_{46})^{(9)}T_{44}^0}{(\mu_2)^{(9)}((R_1)^{(9)}+(r_{44})^{(9)}+(R_2)^{(9)})}\left[e^{((R_1)^{(9)}+(r_{44})^{(9)})t} - e^{-(R_2)^{(9)}t}\right] + T_{46}^0 e^{-(R_2)^{(9)}t}$$

**Definition of** $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$

**Proof :** From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14},t)\right) - (a''_{14})^{(1)}(T_{14},t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

**Definition of** $v^{(1)}$ :- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)}\right) \le \frac{dv^{(1)}}{dt} \le -\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)}\right)$$

From which one obtains

**Definition of** $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$ :-

For $0 < \boxed{(v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}} < (v_1)^{(1)} < (\bar{v}_1)^{(1)}$

$$v^{(1)}(t) \ge \frac{(v_1)^{(1)}+(C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(v_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(v_2)^{(1)}}}$$

it follows $(\nu_0)^{(1)} \leq \nu^{(1)}(t) \leq (\nu_1)^{(1)}$

In the same manner , we get 384

$$\nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{\nu}_1)^{(1)}-(\nu_0)^{(1)}}{(\nu_0)^{(1)}-(\bar{\nu}_2)^{(1)}}}$$

From which we deduce $(\nu_0)^{(1)} \leq \nu^{(1)}(t) \leq (\bar{\nu}_1)^{(1)}$

If $0 < (\nu_1)^{(1)} < (\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0} < (\bar{\nu}_1)^{(1)}$ we find like in the previous case, 385

$$(\nu_1)^{(1)} \leq \frac{(\nu_1)^{(1)}+(C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}} \leq \nu^{(1)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\bar{\nu}_1)^{(1)}$$

If $0 < (\nu_1)^{(1)} \leq (\bar{\nu}_1)^{(1)} \leq \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$ , we obtain 386

$$(\nu_1)^{(1)} \leq \nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\nu_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(1)}(t)$ :-

$$(m_2)^{(1)} \leq \nu^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{\nu^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(1)}(t)$ :-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{13})^{(1)} = (a''_{14})^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)}G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)}T_{14}(t)$ This is an important

consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

**Proof** : From global equations we obtain 387

$$\frac{d\nu^{(2)}}{dt} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right) - (a''_{17})^{(2)}(T_{17}, t)\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

**Definition of** $\nu^{(2)}$ :- $\boxed{\nu^{(2)} = \frac{G_{16}}{G_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \le \frac{d\nu^{(2)}}{dt} \le -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

**Definition of** $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$ :-

For $0 < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \ge \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}}{1 + (C)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(C)^{(2)} = \frac{(\nu_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \le \nu^{(2)}(t) \le (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \le \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \le \nu^{(2)}(t) \le (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \le \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}}{1 + (C)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}} \le \nu^{(2)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \le (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \le (\bar{\nu}_1)^{(2)} \le (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$ , we obtain 394

$$(\nu_1)^{(2)} \le \nu^{(2)}(t) \le \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \le (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(2)}(t)$ :- 395

$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}$, $\boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$

In a completely analogous way, we obtain 396

**Definition of** $u^{(2)}(t)$ :-

$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}$, $\boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :** 397

If $(a''_{16})^{(2)} = (a''_{17})^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)} G_{17}(t)$

Analogously if $(b''_{16})^{(2)} = (b''_{17})^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)} T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

**Proof** : From global equations we obtain 398

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right) - (a''_{21})^{(3)}(T_{21}, t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

**Definition of** $\nu^{(3)}$ :- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right) \leq \frac{d\nu^{(3)}}{dt} \leq -\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right)$$

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From which one obtains

For $0 < (\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\nu_1)^{(3)} < (\bar{\nu}_1)^{(3)}$

$$\nu^{(3)}(t) \geq \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(\nu_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\nu_2)^{(3)}}}$$

it follows $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\nu_1)^{(3)}$

In the same manner , we get 401

$$\nu^{(3)}(t) \le \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{\nu}_1)^{(3)}-(\nu_0)^{(3)}}{(\nu_0)^{(3)}-(\bar{\nu}_2)^{(3)}}}$$

**Definition of** $(\bar{\nu}_1)^{(3)}$ :-

From which we deduce $(\nu_0)^{(3)} \le \nu^{(3)}(t) \le (\bar{\nu}_1)^{(3)}$

If $0 < (\nu_1)^{(3)} < (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (\bar{\nu}_1)^{(3)}$ we find like in the previous case, 402

$$(\nu_1)^{(3)} \le \frac{(\nu_1)^{(3)}+(C)^{(3)}(\nu_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}} \le \nu^{(3)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \le (\bar{\nu}_1)^{(3)}$$

If $0 < (\nu_1)^{(3)} \le (\bar{\nu}_1)^{(3)} \le (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}$ , we obtain 403

$$(\nu_1)^{(3)} \le \nu^{(3)}(t) \le \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \le (\nu_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(3)}(t)$ :-

$$(m_2)^{(3)} \le \nu^{(3)}(t) \le (m_1)^{(3)}, \quad \boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(3)}(t)$ :-

$$(\mu_2)^{(3)} \le u^{(3)}(t) \le (\mu_1)^{(3)}, \quad \boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{20}'')^{(3)} = (a_{21}'')^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)}G_{21}(t)$

Analogously if $(b_{20}'')^{(3)} = (b_{21}'')^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)}T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

**Proof** : From global equations we obtain 404

$$\frac{dv^{(4)}}{dt} = (a_{24})^{(4)} - \left((a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right) - (a''_{25})^{(4)}(T_{25},t)v^{(4)} - (a_{25})^{(4)}v^{(4)}$$

**Definition of** $v^{(4)}$ :- $\boxed{v^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)}\right) \le \frac{dv^{(4)}}{dt} \le -\left((a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_4)^{(4)}v^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$ :-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \ge \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \le \nu^{(4)}(t) \le (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \le \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \le \nu^{(4)}(t) \le (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \le \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}}{1 + (C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}} \le \nu^{(4)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \le (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \le (\bar{\nu}_1)^{(4)} \le \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$ , we obtain 407

$$(\nu_1)^{(4)} \le \nu^{(4)}(t) \le \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \le (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(4)}(t)$ :-

$$(m_2)^{(4)} \le \nu^{(4)}(t) \le (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(4)}(t)$ :-

$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}$, $\boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{24})^{(4)} = (a''_{25})^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)} G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b''_{24})^{(4)} = (b''_{25})^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)} T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

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**Proof** : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a'_{28})^{(5)} - (a'_{29})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right) - (a''_{29})^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

**Definition of** $\nu^{(5)}$ :- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$ :-

For $0 < \boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \le \frac{(\nu_1)^{(5)}+(C)^{(5)}(\nu_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)}-(\nu_2)^{(5)}\right)t\right]}}{1+(C)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)}-(\nu_2)^{(5)}\right)t\right]}} \le \nu^{(5)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \le (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \le (\bar{\nu}_1)^{(5)} \le \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain 411

$$(\nu_1)^{(5)} \le \nu^{(5)}(t) \le \frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \le (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(5)}(t)$ :-

$(m_2)^{(5)} \le \nu^{(5)}(t) \le (m_1)^{(5)}$, $\boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$

In a completely analogous way, we obtain

**Definition of** $u^{(5)}(t)$ :-

$(\mu_2)^{(5)} \le u^{(5)}(t) \le (\mu_1)^{(5)}$, $\boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a_{28}'')^{(5)} = (a_{29}'')^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)}G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b_{28}'')^{(5)} = (b_{29}'')^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)}T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

**Proof** : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a_{32}')^{(6)} - (a_{33}')^{(6)} + (a_{32}'')^{(6)}(T_{33},t)\right) - (a_{33}'')^{(6)}(T_{33},t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

**Definition of** $\nu^{(6)}$ :- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \le \frac{d\nu^{(6)}}{dt} \le -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$ :-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \geq \frac{(\nu_1)^{(6)}+(C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_0)^{(6)}\right)t\right]}}{1+(C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)}-(\nu_0)^{(6)}}{(\nu_0)^{(6)}-(\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\nu_1)^{(6)}$

In the same manner , we get 413

$$\nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{\nu}_1)^{(6)}-(\nu_0)^{(6)}}{(\nu_0)^{(6)}-(\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\bar{\nu}_1)^{(6)}$

If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 414

$$(\nu_1)^{(6)} \leq \frac{(\nu_1)^{(6)}+(C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_2)^{(6)}\right)t\right]}}{1+(C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_2)^{(6)}\right)t\right]}} \leq \nu^{(6)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\bar{\nu}_1)^{(6)}$$

If $0 < (\nu_1)^{(6)} \leq (\bar{\nu}_1)^{(6)} \leq \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$ , we obtain 415

$$(\nu_1)^{(6)} \leq \nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(6)}(t)$ :-

$$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}, \quad \boxed{\nu^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(6)}(t)$ :-

$$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}, \quad \boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)}G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)}T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

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**Proof** : From global equations we obtain

$$\frac{dv^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right) - (a''_{37})^{(7)}(T_{37},t)v^{(7)} - (a_{37})^{(7)}v^{(7)}$$

**Definition of** $v^{(7)}$ :- $\boxed{v^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_2)^{(7)}v^{(7)} - (a_{36})^{(7)}\right) \le \frac{dv^{(7)}}{dt} \le -\left((a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_1)^{(7)}v^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

**Definition of** $(\bar{v}_1)^{(7)}, (v_0)^{(7)}$ :-

For $0 < \boxed{(v_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}} < (v_1)^{(7)} < (\bar{v}_1)^{(7)}$

$$v^{(7)}(t) \ge \frac{(v_1)^{(7)} + (C)^{(7)}(v_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_0)^{(7)}\right)t\right]}}{1 + (C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(v_1)^{(7)} - (v_0)^{(7)}}{(v_0)^{(7)} - (v_2)^{(7)}}}$$

it follows $(v_0)^{(7)} \le v^{(7)}(t) \le (v_1)^{(7)}$

In the same manner , we get 417

$$v^{(7)}(t) \le \frac{(\bar{v}_1)^{(7)} + (\bar{C})^{(7)}(\bar{v}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{v}_1)^{(7)} - (v_0)^{(7)}}{(v_0)^{(7)} - (\bar{v}_2)^{(7)}}}$$

From which we deduce $(v_0)^{(7)} \le v^{(7)}(t) \le (\bar{v}_1)^{(7)}$

If $0 < (v_1)^{(7)} < (v_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}} < (\bar{v}_1)^{(7)}$ we find like in the previous case, 418

$$(v_1)^{(7)} \le \frac{(v_1)^{(7)} + (C)^{(7)}(v_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_2)^{(7)}\right)t\right]}}{1 + (C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_2)^{(7)}\right)t\right]}} \le v^{(7)}(t) \le$$

$$\frac{(\bar{v}_1)^{(7)} + (\bar{C})^{(7)}(\bar{v}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}} \le (\bar{v}_1)^{(7)}$$

If $0 < (v_1)^{(7)} \le (\bar{v}_1)^{(7)} \le \boxed{(v_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$ , we obtain 419

$$(v_1)^{(7)} \le v^{(7)}(t) \le \frac{(\bar{v}_1)^{(7)} + (\bar{C})^{(7)}(\bar{v}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}} \le (v_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $v^{(7)}(t)$ :-

$$(m_2)^{(7)} \leq v^{(7)}(t) \leq (m_1)^{(7)}, \quad \boxed{v^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(7)}(t)$ :- 420

$$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}, \quad \boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $v^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)} G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)} T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

**Proof** : From global equations we obtain 421

$$\frac{dv^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right) - (a''_{41})^{(8)}(T_{41}, t) v^{(8)} - (a_{41})^{(8)} v^{(8)}$$

**Definition of** $v^{(8)}$ :- $\boxed{v^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_2)^{(8)} v^{(8)} - (a_{40})^{(8)}\right) \leq \frac{dv^{(8)}}{dt} \leq -\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_1)^{(8)} v^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$ :-

$$\text{For } 0 < \boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$$

$$v^{(8)}(t) \geq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)}-(\nu_0)^{(8)}}{(\nu_0)^{(8)}-(\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \leq \frac{(\nu_1)^{(8)}+(C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}}{1+(C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}} \leq \nu^{(8)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \leq (\bar{\nu}_1)^{(8)} \leq \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$ , we obtain 424

$$(\nu_1)^{(8)} \leq \nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(8)}(t)$ :-

$$(m_2)^{(8)} \leq \nu^{(8)}(t) \leq (m_1)^{(8)}, \quad \boxed{\nu^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(8)}(t)$ :-

$$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}, \quad \boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $\nu^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)}G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then $(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)}T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

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**Proof** : From 99,20,44,22,23,44 we obtain A

$$\frac{dv^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right) - (a''_{45})^{(9)}(T_{45},t)v^{(9)} - (a_{45})^{(9)}v^{(9)}$$

**Definition of** $v^{(9)}$ :- $\boxed{v^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)}\right) \le \frac{dv^{(9)}}{dt} \le -\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_1)^{(9)}v^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

**Definition of** $(\bar{v}_1)^{(9)}, (v_0)^{(9)}$ :-

For $0 < \boxed{(v_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (v_1)^{(9)} < (\bar{v}_1)^{(9)}$

$$v^{(9)}(t) \ge \frac{(v_1)^{(9)} + (C)^{(9)}(v_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((v_1)^{(9)} - (v_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((v_1)^{(9)} - (v_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(v_1)^{(9)} - (v_0)^{(9)}}{(v_0)^{(9)} - (v_2)^{(9)}}}$$

it follows $(v_0)^{(9)} \le v^{(9)}(t) \le (v_9)^{(9)}$

In the same manner , we get

$$v^{(9)}(t) \le \frac{(\bar{v}_1)^{(9)} + (\bar{C})^{(9)}(\bar{v}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{v}_1)^{(9)} - (\bar{v}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{v}_1)^{(9)} - (\bar{v}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{v}_1)^{(9)} - (v_0)^{(9)}}{(v_0)^{(9)} - (\bar{v}_2)^{(9)}}}$$

From which we deduce $(v_0)^{(9)} \le v^{(9)}(t) \le (\bar{v}_1)^{(9)}$

If $0 < (v_1)^{(9)} < (v_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}} < (\bar{v}_1)^{(9)}$ we find like in the previous case,

$$(v_1)^{(9)} \le \frac{(v_1)^{(9)} + (C)^{(9)}(v_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((v_1)^{(9)} - (v_2)^{(9)}\right)t\right]}}{1 + (C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((v_1)^{(9)} - (v_2)^{(9)}\right)t\right]}} \le v^{(9)}(t) \le$$

$$\frac{(\bar{v}_1)^{(9)} + (\bar{C})^{(9)}(\bar{v}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{v}_1)^{(9)} - (\bar{v}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{v}_1)^{(9)} - (\bar{v}_2)^{(9)}\right)t\right]}} \le (\bar{v}_1)^{(9)}$$

If $0 < (v_1)^{(9)} \le (\bar{v}_1)^{(9)} \le \boxed{(v_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$ , we obtain

$$(v_1)^{(9)} \le v^{(9)}(t) \le \frac{(\bar{v}_1)^{(9)} + (\bar{C})^{(9)}(\bar{v}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{v}_1)^{(9)} - (\bar{v}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{v}_1)^{(9)} - (\bar{v}_2)^{(9)}\right)t\right]}} \le (v_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $v^{(9)}(t)$ :-

$$(m_2)^{(9)} \le v^{(9)}(t) \le (m_1)^{(9)}, \quad \boxed{v^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(9)}(t)$ :-

$(\mu_2)^{(9)} \leq u^{(9)}(t) \leq (\mu_1)^{(9)}$, $\boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)} G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then
$(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)} T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following 425

**Theorem :** If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on $t$ , and the conditions with the notations

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0$ ,

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$

$with\ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a''_i)^{(2)} and\ (b''_i)^{(2)}$ are independent on t , and the conditions with the notations 426

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$ 427

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 428

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$ , 429

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 430

$with\ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a''_i)^{(3)} and\ (b''_i)^{(3)}$ are independent on $t$ , and the conditions with the notations 431

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$ ,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

**Theorem :** If $(a_i'')^{(4)} and\ (b_i'')^{(4)}$ are independent on $t$ , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$ ,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(5)} and\ (b_i'')^{(5)}$ are independent on $t$ , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$ ,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

**Theorem** If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on $t$ , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$ ,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(7)} and\ (b_i'')^{(7)}$ are independent on $t$ , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$ ,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(8)} and\ (b_i'')^{(8)}$ are independent on $t$ , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$ ,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(9)} and\ (b_i'')^{(9)}$ are independent on $t$ , and the conditions (with the notations 436 A
45,46,27,28)

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$ ,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0$ 437

$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0$ 438

$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0$ 439

$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0$ 440

$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ]T_{14} = 0$ 441

$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0$ 442

has a unique positive solution , which is an equilibrium solution for the system

$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0$ 443

$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0$ 444

$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0$ 445

$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\ ]T_{16} = 0$ 446

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\,]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\,]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\,]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\,]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\,]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\big((G_{27})\big)\,]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\big((G_{27})\big)\,]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\big((G_{27})\big)\,]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})\,]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})\,]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})\,]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})\,]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})\,]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})\,]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0 \quad 479$$

$$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0 \quad 480$$

$$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0 \quad 481$$

$$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0 \quad 482$$

$$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0 \quad 483$$

$$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0 \quad 484$$

$$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0 \quad 484\text{A}$$

$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$

$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$

$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$

$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$

$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$

**Proof:** 485

(a) Indeed the first two equations have a nontrivial solution $G_{13}, G_{14}$ if

$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$

**Proof:** 486

(h) Indeed the first two equations have a nontrivial solution $G_{16}, G_{17}$ if

$\mathrm{F}(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$

**Proof:** 487

(a) Indeed the first two equations have a nontrivial solution $G_{20}, G_{21}$ if

$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$

**Proof:** 488

(a) Indeed the first two equations have a nontrivial solution $G_{24}, G_{25}$ if

$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$

**Proof:** 489

(a) Indeed the first two equations have a nontrivial solution $G_{28}, G_{29}$ if

$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$

**Proof:** 490

(a) Indeed the first two equations have a nontrivial solution $G_{32}, G_{33}$ if

$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) +$

$(a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$

**Proof:** 491

(a) Indeed the first two equations have a nontrivial solution $G_{36}, G_{37}$ if

$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$

**Proof:** 492

(a) Indeed the first two equations have a nontrivial solution $G_{40}, G_{41}$ if

$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$

**Proof:** 492 A

(a) Indeed the first two equations have a nontrivial solution $G_{44}, G_{45}$ if

$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$

**Definition and uniqueness of** $T^*_{14}$ :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique $T^*_{14}$ for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

**Definition and uniqueness of** $T^*_{17}$ :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique $T^*_{17}$ for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]}$$ 495

**Definition and uniqueness of** $T^*_{21}$ :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique $T^*_{21}$ for which $f(T^*_{21}) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

**Definition and uniqueness of** $T^*_{25}$ :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that

there exists a unique $T_{25}^{*}$ for which $f(T_{25}^{*})=0$. With this value , we obtain from the three first equations

$$G_{24}=\frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}=\frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T_{25}^{*})]}$$

**Definition and uniqueness of** $T_{29}^{*}$ :- 498

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(5)}(T_{29})$ being increasing, it follows that there exists a unique $T_{29}^{*}$ for which $f(T_{29}^{*})=0$. With this value , we obtain from the three first equations

$$G_{28}=\frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}=\frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T_{29}^{*})]}$$

**Definition and uniqueness of** $T_{33}^{*}$ :- 499

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(6)}(T_{33})$ being increasing, it follows that there exists a unique $T_{33}^{*}$ for which $f(T_{33}^{*})=0$. With this value , we obtain from the three first equations

$$G_{32}=\frac{(a_{32})^{(6)}G_{33}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^{*})]} \quad , \quad G_{34}=\frac{(a_{34})^{(6)}G_{33}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^{*})]}$$

**Definition and uniqueness of** $T_{37}^{*}$ :- 500

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(7)}(T_{37})$ being increasing, it follows that there exists a unique $T_{37}^{*}$ for which $f(T_{37}^{*})=0$. With this value , we obtain from the three first equations

$$G_{36}=\frac{(a_{36})^{(7)}G_{37}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^{*})]} \quad , \quad G_{38}=\frac{(a_{38})^{(7)}G_{37}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^{*})]}$$

**Definition and uniqueness of** $T_{41}^{*}$ :- 501

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique $T_{41}^{*}$ for which $f(T_{41}^{*})=0$. With this value , we obtain from the three first equations

$$G_{40}=\frac{(a_{40})^{(8)}G_{41}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T_{41}^{*})]} \quad , \quad G_{42}=\frac{(a_{42})^{(8)}G_{41}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T_{41}^{*})]}$$

**Definition and uniqueness of** $T_{45}^{*}$ :- 501 A

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(9)}(T_{45})$ being increasing, it follows that there exists a unique $T_{45}^{*}$ for which $f(T_{45}^{*})=0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)} G_{45}}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T^*_{45})]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)} G_{45}}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T^*_{45})]}$$

By the same argument, the equations admit solutions $G_{13}, G_{14}$ if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right] + (b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{14}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{14}$ such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions $G_{16}, G_{17}$ if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right] + (b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ 504
is a decreasing function in $G_{17}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{14}$ such that $\varphi((G_{19})^*) = 0$

By the same argument, the equations admit solutions $G_{20}, G_{21}$ if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right] + (b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{21}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{21}$ such that $\varphi((G_{23})^*) = 0$

By the same argument, the equations admit solutions $G_{24}, G_{25}$ if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right] + (b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{25}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{25}$ such that $\varphi((G_{27})^*) = 0$

By the same argument, the equations admit solutions $G_{28}, G_{29}$ if 507

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right] + (b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{29}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that

there exists a unique $G_{29}^{*}$ such that $\varphi((G_{31})^{*}) = 0$

By the same argument, the equations admit solutions $G_{32}, G_{33}$ if 508
$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$

$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right] + (b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{33}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{33}^{*}$ such that $\varphi(G_{35}{}^{*}) = 0$

By the same argument, the equations admit solutions $G_{36}, G_{37}$ if 509
$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$
$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right] + (b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{37}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{37}^{*}$ such that $\varphi(G_{39}{}^{*}) = 0$

By the same argument, the equations admit solutions $G_{40}, G_{41}$ if 510
$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$
$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right] + (b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{41}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{41}^{*}$ such that $\varphi(G_{43}{}^{*}) = 0$

By the same argument, the equations 92,93 admit solutions $G_{44}, G_{45}$ if
$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$

$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right] + (b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{45}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{45}^{*}$ such that $\varphi((G_{47})^{*}) = 0$

Finally we obtain the unique solution 511

$G_{14}^{*}$ given by $\varphi(G^{*}) = 0$ , $T_{14}^{*}$ given by $f(T_{14}^{*}) = 0$ and

$$G_{13}^{*} = \frac{(a_{13})^{(1)}G_{14}^{*}}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^{*})]} \quad , \quad G_{15}^{*} = \frac{(a_{15})^{(1)}G_{14}^{*}}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^{*})]}$$

$$T_{13}^{*} = \frac{(b_{13})^{(1)}T_{14}^{*}}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^{*})]} \quad , \quad T_{15}^{*} = \frac{(b_{15})^{(1)}T_{14}^{*}}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^{*})]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

$G_{17}^{*}$ given by $\varphi((G_{19})^{*}) = 0$ , $T_{17}^{*}$ given by $f(T_{17}^{*}) = 0$ and 512

$$G_{16}^{*} = \frac{(a_{16})^{(2)} G_{17}^{*}}{[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}^{*})]} \quad , \quad G_{18}^{*} = \frac{(a_{18})^{(2)} G_{17}^{*}}{[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}^{*})]} \tag{513}$$

$$T_{16}^{*} = \frac{(b_{16})^{(2)} T_{17}^{*}}{[(b'_{16})^{(2)} - (b''_{16})^{(2)}((G_{19})^{*})]} \quad , \quad T_{18}^{*} = \frac{(b_{18})^{(2)} T_{17}^{*}}{[(b'_{18})^{(2)} - (b''_{18})^{(2)}((G_{19})^{*})]} \tag{514}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

$G_{21}^{*}$ given by $\varphi((G_{23})^{*}) = 0$ , $T_{21}^{*}$ given by $f(T_{21}^{*}) = 0$ and

$$G_{20}^{*} = \frac{(a_{20})^{(3)} G_{21}^{*}}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^{*})]} \quad , \quad G_{22}^{*} = \frac{(a_{22})^{(3)} G_{21}^{*}}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^{*})]}$$

$$T_{20}^{*} = \frac{(b_{20})^{(3)} T_{21}^{*}}{[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}{}^{*})]} \quad , \quad T_{22}^{*} = \frac{(b_{22})^{(3)} T_{21}^{*}}{[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}{}^{*})]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

$G_{25}^{*}$ given by $\varphi(G_{27}) = 0$ , $T_{25}^{*}$ given by $f(T_{25}^{*}) = 0$ and

$$G_{24}^{*} = \frac{(a_{24})^{(4)} G_{25}^{*}}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}^{*} = \frac{(a_{26})^{(4)} G_{25}^{*}}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^{*})]}$$

$$T_{24}^{*} = \frac{(b_{24})^{(4)} T_{25}^{*}}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^{*})]} \quad , \quad T_{26}^{*} = \frac{(b_{26})^{(4)} T_{25}^{*}}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^{*})]} \tag{517}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

$G_{29}^{*}$ given by $\varphi((G_{31})^{*}) = 0$ , $T_{29}^{*}$ given by $f(T_{29}^{*}) = 0$ and

$$G_{28}^{*} = \frac{(a_{28})^{(5)} G_{29}^{*}}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}^{*} = \frac{(a_{30})^{(5)} G_{29}^{*}}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^{*})]}$$

$$T_{28}^{*} = \frac{(b_{28})^{(5)} T_{29}^{*}}{[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31})^{*})]} \quad , \quad T_{30}^{*} = \frac{(b_{30})^{(5)} T_{29}^{*}}{[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31})^{*})]} \tag{519}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

$G_{33}^{*}$ given by $\varphi((G_{35})^{*}) = 0$ , $T_{33}^{*}$ given by $f(T_{33}^{*}) = 0$ and

$$G_{32}^{*} = \frac{(a_{32})^{(6)} G_{33}^{*}}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^{*})]} \quad , \quad G_{34}^{*} = \frac{(a_{34})^{(6)} G_{33}^{*}}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^{*})]}$$

$$T_{32}^{*} = \frac{(b_{32})^{(6)} T_{33}^{*}}{[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35})^{*})]} \quad , \quad T_{34}^{*} = \frac{(b_{34})^{(6)} T_{33}^{*}}{[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35})^{*})]} \tag{521}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

$G_{37}^*$ given by $\varphi((G_{39})^*) = 0$ , $T_{37}^*$ given by $f(T_{37}^*) = 0$ and

$$G_{36}^* = \frac{(a_{36})^{(7)} G_{37}^*}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^*)]} \quad , \quad G_{38}^* = \frac{(a_{38})^{(7)} G_{37}^*}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^*)]}$$

$$T_{36}^* = \frac{(b_{36})^{(7)} T_{37}^*}{[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39})^*)]} \quad , \quad T_{38}^* = \frac{(b_{38})^{(7)} T_{37}^*}{[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39})^*)]}$$

Finally we obtain the unique solution 523

$G_{41}^*$ given by $\varphi((G_{43})^*) = 0$ , $T_{41}^*$ given by $f(T_{41}^*) = 0$ and

$$G_{40}^* = \frac{(a_{40})^{(8)} G_{41}^*}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42}^* = \frac{(a_{42})^{(8)} G_{41}^*}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

$$T_{40}^* = \frac{(b_{40})^{(8)} T_{41}^*}{[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43})^*)]} \quad , \quad T_{42}^* = \frac{(b_{42})^{(8)} T_{41}^*}{[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43})^*)]}$$

Finally we obtain the unique solution of 89 to 99 523 A

$G_{45}^*$ given by $\varphi((G_{47})^*) = 0$ , $T_{45}^*$ given by $f(T_{45}^*) = 0$ and

$$G_{44}^* = \frac{(a_{44})^{(9)} G_{45}^*}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46}^* = \frac{(a_{46})^{(9)} G_{45}^*}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

$$T_{44}^* = \frac{(b_{44})^{(9)} T_{45}^*}{[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47})^*)]} \quad , \quad T_{46}^* = \frac{(b_{46})^{(9)} T_{45}^*}{[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47})^*)]}$$

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**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

**Proof:** Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a''_{14})^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \frac{\partial (b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a'_{13})^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)}G^*_{13}\mathbb{T}_{14} \quad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a'_{14})^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)}G^*_{14}\mathbb{T}_{14} \quad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a'_{15})^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)}G^*_{15}\mathbb{T}_{14} \quad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b'_{13})^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)}T^*_{13}\mathbb{G}_j\right) \quad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b'_{14})^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)}T^*_{14}\mathbb{G}_j\right) \quad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b'_{15})^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T^*_{15}\mathbb{G}_j\right) \quad 530$$

**ASYMPTOTIC STABILITY ANALYSIS** 531

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(2)}$ and $(b''_i)^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G^*_i + \mathbb{G}_i \quad , T_i = T^*_i + \mathbb{T}_i \quad 532$$

$$\frac{\partial(a''_{17})^{(2)}}{\partial T_{17}}(T^*_{17}) = (q_{17})^{(2)} \quad , \frac{\partial(b''_i)^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \quad 533$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a'_{16})^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G^*_{16}\mathbb{T}_{17} \quad 534$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a'_{17})^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G^*_{17}\mathbb{T}_{17} \quad 535$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a'_{18})^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G^*_{18}\mathbb{T}_{17} \quad 536$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b'_{16})^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T^*_{16}\mathbb{G}_j\right) \quad 537$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b'_{17})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}T^*_{17}\mathbb{G}_j\right) \quad 538$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b'_{18})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}T^*_{18}\mathbb{G}_j\right) \quad 539$$

**ASYMPTOTIC STABILITY ANALYSIS** 540

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(3)}$ *and* $(b''_i)^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{21}'')^{(3)}}{\partial T_{21}}(T_{21}^*) = (q_{21})^{(3)} \quad , \frac{\partial (b_i'')^{(3)}}{\partial G_j}(\,(G_{23})^*\,) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\big((a_{20}')^{(3)} + (p_{20})^{(3)}\big)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G_{20}^*\mathbb{T}_{21} \quad 541$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\big((a_{21}')^{(3)} + (p_{21})^{(3)}\big)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G_{21}^*\mathbb{T}_{21} \quad 542$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\big((a_{22}')^{(3)} + (p_{22})^{(3)}\big)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G_{22}^*\mathbb{T}_{21} \quad 543$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\big((b_{20}')^{(3)} - (r_{20})^{(3)}\big)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\big(s_{(20)(j)}T_{20}^*\mathbb{G}_j\big) \quad 544$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\big((b_{21}')^{(3)} - (r_{21})^{(3)}\big)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\big(s_{(21)(j)}T_{21}^*\mathbb{G}_j\big) \quad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\big((b_{22}')^{(3)} - (r_{22})^{(3)}\big)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\big(s_{(22)(j)}T_{22}^*\mathbb{G}_j\big) \quad 546$$

**ASYMPTOTIC STABILITY ANALYSIS** 547

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}\ and\ (b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{25}'')^{(4)}}{\partial T_{25}}(T_{25}^*) = (q_{25})^{(4)} \quad , \frac{\partial (b_i'')^{(4)}}{\partial G_j}((G_{27})^*\,) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\big((a_{24}')^{(4)} + (p_{24})^{(4)}\big)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G_{24}^*\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\big((a_{25}')^{(4)} + (p_{25})^{(4)}\big)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G_{25}^*\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\big((a_{26}')^{(4)} + (p_{26})^{(4)}\big)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G_{26}^*\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\big((b_{24}')^{(4)} - (r_{24})^{(4)}\big)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\big(s_{(24)(j)}T_{24}^*\mathbb{G}_j\big) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\big((b_{25}')^{(4)} - (r_{25})^{(4)}\big)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\big(s_{(25)(j)}T_{25}^*\mathbb{G}_j\big) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T^*_{26}\mathbb{G}_j\right) \quad 554$$

**ASYMPTOTIC STABILITY ANALYSIS** 555

**Theorem 5:** If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(5)}\ and\ (b''_i)^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 556

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{29})^{(5)}}{\partial T_{29}}(T^*_{29}) = (q_{29})^{(5)} \ , \ \frac{\partial(b''_i)^{(5)}}{\partial G_j}((G_{31})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a'_{28})^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G^*_{28}\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a'_{29})^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G^*_{29}\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a'_{30})^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G^*_{30}\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b'_{28})^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T^*_{28}\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b'_{29})^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T^*_{29}\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b'_{30})^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T^*_{30}\mathbb{G}_j\right) \quad 562$$

**ASYMPTOTIC STABILITY ANALYSIS** 563

**Theorem 6:** If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(6)}\ and\ (b''_i)^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{33})^{(6)}}{\partial T_{33}}(T^*_{33}) = (q_{33})^{(6)} \ , \ \frac{\partial(b''_i)^{(6)}}{\partial G_j}((G_{35})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a'_{32})^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G^*_{32}\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a'_{33})^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G^*_{33}\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a'_{34})^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G^*_{34}\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b'_{32})^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T^*_{32}\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b'_{33})^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T^*_{33}\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b'_{34})^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T^*_{34}\mathbb{G}_j\right) \quad 570$$

**ASYMPTOTIC STABILITY ANALYSIS** 571

**Theorem 7:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 572

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{37})^{(7)}}{\partial T_{37}}(T^*_{37}) = (q_{37})^{(7)} \quad , \frac{\partial(b_i'')^{(7)}}{\partial G_j}\left((G_{39})^{**}\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a'_{36})^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G^*_{36}\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a'_{37})^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G^*_{37}\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a'_{38})^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G^*_{38}\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b'_{36})^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T^*_{36}\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b'_{37})^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T^*_{37}\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b'_{38})^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T^*_{38}\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

**ASYMPTOTIC STABILITY ANALYSIS**

**Theorem 8:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{41}'')^{(8)}}{\partial T_{41}}(T_{41}^*) = (q_{41})^{(8)} \quad , \frac{\partial (b_i'')^{(8)}}{\partial G_j}\left( (G_{43})^* \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a_{40}')^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G_{40}^*\mathbb{T}_{41} \quad (581)$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a_{41}')^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G_{41}^*\mathbb{T}_{41} \quad (582)$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a_{42}')^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G_{42}^*\mathbb{T}_{41} \quad (583)$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b_{40}')^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T_{40}^*\mathbb{G}_j\right) \quad (584)$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b_{41}')^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T_{41}^*\mathbb{G}_j\right) \quad (585)$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b_{42}')^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T_{42}^*\mathbb{G}_j\right) \quad (586)$$

(586 A)

**ASYMPTOTIC STABILITY ANALYSIS**

**Theorem 9:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}$ *and* $(b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.
**Proof:** Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{45}'')^{(9)}}{\partial T_{45}}(T_{45}^*) = (q_{45})^{(9)} \quad , \frac{\partial (b_i'')^{(9)}}{\partial G_j}\left( (G_{47})^* \right) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a_{44}')^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)}G_{44}^*\mathbb{T}_{45} \quad (586\,B)$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a_{45}')^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)}G_{45}^*\mathbb{T}_{45} \quad (586\,C)$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a_{46}')^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)}G_{46}^*\mathbb{T}_{45} \quad (586\,D)$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b_{44}')^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)}T_{44}^*\mathbb{G}_j\right) \quad (586\,E)$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b_{45}')^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)}T_{45}^*\mathbb{G}_j\right) \quad (586\,F)$$

$\frac{d\mathbb{T}_{46}}{dt} = -\left((b'_{46})^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T^*_{46}\mathbb{G}_j\right)$ 586

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**The characteristic equation of this system is** 587

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(q_{13})^{(1)}G^*_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)}T^*_{14} + (b_{14})^{(1)}s_{(13),(14)}T^*_{14}\right)$$

$$+\left(\left((\lambda)^{(1)} + (a'_{14})^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)}G^*_{13} + (a_{13})^{(1)}(q_{14})^{(1)}G^*_{14}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)}T^*_{14} + (b_{14})^{(1)}s_{(13),(13)}T^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((b'_{13})^{(1)} + (b'_{14})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(15)}T^*_{14} + (b_{14})^{(1)}s_{(13),(15)}T^*_{13}\right)\} = 0$$

$+$

$$(\lambda)^{(2)} + (b'_{18})^{(2)} - (r_{18})^{(2)}\{\left((\lambda)^{(2)} + (a'_{18})^{(2)} + (p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^*_{17} + (a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^*_{17}\right)$$

$$+\left(\left((\lambda)^{(2)} + (a'_{17})^{(2)} + (p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^*_{16} + (a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}\right)$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2 + \left((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2 + \left((b'_{16})^{(2)} + (b'_{17})^{(2)} - (r_{16})^{(2)} + (r_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$+\left(\left((\lambda)^{(2)}\right)^2 + \left((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17} + (a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^{*}_{16}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\right)\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left(\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)\right]$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(21)}T^{*}_{21}\right)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^{*}_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^{*}_{21}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(20)}T^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$+\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(22)}T^{*}_{20}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\right)\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left(\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)\right]$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(25)}T^{*}_{25}\right)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^{*}_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^{*}_{25}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(24)}T^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$+\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(26)}T^{*}_{24}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\right)\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(29)}T^{*}_{29}\right)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^{*}_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^{*}_{29}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(28)}T^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$+\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(30)}T^{*}_{28}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\right)\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left(\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)\right]$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(33)}T^{*}_{33}\right)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^{*}_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^{*}_{33}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(32)}T^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$+\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(34)}T^{*}_{32}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\right)\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)\right]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(37)}T^{*}_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^{*}_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^{*}_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(36)}T^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$+\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(38)}T^{*}_{36}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\right)\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(41)}T^{*}_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^{*}_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^{*}_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(40)}T^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$+\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(42)}T^{*}_{40}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\right)\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(45)}T^{*}_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^{*}_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^{*}_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(44)}T^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$+\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(46)}T^{*}_{44}\right)\}=0$$

**And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.**

## SECTION TEN

## Electrical Conductivity Of Microtubules

### INTRODUCTION—VARIABLES USED

**Experimental evaluation of electrical conductivity of microtubules M. Umnov, O. A. Palusinski , P. A. Deymier, R. Guzman, J. Hoying, H. Barnaby, Y. Yang, S. Raghavan Journal of Materials Science January 2007, Volume 42, Issue 1, pp 373-378**

(1) Microtubules (MTs) are (=) self-assembled proteinaceous filaments with nanometer scale diameters

and micrometer scale lengths.

(2) Through conductivity measurements in microchannels authors shed some light on (eb) electrical properties of microtubules.

(3) Measuring electrical resistance they were able to detect (e&eb) the dynamic disassembly of MTs and determine (eb) an upper limit for the electrical conductivity of MT.

(4) The measurements yield (eb) the value of 90 Ω−1 m−1 as (=) the upper limit for the conductivity of MTs, which is in the order of conductivity observed in (eb) inorganic intrinsic semiconductors.

**A critical assessment of the information processing capabilities of neuronal microtubules using coherent excitations Travis John Adrian Craddock, Jack A. Tuszynski Journal of Biological Physics January 2010, Volume 36, Issue 1, pp 53-70 First online: 04 June 2009**

(5) Evidence for signaling, communication, and conductivity in microtubules (MTs) has been shown through both direct and indirect means, and theoretical models predict their potential use in both classical and quantum information processing in (eb) neurons.

(6) The notion of quantum information processing within neurons has been implicated in the phenomena of (e) consciousness; although controversies have arisen in regards to (e&eb) adverse physiological temperature effects on (e&eb) these capabilities.

(7) To investigate the possibility of quantum processes in relation to (e&eb) information processing in MTs, a biophysical MT model is used based on (e) the electrostatic interior of the tubulin protein.

(8) The interior is taken to constitute (e) a double-well potential structure within (eb) which a mobile electron is considered capable of occupying (e) at least two distinct quantum states.

(9) These excitonic states together with MT lattice vibrations determine (eb) the state space of individual tubulin dimers within (eb) the MT lattice.

**(10)** Tubulin dimers are taken as (=) quantum well structures containing (e) an electron that can exist in (eb) either its ground state or first excited state.

## NOTATION

### Module One

Microtubules (MTs) are (=) self-assembled proteinaceous filaments with nanometer scale diameters and micrometer scale lengths

$G_{13}$ : Category one of Microtubules (MTs)

$G_{14}$ : Category two of SAS

$G_{15}$ : Category three of SAS

$T_{13}$ : Category one of self-assembled proteinaceous filaments with nanometer scale diameters and micrometer scale lengths

$T_{14}$ : Category two of SAS

$T_{15}$ : Category three of SAS

### Module Two

Through conductivity measurements in microchannels authors shed some light on (eb) electrical properties of microtubules

$G_{16}$ : Category one of conductivity measurements in microchannels

$G_{17}$ : Category two of SAS

$G_{18}$ : Category three of SAS

$T_{16}$ : Category one of electrical properties of microtubules

$T_{17}$ : Category two of SAS

$T_{18}$ : Category three of SAS

**Module three**

electrical properties of microtubules

$G_{20}$ : Category one of microtubules

$G_{21}$ : Category two of SAS

$G_{22}$ : Category three of SAS

$T_{20}$ : Category one of electrical properties

$T_{21}$ : Category two of SAS

$T_{22}$ : Category three of SAS

**Module four**

Measuring electrical resistance they were able to detect (e&eb) the dynamic disassembly of MTs and determine (eb) an upper limit for the electrical conductivity of MT.

$G_{24}$ : Category one of electrical resistance measurement

$G_{25}$ : Category two of SAS

$G_{26}$ : Category three of SAS

$T_{24}$ : Category one of detection of (e&eb) the dynamic disassembly of MTs and determine (eb) an upper limit for the electrical conductivity of MT.

$T_{25}$ : Category two of SAS

$T_{26}$ : Category three of SAS

**Module five**

Measuring electrical resistance they were able to detect the dynamic disassembly of MTs and determine (eb) an upper limit for the electrical conductivity of MT.

$G_{28}$ : Category one of **Measuring electrical resistance;** detect the dynamic disassembly of MTs and determine (eb) an upper limit for the electrical conductivity of MT.

$G_{29}$ : Category two of SAS

$G_{30}$ : Category three of SAS

$T_{28}$ : Category one of detect the dynamic disassembly of MTs and determine (eb) an upper limit for the electrical conductivity of MT.; **Measuring electrical resistance**

$T_{29}$ : Category two of SAS

$T_{30}$ : Category three of SAS

**Module six**

Measuring electrical resistance they were able to detect the dynamic disassembly of MTs and determine (eb) an upper limit for the electrical conductivity of MT

$G_{32}$ : Category one of dynamic disassembly of MTs

$G_{33}$ : Category two of SAS

$G_{34}$ : Category three of SAS

$T_{32}$ : Category one of upper limit for the electrical conductivity of MT

$T_{33}$ : Category two of SAS

$T_{34}$ : Category three of SAS

**Module seven**

Evidence for signaling, communication, and conductivity in microtubules (MTs) has been shown through both direct and indirect means, and theoretical models predict their potential use in both classical and quantum information processing in (eb) **neurons**

$G_{36}$ : Category one of Evidence for signaling, communication, and conductivity in microtubules (MTs) has been shown through both direct and indirect means, and theoretical models predict their potential use in both classical and quantum information processing; **neurons**

$G_{37}$ : Category two of SAS

$G_{38}$ : Category three of SAS

$T_{36}$ : Category one of **neurons** ; Evidence for signaling, communication, and conductivity in microtubules (MTs) has been shown through both direct and indirect means, and theoretical models predict their potential use in both classical and quantum information processing

$T_{37}$ : Category two of SAS

$T_{38}$ : Category three of SAS

**Module eight**

The notion of quantum information processing within neurons has been implicated in the phenomena of (e) consciousness; although controversies have arisen in regards to (e&eb) adverse physiological temperature effects on (e&eb) these capabilities

$G_{40}$ : Category one of quantum information processing within neurons; phenomena of consciousness

$G_{41}$ : Category two of SAS

$G_{42}$ : Category three of SAS

$T_{40}$ : Category one of phenomena of consciousness ; quantum information processing within neurons

$T_{41}$ : Category two of SAS

$T_{42}$ : Category three of SAS

**Module Nine**

The notion of quantum information processing within neurons has been implicated in the phenomena of consciousness; although controversies have arisen in regards to (e&eb) **adverse physiological temperature effects** on (e&eb) these capabilities

$G_{44}$ : Category one of **adverse physiological temperature effects;** quantum information processing within neurons

$G_{45}$ : Category two of SAS

$G_{46}$ : Category three of SAS

$T_{44}$ : Category one of quantum information processing within neurons; **adverse physiological temperature effects**

$T_{45}$ : Category two of SAS

$T_{46}$ : Category three of SAS

**The Coefficients:**

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}$ , $(b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}$,
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}$, $(b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}\ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$ ,
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)}$,
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)}$,
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)}$,
are Dissipation coefficients

**Module Numbered One**

**The differential system of this model is now (Module Numbered one)**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14},t)\right]G_{13} \quad 1$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14},t)\right]G_{14} \quad 2$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14},t)\right]G_{15} \quad 3$$

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G,t)\right]T_{13} \quad 4$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G,t)\right]T_{14} \quad 5$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G,t)\right]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14},t) =$ First augmentation factor

$-(b''_{13})^{(1)}(G,t) =$ First detritions factor

**Module Numbered Two**

**The differential system of this model is now ( Module numbered two)**

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17},t)\right]G_{16} \quad 7$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17},t)\right]G_{17} \quad 8$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17},t)\right]G_{18} \quad 9$$

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\big((G_{19}),t\big)\right]T_{16} \quad 10$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\big((G_{19}),t\big)\right]T_{17} \quad 11$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\big((G_{19}),t\big)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17},t) =$ First augmentation factor

$-(b''_{16})^{(2)}\big((G_{19}),t\big) =$ First detritions factor

**Module Numbered Three**

**The differential system of this model is now (Module numbered three)**

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21},t)\right]G_{20} \quad 13$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21},t)\right]G_{21} \quad 14$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21},t)\right]G_{22} \quad 15$$

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23},t)\right]T_{20} \quad 16$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23},t)\right]T_{21} \quad 17$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23},t)\right]T_{22} \quad 18$$

$+(a''_{20})^{(3)}(T_{21},t) =$ First augmentation factor

$-(b''_{20})^{(3)}(G_{23},t) =$ First detritions factor

**Module Numbered Four**

**The differential system of this model is now (Module numbered Four)**

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right]G_{24} \quad 19$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25},t)\right]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}, t)\right]G_{26} \quad 21$$
$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27}), t)\right]T_{24} \quad 22$$
$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}((G_{27}), t)\right]T_{25} \quad 23$$
$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27}), t)\right]T_{26} \quad 24$$
$+(a''_{24})^{(4)}(T_{25}, t) =$ **First augmentation factor**
$-(b''_{24})^{(4)}((G_{27}), t) =$ **First detritions factor**

**Module Numbered Five**:

**The differential system of this model is now (Module number five)**

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right]G_{28} \quad 25$$
$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}, t)\right]G_{29} \quad 26$$
$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}, t)\right]G_{30} \quad 27$$
$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31}), t)\right]T_{28} \quad 28$$
$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}((G_{31}), t)\right]T_{29} \quad 29$$
$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31}), t)\right]T_{30} \quad 30$$
$+(a''_{28})^{(5)}(T_{29}, t) =$ **First augmentation factor**
$-(b''_{28})^{(5)}((G_{31}), t) =$ **First detritions factor**

**Module Numbered Six**

**The differential system of this model is now (Module numbered Six)**

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right]G_{32} \quad 31$$
$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}, t)\right]G_{33} \quad 32$$
$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}, t)\right]G_{34} \quad 33$$
$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35}), t)\right]T_{32} \quad 34$$
$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}((G_{35}), t)\right]T_{33} \quad 35$$
$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35}), t)\right]T_{34} \quad 36$$
$+(a''_{32})^{(6)}(T_{33}, t) =$ **First augmentation factor**

**Module Numbered Seven:**

**The differential system of this model is now (Seventh Module)**

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right]G_{36} \quad 37$$
$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37}, t)\right]G_{37} \quad 38$$
$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}, t)\right]G_{38} \quad 39$$
$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39}), t)\right]T_{36} \quad 40$$
$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}((G_{39}), t)\right]T_{37} \quad 41$$
$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39}), t)\right]T_{38} \quad 42$$
$+(a''_{36})^{(7)}(T_{37}, t) =$ **First augmentation factor**

**Module Numbered Eight**

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right]G_{40} \quad 43$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41},t)\right]G_{41} \quad 44$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41},t)\right]G_{42} \quad 45$$

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43}),t)\right]T_{40} \quad 46$$

$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}((G_{43}),t)\right]T_{41} \quad 47$$

$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43}),t)\right]T_{42} \quad 48$$

**Module Numbered Nine**

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47}),t)\right]T_{44} \quad 52$$

$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}((G_{47}),t)\right]T_{45} \quad 53$$

$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47}),t)\right]T_{46} \quad 54$$

$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}((G_{47}),t) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)} & +(a''_{13})^{(1)}(T_{14},t) & +(a''_{16})^{(2,2,)}(T_{17},t) & +(a''_{20})^{(3,3,)}(T_{21},t) \\ +(a''_{24})^{(4,4,4,4,)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{36})^{(7,7)}(T_{37},t) & +(a''_{40})^{(8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)} & +(a''_{14})^{(1)}(T_{14},t) & +(a''_{17})^{(2,2,)}(T_{17},t) & +(a''_{21})^{(3,3,)}(T_{21},t) \\ +(a''_{25})^{(4,4,4,4,)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{37})^{(7,7)}(T_{37},t) & +(a''_{41})^{(8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)} & +(a''_{15})^{(1)}(T_{14},t) & +(a''_{18})^{(2,2,)}(T_{17},t) & +(a''_{22})^{(3,3,)}(T_{21},t) \\ +(a''_{26})^{(4,4,4,4,)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{38})^{(7,7)}(T_{37},t) & +(a''_{42})^{(8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{15} \quad 57$$

Where $(a''_{13})^{(1)}(T_{14},t)$, $(a''_{14})^{(1)}(T_{14},t)$, $(a''_{15})^{(1)}(T_{14},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,)}(T_{17},t)$, $+(a''_{17})^{(2,2,)}(T_{17},t)$, $+(a''_{18})^{(2,2,)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,)}(T_{21},t)$, $+(a''_{21})^{(3,3,)}(T_{21},t)$, $+(a''_{22})^{(3,3,)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{38})^{(7,7)}(T_{37},t)$ $+(a''_{37})^{(7,7)}(T_{37},t)$ $+(a''_{36})^{(7,7)}(T_{37},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{40})^{(8,8)}(T_{41},t)$ $+(a''_{41})^{(8,8)}(T_{41},t)$ $+(a''_{42})^{(8,8)}(T_{41},t)$ are eight augmentation coefficient for 1,2,3

$+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)} & -(b''_{13})^{(1)}(G,t) & -(b''_{16})^{(2,2,)}(G_{19},t) & -(b''_{20})^{(3,3,)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{36})^{(7,7,)}(G_{39},t) & -(b''_{40})^{(8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{13} \quad 58$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)} & -(b''_{14})^{(1)}(G,t) & -(b''_{17})^{(2,2,)}(G_{19},t) & -(b''_{21})^{(3,3,)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{37})^{(7,7,)}(G_{39},t) & -(b''_{41})^{(8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{14} \quad 59$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)} & -(b''_{15})^{(1)}(G,t) & -(b''_{18})^{(2,2,)}(G_{19},t) & -(b''_{22})^{(3,3,)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{38})^{(7,7,)}(G_{39},t) & -(b''_{42})^{(8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{15} \quad 60$$

Where $-(b''_{13})^{(1)}(G,t)$, $-(b''_{14})^{(1)}(G,t)$, $-(b''_{15})^{(1)}(G,t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,)}(G_{19},t)$, $-(b''_{17})^{(2,2,)}(G_{19},t)$, $-(b''_{18})^{(2,2,)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{37})^{(7,7,)}(G_{39},t)$, $-(b''_{36})^{(7,7,)}(G_{39},t)$, $-(b''_{38})^{(7,7,)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8)}(G_{43},t)$ $-(b''_{41})^{(8,8)}(G_{43},t)$ $-(b''_{42})^{(8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[\begin{array}{ccc} (a'_{16})^{(2)} & \boxed{+(a''_{16})^{(2)}(T_{17},t)} & \boxed{+(a''_{13})^{(1,1,)}(T_{14},t)} \quad \boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9)}(T_{45},t)} \end{array}\right] G_{16} \quad (61)$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[\begin{array}{ccc} (a'_{17})^{(2)} & \boxed{+(a''_{17})^{(2)}(T_{17},t)} & \boxed{+(a''_{14})^{(1,1,)}(T_{14},t)} \quad \boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9)}(T_{45},t)} \end{array}\right] G_{17} \quad (62)$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[\begin{array}{ccc} (a'_{18})^{(2)} & \boxed{+(a''_{18})^{(2)}(T_{17},t)} & \boxed{+(a''_{15})^{(1,1,)}(T_{14},t)} \quad \boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9)}(T_{45},t)} \end{array}\right] G_{18} \quad (63)$$

Where $\boxed{+(a''_{16})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,)}(T_{14},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)}$ are eight augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{44})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9)}(T_{45},t)}$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[\begin{array}{ccc} (b'_{16})^{(2)} & \boxed{-(b''_{16})^{(2)}(G_{19},t)} & \boxed{-(b''_{13})^{(1,1,)}(G,t)} \quad \boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9)}(G_{47},t)} \end{array}\right] T_{16} \quad (64)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[\begin{array}{ccc} (b'_{17})^{(2)} & \boxed{-(b''_{17})^{(2)}(G_{19},t)} & \boxed{-(b''_{14})^{(1,1,)}(G,t)} \quad \boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9)}(G_{47},t)} \end{array}\right] T_{17} \quad (65)$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[\begin{array}{ccc} (b'_{18})^{(2)} & \boxed{-(b''_{18})^{(2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,)}(G,t)} \quad \boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9)}(G_{47},t)} \end{array}\right] T_{18} \quad (66)$$

where $-(b''_{16})^{(2)}(G_{19},t)$ , $-(b''_{17})^{(2)}(G_{19},t)$ , $-(b''_{18})^{(2)}(G_{19},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{13})^{(1,1,)}(G,t)$, $-(b''_{14})^{(1,1,)}(G,t)$, $-(b''_{15})^{(1,1,)}(G,t)$ are second detrition coefficients for category 1,2 and 3

$-(b''_{20})^{(3,3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,)}(G_{23},t)$ are third detrition coefficients for category 1,2 and 3

$-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1,2 and 3

$-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1,2 and 3

$-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients for category 1,2 and 3

$-(b''_{36})^{(7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2 and 3

$-(b''_{40})^{(8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1,2 and 3

$-(b''_{44})^{(9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[\begin{array}{c}(a'_{20})^{(3)}\left[+(a''_{20})^{(3)}(T_{21},t)\right]\left[+(a''_{16})^{(2,2,2)}(T_{17},t)\right]\left[+(a''_{13})^{(1,1,1,)}(T_{14},t)\right] \\ \left[+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)\right]\left[+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)\right]\left[+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)\right] \\ \left[+(a''_{36})^{(7,7,7,7)}(T_{37},t)\right]\left[+(a''_{40})^{(8,8,8,8)}(T_{41},t)\right]\left[+(a''_{44})^{(9,9,9)}(T_{45},t)\right]\end{array}\right]G_{20} \quad (67)$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[\begin{array}{c}(a'_{21})^{(3)}\left[+(a''_{21})^{(3)}(T_{21},t)\right]\left[+(a''_{17})^{(2,2,2)}(T_{17},t)\right]\left[+(a''_{14})^{(1,1,1,)}(T_{14},t)\right] \\ \left[+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)\right]\left[+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)\right]\left[+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)\right] \\ \left[+(a''_{37})^{(7,7,7,7)}(T_{37},t)\right]\left[+(a''_{41})^{(8,8,8,8)}(T_{41},t)\right]\left[+(a''_{45})^{(9,9,9)}(T_{45},t)\right]\end{array}\right]G_{21} \quad (68)$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[\begin{array}{c}(a'_{22})^{(3)}\left[+(a''_{22})^{(3)}(T_{21},t)\right]\left[+(a''_{18})^{(2,2,2)}(T_{17},t)\right]\left[+(a''_{15})^{(1,1,1,)}(T_{14},t)\right] \\ \left[+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)\right]\left[+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)\right]\left[+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)\right] \\ \left[+(a''_{38})^{(7,7,7,7)}(T_{37},t)\right]\left[+(a''_{42})^{(8,8,8,8)}(T_{41},t)\right]\left[+(a''_{46})^{(9,9,9)}(T_{45},t)\right]\end{array}\right]G_{22} \quad (69)$$

$+(a''_{20})^{(3)}(T_{21},t)$, $+(a''_{21})^{(3)}(T_{21},t)$, $+(a''_{22})^{(3)}(T_{21},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2)}(T_{17},t)$ are second augmentation coefficients for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,)}(T_{14},t)$ are third augmentation coefficients for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficients for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficients for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficients for category 1, 2 and 3

$+(a''_{36})^{(7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1, 2 and 3

$+(a''_{40})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8)}(T_{41},t)$ are eight augmentation coefficients for category 1, 2 and 3

$+(a''_{44})^{(9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b'_{20})^{(3)} & -(b''_{20})^{(3)}(G_{23},t) & -(b''_{16})^{(2,2,2)}(G_{19},t) & -(b''_{13})^{(1,1,1,)}(G,t) \\ -(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{36})^{(7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{20} \quad (70)$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b'_{21})^{(3)} & -(b''_{21})^{(3)}(G_{23},t) & -(b''_{17})^{(2,2,2)}(G_{19},t) & -(b''_{14})^{(1,1,1,)}(G,t) \\ -(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{37})^{(7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{21} \quad (71)$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)} & -(b''_{22})^{(3)}(G_{23},t) & -(b''_{18})^{(2,2,2)}(G_{19},t) & -(b''_{15})^{(1,1,1,)}(G,t) \\ -(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{38})^{(7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{22} \quad (72)$$

$-(b''_{20})^{(3)}(G_{23},t)$, $-(b''_{21})^{(3)}(G_{23},t)$, $-(b''_{22})^{(3)}(G_{23},t)$ *are first detrition coefficients for category 1, 2 and 3*

$-(b''_{16})^{(2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2)}(G_{19},t)$ *are second detrition coefficients for category 1, 2 and 3*

$-(b''_{13})^{(1,1,1,)}(G,t)$, $-(b''_{14})^{(1,1,1,)}(G,t)$, $-(b''_{15})^{(1,1,1,)}(G,t)$ *are third detrition coefficients for category 1,2 and 3*

$-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)$ *are fourth detrition coefficients for category 1, 2 and 3*

$-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)$ *are fifth detrition coefficients for category 1, 2 and 3*

$-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)$ *are sixth detrition coefficients for category 1, 2 and 3*

$-(b''_{36})^{(7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$-(b''_{46})^{(9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \begin{bmatrix} (a'_{24})^{(4)} & +(a''_{24})^{(4)}(T_{25},t) & +(a''_{28})^{(5,5,)}(T_{29},t) & +(a''_{32})^{(6,6,)}(T_{33},t) \\ +(a''_{13})^{(1,1,1,1)}(T_{14},t) & +(a''_{16})^{(2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3)}(T_{21},t) \\ +(a''_{36})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9)}(T_{45},t) \end{bmatrix} G_{24} \quad (73)$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \begin{bmatrix} (a'_{25})^{(4)} & +(a''_{25})^{(4)}(T_{25},t) & +(a''_{29})^{(5,5,)}(T_{29},t) & +(a''_{33})^{(6,6)}(T_{33},t) \\ +(a''_{14})^{(1,1,1,1)}(T_{14},t) & +(a''_{17})^{(2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3)}(T_{21},t) \\ +(a''_{37})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9)}(T_{45},t) \end{bmatrix} G_{25} \quad (74)$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \begin{bmatrix} (a'_{26})^{(4)} & +(a''_{26})^{(4)}(T_{25},t) & +(a''_{30})^{(5,5,)}(T_{29},t) & +(a''_{34})^{(6,6,)}(T_{33},t) \\ +(a''_{15})^{(1,1,1,1)}(T_{14},t) & +(a''_{18})^{(2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3)}(T_{21},t) \\ +(a''_{38})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9)}(T_{45},t) \end{bmatrix} G_{26} \quad (75)$$

$+(a''_{24})^{(4)}(T_{25},t)$, $+(a''_{25})^{(4)}(T_{25},t)$, $+(a''_{26})^{(4)}(T_{25},t)$ *are first augmentation coefficients category* 1, 2 3

$+(a''_{28})^{(5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,)}(T_{29},t)$ *are second augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{32})^{(6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,)}(T_{33},t)$ *are third augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{13})^{(1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1)}(T_{14},t)$ *are fourth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{16})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2)}(T_{17},t)$ *are fifth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{20})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3)}(T_{21},t)$ *are sixth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)$ *are seventh augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)$ *are eighth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{46})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9)}(T_{45},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \begin{bmatrix} (b'_{24})^{(4)} & -(b''_{24})^{(4)}(G_{27},t) & -(b''_{28})^{(5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,)}(G_{35},t) \\ -(b''_{13})^{(1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3)}(G_{23},t) \\ -(b''_{36})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{24} \quad (76)$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \begin{bmatrix} (b'_{25})^{(4)} & -(b''_{25})^{(4)}(G_{27},t) & -(b''_{29})^{(5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,)}(G_{35},t) \\ -(b''_{14})^{(1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3)}(G_{23},t) \\ -(b''_{37})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{25} \quad (77)$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)} & -(b''_{26})^{(4)}(G_{27},t) & -(b''_{30})^{(5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,)}(G_{35},t) \\ -(b''_{15})^{(1,1,1,1)}(G,t) & -(b''_{18})^{(2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3)}(G_{23},t) \\ -(b''_{38})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{26} \quad (78)$$

$Where\ \boxed{-(b''_{24})^{(4)}(G_{27},t)}, \boxed{-(b''_{25})^{(4)}(G_{27},t)}, \boxed{-(b''_{26})^{(4)}(G_{27},t)}\ are\ first\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b''_{28})^{(5,5,)}(G_{31},t)}, \boxed{-(b''_{29})^{(5,5,)}(G_{31},t)}, \boxed{-(b''_{30})^{(5,5,)}(G_{31},t)}\ are\ second\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b''_{32})^{(6,6,)}(G_{35},t)}, \boxed{-(b''_{33})^{(6,6,)}(G_{35},t)}, \boxed{-(b''_{34})^{(6,6,)}(G_{35},t)}\ are\ third\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b''_{13})^{(1,1,1,1)}(G,t)}, \boxed{-(b''_{14})^{(1,1,1,1)}(G,t)}, \boxed{-(b''_{15})^{(1,1,1,1)}(G,t)}$
$are\ fourth\ \ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2,2)}(G_{19},t)}$
$are\ fifth\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3,3,3,3)}(G_{23},t)}$
$are\ sixth\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)}$
$are\ seventh\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)}$
$are\ eighth\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b''_{46})^{(9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{45})^{(9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{44})^{(9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[\begin{array}{c} (a'_{28})^{(5)}\boxed{+(a''_{28})^{(5)}(T_{29},t)}\ \boxed{+(a''_{24})^{(4,4,)}(T_{25},t)}\ \boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)}\ \boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)}\ \boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{28} \quad 79$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[\begin{array}{c} (a'_{29})^{(5)}\boxed{+(a''_{29})^{(5)}(T_{29},t)}\ \boxed{+(a''_{25})^{(4,4,)}(T_{25},t)}\ \boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)}\ \boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)}\ \boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{29} \quad 80$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[\begin{array}{c} (a'_{30})^{(5)}\boxed{+(a''_{30})^{(5)}(T_{29},t)}\ \boxed{+(a''_{26})^{(4,4,)}(T_{25},t)}\ \boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)}\ \boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)}\ \boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{30} \quad 81$$

$Where\ \boxed{+(a''_{28})^{(5)}(T_{29},t)}, \boxed{+(a''_{29})^{(5)}(T_{29},t)}, \boxed{+(a''_{30})^{(5)}(T_{29},t)}\ are\ first\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$

$And\ \boxed{+(a''_{24})^{(4,4,)}(T_{25},t)}, \boxed{+(a''_{25})^{(4,4,)}(T_{25},t)}, \boxed{+(a''_{26})^{(4,4,)}(T_{25},t)}\ are\ second\ augmentation\ coefficient\ for\ category\ 1,2\ and\ 3$

$\boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)}, \boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)}, \boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)}\ are\ third\ \ augmentation\ coefficient\ for\ category\ 1,2\ and\ 3$

$\boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)}$ are fourth augmentation coefficients for category 1,2, and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)}$ are fifth augmentation coefficients for category 1,2,and 3

$+(a_{20}'')^{(3,3,3,3,3)}(T_{21},t)$, $+(a_{21}'')^{(3,3,3,3,3)}(T_{21},t)$, $+(a_{22}'')^{(3,3,3,3,3)}(T_{21},t)$ are sixth augmentation coefficients for category 1,2, 3

$+(a_{36}'')^{(7,7,7,7,7,7)}(T_{37},t)$ , $+(a_{37}'')^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a_{38}'')^{(7,7,7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1,2, 3

$+(a_{40}'')^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a_{41}'')^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a_{42}'')^{(8,8,8,8,8,8)}(T_{41},t)$ are eighth augmentation coefficients for category 1,2, 3

$+(a_{46}'')^{(9,9,9,9,9)}(T_{45},t)$, $+(a_{45}'')^{(9,9,9,9,9)}(T_{45},t)$, $+(a_{44}'')^{(9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \begin{bmatrix} (b_{28}')^{(5)} & -(b_{28}'')^{(5)}(G_{31},t) & -(b_{24}'')^{(4,4,)}(G_{27},t) & -(b_{32}'')^{(6,6,6)}(G_{35},t) \\ & -(b_{13}'')^{(1,1,1,1,1)}(G,t) & -(b_{16}'')^{(2,2,2,2,2)}(G_{19},t) & -(b_{20}'')^{(3,3,3,3,3)}(G_{23},t) \\ & -(b_{36}'')^{(7,7,7,7,7,7)}(G_{39},t) & -(b_{40}'')^{(8,8,8,8,8,8)}(G_{43},t) & -(b_{44}'')^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{28} \quad 82$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b_{29}')^{(5)} & -(b_{29}'')^{(5)}(G_{31},t) & -(b_{25}'')^{(4,4,)}(G_{27},t) & -(b_{33}'')^{(6,6,6)}(G_{35},t) \\ & -(b_{14}'')^{(1,1,1,1,1)}(G,t) & -(b_{17}'')^{(2,2,2,2,2)}(G_{19},t) & -(b_{21}'')^{(3,3,3,3,3)}(G_{23},t) \\ & -(b_{37}'')^{(7,7,7,7,7,7)}(G_{39},t) & -(b_{41}'')^{(8,8,8,8,8,8)}(G_{43},t) & -(b_{45}'')^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{29} \quad 83$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b_{30}')^{(5)} & -(b_{30}'')^{(5)}(G_{31},t) & -(b_{26}'')^{(4,4,)}(G_{27},t) & -(b_{34}'')^{(6,6,6)}(G_{35},t) \\ & -(b_{15}'')^{(1,1,1,1,1,)}(G,t) & -(b_{18}'')^{(2,2,2,2,2)}(G_{19},t) & -(b_{22}'')^{(3,3,3,3,3)}(G_{23},t) \\ & -(b_{38}'')^{(7,7,7,7,7,7)}(G_{39},t) & -(b_{42}'')^{(8,8,8,8,8,8)}(G_{43},t) & -(b_{46}'')^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{30} \quad 84$$

*where* $-(b_{28}'')^{(5)}(G_{31},t)$ , $-(b_{29}'')^{(5)}(G_{31},t)$ , $-(b_{30}'')^{(5)}(G_{31},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b_{24}'')^{(4,4,)}(G_{27},t)$, $-(b_{25}'')^{(4,4,)}(G_{27},t)$, $-(b_{26}'')^{(4,4,)}(G_{27},t)$ *are second detrition coefficients for category* 1,2 *and* 3

$-(b_{32}'')^{(6,6,6)}(G_{35},t)$, $-(b_{33}'')^{(6,6,6)}(G_{35},t)$, $-(b_{34}'')^{(6,6,6)}(G_{35},t)$ *are third detrition coefficients for category* 1,2 *and* 3

$-(b_{13}'')^{(1,1,1,1,1)}(G,t)$, $-(b_{14}'')^{(1,1,1,1,1)}(G,t)$ , $-(b_{15}'')^{(1,1,1,1,1,)}(G,t)$ are fourth detrition coefficients for category 1,2, and 3

$-(b_{16}'')^{(2,2,2,2,2)}(G_{19},t)$, $-(b_{17}'')^{(2,2,2,2,2)}(G_{19},t)$, $-(b_{18}'')^{(2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1,2, and 3

$-(b_{20}'')^{(3,3,3,3,3)}(G_{23},t)$, $-(b_{21}'')^{(3,3,3,3,3)}(G_{23},t)$, $-(b_{22}'')^{(3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1,2, and 3

$-(b_{36}'')^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b_{37}'')^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b_{38}'')^{(7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2, and 3

$-(b_{42}'')^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b_{41}'')^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b_{40}'')^{(8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1,2, and 3

$-(b_{46}'')^{(9,9,9,9,9)}(G_{47},t)$, $-(b_{45}'')^{(9,9,9,9,9)}(G_{47},t)$, $-(b_{44}'')^{(9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} \qquad 85$$

$$-\left[\begin{matrix}(a'_{32})^{(6)}\boxed{+(a''_{32})^{(6)}(T_{33},t)}\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}\\ \boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}\boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)}\boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)}\boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)}\end{matrix}\right]G_{32}$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[\begin{matrix}(a'_{33})^{(6)}\boxed{+(a''_{33})^{(6)}(T_{33},t)}\boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}\boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}\\ \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}\boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}\boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)}\boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}\boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)}\end{matrix}\right]G_{33} \qquad 86$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[\begin{matrix}(a'_{34})^{(6)}\boxed{+(a''_{34})^{(6)}(T_{33},t)}\boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}\boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}\\ \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}\boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}\boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)}\boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}\boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)}\end{matrix}\right]G_{34} \qquad 87$$

$\boxed{+(a''_{32})^{(6)}(T_{33},t)}, \boxed{+(a''_{33})^{(6)}(T_{33},t)}, \boxed{+(a''_{34})^{(6)}(T_{33},t)}$ *are first augmentation coefficients for category* $1, 2$ *and* $3$

$\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}$ *are second augmentation coefficients for category* $1, 2$ *and* $3$

$\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}$ *are third augmentation coefficients for category* $1, 2$ *and* $3$

$\boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}$ - are fourth augmentation coefficients

$\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}$ - fifth augmentation coefficients

$\boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)}$ sixth augmentation coefficients

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)}$ seventh augmentation coefficients

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}$ Eighth augmentation coefficients

$\boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)}$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[\begin{matrix}(b'_{32})^{(6)}\boxed{-(b''_{32})^{(6)}(G_{35},t)}\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}\\ \boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)}\boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)}\\ \boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)}\boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)}\end{matrix}\right]T_{32} \qquad 88$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[\begin{matrix}(b'_{33})^{(6)}\boxed{-(b''_{33})^{(6)}(G_{35},t)}\boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}\boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}\\ \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}\boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)}\\ \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)}\boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)}\end{matrix}\right]T_{33} \qquad 89$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)}\boxed{-(b''_{34})^{(6)}(G_{35},t)} & \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{34} \quad 90$$

$\boxed{-(b''_{32})^{(6)}(G_{35},t)}, \boxed{-(b''_{33})^{(6)}(G_{35},t)}, \boxed{-(b''_{34})^{(6)}(G_{35},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}$ *are second detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)}$ *are third detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}$ are fourth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \quad 91$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a'_{36})^{(7)}\boxed{+(a''_{36})^{(7)}(T_{37},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \quad 92$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a'_{37})^{(7)}\boxed{+(a''_{37})^{(7)}(T_{37},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{38}}{dt} \quad 93$$

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a'_{38})^{(7)}\boxed{+(a''_{38})^{(7)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{(a''_{36})^{(7)}(T_{37},t)}, \boxed{(a''_{37})^{(7)}(T_{37},t)}, \boxed{(a''_{38})^{(7)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{36}}{dt} =$$



$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b'_{36})^{(7)}\boxed{-(b''_{36})^{(7)}(G_{39},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{37}}{dt} =$$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b'_{37})^{(7)}\boxed{-(b''_{37})^{(7)}(G_{39},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{38}}{dt} =$$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b'_{38})^{(7)}\boxed{-(b''_{38})^{(7)}(G_{39},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition

coefficients for category 1, 2 and 3
$\boxed{-(b_{28}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition
coefficients for category 1, 2 and 3
$\boxed{-(b_{32}'')^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{34}'')^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition
coefficients for category 1, 2 and 3
$\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$
are seventh detrition coefficients for category 1, 2 and 3
$\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition
coefficients for category 1, 2 and 3
$\boxed{-(b_{46}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{45}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{44}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition
coefficients for category 1, 2 and 3



$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \begin{bmatrix} (a_{40}')^{(8)}\boxed{+(a_{40}'')^{(8)}(T_{41},t)} & \boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{20}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{24}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{28}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{32}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{36}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{44}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \begin{bmatrix} (a_{41}')^{(8)}\boxed{+(a_{41}'')^{(8)}(T_{41},t)} & \boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{21}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{25}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{29}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{33}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{37}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{45}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \begin{bmatrix} (a_{42}')^{(8)}\boxed{+(a_{42}'')^{(8)}(T_{41},t)} & \boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{22}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{26}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{30}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{34}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{15}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{38}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{46}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a_{40}'')^{(8)}(T_{41},t)}$, $\boxed{+(a_{41}'')^{(8)}(T_{41},t)}$, $\boxed{+(a_{42}'')^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3
$\boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3
$\boxed{+(a_{20}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{21}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{22}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3
$\boxed{+(a_{24}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{25}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{26}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3
$\boxed{+(a_{28}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{29}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{30}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ $+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ $+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$ are eighth augmentation coefficient for 1,2,3

$+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} =$$

$$(b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)}\left[-(b''_{40})^{(8)}(G_{43},t)\right] & -(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t) & -(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} =$$

$$(b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)}\left[-(b''_{41})^{(8)}(G_{43},t)\right] & -(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t) & -(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} =$$

$$(b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)}\left[-(b''_{42})^{(8)}(G_{43},t)\right] & -(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t) & -(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{15}$$

Where $-(b''_{36})^{(7)}(G_{39},t)$, $-(b''_{37})^{(7)}(G_{39},t)$, $-(b''_{38})^{(7)}(G_{39},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)$, $-(b''_{38})^{(7,7,)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)$ are eighth detrition

coefficients for category 1, 2 and 3
$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3



$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \begin{bmatrix} (a'_{44})^{(9)}\boxed{+(a''_{44})^{(9)}(T_{45},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \begin{bmatrix} (a'_{45})^{(9)}\boxed{+(a''_{45})^{(9)}(T_{45},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \begin{bmatrix} (a'_{46})^{(9)}\boxed{+(a''_{46})^{(9)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a''_{44})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3
$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3
$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3
$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3
$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3
$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3
$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ are Seventh augmentation coefficient for category 1, 2 and 3
$\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3
$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} =$$

$$(b_{44})^{(9)}T_{45} -$$

$$\begin{bmatrix} (b'_{44})^{(9)}\boxed{-(b''_{44})^{(9)}(G_{47},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b'_{45})^{(9)}\boxed{-(b''_{45})^{(9)}(G_{47},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b'_{46})^{(9)}\boxed{-(b''_{46})^{(9)}(G_{47},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{44})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9)}(G_{47},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a'_i)^{(1)}, (a''_i)^{(1)}, (b_i)^{(1)}, (b'_i)^{(1)}, (b''_i)^{(1)} > 0, \quad i,j = 13,14,15 \qquad 97$$

The functions $(a''_i)^{(1)}, (b''_i)^{(1)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a_i'')^{(1)}(T_{14},t) \le (p_i)^{(1)} \le (\hat{A}_{13})^{(1)}$$

$$(b_i'')^{(1)}(G,t) \le (r_i)^{(1)} \le (b_i')^{(1)} \le (\hat{B}_{13})^{(1)}$$

$$lim_{T_2\to\infty}(a_i'')^{(1)}(T_{14},t) = (p_i)^{(1)} \qquad 98$$

$$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = (r_i)^{(1)}$$

**Definition of** $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$ :

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$$|(a_i'')^{(1)}(T_{14}',t) - (a_i'')^{(1)}(T_{14},t)| \le (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G',t) - (b_i'')^{(1)}(G,t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}',t)$ and $(a_i'')^{(1)}(T_{14},t)$ . $(T_{14}',t)$ and $(T_{14},t)$ are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14},t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$ : 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$$

**Definition of** $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$ : 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}$, $(r_i)^{(1)}, i = 13,14,15$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i,j = 16,17,18$$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17},t) \le (p_i)^{(2)} \le \left(\hat{A}_{16}\right)^{(2)} \quad 102$$

$$(b_i'')^{(2)}(G_{19},t) \le (r_i)^{(2)} \le (b_i')^{(2)} \le (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2\to\infty}(a_i'')^{(2)}(T_{17},t) = (p_i)^{(2)} \quad 104$$

$$\lim_{G\to\infty}(b_i'')^{(2)}\left((G_{19}),t\right) = (r_i)^{(2)} \quad 105$$

**Definition of** $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$ : 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}',t) - (a_i'')^{(2)}(T_{17},t)| \le (\hat{k}_{16})^{(2)}|T_{17} - T_{17}'|e^{-(\hat{M}_{16})^{(2)}t} \quad 107$$

$$|(b_i'')^{(2)}((G_{19})',t) - (b_i'')^{(2)}\left((G_{19}),t\right)| < (\hat{k}_{16})^{(2)}||(G_{19}) - (G_{19})'||e^{-(\hat{M}_{16})^{(2)}t} \quad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}',t)$ and$(a_i'')^{(2)}(T_{17},t)$ . $(T_{17}',t)$ and $(T_{17},t)$ are points belonging to the interval $\left[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}\right]$. It is to be noted that $(a_i'')^{(2)}(T_{17},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17},t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$ :

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants 109

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$$

**Definition of** $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$ :

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and\ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1 \quad 110$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(b_i)^{(2)} + (b_i')^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1 \quad 111$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i,j = 20,21,22 \quad 112$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(3)}$, $(\mathrm{r_i})^{(3)}$:

$$(a_i'')^{(3)}(T_{21}, t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23}, t) \leq (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(3)}(T_{21}, t) = (p_i)^{(3)} \quad (113)$$

$$\lim_{G \to \infty}(b_i'')^{(3)}(G_{23}, t) = (r_i)^{(3)}$$

**Definition of** $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$ :

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: (114)

$$|(a_i'')^{(3)}(T_{21}', t) - (a_i'')^{(3)}(T_{21}, t)| \leq (\hat{k}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\hat{M}_{20})^{(3)}t}$$

$$|(b_i'')^{(3)}(G_{23}', t) - (b_i'')^{(3)}(G_{23}, t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}'||e^{-(\hat{M}_{20})^{(3)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}', t)$ and$(a_i'')^{(3)}(T_{21}, t)$ . $(T_{21}', t)$ And $(T_{21}, t)$ are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21}, t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$ : (115)

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with (116)
$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and (\hat{B}_{20})^{(3)}$ and the constants
$(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}, i = 20,21,22,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(a_i)^{(3)} + (a_i')^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(b_i)^{(3)} + (b_i')^{(3)} + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i, j = 24,25,26 \quad (117)$$

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(4)}$, $(r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25},t) \le (p_i)^{(4)} \le (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}\big((G_{27}),t\big) \le (r_i)^{(4)} \le (b_i')^{(4)} \le (\hat{B}_{24})^{(4)}$$

$$lim_{T_2\to\infty}(a_i'')^{(4)}(T_{25},t) = (p_i)^{(4)} \quad (118)$$

$$\lim_{G\to\infty}(b_i'')^{(4)}\big((G_{27}),t\big) = (r_i)^{(4)}$$

**Definition of** $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$ :

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24,25,26}$

They satisfy Lipschitz condition: (119)

$$|(a_i'')^{(4)}(T_{25}',t) - (a_i'')^{(4)}(T_{25},t)| \le (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})',t) - (b_i'')^{(4)}\big((G_{27}),t\big)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}',t)$ and$(a_i'')^{(4)}(T_{25},t)$ . $(T_{25}',t)$ and $(T_{25},t)$ are points belonging to the interval $\big[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}\big]$. It is to be noted that $(a_i'')^{(4)}(T_{25},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25},t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$ : (120)

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$, are positive constants

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}, \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$$

**Definition of** $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$ : (121)

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and\ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}$, $(r_i)^{(4)}, i = 24,25,26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(a_i)^{(4)} + (a_i')^{(4)} + (\hat{A}_{24})^{(4)} + (\hat{P}_{24})^{(4)}(\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(b_i)^{(4)} + (b_i')^{(4)} + (\hat{B}_{24})^{(4)} + (\hat{Q}_{24})^{(4)}(\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i,j = 28,29,30 \quad (122)$$

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(5)}$, $(r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29},t) \le (p_i)^{(5)} \le (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}\big((G_{31}),t\big) \le (r_i)^{(5)} \le (b_i')^{(5)} \le (\hat{B}_{28})^{(5)}$$

$$lim_{T_2\to\infty}(a_i'')^{(5)}(T_{29},t) = (p_i)^{(5)} \qquad 123$$

$$\lim_{G\to\infty}(b_i'')^{(5)}(G_{31},t) = (r_i)^{(5)}$$

**Definition of** $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$ :

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}',t) - (a_i'')^{(5)}(T_{29},t)| \le (\hat{k}_{28})^{(5)}|T_{29} - T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})',t) - (b_i'')^{(5)}\big((G_{31}),t\big)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}',t)$ and$(a_i'')^{(5)}(T_{29},t)$ . $(T_{29}',t)$ and $(T_{29},t)$ are points belonging to the interval $\big[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\big]$. It is to be noted that $(a_i'')^{(5)}(T_{29},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29},t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$ : 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}, \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

**Definition of** $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$ : 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and\ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(a_i)^{(5)} + (a_i')^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)}(\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(b_i)^{(5)} + (b_i')^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)}(\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i,j = 32,33,34 \qquad 127$$

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(6)}$, $(r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33},t) \le (p_i)^{(6)} \le (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}),t) \le (r_i)^{(6)} \le (b_i')^{(6)} \le (\hat{B}_{32})^{(6)}$$

$$lim_{T_2\to\infty}(a_i'')^{(6)}(T_{33},t) = (p_i)^{(6)} \qquad 128$$

$$\lim_{G\to\infty}(b_i'')^{(6)}\left((G_{35}),t\right) = (r_i)^{(6)}$$

**Definition of** $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$ :

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}',t) - (a_i'')^{(6)}(T_{33},t)| \le (\hat{k}_{32})^{(6)}|T_{33} - T_{33}'|e^{-(\hat{M}_{32})^{(6)}t}$$

$$|(b_i'')^{(6)}((G_{35})',t) - (b_i'')^{(6)}\left((G_{35}),t\right)| < (\hat{k}_{32})^{(6)}||(G_{35}) - (G_{35})'||e^{-(\hat{M}_{32})^{(6)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}',t)$ and$(a_i'')^{(6)}(T_{33},t)$ . $(T_{33}',t)$ and $(T_{33},t)$ are points belonging to the interval $\left[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}\right]$. It is to be noted that $(a_i'')^{(6)}(T_{33},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33},t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$ : 129

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$, are positive constants

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$$

**Definition of** $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$ : 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and\ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32,33,34,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(a_i)^{(6)} + (a_i')^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(b_i)^{(6)} + (b_i')^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

(WW) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i,j = 36,37,38$ 131

(XX) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(7)}$, $(r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37},t) \le (p_i)^{(7)} \le (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39},t) \le (r_i)^{(7)} \le (b_i')^{(7)} \le (\hat{B}_{36})^{(7)}$$

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*(YY)* $\lim_{T_2\to\infty} (a_i'')^{(7)}(T_{37},t) = (p_i)^{(7)}$

*(ZZ)*

$\lim_{G\to\infty}(b_i'')^{(7)}\big((G_{39}),t\big) = (r_i)^{(7)}$

**Definition of** $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$ :

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$$|(a_i'')^{(7)}(T_{37}',t) - (a_i'')^{(7)}(T_{37},t)| \le (\hat{k}_{36})^{(7)}|T_{37} - T_{37}'|e^{-(\hat{M}_{36})^{(7)}t}$$

$$|(b_i'')^{(7)}((G_{39})',t) - (b_i'')^{(7)}\big((G_{39}),t\big)| < (\hat{k}_{36})^{(7)}||(G_{39}) - (G_{39})'||e^{-(\hat{M}_{36})^{(7)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}',t)$ and$(a_i'')^{(7)}(T_{37},t)$ . $(T_{37}',t)$ and $(T_{37},t)$ are points belonging to the interval $\big[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}\big]$. It is to be noted that $(a_i'')^{(7)}(T_{37},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37},t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$ : 134

(AAA) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$, are positive constants

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$$

**Definition of** $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$ : 135

(BBB) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)}(\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(b_i)^{(7)} + (b_i')^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)}(\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40,41,42 \tag{136}$$

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

**Definition of** $(p_i)^{(8)}$, $(r_i)^{(8)}$: (137)

$$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)} \tag{138}$$

$$(b_i'')^{(8)}((G_{43}), t) \leq (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)} \tag{139}$$

$$\lim_{T_2 \to \infty} (a_i'')^{(8)}(T_{41}, t) = (p_i)^{(8)} \tag{140}$$

$$\lim_{G \to \infty} (b_i'')^{(8)}((G_{43}), t) = (r_i)^{(8)} \tag{141}$$

**Definition of** $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$ :

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)} |T_{41} - T_{41}'| e^{-(\hat{M}_{40})^{(8)} t} \tag{142}$$

$$|(b_i'')^{(8)}((G_{43})', t) - (b_i'')^{(8)}((G_{43}), t)| < (\hat{k}_{40})^{(8)} ||(G_{43}) - (G_{43})'|| e^{-(\hat{M}_{40})^{(8)} t} \tag{143}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}', t)$ and $(a_i'')^{(8)}(T_{41}, t)$ . $(T_{41}', t)$ and $(T_{41}, t)$ are points belonging to the interval $[(\hat{k}_{40})^{(8)}, (\hat{M}_{40})^{(8)}]$. It is to be noted that $(a_i'')^{(8)}(T_{41}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{40})^{(8)} = 1$ then the function $(a_i'')^{(8)}(T_{41}, t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$ :

$(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$, are positive constants

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}, \frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1 \tag{144}$$

**Definition of** $(\hat{P}_{40})^{(8)}, (\hat{Q}_{40})^{(8)}$ :

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}, (\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)}, (a_i')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}, i = 40,41,42,$

Satisfy the inequalities

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(a_i)^{(8)} + (a_i')^{(8)} + (\hat{A}_{40})^{(8)} + (\hat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}] < 1 \quad 145$$

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(b_i)^{(8)} + (b_i')^{(8)} + (\hat{B}_{40})^{(8)} + (\hat{Q}_{40})^{(8)}(\hat{k}_{40})^{(8)}] < 1 \quad 146$$

Where we suppose

$$(a_i)^{(9)}, (a_i')^{(9)}, (a_i'')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (b_i'')^{(9)} > 0, \quad i,j = 44,45,46 \quad 146A$$

The functions $(a_i'')^{(9)}, (b_i'')^{(9)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(9)}$, $(r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45}, t) \leq (p_i)^{(9)} \leq (\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47}, t) \leq (r_i)^{(9)} \leq (b_i')^{(9)} \leq (\hat{B}_{44})^{(9)}$$

$$\lim_{T_2 \to \infty} (a_i'')^{(9)}(T_{45}, t) = (p_i)^{(9)}$$

$$\lim_{G \to \infty} (b_i'')^{(9)}(G_{47}, t) = (r_i)^{(9)}$$

**Definition of** $(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}$ :

Where $\boxed{(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}}$ are positive constants and $\boxed{i = 44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}', t) - (a_i'')^{(9)}(T_{45}, t)| \leq (\hat{k}_{44})^{(9)}|T_{45} - T_{45}'|e^{-(\hat{M}_{44})^{(9)}t}$$

$$|(b_i'')^{(9)}((G_{47})', t) - (b_i'')^{(9)}((G_{47}), t)| < (\hat{k}_{44})^{(9)}||(G_{47}) - (G_{47})'||e^{-(\hat{M}_{44})^{(9)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}', t)$ and$(a_i'')^{(9)}(T_{45}, t)$ . $(T_{45}', t)$ and $(T_{45}, t)$ are points belonging to the interval $[(\hat{k}_{44})^{(9)}, (\hat{M}_{44})^{(9)}]$. It is to be noted that $(a_i'')^{(9)}(T_{45}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{44})^{(9)} = 1$ then the function $(a_i'')^{(9)}(T_{45}, t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$ :

$(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$, are positive constants

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}, \frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$$

**Definition of** $(\hat{P}_{44})^{(9)}, (\hat{Q}_{44})^{(9)}$ :

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}, (\hat{A}_{44})^{(9)} and (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)}, (a_i')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}, i = 44,45,46,$
satisfy the inequalities

$$\frac{1}{(\widehat{M}_{44})^{(9)}}[(a_i)^{(9)} + (a_i')^{(9)} + (\hat{A}_{44})^{(9)} + (\hat{P}_{44})^{(9)}(\hat{k}_{44})^{(9)}] < 1$$

$$\frac{1}{(\widehat{M}_{44})^{(9)}}[(b_i)^{(9)} + (b_i')^{(9)} + (\hat{B}_{44})^{(9)} + (\hat{Q}_{44})^{(9)}(\hat{k}_{44})^{(9)}] < 1$$

**Theorem 1:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0), T_i(0)$ :

$$G_i(t) \leq (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 2 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

**Definition of** $G_i(0), T_i(0)$

$$G_i(t) \leq (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} \quad , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \leq (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} \quad , \quad T_i(0) = T_i^0 > 0$$

**Theorem 3 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t) \leq (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t} \quad , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \leq (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t} \quad , \quad T_i(0) = T_i^0 > 0$$

**Theorem 4 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

**Definition of** $G_i(0), T_i(0)$ :

$$G_i(t) \leq (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 5 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

**Definition of** $G_i(0), T_i(0)$ :

$$G_i(t) \leq (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 6 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

**Definition of** $G_i(0), T_i(0)$ :

$G_i(t) \le \left(\hat{P}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \le \left(\hat{Q}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Theorem 7:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

**Definition of** $G_i(0), T_i(0)$ :

$G_i(t) \le \left(\hat{P}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \le \left(\hat{Q}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Theorem 8:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

**Definition of** $G_i(0), T_i(0)$ :

$G_i(t) \le \left(\hat{P}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \le \left(\hat{Q}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Theorem 9:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

**Definition of** $G_i(0), T_i(0)$ :

$G_i(t) \le \left(\hat{P}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \le \left(\hat{Q}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Proof:** Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i$: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$G_i(0) = G_i^0$ , $T_i(0) = T_i^0$ , $G_i^0 \le \left(\hat{P}_{13}\right)^{(1)}$ , $T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)}$, 155

$0 \le G_i(t) - G_i^0 \le \left(\hat{P}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t}$ 156

$0 \le T_i(t) - T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t}$ 157

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}(s_{(13)}) - \left((a'_{13})^{(1)} + a''_{13})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{15}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^{0} + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^{0} + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{\mathrm{T}}_{15}(\mathrm{t}) = \mathrm{T}_{15}^{0} + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{15}(s_{(13)})\right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{16})^{(2)}$, $T_i^0 \le (\hat{Q}_{16})^{(2)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

By 160

$$\bar{G}_{16}(t) = G_{16}^{0} + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''_{16})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^{0} + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}(s_{(16)}), s_{(17)})\right) G_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^{0} + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{18}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^{0} + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^{0} + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^{0} + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{18}(s_{(16)})\right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{20})^{(3)}$, $T_i^0 \le (\hat{Q}_{20})^{(3)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

By 161

$$\bar{G}_{20}(t) = G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''_{20})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{22}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^0 + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^0 + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{\mathrm{T}}_{22}(\mathrm{t}) = \mathrm{T}_{22}^0 + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{22}(s_{(20)})\right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{24})^{(4)}, T_i^0 \le (\hat{Q}_{24})^{(4)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

By 162

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{26}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{\mathrm{T}}_{26}(\mathrm{t}) = \mathrm{T}_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{26}(s_{(24)})\right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{28})^{(5)}, T_i^0 \le (\hat{Q}_{28})^{(5)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{28})^{(5)} e^{(\tilde{M}_{28})^{(5)} t}$

By 163

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[ (a_{28})^{(5)} G_{29}(s_{(28)}) - \left( (a'_{28})^{(5)} + a''_{28})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{28}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[ (a_{29})^{(5)} G_{28}(s_{(28)}) - \left( (a'_{29})^{(5)} + (a''_{29})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{29}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[ (a_{30})^{(5)} G_{29}(s_{(28)}) - \left( (a'_{30})^{(5)} + (a''_{30})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{30}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[ (b_{28})^{(5)} T_{29}(s_{(28)}) - \left( (b'_{28})^{(5)} - (b''_{28})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{28}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[ (b_{29})^{(5)} T_{28}(s_{(28)}) - \left( (b'_{29})^{(5)} - (b''_{29})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{29}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{\mathrm{T}}_{30}(\mathrm{t}) = \mathrm{T}_{30}^0 + \int_0^t \left[ (b_{30})^{(5)} T_{29}(s_{(28)}) - \left( (b'_{30})^{(5)} - (b''_{30})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{30}(s_{(28)}) \right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{32})^{(6)}$, $T_i^0 \le (\hat{Q}_{32})^{(6)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\tilde{M}_{32})^{(6)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\tilde{M}_{32})^{(6)} t}$

By 164

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[ (a_{32})^{(6)} G_{33}(s_{(32)}) - \left( (a'_{32})^{(6)} + a''_{32})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[ (a_{33})^{(6)} G_{32}(s_{(32)}) - \left( (a'_{33})^{(6)} + (a''_{33})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[ (a_{34})^{(6)} G_{33}(s_{(32)}) - \left( (a'_{34})^{(6)} + (a''_{34})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{34}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[ (b_{32})^{(6)} T_{33}(s_{(32)}) - \left( (b'_{32})^{(6)} - (b''_{32})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[ (b_{33})^{(6)} T_{32}(s_{(32)}) - \left( (b'_{33})^{(6)} - (b''_{33})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{\mathrm{T}}_{34}(\mathrm{t}) = \mathrm{T}_{34}^0 + \int_0^t \left[ (b_{34})^{(6)} T_{33}(s_{(32)}) - \left( (b'_{34})^{(6)} - (b''_{34})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{34}(s_{(32)}) \right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{36})^{(7)}$, $T_i^0 \leq (\hat{Q}_{36})^{(7)}$,

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} G_{37}(s_{(36)}) - \left((a'_{36})^{(7)} + a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[(a_{37})^{(7)} G_{36}(s_{(36)}) - \left((a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[(a_{38})^{(7)} G_{37}(s_{(36)}) - \left((a'_{38})^{(7)} + (a''_{38})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{38}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[(b_{36})^{(7)} T_{37}(s_{(36)}) - \left((b'_{36})^{(7)} - (b''_{36})^{(7)} (G_{39}(s_{(36)}), s_{(36)})\right) T_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[(b_{37})^{(7)} T_{36}(s_{(36)}) - \left((b'_{37})^{(7)} - (b''_{37})^{(7)} (G_{39}(s_{(36)}), s_{(36)})\right) T_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{38}(t) = T_{38}^0 + \int_0^t \left[(b_{38})^{(7)} T_{37}(s_{(36)}) - \left((b'_{38})^{(7)} - (b''_{38})^{(7)} (G_{39}(s_{(36)}), s_{(36)})\right) T_{38}(s_{(36)})\right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{40})^{(8)}$, $T_i^0 \leq (\hat{Q}_{40})^{(8)}$,

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} G_{41}(s_{(40)}) - \left((a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[(a_{41})^{(8)} G_{40}(s_{(40)}) - \left((a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41}(s_{(40)}), s_{(40)})\right) G_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}(s_{(40)}), s_{(40)})\right) G_{42}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^0 + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43}(s_{(40)}), s_{(40)})\right) T_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^0 + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43}(s_{(40)}), s_{(40)})\right) T_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{\mathrm{T}}_{42}(\mathrm{t}) = \mathrm{T}_{42}^0 + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43}(s_{(40)}), s_{(40)})\right) T_{42}(s_{(40)})\right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** 166

Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ A which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{44})^{(9)},\ T_i^0 \le (\hat{Q}_{44})^{(9)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''_{44})^{(9)}(T_{45}(s_{(44)}), s_{(44)})\right) G_{44}(s_{(44)})\right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45}(s_{(44)}), s_{(44)})\right) G_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}(s_{(44)}), s_{(44)})\right) G_{46}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{44}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{\mathrm{T}}_{46}(\mathrm{t}) = \mathrm{T}_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{46}(s_{(44)})\right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167
$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}}\right)\right] ds_{(13)} =$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left(e^{(\hat{M}_{13})^{(1)} t} - 1\right)$$

From which it follows that 168

$$(G_{13}(t)-G_{13}^{0})e^{-(\hat{M}_{13})^{(1)}t}\leq\frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}}\left[((\hat{P}_{13})^{(1)}+G_{14}^{0})e^{\left(-\frac{(\hat{P}_{13})^{(1)}+G_{14}^{0}}{G_{14}^{0}}\right)}+(\hat{P}_{13})^{(1)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t)\leq G_{16}^{0}+\int_{0}^{t}\left[(a_{16})^{(2)}\left(G_{17}^{0}+(\hat{P}_{16})^{(6)}e^{(\hat{M}_{16})^{(2)}s_{(16)}}\right)\right]ds_{(16)}= \quad 169$$

$$\left(1+(a_{16})^{(2)}t\right)G_{17}^{0}+\frac{(a_{16})^{(2)}(\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}}\left(e^{(\hat{M}_{16})^{(2)}t}-1\right)$$

From which it follows that 170

$$(G_{16}(t)-G_{16}^{0})e^{-(\hat{M}_{16})^{(2)}t}\leq\frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}}\left[((\hat{P}_{16})^{(2)}+G_{17}^{0})e^{\left(-\frac{(\hat{P}_{16})^{(2)}+G_{17}^{0}}{G_{17}^{0}}\right)}+(\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t)\leq G_{20}^{0}+\int_{0}^{t}\left[(a_{20})^{(3)}\left(G_{21}^{0}+(\hat{P}_{20})^{(3)}e^{(\hat{M}_{20})^{(3)}s_{(20)}}\right)\right]ds_{(20)}=$$

$$\left(1+(a_{20})^{(3)}t\right)G_{21}^{0}+\frac{(a_{20})^{(3)}(\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left(e^{(\hat{M}_{20})^{(3)}t}-1\right)$$

From which it follows that 172

$$(G_{20}(t)-G_{20}^{0})e^{-(\hat{M}_{20})^{(3)}t}\leq\frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left[((\hat{P}_{20})^{(3)}+G_{21}^{0})e^{\left(-\frac{(\hat{P}_{20})^{(3)}+G_{21}^{0}}{G_{21}^{0}}\right)}+(\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 173

$$G_{24}(t)\leq G_{24}^{0}+\int_{0}^{t}\left[(a_{24})^{(4)}\left(G_{25}^{0}+(\hat{P}_{24})^{(4)}e^{(\hat{M}_{24})^{(4)}s_{(24)}}\right)\right]ds_{(24)}=$$

$$\left(1+(a_{24})^{(4)}t\right)G_{25}^{0}+\frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left(e^{(\hat{M}_{24})^{(4)}t}-1\right)$$

From which it follows that 174

$$(G_{24}(t)-G_{24}^{0})e^{-(\hat{M}_{24})^{(4)}t}\leq\frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left[((\hat{P}_{24})^{(4)}+G_{25}^{0})e^{\left(-\frac{(\hat{P}_{24})^{(4)}+G_{25}^{0}}{G_{25}^{0}}\right)}+(\hat{P}_{24})^{(4)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t) \le G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} \left(G_{29}^0 + (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} s_{(28)}}\right)\right] ds_{(28)} =$$

$$\left(1 + (a_{28})^{(5)} t\right) G_{29}^0 + \frac{(a_{28})^{(5)} (\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}} \left(e^{(\hat{M}_{28})^{(5)} t} - 1\right)$$

From which it follows that 175

$$(G_{28}(t) - G_{28}^0) e^{-(\hat{M}_{28})^{(5)} t} \le \frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}} \left[\left((\hat{P}_{28})^{(5)} + G_{29}^0\right) e^{\left(-\frac{(\hat{P}_{28})^{(5)} + G_{29}^0}{G_{29}^0}\right)} + (\hat{P}_{28})^{(5)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t) \le G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} \left(G_{33}^0 + (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} s_{(32)}}\right)\right] ds_{(32)} =$$

$$\left(1 + (a_{32})^{(6)} t\right) G_{33}^0 + \frac{(a_{32})^{(6)} (\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}} \left(e^{(\hat{M}_{32})^{(6)} t} - 1\right)$$

From which it follows that 177

$$(G_{32}(t) - G_{32}^0) e^{-(\hat{M}_{32})^{(6)} t} \le \frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}} \left[\left((\hat{P}_{32})^{(6)} + G_{33}^0\right) e^{\left(-\frac{(\hat{P}_{32})^{(6)} + G_{33}^0}{G_{33}^0}\right)} + (\hat{P}_{32})^{(6)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

(i) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t) \le G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} \left(G_{37}^0 + (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} s_{(36)}}\right)\right] ds_{(36)} =$$

$$\left(1 + (a_{36})^{(7)} t\right) G_{37}^0 + \frac{(a_{36})^{(7)} (\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left(e^{(\hat{M}_{36})^{(7)} t} - 1\right)$$

From which it follows that

$$(G_{36}(t) - G_{36}^0) e^{-(\hat{M}_{36})^{(7)} t} \le \frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left[\left((\hat{P}_{36})^{(7)} + G_{37}^0\right) e^{\left(-\frac{(\hat{P}_{36})^{(7)} + G_{37}^0}{G_{37}^0}\right)} + (\hat{P}_{36})^{(7)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t) \le G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} \left(G_{41}^0 + (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} s_{(40)}}\right)\right] ds_{(40)} =$$ 180

$$\left(1+(a_{40})^{(8)}t\right)G_{41}^{0}+\frac{(a_{40})^{(8)}(\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left(e^{(\hat{M}_{40})^{(8)}t}-1\right)$$

From which it follows that 181

$$(G_{40}(t)-G_{40}^{0})e^{-(\hat{M}_{40})^{(8)}t}\leq\frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left[\left((\hat{P}_{40})^{(8)}+G_{41}^{0}\right)e^{\left(-\frac{(\hat{P}_{40})^{(8)}+G_{41}^{0}}{G_{41}^{0}}\right)}+(\hat{P}_{40})^{(8)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 8

Analogous inequalities hold also for $G_{41}, G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t)\leq G_{44}^{0}+\int_{0}^{t}\left[(a_{44})^{(9)}\left(G_{45}^{0}+(\hat{P}_{44})^{(9)}e^{(\hat{M}_{44})^{(9)}s_{(44)}}\right)\right]ds_{(44)}=$$

$$\left(1+(a_{44})^{(9)}t\right)G_{45}^{0}+\frac{(a_{44})^{(9)}(\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}}\left(e^{(\hat{M}_{44})^{(9)}t}-1\right)$$

From which it follows that

$$(G_{44}(t)-G_{44}^{0})e^{-(\hat{M}_{44})^{(9)}t}\leq\frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}}\left[\left((\hat{P}_{44})^{(9)}+G_{45}^{0}\right)e^{\left(-\frac{(\hat{P}_{44})^{(9)}+G_{45}^{0}}{G_{45}^{0}}\right)}+(\hat{P}_{44})^{(9)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 9

Analogous inequalities hold also for $G_{45}, G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}<1$ and to choose 182

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[(\hat{P}_{13})^{(1)}+\left((\hat{P}_{13})^{(1)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{13})^{(1)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{13})^{(1)} \quad 183$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[\left((\hat{Q}_{13})^{(1)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{13})^{(1)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{13})^{(1)}\right]\leq(\hat{Q}_{13})^{(1)} \quad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)},T^{(1)}\right),\left(G^{(2)},T^{(2)}\right)\right)=$$

$$\sup_i\left\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{13})^{(1)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{13})^{(1)}t}\right\}$$

Indeed if we denote

**Definition of** $\tilde{G}, \tilde{T}$ : $\left(\tilde{G}, \tilde{T}\right) = \mathcal{A}^{(1)}(G, T)$

It results

$$\left|\tilde{G}_{13}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{13})^{(1)} \left|G_{14}^{(1)} - G_{14}^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)} s_{(13)}} e^{(\widehat{M}_{13})^{(1)} s_{(13)}}\, ds_{(13)} +$$

$$\int_0^t \{(a'_{13})^{(1)} \left|G_{13}^{(1)} - G_{13}^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)} s_{(13)}} e^{-(\widehat{M}_{13})^{(1)} s_{(13)}} +$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)}, s_{(13)}\right)\left|G_{13}^{(1)} - G_{13}^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)} s_{(13)}} e^{(\widehat{M}_{13})^{(1)} s_{(13)}} +$$

$$G_{13}^{(2)} \left|(a''_{13})^{(1)}\left(T_{14}^{(1)}, s_{(13)}\right) - (a''_{13})^{(1)}\left(T_{14}^{(2)}, s_{(13)}\right)\right|\ e^{-(\widehat{M}_{13})^{(1)} s_{(13)}} e^{(\widehat{M}_{13})^{(1)} s_{(13)}}\} ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|G^{(1)} - G^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)} t} \leq \qquad 186$$
$$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)} + (a'_{13})^{(1)} + (\widehat{A}_{13})^{(1)} + (\widehat{P}_{13})^{(1)}(\widehat{k}_{13})^{(1)}\right) d\left(\left(G^{(1)}, T^{(1)};\ G^{(2)}, T^{(2)}\right)\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 1:** The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)} t}$ *and* $(\widehat{Q}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)} t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(1)}$ and $(b''_i)^{(1)}, i = 13{,}14{,}15$ depend only on $T_{14}$ and respectively on $G$(*and not on* $t$) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any $t$ where $G_i\,(t) = 0$ *and* $T_i\,(t) = 0$

From 19 to 24 it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a'_i)^{(1)} - (a''_i)^{(1)}(T_{14}(s_{(13)}), s_{(13)})\} ds_{(13)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b'_i)^{(1)} t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2$ *and* $\left((\widehat{M}_{13})^{(1)}\right)_3$ : 187

**Remark 3**: if $G_{13}$ is bounded, the same property have also $G_{14}$ *and* $G_{15}$ . indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a'_{14})^{(1)} G_{14}$ and by integrating

$$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1 / (a'_{14})^{(1)}$$

In the same way , one can obtain

$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2/(a'_{15})^{(1)}$

If $G_{14}$ *or* $G_{15}$ is bounded, the same property follows for $G_{13}$ , $G_{15}$ and $G_{13}$ , $G_{14}$ respectively.

**Remark 4:** If $G_{13}$ *is* bounded, from below, the same property holds for $G_{14}$ *and* $G_{15}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{14}$ is bounded from below. 188

**Remark 5:** If $T_{13}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}(G(t),t)) = (b'_{14})^{(1)}$ then $T_{14} \to \infty$. 189

**Definition of** $(m)^{(1)}$ and $\varepsilon_1$ :

Indeed let $t_1$ be so that for $t > t_1$

$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t),t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\frac{2}{\varepsilon_1}$ By taking now $\varepsilon_1$ sufficiently small one sees that $T_{14}$ is unbounded.
The same property holds for $T_{15}$ if $\lim_{t\to\infty}(b''_{15})^{(1)}(G(t),t) = (b'_{15})^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}$ , $\frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}$ *and* $(\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[(\widehat{P}_{16})^{(2)} + \left((\hat{P}_{16})^{(2)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{16})^{(2)}\right] \leq (\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$d\left(\left((G_{19})^{(1)},(T_{19})^{(1)}\right),\left((G_{19})^{(2)},(T_{19})^{(2)}\right)\right) =$

$sup_i\{max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t}, max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t}\}$

Indeed if we denote 195

**Definition of** $\widetilde{G_{19}}, \widetilde{T_{19}}$ : $\left(\widetilde{G_{19}}, \widetilde{T_{19}}\right) = \mathcal{A}^{(2)}(G_{19}, T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{16})^{(2)} \left|G_{17}^{(1)} - G_{17}^{(2)}\right| e^{-(\widehat{M}_{16})^{(2)} s_{(16)}} e^{(\widehat{M}_{16})^{(2)} s_{(16)}}\, ds_{(16)} +$$

$$\int_0^t \{(a'_{16})^{(2)} \left|G_{16}^{(1)} - G_{16}^{(2)}\right| e^{-(\widehat{M}_{16})^{(2)} s_{(16)}} e^{-(\widehat{M}_{16})^{(2)} s_{(16)}} +$$

$$(a''_{16})^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right)\left|G_{16}^{(1)} - G_{16}^{(2)}\right| e^{-(\widehat{M}_{16})^{(2)} s_{(16)}} e^{(\widehat{M}_{16})^{(2)} s_{(16)}} +$$

$$G_{16}^{(2)} \left|(a''_{16})^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right) - (a''_{16})^{(2)}\left(T_{17}^{(2)}, s_{(16)}\right)\right|\ e^{-(\widehat{M}_{16})^{(2)} s_{(16)}} e^{(\widehat{M}_{16})^{(2)} s_{(16)}}\} ds_{(16)}$$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0, t]$ 197

From the hypotheses it follows

$$\left|(G_{19})^{(1)} - (G_{19})^{(2)}\right| e^{-(\widehat{M}_{16})^{(2)} t} \leq$$
$$\frac{1}{(\widehat{M}_{16})^{(2)}}\left((a_{16})^{(2)} + (a'_{16})^{(2)} + (\widehat{A}_{16})^{(2)} + (\widehat{P}_{16})^{(2)}(\widehat{k}_{16})^{(2)}\right) d\left(((G_{19})^{(1)}, (T_{19})^{(1)};\ (G_{19})^{(2)}, (T_{19})^{(2)})\right)$$

And analogous inequalities for $G_i$ and $T_i$. Taking into account the hypothesis the result follows 198

**Remark 6:** The fact that we supposed $(a''_{16})^{(2)}$ and $(b''_{16})^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{16})^{(2)} e^{(\widehat{M}_{16})^{(2)} t}$ and $(\widehat{Q}_{16})^{(2)} e^{(\widehat{M}_{16})^{(2)} t}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i = 16,17,18$ depend only on $T_{17}$ and respectively on $(G_{19})$(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 7:** There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$ 200

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(2)} - (a_i'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})\} ds_{(16)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(2)} t)} > 0 \quad \text{for } t > 0$$

**Definition of** $\left((\widehat{M}_{16})^{(2)}\right)_1$, $\left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$ : 201

**Remark 8:** if $G_{16}$ is bounded, the same property have also $G_{17}$ and $G_{18}$ . indeed if

$G_{16} < (\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt} \leq \left((\widehat{M}_{16})^{(2)}\right)_1 - (a'_{17})^{(2)} G_{17}$ and by integrating

$$G_{17} \leq \left((\widehat{M}_{16})^{(2)}\right)_2 = G_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1 / (a'_{17})^{(2)}$$

In the same way , one can obtain

$G_{18} \leq \left((\widehat{M}_{16})^{(2)}\right)_3 = G_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2/(a'_{18})^{(2)}$

If $G_{17}$ or $G_{18}$ is bounded, the same property follows for $G_{16}$ , $G_{18}$ and $G_{16}$ , $G_{17}$ respectively.

**Remark 9**: If $G_{16}$ is bounded, from below, the same property holds for $G_{17}$ and $G_{18}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{17}$ is bounded from below. 202

**Remark 10:** If $T_{16}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t),t)) = (b'_{17})^{(2)}$ then $T_{17} \to \infty$. 203

**Definition of** $(m)^{(2)}$ and $\varepsilon_2$ :

Indeed let $t_2$ be so that for $t > t_2$

$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t),t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right), \quad t = \log\frac{2}{\varepsilon_2}$ By taking now $\varepsilon_2$ sufficiently small one sees that $T_{17}$ is unbounded. 205
The same property holds for $T_{18}$ if $\lim_{t\to\infty}(b''_{18})^{(2)}\left((G_{19})(t),t\right) = (b'_{18})^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}$ , $\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[(\hat{P}_{20})^{(3)} + \left((\hat{P}_{20})^{(3)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{20})^{(3)}\right] \leq (\hat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself 210

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)},(T_{23})^{(1)}),((G_{23})^{(2)},(T_{23})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{20})^{(3)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

**Definition of** $\widetilde{G_{23}}, \widetilde{T_{23}} :\left( \widetilde{(G_{23})}, \widetilde{(T_{23})} \right) = \mathcal{A}^{(3)}\left((G_{23}), (T_{23})\right)$

It results 213

$$\left|\tilde{G}_{20}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{20})^{(3)} \left|G_{21}^{(1)} - G_{21}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} e^{(\widehat{M}_{20})^{(3)} s_{(20)}}\, ds_{(20)} +$$

$$\int_0^t \{(a'_{20})^{(3)} \left|G_{20}^{(1)} - G_{20}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} +$$

$$(a''_{20})^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right)\left|G_{20}^{(1)} - G_{20}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} e^{(\widehat{M}_{20})^{(3)} s_{(20)}} +$$

$$G_{20}^{(2)} \left|(a''_{20})^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right) - (a''_{20})^{(3)}\left(T_{21}^{(2)}, s_{(20)}\right)\right|\ e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} e^{(\widehat{M}_{20})^{(3)} s_{(20)}}\} ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\left|{G_{23}}^{(1)} - {G_{23}}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)} t} \leq \qquad 214$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)} + (a'_{20})^{(3)} + (\widehat{A}_{20})^{(3)} + (\widehat{P}_{20})^{(3)} (\widehat{k}_{20})^{(3)}\right) d\left(\left((G_{23})^{(1)}, (T_{23})^{(1)};\ (G_{23})^{(2)}, (T_{23})^{(2)}\right)\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 11:** The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)} e^{(\widehat{M}_{20})^{(3)} t}$ *and* $(\widehat{Q}_{20})^{(3)} e^{(\widehat{M}_{20})^{(3)} t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i = 20,21,22$ depend only on $T_{21}$ and respectively on $(G_{23})$(*and not on* $t$) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 12:** There does not exist any $t$ where $G_i(t) = 0$ *and* $T_i(t) = 0$ 216

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(3)} - (a_i'')^{(3)} (T_{21}(s_{(20)}), s_{(20)})\} ds_{(20)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(3)} t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2$ *and* $\left((\widehat{M}_{20})^{(3)}\right)_3$ : 217

**Remark 13:** if $G_{20}$ is bounded, the same property have also $G_{21}$ *and* $G_{22}$ . indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a'_{21})^{(3)} G_{21}$ and by integrating

$$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)} \left((\widehat{M}_{20})^{(3)}\right)_1 / (a'_{21})^{(3)}$$

In the same way , one can obtain

$$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2/(a'_{22})^{(3)}$$

If $G_{21}$ *or* $G_{22}$ is bounded, the same property follows for $G_{20}$ , $G_{22}$ and $G_{20}$ , $G_{21}$ respectively.

**Remark 14:** If $G_{20}$ *is* bounded, from below, the same property holds for $G_{21}$ *and* $G_{22}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{21}$ is bounded from below. 218

**Remark 15:** If $T_{20}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t),t\right) = (b'_{21})^{(3)}$ then $T_{21} \to \infty$. 219

**Definition of** $(m)^{(3)}$ and $\varepsilon_3$ :

Indeed let $t_3$ be so that for $t > t_3$

$$(b_{21})^{(3)} - (b_i'')^{(3)}\left((G_{23})(t),t\right) < \varepsilon_3, T_{20}(t) > (m)^{(3)}$$

Then $\frac{dT_{21}}{dt} \geq (a_{21})^{(3)}(m)^{(3)} - \varepsilon_3 T_{21}$ which leads to 220

$$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t}$$ If we take t such that $e^{-\varepsilon_3 t} = \frac{1}{2}$ it results

$$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right), \quad t = log\frac{2}{\varepsilon_3}$$ By taking now $\varepsilon_3$ sufficiently small one sees that $T_{21}$ is unbounded.
The same property holds for $T_{22}$ if $\lim_{t\to\infty}(b''_{22})^{(3)}\left((G_{23})(t),t\right) = (b'_{22})^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\tilde{M}_{24})^{(4)}}$ , $\frac{(b_i)^{(4)}}{(\tilde{M}_{24})^{(4)}} < 1$ and to choose 221

$(\widehat{P}_{24})^{(4)}$ and $(\widehat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\tilde{M}_{24})^{(4)}}\left[(\widehat{P}_{24})^{(4)} + \left((\hat{P}_{24})^{(4)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{24})^{(4)}$$ 222

$$\frac{(b_i)^{(4)}}{(\tilde{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{24})^{(4)}\right] \leq (\hat{Q}_{24})^{(4)}$$ 223

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself 224

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric 225

$$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\tilde{M}_{24})^{(4)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\tilde{M}_{24})^{(4)}t}\}$$

Indeed if we denote

**Definition of** $\widetilde{(G_{27})}, \widetilde{(T_{27})}$ : $\left(\widetilde{(G_{27})}, \widetilde{(T_{27})}\right) = \mathcal{A}^{(4)}((G_{27}), (T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{24})^{(4)} \left|G_{25}^{(1)} - G_{25}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{(\widehat{M}_{24})^{(4)} s_{(24)}}\, ds_{(24)} +$$

$$\int_0^t \{(a'_{24})^{(4)} \left|G_{24}^{(1)} - G_{24}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} +$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)}, s_{(24)}\right)\left|G_{24}^{(1)} - G_{24}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{(\widehat{M}_{24})^{(4)} s_{(24)}} +$$

$$G_{24}^{(2)} \left|(a''_{24})^{(4)}\left(T_{25}^{(1)}, s_{(24)}\right) - (a''_{24})^{(4)}\left(T_{25}^{(2)}, s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{(\widehat{M}_{24})^{(4)} s_{(24)}}\} ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)} - (G_{27})^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} t} \leq \qquad (226)$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)} + (a'_{24})^{(4)} + (\widehat{A}_{24})^{(4)} + (\widehat{P}_{24})^{(4)} (\widehat{k}_{24})^{(4)}\right) d\left(((G_{27})^{(1)}, (T_{27})^{(1)};\ (G_{27})^{(2)}, (T_{27})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 16:** The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)} t}$ *and* $(\widehat{Q}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)} t}$ respectively of $\mathbb{R}_+$. (227)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i = 24{,}25{,}26$ depend only on $T_{25}$ and respectively on $(G_{27})$(*and not on* $t$) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 17:** There does not exist any $t$ where $G_i(t) = 0$ *and* $T_i(t) = 0$ (228)

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(4)} - (a_i'')^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(4)} t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2$ *and* $\left((\widehat{M}_{24})^{(4)}\right)_3$ : (229)

**Remark 18:** if $G_{24}$ is bounded, the same property have also $G_{25}$ *and* $G_{26}$ . indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a'_{25})^{(4)} G_{25}$ and by integrating

$$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1 / (a'_{25})^{(4)}$$

In the same way , one can obtain

$$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2/(a'_{26})^{(4)}$$

If $G_{25}$ *or* $G_{26}$ is bounded, the same property follows for $G_{24}$ , $G_{26}$ and $G_{24}$ , $G_{25}$ respectively.

**Remark 19:** If $G_{24}$ *is* bounded, from below, the same property holds for $G_{25}$ *and* $G_{26}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{25}$ is bounded from below. 230

**Remark 20:** If $T_{24}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}((G_{27})(t),t)) = (b'_{25})^{(4)}$ then $T_{25} \to \infty$. 231

**Definition of** $(m)^{(4)}$ and $\varepsilon_4$ :

Indeed let $t_4$ be so that for $t > t_4$

$$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t),t) < \varepsilon_4, T_{24}(t) > (m)^{(4)}$$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right)$, $t = log\frac{2}{\varepsilon_4}$ By taking now $\varepsilon_4$ sufficiently small one sees that $T_{25}$ is unbounded.
The same property holds for $T_{26}$ if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t),t\right) = (b'_{26})^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for $G_{29}$ , $G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}$ , $\frac{(b_i)^{(5)}}{(\widehat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\widehat{P}_{28})^{(5)}$ and $(\widehat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}\left[(\widehat{P}_{28})^{(5)} + \left((\widehat{P}_{28})^{(5)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}\left[\left((\widehat{Q}_{28})^{(5)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{28})^{(5)}\right] \leq (\widehat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$$d\left(((G_{31})^{(1)}, (T_{31})^{(1)}), ((G_{31})^{(2)}, (T_{31})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{28})^{(5)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{31}}),(\widetilde{T_{31}})$ : $\left((\widetilde{G_{31}}),(\widetilde{T_{31}})\right)=\mathcal{A}^{(5)}\left((G_{31}),(T_{31})\right)$

It results

$$\left|\tilde{G}_{28}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{28})^{(5)}\left|G_{29}^{(1)}-G_{29}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\,ds_{(28)}+$$

$$\int_0^t\{(a'_{28})^{(5)}\left|G_{28}^{(1)}-G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$(a''_{28})^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)\left|G_{28}^{(1)}-G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$G_{28}^{(2)}\left|(a''_{28})^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)-(a''_{28})^{(5)}\left(T_{29}^{(2)},s_{(28)}\right)\right|\ e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\}ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on it follows

$$\left|(G_{31})^{(1)}-(G_{31})^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}t}\le \qquad 237$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)}+(a'_{28})^{(5)}+(\widehat{A}_{28})^{(5)}+(\widehat{P}_{28})^{(5)}(\widehat{k}_{28})^{(5)}\right)d\left(((G_{31})^{(1)},(T_{31})^{(1)};\ (G_{31})^{(2)},(T_{31})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

**Remark 21:** The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}\ and\ (\widehat{Q}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ respectively of $\mathbb{R}_+$. 238

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i=28,29,30$ depend only on $T_{29}$ and respectively on $(G_{31})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 22:** There does not exist any $t$ where $G_i(t)=0\ and\ T_i(t)=0$ 239

it results

$$G_i(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(5)}-(a_i'')^{(5)}(T_{29}(s_{(28)}),s_{(28)})\}ds_{(28)}\right]}\ge 0$$

$$T_i(t)\ge T_i^0e^{(-(b_i')^{(5)}t)}>0\quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{28})^{(5)}\right)_1,\left((\widehat{M}_{28})^{(5)}\right)_2\ and\ \left((\widehat{M}_{28})^{(5)}\right)_3$ : 240

**Remark 23:** if $G_{28}$ is bounded, the same property have also $G_{29}\ and\ G_{30}$ . indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a'_{29})^{(5)} G_{29}$ and by integrating

$$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1/(a'_{29})^{(5)}$$

In the same way , one can obtain

$$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2/(a'_{30})^{(5)}$$

If $G_{29}$ *or* $G_{30}$ is bounded, the same property follows for $G_{28}$ , $G_{30}$ and $G_{28}$ , $G_{29}$ respectively.

**Remark 24:** If $G_{28}$ *is* bounded, from below, the same property holds for $G_{29}$ *and* $G_{30}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{29}$ is bounded from below. 241

**Remark 25:** If $T_{28}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}((G_{31})(t),t)) = (b'_{29})^{(5)}$ then $T_{29} \to \infty$. 242

**Definition of** $(m)^{(5)}$ and $\varepsilon_5$ :

Indeed let $t_5$ be so that for $t > t_5$

$$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t),t) < \varepsilon_5, T_{28}(t) > (m)^{(5)}$$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right)$, $t = log\frac{2}{\varepsilon_5}$ By taking now $\varepsilon_5$ sufficiently small one sees that $T_{29}$ is unbounded.
The same property holds for $T_{30}$ if $\lim_{t\to\infty}(b_{30}'')^{(5)}\left((G_{31})(t),t\right) = (b'_{30})^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{33}$ , $G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}$ , $\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}} < 1$ and to choose 244

$(\widehat{P}_{32})^{(6)}$ and $(\widehat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\left[(\widehat{P}_{32})^{(6)} + \left((\widehat{P}_{32})^{(6)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{32})^{(6)} \quad 245$$

$$\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\left[\left((\widehat{Q}_{32})^{(6)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{32})^{(6)}\right] \leq (\widehat{Q}_{32})^{(6)} \quad 246$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric 247

$$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{35}}),(\widetilde{T_{35}})$ : $\left((\widetilde{G_{35}}),(\widetilde{T_{35}})\right)=\mathcal{A}^{(6)}\left((G_{35}),(T_{35})\right)$

It results

$$\left|\tilde{G}_{32}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{32})^{(6)}\left|G_{33}^{(1)}-G_{33}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\,ds_{(32)}+$$

$$\int_0^t\{(a'_{32})^{(6)}\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$G_{32}^{(2)}\left|(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)-(a''_{32})^{(6)}\left(T_{33}^{(2)},s_{(32)}\right)\right|\ e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\}ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|(G_{35})^{(1)}-(G_{35})^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}t}\le \qquad 248$$
$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)}+(a'_{32})^{(6)}+(\widehat{A}_{32})^{(6)}+(\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}\right)d\left(((G_{35})^{(1)},(T_{35})^{(1)};\ (G_{35})^{(2)},(T_{35})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 26:** The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ *and* $(\widehat{Q}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ respectively of $\mathbb{R}_+$. 249

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i=32,33,34$ depend only on $T_{33}$ and respectively on $(G_{35})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 27:** There does not exist any $t$ where $G_i(t)=0\ and\ T_i(t)=0$ 250

it results

$$G_i(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(6)}-(a_i'')^{(6)}(T_{33}(s_{(32)}),s_{(32)})\}ds_{(32)}\right]}\ge 0$$

$$T_i(t)\ge T_i^0e^{\left(-(b_i')^{(6)}t\right)}>0\quad \text{for t}>0$$

**Definition of** $\left((\widehat{M}_{32})^{(6)}\right)_1,\left((\widehat{M}_{32})^{(6)}\right)_2$ *and* $\left((\widehat{M}_{32})^{(6)}\right)_3$ : 251

**Remark 28:** if $G_{32}$ is bounded, the same property have also $G_{33}\ and\ G_{34}$ . indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a'_{33})^{(6)}G_{33}$ and by integrating

$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_1/(a'_{33})^{(6)}$

In the same way , one can obtain

$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_2/(a'_{34})^{(6)}$

If $G_{33}\ or\ G_{34}$ is bounded, the same property follows for $G_{32}$ , $G_{34}$ and $G_{32}$ , $G_{33}$ respectively.

**Remark 29:** If $G_{32}$ *is* bounded, from below, the same property holds for $G_{33}\ and\ G_{34}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{33}$ is bounded from below. 252

**Remark 30:** If $T_{32}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}\ ((G_{35})(t),t)) = (b'_{33})^{(6)}$ then $T_{33} \to \infty$. 253

**Definition of** $(m)^{(6)}$ and $\varepsilon_6$ :

Indeed let $t_6$ be so that for $t > t_6$

$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t),t\right) < \varepsilon_6, T_{32}(t) > (m)^{(6)}$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right),\quad t = log\frac{2}{\varepsilon_6}$ By taking now $\varepsilon_6$ sufficiently small one sees that $T_{33}$ is unbounded.
The same property holds for $T_{34}$ if $\lim_{t\to\infty}(b''_{34})^{(6)}\left((G_{35})(t),t(t),t\right) = (b'_{34})^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{37}$ , $G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}$ , $\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}} < 1$ and to choose
$(\widehat{P}_{36})^{(7)}$ and $(\widehat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[(\widehat{P}_{36})^{(7)} + \left((\widehat{P}_{36})^{(7)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{36})^{(7)} \qquad 256$$

$$\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[\left((\widehat{Q}_{36})^{(7)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{36})^{(7)}\right] \leq (\widehat{Q}_{36})^{(7)} \qquad 257$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying

Equations into itself
The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric 258

$$d\left(((G_{39})^{(1)},(T_{39})^{(1)}),((G_{39})^{(2)},(T_{39})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\widehat{M}_{36})^{(7)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\widehat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

**Definition of** $\widetilde{(G_{39})},\widetilde{(T_{39})}:\left(\widetilde{(G_{39})},\widetilde{(T_{39})}\right)=\mathcal{A}^{(7)}((G_{39}),(T_{39}))$
It results

$$|\tilde{G}_{36}^{(1)}-\tilde{G}_i^{(2)}|\le\int_0^t(a_{36})^{(7)}|G_{37}^{(1)}-G_{37}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}ds_{(36)}+$$
$$\int_0^t\{(a'_{36})^{(7)}|G_{36}^{(1)}-G_{36}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$(a''_{36})^{(7)}(T_{37}^{(1)},s_{(36)})|G_{36}^{(1)}-G_{36}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$G_{36}^{(2)}|(a''_{36})^{(7)}(T_{37}^{(1)},s_{(36)})-(a''_{36})^{(7)}(T_{37}^{(2)},s_{(36)})|\ e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\}ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on it follows

$$|(G_{39})^{(1)}-(G_{39})^{(2)}|e^{-(\widehat{M}_{36})^{(7)}t}\le$$ 259
$$\frac{1}{(\widehat{M}_{36})^{(7)}}\left((a_{36})^{(7)}+(a'_{36})^{(7)}+(\widehat{A}_{36})^{(7)}+(\widehat{P}_{36})^{(7)}(\widehat{k}_{36})^{(7)}\right)d\left(((G_{39})^{(1)},(T_{39})^{(1)};\ (G_{39})^{(2)},(T_{39})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 31:** The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ *and* $(\widehat{Q}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ respectively of $\mathbb{R}_+$. 260

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(7)}$ and $(b_i'')^{(7)}, i=36,37,38$ depend only on $T_{37}$ and respectively on $(G_{39})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 32:** There does not exist any $t$ where $G_i(t)=0$ *and* $T_i(t)=0$ 261

it results

$$G_i(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(7)}-(a_i'')^{(7)}(T_{37}(s_{(36)}),s_{(36)})\}ds_{(36)}\right]}\ge 0$$

$$T_i(t)\ge T_i^0e^{(-(b_i')^{(7)}t)}>0\quad \text{for t}>0$$

**Definition of** $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2$ *and* $\left((\widehat{M}_{36})^{(7)}\right)_3$ : 262

**Remark 33:** if $G_{36}$ is bounded, the same property have also $G_{37}$ *and* $G_{38}$ . indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a'_{37})^{(7)} G_{37}$ and by integrating

$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1/(a'_{37})^{(7)}$

In the same way , one can obtain

$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2/(a'_{38})^{(7)}$

If $G_{37}$ *or* $G_{38}$ is bounded, the same property follows for $G_{36}$ , $G_{38}$ and $G_{36}$ , $G_{37}$ respectively.

**Remark 34:** If $G_{36}$ *is* bounded, from below, the same property holds for $G_{37}$ *and* $G_{38}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{37}$ is bounded from below. 263

**Remark 35:** If $T_{36}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(7)}((G_{39})(t),t)) = (b'_{37})^{(7)}$ then $T_{37} \to \infty$. 264

**Definition of** $(m)^{(7)}$ and $\varepsilon_7$ :

Indeed let $t_7$ be so that for $t > t_7$

$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t),t) < \varepsilon_7, T_{36}(t) > (m)^{(7)}$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right), \quad t = log\frac{2}{\varepsilon_7}$ By taking now $\varepsilon_7$ sufficiently small one sees that $T_{37}$ is unbounded.
The same property holds for $T_{38}$ if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t),t\right) = (b'_{38})^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}$ , $\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1$ and to choose 266
$(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ large to have

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[(\hat{P}_{40})^{(8)} + \left((\hat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{40})^{(8)} \quad 267$$

268

$$\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{40})^{(8)}\right]\leq(\hat{Q}_{40})^{(8)}$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)},(T_{43})^{(1)}),((G_{43})^{(2)},(T_{43})^{(2)})\right)= \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

**Definition of** $(\widetilde{G_{43}}),(\widetilde{T_{43}})$ : $\left((\widetilde{G_{43}}),(\widetilde{T_{43}})\right)=\mathcal{A}^{(8)}((G_{43}),(T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{40})^{(8)}\left|G_{41}^{(1)}-G_{41}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\,ds_{(40)}+$$
$$\int_0^t\{(a'_{40})^{(8)}\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$G_{40}^{(2)}\left|(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)-(a''_{40})^{(8)}\left(T_{41}^{(2)},s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\}ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)}-(G_{43})^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}t}\leq \quad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)}+(a'_{40})^{(8)}+(\widehat{A}_{40})^{(8)}+(\widehat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}\right)d\left(((G_{43})^{(1)},(T_{43})^{(1)};\ (G_{43})^{(2)},(T_{43})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 36:** The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ *and* $(\widehat{Q}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ respectively of $\mathbb{R}_+$. 274

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i=40,41,42$ depend only on $\mathrm{T}_{41}$ and respectively on $(G_{43})$(*and not on* $t$) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 37** There does not exist any $t$ where $G_i(t)=0$ *and* $T_i(t)=0$ 275

it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(8)} - (a_i'')^{(8)}(T_{41}(s_{(40)}), s_{(40)})\} ds_{(40)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{(-(b_i')^{(8)} t)} > 0$ for t > 0

**Definition of** $\left((\widehat{M}_{40})^{(8)}\right)_1, \left((\widehat{M}_{40})^{(8)}\right)_2$ *and* $\left((\widehat{M}_{40})^{(8)}\right)_3$ : 276

**Remark 38:** if $G_{40}$ is bounded, the same property have also $G_{41}$ *and* $G_{42}$ . indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a_{41}')^{(8)} G_{41}$ and by integrating

$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)} \left((\widehat{M}_{40})^{(8)}\right)_1 / (a_{41}')^{(8)}$

In the same way , one can obtain

$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)} \left((\widehat{M}_{40})^{(8)}\right)_2 / (a_{42}')^{(8)}$

If $G_{41}$ *or* $G_{42}$ is bounded, the same property follows for $G_{40}$ , $G_{42}$ and $G_{40}$ , $G_{41}$ respectively.

**Remark 39:** If $G_{40}$ *is* bounded, from below, the same property holds for $G_{41}$ *and* $G_{42}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{41}$ is bounded from below. 277

**Remark 40:** If $T_{40}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}((G_{43})(t), t)) = (b_{41}')^{(8)}$ then $T_{41} \to \infty$. 278

**Definition of** $(m)^{(8)}$ and $\varepsilon_8$ :

Indeed let $t_8$ be so that for $t > t_8$

$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t), t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right), \quad t = log\frac{2}{\varepsilon_8}$ By taking now $\varepsilon_8$ sufficiently small one sees that $T_{41}$ is unbounded.
The same property holds for $T_{42}$ if $\lim_{t\to\infty}(b_{42}'')^{(8)}\left((G_{43})(t), t(t), t\right) = (b_{42}')^{(8)}$

It is now sufficient to take $\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}$ , $\frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$ and to choose $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ large to have 279 A

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}\left[(\hat{P}_{44})^{(9)} + \left((\hat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{44})^{(9)} + G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[\left((\hat{Q}_{44})^{(9)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{44})^{(9)}\right]\leq(\hat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)},(T_{47})^{(1)}),\left((G_{47})^{(2)},(T_{47})^{(2)}\right)\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

**Definition of** $\widetilde{(G_{47})},\widetilde{(T_{47})}:\left(\widetilde{(G_{47})},\widetilde{(T_{47})}\right)=\mathcal{A}^{(9)}\left((G_{47}),(T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{44})^{(9)}\left|G_{45}^{(1)}-G_{45}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\,ds_{(44)}+$$
$$\int_0^t\{(a'_{44})^{(9)}\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$G_{44}^{(2)}\left|(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)-(a''_{44})^{(9)}\left(T_{45}^{(2)},s_{(44)}\right)\right|\;e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\}ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}t}\leq$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)}+(a'_{44})^{(9)}+(\widehat{A}_{44})^{(9)}+(\widehat{P}_{44})^{(9)}(\hat{k}_{44})^{(9)}\right)d\left(((G_{47})^{(1)},(T_{47})^{(1)};\;(G_{47})^{(2)},(T_{47})^{(2)})\right)$$

And analogous inequalities for $G_i\;and\;T_i$. Taking into account the hypothesis (39,35,36) the result follows

**Remark 41:** The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}\;and\;(\widehat{Q}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i=44,45,46$ depend only on $T_{45}$ and respectively on $(G_{47})(and\;not\;on\;\;t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 42:** There does not exist any $t$ where $G_i\,(t)=0\;and\;T_i\,(t)=0$

From 99 to 44 it results

$$G_i\,(t)\geq G_i^0e^{\left[-\int_0^t\{(a_i')^{(9)}-(a_i'')^{(9)}(T_{45}(s_{(44)}),s_{(44)})\}ds_{(44)}\right]}\geq 0$$

$T_i(t) \geq T_i^0 e^{(-(b_i')^{(9)}t)} > 0$ for t > 0

**Definition of** $\left((\widehat{M}_{44})^{(9)}\right)_1, \left((\widehat{M}_{44})^{(9)}\right)_2$ *and* $\left((\widehat{M}_{44})^{(9)}\right)_3$ :

**Remark 43:** if $G_{44}$ is bounded, the same property have also $G_{45}$ *and* $G_{46}$ . indeed if

$G_{44} < (\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt} \leq \left((\widehat{M}_{44})^{(9)}\right)_1 - (a_{45}')^{(9)} G_{45}$ and by integrating

$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_1/(a_{45}')^{(9)}$

In the same way , one can obtain

$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_2/(a_{46}')^{(9)}$

If $G_{45}$ *or* $G_{46}$ is bounded, the same property follows for $G_{44}$ , $G_{46}$ and $G_{44}$ , $G_{45}$ respectively.

**Remark 44:** If $G_{44}$ *is* bounded, from below, the same property holds for $G_{45}$ *and* $G_{46}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{45}$ is bounded from below.

**Remark 45:** If $T_{44}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t),t\right) = (b_{45}')^{(9)}$ then $T_{45} \to \infty$.

**Definition of** $(m)^{(9)}$ and $\varepsilon_9$ :

Indeed let $t_9$ be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t),t\right) < \varepsilon_9, T_{44}(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right), \quad t = log\frac{2}{\varepsilon_9}$ By taking now $\varepsilon_9$ sufficiently small one sees that $T_{45}$ is unbounded.
The same property holds for $T_{46}$ if $\lim_{t\to\infty}(b_{46}'')^{(9)}\left((G_{47})(t),t\right) = (b_{46}')^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

**Behavior of the solutions of equation** 280

**Theorem** If we denote and define

**Definition of** $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ :

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)} \leq -(a_{13}')^{(1)} + (a_{14}')^{(1)} - (a_{13}'')^{(1)}(T_{14},t) + (a_{14}'')^{(1)}(T_{14},t) \leq -(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)} \leq -(b_{13}')^{(1)} + (b_{14}')^{(1)} - (b_{13}'')^{(1)}(G,t) - (b_{14}'')^{(1)}(G,t) \leq -(\tau_1)^{(1)}$

**Definition of** $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$ : 281

By $(v_1)^{(1)} > 0$ , $(v_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$ , $(u_2)^{(1)} < 0$ the roots of the equations $(a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

**Definition of** $(\bar{v}_1)^{(1)}$,, $(\bar{v}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$ : 282

By $(\bar{v}_1)^{(1)} > 0$ , $(\bar{v}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$ , $(\bar{u}_2)^{(1)} < 0$ the roots of the equations $(a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

**Definition of** $(m_1)^{(1)}$ , $(m_2)^{(1)}$ , $(\mu_1)^{(1)}, (\mu_2)^{(1)}, (v_0)^{(1)}$ :- 283

If we define $(m_1)^{(1)}$ , $(m_2)^{(1)}$ , $(\mu_1)^{(1)}, (\mu_2)^{(1)}$ by

$(m_2)^{(1)} = (v_0)^{(1)}, (m_1)^{(1)} = (v_1)^{(1)}$, $if\ (v_0)^{(1)} < (v_1)^{(1)}$

$(m_2)^{(1)} = (v_1)^{(1)}, (m_1)^{(1)} = (\bar{v}_1)^{(1)}$ , $if\ (v_1)^{(1)} < (v_0)^{(1)} < (\bar{v}_1)^{(1)}$,

and $\boxed{(v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$

$(m_2)^{(1)} = (v_1)^{(1)}, (m_1)^{(1)} = (v_0)^{(1)}$, $if\ (\bar{v}_1)^{(1)} < (v_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)}$, $if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}$ , $if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)}$,

and $\boxed{(u_0)^{(1)} = \frac{T_{13}^0}{T_{14}^0}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$

are defined

Then the solution of global equations satisfies the inequalities 285

$$G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{13}(t) \le G_{13}^0 e^{(S_1)^{(1)}t}$$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)}t}$$

$$\left(\frac{(a_{15})^{(1)} G_{13}^0}{(m_1)^{(1)}((S_1)^{(1)} - (p_{13})^{(1)} - (S_2)^{(1)})}\left[e^{((S_1)^{(1)} - (p_{13})^{(1)})t} - e^{-(S_2)^{(1)}t}\right] + G_{15}^0 e^{-(S_2)^{(1)}t} \le G_{15}(t) \le \right. \quad 286$$

$$\left. \frac{(a_{15})^{(1)} G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)} - (a'_{15})^{(1)})}\left[e^{(S_1)^{(1)}t} - e^{-(a'_{15})^{(1)}t}\right] + G_{15}^0 e^{-(a'_{15})^{(1)}t}\right)$$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)}t} \le T_{13}(t) \le T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t}} \quad 287$$

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)}t} \le T_{13}(t) \le \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t} \quad 288$$

$\frac{(b_{15})^{(1)}T_{13}^{0}}{(\mu_1)^{(1)}((R_1)^{(1)}-(b'_{15})^{(1)})}\left[e^{(R_1)^{(1)}t}-e^{-(b'_{15})^{(1)}t}\right]+T_{15}^{0}e^{-(b'_{15})^{(1)}t}\leq T_{15}(t)\leq$ 289

$\frac{(a_{15})^{(1)}T_{13}^{0}}{(\mu_2)^{(1)}((R_1)^{(1)}+(r_{13})^{(1)}+(R_2)^{(1)})}\left[e^{((R_1)^{(1)}+(r_{13})^{(1)})t}-e^{-(R_2)^{(1)}t}\right]+T_{15}^{0}e^{-(R_2)^{(1)}t}$

**Definition of** $(S_1)^{(1)},(S_2)^{(1)},(R_1)^{(1)},(R_2)^{(1)}$:- 290

Where $(S_1)^{(1)}=(a_{13})^{(1)}(m_2)^{(1)}-(a'_{13})^{(1)}$

$(S_2)^{(1)}=(a_{15})^{(1)}-(p_{15})^{(1)}$

$(R_1)^{(1)}=(b_{13})^{(1)}(\mu_2)^{(1)}-(b'_{13})^{(1)}$

$(R_2)^{(1)}=(b'_{15})^{(1)}-(r_{15})^{(1)}$

**Behavior of the solutions of equation** 291

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(2)},(\sigma_2)^{(2)},(\tau_1)^{(2)},(\tau_2)^{(2)}$ : 292

$(\sigma_1)^{(2)},(\sigma_2)^{(2)},(\tau_1)^{(2)},(\tau_2)^{(2)}$ four constants satisfying
$-(\sigma_2)^{(2)}\leq-(a'_{16})^{(2)}+(a'_{17})^{(2)}-(a''_{16})^{(2)}(\mathrm{T}_{17},t)+(a''_{17})^{(2)}(\mathrm{T}_{17},t)\leq-(\sigma_1)^{(2)}$ 293

$-(\tau_2)^{(2)}\leq-(b'_{16})^{(2)}+(b'_{17})^{(2)}-(b''_{16})^{(2)}\big((G_{19}),t\big)-(b''_{17})^{(2)}\big((G_{19}),t\big)\leq-(\tau_1)^{(2)}$ 294

**Definition of** $(\nu_1)^{(2)},(\nu_2)^{(2)},(u_1)^{(2)},(u_2)^{(2)}$ : 295

By $(\nu_1)^{(2)}>0$ , $(\nu_2)^{(2)}<0$ and respectively $(u_1)^{(2)}>0$ , $(u_2)^{(2)}<0$ the roots 296

of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2+(\sigma_1)^{(2)}\nu^{(2)}-(a_{16})^{(2)}=0$ 297

and $(b_{14})^{(2)}\big(u^{(2)}\big)^2+(\tau_1)^{(2)}u^{(2)}-(b_{16})^{(2)}=0$ and 298

**Definition of** $(\bar{\nu}_1)^{(2)},,(\bar{\nu}_2)^{(2)},(\bar{u}_1)^{(2)},(\bar{u}_2)^{(2)}$ : 299

By $(\bar{\nu}_1)^{(2)}>0$ , $(\bar{\nu}_2)^{(2)}<0$ and respectively $(\bar{u}_1)^{(2)}>0$ , $(\bar{u}_2)^{(2)}<0$ the 300

roots of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2+(\sigma_2)^{(2)}\nu^{(2)}-(a_{16})^{(2)}=0$ 301

and $(b_{17})^{(2)}\big(u^{(2)}\big)^2+(\tau_2)^{(2)}u^{(2)}-(b_{16})^{(2)}=0$ 302

**Definition of** $(m_1)^{(2)},(m_2)^{(2)},(\mu_1)^{(2)},(\mu_2)^{(2)}$ :- 303

If we define $(m_1)^{(2)},(m_2)^{(2)},(\mu_1)^{(2)},(\mu_2)^{(2)}$ by 304

$(m_2)^{(2)}=(\nu_0)^{(2)},(m_1)^{(2)}=(\nu_1)^{(2)},$ ***if*** $(\nu_0)^{(2)}<(\nu_1)^{(2)}$ 305

$(m_2)^{(2)}=(\nu_1)^{(2)},(m_1)^{(2)}=(\bar{\nu}_1)^{(2)}$ , ***if*** $(\nu_1)^{(2)}<(\nu_0)^{(2)}<(\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)},\ \boldsymbol{if}\ (u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}, \boldsymbol{if}\ (u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)},$

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)},\ \boldsymbol{if}\ (\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)} t}$

$(p_i)^{(2)}$ is defined by equation

$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)} t}$ 311

$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)} t}\right] + G_{18}^0 e^{-(S_2)^{(2)} t} \le G_{18}(t) \le\right.$ 312

$\left.\frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)} t} - e^{-(a'_{18})^{(2)} t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)} t}\right)$

$\boxed{T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}}$ 313

$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}$ 314

$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)} t} - e^{-(b'_{18})^{(2)} t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)} t} \le T_{18}(t) \le$ 315

$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})}\left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)} t}\right] + T_{18}^0 e^{-(R_2)^{(2)} t}$

**Definition of** $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$

$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)}$ 318

$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$

**Behavior of the solutions** 319

**Theorem 3:** If we denote and define

**Definition of** $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ :

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}\big((G_{23}), t\big) \le -(\tau_1)^{(3)}$$

**Definition of** $(v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$ : 320

By $(v_1)^{(3)} > 0$ , $(v_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$ , $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\big(v^{(3)}\big)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{v}_1)^{(3)} > 0$ , $(\bar{v}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$ , $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\big(v^{(3)}\big)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

**Definition of** $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ :- 321

If we define $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (v_0)^{(3)}, (m_1)^{(3)} = (v_1)^{(3)}$, ***if*** $(v_0)^{(3)} < (v_1)^{(3)}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (\bar{v}_1)^{(3)}$ , ***if*** $(v_1)^{(3)} < (v_0)^{(3)} < (\bar{v}_1)^{(3)}$,

and $\boxed{(v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_0)^{(3)}$, ***if*** $(\bar{v}_1)^{(3)} < (v_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}$, ***if*** $(u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$ , ***if*** $(u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}$, and $\boxed{(u_0)^{(3)} = \frac{T^0_{20}}{T^0_{21}}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}$, ***if*** $(\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$$G^0_{20}e^{((s_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G^0_{20}e^{(s_1)^{(3)}t}$$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)}-(p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)}t} \quad 323$$

$$\left(\frac{(a_{22})^{(3)}G_{20}^0}{(m_1)^{(3)}((S_1)^{(3)}-(p_{20})^{(3)}-(S_2)^{(3)})}\left[e^{((S_1)^{(3)}-(p_{20})^{(3)})t} - e^{-(S_2)^{(3)}t}\right] + G_{22}^0 e^{-(S_2)^{(3)}t} \le G_{22}(t) \le \right. \quad 324$$

$$\left. \frac{(a_{22})^{(3)}G_{20}^0}{(m_2)^{(3)}((S_1)^{(3)}-(a'_{22})^{(3)})}\left[e^{(S_1)^{(3)}t} - e^{-(a'_{22})^{(3)}t}\right] + G_{22}^0 e^{-(a'_{22})^{(3)}t}\right)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)}+(r_{20})^{(3)})t}} \quad 325$$

$$\frac{1}{(\mu_1)^{(3)}} T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}} T_{20}^0 e^{((R_1)^{(3)}+(r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)}T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)}-(b'_{22})^{(3)})}\left[e^{(R_1)^{(3)}t} - e^{-(b'_{22})^{(3)}t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)}t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)}T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)}+(r_{20})^{(3)}+(R_2)^{(3)})}\left[e^{((R_1)^{(3)}+(r_{20})^{(3)})t} - e^{-(R_2)^{(3)}t}\right] + T_{22}^0 e^{-(R_2)^{(3)}t}$$

**Definition of** $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$$

$$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$$

$$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$$

**Behavior of the solutions of equation**

**Theorem:** If we denote and define

**Definition of** $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ :

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$$

$$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}((G_{27}), t) - (b''_{25})^{(4)}((G_{27}), t) \le -(\tau_1)^{(4)}$$

**Definition of** $(v_1)^{(4)}, (v_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, v^{(4)}, u^{(4)}$ : 329

By $(v_1)^{(4)} > 0$ , $(v_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$ , $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_1)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

**Definition of** $(\bar{v}_1)^{(4)}, , (\bar{v}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$ : 330

By $(\bar{v}_1)^{(4)} > 0$ , $(\bar{v}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$ , $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

**Definition of** $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}, (\nu_0)^{(4)}$ :-

If we define $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (\nu_0)^{(4)}, (m_1)^{(4)} = (\nu_1)^{(4)}$, ***if*** $(\nu_0)^{(4)} < (\nu_1)^{(4)}$

$(m_2)^{(4)} = (\nu_1)^{(4)}, (m_1)^{(4)} = (\bar{\nu}_1)^{(4)}$, ***if*** $(\nu_4)^{(4)} < (\nu_0)^{(4)} < (\bar{\nu}_1)^{(4)}$,

and $\boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (\nu_4)^{(4)}, (m_1)^{(4)} = (\nu_0)^{(4)}$, ***if*** $(\bar{\nu}_4)^{(4)} < (\nu_0)^{(4)}$

and analogously

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$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)}$, ***if*** $(u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}$, ***if*** $(u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)}$,

and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}$, ***if*** $(\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

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$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)} t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)} t} \quad 333$$

$$\left( \frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})} \left[ e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)} t} \right] + G_{26}^0 e^{-(S_2)^{(4)} t} \le G_{26}(t) \le \right. \quad 334$$

*(a26)4G240(m2)4(S1)4−(a26′)4e(S1)4t−e−(a26′)4t+ G260e−(a26′)4t*

$$\boxed{T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}} T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}} T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t} \quad 335$$

$$\frac{(b_{26})^{(4)} T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)} - (b'_{26})^{(4)})} \left[ e^{(R_1)^{(4)} t} - e^{-(b'_{26})^{(4)} t} \right] + T_{26}^0 e^{-(b'_{26})^{(4)} t} \le T_{26}(t) \le \quad 336$$

$$\frac{(a_{26})^{(4)} T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)} + (r_{24})^{(4)} + (R_2)^{(4)})} \left[ e^{((R_1)^{(4)} + (r_{24})^{(4)})t} - e^{-(R_2)^{(4)} t} \right] + T_{26}^0 e^{-(R_2)^{(4)} t}$$

**Definition of** $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)} (m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$$

$$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$$

**Behavior of the solutions of equation** 338

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ :

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}((G_{31}), t) - (b''_{29})^{(5)}((G_{31}), t) \le -(\tau_1)^{(5)}$$

**Definition of** $(\nu_1)^{(5)}, (\nu_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, \nu^{(5)}, u^{(5)}$ : 339

By $(\nu_1)^{(5)} > 0$ , $(\nu_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$ , $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\left(u^{(5)}\right)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(5)}, , (\bar{\nu}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$ : 340

By $(\bar{\nu}_1)^{(5)} > 0$ , $(\bar{\nu}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$ , $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\left(u^{(5)}\right)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

**Definition of** $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (\nu_0)^{(5)}$ :-

If we define $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$$(m_2)^{(5)} = (\nu_0)^{(5)}, (m_1)^{(5)} = (\nu_1)^{(5)}, \ \boldsymbol{if}\ (\nu_0)^{(5)} < (\nu_1)^{(5)}$$

$$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\bar{\nu}_1)^{(5)}, \boldsymbol{if}\ (\nu_1)^{(5)} < (\nu_0)^{(5)} < (\bar{\nu}_1)^{(5)},$$

and $\boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}}$

$$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$$

and analogously 341

$$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)}, \ \boldsymbol{if}\ (u_0)^{(5)} < (u_1)^{(5)}$$

$$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}, \boldsymbol{if}\ (u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)},$$

and $\boxed{(u_0)^{(5)} = \frac{T^0_{28}}{T^0_{29}}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}, \boldsymbol{if}\ (\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$$G_{28}^{0}e^{((S_1)^{(5)}-(p_{28})^{(5)})t} \le G_{28}(t) \le G_{28}^{0}e^{(S_1)^{(5)}t}$$

where $(p_i)^{(5)}$ is defined by equation

$$\frac{1}{(m_5)^{(5)}}G_{28}^{0}e^{((S_1)^{(5)}-(p_{28})^{(5)})t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}}G_{28}^{0}e^{(S_1)^{(5)}t} \quad 343$$

$$\left(\frac{(a_{30})^{(5)}G_{28}^{0}}{(m_1)^{(5)}((S_1)^{(5)}-(p_{28})^{(5)}-(S_2)^{(5)})}\left[e^{((S_1)^{(5)}-(p_{28})^{(5)})t}-e^{-(S_2)^{(5)}t}\right]+G_{30}^{0}e^{-(S_2)^{(5)}t} \le G_{30}(t) \le \right. \quad 344$$

*(a30)5G280(m2)5(S1)5−(a30′)5e(S1)5t−e−(a30′)5t+ G300e−(a30′)5t*

$$\boxed{T_{28}^{0}e^{(R_1)^{(5)}t} \le T_{28}(t) \le T_{28}^{0}e^{((R_1)^{(5)}+(r_{28})^{(5)})t}} \quad 345$$

$$\frac{1}{(\mu_1)^{(5)}}T_{28}^{0}e^{(R_1)^{(5)}t} \le T_{28}(t) \le \frac{1}{(\mu_2)^{(5)}}T_{28}^{0}e^{((R_1)^{(5)}+(r_{28})^{(5)})t} \quad 346$$

$$\frac{(b_{30})^{(5)}T_{28}^{0}}{(\mu_1)^{(5)}((R_1)^{(5)}-(b'_{30})^{(5)})}\left[e^{(R_1)^{(5)}t}-e^{-(b'_{30})^{(5)}t}\right]+T_{30}^{0}e^{-(b'_{30})^{(5)}t} \le T_{30}(t) \le \quad 347$$

$$\frac{(a_{30})^{(5)}T_{28}^{0}}{(\mu_2)^{(5)}((R_1)^{(5)}+(r_{28})^{(5)}+(R_2)^{(5)})}\left[e^{((R_1)^{(5)}+(r_{28})^{(5)})t}-e^{-(R_2)^{(5)}t}\right]+T_{30}^{0}e^{-(R_2)^{(5)}t}$$

**Definition of** $(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$:- 348

Where $(S_1)^{(5)} = (a_{28})^{(5)}(m_2)^{(5)} - (a'_{28})^{(5)}$

$(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}$

$(R_1)^{(5)} = (b_{28})^{(5)}(\mu_2)^{(5)} - (b'_{28})^{(5)}$

$(R_2)^{(5)} = (b'_{30})^{(5)} - (r_{30})^{(5)}$

**Behavior of the solutions of equation** 349

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ :

$(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)} \le -(a'_{32})^{(6)} + (a'_{33})^{(6)} - (a''_{32})^{(6)}(T_{33}, t) + (a''_{33})^{(6)}(T_{33}, t) \le -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)} \le -(b'_{32})^{(6)} + (b'_{33})^{(6)} - (b''_{32})^{(6)}\big((G_{35}), t\big) - (b''_{33})^{(6)}\big((G_{35}), t\big) \le -(\tau_1)^{(6)}$$

**Definition of** $(v_1)^{(6)}, (v_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, v^{(6)}, u^{(6)}$ : 350

By $(v_1)^{(6)} > 0$ , $(v_2)^{(6)} < 0$ and respectively $(u_1)^{(6)} > 0$ , $(u_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(v^{(6)}\big)^2 + (\sigma_1)^{(6)}v^{(6)} - (a_{32})^{(6)} = 0$

and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$ and

**Definition of** $(\bar{v}_1)^{(6)},,(\bar{v}_2)^{(6)},(\bar{u}_1)^{(6)},(\bar{u}_2)^{(6)}$ : 351

By $(\bar{v}_1)^{(6)} > 0$ , $(\bar{v}_2)^{(6)} < 0$ and respectively $(\bar{u}_1)^{(6)} > 0$ , $(\bar{u}_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\left(v^{(6)}\right)^2 + (\sigma_2)^{(6)}v^{(6)} - (a_{32})^{(6)} = 0$ and $(b_{33})^{(6)}\left(u^{(6)}\right)^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$

**Definition of** $(m_1)^{(6)}$ , $(m_2)^{(6)}$ , $(\mu_1)^{(6)},(\mu_2)^{(6)},(v_0)^{(6)}$ :-

If we define $(m_1)^{(6)}$ , $(m_2)^{(6)}$ , $(\mu_1)^{(6)},(\mu_2)^{(6)}$ by

$(m_2)^{(6)} = (v_0)^{(6)}, (m_1)^{(6)} = (v_1)^{(6)},$ ***if*** $(v_0)^{(6)} < (v_1)^{(6)}$

$(m_2)^{(6)} = (v_1)^{(6)}, (m_1)^{(6)} = (\bar{v}_6)^{(6)}$ , ***if*** $(v_1)^{(6)} < (v_0)^{(6)} < (\bar{v}_1)^{(6)}$,
and $\boxed{(v_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$

$(m_2)^{(6)} = (v_1)^{(6)}, (m_1)^{(6)} = (v_0)^{(6)},$ ***if*** $(\bar{v}_1)^{(6)} < (v_0)^{(6)}$

and analogously 352

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)},$ ***if*** $(u_0)^{(6)} < (u_1)^{(6)}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}$ , ***if*** $(u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)}$,
and $\boxed{(u_0)^{(6)} = \frac{T_{32}^0}{T_{33}^0}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)},$ ***if*** $(\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$

Then the solution of global equations satisfies the inequalities 353

$$G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{32}(t) \le G_{32}^0 e^{(S_1)^{(6)}t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}} G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{33}(t) \le \frac{1}{(m_2)^{(6)}} G_{32}^0 e^{(S_1)^{(6)}t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)} G_{32}^0}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})}\left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)}t}\right] + G_{34}^0 e^{-(S_2)^{(6)}t} \le G_{34}(t) \le \right. \quad 355$$

*(a34)6G320(m2)6(S1)6−(a34′)6e(S1)6t−e−(a34′)6t+ G340e−(a34′)6t*

$$\boxed{T_{32}^0 e^{(R_1)^{(6)}t} \le T_{32}(t) \le T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}} T_{32}^0 e^{(R_1)^{(6)}t} \le T_{32}(t) \le \frac{1}{(\mu_2)^{(6)}} T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)} T_{32}^0}{(\mu_1)^{(6)}((R_1)^{(6)} - (b'_{34})^{(6)})}\left[e^{(R_1)^{(6)}t} - e^{-(b'_{34})^{(6)}t}\right] + T_{34}^0 e^{-(b'_{34})^{(6)}t} \le T_{34}(t) \le \quad 358$$

$$\frac{(a_{34})^{(6)} T_{32}^0}{(\mu_2)^{(6)}((R_1)^{(6)} + (r_{32})^{(6)} + (R_2)^{(6)})}\left[e^{((R_1)^{(6)} + (r_{32})^{(6)})t} - e^{-(R_2)^{(6)}t}\right] + T_{34}^0 e^{-(R_2)^{(6)}t}$$

**Definition of** $(S_1)^{(6)}, (S_2)^{(6)}, (R_1)^{(6)}, (R_2)^{(6)}$:- 359

Where $(S_1)^{(6)} = (a_{32})^{(6)}(m_2)^{(6)} - (a'_{32})^{(6)}$

$(S_2)^{(6)} = (a_{34})^{(6)} - (p_{34})^{(6)}$

$(R_1)^{(6)} = (b_{32})^{(6)}(\mu_2)^{(6)} - (b'_{32})^{(6)}$

$(R_2)^{(6)} = (b'_{34})^{(6)} - (r_{34})^{(6)}$

**Behavior of the solutions of equation**

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$ :

$(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)} \le -(a'_{36})^{(7)} + (a'_{37})^{(7)} - (a''_{36})^{(7)}(T_{37}, t) + (a''_{37})^{(7)}(T_{37}, t) \le -(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)} \le -(b'_{36})^{(7)} + (b'_{37})^{(7)} - (b''_{36})^{(7)}\big((G_{39}), t\big) - (b''_{37})^{(7)}\big((G_{39}), t\big) \le -(\tau_1)^{(7)}$$

**Definition of** $(\nu_1)^{(7)}, (\nu_2)^{(7)}, (u_1)^{(7)}, (u_2)^{(7)}, \nu^{(7)}, u^{(7)}$ : 361

By $(\nu_1)^{(7)} > 0$, $(\nu_2)^{(7)} < 0$ and respectively $(u_1)^{(7)} > 0$, $(u_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_1)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(7)},, (\bar{\nu}_2)^{(7)}, (\bar{u}_1)^{(7)}, (\bar{u}_2)^{(7)}$ : 362

By $(\bar{\nu}_1)^{(7)} > 0$, $(\bar{\nu}_2)^{(7)} < 0$ and respectively $(\bar{u}_1)^{(7)} > 0$, $(\bar{u}_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

**Definition of** $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (\nu_0)^{(7)}$ :-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$(m_2)^{(7)} = (\nu_0)^{(7)}, (m_1)^{(7)} = (\nu_1)^{(7)}$, ***if*** $(\nu_0)^{(7)} < (\nu_1)^{(7)}$

$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\bar{\nu}_1)^{(7)}$, ***if*** $(\nu_1)^{(7)} < (\nu_0)^{(7)} < (\bar{\nu}_1)^{(7)}$,
and $\boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$

$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\nu_0)^{(7)}$, ***if*** $(\bar{\nu}_1)^{(7)} < (\nu_0)^{(7)}$

and analogously 363

$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)},\ \boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$

and $\boxed{(u_0)^{(7)} = \frac{T_{36}^0}{T_{37}^0}}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)}$ where $(u_1)^{(7)}, (\bar{u}_1)^{(7)}$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^0 e^{(S_1)^{(7)} t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)} t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)}((S_1)^{(7)} - (p_{36})^{(7)} - (S_2)^{(7)})}\left[e^{((S_1)^{(7)} - (p_{36})^{(7)})t} - e^{-(S_2)^{(7)} t}\right] + G_{38}^0 e^{-(S_2)^{(7)} t} \le G_{38}(t) \le \right. \quad 366$$

$$\left. \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)}((S_1)^{(7)} - (a'_{38})^{(7)})}\left[e^{(S_1)^{(7)} t} - e^{-(a'_{38})^{(7)} t}\right] + G_{38}^0 e^{-(a'_{38})^{(7)} t}\right)$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t} \quad 368$$

$$\frac{(b_{38})^{(7)} T_{36}^0}{(\mu_1)^{(7)}((R_1)^{(7)} - (b'_{38})^{(7)})}\left[e^{(R_1)^{(7)} t} - e^{-(b'_{38})^{(7)} t}\right] + T_{38}^0 e^{-(b'_{38})^{(7)} t} \le T_{38}(t) \le \quad 369$$

$$\frac{(a_{38})^{(7)} T_{36}^0}{(\mu_2)^{(7)}((R_1)^{(7)} + (r_{36})^{(7)} + (R_2)^{(7)})}\left[e^{((R_1)^{(7)} + (r_{36})^{(7)})t} - e^{-(R_2)^{(7)} t}\right] + T_{38}^0 e^{-(R_2)^{(7)} t}$$

**Definition of** $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$

$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$

$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$

**Behavior of the solutions of equation** 371

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ :

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$$-(\sigma_2)^{(8)} \le -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \le -(\sigma_1)^{(8)}$$

$$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\big((G_{43}), t\big) - (b''_{41})^{(8)}\big((G_{43}), t\big) \le -(\tau_1)^{(8)}$$

**Definition of** $(\nu_1)^{(8)}, (\nu_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, \nu^{(8)}, u^{(8)}$ : 372

By $(\nu_1)^{(8)} > 0, (\nu_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0, (u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(8)}, , (\bar{\nu}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$ :

By $(\bar{\nu}_1)^{(8)} > 0, (\bar{\nu}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0, (\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

**Definition of** $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}, (\nu_0)^{(8)}$ :-

If we define $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$(m_2)^{(8)} = (\nu_0)^{(8)}, (m_1)^{(8)} = (\nu_1)^{(8)}$, ***if*** $(\nu_0)^{(8)} < (\nu_1)^{(8)}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\bar{\nu}_1)^{(8)}$, ***if*** $(\nu_1)^{(8)} < (\nu_0)^{(8)} < (\bar{\nu}_1)^{(8)}$,
and $\boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\nu_0)^{(8)}$, ***if*** $(\bar{\nu}_1)^{(8)} < (\nu_0)^{(8)}$

and analogously 374

$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)}$, ***if*** $(u_0)^{(8)} < (u_1)^{(8)}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}$, ***if*** $(u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)}$,
and $\boxed{(u_0)^{(8)} = \frac{T^0_{40}}{T^0_{41}}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}$, ***if*** $(\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^{0}e^{((S_1)^{(8)}-(p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^{0}e^{(S_1)^{(8)}t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}}G_{40}^{0}e^{((S_1)^{(8)}-(p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}}G_{40}^{0}e^{(S_1)^{(8)}t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)}G_{40}^{0}}{(m_1)^{(8)}((S_1)^{(8)}-(p_{40})^{(8)}-(S_2)^{(8)})}\left[e^{((S_1)^{(8)}-(p_{40})^{(8)})t}-e^{-(S_2)^{(8)}t}\right]+G_{42}^{0}e^{-(S_2)^{(8)}t} \le G_{42}(t) \le \right. \quad 377$$

$$\left.\frac{(a_{42})^{(8)}G_{40}^{0}}{(m_2)^{(8)}((S_1)^{(8)}-(a'_{42})^{(8)})}[e^{(S_1)^{(8)}t}-e^{-(a'_{42})^{(8)}t}]+\ G_{42}^{0}e^{-(a'_{42})^{(8)}t}\right)$$

$$\boxed{T_{40}^{0}e^{(R_1)^{(8)}t} \le T_{40}(t) \le T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}}T_{40}^{0}e^{(R_1)^{(8)}t} \le T_{40}(t) \le \frac{1}{(\mu_2)^{(8)}}T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)}T_{40}^{0}}{(\mu_1)^{(8)}((R_1)^{(8)}-(b'_{42})^{(8)})}\left[e^{(R_1)^{(8)}t}-e^{-(b'_{42})^{(8)}t}\right]+T_{42}^{0}e^{-(b'_{42})^{(8)}t} \le T_{42}(t) \le \quad 380$$

$$\frac{(a_{42})^{(8)}T_{40}^{0}}{(\mu_2)^{(8)}((R_1)^{(8)}+(r_{40})^{(8)}+(R_2)^{(8)})}\left[e^{((R_1)^{(8)}+(r_{40})^{(8)})t}-e^{-(R_2)^{(8)}t}\right]+T_{42}^{0}e^{-(R_2)^{(8)}t}$$

**Definition of** $(S_1)^{(8)}, (S_2)^{(8)}, (R_1)^{(8)}, (R_2)^{(8)}$:- 381

Where $(S_1)^{(8)} = (a_{40})^{(8)}(m_2)^{(8)} - (a'_{40})^{(8)}$

$$(S_2)^{(8)} = (a_{42})^{(8)} - (p_{42})^{(8)}$$

$$(R_1)^{(8)} = (b_{40})^{(8)}(\mu_2)^{(8)} - (b'_{40})^{(8)}$$

$$(R_2)^{(8)} = (b'_{42})^{(8)} - (r_{42})^{(8)}$$

**Behavior of the solutions of equation 37 to 92** 382

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ :

$(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ four constants satisfying

$$-(\sigma_2)^{(9)} \le -(a'_{44})^{(9)} + (a'_{45})^{(9)} - (a''_{44})^{(9)}(T_{45}, t) + (a''_{45})^{(9)}(T_{45}, t) \le -(\sigma_1)^{(9)}$$

$$-(\tau_2)^{(9)} \le -(b'_{44})^{(9)} + (b'_{45})^{(9)} - (b''_{44})^{(9)}\big((G_{47}), t\big) - (b''_{45})^{(9)}\big((G_{47}), t\big) \le -(\tau_1)^{(9)}$$

**Definition of** $(\nu_1)^{(9)}, (\nu_2)^{(9)}, (u_1)^{(9)}, (u_2)^{(9)}, \nu^{(9)}, u^{(9)}$ :

By $(\nu_1)^{(9)} > 0$ , $(\nu_2)^{(9)} < 0$ and respectively $(u_1)^{(9)} > 0$ , $(u_2)^{(9)} < 0$ the roots of the equations

$(a_{45})^{(9)}\left(v^{(9)}\right)^2+(\sigma_1)^{(9)}v^{(9)}-(a_{44})^{(9)}=0$
and $(b_{45})^{(9)}\left(u^{(9)}\right)^2+(\tau_1)^{(9)}u^{(9)}-(b_{44})^{(9)}=0$ and

**Definition of** $(\bar{v}_1)^{(9)},,(\bar{v}_2)^{(9)},(\bar{u}_1)^{(9)},(\bar{u}_2)^{(9)}$ :

By $(\bar{v}_1)^{(9)}>0$ , $(\bar{v}_2)^{(9)}<0$ and respectively $(\bar{u}_1)^{(9)}>0$ , $(\bar{u}_2)^{(9)}<0$ the roots of the equations $(a_{45})^{(9)}\left(v^{(9)}\right)^2+(\sigma_2)^{(9)}v^{(9)}-(a_{44})^{(9)}=0$
and $(b_{45})^{(9)}\left(u^{(9)}\right)^2+(\tau_2)^{(9)}u^{(9)}-(b_{44})^{(9)}=0$

**Definition of** $(m_1)^{(9)}$ , $(m_2)^{(9)}$ , $(\mu_1)^{(9)},(\mu_2)^{(9)},(\nu_0)^{(9)}$ :-

If we define $(m_1)^{(9)}$ , $(m_2)^{(9)}$ , $(\mu_1)^{(9)},(\mu_2)^{(9)}$ by

$(m_2)^{(9)}=(\nu_0)^{(9)},(m_1)^{(9)}=(\nu_1)^{(9)},$ ***if*** $(\nu_0)^{(9)}<(\nu_1)^{(9)}$

$(m_2)^{(9)}=(\nu_1)^{(9)},(m_1)^{(9)}=(\bar{\nu}_1)^{(9)}$ , ***if*** $(\nu_1)^{(9)}<(\nu_0)^{(9)}<(\bar{\nu}_1)^{(9)}$,
and $\boxed{(\nu_0)^{(9)}=\frac{G_{44}^0}{G_{45}^0}}$

$(m_2)^{(9)}=(\nu_1)^{(9)},(m_1)^{(9)}=(\nu_0)^{(9)},$ ***if*** $(\bar{\nu}_1)^{(9)}<(\nu_0)^{(9)}$

and analogously

$(\mu_2)^{(9)}=(u_0)^{(9)},(\mu_1)^{(9)}=(u_1)^{(9)},$ ***if*** $(u_0)^{(9)}<(u_1)^{(9)}$

$(\mu_2)^{(9)}=(u_1)^{(9)},(\mu_1)^{(9)}=(\bar{u}_1)^{(9)}$ , ***if*** $(u_1)^{(9)}<(u_0)^{(9)}<(\bar{u}_1)^{(9)}$,
and $\boxed{(u_0)^{(9)}=\frac{T_{44}^0}{T_{45}^0}}$

$(\mu_2)^{(9)}=(u_1)^{(9)},(\mu_1)^{(9)}=(u_0)^{(9)}$, ***if*** $(\bar{u}_1)^{(9)}<(u_0)^{(9)}$ where $(u_1)^{(9)},(\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G_{44}^0e^{((S_1)^{(9)}-(p_{44})^{(9)})t}\le G_{44}(t)\le G_{44}^0e^{(S_1)^{(9)}t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}}G_{44}^0e^{((S_1)^{(9)}-(p_{44})^{(9)})t}\le G_{45}(t)\le\frac{1}{(m_2)^{(9)}}G_{44}^0e^{(S_1)^{(9)}t}$$

$$\frac{(a_{46})^{(9)}G_{44}^0}{(m_1)^{(9)}((S_1)^{(9)}-(p_{44})^{(9)}-(S_2)^{(9)})}\left[e^{((S_1)^{(9)}-(p_{44})^{(9)})t}-e^{-(S_2)^{(9)}t}\right]+G_{46}^0e^{-(S_2)^{(9)}t}\le G_{46}(t)\le$$

$$\frac{(a_{46})^{(9)}G_{44}^0}{(m_2)^{(9)}((S_1)^{(9)}-(a'_{46})^{(9)})}[e^{(S_1)^{(9)}t}-e^{-(a'_{46})^{(9)}t}]+G_{46}^0e^{-(a'_{46})^{(9)}t})$$

$$\boxed{T_{44}^0e^{(R_1)^{(9)}t}\le T_{44}(t)\le T_{44}^0e^{((R_1)^{(9)}+(r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}}T_{44}^0e^{(R_1)^{(9)}t}\le T_{44}(t)\le\frac{1}{(\mu_2)^{(9)}}T_{44}^0e^{((R_1)^{(9)}+(r_{44})^{(9)})t}$$

$$\frac{(b_{46})^{(9)}T_{44}^{0}}{(\mu_1)^{(9)}((R_1)^{(9)}-(b'_{46})^{(9)})}\left[e^{(R_1)^{(9)}t}-e^{-(b'_{46})^{(9)}t}\right]+T_{46}^{0}e^{-(b'_{46})^{(9)}t}\leq T_{46}(t)\leq$$

$$\frac{(a_{46})^{(9)}T_{44}^{0}}{(\mu_2)^{(9)}((R_1)^{(9)}+(r_{44})^{(9)}+(R_2)^{(9)})}\left[e^{((R_1)^{(9)}+(r_{44})^{(9)})t}-e^{-(R_2)^{(9)}t}\right]+T_{46}^{0}e^{-(R_2)^{(9)}t}$$

**Definition of** $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)}=(a_{44})^{(9)}(m_2)^{(9)}-(a'_{44})^{(9)}$

$(S_2)^{(9)}=(a_{46})^{(9)}-(p_{46})^{(9)}$

$(R_1)^{(9)}=(b_{44})^{(9)}(\mu_2)^{(9)}-(b'_{44})^{(9)}$

$(R_2)^{(9)}=(b'_{46})^{(9)}-(r_{46})^{(9)}$

**Proof :** From global equations we obtain 383

$$\frac{dv^{(1)}}{dt}=(a_{13})^{(1)}-\left((a'_{13})^{(1)}-(a'_{14})^{(1)}+(a''_{13})^{(1)}(T_{14},t)\right)-(a''_{14})^{(1)}(T_{14},t)v^{(1)}-(a_{14})^{(1)}v^{(1)}$$

**Definition of** $v^{(1)}$ :- $\boxed{v^{(1)}=\frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2+(\sigma_2)^{(1)}v^{(1)}-(a_{13})^{(1)}\right)\leq\frac{dv^{(1)}}{dt}\leq-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2+(\sigma_1)^{(1)}v^{(1)}-(a_{13})^{(1)}\right)$$

From which one obtains

**Definition of** $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$ :-

For $0<\boxed{(v_0)^{(1)}=\frac{G_{13}^{0}}{G_{14}^{0}}}<(v_1)^{(1)}<(\bar{v}_1)^{(1)}$

$$v^{(1)}(t)\geq\frac{(v_1)^{(1)}+(C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}}\quad,\quad\boxed{(C)^{(1)}=\frac{(v_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(v_2)^{(1)}}}$$

it follows $(v_0)^{(1)}\leq v^{(1)}(t)\leq(v_1)^{(1)}$

In the same manner , we get 384

$$v^{(1)}(t)\leq\frac{(\bar{v}_1)^{(1)}+(\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}}\quad,\quad\boxed{(\bar{C})^{(1)}=\frac{(\bar{v}_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(\bar{v}_2)^{(1)}}}$$

From which we deduce $(v_0)^{(1)}\leq v^{(1)}(t)\leq(\bar{v}_1)^{(1)}$

If $0<(v_1)^{(1)}<(v_0)^{(1)}=\frac{G_{13}^{0}}{G_{14}^{0}}<(\bar{v}_1)^{(1)}$ we find like in the previous case, 385

$$(\nu_1)^{(1)} \leq \frac{(\nu_1)^{(1)}+(C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}} \leq \nu^{(1)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\bar{\nu}_1)^{(1)}$$

If $0 < (\nu_1)^{(1)} \leq (\bar{\nu}_1)^{(1)} \leq \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$, we obtain 386

$$(\nu_1)^{(1)} \leq \nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\nu_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(1)}(t)$ :-

$$(m_2)^{(1)} \leq \nu^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{\nu^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(1)}(t)$ :-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{13}'')^{(1)} = (a_{14}'')^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)}G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b_{13}'')^{(1)} = (b_{14}'')^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)}T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

**Proof** : From global equations we obtain 387

$$\frac{d\nu^{(2)}}{dt} = (a_{16})^{(2)} - \left((a_{16}')^{(2)} - (a_{17}')^{(2)} + (a_{16}'')^{(2)}(T_{17}, t)\right) - (a_{17}'')^{(2)}(T_{17}, t)\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

**Definition of** $\nu^{(2)}$ :- $\boxed{\nu^{(2)} = \frac{G_{16}}{G_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(v^{(2)}\right)^2+(\sigma_2)^{(2)}v^{(2)}-(a_{16})^{(2)}\right)\leq\frac{dv^{(2)}}{dt}\leq-\left((a_{17})^{(2)}\left(v^{(2)}\right)^2+(\sigma_1)^{(2)}v^{(2)}-(a_{16})^{(2)}\right)$$

From which one obtains 390

**Definition of** $(\bar{\nu}_1)^{(2)},(\nu_0)^{(2)}$ :-

For $0<(\nu_0)^{(2)}=\frac{G_{16}^0}{G_{17}^0}<(\nu_1)^{(2)}<(\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t)\geq\frac{(\nu_1)^{(2)}+(C)^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_0)^{(2)}\right)t\right]}}{1+(C)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_0)^{(2)}\right)t\right]}}\quad,\quad\boxed{(C)^{(2)}=\frac{(\nu_1)^{(2)}-(\nu_0)^{(2)}}{(\nu_0)^{(2)}-(\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)}\leq\nu^{(2)}(t)\leq(\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t)\leq\frac{(\bar{\nu}_1)^{(2)}+(\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}\quad,\quad\boxed{(\bar{C})^{(2)}=\frac{(\bar{\nu}_1)^{(2)}-(\nu_0)^{(2)}}{(\nu_0)^{(2)}-(\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)}\leq\nu^{(2)}(t)\leq(\bar{\nu}_1)^{(2)}$ 392

If $0<(\nu_1)^{(2)}<(\nu_0)^{(2)}=\frac{G_{16}^0}{G_{17}^0}<(\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)}\leq\frac{(\nu_1)^{(2)}+(C)^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}}{1+(C)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}}\leq\nu^{(2)}(t)\leq$$

$$\frac{(\bar{\nu}_1)^{(2)}+(\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}\leq(\bar{\nu}_1)^{(2)}$$

If $0<(\nu_1)^{(2)}\leq(\bar{\nu}_1)^{(2)}\leq(\nu_0)^{(2)}=\frac{G_{16}^0}{G_{17}^0}$ , we obtain 394

$$(\nu_1)^{(2)}\leq\nu^{(2)}(t)\leq\frac{(\bar{\nu}_1)^{(2)}+(\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}\leq(\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(2)}(t)$ :- 395

$$(m_2)^{(2)}\leq\nu^{(2)}(t)\leq(m_1)^{(2)},\quad\boxed{\nu^{(2)}(t)=\frac{G_{16}(t)}{G_{17}(t)}}$$

In a completely analogous way, we obtain 396

**Definition of** $u^{(2)}(t)$ :-

$$(\mu_2)^{(2)}\leq u^{(2)}(t)\leq(\mu_1)^{(2)},\quad\boxed{u^{(2)}(t)=\frac{T_{16}(t)}{T_{17}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :** 397

If $(a''_{16})^{(2)} = (a''_{17})^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)} G_{17}(t)$

Analogously if $(b''_{16})^{(2)} = (b''_{17})^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$ if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)} T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

**Proof** : From global equations we obtain 398

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right) - (a''_{21})^{(3)}(T_{21}, t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

**Definition of** $\nu^{(3)}$ :- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right) \leq \frac{d\nu^{(3)}}{dt} \leq -\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right)$$

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From which one obtains

For $0 < (\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\nu_1)^{(3)} < (\bar{\nu}_1)^{(3)}$

$$\nu^{(3)}(t) \geq \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(\nu_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\nu_2)^{(3)}}}$$

it follows $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\nu_1)^{(3)}$

In the same manner , we get 401

$$\nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)} + (\bar{C})^{(3)}(\bar{\nu}_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)} e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{\nu}_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\bar{\nu}_2)^{(3)}}}$$

**Definition of** $(\bar{\nu}_1)^{(3)}$ :-

From which we deduce $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\bar{\nu}_1)^{(3)}$

If $0 < (\nu_1)^{(3)} < (\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\bar{\nu}_1)^{(3)}$ we find like in the previous case, 402

$$(\nu_1)^{(3)} \le \frac{(\nu_1)^{(3)}+(C)^{(3)}(\nu_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}} \le \nu^{(3)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \le (\bar{\nu}_1)^{(3)}$$

If $0 < (\nu_1)^{(3)} \le (\bar{\nu}_1)^{(3)} \le (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}$ , we obtain 403

$$(\nu_1)^{(3)} \le \nu^{(3)}(t) \le \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \le (\nu_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(3)}(t)$ :-

$$(m_2)^{(3)} \le \nu^{(3)}(t) \le (m_1)^{(3)}, \quad \boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(3)}(t)$ :-

$$(\mu_2)^{(3)} \le u^{(3)}(t) \le (\mu_1)^{(3)}, \quad \boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{20}'')^{(3)} = (a_{21}'')^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)}G_{21}(t)$

Analogously if $(b_{20}'')^{(3)} = (b_{21}'')^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)}T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

**Proof** : From global equations we obtain 404

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a_{24}')^{(4)} - (a_{25}')^{(4)} + (a_{24}'')^{(4)}(T_{25},t)\right) - (a_{25}'')^{(4)}(T_{25},t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

**Definition of** $\nu^{(4)}$ :- $\quad \boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right) \le \frac{d\nu^{(4)}}{dt} \le -\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_4)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$ :-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \geq \frac{(\nu_1)^{(4)}+(C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_0)^{(4)}\right)t\right]}}{4+(C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)}-(\nu_0)^{(4)}}{(\nu_0)^{(4)}-(\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{4+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)}-(\nu_0)^{(4)}}{(\nu_0)^{(4)}-(\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \leq \frac{(\nu_1)^{(4)}+(C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_2)^{(4)}\right)t\right]}}{1+(C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_2)^{(4)}\right)t\right]}} \leq \nu^{(4)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \leq (\bar{\nu}_1)^{(4)} \leq \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$ , we obtain 407

$$(\nu_1)^{(4)} \leq \nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(4)}(t)$ :-

$$(m_2)^{(4)} \leq \nu^{(4)}(t) \leq (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(4)}(t)$ :-

$$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{24})^{(4)} = (a''_{25})^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)}G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b''_{24})^{(4)} = (b''_{25})^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then

$(u_1)^{(4)} = (\bar{u}_4)^{(4)}$if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)}T_{25}(t)$ This is an important consequence of the relation between $(v_1)^{(4)}$ and $(\bar{v}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

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**Proof** : From global equations we obtain

$$\frac{dv^{(5)}}{dt} = (a_{28})^{(5)} - \left((a'_{28})^{(5)} - (a'_{29})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right) - (a''_{29})^{(5)}(T_{29}, t)v^{(5)} - (a_{29})^{(5)}v^{(5)}$$

**Definition of** $v^{(5)}$ :- $\boxed{v^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(v^{(5)}\right)^2 + (\sigma_2)^{(5)}v^{(5)} - (a_{28})^{(5)}\right) \leq \frac{dv^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(v^{(5)}\right)^2 + (\sigma_1)^{(5)}v^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

**Definition of** $(\bar{v}_1)^{(5)}, (v_0)^{(5)}$ :-

For $0 < \boxed{(v_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}} < (v_1)^{(5)} < (\bar{v}_1)^{(5)}$

$$v^{(5)}(t) \geq \frac{(v_1)^{(5)} + (C)^{(5)}(v_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((v_1)^{(5)} - (v_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)}e^{\left[-(a_{29})^{(5)}\left((v_1)^{(5)} - (v_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(v_1)^{(5)} - (v_0)^{(5)}}{(v_0)^{(5)} - (v_2)^{(5)}}}$$

it follows $(v_0)^{(5)} \leq v^{(5)}(t) \leq (v_1)^{(5)}$

In the same manner , we get 409

$$v^{(5)}(t) \leq \frac{(\bar{v}_1)^{(5)} + (\bar{C})^{(5)}(\bar{v}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{v}_1)^{(5)} - (\bar{v}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{v}_1)^{(5)} - (\bar{v}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{v}_1)^{(5)} - (v_0)^{(5)}}{(v_0)^{(5)} - (\bar{v}_2)^{(5)}}}$$

From which we deduce $(v_0)^{(5)} \leq v^{(5)}(t) \leq (\bar{v}_5)^{(5)}$

If $0 < (v_1)^{(5)} < (v_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}} < (\bar{v}_1)^{(5)}$ we find like in the previous case, 410

$$(v_1)^{(5)} \leq \frac{(v_1)^{(5)} + (C)^{(5)}(v_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((v_1)^{(5)} - (v_2)^{(5)}\right)t\right]}}{1 + (C)^{(5)}e^{\left[-(a_{29})^{(5)}\left((v_1)^{(5)} - (v_2)^{(5)}\right)t\right]}} \leq v^{(5)}(t) \leq$$

$$\frac{(\bar{v}_1)^{(5)} + (\bar{C})^{(5)}(\bar{v}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{v}_1)^{(5)} - (\bar{v}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{v}_1)^{(5)} - (\bar{v}_2)^{(5)}\right)t\right]}} \leq (\bar{v}_1)^{(5)}$$

If $0 < (v_1)^{(5)} \leq (\bar{v}_1)^{(5)} \leq \boxed{(v_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}}$ , we obtain 411

$$(v_1)^{(5)} \leq v^{(5)}(t) \leq \frac{(\bar{v}_1)^{(5)} + (\bar{C})^{(5)}(\bar{v}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{v}_1)^{(5)} - (\bar{v}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{v}_1)^{(5)} - (\bar{v}_2)^{(5)}\right)t\right]}} \leq (v_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $v^{(5)}(t)$ :-

$(m_2)^{(5)} \leq v^{(5)}(t) \leq (m_1)^{(5)}$, $\boxed{v^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$

In a completely analogous way, we obtain

**Definition of** $u^{(5)}(t)$ :-

$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}$, $\boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{28})^{(5)} = (a''_{29})^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $v^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)} G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b''_{28})^{(5)} = (b''_{29})^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$ if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)} T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

**Proof** : From global equations we obtain 412

$$\frac{dv^{(6)}}{dt} = (a_{32})^{(6)} - \left((a'_{32})^{(6)} - (a'_{33})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right) - (a''_{33})^{(6)}(T_{33}, t)v^{(6)} - (a_{33})^{(6)}v^{(6)}$$

**Definition of** $v^{(6)}$ :- $\boxed{v^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(v^{(6)}\right)^2 + (\sigma_2)^{(6)}v^{(6)} - (a_{32})^{(6)}\right) \leq \frac{dv^{(6)}}{dt} \leq -\left((a_{33})^{(6)}\left(v^{(6)}\right)^2 + (\sigma_1)^{(6)}v^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$ :-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$v^{(6)}(t) \geq \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \leq v^{(6)}(t) \leq (\nu_1)^{(6)}$

In the same manner , we get 413

$$v^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{\nu}_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\bar{\nu}_1)^{(6)}$

If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 414

$$(\nu_1)^{(6)} \leq \frac{(\nu_1)^{(6)}+(C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_2)^{(6)}\right)t\right]}}{1+(C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_2)^{(6)}\right)t\right]}} \leq \nu^{(6)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\bar{\nu}_1)^{(6)}$$

If $0 < (\nu_1)^{(6)} \leq (\bar{\nu}_1)^{(6)} \leq \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$, we obtain 415

$$(\nu_1)^{(6)} \leq \nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(6)}(t)$ :-

$$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}, \quad \boxed{\nu^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(6)}(t)$ :-

$$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}, \quad \boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)}G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)}T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

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**Proof** : From global equations we obtain

$$\frac{d\nu^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right) - (a''_{37})^{(7)}(T_{37}, t)\nu^{(7)} - (a_{37})^{(7)}\nu^{(7)}$$

**Definition of** $\nu^{(7)}$ :- $\boxed{\nu^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right) \leq \frac{d\nu^{(7)}}{dt} \leq -\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(7)}, (\nu_0)^{(7)}$ :-

$$\text{For } 0 < \boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}} < (\nu_1)^{(7)} < (\bar{\nu}_1)^{(7)}$$

$$\nu^{(7)}(t) \geq \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}}{1 + (C)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(\nu_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\nu_2)^{(7)}}}$$

it follows $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\nu_1)^{(7)}$

In the same manner , we get 417

$$\nu^{(7)}(t) \leq \frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)} e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{\nu}_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\bar{\nu}_2)^{(7)}}}$$

From which we deduce $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\bar{\nu}_1)^{(7)}$

If $0 < (\nu_1)^{(7)} < (\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0} < (\bar{\nu}_1)^{(7)}$ we find like in the previous case, 418

$$(\nu_1)^{(7)} \leq \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_2)^{(7)}\right)t\right]}}{1 + (C)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_2)^{(7)}\right)t\right]}} \leq \nu^{(7)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)} e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \leq (\bar{\nu}_1)^{(7)}$$

If $0 < (\nu_1)^{(7)} \leq (\bar{\nu}_1)^{(7)} \leq \boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$ , we obtain 419

$$(\nu_1)^{(7)} \leq \nu^{(7)}(t) \leq \frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)} e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \leq (\nu_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(7)}(t)$ :-

$$(m_2)^{(7)} \leq \nu^{(7)}(t) \leq (m_1)^{(7)}, \quad \boxed{\nu^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(7)}(t)$ :- 420

$$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}, \quad \boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $\nu^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)} G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)} T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

**Proof** : From global equations we obtain 421

$$\frac{d\nu^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right) - (a''_{41})^{(8)}(T_{41}, t)\nu^{(8)} - (a_{41})^{(8)}\nu^{(8)}$$

**Definition of** $\nu^{(8)}$ :- $\boxed{\nu^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right) \leq \frac{d\nu^{(8)}}{dt} \leq -\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$ :-

For $0 < \boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$

$$\nu^{(8)}(t) \geq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)} e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \leq \frac{(\nu_1)^{(8)}+(C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}}{1+(C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}} \leq \nu^{(8)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \leq (\bar{\nu}_1)^{(8)} \leq \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$ , we obtain

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$$(\nu_1)^{(8)} \leq \nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(8)}(t)$ :-

$$(m_2)^{(8)} \leq \nu^{(8)}(t) \leq (m_1)^{(8)}, \quad \boxed{\nu^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(8)}(t)$ :-

$$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}, \quad \boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $\nu^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)}G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then $(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)}T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

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**Proof** : From 99,20,44,22,23,44 we obtain

$$\frac{d\nu^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45}, t)\right) - (a''_{45})^{(9)}(T_{45}, t)\nu^{(9)} - (a_{45})^{(9)}\nu^{(9)}$$

**Definition of** $\nu^{(9)}$ :- $\quad \boxed{\nu^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(\nu^{(9)}\right)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)}\right) \leq \frac{d\nu^{(9)}}{dt} \leq -\left((a_{45})^{(9)}\left(\nu^{(9)}\right)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$ :-

For $0 < \boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$

$$\nu^{(9)}(t) \geq \frac{(\nu_1)^{(9)}+(C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_0)^{(9)}\right)t\right]}}{1+(C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)}-(\nu_0)^{(9)}}{(\nu_0)^{(9)}-(\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\nu_9)^{(9)}$

In the same manner , we get

$$\nu^{(9)}(t) \leq \frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)}-(\nu_0)^{(9)}}{(\nu_0)^{(9)}-(\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \leq \frac{(\nu_1)^{(9)}+(C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}}{1+(C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}} \leq \nu^{(9)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \leq (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \leq (\bar{\nu}_1)^{(9)} \leq \boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$ , we obtain

$$(\nu_1)^{(9)} \leq \nu^{(9)}(t) \leq \frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \leq (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(9)}(t)$ :-

$$(m_2)^{(9)} \leq \nu^{(9)}(t) \leq (m_1)^{(9)}, \quad \boxed{\nu^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(9)}(t)$ :-

$$(\mu_2)^{(9)} \leq u^{(9)}(t) \leq (\mu_1)^{(9)}, \quad \boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)}G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then
$(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)} T_{45}(t)$ This is an important consequence of the relation between $(v_1)^{(9)}$ and $(\bar{v}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following 425

**Theorem :** If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on $t$ , and the conditions with the notations

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0$ ,

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$

$with\ \ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a''_i)^{(2)} and\ (b''_i)^{(2)}$ are independent on t , and the conditions with the notations 426

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$ 427

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 428

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$ , 429

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 430

$with\ \ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a''_i)^{(3)} and\ (b''_i)^{(3)}$ are independent on $t$ , and the conditions with the notations 431

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$ ,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

**Theorem :** If $(a''_i)^{(4)} and\ (b''_i)^{(4)}$ are independent on $t$ , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$ ,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(5)} and\ (b_i'')^{(5)}$ are independent on $t$ , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$ ,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

**Theorem** If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on $t$ , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$ ,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(7)} and\ (b_i'')^{(7)}$ are independent on $t$ , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$ ,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(8)} and\ (b_i'')^{(8)}$ are independent on $t$ , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$ ,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a''_i)^{(9)} and\ (b''_i)^{(9)}$ are independent on $t$ , and the conditions (with the notations 45,46,27,28) 436 A

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$ ,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0$ 437

$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0$ 438

$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0$ 439

$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0$ 440

$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ]T_{14} = 0$ 441

$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0$ 442

has a unique positive solution , which is an equilibrium solution for the system

$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0$ 443

$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0$ 444

$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0$ 445

$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\ ]T_{16} = 0$ 446

$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\ ]T_{17} = 0$ 447

$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\ ]T_{18} = 0$ 448

has a unique positive solution , which is an equilibrium solution

$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0$ 449

$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0$ 450

$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0$ 451

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\,]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\,]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\,]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\big((G_{27})\big)\,]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\big((G_{27})\big)\,]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\big((G_{27})\big)\,]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})\,]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})\,]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})\,]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})\,]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})\,]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})\,]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0 \quad 479$$

$$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0 \quad 480$$

$$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0 \quad 481$$

$$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0 \quad 482$$

$$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0 \quad 483$$

$$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0 \quad 484$$

$$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0 \quad 484\text{A}$$

$$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$$

$$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$$

$$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$$

$$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$$

$$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$$

**Proof:** 485

(a) Indeed the first two equations have a nontrivial solution $G_{13}, G_{14}$ if

$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$

**Proof:** 486

(i) Indeed the first two equations have a nontrivial solution $G_{16}, G_{17}$ if

$\mathrm{F}(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$

**Proof:** 487

(a) Indeed the first two equations have a nontrivial solution $G_{20}, G_{21}$ if

$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$

**Proof:** 488

(a) Indeed the first two equations have a nontrivial solution $G_{24}, G_{25}$ if

$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$

**Proof:** 489

(a) Indeed the first two equations have a nontrivial solution $G_{28}, G_{29}$ if

$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$

**Proof:** 490

(a) Indeed the first two equations have a nontrivial solution $G_{32}, G_{33}$ if

$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$

**Proof:** 491

(a) Indeed the first two equations have a nontrivial solution $G_{36}, G_{37}$ if

$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$

**Proof:** 492

(a) Indeed the first two equations have a nontrivial solution $G_{40}, G_{41}$ if

$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$

**Proof:** 492 A

(a) Indeed the first two equations have a nontrivial solution $G_{44}, G_{45}$ if

$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$

**Definition and uniqueness of** $T^*_{14}$ :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique $T^*_{14}$ for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

**Definition and uniqueness of** $T^*_{17}$ :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique $T^*_{17}$ for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]}$$ 495

**Definition and uniqueness of** $T^*_{21}$ :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique $T^*_{21}$ for which $f(T^*_{21}) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

**Definition and uniqueness of** $T^*_{25}$ :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that there exists a unique $T^*_{25}$ for which $f(T^*_{25}) = 0$. With this value , we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T^*_{25})]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T^*_{25})]}$$

**Definition and uniqueness of** $T^*_{29}$ :- 498

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(5)}(T_{29})$ being increasing, it follows that there exists a unique $T^*_{29}$ for which $f(T^*_{29}) = 0$. With this value , we obtain from the three first equations

$G_{28} = \frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T^*_{29})]}$ , $G_{30} = \frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T^*_{29})]}$

**Definition and uniqueness of** $T^*_{33}$ :- 499

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(6)}(T_{33})$ being increasing, it follows that there exists a unique $T^*_{33}$ for which $f(T^*_{33}) = 0$. With this value , we obtain from the three first equations

$G_{32} = \frac{(a_{32})^{(6)}G_{33}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T^*_{33})]}$ , $G_{34} = \frac{(a_{34})^{(6)}G_{33}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T^*_{33})]}$

**Definition and uniqueness of** $T^*_{37}$ :- 500

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(7)}(T_{37})$ being increasing, it follows that there exists a unique $T^*_{37}$ for which $f(T^*_{37}) = 0$. With this value , we obtain from the three first equations

$G_{36} = \frac{(a_{36})^{(7)}G_{37}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T^*_{37})]}$ , $G_{38} = \frac{(a_{38})^{(7)}G_{37}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T^*_{37})]}$

**Definition and uniqueness of** $T^*_{41}$ :- 501

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(8)}(T_{41})$ being increasing, it follows that there exists a unique $T^*_{41}$ for which $f(T^*_{41}) = 0$. With this value , we obtain from the three first equations

$G_{40} = \frac{(a_{40})^{(8)}G_{41}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T^*_{41})]}$ , $G_{42} = \frac{(a_{42})^{(8)}G_{41}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T^*_{41})]}$

**Definition and uniqueness of** $T^*_{45}$ :- 501 A

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(9)}(T_{45})$ being increasing, it follows that there exists a unique $T^*_{45}$ for which $f(T^*_{45}) = 0$. With this value , we obtain from the three first equations

$G_{44} = \frac{(a_{44})^{(9)}G_{45}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T^*_{45})]}$ , $G_{46} = \frac{(a_{46})^{(9)}G_{45}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T^*_{45})]}$

By the same argument, the equations admit solutions $G_{13}, G_{14}$ if 502
$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$

$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right]+(b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{14}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{14}$ such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions $G_{16}, G_{17}$ if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right] + (b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that $\varphi$ is a decreasing function in $G_{17}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{14}^{*}$ such that $\varphi((G_{19})^{*}) = 0$ 504

By the same argument, the equations admit solutions $G_{20}, G_{21}$ if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right] + (b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that $\varphi$ is a decreasing function in $G_{21}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{21}^{*}$ such that $\varphi((G_{23})^{*}) = 0$

By the same argument, the equations admit solutions $G_{24}, G_{25}$ if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right] + (b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that $\varphi$ is a decreasing function in $G_{25}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{25}^{*}$ such that $\varphi((G_{27})^{*}) = 0$

By the same argument, the equations admit solutions $G_{28}, G_{29}$ if 507

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right] + (b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that $\varphi$ is a decreasing function in $G_{29}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{29}^{*}$ such that $\varphi((G_{31})^{*}) = 0$

By the same argument, the equations admit solutions $G_{32}, G_{33}$ if 508

$$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$$

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right] + (b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that $\varphi$ is a decreasing function in $G_{33}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{33}^{*}$ such that $\varphi({G_{35}}^{*}) = 0$

By the same argument, the equations admit solutions $G_{36}, G_{37}$ if 509

$$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$$

$$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right]+(b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{37}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{37}$ such that $\varphi(G_{39}{}^*) = 0$

By the same argument, the equations admit solutions $G_{40}, G_{41}$ if 510

$$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$$
$$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right]+(b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{41}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{41}$ such that $\varphi(G_{43}{}^*) = 0$

By the same argument, the equations 92,93 admit solutions $G_{44}, G_{45}$ if

$$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$$

$$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right]+(b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{45}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{45}$ such that $\varphi((G_{47})^*) = 0$

Finally we obtain the unique solution 511

$G^*_{14}$ given by $\varphi(G^*) = 0$ , $T^*_{14}$ given by $f(T^*_{14}) = 0$ and

$$G^*_{13} = \frac{(a_{13})^{(1)}G^*_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G^*_{15} = \frac{(a_{15})^{(1)}G^*_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

$$T^*_{13} = \frac{(b_{13})^{(1)}T^*_{14}}{[(b'_{13})^{(1)}-(b''_{13})^{(1)}(G^*)]} \quad , \quad T^*_{15} = \frac{(b_{15})^{(1)}T^*_{14}}{[(b'_{15})^{(1)}-(b''_{15})^{(1)}(G^*)]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

$\mathrm{G}^*_{17}$ given by $\varphi((G_{19})^*) = 0$ , $\mathrm{T}^*_{17}$ given by $f(\mathrm{T}^*_{17}) = 0$ and 512

$$\mathrm{G}^*_{16} = \frac{(\mathrm{a}_{16})^{(2)}\mathrm{G}^*_{17}}{[(\mathrm{a}'_{16})^{(2)}+(\mathrm{a}''_{16})^{(2)}(\mathrm{T}^*_{17})]} \quad , \quad \mathrm{G}^*_{18} = \frac{(\mathrm{a}_{18})^{(2)}\mathrm{G}^*_{17}}{[(\mathrm{a}'_{18})^{(2)}+(\mathrm{a}''_{18})^{(2)}(\mathrm{T}^*_{17})]} \quad 513$$

$$\mathrm{T}^*_{16} = \frac{(\mathrm{b}_{16})^{(2)}\mathrm{T}^*_{17}}{[(\mathrm{b}'_{16})^{(2)}-(\mathrm{b}''_{16})^{(2)}((G_{19})^*)]} \quad , \quad \mathrm{T}^*_{18} = \frac{(\mathrm{b}_{18})^{(2)}\mathrm{T}^*_{17}}{[(\mathrm{b}'_{18})^{(2)}-(\mathrm{b}''_{18})^{(2)}((G_{19})^*)]} \quad 514$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

$G^*_{21}$ given by $\varphi((G_{23})^*) = 0$ , $T^*_{21}$ given by $f(T^*_{21}) = 0$ and

$$G_{20}^{*} = \frac{(a_{20})^{(3)} G_{21}^{*}}{[(a'_{20})^{(3)} + (a''_{20})^{(3)} (T_{21}^{*})]} \quad , \quad G_{22}^{*} = \frac{(a_{22})^{(3)} G_{21}^{*}}{[(a'_{22})^{(3)} + (a''_{22})^{(3)} (T_{21}^{*})]}$$

$$T_{20}^{*} = \frac{(b_{20})^{(3)} T_{21}^{*}}{[(b'_{20})^{(3)} - (b''_{20})^{(3)} (G_{23}{}^{*})]} \quad , \quad T_{22}^{*} = \frac{(b_{22})^{(3)} T_{21}^{*}}{[(b'_{22})^{(3)} - (b''_{22})^{(3)} (G_{23}{}^{*})]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

$G_{25}^{*}$ given by $\varphi(G_{27}) = 0$ , $T_{25}^{*}$ given by $f(T_{25}^{*}) = 0$ and

$$G_{24}^{*} = \frac{(a_{24})^{(4)} G_{25}^{*}}{[(a'_{24})^{(4)} + (a''_{24})^{(4)} (T_{25}^{*})]} \quad , \quad G_{26}^{*} = \frac{(a_{26})^{(4)} G_{25}^{*}}{[(a'_{26})^{(4)} + (a''_{26})^{(4)} (T_{25}^{*})]}$$

$$T_{24}^{*} = \frac{(b_{24})^{(4)} T_{25}^{*}}{[(b'_{24})^{(4)} - (b''_{24})^{(4)} ((G_{27})^{*})]} \quad , \quad T_{26}^{*} = \frac{(b_{26})^{(4)} T_{25}^{*}}{[(b'_{26})^{(4)} - (b''_{26})^{(4)} ((G_{27})^{*})]} \quad 517$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

$G_{29}^{*}$ given by $\varphi((G_{31})^{*}) = 0$ , $T_{29}^{*}$ given by $f(T_{29}^{*}) = 0$ and

$$G_{28}^{*} = \frac{(a_{28})^{(5)} G_{29}^{*}}{[(a'_{28})^{(5)} + (a''_{28})^{(5)} (T_{29}^{*})]} \quad , \quad G_{30}^{*} = \frac{(a_{30})^{(5)} G_{29}^{*}}{[(a'_{30})^{(5)} + (a''_{30})^{(5)} (T_{29}^{*})]}$$

$$T_{28}^{*} = \frac{(b_{28})^{(5)} T_{29}^{*}}{[(b'_{28})^{(5)} - (b''_{28})^{(5)} ((G_{31})^{*})]} \quad , \quad T_{30}^{*} = \frac{(b_{30})^{(5)} T_{29}^{*}}{[(b'_{30})^{(5)} - (b''_{30})^{(5)} ((G_{31})^{*})]} \quad 519$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

$G_{33}^{*}$ given by $\varphi((G_{35})^{*}) = 0$ , $T_{33}^{*}$ given by $f(T_{33}^{*}) = 0$ and

$$G_{32}^{*} = \frac{(a_{32})^{(6)} G_{33}^{*}}{[(a'_{32})^{(6)} + (a''_{32})^{(6)} (T_{33}^{*})]} \quad , \quad G_{34}^{*} = \frac{(a_{34})^{(6)} G_{33}^{*}}{[(a'_{34})^{(6)} + (a''_{34})^{(6)} (T_{33}^{*})]}$$

$$T_{32}^{*} = \frac{(b_{32})^{(6)} T_{33}^{*}}{[(b'_{32})^{(6)} - (b''_{32})^{(6)} ((G_{35})^{*})]} \quad , \quad T_{34}^{*} = \frac{(b_{34})^{(6)} T_{33}^{*}}{[(b'_{34})^{(6)} - (b''_{34})^{(6)} ((G_{35})^{*})]} \quad 521$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

$G_{37}^{*}$ given by $\varphi((G_{39})^{*}) = 0$ , $T_{37}^{*}$ given by $f(T_{37}^{*}) = 0$ and

$$G_{36}^{*} = \frac{(a_{36})^{(7)} G_{37}^{*}}{[(a'_{36})^{(7)} + (a''_{36})^{(7)} (T_{37}^{*})]} \quad , \quad G_{38}^{*} = \frac{(a_{38})^{(7)} G_{37}^{*}}{[(a'_{38})^{(7)} + (a''_{38})^{(7)} (T_{37}^{*})]}$$

$$T_{36}^{*} = \frac{(b_{36})^{(7)} T_{37}^{*}}{[(b'_{36})^{(7)} - (b''_{36})^{(7)} ((G_{39})^{*})]} \quad , \quad T_{38}^{*} = \frac{(b_{38})^{(7)} T_{37}^{*}}{[(b'_{38})^{(7)} - (b''_{38})^{(7)} ((G_{39})^{*})]}$$

Finally we obtain the unique solution 523

$G_{41}^*$ given by $\varphi((G_{43})^*) = 0$ , $T_{41}^*$ given by $f(T_{41}^*) = 0$ and

$$G_{40}^* = \frac{(a_{40})^{(8)} G_{41}^*}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42}^* = \frac{(a_{42})^{(8)} G_{41}^*}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

$$T_{40}^* = \frac{(b_{40})^{(8)} T_{41}^*}{[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43})^*)]} \quad , \quad T_{42}^* = \frac{(b_{42})^{(8)} T_{41}^*}{[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43})^*)]}$$

Finally we obtain the unique solution of 89 to 99 523 A

$G_{45}^*$ given by $\varphi((G_{47})^*) = 0$ , $T_{45}^*$ given by $f(T_{45}^*) = 0$ and

$$G_{44}^* = \frac{(a_{44})^{(9)} G_{45}^*}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46}^* = \frac{(a_{46})^{(9)} G_{45}^*}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

$$T_{44}^* = \frac{(b_{44})^{(9)} T_{45}^*}{[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47})^*)]} \quad , \quad T_{46}^* = \frac{(b_{46})^{(9)} T_{45}^*}{[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47})^*)]}$$

## ASYMPTOTIC STABILITY ANALYSIS 524

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

**Proof:** Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{14}'')^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \frac{\partial (b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a'_{13})^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)} G_{13}^* \mathbb{T}_{14} \quad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a'_{14})^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)} G_{14}^* \mathbb{T}_{14} \quad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a'_{15})^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)} G_{15}^* \mathbb{T}_{14} \quad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b'_{13})^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)} T_{13}^* \mathbb{G}_j\right) \quad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b'_{14})^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)} T_{14}^* \mathbb{G}_j\right) \quad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b'_{15})^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T^*_{15}\mathbb{G}_j\right) \quad 530$$

**ASYMPTOTIC STABILITY ANALYSIS** 531

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(2)}$ and $(b''_i)^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i \quad 532$$

$$\frac{\partial(a''_{17})^{(2)}}{\partial T_{17}}(T^*_{17}) = (q_{17})^{(2)} \quad , \frac{\partial(b''_i)^{(2)}}{\partial G_j}\left((G_{19})^*\right) = s_{ij} \quad 533$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a'_{16})^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G^*_{16}\mathbb{T}_{17} \quad 534$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a'_{17})^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G^*_{17}\mathbb{T}_{17} \quad 535$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a'_{18})^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G^*_{18}\mathbb{T}_{17} \quad 536$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b'_{16})^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T^*_{16}\mathbb{G}_j\right) \quad 537$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b'_{17})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}T^*_{17}\mathbb{G}_j\right) \quad 538$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b'_{18})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}T^*_{18}\mathbb{G}_j\right) \quad 539$$

**ASYMPTOTIC STABILITY ANALYSIS** 540

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(3)}$ *and* $(b''_i)^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{21})^{(3)}}{\partial T_{21}}(T^*_{21}) = (q_{21})^{(3)} \quad , \frac{\partial(b''_i)^{(3)}}{\partial G_j}\left((G_{23})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a'_{20})^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G^*_{20}\mathbb{T}_{21} \quad 541$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a'_{21})^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G^*_{21}\mathbb{T}_{21} \quad 542$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a'_{22})^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G^*_{22}\mathbb{T}_{21} \quad 543$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b'_{20})^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T^*_{20}\mathbb{G}_j\right) \quad 544$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b'_{21})^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T^*_{21}\mathbb{G}_j\right) \quad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b'_{22})^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T^*_{22}\mathbb{G}_j\right) \quad 546$$

**ASYMPTOTIC STABILITY ANALYSIS** 547

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}$ *and* $(b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{25})^{(4)}}{\partial T_{25}}(T^*_{25}) = (q_{25})^{(4)} \ , \ \frac{\partial(b_i'')^{(4)}}{\partial G_j}((G_{27})^* \ ) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\left((a'_{24})^{(4)} + (p_{24})^{(4)}\right)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G^*_{24}\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\left((a'_{25})^{(4)} + (p_{25})^{(4)}\right)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G^*_{25}\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\left((a'_{26})^{(4)} + (p_{26})^{(4)}\right)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G^*_{26}\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\left((b'_{24})^{(4)} - (r_{24})^{(4)}\right)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(24)(j)}T^*_{24}\mathbb{G}_j\right) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\left((b'_{25})^{(4)} - (r_{25})^{(4)}\right)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\left(s_{(25)(j)}T^*_{25}\mathbb{G}_j\right) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T^*_{26}\mathbb{G}_j\right) \quad 554$$

**ASYMPTOTIC STABILITY ANALYSIS** 555

**Theorem 5:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}$ *and* $(b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 556

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$\frac{\partial(a_{29}'')^{(5)}}{\partial T_{29}}(T_{29}^*) = (q_{29})^{(5)}$ , $\frac{\partial(b_i'')^{(5)}}{\partial G_j}((G_{31})^*) = s_{ij}$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a_{28}')^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G_{28}^*\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a_{29}')^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G_{29}^*\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a_{30}')^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G_{30}^*\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b_{28}')^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T_{28}^*\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b_{29}')^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T_{29}^*\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b_{30}')^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T_{30}^*\mathbb{G}_j\right) \quad 562$$

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**Theorem 6:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}$ *and* $(b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 564

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$\frac{\partial(a_{33}'')^{(6)}}{\partial T_{33}}(T_{33}^*) = (q_{33})^{(6)}$ , $\frac{\partial(b_i'')^{(6)}}{\partial G_j}((G_{35})^*) = s_{ij}$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a_{32}')^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G_{32}^*\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a_{33}')^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G_{33}^*\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a_{34}')^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G_{34}^*\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b_{32}')^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T_{32}^*\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b_{33}')^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T_{33}^*\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b_{34}')^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T_{34}^*\mathbb{G}_j\right) \quad 570$$

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**Theorem 7:** If the conditions of the previous theorem are satisfied and if the functions

$(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 572

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{37}'')^{(7)}}{\partial T_{37}}(T_{37}^*) = (q_{37})^{(7)} \quad , \frac{\partial (b_i'')^{(7)}}{\partial G_j}\left( (G_{39})^{**} \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a_{36}')^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G_{36}^*\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a_{37}')^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G_{37}^*\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a_{38}')^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G_{38}^*\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b_{36}')^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T_{36}^*\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b_{37}')^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T_{37}^*\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b_{38}')^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T_{38}^*\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

## ASYMPTOTIC STABILITY ANALYSIS

**Theorem 8:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 580

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{41}'')^{(8)}}{\partial T_{41}}(T_{41}^*) = (q_{41})^{(8)} \quad , \frac{\partial (b_i'')^{(8)}}{\partial G_j}\left( (G_{43})^{*} \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a_{40}')^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G_{40}^*\mathbb{T}_{41} \quad 581$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a'_{41})^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G^*_{41}\mathbb{T}_{41} \quad 582$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a'_{42})^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G^*_{42}\mathbb{T}_{41} \quad 583$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b'_{40})^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T^*_{40}\mathbb{G}_j\right) \quad 584$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b'_{41})^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T^*_{41}\mathbb{G}_j\right) \quad 585$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b'_{42})^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T^*_{42}\mathbb{G}_j\right) \quad 586$$

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## ASYMPTOTIC STABILITY ANALYSIS

**Theorem 9:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}$ *and* $(b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

**Proof:** Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{45})^{(9)}}{\partial T_{45}}(T^*_{45}) = (q_{45})^{(9)} \quad , \frac{\partial(b_i'')^{(9)}}{\partial G_j}((G_{47})^*) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a'_{44})^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)}G^*_{44}\mathbb{T}_{45} \quad 586\,B$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a'_{45})^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)}G^*_{45}\mathbb{T}_{45} \quad 586\,C$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a'_{46})^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)}G^*_{46}\mathbb{T}_{45} \quad 586\,D$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b'_{44})^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)}T^*_{44}\mathbb{G}_j\right) \quad 586\,E$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b'_{45})^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)}T^*_{45}\mathbb{G}_j\right) \quad 586\,F$$

$$\frac{d\mathbb{T}_{46}}{dt} = -\left((b'_{46})^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T^*_{46}\mathbb{G}_j\right) \quad 586\,G$$

**The characteristic equation of this system is** 587

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(q_{13})^{(1)}G^*_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)}T^*_{14} + (b_{14})^{(1)}s_{(13),(14)}T^*_{14}\right)$$

$$+\left(\left((\lambda)^{(1)}+(a'_{14})^{(1)}+(p_{14})^{(1)}\right)(q_{13})^{(1)}G^{*}_{13}+(a_{13})^{(1)}(q_{14})^{(1)}G^{*}_{14}\right)$$

$$\left(\left((\lambda)^{(1)}+(b'_{13})^{(1)}-(r_{13})^{(1)}\right)s_{(14),(13)}T^{*}_{14}+(b_{14})^{(1)}s_{(13),(13)}T^{*}_{13}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^{2}+\left((a'_{13})^{(1)}+(a'_{14})^{(1)}+(p_{13})^{(1)}+(p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^{2}+\left((b'_{13})^{(1)}+(b'_{14})^{(1)}-(r_{13})^{(1)}+(r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^{2}+\left((a'_{13})^{(1)}+(a'_{14})^{(1)}+(p_{13})^{(1)}+(p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)}+(a'_{13})^{(1)}+(p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^{*}_{14}+(a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^{*}_{13}\right)$$

$$\left(\left((\lambda)^{(1)}+(b'_{13})^{(1)}-(r_{13})^{(1)}\right)s_{(14),(15)}T^{*}_{14}+(b_{14})^{(1)}s_{(13),(15)}T^{*}_{13}\right)\}=0$$

$$+$$

$$(\lambda)^{(2)}+(b'_{18})^{(2)}-(r_{18})^{(2)}\{\left((\lambda)^{(2)}+(a'_{18})^{(2)}+(p_{18})^{(2)}\right)$$

$$\left[\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^{*}_{17}+(a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^{*}_{16}\right)\Big]$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^{*}_{17}\right)$$

$$+\left(\left((\lambda)^{(2)}+(a'_{17})^{(2)}+(p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^{*}_{16}+(a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^{*}_{17}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^{*}_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^{2}+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^{2}+\left((b'_{16})^{(2)}+(b'_{17})^{(2)}-(r_{16})^{(2)}+(r_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$+\left(\left((\lambda)^{(2)}\right)^{2}+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^{*}_{17}+(a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^{*}_{16}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^{*}_{16}\right)\}=0$$

$$+$$

$$(\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)\Big]$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(21)}T^{*}_{21}\right)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^{*}_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^{*}_{21}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(20)}T^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$+\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(22)}T^{*}_{20}\right)\}=0$$

$$+$$

$$(\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left(\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)\right]$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(25)}T^{*}_{25}\right)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^{*}_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^{*}_{25}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(24)}T^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$+\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(26)}T^{*}_{24}\right)\}=0$$

$$+$$

$$(\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(29)}T^{*}_{29}\right)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^{*}_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^{*}_{29}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(28)}T^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$+\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(30)}T^{*}_{28}\right)\}=0$$

$$+$$

$$(\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left(\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)\right]$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(33)}T^{*}_{33}\right)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^{*}_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^{*}_{33}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(32)}T^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$+\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(34)}T^{*}_{32}\right)\}=0$$

$$+$$

$$(\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)\right]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(37)}T^{*}_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^{*}_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^{*}_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(36)}T^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$+\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(38)}T^{*}_{36}\right)\}=0$$

$$+$$

$$(\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)\Big]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(41)}T^{*}_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^{*}_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^{*}_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(40)}T^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$+\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(42)}T^{*}_{40}\right)\}=0$$

$$+$$

$$(\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)\Big]$$

$$\left((\lambda)^{(9)} + (b'_{44})^{(9)} - (r_{44})^{(9)}\right)s_{(45),(45)}T^*_{45} + (b_{45})^{(9)}s_{(44),(45)}T^*_{45}\Big)$$

$$+\left((\lambda)^{(9)} + (a'_{45})^{(9)} + (p_{45})^{(9)}\right)(q_{44})^{(9)}G^*_{44} + (a_{44})^{(9)}(q_{45})^{(9)}G^*_{45}\Big)$$

$$\left((\lambda)^{(9)} + (b'_{44})^{(9)} - (r_{44})^{(9)}\right)s_{(45),(44)}T^*_{45} + (b_{45})^{(9)}s_{(44),(44)}T^*_{44}\Big)$$

$$\left((\lambda)^{(9)}\right)^2 + \left((a'_{44})^{(9)} + (a'_{45})^{(9)} + (p_{44})^{(9)} + (p_{45})^{(9)}\right)(\lambda)^{(9)}\Big)$$

$$\left((\lambda)^{(9)}\right)^2 + \left((b'_{44})^{(9)} + (b'_{45})^{(9)} - (r_{44})^{(9)} + (r_{45})^{(9)}\right)(\lambda)^{(9)}\Big)$$

$$+\left((\lambda)^{(9)}\right)^2 + \left((a'_{44})^{(9)} + (a'_{45})^{(9)} + (p_{44})^{(9)} + (p_{45})^{(9)}\right)(\lambda)^{(9)}\Big)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)} + (a'_{44})^{(9)} + (p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^*_{45} + (a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^*_{44}\right)$$

$$\left((\lambda)^{(9)} + (b'_{44})^{(9)} - (r_{44})^{(9)}\right)s_{(45),(46)}T^*_{45} + (b_{45})^{(9)}s_{(44),(46)}T^*_{44}\Big)\} = 0$$

**And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.**

## SECTION ELEVEN

## Neuronal Microtubules

### INTRODUCTION—VARIABLES USED

**A critical assessment of the information processing capabilities of neuronal microtubules using coherent excitations Travis John Adrian Craddock, Jack A. Tuszynski Journal of Biological Physics January 2010, Volume 36, Issue 1, pp 53-70 First online: 04 June 2009**

(1) Following previous models involving the mechanisms of exciton energy propagation, authors estimate (eb) the strength of exciton and (e&eb) phonon interactions and their effect on (e&eb) the formation and dynamics of coherent exciton domains within MTs.
(2) Also, estimates of energy and timescales for excitons, phonons, their interactions, and thermal effects are presented. Conclusions cast doubt on the possibility of sufficiently long-lived coherent exciton/phonon structures existing at (eb) physiological temperatures in (eb) the absence of thermal isolation mechanisms.
(3) These results are discussed in comparison with (e&eb) previous models based on (e) quantum effects in non-polar hydrophobic regions, which have yet to be disproved.

**Dorian Aur, Mandar Jog, and Roman R. Poznanski, J. Integr Neurosci 10, 413 (2011) DOI:**

**10.1142/S0219635211002865: Computing by physical interaction in neurons**

(4) The electrodynamics of action potentials represents (eb) the fundamental level where (e) information is integrated and processed in neurons.
(5) The Hodgkin–Huxley model cannot explain the non-stereotyped spatial charge density dynamics that occur during (e&eb) action potential propagation.
(6) Revealed in experiments as spike directivity, the non-uniform charge density dynamics within (eb) neurons carry (e&eb) meaningful information and suggest (eb) that fragments of information regarding our memories are endogenously stored in (eb) structural patterns at a molecular level and are revealed (eb) only during spiking activity.
(7) The main conceptual idea is that under the influence of (e&eb) electric fields, efficient computation by interaction occurs between (e&eb) charge densities embedded within (eb) molecular structures and (e&eb) the transient developed flow of electrical charges.
(8) This process of computation underlying (e&eb) electrical interactions and molecular mechanisms at the subcellular level is (=) dissimilar from spiking neuron models that are completely devoid of (e) physical interactions.
**(9)** Computation by interaction describes (eb) a more powerful continuous model of computation than the one that consists of (e) discrete steps as represented in Turing machines.

**NOTATION**

**Module One**

**A critical assessment of the information processing capabilities of neuronal microtubules using coherent excitations Travis John Adrian Craddock, Jack A. Tuszynski Journal of Biological Physics January 2010, Volume 36, Issue 1, pp 53-70 First online: 04 June 2009**

(1) To investigate the possibility of quantum processes in relation to (e&eb) information processing in MTs, a biophysical MT model is used based on (e) the electrostatic interior of the tubulin protein.
(2) The interior is taken to constitute (e) a double-well potential structure within (eb) which a mobile electron is considered capable of occupying (e) at least two distinct quantum states.
(3) These excitonic states together with MT lattice vibrations determine (eb) the state space of individual tubulin dimers within (eb) the MT lattice.
(4) Tubulin dimers are taken as (=) quantum well structures containing (e) an electron that can exist in (eb) either its ground state or first excited state.

The interior is taken to constitute (e) a double-well potential structure within (eb) which a mobile electron is considered capable of occupying (e) at least two distinct quantum states

$G_{13}$ : Category one of double-well potential structure within (eb) which a mobile electron is considered capable of occupying (e) at least two distinct quantum states
$G_{14}$ : Category two of SAS

$G_{15}$ : Category three of SAS

$T_{13}$ : Category one of interior

$T_{14}$ : Category two of SAS

$T_{15}$ : Category three of SAS

**Module Two**

The interior is taken to constitute a double-well potential structure within which a mobile electron is considered capable of occupying (e) at least two distinct quantum states

$G_{16}$ : Category one of two distinct quantum states; interior is taken to constitute a double-well potential structure within which a mobile electron

$G_{17}$ : Category two of SAS

$G_{18}$ : Category three of SAS

$T_{16}$ : Category one of interior is taken to constitute a double-well potential structure within which a mobile electron; two distinct quantum states

$T_{17}$ : Category two of SAS

$T_{18}$ : Category three of SAS

**Module three**

These excitonic states together with MT lattice vibrations determine (eb) the state space of individual tubulin dimers within (eb) the MT lattice

$G_{20}$ : Category one of **excitonic states;** MT lattice vibrations determine (eb) the state space of individual tubulin dimers within (eb) the MT lattice

$G_{21}$ : Category two of SAS

$G_{22}$ : Category three of SAS

$T_{20}$ : Category one of MT lattice vibrations determine (eb) the state space of individual tubulin dimers within (eb) the MT lattice **;excitonic states**

$T_{21}$ : Category two of SAS

$T_{22}$ : Category three of SAS

**Module four**

These excitonic states together with MT lattice vibrations determine (eb) the state space of individual tubulin dimers within (eb) the MT lattice

$G_{24}$ : Category one of excitonic states together with MT lattice vibrations

$G_{25}$ : Category two of SAS

$G_{26}$ : Category three of SAS

$T_{24}$ : Category one of state space of individual tubulin dimers within the MT lattice

$T_{25}$ : Category two of SAS

$T_{26}$ : Category three of SAS

**Module five**

Tubulin dimers are taken as (=) quantum well structures containing (e) an electron that can exist in (eb) either its ground state or first excited state

$G_{28}$ : Category one of Tubulin dimers

$G_{29}$ : Category two of SAS

$G_{30}$ : Category three of SAS

$T_{28}$ : Category one of quantum well structures containing (e) an electron that can exist in (eb) either its ground state or first excited state

$T_{29}$ : Category two of SAS

$T_{30}$ : Category three of SAS

**Module six**

Tubulin dimers are taken as quantum well structures containing an electron that can exist in (eb) **either its ground state or first excited state**

$G_{32}$ : Category one of Tubulin dimers are taken as quantum well structures containing an electron that can exist; **either its ground state or first excited state**

$G_{33}$ : Category two of SAS

$G_{34}$ : Category three of SAS

$T_{32}$ : Category one of **either its ground state or first excited state** ; Tubulin dimers are taken as quantum well structures containing an electron that can exist

$T_{33}$ : Category two of SAS

$T_{34}$ : Category three of SAS

**Module seven**

Following previous models involving the mechanisms of exciton energy propagation, authors estimate the strength of exciton and (e&eb) phonon interactions and their effect on (e&eb) the formation and dynamics of coherent exciton domains within MTs

$G_{36}$ : Category one of previous models involving the mechanisms of exciton energy propagation

$G_{37}$ : Category two of SAS

$G_{38}$ : Category three of SAS

$T_{36}$ : Category one of estimate the strength of exciton and (e&eb) phonon interactions and their effect on (e&eb) the formation and dynamics of coherent exciton domains within MTs

$T_{37}$ : Category two of SAS

$T_{38}$ : Category three of SAS

**Module eight**

estimate the strength of exciton and phonon interactions and their effect on (e&eb) the formation and

dynamics of coherent exciton domains within MTs

$G_{40}$ : Category one of **strength of exciton;** phonon interactions

$G_{41}$ : Category two of SAS

$G_{42}$ : Category three of SAS

$T_{40}$ : Category one of phonon interactions ;**strength of exciton**

$T_{41}$ : Category two of SAS

$T_{42}$ : Category three of SAS

**Module Nine**

**strength of exciton and phonon interactions** and their effect on (e&eb) the formation and dynamics of coherent exciton domains within MTs

$G_{44}$ : Category one of **strength of exciton and phonon interactions;** formation and dynamics of coherent exciton domains within MTs

$G_{45}$ : Category two of SAS

$G_{46}$ : Category three of SAS

$T_{44}$ : Category one of formation and dynamics of coherent exciton domains within MTs ;**strength of exciton and phonon interactions**

$T_{45}$ : Category two of SAS

$T_{46}$ : Category three of SAS

**The Coefficients:**

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}$ , $(b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}$,
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}$, $(b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}\ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$ ,
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)}$,

$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)},$
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)},$
are Dissipation coefficients

**Module Numbered One**

**The differential system of this model is now (Module Numbered one)**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \quad 1$$
$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad 2$$
$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15} \quad 3$$
$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13} \quad 4$$
$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14} \quad 5$$
$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)\right]T_{15} \quad 6$$
$+(a''_{13})^{(1)}(T_{14}, t) =$ First augmentation factor
$-(b''_{13})^{(1)}(G, t) =$ First detritions factor

**Module Numbered Two**

**The differential system of this model is now ( Module numbered two)**

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right]G_{16} \quad 7$$
$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t)\right]G_{17} \quad 8$$
$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t)\right]G_{18} \quad 9$$
$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\left((G_{19}), t\right)\right]T_{16} \quad 10$$
$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\left((G_{19}), t\right)\right]T_{17} \quad 11$$
$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\left((G_{19}), t\right)\right]T_{18} \quad 12$$
$+(a''_{16})^{(2)}(T_{17}, t) =$ First augmentation factor
$-(b''_{16})^{(2)}\left((G_{19}), t\right) =$ First detritions factor

**Module Numbered Three**

**The differential system of this model is now (Module numbered three)**

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right]G_{20} \quad 13$$
$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}, t)\right]G_{21} \quad 14$$
$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}, t)\right]G_{22} \quad 15$$
$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}, t)\right]T_{20} \quad 16$$
$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}, t)\right]T_{21} \quad 17$$
$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}, t)\right]T_{22} \quad 18$$
$+(a''_{20})^{(3)}(T_{21}, t) =$ First augmentation factor
$-(b''_{20})^{(3)}(G_{23}, t) =$ First detritions factor

**Module Numbered Four**

**The differential system of this model is now (Module numbered Four)**

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)\right]G_{24} \quad 19$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}, t)\right]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}, t)\right]G_{26} \quad 21$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27}), t\right)\right]T_{24} \quad 22$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27}), t\right)\right]T_{25} \quad 23$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27}), t\right)\right]T_{26} \quad 24$$

$+(a''_{24})^{(4)}(T_{25}, t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}\left((G_{27}), t\right) =$ **First detritions factor**

**Module Numbered Five**:

**The differential system of this model is now (Module number five)**

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right]G_{28} \quad 25$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}, t)\right]G_{29} \quad 26$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}, t)\right]G_{30} \quad 27$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}\left((G_{31}), t\right)\right]T_{28} \quad 28$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}\left((G_{31}), t\right)\right]T_{29} \quad 29$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}\left((G_{31}), t\right)\right]T_{30} \quad 30$$

$+(a''_{28})^{(5)}(T_{29}, t) =$ **First augmentation factor**

$-(b''_{28})^{(5)}\left((G_{31}), t\right) =$ **First detritions factor**

**Module Numbered Six**

**The differential system of this model is now (Module numbered Six)**

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right]G_{32} \quad 31$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}, t)\right]G_{33} \quad 32$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}, t)\right]G_{34} \quad 33$$

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}\left((G_{35}), t\right)\right]T_{32} \quad 34$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}\left((G_{35}), t\right)\right]T_{33} \quad 35$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}\left((G_{35}), t\right)\right]T_{34} \quad 36$$

$+(a''_{32})^{(6)}(T_{33}, t) =$ **First augmentation factor**

**Module Numbered Seven:**

**The differential system of this model is now (Seventh Module)**

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right]G_{36} \quad 37$$

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37}, t)\right]G_{37} \quad 38$$

$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}, t)\right]G_{38} \quad 39$$

$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}\left((G_{39}), t\right)\right]T_{36} \quad 40$$

$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}\left((G_{39}), t\right)\right]T_{37} \quad 41$$

$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39}),t)\right]T_{38} \quad 42$$
$$+(a''_{36})^{(7)}(T_{37},t) = \textbf{ First augmentation factor}$$

**Module Numbered Eight**

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right]G_{40} \quad 43$$
$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41},t)\right]G_{41} \quad 44$$
$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41},t)\right]G_{42} \quad 45$$
$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43}),t)\right]T_{40} \quad 46$$
$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}((G_{43}),t)\right]T_{41} \quad 47$$
$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43}),t)\right]T_{42} \quad 48$$

**Module Numbered Nine**

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$
$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$
$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$
$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47}),t)\right]T_{44} \quad 52$$
$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}((G_{47}),t)\right]T_{45} \quad 53$$
$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47}),t)\right]T_{46} \quad 54$$
$$+(a''_{44})^{(9)}(T_{45},t) = \textbf{ First augmentation factor}$$
$$-(b''_{44})^{(9)}((G_{47}),t) = \textbf{ First detrition factor}$$

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)} & +(a''_{13})^{(1)}(T_{14},t) & +(a''_{16})^{(2,2,)}(T_{17},t) & +(a''_{20})^{(3,3,)}(T_{21},t) \\ +(a''_{24})^{(4,4,4,4,)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{36})^{(7,7)}(T_{37},t) & +(a''_{40})^{(8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)} & +(a''_{14})^{(1)}(T_{14},t) & +(a''_{17})^{(2,2,)}(T_{17},t) & +(a''_{21})^{(3,3,)}(T_{21},t) \\ +(a''_{25})^{(4,4,4,4,)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{37})^{(7,7)}(T_{37},t) & +(a''_{41})^{(8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)} & +(a''_{15})^{(1)}(T_{14},t) & +(a''_{18})^{(2,2,)}(T_{17},t) & +(a''_{22})^{(3,3,)}(T_{21},t) \\ +(a''_{26})^{(4,4,4,4,)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{38})^{(7,7)}(T_{37},t) & +(a''_{42})^{(8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{15} \quad 57$$

Where $(a''_{13})^{(1)}(T_{14},t)$, $(a''_{14})^{(1)}(T_{14},t)$, $(a''_{15})^{(1)}(T_{14},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,)}(T_{17},t)$, $+(a''_{17})^{(2,2,)}(T_{17},t)$, $+(a''_{18})^{(2,2,)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,)}(T_{21},t)$, $+(a''_{21})^{(3,3,)}(T_{21},t)$, $+(a''_{22})^{(3,3,)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,)}(T_{25},t)$ , $+(a''_{25})^{(4,4,4,4,)}(T_{25},t)$ , $+(a''_{26})^{(4,4,4,4,)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,)}(T_{29},t)$ , $+(a''_{30})^{(5,5,5,5,)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,)}(T_{33},t)$ , $+(a''_{34})^{(6,6,6,6,)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{38})^{(7,7)}(T_{37},t)$ $+(a''_{37})^{(7,7)}(T_{37},t)$ $+(a''_{36})^{(7,7)}(T_{37},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{40})^{(8,8)}(T_{41},t)$ $+(a''_{41})^{(8,8)}(T_{41},t)$ $+(a''_{42})^{(8,8)}(T_{41},t)$ are eight augmentation coefficient for 1,2,3

$+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$ , $+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$ , $+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)} & -(b''_{13})^{(1)}(G,t) & -(b''_{16})^{(2,2,)}(G_{19},t) & -(b''_{20})^{(3,3,)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{36})^{(7,7,)}(G_{39},t) & -(b''_{40})^{(8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{13} \quad 58$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)} & -(b''_{14})^{(1)}(G,t) & -(b''_{17})^{(2,2,)}(G_{19},t) & -(b''_{21})^{(3,3,)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{37})^{(7,7,)}(G_{39},t) & -(b''_{41})^{(8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{14} \quad 59$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)} & -(b''_{15})^{(1)}(G,t) & -(b''_{18})^{(2,2,)}(G_{19},t) & -(b''_{22})^{(3,3,)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{38})^{(7,7,)}(G_{39},t) & -(b''_{42})^{(8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{15} \quad 60$$

Where $-(b''_{13})^{(1)}(G,t)$ , $-(b''_{14})^{(1)}(G,t)$ , $-(b''_{15})^{(1)}(G,t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,)}(G_{19},t)$ , $-(b''_{17})^{(2,2,)}(G_{19},t)$ , $-(b''_{18})^{(2,2,)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,)}(G_{23},t)$ , $-(b''_{21})^{(3,3,)}(G_{23},t)$ , $-(b''_{22})^{(3,3,)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,)}(G_{27},t)$ , $-(b''_{25})^{(4,4,4,4,)}(G_{27},t)$ , $-(b''_{26})^{(4,4,4,4,)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,)}(G_{31},t)$ , $-(b''_{29})^{(5,5,5,5,)}(G_{31},t)$ , $-(b''_{30})^{(5,5,5,5,)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,)}(G_{35},t)$ , $-(b''_{33})^{(6,6,6,6,)}(G_{35},t)$ , $-(b''_{34})^{(6,6,6,6,)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{37})^{(7,7,)}(G_{39},t)$ , $-(b''_{36})^{(7,7,)}(G_{39},t)$ , $-(b''_{38})^{(7,7,)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8)}(G_{43},t)$ $-(b''_{41})^{(8,8)}(G_{43},t)$ $-(b''_{42})^{(8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \begin{bmatrix} (a'_{16})^{(2)} & \boxed{+(a''_{16})^{(2)}(T_{17},t)} & \boxed{+(a''_{13})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)} & \\ \boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9)}(T_{45},t)} & \end{bmatrix} G_{16} \quad (61)$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \begin{bmatrix} (a'_{17})^{(2)} & \boxed{+(a''_{17})^{(2)}(T_{17},t)} & \boxed{+(a''_{14})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)} & \\ \boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9)}(T_{45},t)} & \end{bmatrix} G_{17} \quad (62)$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \begin{bmatrix} (a'_{18})^{(2)} & \boxed{+(a''_{18})^{(2)}(T_{17},t)} & \boxed{+(a''_{15})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)} & \\ \boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9)}(T_{45},t)} & \end{bmatrix} G_{18} \quad (63)$$

Where $\boxed{+(a''_{16})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,)}(T_{14},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)}$ are eight augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{44})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9)}(T_{45},t)}$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \begin{bmatrix} (b'_{16})^{(2)} & \boxed{-(b''_{16})^{(2)}(G_{19},t)} & \boxed{-(b''_{13})^{(1,1,)}(G,t)} & \boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)} & \\ \boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9)}(G_{47},t)} & \end{bmatrix} T_{16} \quad (64)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)} & \boxed{-(b''_{17})^{(2)}(G_{19},t)} & \boxed{-(b''_{14})^{(1,1,)}(G,t)} & \boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)} & \\ \boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9)}(G_{47},t)} & \end{bmatrix} T_{17} \quad (65)$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)}\boxed{-(b''_{18})^{(2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,)}(G,t)} & \boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{18} \quad 66$$

where $\boxed{-(b''_{16})^{(2)}(G_{19},t)}$ , $\boxed{-(b''_{17})^{(2)}(G_{19},t)}$ , $\boxed{-(b''_{18})^{(2)}(G_{19},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,)}(G,t)}$ are second detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{44})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \begin{bmatrix} (a'_{20})^{(3)}\boxed{+(a''_{20})^{(3)}(T_{21},t)} & \boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{20} \quad 67$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \begin{bmatrix} (a'_{21})^{(3)}\boxed{+(a''_{21})^{(3)}(T_{21},t)} & \boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{21} \quad 68$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \begin{bmatrix} (a'_{22})^{(3)}\boxed{+(a''_{22})^{(3)}(T_{21},t)} & \boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{22} \quad 69$$

$\boxed{+(a''_{20})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3)}(T_{21},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}$ are second augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)}$ are third augmentation coefficients for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficients for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficients for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficients for category 1, 2 and 3

$+(a''_{36})^{(7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1, 2 and 3

$+(a''_{40})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8)}(T_{41},t)$ are eight augmentation coefficients for category 1, 2 and 3

$+(a''_{44})^{(9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b'_{20})^{(3)} & -(b''_{20})^{(3)}(G_{23},t) & -(b''_{16})^{(2,2,2)}(G_{19},t) & -(b''_{13})^{(1,1,1,)}(G,t) \\ -(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{36})^{(7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{20} \quad (70)$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b'_{21})^{(3)} & -(b''_{21})^{(3)}(G_{23},t) & -(b''_{17})^{(2,2,2)}(G_{19},t) & -(b''_{14})^{(1,1,1,)}(G,t) \\ -(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{37})^{(7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{21} \quad (71)$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)} & -(b''_{22})^{(3)}(G_{23},t) & -(b''_{18})^{(2,2,2)}(G_{19},t) & -(b''_{15})^{(1,1,1,)}(G,t) \\ -(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{38})^{(7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{22} \quad (72)$$

$-(b''_{20})^{(3)}(G_{23},t)$, $-(b''_{21})^{(3)}(G_{23},t)$, $-(b''_{22})^{(3)}(G_{23},t)$ *are first detrition coefficients for category 1, 2 and 3*

$-(b''_{16})^{(2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2)}(G_{19},t)$ *are second detrition coefficients for category 1, 2 and 3*

$-(b''_{13})^{(1,1,1,)}(G,t)$, $-(b''_{14})^{(1,1,1,)}(G,t)$, $-(b''_{15})^{(1,1,1,)}(G,t)$ *are third detrition coefficients for category 1,2 and 3*

$-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)$ *are fourth detrition coefficients for category 1, 2 and 3*

$-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)$ *are fifth detrition coefficients for category 1, 2 and 3*

$-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)$ *are sixth detrition coefficients for category 1, 2 and 3*

$-(b''_{36})^{(7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt}=(a_{24})^{(4)}G_{25}-\begin{bmatrix}(a'_{24})^{(4)}\boxed{+(a''_{24})^{(4)}(T_{25},t)}&\boxed{+(a''_{28})^{(5,5,)}(T_{29},t)}&\boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}\\ \boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)}&\boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)}&\boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)}&\boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)}&\boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{24} \quad (73)$$

$$\frac{dG_{25}}{dt}=(a_{25})^{(4)}G_{24}-\begin{bmatrix}(a'_{25})^{(4)}\boxed{+(a''_{25})^{(4)}(T_{25},t)}&\boxed{+(a''_{29})^{(5,5,)}(T_{29},t)}&\boxed{+(a''_{33})^{(6,6)}(T_{33},t)}\\ \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)}&\boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)}&\boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)}&\boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)}&\boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{25} \quad (74)$$

$$\frac{dG_{26}}{dt}=(a_{26})^{(4)}G_{25}-\begin{bmatrix}(a'_{26})^{(4)}\boxed{+(a''_{26})^{(4)}(T_{25},t)}&\boxed{+(a''_{30})^{(5,5,)}(T_{29},t)}&\boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}\\ \boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)}&\boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)}&\boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)}&\boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)}&\boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{26} \quad (75)$$

$\boxed{(a''_{24})^{(4)}(T_{25},t)}$, $\boxed{(a''_{25})^{(4)}(T_{25},t)}$, $\boxed{(a''_{26})^{(4)}(T_{25},t)}$ *are first augmentation coefficients category* 1, 2 3

$\boxed{+(a''_{28})^{(5,5,)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,)}(T_{29},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)}$
*are fourth augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)}$
*are fifth augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)}$
*are sixth augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)}$
*are seventh augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)}$
*are eighth augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt}=(b_{24})^{(4)}T_{25}-\begin{bmatrix}(b'_{24})^{(4)}\boxed{-(b''_{24})^{(4)}(G_{27},t)}&\boxed{-(b''_{28})^{(5,5,)}(G_{31},t)}&\boxed{-(b''_{32})^{(6,6,)}(G_{35},t)}\\ \boxed{-(b''_{13})^{(1,1,1,1)}(G,t)}&\boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)}&\boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)}\\ \boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)}&\boxed{-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)}&\boxed{-(b''_{44})^{(9,9,9,9)}(G_{47},t)}\end{bmatrix}T_{24} \quad (76)$$

$$\frac{dT_{25}}{dt}=(b_{25})^{(4)}T_{24}-\begin{bmatrix}(b'_{25})^{(4)}\boxed{-(b''_{25})^{(4)}(G_{27},t)}&\boxed{-(b''_{29})^{(5,5,)}(G_{31},t)}&\boxed{-(b''_{33})^{(6,6,)}(G_{35},t)}\\ \boxed{-(b''_{14})^{(1,1,1,1)}(G,t)}&\boxed{-(b''_{17})^{(2,2,2,2)}(G_{19},t)}&\boxed{-(b''_{21})^{(3,3,3,3)}(G_{23},t)}\\ \boxed{-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)}&\boxed{-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)}&\boxed{-(b''_{45})^{(9,9,9,9)}(G_{47},t)}\end{bmatrix}T_{25} \quad (77)$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)} & -(b''_{26})^{(4)}(G_{27},t) & -(b''_{30})^{(5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1)}(G,t) & -(b''_{18})^{(2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{26} \quad 78$$

*Where* $-(b''_{24})^{(4)}(G_{27},t)$, $-(b''_{25})^{(4)}(G_{27},t)$, $-(b''_{26})^{(4)}(G_{27},t)$ *are first detrition coefficients for category* $1, 2$ *and* $3$

$-(b''_{28})^{(5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,)}(G_{31},t)$ *are second detrition coefficients for category* $1, 2$ *and* $3$

$-(b''_{32})^{(6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,)}(G_{35},t)$ *are third detrition coefficients for category* $1, 2$ *and* $3$

$-(b''_{13})^{(1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1)}(G,t)$

*are fourth detrition coefficients for category* $1, 2$ *and* $3$

$-(b''_{16})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2)}(G_{19},t)$

*are fifth detrition coefficients for category* $1, 2$ *and* $3$

$-(b''_{20})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3)}(G_{23},t)$

*are sixth detrition coefficients for category* $1, 2$ *and* $3$

$-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)$

*are seventh detrition coefficients for category* $1, 2$ *and* $3$

$-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)$

*are eighth detrition coefficients for category* $1, 2$ *and* $3$

$-(b''_{46})^{(9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \begin{bmatrix} (a'_{28})^{(5)} & +(a''_{28})^{(5)}(T_{29},t) & +(a''_{24})^{(4,4,)}(T_{25},t) & +(a''_{32})^{(6,6,6)}(T_{33},t) \\ & +(a''_{13})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{16})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3,3)}(T_{21},t) \\ & +(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{28} \quad 79$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \begin{bmatrix} (a'_{29})^{(5)} & +(a''_{29})^{(5)}(T_{29},t) & +(a''_{25})^{(4,4,)}(T_{25},t) & +(a''_{33})^{(6,6,6)}(T_{33},t) \\ & +(a''_{14})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{17})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3,3)}(T_{21},t) \\ & +(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{29} \quad 80$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \begin{bmatrix} (a'_{30})^{(5)} & +(a''_{30})^{(5)}(T_{29},t) & +(a''_{26})^{(4,4,)}(T_{25},t) & +(a''_{34})^{(6,6,6)}(T_{33},t) \\ & +(a''_{15})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{18})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3,3)}(T_{21},t) \\ & +(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{30} \quad 81$$

*Where* $+(a''_{28})^{(5)}(T_{29},t)$, $+(a''_{29})^{(5)}(T_{29},t)$, $+(a''_{30})^{(5)}(T_{29},t)$ *are first augmentation coefficients for category* $1, 2$ *and* $3$

*And* $+(a''_{24})^{(4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,)}(T_{25},t)$ *are second augmentation coefficient for category* $1, 2$ *and* $3$

$+(a''_{32})^{(6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6)}(T_{33},t)$ *are third augmentation coefficient for category* $1, 2$ *and* $3$

$+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)$ are fourth augmentation coefficients for category 1,2, and 3

$+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)$ are fifth augmentation coefficients for category 1,2,and 3

$+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)$ are sixth augmentation coefficients for category 1,2, 3

$+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)$ , $+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1,2, 3

$+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)$ are eighth augmentation coefficients for category 1,2, 3

$+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \begin{bmatrix} (b'_{28})^{(5)} & -(b''_{28})^{(5)}(G_{31},t) & -(b''_{24})^{(4,4,)}(G_{27},t) & -(b''_{32})^{(6,6,6)}(G_{35},t) \\ & -(b''_{13})^{(1,1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{28} \quad (82)$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b'_{29})^{(5)} & -(b''_{29})^{(5)}(G_{31},t) & -(b''_{25})^{(4,4,)}(G_{27},t) & -(b''_{33})^{(6,6,6)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{29} \quad (83)$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b'_{30})^{(5)} & -(b''_{30})^{(5)}(G_{31},t) & -(b''_{26})^{(4,4,)}(G_{27},t) & -(b''_{34})^{(6,6,6)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1,1,)}(G,t) & -(b''_{18})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{30} \quad (84)$$

*where* $-(b''_{28})^{(5)}(G_{31},t)$ , $-(b''_{29})^{(5)}(G_{31},t)$ , $-(b''_{30})^{(5)}(G_{31},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{24})^{(4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,)}(G_{27},t)$ *are second detrition coefficients for category* 1,2 *and* 3

$-(b''_{32})^{(6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6)}(G_{35},t)$ *are third detrition coefficients for category* 1,2 *and* 3

$-(b''_{13})^{(1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1)}(G,t)$ , $-(b''_{15})^{(1,1,1,1,1,)}(G,t)$ are fourth detrition coefficients for category 1,2, and 3

$-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1,2, and 3

$-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1,2, and 3

$-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2, and 3

$-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} \quad 85$$

$$-\left[\begin{array}{ccc} (a'_{32})^{(6)}\boxed{+(a''_{32})^{(6)}(T_{33},t)} & \boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)} \end{array}\right]G_{32}$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[\begin{array}{ccc} (a'_{33})^{(6)}\boxed{+(a''_{33})^{(6)}(T_{33},t)} & \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)} \end{array}\right]G_{33} \quad 86$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[\begin{array}{ccc} (a'_{34})^{(6)}\boxed{+(a''_{34})^{(6)}(T_{33},t)} & \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)} \end{array}\right]G_{34} \quad 87$$

$\boxed{+(a''_{32})^{(6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6)}(T_{33},t)}$ *are first augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}$ *are second augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}$ *are third augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}$ - are fourth augmentation coefficients

$\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}$ - fifth augmentation coefficients

$\boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)}$ sixth augmentation coefficients

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)}$ seventh augmentation coefficients

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}$ Eighth augmentation coefficients

$\boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)}$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[\begin{array}{ccc} (b'_{32})^{(6)}\boxed{-(b''_{32})^{(6)}(G_{35},t)} & \boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)} \end{array}\right]T_{32} \quad 88$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \begin{bmatrix} (b'_{33})^{(6)}\boxed{-(b''_{33})^{(6)}(G_{35},t)}\boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}\boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}\boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)}\boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{33} \quad (89)$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)}\boxed{-(b''_{34})^{(6)}(G_{35},t)}\boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}\boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}\boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)}\boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{34} \quad (90)$$

$\boxed{-(b''_{32})^{(6)}(G_{35},t)}, \boxed{-(b''_{33})^{(6)}(G_{35},t)}, \boxed{-(b''_{34})^{(6)}(G_{35},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}$ *are second detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)}$ *are third detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}$ are fourth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \quad (91)$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a'_{36})^{(7)}\boxed{+(a''_{36})^{(7)}(T_{37},t)}\boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)}\boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)}\boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)}\boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)}\boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \quad (92)$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a'_{37})^{(7)}\boxed{+(a''_{37})^{(7)}(T_{37},t)}\boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)}\boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)}\boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)}\boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}\boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$\frac{dG_{38}}{dt}$ *93*

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a'_{38})^{(7)}\boxed{+(a''_{38})^{(7)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{(a''_{36})^{(7)}(T_{37},t)}$, $\boxed{(a''_{37})^{(7)}(T_{37},t)}$, $\boxed{(a''_{38})^{(7)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,)}(T_{41},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$\frac{dT_{36}}{dt} =$ 94

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b'_{36})^{(7)}\boxed{-(b''_{36})^{(7)}(G_{39},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$\frac{dT_{37}}{dt} =$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b'_{37})^{(7)}\boxed{-(b''_{37})^{(7)}(G_{39},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$\frac{dT_{38}}{dt} =$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b'_{38})^{(7)}\boxed{-(b''_{38})^{(7)}(G_{39},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3



$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \begin{bmatrix} (a'_{40})^{(8)}\boxed{+(a''_{40})^{(8)}(T_{41},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \begin{bmatrix} (a'_{41})^{(8)}\boxed{+(a''_{41})^{(8)}(T_{41},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \begin{bmatrix} (a'_{42})^{(8)}\boxed{+(a''_{42})^{(8)}(T_{41},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a''_{40})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second

augmentation coefficient for category 1, 2 and 3
$+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)$ , $+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)$ , $+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3
$+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)$ , $+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)$ , $+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3
$+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)$ , $+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3
$+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)$ , $+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3
$+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ $+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ $+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ are seventh augmentation coefficient for 1,2,3
$+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$ , $+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$ , $+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$ are eighth augmentation coefficient for 1,2,3
$+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$ , $+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$ , $+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} =$$

$$(b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)}\left[-(b''_{40})^{(8)}(G_{43},t)\right] & \left[-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} =$$

$$(b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)}\left[-(b''_{41})^{(8)}(G_{43},t)\right] & \left[-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} =$$

$$(b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)}\left[-(b''_{42})^{(8)}(G_{43},t)\right] & \left[-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{15}$$

Where $-(b''_{36})^{(7)}(G_{39},t)$ , $-(b''_{37})^{(7)}(G_{39},t)$ , $-(b''_{38})^{(7)}(G_{39},t)$ are first detrition coefficients for category 1, 2 and 3
$-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$ , $-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$ , $-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3
$-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$ , $-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$ , $-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3
$-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)$ , $-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)$ , $-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{28}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{32}'')^{(6,6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{38}'')^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{36}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{37}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{44}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{45}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{46}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

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$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \begin{bmatrix} (a_{44}')^{(9)}\boxed{+(a_{44}'')^{(9)}(T_{45},t)} & \boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{20}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{24}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{28}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{32}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{36}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{40}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \begin{bmatrix} (a_{45}')^{(9)}\boxed{+(a_{45}'')^{(9)}(T_{45},t)} & \boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{21}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{25}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{29}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{33}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{14}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{37}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{41}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \begin{bmatrix} (a_{46}')^{(9)}\boxed{+(a_{46}'')^{(9)}(T_{37},t)} & \boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{22}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{26}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{30}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{34}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{15}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{38}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{42}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a_{44}'')^{(9)}(T_{45},t)}$, $\boxed{+(a_{45}'')^{(9)}(T_{45},t)}$, $\boxed{+(a_{46}'')^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{20}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{21}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{22}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{24}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{25}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{26}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{28}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{29}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{30}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,1,1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1,1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1,1,1,1,1,1)}(T_{14},t)$ are Seventh augmentation coefficient for category 1, 2 and 3

$+(a''_{38})^{(7,7,7,7,7,7,7,7,7,7)}(T_{37},t)$ $+(a''_{37})^{(7,7,7,7,7,7,7,7,7,7)}(T_{37},t)$ $+(a''_{36})^{(7,7,7,7,7,7,7,7,7,7)}(T_{37},t)$ are eighth augmentation coefficient for 1,2,3

$+(a''_{40})^{(8,8,8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8,8,8,8,8)}(T_{41},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} =$$

$$(b_{44})^{(9)}T_{45} -$$

$$\begin{bmatrix} (b'_{44})^{(9)} & -(b''_{44})^{(9)}(G_{47},t) & -(b''_{16})^{(2,2,2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6,6,6,6,6,6)}(G_{35},t) & \\ -(b''_{13})^{(1,1,1,1,1,1,1,1,1,1)}(G,t) & -(b''_{36})^{(7,7,7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8,8,8,8,8)}(G_{43},t) & \end{bmatrix} T_{13}$$

$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b'_{45})^{(9)} & -(b''_{45})^{(9)}(G_{47},t) & -(b''_{17})^{(2,2,2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6,6,6,6,6,6)}(G_{35},t) & \\ -(b''_{14})^{(1,1,1,1,1,1,1,1,1,1)}(G,t) & -(b''_{37})^{(7,7,7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8,8,8,8,8)}(G_{43},t) & \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b'_{46})^{(9)} & -(b''_{46})^{(9)}(G_{47},t) & -(b''_{18})^{(2,2,2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6,6,6,6,6,6)}(G_{35},t) & \\ -(b''_{15})^{(1,1,1,1,1,1,1,1,1,1)}(G,t) & -(b''_{38})^{(7,7,7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8,8,8,8,8)}(G_{43},t) & \end{bmatrix} T_{15}$$

Where $-(b''_{44})^{(9)}(G_{47},t)$, $-(b''_{45})^{(9)}(G_{47},t)$, $-(b''_{46})^{(9)}(G_{47},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6,6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{15})^{(1,1,1,1,1,1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1,1,1,1,1,1)}(G,t)$, $-(b''_{13})^{(1,1,1,1,1,1,1,1,1,1)}(G,t)$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{37}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{36}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a_i')^{(1)}, (a_i'')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (b_i'')^{(1)} > 0, \quad i,j = 13,14,15 \qquad 97$$

The functions $(a_i'')^{(1)}, (b_i'')^{(1)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a_i'')^{(1)}(T_{14},t) \leq (p_i)^{(1)} \leq (\hat{A}_{13})^{(1)}$$

$$(b_i'')^{(1)}(G,t) \leq (r_i)^{(1)} \leq (b_i')^{(1)} \leq (\hat{B}_{13})^{(1)}$$

$$lim_{T_2\to\infty}(a_i'')^{(1)}(T_{14},t) = (p_i)^{(1)} \qquad 98$$

$$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = (r_i)^{(1)}$$

**Definition of** $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$ :

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$$|(a_i'')^{(1)}(T_{14}',t) - (a_i'')^{(1)}(T_{14},t)| \leq (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G',t) - (b_i'')^{(1)}(G,t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}',t)$ and $(a_i'')^{(1)}(T_{14},t)$ . $(T_{14}',t)$ and $(T_{14},t)$ are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14},t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$ : 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$$

**Definition of** $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$ : 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i,j = 16,17,18$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17}, t) \leq (p_i)^{(2)} \leq (\hat{A}_{16})^{(2)} \quad 102$$

$$(b_i'')^{(2)}(G_{19}, t) \leq (r_i)^{(2)} \leq (b_i')^{(2)} \leq (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2 \to \infty} (a_i'')^{(2)}(T_{17}, t) = (p_i)^{(2)} \quad 104$$

$$\lim_{G \to \infty} (b_i'')^{(2)}((G_{19}), t) = (r_i)^{(2)} \quad 105$$

**Definition of** $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$ : 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}', t) - (a_i'')^{(2)}(T_{17}, t)| \leq (\hat{k}_{16})^{(2)}|T_{17} - T_{17}'|e^{-(\hat{M}_{16})^{(2)}t} \quad 107$$

$$|(b_i'')^{(2)}((G_{19})', t) - (b_i'')^{(2)}((G_{19}), t)| < (\hat{k}_{16})^{(2)}||(G_{19}) - (G_{19})'||e^{-(\hat{M}_{16})^{(2)}t} \quad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}', t)$ and$(a_i'')^{(2)}(T_{17}, t)$ . $(T_{17}', t)$ and $(T_{17}, t)$ are points belonging to the interval $[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}]$ . It is to be noted that $(a_i'')^{(2)}(T_{17}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17}, t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$ :

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants 109

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$$

**Definition of** $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$ :

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1 \quad (110)$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(b_i)^{(2)} + (b_i')^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1 \quad (111)$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i,j = 20,21,22 \quad (112)$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(3)}$, $(r_i)^{(3)}$:

$$(a_i'')^{(3)}(T_{21}, t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23}, t) \leq (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(3)}(T_{21}, t) = (p_i)^{(3)} \quad (113)$$

$$\lim_{G \to \infty}(b_i'')^{(3)}(G_{23}, t) = (r_i)^{(3)}$$

**Definition of** $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$ :

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: (114)

$$|(a_i'')^{(3)}(T_{21}', t) - (a_i'')^{(3)}(T_{21}, t)| \leq (\hat{k}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\hat{M}_{20})^{(3)}t}$$

$$|(b_i'')^{(3)}(G_{23}', t) - (b_i'')^{(3)}(G_{23}, t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}'||e^{-(\hat{M}_{20})^{(3)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}', t)$ and $(a_i'')^{(3)}(T_{21}, t)$ . $(T_{21}', t)$ And $(T_{21}, t)$ are points belonging to the interval $[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}]$. It is to be noted that $(a_i'')^{(3)}(T_{21}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$ : (115)

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$ and the constants $(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}, i = 20,21,22,$ satisfy the inequalities 116

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(a_i)^{(3)} + (a_i')^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(b_i)^{(3)} + (b_i')^{(3)} + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i, j = 24,25,26 \qquad 117$$

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(4)}$, $(r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25}, t) \le (p_i)^{(4)} \le (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}((G_{27}), t) \le (r_i)^{(4)} \le (b_i')^{(4)} \le (\hat{B}_{24})^{(4)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(4)}(T_{25}, t) = (p_i)^{(4)} \qquad 118$$

$$\lim_{G \to \infty}(b_i'')^{(4)}((G_{27}), t) = (r_i)^{(4)}$$

**Definition of** $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$ :

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24,25,26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}', t) - (a_i'')^{(4)}(T_{25}, t)| \le (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})', t) - (b_i'')^{(4)}((G_{27}), t)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}', t)$ and$(a_i'')^{(4)}(T_{25}, t)$ . $(T_{25}', t)$ and $(T_{25}, t)$ are points belonging to the interval $[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}]$. It is to be noted that $(a_i'')^{(4)}(T_{25}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$ : 120

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$, are positive constants

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}, \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$$

**Definition of** $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$ : 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and\ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}, i = 24,25,26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(a_i)^{(4)} + (a_i')^{(4)} + (\hat{A}_{24})^{(4)} + (\hat{P}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(b_i)^{(4)} + (b_i')^{(4)} + (\hat{B}_{24})^{(4)} + (\hat{Q}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i,j = 28,29,30 \qquad 122$$

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(5)}$, $(r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29}, t) \le (p_i)^{(5)} \le (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}((G_{31}), t) \le (r_i)^{(5)} \le (b_i')^{(5)} \le (\hat{B}_{28})^{(5)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(5)}(T_{29}, t) = (p_i)^{(5)} \qquad 123$$

$$\lim_{G \to \infty}(b_i'')^{(5)}(G_{31}, t) = (r_i)^{(5)}$$

**Definition of** $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$ :

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}', t) - (a_i'')^{(5)}(T_{29}, t)| \le (\hat{k}_{28})^{(5)}|T_{29} - T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})', t) - (b_i'')^{(5)}((G_{31}), t)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}', t)$ and $(a_i'')^{(5)}(T_{29}, t)$ . $(T_{29}', t)$ and $(T_{29}, t)$ are points belonging to the interval $[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}]$. It is to be noted that $(a_i'')^{(5)}(T_{29}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29}, t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$ : 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}, \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

**Definition of** $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$ : 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\widehat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and\ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\widehat{M}_{28})^{(5)}}[(a_i)^{(5)} + (a_i')^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\widehat{M}_{28})^{(5)}}[(b_i)^{(5)} + (b_i')^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i, j = 32,33,34 \qquad 127$$

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(6)}$, $(r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33}, t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}), t) \leq (r_i)^{(6)} \leq (b_i')^{(6)} \leq (\hat{B}_{32})^{(6)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(6)}(T_{33}, t) = (p_i)^{(6)} \qquad 128$$

$$\lim_{G \to \infty}(b_i'')^{(6)}((G_{35}), t) = (r_i)^{(6)}$$

**Definition of** $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$ :

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}', t) - (a_i'')^{(6)}(T_{33}, t)| \leq (\hat{k}_{32})^{(6)}|T_{33} - T_{33}'|e^{-(\widehat{M}_{32})^{(6)}t}$$

$$|(b_i'')^{(6)}((G_{35})', t) - (b_i'')^{(6)}((G_{35}), t)| < (\hat{k}_{32})^{(6)}||(G_{35}) - (G_{35})'||e^{-(\widehat{M}_{32})^{(6)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}', t)$ and $(a_i'')^{(6)}(T_{33}, t)$ . $(T_{33}', t)$ and $(T_{33}, t)$ are points belonging to the interval $[(\hat{k}_{32})^{(6)}, (\widehat{M}_{32})^{(6)}]$. It is to be noted that $(a_i'')^{(6)}(T_{33}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33}, t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\widehat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$ : 129

$(\widehat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$, are positive constants

$$\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}} < 1$$

**Definition of** $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$ : 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and\ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32,33,34,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(a_i)^{(6)} + (a_i')^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)} (\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(b_i)^{(6)} + (b_i')^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)} (\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

(CCC) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i, j = 36,37,38$ 131

(DDD) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(7)}$, $(r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37}, t) \leq (p_i)^{(7)} \leq (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39}, t) \leq (r_i)^{(7)} \leq (b_i')^{(7)} \leq (\hat{B}_{36})^{(7)}$$

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*(EEE)* $\lim_{T_2 \to \infty} (a_i'')^{(7)}(T_{37}, t) = (p_i)^{(7)}$

*(FFF)*

$\lim_{G \to \infty} (b_i'')^{(7)}((G_{39}), t) = (r_i)^{(7)}$

**Definition of** $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$ :

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$$|(a_i'')^{(7)}(T_{37}', t) - (a_i'')^{(7)}(T_{37}, t)| \leq (\hat{k}_{36})^{(7)} |T_{37} - T_{37}'| e^{-(\hat{M}_{36})^{(7)} t}$$

$$|(b_i'')^{(7)}((G_{39})', t) - (b_i'')^{(7)}((G_{39}), t)| < (\hat{k}_{36})^{(7)} ||(G_{39}) - (G_{39})'|| e^{-(\hat{M}_{36})^{(7)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}', t)$ and $(a_i'')^{(7)}(T_{37}, t)$ . $(T_{37}', t)$ and $(T_{37}, t)$ are points belonging to the interval $[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}]$ . It is to be noted that $(a_i'')^{(7)}(T_{37}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37}, t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$ : 134

(GGG) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)},$ are positive constants

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$$

**Definition of** $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$ : 135

(HHH) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(b_i)^{(7)} + (b_i')^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40,41,42 \qquad 136$$

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

**Definition of** $(p_i)^{(8)}$, $(r_i)^{(8)}$: 137

$$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)} \qquad 138$$

$$(b_i'')^{(8)}((G_{43}), t) \leq (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)} \qquad 139$$

$$\lim_{T_2 \to \infty} (a_i'')^{(8)}(T_{41}, t) = (p_i)^{(8)} \qquad 140$$

$$\lim_{G \to \infty} (b_i'')^{(8)}\big((G_{43}), t\big) = (r_i)^{(8)} \qquad 141$$

**Definition of** $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$ :

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)} |T_{41} - T_{41}'| e^{-(\hat{M}_{40})^{(8)} t} \qquad 142$$

$$|(b_i'')^{(8)}((G_{43})', t) - (b_i'')^{(8)}\big((G_{43}), t\big)| < (\hat{k}_{40})^{(8)} ||(G_{43}) - (G_{43})'|| e^{-(\hat{M}_{40})^{(8)} t} \qquad 143$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}', t)$ and $(a_i'')^{(8)}(T_{41}, t)$ . $(T_{41}', t)$ and $(T_{41}, t)$ are points belonging to the interval $\big[(\hat{k}_{40})^{(8)}, (\hat{M}_{40})^{(8)}\big]$. It is to be noted that $(a_i'')^{(8)}(T_{41}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{40})^{(8)} = 1$

then the function $(a_i'')^{(8)}(T_{41},t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)}$ :

$(\hat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)},$ are positive constants

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\ ,\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}}<1 \qquad 144$$

**Definition of** $(\hat{P}_{40})^{(8)},(\hat{Q}_{40})^{(8)}$ :

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\hat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)},(\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)},(a_i')^{(8)},(b_i)^{(8)},(b_i')^{(8)},(p_i)^{(8)},\ (r_i)^{(8)},i=40,41,42,$

Satisfy the inequalities

$$\frac{1}{(\hat{M}_{40})^{(8)}}[\,(a_i)^{(8)}+(a_i')^{(8)}+\ (\hat{A}_{40})^{(8)}+\ (\hat{P}_{40})^{(8)}\,(\hat{k}_{40})^{(8)}]<1 \qquad 145$$

$$\frac{1}{(\hat{M}_{40})^{(8)}}[\ (b_i)^{(8)}+(b_i')^{(8)}+\ (\hat{B}_{40})^{(8)}+\ (\hat{Q}_{40})^{(8)}\ \ (\hat{k}_{40})^{(8)}]<1 \qquad 146$$

Where we suppose

$$(a_i)^{(9)},(a_i')^{(9)},(a_i'')^{(9)},(b_i)^{(9)},(b_i')^{(9)},(b_i'')^{(9)}>0,\qquad i,j=44,45,46 \qquad 146\,A$$

The functions $(a_i'')^{(9)},(b_i'')^{(9)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(9)},\ (r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45},t)\le(p_i)^{(9)}\le(\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47},t)\le\ (r_i)^{(9)}\le(b_i')^{(9)}\le(\hat{B}_{44})^{(9)}$$

$$\lim_{T_2\to\infty}(a_i'')^{(9)}(T_{45},t)=(p_i)^{(9)}$$

$$\lim_{G\to\infty}(b_i'')^{(9)}(G_{47},t)=\ (r_i)^{(9)}$$

**Definition of** $(\hat{A}_{44})^{(9)},(\hat{B}_{44})^{(9)}$ :

Where $\boxed{(\hat{A}_{44})^{(9)},(\hat{B}_{44})^{(9)},(p_i)^{(9)},\ (r_i)^{(9)}}$ are positive constants and $\boxed{i=44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}',t)-(a_i'')^{(9)}(T_{45},t)|\le(\hat{k}_{44})^{(9)}|T_{45}-\ T_{45}'|e^{-(\hat{M}_{44})^{(9)}t}$$

$$|(b_i'')^{(9)}((G_{47})',t)-(b_i'')^{(9)}\big((G_{47}),t\big)|<(\hat{k}_{44})^{(9)}||(G_{47})-(G_{47})'||e^{-(\hat{M}_{44})^{(9)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}',t)$ and$(a_i'')^{(9)}(T_{45},t)$ . $(T_{45}',t)$ and $(T_{45},t)$ are points belonging to the interval $\big[(\hat{k}_{44})^{(9)},(\hat{M}_{44})^{(9)}\big]$. It is

to be noted that $(a_i'')^{(9)}(T_{45},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{44})^{(9)}=1$ then the function $(a_i'')^{(9)}(T_{45},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\widehat{M}_{44})^{(9)},(\hat{k}_{44})^{(9)}$ :

$(\widehat{M}_{44})^{(9)},(\hat{k}_{44})^{(9)}$, are positive constants

$$\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\ ,\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}<1$$

**Definition of** $(\hat{P}_{44})^{(9)},(\hat{Q}_{44})^{(9)}$ :

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\widehat{M}_{44})^{(9)},(\hat{k}_{44})^{(9)},(\hat{A}_{44})^{(9)} and\ (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)},(a_i')^{(9)},(b_i)^{(9)},(b_i')^{(9)},(p_i)^{(9)},\ (r_i)^{(9)}, i=44,45,46,$
satisfy the inequalities

$$\frac{1}{(\widehat{M}_{44})^{(9)}}[(a_i)^{(9)}+(a_i')^{(9)}+\ (\hat{A}_{44})^{(9)}+\ (\hat{P}_{44})^{(9)}\ (\hat{k}_{44})^{(9)}]<1$$

$$\frac{1}{(\widehat{M}_{44})^{(9)}}[\ (b_i)^{(9)}+(b_i')^{(9)}+\ (\hat{B}_{44})^{(9)}+\ (\hat{Q}_{44})^{(9)}\ (\hat{k}_{44})^{(9)}]<1$$

**Theorem 1:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0)\ ,T_i(0)$ :

$$G_i(t)\le\ (\hat{P}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}\ ,\ \boxed{G_i(0)=G_i^0>0}$$

$$T_i(t)\le\ (\hat{Q}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}\ ,\ \boxed{T_i(0)=T_i^0>0}$$

**Theorem 2 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

**Definition of** $G_i(0)\ ,T_i(0)$

$$G_i(t)\le\ (\hat{P}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}\ ,\ \ G_i(0)=G_i^0>0$$

$$T_i(t)\le\ (\hat{Q}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}\ ,\ \ T_i(0)=T_i^0>0$$

**Theorem 3 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t)\le\ (\hat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}\ ,\ \ G_i(0)=G_i^0>0$$

$$T_i(t)\le\ (\hat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}\ ,\ \ T_i(0)=T_i^0>0$$

**Theorem 4 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

**Definition of** $G_i(0)\ ,T_i(0)$ :

$G_i(t) \le \left(\hat{P}_{24}\right)^{(4)} e^{(\hat{M}_{24})^{(4)}t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \le \left(\hat{Q}_{24}\right)^{(4)} e^{(\hat{M}_{24})^{(4)}t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Theorem 5 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

**Definition of** $G_i(0)$ , $T_i(0)$ :

$G_i(t) \le \left(\hat{P}_{28}\right)^{(5)} e^{(\hat{M}_{28})^{(5)}t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \le \left(\hat{Q}_{28}\right)^{(5)} e^{(\hat{M}_{28})^{(5)}t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Theorem 6 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

**Definition of** $G_i(0)$ , $T_i(0)$ :

$G_i(t) \le \left(\hat{P}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \le \left(\hat{Q}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Theorem 7:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

**Definition of** $G_i(0)$ , $T_i(0)$ :

$G_i(t) \le \left(\hat{P}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \le \left(\hat{Q}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Theorem 8:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

**Definition of** $G_i(0)$ , $T_i(0)$ :

$G_i(t) \le \left(\hat{P}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \le \left(\hat{Q}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Theorem 9:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

**Definition of** $G_i(0)$ , $T_i(0)$ :

$G_i(t) \le \left(\hat{P}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \le \left(\hat{Q}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Proof:** Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i$: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \le (\hat{P}_{13})^{(1)} \text{ , } T_i^0 \le (\hat{Q}_{13})^{(1)}, \quad 155$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t} \quad 156$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t} \quad 157$$

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}(s_{(13)}) - \left((a'_{13})^{(1)} + a''_{13})^{(1)} (T_{14}(s_{(13)}), s_{(13)})\right) G_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)} (T_{14}(s_{(13)}), s_{(13)})\right) G_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)} (T_{14}(s_{(13)}), s_{(13)})\right) G_{15}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)} (G(s_{(13)}), s_{(13)})\right) T_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)} (G(s_{(13)}), s_{(13)})\right) T_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{15}(t) = T_{15}^0 + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)} (G(s_{(13)}), s_{(13)})\right) T_{15}(s_{(13)})\right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i$: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \le (\hat{P}_{16})^{(2)} \text{ , } T_i^0 \le (\hat{Q}_{16})^{(2)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$$

By 160

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''_{16})^{(2)} (T_{17}(s_{(16)}), s_{(16)})\right) G_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)} (T_{17}(s_{(16)}), s_{(17)})\right) G_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)} (T_{17}(s_{(16)}), s_{(16)})\right) G_{18}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)} (G_{19}(s_{(16)}), s_{(16)})\right) T_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^{0} + \int_0^t \left[ (b_{17})^{(2)} T_{16}(s_{(16)}) - \left( (b'_{17})^{(2)} - (b''_{17})^{(2)} (G_{19}(s_{(16)}), s_{(16)}) \right) T_{17}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^{0} + \int_0^t \left[ (b_{18})^{(2)} T_{17}(s_{(16)}) - \left( (b'_{18})^{(2)} - (b''_{18})^{(2)} (G_{19}(s_{(16)}), s_{(16)}) \right) T_{18}(s_{(16)}) \right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \le (\hat{P}_{20})^{(3)} \text{ , } T_i^0 \le (\hat{Q}_{20})^{(3)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$$

By 161

$$\bar{G}_{20}(t) = G_{20}^0 + \int_0^t \left[ (a_{20})^{(3)} G_{21}(s_{(20)}) - \left( (a'_{20})^{(3)} + a''_{20})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[ (a_{21})^{(3)} G_{20}(s_{(20)}) - \left( (a'_{21})^{(3)} + (a''_{21})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{21}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[ (a_{22})^{(3)} G_{21}(s_{(20)}) - \left( (a'_{22})^{(3)} + (a''_{22})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{22}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^0 + \int_0^t \left[ (b_{20})^{(3)} T_{21}(s_{(20)}) - \left( (b'_{20})^{(3)} - (b''_{20})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^0 + \int_0^t \left[ (b_{21})^{(3)} T_{20}(s_{(20)}) - \left( (b'_{21})^{(3)} - (b''_{21})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{21}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{22}(t) = T_{22}^0 + \int_0^t \left[ (b_{22})^{(3)} T_{21}(s_{(20)}) - \left( (b'_{22})^{(3)} - (b''_{22})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{22}(s_{(20)}) \right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \le (\hat{P}_{24})^{(4)} \text{ , } T_i^0 \le (\hat{Q}_{24})^{(4)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

By 162

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[ (a_{24})^{(4)} G_{25}(s_{(24)}) - \left( (a'_{24})^{(4)} + a''_{24})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{26}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{\mathrm{T}}_{26}(\mathrm{t}) = \mathrm{T}_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{26}(s_{(24)})\right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{28})^{(5)}, T_i^0 \le (\hat{Q}_{28})^{(5)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

By 163

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{30}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{\mathrm{T}}_{30}(\mathrm{t}) = \mathrm{T}_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{30}(s_{(28)})\right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{32})^{(6)}, T_i^0 \le (\hat{Q}_{32})^{(6)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\bar{M}_{32})^{(6)} t}$

By 164

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[ (a_{32})^{(6)} G_{33}(s_{(32)}) - \left( (a'_{32})^{(6)} + a''_{32})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[ (a_{33})^{(6)} G_{32}(s_{(32)}) - \left( (a'_{33})^{(6)} + (a''_{33})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[ (a_{34})^{(6)} G_{33}(s_{(32)}) - \left( (a'_{34})^{(6)} + (a''_{34})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{34}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[ (b_{32})^{(6)} T_{33}(s_{(32)}) - \left( (b'_{32})^{(6)} - (b''_{32})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[ (b_{33})^{(6)} T_{32}(s_{(32)}) - \left( (b'_{33})^{(6)} - (b''_{33})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{\mathrm{T}}_{34}(\mathrm{t}) = \mathrm{T}_{34}^0 + \int_0^t \left[ (b_{34})^{(6)} T_{33}(s_{(32)}) - \left( (b'_{34})^{(6)} - (b''_{34})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{34}(s_{(32)}) \right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[ (a_{36})^{(7)} G_{37}(s_{(36)}) - \left( (a'_{36})^{(7)} + a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[ (a_{37})^{(7)} G_{36}(s_{(36)}) - \left( (a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[ (a_{38})^{(7)} G_{37}(s_{(36)}) - \left( (a'_{38})^{(7)} + (a''_{38})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{38}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[ (b_{36})^{(7)} T_{37}(s_{(36)}) - \left( (b'_{36})^{(7)} - (b''_{36})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[ (b_{37})^{(7)} T_{36}(s_{(36)}) - \left( (b'_{37})^{(7)} - (b''_{37})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^0 + \int_0^t \left[ (b_{38})^{(7)} T_{37}(s_{(36)}) - \left( (b'_{38})^{(7)} - (b''_{38})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{38}(s_{(36)}) \right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0,t)$

**Proof:**

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{40})^{(8)}$, $T_i^0 \le (\hat{Q}_{40})^{(8)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} G_{41}(s_{(40)}) - \left((a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[(a_{41})^{(8)} G_{40}(s_{(40)}) - \left((a'_{41})^{(8)} + (a''_{41})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{42}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^0 + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)} (G_{43}(s_{(40)}), s_{(40)})\right) T_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^0 + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)} (G_{43}(s_{(40)}), s_{(40)})\right) T_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{42}(t) = T_{42}^0 + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)} (G_{43}(s_{(40)}), s_{(40)})\right) T_{42}(s_{(40)})\right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0,t)$

**Proof:** 166
Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ A
which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{44})^{(9)}$, $T_i^0 \le (\hat{Q}_{44})^{(9)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)})\right) G_{44}(s_{(44)})\right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)} (T_{45}(s_{(44)}), s_{(44)})\right) G_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}(s_{(44)}), s_{(44)})\right) G_{46}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{44}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{\mathrm{T}}_{46}(\mathrm{t}) = \mathrm{T}_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{46}(s_{(44)})\right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}}\right)\right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left(e^{(\hat{M}_{13})^{(1)} t} - 1\right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\hat{M}_{13})^{(1)} t} \le \frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{P}_{13})^{(1)} + G_{14}^0\right) e^{\left(-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0}\right)} + (\hat{P}_{13})^{(1)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \le G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}}\right)\right] ds_{(16)} = \quad 169$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1\right)$$

From which it follows that 170

$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \le \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left[\left((\hat{P}_{16})^{(2)} + G_{17}^0\right) e^{\left(-\frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0}\right)} + (\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t) \le G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} \left(G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} s_{(20)}}\right)\right] ds_{(20)} =$$

$$\left(1 + (a_{20})^{(3)} t\right) G_{21}^0 + \frac{(a_{20})^{(3)} (\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left(e^{(\hat{M}_{20})^{(3)} t} - 1\right)$$

From which it follows that 172

$$(G_{20}(t)-G_{20}^{0})e^{-(\hat{M}_{20})^{(3)}t}\leq\frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left[((\hat{P}_{20})^{(3)}+G_{21}^{0})e^{\left(-\frac{(\hat{P}_{20})^{(3)}+G_{21}^{0}}{G_{21}^{0}}\right)}+(\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 173

$$G_{24}(t)\leq G_{24}^{0}+\int_{0}^{t}\left[(a_{24})^{(4)}\left(G_{25}^{0}+(\hat{P}_{24})^{(4)}e^{(\hat{M}_{24})^{(4)}s_{(24)}}\right)\right]ds_{(24)}=$$

$$\left(1+(a_{24})^{(4)}t\right)G_{25}^{0}+\frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left(e^{(\hat{M}_{24})^{(4)}t}-1\right)$$

From which it follows that 174

$$(G_{24}(t)-G_{24}^{0})e^{-(\hat{M}_{24})^{(4)}t}\leq\frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left[((\hat{P}_{24})^{(4)}+G_{25}^{0})e^{\left(-\frac{(\hat{P}_{24})^{(4)}+G_{25}^{0}}{G_{25}^{0}}\right)}+(\hat{P}_{24})^{(4)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t)\leq G_{28}^{0}+\int_{0}^{t}\left[(a_{28})^{(5)}\left(G_{29}^{0}+(\hat{P}_{28})^{(5)}e^{(\hat{M}_{28})^{(5)}s_{(28)}}\right)\right]ds_{(28)}=$$

$$\left(1+(a_{28})^{(5)}t\right)G_{29}^{0}+\frac{(a_{28})^{(5)}(\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left(e^{(\hat{M}_{28})^{(5)}t}-1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t}\leq\frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t)\leq G_{32}^{0}+\int_{0}^{t}\left[(a_{32})^{(6)}\left(G_{33}^{0}+(\hat{P}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\right)\right]ds_{(32)}=$$

$$\left(1+(a_{32})^{(6)}t\right)G_{33}^{0}+\frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left(e^{(\hat{M}_{32})^{(6)}t}-1\right)$$

From which it follows that 177

$$(G_{32}(t)-G_{32}^{0})e^{-(\hat{M}_{32})^{(6)}t}\leq\frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left[((\hat{P}_{32})^{(6)}+G_{33}^{0})e^{\left(-\frac{(\hat{P}_{32})^{(6)}+G_{33}^{0}}{G_{33}^{0}}\right)}+(\hat{P}_{32})^{(6)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

(j) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t) \le G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} \left(G_{37}^0 + (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} s_{(36)}}\right)\right] ds_{(36)} =$$
$$\left(1 + (a_{36})^{(7)} t\right) G_{37}^0 + \frac{(a_{36})^{(7)} (\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left(e^{(\hat{M}_{36})^{(7)} t} - 1\right)$$

From which it follows that

$$(G_{36}(t) - G_{36}^0) e^{-(\hat{M}_{36})^{(7)} t} \le \frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left[((\hat{P}_{36})^{(7)} + G_{37}^0) e^{\left(-\frac{(\hat{P}_{36})^{(7)} + G_{37}^0}{G_{37}^0}\right)} + (\hat{P}_{36})^{(7)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t) \le G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} \left(G_{41}^0 + (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} s_{(40)}}\right)\right] ds_{(40)} = \quad 180$$
$$\left(1 + (a_{40})^{(8)} t\right) G_{41}^0 + \frac{(a_{40})^{(8)} (\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}} \left(e^{(\hat{M}_{40})^{(8)} t} - 1\right)$$

From which it follows that 181

$$(G_{40}(t) - G_{40}^0) e^{-(\hat{M}_{40})^{(8)} t} \le \frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}} \left[((\hat{P}_{40})^{(8)} + G_{41}^0) e^{\left(-\frac{(\hat{P}_{40})^{(8)} + G_{41}^0}{G_{41}^0}\right)} + (\hat{P}_{40})^{(8)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 8

Analogous inequalities hold also for $G_{41}, G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t) \le G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} \left(G_{45}^0 + (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} s_{(44)}}\right)\right] ds_{(44)} =$$
$$\left(1 + (a_{44})^{(9)} t\right) G_{45}^0 + \frac{(a_{44})^{(9)} (\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left(e^{(\hat{M}_{44})^{(9)} t} - 1\right)$$

From which it follows that

$$(G_{44}(t) - G_{44}^0) e^{-(\hat{M}_{44})^{(9)} t} \le \frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left[((\hat{P}_{44})^{(9)} + G_{45}^0) e^{\left(-\frac{(\hat{P}_{44})^{(9)} + G_{45}^0}{G_{45}^0}\right)} + (\hat{P}_{44})^{(9)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 9

Analogous inequalities hold also for $G_{45}, G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$ and to choose 182

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\widehat{M}_{13})^{(1)}}\left[(\widehat{P}_{13})^{(1)}+\left((\hat{P}_{13})^{(1)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{13})^{(1)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{13})^{(1)} \quad 183$$

$$\frac{(b_i)^{(1)}}{(\widehat{M}_{13})^{(1)}}\left[\left((\hat{Q}_{13})^{(1)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{13})^{(1)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{13})^{(1)}\right]\leq(\hat{Q}_{13})^{(1)} \quad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)},T^{(1)}\right),\left(G^{(2)},T^{(2)}\right)\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{13})^{(1)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{13})^{(1)}t}\}$$

Indeed if we denote

**Definition of** $\tilde{G},\tilde{T}:\left(\tilde{G},\tilde{T}\right)=\mathcal{A}^{(1)}(G,T)$

It results

$$\left|\tilde{G}_{13}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{13})^{(1)}\left|G_{14}^{(1)}-G_{14}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\,ds_{(13)}+$$

$$\int_0^t\{(a'_{13})^{(1)}\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$G_{13}^{(2)}\left|(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)-(a''_{13})^{(1)}\left(T_{14}^{(2)},s_{(13)}\right)\right|\ e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses it follows

$$\left|G^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}t}\leq \quad 186$$
$$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)}+(a'_{13})^{(1)}+(\widehat{A}_{13})^{(1)}+(\widehat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}\right)d\left(\left(G^{(1)},T^{(1)};\,G^{(2)},T^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

**Remark 1:** The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}\ and\ (\widehat{Q}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i=13,14,15$ depend only on $\mathrm{T}_{14}$ and respectively on

$G(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$

From 19 to 24 it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(1)} - (a_i'')^{(1)}(T_{14}(s_{(13)}), s_{(13)})\} ds_{(13)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(1)} t)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2\ and\ \left((\widehat{M}_{13})^{(1)}\right)_3$ : 187

**Remark 3**: if $G_{13}$ is bounded, the same property have also $G_{14}\ and\ G_{15}$ . indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a_{14}')^{(1)} G_{14}$ and by integrating

$$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1 / (a_{14}')^{(1)}$$

In the same way , one can obtain

$$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2 / (a_{15}')^{(1)}$$

If $G_{14}\ or\ G_{15}$ is bounded, the same property follows for $G_{13}$ , $G_{15}$ and $G_{13}$ , $G_{14}$ respectively.

**Remark 4:** If $G_{13}$ $is$ bounded, from below, the same property holds for $G_{14}\ and\ G_{15}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{14}$ is bounded from below. 188

**Remark 5:** If $T_{13}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}\,(G(t), t)) = (b_{14}')^{(1)}$ then $T_{14} \to \infty$. 189

**Definition of** $(m)^{(1)}$ and $\varepsilon_1$ :

Indeed let $t_1$ be so that for $t > t_1$

$$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t), t) < \varepsilon_1, T_{13}\,(t) > (m)^{(1)}$$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right)$, $t = log\,\frac{2}{\varepsilon_1}$ By taking now $\varepsilon_1$ sufficiently small one sees that $T_{14}$ is unbounded.
The same property holds for $T_{15}$ if $\lim_{t\to\infty}(b_{15}'')^{(1)}\,(G(t), t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}$ , $\frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}\ and\ (\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[(\widehat{P}_{16})^{(2)}+\left((\hat{P}_{16})^{(2)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{16})^{(2)}\right]\leq(\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

**Definition of** $\widetilde{G_{19}},\widetilde{T_{19}}$ : $\left(\widetilde{G_{19}},\widetilde{T_{19}}\right)=\mathcal{A}^{(2)}(G_{19},T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{16})^{(2)}\left|G_{17}^{(1)}-G_{17}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)}+$$

$$\int_0^t\{(a'_{16})^{(2)}\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$G_{16}^{(2)}\left|(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)-(a''_{16})^{(2)}\left(T_{17}^{(2)},s_{(16)}\right)\right|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0,t]$ 197

From the hypotheses it follows

$$\left|(G_{19})^{(1)}-(G_{19})^{(2)}\right|\mathrm{e}^{-(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}\leq$$
$$\frac{1}{(\widehat{\mathrm{M}}_{16})^{(2)}}\left((a_{16})^{(2)}+(a'_{16})^{(2)}+(\widehat{\mathrm{A}}_{16})^{(2)}+(\widehat{\mathrm{P}}_{16})^{(2)}(\hat{k}_{16})^{(2)}\right)\mathrm{d}\left(((G_{19})^{(1)},(T_{19})^{(1)};\ (G_{19})^{(2)},(T_{19})^{(2)})\right)$$

And analogous inequalities for $G_i$ and $T_i$. Taking into account the hypothesis the result follows 198

**Remark 6:** The fact that we supposed $(a''_{16})^{(2)}$ and $(b''_{16})^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{\mathrm{P}}_{16})^{(2)}\mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}$ and $(\widehat{\mathrm{Q}}_{16})^{(2)}\mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i=16,17,18$ depend only on $\mathrm{T}_{17}$ and respectively on

$(G_{19})$(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 7:** There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$ 200

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(2)} - (a_i'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})\} ds_{(16)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(2)}t)} > 0 \quad \text{for } t > 0$$

**Definition of** $\left((\widehat{M}_{16})^{(2)}\right)_1$, $\left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$: 201

**Remark 8:** if $G_{16}$ is bounded, the same property have also $G_{17}$ and $G_{18}$. indeed if

$G_{16} < (\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt} \leq \left((\widehat{M}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)} G_{17}$ and by integrating

$$G_{17} \leq \left((\widehat{M}_{16})^{(2)}\right)_2 = G_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1 / (a_{17}')^{(2)}$$

In the same way , one can obtain

$$G_{18} \leq \left((\widehat{M}_{16})^{(2)}\right)_3 = G_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2 / (a_{18}')^{(2)}$$

If $G_{17}$ or $G_{18}$ is bounded, the same property follows for $G_{16}$, $G_{18}$ and $G_{16}$, $G_{17}$ respectively.

**Remark 9:** If $G_{16}$ is bounded, from below, the same property holds for $G_{17}$ and $G_{18}$. The proof is analogous with the preceding one. An analogous property is true if $G_{17}$ is bounded from below. 202

**Remark 10:** If $T_{16}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t), t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$. 203

**Definition of** $(m)^{(2)}$ and $\varepsilon_2$ :

Indeed let $t_2$ be so that for $t > t_2$

$$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right)$, $t = \log\frac{2}{\varepsilon_2}$ By taking now $\varepsilon_2$ sufficiently small one sees that $T_{17}$ is unbounded. 205
The same property holds for $T_{18}$ if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t), t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[(\widehat{P}_{20})^{(3)}+\left((\hat{P}_{20})^{(3)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{20})^{(3)}\right]\leq(\hat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself 210

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)},(T_{23})^{(1)}),((G_{23})^{(2)},(T_{23})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

**Definition of** $\widetilde{G_{23}},\widetilde{T_{23}}:(\widetilde{(G_{23})},\widetilde{(T_{23})})=\mathcal{A}^{(3)}((G_{23}),(T_{23}))$

It results 213

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{20})^{(3)}\left|G_{21}^{(1)}-G_{21}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\,ds_{(20)}+$$

$$\int_0^t\{(a'_{20})^{(3)}\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$G_{20}^{(2)}\left|(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)},s_{(20)}\right)\right|\ e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\}ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses it follows

$$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}t}\leq \quad 214$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+(a'_{20})^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\widehat{k}_{20})^{(3)}\right)d\left(((G_{23})^{(1)},(T_{23})^{(1)};\ (G_{23})^{(2)},(T_{23})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

**Remark 11:** The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}\ and\ (\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i=20,21,22$ depend only on $\mathrm{T}_{21}$ and respectively on

$(G_{23})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 12:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 216

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(3)} - (a_i'')^{(3)}(T_{21}(s_{(20)}), s_{(20)})\} ds_{(20)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(3)} t)} > 0$ for t > 0

**Definition of** $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2\ and\ \left((\widehat{M}_{20})^{(3)}\right)_3$ : 217

**Remark 13:** if $G_{20}$ is bounded, the same property have also $G_{21}\ and\ G_{22}$ . indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a_{21}')^{(3)} G_{21}$ and by integrating

$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1 / (a_{21}')^{(3)}$

In the same way , one can obtain

$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2 / (a_{22}')^{(3)}$

If $G_{21}\ or\ G_{22}$ is bounded, the same property follows for $G_{20}$ , $G_{22}$ and $G_{20}$ , $G_{21}$ respectively.

**Remark 14:** If $G_{20}$ *is* bounded, from below, the same property holds for $G_{21}\ and\ G_{22}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{21}$ is bounded from below. 218

**Remark 15:** If $T_{20}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t), t\right)) = (b_{21}')^{(3)}$ then $T_{21} \to \infty$. 219

**Definition of** $(m)^{(3)}$ and $\varepsilon_3$ :

Indeed let $t_3$ be so that for $t > t_3$

$(b_{21})^{(3)} - (b_i'')^{(3)}\left((G_{23})(t), t\right) < \varepsilon_3, T_{20}\,(t) > (m)^{(3)}$

Then $\frac{dT_{21}}{dt} \geq (a_{21})^{(3)}(m)^{(3)} - \varepsilon_3 T_{21}$ which leads to 220

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t} = \frac{1}{2}$ it results

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right),\ \ t = log\,\frac{2}{\varepsilon_3}$ By taking now $\varepsilon_3$ sufficiently small one sees that $T_{21}$ is unbounded.
The same property holds for $T_{22}$ if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t), t\right) = (b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}$ , $\frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}} < 1$ and to choose 221

$(\widehat{P}_{24})^{(4)}$ and $(\widehat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[(\widehat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \tag{222}$$

$$\frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \tag{223}$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself (224)

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric (225)

$$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{24})^{(4)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{27}}),(\widetilde{T_{27}})$ : $\left((\widetilde{G_{27}}),(\widetilde{T_{27}})\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{24})^{(4)}\left|G_{25}^{(1)}-G_{25}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\,ds_{(24)}+$$

$$\int_0^t\{(a'_{24})^{(4)}\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$G_{24}^{(2)}\left|(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)-(a''_{24})^{(4)}\left(T_{25}^{(2)},s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\}ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}t}\leq \tag{226}$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)}+(a'_{24})^{(4)}+(\widehat{A}_{24})^{(4)}+(\widehat{P}_{24})^{(4)}(\hat{k}_{24})^{(4)}\right)d\left(((G_{27})^{(1)},(T_{27})^{(1)};\ (G_{27})^{(2)},(T_{27})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 16:** The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ *and* $(\widehat{Q}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_+$. (227)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i=24,25,26$ depend only on $T_{25}$ and respectively on

$(G_{27})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 17:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 228

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(4)} - (a_i'')^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(4)} t)} > 0$ for t > 0

**Definition of** $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2\ and\ \left((\widehat{M}_{24})^{(4)}\right)_3$ : 229

**Remark 18:** if $G_{24}$ is bounded, the same property have also $G_{25}\ and\ G_{26}$ . indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a_{25}')^{(4)} G_{25}$ and by integrating

$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1 / (a_{25}')^{(4)}$

In the same way , one can obtain

$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2 / (a_{26}')^{(4)}$

If $G_{25}\ or\ G_{26}$ is bounded, the same property follows for $G_{24}$ , $G_{26}$ and $G_{24}$ , $G_{25}$ respectively.

**Remark 19:** If $G_{24}$ *is* bounded, from below, the same property holds for $G_{25}\ and\ G_{26}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{25}$ is bounded from below. 230

**Remark 20:** If $T_{24}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}\ ((G_{27})(t), t)) = (b_{25}')^{(4)}$ then $T_{25} \to \infty$. 231

**Definition of** $(m)^{(4)}$ and $\varepsilon_4$ :

Indeed let $t_4$ be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t), t) < \varepsilon_4, T_{24}\,(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_4}$ By taking now $\varepsilon_4$ sufficiently small one sees that $T_{25}$ is unbounded.
The same property holds for $T_{26}$ if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t), t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for $G_{29}$ , $G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}$ , $\frac{(b_i)^{(5)}}{(\widehat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)}+\left((\hat{P}_{28})^{(5)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{28})^{(5)} \tag{234}$$

$$\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{28})^{(5)}\right]\leq(\hat{Q}_{28})^{(5)} \tag{235}$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric (236)

$$d\left(((G_{31})^{(1)},(T_{31})^{(1)}),((G_{31})^{(2)},(T_{31})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\hat{M}_{28})^{(5)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\hat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{31}}),(\widetilde{T_{31}})$ : $\left((\widetilde{G_{31}}),(\widetilde{T_{31}})\right)=\mathcal{A}^{(5)}\left((G_{31}),(T_{31})\right)$

It results

$$|\tilde{G}_{28}^{(1)}-\tilde{G}_i^{(2)}|\leq\int_0^t(a_{28})^{(5)}|G_{29}^{(1)}-G_{29}^{(2)}|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\,ds_{(28)}+$$

$$\int_0^t\{(a'_{28})^{(5)}|G_{28}^{(1)}-G_{28}^{(2)}|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$(a''_{28})^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)|G_{28}^{(1)}-G_{28}^{(2)}|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$G_{28}^{(2)}|(a''_{28})^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)-(a''_{28})^{(5)}\left(T_{29}^{(2)},s_{(28)}\right)|\;e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\}ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on it follows

$$|(G_{31})^{(1)}-(G_{31})^{(2)}|e^{-(\widehat{M}_{28})^{(5)}t}\leq \tag{237}$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)}+(a'_{28})^{(5)}+(\widehat{A}_{28})^{(5)}+(\widehat{P}_{28})^{(5)}(\widehat{k}_{28})^{(5)}\right)d\left(((G_{31})^{(1)},(T_{31})^{(1)};\;(G_{31})^{(2)},(T_{31})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 21:** The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\hat{P}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ *and* $(\hat{Q}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ respectively of $\mathbb{R}_+$. (238)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on $T_{29}$ and respectively on $(G_{31})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 22:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 239

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(5)} - (a_i'')^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(5)} t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2\ and\ \left((\widehat{M}_{28})^{(5)}\right)_3$ : 240

**Remark 23:** if $G_{28}$ is bounded, the same property have also $G_{29}\ and\ G_{30}$ . indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a_{29}')^{(5)} G_{29}$ and by integrating

$$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1 / (a_{29}')^{(5)}$$

In the same way , one can obtain

$$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2 / (a_{30}')^{(5)}$$

If $G_{29}\ or\ G_{30}$ is bounded, the same property follows for $G_{28}$ , $G_{30}$ and $G_{28}$ , $G_{29}$ respectively.

**Remark 24:** If $G_{28}$ *is* bounded, from below, the same property holds for $G_{29}\ and\ G_{30}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{29}$ is bounded from below. 241

**Remark 25:** If $T_{28}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}\,((G_{31})(t), t)) = (b_{29}')^{(5)}$ then $T_{29} \to \infty$. 242

**Definition of** $(m)^{(5)}$ and $\varepsilon_5$ :

Indeed let $t_5$ be so that for $t > t_5$

$$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t), t) < \varepsilon_5, T_{28}\,(t) > (m)^{(5)}$$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_5}$ By taking now $\varepsilon_5$ sufficiently small one sees that $T_{29}$ is unbounded.
The same property holds for $T_{30}$ if $\lim_{t\to\infty}(b_{30}'')^{(5)}\left((G_{31})(t), t\right) = (b_{30}')^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{33}$ , $G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}$ , $\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}} < 1$ and to choose 244

$(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[(\hat{P}_{32})^{(6)}+\left((\hat{P}_{32})^{(6)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{32})^{(6)} \tag{245}$$

$$\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[\left((\hat{Q}_{32})^{(6)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{32})^{(6)}\right]\leq(\hat{Q}_{32})^{(6)} \tag{246}$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric (247)

$$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\hat{M}_{32})^{(6)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\hat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{35}}),(\widetilde{T_{35}})$ : $\left((\widetilde{G_{35}}),(\widetilde{T_{35}})\right)=\mathcal{A}^{(6)}\left((G_{35}),(T_{35})\right)$

It results

$$|\tilde{G}_{32}^{(1)}-\tilde{G}_i^{(2)}|\leq\int_0^t(a_{32})^{(6)}|G_{33}^{(1)}-G_{33}^{(2)}|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\,ds_{(32)}+$$

$$\int_0^t\{(a'_{32})^{(6)}|G_{32}^{(1)}-G_{32}^{(2)}|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)|G_{32}^{(1)}-G_{32}^{(2)}|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$G_{32}^{(2)}|(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)-(a''_{32})^{(6)}\left(T_{33}^{(2)},s_{(32)}\right)|\;e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\}ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$|(G_{35})^{(1)}-(G_{35})^{(2)}|e^{-(\widehat{M}_{32})^{(6)}t}\leq \tag{248}$$
$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)}+(a'_{32})^{(6)}+(\widehat{A}_{32})^{(6)}+(\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}\right)d\left(((G_{35})^{(1)},(T_{35})^{(1)};\;(G_{35})^{(2)},(T_{35})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 26:** The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\hat{P}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ *and* $(\hat{Q}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ respectively of $\mathbb{R}_+$. (249)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i = 32,33,34$ depend only on $T_{33}$ and respectively on $(G_{35})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 27:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 250

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(6)} - (a_i'')^{(6)}(T_{33}(s_{(32)}), s_{(32)})\} ds_{(32)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(6)} t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{32})^{(6)}\right)_1, \left((\widehat{M}_{32})^{(6)}\right)_2\ and\ \left((\widehat{M}_{32})^{(6)}\right)_3$ : 251

**Remark 28:** if $G_{32}$ is bounded, the same property have also $G_{33}\ and\ G_{34}$ . indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a_{33}')^{(6)} G_{33}$ and by integrating

$$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)} \left((\widehat{M}_{32})^{(6)}\right)_1 / (a_{33}')^{(6)}$$

In the same way , one can obtain

$$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)} \left((\widehat{M}_{32})^{(6)}\right)_2 / (a_{34}')^{(6)}$$

If $G_{33}\ or\ G_{34}$ is bounded, the same property follows for $G_{32}$ , $G_{34}$ and $G_{32}$ , $G_{33}$ respectively.

**Remark 29:** If $G_{32}$ *is* bounded, from below, the same property holds for $G_{33}\ and\ G_{34}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{33}$ is bounded from below. 252

**Remark 30:** If $T_{32}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}\ ((G_{35})(t), t)) = (b_{33}')^{(6)}$ then $T_{33} \to \infty$. 253

**Definition of** $(m)^{(6)}$ and $\varepsilon_6$ :

Indeed let $t_6$ be so that for $t > t_6$

$$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t), t\right) < \varepsilon_6, T_{32}\,(t) > (m)^{(6)}$$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right)$, $t = log\,\frac{2}{\varepsilon_6}$ By taking now $\varepsilon_6$ sufficiently small one sees that $T_{33}$ is unbounded.
The same property holds for $T_{34}$ if $\lim_{t\to\infty}(b_{34}'')^{(6)}\left((G_{35})(t), t(t), t\right) = (b_{34}')^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{37}, G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}$ , $\frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$ and to choose

$(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[(\hat{P}_{36})^{(7)}+\left((\hat{P}_{36})^{(7)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{36})^{(7)} \tag{256}$$

$$\frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[\left((\hat{Q}_{36})^{(7)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{36})^{(7)}\right]\leq(\hat{Q}_{36})^{(7)} \tag{257}$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself
The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric (258)

$$d\left(((G_{39})^{(1)},(T_{39})^{(1)}),((G_{39})^{(2)},(T_{39})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\hat{M}_{36})^{(7)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\hat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{39}}),(\widetilde{T_{39}}):\left((\widetilde{G_{39}}),(\widetilde{T_{39}})\right)=\mathcal{A}^{(7)}((G_{39}),(T_{39}))$
It results

$$|\tilde{G}_{36}^{(1)}-\tilde{G}_i^{(2)}|\leq\int_0^t(a_{36})^{(7)}|G_{37}^{(1)}-G_{37}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\,ds_{(36)}+$$
$$\int_0^t\{(a'_{36})^{(7)}|G_{36}^{(1)}-G_{36}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$(a''_{36})^{(7)}(T_{37}^{(1)},s_{(36)})|G_{36}^{(1)}-G_{36}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$G_{36}^{(2)}|(a''_{36})^{(7)}(T_{37}^{(1)},s_{(36)})-(a''_{36})^{(7)}(T_{37}^{(2)},s_{(36)})|\;e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\}ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on it follows

$$|(G_{39})^{(1)}-(G_{39})^{(2)}|e^{-(\widehat{M}_{36})^{(7)}t}\leq \tag{259}$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}}\left((a_{36})^{(7)}+(a'_{36})^{(7)}+(\widehat{A}_{36})^{(7)}+(\widehat{P}_{36})^{(7)}(\widehat{k}_{36})^{(7)}\right)d\left(((G_{39})^{(1)},(T_{39})^{(1)};\;(G_{39})^{(2)},(T_{39})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 31:** The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ *and* $(\widehat{Q}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ respectively of $\mathbb{R}_+$. (260)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(7)}$ and $(b_i'')^{(7)}, i = 36,37,38$ depend only on $T_{37}$ and respectively on $(G_{39})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 32:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 261

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(7)} - (a_i'')^{(7)}(T_{37}(s_{(36)}), s_{(36)})\} ds_{(36)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(7)} t\right)} > 0$ for t > 0

**Definition of** $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2\ and\ \left((\widehat{M}_{36})^{(7)}\right)_3$ : 262

**Remark 33:** if $G_{36}$ is bounded, the same property have also $G_{37}\ and\ G_{38}$ . indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a_{37}')^{(7)} G_{37}$ and by integrating

$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1 / (a_{37}')^{(7)}$

In the same way , one can obtain

$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2 / (a_{38}')^{(7)}$

If $G_{37}\ or\ G_{38}$ is bounded, the same property follows for $G_{36}$ , $G_{38}$ and $G_{36}$ , $G_{37}$ respectively.

**Remark 34:** If $G_{36}$ *is* bounded, from below, the same property holds for $G_{37}\ and\ G_{38}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{37}$ is bounded from below. 263

**Remark 35:** If $T_{36}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(7)}\ ((G_{39})(t), t)) = (b_{37}')^{(7)}$ then $T_{37} \to \infty$. 264

**Definition of** $(m)^{(7)}$ and $\varepsilon_7$ :

Indeed let $t_7$ be so that for $t > t_7$

$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t), t) < \varepsilon_7, T_{36}\,(t) > (m)^{(7)}$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_7}$ By taking now $\varepsilon_7$ sufficiently small one sees that $T_{37}$ is unbounded.
The same property holds for $T_{38}$ if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t), t\right) = (b_{38}')^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}$ , $\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}} < 1$ and to choose $(\widehat{P}_{40})^{(8)}$ and $(\widehat{Q}_{40})^{(8)}$ large to have 266

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[(\widehat{P}_{40})^{(8)} + \left((\hat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{40})^{(8)} \quad 267$$

$$\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{40})^{(8)}\right] \leq (\hat{Q}_{40})^{(8)} \quad 268$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)}, (T_{43})^{(1)}), ((G_{43})^{(2)}, (T_{43})^{(2)})\right) = \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

**Definition of** $\widetilde{(G_{43})}, \widetilde{(T_{43})}$ : $\left(\widetilde{(G_{43})}, \widetilde{(T_{43})}\right) = \mathcal{A}^{(8)}((G_{43}), (T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{40})^{(8)}\left|G_{41}^{(1)} - G_{41}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\,ds_{(40)} +$$
$$\int_0^t\{(a'_{40})^{(8)}\left|G_{40}^{(1)} - G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{-(\widehat{M}_{40})^{(8)}s_{(40)}} +$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right)\left|G_{40}^{(1)} - G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}} +$$
$$G_{40}^{(2)}\left|(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right) - (a''_{40})^{(8)}\left(T_{41}^{(2)}, s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\}ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0, t]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)} - (G_{43})^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}t} \leq \quad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)} + (a'_{40})^{(8)} + (\widehat{A}_{40})^{(8)} + (\widehat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}\right)d\left(((G_{43})^{(1)}, (T_{43})^{(1)};\ (G_{43})^{(2)}, (T_{43})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

**Remark 36:** The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate 274

condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ *and* $(\widehat{Q}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i = 40,41,42$ depend only on $T_{41}$ and respectively on $(G_{43})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 37** There does not exist any $t$ where $G_i(t) = 0\ and\ T_i(t) = 0$ 275

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(8)} - (a_i'')^{(8)}(T_{41}(s_{(40)}), s_{(40)})\} ds_{(40)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(8)}t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{40})^{(8)}\right)_1, \left((\widehat{M}_{40})^{(8)}\right)_2\ and\ \left((\widehat{M}_{40})^{(8)}\right)_3$ : 276

**Remark 38:** if $G_{40}$ is bounded, the same property have also $G_{41}\ and\ G_{42}$ . indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a_{41}')^{(8)}G_{41}$ and by integrating

$$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1/(a_{41}')^{(8)}$$

In the same way , one can obtain

$$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2/(a_{42}')^{(8)}$$

If $G_{41}\ or\ G_{42}$ is bounded, the same property follows for $G_{40}$ , $G_{42}$ and $G_{40}$ , $G_{41}$ respectively.

**Remark 39:** If $G_{40}$ *is* bounded, from below, the same property holds for $G_{41}\ and\ G_{42}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{41}$ is bounded from below. 277

**Remark 40:** If $T_{40}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}((G_{43})(t), t)) = (b_{41}')^{(8)}$ then $T_{41} \to \infty$. 278

**Definition of** $(m)^{(8)}$ and $\varepsilon_8$ :

Indeed let $t_8$ be so that for $t > t_8$

$$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t), t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right)$, $t = log\frac{2}{\varepsilon_8}$ By taking now $\varepsilon_8$ sufficiently small one sees that $T_{41}$ is unbounded.
The same property holds for $T_{42}$ if $\lim_{t\to\infty}(b''_{42})^{(8)}\left((G_{43})(t), t(t), t\right) = (b'_{42})^{(8)}$



It is now sufficient to take $\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}$ , $\frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$ and to choose $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ large to have

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}\left[(\hat{P}_{44})^{(9)} + \left((\hat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}}\left[\left((\hat{Q}_{44})^{(9)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{44})^{(9)}\right] \leq (\hat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)}, (T_{47})^{(1)}), ((G_{47})^{(2)}, (T_{47})^{(2)})\right) =$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{44})^{(9)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{47}}), (\widetilde{T_{47}}) : \left((\widetilde{G_{47}}), (\widetilde{T_{47}})\right) = \mathcal{A}^{(9)}\left((G_{47}), (T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{44})^{(9)}\left|G_{45}^{(1)} - G_{45}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\,ds_{(44)} +$$
$$\int_0^t\{(a'_{44})^{(9)}\left|G_{44}^{(1)} - G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{-(\widehat{M}_{44})^{(9)}s_{(44)}} +$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right)\left|G_{44}^{(1)} - G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}} +$$
$$G_{44}^{(2)}\left|(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right) - (a''_{44})^{(9)}\left(T_{45}^{(2)}, s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\}ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)} - G^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}t} \leq$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)} + (a'_{44})^{(9)} + (\widehat{A}_{44})^{(9)} + (\widehat{P}_{44})^{(9)}(\hat{k}_{44})^{(9)}\right)d\left(((G_{47})^{(1)}, (T_{47})^{(1)};\ (G_{47})^{(2)}, (T_{47})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis (39,35,36) the result follows

**Remark 41:** The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by

$(\widehat{P}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ *and* $(\widehat{Q}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i = 44{,}45{,}46$ depend only on $T_{45}$ and respectively on $(G_{47})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 42:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$

From 99 to 44 it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t\{(a_i')^{(9)}-(a_i'')^{(9)}(T_{45}(s_{(44)}),s_{(44)})\}ds_{(44)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(9)}t)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{44})^{(9)}\right)_1, \left((\widehat{M}_{44})^{(9)}\right)_2\ and\ \left((\widehat{M}_{44})^{(9)}\right)_3$ :

**Remark 43:** if $G_{44}$ is bounded, the same property have also $G_{45}\ and\ G_{46}$ . indeed if

$G_{44} < (\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt} \leq \left((\widehat{M}_{44})^{(9)}\right)_1 - (a_{45}')^{(9)}G_{45}$ and by integrating

$$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_1/(a_{45}')^{(9)}$$

In the same way , one can obtain

$$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_2/(a_{46}')^{(9)}$$

If $G_{45}\ or\ G_{46}$ is bounded, the same property follows for $G_{44}$ , $G_{46}$ and $G_{44}$ , $G_{45}$ respectively.

**Remark 44:** If $G_{44}$ *is* bounded, from below, the same property holds for $G_{45}\ and\ G_{46}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{45}$ is bounded from below.

**Remark 45:** If $T_{44}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t),t\right) = (b_{45}')^{(9)}$ then $T_{45} \to \infty$.

**Definition of** $(m)^{(9)}$ and $\varepsilon_9$ :

Indeed let $t_9$ be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t),t\right) < \varepsilon_9, T_{44}\,(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right), \quad t = log\frac{2}{\varepsilon_9}$ By taking now $\varepsilon_9$ sufficiently small one sees that $T_{45}$ is unbounded.
The same property holds for $T_{46}$ if $\lim_{t\to\infty}(b_{46}'')^{(9)}\left((G_{47})(t),t\right) = (b_{46}')^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

**Behavior of the solutions of equation** 280

**Theorem** If we denote and define

**Definition of** $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ :

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)} \le -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14}, t) + (a''_{14})^{(1)}(T_{14}, t) \le -(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)} \le -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G, t) - (b''_{14})^{(1)}(G, t) \le -(\tau_1)^{(1)}$

**Definition of** $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$ : 281

By $(\nu_1)^{(1)} > 0$, $(\nu_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$, $(u_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

**Definition of** $(\bar{\nu}_1)^{(1)},, (\bar{\nu}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$ : 282

By $(\bar{\nu}_1)^{(1)} > 0$, $(\bar{\nu}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$, $(\bar{u}_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

**Definition of** $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (\nu_0)^{(1)}$ :- 283

If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by
$(m_2)^{(1)} = (\nu_0)^{(1)}, (m_1)^{(1)} = (\nu_1)^{(1)}$, $if\ (\nu_0)^{(1)} < (\nu_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}, if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)}$,

and $\boxed{(\nu_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)}$, $if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)}$, $if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)}$,

and $\boxed{(u_0)^{(1)} = \frac{T^0_{13}}{T^0_{14}}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$

are defined

Then the solution of global equations satisfies the inequalities 285

$G^0_{13}e^{\left((S_1)^{(1)} - (p_{13})^{(1)}\right)t} \le G_{13}(t) \le G^0_{13}e^{(S_1)^{(1)}t}$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)}-(p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)}t}$$

$$\left(\frac{(a_{15})^{(1)}G_{13}^0}{(m_1)^{(1)}((S_1)^{(1)}-(p_{13})^{(1)}-(S_2)^{(1)})}\left[e^{((S_1)^{(1)}-(p_{13})^{(1)})t} - e^{-(S_2)^{(1)}t}\right] + G_{15}^0 e^{-(S_2)^{(1)}t} \le G_{15}(t) \le \right. \quad (286)$$

$$\left. \frac{(a_{15})^{(1)}G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)}-(a'_{15})^{(1)})}[e^{(S_1)^{(1)}t} - e^{-(a'_{15})^{(1)}t}] + G_{15}^0 e^{-(a'_{15})^{(1)}t}\right)$$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)}t} \le T_{13}(t) \le T_{13}^0 e^{((R_1)^{(1)}+(r_{13})^{(1)})t}} \quad (287)$$

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)}t} \le T_{13}(t) \le \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)}+(r_{13})^{(1)})t} \quad (288)$$

$$\frac{(b_{15})^{(1)}T_{13}^0}{(\mu_1)^{(1)}((R_1)^{(1)}-(b'_{15})^{(1)})}\left[e^{(R_1)^{(1)}t} - e^{-(b'_{15})^{(1)}t}\right] + T_{15}^0 e^{-(b'_{15})^{(1)}t} \le T_{15}(t) \le \quad (289)$$

$$\frac{(a_{15})^{(1)}T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)}+(r_{13})^{(1)}+(R_2)^{(1)})}\left[e^{((R_1)^{(1)}+(r_{13})^{(1)})t} - e^{-(R_2)^{(1)}t}\right] + T_{15}^0 e^{-(R_2)^{(1)}t}$$

**Definition of** $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:- (290)

Where $(S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$

$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$

$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$

$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$

**Behavior of the solutions of equation** (291)

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ : (292)

$(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ four constants satisfying

$$-(\sigma_2)^{(2)} \le -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(T_{17}, t) + (a''_{17})^{(2)}(T_{17}, t) \le -(\sigma_1)^{(2)} \quad (293)$$

$$-(\tau_2)^{(2)} \le -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}((G_{19}), t) - (b''_{17})^{(2)}((G_{19}), t) \le -(\tau_1)^{(2)} \quad (294)$$

**Definition of** $(\nu_1)^{(2)}, (\nu_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$ : (295)

By $(\nu_1)^{(2)} > 0$, $(\nu_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0$, $(u_2)^{(2)} < 0$ the roots (296)

of the equations $(a_{17})^{(2)}(\nu^{(2)})^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ (297)

and $(b_{14})^{(2)}(u^{(2)})^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and (298)

**Definition of** $(\bar{\nu}_1)^{(2)}, , (\bar{\nu}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$ : (299)

By $(\bar{\nu}_1)^{(2)} > 0$, $(\bar{\nu}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0$, $(\bar{u}_2)^{(2)} < 0$ the (300)

roots of the equations $(a_{17})^{(2)}\left(v^{(2)}\right)^2 + (\sigma_2)^{(2)}v^{(2)} - (a_{16})^{(2)} = 0$ 301

and $(b_{17})^{(2)}\left(u^{(2)}\right)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 302

**Definition of** $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ :- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, ***if*** $(u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$, ***if*** $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)}$, ***if*** $(\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)}t}$

$(p_i)^{(2)}$ is defined by equation

$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)}t}$ 311

$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)}t}\right] + G_{18}^0 e^{-(S_2)^{(2)}t} \le G_{18}(t) \le\right.$ 312

$\left.\frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)}t} - e^{-(a'_{18})^{(2)}t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)}t}\right)$

$\boxed{T_{16}^0 e^{(R_1)^{(2)}t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}}$ 313

$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)}t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}$ 314

$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)}t} - e^{-(b'_{18})^{(2)}t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)}t} \le T_{18}(t) \le$ 315

$$\frac{(a_{18})^{(2)}T_{16}^{0}}{(\mu_2)^{(2)}((R_1)^{(2)}+(r_{16})^{(2)}+(R_2)^{(2)})}\left[e^{((R_1)^{(2)}+(r_{16})^{(2)})t}-e^{-(R_2)^{(2)}t}\right]+T_{18}^{0}e^{-(R_2)^{(2)}t}$$

**Definition of** $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$$

$$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)}$$ 318

$$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$$

**Behavior of the solutions** 319

**Theorem 3:** If we denote and define

**Definition of** $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ :

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}\big((G_{23}), t\big) \le -(\tau_1)^{(3)}$$

**Definition of** $(v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$ : 320

By $(v_1)^{(3)} > 0$, $(v_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{v}_1)^{(3)} > 0$, $(\bar{v}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

**Definition of** $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ :- 321

If we define $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (v_0)^{(3)}, (m_1)^{(3)} = (v_1)^{(3)}$, ***if*** $(v_0)^{(3)} < (v_1)^{(3)}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (\bar{v}_1)^{(3)}$, ***if*** $(v_1)^{(3)} < (v_0)^{(3)} < (\bar{v}_1)^{(3)}$,

and $\boxed{(v_0)^{(3)} = \frac{G_{20}^{0}}{G_{21}^{0}}}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_0)^{(3)}$, ***if*** $(\bar{v}_1)^{(3)} < (v_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)},\ \boldsymbol{if}\ (u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}, \boldsymbol{if}\ (u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)},$ and $\boxed{(u_0)^{(3)} = \frac{T_{20}^0}{T_{21}^0}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)},\ \boldsymbol{if}\ (\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G_{20}^0 e^{(S_1)^{(3)} t}$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)} t} \quad 323$$

$$\left(\frac{(a_{22})^{(3)} G_{20}^0}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\left[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)} t}\right] + G_{22}^0 e^{-(S_2)^{(3)} t} \le G_{22}(t) \le \right. \quad 324$$

$$\left. \frac{(a_{22})^{(3)} G_{20}^0}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}\left[e^{(S_1)^{(3)} t} - e^{-(a'_{22})^{(3)} t}\right] + G_{22}^0 e^{-(a'_{22})^{(3)} t}\right)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t}} \quad 325$$

$$\frac{1}{(\mu_1)^{(3)}} T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}} T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)} T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)} - (b'_{22})^{(3)})}\left[e^{(R_1)^{(3)} t} - e^{-(b'_{22})^{(3)} t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)} t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)} T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)} + (r_{20})^{(3)} + (R_2)^{(3)})}\left[e^{((R_1)^{(3)} + (r_{20})^{(3)})t} - e^{-(R_2)^{(3)} t}\right] + T_{22}^0 e^{-(R_2)^{(3)} t}$$

**Definition of** $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

**Behavior of the solutions of equation**

**Theorem:** If we denote and define

**Definition of** $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ :

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$

$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}((G_{27}), t) - (b''_{25})^{(4)}((G_{27}), t) \le -(\tau_1)^{(4)}$

**Definition of** $(v_1)^{(4)}, (v_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, v^{(4)}, u^{(4)}$ : 329

By $(v_1)^{(4)} > 0$ , $(v_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$ , $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_1)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

**Definition of** $(\bar{v}_1)^{(4)},, (\bar{v}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$ : 330

By $(\bar{v}_1)^{(4)} > 0$ , $(\bar{v}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$ , $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

**Definition of** $(m_1)^{(4)}$ , $(m_2)^{(4)}$ , $(\mu_1)^{(4)}, (\mu_2)^{(4)}, (\nu_0)^{(4)}$ :-

If we define $(m_1)^{(4)}$ , $(m_2)^{(4)}$ , $(\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (v_0)^{(4)}, (m_1)^{(4)} = (v_1)^{(4)}$, $\boldsymbol{if}$ $(v_0)^{(4)} < (v_1)^{(4)}$

$(m_2)^{(4)} = (v_1)^{(4)}, (m_1)^{(4)} = (\bar{v}_1)^{(4)}$ , $\boldsymbol{if}$ $(v_4)^{(4)} < (v_0)^{(4)} < (\bar{v}_1)^{(4)}$,
and $\boxed{(v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (v_4)^{(4)}, (m_1)^{(4)} = (v_0)^{(4)}$, $\boldsymbol{if}$ $(\bar{v}_4)^{(4)} < (v_0)^{(4)}$

and analogously

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$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)}$, $\boldsymbol{if}$ $(u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}$ , $\boldsymbol{if}$ $(u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)}$,
and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}, \boldsymbol{if}$ $(\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

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$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)}t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)}t}$$

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$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)}t}\right] + G_{26}^0 e^{-(S_2)^{(4)}t} \le G_{26}(t) \le\right.$$ 334

*(a26)4G240(m2)4(S1)4−(a26′)4e(S1)4t−e−(a26′)4t+ G260e−(a26′)4t*

$$\boxed{T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}}T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}}T_{24}^0 e^{((R_1)^{(4)}+(r_{24})^{(4)})t} \quad 335$$

$$\frac{(b_{26})^{(4)}T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)}-(b'_{26})^{(4)})}\left[e^{(R_1)^{(4)}t} - e^{-(b'_{26})^{(4)}t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)}t} \le T_{26}(t) \le \quad 336$$

$$\frac{(a_{26})^{(4)}T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)}+(r_{24})^{(4)}+(R_2)^{(4)})}\left[e^{((R_1)^{(4)}+(r_{24})^{(4)})t} - e^{-(R_2)^{(4)}t}\right] + T_{26}^0 e^{-(R_2)^{(4)}t}$$

**Definition of** $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

**Behavior of the solutions of equation** 338

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ :

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

**Definition of** $(v_1)^{(5)}, (v_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, v^{(5)}, u^{(5)}$ : 339

By $(v_1)^{(5)} > 0$ , $(v_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$ , $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(v^{(5)}\big)^2 + (\sigma_1)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

**Definition of** $(\bar{v}_1)^{(5)}, , (\bar{v}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$ : 340

By $(\bar{v}_1)^{(5)} > 0$ , $(\bar{v}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$ , $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(v^{(5)}\big)^2 + (\sigma_2)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

**Definition of** $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (v_0)^{(5)}$ :-

If we define $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (v_0)^{(5)}, (m_1)^{(5)} = (v_1)^{(5)}$, ***if*** $(v_0)^{(5)} < (v_1)^{(5)}$

$(m_2)^{(5)} = (v_1)^{(5)}, (m_1)^{(5)} = (\bar{v}_1)^{(5)}$ , ***if*** $(v_1)^{(5)} < (v_0)^{(5)} < (\bar{v}_1)^{(5)}$,
and $\boxed{(v_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)},\ \boldsymbol{if}\ (u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}, \boldsymbol{if}\ (u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)},$

and $\boxed{(u_0)^{(5)} = \frac{T_{28}^0}{T_{29}^0}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}, \boldsymbol{if}\ (\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{28}(t) \le G_{28}^0 e^{(S_1)^{(5)} t}$

where $(p_i)^{(5)}$ is defined by equation

$\frac{1}{(m_5)^{(5)}} G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}} G_{28}^0 e^{(S_1)^{(5)} t}$ 343

$\left(\frac{(a_{30})^{(5)} G_{28}^0}{(m_1)^{(5)}((S_1)^{(5)} - (p_{28})^{(5)} - (S_2)^{(5)})}\left[e^{((S_1)^{(5)} - (p_{28})^{(5)})t} - e^{-(S_2)^{(5)} t}\right] + G_{30}^0 e^{-(S_2)^{(5)} t} \le G_{30}(t) \le\right.$ 344

*(a30)5G280(m2)5(S1)5−(a30′)5e(S1)5t−e−(a30′)5t+ G300e−(a30′)5t*

$\boxed{T_{28}^0 e^{(R_1)^{(5)} t} \le T_{28}(t) \le T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}}$ 345

$\frac{1}{(\mu_1)^{(5)}} T_{28}^0 e^{(R_1)^{(5)} t} \le T_{28}(t) \le \frac{1}{(\mu_2)^{(5)}} T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}$ 346

$\frac{(b_{30})^{(5)} T_{28}^0}{(\mu_1)^{(5)}((R_1)^{(5)} - (b'_{30})^{(5)})}\left[e^{(R_1)^{(5)} t} - e^{-(b'_{30})^{(5)} t}\right] + T_{30}^0 e^{-(b'_{30})^{(5)} t} \le T_{30}(t) \le$ 347

$\frac{(a_{30})^{(5)} T_{28}^0}{(\mu_2)^{(5)}((R_1)^{(5)} + (r_{28})^{(5)} + (R_2)^{(5)})}\left[e^{((R_1)^{(5)} + (r_{28})^{(5)})t} - e^{-(R_2)^{(5)} t}\right] + T_{30}^0 e^{-(R_2)^{(5)} t}$

**Definition of** $(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$:- 348

Where $(S_1)^{(5)} = (a_{28})^{(5)}(m_2)^{(5)} - (a'_{28})^{(5)}$

$(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}$

$(R_1)^{(5)} = (b_{28})^{(5)}(\mu_2)^{(5)} - (b'_{28})^{(5)}$

$(R_2)^{(5)} = (b'_{30})^{(5)} - (r_{30})^{(5)}$

**Behavior of the solutions of equation** 349

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ :

$(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)} \le -(a'_{32})^{(6)} + (a'_{33})^{(6)} - (a''_{32})^{(6)}(T_{33}, t) + (a''_{33})^{(6)}(T_{33}, t) \le -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)} \le -(b'_{32})^{(6)} + (b'_{33})^{(6)} - (b''_{32})^{(6)}\big((G_{35}), t\big) - (b''_{33})^{(6)}\big((G_{35}), t\big) \le -(\tau_1)^{(6)}$$

**Definition of** $(\nu_1)^{(6)}, (\nu_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, \nu^{(6)}, u^{(6)}$ : 350

By $(\nu_1)^{(6)} > 0$, $(\nu_2)^{(6)} < 0$ and respectively $(u_1)^{(6)} > 0$, $(u_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(6)},, (\bar{\nu}_2)^{(6)}, (\bar{u}_1)^{(6)}, (\bar{u}_2)^{(6)}$ : 351

By $(\bar{\nu}_1)^{(6)} > 0$, $(\bar{\nu}_2)^{(6)} < 0$ and respectively $(\bar{u}_1)^{(6)} > 0$, $(\bar{u}_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$

**Definition of** $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}, (\nu_0)^{(6)}$ :-

If we define $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}$ by

$$(m_2)^{(6)} = (\nu_0)^{(6)}, (m_1)^{(6)} = (\nu_1)^{(6)}, \ \boldsymbol{if}\ (\nu_0)^{(6)} < (\nu_1)^{(6)}$$

$$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}, \boldsymbol{if}\ (\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)},$$

and $\boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}}$

$$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$$

and analogously 352

$$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}, \ \boldsymbol{if}\ (u_0)^{(6)} < (u_1)^{(6)}$$

$$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}, \boldsymbol{if}\ (u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)},$$

and $\boxed{(u_0)^{(6)} = \frac{T^0_{32}}{T^0_{33}}}$

$$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}, \boldsymbol{if}\ (\bar{u}_1)^{(6)} < (u_0)^{(6)} \text{ where } (u_1)^{(6)}, (\bar{u}_1)^{(6)}$$

Then the solution of global equations satisfies the inequalities 353

$$G^0_{32}e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{32}(t) \le G^0_{32}e^{(S_1)^{(6)}t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}}G^0_{32}e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{33}(t) \le \frac{1}{(m_2)^{(6)}}G^0_{32}e^{(S_1)^{(6)}t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)}G^0_{32}}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})}\left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)}t}\right] + G^0_{34}e^{-(S_2)^{(6)}t} \le G_{34}(t) \le \right. \quad 355$$

$$\frac{(a_{34})^{(6)}G_{32}^{0}}{(m_2)^{(6)}((S_1)^{(6)}-(a'_{34})^{(6)})}\left[e^{(S_1)^{(6)}t}-e^{-(a'_{34})^{(6)}t}\right]+\ G_{34}^{0}e^{-(a'_{34})^{(6)}t}\Big)$$

$$\boxed{T_{32}^{0}e^{(R_1)^{(6)}t}\le T_{32}(t)\le T_{32}^{0}e^{((R_1)^{(6)}+(r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}}T_{32}^{0}e^{(R_1)^{(6)}t}\le T_{32}(t)\le\frac{1}{(\mu_2)^{(6)}}T_{32}^{0}e^{((R_1)^{(6)}+(r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)}T_{32}^{0}}{(\mu_1)^{(6)}((R_1)^{(6)}-(b'_{34})^{(6)})}\left[e^{(R_1)^{(6)}t}-e^{-(b'_{34})^{(6)}t}\right]+T_{34}^{0}e^{-(b'_{34})^{(6)}t}\le T_{34}(t)\le \quad 358$$

$$\frac{(a_{34})^{(6)}T_{32}^{0}}{(\mu_2)^{(6)}((R_1)^{(6)}+(r_{32})^{(6)}+(R_2)^{(6)})}\left[e^{((R_1)^{(6)}+(r_{32})^{(6)})t}-e^{-(R_2)^{(6)}t}\right]+T_{34}^{0}e^{-(R_2)^{(6)}t}$$

**Definition of** $(S_1)^{(6)},(S_2)^{(6)},(R_1)^{(6)},(R_2)^{(6)}$:- 359

Where $(S_1)^{(6)}=(a_{32})^{(6)}(m_2)^{(6)}-(a'_{32})^{(6)}$

$(S_2)^{(6)}=(a_{34})^{(6)}-(p_{34})^{(6)}$

$(R_1)^{(6)}=(b_{32})^{(6)}(\mu_2)^{(6)}-(b'_{32})^{(6)}$

$(R_2)^{(6)}=(b'_{34})^{(6)}-(r_{34})^{(6)}$

**Behavior of the solutions of equation**

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(7)},(\sigma_2)^{(7)},(\tau_1)^{(7)},(\tau_2)^{(7)}$:

$(\sigma_1)^{(7)},(\sigma_2)^{(7)},(\tau_1)^{(7)},(\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)}\le-(a'_{36})^{(7)}+(a'_{37})^{(7)}-(a''_{36})^{(7)}(T_{37},t)+(a''_{37})^{(7)}(T_{37},t)\le-(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)}\le-(b'_{36})^{(7)}+(b'_{37})^{(7)}-(b''_{36})^{(7)}\big((G_{39}),t\big)-(b''_{37})^{(7)}\big((G_{39}),t\big)\le-(\tau_1)^{(7)}$$

**Definition of** $(v_1)^{(7)},(v_2)^{(7)},(u_1)^{(7)},(u_2)^{(7)},v^{(7)},u^{(7)}$: 361

By $(v_1)^{(7)}>0,(v_2)^{(7)}<0$ and respectively $(u_1)^{(7)}>0,(u_2)^{(7)}<0$ the roots of the equations $(a_{37})^{(7)}\big(v^{(7)}\big)^2+(\sigma_1)^{(7)}v^{(7)}-(a_{36})^{(7)}=0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2+(\tau_1)^{(7)}u^{(7)}-(b_{36})^{(7)}=0$ and

**Definition of** $(\bar{v}_1)^{(7)},,(\bar{v}_2)^{(7)},(\bar{u}_1)^{(7)},(\bar{u}_2)^{(7)}$: 362

By $(\bar{v}_1)^{(7)}>0,(\bar{v}_2)^{(7)}<0$ and respectively $(\bar{u}_1)^{(7)}>0,(\bar{u}_2)^{(7)}<0$ the roots of the equations $(a_{37})^{(7)}\big(v^{(7)}\big)^2+(\sigma_2)^{(7)}v^{(7)}-(a_{36})^{(7)}=0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2+(\tau_2)^{(7)}u^{(7)}-(b_{36})^{(7)}=0$

**Definition of** $(m_1)^{(7)},(m_2)^{(7)},(\mu_1)^{(7)},(\mu_2)^{(7)},(v_0)^{(7)}$:-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$$(m_2)^{(7)} = (\nu_0)^{(7)}, (m_1)^{(7)} = (\nu_1)^{(7)}, \ \boldsymbol{if}\ (\nu_0)^{(7)} < (\nu_1)^{(7)}$$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\bar{\nu}_1)^{(7)}, \boldsymbol{if}\ (\nu_1)^{(7)} < (\nu_0)^{(7)} < (\bar{\nu}_1)^{(7)},$$

and $\boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\nu_0)^{(7)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(7)} < (\nu_0)^{(7)}$$

and analogously 363

$$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}, \ \boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$$

and $\boxed{(u_0)^{(7)} = \frac{T_{36}^0}{T_{37}^0}}$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)} \text{ where } (u_1)^{(7)}, (\bar{u}_1)^{(7)}$$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^0 e^{(S_1)^{(7)} t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)} t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)}((S_1)^{(7)} - (p_{36})^{(7)} - (S_2)^{(7)})}\left[e^{((S_1)^{(7)} - (p_{36})^{(7)})t} - e^{-(S_2)^{(7)} t}\right] + G_{38}^0 e^{-(S_2)^{(7)} t} \le G_{38}(t) \le \right. \quad 366$$

$$\left. \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)}((S_1)^{(7)} - (a'_{38})^{(7)})}\left[e^{(S_1)^{(7)} t} - e^{-(a'_{38})^{(7)} t}\right] + G_{38}^0 e^{-(a'_{38})^{(7)} t}\right)$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t} \quad 368$$

$$\frac{(b_{38})^{(7)} T_{36}^0}{(\mu_1)^{(7)}((R_1)^{(7)} - (b'_{38})^{(7)})}\left[e^{(R_1)^{(7)} t} - e^{-(b'_{38})^{(7)} t}\right] + T_{38}^0 e^{-(b'_{38})^{(7)} t} \le T_{38}(t) \le \quad 369$$

$$\frac{(a_{38})^{(7)} T_{36}^0}{(\mu_2)^{(7)}((R_1)^{(7)} + (r_{36})^{(7)} + (R_2)^{(7)})}\left[e^{((R_1)^{(7)} + (r_{36})^{(7)})t} - e^{-(R_2)^{(7)} t}\right] + T_{38}^0 e^{-(R_2)^{(7)} t}$$

**Definition of** $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$

$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$

$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$

**Behavior of the solutions of equation** 371

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ :

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$-(\sigma_2)^{(8)} \le -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \le -(\sigma_1)^{(8)}$

$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\big((G_{43}), t\big) - (b''_{41})^{(8)}\big((G_{43}), t\big) \le -(\tau_1)^{(8)}$

**Definition of** $(\nu_1)^{(8)}, (\nu_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, \nu^{(8)}, u^{(8)}$ : 372

By $(\nu_1)^{(8)} > 0, (\nu_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0, (u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(8)}, , (\bar{\nu}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$ :

By $(\bar{\nu}_1)^{(8)} > 0, (\bar{\nu}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0, (\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

**Definition of** $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}, (\nu_0)^{(8)}$ :-

If we define $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$(m_2)^{(8)} = (\nu_0)^{(8)}, (m_1)^{(8)} = (\nu_1)^{(8)}$, ***if*** $(\nu_0)^{(8)} < (\nu_1)^{(8)}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\bar{\nu}_1)^{(8)}$, ***if*** $(\nu_1)^{(8)} < (\nu_0)^{(8)} < (\bar{\nu}_1)^{(8)}$,
and $\boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\nu_0)^{(8)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(8)} < (\nu_0)^{(8)}$

and analogously 374

$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)},\ \boldsymbol{if}\ (u_0)^{(8)} < (u_1)^{(8)}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}, \boldsymbol{if}\ (u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)},$

and $\boxed{(u_0)^{(8)} = \frac{T_{40}^0}{T_{41}^0}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}, \boldsymbol{if}\ (\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^0 e^{(S_1)^{(8)} t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}} G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}} G_{40}^0 e^{(S_1)^{(8)} t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)} G_{40}^0}{(m_1)^{(8)}((S_1)^{(8)} - (p_{40})^{(8)} - (S_2)^{(8)})}\left[e^{((S_1)^{(8)} - (p_{40})^{(8)})t} - e^{-(S_2)^{(8)} t}\right] + G_{42}^0 e^{-(S_2)^{(8)} t} \le G_{42}(t) \le \right. \quad 377$$

$$\left. \frac{(a_{42})^{(8)} G_{40}^0}{(m_2)^{(8)}((S_1)^{(8)} - (a'_{42})^{(8)})}\left[e^{(S_1)^{(8)} t} - e^{-(a'_{42})^{(8)} t}\right] + G_{42}^0 e^{-(a'_{42})^{(8)} t}\right)$$

$$\boxed{T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}} T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le \frac{1}{(\mu_2)^{(8)}} T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)} T_{40}^0}{(\mu_1)^{(8)}((R_1)^{(8)} - (b'_{42})^{(8)})}\left[e^{(R_1)^{(8)} t} - e^{-(b'_{42})^{(8)} t}\right] + T_{42}^0 e^{-(b'_{42})^{(8)} t} \le T_{42}(t) \le \quad 380$$

$$\frac{(a_{42})^{(8)} T_{40}^0}{(\mu_2)^{(8)}((R_1)^{(8)} + (r_{40})^{(8)} + (R_2)^{(8)})}\left[e^{((R_1)^{(8)} + (r_{40})^{(8)})t} - e^{-(R_2)^{(8)} t}\right] + T_{42}^0 e^{-(R_2)^{(8)} t}$$

**Definition of** $(S_1)^{(8)}, (S_2)^{(8)}, (R_1)^{(8)}, (R_2)^{(8)}$:- 381

Where $(S_1)^{(8)} = (a_{40})^{(8)}(m_2)^{(8)} - (a'_{40})^{(8)}$

$(S_2)^{(8)} = (a_{42})^{(8)} - (p_{42})^{(8)}$

$(R_1)^{(8)} = (b_{40})^{(8)}(\mu_2)^{(8)} - (b'_{40})^{(8)}$

$(R_2)^{(8)} = (b'_{42})^{(8)} - (r_{42})^{(8)}$

**Behavior of the solutions of equation 37 to 92** 382

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ :

$(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ four constants satisfying

$$-(\sigma_2)^{(9)} \le -(a'_{44})^{(9)} + (a'_{45})^{(9)} - (a''_{44})^{(9)}(T_{45}, t) + (a''_{45})^{(9)}(T_{45}, t) \le -(\sigma_1)^{(9)}$$

$$-(\tau_2)^{(9)} \le -(b'_{44})^{(9)} + (b'_{45})^{(9)} - (b''_{44})^{(9)}\big((G_{47}), t\big) - (b''_{45})^{(9)}\big((G_{47}), t\big) \le -(\tau_1)^{(9)}$$

**Definition of** $(\nu_1)^{(9)}, (\nu_2)^{(9)}, (u_1)^{(9)}, (u_2)^{(9)}, \nu^{(9)}, u^{(9)}$ :

By $(\nu_1)^{(9)} > 0$ , $(\nu_2)^{(9)} < 0$ and respectively $(u_1)^{(9)} > 0$ , $(u_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_1)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(9)}, , (\bar{\nu}_2)^{(9)}, (\bar{u}_1)^{(9)}, (\bar{u}_2)^{(9)}$ :

By $(\bar{\nu}_1)^{(9)} > 0$ , $(\bar{\nu}_2)^{(9)} < 0$ and respectively $(\bar{u}_1)^{(9)} > 0$ , $(\bar{u}_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_2)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$

**Definition of** $(m_1)^{(9)}, (m_2)^{(9)}, (\mu_1)^{(9)}, (\mu_2)^{(9)}, (\nu_0)^{(9)}$ :-

If we define $(m_1)^{(9)}, (m_2)^{(9)}, (\mu_1)^{(9)}, (\mu_2)^{(9)}$ by

$$(m_2)^{(9)} = (\nu_0)^{(9)}, (m_1)^{(9)} = (\nu_1)^{(9)}, \ \boldsymbol{if}\ (\nu_0)^{(9)} < (\nu_1)^{(9)}$$

$$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\bar{\nu}_1)^{(9)}, \boldsymbol{if}\ (\nu_1)^{(9)} < (\nu_0)^{(9)} < (\bar{\nu}_1)^{(9)},$$

and $\boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$

$$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\nu_0)^{(9)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(9)} < (\nu_0)^{(9)}$$

and analogously

$$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)}, \ \boldsymbol{if}\ (u_0)^{(9)} < (u_1)^{(9)}$$

$$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}, \boldsymbol{if}\ (u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)},$$

and $\boxed{(u_0)^{(9)} = \frac{T^0_{44}}{T^0_{45}}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}, \boldsymbol{if}\ (\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G^0_{44} e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{44}(t) \le G^0_{44} e^{(S_1)^{(9)} t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}}G_{44}^0 e^{\left((S_1)^{(9)}-(p_{44})^{(9)}\right)t} \le G_{45}(t) \le \frac{1}{(m_2)^{(9)}}G_{44}^0 e^{(S_1)^{(9)}t}$$

$$\left(\frac{(a_{46})^{(9)}G_{44}^0}{(m_1)^{(9)}\left((S_1)^{(9)}-(p_{44})^{(9)}-(S_2)^{(9)}\right)}\left[e^{\left((S_1)^{(9)}-(p_{44})^{(9)}\right)t}-e^{-(S_2)^{(9)}t}\right]+G_{46}^0 e^{-(S_2)^{(9)}t} \le G_{46}(t) \le \right.$$

$$\left.\frac{(a_{46})^{(9)}G_{44}^0}{(m_2)^{(9)}\left((S_1)^{(9)}-(a'_{46})^{(9)}\right)}\left[e^{(S_1)^{(9)}t}-e^{-(a'_{46})^{(9)}t}\right]+\ G_{46}^0 e^{-(a'_{46})^{(9)}t}\right)$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le T_{44}^0 e^{\left((R_1)^{(9)}+(r_{44})^{(9)}\right)t}}$$

$$\frac{1}{(\mu_1)^{(9)}}T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le \frac{1}{(\mu_2)^{(9)}}T_{44}^0 e^{\left((R_1)^{(9)}+(r_{44})^{(9)}\right)t}$$

$$\frac{(b_{46})^{(9)}T_{44}^0}{(\mu_1)^{(9)}\left((R_1)^{(9)}-(b'_{46})^{(9)}\right)}\left[e^{(R_1)^{(9)}t}-e^{-(b'_{46})^{(9)}t}\right]+T_{46}^0 e^{-(b'_{46})^{(9)}t} \le T_{46}(t) \le$$

$$\frac{(a_{46})^{(9)}T_{44}^0}{(\mu_2)^{(9)}\left((R_1)^{(9)}+(r_{44})^{(9)}+(R_2)^{(9)}\right)}\left[e^{\left((R_1)^{(9)}+(r_{44})^{(9)}\right)t}-e^{-(R_2)^{(9)}t}\right]+T_{46}^0 e^{-(R_2)^{(9)}t}$$

**Definition of** $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$

**Proof :** From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14},t)\right) - (a''_{14})^{(1)}(T_{14},t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

**Definition of** $v^{(1)}$ :- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)}\right) \le \frac{dv^{(1)}}{dt} \le -\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)}\right)$$

From which one obtains

**Definition of** $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$ :-

For $0 < \boxed{(v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}} < (v_1)^{(1)} < (\bar{v}_1)^{(1)}$

$$v^{(1)}(t) \ge \frac{(v_1)^{(1)}+(C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(v_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(v_2)^{(1)}}}$$

it follows $(\nu_0)^{(1)} \leq \nu^{(1)}(t) \leq (\nu_1)^{(1)}$

In the same manner , we get 384

$$\nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{\nu}_1)^{(1)}-(\nu_0)^{(1)}}{(\nu_0)^{(1)}-(\bar{\nu}_2)^{(1)}}}$$

From which we deduce $(\nu_0)^{(1)} \leq \nu^{(1)}(t) \leq (\bar{\nu}_1)^{(1)}$

If $0 < (\nu_1)^{(1)} < (\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0} < (\bar{\nu}_1)^{(1)}$ we find like in the previous case, 385

$$(\nu_1)^{(1)} \leq \frac{(\nu_1)^{(1)}+(C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}} \leq \nu^{(1)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\bar{\nu}_1)^{(1)}$$

If $0 < (\nu_1)^{(1)} \leq (\bar{\nu}_1)^{(1)} \leq \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$ , we obtain 386

$$(\nu_1)^{(1)} \leq \nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\nu_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(1)}(t)$ :-

$$(m_2)^{(1)} \leq \nu^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{\nu^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(1)}(t)$ :-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{13})^{(1)} = (a''_{14})^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)}G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)}T_{14}(t)$ This is an important

consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

**Proof** : From global equations we obtain 387

$$\frac{d\nu^{(2)}}{dt} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right) - (a''_{17})^{(2)}(T_{17}, t)\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

**Definition of** $\nu^{(2)}$ :- $\boxed{\nu^{(2)} = \frac{G_{16}}{G_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \le \frac{d\nu^{(2)}}{dt} \le -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

**Definition of** $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$ :-

For $0 < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \ge \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}}{1 + (C)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(C)^{(2)} = \frac{(\nu_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \le \nu^{(2)}(t) \le (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \le \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \le \nu^{(2)}(t) \le (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \le \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}}{1 + (C)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}} \le \nu^{(2)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \le (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \le (\bar{\nu}_1)^{(2)} \le (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$ , we obtain 394

$$(\nu_1)^{(2)} \le \nu^{(2)}(t) \le \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \le (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(2)}(t)$ :- 395

$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}$, $\boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$

In a completely analogous way, we obtain 396

**Definition of** $u^{(2)}(t)$ :-

$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}$, $\boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :** 397

If $(a''_{16})^{(2)} = (a''_{17})^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)} G_{17}(t)$

Analogously if $(b''_{16})^{(2)} = (b''_{17})^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)} T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

**Proof** : From global equations we obtain 398

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right) - (a''_{21})^{(3)}(T_{21}, t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

**Definition of** $\nu^{(3)}$ :- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right) \leq \frac{d\nu^{(3)}}{dt} \leq -\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right)$$

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From which one obtains

For $0 < (\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\nu_1)^{(3)} < (\bar{\nu}_1)^{(3)}$

$$\nu^{(3)}(t) \geq \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(\nu_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\nu_2)^{(3)}}}$$

it follows $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\nu_1)^{(3)}$

In the same manner , we get 401

$$\nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{\nu}_1)^{(3)}-(\nu_0)^{(3)}}{(\nu_0)^{(3)}-(\bar{\nu}_2)^{(3)}}}$$

**Definition of** $(\bar{\nu}_1)^{(3)}$ :-

From which we deduce $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\bar{\nu}_1)^{(3)}$

If $0 < (\nu_1)^{(3)} < (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (\bar{\nu}_1)^{(3)}$ we find like in the previous case, 402

$$(\nu_1)^{(3)} \leq \frac{(\nu_1)^{(3)}+(C)^{(3)}(\nu_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}} \leq \nu^{(3)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\bar{\nu}_1)^{(3)}$$

If $0 < (\nu_1)^{(3)} \leq (\bar{\nu}_1)^{(3)} \leq (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}$ , we obtain 403

$$(\nu_1)^{(3)} \leq \nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\nu_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(3)}(t)$ :-

$$(m_2)^{(3)} \leq \nu^{(3)}(t) \leq (m_1)^{(3)}, \quad \boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(3)}(t)$ :-

$$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}, \quad \boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{20}'')^{(3)} = (a_{21}'')^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)}G_{21}(t)$

Analogously if $(b_{20}'')^{(3)} = (b_{21}'')^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)}T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

**Proof** : From global equations we obtain 404

$$\frac{dv^{(4)}}{dt} = (a_{24})^{(4)} - \left((a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right) - (a''_{25})^{(4)}(T_{25},t)v^{(4)} - (a_{25})^{(4)}v^{(4)}$$

**Definition of** $v^{(4)}$ :- $\boxed{v^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)}\right) \le \frac{dv^{(4)}}{dt} \le -\left((a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_4)^{(4)}v^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$ :-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \ge \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \le \nu^{(4)}(t) \le (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \le \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \le \nu^{(4)}(t) \le (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \le \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}}{1 + (C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}} \le \nu^{(4)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \le (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \le (\bar{\nu}_1)^{(4)} \le \boxed{(\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}}}$ , we obtain 407

$$(\nu_1)^{(4)} \le \nu^{(4)}(t) \le \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \le (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(4)}(t)$ :-

$$(m_2)^{(4)} \le \nu^{(4)}(t) \le (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(4)}(t)$ :-

$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}$, $\boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{24})^{(4)} = (a''_{25})^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)} G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b''_{24})^{(4)} = (b''_{25})^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)} T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

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**Proof** : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a'_{28})^{(5)} - (a'_{29})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right) - (a''_{29})^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

**Definition of** $\nu^{(5)}$ :- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$ :-

For $0 < \boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \le \frac{(\nu_1)^{(5)}+(C)^{(5)}(\nu_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)}-(\nu_2)^{(5)}\right)t\right]}}{1+(C)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)}-(\nu_2)^{(5)}\right)t\right]}} \le \nu^{(5)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \le (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \le (\bar{\nu}_1)^{(5)} \le \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain 411

$$(\nu_1)^{(5)} \le \nu^{(5)}(t) \le \frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \le (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(5)}(t)$ :-

$(m_2)^{(5)} \le \nu^{(5)}(t) \le (m_1)^{(5)}$, $\boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$

In a completely analogous way, we obtain

**Definition of** $u^{(5)}(t)$ :-

$(\mu_2)^{(5)} \le u^{(5)}(t) \le (\mu_1)^{(5)}$, $\boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a_{28}'')^{(5)} = (a_{29}'')^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)}G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b_{28}'')^{(5)} = (b_{29}'')^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)}T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

**Proof** : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a_{32}')^{(6)} - (a_{33}')^{(6)} + (a_{32}'')^{(6)}(T_{33},t)\right) - (a_{33}'')^{(6)}(T_{33},t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

**Definition of** $\nu^{(6)}$ :- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \le \frac{d\nu^{(6)}}{dt} \le -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$ :-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \geq \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\nu_1)^{(6)}$

In the same manner , we get 413

$$\nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{\nu}_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\bar{\nu}_1)^{(6)}$

If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 414

$$(\nu_1)^{(6)} \leq \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}} \leq \nu^{(6)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\bar{\nu}_1)^{(6)}$$

If $0 < (\nu_1)^{(6)} \leq (\bar{\nu}_1)^{(6)} \leq \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$ , we obtain 415

$$(\nu_1)^{(6)} \leq \nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(6)}(t)$ :-

$$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}, \quad \boxed{\nu^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(6)}(t)$ :-

$$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}, \quad \boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)} G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)} T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

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**Proof** : From global equations we obtain

$$\frac{dv^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right) - (a''_{37})^{(7)}(T_{37},t)v^{(7)} - (a_{37})^{(7)}v^{(7)}$$

**Definition of** $v^{(7)}$ :- $\boxed{v^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_2)^{(7)}v^{(7)} - (a_{36})^{(7)}\right) \le \frac{dv^{(7)}}{dt} \le -\left((a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_1)^{(7)}v^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

**Definition of** $(\bar{v}_1)^{(7)}, (v_0)^{(7)}$ :-

For $0 < \boxed{(v_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}} < (v_1)^{(7)} < (\bar{v}_1)^{(7)}$

$$v^{(7)}(t) \ge \frac{(v_1)^{(7)} + (C)^{(7)}(v_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_0)^{(7)}\right)t\right]}}{1 + (C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(v_1)^{(7)} - (v_0)^{(7)}}{(v_0)^{(7)} - (v_2)^{(7)}}}$$

it follows $(v_0)^{(7)} \le v^{(7)}(t) \le (v_1)^{(7)}$

In the same manner , we get 417

$$v^{(7)}(t) \le \frac{(\bar{v}_1)^{(7)} + (\bar{C})^{(7)}(\bar{v}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{v}_1)^{(7)} - (v_0)^{(7)}}{(v_0)^{(7)} - (\bar{v}_2)^{(7)}}}$$

From which we deduce $(v_0)^{(7)} \le v^{(7)}(t) \le (\bar{v}_1)^{(7)}$

If $0 < (v_1)^{(7)} < (v_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}} < (\bar{v}_1)^{(7)}$ we find like in the previous case, 418

$$(v_1)^{(7)} \le \frac{(v_1)^{(7)} + (C)^{(7)}(v_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_2)^{(7)}\right)t\right]}}{1 + (C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_2)^{(7)}\right)t\right]}} \le v^{(7)}(t) \le$$

$$\frac{(\bar{v}_1)^{(7)} + (\bar{C})^{(7)}(\bar{v}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}} \le (\bar{v}_1)^{(7)}$$

If $0 < (v_1)^{(7)} \le (\bar{v}_1)^{(7)} \le \boxed{(v_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$ , we obtain 419

$$(v_1)^{(7)} \le v^{(7)}(t) \le \frac{(\bar{v}_1)^{(7)} + (\bar{C})^{(7)}(\bar{v}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}} \le (v_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $v^{(7)}(t)$ :-

$$(m_2)^{(7)} \leq v^{(7)}(t) \leq (m_1)^{(7)}, \quad \boxed{v^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(7)}(t)$ :- 420

$$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}, \quad \boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $v^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)} G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)} T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

**Proof** : From global equations we obtain 421

$$\frac{dv^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right) - (a''_{41})^{(8)}(T_{41}, t) v^{(8)} - (a_{41})^{(8)} v^{(8)}$$

**Definition of** $v^{(8)}$ :- $\boxed{v^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_2)^{(8)} v^{(8)} - (a_{40})^{(8)}\right) \leq \frac{dv^{(8)}}{dt} \leq -\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_1)^{(8)} v^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$ :-

$$\text{For } 0 < \boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$$

$$v^{(8)}(t) \geq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)}-(\nu_0)^{(8)}}{(\nu_0)^{(8)}-(\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \leq \frac{(\nu_1)^{(8)}+(C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}}{1+(C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}} \leq \nu^{(8)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \leq (\bar{\nu}_1)^{(8)} \leq \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$ , we obtain 424

$$(\nu_1)^{(8)} \leq \nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(8)}(t)$ :-

$$(m_2)^{(8)} \leq \nu^{(8)}(t) \leq (m_1)^{(8)}, \quad \boxed{\nu^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(8)}(t)$ :-

$$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}, \quad \boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $\nu^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)}G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then $(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)}T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

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**Proof** : From 99,20,44,22,23,44 we obtain A

$$\frac{dv^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right) - (a''_{45})^{(9)}(T_{45},t)v^{(9)} - (a_{45})^{(9)}v^{(9)}$$

**Definition of** $v^{(9)}$ :- $\boxed{v^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)}\right) \le \frac{dv^{(9)}}{dt} \le -\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_1)^{(9)}v^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$ :-

For $0 < \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$

$$v^{(9)}(t) \ge \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \le v^{(9)}(t) \le (\nu_9)^{(9)}$

In the same manner , we get

$$v^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \le v^{(9)}(t) \le (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \le \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}}{1 + (C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}} \le v^{(9)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \le (\bar{\nu}_1)^{(9)} \le \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$ , we obtain

$$(\nu_1)^{(9)} \le v^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $v^{(9)}(t)$ :-

$$(m_2)^{(9)} \le v^{(9)}(t) \le (m_1)^{(9)}, \quad \boxed{v^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(9)}(t)$ :-

$(\mu_2)^{(9)} \leq u^{(9)}(t) \leq (\mu_1)^{(9)}$, $\boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)} G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then
$(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)} T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following 425

**Theorem :** If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on $t$ , and the conditions with the notations

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0$ ,

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$

$with\ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a''_i)^{(2)} and\ (b''_i)^{(2)}$ are independent on t , and the conditions with the notations 426

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$ 427

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 428

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$ , 429

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 430

$with\ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a''_i)^{(3)} and\ (b''_i)^{(3)}$ are independent on $t$ , and the conditions with the notations 431

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$ ,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

**Theorem :** If $(a_i'')^{(4)} and\ (b_i'')^{(4)}$ are independent on $t$ , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$ ,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(5)} and\ (b_i'')^{(5)}$ are independent on $t$ , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$ ,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

**Theorem** If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on $t$ , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$ ,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(7)} and\ (b_i'')^{(7)}$ are independent on $t$ , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$ ,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(8)} and\ (b_i'')^{(8)}$ are independent on $t$ , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$ ,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(9)} and\ (b_i'')^{(9)}$ are independent on $t$ , and the conditions (with the notations 436 A
45,46,27,28)

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$ ,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0$ 437

$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0$ 438

$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0$ 439

$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0$ 440

$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ]T_{14} = 0$ 441

$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0$ 442

has a unique positive solution , which is an equilibrium solution for the system

$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0$ 443

$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0$ 444

$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0$ 445

$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\ ]T_{16} = 0$ 446

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\,]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\,]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\,]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\,]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\,]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\big((G_{27})\big)\,]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\big((G_{27})\big)\,]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\big((G_{27})\big)\,]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})\,]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})\,]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})\,]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})\,]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})\,]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})\,]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0 \quad 479$$

$$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0 \quad 480$$

$$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0 \quad 481$$

$$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0 \quad 482$$

$$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0 \quad 483$$

$$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0 \quad 484$$

$$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0 \quad 484\text{A}$$

$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$

$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$

$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$

$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$

$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$

**Proof:** 485

(a) Indeed the first two equations have a nontrivial solution $G_{13}, G_{14}$ if

$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$

**Proof:** 486

(j) Indeed the first two equations have a nontrivial solution $G_{16}, G_{17}$ if

$\mathrm{F}(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$

**Proof:** 487

(a) Indeed the first two equations have a nontrivial solution $G_{20}, G_{21}$ if

$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$

**Proof:** 488

(a) Indeed the first two equations have a nontrivial solution $G_{24}, G_{25}$ if

$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$

**Proof:** 489

(a) Indeed the first two equations have a nontrivial solution $G_{28}, G_{29}$ if

$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$

**Proof:** 490

(a) Indeed the first two equations have a nontrivial solution $G_{32}, G_{33}$ if

$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) +$

$(a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$

**Proof:** 491

(a) Indeed the first two equations have a nontrivial solution $G_{36}, G_{37}$ if

$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$

**Proof:** 492

(a) Indeed the first two equations have a nontrivial solution $G_{40}, G_{41}$ if

$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$

**Proof:** 492 A

(a) Indeed the first two equations have a nontrivial solution $G_{44}, G_{45}$ if

$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$

**Definition and uniqueness of** $T^*_{14}$ :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique $T^*_{14}$ for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

**Definition and uniqueness of** $T^*_{17}$ :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique $T^*_{17}$ for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]}$$ 495

**Definition and uniqueness of** $T^*_{21}$ :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique $T^*_{21}$ for which $f(T^*_{21}) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

**Definition and uniqueness of** $T^*_{25}$ :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that

there exists a unique $T_{25}^{*}$ for which $f(T_{25}^{*})=0$. With this value , we obtain from the three first equations

$$G_{24}=\frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}=\frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T_{25}^{*})]}$$

**Definition and uniqueness of** $T_{29}^{*}$ :- 498

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(5)}(T_{29})$ being increasing, it follows that there exists a unique $T_{29}^{*}$ for which $f(T_{29}^{*})=0$. With this value , we obtain from the three first equations

$$G_{28}=\frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}=\frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T_{29}^{*})]}$$

**Definition and uniqueness of** $T_{33}^{*}$ :- 499

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(6)}(T_{33})$ being increasing, it follows that there exists a unique $T_{33}^{*}$ for which $f(T_{33}^{*})=0$. With this value , we obtain from the three first equations

$$G_{32}=\frac{(a_{32})^{(6)}G_{33}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^{*})]} \quad , \quad G_{34}=\frac{(a_{34})^{(6)}G_{33}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^{*})]}$$

**Definition and uniqueness of** $T_{37}^{*}$ :- 500

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(7)}(T_{37})$ being increasing, it follows that there exists a unique $T_{37}^{*}$ for which $f(T_{37}^{*})=0$. With this value , we obtain from the three first equations

$$G_{36}=\frac{(a_{36})^{(7)}G_{37}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^{*})]} \quad , \quad G_{38}=\frac{(a_{38})^{(7)}G_{37}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^{*})]}$$

**Definition and uniqueness of** $T_{41}^{*}$ :- 501

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique $T_{41}^{*}$ for which $f(T_{41}^{*})=0$. With this value , we obtain from the three first equations

$$G_{40}=\frac{(a_{40})^{(8)}G_{41}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T_{41}^{*})]} \quad , \quad G_{42}=\frac{(a_{42})^{(8)}G_{41}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T_{41}^{*})]}$$

**Definition and uniqueness of** $T_{45}^{*}$ :- 501 A

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(9)}(T_{45})$ being increasing, it follows that there exists a unique $T_{45}^{*}$ for which $f(T_{45}^{*})=0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)} G_{45}}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T^*_{45})]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)} G_{45}}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T^*_{45})]}$$

By the same argument, the equations admit solutions $G_{13}, G_{14}$ if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right] + (b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{14}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{14}$ such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions $G_{16}, G_{17}$ if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right] + (b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ 504
is a decreasing function in $G_{17}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{14}$ such that $\varphi((G_{19})^*) = 0$

By the same argument, the equations admit solutions $G_{20}, G_{21}$ if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right] + (b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{21}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{21}$ such that $\varphi((G_{23})^*) = 0$

By the same argument, the equations admit solutions $G_{24}, G_{25}$ if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right] + (b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{25}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{25}$ such that $\varphi((G_{27})^*) = 0$

By the same argument, the equations admit solutions $G_{28}, G_{29}$ if 507

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right] + (b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{29}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that

there exists a unique $G_{29}^{*}$ such that $\varphi((G_{31})^{*}) = 0$

By the same argument, the equations admit solutions $G_{32}, G_{33}$ if 508
$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$

$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right] + (b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{33}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{33}^{*}$ such that $\varphi(G_{35}{}^{*}) = 0$

By the same argument, the equations admit solutions $G_{36}, G_{37}$ if 509
$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$
$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right] + (b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{37}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{37}^{*}$ such that $\varphi(G_{39}{}^{*}) = 0$

By the same argument, the equations admit solutions $G_{40}, G_{41}$ if 510
$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$
$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right] + (b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{41}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{41}^{*}$ such that $\varphi(G_{43}{}^{*}) = 0$

By the same argument, the equations 92,93 admit solutions $G_{44}, G_{45}$ if
$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$

$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right] + (b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{45}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{45}^{*}$ such that $\varphi((G_{47})^{*}) = 0$

Finally we obtain the unique solution 511

$G_{14}^{*}$ given by $\varphi(G^{*}) = 0$ , $T_{14}^{*}$ given by $f(T_{14}^{*}) = 0$ and

$$G_{13}^{*} = \frac{(a_{13})^{(1)}G_{14}^{*}}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^{*})]} \quad , \quad G_{15}^{*} = \frac{(a_{15})^{(1)}G_{14}^{*}}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^{*})]}$$

$$T_{13}^{*} = \frac{(b_{13})^{(1)}T_{14}^{*}}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^{*})]} \quad , \quad T_{15}^{*} = \frac{(b_{15})^{(1)}T_{14}^{*}}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^{*})]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

$G_{17}^{*}$ given by $\varphi((G_{19})^{*}) = 0$ , $T_{17}^{*}$ given by $f(T_{17}^{*}) = 0$ and 512

$$G_{16}^{*} = \frac{(a_{16})^{(2)} G_{17}^{*}}{[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}^{*})]} \quad , \quad G_{18}^{*} = \frac{(a_{18})^{(2)} G_{17}^{*}}{[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}^{*})]} \tag{513}$$

$$T_{16}^{*} = \frac{(b_{16})^{(2)} T_{17}^{*}}{[(b'_{16})^{(2)} - (b''_{16})^{(2)}((G_{19})^{*})]} \quad , \quad T_{18}^{*} = \frac{(b_{18})^{(2)} T_{17}^{*}}{[(b'_{18})^{(2)} - (b''_{18})^{(2)}((G_{19})^{*})]} \tag{514}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

$G_{21}^{*}$ given by $\varphi((G_{23})^{*}) = 0$ , $T_{21}^{*}$ given by $f(T_{21}^{*}) = 0$ and

$$G_{20}^{*} = \frac{(a_{20})^{(3)} G_{21}^{*}}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^{*})]} \quad , \quad G_{22}^{*} = \frac{(a_{22})^{(3)} G_{21}^{*}}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^{*})]}$$

$$T_{20}^{*} = \frac{(b_{20})^{(3)} T_{21}^{*}}{[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}{}^{*})]} \quad , \quad T_{22}^{*} = \frac{(b_{22})^{(3)} T_{21}^{*}}{[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}{}^{*})]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

$G_{25}^{*}$ given by $\varphi(G_{27}) = 0$ , $T_{25}^{*}$ given by $f(T_{25}^{*}) = 0$ and

$$G_{24}^{*} = \frac{(a_{24})^{(4)} G_{25}^{*}}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}^{*} = \frac{(a_{26})^{(4)} G_{25}^{*}}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^{*})]}$$

$$T_{24}^{*} = \frac{(b_{24})^{(4)} T_{25}^{*}}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^{*})]} \quad , \quad T_{26}^{*} = \frac{(b_{26})^{(4)} T_{25}^{*}}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^{*})]} \tag{517}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

$G_{29}^{*}$ given by $\varphi((G_{31})^{*}) = 0$ , $T_{29}^{*}$ given by $f(T_{29}^{*}) = 0$ and

$$G_{28}^{*} = \frac{(a_{28})^{(5)} G_{29}^{*}}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}^{*} = \frac{(a_{30})^{(5)} G_{29}^{*}}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^{*})]}$$

$$T_{28}^{*} = \frac{(b_{28})^{(5)} T_{29}^{*}}{[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31})^{*})]} \quad , \quad T_{30}^{*} = \frac{(b_{30})^{(5)} T_{29}^{*}}{[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31})^{*})]} \tag{519}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

$G_{33}^{*}$ given by $\varphi((G_{35})^{*}) = 0$ , $T_{33}^{*}$ given by $f(T_{33}^{*}) = 0$ and

$$G_{32}^{*} = \frac{(a_{32})^{(6)} G_{33}^{*}}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^{*})]} \quad , \quad G_{34}^{*} = \frac{(a_{34})^{(6)} G_{33}^{*}}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^{*})]}$$

$$T_{32}^{*} = \frac{(b_{32})^{(6)} T_{33}^{*}}{[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35})^{*})]} \quad , \quad T_{34}^{*} = \frac{(b_{34})^{(6)} T_{33}^{*}}{[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35})^{*})]} \tag{521}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

$G_{37}^*$ given by $\varphi((G_{39})^*) = 0$ , $T_{37}^*$ given by $f(T_{37}^*) = 0$ and

$$G_{36}^* = \frac{(a_{36})^{(7)} G_{37}^*}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^*)]} \quad , \quad G_{38}^* = \frac{(a_{38})^{(7)} G_{37}^*}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^*)]}$$

$$T_{36}^* = \frac{(b_{36})^{(7)} T_{37}^*}{[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39})^*)]} \quad , \quad T_{38}^* = \frac{(b_{38})^{(7)} T_{37}^*}{[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39})^*)]}$$

Finally we obtain the unique solution 523

$G_{41}^*$ given by $\varphi((G_{43})^*) = 0$ , $T_{41}^*$ given by $f(T_{41}^*) = 0$ and

$$G_{40}^* = \frac{(a_{40})^{(8)} G_{41}^*}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42}^* = \frac{(a_{42})^{(8)} G_{41}^*}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

$$T_{40}^* = \frac{(b_{40})^{(8)} T_{41}^*}{[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43})^*)]} \quad , \quad T_{42}^* = \frac{(b_{42})^{(8)} T_{41}^*}{[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43})^*)]}$$

Finally we obtain the unique solution of 89 to 99 523 A

$G_{45}^*$ given by $\varphi((G_{47})^*) = 0$ , $T_{45}^*$ given by $f(T_{45}^*) = 0$ and

$$G_{44}^* = \frac{(a_{44})^{(9)} G_{45}^*}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46}^* = \frac{(a_{46})^{(9)} G_{45}^*}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

$$T_{44}^* = \frac{(b_{44})^{(9)} T_{45}^*}{[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47})^*)]} \quad , \quad T_{46}^* = \frac{(b_{46})^{(9)} T_{45}^*}{[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47})^*)]}$$

## ASYMPTOTIC STABILITY ANALYSIS 524

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

**Proof:** Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a''_{14})^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \frac{\partial (b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a'_{13})^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)}G^*_{13}\mathbb{T}_{14} \quad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a'_{14})^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)}G^*_{14}\mathbb{T}_{14} \quad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a'_{15})^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)}G^*_{15}\mathbb{T}_{14} \quad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b'_{13})^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)}T^*_{13}\mathbb{G}_j\right) \quad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b'_{14})^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)}T^*_{14}\mathbb{G}_j\right) \quad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b'_{15})^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T^*_{15}\mathbb{G}_j\right) \quad 530$$

**ASYMPTOTIC STABILITY ANALYSIS** 531

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(2)}$ and $(b''_i)^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G^*_i + \mathbb{G}_i \quad , T_i = T^*_i + \mathbb{T}_i \quad 532$$

$$\frac{\partial(a''_{17})^{(2)}}{\partial T_{17}}(T^*_{17}) = (q_{17})^{(2)} \quad , \frac{\partial(b''_i)^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \quad 533$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a'_{16})^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G^*_{16}\mathbb{T}_{17} \quad 534$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a'_{17})^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G^*_{17}\mathbb{T}_{17} \quad 535$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a'_{18})^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G^*_{18}\mathbb{T}_{17} \quad 536$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b'_{16})^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T^*_{16}\mathbb{G}_j\right) \quad 537$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b'_{17})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}T^*_{17}\mathbb{G}_j\right) \quad 538$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b'_{18})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}T^*_{18}\mathbb{G}_j\right) \quad 539$$

**ASYMPTOTIC STABILITY ANALYSIS** 540

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(3)}$ *and* $(b''_i)^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{21}'')^{(3)}}{\partial T_{21}}(T_{21}^*) = (q_{21})^{(3)} \quad , \frac{\partial (b_i'')^{(3)}}{\partial G_j}((G_{23})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a_{20}')^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G_{20}^*\mathbb{T}_{21} \quad 541$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a_{21}')^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G_{21}^*\mathbb{T}_{21} \quad 542$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a_{22}')^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G_{22}^*\mathbb{T}_{21} \quad 543$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b_{20}')^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T_{20}^*\mathbb{G}_j\right) \quad 544$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b_{21}')^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T_{21}^*\mathbb{G}_j\right) \quad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b_{22}')^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T_{22}^*\mathbb{G}_j\right) \quad 546$$

**ASYMPTOTIC STABILITY ANALYSIS** 547

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}$ *and* $(b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{25}'')^{(4)}}{\partial T_{25}}(T_{25}^*) = (q_{25})^{(4)} \quad , \frac{\partial (b_i'')^{(4)}}{\partial G_j}((G_{27})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\left((a_{24}')^{(4)} + (p_{24})^{(4)}\right)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G_{24}^*\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\left((a_{25}')^{(4)} + (p_{25})^{(4)}\right)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G_{25}^*\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\left((a_{26}')^{(4)} + (p_{26})^{(4)}\right)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G_{26}^*\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\left((b_{24}')^{(4)} - (r_{24})^{(4)}\right)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(24)(j)}T_{24}^*\mathbb{G}_j\right) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\left((b_{25}')^{(4)} - (r_{25})^{(4)}\right)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\left(s_{(25)(j)}T_{25}^*\mathbb{G}_j\right) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T^*_{26}\mathbb{G}_j\right) \quad 554$$

**ASYMPTOTIC STABILITY ANALYSIS** 555

**Theorem 5:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}\ and\ (b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 556

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{29})^{(5)}}{\partial T_{29}}(T^*_{29}) = (q_{29})^{(5)} \ , \ \frac{\partial(b_i'')^{(5)}}{\partial G_j}\left((G_{31})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a'_{28})^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G^*_{28}\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a'_{29})^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G^*_{29}\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a'_{30})^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G^*_{30}\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b'_{28})^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T^*_{28}\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b'_{29})^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T^*_{29}\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b'_{30})^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T^*_{30}\mathbb{G}_j\right) \quad 562$$

**ASYMPTOTIC STABILITY ANALYSIS** 563

**Theorem 6:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}\ and\ (b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{33})^{(6)}}{\partial T_{33}}(T^*_{33}) = (q_{33})^{(6)} \ , \ \frac{\partial(b_i'')^{(6)}}{\partial G_j}\left((G_{35})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a'_{32})^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G^*_{32}\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a'_{33})^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G^*_{33}\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a'_{34})^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G^*_{34}\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b'_{32})^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T^*_{32}\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b'_{33})^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T^*_{33}\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b'_{34})^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T^*_{34}\mathbb{G}_j\right) \quad 570$$

**ASYMPTOTIC STABILITY ANALYSIS** 571

**Theorem 7:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 572

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{37})^{(7)}}{\partial T_{37}}(T^*_{37}) = (q_{37})^{(7)} \quad , \frac{\partial(b_i'')^{(7)}}{\partial G_j}\left((G_{39})^{**}\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a'_{36})^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G^*_{36}\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a'_{37})^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G^*_{37}\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a'_{38})^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G^*_{38}\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b'_{36})^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T^*_{36}\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b'_{37})^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T^*_{37}\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b'_{38})^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T^*_{38}\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

**ASYMPTOTIC STABILITY ANALYSIS**

**Theorem 8:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{41}'')^{(8)}}{\partial T_{41}}(T_{41}^*) = (q_{41})^{(8)} \quad , \frac{\partial (b_i'')^{(8)}}{\partial G_j}\left( (G_{43})^* \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a_{40}')^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G_{40}^*\mathbb{T}_{41} \quad (581)$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a_{41}')^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G_{41}^*\mathbb{T}_{41} \quad (582)$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a_{42}')^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G_{42}^*\mathbb{T}_{41} \quad (583)$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b_{40}')^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T_{40}^*\mathbb{G}_j\right) \quad (584)$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b_{41}')^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T_{41}^*\mathbb{G}_j\right) \quad (585)$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b_{42}')^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T_{42}^*\mathbb{G}_j\right) \quad (586)$$

(586 A)

**ASYMPTOTIC STABILITY ANALYSIS**

**Theorem 9:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}$ *and* $(b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

**Proof:** Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{45}'')^{(9)}}{\partial T_{45}}(T_{45}^*) = (q_{45})^{(9)} \quad , \frac{\partial (b_i'')^{(9)}}{\partial G_j}\left( (G_{47})^* \right) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a_{44}')^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)}G_{44}^*\mathbb{T}_{45} \quad (586\,B)$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a_{45}')^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)}G_{45}^*\mathbb{T}_{45} \quad (586\,C)$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a_{46}')^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)}G_{46}^*\mathbb{T}_{45} \quad (586\,D)$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b_{44}')^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)}T_{44}^*\mathbb{G}_j\right) \quad (586\,E)$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b_{45}')^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)}T_{45}^*\mathbb{G}_j\right) \quad (586\,F)$$

$\frac{d\mathbb{T}_{46}}{dt} = -\left((b'_{46})^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T^*_{46}\mathbb{G}_j\right)$ 586

G

587

**The characteristic equation of this system is**

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(q_{13})^{(1)}G^*_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)}T^*_{14} + (b_{14})^{(1)}s_{(13),(14)}T^*_{14}\right)$$

$$+\left(\left((\lambda)^{(1)} + (a'_{14})^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)}G^*_{13} + (a_{13})^{(1)}(q_{14})^{(1)}G^*_{14}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)}T^*_{14} + (b_{14})^{(1)}s_{(13),(13)}T^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((b'_{13})^{(1)} + (b'_{14})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(15)}T^*_{14} + (b_{14})^{(1)}s_{(13),(15)}T^*_{13}\right)\} = 0$$

$+$

$$(\lambda)^{(2)} + (b'_{18})^{(2)} - (r_{18})^{(2)}\{\left((\lambda)^{(2)} + (a'_{18})^{(2)} + (p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^*_{17} + (a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^*_{17}\right)$$

$$+\left(\left((\lambda)^{(2)} + (a'_{17})^{(2)} + (p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^*_{16} + (a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}\right)$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2 + \left((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2 + \left((b'_{16})^{(2)} + (b'_{17})^{(2)} - (r_{16})^{(2)} + (r_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$+\left(\left((\lambda)^{(2)}\right)^2 + \left((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17} + (a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)$$

$$\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^*_{16}\Big)\} = 0$$

$$+$$

$$(\lambda)^{(3)} + (b'_{22})^{(3)} - (r_{22})^{(3)})\{\left((\lambda)^{(3)} + (a'_{22})^{(3)} + (p_{22})^{(3)}\right)$$

$$\left[\left((\lambda)^{(3)} + (a'_{20})^{(3)} + (p_{20})^{(3)}\right)(q_{21})^{(3)}G^*_{21} + (a_{21})^{(3)}(q_{20})^{(3)}G^*_{20}\right)\Big]$$

$$\left((\lambda)^{(3)} + (b'_{20})^{(3)} - (r_{20})^{(3)}\right)s_{(21),(21)}T^*_{21} + (b_{21})^{(3)}s_{(20),(21)}T^*_{21}\Big)$$

$$+\left((\lambda)^{(3)} + (a'_{21})^{(3)} + (p_{21})^{(3)}\right)(q_{20})^{(3)}G^*_{20} + (a_{20})^{(3)}(q_{21})^{(1)}G^*_{21}\Big)$$

$$\left((\lambda)^{(3)} + (b'_{20})^{(3)} - (r_{20})^{(3)}\right)s_{(21),(20)}T^*_{21} + (b_{21})^{(3)}s_{(20),(20)}T^*_{20}\Big)$$

$$\left((\lambda)^{(3)}\right)^2 + \left((a'_{20})^{(3)} + (a'_{21})^{(3)} + (p_{20})^{(3)} + (p_{21})^{(3)}\right)(\lambda)^{(3)}\Big)$$

$$\left((\lambda)^{(3)}\right)^2 + \left((b'_{20})^{(3)} + (b'_{21})^{(3)} - (r_{20})^{(3)} + (r_{21})^{(3)}\right)(\lambda)^{(3)}\Big)$$

$$+\left(\left((\lambda)^{(3)}\right)^2 + \left((a'_{20})^{(3)} + (a'_{21})^{(3)} + (p_{20})^{(3)} + (p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)} + (a'_{20})^{(3)} + (p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^*_{21} + (a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^*_{20}\right)$$

$$\left((\lambda)^{(3)} + (b'_{20})^{(3)} - (r_{20})^{(3)}\right)s_{(21),(22)}T^*_{21} + (b_{21})^{(3)}s_{(20),(22)}T^*_{20}\Big)\} = 0$$

$$+$$

$$(\lambda)^{(4)} + (b'_{26})^{(4)} - (r_{26})^{(4)})\{\left((\lambda)^{(4)} + (a'_{26})^{(4)} + (p_{26})^{(4)}\right)$$

$$\left[\left((\lambda)^{(4)} + (a'_{24})^{(4)} + (p_{24})^{(4)}\right)(q_{25})^{(4)}G^*_{25} + (a_{25})^{(4)}(q_{24})^{(4)}G^*_{24}\right)\Big]$$

$$\left((\lambda)^{(4)} + (b'_{24})^{(4)} - (r_{24})^{(4)}\right)s_{(25),(25)}T^*_{25} + (b_{25})^{(4)}s_{(24),(25)}T^*_{25}\Big)$$

$$+\left((\lambda)^{(4)} + (a'_{25})^{(4)} + (p_{25})^{(4)}\right)(q_{24})^{(4)}G^*_{24} + (a_{24})^{(4)}(q_{25})^{(4)}G^*_{25}\Big)$$

$$\left((\lambda)^{(4)} + (b'_{24})^{(4)} - (r_{24})^{(4)}\right)s_{(25),(24)}T^*_{25} + (b_{25})^{(4)}s_{(24),(24)}T^*_{24}\Big)$$

$$\left((\lambda)^{(4)}\right)^2 + \left((a'_{24})^{(4)} + (a'_{25})^{(4)} + (p_{24})^{(4)} + (p_{25})^{(4)}\right)(\lambda)^{(4)}\Big)$$

$$\left((\lambda)^{(4)}\right)^2 + \left((b'_{24})^{(4)} + (b'_{25})^{(4)} - (r_{24})^{(4)} + (r_{25})^{(4)}\right)(\lambda)^{(4)}\Big)$$

$$+\left(\left((\lambda)^{(4)}\right)^2 + \left((a'_{24})^{(4)} + (a'_{25})^{(4)} + (p_{24})^{(4)} + (p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)} + (a'_{24})^{(4)} + (p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^*_{25} + (a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^*_{24}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(26)}T^{*}_{24}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\right)\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(29)}T^{*}_{29}\right)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^{*}_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^{*}_{29}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(28)}T^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$+\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(30)}T^{*}_{28}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\right)\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left(\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)\right]$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(33)}T^{*}_{33}\right)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^{*}_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^{*}_{33}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(32)}T^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$+\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)$$

$$\left((\lambda)^{(6)} + (b'_{32})^{(6)} - (r_{32})^{(6)}\right)s_{(33),(34)}T^*_{33} + (b_{33})^{(6)}s_{(32),(34)}T^*_{32}\Big)\} = 0$$

$+$

$$(\lambda)^{(7)} + (b'_{38})^{(7)} - (r_{38})^{(7)})\{\left((\lambda)^{(7)} + (a'_{38})^{(7)} + (p_{38})^{(7)}\right)$$

$$\left[\left((\lambda)^{(7)} + (a'_{36})^{(7)} + (p_{36})^{(7)}\right)(q_{37})^{(7)}G^*_{37} + (a_{37})^{(7)}(q_{36})^{(7)}G^*_{36}\right)\Big]$$

$$\left((\lambda)^{(7)} + (b'_{36})^{(7)} - (r_{36})^{(7)}\right)s_{(37),(37)}T^*_{37} + (b_{37})^{(7)}s_{(36),(37)}T^*_{37}\Big)$$

$$+\left((\lambda)^{(7)} + (a'_{37})^{(7)} + (p_{37})^{(7)}\right)(q_{36})^{(7)}G^*_{36} + (a_{36})^{(7)}(q_{37})^{(7)}G^*_{37}\Big)$$

$$\left((\lambda)^{(7)} + (b'_{36})^{(7)} - (r_{36})^{(7)}\right)s_{(37),(36)}T^*_{37} + (b_{37})^{(7)}s_{(36),(36)}T^*_{36}\Big)$$

$$\left((\lambda)^{(7)}\right)^2 + \left((a'_{36})^{(7)} + (a'_{37})^{(7)} + (p_{36})^{(7)} + (p_{37})^{(7)}\right)(\lambda)^{(7)}\Big)$$

$$\left((\lambda)^{(7)}\right)^2 + \left((b'_{36})^{(7)} + (b'_{37})^{(7)} - (r_{36})^{(7)} + (r_{37})^{(7)}\right)(\lambda)^{(7)}\Big)$$

$$+\left(\left((\lambda)^{(7)}\right)^2 + \left((a'_{36})^{(7)} + (a'_{37})^{(7)} + (p_{36})^{(7)} + (p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)} + (a'_{36})^{(7)} + (p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^*_{37} + (a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^*_{36}\right)$$

$$\left((\lambda)^{(7)} + (b'_{36})^{(7)} - (r_{36})^{(7)}\right)s_{(37),(38)}T^*_{37} + (b_{37})^{(7)}s_{(36),(38)}T^*_{36}\Big)\} = 0$$

$+$

$$(\lambda)^{(8)} + (b'_{42})^{(8)} - (r_{42})^{(8)})\{\left((\lambda)^{(8)} + (a'_{42})^{(8)} + (p_{42})^{(8)}\right)$$

$$\left[\left((\lambda)^{(8)} + (a'_{40})^{(8)} + (p_{40})^{(8)}\right)(q_{41})^{(8)}G^*_{41} + (a_{41})^{(8)}(q_{40})^{(8)}G^*_{40}\right)\Big]$$

$$\left((\lambda)^{(8)} + (b'_{40})^{(8)} - (r_{40})^{(8)}\right)s_{(41),(41)}T^*_{41} + (b_{41})^{(8)}s_{(40),(41)}T^*_{41}\Big)$$

$$+\left((\lambda)^{(8)} + (a'_{41})^{(8)} + (p_{41})^{(8)}\right)(q_{40})^{(8)}G^*_{40} + (a_{40})^{(8)}(q_{41})^{(8)}G^*_{41}\Big)$$

$$\left((\lambda)^{(8)} + (b'_{40})^{(8)} - (r_{40})^{(8)}\right)s_{(41),(40)}T^*_{41} + (b_{41})^{(8)}s_{(40),(40)}T^*_{40}\Big)$$

$$\left((\lambda)^{(8)}\right)^2 + \left((a'_{40})^{(8)} + (a'_{41})^{(8)} + (p_{40})^{(8)} + (p_{41})^{(8)}\right)(\lambda)^{(8)}\Big)$$

$$\left((\lambda)^{(8)}\right)^2 + \left((b'_{40})^{(8)} + (b'_{41})^{(8)} - (r_{40})^{(8)} + (r_{41})^{(8)}\right)(\lambda)^{(8)}\Big)$$

$$+\left(\left((\lambda)^{(8)}\right)^2 + \left((a'_{40})^{(8)} + (a'_{41})^{(8)} + (p_{40})^{(8)} + (p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(42)}T^{*}_{40}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\right)\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(45)}T^{*}_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^{*}_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^{*}_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(44)}T^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$+\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(46)}T^{*}_{44}\right)\}=0$$

**And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.**

## SECTION TWELVE

## Physical Interaction In Neurons

### INTRODUCTION—VARIABLES USED

**A critical assessment of the information processing capabilities of neuronal microtubules using coherent excitations Travis John Adrian Craddock, Jack A. Tuszynski Journal of Biological Physics January 2010, Volume 36, Issue 1, pp 53-70 First online: 04 June 2009**

(1) Also, estimates of energy and timescales for excitons, phonons, their interactions, and thermal

effects are presented. Conclusions cast doubt on the possibility of sufficiently long-lived coherent exciton/phonon structures existing at (eb) physiological temperatures in (eb) the absence of thermal isolation mechanisms.

(2) These results are discussed in comparison with (e&eb) previous models based on (e) quantum effects in non-polar hydrophobic regions, which have yet to be disproved.

**Dorian Aur, Mandar Jog, and Roman R. Poznanski, J. Integr Neurosci 10, 413 (2011) DOI: 10.1142/S0219635211002865: Computing by physical interaction in neurons**

(3) The electrodynamics of action potentials represents (eb) the fundamental level where (e) information is integrated and processed in neurons.

(4) The Hodgkin–Huxley model cannot explain the non-stereotyped spatial charge density dynamics that occur during (e&eb) action potential propagation.

(5) Revealed in experiments as spike directivity, the non-uniform charge density dynamics within (eb) neurons carry (e&eb) meaningful information and suggest (eb) that fragments of information regarding our memories are endogenously stored in (eb) structural patterns at a molecular level and are revealed (eb) only during spiking activity.

(6) The main conceptual idea is that under the influence of (e&eb) electric fields, efficient computation by interaction occurs between (e&eb) charge densities embedded within (eb) molecular structures and (e&eb) the transient developed flow of electrical charges.

**(7)** This process of computation underlying (e&eb) electrical interactions and molecular mechanisms at the subcellular level is (=) dissimilar from spiking neuron models that are completely devoid of (e) physical interactions.

**(8)** Computation by interaction describes (eb) a more powerful continuous model of computation than the one that consists of (e) discrete steps as represented in Turing machines

## NOTATION

### Module One

Also, estimates of energy and timescales for excitons, phonons, their interactions, and thermal effects are presented. Conclusions cast doubt on the possibility of sufficiently long-lived coherent exciton/phonon structures existing at (eb) physiological temperatures in (eb) the absence of thermal isolation mechanisms

$G_{13}$ : Category one of Conclusions cast doubt on the possibility of sufficiently long-lived coherent exciton/phonon structures existing at physiological temperatures

$G_{14}$ : Category two of SAS

$G_{15}$ : Category three of SAS

$T_{13}$ : Category one of absence of thermal isolation mechanisms

$T_{14}$ : Category two of SAS

$T_{15}$ : Category three of SAS

### Module Two

These results are discussed in comparison with (e&eb) previous models based on (e) **quantum effects** in non-polar hydrophobic regions, which have yet to be disproved

$G_{16}$ : Category one of **quantum effects;** non-polar hydrophobic regions

$G_{17}$ : Category two of SAS

$G_{18}$ : Category three of SAS

$T_{16}$ : Category one of non-polar hydrophobic regions; **quantum effects**

$T_{17}$ : Category two of SAS

$T_{18}$ : Category three of SAS

**Module three**

The electrodynamics of action potentials represents (eb) the fundamental level where (e) information is integrated and processed in neurons

$G_{20}$ : Category one of electrodynamics of action potentials

$G_{21}$ : Category two of SAS

$G_{22}$ : Category three of SAS

$T_{20}$ : Category one of fundamental level where (e) information is integrated and processed in neurons

$T_{21}$ : Category two of SAS

$T_{22}$ : Category three of SAS

**Module four**

The electrodynamics of action potentials represents the fundamental level where (e) information is integrated and processed in neurons

$G_{24}$ : Category one of information is integrated and processed in neurons

$G_{25}$ : Category two of SAS

$G_{26}$ : Category three of SAS

$T_{24}$ : Category one of electrodynamics of action potentials represents the fundamental level

$T_{25}$ : Category two of SAS

$T_{26}$ : Category three of SAS

**Module five**

The Hodgkin–Huxley model cannot explain the **non-stereotyped spatial charge density dynamics** that occur during (e&eb) action potential propagation

$G_{28}$ : Category one of **non-stereotyped spatial charge density dynamics;** action potential propagation

$G_{29}$ : Category two of SAS

$G_{30}$ : Category three of SAS

$T_{28}$ : Category one of action potential propagation **;non-stereotyped spatial charge density dynamics**

$T_{29}$ : Category two of SAS

$T_{30}$ : Category three of SAS

**Module six**

Revealed in experiments as spike directivity, the **non-uniform charge density dynamics within neurons** carry (e&eb) meaningful information and suggest (eb) that fragments of information regarding our memories are endogenously stored in (eb) structural patterns at a molecular level and are revealed (eb) only during spiking activity

$G_{32}$ : Category one of **non-uniform charge density dynamics within neurons as revealed in experiments as spike directivity;** fragments of information regarding our memories are endogenously stored in (eb) structural patterns at a molecular level and are revealed (eb) only during spiking activity

$G_{33}$ : Category two of SAS

$G_{34}$ : Category three of SAS

$T_{32}$ : Category one of fragments of information regarding our memories are endogenously stored in (eb) structural patterns at a molecular level and are revealed (eb) only during spiking activity; **non-uniform charge density dynamics within neurons as revealed in experiments as spike directivity**

$T_{33}$ : Category two of SAS

$T_{34}$ : Category three of SAS

**Module seven**

the **non-uniform charge density dynamics within neurons** revealed in experiments as spike directivity carry meaningful information and suggest that fragments of information regarding our memories are endogenously stored in (eb) structural patterns at a molecular level and are revealed (eb) only during spiking activity

$G_{36}$ : Category one of Revealed in experiments as spike directivity, the **non-uniform charge density dynamics within neurons** carry meaningful information and suggest that fragments of information regarding our memories are endogenously stored

$G_{37}$ : Category two of SAS

$G_{38}$ : Category three of SAS

$T_{36}$ : Category one of structural patterns at a molecular level and are revealed (eb) only during spiking activity

$T_{37}$ : Category two of SAS

$T_{38}$ : Category three of SAS

**Module eight**

non-uniform charge density dynamics within neurons revealed in experiments as spike directivity carry meaningful information and suggest that fragments of information regarding our memories are endogenously stored in structural patterns at a molecular level and are revealed (eb) only during **spiking activity**

$G_{40}$ : Category one of non-uniform charge density dynamics within neurons revealed in experiments as spike directivity carry meaningful information and suggest that fragments of information regarding our memories are endogenously stored in structural patterns at a molecular level and are revealed; **spiking activity**

$G_{41}$ : Category two of SAS

$G_{42}$ : Category three of SAS

$T_{40}$ : Category one of **spiking activity;** non-uniform charge density dynamics within neurons revealed in experiments as spike directivity carry meaningful information and suggest that fragments of information regarding our memories are endogenously stored in structural patterns at a molecular level and are revealed

$T_{41}$ : Category two of SAS

$T_{42}$ : Category three of SAS

**Module Nine**

The main conceptual idea is that under the influence of (e&eb) electric fields, efficient computation by interaction occurs between (e&eb) charge densities embedded within (eb) molecular structures and (e&eb) the transient developed flow of electrical charges

$G_{44}$ : Category one of **electric fields;** efficient computation by interaction occurs between (e&eb) charge densities embedded within (eb) molecular structures and (e&eb) the transient developed flow of electrical charges

$G_{45}$ : Category two of SAS

$G_{46}$ : Category three of SAS

$T_{44}$ : Category one of efficient computation by interaction occurs between (e&eb) charge densities embedded within (eb) molecular structures and (e&eb) the transient developed flow of electrical charges **;electric fields**

$T_{45}$ : Category two of SAS

$T_{46}$ : Category three of SAS

**The Coefficients:**

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}$ $(a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}$ $(b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}$ , $(b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}$,
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}$, $(b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$

$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}\ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}\ ,$
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)},$
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)},$
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)},$
are Dissipation coefficients

**Module Numbered One**

**The differential system of this model is now (Module Numbered one)**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \quad 1$$
$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad 2$$
$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15} \quad 3$$
$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13} \quad 4$$
$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14} \quad 5$$
$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)\right]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14}, t) =$ First augmentation factor
$-(b''_{13})^{(1)}(G, t) =$ First detritions factor

**Module Numbered Two**

**The differential system of this model is now ( Module numbered two)**

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right]G_{16} \quad 7$$
$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t)\right]G_{17} \quad 8$$
$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t)\right]G_{18} \quad 9$$
$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\big((G_{19}), t\big)\right]T_{16} \quad 10$$
$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\big((G_{19}), t\big)\right]T_{17} \quad 11$$
$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\big((G_{19}), t\big)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17}, t) =$ First augmentation factor
$-(b''_{16})^{(2)}\big((G_{19}), t\big) =$ First detritions factor

**Module Numbered Three**

**The differential system of this model is now (Module numbered three)**

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right]G_{20} \quad 13$$
$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}, t)\right]G_{21} \quad 14$$
$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}, t)\right]G_{22} \quad 15$$
$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}, t)\right]T_{20} \quad 16$$
$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}, t)\right]T_{21} \quad 17$$
$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}, t)\right]T_{22} \quad 18$$

$+(a''_{20})^{(3)}(T_{21},t) =$ First augmentation factor
$-(b''_{20})^{(3)}(G_{23},t) =$ First detritions factor

**Module Numbered Four**

**The differential system of this model is now (Module numbered Four)**

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right]G_{24} \quad 19$$
$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25},t)\right]G_{25} \quad 20$$
$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25},t)\right]G_{26} \quad 21$$
$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27}),t)\right]T_{24} \quad 22$$
$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}((G_{27}),t)\right]T_{25} \quad 23$$
$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27}),t)\right]T_{26} \quad 24$$

$+(a''_{24})^{(4)}(T_{25},t) =$ **First augmentation factor**
$-(b''_{24})^{(4)}((G_{27}),t) =$ **First detritions factor**

**Module Numbered Five**:

**The differential system of this model is now (Module number five)**

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29},t)\right]G_{28} \quad 25$$
$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29},t)\right]G_{29} \quad 26$$
$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29},t)\right]G_{30} \quad 27$$
$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31}),t)\right]T_{28} \quad 28$$
$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}((G_{31}),t)\right]T_{29} \quad 29$$
$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31}),t)\right]T_{30} \quad 30$$

$+(a''_{28})^{(5)}(T_{29},t) =$ **First augmentation factor**
$-(b''_{28})^{(5)}((G_{31}),t) =$ **First detritions factor**

**Module Numbered Six**

**The differential system of this model is now (Module numbered Six)**

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33},t)\right]G_{32} \quad 31$$
$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33},t)\right]G_{33} \quad 32$$
$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33},t)\right]G_{34} \quad 33$$
$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35}),t)\right]T_{32} \quad 34$$
$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}((G_{35}),t)\right]T_{33} \quad 35$$
$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35}),t)\right]T_{34} \quad 36$$

$+(a''_{32})^{(6)}(T_{33},t) =$ **First augmentation factor**

**Module Numbered Seven:**

**The differential system of this model is now (Seventh Module)**

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right]G_{36} \quad 37$$
$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37},t)\right]G_{37} \quad 38$$

$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37},t)\right]G_{38} \quad 39$$
$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39}),t)\right]T_{36} \quad 40$$
$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}((G_{39}),t)\right]T_{37} \quad 41$$
$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39}),t)\right]T_{38} \quad 42$$
$+(a''_{36})^{(7)}(T_{37},t) =$ **First augmentation factor**

**Module Numbered Eight**

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right]G_{40} \quad 43$$
$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41},t)\right]G_{41} \quad 44$$
$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41},t)\right]G_{42} \quad 45$$
$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43}),t)\right]T_{40} \quad 46$$
$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}((G_{43}),t)\right]T_{41} \quad 47$$
$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43}),t)\right]T_{42} \quad 48$$

**Module Numbered Nine**

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$
$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$
$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$
$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47}),t)\right]T_{44} \quad 52$$
$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}((G_{47}),t)\right]T_{45} \quad 53$$
$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47}),t)\right]T_{46} \quad 54$$
$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}((G_{47}),t) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)}\boxed{+(a''_{13})^{(1)}(T_{14},t)}\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7)}(T_{37},t)}\boxed{+(a''_{40})^{(8,8)}(T_{41},t)}\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)}\boxed{+(a''_{14})^{(1)}(T_{14},t)}\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7)}(T_{37},t)}\boxed{+(a''_{41})^{(8,8)}(T_{41},t)}\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)}\boxed{+(a''_{15})^{(1)}(T_{14},t)}\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7)}(T_{37},t)}\boxed{+(a''_{42})^{(8,8)}(T_{41},t)}\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15} \quad 57$$

Where $\boxed{(a''_{13})^{(1)}(T_{14},t)}$, $\boxed{(a''_{14})^{(1)}(T_{14},t)}$, $\boxed{(a''_{15})^{(1)}(T_{14},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}$ are second augmentation coefficient for

category 1, 2 and 3

$+(a''_{20})^{(3,3,)}(T_{21},t)$, $+(a''_{21})^{(3,3,)}(T_{21},t)$, $+(a''_{22})^{(3,3,)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{38})^{(7,7)}(T_{37},t)$ $+(a''_{37})^{(7,7)}(T_{37},t)$ $+(a''_{36})^{(7,7)}(T_{37},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{40})^{(8,8)}(T_{41},t)$ $+(a''_{41})^{(8,8)}(T_{41},t)$ $+(a''_{42})^{(8,8)}(T_{41},t)$ are eight augmentation coefficient for 1,2,3

$+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)} & -(b''_{13})^{(1)}(G,t) & -(b''_{16})^{(2,2,)}(G_{19},t) & -(b''_{20})^{(3,3,)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{36})^{(7,7,)}(G_{39},t) & -(b''_{40})^{(8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{13} \quad 58$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)} & -(b''_{14})^{(1)}(G,t) & -(b''_{17})^{(2,2,)}(G_{19},t) & -(b''_{21})^{(3,3,)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{37})^{(7,7,)}(G_{39},t) & -(b''_{41})^{(8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{14} \quad 59$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)} & -(b''_{15})^{(1)}(G,t) & -(b''_{18})^{(2,2,)}(G_{19},t) & -(b''_{22})^{(3,3,)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{38})^{(7,7,)}(G_{39},t) & -(b''_{42})^{(8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{15} \quad 60$$

Where $-(b''_{13})^{(1)}(G,t)$, $-(b''_{14})^{(1)}(G,t)$, $-(b''_{15})^{(1)}(G,t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,)}(G_{19},t)$, $-(b''_{17})^{(2,2,)}(G_{19},t)$, $-(b''_{18})^{(2,2,)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{37})^{(7,7,)}(G_{39},t)$, $-(b''_{36})^{(7,7,)}(G_{39},t)$, $-(b''_{38})^{(7,7,)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8)}(G_{43},t)}$ $\boxed{-(b''_{41})^{(8,8)}(G_{43},t)}$ $\boxed{-(b''_{42})^{(8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[\begin{array}{ccc} (a'_{16})^{(2)} & \boxed{+(a''_{16})^{(2)}(T_{17},t)} & \boxed{+(a''_{13})^{(1,1,)}(T_{14},t)} \quad \boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9)}(T_{45},t)} \end{array}\right] G_{16} \quad (61)$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[\begin{array}{ccc} (a'_{17})^{(2)} & \boxed{+(a''_{17})^{(2)}(T_{17},t)} & \boxed{+(a''_{14})^{(1,1,)}(T_{14},t)} \quad \boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9)}(T_{45},t)} \end{array}\right] G_{17} \quad (62)$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[\begin{array}{ccc} (a'_{18})^{(2)} & \boxed{+(a''_{18})^{(2)}(T_{17},t)} & \boxed{+(a''_{15})^{(1,1,)}(T_{14},t)} \quad \boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9)}(T_{45},t)} \end{array}\right] G_{18} \quad (63)$$

Where $\boxed{+(a''_{16})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,)}(T_{14},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)}$ are eight augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{44})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9)}(T_{45},t)}$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[\begin{array}{ccc} (b'_{16})^{(2)} & \boxed{-(b''_{16})^{(2)}(G_{19},t)} & \boxed{-(b''_{13})^{(1,1,)}(G,t)} \quad \boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9)}(G_{47},t)} \end{array}\right] T_{16} \quad (64)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)}\boxed{-(b''_{17})^{(2)}(G_{19},t)} & \boxed{-(b''_{14})^{(1,1,)}(G,t)} & \boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{17} \quad (65)$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)}\boxed{-(b''_{18})^{(2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,)}(G,t)} & \boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{18} \quad (66)$$

where $\boxed{-(b''_{16})^{(2)}(G_{19},t)}$ , $\boxed{-(b''_{17})^{(2)}(G_{19},t)}$ , $\boxed{-(b''_{18})^{(2)}(G_{19},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,)}(G,t)}$ are second detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{44})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \begin{bmatrix} (a'_{20})^{(3)}\boxed{+(a''_{20})^{(3)}(T_{21},t)} & \boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{20} \quad (67)$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \begin{bmatrix} (a'_{21})^{(3)}\boxed{+(a''_{21})^{(3)}(T_{21},t)} & \boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{21} \quad (68)$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \begin{bmatrix} (a'_{22})^{(3)}\boxed{+(a''_{22})^{(3)}(T_{21},t)} & \boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{22} \quad (69)$$

$\boxed{+(a''_{20})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3)}(T_{21},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}$ are second augmentation coefficients

for category 1, 2 and 3

$+(a_{13}'')^{(1,1,1,)}(T_{14},t)$, $+(a_{14}'')^{(1,1,1,)}(T_{14},t)$, $+(a_{15}'')^{(1,1,1,)}(T_{14},t)$ are third augmentation coefficients for category 1, 2 and 3

$+(a_{24}'')^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a_{25}'')^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a_{26}'')^{(4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficients for category 1, 2 and 3

$+(a_{28}'')^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a_{29}'')^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a_{30}'')^{(5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficients for category 1, 2 and 3

$+(a_{32}'')^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a_{33}'')^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a_{34}'')^{(6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficients for category 1, 2 and 3

$+(a_{36}'')^{(7,7,7,7)}(T_{37},t)$, $+(a_{37}'')^{(7,7,7,7)}(T_{37},t)$, $+(a_{38}'')^{(7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1, 2 and 3

$+(a_{40}'')^{(8,8,8,8)}(T_{41},t)$, $+(a_{41}'')^{(8,8,8,8)}(T_{41},t)$, $+(a_{42}'')^{(8,8,8,8)}(T_{41},t)$ are eight augmentation coefficients for category 1, 2 and 3

$+(a_{44}'')^{(9,9,9)}(T_{45},t)$, $+(a_{45}'')^{(9,9,9)}(T_{45},t)$, $+(a_{46}'')^{(9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b_{20}')^{(3)} & -(b_{20}'')^{(3)}(G_{23},t) & -(b_{16}'')^{(2,2,2)}(G_{19},t) & -(b_{13}'')^{(1,1,1,)}(G,t) \\ -(b_{24}'')^{(4,4,4,4,4,4)}(G_{27},t) & -(b_{28}'')^{(5,5,5,5,5,5)}(G_{31},t) & -(b_{32}'')^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b_{36}'')^{(7,7,7,7)}(G_{39},t) & -(b_{40}'')^{(8,8,8,8)}(G_{43},t) & -(b_{44}'')^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{20} \quad (70)$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b_{21}')^{(3)} & -(b_{21}'')^{(3)}(G_{23},t) & -(b_{17}'')^{(2,2,2)}(G_{19},t) & -(b_{14}'')^{(1,1,1,)}(G,t) \\ -(b_{25}'')^{(4,4,4,4,4,4)}(G_{27},t) & -(b_{29}'')^{(5,5,5,5,5,5)}(G_{31},t) & -(b_{33}'')^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b_{37}'')^{(7,7,7,7)}(G_{39},t) & -(b_{41}'')^{(8,8,8,8)}(G_{43},t) & -(b_{45}'')^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{21} \quad (71)$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b_{22}')^{(3)} & -(b_{22}'')^{(3)}(G_{23},t) & -(b_{18}'')^{(2,2,2)}(G_{19},t) & -(b_{15}'')^{(1,1,1,)}(G,t) \\ -(b_{26}'')^{(4,4,4,4,4,4)}(G_{27},t) & -(b_{30}'')^{(5,5,5,5,5,5)}(G_{31},t) & -(b_{34}'')^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b_{38}'')^{(7,7,7,7)}(G_{39},t) & -(b_{42}'')^{(8,8,8,8)}(G_{43},t) & -(b_{46}'')^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{22} \quad (72)$$

$-(b_{20}'')^{(3)}(G_{23},t)$, $-(b_{21}'')^{(3)}(G_{23},t)$, $-(b_{22}'')^{(3)}(G_{23},t)$ *are first detrition coefficients for category 1, 2 and 3*

$-(b_{16}'')^{(2,2,2)}(G_{19},t)$, $-(b_{17}'')^{(2,2,2)}(G_{19},t)$, $-(b_{18}'')^{(2,2,2)}(G_{19},t)$ *are second detrition coefficients for category 1, 2 and 3*

$-(b_{13}'')^{(1,1,1,)}(G,t)$, $-(b_{14}'')^{(1,1,1,)}(G,t)$, $-(b_{15}'')^{(1,1,1,)}(G,t)$ *are third detrition coefficients for category 1,2 and 3*

$-(b_{24}'')^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b_{25}'')^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b_{26}'')^{(4,4,4,4,4,4)}(G_{27},t)$ *are fourth detrition coefficients for category 1, 2 and 3*

$-(b_{28}'')^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b_{29}'')^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b_{30}'')^{(5,5,5,5,5,5)}(G_{31},t)$ *are fifth detrition coefficients for category 1, 2 and 3*

$-(b_{32}'')^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b_{33}'')^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b_{34}'')^{(6,6,6,6,6,6)}(G_{35},t)$ *are sixth detrition coefficients for category 1, 2 and 3*

$-(b_{36}'')^{(7,7,7,7)}(G_{39},t)$, $-(b_{37}'')^{(7,7,7,7)}(G_{39},t)$, $-(b_{38}'')^{(7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients

for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt}=(a_{24})^{(4)}G_{25}-\begin{bmatrix}(a'_{24})^{(4)} & \boxed{+(a''_{24})^{(4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}\\ \boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{24} \quad 73$$

$$\frac{dG_{25}}{dt}=(a_{25})^{(4)}G_{24}-\begin{bmatrix}(a'_{25})^{(4)} & \boxed{+(a''_{25})^{(4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6)}(T_{33},t)}\\ \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{25} \quad 74$$

$$\frac{dG_{26}}{dt}=(a_{26})^{(4)}G_{25}-\begin{bmatrix}(a'_{26})^{(4)} & \boxed{+(a''_{26})^{(4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}\\ \boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{26} \quad 75$$

$\boxed{(a''_{24})^{(4)}(T_{25},t)}$, $\boxed{(a''_{25})^{(4)}(T_{25},t)}$, $\boxed{(a''_{26})^{(4)}(T_{25},t)}$ *are first augmentation coefficients category* 1, 2 3

$\boxed{+(a''_{28})^{(5,5,)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,)}(T_{29},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)}$
*are fourth augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)}$
*are fifth augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)}$
*are sixth augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)}$
*are seventh augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)}$
*are eighth augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt}=(b_{24})^{(4)}T_{25}-\begin{bmatrix}(b'_{24})^{(4)} & \boxed{-(b''_{24})^{(4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,)}(G_{35},t)}\\ \boxed{-(b''_{13})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)}\\ \boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9)}(G_{47},t)}\end{bmatrix}T_{24} \quad 76$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \begin{bmatrix} (b'_{25})^{(4)} & -(b''_{25})^{(4)}(G_{27},t) & -(b''_{29})^{(5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{25} \quad (77)$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)} & -(b''_{26})^{(4)}(G_{27},t) & -(b''_{30})^{(5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1)}(G,t) & -(b''_{18})^{(2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{26} \quad (78)$$

*Where* $-(b''_{24})^{(4)}(G_{27},t)$, $-(b''_{25})^{(4)}(G_{27},t)$, $-(b''_{26})^{(4)}(G_{27},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{28})^{(5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,)}(G_{31},t)$ *are second detrition coefficients for category* 1, 2 *and* 3

$-(b''_{32})^{(6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,)}(G_{35},t)$ *are third detrition coefficients for category* 1, 2 *and* 3

$-(b''_{13})^{(1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1)}(G,t)$
*are fourth detrition coefficients for category* 1, 2 *and* 3

$-(b''_{16})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2)}(G_{19},t)$
*are fifth detrition coefficients for category* 1, 2 *and* 3

$-(b''_{20})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3)}(G_{23},t)$
*are sixth detrition coefficients for category* 1, 2 *and* 3

$-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)$
*are seventh detrition coefficients for category* 1, 2 *and* 3

$-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)$
*are eighth detrition coefficients for category* 1, 2 *and* 3

$-(b''_{46})^{(9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \begin{bmatrix} (a'_{28})^{(5)} & +(a''_{28})^{(5)}(T_{29},t) & +(a''_{24})^{(4,4,)}(T_{25},t) & +(a''_{32})^{(6,6,6)}(T_{33},t) \\ & +(a''_{13})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{16})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3,3)}(T_{21},t) \\ & +(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{28} \quad (79)$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \begin{bmatrix} (a'_{29})^{(5)} & +(a''_{29})^{(5)}(T_{29},t) & +(a''_{25})^{(4,4,)}(T_{25},t) & +(a''_{33})^{(6,6,6)}(T_{33},t) \\ & +(a''_{14})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{17})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3,3)}(T_{21},t) \\ & +(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{29} \quad (80)$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \begin{bmatrix} (a'_{30})^{(5)} & +(a''_{30})^{(5)}(T_{29},t) & +(a''_{26})^{(4,4,)}(T_{25},t) & +(a''_{34})^{(6,6,6)}(T_{33},t) \\ & +(a''_{15})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{18})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3,3)}(T_{21},t) \\ & +(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{30} \quad (81)$$

*Where* $+(a''_{28})^{(5)}(T_{29},t)$, $+(a''_{29})^{(5)}(T_{29},t)$, $+(a''_{30})^{(5)}(T_{29},t)$ *are first augmentation coefficients for category* 1, 2 *and* 3

*And* $+(a''_{24})^{(4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,)}(T_{25},t)$ *are second augmentation*

$coefficient\ for\ category\ 1,2\ and\ 3$
$+(a''_{32})^{(6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6)}(T_{33},t)$ $are\ third\ \ augmentation$
$coefficient\ for\ category\ 1,2\ and\ 3$
$+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)$ are fourth augmentation coefficients for category 1,2, and 3
$+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)$ are fifth augmentation coefficients for category 1,2,and 3
$+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)$ are sixth augmentation coefficients for category 1,2, 3
$+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)$ , $+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1,2, 3
$+(a''_{40})^{(8,8\ ,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)$ are eighth augmentation coefficients for category 1,2, 3
$+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt}=(b_{28})^{(5)}T_{29}-\begin{bmatrix}(b'_{28})^{(5)} & -(b''_{28})^{(5)}(G_{31},t) & -(b''_{24})^{(4,4,)}(G_{27},t) & -(b''_{32})^{(6,6,6)}(G_{35},t)\\ & -(b''_{13})^{(1,1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3)}(G_{23},t)\\ & -(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9)}(G_{47},t)\end{bmatrix}T_{28} \quad 82$$

$$\frac{dT_{29}}{dt}=(b_{29})^{(5)}T_{28}-\begin{bmatrix}(b'_{29})^{(5)} & -(b''_{29})^{(5)}(G_{31},t) & -(b''_{25})^{(4,4,)}(G_{27},t) & -(b''_{33})^{(6,6,6)}(G_{35},t)\\ & -(b''_{14})^{(1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3)}(G_{23},t)\\ & -(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9)}(G_{47},t)\end{bmatrix}T_{29} \quad 83$$

$$\frac{dT_{30}}{dt}=(b_{30})^{(5)}T_{29}-\begin{bmatrix}(b'_{30})^{(5)} & -(b''_{30})^{(5)}(G_{31},t) & -(b''_{26})^{(4,4,)}(G_{27},t) & -(b''_{34})^{(6,6,6)}(G_{35},t)\\ & -(b''_{15})^{(1,1,1,1,1,)}(G,t) & -(b''_{18})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3)}(G_{23},t)\\ & -(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9)}(G_{47},t)\end{bmatrix}T_{30} \quad 84$$

$where$ $-(b''_{28})^{(5)}(G_{31},t)$ , $-(b''_{29})^{(5)}(G_{31},t)$ , $-(b''_{30})^{(5)}(G_{31},t)$ $are\ first\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b''_{24})^{(4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,)}(G_{27},t)$ $are\ second\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b''_{32})^{(6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6)}(G_{35},t)$ $are\ \ third\ \ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b''_{13})^{(1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1)}(G,t)$ , $-(b''_{15})^{(1,1,1,1,1,)}(G,t)$ are fourth detrition coefficients for category 1,2, and 3
$-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1,2, and 3
$-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1,2, and 3
$-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[\begin{array}{ccc} (a'_{32})^{(6)}\boxed{+(a''_{32})^{(6)}(T_{33},t)} & \boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{32} \quad (85)$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[\begin{array}{ccc} (a'_{33})^{(6)}\boxed{+(a''_{33})^{(6)}(T_{33},t)} & \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{33} \quad (86)$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[\begin{array}{ccc} (a'_{34})^{(6)}\boxed{+(a''_{34})^{(6)}(T_{33},t)} & \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{34} \quad (87)$$

$\boxed{+(a''_{32})^{(6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6)}(T_{33},t)}$ *are first augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}$ *are second augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}$ *are third augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}$ - are fourth augmentation coefficients

$\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}$ - fifth augmentation coefficients

$\boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)}$ sixth augmentation coefficients

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)}$ seventh augmentation coefficients

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}$ Eighth augmentation coefficients

$\boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)}$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[\begin{array}{ccc} (b'_{32})^{(6)}\boxed{-(b''_{32})^{(6)}(G_{35},t)} & \boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)} \end{array}\right] T_{32} \quad (88)$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \begin{bmatrix} (b'_{33})^{(6)}\boxed{-(b''_{33})^{(6)}(G_{35},t)}\boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}\boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}\boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)}\boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{33} \quad (89)$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)}\boxed{-(b''_{34})^{(6)}(G_{35},t)}\boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}\boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}\boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)}\boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{34} \quad (90)$$

$\boxed{-(b''_{32})^{(6)}(G_{35},t)}, \boxed{-(b''_{33})^{(6)}(G_{35},t)}, \boxed{-(b''_{34})^{(6)}(G_{35},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}$ *are second detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)}$ *are third detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}$ are fourth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \quad (91)$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a'_{36})^{(7)}\boxed{+(a''_{36})^{(7)}(T_{37},t)}\boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)}\boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)}\boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)}\boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)}\boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \quad (92)$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a'_{37})^{(7)}\boxed{+(a''_{37})^{(7)}(T_{37},t)}\boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)}\boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)}\boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)}\boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}\boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$\frac{dG_{38}}{dt}$ *93*

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a'_{38})^{(7)}\boxed{+(a''_{38})^{(7)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{(a''_{36})^{(7)}(T_{37},t)}$, $\boxed{(a''_{37})^{(7)}(T_{37},t)}$, $\boxed{(a''_{38})^{(7)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,)}(T_{41},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$\frac{dT_{36}}{dt} =$ 94

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b'_{36})^{(7)}\boxed{-(b''_{36})^{(7)}(G_{39},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$\frac{dT_{37}}{dt} =$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b'_{37})^{(7)}\boxed{-(b''_{37})^{(7)}(G_{39},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$\frac{dT_{38}}{dt} =$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b'_{38})^{(7)}\boxed{-(b''_{38})^{(7)}(G_{39},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3



$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \begin{bmatrix} (a'_{40})^{(8)}\boxed{+(a''_{40})^{(8)}(T_{41},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \begin{bmatrix} (a'_{41})^{(8)}\boxed{+(a''_{41})^{(8)}(T_{41},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \begin{bmatrix} (a'_{42})^{(8)}\boxed{+(a''_{42})^{(8)}(T_{41},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a''_{40})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second

augmentation coefficient for category 1, 2 and 3
$+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)$ , $+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)$ , $+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3
$+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)$ , $+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)$ , $+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3
$+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)$ , $+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3
$+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)$ , $+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3
$+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ $+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ $+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ are seventh augmentation coefficient for 1,2,3
$+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$ , $+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$ , $+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$ are eighth augmentation coefficient for 1,2,3
$+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$ , $+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$ , $+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} =$$

$$(b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)}\left[-(b''_{40})^{(8)}(G_{43},t)\right] & \left[-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} =$$

$$(b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)}\left[-(b''_{41})^{(8)}(G_{43},t)\right] & \left[-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} =$$

$$(b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)}\left[-(b''_{42})^{(8)}(G_{43},t)\right] & \left[-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{15}$$

Where $-(b''_{36})^{(7)}(G_{39},t)$ , $-(b''_{37})^{(7)}(G_{39},t)$ , $-(b''_{38})^{(7)}(G_{39},t)$ are first detrition coefficients for category 1, 2 and 3
$-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$ , $-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$ , $-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3
$-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$ , $-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$ , $-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3
$-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)$ , $-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)$ , $-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3



$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \begin{bmatrix} (a'_{44})^{(9)}\boxed{+(a''_{44})^{(9)}(T_{45},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \begin{bmatrix} (a'_{45})^{(9)}\boxed{+(a''_{45})^{(9)}(T_{45},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \begin{bmatrix} (a'_{46})^{(9)}\boxed{+(a''_{46})^{(9)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a''_{44})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,1,1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1,1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1,1,1,1,1,1)}(T_{14},t)$ are Seventh augmentation coefficient for category 1, 2 and 3

$+(a''_{38})^{(7,7,7,7,7,7,7,7,7,7)}(T_{37},t)$ $+(a''_{37})^{(7,7,7,7,7,7,7,7,7,7)}(T_{37},t)$ $+(a''_{36})^{(7,7,7,7,7,7,7,7,7,7)}(T_{37},t)$ are eighth augmentation coefficient for 1,2,3

$+(a''_{40})^{(8,8,8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8,8,8,8,8)}(T_{41},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} =$$

$$(b_{44})^{(9)}T_{45} -$$

$$\begin{bmatrix} (b'_{44})^{(9)} & -(b''_{44})^{(9)}(G_{47},t) & -(b''_{16})^{(2,2,2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6,6,6,6,6,6)}(G_{35},t) & \\ -(b''_{13})^{(1,1,1,1,1,1,1,1,1,1)}(G,t) & -(b''_{36})^{(7,7,7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8,8,8,8,8)}(G_{43},t) & \end{bmatrix} T_{13}$$

$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b'_{45})^{(9)} & -(b''_{45})^{(9)}(G_{47},t) & -(b''_{17})^{(2,2,2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6,6,6,6,6,6)}(G_{35},t) & \\ -(b''_{14})^{(1,1,1,1,1,1,1,1,1,1)}(G,t) & -(b''_{37})^{(7,7,7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8,8,8,8,8)}(G_{43},t) & \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b'_{46})^{(9)} & -(b''_{46})^{(9)}(G_{47},t) & -(b''_{18})^{(2,2,2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6,6,6,6,6,6)}(G_{35},t) & \\ -(b''_{15})^{(1,1,1,1,1,1,1,1,1,1)}(G,t) & -(b''_{38})^{(7,7,7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8,8,8,8,8)}(G_{43},t) & \end{bmatrix} T_{15}$$

Where $-(b''_{44})^{(9)}(G_{47},t)$, $-(b''_{45})^{(9)}(G_{47},t)$, $-(b''_{46})^{(9)}(G_{47},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6,6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{15})^{(1,1,1,1,1,1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1,1,1,1,1,1)}(G,t)$, $-(b''_{13})^{(1,1,1,1,1,1,1,1,1,1)}(G,t)$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{37}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{36}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a_i')^{(1)}, (a_i'')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (b_i'')^{(1)} > 0, \quad i,j = 13,14,15 \qquad 97$$

The functions $(a_i'')^{(1)}, (b_i'')^{(1)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a_i'')^{(1)}(T_{14},t) \leq (p_i)^{(1)} \leq (\hat{A}_{13})^{(1)}$$

$$(b_i'')^{(1)}(G,t) \leq (r_i)^{(1)} \leq (b_i')^{(1)} \leq (\hat{B}_{13})^{(1)}$$

$$lim_{T_2\to\infty}(a_i'')^{(1)}(T_{14},t) = (p_i)^{(1)} \qquad 98$$

$$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = (r_i)^{(1)}$$

**Definition of** $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$ :

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$$|(a_i'')^{(1)}(T_{14}',t) - (a_i'')^{(1)}(T_{14},t)| \leq (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G',t) - (b_i'')^{(1)}(G,t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}',t)$ and $(a_i'')^{(1)}(T_{14},t)$ . $(T_{14}',t)$ and $(T_{14},t)$ are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14},t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$ : 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$$

**Definition of** $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$ : 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i,j = 16,17,18$$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17}, t) \leq (p_i)^{(2)} \leq (\hat{A}_{16})^{(2)} \qquad 102$$

$$(b_i'')^{(2)}(G_{19}, t) \leq (r_i)^{(2)} \leq (b_i')^{(2)} \leq (\hat{B}_{16})^{(2)} \qquad 103$$

$$\lim_{T_2 \to \infty} (a_i'')^{(2)}(T_{17}, t) = (p_i)^{(2)} \qquad 104$$

$$\lim_{G \to \infty} (b_i'')^{(2)}((G_{19}), t) = (r_i)^{(2)} \qquad 105$$

**Definition of** $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$ : 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}', t) - (a_i'')^{(2)}(T_{17}, t)| \leq (\hat{k}_{16})^{(2)}|T_{17} - T_{17}'|e^{-(\hat{M}_{16})^{(2)}t} \qquad 107$$

$$|(b_i'')^{(2)}((G_{19})', t) - (b_i'')^{(2)}((G_{19}), t)| < (\hat{k}_{16})^{(2)}||(G_{19}) - (G_{19})'||e^{-(\hat{M}_{16})^{(2)}t} \qquad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}', t)$ and$(a_i'')^{(2)}(T_{17}, t)$ . $(T_{17}', t)$ and $(T_{17}, t)$ are points belonging to the interval $[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}]$ . It is to be noted that $(a_i'')^{(2)}(T_{17}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17}, t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$ :

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants 109

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$$

**Definition of** $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$ :

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1 \quad (110)$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(b_i)^{(2)} + (b_i')^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1 \quad (111)$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i,j = 20,21,22 \quad (112)$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(3)}$, $(r_i)^{(3)}$:

$$(a_i'')^{(3)}(T_{21}, t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23}, t) \leq (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(3)}(T_{21}, t) = (p_i)^{(3)} \quad (113)$$

$$\lim_{G \to \infty}(b_i'')^{(3)}(G_{23}, t) = (r_i)^{(3)}$$

**Definition of** $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$ :

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: (114)

$$|(a_i'')^{(3)}(T_{21}', t) - (a_i'')^{(3)}(T_{21}, t)| \leq (\hat{k}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\hat{M}_{20})^{(3)}t}$$

$$|(b_i'')^{(3)}(G_{23}', t) - (b_i'')^{(3)}(G_{23}, t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}'||e^{-(\hat{M}_{20})^{(3)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}', t)$ and $(a_i'')^{(3)}(T_{21}, t)$ . $(T_{21}', t)$ And $(T_{21}, t)$ are points belonging to the interval $[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}]$. It is to be noted that $(a_i'')^{(3)}(T_{21}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$ : (115)

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$ and the constants $(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}, i = 20,21,22,$ satisfy the inequalities 116

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(a_i)^{(3)} + (a_i')^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(b_i)^{(3)} + (b_i')^{(3)} + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i, j = 24,25,26 \qquad 117$$

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(4)}$, $(r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25}, t) \le (p_i)^{(4)} \le (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}((G_{27}), t) \le (r_i)^{(4)} \le (b_i')^{(4)} \le (\hat{B}_{24})^{(4)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(4)}(T_{25}, t) = (p_i)^{(4)} \qquad 118$$

$$\lim_{G \to \infty}(b_i'')^{(4)}((G_{27}), t) = (r_i)^{(4)}$$

**Definition of** $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$ :

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24,25,26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}', t) - (a_i'')^{(4)}(T_{25}, t)| \le (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})', t) - (b_i'')^{(4)}((G_{27}), t)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}', t)$ and $(a_i'')^{(4)}(T_{25}, t)$ . $(T_{25}', t)$ and $(T_{25}, t)$ are points belonging to the interval $[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}]$. It is to be noted that $(a_i'')^{(4)}(T_{25}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$ : 120

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$, are positive constants

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}, \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$$

**Definition of** $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$ : 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and\ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}, i = 24,25,26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(a_i)^{(4)} + (a_i')^{(4)} + (\hat{A}_{24})^{(4)} + (\hat{P}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(b_i)^{(4)} + (b_i')^{(4)} + (\hat{B}_{24})^{(4)} + (\hat{Q}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i,j = 28,29,30 \qquad 122$$

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(5)}$, $(r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29}, t) \le (p_i)^{(5)} \le (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}((G_{31}), t) \le (r_i)^{(5)} \le (b_i')^{(5)} \le (\hat{B}_{28})^{(5)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(5)}(T_{29}, t) = (p_i)^{(5)} \qquad 123$$

$$\lim_{G \to \infty}(b_i'')^{(5)}(G_{31}, t) = (r_i)^{(5)}$$

**Definition of** $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$ :

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}', t) - (a_i'')^{(5)}(T_{29}, t)| \le (\hat{k}_{28})^{(5)}|T_{29} - T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})', t) - (b_i'')^{(5)}((G_{31}), t)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}', t)$ and $(a_i'')^{(5)}(T_{29}, t)$ . $(T_{29}', t)$ and $(T_{29}, t)$ are points belonging to the interval $[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}]$. It is to be noted that $(a_i'')^{(5)}(T_{29}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29}, t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$ : 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}, \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

**Definition of** $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$ : 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\widehat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and\ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\widehat{M}_{28})^{(5)}}[(a_i)^{(5)} + (a_i')^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\widehat{M}_{28})^{(5)}}[(b_i)^{(5)} + (b_i')^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i,j = 32,33,34$$ 127

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(6)}$, $(r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33}, t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}), t) \leq (r_i)^{(6)} \leq (b_i')^{(6)} \leq (\hat{B}_{32})^{(6)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(6)}(T_{33}, t) = (p_i)^{(6)}$$ 128

$$\lim_{G \to \infty}(b_i'')^{(6)}((G_{35}), t) = (r_i)^{(6)}$$

**Definition of** $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$ :

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}', t) - (a_i'')^{(6)}(T_{33}, t)| \leq (\hat{k}_{32})^{(6)}|T_{33} - T_{33}'|e^{-(\widehat{M}_{32})^{(6)}t}$$

$$|(b_i'')^{(6)}((G_{35})', t) - (b_i'')^{(6)}((G_{35}), t)| < (\hat{k}_{32})^{(6)}||(G_{35}) - (G_{35})'||e^{-(\widehat{M}_{32})^{(6)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}', t)$ and $(a_i'')^{(6)}(T_{33}, t)$ . $(T_{33}', t)$ and $(T_{33}, t)$ are points belonging to the interval $[(\hat{k}_{32})^{(6)}, (\widehat{M}_{32})^{(6)}]$. It is to be noted that $(a_i'')^{(6)}(T_{33}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33}, t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\widehat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$ : 129

$(\widehat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$, are positive constants

$$\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}} < 1$$

**Definition of** $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$ : 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and\ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32,33,34,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(a_i)^{(6)} + (a_i')^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)} (\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(b_i)^{(6)} + (b_i')^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)} (\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

(III) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i, j = 36,37,38$ 131

(JJJ) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(7)}, (r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37}, t) \leq (p_i)^{(7)} \leq (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39}, t) \leq (r_i)^{(7)} \leq (b_i')^{(7)} \leq (\hat{B}_{36})^{(7)}$$

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*(KKK)* $\lim_{T_2 \to \infty} (a_i'')^{(7)}(T_{37}, t) = (p_i)^{(7)}$

*(LLL)*

$\lim_{G \to \infty} (b_i'')^{(7)}((G_{39}), t) = (r_i)^{(7)}$

**Definition of** $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$ :

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$$|(a_i'')^{(7)}(T_{37}', t) - (a_i'')^{(7)}(T_{37}, t)| \leq (\hat{k}_{36})^{(7)} |T_{37} - T_{37}'| e^{-(\hat{M}_{36})^{(7)} t}$$

$$|(b_i'')^{(7)}((G_{39})', t) - (b_i'')^{(7)}((G_{39}), t)| < (\hat{k}_{36})^{(7)} ||(G_{39}) - (G_{39})'|| e^{-(\hat{M}_{36})^{(7)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}', t)$ and $(a_i'')^{(7)}(T_{37}, t)$ . $(T_{37}', t)$ and $(T_{37}, t)$ are points belonging to the interval $[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}]$ . It is to be noted that $(a_i'')^{(7)}(T_{37}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37}, t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$ : 134

(MMM) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)},$ are positive constants

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$$

**Definition of** $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$ : 135

(NNN) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(b_i)^{(7)} + (b_i')^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i, j = 40,41,42 \qquad 136$$

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

**Definition of** $(p_i)^{(8)}$, $(r_i)^{(8)}$: 137

$$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)} \qquad 138$$

$$(b_i'')^{(8)}((G_{43}), t) \leq (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)} \qquad 139$$

$$\lim_{T_2 \to \infty} (a_i'')^{(8)}(T_{41}, t) = (p_i)^{(8)} \qquad 140$$

$$\lim_{G \to \infty} (b_i'')^{(8)}\big((G_{43}), t\big) = (r_i)^{(8)} \qquad 141$$

**Definition of** $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$ :

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)} |T_{41} - T_{41}'| e^{-(\hat{M}_{40})^{(8)} t} \qquad 142$$

$$|(b_i'')^{(8)}((G_{43})', t) - (b_i'')^{(8)}\big((G_{43}), t\big)| < (\hat{k}_{40})^{(8)} ||(G_{43}) - (G_{43})'|| e^{-(\hat{M}_{40})^{(8)} t} \qquad 143$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}', t)$ and $(a_i'')^{(8)}(T_{41}, t)$ . $(T_{41}', t)$ and $(T_{41}, t)$ are points belonging to the interval $\big[(\hat{k}_{40})^{(8)}, (\hat{M}_{40})^{(8)}\big]$. It is to be noted that $(a_i'')^{(8)}(T_{41}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{40})^{(8)} = 1$

then the function $(a_i'')^{(8)}(T_{41},t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)}$ :

$(\hat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)},$ are positive constants

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\ ,\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}}<1 \qquad 144$$

**Definition of** $(\hat{P}_{40})^{(8)},(\hat{Q}_{40})^{(8)}$ :

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\hat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)},(\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)},(a_i')^{(8)},(b_i)^{(8)},(b_i')^{(8)},(p_i)^{(8)},\ (r_i)^{(8)},i=40,41,42,$

Satisfy the inequalities

$$\frac{1}{(\hat{M}_{40})^{(8)}}[\,(a_i)^{(8)}+(a_i')^{(8)}+\ (\hat{A}_{40})^{(8)}+\ (\hat{P}_{40})^{(8)}\,(\hat{k}_{40})^{(8)}]<1 \qquad 145$$

$$\frac{1}{(\hat{M}_{40})^{(8)}}[\ (b_i)^{(8)}+(b_i')^{(8)}+\ (\hat{B}_{40})^{(8)}+\ (\hat{Q}_{40})^{(8)}\ (\hat{k}_{40})^{(8)}]<1 \qquad 146$$

Where we suppose

$$(a_i)^{(9)},(a_i')^{(9)},(a_i'')^{(9)},(b_i)^{(9)},(b_i')^{(9)},(b_i'')^{(9)}>0,\quad i,j=44,45,46 \qquad 146\,A$$

The functions $(a_i'')^{(9)},(b_i'')^{(9)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(9)},\ (r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45},t)\le(p_i)^{(9)}\le(\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47},t)\le\ (r_i)^{(9)}\le(b_i')^{(9)}\le(\hat{B}_{44})^{(9)}$$

$$\lim_{T_2\to\infty}(a_i'')^{(9)}(T_{45},t)=(p_i)^{(9)}$$

$$\lim_{G\to\infty}(b_i'')^{(9)}(G_{47},t)=\ (r_i)^{(9)}$$

**Definition of** $(\hat{A}_{44})^{(9)},(\hat{B}_{44})^{(9)}$ :

Where $\boxed{(\hat{A}_{44})^{(9)},(\hat{B}_{44})^{(9)},(p_i)^{(9)},\ (r_i)^{(9)}}$ are positive constants and $\boxed{i=44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}',t)-(a_i'')^{(9)}(T_{45},t)|\le(\hat{k}_{44})^{(9)}|T_{45}-\ T_{45}'|e^{-(\hat{M}_{44})^{(9)}t}$$

$$|(b_i'')^{(9)}((G_{47})',t)-(b_i'')^{(9)}\big((G_{47}),t\big)|<(\hat{k}_{44})^{(9)}||(G_{47})-(G_{47})'||e^{-(\hat{M}_{44})^{(9)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}',t)$ and$(a_i'')^{(9)}(T_{45},t)$ . $(T_{45}',t)$ and $(T_{45},t)$ are points belonging to the interval $\big[(\hat{k}_{44})^{(9)},(\hat{M}_{44})^{(9)}\big]$. It is

to be noted that $(a_i'')^{(9)}(T_{45},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{44})^{(9)}=1$ then the function $(a_i'')^{(9)}(T_{45},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\widehat{M}_{44})^{(9)},(\hat{k}_{44})^{(9)}$ :

$(\widehat{M}_{44})^{(9)},(\hat{k}_{44})^{(9)}$, are positive constants

$$\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\ ,\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}<1$$

**Definition of** $(\hat{P}_{44})^{(9)},(\hat{Q}_{44})^{(9)}$ :

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\widehat{M}_{44})^{(9)},(\hat{k}_{44})^{(9)},(\hat{A}_{44})^{(9)} and\ (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)},(a_i')^{(9)},(b_i)^{(9)},(b_i')^{(9)},(p_i)^{(9)},\ (r_i)^{(9)}, i=44,45,46,$
satisfy the inequalities

$$\frac{1}{(\widehat{M}_{44})^{(9)}}[(a_i)^{(9)}+(a_i')^{(9)}+\ (\hat{A}_{44})^{(9)}+\ (\hat{P}_{44})^{(9)}\ (\hat{k}_{44})^{(9)}]<1$$

$$\frac{1}{(\widehat{M}_{44})^{(9)}}[\ (b_i)^{(9)}+(b_i')^{(9)}+\ (\hat{B}_{44})^{(9)}+\ (\hat{Q}_{44})^{(9)}\ (\hat{k}_{44})^{(9)}]<1$$

**Theorem 1:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0)\ ,T_i(0)$ :

$$G_i(t)\le\ (\hat{P}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}\ ,\ \boxed{G_i(0)=G_i^0>0}$$

$$T_i(t)\le\ (\hat{Q}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}\ ,\ \boxed{T_i(0)=T_i^0>0}$$

**Theorem 2 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

**Definition of** $G_i(0)\ ,T_i(0)$

$$G_i(t)\le\ (\hat{P}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}\ ,\ G_i(0)=G_i^0>0$$

$$T_i(t)\le\ (\hat{Q}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}\ ,\ T_i(0)=T_i^0>0$$

**Theorem 3 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t)\le\ (\hat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}\ ,\ G_i(0)=G_i^0>0$$

$$T_i(t)\le\ (\hat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}\ ,\ T_i(0)=T_i^0>0$$

**Theorem 4 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

**Definition of** $G_i(0)\ ,T_i(0)$ :

$G_i(t) \leq \left(\hat{P}_{24}\right)^{(4)} e^{(\hat{M}_{24})^{(4)}t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{24}\right)^{(4)} e^{(\hat{M}_{24})^{(4)}t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Theorem 5 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

**Definition of** $G_i(0)$ , $T_i(0)$ :

$G_i(t) \leq \left(\hat{P}_{28}\right)^{(5)} e^{(\hat{M}_{28})^{(5)}t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{28}\right)^{(5)} e^{(\hat{M}_{28})^{(5)}t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Theorem 6 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

**Definition of** $G_i(0)$ , $T_i(0)$ :

$G_i(t) \leq \left(\hat{P}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Theorem 7:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

**Definition of** $G_i(0)$ , $T_i(0)$ :

$G_i(t) \leq \left(\hat{P}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Theorem 8:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

**Definition of** $G_i(0)$ , $T_i(0)$ :

$G_i(t) \leq \left(\hat{P}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Theorem 9:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

**Definition of** $G_i(0)$ , $T_i(0)$ :

$G_i(t) \leq \left(\hat{P}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Proof:** Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i$: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \le (\hat{P}_{13})^{(1)} \text{ , } T_i^0 \le (\hat{Q}_{13})^{(1)}, \quad 155$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t} \quad 156$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t} \quad 157$$

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}(s_{(13)}) - \left((a'_{13})^{(1)} + a''_{13})^{(1)} (T_{14}(s_{(13)}), s_{(13)})\right) G_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)} (T_{14}(s_{(13)}), s_{(13)})\right) G_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)} (T_{14}(s_{(13)}), s_{(13)})\right) G_{15}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)} (G(s_{(13)}), s_{(13)})\right) T_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)} (G(s_{(13)}), s_{(13)})\right) T_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{15}(t) = T_{15}^0 + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)} (G(s_{(13)}), s_{(13)})\right) T_{15}(s_{(13)})\right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i$: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \le (\hat{P}_{16})^{(2)} \text{ , } T_i^0 \le (\hat{Q}_{16})^{(2)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$$

By 160

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''_{16})^{(2)} (T_{17}(s_{(16)}), s_{(16)})\right) G_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)} (T_{17}(s_{(16)}), s_{(17)})\right) G_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)} (T_{17}(s_{(16)}), s_{(16)})\right) G_{18}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)} (G_{19}(s_{(16)}), s_{(16)})\right) T_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^{0} + \int_0^t \left[ (b_{17})^{(2)} T_{16}(s_{(16)}) - \left( (b'_{17})^{(2)} - (b''_{17})^{(2)} (G_{19}(s_{(16)}), s_{(16)}) \right) T_{17}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^{0} + \int_0^t \left[ (b_{18})^{(2)} T_{17}(s_{(16)}) - \left( (b'_{18})^{(2)} - (b''_{18})^{(2)} (G_{19}(s_{(16)}), s_{(16)}) \right) T_{18}(s_{(16)}) \right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \le (\hat{P}_{20})^{(3)} \text{ , } T_i^0 \le (\hat{Q}_{20})^{(3)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$$

By 161

$$\bar{G}_{20}(t) = G_{20}^{0} + \int_0^t \left[ (a_{20})^{(3)} G_{21}(s_{(20)}) - \left( (a'_{20})^{(3)} + a''_{20})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^{0} + \int_0^t \left[ (a_{21})^{(3)} G_{20}(s_{(20)}) - \left( (a'_{21})^{(3)} + (a''_{21})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{21}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^{0} + \int_0^t \left[ (a_{22})^{(3)} G_{21}(s_{(20)}) - \left( (a'_{22})^{(3)} + (a''_{22})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{22}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^{0} + \int_0^t \left[ (b_{20})^{(3)} T_{21}(s_{(20)}) - \left( (b'_{20})^{(3)} - (b''_{20})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^{0} + \int_0^t \left[ (b_{21})^{(3)} T_{20}(s_{(20)}) - \left( (b'_{21})^{(3)} - (b''_{21})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{21}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{\mathrm{T}}_{22}(\mathrm{t}) = \mathrm{T}_{22}^{0} + \int_0^t \left[ (b_{22})^{(3)} T_{21}(s_{(20)}) - \left( (b'_{22})^{(3)} - (b''_{22})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{22}(s_{(20)}) \right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \le (\hat{P}_{24})^{(4)} \text{ , } T_i^0 \le (\hat{Q}_{24})^{(4)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

By 162

$$\bar{G}_{24}(t) = G_{24}^{0} + \int_0^t \left[ (a_{24})^{(4)} G_{25}(s_{(24)}) - \left( (a'_{24})^{(4)} + a''_{24})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{26}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{26}(t) = T_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{26}(s_{(24)})\right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{28})^{(5)}, T_i^0 \le (\hat{Q}_{28})^{(5)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

By 163

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{30}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{30}(t) = T_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{30}(s_{(28)})\right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{32})^{(6)}, T_i^0 \le (\hat{Q}_{32})^{(6)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\bar{M}_{32})^{(6)} t}$

By 164

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[ (a_{32})^{(6)} G_{33}(s_{(32)}) - \left( (a'_{32})^{(6)} + a''_{32})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[ (a_{33})^{(6)} G_{32}(s_{(32)}) - \left( (a'_{33})^{(6)} + (a''_{33})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[ (a_{34})^{(6)} G_{33}(s_{(32)}) - \left( (a'_{34})^{(6)} + (a''_{34})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{34}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[ (b_{32})^{(6)} T_{33}(s_{(32)}) - \left( (b'_{32})^{(6)} - (b''_{32})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[ (b_{33})^{(6)} T_{32}(s_{(32)}) - \left( (b'_{33})^{(6)} - (b''_{33})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{34}(t) = T_{34}^0 + \int_0^t \left[ (b_{34})^{(6)} T_{33}(s_{(32)}) - \left( (b'_{34})^{(6)} - (b''_{34})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{34}(s_{(32)}) \right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[ (a_{36})^{(7)} G_{37}(s_{(36)}) - \left( (a'_{36})^{(7)} + a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[ (a_{37})^{(7)} G_{36}(s_{(36)}) - \left( (a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[ (a_{38})^{(7)} G_{37}(s_{(36)}) - \left( (a'_{38})^{(7)} + (a''_{38})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{38}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[ (b_{36})^{(7)} T_{37}(s_{(36)}) - \left( (b'_{36})^{(7)} - (b''_{36})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[ (b_{37})^{(7)} T_{36}(s_{(36)}) - \left( (b'_{37})^{(7)} - (b''_{37})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{38}(t) = T_{38}^0 + \int_0^t \left[ (b_{38})^{(7)} T_{37}(s_{(36)}) - \left( (b'_{38})^{(7)} - (b''_{38})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{38}(s_{(36)}) \right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0,t)$

**Proof:**

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{40})^{(8)}$, $T_i^0 \le (\hat{Q}_{40})^{(8)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} G_{41}(s_{(40)}) - \left((a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[(a_{41})^{(8)} G_{40}(s_{(40)}) - \left((a'_{41})^{(8)} + (a''_{41})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{42}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^0 + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)} (G_{43}(s_{(40)}), s_{(40)})\right) T_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^0 + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)} (G_{43}(s_{(40)}), s_{(40)})\right) T_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{42}(t) = T_{42}^0 + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)} (G_{43}(s_{(40)}), s_{(40)})\right) T_{42}(s_{(40)})\right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0,t)$

**Proof:** 166
Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ A
which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{44})^{(9)}$, $T_i^0 \le (\hat{Q}_{44})^{(9)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)})\right) G_{44}(s_{(44)})\right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)} (T_{45}(s_{(44)}), s_{(44)})\right) G_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}(s_{(44)}), s_{(44)})\right) G_{46}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{44}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{\mathrm{T}}_{46}(\mathrm{t}) = \mathrm{T}_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{46}(s_{(44)})\right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}}\right)\right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left(e^{(\hat{M}_{13})^{(1)} t} - 1\right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\hat{M}_{13})^{(1)} t} \le \frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{P}_{13})^{(1)} + G_{14}^0\right) e^{\left(-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0}\right)} + (\hat{P}_{13})^{(1)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \le G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}}\right)\right] ds_{(16)} = \quad 169$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1\right)$$

From which it follows that 170

$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \le \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left[\left((\hat{P}_{16})^{(2)} + G_{17}^0\right) e^{\left(-\frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0}\right)} + (\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t) \le G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} \left(G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} s_{(20)}}\right)\right] ds_{(20)} =$$

$$\left(1 + (a_{20})^{(3)} t\right) G_{21}^0 + \frac{(a_{20})^{(3)} (\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left(e^{(\hat{M}_{20})^{(3)} t} - 1\right)$$

From which it follows that 172

$$(G_{20}(t)-G_{20}^{0})e^{-(\hat{M}_{20})^{(3)}t}\leq\frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left[((\hat{P}_{20})^{(3)}+G_{21}^{0})e^{\left(-\frac{(\hat{P}_{20})^{(3)}+G_{21}^{0}}{G_{21}^{0}}\right)}+(\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 173

$$G_{24}(t)\leq G_{24}^{0}+\int_{0}^{t}\left[(a_{24})^{(4)}\left(G_{25}^{0}+(\hat{P}_{24})^{(4)}e^{(\hat{M}_{24})^{(4)}s_{(24)}}\right)\right]ds_{(24)}=$$

$$\left(1+(a_{24})^{(4)}t\right)G_{25}^{0}+\frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left(e^{(\hat{M}_{24})^{(4)}t}-1\right)$$

From which it follows that 174

$$(G_{24}(t)-G_{24}^{0})e^{-(\hat{M}_{24})^{(4)}t}\leq\frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left[((\hat{P}_{24})^{(4)}+G_{25}^{0})e^{\left(-\frac{(\hat{P}_{24})^{(4)}+G_{25}^{0}}{G_{25}^{0}}\right)}+(\hat{P}_{24})^{(4)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t)\leq G_{28}^{0}+\int_{0}^{t}\left[(a_{28})^{(5)}\left(G_{29}^{0}+(\hat{P}_{28})^{(5)}e^{(\hat{M}_{28})^{(5)}s_{(28)}}\right)\right]ds_{(28)}=$$

$$\left(1+(a_{28})^{(5)}t\right)G_{29}^{0}+\frac{(a_{28})^{(5)}(\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left(e^{(\hat{M}_{28})^{(5)}t}-1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t}\leq\frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t)\leq G_{32}^{0}+\int_{0}^{t}\left[(a_{32})^{(6)}\left(G_{33}^{0}+(\hat{P}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\right)\right]ds_{(32)}=$$

$$\left(1+(a_{32})^{(6)}t\right)G_{33}^{0}+\frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left(e^{(\hat{M}_{32})^{(6)}t}-1\right)$$

From which it follows that 177

$$(G_{32}(t)-G_{32}^{0})e^{-(\hat{M}_{32})^{(6)}t}\leq\frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left[((\hat{P}_{32})^{(6)}+G_{33}^{0})e^{\left(-\frac{(\hat{P}_{32})^{(6)}+G_{33}^{0}}{G_{33}^{0}}\right)}+(\hat{P}_{32})^{(6)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}$ , $G_{26}, T_{24}, T_{25}, T_{26}$

(k) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t) \le G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} \left(G_{37}^0 + (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} s_{(36)}}\right)\right] ds_{(36)} =$$
$$\left(1 + (a_{36})^{(7)} t\right) G_{37}^0 + \frac{(a_{36})^{(7)} (\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left(e^{(\hat{M}_{36})^{(7)} t} - 1\right)$$

From which it follows that

$$(G_{36}(t) - G_{36}^0) e^{-(\hat{M}_{36})^{(7)} t} \le \frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left[((\hat{P}_{36})^{(7)} + G_{37}^0) e^{\left(-\frac{(\hat{P}_{36})^{(7)} + G_{37}^0}{G_{37}^0}\right)} + (\hat{P}_{36})^{(7)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t) \le G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} \left(G_{41}^0 + (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} s_{(40)}}\right)\right] ds_{(40)} = \quad 180$$
$$\left(1 + (a_{40})^{(8)} t\right) G_{41}^0 + \frac{(a_{40})^{(8)} (\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}} \left(e^{(\hat{M}_{40})^{(8)} t} - 1\right)$$

From which it follows that 181

$$(G_{40}(t) - G_{40}^0) e^{-(\hat{M}_{40})^{(8)} t} \le \frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}} \left[((\hat{P}_{40})^{(8)} + G_{41}^0) e^{\left(-\frac{(\hat{P}_{40})^{(8)} + G_{41}^0}{G_{41}^0}\right)} + (\hat{P}_{40})^{(8)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 8

Analogous inequalities hold also for $G_{41}$ , $G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t) \le G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} \left(G_{45}^0 + (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} s_{(44)}}\right)\right] ds_{(44)} =$$
$$\left(1 + (a_{44})^{(9)} t\right) G_{45}^0 + \frac{(a_{44})^{(9)} (\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left(e^{(\hat{M}_{44})^{(9)} t} - 1\right)$$

From which it follows that

$$(G_{44}(t) - G_{44}^0) e^{-(\hat{M}_{44})^{(9)} t} \le \frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left[((\hat{P}_{44})^{(9)} + G_{45}^0) e^{\left(-\frac{(\hat{P}_{44})^{(9)} + G_{45}^0}{G_{45}^0}\right)} + (\hat{P}_{44})^{(9)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 9

Analogous inequalities hold also for $G_{45}$ , $G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}$ , $\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$ and to choose 182

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\widehat{M}_{13})^{(1)}}\left[(\widehat{P}_{13})^{(1)}+\left((\hat{P}_{13})^{(1)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{13})^{(1)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{13})^{(1)} \quad 183$$

$$\frac{(b_i)^{(1)}}{(\widehat{M}_{13})^{(1)}}\left[\left((\hat{Q}_{13})^{(1)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{13})^{(1)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{13})^{(1)}\right]\leq(\hat{Q}_{13})^{(1)} \quad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)},T^{(1)}\right),\left(G^{(2)},T^{(2)}\right)\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{13})^{(1)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{13})^{(1)}t}\}$$

Indeed if we denote

**Definition of** $\tilde{G},\tilde{T}:\left(\tilde{G},\tilde{T}\right)=\mathcal{A}^{(1)}(G,T)$

It results

$$\left|\tilde{G}_{13}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{13})^{(1)}\left|G_{14}^{(1)}-G_{14}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\,ds_{(13)}+$$

$$\int_0^t\{(a'_{13})^{(1)}\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$G_{13}^{(2)}\left|(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)-(a''_{13})^{(1)}\left(T_{14}^{(2)},s_{(13)}\right)\right|\ e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses it follows

$$\left|G^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}t}\leq \quad 186$$
$$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)}+(a'_{13})^{(1)}+(\widehat{A}_{13})^{(1)}+(\widehat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}\right)d\left(\left(G^{(1)},T^{(1)};\,G^{(2)},T^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

**Remark 1:** The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}\ and\ (\widehat{Q}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i=13,14,15$ depend only on $\mathrm{T}_{14}$ and respectively on

$G(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$

From 19 to 24 it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(1)} - (a_i'')^{(1)}(T_{14}(s_{(13)}), s_{(13)})\} ds_{(13)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(1)} t)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2\ and\ \left((\widehat{M}_{13})^{(1)}\right)_3$ : 187

**Remark 3**: if $G_{13}$ is bounded, the same property have also $G_{14}\ and\ G_{15}$ . indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a_{14}')^{(1)} G_{14}$ and by integrating

$$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1 / (a_{14}')^{(1)}$$

In the same way , one can obtain

$$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2 / (a_{15}')^{(1)}$$

If $G_{14}\ or\ G_{15}$ is bounded, the same property follows for $G_{13}$ , $G_{15}$ and $G_{13}$ , $G_{14}$ respectively.

**Remark 4:** If $G_{13}$ $is$ bounded, from below, the same property holds for $G_{14}\ and\ G_{15}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{14}$ is bounded from below. 188

**Remark 5:** If $T_{13}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}\,(G(t), t)) = (b_{14}')^{(1)}$ then $T_{14} \to \infty$. 189

**Definition of** $(m)^{(1)}$ and $\varepsilon_1$ :

Indeed let $t_1$ be so that for $t > t_1$

$$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t), t) < \varepsilon_1, T_{13}\,(t) > (m)^{(1)}$$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right)$, $t = log\,\frac{2}{\varepsilon_1}$ By taking now $\varepsilon_1$ sufficiently small one sees that $T_{14}$ is unbounded.
The same property holds for $T_{15}$ if $\lim_{t\to\infty}(b_{15}'')^{(1)}\,(G(t), t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}$ , $\frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}\ and\ (\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[(\widehat{P}_{16})^{(2)}+\left((\hat{P}_{16})^{(2)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{16})^{(2)}\right]\leq(\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

**Definition of** $\widetilde{G_{19}},\widetilde{T_{19}}$ : $\left(\widetilde{G_{19}},\widetilde{T_{19}}\right)=\mathcal{A}^{(2)}(G_{19},T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{16})^{(2)}\left|G_{17}^{(1)}-G_{17}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)}+$$

$$\int_0^t\{(a'_{16})^{(2)}\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$G_{16}^{(2)}\left|(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)-(a''_{16})^{(2)}\left(T_{17}^{(2)},s_{(16)}\right)\right|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0,t]$ 197

From the hypotheses it follows

$$\left|(G_{19})^{(1)}-(G_{19})^{(2)}\right|\mathrm{e}^{-(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}\leq$$
$$\frac{1}{(\widehat{\mathrm{M}}_{16})^{(2)}}\left((a_{16})^{(2)}+(a'_{16})^{(2)}+(\widehat{\mathrm{A}}_{16})^{(2)}+(\widehat{\mathrm{P}}_{16})^{(2)}(\hat{k}_{16})^{(2)}\right)\mathrm{d}\left(((G_{19})^{(1)},(T_{19})^{(1)};\ (G_{19})^{(2)},(T_{19})^{(2)})\right)$$

And analogous inequalities for $G_i$ and $T_i$. Taking into account the hypothesis the result follows 198

**Remark 6:** The fact that we supposed $(a''_{16})^{(2)}$ and $(b''_{16})^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{\mathrm{P}}_{16})^{(2)}\mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}$ and $(\widehat{\mathrm{Q}}_{16})^{(2)}\mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i=16,17,18$ depend only on $\mathrm{T}_{17}$ and respectively on

$(G_{19})$(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 7:** There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$ 200

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(2)} - (a_i'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})\} ds_{(16)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(2)}t)} > 0 \quad \text{for } t > 0$$

**Definition of** $\left((\widehat{M}_{16})^{(2)}\right)_1$, $\left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$: 201

**Remark 8:** if $G_{16}$ is bounded, the same property have also $G_{17}$ and $G_{18}$. indeed if

$G_{16} < (\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt} \leq \left((\widehat{M}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)} G_{17}$ and by integrating

$$G_{17} \leq \left((\widehat{M}_{16})^{(2)}\right)_2 = G_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1 / (a_{17}')^{(2)}$$

In the same way , one can obtain

$$G_{18} \leq \left((\widehat{M}_{16})^{(2)}\right)_3 = G_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2 / (a_{18}')^{(2)}$$

If $G_{17}$ or $G_{18}$ is bounded, the same property follows for $G_{16}$, $G_{18}$ and $G_{16}$, $G_{17}$ respectively.

**Remark 9:** If $G_{16}$ is bounded, from below, the same property holds for $G_{17}$ and $G_{18}$. The proof is analogous with the preceding one. An analogous property is true if $G_{17}$ is bounded from below. 202

**Remark 10:** If $T_{16}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t), t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$. 203

**Definition of** $(m)^{(2)}$ and $\varepsilon_2$ :

Indeed let $t_2$ be so that for $t > t_2$

$$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right)$, $t = \log\frac{2}{\varepsilon_2}$ By taking now $\varepsilon_2$ sufficiently small one sees that $T_{17}$ is unbounded. 205
The same property holds for $T_{18}$ if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t), t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[(\widehat{P}_{20})^{(3)}+\left((\hat{P}_{20})^{(3)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{20})^{(3)}\right]\leq(\hat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself 210

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)},(T_{23})^{(1)}),((G_{23})^{(2)},(T_{23})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

**Definition of** $\widetilde{G_{23}},\widetilde{T_{23}}:(\widetilde{(G_{23})},\widetilde{(T_{23})})=\mathcal{A}^{(3)}((G_{23}),(T_{23}))$

It results 213

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{20})^{(3)}\left|G_{21}^{(1)}-G_{21}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}ds_{(20)}+$$

$$\int_0^t\{(a'_{20})^{(3)}\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$G_{20}^{(2)}\left|(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)},s_{(20)}\right)\right|\ e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\}ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses it follows

$$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}t}\leq \quad 214$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+(a'_{20})^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\widehat{k}_{20})^{(3)}\right)d\left(((G_{23})^{(1)},(T_{23})^{(1)};\ (G_{23})^{(2)},(T_{23})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 11:** The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ *and* $(\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i=20,21,22$ depend only on $T_{21}$ and respectively on

$(G_{23})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 12:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 216

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(3)} - (a_i'')^{(3)}(T_{21}(s_{(20)}), s_{(20)})\} ds_{(20)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(3)} t)} > 0$ for t > 0

**Definition of** $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2\ and\ \left((\widehat{M}_{20})^{(3)}\right)_3$ : 217

**Remark 13:** if $G_{20}$ is bounded, the same property have also $G_{21}\ and\ G_{22}$ . indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a_{21}')^{(3)} G_{21}$ and by integrating

$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1 / (a_{21}')^{(3)}$

In the same way , one can obtain

$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2 / (a_{22}')^{(3)}$

If $G_{21}\ or\ G_{22}$ is bounded, the same property follows for $G_{20}$ , $G_{22}$ and $G_{20}$ , $G_{21}$ respectively.

**Remark 14:** If $G_{20}$ *is* bounded, from below, the same property holds for $G_{21}\ and\ G_{22}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{21}$ is bounded from below. 218

**Remark 15:** If $T_{20}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t), t\right)) = (b_{21}')^{(3)}$ then $T_{21} \to \infty$. 219

**Definition of** $(m)^{(3)}$ and $\varepsilon_3$ :

Indeed let $t_3$ be so that for $t > t_3$

$(b_{21})^{(3)} - (b_i'')^{(3)}\left((G_{23})(t), t\right) < \varepsilon_3, T_{20}\,(t) > (m)^{(3)}$

Then $\frac{dT_{21}}{dt} \geq (a_{21})^{(3)}(m)^{(3)} - \varepsilon_3 T_{21}$ which leads to 220

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t} = \frac{1}{2}$ it results

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_3}$ By taking now $\varepsilon_3$ sufficiently small one sees that $T_{21}$ is unbounded.
The same property holds for $T_{22}$ if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t), t\right) = (b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}$ , $\frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}} < 1$ and to choose 221

$(\widehat{P}_{24})^{(4)}$ and $(\widehat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[(\widehat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \tag{222}$$

$$\frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \tag{223}$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself (224)

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric (225)

$$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{24})^{(4)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{27}}),(\widetilde{T_{27}})$ : $\left((\widetilde{G_{27}}),(\widetilde{T_{27}})\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{24})^{(4)}\left|G_{25}^{(1)}-G_{25}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\,ds_{(24)}+$$

$$\int_0^t\{(a'_{24})^{(4)}\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$G_{24}^{(2)}\left|(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)-(a''_{24})^{(4)}\left(T_{25}^{(2)},s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\}ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}t}\leq \tag{226}$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)}+(a'_{24})^{(4)}+(\widehat{A}_{24})^{(4)}+(\widehat{P}_{24})^{(4)}(\hat{k}_{24})^{(4)}\right)d\left(((G_{27})^{(1)},(T_{27})^{(1)};\ (G_{27})^{(2)},(T_{27})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 16:** The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ *and* $(\widehat{Q}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_+$. (227)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i=24,25,26$ depend only on $T_{25}$ and respectively on

$(G_{27})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 17:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 228

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(4)} - (a_i'')^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(4)} t)} > 0$ for t > 0

**Definition of** $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2\ and\ \left((\widehat{M}_{24})^{(4)}\right)_3$ : 229

**Remark 18:** if $G_{24}$ is bounded, the same property have also $G_{25}\ and\ G_{26}$ . indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a_{25}')^{(4)} G_{25}$ and by integrating

$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1 / (a_{25}')^{(4)}$

In the same way , one can obtain

$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2 / (a_{26}')^{(4)}$

If $G_{25}\ or\ G_{26}$ is bounded, the same property follows for $G_{24}$ , $G_{26}$ and $G_{24}$ , $G_{25}$ respectively.

**Remark 19:** If $G_{24}$ *is* bounded, from below, the same property holds for $G_{25}\ and\ G_{26}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{25}$ is bounded from below. 230

**Remark 20:** If $T_{24}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}\ ((G_{27})(t), t)) = (b_{25}')^{(4)}$ then $T_{25} \to \infty$. 231

**Definition of** $(m)^{(4)}$ and $\varepsilon_4$ :

Indeed let $t_4$ be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t), t) < \varepsilon_4, T_{24}\,(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_4}$ By taking now $\varepsilon_4$ sufficiently small one sees that $T_{25}$ is unbounded.
The same property holds for $T_{26}$ if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t), t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for $G_{29}$ , $G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}$ , $\frac{(b_i)^{(5)}}{(\widehat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}\left[(\widehat{P}_{28})^{(5)}+\left((\hat{P}_{28})^{(5)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{28})^{(5)} \tag{234}$$

$$\frac{(b_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{28})^{(5)}\right]\leq(\hat{Q}_{28})^{(5)} \tag{235}$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric (236)

$$d\left(((G_{31})^{(1)},(T_{31})^{(1)}),((G_{31})^{(2)},(T_{31})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\widehat{M}_{28})^{(5)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\widehat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{31}}),(\widetilde{T_{31}})$ : $\left((\widetilde{G_{31}}),(\widetilde{T_{31}})\right)=\mathcal{A}^{(5)}\left((G_{31}),(T_{31})\right)$

It results

$$|\tilde{G}_{28}^{(1)}-\tilde{G}_i^{(2)}|\leq\int_0^t(a_{28})^{(5)}|G_{29}^{(1)}-G_{29}^{(2)}|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\,ds_{(28)}+$$

$$\int_0^t\{(a'_{28})^{(5)}|G_{28}^{(1)}-G_{28}^{(2)}|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$(a''_{28})^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)|G_{28}^{(1)}-G_{28}^{(2)}|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$G_{28}^{(2)}|(a''_{28})^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)-(a''_{28})^{(5)}\left(T_{29}^{(2)},s_{(28)}\right)|\;e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\}ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on it follows

$$|(G_{31})^{(1)}-(G_{31})^{(2)}|e^{-(\widehat{M}_{28})^{(5)}t}\leq \tag{237}$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)}+(a'_{28})^{(5)}+(\widehat{A}_{28})^{(5)}+(\widehat{P}_{28})^{(5)}(\hat{k}_{28})^{(5)}\right)d\left(((G_{31})^{(1)},(T_{31})^{(1)};\;(G_{31})^{(2)},(T_{31})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 21:** The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ *and* $(\widehat{Q}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ respectively of $\mathbb{R}_+$. (238)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on $T_{29}$ and respectively on $(G_{31})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 22:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 239

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(5)} - (a_i'')^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(5)} t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2\ and\ \left((\widehat{M}_{28})^{(5)}\right)_3$ : 240

**Remark 23:** if $G_{28}$ is bounded, the same property have also $G_{29}\ and\ G_{30}$ . indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a_{29}')^{(5)} G_{29}$ and by integrating

$$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1 / (a_{29}')^{(5)}$$

In the same way , one can obtain

$$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2 / (a_{30}')^{(5)}$$

If $G_{29}\ or\ G_{30}$ is bounded, the same property follows for $G_{28}$ , $G_{30}$ and $G_{28}$ , $G_{29}$ respectively.

**Remark 24:** If $G_{28}$ *is* bounded, from below, the same property holds for $G_{29}\ and\ G_{30}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{29}$ is bounded from below. 241

**Remark 25:** If $T_{28}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}\,((G_{31})(t), t)) = (b_{29}')^{(5)}$ then $T_{29} \to \infty$. 242

**Definition of** $(m)^{(5)}$ and $\varepsilon_5$ :

Indeed let $t_5$ be so that for $t > t_5$

$$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t), t) < \varepsilon_5, T_{28}\,(t) > (m)^{(5)}$$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_5}$ By taking now $\varepsilon_5$ sufficiently small one sees that $T_{29}$ is unbounded.
The same property holds for $T_{30}$ if $\lim_{t\to\infty}(b_{30}'')^{(5)}\left((G_{31})(t), t\right) = (b_{30}')^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{33}$ , $G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}$ , $\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}} < 1$ and to choose 244

$(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[(\hat{P}_{32})^{(6)}+\left((\hat{P}_{32})^{(6)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{32})^{(6)} \tag{245}$$

$$\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[\left((\hat{Q}_{32})^{(6)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{32})^{(6)}\right]\leq(\hat{Q}_{32})^{(6)} \tag{246}$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric (247)

$$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\hat{M}_{32})^{(6)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\hat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{35}}),(\widetilde{T_{35}})$ : $\left((\widetilde{G_{35}}),(\widetilde{T_{35}})\right)=\mathcal{A}^{(6)}\left((G_{35}),(T_{35})\right)$

It results

$$|\tilde{G}_{32}^{(1)}-\tilde{G}_i^{(2)}|\leq\int_0^t(a_{32})^{(6)}|G_{33}^{(1)}-G_{33}^{(2)}|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\,ds_{(32)}+$$

$$\int_0^t\{(a'_{32})^{(6)}|G_{32}^{(1)}-G_{32}^{(2)}|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)|G_{32}^{(1)}-G_{32}^{(2)}|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$G_{32}^{(2)}|(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)-(a''_{32})^{(6)}\left(T_{33}^{(2)},s_{(32)}\right)|\;e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\}ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$|(G_{35})^{(1)}-(G_{35})^{(2)}|e^{-(\widehat{M}_{32})^{(6)}t}\leq \tag{248}$$
$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)}+(a'_{32})^{(6)}+(\widehat{A}_{32})^{(6)}+(\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}\right)d\left(((G_{35})^{(1)},(T_{35})^{(1)};\;(G_{35})^{(2)},(T_{35})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 26:** The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\hat{P}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ *and* $(\hat{Q}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ respectively of $\mathbb{R}_+$. (249)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i = 32,33,34$ depend only on $T_{33}$ and respectively on $(G_{35})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 27:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 250

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(6)} - (a_i'')^{(6)}(T_{33}(s_{(32)}), s_{(32)})\} ds_{(32)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(6)} t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{32})^{(6)}\right)_1, \left((\widehat{M}_{32})^{(6)}\right)_2\ and\ \left((\widehat{M}_{32})^{(6)}\right)_3$ : 251

**Remark 28:** if $G_{32}$ is bounded, the same property have also $G_{33}\ and\ G_{34}$ . indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a_{33}')^{(6)} G_{33}$ and by integrating

$$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_1 / (a_{33}')^{(6)}$$

In the same way , one can obtain

$$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_2 / (a_{34}')^{(6)}$$

If $G_{33}\ or\ G_{34}$ is bounded, the same property follows for $G_{32}$ , $G_{34}$ and $G_{32}$ , $G_{33}$ respectively.

**Remark 29:** If $G_{32}$ *is* bounded, from below, the same property holds for $G_{33}\ and\ G_{34}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{33}$ is bounded from below. 252

**Remark 30:** If $T_{32}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}\ ((G_{35})(t), t)) = (b_{33}')^{(6)}$ then $T_{33} \to \infty$. 253

**Definition of** $(m)^{(6)}$ and $\varepsilon_6$ :

Indeed let $t_6$ be so that for $t > t_6$

$$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t), t\right) < \varepsilon_6, T_{32}\,(t) > (m)^{(6)}$$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right)$, $t = log\,\frac{2}{\varepsilon_6}$ By taking now $\varepsilon_6$ sufficiently small one sees that $T_{33}$ is unbounded.
The same property holds for $T_{34}$ if $\lim_{t\to\infty}(b_{34}'')^{(6)}\left((G_{35})(t), t(t), t\right) = (b_{34}')^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{37}, G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}$ , $\frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$ and to choose

$(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[(\hat{P}_{36})^{(7)}+\left((\hat{P}_{36})^{(7)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{36})^{(7)} \tag{256}$$

$$\frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[\left((\hat{Q}_{36})^{(7)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{36})^{(7)}\right]\leq(\hat{Q}_{36})^{(7)} \tag{257}$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself
The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric (258)

$$d\left(((G_{39})^{(1)},(T_{39})^{(1)}),((G_{39})^{(2)},(T_{39})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\hat{M}_{36})^{(7)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\hat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{39}}),(\widetilde{T_{39}}):\left((\widetilde{G_{39}}),(\widetilde{T_{39}})\right)=\mathcal{A}^{(7)}((G_{39}),(T_{39}))$
It results

$$|\tilde{G}_{36}^{(1)}-\tilde{G}_i^{(2)}|\leq\int_0^t(a_{36})^{(7)}|G_{37}^{(1)}-G_{37}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\,ds_{(36)}+$$
$$\int_0^t\{(a'_{36})^{(7)}|G_{36}^{(1)}-G_{36}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$(a''_{36})^{(7)}(T_{37}^{(1)},s_{(36)})|G_{36}^{(1)}-G_{36}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$G_{36}^{(2)}|(a''_{36})^{(7)}(T_{37}^{(1)},s_{(36)})-(a''_{36})^{(7)}(T_{37}^{(2)},s_{(36)})|\;e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\}ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on it follows

$$|(G_{39})^{(1)}-(G_{39})^{(2)}|e^{-(\widehat{M}_{36})^{(7)}t}\leq \tag{259}$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}}\left((a_{36})^{(7)}+(a'_{36})^{(7)}+(\widehat{A}_{36})^{(7)}+(\widehat{P}_{36})^{(7)}(\widehat{k}_{36})^{(7)}\right)d\left(((G_{39})^{(1)},(T_{39})^{(1)};\;(G_{39})^{(2)},(T_{39})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 31:** The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ *and* $(\widehat{Q}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ respectively of $\mathbb{R}_+$. (260)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(7)}$ and $(b_i'')^{(7)}, i = 36,37,38$ depend only on $T_{37}$ and respectively on $(G_{39})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 32:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 261

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(7)} - (a_i'')^{(7)}(T_{37}(s_{(36)}), s_{(36)})\} ds_{(36)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(7)} t)} > 0$ for t > 0

**Definition of** $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2\ and\ \left((\widehat{M}_{36})^{(7)}\right)_3$ : 262

**Remark 33:** if $G_{36}$ is bounded, the same property have also $G_{37}\ and\ G_{38}$ . indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a_{37}')^{(7)} G_{37}$ and by integrating

$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1 / (a_{37}')^{(7)}$

In the same way , one can obtain

$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2 / (a_{38}')^{(7)}$

If $G_{37}\ or\ G_{38}$ is bounded, the same property follows for $G_{36}$ , $G_{38}$ and $G_{36}$ , $G_{37}$ respectively.

**Remark 34:** If $G_{36}$ *is* bounded, from below, the same property holds for $G_{37}\ and\ G_{38}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{37}$ is bounded from below. 263

**Remark 35:** If $T_{36}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(7)}\ ((G_{39})(t), t)) = (b_{37}')^{(7)}$ then $T_{37} \to \infty$. 264

**Definition of** $(m)^{(7)}$ and $\varepsilon_7$ :

Indeed let $t_7$ be so that for $t > t_7$

$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t), t) < \varepsilon_7, T_{36}\,(t) > (m)^{(7)}$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_7}$ By taking now $\varepsilon_7$ sufficiently small one sees that $T_{37}$ is unbounded.
The same property holds for $T_{38}$ if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t), t\right) = (b_{38}')^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}$ , $\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}} < 1$ and to choose $(\widehat{P}_{40})^{(8)}$ and $(\widehat{Q}_{40})^{(8)}$ large to have 266

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[(\widehat{P}_{40})^{(8)} + \left((\hat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{40})^{(8)} \quad 267$$

$$\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{40})^{(8)}\right] \leq (\hat{Q}_{40})^{(8)} \quad 268$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)}, (T_{43})^{(1)}), ((G_{43})^{(2)}, (T_{43})^{(2)})\right) = \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

**Definition of** $\widetilde{(G_{43})}, \widetilde{(T_{43})}$ : $\left(\widetilde{(G_{43})}, \widetilde{(T_{43})}\right) = \mathcal{A}^{(8)}((G_{43}), (T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{40})^{(8)}\left|G_{41}^{(1)} - G_{41}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\,ds_{(40)} +$$
$$\int_0^t\{(a'_{40})^{(8)}\left|G_{40}^{(1)} - G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{-(\widehat{M}_{40})^{(8)}s_{(40)}} +$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right)\left|G_{40}^{(1)} - G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}} +$$
$$G_{40}^{(2)}\left|(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right) - (a''_{40})^{(8)}\left(T_{41}^{(2)}, s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\}ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0, t]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)} - (G_{43})^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}t} \leq \quad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)} + (a'_{40})^{(8)} + (\widehat{A}_{40})^{(8)} + (\widehat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}\right)d\left(((G_{43})^{(1)}, (T_{43})^{(1)};\ (G_{43})^{(2)}, (T_{43})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

**Remark 36:** The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate 274

condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ *and* $(\widehat{Q}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i = 40,41,42$ depend only on $T_{41}$ and respectively on $(G_{43})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 37** There does not exist any $t$ where $G_i(t) = 0\ and\ T_i(t) = 0$ 275

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(8)} - (a_i'')^{(8)}(T_{41}(s_{(40)}), s_{(40)})\} ds_{(40)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(8)}t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{40})^{(8)}\right)_1, \left((\widehat{M}_{40})^{(8)}\right)_2\ and\ \left((\widehat{M}_{40})^{(8)}\right)_3$ : 276

**Remark 38:** if $G_{40}$ is bounded, the same property have also $G_{41}\ and\ G_{42}$ . indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a_{41}')^{(8)}G_{41}$ and by integrating

$$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1/(a_{41}')^{(8)}$$

In the same way , one can obtain

$$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2/(a_{42}')^{(8)}$$

If $G_{41}\ or\ G_{42}$ is bounded, the same property follows for $G_{40}$ , $G_{42}$ and $G_{40}$ , $G_{41}$ respectively.

**Remark 39:** If $G_{40}$ *is* bounded, from below, the same property holds for $G_{41}\ and\ G_{42}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{41}$ is bounded from below. 277

**Remark 40:** If $T_{40}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}((G_{43})(t), t)) = (b_{41}')^{(8)}$ then $T_{41} \to \infty$. 278

**Definition of** $(m)^{(8)}$ and $\varepsilon_8$ :

Indeed let $t_8$ be so that for $t > t_8$

$$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t), t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right)$, $t = log\frac{2}{\varepsilon_8}$ By taking now $\varepsilon_8$ sufficiently small one sees that $T_{41}$ is unbounded.
The same property holds for $T_{42}$ if $\lim_{t\to\infty}(b''_{42})^{(8)}\left((G_{43})(t), t(t), t\right) = (b'_{42})^{(8)}$



It is now sufficient to take $\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}$ , $\frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$ and to choose $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ large to have

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}\left[(\hat{P}_{44})^{(9)} + \left((\hat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}}\left[\left((\hat{Q}_{44})^{(9)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{44})^{(9)}\right] \leq (\hat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)}, (T_{47})^{(1)}), ((G_{47})^{(2)}, (T_{47})^{(2)})\right) =$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{44})^{(9)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{47}}), (\widetilde{T_{47}}) : \left((\widetilde{G_{47}}), (\widetilde{T_{47}})\right) = \mathcal{A}^{(9)}\left((G_{47}), (T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{44})^{(9)}\left|G_{45}^{(1)} - G_{45}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\,ds_{(44)} +$$
$$\int_0^t\{(a'_{44})^{(9)}\left|G_{44}^{(1)} - G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{-(\widehat{M}_{44})^{(9)}s_{(44)}} +$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right)\left|G_{44}^{(1)} - G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}} +$$
$$G_{44}^{(2)}\left|(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right) - (a''_{44})^{(9)}\left(T_{45}^{(2)}, s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\}ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)} - G^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}t} \leq$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)} + (a'_{44})^{(9)} + (\widehat{A}_{44})^{(9)} + (\widehat{P}_{44})^{(9)}(\hat{k}_{44})^{(9)}\right)d\left(((G_{47})^{(1)}, (T_{47})^{(1)};\ (G_{47})^{(2)}, (T_{47})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis (39,35,36) the result follows

**Remark 41:** The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by

$(\widehat{P}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ *and* $(\widehat{Q}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i = 44{,}45{,}46$ depend only on $T_{45}$ and respectively on $(G_{47})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 42:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$

From 99 to 44 it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t\{(a_i')^{(9)}-(a_i'')^{(9)}(T_{45}(s_{(44)}),s_{(44)})\}ds_{(44)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(9)}t)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{44})^{(9)}\right)_1, \left((\widehat{M}_{44})^{(9)}\right)_2$ *and* $\left((\widehat{M}_{44})^{(9)}\right)_3$ :

**Remark 43:** if $G_{44}$ is bounded, the same property have also $G_{45}\ and\ G_{46}$ . indeed if

$G_{44} < (\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt} \leq \left((\widehat{M}_{44})^{(9)}\right)_1 - (a_{45}')^{(9)}G_{45}$ and by integrating

$$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_1/(a_{45}')^{(9)}$$

In the same way , one can obtain

$$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_2/(a_{46}')^{(9)}$$

If $G_{45}\ or\ G_{46}$ is bounded, the same property follows for $G_{44}$ , $G_{46}$ and $G_{44}$ , $G_{45}$ respectively.

**Remark 44:** If $G_{44}$ *is* bounded, from below, the same property holds for $G_{45}\ and\ G_{46}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{45}$ is bounded from below.

**Remark 45:** If $T_{44}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t),t\right) = (b_{45}')^{(9)}$ then $T_{45} \to \infty$.

**Definition of** $(m)^{(9)}$ and $\varepsilon_9$ :

Indeed let $t_9$ be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t),t\right) < \varepsilon_9, T_{44}\,(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right), \quad t = log\frac{2}{\varepsilon_9}$ By taking now $\varepsilon_9$ sufficiently small one sees that $T_{45}$ is unbounded.
The same property holds for $T_{46}$ if $\lim_{t\to\infty}(b_{46}'')^{(9)}\left((G_{47})(t),t\right) = (b_{46}')^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

**Behavior of the solutions of equation** 280

**Theorem** If we denote and define

**Definition of** $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ :

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)} \le -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14}, t) + (a''_{14})^{(1)}(T_{14}, t) \le -(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)} \le -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G, t) - (b''_{14})^{(1)}(G, t) \le -(\tau_1)^{(1)}$

**Definition of** $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$ : 281

By $(\nu_1)^{(1)} > 0$, $(\nu_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$, $(u_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

**Definition of** $(\bar{\nu}_1)^{(1)},, (\bar{\nu}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$ : 282

By $(\bar{\nu}_1)^{(1)} > 0$, $(\bar{\nu}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$, $(\bar{u}_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

**Definition of** $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (\nu_0)^{(1)}$ :- 283

If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by
$(m_2)^{(1)} = (\nu_0)^{(1)}, (m_1)^{(1)} = (\nu_1)^{(1)}$, $if\ (\nu_0)^{(1)} < (\nu_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}, if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)}$,

and $\boxed{(\nu_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)}$, $if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)}$, $if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)}$,

and $\boxed{(u_0)^{(1)} = \frac{T^0_{13}}{T^0_{14}}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$

are defined

Then the solution of global equations satisfies the inequalities 285

$G^0_{13}e^{\left((S_1)^{(1)} - (p_{13})^{(1)}\right)t} \le G_{13}(t) \le G^0_{13}e^{(S_1)^{(1)}t}$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)}-(p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)}t}$$

$$\left(\frac{(a_{15})^{(1)}G_{13}^0}{(m_1)^{(1)}((S_1)^{(1)}-(p_{13})^{(1)}-(S_2)^{(1)})}\left[e^{((S_1)^{(1)}-(p_{13})^{(1)})t} - e^{-(S_2)^{(1)}t}\right] + G_{15}^0 e^{-(S_2)^{(1)}t} \le G_{15}(t) \le \right. \quad (286)$$

$$\left.\frac{(a_{15})^{(1)}G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)}-(a'_{15})^{(1)})}[e^{(S_1)^{(1)}t} - e^{-(a'_{15})^{(1)}t}] + G_{15}^0 e^{-(a'_{15})^{(1)}t}\right)$$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)}t} \le T_{13}(t) \le T_{13}^0 e^{((R_1)^{(1)}+(r_{13})^{(1)})t}} \quad (287)$$

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)}t} \le T_{13}(t) \le \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)}+(r_{13})^{(1)})t} \quad (288)$$

$$\frac{(b_{15})^{(1)}T_{13}^0}{(\mu_1)^{(1)}((R_1)^{(1)}-(b'_{15})^{(1)})}\left[e^{(R_1)^{(1)}t} - e^{-(b'_{15})^{(1)}t}\right] + T_{15}^0 e^{-(b'_{15})^{(1)}t} \le T_{15}(t) \le \quad (289)$$

$$\frac{(a_{15})^{(1)}T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)}+(r_{13})^{(1)}+(R_2)^{(1)})}\left[e^{((R_1)^{(1)}+(r_{13})^{(1)})t} - e^{-(R_2)^{(1)}t}\right] + T_{15}^0 e^{-(R_2)^{(1)}t}$$

**Definition of** $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:- (290)

Where $(S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$

$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$

$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$

$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$

**Behavior of the solutions of equation** (291)

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ : (292)

$(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ four constants satisfying

$$-(\sigma_2)^{(2)} \le -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(T_{17}, t) + (a''_{17})^{(2)}(T_{17}, t) \le -(\sigma_1)^{(2)} \quad (293)$$

$$-(\tau_2)^{(2)} \le -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}((G_{19}), t) - (b''_{17})^{(2)}((G_{19}), t) \le -(\tau_1)^{(2)} \quad (294)$$

**Definition of** $(\nu_1)^{(2)}, (\nu_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$ : (295)

By $(\nu_1)^{(2)} > 0$, $(\nu_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0$, $(u_2)^{(2)} < 0$ the roots (296)

of the equations $(a_{17})^{(2)}(\nu^{(2)})^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ (297)

and $(b_{14})^{(2)}(u^{(2)})^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and (298)

**Definition of** $(\bar{\nu}_1)^{(2)}, , (\bar{\nu}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$ : (299)

By $(\bar{\nu}_1)^{(2)} > 0$, $(\bar{\nu}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0$, $(\bar{u}_2)^{(2)} < 0$ the (300)

roots of the equations $(a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 301

and $(b_{17})^{(2)}\left(u^{(2)}\right)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 302

**Definition of** $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ :- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, ***if*** $(u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$, ***if*** $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)}$, ***if*** $(\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)}t}$

$(p_i)^{(2)}$ is defined by equation

$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)}t}$ 311

$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)}t}\right] + G_{18}^0 e^{-(S_2)^{(2)}t} \le G_{18}(t) \le\right.$ 312

$\left.\frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)}t} - e^{-(a'_{18})^{(2)}t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)}t}\right)$

$\boxed{T_{16}^0 e^{(R_1)^{(2)}t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}}$ 313

$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)}t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}$ 314

$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)}t} - e^{-(b'_{18})^{(2)}t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)}t} \le T_{18}(t) \le$ 315

$$\frac{(a_{18})^{(2)}T_{16}^{0}}{(\mu_2)^{(2)}((R_1)^{(2)}+(r_{16})^{(2)}+(R_2)^{(2)})}\left[e^{((R_1)^{(2)}+(r_{16})^{(2)})t}-e^{-(R_2)^{(2)}t}\right]+T_{18}^{0}e^{-(R_2)^{(2)}t}$$

**Definition of** $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$$

$$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)}$$ 318

$$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$$

**Behavior of the solutions** 319

**Theorem 3:** If we denote and define

**Definition of** $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ :

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}((G_{23}), t) \le -(\tau_1)^{(3)}$$

**Definition of** $(v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$ : 320

By $(v_1)^{(3)} > 0$, $(v_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations
$(a_{21})^{(3)}(v^{(3)})^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$
and $(b_{21})^{(3)}(u^{(3)})^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{v}_1)^{(3)} > 0$, $(\bar{v}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}(v^{(3)})^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}(u^{(3)})^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

**Definition of** $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ :- 321

If we define $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ by
$(m_2)^{(3)} = (v_0)^{(3)}, (m_1)^{(3)} = (v_1)^{(3)}$, ***if*** $(v_0)^{(3)} < (v_1)^{(3)}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (\bar{v}_1)^{(3)}$, ***if*** $(v_1)^{(3)} < (v_0)^{(3)} < (\bar{v}_1)^{(3)}$,

and $\boxed{(v_0)^{(3)} = \frac{G_{20}^{0}}{G_{21}^{0}}}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_0)^{(3)}$, ***if*** $(\bar{v}_1)^{(3)} < (v_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)},\ \boldsymbol{if}\ (u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}, \boldsymbol{if}\ (u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)},$ and $\boxed{(u_0)^{(3)} = \frac{T_{20}^0}{T_{21}^0}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)},\ \boldsymbol{if}\ (\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G_{20}^0 e^{(S_1)^{(3)} t}$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)} t} \quad 323$$

$$\left(\frac{(a_{22})^{(3)} G_{20}^0}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\left[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)} t}\right] + G_{22}^0 e^{-(S_2)^{(3)} t} \le G_{22}(t) \le \right. \quad 324$$

$$\left. \frac{(a_{22})^{(3)} G_{20}^0}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}\left[e^{(S_1)^{(3)} t} - e^{-(a'_{22})^{(3)} t}\right] + G_{22}^0 e^{-(a'_{22})^{(3)} t}\right)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t}} \quad 325$$

$$\frac{1}{(\mu_1)^{(3)}} T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}} T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)} T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)} - (b'_{22})^{(3)})}\left[e^{(R_1)^{(3)} t} - e^{-(b'_{22})^{(3)} t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)} t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)} T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)} + (r_{20})^{(3)} + (R_2)^{(3)})}\left[e^{((R_1)^{(3)} + (r_{20})^{(3)})t} - e^{-(R_2)^{(3)} t}\right] + T_{22}^0 e^{-(R_2)^{(3)} t}$$

**Definition of** $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

**Behavior of the solutions of equation**

**Theorem:** If we denote and define

**Definition of** $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ :

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$

$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}((G_{27}), t) - (b''_{25})^{(4)}((G_{27}), t) \le -(\tau_1)^{(4)}$

**Definition of** $(v_1)^{(4)}, (v_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, v^{(4)}, u^{(4)}$ : 329

By $(v_1)^{(4)} > 0$ , $(v_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$ , $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_1)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

**Definition of** $(\bar{v}_1)^{(4)},, (\bar{v}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$ : 330

By $(\bar{v}_1)^{(4)} > 0$ , $(\bar{v}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$ , $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

**Definition of** $(m_1)^{(4)}$ , $(m_2)^{(4)}$ , $(\mu_1)^{(4)}, (\mu_2)^{(4)}, (\nu_0)^{(4)}$ :-

If we define $(m_1)^{(4)}$ , $(m_2)^{(4)}$ , $(\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (v_0)^{(4)}, (m_1)^{(4)} = (v_1)^{(4)}$, ***if*** $(v_0)^{(4)} < (v_1)^{(4)}$

$(m_2)^{(4)} = (v_1)^{(4)}, (m_1)^{(4)} = (\bar{v}_1)^{(4)}$ , ***if*** $(v_4)^{(4)} < (v_0)^{(4)} < (\bar{v}_1)^{(4)}$,
and $\boxed{(v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (v_4)^{(4)}, (m_1)^{(4)} = (v_0)^{(4)}$, ***if*** $(\bar{v}_4)^{(4)} < (v_0)^{(4)}$

and analogously

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$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)}$, ***if*** $(u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}$ , ***if*** $(u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)}$,
and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}$, ***if*** $(\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

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$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)}t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)}t}$$

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$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)}t}\right] + G_{26}^0 e^{-(S_2)^{(4)}t} \le G_{26}(t) \le\right.$$ 334

*(a26)4G240(m2)4(S1)4−(a26′)4e(S1)4t−e−(a26′)4t+ G260e−(a26′)4t*

$$\boxed{T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}}T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}}T_{24}^0 e^{((R_1)^{(4)}+(r_{24})^{(4)})t} \quad 335$$

$$\frac{(b_{26})^{(4)}T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)}-(b'_{26})^{(4)})}\left[e^{(R_1)^{(4)}t} - e^{-(b'_{26})^{(4)}t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)}t} \le T_{26}(t) \le \quad 336$$

$$\frac{(a_{26})^{(4)}T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)}+(r_{24})^{(4)}+(R_2)^{(4)})}\left[e^{((R_1)^{(4)}+(r_{24})^{(4)})t} - e^{-(R_2)^{(4)}t}\right] + T_{26}^0 e^{-(R_2)^{(4)}t}$$

**Definition of** $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

**Behavior of the solutions of equation** 338

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ :

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

**Definition of** $(v_1)^{(5)}, (v_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, v^{(5)}, u^{(5)}$ : 339

By $(v_1)^{(5)} > 0$, $(v_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$, $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(v^{(5)}\big)^2 + (\sigma_1)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

**Definition of** $(\bar{v}_1)^{(5)}, , (\bar{v}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$ : 340

By $(\bar{v}_1)^{(5)} > 0$, $(\bar{v}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$, $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(v^{(5)}\big)^2 + (\sigma_2)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

**Definition of** $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (v_0)^{(5)}$ :-

If we define $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (v_0)^{(5)}, (m_1)^{(5)} = (v_1)^{(5)}$, ***if*** $(v_0)^{(5)} < (v_1)^{(5)}$

$(m_2)^{(5)} = (v_1)^{(5)}, (m_1)^{(5)} = (\bar{v}_1)^{(5)}$, ***if*** $(v_1)^{(5)} < (v_0)^{(5)} < (\bar{v}_1)^{(5)}$,
and $\boxed{(v_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)},\ \boldsymbol{if}\ (u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}, \boldsymbol{if}\ (u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)},$

and $\boxed{(u_0)^{(5)} = \frac{T_{28}^0}{T_{29}^0}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}, \boldsymbol{if}\ (\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{28}(t) \le G_{28}^0 e^{(S_1)^{(5)} t}$

where $(p_i)^{(5)}$ is defined by equation

$\frac{1}{(m_5)^{(5)}} G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}} G_{28}^0 e^{(S_1)^{(5)} t}$ 343

$\left(\frac{(a_{30})^{(5)} G_{28}^0}{(m_1)^{(5)}((S_1)^{(5)} - (p_{28})^{(5)} - (S_2)^{(5)})}\left[e^{((S_1)^{(5)} - (p_{28})^{(5)})t} - e^{-(S_2)^{(5)} t}\right] + G_{30}^0 e^{-(S_2)^{(5)} t} \le G_{30}(t) \le\right.$ 344

*(a30)5G280(m2)5(S1)5−(a30′)5e(S1)5t−e−(a30′)5t+ G300e−(a30′)5t*

$\boxed{T_{28}^0 e^{(R_1)^{(5)} t} \le T_{28}(t) \le T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}}$ 345

$\frac{1}{(\mu_1)^{(5)}} T_{28}^0 e^{(R_1)^{(5)} t} \le T_{28}(t) \le \frac{1}{(\mu_2)^{(5)}} T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}$ 346

$\frac{(b_{30})^{(5)} T_{28}^0}{(\mu_1)^{(5)}((R_1)^{(5)} - (b'_{30})^{(5)})}\left[e^{(R_1)^{(5)} t} - e^{-(b'_{30})^{(5)} t}\right] + T_{30}^0 e^{-(b'_{30})^{(5)} t} \le T_{30}(t) \le$ 347

$\frac{(a_{30})^{(5)} T_{28}^0}{(\mu_2)^{(5)}((R_1)^{(5)} + (r_{28})^{(5)} + (R_2)^{(5)})}\left[e^{((R_1)^{(5)} + (r_{28})^{(5)})t} - e^{-(R_2)^{(5)} t}\right] + T_{30}^0 e^{-(R_2)^{(5)} t}$

**Definition of** $(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$:- 348

Where $(S_1)^{(5)} = (a_{28})^{(5)}(m_2)^{(5)} - (a'_{28})^{(5)}$

$(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}$

$(R_1)^{(5)} = (b_{28})^{(5)}(\mu_2)^{(5)} - (b'_{28})^{(5)}$

$(R_2)^{(5)} = (b'_{30})^{(5)} - (r_{30})^{(5)}$

**Behavior of the solutions of equation** 349

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ :

$(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)} \le -(a'_{32})^{(6)} + (a'_{33})^{(6)} - (a''_{32})^{(6)}(T_{33}, t) + (a''_{33})^{(6)}(T_{33}, t) \le -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)} \le -(b'_{32})^{(6)} + (b'_{33})^{(6)} - (b''_{32})^{(6)}\big((G_{35}), t\big) - (b''_{33})^{(6)}\big((G_{35}), t\big) \le -(\tau_1)^{(6)}$$

**Definition of** $(\nu_1)^{(6)}, (\nu_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, \nu^{(6)}, u^{(6)}$ : 350

By $(\nu_1)^{(6)} > 0$, $(\nu_2)^{(6)} < 0$ and respectively $(u_1)^{(6)} > 0$, $(u_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(6)},, (\bar{\nu}_2)^{(6)}, (\bar{u}_1)^{(6)}, (\bar{u}_2)^{(6)}$ : 351

By $(\bar{\nu}_1)^{(6)} > 0$, $(\bar{\nu}_2)^{(6)} < 0$ and respectively $(\bar{u}_1)^{(6)} > 0$, $(\bar{u}_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$

**Definition of** $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}, (\nu_0)^{(6)}$ :-

If we define $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}$ by

$$(m_2)^{(6)} = (\nu_0)^{(6)}, (m_1)^{(6)} = (\nu_1)^{(6)}, \ \boldsymbol{if}\ (\nu_0)^{(6)} < (\nu_1)^{(6)}$$

$$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}, \boldsymbol{if}\ (\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)},$$

and $\boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}}$

$$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$$

and analogously 352

$$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}, \ \boldsymbol{if}\ (u_0)^{(6)} < (u_1)^{(6)}$$

$$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}, \boldsymbol{if}\ (u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)},$$

and $\boxed{(u_0)^{(6)} = \frac{T^0_{32}}{T^0_{33}}}$

$$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}, \boldsymbol{if}\ (\bar{u}_1)^{(6)} < (u_0)^{(6)} \text{ where } (u_1)^{(6)}, (\bar{u}_1)^{(6)}$$

Then the solution of global equations satisfies the inequalities 353

$$G^0_{32}e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{32}(t) \le G^0_{32}e^{(S_1)^{(6)}t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}}G^0_{32}e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{33}(t) \le \frac{1}{(m_2)^{(6)}}G^0_{32}e^{(S_1)^{(6)}t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)}G^0_{32}}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})}\left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)}t}\right] + G^0_{34}e^{-(S_2)^{(6)}t} \le G_{34}(t) \le \right. \quad 355$$

$$\frac{(a_{34})^{(6)}G_{32}^{0}}{(m_2)^{(6)}((S_1)^{(6)}-(a'_{34})^{(6)})}\left[e^{(S_1)^{(6)}t}-e^{-(a'_{34})^{(6)}t}\right]+\ G_{34}^{0}e^{-(a'_{34})^{(6)}t}\Big)$$

$$\boxed{T_{32}^{0}e^{(R_1)^{(6)}t}\le T_{32}(t)\le T_{32}^{0}e^{((R_1)^{(6)}+(r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}}T_{32}^{0}e^{(R_1)^{(6)}t}\le T_{32}(t)\le\frac{1}{(\mu_2)^{(6)}}T_{32}^{0}e^{((R_1)^{(6)}+(r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)}T_{32}^{0}}{(\mu_1)^{(6)}((R_1)^{(6)}-(b'_{34})^{(6)})}\left[e^{(R_1)^{(6)}t}-e^{-(b'_{34})^{(6)}t}\right]+T_{34}^{0}e^{-(b'_{34})^{(6)}t}\le T_{34}(t)\le \quad 358$$

$$\frac{(a_{34})^{(6)}T_{32}^{0}}{(\mu_2)^{(6)}((R_1)^{(6)}+(r_{32})^{(6)}+(R_2)^{(6)})}\left[e^{((R_1)^{(6)}+(r_{32})^{(6)})t}-e^{-(R_2)^{(6)}t}\right]+T_{34}^{0}e^{-(R_2)^{(6)}t}$$

**Definition of** $(S_1)^{(6)},(S_2)^{(6)},(R_1)^{(6)},(R_2)^{(6)}$:- 359

Where $(S_1)^{(6)}=(a_{32})^{(6)}(m_2)^{(6)}-(a'_{32})^{(6)}$

$(S_2)^{(6)}=(a_{34})^{(6)}-(p_{34})^{(6)}$

$(R_1)^{(6)}=(b_{32})^{(6)}(\mu_2)^{(6)}-(b'_{32})^{(6)}$

$(R_2)^{(6)}=(b'_{34})^{(6)}-(r_{34})^{(6)}$

**Behavior of the solutions of equation**

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(7)},(\sigma_2)^{(7)},(\tau_1)^{(7)},(\tau_2)^{(7)}$:

$(\sigma_1)^{(7)},(\sigma_2)^{(7)},(\tau_1)^{(7)},(\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)}\le-(a'_{36})^{(7)}+(a'_{37})^{(7)}-(a''_{36})^{(7)}(T_{37},t)+(a''_{37})^{(7)}(T_{37},t)\le-(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)}\le-(b'_{36})^{(7)}+(b'_{37})^{(7)}-(b''_{36})^{(7)}\big((G_{39}),t\big)-(b''_{37})^{(7)}\big((G_{39}),t\big)\le-(\tau_1)^{(7)}$$

**Definition of** $(v_1)^{(7)},(v_2)^{(7)},(u_1)^{(7)},(u_2)^{(7)},v^{(7)},u^{(7)}$: 361

By $(v_1)^{(7)}>0,(v_2)^{(7)}<0$ and respectively $(u_1)^{(7)}>0,(u_2)^{(7)}<0$ the roots of the equations $(a_{37})^{(7)}\big(v^{(7)}\big)^2+(\sigma_1)^{(7)}v^{(7)}-(a_{36})^{(7)}=0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2+(\tau_1)^{(7)}u^{(7)}-(b_{36})^{(7)}=0$ and

**Definition of** $(\bar{v}_1)^{(7)},,(\bar{v}_2)^{(7)},(\bar{u}_1)^{(7)},(\bar{u}_2)^{(7)}$: 362

By $(\bar{v}_1)^{(7)}>0,(\bar{v}_2)^{(7)}<0$ and respectively $(\bar{u}_1)^{(7)}>0,(\bar{u}_2)^{(7)}<0$ the roots of the equations $(a_{37})^{(7)}\big(v^{(7)}\big)^2+(\sigma_2)^{(7)}v^{(7)}-(a_{36})^{(7)}=0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2+(\tau_2)^{(7)}u^{(7)}-(b_{36})^{(7)}=0$

**Definition of** $(m_1)^{(7)},(m_2)^{(7)},(\mu_1)^{(7)},(\mu_2)^{(7)},(v_0)^{(7)}$:-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$$(m_2)^{(7)} = (\nu_0)^{(7)}, (m_1)^{(7)} = (\nu_1)^{(7)}, \ \boldsymbol{if}\ (\nu_0)^{(7)} < (\nu_1)^{(7)}$$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\bar{\nu}_1)^{(7)}, \boldsymbol{if}\ (\nu_1)^{(7)} < (\nu_0)^{(7)} < (\bar{\nu}_1)^{(7)},$$

and $\boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\nu_0)^{(7)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(7)} < (\nu_0)^{(7)}$$

and analogously 363

$$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}, \ \boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$$

and $\boxed{(u_0)^{(7)} = \frac{T_{36}^0}{T_{37}^0}}$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)} \text{ where } (u_1)^{(7)}, (\bar{u}_1)^{(7)}$$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^0 e^{(S_1)^{(7)} t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)} t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)}((S_1)^{(7)} - (p_{36})^{(7)} - (S_2)^{(7)})}\left[e^{((S_1)^{(7)} - (p_{36})^{(7)})t} - e^{-(S_2)^{(7)} t}\right] + G_{38}^0 e^{-(S_2)^{(7)} t} \le G_{38}(t) \le \right. \quad 366$$

$$\left. \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)}((S_1)^{(7)} - (a'_{38})^{(7)})}\left[e^{(S_1)^{(7)} t} - e^{-(a'_{38})^{(7)} t}\right] + G_{38}^0 e^{-(a'_{38})^{(7)} t}\right)$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t} \quad 368$$

$$\frac{(b_{38})^{(7)} T_{36}^0}{(\mu_1)^{(7)}((R_1)^{(7)} - (b'_{38})^{(7)})}\left[e^{(R_1)^{(7)} t} - e^{-(b'_{38})^{(7)} t}\right] + T_{38}^0 e^{-(b'_{38})^{(7)} t} \le T_{38}(t) \le \quad 369$$

$$\frac{(a_{38})^{(7)} T_{36}^0}{(\mu_2)^{(7)}((R_1)^{(7)} + (r_{36})^{(7)} + (R_2)^{(7)})}\left[e^{((R_1)^{(7)} + (r_{36})^{(7)})t} - e^{-(R_2)^{(7)} t}\right] + T_{38}^0 e^{-(R_2)^{(7)} t}$$

**Definition of** $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$

$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$

$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$

**Behavior of the solutions of equation** 371

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ :

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$-(\sigma_2)^{(8)} \le -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \le -(\sigma_1)^{(8)}$

$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\big((G_{43}), t\big) - (b''_{41})^{(8)}\big((G_{43}), t\big) \le -(\tau_1)^{(8)}$

**Definition of** $(\nu_1)^{(8)}, (\nu_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, \nu^{(8)}, u^{(8)}$ : 372

By $(\nu_1)^{(8)} > 0, (\nu_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0, (u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(8)}, , (\bar{\nu}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$ :

By $(\bar{\nu}_1)^{(8)} > 0, (\bar{\nu}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0, (\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

**Definition of** $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}, (\nu_0)^{(8)}$ :-

If we define $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$(m_2)^{(8)} = (\nu_0)^{(8)}, (m_1)^{(8)} = (\nu_1)^{(8)}$, ***if*** $(\nu_0)^{(8)} < (\nu_1)^{(8)}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\bar{\nu}_1)^{(8)}$, ***if*** $(\nu_1)^{(8)} < (\nu_0)^{(8)} < (\bar{\nu}_1)^{(8)}$,
and $\boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\nu_0)^{(8)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(8)} < (\nu_0)^{(8)}$

and analogously 374

$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)},\ \boldsymbol{if}\ (u_0)^{(8)} < (u_1)^{(8)}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}, \boldsymbol{if}\ (u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)},$

and $\boxed{(u_0)^{(8)} = \frac{T_{40}^0}{T_{41}^0}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}, \boldsymbol{if}\ (\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^0 e^{(S_1)^{(8)} t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}} G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}} G_{40}^0 e^{(S_1)^{(8)} t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)} G_{40}^0}{(m_1)^{(8)}((S_1)^{(8)} - (p_{40})^{(8)} - (S_2)^{(8)})}\left[e^{((S_1)^{(8)} - (p_{40})^{(8)})t} - e^{-(S_2)^{(8)} t}\right] + G_{42}^0 e^{-(S_2)^{(8)} t} \le G_{42}(t) \le \right. \quad 377$$

$$\left. \frac{(a_{42})^{(8)} G_{40}^0}{(m_2)^{(8)}((S_1)^{(8)} - (a'_{42})^{(8)})}\left[e^{(S_1)^{(8)} t} - e^{-(a'_{42})^{(8)} t}\right] + G_{42}^0 e^{-(a'_{42})^{(8)} t}\right)$$

$$\boxed{T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}} T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le \frac{1}{(\mu_2)^{(8)}} T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)} T_{40}^0}{(\mu_1)^{(8)}((R_1)^{(8)} - (b'_{42})^{(8)})}\left[e^{(R_1)^{(8)} t} - e^{-(b'_{42})^{(8)} t}\right] + T_{42}^0 e^{-(b'_{42})^{(8)} t} \le T_{42}(t) \le \quad 380$$

$$\frac{(a_{42})^{(8)} T_{40}^0}{(\mu_2)^{(8)}((R_1)^{(8)} + (r_{40})^{(8)} + (R_2)^{(8)})}\left[e^{((R_1)^{(8)} + (r_{40})^{(8)})t} - e^{-(R_2)^{(8)} t}\right] + T_{42}^0 e^{-(R_2)^{(8)} t}$$

**Definition of** $(S_1)^{(8)}, (S_2)^{(8)}, (R_1)^{(8)}, (R_2)^{(8)}$:- 381

Where $(S_1)^{(8)} = (a_{40})^{(8)}(m_2)^{(8)} - (a'_{40})^{(8)}$

$(S_2)^{(8)} = (a_{42})^{(8)} - (p_{42})^{(8)}$

$(R_1)^{(8)} = (b_{40})^{(8)}(\mu_2)^{(8)} - (b'_{40})^{(8)}$

$(R_2)^{(8)} = (b'_{42})^{(8)} - (r_{42})^{(8)}$

**Behavior of the solutions of equation 37 to 92** 382

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ :

$(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ four constants satisfying

$$-(\sigma_2)^{(9)} \le -(a'_{44})^{(9)} + (a'_{45})^{(9)} - (a''_{44})^{(9)}(T_{45}, t) + (a''_{45})^{(9)}(T_{45}, t) \le -(\sigma_1)^{(9)}$$

$$-(\tau_2)^{(9)} \le -(b'_{44})^{(9)} + (b'_{45})^{(9)} - (b''_{44})^{(9)}\big((G_{47}), t\big) - (b''_{45})^{(9)}\big((G_{47}), t\big) \le -(\tau_1)^{(9)}$$

**Definition of** $(\nu_1)^{(9)}, (\nu_2)^{(9)}, (u_1)^{(9)}, (u_2)^{(9)}, \nu^{(9)}, u^{(9)}$ :

By $(\nu_1)^{(9)} > 0$ , $(\nu_2)^{(9)} < 0$ and respectively $(u_1)^{(9)} > 0$ , $(u_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_1)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(9)}, , (\bar{\nu}_2)^{(9)}, (\bar{u}_1)^{(9)}, (\bar{u}_2)^{(9)}$ :

By $(\bar{\nu}_1)^{(9)} > 0$ , $(\bar{\nu}_2)^{(9)} < 0$ and respectively $(\bar{u}_1)^{(9)} > 0$ , $(\bar{u}_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_2)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$

**Definition of** $(m_1)^{(9)}, (m_2)^{(9)}, (\mu_1)^{(9)}, (\mu_2)^{(9)}, (\nu_0)^{(9)}$ :-

If we define $(m_1)^{(9)}, (m_2)^{(9)}, (\mu_1)^{(9)}, (\mu_2)^{(9)}$ by

$$(m_2)^{(9)} = (\nu_0)^{(9)}, (m_1)^{(9)} = (\nu_1)^{(9)}, \ \boldsymbol{if}\ (\nu_0)^{(9)} < (\nu_1)^{(9)}$$

$$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\bar{\nu}_1)^{(9)}, \boldsymbol{if}\ (\nu_1)^{(9)} < (\nu_0)^{(9)} < (\bar{\nu}_1)^{(9)},$$

and $\boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$

$$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\nu_0)^{(9)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(9)} < (\nu_0)^{(9)}$$

and analogously

$$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)}, \ \boldsymbol{if}\ (u_0)^{(9)} < (u_1)^{(9)}$$

$$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}, \boldsymbol{if}\ (u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)},$$

and $\boxed{(u_0)^{(9)} = \frac{T^0_{44}}{T^0_{45}}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}, \boldsymbol{if}\ (\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G^0_{44} e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{44}(t) \le G^0_{44} e^{(S_1)^{(9)} t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}} G_{44}^0 e^{\left((S_1)^{(9)}-(p_{44})^{(9)}\right)t} \leq G_{45}(t) \leq \frac{1}{(m_2)^{(9)}} G_{44}^0 e^{(S_1)^{(9)}t}$$

$$\left(\frac{(a_{46})^{(9)}G_{44}^0}{(m_1)^{(9)}\left((S_1)^{(9)}-(p_{44})^{(9)}-(S_2)^{(9)}\right)}\left[e^{\left((S_1)^{(9)}-(p_{44})^{(9)}\right)t} - e^{-(S_2)^{(9)}t}\right] + G_{46}^0 e^{-(S_2)^{(9)}t} \leq G_{46}(t) \leq \right.$$

$$\left.\frac{(a_{46})^{(9)}G_{44}^0}{(m_2)^{(9)}\left((S_1)^{(9)}-(a'_{46})^{(9)}\right)}\left[e^{(S_1)^{(9)}t} - e^{-(a'_{46})^{(9)}t}\right] + G_{46}^0 e^{-(a'_{46})^{(9)}t}\right)$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)}t} \leq T_{44}(t) \leq T_{44}^0 e^{\left((R_1)^{(9)}+(r_{44})^{(9)}\right)t}}$$

$$\frac{1}{(\mu_1)^{(9)}} T_{44}^0 e^{(R_1)^{(9)}t} \leq T_{44}(t) \leq \frac{1}{(\mu_2)^{(9)}} T_{44}^0 e^{\left((R_1)^{(9)}+(r_{44})^{(9)}\right)t}$$

$$\frac{(b_{46})^{(9)}T_{44}^0}{(\mu_1)^{(9)}\left((R_1)^{(9)}-(b'_{46})^{(9)}\right)}\left[e^{(R_1)^{(9)}t} - e^{-(b'_{46})^{(9)}t}\right] + T_{46}^0 e^{-(b'_{46})^{(9)}t} \leq T_{46}(t) \leq$$

$$\frac{(a_{46})^{(9)}T_{44}^0}{(\mu_2)^{(9)}\left((R_1)^{(9)}+(r_{44})^{(9)}+(R_2)^{(9)}\right)}\left[e^{\left((R_1)^{(9)}+(r_{44})^{(9)}\right)t} - e^{-(R_2)^{(9)}t}\right] + T_{46}^0 e^{-(R_2)^{(9)}t}$$

**Definition of** $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$

**Proof :** From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right) - (a''_{14})^{(1)}(T_{14}, t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

**Definition of** $v^{(1)}$ :- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)}\right) \leq \frac{dv^{(1)}}{dt} \leq -\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)}\right)$$

From which one obtains

**Definition of** $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$ :-

For $0 < \boxed{(v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}} < (v_1)^{(1)} < (\bar{v}_1)^{(1)}$

$$v^{(1)}(t) \geq \frac{(v_1)^{(1)}+(C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(v_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(v_2)^{(1)}}}$$

it follows $(\nu_0)^{(1)} \leq \nu^{(1)}(t) \leq (\nu_1)^{(1)}$

In the same manner , we get 384

$$\nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{\nu}_1)^{(1)}-(\nu_0)^{(1)}}{(\nu_0)^{(1)}-(\bar{\nu}_2)^{(1)}}}$$

From which we deduce $(\nu_0)^{(1)} \leq \nu^{(1)}(t) \leq (\bar{\nu}_1)^{(1)}$

If $0 < (\nu_1)^{(1)} < (\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0} < (\bar{\nu}_1)^{(1)}$ we find like in the previous case, 385

$$(\nu_1)^{(1)} \leq \frac{(\nu_1)^{(1)}+(C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}} \leq \nu^{(1)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\bar{\nu}_1)^{(1)}$$

If $0 < (\nu_1)^{(1)} \leq (\bar{\nu}_1)^{(1)} \leq \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$ , we obtain 386

$$(\nu_1)^{(1)} \leq \nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\nu_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(1)}(t)$ :-

$$(m_2)^{(1)} \leq \nu^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{\nu^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(1)}(t)$ :-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{13})^{(1)} = (a''_{14})^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)}G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)}T_{14}(t)$ This is an important

consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

**Proof** : From global equations we obtain 387

$$\frac{d\nu^{(2)}}{dt} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right) - (a''_{17})^{(2)}(T_{17}, t)\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

**Definition of** $\nu^{(2)}$ :- $\boxed{\nu^{(2)} = \frac{G_{16}}{G_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \le \frac{d\nu^{(2)}}{dt} \le -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

**Definition of** $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$ :-

For $0 < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \ge \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}}{1 + (C)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(C)^{(2)} = \frac{(\nu_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \le \nu^{(2)}(t) \le (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \le \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \le \nu^{(2)}(t) \le (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \le \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}}{1 + (C)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}} \le \nu^{(2)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \le (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \le (\bar{\nu}_1)^{(2)} \le (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$ , we obtain 394

$$(\nu_1)^{(2)} \le \nu^{(2)}(t) \le \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \le (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(2)}(t)$ :- 395

$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}$, $\boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$

In a completely analogous way, we obtain 396

**Definition of** $u^{(2)}(t)$ :-

$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}$, $\boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :** 397

If $(a''_{16})^{(2)} = (a''_{17})^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)} G_{17}(t)$

Analogously if $(b''_{16})^{(2)} = (b''_{17})^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)} T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

**Proof** : From global equations we obtain 398

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right) - (a''_{21})^{(3)}(T_{21}, t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

**Definition of** $\nu^{(3)}$ :- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right) \leq \frac{d\nu^{(3)}}{dt} \leq -\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right)$$

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From which one obtains

For $0 < (\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\nu_1)^{(3)} < (\bar{\nu}_1)^{(3)}$

$$\nu^{(3)}(t) \geq \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(\nu_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\nu_2)^{(3)}}}$$

it follows $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\nu_1)^{(3)}$

In the same manner , we get 401

$$\nu^{(3)}(t) \le \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{\nu}_1)^{(3)}-(\nu_0)^{(3)}}{(\nu_0)^{(3)}-(\bar{\nu}_2)^{(3)}}}$$

**Definition of** $(\bar{\nu}_1)^{(3)}$ :-

From which we deduce $(\nu_0)^{(3)} \le \nu^{(3)}(t) \le (\bar{\nu}_1)^{(3)}$

If $0 < (\nu_1)^{(3)} < (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (\bar{\nu}_1)^{(3)}$ we find like in the previous case, 402

$$(\nu_1)^{(3)} \le \frac{(\nu_1)^{(3)}+(C)^{(3)}(\nu_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}} \le \nu^{(3)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \le (\bar{\nu}_1)^{(3)}$$

If $0 < (\nu_1)^{(3)} \le (\bar{\nu}_1)^{(3)} \le (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}$ , we obtain 403

$$(\nu_1)^{(3)} \le \nu^{(3)}(t) \le \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \le (\nu_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(3)}(t)$ :-

$$(m_2)^{(3)} \le \nu^{(3)}(t) \le (m_1)^{(3)}, \quad \boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(3)}(t)$ :-

$$(\mu_2)^{(3)} \le u^{(3)}(t) \le (\mu_1)^{(3)}, \quad \boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{20}'')^{(3)} = (a_{21}'')^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)}G_{21}(t)$

Analogously if $(b_{20}'')^{(3)} = (b_{21}'')^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)}T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

**Proof** : From global equations we obtain 404

$$\frac{dv^{(4)}}{dt} = (a_{24})^{(4)} - \left((a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right) - (a''_{25})^{(4)}(T_{25},t)v^{(4)} - (a_{25})^{(4)}v^{(4)}$$

**Definition of** $v^{(4)}$ :- $\boxed{v^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)}\right) \le \frac{dv^{(4)}}{dt} \le -\left((a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_4)^{(4)}v^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$ :-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \ge \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \le \nu^{(4)}(t) \le (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \le \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \le \nu^{(4)}(t) \le (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \le \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}}{1 + (C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}} \le \nu^{(4)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \le (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \le (\bar{\nu}_1)^{(4)} \le \boxed{(\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}}}$ , we obtain 407

$$(\nu_1)^{(4)} \le \nu^{(4)}(t) \le \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \le (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(4)}(t)$ :-

$$(m_2)^{(4)} \le \nu^{(4)}(t) \le (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(4)}(t)$ :-

$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}$, $\boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{24})^{(4)} = (a''_{25})^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)} G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b''_{24})^{(4)} = (b''_{25})^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)} T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

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**Proof** : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a'_{28})^{(5)} - (a'_{29})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right) - (a''_{29})^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

**Definition of** $\nu^{(5)}$ :- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$ :-

For $0 < \boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \le \frac{(\nu_1)^{(5)}+(C)^{(5)}(\nu_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)}-(\nu_2)^{(5)}\right)t\right]}}{1+(C)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)}-(\nu_2)^{(5)}\right)t\right]}} \le \nu^{(5)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \le (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \le (\bar{\nu}_1)^{(5)} \le \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain 411

$$(\nu_1)^{(5)} \le \nu^{(5)}(t) \le \frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \le (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(5)}(t)$ :-

$(m_2)^{(5)} \le \nu^{(5)}(t) \le (m_1)^{(5)}$, $\boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$

In a completely analogous way, we obtain

**Definition of** $u^{(5)}(t)$ :-

$(\mu_2)^{(5)} \le u^{(5)}(t) \le (\mu_1)^{(5)}$, $\boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a_{28}'')^{(5)} = (a_{29}'')^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)}G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b_{28}'')^{(5)} = (b_{29}'')^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)}T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

**Proof** : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a_{32}')^{(6)} - (a_{33}')^{(6)} + (a_{32}'')^{(6)}(T_{33},t)\right) - (a_{33}'')^{(6)}(T_{33},t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

**Definition of** $\nu^{(6)}$ :- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \le \frac{d\nu^{(6)}}{dt} \le -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$ :-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \geq \frac{(\nu_1)^{(6)}+(C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_0)^{(6)}\right)t\right]}}{1+(C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)}-(\nu_0)^{(6)}}{(\nu_0)^{(6)}-(\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\nu_1)^{(6)}$

In the same manner , we get 413

$$\nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{\nu}_1)^{(6)}-(\nu_0)^{(6)}}{(\nu_0)^{(6)}-(\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\bar{\nu}_1)^{(6)}$

If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 414

$$(\nu_1)^{(6)} \leq \frac{(\nu_1)^{(6)}+(C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_2)^{(6)}\right)t\right]}}{1+(C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_2)^{(6)}\right)t\right]}} \leq \nu^{(6)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\bar{\nu}_1)^{(6)}$$

If $0 < (\nu_1)^{(6)} \leq (\bar{\nu}_1)^{(6)} \leq \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$ , we obtain 415

$$(\nu_1)^{(6)} \leq \nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(6)}(t)$ :-

$$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}, \quad \boxed{\nu^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(6)}(t)$ :-

$$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}, \quad \boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)}G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)}T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

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**Proof** : From global equations we obtain

$$\frac{dv^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right) - (a''_{37})^{(7)}(T_{37},t)v^{(7)} - (a_{37})^{(7)}v^{(7)}$$

**Definition of** $v^{(7)}$ :- $\boxed{v^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_2)^{(7)}v^{(7)} - (a_{36})^{(7)}\right) \le \frac{dv^{(7)}}{dt} \le -\left((a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_1)^{(7)}v^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

**Definition of** $(\bar{v}_1)^{(7)}, (v_0)^{(7)}$ :-

For $0 < \boxed{(v_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}} < (v_1)^{(7)} < (\bar{v}_1)^{(7)}$

$$v^{(7)}(t) \ge \frac{(v_1)^{(7)} + (C)^{(7)}(v_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_0)^{(7)}\right)t\right]}}{1 + (C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(v_1)^{(7)} - (v_0)^{(7)}}{(v_0)^{(7)} - (v_2)^{(7)}}}$$

it follows $(v_0)^{(7)} \le v^{(7)}(t) \le (v_1)^{(7)}$

In the same manner , we get 417

$$v^{(7)}(t) \le \frac{(\bar{v}_1)^{(7)} + (\bar{C})^{(7)}(\bar{v}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{v}_1)^{(7)} - (v_0)^{(7)}}{(v_0)^{(7)} - (\bar{v}_2)^{(7)}}}$$

From which we deduce $(v_0)^{(7)} \le v^{(7)}(t) \le (\bar{v}_1)^{(7)}$

If $0 < (v_1)^{(7)} < (v_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}} < (\bar{v}_1)^{(7)}$ we find like in the previous case, 418

$$(v_1)^{(7)} \le \frac{(v_1)^{(7)} + (C)^{(7)}(v_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_2)^{(7)}\right)t\right]}}{1 + (C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_2)^{(7)}\right)t\right]}} \le v^{(7)}(t) \le$$

$$\frac{(\bar{v}_1)^{(7)} + (\bar{C})^{(7)}(\bar{v}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}} \le (\bar{v}_1)^{(7)}$$

If $0 < (v_1)^{(7)} \le (\bar{v}_1)^{(7)} \le \boxed{(v_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$ , we obtain 419

$$(v_1)^{(7)} \le v^{(7)}(t) \le \frac{(\bar{v}_1)^{(7)} + (\bar{C})^{(7)}(\bar{v}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}} \le (v_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $v^{(7)}(t)$ :-

$$(m_2)^{(7)} \leq v^{(7)}(t) \leq (m_1)^{(7)}, \quad \boxed{v^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(7)}(t)$ :- 420

$$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}, \quad \boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $v^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)} G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)} T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

**Proof** : From global equations we obtain 421

$$\frac{dv^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right) - (a''_{41})^{(8)}(T_{41}, t) v^{(8)} - (a_{41})^{(8)} v^{(8)}$$

**Definition of** $v^{(8)}$ :- $\boxed{v^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_2)^{(8)} v^{(8)} - (a_{40})^{(8)}\right) \leq \frac{dv^{(8)}}{dt} \leq -\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_1)^{(8)} v^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$ :-

$$\text{For } 0 < \boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$$

$$v^{(8)}(t) \geq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)}-(\nu_0)^{(8)}}{(\nu_0)^{(8)}-(\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \leq \frac{(\nu_1)^{(8)}+(C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}}{1+(C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}} \leq \nu^{(8)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \leq (\bar{\nu}_1)^{(8)} \leq \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$ , we obtain 424

$$(\nu_1)^{(8)} \leq \nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(8)}(t)$ :-

$$(m_2)^{(8)} \leq \nu^{(8)}(t) \leq (m_1)^{(8)}, \quad \boxed{\nu^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(8)}(t)$ :-

$$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}, \quad \boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $\nu^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)}G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then $(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)}T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

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**Proof** : From 99,20,44,22,23,44 we obtain A

$$\frac{dv^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right) - (a''_{45})^{(9)}(T_{45},t)v^{(9)} - (a_{45})^{(9)}v^{(9)}$$

**Definition of** $v^{(9)}$ :- $\boxed{v^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)}\right) \le \frac{dv^{(9)}}{dt} \le -\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_1)^{(9)}v^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$ :-

For $0 < \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$

$$v^{(9)}(t) \ge \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \le v^{(9)}(t) \le (\nu_9)^{(9)}$

In the same manner , we get

$$v^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \le v^{(9)}(t) \le (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \le \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}}{1 + (C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}} \le v^{(9)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \le (\bar{\nu}_1)^{(9)} \le \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$ , we obtain

$$(\nu_1)^{(9)} \le v^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $v^{(9)}(t)$ :-

$$(m_2)^{(9)} \le v^{(9)}(t) \le (m_1)^{(9)}, \quad \boxed{v^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(9)}(t)$ :-

$(\mu_2)^{(9)} \leq u^{(9)}(t) \leq (\mu_1)^{(9)}$, $\boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)} G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then
$(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)} T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following 425

**Theorem :** If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on $t$ , and the conditions with the notations

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0$ ,

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$

$with\ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a''_i)^{(2)} and\ (b''_i)^{(2)}$ are independent on t , and the conditions with the notations 426

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$ 427

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 428

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$ , 429

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 430

$with\ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a''_i)^{(3)} and\ (b''_i)^{(3)}$ are independent on $t$ , and the conditions with the notations 431

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$ ,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

**Theorem :** If $(a_i'')^{(4)} and\ (b_i'')^{(4)}$ are independent on $t$ , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$ ,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(5)} and\ (b_i'')^{(5)}$ are independent on $t$ , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$ ,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

**Theorem** If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on $t$ , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$ ,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(7)} and\ (b_i'')^{(7)}$ are independent on $t$ , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$ ,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(8)} and\ (b_i'')^{(8)}$ are independent on $t$ , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$ ,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(9)} and\ (b_i'')^{(9)}$ are independent on $t$ , and the conditions (with the notations 436 A
45,46,27,28)

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$ ,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0$ 437

$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0$ 438

$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0$ 439

$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0$ 440

$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ]T_{14} = 0$ 441

$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0$ 442

has a unique positive solution , which is an equilibrium solution for the system

$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0$ 443

$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0$ 444

$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0$ 445

$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\ ]T_{16} = 0$ 446

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\,]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\,]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\,]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\,]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\,]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\big((G_{27})\big)\,]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\big((G_{27})\big)\,]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\big((G_{27})\big)\,]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})\,]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})\,]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})\,]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})\,]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})\,]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})\,]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0 \quad 479$$

$$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0 \quad 480$$

$$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0 \quad 481$$

$$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0 \quad 482$$

$$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0 \quad 483$$

$$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0 \quad 484$$

$$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0 \quad 484\text{A}$$

$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$

$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$

$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$

$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$

$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$

**Proof:** 485

(a) Indeed the first two equations have a nontrivial solution $G_{13}, G_{14}$ if

$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$

**Proof:** 486

(k) Indeed the first two equations have a nontrivial solution $G_{16}, G_{17}$ if

$\mathrm{F}(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$

**Proof:** 487

(a) Indeed the first two equations have a nontrivial solution $G_{20}, G_{21}$ if

$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$

**Proof:** 488

(a) Indeed the first two equations have a nontrivial solution $G_{24}, G_{25}$ if

$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$

**Proof:** 489

(a) Indeed the first two equations have a nontrivial solution $G_{28}, G_{29}$ if

$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$

**Proof:** 490

(a) Indeed the first two equations have a nontrivial solution $G_{32}, G_{33}$ if

$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) +$

$(a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$

**Proof:** 491

(a) Indeed the first two equations have a nontrivial solution $G_{36}, G_{37}$ if

$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$

**Proof:** 492

(a) Indeed the first two equations have a nontrivial solution $G_{40}, G_{41}$ if

$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$

**Proof:** 492 A

(a) Indeed the first two equations have a nontrivial solution $G_{44}, G_{45}$ if

$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$

**Definition and uniqueness of** $T^*_{14}$ :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique $T^*_{14}$ for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

**Definition and uniqueness of** $T^*_{17}$ :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique $T^*_{17}$ for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]}$$ 495

**Definition and uniqueness of** $T^*_{21}$ :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique $T^*_{21}$ for which $f(T^*_{21}) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

**Definition and uniqueness of** $T^*_{25}$ :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that

there exists a unique $T_{25}^{*}$ for which $f(T_{25}^{*})=0$. With this value , we obtain from the three first equations

$$G_{24}=\frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}=\frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T_{25}^{*})]}$$

**Definition and uniqueness of** $T_{29}^{*}$ :- 498

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(5)}(T_{29})$ being increasing, it follows that there exists a unique $T_{29}^{*}$ for which $f(T_{29}^{*})=0$. With this value , we obtain from the three first equations

$$G_{28}=\frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}=\frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T_{29}^{*})]}$$

**Definition and uniqueness of** $T_{33}^{*}$ :- 499

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(6)}(T_{33})$ being increasing, it follows that there exists a unique $T_{33}^{*}$ for which $f(T_{33}^{*})=0$. With this value , we obtain from the three first equations

$$G_{32}=\frac{(a_{32})^{(6)}G_{33}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^{*})]} \quad , \quad G_{34}=\frac{(a_{34})^{(6)}G_{33}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^{*})]}$$

**Definition and uniqueness of** $T_{37}^{*}$ :- 500

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(7)}(T_{37})$ being increasing, it follows that there exists a unique $T_{37}^{*}$ for which $f(T_{37}^{*})=0$. With this value , we obtain from the three first equations

$$G_{36}=\frac{(a_{36})^{(7)}G_{37}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^{*})]} \quad , \quad G_{38}=\frac{(a_{38})^{(7)}G_{37}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^{*})]}$$

**Definition and uniqueness of** $T_{41}^{*}$ :- 501

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique $T_{41}^{*}$ for which $f(T_{41}^{*})=0$. With this value , we obtain from the three first equations

$$G_{40}=\frac{(a_{40})^{(8)}G_{41}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T_{41}^{*})]} \quad , \quad G_{42}=\frac{(a_{42})^{(8)}G_{41}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T_{41}^{*})]}$$

**Definition and uniqueness of** $T_{45}^{*}$ :- 501 A

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(9)}(T_{45})$ being increasing, it follows that there exists a unique $T_{45}^{*}$ for which $f(T_{45}^{*})=0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)} G_{45}}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T^*_{45})]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)} G_{45}}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T^*_{45})]}$$

By the same argument, the equations admit solutions $G_{13}, G_{14}$ if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right] + (b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{14}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{14}$ such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions $G_{16}, G_{17}$ if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right] + (b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ 504
is a decreasing function in $G_{17}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{14}$ such that $\varphi((G_{19})^*) = 0$

By the same argument, the equations admit solutions $G_{20}, G_{21}$ if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right] + (b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{21}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{21}$ such that $\varphi((G_{23})^*) = 0$

By the same argument, the equations admit solutions $G_{24}, G_{25}$ if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right] + (b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{25}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{25}$ such that $\varphi((G_{27})^*) = 0$

By the same argument, the equations admit solutions $G_{28}, G_{29}$ if 507

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right] + (b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{29}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that

there exists a unique $G_{29}^{*}$ such that $\varphi((G_{31})^{*}) = 0$

By the same argument, the equations admit solutions $G_{32}, G_{33}$ if 508
$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$

$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right] + (b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{33}$ taking into account the hypothesis $\varphi(0) > 0\,, \varphi(\infty) < 0$ it follows that there exists a unique $G_{33}^{*}$ such that $\varphi(G_{35}{}^{*}) = 0$

By the same argument, the equations admit solutions $G_{36}, G_{37}$ if 509
$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$
$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right] + (b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{37}$ taking into account the hypothesis $\varphi(0) > 0\,, \varphi(\infty) < 0$ it follows that there exists a unique $G_{37}^{*}$ such that $\varphi(G_{39}{}^{*}) = 0$

By the same argument, the equations admit solutions $G_{40}, G_{41}$ if 510
$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$
$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right] + (b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{41}$ taking into account the hypothesis $\varphi(0) > 0\,, \varphi(\infty) < 0$ it follows that there exists a unique $G_{41}^{*}$ such that $\varphi(G_{43}{}^{*}) = 0$

By the same argument, the equations 92,93 admit solutions $G_{44}, G_{45}$ if
$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$

$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right] + (b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{45}$ taking into account the hypothesis $\varphi(0) > 0\,, \varphi(\infty) < 0$ it follows that there exists a unique $G_{45}^{*}$ such that $\varphi((G_{47})^{*}) = 0$

Finally we obtain the unique solution 511

$G_{14}^{*}$ given by $\varphi(G^{*}) = 0\,, T_{14}^{*}$ given by $f(T_{14}^{*}) = 0$ and

$$G_{13}^{*} = \frac{(a_{13})^{(1)}G_{14}^{*}}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^{*})]} \quad , \quad G_{15}^{*} = \frac{(a_{15})^{(1)}G_{14}^{*}}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^{*})]}$$

$$T_{13}^{*} = \frac{(b_{13})^{(1)}T_{14}^{*}}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^{*})]} \quad , \quad T_{15}^{*} = \frac{(b_{15})^{(1)}T_{14}^{*}}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^{*})]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

$G_{17}^{*}$ given by $\varphi((G_{19})^{*}) = 0$ , $T_{17}^{*}$ given by $f(T_{17}^{*}) = 0$ and 512

$$G_{16}^{*} = \frac{(a_{16})^{(2)} G_{17}^{*}}{[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}^{*})]} \quad , \quad G_{18}^{*} = \frac{(a_{18})^{(2)} G_{17}^{*}}{[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}^{*})]} \qquad 513$$

$$T_{16}^{*} = \frac{(b_{16})^{(2)} T_{17}^{*}}{[(b'_{16})^{(2)} - (b''_{16})^{(2)}((G_{19})^{*})]} \quad , \quad T_{18}^{*} = \frac{(b_{18})^{(2)} T_{17}^{*}}{[(b'_{18})^{(2)} - (b''_{18})^{(2)}((G_{19})^{*})]} \qquad 514$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

$G_{21}^{*}$ given by $\varphi((G_{23})^{*}) = 0$ , $T_{21}^{*}$ given by $f(T_{21}^{*}) = 0$ and

$$G_{20}^{*} = \frac{(a_{20})^{(3)} G_{21}^{*}}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^{*})]} \quad , \quad G_{22}^{*} = \frac{(a_{22})^{(3)} G_{21}^{*}}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^{*})]}$$

$$T_{20}^{*} = \frac{(b_{20})^{(3)} T_{21}^{*}}{[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}{}^{*})]} \quad , \quad T_{22}^{*} = \frac{(b_{22})^{(3)} T_{21}^{*}}{[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}{}^{*})]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

$G_{25}^{*}$ given by $\varphi(G_{27}) = 0$ , $T_{25}^{*}$ given by $f(T_{25}^{*}) = 0$ and

$$G_{24}^{*} = \frac{(a_{24})^{(4)} G_{25}^{*}}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}^{*} = \frac{(a_{26})^{(4)} G_{25}^{*}}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^{*})]}$$

$$T_{24}^{*} = \frac{(b_{24})^{(4)} T_{25}^{*}}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^{*})]} \quad , \quad T_{26}^{*} = \frac{(b_{26})^{(4)} T_{25}^{*}}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^{*})]} \qquad 517$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

$G_{29}^{*}$ given by $\varphi((G_{31})^{*}) = 0$ , $T_{29}^{*}$ given by $f(T_{29}^{*}) = 0$ and

$$G_{28}^{*} = \frac{(a_{28})^{(5)} G_{29}^{*}}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}^{*} = \frac{(a_{30})^{(5)} G_{29}^{*}}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^{*})]}$$

$$T_{28}^{*} = \frac{(b_{28})^{(5)} T_{29}^{*}}{[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31})^{*})]} \quad , \quad T_{30}^{*} = \frac{(b_{30})^{(5)} T_{29}^{*}}{[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31})^{*})]} \qquad 519$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

$G_{33}^{*}$ given by $\varphi((G_{35})^{*}) = 0$ , $T_{33}^{*}$ given by $f(T_{33}^{*}) = 0$ and

$$G_{32}^{*} = \frac{(a_{32})^{(6)} G_{33}^{*}}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^{*})]} \quad , \quad G_{34}^{*} = \frac{(a_{34})^{(6)} G_{33}^{*}}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^{*})]}$$

$$T_{32}^{*} = \frac{(b_{32})^{(6)} T_{33}^{*}}{[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35})^{*})]} \quad , \quad T_{34}^{*} = \frac{(b_{34})^{(6)} T_{33}^{*}}{[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35})^{*})]} \qquad 521$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

$G_{37}^*$ given by $\varphi((G_{39})^*) = 0$ , $T_{37}^*$ given by $f(T_{37}^*) = 0$ and

$$G_{36}^* = \frac{(a_{36})^{(7)} G_{37}^*}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^*)]} \quad , \quad G_{38}^* = \frac{(a_{38})^{(7)} G_{37}^*}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^*)]}$$

$$T_{36}^* = \frac{(b_{36})^{(7)} T_{37}^*}{[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39})^*)]} \quad , \quad T_{38}^* = \frac{(b_{38})^{(7)} T_{37}^*}{[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39})^*)]}$$

Finally we obtain the unique solution 523

$G_{41}^*$ given by $\varphi((G_{43})^*) = 0$ , $T_{41}^*$ given by $f(T_{41}^*) = 0$ and

$$G_{40}^* = \frac{(a_{40})^{(8)} G_{41}^*}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42}^* = \frac{(a_{42})^{(8)} G_{41}^*}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

$$T_{40}^* = \frac{(b_{40})^{(8)} T_{41}^*}{[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43})^*)]} \quad , \quad T_{42}^* = \frac{(b_{42})^{(8)} T_{41}^*}{[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43})^*)]}$$

Finally we obtain the unique solution of 89 to 99 523 A

$G_{45}^*$ given by $\varphi((G_{47})^*) = 0$ , $T_{45}^*$ given by $f(T_{45}^*) = 0$ and

$$G_{44}^* = \frac{(a_{44})^{(9)} G_{45}^*}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46}^* = \frac{(a_{46})^{(9)} G_{45}^*}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

$$T_{44}^* = \frac{(b_{44})^{(9)} T_{45}^*}{[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47})^*)]} \quad , \quad T_{46}^* = \frac{(b_{46})^{(9)} T_{45}^*}{[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47})^*)]}$$

## ASYMPTOTIC STABILITY ANALYSIS 524

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

**Proof:** Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a''_{14})^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \frac{\partial (b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a'_{13})^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)}G^*_{13}\mathbb{T}_{14} \quad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a'_{14})^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)}G^*_{14}\mathbb{T}_{14} \quad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a'_{15})^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)}G^*_{15}\mathbb{T}_{14} \quad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b'_{13})^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)}T^*_{13}\mathbb{G}_j\right) \quad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b'_{14})^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)}T^*_{14}\mathbb{G}_j\right) \quad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b'_{15})^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T^*_{15}\mathbb{G}_j\right) \quad 530$$

**ASYMPTOTIC STABILITY ANALYSIS** 531

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(2)}$ and $(b''_i)^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G^*_i + \mathbb{G}_i \quad , T_i = T^*_i + \mathbb{T}_i \quad 532$$

$$\frac{\partial(a''_{17})^{(2)}}{\partial T_{17}}(T^*_{17}) = (q_{17})^{(2)} \quad , \frac{\partial(b''_i)^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \quad 533$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a'_{16})^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G^*_{16}\mathbb{T}_{17} \quad 534$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a'_{17})^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G^*_{17}\mathbb{T}_{17} \quad 535$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a'_{18})^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G^*_{18}\mathbb{T}_{17} \quad 536$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b'_{16})^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T^*_{16}\mathbb{G}_j\right) \quad 537$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b'_{17})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}T^*_{17}\mathbb{G}_j\right) \quad 538$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b'_{18})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}T^*_{18}\mathbb{G}_j\right) \quad 539$$

**ASYMPTOTIC STABILITY ANALYSIS** 540

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(3)}$ *and* $(b''_i)^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{21}'')^{(3)}}{\partial T_{21}}(T_{21}^*) = (q_{21})^{(3)} \quad , \frac{\partial (b_i'')^{(3)}}{\partial G_j}((G_{23})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a_{20}')^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G_{20}^*\mathbb{T}_{21} \quad 541$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a_{21}')^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G_{21}^*\mathbb{T}_{21} \quad 542$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a_{22}')^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G_{22}^*\mathbb{T}_{21} \quad 543$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b_{20}')^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T_{20}^*\mathbb{G}_j\right) \quad 544$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b_{21}')^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T_{21}^*\mathbb{G}_j\right) \quad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b_{22}')^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T_{22}^*\mathbb{G}_j\right) \quad 546$$

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**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}$ *and* $(b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{25}'')^{(4)}}{\partial T_{25}}(T_{25}^*) = (q_{25})^{(4)} \quad , \frac{\partial (b_i'')^{(4)}}{\partial G_j}((G_{27})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\left((a_{24}')^{(4)} + (p_{24})^{(4)}\right)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G_{24}^*\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\left((a_{25}')^{(4)} + (p_{25})^{(4)}\right)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G_{25}^*\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\left((a_{26}')^{(4)} + (p_{26})^{(4)}\right)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G_{26}^*\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\left((b_{24}')^{(4)} - (r_{24})^{(4)}\right)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(24)(j)}T_{24}^*\mathbb{G}_j\right) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\left((b_{25}')^{(4)} - (r_{25})^{(4)}\right)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\left(s_{(25)(j)}T_{25}^*\mathbb{G}_j\right) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T^*_{26}\mathbb{G}_j\right) \quad 554$$

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**Theorem 5:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}\ and\ (b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 556

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{29})^{(5)}}{\partial T_{29}}(T^*_{29}) = (q_{29})^{(5)} \ , \ \frac{\partial(b_i'')^{(5)}}{\partial G_j}\left((G_{31})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a'_{28})^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G^*_{28}\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a'_{29})^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G^*_{29}\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a'_{30})^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G^*_{30}\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b'_{28})^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T^*_{28}\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b'_{29})^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T^*_{29}\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b'_{30})^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T^*_{30}\mathbb{G}_j\right) \quad 562$$

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**Theorem 6:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}\ and\ (b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{33})^{(6)}}{\partial T_{33}}(T^*_{33}) = (q_{33})^{(6)} \ , \ \frac{\partial(b_i'')^{(6)}}{\partial G_j}\left((G_{35})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a'_{32})^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G^*_{32}\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a'_{33})^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G^*_{33}\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a'_{34})^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G^*_{34}\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b'_{32})^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T^*_{32}\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b'_{33})^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T^*_{33}\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b'_{34})^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T^*_{34}\mathbb{G}_j\right) \quad 570$$

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**Theorem 7:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 572

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{37})^{(7)}}{\partial T_{37}}(T^*_{37}) = (q_{37})^{(7)} \quad , \frac{\partial(b_i'')^{(7)}}{\partial G_j}\left((G_{39})^{**}\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a'_{36})^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G^*_{36}\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a'_{37})^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G^*_{37}\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a'_{38})^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G^*_{38}\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b'_{36})^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T^*_{36}\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b'_{37})^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T^*_{37}\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b'_{38})^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T^*_{38}\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

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**Theorem 8:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{41}'')^{(8)}}{\partial T_{41}}(T_{41}^*) = (q_{41})^{(8)} \quad , \frac{\partial (b_i'')^{(8)}}{\partial G_j}\left( (G_{43})^* \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a_{40}')^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G_{40}^*\mathbb{T}_{41} \quad (581)$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a_{41}')^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G_{41}^*\mathbb{T}_{41} \quad (582)$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a_{42}')^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G_{42}^*\mathbb{T}_{41} \quad (583)$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b_{40}')^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T_{40}^*\mathbb{G}_j\right) \quad (584)$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b_{41}')^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T_{41}^*\mathbb{G}_j\right) \quad (585)$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b_{42}')^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T_{42}^*\mathbb{G}_j\right) \quad (586)$$

(586 A)

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**Theorem 9:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}$ *and* $(b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

**Proof:** Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{45}'')^{(9)}}{\partial T_{45}}(T_{45}^*) = (q_{45})^{(9)} \quad , \frac{\partial (b_i'')^{(9)}}{\partial G_j}\left( (G_{47})^* \right) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a_{44}')^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)}G_{44}^*\mathbb{T}_{45} \quad (586\,B)$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a_{45}')^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)}G_{45}^*\mathbb{T}_{45} \quad (586\,C)$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a_{46}')^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)}G_{46}^*\mathbb{T}_{45} \quad (586\,D)$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b_{44}')^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)}T_{44}^*\mathbb{G}_j\right) \quad (586\,E)$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b_{45}')^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)}T_{45}^*\mathbb{G}_j\right) \quad (586\,F)$$

$\frac{d\mathbb{T}_{46}}{dt} = -\left((b'_{46})^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T^*_{46}\mathbb{G}_j\right)$ 586

G

587

**The characteristic equation of this system is**

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(q_{13})^{(1)}G^*_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)}T^*_{14} + (b_{14})^{(1)}s_{(13),(14)}T^*_{14}\right)$$

$$+\left(\left((\lambda)^{(1)} + (a'_{14})^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)}G^*_{13} + (a_{13})^{(1)}(q_{14})^{(1)}G^*_{14}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)}T^*_{14} + (b_{14})^{(1)}s_{(13),(13)}T^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((b'_{13})^{(1)} + (b'_{14})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(15)}T^*_{14} + (b_{14})^{(1)}s_{(13),(15)}T^*_{13}\right)\} = 0$$

$+$

$$(\lambda)^{(2)} + (b'_{18})^{(2)} - (r_{18})^{(2)}\{\left((\lambda)^{(2)} + (a'_{18})^{(2)} + (p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^*_{17} + (a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^*_{17}\right)$$

$$+\left(\left((\lambda)^{(2)} + (a'_{17})^{(2)} + (p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^*_{16} + (a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}\right)$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2 + \left((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2 + \left((b'_{16})^{(2)} + (b'_{17})^{(2)} - (r_{16})^{(2)} + (r_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$+\left(\left((\lambda)^{(2)}\right)^2 + \left((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17} + (a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^{*}_{16}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\right)\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)\Big]$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(21)}T^{*}_{21}\right)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^{*}_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^{*}_{21}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(20)}T^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$+\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(22)}T^{*}_{20}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\right)\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)\Big]$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(25)}T^{*}_{25}\right)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^{*}_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^{*}_{25}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(24)}T^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$+\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(26)}T^{*}_{24}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\right)\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(29)}T^{*}_{29}\right)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^{*}_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^{*}_{29}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(28)}T^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$+\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(30)}T^{*}_{28}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\right)\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left(\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)\right]$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(33)}T^{*}_{33}\right)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^{*}_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^{*}_{33}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(32)}T^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$+\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(34)}T^{*}_{32}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\right)\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)\right]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(37)}T^{*}_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^{*}_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^{*}_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(36)}T^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$+\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(38)}T^{*}_{36}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\right)\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(41)}T^{*}_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^{*}_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^{*}_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(40)}T^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$+\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(42)}T^{*}_{40}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\right)\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(45)}T^{*}_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^{*}_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^{*}_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(44)}T^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$+\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(46)}T^{*}_{44}\right)\}=0$$

**And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.**

## SECTION THIRTEEN

## Charge Densities Embedded Within Molecular Structures And The Transient Developed Flow Of Electrical Charges

### INTRODUCTION—VARIABLES USED

**Dorian Aur, Mandar Jog, and Roman R. Poznanski, J. Integr Neurosci 10, 413 (2011) DOI: 10.1142/S0219635211002865: Computing by physical interaction in neurons**

(1) The main conceptual idea is that under the influence of (e&eb) electric fields, efficient computation

by interaction occurs between (e&eb) charge densities embedded within (eb) molecular structures and (e&eb) the transient developed flow of electrical charges.

**(2)** This process of computation underlying (e&eb) electrical interactions and molecular mechanisms at the subcellular level is (=) dissimilar from **spiking neuron mode**ls that are completely devoid of (e) physical interactions.

**(3)** Computation by interaction describes (eb) a more powerful continuous model of computation than the one that consists of (e) discrete steps as represented in Turing machines

## NOTATION

### Module One

The main conceptual idea is that under the influence of electric fields, efficient computation by interaction occurs between (e&eb) **charge densities embedded within molecular structures** and (e&eb) the transient developed flow of electrical charges

$G_{13}$ : Category one of charge densities embedded; **molecular structures**

$G_{14}$ : Category two of SAS

$G_{15}$ : Category three of SAS

$T_{13}$ : Category one of **molecular structures** ;charge densities embedded

$T_{14}$ : Category two of SAS

$T_{15}$ : Category three of SAS

### Module Two

$G_{16}$ : Category one of **charge densities embedded within molecular structures;** transient developed flow of electrical charges

$G_{17}$ : Category two of SAS

$G_{18}$ : Category three of SAS

$T_{16}$ : Category one of transient developed flow of electrical charges; **charge densities embedded within molecular structures**

$T_{17}$ : Category two of SAS

$T_{18}$ : Category three of SAS

### Module three

$G_{20}$ : Category one of electric fields; **charge densities embedded within molecular structures and transient developed flow of electrical charges**

$G_{21}$ : Category two of SAS

$G_{22}$ : Category three of SAS

$T_{20}$ : Category one of **charge densities embedded within molecular structures and transient developed flow of electrical charges;** electric fields

$T_{21}$ : Category two of SAS

$T_{22}$ : Category three of SAS

**Module four**

This **process of computation** underlying (e&eb) electrical interactions and molecular mechanisms at the subcellular level is (=) dissimilar from spiking neuron models that are completely devoid of (e) physical interactions

$G_{24}$ : Category one of **process of computation;** electrical interactions and molecular mechanisms at the subcellular level is (=) dissimilar from spiking neuron models that are completely devoid of (e) physical interactions

$G_{25}$ : Category two of SAS

$G_{26}$ : Category three of SAS

$T_{24}$ : Category one of electrical interactions and molecular mechanisms at the subcellular level is (=) dissimilar from spiking neuron models that are completely devoid of (e) physical interactions; **process of computation**

$T_{25}$ : Category two of SAS

$T_{26}$ : Category three of SAS

**Module five**

This process of computation underlying electrical interactions and molecular mechanisms at the subcellular level is (=) dissimilar from spiking neuron models that are completely devoid of (e) physical interactions

$G_{28}$ : Category one of process of computation underlying electrical interactions and molecular mechanisms at the subcellular level

$G_{29}$ : Category two of SAS

$G_{30}$ : Category three of SAS

$T_{28}$ : Category one of dissimilar from spiking neuron models that are completely devoid of (e) physical interactions

$T_{29}$ : Category two of SAS

$T_{30}$ : Category three of SAS

**Module six**

This process of computation underlying electrical interactions and molecular mechanisms at the subcellular level is dissimilar from spiking neuron models that are completely devoid of (e) physical interactions

$G_{32}$ : Category one of physical interactions

$G_{33}$ : Category two of SAS

$G_{34}$ : Category three of SAS

$T_{32}$ : Category one of process of computation underlying electrical interactions and molecular mechanisms at the subcellular level is dissimilar from spiking neuron models

$T_{33}$ : Category two of SAS

$T_{34}$ : Category three of SAS

**Module seven**

Computation by interaction describes (eb) a more powerful continuous model of computation than the one that consists of (e) discrete steps as represented in Turing machines

$G_{36}$ : Category one of Computation by interaction

$G_{37}$ : Category two of SAS

$G_{38}$ : Category three of SAS

$T_{36}$ : Category one of more powerful continuous model of computation than the one that consists of (e) discrete steps as represented in Turing machines

$T_{37}$ : Category two of SAS

$T_{38}$ : Category three of SAS

**Module eight**

Computation by interaction describes a more powerful continuous model of computation than the one that consists of (e) discrete steps as represented in Turing machines

$G_{40}$ : Category one of one that consists of discrete steps as represented in Turing machines

$G_{41}$ : Category two of SAS

$G_{42}$ : Category three of SAS

$T_{40}$ : Category one of Computation by interaction describes a more powerful continuous model of computation

$T_{41}$ : Category two of SAS

$T_{42}$ : Category three of SAS

**Module Nine**

$G_{44}$ : Category one of **spiking neuron models;** physical interaction

$G_{45}$ : Category two of SAS

$G_{46}$ : Category three of SAS

$T_{44}$ : Category one of physical interaction **; spiking neuron models**

$T_{45}$ : Category two of SAS

$T_{46}$ : Category three of SAS

**The Coefficients:**

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}$ , $(b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}$,
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}$, $(b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}\ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$ ,
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)}$,
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)}$,
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)}$,
are Dissipation coefficients

**Module Numbered One**

**The differential system of this model is now (Module Numbered one)**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \quad 1$$
$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad 2$$
$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15} \quad 3$$
$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13} \quad 4$$
$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14} \quad 5$$
$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)\right]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14}, t) =$ First augmentation factor
$-(b''_{13})^{(1)}(G, t) =$ First detritions factor

**Module Numbered Two**

**The differential system of this model is now ( Module numbered two)**

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right]G_{16} \quad 7$$
$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t)\right]G_{17} \quad 8$$
$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t)\right]G_{18} \quad 9$$

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}((G_{19}),t)\right]T_{16} \quad 10$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}((G_{19}),t)\right]T_{17} \quad 11$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}((G_{19}),t)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17},t) =$ First augmentation factor

$-(b''_{16})^{(2)}((G_{19}),t) =$ First detritions factor

**Module Numbered Three**

**The differential system of this model is now (Module numbered three)**

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21},t)\right]G_{20} \quad 13$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21},t)\right]G_{21} \quad 14$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21},t)\right]G_{22} \quad 15$$

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23},t)\right]T_{20} \quad 16$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23},t)\right]T_{21} \quad 17$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23},t)\right]T_{22} \quad 18$$

$+(a''_{20})^{(3)}(T_{21},t) =$ First augmentation factor

$-(b''_{20})^{(3)}(G_{23},t) =$ First detritions factor

**Module Numbered Four**

**The differential system of this model is now (Module numbered Four)**

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right]G_{24} \quad 19$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25},t)\right]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25},t)\right]G_{26} \quad 21$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27}),t)\right]T_{24} \quad 22$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}((G_{27}),t)\right]T_{25} \quad 23$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27}),t)\right]T_{26} \quad 24$$

$+(a''_{24})^{(4)}(T_{25},t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}((G_{27}),t) =$ **First detritions factor**

**Module Numbered Five**:

**The differential system of this model is now (Module number five)**

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29},t)\right]G_{28} \quad 25$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29},t)\right]G_{29} \quad 26$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29},t)\right]G_{30} \quad 27$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31}),t)\right]T_{28} \quad 28$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}((G_{31}),t)\right]T_{29} \quad 29$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31}),t)\right]T_{30} \quad 30$$

$+(a''_{28})^{(5)}(T_{29},t) =$ **First augmentation factor**

$-(b''_{28})^{(5)}((G_{31}),t) =$ **First detritions factor**

**Module Numbered Six**

**The differential system of this model is now (Module numbered Six)**

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right]G_{32} \quad 31$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}, t)\right]G_{33} \quad 32$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}, t)\right]G_{34} \quad 33$$

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35}), t)\right]T_{32} \quad 34$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}((G_{35}), t)\right]T_{33} \quad 35$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35}), t)\right]T_{34} \quad 36$$

$+(a''_{32})^{(6)}(T_{33}, t) =$ **First augmentation factor**

**Module Numbered Seven:**

**The differential system of this model is now (Seventh Module)**

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right]G_{36} \quad 37$$

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37}, t)\right]G_{37} \quad 38$$

$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}, t)\right]G_{38} \quad 39$$

$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39}), t)\right]T_{36} \quad 40$$

$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}((G_{39}), t)\right]T_{37} \quad 41$$

$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39}), t)\right]T_{38} \quad 42$$

$+(a''_{36})^{(7)}(T_{37}, t) =$ **First augmentation factor**

**Module Numbered Eight**

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right]G_{40} \quad 43$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41}, t)\right]G_{41} \quad 44$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}, t)\right]G_{42} \quad 45$$

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43}), t)\right]T_{40} \quad 46$$

$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}((G_{43}), t)\right]T_{41} \quad 47$$

$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43}), t)\right]T_{42} \quad 48$$

**Module Numbered Nine**

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}, t)\right]G_{44} \quad 49$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45}, t)\right]G_{45} \quad 50$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}, t)\right]G_{46} \quad 51$$

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47}), t)\right]T_{44} \quad 52$$

$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}((G_{47}), t)\right]T_{45} \quad 53$$

$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47}), t)\right]T_{46} \quad 54$$

$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**
$-(b''_{44})^{(9)}((G_{47}),t) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)}\boxed{+(a''_{13})^{(1)}(T_{14},t)}\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7)}(T_{37},t)}\boxed{+(a''_{40})^{(8,8)}(T_{41},t)}\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13} \quad (55)$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)}\boxed{+(a''_{14})^{(1)}(T_{14},t)}\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7)}(T_{37},t)}\boxed{+(a''_{41})^{(8,8)}(T_{41},t)}\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14} \quad (56)$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)}\boxed{+(a''_{15})^{(1)}(T_{14},t)}\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7)}(T_{37},t)}\boxed{+(a''_{42})^{(8,8)}(T_{41},t)}\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15} \quad (57)$$

Where $\boxed{(a''_{13})^{(1)}(T_{14},t)}$, $\boxed{(a''_{14})^{(1)}(T_{14},t)}$, $\boxed{(a''_{15})^{(1)}(T_{14},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7)}(T_{37},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8)}(T_{41},t)}$ $\boxed{+(a''_{41})^{(8,8)}(T_{41},t)}$ $\boxed{+(a''_{42})^{(8,8)}(T_{41},t)}$ are eight augmentation coefficient for 1,2,3

$\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)}\boxed{-(b''_{13})^{(1)}(G,t)}\boxed{-(b''_{16})^{(2,2,)}(G_{19},t)}\boxed{-(b''_{20})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)}\boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)}\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,)}(G_{39},t)}\boxed{-(b''_{40})^{(8,8)}(G_{43},t)}\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13} \quad (58)$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)}\boxed{-(b''_{14})^{(1)}(G,t)}\boxed{-(b''_{17})^{(2,2,)}(G_{19},t)}\boxed{-(b''_{21})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)}\boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)}\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,)}(G_{39},t)}\boxed{-(b''_{41})^{(8,8)}(G_{43},t)}\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14} \quad (59)$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)}\boxed{-(b''_{15})^{(1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15} \quad (60)$$

Where $\boxed{-(b''_{13})^{(1)}(G,t)}$, $\boxed{-(b''_{14})^{(1)}(G,t)}$, $\boxed{-(b''_{15})^{(1)}(G,t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8)}(G_{43},t)}$ $\boxed{-(b''_{41})^{(8,8)}(G_{43},t)}$ $\boxed{-(b''_{42})^{(8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \begin{bmatrix} (a'_{16})^{(2)}\boxed{+(a''_{16})^{(2)}(T_{17},t)} & \boxed{+(a''_{13})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{16} \quad (61)$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \begin{bmatrix} (a'_{17})^{(2)}\boxed{+(a''_{17})^{(2)}(T_{17},t)} & \boxed{+(a''_{14})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{17} \quad (62)$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \begin{bmatrix} (a'_{18})^{(2)}\boxed{+(a''_{18})^{(2)}(T_{17},t)} & \boxed{+(a''_{15})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{18} \quad (63)$$

Where $\boxed{+(a''_{16})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,)}(T_{14},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{36})^{(7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7)}(T_{37},t)$ are seventh augmentation coefficient for category 1, 2 and 3

$+(a''_{40})^{(8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8)}(T_{41},t)$ are eight augmentation coefficient for category 1, 2 and 3

$+(a''_{44})^{(9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9)}(T_{45},t)$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \begin{bmatrix} (b'_{16})^{(2)} & -(b''_{16})^{(2)}(G_{19},t) & -(b''_{13})^{(1,1,)}(G,t) & -(b''_{20})^{(3,3,3,)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6)}(G_{35},t) \\ -(b''_{36})^{(7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9)}(G_{47},t) \end{bmatrix} T_{16} \quad 64$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)} & -(b''_{17})^{(2)}(G_{19},t) & -(b''_{14})^{(1,1,)}(G,t) & -(b''_{21})^{(3,3,3,)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6)}(G_{35},t) \\ -(b''_{37})^{(7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9)}(G_{47},t) \end{bmatrix} T_{17} \quad 65$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)} & -(b''_{18})^{(2)}(G_{19},t) & -(b''_{15})^{(1,1,)}(G,t) & -(b''_{22})^{(3,3,3,)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6)}(G_{35},t) \\ -(b''_{38})^{(7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9)}(G_{47},t) \end{bmatrix} T_{18} \quad 66$$

where $-(b''_{16})^{(2)}(G_{19},t)$, $-(b''_{17})^{(2)}(G_{19},t)$, $-(b''_{18})^{(2)}(G_{19},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{13})^{(1,1,)}(G,t)$, $-(b''_{14})^{(1,1,)}(G,t)$, $-(b''_{15})^{(1,1,)}(G,t)$ are second detrition coefficients for category 1,2 and 3

$-(b''_{20})^{(3,3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,)}(G_{23},t)$ are third detrition coefficients for category 1,2 and 3

$-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1,2 and 3

$-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1,2 and 3

$-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients for category 1,2 and 3

$-(b''_{36})^{(7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2 and 3

$-(b''_{40})^{(8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1,2 and 3

$-(b''_{44})^{(9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \begin{bmatrix} (a'_{20})^{(3)} & +(a''_{20})^{(3)}(T_{21},t) & +(a''_{16})^{(2,2,2)}(T_{17},t) & +(a''_{13})^{(1,1,1,)}(T_{14},t) \\ +(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t) & \\ +(a''_{36})^{(7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9)}(T_{45},t) & \end{bmatrix} G_{20} \quad (67)$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \begin{bmatrix} (a'_{21})^{(3)} & +(a''_{21})^{(3)}(T_{21},t) & +(a''_{17})^{(2,2,2)}(T_{17},t) & +(a''_{14})^{(1,1,1,)}(T_{14},t) \\ +(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t) & \\ +(a''_{37})^{(7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9)}(T_{45},t) & \end{bmatrix} G_{21} \quad (68)$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \begin{bmatrix} (a'_{22})^{(3)} & +(a''_{22})^{(3)}(T_{21},t) & +(a''_{18})^{(2,2,2)}(T_{17},t) & +(a''_{15})^{(1,1,1,)}(T_{14},t) \\ +(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t) & \\ +(a''_{38})^{(7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9)}(T_{45},t) & \end{bmatrix} G_{22} \quad (69)$$

$+(a''_{20})^{(3)}(T_{21},t)$, $+(a''_{21})^{(3)}(T_{21},t)$, $+(a''_{22})^{(3)}(T_{21},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2)}(T_{17},t)$ are second augmentation coefficients for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,)}(T_{14},t)$ are third augmentation coefficients for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficients for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficients for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficients for category 1, 2 and 3

$+(a''_{36})^{(7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1, 2 and 3

$+(a''_{40})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8)}(T_{41},t)$ are eight augmentation coefficients for category 1, 2 and 3

$+(a''_{44})^{(9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b'_{20})^{(3)} & -(b''_{20})^{(3)}(G_{23},t) & -(b''_{16})^{(2,2,2)}(G_{19},t) & -(b''_{13})^{(1,1,1,)}(G,t) \\ -(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t) & \\ -(b''_{36})^{(7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9)}(G_{47},t) & \end{bmatrix} T_{20} \quad (70)$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b'_{21})^{(3)} & -(b''_{21})^{(3)}(G_{23},t) & -(b''_{17})^{(2,2,2)}(G_{19},t) & -(b''_{14})^{(1,1,1,)}(G,t) \\ -(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t) & \\ -(b''_{37})^{(7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9)}(G_{47},t) & \end{bmatrix} T_{21} \quad (71)$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)}\boxed{-(b''_{22})^{(3)}(G_{23},t)} & \boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{22} \quad 72$$

$\boxed{-(b''_{20})^{(3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3)}(G_{23},t)}$ *are first detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}$ *are second detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{13})^{(1,1,1,)}(G,t)}, \boxed{-(b''_{14})^{(1,1,1,)}(G,t)}, \boxed{-(b''_{15})^{(1,1,1,)}(G,t)}$ *are third detrition coefficients for category 1,2 and 3*

$\boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)}, \boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)}, \boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)}$ *are fourth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)}, \boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)}, \boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)}$ *are fifth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)}, \boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)}, \boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)}$ *are sixth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{36})^{(7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{37})^{(7,7,7,7)}(G_{39},t)} \boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)}, \boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)}, \boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \begin{bmatrix} (a'_{24})^{(4)}\boxed{+(a''_{24})^{(4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{24} \quad 73$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \begin{bmatrix} (a'_{25})^{(4)}\boxed{+(a''_{25})^{(4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{25} \quad 74$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \begin{bmatrix} (a'_{26})^{(4)}\boxed{+(a''_{26})^{(4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{26} \quad 75$$

$\boxed{(a''_{24})^{(4)}(T_{25},t)}, \boxed{(a''_{25})^{(4)}(T_{25},t)}, \boxed{(a''_{26})^{(4)}(T_{25},t)}$ *are first augmentation coefficients category* $1, 2\ 3$

$\boxed{+(a''_{28})^{(5,5,)}(T_{29},t)}, \boxed{+(a''_{29})^{(5,5,)}(T_{29},t)}, \boxed{+(a''_{30})^{(5,5,)}(T_{29},t)}$ *are second augmentation coefficient for category* $1, 2$ *and* $3$

$\boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}, \boxed{+(a''_{33})^{(6,6,)}(T_{33},t)}, \boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}$ *are third augmentation coefficient for category* $1, 2$ *and* $3$

$\boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)}$

$are\ fourth\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$
$+(a''_{16})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2)}(T_{17},t)$
$are\ fifth\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$
$+(a''_{20})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3)}(T_{21},t)$
$are\ sixth\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$
$+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)$
$are\ seventh\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$
$+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)$
$are\ eighth\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$
$+(a''_{46})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9)}(T_{45},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \begin{bmatrix} (b'_{24})^{(4)}\left[-(b''_{24})^{(4)}(G_{27},t)\right] & \left[-(b''_{28})^{(5,5,)}(G_{31},t)\right] & \left[-(b''_{32})^{(6,6,)}(G_{35},t)\right] \\ \left[-(b''_{13})^{(1,1,1,1)}(G,t)\right] & \left[-(b''_{16})^{(2,2,2,2)}(G_{19},t)\right] & \left[-(b''_{20})^{(3,3,3,3)}(G_{23},t)\right] \\ \left[-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)\right] & \left[-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)\right] & \left[-(b''_{44})^{(9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{24} \quad (76)$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \begin{bmatrix} (b'_{25})^{(4)}\left[-(b''_{25})^{(4)}(G_{27},t)\right] & \left[-(b''_{29})^{(5,5,)}(G_{31},t)\right] & \left[-(b''_{33})^{(6,6,)}(G_{35},t)\right] \\ \left[-(b''_{14})^{(1,1,1,1)}(G,t)\right] & \left[-(b''_{17})^{(2,2,2,2)}(G_{19},t)\right] & \left[-(b''_{21})^{(3,3,3,3)}(G_{23},t)\right] \\ \left[-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)\right] & \left[-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)\right] & \left[-(b''_{45})^{(9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{25} \quad (77)$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)}\left[-(b''_{26})^{(4)}(G_{27},t)\right] & \left[-(b''_{30})^{(5,5,)}(G_{31},t)\right] & \left[-(b''_{34})^{(6,6,)}(G_{35},t)\right] \\ \left[-(b''_{15})^{(1,1,1,1)}(G,t)\right] & \left[-(b''_{18})^{(2,2,2,2)}(G_{19},t)\right] & \left[-(b''_{22})^{(3,3,3,3)}(G_{23},t)\right] \\ \left[-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)\right] & \left[-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)\right] & \left[-(b''_{46})^{(9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{26} \quad (78)$$

$Where$ $-(b''_{24})^{(4)}(G_{27},t)$, $-(b''_{25})^{(4)}(G_{27},t)$, $-(b''_{26})^{(4)}(G_{27},t)$ $are\ first\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b''_{28})^{(5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,)}(G_{31},t)$ $are\ second\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b''_{32})^{(6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,)}(G_{35},t)$ $are\ third\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b''_{13})^{(1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1)}(G,t)$
$are\ fourth\ \ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b''_{16})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2)}(G_{19},t)$
$are\ fifth\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b''_{20})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3)}(G_{23},t)$
$are\ sixth\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)$
$are\ seventh\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)$
$are\ eighth\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b''_{46})^{(9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[\begin{array}{ccc} (a'_{28})^{(5)}\boxed{+(a''_{28})^{(5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{28} \quad (79)$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[\begin{array}{ccc} (a'_{29})^{(5)}\boxed{+(a''_{29})^{(5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{29} \quad (80)$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[\begin{array}{ccc} (a'_{30})^{(5)}\boxed{+(a''_{30})^{(5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{30} \quad (81)$$

*Where* $\boxed{+(a''_{28})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5)}(T_{29},t)}$ *are first augmentation coefficients for category* 1, 2 *and* 3

*And* $\boxed{+(a''_{24})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,)}(T_{25},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)}$ are fourth augmentation coefficients for category 1,2, and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)}$ are fifth augmentation coefficients for category 1,2,and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)}$ are sixth augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)}$ are seventh augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)}$ are eighth augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[\begin{array}{ccc} (b'_{28})^{(5)}\boxed{-(b''_{28})^{(5)}(G_{31},t)} & \boxed{-(b''_{24})^{(4,4,)}(G_{27},t)} & \boxed{-(b''_{32})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)} \end{array}\right] T_{28} \quad (82)$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[\begin{array}{ccc} (b'_{29})^{(5)}\boxed{-(b''_{29})^{(5)}(G_{31},t)} & \boxed{-(b''_{25})^{(4,4,)}(G_{27},t)} & \boxed{-(b''_{33})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)} \end{array}\right] T_{29} \quad (83)$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[\begin{array}{ccc} (b'_{30})^{(5)}\boxed{-(b''_{30})^{(5)}(G_{31},t)} & \boxed{-(b''_{26})^{(4,4,)}(G_{27},t)} & \boxed{-(b''_{34})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)} \end{array}\right] T_{30} \quad (84)$$

$where\ \boxed{-(b''_{28})^{(5)}(G_{31},t)}\ ,\boxed{-(b''_{29})^{(5)}(G_{31},t)}\ ,\boxed{-(b''_{30})^{(5)}(G_{31},t)}\ \ are\ first\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b''_{24})^{(4,4,)}(G_{27},t)}\ ,\boxed{-(b''_{25})^{(4,4,)}(G_{27},t)}\ ,\boxed{-(b''_{26})^{(4,4,)}(G_{27},t)}\ are\ second\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b''_{32})^{(6,6,6)}(G_{35},t)}\ ,\boxed{-(b''_{33})^{(6,6,6)}(G_{35},t)}\ ,\boxed{-(b''_{34})^{(6,6,6)}(G_{35},t)}\ are\ third\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b''_{13})^{(1,1,1,1,1)}(G,t)}\ ,\boxed{-(b''_{14})^{(1,1,1,1,1)}(G,t)}\ ,\boxed{-(b''_{15})^{(1,1,1,1,1,)}(G,t)}$ are fourth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)}\ ,\boxed{-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)}\ ,\boxed{-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)}\ ,\boxed{-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)}\ ,\boxed{-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)}\ ,\boxed{-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)}\ ,\boxed{-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)}\ ,\boxed{-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)}\ ,\boxed{-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)}\ ,\boxed{-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)}\ ,\boxed{-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[\begin{array}{c} (a'_{32})^{(6)}\boxed{+(a''_{32})^{(6)}(T_{33},t)}\ \boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}\ \boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}\ \boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}\ \boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{32} \quad 85$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[\begin{array}{c} (a'_{33})^{(6)}\boxed{+(a''_{33})^{(6)}(T_{33},t)}\ \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}\ \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}\ \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}\ \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{33} \quad 86$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[\begin{array}{c} (a'_{34})^{(6)}\boxed{+(a''_{34})^{(6)}(T_{33},t)}\ \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}\ \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}\ \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}\ \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{34} \quad 87$$

$\boxed{+(a''_{32})^{(6)}(T_{33},t)}\ ,\boxed{+(a''_{33})^{(6)}(T_{33},t)}\ ,\boxed{+(a''_{34})^{(6)}(T_{33},t)}\ are\ first\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}\ ,\boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}\ ,\boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}\ are\ second\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}\ ,\boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}\ ,\boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}\ are\ third\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}\ ,\boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}\ ,\boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}$ - are fourth augmentation coefficients

$\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}\ ,\boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}\ ,\boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}$ - fifth augmentation

coefficients
$+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)$ sixth augmentation coefficients
$+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)$
seventh augmentation coefficients
$+(a''_{40})^{(8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)$
Eighth augmentation coefficients
$+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt}=(b_{32})^{(6)}T_{33}-\begin{bmatrix}(b'_{32})^{(6)}\boxed{-(b''_{32})^{(6)}(G_{35},t)}\ \boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}\ \boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}\\ \boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)}\ \boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)}\ \boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)}\\ \boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)}\ \boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)}\ \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)}\end{bmatrix}T_{32} \quad 88$$

$$\frac{dT_{33}}{dt}=(b_{33})^{(6)}T_{32}-\begin{bmatrix}(b'_{33})^{(6)}\boxed{-(b''_{33})^{(6)}(G_{35},t)}\ \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}\ \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}\\ \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}\ \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}\ \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)}\\ \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)}\ \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)}\ \boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)}\end{bmatrix}T_{33} \quad 89$$

$$\frac{dT_{34}}{dt}=(b_{34})^{(6)}T_{33}-\begin{bmatrix}(b'_{34})^{(6)}\boxed{-(b''_{34})^{(6)}(G_{35},t)}\ \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}\ \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)}\\ \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}\ \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}\ \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)}\\ \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)}\ \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)}\ \boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)}\end{bmatrix}T_{34} \quad 90$$

$\boxed{-(b''_{32})^{(6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6)}(G_{35},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3
$\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}$ *are second detrition coefficients for category* 1, 2 *and* 3
$\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)}$ *are third detrition coefficients for category* 1,2 *and* 3
$\boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}$ are fourth detrition coefficients for category 1, 2, and 3
$\boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1, 2, and 3
$\boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1, 2, and 3
$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2, and 3
$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$
are eighth detrition coefficients for category 1, 2, and 3
$\boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \quad 91$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a'_{36})^{(7)}\boxed{+(a''_{36})^{(7)}(T_{37},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \quad 92$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a'_{37})^{(7)}\boxed{+(a''_{37})^{(7)}(T_{37},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{38}}{dt} \quad 93$$

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a'_{38})^{(7)}\boxed{+(a''_{38})^{(7)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{(a''_{36})^{(7)}(T_{37},t)}$, $\boxed{(a''_{37})^{(7)}(T_{37},t)}$, $\boxed{(a''_{38})^{(7)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{36}}{dt} = \quad 94$$

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b'_{36})^{(7)}\boxed{-(b''_{36})^{(7)}(G_{39},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{37}}{dt} =$$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b'_{37})^{(7)}\boxed{-(b''_{37})^{(7)}(G_{39},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{38}}{dt} =$$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b'_{38})^{(7)}\boxed{-(b''_{38})^{(7)}(G_{39},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{40}}{dt}$$



$$= (a_{40})^{(8)}G_{41} - \begin{bmatrix} (a'_{40})^{(8)}\boxed{+(a''_{40})^{(8)}(T_{41},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{41}}{dt}$$

$$= (a_{41})^{(8)}G_{40} - \begin{bmatrix} (a'_{41})^{(8)}\boxed{+(a''_{41})^{(8)}(T_{41},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \begin{bmatrix} (a'_{42})^{(8)}\boxed{+(a''_{42})^{(8)}(T_{41},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a''_{40})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)}\boxed{-(b''_{40})^{(8)}(G_{43},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)}\boxed{-(b''_{41})^{(8)}(G_{43},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)}\boxed{-(b''_{42})^{(8)}(G_{43},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{44}}{dt} \qquad 96$$

$$= (a_{44})^{(9)}G_{45} - \left[\begin{array}{ccc} (a'_{44})^{(9)}\boxed{+(a''_{44})^{(9)}(T_{45},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{array}\right] G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[\begin{array}{ccc} (a'_{45})^{(9)}\boxed{+(a''_{45})^{(9)}(T_{45},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{array}\right] G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[\begin{array}{ccc} (a'_{46})^{(9)}\boxed{+(a''_{46})^{(9)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{array}\right] G_{15}$$

Where $\boxed{+(a''_{44})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{20}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{21}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{22}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{24}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{25}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{26}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{28}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{29}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{30}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{32}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{33}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{34}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a_{14}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a_{15}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ are Seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{38}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a_{37}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a_{36}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a_{40}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a_{42}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a_{41}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} =$$

$$(b_{44})^{(9)}T_{45} -$$

$$\begin{bmatrix} (b_{44}')^{(9)}\boxed{-(b_{44}'')^{(9)}(G_{47},t)} & \boxed{-(b_{16}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{20}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{24}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{28}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{32}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{13}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{36}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b_{45}')^{(9)}\boxed{-(b_{45}'')^{(9)}(G_{47},t)} & \boxed{-(b_{17}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{21}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{25}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{29}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{33}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{14}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{37}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b_{46}')^{(9)}\boxed{-(b_{46}'')^{(9)}(G_{47},t)} & \boxed{-(b_{18}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{22}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{26}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{30}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{34}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{15}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{38}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b_{44}'')^{(9)}(G_{47},t)}$, $\boxed{-(b_{45}'')^{(9)}(G_{47},t)}$, $\boxed{-(b_{46}'')^{(9)}(G_{47},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{16}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{17}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{18}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{20}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{21}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{22}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{24}'')^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{25}'')^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{26}'')^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{28}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{32}'')^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{34}'')^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{37}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{36}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a_i')^{(1)}, (a_i'')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (b_i'')^{(1)} > 0, \quad i,j = 13,14,15 \qquad 97$$

The functions $(a_i'')^{(1)}, (b_i'')^{(1)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a_i'')^{(1)}(T_{14},t) \leq (p_i)^{(1)} \leq (\hat{A}_{13})^{(1)}$$

$$(b_i'')^{(1)}(G,t) \leq (r_i)^{(1)} \leq (b_i')^{(1)} \leq (\hat{B}_{13})^{(1)}$$

$$lim_{T_2\to\infty}(a_i'')^{(1)}(T_{14},t) = (p_i)^{(1)} \qquad 98$$

$$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = (r_i)^{(1)}$$

**Definition of** $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$ :

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$$|(a_i'')^{(1)}(T_{14}',t) - (a_i'')^{(1)}(T_{14},t)| \leq (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G',t) - (b_i'')^{(1)}(G,t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}',t)$ and $(a_i'')^{(1)}(T_{14},t)$ . $(T_{14}',t)$ and $(T_{14},t)$ are points belonging to the interval $[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}]$. It is to be noted that $(a_i'')^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14},t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$ : 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$

**Definition of** $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$ : 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}$, $(r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)} (\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)} (\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i, j = 16,17,18$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17}, t) \leq (p_i)^{(2)} \leq (\hat{A}_{16})^{(2)} \quad 102$$

$$(b_i'')^{(2)}(G_{19}, t) \leq (r_i)^{(2)} \leq (b_i')^{(2)} \leq (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2 \to \infty} (a_i'')^{(2)}(T_{17}, t) = (p_i)^{(2)} \quad 104$$

$$\lim_{G \to \infty} (b_i'')^{(2)}((G_{19}), t) = (r_i)^{(2)} \quad 105$$

**Definition of** $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$ : 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}', t) - (a_i'')^{(2)}(T_{17}, t)| \leq (\hat{k}_{16})^{(2)} |T_{17} - T_{17}'| e^{-(\hat{M}_{16})^{(2)} t} \quad 107$$

$$|(b_i'')^{(2)}((G_{19})', t) - (b_i'')^{(2)}((G_{19}), t)| < (\hat{k}_{16})^{(2)} ||(G_{19}) - (G_{19})'|| e^{-(\hat{M}_{16})^{(2)} t} \quad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}', t)$ and$(a_i'')^{(2)}(T_{17}, t)$ . $(T_{17}', t)$ and $(T_{17}, t)$ are points belonging to the interval $[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}]$ . It is to be noted that $(a_i'')^{(2)}(T_{17}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17}, t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$ :

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)},$ are positive constants 109

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$$

**Definition of** $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$ :

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and\ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1 \quad 110$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(b_i)^{(2)} + (b_i')^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1 \quad 111$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i, j = 20,21,22 \quad 112$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(3)}$, $(r_i)^{(3)}$:

$$(a_i'')^{(3)}(T_{21}, t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23}, t) \leq (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(3)}(T_{21}, t) = (p_i)^{(3)} \quad 113$$

$$\lim_{G \to \infty}(b_i'')^{(3)}(G_{23}, t) = (r_i)^{(3)}$$

**Definition of** $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$ :

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: 114

$$|(a_i'')^{(3)}(T_{21}', t) - (a_i'')^{(3)}(T_{21}, t)| \leq (\hat{k}_{20})^{(3)} |T_{21} - T_{21}'| e^{-(\hat{M}_{20})^{(3)} t}$$

$$|(b_i'')^{(3)}(G_{23}', t) - (b_i'')^{(3)}(G_{23}, t)| < (\hat{k}_{20})^{(3)} ||G_{23} - G_{23}'|| e^{-(\hat{M}_{20})^{(3)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}', t)$ and $(a_i'')^{(3)}(T_{21}, t)$ . $(T_{21}', t)$ And $(T_{21}, t)$ are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$ : 115

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}$ , $\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with 116
$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$ and the constants
$(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}, i = 20,21,22,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(a_i)^{(3)} + (a_i')^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)} (\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(b_i)^{(3)} + (b_i')^{(3)} + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)} (\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i, j = 24,25,26$$ 117

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(4)}$, $(r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25}, t) \leq (p_i)^{(4)} \leq (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}((G_{27}), t) \leq (r_i)^{(4)} \leq (b_i')^{(4)} \leq (\hat{B}_{24})^{(4)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(4)}(T_{25}, t) = (p_i)^{(4)}$$ 118

$$\lim_{G \to \infty}(b_i'')^{(4)}((G_{27}), t) = (r_i)^{(4)}$$

**Definition of** $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$ :

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24,25,26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}', t) - (a_i'')^{(4)}(T_{25}, t)| \leq (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})', t) - (b_i'')^{(4)}((G_{27}), t)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}', t)$ and$(a_i'')^{(4)}(T_{25}, t)$ . $(T_{25}', t)$ and $(T_{25}, t)$ are points belonging to the interval $[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}]$. It is to be noted that $(a_i'')^{(4)}(T_{25}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25}, t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$ : 120

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)},$ are positive constants

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}, \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$$

**Definition of** $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$ : 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and\ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}, i = 24,25,26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(a_i)^{(4)} + (a_i')^{(4)} + (\hat{A}_{24})^{(4)} + (\hat{P}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(b_i)^{(4)} + (b_i')^{(4)} + (\hat{B}_{24})^{(4)} + (\hat{Q}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i, j = 28,29,30 \qquad 122$$

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(5)}$, $(r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29}, t) \le (p_i)^{(5)} \le (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}((G_{31}), t) \le (r_i)^{(5)} \le (b_i')^{(5)} \le (\hat{B}_{28})^{(5)}$$

$$lim_{T_2 \to \infty} (a_i'')^{(5)} (T_{29}, t) = (p_i)^{(5)} \qquad 123$$

$$\lim_{G \to \infty} (b_i'')^{(5)} (G_{31}, t) = (r_i)^{(5)}$$

**Definition of** $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$ :

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}', t) - (a_i'')^{(5)}(T_{29}, t)| \le (\hat{k}_{28})^{(5)} |T_{29} - T_{29}'| e^{-(\hat{M}_{28})^{(5)} t}$$

$$|(b_i'')^{(5)}((G_{31})', t) - (b_i'')^{(5)}((G_{31}), t)| < (\hat{k}_{28})^{(5)} ||(G_{31}) - (G_{31})'|| e^{-(\hat{M}_{28})^{(5)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}', t)$ and $(a_i'')^{(5)}(T_{29}, t)$ . $(T_{29}', t)$ and $(T_{29}, t)$ are points belonging to the interval $\left[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\right]$. It is to be noted that $(a_i'')^{(5)}(T_{29}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$ : 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)},$ are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}, \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

**Definition of** $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$ : 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and\ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(a_i)^{(5)} + (a_i')^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(b_i)^{(5)} + (b_i')^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i, j = 32,33,34$$ 127

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(6)}$, $(r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33}, t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}), t) \leq (r_i)^{(6)} \leq (b_i')^{(6)} \leq (\hat{B}_{32})^{(6)}$$

$$lim_{T_2 \to \infty} (a_i'')^{(6)}(T_{33}, t) = (p_i)^{(6)}$$ 128

$$\lim_{G \to \infty} (b_i'')^{(6)}((G_{35}), t) = (r_i)^{(6)}$$

**Definition of** $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$ :

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}', t) - (a_i'')^{(6)}(T_{33}, t)| \leq (\hat{k}_{32})^{(6)} |T_{33} - T_{33}'| e^{-(\hat{M}_{32})^{(6)} t}$$

$$|(b_i'')^{(6)}((G_{35})', t) - (b_i'')^{(6)}((G_{35}), t)| < (\hat{k}_{32})^{(6)} ||(G_{35}) - (G_{35})'|| e^{-(\hat{M}_{32})^{(6)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}', t)$ and $(a_i'')^{(6)}(T_{33}, t)$ . $(T_{33}', t)$ and $(T_{33}, t)$ are points belonging to the interval $[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}]$. It is to be noted that $(a_i'')^{(6)}(T_{33}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$ : 129

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)},$ are positive constants

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$$

**Definition of** $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$ : 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and\ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32,33,34,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(a_i)^{(6)} + (a_i')^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)} (\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(b_i)^{(6)} + (b_i')^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)} (\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

(OOO) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i, j = 36,37,38$ 131

(PPP) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(7)}$, $(r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37}, t) \leq (p_i)^{(7)} \leq (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39}, t) \leq (r_i)^{(7)} \leq (b_i')^{(7)} \leq (\hat{B}_{36})^{(7)}$$

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*(QQQ)* $\lim_{T_2 \to \infty} (a_i'')^{(7)}(T_{37}, t) = (p_i)^{(7)}$

*(RRR)*

$\lim_{G \to \infty} (b_i'')^{(7)}((G_{39}), t) = (r_i)^{(7)}$

**Definition of** $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$ :

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$$|(a_i'')^{(7)}(T_{37}', t) - (a_i'')^{(7)}(T_{37}, t)| \leq (\hat{k}_{36})^{(7)} |T_{37} - T_{37}'| e^{-(\hat{M}_{36})^{(7)} t}$$

$$|(b_i'')^{(7)}((G_{39})', t) - (b_i'')^{(7)}((G_{39}), t)| < (\hat{k}_{36})^{(7)} ||(G_{39}) - (G_{39})'|| e^{-(\hat{M}_{36})^{(7)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}', t)$ and $(a_i'')^{(7)}(T_{37}, t)$ . $(T_{37}', t)$ and $(T_{37}, t)$ are points belonging to the interval $[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}]$ . It is to be noted that $(a_i'')^{(7)}(T_{37}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37}, t)$ , the **first augmentation coefficient** attributable to the

system, would be absolutely continuous.

**Definition of** $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$ : 134

(SSS) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)},$ are positive constants

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$$

**Definition of** $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$ : 135

(TTT) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(b_i)^{(7)} + (b_i')^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40,41,42 \qquad 136$$

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

**Definition of** $(p_i)^{(8)}, (r_i)^{(8)}$: 137

$$(a_i'')^{(8)}(T_{41}, t) \le (p_i)^{(8)} \le (\hat{A}_{40})^{(8)} \qquad 138$$

$$(b_i'')^{(8)}((G_{43}), t) \le (r_i)^{(8)} \le (b_i')^{(8)} \le (\hat{B}_{40})^{(8)} \qquad 139$$

$$\lim_{T_2 \to \infty} (a_i'')^{(8)}(T_{41}, t) = (p_i)^{(8)} \qquad 140$$

$$\lim_{G \to \infty} (b_i'')^{(8)}((G_{43}), t) = (r_i)^{(8)} \qquad 141$$

**Definition of** $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$ :

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \le (\hat{k}_{40})^{(8)} |T_{41} - T_{41}'| e^{-(\hat{M}_{40})^{(8)} t} \qquad 142$$

$$|(b_i'')^{(8)}((G_{43})', t) - (b_i'')^{(8)}((G_{43}), t)| < (\hat{k}_{40})^{(8)} ||(G_{43}) - (G_{43})'|| e^{-(\hat{M}_{40})^{(8)} t} \qquad 143$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}', t)$ and $(a_i'')^{(8)}(T_{41}, t)$ . $(T_{41}', t)$ and $(T_{41}, t)$ are points belonging to the interval $\left[(\hat{k}_{40})^{(8)}, (\widehat{M}_{40})^{(8)}\right]$. It is to be noted that $(a_i'')^{(8)}(T_{41}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{40})^{(8)} = 1$ then the function $(a_i'')^{(8)}(T_{41}, t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\widehat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$ :

$(\widehat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)},$ are positive constants

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}, \frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}} < 1 \qquad 144$$

**Definition of** $(\hat{P}_{40})^{(8)}, (\hat{Q}_{40})^{(8)}$ :

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\widehat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}, (\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)}, (a_i')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}, i = 40,41,42,$
Satisfy the inequalities

$$\frac{1}{(\widehat{M}_{40})^{(8)}}[(a_i)^{(8)} + (a_i')^{(8)} + (\hat{A}_{40})^{(8)} + (\hat{P}_{40})^{(8)} (\hat{k}_{40})^{(8)}] < 1 \qquad 145$$

$$\frac{1}{(\widehat{M}_{40})^{(8)}}[(b_i)^{(8)} + (b_i')^{(8)} + (\hat{B}_{40})^{(8)} + (\hat{Q}_{40})^{(8)} (\hat{k}_{40})^{(8)}] < 1 \qquad 146$$

Where we suppose

$$(a_i)^{(9)}, (a_i')^{(9)}, (a_i'')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (b_i'')^{(9)} > 0, \quad i, j = 44,45,46 \qquad 146\text{A}$$

The functions $(a_i'')^{(9)}, (b_i'')^{(9)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(9)}$, $(r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45}, t) \leq (p_i)^{(9)} \leq (\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47}, t) \leq (r_i)^{(9)} \leq (b_i')^{(9)} \leq (\hat{B}_{44})^{(9)}$$

$$\lim_{T_2 \to \infty} (a_i'')^{(9)}(T_{45}, t) = (p_i)^{(9)}$$

$$\lim_{G \to \infty} (b_i'')^{(9)}(G_{47}, t) = (r_i)^{(9)}$$

**Definition of** $(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}$ :

Where $\boxed{(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}}$ are positive constants and $\boxed{i = 44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}', t) - (a_i'')^{(9)}(T_{45}, t)| \leq (\hat{k}_{44})^{(9)} |T_{45} - T_{45}'| e^{-(\widehat{M}_{44})^{(9)} t}$$

$$|(b_i'')^{(9)}((G_{47})', t) - (b_i'')^{(9)}\big((G_{47}), t\big)| < (\hat{k}_{44})^{(9)}||(G_{47}) - (G_{47})'||e^{-(\hat{M}_{44})^{(9)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}', t)$ and$(a_i'')^{(9)}(T_{45}, t)$ . $(T_{45}', t)$ and $(T_{45}, t)$ are points belonging to the interval $\big[(\hat{k}_{44})^{(9)}, (\hat{M}_{44})^{(9)}\big]$. It is to be noted that $(a_i'')^{(9)}(T_{45}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{44})^{(9)} = 1$ then the function $(a_i'')^{(9)}(T_{45}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$ :

$(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$, are positive constants

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}} \ , \frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$$

**Definition of** $(\hat{P}_{44})^{(9)}, (\hat{Q}_{44})^{(9)}$ :

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}, (\hat{A}_{44})^{(9)} and\ (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)}, (a_i')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (p_i)^{(9)}, \ (r_i)^{(9)}, i = 44,45,46,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{44})^{(9)}}[\,(a_i)^{(9)} + (a_i')^{(9)} + \ (\hat{A}_{44})^{(9)} + \ (\hat{P}_{44})^{(9)}\,(\hat{k}_{44})^{(9)}] < 1$$

$$\frac{1}{(\hat{M}_{44})^{(9)}}[\ (b_i)^{(9)} + (b_i')^{(9)} + \ (\hat{B}_{44})^{(9)} + \ (\hat{Q}_{44})^{(9)}\ (\hat{k}_{44})^{(9)}] < 1$$

**Theorem 1:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \big(\hat{P}_{13}\big)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \ , \ \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 2 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

**Definition of** $G_i(0)\,, T_i(0)$

$$G_i(t) \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} \quad , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} \quad , \quad T_i(0) = T_i^0 > 0$$

**Theorem 3 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t) \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t} \quad , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t} \quad , \quad T_i(0) = T_i^0 > 0$$

**Theorem 4 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

**Definition of** $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \left(\hat{P}_{24}\right)^{(4)} e^{(\hat{M}_{24})^{(4)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{24}\right)^{(4)} e^{(\hat{M}_{24})^{(4)}t} \;,\quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 5 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

**Definition of** $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \left(\hat{P}_{28}\right)^{(5)} e^{(\hat{M}_{28})^{(5)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{28}\right)^{(5)} e^{(\hat{M}_{28})^{(5)}t} \;,\quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 6 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

**Definition of** $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \left(\hat{P}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t} \;,\quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 7:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

**Definition of** $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \left(\hat{P}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t} \;,\quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 8:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

**Definition of** $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \left(\hat{P}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t} \;,\quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 9:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

**Definition of** $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \left(\hat{P}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

**Proof:** Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \leq (\hat{P}_{13})^{(1)} \text{ , } T_i^0 \leq (\hat{Q}_{13})^{(1)}, \quad 155$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t} \quad 156$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t} \quad 157$$

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[ (a_{13})^{(1)} G_{14}(s_{(13)}) - \left( (a'_{13})^{(1)} + a''_{13})^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \right) G_{13}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[ (a_{14})^{(1)} G_{13}(s_{(13)}) - \left( (a'_{14})^{(1)} + (a''_{14})^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \right) G_{14}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[ (a_{15})^{(1)} G_{14}(s_{(13)}) - \left( (a'_{15})^{(1)} + (a''_{15})^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \right) G_{15}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[ (b_{13})^{(1)} T_{14}(s_{(13)}) - \left( (b'_{13})^{(1)} - (b''_{13})^{(1)} (G(s_{(13)}), s_{(13)}) \right) T_{13}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[ (b_{14})^{(1)} T_{13}(s_{(13)}) - \left( (b'_{14})^{(1)} - (b''_{14})^{(1)} (G(s_{(13)}), s_{(13)}) \right) T_{14}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{T}_{15}(t) = T_{15}^0 + \int_0^t \left[ (b_{15})^{(1)} T_{14}(s_{(13)}) - \left( (b'_{15})^{(1)} - (b''_{15})^{(1)} (G(s_{(13)}), s_{(13)}) \right) T_{15}(s_{(13)}) \right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \leq (\hat{P}_{16})^{(2)} \text{ , } T_i^0 \leq (\hat{Q}_{16})^{(2)},$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$$

By 160

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[ (a_{16})^{(2)} G_{17}(s_{(16)}) - \left( (a'_{16})^{(2)} + a''_{16})^{(2)} (T_{17}(s_{(16)}), s_{(16)}) \right) G_{16}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[ (a_{17})^{(2)} G_{16}(s_{(16)}) - \left( (a'_{17})^{(2)} + (a''_{17})^{(2)} (T_{17}(s_{(16)}), s_{(17)}) \right) G_{17}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[ (a_{18})^{(2)} G_{17}(s_{(16)}) - \left( (a'_{18})^{(2)} + (a''_{18})^{(2)} (T_{17}(s_{(16)}), s_{(16)}) \right) G_{18}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^{0} + \int_0^t \left[ (b_{16})^{(2)} T_{17}(s_{(16)}) - \left( (b'_{16})^{(2)} - (b''_{16})^{(2)} (G_{19}(s_{(16)}), s_{(16)}) \right) T_{16}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^{0} + \int_0^t \left[ (b_{17})^{(2)} T_{16}(s_{(16)}) - \left( (b'_{17})^{(2)} - (b''_{17})^{(2)} (G_{19}(s_{(16)}), s_{(16)}) \right) T_{17}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^{0} + \int_0^t \left[ (b_{18})^{(2)} T_{17}(s_{(16)}) - \left( (b'_{18})^{(2)} - (b''_{18})^{(2)} (G_{19}(s_{(16)}), s_{(16)}) \right) T_{18}(s_{(16)}) \right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i : \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \le (\hat{P}_{20})^{(3)} \text{ , } T_i^0 \le (\hat{Q}_{20})^{(3)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\bar{M}_{20})^{(3)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\bar{M}_{20})^{(3)} t}$$

By 161

$$\bar{G}_{20}(t) = G_{20}^{0} + \int_0^t \left[ (a_{20})^{(3)} G_{21}(s_{(20)}) - \left( (a'_{20})^{(3)} + a''_{20})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^{0} + \int_0^t \left[ (a_{21})^{(3)} G_{20}(s_{(20)}) - \left( (a'_{21})^{(3)} + (a''_{21})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{21}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^{0} + \int_0^t \left[ (a_{22})^{(3)} G_{21}(s_{(20)}) - \left( (a'_{22})^{(3)} + (a''_{22})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{22}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^{0} + \int_0^t \left[ (b_{20})^{(3)} T_{21}(s_{(20)}) - \left( (b'_{20})^{(3)} - (b''_{20})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^{0} + \int_0^t \left[ (b_{21})^{(3)} T_{20}(s_{(20)}) - \left( (b'_{21})^{(3)} - (b''_{21})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{21}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{22}(t) = T_{22}^{0} + \int_0^t \left[ (b_{22})^{(3)} T_{21}(s_{(20)}) - \left( (b'_{22})^{(3)} - (b''_{22})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{22}(s_{(20)}) \right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i : \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \le (\hat{P}_{24})^{(4)} \text{ , } T_i^0 \le (\hat{Q}_{24})^{(4)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\bar{M}_{24})^{(4)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\bar{M}_{24})^{(4)} t}$$

By 162

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{26}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{\mathrm{T}}_{26}(\mathrm{t}) = \mathrm{T}_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{26}(s_{(24)})\right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{28})^{(5)}$, $T_i^0 \le (\hat{Q}_{28})^{(5)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

By 163

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{30}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{\mathrm{T}}_{30}(\mathrm{t}) = \mathrm{T}_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{30}(s_{(28)})\right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{32})^{(6)}$, $T_i^0 \le (\hat{Q}_{32})^{(6)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\tilde{M}_{32})^{(6)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\tilde{M}_{32})^{(6)} t}$

By 164

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[ (a_{32})^{(6)} G_{33}(s_{(32)}) - \left( (a'_{32})^{(6)} + a''_{32})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[ (a_{33})^{(6)} G_{32}(s_{(32)}) - \left( (a'_{33})^{(6)} + (a''_{33})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[ (a_{34})^{(6)} G_{33}(s_{(32)}) - \left( (a'_{34})^{(6)} + (a''_{34})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{34}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[ (b_{32})^{(6)} T_{33}(s_{(32)}) - \left( (b'_{32})^{(6)} - (b''_{32})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[ (b_{33})^{(6)} T_{32}(s_{(32)}) - \left( (b'_{33})^{(6)} - (b''_{33})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{\mathrm{T}}_{34}(\mathrm{t}) = \mathrm{T}_{34}^0 + \int_0^t \left[ (b_{34})^{(6)} T_{33}(s_{(32)}) - \left( (b'_{34})^{(6)} - (b''_{34})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{34}(s_{(32)}) \right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\tilde{M}_{36})^{(7)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\tilde{M}_{36})^{(7)} t}$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[ (a_{36})^{(7)} G_{37}(s_{(36)}) - \left( (a'_{36})^{(7)} + a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[ (a_{37})^{(7)} G_{36}(s_{(36)}) - \left( (a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[ (a_{38})^{(7)} G_{37}(s_{(36)}) - \left( (a'_{38})^{(7)} + (a''_{38})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{38}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[ (b_{36})^{(7)} T_{37}(s_{(36)}) - \left( (b'_{36})^{(7)} - (b''_{36})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[ (b_{37})^{(7)} T_{36}(s_{(36)}) - \left( (b'_{37})^{(7)} - (b''_{37})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\overline{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^{0} + \int_0^t \left[ (b_{38})^{(7)} T_{37}(s_{(36)}) - \left( (b'_{38})^{(7)} - (b''_{38})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{38}(s_{(36)}) \right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$ , $T_i(0) = T_i^0$ , $G_i^0 \le (\hat{P}_{40})^{(8)}$ , $T_i^0 \le (\hat{Q}_{40})^{(8)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[ (a_{40})^{(8)} G_{41}(s_{(40)}) - \left( (a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[ (a_{41})^{(8)} G_{40}(s_{(40)}) - \left( (a'_{41})^{(8)} + (a''_{41})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[ (a_{42})^{(8)} G_{41}(s_{(40)}) - \left( (a'_{42})^{(8)} + (a''_{42})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{42}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^0 + \int_0^t \left[ (b_{40})^{(8)} T_{41}(s_{(40)}) - \left( (b'_{40})^{(8)} - (b''_{40})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^0 + \int_0^t \left[ (b_{41})^{(8)} T_{40}(s_{(40)}) - \left( (b'_{41})^{(8)} - (b''_{41})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\overline{\mathrm{T}}_{42}(\mathrm{t}) = \mathrm{T}_{42}^{0} + \int_0^t \left[ (b_{42})^{(8)} T_{41}(s_{(40)}) - \left( (b'_{42})^{(8)} - (b''_{42})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{42}(s_{(40)}) \right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** 166
Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy A

$G_i(0) = G_i^0$ , $T_i(0) = T_i^0$ , $G_i^0 \le (\hat{P}_{44})^{(9)}$ , $T_i^0 \le (\hat{Q}_{44})^{(9)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[ (a_{44})^{(9)} G_{45}(s_{(44)}) - \left( (a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45}(s_{(44)}), s_{(44)})\right) G_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}(s_{(44)}), s_{(44)})\right) G_{46}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{44}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{\mathrm{T}}_{46}(\mathrm{t}) = \mathrm{T}_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{46}(s_{(44)})\right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}}\right)\right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left(e^{(\hat{M}_{13})^{(1)} t} - 1\right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\hat{M}_{13})^{(1)} t} \le \frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{P}_{13})^{(1)} + G_{14}^0\right) e^{\left(-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0}\right)} + (\hat{P}_{13})^{(1)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \le G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}}\right)\right] ds_{(16)} = \quad 169$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1\right)$$

From which it follows that 170

$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \le \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left[\left((\hat{P}_{16})^{(2)} + G_{17}^0\right) e^{\left(-\frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0}\right)} + (\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t) \le G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} \left(G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} s_{(20)}}\right)\right] ds_{(20)} =$$

$$\left(1 + (a_{20})^{(3)} t\right) G_{21}^0 + \frac{(a_{20})^{(3)} (\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left(e^{(\hat{M}_{20})^{(3)} t} - 1\right)$$

From which it follows that 172

$$(G_{20}(t)-G_{20}^{0})e^{-(\hat{M}_{20})^{(3)}t}\leq\frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left[((\hat{P}_{20})^{(3)}+G_{21}^{0})e^{\left(-\frac{(\hat{P}_{20})^{(3)}+G_{21}^{0}}{G_{21}^{0}}\right)}+(\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 173

$$G_{24}(t)\leq G_{24}^{0}+\int_{0}^{t}\left[(a_{24})^{(4)}\left(G_{25}^{0}+(\hat{P}_{24})^{(4)}e^{(\hat{M}_{24})^{(4)}s_{(24)}}\right)\right]ds_{(24)}=$$

$$\left(1+(a_{24})^{(4)}t\right)G_{25}^{0}+\frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left(e^{(\hat{M}_{24})^{(4)}t}-1\right)$$

From which it follows that 174

$$(G_{24}(t)-G_{24}^{0})e^{-(\hat{M}_{24})^{(4)}t}\leq\frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left[((\hat{P}_{24})^{(4)}+G_{25}^{0})e^{\left(-\frac{(\hat{P}_{24})^{(4)}+G_{25}^{0}}{G_{25}^{0}}\right)}+(\hat{P}_{24})^{(4)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t)\leq G_{28}^{0}+\int_{0}^{t}\left[(a_{28})^{(5)}\left(G_{29}^{0}+(\hat{P}_{28})^{(5)}e^{(\hat{M}_{28})^{(5)}s_{(28)}}\right)\right]ds_{(28)}=$$

$$\left(1+(a_{28})^{(5)}t\right)G_{29}^{0}+\frac{(a_{28})^{(5)}(\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left(e^{(\hat{M}_{28})^{(5)}t}-1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t}\leq\frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t)\leq G_{32}^{0}+\int_{0}^{t}\left[(a_{32})^{(6)}\left(G_{33}^{0}+(\hat{P}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\right)\right]ds_{(32)}=$$

$$\left(1+(a_{32})^{(6)}t\right)G_{33}^{0}+\frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left(e^{(\hat{M}_{32})^{(6)}t}-1\right)$$

From which it follows that 177

$$(G_{32}(t)-G_{32}^{0})e^{-(\hat{M}_{32})^{(6)}t}\leq\frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left[((\hat{P}_{32})^{(6)}+G_{33}^{0})e^{\left(-\frac{(\hat{P}_{32})^{(6)}+G_{33}^{0}}{G_{33}^{0}}\right)}+(\hat{P}_{32})^{(6)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}$ , $G_{26}, T_{24}, T_{25}, T_{26}$

(l) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t) \le G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} \left(G_{37}^0 + (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} s_{(36)}}\right)\right] ds_{(36)} =$$
$$\left(1 + (a_{36})^{(7)} t\right) G_{37}^0 + \frac{(a_{36})^{(7)} (\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left(e^{(\hat{M}_{36})^{(7)} t} - 1\right)$$

From which it follows that

$$(G_{36}(t) - G_{36}^0) e^{-(\hat{M}_{36})^{(7)} t} \le \frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left[((\hat{P}_{36})^{(7)} + G_{37}^0) e^{\left(-\frac{(\hat{P}_{36})^{(7)} + G_{37}^0}{G_{37}^0}\right)} + (\hat{P}_{36})^{(7)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t) \le G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} \left(G_{41}^0 + (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} s_{(40)}}\right)\right] ds_{(40)} = \quad 180$$
$$\left(1 + (a_{40})^{(8)} t\right) G_{41}^0 + \frac{(a_{40})^{(8)} (\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}} \left(e^{(\hat{M}_{40})^{(8)} t} - 1\right)$$

From which it follows that 181

$$(G_{40}(t) - G_{40}^0) e^{-(\hat{M}_{40})^{(8)} t} \le \frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}} \left[((\hat{P}_{40})^{(8)} + G_{41}^0) e^{\left(-\frac{(\hat{P}_{40})^{(8)} + G_{41}^0}{G_{41}^0}\right)} + (\hat{P}_{40})^{(8)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 8

Analogous inequalities hold also for $G_{41}$ , $G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t) \le G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} \left(G_{45}^0 + (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} s_{(44)}}\right)\right] ds_{(44)} =$$
$$\left(1 + (a_{44})^{(9)} t\right) G_{45}^0 + \frac{(a_{44})^{(9)} (\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left(e^{(\hat{M}_{44})^{(9)} t} - 1\right)$$

From which it follows that

$$(G_{44}(t) - G_{44}^0) e^{-(\hat{M}_{44})^{(9)} t} \le \frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left[((\hat{P}_{44})^{(9)} + G_{45}^0) e^{\left(-\frac{(\hat{P}_{44})^{(9)} + G_{45}^0}{G_{45}^0}\right)} + (\hat{P}_{44})^{(9)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 9

Analogous inequalities hold also for $G_{45}$ , $G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}$ , $\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$ and to choose 182

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[(\hat{P}_{13})^{(1)}+\left((\hat{P}_{13})^{(1)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{13})^{(1)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{13})^{(1)} \quad 183$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[\left((\hat{Q}_{13})^{(1)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{13})^{(1)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{13})^{(1)}\right]\leq(\hat{Q}_{13})^{(1)} \quad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)},T^{(1)}\right),\left(G^{(2)},T^{(2)}\right)\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{13})^{(1)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{13})^{(1)}t}\}$$

Indeed if we denote

**Definition of** $\tilde{G},\tilde{T}:\left(\tilde{G},\tilde{T}\right)=\mathcal{A}^{(1)}(G,T)$

It results

$$\left|\tilde{G}_{13}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{13})^{(1)}\left|G_{14}^{(1)}-G_{14}^{(2)}\right|e^{-(\hat{M}_{13})^{(1)}s_{(13)}}e^{(\hat{M}_{13})^{(1)}s_{(13)}}\,ds_{(13)}+$$

$$\int_0^t\{(a'_{13})^{(1)}\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\hat{M}_{13})^{(1)}s_{(13)}}e^{-(\hat{M}_{13})^{(1)}s_{(13)}}+$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\hat{M}_{13})^{(1)}s_{(13)}}e^{(\hat{M}_{13})^{(1)}s_{(13)}}+$$

$$G_{13}^{(2)}\left|(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)-(a''_{13})^{(1)}\left(T_{14}^{(2)},s_{(13)}\right)\right|\ e^{-(\hat{M}_{13})^{(1)}s_{(13)}}e^{(\hat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|G^{(1)}-G^{(2)}\right|e^{-(\hat{M}_{13})^{(1)}t}\leq \quad 186$$
$$\frac{1}{(\hat{M}_{13})^{(1)}}\left((a_{13})^{(1)}+(a'_{13})^{(1)}+(\hat{A}_{13})^{(1)}+(\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}\right)d\left(\left(G^{(1)},T^{(1)};\,G^{(2)},T^{(2)}\right)\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 1:** The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\hat{P}_{13})^{(1)}e^{(\hat{M}_{13})^{(1)}t}$ *and* $(\hat{Q}_{13})^{(1)}e^{(\hat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i = 13,14,15$ depend only on $T_{14}$ and respectively on $G(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any $t$ where $G_i(t) = 0\ and\ T_i(t) = 0$

From 19 to 24 it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(1)} - (a_i'')^{(1)}(T_{14}(s_{(13)}), s_{(13)})\} ds_{(13)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(1)} t)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2\ and\ \left((\widehat{M}_{13})^{(1)}\right)_3$ : 187

**Remark 3**: if $G_{13}$ is bounded, the same property have also $G_{14}\ and\ G_{15}$ . indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a_{14}')^{(1)} G_{14}$ and by integrating

$$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)} \left((\widehat{M}_{13})^{(1)}\right)_1 / (a_{14}')^{(1)}$$

In the same way , one can obtain

$$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)} \left((\widehat{M}_{13})^{(1)}\right)_2 / (a_{15}')^{(1)}$$

If $G_{14}\ or\ G_{15}$ is bounded, the same property follows for $G_{13}$ , $G_{15}$ and $G_{13}$ , $G_{14}$ respectively.

**Remark 4:** If $G_{13}$ *is* bounded, from below, the same property holds for $G_{14}\ and\ G_{15}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{14}$ is bounded from below. 188

**Remark 5:** If $T_{13}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}(G(t), t)) = (b_{14}')^{(1)}$ then $T_{14} \to \infty$. 189

**Definition of** $(m)^{(1)}$ and $\varepsilon_1$ :

Indeed let $t_1$ be so that for $t > t_1$

$$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t), t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right)$, $t = log\frac{2}{\varepsilon_1}$ By taking now $\varepsilon_1$ sufficiently small one sees that $T_{14}$ is unbounded.
The same property holds for $T_{15}$ if $\lim_{t\to\infty}(b_{15}'')^{(1)}(G(t), t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}$ , $\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}\ and\ (\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[(\widehat{P}_{16})^{(2)}+\left((\hat{P}_{16})^{(2)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{16})^{(2)}\right]\leq(\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

**Definition of** $\widetilde{G_{19}},\widetilde{T_{19}}$ : $\left(\widetilde{G_{19}},\widetilde{T_{19}}\right)=\mathcal{A}^{(2)}(G_{19},T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{16})^{(2)}\left|G_{17}^{(1)}-G_{17}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)}+$$

$$\int_0^t\{(a'_{16})^{(2)}\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$G_{16}^{(2)}\left|(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)-(a''_{16})^{(2)}\left(T_{17}^{(2)},s_{(16)}\right)\right|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0,t]$ 197

From the hypotheses it follows

$$\left|(G_{19})^{(1)}-(G_{19})^{(2)}\right|\mathrm{e}^{-(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}\leq$$
$$\frac{1}{(\widehat{\mathrm{M}}_{16})^{(2)}}\left((a_{16})^{(2)}+(a'_{16})^{(2)}+(\widehat{\mathrm{A}}_{16})^{(2)}+(\widehat{\mathrm{P}}_{16})^{(2)}(\hat{k}_{16})^{(2)}\right)\mathrm{d}\left(((G_{19})^{(1)},(T_{19})^{(1)};\ (G_{19})^{(2)},(T_{19})^{(2)})\right)$$

And analogous inequalities for $G_i$ and $T_i$. Taking into account the hypothesis the result follows 198

**Remark 6:** The fact that we supposed $(a''_{16})^{(2)}$ and $(b''_{16})^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{\mathrm{P}}_{16})^{(2)}\mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}$ and $(\widehat{\mathrm{Q}}_{16})^{(2)}\mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i=16,17,18$ depend only on $\mathrm{T}_{17}$ and respectively on

$(G_{19})$(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 7:** There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$ 200

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(2)} - (a_i'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})\} ds_{(16)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(2)}t)} > 0 \quad \text{for } t > 0$$

**Definition of** $\left((\widehat{M}_{16})^{(2)}\right)_1$, $\left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$: 201

**Remark 8:** if $G_{16}$ is bounded, the same property have also $G_{17}$ and $G_{18}$. indeed if

$G_{16} < (\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt} \leq \left((\widehat{M}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)} G_{17}$ and by integrating

$$G_{17} \leq \left((\widehat{M}_{16})^{(2)}\right)_2 = G_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1 / (a_{17}')^{(2)}$$

In the same way , one can obtain

$$G_{18} \leq \left((\widehat{M}_{16})^{(2)}\right)_3 = G_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2 / (a_{18}')^{(2)}$$

If $G_{17}$ or $G_{18}$ is bounded, the same property follows for $G_{16}$, $G_{18}$ and $G_{16}$, $G_{17}$ respectively.

**Remark 9:** If $G_{16}$ is bounded, from below, the same property holds for $G_{17}$ and $G_{18}$. The proof is analogous with the preceding one. An analogous property is true if $G_{17}$ is bounded from below. 202

**Remark 10:** If $T_{16}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t), t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$. 203

**Definition of** $(m)^{(2)}$ and $\varepsilon_2$ :

Indeed let $t_2$ be so that for $t > t_2$

$$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right)$, $t = \log\frac{2}{\varepsilon_2}$ By taking now $\varepsilon_2$ sufficiently small one sees that $T_{17}$ is unbounded. 205
The same property holds for $T_{18}$ if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t), t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[(\widehat{P}_{20})^{(3)}+\left((\hat{P}_{20})^{(3)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{20})^{(3)}\right]\leq(\hat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself 210

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)},(T_{23})^{(1)}),((G_{23})^{(2)},(T_{23})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

**Definition of** $\widetilde{G_{23}},\widetilde{T_{23}}:(\widetilde{(G_{23})},\widetilde{(T_{23})})=\mathcal{A}^{(3)}((G_{23}),(T_{23}))$

It results 213

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{20})^{(3)}\left|G_{21}^{(1)}-G_{21}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}ds_{(20)}+$$

$$\int_0^t\{(a'_{20})^{(3)}\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$G_{20}^{(2)}\left|(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)},s_{(20)}\right)\right|\ e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\}ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses it follows

$$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}t}\leq \quad 214$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+(a'_{20})^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\widehat{k}_{20})^{(3)}\right)d\left(((G_{23})^{(1)},(T_{23})^{(1)};\ (G_{23})^{(2)},(T_{23})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 11:** The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ *and* $(\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i=20,21,22$ depend only on $T_{21}$ and respectively on

$(G_{23})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 12:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 216

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(3)} - (a_i'')^{(3)}(T_{21}(s_{(20)}), s_{(20)})\} ds_{(20)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(3)} t)} > 0$ for t > 0

**Definition of** $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2\ and\ \left((\widehat{M}_{20})^{(3)}\right)_3$ : 217

**Remark 13:** if $G_{20}$ is bounded, the same property have also $G_{21}\ and\ G_{22}$ . indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a_{21}')^{(3)} G_{21}$ and by integrating

$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1 / (a_{21}')^{(3)}$

In the same way , one can obtain

$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2 / (a_{22}')^{(3)}$

If $G_{21}\ or\ G_{22}$ is bounded, the same property follows for $G_{20}$ , $G_{22}$ and $G_{20}$ , $G_{21}$ respectively.

**Remark 14:** If $G_{20}$ *is* bounded, from below, the same property holds for $G_{21}\ and\ G_{22}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{21}$ is bounded from below. 218

**Remark 15:** If $T_{20}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t), t\right)) = (b_{21}')^{(3)}$ then $T_{21} \to \infty$. 219

**Definition of** $(m)^{(3)}$ and $\varepsilon_3$ :

Indeed let $t_3$ be so that for $t > t_3$

$(b_{21})^{(3)} - (b_i'')^{(3)}\left((G_{23})(t), t\right) < \varepsilon_3, T_{20}\,(t) > (m)^{(3)}$

Then $\frac{dT_{21}}{dt} \geq (a_{21})^{(3)}(m)^{(3)} - \varepsilon_3 T_{21}$ which leads to 220

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t} = \frac{1}{2}$ it results

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_3}$ By taking now $\varepsilon_3$ sufficiently small one sees that $T_{21}$ is unbounded.
The same property holds for $T_{22}$ if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t), t\right) = (b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}$ , $\frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}} < 1$ and to choose 221

$(\widehat{P}_{24})^{(4)}$ and $(\widehat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[(\widehat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \tag{222}$$

$$\frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \tag{223}$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself (224)

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric (225)

$$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{24})^{(4)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{27}}),(\widetilde{T_{27}})$ : $\left((\widetilde{G_{27}}),(\widetilde{T_{27}})\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{24})^{(4)}\left|G_{25}^{(1)}-G_{25}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\,ds_{(24)}+$$

$$\int_0^t\{(a'_{24})^{(4)}\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$G_{24}^{(2)}\left|(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)-(a''_{24})^{(4)}\left(T_{25}^{(2)},s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\}ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}t}\leq \tag{226}$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)}+(a'_{24})^{(4)}+(\widehat{A}_{24})^{(4)}+(\widehat{P}_{24})^{(4)}(\hat{k}_{24})^{(4)}\right)d\left(((G_{27})^{(1)},(T_{27})^{(1)};\ (G_{27})^{(2)},(T_{27})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 16:** The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ *and* $(\widehat{Q}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_+$. (227)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i=24,25,26$ depend only on $T_{25}$ and respectively on

$(G_{27})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 17:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 228

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(4)} - (a_i'')^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(4)} t)} > 0$ for t > 0

**Definition of** $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2\ and\ \left((\widehat{M}_{24})^{(4)}\right)_3$ : 229

**Remark 18:** if $G_{24}$ is bounded, the same property have also $G_{25}\ and\ G_{26}$ . indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a_{25}')^{(4)} G_{25}$ and by integrating

$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1 / (a_{25}')^{(4)}$

In the same way , one can obtain

$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2 / (a_{26}')^{(4)}$

If $G_{25}\ or\ G_{26}$ is bounded, the same property follows for $G_{24}$ , $G_{26}$ and $G_{24}$ , $G_{25}$ respectively.

**Remark 19:** If $G_{24}$ *is* bounded, from below, the same property holds for $G_{25}\ and\ G_{26}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{25}$ is bounded from below. 230

**Remark 20:** If $T_{24}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}\ ((G_{27})(t), t)) = (b_{25}')^{(4)}$ then $T_{25} \to \infty$. 231

**Definition of** $(m)^{(4)}$ and $\varepsilon_4$ :

Indeed let $t_4$ be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t), t) < \varepsilon_4, T_{24}\,(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_4}$ By taking now $\varepsilon_4$ sufficiently small one sees that $T_{25}$ is unbounded.
The same property holds for $T_{26}$ if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t), t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for $G_{29}$ , $G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}$ , $\frac{(b_i)^{(5)}}{(\widehat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)}+\left((\hat{P}_{28})^{(5)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{28})^{(5)}\right]\leq(\hat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$$d\left(((G_{31})^{(1)},(T_{31})^{(1)}),((G_{31})^{(2)},(T_{31})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\hat{M}_{28})^{(5)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\hat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{31}}),(\widetilde{T_{31}})$ : $\left((\widetilde{G_{31}}),(\widetilde{T_{31}})\right)=\mathcal{A}^{(5)}\left((G_{31}),(T_{31})\right)$

It results

$$|\tilde{G}_{28}^{(1)}-\tilde{G}_i^{(2)}|\leq\int_0^t(a_{28})^{(5)}|G_{29}^{(1)}-G_{29}^{(2)}|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\,ds_{(28)}+$$

$$\int_0^t\{(a'_{28})^{(5)}|G_{28}^{(1)}-G_{28}^{(2)}|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$(a''_{28})^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)|G_{28}^{(1)}-G_{28}^{(2)}|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$G_{28}^{(2)}|(a''_{28})^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)-(a''_{28})^{(5)}\left(T_{29}^{(2)},s_{(28)}\right)|\;e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\}ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on it follows

$$|(G_{31})^{(1)}-(G_{31})^{(2)}|e^{-(\widehat{M}_{28})^{(5)}t}\leq \quad 237$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)}+(a'_{28})^{(5)}+(\widehat{A}_{28})^{(5)}+(\widehat{P}_{28})^{(5)}(\widehat{k}_{28})^{(5)}\right)d\left(((G_{31})^{(1)},(T_{31})^{(1)};\;(G_{31})^{(2)},(T_{31})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 21:** The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\hat{P}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ *and* $(\hat{Q}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ respectively of $\mathbb{R}_+$. 238

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on $T_{29}$ and respectively on $(G_{31})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 22:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 239

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(5)} - (a_i'')^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(5)} t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2\ and\ \left((\widehat{M}_{28})^{(5)}\right)_3$ : 240

**Remark 23:** if $G_{28}$ is bounded, the same property have also $G_{29}\ and\ G_{30}$ . indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a_{29}')^{(5)} G_{29}$ and by integrating

$$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1 / (a_{29}')^{(5)}$$

In the same way , one can obtain

$$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2 / (a_{30}')^{(5)}$$

If $G_{29}\ or\ G_{30}$ is bounded, the same property follows for $G_{28}$ , $G_{30}$ and $G_{28}$ , $G_{29}$ respectively.

**Remark 24:** If $G_{28}$ *is* bounded, from below, the same property holds for $G_{29}\ and\ G_{30}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{29}$ is bounded from below. 241

**Remark 25:** If $T_{28}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}\,((G_{31})(t), t)) = (b_{29}')^{(5)}$ then $T_{29} \to \infty$. 242

**Definition of** $(m)^{(5)}$ and $\varepsilon_5$ :

Indeed let $t_5$ be so that for $t > t_5$

$$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t), t) < \varepsilon_5, T_{28}\,(t) > (m)^{(5)}$$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_5}$ By taking now $\varepsilon_5$ sufficiently small one sees that $T_{29}$ is unbounded.
The same property holds for $T_{30}$ if $\lim_{t\to\infty}(b_{30}'')^{(5)}\left((G_{31})(t), t\right) = (b_{30}')^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{33}\,, G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\ , \frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}} < 1$ and to choose 244

$(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[(\hat{P}_{32})^{(6)}+\left((\hat{P}_{32})^{(6)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{32})^{(6)} \tag{245}$$

$$\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[\left((\hat{Q}_{32})^{(6)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{32})^{(6)}\right]\leq(\hat{Q}_{32})^{(6)} \tag{246}$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric (247)

$$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\hat{M}_{32})^{(6)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\hat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{35}}),(\widetilde{T_{35}})$ : $\left((\widetilde{G_{35}}),(\widetilde{T_{35}})\right)=\mathcal{A}^{(6)}\left((G_{35}),(T_{35})\right)$

It results

$$|\tilde{G}_{32}^{(1)}-\tilde{G}_i^{(2)}|\leq\int_0^t(a_{32})^{(6)}|G_{33}^{(1)}-G_{33}^{(2)}|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\,ds_{(32)}+$$

$$\int_0^t\{(a'_{32})^{(6)}|G_{32}^{(1)}-G_{32}^{(2)}|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)|G_{32}^{(1)}-G_{32}^{(2)}|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$G_{32}^{(2)}|(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)-(a''_{32})^{(6)}\left(T_{33}^{(2)},s_{(32)}\right)|\;e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\}ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$|(G_{35})^{(1)}-(G_{35})^{(2)}|e^{-(\widehat{M}_{32})^{(6)}t}\leq \tag{248}$$
$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)}+(a'_{32})^{(6)}+(\widehat{A}_{32})^{(6)}+(\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}\right)d\left(((G_{35})^{(1)},(T_{35})^{(1)};\;(G_{35})^{(2)},(T_{35})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 26:** The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\hat{P}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ *and* $(\hat{Q}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ respectively of $\mathbb{R}_+$. (249)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i = 32,33,34$ depend only on $T_{33}$ and respectively on $(G_{35})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 27:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 250

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(6)} - (a_i'')^{(6)}(T_{33}(s_{(32)}), s_{(32)})\} ds_{(32)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(6)} t\right)} > 0$ for t > 0

**Definition of** $\left((\widehat{M}_{32})^{(6)}\right)_1, \left((\widehat{M}_{32})^{(6)}\right)_2\ and\ \left((\widehat{M}_{32})^{(6)}\right)_3$ : 251

**Remark 28:** if $G_{32}$ is bounded, the same property have also $G_{33}\ and\ G_{34}$ . indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a_{33}')^{(6)} G_{33}$ and by integrating

$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_1 / (a_{33}')^{(6)}$

In the same way , one can obtain

$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_2 / (a_{34}')^{(6)}$

If $G_{33}\ or\ G_{34}$ is bounded, the same property follows for $G_{32}$ , $G_{34}$ and $G_{32}$ , $G_{33}$ respectively.

**Remark 29:** If $G_{32}$ *is* bounded, from below, the same property holds for $G_{33}\ and\ G_{34}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{33}$ is bounded from below. 252

**Remark 30:** If $T_{32}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}\ ((G_{35})(t), t)) = (b_{33}')^{(6)}$ then $T_{33}\ \to \infty$. 253

**Definition of** $(m)^{(6)}$ and $\varepsilon_6$ :

Indeed let $t_6$ be so that for $t\ > t_6$

$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t), t\right) < \varepsilon_6, T_{32}\,(t) > (m)^{(6)}$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33}\ \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33}\ \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_6}$ By taking now $\varepsilon_6$ sufficiently small one sees that $T_{33}$ is unbounded.
The same property holds for $T_{34}$ if $\lim_{t\to\infty}(b_{34}'')^{(6)}\left((G_{35})(t), t(t), t\right) = (b_{34}')^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{37}\ , G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\ , \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$ and to choose

$(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[(\hat{P}_{36})^{(7)}+\left((\hat{P}_{36})^{(7)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{36})^{(7)} \tag{256}$$

$$\frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[\left((\hat{Q}_{36})^{(7)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{36})^{(7)}\right]\leq(\hat{Q}_{36})^{(7)} \tag{257}$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself
The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric (258)

$$d\left(((G_{39})^{(1)},(T_{39})^{(1)}),((G_{39})^{(2)},(T_{39})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\hat{M}_{36})^{(7)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\hat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{39}}),(\widetilde{T_{39}}):\left((\widetilde{G_{39}}),(\widetilde{T_{39}})\right)=\mathcal{A}^{(7)}((G_{39}),(T_{39}))$
It results

$$|\tilde{G}_{36}^{(1)}-\tilde{G}_i^{(2)}|\leq\int_0^t(a_{36})^{(7)}|G_{37}^{(1)}-G_{37}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\,ds_{(36)}+$$
$$\int_0^t\{(a'_{36})^{(7)}|G_{36}^{(1)}-G_{36}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$(a''_{36})^{(7)}(T_{37}^{(1)},s_{(36)})|G_{36}^{(1)}-G_{36}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$G_{36}^{(2)}|(a''_{36})^{(7)}(T_{37}^{(1)},s_{(36)})-(a''_{36})^{(7)}(T_{37}^{(2)},s_{(36)})|\;e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\}ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on it follows

$$|(G_{39})^{(1)}-(G_{39})^{(2)}|e^{-(\widehat{M}_{36})^{(7)}t}\leq \tag{259}$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}}\left((a_{36})^{(7)}+(a'_{36})^{(7)}+(\widehat{A}_{36})^{(7)}+(\widehat{P}_{36})^{(7)}(\widehat{k}_{36})^{(7)}\right)d\left(((G_{39})^{(1)},(T_{39})^{(1)};\;(G_{39})^{(2)},(T_{39})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 31:** The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ *and* $(\widehat{Q}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ respectively of $\mathbb{R}_+$. (260)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(7)}$ and $(b_i'')^{(7)}, i = 36,37,38$ depend only on $T_{37}$ and respectively on $(G_{39})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 32:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 261

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(7)} - (a_i'')^{(7)}(T_{37}(s_{(36)}), s_{(36)})\} ds_{(36)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(7)} t\right)} > 0$ for t > 0

**Definition of** $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2\ and\ \left((\widehat{M}_{36})^{(7)}\right)_3$ : 262

**Remark 33:** if $G_{36}$ is bounded, the same property have also $G_{37}\ and\ G_{38}$ . indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a_{37}')^{(7)} G_{37}$ and by integrating

$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1 / (a_{37}')^{(7)}$

In the same way , one can obtain

$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2 / (a_{38}')^{(7)}$

If $G_{37}\ or\ G_{38}$ is bounded, the same property follows for $G_{36}$ , $G_{38}$ and $G_{36}$ , $G_{37}$ respectively.

**Remark 34:** If $G_{36}$ *is* bounded, from below, the same property holds for $G_{37}\ and\ G_{38}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{37}$ is bounded from below. 263

**Remark 35:** If $T_{36}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(7)}\ ((G_{39})(t), t)) = (b_{37}')^{(7)}$ then $T_{37} \to \infty$. 264

**Definition of** $(m)^{(7)}$ and $\varepsilon_7$ :

Indeed let $t_7$ be so that for $t > t_7$

$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t), t) < \varepsilon_7, T_{36}\,(t) > (m)^{(7)}$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_7}$ By taking now $\varepsilon_7$ sufficiently small one sees that $T_{37}$ is unbounded.
The same property holds for $T_{38}$ if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t), t\right) = (b_{38}')^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}$ , $\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}} < 1$ and to choose $(\widehat{P}_{40})^{(8)}$ and $(\widehat{Q}_{40})^{(8)}$ large to have 266

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[(\widehat{P}_{40})^{(8)} + \left((\hat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{40})^{(8)} \quad 267$$

$$\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{40})^{(8)}\right] \leq (\hat{Q}_{40})^{(8)} \quad 268$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)}, (T_{43})^{(1)}), ((G_{43})^{(2)}, (T_{43})^{(2)})\right) = \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

**Definition of** $\widetilde{(G_{43})}, \widetilde{(T_{43})}$ : $\left(\widetilde{(G_{43})}, \widetilde{(T_{43})}\right) = \mathcal{A}^{(8)}((G_{43}), (T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{40})^{(8)}\left|G_{41}^{(1)} - G_{41}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\,ds_{(40)} +$$
$$\int_0^t\{(a'_{40})^{(8)}\left|G_{40}^{(1)} - G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{-(\widehat{M}_{40})^{(8)}s_{(40)}} +$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right)\left|G_{40}^{(1)} - G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}} +$$
$$G_{40}^{(2)}\left|(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right) - (a''_{40})^{(8)}\left(T_{41}^{(2)}, s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\}ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0, t]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)} - (G_{43})^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}t} \leq \quad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)} + (a'_{40})^{(8)} + (\widehat{A}_{40})^{(8)} + (\widehat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}\right)d\left(((G_{43})^{(1)}, (T_{43})^{(1)};\ (G_{43})^{(2)}, (T_{43})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

**Remark 36:** The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate 274

condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ $and$ $(\widehat{Q}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i = 40,41,42$ depend only on $T_{41}$ and respectively on $(G_{43})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 37** There does not exist any $t$ where $G_i(t) = 0\ and\ T_i(t) = 0$ 275

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(8)} - (a_i'')^{(8)}(T_{41}(s_{(40)}), s_{(40)})\} ds_{(40)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(8)}t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{40})^{(8)}\right)_1, \left((\widehat{M}_{40})^{(8)}\right)_2$ $and$ $\left((\widehat{M}_{40})^{(8)}\right)_3$ : 276

**Remark 38:** if $G_{40}$ is bounded, the same property have also $G_{41}$ $and$ $G_{42}$ . indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a_{41}')^{(8)}G_{41}$ and by integrating

$$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1/(a_{41}')^{(8)}$$

In the same way , one can obtain

$$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2/(a_{42}')^{(8)}$$

If $G_{41}$ $or$ $G_{42}$ is bounded, the same property follows for $G_{40}$ , $G_{42}$ and $G_{40}$ , $G_{41}$ respectively.

**Remark 39:** If $G_{40}$ $is$ bounded, from below, the same property holds for $G_{41}$ $and$ $G_{42}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{41}$ is bounded from below. 277

**Remark 40:** If $T_{40}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}((G_{43})(t), t)) = (b_{41}')^{(8)}$ then $T_{41} \to \infty$. 278

**Definition of** $(m)^{(8)}$ and $\varepsilon_8$ :

Indeed let $t_8$ be so that for $t > t_8$

$$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t), t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right)$, $t = log\frac{2}{\varepsilon_8}$ By taking now $\varepsilon_8$ sufficiently small one sees that $T_{41}$ is unbounded.
The same property holds for $T_{42}$ if $\lim_{t\to\infty}(b''_{42})^{(8)}\left((G_{43})(t), t(t), t\right) = (b'_{42})^{(8)}$



It is now sufficient to take $\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}$ , $\frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$ and to choose $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ large to have

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}\left[(\hat{P}_{44})^{(9)} + \left((\hat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}}\left[\left((\hat{Q}_{44})^{(9)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{44})^{(9)}\right] \leq (\hat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)}, (T_{47})^{(1)}), ((G_{47})^{(2)}, (T_{47})^{(2)})\right) =$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{44})^{(9)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{47}}), (\widetilde{T_{47}}) : \left((\widetilde{G_{47}}), (\widetilde{T_{47}})\right) = \mathcal{A}^{(9)}\left((G_{47}), (T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{44})^{(9)}\left|G_{45}^{(1)} - G_{45}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\,ds_{(44)} +$$
$$\int_0^t\{(a'_{44})^{(9)}\left|G_{44}^{(1)} - G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{-(\widehat{M}_{44})^{(9)}s_{(44)}} +$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right)\left|G_{44}^{(1)} - G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}} +$$
$$G_{44}^{(2)}\left|(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right) - (a''_{44})^{(9)}\left(T_{45}^{(2)}, s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\}ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)} - G^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}t} \leq$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)} + (a'_{44})^{(9)} + (\widehat{A}_{44})^{(9)} + (\widehat{P}_{44})^{(9)}(\hat{k}_{44})^{(9)}\right)d\left(((G_{47})^{(1)}, (T_{47})^{(1)};\ (G_{47})^{(2)}, (T_{47})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis (39,35,36) the result follows

**Remark 41:** The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by

$(\widehat{P}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ *and* $(\widehat{Q}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i = 44{,}45{,}46$ depend only on $T_{45}$ and respectively on $(G_{47})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 42:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$

From 99 to 44 it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t\{(a_i')^{(9)}-(a_i'')^{(9)}(T_{45}(s_{(44)}),s_{(44)})\}ds_{(44)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(9)}t)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{44})^{(9)}\right)_1, \left((\widehat{M}_{44})^{(9)}\right)_2$ *and* $\left((\widehat{M}_{44})^{(9)}\right)_3$ :

**Remark 43:** if $G_{44}$ is bounded, the same property have also $G_{45}\ and\ G_{46}$ . indeed if

$G_{44} < (\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt} \leq \left((\widehat{M}_{44})^{(9)}\right)_1 - (a_{45}')^{(9)}G_{45}$ and by integrating

$$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_1/(a_{45}')^{(9)}$$

In the same way , one can obtain

$$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_2/(a_{46}')^{(9)}$$

If $G_{45}\ or\ G_{46}$ is bounded, the same property follows for $G_{44}$ , $G_{46}$ and $G_{44}$ , $G_{45}$ respectively.

**Remark 44:** If $G_{44}$ *is* bounded, from below, the same property holds for $G_{45}\ and\ G_{46}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{45}$ is bounded from below.

**Remark 45:** If $T_{44}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t),t\right) = (b_{45}')^{(9)}$ then $T_{45} \to \infty$.

**Definition of** $(m)^{(9)}$ and $\varepsilon_9$ :

Indeed let $t_9$ be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t),t\right) < \varepsilon_9, T_{44}\,(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right), \quad t = log\frac{2}{\varepsilon_9}$ By taking now $\varepsilon_9$ sufficiently small one sees that $T_{45}$ is unbounded.
The same property holds for $T_{46}$ if $\lim_{t\to\infty}(b_{46}'')^{(9)}\left((G_{47})(t),t\right) = (b_{46}')^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

**Behavior of the solutions of equation** 280

**Theorem** If we denote and define

**Definition of** $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ :

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)} \le -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14}, t) + (a''_{14})^{(1)}(T_{14}, t) \le -(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)} \le -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G, t) - (b''_{14})^{(1)}(G, t) \le -(\tau_1)^{(1)}$

**Definition of** $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$ : 281

By $(\nu_1)^{(1)} > 0$, $(\nu_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$, $(u_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

**Definition of** $(\bar{\nu}_1)^{(1)},, (\bar{\nu}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$ : 282

By $(\bar{\nu}_1)^{(1)} > 0$, $(\bar{\nu}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$, $(\bar{u}_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

**Definition of** $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (\nu_0)^{(1)}$ :- 283

If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by
$(m_2)^{(1)} = (\nu_0)^{(1)}, (m_1)^{(1)} = (\nu_1)^{(1)}$, $if\ (\nu_0)^{(1)} < (\nu_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}, if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)}$,

and $\boxed{(\nu_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)}$, $if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)}$, $if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)}$,

and $\boxed{(u_0)^{(1)} = \frac{T^0_{13}}{T^0_{14}}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$

are defined

Then the solution of global equations satisfies the inequalities 285

$G^0_{13}e^{\left((S_1)^{(1)} - (p_{13})^{(1)}\right)t} \le G_{13}(t) \le G^0_{13}e^{(S_1)^{(1)}t}$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)}-(p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)}t}$$

$$\left(\frac{(a_{15})^{(1)}G_{13}^0}{(m_1)^{(1)}((S_1)^{(1)}-(p_{13})^{(1)}-(S_2)^{(1)})}\left[e^{((S_1)^{(1)}-(p_{13})^{(1)})t} - e^{-(S_2)^{(1)}t}\right] + G_{15}^0 e^{-(S_2)^{(1)}t} \le G_{15}(t) \le \right. \quad 286$$

$$\left.\frac{(a_{15})^{(1)}G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)}-(a'_{15})^{(1)})}[e^{(S_1)^{(1)}t} - e^{-(a'_{15})^{(1)}t}] + G_{15}^0 e^{-(a'_{15})^{(1)}t}\right)$$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)}t} \le T_{13}(t) \le T_{13}^0 e^{((R_1)^{(1)}+(r_{13})^{(1)})t}} \quad 287$$

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)}t} \le T_{13}(t) \le \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)}+(r_{13})^{(1)})t} \quad 288$$

$$\frac{(b_{15})^{(1)}T_{13}^0}{(\mu_1)^{(1)}((R_1)^{(1)}-(b'_{15})^{(1)})}\left[e^{(R_1)^{(1)}t} - e^{-(b'_{15})^{(1)}t}\right] + T_{15}^0 e^{-(b'_{15})^{(1)}t} \le T_{15}(t) \le \quad 289$$

$$\frac{(a_{15})^{(1)}T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)}+(r_{13})^{(1)}+(R_2)^{(1)})}\left[e^{((R_1)^{(1)}+(r_{13})^{(1)})t} - e^{-(R_2)^{(1)}t}\right] + T_{15}^0 e^{-(R_2)^{(1)}t}$$

**Definition of** $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:- 290

Where $(S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$

$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$

$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$

$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$

**Behavior of the solutions of equation** 291

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ : 292

$(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ four constants satisfying

$-(\sigma_2)^{(2)} \le -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(T_{17}, t) + (a''_{17})^{(2)}(T_{17}, t) \le -(\sigma_1)^{(2)}$ 293

$-(\tau_2)^{(2)} \le -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}((G_{19}), t) - (b''_{17})^{(2)}((G_{19}), t) \le -(\tau_1)^{(2)}$ 294

**Definition of** $(\nu_1)^{(2)}, (\nu_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$ : 295

By $(\nu_1)^{(2)} > 0$, $(\nu_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0$, $(u_2)^{(2)} < 0$ the roots 296

of the equations $(a_{17})^{(2)}(\nu^{(2)})^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 297

and $(b_{14})^{(2)}(u^{(2)})^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and 298

**Definition of** $(\bar{\nu}_1)^{(2)}, , (\bar{\nu}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$ : 299

By $(\bar{\nu}_1)^{(2)} > 0$, $(\bar{\nu}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0$, $(\bar{u}_2)^{(2)} < 0$ the 300

roots of the equations $(a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 301

and $(b_{17})^{(2)}\left(u^{(2)}\right)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 302

**Definition of** $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ :- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, ***if*** $(u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$, ***if*** $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)}$, ***if*** $(\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)}t}$

$(p_i)^{(2)}$ is defined by equation

$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)}t}$ 311

$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)}t}\right] + G_{18}^0 e^{-(S_2)^{(2)}t} \le G_{18}(t) \le\right.$ 312

$\left.\frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)}t} - e^{-(a'_{18})^{(2)}t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)}t}\right)$

$\boxed{T_{16}^0 e^{(R_1)^{(2)}t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}}$ 313

$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)}t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}$ 314

$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)}t} - e^{-(b'_{18})^{(2)}t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)}t} \le T_{18}(t) \le$ 315

$$\frac{(a_{18})^{(2)}T_{16}^{0}}{(\mu_2)^{(2)}((R_1)^{(2)}+(r_{16})^{(2)}+(R_2)^{(2)})}\left[e^{((R_1)^{(2)}+(r_{16})^{(2)})t}-e^{-(R_2)^{(2)}t}\right]+T_{18}^{0}e^{-(R_2)^{(2)}t}$$

**Definition of** $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$$

$$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)}$$ 318

$$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$$

**Behavior of the solutions** 319

**Theorem 3:** If we denote and define

**Definition of** $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ :

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}\big((G_{23}), t\big) \le -(\tau_1)^{(3)}$$

**Definition of** $(v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$ : 320

By $(v_1)^{(3)} > 0$, $(v_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{v}_1)^{(3)} > 0$, $(\bar{v}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

**Definition of** $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ :- 321

If we define $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (v_0)^{(3)}, (m_1)^{(3)} = (v_1)^{(3)}$, ***if*** $(v_0)^{(3)} < (v_1)^{(3)}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (\bar{v}_1)^{(3)}$, ***if*** $(v_1)^{(3)} < (v_0)^{(3)} < (\bar{v}_1)^{(3)}$,

and $\boxed{(v_0)^{(3)} = \frac{G_{20}^{0}}{G_{21}^{0}}}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_0)^{(3)}$, ***if*** $(\bar{v}_1)^{(3)} < (v_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)},\ \boldsymbol{if}\ (u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}, \boldsymbol{if}\ (u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)},$ and $\boxed{(u_0)^{(3)} = \frac{T_{20}^0}{T_{21}^0}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)},\ \boldsymbol{if}\ (\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G_{20}^0 e^{(S_1)^{(3)} t}$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)} t} \quad 323$$

$$\left(\frac{(a_{22})^{(3)} G_{20}^0}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\left[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)} t}\right] + G_{22}^0 e^{-(S_2)^{(3)} t} \le G_{22}(t) \le \right. \quad 324$$

$$\left. \frac{(a_{22})^{(3)} G_{20}^0}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}\left[e^{(S_1)^{(3)} t} - e^{-(a'_{22})^{(3)} t}\right] + G_{22}^0 e^{-(a'_{22})^{(3)} t}\right)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t}} \quad 325$$

$$\frac{1}{(\mu_1)^{(3)}} T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}} T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)} T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)} - (b'_{22})^{(3)})}\left[e^{(R_1)^{(3)} t} - e^{-(b'_{22})^{(3)} t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)} t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)} T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)} + (r_{20})^{(3)} + (R_2)^{(3)})}\left[e^{((R_1)^{(3)} + (r_{20})^{(3)})t} - e^{-(R_2)^{(3)} t}\right] + T_{22}^0 e^{-(R_2)^{(3)} t}$$

**Definition of** $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

**Behavior of the solutions of equation**

**Theorem:** If we denote and define

**Definition of** $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ :

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$

$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}((G_{27}), t) - (b''_{25})^{(4)}((G_{27}), t) \le -(\tau_1)^{(4)}$

**Definition of** $(v_1)^{(4)}, (v_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, v^{(4)}, u^{(4)}$ : 329

By $(v_1)^{(4)} > 0$ , $(v_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$ , $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_1)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

**Definition of** $(\bar{v}_1)^{(4)},, (\bar{v}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$ : 330

By $(\bar{v}_1)^{(4)} > 0$ , $(\bar{v}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$ , $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

**Definition of** $(m_1)^{(4)}$ , $(m_2)^{(4)}$ , $(\mu_1)^{(4)}, (\mu_2)^{(4)}, (\nu_0)^{(4)}$ :-

If we define $(m_1)^{(4)}$ , $(m_2)^{(4)}$ , $(\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (v_0)^{(4)}, (m_1)^{(4)} = (v_1)^{(4)}$, ***if*** $(v_0)^{(4)} < (v_1)^{(4)}$

$(m_2)^{(4)} = (v_1)^{(4)}, (m_1)^{(4)} = (\bar{v}_1)^{(4)}$ , ***if*** $(v_4)^{(4)} < (v_0)^{(4)} < (\bar{v}_1)^{(4)}$,
and $\boxed{(v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (v_4)^{(4)}, (m_1)^{(4)} = (v_0)^{(4)}$, ***if*** $(\bar{v}_4)^{(4)} < (v_0)^{(4)}$

and analogously

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$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)}$, ***if*** $(u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}$ , ***if*** $(u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)}$,
and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}$, ***if*** $(\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

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$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)}t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)}t}$$

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$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)}t}\right] + G_{26}^0 e^{-(S_2)^{(4)}t} \le G_{26}(t) \le\right.$$ 334

*(a26)4G240(m2)4(S1)4−(a26′)4e(S1)4t−e−(a26′)4t+ G260e−(a26′)4t*

$$\boxed{T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}}T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}}T_{24}^0 e^{((R_1)^{(4)}+(r_{24})^{(4)})t} \quad 335$$

$$\frac{(b_{26})^{(4)}T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)}-(b'_{26})^{(4)})}\left[e^{(R_1)^{(4)}t} - e^{-(b'_{26})^{(4)}t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)}t} \le T_{26}(t) \le \quad 336$$

$$\frac{(a_{26})^{(4)}T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)}+(r_{24})^{(4)}+(R_2)^{(4)})}\left[e^{((R_1)^{(4)}+(r_{24})^{(4)})t} - e^{-(R_2)^{(4)}t}\right] + T_{26}^0 e^{-(R_2)^{(4)}t}$$

**Definition of** $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

**Behavior of the solutions of equation** 338

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ :

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

**Definition of** $(v_1)^{(5)}, (v_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, v^{(5)}, u^{(5)}$ : 339

By $(v_1)^{(5)} > 0$, $(v_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$, $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(v^{(5)}\big)^2 + (\sigma_1)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

**Definition of** $(\bar{v}_1)^{(5)}, , (\bar{v}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$ : 340

By $(\bar{v}_1)^{(5)} > 0$, $(\bar{v}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$, $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(v^{(5)}\big)^2 + (\sigma_2)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

**Definition of** $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (v_0)^{(5)}$ :-

If we define $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (v_0)^{(5)}, (m_1)^{(5)} = (v_1)^{(5)}$, ***if*** $(v_0)^{(5)} < (v_1)^{(5)}$

$(m_2)^{(5)} = (v_1)^{(5)}, (m_1)^{(5)} = (\bar{v}_1)^{(5)}$, ***if*** $(v_1)^{(5)} < (v_0)^{(5)} < (\bar{v}_1)^{(5)}$,
and $\boxed{(v_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)},\ \boldsymbol{if}\ (u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}, \boldsymbol{if}\ (u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)},$

and $\boxed{(u_0)^{(5)} = \frac{T_{28}^0}{T_{29}^0}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}, \boldsymbol{if}\ (\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{28}(t) \le G_{28}^0 e^{(S_1)^{(5)} t}$

where $(p_i)^{(5)}$ is defined by equation

$\frac{1}{(m_5)^{(5)}} G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}} G_{28}^0 e^{(S_1)^{(5)} t}$ 343

$\left(\frac{(a_{30})^{(5)} G_{28}^0}{(m_1)^{(5)}((S_1)^{(5)} - (p_{28})^{(5)} - (S_2)^{(5)})}\left[e^{((S_1)^{(5)} - (p_{28})^{(5)})t} - e^{-(S_2)^{(5)} t}\right] + G_{30}^0 e^{-(S_2)^{(5)} t} \le G_{30}(t) \le\right.$ 344

*(a30)5G280(m2)5(S1)5−(a30′)5e(S1)5t−e−(a30′)5t+ G300e−(a30′)5t*

$\boxed{T_{28}^0 e^{(R_1)^{(5)} t} \le T_{28}(t) \le T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}}$ 345

$\frac{1}{(\mu_1)^{(5)}} T_{28}^0 e^{(R_1)^{(5)} t} \le T_{28}(t) \le \frac{1}{(\mu_2)^{(5)}} T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}$ 346

$\frac{(b_{30})^{(5)} T_{28}^0}{(\mu_1)^{(5)}((R_1)^{(5)} - (b'_{30})^{(5)})}\left[e^{(R_1)^{(5)} t} - e^{-(b'_{30})^{(5)} t}\right] + T_{30}^0 e^{-(b'_{30})^{(5)} t} \le T_{30}(t) \le$ 347

$\frac{(a_{30})^{(5)} T_{28}^0}{(\mu_2)^{(5)}((R_1)^{(5)} + (r_{28})^{(5)} + (R_2)^{(5)})}\left[e^{((R_1)^{(5)} + (r_{28})^{(5)})t} - e^{-(R_2)^{(5)} t}\right] + T_{30}^0 e^{-(R_2)^{(5)} t}$

**Definition of** $(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$:- 348

Where $(S_1)^{(5)} = (a_{28})^{(5)}(m_2)^{(5)} - (a'_{28})^{(5)}$

$(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}$

$(R_1)^{(5)} = (b_{28})^{(5)}(\mu_2)^{(5)} - (b'_{28})^{(5)}$

$(R_2)^{(5)} = (b'_{30})^{(5)} - (r_{30})^{(5)}$

**Behavior of the solutions of equation** 349

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ :

$(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)} \le -(a'_{32})^{(6)} + (a'_{33})^{(6)} - (a''_{32})^{(6)}(T_{33}, t) + (a''_{33})^{(6)}(T_{33}, t) \le -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)} \le -(b'_{32})^{(6)} + (b'_{33})^{(6)} - (b''_{32})^{(6)}\big((G_{35}), t\big) - (b''_{33})^{(6)}\big((G_{35}), t\big) \le -(\tau_1)^{(6)}$$

**Definition of** $(\nu_1)^{(6)}, (\nu_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, \nu^{(6)}, u^{(6)}$ : 350

By $(\nu_1)^{(6)} > 0$ , $(\nu_2)^{(6)} < 0$ and respectively $(u_1)^{(6)} > 0$ , $(u_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(6)}, , (\bar{\nu}_2)^{(6)}, (\bar{u}_1)^{(6)}, (\bar{u}_2)^{(6)}$ : 351

By $(\bar{\nu}_1)^{(6)} > 0$ , $(\bar{\nu}_2)^{(6)} < 0$ and respectively $(\bar{u}_1)^{(6)} > 0$ , $(\bar{u}_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$

**Definition of** $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}, (\nu_0)^{(6)}$ :-

If we define $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}$ by

$$(m_2)^{(6)} = (\nu_0)^{(6)}, (m_1)^{(6)} = (\nu_1)^{(6)}, \ \boldsymbol{if}\ (\nu_0)^{(6)} < (\nu_1)^{(6)}$$

$$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}, \boldsymbol{if}\ (\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)},$$

and $\boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}}$

$$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$$

and analogously 352

$$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}, \ \boldsymbol{if}\ (u_0)^{(6)} < (u_1)^{(6)}$$

$$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}, \boldsymbol{if}\ (u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)},$$

and $\boxed{(u_0)^{(6)} = \frac{T^0_{32}}{T^0_{33}}}$

$$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}, \boldsymbol{if}\ (\bar{u}_1)^{(6)} < (u_0)^{(6)} \text{ where } (u_1)^{(6)}, (\bar{u}_1)^{(6)}$$

Then the solution of global equations satisfies the inequalities 353

$$G^0_{32}e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{32}(t) \le G^0_{32}e^{(S_1)^{(6)}t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}}G^0_{32}e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{33}(t) \le \frac{1}{(m_2)^{(6)}}G^0_{32}e^{(S_1)^{(6)}t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)}G^0_{32}}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})}\left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)}t}\right] + G^0_{34}e^{-(S_2)^{(6)}t} \le G_{34}(t) \le \right. \quad 355$$

$$\frac{(a_{34})^{(6)}G_{32}^{0}}{(m_2)^{(6)}((S_1)^{(6)}-(a'_{34})^{(6)})}\left[e^{(S_1)^{(6)}t}-e^{-(a'_{34})^{(6)}t}\right]+\ G_{34}^{0}e^{-(a'_{34})^{(6)}t}\Big)$$

$$\boxed{T_{32}^{0}e^{(R_1)^{(6)}t}\le T_{32}(t)\le T_{32}^{0}e^{((R_1)^{(6)}+(r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}}T_{32}^{0}e^{(R_1)^{(6)}t}\le T_{32}(t)\le\frac{1}{(\mu_2)^{(6)}}T_{32}^{0}e^{((R_1)^{(6)}+(r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)}T_{32}^{0}}{(\mu_1)^{(6)}((R_1)^{(6)}-(b'_{34})^{(6)})}\left[e^{(R_1)^{(6)}t}-e^{-(b'_{34})^{(6)}t}\right]+T_{34}^{0}e^{-(b'_{34})^{(6)}t}\le T_{34}(t)\le \quad 358$$

$$\frac{(a_{34})^{(6)}T_{32}^{0}}{(\mu_2)^{(6)}((R_1)^{(6)}+(r_{32})^{(6)}+(R_2)^{(6)})}\left[e^{((R_1)^{(6)}+(r_{32})^{(6)})t}-e^{-(R_2)^{(6)}t}\right]+T_{34}^{0}e^{-(R_2)^{(6)}t}$$

**Definition of** $(S_1)^{(6)},(S_2)^{(6)},(R_1)^{(6)},(R_2)^{(6)}$:- 359

Where $(S_1)^{(6)}=(a_{32})^{(6)}(m_2)^{(6)}-(a'_{32})^{(6)}$

$(S_2)^{(6)}=(a_{34})^{(6)}-(p_{34})^{(6)}$

$(R_1)^{(6)}=(b_{32})^{(6)}(\mu_2)^{(6)}-(b'_{32})^{(6)}$

$(R_2)^{(6)}=(b'_{34})^{(6)}-(r_{34})^{(6)}$

**Behavior of the solutions of equation**

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(7)},(\sigma_2)^{(7)},(\tau_1)^{(7)},(\tau_2)^{(7)}$ :

$(\sigma_1)^{(7)},(\sigma_2)^{(7)},(\tau_1)^{(7)},(\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)}\le-(a'_{36})^{(7)}+(a'_{37})^{(7)}-(a''_{36})^{(7)}(T_{37},t)+(a''_{37})^{(7)}(T_{37},t)\le-(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)}\le-(b'_{36})^{(7)}+(b'_{37})^{(7)}-(b''_{36})^{(7)}\big((G_{39}),t\big)-(b''_{37})^{(7)}\big((G_{39}),t\big)\le-(\tau_1)^{(7)}$$

**Definition of** $(v_1)^{(7)},(v_2)^{(7)},(u_1)^{(7)},(u_2)^{(7)},v^{(7)},u^{(7)}$ : 361

By $(v_1)^{(7)}>0,(v_2)^{(7)}<0$ and respectively $(u_1)^{(7)}>0,(u_2)^{(7)}<0$ the roots of the equations $(a_{37})^{(7)}\big(v^{(7)}\big)^2+(\sigma_1)^{(7)}v^{(7)}-(a_{36})^{(7)}=0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2+(\tau_1)^{(7)}u^{(7)}-(b_{36})^{(7)}=0$ and

**Definition of** $(\bar{v}_1)^{(7)},,(\bar{v}_2)^{(7)},(\bar{u}_1)^{(7)},(\bar{u}_2)^{(7)}$ : 362

By $(\bar{v}_1)^{(7)}>0,(\bar{v}_2)^{(7)}<0$ and respectively $(\bar{u}_1)^{(7)}>0,(\bar{u}_2)^{(7)}<0$ the roots of the equations $(a_{37})^{(7)}\big(v^{(7)}\big)^2+(\sigma_2)^{(7)}v^{(7)}-(a_{36})^{(7)}=0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2+(\tau_2)^{(7)}u^{(7)}-(b_{36})^{(7)}=0$

**Definition of** $(m_1)^{(7)},(m_2)^{(7)},(\mu_1)^{(7)},(\mu_2)^{(7)},(v_0)^{(7)}$ :-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$$(m_2)^{(7)} = (\nu_0)^{(7)}, (m_1)^{(7)} = (\nu_1)^{(7)}, \ \boldsymbol{if}\ (\nu_0)^{(7)} < (\nu_1)^{(7)}$$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\bar{\nu}_1)^{(7)}, \boldsymbol{if}\ (\nu_1)^{(7)} < (\nu_0)^{(7)} < (\bar{\nu}_1)^{(7)},$$

and $\boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\nu_0)^{(7)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(7)} < (\nu_0)^{(7)}$$

and analogously 363

$$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}, \ \boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$$

and $\boxed{(u_0)^{(7)} = \frac{T_{36}^0}{T_{37}^0}}$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)} \text{ where } (u_1)^{(7)}, (\bar{u}_1)^{(7)}$$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^0 e^{(S_1)^{(7)} t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)} t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)}((S_1)^{(7)} - (p_{36})^{(7)} - (S_2)^{(7)})}\left[e^{((S_1)^{(7)} - (p_{36})^{(7)})t} - e^{-(S_2)^{(7)} t}\right] + G_{38}^0 e^{-(S_2)^{(7)} t} \le G_{38}(t) \le \right. \quad 366$$

$$\left. \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)}((S_1)^{(7)} - (a'_{38})^{(7)})}\left[e^{(S_1)^{(7)} t} - e^{-(a'_{38})^{(7)} t}\right] + G_{38}^0 e^{-(a'_{38})^{(7)} t}\right)$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t} \quad 368$$

$$\frac{(b_{38})^{(7)} T_{36}^0}{(\mu_1)^{(7)}((R_1)^{(7)} - (b'_{38})^{(7)})}\left[e^{(R_1)^{(7)} t} - e^{-(b'_{38})^{(7)} t}\right] + T_{38}^0 e^{-(b'_{38})^{(7)} t} \le T_{38}(t) \le \quad 369$$

$$\frac{(a_{38})^{(7)} T_{36}^0}{(\mu_2)^{(7)}((R_1)^{(7)} + (r_{36})^{(7)} + (R_2)^{(7)})}\left[e^{((R_1)^{(7)} + (r_{36})^{(7)})t} - e^{-(R_2)^{(7)} t}\right] + T_{38}^0 e^{-(R_2)^{(7)} t}$$

**Definition of** $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$

$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$

$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$

**Behavior of the solutions of equation** 371

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ :

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$-(\sigma_2)^{(8)} \le -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \le -(\sigma_1)^{(8)}$

$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\big((G_{43}), t\big) - (b''_{41})^{(8)}\big((G_{43}), t\big) \le -(\tau_1)^{(8)}$

**Definition of** $(\nu_1)^{(8)}, (\nu_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, \nu^{(8)}, u^{(8)}$ : 372

By $(\nu_1)^{(8)} > 0, (\nu_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0, (u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(8)}, , (\bar{\nu}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$ :

By $(\bar{\nu}_1)^{(8)} > 0, (\bar{\nu}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0, (\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

**Definition of** $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}, (\nu_0)^{(8)}$ :-

If we define $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$(m_2)^{(8)} = (\nu_0)^{(8)}, (m_1)^{(8)} = (\nu_1)^{(8)}$, ***if*** $(\nu_0)^{(8)} < (\nu_1)^{(8)}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\bar{\nu}_1)^{(8)}$, ***if*** $(\nu_1)^{(8)} < (\nu_0)^{(8)} < (\bar{\nu}_1)^{(8)}$,
and $\boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\nu_0)^{(8)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(8)} < (\nu_0)^{(8)}$

and analogously 374

$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)},\ \boldsymbol{if}\ (u_0)^{(8)} < (u_1)^{(8)}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}, \boldsymbol{if}\ (u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)},$

and $\boxed{(u_0)^{(8)} = \frac{T_{40}^0}{T_{41}^0}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}, \boldsymbol{if}\ (\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^0 e^{(S_1)^{(8)} t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}} G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}} G_{40}^0 e^{(S_1)^{(8)} t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)} G_{40}^0}{(m_1)^{(8)}((S_1)^{(8)} - (p_{40})^{(8)} - (S_2)^{(8)})}\left[e^{((S_1)^{(8)} - (p_{40})^{(8)})t} - e^{-(S_2)^{(8)} t}\right] + G_{42}^0 e^{-(S_2)^{(8)} t} \le G_{42}(t) \le \right. \quad 377$$

$$\left. \frac{(a_{42})^{(8)} G_{40}^0}{(m_2)^{(8)}((S_1)^{(8)} - (a'_{42})^{(8)})}\left[e^{(S_1)^{(8)} t} - e^{-(a'_{42})^{(8)} t}\right] + G_{42}^0 e^{-(a'_{42})^{(8)} t}\right)$$

$$\boxed{T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}} T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le \frac{1}{(\mu_2)^{(8)}} T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)} T_{40}^0}{(\mu_1)^{(8)}((R_1)^{(8)} - (b'_{42})^{(8)})}\left[e^{(R_1)^{(8)} t} - e^{-(b'_{42})^{(8)} t}\right] + T_{42}^0 e^{-(b'_{42})^{(8)} t} \le T_{42}(t) \le \quad 380$$

$$\frac{(a_{42})^{(8)} T_{40}^0}{(\mu_2)^{(8)}((R_1)^{(8)} + (r_{40})^{(8)} + (R_2)^{(8)})}\left[e^{((R_1)^{(8)} + (r_{40})^{(8)})t} - e^{-(R_2)^{(8)} t}\right] + T_{42}^0 e^{-(R_2)^{(8)} t}$$

**Definition of** $(S_1)^{(8)}, (S_2)^{(8)}, (R_1)^{(8)}, (R_2)^{(8)}$:- 381

Where $(S_1)^{(8)} = (a_{40})^{(8)}(m_2)^{(8)} - (a'_{40})^{(8)}$

$(S_2)^{(8)} = (a_{42})^{(8)} - (p_{42})^{(8)}$

$(R_1)^{(8)} = (b_{40})^{(8)}(\mu_2)^{(8)} - (b'_{40})^{(8)}$

$(R_2)^{(8)} = (b'_{42})^{(8)} - (r_{42})^{(8)}$

**Behavior of the solutions of equation 37 to 92** 382

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ :

$(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ four constants satisfying

$$-(\sigma_2)^{(9)} \le -(a'_{44})^{(9)} + (a'_{45})^{(9)} - (a''_{44})^{(9)}(T_{45}, t) + (a''_{45})^{(9)}(T_{45}, t) \le -(\sigma_1)^{(9)}$$

$$-(\tau_2)^{(9)} \le -(b'_{44})^{(9)} + (b'_{45})^{(9)} - (b''_{44})^{(9)}\big((G_{47}), t\big) - (b''_{45})^{(9)}\big((G_{47}), t\big) \le -(\tau_1)^{(9)}$$

**Definition of** $(\nu_1)^{(9)}, (\nu_2)^{(9)}, (u_1)^{(9)}, (u_2)^{(9)}, \nu^{(9)}, u^{(9)}$ :

By $(\nu_1)^{(9)} > 0$ , $(\nu_2)^{(9)} < 0$ and respectively $(u_1)^{(9)} > 0$ , $(u_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_1)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(9)}, , (\bar{\nu}_2)^{(9)}, (\bar{u}_1)^{(9)}, (\bar{u}_2)^{(9)}$ :

By $(\bar{\nu}_1)^{(9)} > 0$ , $(\bar{\nu}_2)^{(9)} < 0$ and respectively $(\bar{u}_1)^{(9)} > 0$ , $(\bar{u}_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_2)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$

**Definition of** $(m_1)^{(9)}, (m_2)^{(9)}, (\mu_1)^{(9)}, (\mu_2)^{(9)}, (\nu_0)^{(9)}$ :-

If we define $(m_1)^{(9)}, (m_2)^{(9)}, (\mu_1)^{(9)}, (\mu_2)^{(9)}$ by

$$(m_2)^{(9)} = (\nu_0)^{(9)}, (m_1)^{(9)} = (\nu_1)^{(9)}, \ \boldsymbol{if}\ (\nu_0)^{(9)} < (\nu_1)^{(9)}$$

$$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\bar{\nu}_1)^{(9)}, \boldsymbol{if}\ (\nu_1)^{(9)} < (\nu_0)^{(9)} < (\bar{\nu}_1)^{(9)},$$

and $\boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$

$$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\nu_0)^{(9)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(9)} < (\nu_0)^{(9)}$$

and analogously

$$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)}, \ \boldsymbol{if}\ (u_0)^{(9)} < (u_1)^{(9)}$$

$$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}, \boldsymbol{if}\ (u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)},$$

and $\boxed{(u_0)^{(9)} = \frac{T^0_{44}}{T^0_{45}}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}, \boldsymbol{if}\ (\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G^0_{44} e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{44}(t) \le G^0_{44} e^{(S_1)^{(9)} t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}}G_{44}^0 e^{((S_1)^{(9)}-(p_{44})^{(9)})t} \le G_{45}(t) \le \frac{1}{(m_2)^{(9)}}G_{44}^0 e^{(S_1)^{(9)}t}$$

$$\left(\frac{(a_{46})^{(9)}G_{44}^0}{(m_1)^{(9)}((S_1)^{(9)}-(p_{44})^{(9)}-(S_2)^{(9)})}\left[e^{((S_1)^{(9)}-(p_{44})^{(9)})t} - e^{-(S_2)^{(9)}t}\right] + G_{46}^0 e^{-(S_2)^{(9)}t} \le G_{46}(t) \le \right.$$

$$\left.\frac{(a_{46})^{(9)}G_{44}^0}{(m_2)^{(9)}((S_1)^{(9)}-(a'_{46})^{(9)})}[e^{(S_1)^{(9)}t} - e^{-(a'_{46})^{(9)}t}] + G_{46}^0 e^{-(a'_{46})^{(9)}t}\right)$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le T_{44}^0 e^{((R_1)^{(9)}+(r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}}T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le \frac{1}{(\mu_2)^{(9)}}T_{44}^0 e^{((R_1)^{(9)}+(r_{44})^{(9)})t}$$

$$\frac{(b_{46})^{(9)}T_{44}^0}{(\mu_1)^{(9)}((R_1)^{(9)}-(b'_{46})^{(9)})}\left[e^{(R_1)^{(9)}t} - e^{-(b'_{46})^{(9)}t}\right] + T_{46}^0 e^{-(b'_{46})^{(9)}t} \le T_{46}(t) \le$$

$$\frac{(a_{46})^{(9)}T_{44}^0}{(\mu_2)^{(9)}((R_1)^{(9)}+(r_{44})^{(9)}+(R_2)^{(9)})}\left[e^{((R_1)^{(9)}+(r_{44})^{(9)})t} - e^{-(R_2)^{(9)}t}\right] + T_{46}^0 e^{-(R_2)^{(9)}t}$$

**Definition of** $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$

**Proof :** From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14},t)\right) - (a''_{14})^{(1)}(T_{14},t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

**Definition of** $v^{(1)}$ :- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)}\right) \le \frac{dv^{(1)}}{dt} \le -\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(1)}, (\nu_0)^{(1)}$ :-

For $0 < \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}} < (\nu_1)^{(1)} < (\bar{\nu}_1)^{(1)}$

$$\nu^{(1)}(t) \ge \frac{(\nu_1)^{(1)}+(C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_0)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(\nu_1)^{(1)}-(\nu_0)^{(1)}}{(\nu_0)^{(1)}-(\nu_2)^{(1)}}}$$

it follows $(\nu_0)^{(1)} \leq \nu^{(1)}(t) \leq (\nu_1)^{(1)}$

In the same manner , we get 384

$$\nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{\nu}_1)^{(1)}-(\nu_0)^{(1)}}{(\nu_0)^{(1)}-(\bar{\nu}_2)^{(1)}}}$$

From which we deduce $(\nu_0)^{(1)} \leq \nu^{(1)}(t) \leq (\bar{\nu}_1)^{(1)}$

If $0 < (\nu_1)^{(1)} < (\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0} < (\bar{\nu}_1)^{(1)}$ we find like in the previous case, 385

$$(\nu_1)^{(1)} \leq \frac{(\nu_1)^{(1)}+(C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}} \leq \nu^{(1)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\bar{\nu}_1)^{(1)}$$

If $0 < (\nu_1)^{(1)} \leq (\bar{\nu}_1)^{(1)} \leq \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$ , we obtain 386

$$(\nu_1)^{(1)} \leq \nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\nu_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(1)}(t)$ :-

$$(m_2)^{(1)} \leq \nu^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{\nu^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(1)}(t)$ :-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{13})^{(1)} = (a''_{14})^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)}G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)}T_{14}(t)$ This is an important

consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

**Proof** : From global equations we obtain 387

$$\frac{d\nu^{(2)}}{dt} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right) - (a''_{17})^{(2)}(T_{17}, t)\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

**Definition of** $\nu^{(2)}$ :- $\boxed{\nu^{(2)} = \frac{G_{16}}{G_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \le \frac{d\nu^{(2)}}{dt} \le -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

**Definition of** $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$ :-

For $0 < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \ge \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}}{1 + (C)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(C)^{(2)} = \frac{(\nu_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \le \nu^{(2)}(t) \le (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \le \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \le \nu^{(2)}(t) \le (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \le \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}}{1 + (C)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}} \le \nu^{(2)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \le (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \le (\bar{\nu}_1)^{(2)} \le (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$ , we obtain 394

$$(\nu_1)^{(2)} \le \nu^{(2)}(t) \le \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \le (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(2)}(t)$ :- 395

$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}$, $\boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$

In a completely analogous way, we obtain 396

**Definition of** $u^{(2)}(t)$ :-

$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}$, $\boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :** 397

If $(a''_{16})^{(2)} = (a''_{17})^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)} G_{17}(t)$

Analogously if $(b''_{16})^{(2)} = (b''_{17})^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)} T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

**Proof** : From global equations we obtain 398

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right) - (a''_{21})^{(3)}(T_{21}, t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

**Definition of** $\nu^{(3)}$ :- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right) \leq \frac{d\nu^{(3)}}{dt} \leq -\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right)$$

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From which one obtains

For $0 < (\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\nu_1)^{(3)} < (\bar{\nu}_1)^{(3)}$

$$\nu^{(3)}(t) \geq \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(\nu_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\nu_2)^{(3)}}}$$

it follows $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\nu_1)^{(3)}$

In the same manner , we get 401

$$\nu^{(3)}(t) \le \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{\nu}_1)^{(3)}-(\nu_0)^{(3)}}{(\nu_0)^{(3)}-(\bar{\nu}_2)^{(3)}}}$$

**Definition of** $(\bar{\nu}_1)^{(3)}$ :-

From which we deduce $(\nu_0)^{(3)} \le \nu^{(3)}(t) \le (\bar{\nu}_1)^{(3)}$

If $0 < (\nu_1)^{(3)} < (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (\bar{\nu}_1)^{(3)}$ we find like in the previous case, 402

$$(\nu_1)^{(3)} \le \frac{(\nu_1)^{(3)}+(C)^{(3)}(\nu_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}} \le \nu^{(3)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \le (\bar{\nu}_1)^{(3)}$$

If $0 < (\nu_1)^{(3)} \le (\bar{\nu}_1)^{(3)} \le (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}$ , we obtain 403

$$(\nu_1)^{(3)} \le \nu^{(3)}(t) \le \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \le (\nu_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(3)}(t)$ :-

$$(m_2)^{(3)} \le \nu^{(3)}(t) \le (m_1)^{(3)}, \quad \boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(3)}(t)$ :-

$$(\mu_2)^{(3)} \le u^{(3)}(t) \le (\mu_1)^{(3)}, \quad \boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{20}'')^{(3)} = (a_{21}'')^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)}G_{21}(t)$

Analogously if $(b_{20}'')^{(3)} = (b_{21}'')^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)}T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

**Proof** : From global equations we obtain 404

$$\frac{dv^{(4)}}{dt} = (a_{24})^{(4)} - \left((a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right) - (a''_{25})^{(4)}(T_{25},t)v^{(4)} - (a_{25})^{(4)}v^{(4)}$$

**Definition of** $v^{(4)}$ :- $\boxed{v^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)}\right) \le \frac{dv^{(4)}}{dt} \le -\left((a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_4)^{(4)}v^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$ :-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \ge \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \le \nu^{(4)}(t) \le (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \le \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \le \nu^{(4)}(t) \le (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \le \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}}{1 + (C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}} \le \nu^{(4)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \le (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \le (\bar{\nu}_1)^{(4)} \le \boxed{(\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}}}$ , we obtain 407

$$(\nu_1)^{(4)} \le \nu^{(4)}(t) \le \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \le (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(4)}(t)$ :-

$$(m_2)^{(4)} \le \nu^{(4)}(t) \le (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(4)}(t)$ :-

$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}$, $\boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{24})^{(4)} = (a''_{25})^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)} G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b''_{24})^{(4)} = (b''_{25})^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)} T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

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**Proof** : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a'_{28})^{(5)} - (a'_{29})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right) - (a''_{29})^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

**Definition of** $\nu^{(5)}$ :- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$ :-

For $0 < \boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \le \frac{(\nu_1)^{(5)}+(C)^{(5)}(\nu_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)}-(\nu_2)^{(5)}\right)t\right]}}{1+(C)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)}-(\nu_2)^{(5)}\right)t\right]}} \le \nu^{(5)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \le (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \le (\bar{\nu}_1)^{(5)} \le \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain 411

$$(\nu_1)^{(5)} \le \nu^{(5)}(t) \le \frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \le (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(5)}(t)$ :-

$(m_2)^{(5)} \le \nu^{(5)}(t) \le (m_1)^{(5)}$, $\boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$

In a completely analogous way, we obtain

**Definition of** $u^{(5)}(t)$ :-

$(\mu_2)^{(5)} \le u^{(5)}(t) \le (\mu_1)^{(5)}$, $\boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a_{28}'')^{(5)} = (a_{29}'')^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)}G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b_{28}'')^{(5)} = (b_{29}'')^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)}T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

**Proof** : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a_{32}')^{(6)} - (a_{33}')^{(6)} + (a_{32}'')^{(6)}(T_{33},t)\right) - (a_{33}'')^{(6)}(T_{33},t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

**Definition of** $\nu^{(6)}$ :- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \le \frac{d\nu^{(6)}}{dt} \le -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$ :-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \geq \frac{(\nu_1)^{(6)}+(C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_0)^{(6)}\right)t\right]}}{1+(C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)}-(\nu_0)^{(6)}}{(\nu_0)^{(6)}-(\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\nu_1)^{(6)}$

In the same manner , we get 413

$$\nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{\nu}_1)^{(6)}-(\nu_0)^{(6)}}{(\nu_0)^{(6)}-(\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\bar{\nu}_1)^{(6)}$

If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 414

$$(\nu_1)^{(6)} \leq \frac{(\nu_1)^{(6)}+(C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_2)^{(6)}\right)t\right]}}{1+(C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_2)^{(6)}\right)t\right]}} \leq \nu^{(6)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\bar{\nu}_1)^{(6)}$$

If $0 < (\nu_1)^{(6)} \leq (\bar{\nu}_1)^{(6)} \leq \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$ , we obtain 415

$$(\nu_1)^{(6)} \leq \nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(6)}(t)$ :-

$$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}, \quad \boxed{\nu^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(6)}(t)$ :-

$$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}, \quad \boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)}G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)}T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

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**Proof** : From global equations we obtain

$$\frac{dv^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right) - (a''_{37})^{(7)}(T_{37},t)v^{(7)} - (a_{37})^{(7)}v^{(7)}$$

**Definition of** $v^{(7)}$ :- $\boxed{v^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_2)^{(7)}v^{(7)} - (a_{36})^{(7)}\right) \le \frac{dv^{(7)}}{dt} \le -\left((a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_1)^{(7)}v^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

**Definition of** $(\bar{v}_1)^{(7)}, (v_0)^{(7)}$ :-

For $0 < \boxed{(v_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}} < (v_1)^{(7)} < (\bar{v}_1)^{(7)}$

$$v^{(7)}(t) \ge \frac{(v_1)^{(7)} + (C)^{(7)}(v_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_0)^{(7)}\right)t\right]}}{1 + (C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(v_1)^{(7)} - (v_0)^{(7)}}{(v_0)^{(7)} - (v_2)^{(7)}}}$$

it follows $(v_0)^{(7)} \le v^{(7)}(t) \le (v_1)^{(7)}$

In the same manner , we get 417

$$v^{(7)}(t) \le \frac{(\bar{v}_1)^{(7)} + (\bar{C})^{(7)}(\bar{v}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{v}_1)^{(7)} - (v_0)^{(7)}}{(v_0)^{(7)} - (\bar{v}_2)^{(7)}}}$$

From which we deduce $(v_0)^{(7)} \le v^{(7)}(t) \le (\bar{v}_1)^{(7)}$

If $0 < (v_1)^{(7)} < (v_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}} < (\bar{v}_1)^{(7)}$ we find like in the previous case, 418

$$(v_1)^{(7)} \le \frac{(v_1)^{(7)} + (C)^{(7)}(v_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_2)^{(7)}\right)t\right]}}{1 + (C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_2)^{(7)}\right)t\right]}} \le v^{(7)}(t) \le$$

$$\frac{(\bar{v}_1)^{(7)} + (\bar{C})^{(7)}(\bar{v}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}} \le (\bar{v}_1)^{(7)}$$

If $0 < (v_1)^{(7)} \le (\bar{v}_1)^{(7)} \le \boxed{(v_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$ , we obtain 419

$$(v_1)^{(7)} \le v^{(7)}(t) \le \frac{(\bar{v}_1)^{(7)} + (\bar{C})^{(7)}(\bar{v}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}} \le (v_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $v^{(7)}(t)$ :-

$$(m_2)^{(7)} \leq v^{(7)}(t) \leq (m_1)^{(7)}, \quad \boxed{v^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(7)}(t)$ :- 420

$$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}, \quad \boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $v^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)} G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)} T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

**Proof** : From global equations we obtain 421

$$\frac{dv^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right) - (a''_{41})^{(8)}(T_{41}, t) v^{(8)} - (a_{41})^{(8)} v^{(8)}$$

**Definition of** $v^{(8)}$ :- $\boxed{v^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_2)^{(8)} v^{(8)} - (a_{40})^{(8)}\right) \leq \frac{dv^{(8)}}{dt} \leq -\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_1)^{(8)} v^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$ :-

$$\text{For } 0 < \boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$$

$$v^{(8)}(t) \geq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)}-(\nu_0)^{(8)}}{(\nu_0)^{(8)}-(\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \leq \frac{(\nu_1)^{(8)}+(C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}}{1+(C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}} \leq \nu^{(8)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \leq (\bar{\nu}_1)^{(8)} \leq \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$ , we obtain 424

$$(\nu_1)^{(8)} \leq \nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(8)}(t)$ :-

$$(m_2)^{(8)} \leq \nu^{(8)}(t) \leq (m_1)^{(8)}, \quad \boxed{\nu^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(8)}(t)$ :-

$$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}, \quad \boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $\nu^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)}G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then $(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)}T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

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**Proof** : From 99,20,44,22,23,44 we obtain A

$$\frac{dv^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right) - (a''_{45})^{(9)}(T_{45},t)v^{(9)} - (a_{45})^{(9)}v^{(9)}$$

**Definition of** $v^{(9)}$ :- $\boxed{v^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)}\right) \le \frac{dv^{(9)}}{dt} \le -\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_1)^{(9)}v^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$ :-

For $0 < \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$

$$v^{(9)}(t) \ge \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \le v^{(9)}(t) \le (\nu_9)^{(9)}$

In the same manner , we get

$$v^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \le v^{(9)}(t) \le (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \le \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}}{1 + (C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}} \le v^{(9)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \le (\bar{\nu}_1)^{(9)} \le \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$ , we obtain

$$(\nu_1)^{(9)} \le v^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $v^{(9)}(t)$ :-

$$(m_2)^{(9)} \le v^{(9)}(t) \le (m_1)^{(9)}, \quad \boxed{v^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(9)}(t)$ :-

$(\mu_2)^{(9)} \leq u^{(9)}(t) \leq (\mu_1)^{(9)}$, $\boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)} G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then
$(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)} T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following 425

**Theorem :** If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on $t$ , and the conditions with the notations

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0$ ,

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$

$with\ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a''_i)^{(2)} and\ (b''_i)^{(2)}$ are independent on t , and the conditions with the notations 426

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$ 427

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 428

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$ , 429

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 430

$with\ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a''_i)^{(3)} and\ (b''_i)^{(3)}$ are independent on $t$ , and the conditions with the notations 431

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$ ,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

**Theorem :** If $(a_i'')^{(4)} and\ (b_i'')^{(4)}$ are independent on $t$ , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$ ,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(5)} and\ (b_i'')^{(5)}$ are independent on $t$ , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$ ,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

**Theorem** If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on $t$ , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$ ,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(7)} and\ (b_i'')^{(7)}$ are independent on $t$ , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$ ,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(8)} and\ (b_i'')^{(8)}$ are independent on $t$ , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$ ,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(9)} and\ (b_i'')^{(9)}$ are independent on $t$ , and the conditions (with the notations 436 A
45,46,27,28)

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$ ,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0$ 437

$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0$ 438

$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0$ 439

$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0$ 440

$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ]T_{14} = 0$ 441

$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0$ 442

has a unique positive solution , which is an equilibrium solution for the system

$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0$ 443

$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0$ 444

$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0$ 445

$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\ ]T_{16} = 0$ 446

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\,]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\,]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\,]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\,]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\,]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\big((G_{27})\big)\,]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\big((G_{27})\big)\,]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\big((G_{27})\big)\,]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})\,]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})\,]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})\,]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})\,]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})\,]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})\,]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0 \quad 479$$

$$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0 \quad 480$$

$$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0 \quad 481$$

$$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0 \quad 482$$

$$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0 \quad 483$$

$$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0 \quad 484$$

$$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0 \quad 484\text{A}$$

$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$

$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$

$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$

$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$

$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$

**Proof:** 485

(a) Indeed the first two equations have a nontrivial solution $G_{13}, G_{14}$ if

$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$

**Proof:** 486

(l) Indeed the first two equations have a nontrivial solution $G_{16}, G_{17}$ if

$\mathrm{F}(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$

**Proof:** 487

(a) Indeed the first two equations have a nontrivial solution $G_{20}, G_{21}$ if

$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$

**Proof:** 488

(a) Indeed the first two equations have a nontrivial solution $G_{24}, G_{25}$ if

$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$

**Proof:** 489

(a) Indeed the first two equations have a nontrivial solution $G_{28}, G_{29}$ if

$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$

**Proof:** 490

(a) Indeed the first two equations have a nontrivial solution $G_{32}, G_{33}$ if

$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) +$

$(a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$

**Proof:** 491

(a) Indeed the first two equations have a nontrivial solution $G_{36}, G_{37}$ if

$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$

**Proof:** 492

(a) Indeed the first two equations have a nontrivial solution $G_{40}, G_{41}$ if

$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$

**Proof:** 492 A

(a) Indeed the first two equations have a nontrivial solution $G_{44}, G_{45}$ if

$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$

**Definition and uniqueness of** $T^*_{14}$ :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique $T^*_{14}$ for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

**Definition and uniqueness of** $T^*_{17}$ :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique $T^*_{17}$ for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]}$$ 495

**Definition and uniqueness of** $T^*_{21}$ :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique $T^*_{21}$ for which $f(T^*_{21}) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

**Definition and uniqueness of** $T^*_{25}$ :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that

there exists a unique $T_{25}^{*}$ for which $f(T_{25}^{*})=0$. With this value , we obtain from the three first equations

$$G_{24}=\frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}=\frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T_{25}^{*})]}$$

**Definition and uniqueness of** $T_{29}^{*}$ :- 498

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(5)}(T_{29})$ being increasing, it follows that there exists a unique $T_{29}^{*}$ for which $f(T_{29}^{*})=0$. With this value , we obtain from the three first equations

$$G_{28}=\frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}=\frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T_{29}^{*})]}$$

**Definition and uniqueness of** $T_{33}^{*}$ :- 499

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(6)}(T_{33})$ being increasing, it follows that there exists a unique $T_{33}^{*}$ for which $f(T_{33}^{*})=0$. With this value , we obtain from the three first equations

$$G_{32}=\frac{(a_{32})^{(6)}G_{33}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^{*})]} \quad , \quad G_{34}=\frac{(a_{34})^{(6)}G_{33}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^{*})]}$$

**Definition and uniqueness of** $T_{37}^{*}$ :- 500

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(7)}(T_{37})$ being increasing, it follows that there exists a unique $T_{37}^{*}$ for which $f(T_{37}^{*})=0$. With this value , we obtain from the three first equations

$$G_{36}=\frac{(a_{36})^{(7)}G_{37}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^{*})]} \quad , \quad G_{38}=\frac{(a_{38})^{(7)}G_{37}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^{*})]}$$

**Definition and uniqueness of** $T_{41}^{*}$ :- 501

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique $T_{41}^{*}$ for which $f(T_{41}^{*})=0$. With this value , we obtain from the three first equations

$$G_{40}=\frac{(a_{40})^{(8)}G_{41}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T_{41}^{*})]} \quad , \quad G_{42}=\frac{(a_{42})^{(8)}G_{41}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T_{41}^{*})]}$$

**Definition and uniqueness of** $T_{45}^{*}$ :- 501 A

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(9)}(T_{45})$ being increasing, it follows that there exists a unique $T_{45}^{*}$ for which $f(T_{45}^{*})=0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)}G_{45}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T^*_{45})]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)}G_{45}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T^*_{45})]}$$

By the same argument, the equations admit solutions $G_{13}, G_{14}$ if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right]+(b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{14}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{14}$ such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions $G_{16}, G_{17}$ if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right]+(b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ 504
is a decreasing function in $G_{17}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{14}$ such that $\varphi((G_{19})^*) = 0$

By the same argument, the equations admit solutions $G_{20}, G_{21}$ if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right]+(b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{21}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{21}$ such that $\varphi((G_{23})^*) = 0$

By the same argument, the equations admit solutions $G_{24}, G_{25}$ if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right]+(b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{25}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{25}$ such that $\varphi((G_{27})^*) = 0$

By the same argument, the equations admit solutions $G_{28}, G_{29}$ if 507

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right]+(b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{29}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that

there exists a unique $G_{29}^{*}$ such that $\varphi((G_{31})^{*}) = 0$

By the same argument, the equations admit solutions $G_{32}, G_{33}$ if 508
$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$

$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right] + (b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{33}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{33}^{*}$ such that $\varphi(G_{35}{}^{*}) = 0$

By the same argument, the equations admit solutions $G_{36}, G_{37}$ if 509
$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$
$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right] + (b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{37}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{37}^{*}$ such that $\varphi(G_{39}{}^{*}) = 0$

By the same argument, the equations admit solutions $G_{40}, G_{41}$ if 510
$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$
$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right] + (b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{41}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{41}^{*}$ such that $\varphi(G_{43}{}^{*}) = 0$

By the same argument, the equations 92,93 admit solutions $G_{44}, G_{45}$ if
$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$

$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right] + (b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{45}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{45}^{*}$ such that $\varphi((G_{47})^{*}) = 0$

Finally we obtain the unique solution 511

$G_{14}^{*}$ given by $\varphi(G^{*}) = 0$ , $T_{14}^{*}$ given by $f(T_{14}^{*}) = 0$ and

$$G_{13}^{*} = \frac{(a_{13})^{(1)}G_{14}^{*}}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^{*})]} \quad , \quad G_{15}^{*} = \frac{(a_{15})^{(1)}G_{14}^{*}}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^{*})]}$$

$$T_{13}^{*} = \frac{(b_{13})^{(1)}T_{14}^{*}}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^{*})]} \quad , \quad T_{15}^{*} = \frac{(b_{15})^{(1)}T_{14}^{*}}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^{*})]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

$G_{17}^{*}$ given by $\varphi((G_{19})^{*}) = 0$ , $T_{17}^{*}$ given by $f(T_{17}^{*}) = 0$ and 512

$$G_{16}^{*} = \frac{(a_{16})^{(2)} G_{17}^{*}}{[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}^{*})]} \quad , \quad G_{18}^{*} = \frac{(a_{18})^{(2)} G_{17}^{*}}{[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}^{*})]} \qquad 513$$

$$T_{16}^{*} = \frac{(b_{16})^{(2)} T_{17}^{*}}{[(b'_{16})^{(2)} - (b''_{16})^{(2)}((G_{19})^{*})]} \quad , \quad T_{18}^{*} = \frac{(b_{18})^{(2)} T_{17}^{*}}{[(b'_{18})^{(2)} - (b''_{18})^{(2)}((G_{19})^{*})]} \qquad 514$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

$G_{21}^{*}$ given by $\varphi((G_{23})^{*}) = 0$ , $T_{21}^{*}$ given by $f(T_{21}^{*}) = 0$ and

$$G_{20}^{*} = \frac{(a_{20})^{(3)} G_{21}^{*}}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^{*})]} \quad , \quad G_{22}^{*} = \frac{(a_{22})^{(3)} G_{21}^{*}}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^{*})]}$$

$$T_{20}^{*} = \frac{(b_{20})^{(3)} T_{21}^{*}}{[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}{}^{*})]} \quad , \quad T_{22}^{*} = \frac{(b_{22})^{(3)} T_{21}^{*}}{[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}{}^{*})]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

$G_{25}^{*}$ given by $\varphi(G_{27}) = 0$ , $T_{25}^{*}$ given by $f(T_{25}^{*}) = 0$ and

$$G_{24}^{*} = \frac{(a_{24})^{(4)} G_{25}^{*}}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}^{*} = \frac{(a_{26})^{(4)} G_{25}^{*}}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^{*})]}$$

$$T_{24}^{*} = \frac{(b_{24})^{(4)} T_{25}^{*}}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^{*})]} \quad , \quad T_{26}^{*} = \frac{(b_{26})^{(4)} T_{25}^{*}}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^{*})]} \qquad 517$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

$G_{29}^{*}$ given by $\varphi((G_{31})^{*}) = 0$ , $T_{29}^{*}$ given by $f(T_{29}^{*}) = 0$ and

$$G_{28}^{*} = \frac{(a_{28})^{(5)} G_{29}^{*}}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}^{*} = \frac{(a_{30})^{(5)} G_{29}^{*}}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^{*})]}$$

$$T_{28}^{*} = \frac{(b_{28})^{(5)} T_{29}^{*}}{[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31})^{*})]} \quad , \quad T_{30}^{*} = \frac{(b_{30})^{(5)} T_{29}^{*}}{[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31})^{*})]} \qquad 519$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

$G_{33}^{*}$ given by $\varphi((G_{35})^{*}) = 0$ , $T_{33}^{*}$ given by $f(T_{33}^{*}) = 0$ and

$$G_{32}^{*} = \frac{(a_{32})^{(6)} G_{33}^{*}}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^{*})]} \quad , \quad G_{34}^{*} = \frac{(a_{34})^{(6)} G_{33}^{*}}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^{*})]}$$

$$T_{32}^{*} = \frac{(b_{32})^{(6)} T_{33}^{*}}{[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35})^{*})]} \quad , \quad T_{34}^{*} = \frac{(b_{34})^{(6)} T_{33}^{*}}{[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35})^{*})]} \qquad 521$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

$G_{37}^*$ given by $\varphi((G_{39})^*) = 0$ , $T_{37}^*$ given by $f(T_{37}^*) = 0$ and

$$G_{36}^* = \frac{(a_{36})^{(7)} G_{37}^*}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^*)]} \quad , \quad G_{38}^* = \frac{(a_{38})^{(7)} G_{37}^*}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^*)]}$$

$$T_{36}^* = \frac{(b_{36})^{(7)} T_{37}^*}{[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39})^*)]} \quad , \quad T_{38}^* = \frac{(b_{38})^{(7)} T_{37}^*}{[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39})^*)]}$$

Finally we obtain the unique solution 523

$G_{41}^*$ given by $\varphi((G_{43})^*) = 0$ , $T_{41}^*$ given by $f(T_{41}^*) = 0$ and

$$G_{40}^* = \frac{(a_{40})^{(8)} G_{41}^*}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42}^* = \frac{(a_{42})^{(8)} G_{41}^*}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

$$T_{40}^* = \frac{(b_{40})^{(8)} T_{41}^*}{[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43})^*)]} \quad , \quad T_{42}^* = \frac{(b_{42})^{(8)} T_{41}^*}{[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43})^*)]}$$

Finally we obtain the unique solution of 89 to 99 523 A

$G_{45}^*$ given by $\varphi((G_{47})^*) = 0$ , $T_{45}^*$ given by $f(T_{45}^*) = 0$ and

$$G_{44}^* = \frac{(a_{44})^{(9)} G_{45}^*}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46}^* = \frac{(a_{46})^{(9)} G_{45}^*}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

$$T_{44}^* = \frac{(b_{44})^{(9)} T_{45}^*}{[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47})^*)]} \quad , \quad T_{46}^* = \frac{(b_{46})^{(9)} T_{45}^*}{[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47})^*)]}$$

## ASYMPTOTIC STABILITY ANALYSIS 524

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

**Proof:** Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a''_{14})^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \frac{\partial (b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a'_{13})^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)}G^*_{13}\mathbb{T}_{14} \quad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a'_{14})^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)}G^*_{14}\mathbb{T}_{14} \quad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a'_{15})^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)}G^*_{15}\mathbb{T}_{14} \quad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b'_{13})^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)}T^*_{13}\mathbb{G}_j\right) \quad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b'_{14})^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)}T^*_{14}\mathbb{G}_j\right) \quad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b'_{15})^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T^*_{15}\mathbb{G}_j\right) \quad 530$$

**ASYMPTOTIC STABILITY ANALYSIS** 531

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(2)}$ and $(b''_i)^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G^*_i + \mathbb{G}_i \quad , T_i = T^*_i + \mathbb{T}_i \quad 532$$

$$\frac{\partial(a''_{17})^{(2)}}{\partial T_{17}}(T^*_{17}) = (q_{17})^{(2)} \quad , \frac{\partial(b''_i)^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \quad 533$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a'_{16})^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G^*_{16}\mathbb{T}_{17} \quad 534$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a'_{17})^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G^*_{17}\mathbb{T}_{17} \quad 535$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a'_{18})^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G^*_{18}\mathbb{T}_{17} \quad 536$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b'_{16})^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T^*_{16}\mathbb{G}_j\right) \quad 537$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b'_{17})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}T^*_{17}\mathbb{G}_j\right) \quad 538$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b'_{18})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}T^*_{18}\mathbb{G}_j\right) \quad 539$$

**ASYMPTOTIC STABILITY ANALYSIS** 540

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(3)}$ *and* $(b''_i)^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{21}'')^{(3)}}{\partial T_{21}}(T_{21}^*) = (q_{21})^{(3)} \quad , \frac{\partial (b_i'')^{(3)}}{\partial G_j}(\,(G_{23})^*\,) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\big((a_{20}')^{(3)} + (p_{20})^{(3)}\big)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G_{20}^*\mathbb{T}_{21} \qquad 541$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\big((a_{21}')^{(3)} + (p_{21})^{(3)}\big)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G_{21}^*\mathbb{T}_{21} \qquad 542$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\big((a_{22}')^{(3)} + (p_{22})^{(3)}\big)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G_{22}^*\mathbb{T}_{21} \qquad 543$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\big((b_{20}')^{(3)} - (r_{20})^{(3)}\big)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\big(s_{(20)(j)}T_{20}^*\mathbb{G}_j\big) \qquad 544$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\big((b_{21}')^{(3)} - (r_{21})^{(3)}\big)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\big(s_{(21)(j)}T_{21}^*\mathbb{G}_j\big) \qquad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\big((b_{22}')^{(3)} - (r_{22})^{(3)}\big)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\big(s_{(22)(j)}T_{22}^*\mathbb{G}_j\big) \qquad 546$$

**ASYMPTOTIC STABILITY ANALYSIS** 547

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}\ and\ (b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{25}'')^{(4)}}{\partial T_{25}}(T_{25}^*) = (q_{25})^{(4)} \quad , \frac{\partial (b_i'')^{(4)}}{\partial G_j}((G_{27})^*\,) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\big((a_{24}')^{(4)} + (p_{24})^{(4)}\big)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G_{24}^*\mathbb{T}_{25} \qquad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\big((a_{25}')^{(4)} + (p_{25})^{(4)}\big)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G_{25}^*\mathbb{T}_{25} \qquad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\big((a_{26}')^{(4)} + (p_{26})^{(4)}\big)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G_{26}^*\mathbb{T}_{25} \qquad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\big((b_{24}')^{(4)} - (r_{24})^{(4)}\big)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\big(s_{(24)(j)}T_{24}^*\mathbb{G}_j\big) \qquad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\big((b_{25}')^{(4)} - (r_{25})^{(4)}\big)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\big(s_{(25)(j)}T_{25}^*\mathbb{G}_j\big) \qquad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T^*_{26}\mathbb{G}_j\right) \quad 554$$

**ASYMPTOTIC STABILITY ANALYSIS** 555

**Theorem 5:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}\ and\ (b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 556

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{29})^{(5)}}{\partial T_{29}}(T^*_{29}) = (q_{29})^{(5)} \ , \ \frac{\partial(b_i'')^{(5)}}{\partial G_j}\left((G_{31})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a'_{28})^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G^*_{28}\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a'_{29})^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G^*_{29}\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a'_{30})^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G^*_{30}\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b'_{28})^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T^*_{28}\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b'_{29})^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T^*_{29}\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b'_{30})^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T^*_{30}\mathbb{G}_j\right) \quad 562$$

**ASYMPTOTIC STABILITY ANALYSIS** 563

**Theorem 6:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}\ and\ (b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{33})^{(6)}}{\partial T_{33}}(T^*_{33}) = (q_{33})^{(6)} \ , \ \frac{\partial(b_i'')^{(6)}}{\partial G_j}\left((G_{35})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a'_{32})^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G^*_{32}\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a'_{33})^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G^*_{33}\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a'_{34})^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G^*_{34}\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b'_{32})^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T^*_{32}\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b'_{33})^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T^*_{33}\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b'_{34})^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T^*_{34}\mathbb{G}_j\right) \quad 570$$

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**Theorem 7:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 572

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{37})^{(7)}}{\partial T_{37}}(T^*_{37}) = (q_{37})^{(7)} \quad , \frac{\partial(b_i'')^{(7)}}{\partial G_j}\left((G_{39})^{**}\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a'_{36})^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G^*_{36}\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a'_{37})^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G^*_{37}\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a'_{38})^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G^*_{38}\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b'_{36})^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T^*_{36}\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b'_{37})^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T^*_{37}\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b'_{38})^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T^*_{38}\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

**ASYMPTOTIC STABILITY ANALYSIS**

**Theorem 8:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{41}'')^{(8)}}{\partial T_{41}}(T_{41}^*) = (q_{41})^{(8)} \quad , \frac{\partial (b_i'')^{(8)}}{\partial G_j}\left( (G_{43})^* \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a_{40}')^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G_{40}^*\mathbb{T}_{41} \quad (581)$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a_{41}')^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G_{41}^*\mathbb{T}_{41} \quad (582)$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a_{42}')^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G_{42}^*\mathbb{T}_{41} \quad (583)$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b_{40}')^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T_{40}^*\mathbb{G}_j\right) \quad (584)$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b_{41}')^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T_{41}^*\mathbb{G}_j\right) \quad (585)$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b_{42}')^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T_{42}^*\mathbb{G}_j\right) \quad (586)$$

(586 A)

**ASYMPTOTIC STABILITY ANALYSIS**

**Theorem 9:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}$ *and* $(b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.
**Proof:** Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{45}'')^{(9)}}{\partial T_{45}}(T_{45}^*) = (q_{45})^{(9)} \quad , \frac{\partial (b_i'')^{(9)}}{\partial G_j}\left( (G_{47})^* \right) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a_{44}')^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)}G_{44}^*\mathbb{T}_{45} \quad (586\,B)$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a_{45}')^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)}G_{45}^*\mathbb{T}_{45} \quad (586\,C)$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a_{46}')^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)}G_{46}^*\mathbb{T}_{45} \quad (586\,D)$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b_{44}')^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)}T_{44}^*\mathbb{G}_j\right) \quad (586\,E)$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b_{45}')^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)}T_{45}^*\mathbb{G}_j\right) \quad (586\,F)$$

$\frac{d\mathbb{T}_{46}}{dt} = -\left((b'_{46})^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T^*_{46}\mathbb{G}_j\right)$ 586

G

587

**The characteristic equation of this system is**

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(q_{13})^{(1)}G^*_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)}T^*_{14} + (b_{14})^{(1)}s_{(13),(14)}T^*_{14}\right)$$

$$+\left(\left((\lambda)^{(1)} + (a'_{14})^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)}G^*_{13} + (a_{13})^{(1)}(q_{14})^{(1)}G^*_{14}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)}T^*_{14} + (b_{14})^{(1)}s_{(13),(13)}T^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((b'_{13})^{(1)} + (b'_{14})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(15)}T^*_{14} + (b_{14})^{(1)}s_{(13),(15)}T^*_{13}\right)\} = 0$$

$+$

$$(\lambda)^{(2)} + (b'_{18})^{(2)} - (r_{18})^{(2)}\{\left((\lambda)^{(2)} + (a'_{18})^{(2)} + (p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^*_{17} + (a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^*_{17}\right)$$

$$+\left(\left((\lambda)^{(2)} + (a'_{17})^{(2)} + (p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^*_{16} + (a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}\right)$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2 + \left((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2 + \left((b'_{16})^{(2)} + (b'_{17})^{(2)} - (r_{16})^{(2)} + (r_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$+\left(\left((\lambda)^{(2)}\right)^2 + \left((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17} + (a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^{*}_{16}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\right)\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left(\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)\right]$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(21)}T^{*}_{21}\right)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^{*}_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^{*}_{21}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(20)}T^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$+\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(22)}T^{*}_{20}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\right)\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left(\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)\right]$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(25)}T^{*}_{25}\right)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^{*}_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^{*}_{25}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(24)}T^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$+\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(26)}T^{*}_{24}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\right)\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(29)}T^{*}_{29}\right)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^{*}_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^{*}_{29}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(28)}T^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$+\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(30)}T^{*}_{28}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\right)\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left(\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)\right]$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(33)}T^{*}_{33}\right)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^{*}_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^{*}_{33}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(32)}T^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$+\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(34)}T^{*}_{32}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\right)\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)\right]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(37)}T^{*}_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^{*}_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^{*}_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(36)}T^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$+\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(38)}T^{*}_{36}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\right)\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(41)}T^{*}_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^{*}_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^{*}_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(40)}T^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$+\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(42)}T^{*}_{40}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\right)\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(45)}T^{*}_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^{*}_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^{*}_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(44)}T^{*}_{44}\right)$$

$$\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\Big)$$

$$\left((\lambda)^{(9)}\right)^{2}+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\Big)$$

$$+\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(46)}T^{*}_{44}\right)\}=0$$

**And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.**

## SECTION FOURTEEN

## Pathways For Degradation Of Connexins And Gap Junctions

### INTRODUCTION—VARIABLES USED

**Pathways for degradation of connexins and gap junctions Viviana M Berthoud, Peter J Minogue, James G Laing, Eric C Beyer DOI: http://dx.doi.org/10.1016/j.cardiores.2003.12.021 256-267: First**

**published online: 1 May 2004**

(1) Gap junctional proteins, connexins, and gap junctional plaques are short-lived. Three pathways for their degradation have been proposed: (1) misfolded/abnormally oligomerized connexins are (=) retrogradely translocated and degraded by (e) the proteasome through endoplasmic reticulum-associated degradation

(2) (2) Connexins (as monomers or oligomers) may traffic directly from (e) an early secretory compartment to the lysosome for degradation without (e) reaching the plasma membrane;

(3) (3) Connexins within gap junction plaques are degraded by (e) the lysosome after endocytotic internalization.

**(4)** Degradation of gap junction plaques is (=) proteasome-dependent in some cell types. Degradation may be regulated by (e&eb) ubiquitinylation, phosphorylation, or polypeptide domains that act as sorting signals.

**Science 18 April 2008: Vol. 320 no. 5874 pp. 356-358 DOI: 10.1126/science.1154663 Chaotic Dirac Billiard in Graphene Quantum Dots L. A. Ponomarenko1, F. Schedin1, M. I. Katsnelson2, R. Yang1, E. W. Hill1, K. S. Novoselov1,*, A. K. Geim1**

(5) The exceptional electronic properties of graphene, with (e&eb) its charge carriers mimicking (e&eb) relativistic quantum particles and its formidable potential in various applications, have ensured (eb) a rapid growth of interest in this new material.

(6) Authors report on electron transport in quantum dot devices carved entirely from (e) graphene.

(7) At large sizes (>100 nanometers), they behave as (=) conventional single-electron transistors, exhibiting (eb) periodic Coulomb blockade peaks.

(8) For quantum dots smaller than 100 nanometers, the peaks become (=) strongly nonperiodic, indicating (eb) a major contribution of quantum confinement. Random peak spacing and its statistics are well described by (e) the theory of chaotic neutrino billiards.

**(9)** Short constrictions of only a few nanometers in width remain conductive and reveal (eb) a confinement gap of up to 0.5 electron volt, demonstrating (eb) the possibility of molecular-scale electronics based on graphene.

## NOTATION

### Module One

Gap junctional proteins, connexins, and gap junctional plaques are short-lived.

Three pathways for their degradation have been proposed: (1) misfolded/abnormally oligomerized connexins are (=) retrogradely translocated and degraded by (e) the proteasome through endoplasmic reticulum-associated degradation

$G_{13}$ : Category one of misfolded/abnormally oligomerized connexins

$G_{14}$ : Category two of SAS

$G_{15}$ : Category three of SAS

$T_{13}$ : Category one of retrogradely translocated and degraded by (e) the proteasome through endoplasmic reticulum-associated degradation

$T_{14}$ : Category two of SAS

$T_{15}$ : Category three of SAS

**Module Two**

Three pathways for their degradation have been proposed: (1) misfolded/abnormally oligomerized connexins are retrogradely translocated and degraded by (e) the proteasome through endoplasmic reticulum-associated degradation

$G_{16}$ : Category one of proteasome through endoplasmic reticulum-associated degradation

$G_{17}$ : Category two of SAS

$G_{18}$ : Category three of SAS

$T_{16}$ : Category one of misfolded/abnormally oligomerized connexins are retrogradely translocated and degraded

$T_{17}$ : Category two of SAS

$T_{18}$ : Category three of SAS

**Module three**

Three pathways for their degradation have been proposed: (1) **misfolded/abnormally oligomerized connexins are retrogradely translocated and degraded by the proteasome** through endoplasmic reticulum-associated degradation

$G_{20}$ : Category one of **misfolded/abnormally oligomerized connexins are retrogradely translocated and degraded by the proteasome;** endoplasmic reticulum-associated degradation

$G_{21}$ : Category two of SAS

$G_{22}$ : Category three of SAS

$T_{20}$ : Category one of endoplasmic reticulum-associated degradation; **misfolded/abnormally oligomerized connexins are retrogradely translocated and degraded by the proteasome**

$T_{21}$ : Category two of SAS

$T_{22}$ : Category three of SAS

**Module four**

Connexins (as monomers or oligomers) may traffic directly from (e) an early secretory compartment to the lysosome for degradation without (e) reaching the plasma membrane

$G_{24}$ : Category one of early secretory compartment to the lysosome for degradation without (e) reaching the plasma membrane

$G_{25}$ : Category two of SAS

$G_{26}$ : Category three of SAS

$T_{24}$ : Category one of Connexins (as monomers or oligomers) may traffic

$T_{25}$ : Category two of SAS

$T_{26}$ : Category three of SAS

**Module five**

**Connexins (as monomers or oligomers) may traffic directly from an early secretory compartment** to the lysosome for degradation without (e) reaching the plasma membrane

$G_{28}$ : Category one of **Connexins (as monomers or oligomers) may traffic directly from an early secretory compartment;** lysosome for degradation without (e) reaching the plasma membrane

$G_{29}$ : Category two of SAS

$G_{30}$ : Category three of SAS

$T_{28}$ : Category one of lysosome for degradation without (e) reaching the plasma membrane **;Connexins (as monomers or oligomers) may traffic directly from an early secretory compartment**

$T_{29}$ : Category two of SAS

$T_{30}$ : Category three of SAS

**Module six**

Connexins (as monomers or oligomers) may traffic directly from an early secretory compartment to the lysosome for degradation without (e) **reaching the plasma membrane**

$G_{32}$ : Category one of Connexins (as monomers or oligomers) may traffic directly from an early secretory compartment to the lysosome for degradation; **reaching the plasma membrane**

$G_{33}$ : Category two of SAS

$G_{34}$ : Category three of SAS

$T_{32}$ : Category one of **reaching the plasma membrane** ;Connexins (as monomers or oligomers) may traffic directly from an early secretory compartment to the lysosome for degradation

$T_{33}$ : Category two of SAS

$T_{34}$ : Category three of SAS

**Module seven**

Connexins within gap junction plaques are degraded by (e) the lysosome after endocytotic internalization.

$G_{36}$ : Category one of lysosome after endocytotic internalization

$G_{37}$ : Category two of SAS

$G_{38}$ : Category three of SAS

$T_{36}$ : Category one of Connexins within gap junction plaques are degraded

$T_{37}$ : Category two of SAS

$T_{38}$ : Category three of SAS

**Module eight**

Degradation of gap junction plaques is (=) proteasome-dependent in some cell types.

Degradation may be regulated by (e&eb) ubiquitinylation, phosphorylation, or polypeptide domains that act as sorting signals

$G_{40}$ : Category one of Degradation of gap junction plaques

$G_{41}$ : Category two of SAS

$G_{42}$ : Category three of SAS

$T_{40}$ : Category one of proteasome-dependent in some cell types

$T_{41}$ : Category two of SAS

$T_{42}$ : Category three of SAS

**Module Nine**

**Degradation** may be regulated by (e&eb) ubiquitinylation, phosphorylation, or polypeptide domains that act as sorting signals

$G_{44}$ : Category one of **Degradation;** ubiquitinylation, phosphorylation, or polypeptide domains that act as sorting signals

$G_{45}$ : Category two of SAS

$G_{46}$ : Category three of SAS

$T_{44}$ : Category one of ubiquitinylation, phosphorylation, or polypeptide domains that act as sorting signals **;Degradation**

$T_{45}$ : Category two of SAS

$T_{46}$ : Category three of SAS

**The Coefficients:**

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}$ , $(b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)},$
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}, \ (b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}\ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$ ,

$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)},$
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)},$
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)},$
are Dissipation coefficients

**Module Numbered One**

**The differential system of this model is now (Module Numbered one)**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \quad 1$$
$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad 2$$
$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15} \quad 3$$
$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13} \quad 4$$
$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14} \quad 5$$
$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)\right]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14}, t) =$ First augmentation factor
$-(b''_{13})^{(1)}(G, t) =$ First detritions factor

**Module Numbered Two**

**The differential system of this model is now ( Module numbered two)**

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right]G_{16} \quad 7$$
$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t)\right]G_{17} \quad 8$$
$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t)\right]G_{18} \quad 9$$
$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\left((G_{19}), t\right)\right]T_{16} \quad 10$$
$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\left((G_{19}), t\right)\right]T_{17} \quad 11$$
$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\left((G_{19}), t\right)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17}, t) =$ First augmentation factor
$-(b''_{16})^{(2)}\left((G_{19}), t\right) =$ First detritions factor

**Module Numbered Three**

**The differential system of this model is now (Module numbered three)**

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right]G_{20} \quad 13$$
$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}, t)\right]G_{21} \quad 14$$
$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}, t)\right]G_{22} \quad 15$$
$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}, t)\right]T_{20} \quad 16$$
$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}, t)\right]T_{21} \quad 17$$
$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}, t)\right]T_{22} \quad 18$$

$+(a''_{20})^{(3)}(T_{21}, t) =$ First augmentation factor
$-(b''_{20})^{(3)}(G_{23}, t) =$ First detritions factor

**Module Numbered Four**

**The differential system of this model is now (Module numbered Four)**

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right]G_{24} \quad 19$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25},t)\right]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25},t)\right]G_{26} \quad 21$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27}),t)\right]T_{24} \quad 22$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}((G_{27}),t)\right]T_{25} \quad 23$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27}),t)\right]T_{26} \quad 24$$

$+(a''_{24})^{(4)}(T_{25},t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}((G_{27}),t) =$ **First detritions factor**

**Module Numbered Five**:

**The differential system of this model is now (Module number five)**

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29},t)\right]G_{28} \quad 25$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29},t)\right]G_{29} \quad 26$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29},t)\right]G_{30} \quad 27$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31}),t)\right]T_{28} \quad 28$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}((G_{31}),t)\right]T_{29} \quad 29$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31}),t)\right]T_{30} \quad 30$$

$+(a''_{28})^{(5)}(T_{29},t) =$ **First augmentation factor**

$-(b''_{28})^{(5)}((G_{31}),t) =$ **First detritions factor**

**Module Numbered Six**

**The differential system of this model is now (Module numbered Six)**

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33},t)\right]G_{32} \quad 31$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33},t)\right]G_{33} \quad 32$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33},t)\right]G_{34} \quad 33$$

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35}),t)\right]T_{32} \quad 34$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}((G_{35}),t)\right]T_{33} \quad 35$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35}),t)\right]T_{34} \quad 36$$

$+(a''_{32})^{(6)}(T_{33},t) =$ **First augmentation factor**

**Module Numbered Seven:**

**The differential system of this model is now (Seventh Module)**

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right]G_{36} \quad 37$$

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37},t)\right]G_{37} \quad 38$$

$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37},t)\right]G_{38} \quad 39$$

$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39}),t)\right]T_{36} \quad 40$$

$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}((G_{39}),t)\right]T_{37} \quad 41$$

$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39}),t)\right]T_{38} \quad 42$$

$+(a''_{36})^{(7)}(T_{37},t) =$ **First augmentation factor**

**Module Numbered Eight**

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right]G_{40} \quad 43$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41},t)\right]G_{41} \quad 44$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41},t)\right]G_{42} \quad 45$$

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43}),t)\right]T_{40} \quad 46$$

$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}((G_{43}),t)\right]T_{41} \quad 47$$

$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43}),t)\right]T_{42} \quad 48$$

**Module Numbered Nine**

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47}),t)\right]T_{44} \quad 52$$

$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}((G_{47}),t)\right]T_{45} \quad 53$$

$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47}),t)\right]T_{46} \quad 54$$

$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}((G_{47}),t) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)}\boxed{+(a''_{13})^{(1)}(T_{14},t)}\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7)}(T_{37},t)}\boxed{+(a''_{40})^{(8,8)}(T_{41},t)}\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)}\boxed{+(a''_{14})^{(1)}(T_{14},t)}\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7)}(T_{37},t)}\boxed{+(a''_{41})^{(8,8)}(T_{41},t)}\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)}\boxed{+(a''_{15})^{(1)}(T_{14},t)}\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7)}(T_{37},t)}\boxed{+(a''_{42})^{(8,8)}(T_{41},t)}\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15} \quad 57$$

Where $\boxed{(a''_{13})^{(1)}(T_{14},t)}$, $\boxed{(a''_{14})^{(1)}(T_{14},t)}$, $\boxed{(a''_{15})^{(1)}(T_{14},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)}$ are third augmentation coefficient for

category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,)}(T_{25},t)$ , $+(a''_{25})^{(4,4,4,4,)}(T_{25},t)$ , $+(a''_{26})^{(4,4,4,4,)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,)}(T_{29},t)$ , $+(a''_{30})^{(5,5,5,5,)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,)}(T_{33},t)$ , $+(a''_{34})^{(6,6,6,6,)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{38})^{(7,7)}(T_{37},t)$ $+(a''_{37})^{(7,7)}(T_{37},t)$ $+(a''_{36})^{(7,7)}(T_{37},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{40})^{(8,8)}(T_{41},t)$ $+(a''_{41})^{(8,8)}(T_{41},t)$ $+(a''_{42})^{(8,8)}(T_{41},t)$ are eight augmentation coefficient for 1,2,3

$+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)}\boxed{-(b''_{13})^{(1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13} \quad (58)$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)}\boxed{-(b''_{14})^{(1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14} \quad (59)$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)}\boxed{-(b''_{15})^{(1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15} \quad (60)$$

Where $\boxed{-(b''_{13})^{(1)}(G,t)}$, $\boxed{-(b''_{14})^{(1)}(G,t)}$, $\boxed{-(b''_{15})^{(1)}(G,t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8)}(G_{43},t)}$ $\boxed{-(b''_{41})^{(8,8)}(G_{43},t)}$ $\boxed{-(b''_{42})^{(8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \begin{bmatrix} (a'_{16})^{(2)} & \boxed{+(a''_{16})^{(2)}(T_{17},t)} & \boxed{+(a''_{13})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)} & \\ \boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9)}(T_{45},t)} & \end{bmatrix} G_{16} \quad (61)$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \begin{bmatrix} (a'_{17})^{(2)} & \boxed{+(a''_{17})^{(2)}(T_{17},t)} & \boxed{+(a''_{14})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)} & \\ \boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9)}(T_{45},t)} & \end{bmatrix} G_{17} \quad (62)$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \begin{bmatrix} (a'_{18})^{(2)} & \boxed{+(a''_{18})^{(2)}(T_{17},t)} & \boxed{+(a''_{15})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)} & \\ \boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9)}(T_{45},t)} & \end{bmatrix} G_{18} \quad (63)$$

Where $\boxed{+(a''_{16})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,)}(T_{14},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)}$ are eight augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{44})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9)}(T_{45},t)}$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \begin{bmatrix} (b'_{16})^{(2)} & \boxed{-(b''_{16})^{(2)}(G_{19},t)} & \boxed{-(b''_{13})^{(1,1,)}(G,t)} & \boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)} & \\ \boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9)}(G_{47},t)} & \end{bmatrix} T_{16} \quad (64)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)} & \boxed{-(b''_{17})^{(2)}(G_{19},t)} & \boxed{-(b''_{14})^{(1,1,)}(G,t)} & \boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)} & \\ \boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9)}(G_{47},t)} & \end{bmatrix} T_{17} \quad (65)$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)}\boxed{-(b''_{18})^{(2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,)}(G,t)} & \boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{18} \quad 66$$

where $\boxed{-(b''_{16})^{(2)}(G_{19},t)}$ , $\boxed{-(b''_{17})^{(2)}(G_{19},t)}$ , $\boxed{-(b''_{18})^{(2)}(G_{19},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,)}(G,t)}$ are second detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{44})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \begin{bmatrix} (a'_{20})^{(3)}\boxed{+(a''_{20})^{(3)}(T_{21},t)} & \boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{20} \quad 67$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \begin{bmatrix} (a'_{21})^{(3)}\boxed{+(a''_{21})^{(3)}(T_{21},t)} & \boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{21} \quad 68$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \begin{bmatrix} (a'_{22})^{(3)}\boxed{+(a''_{22})^{(3)}(T_{21},t)} & \boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{22} \quad 69$$

$\boxed{+(a''_{20})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3)}(T_{21},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}$ are second augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)}$ are third augmentation coefficients for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficients for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficients for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficients for category 1, 2 and 3

$+(a''_{36})^{(7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1, 2 and 3

$+(a''_{40})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8)}(T_{41},t)$ are eight augmentation coefficients for category 1, 2 and 3

$+(a''_{44})^{(9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b'_{20})^{(3)} & -(b''_{20})^{(3)}(G_{23},t) & -(b''_{16})^{(2,2,2)}(G_{19},t) & -(b''_{13})^{(1,1,1,)}(G,t) \\ -(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{36})^{(7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{20} \quad (70)$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b'_{21})^{(3)} & -(b''_{21})^{(3)}(G_{23},t) & -(b''_{17})^{(2,2,2)}(G_{19},t) & -(b''_{14})^{(1,1,1,)}(G,t) \\ -(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{37})^{(7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{21} \quad (71)$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)} & -(b''_{22})^{(3)}(G_{23},t) & -(b''_{18})^{(2,2,2)}(G_{19},t) & -(b''_{15})^{(1,1,1,)}(G,t) \\ -(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{38})^{(7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{22} \quad (72)$$

$-(b''_{20})^{(3)}(G_{23},t)$, $-(b''_{21})^{(3)}(G_{23},t)$, $-(b''_{22})^{(3)}(G_{23},t)$ *are first detrition coefficients for category 1, 2 and 3*

$-(b''_{16})^{(2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2)}(G_{19},t)$ *are second detrition coefficients for category 1, 2 and 3*

$-(b''_{13})^{(1,1,1,)}(G,t)$, $-(b''_{14})^{(1,1,1,)}(G,t)$, $-(b''_{15})^{(1,1,1,)}(G,t)$ *are third detrition coefficients for category 1,2 and 3*

$-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)$ *are fourth detrition coefficients for category 1, 2 and 3*

$-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)$ *are fifth detrition coefficients for category 1, 2 and 3*

$-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)$ *are sixth detrition coefficients for category 1, 2 and 3*

$-(b''_{36})^{(7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt}=(a_{24})^{(4)}G_{25}-\begin{bmatrix}(a'_{24})^{(4)}\boxed{+(a''_{24})^{(4)}(T_{25},t)}&\boxed{+(a''_{28})^{(5,5,)}(T_{29},t)}&\boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}\\ \boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)}&\boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)}&\boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)}&\boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)}&\boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{24} \quad (73)$$

$$\frac{dG_{25}}{dt}=(a_{25})^{(4)}G_{24}-\begin{bmatrix}(a'_{25})^{(4)}\boxed{+(a''_{25})^{(4)}(T_{25},t)}&\boxed{+(a''_{29})^{(5,5,)}(T_{29},t)}&\boxed{+(a''_{33})^{(6,6)}(T_{33},t)}\\ \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)}&\boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)}&\boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)}&\boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)}&\boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{25} \quad (74)$$

$$\frac{dG_{26}}{dt}=(a_{26})^{(4)}G_{25}-\begin{bmatrix}(a'_{26})^{(4)}\boxed{+(a''_{26})^{(4)}(T_{25},t)}&\boxed{+(a''_{30})^{(5,5,)}(T_{29},t)}&\boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}\\ \boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)}&\boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)}&\boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)}&\boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)}&\boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{26} \quad (75)$$

$\boxed{(a''_{24})^{(4)}(T_{25},t)}$, $\boxed{(a''_{25})^{(4)}(T_{25},t)}$, $\boxed{(a''_{26})^{(4)}(T_{25},t)}$ *are first augmentation coefficients category* 1, 2 3

$\boxed{+(a''_{28})^{(5,5,)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,)}(T_{29},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)}$
*are fourth augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)}$
*are fifth augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)}$
*are sixth augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)}$
*are seventh augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)}$
*are eighth augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt}=(b_{24})^{(4)}T_{25}-\begin{bmatrix}(b'_{24})^{(4)}\boxed{-(b''_{24})^{(4)}(G_{27},t)}&\boxed{-(b''_{28})^{(5,5,)}(G_{31},t)}&\boxed{-(b''_{32})^{(6,6,)}(G_{35},t)}\\ \boxed{-(b''_{13})^{(1,1,1,1)}(G,t)}&\boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)}&\boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)}\\ \boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)}&\boxed{-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)}&\boxed{-(b''_{44})^{(9,9,9,9)}(G_{47},t)}\end{bmatrix}T_{24} \quad (76)$$

$$\frac{dT_{25}}{dt}=(b_{25})^{(4)}T_{24}-\begin{bmatrix}(b'_{25})^{(4)}\boxed{-(b''_{25})^{(4)}(G_{27},t)}&\boxed{-(b''_{29})^{(5,5,)}(G_{31},t)}&\boxed{-(b''_{33})^{(6,6,)}(G_{35},t)}\\ \boxed{-(b''_{14})^{(1,1,1,1)}(G,t)}&\boxed{-(b''_{17})^{(2,2,2,2)}(G_{19},t)}&\boxed{-(b''_{21})^{(3,3,3,3)}(G_{23},t)}\\ \boxed{-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)}&\boxed{-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)}&\boxed{-(b''_{45})^{(9,9,9,9)}(G_{47},t)}\end{bmatrix}T_{25} \quad (77)$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)} & -(b''_{26})^{(4)}(G_{27},t) & -(b''_{30})^{(5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1)}(G,t) & -(b''_{18})^{(2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{26} \quad 78$$

*Where* $-(b''_{24})^{(4)}(G_{27},t)$, $-(b''_{25})^{(4)}(G_{27},t)$, $-(b''_{26})^{(4)}(G_{27},t)$ *are first detrition coefficients for category* $1,2$ *and* $3$

$-(b''_{28})^{(5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,)}(G_{31},t)$ *are second detrition coefficients for category* $1,2$ *and* $3$

$-(b''_{32})^{(6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,)}(G_{35},t)$ *are third detrition coefficients for category* $1,2$ *and* $3$

$-(b''_{13})^{(1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1)}(G,t)$
*are fourth detrition coefficients for category* $1,2$ *and* $3$

$-(b''_{16})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2)}(G_{19},t)$
*are fifth detrition coefficients for category* $1,2$ *and* $3$

$-(b''_{20})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3)}(G_{23},t)$
*are sixth detrition coefficients for category* $1,2$ *and* $3$

$-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)$
*are seventh detrition coefficients for category* $1,2$ *and* $3$

$-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)$
*are eighth detrition coefficients for category* $1,2$ *and* $3$

$-(b''_{46})^{(9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \begin{bmatrix} (a'_{28})^{(5)} & +(a''_{28})^{(5)}(T_{29},t) & +(a''_{24})^{(4,4,)}(T_{25},t) & +(a''_{32})^{(6,6,6)}(T_{33},t) \\ & +(a''_{13})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{16})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3,3)}(T_{21},t) \\ & +(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{28} \quad 79$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \begin{bmatrix} (a'_{29})^{(5)} & +(a''_{29})^{(5)}(T_{29},t) & +(a''_{25})^{(4,4,)}(T_{25},t) & +(a''_{33})^{(6,6,6)}(T_{33},t) \\ & +(a''_{14})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{17})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3,3)}(T_{21},t) \\ & +(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{29} \quad 80$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \begin{bmatrix} (a'_{30})^{(5)} & +(a''_{30})^{(5)}(T_{29},t) & +(a''_{26})^{(4,4,)}(T_{25},t) & +(a''_{34})^{(6,6,6)}(T_{33},t) \\ & +(a''_{15})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{18})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3,3)}(T_{21},t) \\ & +(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{30} \quad 81$$

*Where* $+(a''_{28})^{(5)}(T_{29},t)$, $+(a''_{29})^{(5)}(T_{29},t)$, $+(a''_{30})^{(5)}(T_{29},t)$ *are first augmentation coefficients for category* $1,2$ *and* $3$

*And* $+(a''_{24})^{(4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,)}(T_{25},t)$ *are second augmentation coefficient for category* $1,2$ *and* $3$

$+(a''_{32})^{(6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6)}(T_{33},t)$ *are third augmentation coefficient for category* $1,2$ *and* $3$

$+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)$ are fourth augmentation coefficients for category 1,2, and 3

$+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)$ are fifth augmentation coefficients for category 1,2,and 3

$+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)$ are sixth augmentation coefficients for category 1,2, 3

$+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)$ , $+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1,2, 3

$+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)$ are eighth augmentation coefficients for category 1,2, 3

$+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \begin{bmatrix} (b'_{28})^{(5)} & -(b''_{28})^{(5)}(G_{31},t) & -(b''_{24})^{(4,4,)}(G_{27},t) & -(b''_{32})^{(6,6,6)}(G_{35},t) \\ & -(b''_{13})^{(1,1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{28} \quad 82$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b'_{29})^{(5)} & -(b''_{29})^{(5)}(G_{31},t) & -(b''_{25})^{(4,4,)}(G_{27},t) & -(b''_{33})^{(6,6,6)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{29} \quad 83$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b'_{30})^{(5)} & -(b''_{30})^{(5)}(G_{31},t) & -(b''_{26})^{(4,4,)}(G_{27},t) & -(b''_{34})^{(6,6,6)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1,1,)}(G,t) & -(b''_{18})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{30} \quad 84$$

*where* $-(b''_{28})^{(5)}(G_{31},t)$ , $-(b''_{29})^{(5)}(G_{31},t)$ , $-(b''_{30})^{(5)}(G_{31},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{24})^{(4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,)}(G_{27},t)$ *are second detrition coefficients for category* 1,2 *and* 3

$-(b''_{32})^{(6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6)}(G_{35},t)$ *are third detrition coefficients for category* 1,2 *and* 3

$-(b''_{13})^{(1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1)}(G,t)$ , $-(b''_{15})^{(1,1,1,1,1,)}(G,t)$ are fourth detrition coefficients for category 1,2, and 3

$-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1,2, and 3

$-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1,2, and 3

$-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2, and 3

$-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} \qquad 85$$

$$-\left[\begin{array}{ccc} (a'_{32})^{(6)}\boxed{+(a''_{32})^{(6)}(T_{33},t)} & \boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)} \end{array}\right]G_{32}$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[\begin{array}{ccc} (a'_{33})^{(6)}\boxed{+(a''_{33})^{(6)}(T_{33},t)} & \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)} \end{array}\right]G_{33} \qquad 86$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[\begin{array}{ccc} (a'_{34})^{(6)}\boxed{+(a''_{34})^{(6)}(T_{33},t)} & \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)} \end{array}\right]G_{34} \qquad 87$$

$\boxed{+(a''_{32})^{(6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6)}(T_{33},t)}$ *are first augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}$ *are second augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}$ *are third augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}$ - are fourth augmentation coefficients

$\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}$ - fifth augmentation coefficients

$\boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)}$ sixth augmentation coefficients

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)}$ seventh augmentation coefficients

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}$ Eighth augmentation coefficients

$\boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)}$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[\begin{array}{ccc} (b'_{32})^{(6)}\boxed{-(b''_{32})^{(6)}(G_{35},t)} & \boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)} \end{array}\right]T_{32} \qquad 88$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \begin{bmatrix} (b'_{33})^{(6)}\boxed{-(b''_{33})^{(6)}(G_{35},t)}\boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}\boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}\boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)}\boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{33} \quad (89)$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)}\boxed{-(b''_{34})^{(6)}(G_{35},t)}\boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}\boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}\boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)}\boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{34} \quad (90)$$

$\boxed{-(b''_{32})^{(6)}(G_{35},t)}, \boxed{-(b''_{33})^{(6)}(G_{35},t)}, \boxed{-(b''_{34})^{(6)}(G_{35},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}$ *are second detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)}$ *are third detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}$ are fourth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \quad (91)$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a'_{36})^{(7)}\boxed{+(a''_{36})^{(7)}(T_{37},t)}\boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)}\boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)}\boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)}\boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)}\boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \quad (92)$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a'_{37})^{(7)}\boxed{+(a''_{37})^{(7)}(T_{37},t)}\boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)}\boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)}\boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)}\boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}\boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$\frac{dG_{38}}{dt}$ *93*

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a'_{38})^{(7)}\boxed{+(a''_{38})^{(7)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{(a''_{36})^{(7)}(T_{37},t)}$, $\boxed{(a''_{37})^{(7)}(T_{37},t)}$, $\boxed{(a''_{38})^{(7)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,)}(T_{41},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$\frac{dT_{36}}{dt} =$ 94

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b'_{36})^{(7)}\boxed{-(b''_{36})^{(7)}(G_{39},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$\frac{dT_{37}}{dt} =$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b'_{37})^{(7)}\boxed{-(b''_{37})^{(7)}(G_{39},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$\frac{dT_{38}}{dt} =$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b'_{38})^{(7)}\boxed{-(b''_{38})^{(7)}(G_{39},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b_{36}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{37}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{16}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{17}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{18}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{20}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{21}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{22}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{24}'')^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{25}'')^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{26}'')^{(4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{28}'')^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{32}'')^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{34}'')^{(6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{46}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{45}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{44}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3



$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \begin{bmatrix} (a_{40}')^{(8)}\boxed{+(a_{40}'')^{(8)}(T_{41},t)} & \boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{20}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{24}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{28}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{32}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{36}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{44}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \begin{bmatrix} (a_{41}')^{(8)}\boxed{+(a_{41}'')^{(8)}(T_{41},t)} & \boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{21}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{25}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{29}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{33}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{37}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{45}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \begin{bmatrix} (a_{42}')^{(8)}\boxed{+(a_{42}'')^{(8)}(T_{41},t)} & \boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{22}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{26}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{30}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{34}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{15}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{38}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{46}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a_{40}'')^{(8)}(T_{41},t)}$, $\boxed{+(a_{41}'')^{(8)}(T_{41},t)}$, $\boxed{+(a_{42}'')^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second

augmentation coefficient for category 1, 2 and 3
$+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)$ , $+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)$ , $+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3
$+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)$ , $+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)$ , $+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3
$+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)$ , $+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3
$+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)$ , $+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3
$+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ $+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ $+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ are seventh augmentation coefficient for 1,2,3
$+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$ , $+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$ , $+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$ are eighth augmentation coefficient for 1,2,3
$+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$ , $+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$ , $+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} =$$

$$(b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)}\left[-(b''_{40})^{(8)}(G_{43},t)\right] & \left[-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} =$$

$$(b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)}\left[-(b''_{41})^{(8)}(G_{43},t)\right] & \left[-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} =$$

$$(b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)}\left[-(b''_{42})^{(8)}(G_{43},t)\right] & \left[-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{15}$$

Where $-(b''_{36})^{(7)}(G_{39},t)$ , $-(b''_{37})^{(7)}(G_{39},t)$ , $-(b''_{38})^{(7)}(G_{39},t)$ are first detrition coefficients for category 1, 2 and 3
$-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$ , $-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$ , $-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3
$-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$ , $-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$ , $-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3
$-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)$ , $-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)$ , $-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3



$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \begin{bmatrix} (a'_{44})^{(9)}\boxed{+(a''_{44})^{(9)}(T_{45},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \begin{bmatrix} (a'_{45})^{(9)}\boxed{+(a''_{45})^{(9)}(T_{45},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \begin{bmatrix} (a'_{46})^{(9)}\boxed{+(a''_{46})^{(9)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a''_{44})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,1,1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1,1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1,1,1,1,1,1)}(T_{14},t)$ are Seventh augmentation coefficient for category 1, 2 and 3

$+(a''_{38})^{(7,7,7,7,7,7,7,7,7,7)}(T_{37},t)$ $+(a''_{37})^{(7,7,7,7,7,7,7,7,7,7)}(T_{37},t)$ $+(a''_{36})^{(7,7,7,7,7,7,7,7,7,7)}(T_{37},t)$ are eighth augmentation coefficient for 1,2,3

$+(a''_{40})^{(8,8,8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8,8,8,8,8)}(T_{41},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} =$$

$$(b_{44})^{(9)}T_{45} -$$

$$\begin{bmatrix} (b'_{44})^{(9)} & -(b''_{44})^{(9)}(G_{47},t) & -(b''_{16})^{(2,2,2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6,6,6,6,6,6)}(G_{35},t) & \\ -(b''_{13})^{(1,1,1,1,1,1,1,1,1,1)}(G,t) & -(b''_{36})^{(7,7,7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8,8,8,8,8)}(G_{43},t) & \end{bmatrix} T_{13}$$

$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b'_{45})^{(9)} & -(b''_{45})^{(9)}(G_{47},t) & -(b''_{17})^{(2,2,2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6,6,6,6,6,6)}(G_{35},t) & \\ -(b''_{14})^{(1,1,1,1,1,1,1,1,1,1)}(G,t) & -(b''_{37})^{(7,7,7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8,8,8,8,8)}(G_{43},t) & \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b'_{46})^{(9)} & -(b''_{46})^{(9)}(G_{47},t) & -(b''_{18})^{(2,2,2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6,6,6,6,6,6)}(G_{35},t) & \\ -(b''_{15})^{(1,1,1,1,1,1,1,1,1,1)}(G,t) & -(b''_{38})^{(7,7,7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8,8,8,8,8)}(G_{43},t) & \end{bmatrix} T_{15}$$

Where $-(b''_{44})^{(9)}(G_{47},t)$, $-(b''_{45})^{(9)}(G_{47},t)$, $-(b''_{46})^{(9)}(G_{47},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6,6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{15})^{(1,1,1,1,1,1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1,1,1,1,1,1)}(G,t)$, $-(b''_{13})^{(1,1,1,1,1,1,1,1,1,1)}(G,t)$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{37}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{36}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a_i')^{(1)}, (a_i'')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (b_i'')^{(1)} > 0, \quad i,j = 13,14,15 \qquad 97$$

The functions $(a_i'')^{(1)}, (b_i'')^{(1)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a_i'')^{(1)}(T_{14},t) \leq (p_i)^{(1)} \leq (\hat{A}_{13})^{(1)}$$

$$(b_i'')^{(1)}(G,t) \leq (r_i)^{(1)} \leq (b_i')^{(1)} \leq (\hat{B}_{13})^{(1)}$$

$$lim_{T_2\to\infty}(a_i'')^{(1)}(T_{14},t) = (p_i)^{(1)} \qquad 98$$

$$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = (r_i)^{(1)}$$

**Definition of** $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$ :

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$$|(a_i'')^{(1)}(T_{14}',t) - (a_i'')^{(1)}(T_{14},t)| \leq (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G',t) - (b_i'')^{(1)}(G,t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}',t)$ and $(a_i'')^{(1)}(T_{14},t)$ . $(T_{14}',t)$ and $(T_{14},t)$ are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14},t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$ : 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$$

**Definition of** $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$ : 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i,j = 16,17,18$$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17}, t) \leq (p_i)^{(2)} \leq (\hat{A}_{16})^{(2)} \qquad 102$$

$$(b_i'')^{(2)}(G_{19}, t) \leq (r_i)^{(2)} \leq (b_i')^{(2)} \leq (\hat{B}_{16})^{(2)} \qquad 103$$

$$\lim_{T_2 \to \infty} (a_i'')^{(2)}(T_{17}, t) = (p_i)^{(2)} \qquad 104$$

$$\lim_{G \to \infty} (b_i'')^{(2)}((G_{19}), t) = (r_i)^{(2)} \qquad 105$$

**Definition of** $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$ : 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}', t) - (a_i'')^{(2)}(T_{17}, t)| \leq (\hat{k}_{16})^{(2)}|T_{17} - T_{17}'|e^{-(\hat{M}_{16})^{(2)}t} \qquad 107$$

$$|(b_i'')^{(2)}((G_{19})', t) - (b_i'')^{(2)}((G_{19}), t)| < (\hat{k}_{16})^{(2)}||(G_{19}) - (G_{19})'||e^{-(\hat{M}_{16})^{(2)}t} \qquad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}', t)$ and$(a_i'')^{(2)}(T_{17}, t)$ . $(T_{17}', t)$ and $(T_{17}, t)$ are points belonging to the interval $[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}]$ . It is to be noted that $(a_i'')^{(2)}(T_{17}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17}, t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$ :

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants 109

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$$

**Definition of** $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$ :

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1 \quad (110)$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(b_i)^{(2)} + (b_i')^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1 \quad (111)$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i,j = 20,21,22 \quad (112)$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(3)}$, $(r_i)^{(3)}$:

$$(a_i'')^{(3)}(T_{21}, t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23}, t) \leq (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(3)}(T_{21}, t) = (p_i)^{(3)} \quad (113)$$

$$\lim_{G \to \infty}(b_i'')^{(3)}(G_{23}, t) = (r_i)^{(3)}$$

**Definition of** $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$ :

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: (114)

$$|(a_i'')^{(3)}(T_{21}', t) - (a_i'')^{(3)}(T_{21}, t)| \leq (\hat{k}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\hat{M}_{20})^{(3)}t}$$

$$|(b_i'')^{(3)}(G_{23}', t) - (b_i'')^{(3)}(G_{23}, t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}'||e^{-(\hat{M}_{20})^{(3)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}', t)$ and $(a_i'')^{(3)}(T_{21}, t)$ . $(T_{21}', t)$ And $(T_{21}, t)$ are points belonging to the interval $[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}]$. It is to be noted that $(a_i'')^{(3)}(T_{21}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$ : (115)

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$ and the constants $(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}, i = 20,21,22,$ satisfy the inequalities 116

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(a_i)^{(3)} + (a_i')^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(b_i)^{(3)} + (b_i')^{(3)} + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i, j = 24,25,26 \qquad 117$$

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(4)}$, $(r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25}, t) \le (p_i)^{(4)} \le (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}((G_{27}), t) \le (r_i)^{(4)} \le (b_i')^{(4)} \le (\hat{B}_{24})^{(4)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(4)}(T_{25}, t) = (p_i)^{(4)} \qquad 118$$

$$\lim_{G \to \infty}(b_i'')^{(4)}((G_{27}), t) = (r_i)^{(4)}$$

**Definition of** $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$ :

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24,25,26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}', t) - (a_i'')^{(4)}(T_{25}, t)| \le (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})', t) - (b_i'')^{(4)}((G_{27}), t)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}', t)$ and $(a_i'')^{(4)}(T_{25}, t)$ . $(T_{25}', t)$ and $(T_{25}, t)$ are points belonging to the interval $[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}]$. It is to be noted that $(a_i'')^{(4)}(T_{25}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$ : 120

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$, are positive constants

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}, \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$$

**Definition of** $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$ : 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and\ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}, i = 24,25,26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(a_i)^{(4)} + (a_i')^{(4)} + (\hat{A}_{24})^{(4)} + (\hat{P}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(b_i)^{(4)} + (b_i')^{(4)} + (\hat{B}_{24})^{(4)} + (\hat{Q}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i,j = 28,29,30 \qquad 122$$

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(5)}$, $(r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29}, t) \le (p_i)^{(5)} \le (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}((G_{31}), t) \le (r_i)^{(5)} \le (b_i')^{(5)} \le (\hat{B}_{28})^{(5)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(5)}(T_{29}, t) = (p_i)^{(5)} \qquad 123$$

$$\lim_{G \to \infty}(b_i'')^{(5)}(G_{31}, t) = (r_i)^{(5)}$$

**Definition of** $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$ :

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}', t) - (a_i'')^{(5)}(T_{29}, t)| \le (\hat{k}_{28})^{(5)}|T_{29} - T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})', t) - (b_i'')^{(5)}((G_{31}), t)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}', t)$ and $(a_i'')^{(5)}(T_{29}, t)$ . $(T_{29}', t)$ and $(T_{29}, t)$ are points belonging to the interval $[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}]$. It is to be noted that $(a_i'')^{(5)}(T_{29}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29}, t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$ : 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}, \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

**Definition of** $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$ : 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\widehat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and\ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\widehat{M}_{28})^{(5)}}[(a_i)^{(5)} + (a_i')^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\widehat{M}_{28})^{(5)}}[(b_i)^{(5)} + (b_i')^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i,j = 32,33,34$$ 127

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(6)}$, $(r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33}, t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}), t) \leq (r_i)^{(6)} \leq (b_i')^{(6)} \leq (\hat{B}_{32})^{(6)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(6)}(T_{33}, t) = (p_i)^{(6)}$$ 128

$$\lim_{G \to \infty}(b_i'')^{(6)}((G_{35}), t) = (r_i)^{(6)}$$

**Definition of** $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$ :

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}', t) - (a_i'')^{(6)}(T_{33}, t)| \leq (\hat{k}_{32})^{(6)}|T_{33} - T_{33}'|e^{-(\widehat{M}_{32})^{(6)}t}$$

$$|(b_i'')^{(6)}((G_{35})', t) - (b_i'')^{(6)}((G_{35}), t)| < (\hat{k}_{32})^{(6)}||(G_{35}) - (G_{35})'||e^{-(\widehat{M}_{32})^{(6)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}', t)$ and $(a_i'')^{(6)}(T_{33}, t)$ . $(T_{33}', t)$ and $(T_{33}, t)$ are points belonging to the interval $[(\hat{k}_{32})^{(6)}, (\widehat{M}_{32})^{(6)}]$. It is to be noted that $(a_i'')^{(6)}(T_{33}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33}, t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\widehat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$ : 129

$(\widehat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$, are positive constants

$$\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}} < 1$$

**Definition of** $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$ : 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and\ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32,33,34,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(a_i)^{(6)} + (a_i')^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)} (\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(b_i)^{(6)} + (b_i')^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)} (\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

(UUU) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i, j = 36,37,38$ 131

(VVV) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(7)}$, $(r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37}, t) \leq (p_i)^{(7)} \leq (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39}, t) \leq (r_i)^{(7)} \leq (b_i')^{(7)} \leq (\hat{B}_{36})^{(7)}$$

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*(WWW)* $\lim_{T_2 \to \infty} (a_i'')^{(7)}(T_{37}, t) = (p_i)^{(7)}$

*(XXX)*

$\lim_{G \to \infty} (b_i'')^{(7)}((G_{39}), t) = (r_i)^{(7)}$

**Definition of** $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$ :

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$$|(a_i'')^{(7)}(T_{37}', t) - (a_i'')^{(7)}(T_{37}, t)| \leq (\hat{k}_{36})^{(7)} |T_{37} - T_{37}'| e^{-(\hat{M}_{36})^{(7)} t}$$

$$|(b_i'')^{(7)}((G_{39})', t) - (b_i'')^{(7)}((G_{39}), t)| < (\hat{k}_{36})^{(7)} ||(G_{39}) - (G_{39})'|| e^{-(\hat{M}_{36})^{(7)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}', t)$ and $(a_i'')^{(7)}(T_{37}, t)$ . $(T_{37}', t)$ and $(T_{37}, t)$ are points belonging to the interval $[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}]$ . It is to be noted that $(a_i'')^{(7)}(T_{37}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37}, t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$ : 134

(YYY) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)},$ are positive constants

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$$

**Definition of** $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$ : 135

(ZZZ) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(b_i)^{(7)} + (b_i')^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40,41,42 \qquad 136$$

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

**Definition of** $(p_i)^{(8)}$, $(r_i)^{(8)}$: 137

$$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)} \qquad 138$$

$$(b_i'')^{(8)}((G_{43}), t) \leq (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)} \qquad 139$$

$$\lim_{T_2 \to \infty} (a_i'')^{(8)}(T_{41}, t) = (p_i)^{(8)} \qquad 140$$

$$\lim_{G \to \infty} (b_i'')^{(8)}\big((G_{43}), t\big) = (r_i)^{(8)} \qquad 141$$

**Definition of** $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$ :

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)} |T_{41} - T_{41}'| e^{-(\hat{M}_{40})^{(8)} t} \qquad 142$$

$$|(b_i'')^{(8)}((G_{43})', t) - (b_i'')^{(8)}\big((G_{43}), t\big)| < (\hat{k}_{40})^{(8)} ||(G_{43}) - (G_{43})'|| e^{-(\hat{M}_{40})^{(8)} t} \qquad 143$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}', t)$ and $(a_i'')^{(8)}(T_{41}, t)$ . $(T_{41}', t)$ and $(T_{41}, t)$ are points belonging to the interval $\big[(\hat{k}_{40})^{(8)}, (\hat{M}_{40})^{(8)}\big]$. It is to be noted that $(a_i'')^{(8)}(T_{41}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{40})^{(8)} = 1$

then the function $(a_i'')^{(8)}(T_{41},t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)}$ :

$(\hat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)},$ are positive constants

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\ ,\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}}<1 \qquad 144$$

**Definition of** $(\hat{P}_{40})^{(8)},(\hat{Q}_{40})^{(8)}$ :

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\hat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)},(\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)},(a_i')^{(8)},(b_i)^{(8)},(b_i')^{(8)},(p_i)^{(8)},\ (r_i)^{(8)},i=40,41,42,$

Satisfy the inequalities

$$\frac{1}{(\hat{M}_{40})^{(8)}}[\,(a_i)^{(8)}+(a_i')^{(8)}+\ (\hat{A}_{40})^{(8)}+\ (\hat{P}_{40})^{(8)}\,(\hat{k}_{40})^{(8)}]<1 \qquad 145$$

$$\frac{1}{(\hat{M}_{40})^{(8)}}[\ (b_i)^{(8)}+(b_i')^{(8)}+\ (\hat{B}_{40})^{(8)}+\ (\hat{Q}_{40})^{(8)}\ (\hat{k}_{40})^{(8)}]<1 \qquad 146$$

Where we suppose

$$(a_i)^{(9)},(a_i')^{(9)},(a_i'')^{(9)},(b_i)^{(9)},(b_i')^{(9)},(b_i'')^{(9)}>0,\quad i,j=44,45,46 \qquad 146\,A$$

The functions $(a_i'')^{(9)},(b_i'')^{(9)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(9)},\ (r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45},t)\le(p_i)^{(9)}\le(\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47},t)\le\ (r_i)^{(9)}\le(b_i')^{(9)}\le(\hat{B}_{44})^{(9)}$$

$$\lim_{T_2\to\infty}(a_i'')^{(9)}(T_{45},t)=(p_i)^{(9)}$$

$$\lim_{G\to\infty}(b_i'')^{(9)}(G_{47},t)=\ (r_i)^{(9)}$$

**Definition of** $(\hat{A}_{44})^{(9)},(\hat{B}_{44})^{(9)}$ :

Where $\boxed{(\hat{A}_{44})^{(9)},(\hat{B}_{44})^{(9)},(p_i)^{(9)},\ (r_i)^{(9)}}$ are positive constants and $\boxed{i=44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}',t)-(a_i'')^{(9)}(T_{45},t)|\le(\hat{k}_{44})^{(9)}|T_{45}-\ T_{45}'|e^{-(\hat{M}_{44})^{(9)}t}$$

$$|(b_i'')^{(9)}((G_{47})',t)-(b_i'')^{(9)}\big((G_{47}),t\big)|<(\hat{k}_{44})^{(9)}||(G_{47})-(G_{47})'||e^{-(\hat{M}_{44})^{(9)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}',t)$ and$(a_i'')^{(9)}(T_{45},t)$ . $(T_{45}',t)$ and $(T_{45},t)$ are points belonging to the interval $\big[(\hat{k}_{44})^{(9)},(\hat{M}_{44})^{(9)}\big]$. It is

to be noted that $(a_i'')^{(9)}(T_{45},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{44})^{(9)}=1$ then the function $(a_i'')^{(9)}(T_{45},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\widehat{M}_{44})^{(9)},(\hat{k}_{44})^{(9)}$ :

$(\widehat{M}_{44})^{(9)},(\hat{k}_{44})^{(9)}$, are positive constants

$$\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\ ,\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}<1$$

**Definition of** $(\hat{P}_{44})^{(9)},(\hat{Q}_{44})^{(9)}$ :

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\widehat{M}_{44})^{(9)},(\hat{k}_{44})^{(9)},(\hat{A}_{44})^{(9)} and\ (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)},(a_i')^{(9)},(b_i)^{(9)},(b_i')^{(9)},(p_i)^{(9)},\ (r_i)^{(9)}, i=44,45,46,$ satisfy the inequalities

$$\frac{1}{(\widehat{M}_{44})^{(9)}}[(a_i)^{(9)}+(a_i')^{(9)}+\ (\hat{A}_{44})^{(9)}+\ (\hat{P}_{44})^{(9)}\ (\hat{k}_{44})^{(9)}]<1$$

$$\frac{1}{(\widehat{M}_{44})^{(9)}}[\ (b_i)^{(9)}+(b_i')^{(9)}+\ (\hat{B}_{44})^{(9)}+\ (\hat{Q}_{44})^{(9)}\ (\hat{k}_{44})^{(9)}]<1$$

**Theorem 1:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0)\ ,T_i(0)$ :

$$G_i(t)\le\ (\hat{P}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}\ ,\ \boxed{G_i(0)=G_i^0>0}$$

$$T_i(t)\le\ (\hat{Q}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}\ ,\ \boxed{T_i(0)=T_i^0>0}$$

**Theorem 2 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

**Definition of** $G_i(0)\ ,T_i(0)$

$$G_i(t)\le\ (\hat{P}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}\ ,\ \ G_i(0)=G_i^0>0$$

$$T_i(t)\le\ (\hat{Q}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}\ ,\ \ T_i(0)=T_i^0>0$$

**Theorem 3 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t)\le\ (\hat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}\ ,\ \ G_i(0)=G_i^0>0$$

$$T_i(t)\le\ (\hat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}\ ,\ \ T_i(0)=T_i^0>0$$

**Theorem 4 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

**Definition of** $G_i(0)\ ,T_i(0)$ :

$G_i(t) \leq \left(\hat{P}_{24}\right)^{(4)} e^{(\hat{M}_{24})^{(4)}t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{24}\right)^{(4)} e^{(\hat{M}_{24})^{(4)}t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Theorem 5 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

**Definition of** $G_i(0)$ , $T_i(0)$ :

$G_i(t) \leq \left(\hat{P}_{28}\right)^{(5)} e^{(\hat{M}_{28})^{(5)}t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{28}\right)^{(5)} e^{(\hat{M}_{28})^{(5)}t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Theorem 6 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

**Definition of** $G_i(0)$ , $T_i(0)$ :

$G_i(t) \leq \left(\hat{P}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Theorem 7:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

**Definition of** $G_i(0)$ , $T_i(0)$ :

$G_i(t) \leq \left(\hat{P}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Theorem 8:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

**Definition of** $G_i(0)$ , $T_i(0)$ :

$G_i(t) \leq \left(\hat{P}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Theorem 9:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

**Definition of** $G_i(0)$ , $T_i(0)$ :

$G_i(t) \leq \left(\hat{P}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Proof:** Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i$: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \le (\hat{P}_{13})^{(1)} \text{ , } T_i^0 \le (\hat{Q}_{13})^{(1)}, \quad 155$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t} \quad 156$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t} \quad 157$$

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}(s_{(13)}) - \left((a'_{13})^{(1)} + a''_{13})^{(1)} (T_{14}(s_{(13)}), s_{(13)})\right) G_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)} (T_{14}(s_{(13)}), s_{(13)})\right) G_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)} (T_{14}(s_{(13)}), s_{(13)})\right) G_{15}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)} (G(s_{(13)}), s_{(13)})\right) T_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)} (G(s_{(13)}), s_{(13)})\right) T_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{15}(t) = T_{15}^0 + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)} (G(s_{(13)}), s_{(13)})\right) T_{15}(s_{(13)})\right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i$: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \le (\hat{P}_{16})^{(2)} \text{ , } T_i^0 \le (\hat{Q}_{16})^{(2)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$$

By 160

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''_{16})^{(2)} (T_{17}(s_{(16)}), s_{(16)})\right) G_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)} (T_{17}(s_{(16)}), s_{(17)})\right) G_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)} (T_{17}(s_{(16)}), s_{(16)})\right) G_{18}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)} (G_{19}(s_{(16)}), s_{(16)})\right) T_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^{0} + \int_0^t \left[ (b_{17})^{(2)} T_{16}(s_{(16)}) - \left( (b'_{17})^{(2)} - (b''_{17})^{(2)} (G_{19}(s_{(16)}), s_{(16)}) \right) T_{17}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^{0} + \int_0^t \left[ (b_{18})^{(2)} T_{17}(s_{(16)}) - \left( (b'_{18})^{(2)} - (b''_{18})^{(2)} (G_{19}(s_{(16)}), s_{(16)}) \right) T_{18}(s_{(16)}) \right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \le (\hat{P}_{20})^{(3)} \text{ , } T_i^0 \le (\hat{Q}_{20})^{(3)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$$

By 161

$$\bar{G}_{20}(t) = G_{20}^{0} + \int_0^t \left[ (a_{20})^{(3)} G_{21}(s_{(20)}) - \left( (a'_{20})^{(3)} + a''_{20})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^{0} + \int_0^t \left[ (a_{21})^{(3)} G_{20}(s_{(20)}) - \left( (a'_{21})^{(3)} + (a''_{21})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{21}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^{0} + \int_0^t \left[ (a_{22})^{(3)} G_{21}(s_{(20)}) - \left( (a'_{22})^{(3)} + (a''_{22})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{22}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^{0} + \int_0^t \left[ (b_{20})^{(3)} T_{21}(s_{(20)}) - \left( (b'_{20})^{(3)} - (b''_{20})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^{0} + \int_0^t \left[ (b_{21})^{(3)} T_{20}(s_{(20)}) - \left( (b'_{21})^{(3)} - (b''_{21})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{21}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{\mathrm{T}}_{22}(\mathrm{t}) = \mathrm{T}_{22}^{0} + \int_0^t \left[ (b_{22})^{(3)} T_{21}(s_{(20)}) - \left( (b'_{22})^{(3)} - (b''_{22})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{22}(s_{(20)}) \right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \le (\hat{P}_{24})^{(4)} \text{ , } T_i^0 \le (\hat{Q}_{24})^{(4)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

By 162

$$\bar{G}_{24}(t) = G_{24}^{0} + \int_0^t \left[ (a_{24})^{(4)} G_{25}(s_{(24)}) - \left( (a'_{24})^{(4)} + a''_{24})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{26}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{\mathrm{T}}_{26}(\mathrm{t}) = \mathrm{T}_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{26}(s_{(24)})\right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{28})^{(5)}, T_i^0 \le (\hat{Q}_{28})^{(5)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

By 163

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{30}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{\mathrm{T}}_{30}(\mathrm{t}) = \mathrm{T}_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{30}(s_{(28)})\right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{32})^{(6)}, T_i^0 \le (\hat{Q}_{32})^{(6)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\bar{M}_{32})^{(6)} t}$

By 164

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[ (a_{32})^{(6)} G_{33}(s_{(32)}) - \left( (a'_{32})^{(6)} + a''_{32})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[ (a_{33})^{(6)} G_{32}(s_{(32)}) - \left( (a'_{33})^{(6)} + (a''_{33})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[ (a_{34})^{(6)} G_{33}(s_{(32)}) - \left( (a'_{34})^{(6)} + (a''_{34})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{34}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[ (b_{32})^{(6)} T_{33}(s_{(32)}) - \left( (b'_{32})^{(6)} - (b''_{32})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[ (b_{33})^{(6)} T_{32}(s_{(32)}) - \left( (b'_{33})^{(6)} - (b''_{33})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{34}(t) = T_{34}^0 + \int_0^t \left[ (b_{34})^{(6)} T_{33}(s_{(32)}) - \left( (b'_{34})^{(6)} - (b''_{34})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{34}(s_{(32)}) \right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[ (a_{36})^{(7)} G_{37}(s_{(36)}) - \left( (a'_{36})^{(7)} + a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[ (a_{37})^{(7)} G_{36}(s_{(36)}) - \left( (a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[ (a_{38})^{(7)} G_{37}(s_{(36)}) - \left( (a'_{38})^{(7)} + (a''_{38})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{38}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[ (b_{36})^{(7)} T_{37}(s_{(36)}) - \left( (b'_{36})^{(7)} - (b''_{36})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[ (b_{37})^{(7)} T_{36}(s_{(36)}) - \left( (b'_{37})^{(7)} - (b''_{37})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{38}(t) = T_{38}^0 + \int_0^t \left[ (b_{38})^{(7)} T_{37}(s_{(36)}) - \left( (b'_{38})^{(7)} - (b''_{38})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{38}(s_{(36)}) \right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0,t)$

**Proof:**

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{40})^{(8)}$, $T_i^0 \le (\hat{Q}_{40})^{(8)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} G_{41}(s_{(40)}) - \left((a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[(a_{41})^{(8)} G_{40}(s_{(40)}) - \left((a'_{41})^{(8)} + (a''_{41})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{42}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^0 + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)} (G_{43}(s_{(40)}), s_{(40)})\right) T_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^0 + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)} (G_{43}(s_{(40)}), s_{(40)})\right) T_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{42}(t) = T_{42}^0 + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)} (G_{43}(s_{(40)}), s_{(40)})\right) T_{42}(s_{(40)})\right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0,t)$

**Proof:** 166
Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ A
which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{44})^{(9)}$, $T_i^0 \le (\hat{Q}_{44})^{(9)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)})\right) G_{44}(s_{(44)})\right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)} (T_{45}(s_{(44)}), s_{(44)})\right) G_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}(s_{(44)}), s_{(44)})\right) G_{46}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{44}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{\mathrm{T}}_{46}(\mathrm{t}) = \mathrm{T}_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{46}(s_{(44)})\right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}}\right)\right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left(e^{(\hat{M}_{13})^{(1)} t} - 1\right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\hat{M}_{13})^{(1)} t} \le \frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{P}_{13})^{(1)} + G_{14}^0\right) e^{\left(-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0}\right)} + (\hat{P}_{13})^{(1)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \le G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}}\right)\right] ds_{(16)} = \quad 169$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1\right)$$

From which it follows that 170

$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \le \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left[\left((\hat{P}_{16})^{(2)} + G_{17}^0\right) e^{\left(-\frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0}\right)} + (\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t) \le G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} \left(G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} s_{(20)}}\right)\right] ds_{(20)} =$$

$$\left(1 + (a_{20})^{(3)} t\right) G_{21}^0 + \frac{(a_{20})^{(3)} (\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left(e^{(\hat{M}_{20})^{(3)} t} - 1\right)$$

From which it follows that 172

$$(G_{20}(t)-G_{20}^{0})e^{-(\hat{M}_{20})^{(3)}t}\leq\frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left[((\hat{P}_{20})^{(3)}+G_{21}^{0})e^{\left(-\frac{(\hat{P}_{20})^{(3)}+G_{21}^{0}}{G_{21}^{0}}\right)}+(\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 173

$$G_{24}(t)\leq G_{24}^{0}+\int_{0}^{t}\left[(a_{24})^{(4)}\left(G_{25}^{0}+(\hat{P}_{24})^{(4)}e^{(\hat{M}_{24})^{(4)}s_{(24)}}\right)\right]ds_{(24)}=$$

$$\left(1+(a_{24})^{(4)}t\right)G_{25}^{0}+\frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left(e^{(\hat{M}_{24})^{(4)}t}-1\right)$$

From which it follows that 174

$$(G_{24}(t)-G_{24}^{0})e^{-(\hat{M}_{24})^{(4)}t}\leq\frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left[((\hat{P}_{24})^{(4)}+G_{25}^{0})e^{\left(-\frac{(\hat{P}_{24})^{(4)}+G_{25}^{0}}{G_{25}^{0}}\right)}+(\hat{P}_{24})^{(4)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t)\leq G_{28}^{0}+\int_{0}^{t}\left[(a_{28})^{(5)}\left(G_{29}^{0}+(\hat{P}_{28})^{(5)}e^{(\hat{M}_{28})^{(5)}s_{(28)}}\right)\right]ds_{(28)}=$$

$$\left(1+(a_{28})^{(5)}t\right)G_{29}^{0}+\frac{(a_{28})^{(5)}(\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left(e^{(\hat{M}_{28})^{(5)}t}-1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t}\leq\frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t)\leq G_{32}^{0}+\int_{0}^{t}\left[(a_{32})^{(6)}\left(G_{33}^{0}+(\hat{P}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\right)\right]ds_{(32)}=$$

$$\left(1+(a_{32})^{(6)}t\right)G_{33}^{0}+\frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left(e^{(\hat{M}_{32})^{(6)}t}-1\right)$$

From which it follows that 177

$$(G_{32}(t)-G_{32}^{0})e^{-(\hat{M}_{32})^{(6)}t}\leq\frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left[((\hat{P}_{32})^{(6)}+G_{33}^{0})e^{\left(-\frac{(\hat{P}_{32})^{(6)}+G_{33}^{0}}{G_{33}^{0}}\right)}+(\hat{P}_{32})^{(6)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}$ , $G_{26}, T_{24}, T_{25}, T_{26}$

(m) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t) \le G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} \left(G_{37}^0 + (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} s_{(36)}}\right)\right] ds_{(36)} =$$
$$\left(1 + (a_{36})^{(7)} t\right) G_{37}^0 + \frac{(a_{36})^{(7)} (\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left(e^{(\hat{M}_{36})^{(7)} t} - 1\right)$$

From which it follows that

$$(G_{36}(t) - G_{36}^0) e^{-(\hat{M}_{36})^{(7)} t} \le \frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left[((\hat{P}_{36})^{(7)} + G_{37}^0) e^{\left(-\frac{(\hat{P}_{36})^{(7)} + G_{37}^0}{G_{37}^0}\right)} + (\hat{P}_{36})^{(7)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t) \le G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} \left(G_{41}^0 + (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} s_{(40)}}\right)\right] ds_{(40)} = \quad 180$$
$$\left(1 + (a_{40})^{(8)} t\right) G_{41}^0 + \frac{(a_{40})^{(8)} (\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}} \left(e^{(\hat{M}_{40})^{(8)} t} - 1\right)$$

From which it follows that 181

$$(G_{40}(t) - G_{40}^0) e^{-(\hat{M}_{40})^{(8)} t} \le \frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}} \left[((\hat{P}_{40})^{(8)} + G_{41}^0) e^{\left(-\frac{(\hat{P}_{40})^{(8)} + G_{41}^0}{G_{41}^0}\right)} + (\hat{P}_{40})^{(8)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 8

Analogous inequalities hold also for $G_{41}$ , $G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t) \le G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} \left(G_{45}^0 + (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} s_{(44)}}\right)\right] ds_{(44)} =$$
$$\left(1 + (a_{44})^{(9)} t\right) G_{45}^0 + \frac{(a_{44})^{(9)} (\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left(e^{(\hat{M}_{44})^{(9)} t} - 1\right)$$

From which it follows that

$$(G_{44}(t) - G_{44}^0) e^{-(\hat{M}_{44})^{(9)} t} \le \frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left[((\hat{P}_{44})^{(9)} + G_{45}^0) e^{\left(-\frac{(\hat{P}_{44})^{(9)} + G_{45}^0}{G_{45}^0}\right)} + (\hat{P}_{44})^{(9)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 9

Analogous inequalities hold also for $G_{45}$ , $G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}$ , $\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$ and to choose 182

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\widehat{M}_{13})^{(1)}}\left[(\widehat{P}_{13})^{(1)}+\left((\hat{P}_{13})^{(1)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{13})^{(1)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{13})^{(1)} \quad 183$$

$$\frac{(b_i)^{(1)}}{(\widehat{M}_{13})^{(1)}}\left[\left((\hat{Q}_{13})^{(1)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{13})^{(1)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{13})^{(1)}\right]\leq(\hat{Q}_{13})^{(1)} \quad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)},T^{(1)}\right),\left(G^{(2)},T^{(2)}\right)\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{13})^{(1)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{13})^{(1)}t}\}$$

Indeed if we denote

**Definition of** $\tilde{G},\tilde{T}:\left(\tilde{G},\tilde{T}\right)=\mathcal{A}^{(1)}(G,T)$

It results

$$\left|\tilde{G}_{13}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{13})^{(1)}\left|G_{14}^{(1)}-G_{14}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\,ds_{(13)}+$$

$$\int_0^t\{(a'_{13})^{(1)}\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$G_{13}^{(2)}\left|(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)-(a''_{13})^{(1)}\left(T_{14}^{(2)},s_{(13)}\right)\right|\ e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses it follows

$$\left|G^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}t}\leq \quad 186$$
$$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)}+(a'_{13})^{(1)}+(\widehat{A}_{13})^{(1)}+(\widehat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}\right)d\left(\left(G^{(1)},T^{(1)};\,G^{(2)},T^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

**Remark 1:** The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}\ and\ (\widehat{Q}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i=13,14,15$ depend only on $\mathrm{T}_{14}$ and respectively on

$G(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$

From 19 to 24 it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(1)} - (a_i'')^{(1)}(T_{14}(s_{(13)}), s_{(13)})\} ds_{(13)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(1)} t)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2\ and\ \left((\widehat{M}_{13})^{(1)}\right)_3$ : 187

**Remark 3**: if $G_{13}$ is bounded, the same property have also $G_{14}\ and\ G_{15}$ . indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a_{14}')^{(1)} G_{14}$ and by integrating

$$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1 / (a_{14}')^{(1)}$$

In the same way , one can obtain

$$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2 / (a_{15}')^{(1)}$$

If $G_{14}\ or\ G_{15}$ is bounded, the same property follows for $G_{13}$ , $G_{15}$ and $G_{13}$ , $G_{14}$ respectively.

**Remark 4:** If $G_{13}$ $is$ bounded, from below, the same property holds for $G_{14}\ and\ G_{15}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{14}$ is bounded from below. 188

**Remark 5:** If $T_{13}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}\,(G(t), t)) = (b_{14}')^{(1)}$ then $T_{14} \to \infty$. 189

**Definition of** $(m)^{(1)}$ and $\varepsilon_1$ :

Indeed let $t_1$ be so that for $t > t_1$

$$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t), t) < \varepsilon_1, T_{13}\,(t) > (m)^{(1)}$$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right)$, $t = log\,\frac{2}{\varepsilon_1}$ By taking now $\varepsilon_1$ sufficiently small one sees that $T_{14}$ is unbounded.
The same property holds for $T_{15}$ if $\lim_{t\to\infty}(b_{15}'')^{(1)}\,(G(t), t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}$ , $\frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}\ and\ (\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[(\widehat{P}_{16})^{(2)}+\left((\hat{P}_{16})^{(2)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{16})^{(2)}\right]\leq(\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

**Definition of** $\widetilde{G_{19}},\widetilde{T_{19}}$ : $\left(\widetilde{G_{19}},\widetilde{T_{19}}\right)=\mathcal{A}^{(2)}(G_{19},T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{16})^{(2)}\left|G_{17}^{(1)}-G_{17}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)}+$$

$$\int_0^t\{(a'_{16})^{(2)}\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$G_{16}^{(2)}\left|(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)-(a''_{16})^{(2)}\left(T_{17}^{(2)},s_{(16)}\right)\right|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0,t]$ 197

From the hypotheses it follows

$$\left|(G_{19})^{(1)}-(G_{19})^{(2)}\right|\mathrm{e}^{-(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}\leq$$
$$\frac{1}{(\widehat{\mathrm{M}}_{16})^{(2)}}\left((a_{16})^{(2)}+(a'_{16})^{(2)}+(\widehat{\mathrm{A}}_{16})^{(2)}+(\widehat{\mathrm{P}}_{16})^{(2)}(\hat{k}_{16})^{(2)}\right)\mathrm{d}\left(((G_{19})^{(1)},(T_{19})^{(1)};\ (G_{19})^{(2)},(T_{19})^{(2)})\right)$$

And analogous inequalities for $G_i$ and $T_i$. Taking into account the hypothesis the result follows 198

**Remark 6:** The fact that we supposed $(a''_{16})^{(2)}$ and $(b''_{16})^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{\mathrm{P}}_{16})^{(2)}\mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}$ and $(\widehat{\mathrm{Q}}_{16})^{(2)}\mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i=16,17,18$ depend only on $\mathrm{T}_{17}$ and respectively on

$(G_{19})$(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 7:** There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$ 200

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(2)} - (a_i'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})\} ds_{(16)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(2)}t)} > 0 \quad \text{for } t > 0$$

**Definition of** $\left((\widehat{M}_{16})^{(2)}\right)_1$, $\left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$: 201

**Remark 8:** if $G_{16}$ is bounded, the same property have also $G_{17}$ and $G_{18}$. indeed if

$G_{16} < (\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt} \leq \left((\widehat{M}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)} G_{17}$ and by integrating

$$G_{17} \leq \left((\widehat{M}_{16})^{(2)}\right)_2 = G_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1 / (a_{17}')^{(2)}$$

In the same way , one can obtain

$$G_{18} \leq \left((\widehat{M}_{16})^{(2)}\right)_3 = G_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2 / (a_{18}')^{(2)}$$

If $G_{17}$ or $G_{18}$ is bounded, the same property follows for $G_{16}$, $G_{18}$ and $G_{16}$, $G_{17}$ respectively.

**Remark 9:** If $G_{16}$ is bounded, from below, the same property holds for $G_{17}$ and $G_{18}$. The proof is analogous with the preceding one. An analogous property is true if $G_{17}$ is bounded from below. 202

**Remark 10:** If $T_{16}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t), t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$. 203

**Definition of** $(m)^{(2)}$ and $\varepsilon_2$ :

Indeed let $t_2$ be so that for $t > t_2$

$$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right)$, $t = \log\frac{2}{\varepsilon_2}$ By taking now $\varepsilon_2$ sufficiently small one sees that $T_{17}$ is unbounded. 205
The same property holds for $T_{18}$ if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t), t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[(\widehat{P}_{20})^{(3)}+\left((\hat{P}_{20})^{(3)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{20})^{(3)}\right]\leq(\hat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself 210

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)},(T_{23})^{(1)}),((G_{23})^{(2)},(T_{23})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

**Definition of** $\widetilde{G_{23}},\widetilde{T_{23}}:(\widetilde{(G_{23})},\widetilde{(T_{23})})=\mathcal{A}^{(3)}((G_{23}),(T_{23}))$

It results 213

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{20})^{(3)}\left|G_{21}^{(1)}-G_{21}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}ds_{(20)}+$$

$$\int_0^t\{(a'_{20})^{(3)}\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$G_{20}^{(2)}\left|(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)},s_{(20)}\right)\right|\ e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\}ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses it follows

$$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}t}\leq \quad 214$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+(a'_{20})^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\widehat{k}_{20})^{(3)}\right)d\left(((G_{23})^{(1)},(T_{23})^{(1)};\ (G_{23})^{(2)},(T_{23})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 11:** The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ *and* $(\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i=20,21,22$ depend only on $T_{21}$ and respectively on

$(G_{23})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 12:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 216

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(3)} - (a_i'')^{(3)}(T_{21}(s_{(20)}), s_{(20)})\} ds_{(20)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(3)} t)} > 0$ for t > 0

**Definition of** $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2\ and\ \left((\widehat{M}_{20})^{(3)}\right)_3$ : 217

**Remark 13:** if $G_{20}$ is bounded, the same property have also $G_{21}\ and\ G_{22}$ . indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a_{21}')^{(3)} G_{21}$ and by integrating

$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1 / (a_{21}')^{(3)}$

In the same way , one can obtain

$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2 / (a_{22}')^{(3)}$

If $G_{21}\ or\ G_{22}$ is bounded, the same property follows for $G_{20}$ , $G_{22}$ and $G_{20}$ , $G_{21}$ respectively.

**Remark 14:** If $G_{20}$ *is* bounded, from below, the same property holds for $G_{21}\ and\ G_{22}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{21}$ is bounded from below. 218

**Remark 15:** If $T_{20}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t), t\right)) = (b_{21}')^{(3)}$ then $T_{21} \to \infty$. 219

**Definition of** $(m)^{(3)}$ and $\varepsilon_3$ :

Indeed let $t_3$ be so that for $t > t_3$

$(b_{21})^{(3)} - (b_i'')^{(3)}\left((G_{23})(t), t\right) < \varepsilon_3, T_{20}\,(t) > (m)^{(3)}$

Then $\frac{dT_{21}}{dt} \geq (a_{21})^{(3)}(m)^{(3)} - \varepsilon_3 T_{21}$ which leads to 220

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t} = \frac{1}{2}$ it results

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_3}$ By taking now $\varepsilon_3$ sufficiently small one sees that $T_{21}$ is unbounded.
The same property holds for $T_{22}$ if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t), t\right) = (b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}$ , $\frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}} < 1$ and to choose 221

$(\widehat{P}_{24})^{(4)}$ and $(\widehat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[(\widehat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \tag{222}$$

$$\frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \tag{223}$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself (224)

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric (225)

$$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{24})^{(4)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{27}}),(\widetilde{T_{27}})$ : $\left((\widetilde{G_{27}}),(\widetilde{T_{27}})\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{24})^{(4)}\left|G_{25}^{(1)}-G_{25}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\,ds_{(24)}+$$

$$\int_0^t\{(a'_{24})^{(4)}\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$G_{24}^{(2)}\left|(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)-(a''_{24})^{(4)}\left(T_{25}^{(2)},s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\}ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}t}\leq \tag{226}$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)}+(a'_{24})^{(4)}+(\widehat{A}_{24})^{(4)}+(\widehat{P}_{24})^{(4)}(\hat{k}_{24})^{(4)}\right)d\left(((G_{27})^{(1)},(T_{27})^{(1)};\ (G_{27})^{(2)},(T_{27})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 16:** The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ *and* $(\widehat{Q}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_+$. (227)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i=24,25,26$ depend only on $T_{25}$ and respectively on

$(G_{27})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 17:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 228

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(4)} - (a_i'')^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(4)} t)} > 0$ for t > 0

**Definition of** $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2\ and\ \left((\widehat{M}_{24})^{(4)}\right)_3$ : 229

**Remark 18:** if $G_{24}$ is bounded, the same property have also $G_{25}\ and\ G_{26}$ . indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a_{25}')^{(4)} G_{25}$ and by integrating

$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1 / (a_{25}')^{(4)}$

In the same way , one can obtain

$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2 / (a_{26}')^{(4)}$

If $G_{25}\ or\ G_{26}$ is bounded, the same property follows for $G_{24}$ , $G_{26}$ and $G_{24}$ , $G_{25}$ respectively.

**Remark 19:** If $G_{24}$ *is* bounded, from below, the same property holds for $G_{25}\ and\ G_{26}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{25}$ is bounded from below. 230

**Remark 20:** If $T_{24}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}\ ((G_{27})(t), t)) = (b_{25}')^{(4)}$ then $T_{25}\ \to \infty$. 231

**Definition of** $(m)^{(4)}$ and $\varepsilon_4$ :

Indeed let $t_4$ be so that for $t\ > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t), t) < \varepsilon_4, T_{24}\,(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25}\ \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25}\ \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_4}$ By taking now $\varepsilon_4$ sufficiently small one sees that $T_{25}$ is unbounded.
The same property holds for $T_{26}$ if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t), t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for $G_{29}\ , G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}\ , \frac{(b_i)^{(5)}}{(\widehat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)}+\left((\hat{P}_{28})^{(5)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{28})^{(5)}\right]\leq(\hat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$$d\left(((G_{31})^{(1)},(T_{31})^{(1)}),((G_{31})^{(2)},(T_{31})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\widehat{M}_{28})^{(5)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\widehat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{31}}),(\widetilde{T_{31}})$ : $\left((\widetilde{G_{31}}),(\widetilde{T_{31}})\right)=\mathcal{A}^{(5)}\left((G_{31}),(T_{31})\right)$

It results

$$|\tilde{G}_{28}^{(1)}-\tilde{G}_i^{(2)}|\leq\int_0^t(a_{28})^{(5)}|G_{29}^{(1)}-G_{29}^{(2)}|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\,ds_{(28)}+$$

$$\int_0^t\{(a'_{28})^{(5)}|G_{28}^{(1)}-G_{28}^{(2)}|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$(a''_{28})^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)|G_{28}^{(1)}-G_{28}^{(2)}|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$G_{28}^{(2)}|(a''_{28})^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)-(a''_{28})^{(5)}\left(T_{29}^{(2)},s_{(28)}\right)|\;e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\}ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on it follows

$$|(G_{31})^{(1)}-(G_{31})^{(2)}|e^{-(\widehat{M}_{28})^{(5)}t}\leq \quad 237$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)}+(a'_{28})^{(5)}+(\widehat{A}_{28})^{(5)}+(\hat{P}_{28})^{(5)}(\hat{k}_{28})^{(5)}\right)d\left(((G_{31})^{(1)},(T_{31})^{(1)};\;(G_{31})^{(2)},(T_{31})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 21:** The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\hat{P}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ *and* $(\hat{Q}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ respectively of $\mathbb{R}_+$. 238

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on $T_{29}$ and respectively on $(G_{31})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 22:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 239

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(5)} - (a_i'')^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(5)} t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2\ and\ \left((\widehat{M}_{28})^{(5)}\right)_3$ : 240

**Remark 23:** if $G_{28}$ is bounded, the same property have also $G_{29}\ and\ G_{30}$ . indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a_{29}')^{(5)} G_{29}$ and by integrating

$$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1 / (a_{29}')^{(5)}$$

In the same way , one can obtain

$$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2 / (a_{30}')^{(5)}$$

If $G_{29}\ or\ G_{30}$ is bounded, the same property follows for $G_{28}$ , $G_{30}$ and $G_{28}$ , $G_{29}$ respectively.

**Remark 24:** If $G_{28}$ *is* bounded, from below, the same property holds for $G_{29}\ and\ G_{30}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{29}$ is bounded from below. 241

**Remark 25:** If $T_{28}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}\,((G_{31})(t), t)) = (b_{29}')^{(5)}$ then $T_{29} \to \infty$. 242

**Definition of** $(m)^{(5)}$ and $\varepsilon_5$ :

Indeed let $t_5$ be so that for $t > t_5$

$$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t), t) < \varepsilon_5, T_{28}\,(t) > (m)^{(5)}$$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_5}$ By taking now $\varepsilon_5$ sufficiently small one sees that $T_{29}$ is unbounded.
The same property holds for $T_{30}$ if $\lim_{t\to\infty}(b_{30}'')^{(5)}\left((G_{31})(t), t\right) = (b_{30}')^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{33}$ , $G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}$ , $\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}} < 1$ and to choose 244

$(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[(\hat{P}_{32})^{(6)}+\left((\hat{P}_{32})^{(6)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{32})^{(6)} \tag{245}$$

$$\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[\left((\hat{Q}_{32})^{(6)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{32})^{(6)}\right]\leq(\hat{Q}_{32})^{(6)} \tag{246}$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric (247)

$$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\hat{M}_{32})^{(6)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\hat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

**Definition of** $\widetilde{(G_{35})},\widetilde{(T_{35})}$ : $\left(\widetilde{(G_{35})},\widetilde{(T_{35})}\right)=\mathcal{A}^{(6)}((G_{35}),(T_{35}))$

It results

$$|\tilde{G}_{32}^{(1)}-\tilde{G}_i^{(2)}|\leq\int_0^t(a_{32})^{(6)}|G_{33}^{(1)}-G_{33}^{(2)}|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}ds_{(32)}+$$

$$\int_0^t\{(a'_{32})^{(6)}|G_{32}^{(1)}-G_{32}^{(2)}|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$(a''_{32})^{(6)}(T_{33}^{(1)},s_{(32)})|G_{32}^{(1)}-G_{32}^{(2)}|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$G_{32}^{(2)}|(a''_{32})^{(6)}(T_{33}^{(1)},s_{(32)})-(a''_{32})^{(6)}(T_{33}^{(2)},s_{(32)})|\ e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\}ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$|(G_{35})^{(1)}-(G_{35})^{(2)}|e^{-(\widehat{M}_{32})^{(6)}t}\leq \tag{248}$$
$$\frac{1}{(\widehat{M}_{32})^{(6)}}((a_{32})^{(6)}+(a'_{32})^{(6)}+(\widehat{A}_{32})^{(6)}+(\hat{P}_{32})^{(6)}(\hat{k}_{32})^{(6)})d\left(((G_{35})^{(1)},(T_{35})^{(1)};\ (G_{35})^{(2)},(T_{35})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 26:** The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\hat{P}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ *and* $(\hat{Q}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ respectively of $\mathbb{R}_+$. (249)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i = 32,33,34$ depend only on $T_{33}$ and respectively on $(G_{35})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 27:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 250

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(6)} - (a_i'')^{(6)}(T_{33}(s_{(32)}), s_{(32)})\} ds_{(32)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(6)} t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{32})^{(6)}\right)_1, \left((\widehat{M}_{32})^{(6)}\right)_2\ and\ \left((\widehat{M}_{32})^{(6)}\right)_3$ : 251

**Remark 28:** if $G_{32}$ is bounded, the same property have also $G_{33}\ and\ G_{34}$ . indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a_{33}')^{(6)} G_{33}$ and by integrating

$$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)} \left((\widehat{M}_{32})^{(6)}\right)_1 / (a_{33}')^{(6)}$$

In the same way , one can obtain

$$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)} \left((\widehat{M}_{32})^{(6)}\right)_2 / (a_{34}')^{(6)}$$

If $G_{33}\ or\ G_{34}$ is bounded, the same property follows for $G_{32}$ , $G_{34}$ and $G_{32}$ , $G_{33}$ respectively.

**Remark 29:** If $G_{32}$ *is* bounded, from below, the same property holds for $G_{33}\ and\ G_{34}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{33}$ is bounded from below. 252

**Remark 30:** If $T_{32}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}\ ((G_{35})(t), t)) = (b_{33}')^{(6)}$ then $T_{33} \to \infty$. 253

**Definition of** $(m)^{(6)}$ and $\varepsilon_6$ :

Indeed let $t_6$ be so that for $t > t_6$

$$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t), t\right) < \varepsilon_6, T_{32}\,(t) > (m)^{(6)}$$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right)$, $t = log\,\frac{2}{\varepsilon_6}$ By taking now $\varepsilon_6$ sufficiently small one sees that $T_{33}$ is unbounded.
The same property holds for $T_{34}$ if $\lim_{t\to\infty}(b_{34}'')^{(6)}\left((G_{35})(t), t(t), t\right) = (b_{34}')^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{37}, G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}$ , $\frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$ and to choose

$(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[(\hat{P}_{36})^{(7)}+\left((\hat{P}_{36})^{(7)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{36})^{(7)} \tag{256}$$

$$\frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[\left((\hat{Q}_{36})^{(7)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{36})^{(7)}\right]\leq(\hat{Q}_{36})^{(7)} \tag{257}$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself
The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric (258)

$$d\left(((G_{39})^{(1)},(T_{39})^{(1)}),((G_{39})^{(2)},(T_{39})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\tilde{M}_{36})^{(7)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\tilde{M}_{36})^{(7)}t}\}$$

Indeed if we denote

**Definition of** $\widetilde{(G_{39})},\widetilde{(T_{39})}:(\widetilde{(G_{39})},\widetilde{(T_{39})})=\mathcal{A}^{(7)}((G_{39}),(T_{39}))$
It results

$$|\tilde{G}_{36}^{(1)}-\tilde{G}_i^{(2)}|\leq\int_0^t(a_{36})^{(7)}|G_{37}^{(1)}-G_{37}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\,ds_{(36)}+$$
$$\int_0^t\{(a'_{36})^{(7)}|G_{36}^{(1)}-G_{36}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$(a''_{36})^{(7)}(T_{37}^{(1)},s_{(36)})|G_{36}^{(1)}-G_{36}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$G_{36}^{(2)}|(a''_{36})^{(7)}(T_{37}^{(1)},s_{(36)})-(a''_{36})^{(7)}(T_{37}^{(2)},s_{(36)})|\;e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\}ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on it follows

$$|(G_{39})^{(1)}-(G_{39})^{(2)}|e^{-(\widehat{M}_{36})^{(7)}t}\leq \tag{259}$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}}\left((a_{36})^{(7)}+(a'_{36})^{(7)}+(\widehat{A}_{36})^{(7)}+(\widehat{P}_{36})^{(7)}(\widehat{k}_{36})^{(7)}\right)d\left(((G_{39})^{(1)},(T_{39})^{(1)};\;(G_{39})^{(2)},(T_{39})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 31:** The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ *and* $(\widehat{Q}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ respectively of $\mathbb{R}_+$. (260)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(7)}$ and $(b_i'')^{(7)}, i = 36,37,38$ depend only on $T_{37}$ and respectively on $(G_{39})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 32:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 261

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(7)} - (a_i'')^{(7)}(T_{37}(s_{(36)}), s_{(36)})\} ds_{(36)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(7)} t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2\ and\ \left((\widehat{M}_{36})^{(7)}\right)_3$ : 262

**Remark 33:** if $G_{36}$ is bounded, the same property have also $G_{37}\ and\ G_{38}$ . indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a_{37}')^{(7)} G_{37}$ and by integrating

$$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1 / (a_{37}')^{(7)}$$

In the same way , one can obtain

$$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2 / (a_{38}')^{(7)}$$

If $G_{37}\ or\ G_{38}$ is bounded, the same property follows for $G_{36}$ , $G_{38}$ and $G_{36}$ , $G_{37}$ respectively.

**Remark 34:** If $G_{36}$ *is* bounded, from below, the same property holds for $G_{37}\ and\ G_{38}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{37}$ is bounded from below. 263

**Remark 35:** If $T_{36}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(7)}\ ((G_{39})(t), t)) = (b_{37}')^{(7)}$ then $T_{37} \to \infty$. 264

**Definition of** $(m)^{(7)}$ and $\varepsilon_7$ :

Indeed let $t_7$ be so that for $t > t_7$

$$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t), t) < \varepsilon_7, T_{36}\,(t) > (m)^{(7)}$$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_7}$ By taking now $\varepsilon_7$ sufficiently small one sees that $T_{37}$ is unbounded.
The same property holds for $T_{38}$ if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t), t\right) = (b_{38}')^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}$ , $\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}} < 1$ and to choose $(\widehat{P}_{40})^{(8)}$ and $(\widehat{Q}_{40})^{(8)}$ large to have 266

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[(\widehat{P}_{40})^{(8)} + \left((\hat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{40})^{(8)} \quad 267$$

$$\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{40})^{(8)}\right] \leq (\hat{Q}_{40})^{(8)} \quad 268$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)}, (T_{43})^{(1)}), ((G_{43})^{(2)}, (T_{43})^{(2)})\right) = \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

**Definition of** $\widetilde{(G_{43})}, \widetilde{(T_{43})}$ : $\left(\widetilde{(G_{43})}, \widetilde{(T_{43})}\right) = \mathcal{A}^{(8)}((G_{43}), (T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{40})^{(8)}\left|G_{41}^{(1)} - G_{41}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\,ds_{(40)} +$$
$$\int_0^t\{(a'_{40})^{(8)}\left|G_{40}^{(1)} - G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{-(\widehat{M}_{40})^{(8)}s_{(40)}} +$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right)\left|G_{40}^{(1)} - G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}} +$$
$$G_{40}^{(2)}\left|(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right) - (a''_{40})^{(8)}\left(T_{41}^{(2)}, s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\}ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0, t]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)} - (G_{43})^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}t} \leq \quad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)} + (a'_{40})^{(8)} + (\widehat{A}_{40})^{(8)} + (\widehat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}\right)d\left(((G_{43})^{(1)}, (T_{43})^{(1)};\ (G_{43})^{(2)}, (T_{43})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

**Remark 36:** The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate 274

condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ $and$ $(\widehat{Q}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i = 40,41,42$ depend only on $T_{41}$ and respectively on $(G_{43})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 37** There does not exist any $t$ where $G_i(t) = 0\ and\ T_i(t) = 0$ 275

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(8)} - (a_i'')^{(8)}(T_{41}(s_{(40)}), s_{(40)})\} ds_{(40)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(8)}t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{40})^{(8)}\right)_1, \left((\widehat{M}_{40})^{(8)}\right)_2$ $and$ $\left((\widehat{M}_{40})^{(8)}\right)_3$ : 276

**Remark 38:** if $G_{40}$ is bounded, the same property have also $G_{41}\ and\ G_{42}$ . indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a_{41}')^{(8)}G_{41}$ and by integrating

$$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1/(a_{41}')^{(8)}$$

In the same way , one can obtain

$$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2/(a_{42}')^{(8)}$$

If $G_{41}\ or\ G_{42}$ is bounded, the same property follows for $G_{40}$ , $G_{42}$ and $G_{40}$ , $G_{41}$ respectively.

**Remark 39:** If $G_{40}$ $is$ bounded, from below, the same property holds for $G_{41}\ and\ G_{42}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{41}$ is bounded from below. 277

**Remark 40:** If $T_{40}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}((G_{43})(t), t)) = (b_{41}')^{(8)}$ then $T_{41} \to \infty$. 278

**Definition of** $(m)^{(8)}$ and $\varepsilon_8$ :

Indeed let $t_8$ be so that for $t > t_8$

$$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t), t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right)$, $t = log\frac{2}{\varepsilon_8}$ By taking now $\varepsilon_8$ sufficiently small one sees that $T_{41}$ is unbounded.
The same property holds for $T_{42}$ if $\lim_{t\to\infty}(b''_{42})^{(8)}\left((G_{43})(t), t(t), t\right) = (b'_{42})^{(8)}$



It is now sufficient to take $\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}$ , $\frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$ and to choose $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ large to have

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}\left[(\hat{P}_{44})^{(9)} + \left((\hat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}}\left[\left((\hat{Q}_{44})^{(9)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{44})^{(9)}\right] \leq (\hat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)}, (T_{47})^{(1)}), ((G_{47})^{(2)}, (T_{47})^{(2)})\right) =$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{44})^{(9)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{47}}), (\widetilde{T_{47}}) : \left((\widetilde{G_{47}}), (\widetilde{T_{47}})\right) = \mathcal{A}^{(9)}\left((G_{47}), (T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{44})^{(9)}\left|G_{45}^{(1)} - G_{45}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\,ds_{(44)} +$$
$$\int_0^t\{(a'_{44})^{(9)}\left|G_{44}^{(1)} - G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{-(\widehat{M}_{44})^{(9)}s_{(44)}} +$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right)\left|G_{44}^{(1)} - G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}} +$$
$$G_{44}^{(2)}\left|(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right) - (a''_{44})^{(9)}\left(T_{45}^{(2)}, s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\}ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)} - G^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}t} \leq$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)} + (a'_{44})^{(9)} + (\widehat{A}_{44})^{(9)} + (\widehat{P}_{44})^{(9)}(\hat{k}_{44})^{(9)}\right)d\left(((G_{47})^{(1)}, (T_{47})^{(1)};\ (G_{47})^{(2)}, (T_{47})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis (39,35,36) the result follows

**Remark 41:** The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by

$(\widehat{P}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ *and* $(\widehat{Q}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i = 44{,}45{,}46$ depend only on $T_{45}$ and respectively on $(G_{47})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 42:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$

From 99 to 44 it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t\{(a_i')^{(9)}-(a_i'')^{(9)}(T_{45}(s_{(44)}),s_{(44)})\}ds_{(44)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(9)}t)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{44})^{(9)}\right)_1, \left((\widehat{M}_{44})^{(9)}\right)_2$ *and* $\left((\widehat{M}_{44})^{(9)}\right)_3$ :

**Remark 43:** if $G_{44}$ is bounded, the same property have also $G_{45}\ and\ G_{46}$ . indeed if

$G_{44} < (\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt} \leq \left((\widehat{M}_{44})^{(9)}\right)_1 - (a_{45}')^{(9)}G_{45}$ and by integrating

$$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_1/(a_{45}')^{(9)}$$

In the same way , one can obtain

$$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_2/(a_{46}')^{(9)}$$

If $G_{45}\ or\ G_{46}$ is bounded, the same property follows for $G_{44}$ , $G_{46}$ and $G_{44}$ , $G_{45}$ respectively.

**Remark 44:** If $G_{44}$ *is* bounded, from below, the same property holds for $G_{45}\ and\ G_{46}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{45}$ is bounded from below.

**Remark 45:** If $T_{44}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t),t\right) = (b_{45}')^{(9)}$ then $T_{45} \to \infty$.

**Definition of** $(m)^{(9)}$ and $\varepsilon_9$ :

Indeed let $t_9$ be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t),t\right) < \varepsilon_9, T_{44}\,(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right), \quad t = log\frac{2}{\varepsilon_9}$ By taking now $\varepsilon_9$ sufficiently small one sees that $T_{45}$ is unbounded.
The same property holds for $T_{46}$ if $\lim_{t\to\infty}(b_{46}'')^{(9)}\left((G_{47})(t),t\right) = (b_{46}')^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

**Behavior of the solutions of equation** 280

**Theorem** If we denote and define

**Definition of** $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ :

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)} \le -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14}, t) + (a''_{14})^{(1)}(T_{14}, t) \le -(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)} \le -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G, t) - (b''_{14})^{(1)}(G, t) \le -(\tau_1)^{(1)}$

**Definition of** $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$ : 281

By $(\nu_1)^{(1)} > 0$, $(\nu_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$, $(u_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

**Definition of** $(\bar{\nu}_1)^{(1)},, (\bar{\nu}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$ : 282

By $(\bar{\nu}_1)^{(1)} > 0$, $(\bar{\nu}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$, $(\bar{u}_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

**Definition of** $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (\nu_0)^{(1)}$ :- 283

If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by
$(m_2)^{(1)} = (\nu_0)^{(1)}, (m_1)^{(1)} = (\nu_1)^{(1)}$, $if\ (\nu_0)^{(1)} < (\nu_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}, if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)}$,

and $\boxed{(\nu_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)}$, $if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)}$, $if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)}$,

and $\boxed{(u_0)^{(1)} = \frac{T^0_{13}}{T^0_{14}}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$

are defined

Then the solution of global equations satisfies the inequalities 285

$G^0_{13}e^{\left((S_1)^{(1)} - (p_{13})^{(1)}\right)t} \le G_{13}(t) \le G^0_{13}e^{(S_1)^{(1)}t}$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)}-(p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)}t}$$

$$\left(\frac{(a_{15})^{(1)}G_{13}^0}{(m_1)^{(1)}((S_1)^{(1)}-(p_{13})^{(1)}-(S_2)^{(1)})}\left[e^{((S_1)^{(1)}-(p_{13})^{(1)})t} - e^{-(S_2)^{(1)}t}\right] + G_{15}^0 e^{-(S_2)^{(1)}t} \le G_{15}(t) \le \right. \quad (286)$$

$$\left. \frac{(a_{15})^{(1)}G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)}-(a'_{15})^{(1)})}[e^{(S_1)^{(1)}t} - e^{-(a'_{15})^{(1)}t}] + G_{15}^0 e^{-(a'_{15})^{(1)}t}\right)$$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)}t} \le T_{13}(t) \le T_{13}^0 e^{((R_1)^{(1)}+(r_{13})^{(1)})t}} \quad (287)$$

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)}t} \le T_{13}(t) \le \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)}+(r_{13})^{(1)})t} \quad (288)$$

$$\frac{(b_{15})^{(1)}T_{13}^0}{(\mu_1)^{(1)}((R_1)^{(1)}-(b'_{15})^{(1)})}\left[e^{(R_1)^{(1)}t} - e^{-(b'_{15})^{(1)}t}\right] + T_{15}^0 e^{-(b'_{15})^{(1)}t} \le T_{15}(t) \le \quad (289)$$

$$\frac{(a_{15})^{(1)}T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)}+(r_{13})^{(1)}+(R_2)^{(1)})}\left[e^{((R_1)^{(1)}+(r_{13})^{(1)})t} - e^{-(R_2)^{(1)}t}\right] + T_{15}^0 e^{-(R_2)^{(1)}t}$$

**Definition of** $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:- (290)

Where $(S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$

$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$

$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$

$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$

**Behavior of the solutions of equation** (291)

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$: (292)

$(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ four constants satisfying

$$-(\sigma_2)^{(2)} \le -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(T_{17}, t) + (a''_{17})^{(2)}(T_{17}, t) \le -(\sigma_1)^{(2)} \quad (293)$$

$$-(\tau_2)^{(2)} \le -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}((G_{19}), t) - (b''_{17})^{(2)}((G_{19}), t) \le -(\tau_1)^{(2)} \quad (294)$$

**Definition of** $(\nu_1)^{(2)}, (\nu_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$: (295)

By $(\nu_1)^{(2)} > 0$, $(\nu_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0$, $(u_2)^{(2)} < 0$ the roots (296)

of the equations $(a_{17})^{(2)}(\nu^{(2)})^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ (297)

and $(b_{14})^{(2)}(u^{(2)})^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and (298)

**Definition of** $(\bar{\nu}_1)^{(2)}, , (\bar{\nu}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$: (299)

By $(\bar{\nu}_1)^{(2)} > 0$, $(\bar{\nu}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0$, $(\bar{u}_2)^{(2)} < 0$ the (300)

roots of the equations $(a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 301

and $(b_{17})^{(2)}\left(u^{(2)}\right)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 302

**Definition of** $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ :- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, ***if*** $(u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$, ***if*** $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)}$, ***if*** $(\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)}t}$$

$(p_i)^{(2)}$ is defined by equation

$$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)}t} \quad 311$$

$$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)}t}\right] + G_{18}^0 e^{-(S_2)^{(2)}t} \le G_{18}(t) \le \right. \quad 312$$

$$\left. \frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)}t} - e^{-(a'_{18})^{(2)}t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)}t}\right)$$

$$\boxed{T_{16}^0 e^{(R_1)^{(2)}t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}} \quad 313$$

$$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)}t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t} \quad 314$$

$$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)}t} - e^{-(b'_{18})^{(2)}t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)}t} \le T_{18}(t) \le \quad 315$$

$$\frac{(a_{18})^{(2)}T_{16}^{0}}{(\mu_2)^{(2)}((R_1)^{(2)}+(r_{16})^{(2)}+(R_2)^{(2)})}\left[e^{((R_1)^{(2)}+(r_{16})^{(2)})t}-e^{-(R_2)^{(2)}t}\right]+T_{18}^{0}e^{-(R_2)^{(2)}t}$$

**Definition of** $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$$

$$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)}$$ 318

$$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$$

**Behavior of the solutions** 319

**Theorem 3:** If we denote and define

**Definition of** $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ :

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}\big((G_{23}), t\big) \le -(\tau_1)^{(3)}$$

**Definition of** $(v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$ : 320

By $(v_1)^{(3)} > 0$, $(v_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{v}_1)^{(3)} > 0$, $(\bar{v}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

**Definition of** $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ :- 321

If we define $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (v_0)^{(3)}, (m_1)^{(3)} = (v_1)^{(3)}$, ***if*** $(v_0)^{(3)} < (v_1)^{(3)}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (\bar{v}_1)^{(3)}$, ***if*** $(v_1)^{(3)} < (v_0)^{(3)} < (\bar{v}_1)^{(3)}$,

and $\boxed{(v_0)^{(3)} = \frac{G_{20}^{0}}{G_{21}^{0}}}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_0)^{(3)}$, ***if*** $(\bar{v}_1)^{(3)} < (v_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)},\ \boldsymbol{if}\ (u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}, \boldsymbol{if}\ (u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)},$ and $\boxed{(u_0)^{(3)} = \frac{T_{20}^0}{T_{21}^0}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)},\ \boldsymbol{if}\ (\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G_{20}^0 e^{(S_1)^{(3)} t}$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)} t} \quad 323$$

$$\left(\frac{(a_{22})^{(3)} G_{20}^0}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\left[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)} t}\right] + G_{22}^0 e^{-(S_2)^{(3)} t} \le G_{22}(t) \le \right. \quad 324$$

$$\left. \frac{(a_{22})^{(3)} G_{20}^0}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}\left[e^{(S_1)^{(3)} t} - e^{-(a'_{22})^{(3)} t}\right] + G_{22}^0 e^{-(a'_{22})^{(3)} t}\right)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t}} \quad 325$$

$$\frac{1}{(\mu_1)^{(3)}} T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}} T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)} T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)} - (b'_{22})^{(3)})}\left[e^{(R_1)^{(3)} t} - e^{-(b'_{22})^{(3)} t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)} t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)} T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)} + (r_{20})^{(3)} + (R_2)^{(3)})}\left[e^{((R_1)^{(3)} + (r_{20})^{(3)})t} - e^{-(R_2)^{(3)} t}\right] + T_{22}^0 e^{-(R_2)^{(3)} t}$$

**Definition of** $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

**Behavior of the solutions of equation**

**Theorem:** If we denote and define

**Definition of** $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ :

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$

$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}((G_{27}), t) - (b''_{25})^{(4)}((G_{27}), t) \le -(\tau_1)^{(4)}$

**Definition of** $(v_1)^{(4)}, (v_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, v^{(4)}, u^{(4)}$ : 329

By $(v_1)^{(4)} > 0$ , $(v_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$ , $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_1)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

**Definition of** $(\bar{v}_1)^{(4)},, (\bar{v}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$ : 330

By $(\bar{v}_1)^{(4)} > 0$ , $(\bar{v}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$ , $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

**Definition of** $(m_1)^{(4)}$ , $(m_2)^{(4)}$ , $(\mu_1)^{(4)}, (\mu_2)^{(4)}, (\nu_0)^{(4)}$ :-

If we define $(m_1)^{(4)}$ , $(m_2)^{(4)}$ , $(\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (v_0)^{(4)}, (m_1)^{(4)} = (v_1)^{(4)}$, $\boldsymbol{if}$ $(v_0)^{(4)} < (v_1)^{(4)}$

$(m_2)^{(4)} = (v_1)^{(4)}, (m_1)^{(4)} = (\bar{v}_1)^{(4)}$ , $\boldsymbol{if}$ $(v_4)^{(4)} < (v_0)^{(4)} < (\bar{v}_1)^{(4)}$,
and $\boxed{(v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (v_4)^{(4)}, (m_1)^{(4)} = (v_0)^{(4)}$, $\boldsymbol{if}$ $(\bar{v}_4)^{(4)} < (v_0)^{(4)}$

and analogously

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$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)}$, $\boldsymbol{if}$ $(u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}$ , $\boldsymbol{if}$ $(u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)}$,
and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}, \boldsymbol{if}$ $(\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

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$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)}t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)}t}$$

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$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)}t}\right] + G_{26}^0 e^{-(S_2)^{(4)}t} \le G_{26}(t) \le\right.$$ 334

*(a26)4G240(m2)4(S1)4−(a26′)4e(S1)4t−e−(a26′)4t+ G260e−(a26′)4t*

$$\boxed{T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}}T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}}T_{24}^0 e^{((R_1)^{(4)}+(r_{24})^{(4)})t} \quad 335$$

$$\frac{(b_{26})^{(4)}T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)}-(b'_{26})^{(4)})}\left[e^{(R_1)^{(4)}t} - e^{-(b'_{26})^{(4)}t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)}t} \le T_{26}(t) \le \quad 336$$

$$\frac{(a_{26})^{(4)}T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)}+(r_{24})^{(4)}+(R_2)^{(4)})}\left[e^{((R_1)^{(4)}+(r_{24})^{(4)})t} - e^{-(R_2)^{(4)}t}\right] + T_{26}^0 e^{-(R_2)^{(4)}t}$$

**Definition of** $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

**Behavior of the solutions of equation** 338

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ :

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

**Definition of** $(v_1)^{(5)}, (v_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, v^{(5)}, u^{(5)}$ : 339

By $(v_1)^{(5)} > 0$ , $(v_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$ , $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(v^{(5)}\big)^2 + (\sigma_1)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

**Definition of** $(\bar{v}_1)^{(5)}, , (\bar{v}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$ : 340

By $(\bar{v}_1)^{(5)} > 0$ , $(\bar{v}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$ , $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(v^{(5)}\big)^2 + (\sigma_2)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

**Definition of** $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (v_0)^{(5)}$ :-

If we define $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (v_0)^{(5)}, (m_1)^{(5)} = (v_1)^{(5)}$, ***if*** $(v_0)^{(5)} < (v_1)^{(5)}$

$(m_2)^{(5)} = (v_1)^{(5)}, (m_1)^{(5)} = (\bar{v}_1)^{(5)}$ , ***if*** $(v_1)^{(5)} < (v_0)^{(5)} < (\bar{v}_1)^{(5)}$,
and $\boxed{(v_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)},\ \boldsymbol{if}\ (u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}, \boldsymbol{if}\ (u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)},$

and $\boxed{(u_0)^{(5)} = \frac{T_{28}^0}{T_{29}^0}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}, \boldsymbol{if}\ (\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{28}(t) \le G_{28}^0 e^{(S_1)^{(5)} t}$

where $(p_i)^{(5)}$ is defined by equation

$\frac{1}{(m_5)^{(5)}} G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}} G_{28}^0 e^{(S_1)^{(5)} t}$ 343

$\left(\frac{(a_{30})^{(5)} G_{28}^0}{(m_1)^{(5)}((S_1)^{(5)} - (p_{28})^{(5)} - (S_2)^{(5)})}\left[e^{((S_1)^{(5)} - (p_{28})^{(5)})t} - e^{-(S_2)^{(5)} t}\right] + G_{30}^0 e^{-(S_2)^{(5)} t} \le G_{30}(t) \le\right.$ 344

*(a30)5G280(m2)5(S1)5−(a30′)5e(S1)5t−e−(a30′)5t+ G300e−(a30′)5t*

$\boxed{T_{28}^0 e^{(R_1)^{(5)} t} \le T_{28}(t) \le T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}}$ 345

$\frac{1}{(\mu_1)^{(5)}} T_{28}^0 e^{(R_1)^{(5)} t} \le T_{28}(t) \le \frac{1}{(\mu_2)^{(5)}} T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}$ 346

$\frac{(b_{30})^{(5)} T_{28}^0}{(\mu_1)^{(5)}((R_1)^{(5)} - (b'_{30})^{(5)})}\left[e^{(R_1)^{(5)} t} - e^{-(b'_{30})^{(5)} t}\right] + T_{30}^0 e^{-(b'_{30})^{(5)} t} \le T_{30}(t) \le$ 347

$\frac{(a_{30})^{(5)} T_{28}^0}{(\mu_2)^{(5)}((R_1)^{(5)} + (r_{28})^{(5)} + (R_2)^{(5)})}\left[e^{((R_1)^{(5)} + (r_{28})^{(5)})t} - e^{-(R_2)^{(5)} t}\right] + T_{30}^0 e^{-(R_2)^{(5)} t}$

**Definition of** $(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$:- 348

Where $(S_1)^{(5)} = (a_{28})^{(5)}(m_2)^{(5)} - (a'_{28})^{(5)}$

$(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}$

$(R_1)^{(5)} = (b_{28})^{(5)}(\mu_2)^{(5)} - (b'_{28})^{(5)}$

$(R_2)^{(5)} = (b'_{30})^{(5)} - (r_{30})^{(5)}$

**Behavior of the solutions of equation** 349

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ :

$(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)} \le -(a'_{32})^{(6)} + (a'_{33})^{(6)} - (a''_{32})^{(6)}(T_{33}, t) + (a''_{33})^{(6)}(T_{33}, t) \le -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)} \le -(b'_{32})^{(6)} + (b'_{33})^{(6)} - (b''_{32})^{(6)}\big((G_{35}), t\big) - (b''_{33})^{(6)}\big((G_{35}), t\big) \le -(\tau_1)^{(6)}$$

**Definition of** $(\nu_1)^{(6)}, (\nu_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, \nu^{(6)}, u^{(6)}$ : 350

By $(\nu_1)^{(6)} > 0$ , $(\nu_2)^{(6)} < 0$ and respectively $(u_1)^{(6)} > 0$ , $(u_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(6)}, , (\bar{\nu}_2)^{(6)}, (\bar{u}_1)^{(6)}, (\bar{u}_2)^{(6)}$ : 351

By $(\bar{\nu}_1)^{(6)} > 0$ , $(\bar{\nu}_2)^{(6)} < 0$ and respectively $(\bar{u}_1)^{(6)} > 0$ , $(\bar{u}_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$

**Definition of** $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}, (\nu_0)^{(6)}$ :-

If we define $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}$ by

$$(m_2)^{(6)} = (\nu_0)^{(6)}, (m_1)^{(6)} = (\nu_1)^{(6)}, \ \boldsymbol{if}\ (\nu_0)^{(6)} < (\nu_1)^{(6)}$$

$$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}, \boldsymbol{if}\ (\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)},$$

and $\boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}}$

$$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$$

and analogously 352

$$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}, \ \boldsymbol{if}\ (u_0)^{(6)} < (u_1)^{(6)}$$

$$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}, \boldsymbol{if}\ (u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)},$$

and $\boxed{(u_0)^{(6)} = \frac{T^0_{32}}{T^0_{33}}}$

$$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}, \boldsymbol{if}\ (\bar{u}_1)^{(6)} < (u_0)^{(6)} \text{ where } (u_1)^{(6)}, (\bar{u}_1)^{(6)}$$

Then the solution of global equations satisfies the inequalities 353

$$G^0_{32}e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{32}(t) \le G^0_{32}e^{(S_1)^{(6)}t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}}G^0_{32}e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{33}(t) \le \frac{1}{(m_2)^{(6)}}G^0_{32}e^{(S_1)^{(6)}t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)}G^0_{32}}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})}\left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)}t}\right] + G^0_{34}e^{-(S_2)^{(6)}t} \le G_{34}(t) \le \right. \quad 355$$

$$\frac{(a_{34})^{(6)}G_{32}^{0}}{(m_2)^{(6)}((S_1)^{(6)}-(a'_{34})^{(6)})}\left[e^{(S_1)^{(6)}t}-e^{-(a'_{34})^{(6)}t}\right]+\ G_{34}^{0}e^{-(a'_{34})^{(6)}t}\Big)$$

$$\boxed{T_{32}^{0}e^{(R_1)^{(6)}t}\le T_{32}(t)\le T_{32}^{0}e^{((R_1)^{(6)}+(r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}}T_{32}^{0}e^{(R_1)^{(6)}t}\le T_{32}(t)\le \frac{1}{(\mu_2)^{(6)}}T_{32}^{0}e^{((R_1)^{(6)}+(r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)}T_{32}^{0}}{(\mu_1)^{(6)}((R_1)^{(6)}-(b'_{34})^{(6)})}\left[e^{(R_1)^{(6)}t}-e^{-(b'_{34})^{(6)}t}\right]+T_{34}^{0}e^{-(b'_{34})^{(6)}t}\le T_{34}(t)\le \quad 358$$

$$\frac{(a_{34})^{(6)}T_{32}^{0}}{(\mu_2)^{(6)}((R_1)^{(6)}+(r_{32})^{(6)}+(R_2)^{(6)})}\left[e^{((R_1)^{(6)}+(r_{32})^{(6)})t}-e^{-(R_2)^{(6)}t}\right]+T_{34}^{0}e^{-(R_2)^{(6)}t}$$

**Definition of** $(S_1)^{(6)},(S_2)^{(6)},(R_1)^{(6)},(R_2)^{(6)}$:- 359

Where $(S_1)^{(6)}=(a_{32})^{(6)}(m_2)^{(6)}-(a'_{32})^{(6)}$

$(S_2)^{(6)}=(a_{34})^{(6)}-(p_{34})^{(6)}$

$(R_1)^{(6)}=(b_{32})^{(6)}(\mu_2)^{(6)}-(b'_{32})^{(6)}$

$(R_2)^{(6)}=(b'_{34})^{(6)}-(r_{34})^{(6)}$

**Behavior of the solutions of equation**

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(7)},(\sigma_2)^{(7)},(\tau_1)^{(7)},(\tau_2)^{(7)}$ :

$(\sigma_1)^{(7)},(\sigma_2)^{(7)},(\tau_1)^{(7)},(\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)}\le -(a'_{36})^{(7)}+(a'_{37})^{(7)}-(a''_{36})^{(7)}(T_{37},t)+(a''_{37})^{(7)}(T_{37},t)\le -(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)}\le -(b'_{36})^{(7)}+(b'_{37})^{(7)}-(b''_{36})^{(7)}\big((G_{39}),t\big)-(b''_{37})^{(7)}\big((G_{39}),t\big)\le -(\tau_1)^{(7)}$$

**Definition of** $(v_1)^{(7)},(v_2)^{(7)},(u_1)^{(7)},(u_2)^{(7)},v^{(7)},u^{(7)}$ : 361

By $(v_1)^{(7)}>0,(v_2)^{(7)}<0$ and respectively $(u_1)^{(7)}>0,(u_2)^{(7)}<0$ the roots of the equations $(a_{37})^{(7)}\big(v^{(7)}\big)^2+(\sigma_1)^{(7)}v^{(7)}-(a_{36})^{(7)}=0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2+(\tau_1)^{(7)}u^{(7)}-(b_{36})^{(7)}=0$ and

**Definition of** $(\bar{v}_1)^{(7)},,(\bar{v}_2)^{(7)},(\bar{u}_1)^{(7)},(\bar{u}_2)^{(7)}$ : 362

By $(\bar{v}_1)^{(7)}>0,(\bar{v}_2)^{(7)}<0$ and respectively $(\bar{u}_1)^{(7)}>0,(\bar{u}_2)^{(7)}<0$ the roots of the equations $(a_{37})^{(7)}\big(v^{(7)}\big)^2+(\sigma_2)^{(7)}v^{(7)}-(a_{36})^{(7)}=0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2+(\tau_2)^{(7)}u^{(7)}-(b_{36})^{(7)}=0$

**Definition of** $(m_1)^{(7)},(m_2)^{(7)},(\mu_1)^{(7)},(\mu_2)^{(7)},(v_0)^{(7)}$ :-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$$(m_2)^{(7)} = (\nu_0)^{(7)}, (m_1)^{(7)} = (\nu_1)^{(7)}, \ \boldsymbol{if}\ (\nu_0)^{(7)} < (\nu_1)^{(7)}$$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\bar{\nu}_1)^{(7)}, \boldsymbol{if}\ (\nu_1)^{(7)} < (\nu_0)^{(7)} < (\bar{\nu}_1)^{(7)},$$

and $\boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\nu_0)^{(7)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(7)} < (\nu_0)^{(7)}$$

and analogously 363

$$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}, \ \boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$$

and $\boxed{(u_0)^{(7)} = \frac{T_{36}^0}{T_{37}^0}}$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)} \text{ where } (u_1)^{(7)}, (\bar{u}_1)^{(7)}$$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^0 e^{(S_1)^{(7)} t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)} t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)}((S_1)^{(7)} - (p_{36})^{(7)} - (S_2)^{(7)})}\left[e^{((S_1)^{(7)} - (p_{36})^{(7)})t} - e^{-(S_2)^{(7)} t}\right] + G_{38}^0 e^{-(S_2)^{(7)} t} \le G_{38}(t) \le \right. \quad 366$$

$$\left. \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)}((S_1)^{(7)} - (a'_{38})^{(7)})}\left[e^{(S_1)^{(7)} t} - e^{-(a'_{38})^{(7)} t}\right] + G_{38}^0 e^{-(a'_{38})^{(7)} t}\right)$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t} \quad 368$$

$$\frac{(b_{38})^{(7)} T_{36}^0}{(\mu_1)^{(7)}((R_1)^{(7)} - (b'_{38})^{(7)})}\left[e^{(R_1)^{(7)} t} - e^{-(b'_{38})^{(7)} t}\right] + T_{38}^0 e^{-(b'_{38})^{(7)} t} \le T_{38}(t) \le \quad 369$$

$$\frac{(a_{38})^{(7)} T_{36}^0}{(\mu_2)^{(7)}((R_1)^{(7)} + (r_{36})^{(7)} + (R_2)^{(7)})}\left[e^{((R_1)^{(7)} + (r_{36})^{(7)})t} - e^{-(R_2)^{(7)} t}\right] + T_{38}^0 e^{-(R_2)^{(7)} t}$$

**Definition of** $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$

$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$

$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$

**Behavior of the solutions of equation** 371

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ :

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$-(\sigma_2)^{(8)} \le -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \le -(\sigma_1)^{(8)}$

$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\big((G_{43}), t\big) - (b''_{41})^{(8)}\big((G_{43}), t\big) \le -(\tau_1)^{(8)}$

**Definition of** $(\nu_1)^{(8)}, (\nu_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, \nu^{(8)}, u^{(8)}$ : 372

By $(\nu_1)^{(8)} > 0, (\nu_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0, (u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(8)}, , (\bar{\nu}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$ :

By $(\bar{\nu}_1)^{(8)} > 0, (\bar{\nu}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0, (\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

**Definition of** $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}, (\nu_0)^{(8)}$ :-

If we define $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$(m_2)^{(8)} = (\nu_0)^{(8)}, (m_1)^{(8)} = (\nu_1)^{(8)}$, ***if*** $(\nu_0)^{(8)} < (\nu_1)^{(8)}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\bar{\nu}_1)^{(8)}$, ***if*** $(\nu_1)^{(8)} < (\nu_0)^{(8)} < (\bar{\nu}_1)^{(8)}$,
and $\boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\nu_0)^{(8)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(8)} < (\nu_0)^{(8)}$

and analogously 374

$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)},\ \boldsymbol{if}\ (u_0)^{(8)} < (u_1)^{(8)}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}, \boldsymbol{if}\ (u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)},$

and $\boxed{(u_0)^{(8)} = \frac{T_{40}^0}{T_{41}^0}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}, \boldsymbol{if}\ (\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^0 e^{(S_1)^{(8)} t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}} G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}} G_{40}^0 e^{(S_1)^{(8)} t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)} G_{40}^0}{(m_1)^{(8)}((S_1)^{(8)} - (p_{40})^{(8)} - (S_2)^{(8)})}\left[e^{((S_1)^{(8)} - (p_{40})^{(8)})t} - e^{-(S_2)^{(8)} t}\right] + G_{42}^0 e^{-(S_2)^{(8)} t} \le G_{42}(t) \le \right. \quad 377$$

$$\left. \frac{(a_{42})^{(8)} G_{40}^0}{(m_2)^{(8)}((S_1)^{(8)} - (a'_{42})^{(8)})}\left[e^{(S_1)^{(8)} t} - e^{-(a'_{42})^{(8)} t}\right] + G_{42}^0 e^{-(a'_{42})^{(8)} t}\right)$$

$$\boxed{T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}} T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le \frac{1}{(\mu_2)^{(8)}} T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)} T_{40}^0}{(\mu_1)^{(8)}((R_1)^{(8)} - (b'_{42})^{(8)})}\left[e^{(R_1)^{(8)} t} - e^{-(b'_{42})^{(8)} t}\right] + T_{42}^0 e^{-(b'_{42})^{(8)} t} \le T_{42}(t) \le \quad 380$$

$$\frac{(a_{42})^{(8)} T_{40}^0}{(\mu_2)^{(8)}((R_1)^{(8)} + (r_{40})^{(8)} + (R_2)^{(8)})}\left[e^{((R_1)^{(8)} + (r_{40})^{(8)})t} - e^{-(R_2)^{(8)} t}\right] + T_{42}^0 e^{-(R_2)^{(8)} t}$$

**Definition of** $(S_1)^{(8)}, (S_2)^{(8)}, (R_1)^{(8)}, (R_2)^{(8)}$:- 381

Where $(S_1)^{(8)} = (a_{40})^{(8)}(m_2)^{(8)} - (a'_{40})^{(8)}$

$(S_2)^{(8)} = (a_{42})^{(8)} - (p_{42})^{(8)}$

$(R_1)^{(8)} = (b_{40})^{(8)}(\mu_2)^{(8)} - (b'_{40})^{(8)}$

$(R_2)^{(8)} = (b'_{42})^{(8)} - (r_{42})^{(8)}$

**Behavior of the solutions of equation 37 to 92** 382

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ :

$(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ four constants satisfying

$$-(\sigma_2)^{(9)} \le -(a'_{44})^{(9)} + (a'_{45})^{(9)} - (a''_{44})^{(9)}(T_{45}, t) + (a''_{45})^{(9)}(T_{45}, t) \le -(\sigma_1)^{(9)}$$

$$-(\tau_2)^{(9)} \le -(b'_{44})^{(9)} + (b'_{45})^{(9)} - (b''_{44})^{(9)}\big((G_{47}), t\big) - (b''_{45})^{(9)}\big((G_{47}), t\big) \le -(\tau_1)^{(9)}$$

**Definition of** $(\nu_1)^{(9)}, (\nu_2)^{(9)}, (u_1)^{(9)}, (u_2)^{(9)}, \nu^{(9)}, u^{(9)}$ :

By $(\nu_1)^{(9)} > 0$ , $(\nu_2)^{(9)} < 0$ and respectively $(u_1)^{(9)} > 0$ , $(u_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_1)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(9)}, , (\bar{\nu}_2)^{(9)}, (\bar{u}_1)^{(9)}, (\bar{u}_2)^{(9)}$ :

By $(\bar{\nu}_1)^{(9)} > 0$ , $(\bar{\nu}_2)^{(9)} < 0$ and respectively $(\bar{u}_1)^{(9)} > 0$ , $(\bar{u}_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_2)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$

**Definition of** $(m_1)^{(9)}, (m_2)^{(9)}, (\mu_1)^{(9)}, (\mu_2)^{(9)}, (\nu_0)^{(9)}$ :-

If we define $(m_1)^{(9)}, (m_2)^{(9)}, (\mu_1)^{(9)}, (\mu_2)^{(9)}$ by

$$(m_2)^{(9)} = (\nu_0)^{(9)}, (m_1)^{(9)} = (\nu_1)^{(9)}, \ \boldsymbol{if}\ (\nu_0)^{(9)} < (\nu_1)^{(9)}$$

$$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\bar{\nu}_1)^{(9)}, \boldsymbol{if}\ (\nu_1)^{(9)} < (\nu_0)^{(9)} < (\bar{\nu}_1)^{(9)},$$

and $\boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$

$$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\nu_0)^{(9)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(9)} < (\nu_0)^{(9)}$$

and analogously

$$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)}, \ \boldsymbol{if}\ (u_0)^{(9)} < (u_1)^{(9)}$$

$$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}, \boldsymbol{if}\ (u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)},$$

and $\boxed{(u_0)^{(9)} = \frac{T^0_{44}}{T^0_{45}}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}, \boldsymbol{if}\ (\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G^0_{44} e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{44}(t) \le G^0_{44} e^{(S_1)^{(9)} t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}}G_{44}^0 e^{((S_1)^{(9)}-(p_{44})^{(9)})t} \le G_{45}(t) \le \frac{1}{(m_2)^{(9)}}G_{44}^0 e^{(S_1)^{(9)}t}$$

(

$$\frac{(a_{46})^{(9)}G_{44}^0}{(m_1)^{(9)}((S_1)^{(9)}-(p_{44})^{(9)}-(S_2)^{(9)})}\left[e^{((S_1)^{(9)}-(p_{44})^{(9)})t} - e^{-(S_2)^{(9)}t}\right] + G_{46}^0 e^{-(S_2)^{(9)}t} \le G_{46}(t) \le$$

$$\frac{(a_{46})^{(9)}G_{44}^0}{(m_2)^{(9)}((S_1)^{(9)}-(a'_{46})^{(9)})}[e^{(S_1)^{(9)}t} - e^{-(a'_{46})^{(9)}t}] + G_{46}^0 e^{-(a'_{46})^{(9)}t})$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le T_{44}^0 e^{((R_1)^{(9)}+(r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}}T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le \frac{1}{(\mu_2)^{(9)}}T_{44}^0 e^{((R_1)^{(9)}+(r_{44})^{(9)})t}$$

$$\frac{(b_{46})^{(9)}T_{44}^0}{(\mu_1)^{(9)}((R_1)^{(9)}-(b'_{46})^{(9)})}\left[e^{(R_1)^{(9)}t} - e^{-(b'_{46})^{(9)}t}\right] + T_{46}^0 e^{-(b'_{46})^{(9)}t} \le T_{46}(t) \le$$

$$\frac{(a_{46})^{(9)}T_{44}^0}{(\mu_2)^{(9)}((R_1)^{(9)}+(r_{44})^{(9)}+(R_2)^{(9)})}\left[e^{((R_1)^{(9)}+(r_{44})^{(9)})t} - e^{-(R_2)^{(9)}t}\right] + T_{46}^0 e^{-(R_2)^{(9)}t}$$

**Definition of** $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$

**Proof :** From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14},t)\right) - (a''_{14})^{(1)}(T_{14},t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

**Definition of** $v^{(1)}$ :- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)}\right) \le \frac{dv^{(1)}}{dt} \le -\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)}\right)$$

From which one obtains

**Definition of** $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$ :-

For $0 < \boxed{(v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}} < (v_1)^{(1)} < (\bar{v}_1)^{(1)}$

$$v^{(1)}(t) \ge \frac{(v_1)^{(1)}+(C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(v_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(v_2)^{(1)}}}$$

it follows $(\nu_0)^{(1)} \leq \nu^{(1)}(t) \leq (\nu_1)^{(1)}$

In the same manner , we get 384

$$\nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{\nu}_1)^{(1)}-(\nu_0)^{(1)}}{(\nu_0)^{(1)}-(\bar{\nu}_2)^{(1)}}}$$

From which we deduce $(\nu_0)^{(1)} \leq \nu^{(1)}(t) \leq (\bar{\nu}_1)^{(1)}$

If $0 < (\nu_1)^{(1)} < (\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0} < (\bar{\nu}_1)^{(1)}$ we find like in the previous case, 385

$$(\nu_1)^{(1)} \leq \frac{(\nu_1)^{(1)}+(C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}} \leq \nu^{(1)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\bar{\nu}_1)^{(1)}$$

If $0 < (\nu_1)^{(1)} \leq (\bar{\nu}_1)^{(1)} \leq \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$ , we obtain 386

$$(\nu_1)^{(1)} \leq \nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\nu_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(1)}(t)$ :-

$$(m_2)^{(1)} \leq \nu^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{\nu^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(1)}(t)$ :-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{13})^{(1)} = (a''_{14})^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)}G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)}T_{14}(t)$ This is an important

consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

**Proof** : From global equations we obtain 387

$$\frac{d\nu^{(2)}}{dt} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right) - (a''_{17})^{(2)}(T_{17}, t)\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

**Definition of** $\nu^{(2)}$ :- $\boxed{\nu^{(2)} = \frac{G_{16}}{G_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \le \frac{d\nu^{(2)}}{dt} \le -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

**Definition of** $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$ :-

For $0 < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \ge \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}}{1 + (C)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(C)^{(2)} = \frac{(\nu_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \le \nu^{(2)}(t) \le (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \le \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \le \nu^{(2)}(t) \le (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \le \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}}{1 + (C)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}} \le \nu^{(2)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \le (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \le (\bar{\nu}_1)^{(2)} \le (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$ , we obtain 394

$$(\nu_1)^{(2)} \le \nu^{(2)}(t) \le \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \le (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(2)}(t)$ :- 395

$$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}, \quad \boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$$

In a completely analogous way, we obtain 396

**Definition of** $u^{(2)}(t)$ :-

$$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}, \quad \boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :** 397

If $(a_{16}'')^{(2)} = (a_{17}'')^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)} G_{17}(t)$

Analogously if $(b_{16}'')^{(2)} = (b_{17}'')^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)} T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

**Proof** : From global equations we obtain 398

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a_{20}')^{(3)} - (a_{21}')^{(3)} + (a_{20}'')^{(3)}(T_{21}, t)\right) - (a_{21}'')^{(3)}(T_{21}, t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

**Definition of** $\nu^{(3)}$ :- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right) \leq \frac{d\nu^{(3)}}{dt} \leq -\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right)$$

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From which one obtains

$$\text{For } 0 < (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (\nu_1)^{(3)} < (\bar{\nu}_1)^{(3)}$$

$$\nu^{(3)}(t) \geq \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(\nu_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\nu_2)^{(3)}}}$$

it follows $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\nu_1)^{(3)}$

In the same manner , we get 401

$$\nu^{(3)}(t) \le \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{\nu}_1)^{(3)}-(\nu_0)^{(3)}}{(\nu_0)^{(3)}-(\bar{\nu}_2)^{(3)}}}$$

**Definition of** $(\bar{\nu}_1)^{(3)}$ :-

From which we deduce $(\nu_0)^{(3)} \le \nu^{(3)}(t) \le (\bar{\nu}_1)^{(3)}$

If $0 < (\nu_1)^{(3)} < (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (\bar{\nu}_1)^{(3)}$ we find like in the previous case, 402

$$(\nu_1)^{(3)} \le \frac{(\nu_1)^{(3)}+(C)^{(3)}(\nu_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}} \le \nu^{(3)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \le (\bar{\nu}_1)^{(3)}$$

If $0 < (\nu_1)^{(3)} \le (\bar{\nu}_1)^{(3)} \le (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}$ , we obtain 403

$$(\nu_1)^{(3)} \le \nu^{(3)}(t) \le \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \le (\nu_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(3)}(t)$ :-

$$(m_2)^{(3)} \le \nu^{(3)}(t) \le (m_1)^{(3)}, \quad \boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(3)}(t)$ :-

$$(\mu_2)^{(3)} \le u^{(3)}(t) \le (\mu_1)^{(3)}, \quad \boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{20}'')^{(3)} = (a_{21}'')^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)}G_{21}(t)$

Analogously if $(b_{20}'')^{(3)} = (b_{21}'')^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)}T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

**Proof** : From global equations we obtain 404

$$\frac{dv^{(4)}}{dt} = (a_{24})^{(4)} - \left((a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right) - (a''_{25})^{(4)}(T_{25},t)v^{(4)} - (a_{25})^{(4)}v^{(4)}$$

**Definition of** $v^{(4)}$ :- $\boxed{v^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)}\right) \le \frac{dv^{(4)}}{dt} \le -\left((a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_4)^{(4)}v^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$ :-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \ge \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \le \nu^{(4)}(t) \le (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \le \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \le \nu^{(4)}(t) \le (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \le \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}}{1 + (C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}} \le \nu^{(4)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \le (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \le (\bar{\nu}_1)^{(4)} \le \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$ , we obtain 407

$$(\nu_1)^{(4)} \le \nu^{(4)}(t) \le \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \le (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(4)}(t)$ :-

$$(m_2)^{(4)} \le \nu^{(4)}(t) \le (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(4)}(t)$ :-

$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}$, $\boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{24})^{(4)} = (a''_{25})^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)} G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b''_{24})^{(4)} = (b''_{25})^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)} T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

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**Proof** : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a'_{28})^{(5)} - (a'_{29})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right) - (a''_{29})^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

**Definition of** $\nu^{(5)}$ :- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$ :-

For $0 < \boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \le \frac{(\nu_1)^{(5)}+(C)^{(5)}(\nu_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)}-(\nu_2)^{(5)}\right)t\right]}}{1+(C)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)}-(\nu_2)^{(5)}\right)t\right]}} \le \nu^{(5)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \le (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \le (\bar{\nu}_1)^{(5)} \le \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain 411

$$(\nu_1)^{(5)} \le \nu^{(5)}(t) \le \frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \le (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(5)}(t)$ :-

$(m_2)^{(5)} \le \nu^{(5)}(t) \le (m_1)^{(5)}$, $\boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$

In a completely analogous way, we obtain

**Definition of** $u^{(5)}(t)$ :-

$(\mu_2)^{(5)} \le u^{(5)}(t) \le (\mu_1)^{(5)}$, $\boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a_{28}'')^{(5)} = (a_{29}'')^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)}G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b_{28}'')^{(5)} = (b_{29}'')^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)}T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

**Proof** : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a_{32}')^{(6)} - (a_{33}')^{(6)} + (a_{32}'')^{(6)}(T_{33},t)\right) - (a_{33}'')^{(6)}(T_{33},t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

**Definition of** $\nu^{(6)}$ :- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \le \frac{d\nu^{(6)}}{dt} \le -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$ :-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \geq \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\nu_1)^{(6)}$

In the same manner , we get 413

$$\nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{\nu}_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\bar{\nu}_1)^{(6)}$

If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 414

$$(\nu_1)^{(6)} \leq \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}} \leq \nu^{(6)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\bar{\nu}_1)^{(6)}$$

If $0 < (\nu_1)^{(6)} \leq (\bar{\nu}_1)^{(6)} \leq \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$ , we obtain 415

$$(\nu_1)^{(6)} \leq \nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(6)}(t)$ :-

$$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}, \quad \boxed{\nu^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(6)}(t)$ :-

$$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}, \quad \boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)} G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)} T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

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**Proof** : From global equations we obtain

$$\frac{dv^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right) - (a''_{37})^{(7)}(T_{37},t)v^{(7)} - (a_{37})^{(7)}v^{(7)}$$

**Definition of** $v^{(7)}$ :- $\boxed{v^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_2)^{(7)}v^{(7)} - (a_{36})^{(7)}\right) \le \frac{dv^{(7)}}{dt} \le -\left((a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_1)^{(7)}v^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

**Definition of** $(\bar{v}_1)^{(7)}, (v_0)^{(7)}$ :-

For $0 < \boxed{(v_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}} < (v_1)^{(7)} < (\bar{v}_1)^{(7)}$

$$v^{(7)}(t) \ge \frac{(v_1)^{(7)} + (C)^{(7)}(v_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_0)^{(7)}\right)t\right]}}{1 + (C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(v_1)^{(7)} - (v_0)^{(7)}}{(v_0)^{(7)} - (v_2)^{(7)}}}$$

it follows $(v_0)^{(7)} \le v^{(7)}(t) \le (v_1)^{(7)}$

In the same manner , we get 417

$$v^{(7)}(t) \le \frac{(\bar{v}_1)^{(7)} + (\bar{C})^{(7)}(\bar{v}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{v}_1)^{(7)} - (v_0)^{(7)}}{(v_0)^{(7)} - (\bar{v}_2)^{(7)}}}$$

From which we deduce $(v_0)^{(7)} \le v^{(7)}(t) \le (\bar{v}_1)^{(7)}$

If $0 < (v_1)^{(7)} < (v_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}} < (\bar{v}_1)^{(7)}$ we find like in the previous case, 418

$$(v_1)^{(7)} \le \frac{(v_1)^{(7)} + (C)^{(7)}(v_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_2)^{(7)}\right)t\right]}}{1 + (C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_2)^{(7)}\right)t\right]}} \le v^{(7)}(t) \le$$

$$\frac{(\bar{v}_1)^{(7)} + (\bar{C})^{(7)}(\bar{v}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}} \le (\bar{v}_1)^{(7)}$$

If $0 < (v_1)^{(7)} \le (\bar{v}_1)^{(7)} \le \boxed{(v_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$ , we obtain 419

$$(v_1)^{(7)} \le v^{(7)}(t) \le \frac{(\bar{v}_1)^{(7)} + (\bar{C})^{(7)}(\bar{v}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}} \le (v_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $v^{(7)}(t)$ :-

$$(m_2)^{(7)} \leq v^{(7)}(t) \leq (m_1)^{(7)}, \quad \boxed{v^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(7)}(t)$ :- 420

$$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}, \quad \boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $v^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)} G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)} T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

**Proof** : From global equations we obtain 421

$$\frac{dv^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right) - (a''_{41})^{(8)}(T_{41}, t) v^{(8)} - (a_{41})^{(8)} v^{(8)}$$

**Definition of** $v^{(8)}$ :- $\boxed{v^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_2)^{(8)} v^{(8)} - (a_{40})^{(8)}\right) \leq \frac{dv^{(8)}}{dt} \leq -\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_1)^{(8)} v^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$ :-

$$\text{For } 0 < \boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$$

$$v^{(8)}(t) \geq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)}-(\nu_0)^{(8)}}{(\nu_0)^{(8)}-(\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \leq \frac{(\nu_1)^{(8)}+(C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}}{1+(C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}} \leq \nu^{(8)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \leq (\bar{\nu}_1)^{(8)} \leq \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$ , we obtain 424

$$(\nu_1)^{(8)} \leq \nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(8)}(t)$ :-

$$(m_2)^{(8)} \leq \nu^{(8)}(t) \leq (m_1)^{(8)}, \quad \boxed{\nu^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(8)}(t)$ :-

$$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}, \quad \boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $\nu^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)}G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then $(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)}T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

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**Proof** : From 99,20,44,22,23,44 we obtain A

$$\frac{dv^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right) - (a''_{45})^{(9)}(T_{45},t)v^{(9)} - (a_{45})^{(9)}v^{(9)}$$

**Definition of** $v^{(9)}$ :- $\boxed{v^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)}\right) \le \frac{dv^{(9)}}{dt} \le -\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_1)^{(9)}v^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

**Definition of** $(\bar{v}_1)^{(9)}, (v_0)^{(9)}$ :-

For $0 < \boxed{(v_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (v_1)^{(9)} < (\bar{v}_1)^{(9)}$

$$v^{(9)}(t) \ge \frac{(v_1)^{(9)} + (C)^{(9)}(v_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((v_1)^{(9)} - (v_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((v_1)^{(9)} - (v_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(v_1)^{(9)} - (v_0)^{(9)}}{(v_0)^{(9)} - (v_2)^{(9)}}}$$

it follows $(v_0)^{(9)} \le v^{(9)}(t) \le (v_9)^{(9)}$

In the same manner , we get

$$v^{(9)}(t) \le \frac{(\bar{v}_1)^{(9)} + (\bar{C})^{(9)}(\bar{v}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{v}_1)^{(9)} - (\bar{v}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{v}_1)^{(9)} - (\bar{v}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{v}_1)^{(9)} - (v_0)^{(9)}}{(v_0)^{(9)} - (\bar{v}_2)^{(9)}}}$$

From which we deduce $(v_0)^{(9)} \le v^{(9)}(t) \le (\bar{v}_1)^{(9)}$

If $0 < (v_1)^{(9)} < (v_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}} < (\bar{v}_1)^{(9)}$ we find like in the previous case,

$$(v_1)^{(9)} \le \frac{(v_1)^{(9)} + (C)^{(9)}(v_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((v_1)^{(9)} - (v_2)^{(9)}\right)t\right]}}{1 + (C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((v_1)^{(9)} - (v_2)^{(9)}\right)t\right]}} \le v^{(9)}(t) \le$$

$$\frac{(\bar{v}_1)^{(9)} + (\bar{C})^{(9)}(\bar{v}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{v}_1)^{(9)} - (\bar{v}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{v}_1)^{(9)} - (\bar{v}_2)^{(9)}\right)t\right]}} \le (\bar{v}_1)^{(9)}$$

If $0 < (v_1)^{(9)} \le (\bar{v}_1)^{(9)} \le \boxed{(v_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$ , we obtain

$$(v_1)^{(9)} \le v^{(9)}(t) \le \frac{(\bar{v}_1)^{(9)} + (\bar{C})^{(9)}(\bar{v}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{v}_1)^{(9)} - (\bar{v}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{v}_1)^{(9)} - (\bar{v}_2)^{(9)}\right)t\right]}} \le (v_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $v^{(9)}(t)$ :-

$$(m_2)^{(9)} \le v^{(9)}(t) \le (m_1)^{(9)}, \quad \boxed{v^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(9)}(t)$ :-

$(\mu_2)^{(9)} \leq u^{(9)}(t) \leq (\mu_1)^{(9)}$, $\boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)} G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then $(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)} T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following 425

**Theorem :** If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on $t$ , and the conditions with the notations

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0$ ,

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$

$with\ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a''_i)^{(2)} and\ (b''_i)^{(2)}$ are independent on t , and the conditions with the notations 426

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$ 427

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 428

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$ , 429

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 430

$with\ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a''_i)^{(3)} and\ (b''_i)^{(3)}$ are independent on $t$ , and the conditions with the notations 431

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$ ,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

**Theorem :** If $(a_i'')^{(4)} and\ (b_i'')^{(4)}$ are independent on $t$ , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$ ,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(5)} and\ (b_i'')^{(5)}$ are independent on $t$ , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$ ,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

**Theorem** If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on $t$ , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$ ,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(7)} and\ (b_i'')^{(7)}$ are independent on $t$ , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$ ,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(8)} and\ (b_i'')^{(8)}$ are independent on $t$ , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$ ,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(9)} and\ (b_i'')^{(9)}$ are independent on $t$ , and the conditions (with the notations 436 A
45,46,27,28)

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$ ,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0$ 437

$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0$ 438

$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0$ 439

$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0$ 440

$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ]T_{14} = 0$ 441

$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0$ 442

has a unique positive solution , which is an equilibrium solution for the system

$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0$ 443

$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0$ 444

$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0$ 445

$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\ ]T_{16} = 0$ 446

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\,]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\,]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\,]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\,]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\,]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\big((G_{27})\big)\,]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\big((G_{27})\big)\,]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\big((G_{27})\big)\,]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})\,]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})\,]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})\,]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})\,]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})\,]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})\,]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0 \quad 479$$

$$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0 \quad 480$$

$$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0 \quad 481$$

$$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0 \quad 482$$

$$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0 \quad 483$$

$$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0 \quad 484$$

$$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0 \quad 484\text{A}$$

$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$

$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$

$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$

$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$

$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$

**Proof:** 485

(a) Indeed the first two equations have a nontrivial solution $G_{13}, G_{14}$ if

$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$

**Proof:** 486

(m) Indeed the first two equations have a nontrivial solution $G_{16}, G_{17}$ if

$\mathrm{F}(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$

**Proof:** 487

(a) Indeed the first two equations have a nontrivial solution $G_{20}, G_{21}$ if

$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$

**Proof:** 488

(a) Indeed the first two equations have a nontrivial solution $G_{24}, G_{25}$ if

$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$

**Proof:** 489

(a) Indeed the first two equations have a nontrivial solution $G_{28}, G_{29}$ if

$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$

**Proof:** 490

(a) Indeed the first two equations have a nontrivial solution $G_{32}, G_{33}$ if

$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) +$

$(a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$

**Proof:** 491

(a) Indeed the first two equations have a nontrivial solution $G_{36}, G_{37}$ if

$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$

**Proof:** 492

(a) Indeed the first two equations have a nontrivial solution $G_{40}, G_{41}$ if

$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$

**Proof:** 492 A

(a) Indeed the first two equations have a nontrivial solution $G_{44}, G_{45}$ if

$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$

**Definition and uniqueness of** $T^*_{14}$ :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique $T^*_{14}$ for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

**Definition and uniqueness of** $T^*_{17}$ :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique $T^*_{17}$ for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]}$$ 495

**Definition and uniqueness of** $T^*_{21}$ :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique $T^*_{21}$ for which $f(T^*_{21}) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

**Definition and uniqueness of** $T^*_{25}$ :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that

there exists a unique $T_{25}^{*}$ for which $f(T_{25}^{*})=0$. With this value , we obtain from the three first equations

$$G_{24}=\frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}=\frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T_{25}^{*})]}$$

**Definition and uniqueness of** $T_{29}^{*}$ :- 498

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(5)}(T_{29})$ being increasing, it follows that there exists a unique $T_{29}^{*}$ for which $f(T_{29}^{*})=0$. With this value , we obtain from the three first equations

$$G_{28}=\frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}=\frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T_{29}^{*})]}$$

**Definition and uniqueness of** $T_{33}^{*}$ :- 499

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(6)}(T_{33})$ being increasing, it follows that there exists a unique $T_{33}^{*}$ for which $f(T_{33}^{*})=0$. With this value , we obtain from the three first equations

$$G_{32}=\frac{(a_{32})^{(6)}G_{33}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^{*})]} \quad , \quad G_{34}=\frac{(a_{34})^{(6)}G_{33}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^{*})]}$$

**Definition and uniqueness of** $T_{37}^{*}$ :- 500

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(7)}(T_{37})$ being increasing, it follows that there exists a unique $T_{37}^{*}$ for which $f(T_{37}^{*})=0$. With this value , we obtain from the three first equations

$$G_{36}=\frac{(a_{36})^{(7)}G_{37}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^{*})]} \quad , \quad G_{38}=\frac{(a_{38})^{(7)}G_{37}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^{*})]}$$

**Definition and uniqueness of** $T_{41}^{*}$ :- 501

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique $T_{41}^{*}$ for which $f(T_{41}^{*})=0$. With this value , we obtain from the three first equations

$$G_{40}=\frac{(a_{40})^{(8)}G_{41}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T_{41}^{*})]} \quad , \quad G_{42}=\frac{(a_{42})^{(8)}G_{41}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T_{41}^{*})]}$$

**Definition and uniqueness of** $T_{45}^{*}$ :- 501 A

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(9)}(T_{45})$ being increasing, it follows that there exists a unique $T_{45}^{*}$ for which $f(T_{45}^{*})=0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)}G_{45}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T^*_{45})]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)}G_{45}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T^*_{45})]}$$

By the same argument, the equations admit solutions $G_{13}, G_{14}$ if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right]+(b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{14}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{14}$ such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions $G_{16}, G_{17}$ if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right]+(b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ 504
is a decreasing function in $G_{17}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{14}$ such that $\varphi((G_{19})^*) = 0$

By the same argument, the equations admit solutions $G_{20}, G_{21}$ if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right]+(b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{21}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{21}$ such that $\varphi((G_{23})^*) = 0$

By the same argument, the equations admit solutions $G_{24}, G_{25}$ if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right]+(b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{25}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{25}$ such that $\varphi((G_{27})^*) = 0$

By the same argument, the equations admit solutions $G_{28}, G_{29}$ if 507

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right]+(b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{29}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that

there exists a unique $G_{29}^{*}$ such that $\varphi((G_{31})^{*}) = 0$

By the same argument, the equations admit solutions $G_{32}, G_{33}$ if 508
$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$

$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right] + (b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{33}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{33}^{*}$ such that $\varphi(G_{35}{}^{*}) = 0$

By the same argument, the equations admit solutions $G_{36}, G_{37}$ if 509
$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$
$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right] + (b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{37}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{37}^{*}$ such that $\varphi(G_{39}{}^{*}) = 0$

By the same argument, the equations admit solutions $G_{40}, G_{41}$ if 510
$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$
$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right] + (b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{41}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{41}^{*}$ such that $\varphi(G_{43}{}^{*}) = 0$

By the same argument, the equations 92,93 admit solutions $G_{44}, G_{45}$ if
$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$

$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right] + (b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{45}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{45}^{*}$ such that $\varphi((G_{47})^{*}) = 0$

Finally we obtain the unique solution 511

$G_{14}^{*}$ given by $\varphi(G^{*}) = 0$ , $T_{14}^{*}$ given by $f(T_{14}^{*}) = 0$ and

$$G_{13}^{*} = \frac{(a_{13})^{(1)}G_{14}^{*}}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^{*})]} \quad , \quad G_{15}^{*} = \frac{(a_{15})^{(1)}G_{14}^{*}}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^{*})]}$$

$$T_{13}^{*} = \frac{(b_{13})^{(1)}T_{14}^{*}}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^{*})]} \quad , \quad T_{15}^{*} = \frac{(b_{15})^{(1)}T_{14}^{*}}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^{*})]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

$G_{17}^{*}$ given by $\varphi((G_{19})^{*}) = 0$ , $T_{17}^{*}$ given by $f(T_{17}^{*}) = 0$ and 512

$$G_{16}^{*} = \frac{(a_{16})^{(2)} G_{17}^{*}}{[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}^{*})]} \quad , \quad G_{18}^{*} = \frac{(a_{18})^{(2)} G_{17}^{*}}{[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}^{*})]} \qquad 513$$

$$T_{16}^{*} = \frac{(b_{16})^{(2)} T_{17}^{*}}{[(b'_{16})^{(2)} - (b''_{16})^{(2)}((G_{19})^{*})]} \quad , \quad T_{18}^{*} = \frac{(b_{18})^{(2)} T_{17}^{*}}{[(b'_{18})^{(2)} - (b''_{18})^{(2)}((G_{19})^{*})]} \qquad 514$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

$G_{21}^{*}$ given by $\varphi((G_{23})^{*}) = 0$ , $T_{21}^{*}$ given by $f(T_{21}^{*}) = 0$ and

$$G_{20}^{*} = \frac{(a_{20})^{(3)} G_{21}^{*}}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^{*})]} \quad , \quad G_{22}^{*} = \frac{(a_{22})^{(3)} G_{21}^{*}}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^{*})]}$$

$$T_{20}^{*} = \frac{(b_{20})^{(3)} T_{21}^{*}}{[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}{}^{*})]} \quad , \quad T_{22}^{*} = \frac{(b_{22})^{(3)} T_{21}^{*}}{[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}{}^{*})]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

$G_{25}^{*}$ given by $\varphi(G_{27}) = 0$ , $T_{25}^{*}$ given by $f(T_{25}^{*}) = 0$ and

$$G_{24}^{*} = \frac{(a_{24})^{(4)} G_{25}^{*}}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}^{*} = \frac{(a_{26})^{(4)} G_{25}^{*}}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^{*})]}$$

$$T_{24}^{*} = \frac{(b_{24})^{(4)} T_{25}^{*}}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^{*})]} \quad , \quad T_{26}^{*} = \frac{(b_{26})^{(4)} T_{25}^{*}}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^{*})]} \qquad 517$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

$G_{29}^{*}$ given by $\varphi((G_{31})^{*}) = 0$ , $T_{29}^{*}$ given by $f(T_{29}^{*}) = 0$ and

$$G_{28}^{*} = \frac{(a_{28})^{(5)} G_{29}^{*}}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}^{*} = \frac{(a_{30})^{(5)} G_{29}^{*}}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^{*})]}$$

$$T_{28}^{*} = \frac{(b_{28})^{(5)} T_{29}^{*}}{[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31})^{*})]} \quad , \quad T_{30}^{*} = \frac{(b_{30})^{(5)} T_{29}^{*}}{[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31})^{*})]} \qquad 519$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

$G_{33}^{*}$ given by $\varphi((G_{35})^{*}) = 0$ , $T_{33}^{*}$ given by $f(T_{33}^{*}) = 0$ and

$$G_{32}^{*} = \frac{(a_{32})^{(6)} G_{33}^{*}}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^{*})]} \quad , \quad G_{34}^{*} = \frac{(a_{34})^{(6)} G_{33}^{*}}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^{*})]}$$

$$T_{32}^{*} = \frac{(b_{32})^{(6)} T_{33}^{*}}{[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35})^{*})]} \quad , \quad T_{34}^{*} = \frac{(b_{34})^{(6)} T_{33}^{*}}{[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35})^{*})]} \qquad 521$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

$G_{37}^*$ given by $\varphi((G_{39})^*) = 0$ , $T_{37}^*$ given by $f(T_{37}^*) = 0$ and

$$G_{36}^* = \frac{(a_{36})^{(7)} G_{37}^*}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^*)]} \quad , \quad G_{38}^* = \frac{(a_{38})^{(7)} G_{37}^*}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^*)]}$$

$$T_{36}^* = \frac{(b_{36})^{(7)} T_{37}^*}{[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39})^*)]} \quad , \quad T_{38}^* = \frac{(b_{38})^{(7)} T_{37}^*}{[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39})^*)]}$$

Finally we obtain the unique solution 523

$G_{41}^*$ given by $\varphi((G_{43})^*) = 0$ , $T_{41}^*$ given by $f(T_{41}^*) = 0$ and

$$G_{40}^* = \frac{(a_{40})^{(8)} G_{41}^*}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42}^* = \frac{(a_{42})^{(8)} G_{41}^*}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

$$T_{40}^* = \frac{(b_{40})^{(8)} T_{41}^*}{[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43})^*)]} \quad , \quad T_{42}^* = \frac{(b_{42})^{(8)} T_{41}^*}{[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43})^*)]}$$

Finally we obtain the unique solution of 89 to 99 523 A

$G_{45}^*$ given by $\varphi((G_{47})^*) = 0$ , $T_{45}^*$ given by $f(T_{45}^*) = 0$ and

$$G_{44}^* = \frac{(a_{44})^{(9)} G_{45}^*}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46}^* = \frac{(a_{46})^{(9)} G_{45}^*}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

$$T_{44}^* = \frac{(b_{44})^{(9)} T_{45}^*}{[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47})^*)]} \quad , \quad T_{46}^* = \frac{(b_{46})^{(9)} T_{45}^*}{[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47})^*)]}$$

## ASYMPTOTIC STABILITY ANALYSIS 524

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

**Proof:** Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a''_{14})^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \frac{\partial (b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a'_{13})^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)}G^*_{13}\mathbb{T}_{14} \quad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a'_{14})^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)}G^*_{14}\mathbb{T}_{14} \quad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a'_{15})^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)}G^*_{15}\mathbb{T}_{14} \quad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b'_{13})^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)}T^*_{13}\mathbb{G}_j\right) \quad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b'_{14})^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)}T^*_{14}\mathbb{G}_j\right) \quad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b'_{15})^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T^*_{15}\mathbb{G}_j\right) \quad 530$$

**ASYMPTOTIC STABILITY ANALYSIS** 531

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(2)}$ and $(b''_i)^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G^*_i + \mathbb{G}_i \quad , T_i = T^*_i + \mathbb{T}_i \quad 532$$

$$\frac{\partial(a''_{17})^{(2)}}{\partial T_{17}}(T^*_{17}) = (q_{17})^{(2)} \quad , \frac{\partial(b''_i)^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \quad 533$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a'_{16})^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G^*_{16}\mathbb{T}_{17} \quad 534$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a'_{17})^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G^*_{17}\mathbb{T}_{17} \quad 535$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a'_{18})^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G^*_{18}\mathbb{T}_{17} \quad 536$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b'_{16})^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T^*_{16}\mathbb{G}_j\right) \quad 537$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b'_{17})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}T^*_{17}\mathbb{G}_j\right) \quad 538$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b'_{18})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}T^*_{18}\mathbb{G}_j\right) \quad 539$$

**ASYMPTOTIC STABILITY ANALYSIS** 540

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(3)}$ *and* $(b''_i)^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{21}'')^{(3)}}{\partial T_{21}}(T_{21}^*) = (q_{21})^{(3)} \quad , \frac{\partial (b_i'')^{(3)}}{\partial G_j}(\,(G_{23})^*\,) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\big((a_{20}')^{(3)} + (p_{20})^{(3)}\big)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G_{20}^*\mathbb{T}_{21} \quad 541$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\big((a_{21}')^{(3)} + (p_{21})^{(3)}\big)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G_{21}^*\mathbb{T}_{21} \quad 542$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\big((a_{22}')^{(3)} + (p_{22})^{(3)}\big)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G_{22}^*\mathbb{T}_{21} \quad 543$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\big((b_{20}')^{(3)} - (r_{20})^{(3)}\big)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\big(s_{(20)(j)}T_{20}^*\mathbb{G}_j\big) \quad 544$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\big((b_{21}')^{(3)} - (r_{21})^{(3)}\big)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\big(s_{(21)(j)}T_{21}^*\mathbb{G}_j\big) \quad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\big((b_{22}')^{(3)} - (r_{22})^{(3)}\big)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\big(s_{(22)(j)}T_{22}^*\mathbb{G}_j\big) \quad 546$$

**ASYMPTOTIC STABILITY ANALYSIS** 547

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}\ and\ (b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{25}'')^{(4)}}{\partial T_{25}}(T_{25}^*) = (q_{25})^{(4)} \quad , \frac{\partial (b_i'')^{(4)}}{\partial G_j}((G_{27})^*\,) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\big((a_{24}')^{(4)} + (p_{24})^{(4)}\big)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G_{24}^*\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\big((a_{25}')^{(4)} + (p_{25})^{(4)}\big)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G_{25}^*\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\big((a_{26}')^{(4)} + (p_{26})^{(4)}\big)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G_{26}^*\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\big((b_{24}')^{(4)} - (r_{24})^{(4)}\big)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\big(s_{(24)(j)}T_{24}^*\mathbb{G}_j\big) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\big((b_{25}')^{(4)} - (r_{25})^{(4)}\big)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\big(s_{(25)(j)}T_{25}^*\mathbb{G}_j\big) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T^*_{26}\mathbb{G}_j\right) \quad 554$$

**ASYMPTOTIC STABILITY ANALYSIS** 555

**Theorem 5:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}\ and\ (b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 556

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{29})^{(5)}}{\partial T_{29}}(T^*_{29}) = (q_{29})^{(5)} \ , \ \frac{\partial(b_i'')^{(5)}}{\partial G_j}\left((G_{31})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a'_{28})^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G^*_{28}\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a'_{29})^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G^*_{29}\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a'_{30})^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G^*_{30}\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b'_{28})^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T^*_{28}\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b'_{29})^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T^*_{29}\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b'_{30})^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T^*_{30}\mathbb{G}_j\right) \quad 562$$

**ASYMPTOTIC STABILITY ANALYSIS** 563

**Theorem 6:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}\ and\ (b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{33})^{(6)}}{\partial T_{33}}(T^*_{33}) = (q_{33})^{(6)} \ , \ \frac{\partial(b_i'')^{(6)}}{\partial G_j}\left((G_{35})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a'_{32})^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G^*_{32}\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a'_{33})^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G^*_{33}\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a'_{34})^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G^*_{34}\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b'_{32})^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T^*_{32}\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b'_{33})^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T^*_{33}\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b'_{34})^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T^*_{34}\mathbb{G}_j\right) \quad 570$$

**ASYMPTOTIC STABILITY ANALYSIS** 571

**Theorem 7:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 572

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{37})^{(7)}}{\partial T_{37}}(T^*_{37}) = (q_{37})^{(7)} \quad , \frac{\partial(b_i'')^{(7)}}{\partial G_j}\left((G_{39})^{**}\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a'_{36})^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G^*_{36}\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a'_{37})^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G^*_{37}\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a'_{38})^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G^*_{38}\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b'_{36})^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T^*_{36}\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b'_{37})^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T^*_{37}\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b'_{38})^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T^*_{38}\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

**ASYMPTOTIC STABILITY ANALYSIS**

**Theorem 8:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{41}'')^{(8)}}{\partial T_{41}} (T_{41}^*) = (q_{41})^{(8)} \quad , \frac{\partial (b_i'')^{(8)}}{\partial G_j} ( (G_{43})^* ) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\big((a_{40}')^{(8)} + (p_{40})^{(8)}\big)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)} G_{40}^* \mathbb{T}_{41} \quad (581)$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\big((a_{41}')^{(8)} + (p_{41})^{(8)}\big)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)} G_{41}^* \mathbb{T}_{41} \quad (582)$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\big((a_{42}')^{(8)} + (p_{42})^{(8)}\big)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)} G_{42}^* \mathbb{T}_{41} \quad (583)$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\big((b_{40}')^{(8)} - (r_{40})^{(8)}\big)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\big(s_{(40)(j)} T_{40}^* \mathbb{G}_j\big) \quad (584)$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\big((b_{41}')^{(8)} - (r_{41})^{(8)}\big)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\big(s_{(41)(j)} T_{41}^* \mathbb{G}_j\big) \quad (585)$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\big((b_{42}')^{(8)} - (r_{42})^{(8)}\big)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\big(s_{(42)(j)} T_{42}^* \mathbb{G}_j\big) \quad (586)$$

(586 A)

## ASYMPTOTIC STABILITY ANALYSIS

**Theorem 9:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}$ *and* $(b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

**Proof:** Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{45}'')^{(9)}}{\partial T_{45}} (T_{45}^*) = (q_{45})^{(9)} \quad , \frac{\partial (b_i'')^{(9)}}{\partial G_j} ( (G_{47})^* ) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\big((a_{44}')^{(9)} + (p_{44})^{(9)}\big)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)} G_{44}^* \mathbb{T}_{45} \quad (586\,B)$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\big((a_{45}')^{(9)} + (p_{45})^{(9)}\big)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)} G_{45}^* \mathbb{T}_{45} \quad (586\,C)$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\big((a_{46}')^{(9)} + (p_{46})^{(9)}\big)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)} G_{46}^* \mathbb{T}_{45} \quad (586\,D)$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\big((b_{44}')^{(9)} - (r_{44})^{(9)}\big)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\big(s_{(44)(j)} T_{44}^* \mathbb{G}_j\big) \quad (586\,E)$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\big((b_{45}')^{(9)} - (r_{45})^{(9)}\big)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\big(s_{(45)(j)} T_{45}^* \mathbb{G}_j\big) \quad (586\,F)$$

$\frac{d\mathbb{T}_{46}}{dt} = -\left((b'_{46})^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T^*_{46}\mathbb{G}_j\right)$ 586

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**The characteristic equation of this system is** 587

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(q_{13})^{(1)}G^*_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)}T^*_{14} + (b_{14})^{(1)}s_{(13),(14)}T^*_{14}\right)$$

$$+\left(\left((\lambda)^{(1)} + (a'_{14})^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)}G^*_{13} + (a_{13})^{(1)}(q_{14})^{(1)}G^*_{14}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)}T^*_{14} + (b_{14})^{(1)}s_{(13),(13)}T^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((b'_{13})^{(1)} + (b'_{14})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(15)}T^*_{14} + (b_{14})^{(1)}s_{(13),(15)}T^*_{13}\right)\} = 0$$

$+$

$$(\lambda)^{(2)} + (b'_{18})^{(2)} - (r_{18})^{(2)}\{\left((\lambda)^{(2)} + (a'_{18})^{(2)} + (p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^*_{17} + (a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^*_{17}\right)$$

$$+\left(\left((\lambda)^{(2)} + (a'_{17})^{(2)} + (p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^*_{16} + (a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}\right)$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2 + \left((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2 + \left((b'_{16})^{(2)} + (b'_{17})^{(2)} - (r_{16})^{(2)} + (r_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$+\left(\left((\lambda)^{(2)}\right)^2 + \left((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17} + (a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^{*}_{16}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\right)\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left(\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)\right]$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(21)}T^{*}_{21}\right)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^{*}_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^{*}_{21}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(20)}T^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$+\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(22)}T^{*}_{20}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\right)\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left(\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)\right]$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(25)}T^{*}_{25}\right)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^{*}_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^{*}_{25}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(24)}T^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$+\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(26)}T^{*}_{24}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\right)\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(29)}T^{*}_{29}\right)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^{*}_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^{*}_{29}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(28)}T^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$+\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(30)}T^{*}_{28}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\right)\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left(\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)\right]$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(33)}T^{*}_{33}\right)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^{*}_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^{*}_{33}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(32)}T^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$+\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(34)}T^{*}_{32}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\right)\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)\right]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(37)}T^{*}_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^{*}_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^{*}_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(36)}T^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$+\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(38)}T^{*}_{36}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\right)\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(41)}T^{*}_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^{*}_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^{*}_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(40)}T^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$+\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(42)}T^{*}_{40}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\right)\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(45)}T^{*}_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^{*}_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^{*}_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(44)}T^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$+\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(46)}T^{*}_{44}\right)\}=0$$

**And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.**

## SECTION FIFTEEN

## Neutrino Billiards

### INTRODUCTION—VARIABLES USED

**Neutrino Billiards: Time-Reversal Symmetry-Breaking without Magnetic Fields M. V. Berry, R. J. Mondragon Published 8 July 1987.DOI: 10.1098/rspa.1987.0080 Royal society proceedings**

(1) A Dirac hamiltonian describing massless spin-half particles (`neutrinos') moving (e&eb)in the plane $r = (x, y)$ under (e&eb) the action of a 4-scalar (not electric) potential V(r) is, in position

representation, $H = -i\hbar c\sigma\cdot\nabla + V(r)\sigma_z$, where $\sigma = (\sigma_x, \sigma_y)$ and $\sigma_z$ are the Pauli matrices;

(2) $\hat {H} $ acts on (e&eb) two-component column spinor wavefunctions $\psi(r) = (\psi_1,\psi_2)$ and has (e) eigenvalues $\hbar$ ck$_n$.

(3) H does not (e) possess time-reversal symmetry (T).

(4) If V(r) describes a hard wall bounding a finite domain D (`billiards'), this is equivalent to (=) a novel boundary condition for $\psi_2/\psi_1$. T-breaking is interpreted semiclassically as (eb) a difference of $\pi$ between the phases accumulated by (e&eb) waves travelling in opposite senses round closed geodesics in D with (e&eb)odd numbers of reflections.

(5) Semiclassical (large-k) asymptotics of the eigenvalue counting function (spectral staircase) N(k) are shown to have (e) the `Weyl' leading term Ak$^2$/4$\pi$, where A is the area of D, but zero perimeter correction.

(6) The Dirac equation is transformed to (e&eb) an integral equation round the boundary of D, and forms (eb) the basis of a numerical method for computing the k$_n$.

(7) When D is the unit disc, geodesics are (eb) integrable and the eigenvalues, which satisfy $J_l(k_n) = J_{l+1}(k_n)$, are (=) (locally) Poisson-distributed.

(8) When D is an `Africa' shape (cubic conformal map of the unit disc), (eb) the eigenvalues are (locally) distributed according to (e&eb) the statistics of the Gaussian unitary ensemble of random-matrix theory, as predicted on the basis of (e) T-breaking and lack of geometric symmetry.

**Science 18 April 2008: Vol. 320 no. 5874 pp. 356-358 DOI: 10.1126/science.1154663 Chaotic Dirac Billiard in Graphene Quantum Dots L. A. Ponomarenko1, F. Schedin1, M. I. Katsnelson2, R. Yang1, E. W. Hill1, K. S. Novoselov1,*, A. K. Geim1**

(1) The exceptional electronic properties of graphene, with (e&eb) its charge carriers mimicking (e&eb) relativistic quantum particles and its formidable potential in various applications, have ensured (eb) a rapid growth of interest in this new material.

(2) Authors report on electron transport in quantum dot devices carved entirely from (e) graphene.

(3) At large sizes (>100 nanometers), they behave as (=) conventional single-electron transistors, exhibiting (eb) periodic Coulomb blockade peaks.

**(4)** For quantum dots smaller than 100 nanometers, the peaks become (=) strongly nonperiodic, indicating (eb) a major contribution of quantum confinement. Random peak spacing and its statistics are well described by (e) the theory of chaotic neutrino billiards.

**(5)** Short constrictions of only a few nanometers in width remain conductive and reveal (eb) a confinement gap of up to 0.5 electron volt, demonstrating (eb) the possibility of molecular-scale electronics based on graphene

**Graphene quantum dots: Beyond a Dirac billiard F. Libisch, C. Stampfer, J. Burgdörfer**

(1) Authors present realistic simulations of quantum confinement effects in ballistic graphene quantum dots with linear dimensions of 10 to 40 nm.

(2) Authors determine wavefunctions and energy level statistics in the presence of disorder resulting from edge roughness, charge impurities, or short-ranged scatterers.

(3) Marked deviations from a simple Dirac billiard for massless fermions are found.

**(4)** Authors find a remarkably stable dependence of the nearest-neighbor level spacing on edge roughness suggesting that the roughness of fabricated devices can be potentially characterized by the distribution of measured Coulomb blockade peaks. Subjects: Mesoscale and Nanoscale Physics (cond-mat.mes-hall) Journal reference: Phys. Rev. B, 79, 115423 (2009) DOI: 10.1103/PhysRevB.79.115423 Cite as: arXiv: 0808.3095 [cond-mat.mes-hall]

(or arXiv: 0808.3095v2 [cond-mat.mes-hall] for this version)

## NOTATION

### Module One

A Dirac hamiltonian describing massless spin-half particles (`neutrinos'**)** moving (e&eb) in the plane $r = (x, y)$ under (e&eb) the action of a 4-scalar (not electric) potential V(r) is, in position representation, $H = -i\hbar c\sigma\cdot\nabla + V(r)\sigma_z$, where $\sigma = (\sigma_x, \sigma_y)$ and $\sigma_z$ are the Pauli matrices

$G_{13}$ : Category one of **Dirac hamiltonian describing massless spin-half particles (`neutrinos');** plane $r = (x, y)$ under (e&eb) the action of a 4-scalar (not electric) potential V(r) is, in position representation, $H = -i\hbar c\sigma\cdot\nabla + V(r)\sigma_z$, where $\sigma = (\sigma_x, \sigma_y)$ and $\sigma_z$ are the Pauli matrices

$G_{14}$ : Category two of SAS

$G_{15}$ : Category three of SAS

$T_{13}$ : Category one of plane $r = (x, y)$ under (e&eb) the action of a 4-scalar (not electric) potential V(r) is, in position representation, $H = -i\hbar c\sigma\cdot\nabla + V(r)\sigma_z$, where $\sigma = (\sigma_x, \sigma_y)$ and $\sigma_z$ are the Pauli matrices; **Dirac hamiltonian describing massless spin-half particles (`neutrinos')**

$T_{14}$ : Category two of SAS

$T_{15}$ : Category three of SAS

### Module Two

A Dirac hamiltonian describing massless spin-half particles (`neutrinos') moving in the plane $r = (**x, y)$** under (e&eb) the action of a 4-scalar (not electric) potential V(r) is, in position representation, $H = -i\hbar c\sigma\cdot\nabla + V(r)\sigma_z$, where $\sigma = (\sigma_x, \sigma_y)$ and $\sigma_z$ are the Pauli matrices

$G_{16}$ : Category one of **Dirac hamiltonian describing massless spin-half particles (`neutrinos') moving in the plane $r = (x, y)$;** action of a 4-scalar (not electric) potential V(r) is, in position representation, $H = -i\hbar c\sigma\cdot\nabla + V(r)\sigma_z$, where $\sigma = (\sigma_x, \sigma_y)$ and $\sigma_z$ are the Pauli matrices

$G_{17}$ : Category two of SAS

$G_{18}$ : Category three of SAS

$T_{16}$ : Category one of action of a 4-scalar (not electric) potential V(r) is, in position representation, $H = -i\hbar c\sigma\cdot\nabla + V(r)\sigma_z$, where $\sigma = (\sigma_x, \sigma_y)$ and $\sigma_z$ are the Pauli matrices; **Dirac hamiltonian describing massless spin-half particles (`neutrinos') moving in the plane $r = (x, y)$**

$T_{17}$ : Category two of SAS

$T_{18}$ : Category three of SAS

### Module three

A Dirac hamiltonian describing massless spin-half particles (`neutrinos') moving in the plane $r = (**x, y)$** under the action of a 4-scalar (not electric) potential V(r) is, in position representation, $H = -i\hbar

c\sigma\cdot\nabla + V(r)\sigma_z$, where $\sigma = (\sigma_x, \sigma_y)$ and $\sigma_z$ are the Pauli matrices

$G_{20}$ : Category one of Dirac hamiltonian describing massless spin-half particles (`neutrinos') moving in the plane $r = (**x, y)$** under the action of a 4-scalar (not electric) potential V(r) is, in position representation, $H = -i\hbar c\sigma\cdot\nabla + V(r)\sigma_z$, where $\sigma = (\sigma_x, \sigma_y)$ and $\sigma_z

$G_{21}$ : Category two of SAS

$G_{22}$ : Category three of SAS

$T_{20}$ : Category one of Pauli matrices

$T_{21}$ : Category two of SAS

$T_{22}$ : Category three of SAS

**Module four**

$\hat {H} $ acts on (e&eb) two-component column spinor wavefunctions $\psi(r) = (\psi_1,\psi_2)$ and has (e) eigenvalues $\hbar$ ck$_n$

$G_{24}$ : Category one of **$\hat {H} $;** two-component column spinor wavefunctions $\psi(r) = (\psi_1,\psi_2)$ and has (e) eigenvalues $\hbar$ ck$_n$

$G_{25}$ : Category two of SAS

$G_{26}$ : Category three of SAS

$T_{24}$ : Category one of two-component column spinor wavefunctions $\psi(r) = (\psi_1,\psi_2)$ and has (e) eigenvalues $\hbar$ ck$_n$; **$\hat {H} $**

$T_{25}$ : Category two of SAS

$T_{26}$ : Category three of SAS

**Module five**

$\hat {H} $ acts on two-component column spinor wavefunctions $\psi(r) = (\psi_1,\psi_2)$ and has (e) **eigenvalues $\hbar$ ck$_n$**

$G_{28}$ : Category one of$\hat {H} $ acts on two-component column spinor wavefunctions $\psi(r) = (\psi_1,\psi_2)$; **eigenvalues $\hbar$ ck$_n$**

$G_{29}$ : Category two of SAS

$G_{30}$ : Category three of SAS

$T_{28}$ : Category one of **eigenvalues $\hbar$ ck$_n$ ;** $\hat {H} $ acts on two-component column spinor wavefunctions $\psi(r) = (\psi_1,\psi_2)$

$T_{29}$ : Category two of SAS

$T_{30}$ : Category three of SAS

**Module six**

H does not (e) possess time-reversal symmetry (T)

$G_{32}$ : Category one of possess time-reversal symmetry (T)

$G_{33}$ : Category two of SAS

$G_{34}$ : Category three of SAS

$T_{32}$ : Category one of H (of various experiments or take all the categories as equivalent)

$T_{33}$ : Category two of SAS

$T_{34}$ : Category three of SAS

**Module seven**

If V(r) describes a hard wall bounding a finite domain D (`billiards'), this is equivalent to (=) a novel boundary condition for $\psi_2/\psi_1$.

T-breaking is interpreted semiclassically as (eb) a difference of $\pi$ between the phases accumulated by (e&eb) waves travelling in opposite senses round closed geodesics in D with (e&eb)odd numbers of reflections

$G_{36}$ : Category one of V(r) describes a hard wall bounding a finite domain D (`billiards'),

$G_{37}$ : Category two of SAS

$G_{38}$ : Category three of SAS

$T_{36}$ : Category one of boundary condition for $\psi_2/\psi_1$.

$T_{37}$ : Category two of SAS

$T_{38}$ : Category three of SAS

**Module eight**

T-breaking is interpreted semiclassically as (eb) a difference of $\pi$ between the phases accumulated by (e&eb) waves travelling in opposite senses round closed geodesics in D with (e&eb)odd numbers of reflections

$G_{40}$ : Category one of T-breaking is interpreted semiclassically

$G_{41}$ : Category two of SAS

$G_{42}$ : Category three of SAS

$T_{40}$ : Category one of difference of $\pi$ between the phases accumulated by (e&eb) waves travelling in opposite senses round closed geodesics in D with (e&eb)odd numbers of reflections

$T_{41}$ : Category two of SAS

$T_{42}$ : Category three of SAS

**Module Nine**

**T-breaking is interpreted semiclassically as a difference of $\pi$ between the phases accumulated** by (e&eb) waves travelling in opposite senses round closed geodesics in D with (e&eb)odd numbers of reflections

$G_{44}$ : Category one of **T-breaking is interpreted semiclassically as a difference of $\pi$ between the phases accumulated;** waves travelling in opposite senses round closed geodesics in D with odd numbers of reflections

$G_{45}$ : Category two of SAS

$G_{46}$ : Category three of SAS

$T_{44}$ : Category one of waves travelling in opposite senses round closed geodesics in D with odd numbers of reflections ; **T-breaking is interpreted semiclassically as a difference of $\pi$ between the phases accumulated**

$T_{45}$ : Category two of SAS

$T_{46}$ : Category three of SAS

**The Coefficients:**

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}$ , $(b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)},$
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}$, $(b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}\ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$ ,
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)},$
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)},$
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)},$
are Dissipation coefficients

**Module Numbered One**

**The differential system of this model is now (Module Numbered one)**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14},t)\right]G_{13} \quad 1$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14},t)\right]G_{14} \quad 2$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14},t)\right]G_{15} \quad 3$$

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G,t)\right]T_{13} \quad 4$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G,t)\right]T_{14} \quad 5$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G,t)\right]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14},t) =$ First augmentation factor

$-(b''_{13})^{(1)}(G,t) =$ First detritions factor

**Module Numbered Two**

**The differential system of this model is now ( Module numbered two)**

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17},t)\right]G_{16} \quad 7$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17},t)\right]G_{17} \quad 8$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17},t)\right]G_{18} \quad 9$$

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\left((G_{19}),t\right)\right]T_{16} \quad 10$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\left((G_{19}),t\right)\right]T_{17} \quad 11$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\left((G_{19}),t\right)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17},t) =$ First augmentation factor

$-(b''_{16})^{(2)}\left((G_{19}),t\right) =$ First detritions factor

**Module Numbered Three**

**The differential system of this model is now (Module numbered three)**

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21},t)\right]G_{20} \quad 13$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21},t)\right]G_{21} \quad 14$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21},t)\right]G_{22} \quad 15$$

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23},t)\right]T_{20} \quad 16$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23},t)\right]T_{21} \quad 17$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23},t)\right]T_{22} \quad 18$$

$+(a''_{20})^{(3)}(T_{21},t) =$ First augmentation factor

$-(b''_{20})^{(3)}(G_{23},t) =$ First detritions factor

**Module Numbered Four**

**The differential system of this model is now (Module numbered Four)**

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right]G_{24} \quad 19$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25},t)\right]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25},t)\right]G_{26} \quad 21$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27}),t\right)\right]T_{24} \quad 22$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27}),t\right)\right]T_{25} \quad 23$$

$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27}),t)\right]T_{26}$ 24

$+(a''_{24})^{(4)}(T_{25},t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}\left((G_{27}),t\right) =$ **First detritions factor**

**Module Numbered Five**:

**The differential system of this model is now (Module number five)**

$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29},t)\right]G_{28}$ 25

$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29},t)\right]G_{29}$ 26

$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29},t)\right]G_{30}$ 27

$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31}),t)\right]T_{28}$ 28

$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}((G_{31}),t)\right]T_{29}$ 29

$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31}),t)\right]T_{30}$ 30

$+(a''_{28})^{(5)}(T_{29},t) =$ **First augmentation factor**

$-(b''_{28})^{(5)}\left((G_{31}),t\right) =$ **First detritions factor**

**Module Numbered Six**

**The differential system of this model is now (Module numbered Six)**

$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33},t)\right]G_{32}$ 31

$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33},t)\right]G_{33}$ 32

$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33},t)\right]G_{34}$ 33

$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35}),t)\right]T_{32}$ 34

$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}((G_{35}),t)\right]T_{33}$ 35

$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35}),t)\right]T_{34}$ 36

$+(a''_{32})^{(6)}(T_{33},t) =$ **First augmentation factor**

**Module Numbered Seven:**

**The differential system of this model is now (Seventh Module)**

$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right]G_{36}$ 37

$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37},t)\right]G_{37}$ *38*

$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37},t)\right]G_{38}$ *39*

$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39}),t)\right]T_{36}$ *40*

$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}((G_{39}),t)\right]T_{37}$ *41*

$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39}),t)\right]T_{38}$ *42*

$+(a''_{36})^{(7)}(T_{37},t) =$ **First augmentation factor**

**Module Numbered Eight**

The differential system of this model is now

$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right]G_{40}$ *43*

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41},t)\right]G_{41} \quad 44$$
$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41},t)\right]G_{42} \quad 45$$
$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}\left((G_{43}),t\right)\right]T_{40} \quad 46$$
$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}\left((G_{43}),t\right)\right]T_{41} \quad 47$$
$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}\left((G_{43}),t\right)\right]T_{42} \quad 48$$

**Module Numbered Nine**

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$
$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$
$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$
$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}\left((G_{47}),t\right)\right]T_{44} \quad 52$$
$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}\left((G_{47}),t\right)\right]T_{45} \quad 53$$
$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}\left((G_{47}),t\right)\right]T_{46} \quad 54$$

$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}\left((G_{47}),t\right) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)} & \boxed{+(a''_{13})^{(1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)} & \boxed{+(a''_{14})^{(1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)} & \boxed{+(a''_{15})^{(1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15} \quad 57$$

Where $\boxed{(a''_{13})^{(1)}(T_{14},t)}$, $\boxed{(a''_{14})^{(1)}(T_{14},t)}$, $\boxed{(a''_{15})^{(1)}(T_{14},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{38})^{(7,7)}(T_{37},t)$ $+(a''_{37})^{(7,7)}(T_{37},t)$ $+(a''_{36})^{(7,7)}(T_{37},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{40})^{(8,8)}(T_{41},t)$ $+(a''_{41})^{(8,8)}(T_{41},t)$ $+(a''_{42})^{(8,8)}(T_{41},t)$ are eight augmentation coefficient for 1,2,3

$+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt}=(b_{13})^{(1)}T_{14}-\begin{bmatrix}(b'_{13})^{(1)}\,-(b''_{13})^{(1)}(G,t) & -(b''_{16})^{(2,2,)}(G_{19},t) & -(b''_{20})^{(3,3,)}(G_{23},t)\\ -(b''_{24})^{(4,4,4,4,)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,)}(G_{35},t)\\ -(b''_{36})^{(7,7,)}(G_{39},t) & -(b''_{40})^{(8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)\end{bmatrix}T_{13} \quad (58)$$

$$\frac{dT_{14}}{dt}=(b_{14})^{(1)}T_{13}-\begin{bmatrix}(b'_{14})^{(1)}\,-(b''_{14})^{(1)}(G,t) & -(b''_{17})^{(2,2,)}(G_{19},t) & -(b''_{21})^{(3,3,)}(G_{23},t)\\ -(b''_{25})^{(4,4,4,4,)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,)}(G_{35},t)\\ -(b''_{37})^{(7,7,)}(G_{39},t) & -(b''_{41})^{(8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)\end{bmatrix}T_{14} \quad (59)$$

$$\frac{dT_{15}}{dt}=(b_{15})^{(1)}T_{14}-\begin{bmatrix}(b'_{15})^{(1)}\,-(b''_{15})^{(1)}(G,t) & -(b''_{18})^{(2,2,)}(G_{19},t) & -(b''_{22})^{(3,3,)}(G_{23},t)\\ -(b''_{26})^{(4,4,4,4,)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,)}(G_{35},t)\\ -(b''_{38})^{(7,7,)}(G_{39},t) & -(b''_{42})^{(8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)\end{bmatrix}T_{15} \quad (60)$$

Where $-(b''_{13})^{(1)}(G,t)$, $-(b''_{14})^{(1)}(G,t)$, $-(b''_{15})^{(1)}(G,t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,)}(G_{19},t)$, $-(b''_{17})^{(2,2,)}(G_{19},t)$, $-(b''_{18})^{(2,2,)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{37})^{(7,7,)}(G_{39},t)$, $-(b''_{36})^{(7,7,)}(G_{39},t)$, $-(b''_{38})^{(7,7,)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8)}(G_{43},t)$ $-(b''_{41})^{(8,8)}(G_{43},t)$ $-(b''_{42})^{(8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt}=(a_{16})^{(2)}G_{17}-\begin{bmatrix}(a'_{16})^{(2)} & +(a''_{16})^{(2)}(T_{17},t) & +(a''_{13})^{(1,1,)}(T_{14},t) & +(a''_{20})^{(3,3,3)}(T_{21},t)\\ +(a''_{24})^{(4,4,4,4,4)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,5)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,6)}(T_{33},t)\\ +(a''_{36})^{(7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9)}(T_{45},t)\end{bmatrix}G_{16} \quad (61)$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \begin{bmatrix} (a'_{17})^{(2)}\boxed{+(a''_{17})^{(2)}(T_{17},t)}\ \boxed{+(a''_{14})^{(1,1,)}(T_{14},t)}\ \boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)}\ \boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)}\ \boxed{+(a''_{45})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{17} \quad 62$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \begin{bmatrix} (a'_{18})^{(2)}\boxed{+(a''_{18})^{(2)}(T_{17},t)}\ \boxed{+(a''_{15})^{(1,1,)}(T_{14},t)}\ \boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)}\ \boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)}\ \boxed{+(a''_{46})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{18} \quad 63$$

Where $\boxed{+(a''_{16})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,)}(T_{14},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)}$ are eight augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{44})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9)}(T_{45},t)}$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \begin{bmatrix} (b'_{16})^{(2)}\boxed{-(b''_{16})^{(2)}(G_{19},t)}\ \boxed{-(b''_{13})^{(1,1,)}(G,t)}\ \boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)}\ \boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)}\ \boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)}\ \boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)}\ \boxed{-(b''_{44})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{16} \quad 64$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)}\boxed{-(b''_{17})^{(2)}(G_{19},t)}\ \boxed{-(b''_{14})^{(1,1,)}(G,t)}\ \boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)}\ \boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)}\ \boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)}\ \boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)}\ \boxed{-(b''_{45})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{17} \quad 65$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)}\boxed{-(b''_{18})^{(2)}(G_{19},t)}\ \boxed{-(b''_{15})^{(1,1,)}(G,t)}\ \boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)}\ \boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)}\ \boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)}\ \boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)}\ \boxed{-(b''_{46})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{18} \quad 66$$

where $\boxed{-(b''_{16})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2)}(G_{19},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,)}(G,t)}$ are second detrition coefficients for category

1,2 and 3

$\boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{44})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[\begin{array}{c} (a'_{20})^{(3)}\boxed{+(a''_{20})^{(3)}(T_{21},t)}\ \boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}\ \boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{44})^{(9,9,9)}(T_{45},t)} \end{array}\right] G_{20} \quad (67)$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[\begin{array}{c} (a'_{21})^{(3)}\boxed{+(a''_{21})^{(3)}(T_{21},t)}\ \boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}\ \boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)} \end{array}\right] G_{21} \quad (68)$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[\begin{array}{c} (a'_{22})^{(3)}\boxed{+(a''_{22})^{(3)}(T_{21},t)}\ \boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}\ \boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)} \end{array}\right] G_{22} \quad (69)$$

$\boxed{+(a''_{20})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3)}(T_{21},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}$ are second augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)}$ are third augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)}$ are seventh augmentation

coefficients for category 1, 2 and 3

$+(a''_{40})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8)}(T_{41},t)$ are eight augmentation coefficients for category 1, 2 and 3

$+(a''_{44})^{(9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b'_{20})^{(3)} & -(b''_{20})^{(3)}(G_{23},t) & -(b''_{16})^{(2,2,2)}(G_{19},t) & -(b''_{13})^{(1,1,1,)}(G,t) \\ -(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{36})^{(7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{20} \quad (70)$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b'_{21})^{(3)} & -(b''_{21})^{(3)}(G_{23},t) & -(b''_{17})^{(2,2,2)}(G_{19},t) & -(b''_{14})^{(1,1,1,)}(G,t) \\ -(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{37})^{(7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{21} \quad (71)$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)} & -(b''_{22})^{(3)}(G_{23},t) & -(b''_{18})^{(2,2,2)}(G_{19},t) & -(b''_{15})^{(1,1,1,)}(G,t) \\ -(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{38})^{(7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{22} \quad (72)$$

$-(b''_{20})^{(3)}(G_{23},t)$, $-(b''_{21})^{(3)}(G_{23},t)$, $-(b''_{22})^{(3)}(G_{23},t)$ *are first detrition coefficients for category 1, 2 and 3*

$-(b''_{16})^{(2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2)}(G_{19},t)$ *are second detrition coefficients for category 1, 2 and 3*

$-(b''_{13})^{(1,1,1,)}(G,t)$, $-(b''_{14})^{(1,1,1,)}(G,t)$, $-(b''_{15})^{(1,1,1,)}(G,t)$ *are third detrition coefficients for category 1,2 and 3*

$-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)$ *are fourth detrition coefficients for category 1, 2 and 3*

$-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)$ *are fifth detrition coefficients for category 1, 2 and 3*

$-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)$ *are sixth detrition coefficients for category 1, 2 and 3*

$-(b''_{36})^{(7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$-(b''_{46})^{(9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \begin{bmatrix} (a'_{24})^{(4)} & +(a''_{24})^{(4)}(T_{25},t) & +(a''_{28})^{(5,5,)}(T_{29},t) & +(a''_{32})^{(6,6,)}(T_{33},t) \\ +(a''_{13})^{(1,1,1,1)}(T_{14},t) & +(a''_{16})^{(2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3)}(T_{21},t) \\ +(a''_{36})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9)}(T_{45},t) \end{bmatrix} G_{24} \quad (73)$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \begin{bmatrix} (a'_{25})^{(4)}\boxed{+(a''_{25})^{(4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{25} \quad 74$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \begin{bmatrix} (a'_{26})^{(4)}\boxed{+(a''_{26})^{(4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{26} \quad 75$$

$\boxed{(a''_{24})^{(4)}(T_{25},t)}, \boxed{(a''_{25})^{(4)}(T_{25},t)}, \boxed{(a''_{26})^{(4)}(T_{25},t)}$ *are first augmentation coefficients category* $1, 2\ 3$

$\boxed{+(a''_{28})^{(5,5,)}(T_{29},t)}, \boxed{+(a''_{29})^{(5,5,)}(T_{29},t)}, \boxed{+(a''_{30})^{(5,5,)}(T_{29},t)}$ *are second augmentation coefficient for category* $1, 2$ *and* $3$

$\boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}, \boxed{+(a''_{33})^{(6,6,)}(T_{33},t)}, \boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}$ *are third augmentation coefficient for category* $1, 2$ *and* $3$

$\boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)}$
*are fourth augmentation coefficients for category* $1, 2$ *and* $3$

$\boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)}$
*are fifth augmentation coefficients for category* $1, 2$ *and* $3$

$\boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)}$
*are sixth augmentation coefficients for category* $1, 2$ *and* $3$

$\boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)}$
*are seventh augmentation coefficients for category* $1, 2$ *and* $3$

$\boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)}$
*are eighth augmentation coefficients for category* $1, 2$ *and* $3$

$\boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \begin{bmatrix} (b'_{24})^{(4)}\boxed{-(b''_{24})^{(4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{24} \quad 76$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \begin{bmatrix} (b'_{25})^{(4)}\boxed{-(b''_{25})^{(4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{25} \quad 77$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)}\boxed{-(b''_{26})^{(4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{26} \quad 78$$

*Where* $\boxed{-(b''_{24})^{(4)}(G_{27},t)}, \boxed{-(b''_{25})^{(4)}(G_{27},t)}, \boxed{-(b''_{26})^{(4)}(G_{27},t)}$ *are first detrition coefficients for category* $1, 2$ *and* $3$

$\boxed{-(b''_{28})^{(5,5,)}(G_{31},t)}, \boxed{-(b''_{29})^{(5,5,)}(G_{31},t)}, \boxed{-(b''_{30})^{(5,5,)}(G_{31},t)}$ *are second detrition coefficients for category* $1, 2$ *and* $3$

$\boxed{-(b''_{32})^{(6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,)}(G_{35},t)}$ *are third detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,1)}(G,t)}$
*are fourth detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2)}(G_{19},t)}$
*are fifth detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3)}(G_{23},t)}$
*are sixth detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)}$
*are seventh detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)}$
*are eighth detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{46})^{(9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \begin{bmatrix} (a'_{28})^{(5)} & \boxed{+(a''_{28})^{(5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{28} \quad 79$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \begin{bmatrix} (a'_{29})^{(5)} & \boxed{+(a''_{29})^{(5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{29} \quad 80$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \begin{bmatrix} (a'_{30})^{(5)} & \boxed{+(a''_{30})^{(5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{30} \quad 81$$

*Where* $\boxed{+(a''_{28})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5)}(T_{29},t)}$ *are first augmentation coefficients for category* 1, 2 *and* 3

*And* $\boxed{+(a''_{24})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,)}(T_{25},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)}$ are fourth augmentation coefficients for category 1,2, and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)}$ are fifth augmentation coefficients for category 1,2,and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)}$ are sixth augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)}$ are seventh augmentation coefficients for category 1,2, 3

$+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)$ , $+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)$ , $+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)$ are eighth augmentation coefficients for category 1,2, 3

$+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)$ , $+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)$ , $+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \begin{bmatrix} (b'_{28})^{(5)} & -(b''_{28})^{(5)}(G_{31},t) & -(b''_{24})^{(4,4,)}(G_{27},t) & -(b''_{32})^{(6,6,6)}(G_{35},t) \\ & -(b''_{13})^{(1,1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{28} \quad 82$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b'_{29})^{(5)} & -(b''_{29})^{(5)}(G_{31},t) & -(b''_{25})^{(4,4,)}(G_{27},t) & -(b''_{33})^{(6,6,6)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{29} \quad 83$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b'_{30})^{(5)} & -(b''_{30})^{(5)}(G_{31},t) & -(b''_{26})^{(4,4,)}(G_{27},t) & -(b''_{34})^{(6,6,6)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1,1,)}(G,t) & -(b''_{18})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{30} \quad 84$$

*where* $-(b''_{28})^{(5)}(G_{31},t)$ , $-(b''_{29})^{(5)}(G_{31},t)$ , $-(b''_{30})^{(5)}(G_{31},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{24})^{(4,4,)}(G_{27},t)$ , $-(b''_{25})^{(4,4,)}(G_{27},t)$ , $-(b''_{26})^{(4,4,)}(G_{27},t)$ *are second detrition coefficients for category* 1,2 *and* 3

$-(b''_{32})^{(6,6,6)}(G_{35},t)$ , $-(b''_{33})^{(6,6,6)}(G_{35},t)$ , $-(b''_{34})^{(6,6,6)}(G_{35},t)$ *are third detrition coefficients for category* 1,2 *and* 3

$-(b''_{13})^{(1,1,1,1,1)}(G,t)$ , $-(b''_{14})^{(1,1,1,1,1)}(G,t)$ , $-(b''_{15})^{(1,1,1,1,1,)}(G,t)$ are fourth detrition coefficients for category 1,2, and 3

$-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)$ , $-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)$ , $-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1,2, and 3

$-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)$ , $-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)$ , $-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1,2, and 3

$-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)$ , $-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)$ , $-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2, and 3

$-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)$ , $-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)$ , $-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1,2, and 3

$-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)$ , $-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)$ , $-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} \quad 85$$

$$- \begin{bmatrix} (a'_{32})^{(6)} & +(a''_{32})^{(6)}(T_{33},t) & +(a''_{28})^{(5,5,5)}(T_{29},t) & +(a''_{24})^{(4,4,4,)}(T_{25},t) \\ & +(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t) & +(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t) \\ & +(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{32}$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \begin{bmatrix} (a'_{33})^{(6)} & +(a''_{33})^{(6)}(T_{33},t) & +(a''_{29})^{(5,5,5)}(T_{29},t) & +(a''_{25})^{(4,4,4,)}(T_{25},t) \\ +(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t) & +(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t) \\ +(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{33} \quad 86$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \begin{bmatrix} (a'_{34})^{(6)} & +(a''_{34})^{(6)}(T_{33},t) & +(a''_{30})^{(5,5,5)}(T_{29},t) & +(a''_{26})^{(4,4,4,)}(T_{25},t) \\ +(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t) & +(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t) \\ +(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{34} \quad 87$$

$+(a''_{32})^{(6)}(T_{33},t)$, $+(a''_{33})^{(6)}(T_{33},t)$, $+(a''_{34})^{(6)}(T_{33},t)$ *are first augmentation coefficients for category* $1, 2$ *and* $3$

$+(a''_{28})^{(5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5)}(T_{29},t)$ *are second augmentation coefficients for category* $1, 2$ *and* $3$

$+(a''_{24})^{(4,4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,)}(T_{25},t)$ *are third augmentation coefficients for category* $1, 2$ *and* $3$

$+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)$ - are fourth augmentation coefficients

$+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)$ - fifth augmentation coefficients

$+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)$ sixth augmentation coefficients

$+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)$ seventh augmentation coefficients

$+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)$ Eighth augmentation coefficients

$+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \begin{bmatrix} (b'_{32})^{(6)} & -(b''_{32})^{(6)}(G_{35},t) & -(b''_{28})^{(5,5,5)}(G_{31},t) & -(b''_{24})^{(4,4,4,)}(G_{27},t) \\ -(b''_{13})^{(1,1,1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{32} \quad 88$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \begin{bmatrix} (b'_{33})^{(6)} & -(b''_{33})^{(6)}(G_{35},t) & -(b''_{29})^{(5,5,5)}(G_{31},t) & -(b''_{25})^{(4,4,4,)}(G_{27},t) \\ -(b''_{14})^{(1,1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{33} \quad 89$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)} & -(b''_{34})^{(6)}(G_{35},t) & -(b''_{30})^{(5,5,5)}(G_{31},t) & -(b''_{26})^{(4,4,4,)}(G_{27},t) \\ -(b''_{15})^{(1,1,1,1,1,1)}(G,t) & -(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{34} \quad 90$$

$-(b''_{32})^{(6)}(G_{35},t)$, $-(b''_{33})^{(6)}(G_{35},t)$, $-(b''_{34})^{(6)}(G_{35},t)$ *are first detrition coefficients for category* $1, 2$ *and* $3$

$\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}$ *are second detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)}$ *are third detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}$ are fourth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \qquad 91$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a'_{36})^{(7)}\boxed{+(a''_{36})^{(7)}(T_{37},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \qquad 92$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a'_{37})^{(7)}\boxed{+(a''_{37})^{(7)}(T_{37},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{38}}{dt} \qquad 93$$

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a'_{38})^{(7)}\boxed{+(a''_{38})^{(7)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{(a''_{36})^{(7)}(T_{37},t)}$, $\boxed{(a''_{37})^{(7)}(T_{37},t)}$, $\boxed{(a''_{38})^{(7)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)$ are seventh augmentation coefficient for category 1, 2 and 3

$+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)$ are eighth augmentation coefficient for 1,2,3

$+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{36}}{dt} =$$



$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b'_{36})^{(7)}\left[-(b''_{36})^{(7)}(G_{39},t)\right] & \left[-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)\right] & \left[-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{13}$$

$$\frac{dT_{37}}{dt} =$$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b'_{37})^{(7)}\left[-(b''_{37})^{(7)}(G_{39},t)\right] & \left[-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)\right] & \left[-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{14}$$

$$\frac{dT_{38}}{dt} =$$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b'_{38})^{(7)}\left[-(b''_{38})^{(7)}(G_{39},t)\right] & \left[-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)\right] & \left[-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{15}$$

Where $-(b''_{36})^{(7)}(G_{39},t)$, $-(b''_{37})^{(7)}(G_{39},t)$, $-(b''_{38})^{(7)}(G_{39},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)$, $-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)$

are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3



$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \begin{bmatrix} (a'_{40})^{(8)}\boxed{+(a''_{40})^{(8)}(T_{41},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \begin{bmatrix} (a'_{41})^{(8)}\boxed{+(a''_{41})^{(8)}(T_{41},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \begin{bmatrix} (a'_{42})^{(8)}\boxed{+(a''_{42})^{(8)}(T_{41},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a''_{40})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} =$$

$$(b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)}\left[-(b''_{40})^{(8)}(G_{43},t)\right] & \left[-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} =$$

$$(b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)}\left[-(b''_{41})^{(8)}(G_{43},t)\right] & \left[-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} =$$

$$(b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)}\left[-(b''_{42})^{(8)}(G_{43},t)\right] & \left[-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{15}$$

Where $-(b''_{36})^{(7)}(G_{39},t)$, $-(b''_{37})^{(7)}(G_{39},t)$, $-(b''_{38})^{(7)}(G_{39},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)$, $-(b''_{38})^{(7,7,)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)$ are eighth detrition coefficients for category 1, 2 and 3

$-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[\begin{array}{ccc} (a'_{44})^{(9)}\boxed{+(a''_{44})^{(9)}(T_{45},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{array}\right] G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[\begin{array}{ccc} (a'_{45})^{(9)}\boxed{+(a''_{45})^{(9)}(T_{45},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{array}\right] G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[\begin{array}{ccc} (a'_{46})^{(9)}\boxed{+(a''_{46})^{(9)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{array}\right] G_{15}$$

Where $\boxed{+(a''_{44})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ are Seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} -$$

$$\begin{bmatrix} (b'_{44})^{(9)}\boxed{-(b''_{44})^{(9)}(G_{47},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b'_{45})^{(9)}\boxed{-(b''_{45})^{(9)}(G_{47},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b'_{46})^{(9)}\boxed{-(b''_{46})^{(9)}(G_{47},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{44})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9)}(G_{47},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a'_i)^{(1)}, (a''_i)^{(1)}, (b_i)^{(1)}, (b'_i)^{(1)}, (b''_i)^{(1)} > 0, \quad i,j = 13,14,15 \qquad 97$$

The functions $(a''_i)^{(1)}, (b''_i)^{(1)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$(a_i'')^{(1)}(T_{14},t) \leq (p_i)^{(1)} \leq (\hat{A}_{13})^{(1)}$

$(b_i'')^{(1)}(G,t) \leq (r_i)^{(1)} \leq (b_i')^{(1)} \leq (\hat{B}_{13})^{(1)}$

$lim_{T_2\to\infty}(a_i'')^{(1)}(T_{14},t) = (p_i)^{(1)}$ 98

$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = (r_i)^{(1)}$

**Definition of** $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$ :

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$|(a_i'')^{(1)}(T_{14}',t) - (a_i'')^{(1)}(T_{14},t)| \leq (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$

$|(b_i'')^{(1)}(G',t) - (b_i'')^{(1)}(G,t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}',t)$ and $(a_i'')^{(1)}(T_{14},t)$ . $(T_{14}',t)$ and $(T_{14},t)$ are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14},t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$ : 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}$ , $\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$

**Definition of** $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$ : 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$

$\frac{1}{(\hat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$

Where we suppose

$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i,j = 16,17,18$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17},t) \le (p_i)^{(2)} \le \left(\hat{A}_{16}\right)^{(2)} \quad 102$$

$$(b_i'')^{(2)}(G_{19},t) \le (r_i)^{(2)} \le (b_i')^{(2)} \le (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2\to\infty}(a_i'')^{(2)}(T_{17},t) = (p_i)^{(2)} \quad 104$$

$$\lim_{G\to\infty}(b_i'')^{(2)}\left((G_{19}),t\right) = (r_i)^{(2)} \quad 105$$

**Definition of** $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$ : 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}',t) - (a_i'')^{(2)}(T_{17},t)| \le (\hat{k}_{16})^{(2)}|T_{17} - T_{17}'|e^{-(\hat{M}_{16})^{(2)}t} \quad 107$$

$$|(b_i'')^{(2)}((G_{19})',t) - (b_i'')^{(2)}\left((G_{19}),t\right)| < (\hat{k}_{16})^{(2)}||(G_{19}) - (G_{19})'||e^{-(\hat{M}_{16})^{(2)}t} \quad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}',t)$ and$(a_i'')^{(2)}(T_{17},t)$ . $(T_{17}',t)$ and $(T_{17},t)$ are points belonging to the interval $\left[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}\right]$ . It is to be noted that $(a_i'')^{(2)}(T_{17},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17},t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$ :

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants 109

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$$

**Definition of** $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$ :

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and\ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1 \quad 110$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(b_i)^{(2)} + (b_i')^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1 \quad 111$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i,j = 20,21,22 \quad 112$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(3)}$, $(\mathrm{r_i})^{(3)}$:

$$(a_i'')^{(3)}(T_{21},t) \le (p_i)^{(3)} \le (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23},t) \le (r_i)^{(3)} \le (b_i')^{(3)} \le (\hat{B}_{20})^{(3)}$$

$$lim_{T_2\to\infty}(a_i'')^{(3)}(T_{21},t) = (p_i)^{(3)} \qquad 113$$

$$\lim_{G\to\infty}(b_i'')^{(3)}(G_{23},t) = (r_i)^{(3)}$$

**Definition of** $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$ :

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: 114

$$|(a_i'')^{(3)}(T_{21}',t) - (a_i'')^{(3)}(T_{21},t)| \le (\hat{k}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\hat{M}_{20})^{(3)}t}$$

$$|(b_i'')^{(3)}(G_{23}',t) - (b_i'')^{(3)}(G_{23},t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}'||e^{-(\hat{M}_{20})^{(3)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}',t)$ and$(a_i'')^{(3)}(T_{21},t)$ . $(T_{21}',t)$ And $(T_{21},t)$ are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$ : 115

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$ and the constants $(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}, i = 20,21,22,$ satisfy the inequalities 116

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(a_i)^{(3)} + (a_i')^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(b_i)^{(3)} + (b_i')^{(3)} + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i,j = 24,25,26 \qquad 117$$

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(4)}$, $(r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25},t) \le (p_i)^{(4)} \le (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}\big((G_{27}),t\big) \le (r_i)^{(4)} \le (b_i')^{(4)} \le (\hat{B}_{24})^{(4)}$$

$$lim_{T_2\to\infty}(a_i'')^{(4)}(T_{25},t) = (p_i)^{(4)} \quad (118)$$

$$\lim_{G\to\infty}(b_i'')^{(4)}\big((G_{27}),t\big) = (r_i)^{(4)}$$

**Definition of** $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$ :

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24,25,26}$

They satisfy Lipschitz condition: (119)

$$|(a_i'')^{(4)}(T_{25}',t) - (a_i'')^{(4)}(T_{25},t)| \le (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})',t) - (b_i'')^{(4)}\big((G_{27}),t\big)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}',t)$ and$(a_i'')^{(4)}(T_{25},t)$ . $(T_{25}',t)$ and $(T_{25},t)$ are points belonging to the interval $\big[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}\big]$. It is to be noted that $(a_i'')^{(4)}(T_{25},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25},t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$ : (120)

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$, are positive constants

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}, \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$$

**Definition of** $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$ : (121)

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and\ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}, i = 24,25,26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(a_i)^{(4)} + (a_i')^{(4)} + (\hat{A}_{24})^{(4)} + (\hat{P}_{24})^{(4)}(\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(b_i)^{(4)} + (b_i')^{(4)} + (\hat{B}_{24})^{(4)} + (\hat{Q}_{24})^{(4)}(\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i,j = 28,29,30 \quad (122)$$

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(5)}$, $(r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29},t) \le (p_i)^{(5)} \le (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}\big((G_{31}),t\big) \le (r_i)^{(5)} \le (b_i')^{(5)} \le (\hat{B}_{28})^{(5)}$$

$$lim_{T_2\to\infty}(a_i'')^{(5)}(T_{29},t) = (p_i)^{(5)} \qquad 123$$

$$\lim_{G\to\infty}(b_i'')^{(5)}(G_{31},t) = (r_i)^{(5)}$$

**Definition of** $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$ :

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}',t) - (a_i'')^{(5)}(T_{29},t)| \le (\hat{k}_{28})^{(5)}|T_{29} - T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})',t) - (b_i'')^{(5)}\big((G_{31}),t\big)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}',t)$ and$(a_i'')^{(5)}(T_{29},t)$ . $(T_{29}',t)$ and $(T_{29},t)$ are points belonging to the interval $\big[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\big]$. It is to be noted that $(a_i'')^{(5)}(T_{29},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29},t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$ : 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}, \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

**Definition of** $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$ : 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and\ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(a_i)^{(5)} + (a_i')^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)}(\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(b_i)^{(5)} + (b_i')^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)}(\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i,j = 32,33,34 \qquad 127$$

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(6)}$, $(r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33},t) \le (p_i)^{(6)} \le (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}),t) \le (r_i)^{(6)} \le (b_i')^{(6)} \le (\hat{B}_{32})^{(6)}$$

$$lim_{T_2\to\infty}(a_i'')^{(6)}(T_{33},t) = (p_i)^{(6)} \quad (128)$$

$$\lim_{G\to\infty}(b_i'')^{(6)}\left((G_{35}),t\right) = (r_i)^{(6)}$$

**Definition of** $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$ :

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}',t) - (a_i'')^{(6)}(T_{33},t)| \le (\hat{k}_{32})^{(6)}|T_{33} - T_{33}'|e^{-(\hat{M}_{32})^{(6)}t}$$

$$|(b_i'')^{(6)}((G_{35})',t) - (b_i'')^{(6)}\left((G_{35}),t\right)| < (\hat{k}_{32})^{(6)}||(G_{35}) - (G_{35})'||e^{-(\hat{M}_{32})^{(6)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}',t)$ and$(a_i'')^{(6)}(T_{33},t)$ . $(T_{33}',t)$ and $(T_{33},t)$ are points belonging to the interval $\left[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}\right]$. It is to be noted that $(a_i'')^{(6)}(T_{33},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33},t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$ : (129)

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$, are positive constants

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$$

**Definition of** $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$ : (130)

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and\ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32,33,34,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(a_i)^{(6)} + (a_i')^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(b_i)^{(6)} + (b_i')^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

(AAAA) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i,j = 36,37,38$ (131)

(BBBB) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(7)}$, $(r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37},t) \le (p_i)^{(7)} \le (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39},t) \le (r_i)^{(7)} \le (b_i')^{(7)} \le (\hat{B}_{36})^{(7)} \quad (132)$$

*(CCCC)* $\lim_{T_2\to\infty} (a_i'')^{(7)}(T_{37},t) = (p_i)^{(7)}$

*(DDDD)*

$\lim_{G\to\infty}(b_i'')^{(7)}\big((G_{39}),t\big) = (r_i)^{(7)}$

**Definition of** $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$ :

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: (133)

$$|(a_i'')^{(7)}(T_{37}',t) - (a_i'')^{(7)}(T_{37},t)| \le (\hat{k}_{36})^{(7)}|T_{37} - T_{37}'|e^{-(\hat{M}_{36})^{(7)}t}$$

$$|(b_i'')^{(7)}((G_{39})',t) - (b_i'')^{(7)}\big((G_{39}),t\big)| < (\hat{k}_{36})^{(7)}||(G_{39}) - (G_{39})'||e^{-(\hat{M}_{36})^{(7)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}',t)$ and$(a_i'')^{(7)}(T_{37},t)$ . $(T_{37}',t)$ and $(T_{37},t)$ are points belonging to the interval $\big[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}\big]$. It is to be noted that $(a_i'')^{(7)}(T_{37},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37},t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$ : (134)

(EEEE) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$, are positive constants

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$$

**Definition of** $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$ : (135)

(FFFF) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)}(\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(b_i)^{(7)} + (b_i')^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)}(\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40,41,42 \tag{136}$$

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

**Definition of** $(p_i)^{(8)}$, $(r_i)^{(8)}$: (137)

$$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)} \tag{138}$$

$$(b_i'')^{(8)}((G_{43}), t) \leq (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)} \tag{139}$$

$$\lim_{T_2 \to \infty} (a_i'')^{(8)}(T_{41}, t) = (p_i)^{(8)} \tag{140}$$

$$\lim_{G \to \infty} (b_i'')^{(8)}\left((G_{43}), t\right) = (r_i)^{(8)} \tag{141}$$

**Definition of** $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$ :

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)} |T_{41} - T_{41}'| e^{-(\hat{M}_{40})^{(8)} t} \tag{142}$$

$$|(b_i'')^{(8)}((G_{43})', t) - (b_i'')^{(8)}\left((G_{43}), t\right)| < (\hat{k}_{40})^{(8)} ||(G_{43}) - (G_{43})'|| e^{-(\hat{M}_{40})^{(8)} t} \tag{143}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}', t)$ and $(a_i'')^{(8)}(T_{41}, t)$ . $(T_{41}', t)$ and $(T_{41}, t)$ are points belonging to the interval $\left[(\hat{k}_{40})^{(8)}, (\hat{M}_{40})^{(8)}\right]$. It is to be noted that $(a_i'')^{(8)}(T_{41}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{40})^{(8)} = 1$ then the function $(a_i'')^{(8)}(T_{41}, t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$ :

$(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$, are positive constants

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}, \frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1 \tag{144}$$

**Definition of** $(\hat{P}_{40})^{(8)}, (\hat{Q}_{40})^{(8)}$ :

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}, (\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)}, (a_i')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (p_i)^{(8)}$, $(r_i)^{(8)}, i = 40,41,42,$
Satisfy the inequalities

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(a_i)^{(8)} + (a_i')^{(8)} + (\hat{A}_{40})^{(8)} + (\hat{P}_{40})^{(8)} (\hat{k}_{40})^{(8)}] < 1 \tag{145}$$

$$\frac{1}{(\widehat{M}_{40})^{(8)}}[\ (b_i)^{(8)} + (b_i')^{(8)} + \ (\hat{B}_{40})^{(8)} + \ (\hat{Q}_{40})^{(8)}\ (\hat{k}_{40})^{(8)}] < 1 \qquad 146$$

Where we suppose

$$(a_i)^{(9)}, (a_i')^{(9)}, (a_i'')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (b_i'')^{(9)} > 0, \quad i,j = 44,45,46 \qquad 146\text{A}$$

The functions $(a_i'')^{(9)}, (b_i'')^{(9)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(9)}$, $(r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45}, t) \leq (p_i)^{(9)} \leq (\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47}, t) \leq \ (r_i)^{(9)} \leq (b_i')^{(9)} \leq (\hat{B}_{44})^{(9)}$$

$$\lim_{T_2 \to \infty} (a_i'')^{(9)}(T_{45}, t) = (p_i)^{(9)}$$

$$\lim_{G \to \infty} (b_i'')^{(9)}(G_{47}, t) = \ (r_i)^{(9)}$$

**Definition of** $(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}$ :

Where $\boxed{(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}, (p_i)^{(9)}, \ (r_i)^{(9)}}$ are positive constants and $\boxed{i = 44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}', t) - (a_i'')^{(9)}(T_{45}, t)| \leq (\hat{k}_{44})^{(9)} |T_{45} - T_{45}'| e^{-(\widehat{M}_{44})^{(9)} t}$$

$$|(b_i'')^{(9)}((G_{47})', t) - (b_i'')^{(9)}\big((G_{47}), t\big)| < (\hat{k}_{44})^{(9)} ||(G_{47}) - (G_{47})'|| e^{-(\widehat{M}_{44})^{(9)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}', t)$ and$(a_i'')^{(9)}(T_{45}, t)$ . $(T_{45}', t)$ and $(T_{45}, t)$ are points belonging to the interval $\left[(\hat{k}_{44})^{(9)}, (\widehat{M}_{44})^{(9)}\right]$. It is to be noted that $(a_i'')^{(9)}(T_{45}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{44})^{(9)} = 1$ then the function $(a_i'')^{(9)}(T_{45}, t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\widehat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$ :

$(\widehat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$, are positive constants

$$\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}} \ , \frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}} < 1$$

**Definition of** $(\hat{P}_{44})^{(9)}, (\hat{Q}_{44})^{(9)}$ :

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\widehat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}, (\hat{A}_{44})^{(9)} and\ (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)}, (a_i')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (p_i)^{(9)}, \ (r_i)^{(9)}, i = 44,45,46,$
satisfy the inequalities

$$\frac{1}{(\widehat{M}_{44})^{(9)}}[(a_i)^{(9)} + (a_i')^{(9)} + (\hat{A}_{44})^{(9)} + (\hat{P}_{44})^{(9)}(\hat{k}_{44})^{(9)}] < 1$$

$$\frac{1}{(\widehat{M}_{44})^{(9)}}[(b_i)^{(9)} + (b_i')^{(9)} + (\hat{B}_{44})^{(9)} + (\hat{Q}_{44})^{(9)}(\hat{k}_{44})^{(9)}] < 1$$

**Theorem 1:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0), T_i(0)$ :

$$G_i(t) \le (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 2 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

**Definition of** $G_i(0), T_i(0)$

$$G_i(t) \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} \quad , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} \quad , \quad T_i(0) = T_i^0 > 0$$

**Theorem 3 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t) \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t} \quad , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t} \quad , \quad T_i(0) = T_i^0 > 0$$

**Theorem 4 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

**Definition of** $G_i(0), T_i(0)$ :

$$G_i(t) \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 5 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

**Definition of** $G_i(0), T_i(0)$ :

$$G_i(t) \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 6 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

**Definition of** $G_i(0), T_i(0)$ :

$$G_i(t) \le (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$T_i(t) \leq (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Theorem 7:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

**Definition of** $G_i(0), T_i(0)$ :

$G_i(t) \leq (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Theorem 8:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

**Definition of** $G_i(0), T_i(0)$ :

$G_i(t) \leq (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Theorem 9:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

**Definition of** $G_i(0), T_i(0)$ :

$G_i(t) \leq (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Proof:** Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$G_i(0) = G_i^0$ , $T_i(0) = T_i^0$ , $G_i^0 \leq (\hat{P}_{13})^{(1)}$ , $T_i^0 \leq (\hat{Q}_{13})^{(1)}$, 155

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t}$ 156

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t}$ 157

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[ (a_{13})^{(1)} G_{14}(s_{(13)}) - \left( (a'_{13})^{(1)} + a''_{13})^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \right) G_{13}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[ (a_{14})^{(1)} G_{13}(s_{(13)}) - \left( (a'_{14})^{(1)} + (a''_{14})^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \right) G_{14}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[ (a_{15})^{(1)} G_{14}(s_{(13)}) - \left( (a'_{15})^{(1)} + (a''_{15})^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \right) G_{15}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[ (b_{13})^{(1)} T_{14}(s_{(13)}) - \left( (b'_{13})^{(1)} - (b''_{13})^{(1)} (G(s_{(13)}), s_{(13)}) \right) T_{13}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^{0} + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)}\left(G(s_{(13)}), s_{(13)}\right)\right) T_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{\mathrm{T}}_{15}(\mathrm{t}) = \mathrm{T}_{15}^{0} + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)}\left(G(s_{(13)}), s_{(13)}\right)\right) T_{15}(s_{(13)})\right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{16})^{(2)}, T_i^0 \le (\hat{Q}_{16})^{(2)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$$

By 160

$$\bar{G}_{16}(t) = G_{16}^{0} + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''_{16})^{(2)}\left(T_{17}(s_{(16)}), s_{(16)}\right)\right) G_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^{0} + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)}\left(T_{17}(s_{(16)}), s_{(17)}\right)\right) G_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^{0} + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)}\left(T_{17}(s_{(16)}), s_{(16)}\right)\right) G_{18}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^{0} + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)}\left(G_{19}(s_{(16)}), s_{(16)}\right)\right) T_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^{0} + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)}\left(G_{19}(s_{(16)}), s_{(16)}\right)\right) T_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^{0} + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)}\left(G_{19}(s_{(16)}), s_{(16)}\right)\right) T_{18}(s_{(16)})\right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{20})^{(3)}, T_i^0 \le (\hat{Q}_{20})^{(3)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$$

By 161

$$\bar{G}_{20}(t) = G_{20}^{0} + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''_{20})^{(3)}\left(T_{21}(s_{(20)}), s_{(20)}\right)\right) G_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{22}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^0 + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^0 + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{\mathrm{T}}_{22}(\mathrm{t}) = \mathrm{T}_{22}^0 + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{22}(s_{(20)})\right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{24})^{(4)}, T_i^0 \le (\hat{Q}_{24})^{(4)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

By 162

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{26}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{\mathrm{T}}_{26}(\mathrm{t}) = \mathrm{T}_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{26}(s_{(24)})\right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{28})^{(5)}, T_i^0 \le (\hat{Q}_{28})^{(5)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

By 163

$$\bar{G}_{28}(t) = G_{28}^{0} + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^{0} + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^{0} + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{30}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^{0} + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^{0} + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{\mathrm{T}}_{30}(\mathrm{t}) = \mathrm{T}_{30}^{0} + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{30}(s_{(28)})\right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{32})^{(6)}, T_i^0 \le (\hat{Q}_{32})^{(6)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$$

By 164

$$\bar{G}_{32}(t) = G_{32}^{0} + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a'_{32})^{(6)} + a''_{32})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^{0} + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^{0} + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{34}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^{0} + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^{0} + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{\mathrm{T}}_{34}(\mathrm{t}) = \mathrm{T}_{34}^{0} + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{34}(s_{(32)})\right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$ , $T_i(0) = T_i^0$ , $G_i^0 \leq (\hat{P}_{36})^{(7)}$ , $T_i^0 \leq (\hat{Q}_{36})^{(7)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[ (a_{36})^{(7)} G_{37}(s_{(36)}) - \left( (a'_{36})^{(7)} + a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[ (a_{37})^{(7)} G_{36}(s_{(36)}) - \left( (a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[ (a_{38})^{(7)} G_{37}(s_{(36)}) - \left( (a'_{38})^{(7)} + (a''_{38})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{38}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[ (b_{36})^{(7)} T_{37}(s_{(36)}) - \left( (b'_{36})^{(7)} - (b''_{36})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[ (b_{37})^{(7)} T_{36}(s_{(36)}) - \left( (b'_{37})^{(7)} - (b''_{37})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{38}(t) = T_{38}^0 + \int_0^t \left[ (b_{38})^{(7)} T_{37}(s_{(36)}) - \left( (b'_{38})^{(7)} - (b''_{38})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{38}(s_{(36)}) \right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i$: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$ , $T_i(0) = T_i^0$ , $G_i^0 \leq (\hat{P}_{40})^{(8)}$ , $T_i^0 \leq (\hat{Q}_{40})^{(8)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{40})^{(8)} e^{(\bar{M}_{40})^{(8)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{40})^{(8)} e^{(\bar{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[ (a_{40})^{(8)} G_{41}(s_{(40)}) - \left( (a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[ (a_{41})^{(8)} G_{40}(s_{(40)}) - \left( (a'_{41})^{(8)} + (a''_{41})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}(s_{(40)}), s_{(40)})\right) G_{42}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^0 + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43}(s_{(40)}), s_{(40)})\right) T_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^0 + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43}(s_{(40)}), s_{(40)})\right) T_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{\mathrm{T}}_{42}(\mathrm{t}) = \mathrm{T}_{42}^0 + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43}(s_{(40)}), s_{(40)})\right) T_{42}(s_{(40)})\right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** 166

Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy A

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{44})^{(9)},\ T_i^0 \le (\hat{Q}_{44})^{(9)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''_{44})^{(9)}(T_{45}(s_{(44)}), s_{(44)})\right) G_{44}(s_{(44)})\right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45}(s_{(44)}), s_{(44)})\right) G_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}(s_{(44)}), s_{(44)})\right) G_{46}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{44}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{\mathrm{T}}_{46}(\mathrm{t}) = \mathrm{T}_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{46}(s_{(44)})\right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}}\right)\right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left(e^{(\hat{M}_{13})^{(1)} t} - 1\right)$$

From which it follows that 168

$$(G_{13}(t)-G_{13}^{0})e^{-(\hat{M}_{13})^{(1)}t}\leq\frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}}\left[((\hat{P}_{13})^{(1)}+G_{14}^{0})e^{\left(-\frac{(\hat{P}_{13})^{(1)}+G_{14}^{0}}{G_{14}^{0}}\right)}+(\hat{P}_{13})^{(1)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t)\leq G_{16}^{0}+\int_{0}^{t}\left[(a_{16})^{(2)}\left(G_{17}^{0}+(\hat{P}_{16})^{(6)}e^{(\hat{M}_{16})^{(2)}s_{(16)}}\right)\right]ds_{(16)}= \quad (169)$$

$$\left(1+(a_{16})^{(2)}t\right)G_{17}^{0}+\frac{(a_{16})^{(2)}(\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}}\left(e^{(\hat{M}_{16})^{(2)}t}-1\right)$$

From which it follows that (170)

$$(G_{16}(t)-G_{16}^{0})e^{-(\hat{M}_{16})^{(2)}t}\leq\frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}}\left[((\hat{P}_{16})^{(2)}+G_{17}^{0})e^{\left(-\frac{(\hat{P}_{16})^{(2)}+G_{17}^{0}}{G_{17}^{0}}\right)}+(\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that (171)

$$G_{20}(t)\leq G_{20}^{0}+\int_{0}^{t}\left[(a_{20})^{(3)}\left(G_{21}^{0}+(\hat{P}_{20})^{(3)}e^{(\hat{M}_{20})^{(3)}s_{(20)}}\right)\right]ds_{(20)}=$$

$$\left(1+(a_{20})^{(3)}t\right)G_{21}^{0}+\frac{(a_{20})^{(3)}(\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left(e^{(\hat{M}_{20})^{(3)}t}-1\right)$$

From which it follows that (172)

$$(G_{20}(t)-G_{20}^{0})e^{-(\hat{M}_{20})^{(3)}t}\leq\frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left[((\hat{P}_{20})^{(3)}+G_{21}^{0})e^{\left(-\frac{(\hat{P}_{20})^{(3)}+G_{21}^{0}}{G_{21}^{0}}\right)}+(\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that (173)

$$G_{24}(t)\leq G_{24}^{0}+\int_{0}^{t}\left[(a_{24})^{(4)}\left(G_{25}^{0}+(\hat{P}_{24})^{(4)}e^{(\hat{M}_{24})^{(4)}s_{(24)}}\right)\right]ds_{(24)}=$$

$$\left(1+(a_{24})^{(4)}t\right)G_{25}^{0}+\frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left(e^{(\hat{M}_{24})^{(4)}t}-1\right)$$

From which it follows that (174)

$$(G_{24}(t)-G_{24}^{0})e^{-(\hat{M}_{24})^{(4)}t}\leq\frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left[((\hat{P}_{24})^{(4)}+G_{25}^{0})e^{\left(-\frac{(\hat{P}_{24})^{(4)}+G_{25}^{0}}{G_{25}^{0}}\right)}+(\hat{P}_{24})^{(4)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t) \le G_{28}^0 + \int_0^t \left[ (a_{28})^{(5)} \left( G_{29}^0 + (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} s_{(28)}} \right) \right] ds_{(28)} =$$

$$\left(1 + (a_{28})^{(5)} t\right) G_{29}^0 + \frac{(a_{28})^{(5)} (\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}} \left( e^{(\hat{M}_{28})^{(5)} t} - 1 \right)$$

From which it follows that 175

$$(G_{28}(t) - G_{28}^0) e^{-(\hat{M}_{28})^{(5)} t} \le \frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}} \left[ \left( (\hat{P}_{28})^{(5)} + G_{29}^0 \right) e^{\left( - \frac{(\hat{P}_{28})^{(5)} + G_{29}^0}{G_{29}^0} \right)} + (\hat{P}_{28})^{(5)} \right]$$

$(G_i^0)$ is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t) \le G_{32}^0 + \int_0^t \left[ (a_{32})^{(6)} \left( G_{33}^0 + (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} s_{(32)}} \right) \right] ds_{(32)} =$$

$$\left(1 + (a_{32})^{(6)} t\right) G_{33}^0 + \frac{(a_{32})^{(6)} (\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}} \left( e^{(\hat{M}_{32})^{(6)} t} - 1 \right)$$

From which it follows that 177

$$(G_{32}(t) - G_{32}^0) e^{-(\hat{M}_{32})^{(6)} t} \le \frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}} \left[ \left( (\hat{P}_{32})^{(6)} + G_{33}^0 \right) e^{\left( - \frac{(\hat{P}_{32})^{(6)} + G_{33}^0}{G_{33}^0} \right)} + (\hat{P}_{32})^{(6)} \right]$$

$(G_i^0)$ is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

(n) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t) \le G_{36}^0 + \int_0^t \left[ (a_{36})^{(7)} \left( G_{37}^0 + (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} s_{(36)}} \right) \right] ds_{(36)} =$$
$$\left(1 + (a_{36})^{(7)} t\right) G_{37}^0 + \frac{(a_{36})^{(7)} (\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left( e^{(\hat{M}_{36})^{(7)} t} - 1 \right)$$

From which it follows that

$$(G_{36}(t) - G_{36}^0) e^{-(\hat{M}_{36})^{(7)} t} \le \frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left[ \left( (\hat{P}_{36})^{(7)} + G_{37}^0 \right) e^{\left( - \frac{(\hat{P}_{36})^{(7)} + G_{37}^0}{G_{37}^0} \right)} + (\hat{P}_{36})^{(7)} \right]$$

$(G_i^0)$ is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t) \le G_{40}^0 + \int_0^t \left[ (a_{40})^{(8)} \left( G_{41}^0 + (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} s_{(40)}} \right) \right] ds_{(40)} = \quad 180$$
$$\left(1 + (a_{40})^{(8)} t\right) G_{41}^0 + \frac{(a_{40})^{(8)} (\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}} \left( e^{(\hat{M}_{40})^{(8)} t} - 1 \right)$$

From which it follows that 181

$$(G_{40}(t)-G_{40}^{0})e^{-(\hat{M}_{40})^{(8)}t}\leq\frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left[((\hat{P}_{40})^{(8)}+G_{41}^{0})e^{\left(-\frac{(\hat{P}_{40})^{(8)}+G_{41}^{0}}{G_{41}^{0}}\right)}+(\hat{P}_{40})^{(8)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 8

Analogous inequalities hold also for $G_{41}, G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t)\leq G_{44}^{0}+\int_{0}^{t}\left[(a_{44})^{(9)}\left(G_{45}^{0}+(\hat{P}_{44})^{(9)}e^{(\hat{M}_{44})^{(9)}s_{(44)}}\right)\right]ds_{(44)}=$$

$$\left(1+(a_{44})^{(9)}t\right)G_{45}^{0}+\frac{(a_{44})^{(9)}(\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}}\left(e^{(\hat{M}_{44})^{(9)}t}-1\right)$$

From which it follows that

$$(G_{44}(t)-G_{44}^{0})e^{-(\hat{M}_{44})^{(9)}t}\leq\frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}}\left[((\hat{P}_{44})^{(9)}+G_{45}^{0})e^{\left(-\frac{(\hat{P}_{44})^{(9)}+G_{45}^{0}}{G_{45}^{0}}\right)}+(\hat{P}_{44})^{(9)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 9

Analogous inequalities hold also for $G_{45}, G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}<1$ and to choose 182

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[(\hat{P}_{13})^{(1)}+((\hat{P}_{13})^{(1)}+G_j^0)e^{-\left(\frac{(\hat{P}_{13})^{(1)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{13})^{(1)} \quad 183$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[((\hat{Q}_{13})^{(1)}+T_j^0)e^{-\left(\frac{(\hat{Q}_{13})^{(1)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{13})^{(1)}\right]\leq(\hat{Q}_{13})^{(1)} \quad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left((G^{(1)},T^{(1)}),(G^{(2)},T^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\hat{M}_{13})^{(1)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\hat{M}_{13})^{(1)}t}\}$$

Indeed if we denote

**Definition of** $\tilde{G},\tilde{T}:(\tilde{G},\tilde{T})=\mathcal{A}^{(1)}(G,T)$

It results

$$\left|\tilde{G}_{13}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{13})^{(1)} \left|G_{14}^{(1)}-G_{14}^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)} s_{(13)}} e^{(\widehat{M}_{13})^{(1)} s_{(13)}}\, ds_{(13)} +$$

$$\int_0^t \{(a'_{13})^{(1)} \left|G_{13}^{(1)}-G_{13}^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)} s_{(13)}} e^{-(\widehat{M}_{13})^{(1)} s_{(13)}} +$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)}, s_{(13)}\right)\left|G_{13}^{(1)}-G_{13}^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)} s_{(13)}} e^{(\widehat{M}_{13})^{(1)} s_{(13)}} +$$

$$G_{13}^{(2)} \left|(a''_{13})^{(1)}\left(T_{14}^{(1)}, s_{(13)}\right)-(a''_{13})^{(1)}\left(T_{14}^{(2)}, s_{(13)}\right)\right|\ e^{-(\widehat{M}_{13})^{(1)} s_{(13)}} e^{(\widehat{M}_{13})^{(1)} s_{(13)}}\} ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\left|G^{(1)}-G^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)} t} \leq \qquad (186)$$
$$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)}+\ (a'_{13})^{(1)}+(\widehat{A}_{13})^{(1)}+(\widehat{P}_{13})^{(1)}(\widehat{k}_{13})^{(1)}\right) d\left(\left(G^{(1)}, T^{(1)};\ G^{(2)}, T^{(2)}\right)\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 1:** The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)} t}$ *and* $(\widehat{Q}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)} t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i = 13{,}14{,}15$ depend only on $T_{14}$ and respectively on $G(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any $t$ where $G_i(t) = 0$ *and* $T_i(t) = 0$

From 19 to 24 it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(1)} - (a_i'')^{(1)}(T_{14}(s_{(13)}), s_{(13)})\} ds_{(13)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(1)} t)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2$ *and* $\left((\widehat{M}_{13})^{(1)}\right)_3$ : (187)

**Remark 3**: if $G_{13}$ is bounded, the same property have also $G_{14}$ *and* $G_{15}$ . indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a'_{14})^{(1)} G_{14}$ and by integrating

$$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1 / (a'_{14})^{(1)}$$

In the same way , one can obtain

$$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2 / (a'_{15})^{(1)}$$

If $G_{14}$ *or* $G_{15}$ is bounded, the same property follows for $G_{13}$ , $G_{15}$ and $G_{13}$ , $G_{14}$ respectively.

**Remark 4:** If $G_{13}$ *is* bounded, from below, the same property holds for $G_{14}$ *and* $G_{15}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{14}$ is bounded from below. 188

**Remark 5:** If $T_{13}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}(G(t),t)) = (b_{14}')^{(1)}$ then $T_{14} \to \infty$. 189

**Definition of** $(m)^{(1)}$ and $\varepsilon_1$ :

Indeed let $t_1$ be so that for $t > t_1$

$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t),t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right)$, $t = log\frac{2}{\varepsilon_1}$ By taking now $\varepsilon_1$ sufficiently small one sees that $T_{14}$ is unbounded.
The same property holds for $T_{15}$ if $\lim_{t\to\infty}(b_{15}'')^{(1)}(G(t),t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\tilde{M}_{16})^{(2)}}$ , $\frac{(b_i)^{(2)}}{(\tilde{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}$ *and* $(\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\tilde{M}_{16})^{(2)}}\left[(\hat{P}_{16})^{(2)} + \left((\hat{P}_{16})^{(2)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\tilde{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{16})^{(2)}\right] \leq (\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\tilde{M}_{16})^{(2)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\tilde{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

**Definition of** $\widetilde{G_{19}}, \widetilde{T_{19}}$ : $\left(\widetilde{G_{19}}, \widetilde{T_{19}}\right) = \mathcal{A}^{(2)}(G_{19}, T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{16})^{(2)} \left|G_{17}^{(1)}-G_{17}^{(2)}\right| e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\, ds_{(16)} +$$

$$\int_0^t \{(a'_{16})^{(2)} \left|G_{16}^{(1)}-G_{16}^{(2)}\right| e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} +$$

$$(a''_{16})^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right)\left|G_{16}^{(1)}-G_{16}^{(2)}\right| e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} e^{(\widehat{M}_{16})^{(2)}s_{(16)}} +$$

$$G_{16}^{(2)} \left|(a''_{16})^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right) - (a''_{16})^{(2)}\left(T_{17}^{(2)}, s_{(16)}\right)\right|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\} ds_{(16)}$$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0, t]$ 197

From the hypotheses it follows

$$\left|(G_{19})^{(1)}-(G_{19})^{(2)}\right| e^{-(\widehat{M}_{16})^{(2)}t} \leq$$
$$\frac{1}{(\widehat{M}_{16})^{(2)}}\left((a_{16})^{(2)} + (a'_{16})^{(2)} + (\widehat{A}_{16})^{(2)} + (\widehat{P}_{16})^{(2)}(\widehat{k}_{16})^{(2)}\right) d\left(((G_{19})^{(1)}, (T_{19})^{(1)};\ (G_{19})^{(2)}, (T_{19})^{(2)})\right)$$

And analogous inequalities for $G_i$ and $T_i$. Taking into account the hypothesis the result follows 198

**Remark 6:** The fact that we supposed $(a''_{16})^{(2)}$ and $(b''_{16})^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{16})^{(2)} e^{(\widehat{M}_{16})^{(2)}t}$ and $(\widehat{Q}_{16})^{(2)} e^{(\widehat{M}_{16})^{(2)}t}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i = 16,17,18$ depend only on $T_{17}$ and respectively on $(G_{19})$(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 7:** There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$ 200

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(2)} - (a_i'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})\} ds_{(16)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(2)}t)} > 0 \quad \text{for } t > 0$$

**Definition of** $\left((\widehat{M}_{16})^{(2)}\right)_1$, $\left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$: 201

**Remark 8:** if $G_{16}$ is bounded, the same property have also $G_{17}$ and $G_{18}$ . indeed if

$G_{16} < (\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt} \leq \left((\widehat{M}_{16})^{(2)}\right)_1 - (a'_{17})^{(2)} G_{17}$ and by integrating

$$G_{17} \leq \left((\widehat{M}_{16})^{(2)}\right)_2 = G_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1 / (a'_{17})^{(2)}$$

In the same way , one can obtain

$$G_{18} \leq \left((\widehat{M}_{16})^{(2)}\right)_3 = G_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2 / (a'_{18})^{(2)}$$

If $G_{17}$ or $G_{18}$ is bounded, the same property follows for $G_{16}$, $G_{18}$ and $G_{16}$, $G_{17}$ respectively.

**Remark 9**: If $G_{16}$ is bounded, from below, the same property holds for $G_{17}$ and $G_{18}$. The proof is analogous with the preceding one. An analogous property is true if $G_{17}$ is bounded from below. 202

**Remark 10:** If $T_{16}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t),t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$. 203

**Definition of** $(m)^{(2)}$ and $\varepsilon_2$ :

Indeed let $t_2$ be so that for $t > t_2$

$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t),t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right), \quad t = \log\frac{2}{\varepsilon_2}$ By taking now $\varepsilon_2$ sufficiently small one sees that $T_{17}$ is unbounded. 205
The same property holds for $T_{18}$ if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t),t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\bar{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\bar{M}_{20})^{(3)}} < 1$ and to choose 207

$(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\bar{M}_{20})^{(3)}}\left[(\hat{P}_{20})^{(3)} + \left((\hat{P}_{20})^{(3)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\bar{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{20})^{(3)}\right] \leq (\hat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself 210

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)},(T_{23})^{(1)}),((G_{23})^{(2)},(T_{23})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\bar{M}_{20})^{(3)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\bar{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

**Definition of** $\widetilde{G_{23}}, \widetilde{T_{23}}$ :$\left(\widetilde{(G_{23})},\widetilde{(T_{23})}\right) = \mathcal{A}^{(3)}\left((G_{23}),(T_{23})\right)$

It results 213

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{20})^{(3)} \left|G_{21}^{(1)}-G_{21}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} e^{(\widehat{M}_{20})^{(3)} s_{(20)}}\, ds_{(20)} +$$

$$\int_0^t \{(a'_{20})^{(3)} \left|G_{20}^{(1)}-G_{20}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} +$$

$$(a''_{20})^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} e^{(\widehat{M}_{20})^{(3)} s_{(20)}} +$$

$$G_{20}^{(2)} \left|(a''_{20})^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)}, s_{(20)}\right)\right|\ e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} e^{(\widehat{M}_{20})^{(3)} s_{(20)}}\} ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)} t} \leq \qquad 214$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+\ (a'_{20})^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\widehat{k}_{20})^{(3)}\right) d\left(\left((G_{23})^{(1)}, (T_{23})^{(1)};\ (G_{23})^{(2)}, (T_{23})^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

**Remark 11:** The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)} e^{(\widehat{M}_{20})^{(3)} t}\ and\ (\widehat{Q}_{20})^{(3)} e^{(\widehat{M}_{20})^{(3)} t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i = 20,21,22$ depend only on $T_{21}$ and respectively on $(G_{23})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 12:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 216

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(3)} - (a_i'')^{(3)}(T_{21}(s_{(20)}), s_{(20)})\} ds_{(20)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(3)} t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2\ and\ \left((\widehat{M}_{20})^{(3)}\right)_3$ : 217

**Remark 13:** if $G_{20}$ is bounded, the same property have also $G_{21}\ and\ G_{22}$ . indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a'_{21})^{(3)} G_{21}$ and by integrating

$$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1 / (a'_{21})^{(3)}$$

In the same way , one can obtain

$$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2 / (a'_{22})^{(3)}$$

If $G_{21}$ *or* $G_{22}$ is bounded, the same property follows for $G_{20}$ , $G_{22}$ and $G_{20}$ , $G_{21}$ respectively.

**Remark 14:** If $G_{20}$ *is* bounded, from below, the same property holds for $G_{21}$ *and* $G_{22}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{21}$ is bounded from below. 218

**Remark 15:** If $T_{20}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t),t\right)=(b_{21}')^{(3)}$ then $T_{21}\to\infty$. 219

**Definition of** $(m)^{(3)}$ and $\varepsilon_3$ :

Indeed let $t_3$ be so that for $t > t_3$

$(b_{21})^{(3)}-(b_i'')^{(3)}\left((G_{23})(t),t\right)<\varepsilon_3, T_{20}(t)>(m)^{(3)}$

Then $\frac{dT_{21}}{dt}\geq(a_{21})^{(3)}(m)^{(3)}-\varepsilon_3 T_{21}$ which leads to 220

$T_{21}\geq\left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1-e^{-\varepsilon_3 t})+T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t}=\frac{1}{2}$ it results

$T_{21}\geq\left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right),\quad t=log\frac{2}{\varepsilon_3}$ By taking now $\varepsilon_3$ sufficiently small one sees that $T_{21}$ is unbounded.
The same property holds for $T_{22}$ if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t),t\right)=(b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}$ , $\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}}<1$ and to choose 221

$(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\left[(\hat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \quad 222$$

$$\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \quad 223$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself 224

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric 225

$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$

$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{24})^{(4)}t}\}$

Indeed if we denote

**Definition of** $\widetilde{(G_{27})},\widetilde{(T_{27})}$ : $\left(\widetilde{(G_{27})},\widetilde{(T_{27})}\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{24})^{(4)} \left|G_{25}^{(1)}-G_{25}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)}s_{(24)}} e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\, ds_{(24)} +$$

$$\int_0^t \{(a'_{24})^{(4)}\left|G_{24}^{(1)}-G_{24}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)}s_{(24)}} e^{-(\widehat{M}_{24})^{(4)}s_{(24)}} +$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)}, s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)}s_{(24)}} e^{(\widehat{M}_{24})^{(4)}s_{(24)}} +$$

$$G_{24}^{(2)}\left|(a''_{24})^{(4)}\left(T_{25}^{(1)}, s_{(24)}\right)-(a''_{24})^{(4)}\left(T_{25}^{(2)}, s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)}s_{(24)}} e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\} ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)}t} \leq \qquad (226)$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)}+\ (a'_{24})^{(4)}+(\widehat{A}_{24})^{(4)}+(\widehat{P}_{24})^{(4)}(\widehat{k}_{24})^{(4)}\right) d\left(\left((G_{27})^{(1)}, (T_{27})^{(1)};\ (G_{27})^{(2)}, (T_{27})^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

**Remark 16:** The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}\ and\ (\widehat{Q}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_+$. (227)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i = 24,25,26$ depend only on $T_{25}$ and respectively on $(G_{27})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 17:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ (228)

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(4)}-(a_i'')^{(4)}(T_{25}(s_{(24)}),s_{(24)})\}ds_{(24)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(4)}t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2\ and\ \left((\widehat{M}_{24})^{(4)}\right)_3$ : (229)

**Remark 18:** if $G_{24}$ is bounded, the same property have also $G_{25}\ and\ G_{26}$ . indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a'_{25})^{(4)}G_{25}$ and by integrating

$$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1/(a'_{25})^{(4)}$$

In the same way , one can obtain

$$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2/(a'_{26})^{(4)}$$

If $G_{25}$ *or* $G_{26}$ is bounded, the same property follows for $G_{24}$ , $G_{26}$ and $G_{24}$ , $G_{25}$ respectively.

**Remark 19:** If $G_{24}$ *is* bounded, from below, the same property holds for $G_{25}$ *and* $G_{26}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{25}$ is bounded from below. 230

**Remark 20:** If $T_{24}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}((G_{27})(t),t)) = (b_{25}')^{(4)}$ then $T_{25} \to \infty$. 231

**Definition of** $(m)^{(4)}$ and $\varepsilon_4$ :

Indeed let $t_4$ be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t),t) < \varepsilon_4, T_{24}(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right), \quad t = log\frac{2}{\varepsilon_4}$ By taking now $\varepsilon_4$ sufficiently small one sees that $T_{25}$ is unbounded.
The same property holds for $T_{26}$ if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t),t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for $G_{29}, G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}$ , $\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)} + \left((\hat{P}_{28})^{(5)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{28})^{(5)}\right] \leq (\hat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$$d\left(((G_{31})^{(1)},(T_{31})^{(1)}),((G_{31})^{(2)},(T_{31})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

**Definition of** $\widetilde{(G_{31})}, \widetilde{(T_{31})}$ : $\left(\widetilde{(G_{31})}, \widetilde{(T_{31})}\right) = \mathcal{A}^{(5)}\left((G_{31}), (T_{31})\right)$

It results

$\left|\tilde{G}_{28}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{28})^{(5)} \left|G_{29}^{(1)} - G_{29}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}}\, ds_{(28)} +$

$\int_0^t \{(a'_{28})^{(5)} \left|G_{28}^{(1)} - G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} +$

$(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right)\left|G_{28}^{(1)} - G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}} +$

$G_{28}^{(2)} \left|(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right) - (a''_{28})^{(5)}\left(T_{29}^{(2)}, s_{(28)}\right)\right| \; e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}} \} ds_{(28)}$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$\left|(G_{31})^{(1)} - (G_{31})^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} t} \leq$ 237

$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)} + (a'_{28})^{(5)} + (\widehat{A}_{28})^{(5)} + (\widehat{P}_{28})^{(5)} (\widehat{k}_{28})^{(5)}\right) d\left(\left((G_{31})^{(1)}, (T_{31})^{(1)}; (G_{31})^{(2)}, (T_{31})^{(2)}\right)\right)$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 21:** The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$ *and* $(\widehat{Q}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$ respectively of $\mathbb{R}_+$. 238

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on $T_{29}$ and respectively on $(G_{31})$(*and not on* $t$) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 22:** There does not exist any $t$ where $G_i(t) = 0$ *and* $T_i(t) = 0$ 239

it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(5)} - (a_i'')^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{(-(b_i')^{(5)} t)} > 0$ for t > 0

**Definition of** $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2$ *and* $\left((\widehat{M}_{28})^{(5)}\right)_3$ : 240

**Remark 23:** if $G_{28}$ is bounded, the same property have also $G_{29}$ *and* $G_{30}$ . indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a'_{29})^{(5)} G_{29}$ and by integrating

$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1 / (a'_{29})^{(5)}$

In the same way , one can obtain

$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2/(a'_{30})^{(5)}$

If $G_{29}$ *or* $G_{30}$ is bounded, the same property follows for $G_{28}$, $G_{30}$ and $G_{28}$, $G_{29}$ respectively.

**Remark 24:** If $G_{28}$ *is* bounded, from below, the same property holds for $G_{29}$ *and* $G_{30}$. The proof is analogous with the preceding one. An analogous property is true if $G_{29}$ is bounded from below. 241

**Remark 25:** If $T_{28}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}((G_{31})(t),t)) = (b'_{29})^{(5)}$ then $T_{29} \to \infty$. 242

**Definition of** $(m)^{(5)}$ and $\varepsilon_5$ :

Indeed let $t_5$ be so that for $t > t_5$

$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t),t) < \varepsilon_5, T_{28}(t) > (m)^{(5)}$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\frac{2}{\varepsilon_5}$ By taking now $\varepsilon_5$ sufficiently small one sees that $T_{29}$ is unbounded.
The same property holds for $T_{30}$ if $\lim_{t\to\infty}(b_{30}'')^{(5)}\left((G_{31})(t),t\right) = (b'_{30})^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{33}, G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}$, $\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$ and to choose 244

$(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[(\hat{P}_{32})^{(6)} + \left((\hat{P}_{32})^{(6)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{32})^{(6)} \quad 245$$

$$\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[\left((\hat{Q}_{32})^{(6)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{32})^{(6)}\right] \leq (\hat{Q}_{32})^{(6)} \quad 246$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric 247

$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right) =$

$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{32})^{(6)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{32})^{(6)}t}\}$

Indeed if we denote

**Definition of** $\widetilde{(G_{35})}, \widetilde{(T_{35})}$ : $\left(\widetilde{(G_{35})}, \widetilde{(T_{35})}\right) = \mathcal{A}^{(6)}\left((G_{35}), (T_{35})\right)$

It results

$$\left|\tilde{G}_{32}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{32})^{(6)} \left|G_{33}^{(1)} - G_{33}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}}\, ds_{(32)} +$$

$$\int_0^t \{(a'_{32})^{(6)} \left|G_{32}^{(1)} - G_{32}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} +$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)}, s_{(32)}\right)\left|G_{32}^{(1)} - G_{32}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}} +$$

$$G_{32}^{(2)} \left|(a''_{32})^{(6)}\left(T_{33}^{(1)}, s_{(32)}\right) - (a''_{32})^{(6)}\left(T_{33}^{(2)}, s_{(32)}\right)\right| \; e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}} \} ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|(G_{35})^{(1)} - (G_{35})^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} t} \leq \qquad (248)$$
$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)} + (a'_{32})^{(6)} + (\widehat{A}_{32})^{(6)} + (\widehat{P}_{32})^{(6)} (\widehat{k}_{32})^{(6)}\right) d\left(\left((G_{35})^{(1)}, (T_{35})^{(1)}; \; (G_{35})^{(2)}, (T_{35})^{(2)}\right)\right)$$

And analogous inequalities for $G_i \; and \; T_i$. Taking into account the hypothesis the result follows

**Remark 26:** The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)} t} \; and \; (\widehat{Q}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)} t}$ respectively of $\mathbb{R}_+$. (249)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i = 32{,}33{,}34$ depend only on $T_{33}$ and respectively on $(G_{35})(and \; not \; on \; t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 27:** There does not exist any $t$ where $G_i\,(t) = 0 \; and \; T_i\,(t) = 0$ (250)

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(6)} - (a_i'')^{(6)}(T_{33}(s_{(32)}), s_{(32)})\} ds_{(32)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(6)} t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{32})^{(6)}\right)_1, \left((\widehat{M}_{32})^{(6)}\right)_2 \; and \; \left((\widehat{M}_{32})^{(6)}\right)_3$ : (251)

**Remark 28:** if $G_{32}$ is bounded, the same property have also $G_{33} \; and \; G_{34}$ . indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a'_{33})^{(6)} G_{33}$ and by integrating

$$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_1 / (a'_{33})^{(6)}$$

In the same way , one can obtain

$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_2/(a'_{34})^{(6)}$

If $G_{33}$ *or* $G_{34}$ is bounded, the same property follows for $G_{32}$ , $G_{34}$ and $G_{32}$ , $G_{33}$ respectively.

**Remark 29:** If $G_{32}$ *is* bounded, from below, the same property holds for $G_{33}$ *and* $G_{34}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{33}$ is bounded from below. 252

**Remark 30:** If $T_{32}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}((G_{35})(t),t)) = (b'_{33})^{(6)}$ then $T_{33} \to \infty$. 253

**Definition of** $(m)^{(6)}$ and $\varepsilon_6$ :

Indeed let $t_6$ be so that for $t > t_6$

$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t),t\right) < \varepsilon_6, T_{32}(t) > (m)^{(6)}$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right)$, $t = log\frac{2}{\varepsilon_6}$ By taking now $\varepsilon_6$ sufficiently small one sees that $T_{33}$ is unbounded.
The same property holds for $T_{34}$ if $\lim_{t\to\infty}(b''_{34})^{(6)}\left((G_{35})(t),t(t),t\right) = (b'_{34})^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{37}$ , $G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}$ , $\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}} < 1$ and to choose
$(\widehat{P}_{36})^{(7)}$ and $(\widehat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[(\widehat{P}_{36})^{(7)} + \left((\widehat{P}_{36})^{(7)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{36})^{(7)} \quad 256$$

$$\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[\left((\widehat{Q}_{36})^{(7)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{36})^{(7)}\right] \leq (\widehat{Q}_{36})^{(7)} \quad 257$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself
The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric 258

$d\left(((G_{39})^{(1)},(T_{39})^{(1)}),((G_{39})^{(2)},(T_{39})^{(2)})\right) =$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\tilde{M}_{36})^{(7)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\tilde{M}_{36})^{(7)}t}\}$$

Indeed if we denote

**Definition of** $\widetilde{(G_{39})},\widetilde{(T_{39})}:\left(\widetilde{(G_{39})},\widetilde{(T_{39})}\right)=\mathcal{A}^{(7)}((G_{39}),(T_{39}))$
It results

$$\left|\tilde{G}_{36}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{36})^{(7)}\left|G_{37}^{(1)}-G_{37}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\,ds_{(36)}+$$
$$\int_0^t\{(a'_{36})^{(7)}\left|G_{36}^{(1)}-G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$(a''_{36})^{(7)}\left(T_{37}^{(1)},s_{(36)}\right)\left|G_{36}^{(1)}-G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$G_{36}^{(2)}\left|(a''_{36})^{(7)}\left(T_{37}^{(1)},s_{(36)}\right)-(a''_{36})^{(7)}\left(T_{37}^{(2)},s_{(36)}\right)\right|\ e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\}ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses on it follows

$$\left|(G_{39})^{(1)}-(G_{39})^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}t}\le \qquad 259$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}}\left((a_{36})^{(7)}+(a'_{36})^{(7)}+(\widehat{A}_{36})^{(7)}+(\widehat{P}_{36})^{(7)}(\widehat{k}_{36})^{(7)}\right)d\left(((G_{39})^{(1)},(T_{39})^{(1)};\ (G_{39})^{(2)},(T_{39})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

**Remark 31:** The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}\ and\ (\widehat{Q}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ respectively of $\mathbb{R}_+$. 260

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(7)}$ and $(b_i'')^{(7)}, i=36,37,38$ depend only on $\mathrm{T}_{37}$ and respectively on $(G_{39})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 32:** There does not exist any $t$ where $G_i\,(t)=0\ and\ T_i\,(t)=0$ 261

it results

$$G_i\,(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(7)}-(a_i'')^{(7)}(T_{37}(s_{(36)}),s_{(36)})\}ds_{(36)}\right]}\ge 0$$

$$T_i\,(t)\ge T_i^0e^{\left(-(b_i')^{(7)}t\right)}>0\quad \text{for t}>0$$

**Definition of** $\left((\widehat{M}_{36})^{(7)}\right)_1,\left((\widehat{M}_{36})^{(7)}\right)_2\ and\ \left((\widehat{M}_{36})^{(7)}\right)_3:$ 262

**Remark 33:** if $G_{36}$ is bounded, the same property have also $G_{37}\ and\ G_{38}$ . indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a'_{37})^{(7)} G_{37}$ and by integrating

$$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1/(a'_{37})^{(7)}$$

In the same way , one can obtain

$$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2/(a'_{38})^{(7)}$$

If $G_{37}$ *or* $G_{38}$ is bounded, the same property follows for $G_{36}$ , $G_{38}$ and $G_{36}$ , $G_{37}$ respectively.

**Remark 34:** If $G_{36}$ *is* bounded, from below, the same property holds for $G_{37}$ *and* $G_{38}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{37}$ is bounded from below. 263

**Remark 35:** If $T_{36}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(7)}((G_{39})(t),t)) = (b'_{37})^{(7)}$ then $T_{37} \to \infty$. 264

**Definition of** $(m)^{(7)}$ and $\varepsilon_7$ :

Indeed let $t_7$ be so that for $t > t_7$

$$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t),t) < \varepsilon_7, T_{36}(t) > (m)^{(7)}$$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right), \quad t = log\frac{2}{\varepsilon_7}$$ By taking now $\varepsilon_7$ sufficiently small one sees that $T_{37}$ is unbounded.
The same property holds for $T_{38}$ if $\lim_{t\to\infty}(b''_{38})^{(7)}\left((G_{39})(t),t\right) = (b'_{38})^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}$ , $\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}} < 1$ and to choose 266
$(\widehat{P}_{40})^{(8)}$ and $(\widehat{Q}_{40})^{(8)}$ large to have

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[(\widehat{P}_{40})^{(8)} + \left((\widehat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{40})^{(8)}$$ 267

$$\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[\left((\widehat{Q}_{40})^{(8)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{40})^{(8)}\right] \leq (\widehat{Q}_{40})^{(8)}$$ 268

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)},(T_{43})^{(1)}),((G_{43})^{(2)},(T_{43})^{(2)})\right)= \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

**Definition of** $\widetilde{(G_{43})},\widetilde{(T_{43})}$ : $\left(\widetilde{(G_{43})},\widetilde{(T_{43})}\right)=\mathcal{A}^{(8)}((G_{43}),(T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{40})^{(8)}\left|G_{41}^{(1)}-G_{41}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\,ds_{(40)}+$$
$$\int_0^t\{(a'_{40})^{(8)}\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$G_{40}^{(2)}\left|(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)-(a''_{40})^{(8)}\left(T_{41}^{(2)},s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\}ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0,t]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)}-(G_{43})^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}t}\le \quad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)}+(a'_{40})^{(8)}+(\widehat{A}_{40})^{(8)}+(\widehat{P}_{40})^{(8)}(\widehat{k}_{40})^{(8)}\right)d\left(((G_{43})^{(1)},(T_{43})^{(1)};\ (G_{43})^{(2)},(T_{43})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

**Remark 36:** The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}\ and\ (\widehat{Q}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ respectively of $\mathbb{R}_+$. 274

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i=40,41,42$ depend only on $T_{41}$ and respectively on $(G_{43})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 37** There does not exist any $t$ where $G_i\,(t)=0\ and\ T_i\,(t)=0$ 275

it results

$$G_i\,(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(8)}-(a_i'')^{(8)}(T_{41}(s_{(40)}),s_{(40)})\}ds_{(40)}\right]}\ge 0$$

$$T_i\,(t)\ge T_i^0e^{\left(-(b_i')^{(8)}t\right)}>0\quad \text{for t}>0$$

**Definition of** $\left((\widehat{M}_{40})^{(8)}\right)_1, \left((\widehat{M}_{40})^{(8)}\right)_2$ *and* $\left((\widehat{M}_{40})^{(8)}\right)_3$ : 276

**Remark 38:** if $G_{40}$ is bounded, the same property have also $G_{41}$ *and* $G_{42}$ . indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a'_{41})^{(8)} G_{41}$ and by integrating

$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1/(a'_{41})^{(8)}$

In the same way , one can obtain

$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2/(a'_{42})^{(8)}$

If $G_{41}$ *or* $G_{42}$ is bounded, the same property follows for $G_{40}$ , $G_{42}$ and $G_{40}$ , $G_{41}$ respectively.

**Remark 39:** If $G_{40}$ *is* bounded, from below, the same property holds for $G_{41}$ *and* $G_{42}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{41}$ is bounded from below. 277

**Remark 40:** If $T_{40}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}((G_{43})(t),t)) = (b'_{41})^{(8)}$ then $T_{41} \to \infty$. 278

**Definition of** $(m)^{(8)}$ and $\varepsilon_8$ :

Indeed let $t_8$ be so that for $t > t_8$

$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t),t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right), \quad t = log\frac{2}{\varepsilon_8}$ By taking now $\varepsilon_8$ sufficiently small one sees that $T_{41}$ is unbounded.
The same property holds for $T_{42}$ if $\lim_{t\to\infty}(b''_{42})^{(8)}\left((G_{43})(t),t(t),t\right) = (b'_{42})^{(8)}$

It is now sufficient to take $\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}$ , $\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}} < 1$ and to choose $(\widehat{P}_{44})^{(9)}$ and $(\widehat{Q}_{44})^{(9)}$ large to have 279 A

$$\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[(\widehat{P}_{44})^{(9)} + \left((\widehat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[\left((\widehat{Q}_{44})^{(9)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{44})^{(9)}\right] \leq (\widehat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)},(T_{47})^{(1)}),((G_{47})^{(2)},(T_{47})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{47}}),(\widetilde{T_{47}}):\left((\widetilde{G_{47}}),(\widetilde{T_{47}})\right)=\mathcal{A}^{(9)}\left((G_{47}),(T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{44})^{(9)}\left|G_{45}^{(1)}-G_{45}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\,ds_{(44)}+$$
$$\int_0^t\{(a'_{44})^{(9)}\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$G_{44}^{(2)}\left|(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)-(a''_{44})^{(9)}\left(T_{45}^{(2)},s_{(44)}\right)\right|\;e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\}ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}t}\le$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)}+(a'_{44})^{(9)}+(\widehat{A}_{44})^{(9)}+(\widehat{P}_{44})^{(9)}(\widehat{k}_{44})^{(9)}\right)d\left(((G_{47})^{(1)},(T_{47})^{(1)};\;(G_{47})^{(2)},(T_{47})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis (39,35,36) the result follows

**Remark 41:** The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}\ and\ (\widehat{Q}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i=44,45,46$ depend only on $T_{45}$ and respectively on $(G_{47})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 42:** There does not exist any $t$ where $G_i\,(t)=0\ and\ T_i\,(t)=0$

From 99 to 44 it results

$$G_i\,(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(9)}-(a_i'')^{(9)}(T_{45}(s_{(44)}),s_{(44)})\}ds_{(44)}\right]}\ge 0$$

$$T_i\,(t)\ge T_i^0e^{(-(b_i')^{(9)}t)}>0\quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{44})^{(9)}\right)_1,\left((\widehat{M}_{44})^{(9)}\right)_2\ and\ \left((\widehat{M}_{44})^{(9)}\right)_3$ :

**Remark 43:** if $G_{44}$ is bounded, the same property have also $G_{45}\ and\ G_{46}$ . indeed if

$G_{44} < (\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt} \leq \left((\widehat{M}_{44})^{(9)}\right)_1 - (a'_{45})^{(9)} G_{45}$ and by integrating

$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_1 / (a'_{45})^{(9)}$

In the same way , one can obtain

$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_2 / (a'_{46})^{(9)}$

If $G_{45}$ *or* $G_{46}$ is bounded, the same property follows for $G_{44}$ , $G_{46}$ and $G_{44}$ , $G_{45}$ respectively.

**Remark 44:** If $G_{44}$ *is* bounded, from below, the same property holds for $G_{45}$ *and* $G_{46}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{45}$ is bounded from below.

**Remark 45:** If $T_{44}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t),t\right) = (b'_{45})^{(9)}$ then $T_{45} \to \infty$.

**Definition of** $(m)^{(9)}$ and $\varepsilon_9$ :

Indeed let $t_9$ be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t),t\right) < \varepsilon_9, T_{44}(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right)$, $t = log\frac{2}{\varepsilon_9}$ By taking now $\varepsilon_9$ sufficiently small one sees that $T_{45}$ is unbounded.
The same property holds for $T_{46}$ if $\lim_{t\to\infty}(b''_{46})^{(9)}\left((G_{47})(t),t\right) = (b'_{46})^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

**Behavior of the solutions of equation** 280

**Theorem** If we denote and define

**Definition of** $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ :

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)} \leq -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14},t) + (a''_{14})^{(1)}(T_{14},t) \leq -(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)} \leq -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G,t) - (b''_{14})^{(1)}(G,t) \leq -(\tau_1)^{(1)}$

**Definition of** $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$ : 281

By $(\nu_1)^{(1)} > 0$ , $(\nu_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$ , $(u_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

**Definition of** $(\bar{\nu}_1)^{(1)}, , (\bar{\nu}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$ : 282

By $(\bar{\nu}_1)^{(1)} > 0$ , $(\bar{\nu}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$ , $(\bar{u}_2)^{(1)} < 0$ the roots of the equations

$(a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

**Definition of** $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (v_0)^{(1)}$ :- 283

If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by

$(m_2)^{(1)} = (v_0)^{(1)}, (m_1)^{(1)} = (v_1)^{(1)},\ if\ (v_0)^{(1)} < (v_1)^{(1)}$

$(m_2)^{(1)} = (v_1)^{(1)}, (m_1)^{(1)} = (\bar{v}_1)^{(1)}, if\ (v_1)^{(1)} < (v_0)^{(1)} < (\bar{v}_1)^{(1)},$

and $\boxed{(v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$

$(m_2)^{(1)} = (v_1)^{(1)}, (m_1)^{(1)} = (v_0)^{(1)},\ if\ (\bar{v}_1)^{(1)} < (v_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)},\ if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)},$

and $\boxed{(u_0)^{(1)} = \frac{T_{13}^0}{T_{14}^0}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$

are defined

Then the solution of global equations satisfies the inequalities 285

$$G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{13}(t) \le G_{13}^0 e^{(S_1)^{(1)}t}$$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)}t}$$

$$\left(\frac{(a_{15})^{(1)} G_{13}^0}{(m_1)^{(1)}((S_1)^{(1)} - (p_{13})^{(1)} - (S_2)^{(1)})}\left[e^{((S_1)^{(1)} - (p_{13})^{(1)})t} - e^{-(S_2)^{(1)}t}\right] + G_{15}^0 e^{-(S_2)^{(1)}t} \le G_{15}(t) \le \right. \quad 286$$

$$\left. \frac{(a_{15})^{(1)} G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)} - (a'_{15})^{(1)})}\left[e^{(S_1)^{(1)}t} - e^{-(a'_{15})^{(1)}t}\right] + G_{15}^0 e^{-(a'_{15})^{(1)}t}\right)$$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)}t} \le T_{13}(t) \le T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t}} \quad 287$$

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)}t} \le T_{13}(t) \le \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t} \quad 288$$

$$\frac{(b_{15})^{(1)} T_{13}^0}{(\mu_1)^{(1)}((R_1)^{(1)} - (b'_{15})^{(1)})}\left[e^{(R_1)^{(1)}t} - e^{-(b'_{15})^{(1)}t}\right] + T_{15}^0 e^{-(b'_{15})^{(1)}t} \le T_{15}(t) \le \quad 289$$

$$\frac{(a_{15})^{(1)} T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)} + (r_{13})^{(1)} + (R_2)^{(1)})}\left[e^{((R_1)^{(1)} + (r_{13})^{(1)})t} - e^{-(R_2)^{(1)}t}\right] + T_{15}^0 e^{-(R_2)^{(1)}t}$$

**Definition of** $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:- 290

Where $(S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$

$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$

$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$

$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$

**Behavior of the solutions of equation** 291

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ : 292

$(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ four constants satisfying
$-(\sigma_2)^{(2)} \le -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(T_{17}, t) + (a''_{17})^{(2)}(T_{17}, t) \le -(\sigma_1)^{(2)}$ 293

$-(\tau_2)^{(2)} \le -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}\big((G_{19}), t\big) - (b''_{17})^{(2)}\big((G_{19}), t\big) \le -(\tau_1)^{(2)}$ 294

**Definition of** $(\nu_1)^{(2)}, (\nu_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$ : 295

By $(\nu_1)^{(2)} > 0$ , $(\nu_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0$ , $(u_2)^{(2)} < 0$ the roots 296

of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 297

and $(b_{14})^{(2)}\big(u^{(2)}\big)^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and 298

**Definition of** $(\bar{\nu}_1)^{(2)},, (\bar{\nu}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$ : 299

By $(\bar{\nu}_1)^{(2)} > 0$ , $(\bar{\nu}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0$ , $(\bar{u}_2)^{(2)} < 0$ the 300

roots of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 301

and $(b_{17})^{(2)}\big(u^{(2)}\big)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 302

**Definition of** $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ :- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{G^0_{16}}{G^0_{17}}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)},\ \boldsymbol{if}\ (u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}, \boldsymbol{if}\ (u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)},$

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)},\ \boldsymbol{if}\ (\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)} t}$

$(p_i)^{(2)}$ is defined by equation

$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)} t}$ 311

$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)} t}\right] + G_{18}^0 e^{-(S_2)^{(2)} t} \le G_{18}(t) \le\right.$ 312

$\left.\frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)} t} - e^{-(a'_{18})^{(2)} t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)} t}\right)$

$\boxed{T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}}$ 313

$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}$ 314

$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)} t} - e^{-(b'_{18})^{(2)} t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)} t} \le T_{18}(t) \le$ 315

$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})}\left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)} t}\right] + T_{18}^0 e^{-(R_2)^{(2)} t}$

**Definition of** $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$

$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)}$ 318

$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$

**Behavior of the solutions** 319

**Theorem 3:** If we denote and define

**Definition of** $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ :

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}\big((G_{23}), t\big) \le -(\tau_1)^{(3)}$$

**Definition of** $(\nu_1)^{(3)}, (\nu_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$ : 320

By $(\nu_1)^{(3)} > 0$ , $(\nu_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$ , $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\big(\nu^{(3)}\big)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{\nu}_1)^{(3)} > 0$ , $(\bar{\nu}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$ , $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\big(\nu^{(3)}\big)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

**Definition of** $(m_1)^{(3)}$ , $(m_2)^{(3)}$ , $(\mu_1)^{(3)}, (\mu_2)^{(3)}$ :- 321

If we define $(m_1)^{(3)}$ , $(m_2)^{(3)}$ , $(\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (\nu_0)^{(3)}, (m_1)^{(3)} = (\nu_1)^{(3)}$, ***if*** $(\nu_0)^{(3)} < (\nu_1)^{(3)}$

$(m_2)^{(3)} = (\nu_1)^{(3)}, (m_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ , ***if*** $(\nu_1)^{(3)} < (\nu_0)^{(3)} < (\bar{\nu}_1)^{(3)}$,

and $\boxed{(\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}}$

$(m_2)^{(3)} = (\nu_1)^{(3)}, (m_1)^{(3)} = (\nu_0)^{(3)}$, ***if*** $(\bar{\nu}_1)^{(3)} < (\nu_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}$, ***if*** $(u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$ , ***if*** $(u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}$, and $\boxed{(u_0)^{(3)} = \frac{T^0_{20}}{T^0_{21}}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}$, ***if*** $(\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$$G^0_{20} e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G^0_{20} e^{(S_1)^{(3)} t}$$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}} G^0_{20} e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}} G^0_{20} e^{(S_1)^{(3)} t} \quad 323$$

$$\Big(\frac{(a_{22})^{(3)} G^0_{20}}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\Big[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)} t}\Big] + G^0_{22} e^{-(S_2)^{(3)} t} \le G_{22}(t) \le \quad 324$$

$$\frac{(a_{22})^{(3)} G^0_{20}}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}[e^{(S_1)^{(3)} t} - e^{-(a'_{22})^{(3)} t}] + G^0_{22} e^{-(a'_{22})^{(3)} t}\Big)$$

$$\boxed{T_{20}^{0}e^{(R_1)^{(3)}t} \le T_{20}(t) \le T_{20}^{0}e^{((R_1)^{(3)}+(r_{20})^{(3)})t}} \quad 325$$

$$\frac{1}{(\mu_1)^{(3)}}T_{20}^{0}e^{(R_1)^{(3)}t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}}T_{20}^{0}e^{((R_1)^{(3)}+(r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)}T_{20}^{0}}{(\mu_1)^{(3)}((R_1)^{(3)}-(b'_{22})^{(3)})}\left[e^{(R_1)^{(3)}t} - e^{-(b'_{22})^{(3)}t}\right] + T_{22}^{0}e^{-(b'_{22})^{(3)}t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)}T_{20}^{0}}{(\mu_2)^{(3)}((R_1)^{(3)}+(r_{20})^{(3)}+(R_2)^{(3)})}\left[e^{((R_1)^{(3)}+(r_{20})^{(3)})t} - e^{-(R_2)^{(3)}t}\right] + T_{22}^{0}e^{-(R_2)^{(3)}t}$$

**Definition of** $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

**Behavior of the solutions of equation**

**Theorem:** If we denote and define

**Definition of** $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ :

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$

$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}\big((G_{27}), t\big) - (b''_{25})^{(4)}\big((G_{27}), t\big) \le -(\tau_1)^{(4)}$

**Definition of** $(\nu_1)^{(4)}, (\nu_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, \nu^{(4)}, u^{(4)}$ : 329

By $(\nu_1)^{(4)} > 0$ , $(\nu_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$ , $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\big(\nu^{(4)}\big)^2 + (\sigma_1)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\big(u^{(4)}\big)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(4)},, (\bar{\nu}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$ : 330

By $(\bar{\nu}_1)^{(4)} > 0$ , $(\bar{\nu}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$ , $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\big(\nu^{(4)}\big)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\big(u^{(4)}\big)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

**Definition of** $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}, (\nu_0)^{(4)}$ :-

If we define $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (\nu_0)^{(4)}, (m_1)^{(4)} = (\nu_1)^{(4)}$, ***if*** $(\nu_0)^{(4)} < (\nu_1)^{(4)}$

$(m_2)^{(4)} = (\nu_1)^{(4)}, (m_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ , ***if*** $(\nu_4)^{(4)} < (\nu_0)^{(4)} < (\bar{\nu}_1)^{(4)}$,

and $\boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (\nu_4)^{(4)}, (m_1)^{(4)} = (\nu_0)^{(4)},\ \boldsymbol{if}\ (\bar{\nu}_4)^{(4)} < (\nu_0)^{(4)}$

and analogously

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$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)},\ \boldsymbol{if}\ (u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}, \boldsymbol{if}\ (u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)},$

and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}, \boldsymbol{if}\ (\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

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$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)} t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)} t} \quad 333$$

$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)} t}\right] + G_{26}^0 e^{-(S_2)^{(4)} t} \le G_{26}(t) \le \right. \quad 334$$

*(a26)4G240(m2)4(S1)4−(a26′)4e(S1)4t−e−(a26′)4t+ G260e−(a26′)4t*

$$\boxed{T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}} T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}} T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t} \quad 335$$

$$\frac{(b_{26})^{(4)} T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)} - (b'_{26})^{(4)})}\left[e^{(R_1)^{(4)} t} - e^{-(b'_{26})^{(4)} t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)} t} \le T_{26}(t) \le \quad 336$$

$$\frac{(a_{26})^{(4)} T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)} + (r_{24})^{(4)} + (R_2)^{(4)})}\left[e^{((R_1)^{(4)} + (r_{24})^{(4)})t} - e^{-(R_2)^{(4)} t}\right] + T_{26}^0 e^{-(R_2)^{(4)} t}$$

**Definition of** $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

**Behavior of the solutions of equation** 338

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ :

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

**Definition of** $(\nu_1)^{(5)}, (\nu_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, \nu^{(5)}, u^{(5)}$ : 339

By $(\nu_1)^{(5)} > 0$ , $(\nu_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$ , $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(5)}, , (\bar{\nu}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$ : 340

By $(\bar{\nu}_1)^{(5)} > 0$ , $(\bar{\nu}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$ , $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

**Definition of** $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (\nu_0)^{(5)}$ :-

If we define $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (\nu_0)^{(5)}, (m_1)^{(5)} = (\nu_1)^{(5)}$, ***if*** $(\nu_0)^{(5)} < (\nu_1)^{(5)}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ , ***if*** $(\nu_1)^{(5)} < (\nu_0)^{(5)} < (\bar{\nu}_1)^{(5)}$,
and $\boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)}$, ***if*** $(\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)}$, ***if*** $(u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}$ , ***if*** $(u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)}$,
and $\boxed{(u_0)^{(5)} = \frac{T^0_{28}}{T^0_{29}}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}$, ***if*** $(\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$$G^0_{28} e^{\left((S_1)^{(5)} - (p_{28})^{(5)}\right)t} \le G_{28}(t) \le G^0_{28} e^{(S_1)^{(5)}t}$$

where $(p_i)^{(5)}$ is defined by equation

$$\frac{1}{(m_5)^{(5)}} G^0_{28} e^{\left((S_1)^{(5)} - (p_{28})^{(5)}\right)t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}} G^0_{28} e^{(S_1)^{(5)}t} \quad 343$$

$$\left(\frac{(a_{30})^{(5)}G_{28}^{0}}{(m_1)^{(5)}((S_1)^{(5)}-(p_{28})^{(5)}-(S_2)^{(5)})}\left[e^{((S_1)^{(5)}-(p_{28})^{(5)})t}-e^{-(S_2)^{(5)}t}\right]+G_{30}^{0}e^{-(S_2)^{(5)}t}\leq G_{30}(t)\leq\right. \quad 344$$

*(a30)5G280(m2)5(S1)5−(a30′)5e(S1)5t−e−(a30′)5t+ G300e−(a30′)5t*

$$\boxed{T_{28}^{0}e^{(R_1)^{(5)}t}\leq T_{28}(t)\leq T_{28}^{0}e^{((R_1)^{(5)}+(r_{28})^{(5)})t}} \quad 345$$

$$\frac{1}{(\mu_1)^{(5)}}T_{28}^{0}e^{(R_1)^{(5)}t}\leq T_{28}(t)\leq\frac{1}{(\mu_2)^{(5)}}T_{28}^{0}e^{((R_1)^{(5)}+(r_{28})^{(5)})t} \quad 346$$

$$\frac{(b_{30})^{(5)}T_{28}^{0}}{(\mu_1)^{(5)}((R_1)^{(5)}-(b'_{30})^{(5)})}\left[e^{(R_1)^{(5)}t}-e^{-(b'_{30})^{(5)}t}\right]+T_{30}^{0}e^{-(b'_{30})^{(5)}t}\leq T_{30}(t)\leq \quad 347$$

$$\frac{(a_{30})^{(5)}T_{28}^{0}}{(\mu_2)^{(5)}((R_1)^{(5)}+(r_{28})^{(5)}+(R_2)^{(5)})}\left[e^{((R_1)^{(5)}+(r_{28})^{(5)})t}-e^{-(R_2)^{(5)}t}\right]+T_{30}^{0}e^{-(R_2)^{(5)}t}$$

**Definition of** $(S_1)^{(5)},(S_2)^{(5)},(R_1)^{(5)},(R_2)^{(5)}$:- 348

Where $(S_1)^{(5)}=(a_{28})^{(5)}(m_2)^{(5)}-(a'_{28})^{(5)}$

$(S_2)^{(5)}=(a_{30})^{(5)}-(p_{30})^{(5)}$

$(R_1)^{(5)}=(b_{28})^{(5)}(\mu_2)^{(5)}-(b'_{28})^{(5)}$

$(R_2)^{(5)}=(b'_{30})^{(5)}-(r_{30})^{(5)}$

**Behavior of the solutions of equation** 349

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(6)},(\sigma_2)^{(6)},(\tau_1)^{(6)},(\tau_2)^{(6)}$ :

$(\sigma_1)^{(6)},(\sigma_2)^{(6)},(\tau_1)^{(6)},(\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)}\leq-(a'_{32})^{(6)}+(a'_{33})^{(6)}-(a''_{32})^{(6)}(T_{33},t)+(a''_{33})^{(6)}(T_{33},t)\leq-(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)}\leq-(b'_{32})^{(6)}+(b'_{33})^{(6)}-(b''_{32})^{(6)}\big((G_{35}),t\big)-(b''_{33})^{(6)}\big((G_{35}),t\big)\leq-(\tau_1)^{(6)}$$

**Definition of** $(v_1)^{(6)},(v_2)^{(6)},(u_1)^{(6)},(u_2)^{(6)},v^{(6)},u^{(6)}$ : 350

By $(v_1)^{(6)}>0$ , $(v_2)^{(6)}<0$ and respectively $(u_1)^{(6)}>0$ , $(u_2)^{(6)}<0$ the roots of the equations $(a_{33})^{(6)}\big(v^{(6)}\big)^2+(\sigma_1)^{(6)}v^{(6)}-(a_{32})^{(6)}=0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2+(\tau_1)^{(6)}u^{(6)}-(b_{32})^{(6)}=0$ and

**Definition of** $(\bar{v}_1)^{(6)},,(\bar{v}_2)^{(6)},(\bar{u}_1)^{(6)},(\bar{u}_2)^{(6)}$ : 351

By $(\bar{v}_1)^{(6)}>0$ , $(\bar{v}_2)^{(6)}<0$ and respectively $(\bar{u}_1)^{(6)}>0$ , $(\bar{u}_2)^{(6)}<0$ the roots of the equations $(a_{33})^{(6)}\big(v^{(6)}\big)^2+(\sigma_2)^{(6)}v^{(6)}-(a_{32})^{(6)}=0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2+(\tau_2)^{(6)}u^{(6)}-(b_{32})^{(6)}=0$

**Definition of** $(m_1)^{(6)},(m_2)^{(6)},(\mu_1)^{(6)},(\mu_2)^{(6)},(v_0)^{(6)}$ :-

If we define $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}$ by

$(m_2)^{(6)} = (v_0)^{(6)}, (m_1)^{(6)} = (v_1)^{(6)}$, ***if*** $(v_0)^{(6)} < (v_1)^{(6)}$

$(m_2)^{(6)} = (v_1)^{(6)}, (m_1)^{(6)} = (\bar{v}_6)^{(6)}$, ***if*** $(v_1)^{(6)} < (v_0)^{(6)} < (\bar{v}_1)^{(6)}$,

and $\boxed{(v_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$

$(m_2)^{(6)} = (v_1)^{(6)}, (m_1)^{(6)} = (v_0)^{(6)}$, ***if*** $(\bar{v}_1)^{(6)} < (v_0)^{(6)}$

and analogously 352

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}$, ***if*** $(u_0)^{(6)} < (u_1)^{(6)}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}$, ***if*** $(u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)}$,

and $\boxed{(u_0)^{(6)} = \frac{T_{32}^0}{T_{33}^0}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}$, ***if*** $(\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$

Then the solution of global equations satisfies the inequalities 353

$$G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \leq G_{32}(t) \leq G_{32}^0 e^{(S_1)^{(6)} t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}} G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \leq G_{33}(t) \leq \frac{1}{(m_2)^{(6)}} G_{32}^0 e^{(S_1)^{(6)} t} \quad 354$$

$$\left( \frac{(a_{34})^{(6)} G_{32}^0}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})} \left[ e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)} t} \right] + G_{34}^0 e^{-(S_2)^{(6)} t} \leq G_{34}(t) \leq \right. \quad 355$$

*(a34)6G320(m2)6(S1)6−(a34′)6e(S1)6t−e−(a34′)6t+ G340e−(a34′)6t*

$$\boxed{T_{32}^0 e^{(R_1)^{(6)} t} \leq T_{32}(t) \leq T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}} T_{32}^0 e^{(R_1)^{(6)} t} \leq T_{32}(t) \leq \frac{1}{(\mu_2)^{(6)}} T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)} T_{32}^0}{(\mu_1)^{(6)}((R_1)^{(6)} - (b'_{34})^{(6)})} \left[ e^{(R_1)^{(6)} t} - e^{-(b'_{34})^{(6)} t} \right] + T_{34}^0 e^{-(b'_{34})^{(6)} t} \leq T_{34}(t) \leq \quad 358$$

$$\frac{(a_{34})^{(6)} T_{32}^0}{(\mu_2)^{(6)}((R_1)^{(6)} + (r_{32})^{(6)} + (R_2)^{(6)})} \left[ e^{((R_1)^{(6)} + (r_{32})^{(6)})t} - e^{-(R_2)^{(6)} t} \right] + T_{34}^0 e^{-(R_2)^{(6)} t}$$

**Definition of** $(S_1)^{(6)}, (S_2)^{(6)}, (R_1)^{(6)}, (R_2)^{(6)}$:- 359

Where $(S_1)^{(6)} = (a_{32})^{(6)} (m_2)^{(6)} - (a'_{32})^{(6)}$

$(S_2)^{(6)} = (a_{34})^{(6)} - (p_{34})^{(6)}$

$(R_1)^{(6)} = (b_{32})^{(6)} (\mu_2)^{(6)} - (b'_{32})^{(6)}$

$$(R_2)^{(6)} = (b'_{34})^{(6)} - (r_{34})^{(6)}$$

**Behavior of the solutions of equation**

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$ :

$(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)} \le -(a'_{36})^{(7)} + (a'_{37})^{(7)} - (a''_{36})^{(7)}(T_{37}, t) + (a''_{37})^{(7)}(T_{37}, t) \le -(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)} \le -(b'_{36})^{(7)} + (b'_{37})^{(7)} - (b''_{36})^{(7)}\big((G_{39}), t\big) - (b''_{37})^{(7)}\big((G_{39}), t\big) \le -(\tau_1)^{(7)}$$

**Definition of** $(\nu_1)^{(7)}, (\nu_2)^{(7)}, (u_1)^{(7)}, (u_2)^{(7)}, \nu^{(7)}, u^{(7)}$ : 361

By $(\nu_1)^{(7)} > 0, (\nu_2)^{(7)} < 0$ and respectively $(u_1)^{(7)} > 0, (u_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_1)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(7)},, (\bar{\nu}_2)^{(7)}, (\bar{u}_1)^{(7)}, (\bar{u}_2)^{(7)}$ : 362

By $(\bar{\nu}_1)^{(7)} > 0, (\bar{\nu}_2)^{(7)} < 0$ and respectively $(\bar{u}_1)^{(7)} > 0, (\bar{u}_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

**Definition of** $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (\nu_0)^{(7)}$ :-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$$(m_2)^{(7)} = (\nu_0)^{(7)}, (m_1)^{(7)} = (\nu_1)^{(7)}, \ \boldsymbol{if}\ (\nu_0)^{(7)} < (\nu_1)^{(7)}$$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\bar{\nu}_1)^{(7)}, \boldsymbol{if}\ (\nu_1)^{(7)} < (\nu_0)^{(7)} < (\bar{\nu}_1)^{(7)},$$

and $\boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\nu_0)^{(7)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(7)} < (\nu_0)^{(7)}$$

and analogously 363

$$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}, \ \boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$$

and $\boxed{(u_0)^{(7)} = \frac{T^0_{36}}{T^0_{37}}}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)}$ where $(u_1)^{(7)}, (\bar{u}_1)^{(7)}$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^0 e^{\left((S_1)^{(7)} - (p_{36})^{(7)}\right)t} \leq G_{36}(t) \leq G_{36}^0 e^{(S_1)^{(7)} t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{\left((S_1)^{(7)} - (p_{36})^{(7)}\right)t} \leq G_{37}(t) \leq \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)} t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)}\left((S_1)^{(7)} - (p_{36})^{(7)} - (S_2)^{(7)}\right)}\left[e^{\left((S_1)^{(7)} - (p_{36})^{(7)}\right)t} - e^{-(S_2)^{(7)} t}\right] + G_{38}^0 e^{-(S_2)^{(7)} t} \leq G_{38}(t) \leq \right. \quad 366$$

$$\left. \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)}\left((S_1)^{(7)} - (a'_{38})^{(7)}\right)}\left[e^{(S_1)^{(7)} t} - e^{-(a'_{38})^{(7)} t}\right] + G_{38}^0 e^{-(a'_{38})^{(7)} t}\right)$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)} t} \leq T_{36}(t) \leq T_{36}^0 e^{\left((R_1)^{(7)} + (r_{36})^{(7)}\right)t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)} t} \leq T_{36}(t) \leq \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{\left((R_1)^{(7)} + (r_{36})^{(7)}\right)t} \quad 368$$

$$\frac{(b_{38})^{(7)} T_{36}^0}{(\mu_1)^{(7)}\left((R_1)^{(7)} - (b'_{38})^{(7)}\right)}\left[e^{(R_1)^{(7)} t} - e^{-(b'_{38})^{(7)} t}\right] + T_{38}^0 e^{-(b'_{38})^{(7)} t} \leq T_{38}(t) \leq \quad 369$$

$$\frac{(a_{38})^{(7)} T_{36}^0}{(\mu_2)^{(7)}\left((R_1)^{(7)} + (r_{36})^{(7)} + (R_2)^{(7)}\right)}\left[e^{\left((R_1)^{(7)} + (r_{36})^{(7)}\right)t} - e^{-(R_2)^{(7)} t}\right] + T_{38}^0 e^{-(R_2)^{(7)} t}$$

**Definition of** $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$$

$$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$$

$$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$$

**Behavior of the solutions of equation** 371

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ :

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$$-(\sigma_2)^{(8)} \leq -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \leq -(\sigma_1)^{(8)}$$

$$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\big((G_{43}),t\big) - (b''_{41})^{(8)}\big((G_{43}),t\big) \le -(\tau_1)^{(8)}$$

**Definition of** $(\nu_1)^{(8)}, (\nu_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, \nu^{(8)}, u^{(8)}$ : 372

By $(\nu_1)^{(8)} > 0$ , $(\nu_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0$ , $(u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(8)},, (\bar{\nu}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$ :

By $(\bar{\nu}_1)^{(8)} > 0$ , $(\bar{\nu}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0$ , $(\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

**Definition of** $(m_1)^{(8)}$ , $(m_2)^{(8)}$ , $(\mu_1)^{(8)}, (\mu_2)^{(8)}, (\nu_0)^{(8)}$ :-

If we define $(m_1)^{(8)}$ , $(m_2)^{(8)}$ , $(\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$(m_2)^{(8)} = (\nu_0)^{(8)}, (m_1)^{(8)} = (\nu_1)^{(8)}$, ***if*** $(\nu_0)^{(8)} < (\nu_1)^{(8)}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ , ***if*** $(\nu_1)^{(8)} < (\nu_0)^{(8)} < (\bar{\nu}_1)^{(8)}$,
and $\boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\nu_0)^{(8)}$, ***if*** $(\bar{\nu}_1)^{(8)} < (\nu_0)^{(8)}$

and analogously 374

$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)}$, ***if*** $(u_0)^{(8)} < (u_1)^{(8)}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}$ , ***if*** $(u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)}$,
and $\boxed{(u_0)^{(8)} = \frac{T^0_{40}}{T^0_{41}}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}$, ***if*** $(\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G^0_{40}e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{40}(t) \le G^0_{40}e^{(S_1)^{(8)}t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}} G_{40}^0 e^{((S_1)^{(8)}-(p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}} G_{40}^0 e^{(S_1)^{(8)}t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)}G_{40}^0}{(m_1)^{(8)}((S_1)^{(8)}-(p_{40})^{(8)}-(S_2)^{(8)})}\left[e^{((S_1)^{(8)}-(p_{40})^{(8)})t} - e^{-(S_2)^{(8)}t}\right] + G_{42}^0 e^{-(S_2)^{(8)}t} \le G_{42}(t) \le \right. \quad 377$$

$$\left. \frac{(a_{42})^{(8)}G_{40}^0}{(m_2)^{(8)}((S_1)^{(8)}-(a'_{42})^{(8)})}[e^{(S_1)^{(8)}t} - e^{-(a'_{42})^{(8)}t}] + G_{42}^0 e^{-(a'_{42})^{(8)}t}\right)$$

$$\boxed{T_{40}^0 e^{(R_1)^{(8)}t} \le T_{40}(t) \le T_{40}^0 e^{((R_1)^{(8)}+(r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}} T_{40}^0 e^{(R_1)^{(8)}t} \le T_{40}(t) \le \frac{1}{(\mu_2)^{(8)}} T_{40}^0 e^{((R_1)^{(8)}+(r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)}T_{40}^0}{(\mu_1)^{(8)}((R_1)^{(8)}-(b'_{42})^{(8)})}\left[e^{(R_1)^{(8)}t} - e^{-(b'_{42})^{(8)}t}\right] + T_{42}^0 e^{-(b'_{42})^{(8)}t} \le T_{42}(t) \le \quad 380$$

$$\frac{(a_{42})^{(8)}T_{40}^0}{(\mu_2)^{(8)}((R_1)^{(8)}+(r_{40})^{(8)}+(R_2)^{(8)})}\left[e^{((R_1)^{(8)}+(r_{40})^{(8)})t} - e^{-(R_2)^{(8)}t}\right] + T_{42}^0 e^{-(R_2)^{(8)}t}$$

**Definition of** $(S_1)^{(8)}, (S_2)^{(8)}, (R_1)^{(8)}, (R_2)^{(8)}$:- 381

Where $(S_1)^{(8)} = (a_{40})^{(8)}(m_2)^{(8)} - (a'_{40})^{(8)}$

$(S_2)^{(8)} = (a_{42})^{(8)} - (p_{42})^{(8)}$

$(R_1)^{(8)} = (b_{40})^{(8)}(\mu_2)^{(8)} - (b'_{40})^{(8)}$

$(R_2)^{(8)} = (b'_{42})^{(8)} - (r_{42})^{(8)}$

**Behavior of the solutions of equation 37 to 92** 382

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ :

$(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ four constants satisfying

$$-(\sigma_2)^{(9)} \le -(a'_{44})^{(9)} + (a'_{45})^{(9)} - (a''_{44})^{(9)}(T_{45}, t) + (a''_{45})^{(9)}(T_{45}, t) \le -(\sigma_1)^{(9)}$$

$$-(\tau_2)^{(9)} \le -(b'_{44})^{(9)} + (b'_{45})^{(9)} - (b''_{44})^{(9)}\big((G_{47}), t\big) - (b''_{45})^{(9)}\big((G_{47}), t\big) \le -(\tau_1)^{(9)}$$

**Definition of** $(\nu_1)^{(9)}, (\nu_2)^{(9)}, (u_1)^{(9)}, (u_2)^{(9)}, \nu^{(9)}, u^{(9)}$ :

By $(\nu_1)^{(9)} > 0$ , $(\nu_2)^{(9)} < 0$ and respectively $(u_1)^{(9)} > 0$ , $(u_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_1)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(9)}, , (\bar{\nu}_2)^{(9)}, (\bar{u}_1)^{(9)}, (\bar{u}_2)^{(9)}$ :

By $(\bar{\nu}_1)^{(9)} > 0$ , $(\bar{\nu}_2)^{(9)} < 0$ and respectively $(\bar{u}_1)^{(9)} > 0$ , $(\bar{u}_2)^{(9)} < 0$ the

roots of the equations $(a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\left(u^{(9)}\right)^2 + (\tau_2)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$

**Definition of** $(m_1)^{(9)}, (m_2)^{(9)}, (\mu_1)^{(9)}, (\mu_2)^{(9)}, (v_0)^{(9)}$ :-

If we define $(m_1)^{(9)}, (m_2)^{(9)}, (\mu_1)^{(9)}, (\mu_2)^{(9)}$ by

$(m_2)^{(9)} = (v_0)^{(9)}, (m_1)^{(9)} = (v_1)^{(9)}$, ***if*** $(v_0)^{(9)} < (v_1)^{(9)}$

$(m_2)^{(9)} = (v_1)^{(9)}, (m_1)^{(9)} = (\bar{v}_1)^{(9)}$, ***if*** $(v_1)^{(9)} < (v_0)^{(9)} < (\bar{v}_1)^{(9)}$,
and $\boxed{(v_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$

$(m_2)^{(9)} = (v_1)^{(9)}, (m_1)^{(9)} = (v_0)^{(9)}$, ***if*** $(\bar{v}_1)^{(9)} < (v_0)^{(9)}$

and analogously

$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)}$, ***if*** $(u_0)^{(9)} < (u_1)^{(9)}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}$, ***if*** $(u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)}$,
and $\boxed{(u_0)^{(9)} = \frac{T_{44}^0}{T_{45}^0}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}$, ***if*** $(\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{44}(t) \le G_{44}^0 e^{(S_1)^{(9)}t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}} G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{45}(t) \le \frac{1}{(m_2)^{(9)}} G_{44}^0 e^{(S_1)^{(9)}t}$$

$$\left( \frac{(a_{46})^{(9)} G_{44}^0}{(m_1)^{(9)}((S_1)^{(9)} - (p_{44})^{(9)} - (S_2)^{(9)})} \left[ e^{((S_1)^{(9)} - (p_{44})^{(9)})t} - e^{-(S_2)^{(9)}t} \right] + G_{46}^0 e^{-(S_2)^{(9)}t} \le G_{46}(t) \le \right.$$
$$\left. \frac{(a_{46})^{(9)} G_{44}^0}{(m_2)^{(9)}((S_1)^{(9)} - (a'_{46})^{(9)})} [e^{(S_1)^{(9)}t} - e^{-(a'_{46})^{(9)}t}] + G_{46}^0 e^{-(a'_{46})^{(9)}t} \right)$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}} T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le \frac{1}{(\mu_2)^{(9)}} T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}$$

$$\frac{(b_{46})^{(9)} T_{44}^0}{(\mu_1)^{(9)}((R_1)^{(9)} - (b'_{46})^{(9)})} \left[ e^{(R_1)^{(9)}t} - e^{-(b'_{46})^{(9)}t} \right] + T_{46}^0 e^{-(b'_{46})^{(9)}t} \le T_{46}(t) \le$$

$$\frac{(a_{46})^{(9)} T_{44}^0}{(\mu_2)^{(9)}((R_1)^{(9)} + (r_{44})^{(9)} + (R_2)^{(9)})} \left[ e^{((R_1)^{(9)} + (r_{44})^{(9)})t} - e^{-(R_2)^{(9)}t} \right] + T_{46}^0 e^{-(R_2)^{(9)}t}$$

**Definition of** $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$

**Proof :** From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right) - (a''_{14})^{(1)}(T_{14}, t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

**Definition of** $v^{(1)}$ :- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)}\right) \le \frac{dv^{(1)}}{dt} \le -\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)}\right)$$

From which one obtains

**Definition of** $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$ :-

For $0 < \boxed{(v_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}} < (v_1)^{(1)} < (\bar{v}_1)^{(1)}$

$$v^{(1)}(t) \ge \frac{(v_1)^{(1)} + (C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_0)^{(1)}\right)t\right]}}{1 + (C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(v_1)^{(1)} - (v_0)^{(1)}}{(v_0)^{(1)} - (v_2)^{(1)}}}$$

it follows $(v_0)^{(1)} \le v^{(1)}(t) \le (v_1)^{(1)}$

In the same manner , we get 384

$$v^{(1)}(t) \le \frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{v}_1)^{(1)} - (v_0)^{(1)}}{(v_0)^{(1)} - (\bar{v}_2)^{(1)}}}$$

From which we deduce $(v_0)^{(1)} \le v^{(1)}(t) \le (\bar{v}_1)^{(1)}$

If $0 < (v_1)^{(1)} < (v_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}} < (\bar{v}_1)^{(1)}$ we find like in the previous case, 385

$$(v_1)^{(1)} \le \frac{(v_1)^{(1)} + (C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_2)^{(1)}\right)t\right]}}{1 + (C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_2)^{(1)}\right)t\right]}} \le v^{(1)}(t) \le$$

$$\frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}} \le (\bar{v}_1)^{(1)}$$

If $0 < (\nu_1)^{(1)} \leq (\bar{\nu}_1)^{(1)} \leq \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$ , we obtain 386

$$(\nu_1)^{(1)} \leq \nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)} + (\bar{C})^{(1)}(\bar{\nu}_2)^{(1)} e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)} - (\bar{\nu}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)} e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)} - (\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\nu_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(1)}(t)$ :-

$$(m_2)^{(1)} \leq \nu^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{\nu^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(1)}(t)$ :-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{13}'')^{(1)} = (a_{14}'')^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)} G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b_{13}'')^{(1)} = (b_{14}'')^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)} T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

**Proof** : From global equations we obtain 387

$$\frac{d\nu^{(2)}}{dt} = (a_{16})^{(2)} - \left((a_{16}')^{(2)} - (a_{17}')^{(2)} + (a_{16}'')^{(2)}(T_{17}, t)\right) - (a_{17}'')^{(2)}(T_{17}, t)\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

**Definition of** $\nu^{(2)}$ :- $\boxed{\nu^{(2)} = \frac{G_{16}}{G_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \leq \frac{d\nu^{(2)}}{dt} \leq -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

**Definition of** $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$ :-

For $0 < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \geq \frac{(\nu_1)^{(2)}+(\mathrm{C})^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_0)^{(2)}\right)t\right]}}{1+(\mathrm{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\mathrm{C})^{(2)}=\frac{(\nu_1)^{(2)}-(\nu_0)^{(2)}}{(\nu_0)^{(2)}-(\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)}+(\bar{\mathrm{C}})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{\mathrm{C}})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{\mathrm{C}})^{(2)}=\frac{(\bar{\nu}_1)^{(2)}-(\nu_0)^{(2)}}{(\nu_0)^{(2)}-(\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{\mathrm{G}_{16}^0}{\mathrm{G}_{17}^0} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \leq \frac{(\nu_1)^{(2)}+(\mathrm{C})^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}}{1+(\mathrm{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}} \leq \nu^{(2)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(2)}+(\bar{\mathrm{C}})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{\mathrm{C}})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \leq (\bar{\nu}_1)^{(2)} \leq (\nu_0)^{(2)} = \frac{\mathrm{G}_{16}^0}{\mathrm{G}_{17}^0}$ , we obtain 394

$$(\nu_1)^{(2)} \leq \nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)}+(\bar{\mathrm{C}})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{\mathrm{C}})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(2)}(t)$ :- 395

$$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}, \quad \boxed{\nu^{(2)}(t)=\frac{G_{16}(t)}{G_{17}(t)}}$$

In a completely analogous way, we obtain 396

**Definition of** $u^{(2)}(t)$ :-

$$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}, \quad \boxed{u^{(2)}(t)=\frac{T_{16}(t)}{T_{17}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :** 397

If $(a_{16}'')^{(2)} = (a_{17}'')^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)}G_{17}(t)$

Analogously if $(b''_{16})^{(2)} = (b''_{17})^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)}T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

**Proof** : From global equations we obtain 398

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right) - (a''_{21})^{(3)}(T_{21}, t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

**Definition of** $\nu^{(3)}$ :- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right) \leq \frac{d\nu^{(3)}}{dt} \leq -\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right)$$

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From which one obtains

For $0 < (\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\nu_1)^{(3)} < (\bar{\nu}_1)^{(3)}$

$$\nu^{(3)}(t) \geq \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(\nu_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\nu_2)^{(3)}}}$$

it follows $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\nu_1)^{(3)}$

In the same manner , we get 401

$$\nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)} + (\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{\nu}_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\bar{\nu}_2)^{(3)}}}$$

**Definition of** $(\bar{\nu}_1)^{(3)}$ :-

From which we deduce $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\bar{\nu}_1)^{(3)}$

If $0 < (\nu_1)^{(3)} < (\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\bar{\nu}_1)^{(3)}$ we find like in the previous case, 402

$$(\nu_1)^{(3)} \leq \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_2)^{(3)}\right)t\right]}}{1 + (C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_2)^{(3)}\right)t\right]}} \leq \nu^{(3)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(3)} + (\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\bar{\nu}_1)^{(3)}$$

If $0 < (\nu_1)^{(3)} \leq (\bar{\nu}_1)^{(3)} \leq (\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}$ , we obtain 403

$$(\nu_1)^{(3)} \leq \nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\nu_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(3)}(t)$ :-

$$(m_2)^{(3)} \leq \nu^{(3)}(t) \leq (m_1)^{(3)}, \quad \boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(3)}(t)$ :-

$$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}, \quad \boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{20})^{(3)} = (a''_{21})^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)}G_{21}(t)$

Analogously if $(b''_{20})^{(3)} = (b''_{21})^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)}T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

**Proof** : From global equations we obtain 404

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)\right) - (a''_{25})^{(4)}(T_{25}, t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

**Definition of** $\nu^{(4)}$ :- $\quad \boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right) \leq \frac{d\nu^{(4)}}{dt} \leq -\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_4)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$ :-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \geq \frac{(\nu_1)^{(4)}+(C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_0)^{(4)}\right)t\right]}}{4+(C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)}-(\nu_0)^{(4)}}{(\nu_0)^{(4)}-(\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{4+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)}-(\nu_0)^{(4)}}{(\nu_0)^{(4)}-(\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \leq \frac{(\nu_1)^{(4)}+(C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_2)^{(4)}\right)t\right]}}{1+(C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_2)^{(4)}\right)t\right]}} \leq \nu^{(4)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \leq (\bar{\nu}_1)^{(4)} \leq \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$ , we obtain 407

$$(\nu_1)^{(4)} \leq \nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(4)}(t)$ :-

$$(m_2)^{(4)} \leq \nu^{(4)}(t) \leq (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(4)}(t)$ :-

$$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a_{24}'')^{(4)} = (a_{25}'')^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)}G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b_{24}'')^{(4)} = (b_{25}'')^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)}T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

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**Proof** : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a_{28}')^{(5)} - (a_{29}')^{(5)} + (a_{28}'')^{(5)}(T_{29}, t)\right) - (a_{29}'')^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

**Definition of** $\nu^{(5)}$ :- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$ :-

For $0 < \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \leq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}}{1 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}} \leq \nu^{(5)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \leq (\bar{\nu}_1)^{(5)} \leq \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$ , we obtain 411

$$(\nu_1)^{(5)} \leq \nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(5)}(t)$ :-

$(m_2)^{(5)} \leq \nu^{(5)}(t) \leq (m_1)^{(5)}$, $\boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$

In a completely analogous way, we obtain

**Definition of** $u^{(5)}(t)$ :-

$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}$, $\boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{28})^{(5)} = (a''_{29})^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)} G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b''_{28})^{(5)} = (b''_{29})^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$ if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)} T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

**Proof** : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a'_{32})^{(6)} - (a'_{33})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right) - (a''_{33})^{(6)}(T_{33}, t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

**Definition of** $\nu^{(6)}$ :- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \leq \frac{d\nu^{(6)}}{dt} \leq -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$ :-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \geq \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\nu_1)^{(6)}$

In the same manner , we get 413

$$\nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{\nu}_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\bar{\nu}_1)^{(6)}$

If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 414

$$(\nu_1)^{(6)} \leq \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}} \leq \nu^{(6)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\bar{\nu}_1)^{(6)}$$

If $0 < (\nu_1)^{(6)} \leq (\bar{\nu}_1)^{(6)} \leq \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$, we obtain

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$$(\nu_1)^{(6)} \leq \nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(6)}(t)$ :-

$$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}, \quad \boxed{\nu^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(6)}(t)$ :-

$$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}, \quad \boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)}G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)}T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

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**Proof** : From global equations we obtain

$$\frac{d\nu^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right) - (a''_{37})^{(7)}(T_{37}, t)\nu^{(7)} - (a_{37})^{(7)}\nu^{(7)}$$

**Definition of** $\nu^{(7)}$ :- $\boxed{\nu^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right) \leq \frac{d\nu^{(7)}}{dt} \leq -\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(7)}, (\nu_0)^{(7)}$ :-

For $0 < \boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}} < (\nu_1)^{(7)} < (\bar{\nu}_1)^{(7)}$

$$\nu^{(7)}(t) \geq \frac{(\nu_1)^{(7)}+(C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_0)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)}=\frac{(\nu_1)^{(7)}-(\nu_0)^{(7)}}{(\nu_0)^{(7)}-(\nu_2)^{(7)}}}$$

it follows $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\nu_1)^{(7)}$

In the same manner , we get 417

$$\nu^{(7)}(t) \leq \frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)}=\frac{(\bar{\nu}_1)^{(7)}-(\nu_0)^{(7)}}{(\nu_0)^{(7)}-(\bar{\nu}_2)^{(7)}}}$$

From which we deduce $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\bar{\nu}_1)^{(7)}$

If $0 < (\nu_1)^{(7)} < (\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0} < (\bar{\nu}_1)^{(7)}$ we find like in the previous case, 418

$$(\nu_1)^{(7)} \leq \frac{(\nu_1)^{(7)}+(C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}} \leq \nu^{(7)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \leq (\bar{\nu}_1)^{(7)}$$

If $0 < (\nu_1)^{(7)} \leq (\bar{\nu}_1)^{(7)} \leq \boxed{(\nu_0)^{(7)}=\frac{G_{36}^0}{G_{37}^0}}$ , we obtain 419

$$(\nu_1)^{(7)} \leq \nu^{(7)}(t) \leq \frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \leq (\nu_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have
**Definition of** $\nu^{(7)}(t)$ :-

$$(m_2)^{(7)} \leq \nu^{(7)}(t) \leq (m_1)^{(7)}, \quad \boxed{\nu^{(7)}(t)=\frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(7)}(t)$ :- 420

$$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}, \quad \boxed{u^{(7)}(t)=\frac{T_{36}(t)}{T_{37}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $\nu^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)}G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for**

**the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)}T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

**Proof** : From global equations we obtain 421

$$\frac{d\nu^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right) - (a''_{41})^{(8)}(T_{41}, t)\nu^{(8)} - (a_{41})^{(8)}\nu^{(8)}$$

**Definition of** $\nu^{(8)}$ :- $\boxed{\nu^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right) \le \frac{d\nu^{(8)}}{dt} \le -\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$ :-

For $0 < \boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$

$$\nu^{(8)}(t) \ge \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \le \nu^{(8)}(t) \le (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \le \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \le \nu^{(8)}(t) \le (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \le \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}} \le \nu^{(8)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \le (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \le (\bar{\nu}_1)^{(8)} \le \boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}}$ , we obtain 424

$$(v_1)^{(8)} \leq v^{(8)}(t) \leq \frac{(\bar{v}_1)^{(8)}+(\bar{C})^{(8)}(\bar{v}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{v}_1)^{(8)}-(\bar{v}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{v}_1)^{(8)}-(\bar{v}_2)^{(8)}\right)t\right]}} \leq (v_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $v^{(8)}(t)$ :-

$$(m_2)^{(8)} \leq v^{(8)}(t) \leq (m_1)^{(8)}, \quad \boxed{v^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(8)}(t)$ :-

$$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}, \quad \boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(v_1)^{(8)} = (\bar{v}_1)^{(8)}$ if in addition $(v_0)^{(8)} = (v_1)^{(8)}$ then $v^{(8)}(t) = (v_0)^{(8)}$ and as a consequence $G_{40}(t) = (v_0)^{(8)}G_{41}(t)$ **this also defines** $(v_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then $(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)}T_{41}(t)$ This is an important consequence of the relation between $(v_1)^{(8)}$ and $(\bar{v}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

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**Proof** : From 99,20,44,22,23,44 we obtain

$$\frac{dv^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45}, t)\right) - (a''_{45})^{(9)}(T_{45}, t)v^{(9)} - (a_{45})^{(9)}v^{(9)}$$

**Definition of** $v^{(9)}$ :- $\boxed{v^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)}\right) \leq \frac{dv^{(9)}}{dt} \leq -\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_1)^{(9)}v^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

**Definition of** $(\bar{v}_1)^{(9)}, (v_0)^{(9)}$ :-

For $0 < \boxed{(v_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (v_1)^{(9)} < (\bar{v}_1)^{(9)}$

$$v^{(9)}(t) \geq \frac{(v_1)^{(9)}+(C)^{(9)}(v_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((v_1)^{(9)}-(v_0)^{(9)}\right)t\right]}}{1+(C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((v_1)^{(9)}-(v_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(v_1)^{(9)}-(v_0)^{(9)}}{(v_0)^{(9)}-(v_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\nu_9)^{(9)}$

In the same manner , we get

$$\nu^{(9)}(t) \leq \frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)}-(\nu_0)^{(9)}}{(\nu_0)^{(9)}-(\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \leq \frac{(\nu_1)^{(9)}+(C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}}{1+(C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}} \leq \nu^{(9)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \leq (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \leq (\bar{\nu}_1)^{(9)} \leq \boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$ , we obtain

$$(\nu_1)^{(9)} \leq \nu^{(9)}(t) \leq \frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \leq (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(9)}(t)$ :-

$$(m_2)^{(9)} \leq \nu^{(9)}(t) \leq (m_1)^{(9)}, \quad \boxed{\nu^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(9)}(t)$ :-

$$(\mu_2)^{(9)} \leq u^{(9)}(t) \leq (\mu_1)^{(9)}, \quad \boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)}G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then $(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)}T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following

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**Theorem :** If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on $t$ , and the conditions with the notations

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0$ ,

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$

$with\ \ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(2)} and\ (b_i'')^{(2)}$ are independent on t , and the conditions with the notations 426

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$ 427

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 428

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$ , 429

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 430

$with\ \ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(3)} and\ (b_i'')^{(3)}$ are independent on $t$ , and the conditions with the notations 431

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$ ,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

**Theorem :** If $(a_i'')^{(4)} and\ (b_i'')^{(4)}$ are independent on $t$ , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$ ,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(5)} and\ (b_i'')^{(5)}$ are independent on $t$ , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$ ,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

**Theorem** If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on $t$ , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$ ,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(7)} and\ (b_i'')^{(7)}$ are independent on $t$ , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$ ,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(8)} and\ (b_i'')^{(8)}$ are independent on $t$ , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$ ,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(9)} and\ (b_i'')^{(9)}$ are independent on $t$ , and the conditions (with the notations 436 A
45,46,27,28)

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$ ,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0 \quad 437$$

$$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0 \quad 438$$

$$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0 \quad 439$$

$$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0 \quad 440$$

$$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ]T_{14} = 0 \quad 441$$

$$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0 \quad 442$$

has a unique positive solution , which is an equilibrium solution for the system

$$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0 \quad 443$$

$$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0 \quad 444$$

$$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0 \quad 445$$

$$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\ ]T_{16} = 0 \quad 446$$

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\ ]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\ ]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\ ]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\ ]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\ ]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27})\right)]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27})\right)]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27})\right)]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0 \quad 479$$

$$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0 \quad 480$$

$$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0 \quad 481$$

$$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0 \quad 482$$

$$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0 \quad 483$$

$$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0 \quad 484$$

$$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0 \quad 484\text{A}$$

$$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$$

$$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$$

$$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$$

$$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$$

$$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$$

**Proof:** 485

(a) Indeed the first two equations have a nontrivial solution $G_{13}, G_{14}$ if

$$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$$

**Proof:** 486

(n) Indeed the first two equations have a nontrivial solution $G_{16}, G_{17}$ if

$$F(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$$

**Proof:** 487

(a) Indeed the first two equations have a nontrivial solution $G_{20}, G_{21}$ if

$$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$$

**Proof:** 488

(a) Indeed the first two equations have a nontrivial solution $G_{24}, G_{25}$ if

$$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$$

**Proof:** 489

(a) Indeed the first two equations have a nontrivial solution $G_{28}, G_{29}$ if

$$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$$

**Proof:** 490

(a) Indeed the first two equations have a nontrivial solution $G_{32}, G_{33}$ if

$$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$$

**Proof:** 491

(a) Indeed the first two equations have a nontrivial solution $G_{36}, G_{37}$ if

$$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$$

**Proof:** 492

(a) Indeed the first two equations have a nontrivial solution $G_{40}, G_{41}$ if

$$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$$

**Proof:** 492 A

(a) Indeed the first two equations have a nontrivial solution $G_{44}, G_{45}$ if

$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$

**Definition and uniqueness of** $T^*_{14}$ :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique $T^*_{14}$ for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

**Definition and uniqueness of** $T^*_{17}$ :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique $T^*_{17}$ for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]} \qquad 495$$

**Definition and uniqueness of** $T^*_{21}$ :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique $T^*_{21}$ for which $f(T^*_{21}) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

**Definition and uniqueness of** $T^*_{25}$ :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that there exists a unique $T^*_{25}$ for which $f(T^*_{25}) = 0$. With this value , we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T^*_{25})]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T^*_{25})]}$$

**Definition and uniqueness of** $T^*_{29}$ :- 498

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(5)}(T_{29})$ being increasing, it follows that there exists a unique $T^*_{29}$ for which $f(T^*_{29}) = 0$. With this value , we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T^*_{29})]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T^*_{29})]}$$

**Definition and uniqueness of** $T^*_{33}$ :- 499

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(6)}(T_{33})$ being increasing, it follows that

there exists a unique $T_{33}^{*}$ for which $f(T_{33}^{*})=0$. With this value , we obtain from the three first equations

$$G_{32}=\frac{(a_{32})^{(6)}G_{33}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^{*})]} \quad , \quad G_{34}=\frac{(a_{34})^{(6)}G_{33}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^{*})]}$$

**Definition and uniqueness of** $T_{37}^{*}$ :- 500

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(7)}(T_{37})$ being increasing, it follows that there exists a unique $T_{37}^{*}$ for which $f(T_{37}^{*})=0$. With this value , we obtain from the three first equations

$$G_{36}=\frac{(a_{36})^{(7)}G_{37}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^{*})]} \quad , \quad G_{38}=\frac{(a_{38})^{(7)}G_{37}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^{*})]}$$

**Definition and uniqueness of** $T_{41}^{*}$ :- 501

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique $T_{41}^{*}$ for which $f(T_{41}^{*})=0$. With this value , we obtain from the three first equations

$$G_{40}=\frac{(a_{40})^{(8)}G_{41}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T_{41}^{*})]} \quad , \quad G_{42}=\frac{(a_{42})^{(8)}G_{41}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T_{41}^{*})]}$$

**Definition and uniqueness of** $T_{45}^{*}$ :- 501 A

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(9)}(T_{45})$ being increasing, it follows that there exists a unique $T_{45}^{*}$ for which $f(T_{45}^{*})=0$. With this value , we obtain from the three first equations

$$G_{44}=\frac{(a_{44})^{(9)}G_{45}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T_{45}^{*})]} \quad , \quad G_{46}=\frac{(a_{46})^{(9)}G_{45}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T_{45}^{*})]}$$

By the same argument, the equations admit solutions $G_{13}, G_{14}$ if 502

$$\varphi(G)=(b'_{13})^{(1)}(b'_{14})^{(1)}-(b_{13})^{(1)}(b_{14})^{(1)}-$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G)+(b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right]+(b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G)=0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{14}$ taking into account the hypothesis $\varphi(0)>0$ , $\varphi(\infty)<0$ it follows that there exists a unique $G_{14}^{*}$ such that $\varphi(G^{*})=0$

By the same argument, the equations admit solutions $G_{16}, G_{17}$ if 503

$$\varphi(G_{19})=(b'_{16})^{(2)}(b'_{17})^{(2)}-(b_{16})^{(2)}(b_{17})^{(2)}-$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19})+(b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right]+(b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19})=0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{17}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{14}^{*}$ such that $\varphi((G_{19})^{*}) = 0$ 504

By the same argument, the equations admit solutions $G_{20}, G_{21}$ if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right] + (b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{21}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{21}^{*}$ such that $\varphi((G_{23})^{*}) = 0$

By the same argument, the equations admit solutions $G_{24}, G_{25}$ if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right] + (b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{25}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{25}^{*}$ such that $\varphi((G_{27})^{*}) = 0$

By the same argument, the equations admit solutions $G_{28}, G_{29}$ if 507

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right] + (b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{29}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{29}^{*}$ such that $\varphi((G_{31})^{*}) = 0$

By the same argument, the equations admit solutions $G_{32}, G_{33}$ if 508

$$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$$

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right] + (b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{33}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{33}^{*}$ such that $\varphi({G_{35}}^{*}) = 0$

By the same argument, the equations admit solutions $G_{36}, G_{37}$ if 509

$$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$$

$$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right] + (b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{37}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{37}^{*}$ such that $\varphi({G_{39}}^{*}) = 0$

By the same argument, the equations admit solutions $G_{40}, G_{41}$ if 510

$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$
$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right] + (b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{41}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{41}$ such that $\varphi(G_{43}{}^*) = 0$

By the same argument, the equations 92,93 admit solutions $G_{44}, G_{45}$ if
$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$

$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right] + (b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{45}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{45}$ such that $\varphi((G_{47})^*) = 0$

Finally we obtain the unique solution 511

$G^*_{14}$ given by $\varphi(G^*) = 0$ , $T^*_{14}$ given by $f(T^*_{14}) = 0$ and

$$G^*_{13} = \frac{(a_{13})^{(1)}G^*_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G^*_{15} = \frac{(a_{15})^{(1)}G^*_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

$$T^*_{13} = \frac{(b_{13})^{(1)}T^*_{14}}{[(b'_{13})^{(1)}-(b''_{13})^{(1)}(G^*)]} \quad , \quad T^*_{15} = \frac{(b_{15})^{(1)}T^*_{14}}{[(b'_{15})^{(1)}-(b''_{15})^{(1)}(G^*)]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

$G^*_{17}$ given by $\varphi((G_{19})^*) = 0$ , $T^*_{17}$ given by $f(T^*_{17}) = 0$ and 512

$$G^*_{16} = \frac{(a_{16})^{(2)}G^*_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G^*_{18} = \frac{(a_{18})^{(2)}G^*_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]} \quad 513$$

$$T^*_{16} = \frac{(b_{16})^{(2)}T^*_{17}}{[(b'_{16})^{(2)}-(b''_{16})^{(2)}((G_{19})^*)]} \quad , \quad T^*_{18} = \frac{(b_{18})^{(2)}T^*_{17}}{[(b'_{18})^{(2)}-(b''_{18})^{(2)}((G_{19})^*)]} \quad 514$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

$G^*_{21}$ given by $\varphi((G_{23})^*) = 0$ , $T^*_{21}$ given by $f(T^*_{21}) = 0$ and

$$G^*_{20} = \frac{(a_{20})^{(3)}G^*_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G^*_{22} = \frac{(a_{22})^{(3)}G^*_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

$$T^*_{20} = \frac{(b_{20})^{(3)}T^*_{21}}{[(b'_{20})^{(3)}-(b''_{20})^{(3)}(G_{23}{}^*)]} \quad , \quad T^*_{22} = \frac{(b_{22})^{(3)}T^*_{21}}{[(b'_{22})^{(3)}-(b''_{22})^{(3)}(G_{23}{}^*)]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

$G_{25}^{*}$ given by $\varphi(G_{27}) = 0$ , $T_{25}^{*}$ given by $f(T_{25}^{*}) = 0$ and

$$G_{24}^{*} = \frac{(a_{24})^{(4)} G_{25}^{*}}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}^{*} = \frac{(a_{26})^{(4)} G_{25}^{*}}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^{*})]}$$

$$T_{24}^{*} = \frac{(b_{24})^{(4)} T_{25}^{*}}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^{*})]} \quad , \quad T_{26}^{*} = \frac{(b_{26})^{(4)} T_{25}^{*}}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^{*})]} \qquad 517$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

$G_{29}^{*}$ given by $\varphi((G_{31})^{*}) = 0$ , $T_{29}^{*}$ given by $f(T_{29}^{*}) = 0$ and

$$G_{28}^{*} = \frac{(a_{28})^{(5)} G_{29}^{*}}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}^{*} = \frac{(a_{30})^{(5)} G_{29}^{*}}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^{*})]}$$

$$T_{28}^{*} = \frac{(b_{28})^{(5)} T_{29}^{*}}{[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31})^{*})]} \quad , \quad T_{30}^{*} = \frac{(b_{30})^{(5)} T_{29}^{*}}{[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31})^{*})]} \qquad 519$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

$G_{33}^{*}$ given by $\varphi((G_{35})^{*}) = 0$ , $T_{33}^{*}$ given by $f(T_{33}^{*}) = 0$ and

$$G_{32}^{*} = \frac{(a_{32})^{(6)} G_{33}^{*}}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^{*})]} \quad , \quad G_{34}^{*} = \frac{(a_{34})^{(6)} G_{33}^{*}}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^{*})]}$$

$$T_{32}^{*} = \frac{(b_{32})^{(6)} T_{33}^{*}}{[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35})^{*})]} \quad , \quad T_{34}^{*} = \frac{(b_{34})^{(6)} T_{33}^{*}}{[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35})^{*})]} \qquad 521$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

$G_{37}^{*}$ given by $\varphi((G_{39})^{*}) = 0$ , $T_{37}^{*}$ given by $f(T_{37}^{*}) = 0$ and

$$G_{36}^{*} = \frac{(a_{36})^{(7)} G_{37}^{*}}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^{*})]} \quad , \quad G_{38}^{*} = \frac{(a_{38})^{(7)} G_{37}^{*}}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^{*})]}$$

$$T_{36}^{*} = \frac{(b_{36})^{(7)} T_{37}^{*}}{[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39})^{*})]} \quad , \quad T_{38}^{*} = \frac{(b_{38})^{(7)} T_{37}^{*}}{[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39})^{*})]}$$

Finally we obtain the unique solution 523

$G_{41}^{*}$ given by $\varphi((G_{43})^{*}) = 0$ , $T_{41}^{*}$ given by $f(T_{41}^{*}) = 0$ and

$$G_{40}^{*} = \frac{(a_{40})^{(8)} G_{41}^{*}}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^{*})]} \quad , \quad G_{42}^{*} = \frac{(a_{42})^{(8)} G_{41}^{*}}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^{*})]}$$

$$T_{40}^{*}=\frac{(b_{40})^{(8)}T_{41}^{*}}{[(b'_{40})^{(8)}-(b''_{40})^{(8)}((G_{43})^{*})]} \quad , \quad T_{42}^{*}=\frac{(b_{42})^{(8)}T_{41}^{*}}{[(b'_{42})^{(8)}-(b''_{42})^{(8)}((G_{43})^{*})]}$$

Finally we obtain the unique solution of 89 to 99 523

A

$G_{45}^{*}$ given by $\varphi((G_{47})^{*})=0$ , $T_{45}^{*}$ given by $f(T_{45}^{*})=0$ and

$$G_{44}^{*}=\frac{(a_{44})^{(9)}G_{45}^{*}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T_{45}^{*})]} \quad , \quad G_{46}^{*}=\frac{(a_{46})^{(9)}G_{45}^{*}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T_{45}^{*})]}$$

$$T_{44}^{*}=\frac{(b_{44})^{(9)}T_{45}^{*}}{[(b'_{44})^{(9)}-(b''_{44})^{(9)}((G_{47})^{*})]} \quad , \quad T_{46}^{*}=\frac{(b_{46})^{(9)}T_{45}^{*}}{[(b'_{46})^{(9)}-(b''_{46})^{(9)}((G_{47})^{*})]}$$

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**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}\ and\ (b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

**Proof:** Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i=G_i^{*}+\mathbb{G}_i \qquad , T_i=T_i^{*}+\mathbb{T}_i$$

$$\frac{\partial(a''_{14})^{(1)}}{\partial T_{14}}(T_{14}^{*})=(q_{14})^{(1)} \quad , \frac{\partial(b_i'')^{(1)}}{\partial G_j}(G^{*})=s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt}=-\left((a'_{13})^{(1)}+(p_{13})^{(1)}\right)\mathbb{G}_{13}+(a_{13})^{(1)}\mathbb{G}_{14}-(q_{13})^{(1)}G_{13}^{*}\mathbb{T}_{14} \quad 525$$

$$\frac{d\mathbb{G}_{14}}{dt}=-\left((a'_{14})^{(1)}+(p_{14})^{(1)}\right)\mathbb{G}_{14}+(a_{14})^{(1)}\mathbb{G}_{13}-(q_{14})^{(1)}G_{14}^{*}\mathbb{T}_{14} \quad 526$$

$$\frac{d\mathbb{G}_{15}}{dt}=-\left((a'_{15})^{(1)}+(p_{15})^{(1)}\right)\mathbb{G}_{15}+(a_{15})^{(1)}\mathbb{G}_{14}-(q_{15})^{(1)}G_{15}^{*}\mathbb{T}_{14} \quad 527$$

$$\frac{d\mathbb{T}_{13}}{dt}=-\left((b'_{13})^{(1)}-(r_{13})^{(1)}\right)\mathbb{T}_{13}+(b_{13})^{(1)}\mathbb{T}_{14}+\sum_{j=13}^{15}\left(s_{(13)(j)}T_{13}^{*}\mathbb{G}_j\right) \quad 528$$

$$\frac{d\mathbb{T}_{14}}{dt}=-\left((b'_{14})^{(1)}-(r_{14})^{(1)}\right)\mathbb{T}_{14}+(b_{14})^{(1)}\mathbb{T}_{13}+\sum_{j=13}^{15}\left(s_{(14)(j)}T_{14}^{*}\mathbb{G}_j\right) \quad 529$$

$$\frac{d\mathbb{T}_{15}}{dt}=-\left((b'_{15})^{(1)}-(r_{15})^{(1)}\right)\mathbb{T}_{15}+(b_{15})^{(1)}\mathbb{T}_{14}+\sum_{j=13}^{15}\left(s_{(15)(j)}T_{15}^{*}\mathbb{G}_j\right) \quad 530$$

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**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(2)}$ and $(b_i'')^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i \tag{532}$$

$$\frac{\partial(a_{17}'')^{(2)}}{\partial T_{17}}(T_{17}^*) = (q_{17})^{(2)} \quad , \frac{\partial(b_i'')^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \tag{533}$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a_{16}')^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G_{16}^*\mathbb{T}_{17} \tag{534}$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a_{17}')^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G_{17}^*\mathbb{T}_{17} \tag{535}$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a_{18}')^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G_{18}^*\mathbb{T}_{17} \tag{536}$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b_{16}')^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T_{16}^*\mathbb{G}_j\right) \tag{537}$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b_{17}')^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}T_{17}^*\mathbb{G}_j\right) \tag{538}$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b_{18}')^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}T_{18}^*\mathbb{G}_j\right) \tag{539}$$

**ASYMPTOTIC STABILITY ANALYSIS** 540

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(3)}$ *and* $(b_i'')^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{21}'')^{(3)}}{\partial T_{21}}(T_{21}^*) = (q_{21})^{(3)} \quad , \frac{\partial(b_i'')^{(3)}}{\partial G_j}((G_{23})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a_{20}')^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G_{20}^*\mathbb{T}_{21} \tag{541}$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a_{21}')^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G_{21}^*\mathbb{T}_{21} \tag{542}$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a_{22}')^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G_{22}^*\mathbb{T}_{21} \tag{543}$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b_{20}')^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T_{20}^*\mathbb{G}_j\right) \tag{544}$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b_{21}')^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T_{21}^*\mathbb{G}_j\right) \tag{545}$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b'_{22})^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T^*_{22}\mathbb{G}_j\right) \quad 546$$

**ASYMPTOTIC STABILITY ANALYSIS** 547

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}\ and\ (b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{25})^{(4)}}{\partial T_{25}}(T^*_{25}) = (q_{25})^{(4)} \ , \ \frac{\partial(b_i'')^{(4)}}{\partial G_j}((G_{27})^*\ ) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\left((a'_{24})^{(4)} + (p_{24})^{(4)}\right)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G^*_{24}\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\left((a'_{25})^{(4)} + (p_{25})^{(4)}\right)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G^*_{25}\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\left((a'_{26})^{(4)} + (p_{26})^{(4)}\right)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G^*_{26}\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\left((b'_{24})^{(4)} - (r_{24})^{(4)}\right)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(24)(j)}T^*_{24}\mathbb{G}_j\right) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\left((b'_{25})^{(4)} - (r_{25})^{(4)}\right)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\left(s_{(25)(j)}T^*_{25}\mathbb{G}_j\right) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T^*_{26}\mathbb{G}_j\right) \quad 554$$

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**Theorem 5:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}\ and\ (b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 556

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{29})^{(5)}}{\partial T_{29}}(T^*_{29}) = (q_{29})^{(5)} \ , \ \frac{\partial(b_i'')^{(5)}}{\partial G_j}(\ (G_{31})^*\ ) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a'_{28})^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G^*_{28}\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a'_{29})^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G^*_{29}\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a'_{30})^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G^*_{30}\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b'_{28})^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T^*_{28}\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b'_{29})^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T^*_{29}\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b'_{30})^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T^*_{30}\mathbb{G}_j\right) \quad 562$$

**ASYMPTOTIC STABILITY ANALYSIS** 563

**Theorem 6:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}$ *and* $(b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{33})^{(6)}}{\partial T_{33}}(T^*_{33}) = (q_{33})^{(6)} \quad , \frac{\partial(b_i'')^{(6)}}{\partial G_j}\left((G_{35})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a'_{32})^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G^*_{32}\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a'_{33})^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G^*_{33}\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a'_{34})^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G^*_{34}\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b'_{32})^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T^*_{32}\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b'_{33})^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T^*_{33}\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b'_{34})^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T^*_{34}\mathbb{G}_j\right) \quad 570$$

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**Theorem 7:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 572

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{37}'')^{(7)}}{\partial T_{37}}(T_{37}^{*}) = (q_{37})^{(7)} \quad , \frac{\partial(b_i'')^{(7)}}{\partial G_j}\left( (G_{39})^{**} \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a_{36}')^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G_{36}^{*}\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a_{37}')^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G_{37}^{*}\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a_{38}')^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G_{38}^{*}\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b_{36}')^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T_{36}^{*}\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b_{37}')^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T_{37}^{*}\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b_{38}')^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T_{38}^{*}\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

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**Theorem 8:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 580

$$G_i = G_i^{*} + \mathbb{G}_i \qquad , T_i = T_i^{*} + \mathbb{T}_i$$

$$\frac{\partial(a_{41}'')^{(8)}}{\partial T_{41}}(T_{41}^{*}) = (q_{41})^{(8)} \quad , \frac{\partial(b_i'')^{(8)}}{\partial G_j}\left( (G_{43})^{*} \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a_{40}')^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G_{40}^{*}\mathbb{T}_{41} \quad 581$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a_{41}')^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G_{41}^{*}\mathbb{T}_{41} \quad 582$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a_{42}')^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G_{42}^{*}\mathbb{T}_{41} \quad 583$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b_{40}')^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T_{40}^{*}\mathbb{G}_j\right) \quad 584$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b_{41}')^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T_{41}^{*}\mathbb{G}_j\right) \quad 585$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b'_{42})^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T^*_{42}\mathbb{G}_j\right) \quad 586$$

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**ASYMPTOTIC STABILITY ANALYSIS** A

**Theorem 9:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}\ and\ (b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

**Proof: Denote**

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{45})^{(9)}}{\partial T_{45}}(T^*_{45}) = (q_{45})^{(9)} \ , \ \frac{\partial(b_i'')^{(9)}}{\partial G_j}(\ (G_{47})^*\ ) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a'_{44})^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)}G^*_{44}\mathbb{T}_{45} \quad 586\ B$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a'_{45})^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)}G^*_{45}\mathbb{T}_{45} \quad 586\ C$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a'_{46})^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)}G^*_{46}\mathbb{T}_{45} \quad 586\ D$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b'_{44})^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)}T^*_{44}\mathbb{G}_j\right) \quad 586\ E$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b'_{45})^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)}T^*_{45}\mathbb{G}_j\right) \quad 586\ F$$

$$\frac{d\mathbb{T}_{46}}{dt} = -\left((b'_{46})^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T^*_{46}\mathbb{G}_j\right) \quad 586\ G$$

**The characteristic equation of this system is** 587

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(q_{13})^{(1)}G^*_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)}T^*_{14} + (b_{14})^{(1)}s_{(13),(14)}T^*_{14}\right)$$

$$+\left(\left((\lambda)^{(1)} + (a'_{14})^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)}G^*_{13} + (a_{13})^{(1)}(q_{14})^{(1)}G^*_{14}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)}T^*_{14} + (b_{14})^{(1)}s_{(13),(13)}T^*_{13}\right)$$

$$\left((\lambda)^{(1)}\right)^2 + \left(\ (a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\Big)$$

$$\left((\lambda)^{(1)}\right)^2 + \left(\ (b'_{13})^{(1)} + (b'_{14})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)(\lambda)^{(1)}\Big)$$

$$+\left(\left((\lambda)^{(1)}\right)^2+\left((a'_{13})^{(1)}+(a'_{14})^{(1)}+(p_{13})^{(1)}+(p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)}+(a'_{13})^{(1)}+(p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^*_{14}+(a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}+(b'_{13})^{(1)}-(r_{13})^{(1)}\right)s_{(14),(15)}T^*_{14}+(b_{14})^{(1)}s_{(13),(15)}T^*_{13}\right)\}=0$$

$$+$$

$$(\lambda)^{(2)}+(b'_{18})^{(2)}-(r_{18})^{(2)}\}\{\left((\lambda)^{(2)}+(a'_{18})^{(2)}+(p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^*_{17}+(a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^*_{17}\right)$$

$$+\left(\left((\lambda)^{(2)}+(a'_{17})^{(2)}+(p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^*_{16}+(a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2+\left((b'_{16})^{(2)}+(b'_{17})^{(2)}-(r_{16})^{(2)}+(r_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$+\left(\left((\lambda)^{(2)}\right)^2+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}+(a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^*_{16}\right)\}=0$$

$$+$$

$$(\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\}\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left(\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^*_{20}\right)\right]$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^*_{21}+(b_{21})^{(3)}s_{(20),(21)}T^*_{21}\right)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^*_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^*_{21}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^*_{21}+(b_{21})^{(3)}s_{(20),(20)}T^*_{20}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^2+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$+\left(\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^*_{20}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^*_{21}+(b_{21})^{(3)}s_{(20),(22)}T^*_{20}\right)\}=0$$

$$+$$

$$(\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left(\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^*_{24}\right)\right]$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^*_{25}+(b_{25})^{(4)}s_{(24),(25)}T^*_{25}\right)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^*_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^*_{25}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^*_{25}+(b_{25})^{(4)}s_{(24),(24)}T^*_{24}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^2+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$+\left(\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^*_{24}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^*_{25}+(b_{25})^{(4)}s_{(24),(26)}T^*_{24}\right)\}=0$$

$$+$$

$$(\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^*_{28}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^*_{29}+(b_{29})^{(5)}s_{(28),(29)}T^*_{29}\right)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^*_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^*_{29}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^*_{29}+(b_{29})^{(5)}s_{(28),(28)}T^*_{28}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^2+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$+\left(\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^*_{28}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^*_{29}+(b_{29})^{(5)}s_{(28),(30)}T^*_{28}\right)\}=0$$

$$+$$

$$(\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left(\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^*_{32}\right)\right]$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^*_{33}+(b_{33})^{(6)}s_{(32),(33)}T^*_{33}\right)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^*_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^*_{33}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^*_{33}+(b_{33})^{(6)}s_{(32),(32)}T^*_{32}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^2+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$+\left(\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^*_{32}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^*_{33}+(b_{33})^{(6)}s_{(32),(34)}T^*_{32}\right)\}=0$$

$$+$$

$$(\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^*_{36}\right)\right]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^*_{37}+(b_{37})^{(7)}s_{(36),(37)}T^*_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^*_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^*_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^*_{37}+(b_{37})^{(7)}s_{(36),(36)}T^*_{36}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^2+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$+\left(\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^*_{36}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^*_{37}+(b_{37})^{(7)}s_{(36),(38)}T^*_{36}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\right)\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^*_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^*_{41}+(b_{41})^{(8)}s_{(40),(41)}T^*_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^*_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^*_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^*_{41}+(b_{41})^{(8)}s_{(40),(40)}T^*_{40}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^2+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$+\left(\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^*_{40}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^*_{41}+(b_{41})^{(8)}s_{(40),(42)}T^*_{40}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\right)\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^*_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^*_{45}+(b_{45})^{(9)}s_{(44),(45)}T^*_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^*_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^*_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^*_{45}+(b_{45})^{(9)}s_{(44),(44)}T^*_{44}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^2+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$+\left(\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^*_{44}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^*_{45}+(b_{45})^{(9)}s_{(44),(46)}T^*_{44}\right)\}=0$$

**And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.**

## SECTION SIXTEEN

## Weyl' Leading Term Ak$^2$/4$\Pi$,

### INTRODUCTION—VARIABLES USED

**Neutrino Billiards: Time-Reversal Symmetry-Breaking without Magnetic Fields M. V. Berry, R. J. Mondragon Published 8 July 1987.DOI: 10.1098/rspa.1987.0080 Royal society proceedings**

(1) Semiclassical (large-k) asymptotics of the eigenvalue counting function (spectral staircase) N(k) are shown to have (e) the `Weyl' leading term Ak$^2$/4$\pi$, where A is the area of D, but zero perimeter correction.
(2) The Dirac equation is transformed to (e&eb) an integral equation round the boundary of D, and forms (eb) the basis of a numerical method for computing the k$_n$.
(3) When D is the unit disc, geodesics are (eb) integrable and the eigenvalues, which satisfy $J_l(k_n) = J_{l+1}(k_n)$, are (=) (locally) Poisson-distributed.
(4) When D is an `Africa' shape (cubic conformal map of the unit disc), (eb) the eigenvalues are (locally) distributed according to (e&eb) the statistics of the Gaussian unitary ensemble of random-matrix theory, as predicted on the basis of (e) T-breaking and lack of geometric symmetry.

**Science 18 April 2008: Vol. 320 no. 5874 pp. 356-358 DOI: 10.1126/science.1154663 Chaotic Dirac Billiard in Graphene Quantum Dots L. A. Ponomarenko1, F. Schedin1, M. I. Katsnelson2, R. Yang1, E. W. Hill1, K. S. Novoselov1,*, A. K. Geim1**

(6) The exceptional electronic properties of graphene, with (e&eb) its charge carriers mimicking (e&eb) relativistic quantum particles and its formidable potential in various applications, have ensured (eb) a rapid growth of interest in this new material.
(7) Authors report on electron transport in quantum dot devices carved entirely from (e) graphene.
(8) At large sizes (>100 nanometers), they behave as (=) conventional single-electron transistors, exhibiting (eb) periodic Coulomb blockade peaks.
**(9)** For quantum dots smaller than 100 nanometers, the peaks become (=) strongly nonperiodic,

indicating (eb) a major contribution of quantum confinement. Random peak spacing and its statistics are well described by (e) the theory of chaotic neutrino billiards.

**(10)** Short constrictions of only a few nanometers in width remain conductive and reveal (eb) a confinement gap of up to 0.5 electron volt, demonstrating (eb) the possibility of molecular-scale electronics based on graphene

**Graphene quantum dots: Beyond a Dirac billiard F. Libisch, C. Stampfer, J. Burgdörfer**

(5) Authors present realistic simulations of quantum confinement effects in ballistic graphene quantum dots with linear dimensions of 10 to 40 nm.

(6) Authors determine wavefunctions and energy level statistics in the presence of disorder resulting from edge roughness, charge impurities, or short-ranged scatterers.

**(7)** Marked deviations from a simple Dirac billiard for massless fermions are found.

**(8)** Authors find a remarkably stable dependence of the nearest-neighbor level spacing on edge roughness suggesting that the roughness of fabricated devices can be potentially characterized by the distribution of measured Coulomb blockade peaks. Subjects: Mesoscale and Nanoscale Physics (cond-mat.mes-hall) Journal reference: Phys. Rev. B, 79, 115423 (2009) DOI: 10.1103/PhysRevB.79.115423 Cite as: arXiv: 0808.3095 [cond-mat.mes-hall] (or arXiv: 0808.3095v2 [cond-mat.mes-hall] for this version)

## NOTATION

### Module One

Semiclassical (large-k) asymptotics of the eigenvalue counting function (spectral staircase) N(k) are shown to have (e) the `Weyl' leading term Ak$^2$/4$\pi$, where A is the area of D, but zero perimeter correction

$G_{13}$ : Category one of `Weyl' leading term Ak$^2$/4$\pi$, where A is the area of D, but zero perimeter correction

$G_{14}$ : Category two of SAS

$G_{15}$ : Category three of SAS

$T_{13}$ : Category one of Semiclassical (large-k) asymptotics of the eigenvalue counting function (spectral staircase) N(k)

$T_{14}$ : Category two of SAS

$T_{15}$ : Category three of SAS

### Module Two

The Dirac equation is transformed to (e&eb) an integral equation round the boundary of D, and forms (eb) the basis of a numerical method for computing the k$_n$.

$G_{16}$ : Category one of **Dirac equation;** integral equation round the boundary of D, and forms (eb) the basis of a numerical method for computing the k$_n$.

$G_{17}$ : Category two of SAS

$G_{18}$ : Category three of SAS

$T_{16}$ : Category one of integral equation round the boundary of D, and forms (eb) the basis of a numerical

method for computing the k$_n$.; **Dirac equation**

$T_{17}$ : Category two of SAS

$T_{18}$ : Category three of SAS

**Module three**

The Dirac equation is transformed to an integral equation round the boundary of D, and forms (eb) the basis of a numerical method for computing the k$_n$.

$G_{20}$ : Category one of Dirac equation is transformed to an integral equation round the boundary of D

$G_{21}$ : Category two of SAS

$G_{22}$ : Category three of SAS

$T_{20}$ : Category one of basis of a numerical method for computing the k$_n$.

$T_{21}$ : Category two of SAS

$T_{22}$ : Category three of SAS

**Module four**

When D is the unit disc, geodesics are (eb) integrable and the eigenvalues, which satisfy $J_l(k_n) = J_{l+1}(k_n)$, are (=) (locally) Poisson-distributed

$G_{24}$ : Category one of D is the unit disc

$G_{25}$ : Category two of SAS

$G_{26}$ : Category three of SAS

$T_{24}$ : Category one of geodesics are (eb) integrable and the eigenvalues, which satisfy $J_l(k_n) = J_{l+1}(k_n)$, are (=) (locally) Poisson-distributed

$T_{25}$ : Category two of SAS

$T_{26}$ : Category three of SAS

**Module five**

When D is the unit disc, geodesics are integrable and the eigenvalues, which satisfy $J_l(k_n) = J_{l+1}(k_n)$, are (=) (locally) Poisson-distributed

$G_{28}$ : Category one of D is the unit disc, geodesics are integrable

$G_{29}$ : Category two of SAS

$G_{30}$ : Category three of SAS

$T_{28}$ : Category one of eigenvalues, which satisfy $J_l(k_n) = J_{l+1}(k_n)$, are (=) (locally) Poisson-distributed

$T_{29}$ : Category two of SAS

$T_{30}$ : Category three of SAS

**Module six**

When D is an `Africa' shape (cubic conformal map of the unit disc), (eb) the eigenvalues are (locally) distributed according to (e&eb) the statistics of the Gaussian unitary ensemble of random-matrix theory, as predicted on the basis of (e) T-breaking and lack of geometric symmetry

$G_{32}$ : Category one of D is an `Africa' shape (cubic conformal map of the unit disc),

$G_{33}$ : Category two of SAS

$G_{34}$ : Category three of SAS

$T_{32}$ : Category one of eigenvalues are (locally) distributed according to (e&eb) the statistics of the Gaussian unitary ensemble of random-matrix theory, as predicted on the basis of (e) T-breaking and lack of geometric symmetry

$T_{33}$ : Category two of SAS

$T_{34}$ : Category three of SAS

**Module seven**

When D is an `Africa' shape (cubic conformal map of the unit disc), the eigenvalues are (locally) distributed according to (e&eb) the statistics of the Gaussian unitary ensemble of random-matrix theory, as predicted on the basis of (e) T-breaking and lack of geometric symmetry

$G_{36}$ : Category one of **D is an `Africa' shape (cubic conformal map of the unit disc), the eigenvalues are (locally) distributed;** statistics of the Gaussian unitary ensemble of random-matrix theory, as predicted on the basis of (e) T-breaking and lack of geometric symmetry

$G_{37}$ : Category two of SAS

$G_{38}$ : Category three of SAS

$T_{36}$ : Category one of statistics of the Gaussian unitary ensemble of random-matrix theory, as predicted on the basis of (e) T-breaking and lack of geometric symmetry; **D is an `Africa' shape (cubic conformal map of the unit disc), the eigenvalues are (locally) distributed**

$T_{37}$ : Category two of SAS

$T_{38}$ : Category three of SAS

**Module eight**

When D is an `Africa' shape (cubic conformal map of the unit disc), the eigenvalues are (locally) distributed according to the statistics of the Gaussian unitary ensemble of random-matrix theory, as predicted on the basis of (e) T-breaking and lack of geometric symmetry

$G_{40}$ : Category one of T-breaking and lack of geometric symmetry

$G_{41}$ : Category two of SAS

$G_{42}$ : Category three of SAS

$T_{40}$ : Category one of D is an `Africa' shape (cubic conformal map of the unit disc), the eigenvalues are (locally) distributed according to the statistics of the Gaussian unitary ensemble of random-matrix theory, as predicted on the basis

$T_{41}$ : Category two of SAS

$T_{42}$ : Category three of SAS

**Module Nine**

Authors report on electron transport in quantum dot devices carved entirely from (e) graphene

$G_{44}$ : Category one of graphene

$G_{45}$ : Category two of SAS

$G_{46}$ : Category three of SAS

$T_{44}$ : Category one of electron transport in quantum dot devices

$T_{45}$ : Category two of SAS

$T_{46}$ : Category three of SAS

**The Coefficients:**

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}$ , $(b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}$,
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}$, $(b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}\ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$ ,
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)}$,
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)}$,
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)}$,
are Dissipation coefficients

**Module Numbered One**

**The differential system of this model is now (Module Numbered one)**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14},t)\right]G_{13} \quad 1$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14},t)\right]G_{14} \quad 2$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14},t)\right]G_{15} \quad 3$$

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G,t)\right]T_{13} \quad 4$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G,t)\right]T_{14} \quad 5$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G,t)\right]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14},t) =$ First augmentation factor

$-(b''_{13})^{(1)}(G,t) =$ First detritions factor

**Module Numbered Two**

**The differential system of this model is now ( Module numbered two)**

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17},t)\right]G_{16} \quad 7$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17},t)\right]G_{17} \quad 8$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17},t)\right]G_{18} \quad 9$$

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\big((G_{19}),t\big)\right]T_{16} \quad 10$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\big((G_{19}),t\big)\right]T_{17} \quad 11$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\big((G_{19}),t\big)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17},t) =$ First augmentation factor

$-(b''_{16})^{(2)}\big((G_{19}),t\big) =$ First detritions factor

**Module Numbered Three**

**The differential system of this model is now (Module numbered three)**

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21},t)\right]G_{20} \quad 13$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21},t)\right]G_{21} \quad 14$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21},t)\right]G_{22} \quad 15$$

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23},t)\right]T_{20} \quad 16$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23},t)\right]T_{21} \quad 17$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23},t)\right]T_{22} \quad 18$$

$+(a''_{20})^{(3)}(T_{21},t) =$ First augmentation factor

$-(b''_{20})^{(3)}(G_{23},t) =$ First detritions factor

**Module Numbered Four**

**The differential system of this model is now (Module numbered Four)**

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right]G_{24} \quad 19$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25},t)\right]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}, t)\right]G_{26} \quad 21$$
$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27}), t\right)\right]T_{24} \quad 22$$
$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27}), t\right)\right]T_{25} \quad 23$$
$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27}), t\right)\right]T_{26} \quad 24$$
$+(a''_{24})^{(4)}(T_{25}, t) =$ **First augmentation factor**
$-(b''_{24})^{(4)}\left((G_{27}), t\right) =$ **First detritions factor**

**Module Numbered Five**:

**The differential system of this model is now (Module number five)**

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right]G_{28} \quad 25$$
$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}, t)\right]G_{29} \quad 26$$
$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}, t)\right]G_{30} \quad 27$$
$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}\left((G_{31}), t\right)\right]T_{28} \quad 28$$
$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}\left((G_{31}), t\right)\right]T_{29} \quad 29$$
$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}\left((G_{31}), t\right)\right]T_{30} \quad 30$$
$+(a''_{28})^{(5)}(T_{29}, t) =$ **First augmentation factor**
$-(b''_{28})^{(5)}\left((G_{31}), t\right) =$ **First detritions factor**

**Module Numbered Six**

**The differential system of this model is now (Module numbered Six)**

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right]G_{32} \quad 31$$
$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}, t)\right]G_{33} \quad 32$$
$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}, t)\right]G_{34} \quad 33$$
$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}\left((G_{35}), t\right)\right]T_{32} \quad 34$$
$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}\left((G_{35}), t\right)\right]T_{33} \quad 35$$
$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}\left((G_{35}), t\right)\right]T_{34} \quad 36$$
$+(a''_{32})^{(6)}(T_{33}, t) =$ **First augmentation factor**

**Module Numbered Seven:**

**The differential system of this model is now (Seventh Module)**

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right]G_{36} \quad 37$$
$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37}, t)\right]G_{37} \quad 38$$
$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}, t)\right]G_{38} \quad 39$$
$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}\left((G_{39}), t\right)\right]T_{36} \quad 40$$
$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}\left((G_{39}), t\right)\right]T_{37} \quad 41$$
$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}\left((G_{39}), t\right)\right]T_{38} \quad 42$$
$+(a''_{36})^{(7)}(T_{37}, t) =$ **First augmentation factor**

**Module Numbered Eight**

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right]G_{40} \quad 43$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41},t)\right]G_{41} \quad 44$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41},t)\right]G_{42} \quad 45$$

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}\left((G_{43}),t\right)\right]T_{40} \quad 46$$

$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}\left((G_{43}),t\right)\right]T_{41} \quad 47$$

$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}\left((G_{43}),t\right)\right]T_{42} \quad 48$$

**Module Numbered Nine**

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}\left((G_{47}),t\right)\right]T_{44} \quad 52$$

$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}\left((G_{47}),t\right)\right]T_{45} \quad 53$$

$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}\left((G_{47}),t\right)\right]T_{46} \quad 54$$

$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}\left((G_{47}),t\right) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)} & +(a''_{13})^{(1)}(T_{14},t) & +(a''_{16})^{(2,2,)}(T_{17},t) & +(a''_{20})^{(3,3,)}(T_{21},t) \\ +(a''_{24})^{(4,4,4,4,)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{36})^{(7,7)}(T_{37},t) & +(a''_{40})^{(8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)} & +(a''_{14})^{(1)}(T_{14},t) & +(a''_{17})^{(2,2,)}(T_{17},t) & +(a''_{21})^{(3,3,)}(T_{21},t) \\ +(a''_{25})^{(4,4,4,4,)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{37})^{(7,7)}(T_{37},t) & +(a''_{41})^{(8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)} & +(a''_{15})^{(1)}(T_{14},t) & +(a''_{18})^{(2,2,)}(T_{17},t) & +(a''_{22})^{(3,3,)}(T_{21},t) \\ +(a''_{26})^{(4,4,4,4,)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{38})^{(7,7)}(T_{37},t) & +(a''_{42})^{(8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{15} \quad 57$$

Where $(a''_{13})^{(1)}(T_{14},t)$, $(a''_{14})^{(1)}(T_{14},t)$, $(a''_{15})^{(1)}(T_{14},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,)}(T_{17},t)$, $+(a''_{17})^{(2,2,)}(T_{17},t)$, $+(a''_{18})^{(2,2,)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,)}(T_{21},t)$, $+(a''_{21})^{(3,3,)}(T_{21},t)$, $+(a''_{22})^{(3,3,)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{38})^{(7,7)}(T_{37},t)$ $+(a''_{37})^{(7,7)}(T_{37},t)$ $+(a''_{36})^{(7,7)}(T_{37},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{40})^{(8,8)}(T_{41},t)$ $+(a''_{41})^{(8,8)}(T_{41},t)$ $+(a''_{42})^{(8,8)}(T_{41},t)$ are eight augmentation coefficient for 1,2,3

$+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)} & -(b''_{13})^{(1)}(G,t) & -(b''_{16})^{(2,2,)}(G_{19},t) & -(b''_{20})^{(3,3,)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{36})^{(7,7,)}(G_{39},t) & -(b''_{40})^{(8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{13} \quad 58$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)} & -(b''_{14})^{(1)}(G,t) & -(b''_{17})^{(2,2,)}(G_{19},t) & -(b''_{21})^{(3,3,)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{37})^{(7,7,)}(G_{39},t) & -(b''_{41})^{(8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{14} \quad 59$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)} & -(b''_{15})^{(1)}(G,t) & -(b''_{18})^{(2,2,)}(G_{19},t) & -(b''_{22})^{(3,3,)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{38})^{(7,7,)}(G_{39},t) & -(b''_{42})^{(8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{15} \quad 60$$

Where $-(b''_{13})^{(1)}(G,t)$, $-(b''_{14})^{(1)}(G,t)$, $-(b''_{15})^{(1)}(G,t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,)}(G_{19},t)$, $-(b''_{17})^{(2,2,)}(G_{19},t)$, $-(b''_{18})^{(2,2,)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{37})^{(7,7,)}(G_{39},t)$, $-(b''_{36})^{(7,7,)}(G_{39},t)$, $-(b''_{38})^{(7,7,)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8)}(G_{43},t)$ $-(b''_{41})^{(8,8)}(G_{43},t)$ $-(b''_{42})^{(8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[\begin{array}{c} (a'_{16})^{(2)}\boxed{+(a''_{16})^{(2)}(T_{17},t)}\ \boxed{+(a''_{13})^{(1,1,)}(T_{14},t)}\ \boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)}\ \boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)}\ \boxed{+(a''_{44})^{(9,9)}(T_{45},t)} \end{array}\right] G_{16} \quad (61)$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[\begin{array}{c} (a'_{17})^{(2)}\boxed{+(a''_{17})^{(2)}(T_{17},t)}\ \boxed{+(a''_{14})^{(1,1,)}(T_{14},t)}\ \boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)}\ \boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)}\ \boxed{+(a''_{45})^{(9,9)}(T_{45},t)} \end{array}\right] G_{17} \quad (62)$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[\begin{array}{c} (a'_{18})^{(2)}\boxed{+(a''_{18})^{(2)}(T_{17},t)}\ \boxed{+(a''_{15})^{(1,1,)}(T_{14},t)}\ \boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)}\ \boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)}\ \boxed{+(a''_{46})^{(9,9)}(T_{45},t)} \end{array}\right] G_{18} \quad (63)$$

Where $\boxed{+(a''_{16})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,)}(T_{14},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)}$ are eight augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{44})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9)}(T_{45},t)}$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[\begin{array}{c} (b'_{16})^{(2)}\boxed{-(b''_{16})^{(2)}(G_{19},t)}\ \boxed{-(b''_{13})^{(1,1,)}(G,t)}\ \boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)}\ \boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)}\ \boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)}\ \boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)}\ \boxed{-(b''_{44})^{(9,9)}(G_{47},t)} \end{array}\right] T_{16} \quad (64)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[\begin{array}{c} (b'_{17})^{(2)}\boxed{-(b''_{17})^{(2)}(G_{19},t)}\ \boxed{-(b''_{14})^{(1,1,)}(G,t)}\ \boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)}\ \boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)}\ \boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)}\ \boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)}\ \boxed{-(b''_{45})^{(9,9)}(G_{47},t)} \end{array}\right] T_{17} \quad (65)$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[\begin{array}{c} (b'_{18})^{(2)}\boxed{-(b''_{18})^{(2)}(G_{19},t)}\ \boxed{-(b''_{15})^{(1,1,)}(G,t)}\ \boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)}\ \boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)}\ \boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)}\ \boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)}\ \boxed{-(b''_{46})^{(9,9)}(G_{47},t)} \end{array}\right] T_{18} \quad (66)$$

where $-(b''_{16})^{(2)}(G_{19},t)$ , $-(b''_{17})^{(2)}(G_{19},t)$ , $-(b''_{18})^{(2)}(G_{19},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{13})^{(1,1,)}(G,t)$, $-(b''_{14})^{(1,1,)}(G,t)$, $-(b''_{15})^{(1,1,)}(G,t)$ are second detrition coefficients for category 1,2 and 3

$-(b''_{20})^{(3,3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,)}(G_{23},t)$ are third detrition coefficients for category 1,2 and 3

$-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1,2 and 3

$-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1,2 and 3

$-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients for category 1,2 and 3

$-(b''_{36})^{(7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2 and 3

$-(b''_{40})^{(8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1,2 and 3

$-(b''_{44})^{(9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[\begin{array}{c} (a'_{20})^{(3)}\left[+(a''_{20})^{(3)}(T_{21},t)\right]\left[+(a''_{16})^{(2,2,2)}(T_{17},t)\right]\left[+(a''_{13})^{(1,1,1,)}(T_{14},t)\right] \\ \left[+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)\right]\left[+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)\right]\left[+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)\right] \\ \left[+(a''_{36})^{(7,7,7,7)}(T_{37},t)\right]\left[+(a''_{40})^{(8,8,8,8)}(T_{41},t)\right]\left[+(a''_{44})^{(9,9,9)}(T_{45},t)\right] \end{array}\right] G_{20} \quad (67)$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[\begin{array}{c} (a'_{21})^{(3)}\left[+(a''_{21})^{(3)}(T_{21},t)\right]\left[+(a''_{17})^{(2,2,2)}(T_{17},t)\right]\left[+(a''_{14})^{(1,1,1,)}(T_{14},t)\right] \\ \left[+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)\right]\left[+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)\right]\left[+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)\right] \\ \left[+(a''_{37})^{(7,7,7,7)}(T_{37},t)\right]\left[+(a''_{41})^{(8,8,8,8)}(T_{41},t)\right]\left[+(a''_{45})^{(9,9,9)}(T_{45},t)\right] \end{array}\right] G_{21} \quad (68)$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[\begin{array}{c} (a'_{22})^{(3)}\left[+(a''_{22})^{(3)}(T_{21},t)\right]\left[+(a''_{18})^{(2,2,2)}(T_{17},t)\right]\left[+(a''_{15})^{(1,1,1,)}(T_{14},t)\right] \\ \left[+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)\right]\left[+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)\right]\left[+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)\right] \\ \left[+(a''_{38})^{(7,7,7,7)}(T_{37},t)\right]\left[+(a''_{42})^{(8,8,8,8)}(T_{41},t)\right]\left[+(a''_{46})^{(9,9,9)}(T_{45},t)\right] \end{array}\right] G_{22} \quad (69)$$

$+(a''_{20})^{(3)}(T_{21},t)$, $+(a''_{21})^{(3)}(T_{21},t)$, $+(a''_{22})^{(3)}(T_{21},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2)}(T_{17},t)$ are second augmentation coefficients for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,)}(T_{14},t)$ are third augmentation coefficients for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficients for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficients for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficients for category 1, 2 and 3

$+(a''_{36})^{(7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1, 2 and 3

$+(a''_{40})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8)}(T_{41},t)$ are eight augmentation coefficients for category 1, 2 and 3

$+(a''_{44})^{(9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b'_{20})^{(3)} & -(b''_{20})^{(3)}(G_{23},t) & -(b''_{16})^{(2,2,2)}(G_{19},t) & -(b''_{13})^{(1,1,1,)}(G,t) \\ -(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{36})^{(7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{20} \quad (70)$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b'_{21})^{(3)} & -(b''_{21})^{(3)}(G_{23},t) & -(b''_{17})^{(2,2,2)}(G_{19},t) & -(b''_{14})^{(1,1,1,)}(G,t) \\ -(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{37})^{(7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{21} \quad (71)$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)} & -(b''_{22})^{(3)}(G_{23},t) & -(b''_{18})^{(2,2,2)}(G_{19},t) & -(b''_{15})^{(1,1,1,)}(G,t) \\ -(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{38})^{(7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{22} \quad (72)$$

$-(b''_{20})^{(3)}(G_{23},t)$, $-(b''_{21})^{(3)}(G_{23},t)$, $-(b''_{22})^{(3)}(G_{23},t)$ *are first detrition coefficients for category 1, 2 and 3*

$-(b''_{16})^{(2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2)}(G_{19},t)$ *are second detrition coefficients for category 1, 2 and 3*

$-(b''_{13})^{(1,1,1,)}(G,t)$, $-(b''_{14})^{(1,1,1,)}(G,t)$, $-(b''_{15})^{(1,1,1,)}(G,t)$ *are third detrition coefficients for category 1,2 and 3*

$-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)$ *are fourth detrition coefficients for category 1, 2 and 3*

$-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)$ *are fifth detrition coefficients for category 1, 2 and 3*

$-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)$ *are sixth detrition coefficients for category 1, 2 and 3*

$-(b''_{36})^{(7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$-(b''_{46})^{(9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \begin{bmatrix} (a'_{24})^{(4)} & +(a''_{24})^{(4)}(T_{25},t) & +(a''_{28})^{(5,5,)}(T_{29},t) & +(a''_{32})^{(6,6,)}(T_{33},t) \\ +(a''_{13})^{(1,1,1,1)}(T_{14},t) & +(a''_{16})^{(2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3)}(T_{21},t) \\ +(a''_{36})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9)}(T_{45},t) \end{bmatrix} G_{24} \quad (73)$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \begin{bmatrix} (a'_{25})^{(4)} & +(a''_{25})^{(4)}(T_{25},t) & +(a''_{29})^{(5,5,)}(T_{29},t) & +(a''_{33})^{(6,6)}(T_{33},t) \\ +(a''_{14})^{(1,1,1,1)}(T_{14},t) & +(a''_{17})^{(2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3)}(T_{21},t) \\ +(a''_{37})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9)}(T_{45},t) \end{bmatrix} G_{25} \quad (74)$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \begin{bmatrix} (a'_{26})^{(4)} & +(a''_{26})^{(4)}(T_{25},t) & +(a''_{30})^{(5,5,)}(T_{29},t) & +(a''_{34})^{(6,6,)}(T_{33},t) \\ +(a''_{15})^{(1,1,1,1)}(T_{14},t) & +(a''_{18})^{(2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3)}(T_{21},t) \\ +(a''_{38})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9)}(T_{45},t) \end{bmatrix} G_{26} \quad (75)$$

$+(a''_{24})^{(4)}(T_{25},t)$, $+(a''_{25})^{(4)}(T_{25},t)$, $+(a''_{26})^{(4)}(T_{25},t)$ *are first augmentation coefficients category* 1, 2 3

$+(a''_{28})^{(5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,)}(T_{29},t)$ *are second augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{32})^{(6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,)}(T_{33},t)$ *are third augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{13})^{(1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1)}(T_{14},t)$ *are fourth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{16})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2)}(T_{17},t)$ *are fifth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{20})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3)}(T_{21},t)$ *are sixth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)$ *are seventh augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)$ *are eighth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{46})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9)}(T_{45},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \begin{bmatrix} (b'_{24})^{(4)} & -(b''_{24})^{(4)}(G_{27},t) & -(b''_{28})^{(5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,)}(G_{35},t) \\ -(b''_{13})^{(1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3)}(G_{23},t) \\ -(b''_{36})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{24} \quad (76)$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \begin{bmatrix} (b'_{25})^{(4)} & -(b''_{25})^{(4)}(G_{27},t) & -(b''_{29})^{(5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,)}(G_{35},t) \\ -(b''_{14})^{(1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3)}(G_{23},t) \\ -(b''_{37})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{25} \quad (77)$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)} & -(b''_{26})^{(4)}(G_{27},t) & -(b''_{30})^{(5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,)}(G_{35},t) \\ -(b''_{15})^{(1,1,1,1)}(G,t) & -(b''_{18})^{(2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3)}(G_{23},t) \\ -(b''_{38})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{26} \quad (78)$$

$Where\ \boxed{-(b_{24}'')^{(4)}(G_{27},t)}, \boxed{-(b_{25}'')^{(4)}(G_{27},t)}, \boxed{-(b_{26}'')^{(4)}(G_{27},t)}\ are\ first\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b_{28}'')^{(5,5,)}(G_{31},t)}, \boxed{-(b_{29}'')^{(5,5,)}(G_{31},t)}, \boxed{-(b_{30}'')^{(5,5,)}(G_{31},t)}\ are\ second\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b_{32}'')^{(6,6,)}(G_{35},t)}, \boxed{-(b_{33}'')^{(6,6,)}(G_{35},t)}, \boxed{-(b_{34}'')^{(6,6,)}(G_{35},t)}\ are\ third\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b_{13}'')^{(1,1,1,1)}(G,t)}, \boxed{-(b_{14}'')^{(1,1,1,1)}(G,t)}, \boxed{-(b_{15}'')^{(1,1,1,1)}(G,t)}$

$are\ fourth\ \ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b_{16}'')^{(2,2,2,2)}(G_{19},t)}, \boxed{-(b_{17}'')^{(2,2,2,2)}(G_{19},t)}, \boxed{-(b_{18}'')^{(2,2,2,2)}(G_{19},t)}$

$are\ fifth\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b_{20}'')^{(3,3,3,3)}(G_{23},t)}, \boxed{-(b_{21}'')^{(3,3,3,3)}(G_{23},t)}, \boxed{-(b_{22}'')^{(3,3,3,3)}(G_{23},t)}$

$are\ sixth\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b_{36}'')^{(7,7,7,7,7)}(G_{39},t)}, \boxed{-(b_{37}'')^{(7,7,7,7,7)}(G_{39},t)}, \boxed{-(b_{38}'')^{(7,7,7,7,7)}(G_{39},t)}$

$are\ seventh\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b_{40}'')^{(8,8,8,8,8)}(G_{43},t)}, \boxed{-(b_{41}'')^{(8,8,8,8,8)}(G_{43},t)}, \boxed{-(b_{42}'')^{(8,8,8,8,8)}(G_{43},t)}$

$are\ eighth\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b_{46}'')^{(9,9,9,9)}(G_{47},t)}, \boxed{-(b_{45}'')^{(9,9,9,9)}(G_{47},t)}, \boxed{-(b_{44}'')^{(9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[\begin{array}{c} (a_{28}')^{(5)}\boxed{+(a_{28}'')^{(5)}(T_{29},t)}\ \boxed{+(a_{24}'')^{(4,4,)}(T_{25},t)}\ \boxed{+(a_{32}'')^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1)}(T_{14},t)}\ \boxed{+(a_{16}'')^{(2,2,2,2,2)}(T_{17},t)}\ \boxed{+(a_{20}'')^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{36}'')^{(7,7,7,7,7,7)}(T_{37},t)}\ \boxed{+(a_{40}'')^{(8,8,8,8,8,8)}(T_{41},t)}\ \boxed{+(a_{44}'')^{(9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{28} \qquad 79$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[\begin{array}{c} (a_{29}')^{(5)}\boxed{+(a_{29}'')^{(5)}(T_{29},t)}\ \boxed{+(a_{25}'')^{(4,4,)}(T_{25},t)}\ \boxed{+(a_{33}'')^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a_{14}'')^{(1,1,1,1,1)}(T_{14},t)}\ \boxed{+(a_{17}'')^{(2,2,2,2,2)}(T_{17},t)}\ \boxed{+(a_{21}'')^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{37}'')^{(7,7,7,7,7,7)}(T_{37},t)}\ \boxed{+(a_{41}'')^{(8,8,8,8,8,8)}(T_{41},t)}\ \boxed{+(a_{45}'')^{(9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{29} \qquad 80$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[\begin{array}{c} (a_{30}')^{(5)}\boxed{+(a_{30}'')^{(5)}(T_{29},t)}\ \boxed{+(a_{26}'')^{(4,4,)}(T_{25},t)}\ \boxed{+(a_{34}'')^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a_{15}'')^{(1,1,1,1,1)}(T_{14},t)}\ \boxed{+(a_{18}'')^{(2,2,2,2,2)}(T_{17},t)}\ \boxed{+(a_{22}'')^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{38}'')^{(7,7,7,7,7,7)}(T_{37},t)}\ \boxed{+(a_{42}'')^{(8,8,8,8,8,8)}(T_{41},t)}\ \boxed{+(a_{46}'')^{(9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{30} \qquad 81$$

$Where\ \boxed{+(a_{28}'')^{(5)}(T_{29},t)}, \boxed{+(a_{29}'')^{(5)}(T_{29},t)}, \boxed{+(a_{30}'')^{(5)}(T_{29},t)}\ are\ first\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$

$And\ \boxed{+(a_{24}'')^{(4,4,)}(T_{25},t)}, \boxed{+(a_{25}'')^{(4,4,)}(T_{25},t)}, \boxed{+(a_{26}'')^{(4,4,)}(T_{25},t)}\ are\ second\ augmentation\ coefficient\ for\ category\ 1,2\ and\ 3$

$\boxed{+(a_{32}'')^{(6,6,6)}(T_{33},t)}, \boxed{+(a_{33}'')^{(6,6,6)}(T_{33},t)}, \boxed{+(a_{34}'')^{(6,6,6)}(T_{33},t)}\ are\ third\ \ augmentation\ coefficient\ for\ category\ 1,2\ and\ 3$

$\boxed{+(a_{13}'')^{(1,1,1,1,1)}(T_{14},t)}, \boxed{+(a_{14}'')^{(1,1,1,1,1)}(T_{14},t)}, \boxed{+(a_{15}'')^{(1,1,1,1,1)}(T_{14},t)}$ are fourth augmentation coefficients for category 1,2, and 3

$\boxed{+(a_{16}'')^{(2,2,2,2,2)}(T_{17},t)}, \boxed{+(a_{17}'')^{(2,2,2,2,2)}(T_{17},t)}, \boxed{+(a_{18}'')^{(2,2,2,2,2)}(T_{17},t)}$ are fifth augmentation coefficients for category 1,2,and 3

$+(a_{20}'')^{(3,3,3,3,3)}(T_{21},t)$, $+(a_{21}'')^{(3,3,3,3,3)}(T_{21},t)$, $+(a_{22}'')^{(3,3,3,3,3)}(T_{21},t)$ are sixth augmentation coefficients for category 1,2, 3

$+(a_{36}'')^{(7,7,7,7,7,7)}(T_{37},t)$ , $+(a_{37}'')^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a_{38}'')^{(7,7,7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1,2, 3

$+(a_{40}'')^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a_{41}'')^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a_{42}'')^{(8,8,8,8,8,8)}(T_{41},t)$ are eighth augmentation coefficients for category 1,2, 3

$+(a_{46}'')^{(9,9,9,9,9)}(T_{45},t)$, $+(a_{45}'')^{(9,9,9,9,9)}(T_{45},t)$, $+(a_{44}'')^{(9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \begin{bmatrix} (b_{28}')^{(5)} & -(b_{28}'')^{(5)}(G_{31},t) & -(b_{24}'')^{(4,4,)}(G_{27},t) & -(b_{32}'')^{(6,6,6)}(G_{35},t) \\ -(b_{13}'')^{(1,1,1,1,1)}(G,t) & -(b_{16}'')^{(2,2,2,2,2)}(G_{19},t) & -(b_{20}'')^{(3,3,3,3,3)}(G_{23},t) & \\ -(b_{36}'')^{(7,7,7,7,7,7)}(G_{39},t) & -(b_{40}'')^{(8,8,8,8,8,8)}(G_{43},t) & -(b_{44}'')^{(9,9,9,9,9)}(G_{47},t) & \end{bmatrix} T_{28} \quad 82$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b_{29}')^{(5)} & -(b_{29}'')^{(5)}(G_{31},t) & -(b_{25}'')^{(4,4,)}(G_{27},t) & -(b_{33}'')^{(6,6,6)}(G_{35},t) \\ -(b_{14}'')^{(1,1,1,1,1)}(G,t) & -(b_{17}'')^{(2,2,2,2,2)}(G_{19},t) & -(b_{21}'')^{(3,3,3,3,3)}(G_{23},t) & \\ -(b_{37}'')^{(7,7,7,7,7,7)}(G_{39},t) & -(b_{41}'')^{(8,8,8,8,8,8)}(G_{43},t) & -(b_{45}'')^{(9,9,9,9,9)}(G_{47},t) & \end{bmatrix} T_{29} \quad 83$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b_{30}')^{(5)} & -(b_{30}'')^{(5)}(G_{31},t) & -(b_{26}'')^{(4,4,)}(G_{27},t) & -(b_{34}'')^{(6,6,6)}(G_{35},t) \\ -(b_{15}'')^{(1,1,1,1,1,)}(G,t) & -(b_{18}'')^{(2,2,2,2,2)}(G_{19},t) & -(b_{22}'')^{(3,3,3,3,3)}(G_{23},t) & \\ -(b_{38}'')^{(7,7,7,7,7,7)}(G_{39},t) & -(b_{42}'')^{(8,8,8,8,8,8)}(G_{43},t) & -(b_{46}'')^{(9,9,9,9,9)}(G_{47},t) & \end{bmatrix} T_{30} \quad 84$$

*where* $-(b_{28}'')^{(5)}(G_{31},t)$ , $-(b_{29}'')^{(5)}(G_{31},t)$ , $-(b_{30}'')^{(5)}(G_{31},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b_{24}'')^{(4,4,)}(G_{27},t)$, $-(b_{25}'')^{(4,4,)}(G_{27},t)$, $-(b_{26}'')^{(4,4,)}(G_{27},t)$ *are second detrition coefficients for category* 1,2 *and* 3

$-(b_{32}'')^{(6,6,6)}(G_{35},t)$, $-(b_{33}'')^{(6,6,6)}(G_{35},t)$, $-(b_{34}'')^{(6,6,6)}(G_{35},t)$ *are third detrition coefficients for category* 1,2 *and* 3

$-(b_{13}'')^{(1,1,1,1,1)}(G,t)$, $-(b_{14}'')^{(1,1,1,1,1)}(G,t)$ , $-(b_{15}'')^{(1,1,1,1,1,)}(G,t)$ are fourth detrition coefficients for category 1,2, and 3

$-(b_{16}'')^{(2,2,2,2,2)}(G_{19},t)$, $-(b_{17}'')^{(2,2,2,2,2)}(G_{19},t)$, $-(b_{18}'')^{(2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1,2, and 3

$-(b_{20}'')^{(3,3,3,3,3)}(G_{23},t)$, $-(b_{21}'')^{(3,3,3,3,3)}(G_{23},t)$, $-(b_{22}'')^{(3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1,2, and 3

$-(b_{36}'')^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b_{37}'')^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b_{38}'')^{(7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2, and 3

$-(b_{42}'')^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b_{41}'')^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b_{40}'')^{(8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1,2, and 3

$-(b_{46}'')^{(9,9,9,9,9)}(G_{47},t)$, $-(b_{45}'')^{(9,9,9,9,9)}(G_{47},t)$, $-(b_{44}'')^{(9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} \qquad 85$$

$$-\left[\begin{matrix}(a'_{32})^{(6)}\boxed{+(a''_{32})^{(6)}(T_{33},t)}\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}\\ \boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}\boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)}\boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)}\boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)}\end{matrix}\right]G_{32}$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[\begin{matrix}(a'_{33})^{(6)}\boxed{+(a''_{33})^{(6)}(T_{33},t)}\boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}\boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}\\ \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}\boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}\boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)}\boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}\boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)}\end{matrix}\right]G_{33} \qquad 86$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[\begin{matrix}(a'_{34})^{(6)}\boxed{+(a''_{34})^{(6)}(T_{33},t)}\boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}\boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}\\ \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}\boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}\boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)}\boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}\boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)}\end{matrix}\right]G_{34} \qquad 87$$

$\boxed{+(a''_{32})^{(6)}(T_{33},t)}, \boxed{+(a''_{33})^{(6)}(T_{33},t)}, \boxed{+(a''_{34})^{(6)}(T_{33},t)}$ *are first augmentation coefficients for category* $1, 2$ *and* $3$

$\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}$ *are second augmentation coefficients for category* $1, 2$ *and* $3$

$\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}$ *are third augmentation coefficients for category* $1, 2$ *and* $3$

$\boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}$ - are fourth augmentation coefficients

$\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}$ - fifth augmentation coefficients

$\boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)}$ sixth augmentation coefficients

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)}$ seventh augmentation coefficients

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}$ Eighth augmentation coefficients

$\boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)}$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[\begin{matrix}(b'_{32})^{(6)}\boxed{-(b''_{32})^{(6)}(G_{35},t)}\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}\\ \boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)}\boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)}\\ \boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)}\boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)}\end{matrix}\right]T_{32} \qquad 88$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[\begin{matrix}(b'_{33})^{(6)}\boxed{-(b''_{33})^{(6)}(G_{35},t)}\boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}\boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}\\ \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}\boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)}\\ \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)}\boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)}\end{matrix}\right]T_{33} \qquad 89$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)}\boxed{-(b''_{34})^{(6)}(G_{35},t)} & \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{34} \quad 90$$

$\boxed{-(b''_{32})^{(6)}(G_{35},t)}, \boxed{-(b''_{33})^{(6)}(G_{35},t)}, \boxed{-(b''_{34})^{(6)}(G_{35},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}$ *are second detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)}$ *are third detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}$ are fourth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \quad 91$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a'_{36})^{(7)}\boxed{+(a''_{36})^{(7)}(T_{37},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \quad 92$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a'_{37})^{(7)}\boxed{+(a''_{37})^{(7)}(T_{37},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{38}}{dt} \quad 93$$

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a'_{38})^{(7)}\boxed{+(a''_{38})^{(7)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{(a''_{36})^{(7)}(T_{37},t)}, \boxed{(a''_{37})^{(7)}(T_{37},t)}, \boxed{(a''_{38})^{(7)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a_{16}'')^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{17}'')^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{18}'')^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{20}'')^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{21}'')^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{22}'')^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{24}'')^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{25}'')^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{26}'')^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{28}'')^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{29}'')^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{30}'')^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{32}'')^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{33}'')^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{34}'')^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{13}'')^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a_{13}'')^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a_{15}'')^{(1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{42}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a_{41}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a_{40}'')^{(8,8,8,8,8,8,8,8,)}(T_{41},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a_{46}'')^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a_{45}'')^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a_{44}'')^{(9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{36}}{dt} =$$



$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b_{36}')^{(7)}\boxed{-(b_{36}'')^{(7)}(G_{39},t)} & \boxed{-(b_{16}'')^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{20}'')^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{24}'')^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{28}'')^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{32}'')^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{13}'')^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b_{44}'')^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{37}}{dt} =$$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b_{37}')^{(7)}\boxed{-(b_{37}'')^{(7)}(G_{39},t)} & \boxed{-(b_{17}'')^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{21}'')^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{25}'')^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{29}'')^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{33}'')^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{14}'')^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b_{45}'')^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{38}}{dt} =$$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b_{38}')^{(7)}\boxed{-(b_{38}'')^{(7)}(G_{39},t)} & \boxed{-(b_{18}'')^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{22}'')^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{26}'')^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{30}'')^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{34}'')^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{15}'')^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b_{46}'')^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b_{36}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{37}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{16}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{17}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{18}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{20}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{21}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{22}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{24}'')^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{25}'')^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{26}'')^{(4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition

coefficients for category 1, 2 and 3
$\boxed{-(b_{28}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition
coefficients for category 1, 2 and 3
$\boxed{-(b_{32}'')^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{34}'')^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition
coefficients for category 1, 2 and 3
$\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$
are seventh detrition coefficients for category 1, 2 and 3
$\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition
coefficients for category 1, 2 and 3
$\boxed{-(b_{46}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{45}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{44}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition
coefficients for category 1, 2 and 3



$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \begin{bmatrix} (a_{40}')^{(8)}\boxed{+(a_{40}'')^{(8)}(T_{41},t)} & \boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{20}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{24}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{28}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{32}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{36}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{44}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \begin{bmatrix} (a_{41}')^{(8)}\boxed{+(a_{41}'')^{(8)}(T_{41},t)} & \boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{21}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{25}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{29}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{33}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{37}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{45}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \begin{bmatrix} (a_{42}')^{(8)}\boxed{+(a_{42}'')^{(8)}(T_{41},t)} & \boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{22}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{26}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{30}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{34}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{15}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{38}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{46}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a_{40}'')^{(8)}(T_{41},t)}$, $\boxed{+(a_{41}'')^{(8)}(T_{41},t)}$, $\boxed{+(a_{42}'')^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3
$\boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3
$\boxed{+(a_{20}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{21}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{22}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3
$\boxed{+(a_{24}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{25}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{26}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3
$\boxed{+(a_{28}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{29}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{30}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ $+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ $+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$ are eighth augmentation coefficient for 1,2,3

$+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} =$$

$$(b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)}\left[-(b''_{40})^{(8)}(G_{43},t)\right] & \left[-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} =$$

$$(b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)}\left[-(b''_{41})^{(8)}(G_{43},t)\right] & \left[-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} =$$

$$(b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)}\left[-(b''_{42})^{(8)}(G_{43},t)\right] & \left[-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{15}$$

Where $-(b''_{36})^{(7)}(G_{39},t)$, $-(b''_{37})^{(7)}(G_{39},t)$, $-(b''_{38})^{(7)}(G_{39},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)$, $-(b''_{38})^{(7,7,)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)$ are eighth detrition

coefficients for category 1, 2 and 3
$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3



$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \begin{bmatrix} (a'_{44})^{(9)}\boxed{+(a''_{44})^{(9)}(T_{45},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \begin{bmatrix} (a'_{45})^{(9)}\boxed{+(a''_{45})^{(9)}(T_{45},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \begin{bmatrix} (a'_{46})^{(9)}\boxed{+(a''_{46})^{(9)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a''_{44})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3
$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3
$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3
$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3
$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3
$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3
$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ are Seventh augmentation coefficient for category 1, 2 and 3
$\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3
$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} =$$

$$(b_{44})^{(9)}T_{45} -$$

$$\begin{bmatrix} (b'_{44})^{(9)}\boxed{-(b''_{44})^{(9)}(G_{47},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b'_{45})^{(9)}\boxed{-(b''_{45})^{(9)}(G_{47},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b'_{46})^{(9)}\boxed{-(b''_{46})^{(9)}(G_{47},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{44})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9)}(G_{47},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a'_i)^{(1)}, (a''_i)^{(1)}, (b_i)^{(1)}, (b'_i)^{(1)}, (b''_i)^{(1)} > 0, \quad i,j = 13,14,15 \qquad 97$$

The functions $(a''_i)^{(1)}, (b''_i)^{(1)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a_i'')^{(1)}(T_{14},t) \le (p_i)^{(1)} \le (\hat{A}_{13})^{(1)}$$

$$(b_i'')^{(1)}(G,t) \le (r_i)^{(1)} \le (b_i')^{(1)} \le (\hat{B}_{13})^{(1)}$$

$$lim_{T_2\to\infty}(a_i'')^{(1)}(T_{14},t) = (p_i)^{(1)} \qquad 98$$

$$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = (r_i)^{(1)}$$

**Definition of** $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$ :

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$$|(a_i'')^{(1)}(T_{14}',t) - (a_i'')^{(1)}(T_{14},t)| \le (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G',t) - (b_i'')^{(1)}(G,t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}',t)$ and $(a_i'')^{(1)}(T_{14},t)$ . $(T_{14}',t)$ and $(T_{14},t)$ are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14},t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$ : 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$$

**Definition of** $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$ : 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}$, $(r_i)^{(1)}, i = 13,14,15$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i,j = 16,17,18$$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17},t) \le (p_i)^{(2)} \le \left(\hat{A}_{16}\right)^{(2)} \quad 102$$

$$(b_i'')^{(2)}(G_{19},t) \le (r_i)^{(2)} \le (b_i')^{(2)} \le (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2\to\infty}(a_i'')^{(2)}(T_{17},t) = (p_i)^{(2)} \quad 104$$

$$\lim_{G\to\infty}(b_i'')^{(2)}\left((G_{19}),t\right) = (r_i)^{(2)} \quad 105$$

**Definition of** $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$ : 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}',t) - (a_i'')^{(2)}(T_{17},t)| \le (\hat{k}_{16})^{(2)}|T_{17} - T_{17}'|e^{-(\hat{M}_{16})^{(2)}t} \quad 107$$

$$|(b_i'')^{(2)}((G_{19})',t) - (b_i'')^{(2)}\left((G_{19}),t\right)| < (\hat{k}_{16})^{(2)}||(G_{19}) - (G_{19})'||e^{-(\hat{M}_{16})^{(2)}t} \quad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}',t)$ and$(a_i'')^{(2)}(T_{17},t)$ . $(T_{17}',t)$ and $(T_{17},t)$ are points belonging to the interval $\left[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}\right]$. It is to be noted that $(a_i'')^{(2)}(T_{17},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17},t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$ :

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants 109

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$$

**Definition of** $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$ :

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and\ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1 \quad 110$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(b_i)^{(2)} + (b_i')^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1 \quad 111$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i,j = 20,21,22 \quad 112$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(3)}$, $(\mathrm{r_i})^{(3)}$:

$$(a_i'')^{(3)}(T_{21}, t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23}, t) \leq (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(3)}(T_{21}, t) = (p_i)^{(3)} \quad (113)$$

$$\lim_{G \to \infty}(b_i'')^{(3)}(G_{23}, t) = (r_i)^{(3)}$$

**Definition of** $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$ :

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: (114)

$$|(a_i'')^{(3)}(T_{21}', t) - (a_i'')^{(3)}(T_{21}, t)| \leq (\hat{k}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\hat{M}_{20})^{(3)}t}$$

$$|(b_i'')^{(3)}(G_{23}', t) - (b_i'')^{(3)}(G_{23}, t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}'||e^{-(\hat{M}_{20})^{(3)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}', t)$ and$(a_i'')^{(3)}(T_{21}, t)$ . $(T_{21}', t)$ And $(T_{21}, t)$ are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21}, t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$ : (115)

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with (116)
$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$ and the constants
$(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}, i = 20,21,22,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(a_i)^{(3)} + (a_i')^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(b_i)^{(3)} + (b_i')^{(3)} + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i, j = 24,25,26 \quad (117)$$

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(4)}$, $(r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25},t) \le (p_i)^{(4)} \le (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}\big((G_{27}),t\big) \le (r_i)^{(4)} \le (b_i')^{(4)} \le (\hat{B}_{24})^{(4)}$$

$$lim_{T_2\to\infty}(a_i'')^{(4)}(T_{25},t) = (p_i)^{(4)} \quad (118)$$

$$\lim_{G\to\infty}(b_i'')^{(4)}\big((G_{27}),t\big) = (r_i)^{(4)}$$

**Definition of** $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$ :

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24,25,26}$

They satisfy Lipschitz condition: (119)

$$|(a_i'')^{(4)}(T_{25}',t) - (a_i'')^{(4)}(T_{25},t)| \le (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})',t) - (b_i'')^{(4)}\big((G_{27}),t\big)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}',t)$ and$(a_i'')^{(4)}(T_{25},t)$ . $(T_{25}',t)$ and $(T_{25},t)$ are points belonging to the interval $\big[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}\big]$. It is to be noted that $(a_i'')^{(4)}(T_{25},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25},t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$ : (120)

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$, are positive constants

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}, \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$$

**Definition of** $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$ : (121)

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and\ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}$, $(r_i)^{(4)}, i = 24,25,26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(a_i)^{(4)} + (a_i')^{(4)} + (\hat{A}_{24})^{(4)} + (\hat{P}_{24})^{(4)}(\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(b_i)^{(4)} + (b_i')^{(4)} + (\hat{B}_{24})^{(4)} + (\hat{Q}_{24})^{(4)}(\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i,j = 28,29,30 \quad (122)$$

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(5)}$, $(r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29},t) \le (p_i)^{(5)} \le (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}\big((G_{31}),t\big) \le (r_i)^{(5)} \le (b_i')^{(5)} \le (\hat{B}_{28})^{(5)}$$

$$lim_{T_2\to\infty}(a_i'')^{(5)}(T_{29},t) = (p_i)^{(5)} \qquad 123$$

$$\lim_{G\to\infty}(b_i'')^{(5)}(G_{31},t) = (r_i)^{(5)}$$

**Definition of** $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$ :

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}',t) - (a_i'')^{(5)}(T_{29},t)| \le (\hat{k}_{28})^{(5)}|T_{29} - T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})',t) - (b_i'')^{(5)}\big((G_{31}),t\big)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}',t)$ and$(a_i'')^{(5)}(T_{29},t)$ . $(T_{29}',t)$ and $(T_{29},t)$ are points belonging to the interval $\big[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\big]$. It is to be noted that $(a_i'')^{(5)}(T_{29},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29},t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$ : 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}, \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

**Definition of** $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$ : 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and\ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(a_i)^{(5)} + (a_i')^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)}(\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(b_i)^{(5)} + (b_i')^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)}(\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i,j = 32,33,34 \qquad 127$$

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(6)}$, $(r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33},t) \le (p_i)^{(6)} \le (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}),t) \le (r_i)^{(6)} \le (b_i')^{(6)} \le (\hat{B}_{32})^{(6)}$$

$$lim_{T_2\to\infty}(a_i'')^{(6)}(T_{33},t) = (p_i)^{(6)} \qquad 128$$

$$\lim_{G\to\infty}(b_i'')^{(6)}\left((G_{35}),t\right) = (r_i)^{(6)}$$

**Definition of** $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$ :

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}',t) - (a_i'')^{(6)}(T_{33},t)| \le (\hat{k}_{32})^{(6)}|T_{33} - T_{33}'|e^{-(\hat{M}_{32})^{(6)}t}$$

$$|(b_i'')^{(6)}((G_{35})',t) - (b_i'')^{(6)}\left((G_{35}),t\right)| < (\hat{k}_{32})^{(6)}||(G_{35}) - (G_{35})'||e^{-(\hat{M}_{32})^{(6)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}',t)$ and$(a_i'')^{(6)}(T_{33},t)$ . $(T_{33}',t)$ and $(T_{33},t)$ are points belonging to the interval $\left[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}\right]$. It is to be noted that $(a_i'')^{(6)}(T_{33},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33},t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$ : 129

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$, are positive constants

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$$

**Definition of** $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$ : 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and\ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32,33,34,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(a_i)^{(6)} + (a_i')^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(b_i)^{(6)} + (b_i')^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

(GGGG) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i,j = 36,37,38$ 131

(HHHH)The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(7)}$, $(r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37},t) \le (p_i)^{(7)} \le (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39},t) \le (r_i)^{(7)} \le (b_i')^{(7)} \le (\hat{B}_{36})^{(7)} \quad (132)$$

*(IIII)* $\lim_{T_2\to\infty} (a_i'')^{(7)}(T_{37},t) = (p_i)^{(7)}$

*(JJJJ)*

$\lim_{G\to\infty}(b_i'')^{(7)}\big((G_{39}),t\big) = (r_i)^{(7)}$

**Definition of** $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$ :

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: (133)

$$|(a_i'')^{(7)}(T_{37}',t) - (a_i'')^{(7)}(T_{37},t)| \le (\hat{k}_{36})^{(7)}|T_{37} - T_{37}'|e^{-(\hat{M}_{36})^{(7)}t}$$

$$|(b_i'')^{(7)}((G_{39})',t) - (b_i'')^{(7)}\big((G_{39}),t\big)| < (\hat{k}_{36})^{(7)}||(G_{39}) - (G_{39})'||e^{-(\hat{M}_{36})^{(7)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}',t)$ and$(a_i'')^{(7)}(T_{37},t)$ . $(T_{37}',t)$ and $(T_{37},t)$ are points belonging to the interval $\big[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}\big]$. It is to be noted that $(a_i'')^{(7)}(T_{37},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37},t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$ : (134)

(KKKK) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$, are positive constants

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$$

**Definition of** $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$ : (135)

(LLLL) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)}(\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(b_i)^{(7)} + (b_i')^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)}(\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40,41,42 \tag{136}$$

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

**Definition of** $(p_i)^{(8)}$, $(r_i)^{(8)}$: (137)

$$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)} \tag{138}$$

$$(b_i'')^{(8)}((G_{43}), t) \leq (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)} \tag{139}$$

$$\lim_{T_2 \to \infty} (a_i'')^{(8)}(T_{41}, t) = (p_i)^{(8)} \tag{140}$$

$$\lim_{G \to \infty} (b_i'')^{(8)}((G_{43}), t) = (r_i)^{(8)} \tag{141}$$

**Definition of** $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$ :

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)} |T_{41} - T_{41}'| e^{-(\hat{M}_{40})^{(8)} t} \tag{142}$$

$$|(b_i'')^{(8)}((G_{43})', t) - (b_i'')^{(8)}((G_{43}), t)| < (\hat{k}_{40})^{(8)} ||(G_{43}) - (G_{43})'|| e^{-(\hat{M}_{40})^{(8)} t} \tag{143}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}', t)$ and $(a_i'')^{(8)}(T_{41}, t)$ . $(T_{41}', t)$ and $(T_{41}, t)$ are points belonging to the interval $[(\hat{k}_{40})^{(8)}, (\hat{M}_{40})^{(8)}]$. It is to be noted that $(a_i'')^{(8)}(T_{41}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{40})^{(8)} = 1$ then the function $(a_i'')^{(8)}(T_{41}, t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$ :

$(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$, are positive constants

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}, \frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1 \tag{144}$$

**Definition of** $(\hat{P}_{40})^{(8)}, (\hat{Q}_{40})^{(8)}$ :

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}, (\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)}, (a_i')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}, i = 40,41,42,$

Satisfy the inequalities

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(a_i)^{(8)} + (a_i')^{(8)} + (\hat{A}_{40})^{(8)} + (\hat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}] < 1 \quad 145$$

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(b_i)^{(8)} + (b_i')^{(8)} + (\hat{B}_{40})^{(8)} + (\hat{Q}_{40})^{(8)}(\hat{k}_{40})^{(8)}] < 1 \quad 146$$

Where we suppose

$$(a_i)^{(9)}, (a_i')^{(9)}, (a_i'')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (b_i'')^{(9)} > 0, \quad i,j = 44,45,46 \quad 146A$$

The functions $(a_i'')^{(9)}, (b_i'')^{(9)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(9)}$, $(r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45}, t) \leq (p_i)^{(9)} \leq (\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47}, t) \leq (r_i)^{(9)} \leq (b_i')^{(9)} \leq (\hat{B}_{44})^{(9)}$$

$$\lim_{T_2 \to \infty} (a_i'')^{(9)}(T_{45}, t) = (p_i)^{(9)}$$

$$\lim_{G \to \infty} (b_i'')^{(9)}(G_{47}, t) = (r_i)^{(9)}$$

**Definition of** $(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}$ :

Where $\boxed{(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}}$ are positive constants and $\boxed{i = 44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}', t) - (a_i'')^{(9)}(T_{45}, t)| \leq (\hat{k}_{44})^{(9)}|T_{45} - T_{45}'|e^{-(\hat{M}_{44})^{(9)}t}$$

$$|(b_i'')^{(9)}((G_{47})', t) - (b_i'')^{(9)}((G_{47}), t)| < (\hat{k}_{44})^{(9)}||(G_{47}) - (G_{47})'||e^{-(\hat{M}_{44})^{(9)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}', t)$ and$(a_i'')^{(9)}(T_{45}, t)$ . $(T_{45}', t)$ and $(T_{45}, t)$ are points belonging to the interval $[(\hat{k}_{44})^{(9)}, (\hat{M}_{44})^{(9)}]$. It is to be noted that $(a_i'')^{(9)}(T_{45}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{44})^{(9)} = 1$ then the function $(a_i'')^{(9)}(T_{45}, t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$ :

$(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$, are positive constants

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}, \frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$$

**Definition of** $(\hat{P}_{44})^{(9)}, (\hat{Q}_{44})^{(9)}$ :

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}, (\hat{A}_{44})^{(9)} and (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)}, (a_i')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}, i = 44,45,46,$
satisfy the inequalities

$$\frac{1}{(\widehat{M}_{44})^{(9)}}[(a_i)^{(9)} + (a_i')^{(9)} + (\hat{A}_{44})^{(9)} + (\hat{P}_{44})^{(9)}(\hat{k}_{44})^{(9)}] < 1$$

$$\frac{1}{(\widehat{M}_{44})^{(9)}}[(b_i)^{(9)} + (b_i')^{(9)} + (\hat{B}_{44})^{(9)} + (\hat{Q}_{44})^{(9)}(\hat{k}_{44})^{(9)}] < 1$$

**Theorem 1:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0), T_i(0)$ :

$$G_i(t) \leq (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 2 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

**Definition of** $G_i(0), T_i(0)$

$$G_i(t) \leq (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} \quad , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \leq (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} \quad , \quad T_i(0) = T_i^0 > 0$$

**Theorem 3 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t) \leq (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t} \quad , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \leq (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t} \quad , \quad T_i(0) = T_i^0 > 0$$

**Theorem 4 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

**Definition of** $G_i(0), T_i(0)$ :

$$G_i(t) \leq (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 5 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

**Definition of** $G_i(0), T_i(0)$ :

$$G_i(t) \leq (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 6 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

**Definition of** $G_i(0), T_i(0)$ :

$G_i(t) \leq \left(\hat{P}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Theorem 7:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

**Definition of** $G_i(0), T_i(0)$ :

$G_i(t) \leq \left(\hat{P}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Theorem 8:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

**Definition of** $G_i(0), T_i(0)$ :

$G_i(t) \leq \left(\hat{P}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Theorem 9:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

**Definition of** $G_i(0), T_i(0)$ :

$G_i(t) \leq \left(\hat{P}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Proof:** Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i$: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$G_i(0) = G_i^0$ , $T_i(0) = T_i^0$ , $G_i^0 \leq \left(\hat{P}_{13}\right)^{(1)}, T_i^0 \leq \left(\hat{Q}_{13}\right)^{(1)}$, 155

$0 \leq G_i(t) - G_i^0 \leq \left(\hat{P}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t}$ 156

$0 \leq T_i(t) - T_i^0 \leq \left(\hat{Q}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t}$ 157

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}(s_{(13)}) - \left((a'_{13})^{(1)} + a''_{13})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{15}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^{0} + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^{0} + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{\mathrm{T}}_{15}(\mathrm{t}) = \mathrm{T}_{15}^{0} + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{15}(s_{(13)})\right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{16})^{(2)}$, $T_i^0 \le (\hat{Q}_{16})^{(2)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

By 160

$$\bar{G}_{16}(t) = G_{16}^{0} + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''_{16})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^{0} + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}(s_{(16)}), s_{(17)})\right) G_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^{0} + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{18}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^{0} + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^{0} + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^{0} + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{18}(s_{(16)})\right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{20})^{(3)}$, $T_i^0 \le (\hat{Q}_{20})^{(3)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

By 161

$$\bar{G}_{20}(t) = G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''_{20})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{22}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^0 + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^0 + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{\mathrm{T}}_{22}(\mathrm{t}) = \mathrm{T}_{22}^0 + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{22}(s_{(20)})\right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{24})^{(4)}, T_i^0 \le (\hat{Q}_{24})^{(4)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

By 162

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{26}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{\mathrm{T}}_{26}(\mathrm{t}) = \mathrm{T}_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{26}(s_{(24)})\right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{28})^{(5)}, T_i^0 \le (\hat{Q}_{28})^{(5)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{28})^{(5)} e^{(\tilde{M}_{28})^{(5)} t}$

By 163

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[ (a_{28})^{(5)} G_{29}(s_{(28)}) - \left( (a'_{28})^{(5)} + a''_{28})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{28}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[ (a_{29})^{(5)} G_{28}(s_{(28)}) - \left( (a'_{29})^{(5)} + (a''_{29})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{29}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[ (a_{30})^{(5)} G_{29}(s_{(28)}) - \left( (a'_{30})^{(5)} + (a''_{30})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{30}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[ (b_{28})^{(5)} T_{29}(s_{(28)}) - \left( (b'_{28})^{(5)} - (b''_{28})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{28}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[ (b_{29})^{(5)} T_{28}(s_{(28)}) - \left( (b'_{29})^{(5)} - (b''_{29})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{29}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{\mathrm{T}}_{30}(\mathrm{t}) = \mathrm{T}_{30}^0 + \int_0^t \left[ (b_{30})^{(5)} T_{29}(s_{(28)}) - \left( (b'_{30})^{(5)} - (b''_{30})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{30}(s_{(28)}) \right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{32})^{(6)}$, $T_i^0 \le (\hat{Q}_{32})^{(6)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\tilde{M}_{32})^{(6)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\tilde{M}_{32})^{(6)} t}$

By 164

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[ (a_{32})^{(6)} G_{33}(s_{(32)}) - \left( (a'_{32})^{(6)} + a''_{32})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[ (a_{33})^{(6)} G_{32}(s_{(32)}) - \left( (a'_{33})^{(6)} + (a''_{33})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[ (a_{34})^{(6)} G_{33}(s_{(32)}) - \left( (a'_{34})^{(6)} + (a''_{34})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{34}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[ (b_{32})^{(6)} T_{33}(s_{(32)}) - \left( (b'_{32})^{(6)} - (b''_{32})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[ (b_{33})^{(6)} T_{32}(s_{(32)}) - \left( (b'_{33})^{(6)} - (b''_{33})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{\mathrm{T}}_{34}(\mathrm{t}) = \mathrm{T}_{34}^0 + \int_0^t \left[ (b_{34})^{(6)} T_{33}(s_{(32)}) - \left( (b'_{34})^{(6)} - (b''_{34})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{34}(s_{(32)}) \right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} G_{37}(s_{(36)}) - \left((a'_{36})^{(7)} + a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[(a_{37})^{(7)} G_{36}(s_{(36)}) - \left((a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[(a_{38})^{(7)} G_{37}(s_{(36)}) - \left((a'_{38})^{(7)} + (a''_{38})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{38}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[(b_{36})^{(7)} T_{37}(s_{(36)}) - \left((b'_{36})^{(7)} - (b''_{36})^{(7)} (G_{39}(s_{(36)}), s_{(36)})\right) T_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[(b_{37})^{(7)} T_{36}(s_{(36)}) - \left((b'_{37})^{(7)} - (b''_{37})^{(7)} (G_{39}(s_{(36)}), s_{(36)})\right) T_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{38}(t) = T_{38}^0 + \int_0^t \left[(b_{38})^{(7)} T_{37}(s_{(36)}) - \left((b'_{38})^{(7)} - (b''_{38})^{(7)} (G_{39}(s_{(36)}), s_{(36)})\right) T_{38}(s_{(36)})\right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{40})^{(8)}$, $T_i^0 \le (\hat{Q}_{40})^{(8)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} G_{41}(s_{(40)}) - \left((a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[(a_{41})^{(8)} G_{40}(s_{(40)}) - \left((a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41}(s_{(40)}), s_{(40)})\right) G_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}(s_{(40)}), s_{(40)})\right) G_{42}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^0 + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43}(s_{(40)}), s_{(40)})\right) T_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^0 + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43}(s_{(40)}), s_{(40)})\right) T_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{\mathrm{T}}_{42}(\mathrm{t}) = \mathrm{T}_{42}^0 + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43}(s_{(40)}), s_{(40)})\right) T_{42}(s_{(40)})\right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** 166
Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ A
which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{44})^{(9)},\ T_i^0 \le (\hat{Q}_{44})^{(9)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''_{44})^{(9)}(T_{45}(s_{(44)}), s_{(44)})\right) G_{44}(s_{(44)})\right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45}(s_{(44)}), s_{(44)})\right) G_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}(s_{(44)}), s_{(44)})\right) G_{46}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{44}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{\mathrm{T}}_{46}(\mathrm{t}) = \mathrm{T}_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{46}(s_{(44)})\right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167
$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}}\right)\right] ds_{(13)} =$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left(e^{(\hat{M}_{13})^{(1)} t} - 1\right)$$

From which it follows that 168

$$(G_{13}(t)-G_{13}^{0})e^{-(\hat{M}_{13})^{(1)}t}\leq\frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}}\left[((\hat{P}_{13})^{(1)}+G_{14}^{0})e^{\left(-\frac{(\hat{P}_{13})^{(1)}+G_{14}^{0}}{G_{14}^{0}}\right)}+(\hat{P}_{13})^{(1)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t)\leq G_{16}^{0}+\int_{0}^{t}\left[(a_{16})^{(2)}\left(G_{17}^{0}+(\hat{P}_{16})^{(6)}e^{(\hat{M}_{16})^{(2)}s_{(16)}}\right)\right]ds_{(16)}= \quad 169$$

$$\left(1+(a_{16})^{(2)}t\right)G_{17}^{0}+\frac{(a_{16})^{(2)}(\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}}\left(e^{(\hat{M}_{16})^{(2)}t}-1\right)$$

From which it follows that 170

$$(G_{16}(t)-G_{16}^{0})e^{-(\hat{M}_{16})^{(2)}t}\leq\frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}}\left[((\hat{P}_{16})^{(2)}+G_{17}^{0})e^{\left(-\frac{(\hat{P}_{16})^{(2)}+G_{17}^{0}}{G_{17}^{0}}\right)}+(\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t)\leq G_{20}^{0}+\int_{0}^{t}\left[(a_{20})^{(3)}\left(G_{21}^{0}+(\hat{P}_{20})^{(3)}e^{(\hat{M}_{20})^{(3)}s_{(20)}}\right)\right]ds_{(20)}=$$

$$\left(1+(a_{20})^{(3)}t\right)G_{21}^{0}+\frac{(a_{20})^{(3)}(\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left(e^{(\hat{M}_{20})^{(3)}t}-1\right)$$

From which it follows that 172

$$(G_{20}(t)-G_{20}^{0})e^{-(\hat{M}_{20})^{(3)}t}\leq\frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left[((\hat{P}_{20})^{(3)}+G_{21}^{0})e^{\left(-\frac{(\hat{P}_{20})^{(3)}+G_{21}^{0}}{G_{21}^{0}}\right)}+(\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 173

$$G_{24}(t)\leq G_{24}^{0}+\int_{0}^{t}\left[(a_{24})^{(4)}\left(G_{25}^{0}+(\hat{P}_{24})^{(4)}e^{(\hat{M}_{24})^{(4)}s_{(24)}}\right)\right]ds_{(24)}=$$

$$\left(1+(a_{24})^{(4)}t\right)G_{25}^{0}+\frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left(e^{(\hat{M}_{24})^{(4)}t}-1\right)$$

From which it follows that 174

$$(G_{24}(t)-G_{24}^{0})e^{-(\hat{M}_{24})^{(4)}t}\leq\frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left[((\hat{P}_{24})^{(4)}+G_{25}^{0})e^{\left(-\frac{(\hat{P}_{24})^{(4)}+G_{25}^{0}}{G_{25}^{0}}\right)}+(\hat{P}_{24})^{(4)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t) \le G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} \left(G_{29}^0 + (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} s_{(28)}}\right)\right] ds_{(28)} =$$

$$\left(1 + (a_{28})^{(5)} t\right) G_{29}^0 + \frac{(a_{28})^{(5)} (\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}} \left(e^{(\hat{M}_{28})^{(5)} t} - 1\right)$$

From which it follows that 175

$$(G_{28}(t) - G_{28}^0) e^{-(\hat{M}_{28})^{(5)} t} \le \frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}} \left[\left((\hat{P}_{28})^{(5)} + G_{29}^0\right) e^{\left(-\frac{(\hat{P}_{28})^{(5)} + G_{29}^0}{G_{29}^0}\right)} + (\hat{P}_{28})^{(5)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t) \le G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} \left(G_{33}^0 + (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} s_{(32)}}\right)\right] ds_{(32)} =$$

$$\left(1 + (a_{32})^{(6)} t\right) G_{33}^0 + \frac{(a_{32})^{(6)} (\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}} \left(e^{(\hat{M}_{32})^{(6)} t} - 1\right)$$

From which it follows that 177

$$(G_{32}(t) - G_{32}^0) e^{-(\hat{M}_{32})^{(6)} t} \le \frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}} \left[\left((\hat{P}_{32})^{(6)} + G_{33}^0\right) e^{\left(-\frac{(\hat{P}_{32})^{(6)} + G_{33}^0}{G_{33}^0}\right)} + (\hat{P}_{32})^{(6)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

(o) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t) \le G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} \left(G_{37}^0 + (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} s_{(36)}}\right)\right] ds_{(36)} =$$

$$\left(1 + (a_{36})^{(7)} t\right) G_{37}^0 + \frac{(a_{36})^{(7)} (\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left(e^{(\hat{M}_{36})^{(7)} t} - 1\right)$$

From which it follows that

$$(G_{36}(t) - G_{36}^0) e^{-(\hat{M}_{36})^{(7)} t} \le \frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left[\left((\hat{P}_{36})^{(7)} + G_{37}^0\right) e^{\left(-\frac{(\hat{P}_{36})^{(7)} + G_{37}^0}{G_{37}^0}\right)} + (\hat{P}_{36})^{(7)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t) \le G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} \left(G_{41}^0 + (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} s_{(40)}}\right)\right] ds_{(40)} =$$ 180

$$\left(1+(a_{40})^{(8)}t\right)G_{41}^{0}+\frac{(a_{40})^{(8)}(\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left(e^{(\hat{M}_{40})^{(8)}t}-1\right)$$

From which it follows that 181

$$(G_{40}(t)-G_{40}^{0})e^{-(\hat{M}_{40})^{(8)}t}\leq\frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left[\left((\hat{P}_{40})^{(8)}+G_{41}^{0}\right)e^{\left(-\frac{(\hat{P}_{40})^{(8)}+G_{41}^{0}}{G_{41}^{0}}\right)}+(\hat{P}_{40})^{(8)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 8

Analogous inequalities hold also for $G_{41}, G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t)\leq G_{44}^{0}+\int_{0}^{t}\left[(a_{44})^{(9)}\left(G_{45}^{0}+(\hat{P}_{44})^{(9)}e^{(\hat{M}_{44})^{(9)}s_{(44)}}\right)\right]ds_{(44)}=$$

$$\left(1+(a_{44})^{(9)}t\right)G_{45}^{0}+\frac{(a_{44})^{(9)}(\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}}\left(e^{(\hat{M}_{44})^{(9)}t}-1\right)$$

From which it follows that

$$(G_{44}(t)-G_{44}^{0})e^{-(\hat{M}_{44})^{(9)}t}\leq\frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}}\left[\left((\hat{P}_{44})^{(9)}+G_{45}^{0}\right)e^{\left(-\frac{(\hat{P}_{44})^{(9)}+G_{45}^{0}}{G_{45}^{0}}\right)}+(\hat{P}_{44})^{(9)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 9

Analogous inequalities hold also for $G_{45}, G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}<1$ and to choose 182

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[(\hat{P}_{13})^{(1)}+\left((\hat{P}_{13})^{(1)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{13})^{(1)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{13})^{(1)} \quad 183$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[\left((\hat{Q}_{13})^{(1)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{13})^{(1)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{13})^{(1)}\right]\leq(\hat{Q}_{13})^{(1)} \quad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)},T^{(1)}\right),\left(G^{(2)},T^{(2)}\right)\right)=$$

$$\sup_i\left\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{13})^{(1)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{13})^{(1)}t}\right\}$$

Indeed if we denote

**Definition of** $\tilde{G}, \tilde{T}$ : $\left(\tilde{G}, \tilde{T}\right) = \mathcal{A}^{(1)}(G, T)$

It results

$$\left|\tilde{G}_{13}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{13})^{(1)} \left|G_{14}^{(1)} - G_{14}^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)} s_{(13)}} e^{(\widehat{M}_{13})^{(1)} s_{(13)}}\, ds_{(13)} +$$

$$\int_0^t \{(a'_{13})^{(1)} \left|G_{13}^{(1)} - G_{13}^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)} s_{(13)}} e^{-(\widehat{M}_{13})^{(1)} s_{(13)}} +$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)}, s_{(13)}\right)\left|G_{13}^{(1)} - G_{13}^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)} s_{(13)}} e^{(\widehat{M}_{13})^{(1)} s_{(13)}} +$$

$$G_{13}^{(2)} \left|(a''_{13})^{(1)}\left(T_{14}^{(1)}, s_{(13)}\right) - (a''_{13})^{(1)}\left(T_{14}^{(2)}, s_{(13)}\right)\right|\ e^{-(\widehat{M}_{13})^{(1)} s_{(13)}} e^{(\widehat{M}_{13})^{(1)} s_{(13)}}\} ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|G^{(1)} - G^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)} t} \leq \qquad 186$$
$$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)} + (a'_{13})^{(1)} + (\widehat{A}_{13})^{(1)} + (\widehat{P}_{13})^{(1)}(\widehat{k}_{13})^{(1)}\right) d\left(\left(G^{(1)}, T^{(1)};\ G^{(2)}, T^{(2)}\right)\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 1:** The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)} t}$ *and* $(\widehat{Q}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)} t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(1)}$ and $(b''_i)^{(1)}, i = 13{,}14{,}15$ depend only on $T_{14}$ and respectively on $G$(*and not on* $t$) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any $t$ where $G_i(t) = 0$ *and* $T_i(t) = 0$

From 19 to 24 it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a'_i)^{(1)} - (a''_i)^{(1)}(T_{14}(s_{(13)}), s_{(13)})\} ds_{(13)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b'_i)^{(1)} t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2$ *and* $\left((\widehat{M}_{13})^{(1)}\right)_3$ : 187

**Remark 3**: if $G_{13}$ is bounded, the same property have also $G_{14}$ *and* $G_{15}$ . indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a'_{14})^{(1)} G_{14}$ and by integrating

$$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1 / (a'_{14})^{(1)}$$

In the same way , one can obtain

$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2/(a'_{15})^{(1)}$

If $G_{14}$ *or* $G_{15}$ is bounded, the same property follows for $G_{13}$ , $G_{15}$ and $G_{13}$ , $G_{14}$ respectively.

**Remark 4:** If $G_{13}$ *is* bounded, from below, the same property holds for $G_{14}$ *and* $G_{15}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{14}$ is bounded from below. 188

**Remark 5:** If $T_{13}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}(G(t),t)) = (b'_{14})^{(1)}$ then $T_{14} \to \infty$. 189

**Definition of** $(m)^{(1)}$ and $\varepsilon_1$ :

Indeed let $t_1$ be so that for $t > t_1$

$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t),t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\frac{2}{\varepsilon_1}$ By taking now $\varepsilon_1$ sufficiently small one sees that $T_{14}$ is unbounded.
The same property holds for $T_{15}$ if $\lim_{t\to\infty}(b''_{15})^{(1)}(G(t),t) = (b'_{15})^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}$ , $\frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}$ *and* $(\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[(\widehat{P}_{16})^{(2)} + \left((\hat{P}_{16})^{(2)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{16})^{(2)}\right] \leq (\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$d\left(\left((G_{19})^{(1)},(T_{19})^{(1)}\right),\left((G_{19})^{(2)},(T_{19})^{(2)}\right)\right) =$

$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t}\}$

Indeed if we denote 195

**Definition of** $\widetilde{G_{19}}, \widetilde{T_{19}}$ : $\left(\widetilde{G_{19}}, \widetilde{T_{19}}\right) = \mathcal{A}^{(2)}(G_{19}, T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{16})^{(2)} \left|G_{17}^{(1)} - G_{17}^{(2)}\right| e^{-(\widehat{M}_{16})^{(2)} s_{(16)}} e^{(\widehat{M}_{16})^{(2)} s_{(16)}}\, ds_{(16)} +$$

$$\int_0^t \{(a'_{16})^{(2)} \left|G_{16}^{(1)} - G_{16}^{(2)}\right| e^{-(\widehat{M}_{16})^{(2)} s_{(16)}} e^{-(\widehat{M}_{16})^{(2)} s_{(16)}} +$$

$$(a''_{16})^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right)\left|G_{16}^{(1)} - G_{16}^{(2)}\right| e^{-(\widehat{M}_{16})^{(2)} s_{(16)}} e^{(\widehat{M}_{16})^{(2)} s_{(16)}} +$$

$$G_{16}^{(2)} \left|(a''_{16})^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right) - (a''_{16})^{(2)}\left(T_{17}^{(2)}, s_{(16)}\right)\right|\ e^{-(\widehat{M}_{16})^{(2)} s_{(16)}} e^{(\widehat{M}_{16})^{(2)} s_{(16)}}\} ds_{(16)}$$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0, t]$ 197

From the hypotheses it follows

$$\left|(G_{19})^{(1)} - (G_{19})^{(2)}\right| e^{-(\widehat{M}_{16})^{(2)} t} \leq$$
$$\frac{1}{(\widehat{M}_{16})^{(2)}}\left((a_{16})^{(2)} + (a'_{16})^{(2)} + (\widehat{A}_{16})^{(2)} + (\widehat{P}_{16})^{(2)}(\hat{k}_{16})^{(2)}\right) d\left(((G_{19})^{(1)}, (T_{19})^{(1)};\ (G_{19})^{(2)}, (T_{19})^{(2)})\right)$$

And analogous inequalities for $G_i$ and $T_i$. Taking into account the hypothesis the result follows 198

**Remark 6:** The fact that we supposed $(a''_{16})^{(2)}$ and $(b''_{16})^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{16})^{(2)} e^{(\widehat{M}_{16})^{(2)} t}$ and $(\widehat{Q}_{16})^{(2)} e^{(\widehat{M}_{16})^{(2)} t}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i = 16,17,18$ depend only on $T_{17}$ and respectively on $(G_{19})$(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 7:** There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$ 200

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(2)} - (a_i'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})\} ds_{(16)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(2)} t)} > 0 \quad \text{for } t > 0$$

**Definition of** $\left((\widehat{M}_{16})^{(2)}\right)_1$, $\left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$ : 201

**Remark 8:** if $G_{16}$ is bounded, the same property have also $G_{17}$ and $G_{18}$ . indeed if

$G_{16} < (\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt} \leq \left((\widehat{M}_{16})^{(2)}\right)_1 - (a'_{17})^{(2)} G_{17}$ and by integrating

$$G_{17} \leq \left((\widehat{M}_{16})^{(2)}\right)_2 = G_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1 / (a'_{17})^{(2)}$$

In the same way , one can obtain

$G_{18} \leq \left((\widehat{M}_{16})^{(2)}\right)_3 = G_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2/(a'_{18})^{(2)}$

If $G_{17}$ or $G_{18}$ is bounded, the same property follows for $G_{16}$ , $G_{18}$ and $G_{16}$ , $G_{17}$ respectively.

**Remark 9**: If $G_{16}$ is bounded, from below, the same property holds for $G_{17}$ and $G_{18}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{17}$ is bounded from below. 202

**Remark 10:** If $T_{16}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t),t)) = (b'_{17})^{(2)}$ then $T_{17} \to \infty$. 203

**Definition of** $(m)^{(2)}$ and $\varepsilon_2$ :

Indeed let $t_2$ be so that for $t > t_2$

$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t),t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right), \quad t = \log\frac{2}{\varepsilon_2}$ By taking now $\varepsilon_2$ sufficiently small one sees that $T_{17}$ is unbounded. 205
The same property holds for $T_{18}$ if $\lim_{t\to\infty}(b''_{18})^{(2)}\left((G_{19})(t),t\right) = (b'_{18})^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}$ , $\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[(\hat{P}_{20})^{(3)} + \left((\hat{P}_{20})^{(3)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{20})^{(3)}\right] \leq (\hat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself 210

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)},(T_{23})^{(1)}),((G_{23})^{(2)},(T_{23})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{20})^{(3)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

**Definition of** $\widetilde{G_{23}}, \widetilde{T_{23}} :\left( \widetilde{(G_{23})}, \widetilde{(T_{23})} \right) = \mathcal{A}^{(3)}\left((G_{23}), (T_{23})\right)$

It results 213

$$\left|\tilde{G}_{20}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{20})^{(3)} \left|G_{21}^{(1)} - G_{21}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} e^{(\widehat{M}_{20})^{(3)} s_{(20)}}\, ds_{(20)} +$$

$$\int_0^t \{(a'_{20})^{(3)} \left|G_{20}^{(1)} - G_{20}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} +$$

$$(a''_{20})^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right)\left|G_{20}^{(1)} - G_{20}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} e^{(\widehat{M}_{20})^{(3)} s_{(20)}} +$$

$$G_{20}^{(2)} \left|(a''_{20})^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right) - (a''_{20})^{(3)}\left(T_{21}^{(2)}, s_{(20)}\right)\right|\ e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} e^{(\widehat{M}_{20})^{(3)} s_{(20)}}\} ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\left|G_{23}{}^{(1)} - G_{23}{}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)} t} \leq \qquad 214$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)} + (a'_{20})^{(3)} + (\widehat{A}_{20})^{(3)} + (\widehat{P}_{20})^{(3)} (\widehat{k}_{20})^{(3)}\right) d\left(\left((G_{23})^{(1)}, (T_{23})^{(1)};\ (G_{23})^{(2)}, (T_{23})^{(2)}\right)\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 11:** The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)} e^{(\widehat{M}_{20})^{(3)} t}$ *and* $(\widehat{Q}_{20})^{(3)} e^{(\widehat{M}_{20})^{(3)} t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i = 20,21,22$ depend only on $T_{21}$ and respectively on $(G_{23})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 12:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 216

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(3)} - (a_i'')^{(3)}(T_{21}(s_{(20)}), s_{(20)})\} ds_{(20)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(3)} t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2\ and\ \left((\widehat{M}_{20})^{(3)}\right)_3$ : 217

**Remark 13:** if $G_{20}$ is bounded, the same property have also $G_{21}\ and\ G_{22}$ . indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a'_{21})^{(3)} G_{21}$ and by integrating

$$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1 / (a'_{21})^{(3)}$$

In the same way , one can obtain

$$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2/(a'_{22})^{(3)}$$

If $G_{21}$ *or* $G_{22}$ is bounded, the same property follows for $G_{20}$ , $G_{22}$ and $G_{20}$ , $G_{21}$ respectively.

**Remark 14:** If $G_{20}$ *is* bounded, from below, the same property holds for $G_{21}$ *and* $G_{22}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{21}$ is bounded from below. 218

**Remark 15:** If $T_{20}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t),t\right) = (b'_{21})^{(3)}$ then $T_{21} \to \infty$. 219

**Definition of** $(m)^{(3)}$ and $\varepsilon_3$ :

Indeed let $t_3$ be so that for $t > t_3$

$$(b_{21})^{(3)} - (b_i'')^{(3)}\left((G_{23})(t),t\right) < \varepsilon_3, T_{20}(t) > (m)^{(3)}$$

Then $\frac{dT_{21}}{dt} \geq (a_{21})^{(3)}(m)^{(3)} - \varepsilon_3 T_{21}$ which leads to 220

$$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t}$$ If we take t such that $e^{-\varepsilon_3 t} = \frac{1}{2}$ it results

$$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right), \quad t = log\frac{2}{\varepsilon_3}$$ By taking now $\varepsilon_3$ sufficiently small one sees that $T_{21}$ is unbounded.
The same property holds for $T_{22}$ if $\lim_{t\to\infty}(b''_{22})^{(3)}\left((G_{23})(t),t\right) = (b'_{22})^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\tilde{M}_{24})^{(4)}}$ , $\frac{(b_i)^{(4)}}{(\tilde{M}_{24})^{(4)}} < 1$ and to choose 221

$(\widehat{P}_{24})^{(4)}$ and $(\widehat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\tilde{M}_{24})^{(4)}}\left[(\widehat{P}_{24})^{(4)} + \left((\hat{P}_{24})^{(4)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{24})^{(4)}$$ 222

$$\frac{(b_i)^{(4)}}{(\tilde{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{24})^{(4)}\right] \leq (\hat{Q}_{24})^{(4)}$$ 223

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself 224

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric 225

$$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\tilde{M}_{24})^{(4)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\tilde{M}_{24})^{(4)}t}\}$$

Indeed if we denote

**Definition of** $\widetilde{(G_{27})}, \widetilde{(T_{27})}$ : $\left(\widetilde{(G_{27})}, \widetilde{(T_{27})}\right) = \mathcal{A}^{(4)}((G_{27}), (T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{24})^{(4)} \left|G_{25}^{(1)} - G_{25}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{(\widehat{M}_{24})^{(4)} s_{(24)}}\, ds_{(24)} +$$

$$\int_0^t \{(a'_{24})^{(4)} \left|G_{24}^{(1)} - G_{24}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} +$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)}, s_{(24)}\right)\left|G_{24}^{(1)} - G_{24}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{(\widehat{M}_{24})^{(4)} s_{(24)}} +$$

$$G_{24}^{(2)} \left|(a''_{24})^{(4)}\left(T_{25}^{(1)}, s_{(24)}\right) - (a''_{24})^{(4)}\left(T_{25}^{(2)}, s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{(\widehat{M}_{24})^{(4)} s_{(24)}}\} ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)} - (G_{27})^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} t} \leq \qquad (226)$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)} + (a'_{24})^{(4)} + (\widehat{A}_{24})^{(4)} + (\widehat{P}_{24})^{(4)} (\widehat{k}_{24})^{(4)}\right) d\left(((G_{27})^{(1)}, (T_{27})^{(1)};\ (G_{27})^{(2)}, (T_{27})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 16:** The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)} t}$ *and* $(\widehat{Q}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)} t}$ respectively of $\mathbb{R}_+$. (227)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i = 24{,}25{,}26$ depend only on $T_{25}$ and respectively on $(G_{27})$(*and not on* $t$) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 17:** There does not exist any $t$ where $G_i(t) = 0$ *and* $T_i(t) = 0$ (228)

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(4)} - (a_i'')^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(4)} t)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2$ *and* $\left((\widehat{M}_{24})^{(4)}\right)_3$ : (229)

**Remark 18:** if $G_{24}$ is bounded, the same property have also $G_{25}$ *and* $G_{26}$ . indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a'_{25})^{(4)} G_{25}$ and by integrating

$$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1 / (a'_{25})^{(4)}$$

In the same way , one can obtain

$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2/(a'_{26})^{(4)}$

If $G_{25}$ *or* $G_{26}$ is bounded, the same property follows for $G_{24}$ , $G_{26}$ and $G_{24}$ , $G_{25}$ respectively.

**Remark 19:** If $G_{24}$ *is* bounded, from below, the same property holds for $G_{25}$ *and* $G_{26}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{25}$ is bounded from below. 230

**Remark 20:** If $T_{24}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}((G_{27})(t),t)) = (b'_{25})^{(4)}$ then $T_{25} \to \infty$. 231

**Definition of** $(m)^{(4)}$ and $\varepsilon_4$ :

Indeed let $t_4$ be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t),t) < \varepsilon_4, T_{24}(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right), \quad t = log\frac{2}{\varepsilon_4}$ By taking now $\varepsilon_4$ sufficiently small one sees that $T_{25}$ is unbounded.
The same property holds for $T_{26}$ if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t),t\right) = (b'_{26})^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for $G_{29}$ , $G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}$ , $\frac{(b_i)^{(5)}}{(\widehat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\widehat{P}_{28})^{(5)}$ and $(\widehat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}\left[(\widehat{P}_{28})^{(5)} + \left((\widehat{P}_{28})^{(5)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}\left[\left((\widehat{Q}_{28})^{(5)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{28})^{(5)}\right] \leq (\widehat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$d\left(((G_{31})^{(1)}, (T_{31})^{(1)}), ((G_{31})^{(2)}, (T_{31})^{(2)})\right) =$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{28})^{(5)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{31}}),(\widetilde{T_{31}})$ : $\left((\widetilde{G_{31}}),(\widetilde{T_{31}})\right)=\mathcal{A}^{(5)}\left((G_{31}),(T_{31})\right)$

It results

$$\left|\tilde{G}_{28}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{28})^{(5)}\left|G_{29}^{(1)}-G_{29}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\,ds_{(28)}+$$

$$\int_0^t\{(a'_{28})^{(5)}\left|G_{28}^{(1)}-G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$(a''_{28})^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)\left|G_{28}^{(1)}-G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$G_{28}^{(2)}\left|(a''_{28})^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)-(a''_{28})^{(5)}\left(T_{29}^{(2)},s_{(28)}\right)\right|\ e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\}ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on it follows

$$\left|(G_{31})^{(1)}-(G_{31})^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}t}\le \qquad 237$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)}+(a'_{28})^{(5)}+(\widehat{A}_{28})^{(5)}+(\widehat{P}_{28})^{(5)}(\widehat{k}_{28})^{(5)}\right)d\left(((G_{31})^{(1)},(T_{31})^{(1)};\ (G_{31})^{(2)},(T_{31})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

**Remark 21:** The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}\ and\ (\widehat{Q}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ respectively of $\mathbb{R}_+$. 238

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i=28,29,30$ depend only on $T_{29}$ and respectively on $(G_{31})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 22:** There does not exist any $t$ where $G_i(t)=0\ and\ T_i(t)=0$ 239

it results

$$G_i(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(5)}-(a_i'')^{(5)}(T_{29}(s_{(28)}),s_{(28)})\}ds_{(28)}\right]}\ge 0$$

$$T_i(t)\ge T_i^0e^{(-(b_i')^{(5)}t)}>0\quad \text{for t}>0$$

**Definition of** $\left((\widehat{M}_{28})^{(5)}\right)_1,\left((\widehat{M}_{28})^{(5)}\right)_2\ and\ \left((\widehat{M}_{28})^{(5)}\right)_3$ : 240

**Remark 23:** if $G_{28}$ is bounded, the same property have also $G_{29}\ and\ G_{30}$ . indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a'_{29})^{(5)} G_{29}$ and by integrating

$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1/(a'_{29})^{(5)}$

In the same way , one can obtain

$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2/(a'_{30})^{(5)}$

If $G_{29}$ *or* $G_{30}$ is bounded, the same property follows for $G_{28}$ , $G_{30}$ and $G_{28}$ , $G_{29}$ respectively.

**Remark 24:** If $G_{28}$ *is* bounded, from below, the same property holds for $G_{29}$ *and* $G_{30}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{29}$ is bounded from below. 241

**Remark 25:** If $T_{28}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}((G_{31})(t),t)) = (b'_{29})^{(5)}$ then $T_{29} \to \infty$. 242

**Definition of** $(m)^{(5)}$ and $\varepsilon_5$ :

Indeed let $t_5$ be so that for $t > t_5$

$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t),t) < \varepsilon_5, T_{28}(t) > (m)^{(5)}$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\frac{2}{\varepsilon_5}$ By taking now $\varepsilon_5$ sufficiently small one sees that $T_{29}$ is unbounded.
The same property holds for $T_{30}$ if $\lim_{t\to\infty}(b_{30}'')^{(5)}\left((G_{31})(t),t\right) = (b'_{30})^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{33}$ , $G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}$ , $\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}} < 1$ and to choose 244

$(\widehat{P}_{32})^{(6)}$ and $(\widehat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\left[(\widehat{P}_{32})^{(6)} + \left((\widehat{P}_{32})^{(6)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{32})^{(6)} \quad 245$$

$$\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\left[\left((\widehat{Q}_{32})^{(6)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{32})^{(6)}\right] \leq (\widehat{Q}_{32})^{(6)} \quad 246$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric 247

$$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{35}}),(\widetilde{T_{35}})$ : $\left((\widetilde{G_{35}}),(\widetilde{T_{35}})\right)=\mathcal{A}^{(6)}\left((G_{35}),(T_{35})\right)$

It results

$$\left|\tilde{G}_{32}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{32})^{(6)}\left|G_{33}^{(1)}-G_{33}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\,ds_{(32)}+$$

$$\int_0^t\{(a'_{32})^{(6)}\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$G_{32}^{(2)}\left|(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)-(a''_{32})^{(6)}\left(T_{33}^{(2)},s_{(32)}\right)\right|\ e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\}ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|(G_{35})^{(1)}-(G_{35})^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}t}\le \qquad 248$$
$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)}+(a'_{32})^{(6)}+(\widehat{A}_{32})^{(6)}+(\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}\right)d\left(((G_{35})^{(1)},(T_{35})^{(1)};\ (G_{35})^{(2)},(T_{35})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 26:** The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ *and* $(\widehat{Q}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ respectively of $\mathbb{R}_+$. 249

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i=32,33,34$ depend only on $T_{33}$ and respectively on $(G_{35})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 27:** There does not exist any $t$ where $G_i(t)=0$ *and* $T_i(t)=0$ 250

it results

$$G_i(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(6)}-(a_i'')^{(6)}(T_{33}(s_{(32)}),s_{(32)})\}ds_{(32)}\right]}\ge 0$$

$$T_i(t)\ge T_i^0e^{\left(-(b_i')^{(6)}t\right)}>0\quad \text{for t}>0$$

**Definition of** $\left((\widehat{M}_{32})^{(6)}\right)_1,\left((\widehat{M}_{32})^{(6)}\right)_2$ *and* $\left((\widehat{M}_{32})^{(6)}\right)_3$ : 251

**Remark 28:** if $G_{32}$ is bounded, the same property have also $G_{33}\ and\ G_{34}$ . indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a'_{33})^{(6)}G_{33}$ and by integrating

$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_1/(a'_{33})^{(6)}$

In the same way , one can obtain

$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_2/(a'_{34})^{(6)}$

If $G_{33}\ or\ G_{34}$ is bounded, the same property follows for $G_{32}$ , $G_{34}$ and $G_{32}$ , $G_{33}$ respectively.

**Remark 29:** If $G_{32}$ *is* bounded, from below, the same property holds for $G_{33}\ and\ G_{34}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{33}$ is bounded from below. 252

**Remark 30:** If $T_{32}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}\ ((G_{35})(t),t)) = (b'_{33})^{(6)}$ then $T_{33} \to \infty$. 253

**Definition of** $(m)^{(6)}$ and $\varepsilon_6$ :

Indeed let $t_6$ be so that for $t > t_6$

$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t),t\right) < \varepsilon_6, T_{32}(t) > (m)^{(6)}$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right),\quad t = log\frac{2}{\varepsilon_6}$ By taking now $\varepsilon_6$ sufficiently small one sees that $T_{33}$ is unbounded.
The same property holds for $T_{34}$ if $\lim_{t\to\infty}(b''_{34})^{(6)}\left((G_{35})(t),t(t),t\right) = (b'_{34})^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{37}$ , $G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}$ , $\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}} < 1$ and to choose
$(\widehat{P}_{36})^{(7)}$ and $(\widehat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[(\widehat{P}_{36})^{(7)} + \left((\widehat{P}_{36})^{(7)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{36})^{(7)} \quad 256$$

$$\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[\left((\widehat{Q}_{36})^{(7)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{36})^{(7)}\right] \leq (\widehat{Q}_{36})^{(7)} \quad 257$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying

Equations into itself

The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric 258

$$d\left(((G_{39})^{(1)},(T_{39})^{(1)}),((G_{39})^{(2)},(T_{39})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\widehat{M}_{36})^{(7)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\widehat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

**Definition of** $\widetilde{(G_{39})},\widetilde{(T_{39})}:\left(\widetilde{(G_{39})},\widetilde{(T_{39})}\right)=\mathcal{A}^{(7)}((G_{39}),(T_{39}))$

It results

$$|\tilde{G}_{36}^{(1)}-\tilde{G}_i^{(2)}|\le\int_0^t(a_{36})^{(7)}|G_{37}^{(1)}-G_{37}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}ds_{(36)}+$$
$$\int_0^t\{(a'_{36})^{(7)}|G_{36}^{(1)}-G_{36}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$(a''_{36})^{(7)}(T_{37}^{(1)},s_{(36)})|G_{36}^{(1)}-G_{36}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$G_{36}^{(2)}|(a''_{36})^{(7)}(T_{37}^{(1)},s_{(36)})-(a''_{36})^{(7)}(T_{37}^{(2)},s_{(36)})|\ e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\}ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on it follows

$$|(G_{39})^{(1)}-(G_{39})^{(2)}|e^{-(\widehat{M}_{36})^{(7)}t}\le$$ 259
$$\frac{1}{(\widehat{M}_{36})^{(7)}}((a_{36})^{(7)}+(a'_{36})^{(7)}+(\widehat{A}_{36})^{(7)}+(\widehat{P}_{36})^{(7)}(\widehat{k}_{36})^{(7)})d\left(((G_{39})^{(1)},(T_{39})^{(1)};\ (G_{39})^{(2)},(T_{39})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

**Remark 31:** The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}\ and\ (\widehat{Q}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ respectively of $\mathbb{R}_+$. 260

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(7)}$ and $(b_i'')^{(7)}, i=36,37,38$ depend only on $T_{37}$ and respectively on $(G_{39})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 32:** There does not exist any $t$ where $G_i(t)=0\ and\ T_i(t)=0$ 261

it results

$$G_i(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(7)}-(a_i'')^{(7)}(T_{37}(s_{(36)}),s_{(36)})\}ds_{(36)}\right]}\ge 0$$

$$T_i(t)\ge T_i^0e^{(-(b_i')^{(7)}t)}>0\quad \text{for t}>0$$

**Definition of** $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2$ *and* $\left((\widehat{M}_{36})^{(7)}\right)_3$ : 262

**Remark 33:** if $G_{36}$ is bounded, the same property have also $G_{37}$ *and* $G_{38}$ . indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a'_{37})^{(7)} G_{37}$ and by integrating

$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1/(a'_{37})^{(7)}$

In the same way , one can obtain

$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2/(a'_{38})^{(7)}$

If $G_{37}$ *or* $G_{38}$ is bounded, the same property follows for $G_{36}$ , $G_{38}$ and $G_{36}$ , $G_{37}$ respectively.

**Remark 34:** If $G_{36}$ *is* bounded, from below, the same property holds for $G_{37}$ *and* $G_{38}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{37}$ is bounded from below. 263

**Remark 35:** If $T_{36}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(7)}((G_{39})(t),t)) = (b'_{37})^{(7)}$ then $T_{37} \to \infty$. 264

**Definition of** $(m)^{(7)}$ and $\varepsilon_7$ :

Indeed let $t_7$ be so that for $t > t_7$

$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t),t) < \varepsilon_7, T_{36}(t) > (m)^{(7)}$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right), \quad t = log\frac{2}{\varepsilon_7}$ By taking now $\varepsilon_7$ sufficiently small one sees that $T_{37}$ is unbounded.
The same property holds for $T_{38}$ if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t),t\right) = (b'_{38})^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}$ , $\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1$ and to choose 266
$(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ large to have

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[(\hat{P}_{40})^{(8)} + \left((\hat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{40})^{(8)} \quad 267$$

268

$$\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{40})^{(8)}\right]\leq(\hat{Q}_{40})^{(8)}$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)},(T_{43})^{(1)}),((G_{43})^{(2)},(T_{43})^{(2)})\right)= \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

**Definition of** $(\widetilde{G_{43}}),(\widetilde{T_{43}})$ : $\left((\widetilde{G_{43}}),(\widetilde{T_{43}})\right)=\mathcal{A}^{(8)}((G_{43}),(T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{40})^{(8)}\left|G_{41}^{(1)}-G_{41}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\,ds_{(40)}+$$
$$\int_0^t\{(a'_{40})^{(8)}\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$G_{40}^{(2)}\left|(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)-(a''_{40})^{(8)}\left(T_{41}^{(2)},s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\}ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)}-(G_{43})^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}t}\leq \quad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)}+(a'_{40})^{(8)}+(\widehat{A}_{40})^{(8)}+(\widehat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}\right)d\left(((G_{43})^{(1)},(T_{43})^{(1)};\ (G_{43})^{(2)},(T_{43})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 36:** The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ *and* $(\widehat{Q}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ respectively of $\mathbb{R}_+$. 274

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i=40,41,42$ depend only on $\mathrm{T}_{41}$ and respectively on $(G_{43})$(*and not on* $t$) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 37** There does not exist any $t$ where $G_i(t)=0$ *and* $T_i(t)=0$ 275

it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(8)} - (a_i'')^{(8)}(T_{41}(s_{(40)}), s_{(40)})\} ds_{(40)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{(-(b_i')^{(8)} t)} > 0$ for t > 0

**Definition of** $\left((\widehat{M}_{40})^{(8)}\right)_1, \left((\widehat{M}_{40})^{(8)}\right)_2$ *and* $\left((\widehat{M}_{40})^{(8)}\right)_3$ : 276

**Remark 38:** if $G_{40}$ is bounded, the same property have also $G_{41}$ *and* $G_{42}$ . indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a_{41}')^{(8)} G_{41}$ and by integrating

$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)} \left((\widehat{M}_{40})^{(8)}\right)_1 / (a_{41}')^{(8)}$

In the same way , one can obtain

$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)} \left((\widehat{M}_{40})^{(8)}\right)_2 / (a_{42}')^{(8)}$

If $G_{41}$ *or* $G_{42}$ is bounded, the same property follows for $G_{40}$ , $G_{42}$ and $G_{40}$ , $G_{41}$ respectively.

**Remark 39:** If $G_{40}$ *is* bounded, from below, the same property holds for $G_{41}$ *and* $G_{42}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{41}$ is bounded from below. 277

**Remark 40:** If $T_{40}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}((G_{43})(t), t)) = (b_{41}')^{(8)}$ then $T_{41} \to \infty$. 278

**Definition of** $(m)^{(8)}$ and $\varepsilon_8$ :

Indeed let $t_8$ be so that for $t > t_8$

$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t), t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right), \quad t = log\frac{2}{\varepsilon_8}$ By taking now $\varepsilon_8$ sufficiently small one sees that $T_{41}$ is unbounded.
The same property holds for $T_{42}$ if $\lim_{t\to\infty}(b_{42}'')^{(8)}\left((G_{43})(t), t(t), t\right) = (b_{42}')^{(8)}$

It is now sufficient to take $\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}$ , $\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}} < 1$ and to choose $(\widehat{P}_{44})^{(9)}$ and $(\widehat{Q}_{44})^{(9)}$ large to have 279 A

$$\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[(\widehat{P}_{44})^{(9)} + \left((\widehat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{44})^{(9)} + G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[\left((\hat{Q}_{44})^{(9)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{44})^{(9)}\right]\leq(\hat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)},(T_{47})^{(1)}),\left((G_{47})^{(2)},(T_{47})^{(2)}\right)\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

**Definition of** $\widetilde{(G_{47})},\widetilde{(T_{47})}:\left(\widetilde{(G_{47})},\widetilde{(T_{47})}\right)=\mathcal{A}^{(9)}\left((G_{47}),(T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{44})^{(9)}\left|G_{45}^{(1)}-G_{45}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\,ds_{(44)}+$$
$$\int_0^t\{(a'_{44})^{(9)}\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$G_{44}^{(2)}\left|(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)-(a''_{44})^{(9)}\left(T_{45}^{(2)},s_{(44)}\right)\right|\;e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\}ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}t}\leq$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)}+(a'_{44})^{(9)}+(\widehat{A}_{44})^{(9)}+(\widehat{P}_{44})^{(9)}(\hat{k}_{44})^{(9)}\right)d\left(((G_{47})^{(1)},(T_{47})^{(1)};\;(G_{47})^{(2)},(T_{47})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis (39,35,36) the result follows

**Remark 41:** The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}\ and\ (\widehat{Q}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i=44,45,46$ depend only on $T_{45}$ and respectively on $(G_{47})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 42:** There does not exist any $t$ where $G_i\,(t)=0\ and\ T_i\,(t)=0$

From 99 to 44 it results

$$G_i\,(t)\geq G_i^0e^{\left[-\int_0^t\{(a_i')^{(9)}-(a_i'')^{(9)}(T_{45}(s_{(44)}),s_{(44)})\}ds_{(44)}\right]}\geq 0$$

$T_i(t) \geq T_i^0 e^{(-(b_i')^{(9)}t)} > 0$ for t > 0

**Definition of** $\left((\widehat{M}_{44})^{(9)}\right)_1, \left((\widehat{M}_{44})^{(9)}\right)_2$ *and* $\left((\widehat{M}_{44})^{(9)}\right)_3$ :

**Remark 43:** if $G_{44}$ is bounded, the same property have also $G_{45}$ *and* $G_{46}$ . indeed if

$G_{44} < (\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt} \leq \left((\widehat{M}_{44})^{(9)}\right)_1 - (a_{45}')^{(9)} G_{45}$ and by integrating

$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_1/(a_{45}')^{(9)}$

In the same way , one can obtain

$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_2/(a_{46}')^{(9)}$

If $G_{45}$ *or* $G_{46}$ is bounded, the same property follows for $G_{44}$ , $G_{46}$ and $G_{44}$ , $G_{45}$ respectively.

**Remark 44:** If $G_{44}$ *is* bounded, from below, the same property holds for $G_{45}$ *and* $G_{46}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{45}$ is bounded from below.

**Remark 45:** If $T_{44}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t),t\right) = (b_{45}')^{(9)}$ then $T_{45} \to \infty$.

**Definition of** $(m)^{(9)}$ and $\varepsilon_9$ :

Indeed let $t_9$ be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t),t\right) < \varepsilon_9, T_{44}(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right), \quad t = log\frac{2}{\varepsilon_9}$ By taking now $\varepsilon_9$ sufficiently small one sees that $T_{45}$ is unbounded.
The same property holds for $T_{46}$ if $\lim_{t\to\infty}(b_{46}'')^{(9)}\left((G_{47})(t),t\right) = (b_{46}')^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

**Behavior of the solutions of equation** 280

**Theorem** If we denote and define

**Definition of** $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ :

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)} \leq -(a_{13}')^{(1)} + (a_{14}')^{(1)} - (a_{13}'')^{(1)}(T_{14},t) + (a_{14}'')^{(1)}(T_{14},t) \leq -(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)} \leq -(b_{13}')^{(1)} + (b_{14}')^{(1)} - (b_{13}'')^{(1)}(G,t) - (b_{14}'')^{(1)}(G,t) \leq -(\tau_1)^{(1)}$

**Definition of** $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$ : 281

By $(v_1)^{(1)} > 0$ , $(v_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$ , $(u_2)^{(1)} < 0$ the roots of the equations $(a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

**Definition of** $(\bar{v}_1)^{(1)}$,, $(\bar{v}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$ : 282

By $(\bar{v}_1)^{(1)} > 0$ , $(\bar{v}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$ , $(\bar{u}_2)^{(1)} < 0$ the roots of the equations $(a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

**Definition of** $(m_1)^{(1)}$ , $(m_2)^{(1)}$ , $(\mu_1)^{(1)}, (\mu_2)^{(1)}, (v_0)^{(1)}$ :- 283

If we define $(m_1)^{(1)}$ , $(m_2)^{(1)}$ , $(\mu_1)^{(1)}, (\mu_2)^{(1)}$ by

$(m_2)^{(1)} = (v_0)^{(1)}, (m_1)^{(1)} = (v_1)^{(1)}$, $if\ (v_0)^{(1)} < (v_1)^{(1)}$

$(m_2)^{(1)} = (v_1)^{(1)}, (m_1)^{(1)} = (\bar{v}_1)^{(1)}$ , $if\ (v_1)^{(1)} < (v_0)^{(1)} < (\bar{v}_1)^{(1)}$,

and $\boxed{(v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$

$(m_2)^{(1)} = (v_1)^{(1)}, (m_1)^{(1)} = (v_0)^{(1)}$, $if\ (\bar{v}_1)^{(1)} < (v_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)}$, $if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}$ , $if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)}$,

and $\boxed{(u_0)^{(1)} = \frac{T_{13}^0}{T_{14}^0}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$

are defined

Then the solution of global equations satisfies the inequalities 285

$$G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{13}(t) \le G_{13}^0 e^{(S_1)^{(1)}t}$$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)}t}$$

$$\left(\frac{(a_{15})^{(1)} G_{13}^0}{(m_1)^{(1)}((S_1)^{(1)} - (p_{13})^{(1)} - (S_2)^{(1)})}\left[e^{((S_1)^{(1)} - (p_{13})^{(1)})t} - e^{-(S_2)^{(1)}t}\right] + G_{15}^0 e^{-(S_2)^{(1)}t} \le G_{15}(t) \le\right. \quad 286$$

$$\left.\frac{(a_{15})^{(1)} G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)} - (a'_{15})^{(1)})}\left[e^{(S_1)^{(1)}t} - e^{-(a'_{15})^{(1)}t}\right] + G_{15}^0 e^{-(a'_{15})^{(1)}t}\right)$$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)}t} \le T_{13}(t) \le T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t}} \quad 287$$

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)}t} \le T_{13}(t) \le \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t} \quad 288$$

$\frac{(b_{15})^{(1)}T_{13}^{0}}{(\mu_1)^{(1)}((R_1)^{(1)}-(b'_{15})^{(1)})}\left[e^{(R_1)^{(1)}t}-e^{-(b'_{15})^{(1)}t}\right]+T_{15}^{0}e^{-(b'_{15})^{(1)}t}\leq T_{15}(t)\leq$ 289

$\frac{(a_{15})^{(1)}T_{13}^{0}}{(\mu_2)^{(1)}((R_1)^{(1)}+(r_{13})^{(1)}+(R_2)^{(1)})}\left[e^{((R_1)^{(1)}+(r_{13})^{(1)})t}-e^{-(R_2)^{(1)}t}\right]+T_{15}^{0}e^{-(R_2)^{(1)}t}$

**Definition of** $(S_1)^{(1)},(S_2)^{(1)},(R_1)^{(1)},(R_2)^{(1)}$:- 290

Where $(S_1)^{(1)}=(a_{13})^{(1)}(m_2)^{(1)}-(a'_{13})^{(1)}$

$(S_2)^{(1)}=(a_{15})^{(1)}-(p_{15})^{(1)}$

$(R_1)^{(1)}=(b_{13})^{(1)}(\mu_2)^{(1)}-(b'_{13})^{(1)}$

$(R_2)^{(1)}=(b'_{15})^{(1)}-(r_{15})^{(1)}$

**Behavior of the solutions of equation** 291

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(2)},(\sigma_2)^{(2)},(\tau_1)^{(2)},(\tau_2)^{(2)}$ : 292

$(\sigma_1)^{(2)},(\sigma_2)^{(2)},(\tau_1)^{(2)},(\tau_2)^{(2)}$ four constants satisfying
$-(\sigma_2)^{(2)}\leq-(a'_{16})^{(2)}+(a'_{17})^{(2)}-(a''_{16})^{(2)}(\mathrm{T}_{17},t)+(a''_{17})^{(2)}(\mathrm{T}_{17},t)\leq-(\sigma_1)^{(2)}$ 293

$-(\tau_2)^{(2)}\leq-(b'_{16})^{(2)}+(b'_{17})^{(2)}-(b''_{16})^{(2)}\big((G_{19}),t\big)-(b''_{17})^{(2)}\big((G_{19}),t\big)\leq-(\tau_1)^{(2)}$ 294

**Definition of** $(\nu_1)^{(2)},(\nu_2)^{(2)},(u_1)^{(2)},(u_2)^{(2)}$ : 295

By $(\nu_1)^{(2)}>0$ , $(\nu_2)^{(2)}<0$ and respectively $(u_1)^{(2)}>0$ , $(u_2)^{(2)}<0$ the roots 296

of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2+(\sigma_1)^{(2)}\nu^{(2)}-(a_{16})^{(2)}=0$ 297

and $(b_{14})^{(2)}\big(u^{(2)}\big)^2+(\tau_1)^{(2)}u^{(2)}-(b_{16})^{(2)}=0$ and 298

**Definition of** $(\bar{\nu}_1)^{(2)},,(\bar{\nu}_2)^{(2)},(\bar{u}_1)^{(2)},(\bar{u}_2)^{(2)}$ : 299

By $(\bar{\nu}_1)^{(2)}>0$ , $(\bar{\nu}_2)^{(2)}<0$ and respectively $(\bar{u}_1)^{(2)}>0$ , $(\bar{u}_2)^{(2)}<0$ the 300

roots of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2+(\sigma_2)^{(2)}\nu^{(2)}-(a_{16})^{(2)}=0$ 301

and $(b_{17})^{(2)}\big(u^{(2)}\big)^2+(\tau_2)^{(2)}u^{(2)}-(b_{16})^{(2)}=0$ 302

**Definition of** $(m_1)^{(2)},(m_2)^{(2)},(\mu_1)^{(2)},(\mu_2)^{(2)}$ :- 303

If we define $(m_1)^{(2)},(m_2)^{(2)},(\mu_1)^{(2)},(\mu_2)^{(2)}$ by 304

$(m_2)^{(2)}=(\nu_0)^{(2)},(m_1)^{(2)}=(\nu_1)^{(2)},$ ***if*** $(\nu_0)^{(2)}<(\nu_1)^{(2)}$ 305

$(m_2)^{(2)}=(\nu_1)^{(2)},(m_1)^{(2)}=(\bar{\nu}_1)^{(2)}$ , ***if*** $(\nu_1)^{(2)}<(\nu_0)^{(2)}<(\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)},\ \boldsymbol{if}\ (u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}, \boldsymbol{if}\ (u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)},$

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)},\ \boldsymbol{if}\ (\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)} t}$

$(p_i)^{(2)}$ is defined by equation

$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)} t}$ 311

$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)} t}\right] + G_{18}^0 e^{-(S_2)^{(2)} t} \le G_{18}(t) \le\right.$ 312

$\left.\frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)} t} - e^{-(a'_{18})^{(2)} t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)} t}\right)$

$\boxed{T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}}$ 313

$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}$ 314

$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)} t} - e^{-(b'_{18})^{(2)} t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)} t} \le T_{18}(t) \le$ 315

$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})}\left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)} t}\right] + T_{18}^0 e^{-(R_2)^{(2)} t}$

**Definition of** $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$

$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)}$ 318

$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$

**Behavior of the solutions** 319

**Theorem 3:** If we denote and define

**Definition of** $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ :

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}\big((G_{23}), t\big) \le -(\tau_1)^{(3)}$$

**Definition of** $(v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$ : 320

By $(v_1)^{(3)} > 0$ , $(v_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$ , $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\big(v^{(3)}\big)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{v}_1)^{(3)} > 0$ , $(\bar{v}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$ , $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\big(v^{(3)}\big)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

**Definition of** $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ :- 321

If we define $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (v_0)^{(3)}, (m_1)^{(3)} = (v_1)^{(3)}$, ***if*** $(v_0)^{(3)} < (v_1)^{(3)}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (\bar{v}_1)^{(3)}$ , ***if*** $(v_1)^{(3)} < (v_0)^{(3)} < (\bar{v}_1)^{(3)}$,

and $\boxed{(v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_0)^{(3)}$, ***if*** $(\bar{v}_1)^{(3)} < (v_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}$, ***if*** $(u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$ , ***if*** $(u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}$, and $\boxed{(u_0)^{(3)} = \frac{T^0_{20}}{T^0_{21}}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}$, ***if*** $(\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$$G^0_{20}e^{((s_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G^0_{20}e^{(s_1)^{(3)}t}$$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)}-(p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)}t} \quad 323$$

$$\left(\frac{(a_{22})^{(3)}G_{20}^0}{(m_1)^{(3)}((S_1)^{(3)}-(p_{20})^{(3)}-(S_2)^{(3)})}\left[e^{((S_1)^{(3)}-(p_{20})^{(3)})t} - e^{-(S_2)^{(3)}t}\right] + G_{22}^0 e^{-(S_2)^{(3)}t} \le G_{22}(t) \le \right. \quad 324$$

$$\left. \frac{(a_{22})^{(3)}G_{20}^0}{(m_2)^{(3)}((S_1)^{(3)}-(a'_{22})^{(3)})}[e^{(S_1)^{(3)}t} - e^{-(a'_{22})^{(3)}t}] + G_{22}^0 e^{-(a'_{22})^{(3)}t}\right)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)}+(r_{20})^{(3)})t}} \quad 325$$

$$\frac{1}{(\mu_1)^{(3)}} T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}} T_{20}^0 e^{((R_1)^{(3)}+(r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)}T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)}-(b'_{22})^{(3)})}\left[e^{(R_1)^{(3)}t} - e^{-(b'_{22})^{(3)}t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)}t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)}T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)}+(r_{20})^{(3)}+(R_2)^{(3)})}\left[e^{((R_1)^{(3)}+(r_{20})^{(3)})t} - e^{-(R_2)^{(3)}t}\right] + T_{22}^0 e^{-(R_2)^{(3)}t}$$

**Definition of** $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$$

$$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$$

$$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$$

**Behavior of the solutions of equation**

**Theorem:** If we denote and define

**Definition of** $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ :

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$$

$$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}((G_{27}), t) - (b''_{25})^{(4)}((G_{27}), t) \le -(\tau_1)^{(4)}$$

**Definition of** $(v_1)^{(4)}, (v_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, v^{(4)}, u^{(4)}$ : 329

By $(v_1)^{(4)} > 0$ , $(v_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$ , $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_1)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$

and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

**Definition of** $(\bar{v}_1)^{(4)}, , (\bar{v}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$ : 330

By $(\bar{v}_1)^{(4)} > 0$ , $(\bar{v}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$ , $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$

and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

**Definition of** $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}, (\nu_0)^{(4)}$ :-

If we define $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (\nu_0)^{(4)}, (m_1)^{(4)} = (\nu_1)^{(4)}$, ***if*** $(\nu_0)^{(4)} < (\nu_1)^{(4)}$

$(m_2)^{(4)} = (\nu_1)^{(4)}, (m_1)^{(4)} = (\bar{\nu}_1)^{(4)}$, ***if*** $(\nu_4)^{(4)} < (\nu_0)^{(4)} < (\bar{\nu}_1)^{(4)}$,

and $\boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (\nu_4)^{(4)}, (m_1)^{(4)} = (\nu_0)^{(4)}$, ***if*** $(\bar{\nu}_4)^{(4)} < (\nu_0)^{(4)}$

and analogously

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$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)}$, ***if*** $(u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}$, ***if*** $(u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)}$,

and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}$, ***if*** $(\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

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$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)} t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)} t} \quad 333$$

$$\left( \frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})} \left[ e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)} t} \right] + G_{26}^0 e^{-(S_2)^{(4)} t} \le G_{26}(t) \le \right. \quad 334$$

*(a26)4G240(m2)4(S1)4−(a26′)4e(S1)4t−e−(a26′)4t+ G260e−(a26′)4t*

$$\boxed{T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}} T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}} T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t} \quad 335$$

$$\frac{(b_{26})^{(4)} T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)} - (b'_{26})^{(4)})} \left[ e^{(R_1)^{(4)} t} - e^{-(b'_{26})^{(4)} t} \right] + T_{26}^0 e^{-(b'_{26})^{(4)} t} \le T_{26}(t) \le \quad 336$$

$$\frac{(a_{26})^{(4)} T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)} + (r_{24})^{(4)} + (R_2)^{(4)})} \left[ e^{((R_1)^{(4)} + (r_{24})^{(4)})t} - e^{-(R_2)^{(4)} t} \right] + T_{26}^0 e^{-(R_2)^{(4)} t}$$

**Definition of** $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)} (m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$$

$$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$$

**Behavior of the solutions of equation** 338

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ :

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}((G_{31}), t) - (b''_{29})^{(5)}((G_{31}), t) \le -(\tau_1)^{(5)}$$

**Definition of** $(\nu_1)^{(5)}, (\nu_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, \nu^{(5)}, u^{(5)}$ : 339

By $(\nu_1)^{(5)} > 0$ , $(\nu_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$ , $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\left(u^{(5)}\right)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(5)}, , (\bar{\nu}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$ : 340

By $(\bar{\nu}_1)^{(5)} > 0$ , $(\bar{\nu}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$ , $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\left(u^{(5)}\right)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

**Definition of** $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (\nu_0)^{(5)}$ :-

If we define $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (\nu_0)^{(5)}, (m_1)^{(5)} = (\nu_1)^{(5)}$, ***if*** $(\nu_0)^{(5)} < (\nu_1)^{(5)}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ , ***if*** $(\nu_1)^{(5)} < (\nu_0)^{(5)} < (\bar{\nu}_1)^{(5)}$,
and $\boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)}$, ***if*** $(\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)}$, ***if*** $(u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}$ , ***if*** $(u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)}$,
and $\boxed{(u_0)^{(5)} = \frac{T^0_{28}}{T^0_{29}}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}$, ***if*** $(\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$$G_{28}^{0}e^{((S_1)^{(5)}-(p_{28})^{(5)})t} \le G_{28}(t) \le G_{28}^{0}e^{(S_1)^{(5)}t}$$

where $(p_i)^{(5)}$ is defined by equation

$$\frac{1}{(m_5)^{(5)}}G_{28}^{0}e^{((S_1)^{(5)}-(p_{28})^{(5)})t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}}G_{28}^{0}e^{(S_1)^{(5)}t} \quad 343$$

$$\left(\frac{(a_{30})^{(5)}G_{28}^{0}}{(m_1)^{(5)}((S_1)^{(5)}-(p_{28})^{(5)}-(S_2)^{(5)})}\left[e^{((S_1)^{(5)}-(p_{28})^{(5)})t}-e^{-(S_2)^{(5)}t}\right]+G_{30}^{0}e^{-(S_2)^{(5)}t} \le G_{30}(t) \le \right. \quad 344$$

*(a30)5G280(m2)5(S1)5−(a30′)5e(S1)5t−e−(a30′)5t+ G300e−(a30′)5t*

$$\boxed{T_{28}^{0}e^{(R_1)^{(5)}t} \le T_{28}(t) \le T_{28}^{0}e^{((R_1)^{(5)}+(r_{28})^{(5)})t}} \quad 345$$

$$\frac{1}{(\mu_1)^{(5)}}T_{28}^{0}e^{(R_1)^{(5)}t} \le T_{28}(t) \le \frac{1}{(\mu_2)^{(5)}}T_{28}^{0}e^{((R_1)^{(5)}+(r_{28})^{(5)})t} \quad 346$$

$$\frac{(b_{30})^{(5)}T_{28}^{0}}{(\mu_1)^{(5)}((R_1)^{(5)}-(b'_{30})^{(5)})}\left[e^{(R_1)^{(5)}t}-e^{-(b'_{30})^{(5)}t}\right]+T_{30}^{0}e^{-(b'_{30})^{(5)}t} \le T_{30}(t) \le \quad 347$$

$$\frac{(a_{30})^{(5)}T_{28}^{0}}{(\mu_2)^{(5)}((R_1)^{(5)}+(r_{28})^{(5)}+(R_2)^{(5)})}\left[e^{((R_1)^{(5)}+(r_{28})^{(5)})t}-e^{-(R_2)^{(5)}t}\right]+T_{30}^{0}e^{-(R_2)^{(5)}t}$$

**Definition of** $(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$:- 348

Where $(S_1)^{(5)} = (a_{28})^{(5)}(m_2)^{(5)} - (a'_{28})^{(5)}$

$(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}$

$(R_1)^{(5)} = (b_{28})^{(5)}(\mu_2)^{(5)} - (b'_{28})^{(5)}$

$(R_2)^{(5)} = (b'_{30})^{(5)} - (r_{30})^{(5)}$

**Behavior of the solutions of equation** 349

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ :

$(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)} \le -(a'_{32})^{(6)} + (a'_{33})^{(6)} - (a''_{32})^{(6)}(T_{33}, t) + (a''_{33})^{(6)}(T_{33}, t) \le -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)} \le -(b'_{32})^{(6)} + (b'_{33})^{(6)} - (b''_{32})^{(6)}\big((G_{35}), t\big) - (b''_{33})^{(6)}\big((G_{35}), t\big) \le -(\tau_1)^{(6)}$$

**Definition of** $(v_1)^{(6)}, (v_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, v^{(6)}, u^{(6)}$ : 350

By $(v_1)^{(6)} > 0$ , $(v_2)^{(6)} < 0$ and respectively $(u_1)^{(6)} > 0$ , $(u_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(v^{(6)}\big)^2 + (\sigma_1)^{(6)}v^{(6)} - (a_{32})^{(6)} = 0$

and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$ and

**Definition of** $(\bar{v}_1)^{(6)},,(\bar{v}_2)^{(6)},(\bar{u}_1)^{(6)},(\bar{u}_2)^{(6)}$ : 351

By $(\bar{v}_1)^{(6)} > 0$ , $(\bar{v}_2)^{(6)} < 0$ and respectively $(\bar{u}_1)^{(6)} > 0$ , $(\bar{u}_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\left(v^{(6)}\right)^2 + (\sigma_2)^{(6)}v^{(6)} - (a_{32})^{(6)} = 0$ and $(b_{33})^{(6)}\left(u^{(6)}\right)^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$

**Definition of** $(m_1)^{(6)}$ , $(m_2)^{(6)}$ , $(\mu_1)^{(6)},(\mu_2)^{(6)},(v_0)^{(6)}$ :-

If we define $(m_1)^{(6)}$ , $(m_2)^{(6)}$ , $(\mu_1)^{(6)},(\mu_2)^{(6)}$ by

$(m_2)^{(6)} = (v_0)^{(6)}, (m_1)^{(6)} = (v_1)^{(6)},$ ***if*** $(v_0)^{(6)} < (v_1)^{(6)}$

$(m_2)^{(6)} = (v_1)^{(6)}, (m_1)^{(6)} = (\bar{v}_6)^{(6)}$ , ***if*** $(v_1)^{(6)} < (v_0)^{(6)} < (\bar{v}_1)^{(6)}$,
and $\boxed{(v_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$

$(m_2)^{(6)} = (v_1)^{(6)}, (m_1)^{(6)} = (v_0)^{(6)},$ ***if*** $(\bar{v}_1)^{(6)} < (v_0)^{(6)}$

and analogously 352

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)},$ ***if*** $(u_0)^{(6)} < (u_1)^{(6)}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}$ , ***if*** $(u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)}$,
and $\boxed{(u_0)^{(6)} = \frac{T_{32}^0}{T_{33}^0}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)},$ ***if*** $(\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$

Then the solution of global equations satisfies the inequalities 353

$$G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{32}(t) \le G_{32}^0 e^{(S_1)^{(6)}t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}} G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{33}(t) \le \frac{1}{(m_2)^{(6)}} G_{32}^0 e^{(S_1)^{(6)}t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)} G_{32}^0}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})}\left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)}t}\right] + G_{34}^0 e^{-(S_2)^{(6)}t} \le G_{34}(t) \le \right. \quad 355$$

*(a34)6G320(m2)6(S1)6−(a34′)6e(S1)6t−e−(a34′)6t+ G340e−(a34′)6t*

$$\boxed{T_{32}^0 e^{(R_1)^{(6)}t} \le T_{32}(t) \le T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}} T_{32}^0 e^{(R_1)^{(6)}t} \le T_{32}(t) \le \frac{1}{(\mu_2)^{(6)}} T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)} T_{32}^0}{(\mu_1)^{(6)}((R_1)^{(6)} - (b'_{34})^{(6)})}\left[e^{(R_1)^{(6)}t} - e^{-(b'_{34})^{(6)}t}\right] + T_{34}^0 e^{-(b'_{34})^{(6)}t} \le T_{34}(t) \le \quad 358$$

$$\frac{(a_{34})^{(6)} T_{32}^0}{(\mu_2)^{(6)}((R_1)^{(6)} + (r_{32})^{(6)} + (R_2)^{(6)})}\left[e^{((R_1)^{(6)} + (r_{32})^{(6)})t} - e^{-(R_2)^{(6)}t}\right] + T_{34}^0 e^{-(R_2)^{(6)}t}$$

**Definition of** $(S_1)^{(6)}, (S_2)^{(6)}, (R_1)^{(6)}, (R_2)^{(6)}$:- 359

Where $(S_1)^{(6)} = (a_{32})^{(6)}(m_2)^{(6)} - (a'_{32})^{(6)}$

$(S_2)^{(6)} = (a_{34})^{(6)} - (p_{34})^{(6)}$

$(R_1)^{(6)} = (b_{32})^{(6)}(\mu_2)^{(6)} - (b'_{32})^{(6)}$

$(R_2)^{(6)} = (b'_{34})^{(6)} - (r_{34})^{(6)}$

**Behavior of the solutions of equation**

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$ :

$(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$ four constants satisfying

$-(\sigma_2)^{(7)} \le -(a'_{36})^{(7)} + (a'_{37})^{(7)} - (a''_{36})^{(7)}(T_{37}, t) + (a''_{37})^{(7)}(T_{37}, t) \le -(\sigma_1)^{(7)}$

$-(\tau_2)^{(7)} \le -(b'_{36})^{(7)} + (b'_{37})^{(7)} - (b''_{36})^{(7)}((G_{39}), t) - (b''_{37})^{(7)}((G_{39}), t) \le -(\tau_1)^{(7)}$

**Definition of** $(\nu_1)^{(7)}, (\nu_2)^{(7)}, (u_1)^{(7)}, (u_2)^{(7)}, \nu^{(7)}, u^{(7)}$ : 361

By $(\nu_1)^{(7)} > 0$, $(\nu_2)^{(7)} < 0$ and respectively $(u_1)^{(7)} > 0$, $(u_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}(\nu^{(7)})^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}(u^{(7)})^2 + (\tau_1)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(7)},, (\bar{\nu}_2)^{(7)}, (\bar{u}_1)^{(7)}, (\bar{u}_2)^{(7)}$ : 362

By $(\bar{\nu}_1)^{(7)} > 0$, $(\bar{\nu}_2)^{(7)} < 0$ and respectively $(\bar{u}_1)^{(7)} > 0$, $(\bar{u}_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}(\nu^{(7)})^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}(u^{(7)})^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

**Definition of** $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (\nu_0)^{(7)}$ :-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$(m_2)^{(7)} = (\nu_0)^{(7)}, (m_1)^{(7)} = (\nu_1)^{(7)}$, ***if*** $(\nu_0)^{(7)} < (\nu_1)^{(7)}$

$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\bar{\nu}_1)^{(7)}$, ***if*** $(\nu_1)^{(7)} < (\nu_0)^{(7)} < (\bar{\nu}_1)^{(7)}$,
and $\boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$

$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\nu_0)^{(7)}$, ***if*** $(\bar{\nu}_1)^{(7)} < (\nu_0)^{(7)}$

and analogously 363

$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)},\ \boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$

and $\boxed{(u_0)^{(7)} = \frac{T_{36}^0}{T_{37}^0}}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)}$ where $(u_1)^{(7)}, (\bar{u}_1)^{(7)}$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^0 e^{(S_1)^{(7)} t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)} t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)}((S_1)^{(7)} - (p_{36})^{(7)} - (S_2)^{(7)})}\left[e^{((S_1)^{(7)} - (p_{36})^{(7)})t} - e^{-(S_2)^{(7)} t}\right] + G_{38}^0 e^{-(S_2)^{(7)} t} \le G_{38}(t) \le \right. \quad 366$$

$$\left. \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)}((S_1)^{(7)} - (a'_{38})^{(7)})}\left[e^{(S_1)^{(7)} t} - e^{-(a'_{38})^{(7)} t}\right] + G_{38}^0 e^{-(a'_{38})^{(7)} t}\right)$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t} \quad 368$$

$$\frac{(b_{38})^{(7)} T_{36}^0}{(\mu_1)^{(7)}((R_1)^{(7)} - (b'_{38})^{(7)})}\left[e^{(R_1)^{(7)} t} - e^{-(b'_{38})^{(7)} t}\right] + T_{38}^0 e^{-(b'_{38})^{(7)} t} \le T_{38}(t) \le \quad 369$$

$$\frac{(a_{38})^{(7)} T_{36}^0}{(\mu_2)^{(7)}((R_1)^{(7)} + (r_{36})^{(7)} + (R_2)^{(7)})}\left[e^{((R_1)^{(7)} + (r_{36})^{(7)})t} - e^{-(R_2)^{(7)} t}\right] + T_{38}^0 e^{-(R_2)^{(7)} t}$$

**Definition of** $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$

$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$

$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$

**Behavior of the solutions of equation** 371

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ :

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$$-(\sigma_2)^{(8)} \le -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \le -(\sigma_1)^{(8)}$$

$$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\big((G_{43}), t\big) - (b''_{41})^{(8)}\big((G_{43}), t\big) \le -(\tau_1)^{(8)}$$

**Definition of** $(\nu_1)^{(8)}, (\nu_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, \nu^{(8)}, u^{(8)}$ : 372

By $(\nu_1)^{(8)} > 0, (\nu_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0, (u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(8)}, , (\bar{\nu}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$ :

By $(\bar{\nu}_1)^{(8)} > 0, (\bar{\nu}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0, (\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

**Definition of** $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}, (\nu_0)^{(8)}$ :-

If we define $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$(m_2)^{(8)} = (\nu_0)^{(8)}, (m_1)^{(8)} = (\nu_1)^{(8)}$, ***if*** $(\nu_0)^{(8)} < (\nu_1)^{(8)}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\bar{\nu}_1)^{(8)}$, ***if*** $(\nu_1)^{(8)} < (\nu_0)^{(8)} < (\bar{\nu}_1)^{(8)}$,
and $\boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\nu_0)^{(8)}$, ***if*** $(\bar{\nu}_1)^{(8)} < (\nu_0)^{(8)}$

and analogously 374

$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)}$, ***if*** $(u_0)^{(8)} < (u_1)^{(8)}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}$, ***if*** $(u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)}$,
and $\boxed{(u_0)^{(8)} = \frac{T^0_{40}}{T^0_{41}}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}$, ***if*** $(\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^{0}e^{((S_1)^{(8)}-(p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^{0}e^{(S_1)^{(8)}t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}}G_{40}^{0}e^{((S_1)^{(8)}-(p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}}G_{40}^{0}e^{(S_1)^{(8)}t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)}G_{40}^{0}}{(m_1)^{(8)}((S_1)^{(8)}-(p_{40})^{(8)}-(S_2)^{(8)})}\left[e^{((S_1)^{(8)}-(p_{40})^{(8)})t}-e^{-(S_2)^{(8)}t}\right]+G_{42}^{0}e^{-(S_2)^{(8)}t} \le G_{42}(t) \le \right. \quad 377$$

$$\left.\frac{(a_{42})^{(8)}G_{40}^{0}}{(m_2)^{(8)}((S_1)^{(8)}-(a'_{42})^{(8)})}[e^{(S_1)^{(8)}t}-e^{-(a'_{42})^{(8)}t}]+\ G_{42}^{0}e^{-(a'_{42})^{(8)}t}\right)$$

$$\boxed{T_{40}^{0}e^{(R_1)^{(8)}t} \le T_{40}(t) \le T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}}T_{40}^{0}e^{(R_1)^{(8)}t} \le T_{40}(t) \le \frac{1}{(\mu_2)^{(8)}}T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)}T_{40}^{0}}{(\mu_1)^{(8)}((R_1)^{(8)}-(b'_{42})^{(8)})}\left[e^{(R_1)^{(8)}t}-e^{-(b'_{42})^{(8)}t}\right]+T_{42}^{0}e^{-(b'_{42})^{(8)}t} \le T_{42}(t) \le \quad 380$$

$$\frac{(a_{42})^{(8)}T_{40}^{0}}{(\mu_2)^{(8)}((R_1)^{(8)}+(r_{40})^{(8)}+(R_2)^{(8)})}\left[e^{((R_1)^{(8)}+(r_{40})^{(8)})t}-e^{-(R_2)^{(8)}t}\right]+T_{42}^{0}e^{-(R_2)^{(8)}t}$$

**Definition of** $(S_1)^{(8)}, (S_2)^{(8)}, (R_1)^{(8)}, (R_2)^{(8)}$:- 381

Where $(S_1)^{(8)} = (a_{40})^{(8)}(m_2)^{(8)} - (a'_{40})^{(8)}$

$$(S_2)^{(8)} = (a_{42})^{(8)} - (p_{42})^{(8)}$$

$$(R_1)^{(8)} = (b_{40})^{(8)}(\mu_2)^{(8)} - (b'_{40})^{(8)}$$

$$(R_2)^{(8)} = (b'_{42})^{(8)} - (r_{42})^{(8)}$$

**Behavior of the solutions of equation 37 to 92** 382

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ :

$(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ four constants satisfying

$$-(\sigma_2)^{(9)} \le -(a'_{44})^{(9)} + (a'_{45})^{(9)} - (a''_{44})^{(9)}(T_{45}, t) + (a''_{45})^{(9)}(T_{45}, t) \le -(\sigma_1)^{(9)}$$

$$-(\tau_2)^{(9)} \le -(b'_{44})^{(9)} + (b'_{45})^{(9)} - (b''_{44})^{(9)}\big((G_{47}), t\big) - (b''_{45})^{(9)}\big((G_{47}), t\big) \le -(\tau_1)^{(9)}$$

**Definition of** $(\nu_1)^{(9)}, (\nu_2)^{(9)}, (u_1)^{(9)}, (u_2)^{(9)}, \nu^{(9)}, u^{(9)}$ :

By $(\nu_1)^{(9)} > 0$ , $(\nu_2)^{(9)} < 0$ and respectively $(u_1)^{(9)} > 0$ , $(u_2)^{(9)} < 0$ the roots of the equations

$(a_{45})^{(9)}\left(v^{(9)}\right)^2+(\sigma_1)^{(9)}v^{(9)}-(a_{44})^{(9)}=0$

and $(b_{45})^{(9)}\left(u^{(9)}\right)^2+(\tau_1)^{(9)}u^{(9)}-(b_{44})^{(9)}=0$ and

**Definition of** $(\bar{v}_1)^{(9)},,(\bar{v}_2)^{(9)},(\bar{u}_1)^{(9)},(\bar{u}_2)^{(9)}$ :

By $(\bar{v}_1)^{(9)}>0$ , $(\bar{v}_2)^{(9)}<0$ and respectively $(\bar{u}_1)^{(9)}>0$ , $(\bar{u}_2)^{(9)}<0$ the roots of the equations $(a_{45})^{(9)}\left(v^{(9)}\right)^2+(\sigma_2)^{(9)}v^{(9)}-(a_{44})^{(9)}=0$

and $(b_{45})^{(9)}\left(u^{(9)}\right)^2+(\tau_2)^{(9)}u^{(9)}-(b_{44})^{(9)}=0$

**Definition of** $(m_1)^{(9)}$ , $(m_2)^{(9)}$ , $(\mu_1)^{(9)},(\mu_2)^{(9)},(\nu_0)^{(9)}$ :-

If we define $(m_1)^{(9)}$ , $(m_2)^{(9)}$ , $(\mu_1)^{(9)},(\mu_2)^{(9)}$ by

$(m_2)^{(9)}=(\nu_0)^{(9)},(m_1)^{(9)}=(\nu_1)^{(9)},$ $\boldsymbol{if}$ $(\nu_0)^{(9)}<(\nu_1)^{(9)}$

$(m_2)^{(9)}=(\nu_1)^{(9)},(m_1)^{(9)}=(\bar{\nu}_1)^{(9)}$ , $\boldsymbol{if}$ $(\nu_1)^{(9)}<(\nu_0)^{(9)}<(\bar{\nu}_1)^{(9)}$,

and $\boxed{(\nu_0)^{(9)}=\frac{G_{44}^0}{G_{45}^0}}$

$(m_2)^{(9)}=(\nu_1)^{(9)},(m_1)^{(9)}=(\nu_0)^{(9)},$ $\boldsymbol{if}$ $(\bar{\nu}_1)^{(9)}<(\nu_0)^{(9)}$

and analogously

$(\mu_2)^{(9)}=(u_0)^{(9)},(\mu_1)^{(9)}=(u_1)^{(9)},$ $\boldsymbol{if}$ $(u_0)^{(9)}<(u_1)^{(9)}$

$(\mu_2)^{(9)}=(u_1)^{(9)},(\mu_1)^{(9)}=(\bar{u}_1)^{(9)}$ , $\boldsymbol{if}$ $(u_1)^{(9)}<(u_0)^{(9)}<(\bar{u}_1)^{(9)}$,

and $\boxed{(u_0)^{(9)}=\frac{T_{44}^0}{T_{45}^0}}$

$(\mu_2)^{(9)}=(u_1)^{(9)},(\mu_1)^{(9)}=(u_0)^{(9)},\boldsymbol{if}$ $(\bar{u}_1)^{(9)}<(u_0)^{(9)}$ where $(u_1)^{(9)},(\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G_{44}^0e^{((S_1)^{(9)}-(p_{44})^{(9)})t}\le G_{44}(t)\le G_{44}^0e^{(S_1)^{(9)}t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}}G_{44}^0e^{((S_1)^{(9)}-(p_{44})^{(9)})t}\le G_{45}(t)\le\frac{1}{(m_2)^{(9)}}G_{44}^0e^{(S_1)^{(9)}t}$$

$$\frac{(a_{46})^{(9)}G_{44}^0}{(m_1)^{(9)}((S_1)^{(9)}-(p_{44})^{(9)}-(S_2)^{(9)})}\left[e^{((S_1)^{(9)}-(p_{44})^{(9)})t}-e^{-(S_2)^{(9)}t}\right]+G_{46}^0e^{-(S_2)^{(9)}t}\le G_{46}(t)\le$$

$$\frac{(a_{46})^{(9)}G_{44}^0}{(m_2)^{(9)}((S_1)^{(9)}-(a'_{46})^{(9)})}[e^{(S_1)^{(9)}t}-e^{-(a'_{46})^{(9)}t}]+G_{46}^0e^{-(a'_{46})^{(9)}t})$$

$$\boxed{T_{44}^0e^{(R_1)^{(9)}t}\le T_{44}(t)\le T_{44}^0e^{((R_1)^{(9)}+(r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}}T_{44}^0e^{(R_1)^{(9)}t}\le T_{44}(t)\le\frac{1}{(\mu_2)^{(9)}}T_{44}^0e^{((R_1)^{(9)}+(r_{44})^{(9)})t}$$

$$\frac{(b_{46})^{(9)}T_{44}^{0}}{(\mu_1)^{(9)}((R_1)^{(9)}-(b'_{46})^{(9)})}\left[e^{(R_1)^{(9)}t}-e^{-(b'_{46})^{(9)}t}\right]+T_{46}^{0}e^{-(b'_{46})^{(9)}t}\leq T_{46}(t)\leq$$

$$\frac{(a_{46})^{(9)}T_{44}^{0}}{(\mu_2)^{(9)}((R_1)^{(9)}+(r_{44})^{(9)}+(R_2)^{(9)})}\left[e^{((R_1)^{(9)}+(r_{44})^{(9)})t}-e^{-(R_2)^{(9)}t}\right]+T_{46}^{0}e^{-(R_2)^{(9)}t}$$

**Definition of** $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)}=(a_{44})^{(9)}(m_2)^{(9)}-(a'_{44})^{(9)}$

$(S_2)^{(9)}=(a_{46})^{(9)}-(p_{46})^{(9)}$

$(R_1)^{(9)}=(b_{44})^{(9)}(\mu_2)^{(9)}-(b'_{44})^{(9)}$

$(R_2)^{(9)}=(b'_{46})^{(9)}-(r_{46})^{(9)}$

**Proof :** From global equations we obtain 383

$$\frac{dv^{(1)}}{dt}=(a_{13})^{(1)}-\left((a'_{13})^{(1)}-(a'_{14})^{(1)}+(a''_{13})^{(1)}(T_{14},t)\right)-(a''_{14})^{(1)}(T_{14},t)v^{(1)}-(a_{14})^{(1)}v^{(1)}$$

**Definition of** $v^{(1)}$ :- $\boxed{v^{(1)}=\frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2+(\sigma_2)^{(1)}v^{(1)}-(a_{13})^{(1)}\right)\leq\frac{dv^{(1)}}{dt}\leq-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2+(\sigma_1)^{(1)}v^{(1)}-(a_{13})^{(1)}\right)$$

From which one obtains

**Definition of** $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$ :-

For $0<\boxed{(v_0)^{(1)}=\frac{G_{13}^{0}}{G_{14}^{0}}}<(v_1)^{(1)}<(\bar{v}_1)^{(1)}$

$$v^{(1)}(t)\geq\frac{(v_1)^{(1)}+(C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}}\quad,\quad\boxed{(C)^{(1)}=\frac{(v_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(v_2)^{(1)}}}$$

it follows $(v_0)^{(1)}\leq v^{(1)}(t)\leq(v_1)^{(1)}$

In the same manner , we get 384

$$v^{(1)}(t)\leq\frac{(\bar{v}_1)^{(1)}+(\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}}\quad,\quad\boxed{(\bar{C})^{(1)}=\frac{(\bar{v}_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(\bar{v}_2)^{(1)}}}$$

From which we deduce $(v_0)^{(1)}\leq v^{(1)}(t)\leq(\bar{v}_1)^{(1)}$

If $0<(v_1)^{(1)}<(v_0)^{(1)}=\frac{G_{13}^{0}}{G_{14}^{0}}<(\bar{v}_1)^{(1)}$ we find like in the previous case, 385

$$(\nu_1)^{(1)} \leq \frac{(\nu_1)^{(1)}+(C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}} \leq \nu^{(1)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\bar{\nu}_1)^{(1)}$$

If $0 < (\nu_1)^{(1)} \leq (\bar{\nu}_1)^{(1)} \leq \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$, we obtain 386

$$(\nu_1)^{(1)} \leq \nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\nu_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(1)}(t)$ :-

$$(m_2)^{(1)} \leq \nu^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{\nu^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(1)}(t)$ :-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{13}'')^{(1)} = (a_{14}'')^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)}G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b_{13}'')^{(1)} = (b_{14}'')^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)}T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

**Proof** : From global equations we obtain 387

$$\frac{d\nu^{(2)}}{dt} = (a_{16})^{(2)} - \left((a_{16}')^{(2)} - (a_{17}')^{(2)} + (a_{16}'')^{(2)}(T_{17}, t)\right) - (a_{17}'')^{(2)}(T_{17}, t)\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

**Definition of** $\nu^{(2)}$ :- $\boxed{\nu^{(2)} = \frac{G_{16}}{G_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(v^{(2)}\right)^2+(\sigma_2)^{(2)}v^{(2)}-(a_{16})^{(2)}\right)\le\frac{dv^{(2)}}{dt}\le-\left((a_{17})^{(2)}\left(v^{(2)}\right)^2+(\sigma_1)^{(2)}v^{(2)}-(a_{16})^{(2)}\right)$$

From which one obtains 390

**Definition of** $(\bar{\nu}_1)^{(2)},(\nu_0)^{(2)}$ :-

For $0<(\nu_0)^{(2)}=\frac{G_{16}^0}{G_{17}^0}<(\nu_1)^{(2)}<(\bar{\nu}_1)^{(2)}$

$$v^{(2)}(t)\ge\frac{(\nu_1)^{(2)}+(C)^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_0)^{(2)}\right)t\right]}}{1+(C)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_0)^{(2)}\right)t\right]}}\quad,\quad\boxed{(C)^{(2)}=\frac{(\nu_1)^{(2)}-(\nu_0)^{(2)}}{(\nu_0)^{(2)}-(\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)}\le v^{(2)}(t)\le(\nu_1)^{(2)}$

In the same manner , we get 391

$$v^{(2)}(t)\le\frac{(\bar{\nu}_1)^{(2)}+(\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}\quad,\quad\boxed{(\bar{C})^{(2)}=\frac{(\bar{\nu}_1)^{(2)}-(\nu_0)^{(2)}}{(\nu_0)^{(2)}-(\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)}\le v^{(2)}(t)\le(\bar{\nu}_1)^{(2)}$ 392

If $0<(\nu_1)^{(2)}<(\nu_0)^{(2)}=\frac{G_{16}^0}{G_{17}^0}<(\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)}\le\frac{(\nu_1)^{(2)}+(C)^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}}{1+(C)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}}\le v^{(2)}(t)\le$$

$$\frac{(\bar{\nu}_1)^{(2)}+(\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}\le(\bar{\nu}_1)^{(2)}$$

If $0<(\nu_1)^{(2)}\le(\bar{\nu}_1)^{(2)}\le(\nu_0)^{(2)}=\frac{G_{16}^0}{G_{17}^0}$ , we obtain 394

$$(\nu_1)^{(2)}\le v^{(2)}(t)\le\frac{(\bar{\nu}_1)^{(2)}+(\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}\le(\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $v^{(2)}(t)$ :- 395

$$(m_2)^{(2)}\le v^{(2)}(t)\le(m_1)^{(2)},\quad\boxed{v^{(2)}(t)=\frac{G_{16}(t)}{G_{17}(t)}}$$

In a completely analogous way, we obtain 396

**Definition of** $u^{(2)}(t)$ :-

$$(\mu_2)^{(2)}\le u^{(2)}(t)\le(\mu_1)^{(2)},\quad\boxed{u^{(2)}(t)=\frac{T_{16}(t)}{T_{17}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :** 397

If $(a''_{16})^{(2)} = (a''_{17})^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)} G_{17}(t)$

Analogously if $(b''_{16})^{(2)} = (b''_{17})^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$ if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)} T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

**Proof** : From global equations we obtain 398

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right) - (a''_{21})^{(3)}(T_{21}, t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

**Definition of** $\nu^{(3)}$ :- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right) \leq \frac{d\nu^{(3)}}{dt} \leq -\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right)$$

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From which one obtains

For $0 < (\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\nu_1)^{(3)} < (\bar{\nu}_1)^{(3)}$

$$\nu^{(3)}(t) \geq \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(\nu_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\nu_2)^{(3)}}}$$

it follows $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\nu_1)^{(3)}$

In the same manner , we get 401

$$\nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)} + (\bar{C})^{(3)}(\bar{\nu}_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)} e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{\nu}_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\bar{\nu}_2)^{(3)}}}$$

**Definition of** $(\bar{\nu}_1)^{(3)}$ :-

From which we deduce $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\bar{\nu}_1)^{(3)}$

If $0 < (\nu_1)^{(3)} < (\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\bar{\nu}_1)^{(3)}$ we find like in the previous case, 402

$$(\nu_1)^{(3)} \le \frac{(\nu_1)^{(3)}+(C)^{(3)}(\nu_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}} \le \nu^{(3)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \le (\bar{\nu}_1)^{(3)}$$

If $0 < (\nu_1)^{(3)} \le (\bar{\nu}_1)^{(3)} \le (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}$ , we obtain 403

$$(\nu_1)^{(3)} \le \nu^{(3)}(t) \le \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \le (\nu_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(3)}(t)$ :-

$$(m_2)^{(3)} \le \nu^{(3)}(t) \le (m_1)^{(3)}, \quad \boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(3)}(t)$ :-

$$(\mu_2)^{(3)} \le u^{(3)}(t) \le (\mu_1)^{(3)}, \quad \boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{20}'')^{(3)} = (a_{21}'')^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)}G_{21}(t)$

Analogously if $(b_{20}'')^{(3)} = (b_{21}'')^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)}T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

**Proof** : From global equations we obtain 404

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a_{24}')^{(4)} - (a_{25}')^{(4)} + (a_{24}'')^{(4)}(T_{25},t)\right) - (a_{25}'')^{(4)}(T_{25},t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

**Definition of** $\nu^{(4)}$ :- $\quad \boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right) \le \frac{d\nu^{(4)}}{dt} \le -\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_4)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$ :-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \geq \frac{(\nu_1)^{(4)}+(C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_0)^{(4)}\right)t\right]}}{4+(C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)}-(\nu_0)^{(4)}}{(\nu_0)^{(4)}-(\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{4+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)}-(\nu_0)^{(4)}}{(\nu_0)^{(4)}-(\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \leq \frac{(\nu_1)^{(4)}+(C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_2)^{(4)}\right)t\right]}}{1+(C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_2)^{(4)}\right)t\right]}} \leq \nu^{(4)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \leq (\bar{\nu}_1)^{(4)} \leq \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$ , we obtain 407

$$(\nu_1)^{(4)} \leq \nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(4)}(t)$ :-

$$(m_2)^{(4)} \leq \nu^{(4)}(t) \leq (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(4)}(t)$ :-

$$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{24})^{(4)} = (a''_{25})^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)}G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b''_{24})^{(4)} = (b''_{25})^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then

$(u_1)^{(4)} = (\bar{u}_4)^{(4)}$if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)}T_{25}(t)$ This is an important consequence of the relation between $(v_1)^{(4)}$ and $(\bar{v}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

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**Proof** : From global equations we obtain

$$\frac{dv^{(5)}}{dt} = (a_{28})^{(5)} - \left((a'_{28})^{(5)} - (a'_{29})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right) - (a''_{29})^{(5)}(T_{29}, t)v^{(5)} - (a_{29})^{(5)}v^{(5)}$$

**Definition of** $v^{(5)}$ :- $\boxed{v^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(v^{(5)}\right)^2 + (\sigma_2)^{(5)}v^{(5)} - (a_{28})^{(5)}\right) \leq \frac{dv^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(v^{(5)}\right)^2 + (\sigma_1)^{(5)}v^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

**Definition of** $(\bar{v}_1)^{(5)}, (v_0)^{(5)}$ :-

For $0 < \boxed{(v_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}} < (v_1)^{(5)} < (\bar{v}_1)^{(5)}$

$$v^{(5)}(t) \geq \frac{(v_1)^{(5)} + (C)^{(5)}(v_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((v_1)^{(5)} - (v_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)}e^{\left[-(a_{29})^{(5)}\left((v_1)^{(5)} - (v_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(v_1)^{(5)} - (v_0)^{(5)}}{(v_0)^{(5)} - (v_2)^{(5)}}}$$

it follows $(v_0)^{(5)} \leq v^{(5)}(t) \leq (v_1)^{(5)}$

In the same manner , we get 409

$$v^{(5)}(t) \leq \frac{(\bar{v}_1)^{(5)} + (\bar{C})^{(5)}(\bar{v}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{v}_1)^{(5)} - (\bar{v}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{v}_1)^{(5)} - (\bar{v}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{v}_1)^{(5)} - (v_0)^{(5)}}{(v_0)^{(5)} - (\bar{v}_2)^{(5)}}}$$

From which we deduce $(v_0)^{(5)} \leq v^{(5)}(t) \leq (\bar{v}_5)^{(5)}$

If $0 < (v_1)^{(5)} < (v_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}} < (\bar{v}_1)^{(5)}$ we find like in the previous case, 410

$$(v_1)^{(5)} \leq \frac{(v_1)^{(5)} + (C)^{(5)}(v_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((v_1)^{(5)} - (v_2)^{(5)}\right)t\right]}}{1 + (C)^{(5)}e^{\left[-(a_{29})^{(5)}\left((v_1)^{(5)} - (v_2)^{(5)}\right)t\right]}} \leq v^{(5)}(t) \leq$$

$$\frac{(\bar{v}_1)^{(5)} + (\bar{C})^{(5)}(\bar{v}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{v}_1)^{(5)} - (\bar{v}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{v}_1)^{(5)} - (\bar{v}_2)^{(5)}\right)t\right]}} \leq (\bar{v}_1)^{(5)}$$

If $0 < (v_1)^{(5)} \leq (\bar{v}_1)^{(5)} \leq \boxed{(v_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}}$ , we obtain 411

$$(v_1)^{(5)} \leq v^{(5)}(t) \leq \frac{(\bar{v}_1)^{(5)} + (\bar{C})^{(5)}(\bar{v}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{v}_1)^{(5)} - (\bar{v}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{v}_1)^{(5)} - (\bar{v}_2)^{(5)}\right)t\right]}} \leq (v_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $v^{(5)}(t)$ :-

$(m_2)^{(5)} \leq v^{(5)}(t) \leq (m_1)^{(5)}$, $\boxed{v^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$

In a completely analogous way, we obtain

**Definition of** $u^{(5)}(t)$ :-

$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}$, $\boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{28})^{(5)} = (a''_{29})^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $v^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)} G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b''_{28})^{(5)} = (b''_{29})^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$ if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)} T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

**Proof** : From global equations we obtain 412

$$\frac{dv^{(6)}}{dt} = (a_{32})^{(6)} - \left((a'_{32})^{(6)} - (a'_{33})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right) - (a''_{33})^{(6)}(T_{33}, t) v^{(6)} - (a_{33})^{(6)} v^{(6)}$$

**Definition of** $v^{(6)}$ :- $\boxed{v^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(v^{(6)}\right)^2 + (\sigma_2)^{(6)} v^{(6)} - (a_{32})^{(6)}\right) \leq \frac{dv^{(6)}}{dt} \leq -\left((a_{33})^{(6)}\left(v^{(6)}\right)^2 + (\sigma_1)^{(6)} v^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$ :-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$v^{(6)}(t) \geq \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \leq v^{(6)}(t) \leq (\nu_1)^{(6)}$

In the same manner , we get 413

$$v^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{\nu}_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \le \nu^{(6)}(t) \le (\bar{\nu}_1)^{(6)}$

If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 414

$$(\nu_1)^{(6)} \le \frac{(\nu_1)^{(6)}+(C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_2)^{(6)}\right)t\right]}}{1+(C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_2)^{(6)}\right)t\right]}} \le \nu^{(6)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \le (\bar{\nu}_1)^{(6)}$$

If $0 < (\nu_1)^{(6)} \le (\bar{\nu}_1)^{(6)} \le \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$, we obtain 415

$$(\nu_1)^{(6)} \le \nu^{(6)}(t) \le \frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \le (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(6)}(t)$ :-

$$(m_2)^{(6)} \le \nu^{(6)}(t) \le (m_1)^{(6)}, \quad \boxed{\nu^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(6)}(t)$ :-

$$(\mu_2)^{(6)} \le u^{(6)}(t) \le (\mu_1)^{(6)}, \quad \boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)}G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)}T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

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**Proof** : From global equations we obtain

$$\frac{d\nu^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right) - (a''_{37})^{(7)}(T_{37}, t)\nu^{(7)} - (a_{37})^{(7)}\nu^{(7)}$$

**Definition of** $\nu^{(7)}$ :- $\boxed{\nu^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right) \le \frac{d\nu^{(7)}}{dt} \le -\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(7)}, (\nu_0)^{(7)}$ :-

$$\text{For } 0 < \boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}} < (\nu_1)^{(7)} < (\bar{\nu}_1)^{(7)}$$

$$\nu^{(7)}(t) \geq \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}}{1 + (C)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(\nu_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\nu_2)^{(7)}}}$$

it follows $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\nu_1)^{(7)}$

In the same manner , we get 417

$$\nu^{(7)}(t) \leq \frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)} e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{\nu}_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\bar{\nu}_2)^{(7)}}}$$

From which we deduce $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\bar{\nu}_1)^{(7)}$

If $0 < (\nu_1)^{(7)} < (\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0} < (\bar{\nu}_1)^{(7)}$ we find like in the previous case, 418

$$(\nu_1)^{(7)} \leq \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_2)^{(7)}\right)t\right]}}{1 + (C)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_2)^{(7)}\right)t\right]}} \leq \nu^{(7)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)} e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \leq (\bar{\nu}_1)^{(7)}$$

If $0 < (\nu_1)^{(7)} \leq (\bar{\nu}_1)^{(7)} \leq \boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$ , we obtain 419

$$(\nu_1)^{(7)} \leq \nu^{(7)}(t) \leq \frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)} e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \leq (\nu_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(7)}(t)$ :-

$$(m_2)^{(7)} \leq \nu^{(7)}(t) \leq (m_1)^{(7)}, \quad \boxed{\nu^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(7)}(t)$ :- 420

$$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}, \quad \boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $\nu^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)} G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)} T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

**Proof** : From global equations we obtain 421

$$\frac{d\nu^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right) - (a''_{41})^{(8)}(T_{41}, t)\nu^{(8)} - (a_{41})^{(8)}\nu^{(8)}$$

**Definition of** $\nu^{(8)}$ :- $\boxed{\nu^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right) \leq \frac{d\nu^{(8)}}{dt} \leq -\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$ :-

For $0 < \boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$

$$\nu^{(8)}(t) \geq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)} e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \leq \frac{(\nu_1)^{(8)}+(C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}}{1+(C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}} \leq \nu^{(8)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \leq (\bar{\nu}_1)^{(8)} \leq \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$, we obtain

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$$(\nu_1)^{(8)} \leq \nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(8)}(t)$ :-

$$(m_2)^{(8)} \leq \nu^{(8)}(t) \leq (m_1)^{(8)}, \quad \boxed{\nu^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(8)}(t)$ :-

$$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}, \quad \boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $\nu^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)}G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then $(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)}T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

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**Proof** : From 99,20,44,22,23,44 we obtain

$$\frac{d\nu^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45}, t)\right) - (a''_{45})^{(9)}(T_{45}, t)\nu^{(9)} - (a_{45})^{(9)}\nu^{(9)}$$

**Definition of** $\nu^{(9)}$ :- $\quad \boxed{\nu^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(\nu^{(9)}\right)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)}\right) \leq \frac{d\nu^{(9)}}{dt} \leq -\left((a_{45})^{(9)}\left(\nu^{(9)}\right)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$ :-

For $0 < \boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$

$$\nu^{(9)}(t) \geq \frac{(\nu_1)^{(9)}+(C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_0)^{(9)}\right)t\right]}}{1+(C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)}-(\nu_0)^{(9)}}{(\nu_0)^{(9)}-(\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\nu_9)^{(9)}$

In the same manner , we get

$$\nu^{(9)}(t) \leq \frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)}-(\nu_0)^{(9)}}{(\nu_0)^{(9)}-(\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \leq \frac{(\nu_1)^{(9)}+(C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}}{1+(C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}} \leq \nu^{(9)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \leq (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \leq (\bar{\nu}_1)^{(9)} \leq \boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$ , we obtain

$$(\nu_1)^{(9)} \leq \nu^{(9)}(t) \leq \frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \leq (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(9)}(t)$ :-

$$(m_2)^{(9)} \leq \nu^{(9)}(t) \leq (m_1)^{(9)}, \quad \boxed{\nu^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(9)}(t)$ :-

$$(\mu_2)^{(9)} \leq u^{(9)}(t) \leq (\mu_1)^{(9)}, \quad \boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)}G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then
$(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)}T_{45}(t)$ This is an important consequence of the relation between $(v_1)^{(9)}$ and $(\bar{v}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following 425

**Theorem :** If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on $t$ , and the conditions with the notations

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0$ ,

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$

$with\ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a''_i)^{(2)} and\ (b''_i)^{(2)}$ are independent on t , and the conditions with the notations 426

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$ 427

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 428

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$ , 429

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 430

$with\ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a''_i)^{(3)} and\ (b''_i)^{(3)}$ are independent on $t$ , and the conditions with the notations 431

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$ ,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

**Theorem :** If $(a''_i)^{(4)} and\ (b''_i)^{(4)}$ are independent on $t$ , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$ ,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(5)} and\ (b_i'')^{(5)}$ are independent on $t$ , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$ ,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

**Theorem** If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on $t$ , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$ ,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(7)} and\ (b_i'')^{(7)}$ are independent on $t$ , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$ ,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(8)} and\ (b_i'')^{(8)}$ are independent on $t$ , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$ ,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a''_i)^{(9)} and\ (b''_i)^{(9)}$ are independent on $t$ , and the conditions (with the notations 45,46,27,28) 436 A

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$ ,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0$ 437

$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0$ 438

$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0$ 439

$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0$ 440

$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ]T_{14} = 0$ 441

$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0$ 442

has a unique positive solution , which is an equilibrium solution for the system

$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0$ 443

$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0$ 444

$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0$ 445

$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\ ]T_{16} = 0$ 446

$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\ ]T_{17} = 0$ 447

$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\ ]T_{18} = 0$ 448

has a unique positive solution , which is an equilibrium solution

$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0$ 449

$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0$ 450

$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0$ 451

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\,]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\,]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\,]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\big((G_{27})\big)\,]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\big((G_{27})\big)\,]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\big((G_{27})\big)\,]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})\,]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})\,]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})\,]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})\,]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})\,]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})\,]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0 \quad 479$$

$$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0 \quad 480$$

$$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0 \quad 481$$

$$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0 \quad 482$$

$$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0 \quad 483$$

$$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0 \quad 484$$

$$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0 \quad 484\text{A}$$

$$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$$

$$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$$

$$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$$

$$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$$

$$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$$

**Proof:** 485

(a) Indeed the first two equations have a nontrivial solution $G_{13}, G_{14}$ if

$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$

**Proof:** 486

(o) Indeed the first two equations have a nontrivial solution $G_{16}, G_{17}$ if

$\mathrm{F}(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$

**Proof:** 487

(a) Indeed the first two equations have a nontrivial solution $G_{20}, G_{21}$ if

$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$

**Proof:** 488

(a) Indeed the first two equations have a nontrivial solution $G_{24}, G_{25}$ if

$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$

**Proof:** 489

(a) Indeed the first two equations have a nontrivial solution $G_{28}, G_{29}$ if

$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$

**Proof:** 490

(a) Indeed the first two equations have a nontrivial solution $G_{32}, G_{33}$ if

$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$

**Proof:** 491

(a) Indeed the first two equations have a nontrivial solution $G_{36}, G_{37}$ if

$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$

**Proof:** 492

(a) Indeed the first two equations have a nontrivial solution $G_{40}, G_{41}$ if

$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$

**Proof:** 492 A

(a) Indeed the first two equations have a nontrivial solution $G_{44}, G_{45}$ if

$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$

**Definition and uniqueness of** $T^*_{14}$ :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique $T^*_{14}$ for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

**Definition and uniqueness of** $T^*_{17}$ :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique $T^*_{17}$ for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]}$$ 495

**Definition and uniqueness of** $T^*_{21}$ :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique $T^*_{21}$ for which $f(T^*_{21}) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

**Definition and uniqueness of** $T^*_{25}$ :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that there exists a unique $T^*_{25}$ for which $f(T^*_{25}) = 0$. With this value , we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T^*_{25})]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T^*_{25})]}$$

**Definition and uniqueness of** $T^*_{29}$ :- 498

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(5)}(T_{29})$ being increasing, it follows that there exists a unique $T^*_{29}$ for which $f(T^*_{29}) = 0$. With this value , we obtain from the three first equations

$G_{28} = \frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T^*_{29})]}$ , $G_{30} = \frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T^*_{29})]}$

**Definition and uniqueness of** $T^*_{33}$ :- 499

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(6)}(T_{33})$ being increasing, it follows that there exists a unique $T^*_{33}$ for which $f(T^*_{33}) = 0$. With this value , we obtain from the three first equations

$G_{32} = \frac{(a_{32})^{(6)}G_{33}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T^*_{33})]}$ , $G_{34} = \frac{(a_{34})^{(6)}G_{33}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T^*_{33})]}$

**Definition and uniqueness of** $T^*_{37}$ :- 500

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(7)}(T_{37})$ being increasing, it follows that there exists a unique $T^*_{37}$ for which $f(T^*_{37}) = 0$. With this value , we obtain from the three first equations

$G_{36} = \frac{(a_{36})^{(7)}G_{37}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T^*_{37})]}$ , $G_{38} = \frac{(a_{38})^{(7)}G_{37}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T^*_{37})]}$

**Definition and uniqueness of** $T^*_{41}$ :- 501

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(8)}(T_{41})$ being increasing, it follows that there exists a unique $T^*_{41}$ for which $f(T^*_{41}) = 0$. With this value , we obtain from the three first equations

$G_{40} = \frac{(a_{40})^{(8)}G_{41}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T^*_{41})]}$ , $G_{42} = \frac{(a_{42})^{(8)}G_{41}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T^*_{41})]}$

**Definition and uniqueness of** $T^*_{45}$ :- 501 A

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(9)}(T_{45})$ being increasing, it follows that there exists a unique $T^*_{45}$ for which $f(T^*_{45}) = 0$. With this value , we obtain from the three first equations

$G_{44} = \frac{(a_{44})^{(9)}G_{45}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T^*_{45})]}$ , $G_{46} = \frac{(a_{46})^{(9)}G_{45}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T^*_{45})]}$

By the same argument, the equations admit solutions $G_{13}, G_{14}$ if 502
$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$

$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right]+(b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{14}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{14}$ such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions $G_{16}, G_{17}$ if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right] + (b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that $\varphi$ is a decreasing function in $G_{17}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{14}^{*}$ such that $\varphi((G_{19})^{*}) = 0$ 504

By the same argument, the equations admit solutions $G_{20}, G_{21}$ if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right] + (b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that $\varphi$ is a decreasing function in $G_{21}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{21}^{*}$ such that $\varphi((G_{23})^{*}) = 0$

By the same argument, the equations admit solutions $G_{24}, G_{25}$ if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right] + (b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that $\varphi$ is a decreasing function in $G_{25}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{25}^{*}$ such that $\varphi((G_{27})^{*}) = 0$

By the same argument, the equations admit solutions $G_{28}, G_{29}$ if 507

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right] + (b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that $\varphi$ is a decreasing function in $G_{29}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{29}^{*}$ such that $\varphi((G_{31})^{*}) = 0$

By the same argument, the equations admit solutions $G_{32}, G_{33}$ if 508

$$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$$

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right] + (b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that $\varphi$ is a decreasing function in $G_{33}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{33}^{*}$ such that $\varphi({G_{35}}^{*}) = 0$

By the same argument, the equations admit solutions $G_{36}, G_{37}$ if 509

$$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$$

$$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right]+(b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{37}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{37}$ such that $\varphi(G_{39}{}^*) = 0$

By the same argument, the equations admit solutions $G_{40}, G_{41}$ if 510

$$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$$
$$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right]+(b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{41}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{41}$ such that $\varphi(G_{43}{}^*) = 0$

By the same argument, the equations 92,93 admit solutions $G_{44}, G_{45}$ if

$$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$$

$$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right]+(b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{45}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{45}$ such that $\varphi((G_{47})^*) = 0$

Finally we obtain the unique solution 511

$G^*_{14}$ given by $\varphi(G^*) = 0$ , $T^*_{14}$ given by $f(T^*_{14}) = 0$ and

$$G^*_{13} = \frac{(a_{13})^{(1)}G^*_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G^*_{15} = \frac{(a_{15})^{(1)}G^*_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

$$T^*_{13} = \frac{(b_{13})^{(1)}T^*_{14}}{[(b'_{13})^{(1)}-(b''_{13})^{(1)}(G^*)]} \quad , \quad T^*_{15} = \frac{(b_{15})^{(1)}T^*_{14}}{[(b'_{15})^{(1)}-(b''_{15})^{(1)}(G^*)]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

$\mathrm{G}^*_{17}$ given by $\varphi((G_{19})^*) = 0$ , $\mathrm{T}^*_{17}$ given by $f(\mathrm{T}^*_{17}) = 0$ and 512

$$\mathrm{G}^*_{16} = \frac{(\mathrm{a}_{16})^{(2)}\mathrm{G}^*_{17}}{[(\mathrm{a}'_{16})^{(2)}+(\mathrm{a}''_{16})^{(2)}(\mathrm{T}^*_{17})]} \quad , \quad \mathrm{G}^*_{18} = \frac{(\mathrm{a}_{18})^{(2)}\mathrm{G}^*_{17}}{[(\mathrm{a}'_{18})^{(2)}+(\mathrm{a}''_{18})^{(2)}(\mathrm{T}^*_{17})]} \quad 513$$

$$\mathrm{T}^*_{16} = \frac{(\mathrm{b}_{16})^{(2)}\mathrm{T}^*_{17}}{[(\mathrm{b}'_{16})^{(2)}-(\mathrm{b}''_{16})^{(2)}((G_{19})^*)]} \quad , \quad \mathrm{T}^*_{18} = \frac{(\mathrm{b}_{18})^{(2)}\mathrm{T}^*_{17}}{[(\mathrm{b}'_{18})^{(2)}-(\mathrm{b}''_{18})^{(2)}((G_{19})^*)]} \quad 514$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

$G^*_{21}$ given by $\varphi((G_{23})^*) = 0$ , $T^*_{21}$ given by $f(T^*_{21}) = 0$ and

$$G_{20}^{*} = \frac{(a_{20})^{(3)}G_{21}^{*}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T_{21}^{*})]} \quad , \quad G_{22}^{*} = \frac{(a_{22})^{(3)}G_{21}^{*}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T_{21}^{*})]}$$

$$T_{20}^{*} = \frac{(b_{20})^{(3)}T_{21}^{*}}{[(b'_{20})^{(3)}-(b''_{20})^{(3)}(G_{23}{}^{*})]} \quad , \quad T_{22}^{*} = \frac{(b_{22})^{(3)}T_{21}^{*}}{[(b'_{22})^{(3)}-(b''_{22})^{(3)}(G_{23}{}^{*})]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

$G_{25}^{*}$ given by $\varphi(G_{27}) = 0$ , $T_{25}^{*}$ given by $f(T_{25}^{*}) = 0$ and

$$G_{24}^{*} = \frac{(a_{24})^{(4)}G_{25}^{*}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}^{*} = \frac{(a_{26})^{(4)}G_{25}^{*}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T_{25}^{*})]}$$

$$T_{24}^{*} = \frac{(b_{24})^{(4)}T_{25}^{*}}{[(b'_{24})^{(4)}-(b''_{24})^{(4)}((G_{27})^{*})]} \quad , \quad T_{26}^{*} = \frac{(b_{26})^{(4)}T_{25}^{*}}{[(b'_{26})^{(4)}-(b''_{26})^{(4)}((G_{27})^{*})]} \qquad 517$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

$G_{29}^{*}$ given by $\varphi((G_{31})^{*}) = 0$ , $T_{29}^{*}$ given by $f(T_{29}^{*}) = 0$ and

$$G_{28}^{*} = \frac{(a_{28})^{(5)}G_{29}^{*}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}^{*} = \frac{(a_{30})^{(5)}G_{29}^{*}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T_{29}^{*})]}$$

$$T_{28}^{*} = \frac{(b_{28})^{(5)}T_{29}^{*}}{[(b'_{28})^{(5)}-(b''_{28})^{(5)}((G_{31})^{*})]} \quad , \quad T_{30}^{*} = \frac{(b_{30})^{(5)}T_{29}^{*}}{[(b'_{30})^{(5)}-(b''_{30})^{(5)}((G_{31})^{*})]} \qquad 519$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

$G_{33}^{*}$ given by $\varphi((G_{35})^{*}) = 0$ , $T_{33}^{*}$ given by $f(T_{33}^{*}) = 0$ and

$$G_{32}^{*} = \frac{(a_{32})^{(6)}G_{33}^{*}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^{*})]} \quad , \quad G_{34}^{*} = \frac{(a_{34})^{(6)}G_{33}^{*}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^{*})]}$$

$$T_{32}^{*} = \frac{(b_{32})^{(6)}T_{33}^{*}}{[(b'_{32})^{(6)}-(b''_{32})^{(6)}((G_{35})^{*})]} \quad , \quad T_{34}^{*} = \frac{(b_{34})^{(6)}T_{33}^{*}}{[(b'_{34})^{(6)}-(b''_{34})^{(6)}((G_{35})^{*})]} \qquad 521$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

$G_{37}^{*}$ given by $\varphi((G_{39})^{*}) = 0$ , $T_{37}^{*}$ given by $f(T_{37}^{*}) = 0$ and

$$G_{36}^{*} = \frac{(a_{36})^{(7)}G_{37}^{*}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^{*})]} \quad , \quad G_{38}^{*} = \frac{(a_{38})^{(7)}G_{37}^{*}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^{*})]}$$

$$T_{36}^{*} = \frac{(b_{36})^{(7)}T_{37}^{*}}{[(b'_{36})^{(7)}-(b''_{36})^{(7)}((G_{39})^{*})]} \quad , \quad T_{38}^{*} = \frac{(b_{38})^{(7)}T_{37}^{*}}{[(b'_{38})^{(7)}-(b''_{38})^{(7)}((G_{39})^{*})]}$$

Finally we obtain the unique solution 523

$G_{41}^*$ given by $\varphi((G_{43})^*) = 0$ , $T_{41}^*$ given by $f(T_{41}^*) = 0$ and

$$G_{40}^* = \frac{(a_{40})^{(8)} G_{41}^*}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42}^* = \frac{(a_{42})^{(8)} G_{41}^*}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

$$T_{40}^* = \frac{(b_{40})^{(8)} T_{41}^*}{[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43})^*)]} \quad , \quad T_{42}^* = \frac{(b_{42})^{(8)} T_{41}^*}{[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43})^*)]}$$

Finally we obtain the unique solution of 89 to 99 523 A

$G_{45}^*$ given by $\varphi((G_{47})^*) = 0$ , $T_{45}^*$ given by $f(T_{45}^*) = 0$ and

$$G_{44}^* = \frac{(a_{44})^{(9)} G_{45}^*}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46}^* = \frac{(a_{46})^{(9)} G_{45}^*}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

$$T_{44}^* = \frac{(b_{44})^{(9)} T_{45}^*}{[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47})^*)]} \quad , \quad T_{46}^* = \frac{(b_{46})^{(9)} T_{45}^*}{[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47})^*)]}$$

## ASYMPTOTIC STABILITY ANALYSIS 524

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

**Proof:** Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{14}'')^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \frac{\partial (b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a'_{13})^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)} G_{13}^* \mathbb{T}_{14} \quad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a'_{14})^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)} G_{14}^* \mathbb{T}_{14} \quad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a'_{15})^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)} G_{15}^* \mathbb{T}_{14} \quad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b'_{13})^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)} T_{13}^* \mathbb{G}_j\right) \quad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b'_{14})^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)} T_{14}^* \mathbb{G}_j\right) \quad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b'_{15})^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T^*_{15}\mathbb{G}_j\right) \quad 530$$

**ASYMPTOTIC STABILITY ANALYSIS** 531

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(2)}$ and $(b''_i)^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i \quad 532$$

$$\frac{\partial(a''_{17})^{(2)}}{\partial T_{17}}(T^*_{17}) = (q_{17})^{(2)} \quad , \frac{\partial(b''_i)^{(2)}}{\partial G_j}\left((G_{19})^*\right) = s_{ij} \quad 533$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a'_{16})^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G^*_{16}\mathbb{T}_{17} \quad 534$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a'_{17})^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G^*_{17}\mathbb{T}_{17} \quad 535$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a'_{18})^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G^*_{18}\mathbb{T}_{17} \quad 536$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b'_{16})^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T^*_{16}\mathbb{G}_j\right) \quad 537$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b'_{17})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}T^*_{17}\mathbb{G}_j\right) \quad 538$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b'_{18})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}T^*_{18}\mathbb{G}_j\right) \quad 539$$

**ASYMPTOTIC STABILITY ANALYSIS** 540

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(3)}$ *and* $(b''_i)^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{21})^{(3)}}{\partial T_{21}}(T^*_{21}) = (q_{21})^{(3)} \quad , \frac{\partial(b''_i)^{(3)}}{\partial G_j}\left((G_{23})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a'_{20})^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G^*_{20}\mathbb{T}_{21} \quad 541$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a'_{21})^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G^*_{21}\mathbb{T}_{21} \quad 542$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a'_{22})^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G^*_{22}\mathbb{T}_{21} \quad 543$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b'_{20})^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T^*_{20}\mathbb{G}_j\right) \quad 544$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b'_{21})^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T^*_{21}\mathbb{G}_j\right) \quad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b'_{22})^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T^*_{22}\mathbb{G}_j\right) \quad 546$$

**ASYMPTOTIC STABILITY ANALYSIS** 547

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}$ *and* $(b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{25})^{(4)}}{\partial T_{25}}(T^*_{25}) = (q_{25})^{(4)} \ , \ \frac{\partial(b_i'')^{(4)}}{\partial G_j}((G_{27})^* \ ) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\left((a'_{24})^{(4)} + (p_{24})^{(4)}\right)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G^*_{24}\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\left((a'_{25})^{(4)} + (p_{25})^{(4)}\right)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G^*_{25}\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\left((a'_{26})^{(4)} + (p_{26})^{(4)}\right)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G^*_{26}\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\left((b'_{24})^{(4)} - (r_{24})^{(4)}\right)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(24)(j)}T^*_{24}\mathbb{G}_j\right) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\left((b'_{25})^{(4)} - (r_{25})^{(4)}\right)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\left(s_{(25)(j)}T^*_{25}\mathbb{G}_j\right) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T^*_{26}\mathbb{G}_j\right) \quad 554$$

**ASYMPTOTIC STABILITY ANALYSIS** 555

**Theorem 5:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}$ *and* $(b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 556

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$\frac{\partial(a_{29}'')^{(5)}}{\partial T_{29}}(T_{29}^*) = (q_{29})^{(5)}$ , $\frac{\partial(b_i'')^{(5)}}{\partial G_j}((G_{31})^*) = s_{ij}$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a_{28}')^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G_{28}^*\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a_{29}')^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G_{29}^*\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a_{30}')^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G_{30}^*\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b_{28}')^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T_{28}^*\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b_{29}')^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T_{29}^*\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b_{30}')^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T_{30}^*\mathbb{G}_j\right) \quad 562$$

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**Theorem 6:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}$ *and* $(b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 564

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$\frac{\partial(a_{33}'')^{(6)}}{\partial T_{33}}(T_{33}^*) = (q_{33})^{(6)}$ , $\frac{\partial(b_i'')^{(6)}}{\partial G_j}((G_{35})^*) = s_{ij}$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a_{32}')^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G_{32}^*\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a_{33}')^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G_{33}^*\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a_{34}')^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G_{34}^*\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b_{32}')^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T_{32}^*\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b_{33}')^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T_{33}^*\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b_{34}')^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T_{34}^*\mathbb{G}_j\right) \quad 570$$

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**Theorem 7:** If the conditions of the previous theorem are satisfied and if the functions

$(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 572

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{37}'')^{(7)}}{\partial T_{37}}(T_{37}^*) = (q_{37})^{(7)} \quad , \frac{\partial (b_i'')^{(7)}}{\partial G_j}\left( (G_{39})^{**} \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a_{36}')^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G_{36}^*\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a_{37}')^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G_{37}^*\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a_{38}')^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G_{38}^*\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b_{36}')^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T_{36}^*\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b_{37}')^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T_{37}^*\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b_{38}')^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T_{38}^*\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

## ASYMPTOTIC STABILITY ANALYSIS

**Theorem 8:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 580

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{41}'')^{(8)}}{\partial T_{41}}(T_{41}^*) = (q_{41})^{(8)} \quad , \frac{\partial (b_i'')^{(8)}}{\partial G_j}\left( (G_{43})^{*} \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a_{40}')^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G_{40}^*\mathbb{T}_{41} \quad 581$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a'_{41})^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G^*_{41}\mathbb{T}_{41} \quad 582$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a'_{42})^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G^*_{42}\mathbb{T}_{41} \quad 583$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b'_{40})^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T^*_{40}\mathbb{G}_j\right) \quad 584$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b'_{41})^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T^*_{41}\mathbb{G}_j\right) \quad 585$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b'_{42})^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T^*_{42}\mathbb{G}_j\right) \quad 586$$

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## ASYMPTOTIC STABILITY ANALYSIS

**Theorem 9:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}$ *and* $(b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

**Proof:** Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{45})^{(9)}}{\partial T_{45}}(T^*_{45}) = (q_{45})^{(9)} \quad , \frac{\partial(b_i'')^{(9)}}{\partial G_j}((G_{47})^*) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a'_{44})^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)}G^*_{44}\mathbb{T}_{45} \quad 586\,B$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a'_{45})^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)}G^*_{45}\mathbb{T}_{45} \quad 586\,C$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a'_{46})^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)}G^*_{46}\mathbb{T}_{45} \quad 586\,D$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b'_{44})^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)}T^*_{44}\mathbb{G}_j\right) \quad 586\,E$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b'_{45})^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)}T^*_{45}\mathbb{G}_j\right) \quad 586\,F$$

$$\frac{d\mathbb{T}_{46}}{dt} = -\left((b'_{46})^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T^*_{46}\mathbb{G}_j\right) \quad 586\,G$$

**The characteristic equation of this system is** 587

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(q_{13})^{(1)}G^*_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)}T^*_{14} + (b_{14})^{(1)}s_{(13),(14)}T^*_{14}\right)$$

$$+\left(\left((\lambda)^{(1)}+(a'_{14})^{(1)}+(p_{14})^{(1)}\right)(q_{13})^{(1)}G^{*}_{13}+(a_{13})^{(1)}(q_{14})^{(1)}G^{*}_{14}\right)$$

$$\left(\left((\lambda)^{(1)}+(b'_{13})^{(1)}-(r_{13})^{(1)}\right)s_{(14),(13)}T^{*}_{14}+(b_{14})^{(1)}s_{(13),(13)}T^{*}_{13}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^{2}+\left((a'_{13})^{(1)}+(a'_{14})^{(1)}+(p_{13})^{(1)}+(p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^{2}+\left((b'_{13})^{(1)}+(b'_{14})^{(1)}-(r_{13})^{(1)}+(r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^{2}+\left((a'_{13})^{(1)}+(a'_{14})^{(1)}+(p_{13})^{(1)}+(p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)}+(a'_{13})^{(1)}+(p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^{*}_{14}+(a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^{*}_{13}\right)$$

$$\left(\left((\lambda)^{(1)}+(b'_{13})^{(1)}-(r_{13})^{(1)}\right)s_{(14),(15)}T^{*}_{14}+(b_{14})^{(1)}s_{(13),(15)}T^{*}_{13}\right)\}=0$$

$$+$$

$$(\lambda)^{(2)}+(b'_{18})^{(2)}-(r_{18})^{(2)}\{\left((\lambda)^{(2)}+(a'_{18})^{(2)}+(p_{18})^{(2)}\right)$$

$$\left[\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^{*}_{17}+(a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^{*}_{16}\right)\Big]$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^{*}_{17}\right)$$

$$+\left(\left((\lambda)^{(2)}+(a'_{17})^{(2)}+(p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^{*}_{16}+(a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^{*}_{17}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^{*}_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^{2}+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^{2}+\left((b'_{16})^{(2)}+(b'_{17})^{(2)}-(r_{16})^{(2)}+(r_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$+\left(\left((\lambda)^{(2)}\right)^{2}+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^{*}_{17}+(a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^{*}_{16}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^{*}_{16}\right)\}=0$$

$$+$$

$$(\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)\Big]$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(21)}T^{*}_{21}\right)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^{*}_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^{*}_{21}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(20)}T^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$+\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(22)}T^{*}_{20}\right)\}=0$$

$$+$$

$$(\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left(\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)\right]$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(25)}T^{*}_{25}\right)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^{*}_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^{*}_{25}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(24)}T^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$+\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(26)}T^{*}_{24}\right)\}=0$$

$$+$$

$$(\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(29)}T^{*}_{29}\right)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^{*}_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^{*}_{29}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(28)}T^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$+\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(30)}T^{*}_{28}\right)\}=0$$

$$+$$

$$(\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left(\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)\right]$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(33)}T^{*}_{33}\right)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^{*}_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^{*}_{33}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(32)}T^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$+\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(34)}T^{*}_{32}\right)\}=0$$

$$+$$

$$(\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)\right]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(37)}T^{*}_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^{*}_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^{*}_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(36)}T^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$+\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(38)}T^{*}_{36}\right)\}=0$$

$$+$$

$$(\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(41)}T^{*}_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^{*}_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^{*}_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(40)}T^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$+\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(42)}T^{*}_{40}\right)\}=0$$

$$+$$

$$(\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)\right]$$

$$\left((\lambda)^{(9)} + (b'_{44})^{(9)} - (r_{44})^{(9)}\right)s_{(45),(45)}T^*_{45} + (b_{45})^{(9)}s_{(44),(45)}T^*_{45}\Big)$$

$$+\left((\lambda)^{(9)} + (a'_{45})^{(9)} + (p_{45})^{(9)}\right)(q_{44})^{(9)}G^*_{44} + (a_{44})^{(9)}(q_{45})^{(9)}G^*_{45}\Big)$$

$$\left((\lambda)^{(9)} + (b'_{44})^{(9)} - (r_{44})^{(9)}\right)s_{(45),(44)}T^*_{45} + (b_{45})^{(9)}s_{(44),(44)}T^*_{44}\Big)$$

$$\left((\lambda)^{(9)}\right)^2 + \left((a'_{44})^{(9)} + (a'_{45})^{(9)} + (p_{44})^{(9)} + (p_{45})^{(9)}\right)(\lambda)^{(9)}\Big)$$

$$\left((\lambda)^{(9)}\right)^2 + \left((b'_{44})^{(9)} + (b'_{45})^{(9)} - (r_{44})^{(9)} + (r_{45})^{(9)}\right)(\lambda)^{(9)}\Big)$$

$$+\left((\lambda)^{(9)}\right)^2 + \left((a'_{44})^{(9)} + (a'_{45})^{(9)} + (p_{44})^{(9)} + (p_{45})^{(9)}\right)(\lambda)^{(9)}\Big)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)} + (a'_{44})^{(9)} + (p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^*_{45} + (a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^*_{44}\right)$$

$$\left((\lambda)^{(9)} + (b'_{44})^{(9)} - (r_{44})^{(9)}\right)s_{(45),(46)}T^*_{45} + (b_{45})^{(9)}s_{(44),(46)}T^*_{44}\Big)\} = 0$$

**And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.**

## SECTION SEVENTEEN

## Quadratic Polynomials Of Small Modulus

### INTRODUCTION—VARIABLES USED

**Quadratic polynomials of small modulus cannot represent OR Holden Lee (Submitted on 29 Sep 2015 (v1), last revised 12 Nov 2015 (this version, v2))**

(1) An open problem in complexity theory is to find the minimal degree of (e) a polynomial representing (eb) the n-bit OR function modulo composite m.
(2) This problem is related to (e&eb) understanding the power of circuits with MODm gates where (e) m is composite.
(3) The OR function is of particular interest because (e) it is the simplest function not (e) amenable to bounds from (e) communication complexity.
(4) Tardos and Barrington established a lower bound of Ω ((logn) Om (1)), and Barrington, Beigel, and Rudich established an upper bound of nOm(1).No progress has been made on closing this gap for twenty years, and progress will likely require new techniques.
(5) Authors make progress on this question viewed from a different perspective: rather than fixing the modulus m and bounding the minimum degree d in terms of (e&eb) the number of variables n, authors fix the degree d and bound n in terms of (e&eb) the modulus m.

(6) For degree d=2, Authors prove a Quasipolynomial bound of n≤mO(d)≤mO (logm), improving the previous best bound of 2O(m) implied by Tardos and Barrington's general bound.

(7) To understand the computational power of quadratic polynomials modulo m, authors introduce a certain dichotomy which may be of independent interest.

(8) Namely, authors define a notion of Boolean rank of (e) a quadratic polynomial f and relate it to (e&eb) the notion of diagonal rigidity.

(9) Using additive combinatorics, Authors show that (eb) when the rank is low, f(x) =0 must have (e) many solutions.

**(10)** Using techniques from exponential sums, authors show that when the rank of f is high, (eb)f is close to equidistributed. In either case, f cannot (e) represent the OR function in many variables. Subjects: Computational Complexity (cs.CC) Cite as: arXiv: 1509.08896 [cs.CC] (or arXiv:1509.08896v2 [cs.CC] for this version)

## NOTATION

### Module One

An open problem in complexity theory is to find the minimal degree of (e) a polynomial representing (eb) the n-bit OR function modulo composite m

$G_{13}$ : Category one of polynomial representing (eb) the n-bit OR function modulo composite m

$G_{14}$ : Category two of SAS

$G_{15}$ : Category three of SAS

$T_{13}$ : Category one of minimal degree

$T_{14}$ : Category two of SAS

$T_{15}$ : Category three of SAS

### Module Two

An open problem in complexity theory is to find the minimal degree of a polynomial representing (eb) the n-bit OR function modulo composite m

$G_{16}$ : Category one of minimal degree of a polynomial

$G_{17}$ : Category two of SAS

$G_{18}$ : Category three of SAS

$T_{16}$ : Category one of n-bit OR function modulo composite m

$T_{17}$ : Category two of SAS

$T_{18}$ : Category three of SAS

### Module three

An open problem in complexity theory is to find the minimal degree of a polynomial representing the n-bit OR function modulo composite m

This problem is related to (e&eb) understanding the power of circuits with MODm gates where (e) m is composite

$G_{20}$ : Category one of **finding the minimal degree of a polynomial representing the n-bit OR function modulo composite m;** understanding the power of circuits with MODm gates where m is composite

$G_{21}$ : Category two of SAS

$G_{22}$ : Category three of SAS

$T_{20}$ : Category one of understanding the power of circuits with MODm gates where (e) m is composite **;finding the minimal degree of a polynomial representing the n-bit OR function modulo composite m**

$T_{21}$ : Category two of SAS

$T_{22}$ : Category three of SAS

**Module four**

The OR function is of particular interest because (e) it is the simplest function not (e) amenable to bounds from (e) communication complexity.

Tardos and Barrington established a lower bound of Ω ((logn) Om (1)), and Barrington, Beigel, and Rudich established an upper bound of nOm(1).No progress has been made on closing this gap for twenty years, and progress will likely require new techniques

$G_{24}$ : Category one of simplest function not (e) amenable to bounds from (e) communication complexity.

$G_{25}$ : Category two of SAS

$G_{26}$ : Category three of SAS

$T_{24}$ : Category one of OR function is of particular interest

$T_{25}$ : Category two of SAS

$T_{26}$ : Category three of SAS

**Module five**

OR function is of particular interest because it is the simplest function not (e) amenable to bounds from (e) communication complexity

$G_{28}$ : Category one of communication complexity

$G_{29}$ : Category two of SAS

$G_{30}$ : Category three of SAS

$T_{28}$ : Category one of OR function is of particular interest because it is the simplest function not amenable to bounds

$T_{29}$ : Category two of SAS

$T_{30}$ : Category three of SAS

**Module six**

Authors make progress on this question viewed from a different perspective: rather than fixing the modulus m and bounding the minimum degree d in terms of (e&eb) the number of variables n, authors fix the degree d and bound n in terms of (e&eb) the modulus m

$G_{32}$ : Category one of **fixing the modulus m and bounding the minimum degree d;** number of variables n

$G_{33}$ : Category two of SAS

$G_{34}$ : Category three of SAS

$T_{32}$ : Category one of number of variables n **;fixing the modulus m and bounding the minimum degree d**

$T_{33}$ : Category two of SAS

$T_{34}$ : Category three of SAS

**Module seven**

Authors make progress on this question viewed from a different perspective: rather than **fixing the modulus m and bounding the minimum degree d in terms of the number of variables n, authors fix the degree d and bound n** in terms of (e&eb) the modulus m

$G_{36}$ : Category one of **fixing the modulus m and bounding the minimum degree d in terms of the number of variables n, authors fix the degree d and bound n;** modulus m

$G_{37}$ : Category two of SAS

$G_{38}$ : Category three of SAS

$T_{36}$ : Category one of modulus m **; fixing the modulus m and bounding the minimum degree d in terms of the number of variables n, authors fix the degree d and bound n**

$T_{37}$ : Category two of SAS

$T_{38}$ : Category three of SAS

**Module eight**

For degree d=2, Authors prove a Quasipolynomial bound of n≤mO(d)≤mO (logm), improving the previous best bound of 2O(m) implied by Tardos and Barrington's general bound.

$G_{40}$ : Category one of best bound of 2O(m) implied by Tardos and Barrington's general bound

$G_{41}$ : Category two of SAS

$G_{42}$ : Category three of SAS

$T_{40}$ : Category one of Quasipolynomial bound of n≤mO(d)≤mO (logm),

$T_{41}$ : Category two of SAS

$T_{42}$ : Category three of SAS

**Module Nine**

Namely, authors define a notion of **Boolean rank of a quadratic polynomial f** and relate it to (e&eb) the notion of diagonal rigidity

$G_{44}$ : Category one of **Boolean rank of a quadratic polynomial f;** notion of diagonal rigidity

$G_{45}$ : Category two of SAS

$G_{46}$ : Category three of SAS

$T_{44}$ : Category one of notion of diagonal rigidity **;Boolean rank of a quadratic polynomial f**

$T_{45}$ : Category two of SAS

$T_{46}$ : Category three of SAS

## SECTION EIGHTEEN

## Dynamical Localization In Atomic Momentum Transfer

### INTRODUCTION—VARIABLES USED

**Observation of Dynamical Localization in Atomic Momentum Transfer: A New Testing Ground for Quantum Chaos F. L. Moore, J. C. Robinson, C. Bharucha, P. E. Williams, and M. G. Raizen Phys. Rev. Lett. 73, 2974 – Published 28 November 1994**

(1) Authors report a direct observation of (e) dynamical localization. Dynamical localization effect is (=) a quantum suppression of diffusion in a system that is (=) classically chaotic.
(2) experiment measures (eb) the momentum transferred from a modulated standing wave of a near-resonant laser to (e&eb) a sample of ultracold atoms and is (=) a realization of a periodically driven rotor.
**(3)** The conceptual simplicity and experimental control over (e&eb) the time dependent Hamiltonian make this system an (=) ideal testing ground for the predictions of quantum chaos. **http://dx.doi.org/10.1103/PhysRevLett.73.2974©American Physical Society**

**Quantum Chaos on Graphs Tsampikos Kottos and Uzy Smilansky Phys. Rev. Lett 79, 4794 – Published 15 December 1997**

(4) Authors quantize (eb) graphs (networks) which consist of (e) a finite number of bonds and nodes. They show that their spectral statistics is well reproduced by (e) random matrix theory.
(5) They also define a classical phase space for (e) the graph, where (e) the dynamics is mixing and the periodic orbits (loops on the graph) proliferate (eb) exponentially.
(6) An exact trace formula for the quantum spectrum is developed and used to (e) investigate the origin of the connection between random matrix theory and (e&eb) the underlying chaotic classical dynamics.
**(7)** Being an exact theory, and due to its relative simplicity, it offers (eb) new insights into this problem which is at the forefront of the research in quantum chaos and related fields. **http://dx.doi.org/10.1103/PhysRevLett.79.4794©American Physical Society**

### NOTATION

**Module One**

**Quadratic polynomials of small modulus cannot represent OR Holden Lee (Submitted on 29 Sep 2015 (v1), last revised 12 Nov 2015 (this version, v2))**

(1) Using additive combinatorics, Authors show that (eb) when the rank is low, f(x) =0 must have (e) many solutions.
(2) Using techniques from exponential sums, authors show that when the rank of f is high, (eb) f is close to equidistributed. In either case, f cannot (e) represent the OR function in many variables. Subjects: Computational Complexity (cs.CC) Cite as: arXiv: 1509.08896 [cs.CC] (or arXiv:1509.08896v2 [cs.CC] for this version)

Using additive combinatorics, Authors show that (eb) when the rank is low, f(x) =0 must have (e) many solutions

$G_{13}$ : Category one of rank is low

$G_{14}$ : Category two of SAS

$G_{15}$ : Category three of SAS

$T_{13}$ : Category one of, f(x) =0 must have many solutions

$T_{14}$ : Category two of SAS

$T_{15}$ : Category three of SAS

**Module Two**

Using techniques from exponential sums, authors show that when the rank of f is high, (eb) f is close to equidistributed.

In either case, f cannot (e) represent the OR function in many variables. Subjects: Computational Complexity (cs.CC) Cite as: arXiv: 1509.08896 [cs.CC] (or arXiv:1509.08896v2 [cs.CC] for this version)

$G_{16}$ : Category one of rank of f is high

$G_{17}$ : Category two of SAS

$G_{18}$ : Category three of SAS

$T_{16}$ : Category one of f is close to equidistributed

$T_{17}$ : Category two of SAS

$T_{18}$ : Category three of SAS

**Module three**

Authors report a direct observation of (e) dynamical localization.

Dynamical localization effect is (=) a quantum suppression of diffusion in a system that is (=) classically chaotic

$G_{20}$ : Category one of Dynamical localization effect

$G_{21}$ : Category two of SAS

$G_{22}$ : Category three of SAS

$T_{20}$ : Category one of quantum suppression of diffusion in a system that is classically chaotic

$T_{21}$ : Category two of SAS

$T_{22}$ : Category three of SAS

**Module four**

experiment measures (eb) the **momentum transferred from a modulated standing wave of a near-resonant laser** to (e&eb) a sample of ultracold atoms and is (=) a realization of a periodically driven rotor
$G_{24}$ : Category one of **momentum transferred from a modulated standing wave of a near-resonant laser;** sample of ultracold atoms and is a realization of a periodically driven rotor

$G_{25}$ : Category two of SAS

$G_{26}$ : Category three of SAS

$T_{24}$ : Category one of sample of ultracold atoms and is a realization of a periodically driven rotor; **momentum transferred from a modulated standing wave of a near-resonant laser**

$T_{25}$ : Category two of SAS

$T_{26}$ : Category three of SAS

**Module five**

The conceptual simplicity and experimental control over the time dependent Hamiltonian make this system an (=) ideal testing ground for the predictions of quantum chaos.

**http://dx.doi.org/10.1103/PhysRevLett.73.2974©American Physical Society**

$G_{28}$ : Category one of conceptual simplicity and experimental control over the time dependent Hamiltonian

$G_{29}$ : Category two of SAS

$G_{30}$ : Category three of SAS

$T_{28}$ : Category one of testing ground for the predictions of quantum chaos

$T_{29}$ : Category two of SAS

$T_{30}$ : Category three of SAS

**Module six**

Authors quantize graphs (networks) which consist of (e) a finite number of bonds and nodes.

They show that their spectral statistics is well reproduced by (e) random matrix theory

$G_{32}$ : Category one of finite number of bonds and nodes

$G_{33}$ : Category two of SAS

$G_{34}$ : Category three of SAS

$T_{32}$ : Category one of graphs (networks)

$T_{33}$ : Category two of SAS

$T_{34}$ : Category three of SAS

**Module seven**

**spectral statistics** is well reproduced by (e) random matrix theory

**Source: Wikipedia**

Gaussian unitary ensemble GUE(n) is described by the Gaussian measure with density

$$\frac{1}{Z_{\mathrm{GUE}(n)}} e^{-\frac{n}{2}\mathrm{tr}H^2}$$

Gaussian orthogonal ensemble GOE(n) is described by the Gaussian measure with density

$$\frac{1}{Z_{\mathrm{GOE}(n)}} e^{-\frac{n}{4}\mathrm{tr}H^2}$$

The Gaussian symplectic ensemble GSE(n) is described by the Gaussian measure with density

$$\frac{1}{Z_{\mathrm{GSE}(n)}} e^{-n\mathrm{tr}H^2}$$

he joint probability density for the eigenvalues λ1,λ2,...,λn of GUE/GOE/GSE is given by

$$\frac{1}{Z_{\beta,n}} \prod_{k=1}^{n} e^{-\frac{\beta n}{4}\lambda_k^2} \prod_{i<j} |\lambda_j - \lambda_i|^{\beta} \quad , \qquad (1)$$

$G_{36}$ : Category one of **spectral statistics;** random matrix theory

$G_{37}$ : Category two of SAS

$G_{38}$ : Category three of SAS

$T_{36}$ : Category one of random matrix theory; **spectral statistics**

$T_{37}$ : Category two of SAS

$T_{38}$ : Category three of SAS

**Module eight**

They also define a classical phase space for the graph, where (e) the dynamics is mixing and the periodic orbits (loops on the graph) proliferate (eb) exponentially

$G_{40}$ : Category one of dynamics is mixing and the periodic orbits (loops on the graph) proliferate exponentially; **classical phase space for the graph**

$G_{41}$ : Category two of SAS

$G_{42}$ : Category three of SAS

$T_{40}$ : Category one of **classical phase space for the graph**; dynamics is mixing and the periodic orbits (loops on the graph) proliferate (eb) exponentially

$T_{41}$ : Category two of SAS

$T_{42}$ : Category three of SAS

**Module Nine**

An exact trace formula for the quantum spectrum is developed and used to investigate the origin of the connection between **random matrix theory** and (e&eb) the underlying chaotic classical dynamics.

$G_{44}$ : Category one of **random matrix theory;** underlying chaotic classical dynamics

$G_{45}$ : Category two of SAS

$G_{46}$ : Category three of SAS

$T_{44}$ : Category one of underlying chaotic classical dynamics; **random matrix theory**

$T_{45}$ : Category two of SAS

$T_{46}$ : Category three of SAS

**The Coefficients:**

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}$ $(a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}$ $(b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}$ , $(b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}$,
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}$, $(b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}$ $(a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$ ,
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$

$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)},$
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)},$
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)},$
are Dissipation coefficients

**Module Numbered One**

**The differential system of this model is now (Module Numbered one)**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \quad 1$$
$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad 2$$
$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15} \quad 3$$
$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13} \quad 4$$
$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14} \quad 5$$
$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)\right]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14}, t) =$ First augmentation factor
$-(b''_{13})^{(1)}(G, t) =$ First detritions factor

**Module Numbered Two**

**The differential system of this model is now ( Module numbered two)**

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right]G_{16} \quad 7$$
$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t)\right]G_{17} \quad 8$$
$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t)\right]G_{18} \quad 9$$
$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\left((G_{19}), t\right)\right]T_{16} \quad 10$$
$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\left((G_{19}), t\right)\right]T_{17} \quad 11$$
$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\left((G_{19}), t\right)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17}, t) =$ First augmentation factor
$-(b''_{16})^{(2)}\left((G_{19}), t\right) =$ First detritions factor

**Module Numbered Three**

**The differential system of this model is now (Module numbered three)**

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right]G_{20} \quad 13$$
$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}, t)\right]G_{21} \quad 14$$
$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}, t)\right]G_{22} \quad 15$$
$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}, t)\right]T_{20} \quad 16$$
$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}, t)\right]T_{21} \quad 17$$
$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}, t)\right]T_{22} \quad 18$$

$+(a''_{20})^{(3)}(T_{21}, t) =$ First augmentation factor
$-(b''_{20})^{(3)}(G_{23}, t) =$ First detritions factor

**Module Numbered Four**

**The differential system of this model is now (Module numbered Four)**

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right]G_{24} \quad 19$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25},t)\right]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25},t)\right]G_{26} \quad 21$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27}),t\right)\right]T_{24} \quad 22$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27}),t\right)\right]T_{25} \quad 23$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27}),t\right)\right]T_{26} \quad 24$$

$+(a''_{24})^{(4)}(T_{25},t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}\left((G_{27}),t\right) =$ **First detritions factor**

**Module Numbered Five**:

**The differential system of this model is now (Module number five)**

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29},t)\right]G_{28} \quad 25$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29},t)\right]G_{29} \quad 26$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29},t)\right]G_{30} \quad 27$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}\left((G_{31}),t\right)\right]T_{28} \quad 28$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}\left((G_{31}),t\right)\right]T_{29} \quad 29$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}\left((G_{31}),t\right)\right]T_{30} \quad 30$$

$+(a''_{28})^{(5)}(T_{29},t) =$ **First augmentation factor**

$-(b''_{28})^{(5)}\left((G_{31}),t\right) =$ **First detritions factor**

**Module Numbered Six**

**The differential system of this model is now (Module numbered Six)**

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33},t)\right]G_{32} \quad 31$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33},t)\right]G_{33} \quad 32$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33},t)\right]G_{34} \quad 33$$

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}\left((G_{35}),t\right)\right]T_{32} \quad 34$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}\left((G_{35}),t\right)\right]T_{33} \quad 35$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}\left((G_{35}),t\right)\right]T_{34} \quad 36$$

$+(a''_{32})^{(6)}(T_{33},t) =$ **First augmentation factor**

**Module Numbered Seven:**

**The differential system of this model is now (Seventh Module)**

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right]G_{36} \quad 37$$

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37},t)\right]G_{37} \quad 38$$

$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37},t)\right]G_{38} \quad 39$$

$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}\left((G_{39}),t\right)\right]T_{36} \quad 40$$

$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}\left((G_{39}),t\right)\right]T_{37} \quad 41$$

$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39}),t)\right]T_{38} \quad 42$$
$$+(a''_{36})^{(7)}(T_{37},t) = \textbf{First augmentation factor}$$

**Module Numbered Eight**

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right]G_{40} \quad 43$$
$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41},t)\right]G_{41} \quad 44$$
$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41},t)\right]G_{42} \quad 45$$
$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43}),t)\right]T_{40} \quad 46$$
$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}((G_{43}),t)\right]T_{41} \quad 47$$
$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43}),t)\right]T_{42} \quad 48$$

**Module Numbered Nine**

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$
$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$
$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$
$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47}),t)\right]T_{44} \quad 52$$
$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}((G_{47}),t)\right]T_{45} \quad 53$$
$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47}),t)\right]T_{46} \quad 54$$
$$+(a''_{44})^{(9)}(T_{45},t) = \textbf{First augmentation factor}$$
$$-(b''_{44})^{(9)}((G_{47}),t) = \textbf{First detrition factor}$$

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)} & +(a''_{13})^{(1)}(T_{14},t) & +(a''_{16})^{(2,2,)}(T_{17},t) & +(a''_{20})^{(3,3,)}(T_{21},t) \\ +(a''_{24})^{(4,4,4,4,)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{36})^{(7,7)}(T_{37},t) & +(a''_{40})^{(8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)} & +(a''_{14})^{(1)}(T_{14},t) & +(a''_{17})^{(2,2,)}(T_{17},t) & +(a''_{21})^{(3,3,)}(T_{21},t) \\ +(a''_{25})^{(4,4,4,4,)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{37})^{(7,7)}(T_{37},t) & +(a''_{41})^{(8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)} & +(a''_{15})^{(1)}(T_{14},t) & +(a''_{18})^{(2,2,)}(T_{17},t) & +(a''_{22})^{(3,3,)}(T_{21},t) \\ +(a''_{26})^{(4,4,4,4,)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{38})^{(7,7)}(T_{37},t) & +(a''_{42})^{(8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{15} \quad 57$$

Where $(a''_{13})^{(1)}(T_{14},t)$, $(a''_{14})^{(1)}(T_{14},t)$, $(a''_{15})^{(1)}(T_{14},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,)}(T_{17},t)$, $+(a''_{17})^{(2,2,)}(T_{17},t)$, $+(a''_{18})^{(2,2,)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,)}(T_{21},t)$, $+(a''_{21})^{(3,3,)}(T_{21},t)$, $+(a''_{22})^{(3,3,)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,)}(T_{25},t)$ , $+(a''_{25})^{(4,4,4,4,)}(T_{25},t)$ , $+(a''_{26})^{(4,4,4,4,)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,)}(T_{29},t)$ , $+(a''_{30})^{(5,5,5,5,)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,)}(T_{33},t)$ , $+(a''_{34})^{(6,6,6,6,)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{38})^{(7,7)}(T_{37},t)$ $+(a''_{37})^{(7,7)}(T_{37},t)$ $+(a''_{36})^{(7,7)}(T_{37},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{40})^{(8,8)}(T_{41},t)$ $+(a''_{41})^{(8,8)}(T_{41},t)$ $+(a''_{42})^{(8,8)}(T_{41},t)$ are eight augmentation coefficient for 1,2,3

$+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$ , $+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$ , $+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)} & -(b''_{13})^{(1)}(G,t) & -(b''_{16})^{(2,2,)}(G_{19},t) & -(b''_{20})^{(3,3,)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{36})^{(7,7,)}(G_{39},t) & -(b''_{40})^{(8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{13} \quad 58$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)} & -(b''_{14})^{(1)}(G,t) & -(b''_{17})^{(2,2,)}(G_{19},t) & -(b''_{21})^{(3,3,)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{37})^{(7,7,)}(G_{39},t) & -(b''_{41})^{(8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{14} \quad 59$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)} & -(b''_{15})^{(1)}(G,t) & -(b''_{18})^{(2,2,)}(G_{19},t) & -(b''_{22})^{(3,3,)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{38})^{(7,7,)}(G_{39},t) & -(b''_{42})^{(8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{15} \quad 60$$

Where $-(b''_{13})^{(1)}(G,t)$ , $-(b''_{14})^{(1)}(G,t)$ , $-(b''_{15})^{(1)}(G,t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,)}(G_{19},t)$ , $-(b''_{17})^{(2,2,)}(G_{19},t)$ , $-(b''_{18})^{(2,2,)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,)}(G_{23},t)$ , $-(b''_{21})^{(3,3,)}(G_{23},t)$ , $-(b''_{22})^{(3,3,)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,)}(G_{27},t)$ , $-(b''_{25})^{(4,4,4,4,)}(G_{27},t)$ , $-(b''_{26})^{(4,4,4,4,)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,)}(G_{31},t)$ , $-(b''_{29})^{(5,5,5,5,)}(G_{31},t)$ , $-(b''_{30})^{(5,5,5,5,)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,)}(G_{35},t)$ , $-(b''_{33})^{(6,6,6,6,)}(G_{35},t)$ , $-(b''_{34})^{(6,6,6,6,)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{37})^{(7,7,)}(G_{39},t)$ , $-(b''_{36})^{(7,7,)}(G_{39},t)$ , $-(b''_{38})^{(7,7,)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8)}(G_{43},t)$ $-(b''_{41})^{(8,8)}(G_{43},t)$ $-(b''_{42})^{(8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \begin{bmatrix} (a'_{16})^{(2)} & \boxed{+(a''_{16})^{(2)}(T_{17},t)} & \boxed{+(a''_{13})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)} & \\ \boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9)}(T_{45},t)} & \end{bmatrix} G_{16} \quad (61)$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \begin{bmatrix} (a'_{17})^{(2)} & \boxed{+(a''_{17})^{(2)}(T_{17},t)} & \boxed{+(a''_{14})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)} & \\ \boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9)}(T_{45},t)} & \end{bmatrix} G_{17} \quad (62)$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \begin{bmatrix} (a'_{18})^{(2)} & \boxed{+(a''_{18})^{(2)}(T_{17},t)} & \boxed{+(a''_{15})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)} & \\ \boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9)}(T_{45},t)} & \end{bmatrix} G_{18} \quad (63)$$

Where $\boxed{+(a''_{16})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,)}(T_{14},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)}$ are eight augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{44})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9)}(T_{45},t)}$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \begin{bmatrix} (b'_{16})^{(2)} & \boxed{-(b''_{16})^{(2)}(G_{19},t)} & \boxed{-(b''_{13})^{(1,1,)}(G,t)} & \boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)} & \\ \boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9)}(G_{47},t)} & \end{bmatrix} T_{16} \quad (64)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)} & \boxed{-(b''_{17})^{(2)}(G_{19},t)} & \boxed{-(b''_{14})^{(1,1,)}(G,t)} & \boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)} & \\ \boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9)}(G_{47},t)} & \end{bmatrix} T_{17} \quad (65)$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)}\boxed{-(b''_{18})^{(2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,)}(G,t)} & \boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{18} \quad 66$$

where $\boxed{-(b''_{16})^{(2)}(G_{19},t)}$ , $\boxed{-(b''_{17})^{(2)}(G_{19},t)}$ , $\boxed{-(b''_{18})^{(2)}(G_{19},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,)}(G,t)}$ are second detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{44})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \begin{bmatrix} (a'_{20})^{(3)}\boxed{+(a''_{20})^{(3)}(T_{21},t)} & \boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{20} \quad 67$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \begin{bmatrix} (a'_{21})^{(3)}\boxed{+(a''_{21})^{(3)}(T_{21},t)} & \boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{21} \quad 68$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \begin{bmatrix} (a'_{22})^{(3)}\boxed{+(a''_{22})^{(3)}(T_{21},t)} & \boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{22} \quad 69$$

$\boxed{+(a''_{20})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3)}(T_{21},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}$ are second augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)}$ are third augmentation coefficients for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficients for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficients for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficients for category 1, 2 and 3

$+(a''_{36})^{(7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1, 2 and 3

$+(a''_{40})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8)}(T_{41},t)$ are eight augmentation coefficients for category 1, 2 and 3

$+(a''_{44})^{(9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b'_{20})^{(3)} & -(b''_{20})^{(3)}(G_{23},t) & -(b''_{16})^{(2,2,2)}(G_{19},t) & -(b''_{13})^{(1,1,1,)}(G,t) \\ -(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{36})^{(7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{20} \quad (70)$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b'_{21})^{(3)} & -(b''_{21})^{(3)}(G_{23},t) & -(b''_{17})^{(2,2,2)}(G_{19},t) & -(b''_{14})^{(1,1,1,)}(G,t) \\ -(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{37})^{(7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{21} \quad (71)$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)} & -(b''_{22})^{(3)}(G_{23},t) & -(b''_{18})^{(2,2,2)}(G_{19},t) & -(b''_{15})^{(1,1,1,)}(G,t) \\ -(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{38})^{(7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{22} \quad (72)$$

$-(b''_{20})^{(3)}(G_{23},t)$, $-(b''_{21})^{(3)}(G_{23},t)$, $-(b''_{22})^{(3)}(G_{23},t)$ *are first detrition coefficients for category 1, 2 and 3*

$-(b''_{16})^{(2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2)}(G_{19},t)$ *are second detrition coefficients for category 1, 2 and 3*

$-(b''_{13})^{(1,1,1,)}(G,t)$, $-(b''_{14})^{(1,1,1,)}(G,t)$, $-(b''_{15})^{(1,1,1,)}(G,t)$ *are third detrition coefficients for category 1,2 and 3*

$-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)$ *are fourth detrition coefficients for category 1, 2 and 3*

$-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)$ *are fifth detrition coefficients for category 1, 2 and 3*

$-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)$ *are sixth detrition coefficients for category 1, 2 and 3*

$-(b''_{36})^{(7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt}=(a_{24})^{(4)}G_{25}-\begin{bmatrix}(a'_{24})^{(4)}\boxed{+(a''_{24})^{(4)}(T_{25},t)}&\boxed{+(a''_{28})^{(5,5,)}(T_{29},t)}&\boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}\\ \boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)}&\boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)}&\boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)}&\boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)}&\boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{24} \quad (73)$$

$$\frac{dG_{25}}{dt}=(a_{25})^{(4)}G_{24}-\begin{bmatrix}(a'_{25})^{(4)}\boxed{+(a''_{25})^{(4)}(T_{25},t)}&\boxed{+(a''_{29})^{(5,5,)}(T_{29},t)}&\boxed{+(a''_{33})^{(6,6)}(T_{33},t)}\\ \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)}&\boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)}&\boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)}&\boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)}&\boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{25} \quad (74)$$

$$\frac{dG_{26}}{dt}=(a_{26})^{(4)}G_{25}-\begin{bmatrix}(a'_{26})^{(4)}\boxed{+(a''_{26})^{(4)}(T_{25},t)}&\boxed{+(a''_{30})^{(5,5,)}(T_{29},t)}&\boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}\\ \boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)}&\boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)}&\boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)}&\boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)}&\boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{26} \quad (75)$$

$\boxed{(a''_{24})^{(4)}(T_{25},t)}$, $\boxed{(a''_{25})^{(4)}(T_{25},t)}$, $\boxed{(a''_{26})^{(4)}(T_{25},t)}$ *are first augmentation coefficients category* 1, 2 3

$\boxed{+(a''_{28})^{(5,5,)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,)}(T_{29},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)}$
*are fourth augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)}$
*are fifth augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)}$
*are sixth augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)}$
*are seventh augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)}$
*are eighth augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt}=(b_{24})^{(4)}T_{25}-\begin{bmatrix}(b'_{24})^{(4)}\boxed{-(b''_{24})^{(4)}(G_{27},t)}&\boxed{-(b''_{28})^{(5,5,)}(G_{31},t)}&\boxed{-(b''_{32})^{(6,6,)}(G_{35},t)}\\ \boxed{-(b''_{13})^{(1,1,1,1)}(G,t)}&\boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)}&\boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)}\\ \boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)}&\boxed{-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)}&\boxed{-(b''_{44})^{(9,9,9,9)}(G_{47},t)}\end{bmatrix}T_{24} \quad (76)$$

$$\frac{dT_{25}}{dt}=(b_{25})^{(4)}T_{24}-\begin{bmatrix}(b'_{25})^{(4)}\boxed{-(b''_{25})^{(4)}(G_{27},t)}&\boxed{-(b''_{29})^{(5,5,)}(G_{31},t)}&\boxed{-(b''_{33})^{(6,6,)}(G_{35},t)}\\ \boxed{-(b''_{14})^{(1,1,1,1)}(G,t)}&\boxed{-(b''_{17})^{(2,2,2,2)}(G_{19},t)}&\boxed{-(b''_{21})^{(3,3,3,3)}(G_{23},t)}\\ \boxed{-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)}&\boxed{-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)}&\boxed{-(b''_{45})^{(9,9,9,9)}(G_{47},t)}\end{bmatrix}T_{25} \quad (77)$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)}\boxed{-(b''_{26})^{(4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{26} \quad 78$$

*Where* $\boxed{-(b''_{24})^{(4)}(G_{27},t)}, \boxed{-(b''_{25})^{(4)}(G_{27},t)}, \boxed{-(b''_{26})^{(4)}(G_{27},t)}$ *are first detrition coefficients for category* $1, 2$ *and* $3$

$\boxed{-(b''_{28})^{(5,5,)}(G_{31},t)}, \boxed{-(b''_{29})^{(5,5,)}(G_{31},t)}, \boxed{-(b''_{30})^{(5,5,)}(G_{31},t)}$ *are second detrition coefficients for category* $1, 2$ *and* $3$

$\boxed{-(b''_{32})^{(6,6,)}(G_{35},t)}, \boxed{-(b''_{33})^{(6,6,)}(G_{35},t)}, \boxed{-(b''_{34})^{(6,6,)}(G_{35},t)}$ *are third detrition coefficients for category* $1, 2$ *and* $3$

$\boxed{-(b''_{13})^{(1,1,1,1)}(G,t)}, \boxed{-(b''_{14})^{(1,1,1,1)}(G,t)}, \boxed{-(b''_{15})^{(1,1,1,1)}(G,t)}$

*are fourth detrition coefficients for category* $1, 2$ *and* $3$

$\boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2,2)}(G_{19},t)}$

*are fifth detrition coefficients for category* $1, 2$ *and* $3$

$\boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3,3,3,3)}(G_{23},t)}$

*are sixth detrition coefficients for category* $1, 2$ *and* $3$

$\boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)}$

*are seventh detrition coefficients for category* $1, 2$ *and* $3$

$\boxed{-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)}$

*are eighth detrition coefficients for category* $1, 2$ *and* $3$

$\boxed{-(b''_{46})^{(9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{45})^{(9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{44})^{(9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \begin{bmatrix} (a'_{28})^{(5)}\boxed{+(a''_{28})^{(5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{28} \quad 79$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \begin{bmatrix} (a'_{29})^{(5)}\boxed{+(a''_{29})^{(5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{29} \quad 80$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \begin{bmatrix} (a'_{30})^{(5)}\boxed{+(a''_{30})^{(5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{30} \quad 81$$

*Where* $\boxed{+(a''_{28})^{(5)}(T_{29},t)}, \boxed{+(a''_{29})^{(5)}(T_{29},t)}, \boxed{+(a''_{30})^{(5)}(T_{29},t)}$ *are first augmentation coefficients for category* $1, 2$ *and* $3$

*And* $\boxed{+(a''_{24})^{(4,4,)}(T_{25},t)}, \boxed{+(a''_{25})^{(4,4,)}(T_{25},t)}, \boxed{+(a''_{26})^{(4,4,)}(T_{25},t)}$ *are second augmentation coefficient for category* $1, 2$ *and* $3$

$\boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)}, \boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)}, \boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)}$ *are third augmentation coefficient for category* $1, 2$ *and* $3$

$+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)$ are fourth augmentation coefficients for category 1,2, and 3

$+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)$ are fifth augmentation coefficients for category 1,2,and 3

$+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)$ are sixth augmentation coefficients for category 1,2, 3

$+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)$ , $+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1,2, 3

$+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)$ are eighth augmentation coefficients for category 1,2, 3

$+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \begin{bmatrix} (b'_{28})^{(5)} & -(b''_{28})^{(5)}(G_{31},t) & -(b''_{24})^{(4,4,)}(G_{27},t) & -(b''_{32})^{(6,6,6)}(G_{35},t) \\ & -(b''_{13})^{(1,1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{28} \quad 82$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b'_{29})^{(5)} & -(b''_{29})^{(5)}(G_{31},t) & -(b''_{25})^{(4,4,)}(G_{27},t) & -(b''_{33})^{(6,6,6)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{29} \quad 83$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b'_{30})^{(5)} & -(b''_{30})^{(5)}(G_{31},t) & -(b''_{26})^{(4,4,)}(G_{27},t) & -(b''_{34})^{(6,6,6)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1,1,)}(G,t) & -(b''_{18})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{30} \quad 84$$

*where* $-(b''_{28})^{(5)}(G_{31},t)$ , $-(b''_{29})^{(5)}(G_{31},t)$ , $-(b''_{30})^{(5)}(G_{31},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{24})^{(4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,)}(G_{27},t)$ *are second detrition coefficients for category* 1,2 *and* 3

$-(b''_{32})^{(6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6)}(G_{35},t)$ *are third detrition coefficients for category* 1,2 *and* 3

$-(b''_{13})^{(1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1)}(G,t)$ , $-(b''_{15})^{(1,1,1,1,1,)}(G,t)$ are fourth detrition coefficients for category 1,2, and 3

$-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1,2, and 3

$-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1,2, and 3

$-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2, and 3

$-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} \quad 85$$

$$-\left[\begin{array}{ccc} (a'_{32})^{(6)}\boxed{+(a''_{32})^{(6)}(T_{33},t)} & \boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)} \end{array}\right]G_{32}$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[\begin{array}{ccc} (a'_{33})^{(6)}\boxed{+(a''_{33})^{(6)}(T_{33},t)} & \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)} \end{array}\right]G_{33} \quad 86$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[\begin{array}{ccc} (a'_{34})^{(6)}\boxed{+(a''_{34})^{(6)}(T_{33},t)} & \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)} \end{array}\right]G_{34} \quad 87$$

$\boxed{+(a''_{32})^{(6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6)}(T_{33},t)}$ *are first augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}$ *are second augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}$ *are third augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}$ - are fourth augmentation coefficients

$\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}$ - fifth augmentation coefficients

$\boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)}$ sixth augmentation coefficients

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)}$ seventh augmentation coefficients

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}$ Eighth augmentation coefficients

$\boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)}$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[\begin{array}{ccc} (b'_{32})^{(6)}\boxed{-(b''_{32})^{(6)}(G_{35},t)} & \boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)} \end{array}\right]T_{32} \quad 88$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \begin{bmatrix} (b'_{33})^{(6)}\boxed{-(b''_{33})^{(6)}(G_{35},t)}\boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}\boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}\boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)}\boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{33} \quad (89)$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)}\boxed{-(b''_{34})^{(6)}(G_{35},t)}\boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}\boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}\boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)}\boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{34} \quad (90)$$

$\boxed{-(b''_{32})^{(6)}(G_{35},t)}, \boxed{-(b''_{33})^{(6)}(G_{35},t)}, \boxed{-(b''_{34})^{(6)}(G_{35},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}$ *are second detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)}$ *are third detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}$ are fourth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \quad (91)$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a'_{36})^{(7)}\boxed{+(a''_{36})^{(7)}(T_{37},t)}\boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)}\boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)}\boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)}\boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)}\boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \quad (92)$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a'_{37})^{(7)}\boxed{+(a''_{37})^{(7)}(T_{37},t)}\boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)}\boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)}\boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)}\boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}\boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{38}}{dt} \qquad 93$$

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a'_{38})^{(7)}\boxed{+(a''_{38})^{(7)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{(a''_{36})^{(7)}(T_{37},t)}$, $\boxed{(a''_{37})^{(7)}(T_{37},t)}$, $\boxed{(a''_{38})^{(7)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,)}(T_{41},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{36}}{dt} = \qquad 94$$

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b'_{36})^{(7)}\boxed{-(b''_{36})^{(7)}(G_{39},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{37}}{dt} =$$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b'_{37})^{(7)}\boxed{-(b''_{37})^{(7)}(G_{39},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{38}}{dt} =$$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b'_{38})^{(7)}\boxed{-(b''_{38})^{(7)}(G_{39},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b_{36}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{37}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{16}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{17}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{18}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{20}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{21}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{22}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{24}'')^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{25}'')^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{26}'')^{(4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{28}'')^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{32}'')^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{34}'')^{(6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{46}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{45}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{44}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3



$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \begin{bmatrix} (a_{40}')^{(8)}\boxed{+(a_{40}'')^{(8)}(T_{41},t)} & \boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{20}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{24}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{28}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{32}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{36}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{44}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \begin{bmatrix} (a_{41}')^{(8)}\boxed{+(a_{41}'')^{(8)}(T_{41},t)} & \boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{21}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{25}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{29}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{33}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{37}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{45}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \begin{bmatrix} (a_{42}')^{(8)}\boxed{+(a_{42}'')^{(8)}(T_{41},t)} & \boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{22}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{26}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{30}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{34}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{15}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{38}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{46}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a_{40}'')^{(8)}(T_{41},t)}$, $\boxed{+(a_{41}'')^{(8)}(T_{41},t)}$, $\boxed{+(a_{42}'')^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second

augmentation coefficient for category 1, 2 and 3
$+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)$ , $+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)$ , $+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3
$+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)$ , $+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)$ , $+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3
$+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)$ , $+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3
$+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)$ , $+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3
$+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ $+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ $+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ are seventh augmentation coefficient for 1,2,3
$+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$ , $+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$ , $+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$ are eighth augmentation coefficient for 1,2,3
$+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$ , $+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$ , $+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} =$$

$$(b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)}\left[-(b''_{40})^{(8)}(G_{43},t)\right] & \left[-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} =$$

$$(b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)}\left[-(b''_{41})^{(8)}(G_{43},t)\right] & \left[-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} =$$

$$(b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)}\left[-(b''_{42})^{(8)}(G_{43},t)\right] & \left[-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{15}$$

Where $-(b''_{36})^{(7)}(G_{39},t)$ , $-(b''_{37})^{(7)}(G_{39},t)$ , $-(b''_{38})^{(7)}(G_{39},t)$ are first detrition coefficients for category 1, 2 and 3
$-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$ , $-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$ , $-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3
$-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$ , $-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$ , $-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3
$-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)$ , $-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)$ , $-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3



$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \begin{bmatrix} (a'_{44})^{(9)}\boxed{+(a''_{44})^{(9)}(T_{45},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \begin{bmatrix} (a'_{45})^{(9)}\boxed{+(a''_{45})^{(9)}(T_{45},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \begin{bmatrix} (a'_{46})^{(9)}\boxed{+(a''_{46})^{(9)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a''_{44})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$ are Seventh augmentation coefficient for category 1, 2 and 3

$+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ $+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ $+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ are eighth augmentation coefficient for 1,2,3

$+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} =$$

$$(b_{44})^{(9)}T_{45} -$$

$$\begin{bmatrix} (b'_{44})^{(9)} & -(b''_{44})^{(9)}(G_{47},t) & -(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t) & \\ -(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t) & -(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t) & \end{bmatrix} T_{13}$$

$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b'_{45})^{(9)} & -(b''_{45})^{(9)}(G_{47},t) & -(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t) & \\ -(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t) & -(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t) & \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b'_{46})^{(9)} & -(b''_{46})^{(9)}(G_{47},t) & -(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t) & \\ -(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t) & -(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t) & \end{bmatrix} T_{15}$$

Where $-(b''_{44})^{(9)}(G_{47},t)$, $-(b''_{45})^{(9)}(G_{47},t)$, $-(b''_{46})^{(9)}(G_{47},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)$, $-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{37}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{36}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a_i')^{(1)}, (a_i'')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (b_i'')^{(1)} > 0, \quad i,j = 13,14,15 \qquad 97$$

The functions $(a_i'')^{(1)}, (b_i'')^{(1)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a_i'')^{(1)}(T_{14},t) \leq (p_i)^{(1)} \leq (\hat{A}_{13})^{(1)}$$

$$(b_i'')^{(1)}(G,t) \leq (r_i)^{(1)} \leq (b_i')^{(1)} \leq (\hat{B}_{13})^{(1)}$$

$$lim_{T_2\to\infty}(a_i'')^{(1)}(T_{14},t) = (p_i)^{(1)} \qquad 98$$

$$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = (r_i)^{(1)}$$

**Definition of** $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$ :

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$$|(a_i'')^{(1)}(T_{14}',t) - (a_i'')^{(1)}(T_{14},t)| \leq (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G',t) - (b_i'')^{(1)}(G,t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}',t)$ and $(a_i'')^{(1)}(T_{14},t)$ . $(T_{14}',t)$ and $(T_{14},t)$ are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14},t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$ : 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$$

**Definition of** $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$ : 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i,j = 16,17,18$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17}, t) \leq (p_i)^{(2)} \leq (\hat{A}_{16})^{(2)} \quad 102$$

$$(b_i'')^{(2)}(G_{19}, t) \leq (r_i)^{(2)} \leq (b_i')^{(2)} \leq (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2 \to \infty} (a_i'')^{(2)}(T_{17}, t) = (p_i)^{(2)} \quad 104$$

$$\lim_{G \to \infty} (b_i'')^{(2)}((G_{19}), t) = (r_i)^{(2)} \quad 105$$

**Definition of** $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$ : 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}', t) - (a_i'')^{(2)}(T_{17}, t)| \leq (\hat{k}_{16})^{(2)}|T_{17} - T_{17}'|e^{-(\hat{M}_{16})^{(2)}t} \quad 107$$

$$|(b_i'')^{(2)}((G_{19})', t) - (b_i'')^{(2)}((G_{19}), t)| < (\hat{k}_{16})^{(2)}||(G_{19}) - (G_{19})'||e^{-(\hat{M}_{16})^{(2)}t} \quad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}', t)$ and$(a_i'')^{(2)}(T_{17}, t)$ . $(T_{17}', t)$ and $(T_{17}, t)$ are points belonging to the interval $[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}]$ . It is to be noted that $(a_i'')^{(2)}(T_{17}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17}, t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$ :

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants 109

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$$

**Definition of** $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$ :

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1 \quad (110)$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(b_i)^{(2)} + (b_i')^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1 \quad (111)$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i,j = 20,21,22 \quad (112)$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(3)}$, $(r_i)^{(3)}$:

$$(a_i'')^{(3)}(T_{21}, t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23}, t) \leq (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(3)}(T_{21}, t) = (p_i)^{(3)} \quad (113)$$

$$\lim_{G \to \infty}(b_i'')^{(3)}(G_{23}, t) = (r_i)^{(3)}$$

**Definition of** $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$ :

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: (114)

$$|(a_i'')^{(3)}(T_{21}', t) - (a_i'')^{(3)}(T_{21}, t)| \leq (\hat{k}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\hat{M}_{20})^{(3)}t}$$

$$|(b_i'')^{(3)}(G_{23}', t) - (b_i'')^{(3)}(G_{23}, t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}'||e^{-(\hat{M}_{20})^{(3)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}', t)$ and $(a_i'')^{(3)}(T_{21}, t)$ . $(T_{21}', t)$ And $(T_{21}, t)$ are points belonging to the interval $[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}]$. It is to be noted that $(a_i'')^{(3)}(T_{21}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$ : (115)

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$ and the constants $(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}, i = 20,21,22,$ satisfy the inequalities 116

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(a_i)^{(3)} + (a_i')^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(b_i)^{(3)} + (b_i')^{(3)} + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i, j = 24,25,26 \qquad 117$$

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(4)}$, $(r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25}, t) \le (p_i)^{(4)} \le (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}((G_{27}), t) \le (r_i)^{(4)} \le (b_i')^{(4)} \le (\hat{B}_{24})^{(4)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(4)}(T_{25}, t) = (p_i)^{(4)} \qquad 118$$

$$\lim_{G \to \infty}(b_i'')^{(4)}((G_{27}), t) = (r_i)^{(4)}$$

**Definition of** $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$ :

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24,25,26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}', t) - (a_i'')^{(4)}(T_{25}, t)| \le (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})', t) - (b_i'')^{(4)}((G_{27}), t)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}', t)$ and $(a_i'')^{(4)}(T_{25}, t)$ . $(T_{25}', t)$ and $(T_{25}, t)$ are points belonging to the interval $[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}]$. It is to be noted that $(a_i'')^{(4)}(T_{25}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$ : 120

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$, are positive constants

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}, \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$$

**Definition of** $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$ : 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and\ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}, i = 24,25,26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(a_i)^{(4)} + (a_i')^{(4)} + (\hat{A}_{24})^{(4)} + (\hat{P}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(b_i)^{(4)} + (b_i')^{(4)} + (\hat{B}_{24})^{(4)} + (\hat{Q}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i,j = 28,29,30 \qquad 122$$

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(5)}$, $(r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29}, t) \le (p_i)^{(5)} \le (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}((G_{31}), t) \le (r_i)^{(5)} \le (b_i')^{(5)} \le (\hat{B}_{28})^{(5)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(5)}(T_{29}, t) = (p_i)^{(5)} \qquad 123$$

$$\lim_{G \to \infty}(b_i'')^{(5)}(G_{31}, t) = (r_i)^{(5)}$$

**Definition of** $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$ :

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}', t) - (a_i'')^{(5)}(T_{29}, t)| \le (\hat{k}_{28})^{(5)}|T_{29} - T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})', t) - (b_i'')^{(5)}((G_{31}), t)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}', t)$ and $(a_i'')^{(5)}(T_{29}, t)$ . $(T_{29}', t)$ and $(T_{29}, t)$ are points belonging to the interval $[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}]$. It is to be noted that $(a_i'')^{(5)}(T_{29}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29}, t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$ : 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}, \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

**Definition of** $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$ : 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\widehat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and\ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\widehat{M}_{28})^{(5)}}[(a_i)^{(5)} + (a_i')^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\widehat{M}_{28})^{(5)}}[(b_i)^{(5)} + (b_i')^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i, j = 32,33,34$$ 127

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(6)}$, $(r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33}, t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}), t) \leq (r_i)^{(6)} \leq (b_i')^{(6)} \leq (\hat{B}_{32})^{(6)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(6)}(T_{33}, t) = (p_i)^{(6)}$$ 128

$$\lim_{G \to \infty}(b_i'')^{(6)}((G_{35}), t) = (r_i)^{(6)}$$

**Definition of** $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$ :

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}', t) - (a_i'')^{(6)}(T_{33}, t)| \leq (\hat{k}_{32})^{(6)}|T_{33} - T_{33}'|e^{-(\widehat{M}_{32})^{(6)}t}$$

$$|(b_i'')^{(6)}((G_{35})', t) - (b_i'')^{(6)}((G_{35}), t)| < (\hat{k}_{32})^{(6)}||(G_{35}) - (G_{35})'||e^{-(\widehat{M}_{32})^{(6)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}', t)$ and $(a_i'')^{(6)}(T_{33}, t)$ . $(T_{33}', t)$ and $(T_{33}, t)$ are points belonging to the interval $[(\hat{k}_{32})^{(6)}, (\widehat{M}_{32})^{(6)}]$. It is to be noted that $(a_i'')^{(6)}(T_{33}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33}, t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\widehat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$ : 129

$(\widehat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$, are positive constants

$$\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}} < 1$$

**Definition of** $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$ : 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and\ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32,33,34,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(a_i)^{(6)} + (a_i')^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)} (\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(b_i)^{(6)} + (b_i')^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)} (\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

(MMMM) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i, j = 36,37,38$ 131

(NNNN) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(7)}, (r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37}, t) \leq (p_i)^{(7)} \leq (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39}, t) \leq (r_i)^{(7)} \leq (b_i')^{(7)} \leq (\hat{B}_{36})^{(7)}$$

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*(OOOO)* $\lim_{T_2 \to \infty} (a_i'')^{(7)}(T_{37}, t) = (p_i)^{(7)}$

*(PPPP)*

$\lim_{G \to \infty} (b_i'')^{(7)}((G_{39}), t) = (r_i)^{(7)}$

**Definition of** $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$ :

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$$|(a_i'')^{(7)}(T_{37}', t) - (a_i'')^{(7)}(T_{37}, t)| \leq (\hat{k}_{36})^{(7)} |T_{37} - T_{37}'| e^{-(\hat{M}_{36})^{(7)} t}$$

$$|(b_i'')^{(7)}((G_{39})', t) - (b_i'')^{(7)}((G_{39}), t)| < (\hat{k}_{36})^{(7)} ||(G_{39}) - (G_{39})'|| e^{-(\hat{M}_{36})^{(7)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}', t)$ and $(a_i'')^{(7)}(T_{37}, t)$ . $(T_{37}', t)$ and $(T_{37}, t)$ are points belonging to the interval $[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}]$ . It is to be noted that $(a_i'')^{(7)}(T_{37}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37}, t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$ : 134

(QQQQ) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)},$ are positive constants

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$$

**Definition of** $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$ : 135

(RRRR) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(b_i)^{(7)} + (b_i')^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i, j = 40,41,42 \qquad 136$$

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

**Definition of** $(p_i)^{(8)}$, $(r_i)^{(8)}$: 137

$$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)} \qquad 138$$

$$(b_i'')^{(8)}((G_{43}), t) \leq (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)} \qquad 139$$

$$\lim_{T_2 \to \infty} (a_i'')^{(8)}(T_{41}, t) = (p_i)^{(8)} \qquad 140$$

$$\lim_{G \to \infty} (b_i'')^{(8)}\big((G_{43}), t\big) = (r_i)^{(8)} \qquad 141$$

**Definition of** $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$ :

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)} |T_{41} - T_{41}'| e^{-(\hat{M}_{40})^{(8)} t} \qquad 142$$

$$|(b_i'')^{(8)}((G_{43})', t) - (b_i'')^{(8)}\big((G_{43}), t\big)| < (\hat{k}_{40})^{(8)} ||(G_{43}) - (G_{43})'|| e^{-(\hat{M}_{40})^{(8)} t} \qquad 143$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}', t)$ and $(a_i'')^{(8)}(T_{41}, t)$ . $(T_{41}', t)$ and $(T_{41}, t)$ are points belonging to the interval $\big[(\hat{k}_{40})^{(8)}, (\hat{M}_{40})^{(8)}\big]$. It is to be noted that $(a_i'')^{(8)}(T_{41}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{40})^{(8)} = 1$

then the function $(a_i'')^{(8)}(T_{41},t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)}$ :

$(\hat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)},$ are positive constants

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\ ,\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}}<1 \qquad 144$$

**Definition of** $(\hat{P}_{40})^{(8)},(\hat{Q}_{40})^{(8)}$ :

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\hat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)},(\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)},(a_i')^{(8)},(b_i)^{(8)},(b_i')^{(8)},(p_i)^{(8)},\ (r_i)^{(8)},i=40,41,42,$

Satisfy the inequalities

$$\frac{1}{(\hat{M}_{40})^{(8)}}[\,(a_i)^{(8)}+(a_i')^{(8)}+\ (\hat{A}_{40})^{(8)}+\ (\hat{P}_{40})^{(8)}\,(\hat{k}_{40})^{(8)}]<1 \qquad 145$$

$$\frac{1}{(\hat{M}_{40})^{(8)}}[\ (b_i)^{(8)}+(b_i')^{(8)}+\ (\hat{B}_{40})^{(8)}+\ (\hat{Q}_{40})^{(8)}\ (\hat{k}_{40})^{(8)}]<1 \qquad 146$$

Where we suppose

$$(a_i)^{(9)},(a_i')^{(9)},(a_i'')^{(9)},(b_i)^{(9)},(b_i')^{(9)},(b_i'')^{(9)}>0,\quad i,j=44,45,46 \qquad 146\,A$$

The functions $(a_i'')^{(9)},(b_i'')^{(9)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(9)},\ (r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45},t)\le(p_i)^{(9)}\le(\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47},t)\le\ (r_i)^{(9)}\le(b_i')^{(9)}\le(\hat{B}_{44})^{(9)}$$

$$\lim_{T_2\to\infty}(a_i'')^{(9)}(T_{45},t)=(p_i)^{(9)}$$

$$\lim_{G\to\infty}(b_i'')^{(9)}(G_{47},t)=\ (r_i)^{(9)}$$

**Definition of** $(\hat{A}_{44})^{(9)},(\hat{B}_{44})^{(9)}$ :

Where $\boxed{(\hat{A}_{44})^{(9)},(\hat{B}_{44})^{(9)},(p_i)^{(9)},\ (r_i)^{(9)}}$ are positive constants and $\boxed{i=44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}',t)-(a_i'')^{(9)}(T_{45},t)|\le(\hat{k}_{44})^{(9)}|T_{45}-\ T_{45}'|e^{-(\hat{M}_{44})^{(9)}t}$$

$$|(b_i'')^{(9)}((G_{47})',t)-(b_i'')^{(9)}\big((G_{47}),t\big)|<(\hat{k}_{44})^{(9)}||(G_{47})-(G_{47})'||e^{-(\hat{M}_{44})^{(9)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}',t)$ and$(a_i'')^{(9)}(T_{45},t)$ . $(T_{45}',t)$ and $(T_{45},t)$ are points belonging to the interval $\big[(\hat{k}_{44})^{(9)},(\hat{M}_{44})^{(9)}\big]$. It is

to be noted that $(a_i'')^{(9)}(T_{45},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{44})^{(9)}=1$ then the function $(a_i'')^{(9)}(T_{45},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\widehat{M}_{44})^{(9)},(\hat{k}_{44})^{(9)}$ :

$(\widehat{M}_{44})^{(9)},(\hat{k}_{44})^{(9)}$, are positive constants

$$\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\ ,\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}<1$$

**Definition of** $(\hat{P}_{44})^{(9)},(\hat{Q}_{44})^{(9)}$ :

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\widehat{M}_{44})^{(9)},(\hat{k}_{44})^{(9)},(\hat{A}_{44})^{(9)} and\ (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)},(a_i')^{(9)},(b_i)^{(9)},(b_i')^{(9)},(p_i)^{(9)},\ (r_i)^{(9)}, i=44,45,46,$ satisfy the inequalities

$$\frac{1}{(\widehat{M}_{44})^{(9)}}[(a_i)^{(9)}+(a_i')^{(9)}+\ (\hat{A}_{44})^{(9)}+\ (\hat{P}_{44})^{(9)}\ (\hat{k}_{44})^{(9)}]<1$$

$$\frac{1}{(\widehat{M}_{44})^{(9)}}[\ (b_i)^{(9)}+(b_i')^{(9)}+\ (\hat{B}_{44})^{(9)}+\ (\hat{Q}_{44})^{(9)}\ (\hat{k}_{44})^{(9)}]<1$$

**Theorem 1:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0)\ ,T_i(0)$ :

$$G_i(t)\le\ (\hat{P}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}\ ,\ \boxed{G_i(0)=G_i^0>0}$$

$$T_i(t)\le\ (\hat{Q}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}\ ,\ \boxed{T_i(0)=T_i^0>0}$$

**Theorem 2 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

**Definition of** $G_i(0)\ ,T_i(0)$

$$G_i(t)\le\ (\hat{P}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}\ ,\ \ G_i(0)=G_i^0>0$$

$$T_i(t)\le\ (\hat{Q}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}\ ,\ \ T_i(0)=T_i^0>0$$

**Theorem 3 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t)\le\ (\hat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}\ ,\ \ G_i(0)=G_i^0>0$$

$$T_i(t)\le\ (\hat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}\ ,\ \ T_i(0)=T_i^0>0$$

**Theorem 4 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

**Definition of** $G_i(0)\ ,T_i(0)$ :

$G_i(t) \leq \left(\hat{P}_{24}\right)^{(4)} e^{(\hat{M}_{24})^{(4)}t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{24}\right)^{(4)} e^{(\hat{M}_{24})^{(4)}t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Theorem 5 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

**Definition of** $G_i(0)$ , $T_i(0)$ :

$G_i(t) \leq \left(\hat{P}_{28}\right)^{(5)} e^{(\hat{M}_{28})^{(5)}t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{28}\right)^{(5)} e^{(\hat{M}_{28})^{(5)}t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Theorem 6 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

**Definition of** $G_i(0)$ , $T_i(0)$ :

$G_i(t) \leq \left(\hat{P}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Theorem 7:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

**Definition of** $G_i(0)$ , $T_i(0)$ :

$G_i(t) \leq \left(\hat{P}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Theorem 8:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

**Definition of** $G_i(0)$ , $T_i(0)$ :

$G_i(t) \leq \left(\hat{P}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Theorem 9:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

**Definition of** $G_i(0)$ , $T_i(0)$ :

$G_i(t) \leq \left(\hat{P}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Proof:** Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i$: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \le (\hat{P}_{13})^{(1)} \text{ , } T_i^0 \le (\hat{Q}_{13})^{(1)}, \quad 155$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t} \quad 156$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t} \quad 157$$

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}(s_{(13)}) - \left((a'_{13})^{(1)} + a''_{13})^{(1)} (T_{14}(s_{(13)}), s_{(13)})\right) G_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)} (T_{14}(s_{(13)}), s_{(13)})\right) G_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)} (T_{14}(s_{(13)}), s_{(13)})\right) G_{15}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)} (G(s_{(13)}), s_{(13)})\right) T_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)} (G(s_{(13)}), s_{(13)})\right) T_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{15}(t) = T_{15}^0 + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)} (G(s_{(13)}), s_{(13)})\right) T_{15}(s_{(13)})\right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i$: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \le (\hat{P}_{16})^{(2)} \text{ , } T_i^0 \le (\hat{Q}_{16})^{(2)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$$

By 160

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''_{16})^{(2)} (T_{17}(s_{(16)}), s_{(16)})\right) G_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)} (T_{17}(s_{(16)}), s_{(17)})\right) G_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)} (T_{17}(s_{(16)}), s_{(16)})\right) G_{18}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)} (G_{19}(s_{(16)}), s_{(16)})\right) T_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^{0} + \int_0^t \left[ (b_{17})^{(2)} T_{16}(s_{(16)}) - \left( (b'_{17})^{(2)} - (b''_{17})^{(2)} (G_{19}(s_{(16)}), s_{(16)}) \right) T_{17}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^{0} + \int_0^t \left[ (b_{18})^{(2)} T_{17}(s_{(16)}) - \left( (b'_{18})^{(2)} - (b''_{18})^{(2)} (G_{19}(s_{(16)}), s_{(16)}) \right) T_{18}(s_{(16)}) \right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i$: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$ , $T_i(0) = T_i^0$ , $G_i^0 \le (\hat{P}_{20})^{(3)}$ , $T_i^0 \le (\hat{Q}_{20})^{(3)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

By 161

$$\bar{G}_{20}(t) = G_{20}^{0} + \int_0^t \left[ (a_{20})^{(3)} G_{21}(s_{(20)}) - \left( (a'_{20})^{(3)} + a''_{20})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^{0} + \int_0^t \left[ (a_{21})^{(3)} G_{20}(s_{(20)}) - \left( (a'_{21})^{(3)} + (a''_{21})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{21}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^{0} + \int_0^t \left[ (a_{22})^{(3)} G_{21}(s_{(20)}) - \left( (a'_{22})^{(3)} + (a''_{22})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{22}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^{0} + \int_0^t \left[ (b_{20})^{(3)} T_{21}(s_{(20)}) - \left( (b'_{20})^{(3)} - (b''_{20})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^{0} + \int_0^t \left[ (b_{21})^{(3)} T_{20}(s_{(20)}) - \left( (b'_{21})^{(3)} - (b''_{21})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{21}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{\mathrm{T}}_{22}(\mathrm{t}) = \mathrm{T}_{22}^{0} + \int_0^t \left[ (b_{22})^{(3)} T_{21}(s_{(20)}) - \left( (b'_{22})^{(3)} - (b''_{22})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{22}(s_{(20)}) \right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i$: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$ , $T_i(0) = T_i^0$ , $G_i^0 \le (\hat{P}_{24})^{(4)}$ , $T_i^0 \le (\hat{Q}_{24})^{(4)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

By 162

$$\bar{G}_{24}(t) = G_{24}^{0} + \int_0^t \left[ (a_{24})^{(4)} G_{25}(s_{(24)}) - \left( (a'_{24})^{(4)} + a''_{24})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{26}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{26}(t) = T_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{26}(s_{(24)})\right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{28})^{(5)}, T_i^0 \le (\hat{Q}_{28})^{(5)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

By 163

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{30}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{30}(t) = T_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{30}(s_{(28)})\right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{32})^{(6)}, T_i^0 \le (\hat{Q}_{32})^{(6)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\bar{M}_{32})^{(6)} t}$

By 164

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[ (a_{32})^{(6)} G_{33}(s_{(32)}) - \left( (a'_{32})^{(6)} + a''_{32})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[ (a_{33})^{(6)} G_{32}(s_{(32)}) - \left( (a'_{33})^{(6)} + (a''_{33})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[ (a_{34})^{(6)} G_{33}(s_{(32)}) - \left( (a'_{34})^{(6)} + (a''_{34})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{34}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[ (b_{32})^{(6)} T_{33}(s_{(32)}) - \left( (b'_{32})^{(6)} - (b''_{32})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[ (b_{33})^{(6)} T_{32}(s_{(32)}) - \left( (b'_{33})^{(6)} - (b''_{33})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{34}(t) = T_{34}^0 + \int_0^t \left[ (b_{34})^{(6)} T_{33}(s_{(32)}) - \left( (b'_{34})^{(6)} - (b''_{34})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{34}(s_{(32)}) \right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[ (a_{36})^{(7)} G_{37}(s_{(36)}) - \left( (a'_{36})^{(7)} + a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[ (a_{37})^{(7)} G_{36}(s_{(36)}) - \left( (a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[ (a_{38})^{(7)} G_{37}(s_{(36)}) - \left( (a'_{38})^{(7)} + (a''_{38})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{38}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[ (b_{36})^{(7)} T_{37}(s_{(36)}) - \left( (b'_{36})^{(7)} - (b''_{36})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[ (b_{37})^{(7)} T_{36}(s_{(36)}) - \left( (b'_{37})^{(7)} - (b''_{37})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{38}(t) = T_{38}^0 + \int_0^t \left[ (b_{38})^{(7)} T_{37}(s_{(36)}) - \left( (b'_{38})^{(7)} - (b''_{38})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{38}(s_{(36)}) \right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0,t)$

**Proof:**

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{40})^{(8)}$, $T_i^0 \le (\hat{Q}_{40})^{(8)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} G_{41}(s_{(40)}) - \left((a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[(a_{41})^{(8)} G_{40}(s_{(40)}) - \left((a'_{41})^{(8)} + (a''_{41})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{42}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^0 + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)} (G_{43}(s_{(40)}), s_{(40)})\right) T_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^0 + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)} (G_{43}(s_{(40)}), s_{(40)})\right) T_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{42}(t) = T_{42}^0 + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)} (G_{43}(s_{(40)}), s_{(40)})\right) T_{42}(s_{(40)})\right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0,t)$

**Proof:** 166
Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ A which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{44})^{(9)}$, $T_i^0 \le (\hat{Q}_{44})^{(9)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)})\right) G_{44}(s_{(44)})\right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)} (T_{45}(s_{(44)}), s_{(44)})\right) G_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}(s_{(44)}), s_{(44)})\right) G_{46}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{44}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{\mathrm{T}}_{46}(\mathrm{t}) = \mathrm{T}_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{46}(s_{(44)})\right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}}\right)\right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left(e^{(\hat{M}_{13})^{(1)} t} - 1\right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\hat{M}_{13})^{(1)} t} \le \frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{P}_{13})^{(1)} + G_{14}^0\right) e^{\left(-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0}\right)} + (\hat{P}_{13})^{(1)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \le G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}}\right)\right] ds_{(16)} = \quad 169$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1\right)$$

From which it follows that 170

$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \le \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left[\left((\hat{P}_{16})^{(2)} + G_{17}^0\right) e^{\left(-\frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0}\right)} + (\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t) \le G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} \left(G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} s_{(20)}}\right)\right] ds_{(20)} =$$

$$\left(1 + (a_{20})^{(3)} t\right) G_{21}^0 + \frac{(a_{20})^{(3)} (\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left(e^{(\hat{M}_{20})^{(3)} t} - 1\right)$$

From which it follows that 172

$$(G_{20}(t)-G_{20}^{0})e^{-(\hat{M}_{20})^{(3)}t}\leq\frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left[((\hat{P}_{20})^{(3)}+G_{21}^{0})e^{\left(-\frac{(\hat{P}_{20})^{(3)}+G_{21}^{0}}{G_{21}^{0}}\right)}+(\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 173

$$G_{24}(t)\leq G_{24}^{0}+\int_{0}^{t}\left[(a_{24})^{(4)}\left(G_{25}^{0}+(\hat{P}_{24})^{(4)}e^{(\hat{M}_{24})^{(4)}s_{(24)}}\right)\right]ds_{(24)}=$$

$$\left(1+(a_{24})^{(4)}t\right)G_{25}^{0}+\frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left(e^{(\hat{M}_{24})^{(4)}t}-1\right)$$

From which it follows that 174

$$(G_{24}(t)-G_{24}^{0})e^{-(\hat{M}_{24})^{(4)}t}\leq\frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left[((\hat{P}_{24})^{(4)}+G_{25}^{0})e^{\left(-\frac{(\hat{P}_{24})^{(4)}+G_{25}^{0}}{G_{25}^{0}}\right)}+(\hat{P}_{24})^{(4)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t)\leq G_{28}^{0}+\int_{0}^{t}\left[(a_{28})^{(5)}\left(G_{29}^{0}+(\hat{P}_{28})^{(5)}e^{(\hat{M}_{28})^{(5)}s_{(28)}}\right)\right]ds_{(28)}=$$

$$\left(1+(a_{28})^{(5)}t\right)G_{29}^{0}+\frac{(a_{28})^{(5)}(\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left(e^{(\hat{M}_{28})^{(5)}t}-1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t}\leq\frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t)\leq G_{32}^{0}+\int_{0}^{t}\left[(a_{32})^{(6)}\left(G_{33}^{0}+(\hat{P}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\right)\right]ds_{(32)}=$$

$$\left(1+(a_{32})^{(6)}t\right)G_{33}^{0}+\frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left(e^{(\hat{M}_{32})^{(6)}t}-1\right)$$

From which it follows that 177

$$(G_{32}(t)-G_{32}^{0})e^{-(\hat{M}_{32})^{(6)}t}\leq\frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left[((\hat{P}_{32})^{(6)}+G_{33}^{0})e^{\left(-\frac{(\hat{P}_{32})^{(6)}+G_{33}^{0}}{G_{33}^{0}}\right)}+(\hat{P}_{32})^{(6)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}$ , $G_{26}, T_{24}, T_{25}, T_{26}$

(p) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t) \le G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} \left(G_{37}^0 + (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} s_{(36)}}\right)\right] ds_{(36)} =$$
$$\left(1 + (a_{36})^{(7)} t\right) G_{37}^0 + \frac{(a_{36})^{(7)} (\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left(e^{(\hat{M}_{36})^{(7)} t} - 1\right)$$

From which it follows that

$$(G_{36}(t) - G_{36}^0) e^{-(\hat{M}_{36})^{(7)} t} \le \frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left[((\hat{P}_{36})^{(7)} + G_{37}^0) e^{\left(-\frac{(\hat{P}_{36})^{(7)} + G_{37}^0}{G_{37}^0}\right)} + (\hat{P}_{36})^{(7)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t) \le G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} \left(G_{41}^0 + (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} s_{(40)}}\right)\right] ds_{(40)} = \quad 180$$
$$\left(1 + (a_{40})^{(8)} t\right) G_{41}^0 + \frac{(a_{40})^{(8)} (\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}} \left(e^{(\hat{M}_{40})^{(8)} t} - 1\right)$$

From which it follows that 181

$$(G_{40}(t) - G_{40}^0) e^{-(\hat{M}_{40})^{(8)} t} \le \frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}} \left[((\hat{P}_{40})^{(8)} + G_{41}^0) e^{\left(-\frac{(\hat{P}_{40})^{(8)} + G_{41}^0}{G_{41}^0}\right)} + (\hat{P}_{40})^{(8)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 8

Analogous inequalities hold also for $G_{41}$ , $G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t) \le G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} \left(G_{45}^0 + (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} s_{(44)}}\right)\right] ds_{(44)} =$$
$$\left(1 + (a_{44})^{(9)} t\right) G_{45}^0 + \frac{(a_{44})^{(9)} (\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left(e^{(\hat{M}_{44})^{(9)} t} - 1\right)$$

From which it follows that

$$(G_{44}(t) - G_{44}^0) e^{-(\hat{M}_{44})^{(9)} t} \le \frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left[((\hat{P}_{44})^{(9)} + G_{45}^0) e^{\left(-\frac{(\hat{P}_{44})^{(9)} + G_{45}^0}{G_{45}^0}\right)} + (\hat{P}_{44})^{(9)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 9

Analogous inequalities hold also for $G_{45}$ , $G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}$ , $\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$ and to choose 182

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\widehat{M}_{13})^{(1)}}\left[(\widehat{P}_{13})^{(1)}+\left((\hat{P}_{13})^{(1)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{13})^{(1)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{13})^{(1)} \quad 183$$

$$\frac{(b_i)^{(1)}}{(\widehat{M}_{13})^{(1)}}\left[\left((\hat{Q}_{13})^{(1)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{13})^{(1)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{13})^{(1)}\right]\leq(\hat{Q}_{13})^{(1)} \quad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)},T^{(1)}\right),\left(G^{(2)},T^{(2)}\right)\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{13})^{(1)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{13})^{(1)}t}\}$$

Indeed if we denote

**Definition of** $\tilde{G},\tilde{T}:\left(\tilde{G},\tilde{T}\right)=\mathcal{A}^{(1)}(G,T)$

It results

$$\left|\tilde{G}_{13}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{13})^{(1)}\left|G_{14}^{(1)}-G_{14}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\,ds_{(13)}+$$

$$\int_0^t\{(a'_{13})^{(1)}\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$G_{13}^{(2)}\left|(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)-(a''_{13})^{(1)}\left(T_{14}^{(2)},s_{(13)}\right)\right|\ e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses it follows

$$\left|G^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}t}\leq \quad 186$$
$$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)}+(a'_{13})^{(1)}+(\widehat{A}_{13})^{(1)}+(\widehat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}\right)d\left(\left(G^{(1)},T^{(1)};\,G^{(2)},T^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

**Remark 1:** The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}\ and\ (\widehat{Q}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i=13,14,15$ depend only on $\mathrm{T}_{14}$ and respectively on

$G(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$

From 19 to 24 it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(1)} - (a_i'')^{(1)}(T_{14}(s_{(13)}), s_{(13)})\} ds_{(13)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(1)} t)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2\ and\ \left((\widehat{M}_{13})^{(1)}\right)_3$ : 187

**Remark 3**: if $G_{13}$ is bounded, the same property have also $G_{14}\ and\ G_{15}$ . indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a_{14}')^{(1)} G_{14}$ and by integrating

$$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1 / (a_{14}')^{(1)}$$

In the same way , one can obtain

$$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2 / (a_{15}')^{(1)}$$

If $G_{14}\ or\ G_{15}$ is bounded, the same property follows for $G_{13}$ , $G_{15}$ and $G_{13}$ , $G_{14}$ respectively.

**Remark 4:** If $G_{13}$ $is$ bounded, from below, the same property holds for $G_{14}\ and\ G_{15}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{14}$ is bounded from below. 188

**Remark 5:** If $T_{13}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}\,(G(t), t)) = (b_{14}')^{(1)}$ then $T_{14} \to \infty$. 189

**Definition of** $(m)^{(1)}$ and $\varepsilon_1$ :

Indeed let $t_1$ be so that for $t > t_1$

$$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t), t) < \varepsilon_1, T_{13}\,(t) > (m)^{(1)}$$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_1}$ By taking now $\varepsilon_1$ sufficiently small one sees that $T_{14}$ is unbounded.
The same property holds for $T_{15}$ if $\lim_{t\to\infty}(b_{15}'')^{(1)}\,(G(t), t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}$ , $\frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}\ and\ (\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[(\widehat{P}_{16})^{(2)}+\left((\hat{P}_{16})^{(2)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{16})^{(2)}\right]\leq(\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

**Definition of** $\widetilde{G_{19}},\widetilde{T_{19}}$ : $\left(\widetilde{G_{19}},\widetilde{T_{19}}\right)=\mathcal{A}^{(2)}(G_{19},T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{16})^{(2)}\left|G_{17}^{(1)}-G_{17}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)}+$$

$$\int_0^t\{(a'_{16})^{(2)}\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$G_{16}^{(2)}\left|(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)-(a''_{16})^{(2)}\left(T_{17}^{(2)},s_{(16)}\right)\right|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0,t]$ 197

From the hypotheses it follows

$$\left|(G_{19})^{(1)}-(G_{19})^{(2)}\right|\mathrm{e}^{-(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}\leq$$
$$\frac{1}{(\widehat{\mathrm{M}}_{16})^{(2)}}\left((a_{16})^{(2)}+(a'_{16})^{(2)}+(\widehat{\mathrm{A}}_{16})^{(2)}+(\widehat{\mathrm{P}}_{16})^{(2)}(\hat{k}_{16})^{(2)}\right)\mathrm{d}\left(((G_{19})^{(1)},(T_{19})^{(1)};\ (G_{19})^{(2)},(T_{19})^{(2)})\right)$$

And analogous inequalities for $G_i$ and $T_i$. Taking into account the hypothesis the result follows 198

**Remark 6:** The fact that we supposed $(a''_{16})^{(2)}$ and $(b''_{16})^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{\mathrm{P}}_{16})^{(2)}\mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}$ and $(\widehat{\mathrm{Q}}_{16})^{(2)}\mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i=16,17,18$ depend only on $\mathrm{T}_{17}$ and respectively on

$(G_{19})$(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 7:** There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$ 200

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(2)} - (a_i'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})\} ds_{(16)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(2)}t)} > 0 \quad \text{for } t > 0$$

**Definition of** $\left((\widehat{M}_{16})^{(2)}\right)_1$, $\left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$: 201

**Remark 8:** if $G_{16}$ is bounded, the same property have also $G_{17}$ and $G_{18}$. indeed if

$G_{16} < (\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt} \leq \left((\widehat{M}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)} G_{17}$ and by integrating

$$G_{17} \leq \left((\widehat{M}_{16})^{(2)}\right)_2 = G_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1 / (a_{17}')^{(2)}$$

In the same way , one can obtain

$$G_{18} \leq \left((\widehat{M}_{16})^{(2)}\right)_3 = G_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2 / (a_{18}')^{(2)}$$

If $G_{17}$ or $G_{18}$ is bounded, the same property follows for $G_{16}$, $G_{18}$ and $G_{16}$, $G_{17}$ respectively.

**Remark 9:** If $G_{16}$ is bounded, from below, the same property holds for $G_{17}$ and $G_{18}$. The proof is analogous with the preceding one. An analogous property is true if $G_{17}$ is bounded from below. 202

**Remark 10:** If $T_{16}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t), t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$. 203

**Definition of** $(m)^{(2)}$ and $\varepsilon_2$ :

Indeed let $t_2$ be so that for $t > t_2$

$$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right)$, $t = \log\frac{2}{\varepsilon_2}$ By taking now $\varepsilon_2$ sufficiently small one sees that $T_{17}$ is unbounded. 205
The same property holds for $T_{18}$ if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t), t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[(\widehat{P}_{20})^{(3)}+\left((\hat{P}_{20})^{(3)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{20})^{(3)}\right]\leq(\hat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself 210

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)},(T_{23})^{(1)}),((G_{23})^{(2)},(T_{23})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

**Definition of** $\widetilde{G_{23}},\widetilde{T_{23}}:(\widetilde{(G_{23})},\widetilde{(T_{23})})=\mathcal{A}^{(3)}((G_{23}),(T_{23}))$

It results 213

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{20})^{(3)}\left|G_{21}^{(1)}-G_{21}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\,ds_{(20)}+$$

$$\int_0^t\{(a'_{20})^{(3)}\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$G_{20}^{(2)}\left|(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)},s_{(20)}\right)\right|\ e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\}ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses it follows

$$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}t}\leq \quad 214$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+(a'_{20})^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\widehat{k}_{20})^{(3)}\right)d\left(((G_{23})^{(1)},(T_{23})^{(1)};\ (G_{23})^{(2)},(T_{23})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 11:** The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ *and* $(\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i=20,21,22$ depend only on $T_{21}$ and respectively on

$(G_{23})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 12:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 216

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(3)} - (a_i'')^{(3)}(T_{21}(s_{(20)}), s_{(20)})\} ds_{(20)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(3)} t)} > 0$ for t > 0

**Definition of** $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2\ and\ \left((\widehat{M}_{20})^{(3)}\right)_3$ : 217

**Remark 13:** if $G_{20}$ is bounded, the same property have also $G_{21}\ and\ G_{22}$ . indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a_{21}')^{(3)} G_{21}$ and by integrating

$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1 / (a_{21}')^{(3)}$

In the same way , one can obtain

$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2 / (a_{22}')^{(3)}$

If $G_{21}\ or\ G_{22}$ is bounded, the same property follows for $G_{20}$ , $G_{22}$ and $G_{20}$ , $G_{21}$ respectively.

**Remark 14:** If $G_{20}$ *is* bounded, from below, the same property holds for $G_{21}\ and\ G_{22}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{21}$ is bounded from below. 218

**Remark 15:** If $T_{20}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t), t\right)) = (b_{21}')^{(3)}$ then $T_{21} \to \infty$. 219

**Definition of** $(m)^{(3)}$ and $\varepsilon_3$ :

Indeed let $t_3$ be so that for $t > t_3$

$(b_{21})^{(3)} - (b_i'')^{(3)}\left((G_{23})(t), t\right) < \varepsilon_3, T_{20}\,(t) > (m)^{(3)}$

Then $\frac{dT_{21}}{dt} \geq (a_{21})^{(3)}(m)^{(3)} - \varepsilon_3 T_{21}$ which leads to 220

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t} = \frac{1}{2}$ it results

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_3}$ By taking now $\varepsilon_3$ sufficiently small one sees that $T_{21}$ is unbounded.
The same property holds for $T_{22}$ if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t), t\right) = (b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}$ , $\frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}} < 1$ and to choose 221

$(\widehat{P}_{24})^{(4)}$ and $(\widehat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[(\widehat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \tag{222}$$

$$\frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \tag{223}$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself (224)

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric (225)

$$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{24})^{(4)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{27}}),(\widetilde{T_{27}})$ : $\left((\widetilde{G_{27}}),(\widetilde{T_{27}})\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{24})^{(4)}\left|G_{25}^{(1)}-G_{25}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\,ds_{(24)}+$$

$$\int_0^t\{(a'_{24})^{(4)}\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$G_{24}^{(2)}\left|(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)-(a''_{24})^{(4)}\left(T_{25}^{(2)},s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\}ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}t}\leq \tag{226}$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)}+(a'_{24})^{(4)}+(\widehat{A}_{24})^{(4)}+(\widehat{P}_{24})^{(4)}(\hat{k}_{24})^{(4)}\right)d\left(((G_{27})^{(1)},(T_{27})^{(1)};\ (G_{27})^{(2)},(T_{27})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 16:** The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ *and* $(\widehat{Q}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_+$. (227)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i=24,25,26$ depend only on $T_{25}$ and respectively on

$(G_{27})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 17:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 228

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(4)} - (a_i'')^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(4)} t)} > 0$ for t > 0

**Definition of** $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2\ and\ \left((\widehat{M}_{24})^{(4)}\right)_3$ : 229

**Remark 18:** if $G_{24}$ is bounded, the same property have also $G_{25}\ and\ G_{26}$ . indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a_{25}')^{(4)} G_{25}$ and by integrating

$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1 / (a_{25}')^{(4)}$

In the same way , one can obtain

$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2 / (a_{26}')^{(4)}$

If $G_{25}\ or\ G_{26}$ is bounded, the same property follows for $G_{24}$ , $G_{26}$ and $G_{24}$ , $G_{25}$ respectively.

**Remark 19:** If $G_{24}$ *is* bounded, from below, the same property holds for $G_{25}\ and\ G_{26}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{25}$ is bounded from below. 230

**Remark 20:** If $T_{24}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}\ ((G_{27})(t), t)) = (b_{25}')^{(4)}$ then $T_{25} \to \infty$. 231

**Definition of** $(m)^{(4)}$ and $\varepsilon_4$ :

Indeed let $t_4$ be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t), t) < \varepsilon_4, T_{24}\,(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_4}$ By taking now $\varepsilon_4$ sufficiently small one sees that $T_{25}$ is unbounded.
The same property holds for $T_{26}$ if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t), t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for $G_{29}$ , $G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}$ , $\frac{(b_i)^{(5)}}{(\widehat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)}+\left((\hat{P}_{28})^{(5)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{28})^{(5)}\right]\leq(\hat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$$d\left(((G_{31})^{(1)},(T_{31})^{(1)}),((G_{31})^{(2)},(T_{31})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\hat{M}_{28})^{(5)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\hat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{31}}),(\widetilde{T_{31}})$ : $\left((\widetilde{G_{31}}),(\widetilde{T_{31}})\right)=\mathcal{A}^{(5)}\left((G_{31}),(T_{31})\right)$

It results

$$|\tilde{G}_{28}^{(1)}-\tilde{G}_i^{(2)}|\leq\int_0^t(a_{28})^{(5)}|G_{29}^{(1)}-G_{29}^{(2)}|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\,ds_{(28)}+$$

$$\int_0^t\{(a'_{28})^{(5)}|G_{28}^{(1)}-G_{28}^{(2)}|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$(a''_{28})^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)|G_{28}^{(1)}-G_{28}^{(2)}|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$G_{28}^{(2)}|(a''_{28})^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)-(a''_{28})^{(5)}\left(T_{29}^{(2)},s_{(28)}\right)|\;e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\}ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on it follows

$$|(G_{31})^{(1)}-(G_{31})^{(2)}|e^{-(\widehat{M}_{28})^{(5)}t}\leq \quad 237$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)}+(a'_{28})^{(5)}+(\widehat{A}_{28})^{(5)}+(\widehat{P}_{28})^{(5)}(\widehat{k}_{28})^{(5)}\right)d\left(((G_{31})^{(1)},(T_{31})^{(1)};\;(G_{31})^{(2)},(T_{31})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 21:** The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\hat{P}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ *and* $(\hat{Q}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ respectively of $\mathbb{R}_+$. 238

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on $T_{29}$ and respectively on $(G_{31})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 22:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 239

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(5)} - (a_i'')^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(5)} t\right)} > 0$ for t > 0

**Definition of** $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2\ and\ \left((\widehat{M}_{28})^{(5)}\right)_3$ : 240

**Remark 23:** if $G_{28}$ is bounded, the same property have also $G_{29}\ and\ G_{30}$ . indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a_{29}')^{(5)} G_{29}$ and by integrating

$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1 / (a_{29}')^{(5)}$

In the same way , one can obtain

$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2 / (a_{30}')^{(5)}$

If $G_{29}\ or\ G_{30}$ is bounded, the same property follows for $G_{28}$ , $G_{30}$ and $G_{28}$ , $G_{29}$ respectively.

**Remark 24:** If $G_{28}$ *is* bounded, from below, the same property holds for $G_{29}\ and\ G_{30}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{29}$ is bounded from below. 241

**Remark 25:** If $T_{28}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}\,((G_{31})(t), t)) = (b_{29}')^{(5)}$ then $T_{29} \to \infty$. 242

**Definition of** $(m)^{(5)}$ and $\varepsilon_5$ :

Indeed let $t_5$ be so that for $t > t_5$

$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t), t) < \varepsilon_5, T_{28}\,(t) > (m)^{(5)}$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_5}$ By taking now $\varepsilon_5$ sufficiently small one sees that $T_{29}$ is unbounded.
The same property holds for $T_{30}$ if $\lim_{t\to\infty}(b_{30}'')^{(5)}\left((G_{31})(t), t\right) = (b_{30}')^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{33}\ , G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\ , \frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}} < 1$ and to choose 244

$(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[(\hat{P}_{32})^{(6)}+\left((\hat{P}_{32})^{(6)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{32})^{(6)} \tag{245}$$

$$\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[\left((\hat{Q}_{32})^{(6)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{32})^{(6)}\right]\leq(\hat{Q}_{32})^{(6)} \tag{246}$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric (247)

$$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\hat{M}_{32})^{(6)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\hat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{35}}),(\widetilde{T_{35}})$ : $\left((\widetilde{G_{35}}),(\widetilde{T_{35}})\right)=\mathcal{A}^{(6)}\left((G_{35}),(T_{35})\right)$

It results

$$|\tilde{G}_{32}^{(1)}-\tilde{G}_i^{(2)}|\leq\int_0^t(a_{32})^{(6)}|G_{33}^{(1)}-G_{33}^{(2)}|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\,ds_{(32)}+$$

$$\int_0^t\{(a'_{32})^{(6)}|G_{32}^{(1)}-G_{32}^{(2)}|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)|G_{32}^{(1)}-G_{32}^{(2)}|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$G_{32}^{(2)}|(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)-(a''_{32})^{(6)}\left(T_{33}^{(2)},s_{(32)}\right)|\;e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\}ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$|(G_{35})^{(1)}-(G_{35})^{(2)}|e^{-(\widehat{M}_{32})^{(6)}t}\leq \tag{248}$$
$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)}+(a'_{32})^{(6)}+(\widehat{A}_{32})^{(6)}+(\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}\right)d\left(((G_{35})^{(1)},(T_{35})^{(1)};\;(G_{35})^{(2)},(T_{35})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 26:** The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\hat{P}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ *and* $(\hat{Q}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ respectively of $\mathbb{R}_+$. (249)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i = 32,33,34$ depend only on $T_{33}$ and respectively on $(G_{35})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 27:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 250

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(6)} - (a_i'')^{(6)}(T_{33}(s_{(32)}), s_{(32)})\} ds_{(32)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(6)} t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{32})^{(6)}\right)_1, \left((\widehat{M}_{32})^{(6)}\right)_2\ and\ \left((\widehat{M}_{32})^{(6)}\right)_3$ : 251

**Remark 28:** if $G_{32}$ is bounded, the same property have also $G_{33}\ and\ G_{34}$ . indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a_{33}')^{(6)} G_{33}$ and by integrating

$$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)} \left((\widehat{M}_{32})^{(6)}\right)_1 / (a_{33}')^{(6)}$$

In the same way , one can obtain

$$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)} \left((\widehat{M}_{32})^{(6)}\right)_2 / (a_{34}')^{(6)}$$

If $G_{33}\ or\ G_{34}$ is bounded, the same property follows for $G_{32}$ , $G_{34}$ and $G_{32}$ , $G_{33}$ respectively.

**Remark 29:** If $G_{32}$ *is* bounded, from below, the same property holds for $G_{33}\ and\ G_{34}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{33}$ is bounded from below. 252

**Remark 30:** If $T_{32}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}\ ((G_{35})(t), t)) = (b_{33}')^{(6)}$ then $T_{33} \to \infty$. 253

**Definition of** $(m)^{(6)}$ and $\varepsilon_6$ :

Indeed let $t_6$ be so that for $t > t_6$

$$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t), t\right) < \varepsilon_6, T_{32}\,(t) > (m)^{(6)}$$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right)$, $t = log\,\frac{2}{\varepsilon_6}$ By taking now $\varepsilon_6$ sufficiently small one sees that $T_{33}$ is unbounded.
The same property holds for $T_{34}$ if $\lim_{t\to\infty}(b_{34}'')^{(6)}\left((G_{35})(t), t(t), t\right) = (b_{34}')^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{37}, G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}$ , $\frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$ and to choose

$(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[(\hat{P}_{36})^{(7)}+\left((\hat{P}_{36})^{(7)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{36})^{(7)} \tag{256}$$

$$\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[\left((\hat{Q}_{36})^{(7)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{36})^{(7)}\right]\leq(\hat{Q}_{36})^{(7)} \tag{257}$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself
The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric (258)

$$d\left(((G_{39})^{(1)},(T_{39})^{(1)}),((G_{39})^{(2)},(T_{39})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\tilde{M}_{36})^{(7)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\tilde{M}_{36})^{(7)}t}\}$$

Indeed if we denote

**Definition of** $\widetilde{(G_{39})},\widetilde{(T_{39})}:(\widetilde{(G_{39})},\widetilde{(T_{39})})=\mathcal{A}^{(7)}((G_{39}),(T_{39}))$
It results

$$|\tilde{G}_{36}^{(1)}-\tilde{G}_i^{(2)}|\leq\int_0^t(a_{36})^{(7)}|G_{37}^{(1)}-G_{37}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\,ds_{(36)}+$$
$$\int_0^t\{(a'_{36})^{(7)}|G_{36}^{(1)}-G_{36}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$(a''_{36})^{(7)}(T_{37}^{(1)},s_{(36)})|G_{36}^{(1)}-G_{36}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$G_{36}^{(2)}|(a''_{36})^{(7)}(T_{37}^{(1)},s_{(36)})-(a''_{36})^{(7)}(T_{37}^{(2)},s_{(36)})|\;e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\}ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on it follows

$$|(G_{39})^{(1)}-(G_{39})^{(2)}|e^{-(\widehat{M}_{36})^{(7)}t}\leq \tag{259}$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}}\left((a_{36})^{(7)}+(a'_{36})^{(7)}+(\hat{A}_{36})^{(7)}+(\hat{P}_{36})^{(7)}(\hat{k}_{36})^{(7)}\right)d\left(((G_{39})^{(1)},(T_{39})^{(1)};\;(G_{39})^{(2)},(T_{39})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 31:** The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\hat{P}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ *and* $(\hat{Q}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ respectively of $\mathbb{R}_+$. (260)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(7)}$ and $(b_i'')^{(7)}, i = 36,37,38$ depend only on $T_{37}$ and respectively on $(G_{39})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 32:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 261

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(7)} - (a_i'')^{(7)}(T_{37}(s_{(36)}), s_{(36)})\} ds_{(36)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(7)} t\right)} > 0$ for t > 0

**Definition of** $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2\ and\ \left((\widehat{M}_{36})^{(7)}\right)_3$ : 262

**Remark 33:** if $G_{36}$ is bounded, the same property have also $G_{37}\ and\ G_{38}$ . indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a_{37}')^{(7)} G_{37}$ and by integrating

$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1 / (a_{37}')^{(7)}$

In the same way , one can obtain

$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2 / (a_{38}')^{(7)}$

If $G_{37}\ or\ G_{38}$ is bounded, the same property follows for $G_{36}$ , $G_{38}$ and $G_{36}$ , $G_{37}$ respectively.

**Remark 34:** If $G_{36}$ *is* bounded, from below, the same property holds for $G_{37}\ and\ G_{38}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{37}$ is bounded from below. 263

**Remark 35:** If $T_{36}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(7)}\ ((G_{39})(t), t)) = (b_{37}')^{(7)}$ then $T_{37} \to \infty$. 264

**Definition of** $(m)^{(7)}$ and $\varepsilon_7$ :

Indeed let $t_7$ be so that for $t > t_7$

$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t), t) < \varepsilon_7, T_{36}\,(t) > (m)^{(7)}$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_7}$ By taking now $\varepsilon_7$ sufficiently small one sees that $T_{37}$ is unbounded.
The same property holds for $T_{38}$ if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t), t\right) = (b_{38}')^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}$ , $\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}} < 1$ and to choose $(\widehat{P}_{40})^{(8)}$ and $(\widehat{Q}_{40})^{(8)}$ large to have 266

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[(\widehat{P}_{40})^{(8)} + \left((\hat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{40})^{(8)} \quad 267$$

$$\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{40})^{(8)}\right] \leq (\hat{Q}_{40})^{(8)} \quad 268$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)}, (T_{43})^{(1)}), ((G_{43})^{(2)}, (T_{43})^{(2)})\right) = \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

**Definition of** $\widetilde{(G_{43})}, \widetilde{(T_{43})}$ : $\left(\widetilde{(G_{43})}, \widetilde{(T_{43})}\right) = \mathcal{A}^{(8)}((G_{43}), (T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{40})^{(8)}\left|G_{41}^{(1)} - G_{41}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\,ds_{(40)} +$$
$$\int_0^t\{(a'_{40})^{(8)}\left|G_{40}^{(1)} - G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{-(\widehat{M}_{40})^{(8)}s_{(40)}} +$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right)\left|G_{40}^{(1)} - G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}} +$$
$$G_{40}^{(2)}\left|(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right) - (a''_{40})^{(8)}\left(T_{41}^{(2)}, s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\}ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0, t]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)} - (G_{43})^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}t} \leq \quad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)} + (a'_{40})^{(8)} + (\widehat{A}_{40})^{(8)} + (\widehat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}\right)d\left(((G_{43})^{(1)}, (T_{43})^{(1)};\ (G_{43})^{(2)}, (T_{43})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

**Remark 36:** The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate 274

condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ *and* $(\widehat{Q}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i = 40,41,42$ depend only on $T_{41}$ and respectively on $(G_{43})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 37** There does not exist any $t$ where $G_i(t) = 0\ and\ T_i(t) = 0$ 275

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(8)} - (a_i'')^{(8)}(T_{41}(s_{(40)}), s_{(40)})\} ds_{(40)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(8)}t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{40})^{(8)}\right)_1, \left((\widehat{M}_{40})^{(8)}\right)_2\ and\ \left((\widehat{M}_{40})^{(8)}\right)_3$ : 276

**Remark 38:** if $G_{40}$ is bounded, the same property have also $G_{41}\ and\ G_{42}$ . indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a_{41}')^{(8)}G_{41}$ and by integrating

$$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1/(a_{41}')^{(8)}$$

In the same way , one can obtain

$$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2/(a_{42}')^{(8)}$$

If $G_{41}\ or\ G_{42}$ is bounded, the same property follows for $G_{40}$ , $G_{42}$ and $G_{40}$ , $G_{41}$ respectively.

**Remark 39:** If $G_{40}$ *is* bounded, from below, the same property holds for $G_{41}\ and\ G_{42}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{41}$ is bounded from below. 277

**Remark 40:** If $T_{40}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}((G_{43})(t), t)) = (b_{41}')^{(8)}$ then $T_{41} \to \infty$. 278

**Definition of** $(m)^{(8)}$ and $\varepsilon_8$ :

Indeed let $t_8$ be so that for $t > t_8$

$$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t), t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right)$, $t = log\frac{2}{\varepsilon_8}$ By taking now $\varepsilon_8$ sufficiently small one sees that $T_{41}$ is unbounded.
The same property holds for $T_{42}$ if $\lim_{t\to\infty}(b''_{42})^{(8)}\left((G_{43})(t), t(t), t\right) = (b'_{42})^{(8)}$



It is now sufficient to take $\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}$ , $\frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$ and to choose $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ large to have

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}\left[(\hat{P}_{44})^{(9)} + \left((\hat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}}\left[\left((\hat{Q}_{44})^{(9)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{44})^{(9)}\right] \leq (\hat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)}, (T_{47})^{(1)}), ((G_{47})^{(2)}, (T_{47})^{(2)})\right) =$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{44})^{(9)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{47}}), (\widetilde{T_{47}}) : \left((\widetilde{G_{47}}), (\widetilde{T_{47}})\right) = \mathcal{A}^{(9)}\left((G_{47}), (T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{44})^{(9)}\left|G_{45}^{(1)} - G_{45}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\,ds_{(44)} +$$
$$\int_0^t\{(a'_{44})^{(9)}\left|G_{44}^{(1)} - G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{-(\widehat{M}_{44})^{(9)}s_{(44)}} +$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right)\left|G_{44}^{(1)} - G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}} +$$
$$G_{44}^{(2)}\left|(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right) - (a''_{44})^{(9)}\left(T_{45}^{(2)}, s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\}ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)} - G^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}t} \leq$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)} + (a'_{44})^{(9)} + (\widehat{A}_{44})^{(9)} + (\widehat{P}_{44})^{(9)}(\hat{k}_{44})^{(9)}\right)d\left(((G_{47})^{(1)}, (T_{47})^{(1)};\ (G_{47})^{(2)}, (T_{47})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis (39,35,36) the result follows

**Remark 41:** The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by

$(\widehat{P}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ *and* $(\widehat{Q}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i = 44{,}45{,}46$ depend only on $T_{45}$ and respectively on $(G_{47})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 42:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$

From 99 to 44 it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t\{(a_i')^{(9)}-(a_i'')^{(9)}(T_{45}(s_{(44)}),s_{(44)})\}ds_{(44)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(9)}t)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{44})^{(9)}\right)_1, \left((\widehat{M}_{44})^{(9)}\right)_2\ and\ \left((\widehat{M}_{44})^{(9)}\right)_3$ :

**Remark 43:** if $G_{44}$ is bounded, the same property have also $G_{45}\ and\ G_{46}$ . indeed if

$G_{44} < (\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt} \leq \left((\widehat{M}_{44})^{(9)}\right)_1 - (a_{45}')^{(9)}G_{45}$ and by integrating

$$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_1/(a_{45}')^{(9)}$$

In the same way , one can obtain

$$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_2/(a_{46}')^{(9)}$$

If $G_{45}\ or\ G_{46}$ is bounded, the same property follows for $G_{44}$ , $G_{46}$ and $G_{44}$ , $G_{45}$ respectively.

**Remark 44:** If $G_{44}$ *is* bounded, from below, the same property holds for $G_{45}\ and\ G_{46}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{45}$ is bounded from below.

**Remark 45:** If $T_{44}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t),t\right) = (b_{45}')^{(9)}$ then $T_{45} \to \infty$.

**Definition of** $(m)^{(9)}$ and $\varepsilon_9$ :

Indeed let $t_9$ be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t),t\right) < \varepsilon_9, T_{44}\,(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right), \quad t = log\frac{2}{\varepsilon_9}$ By taking now $\varepsilon_9$ sufficiently small one sees that $T_{45}$ is unbounded.
The same property holds for $T_{46}$ if $\lim_{t\to\infty}(b_{46}'')^{(9)}\left((G_{47})(t),t\right) = (b_{46}')^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

**Behavior of the solutions of equation** 280

**Theorem** If we denote and define

**Definition of** $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ :

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)} \le -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14}, t) + (a''_{14})^{(1)}(T_{14}, t) \le -(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)} \le -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G, t) - (b''_{14})^{(1)}(G, t) \le -(\tau_1)^{(1)}$

**Definition of** $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$ : 281

By $(\nu_1)^{(1)} > 0$, $(\nu_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$, $(u_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

**Definition of** $(\bar{\nu}_1)^{(1)},, (\bar{\nu}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$ : 282

By $(\bar{\nu}_1)^{(1)} > 0$, $(\bar{\nu}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$, $(\bar{u}_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

**Definition of** $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (\nu_0)^{(1)}$ :- 283

If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by
$(m_2)^{(1)} = (\nu_0)^{(1)}, (m_1)^{(1)} = (\nu_1)^{(1)}$, $if\ (\nu_0)^{(1)} < (\nu_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}, if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)}$,

and $\boxed{(\nu_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)}$, $if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)}$, $if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)}$,

and $\boxed{(u_0)^{(1)} = \frac{T^0_{13}}{T^0_{14}}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$

are defined

Then the solution of global equations satisfies the inequalities 285

$G^0_{13}e^{\left((S_1)^{(1)} - (p_{13})^{(1)}\right)t} \le G_{13}(t) \le G^0_{13}e^{(S_1)^{(1)}t}$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)}-(p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)}t}$$

$$\left(\frac{(a_{15})^{(1)}G_{13}^0}{(m_1)^{(1)}((S_1)^{(1)}-(p_{13})^{(1)}-(S_2)^{(1)})}\left[e^{((S_1)^{(1)}-(p_{13})^{(1)})t} - e^{-(S_2)^{(1)}t}\right] + G_{15}^0 e^{-(S_2)^{(1)}t} \le G_{15}(t) \le \right. \quad (286)$$

$$\left. \frac{(a_{15})^{(1)}G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)}-(a'_{15})^{(1)})}[e^{(S_1)^{(1)}t} - e^{-(a'_{15})^{(1)}t}] + G_{15}^0 e^{-(a'_{15})^{(1)}t}\right)$$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)}t} \le T_{13}(t) \le T_{13}^0 e^{((R_1)^{(1)}+(r_{13})^{(1)})t}} \quad (287)$$

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)}t} \le T_{13}(t) \le \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)}+(r_{13})^{(1)})t} \quad (288)$$

$$\frac{(b_{15})^{(1)}T_{13}^0}{(\mu_1)^{(1)}((R_1)^{(1)}-(b'_{15})^{(1)})}\left[e^{(R_1)^{(1)}t} - e^{-(b'_{15})^{(1)}t}\right] + T_{15}^0 e^{-(b'_{15})^{(1)}t} \le T_{15}(t) \le \quad (289)$$

$$\frac{(a_{15})^{(1)}T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)}+(r_{13})^{(1)}+(R_2)^{(1)})}\left[e^{((R_1)^{(1)}+(r_{13})^{(1)})t} - e^{-(R_2)^{(1)}t}\right] + T_{15}^0 e^{-(R_2)^{(1)}t}$$

**Definition of** $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:- (290)

Where $(S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$

$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$

$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$

$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$

**Behavior of the solutions of equation** (291)

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ : (292)

$(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ four constants satisfying

$$-(\sigma_2)^{(2)} \le -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(T_{17}, t) + (a''_{17})^{(2)}(T_{17}, t) \le -(\sigma_1)^{(2)} \quad (293)$$

$$-(\tau_2)^{(2)} \le -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}((G_{19}), t) - (b''_{17})^{(2)}((G_{19}), t) \le -(\tau_1)^{(2)} \quad (294)$$

**Definition of** $(\nu_1)^{(2)}, (\nu_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$ : (295)

By $(\nu_1)^{(2)} > 0$, $(\nu_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0$, $(u_2)^{(2)} < 0$ the roots (296)

of the equations $(a_{17})^{(2)}(\nu^{(2)})^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ (297)

and $(b_{14})^{(2)}(u^{(2)})^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and (298)

**Definition of** $(\bar{\nu}_1)^{(2)}, , (\bar{\nu}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$ : (299)

By $(\bar{\nu}_1)^{(2)} > 0$, $(\bar{\nu}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0$, $(\bar{u}_2)^{(2)} < 0$ the (300)

roots of the equations $(a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 301

and $(b_{17})^{(2)}\left(u^{(2)}\right)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 302

**Definition of** $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ :- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, ***if*** $(u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$, ***if*** $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)}$, ***if*** $(\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)}t}$

$(p_i)^{(2)}$ is defined by equation

$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)}t}$ 311

$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)}t}\right] + G_{18}^0 e^{-(S_2)^{(2)}t} \le G_{18}(t) \le\right.$ 312

$\left.\frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)}t} - e^{-(a'_{18})^{(2)}t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)}t}\right)$

$\boxed{T_{16}^0 e^{(R_1)^{(2)}t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}}$ 313

$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)}t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}$ 314

$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)}t} - e^{-(b'_{18})^{(2)}t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)}t} \le T_{18}(t) \le$ 315

$$\frac{(a_{18})^{(2)}T_{16}^{0}}{(\mu_2)^{(2)}((R_1)^{(2)}+(r_{16})^{(2)}+(R_2)^{(2)})}\left[e^{((R_1)^{(2)}+(r_{16})^{(2)})t}-e^{-(R_2)^{(2)}t}\right]+T_{18}^{0}e^{-(R_2)^{(2)}t}$$

**Definition of** $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$$

$$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)}$$ 318

$$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$$

**Behavior of the solutions** 319

**Theorem 3:** If we denote and define

**Definition of** $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ :

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}\big((G_{23}), t\big) \le -(\tau_1)^{(3)}$$

**Definition of** $(v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$ : 320

By $(v_1)^{(3)} > 0$, $(v_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{v}_1)^{(3)} > 0$, $(\bar{v}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

**Definition of** $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ :- 321

If we define $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (v_0)^{(3)}, (m_1)^{(3)} = (v_1)^{(3)}$, ***if*** $(v_0)^{(3)} < (v_1)^{(3)}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (\bar{v}_1)^{(3)}$, ***if*** $(v_1)^{(3)} < (v_0)^{(3)} < (\bar{v}_1)^{(3)}$,

and $\boxed{(v_0)^{(3)} = \frac{G_{20}^{0}}{G_{21}^{0}}}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_0)^{(3)}$, ***if*** $(\bar{v}_1)^{(3)} < (v_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)},\ \boldsymbol{if}\ (u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}, \boldsymbol{if}\ (u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)},$ and $\boxed{(u_0)^{(3)} = \frac{T_{20}^0}{T_{21}^0}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)},\ \boldsymbol{if}\ (\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G_{20}^0 e^{(S_1)^{(3)} t}$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)} t} \quad 323$$

$$\left(\frac{(a_{22})^{(3)} G_{20}^0}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\left[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)} t}\right] + G_{22}^0 e^{-(S_2)^{(3)} t} \le G_{22}(t) \le \right. \quad 324$$

$$\left. \frac{(a_{22})^{(3)} G_{20}^0}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}\left[e^{(S_1)^{(3)} t} - e^{-(a'_{22})^{(3)} t}\right] + G_{22}^0 e^{-(a'_{22})^{(3)} t}\right)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t}} \quad 325$$

$$\frac{1}{(\mu_1)^{(3)}} T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}} T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)} T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)} - (b'_{22})^{(3)})}\left[e^{(R_1)^{(3)} t} - e^{-(b'_{22})^{(3)} t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)} t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)} T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)} + (r_{20})^{(3)} + (R_2)^{(3)})}\left[e^{((R_1)^{(3)} + (r_{20})^{(3)})t} - e^{-(R_2)^{(3)} t}\right] + T_{22}^0 e^{-(R_2)^{(3)} t}$$

**Definition of** $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

**Behavior of the solutions of equation**

**Theorem:** If we denote and define

**Definition of** $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ :

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$

$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}((G_{27}), t) - (b''_{25})^{(4)}((G_{27}), t) \le -(\tau_1)^{(4)}$

**Definition of** $(v_1)^{(4)}, (v_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, v^{(4)}, u^{(4)}$ : 329

By $(v_1)^{(4)} > 0$ , $(v_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$ , $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_1)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

**Definition of** $(\bar{v}_1)^{(4)},, (\bar{v}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$ : 330

By $(\bar{v}_1)^{(4)} > 0$ , $(\bar{v}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$ , $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

**Definition of** $(m_1)^{(4)}$ , $(m_2)^{(4)}$ , $(\mu_1)^{(4)}, (\mu_2)^{(4)}, (\nu_0)^{(4)}$ :-

If we define $(m_1)^{(4)}$ , $(m_2)^{(4)}$ , $(\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (v_0)^{(4)}, (m_1)^{(4)} = (v_1)^{(4)}$, ***if*** $(v_0)^{(4)} < (v_1)^{(4)}$

$(m_2)^{(4)} = (v_1)^{(4)}, (m_1)^{(4)} = (\bar{v}_1)^{(4)}$ , ***if*** $(v_4)^{(4)} < (v_0)^{(4)} < (\bar{v}_1)^{(4)}$,
and $\boxed{(v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (v_4)^{(4)}, (m_1)^{(4)} = (v_0)^{(4)}$, ***if*** $(\bar{v}_4)^{(4)} < (v_0)^{(4)}$

and analogously

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$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)}$, ***if*** $(u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}$ , ***if*** $(u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)}$,
and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}$, ***if*** $(\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

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$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)}t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)}t}$$

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$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)}t}\right] + G_{26}^0 e^{-(S_2)^{(4)}t} \le G_{26}(t) \le\right.$$ 334

*(a26)4G240(m2)4(S1)4−(a26′)4e(S1)4t−e−(a26′)4t+ G260e−(a26′)4t*

$$\boxed{T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}}T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}}T_{24}^0 e^{((R_1)^{(4)}+(r_{24})^{(4)})t} \quad 335$$

$$\frac{(b_{26})^{(4)}T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)}-(b'_{26})^{(4)})}\left[e^{(R_1)^{(4)}t} - e^{-(b'_{26})^{(4)}t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)}t} \le T_{26}(t) \le \quad 336$$

$$\frac{(a_{26})^{(4)}T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)}+(r_{24})^{(4)}+(R_2)^{(4)})}\left[e^{((R_1)^{(4)}+(r_{24})^{(4)})t} - e^{-(R_2)^{(4)}t}\right] + T_{26}^0 e^{-(R_2)^{(4)}t}$$

**Definition of** $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

**Behavior of the solutions of equation** 338

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ :

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

**Definition of** $(v_1)^{(5)}, (v_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, v^{(5)}, u^{(5)}$ : 339

By $(v_1)^{(5)} > 0$ , $(v_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$ , $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(v^{(5)}\big)^2 + (\sigma_1)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

**Definition of** $(\bar{v}_1)^{(5)}, , (\bar{v}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$ : 340

By $(\bar{v}_1)^{(5)} > 0$ , $(\bar{v}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$ , $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(v^{(5)}\big)^2 + (\sigma_2)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

**Definition of** $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (v_0)^{(5)}$ :-

If we define $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (v_0)^{(5)}, (m_1)^{(5)} = (v_1)^{(5)}$, ***if*** $(v_0)^{(5)} < (v_1)^{(5)}$

$(m_2)^{(5)} = (v_1)^{(5)}, (m_1)^{(5)} = (\bar{v}_1)^{(5)}$ , ***if*** $(v_1)^{(5)} < (v_0)^{(5)} < (\bar{v}_1)^{(5)}$,
and $\boxed{(v_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)},\ \boldsymbol{if}\ (u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}, \boldsymbol{if}\ (u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)},$

and $\boxed{(u_0)^{(5)} = \frac{T_{28}^0}{T_{29}^0}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}, \boldsymbol{if}\ (\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{28}(t) \le G_{28}^0 e^{(S_1)^{(5)} t}$

where $(p_i)^{(5)}$ is defined by equation

$\frac{1}{(m_5)^{(5)}} G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}} G_{28}^0 e^{(S_1)^{(5)} t}$ 343

$\left(\frac{(a_{30})^{(5)} G_{28}^0}{(m_1)^{(5)}((S_1)^{(5)} - (p_{28})^{(5)} - (S_2)^{(5)})}\left[e^{((S_1)^{(5)} - (p_{28})^{(5)})t} - e^{-(S_2)^{(5)} t}\right] + G_{30}^0 e^{-(S_2)^{(5)} t} \le G_{30}(t) \le\right.$ 344

*(a30)5G280(m2)5(S1)5−(a30′)5e(S1)5t−e−(a30′)5t+ G300e−(a30′)5t*

$\boxed{T_{28}^0 e^{(R_1)^{(5)} t} \le T_{28}(t) \le T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}}$ 345

$\frac{1}{(\mu_1)^{(5)}} T_{28}^0 e^{(R_1)^{(5)} t} \le T_{28}(t) \le \frac{1}{(\mu_2)^{(5)}} T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}$ 346

$\frac{(b_{30})^{(5)} T_{28}^0}{(\mu_1)^{(5)}((R_1)^{(5)} - (b'_{30})^{(5)})}\left[e^{(R_1)^{(5)} t} - e^{-(b'_{30})^{(5)} t}\right] + T_{30}^0 e^{-(b'_{30})^{(5)} t} \le T_{30}(t) \le$ 347

$\frac{(a_{30})^{(5)} T_{28}^0}{(\mu_2)^{(5)}((R_1)^{(5)} + (r_{28})^{(5)} + (R_2)^{(5)})}\left[e^{((R_1)^{(5)} + (r_{28})^{(5)})t} - e^{-(R_2)^{(5)} t}\right] + T_{30}^0 e^{-(R_2)^{(5)} t}$

**Definition of** $(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$:- 348

Where $(S_1)^{(5)} = (a_{28})^{(5)}(m_2)^{(5)} - (a'_{28})^{(5)}$

$(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}$

$(R_1)^{(5)} = (b_{28})^{(5)}(\mu_2)^{(5)} - (b'_{28})^{(5)}$

$(R_2)^{(5)} = (b'_{30})^{(5)} - (r_{30})^{(5)}$

**Behavior of the solutions of equation** 349

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ :

$(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)} \le -(a'_{32})^{(6)} + (a'_{33})^{(6)} - (a''_{32})^{(6)}(T_{33}, t) + (a''_{33})^{(6)}(T_{33}, t) \le -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)} \le -(b'_{32})^{(6)} + (b'_{33})^{(6)} - (b''_{32})^{(6)}\big((G_{35}), t\big) - (b''_{33})^{(6)}\big((G_{35}), t\big) \le -(\tau_1)^{(6)}$$

**Definition of** $(\nu_1)^{(6)}, (\nu_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, \nu^{(6)}, u^{(6)}$ : 350

By $(\nu_1)^{(6)} > 0$, $(\nu_2)^{(6)} < 0$ and respectively $(u_1)^{(6)} > 0$, $(u_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(6)},, (\bar{\nu}_2)^{(6)}, (\bar{u}_1)^{(6)}, (\bar{u}_2)^{(6)}$ : 351

By $(\bar{\nu}_1)^{(6)} > 0$, $(\bar{\nu}_2)^{(6)} < 0$ and respectively $(\bar{u}_1)^{(6)} > 0$, $(\bar{u}_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$

**Definition of** $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}, (\nu_0)^{(6)}$ :-

If we define $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}$ by

$$(m_2)^{(6)} = (\nu_0)^{(6)}, (m_1)^{(6)} = (\nu_1)^{(6)}, \ \boldsymbol{if}\ (\nu_0)^{(6)} < (\nu_1)^{(6)}$$

$$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}, \boldsymbol{if}\ (\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)},$$

and $\boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}}$

$$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$$

and analogously 352

$$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}, \ \boldsymbol{if}\ (u_0)^{(6)} < (u_1)^{(6)}$$

$$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}, \boldsymbol{if}\ (u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)},$$

and $\boxed{(u_0)^{(6)} = \frac{T^0_{32}}{T^0_{33}}}$

$$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}, \boldsymbol{if}\ (\bar{u}_1)^{(6)} < (u_0)^{(6)} \text{ where } (u_1)^{(6)}, (\bar{u}_1)^{(6)}$$

Then the solution of global equations satisfies the inequalities 353

$$G^0_{32}e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{32}(t) \le G^0_{32}e^{(S_1)^{(6)}t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}}G^0_{32}e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{33}(t) \le \frac{1}{(m_2)^{(6)}}G^0_{32}e^{(S_1)^{(6)}t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)}G^0_{32}}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})}\left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)}t}\right] + G^0_{34}e^{-(S_2)^{(6)}t} \le G_{34}(t) \le \right. \quad 355$$

$$\frac{(a_{34})^{(6)}G_{32}^{0}}{(m_2)^{(6)}((S_1)^{(6)}-(a'_{34})^{(6)})}\left[e^{(S_1)^{(6)}t}-e^{-(a'_{34})^{(6)}t}\right]+\ G_{34}^{0}e^{-(a'_{34})^{(6)}t}\Big)$$

$$\boxed{T_{32}^{0}e^{(R_1)^{(6)}t}\le T_{32}(t)\le T_{32}^{0}e^{((R_1)^{(6)}+(r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}}T_{32}^{0}e^{(R_1)^{(6)}t}\le T_{32}(t)\le\frac{1}{(\mu_2)^{(6)}}T_{32}^{0}e^{((R_1)^{(6)}+(r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)}T_{32}^{0}}{(\mu_1)^{(6)}((R_1)^{(6)}-(b'_{34})^{(6)})}\left[e^{(R_1)^{(6)}t}-e^{-(b'_{34})^{(6)}t}\right]+T_{34}^{0}e^{-(b'_{34})^{(6)}t}\le T_{34}(t)\le \quad 358$$

$$\frac{(a_{34})^{(6)}T_{32}^{0}}{(\mu_2)^{(6)}((R_1)^{(6)}+(r_{32})^{(6)}+(R_2)^{(6)})}\left[e^{((R_1)^{(6)}+(r_{32})^{(6)})t}-e^{-(R_2)^{(6)}t}\right]+T_{34}^{0}e^{-(R_2)^{(6)}t}$$

**Definition of** $(S_1)^{(6)},(S_2)^{(6)},(R_1)^{(6)},(R_2)^{(6)}$:- 359

Where $(S_1)^{(6)}=(a_{32})^{(6)}(m_2)^{(6)}-(a'_{32})^{(6)}$

$(S_2)^{(6)}=(a_{34})^{(6)}-(p_{34})^{(6)}$

$(R_1)^{(6)}=(b_{32})^{(6)}(\mu_2)^{(6)}-(b'_{32})^{(6)}$

$(R_2)^{(6)}=(b'_{34})^{(6)}-(r_{34})^{(6)}$

**Behavior of the solutions of equation**

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(7)},(\sigma_2)^{(7)},(\tau_1)^{(7)},(\tau_2)^{(7)}$:

$(\sigma_1)^{(7)},(\sigma_2)^{(7)},(\tau_1)^{(7)},(\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)}\le-(a'_{36})^{(7)}+(a'_{37})^{(7)}-(a''_{36})^{(7)}(T_{37},t)+(a''_{37})^{(7)}(T_{37},t)\le-(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)}\le-(b'_{36})^{(7)}+(b'_{37})^{(7)}-(b''_{36})^{(7)}\big((G_{39}),t\big)-(b''_{37})^{(7)}\big((G_{39}),t\big)\le-(\tau_1)^{(7)}$$

**Definition of** $(v_1)^{(7)},(v_2)^{(7)},(u_1)^{(7)},(u_2)^{(7)},v^{(7)},u^{(7)}$: 361

By $(v_1)^{(7)}>0,(v_2)^{(7)}<0$ and respectively $(u_1)^{(7)}>0,(u_2)^{(7)}<0$ the roots of the equations $(a_{37})^{(7)}\big(v^{(7)}\big)^2+(\sigma_1)^{(7)}v^{(7)}-(a_{36})^{(7)}=0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2+(\tau_1)^{(7)}u^{(7)}-(b_{36})^{(7)}=0$ and

**Definition of** $(\bar{v}_1)^{(7)},,(\bar{v}_2)^{(7)},(\bar{u}_1)^{(7)},(\bar{u}_2)^{(7)}$: 362

By $(\bar{v}_1)^{(7)}>0,(\bar{v}_2)^{(7)}<0$ and respectively $(\bar{u}_1)^{(7)}>0,(\bar{u}_2)^{(7)}<0$ the roots of the equations $(a_{37})^{(7)}\big(v^{(7)}\big)^2+(\sigma_2)^{(7)}v^{(7)}-(a_{36})^{(7)}=0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2+(\tau_2)^{(7)}u^{(7)}-(b_{36})^{(7)}=0$

**Definition of** $(m_1)^{(7)},(m_2)^{(7)},(\mu_1)^{(7)},(\mu_2)^{(7)},(v_0)^{(7)}$:-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$$(m_2)^{(7)} = (\nu_0)^{(7)}, (m_1)^{(7)} = (\nu_1)^{(7)}, \ \boldsymbol{if}\ (\nu_0)^{(7)} < (\nu_1)^{(7)}$$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\bar{\nu}_1)^{(7)}, \boldsymbol{if}\ (\nu_1)^{(7)} < (\nu_0)^{(7)} < (\bar{\nu}_1)^{(7)},$$

and $\boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\nu_0)^{(7)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(7)} < (\nu_0)^{(7)}$$

and analogously 363

$$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}, \ \boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$$

and $\boxed{(u_0)^{(7)} = \frac{T_{36}^0}{T_{37}^0}}$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)} \text{ where } (u_1)^{(7)}, (\bar{u}_1)^{(7)}$$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^0 e^{(S_1)^{(7)} t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)} t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)}((S_1)^{(7)} - (p_{36})^{(7)} - (S_2)^{(7)})}\left[e^{((S_1)^{(7)} - (p_{36})^{(7)})t} - e^{-(S_2)^{(7)} t}\right] + G_{38}^0 e^{-(S_2)^{(7)} t} \le G_{38}(t) \le \right. \quad 366$$

$$\left. \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)}((S_1)^{(7)} - (a'_{38})^{(7)})}\left[e^{(S_1)^{(7)} t} - e^{-(a'_{38})^{(7)} t}\right] + G_{38}^0 e^{-(a'_{38})^{(7)} t}\right)$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t} \quad 368$$

$$\frac{(b_{38})^{(7)} T_{36}^0}{(\mu_1)^{(7)}((R_1)^{(7)} - (b'_{38})^{(7)})}\left[e^{(R_1)^{(7)} t} - e^{-(b'_{38})^{(7)} t}\right] + T_{38}^0 e^{-(b'_{38})^{(7)} t} \le T_{38}(t) \le \quad 369$$

$$\frac{(a_{38})^{(7)} T_{36}^0}{(\mu_2)^{(7)}((R_1)^{(7)} + (r_{36})^{(7)} + (R_2)^{(7)})}\left[e^{((R_1)^{(7)} + (r_{36})^{(7)})t} - e^{-(R_2)^{(7)} t}\right] + T_{38}^0 e^{-(R_2)^{(7)} t}$$

**Definition of** $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$

$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$

$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$

**Behavior of the solutions of equation** 371

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ :

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$-(\sigma_2)^{(8)} \le -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \le -(\sigma_1)^{(8)}$

$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\big((G_{43}), t\big) - (b''_{41})^{(8)}\big((G_{43}), t\big) \le -(\tau_1)^{(8)}$

**Definition of** $(\nu_1)^{(8)}, (\nu_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, \nu^{(8)}, u^{(8)}$ : 372

By $(\nu_1)^{(8)} > 0, (\nu_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0, (u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(8)}, , (\bar{\nu}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$ :

By $(\bar{\nu}_1)^{(8)} > 0, (\bar{\nu}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0, (\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

**Definition of** $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}, (\nu_0)^{(8)}$ :-

If we define $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$(m_2)^{(8)} = (\nu_0)^{(8)}, (m_1)^{(8)} = (\nu_1)^{(8)}$, ***if*** $(\nu_0)^{(8)} < (\nu_1)^{(8)}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\bar{\nu}_1)^{(8)}$, ***if*** $(\nu_1)^{(8)} < (\nu_0)^{(8)} < (\bar{\nu}_1)^{(8)}$,
and $\boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}}$

$$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\nu_0)^{(8)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(8)} < (\nu_0)^{(8)}$$

and analogously 374

$$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)}, \ \boldsymbol{if}\ (u_0)^{(8)} < (u_1)^{(8)}$$

$$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}, \boldsymbol{if}\ (u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)},$$

and $\boxed{(u_0)^{(8)} = \frac{T_{40}^0}{T_{41}^0}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}, \boldsymbol{if}\ (\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^0 e^{(S_1)^{(8)} t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}} G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}} G_{40}^0 e^{(S_1)^{(8)} t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)} G_{40}^0}{(m_1)^{(8)}((S_1)^{(8)} - (p_{40})^{(8)} - (S_2)^{(8)})}\left[e^{((S_1)^{(8)} - (p_{40})^{(8)})t} - e^{-(S_2)^{(8)} t}\right] + G_{42}^0 e^{-(S_2)^{(8)} t} \le G_{42}(t) \le \right. \quad 377$$

$$\left. \frac{(a_{42})^{(8)} G_{40}^0}{(m_2)^{(8)}((S_1)^{(8)} - (a'_{42})^{(8)})}\left[e^{(S_1)^{(8)} t} - e^{-(a'_{42})^{(8)} t}\right] + G_{42}^0 e^{-(a'_{42})^{(8)} t}\right)$$

$$\boxed{T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}} T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le \frac{1}{(\mu_2)^{(8)}} T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)} T_{40}^0}{(\mu_1)^{(8)}((R_1)^{(8)} - (b'_{42})^{(8)})}\left[e^{(R_1)^{(8)} t} - e^{-(b'_{42})^{(8)} t}\right] + T_{42}^0 e^{-(b'_{42})^{(8)} t} \le T_{42}(t) \le \quad 380$$

$$\frac{(a_{42})^{(8)} T_{40}^0}{(\mu_2)^{(8)}((R_1)^{(8)} + (r_{40})^{(8)} + (R_2)^{(8)})}\left[e^{((R_1)^{(8)} + (r_{40})^{(8)})t} - e^{-(R_2)^{(8)} t}\right] + T_{42}^0 e^{-(R_2)^{(8)} t}$$

**Definition of** $(S_1)^{(8)}, (S_2)^{(8)}, (R_1)^{(8)}, (R_2)^{(8)}$:- 381

Where $(S_1)^{(8)} = (a_{40})^{(8)}(m_2)^{(8)} - (a'_{40})^{(8)}$

$$(S_2)^{(8)} = (a_{42})^{(8)} - (p_{42})^{(8)}$$

$$(R_1)^{(8)} = (b_{40})^{(8)}(\mu_2)^{(8)} - (b'_{40})^{(8)}$$

$$(R_2)^{(8)} = (b'_{42})^{(8)} - (r_{42})^{(8)}$$

**Behavior of the solutions of equation 37 to 92** 382

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ :

$(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ four constants satisfying

$$-(\sigma_2)^{(9)} \le -(a'_{44})^{(9)} + (a'_{45})^{(9)} - (a''_{44})^{(9)}(T_{45}, t) + (a''_{45})^{(9)}(T_{45}, t) \le -(\sigma_1)^{(9)}$$

$$-(\tau_2)^{(9)} \le -(b'_{44})^{(9)} + (b'_{45})^{(9)} - (b''_{44})^{(9)}\big((G_{47}), t\big) - (b''_{45})^{(9)}\big((G_{47}), t\big) \le -(\tau_1)^{(9)}$$

**Definition of** $(\nu_1)^{(9)}, (\nu_2)^{(9)}, (u_1)^{(9)}, (u_2)^{(9)}, \nu^{(9)}, u^{(9)}$ :

By $(\nu_1)^{(9)} > 0$ , $(\nu_2)^{(9)} < 0$ and respectively $(u_1)^{(9)} > 0$ , $(u_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_1)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(9)}, , (\bar{\nu}_2)^{(9)}, (\bar{u}_1)^{(9)}, (\bar{u}_2)^{(9)}$ :

By $(\bar{\nu}_1)^{(9)} > 0$ , $(\bar{\nu}_2)^{(9)} < 0$ and respectively $(\bar{u}_1)^{(9)} > 0$ , $(\bar{u}_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_2)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$

**Definition of** $(m_1)^{(9)}, (m_2)^{(9)}, (\mu_1)^{(9)}, (\mu_2)^{(9)}, (\nu_0)^{(9)}$ :-

If we define $(m_1)^{(9)}, (m_2)^{(9)}, (\mu_1)^{(9)}, (\mu_2)^{(9)}$ by

$$(m_2)^{(9)} = (\nu_0)^{(9)}, (m_1)^{(9)} = (\nu_1)^{(9)}, \ \boldsymbol{if}\ (\nu_0)^{(9)} < (\nu_1)^{(9)}$$

$$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\bar{\nu}_1)^{(9)}, \boldsymbol{if}\ (\nu_1)^{(9)} < (\nu_0)^{(9)} < (\bar{\nu}_1)^{(9)},$$

and $\boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$

$$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\nu_0)^{(9)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(9)} < (\nu_0)^{(9)}$$

and analogously

$$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)}, \ \boldsymbol{if}\ (u_0)^{(9)} < (u_1)^{(9)}$$

$$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}, \boldsymbol{if}\ (u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)},$$

and $\boxed{(u_0)^{(9)} = \frac{T^0_{44}}{T^0_{45}}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}, \boldsymbol{if}\ (\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G^0_{44} e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{44}(t) \le G^0_{44} e^{(S_1)^{(9)} t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}}G_{44}^0 e^{((S_1)^{(9)}-(p_{44})^{(9)})t} \le G_{45}(t) \le \frac{1}{(m_2)^{(9)}}G_{44}^0 e^{(S_1)^{(9)}t}$$

$$\left(\frac{(a_{46})^{(9)}G_{44}^0}{(m_1)^{(9)}((S_1)^{(9)}-(p_{44})^{(9)}-(S_2)^{(9)})}\left[e^{((S_1)^{(9)}-(p_{44})^{(9)})t} - e^{-(S_2)^{(9)}t}\right] + G_{46}^0 e^{-(S_2)^{(9)}t} \le G_{46}(t) \le \right.$$

$$\left.\frac{(a_{46})^{(9)}G_{44}^0}{(m_2)^{(9)}((S_1)^{(9)}-(a'_{46})^{(9)})}[e^{(S_1)^{(9)}t} - e^{-(a'_{46})^{(9)}t}] + G_{46}^0 e^{-(a'_{46})^{(9)}t}\right)$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le T_{44}^0 e^{((R_1)^{(9)}+(r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}}T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le \frac{1}{(\mu_2)^{(9)}}T_{44}^0 e^{((R_1)^{(9)}+(r_{44})^{(9)})t}$$

$$\frac{(b_{46})^{(9)}T_{44}^0}{(\mu_1)^{(9)}((R_1)^{(9)}-(b'_{46})^{(9)})}\left[e^{(R_1)^{(9)}t} - e^{-(b'_{46})^{(9)}t}\right] + T_{46}^0 e^{-(b'_{46})^{(9)}t} \le T_{46}(t) \le$$

$$\frac{(a_{46})^{(9)}T_{44}^0}{(\mu_2)^{(9)}((R_1)^{(9)}+(r_{44})^{(9)}+(R_2)^{(9)})}\left[e^{((R_1)^{(9)}+(r_{44})^{(9)})t} - e^{-(R_2)^{(9)}t}\right] + T_{46}^0 e^{-(R_2)^{(9)}t}$$

**Definition of** $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$

**Proof :** From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14},t)\right) - (a''_{14})^{(1)}(T_{14},t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

**Definition of** $v^{(1)}$ :- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)}\right) \le \frac{dv^{(1)}}{dt} \le -\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)}\right)$$

From which one obtains

**Definition of** $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$ :-

For $0 < \boxed{(v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}} < (v_1)^{(1)} < (\bar{v}_1)^{(1)}$

$$v^{(1)}(t) \ge \frac{(v_1)^{(1)}+(C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(v_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(v_2)^{(1)}}}$$

it follows $(\nu_0)^{(1)} \leq \nu^{(1)}(t) \leq (\nu_1)^{(1)}$

In the same manner , we get 384

$$\nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{\nu}_1)^{(1)}-(\nu_0)^{(1)}}{(\nu_0)^{(1)}-(\bar{\nu}_2)^{(1)}}}$$

From which we deduce $(\nu_0)^{(1)} \leq \nu^{(1)}(t) \leq (\bar{\nu}_1)^{(1)}$

If $0 < (\nu_1)^{(1)} < (\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0} < (\bar{\nu}_1)^{(1)}$ we find like in the previous case, 385

$$(\nu_1)^{(1)} \leq \frac{(\nu_1)^{(1)}+(C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}} \leq \nu^{(1)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\bar{\nu}_1)^{(1)}$$

If $0 < (\nu_1)^{(1)} \leq (\bar{\nu}_1)^{(1)} \leq \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$ , we obtain 386

$$(\nu_1)^{(1)} \leq \nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\nu_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(1)}(t)$ :-

$$(m_2)^{(1)} \leq \nu^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{\nu^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(1)}(t)$ :-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{13})^{(1)} = (a''_{14})^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)}G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)}T_{14}(t)$ This is an important

consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

**Proof** : From global equations we obtain 387

$$\frac{d\nu^{(2)}}{dt} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right) - (a''_{17})^{(2)}(T_{17}, t)\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

**Definition of** $\nu^{(2)}$ :- $\boxed{\nu^{(2)} = \frac{G_{16}}{G_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \le \frac{d\nu^{(2)}}{dt} \le -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

**Definition of** $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$ :-

For $0 < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \ge \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}}{1 + (C)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(C)^{(2)} = \frac{(\nu_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \le \nu^{(2)}(t) \le (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \le \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \le \nu^{(2)}(t) \le (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \le \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}}{1 + (C)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}} \le \nu^{(2)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \le (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \le (\bar{\nu}_1)^{(2)} \le (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$ , we obtain 394

$$(\nu_1)^{(2)} \le \nu^{(2)}(t) \le \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \le (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(2)}(t)$ :- 395

$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}$, $\boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$

In a completely analogous way, we obtain 396

**Definition of** $u^{(2)}(t)$ :-

$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}$, $\boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :** 397

If $(a_{16}'')^{(2)} = (a_{17}'')^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)} G_{17}(t)$

Analogously if $(b_{16}'')^{(2)} = (b_{17}'')^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)} T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

**Proof** : From global equations we obtain 398

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a_{20}')^{(3)} - (a_{21}')^{(3)} + (a_{20}'')^{(3)}(T_{21}, t)\right) - (a_{21}'')^{(3)}(T_{21}, t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

**Definition of** $\nu^{(3)}$ :- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right) \leq \frac{d\nu^{(3)}}{dt} \leq -\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right)$$

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From which one obtains

For $0 < (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (\nu_1)^{(3)} < (\bar{\nu}_1)^{(3)}$

$$\nu^{(3)}(t) \geq \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(\nu_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\nu_2)^{(3)}}}$$

it follows $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\nu_1)^{(3)}$

In the same manner , we get 401

$$\nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{\nu}_1)^{(3)}-(\nu_0)^{(3)}}{(\nu_0)^{(3)}-(\bar{\nu}_2)^{(3)}}}$$

**Definition of** $(\bar{\nu}_1)^{(3)}$ :-

From which we deduce $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\bar{\nu}_1)^{(3)}$

If $0 < (\nu_1)^{(3)} < (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (\bar{\nu}_1)^{(3)}$ we find like in the previous case, 402

$$(\nu_1)^{(3)} \leq \frac{(\nu_1)^{(3)}+(C)^{(3)}(\nu_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}} \leq \nu^{(3)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\bar{\nu}_1)^{(3)}$$

If $0 < (\nu_1)^{(3)} \leq (\bar{\nu}_1)^{(3)} \leq (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}$ , we obtain 403

$$(\nu_1)^{(3)} \leq \nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\nu_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(3)}(t)$ :-

$$(m_2)^{(3)} \leq \nu^{(3)}(t) \leq (m_1)^{(3)}, \quad \boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(3)}(t)$ :-

$$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}, \quad \boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{20}'')^{(3)} = (a_{21}'')^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)}G_{21}(t)$

Analogously if $(b_{20}'')^{(3)} = (b_{21}'')^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)}T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

**Proof** : From global equations we obtain 404

$$\frac{dv^{(4)}}{dt} = (a_{24})^{(4)} - \left((a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right) - (a''_{25})^{(4)}(T_{25},t)v^{(4)} - (a_{25})^{(4)}v^{(4)}$$

**Definition of** $v^{(4)}$ :- $\boxed{v^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)}\right) \le \frac{dv^{(4)}}{dt} \le -\left((a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_4)^{(4)}v^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$ :-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \ge \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \le \nu^{(4)}(t) \le (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \le \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \le \nu^{(4)}(t) \le (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \le \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}}{1 + (C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}} \le \nu^{(4)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \le (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \le (\bar{\nu}_1)^{(4)} \le \boxed{(\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}}}$ , we obtain 407

$$(\nu_1)^{(4)} \le \nu^{(4)}(t) \le \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \le (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(4)}(t)$ :-

$$(m_2)^{(4)} \le \nu^{(4)}(t) \le (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(4)}(t)$ :-

$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}$, $\boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{24})^{(4)} = (a''_{25})^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)} G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b''_{24})^{(4)} = (b''_{25})^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)} T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

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**Proof** : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a'_{28})^{(5)} - (a'_{29})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right) - (a''_{29})^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

**Definition of** $\nu^{(5)}$ :- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$ :-

For $0 < \boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \le \frac{(\nu_1)^{(5)}+(C)^{(5)}(\nu_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)}-(\nu_2)^{(5)}\right)t\right]}}{1+(C)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)}-(\nu_2)^{(5)}\right)t\right]}} \le \nu^{(5)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \le (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \le (\bar{\nu}_1)^{(5)} \le \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain 411

$$(\nu_1)^{(5)} \le \nu^{(5)}(t) \le \frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \le (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(5)}(t)$ :-

$(m_2)^{(5)} \le \nu^{(5)}(t) \le (m_1)^{(5)}$, $\boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$

In a completely analogous way, we obtain

**Definition of** $u^{(5)}(t)$ :-

$(\mu_2)^{(5)} \le u^{(5)}(t) \le (\mu_1)^{(5)}$, $\boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a_{28}'')^{(5)} = (a_{29}'')^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)}G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b_{28}'')^{(5)} = (b_{29}'')^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)}T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

**Proof** : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a_{32}')^{(6)} - (a_{33}')^{(6)} + (a_{32}'')^{(6)}(T_{33},t)\right) - (a_{33}'')^{(6)}(T_{33},t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

**Definition of** $\nu^{(6)}$ :- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \le \frac{d\nu^{(6)}}{dt} \le -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$ :-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \geq \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\nu_1)^{(6)}$

In the same manner , we get 413

$$\nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{\nu}_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\bar{\nu}_1)^{(6)}$

If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 414

$$(\nu_1)^{(6)} \leq \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}} \leq \nu^{(6)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\bar{\nu}_1)^{(6)}$$

If $0 < (\nu_1)^{(6)} \leq (\bar{\nu}_1)^{(6)} \leq \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$ , we obtain 415

$$(\nu_1)^{(6)} \leq \nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(6)}(t)$ :-

$$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}, \quad \boxed{\nu^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(6)}(t)$ :-

$$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}, \quad \boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)} G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)} T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

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**Proof** : From global equations we obtain

$$\frac{dv^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right) - (a''_{37})^{(7)}(T_{37},t)v^{(7)} - (a_{37})^{(7)}v^{(7)}$$

**Definition of** $v^{(7)}$ :- $\boxed{v^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_2)^{(7)}v^{(7)} - (a_{36})^{(7)}\right) \le \frac{dv^{(7)}}{dt} \le -\left((a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_1)^{(7)}v^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

**Definition of** $(\bar{v}_1)^{(7)}, (v_0)^{(7)}$ :-

For $0 < \boxed{(v_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}} < (v_1)^{(7)} < (\bar{v}_1)^{(7)}$

$$v^{(7)}(t) \ge \frac{(v_1)^{(7)} + (C)^{(7)}(v_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_0)^{(7)}\right)t\right]}}{1 + (C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(v_1)^{(7)} - (v_0)^{(7)}}{(v_0)^{(7)} - (v_2)^{(7)}}}$$

it follows $(v_0)^{(7)} \le v^{(7)}(t) \le (v_1)^{(7)}$

In the same manner , we get 417

$$v^{(7)}(t) \le \frac{(\bar{v}_1)^{(7)} + (\bar{C})^{(7)}(\bar{v}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{v}_1)^{(7)} - (v_0)^{(7)}}{(v_0)^{(7)} - (\bar{v}_2)^{(7)}}}$$

From which we deduce $(v_0)^{(7)} \le v^{(7)}(t) \le (\bar{v}_1)^{(7)}$

If $0 < (v_1)^{(7)} < (v_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}} < (\bar{v}_1)^{(7)}$ we find like in the previous case, 418

$$(v_1)^{(7)} \le \frac{(v_1)^{(7)} + (C)^{(7)}(v_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_2)^{(7)}\right)t\right]}}{1 + (C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_2)^{(7)}\right)t\right]}} \le v^{(7)}(t) \le$$

$$\frac{(\bar{v}_1)^{(7)} + (\bar{C})^{(7)}(\bar{v}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}} \le (\bar{v}_1)^{(7)}$$

If $0 < (v_1)^{(7)} \le (\bar{v}_1)^{(7)} \le \boxed{(v_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$ , we obtain 419

$$(v_1)^{(7)} \le v^{(7)}(t) \le \frac{(\bar{v}_1)^{(7)} + (\bar{C})^{(7)}(\bar{v}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}} \le (v_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $v^{(7)}(t)$ :-

$$(m_2)^{(7)} \leq v^{(7)}(t) \leq (m_1)^{(7)}, \quad \boxed{v^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(7)}(t)$ :- 420

$$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}, \quad \boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $v^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)} G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)} T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

**Proof** : From global equations we obtain 421

$$\frac{dv^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right) - (a''_{41})^{(8)}(T_{41}, t) v^{(8)} - (a_{41})^{(8)} v^{(8)}$$

**Definition of** $v^{(8)}$ :- $\boxed{v^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_2)^{(8)} v^{(8)} - (a_{40})^{(8)}\right) \leq \frac{dv^{(8)}}{dt} \leq -\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_1)^{(8)} v^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$ :-

$$\text{For } 0 < \boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$$

$$v^{(8)}(t) \geq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)}-(\nu_0)^{(8)}}{(\nu_0)^{(8)}-(\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \leq \frac{(\nu_1)^{(8)}+(C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}}{1+(C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}} \leq \nu^{(8)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \leq (\bar{\nu}_1)^{(8)} \leq \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$ , we obtain 424

$$(\nu_1)^{(8)} \leq \nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(8)}(t)$ :-

$$(m_2)^{(8)} \leq \nu^{(8)}(t) \leq (m_1)^{(8)}, \quad \boxed{\nu^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(8)}(t)$ :-

$$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}, \quad \boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $\nu^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)}G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then $(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)}T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

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**Proof** : From 99,20,44,22,23,44 we obtain A

$$\frac{dv^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right) - (a''_{45})^{(9)}(T_{45},t)v^{(9)} - (a_{45})^{(9)}v^{(9)}$$

**Definition of** $v^{(9)}$ :- $\boxed{v^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)}\right) \le \frac{dv^{(9)}}{dt} \le -\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_1)^{(9)}v^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$ :-

For $0 < \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$

$$v^{(9)}(t) \ge \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \le v^{(9)}(t) \le (\nu_9)^{(9)}$

In the same manner , we get

$$v^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \le v^{(9)}(t) \le (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \le \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}}{1 + (C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}} \le v^{(9)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \le (\bar{\nu}_1)^{(9)} \le \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$ , we obtain

$$(\nu_1)^{(9)} \le v^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $v^{(9)}(t)$ :-

$$(m_2)^{(9)} \le v^{(9)}(t) \le (m_1)^{(9)}, \quad \boxed{v^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(9)}(t)$ :-

$(\mu_2)^{(9)} \leq u^{(9)}(t) \leq (\mu_1)^{(9)}$, $\boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)} G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then
$(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)} T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following 425

**Theorem :** If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on $t$ , and the conditions with the notations

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0$ ,

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$

$with\ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a''_i)^{(2)} and\ (b''_i)^{(2)}$ are independent on t , and the conditions with the notations 426

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$ 427

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 428

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$ , 429

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 430

$with\ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a''_i)^{(3)} and\ (b''_i)^{(3)}$ are independent on $t$ , and the conditions with the notations 431

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$ ,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

**Theorem :** If $(a_i'')^{(4)} and\ (b_i'')^{(4)}$ are independent on $t$ , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$ ,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(5)} and\ (b_i'')^{(5)}$ are independent on $t$ , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$ ,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

**Theorem** If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on $t$ , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$ ,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(7)} and\ (b_i'')^{(7)}$ are independent on $t$ , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$ ,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(8)} and\ (b_i'')^{(8)}$ are independent on $t$ , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$ ,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(9)} and\ (b_i'')^{(9)}$ are independent on $t$ , and the conditions (with the notations 436 A
45,46,27,28)

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$ ,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0$ 437

$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0$ 438

$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0$ 439

$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0$ 440

$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ]T_{14} = 0$ 441

$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0$ 442

has a unique positive solution , which is an equilibrium solution for the system

$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0$ 443

$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0$ 444

$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0$ 445

$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\ ]T_{16} = 0$ 446

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\,]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\,]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\,]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\,]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\,]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\big((G_{27})\big)\,]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\big((G_{27})\big)\,]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\big((G_{27})\big)\,]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})\,]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})\,]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})\,]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})\,]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})\,]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})\,]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0 \quad 479$$

$$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0 \quad 480$$

$$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0 \quad 481$$

$$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0 \quad 482$$

$$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0 \quad 483$$

$$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0 \quad 484$$

$$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0 \quad 484\text{A}$$

$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$

$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$

$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$

$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$

$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$

**Proof:** 485

(a) Indeed the first two equations have a nontrivial solution $G_{13}, G_{14}$ if

$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$

**Proof:** 486

(p) Indeed the first two equations have a nontrivial solution $G_{16}, G_{17}$ if

$\mathrm{F}(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$

**Proof:** 487

(a) Indeed the first two equations have a nontrivial solution $G_{20}, G_{21}$ if

$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$

**Proof:** 488

(a) Indeed the first two equations have a nontrivial solution $G_{24}, G_{25}$ if

$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$

**Proof:** 489

(a) Indeed the first two equations have a nontrivial solution $G_{28}, G_{29}$ if

$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$

**Proof:** 490

(a) Indeed the first two equations have a nontrivial solution $G_{32}, G_{33}$ if

$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) +$

$(a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$

**Proof:** 491

(a) Indeed the first two equations have a nontrivial solution $G_{36}, G_{37}$ if

$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$

**Proof:** 492

(a) Indeed the first two equations have a nontrivial solution $G_{40}, G_{41}$ if

$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$

**Proof:** 492 A

(a) Indeed the first two equations have a nontrivial solution $G_{44}, G_{45}$ if

$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$

**Definition and uniqueness of** $T^*_{14}$ :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique $T^*_{14}$ for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

**Definition and uniqueness of** $T^*_{17}$ :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique $T^*_{17}$ for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]}$$ 495

**Definition and uniqueness of** $T^*_{21}$ :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique $T^*_{21}$ for which $f(T^*_{21}) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

**Definition and uniqueness of** $T^*_{25}$ :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that

there exists a unique $T_{25}^{*}$ for which $f(T_{25}^{*})=0$. With this value , we obtain from the three first equations

$$G_{24}=\frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}=\frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T_{25}^{*})]}$$

**Definition and uniqueness of** $T_{29}^{*}$ :- 498

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(5)}(T_{29})$ being increasing, it follows that there exists a unique $T_{29}^{*}$ for which $f(T_{29}^{*})=0$. With this value , we obtain from the three first equations

$$G_{28}=\frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}=\frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T_{29}^{*})]}$$

**Definition and uniqueness of** $T_{33}^{*}$ :- 499

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(6)}(T_{33})$ being increasing, it follows that there exists a unique $T_{33}^{*}$ for which $f(T_{33}^{*})=0$. With this value , we obtain from the three first equations

$$G_{32}=\frac{(a_{32})^{(6)}G_{33}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^{*})]} \quad , \quad G_{34}=\frac{(a_{34})^{(6)}G_{33}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^{*})]}$$

**Definition and uniqueness of** $T_{37}^{*}$ :- 500

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(7)}(T_{37})$ being increasing, it follows that there exists a unique $T_{37}^{*}$ for which $f(T_{37}^{*})=0$. With this value , we obtain from the three first equations

$$G_{36}=\frac{(a_{36})^{(7)}G_{37}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^{*})]} \quad , \quad G_{38}=\frac{(a_{38})^{(7)}G_{37}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^{*})]}$$

**Definition and uniqueness of** $T_{41}^{*}$ :- 501

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique $T_{41}^{*}$ for which $f(T_{41}^{*})=0$. With this value , we obtain from the three first equations

$$G_{40}=\frac{(a_{40})^{(8)}G_{41}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T_{41}^{*})]} \quad , \quad G_{42}=\frac{(a_{42})^{(8)}G_{41}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T_{41}^{*})]}$$

**Definition and uniqueness of** $T_{45}^{*}$ :- 501 A

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(9)}(T_{45})$ being increasing, it follows that there exists a unique $T_{45}^{*}$ for which $f(T_{45}^{*})=0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)}G_{45}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T^*_{45})]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)}G_{45}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T^*_{45})]}$$

By the same argument, the equations admit solutions $G_{13}, G_{14}$ if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right]+(b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{14}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{14}$ such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions $G_{16}, G_{17}$ if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right]+(b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ 504
is a decreasing function in $G_{17}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{14}$ such that $\varphi((G_{19})^*) = 0$

By the same argument, the equations admit solutions $G_{20}, G_{21}$ if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right]+(b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{21}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{21}$ such that $\varphi((G_{23})^*) = 0$

By the same argument, the equations admit solutions $G_{24}, G_{25}$ if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right]+(b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{25}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{25}$ such that $\varphi((G_{27})^*) = 0$

By the same argument, the equations admit solutions $G_{28}, G_{29}$ if 507

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right]+(b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{29}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that

there exists a unique $G_{29}^{*}$ such that $\varphi((G_{31})^{*}) = 0$

By the same argument, the equations admit solutions $G_{32}, G_{33}$ if 508
$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$

$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right] + (b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{33}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{33}^{*}$ such that $\varphi(G_{35}{}^{*}) = 0$

By the same argument, the equations admit solutions $G_{36}, G_{37}$ if 509
$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$
$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right] + (b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{37}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{37}^{*}$ such that $\varphi(G_{39}{}^{*}) = 0$

By the same argument, the equations admit solutions $G_{40}, G_{41}$ if 510
$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$
$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right] + (b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{41}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{41}^{*}$ such that $\varphi(G_{43}{}^{*}) = 0$

By the same argument, the equations 92,93 admit solutions $G_{44}, G_{45}$ if
$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$

$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right] + (b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{45}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{45}^{*}$ such that $\varphi((G_{47})^{*}) = 0$

Finally we obtain the unique solution 511

$G_{14}^{*}$ given by $\varphi(G^{*}) = 0$ , $T_{14}^{*}$ given by $f(T_{14}^{*}) = 0$ and

$$G_{13}^{*} = \frac{(a_{13})^{(1)}G_{14}^{*}}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^{*})]} \quad , \quad G_{15}^{*} = \frac{(a_{15})^{(1)}G_{14}^{*}}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^{*})]}$$

$$T_{13}^{*} = \frac{(b_{13})^{(1)}T_{14}^{*}}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^{*})]} \quad , \quad T_{15}^{*} = \frac{(b_{15})^{(1)}T_{14}^{*}}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^{*})]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

$G_{17}^{*}$ given by $\varphi((G_{19})^{*}) = 0$ , $T_{17}^{*}$ given by $f(T_{17}^{*}) = 0$ and 512

$$G_{16}^{*} = \frac{(a_{16})^{(2)} G_{17}^{*}}{[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}^{*})]} \quad , \quad G_{18}^{*} = \frac{(a_{18})^{(2)} G_{17}^{*}}{[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}^{*})]} \tag{513}$$

$$T_{16}^{*} = \frac{(b_{16})^{(2)} T_{17}^{*}}{[(b'_{16})^{(2)} - (b''_{16})^{(2)}((G_{19})^{*})]} \quad , \quad T_{18}^{*} = \frac{(b_{18})^{(2)} T_{17}^{*}}{[(b'_{18})^{(2)} - (b''_{18})^{(2)}((G_{19})^{*})]} \tag{514}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

$G_{21}^{*}$ given by $\varphi((G_{23})^{*}) = 0$ , $T_{21}^{*}$ given by $f(T_{21}^{*}) = 0$ and

$$G_{20}^{*} = \frac{(a_{20})^{(3)} G_{21}^{*}}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^{*})]} \quad , \quad G_{22}^{*} = \frac{(a_{22})^{(3)} G_{21}^{*}}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^{*})]}$$

$$T_{20}^{*} = \frac{(b_{20})^{(3)} T_{21}^{*}}{[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}{}^{*})]} \quad , \quad T_{22}^{*} = \frac{(b_{22})^{(3)} T_{21}^{*}}{[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}{}^{*})]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

$G_{25}^{*}$ given by $\varphi(G_{27}) = 0$ , $T_{25}^{*}$ given by $f(T_{25}^{*}) = 0$ and

$$G_{24}^{*} = \frac{(a_{24})^{(4)} G_{25}^{*}}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}^{*} = \frac{(a_{26})^{(4)} G_{25}^{*}}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^{*})]}$$

$$T_{24}^{*} = \frac{(b_{24})^{(4)} T_{25}^{*}}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^{*})]} \quad , \quad T_{26}^{*} = \frac{(b_{26})^{(4)} T_{25}^{*}}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^{*})]} \tag{517}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

$G_{29}^{*}$ given by $\varphi((G_{31})^{*}) = 0$ , $T_{29}^{*}$ given by $f(T_{29}^{*}) = 0$ and

$$G_{28}^{*} = \frac{(a_{28})^{(5)} G_{29}^{*}}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}^{*} = \frac{(a_{30})^{(5)} G_{29}^{*}}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^{*})]}$$

$$T_{28}^{*} = \frac{(b_{28})^{(5)} T_{29}^{*}}{[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31})^{*})]} \quad , \quad T_{30}^{*} = \frac{(b_{30})^{(5)} T_{29}^{*}}{[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31})^{*})]} \tag{519}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

$G_{33}^{*}$ given by $\varphi((G_{35})^{*}) = 0$ , $T_{33}^{*}$ given by $f(T_{33}^{*}) = 0$ and

$$G_{32}^{*} = \frac{(a_{32})^{(6)} G_{33}^{*}}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^{*})]} \quad , \quad G_{34}^{*} = \frac{(a_{34})^{(6)} G_{33}^{*}}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^{*})]}$$

$$T_{32}^{*} = \frac{(b_{32})^{(6)} T_{33}^{*}}{[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35})^{*})]} \quad , \quad T_{34}^{*} = \frac{(b_{34})^{(6)} T_{33}^{*}}{[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35})^{*})]} \tag{521}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

$G_{37}^*$ given by $\varphi((G_{39})^*) = 0$ , $T_{37}^*$ given by $f(T_{37}^*) = 0$ and

$$G_{36}^* = \frac{(a_{36})^{(7)} G_{37}^*}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^*)]} \quad , \quad G_{38}^* = \frac{(a_{38})^{(7)} G_{37}^*}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^*)]}$$

$$T_{36}^* = \frac{(b_{36})^{(7)} T_{37}^*}{[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39})^*)]} \quad , \quad T_{38}^* = \frac{(b_{38})^{(7)} T_{37}^*}{[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39})^*)]}$$

Finally we obtain the unique solution 523

$G_{41}^*$ given by $\varphi((G_{43})^*) = 0$ , $T_{41}^*$ given by $f(T_{41}^*) = 0$ and

$$G_{40}^* = \frac{(a_{40})^{(8)} G_{41}^*}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42}^* = \frac{(a_{42})^{(8)} G_{41}^*}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

$$T_{40}^* = \frac{(b_{40})^{(8)} T_{41}^*}{[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43})^*)]} \quad , \quad T_{42}^* = \frac{(b_{42})^{(8)} T_{41}^*}{[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43})^*)]}$$

Finally we obtain the unique solution of 89 to 99 523 A

$G_{45}^*$ given by $\varphi((G_{47})^*) = 0$ , $T_{45}^*$ given by $f(T_{45}^*) = 0$ and

$$G_{44}^* = \frac{(a_{44})^{(9)} G_{45}^*}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46}^* = \frac{(a_{46})^{(9)} G_{45}^*}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

$$T_{44}^* = \frac{(b_{44})^{(9)} T_{45}^*}{[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47})^*)]} \quad , \quad T_{46}^* = \frac{(b_{46})^{(9)} T_{45}^*}{[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47})^*)]}$$

## ASYMPTOTIC STABILITY ANALYSIS 524

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

**Proof:** Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a''_{14})^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \frac{\partial (b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a'_{13})^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)}G^*_{13}\mathbb{T}_{14} \quad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a'_{14})^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)}G^*_{14}\mathbb{T}_{14} \quad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a'_{15})^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)}G^*_{15}\mathbb{T}_{14} \quad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b'_{13})^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)}T^*_{13}\mathbb{G}_j\right) \quad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b'_{14})^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)}T^*_{14}\mathbb{G}_j\right) \quad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b'_{15})^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T^*_{15}\mathbb{G}_j\right) \quad 530$$

**ASYMPTOTIC STABILITY ANALYSIS** 531

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(2)}$ and $(b''_i)^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G^*_i + \mathbb{G}_i \quad , T_i = T^*_i + \mathbb{T}_i \quad 532$$

$$\frac{\partial(a''_{17})^{(2)}}{\partial T_{17}}(T^*_{17}) = (q_{17})^{(2)} \quad , \frac{\partial(b''_i)^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \quad 533$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a'_{16})^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G^*_{16}\mathbb{T}_{17} \quad 534$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a'_{17})^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G^*_{17}\mathbb{T}_{17} \quad 535$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a'_{18})^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G^*_{18}\mathbb{T}_{17} \quad 536$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b'_{16})^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T^*_{16}\mathbb{G}_j\right) \quad 537$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b'_{17})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}T^*_{17}\mathbb{G}_j\right) \quad 538$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b'_{18})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}T^*_{18}\mathbb{G}_j\right) \quad 539$$

**ASYMPTOTIC STABILITY ANALYSIS** 540

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(3)}$ *and* $(b''_i)^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{21}'')^{(3)}}{\partial T_{21}}(T_{21}^*) = (q_{21})^{(3)} \quad , \frac{\partial (b_i'')^{(3)}}{\partial G_j}(\,(G_{23})^*\,) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\big((a_{20}')^{(3)} + (p_{20})^{(3)}\big)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G_{20}^*\mathbb{T}_{21} \quad 541$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\big((a_{21}')^{(3)} + (p_{21})^{(3)}\big)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G_{21}^*\mathbb{T}_{21} \quad 542$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\big((a_{22}')^{(3)} + (p_{22})^{(3)}\big)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G_{22}^*\mathbb{T}_{21} \quad 543$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\big((b_{20}')^{(3)} - (r_{20})^{(3)}\big)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\big(s_{(20)(j)}T_{20}^*\mathbb{G}_j\big) \quad 544$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\big((b_{21}')^{(3)} - (r_{21})^{(3)}\big)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\big(s_{(21)(j)}T_{21}^*\mathbb{G}_j\big) \quad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\big((b_{22}')^{(3)} - (r_{22})^{(3)}\big)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\big(s_{(22)(j)}T_{22}^*\mathbb{G}_j\big) \quad 546$$

**ASYMPTOTIC STABILITY ANALYSIS** 547

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}\ and\ (b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{25}'')^{(4)}}{\partial T_{25}}(T_{25}^*) = (q_{25})^{(4)} \quad , \frac{\partial (b_i'')^{(4)}}{\partial G_j}((G_{27})^*\,) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\big((a_{24}')^{(4)} + (p_{24})^{(4)}\big)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G_{24}^*\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\big((a_{25}')^{(4)} + (p_{25})^{(4)}\big)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G_{25}^*\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\big((a_{26}')^{(4)} + (p_{26})^{(4)}\big)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G_{26}^*\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\big((b_{24}')^{(4)} - (r_{24})^{(4)}\big)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\big(s_{(24)(j)}T_{24}^*\mathbb{G}_j\big) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\big((b_{25}')^{(4)} - (r_{25})^{(4)}\big)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\big(s_{(25)(j)}T_{25}^*\mathbb{G}_j\big) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T^*_{26}\mathbb{G}_j\right) \quad 554$$

**ASYMPTOTIC STABILITY ANALYSIS** 555

**Theorem 5:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}\ and\ (b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 556

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{29})^{(5)}}{\partial T_{29}}(T^*_{29}) = (q_{29})^{(5)} \ , \ \frac{\partial(b_i'')^{(5)}}{\partial G_j}((G_{31})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a'_{28})^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G^*_{28}\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a'_{29})^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G^*_{29}\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a'_{30})^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G^*_{30}\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b'_{28})^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T^*_{28}\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b'_{29})^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T^*_{29}\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b'_{30})^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T^*_{30}\mathbb{G}_j\right) \quad 562$$

**ASYMPTOTIC STABILITY ANALYSIS** 563

**Theorem 6:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}\ and\ (b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{33})^{(6)}}{\partial T_{33}}(T^*_{33}) = (q_{33})^{(6)} \ , \ \frac{\partial(b_i'')^{(6)}}{\partial G_j}((G_{35})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a'_{32})^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G^*_{32}\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a'_{33})^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G^*_{33}\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a'_{34})^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G^*_{34}\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b'_{32})^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T^*_{32}\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b'_{33})^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T^*_{33}\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b'_{34})^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T^*_{34}\mathbb{G}_j\right) \quad 570$$

**ASYMPTOTIC STABILITY ANALYSIS** 571

**Theorem 7:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 572

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{37})^{(7)}}{\partial T_{37}}(T^*_{37}) = (q_{37})^{(7)} \quad , \frac{\partial(b_i'')^{(7)}}{\partial G_j}\left((G_{39})^{**}\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a'_{36})^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G^*_{36}\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a'_{37})^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G^*_{37}\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a'_{38})^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G^*_{38}\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b'_{36})^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T^*_{36}\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b'_{37})^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T^*_{37}\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b'_{38})^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T^*_{38}\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

**ASYMPTOTIC STABILITY ANALYSIS**

**Theorem 8:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{41}'')^{(8)}}{\partial T_{41}}(T_{41}^*) = (q_{41})^{(8)} \quad , \frac{\partial (b_i'')^{(8)}}{\partial G_j}\left( (G_{43})^* \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a_{40}')^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G_{40}^*\mathbb{T}_{41} \quad (581)$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a_{41}')^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G_{41}^*\mathbb{T}_{41} \quad (582)$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a_{42}')^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G_{42}^*\mathbb{T}_{41} \quad (583)$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b_{40}')^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T_{40}^*\mathbb{G}_j\right) \quad (584)$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b_{41}')^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T_{41}^*\mathbb{G}_j\right) \quad (585)$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b_{42}')^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T_{42}^*\mathbb{G}_j\right) \quad (586)$$

(586 A)

## ASYMPTOTIC STABILITY ANALYSIS

**Theorem 9:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}$ *and* $(b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

**Proof:** Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{45}'')^{(9)}}{\partial T_{45}}(T_{45}^*) = (q_{45})^{(9)} \quad , \frac{\partial (b_i'')^{(9)}}{\partial G_j}\left( (G_{47})^* \right) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a_{44}')^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)}G_{44}^*\mathbb{T}_{45} \quad (586\,B)$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a_{45}')^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)}G_{45}^*\mathbb{T}_{45} \quad (586\,C)$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a_{46}')^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)}G_{46}^*\mathbb{T}_{45} \quad (586\,D)$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b_{44}')^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)}T_{44}^*\mathbb{G}_j\right) \quad (586\,E)$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b_{45}')^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)}T_{45}^*\mathbb{G}_j\right) \quad (586\,F)$$

$\frac{d\mathbb{T}_{46}}{dt} = -\left((b'_{46})^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T^*_{46}\mathbb{G}_j\right)$ 586

G

**The characteristic equation of this system is** 587

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(q_{13})^{(1)}G^*_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)}T^*_{14} + (b_{14})^{(1)}s_{(13),(14)}T^*_{14}\right)$$

$$+\left(\left((\lambda)^{(1)} + (a'_{14})^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)}G^*_{13} + (a_{13})^{(1)}(q_{14})^{(1)}G^*_{14}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)}T^*_{14} + (b_{14})^{(1)}s_{(13),(13)}T^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((b'_{13})^{(1)} + (b'_{14})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(15)}T^*_{14} + (b_{14})^{(1)}s_{(13),(15)}T^*_{13}\right)\} = 0$$

$+$

$$(\lambda)^{(2)} + (b'_{18})^{(2)} - (r_{18})^{(2)}\{\left((\lambda)^{(2)} + (a'_{18})^{(2)} + (p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^*_{17} + (a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^*_{17}\right)$$

$$+\left(\left((\lambda)^{(2)} + (a'_{17})^{(2)} + (p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^*_{16} + (a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}\right)$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2 + \left((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2 + \left((b'_{16})^{(2)} + (b'_{17})^{(2)} - (r_{16})^{(2)} + (r_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$+\left(\left((\lambda)^{(2)}\right)^2 + \left((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17} + (a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^{*}_{16}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\right)\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left(\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)\right]$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(21)}T^{*}_{21}\right)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^{*}_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^{*}_{21}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(20)}T^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$+\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(22)}T^{*}_{20}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\right)\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left(\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)\right]$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(25)}T^{*}_{25}\right)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^{*}_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^{*}_{25}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(24)}T^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$+\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(26)}T^{*}_{24}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\right)\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(29)}T^{*}_{29}\right)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^{*}_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^{*}_{29}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(28)}T^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$+\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(30)}T^{*}_{28}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\right)\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left(\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)\right]$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(33)}T^{*}_{33}\right)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^{*}_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^{*}_{33}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(32)}T^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$+\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(34)}T^{*}_{32}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\right)\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)\right]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(37)}T^{*}_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^{*}_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^{*}_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(36)}T^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$+\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(38)}T^{*}_{36}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\right)\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(41)}T^{*}_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^{*}_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^{*}_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(40)}T^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$+\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(42)}T^{*}_{40}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\right)\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(45)}T^{*}_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^{*}_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^{*}_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(44)}T^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$+\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(46)}T^{*}_{44}\right)\}=0$$

**And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.**

## SECTION NINETEEN

## Quantum Chaos With Quantum Tomography

### INTRODUCTION—VARIABLES USED

**Quantum Chaos and Quantum Computing Structures Carlos Pedro Gonçalves (Submitted on 13 Aug 2012)**

(1) A system of quantum computing structures is introduced and proven capable of making emerge, on average, the orbits of classical bounded nonlinear maps on (e&eb) \mathbb{C} through (e&eb)the

iterative action of path-dependent quantum gates.

(2) The effects of emerging nonlinear dynamics and chaos upon the (e&eb) quantum averages of relevant observables and quantum probabilities are exemplified for a version of Chirikov's standard map on \mathbb{C} .

(3) Both the individual orbits and ensemble properties are addressed so that (e) the Poincar\'e map for (e) Chirikov's standard map, in the current quantum setting, is reinterpreted in terms of (e&eb) a quantum ensemble which is then formally introduced within (eb) the formalized system of quantum computing structures, in terms of (e&eb) quantum register machines, revealing (eb) three phases of quantum ensemble dynamics: the regular, the chaotic and an intermediate phase called **complex quantum stochastic phase** which shares similarities to (e&eb) the edge of chaos notion from (e) classical cellular automata and classical random boolean networks' evolutionary computation. Subjects: Chaotic Dynamics (nlin.CD); Statistical Mechanics (cond-mat.stat-mech); Quantum Physics (quant-ph) MSC classes: 81Q50, 81P68 Cite as: arXiv: 1208.2610 [nlin.CD] (or arXiv: 1208.2610v1 [nlin.CD] for this version)

**Quantum Computing of Quantum Chaos in the Kicked Rotator Model B. Levi, B. Georgeot, D.L. Shepelyansky (Univ. P.Sabatier/CNRS, Toulouse)**

**(4)** **Authors** investigate a quantum algorithm which simulates efficiently the quantum kicked rotator model, a system which displays rich physical properties, and enables to study problems of quantum chaos, atomic physics and localization of electrons in solids.

**(5)** The effects of errors in gate operations are tested on this algorithm in numerical simulations with up to 20 qubits. In this way various physical quantities are investigated.

**(6)** Some of them, such as second moment of probability distribution and tunneling transitions through invariant curves are shown to be particularly sensitive to errors.

**(7)** However, investigations of the fidelity and Wigner and Husimi distributions show that these physical quantities are robust in presence of imperfections.

**(8)** This implies that the algorithm can simulate the dynamics of quantum chaos in presence of a moderate amount of noise. Subjects: Quantum Physics (quant-ph); Condensed Matter (cond-mat); Chaotic Dynamics (nlin.CD) Journal reference:Phys. Rev. E 67, 046220 (2003) DOI:10.1103/PhysRevE.67.046220 Cite as: arXiv:quant-ph/0210154 (or arXiv:quant-ph/0210154v2 for this version)

**Characterizing and Quantifying Quantum Chaos with Quantum Tomography Vaibhav Madhok, Carlos A. Riofrío, Ivan H. Deutsch**

(1) Authors explore quantum signatures of (e) classical chaos by studying the rate of information gain in (eb) quantum tomography.

(2) The tomographic record consists of (e) a time series of expectation values of (e) a Hermitian operator evolving under (e&eb) application of the Floquet operator of a quantum map that possesses (or lacks) (e&eb) time reversal symmetry.

(3) They find that the rate of information gain, and hence the fidelity of quantum state reconstruction, depends on (=) the symmetry class of the quantum map involved.

**(4)** Moreover, authors find an increase in information gain and hence higher reconstruction fidelities when (e) the Floquet maps employed increase in (eb) chaoticity.

**(5)** Authors make predictions for (e) the information gain and show that (eb) these results are well described by (e) random matrix theory in (eb) the fully chaotic regime.

**(6)** Authors derive analytical expressions for (e) bounds on information gain using (e) random matrix theory for (e) different class of maps and show that these bounds are realized by (e) fully chaotic quantum systems. **Cite as: arXiv: 1506.02708 [quant-ph] (or arXiv: 1506.02708v1 [quant-**

**ph] for this version). ©American Physical Society**

**Quantum chaos in ultracold collisions between Yb(1S0) and Yb(3P2) Dermot G. Green, Christophe L. Vaillant, Matthew D. Frye, Masato Morita, Jeremy M. Hutson**

(7) Authors calculate and analyze Feshbach resonance spectra for (e) ultracold Yb (1S0) + Yb (3P2) collisions as a function of (e) an interatomic potential scaling factor λ and external magnetic field.
(8) Authors show that, at zero fields, the resonances are distributed randomly in (eb) λ, but that signatures of quantum chaos emerge (eb) as a field is applied.
**(9)** The random zero-field distribution arises from (e) superposition of structured spectra associated with (e&eb) individual total angular momenta.
**(10)** In addition, they show that the resonances in magnetic field in the experimentally accessible range 400 to 2000~G are (=) chaotically distributed, with (e&eb) strong level repulsion that is (=) characteristic of quantum chaos. **Cite as: arXiv: 1510.06992 [physics.atom-ph] (or arXiv: 1510.06992v1 [physics.atom-ph] for this version) ©American Physical Society**

**NOTATION**

**Module One**

A system of quantum computing structures is introduced and proven capable of making emerge, on average, the orbits of classical bounded nonlinear maps on (e&eb) \mathbb{C} through (e&eb)the iterative action of path-dependent quantum gates

$G_{13}$ : Category one of system of quantum computing structures is introduced making emerge

$G_{14}$ : Category two of SAS

$G_{15}$ : Category three of SAS

$T_{13}$ : Category one of orbits of classical bounded nonlinear maps on (e&eb) \mathbb{C} through (e&eb)the iterative action of path-dependent quantum gates

$T_{14}$ : Category two of SAS

$T_{15}$ : Category three of SAS

**Module Two**

A system of quantum computing structures is introduced and proven capable of making emerge, on average, the orbits of classical bounded nonlinear maps on (e&eb) \mathbb{C} through (e&eb)the iterative action of path-dependent quantum gates

$G_{16}$ : Category one of **system of quantum computing structures is introduced and proven capable of making emerge, on average, the orbits of classical bounded nonlinear maps;** \mathbb{C} through (e&eb)the iterative action of path-dependent quantum gates

$G_{17}$ : Category two of SAS

$G_{18}$ : Category three of SAS

$T_{16}$ : Category one of\mathbb{C} through (e&eb)the iterative action of path-dependent quantum gates; **system of quantum computing structures is introduced and proven capable of making emerge, on average, the orbits of classical bounded nonlinear maps**

$T_{17}$ : Category two of SAS

$T_{18}$ : Category three of SAS

**Module three**

The effects of emerging nonlinear dynamics and chaos upon the (e&eb) quantum averages of relevant observables and quantum probabilities are exemplified for a version of Chirikov's standard map on \mathbb{C}

$G_{20}$ : Category one of **emerging nonlinear dynamics and chaos;** quantum averages of relevant observables and quantum probabilities are exemplified for a version of Chirikov's standard map on \mathbb{C}

$G_{21}$ : Category two of SAS

$G_{22}$ : Category three of SAS

$T_{20}$ : Category one of quantum averages of relevant observables and quantum probabilities are exemplified for a version of Chirikov's standard map on \mathbb{C} ;**emerging nonlinear dynamics and chaos**

$T_{21}$ : Category two of SAS

$T_{22}$ : Category three of SAS

**Module four**

Both the individual orbits and ensemble properties are addressed so that (e) the Poincar\'e map for (e) Chirikov's standard map, in the current quantum setting, is reinterpreted in terms of (e&eb) a quantum ensemble which is then formally introduced within (eb) the formalized system of quantum computing structures, in terms of (e&eb) quantum register machines, revealing (eb) three phases of quantum ensemble dynamics: the regular, the chaotic and an intermediate phase called **complex quantum stochastic phase** which shares similarities to (e&eb) the edge of chaos notion from (e) classical cellular automata and classical random boolean networks' evolutionary computation.

Subjects: Chaotic Dynamics (nlin.CD); Statistical Mechanics (cond-mat.stat-mech); Quantum Physics (quant-ph) MSC classes: 81Q50, 81P68 Cite as: arXiv: 1208.2610 [nlin.CD] (or arXiv: 1208.2610v1 [nlin.CD] for this version)

$G_{24}$ : Category one of Poincar\'e map for (e) Chirikov's standard map, in the current quantum setting, is reinterpreted in terms of (e&eb) a quantum ensemble which is then formally introduced within (eb) the formalized system of quantum computing structures, in terms of (e&eb) quantum register machines, revealing (eb) three phases of quantum ensemble dynamics: the regular, the chaotic and an intermediate phase called **complex quantum stochastic phase** which shares similarities to (e&eb) the edge of chaos notion from (e) classical cellular automata and classical random boolean networks' evolutionary computation.

$G_{25}$ : Category two of SAS

$G_{26}$ : Category three of SAS

$T_{24}$ : Category one of individual orbits and ensemble properties are addressed

$T_{25}$ : Category two of SAS

$T_{26}$ : Category three of SAS

**Module five**

Both the individual orbits and ensemble properties are addressed so that the Poincar\'e map for Chirikov's standard map, in the current quantum setting, is reinterpreted in terms of (e&eb) a quantum ensemble which is then formally introduced within (eb) the formalized system of quantum computing structures, in terms of (e&eb) quantum register machines, revealing (eb) three phases of quantum ensemble dynamics: the regular, the chaotic and an intermediate phase called **complex quantum stochastic phase** which shares similarities to (e&eb) the edge of chaos notion from (e) classical cellular automata and classical random boolean networks' evolutionary computation.

Subjects: Chaotic Dynamics (nlin.CD); Statistical Mechanics (cond-mat.stat-mech); Quantum Physics (quant-ph) MSC classes: 81Q50, 81P68 Cite as: arXiv: 1208.2610 [nlin.CD] (or arXiv: 1208.2610v1 [nlin.CD] for this version)

$G_{28}$ : Category one of **individual orbits and ensemble properties are addressed so that the Poincar\'e map for Chirikov's standard map, in the current quantum setting;** quantum ensemble which is then formally introduced within (eb) the formalized system of quantum computing structures, in terms of (e&eb) quantum register machines, revealing (eb) three phases of quantum ensemble dynamics: the regular, the chaotic and an intermediate phase called **complex quantum stochastic phase** which shares similarities to (e&eb) the edge of chaos notion from (e) classical cellular automata and classical random boolean networks' evolutionary computation

$G_{29}$ : Category two of SAS

$G_{30}$ : Category three of SAS

$T_{28}$ : Category one of quantum ensemble which is then formally introduced within (eb) the formalized system of quantum computing structures, in terms of (e&eb) quantum register machines, revealing (eb) three phases of quantum ensemble dynamics: the regular, the chaotic and an intermediate phase called **complex quantum stochastic phase** which shares similarities to (e&eb) the edge of chaos notion from (e) classical cellular automata and classical random boolean networks' evolutionary computation; **individual orbits and ensemble properties are addressed so that the Poincar\'e map for Chirikov's standard map, in the current quantum setting**

$T_{29}$ : Category two of SAS

$T_{30}$ : Category three of SAS

**Module six**

Both the individual orbits and ensemble properties are addressed so that the Poincar\'e map for Chirikov's standard map, in the current quantum setting, is reinterpreted in terms of a quantum ensemble which is then formally introduced within (eb) the formalized system of quantum computing structures, in terms of (e&eb) quantum register machines, revealing (eb) three phases of quantum ensemble dynamics: the regular, the chaotic and an intermediate phase called **complex quantum stochastic phase** which shares similarities to (e&eb) the edge of chaos notion from (e) classical cellular automata and classical random boolean networks' evolutionary computation.

Subjects: Chaotic Dynamics (nlin.CD); Statistical Mechanics (cond-mat.stat-mech); Quantum

Physics (quant-ph) MSC classes: 81Q50, 81P68 Cite as: arXiv: 1208.2610 [nlin.CD] (or arXiv: 1208.2610v1 [nlin.CD] for this version)

$G_{32}$ : Category one of individual orbits and ensemble properties are addressed so that the Poincar\'e map for Chirikov's standard map, in the current quantum setting, is reinterpreted in terms of a quantum ensemble which is then formally introduced

$G_{33}$ : Category two of SAS

$G_{34}$ : Category three of SAS

$T_{32}$ : Category one of formalized system of quantum computing structures, in terms of (e&eb) quantum register machines, revealing (eb) three phases of quantum ensemble dynamics: the regular, the chaotic and an intermediate phase called **complex quantum stochastic phase** which shares similarities to (e&eb) the edge of chaos notion from (e) classical cellular automata and classical random boolean networks' evolutionary computation

$T_{33}$ : Category two of SAS

$T_{34}$ : Category three of SAS

**Module seven**

Both the individual orbits and ensemble properties are addressed so that the Poincar\'e map for Chirikov's standard map, in the current quantum setting, is reinterpreted in terms of a quantum ensemble which is then formally introduced within the formalized system of quantum computing structures, in terms of (e&eb) quantum register machines, revealing (eb) three phases of quantum ensemble dynamics: the regular, the chaotic and an intermediate phase called **complex quantum stochastic phase** which shares similarities to (e&eb) the edge of chaos notion from (e) classical cellular automata and classical random boolean networks' evolutionary computation.

Subjects: Chaotic Dynamics (nlin.CD); Statistical Mechanics (cond-mat.stat-mech); Quantum Physics (quant-ph) MSC classes: 81Q50, 81P68 Cite as: arXiv: 1208.2610 [nlin.CD] (or arXiv: 1208.2610v1 [nlin.CD] for this version)

$G_{36}$ : Category one of **individual orbits and ensemble properties are addressed so that the Poincar\'e map for Chirikov's standard map, in the current quantum setting, is reinterpreted in terms of a quantum ensemble which is then formally introduced within the formalized system of quantum computing structures;** quantum register machines, revealing (eb) three phases of quantum ensemble dynamics: the regular, the chaotic and an intermediate phase called **complex quantum stochastic phase** which shares similarities to (e&eb) the edge of chaos notion from (e) classical cellular automata and classical random boolean networks' evolutionary computation

$G_{37}$ : Category two of SAS

$G_{38}$ : Category three of SAS

$T_{36}$ : Category one of quantum register machines, revealing (eb) three phases of quantum ensemble dynamics: the regular, the chaotic and an intermediate phase called **complex quantum stochastic phase** which shares similarities to (e&eb) the edge of chaos notion from (e) classical cellular automata and classical random boolean networks' evolutionary computation; **individual orbits and ensemble properties**

**are addressed so that the Poincar\'e map for Chirikov's standard map, in the current quantum setting, is reinterpreted in terms of a quantum ensemble which is then formally introduced within the formalized system of quantum computing structures**

$T_{37}$ : Category two of SAS

$T_{38}$ : Category three of SAS

**Module eight**

Both the individual orbits and ensemble properties are addressed so that the Poincar\'e map for Chirikov's standard map, in the current quantum setting, is reinterpreted in terms of a quantum ensemble which is then formally introduced within the formalized system of quantum computing structures, in terms of quantum register machines, revealing (eb) three phases of quantum ensemble dynamics: the regular, the chaotic and an intermediate phase called **complex quantum stochastic phase** which shares similarities to (e&eb) the edge of chaos notion from (e) classical cellular automata and classical random boolean networks' evolutionary computation.

Subjects: Chaotic Dynamics (nlin.CD); Statistical Mechanics (cond-mat.stat-mech); Quantum Physics (quant-ph) MSC classes: 81Q50, 81P68 Cite as: arXiv: 1208.2610 [nlin.CD] (or arXiv: 1208.2610v1 [nlin.CD] for this version)

$G_{40}$ : Category one of individual orbits and ensemble properties are addressed so that the Poincar\'e map for Chirikov's standard map, in the current quantum setting, is reinterpreted in terms of a quantum ensemble which is then formally introduced within the formalized system of quantum computing structures, in terms of quantum register machines

$G_{41}$ : Category two of SAS

$G_{42}$ : Category three of SAS

$T_{40}$ : Category one of three phases of quantum ensemble dynamics: the regular, the chaotic and an intermediate phase called **complex quantum stochastic phase** which shares similarities to (e&eb) the edge of chaos notion from (e) classical cellular automata and classical random boolean networks' evolutionary computation.

$T_{41}$ : Category two of SAS

$T_{42}$ : Category three of SAS

**Module Nine**

Both the individual orbits and ensemble properties are addressed so that the Poincar\'e map for Chirikov's standard map, in the current quantum setting, is reinterpreted in terms of a quantum ensemble which is then formally introduced within the formalized system of quantum computing structures, in terms of quantum register machines, revealing three phases of quantum ensemble dynamics: the regular, the chaotic and an intermediate phase called **complex quantum stochastic phase** which shares similarities to (e&eb) the edge of chaos notion from classical cellular automata and classical random boolean networks' evolutionary computation.

Subjects: Chaotic Dynamics (nlin.CD); Statistical Mechanics (cond-mat.stat-mech); Quantum

Physics (quant-ph) MSC classes: 81Q50, 81P68 Cite as: arXiv: 1208.2610 [nlin.CD] (or arXiv: 1208.2610v1 [nlin.CD] for this version)

$G_{44}$ : Category one of individual orbits and ensemble properties are addressed so that the Poincar\'e map for Chirikov's standard map, in the current quantum setting, is reinterpreted in terms of a quantum ensemble which is then formally introduced within the formalized system of quantum computing structures, in terms of quantum register machines, revealing three phases of quantum ensemble dynamics: the regular, the chaotic and an intermediate phase called **complex quantum stochastic phase** which shares similarities; **edge of chaos notion from classical cellular automata and classical random boolean networks' evolutionary computation**

$G_{45}$ : Category two of SAS

$G_{46}$ : Category three of SAS

$T_{44}$ : Category one of **edge of chaos notion from classical cellular automata and classical random boolean networks' evolutionary computation;** individual orbits and ensemble properties are addressed so that the Poincar\'e map for Chirikov's standard map, in the current quantum setting, is reinterpreted in terms of a quantum ensemble which is then formally introduced within the formalized system of quantum computing structures, in terms of quantum register machines, revealing three phases of quantum ensemble dynamics: the regular, the chaotic and an intermediate phase called **complex quantum stochastic phase** which shares similarities

$T_{45}$ : Category two of SAS

$T_{46}$ : Category three of SAS

**The Coefficients:**

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}$ $(a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}$ $(b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}$ , $(b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}$,
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}$, $(b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}$ $(a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$ ,
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)}$,
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)}$,
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)}$,
are Dissipation coefficients

**Module Numbered One**

**The differential system of this model is now (Module Numbered one)**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14},t)\right]G_{13} \quad 1$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14},t)\right]G_{14} \quad 2$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14},t)\right]G_{15} \quad 3$$

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G,t)\right]T_{13} \quad 4$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G,t)\right]T_{14} \quad 5$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G,t)\right]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14},t) =$ First augmentation factor

$-(b''_{13})^{(1)}(G,t) =$ First detritions factor

**Module Numbered Two**

**The differential system of this model is now ( Module numbered two)**

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17},t)\right]G_{16} \quad 7$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17},t)\right]G_{17} \quad 8$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17},t)\right]G_{18} \quad 9$$

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\big((G_{19}),t\big)\right]T_{16} \quad 10$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\big((G_{19}),t\big)\right]T_{17} \quad 11$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\big((G_{19}),t\big)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17},t) =$ First augmentation factor

$-(b''_{16})^{(2)}\big((G_{19}),t\big) =$ First detritions factor

**Module Numbered Three**

**The differential system of this model is now (Module numbered three)**

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21},t)\right]G_{20} \quad 13$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21},t)\right]G_{21} \quad 14$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21},t)\right]G_{22} \quad 15$$

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23},t)\right]T_{20} \quad 16$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23},t)\right]T_{21} \quad 17$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23},t)\right]T_{22} \quad 18$$

$+(a''_{20})^{(3)}(T_{21},t) =$ First augmentation factor

$-(b''_{20})^{(3)}(G_{23},t) =$ First detritions factor

**Module Numbered Four**

**The differential system of this model is now (Module numbered Four)**

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right]G_{24} \quad 19$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25},t)\right]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}, t)\right]G_{26} \quad 21$$
$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27}), t\right)\right]T_{24} \quad 22$$
$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27}), t\right)\right]T_{25} \quad 23$$
$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27}), t\right)\right]T_{26} \quad 24$$
$+(a''_{24})^{(4)}(T_{25}, t) =$ **First augmentation factor**
$-(b''_{24})^{(4)}\left((G_{27}), t\right) =$ **First detritions factor**

**Module Numbered Five**:

**The differential system of this model is now (Module number five)**

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right]G_{28} \quad 25$$
$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}, t)\right]G_{29} \quad 26$$
$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}, t)\right]G_{30} \quad 27$$
$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}\left((G_{31}), t\right)\right]T_{28} \quad 28$$
$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}\left((G_{31}), t\right)\right]T_{29} \quad 29$$
$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}\left((G_{31}), t\right)\right]T_{30} \quad 30$$
$+(a''_{28})^{(5)}(T_{29}, t) =$ **First augmentation factor**
$-(b''_{28})^{(5)}\left((G_{31}), t\right) =$ **First detritions factor**

**Module Numbered Six**

**The differential system of this model is now (Module numbered Six)**

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right]G_{32} \quad 31$$
$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}, t)\right]G_{33} \quad 32$$
$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}, t)\right]G_{34} \quad 33$$
$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}\left((G_{35}), t\right)\right]T_{32} \quad 34$$
$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}\left((G_{35}), t\right)\right]T_{33} \quad 35$$
$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}\left((G_{35}), t\right)\right]T_{34} \quad 36$$
$+(a''_{32})^{(6)}(T_{33}, t) =$ **First augmentation factor**

**Module Numbered Seven:**

**The differential system of this model is now (Seventh Module)**

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right]G_{36} \quad 37$$
$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37}, t)\right]G_{37} \quad 38$$
$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}, t)\right]G_{38} \quad 39$$
$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}\left((G_{39}), t\right)\right]T_{36} \quad 40$$
$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}\left((G_{39}), t\right)\right]T_{37} \quad 41$$
$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}\left((G_{39}), t\right)\right]T_{38} \quad 42$$
$+(a''_{36})^{(7)}(T_{37}, t) =$ **First augmentation factor**

**Module Numbered Eight**

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right]G_{40} \quad 43$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41},t)\right]G_{41} \quad 44$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41},t)\right]G_{42} \quad 45$$

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}\left((G_{43}),t\right)\right]T_{40} \quad 46$$

$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}\left((G_{43}),t\right)\right]T_{41} \quad 47$$

$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}\left((G_{43}),t\right)\right]T_{42} \quad 48$$

**Module Numbered Nine**

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}\left((G_{47}),t\right)\right]T_{44} \quad 52$$

$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}\left((G_{47}),t\right)\right]T_{45} \quad 53$$

$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}\left((G_{47}),t\right)\right]T_{46} \quad 54$$

$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}\left((G_{47}),t\right) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)} & +(a''_{13})^{(1)}(T_{14},t) & +(a''_{16})^{(2,2,)}(T_{17},t) & +(a''_{20})^{(3,3,)}(T_{21},t) \\ +(a''_{24})^{(4,4,4,4,)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{36})^{(7,7)}(T_{37},t) & +(a''_{40})^{(8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)} & +(a''_{14})^{(1)}(T_{14},t) & +(a''_{17})^{(2,2,)}(T_{17},t) & +(a''_{21})^{(3,3,)}(T_{21},t) \\ +(a''_{25})^{(4,4,4,4,)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{37})^{(7,7)}(T_{37},t) & +(a''_{41})^{(8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)} & +(a''_{15})^{(1)}(T_{14},t) & +(a''_{18})^{(2,2,)}(T_{17},t) & +(a''_{22})^{(3,3,)}(T_{21},t) \\ +(a''_{26})^{(4,4,4,4,)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{38})^{(7,7)}(T_{37},t) & +(a''_{42})^{(8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{15} \quad 57$$

Where $(a''_{13})^{(1)}(T_{14},t)$, $(a''_{14})^{(1)}(T_{14},t)$, $(a''_{15})^{(1)}(T_{14},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,)}(T_{17},t)$, $+(a''_{17})^{(2,2,)}(T_{17},t)$, $+(a''_{18})^{(2,2,)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,)}(T_{21},t)$, $+(a''_{21})^{(3,3,)}(T_{21},t)$, $+(a''_{22})^{(3,3,)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{38})^{(7,7)}(T_{37},t)$ $+(a''_{37})^{(7,7)}(T_{37},t)$ $+(a''_{36})^{(7,7)}(T_{37},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{40})^{(8,8)}(T_{41},t)$ $+(a''_{41})^{(8,8)}(T_{41},t)$ $+(a''_{42})^{(8,8)}(T_{41},t)$ are eight augmentation coefficient for 1,2,3

$+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)} & -(b''_{13})^{(1)}(G,t) & -(b''_{16})^{(2,2,)}(G_{19},t) & -(b''_{20})^{(3,3,)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{36})^{(7,7,)}(G_{39},t) & -(b''_{40})^{(8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{13} \quad 58$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)} & -(b''_{14})^{(1)}(G,t) & -(b''_{17})^{(2,2,)}(G_{19},t) & -(b''_{21})^{(3,3,)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{37})^{(7,7,)}(G_{39},t) & -(b''_{41})^{(8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{14} \quad 59$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)} & -(b''_{15})^{(1)}(G,t) & -(b''_{18})^{(2,2,)}(G_{19},t) & -(b''_{22})^{(3,3,)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{38})^{(7,7,)}(G_{39},t) & -(b''_{42})^{(8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{15} \quad 60$$

Where $-(b''_{13})^{(1)}(G,t)$, $-(b''_{14})^{(1)}(G,t)$, $-(b''_{15})^{(1)}(G,t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,)}(G_{19},t)$, $-(b''_{17})^{(2,2,)}(G_{19},t)$, $-(b''_{18})^{(2,2,)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{37})^{(7,7,)}(G_{39},t)$, $-(b''_{36})^{(7,7,)}(G_{39},t)$, $-(b''_{38})^{(7,7,)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8)}(G_{43},t)$ $-(b''_{41})^{(8,8)}(G_{43},t)$ $-(b''_{42})^{(8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[\begin{array}{c} (a'_{16})^{(2)}\boxed{+(a''_{16})^{(2)}(T_{17},t)}\ \boxed{+(a''_{13})^{(1,1,)}(T_{14},t)}\ \boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)}\ \boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)}\ \boxed{+(a''_{44})^{(9,9)}(T_{45},t)} \end{array}\right] G_{16} \quad (61)$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[\begin{array}{c} (a'_{17})^{(2)}\boxed{+(a''_{17})^{(2)}(T_{17},t)}\ \boxed{+(a''_{14})^{(1,1,)}(T_{14},t)}\ \boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)}\ \boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)}\ \boxed{+(a''_{45})^{(9,9)}(T_{45},t)} \end{array}\right] G_{17} \quad (62)$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[\begin{array}{c} (a'_{18})^{(2)}\boxed{+(a''_{18})^{(2)}(T_{17},t)}\ \boxed{+(a''_{15})^{(1,1,)}(T_{14},t)}\ \boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)}\ \boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)}\ \boxed{+(a''_{46})^{(9,9)}(T_{45},t)} \end{array}\right] G_{18} \quad (63)$$

Where $\boxed{+(a''_{16})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,)}(T_{14},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)}$ are eight augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{44})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9)}(T_{45},t)}$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[\begin{array}{c} (b'_{16})^{(2)}\boxed{-(b''_{16})^{(2)}(G_{19},t)}\ \boxed{-(b''_{13})^{(1,1,)}(G,t)}\ \boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)}\ \boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)}\ \boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)}\ \boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)}\ \boxed{-(b''_{44})^{(9,9)}(G_{47},t)} \end{array}\right] T_{16} \quad (64)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[\begin{array}{c} (b'_{17})^{(2)}\boxed{-(b''_{17})^{(2)}(G_{19},t)}\ \boxed{-(b''_{14})^{(1,1,)}(G,t)}\ \boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)}\ \boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)}\ \boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)}\ \boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)}\ \boxed{-(b''_{45})^{(9,9)}(G_{47},t)} \end{array}\right] T_{17} \quad (65)$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[\begin{array}{c} (b'_{18})^{(2)}\boxed{-(b''_{18})^{(2)}(G_{19},t)}\ \boxed{-(b''_{15})^{(1,1,)}(G,t)}\ \boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)}\ \boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)}\ \boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)}\ \boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)}\ \boxed{-(b''_{46})^{(9,9)}(G_{47},t)} \end{array}\right] T_{18} \quad (66)$$

where $-(b''_{16})^{(2)}(G_{19},t)$ , $-(b''_{17})^{(2)}(G_{19},t)$ , $-(b''_{18})^{(2)}(G_{19},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{13})^{(1,1,)}(G,t)$, $-(b''_{14})^{(1,1,)}(G,t)$, $-(b''_{15})^{(1,1,)}(G,t)$ are second detrition coefficients for category 1,2 and 3

$-(b''_{20})^{(3,3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,)}(G_{23},t)$ are third detrition coefficients for category 1,2 and 3

$-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1,2 and 3

$-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1,2 and 3

$-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients for category 1,2 and 3

$-(b''_{36})^{(7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2 and 3

$-(b''_{40})^{(8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1,2 and 3

$-(b''_{44})^{(9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[\begin{array}{c} (a'_{20})^{(3)}\left[+(a''_{20})^{(3)}(T_{21},t)\right]\left[+(a''_{16})^{(2,2,2)}(T_{17},t)\right]\left[+(a''_{13})^{(1,1,1,)}(T_{14},t)\right] \\ \left[+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)\right]\left[+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)\right]\left[+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)\right] \\ \left[+(a''_{36})^{(7,7,7,7)}(T_{37},t)\right]\left[+(a''_{40})^{(8,8,8,8)}(T_{41},t)\right]\left[+(a''_{44})^{(9,9,9)}(T_{45},t)\right] \end{array}\right] G_{20} \quad (67)$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[\begin{array}{c} (a'_{21})^{(3)}\left[+(a''_{21})^{(3)}(T_{21},t)\right]\left[+(a''_{17})^{(2,2,2)}(T_{17},t)\right]\left[+(a''_{14})^{(1,1,1,)}(T_{14},t)\right] \\ \left[+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)\right]\left[+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)\right]\left[+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)\right] \\ \left[+(a''_{37})^{(7,7,7,7)}(T_{37},t)\right]\left[+(a''_{41})^{(8,8,8,8)}(T_{41},t)\right]\left[+(a''_{45})^{(9,9,9)}(T_{45},t)\right] \end{array}\right] G_{21} \quad (68)$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[\begin{array}{c} (a'_{22})^{(3)}\left[+(a''_{22})^{(3)}(T_{21},t)\right]\left[+(a''_{18})^{(2,2,2)}(T_{17},t)\right]\left[+(a''_{15})^{(1,1,1,)}(T_{14},t)\right] \\ \left[+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)\right]\left[+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)\right]\left[+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)\right] \\ \left[+(a''_{38})^{(7,7,7,7)}(T_{37},t)\right]\left[+(a''_{42})^{(8,8,8,8)}(T_{41},t)\right]\left[+(a''_{46})^{(9,9,9)}(T_{45},t)\right] \end{array}\right] G_{22} \quad (69)$$

$+(a''_{20})^{(3)}(T_{21},t)$, $+(a''_{21})^{(3)}(T_{21},t)$, $+(a''_{22})^{(3)}(T_{21},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2)}(T_{17},t)$ are second augmentation coefficients for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,)}(T_{14},t)$ are third augmentation coefficients for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficients for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficients for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficients for category 1, 2 and 3

$+(a''_{36})^{(7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1, 2 and 3

$+(a''_{40})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8)}(T_{41},t)$ are eight augmentation coefficients for category 1, 2 and 3

$+(a''_{44})^{(9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b'_{20})^{(3)} & -(b''_{20})^{(3)}(G_{23},t) & -(b''_{16})^{(2,2,2)}(G_{19},t) & -(b''_{13})^{(1,1,1,)}(G,t) \\ -(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{36})^{(7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{20} \quad (70)$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b'_{21})^{(3)} & -(b''_{21})^{(3)}(G_{23},t) & -(b''_{17})^{(2,2,2)}(G_{19},t) & -(b''_{14})^{(1,1,1,)}(G,t) \\ -(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{37})^{(7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{21} \quad (71)$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)} & -(b''_{22})^{(3)}(G_{23},t) & -(b''_{18})^{(2,2,2)}(G_{19},t) & -(b''_{15})^{(1,1,1,)}(G,t) \\ -(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{38})^{(7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{22} \quad (72)$$

$-(b''_{20})^{(3)}(G_{23},t)$, $-(b''_{21})^{(3)}(G_{23},t)$, $-(b''_{22})^{(3)}(G_{23},t)$ *are first detrition coefficients for category 1, 2 and 3*

$-(b''_{16})^{(2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2)}(G_{19},t)$ *are second detrition coefficients for category 1, 2 and 3*

$-(b''_{13})^{(1,1,1,)}(G,t)$, $-(b''_{14})^{(1,1,1,)}(G,t)$, $-(b''_{15})^{(1,1,1,)}(G,t)$ *are third detrition coefficients for category 1,2 and 3*

$-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)$ *are fourth detrition coefficients for category 1, 2 and 3*

$-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)$ *are fifth detrition coefficients for category 1, 2 and 3*

$-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)$ *are sixth detrition coefficients for category 1, 2 and 3*

$-(b''_{36})^{(7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$-(b''_{46})^{(9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \begin{bmatrix} (a'_{24})^{(4)} & +(a''_{24})^{(4)}(T_{25},t) & +(a''_{28})^{(5,5,)}(T_{29},t) & +(a''_{32})^{(6,6,)}(T_{33},t) \\ +(a''_{13})^{(1,1,1,1)}(T_{14},t) & +(a''_{16})^{(2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3)}(T_{21},t) \\ +(a''_{36})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9)}(T_{45},t) \end{bmatrix} G_{24} \quad (73)$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \begin{bmatrix} (a'_{25})^{(4)} & +(a''_{25})^{(4)}(T_{25},t) & +(a''_{29})^{(5,5,)}(T_{29},t) & +(a''_{33})^{(6,6)}(T_{33},t) \\ +(a''_{14})^{(1,1,1,1)}(T_{14},t) & +(a''_{17})^{(2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3)}(T_{21},t) \\ +(a''_{37})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9)}(T_{45},t) \end{bmatrix} G_{25} \quad (74)$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \begin{bmatrix} (a'_{26})^{(4)} & +(a''_{26})^{(4)}(T_{25},t) & +(a''_{30})^{(5,5,)}(T_{29},t) & +(a''_{34})^{(6,6,)}(T_{33},t) \\ +(a''_{15})^{(1,1,1,1)}(T_{14},t) & +(a''_{18})^{(2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3)}(T_{21},t) \\ +(a''_{38})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9)}(T_{45},t) \end{bmatrix} G_{26} \quad (75)$$

$+(a''_{24})^{(4)}(T_{25},t)$, $+(a''_{25})^{(4)}(T_{25},t)$, $+(a''_{26})^{(4)}(T_{25},t)$ *are first augmentation coefficients category* 1, 2 3

$+(a''_{28})^{(5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,)}(T_{29},t)$ *are second augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{32})^{(6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,)}(T_{33},t)$ *are third augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{13})^{(1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1)}(T_{14},t)$ *are fourth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{16})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2)}(T_{17},t)$ *are fifth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{20})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3)}(T_{21},t)$ *are sixth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)$ *are seventh augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)$ *are eighth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{46})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9)}(T_{45},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \begin{bmatrix} (b'_{24})^{(4)} & -(b''_{24})^{(4)}(G_{27},t) & -(b''_{28})^{(5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,)}(G_{35},t) \\ -(b''_{13})^{(1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3)}(G_{23},t) \\ -(b''_{36})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{24} \quad (76)$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \begin{bmatrix} (b'_{25})^{(4)} & -(b''_{25})^{(4)}(G_{27},t) & -(b''_{29})^{(5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,)}(G_{35},t) \\ -(b''_{14})^{(1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3)}(G_{23},t) \\ -(b''_{37})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{25} \quad (77)$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)} & -(b''_{26})^{(4)}(G_{27},t) & -(b''_{30})^{(5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,)}(G_{35},t) \\ -(b''_{15})^{(1,1,1,1)}(G,t) & -(b''_{18})^{(2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3)}(G_{23},t) \\ -(b''_{38})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{26} \quad (78)$$

$Where\ \boxed{-(b''_{24})^{(4)}(G_{27},t)}, \boxed{-(b''_{25})^{(4)}(G_{27},t)}, \boxed{-(b''_{26})^{(4)}(G_{27},t)}\ are\ first\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b''_{28})^{(5,5,)}(G_{31},t)}, \boxed{-(b''_{29})^{(5,5,)}(G_{31},t)}, \boxed{-(b''_{30})^{(5,5,)}(G_{31},t)}\ are\ second\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b''_{32})^{(6,6,)}(G_{35},t)}, \boxed{-(b''_{33})^{(6,6,)}(G_{35},t)}, \boxed{-(b''_{34})^{(6,6,)}(G_{35},t)}\ are\ third\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b''_{13})^{(1,1,1,1)}(G,t)}, \boxed{-(b''_{14})^{(1,1,1,1)}(G,t)}, \boxed{-(b''_{15})^{(1,1,1,1)}(G,t)}$
$are\ fourth\ \ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2,2)}(G_{19},t)}$
$are\ fifth\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3,3,3,3)}(G_{23},t)}$
$are\ sixth\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)}$
$are\ seventh\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)}$
$are\ eighth\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b''_{46})^{(9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{45})^{(9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{44})^{(9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[\begin{array}{c} (a'_{28})^{(5)}\boxed{+(a''_{28})^{(5)}(T_{29},t)}\ \boxed{+(a''_{24})^{(4,4,)}(T_{25},t)}\ \boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)}\ \boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)}\ \boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{28} \quad 79$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[\begin{array}{c} (a'_{29})^{(5)}\boxed{+(a''_{29})^{(5)}(T_{29},t)}\ \boxed{+(a''_{25})^{(4,4,)}(T_{25},t)}\ \boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)}\ \boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)}\ \boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{29} \quad 80$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[\begin{array}{c} (a'_{30})^{(5)}\boxed{+(a''_{30})^{(5)}(T_{29},t)}\ \boxed{+(a''_{26})^{(4,4,)}(T_{25},t)}\ \boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)}\ \boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)}\ \boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{30} \quad 81$$

$Where\ \boxed{+(a''_{28})^{(5)}(T_{29},t)}, \boxed{+(a''_{29})^{(5)}(T_{29},t)}, \boxed{+(a''_{30})^{(5)}(T_{29},t)}\ are\ first\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$

$And\ \boxed{+(a''_{24})^{(4,4,)}(T_{25},t)}, \boxed{+(a''_{25})^{(4,4,)}(T_{25},t)}, \boxed{+(a''_{26})^{(4,4,)}(T_{25},t)}\ are\ second\ augmentation\ coefficient\ for\ category\ 1,2\ and\ 3$

$\boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)}, \boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)}, \boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)}\ are\ third\ \ augmentation\ coefficient\ for\ category\ 1,2\ and\ 3$

$\boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)}$ are fourth augmentation coefficients for category 1,2, and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)}$ are fifth augmentation coefficients for category 1,2,and 3

$+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)$ are sixth augmentation coefficients for category 1,2, 3

$+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)$ , $+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1,2, 3

$+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)$ are eighth augmentation coefficients for category 1,2, 3

$+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \begin{bmatrix} (b'_{28})^{(5)} & -(b''_{28})^{(5)}(G_{31},t) & -(b''_{24})^{(4,4,)}(G_{27},t) & -(b''_{32})^{(6,6,6)}(G_{35},t) \\ -(b''_{13})^{(1,1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3)}(G_{23},t) & \\ -(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9)}(G_{47},t) & \end{bmatrix} T_{28} \quad 82$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b'_{29})^{(5)} & -(b''_{29})^{(5)}(G_{31},t) & -(b''_{25})^{(4,4,)}(G_{27},t) & -(b''_{33})^{(6,6,6)}(G_{35},t) \\ -(b''_{14})^{(1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3)}(G_{23},t) & \\ -(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9)}(G_{47},t) & \end{bmatrix} T_{29} \quad 83$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b'_{30})^{(5)} & -(b''_{30})^{(5)}(G_{31},t) & -(b''_{26})^{(4,4,)}(G_{27},t) & -(b''_{34})^{(6,6,6)}(G_{35},t) \\ -(b''_{15})^{(1,1,1,1,1,)}(G,t) & -(b''_{18})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3)}(G_{23},t) & \\ -(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9)}(G_{47},t) & \end{bmatrix} T_{30} \quad 84$$

*where* $-(b''_{28})^{(5)}(G_{31},t)$ , $-(b''_{29})^{(5)}(G_{31},t)$ , $-(b''_{30})^{(5)}(G_{31},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{24})^{(4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,)}(G_{27},t)$ *are second detrition coefficients for category* 1,2 *and* 3

$-(b''_{32})^{(6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6)}(G_{35},t)$ *are third detrition coefficients for category* 1,2 *and* 3

$-(b''_{13})^{(1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1)}(G,t)$ , $-(b''_{15})^{(1,1,1,1,1,)}(G,t)$ are fourth detrition coefficients for category 1,2, and 3

$-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1,2, and 3

$-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1,2, and 3

$-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2, and 3

$-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1,2, and 3

$-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} \qquad 85$$

$$-\left[\begin{matrix}(a'_{32})^{(6)}\boxed{+(a''_{32})^{(6)}(T_{33},t)}\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}\\ \boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}\boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)}\boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)}\boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)}\end{matrix}\right]G_{32}$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[\begin{matrix}(a'_{33})^{(6)}\boxed{+(a''_{33})^{(6)}(T_{33},t)}\boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}\boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}\\ \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}\boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}\boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)}\boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}\boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)}\end{matrix}\right]G_{33} \qquad 86$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[\begin{matrix}(a'_{34})^{(6)}\boxed{+(a''_{34})^{(6)}(T_{33},t)}\boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}\boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}\\ \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}\boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}\boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)}\boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}\boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)}\end{matrix}\right]G_{34} \qquad 87$$

$\boxed{+(a''_{32})^{(6)}(T_{33},t)}, \boxed{+(a''_{33})^{(6)}(T_{33},t)}, \boxed{+(a''_{34})^{(6)}(T_{33},t)}$ *are first augmentation coefficients for category* $1, 2$ *and* $3$

$\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}$ *are second augmentation coefficients for category* $1, 2$ *and* $3$

$\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}$ *are third augmentation coefficients for category* $1, 2$ *and* $3$

$\boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}$ - are fourth augmentation coefficients

$\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}$ - fifth augmentation coefficients

$\boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)}$ sixth augmentation coefficients

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)}$ seventh augmentation coefficients

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}$ Eighth augmentation coefficients

$\boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)}$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[\begin{matrix}(b'_{32})^{(6)}\boxed{-(b''_{32})^{(6)}(G_{35},t)}\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}\\ \boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)}\boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)}\\ \boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)}\boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)}\end{matrix}\right]T_{32} \qquad 88$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[\begin{matrix}(b'_{33})^{(6)}\boxed{-(b''_{33})^{(6)}(G_{35},t)}\boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}\boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}\\ \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}\boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)}\\ \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)}\boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)}\end{matrix}\right]T_{33} \qquad 89$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)}\boxed{-(b''_{34})^{(6)}(G_{35},t)} & \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{34} \quad 90$$

$\boxed{-(b''_{32})^{(6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6)}(G_{35},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}$ *are second detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)}$ *are third detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}$ are fourth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \quad 91$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a'_{36})^{(7)}\boxed{+(a''_{36})^{(7)}(T_{37},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \quad 92$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a'_{37})^{(7)}\boxed{+(a''_{37})^{(7)}(T_{37},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{38}}{dt} \quad 93$$

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a'_{38})^{(7)}\boxed{+(a''_{38})^{(7)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{(a''_{36})^{(7)}(T_{37},t)}$, $\boxed{(a''_{37})^{(7)}(T_{37},t)}$, $\boxed{(a''_{38})^{(7)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$+(a_{16}'')^{(2,2,2,2,2,2,2)}(T_{17},t)$, $+(a_{17}'')^{(2,2,2,2,2,2,2)}(T_{17},t)$, $+(a_{18}'')^{(2,2,2,2,2,2,2)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a_{20}'')^{(3,3,3,3,3,3,3)}(T_{21},t)$, $+(a_{21}'')^{(3,3,3,3,3,3,3)}(T_{21},t)$, $+(a_{22}'')^{(3,3,3,3,3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a_{24}'')^{(4,4,4,4,4,4,4)}(T_{25},t)$, $+(a_{25}'')^{(4,4,4,4,4,4,4)}(T_{25},t)$, $+(a_{26}'')^{(4,4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a_{28}'')^{(5,5,5,5,5,5,5)}(T_{29},t)$, $+(a_{29}'')^{(5,5,5,5,5,5,5)}(T_{29},t)$, $+(a_{30}'')^{(5,5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a_{32}'')^{(6,6,6,6,6,6,6)}(T_{33},t)$, $+(a_{33}'')^{(6,6,6,6,6,6,6)}(T_{33},t)$, $+(a_{34}'')^{(6,6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a_{13}'')^{(1,1,1,1,1,1,1)}(T_{14},t)$, $+(a_{13}'')^{(1,1,1,1,1,1,1)}(T_{14},t)$, $+(a_{15}'')^{(1,1,1,1,1,1,1)}(T_{14},t)$ are seventh augmentation coefficient for category 1, 2 and 3

$+(a_{42}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a_{41}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a_{40}'')^{(8,8,8,8,8,8,8,8,)}(T_{41},t)$ are eighth augmentation coefficient for 1,2,3

$+(a_{46}'')^{(9,9,9,9,9,9,9)}(T_{45},t)$, $+(a_{45}'')^{(9,9,9,9,9,9,9)}(T_{45},t)$, $+(a_{44}'')^{(9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{36}}{dt} =$$



$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b_{36}')^{(7)}\left[-(b_{36}'')^{(7)}(G_{39},t)\right] & \left[-(b_{16}'')^{(2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b_{20}'')^{(3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b_{24}'')^{(4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b_{28}'')^{(5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b_{32}'')^{(6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b_{13}'')^{(1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b_{40}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)\right] & \left[-(b_{44}'')^{(9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{13}$$

$$\frac{dT_{37}}{dt} =$$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b_{37}')^{(7)}\left[-(b_{37}'')^{(7)}(G_{39},t)\right] & \left[-(b_{17}'')^{(2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b_{21}'')^{(3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b_{25}'')^{(4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b_{29}'')^{(5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b_{33}'')^{(6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b_{14}'')^{(1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b_{41}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)\right] & \left[-(b_{45}'')^{(9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{14}$$

$$\frac{dT_{38}}{dt} =$$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b_{38}')^{(7)}\left[-(b_{38}'')^{(7)}(G_{39},t)\right] & \left[-(b_{18}'')^{(2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b_{22}'')^{(3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b_{26}'')^{(4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b_{30}'')^{(5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b_{34}'')^{(6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b_{15}'')^{(1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b_{42}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)\right] & \left[-(b_{46}'')^{(9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{15}$$

Where $-(b_{36}'')^{(7)}(G_{39},t)$, $-(b_{37}'')^{(7)}(G_{39},t)$, $-(b_{38}'')^{(7)}(G_{39},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b_{16}'')^{(2,2,2,2,2,2,2)}(G_{19},t)$, $-(b_{17}'')^{(2,2,2,2,2,2,2)}(G_{19},t)$, $-(b_{18}'')^{(2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b_{20}'')^{(3,3,3,3,3,3,3)}(G_{23},t)$, $-(b_{21}'')^{(3,3,3,3,3,3,3)}(G_{23},t)$, $-(b_{22}'')^{(3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b_{24}'')^{(4,4,4,4,4,4,4)}(G_{27},t)$, $-(b_{25}'')^{(4,4,4,4,4,4,4)}(G_{27},t)$, $-(b_{26}'')^{(4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition

coefficients for category 1, 2 and 3
$\boxed{-(b_{28}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition
coefficients for category 1, 2 and 3
$\boxed{-(b_{32}'')^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{34}'')^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition
coefficients for category 1, 2 and 3
$\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$
are seventh detrition coefficients for category 1, 2 and 3
$\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition
coefficients for category 1, 2 and 3
$\boxed{-(b_{46}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{45}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{44}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition
coefficients for category 1, 2 and 3



$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \begin{bmatrix} (a_{40}')^{(8)}\boxed{+(a_{40}'')^{(8)}(T_{41},t)} & \boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{20}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{24}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{28}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{32}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{36}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{44}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \begin{bmatrix} (a_{41}')^{(8)}\boxed{+(a_{41}'')^{(8)}(T_{41},t)} & \boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{21}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{25}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{29}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{33}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{37}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{45}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \begin{bmatrix} (a_{42}')^{(8)}\boxed{+(a_{42}'')^{(8)}(T_{41},t)} & \boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{22}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{26}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{30}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{34}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{15}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{38}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{46}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a_{40}'')^{(8)}(T_{41},t)}$, $\boxed{+(a_{41}'')^{(8)}(T_{41},t)}$, $\boxed{+(a_{42}'')^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3
$\boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3
$\boxed{+(a_{20}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{21}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{22}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3
$\boxed{+(a_{24}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{25}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{26}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3
$\boxed{+(a_{28}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{29}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{30}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ $+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ $+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$ are eighth augmentation coefficient for 1,2,3

$+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} =$$

$$(b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)}\left[-(b''_{40})^{(8)}(G_{43},t)\right] & \left[-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} =$$

$$(b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)}\left[-(b''_{41})^{(8)}(G_{43},t)\right] & \left[-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} =$$

$$(b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)}\left[-(b''_{42})^{(8)}(G_{43},t)\right] & \left[-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{15}$$

Where $-(b''_{36})^{(7)}(G_{39},t)$, $-(b''_{37})^{(7)}(G_{39},t)$, $-(b''_{38})^{(7)}(G_{39},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)$, $-(b''_{38})^{(7,7,)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)$ are eighth detrition

coefficients for category 1, 2 and 3
$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3



$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \begin{bmatrix} (a'_{44})^{(9)}\boxed{+(a''_{44})^{(9)}(T_{45},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \begin{bmatrix} (a'_{45})^{(9)}\boxed{+(a''_{45})^{(9)}(T_{45},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \begin{bmatrix} (a'_{46})^{(9)}\boxed{+(a''_{46})^{(9)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a''_{44})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3
$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3
$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3
$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3
$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3
$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3
$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ are Seventh augmentation coefficient for category 1, 2 and 3
$\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3
$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} =$$

$$(b_{44})^{(9)}T_{45} -$$

$$\begin{bmatrix} (b'_{44})^{(9)}\boxed{-(b''_{44})^{(9)}(G_{47},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b'_{45})^{(9)}\boxed{-(b''_{45})^{(9)}(G_{47},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b'_{46})^{(9)}\boxed{-(b''_{46})^{(9)}(G_{47},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{44})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9)}(G_{47},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a'_i)^{(1)}, (a''_i)^{(1)}, (b_i)^{(1)}, (b'_i)^{(1)}, (b''_i)^{(1)} > 0, \quad i,j = 13,14,15 \qquad 97$$

The functions $(a''_i)^{(1)}, (b''_i)^{(1)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a_i'')^{(1)}(T_{14},t) \le (p_i)^{(1)} \le (\hat{A}_{13})^{(1)}$$

$$(b_i'')^{(1)}(G,t) \le (r_i)^{(1)} \le (b_i')^{(1)} \le (\hat{B}_{13})^{(1)}$$

$$lim_{T_2\to\infty}(a_i'')^{(1)}(T_{14},t) = (p_i)^{(1)} \qquad 98$$

$$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = (r_i)^{(1)}$$

**Definition of** $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$ :

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$$|(a_i'')^{(1)}(T_{14}',t) - (a_i'')^{(1)}(T_{14},t)| \le (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G',t) - (b_i'')^{(1)}(G,t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}',t)$ and $(a_i'')^{(1)}(T_{14},t)$ . $(T_{14}',t)$ and $(T_{14},t)$ are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14},t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$ : 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$$

**Definition of** $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$ : 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}$, $(r_i)^{(1)}, i = 13,14,15$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i,j = 16,17,18$$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17},t) \le (p_i)^{(2)} \le \left(\hat{A}_{16}\right)^{(2)} \quad 102$$

$$(b_i'')^{(2)}(G_{19},t) \le (r_i)^{(2)} \le (b_i')^{(2)} \le (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2\to\infty}(a_i'')^{(2)}(T_{17},t) = (p_i)^{(2)} \quad 104$$

$$\lim_{G\to\infty}(b_i'')^{(2)}\left((G_{19}),t\right) = (r_i)^{(2)} \quad 105$$

**Definition of** $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$ : 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}',t) - (a_i'')^{(2)}(T_{17},t)| \le (\hat{k}_{16})^{(2)}|T_{17} - T_{17}'|e^{-(\hat{M}_{16})^{(2)}t} \quad 107$$

$$|(b_i'')^{(2)}((G_{19})',t) - (b_i'')^{(2)}\left((G_{19}),t\right)| < (\hat{k}_{16})^{(2)}||(G_{19}) - (G_{19})'||e^{-(\hat{M}_{16})^{(2)}t} \quad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}',t)$ and$(a_i'')^{(2)}(T_{17},t)$ . $(T_{17}',t)$ and $(T_{17},t)$ are points belonging to the interval $\left[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}\right]$. It is to be noted that $(a_i'')^{(2)}(T_{17},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17},t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$ :

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants 109

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$$

**Definition of** $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$ :

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and\ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1 \quad 110$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(b_i)^{(2)} + (b_i')^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1 \quad 111$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i,j = 20,21,22 \quad 112$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(3)}$, $(\mathrm{r_i})^{(3)}$:

$$(a_i'')^{(3)}(T_{21}, t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23}, t) \leq (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(3)}(T_{21}, t) = (p_i)^{(3)} \quad (113)$$

$$\lim_{G \to \infty}(b_i'')^{(3)}(G_{23}, t) = (r_i)^{(3)}$$

**Definition of** $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$ :

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: (114)

$$|(a_i'')^{(3)}(T_{21}', t) - (a_i'')^{(3)}(T_{21}, t)| \leq (\hat{k}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\hat{M}_{20})^{(3)}t}$$

$$|(b_i'')^{(3)}(G_{23}', t) - (b_i'')^{(3)}(G_{23}, t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}'||e^{-(\hat{M}_{20})^{(3)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}', t)$ and$(a_i'')^{(3)}(T_{21}, t)$ . $(T_{21}', t)$ And $(T_{21}, t)$ are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21}, t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$ : (115)

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with (116)
$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$ and the constants
$(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}, i = 20,21,22,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(a_i)^{(3)} + (a_i')^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(b_i)^{(3)} + (b_i')^{(3)} + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i, j = 24,25,26 \quad (117)$$

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(4)}$, $(r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25},t) \le (p_i)^{(4)} \le (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}\big((G_{27}),t\big) \le (r_i)^{(4)} \le (b_i')^{(4)} \le (\hat{B}_{24})^{(4)}$$

$$lim_{T_2\to\infty}(a_i'')^{(4)}(T_{25},t) = (p_i)^{(4)} \quad (118)$$

$$\lim_{G\to\infty}(b_i'')^{(4)}\big((G_{27}),t\big) = (r_i)^{(4)}$$

**Definition of** $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$ :

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24,25,26}$

They satisfy Lipschitz condition: (119)

$$|(a_i'')^{(4)}(T_{25}',t) - (a_i'')^{(4)}(T_{25},t)| \le (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})',t) - (b_i'')^{(4)}\big((G_{27}),t\big)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}',t)$ and$(a_i'')^{(4)}(T_{25},t)$ . $(T_{25}',t)$ and $(T_{25},t)$ are points belonging to the interval $\big[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}\big]$. It is to be noted that $(a_i'')^{(4)}(T_{25},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25},t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$ : (120)

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$, are positive constants

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}, \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$$

**Definition of** $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$ : (121)

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and\ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}, i = 24,25,26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(a_i)^{(4)} + (a_i')^{(4)} + (\hat{A}_{24})^{(4)} + (\hat{P}_{24})^{(4)}(\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(b_i)^{(4)} + (b_i')^{(4)} + (\hat{B}_{24})^{(4)} + (\hat{Q}_{24})^{(4)}(\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i,j = 28,29,30 \quad (122)$$

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(5)}$, $(r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29},t) \le (p_i)^{(5)} \le (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}\big((G_{31}),t\big) \le (r_i)^{(5)} \le (b_i')^{(5)} \le (\hat{B}_{28})^{(5)}$$

$$lim_{T_2\to\infty}(a_i'')^{(5)}(T_{29},t) = (p_i)^{(5)} \qquad 123$$

$$\lim_{G\to\infty}(b_i'')^{(5)}(G_{31},t) = (r_i)^{(5)}$$

**Definition of** $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$ :

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}',t) - (a_i'')^{(5)}(T_{29},t)| \le (\hat{k}_{28})^{(5)}|T_{29} - T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})',t) - (b_i'')^{(5)}\big((G_{31}),t\big)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}',t)$ and$(a_i'')^{(5)}(T_{29},t)$ . $(T_{29}',t)$ and $(T_{29},t)$ are points belonging to the interval $\big[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\big]$. It is to be noted that $(a_i'')^{(5)}(T_{29},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29},t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$ : 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}, \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

**Definition of** $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$ : 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and\ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(a_i)^{(5)} + (a_i')^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)}(\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(b_i)^{(5)} + (b_i')^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)}(\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i,j = 32,33,34 \qquad 127$$

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(6)}$, $(r_i)^{(6)}$:

$(a_i'')^{(6)}(T_{33},t) \le (p_i)^{(6)} \le (\hat{A}_{32})^{(6)}$

$(b_i'')^{(6)}((G_{35}),t) \le (r_i)^{(6)} \le (b_i')^{(6)} \le (\hat{B}_{32})^{(6)}$

$lim_{T_2\to\infty}(a_i'')^{(6)}(T_{33},t) = (p_i)^{(6)}$ 128

$\lim_{G\to\infty}(b_i'')^{(6)}((G_{35}),t) = (r_i)^{(6)}$

**Definition of** $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$ :

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$|(a_i'')^{(6)}(T_{33}',t) - (a_i'')^{(6)}(T_{33},t)| \le (\hat{k}_{32})^{(6)}|T_{33} - T_{33}'|e^{-(\hat{M}_{32})^{(6)}t}$

$|(b_i'')^{(6)}((G_{35})',t) - (b_i'')^{(6)}((G_{35}),t)| < (\hat{k}_{32})^{(6)}||(G_{35}) - (G_{35})'||e^{-(\hat{M}_{32})^{(6)}t}$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}',t)$ and$(a_i'')^{(6)}(T_{33},t)$ . $(T_{33}',t)$ and $(T_{33},t)$ are points belonging to the interval $\left[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}\right]$. It is to be noted that $(a_i'')^{(6)}(T_{33},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33},t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$ : 129

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$, are positive constants

$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}$ , $\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$

**Definition of** $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$ : 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and\ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32,33,34,$ satisfy the inequalities

$\frac{1}{(\hat{M}_{32})^{(6)}}[(a_i)^{(6)} + (a_i')^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$

$\frac{1}{(\hat{M}_{32})^{(6)}}[(b_i)^{(6)} + (b_i')^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$

Where we suppose

(SSSS) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i,j = 36,37,38$ 131

(TTTT) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(7)}$, $(r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37},t) \le (p_i)^{(7)} \le (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39},t) \le (r_i)^{(7)} \le (b_i')^{(7)} \le (\hat{B}_{36})^{(7)} \quad (132)$$

*(UUUU)* $\lim_{T_2\to\infty} (a_i'')^{(7)}(T_{37},t) = (p_i)^{(7)}$

*(VVVV)*

$\lim_{G\to\infty}(b_i'')^{(7)}\big((G_{39}),t\big) = (r_i)^{(7)}$

**Definition of** $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$ :

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: (133)

$$|(a_i'')^{(7)}(T_{37}',t) - (a_i'')^{(7)}(T_{37},t)| \le (\hat{k}_{36})^{(7)}|T_{37} - T_{37}'|e^{-(\hat{M}_{36})^{(7)}t}$$

$$|(b_i'')^{(7)}((G_{39})',t) - (b_i'')^{(7)}\big((G_{39}),t\big)| < (\hat{k}_{36})^{(7)}||(G_{39}) - (G_{39})'||e^{-(\hat{M}_{36})^{(7)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}',t)$ and$(a_i'')^{(7)}(T_{37},t)$ . $(T_{37}',t)$ and $(T_{37},t)$ are points belonging to the interval $\big[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}\big]$. It is to be noted that $(a_i'')^{(7)}(T_{37},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37},t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$ : (134)

(WWWW) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$, are positive constants

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$$

**Definition of** $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$ : (135)

(XXXX) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)}(\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(b_i)^{(7)} + (b_i')^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)}(\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40,41,42 \tag{136}$$

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

**Definition of** $(p_i)^{(8)}$, $(r_i)^{(8)}$: (137)

$$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)} \tag{138}$$

$$(b_i'')^{(8)}((G_{43}), t) \leq (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)} \tag{139}$$

$$\lim_{T_2 \to \infty} (a_i'')^{(8)}(T_{41}, t) = (p_i)^{(8)} \tag{140}$$

$$\lim_{G \to \infty} (b_i'')^{(8)}\big((G_{43}), t\big) = (r_i)^{(8)} \tag{141}$$

**Definition of** $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$ :

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)} |T_{41} - T_{41}'| e^{-(\hat{M}_{40})^{(8)} t} \tag{142}$$

$$|(b_i'')^{(8)}((G_{43})', t) - (b_i'')^{(8)}\big((G_{43}), t\big)| < (\hat{k}_{40})^{(8)} ||(G_{43}) - (G_{43})'|| e^{-(\hat{M}_{40})^{(8)} t} \tag{143}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}', t)$ and $(a_i'')^{(8)}(T_{41}, t)$ . $(T_{41}', t)$ and $(T_{41}, t)$ are points belonging to the interval $\left[(\hat{k}_{40})^{(8)}, (\hat{M}_{40})^{(8)}\right]$. It is to be noted that $(a_i'')^{(8)}(T_{41}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{40})^{(8)} = 1$ then the function $(a_i'')^{(8)}(T_{41}, t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$ :

$(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$, are positive constants

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}, \frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1 \tag{144}$$

**Definition of** $(\hat{P}_{40})^{(8)}, (\hat{Q}_{40})^{(8)}$ :

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}, (\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)}, (a_i')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}, i = 40,41,42,$

Satisfy the inequalities

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(a_i)^{(8)} + (a_i')^{(8)} + (\hat{A}_{40})^{(8)} + (\hat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}] < 1 \quad 145$$

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(b_i)^{(8)} + (b_i')^{(8)} + (\hat{B}_{40})^{(8)} + (\hat{Q}_{40})^{(8)}(\hat{k}_{40})^{(8)}] < 1 \quad 146$$

Where we suppose

$$(a_i)^{(9)}, (a_i')^{(9)}, (a_i'')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (b_i'')^{(9)} > 0, \quad i,j = 44,45,46 \quad 146A$$

The functions $(a_i'')^{(9)}, (b_i'')^{(9)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(9)}$, $(r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45}, t) \leq (p_i)^{(9)} \leq (\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47}, t) \leq (r_i)^{(9)} \leq (b_i')^{(9)} \leq (\hat{B}_{44})^{(9)}$$

$$\lim_{T_2 \to \infty} (a_i'')^{(9)}(T_{45}, t) = (p_i)^{(9)}$$

$$\lim_{G \to \infty} (b_i'')^{(9)}(G_{47}, t) = (r_i)^{(9)}$$

**Definition of** $(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}$ :

Where $\boxed{(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}}$ are positive constants and $\boxed{i = 44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}', t) - (a_i'')^{(9)}(T_{45}, t)| \leq (\hat{k}_{44})^{(9)}|T_{45} - T_{45}'|e^{-(\hat{M}_{44})^{(9)}t}$$

$$|(b_i'')^{(9)}((G_{47})', t) - (b_i'')^{(9)}((G_{47}), t)| < (\hat{k}_{44})^{(9)}||(G_{47}) - (G_{47})'||e^{-(\hat{M}_{44})^{(9)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}', t)$ and$(a_i'')^{(9)}(T_{45}, t)$ . $(T_{45}', t)$ and $(T_{45}, t)$ are points belonging to the interval $[(\hat{k}_{44})^{(9)}, (\hat{M}_{44})^{(9)}]$. It is to be noted that $(a_i'')^{(9)}(T_{45}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{44})^{(9)} = 1$ then the function $(a_i'')^{(9)}(T_{45}, t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$ :

$(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$, are positive constants

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}, \frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$$

**Definition of** $(\hat{P}_{44})^{(9)}, (\hat{Q}_{44})^{(9)}$ :

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}, (\hat{A}_{44})^{(9)} and (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)}, (a_i')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}, i = 44,45,46,$
satisfy the inequalities

$$\frac{1}{(\widehat{M}_{44})^{(9)}}[(a_i)^{(9)} + (a_i')^{(9)} + (\hat{A}_{44})^{(9)} + (\hat{P}_{44})^{(9)}(\hat{k}_{44})^{(9)}] < 1$$

$$\frac{1}{(\widehat{M}_{44})^{(9)}}[(b_i)^{(9)} + (b_i')^{(9)} + (\hat{B}_{44})^{(9)} + (\hat{Q}_{44})^{(9)}(\hat{k}_{44})^{(9)}] < 1$$

**Theorem 1:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0), T_i(0)$ :

$$G_i(t) \leq (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 2 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

**Definition of** $G_i(0), T_i(0)$

$$G_i(t) \leq (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} \quad , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \leq (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} \quad , \quad T_i(0) = T_i^0 > 0$$

**Theorem 3 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t) \leq (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t} \quad , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \leq (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t} \quad , \quad T_i(0) = T_i^0 > 0$$

**Theorem 4 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

**Definition of** $G_i(0), T_i(0)$ :

$$G_i(t) \leq (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 5 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

**Definition of** $G_i(0), T_i(0)$ :

$$G_i(t) \leq (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 6 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

**Definition of** $G_i(0), T_i(0)$ :

$G_i(t) \le \left(\hat{P}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \le \left(\hat{Q}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Theorem 7:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

**Definition of** $G_i(0), T_i(0)$ :

$G_i(t) \le \left(\hat{P}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \le \left(\hat{Q}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Theorem 8:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

**Definition of** $G_i(0), T_i(0)$ :

$G_i(t) \le \left(\hat{P}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \le \left(\hat{Q}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Theorem 9:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

**Definition of** $G_i(0), T_i(0)$ :

$G_i(t) \le \left(\hat{P}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \le \left(\hat{Q}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Proof:** Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i$: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$G_i(0) = G_i^0$ , $T_i(0) = T_i^0$ , $G_i^0 \le \left(\hat{P}_{13}\right)^{(1)}$ , $T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)}$, 155

$0 \le G_i(t) - G_i^0 \le \left(\hat{P}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t}$ 156

$0 \le T_i(t) - T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t}$ 157

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}(s_{(13)}) - \left((a'_{13})^{(1)} + a''_{13})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{15}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^{0} + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^{0} + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{\mathrm{T}}_{15}(\mathrm{t}) = \mathrm{T}_{15}^{0} + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{15}(s_{(13)})\right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{16})^{(2)}$, $T_i^0 \le (\hat{Q}_{16})^{(2)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

By 160

$$\bar{G}_{16}(t) = G_{16}^{0} + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''_{16})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^{0} + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}(s_{(16)}), s_{(17)})\right) G_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^{0} + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{18}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^{0} + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^{0} + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^{0} + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{18}(s_{(16)})\right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{20})^{(3)}$, $T_i^0 \le (\hat{Q}_{20})^{(3)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

By 161

$$\bar{G}_{20}(t) = G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''_{20})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{22}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^0 + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^0 + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{\mathrm{T}}_{22}(\mathrm{t}) = \mathrm{T}_{22}^0 + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{22}(s_{(20)})\right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{24})^{(4)}, T_i^0 \le (\hat{Q}_{24})^{(4)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

By 162

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{26}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{\mathrm{T}}_{26}(\mathrm{t}) = \mathrm{T}_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{26}(s_{(24)})\right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{28})^{(5)}, T_i^0 \le (\hat{Q}_{28})^{(5)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{28})^{(5)} e^{(\tilde{M}_{28})^{(5)} t}$

By 163

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[ (a_{28})^{(5)} G_{29}(s_{(28)}) - \left( (a'_{28})^{(5)} + a''_{28})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{28}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[ (a_{29})^{(5)} G_{28}(s_{(28)}) - \left( (a'_{29})^{(5)} + (a''_{29})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{29}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[ (a_{30})^{(5)} G_{29}(s_{(28)}) - \left( (a'_{30})^{(5)} + (a''_{30})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{30}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[ (b_{28})^{(5)} T_{29}(s_{(28)}) - \left( (b'_{28})^{(5)} - (b''_{28})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{28}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[ (b_{29})^{(5)} T_{28}(s_{(28)}) - \left( (b'_{29})^{(5)} - (b''_{29})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{29}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{\mathrm{T}}_{30}(\mathrm{t}) = \mathrm{T}_{30}^0 + \int_0^t \left[ (b_{30})^{(5)} T_{29}(s_{(28)}) - \left( (b'_{30})^{(5)} - (b''_{30})^{(5)} (G_{31}(s_{(28)}), s_{(28)}) \right) T_{30}(s_{(28)}) \right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{32})^{(6)}$, $T_i^0 \le (\hat{Q}_{32})^{(6)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\tilde{M}_{32})^{(6)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\tilde{M}_{32})^{(6)} t}$

By 164

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[ (a_{32})^{(6)} G_{33}(s_{(32)}) - \left( (a'_{32})^{(6)} + a''_{32})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[ (a_{33})^{(6)} G_{32}(s_{(32)}) - \left( (a'_{33})^{(6)} + (a''_{33})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[ (a_{34})^{(6)} G_{33}(s_{(32)}) - \left( (a'_{34})^{(6)} + (a''_{34})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{34}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[ (b_{32})^{(6)} T_{33}(s_{(32)}) - \left( (b'_{32})^{(6)} - (b''_{32})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[ (b_{33})^{(6)} T_{32}(s_{(32)}) - \left( (b'_{33})^{(6)} - (b''_{33})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{\mathrm{T}}_{34}(\mathrm{t}) = \mathrm{T}_{34}^0 + \int_0^t \left[ (b_{34})^{(6)} T_{33}(s_{(32)}) - \left( (b'_{34})^{(6)} - (b''_{34})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{34}(s_{(32)}) \right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{36})^{(7)}$, $T_i^0 \leq (\hat{Q}_{36})^{(7)}$,

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} G_{37}(s_{(36)}) - \left((a'_{36})^{(7)} + a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[(a_{37})^{(7)} G_{36}(s_{(36)}) - \left((a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[(a_{38})^{(7)} G_{37}(s_{(36)}) - \left((a'_{38})^{(7)} + (a''_{38})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{38}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[(b_{36})^{(7)} T_{37}(s_{(36)}) - \left((b'_{36})^{(7)} - (b''_{36})^{(7)} (G_{39}(s_{(36)}), s_{(36)})\right) T_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[(b_{37})^{(7)} T_{36}(s_{(36)}) - \left((b'_{37})^{(7)} - (b''_{37})^{(7)} (G_{39}(s_{(36)}), s_{(36)})\right) T_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{38}(t) = T_{38}^0 + \int_0^t \left[(b_{38})^{(7)} T_{37}(s_{(36)}) - \left((b'_{38})^{(7)} - (b''_{38})^{(7)} (G_{39}(s_{(36)}), s_{(36)})\right) T_{38}(s_{(36)})\right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{40})^{(8)}$, $T_i^0 \leq (\hat{Q}_{40})^{(8)}$,

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} G_{41}(s_{(40)}) - \left((a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[(a_{41})^{(8)} G_{40}(s_{(40)}) - \left((a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41}(s_{(40)}), s_{(40)})\right) G_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}(s_{(40)}), s_{(40)})\right) G_{42}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^0 + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43}(s_{(40)}), s_{(40)})\right) T_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^0 + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43}(s_{(40)}), s_{(40)})\right) T_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{\mathrm{T}}_{42}(\mathrm{t}) = \mathrm{T}_{42}^0 + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43}(s_{(40)}), s_{(40)})\right) T_{42}(s_{(40)})\right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** 166

Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ A which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{44})^{(9)},\ T_i^0 \le (\hat{Q}_{44})^{(9)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''_{44})^{(9)}(T_{45}(s_{(44)}), s_{(44)})\right) G_{44}(s_{(44)})\right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45}(s_{(44)}), s_{(44)})\right) G_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}(s_{(44)}), s_{(44)})\right) G_{46}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{44}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{\mathrm{T}}_{46}(\mathrm{t}) = \mathrm{T}_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{46}(s_{(44)})\right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167
$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}}\right)\right] ds_{(13)} =$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left(e^{(\hat{M}_{13})^{(1)} t} - 1\right)$$

From which it follows that 168

$$(G_{13}(t)-G_{13}^{0})e^{-(\hat{M}_{13})^{(1)}t}\leq\frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}}\left[((\hat{P}_{13})^{(1)}+G_{14}^{0})e^{\left(-\frac{(\hat{P}_{13})^{(1)}+G_{14}^{0}}{G_{14}^{0}}\right)}+(\hat{P}_{13})^{(1)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t)\leq G_{16}^{0}+\int_{0}^{t}\left[(a_{16})^{(2)}\left(G_{17}^{0}+(\hat{P}_{16})^{(6)}e^{(\hat{M}_{16})^{(2)}s_{(16)}}\right)\right]ds_{(16)}= \quad 169$$

$$\left(1+(a_{16})^{(2)}t\right)G_{17}^{0}+\frac{(a_{16})^{(2)}(\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}}\left(e^{(\hat{M}_{16})^{(2)}t}-1\right)$$

From which it follows that 170

$$(G_{16}(t)-G_{16}^{0})e^{-(\hat{M}_{16})^{(2)}t}\leq\frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}}\left[((\hat{P}_{16})^{(2)}+G_{17}^{0})e^{\left(-\frac{(\hat{P}_{16})^{(2)}+G_{17}^{0}}{G_{17}^{0}}\right)}+(\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t)\leq G_{20}^{0}+\int_{0}^{t}\left[(a_{20})^{(3)}\left(G_{21}^{0}+(\hat{P}_{20})^{(3)}e^{(\hat{M}_{20})^{(3)}s_{(20)}}\right)\right]ds_{(20)}=$$

$$\left(1+(a_{20})^{(3)}t\right)G_{21}^{0}+\frac{(a_{20})^{(3)}(\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left(e^{(\hat{M}_{20})^{(3)}t}-1\right)$$

From which it follows that 172

$$(G_{20}(t)-G_{20}^{0})e^{-(\hat{M}_{20})^{(3)}t}\leq\frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left[((\hat{P}_{20})^{(3)}+G_{21}^{0})e^{\left(-\frac{(\hat{P}_{20})^{(3)}+G_{21}^{0}}{G_{21}^{0}}\right)}+(\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 173

$$G_{24}(t)\leq G_{24}^{0}+\int_{0}^{t}\left[(a_{24})^{(4)}\left(G_{25}^{0}+(\hat{P}_{24})^{(4)}e^{(\hat{M}_{24})^{(4)}s_{(24)}}\right)\right]ds_{(24)}=$$

$$\left(1+(a_{24})^{(4)}t\right)G_{25}^{0}+\frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left(e^{(\hat{M}_{24})^{(4)}t}-1\right)$$

From which it follows that 174

$$(G_{24}(t)-G_{24}^{0})e^{-(\hat{M}_{24})^{(4)}t}\leq\frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left[((\hat{P}_{24})^{(4)}+G_{25}^{0})e^{\left(-\frac{(\hat{P}_{24})^{(4)}+G_{25}^{0}}{G_{25}^{0}}\right)}+(\hat{P}_{24})^{(4)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t) \le G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} \left(G_{29}^0 + (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} s_{(28)}}\right)\right] ds_{(28)} =$$

$$\left(1 + (a_{28})^{(5)} t\right) G_{29}^0 + \frac{(a_{28})^{(5)} (\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}} \left(e^{(\hat{M}_{28})^{(5)} t} - 1\right)$$

From which it follows that 175

$$(G_{28}(t) - G_{28}^0) e^{-(\hat{M}_{28})^{(5)} t} \le \frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}} \left[\left((\hat{P}_{28})^{(5)} + G_{29}^0\right) e^{\left(-\frac{(\hat{P}_{28})^{(5)} + G_{29}^0}{G_{29}^0}\right)} + (\hat{P}_{28})^{(5)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t) \le G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} \left(G_{33}^0 + (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} s_{(32)}}\right)\right] ds_{(32)} =$$

$$\left(1 + (a_{32})^{(6)} t\right) G_{33}^0 + \frac{(a_{32})^{(6)} (\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}} \left(e^{(\hat{M}_{32})^{(6)} t} - 1\right)$$

From which it follows that 177

$$(G_{32}(t) - G_{32}^0) e^{-(\hat{M}_{32})^{(6)} t} \le \frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}} \left[\left((\hat{P}_{32})^{(6)} + G_{33}^0\right) e^{\left(-\frac{(\hat{P}_{32})^{(6)} + G_{33}^0}{G_{33}^0}\right)} + (\hat{P}_{32})^{(6)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

(q) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t) \le G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} \left(G_{37}^0 + (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} s_{(36)}}\right)\right] ds_{(36)} =$$

$$\left(1 + (a_{36})^{(7)} t\right) G_{37}^0 + \frac{(a_{36})^{(7)} (\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left(e^{(\hat{M}_{36})^{(7)} t} - 1\right)$$

From which it follows that

$$(G_{36}(t) - G_{36}^0) e^{-(\hat{M}_{36})^{(7)} t} \le \frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left[\left((\hat{P}_{36})^{(7)} + G_{37}^0\right) e^{\left(-\frac{(\hat{P}_{36})^{(7)} + G_{37}^0}{G_{37}^0}\right)} + (\hat{P}_{36})^{(7)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t) \le G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} \left(G_{41}^0 + (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} s_{(40)}}\right)\right] ds_{(40)} =$$ 180

$$\left(1+(a_{40})^{(8)}t\right)G_{41}^{0}+\frac{(a_{40})^{(8)}(\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left(e^{(\hat{M}_{40})^{(8)}t}-1\right)$$

From which it follows that 181

$$(G_{40}(t)-G_{40}^{0})e^{-(\hat{M}_{40})^{(8)}t}\leq\frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left[\left((\hat{P}_{40})^{(8)}+G_{41}^{0}\right)e^{\left(-\frac{(\hat{P}_{40})^{(8)}+G_{41}^{0}}{G_{41}^{0}}\right)}+(\hat{P}_{40})^{(8)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 8

Analogous inequalities hold also for $G_{41}, G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t)\leq G_{44}^{0}+\int_{0}^{t}\left[(a_{44})^{(9)}\left(G_{45}^{0}+(\hat{P}_{44})^{(9)}e^{(\hat{M}_{44})^{(9)}s_{(44)}}\right)\right]ds_{(44)}=$$

$$\left(1+(a_{44})^{(9)}t\right)G_{45}^{0}+\frac{(a_{44})^{(9)}(\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}}\left(e^{(\hat{M}_{44})^{(9)}t}-1\right)$$

From which it follows that

$$(G_{44}(t)-G_{44}^{0})e^{-(\hat{M}_{44})^{(9)}t}\leq\frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}}\left[\left((\hat{P}_{44})^{(9)}+G_{45}^{0}\right)e^{\left(-\frac{(\hat{P}_{44})^{(9)}+G_{45}^{0}}{G_{45}^{0}}\right)}+(\hat{P}_{44})^{(9)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 9

Analogous inequalities hold also for $G_{45}, G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}<1$ and to choose 182

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[(\hat{P}_{13})^{(1)}+\left((\hat{P}_{13})^{(1)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{13})^{(1)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{13})^{(1)} \quad 183$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[\left((\hat{Q}_{13})^{(1)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{13})^{(1)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{13})^{(1)}\right]\leq(\hat{Q}_{13})^{(1)} \quad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)},T^{(1)}\right),\left(G^{(2)},T^{(2)}\right)\right)=$$

$$\sup_i\left\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{13})^{(1)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{13})^{(1)}t}\right\}$$

Indeed if we denote

**Definition of** $\tilde{G}, \tilde{T}$ : $\left(\tilde{G}, \tilde{T}\right) = \mathcal{A}^{(1)}(G, T)$

It results

$$\left|\tilde{G}_{13}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{13})^{(1)} \left|G_{14}^{(1)} - G_{14}^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)} s_{(13)}} e^{(\widehat{M}_{13})^{(1)} s_{(13)}}\, ds_{(13)} +$$

$$\int_0^t \{(a'_{13})^{(1)} \left|G_{13}^{(1)} - G_{13}^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)} s_{(13)}} e^{-(\widehat{M}_{13})^{(1)} s_{(13)}} +$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)}, s_{(13)}\right)\left|G_{13}^{(1)} - G_{13}^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)} s_{(13)}} e^{(\widehat{M}_{13})^{(1)} s_{(13)}} +$$

$$G_{13}^{(2)} \left|(a''_{13})^{(1)}\left(T_{14}^{(1)}, s_{(13)}\right) - (a''_{13})^{(1)}\left(T_{14}^{(2)}, s_{(13)}\right)\right|\ e^{-(\widehat{M}_{13})^{(1)} s_{(13)}} e^{(\widehat{M}_{13})^{(1)} s_{(13)}}\} ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|G^{(1)} - G^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)} t} \leq \qquad (186)$$
$$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)} + (a'_{13})^{(1)} + (\widehat{A}_{13})^{(1)} + (\widehat{P}_{13})^{(1)}(\widehat{k}_{13})^{(1)}\right) d\left(\left(G^{(1)}, T^{(1)};\ G^{(2)}, T^{(2)}\right)\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 1:** The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)} t}$ *and* $(\widehat{Q}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)} t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(1)}$ and $(b''_i)^{(1)}, i = 13{,}14{,}15$ depend only on $T_{14}$ and respectively on $G$(*and not on* $t$) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any $t$ where $G_i(t) = 0$ *and* $T_i(t) = 0$

From 19 to 24 it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a'_i)^{(1)} - (a''_i)^{(1)}(T_{14}(s_{(13)}), s_{(13)})\} ds_{(13)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b'_i)^{(1)} t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2$ *and* $\left((\widehat{M}_{13})^{(1)}\right)_3$ : (187)

**Remark 3**: if $G_{13}$ is bounded, the same property have also $G_{14}$ *and* $G_{15}$ . indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a'_{14})^{(1)} G_{14}$ and by integrating

$$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1 / (a'_{14})^{(1)}$$

In the same way , one can obtain

$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2/(a'_{15})^{(1)}$

If $G_{14}$ *or* $G_{15}$ is bounded, the same property follows for $G_{13}$ , $G_{15}$ and $G_{13}$ , $G_{14}$ respectively.

**Remark 4:** If $G_{13}$ *is* bounded, from below, the same property holds for $G_{14}$ *and* $G_{15}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{14}$ is bounded from below. 188

**Remark 5:** If $T_{13}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}(G(t),t)) = (b'_{14})^{(1)}$ then $T_{14} \to \infty$. 189

**Definition of** $(m)^{(1)}$ and $\varepsilon_1$ :

Indeed let $t_1$ be so that for $t > t_1$

$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t),t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\frac{2}{\varepsilon_1}$ By taking now $\varepsilon_1$ sufficiently small one sees that $T_{14}$ is unbounded.
The same property holds for $T_{15}$ if $\lim_{t\to\infty}(b''_{15})^{(1)}(G(t),t) = (b'_{15})^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}$ , $\frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}$ *and* $(\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[(\widehat{P}_{16})^{(2)} + \left((\hat{P}_{16})^{(2)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{16})^{(2)}\right] \leq (\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$d\left(\left((G_{19})^{(1)},(T_{19})^{(1)}\right),\left((G_{19})^{(2)},(T_{19})^{(2)}\right)\right) =$

$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t}\}$

Indeed if we denote 195

**Definition of** $\widetilde{G_{19}}, \widetilde{T_{19}}$ : $\left(\widetilde{G_{19}}, \widetilde{T_{19}}\right) = \mathcal{A}^{(2)}(G_{19}, T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{16})^{(2)} \left|G_{17}^{(1)} - G_{17}^{(2)}\right| e^{-(\widehat{M}_{16})^{(2)} s_{(16)}} e^{(\widehat{M}_{16})^{(2)} s_{(16)}}\, ds_{(16)} +$$

$$\int_0^t \{(a'_{16})^{(2)} \left|G_{16}^{(1)} - G_{16}^{(2)}\right| e^{-(\widehat{M}_{16})^{(2)} s_{(16)}} e^{-(\widehat{M}_{16})^{(2)} s_{(16)}} +$$

$$(a''_{16})^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right)\left|G_{16}^{(1)} - G_{16}^{(2)}\right| e^{-(\widehat{M}_{16})^{(2)} s_{(16)}} e^{(\widehat{M}_{16})^{(2)} s_{(16)}} +$$

$$G_{16}^{(2)} \left|(a''_{16})^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right) - (a''_{16})^{(2)}\left(T_{17}^{(2)}, s_{(16)}\right)\right| \; e^{-(\widehat{M}_{16})^{(2)} s_{(16)}} e^{(\widehat{M}_{16})^{(2)} s_{(16)}}\} ds_{(16)}$$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0, t]$ 197

From the hypotheses it follows

$$\left|(G_{19})^{(1)} - (G_{19})^{(2)}\right| \mathrm{e}^{-(\widehat{\mathrm{M}}_{16})^{(2)} \mathrm{t}} \leq$$
$$\frac{1}{(\widehat{\mathrm{M}}_{16})^{(2)}}\left((a_{16})^{(2)} + (a'_{16})^{(2)} + (\widehat{\mathrm{A}}_{16})^{(2)} + (\widehat{\mathrm{P}}_{16})^{(2)}(\hat{k}_{16})^{(2)}\right) \mathrm{d}\left(((G_{19})^{(1)}, (T_{19})^{(1)};\ (G_{19})^{(2)}, (T_{19})^{(2)})\right)$$

And analogous inequalities for $\mathrm{G}_i$ and $\mathrm{T}_i$. Taking into account the hypothesis the result follows 198

**Remark 6:** The fact that we supposed $(a''_{16})^{(2)}$ and $(b''_{16})^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{\mathrm{P}}_{16})^{(2)} \mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)} \mathrm{t}}$ and $(\widehat{\mathrm{Q}}_{16})^{(2)} \mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)} \mathrm{t}}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i = 16{,}17{,}18$ depend only on $\mathrm{T}_{17}$ and respectively on $(G_{19})$(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 7:** There does not exist any t where $\mathrm{G}_i\,(\mathrm{t}) = 0$ and $\mathrm{T}_i\,(\mathrm{t}) = 0$ 200

it results

$$\mathrm{G}_i\,(\mathrm{t}) \geq \mathrm{G}_i^0 \mathrm{e}^{\left[-\int_0^{\mathrm{t}} \{(a_i')^{(2)} - (a_i'')^{(2)}(\mathrm{T}_{17}(s_{(16)}), s_{(16)})\} \mathrm{d}s_{(16)}\right]} \geq 0$$

$$\mathrm{T}_i\,(\mathrm{t}) \geq \mathrm{T}_i^0 \mathrm{e}^{(-(b_i')^{(2)} \mathrm{t})} > 0 \quad \text{for } \mathrm{t} > 0$$

**Definition of** $\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_1$, $\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_2$ and $\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_3$ : 201

**Remark 8:** if $\mathrm{G}_{16}$ is bounded, the same property have also $\mathrm{G}_{17}$ and $\mathrm{G}_{18}$ . indeed if

$\mathrm{G}_{16} < (\widehat{\mathrm{M}}_{16})^{(2)}$ it follows $\frac{\mathrm{dG}_{17}}{\mathrm{dt}} \leq \left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_1 - (a'_{17})^{(2)} \mathrm{G}_{17}$ and by integrating

$$\mathrm{G}_{17} \leq \left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_2 = \mathrm{G}_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_1 / (a'_{17})^{(2)}$$

In the same way , one can obtain

$G_{18} \leq \left((\widehat{M}_{16})^{(2)}\right)_3 = G_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2/(a'_{18})^{(2)}$

If $G_{17}$ or $G_{18}$ is bounded, the same property follows for $G_{16}$ , $G_{18}$ and $G_{16}$ , $G_{17}$ respectively.

**Remark 9**: If $G_{16}$ is bounded, from below, the same property holds for $G_{17}$ and $G_{18}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{17}$ is bounded from below. 202

**Remark 10:** If $T_{16}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t),t)) = (b'_{17})^{(2)}$ then $T_{17} \to \infty$. 203

**Definition of** $(m)^{(2)}$ and $\varepsilon_2$ :

Indeed let $t_2$ be so that for $t > t_2$

$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t),t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right), \quad t = \log\frac{2}{\varepsilon_2}$ By taking now $\varepsilon_2$ sufficiently small one sees that $T_{17}$ is unbounded. 205
The same property holds for $T_{18}$ if $\lim_{t\to\infty}(b''_{18})^{(2)}\left((G_{19})(t),t\right) = (b'_{18})^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}$ , $\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[(\hat{P}_{20})^{(3)} + \left((\hat{P}_{20})^{(3)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{20})^{(3)}\right] \leq (\hat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself 210

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)},(T_{23})^{(1)}),((G_{23})^{(2)},(T_{23})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{20})^{(3)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

**Definition of** $\widetilde{G_{23}}, \widetilde{T_{23}} :\left( \widetilde{(G_{23})}, \widetilde{(T_{23})} \right) = \mathcal{A}^{(3)}\left((G_{23}), (T_{23})\right)$

It results 213

$$\left|\tilde{G}_{20}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{20})^{(3)} \left|G_{21}^{(1)} - G_{21}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} e^{(\widehat{M}_{20})^{(3)} s_{(20)}}\, ds_{(20)} +$$

$$\int_0^t \{(a'_{20})^{(3)} \left|G_{20}^{(1)} - G_{20}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} +$$

$$(a''_{20})^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right)\left|G_{20}^{(1)} - G_{20}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} e^{(\widehat{M}_{20})^{(3)} s_{(20)}} +$$

$$G_{20}^{(2)} \left|(a''_{20})^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right) - (a''_{20})^{(3)}\left(T_{21}^{(2)}, s_{(20)}\right)\right|\ e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} e^{(\widehat{M}_{20})^{(3)} s_{(20)}}\} ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|{G_{23}}^{(1)} - {G_{23}}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)} t} \leq \qquad 214$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)} + (a'_{20})^{(3)} + (\widehat{A}_{20})^{(3)} + (\widehat{P}_{20})^{(3)} (\widehat{k}_{20})^{(3)}\right) d\left(\left((G_{23})^{(1)}, (T_{23})^{(1)};\ (G_{23})^{(2)}, (T_{23})^{(2)}\right)\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 11:** The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)} e^{(\widehat{M}_{20})^{(3)} t}$ *and* $(\widehat{Q}_{20})^{(3)} e^{(\widehat{M}_{20})^{(3)} t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i = 20,21,22$ depend only on $T_{21}$ and respectively on $(G_{23})$(*and not on* $t$) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 12:** There does not exist any $t$ where $G_i(t) = 0$ *and* $T_i(t) = 0$ 216

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(3)} - (a_i'')^{(3)}(T_{21}(s_{(20)}), s_{(20)})\} ds_{(20)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(3)} t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2$ *and* $\left((\widehat{M}_{20})^{(3)}\right)_3$ : 217

**Remark 13:** if $G_{20}$ is bounded, the same property have also $G_{21}$ *and* $G_{22}$ . indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a'_{21})^{(3)} G_{21}$ and by integrating

$$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1 / (a'_{21})^{(3)}$$

In the same way , one can obtain

$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2/(a'_{22})^{(3)}$

If $G_{21}$ *or* $G_{22}$ is bounded, the same property follows for $G_{20}$ , $G_{22}$ and $G_{20}$ , $G_{21}$ respectively.

**Remark 14:** If $G_{20}$ *is* bounded, from below, the same property holds for $G_{21}$ *and* $G_{22}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{21}$ is bounded from below. 218

**Remark 15:** If $T_{20}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t),t\right) = (b'_{21})^{(3)}$ then $T_{21} \to \infty$. 219

**Definition of** $(m)^{(3)}$ and $\varepsilon_3$ :

Indeed let $t_3$ be so that for $t > t_3$

$(b_{21})^{(3)} - (b_i'')^{(3)}\left((G_{23})(t),t\right) < \varepsilon_3, T_{20}(t) > (m)^{(3)}$

Then $\frac{dT_{21}}{dt} \geq (a_{21})^{(3)}(m)^{(3)} - \varepsilon_3 T_{21}$ which leads to 220

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t} = \frac{1}{2}$ it results

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right), \quad t = log\frac{2}{\varepsilon_3}$ By taking now $\varepsilon_3$ sufficiently small one sees that $T_{21}$ is unbounded.
The same property holds for $T_{22}$ if $\lim_{t\to\infty}(b''_{22})^{(3)}\left((G_{23})(t),t\right) = (b'_{22})^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\tilde{M}_{24})^{(4)}}$ , $\frac{(b_i)^{(4)}}{(\tilde{M}_{24})^{(4)}} < 1$ and to choose 221

$(\widehat{P}_{24})^{(4)}$ and $(\widehat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\tilde{M}_{24})^{(4)}}\left[(\widehat{P}_{24})^{(4)} + \left((\hat{P}_{24})^{(4)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{24})^{(4)} \quad 222$$

$$\frac{(b_i)^{(4)}}{(\tilde{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{24})^{(4)}\right] \leq (\hat{Q}_{24})^{(4)} \quad 223$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself 224

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric 225

$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right) =$

$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\tilde{M}_{24})^{(4)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\tilde{M}_{24})^{(4)}t}\}$

Indeed if we denote

**Definition of** $\widetilde{(G_{27})}, \widetilde{(T_{27})}$ : $\left(\widetilde{(G_{27})}, \widetilde{(T_{27})}\right) = \mathcal{A}^{(4)}((G_{27}), (T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{24})^{(4)} \left|G_{25}^{(1)} - G_{25}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{(\widehat{M}_{24})^{(4)} s_{(24)}}\, ds_{(24)} +$$

$$\int_0^t \{(a'_{24})^{(4)} \left|G_{24}^{(1)} - G_{24}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} +$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)}, s_{(24)}\right)\left|G_{24}^{(1)} - G_{24}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{(\widehat{M}_{24})^{(4)} s_{(24)}} +$$

$$G_{24}^{(2)} \left|(a''_{24})^{(4)}\left(T_{25}^{(1)}, s_{(24)}\right) - (a''_{24})^{(4)}\left(T_{25}^{(2)}, s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{(\widehat{M}_{24})^{(4)} s_{(24)}}\} ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)} - (G_{27})^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} t} \leq \qquad (226)$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)} + (a'_{24})^{(4)} + (\widehat{A}_{24})^{(4)} + (\widehat{P}_{24})^{(4)} (\widehat{k}_{24})^{(4)}\right) d\left(((G_{27})^{(1)}, (T_{27})^{(1)};\ (G_{27})^{(2)}, (T_{27})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 16:** The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)} t}$ *and* $(\widehat{Q}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)} t}$ respectively of $\mathbb{R}_+$. (227)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i = 24{,}25{,}26$ depend only on $T_{25}$ and respectively on $(G_{27})$(*and not on* $t$) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 17:** There does not exist any $t$ where $G_i(t) = 0$ *and* $T_i(t) = 0$ (228)

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(4)} - (a_i'')^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(4)} t)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2$ *and* $\left((\widehat{M}_{24})^{(4)}\right)_3$ : (229)

**Remark 18:** if $G_{24}$ is bounded, the same property have also $G_{25}$ *and* $G_{26}$ . indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a'_{25})^{(4)} G_{25}$ and by integrating

$$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1 / (a'_{25})^{(4)}$$

In the same way , one can obtain

$$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2/(a'_{26})^{(4)}$$

If $G_{25}$ *or* $G_{26}$ is bounded, the same property follows for $G_{24}$ , $G_{26}$ and $G_{24}$ , $G_{25}$ respectively.

**Remark 19:** If $G_{24}$ *is* bounded, from below, the same property holds for $G_{25}$ *and* $G_{26}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{25}$ is bounded from below. 230

**Remark 20:** If $T_{24}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}((G_{27})(t),t)) = (b'_{25})^{(4)}$ then $T_{25} \to \infty$. 231

**Definition of** $(m)^{(4)}$ and $\varepsilon_4$ :

Indeed let $t_4$ be so that for $t > t_4$

$$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t),t) < \varepsilon_4, T_{24}(t) > (m)^{(4)}$$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right)$, $t = log\frac{2}{\varepsilon_4}$ By taking now $\varepsilon_4$ sufficiently small one sees that $T_{25}$ is unbounded.
The same property holds for $T_{26}$ if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t),t\right) = (b'_{26})^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for $G_{29}$ , $G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}$ , $\frac{(b_i)^{(5)}}{(\widehat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\widehat{P}_{28})^{(5)}$ and $(\widehat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}\left[(\widehat{P}_{28})^{(5)} + \left((\widehat{P}_{28})^{(5)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}\left[\left((\widehat{Q}_{28})^{(5)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{28})^{(5)}\right] \leq (\widehat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$$d\left(((G_{31})^{(1)}, (T_{31})^{(1)}), ((G_{31})^{(2)}, (T_{31})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{28})^{(5)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{31}}),(\widetilde{T_{31}})$ : $\left((\widetilde{G_{31}}),(\widetilde{T_{31}})\right)=\mathcal{A}^{(5)}\left((G_{31}),(T_{31})\right)$

It results

$$\left|\tilde{G}_{28}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{28})^{(5)}\left|G_{29}^{(1)}-G_{29}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\,ds_{(28)}+$$

$$\int_0^t\{(a'_{28})^{(5)}\left|G_{28}^{(1)}-G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$(a''_{28})^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)\left|G_{28}^{(1)}-G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$G_{28}^{(2)}\left|(a''_{28})^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)-(a''_{28})^{(5)}\left(T_{29}^{(2)},s_{(28)}\right)\right|\ e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\}ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on it follows

$$\left|(G_{31})^{(1)}-(G_{31})^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}t}\le \qquad 237$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)}+(a'_{28})^{(5)}+(\widehat{A}_{28})^{(5)}+(\widehat{P}_{28})^{(5)}(\widehat{k}_{28})^{(5)}\right)d\left(((G_{31})^{(1)},(T_{31})^{(1)};\ (G_{31})^{(2)},(T_{31})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

**Remark 21:** The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}\ and\ (\widehat{Q}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ respectively of $\mathbb{R}_+$. 238

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i=28,29,30$ depend only on $T_{29}$ and respectively on $(G_{31})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 22:** There does not exist any $t$ where $G_i(t)=0\ and\ T_i(t)=0$ 239

it results

$$G_i(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(5)}-(a_i'')^{(5)}(T_{29}(s_{(28)}),s_{(28)})\}ds_{(28)}\right]}\ge 0$$

$$T_i(t)\ge T_i^0e^{(-(b_i')^{(5)}t)}>0\quad \text{for t}>0$$

**Definition of** $\left((\widehat{M}_{28})^{(5)}\right)_1,\left((\widehat{M}_{28})^{(5)}\right)_2\ and\ \left((\widehat{M}_{28})^{(5)}\right)_3$ : 240

**Remark 23:** if $G_{28}$ is bounded, the same property have also $G_{29}\ and\ G_{30}$ . indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a'_{29})^{(5)} G_{29}$ and by integrating

$$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1/(a'_{29})^{(5)}$$

In the same way , one can obtain

$$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2/(a'_{30})^{(5)}$$

If $G_{29}$ *or* $G_{30}$ is bounded, the same property follows for $G_{28}$ , $G_{30}$ and $G_{28}$ , $G_{29}$ respectively.

**Remark 24:** If $G_{28}$ *is* bounded, from below, the same property holds for $G_{29}$ *and* $G_{30}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{29}$ is bounded from below. 241

**Remark 25:** If $T_{28}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}((G_{31})(t),t)) = (b'_{29})^{(5)}$ then $T_{29} \to \infty$. 242

**Definition of** $(m)^{(5)}$ and $\varepsilon_5$ :

Indeed let $t_5$ be so that for $t > t_5$

$$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t),t) < \varepsilon_5, T_{28}(t) > (m)^{(5)}$$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right)$, $t = log\frac{2}{\varepsilon_5}$ By taking now $\varepsilon_5$ sufficiently small one sees that $T_{29}$ is unbounded.
The same property holds for $T_{30}$ if $\lim_{t\to\infty}(b''_{30})^{(5)}\left((G_{31})(t),t\right) = (b'_{30})^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{33}$ , $G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}$ , $\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}} < 1$ and to choose 244

$(\widehat{P}_{32})^{(6)}$ and $(\widehat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\left[(\widehat{P}_{32})^{(6)} + \left((\widehat{P}_{32})^{(6)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{32})^{(6)} \quad 245$$

$$\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\left[\left((\widehat{Q}_{32})^{(6)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{32})^{(6)}\right] \leq (\widehat{Q}_{32})^{(6)} \quad 246$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric 247

$$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{35}}),(\widetilde{T_{35}})$ : $\left((\widetilde{G_{35}}),(\widetilde{T_{35}})\right)=\mathcal{A}^{(6)}\left((G_{35}),(T_{35})\right)$

It results

$$\left|\tilde{G}_{32}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{32})^{(6)}\left|G_{33}^{(1)}-G_{33}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\,ds_{(32)}+$$

$$\int_0^t\{(a'_{32})^{(6)}\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$G_{32}^{(2)}\left|(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)-(a''_{32})^{(6)}\left(T_{33}^{(2)},s_{(32)}\right)\right|\ e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\}ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|(G_{35})^{(1)}-(G_{35})^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}t}\le \qquad 248$$
$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)}+(a'_{32})^{(6)}+(\widehat{A}_{32})^{(6)}+(\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}\right)d\left(((G_{35})^{(1)},(T_{35})^{(1)};\ (G_{35})^{(2)},(T_{35})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 26:** The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ *and* $(\widehat{Q}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ respectively of $\mathbb{R}_+$. 249

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i=32,33,34$ depend only on $T_{33}$ and respectively on $(G_{35})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 27:** There does not exist any $t$ where $G_i(t)=0\ and\ T_i(t)=0$ 250

it results

$$G_i(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(6)}-(a_i'')^{(6)}(T_{33}(s_{(32)}),s_{(32)})\}ds_{(32)}\right]}\ge 0$$

$$T_i(t)\ge T_i^0e^{\left(-(b_i')^{(6)}t\right)}>0\quad \text{for t}>0$$

**Definition of** $\left((\widehat{M}_{32})^{(6)}\right)_1,\left((\widehat{M}_{32})^{(6)}\right)_2$ *and* $\left((\widehat{M}_{32})^{(6)}\right)_3$ : 251

**Remark 28:** if $G_{32}$ is bounded, the same property have also $G_{33}\ and\ G_{34}$ . indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a'_{33})^{(6)}G_{33}$ and by integrating

$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_1/(a'_{33})^{(6)}$

In the same way , one can obtain

$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_2/(a'_{34})^{(6)}$

If $G_{33}\ or\ G_{34}$ is bounded, the same property follows for $G_{32}$ , $G_{34}$ and $G_{32}$ , $G_{33}$ respectively.

**Remark 29:** If $G_{32}$ *is* bounded, from below, the same property holds for $G_{33}\ and\ G_{34}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{33}$ is bounded from below. 252

**Remark 30:** If $T_{32}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}\ ((G_{35})(t),t)) = (b'_{33})^{(6)}$ then $T_{33} \to \infty$. 253

**Definition of** $(m)^{(6)}$ and $\varepsilon_6$ :

Indeed let $t_6$ be so that for $t > t_6$

$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t),t\right) < \varepsilon_6, T_{32}(t) > (m)^{(6)}$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right),\quad t = log\frac{2}{\varepsilon_6}$ By taking now $\varepsilon_6$ sufficiently small one sees that $T_{33}$ is unbounded.
The same property holds for $T_{34}$ if $\lim_{t\to\infty}(b''_{34})^{(6)}\left((G_{35})(t),t(t),t\right) = (b'_{34})^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{37}, G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}$ , $\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}} < 1$ and to choose
$(\widehat{P}_{36})^{(7)}$ and $(\widehat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[(\widehat{P}_{36})^{(7)} + \left((\widehat{P}_{36})^{(7)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{36})^{(7)} \qquad 256$$

$$\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[\left((\widehat{Q}_{36})^{(7)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{36})^{(7)}\right] \leq (\widehat{Q}_{36})^{(7)} \qquad 257$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying

Equations into itself
The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric 258

$$d\left(((G_{39})^{(1)},(T_{39})^{(1)}),((G_{39})^{(2)},(T_{39})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\widehat{M}_{36})^{(7)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\widehat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

**Definition of** $\widetilde{(G_{39})},\widetilde{(T_{39})}:\left(\widetilde{(G_{39})},\widetilde{(T_{39})}\right)=\mathcal{A}^{(7)}((G_{39}),(T_{39}))$
It results

$$|\tilde{G}_{36}^{(1)}-\tilde{G}_i^{(2)}|\le\int_0^t(a_{36})^{(7)}|G_{37}^{(1)}-G_{37}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}ds_{(36)}+$$
$$\int_0^t\{(a'_{36})^{(7)}|G_{36}^{(1)}-G_{36}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$(a''_{36})^{(7)}(T_{37}^{(1)},s_{(36)})|G_{36}^{(1)}-G_{36}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$G_{36}^{(2)}|(a''_{36})^{(7)}(T_{37}^{(1)},s_{(36)})-(a''_{36})^{(7)}(T_{37}^{(2)},s_{(36)})|\ e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\}ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on it follows

$$|(G_{39})^{(1)}-(G_{39})^{(2)}|e^{-(\widehat{M}_{36})^{(7)}t}\le$$ 259
$$\frac{1}{(\widehat{M}_{36})^{(7)}}((a_{36})^{(7)}+(a'_{36})^{(7)}+(\widehat{A}_{36})^{(7)}+(\widehat{P}_{36})^{(7)}(\widehat{k}_{36})^{(7)})d\left(((G_{39})^{(1)},(T_{39})^{(1)};\ (G_{39})^{(2)},(T_{39})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 31:** The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ *and* $(\widehat{Q}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ respectively of $\mathbb{R}_+$. 260

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(7)}$ and $(b_i'')^{(7)}, i=36,37,38$ depend only on $T_{37}$ and respectively on $(G_{39})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 32:** There does not exist any $t$ where $G_i(t)=0\ and\ T_i(t)=0$ 261

it results

$$G_i(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(7)}-(a_i'')^{(7)}(T_{37}(s_{(36)}),s_{(36)})\}ds_{(36)}\right]}\ge 0$$

$$T_i(t)\ge T_i^0e^{(-(b_i')^{(7)}t)}>0\quad \text{for t}>0$$

**Definition of** $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2$ *and* $\left((\widehat{M}_{36})^{(7)}\right)_3$ : 262

**Remark 33:** if $G_{36}$ is bounded, the same property have also $G_{37}$ *and* $G_{38}$ . indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a'_{37})^{(7)} G_{37}$ and by integrating

$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1/(a'_{37})^{(7)}$

In the same way , one can obtain

$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2/(a'_{38})^{(7)}$

If $G_{37}$ *or* $G_{38}$ is bounded, the same property follows for $G_{36}$ , $G_{38}$ and $G_{36}$ , $G_{37}$ respectively.

**Remark 34:** If $G_{36}$ *is* bounded, from below, the same property holds for $G_{37}$ *and* $G_{38}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{37}$ is bounded from below. 263

**Remark 35:** If $T_{36}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(7)}((G_{39})(t),t)) = (b'_{37})^{(7)}$ then $T_{37} \to \infty$. 264

**Definition of** $(m)^{(7)}$ and $\varepsilon_7$ :

Indeed let $t_7$ be so that for $t > t_7$

$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t),t) < \varepsilon_7, T_{36}(t) > (m)^{(7)}$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right), \quad t = log\frac{2}{\varepsilon_7}$ By taking now $\varepsilon_7$ sufficiently small one sees that $T_{37}$ is unbounded.
The same property holds for $T_{38}$ if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t),t\right) = (b'_{38})^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}$ , $\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1$ and to choose 266
$(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ large to have

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[(\hat{P}_{40})^{(8)} + \left((\hat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{40})^{(8)} \quad 267$$

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$$\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{40})^{(8)}\right]\leq(\hat{Q}_{40})^{(8)}$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)},(T_{43})^{(1)}),((G_{43})^{(2)},(T_{43})^{(2)})\right)= \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

**Definition of** $(\widetilde{G_{43}}),(\widetilde{T_{43}})$ : $\left((\widetilde{G_{43}}),(\widetilde{T_{43}})\right)=\mathcal{A}^{(8)}((G_{43}),(T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{40})^{(8)}\left|G_{41}^{(1)}-G_{41}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\,ds_{(40)}+$$
$$\int_0^t\{(a'_{40})^{(8)}\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$G_{40}^{(2)}\left|(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)-(a''_{40})^{(8)}\left(T_{41}^{(2)},s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\}ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)}-(G_{43})^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}t}\leq \quad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)}+(a'_{40})^{(8)}+(\widehat{A}_{40})^{(8)}+(\widehat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}\right)d\left(((G_{43})^{(1)},(T_{43})^{(1)};\ (G_{43})^{(2)},(T_{43})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 36:** The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ *and* $(\widehat{Q}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ respectively of $\mathbb{R}_+$. 274

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i=40{,}41{,}42$ depend only on $\mathrm{T}_{41}$ and respectively on $(G_{43})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 37** There does not exist any $t$ where $G_i(t)=0$ *and* $T_i(t)=0$ 275

it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(8)} - (a_i'')^{(8)}(T_{41}(s_{(40)}), s_{(40)})\} ds_{(40)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{(-(b_i')^{(8)} t)} > 0$ for t > 0

**Definition of** $\left((\widehat{M}_{40})^{(8)}\right)_1, \left((\widehat{M}_{40})^{(8)}\right)_2$ *and* $\left((\widehat{M}_{40})^{(8)}\right)_3$ : 276

**Remark 38:** if $G_{40}$ is bounded, the same property have also $G_{41}$ *and* $G_{42}$ . indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a_{41}')^{(8)} G_{41}$ and by integrating

$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)} \left((\widehat{M}_{40})^{(8)}\right)_1 / (a_{41}')^{(8)}$

In the same way , one can obtain

$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)} \left((\widehat{M}_{40})^{(8)}\right)_2 / (a_{42}')^{(8)}$

If $G_{41}$ *or* $G_{42}$ is bounded, the same property follows for $G_{40}$ , $G_{42}$ and $G_{40}$ , $G_{41}$ respectively.

**Remark 39:** If $G_{40}$ *is* bounded, from below, the same property holds for $G_{41}$ *and* $G_{42}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{41}$ is bounded from below. 277

**Remark 40:** If $T_{40}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}((G_{43})(t), t)) = (b_{41}')^{(8)}$ then $T_{41} \to \infty$. 278

**Definition of** $(m)^{(8)}$ and $\varepsilon_8$ :

Indeed let $t_8$ be so that for $t > t_8$

$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t), t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right), \quad t = log\frac{2}{\varepsilon_8}$ By taking now $\varepsilon_8$ sufficiently small one sees that $T_{41}$ is unbounded.
The same property holds for $T_{42}$ if $\lim_{t\to\infty}(b_{42}'')^{(8)}\left((G_{43})(t), t(t), t\right) = (b_{42}')^{(8)}$

It is now sufficient to take $\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}$ , $\frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$ and to choose $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ large to have 279 A

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}\left[(\hat{P}_{44})^{(9)} + \left((\hat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{44})^{(9)} + G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[\left((\hat{Q}_{44})^{(9)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{44})^{(9)}\right]\leq(\hat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)},(T_{47})^{(1)}),\left((G_{47})^{(2)},(T_{47})^{(2)}\right)\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

**Definition of** $\widetilde{(G_{47})},\widetilde{(T_{47})}:\left(\widetilde{(G_{47})},\widetilde{(T_{47})}\right)=\mathcal{A}^{(9)}\left((G_{47}),(T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{44})^{(9)}\left|G_{45}^{(1)}-G_{45}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\,ds_{(44)}+$$
$$\int_0^t\{(a'_{44})^{(9)}\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$G_{44}^{(2)}\left|(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)-(a''_{44})^{(9)}\left(T_{45}^{(2)},s_{(44)}\right)\right|\;e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\}ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}t}\leq$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)}+(a'_{44})^{(9)}+(\widehat{A}_{44})^{(9)}+(\widehat{P}_{44})^{(9)}(\hat{k}_{44})^{(9)}\right)d\left(((G_{47})^{(1)},(T_{47})^{(1)};\;(G_{47})^{(2)},(T_{47})^{(2)})\right)$$

And analogous inequalities for $G_i\;and\;T_i$. Taking into account the hypothesis (39,35,36) the result follows

**Remark 41:** The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}\;and\;(\widehat{Q}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i=44,45,46$ depend only on $T_{45}$ and respectively on $(G_{47})(and\;not\;on\;\;t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 42:** There does not exist any $t$ where $G_i\,(t)=0\;and\;T_i\,(t)=0$

From 99 to 44 it results

$$G_i\,(t)\geq G_i^0e^{\left[-\int_0^t\{(a_i')^{(9)}-(a_i'')^{(9)}(T_{45}(s_{(44)}),s_{(44)})\}ds_{(44)}\right]}\geq 0$$

$T_i(t) \geq T_i^0 e^{(-(b_i')^{(9)}t)} > 0$ for t > 0

**Definition of** $\left((\widehat{M}_{44})^{(9)}\right)_1, \left((\widehat{M}_{44})^{(9)}\right)_2$ *and* $\left((\widehat{M}_{44})^{(9)}\right)_3$ :

**Remark 43:** if $G_{44}$ is bounded, the same property have also $G_{45}$ *and* $G_{46}$ . indeed if

$G_{44} < (\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt} \leq \left((\widehat{M}_{44})^{(9)}\right)_1 - (a_{45}')^{(9)} G_{45}$ and by integrating

$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_1/(a_{45}')^{(9)}$

In the same way , one can obtain

$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_2/(a_{46}')^{(9)}$

If $G_{45}$ *or* $G_{46}$ is bounded, the same property follows for $G_{44}$ , $G_{46}$ and $G_{44}$ , $G_{45}$ respectively.

**Remark 44:** If $G_{44}$ *is* bounded, from below, the same property holds for $G_{45}$ *and* $G_{46}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{45}$ is bounded from below.

**Remark 45:** If $T_{44}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t),t\right) = (b_{45}')^{(9)}$ then $T_{45} \to \infty$.

**Definition of** $(m)^{(9)}$ and $\varepsilon_9$ :

Indeed let $t_9$ be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t),t\right) < \varepsilon_9, T_{44}(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right), \quad t = log\frac{2}{\varepsilon_9}$ By taking now $\varepsilon_9$ sufficiently small one sees that $T_{45}$ is unbounded.
The same property holds for $T_{46}$ if $\lim_{t\to\infty}(b_{46}'')^{(9)}\left((G_{47})(t),t\right) = (b_{46}')^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

**Behavior of the solutions of equation** 280

**Theorem** If we denote and define

**Definition of** $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ :

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)} \leq -(a_{13}')^{(1)} + (a_{14}')^{(1)} - (a_{13}'')^{(1)}(T_{14},t) + (a_{14}'')^{(1)}(T_{14},t) \leq -(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)} \leq -(b_{13}')^{(1)} + (b_{14}')^{(1)} - (b_{13}'')^{(1)}(G,t) - (b_{14}'')^{(1)}(G,t) \leq -(\tau_1)^{(1)}$

**Definition of** $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$ : 281

By $(v_1)^{(1)} > 0$ , $(v_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$ , $(u_2)^{(1)} < 0$ the roots of the equations $(a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

**Definition of** $(\bar{v}_1)^{(1)}$,, $(\bar{v}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$ : 282

By $(\bar{v}_1)^{(1)} > 0$ , $(\bar{v}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$ , $(\bar{u}_2)^{(1)} < 0$ the roots of the equations $(a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

**Definition of** $(m_1)^{(1)}$ , $(m_2)^{(1)}$ , $(\mu_1)^{(1)}, (\mu_2)^{(1)}, (v_0)^{(1)}$ :- 283

If we define $(m_1)^{(1)}$ , $(m_2)^{(1)}$ , $(\mu_1)^{(1)}, (\mu_2)^{(1)}$ by

$(m_2)^{(1)} = (v_0)^{(1)}, (m_1)^{(1)} = (v_1)^{(1)}$, $if\ (v_0)^{(1)} < (v_1)^{(1)}$

$(m_2)^{(1)} = (v_1)^{(1)}, (m_1)^{(1)} = (\bar{v}_1)^{(1)}$ , $if\ (v_1)^{(1)} < (v_0)^{(1)} < (\bar{v}_1)^{(1)}$,

and $\boxed{(v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$

$(m_2)^{(1)} = (v_1)^{(1)}, (m_1)^{(1)} = (v_0)^{(1)}$, $if\ (\bar{v}_1)^{(1)} < (v_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)}$, $if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}$ , $if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)}$,

and $\boxed{(u_0)^{(1)} = \frac{T_{13}^0}{T_{14}^0}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$

are defined

Then the solution of global equations satisfies the inequalities 285

$$G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{13}(t) \le G_{13}^0 e^{(S_1)^{(1)}t}$$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)}t}$$

$$\left(\frac{(a_{15})^{(1)} G_{13}^0}{(m_1)^{(1)}((S_1)^{(1)} - (p_{13})^{(1)} - (S_2)^{(1)})}\left[e^{((S_1)^{(1)} - (p_{13})^{(1)})t} - e^{-(S_2)^{(1)}t}\right] + G_{15}^0 e^{-(S_2)^{(1)}t} \le G_{15}(t) \le\right. \quad 286$$

$$\left.\frac{(a_{15})^{(1)} G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)} - (a'_{15})^{(1)})}\left[e^{(S_1)^{(1)}t} - e^{-(a'_{15})^{(1)}t}\right] + G_{15}^0 e^{-(a'_{15})^{(1)}t}\right)$$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)}t} \le T_{13}(t) \le T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t}} \quad 287$$

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)}t} \le T_{13}(t) \le \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t} \quad 288$$

$\frac{(b_{15})^{(1)}T_{13}^{0}}{(\mu_1)^{(1)}((R_1)^{(1)}-(b'_{15})^{(1)})}\left[e^{(R_1)^{(1)}t}-e^{-(b'_{15})^{(1)}t}\right]+T_{15}^{0}e^{-(b'_{15})^{(1)}t}\leq T_{15}(t)\leq$ 289

$\frac{(a_{15})^{(1)}T_{13}^{0}}{(\mu_2)^{(1)}((R_1)^{(1)}+(r_{13})^{(1)}+(R_2)^{(1)})}\left[e^{((R_1)^{(1)}+(r_{13})^{(1)})t}-e^{-(R_2)^{(1)}t}\right]+T_{15}^{0}e^{-(R_2)^{(1)}t}$

**Definition of** $(S_1)^{(1)},(S_2)^{(1)},(R_1)^{(1)},(R_2)^{(1)}$:- 290

Where $(S_1)^{(1)}=(a_{13})^{(1)}(m_2)^{(1)}-(a'_{13})^{(1)}$

$(S_2)^{(1)}=(a_{15})^{(1)}-(p_{15})^{(1)}$

$(R_1)^{(1)}=(b_{13})^{(1)}(\mu_2)^{(1)}-(b'_{13})^{(1)}$

$(R_2)^{(1)}=(b'_{15})^{(1)}-(r_{15})^{(1)}$

**Behavior of the solutions of equation** 291

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(2)},(\sigma_2)^{(2)},(\tau_1)^{(2)},(\tau_2)^{(2)}$ : 292

$(\sigma_1)^{(2)},(\sigma_2)^{(2)},(\tau_1)^{(2)},(\tau_2)^{(2)}$ four constants satisfying
$-(\sigma_2)^{(2)}\leq-(a'_{16})^{(2)}+(a'_{17})^{(2)}-(a''_{16})^{(2)}(\mathrm{T}_{17},t)+(a''_{17})^{(2)}(\mathrm{T}_{17},t)\leq-(\sigma_1)^{(2)}$ 293

$-(\tau_2)^{(2)}\leq-(b'_{16})^{(2)}+(b'_{17})^{(2)}-(b''_{16})^{(2)}\big((G_{19}),t\big)-(b''_{17})^{(2)}\big((G_{19}),t\big)\leq-(\tau_1)^{(2)}$ 294

**Definition of** $(\nu_1)^{(2)},(\nu_2)^{(2)},(u_1)^{(2)},(u_2)^{(2)}$ : 295

By $(\nu_1)^{(2)}>0$ , $(\nu_2)^{(2)}<0$ and respectively $(u_1)^{(2)}>0$ , $(u_2)^{(2)}<0$ the roots 296

of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2+(\sigma_1)^{(2)}\nu^{(2)}-(a_{16})^{(2)}=0$ 297

and $(b_{14})^{(2)}\big(u^{(2)}\big)^2+(\tau_1)^{(2)}u^{(2)}-(b_{16})^{(2)}=0$ and 298

**Definition of** $(\bar{\nu}_1)^{(2)},,(\bar{\nu}_2)^{(2)},(\bar{u}_1)^{(2)},(\bar{u}_2)^{(2)}$ : 299

By $(\bar{\nu}_1)^{(2)}>0$ , $(\bar{\nu}_2)^{(2)}<0$ and respectively $(\bar{u}_1)^{(2)}>0$ , $(\bar{u}_2)^{(2)}<0$ the 300

roots of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2+(\sigma_2)^{(2)}\nu^{(2)}-(a_{16})^{(2)}=0$ 301

and $(b_{17})^{(2)}\big(u^{(2)}\big)^2+(\tau_2)^{(2)}u^{(2)}-(b_{16})^{(2)}=0$ 302

**Definition of** $(m_1)^{(2)},(m_2)^{(2)},(\mu_1)^{(2)},(\mu_2)^{(2)}$ :- 303

If we define $(m_1)^{(2)},(m_2)^{(2)},(\mu_1)^{(2)},(\mu_2)^{(2)}$ by 304

$(m_2)^{(2)}=(\nu_0)^{(2)},(m_1)^{(2)}=(\nu_1)^{(2)},$ ***if*** $(\nu_0)^{(2)}<(\nu_1)^{(2)}$ 305

$(m_2)^{(2)}=(\nu_1)^{(2)},(m_1)^{(2)}=(\bar{\nu}_1)^{(2)}$ , ***if*** $(\nu_1)^{(2)}<(\nu_0)^{(2)}<(\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)},\ \boldsymbol{if}\ (u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}, \boldsymbol{if}\ (u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)},$

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)},\ \boldsymbol{if}\ (\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)} t}$

$(p_i)^{(2)}$ is defined by equation

$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)} t}$ 311

$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)} t}\right] + G_{18}^0 e^{-(S_2)^{(2)} t} \le G_{18}(t) \le\right.$ 312

$\left.\frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)} t} - e^{-(a'_{18})^{(2)} t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)} t}\right)$

$\boxed{T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}}$ 313

$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}$ 314

$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)} t} - e^{-(b'_{18})^{(2)} t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)} t} \le T_{18}(t) \le$ 315

$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})}\left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)} t}\right] + T_{18}^0 e^{-(R_2)^{(2)} t}$

**Definition of** $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$

$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)}$ 318

$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$

**Behavior of the solutions** 319

**Theorem 3:** If we denote and define

**Definition of** $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ :

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}\big((G_{23}), t\big) \le -(\tau_1)^{(3)}$$

**Definition of** $(v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$ : 320

By $(v_1)^{(3)} > 0$ , $(v_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$ , $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\big(v^{(3)}\big)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{v}_1)^{(3)} > 0$ , $(\bar{v}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$ , $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\big(v^{(3)}\big)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

**Definition of** $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ :- 321

If we define $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (v_0)^{(3)}, (m_1)^{(3)} = (v_1)^{(3)}$, ***if*** $(v_0)^{(3)} < (v_1)^{(3)}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (\bar{v}_1)^{(3)}$ , ***if*** $(v_1)^{(3)} < (v_0)^{(3)} < (\bar{v}_1)^{(3)}$,

and $\boxed{(v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_0)^{(3)}$, ***if*** $(\bar{v}_1)^{(3)} < (v_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}$, ***if*** $(u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$ , ***if*** $(u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}$, and $\boxed{(u_0)^{(3)} = \frac{T^0_{20}}{T^0_{21}}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}$, ***if*** $(\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$$G^0_{20}e^{((s_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G^0_{20}e^{(s_1)^{(3)}t}$$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)}-(p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)}t} \quad 323$$

$$\left(\frac{(a_{22})^{(3)}G_{20}^0}{(m_1)^{(3)}((S_1)^{(3)}-(p_{20})^{(3)}-(S_2)^{(3)})}\left[e^{((S_1)^{(3)}-(p_{20})^{(3)})t} - e^{-(S_2)^{(3)}t}\right] + G_{22}^0 e^{-(S_2)^{(3)}t} \le G_{22}(t) \le \right. \quad 324$$

$$\left. \frac{(a_{22})^{(3)}G_{20}^0}{(m_2)^{(3)}((S_1)^{(3)}-(a'_{22})^{(3)})}\left[e^{(S_1)^{(3)}t} - e^{-(a'_{22})^{(3)}t}\right] + G_{22}^0 e^{-(a'_{22})^{(3)}t}\right)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)}+(r_{20})^{(3)})t}} \quad 325$$

$$\frac{1}{(\mu_1)^{(3)}} T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}} T_{20}^0 e^{((R_1)^{(3)}+(r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)}T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)}-(b'_{22})^{(3)})}\left[e^{(R_1)^{(3)}t} - e^{-(b'_{22})^{(3)}t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)}t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)}T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)}+(r_{20})^{(3)}+(R_2)^{(3)})}\left[e^{((R_1)^{(3)}+(r_{20})^{(3)})t} - e^{-(R_2)^{(3)}t}\right] + T_{22}^0 e^{-(R_2)^{(3)}t}$$

**Definition of** $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

**Behavior of the solutions of equation**

**Theorem:** If we denote and define

**Definition of** $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ :

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$

$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}((G_{27}), t) - (b''_{25})^{(4)}((G_{27}), t) \le -(\tau_1)^{(4)}$

**Definition of** $(v_1)^{(4)}, (v_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, v^{(4)}, u^{(4)}$ : 329

By $(v_1)^{(4)} > 0$ , $(v_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$ , $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_1)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

**Definition of** $(\bar{v}_1)^{(4)}, , (\bar{v}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$ : 330

By $(\bar{v}_1)^{(4)} > 0$ , $(\bar{v}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$ , $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

**Definition of** $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}, (\nu_0)^{(4)}$ :-

If we define $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (\nu_0)^{(4)}, (m_1)^{(4)} = (\nu_1)^{(4)}$, ***if*** $(\nu_0)^{(4)} < (\nu_1)^{(4)}$

$(m_2)^{(4)} = (\nu_1)^{(4)}, (m_1)^{(4)} = (\bar{\nu}_1)^{(4)}$, ***if*** $(\nu_4)^{(4)} < (\nu_0)^{(4)} < (\bar{\nu}_1)^{(4)}$,

and $\boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (\nu_4)^{(4)}, (m_1)^{(4)} = (\nu_0)^{(4)}$, ***if*** $(\bar{\nu}_4)^{(4)} < (\nu_0)^{(4)}$

and analogously

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$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)}$, ***if*** $(u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}$, ***if*** $(u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)}$,

and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}$, ***if*** $(\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

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$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)} t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)} t} \quad 333$$

$$\left( \frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})} \left[ e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)} t} \right] + G_{26}^0 e^{-(S_2)^{(4)} t} \le G_{26}(t) \le \right. \quad 334$$

*(a26)4G240(m2)4(S1)4−(a26′)4e(S1)4t−e−(a26′)4t+ G260e−(a26′)4t*

$$\boxed{T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}} T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}} T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t} \quad 335$$

$$\frac{(b_{26})^{(4)} T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)} - (b'_{26})^{(4)})} \left[ e^{(R_1)^{(4)} t} - e^{-(b'_{26})^{(4)} t} \right] + T_{26}^0 e^{-(b'_{26})^{(4)} t} \le T_{26}(t) \le \quad 336$$

$$\frac{(a_{26})^{(4)} T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)} + (r_{24})^{(4)} + (R_2)^{(4)})} \left[ e^{((R_1)^{(4)} + (r_{24})^{(4)})t} - e^{-(R_2)^{(4)} t} \right] + T_{26}^0 e^{-(R_2)^{(4)} t}$$

**Definition of** $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)} (m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$$

$$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$$

**Behavior of the solutions of equation** 338

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ :

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}((G_{31}), t) - (b''_{29})^{(5)}((G_{31}), t) \le -(\tau_1)^{(5)}$$

**Definition of** $(\nu_1)^{(5)}, (\nu_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, \nu^{(5)}, u^{(5)}$ : 339

By $(\nu_1)^{(5)} > 0$ , $(\nu_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$ , $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\left(u^{(5)}\right)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(5)}, , (\bar{\nu}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$ : 340

By $(\bar{\nu}_1)^{(5)} > 0$ , $(\bar{\nu}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$ , $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\left(u^{(5)}\right)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

**Definition of** $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (\nu_0)^{(5)}$ :-

If we define $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$$(m_2)^{(5)} = (\nu_0)^{(5)}, (m_1)^{(5)} = (\nu_1)^{(5)}, \ \boldsymbol{if}\ (\nu_0)^{(5)} < (\nu_1)^{(5)}$$

$$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\bar{\nu}_1)^{(5)}, \boldsymbol{if}\ (\nu_1)^{(5)} < (\nu_0)^{(5)} < (\bar{\nu}_1)^{(5)},$$
and $\boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}}$

$$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$$

and analogously 341

$$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)}, \ \boldsymbol{if}\ (u_0)^{(5)} < (u_1)^{(5)}$$

$$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}, \boldsymbol{if}\ (u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)},$$
and $\boxed{(u_0)^{(5)} = \frac{T^0_{28}}{T^0_{29}}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}, \boldsymbol{if}\ (\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$$G_{28}^{0}e^{((S_1)^{(5)}-(p_{28})^{(5)})t} \le G_{28}(t) \le G_{28}^{0}e^{(S_1)^{(5)}t}$$

where $(p_i)^{(5)}$ is defined by equation

$$\frac{1}{(m_5)^{(5)}}G_{28}^{0}e^{((S_1)^{(5)}-(p_{28})^{(5)})t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}}G_{28}^{0}e^{(S_1)^{(5)}t} \quad 343$$

$$\left(\frac{(a_{30})^{(5)}G_{28}^{0}}{(m_1)^{(5)}((S_1)^{(5)}-(p_{28})^{(5)}-(S_2)^{(5)})}\left[e^{((S_1)^{(5)}-(p_{28})^{(5)})t}-e^{-(S_2)^{(5)}t}\right]+G_{30}^{0}e^{-(S_2)^{(5)}t} \le G_{30}(t) \le \right. \quad 344$$

*(a30)5G280(m2)5(S1)5−(a30′)5e(S1)5t−e−(a30′)5t+ G300e−(a30′)5t*

$$\boxed{T_{28}^{0}e^{(R_1)^{(5)}t} \le T_{28}(t) \le T_{28}^{0}e^{((R_1)^{(5)}+(r_{28})^{(5)})t}} \quad 345$$

$$\frac{1}{(\mu_1)^{(5)}}T_{28}^{0}e^{(R_1)^{(5)}t} \le T_{28}(t) \le \frac{1}{(\mu_2)^{(5)}}T_{28}^{0}e^{((R_1)^{(5)}+(r_{28})^{(5)})t} \quad 346$$

$$\frac{(b_{30})^{(5)}T_{28}^{0}}{(\mu_1)^{(5)}((R_1)^{(5)}-(b'_{30})^{(5)})}\left[e^{(R_1)^{(5)}t}-e^{-(b'_{30})^{(5)}t}\right]+T_{30}^{0}e^{-(b'_{30})^{(5)}t} \le T_{30}(t) \le \quad 347$$

$$\frac{(a_{30})^{(5)}T_{28}^{0}}{(\mu_2)^{(5)}((R_1)^{(5)}+(r_{28})^{(5)}+(R_2)^{(5)})}\left[e^{((R_1)^{(5)}+(r_{28})^{(5)})t}-e^{-(R_2)^{(5)}t}\right]+T_{30}^{0}e^{-(R_2)^{(5)}t}$$

**Definition of** $(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$:- 348

Where $(S_1)^{(5)} = (a_{28})^{(5)}(m_2)^{(5)} - (a'_{28})^{(5)}$

$(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}$

$(R_1)^{(5)} = (b_{28})^{(5)}(\mu_2)^{(5)} - (b'_{28})^{(5)}$

$(R_2)^{(5)} = (b'_{30})^{(5)} - (r_{30})^{(5)}$

**Behavior of the solutions of equation** 349

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ :

$(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)} \le -(a'_{32})^{(6)} + (a'_{33})^{(6)} - (a''_{32})^{(6)}(T_{33}, t) + (a''_{33})^{(6)}(T_{33}, t) \le -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)} \le -(b'_{32})^{(6)} + (b'_{33})^{(6)} - (b''_{32})^{(6)}\big((G_{35}), t\big) - (b''_{33})^{(6)}\big((G_{35}), t\big) \le -(\tau_1)^{(6)}$$

**Definition of** $(v_1)^{(6)}, (v_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, v^{(6)}, u^{(6)}$ : 350

By $(v_1)^{(6)} > 0$ , $(v_2)^{(6)} < 0$ and respectively $(u_1)^{(6)} > 0$ , $(u_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(v^{(6)}\big)^2 + (\sigma_1)^{(6)}v^{(6)} - (a_{32})^{(6)} = 0$

and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$ and

**Definition of** $(\bar{v}_1)^{(6)},,(\bar{v}_2)^{(6)},(\bar{u}_1)^{(6)},(\bar{u}_2)^{(6)}$ : 351

By $(\bar{v}_1)^{(6)} > 0$ , $(\bar{v}_2)^{(6)} < 0$ and respectively $(\bar{u}_1)^{(6)} > 0$ , $(\bar{u}_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\left(v^{(6)}\right)^2 + (\sigma_2)^{(6)}v^{(6)} - (a_{32})^{(6)} = 0$ and $(b_{33})^{(6)}\left(u^{(6)}\right)^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$

**Definition of** $(m_1)^{(6)}$ , $(m_2)^{(6)}$ , $(\mu_1)^{(6)},(\mu_2)^{(6)},(v_0)^{(6)}$ :-

If we define $(m_1)^{(6)}$ , $(m_2)^{(6)}$ , $(\mu_1)^{(6)},(\mu_2)^{(6)}$ by

$(m_2)^{(6)} = (v_0)^{(6)}, (m_1)^{(6)} = (v_1)^{(6)}$, ***if*** $(v_0)^{(6)} < (v_1)^{(6)}$

$(m_2)^{(6)} = (v_1)^{(6)}, (m_1)^{(6)} = (\bar{v}_6)^{(6)}$ , ***if*** $(v_1)^{(6)} < (v_0)^{(6)} < (\bar{v}_1)^{(6)}$,
and $\boxed{(v_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$

$(m_2)^{(6)} = (v_1)^{(6)}, (m_1)^{(6)} = (v_0)^{(6)}$, ***if*** $(\bar{v}_1)^{(6)} < (v_0)^{(6)}$

and analogously 352

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}$, ***if*** $(u_0)^{(6)} < (u_1)^{(6)}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}$ , ***if*** $(u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)}$,
and $\boxed{(u_0)^{(6)} = \frac{T_{32}^0}{T_{33}^0}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}$, ***if*** $(\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$

Then the solution of global equations satisfies the inequalities 353

$$G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{32}(t) \le G_{32}^0 e^{(S_1)^{(6)}t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}} G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{33}(t) \le \frac{1}{(m_2)^{(6)}} G_{32}^0 e^{(S_1)^{(6)}t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)} G_{32}^0}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})}\left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)}t}\right] + G_{34}^0 e^{-(S_2)^{(6)}t} \le G_{34}(t) \le \right. \quad 355$$

*(a34)6G320(m2)6(S1)6−(a34′)6e(S1)6t−e−(a34′)6t+ G340e−(a34′)6t*

$$\boxed{T_{32}^0 e^{(R_1)^{(6)}t} \le T_{32}(t) \le T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}} T_{32}^0 e^{(R_1)^{(6)}t} \le T_{32}(t) \le \frac{1}{(\mu_2)^{(6)}} T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)} T_{32}^0}{(\mu_1)^{(6)}((R_1)^{(6)} - (b'_{34})^{(6)})}\left[e^{(R_1)^{(6)}t} - e^{-(b'_{34})^{(6)}t}\right] + T_{34}^0 e^{-(b'_{34})^{(6)}t} \le T_{34}(t) \le \quad 358$$

$$\frac{(a_{34})^{(6)} T_{32}^0}{(\mu_2)^{(6)}((R_1)^{(6)} + (r_{32})^{(6)} + (R_2)^{(6)})}\left[e^{((R_1)^{(6)} + (r_{32})^{(6)})t} - e^{-(R_2)^{(6)}t}\right] + T_{34}^0 e^{-(R_2)^{(6)}t}$$

**Definition of** $(S_1)^{(6)}, (S_2)^{(6)}, (R_1)^{(6)}, (R_2)^{(6)}$:- 359

Where $(S_1)^{(6)} = (a_{32})^{(6)}(m_2)^{(6)} - (a'_{32})^{(6)}$

$(S_2)^{(6)} = (a_{34})^{(6)} - (p_{34})^{(6)}$

$(R_1)^{(6)} = (b_{32})^{(6)}(\mu_2)^{(6)} - (b'_{32})^{(6)}$

$(R_2)^{(6)} = (b'_{34})^{(6)} - (r_{34})^{(6)}$

**Behavior of the solutions of equation**

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$ :

$(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)} \le -(a'_{36})^{(7)} + (a'_{37})^{(7)} - (a''_{36})^{(7)}(T_{37}, t) + (a''_{37})^{(7)}(T_{37}, t) \le -(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)} \le -(b'_{36})^{(7)} + (b'_{37})^{(7)} - (b''_{36})^{(7)}\big((G_{39}), t\big) - (b''_{37})^{(7)}\big((G_{39}), t\big) \le -(\tau_1)^{(7)}$$

**Definition of** $(\nu_1)^{(7)}, (\nu_2)^{(7)}, (u_1)^{(7)}, (u_2)^{(7)}, \nu^{(7)}, u^{(7)}$ : 361

By $(\nu_1)^{(7)} > 0$, $(\nu_2)^{(7)} < 0$ and respectively $(u_1)^{(7)} > 0$, $(u_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_1)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(7)},, (\bar{\nu}_2)^{(7)}, (\bar{u}_1)^{(7)}, (\bar{u}_2)^{(7)}$ : 362

By $(\bar{\nu}_1)^{(7)} > 0$, $(\bar{\nu}_2)^{(7)} < 0$ and respectively $(\bar{u}_1)^{(7)} > 0$, $(\bar{u}_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

**Definition of** $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (\nu_0)^{(7)}$ :-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$(m_2)^{(7)} = (\nu_0)^{(7)}, (m_1)^{(7)} = (\nu_1)^{(7)}$, ***if*** $(\nu_0)^{(7)} < (\nu_1)^{(7)}$

$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\bar{\nu}_1)^{(7)}$, ***if*** $(\nu_1)^{(7)} < (\nu_0)^{(7)} < (\bar{\nu}_1)^{(7)}$,
and $\boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$

$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\nu_0)^{(7)}$, ***if*** $(\bar{\nu}_1)^{(7)} < (\nu_0)^{(7)}$

and analogously 363

$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)},\ \boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$

and $\boxed{(u_0)^{(7)} = \frac{T_{36}^0}{T_{37}^0}}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)}$ where $(u_1)^{(7)}, (\bar{u}_1)^{(7)}$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^0 e^{(S_1)^{(7)} t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)} t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)}((S_1)^{(7)} - (p_{36})^{(7)} - (S_2)^{(7)})}\left[e^{((S_1)^{(7)} - (p_{36})^{(7)})t} - e^{-(S_2)^{(7)} t}\right] + G_{38}^0 e^{-(S_2)^{(7)} t} \le G_{38}(t) \le \right. \quad 366$$

$$\left. \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)}((S_1)^{(7)} - (a'_{38})^{(7)})}\left[e^{(S_1)^{(7)} t} - e^{-(a'_{38})^{(7)} t}\right] + G_{38}^0 e^{-(a'_{38})^{(7)} t}\right)$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t} \quad 368$$

$$\frac{(b_{38})^{(7)} T_{36}^0}{(\mu_1)^{(7)}((R_1)^{(7)} - (b'_{38})^{(7)})}\left[e^{(R_1)^{(7)} t} - e^{-(b'_{38})^{(7)} t}\right] + T_{38}^0 e^{-(b'_{38})^{(7)} t} \le T_{38}(t) \le \quad 369$$

$$\frac{(a_{38})^{(7)} T_{36}^0}{(\mu_2)^{(7)}((R_1)^{(7)} + (r_{36})^{(7)} + (R_2)^{(7)})}\left[e^{((R_1)^{(7)} + (r_{36})^{(7)})t} - e^{-(R_2)^{(7)} t}\right] + T_{38}^0 e^{-(R_2)^{(7)} t}$$

**Definition of** $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$

$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$

$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$

**Behavior of the solutions of equation** 371

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ :

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$$-(\sigma_2)^{(8)} \le -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \le -(\sigma_1)^{(8)}$$

$$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\big((G_{43}), t\big) - (b''_{41})^{(8)}\big((G_{43}), t\big) \le -(\tau_1)^{(8)}$$

**Definition of** $(\nu_1)^{(8)}, (\nu_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, \nu^{(8)}, u^{(8)}$ : 372

By $(\nu_1)^{(8)} > 0, (\nu_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0, (u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(8)}, , (\bar{\nu}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$ :

By $(\bar{\nu}_1)^{(8)} > 0, (\bar{\nu}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0, (\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

**Definition of** $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}, (\nu_0)^{(8)}$ :-

If we define $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$(m_2)^{(8)} = (\nu_0)^{(8)}, (m_1)^{(8)} = (\nu_1)^{(8)}$, ***if*** $(\nu_0)^{(8)} < (\nu_1)^{(8)}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\bar{\nu}_1)^{(8)}$, ***if*** $(\nu_1)^{(8)} < (\nu_0)^{(8)} < (\bar{\nu}_1)^{(8)}$,
and $\boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\nu_0)^{(8)}$, ***if*** $(\bar{\nu}_1)^{(8)} < (\nu_0)^{(8)}$

and analogously 374

$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)}$, ***if*** $(u_0)^{(8)} < (u_1)^{(8)}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}$, ***if*** $(u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)}$,
and $\boxed{(u_0)^{(8)} = \frac{T^0_{40}}{T^0_{41}}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}$, ***if*** $(\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^{0}e^{((S_1)^{(8)}-(p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^{0}e^{(S_1)^{(8)}t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}}G_{40}^{0}e^{((S_1)^{(8)}-(p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}}G_{40}^{0}e^{(S_1)^{(8)}t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)}G_{40}^{0}}{(m_1)^{(8)}((S_1)^{(8)}-(p_{40})^{(8)}-(S_2)^{(8)})}\left[e^{((S_1)^{(8)}-(p_{40})^{(8)})t}-e^{-(S_2)^{(8)}t}\right]+G_{42}^{0}e^{-(S_2)^{(8)}t} \le G_{42}(t) \le \right. \quad 377$$

$$\left.\frac{(a_{42})^{(8)}G_{40}^{0}}{(m_2)^{(8)}((S_1)^{(8)}-(a'_{42})^{(8)})}[e^{(S_1)^{(8)}t}-e^{-(a'_{42})^{(8)}t}]+\ G_{42}^{0}e^{-(a'_{42})^{(8)}t}\right)$$

$$\boxed{T_{40}^{0}e^{(R_1)^{(8)}t} \le T_{40}(t) \le T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}}T_{40}^{0}e^{(R_1)^{(8)}t} \le T_{40}(t) \le \frac{1}{(\mu_2)^{(8)}}T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)}T_{40}^{0}}{(\mu_1)^{(8)}((R_1)^{(8)}-(b'_{42})^{(8)})}\left[e^{(R_1)^{(8)}t}-e^{-(b'_{42})^{(8)}t}\right]+T_{42}^{0}e^{-(b'_{42})^{(8)}t} \le T_{42}(t) \le \quad 380$$

$$\frac{(a_{42})^{(8)}T_{40}^{0}}{(\mu_2)^{(8)}((R_1)^{(8)}+(r_{40})^{(8)}+(R_2)^{(8)})}\left[e^{((R_1)^{(8)}+(r_{40})^{(8)})t}-e^{-(R_2)^{(8)}t}\right]+T_{42}^{0}e^{-(R_2)^{(8)}t}$$

**Definition of** $(S_1)^{(8)}, (S_2)^{(8)}, (R_1)^{(8)}, (R_2)^{(8)}$:- 381

Where $(S_1)^{(8)} = (a_{40})^{(8)}(m_2)^{(8)} - (a'_{40})^{(8)}$

$$(S_2)^{(8)} = (a_{42})^{(8)} - (p_{42})^{(8)}$$

$$(R_1)^{(8)} = (b_{40})^{(8)}(\mu_2)^{(8)} - (b'_{40})^{(8)}$$

$$(R_2)^{(8)} = (b'_{42})^{(8)} - (r_{42})^{(8)}$$

**Behavior of the solutions of equation 37 to 92** 382

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ :

$(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ four constants satisfying

$$-(\sigma_2)^{(9)} \le -(a'_{44})^{(9)} + (a'_{45})^{(9)} - (a''_{44})^{(9)}(T_{45}, t) + (a''_{45})^{(9)}(T_{45}, t) \le -(\sigma_1)^{(9)}$$

$$-(\tau_2)^{(9)} \le -(b'_{44})^{(9)} + (b'_{45})^{(9)} - (b''_{44})^{(9)}\big((G_{47}), t\big) - (b''_{45})^{(9)}\big((G_{47}), t\big) \le -(\tau_1)^{(9)}$$

**Definition of** $(\nu_1)^{(9)}, (\nu_2)^{(9)}, (u_1)^{(9)}, (u_2)^{(9)}, \nu^{(9)}, u^{(9)}$ :

By $(\nu_1)^{(9)} > 0$ , $(\nu_2)^{(9)} < 0$ and respectively $(u_1)^{(9)} > 0$ , $(u_2)^{(9)} < 0$ the roots of the equations

$(a_{45})^{(9)}\left(v^{(9)}\right)^2+(\sigma_1)^{(9)}v^{(9)}-(a_{44})^{(9)}=0$

and $(b_{45})^{(9)}\left(u^{(9)}\right)^2+(\tau_1)^{(9)}u^{(9)}-(b_{44})^{(9)}=0$ and

**Definition of** $(\bar{v}_1)^{(9)},,(\bar{v}_2)^{(9)},(\bar{u}_1)^{(9)},(\bar{u}_2)^{(9)}$ :

By $(\bar{v}_1)^{(9)}>0$ , $(\bar{v}_2)^{(9)}<0$ and respectively $(\bar{u}_1)^{(9)}>0$ , $(\bar{u}_2)^{(9)}<0$ the roots of the equations $(a_{45})^{(9)}\left(v^{(9)}\right)^2+(\sigma_2)^{(9)}v^{(9)}-(a_{44})^{(9)}=0$ and $(b_{45})^{(9)}\left(u^{(9)}\right)^2+(\tau_2)^{(9)}u^{(9)}-(b_{44})^{(9)}=0$

**Definition of** $(m_1)^{(9)}$ , $(m_2)^{(9)}$ , $(\mu_1)^{(9)},(\mu_2)^{(9)},(\nu_0)^{(9)}$ :-

If we define $(m_1)^{(9)}$ , $(m_2)^{(9)}$ , $(\mu_1)^{(9)},(\mu_2)^{(9)}$ by

$(m_2)^{(9)}=(\nu_0)^{(9)},(m_1)^{(9)}=(\nu_1)^{(9)},$ $\boldsymbol{if}$ $(\nu_0)^{(9)}<(\nu_1)^{(9)}$

$(m_2)^{(9)}=(\nu_1)^{(9)},(m_1)^{(9)}=(\bar{\nu}_1)^{(9)}$ , $\boldsymbol{if}$ $(\nu_1)^{(9)}<(\nu_0)^{(9)}<(\bar{\nu}_1)^{(9)}$,

and $\boxed{(\nu_0)^{(9)}=\frac{G_{44}^0}{G_{45}^0}}$

$(m_2)^{(9)}=(\nu_1)^{(9)},(m_1)^{(9)}=(\nu_0)^{(9)},$ $\boldsymbol{if}$ $(\bar{\nu}_1)^{(9)}<(\nu_0)^{(9)}$

and analogously

$(\mu_2)^{(9)}=(u_0)^{(9)},(\mu_1)^{(9)}=(u_1)^{(9)},$ $\boldsymbol{if}$ $(u_0)^{(9)}<(u_1)^{(9)}$

$(\mu_2)^{(9)}=(u_1)^{(9)},(\mu_1)^{(9)}=(\bar{u}_1)^{(9)}$ , $\boldsymbol{if}$ $(u_1)^{(9)}<(u_0)^{(9)}<(\bar{u}_1)^{(9)}$,

and $\boxed{(u_0)^{(9)}=\frac{T_{44}^0}{T_{45}^0}}$

$(\mu_2)^{(9)}=(u_1)^{(9)},(\mu_1)^{(9)}=(u_0)^{(9)},\boldsymbol{if}$ $(\bar{u}_1)^{(9)}<(u_0)^{(9)}$ where $(u_1)^{(9)},(\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G_{44}^0e^{((S_1)^{(9)}-(p_{44})^{(9)})t}\le G_{44}(t)\le G_{44}^0e^{(S_1)^{(9)}t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}}G_{44}^0e^{((S_1)^{(9)}-(p_{44})^{(9)})t}\le G_{45}(t)\le\frac{1}{(m_2)^{(9)}}G_{44}^0e^{(S_1)^{(9)}t}$$

$$\frac{(a_{46})^{(9)}G_{44}^0}{(m_1)^{(9)}((S_1)^{(9)}-(p_{44})^{(9)}-(S_2)^{(9)})}\left[e^{((S_1)^{(9)}-(p_{44})^{(9)})t}-e^{-(S_2)^{(9)}t}\right]+G_{46}^0e^{-(S_2)^{(9)}t}\le G_{46}(t)\le$$

$$\frac{(a_{46})^{(9)}G_{44}^0}{(m_2)^{(9)}((S_1)^{(9)}-(a'_{46})^{(9)})}[e^{(S_1)^{(9)}t}-e^{-(a'_{46})^{(9)}t}]+G_{46}^0e^{-(a'_{46})^{(9)}t})$$

$$\boxed{T_{44}^0e^{(R_1)^{(9)}t}\le T_{44}(t)\le T_{44}^0e^{((R_1)^{(9)}+(r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}}T_{44}^0e^{(R_1)^{(9)}t}\le T_{44}(t)\le\frac{1}{(\mu_2)^{(9)}}T_{44}^0e^{((R_1)^{(9)}+(r_{44})^{(9)})t}$$

$$\frac{(b_{46})^{(9)}T_{44}^{0}}{(\mu_1)^{(9)}((R_1)^{(9)}-(b'_{46})^{(9)})}\left[e^{(R_1)^{(9)}t}-e^{-(b'_{46})^{(9)}t}\right]+T_{46}^{0}e^{-(b'_{46})^{(9)}t}\leq T_{46}(t)\leq$$

$$\frac{(a_{46})^{(9)}T_{44}^{0}}{(\mu_2)^{(9)}((R_1)^{(9)}+(r_{44})^{(9)}+(R_2)^{(9)})}\left[e^{((R_1)^{(9)}+(r_{44})^{(9)})t}-e^{-(R_2)^{(9)}t}\right]+T_{46}^{0}e^{-(R_2)^{(9)}t}$$

**Definition of** $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)}=(a_{44})^{(9)}(m_2)^{(9)}-(a'_{44})^{(9)}$

$(S_2)^{(9)}=(a_{46})^{(9)}-(p_{46})^{(9)}$

$(R_1)^{(9)}=(b_{44})^{(9)}(\mu_2)^{(9)}-(b'_{44})^{(9)}$

$(R_2)^{(9)}=(b'_{46})^{(9)}-(r_{46})^{(9)}$

**Proof :** From global equations we obtain 383

$$\frac{dv^{(1)}}{dt}=(a_{13})^{(1)}-\left((a'_{13})^{(1)}-(a'_{14})^{(1)}+(a''_{13})^{(1)}(T_{14},t)\right)-(a''_{14})^{(1)}(T_{14},t)v^{(1)}-(a_{14})^{(1)}v^{(1)}$$

**Definition of** $v^{(1)}$ :- $\boxed{v^{(1)}=\frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2+(\sigma_2)^{(1)}v^{(1)}-(a_{13})^{(1)}\right)\leq\frac{dv^{(1)}}{dt}\leq-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2+(\sigma_1)^{(1)}v^{(1)}-(a_{13})^{(1)}\right)$$

From which one obtains

**Definition of** $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$ :-

For $0<\boxed{(v_0)^{(1)}=\frac{G_{13}^{0}}{G_{14}^{0}}}<(v_1)^{(1)}<(\bar{v}_1)^{(1)}$

$$v^{(1)}(t)\geq\frac{(v_1)^{(1)}+(C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}}\quad,\quad\boxed{(C)^{(1)}=\frac{(v_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(v_2)^{(1)}}}$$

it follows $(v_0)^{(1)}\leq v^{(1)}(t)\leq(v_1)^{(1)}$

In the same manner , we get 384

$$v^{(1)}(t)\leq\frac{(\bar{v}_1)^{(1)}+(\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}}\quad,\quad\boxed{(\bar{C})^{(1)}=\frac{(\bar{v}_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(\bar{v}_2)^{(1)}}}$$

From which we deduce $(v_0)^{(1)}\leq v^{(1)}(t)\leq(\bar{v}_1)^{(1)}$

If $0<(v_1)^{(1)}<(v_0)^{(1)}=\frac{G_{13}^{0}}{G_{14}^{0}}<(\bar{v}_1)^{(1)}$ we find like in the previous case, 385

$$(\nu_1)^{(1)} \le \frac{(\nu_1)^{(1)}+(C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}} \le \nu^{(1)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \le (\bar{\nu}_1)^{(1)}$$

If $0 < (\nu_1)^{(1)} \le (\bar{\nu}_1)^{(1)} \le \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$, we obtain 386

$$(\nu_1)^{(1)} \le \nu^{(1)}(t) \le \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \le (\nu_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(1)}(t)$ :-

$$(m_2)^{(1)} \le \nu^{(1)}(t) \le (m_1)^{(1)}, \quad \boxed{\nu^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(1)}(t)$ :-

$$(\mu_2)^{(1)} \le u^{(1)}(t) \le (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{13}'')^{(1)} = (a_{14}'')^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)}G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b_{13}'')^{(1)} = (b_{14}'')^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)}T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

**Proof** : From global equations we obtain 387

$$\frac{d\nu^{(2)}}{dt} = (a_{16})^{(2)} - \left((a_{16}')^{(2)} - (a_{17}')^{(2)} + (a_{16}'')^{(2)}(T_{17}, t)\right) - (a_{17}'')^{(2)}(T_{17}, t)\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

**Definition of** $\nu^{(2)}$ :- $\boxed{\nu^{(2)} = \frac{G_{16}}{G_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(v^{(2)}\right)^2+(\sigma_2)^{(2)}v^{(2)}-(a_{16})^{(2)}\right)\leq\frac{dv^{(2)}}{dt}\leq-\left((a_{17})^{(2)}\left(v^{(2)}\right)^2+(\sigma_1)^{(2)}v^{(2)}-(a_{16})^{(2)}\right)$$

From which one obtains 390

**Definition of** $(\bar{v}_1)^{(2)},(v_0)^{(2)}$ :-

For $0<(v_0)^{(2)}=\frac{G_{16}^0}{G_{17}^0}<(v_1)^{(2)}<(\bar{v}_1)^{(2)}$

$$v^{(2)}(t)\geq\frac{(v_1)^{(2)}+(C)^{(2)}(v_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((v_1)^{(2)}-(v_0)^{(2)}\right)t\right]}}{1+(C)^{(2)}e^{\left[-(a_{17})^{(2)}\left((v_1)^{(2)}-(v_0)^{(2)}\right)t\right]}}\quad,\quad\boxed{(C)^{(2)}=\frac{(v_1)^{(2)}-(v_0)^{(2)}}{(v_0)^{(2)}-(v_2)^{(2)}}}$$

it follows $(v_0)^{(2)}\leq v^{(2)}(t)\leq(v_1)^{(2)}$

In the same manner , we get 391

$$v^{(2)}(t)\leq\frac{(\bar{v}_1)^{(2)}+(\bar{C})^{(2)}(\bar{v}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{v}_1)^{(2)}-(\bar{v}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{v}_1)^{(2)}-(\bar{v}_2)^{(2)}\right)t\right]}}\quad,\quad\boxed{(\bar{C})^{(2)}=\frac{(\bar{v}_1)^{(2)}-(v_0)^{(2)}}{(v_0)^{(2)}-(\bar{v}_2)^{(2)}}}$$

From which we deduce $(v_0)^{(2)}\leq v^{(2)}(t)\leq(\bar{v}_1)^{(2)}$ 392

If $0<(v_1)^{(2)}<(v_0)^{(2)}=\frac{G_{16}^0}{G_{17}^0}<(\bar{v}_1)^{(2)}$ we find like in the previous case, 393

$$(v_1)^{(2)}\leq\frac{(v_1)^{(2)}+(C)^{(2)}(v_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((v_1)^{(2)}-(v_2)^{(2)}\right)t\right]}}{1+(C)^{(2)}e^{\left[-(a_{17})^{(2)}\left((v_1)^{(2)}-(v_2)^{(2)}\right)t\right]}}\leq v^{(2)}(t)\leq$$

$$\frac{(\bar{v}_1)^{(2)}+(\bar{C})^{(2)}(\bar{v}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{v}_1)^{(2)}-(\bar{v}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{v}_1)^{(2)}-(\bar{v}_2)^{(2)}\right)t\right]}}\leq(\bar{v}_1)^{(2)}$$

If $0<(v_1)^{(2)}\leq(\bar{v}_1)^{(2)}\leq(v_0)^{(2)}=\frac{G_{16}^0}{G_{17}^0}$ , we obtain 394

$$(v_1)^{(2)}\leq v^{(2)}(t)\leq\frac{(\bar{v}_1)^{(2)}+(\bar{C})^{(2)}(\bar{v}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{v}_1)^{(2)}-(\bar{v}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{v}_1)^{(2)}-(\bar{v}_2)^{(2)}\right)t\right]}}\leq(v_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $v^{(2)}(t)$ :- 395

$$(m_2)^{(2)}\leq v^{(2)}(t)\leq(m_1)^{(2)},\quad\boxed{v^{(2)}(t)=\frac{G_{16}(t)}{G_{17}(t)}}$$

In a completely analogous way, we obtain 396

**Definition of** $u^{(2)}(t)$ :-

$$(\mu_2)^{(2)}\leq u^{(2)}(t)\leq(\mu_1)^{(2)},\quad\boxed{u^{(2)}(t)=\frac{T_{16}(t)}{T_{17}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :** 397

If $(a''_{16})^{(2)} = (a''_{17})^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)} G_{17}(t)$

Analogously if $(b''_{16})^{(2)} = (b''_{17})^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$ if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)} T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

**Proof** : From global equations we obtain 398

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right) - (a''_{21})^{(3)}(T_{21}, t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

**Definition of** $\nu^{(3)}$ :- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right) \leq \frac{d\nu^{(3)}}{dt} \leq -\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right)$$

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From which one obtains

For $0 < (\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\nu_1)^{(3)} < (\bar{\nu}_1)^{(3)}$

$$\nu^{(3)}(t) \geq \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(\nu_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\nu_2)^{(3)}}}$$

it follows $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\nu_1)^{(3)}$

In the same manner , we get 401

$$\nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)} + (\bar{C})^{(3)}(\bar{\nu}_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)} e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{\nu}_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\bar{\nu}_2)^{(3)}}}$$

**Definition of** $(\bar{\nu}_1)^{(3)}$ :-

From which we deduce $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\bar{\nu}_1)^{(3)}$

If $0 < (\nu_1)^{(3)} < (\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\bar{\nu}_1)^{(3)}$ we find like in the previous case, 402

$$(\nu_1)^{(3)} \le \frac{(\nu_1)^{(3)}+(C)^{(3)}(\nu_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}} \le \nu^{(3)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \le (\bar{\nu}_1)^{(3)}$$

If $0 < (\nu_1)^{(3)} \le (\bar{\nu}_1)^{(3)} \le (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}$ , we obtain 403

$$(\nu_1)^{(3)} \le \nu^{(3)}(t) \le \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \le (\nu_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(3)}(t)$ :-

$$(m_2)^{(3)} \le \nu^{(3)}(t) \le (m_1)^{(3)}, \quad \boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(3)}(t)$ :-

$$(\mu_2)^{(3)} \le u^{(3)}(t) \le (\mu_1)^{(3)}, \quad \boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{20}'')^{(3)} = (a_{21}'')^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)}G_{21}(t)$

Analogously if $(b_{20}'')^{(3)} = (b_{21}'')^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)}T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

**Proof** : From global equations we obtain 404

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a_{24}')^{(4)} - (a_{25}')^{(4)} + (a_{24}'')^{(4)}(T_{25},t)\right) - (a_{25}'')^{(4)}(T_{25},t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

**Definition of** $\nu^{(4)}$ :- $\quad \boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right) \le \frac{d\nu^{(4)}}{dt} \le -\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_4)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$ :-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \geq \frac{(\nu_1)^{(4)}+(C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_0)^{(4)}\right)t\right]}}{4+(C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)}-(\nu_0)^{(4)}}{(\nu_0)^{(4)}-(\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{4+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)}-(\nu_0)^{(4)}}{(\nu_0)^{(4)}-(\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \leq \frac{(\nu_1)^{(4)}+(C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_2)^{(4)}\right)t\right]}}{1+(C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_2)^{(4)}\right)t\right]}} \leq \nu^{(4)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \leq (\bar{\nu}_1)^{(4)} \leq \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$ , we obtain 407

$$(\nu_1)^{(4)} \leq \nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(4)}(t)$ :-

$$(m_2)^{(4)} \leq \nu^{(4)}(t) \leq (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(4)}(t)$ :-

$$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{24})^{(4)} = (a''_{25})^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)}G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b''_{24})^{(4)} = (b''_{25})^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then

$(u_1)^{(4)} = (\bar{u}_4)^{(4)}$if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)}T_{25}(t)$ This is an important consequence of the relation between $(v_1)^{(4)}$ and $(\bar{v}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

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**Proof** : From global equations we obtain

$$\frac{dv^{(5)}}{dt} = (a_{28})^{(5)} - \left((a'_{28})^{(5)} - (a'_{29})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right) - (a''_{29})^{(5)}(T_{29}, t)v^{(5)} - (a_{29})^{(5)}v^{(5)}$$

**Definition of** $v^{(5)}$ :- $\boxed{v^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(v^{(5)}\right)^2 + (\sigma_2)^{(5)}v^{(5)} - (a_{28})^{(5)}\right) \leq \frac{dv^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(v^{(5)}\right)^2 + (\sigma_1)^{(5)}v^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

**Definition of** $(\bar{v}_1)^{(5)}, (v_0)^{(5)}$ :-

For $0 < \boxed{(v_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}} < (v_1)^{(5)} < (\bar{v}_1)^{(5)}$

$$v^{(5)}(t) \geq \frac{(v_1)^{(5)} + (C)^{(5)}(v_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((v_1)^{(5)} - (v_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)}e^{\left[-(a_{29})^{(5)}\left((v_1)^{(5)} - (v_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(v_1)^{(5)} - (v_0)^{(5)}}{(v_0)^{(5)} - (v_2)^{(5)}}}$$

it follows $(v_0)^{(5)} \leq v^{(5)}(t) \leq (v_1)^{(5)}$

In the same manner , we get 409

$$v^{(5)}(t) \leq \frac{(\bar{v}_1)^{(5)} + (\bar{C})^{(5)}(\bar{v}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{v}_1)^{(5)} - (\bar{v}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{v}_1)^{(5)} - (\bar{v}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{v}_1)^{(5)} - (v_0)^{(5)}}{(v_0)^{(5)} - (\bar{v}_2)^{(5)}}}$$

From which we deduce $(v_0)^{(5)} \leq v^{(5)}(t) \leq (\bar{v}_5)^{(5)}$

If $0 < (v_1)^{(5)} < (v_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}} < (\bar{v}_1)^{(5)}$ we find like in the previous case, 410

$$(v_1)^{(5)} \leq \frac{(v_1)^{(5)} + (C)^{(5)}(v_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((v_1)^{(5)} - (v_2)^{(5)}\right)t\right]}}{1 + (C)^{(5)}e^{\left[-(a_{29})^{(5)}\left((v_1)^{(5)} - (v_2)^{(5)}\right)t\right]}} \leq v^{(5)}(t) \leq$$

$$\frac{(\bar{v}_1)^{(5)} + (\bar{C})^{(5)}(\bar{v}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{v}_1)^{(5)} - (\bar{v}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{v}_1)^{(5)} - (\bar{v}_2)^{(5)}\right)t\right]}} \leq (\bar{v}_1)^{(5)}$$

If $0 < (v_1)^{(5)} \leq (\bar{v}_1)^{(5)} \leq \boxed{(v_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}}$ , we obtain 411

$$(v_1)^{(5)} \leq v^{(5)}(t) \leq \frac{(\bar{v}_1)^{(5)} + (\bar{C})^{(5)}(\bar{v}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{v}_1)^{(5)} - (\bar{v}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{v}_1)^{(5)} - (\bar{v}_2)^{(5)}\right)t\right]}} \leq (v_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $v^{(5)}(t)$ :-

$(m_2)^{(5)} \leq v^{(5)}(t) \leq (m_1)^{(5)}$, $\boxed{v^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$

In a completely analogous way, we obtain

**Definition of** $u^{(5)}(t)$ :-

$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}$, $\boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{28})^{(5)} = (a''_{29})^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $v^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)} G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b''_{28})^{(5)} = (b''_{29})^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$ if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)} T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

**Proof** : From global equations we obtain 412

$$\frac{dv^{(6)}}{dt} = (a_{32})^{(6)} - \left((a'_{32})^{(6)} - (a'_{33})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right) - (a''_{33})^{(6)}(T_{33}, t)v^{(6)} - (a_{33})^{(6)}v^{(6)}$$

**Definition of** $v^{(6)}$ :- $\boxed{v^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(v^{(6)}\right)^2 + (\sigma_2)^{(6)}v^{(6)} - (a_{32})^{(6)}\right) \leq \frac{dv^{(6)}}{dt} \leq -\left((a_{33})^{(6)}\left(v^{(6)}\right)^2 + (\sigma_1)^{(6)}v^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$ :-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$v^{(6)}(t) \geq \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \leq v^{(6)}(t) \leq (\nu_1)^{(6)}$

In the same manner , we get 413

$$v^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{\nu}_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \le \nu^{(6)}(t) \le (\bar{\nu}_1)^{(6)}$

If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 414

$$(\nu_1)^{(6)} \le \frac{(\nu_1)^{(6)}+(C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_2)^{(6)}\right)t\right]}}{1+(C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_2)^{(6)}\right)t\right]}} \le \nu^{(6)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \le (\bar{\nu}_1)^{(6)}$$

If $0 < (\nu_1)^{(6)} \le (\bar{\nu}_1)^{(6)} \le \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$, we obtain 415

$$(\nu_1)^{(6)} \le \nu^{(6)}(t) \le \frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \le (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(6)}(t)$ :-

$$(m_2)^{(6)} \le \nu^{(6)}(t) \le (m_1)^{(6)}, \quad \boxed{\nu^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(6)}(t)$ :-

$$(\mu_2)^{(6)} \le u^{(6)}(t) \le (\mu_1)^{(6)}, \quad \boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)}G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)}T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

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**Proof** : From global equations we obtain

$$\frac{d\nu^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right) - (a''_{37})^{(7)}(T_{37}, t)\nu^{(7)} - (a_{37})^{(7)}\nu^{(7)}$$

**Definition of** $\nu^{(7)}$ :- $\boxed{\nu^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right) \le \frac{d\nu^{(7)}}{dt} \le -\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(7)}, (\nu_0)^{(7)}$ :-

$$\text{For } 0 < \boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}} < (\nu_1)^{(7)} < (\bar{\nu}_1)^{(7)}$$

$$\nu^{(7)}(t) \geq \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}}{1 + (C)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(\nu_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\nu_2)^{(7)}}}$$

it follows $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\nu_1)^{(7)}$

In the same manner , we get 417

$$\nu^{(7)}(t) \leq \frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)} e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{\nu}_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\bar{\nu}_2)^{(7)}}}$$

From which we deduce $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\bar{\nu}_1)^{(7)}$

If $0 < (\nu_1)^{(7)} < (\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0} < (\bar{\nu}_1)^{(7)}$ we find like in the previous case, 418

$$(\nu_1)^{(7)} \leq \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_2)^{(7)}\right)t\right]}}{1 + (C)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_2)^{(7)}\right)t\right]}} \leq \nu^{(7)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)} e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \leq (\bar{\nu}_1)^{(7)}$$

If $0 < (\nu_1)^{(7)} \leq (\bar{\nu}_1)^{(7)} \leq \boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$ , we obtain 419

$$(\nu_1)^{(7)} \leq \nu^{(7)}(t) \leq \frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)} e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \leq (\nu_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(7)}(t)$ :-

$$(m_2)^{(7)} \leq \nu^{(7)}(t) \leq (m_1)^{(7)}, \quad \boxed{\nu^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(7)}(t)$ :- 420

$$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}, \quad \boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $\nu^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)} G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)} T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

**Proof** : From global equations we obtain 421

$$\frac{d\nu^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right) - (a''_{41})^{(8)}(T_{41}, t)\nu^{(8)} - (a_{41})^{(8)}\nu^{(8)}$$

**Definition of** $\nu^{(8)}$ :- $\boxed{\nu^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right) \leq \frac{d\nu^{(8)}}{dt} \leq -\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$ :-

For $0 < \boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$

$$\nu^{(8)}(t) \geq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)} e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \leq \frac{(\nu_1)^{(8)}+(C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}}{1+(C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}} \leq \nu^{(8)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \leq (\bar{\nu}_1)^{(8)} \leq \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$ , we obtain

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$$(\nu_1)^{(8)} \leq \nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(8)}(t)$ :-

$$(m_2)^{(8)} \leq \nu^{(8)}(t) \leq (m_1)^{(8)}, \quad \boxed{\nu^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(8)}(t)$ :-

$$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}, \quad \boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $\nu^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)}G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then $(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)}T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

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**Proof** : From 99,20,44,22,23,44 we obtain

$$\frac{d\nu^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45}, t)\right) - (a''_{45})^{(9)}(T_{45}, t)\nu^{(9)} - (a_{45})^{(9)}\nu^{(9)}$$

**Definition of** $\nu^{(9)}$ :- $\boxed{\nu^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(\nu^{(9)}\right)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)}\right) \leq \frac{d\nu^{(9)}}{dt} \leq -\left((a_{45})^{(9)}\left(\nu^{(9)}\right)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$ :-

For $0 < \boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$

$$\nu^{(9)}(t) \geq \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\nu_9)^{(9)}$

In the same manner , we get

$$\nu^{(9)}(t) \leq \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \leq \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}}{1 + (C)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}} \leq \nu^{(9)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \leq (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \leq (\bar{\nu}_1)^{(9)} \leq \boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$ , we obtain

$$(\nu_1)^{(9)} \leq \nu^{(9)}(t) \leq \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \leq (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(9)}(t)$ :-

$$(m_2)^{(9)} \leq \nu^{(9)}(t) \leq (m_1)^{(9)}, \quad \boxed{\nu^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(9)}(t)$ :-

$$(\mu_2)^{(9)} \leq u^{(9)}(t) \leq (\mu_1)^{(9)}, \quad \boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)} G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then
$(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)} T_{45}(t)$ This is an important consequence of the relation between $(v_1)^{(9)}$ and $(\bar{v}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following 425

**Theorem :** If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on $t$ , and the conditions with the notations

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0$ ,

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$

$with\ \ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a''_i)^{(2)} and\ (b''_i)^{(2)}$ are independent on t , and the conditions with the notations 426

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$ 427

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 428

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$ , 429

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 430

$with\ \ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a''_i)^{(3)} and\ (b''_i)^{(3)}$ are independent on $t$ , and the conditions with the notations 431

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$ ,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

**Theorem :** If $(a''_i)^{(4)} and\ (b''_i)^{(4)}$ are independent on $t$ , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$ ,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(5)} and\ (b_i'')^{(5)}$ are independent on $t$ , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$ ,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

**Theorem** If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on $t$ , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$ ,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(7)} and\ (b_i'')^{(7)}$ are independent on $t$ , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$ ,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(8)} and\ (b_i'')^{(8)}$ are independent on $t$ , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$ ,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a''_i)^{(9)} and\ (b''_i)^{(9)}$ are independent on $t$ , and the conditions (with the notations 45,46,27,28) 436 A

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$ ,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0$ 437

$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0$ 438

$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0$ 439

$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0$ 440

$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ]T_{14} = 0$ 441

$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0$ 442

has a unique positive solution , which is an equilibrium solution for the system

$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0$ 443

$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0$ 444

$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0$ 445

$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\ ]T_{16} = 0$ 446

$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\ ]T_{17} = 0$ 447

$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\ ]T_{18} = 0$ 448

has a unique positive solution , which is an equilibrium solution

$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0$ 449

$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0$ 450

$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0$ 451

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\,]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\,]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\,]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\big((G_{27})\big)\,]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\big((G_{27})\big)\,]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\big((G_{27})\big)\,]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})\,]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})\,]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})\,]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})\,]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})\,]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})\,]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0 \quad 479$$

$$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0 \quad 480$$

$$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0 \quad 481$$

$$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0 \quad 482$$

$$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0 \quad 483$$

$$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0 \quad 484$$

$$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0 \quad 484\text{A}$$

$$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$$

$$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$$

$$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$$

$$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$$

$$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$$

**Proof:** 485

(a) Indeed the first two equations have a nontrivial solution $G_{13}, G_{14}$ if

$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$

**Proof:** 486

(q) Indeed the first two equations have a nontrivial solution $G_{16}, G_{17}$ if

$\mathrm{F}(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$

**Proof:** 487

(a) Indeed the first two equations have a nontrivial solution $G_{20}, G_{21}$ if

$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$

**Proof:** 488

(a) Indeed the first two equations have a nontrivial solution $G_{24}, G_{25}$ if

$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$

**Proof:** 489

(a) Indeed the first two equations have a nontrivial solution $G_{28}, G_{29}$ if

$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$

**Proof:** 490

(a) Indeed the first two equations have a nontrivial solution $G_{32}, G_{33}$ if

$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$

**Proof:** 491

(a) Indeed the first two equations have a nontrivial solution $G_{36}, G_{37}$ if

$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$

**Proof:** 492

(a) Indeed the first two equations have a nontrivial solution $G_{40}, G_{41}$ if

$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$

**Proof:** 492 A

(a) Indeed the first two equations have a nontrivial solution $G_{44}, G_{45}$ if

$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$

**Definition and uniqueness of** $T^*_{14}$ :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique $T^*_{14}$ for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

**Definition and uniqueness of** $T^*_{17}$ :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique $T^*_{17}$ for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]}$$ 495

**Definition and uniqueness of** $T^*_{21}$ :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique $T^*_{21}$ for which $f(T^*_{21}) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

**Definition and uniqueness of** $T^*_{25}$ :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that there exists a unique $T^*_{25}$ for which $f(T^*_{25}) = 0$. With this value , we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T^*_{25})]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T^*_{25})]}$$

**Definition and uniqueness of** $T^*_{29}$ :- 498

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(5)}(T_{29})$ being increasing, it follows that there exists a unique $T^*_{29}$ for which $f(T^*_{29}) = 0$. With this value , we obtain from the three first equations

$G_{28} = \frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T^*_{29})]}$ , $G_{30} = \frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T^*_{29})]}$

**Definition and uniqueness of** $T^*_{33}$ :- 499

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(6)}(T_{33})$ being increasing, it follows that there exists a unique $T^*_{33}$ for which $f(T^*_{33}) = 0$. With this value , we obtain from the three first equations

$G_{32} = \frac{(a_{32})^{(6)}G_{33}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T^*_{33})]}$ , $G_{34} = \frac{(a_{34})^{(6)}G_{33}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T^*_{33})]}$

**Definition and uniqueness of** $T^*_{37}$ :- 500

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(7)}(T_{37})$ being increasing, it follows that there exists a unique $T^*_{37}$ for which $f(T^*_{37}) = 0$. With this value , we obtain from the three first equations

$G_{36} = \frac{(a_{36})^{(7)}G_{37}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T^*_{37})]}$ , $G_{38} = \frac{(a_{38})^{(7)}G_{37}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T^*_{37})]}$

**Definition and uniqueness of** $T^*_{41}$ :- 501

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(8)}(T_{41})$ being increasing, it follows that there exists a unique $T^*_{41}$ for which $f(T^*_{41}) = 0$. With this value , we obtain from the three first equations

$G_{40} = \frac{(a_{40})^{(8)}G_{41}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T^*_{41})]}$ , $G_{42} = \frac{(a_{42})^{(8)}G_{41}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T^*_{41})]}$

**Definition and uniqueness of** $T^*_{45}$ :- 501 A

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(9)}(T_{45})$ being increasing, it follows that there exists a unique $T^*_{45}$ for which $f(T^*_{45}) = 0$. With this value , we obtain from the three first equations

$G_{44} = \frac{(a_{44})^{(9)}G_{45}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T^*_{45})]}$ , $G_{46} = \frac{(a_{46})^{(9)}G_{45}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T^*_{45})]}$

By the same argument, the equations admit solutions $G_{13}, G_{14}$ if 502
$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$

$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right]+(b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{14}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{14}$ such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions $G_{16}, G_{17}$ if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right] + (b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that $\varphi$ is a decreasing function in $G_{17}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{14}^{*}$ such that $\varphi((G_{19})^{*}) = 0$ 504

By the same argument, the equations admit solutions $G_{20}, G_{21}$ if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right] + (b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that $\varphi$ is a decreasing function in $G_{21}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{21}^{*}$ such that $\varphi((G_{23})^{*}) = 0$

By the same argument, the equations admit solutions $G_{24}, G_{25}$ if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right] + (b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that $\varphi$ is a decreasing function in $G_{25}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{25}^{*}$ such that $\varphi((G_{27})^{*}) = 0$

By the same argument, the equations admit solutions $G_{28}, G_{29}$ if 507

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right] + (b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that $\varphi$ is a decreasing function in $G_{29}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{29}^{*}$ such that $\varphi((G_{31})^{*}) = 0$

By the same argument, the equations admit solutions $G_{32}, G_{33}$ if 508

$$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$$

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right] + (b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that $\varphi$ is a decreasing function in $G_{33}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{33}^{*}$ such that $\varphi({G_{35}}^{*}) = 0$

By the same argument, the equations admit solutions $G_{36}, G_{37}$ if 509

$$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$$

$$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right]+(b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{37}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{37}$ such that $\varphi(G_{39}{}^*) = 0$

By the same argument, the equations admit solutions $G_{40}, G_{41}$ if 510

$$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$$
$$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right]+(b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{41}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{41}$ such that $\varphi(G_{43}{}^*) = 0$

By the same argument, the equations 92,93 admit solutions $G_{44}, G_{45}$ if

$$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$$

$$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right]+(b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{45}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{45}$ such that $\varphi((G_{47})^*) = 0$

Finally we obtain the unique solution 511

$G^*_{14}$ given by $\varphi(G^*) = 0$ , $T^*_{14}$ given by $f(T^*_{14}) = 0$ and

$$G^*_{13} = \frac{(a_{13})^{(1)}G^*_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G^*_{15} = \frac{(a_{15})^{(1)}G^*_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

$$T^*_{13} = \frac{(b_{13})^{(1)}T^*_{14}}{[(b'_{13})^{(1)}-(b''_{13})^{(1)}(G^*)]} \quad , \quad T^*_{15} = \frac{(b_{15})^{(1)}T^*_{14}}{[(b'_{15})^{(1)}-(b''_{15})^{(1)}(G^*)]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

$G^*_{17}$ given by $\varphi((G_{19})^*) = 0$ , $T^*_{17}$ given by $f(T^*_{17}) = 0$ and 512

$$G^*_{16} = \frac{(a_{16})^{(2)}G^*_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G^*_{18} = \frac{(a_{18})^{(2)}G^*_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]} \quad 513$$

$$T^*_{16} = \frac{(b_{16})^{(2)}T^*_{17}}{[(b'_{16})^{(2)}-(b''_{16})^{(2)}((G_{19})^*)]} \quad , \quad T^*_{18} = \frac{(b_{18})^{(2)}T^*_{17}}{[(b'_{18})^{(2)}-(b''_{18})^{(2)}((G_{19})^*)]} \quad 514$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

$G^*_{21}$ given by $\varphi((G_{23})^*) = 0$ , $T^*_{21}$ given by $f(T^*_{21}) = 0$ and

$$G_{20}^{*}=\frac{(a_{20})^{(3)}G_{21}^{*}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T_{21}^{*})]}\quad,\quad G_{22}^{*}=\frac{(a_{22})^{(3)}G_{21}^{*}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T_{21}^{*})]}$$

$$T_{20}^{*}=\frac{(b_{20})^{(3)}T_{21}^{*}}{[(b'_{20})^{(3)}-(b''_{20})^{(3)}(G_{23}{}^{*})]}\quad,\quad T_{22}^{*}=\frac{(b_{22})^{(3)}T_{21}^{*}}{[(b'_{22})^{(3)}-(b''_{22})^{(3)}(G_{23}{}^{*})]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

$G_{25}^{*}$ given by $\varphi(G_{27})=0$ , $T_{25}^{*}$ given by $f(T_{25}^{*})=0$ and

$$G_{24}^{*}=\frac{(a_{24})^{(4)}G_{25}^{*}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T_{25}^{*})]}\quad,\quad G_{26}^{*}=\frac{(a_{26})^{(4)}G_{25}^{*}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T_{25}^{*})]}$$

$$T_{24}^{*}=\frac{(b_{24})^{(4)}T_{25}^{*}}{[(b'_{24})^{(4)}-(b''_{24})^{(4)}((G_{27})^{*})]}\quad,\quad T_{26}^{*}=\frac{(b_{26})^{(4)}T_{25}^{*}}{[(b'_{26})^{(4)}-(b''_{26})^{(4)}((G_{27})^{*})]} \qquad 517$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

$G_{29}^{*}$ given by $\varphi((G_{31})^{*})=0$ , $T_{29}^{*}$ given by $f(T_{29}^{*})=0$ and

$$G_{28}^{*}=\frac{(a_{28})^{(5)}G_{29}^{*}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T_{29}^{*})]}\quad,\quad G_{30}^{*}=\frac{(a_{30})^{(5)}G_{29}^{*}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T_{29}^{*})]}$$

$$T_{28}^{*}=\frac{(b_{28})^{(5)}T_{29}^{*}}{[(b'_{28})^{(5)}-(b''_{28})^{(5)}((G_{31})^{*})]}\quad,\quad T_{30}^{*}=\frac{(b_{30})^{(5)}T_{29}^{*}}{[(b'_{30})^{(5)}-(b''_{30})^{(5)}((G_{31})^{*})]} \qquad 519$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

$G_{33}^{*}$ given by $\varphi((G_{35})^{*})=0$ , $T_{33}^{*}$ given by $f(T_{33}^{*})=0$ and

$$G_{32}^{*}=\frac{(a_{32})^{(6)}G_{33}^{*}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^{*})]}\quad,\quad G_{34}^{*}=\frac{(a_{34})^{(6)}G_{33}^{*}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^{*})]}$$

$$T_{32}^{*}=\frac{(b_{32})^{(6)}T_{33}^{*}}{[(b'_{32})^{(6)}-(b''_{32})^{(6)}((G_{35})^{*})]}\quad,\quad T_{34}^{*}=\frac{(b_{34})^{(6)}T_{33}^{*}}{[(b'_{34})^{(6)}-(b''_{34})^{(6)}((G_{35})^{*})]} \qquad 521$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

$G_{37}^{*}$ given by $\varphi((G_{39})^{*})=0$ , $T_{37}^{*}$ given by $f(T_{37}^{*})=0$ and

$$G_{36}^{*}=\frac{(a_{36})^{(7)}G_{37}^{*}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^{*})]}\quad,\quad G_{38}^{*}=\frac{(a_{38})^{(7)}G_{37}^{*}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^{*})]}$$

$$T_{36}^{*}=\frac{(b_{36})^{(7)}T_{37}^{*}}{[(b'_{36})^{(7)}-(b''_{36})^{(7)}((G_{39})^{*})]}\quad,\quad T_{38}^{*}=\frac{(b_{38})^{(7)}T_{37}^{*}}{[(b'_{38})^{(7)}-(b''_{38})^{(7)}((G_{39})^{*})]}$$

Finally we obtain the unique solution 523

$G_{41}^*$ given by $\varphi((G_{43})^*) = 0$ , $T_{41}^*$ given by $f(T_{41}^*) = 0$ and

$$G_{40}^* = \frac{(a_{40})^{(8)} G_{41}^*}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42}^* = \frac{(a_{42})^{(8)} G_{41}^*}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

$$T_{40}^* = \frac{(b_{40})^{(8)} T_{41}^*}{[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43})^*)]} \quad , \quad T_{42}^* = \frac{(b_{42})^{(8)} T_{41}^*}{[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43})^*)]}$$

Finally we obtain the unique solution of 89 to 99 523 A

$G_{45}^*$ given by $\varphi((G_{47})^*) = 0$ , $T_{45}^*$ given by $f(T_{45}^*) = 0$ and

$$G_{44}^* = \frac{(a_{44})^{(9)} G_{45}^*}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46}^* = \frac{(a_{46})^{(9)} G_{45}^*}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

$$T_{44}^* = \frac{(b_{44})^{(9)} T_{45}^*}{[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47})^*)]} \quad , \quad T_{46}^* = \frac{(b_{46})^{(9)} T_{45}^*}{[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47})^*)]}$$

## ASYMPTOTIC STABILITY ANALYSIS 524

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

**Proof:** Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{14}'')^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \frac{\partial (b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a'_{13})^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)} G_{13}^* \mathbb{T}_{14} \quad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a'_{14})^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)} G_{14}^* \mathbb{T}_{14} \quad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a'_{15})^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)} G_{15}^* \mathbb{T}_{14} \quad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b'_{13})^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)} T_{13}^* \mathbb{G}_j\right) \quad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b'_{14})^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)} T_{14}^* \mathbb{G}_j\right) \quad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b'_{15})^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T^*_{15}\mathbb{G}_j\right) \quad 530$$

**ASYMPTOTIC STABILITY ANALYSIS** 531

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(2)}$ and $(b''_i)^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i \quad 532$$

$$\frac{\partial(a''_{17})^{(2)}}{\partial T_{17}}(T^*_{17}) = (q_{17})^{(2)} \quad , \frac{\partial(b''_i)^{(2)}}{\partial G_j}\left((G_{19})^*\right) = s_{ij} \quad 533$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a'_{16})^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G^*_{16}\mathbb{T}_{17} \quad 534$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a'_{17})^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G^*_{17}\mathbb{T}_{17} \quad 535$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a'_{18})^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G^*_{18}\mathbb{T}_{17} \quad 536$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b'_{16})^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T^*_{16}\mathbb{G}_j\right) \quad 537$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b'_{17})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}T^*_{17}\mathbb{G}_j\right) \quad 538$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b'_{18})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}T^*_{18}\mathbb{G}_j\right) \quad 539$$

**ASYMPTOTIC STABILITY ANALYSIS** 540

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(3)}$ *and* $(b''_i)^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{21})^{(3)}}{\partial T_{21}}(T^*_{21}) = (q_{21})^{(3)} \quad , \frac{\partial(b''_i)^{(3)}}{\partial G_j}\left((G_{23})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a'_{20})^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G^*_{20}\mathbb{T}_{21} \quad 541$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a'_{21})^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G^*_{21}\mathbb{T}_{21} \quad 542$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a'_{22})^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G^*_{22}\mathbb{T}_{21} \quad 543$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b'_{20})^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T^*_{20}\mathbb{G}_j\right) \quad 544$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b'_{21})^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T^*_{21}\mathbb{G}_j\right) \quad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b'_{22})^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T^*_{22}\mathbb{G}_j\right) \quad 546$$

**ASYMPTOTIC STABILITY ANALYSIS** 547

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}$ *and* $(b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{25})^{(4)}}{\partial T_{25}}(T^*_{25}) = (q_{25})^{(4)} \ , \ \frac{\partial(b_i'')^{(4)}}{\partial G_j}((G_{27})^* \ ) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\left((a'_{24})^{(4)} + (p_{24})^{(4)}\right)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G^*_{24}\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\left((a'_{25})^{(4)} + (p_{25})^{(4)}\right)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G^*_{25}\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\left((a'_{26})^{(4)} + (p_{26})^{(4)}\right)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G^*_{26}\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\left((b'_{24})^{(4)} - (r_{24})^{(4)}\right)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(24)(j)}T^*_{24}\mathbb{G}_j\right) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\left((b'_{25})^{(4)} - (r_{25})^{(4)}\right)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\left(s_{(25)(j)}T^*_{25}\mathbb{G}_j\right) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T^*_{26}\mathbb{G}_j\right) \quad 554$$

**ASYMPTOTIC STABILITY ANALYSIS** 555

**Theorem 5:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}$ *and* $(b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 556

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$\frac{\partial(a_{29}'')^{(5)}}{\partial T_{29}}(T_{29}^*) = (q_{29})^{(5)}$ , $\frac{\partial(b_i'')^{(5)}}{\partial G_j}((G_{31})^*) = s_{ij}$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a_{28}')^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G_{28}^*\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a_{29}')^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G_{29}^*\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a_{30}')^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G_{30}^*\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b_{28}')^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T_{28}^*\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b_{29}')^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T_{29}^*\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b_{30}')^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T_{30}^*\mathbb{G}_j\right) \quad 562$$

**ASYMPTOTIC STABILITY ANALYSIS** 563

**Theorem 6:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}$ *and* $(b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 564

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$\frac{\partial(a_{33}'')^{(6)}}{\partial T_{33}}(T_{33}^*) = (q_{33})^{(6)}$ , $\frac{\partial(b_i'')^{(6)}}{\partial G_j}((G_{35})^*) = s_{ij}$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a_{32}')^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G_{32}^*\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a_{33}')^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G_{33}^*\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a_{34}')^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G_{34}^*\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b_{32}')^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T_{32}^*\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b_{33}')^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T_{33}^*\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b_{34}')^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T_{34}^*\mathbb{G}_j\right) \quad 570$$

**ASYMPTOTIC STABILITY ANALYSIS** 571

**Theorem 7:** If the conditions of the previous theorem are satisfied and if the functions

$(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 572

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{37}'')^{(7)}}{\partial T_{37}}(T_{37}^*) = (q_{37})^{(7)} \quad , \frac{\partial (b_i'')^{(7)}}{\partial G_j}\left( (G_{39})^{**} \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a_{36}')^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G_{36}^*\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a_{37}')^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G_{37}^*\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a_{38}')^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G_{38}^*\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b_{36}')^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T_{36}^*\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b_{37}')^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T_{37}^*\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b_{38}')^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T_{38}^*\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

## ASYMPTOTIC STABILITY ANALYSIS

**Theorem 8:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 580

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{41}'')^{(8)}}{\partial T_{41}}(T_{41}^*) = (q_{41})^{(8)} \quad , \frac{\partial (b_i'')^{(8)}}{\partial G_j}\left( (G_{43})^{*} \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a_{40}')^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G_{40}^*\mathbb{T}_{41} \quad 581$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a'_{41})^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G^*_{41}\mathbb{T}_{41} \quad 582$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a'_{42})^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G^*_{42}\mathbb{T}_{41} \quad 583$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b'_{40})^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T^*_{40}\mathbb{G}_j\right) \quad 584$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b'_{41})^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T^*_{41}\mathbb{G}_j\right) \quad 585$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b'_{42})^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T^*_{42}\mathbb{G}_j\right) \quad 586$$

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## ASYMPTOTIC STABILITY ANALYSIS

**Theorem 9:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}$ *and* $(b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

**Proof:** Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{45})^{(9)}}{\partial T_{45}}(T^*_{45}) = (q_{45})^{(9)} \quad , \frac{\partial(b_i'')^{(9)}}{\partial G_j}((G_{47})^*) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a'_{44})^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)}G^*_{44}\mathbb{T}_{45} \quad 586\,B$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a'_{45})^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)}G^*_{45}\mathbb{T}_{45} \quad 586\,C$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a'_{46})^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)}G^*_{46}\mathbb{T}_{45} \quad 586\,D$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b'_{44})^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)}T^*_{44}\mathbb{G}_j\right) \quad 586\,E$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b'_{45})^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)}T^*_{45}\mathbb{G}_j\right) \quad 586\,F$$

$$\frac{d\mathbb{T}_{46}}{dt} = -\left((b'_{46})^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T^*_{46}\mathbb{G}_j\right) \quad 586\,G$$

**The characteristic equation of this system is** 587

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(q_{13})^{(1)}G^*_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)}T^*_{14} + (b_{14})^{(1)}s_{(13),(14)}T^*_{14}\right)$$

$$+\left(\left((\lambda)^{(1)}+(a'_{14})^{(1)}+(p_{14})^{(1)}\right)(q_{13})^{(1)}G^{*}_{13}+(a_{13})^{(1)}(q_{14})^{(1)}G^{*}_{14}\right)$$

$$\left(\left((\lambda)^{(1)}+(b'_{13})^{(1)}-(r_{13})^{(1)}\right)s_{(14),(13)}T^{*}_{14}+(b_{14})^{(1)}s_{(13),(13)}T^{*}_{13}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^{2}+\left((a'_{13})^{(1)}+(a'_{14})^{(1)}+(p_{13})^{(1)}+(p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^{2}+\left((b'_{13})^{(1)}+(b'_{14})^{(1)}-(r_{13})^{(1)}+(r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^{2}+\left((a'_{13})^{(1)}+(a'_{14})^{(1)}+(p_{13})^{(1)}+(p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)}+(a'_{13})^{(1)}+(p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^{*}_{14}+(a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^{*}_{13}\right)$$

$$\left(\left((\lambda)^{(1)}+(b'_{13})^{(1)}-(r_{13})^{(1)}\right)s_{(14),(15)}T^{*}_{14}+(b_{14})^{(1)}s_{(13),(15)}T^{*}_{13}\right)\}=0$$

$$+$$

$$(\lambda)^{(2)}+(b'_{18})^{(2)}-(r_{18})^{(2)}\{\left((\lambda)^{(2)}+(a'_{18})^{(2)}+(p_{18})^{(2)}\right)$$

$$\left[\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^{*}_{17}+(a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^{*}_{16}\right)\Big]$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^{*}_{17}\right)$$

$$+\left(\left((\lambda)^{(2)}+(a'_{17})^{(2)}+(p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^{*}_{16}+(a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^{*}_{17}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^{*}_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^{2}+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^{2}+\left((b'_{16})^{(2)}+(b'_{17})^{(2)}-(r_{16})^{(2)}+(r_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$+\left(\left((\lambda)^{(2)}\right)^{2}+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^{*}_{17}+(a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^{*}_{16}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^{*}_{16}\right)\}=0$$

$$+$$

$$(\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)\Big]$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(21)}T^{*}_{21}\right)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^{*}_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^{*}_{21}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(20)}T^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$+\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(22)}T^{*}_{20}\right)\}=0$$

$$+$$

$$(\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left(\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)\right]$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(25)}T^{*}_{25}\right)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^{*}_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^{*}_{25}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(24)}T^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$+\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(26)}T^{*}_{24}\right)\}=0$$

$$+$$

$$(\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(29)}T^{*}_{29}\right)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^{*}_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^{*}_{29}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(28)}T^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$+\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(30)}T^{*}_{28}\right)\}=0$$

$$+$$

$$(\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left(\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)\right]$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(33)}T^{*}_{33}\right)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^{*}_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^{*}_{33}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(32)}T^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$+\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(34)}T^{*}_{32}\right)\}=0$$

$$+$$

$$(\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)\right]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(37)}T^{*}_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^{*}_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^{*}_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(36)}T^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$+\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(38)}T^{*}_{36}\right)\}=0$$

$$+$$

$$(\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(41)}T^{*}_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^{*}_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^{*}_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(40)}T^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$+\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(42)}T^{*}_{40}\right)\}=0$$

$$+$$

$$(\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)\right]$$

$$\left((\lambda)^{(9)} + (b'_{44})^{(9)} - (r_{44})^{(9)}\right)s_{(45),(45)}T^*_{45} + (b_{45})^{(9)}s_{(44),(45)}T^*_{45}\Big)$$

$$+\left((\lambda)^{(9)} + (a'_{45})^{(9)} + (p_{45})^{(9)}\right)(q_{44})^{(9)}G^*_{44} + (a_{44})^{(9)}(q_{45})^{(9)}G^*_{45}\Big)$$

$$\left((\lambda)^{(9)} + (b'_{44})^{(9)} - (r_{44})^{(9)}\right)s_{(45),(44)}T^*_{45} + (b_{45})^{(9)}s_{(44),(44)}T^*_{44}\Big)$$

$$\left((\lambda)^{(9)}\right)^2 + \left((a'_{44})^{(9)} + (a'_{45})^{(9)} + (p_{44})^{(9)} + (p_{45})^{(9)}\right)(\lambda)^{(9)}\Big)$$

$$\left((\lambda)^{(9)}\right)^2 + \left((b'_{44})^{(9)} + (b'_{45})^{(9)} - (r_{44})^{(9)} + (r_{45})^{(9)}\right)(\lambda)^{(9)}\Big)$$

$$+\left((\lambda)^{(9)}\right)^2 + \left((a'_{44})^{(9)} + (a'_{45})^{(9)} + (p_{44})^{(9)} + (p_{45})^{(9)}\right)(\lambda)^{(9)}\Big)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)} + (a'_{44})^{(9)} + (p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^*_{45} + (a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^*_{44}\right)$$

$$\left((\lambda)^{(9)} + (b'_{44})^{(9)} - (r_{44})^{(9)}\right)s_{(45),(46)}T^*_{45} + (b_{45})^{(9)}s_{(44),(46)}T^*_{44}\Big)\} = 0$$

**And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.**

## SECTION TWENTY

## Quantum Fidelity Amplitude

### INTRODUCTION—VARIABLES USED

**Loschmidt echo Arseni Goussev et al. (2012), Scholarpedia, 7(8):11687.Following is written in model able language, complete exposition of which is not done ascribable to spatial restraints doi:10.4249/scholarpedia.11687revision #127578 [link to/cite this article] Post-publication activity Curator: Rodolfo A. Jalabert Dr. Arseni Goussev, Max Planck Institute for the Physics of Complex Systems, Dresden, Germany Prof. Rodolfo A. Jalabert, Institut de Physique et Chimie des Matériaux de Strasbourg, CNRS, Université de Strasbourg, France Prof. Horacio M. Pastawski, Facultad de Matemática, Astronomía y Física and Instituto de Física Enrique Gaviola, Universidad Nacional de Córdoba, Córdoba, Argentina Prof. Diego Ariel Wisniacki, Dpto. De Física, University of Buenos Aires, Capital Federal, Buenos Aires, Argentina (Scholarpedia): For figures kindly see Scholarpedia.**

(1) The Loschmidt echo is (=) a measure of the revival occurring when (e) an imperfect time-reversal procedure is applied to (e&eb) a complex quantum system.
(2) It allows to quantify the sensitivity of quantum evolution to (e&eb) perturbations.
(3) An initial quantum state $|\psi 0\rangle$ evolves during a time t under (e&eb) a Hamiltonian H1 reaching the state $|\psi t\rangle$.

(4) Aiming to recover the initial state $|\psi 0\rangle$ a new Hamiltonian $-H2$ is applied between (e&eb) t and 2t.
(5) Perfect recover of $|\psi 0\rangle$ would be achieved by (e&eb) choosing H2 to be (=) equal to H1.
(6) This is not possible in realistic setups, and there always appears (eb) a difference between H2 and H1, leading (eb) to a non-perfect recovery of the initial state.
(7) The forward evolution between t and 2t under (e&eb) the Hamiltonian $-H2$ is equivalent to (eb) a backward evolution from t to 0 under H2, which embodies (e) the notion of time-reversal.
(8) The Loschmidt echo studies are focused on the cases where (e) the dynamics induced by the Hamiltonians H1 and H2 are (=) non-trivial, or sufficiently complex (like that of a classically chaotic one-particle system or a many-body system).

**Basic concept Refs. (Loschmidt 1876; Boltzmann 1877)**

(9) The concept of time-reversal has captured the imagination of physicists for centuries, leading to numerous vivid discussions. An emblematic example of these was the controversy around the second law of thermodynamics between Joseph Loschmidt and Ludwig Boltzmann, currently known as the Loschmidt paradox. When Boltzmann was trying to develop the microscopic theory of the second law of thermodynamics, Loschmidt raised an objection that had profound influence on the subsequent development of the theory. He argued that, due to the time-reversal invariance of classical mechanics, evolution in (eb) which the entropy can decrease must exist.
(10) These states could be reached by reversing (e&eb) velocities of all molecules of the system.
(11) After such a reversal the entropy would then no longer grow but decrease, seemingly violating (e) the second law of thermodynamics.
(12) As a response Boltzmann argued that such a time-reversal experiment would be impossible and put forward a statistical interpretation of the second law. Undoubtedly, Boltzmann's argument is a valid approach to the problem of the "arrow of time" for generic macroscopic systems. However, in quantum systems with few degrees of freedom, today's technological advances make it meaningful to address time-reversal experiments.

**Pioneering experiments Refs. (Hahn 1950; Rhim et al. 1971; Brewer et al. 1984; Zhang et al. 1992; Levstein et al. 1998)**

(13) The first controlled experimental implementation of time-reversal was achieved in the fifties with the inversion of nuclear spins precessing around (e&eb) a magnetic field.
(14) Such a reversal was able to compensate (e&eb) the local-field inhomogeneities responsible for (e) the decay of (e) the free induction signal.
(15) The time-reversal was implemented through (e&eb) a radio-frequency pulse, leading to (eb) the formation of a revival in the induction signal, known as (=) spin echo or Hahn echo.
(16) Early on Erwin Hahn recognized that his procedure, which he viewed as a change in the sign of the system Hamiltonian, provided (eb) a quantum implementation of the Loschmidt proposal.
(17) Interactions between spins, not reversed in Hahn's procedure, were the main cause of (e) the decay of the echo, within a time scale known as T2.
(18) This experiment was followed by a number of variants, both within magnetic resonance as well in other time-dependent spectroscopes'. In particular, the dynamical decoupling techniques implemented in quantum registries to isolate them from (e) their environment are (=)variants of Hahn's original experiment.
(19) The next level of complexity was developed with (e&eb) the reversal of the many-body interactions.
(20) No general recipe is available in this case, but the sign of the truncated spin-spin dipolar interaction, which is associated with (e&eb) the cosine of the angle between the inter-spin vector and the

quantizing magnetic field, can be reversed by (e) rotating the spins into a quantization axis. This was the proposal of the magic echo procedure, implemented by Won-Kyu Rhim, Alex Pines and John Stewart Waugh in the seventies.

(21) Substantially simpler is the polarization echo sequence, implemented by Richard Ernst and collaborators in the nineties. In this last case the initial state has (e) a local nature as it is labeled by the presence of a rare 13C which plays (e&eb) the role of a local probe to inject and later on detect the polarization of a nearby 1H immersed in a 1H network.

(22) This idea was further exploited by Patricia Levstein, Horacio Pastawski and Gonzalo Usaj to test the stability of (e) many-body dynamics.

(23) They suggested that the inefficiency of the time-reversal procedure found in all the previous experiments has a connection with (e&eb) quantum chaos and the inherent dynamical complexity of the many-body spin system.

**Definition Refs. (Peres 1984; Jalabert et al. 2001)**

(24) The Loschmidt echo is defined as M(t)=||⟨ψ0|eiH2t/ħe−iH1t/ħ|ψ0⟩||2,(1) where |ψ0⟩ is the state of the system at time 0, H1 is the Hamiltonian governing (e&eb) the forward evolution, H2 is the Hamiltonian governing (e&eb) the backward evolution, t is the instant at which the reversal takes place. Figure 1: Schematic flow of the time-evolution for (a) the Loschmidt echo and (b) the fidelity.

(25) The time evolution appearing in Eq. (1) is schematically represented in Figure 1 (a), where the **Loschmidt echo quantifies (eb) the degree of irreversibility**.

(26) Alternatively, Eq. (1) can be interpreted as the overlap at time t of two states evolved from (e) |ψ0⟩ under the action of (e&eb) the Hamiltonian operators H1 and H2.

(27) In this case the Loschmidt echo is (=) a measure of the sensitivity of quantum evolution to (e&eb) perturbations.

(28) The quantity M (t) interpreted in this manner is illustrated in Figure 1 (b) and is usually referred to as (=) fidelity.

(29) The equivalence between Loschmidt echo and fidelity is displayed in Figure 2 for the example of an initially localized wave-packet in (eb) a Lorentz gas.

(30) The probability density of the evolved state under H1 (H2) is plotted in Figure 2 (b) (c), while Figure 2 (d) presents the state resulting from (e) the combined evolution H1 and then −H2 both for a time t.

(31) The overlap squared between the states (a) and (d) defines (eb) the Loschmidt echo, while (e) the overlap squared between the states (b) and (c) defines (eb) the fidelity.

(32) It is important to remark that even though the probability density distributions in figures (b) and (c) are seemingly identical, the phase randomization due to (e) the difference between the two Hamiltonians leads to (eb) a weak Loschmidt echo.

(33) Figure 2: Wave-packet evolution in a Lorentz gas. (a) Initial state at time t=0 with momentum pointing to the left. (b) State evolved with the Hamiltonian H1 in (eb) the interval (0, t). (c) State evolved with (e&eb) the Hamiltonian H2 in the same time interval. (d) State evolved from that depicted in panel (b) with the −H2 for the time interval (t, 2t). In panels (b) and (c) the classical trajectories corresponding to (e&eb) three initial positions within (eb) the original wave-packet are shown for reference. The square of the overlap between the states (a) and (d), the Loschmidt Echo, is (=) M (t) =0.09, the same (=) as that between the states of panels (b) and (c). Adapted from Ref. (Cucchietti et al. 2002b). Copyright (2002), American Physical Society.

(34) The definition given by Eq. (1) assumes (eb) that the Hamiltonian operators H1 and H2 are (=) independent of time.

(35) A generalization to the case of time dependent Hamiltonians is straightforward. The case of **non-**

Hermitian operators H1 and H2 can also be considered, but in such a case the equivalence between the Loschmidt echo and≠ fidelity would not hold.

**Loschmidt echo and decoherence Refs. (Zurek 2001, 2003; Cucchietti et al. 2003, 2004; Casabone et al. 2010; Bonança 2011)**

(36) In an isolated quantum system the evolution is (=) unitary; initially pure states remain pure.
(37) When the system is connected to (e&eb) the external world the purity of an initial state is generically lost (e) in the course of (e&eb) time evolution.
(38) This ubiquitous process is referred to as (=) decoherence.
(39) The coupling to the environment degrees of freedom, or alternatively the lack of complete knowledge of the system Hamiltonian, (e) typically washes the specific interference properties that would be observed within (eb) a unitary evolution.
(40) Since interference is (=) one of the most important signatures of quantum mechanics, decoherence has both, fundamental and practical interest.
(41) Concerning the former aspect, decoherence has been proposed as (=) a road towards the classical behavior observed in (eb) macroscopic systems.
(42) On the second aspect, it is clear that decoherence represents (eb) a limitation in the implementation of quantum computers or nanoscopic devices based on (e) quantum effects, especially when (e) the number of qubits is scaled up, increasing (eb) the complexity of the system.
(43) The Loschmidt echo, by including (e) a non-controlled part of the Hamiltonian, or the noisy effects of the environmental degrees of freedom, gives a way (eb) to **quantify decoherence effect**.
(44) By attempting the time-reversal of the controlled part of the Hamiltonian, (eb) one can quantify at which rate the neighborhood of the system (either a unitary part of larger system Hamiltonian or though the effect of (e) infinite uncontrolled environmental degrees of freedom) acts as (=) decoherence.
(45) Moreover, other quantifiers of decoherence, such as for instance the purity, can be shown, under certain assumptions, to be characterized by (e) the same dependence on the underlying classical dynamics as the Loschmidt echo.

**Loschmidt echo and fidelity of quantum computing Ref. (Georgeot et al. 2001, 2002; Frahm et al. 2004; García-Mata et al. 2008)**

(46) A realistic quantum computer (see e.g., Nielsen & Chuang 2000) will always operate in (e&eb) presence of noise, decoherence and residual static imperfections and inter-qubit couplings between (e&eb) qubits forming (eb) a quantum computer hardware.
(47) These imperfections affect (e&eb) the accuracy of quantum computations being linked with (e&eb) fidelity decay.
(48) The dependence of fidelity decay for quantum algorithms simulating (e&eb) classical and quantum chaotic dynamics are analyzed in above papers and the universal law for fidelity decay in quantum computations of (e) complex dynamics in (eb) presence of realistic imperfections is established on (e) the basis of analytical theory and extensive numerical simulations with up to 30 qubits.

**Broad interest of the Loschmidt echo**

(49) Since the beginning of this century, the Loschmidt echo has been a subject of intensive studies by researchers from different scientific communities. A list of problems, in which the concept of the Loschmidt echo appears naturally, includes
(i) Quantum chaos, or the quantum theory of classically chaotic systems
(ii) Decoherence or the emergence of "classical world" (open quantum systems)

(iii) Quantum computation and quantum information
(iv) Spin echo in Nuclear Magnetic Resonance
(v) Linear waves (Elastic waves, microwaves, time-reversal mirrors, etc.)
(vi) Nonlinear waves (Bose-Einstein condensates, Loschmidt cooling, etc.)
(vii) Statistical mechanics of small systems
(viii) Quantum chemistry and molecular dynamics
(ix) Quantum phase transitions and quantum criticality

**Mathematical aspects of the Loschmidt echo**

(50) Such broad interest stems from the fact that the Loschmidt echo is (=) a measurable quantity exhibiting (eb), in certain regimes, a robust and universal behavior.
(51) It is by means of the forward and backward time-evolutions that the Loschmidt echo filters out (e&eb) some uninteresting effects, while (e) amplifying other, more important physical processes taking place in complex quantum systems.

**Loschmidt echo: techniques and main results**

(52) The Loschmidt echo is (=) typically a decreasing function of t.
(53) It is therefore of foremost importance to determine (eb) the form of the decay and the associated characteristic times in various physical situations as (=) a function of the characteristic quantities of the problem.
(54) Even though the Loschmidt echo has been addressed in a variety of scenarios, the main effort has been directed towards the simplest set-up of one-body dynamics. Well established results for the case of one-body systems, together with techniques adapted from the field of quantum chaos, are summarized in this section.

**Characteristic quantities**

(55) A number of physical quantities, describing (eb) characteristic time and energy scales of (e) a system under consideration, prove to be (=) especially important in the theoretical studies of the Loschmidt echo. These quantities include:

(i) g, mean density of states -- By definition, $g(E)=1(2\pi\hbar)dV(E)E$, where V(E) is the volume of the phase-space enclosed by (e&eb) a surface of constant energy E.
(ii) The mean level spacing $\Delta$ is given by (=) an inverse of the density of states, $\Delta=1/g$.
(iii) Note that g is the smooth part (or average) of the full density of states, $g_{full}(E) = \sum_n \delta(E-E_n)$, where $\{E_n\}$ are (=) the eigenenergies of the Hamiltonian.
(iv) glocal, local density of states. -- Let $|\psi\rangle$ be a reference quantum state. The local density of states with respect to $|\psi\rangle$ is defined as (=) $g_{local}(E,|\psi\rangle)=\sum_n |\langle n|\psi\rangle|^2\delta(E-E_n)$, where $\{|n\rangle\}$ and $\{E_n\}$ are respectively the **eigenstates and eigenenergies** of the Hamiltonian in question.
(v) N, effective size of the Hilbert space for a quantum state. -- An initial quantum state $|\psi 0\rangle$ of a closed quantum system can be viewed as (=) a linear combination of N eigenstates of the Hamiltonian.
(vi) In a d-dimensional system, N is given by (=) the volume of the phase space, accessible to (e) $|\psi 0\rangle$ in the course of its time evolution, divided by/ the volume of the Planck's cell, $(2\pi\hbar)$ d.
(vii) $\lambda$, mean **Lyapunov exponent**. -- In classical systems with chaotic dynamics, a distance between (e&eb) two typical trajectories, initiating at (eb) infinitesimally close points in (eb) phase-space, grows (eb) exponentially with time.
(viii) The rate of this growth, averaged over (e&eb) all trajectory pairs at a given energy, is (=) the

mean Lyapunov exponent λ.

(ix) The latter characterizes (eb) dynamical instability of a classical system with respect to (e&eb) perturbations of its initial conditions.

(x) tE, Ehrenfest time. -- The center of an initially localized wave packet stays close to (e) the phase-space trajectory of (e) the corresponding classical particle for (e) times shorter than the so-called Ehrenfest time, tE.

(xi) The latter, in chaotic systems, can be estimated as (=) tE=1λlnLσ, where λ is (=) the mean Lyapunov exponent of the classical system, L is (=) the characteristic linear size of the system, and σ is (=) the initial dispersion of the wave packet.

(xii) **tH, Heisenberg time**. -- The discreteness of the energy spectrum of a closed quantum system becomes dynamically important at times comparable to (e&eb) (and longer than) the so-called Heisenberg time, tH=ħ/Δ=ħg, where (e) Δ and g are the mean energy level spacing and the mean density of states, respectively. Semiclassical (short-wavelength) approximations to quantum dynamics are known to break down beyond (e) the Heisenberg time.

**Calculational techniques Semi classics Refs. (Jalabert 2001; Vanícek et al. 2003; Cucchietti et al. 2004; Vanícek 2004, 2006; Gutkin et al. 2010; Zambrano et al. 2011)**

(56) In this context, semi classics stand for (=) the short-wavelength approximation to the quantum evolution in terms of (e&eb) classical trajectories.

(57) The key ingredient is the **Van Vleck-Gutzwiller approximation** to (e) the propagator (matrix element of the evolution operator in (eb) the position representation) ⟨q′|e−iHt/ħ|q⟩=(12πiħ)d/2∑γ(q→q′,t)C1/2γexp(iħRγ−iπ2νγ)(2)

(i) γ stands for the classical trajectories going from q to q′ in time t.

(ii) Rγ=∫t0dτLγ is the Hamilton's principal function for the trajectory γ.

(iii) Here, Lγ denotes the Lagrangian along γ.

(iv) Cγ=|det(−∇q′∇qRγ)| is the stability factor of γ

(v) νγ is the number of conjugate points along γ

(vi) d is the number of dimensions of the position space.

Figure 3: Semiclassical approximation to the Loschmidt echo.

(58) A combination of Eqs. (1) and (2) leads to the semiclassical approximation to the Loschmidt echo shown in Figure 3. Here, the summand is a product of terms with the following pictorial representation:

(59) A red circles at a point qj corresponds to (e&eb) ⟨qj|ψ⟩

(60) A blue circles at a point qj correspond to ⟨ψ|qj⟩.

(61) That is, every blue circle is a complex conjugate of the corresponding red circle

(62) A solid curve leading from qj to qk along a trajectory γ corresponds to (2πiħ)−d/2 C1/2γexp(iRγ/ħ−iπνγ/2). The Hamiltonian H1 is used if γ equals α1, and the Hamiltonian H2 is used if γ equals β2.

(63) A dashed curve leading from qj to qk along a trajectory γ corresponds to (e&eb) (−2πiħ)−d/2 C1/2γexp(−iRγ/ħ+iπνγ/2)

(64) The Hamiltonian H1 is used if γ equals (=) α2 and the Hamiltonian H2 is used if (e) γ equals β1. In other words, every dashed curve is a complex conjugate of the corresponding solid curve.

(65) In Loschmidt echo studies one is typically in interested in the case of H2 being close to H1.

(66) It is common to describe the perturbation by an operator defined as κΣ=H2−H1,(3) where κ parametrizes its strength.

(67) The expression in Figure 1 is, in general, extremely complicated to calculate as it involves six

spacial integrals and the sum over four classical trajectories. Semiclassical evaluations of the Loschmidt echo deal with different approximations to the expression in Figure 1. The following steps are usually implemented in the semiclassical calculations in order to obtain meaningful analytical results:

(a) If the initial state is spatially localized around a point q0, the diagram of Figure 1 can be approximated by a simpler diagram in which the points q1, q2, q3, and q4 merge into (e&eb) q0. Within this approximation there are only two spacial integrals to be done (over q5, and q6).

(b) Assuming that the perturbation $\kappa\Sigma$ is classically small but quantum mechanically significant (which means that the perturbation does not (e) change the topology of the trajectories but introduces a phase difference) trajectories $\alpha$ j and $\beta$j are identified for j=1, 2.

(c) This is the so-called diagonal approximation. The shadowing theorem ensures (eb) that the identification of classical trajectories $\alpha$ and $\beta$ is always possible and that the resulting action difference can be obtained from (e) the accumulated phase of the perturbation along one of the trajectories.

(d) For instance, in the case where $\Sigma$ only depends on position coordinates $\Delta R\gamma=-\kappa\int t0 d\tau\ \Sigma\ (q\gamma\ (\tau))$, (4) for $\gamma=\alpha1, \alpha2$. The diagonal approximation reduces the expression in Figure 1 to (e&eb) a sum over only two trajectories, $\alpha1$ and $\alpha2$, of the unperturbed system.

(e) A key step that allows to make further progress in the semiclassical calculation is (=) to regroup the resulting pair of trajectories ($\alpha1$,$\alpha2$) into two different families:

(i) Trajectories where q5 is near q6 and $\alpha1$ evolves close to (e) $\alpha2$.

(j) The rest of pairs, where (e) the two trajectories are uncorrelated

(k) The semiclassical calculation usually proceeds by estimating the accumulated phases along (e&eb) the trajectories for (e) the two kinds of resulting pairs, and then performing some kind of average of M(t) (over the perturbation, the initial conditions or the evolution of the classical trajectories).

(l) In the case of complex classical dynamics these averages are justified and allow (eb) to obtain the mean value $\overline{M(t)}$ of the Loschmidt echo in terms of (e&eb) the main parameters of problem.

(68) An alternative semiclassical formulation for fidelity amplitude m(t) (defined through $M(t)=|m(t)|2$) which avoids (e) the usual trajectory-search problem of the standard semi classics, is (=) the so called dephasing representation $m(t)=\int dqdp\exp(-i\hbar\Delta R(q,p,t))W0(q,p)(5)$ where $W0(q,p)=1(2\pi\hbar)d\int d(\delta q)\exp(-i\hbar p\cdot\delta q)\langle q+\delta q2|\psi0\rangle\langle\psi0|q-\delta q2\rangle(6)$ is (=) the **Wigner function** of the initial state $|\psi0\rangle$ and $\Delta R(q,p,t)=\int t0 d\tau[L2(\bar{q}(\tau),\bar{p}(\tau))-L1(\bar{q}(\tau),\bar{p}(\tau))](7)$is (=) the action difference evaluated along (e&eb) the phase-space trajectory $(\bar{q}(\tau),\bar{p}(\tau))$ evolved from (q,p) under (e&eb) the unperturbed Hamiltonian H1.

(69) In this way the decay can be attributed to the dephasing produced by (e) the perturbation of the actions --thus the name dephasing representation.

(70) In the case where $\Sigma$ only depends on position coordinates (as in Eq. (4)) the phase difference (7) reads $\Delta R\ (q, p, t) =-\kappa\int t0 d\tau\ \Sigma\ (\bar{q}\ (\tau))$. (8)

(71) For generic chaotic systems and initially localized states, the calculation of M (t) using the dephasing representation follows (e&eb) the same lines of the standard semi classics.

(72) The standard semiclassical approach becomes (=) an initial-value problem once the integration over the final position is traded by (e&eb) the integration over the initial momentum using (e) the stability factor C$\gamma$ as the Jacobian of the transformation.

(73) However, Eq. (5) bears the advantage that it regards m (t) as the solution of an initial-value problem, as opposed to (e&eb) a boundary-value problem.

(74) This proves especially convenient in situations when (e) the sums over classical trajectories are evaluated explicitly.

**Random matrix theory Refs. (Cucchietti et al. 2002a; Cerruti et al. 2003; Frahm et al. 2004; Gorin et al. 2004; Stöckmann et al. 2004, 2005; Kohler et al. 2008, 2011, 2012)**

(75) Random matrix theory is a powerful technique to understand the statistical behavior of (e) quantum complex systems.

(76) Among the latter quantum systems exhibiting (eb) chaotic dynamics in the classical limit are particularly important, as their spectral properties are well described by (e) averages taken over appropriate ensembles of (e) matrices that satisfy certain general symmetry restrictions.

(77) The simplest Hamiltonian ensemble is (=) that of real symmetric N×N matrices (appropriate for cases with time-reversal symmetries) usually referred to as Gaussian Orthogonal Ensemble.

(78) The invariance of the probability distribution under (e&eb) orthogonal transformations and the statistical independence of the matrix elements lead to (eb) a joint probability distribution of the matrix elements of the Hamiltonian, PN(H)=KNexp(−tr{H2}4v),(9) where (e) v defines the scale of the matrix elements and KN is a normalization constant. Each of the N (N+1)/2 independent matrix element Hij is (=) a zero-centered Gaussian such that Hij¯¯¯¯¯=H2ij¯¯¯¯¯=0(1+δij) v2i≤j,i≤j.

(79) The averages are taken over (e&eb) the matrix ensemble.

(80) The Loschmidt echo in complex systems can be addressed by imposing (e&eb) the Random Matrix hypothesis for (e) the forward evolution Hamiltonian H1, for (e) the perturbation κΣ representing (eb) the imperfection of the time-reversal, or for both.

(81) Generically, the same results are obtained, independently of (e) the choice of the matrix considered to be random.

(82) Averages of the Loschmidt echo over matrix ensembles are justified for (e) ergodic classical dynamics.

(83) The absence of finite classical time scales, like the inverse Lyapunov exponent or the escape rate, in the Random matrix theory hinders (e) the description of the Loschmidt echo decays that depend on ∝ those quantities.

(84) Recently, using the Random-Matrix-Theory approach, Kohler and coworkers have established relations between the averaged fidelity decay and (e&eb) the so-called cross-form factor, characterizing (eb) parametric correlations of the energy spectra of (e) the unperturbed and perturbed systems.

(85) Notably, these relations exist not only in the case of (e) fully chaotic unperturbed and perturbed systems, but also in (eb) the case of a regular system perturbed by (e) a chaotic perturbation. General dependence of the fidelity decay on (=) Heisenberg and Thouless time scales is established by Frahm et al.

**Numerical simulations Refs. (Tal-Ezer et al. 1984; De Raedt 1996; van Dijk et al. 2011)**

(86) A numerical simulation of (e) the time-evolution of initially localized wave-packets under slightly different Hamiltonians, H1 and H2, is (=) a valuable tool in understanding the behavior of the Loschmidt echo in (eb) different systems and various regimes.

(87) Even though there exist numerous approaches to solving the **time-dependent Schrödinger equation** on a computer, the majority of numerical studies of the Loschmidt echo have been concerned with (e&eb) the following two methods. Both methods provide accurate, efficient, and stable approximations to (e) the evolution operator e−iAτ, where (e) the operator A corresponds to (e&eb) a (properly rescaled) Hamiltonian, and τ denotes (eb) a sufficiently short propagation time-step.

(88) **Trotter-Suzuki algorithm**. -- This method involves three implementation stages. First, one decomposes A into (e&eb) a finite (and practically small) number of components, A=A1+A2+…+An, such that (e) the operator exp (−iAjt) can be constructed analytically for (e) all

j=1, 2… n.

(89) Second, one defines (eb) the symmetric operator U2 (τ) =e−iAnτ/2…e−iA2τ/2e−iA1τe−iA2τ/2…e−iAnτ/2.

(90) Third, one constructs the operator U4(τ)=U2(pτ)U2(pτ)U2((1−4p)τ)U2(pτ)U2(pτ) with (e&eb) p=1/(4−41/3).

(91) This provides a unitary approximation to (e) the original propagator accurate up to (e) the 4th order in τ, i.e., e−iAτ=U4 (τ) +O (τ5).

(92) **Chebyshev-polynomial expansion**. -- This approach requires (e) that the operator A is (=) normalized in such a way that all its eigenvalues lie (eb) in the interval between -1 and 1.

(93) The method involves constructing the operator SN(τ)=J0(τ)+2∑Nn=1(−i)nJn(τ)Tn(A), where (e) {Jn} and {Tn} are, respectively, the **Bessel functions** of the first kind and the Chebyshev polynomials of the first kind.

(94) In actual computations, one makes use of the recurrence relation Tn+1(A)=2ATn(A)−Tn−1(A), with (e&eb) T0(A)=1 and T1(A)=A, to calculate (eb) SN(τ).

(95) Then, for large values of n and fixed τ, the Bessel function Jn (τ) rapidly decays as (e) a function of n.

(96) In fact, |Jn (τ)|∼(τ/2)n/n! as (e) n→∞.

(97) This is why SN (τ), with (e&eb) a sufficiently large N, provides (eb) an extremely accurate approximation to (e) the propagator in question, i.e., e−iAτ=SN (τ) +O ((τ/2) N/N!).

**Decay of the Loschmidt echo: different regimes**

(98) The decay of the Loschmidt echo mainly depends upon ∝ the underlying classical dynamics of the system, the initial state, and the nature and strength of the perturbation κΣ.

(99) The behavior of the Loschmidt echo is fairly well understood for (e) single-particle quantum systems whose dynamics is (=) fully chaotic in the classical limit. Some progress has been done towards the theory of the Loschmidt echo in systems with regular and mixed phase space.

**Chaotic dynamics Refs. (Shepelyansky 1983; Jalabert et al. 2001; Jacquod et al. 2001; Cerruti et al. 2002, 2003; Wisniacki 2002, 2003; Cucchietti et al. 2004; Gorin et al. 2004; Gutiérrez et al. 2009)**

(100) Figure 4: Decay regimes of the Loschmidt echo in quantum systems with (e&eb) chaotic classical limit.

(101) Characteristic crossover times are (=) t≃ħ/η, signaling (eb) the end of the initial parabolic decay, and ts (≃Γ−1lnN for (e) the non-perturbative/exponential regime) indicating (eb) the onset of the saturation.

(102) For classical chaotic dynamics the errors grow exponentially with (e&eb) time and thus the time reversibility of Loschmidt echo is completely broken (e) after logarithmically short times.

(103) This resolves (e) the Loschmidt paradox in the world of classical mechanics.

(104) For a quantum evolution, even if its corresponding classical dynamics is fully chaotic, small errors grow (eb) only polynomially with time in agreement with (e&eb) the quantum perturbation theory and hence a deeper analysis should be performed here.

(105) The time-decay of the Loschmidt echo, averaged over (e&eb) an ensemble of initial states, Hamilton operators, or perturbations, typically exhibits (eb) three consecutive stages (see Figure 4):

(106) **Short-time parabolic decay**: M (t) $\overline{\phantom{xx}}$ ≃1− (ηt/ħ) 2.

(107) It is a short, initial stage of the Loschmidt echo decays in (eb) all quantum systems.

(108) Here, η is (=) an average dispersion of the perturbation operator evaluated with respect to (e&eb) the initial state, i.e., (η/κ) 2=⟨ψ|Σ2|ψ⟩ $\overline{\phantom{xxxx}}$ −⟨ψ|Σ|ψ⟩2 $\overline{\phantom{xxxx}}$.

(109) The parabolic decay holds for (e) times t short enough for (e) the propagators of (e) the

unperturbed and (e&eb) perturbed systems, exp (−iHjt/ħ) with j=1, 2, to be reliably approximated by (e) their second order Taylor expansions, 1−iHjt/ħ− (Hjt) 2/(2ħ2).

(110) **Intermediate-time asymptotic decay:** The short-time parabolic decay is typically followed by (e) an "asymptotic" decay regime, whose functional form depends on ∝ the strength of the perturbation.

(111) **Perturbative/Gaussian regime**, M (t) ¯¯¯

trajectories, while the second term is (=) a contribution of correlated trajectory pairs, such that (e) α1 evolves close to α2 (see section Semi classics).

(125) Unlike the constant c, the prefactor b generally exhibits (eb) an explicit dependence on both t and κ.

(126) However, this dependence is (=) sub-exponential, and, as a consequence, (eb) the decay rate Γ of the Loschmidt echo is dominated by (e) the minimum of cκ2 and λ, leading to (eb) a commonly accepted (but oversimplified) interpretation of (e) the exponential decay (see Figure 6).

(127) For perturbation strengths κ that are weak compared to (e&eb) a critical strength κc, the dependence is (=) parabolic, Γ=cκ2.

(128) The semiclassical calculations give the expression of c in terms of (e&eb) correlation functions of the perturbation, and the random matrix approach leads to (eb) the general dependence cκ2=2πη2/(ħΔ), where η is (=) given by the mean value of the width of the local density of states with respect to (e&eb) the initial state |ψ0⟩.

(129) Such a behavior is commonly referred to as (=) the Fermi-golden-rule regime.

(130) For perturbations that are stronger than κc, but weaker than a certain bound value κb, the decay rate Γ is (=) independent of κ and equals (eb) the average Lyapunov exponent λ of the underlying classical system.

(131) This decay is known as (=) the perturbation-independent or Lyapunov regime.

(132) This regime holds for (e) perturbations up to the breakdown strength κb, beyond (e) which it is no longer possible to regard every trajectory of the perturbed system as a result of (e) a continuous deformation of (e) the corresponding unperturbed trajectory, so that bifurcations have to be (=) taken into account.

(133) The Lyapunov regime is especially remarkable, as the decay rate is (=) totally independent of (e) the strength of the perturbation which is (=) at the origin of the decay.

(134) The exponential decay of the Loschmidt echo is generally followed in time by (e) saturation at a value on the order of N−1, where (e) N is the size of the effective Hilbert space.

(135) This implies that (eb) in the non-perturbative regime, the saturation occurs (eb) at time ts≃Γ−1lnN.

(136) In the case of a two-dimensional chaotic billiard (introduced above), the saturation time is (=) given by ts≃Γ−1ln (|p0|A/ (ħσ)) − (2Γ) −1ln (2π).

(137) It is interesting to note that the saturation time can, in principle, be (=) arbitrarily long.

(138) In particular, ts can exceed other important time scales of quantum dynamics, such as the Heisenberg time tH≡ħg(E) with g(E) denoting the density of states. Indeed, in two-dimensional billiards the saturation time ts→∞ as |p0|→∞, while (e) the Heisenberg time tH is independent of the particle's momentum.

(139) The exponential decay, M (t) ¯¯¯¯¯¯¯¯¯¯≃exp (−Γt), only holds for (e) perturbations weaker than κb.

(140) Beyond this threshold, the exponential regime breaks down and gives (e&eb) way to another regime, in which the Loschmidt echo exhibits (eb) a Gaussian dependence on time.

(141) Some of the mentioned regimes can be obtained using alternative approaches relating the Loschmidt echo with (e&eb) the two-point time auto-correlation function of the generator of the perturbation.

**Local perturbations Refs. (Goussev et al. 2007, 2008; Höhmann et al. 2008; Ares et al. 2009; Köber et al. 2011)**

(142) A Hamiltonian perturbation is said to be "local" if (e) it is concentrated in a small region of (e) the phase space accessible to the system.

(143) In the case of chaotic dynamics, the phase space extent of a local perturbation can be characterized by (e) a rate γ, known as the escape rate, which is defined as (=) the rate at which trajectories of the corresponding classical system visit (e&eb) the perturbation region.

(144) For local perturbations, the escape rate is small compared to (e&eb) the characteristic rate at which a typical trajectory "explores" (e&eb) the dynamically available phase space, and also small compared to (e&eb) the average Lyapunov exponent of the system.

Figure 7: Exponential decay rate Γ as a function of the perturbation strength κ in the case of a local Hamiltonian perturbation.

(145) Figure 7 illustrates the characteristic dependence of the decay rate Γ on the perturbation strength κ. The function Γ (κ) exhibits (eb) a number of distinctive features.

(146) Thus, for sufficiently weak perturbations, the decay rate grows quadratically with (e&eb) the perturbations strength, Γ∼κ2, demonstrating (eb) the Fermi-golden-rule regime.

(147) In the limit of strong perturbations, κ→∞, the decay rate Γ saturates at (eb) a perturbation-independent value 2γ.

(148) This saturation is known as (=) the escape-rate regime.

(149) The crossover from the Fermi-golden-rule to (e&eb) the escape rate regime is (=) non-monotonic, and Γ (κ) generally exhibits (eb) well-pronounced oscillations.

(150) The amplitude and frequency of these oscillations depend on ∝ the nature and physical properties of (e) the particular Hamiltonian perturbation.

(151) An important point is that the function Γ (κ) is given for (e) all strengths κ by the width of (e) the local density of states.

(152) The exponential decay, M (t) $\overline{\qquad}$≃exp (−Γt) with (e&eb) the non-monotonic function Γ (κ) described above and illustrated in Figure 7, has been obtained by (e) semiclassical analysis. (Its existence has now been observed in (eb) numerical and laboratory experiments).

(153) The key assumption of any semiclassical reasoning is (=) that the de Broglie wavelength corresponds to (e&eb) the shortest length scale of the system.

(154) This assumption is obviously violated for (e) systems, in which the spatial linear size of the Hamiltonian perturbation is small, compared to (e&eb) the de Broglie wavelength.

(155) Generally, the decay of the Loschmidt echo in such systems is different from (e) the exponential decay.

(156) For instance, two-dimensional chaotic systems in (eb) the limit of point-like Hamiltonian perturbations, for (e) which γ→0, exhibit (eb) the algebraic, inverse-quadratic decay of (e) the Loschmidt echo, M (t) $\overline{\qquad}$∼t−2.

**Regular and mixed dynamics Refs. (Prosen 2002; Jacquod et al. 2003; Prosen et al. 2003; Sankaranarayanan et al. 2003; Weinstein et al. 2005; Wang et al. 2007; Goussev 2011)**

(157) Time dependence of the Loschmidt echo in quantum system with (e&eb) regular or mixed classical dynamics is typically more complex than (e) that in fully chaotic systems.

(158) As a result of this complexity, a comprehensive classification of decay regimes is still lacking. Nevertheless, a number of important results have been established for the Loschmidt echo decay in systems whose phase space is (eb) predominantly regular in the classical limit.

(159) In the limit of weak Hamiltonian perturbations, κ→0, the average Loschmidt echo generally exhibits (eb) a Gaussian decay.

(160) The duration tG of this decay is inversely proportional to ∝ the perturbation strength, tG∼κ−1.

(161) The existence of the Gaussian decay requires (e) the perturbation to be sufficiently weak

such that (e) the decay time tG is long in comparison with (e&eb) any relaxation (averaging) time scale of the system.

(162) If the Hamiltonian perturbation is sufficiently strong and varies (e&eb) rapidly (or is almost "random") along (e&eb) a typical classical trajectory of (e) the unperturbed Hamiltonian, then (eb) the average Loschmidt echo is known to exhibit (eb) the algebraic decay $\overline{M(t)}\sim t^{-3d/2}$, with d being the dimensionality of the system.

(163) This power-law decay is faster than (e) the decay of the overlap of the corresponding classical phase-space densities, $t^{-d}$.

(164) Numerous case studies of the unaveraged (individual-realization) Loschmidt echo M (t) have revealed that (eb) the functional form of the decay depends strongly upon ∝ the location of the initial quantum state with respect to (e&eb) the phase space of the underlying, unperturbed and perturbed, classical systems.

(165) In addition, the decay is sensitive to (e&eb) certain properties of the Hamiltonian perturbation.

(166) For instance, the Loschmidt echo decays differently depending on ∝ whether the perturbation is global or local in phase space, whether it vanishes under (e&eb) averaging over time, or whether it preserves or destroys (e&eb) integrability of the underlying classical system.

(167) A number of different regimes, such as the exponential decay and power-law decay with vary (e&eb) decay exponents have been observed in (eb) numerical simulations.

(168) Interesting phenomena of quantum revivals, closely related to (e&eb) (quasi-)periodicity of the underlying (e&eb) classical motion, and temporary quantum freeze of (e) the Loschmidt echo have been reported in various studies.

(169) However, a comprehensive classification of all decay regimes of (e) the Loschmidt echo in regular systems still remains a challenge in the field. Model attempts to alleviate the complexity.

**Numerical observation of various decay regimes Refs. (Jacquod et al. 2001; Cucchietti et al. 2002a, 2002b, 2004; Goussev et al. 2007, 2008; Ares et al. 2009; García-Mata et al. 2011b)**

(170) Figure 8: Decay rate of the Loschmidt echo as (=) a function of the perturbation strength κ for (e) the three concentrations of impurities in a Lorentz gas.

(171) The rates Γ are normalized to (e&eb) the group velocity of (e) the initial wave packet.

(172) The solid lines are (=) the corresponding classical Lyapunov exponents and the dashed lines are fits to the quadratic behavior. Inset: a log-log scale of the same data showing the quadratic increase of Γ for small perturbations. Adapted from Ref. (Cucchietti et al. 2004). Copyright (2004), American Physical Society.

(173) Figure 9: The Loschmidt echo decay in the desymmetrized diamond billiard for (e) four different values of the curvature radius r of (e) the arc boundary deformation.

(174) The solid straight line gives (eb) the trend of the exp (−2γt) decay. Inset: Forward-time wave packet evolution in the **unperturbed billiard** followed by (e&eb) the reversed-time evolution in the **perturbed billiard**.

(175) The arrow shows the momentum direction of (e) the initial wave packet. The propagation time corresponds to approximately 10 collisions of (e) the classical particle. Adapted from Ref. (Goussev et al. 2007). Copyright (2007), American Physical Society.

(176) Numerical simulations have been of foremost importance for establishing the results summarized in sections Chaotic dynamics and Regular and mixed dynamics. The main body of numerical work has been devoted to one-particle systems. Various chaotic dynamical systems, globally perturbed, have been studied numerically; among them the Lorentz gas, two-dimensional hard-wall and soft-wall billiards. As a prominent example, Figure 8 shows the crossover from (e) the perturbative Fermi-golden-rule regime to (e&eb) the Lyapunov regime of the decay rate of the

Loschmidt echo as (=) a function of the perturbation strength in the Lorentz gas.

(177) The predictions for local perturbations, described in section Local perturbations, were also tested in (eb) a chaotic billiard system. In this case, a deformation of a small region of (e) the boundary was considered as (=) the perturbation (see Figure 9).

(178) Figure 10: Decay Γ of the Loschmidt echo of a cat map perturbed locally with (e&eb) a shear in momentum as (=) a function of the scaled perturbation strength κ.

(179) The symbols correspond to (e&eb) the following fraction of the phase space that is perturbed: ▽ (40%), ∆ (30%), + (20%) and ∗ (10%).

(180) The width η of the local density of states is also plotted with solid lines. This width gives (=) the **spread of the states** of the unperturbed system in the bases of (e) the perturbed one

(181) It is noticeable that, the smaller is the perturbed region,(eb) the more similar to η Γ is. Adapted from Ref. (Ares at al. 2009). Copyright (2009), American Physical Society.

(182) Other enlightening numerical studies were carried in quantum maps. These systems possess (e) all the essential ingredients of the chaotic dynamics and are, at the same time, extremely simple from a numerical point of view, both at the classical and at the quantum levels.

(183) The predictions for locally perturbed chaotic systems were (=) also observed in the paradigmatic cat map.

(184) Figure 10 shows the decay rate of the Loschmidt echo as (=) a function of the perturbation strength κ in the cat maps under (e&eb) the action of a local perturbation.

(185) When the map was perturbed in all its phase space, three decay regimes (instead of two) were observed depending on ∝ the perturbation strength: (i) the Fermi-golden-rule regime for (e) weak perturbations, (ii) a regime of an oscillatory dependence of (e) the decay rate on κ at intermediate perturbation strengths, and (iii) the Lyapunov regime for (e) strong perturbations. The complete understanding of the crossover between (e&eb) the last two regimes remains an open problem.

**Beyond the "standard" picture: fluctuations, disorder, many-body systems Refs. (Adamov et al. 2003; Silvestrov et al. 2003; Petitjean et al. 2005; Manfredi et al. 2006, 2008; Quan et al. 2006; Pozzo et al. 2007; Zangara et al. 2012)**

(186) Decay regimes of the average Loschmidt echo, discussed above, are the ones most commonly observed, and generally regarded as (=) "standard", in (eb) low-dimensional quantum systems.

(187) However, when analyzing the Loschmidt echo beyond (e) a simple averaging over the initial state or perturbation, or in a more complex setup, one often encounters (e&eb) departures from (e) the standard picture.

(188) It was fist pointed out by Silvestrov and coworkers that, for sufficiently strong perturbations and on time scales short compared to the Ehrenfest time, statistical fluctuations generally play an important role (e&eb) in the problem of the Loschmidt echo decay.

(189) In particular, the average Loschmidt echo, M (t) ¯¯¯¯¯¯¯¯, is typically dominated by (e) rare fluctuations, characteristic of (e) only a small fraction of the chaotic phase space.

(190) In the main part of the phase space, however, the unaveraged Loschmidt echo, M(t), decays much faster than (e&eb) M(t)¯¯¯¯¯¯¯¯: this decay can be as (=) fast as double-exponential, M(t)∼exp(−constant×e2λt).

(191) It is only after the Ehrenfest time that M (t) follows M (t) ¯¯¯¯¯¯¯¯ in the main part of the phase space.

(192) The variance of the Loschmidt echo, M2¯¯¯¯¯¯¯−M¯¯¯¯2, was addressed by Petitjean and Jacquod within (e&eb) both a semiclassical and a Random Matrix Theory approach.

(193) The variance was shown to exhibit (eb) a rich nonmonotonous dependence on time, characterized by (e) an algebraic growth at short times followed by (e&eb) an exponential decay at long times.

(194) In disordered systems, the Loschmidt echo may exhibit (eb) even richer decay than that in low-dimensional dynamical systems, since (e) the elastic mean-free path and the disorder correlation length enter (e&eb) as relevant scales.

(195) For long-range disorder (small-angle scattering) an asymptotic regime governed by (e&eb) the classical Lyapunov exponent emerges.

(196) A Josephson flux qubit operated at high energies leads to (eb) a chaotic dynamics, and the Lyapunov regime can be obtained when (e) a Loschmidt setup is considered in such a system.

(197) The Loschmidt echo studies in (eb) many-body systems are comparatively less developed than those in one-body cases. Model throws light on the issue

(198) The numerical calculation of the many-body Loschmidt echo is a highly demanding computational task, and the standard approximations are in difficulty for (e) describing the small difference obtained after (e) the forward and backward time-evolutions.

(199) For this reason, the most reliable results of Loschmidt echo in many-body systems are (=) those of a one dimensional spin chain or in an Ising model with transverse field, where (e) the influence of criticality in the fidelity decay has been established.

(200) Trapped Bose-Einstein condensates have been treated as (=) a many-body Loschmidt echo setup using (e) mean-field approaches.

**Phase-space representation and classical fidelity Refs. (Benenti et al. 2002, 2003; Eckhardt 2003; García-Mata et al. 2003; Cucchietti et al. 2004; Veble et al. 2004)**

(201) An intuitively appealing representation of the **Loschmidt echo in phase space** is obtained by (e) expressing Eq. (1) as $M(t)=1(2\pi\hbar)d\int dq\int dp\ WH1(q,p;t)\ WH2(q,p;t)(11)$ where (e) WH1 and WH2 are the Wigner functions resulting from (e) the evolution of W0(q,p), introduced in Eq. (6), under (e&eb) the action of the Hamiltonian operators H1 and H2 respectively.

(202) This approach is naturally connected to (e&eb) the dephasing representation of section Semi classics and provides (eb) a particularly useful framework in the study of decoherence, as the Wigner function is a privileged tool to (e) understand the connection between quantum and (e&eb) classical dynamics.

(203) An initial Gaussian wave-packet is associated with (e&eb) a Gaussian Wigner function that will develop (eb) in phase-space non-positive structures and be deformed under (e&eb) the evolution of H1 and H2.

(204) The sensitivity of the overlap (11) to the non-positive parts and (e&eb) to the deformations of the Wigner function can be related with (e&eb) the different decay regimes discussed in section Chaotic dynamics.

(205) The form (11) of the quantum fidelity allows (eb) to define a classical fidelity by using in (e&eb) a classical problem the **Liouville distributions** LH1 and LH2 instead of (e&eb) WH1 and WH2 respectively.

(206) This definition follows from the logical representation of the Liouville distribution as (=) the classical limit of the Wigner function.

(207) However, the definition by an overlap of distributions does not (e) convey the sense of classical reversibility: a trajectory evolving forward in time with H1 and backwards with H2 would give a considerable contribution to (e&eb) the overlap not only if it ends up close to the initial point, but whenever (e) the final point is in the neighborhood of those of the initial distribution.

(208) The labeling of trajectories and particles, characteristic of classical mechanics, is (=) then not taken into account in this definition of classical fidelity.

(209) The correspondence principle dictates that (e&eb) the quantum fidelity follows its (e&eb) classical counterpart up to (e) the Ehrenfest time.

(210) However, such a correspondence does not imply that, in the intermediate-time asymptotic decay of globally perturbed chaotic systems, the Lyapunov regime is only present up to this characteristic time scale.

(211) In fact, the Lyapunov regime is not simply an effect of the classical-quantum correspondence. For instance, in a two-dimensional billiard, the saturation time tS (see section Chaotic dynamics) that would signal the end of the asymptotic decay (and so the end of the Lyapunov decay if one is in the appropriate regime) can be much larger than the Ehrenfest time.

**Experiments Nuclear Magnetic Resonance Ref. (Slichter 1990). See particularly Sec. 8.8 and App. E.**

(212) Nuclear magnetic resonance has been the main tool to study echoes generated by different time-reversal procedures since the fifties.

(213) After the time-reversal of individual spin precession implemented by Erwin Hahn in the spin echo, one had to wait until the seventies, when Rhim and Kessemeir achieved (eb) the reversion of (e) the spin-spin interactions.

(214) The resulting reversal of the macroscopic polarization is known as (=) magic echo and was thoroughly studied by Rhim, Pines and Waugh. In the nineties Ernst et al. implemented a strategy to address a localized spin excitation and Levstein et al. realized that such study was (=) optimal for a theoretical description.

(215) They used crystal structures with abundant of interacting 1H spins, where (e) the dipolar coupling produces (eb) the "diffusion" of the initial polarization.

(216) The time-reversal procedure is then able to refocus the excitation generating (eb) a polarization echo. These last experiments have a direct connection with the concepts discussed in this review.

**Basic concepts on spin dynamics Refs. (Pastawski et al. 1995; Mádi et al. 1997; Levstein et al. 1998; Danieli et al. 2004; Zangara et al. 2012)**

(217) A typical experiment starts with a sample which constitutes a network with about 1023 interacting spins in (eb) thermal equilibrium in (eb) presence of an external magnetic field B0.

(218) This many-spin state is denoted as (=) $|\Psi eq\rangle$.

(219) In this state every spin has an almost equal probability of being up or down since (e) the thermal energy, kBT is much higher than (e) any other relevant energy scale of the system.

(220) At time t=0 a sequence of radiofrequency pulses ensures (eb) that spin at site 0th is oriented along (e&eb) z direction.

(221) This is represented by the action of the spin operator S+0 on (e&eb) $|\psi eq\rangle$.

In a system with m+1 spins, we represent this initially excited state as (=) $|\Psi 0\rangle=S+0|\Psi eq\rangle||\langle\Psi eq|S-0S+0||\Psi eq\rangle||1/2=\sum r=12mei\phi r2m/2|\uparrow 0\rangle\otimes|\beta r\rangle$,(12) where (e) the denominator $||\langle\Psi eq|S-0S+0||\Psi eq\rangle||1/2$ ensures (eb) that the initially excited state has (e) a proper normalization and $\phi r$ is (=) a random phase that describe (eb) a mixture of states of the form (eb) $|\beta r\rangle=|s1\rangle\otimes|s2\rangle\otimes|s3\rangle\otimes...\otimes|sm\rangle$with$|sk\rangle\in\{|\uparrow\rangle,|\downarrow\rangle\}$.(13

(223) The preparation of this state involves the use of (e &eb) a 13C nucleus as a "spy" to inject and detect (e&eb) polarization at the directly bonded 1H spin (0th).

(224) Then, the system evolves under (e&eb) mutual many-body interaction described by (e) H1 for a period t1.

(225) In the polarization echo experiment, this is described by effective dipolar interaction Hamiltonian H1, truncated by (e) the Zeeman field,

H1≡PolarizationEchoExperiment∑i,j[2SziSzj−12(S+iS−j+S−iS+j)].(14)

(226) H1 contains (e) flip-flop or XY operators of (e) the form S+iS−j+S−iS+j, as well as Ising terms of the form SziSzj.

(227) The dipolar interaction di, j constant decay with (e&eb) the third power of the distance between sites i and j.

(228) H1 produces (eb) the spread of (e) the initially localized excitation.

(229) Since total polarization is conserved under (e&eb) H1, such process is commonly known as (=) "spin diffusion".

(230) Then, a new pulse sequence rotates (e&eb) all the spins 90 degrees and the irradiation with (e&eb) r.f. field produces (eb) a further truncation of the dipolar interaction along (e&eb) the rotating field.

(231) The resulting effective Hamiltonian −H2=− (H1+Σ) acts for (e&eb) another period t2.

(232) Observe that the reversal is not perfect but (e) Σ is a small term which is due to (e) truncations required to form (eb) the effective Hamiltonian.

(233) This constitutes (e) the "backwards" evolution period after which the local polarization at site 0th is measured.

(234) We notice that Σ acts as (=) a self-energy operator that may account for (e) interactions within (eb) the same Hilbert space, and thus is (=) just a Hermitian effective potential.

(235) However, it also may describe (eb) the interaction with (e&eb) a different subsystem that constitutes (e) the "environment".

(236) In the last case Σ=Δ−iΓ would also have (e) an imaginary (non-Hermitian) component.

(237) In this last case, evolution of observables should be described by (e) two coupled Lindblad or Keldysh equations for the density matrix requiring (e) a self-consistent evaluation of dynamics of the system and the environment.

(238) This last situation would make the analysis less straightforward and might hinder (e) the **essential physics**.

(239) Thus, we first focus on perturbations that ensure unitary evolution. An example discussed by Zangara et al. of this situation is (=) a second frozen spin chain interacting with (e&eb) the first through (e&eb) an Ising interaction.

(240) In these cases, a polarization echo is formed when (e) t2=t1, i.e. after a total evolution time 2t=t2+t1.

(241) The normalized amplitude is given by the spin autocorrelation function that accounts for (e) the observed local polarization, MPE(t)=⟨Ψeq|S−0(2t) S+0|Ψeq⟩⟨Ψeq|S−0S+0|Ψeq⟩,(15) where S−0(2t)=eiH1t/ħe−iH2t/ħS−0eiH2t/ħe−iH1t/ħ is (=) the local spin operator, expressed in the Heisenberg representation respect to (e&eb) the acting Hamiltonian H(τ)=H1θ(t−τ)−H2θ(τ−t). An equivalent definition of MPE (t) in terms of (e&eb) Sz0 (2t) Sz0 was also proved useful.

(242) The connection of this experimental observable with (e&eb) the concepts discussed so far are already hinted but not yet clarified by Eq. (15).

(243) Thus, we consider that the m+1 spins are arranged in (eb) a linear chain or in an odd sized ring where (e) spin-spin interaction is restricted to XY terms acting on (e&eb) nearest-neighbors.

(244) In this case the Wigner-Jordan spin-fermion mapping allows to (eb) describe the spins system as (=) a gas of non-interacting fermions in a 1-d lattice.

(245) In that problem, the initial excitation propagates as (e&eb) a single spinless fermion.

(246) In turn, the observed local polarization can be written back in terms of (e&eb) the evolution of (e) an initially localized **"spin wave":** |ψ0⟩=|↑0⟩⊗|↓1⟩⊗|↓2⟩⊗|↓3⟩⊗...⊗|↓m⟩,(16) where| |↑n⟩ (|↓n⟩) means that the n-th site is occupied (unoccupied).

(247) Thus, it is clear that the excitation at site 0-th can spread along (e&eb) the 1-d chain

through (e&eb) the nearest neighbor flip-flop interaction.

(248) This correspondence between many-body dynamics and (e&eb) spin wave behavior was worked out in detail by Danieli et al.

(249) This procedure was originally employed by (e) Pastawski et al. in the context of time-reversal experiments to predict (eb) the presence of Poincaré recurrences (mesoscopic echoes) which were clearly observed by (e) Mádi et al. at the laboratory of Richard Ernst in Zurich.

(250) In this condition, the polarization detected (eb) after the time-reversal procedure results: MPE(t)≡1dsystemwithXYinteractionM(t)=||⟨ψ0|eiH2t/ħe−iH1t/ħ|ψ0⟩||2.(17)

(251) Notice that here |ψ0⟩ denotes (=) a single particle wave function, there are no other operators than the propagators and (e&eb) the modulus square makes explicit the positivity of (e) MPE (t). These features that were not obvious in Eq. (15) and make it to agree with the definition of the Loschmidt echo of Eq. (1).

(252) Many other effective many-body Hamiltonians can be experimentally reversed to obtain (eb) the revival of different observables that reduce to (e) a detectable polarization.

(253) Thus it is now a common practice to loosely call Loschmidt echoes to (e&eb) the polarization resulting from (e) any of these possible spin dynamics reversal procedures, regardless of the applicability of (e&eb) the single particle correspondence, with (e&eb) an explicit mention of the observable used.

**Time-reversal of the dipolar many-spin interaction Refs. (Levstein et al. 1998; Usaj et al. 1998; Pastawski et al. 2000; Zurek et al. 2007)**

(254) Figure 11: Ferrocene crystal showing (eb) the 1H (small dots), C atoms (medium-sized dots) and Fe (large dots).

(255) A rare 13C nucleus (usually called a "spy" nucleus), acting as (e&eb) a local probe, is shown by the arrow.

(256) Polarization evolves in (eb) the H lattice. Fe atoms at the center are not (e) magnetically active, but when replaced by (e) Co atoms relaxation of polarization is achieved. Adapted from Ref. (Levstein et al. 1998). Copyright (1998), American Institute of Physics.

(257) The experiments have been carried out (e&eb) in different molecular crystals of the cyclopentadienyl (C5H5) family.

(258) There, 1H nuclei are arranged in five fold rings with (e&eb) strong intra-ring dipolar interactions which nevertheless are not (e) constrained to the molecule but also yield (eb) appreciable intermolecular couplings.

(259) Polarization can be injected on (e&eb) those 1H which are close to a "spy" 13C nucleus that acts as (e&eb) a local probe used to (e) inject and eventually detect (e&eb) local polarization as represented in Figure 11.

(260) Spreading of the initially localized spin excitation proceeds through (e&eb) a dipolar interaction which is truncated (e) either in the basis of external magnetic field (Laboratory frame) or in the rotating frame of a radio frequency field.

(261) This choice is crucial in order to achieve (eb) the change of the sign and the eventual scale down of (e) the effective Hamiltonian.

(262) Thus a spin dynamics with H1 in the Laboratory frame is reversed by (e&eb)a dynamics of H2 in the rotating frame or vice versa.

(263) The role of environment might be played by paramagnetic Co(II) or quadrupolar Mn nuclei that replace (e&eb) the Fe and thus they are absent for (e) pure ferrocene crystals.

(264) Besides, in pure ferrocene truncation in the rotating frame give rise to (eb) non-inverted non-secular terms which, being suppressed by (e) the corresponding Zeeman energy, are at most of the order of a few percent of the matrix elements of H2.

(265) This would yield (eb) a Hermitian Σ

(266) The strength of these terms is inversely proportional to (=) B1, the strength of the r.f. pulse, and thus one has the possibility to experimentally reduce (e) its importance by (e) increasing the r.f. power.

(267) Figure 12: Formation and fading away of the Loschmidt echo with reversal time t2= = elapsed after (e) different periods t1 of forward evolution.

(268) Notice that the tails of the first Loschmidt echo reflect (e&eb) the forward dynamics. At longer times there is a first revival associated with (e&eb) the neighbors and a small revival associated with (e&eb) the mesoscopic echo resulting from (e) the dynamics within the ring.

(269) Superimposed with the Loschmidt echo there are (e&eb) high frequency oscillations.

(270) These are coherent interferences resulting from (e) an incomplete transfer between the 13C, used as (=) a local probe, and the nearby 1H nuclei. Adapted from Ref. (Levstein et al. 1998). Copyright (1998), American Institute of Physics.

(271) The results showing the buildup of the Loschmidt echoes, under (e&eb) H2 after different periods of evolution with H1, are presented in Figure 12.

(272) The universal features of the Loschmidt echo appear when one studies the maximum recovered polarization (Loschmidt echo intensity) as (=) a function of t, the evolution time before reversal.

(273) In Ferrocene crystals, once one substrates the background noise it is clear that the Loschmidt echo follows a clear Gaussian law, M (t) =exp [−12 (tT3)2], (18) for over two orders of magnitude and serves as (=) a definition of the characteristic time T3.

(274) The same decay law is observed when, using (e) a specially tailored pulse sequence, the Hamiltonians are scaled down by (er) a factor n=1, 2, 8 and 16 respectively. i.e. the characteristic decay time is increased by (e) the factor n.

(275) The scaling of 1/T3 with the Hamiltonian strength is also observed when (e) H1 and H2 are scaled down by using different crystal orientations.

(276) Figure 13: Attenuation of the polarization echo in the ferrocene single crystal as a function of t for n=1 (REPE sequence). The line represents (eb) a Gaussian fitting. The inset shows the PE attenuation for progressively reduced (e) dipolar dynamics, n= 1, 2, 8, and 16. No asymptotic regime is reached within (eb) the experimental timescale. The solid lines are (=) Gaussian fittings. Adapted from Ref. (Usaj et al. 1998).

(277) In contrast to Ferrocene, in Cobaltocene crystals, one starts with a Gaussian decay, and as the dipolar Hamiltonian is scaled down, (eb) an exponential decay of the Loschmidt echo is developed and became stable with (e&eb) a characteristic time τSE.

(278) Figure 14: Attenuation of the polarization echo in the cobaltocene sample as (=) a function of t for n = 1, 2, 5, 8, and 16.

(279) The experimental data show a clear crossover between a dominant Gaussian attenuation to an exponential one. The solid lines represent fittings of the whole set of data to Eq. (19). Adapted from Ref. (Usaj et al. 1998).

(280) Thus, the overall behavior of the recovered local polarization is (=) $M(t)=\exp[-12(tnT3)2-t\tau SE]$.(19)

(281) One may assign this asymptotic exponential decay rate 1/τSE to (e) a Fermi Golden Rule decoherence rate induced by (e) the perturbation induced by (e) the paramagnetic nature of the Co(II) nuclei which acts as (e&eb) an environment.

(282) Further experiments by Pastawski et al. in crystals free of magnetic impurities confirmed (eb) the Gaussian decay with a rate 1/T3 that depends only weakly on (=) the r.f. power and, when (e) truncation terms becomes (=) too small, saturates to an intrinsic value that scales with (e&eb) the strength of the reverted Hamiltonian.

(283) In summary, these experiments hinted that there is (=) an intrinsic decoherence rate which is fixed by (e) the inverted Hamiltonian.

(284) Thus, other than the difference between the Gaussian and exponential decay, one might say that 1/T3 plays a role (e&eb) similar to the Lyapunov exponent in (eb) the reversal of chaotic one body systems.

(285) This surprising finding suggested that, even in absence of any important perturbation, in the experimental limit of $m\to\infty$, even very small residual terms, in presence of (e) the dynamical instability of a very complex many-body dynamics, are efficient enough to (e) set the Loschmidt echo decay into (e&eb) a perturbation independent decay regime.

(286) This plays the role of (e&eb) an "**intrinsic decoherence**" with (e&eb) a time scale determined by the inverted Hamiltonian.

(287) It was precisely the hypothesis of an "intrinsic decoherence rate" what triggered the (eb) theoretical analysis of (e) time-reversal in chaotic systems.

(288) Indeed, while dynamics of one body systems in 1D, described by Eq. (17), cannot be chaotic, one deems that disorder and the more complex many-body dynamics present in higher dimensional systems, have (e) mixing properties which makes them assimilable to (e) chaotic systems.

(289) This observation led to (eb) G. Usaj et al. to propose that quantum chaos contains (e) the underlying physics of (e) a time-reversal experiment in a many-body system.

(290) It was also argued that MPE (tR) constitutes (e) an entropy measure.

(291) These ideas finally boiled down into (e&eb) the model proposed by Jalabert and Pastawski.

(292) While Gaussian decays appears quite naturally as a consequence of (e) the large number statistics of many-spin states that are progressively incorporated into (e) the dynamics, the perturbation independent decay have (e) not found yet a straightforward explanation in this context.

**Further time-reversal Experiments Refs. (Levstein et al. 2004; Sánchez et al. 2007, 2009; Rufeil-Fiori et al. 2009; Ramanathan et al. 2011)**

(293) While time-reversal of different interactions is (=) a ubiquitous experimental tool in NMR, there are not so many studies devoted to grasp at the origins of the (in) efficiency of (e) such procedures.

(294) In particular, time-reversal was implemented using (e) different procedures and systems that range from 3d and quasi-1d crystals to (e&eb) molecules in oriented liquid crystalline phases. In these cases, also different initial states and effective Hamiltonians were studied.

(295) Of particular interest are the double quantum (DQ) Hamiltonians, $H1\equiv LoschmidtEchoforDQHDQ=\sum i,j\tilde{d}i,j[S+iS+j+S-iS-j].(20)$

(296) The notation $\tilde{d}$ for the interaction strength recalls at it is an effective Hamiltonians built up from (e) dipole-dipole interaction by (e) suitable pulse sequences whose effect is described with (e&eb) the help of the average Hamiltonian theory.

(297) Since HDQ does not commute with (e&eb) the polarization described by $\sum iSzi$, it gives (eb) a more mixing as dynamics explores (e&eb) different subspaces of the Hilbert space inducing (eb) the multiple quantum coherences.

(298) Besides HDQ, there are (=) high order terms that act as (e&eb) an environment whose instantaneous decoherence could be described by (e&eb) a Fermi Golden Rule.

(299) One experimental observation is that the larger the number of subspaces correlated by the interaction (coherences of high order) (eb) faster the decoherence.

(300) One of the interesting properties of 1d systems is that the dynamics induced by (e) a HDQ has (e) a precise correspondence with (e&eb) an XY chain.

(301) Thus, the Loschmidt echo is given by Eq. (17). In real systems, such as Hydroxyapatite (HAp), inter-chain interactions are (=) significative, quantum Zeno effect induced by (e) a strong dynamics along the chain prevents (e) the development of coherences beyond second order and Eq. (17) remains valid.

(302) This explains why the Loschmidt echo observed by Rufeil-Fiori et al. for the double quantum Hamiltonian shows (eb) a Loschmidt echo with (e&eb)an exponential decay described by (e) the Fermi golden rule.

(303) Figure 15: Loschmidt Echo experiment based on (e) HDQ and -HDQ, for (e) hydroxyapatite (HAp) (quasi-1d crystal) and Adamantane, a fully 3-d crystal. Here, the source of Σ is very small high order terms in (eb) the effective Hamiltonian.

(304) This decay contrasts with (e&eb) results for an Adamantane crystal, a highly connected 3d system.

(305) There, a Fermi-function like decay is indicative of (e) the two time scales. Each molecule has 16 spins which do not (e) have direct interactions, thus remain (eb) independent for short times until they interact through (e&eb) neighbor molecules.

(306) In this case decoherence remains weak while (e) intermolecular correlation builds up. Once neighboring molecules become fully coupled (eb) the density of directly connected states becomes (=) very high and the Fermi-golden-rule controlled (er&eb) exponential decay takes over.

(307) Spin dynamics in various liquid crystals was studied in series of papers whose aim was to reduce (e) the number of interacting spins to (e) those within each molecule.

(308) However, the experiences have shown that a number of residual interactions remain (eb) significative and this strongly compromises the effectiveness of the Loschmidt echo sequences.

**Microwave billiards Refs. (Schäfer et al. 2005a, 2005b; Höhmann et al. 2008; Bodyfelt et al. 2009; Köber et al. 2010, 2011)**

(309) Microwave-frequency electromagnetic waves in quasi-two-dimensional cavities are effectively governed by (e&eb) the Helmholtz equation, which is equivalent to (=) the stationary Schrödinger equation. This equivalence allows one to address the Loschmidt echo in laboratory experiments with microwave billiards.

(310) Quantities directly measured in microwave experiments are frequency-dependent scattering matrix elements, Sab (ν) and S′ab(ν), corresponding to (e&eb) the unperturbed and perturbed chaotic billiard systems, respectively

(311) Here, the indices a and b refer to the antennae (or scattering channels) involved in (e&eb) the experiment.

(312) The change from the frequency domain to (e&eb) the time domain is achieved by (e) the Fourier transforms, S^ab (t) =∫dνe−2πivtSab (ν) and S^′ab (t) =∫dνe−2πivtS′ab (ν), performed over (e&eb) an appropriate frequency window.

(313) The sensitivity of a scattering process to system perturbations are then quantified by (e) the scattering fidelity amplitude

(314) fab(t)=⟨S^∗ab(t)S^′ab(t)⟩⟨|S^ab(t)|2⟩⟨|S^ab(t)|2⟩−−−−−−−−−−−−−−−−−√,(21) where (e) the asterisk stands for the complex conjugation, and the angular brackets denote (eb) an ensemble average over different realizations of the experiment (e.g., different positions of scatterers, antennae, etc.).

(315) The scattering fidelity itself is (=) a real-valued quantity defined as Fab (t) =|fab (t)|2.

(316) The numerator in the right-hand-side of Eq. (21) quantifies (eb) correlations between scattering matrix elements of the unperturbed and (e&eb) perturbed system.

(317) The decay of the numerator however is dominated by (e) the decay of the autocorrelations, so that (e) the denominator is introduced to (e) compensate for the autocorrelation

contributions to the correlation decay.

(318) Figure 16: Correlation function C^ [S∗11, S′11] ∝⟨S^∗11 (t) S^′11 (t) ⟩. The experimental results for the autocorrelation are shown in black, while (e) the correlations of perturbed and unperturbed system are shown in gray/orange.

(319) The smooth solid curve corresponds to (e&eb) the theoretical autocorrelation function, and the dashed curve to (e) the product of autocorrelation function and fidelity amplitude. The inset shows (eb) the billiard geometry used. Movable parts are marked with an arrow. Adapted from Ref. (Schäfer et al. 2005b). Copyright (2005), American Physical Society.

(320) Figure 17: Decay rate of the Loschmidt echo in a microwave billiard as (=) a function of the displacement h of (e) a small flat part of (e) the billiard boundary.

(321) The solid (dashed) curve corresponds to (e&eb) the decay rate predicted by (e) the semiclassical theory taking (not taking) into account classical trajectories multiply scattered within (e&eb) the boundary deformation region. Adapted from Ref. (Köber et al. 2011). Copyright (2011), American Physical Society.

(322) In chaotic systems and in the case of a weak coupling of the measuring antennae to (e&eb) the system, the scattering fidelity is known to approach (e&eb) the Loschmidt echo for (e) a random initial state.

(323) This makes microwave billiards well suited for (e) experimental studies in the field.

(324) In particular, microwave studies provided compelling experimental evidence for the validity of some of the known decay regimes of (e) the Loschmidt echo for global and local Hamiltonian perturbations.

**Elastic waves Refs. (Lobkis et al. 2003; Gorin et al. 2006)**

(325) Sound waves that travel through (e&eb) an elastic medium are multiple scattered by (e&eb) its inhomogeneities generating (eb) a slowly decaying wave diffusion.

(326) **If no change occurs in the medium over time, (eb) the usually called coda waves are highly repeatable**, that is, for identical excitations the waveforms are (=) indistinguishable.

(327) While if the medium is perturbed, the change in the multiple scattered waves will result in (eb) an observable change in the **coda waves**.

(328) Lobkis and Weaver have measured the sensitivity to (e&eb) temperature changes of (e) elastic coda waves in (eb) aluminum alloy blocks.

(329) They used the cross correlation function between two signals obtained at (eb) different temperatures T1 and T2 $X(\varepsilon)=\int dtST1(t)ST2(t(1+\epsilon))\int dtS2T1(t)\int dtS2T2(t(1+\epsilon))-----------\sqrt{}$.(22) to evaluate the distortion that is defined as (=) $D(t)=-\ln(Xmax)$, where (e) the time dependence is (=) given by the "age" of the signal.

(330) If formulated as a scattering process, it is shown that $D(t) =-\ln [f(t)]$, where f (t) is the scattering fidelity that was introduced to analyze the fidelity decay in microwave billiards.

(331) For sufficiently chaotic dynamics in systems weakly coupled to (e&eb) decay channels, the scattering fidelity approaches (eb) the standard fidelity amplitude (the Loschmidt echo is (=) the absolute square of the fidelity amplitude).

(332) Figure 18: The distortion as a function of time for coda waves that travel in an aluminum alloy block at different temperatures. The thin lines correspond to (e&eb) measurements in different frequency ranges.

(333) The thick lines show the corresponding theoretical curves obtained using (e) Random matrix theory. Inset: Laboratory configuration. An aluminum specimen, typically with nonparallel faces and a defocusing cylindrical hole, is cooled in (eb) a vacuum as temperature and ultrasonic response are monitored.

(334) A reed relay isolates the response from the pulser. Adapted from Refs. (Lobkis et al. 2003; Gorin at al. 2006). Copyright (2003, 2006), American Physical Society. The results obtained in acoustic signals traveling in an aluminum blocks were understood using (e) the random matrix predictions for the standard fidelity (see Figure 18).

(335) A surprising and unexpected finding is that the scattering fidelity decay for blocks with (e&eb) chaotic and regular classical dynamics is explained by (e) the same random matrix expressions.

**Cold atoms Refs. (Andersen et al. 2003, 2004, 2006; Wimberger et al. 2006; Martin et al. 2008; Wu et al. 2009; Krivolapov et al. 2011; Ullah et al. 2011; Dubertrand et al. 2012)**

(336) The Loschmidt echo, or rather a quantity closely related to the Loschmidt echo, has been carefully studied in laboratory experiments with (e&eb) cold atoms trapped inside (e&eb) optical cavities.

(337) These experiments offer an atom-optics realization of the echo in (eb) two-dimensional quantum billiards with (e&eb) underlying chaotic or mixed classical dynamics.

(338) The basic idea underlying the experiments with atom-optics billiards is as follows. An effectively two-level atom, with (e&eb) the internal states denoted by $|1\rangle$ and $|2\rangle$, is initially prepared in a state $|1\rangle\otimes|\psi\rangle$, where (e) $|\psi\rangle$ stands for the spatial component of the initial state.

(339) A laboratory observation of the echo involves first exposing the atom to (e&eb) a sequence of microwave-frequency electromagnetic pulses and then measuring (eb) the probability P2 of finding the atom in the internal state $|2\rangle$.

(340) In the protocol targeting the Loschmidt echo, $||\langle\psi|eiH2t/\hbar e{-}iH1t/\hbar|\psi\rangle||2$, an atom is first irradiated with (e&eb) a so-called $\pi/2$ pulse, which changes (e&eb) the atomic state into (e&eb) an equiprobable superposition of $|1\rangle\otimes|\psi\rangle$ and $|2\rangle\otimes|\psi\rangle$.

(341) The pulse is practically instantaneous and introduces almost no change to (e) the spatial component of the state.

(342) Then, the atom is left to evolve in an optical trap for (e) a time t, during which components $|1\rangle$ and $|2\rangle$ of the atomic state propagate under (e&eb) Hamiltonians H1 and H2 respectively.

(343) The difference between H1 and H2 originates from (e) a difference in dipole interaction potentials exerted upon (e&eb) the states $|1\rangle$ and $|2\rangle$ by (e) the optical trap.

(344) Finally, another $\pi/2$ pulse is applied to (e&eb) the atom and the probability P2 of finding the atom in the internal state $|2\rangle$ is (=) measured; P2 turns out to be a quantity closely related to (e&eb) the Loschmidt echo.

(345) Realistic experiments however deal with (e&eb) thermal incoherent mixtures of initial states, rather than with pure states.

(346) As a result (eb) the echo amplitude, due to (e) an individual state of this thermal mixture, contributes to (e&eb) P2 with (e&eb) an effectively random phase, smearing out (e) the echo signal.

(347) In order to overcome (e) this difficulty, Andersen, Davidson, Grünzweig, and Kaplan addressed a new echo measure, $||\langle\psi|eiH2t/2\hbar eiH1t/2\hbar e{-}iH2t/2\hbar e{-}iH1t/2\hbar|\psi\rangle||2$, closely related to (e&eb) the Loschmidt echo. In their experiments, "two-level" atoms, initially prepared in (eb) the state $|1\rangle\otimes|\psi\rangle$, were successively exposed to (e&eb) (i) a $\pi/2$ pulse, (ii) evolution for (e) a time t/2, (iii) a $\pi$ pulse swapping the population of (e) the internal states $|1\rangle$ and $|2\rangle$, (iv) another evolution for a time t/2, (v) a $\pi/2$ pulse, and finally (vi) a measurement of P2.

(348) Under this protocol, each state of the thermal mixture contributed to (e&eb) the probability P2 with the same phase.

(349) This allowed for a reliable measurement of (e) the echo even for ensembles of (e) more than a million of thermally populated states.

(350) More recently, atom interferometry has also been used to investigate several aspects of time-reversal in (eb) quantum kicked rotor systems.

(351) Wu, Tonyushkin, and Prentiss studied the dynamics of (e) laser-cooled rubidium atoms subjected to (e&eb) periodically pulsed optical standing waves, as (=) an atom-optics realization of the kicked rotor.

(352) Their experiments have demonstrated that quantum fidelity of a system, that is (=) chaotic in the classical limit, can survive (eb) strong perturbations over long time without decay.

(353) In a similar setup, Ullah and Hoogerland have performed an experiment that demonstrated the possibility of using time-reversal evolution for (e) cooling atomic matter waves.

(354) The problem of fidelity decay in (eb)&e kicked atom-optics systems has recently drawn considerable interest among theoreticians.

**Time-reversal mirrors Refs. (Fink 1999; Calvo et al. 2008; Taddese et al. 2009)**

(355) Another time-reversal attempt conceptually close to (e&eb) the Loschmidt echo is that of (e) time-reversal mirrors, developed in (eb) the last twenty years.

(356) Such a procedure has been successfully implemented in various setups where (e0 classical waves propagate through (e&eb) a complex media (going from acoustic to electromagnetic waves).

(357) In the time-reversal mirror protocol an initially localized pulse is recorded by (e) a collection of receiver-emitter transducers during (e&eb) a time interval where (e) the wave suffers multiple scattering.

(358) The later re-emission of (e) the time-reversed signal by each transducer during (e&eb) an interval of time equal to (=) the recording one leads to (eb) the refocusing of the signal in the region of the original excitation.

(359) Two features of this protocol have lead to (eb) considerable surprise among the practitioners: even one transducer is enough to obtain (eb) a good reproduction of the original signal

(360) The quality of the refocusing was improved by (e) the complexity of the media yielding (eb) the multiple scattering of the waves.

(361) A semiclassical theory of time-reversal focusing can be built up in terms of propagators and classical trajectories. The two previous features can be acknowledged by the semiclassical theory.

(362) Time-reversal mirrors and (e&eb: AC=DC=0) Loschmidt echos differ (e&eb) since (e) the former aims (eb) the refocalization of a wave which is (=) localized in space and time, while (e) the latter attempt to (e) time-reverse a quantum state.

(363) A common aspect between both protocols is *(=) the fact that the Hamiltonian for the forward and backward evolutions can differ due to (e) modifications of the environment during (e&eb) the process.

(364) Time-reversal mirrors are not only conceptually important, but also have (e) very important technological applications as for example brain therapy, lithotripsy, nondestructive testing and telecommunications.

(365) Recently a sensor of perturbations was proposed and demonstrated combining the ideas of the Loschmidt echo and (e&eb) time reversal mirror in classical waves.

**Past, present, and future of the Loschmidt echo studies**

(366) Since the early discussions between Boltzmann and Loschmidt on irreversibility and (e&eb) time-reversal, it has been clearly established that chaos is (=) the source of the irreversibility in statistical mechanics.

(367) However, when considering the sources of irreversibility in (eb) a quantum system with (e&eb) a few degrees of freedom Quantum Mechanics, which is (=) the fundamental theory of the microscopic world, does not (e) allow for chaotic behavior in the sense in which the latter appears in Classical Mechanics.

(368) That is, there is no exponential separation between states with nearby initial conditions since (e) **quantum evolution is unitary**.

(369) Trying to understand the origin of (e) irreversibility in quantum mechanics, Asher Peres proposed in 1984 as an alternative, to study the stability of quantum motion owing to (e) perturbations in the Hamiltonian.

(370) In his seminal paper, Peres considered M (t), later called Loschmidt echo, as (=) a measure of sensibility and (e&eb) reversibility of quantum evolutions.

(371) He aimed to distinguish classically chaotic or integrable dynamics according to (e&eb) the speed at which the fidelity decays.

(372) Peres reached the conclusion that the long-time behavior of the fidelity (or saturation, following the terminology of section Chaotic dynamics) in classically chaotic systems is (=) characterized by (e) smaller values and smaller fluctuations as compared to (e&eb) the case of regular dynamics.

(373) In view of numerous possibilities of different behaviors that are allowed (eb) in regular systems, such a conclusion does not always hold.

(374) It is interesting to notice that Peres' article appeared almost at the same time than two other seminal works studying the relevance of the underlying classical dynamics for quantum properties. These are the random-matrix description of the statistical properties of classically chaotic systems by Oriol Bohigas and collaborators; and the understanding of the spectral rigidity from (e) a semiclassical analysis of periodic orbits proposed by Michael Berry.

(375) The field of Quantum Chaos developed building upon these founding ideas and most of the subsequent works were concerned with the spectral properties of (e) classically chaotic systems. Comparatively very little work was done along the Peres' proposal in the decade following it.

(376) Motivated by the puzzles posed by (e) echo experiments in extended dipolar coupled nuclear spin systems, Rodolfo Jalabert and Horacio Pastawski studied in 2001 the behavior of the Loschmidt echo for (e) classically chaotic systems.

(377) They found that, depending on the perturbation strength, the decay of the Loschmidt echo exhibits (eb) mainly two different behaviors

(378) For weak perturbations, the decay is exponential with (e&eb) the rate that depends on the perturbation strength and that is (=) given by the width of the local density of states (usually called the Fermi-golden-rule regime).

(379) For stronger perturbations, however, there is a crossover to (e&eb) a perturbation-independent regime, characterized by (e) an exponential decay with the rate equal to (eb)the average Lyapunov exponent of the underlying (e&eb) classical system.

(380) The connection of the quantum Loschmidt echo with (e&eb) the classical chaos generated (eb) a great activity in the field.

(381) Researches from different fields, such as quantum chaos, solid state physics, acoustics, and cold atom physics, have made important contributions towards understanding various aspects of the Loschmidt echo. During the first years of the last decade the studies were mainly focused on one-body aspects of the problem and on the connection between the Loschmidt echo and (e&eb)decoherence; many interesting experiments were also performed.

(382) In the last years, the interest has primarily been focused on various many-body aspects of the Loschmidt echo. For example, there are studies that consider the decay of the Loschmidt echo as (=) a signature of a quantum phase transition, or that concentrate on the relation between the

Loschmidt echo and (e&eb) the statistics of the work done by a quantum critical system when (e) a control parameter is quenched. In view of recent improvements in experimental many-body system techniques, the issue of the Loschmidt echo decay has become more concrete.

**Some of the more important open questions in the field are:**

(383) The Lyapunov (perturbation-independent) decay regime has been one of the most influential breakthroughs in (eb) the theory of the Loschmidt echo. However, an experimental observation of the regime in simple well-controlled systems is still lacking.

(384) The first experimental measure of the Loschmidt echo was done in (eb) a many-body spin system using (e) NMR techniques.

(385) However, theoretically little is known about (e&eb) the behavior of the Loschmidt echo in (eb) many-body systems. Very interesting experiments are now being carried on using NMR.

(386) The semiclassical theory of the Loschmidt echo has proved to be (=) a powerful tool to understand many aspects of (e) its behavior.

(387) But this approach has some well-recognized difficulties, such as the root search problem or the exponentially growing number of classical orbits needed for (e) the semiclassical expansion.

(388) Recently, a simple semiclassical dephasing representation of the Loschmidt echo was proposed that does not (e) suffer of the usual problems of the semiclassical theories. The range of validity is however unknown.

(389) There is a vast amount of work characterizing (eb) the decay regimes of (e) the Loschmidt echo as universal.

(390) For chaotic systems perturbed with (e&eb) global or local perturbations the behavior of the decay rate of the Loschmidt echo is (=) well established (see section Decay of the Loschmidt echo: different regimes).

(391) However, recent works in quantum maps that are (=) perturbed in all phase space, have (e) reported non-universal oscillatory behavior in (eb) the decay rate of the Loschmidt echo as (=) a function of (e) the perturbation strength.

(392) Deviations from (e) the perturbation independence are found usually in the form of (e&eb) oscillations around the Lyapunov exponent.

(393) Moreover, there are cases where deviations are considerably large rendering (eb) the Lyapunov regime non-existing.

(394) The fully understanding of this non-universal behavior (eb) that was also observed in a Josephson flux quit, is still lacking (Pozzo et al. 2007; García-Mata et al. 2011b).

(395) The semiclassical theory of the Lyapunov decay of the Loschmidt echo is based on (e) highly localized initial states and (e&eb) the diagonal approximation of the expression for M(t), shown in Figure 1.

(396) But recent results using (e) the semiclassical dephasing representation have shown a Lyapunov regime in (eb) the mean value of the fidelity amplitude ⟨m (t) ⟩ (García-Mata et al. 2011a; Zambrano et al. 2011).

(397) Systems with regular or fully chaotic dynamics are (=) rather exceptional in nature.

(398) A generic system has mixed phase space consisting of integrable islands immersed in (eb) a chaotic sea. Few general results are known about the Loschmidt echo in this scenario.

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**Path integral approach to the quantum fidelity amplitude Jiri Vanicek, Doron Cohen**

(1) The Loschmidt echo is (=) a measure of quantum irreversibility and is determined by (e) the fidelity amplitude of (e) an imperfect time-reversal protocol.
(2) Fidelity amplitude plays an important role both in (e&eb) the foundations of quantum mechanics and its applications, such as (=) time-resolved electronic spectroscopy.
(3) Authors derive an exact path integral formula for (e) the fidelity amplitude and use it to obtain (eb) a series of increasingly accurate semiclassical approximations by (e) truncating an exact expansion of the path integral exponent.
(4) While the zeroth-order expansion results in (eb) a remarkably simple, yet nontrivial approximation for the fidelity amplitude, the first-order expansion yields (eb) an alternative derivation of the so-called "dephasing representation", circumventing (e) the use of semiclassical propagator as (=) in the original derivation.
(5) Authors also obtain approximate expression for fidelity based on (e) the second-order expansion, which resolves (e) several shortcomings of the dephasing representation.
**(6)** The rigorous derivation from the path integral permits (eb) the identification of sufficient conditions under which various approximations obtained becomes exact. Cite **as:arXiv: 1510.03140 [quant-ph] (or arXiv: 1510.03140v1 [quant-ph] for this version**) **©American Physical Society**

**Characterizing and Quantifying Quantum Chaos with Quantum Tomography Vaibhav Madhok, Carlos A. Riofrío, Ivan H. Deutsch**

(1) Authors explore quantum signatures of (e) classical chaos by studying the rate of information gain in (eb) quantum tomography.
(2) The tomographic record consists of (e) a time series of expectation values of (e) a Hermitian operator evolving under (e&eb) application of the Floquet operator of a quantum map that possesses (or lacks) (e&eb) time reversal symmetry.
(3) They find that the rate of information gain, and hence the fidelity of quantum state reconstruction, depends on (=) the symmetry class of the quantum map involved.
**(4)** Moreover, authors find an increase in information gain and hence higher reconstruction fidelities when (e) the Floquet maps employed increase in (eb) chaoticity.
**(5)** Authors make predictions for (e) the information gain and show that (eb) these results are well described by (e) random matrix theory in (eb) the fully chaotic regime.
**(6)** Authors derive analytical expressions for (e) bounds on information gain using (e) random matrix theory for (e) different class of maps and show that these bounds are realized by (e) fully chaotic quantum systems. **Cite as: arXiv: 1506.02708 [quant-ph] (or arXiv: 1506.02708v1 [quant-ph] for this version). ©American Physical Society**

**NOTATION**

**Module One**

The Loschmidt echo is (=) a measure of quantum irreversibility and is determined by (e) the fidelity amplitude of (e) an imperfect time-reversal protocol

$G_{13}$ : Category one of Loschmidt echo

$G_{14}$ : Category two of SAS

$G_{15}$ : Category three of SAS

$T_{13}$ : Category one of measure of quantum irreversibility and is determined by (e) the fidelity amplitude of (e) an imperfect time-reversal protocol

$T_{14}$ : Category two of SAS

$T_{15}$ : Category three of SAS

**Module Two**

The Loschmidt echo is a measure of quantum irreversibility and is determined by (e) the fidelity amplitude of (e) an imperfect time-reversal protocol

$G_{16}$ : Category one of Loschmidt echo is a measure of quantum irreversibility and is determined; **fidelity amplitude of (e) an imperfect time-reversal protocol**

$G_{17}$ : Category two of SAS

$G_{18}$ : Category three of SAS

$T_{16}$ : Category one of **fidelity amplitude of (e) an imperfect time-reversal protocol** ;Loschmidt echo is a measure of quantum irreversibility and is determined

$T_{17}$ : Category two of SAS

$T_{18}$ : Category three of SAS

**Module three**

Loschmidt echo is a measure of quantum irreversibility and is determined by the fidelity amplitude of (e) an imperfect time-reversal protocol

$G_{20}$ : Category one of Loschmidt echo

$G_{21}$ : Category two of SAS

$G_{22}$ : Category three of SAS

$T_{20}$ : Category one of measure of quantum irreversibility and is determined by the fidelity amplitude of (e) an imperfect time-reversal protocol

$T_{21}$ : Category two of SAS

$T_{22}$ : Category three of SAS

**Module four**

Loschmidt echo is a measure of quantum irreversibility and is determined by the fidelity amplitude of (e) an **imperfect time-reversal protocol**

$G_{24}$ : Category one of Loschmidt echo is a measure of quantum irreversibility and is determined by the fidelity amplitude; **imperfect time-reversal protocol**

$G_{25}$ : Category two of SAS

$G_{26}$ : Category three of SAS

$T_{24}$ : Category one of **imperfect time-reversal protocol;** Loschmidt echo is a measure of quantum irreversibility and is determined by the fidelity amplitude

$T_{25}$ : Category two of SAS

$T_{26}$ : Category three of SAS

**Module five**

**Fidelity amplitude** plays an important role both in (e&eb) the foundations of quantum mechanics and its applications, such as (=) time-resolved electronic spectroscopy

$G_{28}$ : Category one of **Fidelity amplitude;** time-resolved electronic spectroscopy

$G_{29}$ : Category two of SAS

$G_{30}$ : Category three of SAS

$T_{28}$ : Category one of time-resolved electronic spectroscopy **;Fidelity amplitude**

$T_{29}$ : Category two of SAS

$T_{30}$ : Category three of SAS

**Module six**

Authors derive an exact path integral formula for (e) the fidelity amplitude and use it to obtain (eb) a series of increasingly accurate semiclassical approximations by (e) truncating an exact expansion of the path integral exponent

$G_{32}$ : Category one of fidelity amplitude and use it to obtain (eb) a series of increasingly accurate semiclassical approximations by (e) truncating an exact expansion of the path integral exponent

$G_{33}$ : Category two of SAS

$G_{34}$ : Category three of SAS

$T_{32}$ : Category one of exact path integral formula

$T_{33}$ : Category two of SAS

$T_{34}$ : Category three of SAS

**Module seven**

Authors derive an exact path integral formula for the fidelity amplitude and use it to obtain (eb) a series of increasingly accurate semiclassical approximations by (e) truncating an exact expansion of the path integral exponent

$G_{36}$ : Category one of exact path integral formula for the fidelity amplitude

$G_{37}$ : Category two of SAS

$G_{38}$ : Category three of SAS

$T_{36}$ : Category one of series of increasingly accurate semiclassical approximations by (e) truncating an exact expansion of the path integral exponent

$T_{37}$ : Category two of SAS

$T_{38}$ : Category three of SAS

**Module eight**

Authors derive an exact path integral formula for the fidelity amplitude and use it to obtain a series of increasingly accurate semiclassical approximations by (e) **truncating an exact expansion of the path integral exponent**

$G_{40}$ : Category one of exact path integral formula for the fidelity amplitude and use it to obtain a series of increasingly accurate semiclassical approximations; **truncating an exact expansion of the path integral exponent**

$G_{41}$ : Category two of SAS

$G_{42}$ : Category three of SAS

$T_{40}$ : Category one of **truncating an exact expansion of the path integral exponent;** exact path integral formula for the fidelity amplitude and use it to obtain a series of increasingly accurate semiclassical approximations

$T_{41}$ : Category two of SAS

$T_{42}$ : Category three of SAS

**Module Nine**

While the zeroth-order expansion results in (eb) a remarkably simple, yet nontrivial approximation for the fidelity amplitude, the first-order expansion yields (eb) an alternative derivation of the so-called "dephasing representation", circumventing (e) the use of semiclassical propagator as (=) in the original derivation

$G_{44}$ : Category one of first-order expansion

$G_{45}$ : Category two of SAS

$G_{46}$ : Category three of SAS

$T_{44}$ : Category one of alternative derivation of the so-called "dephasing representation", circumventing (e) the use of semiclassical propagator as (=) in the original derivation

$T_{45}$ : Category two of SAS

$T_{46}$ : Category three of SAS

**The Coefficients:**

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}\ ,\ (b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)},$
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)},\ \ (b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}\ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}\ ,$
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)},$
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)},$
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)},$
are Dissipation coefficients

**Module Numbered One**

**The differential system of this model is now (Module Numbered one)**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \quad 1$$
$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad 2$$
$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15} \quad 3$$
$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13} \quad 4$$
$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14} \quad 5$$
$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)\right]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14}, t) =$ First augmentation factor
$-(b''_{13})^{(1)}(G, t) =$ First detritions factor

**Module Numbered Two**

**The differential system of this model is now ( Module numbered two)**

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right]G_{16} \quad 7$$
$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t)\right]G_{17} \quad 8$$
$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t)\right]G_{18} \quad 9$$
$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\big((G_{19}), t\big)\right]T_{16} \quad 10$$
$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\big((G_{19}), t\big)\right]T_{17} \quad 11$$
$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\big((G_{19}), t\big)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17}, t) =$ First augmentation factor
$-(b''_{16})^{(2)}\big((G_{19}), t\big) =$ First detritions factor

**Module Numbered Three**

**The differential system of this model is now (Module numbered three)**

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21},t)\right]G_{20} \quad 13$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21},t)\right]G_{21} \quad 14$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21},t)\right]G_{22} \quad 15$$

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23},t)\right]T_{20} \quad 16$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23},t)\right]T_{21} \quad 17$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23},t)\right]T_{22} \quad 18$$

$+(a''_{20})^{(3)}(T_{21},t) =$ First augmentation factor

$-(b''_{20})^{(3)}(G_{23},t) =$ First detritions factor

**Module Numbered Four**

**The differential system of this model is now (Module numbered Four)**

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right]G_{24} \quad 19$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25},t)\right]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25},t)\right]G_{26} \quad 21$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27}),t\right)\right]T_{24} \quad 22$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27}),t\right)\right]T_{25} \quad 23$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27}),t\right)\right]T_{26} \quad 24$$

$+(a''_{24})^{(4)}(T_{25},t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}\left((G_{27}),t\right) =$ **First detritions factor**

**Module Numbered Five**:

**The differential system of this model is now (Module number five)**

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29},t)\right]G_{28} \quad 25$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29},t)\right]G_{29} \quad 26$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29},t)\right]G_{30} \quad 27$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}\left((G_{31}),t\right)\right]T_{28} \quad 28$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}\left((G_{31}),t\right)\right]T_{29} \quad 29$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}\left((G_{31}),t\right)\right]T_{30} \quad 30$$

$+(a''_{28})^{(5)}(T_{29},t) =$ **First augmentation factor**

$-(b''_{28})^{(5)}\left((G_{31}),t\right) =$ **First detritions factor**

**Module Numbered Six**

**The differential system of this model is now (Module numbered Six)**

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33},t)\right]G_{32} \quad 31$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33},t)\right]G_{33} \quad 32$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}, t)\right]G_{34} \quad 33$$
$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}\left((G_{35}), t\right)\right]T_{32} \quad 34$$
$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}\left((G_{35}), t\right)\right]T_{33} \quad 35$$
$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}\left((G_{35}), t\right)\right]T_{34} \quad 36$$
$+(a''_{32})^{(6)}(T_{33}, t) =$ **First augmentation factor**

**Module Numbered Seven:**

**The differential system of this model is now (Seventh Module)**

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right]G_{36} \quad 37$$
$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37}, t)\right]G_{37} \quad 38$$
$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}, t)\right]G_{38} \quad 39$$
$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}\left((G_{39}), t\right)\right]T_{36} \quad 40$$
$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}\left((G_{39}), t\right)\right]T_{37} \quad 41$$
$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}\left((G_{39}), t\right)\right]T_{38} \quad 42$$
$+(a''_{36})^{(7)}(T_{37}, t) =$ **First augmentation factor**

**Module Numbered Eight**

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right]G_{40} \quad 43$$
$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41}, t)\right]G_{41} \quad 44$$
$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}, t)\right]G_{42} \quad 45$$
$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}\left((G_{43}), t\right)\right]T_{40} \quad 46$$
$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}\left((G_{43}), t\right)\right]T_{41} \quad 47$$
$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}\left((G_{43}), t\right)\right]T_{42} \quad 48$$

**Module Numbered Nine**

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}, t)\right]G_{44} \quad 49$$
$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45}, t)\right]G_{45} \quad 50$$
$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}, t)\right]G_{46} \quad 51$$
$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}\left((G_{47}), t\right)\right]T_{44} \quad 52$$
$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}\left((G_{47}), t\right)\right]T_{45} \quad 53$$
$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}\left((G_{47}), t\right)\right]T_{46} \quad 54$$
$+(a''_{44})^{(9)}(T_{45}, t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}\left((G_{47}), t\right) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[\begin{array}{ccc} (a'_{13})^{(1)}\boxed{+(a''_{13})^{(1)}(T_{14}, t)} & \boxed{+(a''_{16})^{(2,2,)}(T_{17}, t)} & \boxed{+(a''_{20})^{(3,3,)}(T_{21}, t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25}, t)} & \boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29}, t)} & \boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33}, t)} \\ \boxed{+(a''_{36})^{(7,7)}(T_{37}, t)} & \boxed{+(a''_{40})^{(8,8)}(T_{41}, t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45}, t)} \end{array}\right]G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)}\boxed{+(a''_{14})^{(1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14} \quad (56)$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)}\boxed{+(a''_{15})^{(1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15} \quad (57)$$

Where $\boxed{(a''_{13})^{(1)}(T_{14},t)}$, $\boxed{(a''_{14})^{(1)}(T_{14},t)}$, $\boxed{(a''_{15})^{(1)}(T_{14},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7)}(T_{37},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8)}(T_{41},t)}$ $\boxed{+(a''_{41})^{(8,8)}(T_{41},t)}$ $\boxed{+(a''_{42})^{(8,8)}(T_{41},t)}$ are eight augmentation coefficient for 1,2,3

$\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)}\boxed{-(b''_{13})^{(1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13} \quad (58)$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)}\boxed{-(b''_{14})^{(1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14} \quad (59)$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)}\boxed{-(b''_{15})^{(1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15} \quad (60)$$

Where $\boxed{-(b''_{13})^{(1)}(G,t)}$, $\boxed{-(b''_{14})^{(1)}(G,t)}$, $\boxed{-(b''_{15})^{(1)}(G,t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$-(b_{20}'')^{(3,3,)}(G_{23},t)$, $-(b_{21}'')^{(3,3,)}(G_{23},t)$, $-(b_{22}'')^{(3,3,)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b_{24}'')^{(4,4,4,4,)}(G_{27},t)$, $-(b_{25}'')^{(4,4,4,4,)}(G_{27},t)$, $-(b_{26}'')^{(4,4,4,4,)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b_{28}'')^{(5,5,5,5,)}(G_{31},t)$, $-(b_{29}'')^{(5,5,5,5,)}(G_{31},t)$, $-(b_{30}'')^{(5,5,5,5,)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b_{32}'')^{(6,6,6,6,)}(G_{35},t)$, $-(b_{33}'')^{(6,6,6,6,)}(G_{35},t)$, $-(b_{34}'')^{(6,6,6,6,)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b_{37}'')^{(7,7,)}(G_{39},t)$, $-(b_{36}'')^{(7,7,)}(G_{39},t)$, $-(b_{38}'')^{(7,7,)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b_{40}'')^{(8,8)}(G_{43},t)$ $-(b_{41}'')^{(8,8)}(G_{43},t)$ $-(b_{42}'')^{(8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$-(b_{44}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b_{45}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b_{46}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[\begin{array}{ccc} (a_{16}')^{(2)} & +(a_{16}'')^{(2)}(T_{17},t) & +(a_{13}'')^{(1,1,)}(T_{14},t) \quad +(a_{20}'')^{(3,3,3)}(T_{21},t) \\ +(a_{24}'')^{(4,4,4,4,4)}(T_{25},t) & +(a_{28}'')^{(5,5,5,5,5)}(T_{29},t) & +(a_{32}'')^{(6,6,6,6,6)}(T_{33},t) \\ +(a_{36}'')^{(7,7,7)}(T_{37},t) & +(a_{40}'')^{(8,8,8)}(T_{41},t) & +(a_{44}'')^{(9,9)}(T_{45},t) \end{array}\right] G_{16} \quad (61)$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[\begin{array}{ccc} (a_{17}')^{(2)} & +(a_{17}'')^{(2)}(T_{17},t) & +(a_{14}'')^{(1,1,)}(T_{14},t) \quad +(a_{21}'')^{(3,3,3)}(T_{21},t) \\ +(a_{25}'')^{(4,4,4,4,4)}(T_{25},t) & +(a_{29}'')^{(5,5,5,5,5)}(T_{29},t) & +(a_{33}'')^{(6,6,6,6,6)}(T_{33},t) \\ +(a_{37}'')^{(7,7,7)}(T_{37},t) & +(a_{41}'')^{(8,8,8)}(T_{41},t) & +(a_{45}'')^{(9,9)}(T_{45},t) \end{array}\right] G_{17} \quad (62)$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[\begin{array}{ccc} (a_{18}')^{(2)} & +(a_{18}'')^{(2)}(T_{17},t) & +(a_{15}'')^{(1,1,)}(T_{14},t) \quad +(a_{22}'')^{(3,3,3)}(T_{21},t) \\ +(a_{26}'')^{(4,4,4,4,4)}(T_{25},t) & +(a_{30}'')^{(5,5,5,5,5)}(T_{29},t) & +(a_{34}'')^{(6,6,6,6,6)}(T_{33},t) \\ +(a_{38}'')^{(7,7,7)}(T_{37},t) & +(a_{42}'')^{(8,8,8)}(T_{41},t) & +(a_{46}'')^{(9,9)}(T_{45},t) \end{array}\right] G_{18} \quad (63)$$

Where $+(a_{16}'')^{(2)}(T_{17},t)$, $+(a_{17}'')^{(2)}(T_{17},t)$, $+(a_{18}'')^{(2)}(T_{17},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a_{13}'')^{(1,1,)}(T_{14},t)$, $+(a_{14}'')^{(1,1,)}(T_{14},t)$, $+(a_{15}'')^{(1,1,)}(T_{14},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a_{20}'')^{(3,3,3)}(T_{21},t)$, $+(a_{21}'')^{(3,3,3)}(T_{21},t)$, $+(a_{22}'')^{(3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a_{24}'')^{(4,4,4,4,4)}(T_{25},t)$, $+(a_{25}'')^{(4,4,4,4,4)}(T_{25},t)$, $+(a_{26}'')^{(4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a_{28}'')^{(5,5,5,5,5)}(T_{29},t)$, $+(a_{29}'')^{(5,5,5,5,5)}(T_{29},t)$, $+(a_{30}'')^{(5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a_{32}'')^{(6,6,6,6,6)}(T_{33},t)$, $+(a_{33}'')^{(6,6,6,6,6)}(T_{33},t)$, $+(a_{34}'')^{(6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a_{36}'')^{(7,7,7)}(T_{37},t)$, $+(a_{37}'')^{(7,7,7)}(T_{37},t)$, $+(a_{38}'')^{(7,7,7)}(T_{37},t)$ are seventh augmentation coefficient for category 1, 2 and 3

$+(a''_{40})^{(8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8)}(T_{41},t)$ are eight augmentation coefficient for category 1, 2 and 3

$+(a''_{44})^{(9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9)}(T_{45},t)$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \begin{bmatrix} (b'_{16})^{(2)} & -(b''_{16})^{(2)}(G_{19},t) & -(b''_{13})^{(1,1,)}(G,t) & -(b''_{20})^{(3,3,3,)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6)}(G_{35},t) & \\ -(b''_{36})^{(7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9)}(G_{47},t) & \end{bmatrix} T_{16} \quad (64)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)} & -(b''_{17})^{(2)}(G_{19},t) & -(b''_{14})^{(1,1,)}(G,t) & -(b''_{21})^{(3,3,3,)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6)}(G_{35},t) & \\ -(b''_{37})^{(7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9)}(G_{47},t) & \end{bmatrix} T_{17} \quad (65)$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)} & -(b''_{18})^{(2)}(G_{19},t) & -(b''_{15})^{(1,1,)}(G,t) & -(b''_{22})^{(3,3,3,)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6)}(G_{35},t) & \\ -(b''_{38})^{(7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9)}(G_{47},t) & \end{bmatrix} T_{18} \quad (66)$$

where $-(b''_{16})^{(2)}(G_{19},t)$, $-(b''_{17})^{(2)}(G_{19},t)$, $-(b''_{18})^{(2)}(G_{19},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{13})^{(1,1,)}(G,t)$, $-(b''_{14})^{(1,1,)}(G,t)$, $-(b''_{15})^{(1,1,)}(G,t)$ are second detrition coefficients for category 1,2 and 3

$-(b''_{20})^{(3,3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,)}(G_{23},t)$ are third detrition coefficients for category 1,2 and 3

$-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1,2 and 3

$-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1,2 and 3

$-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients for category 1,2 and 3

$-(b''_{36})^{(7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2 and 3

$-(b''_{40})^{(8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1,2 and 3

$-(b''_{44})^{(9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \begin{bmatrix} (a'_{20})^{(3)} & +(a''_{20})^{(3)}(T_{21},t) & +(a''_{16})^{(2,2,2)}(T_{17},t) & +(a''_{13})^{(1,1,1,)}(T_{14},t) \\ +(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t) & \\ +(a''_{36})^{(7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9)}(T_{45},t) & \end{bmatrix} G_{20} \quad (67)$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \begin{bmatrix} (a'_{21})^{(3)}\boxed{+(a''_{21})^{(3)}(T_{21},t)}\ \boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}\ \boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{21} \quad 68$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \begin{bmatrix} (a'_{22})^{(3)}\boxed{+(a''_{22})^{(3)}(T_{21},t)}\ \boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}\ \boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{22} \quad 69$$

$\boxed{+(a''_{20})^{(3)}(T_{21},t)}, \boxed{+(a''_{21})^{(3)}(T_{21},t)}, \boxed{+(a''_{22})^{(3)}(T_{21},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}, \boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}, \boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}$ are second augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)}$ are third augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)}, \boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)}, \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)}, \boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)}, \boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)}, \boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)}, \boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)}$ are seventh augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)}$ are eight augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{44})^{(9,9,9)}(T_{45},t)}, \boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)}, \boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)}$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b'_{20})^{(3)}\boxed{-(b''_{20})^{(3)}(G_{23},t)}\ \boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}\ \boxed{-(b''_{13})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)}\ \boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)}\ \boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7)}(G_{39},t)}\ \boxed{-(b''_{40})^{(8,8,8,8)}(G_{43},t)}\ \boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{20} \quad 70$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b'_{21})^{(3)}\boxed{-(b''_{21})^{(3)}(G_{23},t)}\ \boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}\ \boxed{-(b''_{14})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)}\ \boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)}\ \boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7)}(G_{39},t)}\ \boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)}\ \boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{21} \quad 71$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)}\boxed{-(b''_{22})^{(3)}(G_{23},t)}\ \boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}\ \boxed{-(b''_{15})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)}\ \boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)}\ \boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)}\ \boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)}\ \boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{22} \quad 72$$

$\boxed{-(b''_{20})^{(3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3)}(G_{23},t)}$ *are first detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}$ *are second detrition coefficients for*

*category 1, 2 and 3*

$\boxed{-(b_{13}'')^{(1,1,1,)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,)}(G,t)}$, $\boxed{-(b_{15}'')^{(1,1,1,)}(G,t)}$ *are third detrition coefficients for category 1,2 and 3*

$\boxed{-(b_{24}'')^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{25}'')^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{26}'')^{(4,4,4,4,4,4)}(G_{27},t)}$ *are fourth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b_{28}'')^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5)}(G_{31},t)}$ *are fifth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b_{32}'')^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{34}'')^{(6,6,6,6,6,6)}(G_{35},t)}$ *are sixth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b_{36}'')^{(7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{37}'')^{(7,7,7,7)}(G_{39},t)}$ $\boxed{-(b_{38}'')^{(7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{40}'')^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{42}'')^{(8,8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{46}'')^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b_{45}'')^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b_{44}'')^{(9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[\begin{array}{ccc} (a_{24}')^{(4)} & \boxed{+(a_{24}'')^{(4)}(T_{25},t)} & \boxed{+(a_{28}'')^{(5,5,)}(T_{29},t)} \quad \boxed{+(a_{32}'')^{(6,6,)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a_{16}'')^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a_{20}'')^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{36}'')^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{40}'')^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a_{44}'')^{(9,9,9,9)}(T_{45},t)} \end{array}\right] G_{24} \quad 73$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[\begin{array}{ccc} (a_{25}')^{(4)} & \boxed{+(a_{25}'')^{(4)}(T_{25},t)} & \boxed{+(a_{29}'')^{(5,5,)}(T_{29},t)} \quad \boxed{+(a_{33}'')^{(6,6)}(T_{33},t)} \\ \boxed{+(a_{14}'')^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a_{17}'')^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a_{21}'')^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{37}'')^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{41}'')^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a_{45}'')^{(9,9,9,9)}(T_{45},t)} \end{array}\right] G_{25} \quad 74$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[\begin{array}{ccc} (a_{26}')^{(4)} & \boxed{+(a_{26}'')^{(4)}(T_{25},t)} & \boxed{+(a_{30}'')^{(5,5,)}(T_{29},t)} \quad \boxed{+(a_{34}'')^{(6,6,)}(T_{33},t)} \\ \boxed{+(a_{15}'')^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a_{18}'')^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a_{22}'')^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{38}'')^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{42}'')^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a_{46}'')^{(9,9,9,9)}(T_{45},t)} \end{array}\right] G_{26} \quad 75$$

$\boxed{(a_{24}'')^{(4)}(T_{25},t)}$, $\boxed{(a_{25}'')^{(4)}(T_{25},t)}$, $\boxed{(a_{26}'')^{(4)}(T_{25},t)}$ *are first augmentation coefficients category* $1, 2$ $3$

$\boxed{+(a_{28}'')^{(5,5,)}(T_{29},t)}$, $\boxed{+(a_{29}'')^{(5,5,)}(T_{29},t)}$, $\boxed{+(a_{30}'')^{(5,5,)}(T_{29},t)}$ *are second augmentation coefficient for category* $1, 2$ *and* $3$

$\boxed{+(a_{32}'')^{(6,6,)}(T_{33},t)}$, $\boxed{+(a_{33}'')^{(6,6,)}(T_{33},t)}$, $\boxed{+(a_{34}'')^{(6,6,)}(T_{33},t)}$ *are third augmentation coefficient for category* $1, 2$ *and* $3$

$\boxed{+(a_{13}'')^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a_{14}'')^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a_{15}'')^{(1,1,1,1)}(T_{14},t)}$ *are fourth augmentation coefficients for category* $1, 2$ *and* $3$

$\boxed{+(a_{16}'')^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{17}'')^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{18}'')^{(2,2,2,2)}(T_{17},t)}$ *are fifth augmentation coefficients for category* $1, 2$ *and* $3$

$\boxed{+(a_{20}'')^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{21}'')^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{22}'')^{(3,3,3,3)}(T_{21},t)}$ *are sixth augmentation coefficients for category* $1, 2$ *and* $3$

$\boxed{+(a_{36}'')^{(7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a_{37}'')^{(7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a_{38}'')^{(7,7,7,7,7)}(T_{37},t)}$ *are seventh augmentation coefficients for category* $1, 2$ *and* $3$

$+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)$

*are eighth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{46})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9)}(T_{45},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \begin{bmatrix} (b'_{24})^{(4)} & -(b''_{24})^{(4)}(G_{27},t) & -(b''_{28})^{(5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,)}(G_{35},t) \\ & -(b''_{13})^{(1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{24} \quad (76)$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \begin{bmatrix} (b'_{25})^{(4)} & -(b''_{25})^{(4)}(G_{27},t) & -(b''_{29})^{(5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{25} \quad (77)$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)} & -(b''_{26})^{(4)}(G_{27},t) & -(b''_{30})^{(5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1)}(G,t) & -(b''_{18})^{(2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{26} \quad (78)$$

*Where* $-(b''_{24})^{(4)}(G_{27},t)$, $-(b''_{25})^{(4)}(G_{27},t)$, $-(b''_{26})^{(4)}(G_{27},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{28})^{(5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,)}(G_{31},t)$ *are second detrition coefficients for category* 1, 2 *and* 3

$-(b''_{32})^{(6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,)}(G_{35},t)$ *are third detrition coefficients for category* 1, 2 *and* 3

$-(b''_{13})^{(1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1)}(G,t)$

*are fourth detrition coefficients for category* 1, 2 *and* 3

$-(b''_{16})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2)}(G_{19},t)$

*are fifth detrition coefficients for category* 1, 2 *and* 3

$-(b''_{20})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3)}(G_{23},t)$

*are sixth detrition coefficients for category* 1, 2 *and* 3

$-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)$

*are seventh detrition coefficients for category* 1, 2 *and* 3

$-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)$

*are eighth detrition coefficients for category* 1, 2 *and* 3

$-(b''_{46})^{(9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \begin{bmatrix} (a'_{28})^{(5)} & +(a''_{28})^{(5)}(T_{29},t) & +(a''_{24})^{(4,4,)}(T_{25},t) & +(a''_{32})^{(6,6,6)}(T_{33},t) \\ & +(a''_{13})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{16})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3,3)}(T_{21},t) \\ & +(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{28} \quad (79)$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \begin{bmatrix} (a'_{29})^{(5)} & +(a''_{29})^{(5)}(T_{29},t) & +(a''_{25})^{(4,4,)}(T_{25},t) & +(a''_{33})^{(6,6,6)}(T_{33},t) \\ +(a''_{14})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{17})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3,3)}(T_{21},t) \\ +(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{29} \quad 80$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \begin{bmatrix} (a'_{30})^{(5)} & +(a''_{30})^{(5)}(T_{29},t) & +(a''_{26})^{(4,4,)}(T_{25},t) & +(a''_{34})^{(6,6,6)}(T_{33},t) \\ +(a''_{15})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{18})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3,3)}(T_{21},t) \\ +(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{30} \quad 81$$

*Where* $+(a''_{28})^{(5)}(T_{29},t)$, $+(a''_{29})^{(5)}(T_{29},t)$, $+(a''_{30})^{(5)}(T_{29},t)$ *are first augmentation coefficients for category* 1, 2 *and* 3

*And* $+(a''_{24})^{(4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,)}(T_{25},t)$ *are second augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{32})^{(6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6)}(T_{33},t)$ *are third augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)$ are fourth augmentation coefficients for category 1,2, and 3

$+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)$ are fifth augmentation coefficients for category 1,2,and 3

$+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)$ are sixth augmentation coefficients for category 1,2, 3

$+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)$ , $+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1,2, 3

$+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)$ are eighth augmentation coefficients for category 1,2, 3

$+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \begin{bmatrix} (b'_{28})^{(5)} & -(b''_{28})^{(5)}(G_{31},t) & -(b''_{24})^{(4,4,)}(G_{27},t) & -(b''_{32})^{(6,6,6)}(G_{35},t) \\ -(b''_{13})^{(1,1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3)}(G_{23},t) \\ -(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{28} \quad 82$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b'_{29})^{(5)} & -(b''_{29})^{(5)}(G_{31},t) & -(b''_{25})^{(4,4,)}(G_{27},t) & -(b''_{33})^{(6,6,6)}(G_{35},t) \\ -(b''_{14})^{(1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3)}(G_{23},t) \\ -(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{29} \quad 83$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b'_{30})^{(5)} & -(b''_{30})^{(5)}(G_{31},t) & -(b''_{26})^{(4,4,)}(G_{27},t) & -(b''_{34})^{(6,6,6)}(G_{35},t) \\ -(b''_{15})^{(1,1,1,1,1,)}(G,t) & -(b''_{18})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3)}(G_{23},t) \\ -(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{30} \quad 84$$

*where* $-(b''_{28})^{(5)}(G_{31},t)$ , $-(b''_{29})^{(5)}(G_{31},t)$ , $-(b''_{30})^{(5)}(G_{31},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{24})^{(4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,)}(G_{27},t)$ *are second detrition coefficients*

$for\ category\ 1,2\ and\ 3$

$\boxed{-(b''_{32})^{(6,6,6)}(G_{35},t)}, \boxed{-(b''_{33})^{(6,6,6)}(G_{35},t)}, \boxed{-(b''_{34})^{(6,6,6)}(G_{35},t)}$ $are\ third\ detrition\ coefficients$

$for\ category\ 1,2\ and\ 3$

$\boxed{-(b''_{13})^{(1,1,1,1,1)}(G,t)}, \boxed{-(b''_{14})^{(1,1,1,1,1)}(G,t)}, \boxed{-(b''_{15})^{(1,1,1,1,1,)}(G,t)}$ are fourth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[\begin{array}{c} (a'_{32})^{(6)}\boxed{+(a''_{32})^{(6)}(T_{33},t)}\ \boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}\ \boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}\ \boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}\ \boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{32} \quad 85$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[\begin{array}{c} (a'_{33})^{(6)}\boxed{+(a''_{33})^{(6)}(T_{33},t)}\ \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}\ \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}\ \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}\ \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{33} \quad 86$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[\begin{array}{c} (a'_{34})^{(6)}\boxed{+(a''_{34})^{(6)}(T_{33},t)}\ \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}\ \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}\ \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}\ \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{34} \quad 87$$

$\boxed{+(a''_{32})^{(6)}(T_{33},t)}, \boxed{+(a''_{33})^{(6)}(T_{33},t)}, \boxed{+(a''_{34})^{(6)}(T_{33},t)}$ $are\ first\ augmentation\ coefficients$

$for\ category\ 1,2\ and\ 3$

$\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}$ $are\ second\ augmentation$

$coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}$ $are\ third\ augmentation$

$coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}$ - are fourth augmentation coefficients

$\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}$ - fifth augmentation coefficients

$\boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)}$ sixth augmentation coefficients

$+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)$
seventh augmentation coefficients
$+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)$
Eighth augmentation coefficients
$+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \begin{bmatrix} (b'_{32})^{(6)} & -(b''_{32})^{(6)}(G_{35},t) & -(b''_{28})^{(5,5,5)}(G_{31},t) & -(b''_{24})^{(4,4,4,)}(G_{27},t) \\ & -(b''_{13})^{(1,1,1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{32} \quad (88)$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \begin{bmatrix} (b'_{33})^{(6)} & -(b''_{33})^{(6)}(G_{35},t) & -(b''_{29})^{(5,5,5)}(G_{31},t) & -(b''_{25})^{(4,4,4,)}(G_{27},t) \\ & -(b''_{14})^{(1,1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{33} \quad (89)$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)} & -(b''_{34})^{(6)}(G_{35},t) & -(b''_{30})^{(5,5,5)}(G_{31},t) & -(b''_{26})^{(4,4,4,)}(G_{27},t) \\ & -(b''_{15})^{(1,1,1,1,1,1)}(G,t) & -(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{34} \quad (90)$$

$-(b''_{32})^{(6)}(G_{35},t)$, $-(b''_{33})^{(6)}(G_{35},t)$, $-(b''_{34})^{(6)}(G_{35},t)$ *are first detrition coefficients for category* 1, 2 *and* 3
$-(b''_{28})^{(5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5)}(G_{31},t)$ *are second detrition coefficients for category* 1, 2 *and* 3
$-(b''_{24})^{(4,4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,)}(G_{27},t)$ *are third detrition coefficients for category* 1,2 *and* 3
$-(b''_{13})^{(1,1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1,1,1)}(G,t)$ are fourth detrition coefficients for category 1, 2, and 3
$-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1, 2, and 3
$-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1, 2, and 3
$-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2, and 3
$-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)$
are eighth detrition coefficients for category 1, 2, and 3
$-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \qquad 91$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a'_{36})^{(7)}\boxed{+(a''_{36})^{(7)}(T_{37},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \qquad 92$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a'_{37})^{(7)}\boxed{+(a''_{37})^{(7)}(T_{37},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{38}}{dt} \qquad 93$$

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a'_{38})^{(7)}\boxed{+(a''_{38})^{(7)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{(a''_{36})^{(7)}(T_{37},t)}$, $\boxed{(a''_{37})^{(7)}(T_{37},t)}$, $\boxed{(a''_{38})^{(7)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{36}}{dt} = \qquad 94$$

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b'_{36})^{(7)}\boxed{-(b''_{36})^{(7)}(G_{39},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{37}}{dt} =$$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b'_{37})^{(7)}\boxed{-(b''_{37})^{(7)}(G_{39},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{38}}{dt} =$$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b'_{38})^{(7)}\boxed{-(b''_{38})^{(7)}(G_{39},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{40}}{dt}$$



$$= (a_{40})^{(8)}G_{41} - \begin{bmatrix} (a'_{40})^{(8)}\boxed{+(a''_{40})^{(8)}(T_{41},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{41}}{dt}$$

$$= (a_{41})^{(8)}G_{40} - \begin{bmatrix} (a'_{41})^{(8)}\boxed{+(a''_{41})^{(8)}(T_{41},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \begin{bmatrix} (a'_{42})^{(8)}\boxed{+(a''_{42})^{(8)}(T_{41},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a''_{40})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)}\boxed{-(b''_{40})^{(8)}(G_{43},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)}\boxed{-(b''_{41})^{(8)}(G_{43},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)}\boxed{-(b''_{42})^{(8)}(G_{43},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b_{36}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{37}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{16}'')^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{17}'')^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{18}'')^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{20}'')^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{21}'')^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{22}'')^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{24}'')^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{25}'')^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{26}'')^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{28}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{32}'')^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{38}'')^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{36}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{37}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{44}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{45}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{46}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{44}}{dt} \qquad 96$$

$$= (a_{44})^{(9)}G_{45} - \left[\begin{array}{c} (a_{44}')^{(9)}\boxed{+(a_{44}'')^{(9)}(T_{45},t)}\ \boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}\ \boxed{+(a_{20}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{24}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a_{28}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a_{32}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}\ \boxed{+(a_{36}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}\ \boxed{+(a_{40}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{array}\right] G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[\begin{array}{c} (a_{45}')^{(9)}\boxed{+(a_{45}'')^{(9)}(T_{45},t)}\ \boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}\ \boxed{+(a_{21}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{25}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a_{29}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a_{33}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{14}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}\ \boxed{+(a_{37}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}\ \boxed{+(a_{41}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{array}\right] G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[\begin{array}{c} (a_{46}')^{(9)}\boxed{+(a_{46}'')^{(9)}(T_{37},t)}\ \boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}\ \boxed{+(a_{22}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{26}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a_{30}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a_{34}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{15}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}\ \boxed{+(a_{38}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}\ \boxed{+(a_{42}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{array}\right] G_{15}$$

Where $\boxed{+(a_{44}'')^{(9)}(T_{45},t)}$, $\boxed{+(a_{45}'')^{(9)}(T_{45},t)}$, $\boxed{+(a_{46}'')^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{20}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{21}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{22}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{24}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{25}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{26}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{28}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{29}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{30}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{32}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{33}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{34}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a_{14}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a_{15}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ are Seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{38}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a_{37}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a_{36}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a_{40}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a_{42}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a_{41}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} =$$

$$(b_{44})^{(9)}T_{45} -$$

$$\begin{bmatrix} (b_{44}')^{(9)}\boxed{-(b_{44}'')^{(9)}(G_{47},t)} & \boxed{-(b_{16}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{20}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{24}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{28}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{32}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{13}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{36}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b_{45}')^{(9)}\boxed{-(b_{45}'')^{(9)}(G_{47},t)} & \boxed{-(b_{17}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{21}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{25}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{29}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{33}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{14}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{37}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b_{46}')^{(9)}\boxed{-(b_{46}'')^{(9)}(G_{47},t)} & \boxed{-(b_{18}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{22}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{26}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{30}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{34}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{15}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{38}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b_{44}'')^{(9)}(G_{47},t)}$, $\boxed{-(b_{45}'')^{(9)}(G_{47},t)}$, $\boxed{-(b_{46}'')^{(9)}(G_{47},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{16}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{17}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{18}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{20}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{21}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{22}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{24}'')^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{25}'')^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{26}'')^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{28}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{32}'')^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{34}'')^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{37}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{36}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a_i')^{(1)}, (a_i'')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (b_i'')^{(1)} > 0, \quad i,j = 13,14,15 \qquad 97$$

The functions $(a_i'')^{(1)}, (b_i'')^{(1)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a_i'')^{(1)}(T_{14},t) \le (p_i)^{(1)} \le (\hat{A}_{13})^{(1)}$$

$$(b_i'')^{(1)}(G,t) \le (r_i)^{(1)} \le (b_i')^{(1)} \le (\hat{B}_{13})^{(1)}$$

$$lim_{T_2\to\infty}(a_i'')^{(1)}(T_{14},t) = (p_i)^{(1)} \qquad 98$$

$$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = (r_i)^{(1)}$$

**Definition of** $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$ :

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$$|(a_i'')^{(1)}(T_{14}',t) - (a_i'')^{(1)}(T_{14},t)| \le (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G',t) - (b_i'')^{(1)}(G,t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}',t)$ and $(a_i'')^{(1)}(T_{14},t)$ . $(T_{14}',t)$ and $(T_{14},t)$ are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14},t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$ : 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$

**Definition of** $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$ : 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}$, $(r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)} (\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)} (\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i, j = 16,17,18$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17}, t) \leq (p_i)^{(2)} \leq (\hat{A}_{16})^{(2)} \quad 102$$

$$(b_i'')^{(2)}(G_{19}, t) \leq (r_i)^{(2)} \leq (b_i')^{(2)} \leq (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2 \to \infty} (a_i'')^{(2)}(T_{17}, t) = (p_i)^{(2)} \quad 104$$

$$\lim_{G \to \infty} (b_i'')^{(2)}((G_{19}), t) = (r_i)^{(2)} \quad 105$$

**Definition of** $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$ : 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}', t) - (a_i'')^{(2)}(T_{17}, t)| \leq (\hat{k}_{16})^{(2)} |T_{17} - T_{17}'| e^{-(\hat{M}_{16})^{(2)} t} \quad 107$$

$$|(b_i'')^{(2)}((G_{19})', t) - (b_i'')^{(2)}((G_{19}), t)| < (\hat{k}_{16})^{(2)} ||(G_{19}) - (G_{19})'|| e^{-(\hat{M}_{16})^{(2)} t} \quad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}', t)$ and$(a_i'')^{(2)}(T_{17}, t)$ . $(T_{17}', t)$ and $(T_{17}, t)$ are points belonging to the interval $[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}]$ . It is to be noted that $(a_i'')^{(2)}(T_{17}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17}, t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$ :

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)},$ are positive constants 109

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$$

**Definition of** $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$ :

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and\ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1 \quad 110$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(b_i)^{(2)} + (b_i')^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1 \quad 111$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i, j = 20,21,22 \quad 112$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(3)}$, $(r_i)^{(3)}$:

$$(a_i'')^{(3)}(T_{21}, t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23}, t) \leq (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(3)}(T_{21}, t) = (p_i)^{(3)} \quad 113$$

$$\lim_{G \to \infty}(b_i'')^{(3)}(G_{23}, t) = (r_i)^{(3)}$$

**Definition of** $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$ :

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: 114

$$|(a_i'')^{(3)}(T_{21}', t) - (a_i'')^{(3)}(T_{21}, t)| \leq (\hat{k}_{20})^{(3)} |T_{21} - T_{21}'| e^{-(\hat{M}_{20})^{(3)} t}$$

$$|(b_i'')^{(3)}(G_{23}', t) - (b_i'')^{(3)}(G_{23}, t)| < (\hat{k}_{20})^{(3)} ||G_{23} - G_{23}'|| e^{-(\hat{M}_{20})^{(3)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}', t)$ and $(a_i'')^{(3)}(T_{21}, t)$ . $(T_{21}', t)$ And $(T_{21}, t)$ are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21}, t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$ : 115

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}$ , $\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with 116
$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$ and the constants
$(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}, i = 20,21,22,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(a_i)^{(3)} + (a_i')^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)} (\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(b_i)^{(3)} + (b_i')^{(3)} + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)} (\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i, j = 24,25,26$$ 117

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(4)}$, $(r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25}, t) \leq (p_i)^{(4)} \leq (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}((G_{27}), t) \leq (r_i)^{(4)} \leq (b_i')^{(4)} \leq (\hat{B}_{24})^{(4)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(4)}(T_{25}, t) = (p_i)^{(4)}$$ 118

$$\lim_{G \to \infty}(b_i'')^{(4)}((G_{27}), t) = (r_i)^{(4)}$$

**Definition of** $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$ :

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24,25,26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}', t) - (a_i'')^{(4)}(T_{25}, t)| \leq (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})', t) - (b_i'')^{(4)}((G_{27}), t)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}', t)$ and$(a_i'')^{(4)}(T_{25}, t)$ . $(T_{25}', t)$ and $(T_{25}, t)$ are points belonging to the interval $[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}]$. It is to be noted that $(a_i'')^{(4)}(T_{25}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25}, t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$ : 120

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)},$ are positive constants

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}, \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$$

**Definition of** $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$ : 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and\ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}, i = 24,25,26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(a_i)^{(4)} + (a_i')^{(4)} + (\hat{A}_{24})^{(4)} + (\hat{P}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(b_i)^{(4)} + (b_i')^{(4)} + (\hat{B}_{24})^{(4)} + (\hat{Q}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i, j = 28,29,30$$ 122

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(5)}$, $(r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29}, t) \le (p_i)^{(5)} \le (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}((G_{31}), t) \le (r_i)^{(5)} \le (b_i')^{(5)} \le (\hat{B}_{28})^{(5)}$$

$$lim_{T_2 \to \infty} (a_i'')^{(5)}(T_{29}, t) = (p_i)^{(5)}$$ 123

$$\lim_{G \to \infty} (b_i'')^{(5)}(G_{31}, t) = (r_i)^{(5)}$$

**Definition of** $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$ :

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}', t) - (a_i'')^{(5)}(T_{29}, t)| \le (\hat{k}_{28})^{(5)} |T_{29} - T_{29}'| e^{-(\hat{M}_{28})^{(5)} t}$$

$$|(b_i'')^{(5)}((G_{31})', t) - (b_i'')^{(5)}((G_{31}), t)| < (\hat{k}_{28})^{(5)} ||(G_{31}) - (G_{31})'|| e^{-(\hat{M}_{28})^{(5)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}', t)$ and $(a_i'')^{(5)}(T_{29}, t)$ . $(T_{29}', t)$ and $(T_{29}, t)$ are points belonging to the interval $\left[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\right]$. It is to be noted that $(a_i'')^{(5)}(T_{29}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$ : 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)},$ are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}, \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

**Definition of** $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$ : 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and\ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(a_i)^{(5)} + (a_i')^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(b_i)^{(5)} + (b_i')^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i, j = 32,33,34$$ 127

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(6)}$, $(r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33}, t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}), t) \leq (r_i)^{(6)} \leq (b_i')^{(6)} \leq (\hat{B}_{32})^{(6)}$$

$$lim_{T_2 \to \infty} (a_i'')^{(6)} (T_{33}, t) = (p_i)^{(6)}$$ 128

$$\lim_{G \to \infty} (b_i'')^{(6)} ((G_{35}), t) = (r_i)^{(6)}$$

**Definition of** $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$ :

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}', t) - (a_i'')^{(6)}(T_{33}, t)| \leq (\hat{k}_{32})^{(6)} |T_{33} - T_{33}'| e^{-(\hat{M}_{32})^{(6)} t}$$

$$|(b_i'')^{(6)}((G_{35})', t) - (b_i'')^{(6)}((G_{35}), t)| < (\hat{k}_{32})^{(6)} ||(G_{35}) - (G_{35})'|| e^{-(\hat{M}_{32})^{(6)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}', t)$ and $(a_i'')^{(6)}(T_{33}, t)$ . $(T_{33}', t)$ and $(T_{33}, t)$ are points belonging to the interval $[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}]$. It is to be noted that $(a_i'')^{(6)}(T_{33}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$ : 129

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)},$ are positive constants

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$$

**Definition of** $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$ : 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and\ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32,33,34,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(a_i)^{(6)} + (a_i')^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)} (\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(b_i)^{(6)} + (b_i')^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)} (\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

(YYYY) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i, j = 36,37,38$ 131

(ZZZZ) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(7)}$, $(r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37}, t) \le (p_i)^{(7)} \le (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39}, t) \le (r_i)^{(7)} \le (b_i')^{(7)} \le (\hat{B}_{36})^{(7)}$$

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*(AAAAA)* $\lim_{T_2 \to \infty} (a_i'')^{(7)}(T_{37}, t) = (p_i)^{(7)}$

*(BBBBB)*

$\lim_{G \to \infty} (b_i'')^{(7)}((G_{39}), t) = (r_i)^{(7)}$

**Definition of** $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$ :

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$$|(a_i'')^{(7)}(T_{37}', t) - (a_i'')^{(7)}(T_{37}, t)| \le (\hat{k}_{36})^{(7)} |T_{37} - T_{37}'| e^{-(\hat{M}_{36})^{(7)} t}$$

$$|(b_i'')^{(7)}((G_{39})', t) - (b_i'')^{(7)}((G_{39}), t)| < (\hat{k}_{36})^{(7)} ||(G_{39}) - (G_{39})'|| e^{-(\hat{M}_{36})^{(7)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}', t)$ and $(a_i'')^{(7)}(T_{37}, t)$ . $(T_{37}', t)$ and $(T_{37}, t)$ are points belonging to the interval $[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}]$ . It is to be noted that $(a_i'')^{(7)}(T_{37}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37}, t)$ , the **first augmentation coefficient** attributable to the

system, would be absolutely continuous.

**Definition of** $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$ : 134

(CCCCC) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)},$ are positive constants

$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$

**Definition of** $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$ : 135

(DDDDD) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(b_i)^{(7)} + (b_i')^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40,41,42$ 136

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

**Definition of** $(p_i)^{(8)}, (r_i)^{(8)}$: 137

$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)}$ 138

$(b_i'')^{(8)}((G_{43}), t) \leq (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)}$ 139

$\lim_{T_2 \to \infty} (a_i'')^{(8)}(T_{41}, t) = (p_i)^{(8)}$ 140

$\lim_{G \to \infty} (b_i'')^{(8)}((G_{43}), t) = (r_i)^{(8)}$ 141

**Definition of** $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$ :

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)} |T_{41} - T_{41}'| e^{-(\hat{M}_{40})^{(8)} t}$ 142

$|(b_i'')^{(8)}((G_{43})', t) - (b_i'')^{(8)}((G_{43}), t)| < (\hat{k}_{40})^{(8)} ||(G_{43}) - (G_{43})'|| e^{-(\hat{M}_{40})^{(8)} t}$ 143

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}', t)$ and $(a_i'')^{(8)}(T_{41}, t)$ . $(T_{41}', t)$ and $(T_{41}, t)$ are points belonging to the interval $\left[(\hat{k}_{40})^{(8)}, (\widehat{M}_{40})^{(8)}\right]$. It is to be noted that $(a_i'')^{(8)}(T_{41}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{40})^{(8)} = 1$ then the function $(a_i'')^{(8)}(T_{41}, t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\widehat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$ :

$(\widehat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)},$ are positive constants

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}, \frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}} < 1 \qquad 144$$

**Definition of** $(\hat{P}_{40})^{(8)}, (\hat{Q}_{40})^{(8)}$ :

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\widehat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}, (\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)}, (a_i')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}, i = 40,41,42,$
Satisfy the inequalities

$$\frac{1}{(\widehat{M}_{40})^{(8)}}[(a_i)^{(8)} + (a_i')^{(8)} + (\hat{A}_{40})^{(8)} + (\hat{P}_{40})^{(8)} (\hat{k}_{40})^{(8)}] < 1 \qquad 145$$

$$\frac{1}{(\widehat{M}_{40})^{(8)}}[(b_i)^{(8)} + (b_i')^{(8)} + (\hat{B}_{40})^{(8)} + (\hat{Q}_{40})^{(8)} (\hat{k}_{40})^{(8)}] < 1 \qquad 146$$

Where we suppose

$$(a_i)^{(9)}, (a_i')^{(9)}, (a_i'')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (b_i'')^{(9)} > 0, \quad i, j = 44,45,46 \qquad 146\,A$$

The functions $(a_i'')^{(9)}, (b_i'')^{(9)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(9)}$, $(r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45}, t) \leq (p_i)^{(9)} \leq (\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47}, t) \leq (r_i)^{(9)} \leq (b_i')^{(9)} \leq (\hat{B}_{44})^{(9)}$$

$$\lim_{T_2 \to \infty} (a_i'')^{(9)}(T_{45}, t) = (p_i)^{(9)}$$

$$\lim_{G \to \infty} (b_i'')^{(9)}(G_{47}, t) = (r_i)^{(9)}$$

**Definition of** $(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}$ :

Where $\boxed{(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}}$ are positive constants and $\boxed{i = 44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}', t) - (a_i'')^{(9)}(T_{45}, t)| \leq (\hat{k}_{44})^{(9)} |T_{45} - T_{45}'| e^{-(\widehat{M}_{44})^{(9)} t}$$

$$|(b_i'')^{(9)}((G_{47})', t) - (b_i'')^{(9)}\big((G_{47}), t\big)| < (\hat{k}_{44})^{(9)}||(G_{47}) - (G_{47})'||e^{-(\hat{M}_{44})^{(9)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}', t)$ and$(a_i'')^{(9)}(T_{45}, t)$ . $(T_{45}', t)$ and $(T_{45}, t)$ are points belonging to the interval $\big[(\hat{k}_{44})^{(9)}, (\hat{M}_{44})^{(9)}\big]$. It is to be noted that $(a_i'')^{(9)}(T_{45}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{44})^{(9)} = 1$ then the function $(a_i'')^{(9)}(T_{45}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$ :

$(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$, are positive constants

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}} \ , \frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$$

**Definition of** $(\hat{P}_{44})^{(9)}, (\hat{Q}_{44})^{(9)}$ :

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}, (\hat{A}_{44})^{(9)} and\ (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)}, (a_i')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (p_i)^{(9)}, \ (r_i)^{(9)}, i = 44,45,46,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{44})^{(9)}}[\,(a_i)^{(9)} + (a_i')^{(9)} + \ (\hat{A}_{44})^{(9)} + \ (\hat{P}_{44})^{(9)}\,(\hat{k}_{44})^{(9)}] < 1$$

$$\frac{1}{(\hat{M}_{44})^{(9)}}[\ (b_i)^{(9)} + (b_i')^{(9)} + \ (\hat{B}_{44})^{(9)} + \ (\hat{Q}_{44})^{(9)}\ (\hat{k}_{44})^{(9)}] < 1$$

**Theorem 1:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \big(\hat{P}_{13}\big)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \ , \ \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 2 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

**Definition of** $G_i(0)\,, T_i(0)$

$$G_i(t) \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} \quad , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} \quad , \quad T_i(0) = T_i^0 > 0$$

**Theorem 3 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t) \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t} \quad , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t} \quad , \quad T_i(0) = T_i^0 > 0$$

**Theorem 4 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

**Definition of** $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \left(\hat{P}_{24}\right)^{(4)} e^{(\hat{M}_{24})^{(4)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{24}\right)^{(4)} e^{(\hat{M}_{24})^{(4)}t} \;,\quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 5 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

**Definition of** $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \left(\hat{P}_{28}\right)^{(5)} e^{(\hat{M}_{28})^{(5)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{28}\right)^{(5)} e^{(\hat{M}_{28})^{(5)}t} \;,\quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 6 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

**Definition of** $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \left(\hat{P}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t} \;,\quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 7:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

**Definition of** $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \left(\hat{P}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t} \;,\quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 8:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

**Definition of** $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \left(\hat{P}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t} \;,\quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 9:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

**Definition of** $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \left(\hat{P}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

**Proof:** Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \leq (\hat{P}_{13})^{(1)} \text{ , } T_i^0 \leq (\hat{Q}_{13})^{(1)}, \quad 155$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t} \quad 156$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t} \quad 157$$

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[ (a_{13})^{(1)} G_{14}(s_{(13)}) - \left( (a'_{13})^{(1)} + a''_{13})^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \right) G_{13}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[ (a_{14})^{(1)} G_{13}(s_{(13)}) - \left( (a'_{14})^{(1)} + (a''_{14})^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \right) G_{14}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[ (a_{15})^{(1)} G_{14}(s_{(13)}) - \left( (a'_{15})^{(1)} + (a''_{15})^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \right) G_{15}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[ (b_{13})^{(1)} T_{14}(s_{(13)}) - \left( (b'_{13})^{(1)} - (b''_{13})^{(1)} (G(s_{(13)}), s_{(13)}) \right) T_{13}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[ (b_{14})^{(1)} T_{13}(s_{(13)}) - \left( (b'_{14})^{(1)} - (b''_{14})^{(1)} (G(s_{(13)}), s_{(13)}) \right) T_{14}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{T}_{15}(t) = T_{15}^0 + \int_0^t \left[ (b_{15})^{(1)} T_{14}(s_{(13)}) - \left( (b'_{15})^{(1)} - (b''_{15})^{(1)} (G(s_{(13)}), s_{(13)}) \right) T_{15}(s_{(13)}) \right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \leq (\hat{P}_{16})^{(2)} \text{ , } T_i^0 \leq (\hat{Q}_{16})^{(2)},$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$$

By 160

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[ (a_{16})^{(2)} G_{17}(s_{(16)}) - \left( (a'_{16})^{(2)} + a''_{16})^{(2)} (T_{17}(s_{(16)}), s_{(16)}) \right) G_{16}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[ (a_{17})^{(2)} G_{16}(s_{(16)}) - \left( (a'_{17})^{(2)} + (a''_{17})^{(2)} (T_{17}(s_{(16)}), s_{(17)}) \right) G_{17}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[ (a_{18})^{(2)} G_{17}(s_{(16)}) - \left( (a'_{18})^{(2)} + (a''_{18})^{(2)} (T_{17}(s_{(16)}), s_{(16)}) \right) G_{18}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^{0} + \int_0^t \left[ (b_{16})^{(2)} T_{17}(s_{(16)}) - \left( (b'_{16})^{(2)} - (b''_{16})^{(2)} (G_{19}(s_{(16)}), s_{(16)}) \right) T_{16}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^{0} + \int_0^t \left[ (b_{17})^{(2)} T_{16}(s_{(16)}) - \left( (b'_{17})^{(2)} - (b''_{17})^{(2)} (G_{19}(s_{(16)}), s_{(16)}) \right) T_{17}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^{0} + \int_0^t \left[ (b_{18})^{(2)} T_{17}(s_{(16)}) - \left( (b'_{18})^{(2)} - (b''_{18})^{(2)} (G_{19}(s_{(16)}), s_{(16)}) \right) T_{18}(s_{(16)}) \right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i : \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \le (\hat{P}_{20})^{(3)} \text{ , } T_i^0 \le (\hat{Q}_{20})^{(3)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\bar{M}_{20})^{(3)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\bar{M}_{20})^{(3)} t}$$

By 161

$$\bar{G}_{20}(t) = G_{20}^{0} + \int_0^t \left[ (a_{20})^{(3)} G_{21}(s_{(20)}) - \left( (a'_{20})^{(3)} + a''_{20})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^{0} + \int_0^t \left[ (a_{21})^{(3)} G_{20}(s_{(20)}) - \left( (a'_{21})^{(3)} + (a''_{21})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{21}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^{0} + \int_0^t \left[ (a_{22})^{(3)} G_{21}(s_{(20)}) - \left( (a'_{22})^{(3)} + (a''_{22})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{22}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^{0} + \int_0^t \left[ (b_{20})^{(3)} T_{21}(s_{(20)}) - \left( (b'_{20})^{(3)} - (b''_{20})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^{0} + \int_0^t \left[ (b_{21})^{(3)} T_{20}(s_{(20)}) - \left( (b'_{21})^{(3)} - (b''_{21})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{21}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{22}(t) = T_{22}^{0} + \int_0^t \left[ (b_{22})^{(3)} T_{21}(s_{(20)}) - \left( (b'_{22})^{(3)} - (b''_{22})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{22}(s_{(20)}) \right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i : \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \le (\hat{P}_{24})^{(4)} \text{ , } T_i^0 \le (\hat{Q}_{24})^{(4)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\bar{M}_{24})^{(4)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\bar{M}_{24})^{(4)} t}$$

By 162

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{26}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{26}(t) = T_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{26}(s_{(24)})\right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{28})^{(5)}$, $T_i^0 \le (\hat{Q}_{28})^{(5)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

By 163

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{30}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{30}(t) = T_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{30}(s_{(28)})\right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{32})^{(6)}$, $T_i^0 \le (\hat{Q}_{32})^{(6)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\bar{M}_{32})^{(6)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\bar{M}_{32})^{(6)} t}$

By 164

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[ (a_{32})^{(6)} G_{33}(s_{(32)}) - \left( (a'_{32})^{(6)} + a''_{32})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[ (a_{33})^{(6)} G_{32}(s_{(32)}) - \left( (a'_{33})^{(6)} + (a''_{33})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[ (a_{34})^{(6)} G_{33}(s_{(32)}) - \left( (a'_{34})^{(6)} + (a''_{34})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{34}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[ (b_{32})^{(6)} T_{33}(s_{(32)}) - \left( (b'_{32})^{(6)} - (b''_{32})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[ (b_{33})^{(6)} T_{32}(s_{(32)}) - \left( (b'_{33})^{(6)} - (b''_{33})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{34}(t) = T_{34}^0 + \int_0^t \left[ (b_{34})^{(6)} T_{33}(s_{(32)}) - \left( (b'_{34})^{(6)} - (b''_{34})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{34}(s_{(32)}) \right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[ (a_{36})^{(7)} G_{37}(s_{(36)}) - \left( (a'_{36})^{(7)} + a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[ (a_{37})^{(7)} G_{36}(s_{(36)}) - \left( (a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[ (a_{38})^{(7)} G_{37}(s_{(36)}) - \left( (a'_{38})^{(7)} + (a''_{38})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{38}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[ (b_{36})^{(7)} T_{37}(s_{(36)}) - \left( (b'_{36})^{(7)} - (b''_{36})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[ (b_{37})^{(7)} T_{36}(s_{(36)}) - \left( (b'_{37})^{(7)} - (b''_{37})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\overline{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^{0} + \int_0^t \left[ (b_{38})^{(7)} T_{37}(s_{(36)}) - \left( (b'_{38})^{(7)} - (b''_{38})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{38}(s_{(36)}) \right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$ , $T_i(0) = T_i^0$ , $G_i^0 \le (\hat{P}_{40})^{(8)}$ , $T_i^0 \le (\hat{Q}_{40})^{(8)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[ (a_{40})^{(8)} G_{41}(s_{(40)}) - \left( (a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[ (a_{41})^{(8)} G_{40}(s_{(40)}) - \left( (a'_{41})^{(8)} + (a''_{41})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[ (a_{42})^{(8)} G_{41}(s_{(40)}) - \left( (a'_{42})^{(8)} + (a''_{42})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{42}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^0 + \int_0^t \left[ (b_{40})^{(8)} T_{41}(s_{(40)}) - \left( (b'_{40})^{(8)} - (b''_{40})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^0 + \int_0^t \left[ (b_{41})^{(8)} T_{40}(s_{(40)}) - \left( (b'_{41})^{(8)} - (b''_{41})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\overline{\mathrm{T}}_{42}(\mathrm{t}) = \mathrm{T}_{42}^{0} + \int_0^t \left[ (b_{42})^{(8)} T_{41}(s_{(40)}) - \left( (b'_{42})^{(8)} - (b''_{42})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{42}(s_{(40)}) \right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** 166
Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy A

$G_i(0) = G_i^0$ , $T_i(0) = T_i^0$ , $G_i^0 \le (\hat{P}_{44})^{(9)}$ , $T_i^0 \le (\hat{Q}_{44})^{(9)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[ (a_{44})^{(9)} G_{45}(s_{(44)}) - \left( (a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45}(s_{(44)}), s_{(44)})\right) G_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}(s_{(44)}), s_{(44)})\right) G_{46}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{44}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{46}(t) = T_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{46}(s_{(44)})\right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}}\right)\right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left(e^{(\hat{M}_{13})^{(1)} t} - 1\right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\hat{M}_{13})^{(1)} t} \le \frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{P}_{13})^{(1)} + G_{14}^0\right) e^{\left(-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0}\right)} + (\hat{P}_{13})^{(1)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \le G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}}\right)\right] ds_{(16)} = \quad 169$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1\right)$$

From which it follows that 170

$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \le \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left[\left((\hat{P}_{16})^{(2)} + G_{17}^0\right) e^{\left(-\frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0}\right)} + (\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t) \le G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} \left(G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} s_{(20)}}\right)\right] ds_{(20)} =$$

$$\left(1 + (a_{20})^{(3)} t\right) G_{21}^0 + \frac{(a_{20})^{(3)} (\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left(e^{(\hat{M}_{20})^{(3)} t} - 1\right)$$

From which it follows that 172

$$(G_{20}(t)-G_{20}^{0})e^{-(\hat{M}_{20})^{(3)}t}\leq\frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left[((\hat{P}_{20})^{(3)}+G_{21}^{0})e^{\left(-\frac{(\hat{P}_{20})^{(3)}+G_{21}^{0}}{G_{21}^{0}}\right)}+(\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 173

$$G_{24}(t)\leq G_{24}^{0}+\int_{0}^{t}\left[(a_{24})^{(4)}\left(G_{25}^{0}+(\hat{P}_{24})^{(4)}e^{(\hat{M}_{24})^{(4)}s_{(24)}}\right)\right]ds_{(24)}=$$

$$\left(1+(a_{24})^{(4)}t\right)G_{25}^{0}+\frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left(e^{(\hat{M}_{24})^{(4)}t}-1\right)$$

From which it follows that 174

$$(G_{24}(t)-G_{24}^{0})e^{-(\hat{M}_{24})^{(4)}t}\leq\frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left[((\hat{P}_{24})^{(4)}+G_{25}^{0})e^{\left(-\frac{(\hat{P}_{24})^{(4)}+G_{25}^{0}}{G_{25}^{0}}\right)}+(\hat{P}_{24})^{(4)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t)\leq G_{28}^{0}+\int_{0}^{t}\left[(a_{28})^{(5)}\left(G_{29}^{0}+(\hat{P}_{28})^{(5)}e^{(\hat{M}_{28})^{(5)}s_{(28)}}\right)\right]ds_{(28)}=$$

$$\left(1+(a_{28})^{(5)}t\right)G_{29}^{0}+\frac{(a_{28})^{(5)}(\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left(e^{(\hat{M}_{28})^{(5)}t}-1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t}\leq\frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t)\leq G_{32}^{0}+\int_{0}^{t}\left[(a_{32})^{(6)}\left(G_{33}^{0}+(\hat{P}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\right)\right]ds_{(32)}=$$

$$\left(1+(a_{32})^{(6)}t\right)G_{33}^{0}+\frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left(e^{(\hat{M}_{32})^{(6)}t}-1\right)$$

From which it follows that 177

$$(G_{32}(t)-G_{32}^{0})e^{-(\hat{M}_{32})^{(6)}t}\leq\frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left[((\hat{P}_{32})^{(6)}+G_{33}^{0})e^{\left(-\frac{(\hat{P}_{32})^{(6)}+G_{33}^{0}}{G_{33}^{0}}\right)}+(\hat{P}_{32})^{(6)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

(r) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t) \le G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} \left(G_{37}^0 + (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} s_{(36)}}\right)\right] ds_{(36)} =$$
$$\left(1 + (a_{36})^{(7)} t\right) G_{37}^0 + \frac{(a_{36})^{(7)} (\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left(e^{(\hat{M}_{36})^{(7)} t} - 1\right)$$

From which it follows that

$$(G_{36}(t) - G_{36}^0) e^{-(\hat{M}_{36})^{(7)} t} \le \frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left[((\hat{P}_{36})^{(7)} + G_{37}^0) e^{\left(-\frac{(\hat{P}_{36})^{(7)} + G_{37}^0}{G_{37}^0}\right)} + (\hat{P}_{36})^{(7)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t) \le G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} \left(G_{41}^0 + (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} s_{(40)}}\right)\right] ds_{(40)} = \quad 180$$
$$\left(1 + (a_{40})^{(8)} t\right) G_{41}^0 + \frac{(a_{40})^{(8)} (\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}} \left(e^{(\hat{M}_{40})^{(8)} t} - 1\right)$$

From which it follows that 181

$$(G_{40}(t) - G_{40}^0) e^{-(\hat{M}_{40})^{(8)} t} \le \frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}} \left[((\hat{P}_{40})^{(8)} + G_{41}^0) e^{\left(-\frac{(\hat{P}_{40})^{(8)} + G_{41}^0}{G_{41}^0}\right)} + (\hat{P}_{40})^{(8)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 8

Analogous inequalities hold also for $G_{41}, G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t) \le G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} \left(G_{45}^0 + (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} s_{(44)}}\right)\right] ds_{(44)} =$$
$$\left(1 + (a_{44})^{(9)} t\right) G_{45}^0 + \frac{(a_{44})^{(9)} (\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left(e^{(\hat{M}_{44})^{(9)} t} - 1\right)$$

From which it follows that

$$(G_{44}(t) - G_{44}^0) e^{-(\hat{M}_{44})^{(9)} t} \le \frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left[((\hat{P}_{44})^{(9)} + G_{45}^0) e^{\left(-\frac{(\hat{P}_{44})^{(9)} + G_{45}^0}{G_{45}^0}\right)} + (\hat{P}_{44})^{(9)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 9

Analogous inequalities hold also for $G_{45}, G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$ and to choose 182

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[(\hat{P}_{13})^{(1)}+\left((\hat{P}_{13})^{(1)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{13})^{(1)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{13})^{(1)} \quad 183$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[\left((\hat{Q}_{13})^{(1)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{13})^{(1)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{13})^{(1)}\right]\leq(\hat{Q}_{13})^{(1)} \quad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)},T^{(1)}\right),\left(G^{(2)},T^{(2)}\right)\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{13})^{(1)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{13})^{(1)}t}\}$$

Indeed if we denote

**Definition of** $\tilde{G},\tilde{T}:\left(\tilde{G},\tilde{T}\right)=\mathcal{A}^{(1)}(G,T)$

It results

$$\left|\tilde{G}_{13}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{13})^{(1)}\left|G_{14}^{(1)}-G_{14}^{(2)}\right|e^{-(\hat{M}_{13})^{(1)}s_{(13)}}e^{(\hat{M}_{13})^{(1)}s_{(13)}}ds_{(13)}+$$

$$\int_0^t\{(a'_{13})^{(1)}\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\hat{M}_{13})^{(1)}s_{(13)}}e^{-(\hat{M}_{13})^{(1)}s_{(13)}}+$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\hat{M}_{13})^{(1)}s_{(13)}}e^{(\hat{M}_{13})^{(1)}s_{(13)}}+$$

$$G_{13}^{(2)}\left|(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)-(a''_{13})^{(1)}\left(T_{14}^{(2)},s_{(13)}\right)\right|\ e^{-(\hat{M}_{13})^{(1)}s_{(13)}}e^{(\hat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|G^{(1)}-G^{(2)}\right|e^{-(\hat{M}_{13})^{(1)}t}\leq \quad 186$$
$$\frac{1}{(\hat{M}_{13})^{(1)}}\left((a_{13})^{(1)}+(a'_{13})^{(1)}+(\hat{A}_{13})^{(1)}+(\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}\right)d\left(\left(G^{(1)},T^{(1)};\ G^{(2)},T^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

**Remark 1:** The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\hat{P}_{13})^{(1)}e^{(\hat{M}_{13})^{(1)}t}\ and\ (\hat{Q}_{13})^{(1)}e^{(\hat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i = 13,14,15$ depend only on $T_{14}$ and respectively on $G(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any $t$ where $G_i(t) = 0\ and\ T_i(t) = 0$

From 19 to 24 it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(1)} - (a_i'')^{(1)}(T_{14}(s_{(13)}), s_{(13)})\} ds_{(13)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(1)} t)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2\ and\ \left((\widehat{M}_{13})^{(1)}\right)_3$ : 187

**Remark 3**: if $G_{13}$ is bounded, the same property have also $G_{14}\ and\ G_{15}$ . indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a_{14}')^{(1)} G_{14}$ and by integrating

$$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)} \left((\widehat{M}_{13})^{(1)}\right)_1 / (a_{14}')^{(1)}$$

In the same way , one can obtain

$$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)} \left((\widehat{M}_{13})^{(1)}\right)_2 / (a_{15}')^{(1)}$$

If $G_{14}\ or\ G_{15}$ is bounded, the same property follows for $G_{13}$ , $G_{15}$ and $G_{13}$ , $G_{14}$ respectively.

**Remark 4:** If $G_{13}$ *is* bounded, from below, the same property holds for $G_{14}\ and\ G_{15}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{14}$ is bounded from below. 188

**Remark 5:** If $T_{13}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}(G(t), t)) = (b_{14}')^{(1)}$ then $T_{14} \to \infty$. 189

**Definition of** $(m)^{(1)}$ and $\varepsilon_1$ :

Indeed let $t_1$ be so that for $t > t_1$

$$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t), t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right)$, $t = log\frac{2}{\varepsilon_1}$ By taking now $\varepsilon_1$ sufficiently small one sees that $T_{14}$ is unbounded.
The same property holds for $T_{15}$ if $\lim_{t\to\infty}(b_{15}'')^{(1)}(G(t), t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}$ , $\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}\ and\ (\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[(\widehat{P}_{16})^{(2)}+\left((\hat{P}_{16})^{(2)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{16})^{(2)}\right]\leq(\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

**Definition of** $\widetilde{G_{19}},\widetilde{T_{19}}$ : $\left(\widetilde{G_{19}},\widetilde{T_{19}}\right)=\mathcal{A}^{(2)}(G_{19},T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{16})^{(2)}\left|G_{17}^{(1)}-G_{17}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)}+$$

$$\int_0^t\{(a'_{16})^{(2)}\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$G_{16}^{(2)}\left|(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)-(a''_{16})^{(2)}\left(T_{17}^{(2)},s_{(16)}\right)\right|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0,t]$ 197

From the hypotheses it follows

$$\left|(G_{19})^{(1)}-(G_{19})^{(2)}\right|\mathrm{e}^{-(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}\leq$$
$$\frac{1}{(\widehat{\mathrm{M}}_{16})^{(2)}}\left((a_{16})^{(2)}+(a'_{16})^{(2)}+(\widehat{\mathrm{A}}_{16})^{(2)}+(\widehat{\mathrm{P}}_{16})^{(2)}(\hat{k}_{16})^{(2)}\right)\mathrm{d}\left(((G_{19})^{(1)},(T_{19})^{(1)};\ (G_{19})^{(2)},(T_{19})^{(2)})\right)$$

And analogous inequalities for $G_i$ and $T_i$. Taking into account the hypothesis the result follows 198

**Remark 6:** The fact that we supposed $(a''_{16})^{(2)}$ and $(b''_{16})^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{\mathrm{P}}_{16})^{(2)}\mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}$ and $(\widehat{\mathrm{Q}}_{16})^{(2)}\mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i=16,17,18$ depend only on $\mathrm{T}_{17}$ and respectively on

$(G_{19})$(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 7:** There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$ 200

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(2)} - (a_i'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})\} ds_{(16)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(2)}t)} > 0 \quad \text{for } t > 0$$

**Definition of** $\left((\widehat{M}_{16})^{(2)}\right)_1$, $\left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$: 201

**Remark 8:** if $G_{16}$ is bounded, the same property have also $G_{17}$ and $G_{18}$. indeed if

$G_{16} < (\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt} \leq \left((\widehat{M}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)} G_{17}$ and by integrating

$$G_{17} \leq \left((\widehat{M}_{16})^{(2)}\right)_2 = G_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1 / (a_{17}')^{(2)}$$

In the same way , one can obtain

$$G_{18} \leq \left((\widehat{M}_{16})^{(2)}\right)_3 = G_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2 / (a_{18}')^{(2)}$$

If $G_{17}$ or $G_{18}$ is bounded, the same property follows for $G_{16}$, $G_{18}$ and $G_{16}$, $G_{17}$ respectively.

**Remark 9:** If $G_{16}$ is bounded, from below, the same property holds for $G_{17}$ and $G_{18}$. The proof is analogous with the preceding one. An analogous property is true if $G_{17}$ is bounded from below. 202

**Remark 10:** If $T_{16}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t), t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$. 203

**Definition of** $(m)^{(2)}$ and $\varepsilon_2$ :

Indeed let $t_2$ be so that for $t > t_2$

$$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right)$, $t = \log\frac{2}{\varepsilon_2}$ By taking now $\varepsilon_2$ sufficiently small one sees that $T_{17}$ is unbounded. 205
The same property holds for $T_{18}$ if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t), t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[(\widehat{P}_{20})^{(3)}+\left((\hat{P}_{20})^{(3)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{20})^{(3)}\right]\leq(\hat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself 210

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)},(T_{23})^{(1)}),((G_{23})^{(2)},(T_{23})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

**Definition of** $\widetilde{G_{23}},\widetilde{T_{23}}:(\widetilde{(G_{23})},\widetilde{(T_{23})})=\mathcal{A}^{(3)}((G_{23}),(T_{23}))$

It results 213

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{20})^{(3)}\left|G_{21}^{(1)}-G_{21}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\,ds_{(20)}+$$

$$\int_0^t\{(a'_{20})^{(3)}\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$G_{20}^{(2)}\left|(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)},s_{(20)}\right)\right|\ e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\}ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses it follows

$$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}t}\leq \quad 214$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+(a'_{20})^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\widehat{k}_{20})^{(3)}\right)d\left(((G_{23})^{(1)},(T_{23})^{(1)};\ (G_{23})^{(2)},(T_{23})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

**Remark 11:** The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}\ and\ (\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i=20,21,22$ depend only on $\mathrm{T}_{21}$ and respectively on

$(G_{23})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 12:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 216

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(3)} - (a_i'')^{(3)}(T_{21}(s_{(20)}), s_{(20)})\} ds_{(20)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(3)} t)} > 0$ for t > 0

**Definition of** $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2\ and\ \left((\widehat{M}_{20})^{(3)}\right)_3$ : 217

**Remark 13:** if $G_{20}$ is bounded, the same property have also $G_{21}\ and\ G_{22}$ . indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a_{21}')^{(3)} G_{21}$ and by integrating

$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1 / (a_{21}')^{(3)}$

In the same way , one can obtain

$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2 / (a_{22}')^{(3)}$

If $G_{21}\ or\ G_{22}$ is bounded, the same property follows for $G_{20}$ , $G_{22}$ and $G_{20}$ , $G_{21}$ respectively.

**Remark 14:** If $G_{20}\ \ is$ bounded, from below, the same property holds for $G_{21}\ and\ G_{22}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{21}$ is bounded from below. 218

**Remark 15:** If $T_{20}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t), t\right)) = (b_{21}')^{(3)}$ then $T_{21} \to \infty$. 219

**Definition of** $(m)^{(3)}$ and $\varepsilon_3$ :

Indeed let $t_3$ be so that for $t > t_3$

$(b_{21})^{(3)} - (b_i'')^{(3)}\left((G_{23})(t), t\right) < \varepsilon_3, T_{20}\,(t) > (m)^{(3)}$

Then $\frac{dT_{21}}{dt} \geq (a_{21})^{(3)}(m)^{(3)} - \varepsilon_3 T_{21}$ which leads to 220

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t} = \frac{1}{2}$ it results

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right),\ \ t = log\,\frac{2}{\varepsilon_3}$ By taking now $\varepsilon_3$ sufficiently small one sees that $T_{21}$ is unbounded.
The same property holds for $T_{22}$ if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t), t\right) = (b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}$ , $\frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}} < 1$ and to choose 221

$(\widehat{P}_{24})^{(4)}$ and $(\widehat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[(\widehat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \tag{222}$$

$$\frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \tag{223}$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself (224)

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric (225)

$$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{24})^{(4)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{27}}),(\widetilde{T_{27}})$ : $\left((\widetilde{G_{27}}),(\widetilde{T_{27}})\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{24})^{(4)}\left|G_{25}^{(1)}-G_{25}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\,ds_{(24)}+$$

$$\int_0^t\{(a'_{24})^{(4)}\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$G_{24}^{(2)}\left|(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)-(a''_{24})^{(4)}\left(T_{25}^{(2)},s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\}ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}t}\leq \tag{226}$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)}+(a'_{24})^{(4)}+(\widehat{A}_{24})^{(4)}+(\widehat{P}_{24})^{(4)}(\hat{k}_{24})^{(4)}\right)d\left(((G_{27})^{(1)},(T_{27})^{(1)};\ (G_{27})^{(2)},(T_{27})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 16:** The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ *and* $(\widehat{Q}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_+$. (227)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i=24,25,26$ depend only on $T_{25}$ and respectively on

$(G_{27})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 17:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 228

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(4)} - (a_i'')^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(4)} t)} > 0$ for t > 0

**Definition of** $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2\ and\ \left((\widehat{M}_{24})^{(4)}\right)_3$ : 229

**Remark 18:** if $G_{24}$ is bounded, the same property have also $G_{25}\ and\ G_{26}$ . indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a_{25}')^{(4)} G_{25}$ and by integrating

$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1 / (a_{25}')^{(4)}$

In the same way , one can obtain

$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2 / (a_{26}')^{(4)}$

If $G_{25}\ or\ G_{26}$ is bounded, the same property follows for $G_{24}$ , $G_{26}$ and $G_{24}$ , $G_{25}$ respectively.

**Remark 19:** If $G_{24}$ *is* bounded, from below, the same property holds for $G_{25}\ and\ G_{26}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{25}$ is bounded from below. 230

**Remark 20:** If $T_{24}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}\ ((G_{27})(t), t)) = (b_{25}')^{(4)}$ then $T_{25}\ \to \infty$. 231

**Definition of** $(m)^{(4)}$ and $\varepsilon_4$ :

Indeed let $t_4$ be so that for $t\ > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t), t) < \varepsilon_4, T_{24}\,(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25}\ \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25}\ \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_4}$ By taking now $\varepsilon_4$ sufficiently small one sees that $T_{25}$ is unbounded.
The same property holds for $T_{26}$ if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t), t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for $G_{29}$ , $G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}$ , $\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)}+\left((\hat{P}_{28})^{(5)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{28})^{(5)}\right]\leq(\hat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$$d\left(((G_{31})^{(1)},(T_{31})^{(1)}),((G_{31})^{(2)},(T_{31})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\widehat{M}_{28})^{(5)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\widehat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{31}}),(\widetilde{T_{31}})$ : $\left((\widetilde{G_{31}}),(\widetilde{T_{31}})\right)=\mathcal{A}^{(5)}\left((G_{31}),(T_{31})\right)$

It results

$$|\tilde{G}_{28}^{(1)}-\tilde{G}_i^{(2)}|\leq\int_0^t(a_{28})^{(5)}|G_{29}^{(1)}-G_{29}^{(2)}|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\,ds_{(28)}+$$

$$\int_0^t\{(a'_{28})^{(5)}|G_{28}^{(1)}-G_{28}^{(2)}|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$(a''_{28})^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)|G_{28}^{(1)}-G_{28}^{(2)}|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$G_{28}^{(2)}|(a''_{28})^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)-(a''_{28})^{(5)}\left(T_{29}^{(2)},s_{(28)}\right)|\;e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\}ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on it follows

$$|(G_{31})^{(1)}-(G_{31})^{(2)}|e^{-(\widehat{M}_{28})^{(5)}t}\leq \quad 237$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)}+(a'_{28})^{(5)}+(\widehat{A}_{28})^{(5)}+(\hat{P}_{28})^{(5)}(\hat{k}_{28})^{(5)}\right)d\left(((G_{31})^{(1)},(T_{31})^{(1)};\;(G_{31})^{(2)},(T_{31})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 21:** The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\hat{P}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ *and* $(\hat{Q}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ respectively of $\mathbb{R}_+$. 238

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on $T_{29}$ and respectively on $(G_{31})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 22:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 239

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(5)} - (a_i'')^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(5)} t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2\ and\ \left((\widehat{M}_{28})^{(5)}\right)_3$ : 240

**Remark 23:** if $G_{28}$ is bounded, the same property have also $G_{29}\ and\ G_{30}$ . indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a_{29}')^{(5)} G_{29}$ and by integrating

$$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1 / (a_{29}')^{(5)}$$

In the same way , one can obtain

$$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2 / (a_{30}')^{(5)}$$

If $G_{29}\ or\ G_{30}$ is bounded, the same property follows for $G_{28}$ , $G_{30}$ and $G_{28}$ , $G_{29}$ respectively.

**Remark 24:** If $G_{28}$ *is* bounded, from below, the same property holds for $G_{29}\ and\ G_{30}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{29}$ is bounded from below. 241

**Remark 25:** If $T_{28}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}\,((G_{31})(t), t)) = (b_{29}')^{(5)}$ then $T_{29} \to \infty$. 242

**Definition of** $(m)^{(5)}$ and $\varepsilon_5$ :

Indeed let $t_5$ be so that for $t > t_5$

$$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t), t) < \varepsilon_5, T_{28}\,(t) > (m)^{(5)}$$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_5}$ By taking now $\varepsilon_5$ sufficiently small one sees that $T_{29}$ is unbounded.
The same property holds for $T_{30}$ if $\lim_{t\to\infty}(b_{30}'')^{(5)}\left((G_{31})(t), t\right) = (b_{30}')^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{33}$ , $G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}$ , $\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}} < 1$ and to choose 244

$(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[(\hat{P}_{32})^{(6)}+\left((\hat{P}_{32})^{(6)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{32})^{(6)} \tag{245}$$

$$\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[\left((\hat{Q}_{32})^{(6)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{32})^{(6)}\right]\leq(\hat{Q}_{32})^{(6)} \tag{246}$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric (247)

$$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\hat{M}_{32})^{(6)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\hat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{35}}),(\widetilde{T_{35}})$ : $\left((\widetilde{G_{35}}),(\widetilde{T_{35}})\right)=\mathcal{A}^{(6)}\left((G_{35}),(T_{35})\right)$

It results

$$|\tilde{G}_{32}^{(1)}-\tilde{G}_i^{(2)}|\leq\int_0^t(a_{32})^{(6)}|G_{33}^{(1)}-G_{33}^{(2)}|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\,ds_{(32)}+$$

$$\int_0^t\{(a'_{32})^{(6)}|G_{32}^{(1)}-G_{32}^{(2)}|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)|G_{32}^{(1)}-G_{32}^{(2)}|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$G_{32}^{(2)}|(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)-(a''_{32})^{(6)}\left(T_{33}^{(2)},s_{(32)}\right)|\;e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\}ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$|(G_{35})^{(1)}-(G_{35})^{(2)}|e^{-(\widehat{M}_{32})^{(6)}t}\leq \tag{248}$$
$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)}+(a'_{32})^{(6)}+(\widehat{A}_{32})^{(6)}+(\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}\right)d\left(((G_{35})^{(1)},(T_{35})^{(1)};\;(G_{35})^{(2)},(T_{35})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 26:** The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\hat{P}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ *and* $(\hat{Q}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ respectively of $\mathbb{R}_+$. (249)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i = 32,33,34$ depend only on $T_{33}$ and respectively on $(G_{35})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 27:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 250

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(6)} - (a_i'')^{(6)}(T_{33}(s_{(32)}), s_{(32)})\} ds_{(32)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(6)} t\right)} > 0$ for t > 0

**Definition of** $\left((\widehat{M}_{32})^{(6)}\right)_1, \left((\widehat{M}_{32})^{(6)}\right)_2\ and\ \left((\widehat{M}_{32})^{(6)}\right)_3$ : 251

**Remark 28:** if $G_{32}$ is bounded, the same property have also $G_{33}\ and\ G_{34}$ . indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a_{33}')^{(6)} G_{33}$ and by integrating

$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_1 / (a_{33}')^{(6)}$

In the same way , one can obtain

$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_2 / (a_{34}')^{(6)}$

If $G_{33}\ or\ G_{34}$ is bounded, the same property follows for $G_{32}$ , $G_{34}$ and $G_{32}$ , $G_{33}$ respectively.

**Remark 29:** If $G_{32}$ *is* bounded, from below, the same property holds for $G_{33}\ and\ G_{34}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{33}$ is bounded from below. 252

**Remark 30:** If $T_{32}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}\ ((G_{35})(t), t)) = (b_{33}')^{(6)}$ then $T_{33} \to \infty$. 253

**Definition of** $(m)^{(6)}$ and $\varepsilon_6$ :

Indeed let $t_6$ be so that for $t > t_6$

$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t), t\right) < \varepsilon_6, T_{32}\,(t) > (m)^{(6)}$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_6}$ By taking now $\varepsilon_6$ sufficiently small one sees that $T_{33}$ is unbounded.
The same property holds for $T_{34}$ if $\lim_{t\to\infty}(b_{34}'')^{(6)}\left((G_{35})(t), t(t), t\right) = (b_{34}')^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{37}, G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}$ , $\frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$ and to choose

$(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[(\hat{P}_{36})^{(7)}+\left((\hat{P}_{36})^{(7)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{36})^{(7)} \tag{256}$$

$$\frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[\left((\hat{Q}_{36})^{(7)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{36})^{(7)}\right]\leq(\hat{Q}_{36})^{(7)} \tag{257}$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself
The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric (258)

$$d\left(((G_{39})^{(1)},(T_{39})^{(1)}),((G_{39})^{(2)},(T_{39})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\hat{M}_{36})^{(7)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\hat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

**Definition of** $\widetilde{(G_{39})},\widetilde{(T_{39})}:(\widetilde{(G_{39})},\widetilde{(T_{39})})=\mathcal{A}^{(7)}((G_{39}),(T_{39}))$
It results

$$|\tilde{G}_{36}^{(1)}-\tilde{G}_i^{(2)}|\leq\int_0^t(a_{36})^{(7)}|G_{37}^{(1)}-G_{37}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\,ds_{(36)}+$$
$$\int_0^t\{(a'_{36})^{(7)}|G_{36}^{(1)}-G_{36}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$(a''_{36})^{(7)}(T_{37}^{(1)},s_{(36)})|G_{36}^{(1)}-G_{36}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$G_{36}^{(2)}|(a''_{36})^{(7)}(T_{37}^{(1)},s_{(36)})-(a''_{36})^{(7)}(T_{37}^{(2)},s_{(36)})|\;e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\}ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on it follows

$$|(G_{39})^{(1)}-(G_{39})^{(2)}|e^{-(\widehat{M}_{36})^{(7)}t}\leq \tag{259}$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}}\left((a_{36})^{(7)}+(a'_{36})^{(7)}+(\widehat{A}_{36})^{(7)}+(\widehat{P}_{36})^{(7)}(\widehat{k}_{36})^{(7)}\right)d\left(((G_{39})^{(1)},(T_{39})^{(1)};\;(G_{39})^{(2)},(T_{39})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 31:** The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ *and* $(\widehat{Q}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ respectively of $\mathbb{R}_+$. (260)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(7)}$ and $(b_i'')^{(7)}, i = 36,37,38$ depend only on $T_{37}$ and respectively on $(G_{39})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 32:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 261

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(7)} - (a_i'')^{(7)}(T_{37}(s_{(36)}), s_{(36)})\} ds_{(36)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(7)} t\right)} > 0$ for t > 0

**Definition of** $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2\ and\ \left((\widehat{M}_{36})^{(7)}\right)_3$ : 262

**Remark 33:** if $G_{36}$ is bounded, the same property have also $G_{37}\ and\ G_{38}$ . indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a_{37}')^{(7)} G_{37}$ and by integrating

$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1 / (a_{37}')^{(7)}$

In the same way , one can obtain

$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2 / (a_{38}')^{(7)}$

If $G_{37}\ or\ G_{38}$ is bounded, the same property follows for $G_{36}$ , $G_{38}$ and $G_{36}$ , $G_{37}$ respectively.

**Remark 34:** If $G_{36}$ *is* bounded, from below, the same property holds for $G_{37}\ and\ G_{38}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{37}$ is bounded from below. 263

**Remark 35:** If $T_{36}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(7)}\ ((G_{39})(t), t)) = (b_{37}')^{(7)}$ then $T_{37} \to \infty$. 264

**Definition of** $(m)^{(7)}$ and $\varepsilon_7$ :

Indeed let $t_7$ be so that for $t > t_7$

$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t), t) < \varepsilon_7, T_{36}\,(t) > (m)^{(7)}$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_7}$ By taking now $\varepsilon_7$ sufficiently small one sees that $T_{37}$ is unbounded.
The same property holds for $T_{38}$ if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t), t\right) = (b_{38}')^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}$ , $\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}} < 1$ and to choose $(\widehat{P}_{40})^{(8)}$ and $(\widehat{Q}_{40})^{(8)}$ large to have 266

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[(\widehat{P}_{40})^{(8)} + \left((\hat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{40})^{(8)} \quad 267$$

$$\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{40})^{(8)}\right] \leq (\hat{Q}_{40})^{(8)} \quad 268$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)}, (T_{43})^{(1)}), ((G_{43})^{(2)}, (T_{43})^{(2)})\right) = \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

**Definition of** $\widetilde{(G_{43})}, \widetilde{(T_{43})}$ : $\left(\widetilde{(G_{43})}, \widetilde{(T_{43})}\right) = \mathcal{A}^{(8)}((G_{43}), (T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{40})^{(8)}\left|G_{41}^{(1)} - G_{41}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\,ds_{(40)} +$$
$$\int_0^t\{(a'_{40})^{(8)}\left|G_{40}^{(1)} - G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{-(\widehat{M}_{40})^{(8)}s_{(40)}} +$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right)\left|G_{40}^{(1)} - G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}} +$$
$$G_{40}^{(2)}\left|(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right) - (a''_{40})^{(8)}\left(T_{41}^{(2)}, s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\}ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0, t]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)} - (G_{43})^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}t} \leq \quad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)} + (a'_{40})^{(8)} + (\widehat{A}_{40})^{(8)} + (\widehat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}\right)d\left(((G_{43})^{(1)}, (T_{43})^{(1)};\ (G_{43})^{(2)}, (T_{43})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

**Remark 36:** The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate 274

condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ $and$ $(\widehat{Q}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i = 40,41,42$ depend only on $T_{41}$ and respectively on $(G_{43})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 37** There does not exist any $t$ where $G_i(t) = 0\ and\ T_i(t) = 0$ 275

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(8)} - (a_i'')^{(8)}(T_{41}(s_{(40)}), s_{(40)})\} ds_{(40)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(8)}t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{40})^{(8)}\right)_1, \left((\widehat{M}_{40})^{(8)}\right)_2$ $and$ $\left((\widehat{M}_{40})^{(8)}\right)_3$ : 276

**Remark 38:** if $G_{40}$ is bounded, the same property have also $G_{41}\ and\ G_{42}$ . indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a_{41}')^{(8)}G_{41}$ and by integrating

$$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1/(a_{41}')^{(8)}$$

In the same way , one can obtain

$$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2/(a_{42}')^{(8)}$$

If $G_{41}\ or\ G_{42}$ is bounded, the same property follows for $G_{40}$ , $G_{42}$ and $G_{40}$ , $G_{41}$ respectively.

**Remark 39:** If $G_{40}$ $is$ bounded, from below, the same property holds for $G_{41}\ and\ G_{42}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{41}$ is bounded from below. 277

**Remark 40:** If $T_{40}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}((G_{43})(t), t)) = (b_{41}')^{(8)}$ then $T_{41} \to \infty$. 278

**Definition of** $(m)^{(8)}$ and $\varepsilon_8$ :

Indeed let $t_8$ be so that for $t > t_8$

$$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t), t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right)$, $t = log\frac{2}{\varepsilon_8}$ By taking now $\varepsilon_8$ sufficiently small one sees that $T_{41}$ is unbounded.
The same property holds for $T_{42}$ if $\lim_{t\to\infty}(b''_{42})^{(8)}\left((G_{43})(t), t(t), t\right) = (b'_{42})^{(8)}$



It is now sufficient to take $\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}$ , $\frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$ and to choose $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ large to have

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}\left[(\hat{P}_{44})^{(9)} + \left((\hat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}}\left[\left((\hat{Q}_{44})^{(9)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{44})^{(9)}\right] \leq (\hat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)}, (T_{47})^{(1)}), ((G_{47})^{(2)}, (T_{47})^{(2)})\right) =$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{44})^{(9)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

**Definition of** $\widetilde{(G_{47})}, \widetilde{(T_{47})} : \left(\widetilde{(G_{47})}, \widetilde{(T_{47})}\right) = \mathcal{A}^{(9)}\left((G_{47}), (T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{44})^{(9)}\left|G_{45}^{(1)} - G_{45}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\,ds_{(44)} +$$
$$\int_0^t\{(a'_{44})^{(9)}\left|G_{44}^{(1)} - G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{-(\widehat{M}_{44})^{(9)}s_{(44)}} +$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right)\left|G_{44}^{(1)} - G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}} +$$
$$G_{44}^{(2)}\left|(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right) - (a''_{44})^{(9)}\left(T_{45}^{(2)}, s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\}ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)} - G^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}t} \leq$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)} + (a'_{44})^{(9)} + (\widehat{A}_{44})^{(9)} + (\widehat{P}_{44})^{(9)}(\hat{k}_{44})^{(9)}\right)d\left(((G_{47})^{(1)}, (T_{47})^{(1)};\ (G_{47})^{(2)}, (T_{47})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis (39,35,36) the result follows

**Remark 41:** The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by

$(\widehat{P}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ *and* $(\widehat{Q}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i = 44{,}45{,}46$ depend only on $T_{45}$ and respectively on $(G_{47})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 42:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$

From 99 to 44 it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t\{(a_i')^{(9)}-(a_i'')^{(9)}(T_{45}(s_{(44)}),s_{(44)})\}ds_{(44)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(9)}t)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{44})^{(9)}\right)_1, \left((\widehat{M}_{44})^{(9)}\right)_2$ *and* $\left((\widehat{M}_{44})^{(9)}\right)_3$ :

**Remark 43:** if $G_{44}$ is bounded, the same property have also $G_{45}\ and\ G_{46}$ . indeed if

$G_{44} < (\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt} \leq \left((\widehat{M}_{44})^{(9)}\right)_1 - (a_{45}')^{(9)}G_{45}$ and by integrating

$$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_1/(a_{45}')^{(9)}$$

In the same way , one can obtain

$$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_2/(a_{46}')^{(9)}$$

If $G_{45}\ or\ G_{46}$ is bounded, the same property follows for $G_{44}$ , $G_{46}$ and $G_{44}$ , $G_{45}$ respectively.

**Remark 44:** If $G_{44}$ *is* bounded, from below, the same property holds for $G_{45}\ and\ G_{46}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{45}$ is bounded from below.

**Remark 45:** If $T_{44}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t),t\right) = (b_{45}')^{(9)}$ then $T_{45} \to \infty$.

**Definition of** $(m)^{(9)}$ and $\varepsilon_9$ :

Indeed let $t_9$ be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t),t\right) < \varepsilon_9, T_{44}\,(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right), \quad t = log\frac{2}{\varepsilon_9}$ By taking now $\varepsilon_9$ sufficiently small one sees that $T_{45}$ is unbounded.
The same property holds for $T_{46}$ if $\lim_{t\to\infty}(b_{46}'')^{(9)}\left((G_{47})(t),t\right) = (b_{46}')^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

**Behavior of the solutions of equation** 280

**Theorem** If we denote and define

**Definition of** $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ :

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)} \le -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14}, t) + (a''_{14})^{(1)}(T_{14}, t) \le -(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)} \le -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G, t) - (b''_{14})^{(1)}(G, t) \le -(\tau_1)^{(1)}$

**Definition of** $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$ : 281

By $(\nu_1)^{(1)} > 0$, $(\nu_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$, $(u_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

**Definition of** $(\bar{\nu}_1)^{(1)},, (\bar{\nu}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$ : 282

By $(\bar{\nu}_1)^{(1)} > 0$, $(\bar{\nu}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$, $(\bar{u}_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

**Definition of** $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (\nu_0)^{(1)}$ :- 283

If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by
$(m_2)^{(1)} = (\nu_0)^{(1)}, (m_1)^{(1)} = (\nu_1)^{(1)}$, $if\ (\nu_0)^{(1)} < (\nu_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}, if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)}$,

and $\boxed{(\nu_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)}$, $if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)}$, $if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)}$,

and $\boxed{(u_0)^{(1)} = \frac{T^0_{13}}{T^0_{14}}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$

are defined

Then the solution of global equations satisfies the inequalities 285

$G^0_{13}e^{\left((S_1)^{(1)} - (p_{13})^{(1)}\right)t} \le G_{13}(t) \le G^0_{13}e^{(S_1)^{(1)}t}$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)}-(p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)}t}$$

$$\left(\frac{(a_{15})^{(1)}G_{13}^0}{(m_1)^{(1)}((S_1)^{(1)}-(p_{13})^{(1)}-(S_2)^{(1)})}\left[e^{((S_1)^{(1)}-(p_{13})^{(1)})t} - e^{-(S_2)^{(1)}t}\right] + G_{15}^0 e^{-(S_2)^{(1)}t} \le G_{15}(t) \le \right. \quad (286)$$

$$\left. \frac{(a_{15})^{(1)}G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)}-(a'_{15})^{(1)})}[e^{(S_1)^{(1)}t} - e^{-(a'_{15})^{(1)}t}] + G_{15}^0 e^{-(a'_{15})^{(1)}t}\right)$$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)}t} \le T_{13}(t) \le T_{13}^0 e^{((R_1)^{(1)}+(r_{13})^{(1)})t}} \quad (287)$$

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)}t} \le T_{13}(t) \le \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)}+(r_{13})^{(1)})t} \quad (288)$$

$$\frac{(b_{15})^{(1)}T_{13}^0}{(\mu_1)^{(1)}((R_1)^{(1)}-(b'_{15})^{(1)})}\left[e^{(R_1)^{(1)}t} - e^{-(b'_{15})^{(1)}t}\right] + T_{15}^0 e^{-(b'_{15})^{(1)}t} \le T_{15}(t) \le \quad (289)$$

$$\frac{(a_{15})^{(1)}T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)}+(r_{13})^{(1)}+(R_2)^{(1)})}\left[e^{((R_1)^{(1)}+(r_{13})^{(1)})t} - e^{-(R_2)^{(1)}t}\right] + T_{15}^0 e^{-(R_2)^{(1)}t}$$

**Definition of** $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:- (290)

Where $(S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$

$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$

$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$

$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$

**Behavior of the solutions of equation** (291)

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ : (292)

$(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ four constants satisfying

$$-(\sigma_2)^{(2)} \le -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(T_{17}, t) + (a''_{17})^{(2)}(T_{17}, t) \le -(\sigma_1)^{(2)} \quad (293)$$

$$-(\tau_2)^{(2)} \le -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}((G_{19}), t) - (b''_{17})^{(2)}((G_{19}), t) \le -(\tau_1)^{(2)} \quad (294)$$

**Definition of** $(\nu_1)^{(2)}, (\nu_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$ : (295)

By $(\nu_1)^{(2)} > 0$, $(\nu_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0$, $(u_2)^{(2)} < 0$ the roots (296)

of the equations $(a_{17})^{(2)}(\nu^{(2)})^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ (297)

and $(b_{14})^{(2)}(u^{(2)})^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and (298)

**Definition of** $(\bar{\nu}_1)^{(2)}, , (\bar{\nu}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$ : (299)

By $(\bar{\nu}_1)^{(2)} > 0$, $(\bar{\nu}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0$, $(\bar{u}_2)^{(2)} < 0$ the (300)

roots of the equations $(a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 301

and $(b_{17})^{(2)}\left(u^{(2)}\right)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 302

**Definition of** $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ :- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, ***if*** $(u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$, ***if*** $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)}$, ***if*** $(\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)}t}$$

$(p_i)^{(2)}$ is defined by equation

$$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)}t} \quad 311$$

$$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)}t}\right] + G_{18}^0 e^{-(S_2)^{(2)}t} \le G_{18}(t) \le \right. \quad 312$$

$$\left. \frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)}t} - e^{-(a'_{18})^{(2)}t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)}t}\right)$$

$$\boxed{T_{16}^0 e^{(R_1)^{(2)}t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}} \quad 313$$

$$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)}t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t} \quad 314$$

$$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)}t} - e^{-(b'_{18})^{(2)}t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)}t} \le T_{18}(t) \le \quad 315$$

$$\frac{(a_{18})^{(2)}T_{16}^{0}}{(\mu_2)^{(2)}((R_1)^{(2)}+(r_{16})^{(2)}+(R_2)^{(2)})}\left[e^{((R_1)^{(2)}+(r_{16})^{(2)})t}-e^{-(R_2)^{(2)}t}\right]+T_{18}^{0}e^{-(R_2)^{(2)}t}$$

**Definition of** $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$$

$$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)}$$ 318

$$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$$

**Behavior of the solutions** 319

**Theorem 3:** If we denote and define

**Definition of** $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ :

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}\big((G_{23}), t\big) \le -(\tau_1)^{(3)}$$

**Definition of** $(v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$ : 320

By $(v_1)^{(3)} > 0$, $(v_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{v}_1)^{(3)} > 0$, $(\bar{v}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

**Definition of** $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ :- 321

If we define $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (v_0)^{(3)}, (m_1)^{(3)} = (v_1)^{(3)}$, ***if*** $(v_0)^{(3)} < (v_1)^{(3)}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (\bar{v}_1)^{(3)}$, ***if*** $(v_1)^{(3)} < (v_0)^{(3)} < (\bar{v}_1)^{(3)}$,

and $\boxed{(v_0)^{(3)} = \frac{G_{20}^{0}}{G_{21}^{0}}}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_0)^{(3)}$, ***if*** $(\bar{v}_1)^{(3)} < (v_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)},$ ***if*** $(u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$ , ***if*** $(u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)},$ and $\boxed{(u_0)^{(3)} = \frac{T_{20}^0}{T_{21}^0}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)},$ ***if*** $(\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G_{20}^0 e^{(S_1)^{(3)} t}$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)} t} \quad 323$$

$$\left(\frac{(a_{22})^{(3)} G_{20}^0}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\left[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)} t}\right] + G_{22}^0 e^{-(S_2)^{(3)} t} \le G_{22}(t) \le \right. \quad 324$$

$$\left. \frac{(a_{22})^{(3)} G_{20}^0}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}\left[e^{(S_1)^{(3)} t} - e^{-(a'_{22})^{(3)} t}\right] + G_{22}^0 e^{-(a'_{22})^{(3)} t}\right)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t}} \quad 325$$

$$\frac{1}{(\mu_1)^{(3)}} T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}} T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)} T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)} - (b'_{22})^{(3)})}\left[e^{(R_1)^{(3)} t} - e^{-(b'_{22})^{(3)} t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)} t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)} T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)} + (r_{20})^{(3)} + (R_2)^{(3)})}\left[e^{((R_1)^{(3)} + (r_{20})^{(3)})t} - e^{-(R_2)^{(3)} t}\right] + T_{22}^0 e^{-(R_2)^{(3)} t}$$

**Definition of** $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

**Behavior of the solutions of equation**

**Theorem:** If we denote and define

**Definition of** $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ :

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$

$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}((G_{27}), t) - (b''_{25})^{(4)}((G_{27}), t) \le -(\tau_1)^{(4)}$

**Definition of** $(v_1)^{(4)}, (v_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, v^{(4)}, u^{(4)}$ : 329

By $(v_1)^{(4)} > 0$ , $(v_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$ , $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_1)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

**Definition of** $(\bar{v}_1)^{(4)},, (\bar{v}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$ : 330

By $(\bar{v}_1)^{(4)} > 0$ , $(\bar{v}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$ , $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

**Definition of** $(m_1)^{(4)}$ , $(m_2)^{(4)}$ , $(\mu_1)^{(4)}, (\mu_2)^{(4)}, (\nu_0)^{(4)}$ :-

If we define $(m_1)^{(4)}$ , $(m_2)^{(4)}$ , $(\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (v_0)^{(4)}, (m_1)^{(4)} = (v_1)^{(4)}$, $\boldsymbol{if}$ $(v_0)^{(4)} < (v_1)^{(4)}$

$(m_2)^{(4)} = (v_1)^{(4)}, (m_1)^{(4)} = (\bar{v}_1)^{(4)}$ , $\boldsymbol{if}$ $(v_4)^{(4)} < (v_0)^{(4)} < (\bar{v}_1)^{(4)}$,
and $\boxed{(v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (v_4)^{(4)}, (m_1)^{(4)} = (v_0)^{(4)}$, $\boldsymbol{if}$ $(\bar{v}_4)^{(4)} < (v_0)^{(4)}$

and analogously

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$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)}$, $\boldsymbol{if}$ $(u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}$ , $\boldsymbol{if}$ $(u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)}$,
and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}, \boldsymbol{if}$ $(\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

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$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)}t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)}t}$$

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$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)}t}\right] + G_{26}^0 e^{-(S_2)^{(4)}t} \le G_{26}(t) \le\right.$$ 334

*(a26)4G240(m2)4(S1)4−(a26′)4e(S1)4t−e−(a26′)4t+ G260e−(a26′)4t*

$$\boxed{T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}}T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}}T_{24}^0 e^{((R_1)^{(4)}+(r_{24})^{(4)})t} \quad 335$$

$$\frac{(b_{26})^{(4)}T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)}-(b'_{26})^{(4)})}\left[e^{(R_1)^{(4)}t} - e^{-(b'_{26})^{(4)}t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)}t} \le T_{26}(t) \le \quad 336$$

$$\frac{(a_{26})^{(4)}T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)}+(r_{24})^{(4)}+(R_2)^{(4)})}\left[e^{((R_1)^{(4)}+(r_{24})^{(4)})t} - e^{-(R_2)^{(4)}t}\right] + T_{26}^0 e^{-(R_2)^{(4)}t}$$

**Definition of** $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

**Behavior of the solutions of equation** 338

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ :

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

**Definition of** $(v_1)^{(5)}, (v_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, v^{(5)}, u^{(5)}$ : 339

By $(v_1)^{(5)} > 0$, $(v_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$, $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(v^{(5)}\big)^2 + (\sigma_1)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

**Definition of** $(\bar{v}_1)^{(5)}, , (\bar{v}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$ : 340

By $(\bar{v}_1)^{(5)} > 0$, $(\bar{v}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$, $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(v^{(5)}\big)^2 + (\sigma_2)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

**Definition of** $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (v_0)^{(5)}$ :-

If we define $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (v_0)^{(5)}, (m_1)^{(5)} = (v_1)^{(5)}$, ***if*** $(v_0)^{(5)} < (v_1)^{(5)}$

$(m_2)^{(5)} = (v_1)^{(5)}, (m_1)^{(5)} = (\bar{v}_1)^{(5)}$, ***if*** $(v_1)^{(5)} < (v_0)^{(5)} < (\bar{v}_1)^{(5)}$,
and $\boxed{(v_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)},\ \boldsymbol{if}\ (u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}, \boldsymbol{if}\ (u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)},$

and $\boxed{(u_0)^{(5)} = \frac{T_{28}^0}{T_{29}^0}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}, \boldsymbol{if}\ (\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{28}(t) \le G_{28}^0 e^{(S_1)^{(5)} t}$

where $(p_i)^{(5)}$ is defined by equation

$\frac{1}{(m_5)^{(5)}} G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}} G_{28}^0 e^{(S_1)^{(5)} t}$ 343

$\left(\frac{(a_{30})^{(5)} G_{28}^0}{(m_1)^{(5)}((S_1)^{(5)} - (p_{28})^{(5)} - (S_2)^{(5)})}\left[e^{((S_1)^{(5)} - (p_{28})^{(5)})t} - e^{-(S_2)^{(5)} t}\right] + G_{30}^0 e^{-(S_2)^{(5)} t} \le G_{30}(t) \le\right.$ 344

*(a30)5G280(m2)5(S1)5−(a30′)5e(S1)5t−e−(a30′)5t+ G300e−(a30′)5t*

$\boxed{T_{28}^0 e^{(R_1)^{(5)} t} \le T_{28}(t) \le T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}}$ 345

$\frac{1}{(\mu_1)^{(5)}} T_{28}^0 e^{(R_1)^{(5)} t} \le T_{28}(t) \le \frac{1}{(\mu_2)^{(5)}} T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}$ 346

$\frac{(b_{30})^{(5)} T_{28}^0}{(\mu_1)^{(5)}((R_1)^{(5)} - (b'_{30})^{(5)})}\left[e^{(R_1)^{(5)} t} - e^{-(b'_{30})^{(5)} t}\right] + T_{30}^0 e^{-(b'_{30})^{(5)} t} \le T_{30}(t) \le$ 347

$\frac{(a_{30})^{(5)} T_{28}^0}{(\mu_2)^{(5)}((R_1)^{(5)} + (r_{28})^{(5)} + (R_2)^{(5)})}\left[e^{((R_1)^{(5)} + (r_{28})^{(5)})t} - e^{-(R_2)^{(5)} t}\right] + T_{30}^0 e^{-(R_2)^{(5)} t}$

**Definition of** $(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$:- 348

Where $(S_1)^{(5)} = (a_{28})^{(5)}(m_2)^{(5)} - (a'_{28})^{(5)}$

$(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}$

$(R_1)^{(5)} = (b_{28})^{(5)}(\mu_2)^{(5)} - (b'_{28})^{(5)}$

$(R_2)^{(5)} = (b'_{30})^{(5)} - (r_{30})^{(5)}$

**Behavior of the solutions of equation** 349

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ :

$(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)} \le -(a'_{32})^{(6)} + (a'_{33})^{(6)} - (a''_{32})^{(6)}(T_{33}, t) + (a''_{33})^{(6)}(T_{33}, t) \le -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)} \le -(b'_{32})^{(6)} + (b'_{33})^{(6)} - (b''_{32})^{(6)}\big((G_{35}), t\big) - (b''_{33})^{(6)}\big((G_{35}), t\big) \le -(\tau_1)^{(6)}$$

**Definition of** $(\nu_1)^{(6)}, (\nu_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, \nu^{(6)}, u^{(6)}$ : 350

By $(\nu_1)^{(6)} > 0$, $(\nu_2)^{(6)} < 0$ and respectively $(u_1)^{(6)} > 0$, $(u_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(6)},, (\bar{\nu}_2)^{(6)}, (\bar{u}_1)^{(6)}, (\bar{u}_2)^{(6)}$ : 351

By $(\bar{\nu}_1)^{(6)} > 0$, $(\bar{\nu}_2)^{(6)} < 0$ and respectively $(\bar{u}_1)^{(6)} > 0$, $(\bar{u}_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$

**Definition of** $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}, (\nu_0)^{(6)}$ :-

If we define $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}$ by

$$(m_2)^{(6)} = (\nu_0)^{(6)}, (m_1)^{(6)} = (\nu_1)^{(6)}, \ \boldsymbol{if}\ (\nu_0)^{(6)} < (\nu_1)^{(6)}$$

$$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}, \boldsymbol{if}\ (\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)},$$

and $\boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}}$

$$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$$

and analogously 352

$$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}, \ \boldsymbol{if}\ (u_0)^{(6)} < (u_1)^{(6)}$$

$$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}, \boldsymbol{if}\ (u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)},$$

and $\boxed{(u_0)^{(6)} = \frac{T^0_{32}}{T^0_{33}}}$

$$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}, \boldsymbol{if}\ (\bar{u}_1)^{(6)} < (u_0)^{(6)} \text{ where } (u_1)^{(6)}, (\bar{u}_1)^{(6)}$$

Then the solution of global equations satisfies the inequalities 353

$$G^0_{32}e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{32}(t) \le G^0_{32}e^{(S_1)^{(6)}t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}}G^0_{32}e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{33}(t) \le \frac{1}{(m_2)^{(6)}}G^0_{32}e^{(S_1)^{(6)}t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)}G^0_{32}}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})}\left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)}t}\right] + G^0_{34}e^{-(S_2)^{(6)}t} \le G_{34}(t) \le \right. \quad 355$$

$$\frac{(a_{34})^{(6)}G_{32}^{0}}{(m_2)^{(6)}((S_1)^{(6)}-(a'_{34})^{(6)})}\left[e^{(S_1)^{(6)}t}-e^{-(a'_{34})^{(6)}t}\right]+\ G_{34}^{0}e^{-(a'_{34})^{(6)}t}\Big)$$

$$\boxed{T_{32}^{0}e^{(R_1)^{(6)}t}\le T_{32}(t)\le T_{32}^{0}e^{((R_1)^{(6)}+(r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}}T_{32}^{0}e^{(R_1)^{(6)}t}\le T_{32}(t)\le\frac{1}{(\mu_2)^{(6)}}T_{32}^{0}e^{((R_1)^{(6)}+(r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)}T_{32}^{0}}{(\mu_1)^{(6)}((R_1)^{(6)}-(b'_{34})^{(6)})}\left[e^{(R_1)^{(6)}t}-e^{-(b'_{34})^{(6)}t}\right]+T_{34}^{0}e^{-(b'_{34})^{(6)}t}\le T_{34}(t)\le \quad 358$$

$$\frac{(a_{34})^{(6)}T_{32}^{0}}{(\mu_2)^{(6)}((R_1)^{(6)}+(r_{32})^{(6)}+(R_2)^{(6)})}\left[e^{((R_1)^{(6)}+(r_{32})^{(6)})t}-e^{-(R_2)^{(6)}t}\right]+T_{34}^{0}e^{-(R_2)^{(6)}t}$$

**Definition of** $(S_1)^{(6)},(S_2)^{(6)},(R_1)^{(6)},(R_2)^{(6)}$:- 359

Where $(S_1)^{(6)}=(a_{32})^{(6)}(m_2)^{(6)}-(a'_{32})^{(6)}$

$(S_2)^{(6)}=(a_{34})^{(6)}-(p_{34})^{(6)}$

$(R_1)^{(6)}=(b_{32})^{(6)}(\mu_2)^{(6)}-(b'_{32})^{(6)}$

$(R_2)^{(6)}=(b'_{34})^{(6)}-(r_{34})^{(6)}$

**Behavior of the solutions of equation**

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(7)},(\sigma_2)^{(7)},(\tau_1)^{(7)},(\tau_2)^{(7)}$:

$(\sigma_1)^{(7)},(\sigma_2)^{(7)},(\tau_1)^{(7)},(\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)}\le-(a'_{36})^{(7)}+(a'_{37})^{(7)}-(a''_{36})^{(7)}(T_{37},t)+(a''_{37})^{(7)}(T_{37},t)\le-(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)}\le-(b'_{36})^{(7)}+(b'_{37})^{(7)}-(b''_{36})^{(7)}\big((G_{39}),t\big)-(b''_{37})^{(7)}\big((G_{39}),t\big)\le-(\tau_1)^{(7)}$$

**Definition of** $(v_1)^{(7)},(v_2)^{(7)},(u_1)^{(7)},(u_2)^{(7)},v^{(7)},u^{(7)}$: 361

By $(v_1)^{(7)}>0,(v_2)^{(7)}<0$ and respectively $(u_1)^{(7)}>0,(u_2)^{(7)}<0$ the roots of the equations $(a_{37})^{(7)}\big(v^{(7)}\big)^2+(\sigma_1)^{(7)}v^{(7)}-(a_{36})^{(7)}=0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2+(\tau_1)^{(7)}u^{(7)}-(b_{36})^{(7)}=0$ and

**Definition of** $(\bar{v}_1)^{(7)},,(\bar{v}_2)^{(7)},(\bar{u}_1)^{(7)},(\bar{u}_2)^{(7)}$: 362

By $(\bar{v}_1)^{(7)}>0,(\bar{v}_2)^{(7)}<0$ and respectively $(\bar{u}_1)^{(7)}>0,(\bar{u}_2)^{(7)}<0$ the roots of the equations $(a_{37})^{(7)}\big(v^{(7)}\big)^2+(\sigma_2)^{(7)}v^{(7)}-(a_{36})^{(7)}=0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2+(\tau_2)^{(7)}u^{(7)}-(b_{36})^{(7)}=0$

**Definition of** $(m_1)^{(7)},(m_2)^{(7)},(\mu_1)^{(7)},(\mu_2)^{(7)},(v_0)^{(7)}$:-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$$(m_2)^{(7)} = (v_0)^{(7)}, (m_1)^{(7)} = (v_1)^{(7)}, \ \boldsymbol{if}\ (v_0)^{(7)} < (v_1)^{(7)}$$

$$(m_2)^{(7)} = (v_1)^{(7)}, (m_1)^{(7)} = (\bar{v}_1)^{(7)}, \boldsymbol{if}\ (v_1)^{(7)} < (v_0)^{(7)} < (\bar{v}_1)^{(7)},$$

and $\boxed{(v_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$

$$(m_2)^{(7)} = (v_1)^{(7)}, (m_1)^{(7)} = (v_0)^{(7)}, \ \boldsymbol{if}\ (\bar{v}_1)^{(7)} < (v_0)^{(7)}$$

and analogously 363

$$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}, \ \boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$$

and $\boxed{(u_0)^{(7)} = \frac{T_{36}^0}{T_{37}^0}}$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)} \text{ where } (u_1)^{(7)}, (\bar{u}_1)^{(7)}$$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^0 e^{(S_1)^{(7)} t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)} t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)}((S_1)^{(7)} - (p_{36})^{(7)} - (S_2)^{(7)})}\left[e^{((S_1)^{(7)} - (p_{36})^{(7)})t} - e^{-(S_2)^{(7)} t}\right] + G_{38}^0 e^{-(S_2)^{(7)} t} \le G_{38}(t) \le \right. \quad 366$$

$$\left. \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)}((S_1)^{(7)} - (a'_{38})^{(7)})}\left[e^{(S_1)^{(7)} t} - e^{-(a'_{38})^{(7)} t}\right] + G_{38}^0 e^{-(a'_{38})^{(7)} t}\right)$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t} \quad 368$$

$$\frac{(b_{38})^{(7)} T_{36}^0}{(\mu_1)^{(7)}((R_1)^{(7)} - (b'_{38})^{(7)})}\left[e^{(R_1)^{(7)} t} - e^{-(b'_{38})^{(7)} t}\right] + T_{38}^0 e^{-(b'_{38})^{(7)} t} \le T_{38}(t) \le \quad 369$$

$$\frac{(a_{38})^{(7)} T_{36}^0}{(\mu_2)^{(7)}((R_1)^{(7)} + (r_{36})^{(7)} + (R_2)^{(7)})}\left[e^{((R_1)^{(7)} + (r_{36})^{(7)})t} - e^{-(R_2)^{(7)} t}\right] + T_{38}^0 e^{-(R_2)^{(7)} t}$$

**Definition of** $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$

$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$

$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$

**Behavior of the solutions of equation** 371

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ :

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$-(\sigma_2)^{(8)} \le -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \le -(\sigma_1)^{(8)}$

$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\big((G_{43}), t\big) - (b''_{41})^{(8)}\big((G_{43}), t\big) \le -(\tau_1)^{(8)}$

**Definition of** $(v_1)^{(8)}, (v_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, v^{(8)}, u^{(8)}$ : 372

By $(v_1)^{(8)} > 0, (v_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0, (u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\big(v^{(8)}\big)^2 + (\sigma_1)^{(8)}v^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

**Definition of** $(\bar{v}_1)^{(8)}, , (\bar{v}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$ :

By $(\bar{v}_1)^{(8)} > 0, (\bar{v}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0, (\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\big(v^{(8)}\big)^2 + (\sigma_2)^{(8)}v^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

**Definition of** $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}, (v_0)^{(8)}$ :-

If we define $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$(m_2)^{(8)} = (v_0)^{(8)}, (m_1)^{(8)} = (v_1)^{(8)}$, ***if*** $(v_0)^{(8)} < (v_1)^{(8)}$

$(m_2)^{(8)} = (v_1)^{(8)}, (m_1)^{(8)} = (\bar{v}_1)^{(8)}$, ***if*** $(v_1)^{(8)} < (v_0)^{(8)} < (\bar{v}_1)^{(8)}$,
and $\boxed{(v_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\nu_0)^{(8)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(8)} < (\nu_0)^{(8)}$

and analogously 374

$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)},\ \boldsymbol{if}\ (u_0)^{(8)} < (u_1)^{(8)}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}, \boldsymbol{if}\ (u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)},$

and $\boxed{(u_0)^{(8)} = \frac{T_{40}^0}{T_{41}^0}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}, \boldsymbol{if}\ (\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^0 e^{(S_1)^{(8)} t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}} G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}} G_{40}^0 e^{(S_1)^{(8)} t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)} G_{40}^0}{(m_1)^{(8)}((S_1)^{(8)} - (p_{40})^{(8)} - (S_2)^{(8)})}\left[e^{((S_1)^{(8)} - (p_{40})^{(8)})t} - e^{-(S_2)^{(8)} t}\right] + G_{42}^0 e^{-(S_2)^{(8)} t} \le G_{42}(t) \le \right. \quad 377$$

$$\left. \frac{(a_{42})^{(8)} G_{40}^0}{(m_2)^{(8)}((S_1)^{(8)} - (a'_{42})^{(8)})}\left[e^{(S_1)^{(8)} t} - e^{-(a'_{42})^{(8)} t}\right] + G_{42}^0 e^{-(a'_{42})^{(8)} t}\right)$$

$$\boxed{T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}} T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le \frac{1}{(\mu_2)^{(8)}} T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)} T_{40}^0}{(\mu_1)^{(8)}((R_1)^{(8)} - (b'_{42})^{(8)})}\left[e^{(R_1)^{(8)} t} - e^{-(b'_{42})^{(8)} t}\right] + T_{42}^0 e^{-(b'_{42})^{(8)} t} \le T_{42}(t) \le \quad 380$$

$$\frac{(a_{42})^{(8)} T_{40}^0}{(\mu_2)^{(8)}((R_1)^{(8)} + (r_{40})^{(8)} + (R_2)^{(8)})}\left[e^{((R_1)^{(8)} + (r_{40})^{(8)})t} - e^{-(R_2)^{(8)} t}\right] + T_{42}^0 e^{-(R_2)^{(8)} t}$$

**Definition of** $(S_1)^{(8)}, (S_2)^{(8)}, (R_1)^{(8)}, (R_2)^{(8)}$:- 381

Where $(S_1)^{(8)} = (a_{40})^{(8)}(m_2)^{(8)} - (a'_{40})^{(8)}$

$(S_2)^{(8)} = (a_{42})^{(8)} - (p_{42})^{(8)}$

$(R_1)^{(8)} = (b_{40})^{(8)}(\mu_2)^{(8)} - (b'_{40})^{(8)}$

$(R_2)^{(8)} = (b'_{42})^{(8)} - (r_{42})^{(8)}$

**Behavior of the solutions of equation 37 to 92** 382

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ :

$(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ four constants satisfying

$$-(\sigma_2)^{(9)} \le -(a'_{44})^{(9)} + (a'_{45})^{(9)} - (a''_{44})^{(9)}(T_{45}, t) + (a''_{45})^{(9)}(T_{45}, t) \le -(\sigma_1)^{(9)}$$

$$-(\tau_2)^{(9)} \le -(b'_{44})^{(9)} + (b'_{45})^{(9)} - (b''_{44})^{(9)}\big((G_{47}), t\big) - (b''_{45})^{(9)}\big((G_{47}), t\big) \le -(\tau_1)^{(9)}$$

**Definition of** $(\nu_1)^{(9)}, (\nu_2)^{(9)}, (u_1)^{(9)}, (u_2)^{(9)}, \nu^{(9)}, u^{(9)}$ :

By $(\nu_1)^{(9)} > 0$ , $(\nu_2)^{(9)} < 0$ and respectively $(u_1)^{(9)} > 0$ , $(u_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_1)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(9)}, , (\bar{\nu}_2)^{(9)}, (\bar{u}_1)^{(9)}, (\bar{u}_2)^{(9)}$ :

By $(\bar{\nu}_1)^{(9)} > 0$ , $(\bar{\nu}_2)^{(9)} < 0$ and respectively $(\bar{u}_1)^{(9)} > 0$ , $(\bar{u}_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_2)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$

**Definition of** $(m_1)^{(9)}, (m_2)^{(9)}, (\mu_1)^{(9)}, (\mu_2)^{(9)}, (\nu_0)^{(9)}$ :-

If we define $(m_1)^{(9)}, (m_2)^{(9)}, (\mu_1)^{(9)}, (\mu_2)^{(9)}$ by

$$(m_2)^{(9)} = (\nu_0)^{(9)}, (m_1)^{(9)} = (\nu_1)^{(9)}, \ \boldsymbol{if}\ (\nu_0)^{(9)} < (\nu_1)^{(9)}$$

$$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\bar{\nu}_1)^{(9)}, \boldsymbol{if}\ (\nu_1)^{(9)} < (\nu_0)^{(9)} < (\bar{\nu}_1)^{(9)},$$

and $\boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$

$$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\nu_0)^{(9)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(9)} < (\nu_0)^{(9)}$$

and analogously

$$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)}, \ \boldsymbol{if}\ (u_0)^{(9)} < (u_1)^{(9)}$$

$$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}, \boldsymbol{if}\ (u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)},$$

and $\boxed{(u_0)^{(9)} = \frac{T^0_{44}}{T^0_{45}}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}, \boldsymbol{if}\ (\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G^0_{44} e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{44}(t) \le G^0_{44} e^{(S_1)^{(9)} t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}}G_{44}^0 e^{((S_1)^{(9)}-(p_{44})^{(9)})t} \le G_{45}(t) \le \frac{1}{(m_2)^{(9)}}G_{44}^0 e^{(S_1)^{(9)}t}$$

$$\left(\frac{(a_{46})^{(9)}G_{44}^0}{(m_1)^{(9)}((S_1)^{(9)}-(p_{44})^{(9)}-(S_2)^{(9)})}\left[e^{((S_1)^{(9)}-(p_{44})^{(9)})t} - e^{-(S_2)^{(9)}t}\right] + G_{46}^0 e^{-(S_2)^{(9)}t} \le G_{46}(t) \le \right.$$

$$\left.\frac{(a_{46})^{(9)}G_{44}^0}{(m_2)^{(9)}((S_1)^{(9)}-(a'_{46})^{(9)})}[e^{(S_1)^{(9)}t} - e^{-(a'_{46})^{(9)}t}] + G_{46}^0 e^{-(a'_{46})^{(9)}t}\right)$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le T_{44}^0 e^{((R_1)^{(9)}+(r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}}T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le \frac{1}{(\mu_2)^{(9)}}T_{44}^0 e^{((R_1)^{(9)}+(r_{44})^{(9)})t}$$

$$\frac{(b_{46})^{(9)}T_{44}^0}{(\mu_1)^{(9)}((R_1)^{(9)}-(b'_{46})^{(9)})}\left[e^{(R_1)^{(9)}t} - e^{-(b'_{46})^{(9)}t}\right] + T_{46}^0 e^{-(b'_{46})^{(9)}t} \le T_{46}(t) \le$$

$$\frac{(a_{46})^{(9)}T_{44}^0}{(\mu_2)^{(9)}((R_1)^{(9)}+(r_{44})^{(9)}+(R_2)^{(9)})}\left[e^{((R_1)^{(9)}+(r_{44})^{(9)})t} - e^{-(R_2)^{(9)}t}\right] + T_{46}^0 e^{-(R_2)^{(9)}t}$$

**Definition of** $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$

**Proof :** From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14},t)\right) - (a''_{14})^{(1)}(T_{14},t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

**Definition of** $v^{(1)}$ :- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)}\right) \le \frac{dv^{(1)}}{dt} \le -\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)}\right)$$

From which one obtains

**Definition of** $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$ :-

For $0 < \boxed{(v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}} < (v_1)^{(1)} < (\bar{v}_1)^{(1)}$

$$v^{(1)}(t) \ge \frac{(v_1)^{(1)}+(C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(v_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(v_2)^{(1)}}}$$

it follows $(\nu_0)^{(1)} \leq \nu^{(1)}(t) \leq (\nu_1)^{(1)}$

In the same manner , we get 384

$$\nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{\nu}_1)^{(1)}-(\nu_0)^{(1)}}{(\nu_0)^{(1)}-(\bar{\nu}_2)^{(1)}}}$$

From which we deduce $(\nu_0)^{(1)} \leq \nu^{(1)}(t) \leq (\bar{\nu}_1)^{(1)}$

If $0 < (\nu_1)^{(1)} < (\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0} < (\bar{\nu}_1)^{(1)}$ we find like in the previous case, 385

$$(\nu_1)^{(1)} \leq \frac{(\nu_1)^{(1)}+(C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}} \leq \nu^{(1)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\bar{\nu}_1)^{(1)}$$

If $0 < (\nu_1)^{(1)} \leq (\bar{\nu}_1)^{(1)} \leq \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$ , we obtain 386

$$(\nu_1)^{(1)} \leq \nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\nu_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(1)}(t)$ :-

$$(m_2)^{(1)} \leq \nu^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{\nu^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(1)}(t)$ :-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{13})^{(1)} = (a''_{14})^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)}G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)}T_{14}(t)$ This is an important

consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

**Proof** : From global equations we obtain 387

$$\frac{d\nu^{(2)}}{dt} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right) - (a''_{17})^{(2)}(T_{17}, t)\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

**Definition of** $\nu^{(2)}$ :- $\boxed{\nu^{(2)} = \frac{G_{16}}{G_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \le \frac{d\nu^{(2)}}{dt} \le -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

**Definition of** $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$ :-

For $0 < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \ge \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}}{1 + (C)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(C)^{(2)} = \frac{(\nu_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \le \nu^{(2)}(t) \le (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \le \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \le \nu^{(2)}(t) \le (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \le \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}}{1 + (C)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}} \le \nu^{(2)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \le (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \le (\bar{\nu}_1)^{(2)} \le (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$ , we obtain 394

$$(\nu_1)^{(2)} \le \nu^{(2)}(t) \le \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \le (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(2)}(t)$ :- 395

$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}$, $\boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$

In a completely analogous way, we obtain 396

**Definition of** $u^{(2)}(t)$ :-

$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}$, $\boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :** 397

If $(a''_{16})^{(2)} = (a''_{17})^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)} G_{17}(t)$

Analogously if $(b''_{16})^{(2)} = (b''_{17})^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)} T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

**Proof** : From global equations we obtain 398

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right) - (a''_{21})^{(3)}(T_{21}, t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

**Definition of** $\nu^{(3)}$ :- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right) \leq \frac{d\nu^{(3)}}{dt} \leq -\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right)$$

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From which one obtains

For $0 < (\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\nu_1)^{(3)} < (\bar{\nu}_1)^{(3)}$

$$\nu^{(3)}(t) \geq \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(\nu_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\nu_2)^{(3)}}}$$

it follows $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\nu_1)^{(3)}$

In the same manner , we get 401

$$\nu^{(3)}(t) \le \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{\nu}_1)^{(3)}-(\nu_0)^{(3)}}{(\nu_0)^{(3)}-(\bar{\nu}_2)^{(3)}}}$$

**Definition of** $(\bar{\nu}_1)^{(3)}$ :-

From which we deduce $(\nu_0)^{(3)} \le \nu^{(3)}(t) \le (\bar{\nu}_1)^{(3)}$

If $0 < (\nu_1)^{(3)} < (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (\bar{\nu}_1)^{(3)}$ we find like in the previous case, 402

$$(\nu_1)^{(3)} \le \frac{(\nu_1)^{(3)}+(C)^{(3)}(\nu_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}} \le \nu^{(3)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \le (\bar{\nu}_1)^{(3)}$$

If $0 < (\nu_1)^{(3)} \le (\bar{\nu}_1)^{(3)} \le (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}$ , we obtain 403

$$(\nu_1)^{(3)} \le \nu^{(3)}(t) \le \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \le (\nu_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(3)}(t)$ :-

$$(m_2)^{(3)} \le \nu^{(3)}(t) \le (m_1)^{(3)}, \quad \boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(3)}(t)$ :-

$$(\mu_2)^{(3)} \le u^{(3)}(t) \le (\mu_1)^{(3)}, \quad \boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{20}'')^{(3)} = (a_{21}'')^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)}G_{21}(t)$

Analogously if $(b_{20}'')^{(3)} = (b_{21}'')^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)}T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

**Proof** : From global equations we obtain 404

$$\frac{dv^{(4)}}{dt} = (a_{24})^{(4)} - \left((a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right) - (a''_{25})^{(4)}(T_{25},t)v^{(4)} - (a_{25})^{(4)}v^{(4)}$$

**Definition of** $v^{(4)}$ :- $\boxed{v^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)}\right) \le \frac{dv^{(4)}}{dt} \le -\left((a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_4)^{(4)}v^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$ :-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \ge \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \le \nu^{(4)}(t) \le (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \le \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \le \nu^{(4)}(t) \le (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \le \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}}{1 + (C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}} \le \nu^{(4)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \le (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \le (\bar{\nu}_1)^{(4)} \le \boxed{(\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}}}$ , we obtain 407

$$(\nu_1)^{(4)} \le \nu^{(4)}(t) \le \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \le (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(4)}(t)$ :-

$$(m_2)^{(4)} \le \nu^{(4)}(t) \le (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(4)}(t)$ :-

$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}$, $\boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{24})^{(4)} = (a''_{25})^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)} G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b''_{24})^{(4)} = (b''_{25})^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)} T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

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**Proof** : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a'_{28})^{(5)} - (a'_{29})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right) - (a''_{29})^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

**Definition of** $\nu^{(5)}$ :- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$ :-

For $0 < \boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \le \frac{(\nu_1)^{(5)}+(C)^{(5)}(\nu_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)}-(\nu_2)^{(5)}\right)t\right]}}{1+(C)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)}-(\nu_2)^{(5)}\right)t\right]}} \le \nu^{(5)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \le (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \le (\bar{\nu}_1)^{(5)} \le \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain 411

$$(\nu_1)^{(5)} \le \nu^{(5)}(t) \le \frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \le (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(5)}(t)$ :-

$(m_2)^{(5)} \le \nu^{(5)}(t) \le (m_1)^{(5)}$, $\boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$

In a completely analogous way, we obtain

**Definition of** $u^{(5)}(t)$ :-

$(\mu_2)^{(5)} \le u^{(5)}(t) \le (\mu_1)^{(5)}$, $\boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a_{28}'')^{(5)} = (a_{29}'')^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)}G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b_{28}'')^{(5)} = (b_{29}'')^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)}T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

**Proof** : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a_{32}')^{(6)} - (a_{33}')^{(6)} + (a_{32}'')^{(6)}(T_{33},t)\right) - (a_{33}'')^{(6)}(T_{33},t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

**Definition of** $\nu^{(6)}$ :- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \le \frac{d\nu^{(6)}}{dt} \le -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$ :-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \geq \frac{(\nu_1)^{(6)}+(C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_0)^{(6)}\right)t\right]}}{1+(C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)}-(\nu_0)^{(6)}}{(\nu_0)^{(6)}-(\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\nu_1)^{(6)}$

In the same manner , we get 413

$$\nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{\nu}_1)^{(6)}-(\nu_0)^{(6)}}{(\nu_0)^{(6)}-(\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\bar{\nu}_1)^{(6)}$

If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 414

$$(\nu_1)^{(6)} \leq \frac{(\nu_1)^{(6)}+(C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_2)^{(6)}\right)t\right]}}{1+(C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_2)^{(6)}\right)t\right]}} \leq \nu^{(6)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\bar{\nu}_1)^{(6)}$$

If $0 < (\nu_1)^{(6)} \leq (\bar{\nu}_1)^{(6)} \leq \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$ , we obtain 415

$$(\nu_1)^{(6)} \leq \nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(6)}(t)$ :-

$$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}, \quad \boxed{\nu^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(6)}(t)$ :-

$$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}, \quad \boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)}G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)}T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

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**Proof** : From global equations we obtain

$$\frac{dv^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right) - (a''_{37})^{(7)}(T_{37},t)v^{(7)} - (a_{37})^{(7)}v^{(7)}$$

**Definition of** $v^{(7)}$ :- $\boxed{v^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_2)^{(7)}v^{(7)} - (a_{36})^{(7)}\right) \le \frac{dv^{(7)}}{dt} \le -\left((a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_1)^{(7)}v^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

**Definition of** $(\bar{v}_1)^{(7)}, (v_0)^{(7)}$ :-

For $0 < \boxed{(v_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}} < (v_1)^{(7)} < (\bar{v}_1)^{(7)}$

$$v^{(7)}(t) \ge \frac{(v_1)^{(7)} + (C)^{(7)}(v_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_0)^{(7)}\right)t\right]}}{1 + (C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(v_1)^{(7)} - (v_0)^{(7)}}{(v_0)^{(7)} - (v_2)^{(7)}}}$$

it follows $(v_0)^{(7)} \le v^{(7)}(t) \le (v_1)^{(7)}$

In the same manner , we get 417

$$v^{(7)}(t) \le \frac{(\bar{v}_1)^{(7)} + (\bar{C})^{(7)}(\bar{v}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{v}_1)^{(7)} - (v_0)^{(7)}}{(v_0)^{(7)} - (\bar{v}_2)^{(7)}}}$$

From which we deduce $(v_0)^{(7)} \le v^{(7)}(t) \le (\bar{v}_1)^{(7)}$

If $0 < (v_1)^{(7)} < (v_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}} < (\bar{v}_1)^{(7)}$ we find like in the previous case, 418

$$(v_1)^{(7)} \le \frac{(v_1)^{(7)} + (C)^{(7)}(v_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_2)^{(7)}\right)t\right]}}{1 + (C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_2)^{(7)}\right)t\right]}} \le v^{(7)}(t) \le$$

$$\frac{(\bar{v}_1)^{(7)} + (\bar{C})^{(7)}(\bar{v}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}} \le (\bar{v}_1)^{(7)}$$

If $0 < (v_1)^{(7)} \le (\bar{v}_1)^{(7)} \le \boxed{(v_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$ , we obtain 419

$$(v_1)^{(7)} \le v^{(7)}(t) \le \frac{(\bar{v}_1)^{(7)} + (\bar{C})^{(7)}(\bar{v}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}} \le (v_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $v^{(7)}(t)$ :-

$$(m_2)^{(7)} \leq v^{(7)}(t) \leq (m_1)^{(7)}, \quad \boxed{v^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(7)}(t)$ :- 420

$$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}, \quad \boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $v^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)} G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)} T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

**Proof** : From global equations we obtain 421

$$\frac{dv^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right) - (a''_{41})^{(8)}(T_{41}, t) v^{(8)} - (a_{41})^{(8)} v^{(8)}$$

**Definition of** $v^{(8)}$ :- $\boxed{v^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_2)^{(8)} v^{(8)} - (a_{40})^{(8)}\right) \leq \frac{dv^{(8)}}{dt} \leq -\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_1)^{(8)} v^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$ :-

$$\text{For } 0 < \boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$$

$$v^{(8)}(t) \geq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)}-(\nu_0)^{(8)}}{(\nu_0)^{(8)}-(\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \leq \frac{(\nu_1)^{(8)}+(C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}}{1+(C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}} \leq \nu^{(8)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \leq (\bar{\nu}_1)^{(8)} \leq \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$ , we obtain 424

$$(\nu_1)^{(8)} \leq \nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(8)}(t)$ :-

$$(m_2)^{(8)} \leq \nu^{(8)}(t) \leq (m_1)^{(8)}, \quad \boxed{\nu^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(8)}(t)$ :-

$$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}, \quad \boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $\nu^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)}G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then $(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)}T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

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**Proof** : From 99,20,44,22,23,44 we obtain A

$$\frac{dv^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right) - (a''_{45})^{(9)}(T_{45},t)v^{(9)} - (a_{45})^{(9)}v^{(9)}$$

**Definition of** $v^{(9)}$ :- $\boxed{v^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)}\right) \le \frac{dv^{(9)}}{dt} \le -\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_1)^{(9)}v^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$ :-

For $0 < \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$

$$v^{(9)}(t) \ge \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \le v^{(9)}(t) \le (\nu_9)^{(9)}$

In the same manner , we get

$$v^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \le v^{(9)}(t) \le (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \le \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}}{1 + (C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}} \le v^{(9)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \le (\bar{\nu}_1)^{(9)} \le \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$ , we obtain

$$(\nu_1)^{(9)} \le v^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $v^{(9)}(t)$ :-

$$(m_2)^{(9)} \le v^{(9)}(t) \le (m_1)^{(9)}, \quad \boxed{v^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(9)}(t)$ :-

$(\mu_2)^{(9)} \leq u^{(9)}(t) \leq (\mu_1)^{(9)}$, $\boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)} G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then
$(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)} T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following 425

**Theorem :** If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on $t$ , and the conditions with the notations

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0$ ,

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$

$with\ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a''_i)^{(2)} and\ (b''_i)^{(2)}$ are independent on t , and the conditions with the notations 426

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$ 427

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 428

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$ , 429

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 430

$with\ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a''_i)^{(3)} and\ (b''_i)^{(3)}$ are independent on $t$ , and the conditions with the notations 431

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$ ,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

**Theorem :** If $(a_i'')^{(4)} and\ (b_i'')^{(4)}$ are independent on $t$ , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$ ,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(5)} and\ (b_i'')^{(5)}$ are independent on $t$ , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$ ,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

**Theorem** If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on $t$ , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$ ,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(7)} and\ (b_i'')^{(7)}$ are independent on $t$ , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$ ,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(8)} and\ (b_i'')^{(8)}$ are independent on $t$ , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$ ,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(9)} and\ (b_i'')^{(9)}$ are independent on $t$ , and the conditions (with the notations 436 A
45,46,27,28)

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$ ,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0$ 437

$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0$ 438

$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0$ 439

$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0$ 440

$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ]T_{14} = 0$ 441

$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0$ 442

has a unique positive solution , which is an equilibrium solution for the system

$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0$ 443

$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0$ 444

$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0$ 445

$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\ ]T_{16} = 0$ 446

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\,]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\,]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\,]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\,]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\,]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\big((G_{27})\big)\,]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\big((G_{27})\big)\,]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\big((G_{27})\big)\,]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})\,]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})\,]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})\,]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})\,]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})\,]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})\,]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0 \quad 479$$

$$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0 \quad 480$$

$$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0 \quad 481$$

$$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0 \quad 482$$

$$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0 \quad 483$$

$$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0 \quad 484$$

$$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0 \quad 484\text{A}$$

$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$

$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$

$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$

$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$

$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$

**Proof:** 485

(a) Indeed the first two equations have a nontrivial solution $G_{13}, G_{14}$ if

$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$

**Proof:** 486

(r) Indeed the first two equations have a nontrivial solution $G_{16}, G_{17}$ if

$\mathrm{F}(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$

**Proof:** 487

(a) Indeed the first two equations have a nontrivial solution $G_{20}, G_{21}$ if

$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$

**Proof:** 488

(a) Indeed the first two equations have a nontrivial solution $G_{24}, G_{25}$ if

$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$

**Proof:** 489

(a) Indeed the first two equations have a nontrivial solution $G_{28}, G_{29}$ if

$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$

**Proof:** 490

(a) Indeed the first two equations have a nontrivial solution $G_{32}, G_{33}$ if

$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) +$

$(a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$

**Proof:** 491

(a) Indeed the first two equations have a nontrivial solution $G_{36}, G_{37}$ if

$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$

**Proof:** 492

(a) Indeed the first two equations have a nontrivial solution $G_{40}, G_{41}$ if

$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$

**Proof:** 492 A

(a) Indeed the first two equations have a nontrivial solution $G_{44}, G_{45}$ if

$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$

**Definition and uniqueness of** $T^*_{14}$ :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique $T^*_{14}$ for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

**Definition and uniqueness of** $T^*_{17}$ :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique $T^*_{17}$ for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]}$$ 495

**Definition and uniqueness of** $T^*_{21}$ :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique $T^*_{21}$ for which $f(T^*_{21}) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

**Definition and uniqueness of** $T^*_{25}$ :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that

there exists a unique $T_{25}^{*}$ for which $f(T_{25}^{*})=0$. With this value , we obtain from the three first equations

$$G_{24}=\frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}=\frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T_{25}^{*})]}$$

**Definition and uniqueness of** $T_{29}^{*}$ :- 498

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(5)}(T_{29})$ being increasing, it follows that there exists a unique $T_{29}^{*}$ for which $f(T_{29}^{*})=0$. With this value , we obtain from the three first equations

$$G_{28}=\frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}=\frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T_{29}^{*})]}$$

**Definition and uniqueness of** $T_{33}^{*}$ :- 499

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(6)}(T_{33})$ being increasing, it follows that there exists a unique $T_{33}^{*}$ for which $f(T_{33}^{*})=0$. With this value , we obtain from the three first equations

$$G_{32}=\frac{(a_{32})^{(6)}G_{33}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^{*})]} \quad , \quad G_{34}=\frac{(a_{34})^{(6)}G_{33}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^{*})]}$$

**Definition and uniqueness of** $T_{37}^{*}$ :- 500

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(7)}(T_{37})$ being increasing, it follows that there exists a unique $T_{37}^{*}$ for which $f(T_{37}^{*})=0$. With this value , we obtain from the three first equations

$$G_{36}=\frac{(a_{36})^{(7)}G_{37}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^{*})]} \quad , \quad G_{38}=\frac{(a_{38})^{(7)}G_{37}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^{*})]}$$

**Definition and uniqueness of** $T_{41}^{*}$ :- 501

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique $T_{41}^{*}$ for which $f(T_{41}^{*})=0$. With this value , we obtain from the three first equations

$$G_{40}=\frac{(a_{40})^{(8)}G_{41}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T_{41}^{*})]} \quad , \quad G_{42}=\frac{(a_{42})^{(8)}G_{41}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T_{41}^{*})]}$$

**Definition and uniqueness of** $T_{45}^{*}$ :- 501 A

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(9)}(T_{45})$ being increasing, it follows that there exists a unique $T_{45}^{*}$ for which $f(T_{45}^{*})=0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)}G_{45}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T^*_{45})]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)}G_{45}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T^*_{45})]}$$

By the same argument, the equations admit solutions $G_{13}, G_{14}$ if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right]+(b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{14}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{14}$ such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions $G_{16}, G_{17}$ if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right]+(b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ 504
is a decreasing function in $G_{17}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{14}$ such that $\varphi((G_{19})^*) = 0$

By the same argument, the equations admit solutions $G_{20}, G_{21}$ if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right]+(b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{21}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{21}$ such that $\varphi((G_{23})^*) = 0$

By the same argument, the equations admit solutions $G_{24}, G_{25}$ if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right]+(b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{25}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{25}$ such that $\varphi((G_{27})^*) = 0$

By the same argument, the equations admit solutions $G_{28}, G_{29}$ if 507

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right]+(b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{29}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that

there exists a unique $G_{29}^{*}$ such that $\varphi((G_{31})^{*}) = 0$

By the same argument, the equations admit solutions $G_{32}, G_{33}$ if 508
$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$

$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right] + (b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{33}$ taking into account the hypothesis $\varphi(0) > 0\,, \varphi(\infty) < 0$ it follows that there exists a unique $G_{33}^{*}$ such that $\varphi(G_{35}{}^{*}) = 0$

By the same argument, the equations admit solutions $G_{36}, G_{37}$ if 509
$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$
$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right] + (b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{37}$ taking into account the hypothesis $\varphi(0) > 0\,, \varphi(\infty) < 0$ it follows that there exists a unique $G_{37}^{*}$ such that $\varphi(G_{39}{}^{*}) = 0$

By the same argument, the equations admit solutions $G_{40}, G_{41}$ if 510
$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$
$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right] + (b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{41}$ taking into account the hypothesis $\varphi(0) > 0\,, \varphi(\infty) < 0$ it follows that there exists a unique $G_{41}^{*}$ such that $\varphi(G_{43}{}^{*}) = 0$

By the same argument, the equations 92,93 admit solutions $G_{44}, G_{45}$ if
$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$

$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right] + (b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{45}$ taking into account the hypothesis $\varphi(0) > 0\,, \varphi(\infty) < 0$ it follows that there exists a unique $G_{45}^{*}$ such that $\varphi((G_{47})^{*}) = 0$

Finally we obtain the unique solution 511

$G_{14}^{*}$ given by $\varphi(G^{*}) = 0\,, T_{14}^{*}$ given by $f(T_{14}^{*}) = 0$ and

$$G_{13}^{*} = \frac{(a_{13})^{(1)}G_{14}^{*}}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^{*})]} \quad , \quad G_{15}^{*} = \frac{(a_{15})^{(1)}G_{14}^{*}}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^{*})]}$$

$$T_{13}^{*} = \frac{(b_{13})^{(1)}T_{14}^{*}}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^{*})]} \quad , \quad T_{15}^{*} = \frac{(b_{15})^{(1)}T_{14}^{*}}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^{*})]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

$G_{17}^{*}$ given by $\varphi((G_{19})^{*}) = 0$ , $T_{17}^{*}$ given by $f(T_{17}^{*}) = 0$ and 512

$$G_{16}^{*} = \frac{(a_{16})^{(2)} G_{17}^{*}}{[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}^{*})]} \quad , \quad G_{18}^{*} = \frac{(a_{18})^{(2)} G_{17}^{*}}{[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}^{*})]} \qquad 513$$

$$T_{16}^{*} = \frac{(b_{16})^{(2)} T_{17}^{*}}{[(b'_{16})^{(2)} - (b''_{16})^{(2)}((G_{19})^{*})]} \quad , \quad T_{18}^{*} = \frac{(b_{18})^{(2)} T_{17}^{*}}{[(b'_{18})^{(2)} - (b''_{18})^{(2)}((G_{19})^{*})]} \qquad 514$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

$G_{21}^{*}$ given by $\varphi((G_{23})^{*}) = 0$ , $T_{21}^{*}$ given by $f(T_{21}^{*}) = 0$ and

$$G_{20}^{*} = \frac{(a_{20})^{(3)} G_{21}^{*}}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^{*})]} \quad , \quad G_{22}^{*} = \frac{(a_{22})^{(3)} G_{21}^{*}}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^{*})]}$$

$$T_{20}^{*} = \frac{(b_{20})^{(3)} T_{21}^{*}}{[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}{}^{*})]} \quad , \quad T_{22}^{*} = \frac{(b_{22})^{(3)} T_{21}^{*}}{[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}{}^{*})]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

$G_{25}^{*}$ given by $\varphi(G_{27}) = 0$ , $T_{25}^{*}$ given by $f(T_{25}^{*}) = 0$ and

$$G_{24}^{*} = \frac{(a_{24})^{(4)} G_{25}^{*}}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}^{*} = \frac{(a_{26})^{(4)} G_{25}^{*}}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^{*})]}$$

$$T_{24}^{*} = \frac{(b_{24})^{(4)} T_{25}^{*}}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^{*})]} \quad , \quad T_{26}^{*} = \frac{(b_{26})^{(4)} T_{25}^{*}}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^{*})]} \qquad 517$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

$G_{29}^{*}$ given by $\varphi((G_{31})^{*}) = 0$ , $T_{29}^{*}$ given by $f(T_{29}^{*}) = 0$ and

$$G_{28}^{*} = \frac{(a_{28})^{(5)} G_{29}^{*}}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}^{*} = \frac{(a_{30})^{(5)} G_{29}^{*}}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^{*})]}$$

$$T_{28}^{*} = \frac{(b_{28})^{(5)} T_{29}^{*}}{[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31})^{*})]} \quad , \quad T_{30}^{*} = \frac{(b_{30})^{(5)} T_{29}^{*}}{[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31})^{*})]} \qquad 519$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

$G_{33}^{*}$ given by $\varphi((G_{35})^{*}) = 0$ , $T_{33}^{*}$ given by $f(T_{33}^{*}) = 0$ and

$$G_{32}^{*} = \frac{(a_{32})^{(6)} G_{33}^{*}}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^{*})]} \quad , \quad G_{34}^{*} = \frac{(a_{34})^{(6)} G_{33}^{*}}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^{*})]}$$

$$T_{32}^{*} = \frac{(b_{32})^{(6)} T_{33}^{*}}{[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35})^{*})]} \quad , \quad T_{34}^{*} = \frac{(b_{34})^{(6)} T_{33}^{*}}{[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35})^{*})]} \qquad 521$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

$G_{37}^*$ given by $\varphi((G_{39})^*) = 0$ , $T_{37}^*$ given by $f(T_{37}^*) = 0$ and

$$G_{36}^* = \frac{(a_{36})^{(7)} G_{37}^*}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^*)]} \quad , \quad G_{38}^* = \frac{(a_{38})^{(7)} G_{37}^*}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^*)]}$$

$$T_{36}^* = \frac{(b_{36})^{(7)} T_{37}^*}{[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39})^*)]} \quad , \quad T_{38}^* = \frac{(b_{38})^{(7)} T_{37}^*}{[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39})^*)]}$$

Finally we obtain the unique solution 523

$G_{41}^*$ given by $\varphi((G_{43})^*) = 0$ , $T_{41}^*$ given by $f(T_{41}^*) = 0$ and

$$G_{40}^* = \frac{(a_{40})^{(8)} G_{41}^*}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42}^* = \frac{(a_{42})^{(8)} G_{41}^*}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

$$T_{40}^* = \frac{(b_{40})^{(8)} T_{41}^*}{[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43})^*)]} \quad , \quad T_{42}^* = \frac{(b_{42})^{(8)} T_{41}^*}{[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43})^*)]}$$

Finally we obtain the unique solution of 89 to 99 523 A

$G_{45}^*$ given by $\varphi((G_{47})^*) = 0$ , $T_{45}^*$ given by $f(T_{45}^*) = 0$ and

$$G_{44}^* = \frac{(a_{44})^{(9)} G_{45}^*}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46}^* = \frac{(a_{46})^{(9)} G_{45}^*}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

$$T_{44}^* = \frac{(b_{44})^{(9)} T_{45}^*}{[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47})^*)]} \quad , \quad T_{46}^* = \frac{(b_{46})^{(9)} T_{45}^*}{[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47})^*)]}$$

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**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

**Proof:** Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a''_{14})^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \frac{\partial (b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a'_{13})^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)}G^*_{13}\mathbb{T}_{14} \quad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a'_{14})^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)}G^*_{14}\mathbb{T}_{14} \quad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a'_{15})^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)}G^*_{15}\mathbb{T}_{14} \quad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b'_{13})^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)}T^*_{13}\mathbb{G}_j\right) \quad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b'_{14})^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)}T^*_{14}\mathbb{G}_j\right) \quad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b'_{15})^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T^*_{15}\mathbb{G}_j\right) \quad 530$$

**ASYMPTOTIC STABILITY ANALYSIS** 531

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(2)}$ and $(b''_i)^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G^*_i + \mathbb{G}_i \quad , T_i = T^*_i + \mathbb{T}_i \quad 532$$

$$\frac{\partial(a''_{17})^{(2)}}{\partial T_{17}}(T^*_{17}) = (q_{17})^{(2)} \quad , \frac{\partial(b''_i)^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \quad 533$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a'_{16})^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G^*_{16}\mathbb{T}_{17} \quad 534$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a'_{17})^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G^*_{17}\mathbb{T}_{17} \quad 535$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a'_{18})^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G^*_{18}\mathbb{T}_{17} \quad 536$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b'_{16})^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T^*_{16}\mathbb{G}_j\right) \quad 537$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b'_{17})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}T^*_{17}\mathbb{G}_j\right) \quad 538$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b'_{18})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}T^*_{18}\mathbb{G}_j\right) \quad 539$$

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**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(3)}$ *and* $(b''_i)^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{21}'')^{(3)}}{\partial T_{21}}(T_{21}^*) = (q_{21})^{(3)} \quad , \frac{\partial (b_i'')^{(3)}}{\partial G_j}(\,(G_{23})^*\,) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\big((a_{20}')^{(3)} + (p_{20})^{(3)}\big)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G_{20}^*\mathbb{T}_{21} \quad 541$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\big((a_{21}')^{(3)} + (p_{21})^{(3)}\big)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G_{21}^*\mathbb{T}_{21} \quad 542$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\big((a_{22}')^{(3)} + (p_{22})^{(3)}\big)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G_{22}^*\mathbb{T}_{21} \quad 543$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\big((b_{20}')^{(3)} - (r_{20})^{(3)}\big)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\big(s_{(20)(j)}T_{20}^*\mathbb{G}_j\big) \quad 544$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\big((b_{21}')^{(3)} - (r_{21})^{(3)}\big)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\big(s_{(21)(j)}T_{21}^*\mathbb{G}_j\big) \quad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\big((b_{22}')^{(3)} - (r_{22})^{(3)}\big)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\big(s_{(22)(j)}T_{22}^*\mathbb{G}_j\big) \quad 546$$

**ASYMPTOTIC STABILITY ANALYSIS** 547

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}\ and\ (b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{25}'')^{(4)}}{\partial T_{25}}(T_{25}^*) = (q_{25})^{(4)} \quad , \frac{\partial (b_i'')^{(4)}}{\partial G_j}((G_{27})^*\,) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\big((a_{24}')^{(4)} + (p_{24})^{(4)}\big)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G_{24}^*\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\big((a_{25}')^{(4)} + (p_{25})^{(4)}\big)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G_{25}^*\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\big((a_{26}')^{(4)} + (p_{26})^{(4)}\big)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G_{26}^*\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\big((b_{24}')^{(4)} - (r_{24})^{(4)}\big)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\big(s_{(24)(j)}T_{24}^*\mathbb{G}_j\big) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\big((b_{25}')^{(4)} - (r_{25})^{(4)}\big)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\big(s_{(25)(j)}T_{25}^*\mathbb{G}_j\big) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T^*_{26}\mathbb{G}_j\right) \quad 554$$

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**Theorem 5:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}\ and\ (b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 556

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{29})^{(5)}}{\partial T_{29}}(T^*_{29}) = (q_{29})^{(5)} \ , \ \frac{\partial(b_i'')^{(5)}}{\partial G_j}\left((G_{31})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a'_{28})^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G^*_{28}\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a'_{29})^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G^*_{29}\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a'_{30})^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G^*_{30}\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b'_{28})^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T^*_{28}\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b'_{29})^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T^*_{29}\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b'_{30})^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T^*_{30}\mathbb{G}_j\right) \quad 562$$

**ASYMPTOTIC STABILITY ANALYSIS** 563

**Theorem 6:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}\ and\ (b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{33})^{(6)}}{\partial T_{33}}(T^*_{33}) = (q_{33})^{(6)} \ , \ \frac{\partial(b_i'')^{(6)}}{\partial G_j}\left((G_{35})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a'_{32})^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G^*_{32}\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a'_{33})^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G^*_{33}\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a'_{34})^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G^*_{34}\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b'_{32})^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T^*_{32}\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b'_{33})^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T^*_{33}\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b'_{34})^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T^*_{34}\mathbb{G}_j\right) \quad 570$$

**ASYMPTOTIC STABILITY ANALYSIS** 571

**Theorem 7:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 572

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{37})^{(7)}}{\partial T_{37}}(T^*_{37}) = (q_{37})^{(7)} \quad , \frac{\partial(b_i'')^{(7)}}{\partial G_j}\left((G_{39})^{**}\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a'_{36})^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G^*_{36}\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a'_{37})^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G^*_{37}\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a'_{38})^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G^*_{38}\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b'_{36})^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T^*_{36}\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b'_{37})^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T^*_{37}\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b'_{38})^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T^*_{38}\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

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**Theorem 8:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{41}'')^{(8)}}{\partial T_{41}} (T_{41}^*) = (q_{41})^{(8)} \quad , \frac{\partial (b_i'')^{(8)}}{\partial G_j} ( (G_{43})^* ) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\big((a_{40}')^{(8)} + (p_{40})^{(8)}\big)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)} G_{40}^* \mathbb{T}_{41} \quad (581)$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\big((a_{41}')^{(8)} + (p_{41})^{(8)}\big)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)} G_{41}^* \mathbb{T}_{41} \quad (582)$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\big((a_{42}')^{(8)} + (p_{42})^{(8)}\big)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)} G_{42}^* \mathbb{T}_{41} \quad (583)$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\big((b_{40}')^{(8)} - (r_{40})^{(8)}\big)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\big(s_{(40)(j)} T_{40}^* \mathbb{G}_j\big) \quad (584)$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\big((b_{41}')^{(8)} - (r_{41})^{(8)}\big)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\big(s_{(41)(j)} T_{41}^* \mathbb{G}_j\big) \quad (585)$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\big((b_{42}')^{(8)} - (r_{42})^{(8)}\big)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\big(s_{(42)(j)} T_{42}^* \mathbb{G}_j\big) \quad (586)$$

(586 A)

## ASYMPTOTIC STABILITY ANALYSIS

**Theorem 9:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}$ *and* $(b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

**Proof:** Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{45}'')^{(9)}}{\partial T_{45}} (T_{45}^*) = (q_{45})^{(9)} \quad , \frac{\partial (b_i'')^{(9)}}{\partial G_j} ( (G_{47})^* ) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\big((a_{44}')^{(9)} + (p_{44})^{(9)}\big)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)} G_{44}^* \mathbb{T}_{45} \quad (586\,B)$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\big((a_{45}')^{(9)} + (p_{45})^{(9)}\big)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)} G_{45}^* \mathbb{T}_{45} \quad (586\,C)$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\big((a_{46}')^{(9)} + (p_{46})^{(9)}\big)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)} G_{46}^* \mathbb{T}_{45} \quad (586\,D)$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\big((b_{44}')^{(9)} - (r_{44})^{(9)}\big)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\big(s_{(44)(j)} T_{44}^* \mathbb{G}_j\big) \quad (586\,E)$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\big((b_{45}')^{(9)} - (r_{45})^{(9)}\big)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\big(s_{(45)(j)} T_{45}^* \mathbb{G}_j\big) \quad (586\,F)$$

$\frac{d\mathbb{T}_{46}}{dt} = -\left((b'_{46})^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T^*_{46}\mathbb{G}_j\right)$ 586

G

587

**The characteristic equation of this system is**

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(q_{13})^{(1)}G^*_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)}T^*_{14} + (b_{14})^{(1)}s_{(13),(14)}T^*_{14}\right)$$

$$+\left(\left((\lambda)^{(1)} + (a'_{14})^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)}G^*_{13} + (a_{13})^{(1)}(q_{14})^{(1)}G^*_{14}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)}T^*_{14} + (b_{14})^{(1)}s_{(13),(13)}T^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((b'_{13})^{(1)} + (b'_{14})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(15)}T^*_{14} + (b_{14})^{(1)}s_{(13),(15)}T^*_{13}\right)\} = 0$$

$+$

$$(\lambda)^{(2)} + (b'_{18})^{(2)} - (r_{18})^{(2)}\{\left((\lambda)^{(2)} + (a'_{18})^{(2)} + (p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^*_{17} + (a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^*_{17}\right)$$

$$+\left(\left((\lambda)^{(2)} + (a'_{17})^{(2)} + (p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^*_{16} + (a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}\right)$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2 + \left((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2 + \left((b'_{16})^{(2)} + (b'_{17})^{(2)} - (r_{16})^{(2)} + (r_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$+\left(\left((\lambda)^{(2)}\right)^2 + \left((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17} + (a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^{*}_{16}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\right)\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left(\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)\right]$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(21)}T^{*}_{21}\right)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^{*}_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^{*}_{21}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(20)}T^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$+\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(22)}T^{*}_{20}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\right)\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left(\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)\right]$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(25)}T^{*}_{25}\right)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^{*}_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^{*}_{25}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(24)}T^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$+\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(26)}T^{*}_{24}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\right)\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(29)}T^{*}_{29}\right)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^{*}_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^{*}_{29}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(28)}T^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$+\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(30)}T^{*}_{28}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\right)\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left(\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)\right]$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(33)}T^{*}_{33}\right)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^{*}_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^{*}_{33}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(32)}T^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$+\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(34)}T^{*}_{32}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\right)\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)\right]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(37)}T^{*}_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^{*}_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^{*}_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(36)}T^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$+\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(38)}T^{*}_{36}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\right)\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(41)}T^{*}_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^{*}_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^{*}_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(40)}T^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$+\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(42)}T^{*}_{40}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\right)\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(45)}T^{*}_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^{*}_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^{*}_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(44)}T^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$+\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(46)}T^{*}_{44}\right)\}=0$$

**And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.**

## SECTION TWENTY ONE

## Thermodynamics Of Quantum Transport

### INTRODUCTION—VARIABLES USED

**Scattering approach to the thermodynamics of quantum transport Pierre Gaspard Published 2 April 2015 • © 2015 IOP Publishing Ltd and Deutsche Physikalische Gesellschaft • New Journal of Physics, Volume 17, April 2015 Focus on Quantum Thermodynamics**

(1) The thermodynamic entropy production for the scattering processes of (e) noninteracting bosons

and fermions in mesoscopic systems is shown to be related to (e&eb) the difference between the Connes–Narnhofer–Thirring entropy per unit time, characterizing (eb) temporal disorder in (eb) the motion of quantum particles, and the associated (e&eb) time-reversed co entropy per unit time.

(2) Under nonequilibrium conditions, the positivity of (e) thermodynamic entropy production can thus be interpreted as (eb) a time-reversal symmetry breaking in (eb) the temporal disorder of the quantum transport process.

(3) Moreover, the full counting statistics of both fermionic and bosonic quantum transport is formulated in relation with (e&eb) the energy and particle currents producing (eb) thermodynamic entropy in (eb) nonequilibrium steady states. **Content from this work may be used under the terms of the Creative Commons Attribution 3.0 licence. Any further distribution of this work must maintain attribution to the author(s) and the title of the work, journal citation and DOI**.

**Geometry of Quantum Observables and Thermodynamics of Small Systems Maxim Olshanii Phys. Rev. Lett 114, 060401 – Published 11 February 2015**

(4) Authors show that for classical and quantum observables, the integrability-to (e&eb)-Ergodicity transition leave (eb) constant the sum of (a) the ensemble variance of (e) the temporal average and (b) the ensemble average of (e) temporal variance.

(5) The induced Frobenius (Hilbert-Schmidt) geometry of (e) quantum observables encodes (e) how eigenstate thermalization appears, the inverse participation ratio decreases, and the integrals of motion disappear during (e&eb) the transition.

**(6)** Authors use it (e) to optimize the set of conserved quantities entering (eb) the generalized Gibbs ensemble for (e) integrable, near-integrable, or mesoscopic systems. **http://dx.doi.org/10.1103/PhysRevLett.114.060401©American Physical Society**

## NOTATION

### Module One

The thermodynamic entropy production for the scattering processes of (e) noninteracting bosons and fermions in mesoscopic systems is shown to be related to (e&eb) the difference between the Connes–Narnhofer–Thirring entropy per unit time, characterizing (eb) temporal disorder in (eb) the motion of quantum particles, and the associated (e&eb) time-reversed co entropy per unit time

$G_{13}$ : Category one of noninteracting bosons and fermions in mesoscopic systems is shown to be related to (e&eb) the difference between the Connes–Narnhofer–Thirring entropy per unit time, characterizing (eb) temporal disorder in (eb) the motion of quantum particles, and the associated (e&eb) time-reversed co entropy per unit time

$G_{14}$ : Category two of SAS

$G_{15}$ : Category three of SAS

$T_{13}$ : Category one of thermodynamic entropy production for the scattering processes

$T_{14}$ : Category two of SAS

$T_{15}$ : Category three of SAS

### Module Two

The **thermodynamic entropy production for the scattering processes of noninteracting bosons and fermions in mesoscopic systems** is shown to be related to (e&eb) the difference between the Connes–Narnhofer–Thirring entropy per unit time, characterizing (eb) temporal disorder in (eb) the motion of quantum particles, and the associated (e&eb) time-reversed co entropy per unit time

$G_{16}$ : Category one of **thermodynamic entropy production for the scattering processes of noninteracting bosons and fermions in mesoscopic systems;** difference between the Connes–Narnhofer–Thirring entropy per unit time, characterizing (eb) temporal disorder in (eb) the motion of quantum particles, and the associated (e&eb) time-reversed co entropy per unit time

$G_{17}$ : Category two of SAS

$G_{18}$ : Category three of SAS

$T_{16}$ : Category one of difference between the Connes–Narnhofer–Thirring entropy per unit time, characterizing (eb) temporal disorder in (eb) the motion of quantum particles, and the associated (e&eb) time-reversed co entropy per unit time; **thermodynamic entropy production for the scattering processes of noninteracting bosons and fermions in mesoscopic systems**

$T_{17}$ : Category two of SAS

$T_{18}$ : Category three of SAS

**Module three**

The thermodynamic entropy production for the scattering processes of noninteracting bosons and fermions in mesoscopic systems is shown to be related to the difference between the Connes–Narnhofer–Thirring entropy per unit time, characterizing (eb) temporal disorder in (eb) the motion of quantum particles, and the associated (e&eb) time-reversed co entropy per unit time

$G_{20}$ : Category one of thermodynamic entropy production for the scattering processes of noninteracting bosons and fermions in mesoscopic systems is shown to be related to the difference between the Connes–Narnhofer–Thirring entropy per unit time

$G_{21}$ : Category two of SAS

$G_{22}$ : Category three of SAS

$T_{20}$ : Category one of temporal disorder in (eb) the motion of quantum particles, and the associated (e&eb) time-reversed co entropy per unit time

$T_{21}$ : Category two of SAS

$T_{22}$ : Category three of SAS

**Module four**

The thermodynamic entropy production for the scattering processes of noninteracting bosons and fermions in mesoscopic systems is shown to be related to the difference between the Connes–Narnhofer–Thirring entropy per unit time, characterizing temporal disorder in the motion of quantum particles, and the associated (e&eb) **time-reversed co entropy per unit time**

$G_{24}$ : Category one of thermodynamic entropy production for the scattering processes of noninteracting bosons and fermions in mesoscopic systems is shown to be related to the difference between the Connes–Narnhofer–Thirring entropy per unit time, characterizing temporal disorder in the motion of quantum particles; **time-reversed co entropy per unit time**

$G_{25}$ : Category two of SAS

$G_{26}$ : Category three of SAS

$T_{24}$ : Category one of **time-reversed co entropy per unit time;** thermodynamic entropy production for the scattering processes of noninteracting bosons and fermions in mesoscopic systems is shown to be related to the difference between the Connes–Narnhofer–Thirring entropy per unit time, characterizing temporal disorder in the motion of quantum particles

$T_{25}$ : Category two of SAS

$T_{26}$ : Category three of SAS

**Module five**

the positivity of thermodynamic entropy production under nonequilibrium conditions can thus be interpreted as (eb) a time-reversal symmetry breaking in (eb) the temporal disorder of the quantum transport process

$G_{28}$ : Category one of positivity of thermodynamic entropy production under nonequilibrium conditions

$G_{29}$ : Category two of SAS

$G_{30}$ : Category three of SAS

$T_{28}$ : Category one of time-reversal symmetry breaking in (eb) the temporal disorder of the quantum transport process

$T_{29}$ : Category two of SAS

$T_{30}$ : Category three of SAS

**Module six**

the positivity of thermodynamic entropy production under nonequilibrium conditions can thus be interpreted as a time-reversal symmetry breaking in (eb) the temporal disorder of the quantum transport process

$G_{32}$ : Category one of positivity of thermodynamic entropy production under nonequilibrium conditions can thus be interpreted as a time-reversal symmetry breaking

$G_{33}$ : Category two of SAS

$G_{34}$ : Category three of SAS

$T_{32}$ : Category one of temporal disorder of the quantum transport process

$T_{33}$ : Category two of SAS

$T_{34}$ : Category three of SAS

**Module seven**

Moreover, the **full counting statistics of both fermionic and bosonic quantum transport** is formulated in relation with (e&eb) the energy and particle currents producing (eb) thermodynamic entropy in (eb) nonequilibrium steady states.

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**of the work, journal citation and DOI**

$G_{36}$ : Category one of **full counting statistics of both fermionic and bosonic quantum transport;** energy and particle currents producing (eb) thermodynamic entropy in (eb) nonequilibrium steady states.

$G_{37}$ : Category two of SAS

$G_{38}$ : Category three of SAS

$T_{36}$ : Category one of energy and particle currents producing (eb) thermodynamic entropy in (eb) nonequilibrium steady states; **full counting statistics of both fermionic and bosonic quantum transport**

$T_{37}$ : Category two of SAS

$T_{38}$ : Category three of SAS

**Module eight**

Moreover, the full counting statistics of both fermionic and bosonic quantum transport is formulated in relation with the energy and particle currents producing (eb) thermodynamic entropy in (eb) nonequilibrium steady states

$G_{40}$ : Category one of full counting statistics of both fermionic and bosonic quantum transport is formulated in relation with the energy and particle currents

$G_{41}$ : Category two of SAS

$G_{42}$ : Category three of SAS

$T_{40}$ : Category one of thermodynamic entropy in (eb) nonequilibrium steady states

$T_{41}$ : Category two of SAS

$T_{42}$ : Category three of SAS

**Module Nine**

thermodynamic entropy in (eb) nonequilibrium steady states

$G_{44}$ : Category one of thermodynamic entropy

$G_{45}$ : Category two of SAS

$G_{46}$ : Category three of SAS

$T_{44}$ : Category one of nonequilibrium steady states

$T_{45}$ : Category two of SAS

$T_{46}$ : Category three of SAS

**The Coefficients:**

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}\ ,\ (b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)},$
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)},\ \ (b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}\ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}\ ,$
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)},$
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)},$
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)},$
are Dissipation coefficients

**Module Numbered One**

**The differential system of this model is now (Module Numbered one)**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \quad 1$$
$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad 2$$
$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15} \quad 3$$
$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13} \quad 4$$
$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14} \quad 5$$
$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)\right]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14}, t) =$ First augmentation factor
$-(b''_{13})^{(1)}(G, t) =$ First detritions factor

**Module Numbered Two**

**The differential system of this model is now ( Module numbered two)**

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right]G_{16} \quad 7$$
$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t)\right]G_{17} \quad 8$$
$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t)\right]G_{18} \quad 9$$
$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\big((G_{19}), t\big)\right]T_{16} \quad 10$$
$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\big((G_{19}), t\big)\right]T_{17} \quad 11$$
$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\big((G_{19}), t\big)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17}, t) =$ First augmentation factor
$-(b''_{16})^{(2)}\big((G_{19}), t\big) =$ First detritions factor

**Module Numbered Three**

**The differential system of this model is now (Module numbered three)**

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21},t)\right]G_{20} \quad 13$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21},t)\right]G_{21} \quad 14$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21},t)\right]G_{22} \quad 15$$

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23},t)\right]T_{20} \quad 16$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23},t)\right]T_{21} \quad 17$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23},t)\right]T_{22} \quad 18$$

$+(a''_{20})^{(3)}(T_{21},t) =$ First augmentation factor

$-(b''_{20})^{(3)}(G_{23},t) =$ First detritions factor

**Module Numbered Four**

**The differential system of this model is now (Module numbered Four)**

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right]G_{24} \quad 19$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25},t)\right]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25},t)\right]G_{26} \quad 21$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}\big((G_{27}),t\big)\right]T_{24} \quad 22$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}\big((G_{27}),t\big)\right]T_{25} \quad 23$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}\big((G_{27}),t\big)\right]T_{26} \quad 24$$

$+(a''_{24})^{(4)}(T_{25},t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}\big((G_{27}),t\big) =$ **First detritions factor**

**Module Numbered Five**:

**The differential system of this model is now (Module number five)**

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29},t)\right]G_{28} \quad 25$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29},t)\right]G_{29} \quad 26$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29},t)\right]G_{30} \quad 27$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}\big((G_{31}),t\big)\right]T_{28} \quad 28$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}\big((G_{31}),t\big)\right]T_{29} \quad 29$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}\big((G_{31}),t\big)\right]T_{30} \quad 30$$

$+(a''_{28})^{(5)}(T_{29},t) =$ **First augmentation factor**

$-(b''_{28})^{(5)}\big((G_{31}),t\big) =$ **First detritions factor**

**Module Numbered Six**

**The differential system of this model is now (Module numbered Six)**

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33},t)\right]G_{32} \quad 31$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33},t)\right]G_{33} \quad 32$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33},t)\right]G_{34} \quad 33$$

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35}),t)\right]T_{32} \quad 34$$
$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}((G_{35}),t)\right]T_{33} \quad 35$$
$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35}),t)\right]T_{34} \quad 36$$
$+(a''_{32})^{(6)}(T_{33},t) =$ **First augmentation factor**

**Module Numbered Seven:**

**The differential system of this model is now (Seventh Module)**

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right]G_{36} \quad 37$$
$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37},t)\right]G_{37} \quad 38$$
$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37},t)\right]G_{38} \quad 39$$
$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39}),t)\right]T_{36} \quad 40$$
$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}((G_{39}),t)\right]T_{37} \quad 41$$
$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39}),t)\right]T_{38} \quad 42$$
$+(a''_{36})^{(7)}(T_{37},t) =$ **First augmentation factor**

**Module Numbered Eight**

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right]G_{40} \quad 43$$
$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41},t)\right]G_{41} \quad 44$$
$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41},t)\right]G_{42} \quad 45$$
$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43}),t)\right]T_{40} \quad 46$$
$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}((G_{43}),t)\right]T_{41} \quad 47$$
$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43}),t)\right]T_{42} \quad 48$$

**Module Numbered Nine**

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$
$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$
$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$
$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47}),t)\right]T_{44} \quad 52$$
$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}((G_{47}),t)\right]T_{45} \quad 53$$
$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47}),t)\right]T_{46} \quad 54$$
$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**
$-(b''_{44})^{(9)}((G_{47}),t) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)} & +(a''_{13})^{(1)}(T_{14},t) & +(a''_{16})^{(2,2,)}(T_{17},t) & +(a''_{20})^{(3,3,)}(T_{21},t) \\ +(a''_{24})^{(4,4,4,4,)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,)}(T_{33},t) & \\ +(a''_{36})^{(7,7)}(T_{37},t) & +(a''_{40})^{(8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) & \end{bmatrix} G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)}\boxed{+(a''_{14})^{(1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14} \quad (56)$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)}\boxed{+(a''_{15})^{(1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15} \quad (57)$$

Where $\boxed{(a''_{13})^{(1)}(T_{14},t)}$, $\boxed{(a''_{14})^{(1)}(T_{14},t)}$, $\boxed{(a''_{15})^{(1)}(T_{14},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7)}(T_{37},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8)}(T_{41},t)}$ $\boxed{+(a''_{41})^{(8,8)}(T_{41},t)}$ $\boxed{+(a''_{42})^{(8,8)}(T_{41},t)}$ are eight augmentation coefficient for 1,2,3

$\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)}\boxed{-(b''_{13})^{(1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13} \quad (58)$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)}\boxed{-(b''_{14})^{(1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14} \quad (59)$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)}\boxed{-(b''_{15})^{(1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15} \quad (60)$$

Where $\boxed{-(b''_{13})^{(1)}(G,t)}$, $\boxed{-(b''_{14})^{(1)}(G,t)}$, $\boxed{-(b''_{15})^{(1)}(G,t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8)}(G_{43},t)}$ $\boxed{-(b''_{41})^{(8,8)}(G_{43},t)}$ $\boxed{-(b''_{42})^{(8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[\begin{array}{ccc} (a'_{16})^{(2)}\boxed{+(a''_{16})^{(2)}(T_{17},t)} & \boxed{+(a''_{13})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9)}(T_{45},t)} \end{array}\right] G_{16} \quad (61)$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[\begin{array}{ccc} (a'_{17})^{(2)}\boxed{+(a''_{17})^{(2)}(T_{17},t)} & \boxed{+(a''_{14})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9)}(T_{45},t)} \end{array}\right] G_{17} \quad (62)$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[\begin{array}{ccc} (a'_{18})^{(2)}\boxed{+(a''_{18})^{(2)}(T_{17},t)} & \boxed{+(a''_{15})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9)}(T_{45},t)} \end{array}\right] G_{18} \quad (63)$$

Where $\boxed{+(a''_{16})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,)}(T_{14},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$+(a''_{40})^{(8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8)}(T_{41},t)$ are eight augmentation coefficient for category 1, 2 and 3

$+(a''_{44})^{(9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9)}(T_{45},t)$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \begin{bmatrix} (b'_{16})^{(2)} & -(b''_{16})^{(2)}(G_{19},t) & -(b''_{13})^{(1,1,)}(G,t) & -(b''_{20})^{(3,3,3,)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6)}(G_{35},t) & \\ -(b''_{36})^{(7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9)}(G_{47},t) & \end{bmatrix} T_{16} \quad (64)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)} & -(b''_{17})^{(2)}(G_{19},t) & -(b''_{14})^{(1,1,)}(G,t) & -(b''_{21})^{(3,3,3,)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6)}(G_{35},t) & \\ -(b''_{37})^{(7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9)}(G_{47},t) & \end{bmatrix} T_{17} \quad (65)$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)} & -(b''_{18})^{(2)}(G_{19},t) & -(b''_{15})^{(1,1,)}(G,t) & -(b''_{22})^{(3,3,3,)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6)}(G_{35},t) & \\ -(b''_{38})^{(7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9)}(G_{47},t) & \end{bmatrix} T_{18} \quad (66)$$

where $-(b''_{16})^{(2)}(G_{19},t)$, $-(b''_{17})^{(2)}(G_{19},t)$, $-(b''_{18})^{(2)}(G_{19},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{13})^{(1,1,)}(G,t)$, $-(b''_{14})^{(1,1,)}(G,t)$, $-(b''_{15})^{(1,1,)}(G,t)$ are second detrition coefficients for category 1,2 and 3

$-(b''_{20})^{(3,3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,)}(G_{23},t)$ are third detrition coefficients for category 1,2 and 3

$-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1,2 and 3

$-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1,2 and 3

$-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients for category 1,2 and 3

$-(b''_{36})^{(7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2 and 3

$-(b''_{40})^{(8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1,2 and 3

$-(b''_{44})^{(9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \begin{bmatrix} (a'_{20})^{(3)} & +(a''_{20})^{(3)}(T_{21},t) & +(a''_{16})^{(2,2,2)}(T_{17},t) & +(a''_{13})^{(1,1,1,)}(T_{14},t) \\ +(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t) & \\ +(a''_{36})^{(7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9)}(T_{45},t) & \end{bmatrix} G_{20} \quad (67)$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \begin{bmatrix} (a'_{21})^{(3)}\boxed{+(a''_{21})^{(3)}(T_{21},t)}\ \boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}\ \boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{21} \quad (68)$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \begin{bmatrix} (a'_{22})^{(3)}\boxed{+(a''_{22})^{(3)}(T_{21},t)}\ \boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}\ \boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{22} \quad (69)$$

$\boxed{+(a''_{20})^{(3)}(T_{21},t)}, \boxed{+(a''_{21})^{(3)}(T_{21},t)}, \boxed{+(a''_{22})^{(3)}(T_{21},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}, \boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}, \boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}$ are second augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)}$ are third augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)}, \boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)}, \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)}, \boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)}, \boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)}, \boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)}, \boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)}$ are seventh augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)}$ are eight augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{44})^{(9,9,9)}(T_{45},t)}, \boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)}, \boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)}$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b'_{20})^{(3)}\boxed{-(b''_{20})^{(3)}(G_{23},t)}\ \boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}\ \boxed{-(b''_{13})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)}\ \boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)}\ \boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7)}(G_{39},t)}\ \boxed{-(b''_{40})^{(8,8,8,8)}(G_{43},t)}\ \boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{20} \quad (70)$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b'_{21})^{(3)}\boxed{-(b''_{21})^{(3)}(G_{23},t)}\ \boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}\ \boxed{-(b''_{14})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)}\ \boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)}\ \boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7)}(G_{39},t)}\ \boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)}\ \boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{21} \quad (71)$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)}\boxed{-(b''_{22})^{(3)}(G_{23},t)}\ \boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}\ \boxed{-(b''_{15})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)}\ \boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)}\ \boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)}\ \boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)}\ \boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{22} \quad (72)$$

$\boxed{-(b''_{20})^{(3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3)}(G_{23},t)}$ *are first detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}$ *are second detrition coefficients for*

*category 1, 2 and 3*

$\boxed{-(b_{13}'')^{(1,1,1,)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,)}(G,t)}$, $\boxed{-(b_{15}'')^{(1,1,1,)}(G,t)}$ *are third detrition coefficients for category 1,2 and 3*

$\boxed{-(b_{24}'')^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{25}'')^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{26}'')^{(4,4,4,4,4,4)}(G_{27},t)}$ *are fourth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b_{28}'')^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5)}(G_{31},t)}$ *are fifth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b_{32}'')^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{34}'')^{(6,6,6,6,6,6)}(G_{35},t)}$ *are sixth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b_{36}'')^{(7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{37}'')^{(7,7,7,7)}(G_{39},t)}$ $\boxed{-(b_{38}'')^{(7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{40}'')^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{42}'')^{(8,8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{46}'')^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b_{45}'')^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b_{44}'')^{(9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[\begin{array}{ccc} (a_{24}')^{(4)} & \boxed{+(a_{24}'')^{(4)}(T_{25},t)} & \boxed{+(a_{28}'')^{(5,5,)}(T_{29},t)} \quad \boxed{+(a_{32}'')^{(6,6,)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a_{16}'')^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a_{20}'')^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{36}'')^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{40}'')^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a_{44}'')^{(9,9,9,9)}(T_{45},t)} \end{array}\right] G_{24} \quad (73)$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[\begin{array}{ccc} (a_{25}')^{(4)} & \boxed{+(a_{25}'')^{(4)}(T_{25},t)} & \boxed{+(a_{29}'')^{(5,5,)}(T_{29},t)} \quad \boxed{+(a_{33}'')^{(6,6)}(T_{33},t)} \\ \boxed{+(a_{14}'')^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a_{17}'')^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a_{21}'')^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{37}'')^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{41}'')^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a_{45}'')^{(9,9,9,9)}(T_{45},t)} \end{array}\right] G_{25} \quad (74)$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[\begin{array}{ccc} (a_{26}')^{(4)} & \boxed{+(a_{26}'')^{(4)}(T_{25},t)} & \boxed{+(a_{30}'')^{(5,5,)}(T_{29},t)} \quad \boxed{+(a_{34}'')^{(6,6,)}(T_{33},t)} \\ \boxed{+(a_{15}'')^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a_{18}'')^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a_{22}'')^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{38}'')^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{42}'')^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a_{46}'')^{(9,9,9,9)}(T_{45},t)} \end{array}\right] G_{26} \quad (75)$$

$\boxed{(a_{24}'')^{(4)}(T_{25},t)}$, $\boxed{(a_{25}'')^{(4)}(T_{25},t)}$, $\boxed{(a_{26}'')^{(4)}(T_{25},t)}$ *are first augmentation coefficients category* $1, 2$ $3$

$\boxed{+(a_{28}'')^{(5,5,)}(T_{29},t)}$, $\boxed{+(a_{29}'')^{(5,5,)}(T_{29},t)}$, $\boxed{+(a_{30}'')^{(5,5,)}(T_{29},t)}$ *are second augmentation coefficient for category* $1, 2$ *and* $3$

$\boxed{+(a_{32}'')^{(6,6,)}(T_{33},t)}$, $\boxed{+(a_{33}'')^{(6,6,)}(T_{33},t)}$, $\boxed{+(a_{34}'')^{(6,6,)}(T_{33},t)}$ *are third augmentation coefficient for category* $1, 2$ *and* $3$

$\boxed{+(a_{13}'')^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a_{14}'')^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a_{15}'')^{(1,1,1,1)}(T_{14},t)}$ *are fourth augmentation coefficients for category* $1, 2$ *and* $3$

$\boxed{+(a_{16}'')^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{17}'')^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{18}'')^{(2,2,2,2)}(T_{17},t)}$ *are fifth augmentation coefficients for category* $1, 2$ *and* $3$

$\boxed{+(a_{20}'')^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{21}'')^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{22}'')^{(3,3,3,3)}(T_{21},t)}$ *are sixth augmentation coefficients for category* $1, 2$ *and* $3$

$\boxed{+(a_{36}'')^{(7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a_{37}'')^{(7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a_{38}'')^{(7,7,7,7,7)}(T_{37},t)}$ *are seventh augmentation coefficients for category* $1, 2$ *and* $3$

$+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)$

*are eighth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{46})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9)}(T_{45},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \begin{bmatrix} (b'_{24})^{(4)} & -(b''_{24})^{(4)}(G_{27},t) & -(b''_{28})^{(5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,)}(G_{35},t) \\ & -(b''_{13})^{(1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{24} \quad (76)$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \begin{bmatrix} (b'_{25})^{(4)} & -(b''_{25})^{(4)}(G_{27},t) & -(b''_{29})^{(5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{25} \quad (77)$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)} & -(b''_{26})^{(4)}(G_{27},t) & -(b''_{30})^{(5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1)}(G,t) & -(b''_{18})^{(2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{26} \quad (78)$$

*Where* $-(b''_{24})^{(4)}(G_{27},t)$, $-(b''_{25})^{(4)}(G_{27},t)$, $-(b''_{26})^{(4)}(G_{27},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{28})^{(5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,)}(G_{31},t)$ *are second detrition coefficients for category* 1, 2 *and* 3

$-(b''_{32})^{(6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,)}(G_{35},t)$ *are third detrition coefficients for category* 1, 2 *and* 3

$-(b''_{13})^{(1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1)}(G,t)$

*are fourth detrition coefficients for category* 1, 2 *and* 3

$-(b''_{16})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2)}(G_{19},t)$

*are fifth detrition coefficients for category* 1, 2 *and* 3

$-(b''_{20})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3)}(G_{23},t)$

*are sixth detrition coefficients for category* 1, 2 *and* 3

$-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)$

*are seventh detrition coefficients for category* 1, 2 *and* 3

$-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)$

*are eighth detrition coefficients for category* 1, 2 *and* 3

$-(b''_{46})^{(9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \begin{bmatrix} (a'_{28})^{(5)} & +(a''_{28})^{(5)}(T_{29},t) & +(a''_{24})^{(4,4,)}(T_{25},t) & +(a''_{32})^{(6,6,6)}(T_{33},t) \\ & +(a''_{13})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{16})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3,3)}(T_{21},t) \\ & +(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{28} \quad (79)$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \begin{bmatrix} (a'_{29})^{(5)} & +(a''_{29})^{(5)}(T_{29},t) & +(a''_{25})^{(4,4,)}(T_{25},t) & +(a''_{33})^{(6,6,6)}(T_{33},t) \\ +(a''_{14})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{17})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3,3)}(T_{21},t) \\ +(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{29} \quad (80)$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \begin{bmatrix} (a'_{30})^{(5)} & +(a''_{30})^{(5)}(T_{29},t) & +(a''_{26})^{(4,4,)}(T_{25},t) & +(a''_{34})^{(6,6,6)}(T_{33},t) \\ +(a''_{15})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{18})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3,3)}(T_{21},t) \\ +(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{30} \quad (81)$$

*Where* $+(a''_{28})^{(5)}(T_{29},t)$, $+(a''_{29})^{(5)}(T_{29},t)$, $+(a''_{30})^{(5)}(T_{29},t)$ *are first augmentation coefficients for category* 1, 2 *and* 3

*And* $+(a''_{24})^{(4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,)}(T_{25},t)$ *are second augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{32})^{(6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6)}(T_{33},t)$ *are third augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)$ are fourth augmentation coefficients for category 1,2, and 3

$+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)$ are fifth augmentation coefficients for category 1,2,and 3

$+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)$ are sixth augmentation coefficients for category 1,2, 3

$+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)$ , $+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1,2, 3

$+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)$ are eighth augmentation coefficients for category 1,2, 3

$+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \begin{bmatrix} (b'_{28})^{(5)} & -(b''_{28})^{(5)}(G_{31},t) & -(b''_{24})^{(4,4,)}(G_{27},t) & -(b''_{32})^{(6,6,6)}(G_{35},t) \\ -(b''_{13})^{(1,1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3)}(G_{23},t) \\ -(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{28} \quad (82)$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b'_{29})^{(5)} & -(b''_{29})^{(5)}(G_{31},t) & -(b''_{25})^{(4,4,)}(G_{27},t) & -(b''_{33})^{(6,6,6)}(G_{35},t) \\ -(b''_{14})^{(1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3)}(G_{23},t) \\ -(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{29} \quad (83)$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b'_{30})^{(5)} & -(b''_{30})^{(5)}(G_{31},t) & -(b''_{26})^{(4,4,)}(G_{27},t) & -(b''_{34})^{(6,6,6)}(G_{35},t) \\ -(b''_{15})^{(1,1,1,1,1,)}(G,t) & -(b''_{18})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3)}(G_{23},t) \\ -(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{30} \quad (84)$$

*where* $-(b''_{28})^{(5)}(G_{31},t)$ , $-(b''_{29})^{(5)}(G_{31},t)$ , $-(b''_{30})^{(5)}(G_{31},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{24})^{(4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,)}(G_{27},t)$ *are second detrition coefficients*

$for\ category\ 1,2\ and\ 3$

$\boxed{-(b''_{32})^{(6,6,6)}(G_{35},t)}, \boxed{-(b''_{33})^{(6,6,6)}(G_{35},t)}, \boxed{-(b''_{34})^{(6,6,6)}(G_{35},t)}$ $are\ third\ detrition\ coefficients$

$for\ category\ 1,2\ and\ 3$

$\boxed{-(b''_{13})^{(1,1,1,1,1)}(G,t)}, \boxed{-(b''_{14})^{(1,1,1,1,1)}(G,t)}, \boxed{-(b''_{15})^{(1,1,1,1,1,)}(G,t)}$ are fourth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[\begin{array}{c} (a'_{32})^{(6)}\boxed{+(a''_{32})^{(6)}(T_{33},t)}\ \boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}\ \boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}\ \boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}\ \boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{32} \quad (85)$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[\begin{array}{c} (a'_{33})^{(6)}\boxed{+(a''_{33})^{(6)}(T_{33},t)}\ \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}\ \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}\ \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}\ \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{33} \quad (86)$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[\begin{array}{c} (a'_{34})^{(6)}\boxed{+(a''_{34})^{(6)}(T_{33},t)}\ \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}\ \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}\ \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}\ \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{34} \quad (87)$$

$\boxed{+(a''_{32})^{(6)}(T_{33},t)}, \boxed{+(a''_{33})^{(6)}(T_{33},t)}, \boxed{+(a''_{34})^{(6)}(T_{33},t)}$ $are\ first\ augmentation\ coefficients$

$for\ category\ 1,2\ and\ 3$

$\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}$ $are\ second\ augmentation$

$coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}$ $are\ third\ augmentation$

$coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}$ - are fourth augmentation coefficients

$\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}$ - fifth augmentation coefficients

$\boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)}$ sixth augmentation coefficients

$+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)$
seventh augmentation coefficients
$+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)$
Eighth augmentation coefficients
$+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \begin{bmatrix} (b'_{32})^{(6)} & -(b''_{32})^{(6)}(G_{35},t) & -(b''_{28})^{(5,5,5)}(G_{31},t) & -(b''_{24})^{(4,4,4,)}(G_{27},t) \\ & -(b''_{13})^{(1,1,1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{32} \quad (88)$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \begin{bmatrix} (b'_{33})^{(6)} & -(b''_{33})^{(6)}(G_{35},t) & -(b''_{29})^{(5,5,5)}(G_{31},t) & -(b''_{25})^{(4,4,4,)}(G_{27},t) \\ & -(b''_{14})^{(1,1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{33} \quad (89)$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)} & -(b''_{34})^{(6)}(G_{35},t) & -(b''_{30})^{(5,5,5)}(G_{31},t) & -(b''_{26})^{(4,4,4,)}(G_{27},t) \\ & -(b''_{15})^{(1,1,1,1,1,1)}(G,t) & -(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{34} \quad (90)$$

$-(b''_{32})^{(6)}(G_{35},t)$, $-(b''_{33})^{(6)}(G_{35},t)$, $-(b''_{34})^{(6)}(G_{35},t)$ *are first detrition coefficients for category* 1, 2 *and* 3
$-(b''_{28})^{(5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5)}(G_{31},t)$ *are second detrition coefficients for category* 1, 2 *and* 3
$-(b''_{24})^{(4,4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,)}(G_{27},t)$ *are third detrition coefficients for category* 1,2 *and* 3
$-(b''_{13})^{(1,1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1,1,1)}(G,t)$ are fourth detrition coefficients for category 1, 2, and 3
$-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1, 2, and 3
$-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1, 2, and 3
$-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2, and 3
$-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)$
are eighth detrition coefficients for category 1, 2, and 3
$-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \qquad 91$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a'_{36})^{(7)}\boxed{+(a''_{36})^{(7)}(T_{37},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \qquad 92$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a'_{37})^{(7)}\boxed{+(a''_{37})^{(7)}(T_{37},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{38}}{dt} \qquad 93$$

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a'_{38})^{(7)}\boxed{+(a''_{38})^{(7)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{(a''_{36})^{(7)}(T_{37},t)}$, $\boxed{(a''_{37})^{(7)}(T_{37},t)}$, $\boxed{(a''_{38})^{(7)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{36}}{dt} = \qquad 94$$

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b'_{36})^{(7)}\boxed{-(b''_{36})^{(7)}(G_{39},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{37}}{dt} =$$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b'_{37})^{(7)}\boxed{-(b''_{37})^{(7)}(G_{39},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{38}}{dt} =$$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b'_{38})^{(7)}\boxed{-(b''_{38})^{(7)}(G_{39},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{40}}{dt}$$



$$= (a_{40})^{(8)}G_{41} - \begin{bmatrix} (a'_{40})^{(8)}\boxed{+(a''_{40})^{(8)}(T_{41},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{41}}{dt}$$

$$= (a_{41})^{(8)}G_{40} - \begin{bmatrix} (a'_{41})^{(8)}\boxed{+(a''_{41})^{(8)}(T_{41},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \begin{bmatrix} (a'_{42})^{(8)}\boxed{+(a''_{42})^{(8)}(T_{41},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a''_{40})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)}\boxed{-(b''_{40})^{(8)}(G_{43},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)}\boxed{-(b''_{41})^{(8)}(G_{43},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)}\boxed{-(b''_{42})^{(8)}(G_{43},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{44}}{dt} \qquad 96$$

$$= (a_{44})^{(9)}G_{45} - \left[\begin{array}{ccc} (a'_{44})^{(9)}\boxed{+(a''_{44})^{(9)}(T_{45},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} \end{array}\right] G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[\begin{array}{ccc} (a'_{45})^{(9)}\boxed{+(a''_{45})^{(9)}(T_{45},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} \end{array}\right] G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[\begin{array}{ccc} (a'_{46})^{(9)}\boxed{+(a''_{46})^{(9)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} \end{array}\right] G_{15}$$

Where $\boxed{+(a''_{44})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{20}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{21}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{22}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{24}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{25}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{26}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{28}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{29}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{30}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{32}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{33}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{34}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a_{14}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a_{15}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ are Seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{38}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a_{37}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a_{36}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a_{40}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a_{42}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a_{41}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} =$$

$$(b_{44})^{(9)}T_{45} -$$

$$\begin{bmatrix} (b_{44}')^{(9)}\boxed{-(b_{44}'')^{(9)}(G_{47},t)} & \boxed{-(b_{16}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{20}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{24}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{28}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{32}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{13}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{36}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b_{45}')^{(9)}\boxed{-(b_{45}'')^{(9)}(G_{47},t)} & \boxed{-(b_{17}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{21}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{25}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{29}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{33}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{14}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{37}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b_{46}')^{(9)}\boxed{-(b_{46}'')^{(9)}(G_{47},t)} & \boxed{-(b_{18}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{22}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{26}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{30}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{34}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{15}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{38}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b_{44}'')^{(9)}(G_{47},t)}$, $\boxed{-(b_{45}'')^{(9)}(G_{47},t)}$, $\boxed{-(b_{46}'')^{(9)}(G_{47},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{16}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{17}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{18}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{20}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{21}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{22}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{24}'')^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{25}'')^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{26}'')^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{28}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{32}'')^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{34}'')^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{37}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{36}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a_i')^{(1)}, (a_i'')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (b_i'')^{(1)} > 0, \quad i,j = 13,14,15 \qquad 97$$

The functions $(a_i'')^{(1)}, (b_i'')^{(1)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a_i'')^{(1)}(T_{14},t) \leq (p_i)^{(1)} \leq (\hat{A}_{13})^{(1)}$$

$$(b_i'')^{(1)}(G,t) \leq (r_i)^{(1)} \leq (b_i')^{(1)} \leq (\hat{B}_{13})^{(1)}$$

$$lim_{T_2\to\infty}(a_i'')^{(1)}(T_{14},t) = (p_i)^{(1)} \qquad 98$$

$$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = (r_i)^{(1)}$$

**Definition of** $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$ :

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$$|(a_i'')^{(1)}(T_{14}',t) - (a_i'')^{(1)}(T_{14},t)| \leq (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G',t) - (b_i'')^{(1)}(G,t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}',t)$ and $(a_i'')^{(1)}(T_{14},t)$ . $(T_{14}',t)$ and $(T_{14},t)$ are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14},t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$ : 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$

**Definition of** $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$ : 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}$, $(r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)} (\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)} (\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i, j = 16,17,18$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17}, t) \leq (p_i)^{(2)} \leq (\hat{A}_{16})^{(2)} \quad 102$$

$$(b_i'')^{(2)}(G_{19}, t) \leq (r_i)^{(2)} \leq (b_i')^{(2)} \leq (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2 \to \infty} (a_i'')^{(2)}(T_{17}, t) = (p_i)^{(2)} \quad 104$$

$$\lim_{G \to \infty} (b_i'')^{(2)}((G_{19}), t) = (r_i)^{(2)} \quad 105$$

**Definition of** $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$ : 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}', t) - (a_i'')^{(2)}(T_{17}, t)| \leq (\hat{k}_{16})^{(2)} |T_{17} - T_{17}'| e^{-(\hat{M}_{16})^{(2)} t} \quad 107$$

$$|(b_i'')^{(2)}((G_{19})', t) - (b_i'')^{(2)}((G_{19}), t)| < (\hat{k}_{16})^{(2)} ||(G_{19}) - (G_{19})'|| e^{-(\hat{M}_{16})^{(2)} t} \quad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}', t)$ and$(a_i'')^{(2)}(T_{17}, t)$ . $(T_{17}', t)$ and $(T_{17}, t)$ are points belonging to the interval $[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}]$ . It is to be noted that $(a_i'')^{(2)}(T_{17}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17}, t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$ :

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)},$ are positive constants 109

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$$

**Definition of** $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$ :

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and\ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1 \quad 110$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(b_i)^{(2)} + (b_i')^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1 \quad 111$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i, j = 20,21,22 \quad 112$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(3)}$, $(r_i)^{(3)}$:

$$(a_i'')^{(3)}(T_{21}, t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23}, t) \leq (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(3)}(T_{21}, t) = (p_i)^{(3)} \quad 113$$

$$\lim_{G \to \infty}(b_i'')^{(3)}(G_{23}, t) = (r_i)^{(3)}$$

**Definition of** $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$ :

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: 114

$$|(a_i'')^{(3)}(T_{21}', t) - (a_i'')^{(3)}(T_{21}, t)| \leq (\hat{k}_{20})^{(3)} |T_{21} - T_{21}'| e^{-(\hat{M}_{20})^{(3)} t}$$

$$|(b_i'')^{(3)}(G_{23}', t) - (b_i'')^{(3)}(G_{23}, t)| < (\hat{k}_{20})^{(3)} ||G_{23} - G_{23}'|| e^{-(\hat{M}_{20})^{(3)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}', t)$ and$(a_i'')^{(3)}(T_{21}, t)$ . $(T_{21}', t)$ And $(T_{21}, t)$ are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21}, t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$ : 115

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}$ , $\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with 116
$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$ and the constants
$(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}, i = 20,21,22,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(a_i)^{(3)} + (a_i')^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)} (\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(b_i)^{(3)} + (b_i')^{(3)} + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)} (\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i, j = 24,25,26$$ 117

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(4)}$, $(r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25}, t) \leq (p_i)^{(4)} \leq (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}((G_{27}), t) \leq (r_i)^{(4)} \leq (b_i')^{(4)} \leq (\hat{B}_{24})^{(4)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(4)}(T_{25}, t) = (p_i)^{(4)}$$ 118

$$\lim_{G \to \infty}(b_i'')^{(4)}((G_{27}), t) = (r_i)^{(4)}$$

**Definition of** $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$ :

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24,25,26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}', t) - (a_i'')^{(4)}(T_{25}, t)| \leq (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})', t) - (b_i'')^{(4)}((G_{27}), t)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}', t)$ and$(a_i'')^{(4)}(T_{25}, t)$ . $(T_{25}', t)$ and $(T_{25}, t)$ are points belonging to the interval $[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}]$. It is to be noted that $(a_i'')^{(4)}(T_{25}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25}, t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$ : 120

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)},$ are positive constants

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}, \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$$

**Definition of** $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$ : 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and\ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}, i = 24,25,26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(a_i)^{(4)} + (a_i')^{(4)} + (\hat{A}_{24})^{(4)} + (\hat{P}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(b_i)^{(4)} + (b_i')^{(4)} + (\hat{B}_{24})^{(4)} + (\hat{Q}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i, j = 28,29,30$$ 122

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(5)}$, $(r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29}, t) \le (p_i)^{(5)} \le (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}((G_{31}), t) \le (r_i)^{(5)} \le (b_i')^{(5)} \le (\hat{B}_{28})^{(5)}$$

$$lim_{T_2 \to \infty} (a_i'')^{(5)} (T_{29}, t) = (p_i)^{(5)}$$ 123

$$\lim_{G \to \infty} (b_i'')^{(5)} (G_{31}, t) = (r_i)^{(5)}$$

**Definition of** $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$ :

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}', t) - (a_i'')^{(5)}(T_{29}, t)| \le (\hat{k}_{28})^{(5)} |T_{29} - T_{29}'| e^{-(\hat{M}_{28})^{(5)} t}$$

$$|(b_i'')^{(5)}((G_{31})', t) - (b_i'')^{(5)}((G_{31}), t)| < (\hat{k}_{28})^{(5)} ||(G_{31}) - (G_{31})'|| e^{-(\hat{M}_{28})^{(5)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}', t)$ and $(a_i'')^{(5)}(T_{29}, t)$ . $(T_{29}', t)$ and $(T_{29}, t)$ are points belonging to the interval $\left[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\right]$. It is to be noted that $(a_i'')^{(5)}(T_{29}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$ : 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)},$ are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}, \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

**Definition of** $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$ : 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and\ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(a_i)^{(5)} + (a_i')^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(b_i)^{(5)} + (b_i')^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i, j = 32,33,34$$ 127

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(6)}$, $(r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33}, t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}), t) \leq (r_i)^{(6)} \leq (b_i')^{(6)} \leq (\hat{B}_{32})^{(6)}$$

$$lim_{T_2 \to \infty} (a_i'')^{(6)} (T_{33}, t) = (p_i)^{(6)}$$ 128

$$\lim_{G \to \infty} (b_i'')^{(6)} \left((G_{35}), t\right) = (r_i)^{(6)}$$

**Definition of** $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$ :

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}', t) - (a_i'')^{(6)}(T_{33}, t)| \leq (\hat{k}_{32})^{(6)} |T_{33} - T_{33}'| e^{-(\hat{M}_{32})^{(6)} t}$$

$$|(b_i'')^{(6)}((G_{35})', t) - (b_i'')^{(6)}\left((G_{35}), t\right)| < (\hat{k}_{32})^{(6)} ||(G_{35}) - (G_{35})'|| e^{-(\hat{M}_{32})^{(6)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}', t)$ and $(a_i'')^{(6)}(T_{33}, t)$ . $(T_{33}', t)$ and $(T_{33}, t)$ are points belonging to the interval $\left[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}\right]$. It is to be noted that $(a_i'')^{(6)}(T_{33}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$ : 129

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)},$ are positive constants

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$$

**Definition of** $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$ : 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and\ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32,33,34,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(a_i)^{(6)} + (a_i')^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)} (\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(b_i)^{(6)} + (b_i')^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)} (\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

(EEEEE) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i, j = 36,37,38$ 131

(FFFFF) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(7)}$, $(r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37}, t) \leq (p_i)^{(7)} \leq (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39}, t) \leq (r_i)^{(7)} \leq (b_i')^{(7)} \leq (\hat{B}_{36})^{(7)}$$

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*(GGGGG)* $\lim_{T_2 \to \infty} (a_i'')^{(7)}(T_{37}, t) = (p_i)^{(7)}$

*(HHHHH)*

$\lim_{G \to \infty} (b_i'')^{(7)}((G_{39}), t) = (r_i)^{(7)}$

**Definition of** $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$ :

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$$|(a_i'')^{(7)}(T_{37}', t) - (a_i'')^{(7)}(T_{37}, t)| \leq (\hat{k}_{36})^{(7)} |T_{37} - T_{37}'| e^{-(\hat{M}_{36})^{(7)} t}$$

$$|(b_i'')^{(7)}((G_{39})', t) - (b_i'')^{(7)}((G_{39}), t)| < (\hat{k}_{36})^{(7)} ||(G_{39}) - (G_{39})'|| e^{-(\hat{M}_{36})^{(7)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}', t)$ and $(a_i'')^{(7)}(T_{37}, t)$ . $(T_{37}', t)$ and $(T_{37}, t)$ are points belonging to the interval $[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}]$ . It is to be noted that $(a_i'')^{(7)}(T_{37}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37}, t)$ , the **first augmentation coefficient** attributable to the

system, would be absolutely continuous.

**Definition of** $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$ : 134

(IIIII) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)},$ are positive constants

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$$

**Definition of** $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$ : 135

(JJJJJ) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(b_i)^{(7)} + (b_i')^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40,41,42 \qquad 136$$

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

**Definition of** $(p_i)^{(8)}$, $(r_i)^{(8)}$: 137

$$(a_i'')^{(8)}(T_{41}, t) \le (p_i)^{(8)} \le (\hat{A}_{40})^{(8)} \qquad 138$$

$$(b_i'')^{(8)}((G_{43}), t) \le (r_i)^{(8)} \le (b_i')^{(8)} \le (\hat{B}_{40})^{(8)} \qquad 139$$

$$\lim_{T_2 \to \infty} (a_i'')^{(8)}(T_{41}, t) = (p_i)^{(8)} \qquad 140$$

$$\lim_{G \to \infty} (b_i'')^{(8)}((G_{43}), t) = (r_i)^{(8)} \qquad 141$$

**Definition of** $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$ :

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \le (\hat{k}_{40})^{(8)} |T_{41} - T_{41}'| e^{-(\hat{M}_{40})^{(8)} t} \qquad 142$$

$$|(b_i'')^{(8)}((G_{43})', t) - (b_i'')^{(8)}((G_{43}), t)| < (\hat{k}_{40})^{(8)} ||(G_{43}) - (G_{43})'|| e^{-(\hat{M}_{40})^{(8)} t} \qquad 143$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}', t)$ and $(a_i'')^{(8)}(T_{41}, t)$ . $(T_{41}', t)$ and $(T_{41}, t)$ are points belonging to the interval $\left[(\hat{k}_{40})^{(8)}, (\widehat{M}_{40})^{(8)}\right]$. It is to be noted that $(a_i'')^{(8)}(T_{41}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{40})^{(8)} = 1$ then the function $(a_i'')^{(8)}(T_{41}, t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\widehat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$ :

$(\widehat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)},$ are positive constants

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}, \frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}} < 1 \qquad 144$$

**Definition of** $(\hat{P}_{40})^{(8)}, (\hat{Q}_{40})^{(8)}$ :

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\widehat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}, (\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)}, (a_i')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}, i = 40,41,42,$
Satisfy the inequalities

$$\frac{1}{(\widehat{M}_{40})^{(8)}}[(a_i)^{(8)} + (a_i')^{(8)} + (\hat{A}_{40})^{(8)} + (\hat{P}_{40})^{(8)} (\hat{k}_{40})^{(8)}] < 1 \qquad 145$$

$$\frac{1}{(\widehat{M}_{40})^{(8)}}[(b_i)^{(8)} + (b_i')^{(8)} + (\hat{B}_{40})^{(8)} + (\hat{Q}_{40})^{(8)} (\hat{k}_{40})^{(8)}] < 1 \qquad 146$$

Where we suppose

$$(a_i)^{(9)}, (a_i')^{(9)}, (a_i'')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (b_i'')^{(9)} > 0, \quad i, j = 44,45,46 \qquad 146\text{A}$$

The functions $(a_i'')^{(9)}, (b_i'')^{(9)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(9)}$, $(r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45}, t) \leq (p_i)^{(9)} \leq (\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47}, t) \leq (r_i)^{(9)} \leq (b_i')^{(9)} \leq (\hat{B}_{44})^{(9)}$$

$$\lim_{T_2 \to \infty} (a_i'')^{(9)}(T_{45}, t) = (p_i)^{(9)}$$

$$\lim_{G \to \infty} (b_i'')^{(9)}(G_{47}, t) = (r_i)^{(9)}$$

**Definition of** $(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}$ :

Where $\boxed{(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}}$ are positive constants and $\boxed{i = 44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}', t) - (a_i'')^{(9)}(T_{45}, t)| \leq (\hat{k}_{44})^{(9)} |T_{45} - T_{45}'| e^{-(\widehat{M}_{44})^{(9)} t}$$

$$|(b_i'')^{(9)}((G_{47})', t) - (b_i'')^{(9)}\big((G_{47}), t\big)| < (\hat{k}_{44})^{(9)}||(G_{47}) - (G_{47})'||e^{-(\hat{M}_{44})^{(9)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}', t)$ and$(a_i'')^{(9)}(T_{45}, t)$ . $(T_{45}', t)$ and $(T_{45}, t)$ are points belonging to the interval $\big[(\hat{k}_{44})^{(9)}, (\hat{M}_{44})^{(9)}\big]$. It is to be noted that $(a_i'')^{(9)}(T_{45}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{44})^{(9)} = 1$ then the function $(a_i'')^{(9)}(T_{45}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$ :

$(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$, are positive constants

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}} \ , \frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$$

**Definition of** $(\hat{P}_{44})^{(9)}, (\hat{Q}_{44})^{(9)}$ :

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}, (\hat{A}_{44})^{(9)} and\ (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)}, (a_i')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (p_i)^{(9)}, \ (r_i)^{(9)}, i = 44,45,46,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{44})^{(9)}}[\,(a_i)^{(9)} + (a_i')^{(9)} + \ (\hat{A}_{44})^{(9)} + \ (\hat{P}_{44})^{(9)}\,(\hat{k}_{44})^{(9)}] < 1$$

$$\frac{1}{(\hat{M}_{44})^{(9)}}[\ (b_i)^{(9)} + (b_i')^{(9)} + \ (\hat{B}_{44})^{(9)} + \ (\hat{Q}_{44})^{(9)}\ (\hat{k}_{44})^{(9)}] < 1$$

**Theorem 1:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \big(\hat{P}_{13}\big)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \ , \ \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 2 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

**Definition of** $G_i(0)\,, T_i(0)$

$$G_i(t) \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} \quad , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} \quad , \quad T_i(0) = T_i^0 > 0$$

**Theorem 3 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t) \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t} \quad , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t} \quad , \quad T_i(0) = T_i^0 > 0$$

**Theorem 4 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

**Definition of** $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \left(\hat{P}_{24}\right)^{(4)} e^{(\hat{M}_{24})^{(4)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{24}\right)^{(4)} e^{(\hat{M}_{24})^{(4)}t} \;,\quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 5 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

**Definition of** $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \left(\hat{P}_{28}\right)^{(5)} e^{(\hat{M}_{28})^{(5)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{28}\right)^{(5)} e^{(\hat{M}_{28})^{(5)}t} \;,\quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 6 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

**Definition of** $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \left(\hat{P}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t} \;,\quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 7:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

**Definition of** $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \left(\hat{P}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t} \;,\quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 8:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

**Definition of** $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \left(\hat{P}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t} \;,\quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 9:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

**Definition of** $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \left(\hat{P}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

**Proof:** Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$$G_i(0) = G_i^0 \, , \, T_i(0) = T_i^0 \, , \, G_i^0 \leq (\hat{P}_{13})^{(1)} \, , T_i^0 \leq (\hat{Q}_{13})^{(1)}, \quad 155$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t} \quad 156$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t} \quad 157$$

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[ (a_{13})^{(1)} G_{14}(s_{(13)}) - \left( (a'_{13})^{(1)} + a''_{13})^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \right) G_{13}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[ (a_{14})^{(1)} G_{13}(s_{(13)}) - \left( (a'_{14})^{(1)} + (a''_{14})^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \right) G_{14}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[ (a_{15})^{(1)} G_{14}(s_{(13)}) - \left( (a'_{15})^{(1)} + (a''_{15})^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \right) G_{15}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[ (b_{13})^{(1)} T_{14}(s_{(13)}) - \left( (b'_{13})^{(1)} - (b''_{13})^{(1)} (G(s_{(13)}), s_{(13)}) \right) T_{13}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[ (b_{14})^{(1)} T_{13}(s_{(13)}) - \left( (b'_{14})^{(1)} - (b''_{14})^{(1)} (G(s_{(13)}), s_{(13)}) \right) T_{14}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{T}_{15}(t) = T_{15}^0 + \int_0^t \left[ (b_{15})^{(1)} T_{14}(s_{(13)}) - \left( (b'_{15})^{(1)} - (b''_{15})^{(1)} (G(s_{(13)}), s_{(13)}) \right) T_{15}(s_{(13)}) \right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0 \, , \, T_i(0) = T_i^0 \, , \, G_i^0 \leq (\hat{P}_{16})^{(2)} \, , T_i^0 \leq (\hat{Q}_{16})^{(2)},$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$$

By 160

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[ (a_{16})^{(2)} G_{17}(s_{(16)}) - \left( (a'_{16})^{(2)} + a''_{16})^{(2)} (T_{17}(s_{(16)}), s_{(16)}) \right) G_{16}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[ (a_{17})^{(2)} G_{16}(s_{(16)}) - \left( (a'_{17})^{(2)} + (a''_{17})^{(2)} (T_{17}(s_{(16)}), s_{(17)}) \right) G_{17}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[ (a_{18})^{(2)} G_{17}(s_{(16)}) - \left( (a'_{18})^{(2)} + (a''_{18})^{(2)} (T_{17}(s_{(16)}), s_{(16)}) \right) G_{18}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^{0} + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^{0} + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^{0} + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{18}(s_{(16)})\right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{20})^{(3)}$, $T_i^0 \le (\hat{Q}_{20})^{(3)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\tilde{M}_{20})^{(3)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\tilde{M}_{20})^{(3)} t}$

By 161

$$\bar{G}_{20}(t) = G_{20}^{0} + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''_{20})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^{0} + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^{0} + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{22}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^{0} + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^{0} + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{22}(t) = T_{22}^{0} + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{22}(s_{(20)})\right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{24})^{(4)}$, $T_i^0 \le (\hat{Q}_{24})^{(4)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\tilde{M}_{24})^{(4)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\tilde{M}_{24})^{(4)} t}$

By 162

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{26}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{\mathrm{T}}_{26}(\mathrm{t}) = \mathrm{T}_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{26}(s_{(24)})\right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{28})^{(5)}, T_i^0 \le (\hat{Q}_{28})^{(5)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

By 163

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{30}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{\mathrm{T}}_{30}(\mathrm{t}) = \mathrm{T}_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{30}(s_{(28)})\right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{32})^{(6)}, T_i^0 \le (\hat{Q}_{32})^{(6)},$$

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\bar{M}_{32})^{(6)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\bar{M}_{32})^{(6)} t}$

By 164

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[ (a_{32})^{(6)} G_{33}(s_{(32)}) - \left( (a'_{32})^{(6)} + a''_{32})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[ (a_{33})^{(6)} G_{32}(s_{(32)}) - \left( (a'_{33})^{(6)} + (a''_{33})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[ (a_{34})^{(6)} G_{33}(s_{(32)}) - \left( (a'_{34})^{(6)} + (a''_{34})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{34}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[ (b_{32})^{(6)} T_{33}(s_{(32)}) - \left( (b'_{32})^{(6)} - (b''_{32})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[ (b_{33})^{(6)} T_{32}(s_{(32)}) - \left( (b'_{33})^{(6)} - (b''_{33})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{34}(t) = T_{34}^0 + \int_0^t \left[ (b_{34})^{(6)} T_{33}(s_{(32)}) - \left( (b'_{34})^{(6)} - (b''_{34})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{34}(s_{(32)}) \right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[ (a_{36})^{(7)} G_{37}(s_{(36)}) - \left( (a'_{36})^{(7)} + a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[ (a_{37})^{(7)} G_{36}(s_{(36)}) - \left( (a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[ (a_{38})^{(7)} G_{37}(s_{(36)}) - \left( (a'_{38})^{(7)} + (a''_{38})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{38}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[ (b_{36})^{(7)} T_{37}(s_{(36)}) - \left( (b'_{36})^{(7)} - (b''_{36})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[ (b_{37})^{(7)} T_{36}(s_{(36)}) - \left( (b'_{37})^{(7)} - (b''_{37})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\overline{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^{0} + \int_0^t \left[ (b_{38})^{(7)} T_{37}(s_{(36)}) - \left( (b'_{38})^{(7)} - (b''_{38})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{38}(s_{(36)}) \right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{40})^{(8)}, T_i^0 \le (\hat{Q}_{40})^{(8)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[ (a_{40})^{(8)} G_{41}(s_{(40)}) - \left( (a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[ (a_{41})^{(8)} G_{40}(s_{(40)}) - \left( (a'_{41})^{(8)} + (a''_{41})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[ (a_{42})^{(8)} G_{41}(s_{(40)}) - \left( (a'_{42})^{(8)} + (a''_{42})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{42}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^0 + \int_0^t \left[ (b_{40})^{(8)} T_{41}(s_{(40)}) - \left( (b'_{40})^{(8)} - (b''_{40})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^0 + \int_0^t \left[ (b_{41})^{(8)} T_{40}(s_{(40)}) - \left( (b'_{41})^{(8)} - (b''_{41})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\overline{\mathrm{T}}_{42}(\mathrm{t}) = \mathrm{T}_{42}^{0} + \int_0^t \left[ (b_{42})^{(8)} T_{41}(s_{(40)}) - \left( (b'_{42})^{(8)} - (b''_{42})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{42}(s_{(40)}) \right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** 166

Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy A

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{44})^{(9)}, T_i^0 \le (\hat{Q}_{44})^{(9)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[ (a_{44})^{(9)} G_{45}(s_{(44)}) - \left( (a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45}(s_{(44)}), s_{(44)})\right) G_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}(s_{(44)}), s_{(44)})\right) G_{46}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{44}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{46}(t) = T_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{46}(s_{(44)})\right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}}\right)\right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left(e^{(\hat{M}_{13})^{(1)} t} - 1\right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\hat{M}_{13})^{(1)} t} \le \frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{P}_{13})^{(1)} + G_{14}^0\right) e^{\left(-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0}\right)} + (\hat{P}_{13})^{(1)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \le G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}}\right)\right] ds_{(16)} = \quad 169$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1\right)$$

From which it follows that 170

$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \le \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left[\left((\hat{P}_{16})^{(2)} + G_{17}^0\right) e^{\left(-\frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0}\right)} + (\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t) \le G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} \left(G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} s_{(20)}}\right)\right] ds_{(20)} =$$

$$\left(1 + (a_{20})^{(3)} t\right) G_{21}^0 + \frac{(a_{20})^{(3)} (\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left(e^{(\hat{M}_{20})^{(3)} t} - 1\right)$$

From which it follows that 172

$$(G_{20}(t)-G_{20}^{0})e^{-(\hat{M}_{20})^{(3)}t}\leq\frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left[((\hat{P}_{20})^{(3)}+G_{21}^{0})e^{\left(-\frac{(\hat{P}_{20})^{(3)}+G_{21}^{0}}{G_{21}^{0}}\right)}+(\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 173

$$G_{24}(t)\leq G_{24}^{0}+\int_{0}^{t}\left[(a_{24})^{(4)}\left(G_{25}^{0}+(\hat{P}_{24})^{(4)}e^{(\hat{M}_{24})^{(4)}s_{(24)}}\right)\right]ds_{(24)}=$$

$$\left(1+(a_{24})^{(4)}t\right)G_{25}^{0}+\frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left(e^{(\hat{M}_{24})^{(4)}t}-1\right)$$

From which it follows that 174

$$(G_{24}(t)-G_{24}^{0})e^{-(\hat{M}_{24})^{(4)}t}\leq\frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left[((\hat{P}_{24})^{(4)}+G_{25}^{0})e^{\left(-\frac{(\hat{P}_{24})^{(4)}+G_{25}^{0}}{G_{25}^{0}}\right)}+(\hat{P}_{24})^{(4)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t)\leq G_{28}^{0}+\int_{0}^{t}\left[(a_{28})^{(5)}\left(G_{29}^{0}+(\hat{P}_{28})^{(5)}e^{(\hat{M}_{28})^{(5)}s_{(28)}}\right)\right]ds_{(28)}=$$

$$\left(1+(a_{28})^{(5)}t\right)G_{29}^{0}+\frac{(a_{28})^{(5)}(\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left(e^{(\hat{M}_{28})^{(5)}t}-1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t}\leq\frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t)\leq G_{32}^{0}+\int_{0}^{t}\left[(a_{32})^{(6)}\left(G_{33}^{0}+(\hat{P}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\right)\right]ds_{(32)}=$$

$$\left(1+(a_{32})^{(6)}t\right)G_{33}^{0}+\frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left(e^{(\hat{M}_{32})^{(6)}t}-1\right)$$

From which it follows that 177

$$(G_{32}(t)-G_{32}^{0})e^{-(\hat{M}_{32})^{(6)}t}\leq\frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left[((\hat{P}_{32})^{(6)}+G_{33}^{0})e^{\left(-\frac{(\hat{P}_{32})^{(6)}+G_{33}^{0}}{G_{33}^{0}}\right)}+(\hat{P}_{32})^{(6)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}$ , $G_{26}, T_{24}, T_{25}, T_{26}$

(s) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t) \le G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} \left(G_{37}^0 + (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} s_{(36)}}\right)\right] ds_{(36)} =$$
$$\left(1 + (a_{36})^{(7)} t\right) G_{37}^0 + \frac{(a_{36})^{(7)} (\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left(e^{(\hat{M}_{36})^{(7)} t} - 1\right)$$

From which it follows that

$$(G_{36}(t) - G_{36}^0) e^{-(\hat{M}_{36})^{(7)} t} \le \frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left[((\hat{P}_{36})^{(7)} + G_{37}^0) e^{\left(-\frac{(\hat{P}_{36})^{(7)} + G_{37}^0}{G_{37}^0}\right)} + (\hat{P}_{36})^{(7)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t) \le G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} \left(G_{41}^0 + (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} s_{(40)}}\right)\right] ds_{(40)} = \quad 180$$
$$\left(1 + (a_{40})^{(8)} t\right) G_{41}^0 + \frac{(a_{40})^{(8)} (\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}} \left(e^{(\hat{M}_{40})^{(8)} t} - 1\right)$$

From which it follows that 181

$$(G_{40}(t) - G_{40}^0) e^{-(\hat{M}_{40})^{(8)} t} \le \frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}} \left[((\hat{P}_{40})^{(8)} + G_{41}^0) e^{\left(-\frac{(\hat{P}_{40})^{(8)} + G_{41}^0}{G_{41}^0}\right)} + (\hat{P}_{40})^{(8)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 8

Analogous inequalities hold also for $G_{41}$ , $G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t) \le G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} \left(G_{45}^0 + (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} s_{(44)}}\right)\right] ds_{(44)} =$$
$$\left(1 + (a_{44})^{(9)} t\right) G_{45}^0 + \frac{(a_{44})^{(9)} (\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left(e^{(\hat{M}_{44})^{(9)} t} - 1\right)$$

From which it follows that

$$(G_{44}(t) - G_{44}^0) e^{-(\hat{M}_{44})^{(9)} t} \le \frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left[((\hat{P}_{44})^{(9)} + G_{45}^0) e^{\left(-\frac{(\hat{P}_{44})^{(9)} + G_{45}^0}{G_{45}^0}\right)} + (\hat{P}_{44})^{(9)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 9

Analogous inequalities hold also for $G_{45}$ , $G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}$ , $\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$ and to choose 182

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[(\hat{P}_{13})^{(1)}+\left((\hat{P}_{13})^{(1)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{13})^{(1)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{13})^{(1)} \quad 183$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[\left((\hat{Q}_{13})^{(1)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{13})^{(1)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{13})^{(1)}\right]\leq(\hat{Q}_{13})^{(1)} \quad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)},T^{(1)}\right),\left(G^{(2)},T^{(2)}\right)\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{13})^{(1)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{13})^{(1)}t}\}$$

Indeed if we denote

**Definition of** $\tilde{G},\tilde{T}:\left(\tilde{G},\tilde{T}\right)=\mathcal{A}^{(1)}(G,T)$

It results

$$\left|\tilde{G}_{13}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{13})^{(1)}\left|G_{14}^{(1)}-G_{14}^{(2)}\right|e^{-(\hat{M}_{13})^{(1)}s_{(13)}}e^{(\hat{M}_{13})^{(1)}s_{(13)}}ds_{(13)}+$$

$$\int_0^t\{(a'_{13})^{(1)}\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\hat{M}_{13})^{(1)}s_{(13)}}e^{-(\hat{M}_{13})^{(1)}s_{(13)}}+$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\hat{M}_{13})^{(1)}s_{(13)}}e^{(\hat{M}_{13})^{(1)}s_{(13)}}+$$

$$G_{13}^{(2)}\left|(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)-(a''_{13})^{(1)}\left(T_{14}^{(2)},s_{(13)}\right)\right|\ e^{-(\hat{M}_{13})^{(1)}s_{(13)}}e^{(\hat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|G^{(1)}-G^{(2)}\right|e^{-(\hat{M}_{13})^{(1)}t}\leq \quad 186$$
$$\frac{1}{(\hat{M}_{13})^{(1)}}\left((a_{13})^{(1)}+(a'_{13})^{(1)}+(\hat{A}_{13})^{(1)}+(\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}\right)d\left(\left(G^{(1)},T^{(1)};\ G^{(2)},T^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

**Remark 1:** The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\hat{P}_{13})^{(1)}e^{(\hat{M}_{13})^{(1)}t}\ and\ (\hat{Q}_{13})^{(1)}e^{(\hat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i = 13,14,15$ depend only on $T_{14}$ and respectively on $G(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any $t$ where $G_i(t) = 0\ and\ T_i(t) = 0$

From 19 to 24 it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(1)} - (a_i'')^{(1)}(T_{14}(s_{(13)}), s_{(13)})\} ds_{(13)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(1)} t)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2\ and\ \left((\widehat{M}_{13})^{(1)}\right)_3$ : 187

**Remark 3**: if $G_{13}$ is bounded, the same property have also $G_{14}\ and\ G_{15}$ . indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a_{14}')^{(1)} G_{14}$ and by integrating

$$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)} \left((\widehat{M}_{13})^{(1)}\right)_1 / (a_{14}')^{(1)}$$

In the same way , one can obtain

$$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)} \left((\widehat{M}_{13})^{(1)}\right)_2 / (a_{15}')^{(1)}$$

If $G_{14}\ or\ G_{15}$ is bounded, the same property follows for $G_{13}$ , $G_{15}$ and $G_{13}$ , $G_{14}$ respectively.

**Remark 4:** If $G_{13}$ *is* bounded, from below, the same property holds for $G_{14}\ and\ G_{15}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{14}$ is bounded from below. 188

**Remark 5:** If $T_{13}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}(G(t), t)) = (b_{14}')^{(1)}$ then $T_{14} \to \infty$. 189

**Definition of** $(m)^{(1)}$ and $\varepsilon_1$ :

Indeed let $t_1$ be so that for $t > t_1$

$$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t), t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right)$, $t = log\frac{2}{\varepsilon_1}$ By taking now $\varepsilon_1$ sufficiently small one sees that $T_{14}$ is unbounded.
The same property holds for $T_{15}$ if $\lim_{t\to\infty}(b_{15}'')^{(1)}(G(t), t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}$ , $\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}\ and\ (\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[(\widehat{P}_{16})^{(2)}+\left((\hat{P}_{16})^{(2)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{16})^{(2)}\right]\leq(\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

**Definition of** $\widetilde{G_{19}},\widetilde{T_{19}}$ : $\left(\widetilde{G_{19}},\widetilde{T_{19}}\right)=\mathcal{A}^{(2)}(G_{19},T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{16})^{(2)}\left|G_{17}^{(1)}-G_{17}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)}+$$

$$\int_0^t\{(a'_{16})^{(2)}\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$G_{16}^{(2)}\left|(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)-(a''_{16})^{(2)}\left(T_{17}^{(2)},s_{(16)}\right)\right|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0,t]$ 197

From the hypotheses it follows

$$\left|(G_{19})^{(1)}-(G_{19})^{(2)}\right|\mathrm{e}^{-(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}\leq$$
$$\frac{1}{(\widehat{\mathrm{M}}_{16})^{(2)}}\left((a_{16})^{(2)}+(a'_{16})^{(2)}+(\widehat{\mathrm{A}}_{16})^{(2)}+(\widehat{\mathrm{P}}_{16})^{(2)}(\hat{k}_{16})^{(2)}\right)\mathrm{d}\left(((G_{19})^{(1)},(T_{19})^{(1)};\ (G_{19})^{(2)},(T_{19})^{(2)})\right)$$

And analogous inequalities for $G_i$ and $T_i$. Taking into account the hypothesis the result follows 198

**Remark 6:** The fact that we supposed $(a''_{16})^{(2)}$ and $(b''_{16})^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{\mathrm{P}}_{16})^{(2)}\mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}$ and $(\widehat{\mathrm{Q}}_{16})^{(2)}\mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i=16,17,18$ depend only on $\mathrm{T}_{17}$ and respectively on

$(G_{19})$(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 7:** There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$ 200

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(2)} - (a_i'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})\} ds_{(16)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(2)}t)} > 0 \quad \text{for } t > 0$$

**Definition of** $\left((\widehat{M}_{16})^{(2)}\right)_1$, $\left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$: 201

**Remark 8:** if $G_{16}$ is bounded, the same property have also $G_{17}$ and $G_{18}$. indeed if

$G_{16} < (\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt} \leq \left((\widehat{M}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)} G_{17}$ and by integrating

$$G_{17} \leq \left((\widehat{M}_{16})^{(2)}\right)_2 = G_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1 / (a_{17}')^{(2)}$$

In the same way , one can obtain

$$G_{18} \leq \left((\widehat{M}_{16})^{(2)}\right)_3 = G_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2 / (a_{18}')^{(2)}$$

If $G_{17}$ or $G_{18}$ is bounded, the same property follows for $G_{16}$, $G_{18}$ and $G_{16}$, $G_{17}$ respectively.

**Remark 9:** If $G_{16}$ is bounded, from below, the same property holds for $G_{17}$ and $G_{18}$. The proof is analogous with the preceding one. An analogous property is true if $G_{17}$ is bounded from below. 202

**Remark 10:** If $T_{16}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t), t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$. 203

**Definition of** $(m)^{(2)}$ and $\varepsilon_2$ :

Indeed let $t_2$ be so that for $t > t_2$

$$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right)$, $t = \log\frac{2}{\varepsilon_2}$ By taking now $\varepsilon_2$ sufficiently small one sees that $T_{17}$ is unbounded. 205
The same property holds for $T_{18}$ if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t), t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[(\widehat{P}_{20})^{(3)}+\left((\hat{P}_{20})^{(3)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{20})^{(3)}\right]\leq(\hat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself 210

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)},(T_{23})^{(1)}),((G_{23})^{(2)},(T_{23})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

**Definition of** $\widetilde{G_{23}},\widetilde{T_{23}}:(\widetilde{(G_{23})},\widetilde{(T_{23})})=\mathcal{A}^{(3)}((G_{23}),(T_{23}))$

It results 213

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{20})^{(3)}\left|G_{21}^{(1)}-G_{21}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}ds_{(20)}+$$

$$\int_0^t\{(a'_{20})^{(3)}\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$G_{20}^{(2)}\left|(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)},s_{(20)}\right)\right|\ e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\}ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses it follows

$$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}t}\leq \quad 214$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+(a'_{20})^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\widehat{k}_{20})^{(3)}\right)d\left(((G_{23})^{(1)},(T_{23})^{(1)};\ (G_{23})^{(2)},(T_{23})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 11:** The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ *and* $(\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i=20,21,22$ depend only on $T_{21}$ and respectively on

$(G_{23})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 12:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 216

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(3)} - (a_i'')^{(3)}(T_{21}(s_{(20)}), s_{(20)})\} ds_{(20)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(3)} t)} > 0$ for t > 0

**Definition of** $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2\ and\ \left((\widehat{M}_{20})^{(3)}\right)_3$ : 217

**Remark 13:** if $G_{20}$ is bounded, the same property have also $G_{21}\ and\ G_{22}$ . indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a_{21}')^{(3)} G_{21}$ and by integrating

$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1 / (a_{21}')^{(3)}$

In the same way , one can obtain

$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2 / (a_{22}')^{(3)}$

If $G_{21}\ or\ G_{22}$ is bounded, the same property follows for $G_{20}$ , $G_{22}$ and $G_{20}$ , $G_{21}$ respectively.

**Remark 14:** If $G_{20}\ \ is$ bounded, from below, the same property holds for $G_{21}\ and\ G_{22}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{21}$ is bounded from below. 218

**Remark 15:** If $T_{20}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t), t\right)) = (b_{21}')^{(3)}$ then $T_{21} \to \infty$. 219

**Definition of** $(m)^{(3)}$ and $\varepsilon_3$ :

Indeed let $t_3$ be so that for $t > t_3$

$(b_{21})^{(3)} - (b_i'')^{(3)}\left((G_{23})(t), t\right) < \varepsilon_3, T_{20}\,(t) > (m)^{(3)}$

Then $\frac{dT_{21}}{dt} \geq (a_{21})^{(3)}(m)^{(3)} - \varepsilon_3 T_{21}$ which leads to 220

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t} = \frac{1}{2}$ it results

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right),\ \ t = log\,\frac{2}{\varepsilon_3}$ By taking now $\varepsilon_3$ sufficiently small one sees that $T_{21}$ is unbounded.
The same property holds for $T_{22}$ if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t), t\right) = (b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}$ , $\frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}} < 1$ and to choose 221

$(\widehat{P}_{24})^{(4)}$ and $(\widehat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[(\widehat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \tag{222}$$

$$\frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \tag{223}$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself (224)

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric (225)

$$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{24})^{(4)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{27}}),(\widetilde{T_{27}})$ : $\left((\widetilde{G_{27}}),(\widetilde{T_{27}})\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{24})^{(4)}\left|G_{25}^{(1)}-G_{25}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\,ds_{(24)}+$$

$$\int_0^t\{(a'_{24})^{(4)}\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$G_{24}^{(2)}\left|(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)-(a''_{24})^{(4)}\left(T_{25}^{(2)},s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\}ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}t}\leq \tag{226}$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)}+(a'_{24})^{(4)}+(\widehat{A}_{24})^{(4)}+(\widehat{P}_{24})^{(4)}(\hat{k}_{24})^{(4)}\right)d\left(((G_{27})^{(1)},(T_{27})^{(1)};\ (G_{27})^{(2)},(T_{27})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 16:** The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ *and* $(\widehat{Q}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_+$. (227)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i=24,25,26$ depend only on $T_{25}$ and respectively on

$(G_{27})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 17:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 228

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(4)} - (a_i'')^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(4)} t)} > 0$ for t > 0

**Definition of** $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2\ and\ \left((\widehat{M}_{24})^{(4)}\right)_3$ : 229

**Remark 18:** if $G_{24}$ is bounded, the same property have also $G_{25}\ and\ G_{26}$ . indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a_{25}')^{(4)} G_{25}$ and by integrating

$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1 / (a_{25}')^{(4)}$

In the same way , one can obtain

$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2 / (a_{26}')^{(4)}$

If $G_{25}\ or\ G_{26}$ is bounded, the same property follows for $G_{24}$ , $G_{26}$ and $G_{24}$ , $G_{25}$ respectively.

**Remark 19:** If $G_{24}$ *is* bounded, from below, the same property holds for $G_{25}\ and\ G_{26}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{25}$ is bounded from below. 230

**Remark 20:** If $T_{24}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}\ ((G_{27})(t), t)) = (b_{25}')^{(4)}$ then $T_{25} \to \infty$. 231

**Definition of** $(m)^{(4)}$ and $\varepsilon_4$ :

Indeed let $t_4$ be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t), t) < \varepsilon_4, T_{24}\,(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_4}$ By taking now $\varepsilon_4$ sufficiently small one sees that $T_{25}$ is unbounded.
The same property holds for $T_{26}$ if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t), t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for $G_{29}$ , $G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}$ , $\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)}+\left((\hat{P}_{28})^{(5)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{28})^{(5)}\right]\leq(\hat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$$d\left(((G_{31})^{(1)},(T_{31})^{(1)}),((G_{31})^{(2)},(T_{31})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\hat{M}_{28})^{(5)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\hat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{31}}),(\widetilde{T_{31}})$ : $\left((\widetilde{G_{31}}),(\widetilde{T_{31}})\right)=\mathcal{A}^{(5)}\left((G_{31}),(T_{31})\right)$

It results

$$|\tilde{G}_{28}^{(1)}-\tilde{G}_i^{(2)}|\leq\int_0^t(a_{28})^{(5)}|G_{29}^{(1)}-G_{29}^{(2)}|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\,ds_{(28)}+$$

$$\int_0^t\{(a'_{28})^{(5)}|G_{28}^{(1)}-G_{28}^{(2)}|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$(a''_{28})^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)|G_{28}^{(1)}-G_{28}^{(2)}|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$G_{28}^{(2)}|(a''_{28})^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)-(a''_{28})^{(5)}\left(T_{29}^{(2)},s_{(28)}\right)|\;e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\}ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on it follows

$$|(G_{31})^{(1)}-(G_{31})^{(2)}|e^{-(\widehat{M}_{28})^{(5)}t}\leq \quad 237$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)}+(a'_{28})^{(5)}+(\widehat{A}_{28})^{(5)}+(\widehat{P}_{28})^{(5)}(\widehat{k}_{28})^{(5)}\right)d\left(((G_{31})^{(1)},(T_{31})^{(1)};\;(G_{31})^{(2)},(T_{31})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 21:** The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\hat{P}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ *and* $(\hat{Q}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ respectively of $\mathbb{R}_+$. 238

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on $T_{29}$ and respectively on $(G_{31})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 22:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 239

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(5)} - (a_i'')^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(5)} t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2\ and\ \left((\widehat{M}_{28})^{(5)}\right)_3$ : 240

**Remark 23:** if $G_{28}$ is bounded, the same property have also $G_{29}\ and\ G_{30}$ . indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a_{29}')^{(5)} G_{29}$ and by integrating

$$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1 / (a_{29}')^{(5)}$$

In the same way , one can obtain

$$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2 / (a_{30}')^{(5)}$$

If $G_{29}\ or\ G_{30}$ is bounded, the same property follows for $G_{28}$ , $G_{30}$ and $G_{28}$ , $G_{29}$ respectively.

**Remark 24:** If $G_{28}$ *is* bounded, from below, the same property holds for $G_{29}\ and\ G_{30}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{29}$ is bounded from below. 241

**Remark 25:** If $T_{28}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}\,((G_{31})(t), t)) = (b_{29}')^{(5)}$ then $T_{29} \to \infty$. 242

**Definition of** $(m)^{(5)}$ and $\varepsilon_5$ :

Indeed let $t_5$ be so that for $t > t_5$

$$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t), t) < \varepsilon_5, T_{28}\,(t) > (m)^{(5)}$$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_5}$ By taking now $\varepsilon_5$ sufficiently small one sees that $T_{29}$ is unbounded.
The same property holds for $T_{30}$ if $\lim_{t\to\infty}(b_{30}'')^{(5)}\left((G_{31})(t), t\right) = (b_{30}')^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{33}$ , $G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}$ , $\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}} < 1$ and to choose 244

$(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[(\hat{P}_{32})^{(6)}+\left((\hat{P}_{32})^{(6)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{32})^{(6)} \tag{245}$$

$$\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[\left((\hat{Q}_{32})^{(6)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{32})^{(6)}\right]\leq(\hat{Q}_{32})^{(6)} \tag{246}$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric (247)

$$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\hat{M}_{32})^{(6)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\hat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{35}}),(\widetilde{T_{35}})$ : $\left((\widetilde{G_{35}}),(\widetilde{T_{35}})\right)=\mathcal{A}^{(6)}\left((G_{35}),(T_{35})\right)$

It results

$$|\tilde{G}_{32}^{(1)}-\tilde{G}_i^{(2)}|\leq\int_0^t(a_{32})^{(6)}|G_{33}^{(1)}-G_{33}^{(2)}|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\,ds_{(32)}+$$

$$\int_0^t\{(a'_{32})^{(6)}|G_{32}^{(1)}-G_{32}^{(2)}|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)|G_{32}^{(1)}-G_{32}^{(2)}|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$G_{32}^{(2)}|(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)-(a''_{32})^{(6)}\left(T_{33}^{(2)},s_{(32)}\right)|\;e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\}ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$|(G_{35})^{(1)}-(G_{35})^{(2)}|e^{-(\widehat{M}_{32})^{(6)}t}\leq \tag{248}$$
$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)}+(a'_{32})^{(6)}+(\widehat{A}_{32})^{(6)}+(\hat{P}_{32})^{(6)}(\hat{k}_{32})^{(6)}\right)d\left(((G_{35})^{(1)},(T_{35})^{(1)};\;(G_{35})^{(2)},(T_{35})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 26:** The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\hat{P}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ *and* $(\hat{Q}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ respectively of $\mathbb{R}_+$. (249)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i = 32,33,34$ depend only on $T_{33}$ and respectively on $(G_{35})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 27:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 250

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(6)} - (a_i'')^{(6)}(T_{33}(s_{(32)}), s_{(32)})\} ds_{(32)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(6)} t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{32})^{(6)}\right)_1, \left((\widehat{M}_{32})^{(6)}\right)_2\ and\ \left((\widehat{M}_{32})^{(6)}\right)_3$ : 251

**Remark 28:** if $G_{32}$ is bounded, the same property have also $G_{33}\ and\ G_{34}$ . indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a_{33}')^{(6)} G_{33}$ and by integrating

$$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)} \left((\widehat{M}_{32})^{(6)}\right)_1 / (a_{33}')^{(6)}$$

In the same way , one can obtain

$$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)} \left((\widehat{M}_{32})^{(6)}\right)_2 / (a_{34}')^{(6)}$$

If $G_{33}\ or\ G_{34}$ is bounded, the same property follows for $G_{32}$ , $G_{34}$ and $G_{32}$ , $G_{33}$ respectively.

**Remark 29:** If $G_{32}$ *is* bounded, from below, the same property holds for $G_{33}\ and\ G_{34}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{33}$ is bounded from below. 252

**Remark 30:** If $T_{32}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}\ ((G_{35})(t), t)) = (b_{33}')^{(6)}$ then $T_{33} \to \infty$. 253

**Definition of** $(m)^{(6)}$ and $\varepsilon_6$ :

Indeed let $t_6$ be so that for $t > t_6$

$$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t), t\right) < \varepsilon_6, T_{32}\,(t) > (m)^{(6)}$$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right)$, $t = log\,\frac{2}{\varepsilon_6}$ By taking now $\varepsilon_6$ sufficiently small one sees that $T_{33}$ is unbounded.
The same property holds for $T_{34}$ if $\lim_{t\to\infty}(b_{34}'')^{(6)}\left((G_{35})(t), t(t), t\right) = (b_{34}')^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{37}, G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}$ , $\frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$ and to choose

$(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[(\hat{P}_{36})^{(7)}+\left((\hat{P}_{36})^{(7)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{36})^{(7)} \tag{256}$$

$$\frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[\left((\hat{Q}_{36})^{(7)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{36})^{(7)}\right]\leq(\hat{Q}_{36})^{(7)} \tag{257}$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself
The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric (258)

$$d\left(((G_{39})^{(1)},(T_{39})^{(1)}),((G_{39})^{(2)},(T_{39})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\hat{M}_{36})^{(7)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\hat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

**Definition of** $\widetilde{(G_{39})},\widetilde{(T_{39})}:(\widetilde{(G_{39})},\widetilde{(T_{39})})=\mathcal{A}^{(7)}((G_{39}),(T_{39}))$
It results

$$|\tilde{G}_{36}^{(1)}-\tilde{G}_i^{(2)}|\leq\int_0^t(a_{36})^{(7)}|G_{37}^{(1)}-G_{37}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\,ds_{(36)}+$$
$$\int_0^t\{(a'_{36})^{(7)}|G_{36}^{(1)}-G_{36}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$(a''_{36})^{(7)}(T_{37}^{(1)},s_{(36)})|G_{36}^{(1)}-G_{36}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$G_{36}^{(2)}|(a''_{36})^{(7)}(T_{37}^{(1)},s_{(36)})-(a''_{36})^{(7)}(T_{37}^{(2)},s_{(36)})|\;e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\}ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on it follows

$$|(G_{39})^{(1)}-(G_{39})^{(2)}|e^{-(\widehat{M}_{36})^{(7)}t}\leq \tag{259}$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}}\left((a_{36})^{(7)}+(a'_{36})^{(7)}+(\hat{A}_{36})^{(7)}+(\hat{P}_{36})^{(7)}(\hat{k}_{36})^{(7)}\right)d\left(((G_{39})^{(1)},(T_{39})^{(1)};\;(G_{39})^{(2)},(T_{39})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 31:** The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\hat{P}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ *and* $(\hat{Q}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ respectively of $\mathbb{R}_+$. (260)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(7)}$ and $(b_i'')^{(7)}, i = 36,37,38$ depend only on $T_{37}$ and respectively on $(G_{39})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 32:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 261

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(7)} - (a_i'')^{(7)}(T_{37}(s_{(36)}), s_{(36)})\} ds_{(36)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(7)} t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2\ and\ \left((\widehat{M}_{36})^{(7)}\right)_3$ : 262

**Remark 33:** if $G_{36}$ is bounded, the same property have also $G_{37}\ and\ G_{38}$ . indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a_{37}')^{(7)} G_{37}$ and by integrating

$$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1 / (a_{37}')^{(7)}$$

In the same way , one can obtain

$$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2 / (a_{38}')^{(7)}$$

If $G_{37}\ or\ G_{38}$ is bounded, the same property follows for $G_{36}$ , $G_{38}$ and $G_{36}$ , $G_{37}$ respectively.

**Remark 34:** If $G_{36}$ *is* bounded, from below, the same property holds for $G_{37}\ and\ G_{38}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{37}$ is bounded from below. 263

**Remark 35:** If $T_{36}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(7)}\ ((G_{39})(t), t)) = (b_{37}')^{(7)}$ then $T_{37} \to \infty$. 264

**Definition of** $(m)^{(7)}$ and $\varepsilon_7$ :

Indeed let $t_7$ be so that for $t > t_7$

$$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t), t) < \varepsilon_7, T_{36}\,(t) > (m)^{(7)}$$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_7}$ By taking now $\varepsilon_7$ sufficiently small one sees that $T_{37}$ is unbounded.
The same property holds for $T_{38}$ if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t), t\right) = (b_{38}')^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}$ , $\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}} < 1$ and to choose $(\widehat{P}_{40})^{(8)}$ and $(\widehat{Q}_{40})^{(8)}$ large to have 266

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[(\widehat{P}_{40})^{(8)} + \left((\hat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{40})^{(8)} \quad 267$$

$$\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{40})^{(8)}\right] \leq (\hat{Q}_{40})^{(8)} \quad 268$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)}, (T_{43})^{(1)}), ((G_{43})^{(2)}, (T_{43})^{(2)})\right) = \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

**Definition of** $\widetilde{(G_{43})}, \widetilde{(T_{43})}$ : $\left(\widetilde{(G_{43})}, \widetilde{(T_{43})}\right) = \mathcal{A}^{(8)}((G_{43}), (T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{40})^{(8)}\left|G_{41}^{(1)} - G_{41}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\,ds_{(40)} +$$
$$\int_0^t\{(a'_{40})^{(8)}\left|G_{40}^{(1)} - G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{-(\widehat{M}_{40})^{(8)}s_{(40)}} +$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right)\left|G_{40}^{(1)} - G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}} +$$
$$G_{40}^{(2)}\left|(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right) - (a''_{40})^{(8)}\left(T_{41}^{(2)}, s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\}ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0, t]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)} - (G_{43})^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}t} \leq \quad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)} + (a'_{40})^{(8)} + (\widehat{A}_{40})^{(8)} + (\widehat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}\right)d\left(((G_{43})^{(1)}, (T_{43})^{(1)};\ (G_{43})^{(2)}, (T_{43})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

**Remark 36:** The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate 274

condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ *and* $(\widehat{Q}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i = 40{,}41{,}42$ depend only on $T_{41}$ and respectively on $(G_{43})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 37** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 275

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t\{(a_i')^{(8)}-(a_i'')^{(8)}(T_{41}(s_{(40)}),s_{(40)})\}ds_{(40)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(8)}t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{40})^{(8)}\right)_1, \left((\widehat{M}_{40})^{(8)}\right)_2\ and\ \left((\widehat{M}_{40})^{(8)}\right)_3$ : 276

**Remark 38:** if $G_{40}$ is bounded, the same property have also $G_{41}\ and\ G_{42}$ . indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a_{41}')^{(8)}G_{41}$ and by integrating

$$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1/(a_{41}')^{(8)}$$

In the same way , one can obtain

$$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2/(a_{42}')^{(8)}$$

If $G_{41}\ or\ G_{42}$ is bounded, the same property follows for $G_{40}$ , $G_{42}$ and $G_{40}$ , $G_{41}$ respectively.

**Remark 39:** If $G_{40}$ *is* bounded, from below, the same property holds for $G_{41}\ and\ G_{42}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{41}$ is bounded from below. 277

**Remark 40:** If $T_{40}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}\ ((G_{43})(t),t)) = (b_{41}')^{(8)}$ then $T_{41} \to \infty$. 278

**Definition of** $(m)^{(8)}$ and $\varepsilon_8$ :

Indeed let $t_8$ be so that for $t\ > t_8$

$$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t),t\right) < \varepsilon_8, T_{40}\,(t) > (m)^{(8)}$$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right)$, $t = log\frac{2}{\varepsilon_8}$ By taking now $\varepsilon_8$ sufficiently small one sees that $T_{41}$ is unbounded.
The same property holds for $T_{42}$ if $\lim_{t\to\infty}(b''_{42})^{(8)}\left((G_{43})(t), t(t), t\right) = (b'_{42})^{(8)}$



It is now sufficient to take $\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}$ , $\frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$ and to choose $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ large to have

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}\left[(\hat{P}_{44})^{(9)} + \left((\hat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}}\left[\left((\hat{Q}_{44})^{(9)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{44})^{(9)}\right] \leq (\hat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)}, (T_{47})^{(1)}), ((G_{47})^{(2)}, (T_{47})^{(2)})\right) =$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{44})^{(9)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{47}}), (\widetilde{T_{47}}) : \left((\widetilde{G_{47}}), (\widetilde{T_{47}})\right) = \mathcal{A}^{(9)}\left((G_{47}), (T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{44})^{(9)}\left|G_{45}^{(1)} - G_{45}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\,ds_{(44)} +$$
$$\int_0^t\{(a'_{44})^{(9)}\left|G_{44}^{(1)} - G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{-(\widehat{M}_{44})^{(9)}s_{(44)}} +$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right)\left|G_{44}^{(1)} - G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}} +$$
$$G_{44}^{(2)}\left|(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right) - (a''_{44})^{(9)}\left(T_{45}^{(2)}, s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\}ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)} - G^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}t} \leq$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)} + (a'_{44})^{(9)} + (\widehat{A}_{44})^{(9)} + (\widehat{P}_{44})^{(9)}(\hat{k}_{44})^{(9)}\right)d\left(((G_{47})^{(1)}, (T_{47})^{(1)};\ (G_{47})^{(2)}, (T_{47})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis (39,35,36) the result follows

**Remark 41:** The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by

$(\widehat{P}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ *and* $(\widehat{Q}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i = 44{,}45{,}46$ depend only on $T_{45}$ and respectively on $(G_{47})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 42:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$

From 99 to 44 it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t\{(a_i')^{(9)}-(a_i'')^{(9)}(T_{45}(s_{(44)}),s_{(44)})\}ds_{(44)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(9)}t)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{44})^{(9)}\right)_1, \left((\widehat{M}_{44})^{(9)}\right)_2$ *and* $\left((\widehat{M}_{44})^{(9)}\right)_3$ :

**Remark 43:** if $G_{44}$ is bounded, the same property have also $G_{45}\ and\ G_{46}$ . indeed if

$G_{44} < (\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt} \leq \left((\widehat{M}_{44})^{(9)}\right)_1 - (a_{45}')^{(9)}G_{45}$ and by integrating

$$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_1/(a_{45}')^{(9)}$$

In the same way , one can obtain

$$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_2/(a_{46}')^{(9)}$$

If $G_{45}\ or\ G_{46}$ is bounded, the same property follows for $G_{44}$ , $G_{46}$ and $G_{44}$ , $G_{45}$ respectively.

**Remark 44:** If $G_{44}$ *is* bounded, from below, the same property holds for $G_{45}\ and\ G_{46}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{45}$ is bounded from below.

**Remark 45:** If $T_{44}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t),t\right) = (b_{45}')^{(9)}$ then $T_{45} \to \infty$.

**Definition of** $(m)^{(9)}$ and $\varepsilon_9$ :

Indeed let $t_9$ be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t),t\right) < \varepsilon_9, T_{44}\,(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right), \quad t = log\frac{2}{\varepsilon_9}$ By taking now $\varepsilon_9$ sufficiently small one sees that $T_{45}$ is unbounded.
The same property holds for $T_{46}$ if $\lim_{t\to\infty}(b_{46}'')^{(9)}\left((G_{47})(t),t\right) = (b_{46}')^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

**Behavior of the solutions of equation** 280

**Theorem** If we denote and define

**Definition of** $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ :

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)} \le -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14}, t) + (a''_{14})^{(1)}(T_{14}, t) \le -(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)} \le -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G, t) - (b''_{14})^{(1)}(G, t) \le -(\tau_1)^{(1)}$

**Definition of** $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$ : 281

By $(\nu_1)^{(1)} > 0$, $(\nu_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$, $(u_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

**Definition of** $(\bar{\nu}_1)^{(1)},, (\bar{\nu}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$ : 282

By $(\bar{\nu}_1)^{(1)} > 0$, $(\bar{\nu}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$, $(\bar{u}_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

**Definition of** $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (\nu_0)^{(1)}$ :- 283

If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by
$(m_2)^{(1)} = (\nu_0)^{(1)}, (m_1)^{(1)} = (\nu_1)^{(1)}$, $if\ (\nu_0)^{(1)} < (\nu_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}, if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)}$,

and $\boxed{(\nu_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)}$, $if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)}$, $if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)}$,

and $\boxed{(u_0)^{(1)} = \frac{T^0_{13}}{T^0_{14}}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$

are defined

Then the solution of global equations satisfies the inequalities 285

$G^0_{13}e^{\left((S_1)^{(1)} - (p_{13})^{(1)}\right)t} \le G_{13}(t) \le G^0_{13}e^{(S_1)^{(1)}t}$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)}-(p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)}t}$$

$$\left(\frac{(a_{15})^{(1)}G_{13}^0}{(m_1)^{(1)}((S_1)^{(1)}-(p_{13})^{(1)}-(S_2)^{(1)})}\left[e^{((S_1)^{(1)}-(p_{13})^{(1)})t} - e^{-(S_2)^{(1)}t}\right] + G_{15}^0 e^{-(S_2)^{(1)}t} \le G_{15}(t) \le \right. \quad (286)$$

$$\left. \frac{(a_{15})^{(1)}G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)}-(a'_{15})^{(1)})}\left[e^{(S_1)^{(1)}t} - e^{-(a'_{15})^{(1)}t}\right] + G_{15}^0 e^{-(a'_{15})^{(1)}t}\right)$$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)}t} \le T_{13}(t) \le T_{13}^0 e^{((R_1)^{(1)}+(r_{13})^{(1)})t}} \quad (287)$$

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)}t} \le T_{13}(t) \le \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)}+(r_{13})^{(1)})t} \quad (288)$$

$$\frac{(b_{15})^{(1)}T_{13}^0}{(\mu_1)^{(1)}((R_1)^{(1)}-(b'_{15})^{(1)})}\left[e^{(R_1)^{(1)}t} - e^{-(b'_{15})^{(1)}t}\right] + T_{15}^0 e^{-(b'_{15})^{(1)}t} \le T_{15}(t) \le \quad (289)$$

$$\frac{(a_{15})^{(1)}T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)}+(r_{13})^{(1)}+(R_2)^{(1)})}\left[e^{((R_1)^{(1)}+(r_{13})^{(1)})t} - e^{-(R_2)^{(1)}t}\right] + T_{15}^0 e^{-(R_2)^{(1)}t}$$

**Definition of** $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:- (290)

Where $(S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$

$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$

$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$

$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$

**Behavior of the solutions of equation** (291)

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ : (292)

$(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ four constants satisfying

$-(\sigma_2)^{(2)} \le -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(T_{17}, t) + (a''_{17})^{(2)}(T_{17}, t) \le -(\sigma_1)^{(2)}$ (293)

$-(\tau_2)^{(2)} \le -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}((G_{19}), t) - (b''_{17})^{(2)}((G_{19}), t) \le -(\tau_1)^{(2)}$ (294)

**Definition of** $(\nu_1)^{(2)}, (\nu_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$ : (295)

By $(\nu_1)^{(2)} > 0$, $(\nu_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0$, $(u_2)^{(2)} < 0$ the roots (296)

of the equations $(a_{17})^{(2)}(\nu^{(2)})^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ (297)

and $(b_{14})^{(2)}(u^{(2)})^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and (298)

**Definition of** $(\bar{\nu}_1)^{(2)}, , (\bar{\nu}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$ : (299)

By $(\bar{\nu}_1)^{(2)} > 0$, $(\bar{\nu}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0$, $(\bar{u}_2)^{(2)} < 0$ the (300)

roots of the equations $(a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 301

and $(b_{17})^{(2)}\left(u^{(2)}\right)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 302

**Definition of** $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ :- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, ***if*** $(u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$, ***if*** $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)}$, ***if*** $(\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)}t}$

$(p_i)^{(2)}$ is defined by equation

$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)}t}$ 311

$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)}t}\right] + G_{18}^0 e^{-(S_2)^{(2)}t} \le G_{18}(t) \le\right.$ 312

$\left.\frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)}t} - e^{-(a'_{18})^{(2)}t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)}t}\right)$

$\boxed{T_{16}^0 e^{(R_1)^{(2)}t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}}$ 313

$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)}t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}$ 314

$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)}t} - e^{-(b'_{18})^{(2)}t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)}t} \le T_{18}(t) \le$ 315

$$\frac{(a_{18})^{(2)}T_{16}^{0}}{(\mu_2)^{(2)}((R_1)^{(2)}+(r_{16})^{(2)}+(R_2)^{(2)})}\left[e^{((R_1)^{(2)}+(r_{16})^{(2)})t}-e^{-(R_2)^{(2)}t}\right]+T_{18}^{0}e^{-(R_2)^{(2)}t}$$

**Definition of** $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$$

$$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)}$$ 318

$$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$$

**Behavior of the solutions** 319

**Theorem 3:** If we denote and define

**Definition of** $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ :

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}\big((G_{23}), t\big) \le -(\tau_1)^{(3)}$$

**Definition of** $(v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$ : 320

By $(v_1)^{(3)} > 0$, $(v_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{v}_1)^{(3)} > 0$, $(\bar{v}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

**Definition of** $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ :- 321

If we define $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (v_0)^{(3)}, (m_1)^{(3)} = (v_1)^{(3)}$, ***if*** $(v_0)^{(3)} < (v_1)^{(3)}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (\bar{v}_1)^{(3)}$, ***if*** $(v_1)^{(3)} < (v_0)^{(3)} < (\bar{v}_1)^{(3)}$,

and $\boxed{(v_0)^{(3)} = \frac{G_{20}^{0}}{G_{21}^{0}}}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_0)^{(3)}$, ***if*** $(\bar{v}_1)^{(3)} < (v_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)},\ \boldsymbol{if}\ (u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}, \boldsymbol{if}\ (u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)},$ and $\boxed{(u_0)^{(3)} = \frac{T_{20}^0}{T_{21}^0}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)},\ \boldsymbol{if}\ (\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G_{20}^0 e^{(S_1)^{(3)} t}$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)} t} \quad 323$$

$$\left(\frac{(a_{22})^{(3)} G_{20}^0}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\left[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)} t}\right] + G_{22}^0 e^{-(S_2)^{(3)} t} \le G_{22}(t) \le \right. \quad 324$$

$$\left. \frac{(a_{22})^{(3)} G_{20}^0}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}\left[e^{(S_1)^{(3)} t} - e^{-(a'_{22})^{(3)} t}\right] + G_{22}^0 e^{-(a'_{22})^{(3)} t}\right)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t}} \quad 325$$

$$\frac{1}{(\mu_1)^{(3)}} T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}} T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)} T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)} - (b'_{22})^{(3)})}\left[e^{(R_1)^{(3)} t} - e^{-(b'_{22})^{(3)} t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)} t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)} T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)} + (r_{20})^{(3)} + (R_2)^{(3)})}\left[e^{((R_1)^{(3)} + (r_{20})^{(3)})t} - e^{-(R_2)^{(3)} t}\right] + T_{22}^0 e^{-(R_2)^{(3)} t}$$

**Definition of** $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

**Behavior of the solutions of equation**

**Theorem:** If we denote and define

**Definition of** $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ :

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$

$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}((G_{27}), t) - (b''_{25})^{(4)}((G_{27}), t) \le -(\tau_1)^{(4)}$

**Definition of** $(v_1)^{(4)}, (v_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, v^{(4)}, u^{(4)}$ : 329

By $(v_1)^{(4)} > 0$ , $(v_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$ , $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_1)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

**Definition of** $(\bar{v}_1)^{(4)},, (\bar{v}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$ : 330

By $(\bar{v}_1)^{(4)} > 0$ , $(\bar{v}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$ , $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

**Definition of** $(m_1)^{(4)}$ , $(m_2)^{(4)}$ , $(\mu_1)^{(4)}, (\mu_2)^{(4)}, (\nu_0)^{(4)}$ :-

If we define $(m_1)^{(4)}$ , $(m_2)^{(4)}$ , $(\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (v_0)^{(4)}, (m_1)^{(4)} = (v_1)^{(4)}$, ***if*** $(v_0)^{(4)} < (v_1)^{(4)}$

$(m_2)^{(4)} = (v_1)^{(4)}, (m_1)^{(4)} = (\bar{v}_1)^{(4)}$ , ***if*** $(v_4)^{(4)} < (v_0)^{(4)} < (\bar{v}_1)^{(4)}$,
and $\boxed{(v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (v_4)^{(4)}, (m_1)^{(4)} = (v_0)^{(4)}$, ***if*** $(\bar{v}_4)^{(4)} < (v_0)^{(4)}$

and analogously

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$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)}$, ***if*** $(u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}$ , ***if*** $(u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)}$,
and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}$, ***if*** $(\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

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$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)}t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)}t}$$

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$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)}t}\right] + G_{26}^0 e^{-(S_2)^{(4)}t} \le G_{26}(t) \le\right.$$ 334

*(a26)4G240(m2)4(S1)4−(a26′)4e(S1)4t−e−(a26′)4t+ G260e−(a26′)4t*

$$\boxed{T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}}T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}}T_{24}^0 e^{((R_1)^{(4)}+(r_{24})^{(4)})t} \quad 335$$

$$\frac{(b_{26})^{(4)}T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)}-(b'_{26})^{(4)})}\left[e^{(R_1)^{(4)}t} - e^{-(b'_{26})^{(4)}t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)}t} \le T_{26}(t) \le \quad 336$$

$$\frac{(a_{26})^{(4)}T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)}+(r_{24})^{(4)}+(R_2)^{(4)})}\left[e^{((R_1)^{(4)}+(r_{24})^{(4)})t} - e^{-(R_2)^{(4)}t}\right] + T_{26}^0 e^{-(R_2)^{(4)}t}$$

**Definition of** $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

**Behavior of the solutions of equation** 338

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ :

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

**Definition of** $(v_1)^{(5)}, (v_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, v^{(5)}, u^{(5)}$ : 339

By $(v_1)^{(5)} > 0$ , $(v_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$ , $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(v^{(5)}\big)^2 + (\sigma_1)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

**Definition of** $(\bar{v}_1)^{(5)}, , (\bar{v}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$ : 340

By $(\bar{v}_1)^{(5)} > 0$ , $(\bar{v}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$ , $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(v^{(5)}\big)^2 + (\sigma_2)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

**Definition of** $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (v_0)^{(5)}$ :-

If we define $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (v_0)^{(5)}, (m_1)^{(5)} = (v_1)^{(5)}$, ***if*** $(v_0)^{(5)} < (v_1)^{(5)}$

$(m_2)^{(5)} = (v_1)^{(5)}, (m_1)^{(5)} = (\bar{v}_1)^{(5)}$ , ***if*** $(v_1)^{(5)} < (v_0)^{(5)} < (\bar{v}_1)^{(5)}$,
and $\boxed{(v_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)},\ \boldsymbol{if}\ (u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}, \boldsymbol{if}\ (u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)},$

and $\boxed{(u_0)^{(5)} = \frac{T_{28}^0}{T_{29}^0}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}, \boldsymbol{if}\ (\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{28}(t) \le G_{28}^0 e^{(S_1)^{(5)} t}$

where $(p_i)^{(5)}$ is defined by equation

$\frac{1}{(m_5)^{(5)}} G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}} G_{28}^0 e^{(S_1)^{(5)} t}$ 343

$\left(\frac{(a_{30})^{(5)} G_{28}^0}{(m_1)^{(5)}((S_1)^{(5)} - (p_{28})^{(5)} - (S_2)^{(5)})}\left[e^{((S_1)^{(5)} - (p_{28})^{(5)})t} - e^{-(S_2)^{(5)} t}\right] + G_{30}^0 e^{-(S_2)^{(5)} t} \le G_{30}(t) \le\right.$ 344

*(a30)5G280(m2)5(S1)5−(a30′)5e(S1)5t−e−(a30′)5t+ G300e−(a30′)5t*

$\boxed{T_{28}^0 e^{(R_1)^{(5)} t} \le T_{28}(t) \le T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}}$ 345

$\frac{1}{(\mu_1)^{(5)}} T_{28}^0 e^{(R_1)^{(5)} t} \le T_{28}(t) \le \frac{1}{(\mu_2)^{(5)}} T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}$ 346

$\frac{(b_{30})^{(5)} T_{28}^0}{(\mu_1)^{(5)}((R_1)^{(5)} - (b'_{30})^{(5)})}\left[e^{(R_1)^{(5)} t} - e^{-(b'_{30})^{(5)} t}\right] + T_{30}^0 e^{-(b'_{30})^{(5)} t} \le T_{30}(t) \le$ 347

$\frac{(a_{30})^{(5)} T_{28}^0}{(\mu_2)^{(5)}((R_1)^{(5)} + (r_{28})^{(5)} + (R_2)^{(5)})}\left[e^{((R_1)^{(5)} + (r_{28})^{(5)})t} - e^{-(R_2)^{(5)} t}\right] + T_{30}^0 e^{-(R_2)^{(5)} t}$

**Definition of** $(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$:- 348

Where $(S_1)^{(5)} = (a_{28})^{(5)}(m_2)^{(5)} - (a'_{28})^{(5)}$

$(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}$

$(R_1)^{(5)} = (b_{28})^{(5)}(\mu_2)^{(5)} - (b'_{28})^{(5)}$

$(R_2)^{(5)} = (b'_{30})^{(5)} - (r_{30})^{(5)}$

**Behavior of the solutions of equation** 349

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ :

$(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)} \le -(a'_{32})^{(6)} + (a'_{33})^{(6)} - (a''_{32})^{(6)}(T_{33}, t) + (a''_{33})^{(6)}(T_{33}, t) \le -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)} \le -(b'_{32})^{(6)} + (b'_{33})^{(6)} - (b''_{32})^{(6)}\big((G_{35}), t\big) - (b''_{33})^{(6)}\big((G_{35}), t\big) \le -(\tau_1)^{(6)}$$

**Definition of** $(\nu_1)^{(6)}, (\nu_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, \nu^{(6)}, u^{(6)}$ : 350

By $(\nu_1)^{(6)} > 0$, $(\nu_2)^{(6)} < 0$ and respectively $(u_1)^{(6)} > 0$, $(u_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(6)},, (\bar{\nu}_2)^{(6)}, (\bar{u}_1)^{(6)}, (\bar{u}_2)^{(6)}$ : 351

By $(\bar{\nu}_1)^{(6)} > 0$, $(\bar{\nu}_2)^{(6)} < 0$ and respectively $(\bar{u}_1)^{(6)} > 0$, $(\bar{u}_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$

**Definition of** $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}, (\nu_0)^{(6)}$ :-

If we define $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}$ by

$$(m_2)^{(6)} = (\nu_0)^{(6)}, (m_1)^{(6)} = (\nu_1)^{(6)}, \ \boldsymbol{if}\ (\nu_0)^{(6)} < (\nu_1)^{(6)}$$

$$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}, \boldsymbol{if}\ (\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)},$$

and $\boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}}$

$$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$$

and analogously 352

$$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}, \ \boldsymbol{if}\ (u_0)^{(6)} < (u_1)^{(6)}$$

$$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}, \boldsymbol{if}\ (u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)},$$

and $\boxed{(u_0)^{(6)} = \frac{T^0_{32}}{T^0_{33}}}$

$$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}, \boldsymbol{if}\ (\bar{u}_1)^{(6)} < (u_0)^{(6)} \text{ where } (u_1)^{(6)}, (\bar{u}_1)^{(6)}$$

Then the solution of global equations satisfies the inequalities 353

$$G^0_{32}e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{32}(t) \le G^0_{32}e^{(S_1)^{(6)}t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}}G^0_{32}e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{33}(t) \le \frac{1}{(m_2)^{(6)}}G^0_{32}e^{(S_1)^{(6)}t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)}G^0_{32}}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})}\left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)}t}\right] + G^0_{34}e^{-(S_2)^{(6)}t} \le G_{34}(t) \le \right. \quad 355$$

$$\frac{(a_{34})^{(6)}G_{32}^{0}}{(m_2)^{(6)}((S_1)^{(6)}-(a'_{34})^{(6)})}\left[e^{(S_1)^{(6)}t}-e^{-(a'_{34})^{(6)}t}\right]+\ G_{34}^{0}e^{-(a'_{34})^{(6)}t}\Big)$$

$$\boxed{T_{32}^{0}e^{(R_1)^{(6)}t}\le T_{32}(t)\le T_{32}^{0}e^{((R_1)^{(6)}+(r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}}T_{32}^{0}e^{(R_1)^{(6)}t}\le T_{32}(t)\le\frac{1}{(\mu_2)^{(6)}}T_{32}^{0}e^{((R_1)^{(6)}+(r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)}T_{32}^{0}}{(\mu_1)^{(6)}((R_1)^{(6)}-(b'_{34})^{(6)})}\left[e^{(R_1)^{(6)}t}-e^{-(b'_{34})^{(6)}t}\right]+T_{34}^{0}e^{-(b'_{34})^{(6)}t}\le T_{34}(t)\le \quad 358$$

$$\frac{(a_{34})^{(6)}T_{32}^{0}}{(\mu_2)^{(6)}((R_1)^{(6)}+(r_{32})^{(6)}+(R_2)^{(6)})}\left[e^{((R_1)^{(6)}+(r_{32})^{(6)})t}-e^{-(R_2)^{(6)}t}\right]+T_{34}^{0}e^{-(R_2)^{(6)}t}$$

**Definition of** $(S_1)^{(6)},(S_2)^{(6)},(R_1)^{(6)},(R_2)^{(6)}$:- 359

Where $(S_1)^{(6)}=(a_{32})^{(6)}(m_2)^{(6)}-(a'_{32})^{(6)}$

$(S_2)^{(6)}=(a_{34})^{(6)}-(p_{34})^{(6)}$

$(R_1)^{(6)}=(b_{32})^{(6)}(\mu_2)^{(6)}-(b'_{32})^{(6)}$

$(R_2)^{(6)}=(b'_{34})^{(6)}-(r_{34})^{(6)}$

**Behavior of the solutions of equation**

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(7)},(\sigma_2)^{(7)},(\tau_1)^{(7)},(\tau_2)^{(7)}$ :

$(\sigma_1)^{(7)},(\sigma_2)^{(7)},(\tau_1)^{(7)},(\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)}\le-(a'_{36})^{(7)}+(a'_{37})^{(7)}-(a''_{36})^{(7)}(T_{37},t)+(a''_{37})^{(7)}(T_{37},t)\le-(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)}\le-(b'_{36})^{(7)}+(b'_{37})^{(7)}-(b''_{36})^{(7)}\big((G_{39}),t\big)-(b''_{37})^{(7)}\big((G_{39}),t\big)\le-(\tau_1)^{(7)}$$

**Definition of** $(v_1)^{(7)},(v_2)^{(7)},(u_1)^{(7)},(u_2)^{(7)},v^{(7)},u^{(7)}$ : 361

By $(v_1)^{(7)}>0,(v_2)^{(7)}<0$ and respectively $(u_1)^{(7)}>0,(u_2)^{(7)}<0$ the roots of the equations $(a_{37})^{(7)}\big(v^{(7)}\big)^2+(\sigma_1)^{(7)}v^{(7)}-(a_{36})^{(7)}=0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2+(\tau_1)^{(7)}u^{(7)}-(b_{36})^{(7)}=0$ and

**Definition of** $(\bar{v}_1)^{(7)},,(\bar{v}_2)^{(7)},(\bar{u}_1)^{(7)},(\bar{u}_2)^{(7)}$ : 362

By $(\bar{v}_1)^{(7)}>0,(\bar{v}_2)^{(7)}<0$ and respectively $(\bar{u}_1)^{(7)}>0,(\bar{u}_2)^{(7)}<0$ the roots of the equations $(a_{37})^{(7)}\big(v^{(7)}\big)^2+(\sigma_2)^{(7)}v^{(7)}-(a_{36})^{(7)}=0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2+(\tau_2)^{(7)}u^{(7)}-(b_{36})^{(7)}=0$

**Definition of** $(m_1)^{(7)},(m_2)^{(7)},(\mu_1)^{(7)},(\mu_2)^{(7)},(v_0)^{(7)}$ :-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$$(m_2)^{(7)} = (\nu_0)^{(7)}, (m_1)^{(7)} = (\nu_1)^{(7)}, \ \boldsymbol{if}\ (\nu_0)^{(7)} < (\nu_1)^{(7)}$$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\bar{\nu}_1)^{(7)}, \boldsymbol{if}\ (\nu_1)^{(7)} < (\nu_0)^{(7)} < (\bar{\nu}_1)^{(7)},$$

and $\boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\nu_0)^{(7)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(7)} < (\nu_0)^{(7)}$$

and analogously 363

$$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}, \ \boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$$

and $\boxed{(u_0)^{(7)} = \frac{T_{36}^0}{T_{37}^0}}$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)} \text{ where } (u_1)^{(7)}, (\bar{u}_1)^{(7)}$$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^0 e^{(S_1)^{(7)} t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)} t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)}((S_1)^{(7)} - (p_{36})^{(7)} - (S_2)^{(7)})}\left[e^{((S_1)^{(7)} - (p_{36})^{(7)})t} - e^{-(S_2)^{(7)} t}\right] + G_{38}^0 e^{-(S_2)^{(7)} t} \le G_{38}(t) \le \right. \quad 366$$

$$\left. \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)}((S_1)^{(7)} - (a'_{38})^{(7)})}\left[e^{(S_1)^{(7)} t} - e^{-(a'_{38})^{(7)} t}\right] + G_{38}^0 e^{-(a'_{38})^{(7)} t}\right)$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t} \quad 368$$

$$\frac{(b_{38})^{(7)} T_{36}^0}{(\mu_1)^{(7)}((R_1)^{(7)} - (b'_{38})^{(7)})}\left[e^{(R_1)^{(7)} t} - e^{-(b'_{38})^{(7)} t}\right] + T_{38}^0 e^{-(b'_{38})^{(7)} t} \le T_{38}(t) \le \quad 369$$

$$\frac{(a_{38})^{(7)} T_{36}^0}{(\mu_2)^{(7)}((R_1)^{(7)} + (r_{36})^{(7)} + (R_2)^{(7)})}\left[e^{((R_1)^{(7)} + (r_{36})^{(7)})t} - e^{-(R_2)^{(7)} t}\right] + T_{38}^0 e^{-(R_2)^{(7)} t}$$

**Definition of** $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$

$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$

$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$

**Behavior of the solutions of equation** 371

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ :

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$-(\sigma_2)^{(8)} \le -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \le -(\sigma_1)^{(8)}$

$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\big((G_{43}), t\big) - (b''_{41})^{(8)}\big((G_{43}), t\big) \le -(\tau_1)^{(8)}$

**Definition of** $(\nu_1)^{(8)}, (\nu_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, \nu^{(8)}, u^{(8)}$ : 372

By $(\nu_1)^{(8)} > 0, (\nu_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0, (u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(8)}, , (\bar{\nu}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$ :

By $(\bar{\nu}_1)^{(8)} > 0, (\bar{\nu}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0, (\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

**Definition of** $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}, (\nu_0)^{(8)}$ :-

If we define $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$(m_2)^{(8)} = (\nu_0)^{(8)}, (m_1)^{(8)} = (\nu_1)^{(8)}$, ***if*** $(\nu_0)^{(8)} < (\nu_1)^{(8)}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\bar{\nu}_1)^{(8)}$, ***if*** $(\nu_1)^{(8)} < (\nu_0)^{(8)} < (\bar{\nu}_1)^{(8)}$,
and $\boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\nu_0)^{(8)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(8)} < (\nu_0)^{(8)}$

and analogously 374

$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)},\ \boldsymbol{if}\ (u_0)^{(8)} < (u_1)^{(8)}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}, \boldsymbol{if}\ (u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)},$

and $\boxed{(u_0)^{(8)} = \frac{T_{40}^0}{T_{41}^0}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}, \boldsymbol{if}\ (\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^0 e^{(S_1)^{(8)} t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}} G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}} G_{40}^0 e^{(S_1)^{(8)} t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)} G_{40}^0}{(m_1)^{(8)}((S_1)^{(8)} - (p_{40})^{(8)} - (S_2)^{(8)})}\left[e^{((S_1)^{(8)} - (p_{40})^{(8)})t} - e^{-(S_2)^{(8)} t}\right] + G_{42}^0 e^{-(S_2)^{(8)} t} \le G_{42}(t) \le \right. \quad 377$$

$$\left. \frac{(a_{42})^{(8)} G_{40}^0}{(m_2)^{(8)}((S_1)^{(8)} - (a'_{42})^{(8)})}\left[e^{(S_1)^{(8)} t} - e^{-(a'_{42})^{(8)} t}\right] + G_{42}^0 e^{-(a'_{42})^{(8)} t}\right)$$

$$\boxed{T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}} T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le \frac{1}{(\mu_2)^{(8)}} T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)} T_{40}^0}{(\mu_1)^{(8)}((R_1)^{(8)} - (b'_{42})^{(8)})}\left[e^{(R_1)^{(8)} t} - e^{-(b'_{42})^{(8)} t}\right] + T_{42}^0 e^{-(b'_{42})^{(8)} t} \le T_{42}(t) \le \quad 380$$

$$\frac{(a_{42})^{(8)} T_{40}^0}{(\mu_2)^{(8)}((R_1)^{(8)} + (r_{40})^{(8)} + (R_2)^{(8)})}\left[e^{((R_1)^{(8)} + (r_{40})^{(8)})t} - e^{-(R_2)^{(8)} t}\right] + T_{42}^0 e^{-(R_2)^{(8)} t}$$

**Definition of** $(S_1)^{(8)}, (S_2)^{(8)}, (R_1)^{(8)}, (R_2)^{(8)}$:- 381

Where $(S_1)^{(8)} = (a_{40})^{(8)}(m_2)^{(8)} - (a'_{40})^{(8)}$

$(S_2)^{(8)} = (a_{42})^{(8)} - (p_{42})^{(8)}$

$(R_1)^{(8)} = (b_{40})^{(8)}(\mu_2)^{(8)} - (b'_{40})^{(8)}$

$(R_2)^{(8)} = (b'_{42})^{(8)} - (r_{42})^{(8)}$

**Behavior of the solutions of equation 37 to 92** 382

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ :

$(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ four constants satisfying

$$-(\sigma_2)^{(9)} \le -(a'_{44})^{(9)} + (a'_{45})^{(9)} - (a''_{44})^{(9)}(T_{45}, t) + (a''_{45})^{(9)}(T_{45}, t) \le -(\sigma_1)^{(9)}$$

$$-(\tau_2)^{(9)} \le -(b'_{44})^{(9)} + (b'_{45})^{(9)} - (b''_{44})^{(9)}\big((G_{47}), t\big) - (b''_{45})^{(9)}\big((G_{47}), t\big) \le -(\tau_1)^{(9)}$$

**Definition of** $(\nu_1)^{(9)}, (\nu_2)^{(9)}, (u_1)^{(9)}, (u_2)^{(9)}, \nu^{(9)}, u^{(9)}$ :

By $(\nu_1)^{(9)} > 0$ , $(\nu_2)^{(9)} < 0$ and respectively $(u_1)^{(9)} > 0$ , $(u_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_1)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(9)}, , (\bar{\nu}_2)^{(9)}, (\bar{u}_1)^{(9)}, (\bar{u}_2)^{(9)}$ :

By $(\bar{\nu}_1)^{(9)} > 0$ , $(\bar{\nu}_2)^{(9)} < 0$ and respectively $(\bar{u}_1)^{(9)} > 0$ , $(\bar{u}_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_2)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$

**Definition of** $(m_1)^{(9)}, (m_2)^{(9)}, (\mu_1)^{(9)}, (\mu_2)^{(9)}, (\nu_0)^{(9)}$ :-

If we define $(m_1)^{(9)}, (m_2)^{(9)}, (\mu_1)^{(9)}, (\mu_2)^{(9)}$ by

$$(m_2)^{(9)} = (\nu_0)^{(9)}, (m_1)^{(9)} = (\nu_1)^{(9)}, \ \boldsymbol{if}\ (\nu_0)^{(9)} < (\nu_1)^{(9)}$$

$$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\bar{\nu}_1)^{(9)}, \boldsymbol{if}\ (\nu_1)^{(9)} < (\nu_0)^{(9)} < (\bar{\nu}_1)^{(9)},$$

and $\boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$

$$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\nu_0)^{(9)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(9)} < (\nu_0)^{(9)}$$

and analogously

$$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)}, \ \boldsymbol{if}\ (u_0)^{(9)} < (u_1)^{(9)}$$

$$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}, \boldsymbol{if}\ (u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)},$$

and $\boxed{(u_0)^{(9)} = \frac{T^0_{44}}{T^0_{45}}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}, \boldsymbol{if}\ (\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G^0_{44} e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{44}(t) \le G^0_{44} e^{(S_1)^{(9)} t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}}G_{44}^0 e^{\left((S_1)^{(9)}-(p_{44})^{(9)}\right)t} \le G_{45}(t) \le \frac{1}{(m_2)^{(9)}}G_{44}^0 e^{(S_1)^{(9)}t}$$

$$\left(\frac{(a_{46})^{(9)}G_{44}^0}{(m_1)^{(9)}\left((S_1)^{(9)}-(p_{44})^{(9)}-(S_2)^{(9)}\right)}\left[e^{\left((S_1)^{(9)}-(p_{44})^{(9)}\right)t}-e^{-(S_2)^{(9)}t}\right]+G_{46}^0 e^{-(S_2)^{(9)}t} \le G_{46}(t) \le \right.$$

$$\left.\frac{(a_{46})^{(9)}G_{44}^0}{(m_2)^{(9)}\left((S_1)^{(9)}-(a'_{46})^{(9)}\right)}\left[e^{(S_1)^{(9)}t}-e^{-(a'_{46})^{(9)}t}\right]+\ G_{46}^0 e^{-(a'_{46})^{(9)}t}\right)$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le T_{44}^0 e^{\left((R_1)^{(9)}+(r_{44})^{(9)}\right)t}}$$

$$\frac{1}{(\mu_1)^{(9)}}T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le \frac{1}{(\mu_2)^{(9)}}T_{44}^0 e^{\left((R_1)^{(9)}+(r_{44})^{(9)}\right)t}$$

$$\frac{(b_{46})^{(9)}T_{44}^0}{(\mu_1)^{(9)}\left((R_1)^{(9)}-(b'_{46})^{(9)}\right)}\left[e^{(R_1)^{(9)}t}-e^{-(b'_{46})^{(9)}t}\right]+T_{46}^0 e^{-(b'_{46})^{(9)}t} \le T_{46}(t) \le$$

$$\frac{(a_{46})^{(9)}T_{44}^0}{(\mu_2)^{(9)}\left((R_1)^{(9)}+(r_{44})^{(9)}+(R_2)^{(9)}\right)}\left[e^{\left((R_1)^{(9)}+(r_{44})^{(9)}\right)t}-e^{-(R_2)^{(9)}t}\right]+T_{46}^0 e^{-(R_2)^{(9)}t}$$

**Definition of** $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$

**Proof :** From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14},t)\right) - (a''_{14})^{(1)}(T_{14},t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

**Definition of** $v^{(1)}$ :- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)}\right) \le \frac{dv^{(1)}}{dt} \le -\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)}\right)$$

From which one obtains

**Definition of** $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$ :-

For $0 < \boxed{(v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}} < (v_1)^{(1)} < (\bar{v}_1)^{(1)}$

$$v^{(1)}(t) \ge \frac{(v_1)^{(1)}+(C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(v_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(v_2)^{(1)}}}$$

it follows $(\nu_0)^{(1)} \leq \nu^{(1)}(t) \leq (\nu_1)^{(1)}$

In the same manner , we get 384

$$\nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{\nu}_1)^{(1)}-(\nu_0)^{(1)}}{(\nu_0)^{(1)}-(\bar{\nu}_2)^{(1)}}}$$

From which we deduce $(\nu_0)^{(1)} \leq \nu^{(1)}(t) \leq (\bar{\nu}_1)^{(1)}$

If $0 < (\nu_1)^{(1)} < (\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0} < (\bar{\nu}_1)^{(1)}$ we find like in the previous case, 385

$$(\nu_1)^{(1)} \leq \frac{(\nu_1)^{(1)}+(C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}} \leq \nu^{(1)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\bar{\nu}_1)^{(1)}$$

If $0 < (\nu_1)^{(1)} \leq (\bar{\nu}_1)^{(1)} \leq \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$ , we obtain 386

$$(\nu_1)^{(1)} \leq \nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\nu_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(1)}(t)$ :-

$$(m_2)^{(1)} \leq \nu^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{\nu^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(1)}(t)$ :-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{13})^{(1)} = (a''_{14})^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)}G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)}T_{14}(t)$ This is an important

consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

**Proof** : From global equations we obtain 387

$$\frac{d\nu^{(2)}}{dt} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right) - (a''_{17})^{(2)}(T_{17}, t)\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

**Definition of** $\nu^{(2)}$ :- $\boxed{\nu^{(2)} = \frac{G_{16}}{G_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \le \frac{d\nu^{(2)}}{dt} \le -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

**Definition of** $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$ :-

For $0 < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \ge \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}}{1 + (C)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(C)^{(2)} = \frac{(\nu_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \le \nu^{(2)}(t) \le (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \le \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \le \nu^{(2)}(t) \le (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \le \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}}{1 + (C)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}} \le \nu^{(2)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \le (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \le (\bar{\nu}_1)^{(2)} \le (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$ , we obtain 394

$$(\nu_1)^{(2)} \le \nu^{(2)}(t) \le \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \le (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(2)}(t)$ :- 395

$$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}, \quad \boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$$

In a completely analogous way, we obtain 396

**Definition of** $u^{(2)}(t)$ :-

$$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}, \quad \boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :** 397

If $(a''_{16})^{(2)} = (a''_{17})^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)} G_{17}(t)$

Analogously if $(b''_{16})^{(2)} = (b''_{17})^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)} T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

**Proof** : From global equations we obtain 398

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right) - (a''_{21})^{(3)}(T_{21}, t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

**Definition of** $\nu^{(3)}$ :- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right) \leq \frac{d\nu^{(3)}}{dt} \leq -\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right)$$

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From which one obtains

For $0 < (\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\nu_1)^{(3)} < (\bar{\nu}_1)^{(3)}$

$$\nu^{(3)}(t) \geq \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(\nu_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\nu_2)^{(3)}}}$$

it follows $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\nu_1)^{(3)}$

In the same manner , we get 401

$$\nu^{(3)}(t) \le \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{\nu}_1)^{(3)}-(\nu_0)^{(3)}}{(\nu_0)^{(3)}-(\bar{\nu}_2)^{(3)}}}$$

**Definition of** $(\bar{\nu}_1)^{(3)}$ :-

From which we deduce $(\nu_0)^{(3)} \le \nu^{(3)}(t) \le (\bar{\nu}_1)^{(3)}$

If $0 < (\nu_1)^{(3)} < (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (\bar{\nu}_1)^{(3)}$ we find like in the previous case, 402

$$(\nu_1)^{(3)} \le \frac{(\nu_1)^{(3)}+(C)^{(3)}(\nu_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}} \le \nu^{(3)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \le (\bar{\nu}_1)^{(3)}$$

If $0 < (\nu_1)^{(3)} \le (\bar{\nu}_1)^{(3)} \le (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}$ , we obtain 403

$$(\nu_1)^{(3)} \le \nu^{(3)}(t) \le \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \le (\nu_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(3)}(t)$ :-

$$(m_2)^{(3)} \le \nu^{(3)}(t) \le (m_1)^{(3)}, \quad \boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(3)}(t)$ :-

$$(\mu_2)^{(3)} \le u^{(3)}(t) \le (\mu_1)^{(3)}, \quad \boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{20}'')^{(3)} = (a_{21}'')^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)}G_{21}(t)$

Analogously if $(b_{20}'')^{(3)} = (b_{21}'')^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)}T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

**Proof** : From global equations we obtain 404

$$\frac{dv^{(4)}}{dt} = (a_{24})^{(4)} - \left((a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right) - (a''_{25})^{(4)}(T_{25},t)v^{(4)} - (a_{25})^{(4)}v^{(4)}$$

**Definition of** $v^{(4)}$ :- $\boxed{v^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)}\right) \le \frac{dv^{(4)}}{dt} \le -\left((a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_4)^{(4)}v^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$ :-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \ge \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \le \nu^{(4)}(t) \le (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \le \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \le \nu^{(4)}(t) \le (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \le \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}}{1 + (C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}} \le \nu^{(4)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \le (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \le (\bar{\nu}_1)^{(4)} \le \boxed{(\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}}}$ , we obtain 407

$$(\nu_1)^{(4)} \le \nu^{(4)}(t) \le \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \le (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(4)}(t)$ :-

$$(m_2)^{(4)} \le \nu^{(4)}(t) \le (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(4)}(t)$ :-

$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}$, $\boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{24})^{(4)} = (a''_{25})^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)} G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b''_{24})^{(4)} = (b''_{25})^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)} T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

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**Proof** : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a'_{28})^{(5)} - (a'_{29})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right) - (a''_{29})^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

**Definition of** $\nu^{(5)}$ :- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$ :-

For $0 < \boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \le \frac{(\nu_1)^{(5)}+(C)^{(5)}(\nu_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)}-(\nu_2)^{(5)}\right)t\right]}}{1+(C)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)}-(\nu_2)^{(5)}\right)t\right]}} \le \nu^{(5)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \le (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \le (\bar{\nu}_1)^{(5)} \le \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain 411

$$(\nu_1)^{(5)} \le \nu^{(5)}(t) \le \frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \le (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(5)}(t)$ :-

$(m_2)^{(5)} \le \nu^{(5)}(t) \le (m_1)^{(5)}$, $\boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$

In a completely analogous way, we obtain

**Definition of** $u^{(5)}(t)$ :-

$(\mu_2)^{(5)} \le u^{(5)}(t) \le (\mu_1)^{(5)}$, $\boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a_{28}'')^{(5)} = (a_{29}'')^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)}G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b_{28}'')^{(5)} = (b_{29}'')^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)}T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

**Proof** : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a_{32}')^{(6)} - (a_{33}')^{(6)} + (a_{32}'')^{(6)}(T_{33},t)\right) - (a_{33}'')^{(6)}(T_{33},t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

**Definition of** $\nu^{(6)}$ :- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \le \frac{d\nu^{(6)}}{dt} \le -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$ :-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \geq \frac{(\nu_1)^{(6)}+(C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_0)^{(6)}\right)t\right]}}{1+(C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)}-(\nu_0)^{(6)}}{(\nu_0)^{(6)}-(\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\nu_1)^{(6)}$

In the same manner , we get 413

$$\nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{\nu}_1)^{(6)}-(\nu_0)^{(6)}}{(\nu_0)^{(6)}-(\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\bar{\nu}_1)^{(6)}$

If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 414

$$(\nu_1)^{(6)} \leq \frac{(\nu_1)^{(6)}+(C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_2)^{(6)}\right)t\right]}}{1+(C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_2)^{(6)}\right)t\right]}} \leq \nu^{(6)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\bar{\nu}_1)^{(6)}$$

If $0 < (\nu_1)^{(6)} \leq (\bar{\nu}_1)^{(6)} \leq \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$ , we obtain 415

$$(\nu_1)^{(6)} \leq \nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(6)}(t)$ :-

$$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}, \quad \boxed{\nu^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(6)}(t)$ :-

$$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}, \quad \boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)}G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)}T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

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**Proof** : From global equations we obtain

$$\frac{dv^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right) - (a''_{37})^{(7)}(T_{37},t)v^{(7)} - (a_{37})^{(7)}v^{(7)}$$

**Definition of** $v^{(7)}$ :- $\boxed{v^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_2)^{(7)}v^{(7)} - (a_{36})^{(7)}\right) \le \frac{dv^{(7)}}{dt} \le -\left((a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_1)^{(7)}v^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

**Definition of** $(\bar{v}_1)^{(7)}, (v_0)^{(7)}$ :-

For $0 < \boxed{(v_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}} < (v_1)^{(7)} < (\bar{v}_1)^{(7)}$

$$v^{(7)}(t) \ge \frac{(v_1)^{(7)} + (C)^{(7)}(v_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_0)^{(7)}\right)t\right]}}{1 + (C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(v_1)^{(7)} - (v_0)^{(7)}}{(v_0)^{(7)} - (v_2)^{(7)}}}$$

it follows $(v_0)^{(7)} \le v^{(7)}(t) \le (v_1)^{(7)}$

In the same manner , we get 417

$$v^{(7)}(t) \le \frac{(\bar{v}_1)^{(7)} + (\bar{C})^{(7)}(\bar{v}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{v}_1)^{(7)} - (v_0)^{(7)}}{(v_0)^{(7)} - (\bar{v}_2)^{(7)}}}$$

From which we deduce $(v_0)^{(7)} \le v^{(7)}(t) \le (\bar{v}_1)^{(7)}$

If $0 < (v_1)^{(7)} < (v_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}} < (\bar{v}_1)^{(7)}$ we find like in the previous case, 418

$$(v_1)^{(7)} \le \frac{(v_1)^{(7)} + (C)^{(7)}(v_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_2)^{(7)}\right)t\right]}}{1 + (C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_2)^{(7)}\right)t\right]}} \le v^{(7)}(t) \le$$

$$\frac{(\bar{v}_1)^{(7)} + (\bar{C})^{(7)}(\bar{v}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}} \le (\bar{v}_1)^{(7)}$$

If $0 < (v_1)^{(7)} \le (\bar{v}_1)^{(7)} \le \boxed{(v_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$ , we obtain 419

$$(v_1)^{(7)} \le v^{(7)}(t) \le \frac{(\bar{v}_1)^{(7)} + (\bar{C})^{(7)}(\bar{v}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}} \le (v_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $v^{(7)}(t)$ :-

$$(m_2)^{(7)} \leq v^{(7)}(t) \leq (m_1)^{(7)}, \quad \boxed{v^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(7)}(t)$ :- 420

$$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}, \quad \boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $v^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)} G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)} T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

**Proof** : From global equations we obtain 421

$$\frac{dv^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right) - (a''_{41})^{(8)}(T_{41}, t) v^{(8)} - (a_{41})^{(8)} v^{(8)}$$

**Definition of** $v^{(8)}$ :- $\boxed{v^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_2)^{(8)} v^{(8)} - (a_{40})^{(8)}\right) \leq \frac{dv^{(8)}}{dt} \leq -\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_1)^{(8)} v^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$ :-

$$\text{For } 0 < \boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$$

$$v^{(8)}(t) \geq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)}-(\nu_0)^{(8)}}{(\nu_0)^{(8)}-(\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \leq \frac{(\nu_1)^{(8)}+(C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}}{1+(C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}} \leq \nu^{(8)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \leq (\bar{\nu}_1)^{(8)} \leq \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$ , we obtain 424

$$(\nu_1)^{(8)} \leq \nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(8)}(t)$ :-

$$(m_2)^{(8)} \leq \nu^{(8)}(t) \leq (m_1)^{(8)}, \quad \boxed{\nu^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(8)}(t)$ :-

$$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}, \quad \boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $\nu^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)}G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then $(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)}T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

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**Proof** : From 99,20,44,22,23,44 we obtain A

$$\frac{dv^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right) - (a''_{45})^{(9)}(T_{45},t)v^{(9)} - (a_{45})^{(9)}v^{(9)}$$

**Definition of** $v^{(9)}$ :- $\boxed{v^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)}\right) \le \frac{dv^{(9)}}{dt} \le -\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_1)^{(9)}v^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

**Definition of** $(\bar{v}_1)^{(9)}, (v_0)^{(9)}$ :-

For $0 < \boxed{(v_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (v_1)^{(9)} < (\bar{v}_1)^{(9)}$

$$v^{(9)}(t) \ge \frac{(v_1)^{(9)} + (C)^{(9)}(v_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((v_1)^{(9)} - (v_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((v_1)^{(9)} - (v_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(v_1)^{(9)} - (v_0)^{(9)}}{(v_0)^{(9)} - (v_2)^{(9)}}}$$

it follows $(v_0)^{(9)} \le v^{(9)}(t) \le (v_9)^{(9)}$

In the same manner , we get

$$v^{(9)}(t) \le \frac{(\bar{v}_1)^{(9)} + (\bar{C})^{(9)}(\bar{v}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{v}_1)^{(9)} - (\bar{v}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{v}_1)^{(9)} - (\bar{v}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{v}_1)^{(9)} - (v_0)^{(9)}}{(v_0)^{(9)} - (\bar{v}_2)^{(9)}}}$$

From which we deduce $(v_0)^{(9)} \le v^{(9)}(t) \le (\bar{v}_1)^{(9)}$

If $0 < (v_1)^{(9)} < (v_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}} < (\bar{v}_1)^{(9)}$ we find like in the previous case,

$$(v_1)^{(9)} \le \frac{(v_1)^{(9)} + (C)^{(9)}(v_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((v_1)^{(9)} - (v_2)^{(9)}\right)t\right]}}{1 + (C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((v_1)^{(9)} - (v_2)^{(9)}\right)t\right]}} \le v^{(9)}(t) \le$$

$$\frac{(\bar{v}_1)^{(9)} + (\bar{C})^{(9)}(\bar{v}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{v}_1)^{(9)} - (\bar{v}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{v}_1)^{(9)} - (\bar{v}_2)^{(9)}\right)t\right]}} \le (\bar{v}_1)^{(9)}$$

If $0 < (v_1)^{(9)} \le (\bar{v}_1)^{(9)} \le \boxed{(v_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$ , we obtain

$$(v_1)^{(9)} \le v^{(9)}(t) \le \frac{(\bar{v}_1)^{(9)} + (\bar{C})^{(9)}(\bar{v}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{v}_1)^{(9)} - (\bar{v}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{v}_1)^{(9)} - (\bar{v}_2)^{(9)}\right)t\right]}} \le (v_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $v^{(9)}(t)$ :-

$$(m_2)^{(9)} \le v^{(9)}(t) \le (m_1)^{(9)}, \quad \boxed{v^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(9)}(t)$ :-

$(\mu_2)^{(9)} \leq u^{(9)}(t) \leq (\mu_1)^{(9)}$, $\boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)} G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then
$(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)} T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following 425

**Theorem :** If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on $t$ , and the conditions with the notations

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0$ ,

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$

$with\ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a''_i)^{(2)} and\ (b''_i)^{(2)}$ are independent on t , and the conditions with the notations 426

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$ 427

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 428

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$ , 429

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 430

$with\ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a''_i)^{(3)} and\ (b''_i)^{(3)}$ are independent on $t$ , and the conditions with the notations 431

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$ ,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

**Theorem :** If $(a_i'')^{(4)} and\ (b_i'')^{(4)}$ are independent on $t$ , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$ ,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(5)} and\ (b_i'')^{(5)}$ are independent on $t$ , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$ ,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

**Theorem** If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on $t$ , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$ ,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(7)} and\ (b_i'')^{(7)}$ are independent on $t$ , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$ ,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(8)} and\ (b_i'')^{(8)}$ are independent on $t$ , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$ ,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(9)} and\ (b_i'')^{(9)}$ are independent on $t$ , and the conditions (with the notations 436 A
45,46,27,28)

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$ ,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0$ 437

$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0$ 438

$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0$ 439

$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0$ 440

$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ]T_{14} = 0$ 441

$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0$ 442

has a unique positive solution , which is an equilibrium solution for the system

$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0$ 443

$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0$ 444

$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0$ 445

$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\ ]T_{16} = 0$ 446

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\,]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\,]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\,]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\,]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\,]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\big((G_{27})\big)\,]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\big((G_{27})\big)\,]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\big((G_{27})\big)\,]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})\,]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})\,]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})\,]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})\,]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})\,]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})\,]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0 \quad 479$$

$$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0 \quad 480$$

$$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0 \quad 481$$

$$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0 \quad 482$$

$$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0 \quad 483$$

$$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0 \quad 484$$

$$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0 \quad 484\text{A}$$

$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$

$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$

$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$

$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$

$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$

**Proof:** 485

(a) Indeed the first two equations have a nontrivial solution $G_{13}, G_{14}$ if

$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$

**Proof:** 486

(s) Indeed the first two equations have a nontrivial solution $G_{16}, G_{17}$ if

$\mathrm{F}(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$

**Proof:** 487

(a) Indeed the first two equations have a nontrivial solution $G_{20}, G_{21}$ if

$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$

**Proof:** 488

(a) Indeed the first two equations have a nontrivial solution $G_{24}, G_{25}$ if

$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$

**Proof:** 489

(a) Indeed the first two equations have a nontrivial solution $G_{28}, G_{29}$ if

$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$

**Proof:** 490

(a) Indeed the first two equations have a nontrivial solution $G_{32}, G_{33}$ if

$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) +$

$(a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$

**Proof:** 491

(a) Indeed the first two equations have a nontrivial solution $G_{36}, G_{37}$ if

$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$

**Proof:** 492

(a) Indeed the first two equations have a nontrivial solution $G_{40}, G_{41}$ if

$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$

**Proof:** 492 A

(a) Indeed the first two equations have a nontrivial solution $G_{44}, G_{45}$ if

$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$

**Definition and uniqueness of** $T^*_{14}$ :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique $T^*_{14}$ for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

**Definition and uniqueness of** $T^*_{17}$ :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique $T^*_{17}$ for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]}$$ 495

**Definition and uniqueness of** $T^*_{21}$ :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique $T^*_{21}$ for which $f(T^*_{21}) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

**Definition and uniqueness of** $T^*_{25}$ :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that

there exists a unique $T_{25}^{*}$ for which $f(T_{25}^{*})=0$. With this value , we obtain from the three first equations

$$G_{24}=\frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}=\frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T_{25}^{*})]}$$

**Definition and uniqueness of** $T_{29}^{*}$ :- 498

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(5)}(T_{29})$ being increasing, it follows that there exists a unique $T_{29}^{*}$ for which $f(T_{29}^{*})=0$. With this value , we obtain from the three first equations

$$G_{28}=\frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}=\frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T_{29}^{*})]}$$

**Definition and uniqueness of** $T_{33}^{*}$ :- 499

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(6)}(T_{33})$ being increasing, it follows that there exists a unique $T_{33}^{*}$ for which $f(T_{33}^{*})=0$. With this value , we obtain from the three first equations

$$G_{32}=\frac{(a_{32})^{(6)}G_{33}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^{*})]} \quad , \quad G_{34}=\frac{(a_{34})^{(6)}G_{33}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^{*})]}$$

**Definition and uniqueness of** $T_{37}^{*}$ :- 500

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(7)}(T_{37})$ being increasing, it follows that there exists a unique $T_{37}^{*}$ for which $f(T_{37}^{*})=0$. With this value , we obtain from the three first equations

$$G_{36}=\frac{(a_{36})^{(7)}G_{37}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^{*})]} \quad , \quad G_{38}=\frac{(a_{38})^{(7)}G_{37}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^{*})]}$$

**Definition and uniqueness of** $T_{41}^{*}$ :- 501

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique $T_{41}^{*}$ for which $f(T_{41}^{*})=0$. With this value , we obtain from the three first equations

$$G_{40}=\frac{(a_{40})^{(8)}G_{41}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T_{41}^{*})]} \quad , \quad G_{42}=\frac{(a_{42})^{(8)}G_{41}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T_{41}^{*})]}$$

**Definition and uniqueness of** $T_{45}^{*}$ :- 501 A

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(9)}(T_{45})$ being increasing, it follows that there exists a unique $T_{45}^{*}$ for which $f(T_{45}^{*})=0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)} G_{45}}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T^*_{45})]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)} G_{45}}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T^*_{45})]}$$

By the same argument, the equations admit solutions $G_{13}, G_{14}$ if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right] + (b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{14}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{14}$ such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions $G_{16}, G_{17}$ if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right] + (b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ 504
is a decreasing function in $G_{17}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{14}$ such that $\varphi((G_{19})^*) = 0$

By the same argument, the equations admit solutions $G_{20}, G_{21}$ if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right] + (b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{21}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{21}$ such that $\varphi((G_{23})^*) = 0$

By the same argument, the equations admit solutions $G_{24}, G_{25}$ if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right] + (b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{25}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{25}$ such that $\varphi((G_{27})^*) = 0$

By the same argument, the equations admit solutions $G_{28}, G_{29}$ if 507

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right] + (b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{29}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that

there exists a unique $G_{29}^{*}$ such that $\varphi((G_{31})^{*}) = 0$

By the same argument, the equations admit solutions $G_{32}, G_{33}$ if 508
$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$

$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right] + (b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{33}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{33}^{*}$ such that $\varphi(G_{35}{}^{*}) = 0$

By the same argument, the equations admit solutions $G_{36}, G_{37}$ if 509
$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$
$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right] + (b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{37}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{37}^{*}$ such that $\varphi(G_{39}{}^{*}) = 0$

By the same argument, the equations admit solutions $G_{40}, G_{41}$ if 510
$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$
$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right] + (b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{41}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{41}^{*}$ such that $\varphi(G_{43}{}^{*}) = 0$

By the same argument, the equations 92,93 admit solutions $G_{44}, G_{45}$ if
$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$

$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right] + (b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{45}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{45}^{*}$ such that $\varphi((G_{47})^{*}) = 0$

Finally we obtain the unique solution 511

$G_{14}^{*}$ given by $\varphi(G^{*}) = 0$ , $T_{14}^{*}$ given by $f(T_{14}^{*}) = 0$ and

$$G_{13}^{*} = \frac{(a_{13})^{(1)}G_{14}^{*}}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^{*})]} \quad , \quad G_{15}^{*} = \frac{(a_{15})^{(1)}G_{14}^{*}}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^{*})]}$$

$$T_{13}^{*} = \frac{(b_{13})^{(1)}T_{14}^{*}}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^{*})]} \quad , \quad T_{15}^{*} = \frac{(b_{15})^{(1)}T_{14}^{*}}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^{*})]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

$G_{17}^{*}$ given by $\varphi((G_{19})^{*}) = 0$ , $T_{17}^{*}$ given by $f(T_{17}^{*}) = 0$ and 512

$$G_{16}^{*} = \frac{(a_{16})^{(2)} G_{17}^{*}}{[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}^{*})]} \quad , \quad G_{18}^{*} = \frac{(a_{18})^{(2)} G_{17}^{*}}{[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}^{*})]} \qquad 513$$

$$T_{16}^{*} = \frac{(b_{16})^{(2)} T_{17}^{*}}{[(b'_{16})^{(2)} - (b''_{16})^{(2)}((G_{19})^{*})]} \quad , \quad T_{18}^{*} = \frac{(b_{18})^{(2)} T_{17}^{*}}{[(b'_{18})^{(2)} - (b''_{18})^{(2)}((G_{19})^{*})]} \qquad 514$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

$G_{21}^{*}$ given by $\varphi((G_{23})^{*}) = 0$ , $T_{21}^{*}$ given by $f(T_{21}^{*}) = 0$ and

$$G_{20}^{*} = \frac{(a_{20})^{(3)} G_{21}^{*}}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^{*})]} \quad , \quad G_{22}^{*} = \frac{(a_{22})^{(3)} G_{21}^{*}}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^{*})]}$$

$$T_{20}^{*} = \frac{(b_{20})^{(3)} T_{21}^{*}}{[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}{}^{*})]} \quad , \quad T_{22}^{*} = \frac{(b_{22})^{(3)} T_{21}^{*}}{[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}{}^{*})]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

$G_{25}^{*}$ given by $\varphi(G_{27}) = 0$ , $T_{25}^{*}$ given by $f(T_{25}^{*}) = 0$ and

$$G_{24}^{*} = \frac{(a_{24})^{(4)} G_{25}^{*}}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}^{*} = \frac{(a_{26})^{(4)} G_{25}^{*}}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^{*})]}$$

$$T_{24}^{*} = \frac{(b_{24})^{(4)} T_{25}^{*}}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^{*})]} \quad , \quad T_{26}^{*} = \frac{(b_{26})^{(4)} T_{25}^{*}}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^{*})]} \qquad 517$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

$G_{29}^{*}$ given by $\varphi((G_{31})^{*}) = 0$ , $T_{29}^{*}$ given by $f(T_{29}^{*}) = 0$ and

$$G_{28}^{*} = \frac{(a_{28})^{(5)} G_{29}^{*}}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}^{*} = \frac{(a_{30})^{(5)} G_{29}^{*}}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^{*})]}$$

$$T_{28}^{*} = \frac{(b_{28})^{(5)} T_{29}^{*}}{[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31})^{*})]} \quad , \quad T_{30}^{*} = \frac{(b_{30})^{(5)} T_{29}^{*}}{[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31})^{*})]} \qquad 519$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

$G_{33}^{*}$ given by $\varphi((G_{35})^{*}) = 0$ , $T_{33}^{*}$ given by $f(T_{33}^{*}) = 0$ and

$$G_{32}^{*} = \frac{(a_{32})^{(6)} G_{33}^{*}}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^{*})]} \quad , \quad G_{34}^{*} = \frac{(a_{34})^{(6)} G_{33}^{*}}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^{*})]}$$

$$T_{32}^{*} = \frac{(b_{32})^{(6)} T_{33}^{*}}{[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35})^{*})]} \quad , \quad T_{34}^{*} = \frac{(b_{34})^{(6)} T_{33}^{*}}{[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35})^{*})]} \qquad 521$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

$G_{37}^*$ given by $\varphi((G_{39})^*) = 0$ , $T_{37}^*$ given by $f(T_{37}^*) = 0$ and

$$G_{36}^* = \frac{(a_{36})^{(7)} G_{37}^*}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^*)]} \quad , \quad G_{38}^* = \frac{(a_{38})^{(7)} G_{37}^*}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^*)]}$$

$$T_{36}^* = \frac{(b_{36})^{(7)} T_{37}^*}{[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39})^*)]} \quad , \quad T_{38}^* = \frac{(b_{38})^{(7)} T_{37}^*}{[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39})^*)]}$$

Finally we obtain the unique solution 523

$G_{41}^*$ given by $\varphi((G_{43})^*) = 0$ , $T_{41}^*$ given by $f(T_{41}^*) = 0$ and

$$G_{40}^* = \frac{(a_{40})^{(8)} G_{41}^*}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42}^* = \frac{(a_{42})^{(8)} G_{41}^*}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

$$T_{40}^* = \frac{(b_{40})^{(8)} T_{41}^*}{[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43})^*)]} \quad , \quad T_{42}^* = \frac{(b_{42})^{(8)} T_{41}^*}{[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43})^*)]}$$

Finally we obtain the unique solution of 89 to 99 523 A

$G_{45}^*$ given by $\varphi((G_{47})^*) = 0$ , $T_{45}^*$ given by $f(T_{45}^*) = 0$ and

$$G_{44}^* = \frac{(a_{44})^{(9)} G_{45}^*}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46}^* = \frac{(a_{46})^{(9)} G_{45}^*}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

$$T_{44}^* = \frac{(b_{44})^{(9)} T_{45}^*}{[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47})^*)]} \quad , \quad T_{46}^* = \frac{(b_{46})^{(9)} T_{45}^*}{[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47})^*)]}$$

## ASYMPTOTIC STABILITY ANALYSIS 524

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

**Proof:** Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a''_{14})^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \frac{\partial (b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a'_{13})^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)}G^*_{13}\mathbb{T}_{14} \quad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a'_{14})^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)}G^*_{14}\mathbb{T}_{14} \quad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a'_{15})^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)}G^*_{15}\mathbb{T}_{14} \quad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b'_{13})^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)}T^*_{13}\mathbb{G}_j\right) \quad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b'_{14})^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)}T^*_{14}\mathbb{G}_j\right) \quad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b'_{15})^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T^*_{15}\mathbb{G}_j\right) \quad 530$$

**ASYMPTOTIC STABILITY ANALYSIS** 531

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(2)}$ and $(b''_i)^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G^*_i + \mathbb{G}_i \quad , T_i = T^*_i + \mathbb{T}_i \quad 532$$

$$\frac{\partial(a''_{17})^{(2)}}{\partial T_{17}}(T^*_{17}) = (q_{17})^{(2)} \quad , \frac{\partial(b''_i)^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \quad 533$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a'_{16})^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G^*_{16}\mathbb{T}_{17} \quad 534$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a'_{17})^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G^*_{17}\mathbb{T}_{17} \quad 535$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a'_{18})^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G^*_{18}\mathbb{T}_{17} \quad 536$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b'_{16})^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T^*_{16}\mathbb{G}_j\right) \quad 537$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b'_{17})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}T^*_{17}\mathbb{G}_j\right) \quad 538$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b'_{18})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}T^*_{18}\mathbb{G}_j\right) \quad 539$$

**ASYMPTOTIC STABILITY ANALYSIS** 540

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(3)}$ *and* $(b''_i)^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{21}'')^{(3)}}{\partial T_{21}}(T_{21}^*) = (q_{21})^{(3)} \quad , \frac{\partial (b_i'')^{(3)}}{\partial G_j}(\,(G_{23})^*\,) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\big((a_{20}')^{(3)} + (p_{20})^{(3)}\big)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G_{20}^*\mathbb{T}_{21} \qquad 541$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\big((a_{21}')^{(3)} + (p_{21})^{(3)}\big)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G_{21}^*\mathbb{T}_{21} \qquad 542$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\big((a_{22}')^{(3)} + (p_{22})^{(3)}\big)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G_{22}^*\mathbb{T}_{21} \qquad 543$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\big((b_{20}')^{(3)} - (r_{20})^{(3)}\big)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\big(s_{(20)(j)}T_{20}^*\mathbb{G}_j\big) \qquad 544$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\big((b_{21}')^{(3)} - (r_{21})^{(3)}\big)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\big(s_{(21)(j)}T_{21}^*\mathbb{G}_j\big) \qquad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\big((b_{22}')^{(3)} - (r_{22})^{(3)}\big)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\big(s_{(22)(j)}T_{22}^*\mathbb{G}_j\big) \qquad 546$$

**ASYMPTOTIC STABILITY ANALYSIS** 547

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}\ and\ (b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{25}'')^{(4)}}{\partial T_{25}}(T_{25}^*) = (q_{25})^{(4)} \quad , \frac{\partial (b_i'')^{(4)}}{\partial G_j}((G_{27})^*\,) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\big((a_{24}')^{(4)} + (p_{24})^{(4)}\big)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G_{24}^*\mathbb{T}_{25} \qquad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\big((a_{25}')^{(4)} + (p_{25})^{(4)}\big)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G_{25}^*\mathbb{T}_{25} \qquad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\big((a_{26}')^{(4)} + (p_{26})^{(4)}\big)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G_{26}^*\mathbb{T}_{25} \qquad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\big((b_{24}')^{(4)} - (r_{24})^{(4)}\big)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\big(s_{(24)(j)}T_{24}^*\mathbb{G}_j\big) \qquad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\big((b_{25}')^{(4)} - (r_{25})^{(4)}\big)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\big(s_{(25)(j)}T_{25}^*\mathbb{G}_j\big) \qquad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T^*_{26}\mathbb{G}_j\right) \quad 554$$

**ASYMPTOTIC STABILITY ANALYSIS** 555

**Theorem 5:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}\ and\ (b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 556

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{29})^{(5)}}{\partial T_{29}}(T^*_{29}) = (q_{29})^{(5)} \ , \ \frac{\partial(b_i'')^{(5)}}{\partial G_j}((G_{31})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a'_{28})^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G^*_{28}\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a'_{29})^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G^*_{29}\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a'_{30})^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G^*_{30}\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b'_{28})^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T^*_{28}\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b'_{29})^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T^*_{29}\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b'_{30})^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T^*_{30}\mathbb{G}_j\right) \quad 562$$

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**Theorem 6:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}\ and\ (b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{33})^{(6)}}{\partial T_{33}}(T^*_{33}) = (q_{33})^{(6)} \ , \ \frac{\partial(b_i'')^{(6)}}{\partial G_j}((G_{35})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a'_{32})^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G^*_{32}\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a'_{33})^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G^*_{33}\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a'_{34})^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G^*_{34}\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b'_{32})^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T^*_{32}\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b'_{33})^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T^*_{33}\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b'_{34})^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T^*_{34}\mathbb{G}_j\right) \quad 570$$

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**Theorem 7:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 572

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{37})^{(7)}}{\partial T_{37}}(T^*_{37}) = (q_{37})^{(7)} \quad , \frac{\partial(b_i'')^{(7)}}{\partial G_j}\left((G_{39})^{**}\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a'_{36})^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G^*_{36}\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a'_{37})^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G^*_{37}\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a'_{38})^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G^*_{38}\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b'_{36})^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T^*_{36}\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b'_{37})^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T^*_{37}\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b'_{38})^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T^*_{38}\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

**ASYMPTOTIC STABILITY ANALYSIS**

**Theorem 8:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{41}'')^{(8)}}{\partial T_{41}}(T_{41}^*) = (q_{41})^{(8)} \quad , \frac{\partial (b_i'')^{(8)}}{\partial G_j}\left( (G_{43})^* \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a_{40}')^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G_{40}^*\mathbb{T}_{41} \quad (581)$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a_{41}')^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G_{41}^*\mathbb{T}_{41} \quad (582)$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a_{42}')^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G_{42}^*\mathbb{T}_{41} \quad (583)$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b_{40}')^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T_{40}^*\mathbb{G}_j\right) \quad (584)$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b_{41}')^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T_{41}^*\mathbb{G}_j\right) \quad (585)$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b_{42}')^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T_{42}^*\mathbb{G}_j\right) \quad (586)$$

(586 A)

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**Theorem 9:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}$ *and* $(b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.
**Proof:** Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{45}'')^{(9)}}{\partial T_{45}}(T_{45}^*) = (q_{45})^{(9)} \quad , \frac{\partial (b_i'')^{(9)}}{\partial G_j}\left( (G_{47})^* \right) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a_{44}')^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)}G_{44}^*\mathbb{T}_{45} \quad (586\,B)$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a_{45}')^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)}G_{45}^*\mathbb{T}_{45} \quad (586\,C)$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a_{46}')^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)}G_{46}^*\mathbb{T}_{45} \quad (586\,D)$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b_{44}')^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)}T_{44}^*\mathbb{G}_j\right) \quad (586\,E)$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b_{45}')^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)}T_{45}^*\mathbb{G}_j\right) \quad (586\,F)$$

$\frac{d\mathbb{T}_{46}}{dt} = -\left((b'_{46})^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T^*_{46}\mathbb{G}_j\right)$ 586

G

587

**The characteristic equation of this system is**

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(q_{13})^{(1)}G^*_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)}T^*_{14} + (b_{14})^{(1)}s_{(13),(14)}T^*_{14}\right)$$

$$+\left(\left((\lambda)^{(1)} + (a'_{14})^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)}G^*_{13} + (a_{13})^{(1)}(q_{14})^{(1)}G^*_{14}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)}T^*_{14} + (b_{14})^{(1)}s_{(13),(13)}T^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((b'_{13})^{(1)} + (b'_{14})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(15)}T^*_{14} + (b_{14})^{(1)}s_{(13),(15)}T^*_{13}\right)\} = 0$$

$$+$$

$$(\lambda)^{(2)} + (b'_{18})^{(2)} - (r_{18})^{(2)}\{\left((\lambda)^{(2)} + (a'_{18})^{(2)} + (p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^*_{17} + (a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^*_{17}\right)$$

$$+\left(\left((\lambda)^{(2)} + (a'_{17})^{(2)} + (p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^*_{16} + (a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}\right)$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2 + \left((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2 + \left((b'_{16})^{(2)} + (b'_{17})^{(2)} - (r_{16})^{(2)} + (r_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$+\left(\left((\lambda)^{(2)}\right)^2 + \left((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17} + (a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^{*}_{16}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\right)\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left(\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)\right]$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(21)}T^{*}_{21}\right)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^{*}_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^{*}_{21}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(20)}T^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$+\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(22)}T^{*}_{20}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\right)\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left(\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)\right]$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(25)}T^{*}_{25}\right)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^{*}_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^{*}_{25}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(24)}T^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$+\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(26)}T^{*}_{24}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\right)\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(29)}T^{*}_{29}\right)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^{*}_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^{*}_{29}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(28)}T^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$+\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(30)}T^{*}_{28}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\right)\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left(\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)\right]$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(33)}T^{*}_{33}\right)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^{*}_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^{*}_{33}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(32)}T^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$+\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(34)}T^{*}_{32}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\right)\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)\right]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(37)}T^{*}_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^{*}_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^{*}_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(36)}T^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$+\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(38)}T^{*}_{36}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\right)\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(41)}T^{*}_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^{*}_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^{*}_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(40)}T^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$+\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(42)}T^{*}_{40}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\right)\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(45)}T^{*}_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^{*}_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^{*}_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(44)}T^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$+\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(46)}T^{*}_{44}\right)\}=0$$

**And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.**

## SECTION TWENTY TWO

## Geometry Of Quantum Observables

### INTRODUCTION—VARIABLES USED

**Geometry of Quantum Observables and Thermodynamics of Small Systems Maxim Olshanii Phys. Rev. Lett 114, 060401 – Published 11 February 2015**

(1) Authors show that for classical and quantum observables, the integrability-to (e&eb)-Ergodicity transition leave (eb) constant the sum of (a) the ensemble variance of (e) the temporal average and

(b) the ensemble average of (e) temporal variance.

**(2)** The induced Frobenius (Hilbert-Schmidt) geometry of (e) quantum observables encodes (e) how eigenstate thermalization appears, the inverse participation ratio decreases, and the integrals of motion disappear during (e&eb) the transition.

**(3)** Authors use it (e) to optimize the set of conserved quantities entering (eb) the generalized Gibbs ensemble for (e) integrable, near-integrable, or mesoscopic systems. **http://dx.doi.org/10.1103/PhysRevLett.114.060401©American Physical Society**

## NOTATION

### Module One

Authors show that for classical and quantum observables, the **integrability**-to (e&eb)-Ergodicity transition leave (eb) constant the sum of (a) the ensemble variance of (e) the temporal average and (b) the ensemble average of (e) temporal variance

$G_{13}$ : Category one of **integrability;** Ergodicity transition leave (eb) constant the sum of (a) the ensemble variance of (e) the temporal average and (b) the ensemble average of (e) temporal variance

$G_{14}$ : Category two of SAS

$G_{15}$ : Category three of SAS

$T_{13}$ : Category one of Ergodicity transition leave (eb) constant the sum of (a) the ensemble variance of (e) the temporal average and (b) the ensemble average of (e) temporal variance; **integrability**

$T_{14}$ : Category two of SAS

$T_{15}$ : Category three of SAS

### Module Two

Authors show that the **integrability**-to -Ergodicity transition for classical and quantum observables leave (eb) constant the sum of (a) the ensemble variance of (e) the temporal average and (b) the ensemble average of (e) temporal variance

$G_{16}$ : Category one of **integrability**-to -Ergodicity transition for classical and quantum observables

$G_{17}$ : Category two of SAS

$G_{18}$ : Category three of SAS

$T_{16}$ : Category one of constant the sum of (a) the ensemble variance of (e) the temporal average and (b) the ensemble average of (e) temporal variance

$T_{17}$ : Category two of SAS

$T_{18}$ : Category three of SAS

### Module three

Authors show that the **integrability**-to -Ergodicity transition for classical and quantum observables leave constant the sum of (a) the ensemble variance of (e) the temporal average and (b) the ensemble average of (e) temporal variance

$G_{20}$ : Category one of temporal average

$G_{21}$ : Category two of SAS

$G_{22}$ : Category three of SAS

$T_{20}$ : Category one of ensemble variance

$T_{21}$ : Category two of SAS

$T_{22}$ : Category three of SAS

**Module four**

ensemble average of (e) **temporal variance**

$G_{24}$ : Category one of ensemble average; **temporal variance**

$G_{25}$ : Category two of SAS

$G_{26}$ : Category three of SAS

$T_{24}$ : Category one of **temporal variance** ; ensemble average

$T_{25}$ : Category two of SAS

$T_{26}$ : Category three of SAS

**Module five**

The induced Frobenius (Hilbert-Schmidt) geometry of (e) **quantum observables** encodes (e) how eigenstate thermalization appears, the inverse participation ratio decreases, and the integrals of motion disappear during (e&eb) the transition

$G_{28}$ : Category one of induced Frobenius (Hilbert-Schmidt) geometry; **quantum observables**

$G_{29}$ : Category two of SAS

$G_{30}$ : Category three of SAS

$T_{28}$ : Category one of **quantum observables;** induced Frobenius (Hilbert-Schmidt) geometry

$T_{29}$ : Category two of SAS

$T_{30}$ : Category three of SAS

**Module six**

The induced Frobenius (Hilbert-Schmidt) geometry of quantum observables encodes (e) how eigenstate thermalization appears, the inverse participation ratio decreases, and the integrals of motion disappear during (e&eb) the transition

$G_{32}$ : Category one of induced Frobenius (Hilbert-Schmidt) geometry of **quantum observables; appearance of eigen state thermalisation**

$G_{33}$ : Category two of SAS

$G_{34}$ : Category three of SAS

$T_{32}$ : Category one of **appearance of eigen state thermalisation;** induced Frobenius (Hilbert-Schmidt) geometry of **quantum observables**

$T_{33}$ : Category two of SAS

$T_{34}$ : Category three of SAS

**Module seven**

The induced Frobenius (Hilbert-Schmidt) geometry of quantum observables encodes (e) how eigenstate thermalization appears, the inverse participation ratio decreases, and the integrals of motion disappear during (e&eb) the transition

$G_{36}$ : Category one of **induced Frobenius (Hilbert-Schmidt) geometry of quantum observables; decrease of** inverse participation ratio

$G_{37}$ : Category two of SAS

$G_{38}$ : Category three of SAS

$T_{36}$ : Category one of **decrease of** inverse participation ratio **; induced Frobenius (Hilbert-Schmidt) geometry of quantum observables**

$T_{37}$ : Category two of SAS

$T_{38}$ : Category three of SAS

**Module eight**

The **induced Frobenius (Hilbert-Schmidt) geometry of quantum observables** encodes (e) how eigenstate thermalization appears, the inverse participation ratio decreases, and the integrals of motion disappear during the transition

$G_{40}$ : Category one of **induced Frobenius (Hilbert-Schmidt) geometry of quantum observables;** integrals of motion disappear during the transition

$G_{41}$ : Category two of SAS

$G_{42}$ : Category three of SAS

$T_{40}$ : Category one of integrals of motion disappear during the transition; **induced Frobenius (Hilbert-Schmidt) geometry of quantum observables**

$T_{41}$ : Category two of SAS

$T_{42}$ : Category three of SAS

**Module Nine**

Authors use it (e) to **optimize the set of conserved quantities** entering (eb) the generalized Gibbs ensemble for integrable, near-integrable, or mesoscopic systems.

**http://dx.doi.org/10.1103/PhysRevLett.114.060401©American Physical Society**

$G_{44}$ : Category one of **optimize the set of conserved quantities;** generalized Gibbs ensemble for integrable, near-integrable, or mesoscopic systems.

$G_{45}$ : Category two of SAS

$G_{46}$ : Category three of SAS

$T_{44}$ : Category one of generalized Gibbs ensemble for integrable, near-integrable, or mesoscopic systems. ;**optimize the set of conserved quantities**

$T_{45}$ : Category two of SAS

$T_{46}$ : Category three of SAS

**The Coefficients:**

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}$ , $(b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}$,
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}$, $(b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}\ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$ ,
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)}$,
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)}$,
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)}$,
are Dissipation coefficients
**Module Numbered One**

**The differential system of this model is now (Module Numbered one)**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \quad 1$$
$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad 2$$
$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15} \quad 3$$
$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13} \quad 4$$
$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14} \quad 5$$

$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G,t)\right]T_{15}$ 6

$+(a''_{13})^{(1)}(T_{14},t) =$ First augmentation factor

$-(b''_{13})^{(1)}(G,t) =$ First detritions factor

**Module Numbered Two**

**The differential system of this model is now ( Module numbered two)**

$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17},t)\right]G_{16}$ 7

$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17},t)\right]G_{17}$ 8

$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17},t)\right]G_{18}$ 9

$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\left((G_{19}),t\right)\right]T_{16}$ 10

$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\left((G_{19}),t\right)\right]T_{17}$ 11

$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\left((G_{19}),t\right)\right]T_{18}$ 12

$+(a''_{16})^{(2)}(T_{17},t) =$ First augmentation factor

$-(b''_{16})^{(2)}\left((G_{19}),t\right) =$ First detritions factor

**Module Numbered Three**

**The differential system of this model is now (Module numbered three)**

$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21},t)\right]G_{20}$ 13

$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21},t)\right]G_{21}$ 14

$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21},t)\right]G_{22}$ 15

$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23},t)\right]T_{20}$ 16

$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23},t)\right]T_{21}$ 17

$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23},t)\right]T_{22}$ 18

$+(a''_{20})^{(3)}(T_{21},t) =$ First augmentation factor

$-(b''_{20})^{(3)}(G_{23},t) =$ First detritions factor

**Module Numbered Four**

**The differential system of this model is now (Module numbered Four)**

$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right]G_{24}$ 19

$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25},t)\right]G_{25}$ 20

$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25},t)\right]G_{26}$ 21

$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27}),t\right)\right]T_{24}$ 22

$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27}),t\right)\right]T_{25}$ 23

$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27}),t\right)\right]T_{26}$ 24

$+(a''_{24})^{(4)}(T_{25},t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}\left((G_{27}),t\right) =$ **First detritions factor**

**Module Numbered Five**:

**The differential system of this model is now (Module number five)**

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right]G_{28} \quad 25$$
$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}, t)\right]G_{29} \quad 26$$
$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}, t)\right]G_{30} \quad 27$$
$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31}), t)\right]T_{28} \quad 28$$
$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}((G_{31}), t)\right]T_{29} \quad 29$$
$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31}), t)\right]T_{30} \quad 30$$

$+(a''_{28})^{(5)}(T_{29}, t) =$ **First augmentation factor**

$-(b''_{28})^{(5)}((G_{31}), t) =$ **First detritions factor**

**Module Numbered Six**

**The differential system of this model is now (Module numbered Six)**

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right]G_{32} \quad 31$$
$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}, t)\right]G_{33} \quad 32$$
$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}, t)\right]G_{34} \quad 33$$
$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35}), t)\right]T_{32} \quad 34$$
$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}((G_{35}), t)\right]T_{33} \quad 35$$
$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35}), t)\right]T_{34} \quad 36$$

$+(a''_{32})^{(6)}(T_{33}, t) =$ **First augmentation factor**

**Module Numbered Seven:**

**The differential system of this model is now (Seventh Module)**

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right]G_{36} \quad 37$$
$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37}, t)\right]G_{37} \quad 38$$
$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}, t)\right]G_{38} \quad 39$$
$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39}), t)\right]T_{36} \quad 40$$
$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}((G_{39}), t)\right]T_{37} \quad 41$$
$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39}), t)\right]T_{38} \quad 42$$

$+(a''_{36})^{(7)}(T_{37}, t) =$ **First augmentation factor**

**Module Numbered Eight**

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right]G_{40} \quad 43$$
$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41}, t)\right]G_{41} \quad 44$$
$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}, t)\right]G_{42} \quad 45$$
$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43}), t)\right]T_{40} \quad 46$$
$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}((G_{43}), t)\right]T_{41} \quad 47$$
$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43}), t)\right]T_{42} \quad 48$$

**Module Numbered Nine**

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}\left((G_{47}),t\right)\right]T_{44} \quad 52$$

$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}\left((G_{47}),t\right)\right]T_{45} \quad 53$$

$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}\left((G_{47}),t\right)\right]T_{46} \quad 54$$

$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}\left((G_{47}),t\right) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)}\boxed{+(a''_{13})^{(1)}(T_{14},t)}\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7)}(T_{37},t)}\boxed{+(a''_{40})^{(8,8)}(T_{41},t)}\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)}\boxed{+(a''_{14})^{(1)}(T_{14},t)}\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7)}(T_{37},t)}\boxed{+(a''_{41})^{(8,8)}(T_{41},t)}\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)}\boxed{+(a''_{15})^{(1)}(T_{14},t)}\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7)}(T_{37},t)}\boxed{+(a''_{42})^{(8,8)}(T_{41},t)}\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15} \quad 57$$

Where $\boxed{(a''_{13})^{(1)}(T_{14},t)}$, $\boxed{(a''_{14})^{(1)}(T_{14},t)}$, $\boxed{(a''_{15})^{(1)}(T_{14},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7)}(T_{37},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8)}(T_{41},t)}$ $\boxed{+(a''_{41})^{(8,8)}(T_{41},t)}$ $\boxed{+(a''_{42})^{(8,8)}(T_{41},t)}$ are eight augmentation coefficient for 1,2,3

$\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)}\boxed{-(b''_{13})^{(1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13} \quad (58)$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)}\boxed{-(b''_{14})^{(1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14} \quad (59)$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)}\boxed{-(b''_{15})^{(1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15} \quad (60)$$

Where $\boxed{-(b''_{13})^{(1)}(G,t)}$, $\boxed{-(b''_{14})^{(1)}(G,t)}$, $\boxed{-(b''_{15})^{(1)}(G,t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8)}(G_{43},t)}$ $\boxed{-(b''_{41})^{(8,8)}(G_{43},t)}$ $\boxed{-(b''_{42})^{(8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \begin{bmatrix} (a'_{16})^{(2)}\boxed{+(a''_{16})^{(2)}(T_{17},t)} & \boxed{+(a''_{13})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{16} \quad (61)$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \begin{bmatrix} (a'_{17})^{(2)}\boxed{+(a''_{17})^{(2)}(T_{17},t)} & \boxed{+(a''_{14})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{17} \quad (62)$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \begin{bmatrix} (a'_{18})^{(2)}\boxed{+(a''_{18})^{(2)}(T_{17},t)} & \boxed{+(a''_{15})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{18} \quad (63)$$

Where $\boxed{+(a''_{16})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,)}(T_{14},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)}$ are eight augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{44})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9)}(T_{45},t)}$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \begin{bmatrix} (b'_{16})^{(2)}\boxed{-(b''_{16})^{(2)}(G_{19},t)} & \boxed{-(b''_{13})^{(1,1,)}(G,t)} & \boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{16} \quad (64)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)}\boxed{-(b''_{17})^{(2)}(G_{19},t)} & \boxed{-(b''_{14})^{(1,1,)}(G,t)} & \boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{17} \quad (65)$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)}\boxed{-(b''_{18})^{(2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,)}(G,t)} & \boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{18} \quad (66)$$

where $\boxed{-(b''_{16})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2)}(G_{19},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,)}(G,t)}$ are second detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)}$ are fourth detrition

coefficients for category 1,2 and 3

$-(b_{28}'')^{(5,5,5,5,5)}(G_{31},t)$, $-(b_{29}'')^{(5,5,5,5,5)}(G_{31},t)$, $-(b_{30}'')^{(5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1,2 and 3

$-(b_{32}'')^{(6,6,6,6,6)}(G_{35},t)$, $-(b_{33}'')^{(6,6,6,6,6)}(G_{35},t)$, $-(b_{34}'')^{(6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients for category 1,2 and 3

$-(b_{36}'')^{(7,7,7)}(G_{39},t)$, $-(b_{37}'')^{(7,7,7)}(G_{39},t)$, $-(b_{38}'')^{(7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2 and 3

$-(b_{40}'')^{(8,8,8)}(G_{43},t)$, $-(b_{41}'')^{(8,8,8)}(G_{43},t)$, $-(b_{42}'')^{(8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1,2 and 3

$-(b_{44}'')^{(9,9)}(G_{47},t)$, $-(b_{46}'')^{(9,9)}(G_{47},t)$, $-(b_{45}'')^{(9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[\begin{array}{c}(a_{20}')^{(3)}\,+(a_{20}'')^{(3)}(T_{21},t)\;+(a_{16}'')^{(2,2,2)}(T_{17},t)\;+(a_{13}'')^{(1,1,1,)}(T_{14},t)\\ +(a_{24}'')^{(4,4,4,4,4,4)}(T_{25},t)\;+(a_{28}'')^{(5,5,5,5,5,5)}(T_{29},t)\;+(a_{32}'')^{(6,6,6,6,6,6)}(T_{33},t)\\ +(a_{36}'')^{(7,7,7,7)}(T_{37},t)\;+(a_{40}'')^{(8,8,8,8)}(T_{41},t)\;+(a_{44}'')^{(9,9,9)}(T_{45},t)\end{array}\right]G_{20} \quad 67$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[\begin{array}{c}(a_{21}')^{(3)}\,+(a_{21}'')^{(3)}(T_{21},t)\;+(a_{17}'')^{(2,2,2)}(T_{17},t)\;+(a_{14}'')^{(1,1,1,)}(T_{14},t)\\ +(a_{25}'')^{(4,4,4,4,4,4)}(T_{25},t)\;+(a_{29}'')^{(5,5,5,5,5,5)}(T_{29},t)\;+(a_{33}'')^{(6,6,6,6,6,6)}(T_{33},t)\\ +(a_{37}'')^{(7,7,7,7)}(T_{37},t)\;+(a_{41}'')^{(8,8,8,8)}(T_{41},t)\;+(a_{45}'')^{(9,9,9)}(T_{45},t)\end{array}\right]G_{21} \quad 68$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[\begin{array}{c}(a_{22}')^{(3)}\,+(a_{22}'')^{(3)}(T_{21},t)\;+(a_{18}'')^{(2,2,2)}(T_{17},t)\;+(a_{15}'')^{(1,1,1,)}(T_{14},t)\\ +(a_{26}'')^{(4,4,4,4,4,4)}(T_{25},t)\;+(a_{30}'')^{(5,5,5,5,5,5)}(T_{29},t)\;+(a_{34}'')^{(6,6,6,6,6,6)}(T_{33},t)\\ +(a_{38}'')^{(7,7,7,7)}(T_{37},t)\;+(a_{42}'')^{(8,8,8,8)}(T_{41},t)\;+(a_{46}'')^{(9,9,9)}(T_{45},t)\end{array}\right]G_{22} \quad 69$$

$+(a_{20}'')^{(3)}(T_{21},t)$, $+(a_{21}'')^{(3)}(T_{21},t)$, $+(a_{22}'')^{(3)}(T_{21},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a_{16}'')^{(2,2,2)}(T_{17},t)$, $+(a_{17}'')^{(2,2,2)}(T_{17},t)$, $+(a_{18}'')^{(2,2,2)}(T_{17},t)$ are second augmentation coefficients for category 1, 2 and 3

$+(a_{13}'')^{(1,1,1,)}(T_{14},t)$, $+(a_{14}'')^{(1,1,1,)}(T_{14},t)$, $+(a_{15}'')^{(1,1,1,)}(T_{14},t)$ are third augmentation coefficients for category 1, 2 and 3

$+(a_{24}'')^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a_{25}'')^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a_{26}'')^{(4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficients for category 1, 2 and 3

$+(a_{28}'')^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a_{29}'')^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a_{30}'')^{(5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficients for category 1, 2 and 3

$+(a_{32}'')^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a_{33}'')^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a_{34}'')^{(6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficients for category 1, 2 and 3

$+(a_{36}'')^{(7,7,7,7)}(T_{37},t)$, $+(a_{37}'')^{(7,7,7,7)}(T_{37},t)$, $+(a_{38}'')^{(7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1, 2 and 3

$+(a_{40}'')^{(8,8,8,8)}(T_{41},t)$, $+(a_{41}'')^{(8,8,8,8)}(T_{41},t)$, $+(a_{42}'')^{(8,8,8,8)}(T_{41},t)$ are eight augmentation coefficients for category 1, 2 and 3

$+(a_{44}'')^{(9,9,9)}(T_{45},t)$, $+(a_{45}'')^{(9,9,9)}(T_{45},t)$, $+(a_{46}'')^{(9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for

category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b'_{20})^{(3)}\boxed{-(b''_{20})^{(3)}(G_{23},t)}\boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}\boxed{-(b''_{13})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)}\boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)}\boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7)}(G_{39},t)}\boxed{-(b''_{40})^{(8,8,8,8)}(G_{43},t)}\boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{20} \quad 70$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b'_{21})^{(3)}\boxed{-(b''_{21})^{(3)}(G_{23},t)}\boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}\boxed{-(b''_{14})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)}\boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)}\boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7)}(G_{39},t)}\boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)}\boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{21} \quad 71$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)}\boxed{-(b''_{22})^{(3)}(G_{23},t)}\boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}\boxed{-(b''_{15})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)}\boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)}\boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)}\boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)}\boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{22} \quad 72$$

$\boxed{-(b''_{20})^{(3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3)}(G_{23},t)}$ *are first detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}$ *are second detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{13})^{(1,1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,)}(G,t)}$ *are third detrition coefficients for category 1,2 and 3*

$\boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)}$ *are fourth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)}$ *are fifth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)}$ *are sixth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{36})^{(7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7)}(G_{39},t)}$ $\boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \begin{bmatrix} (a'_{24})^{(4)}\boxed{+(a''_{24})^{(4)}(T_{25},t)}\boxed{+(a''_{28})^{(5,5,)}(T_{29},t)}\boxed{+(a''_{32})^{(6,6,)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)}\boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)}\boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)}\boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)}\boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{24} \quad 73$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \begin{bmatrix} (a'_{25})^{(4)}\boxed{+(a''_{25})^{(4)}(T_{25},t)}\boxed{+(a''_{29})^{(5,5,)}(T_{29},t)}\boxed{+(a''_{33})^{(6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)}\boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)}\boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)}\boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)}\boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{25} \quad 74$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[\begin{array}{ccc} (a'_{26})^{(4)} & +(a''_{26})^{(4)}(T_{25},t) & +(a''_{30})^{(5,5,)}(T_{29},t) \quad +(a''_{34})^{(6,6,)}(T_{33},t) \\ +(a''_{15})^{(1,1,1,1)}(T_{14},t) & +(a''_{18})^{(2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3)}(T_{21},t) \\ +(a''_{38})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9)}(T_{45},t) \end{array}\right] G_{26} \quad 75$$

$(a''_{24})^{(4)}(T_{25},t)$, $(a''_{25})^{(4)}(T_{25},t)$, $(a''_{26})^{(4)}(T_{25},t)$ *are first augmentation coefficients category* 1, 2 3

$+(a''_{28})^{(5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,)}(T_{29},t)$ *are second augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{32})^{(6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,)}(T_{33},t)$ *are third augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{13})^{(1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1)}(T_{14},t)$ *are fourth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{16})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2)}(T_{17},t)$ *are fifth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{20})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3)}(T_{21},t)$ *are sixth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)$ *are seventh augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)$ *are eighth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{46})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9)}(T_{45},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[\begin{array}{ccc} (b'_{24})^{(4)} & -(b''_{24})^{(4)}(G_{27},t) & -(b''_{28})^{(5,5,)}(G_{31},t) \quad -(b''_{32})^{(6,6,)}(G_{35},t) \\ -(b''_{13})^{(1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3)}(G_{23},t) \\ -(b''_{36})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9)}(G_{47},t) \end{array}\right] T_{24} \quad 76$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[\begin{array}{ccc} (b'_{25})^{(4)} & -(b''_{25})^{(4)}(G_{27},t) & -(b''_{29})^{(5,5,)}(G_{31},t) \quad -(b''_{33})^{(6,6,)}(G_{35},t) \\ -(b''_{14})^{(1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3)}(G_{23},t) \\ -(b''_{37})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9)}(G_{47},t) \end{array}\right] T_{25} \quad 77$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[\begin{array}{ccc} (b'_{26})^{(4)} & -(b''_{26})^{(4)}(G_{27},t) & -(b''_{30})^{(5,5,)}(G_{31},t) \quad -(b''_{34})^{(6,6,)}(G_{35},t) \\ -(b''_{15})^{(1,1,1,1)}(G,t) & -(b''_{18})^{(2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3)}(G_{23},t) \\ -(b''_{38})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9)}(G_{47},t) \end{array}\right] T_{26} \quad 78$$

*Where* $-(b''_{24})^{(4)}(G_{27},t)$, $-(b''_{25})^{(4)}(G_{27},t)$, $-(b''_{26})^{(4)}(G_{27},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{28})^{(5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,)}(G_{31},t)$ *are second detrition coefficients for category* 1, 2 *and* 3

$-(b''_{32})^{(6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,)}(G_{35},t)$ *are third detrition coefficients for category* 1, 2 *and* 3

$-(b''_{13})^{(1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1)}(G,t)$

*are fourth detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2,2)}(G_{19},t)}$

*are fifth detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3,3,3,3)}(G_{23},t)}$

*are sixth detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)}$

*are seventh detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)}$

*are eighth detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{46})^{(9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{45})^{(9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{44})^{(9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[\begin{array}{ccc} (a'_{28})^{(5)}\boxed{+(a''_{28})^{(5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{28} \quad (79)$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[\begin{array}{ccc} (a'_{29})^{(5)}\boxed{+(a''_{29})^{(5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{29} \quad (80)$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[\begin{array}{ccc} (a'_{30})^{(5)}\boxed{+(a''_{30})^{(5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{30} \quad (81)$$

*Where* $\boxed{+(a''_{28})^{(5)}(T_{29},t)}, \boxed{+(a''_{29})^{(5)}(T_{29},t)}, \boxed{+(a''_{30})^{(5)}(T_{29},t)}$ *are first augmentation coefficients for category* 1, 2 *and* 3

*And* $\boxed{+(a''_{24})^{(4,4,)}(T_{25},t)}, \boxed{+(a''_{25})^{(4,4,)}(T_{25},t)}, \boxed{+(a''_{26})^{(4,4,)}(T_{25},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)}, \boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)}, \boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)}$ are fourth augmentation coefficients for category 1,2, and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)}$ are fifth augmentation coefficients for category 1,2,and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)}$ are sixth augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)}$ are seventh augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)}$ are eighth augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation

coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \begin{bmatrix} (b'_{28})^{(5)}\boxed{-(b''_{28})^{(5)}(G_{31},t)} & \boxed{-(b''_{24})^{(4,4,)}(G_{27},t)} & \boxed{-(b''_{32})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{28} \quad (82)$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b'_{29})^{(5)}\boxed{-(b''_{29})^{(5)}(G_{31},t)} & \boxed{-(b''_{25})^{(4,4,)}(G_{27},t)} & \boxed{-(b''_{33})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{29} \quad (83)$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b'_{30})^{(5)}\boxed{-(b''_{30})^{(5)}(G_{31},t)} & \boxed{-(b''_{26})^{(4,4,)}(G_{27},t)} & \boxed{-(b''_{34})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{30} \quad (84)$$

*where* $\boxed{-(b''_{28})^{(5)}(G_{31},t)}$ , $\boxed{-(b''_{29})^{(5)}(G_{31},t)}$ , $\boxed{-(b''_{30})^{(5)}(G_{31},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{24})^{(4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,)}(G_{27},t)}$ *are second detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{32})^{(6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6)}(G_{35},t)}$ *are third detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1)}(G,t)}$ , $\boxed{-(b''_{15})^{(1,1,1,1,1,)}(G,t)}$ are fourth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)}$ , $\boxed{-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)}$ , $\boxed{-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)}$ , $\boxed{-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)}$ , $\boxed{-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)}$ , $\boxed{-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)}$ , $\boxed{-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \begin{bmatrix} (a'_{32})^{(6)}\boxed{+(a''_{32})^{(6)}(T_{33},t)} & \boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{32} \quad (85)$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \begin{bmatrix} (a'_{33})^{(6)}\boxed{+(a''_{33})^{(6)}(T_{33},t)} & \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{33} \quad (86)$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \begin{bmatrix} (a'_{34})^{(6)}\boxed{+(a''_{34})^{(6)}(T_{33},t)} & \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{34} \quad 87$$

$\boxed{+(a''_{32})^{(6)}(T_{33},t)}, \boxed{+(a''_{33})^{(6)}(T_{33},t)}, \boxed{+(a''_{34})^{(6)}(T_{33},t)}$ *are first augmentation coefficients for category* $1, 2$ *and* $3$

$\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}$ *are second augmentation coefficients for category* $1, 2$ *and* $3$

$\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}$ *are third augmentation coefficients for category* $1, 2$ *and* $3$

$\boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}$ - are fourth augmentation coefficients

$\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}$ - fifth augmentation coefficients

$\boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)}$ sixth augmentation coefficients

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)}$ seventh augmentation coefficients

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}$ Eighth augmentation coefficients

$\boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)}$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \begin{bmatrix} (b'_{32})^{(6)}\boxed{-(b''_{32})^{(6)}(G_{35},t)} & \boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{32} \quad 88$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \begin{bmatrix} (b'_{33})^{(6)}\boxed{-(b''_{33})^{(6)}(G_{35},t)} & \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{33} \quad 89$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)}\boxed{-(b''_{34})^{(6)}(G_{35},t)} & \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{34} \quad 90$$

$\boxed{-(b''_{32})^{(6)}(G_{35},t)}, \boxed{-(b''_{33})^{(6)}(G_{35},t)}, \boxed{-(b''_{34})^{(6)}(G_{35},t)}$ *are first detrition coefficients for category* $1, 2$ *and* $3$

$\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}$ *are second detrition coefficients for category* $1, 2$ *and* $3$

$\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)}$ *are third detrition coefficients*

$for\ category\ 1,2\ and\ 3$

$-(b_{13}'')^{(1,1,1,1,1,1,1)}(G,t)$, $-(b_{14}'')^{(1,1,1,1,1,1,1)}(G,t)$, $-(b_{15}'')^{(1,1,1,1,1,1,1)}(G,t)$ are fourth detrition coefficients for category 1, 2, and 3

$-(b_{16}'')^{(2,2,2,2,2,2,2)}(G_{19},t)$, $-(b_{17}'')^{(2,2,2,2,2,2,2)}(G_{19},t)$, $-(b_{18}'')^{(2,2,2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1, 2, and 3

$-(b_{20}'')^{(3,3,3,3,3,3,3)}(G_{23},t)$, $-(b_{21}'')^{(3,3,3,3,3,3,3)}(G_{23},t)$, $-(b_{22}'')^{(3,3,3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1, 2, and 3

$-(b_{36}'')^{(7,7,7,7,7,7,7)}(G_{39},t)$, $-(b_{37}'')^{(7,7,7,7,7,7,7)}(G_{39},t)$, $-(b_{38}'')^{(7,7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2, and 3

$-(b_{40}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)$, $-(b_{41}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)$, $-(b_{42}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1, 2, and 3

$-(b_{46}'')^{(9,9,9,9,9,9,9)}(G_{47},t)$, $-(b_{45}'')^{(9,9,9,9,9,9,9)}(G_{47},t)$, $-(b_{44}'')^{(9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \qquad 91$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a_{36}')^{(7)} & +(a_{36}'')^{(7)}(T_{37},t) & +(a_{16}'')^{(2,2,2,2,2,2,2)}(T_{17},t) & +(a_{20}'')^{(3,3,3,3,3,3,3)}(T_{21},t) \\ & +(a_{24}'')^{(4,4,4,4,4,4,4)}(T_{25},t) & +(a_{28}'')^{(5,5,5,5,5,5,5)}(T_{29},t) & +(a_{32}'')^{(6,6,6,6,6,6,6)}(T_{33},t) \\ & +(a_{13}'')^{(1,1,1,1,1,1,1)}(T_{14},t) & +(a_{40}'')^{(8,8,8,8,8,8,8,8,)}(T_{41},t) & +(a_{44}'')^{(9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \qquad 92$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a_{37}')^{(7)} & +(a_{37}'')^{(7)}(T_{37},t) & +(a_{17}'')^{(2,2,2,2,2,2,2)}(T_{17},t) & +(a_{21}'')^{(3,3,3,3,3,3,3)}(T_{21},t) \\ & +(a_{25}'')^{(4,4,4,4,4,4,4)}(T_{25},t) & +(a_{29}'')^{(5,5,5,5,5,5,5)}(T_{29},t) & +(a_{33}'')^{(6,6,6,6,6,6,6)}(T_{33},t) \\ & +(a_{13}'')^{(1,1,1,1,1,1,1)}(T_{14},t) & +(a_{41}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t) & +(a_{45}'')^{(9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{14}$$

$$\frac{dG_{38}}{dt} \qquad 93$$

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a_{38}')^{(7)} & +(a_{38}'')^{(7)}(T_{37},t) & +(a_{18}'')^{(2,2,2,2,2,2,2)}(T_{17},t) & +(a_{22}'')^{(3,3,3,3,3,3,3)}(T_{21},t) \\ & +(a_{26}'')^{(4,4,4,4,4,4,4)}(T_{25},t) & +(a_{30}'')^{(5,5,5,5,5,5,5)}(T_{29},t) & +(a_{34}'')^{(6,6,6,6,6,6,6)}(T_{33},t) \\ & +(a_{15}'')^{(1,1,1,1,1,1,1)}(T_{14},t) & +(a_{42}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t) & +(a_{46}'')^{(9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{15}$$

Where $(a_{36}'')^{(7)}(T_{37},t)$, $(a_{37}'')^{(7)}(T_{37},t)$, $(a_{38}'')^{(7)}(T_{37},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a_{16}'')^{(2,2,2,2,2,2,2)}(T_{17},t)$, $+(a_{17}'')^{(2,2,2,2,2,2,2)}(T_{17},t)$, $+(a_{18}'')^{(2,2,2,2,2,2,2)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a_{20}'')^{(3,3,3,3,3,3,3)}(T_{21},t)$, $+(a_{21}'')^{(3,3,3,3,3,3,3)}(T_{21},t)$, $+(a_{22}'')^{(3,3,3,3,3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a_{24}'')^{(4,4,4,4,4,4,4)}(T_{25},t)$, $+(a_{25}'')^{(4,4,4,4,4,4,4)}(T_{25},t)$, $+(a_{26}'')^{(4,4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a_{28}'')^{(5,5,5,5,5,5,5)}(T_{29},t)$, $+(a_{29}'')^{(5,5,5,5,5,5,5)}(T_{29},t)$, $+(a_{30}'')^{(5,5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a_{32}'')^{(6,6,6,6,6,6,6)}(T_{33},t)$, $+(a_{33}'')^{(6,6,6,6,6,6,6)}(T_{33},t)$, $+(a_{34}'')^{(6,6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation

coefficient for category 1, 2 and 3
$+(a_{13}'')^{(1,1,1,1,1,1,1)}(T_{14},t)$, $+(a_{13}'')^{(1,1,1,1,1,1,1)}(T_{14},t)$, $+(a_{15}'')^{(1,1,1,1,1,1,1)}(T_{14},t)$ are seventh augmentation coefficient for category 1, 2 and 3
$+(a_{42}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a_{41}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a_{40}'')^{(8,8,8,8,8,8,8,8,)}(T_{41},t)$ are eighth augmentation coefficient for 1,2,3
$+(a_{46}'')^{(9,9,9,9,9,9,9)}(T_{45},t)$, $+(a_{45}'')^{(9,9,9,9,9,9,9)}(T_{45},t)$, $+(a_{44}'')^{(9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{36}}{dt} =$$



$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b_{36}')^{(7)}\boxed{-(b_{36}'')^{(7)}(G_{39},t)} & \boxed{-(b_{16}'')^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{20}'')^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{24}'')^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{28}'')^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{32}'')^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{13}'')^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b_{44}'')^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{37}}{dt} =$$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b_{37}')^{(7)}\boxed{-(b_{37}'')^{(7)}(G_{39},t)} & \boxed{-(b_{17}'')^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{21}'')^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{25}'')^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{29}'')^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{33}'')^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{14}'')^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b_{45}'')^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{38}}{dt} =$$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b_{38}')^{(7)}\boxed{-(b_{38}'')^{(7)}(G_{39},t)} & \boxed{-(b_{18}'')^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{22}'')^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{26}'')^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{30}'')^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{34}'')^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{15}'')^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b_{46}'')^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b_{36}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{37}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3
$\boxed{-(b_{16}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{17}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{18}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3
$\boxed{-(b_{20}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{21}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{22}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3
$\boxed{-(b_{24}'')^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{25}'')^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{26}'')^{(4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3
$\boxed{-(b_{28}'')^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3
$\boxed{-(b_{32}'')^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{34}'')^{(6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3
$\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3
$\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3



$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \begin{bmatrix} (a'_{40})^{(8)}\boxed{+(a''_{40})^{(8)}(T_{41},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \begin{bmatrix} (a'_{41})^{(8)}\boxed{+(a''_{41})^{(8)}(T_{41},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \begin{bmatrix} (a'_{42})^{(8)}\boxed{+(a''_{42})^{(8)}(T_{41},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a''_{40})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} =$$

$$(b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)}\boxed{-(b''_{40})^{(8)}(G_{43},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} =$$

$$(b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)}\boxed{-(b''_{41})^{(8)}(G_{43},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} =$$

$$(b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)}\boxed{-(b''_{42})^{(8)}(G_{43},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3



$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \begin{bmatrix} (a'_{44})^{(9)}\boxed{+(a''_{44})^{(9)}(T_{45},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \begin{bmatrix} (a'_{45})^{(9)}\boxed{+(a''_{45})^{(9)}(T_{45},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \begin{bmatrix} (a'_{46})^{(9)}\boxed{+(a''_{46})^{(9)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a''_{44})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ are Seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} -$$

$$\begin{bmatrix} (b'_{44})^{(9)}\boxed{-(b''_{44})^{(9)}(G_{47},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b'_{45})^{(9)}\boxed{-(b''_{45})^{(9)}(G_{47},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b'_{46})^{(9)}\boxed{-(b''_{46})^{(9)}(G_{47},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{44})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9)}(G_{47},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a'_i)^{(1)}, (a''_i)^{(1)}, (b_i)^{(1)}, (b'_i)^{(1)}, (b''_i)^{(1)} > 0, \quad i,j = 13,14,15$$ *97*

The functions $(a''_i)^{(1)}, (b''_i)^{(1)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$(a_i'')^{(1)}(T_{14},t) \leq (p_i)^{(1)} \leq (\hat{A}_{13})^{(1)}$

$(b_i'')^{(1)}(G,t) \leq (r_i)^{(1)} \leq (b_i')^{(1)} \leq (\hat{B}_{13})^{(1)}$

$lim_{T_2\to\infty}(a_i'')^{(1)}(T_{14},t) = (p_i)^{(1)}$ 98

$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = (r_i)^{(1)}$

**Definition of** $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$ :

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$|(a_i'')^{(1)}(T_{14}',t) - (a_i'')^{(1)}(T_{14},t)| \leq (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$

$|(b_i'')^{(1)}(G',t) - (b_i'')^{(1)}(G,t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}',t)$ and $(a_i'')^{(1)}(T_{14},t)$ . $(T_{14}',t)$ and $(T_{14},t)$ are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14},t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$ : 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}$ , $\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$

**Definition of** $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$ : 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$

$\frac{1}{(\hat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$

Where we suppose

$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i,j = 16,17,18$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17},t) \le (p_i)^{(2)} \le \left(\hat{A}_{16}\right)^{(2)} \quad 102$$

$$(b_i'')^{(2)}(G_{19},t) \le (r_i)^{(2)} \le (b_i')^{(2)} \le (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2\to\infty}(a_i'')^{(2)}(T_{17},t) = (p_i)^{(2)} \quad 104$$

$$\lim_{G\to\infty}(b_i'')^{(2)}\left((G_{19}),t\right) = (r_i)^{(2)} \quad 105$$

**Definition of** $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$ : 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}',t) - (a_i'')^{(2)}(T_{17},t)| \le (\hat{k}_{16})^{(2)}|T_{17} - T_{17}'|e^{-(\hat{M}_{16})^{(2)}t} \quad 107$$

$$|(b_i'')^{(2)}((G_{19})',t) - (b_i'')^{(2)}\left((G_{19}),t\right)| < (\hat{k}_{16})^{(2)}||(G_{19}) - (G_{19})'||e^{-(\hat{M}_{16})^{(2)}t} \quad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}',t)$ and$(a_i'')^{(2)}(T_{17},t)$ . $(T_{17}',t)$ and $(T_{17},t)$ are points belonging to the interval $\left[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}\right]$ . It is to be noted that $(a_i'')^{(2)}(T_{17},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17},t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$ :

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants 109

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$$

**Definition of** $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$ :

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1 \quad 110$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(b_i)^{(2)} + (b_i')^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1 \quad 111$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i,j = 20,21,22 \quad 112$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(3)}$, $(\mathrm{r_i})^{(3)}$:

$$(a_i'')^{(3)}(T_{21},t) \le (p_i)^{(3)} \le (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23},t) \le (r_i)^{(3)} \le (b_i')^{(3)} \le (\hat{B}_{20})^{(3)}$$

$$lim_{T_2\to\infty}(a_i'')^{(3)}(T_{21},t) = (p_i)^{(3)} \qquad 113$$

$$\lim_{G\to\infty}(b_i'')^{(3)}(G_{23},t) = (r_i)^{(3)}$$

**Definition of** $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$ :

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: 114

$$|(a_i'')^{(3)}(T_{21}',t) - (a_i'')^{(3)}(T_{21},t)| \le (\hat{k}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\hat{M}_{20})^{(3)}t}$$

$$|(b_i'')^{(3)}(G_{23}',t) - (b_i'')^{(3)}(G_{23},t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}'||e^{-(\hat{M}_{20})^{(3)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}',t)$ and$(a_i'')^{(3)}(T_{21},t)$ . $(T_{21}',t)$ And $(T_{21},t)$ are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$ : 115

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$ and the constants $(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}, i = 20,21,22,$ satisfy the inequalities 116

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(a_i)^{(3)} + (a_i')^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(b_i)^{(3)} + (b_i')^{(3)} + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i,j = 24,25,26 \qquad 117$$

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(4)}$, $(r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25},t) \le (p_i)^{(4)} \le (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}\big((G_{27}),t\big) \le (r_i)^{(4)} \le (b_i')^{(4)} \le (\hat{B}_{24})^{(4)}$$

$$lim_{T_2\to\infty}(a_i'')^{(4)}(T_{25},t) = (p_i)^{(4)} \quad (118)$$

$$\lim_{G\to\infty}(b_i'')^{(4)}\big((G_{27}),t\big) = (r_i)^{(4)}$$

**Definition of** $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$ :

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24,25,26}$

They satisfy Lipschitz condition: (119)

$$|(a_i'')^{(4)}(T_{25}',t) - (a_i'')^{(4)}(T_{25},t)| \le (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})',t) - (b_i'')^{(4)}\big((G_{27}),t\big)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}',t)$ and$(a_i'')^{(4)}(T_{25},t)$ . $(T_{25}',t)$ and $(T_{25},t)$ are points belonging to the interval $\left[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}\right]$. It is to be noted that $(a_i'')^{(4)}(T_{25},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25},t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$ : (120)

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$, are positive constants

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}, \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$$

**Definition of** $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$ : (121)

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and\ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}, i = 24,25,26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(a_i)^{(4)} + (a_i')^{(4)} + (\hat{A}_{24})^{(4)} + (\hat{P}_{24})^{(4)}(\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(b_i)^{(4)} + (b_i')^{(4)} + (\hat{B}_{24})^{(4)} + (\hat{Q}_{24})^{(4)}(\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i,j = 28,29,30 \quad (122)$$

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(5)}$, $(r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29},t) \le (p_i)^{(5)} \le (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}\big((G_{31}),t\big) \le (r_i)^{(5)} \le (b_i')^{(5)} \le (\hat{B}_{28})^{(5)}$$

$$lim_{T_2\to\infty}(a_i'')^{(5)}(T_{29},t) = (p_i)^{(5)} \qquad 123$$

$$\lim_{G\to\infty}(b_i'')^{(5)}(G_{31},t) = (r_i)^{(5)}$$

**Definition of** $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$ :

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}',t) - (a_i'')^{(5)}(T_{29},t)| \le (\hat{k}_{28})^{(5)}|T_{29} - T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})',t) - (b_i'')^{(5)}\big((G_{31}),t\big)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}',t)$ and$(a_i'')^{(5)}(T_{29},t)$ . $(T_{29}',t)$ and $(T_{29},t)$ are points belonging to the interval $\big[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\big]$. It is to be noted that $(a_i'')^{(5)}(T_{29},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29},t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$ : 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}, \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

**Definition of** $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$ : 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and\ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(a_i)^{(5)} + (a_i')^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)}(\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(b_i)^{(5)} + (b_i')^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)}(\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i,j = 32,33,34 \qquad 127$$

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(6)}$, $(r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33},t) \le (p_i)^{(6)} \le (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}),t) \le (r_i)^{(6)} \le (b_i')^{(6)} \le (\hat{B}_{32})^{(6)}$$

$$lim_{T_2\to\infty}(a_i'')^{(6)}(T_{33},t) = (p_i)^{(6)} \quad (128)$$

$$\lim_{G\to\infty}(b_i'')^{(6)}\left((G_{35}),t\right) = (r_i)^{(6)}$$

**Definition of** $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$ :

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}',t) - (a_i'')^{(6)}(T_{33},t)| \le (\hat{k}_{32})^{(6)}|T_{33} - T_{33}'|e^{-(\hat{M}_{32})^{(6)}t}$$

$$|(b_i'')^{(6)}((G_{35})',t) - (b_i'')^{(6)}\left((G_{35}),t\right)| < (\hat{k}_{32})^{(6)}||(G_{35}) - (G_{35})'||e^{-(\hat{M}_{32})^{(6)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}',t)$ and$(a_i'')^{(6)}(T_{33},t)$ . $(T_{33}',t)$ and $(T_{33},t)$ are points belonging to the interval $\left[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}\right]$. It is to be noted that $(a_i'')^{(6)}(T_{33},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33},t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$ : (129)

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$, are positive constants

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}} \ , \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$$

**Definition of** $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$ : (130)

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and\ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32,33,34,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(a_i)^{(6)} + (a_i')^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(b_i)^{(6)} + (b_i')^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

(KKKKK) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i,j = 36,37,38$ (131)

(LLLLL) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(7)}$, $(r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37},t) \le (p_i)^{(7)} \le (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39},t) \le (r_i)^{(7)} \le (b_i')^{(7)} \le (\hat{B}_{36})^{(7)}$$

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*(MMMMM)* $\lim_{T_2\to\infty} (a_i'')^{(7)}(T_{37},t) = (p_i)^{(7)}$

*(NNNNN)*

$\lim_{G\to\infty}(b_i'')^{(7)}\big((G_{39}),t\big) = (r_i)^{(7)}$

**Definition of** $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$ :

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$$|(a_i'')^{(7)}(T_{37}',t) - (a_i'')^{(7)}(T_{37},t)| \le (\hat{k}_{36})^{(7)}|T_{37} - T_{37}'|e^{-(\hat{M}_{36})^{(7)}t}$$

$$|(b_i'')^{(7)}((G_{39})',t) - (b_i'')^{(7)}\big((G_{39}),t\big)| < (\hat{k}_{36})^{(7)}||(G_{39}) - (G_{39})'||e^{-(\hat{M}_{36})^{(7)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}',t)$ and$(a_i'')^{(7)}(T_{37},t)$ . $(T_{37}',t)$ and $(T_{37},t)$ are points belonging to the interval $\big[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}\big]$. It is to be noted that $(a_i'')^{(7)}(T_{37},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37},t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$ : 134

(OOOOO) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$, are positive constants

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$$

**Definition of** $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$ : 135

(PPPPP) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)}(\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(b_i)^{(7)} + (b_i')^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)}(\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40,41,42 \tag{136}$$

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

**Definition of** $(p_i)^{(8)}$, $(r_i)^{(8)}$: (137)

$$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)} \tag{138}$$

$$(b_i'')^{(8)}((G_{43}), t) \leq (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)} \tag{139}$$

$$\lim_{T_2 \to \infty} (a_i'')^{(8)}(T_{41}, t) = (p_i)^{(8)} \tag{140}$$

$$\lim_{G \to \infty} (b_i'')^{(8)}\left((G_{43}), t\right) = (r_i)^{(8)} \tag{141}$$

**Definition of** $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$ :

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)} |T_{41} - T_{41}'| e^{-(\hat{M}_{40})^{(8)} t} \tag{142}$$

$$|(b_i'')^{(8)}((G_{43})', t) - (b_i'')^{(8)}\left((G_{43}), t\right)| < (\hat{k}_{40})^{(8)} ||(G_{43}) - (G_{43})'|| e^{-(\hat{M}_{40})^{(8)} t} \tag{143}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}', t)$ and $(a_i'')^{(8)}(T_{41}, t)$ . $(T_{41}', t)$ and $(T_{41}, t)$ are points belonging to the interval $\left[(\hat{k}_{40})^{(8)}, (\hat{M}_{40})^{(8)}\right]$. It is to be noted that $(a_i'')^{(8)}(T_{41}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{40})^{(8)} = 1$ then the function $(a_i'')^{(8)}(T_{41}, t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$ :

$(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$, are positive constants

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}, \frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1 \tag{144}$$

**Definition of** $(\hat{P}_{40})^{(8)}, (\hat{Q}_{40})^{(8)}$ :

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}, (\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)}, (a_i')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (p_i)^{(8)}$, $(r_i)^{(8)}, i = 40,41,42,$
Satisfy the inequalities

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(a_i)^{(8)} + (a_i')^{(8)} + (\hat{A}_{40})^{(8)} + (\hat{P}_{40})^{(8)} (\hat{k}_{40})^{(8)}] < 1 \tag{145}$$

$$\frac{1}{(\widehat{M}_{40})^{(8)}}[\ (b_i)^{(8)} + (b_i')^{(8)} + \ (\hat{B}_{40})^{(8)} + \ (\hat{Q}_{40})^{(8)}\ (\hat{k}_{40})^{(8)}] < 1 \qquad 146$$

Where we suppose

$$(a_i)^{(9)}, (a_i')^{(9)}, (a_i'')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (b_i'')^{(9)} > 0, \quad i,j = 44,45,46 \qquad 146\ A$$

The functions $(a_i'')^{(9)}, (b_i'')^{(9)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(9)}$, $(r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45}, t) \leq (p_i)^{(9)} \leq (\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47}, t) \leq \ (r_i)^{(9)} \leq (b_i')^{(9)} \leq (\hat{B}_{44})^{(9)}$$

$$\lim_{T_2 \to \infty} (a_i'')^{(9)}(T_{45}, t) = (p_i)^{(9)}$$

$$\lim_{G \to \infty} (b_i'')^{(9)}(G_{47}, t) = \ (r_i)^{(9)}$$

**Definition of** $(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}$ :

Where $\boxed{(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}, (p_i)^{(9)}, \ (r_i)^{(9)}}$ are positive constants and $\boxed{i = 44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}', t) - (a_i'')^{(9)}(T_{45}, t)| \leq (\hat{k}_{44})^{(9)} |T_{45} - T_{45}'| e^{-(\widehat{M}_{44})^{(9)} t}$$

$$|(b_i'')^{(9)}((G_{47})', t) - (b_i'')^{(9)}((G_{47}), t)| < (\hat{k}_{44})^{(9)} ||(G_{47}) - (G_{47})'|| e^{-(\widehat{M}_{44})^{(9)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}', t)$ and$(a_i'')^{(9)}(T_{45}, t)$ . $(T_{45}', t)$ and $(T_{45}, t)$ are points belonging to the interval $\left[(\hat{k}_{44})^{(9)}, (\widehat{M}_{44})^{(9)}\right]$. It is to be noted that $(a_i'')^{(9)}(T_{45}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{44})^{(9)} = 1$ then the function $(a_i'')^{(9)}(T_{45}, t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\widehat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$ :

$(\widehat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$, are positive constants

$$\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}} \ , \frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}} < 1$$

**Definition of** $(\hat{P}_{44})^{(9)}, (\hat{Q}_{44})^{(9)}$ :

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\widehat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}, (\hat{A}_{44})^{(9)} and\ (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)}, (a_i')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (p_i)^{(9)}, \ (r_i)^{(9)}, i = 44,45,46,$
satisfy the inequalities

$$\frac{1}{(\widehat{M}_{44})^{(9)}}[(a_i)^{(9)} + (a_i')^{(9)} + (\hat{A}_{44})^{(9)} + (\hat{P}_{44})^{(9)}(\hat{k}_{44})^{(9)}] < 1$$

$$\frac{1}{(\widehat{M}_{44})^{(9)}}[(b_i)^{(9)} + (b_i')^{(9)} + (\hat{B}_{44})^{(9)} + (\hat{Q}_{44})^{(9)}(\hat{k}_{44})^{(9)}] < 1$$

**Theorem 1:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0), T_i(0)$ :

$$G_i(t) \le (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 2 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

**Definition of** $G_i(0), T_i(0)$

$$G_i(t) \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} \quad , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} \quad , \quad T_i(0) = T_i^0 > 0$$

**Theorem 3 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t) \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t} \quad , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t} \quad , \quad T_i(0) = T_i^0 > 0$$

**Theorem 4 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

**Definition of** $G_i(0), T_i(0)$ :

$$G_i(t) \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 5 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

**Definition of** $G_i(0), T_i(0)$ :

$$G_i(t) \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 6 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

**Definition of** $G_i(0), T_i(0)$ :

$$G_i(t) \le (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$T_i(t) \leq (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Theorem 7:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

**Definition of** $G_i(0), T_i(0)$ :

$G_i(t) \leq (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Theorem 8:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

**Definition of** $G_i(0), T_i(0)$ :

$G_i(t) \leq (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Theorem 9:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

**Definition of** $G_i(0), T_i(0)$ :

$G_i(t) \leq (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Proof:** Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$G_i(0) = G_i^0$ , $T_i(0) = T_i^0$ , $G_i^0 \leq (\hat{P}_{13})^{(1)}$ , $T_i^0 \leq (\hat{Q}_{13})^{(1)}$, 155

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t}$ 156

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t}$ 157

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[ (a_{13})^{(1)} G_{14}(s_{(13)}) - \left( (a'_{13})^{(1)} + a''_{13})^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \right) G_{13}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[ (a_{14})^{(1)} G_{13}(s_{(13)}) - \left( (a'_{14})^{(1)} + (a''_{14})^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \right) G_{14}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[ (a_{15})^{(1)} G_{14}(s_{(13)}) - \left( (a'_{15})^{(1)} + (a''_{15})^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \right) G_{15}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[ (b_{13})^{(1)} T_{14}(s_{(13)}) - \left( (b'_{13})^{(1)} - (b''_{13})^{(1)} (G(s_{(13)}), s_{(13)}) \right) T_{13}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^{0} + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)}\left(G(s_{(13)}), s_{(13)}\right)\right) T_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{\mathrm{T}}_{15}(\mathrm{t}) = \mathrm{T}_{15}^{0} + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)}\left(G(s_{(13)}), s_{(13)}\right)\right) T_{15}(s_{(13)})\right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{16})^{(2)}, T_i^0 \le (\hat{Q}_{16})^{(2)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$$

By 160

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''_{16})^{(2)}\left(T_{17}(s_{(16)}), s_{(16)}\right)\right) G_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)}\left(T_{17}(s_{(16)}), s_{(17)}\right)\right) G_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)}\left(T_{17}(s_{(16)}), s_{(16)}\right)\right) G_{18}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)}\left(G_{19}(s_{(16)}), s_{(16)}\right)\right) T_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^0 + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)}\left(G_{19}(s_{(16)}), s_{(16)}\right)\right) T_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^0 + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)}\left(G_{19}(s_{(16)}), s_{(16)}\right)\right) T_{18}(s_{(16)})\right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{20})^{(3)}, T_i^0 \le (\hat{Q}_{20})^{(3)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$$

By 161

$$\bar{G}_{20}(t) = G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''_{20})^{(3)}\left(T_{21}(s_{(20)}), s_{(20)}\right)\right) G_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{22}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^0 + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^0 + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{\mathrm{T}}_{22}(\mathrm{t}) = \mathrm{T}_{22}^0 + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{22}(s_{(20)})\right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{24})^{(4)}, T_i^0 \le (\hat{Q}_{24})^{(4)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

By



$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{26}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{\mathrm{T}}_{26}(\mathrm{t}) = \mathrm{T}_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{26}(s_{(24)})\right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{28})^{(5)}, T_i^0 \le (\hat{Q}_{28})^{(5)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

By 163

$$\bar{G}_{28}(t) = G_{28}^{0} + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^{0} + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^{0} + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{30}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^{0} + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^{0} + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{\mathrm{T}}_{30}(\mathrm{t}) = \mathrm{T}_{30}^{0} + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{30}(s_{(28)})\right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{32})^{(6)}, T_i^0 \le (\hat{Q}_{32})^{(6)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$$

By 164

$$\bar{G}_{32}(t) = G_{32}^{0} + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a'_{32})^{(6)} + a''_{32})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^{0} + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^{0} + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{34}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^{0} + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^{0} + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{\mathrm{T}}_{34}(\mathrm{t}) = \mathrm{T}_{34}^{0} + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{34}(s_{(32)})\right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$ , $T_i(0) = T_i^0$ , $G_i^0 \leq (\hat{P}_{36})^{(7)}$ , $T_i^0 \leq (\hat{Q}_{36})^{(7)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[ (a_{36})^{(7)} G_{37}(s_{(36)}) - \left( (a'_{36})^{(7)} + a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[ (a_{37})^{(7)} G_{36}(s_{(36)}) - \left( (a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[ (a_{38})^{(7)} G_{37}(s_{(36)}) - \left( (a'_{38})^{(7)} + (a''_{38})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{38}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[ (b_{36})^{(7)} T_{37}(s_{(36)}) - \left( (b'_{36})^{(7)} - (b''_{36})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[ (b_{37})^{(7)} T_{36}(s_{(36)}) - \left( (b'_{37})^{(7)} - (b''_{37})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{38}(t) = T_{38}^0 + \int_0^t \left[ (b_{38})^{(7)} T_{37}(s_{(36)}) - \left( (b'_{38})^{(7)} - (b''_{38})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{38}(s_{(36)}) \right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i$: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$ , $T_i(0) = T_i^0$ , $G_i^0 \leq (\hat{P}_{40})^{(8)}$ , $T_i^0 \leq (\hat{Q}_{40})^{(8)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{40})^{(8)} e^{(\bar{M}_{40})^{(8)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{40})^{(8)} e^{(\bar{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[ (a_{40})^{(8)} G_{41}(s_{(40)}) - \left( (a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[ (a_{41})^{(8)} G_{40}(s_{(40)}) - \left( (a'_{41})^{(8)} + (a''_{41})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}(s_{(40)}), s_{(40)})\right) G_{42}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^0 + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43}(s_{(40)}), s_{(40)})\right) T_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^0 + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43}(s_{(40)}), s_{(40)})\right) T_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{\mathrm{T}}_{42}(\mathrm{t}) = \mathrm{T}_{42}^0 + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43}(s_{(40)}), s_{(40)})\right) T_{42}(s_{(40)})\right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** 166

Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy A

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{44})^{(9)},\ T_i^0 \le (\hat{Q}_{44})^{(9)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''_{44})^{(9)}(T_{45}(s_{(44)}), s_{(44)})\right) G_{44}(s_{(44)})\right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45}(s_{(44)}), s_{(44)})\right) G_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}(s_{(44)}), s_{(44)})\right) G_{46}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{44}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{\mathrm{T}}_{46}(\mathrm{t}) = \mathrm{T}_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{46}(s_{(44)})\right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}}\right)\right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left(e^{(\hat{M}_{13})^{(1)} t} - 1\right)$$

From which it follows that 168

$$(G_{13}(t)-G_{13}^0)e^{-(\hat{M}_{13})^{(1)}t} \le \frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}}\left[((\hat{P}_{13})^{(1)}+G_{14}^0)e^{\left(-\frac{(\hat{P}_{13})^{(1)}+G_{14}^0}{G_{14}^0}\right)}+(\hat{P}_{13})^{(1)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \le G_{16}^0 + \int_0^t \left[(a_{16})^{(2)}\left(G_{17}^0+(\hat{P}_{16})^{(6)}e^{(\hat{M}_{16})^{(2)}s_{(16)}}\right)\right] ds_{(16)} = \quad (169)$$

$$\left(1+(a_{16})^{(2)}t\right)G_{17}^0 + \frac{(a_{16})^{(2)}(\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}}\left(e^{(\hat{M}_{16})^{(2)}t}-1\right)$$

From which it follows that (170)

$$(G_{16}(t)-G_{16}^0)e^{-(\hat{M}_{16})^{(2)}t} \le \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}}\left[((\hat{P}_{16})^{(2)}+G_{17}^0)e^{\left(-\frac{(\hat{P}_{16})^{(2)}+G_{17}^0}{G_{17}^0}\right)}+(\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that (171)

$$G_{20}(t) \le G_{20}^0 + \int_0^t \left[(a_{20})^{(3)}\left(G_{21}^0+(\hat{P}_{20})^{(3)}e^{(\hat{M}_{20})^{(3)}s_{(20)}}\right)\right] ds_{(20)} =$$

$$\left(1+(a_{20})^{(3)}t\right)G_{21}^0 + \frac{(a_{20})^{(3)}(\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left(e^{(\hat{M}_{20})^{(3)}t}-1\right)$$

From which it follows that (172)

$$(G_{20}(t)-G_{20}^0)e^{-(\hat{M}_{20})^{(3)}t} \le \frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left[((\hat{P}_{20})^{(3)}+G_{21}^0)e^{\left(-\frac{(\hat{P}_{20})^{(3)}+G_{21}^0}{G_{21}^0}\right)}+(\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that (173)

$$G_{24}(t) \le G_{24}^0 + \int_0^t \left[(a_{24})^{(4)}\left(G_{25}^0+(\hat{P}_{24})^{(4)}e^{(\hat{M}_{24})^{(4)}s_{(24)}}\right)\right] ds_{(24)} =$$

$$\left(1+(a_{24})^{(4)}t\right)G_{25}^0 + \frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left(e^{(\hat{M}_{24})^{(4)}t}-1\right)$$

From which it follows that (174)

$$(G_{24}(t)-G_{24}^0)e^{-(\hat{M}_{24})^{(4)}t} \le \frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left[((\hat{P}_{24})^{(4)}+G_{25}^0)e^{\left(-\frac{(\hat{P}_{24})^{(4)}+G_{25}^0}{G_{25}^0}\right)}+(\hat{P}_{24})^{(4)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t) \le G_{28}^0 + \int_0^t \left[ (a_{28})^{(5)} \left( G_{29}^0 + (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} s_{(28)}} \right) \right] ds_{(28)} =$$

$$\left(1 + (a_{28})^{(5)} t\right) G_{29}^0 + \frac{(a_{28})^{(5)} (\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}} \left( e^{(\hat{M}_{28})^{(5)} t} - 1 \right)$$

From which it follows that 175

$$(G_{28}(t) - G_{28}^0) e^{-(\hat{M}_{28})^{(5)} t} \le \frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}} \left[ \left( (\hat{P}_{28})^{(5)} + G_{29}^0 \right) e^{\left( - \frac{(\hat{P}_{28})^{(5)} + G_{29}^0}{G_{29}^0} \right)} + (\hat{P}_{28})^{(5)} \right]$$

$(G_i^0)$ is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t) \le G_{32}^0 + \int_0^t \left[ (a_{32})^{(6)} \left( G_{33}^0 + (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} s_{(32)}} \right) \right] ds_{(32)} =$$

$$\left(1 + (a_{32})^{(6)} t\right) G_{33}^0 + \frac{(a_{32})^{(6)} (\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}} \left( e^{(\hat{M}_{32})^{(6)} t} - 1 \right)$$

From which it follows that 177

$$(G_{32}(t) - G_{32}^0) e^{-(\hat{M}_{32})^{(6)} t} \le \frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}} \left[ \left( (\hat{P}_{32})^{(6)} + G_{33}^0 \right) e^{\left( - \frac{(\hat{P}_{32})^{(6)} + G_{33}^0}{G_{33}^0} \right)} + (\hat{P}_{32})^{(6)} \right]$$

$(G_i^0)$ is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

(t) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t) \le G_{36}^0 + \int_0^t \left[ (a_{36})^{(7)} \left( G_{37}^0 + (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} s_{(36)}} \right) \right] ds_{(36)} =$$

$$\left(1 + (a_{36})^{(7)} t\right) G_{37}^0 + \frac{(a_{36})^{(7)} (\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left( e^{(\hat{M}_{36})^{(7)} t} - 1 \right)$$

From which it follows that

$$(G_{36}(t) - G_{36}^0) e^{-(\hat{M}_{36})^{(7)} t} \le \frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left[ \left( (\hat{P}_{36})^{(7)} + G_{37}^0 \right) e^{\left( - \frac{(\hat{P}_{36})^{(7)} + G_{37}^0}{G_{37}^0} \right)} + (\hat{P}_{36})^{(7)} \right]$$

$(G_i^0)$ is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t) \le G_{40}^0 + \int_0^t \left[ (a_{40})^{(8)} \left( G_{41}^0 + (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} s_{(40)}} \right) \right] ds_{(40)} = \quad 180$$

$$\left(1 + (a_{40})^{(8)} t\right) G_{41}^0 + \frac{(a_{40})^{(8)} (\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}} \left( e^{(\hat{M}_{40})^{(8)} t} - 1 \right)$$

From which it follows that 181

$$(G_{40}(t)-G_{40}^{0})e^{-(\hat{M}_{40})^{(8)}t}\leq\frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left[((\hat{P}_{40})^{(8)}+G_{41}^{0})e^{\left(-\frac{(\hat{P}_{40})^{(8)}+G_{41}^{0}}{G_{41}^{0}}\right)}+(\hat{P}_{40})^{(8)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 8

Analogous inequalities hold also for $G_{41}, G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t)\leq G_{44}^{0}+\int_{0}^{t}\left[(a_{44})^{(9)}\left(G_{45}^{0}+(\hat{P}_{44})^{(9)}e^{(\hat{M}_{44})^{(9)}s_{(44)}}\right)\right]ds_{(44)}=$$

$$\left(1+(a_{44})^{(9)}t\right)G_{45}^{0}+\frac{(a_{44})^{(9)}(\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}}\left(e^{(\hat{M}_{44})^{(9)}t}-1\right)$$

From which it follows that

$$(G_{44}(t)-G_{44}^{0})e^{-(\hat{M}_{44})^{(9)}t}\leq\frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}}\left[((\hat{P}_{44})^{(9)}+G_{45}^{0})e^{\left(-\frac{(\hat{P}_{44})^{(9)}+G_{45}^{0}}{G_{45}^{0}}\right)}+(\hat{P}_{44})^{(9)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 9

Analogous inequalities hold also for $G_{45}, G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}<1$ and to choose 182

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[(\hat{P}_{13})^{(1)}+((\hat{P}_{13})^{(1)}+G_j^0)e^{-\left(\frac{(\hat{P}_{13})^{(1)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{13})^{(1)} \quad 183$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[((\hat{Q}_{13})^{(1)}+T_j^0)e^{-\left(\frac{(\hat{Q}_{13})^{(1)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{13})^{(1)}\right]\leq(\hat{Q}_{13})^{(1)} \quad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left((G^{(1)},T^{(1)}),(G^{(2)},T^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\hat{M}_{13})^{(1)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\hat{M}_{13})^{(1)}t}\}$$

Indeed if we denote

**Definition of** $\tilde{G},\tilde{T}:(\tilde{G},\tilde{T})=\mathcal{A}^{(1)}(G,T)$

It results

$$\left|\tilde{G}_{13}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{13})^{(1)} \left|G_{14}^{(1)}-G_{14}^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\, ds_{(13)} +$$

$$\int_0^t \{(a'_{13})^{(1)} \left|G_{13}^{(1)}-G_{13}^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} +$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)}, s_{(13)}\right)\left|G_{13}^{(1)}-G_{13}^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} e^{(\widehat{M}_{13})^{(1)}s_{(13)}} +$$

$$G_{13}^{(2)} \left|(a''_{13})^{(1)}\left(T_{14}^{(1)}, s_{(13)}\right) - (a''_{13})^{(1)}\left(T_{14}^{(2)}, s_{(13)}\right)\right|\ e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\} ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\left|G^{(1)}-G^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)}t} \leq \qquad (186)$$
$$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)} + (a'_{13})^{(1)} + (\widehat{A}_{13})^{(1)} + (\widehat{P}_{13})^{(1)}(\widehat{k}_{13})^{(1)}\right) d\left(\left(G^{(1)}, T^{(1)};\ G^{(2)}, T^{(2)}\right)\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 1:** The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t}$ *and* $(\widehat{Q}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i = 13,14,15$ depend only on $T_{14}$ and respectively on $G(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any $t$ where $G_i(t) = 0$ *and* $T_i(t) = 0$

From 19 to 24 it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(1)} - (a_i'')^{(1)}(T_{14}(s_{(13)}), s_{(13)})\} ds_{(13)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(1)}t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2$ *and* $\left((\widehat{M}_{13})^{(1)}\right)_3$ : (187)

**Remark 3**: if $G_{13}$ is bounded, the same property have also $G_{14}$ *and* $G_{15}$ . indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a'_{14})^{(1)} G_{14}$ and by integrating

$$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1 / (a'_{14})^{(1)}$$

In the same way , one can obtain

$$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2 / (a'_{15})^{(1)}$$

If $G_{14}$ *or* $G_{15}$ is bounded, the same property follows for $G_{13}$ , $G_{15}$ and $G_{13}$ , $G_{14}$ respectively.

**Remark 4:** If $G_{13}$ *is* bounded, from below, the same property holds for $G_{14}$ *and* $G_{15}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{14}$ is bounded from below. 188

**Remark 5:** If $T_{13}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}\,(G(t),t)) = (b_{14}')^{(1)}$ then $T_{14} \to \infty$. 189

**Definition of** $(m)^{(1)}$ and $\varepsilon_1$ :

Indeed let $t_1$ be so that for $t > t_1$

$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t),t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_1}$ By taking now $\varepsilon_1$ sufficiently small one sees that $T_{14}$ is unbounded.
The same property holds for $T_{15}$ if $\lim_{t\to\infty}(b_{15}'')^{(1)}\,(G(t),t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}$ , $\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}$ *and* $(\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[(\hat{P}_{16})^{(2)} + \left((\hat{P}_{16})^{(2)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{16})^{(2)}\right] \leq (\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

**Definition of** $\widetilde{G_{19}}, \widetilde{T_{19}}$ : $\left(\widetilde{G_{19}}, \widetilde{T_{19}}\right) = \mathcal{A}^{(2)}(G_{19}, T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{16})^{(2)} \left|G_{17}^{(1)}-G_{17}^{(2)}\right| e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\, ds_{(16)} +$$

$$\int_0^t \{(a'_{16})^{(2)} \left|G_{16}^{(1)}-G_{16}^{(2)}\right| e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} +$$

$$(a''_{16})^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right)\left|G_{16}^{(1)}-G_{16}^{(2)}\right| e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} e^{(\widehat{M}_{16})^{(2)}s_{(16)}} +$$

$$G_{16}^{(2)} \left|(a''_{16})^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right)-(a''_{16})^{(2)}\left(T_{17}^{(2)}, s_{(16)}\right)\right|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\} ds_{(16)}$$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0, t]$ 197

From the hypotheses it follows

$$\left|(G_{19})^{(1)}-(G_{19})^{(2)}\right| e^{-(\widehat{M}_{16})^{(2)}t} \leq$$
$$\frac{1}{(\widehat{M}_{16})^{(2)}}\left((a_{16})^{(2)}+\ (a'_{16})^{(2)}+(\widehat{A}_{16})^{(2)}+(\widehat{P}_{16})^{(2)}(\widehat{k}_{16})^{(2)}\right) d\left(\left((G_{19})^{(1)}, (T_{19})^{(1)};\ (G_{19})^{(2)}, (T_{19})^{(2)}\right)\right)$$

And analogous inequalities for $G_i$ and $T_i$. Taking into account the hypothesis the result follows 198

**Remark 6:** The fact that we supposed $(a''_{16})^{(2)}$ and $(b''_{16})^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{16})^{(2)} e^{(\widehat{M}_{16})^{(2)}t}$ and $(\widehat{Q}_{16})^{(2)} e^{(\widehat{M}_{16})^{(2)}t}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i = 16,17,18$ depend only on $T_{17}$ and respectively on $(G_{19})$(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 7:** There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$ 200

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(2)}-(a_i'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})\} ds_{(16)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(2)}t)} > 0 \quad \text{for } t > 0$$

**Definition of** $\left((\widehat{M}_{16})^{(2)}\right)_1$, $\left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$: 201

**Remark 8:** if $G_{16}$ is bounded, the same property have also $G_{17}$ and $G_{18}$ . indeed if

$G_{16} < (\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt} \leq \left((\widehat{M}_{16})^{(2)}\right)_1 - (a'_{17})^{(2)} G_{17}$ and by integrating

$$G_{17} \leq \left((\widehat{M}_{16})^{(2)}\right)_2 = G_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1 / (a'_{17})^{(2)}$$

In the same way , one can obtain

$$G_{18} \leq \left((\widehat{M}_{16})^{(2)}\right)_3 = G_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2 / (a'_{18})^{(2)}$$

If $G_{17}$ or $G_{18}$ is bounded, the same property follows for $G_{16}$, $G_{18}$ and $G_{16}$, $G_{17}$ respectively.

**Remark 9**: If $G_{16}$ is bounded, from below, the same property holds for $G_{17}$ and $G_{18}$. The proof is analogous with the preceding one. An analogous property is true if $G_{17}$ is bounded from below. 202

**Remark 10:** If $T_{16}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t),t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$. 203

**Definition of** $(m)^{(2)}$ and $\varepsilon_2$ :

Indeed let $t_2$ be so that for $t > t_2$

$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t),t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right), \quad t = \log\frac{2}{\varepsilon_2}$ By taking now $\varepsilon_2$ sufficiently small one sees that $T_{17}$ is unbounded. 205
The same property holds for $T_{18}$ if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t),t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\bar{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\bar{M}_{20})^{(3)}} < 1$ and to choose 207

$(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\bar{M}_{20})^{(3)}}\left[(\hat{P}_{20})^{(3)} + \left((\hat{P}_{20})^{(3)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\bar{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{20})^{(3)}\right] \leq (\hat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself 210

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)},(T_{23})^{(1)}),((G_{23})^{(2)},(T_{23})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\bar{M}_{20})^{(3)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\bar{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

**Definition of** $\widetilde{G_{23}}, \widetilde{T_{23}}$ :$\left(\widetilde{(G_{23})},\widetilde{(T_{23})}\right) = \mathcal{A}^{(3)}\left((G_{23}),(T_{23})\right)$

It results 213

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_{0}^{t}(a_{20})^{(3)}\left|G_{21}^{(1)}-G_{21}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\, ds_{(20)} +$$

$$\int_{0}^{t}\{(a'_{20})^{(3)}\left|G_{20}^{(1)}-G_{20}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} +$$

$$(a''_{20})^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{(\widehat{M}_{20})^{(3)}s_{(20)}} +$$

$$G_{20}^{(2)}\left|(a''_{20})^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)}, s_{(20)}\right)\right|\ e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\} ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}t} \leq \qquad 214$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+\ (a'_{20})^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\widehat{k}_{20})^{(3)}\right) d\left(\left((G_{23})^{(1)},(T_{23})^{(1)};\ (G_{23})^{(2)},(T_{23})^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

**Remark 11:** The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}\ and\ (\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_{+}$. 215

If instead of proving the existence of the solution on $\mathbb{R}_{+}$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i = 20,21,22$ depend only on $T_{21}$ and respectively on $(G_{23})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 12:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 216

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t\{(a_i')^{(3)}-(a_i'')^{(3)}(T_{21}(s_{(20)}),s_{(20)})\}ds_{(20)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(3)}t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2\ and\ \left((\widehat{M}_{20})^{(3)}\right)_3$ : 217

**Remark 13:** if $G_{20}$ is bounded, the same property have also $G_{21}\ and\ G_{22}$ . indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a'_{21})^{(3)}G_{21}$ and by integrating

$$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1/(a'_{21})^{(3)}$$

In the same way , one can obtain

$$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2/(a'_{22})^{(3)}$$

If $G_{21}$ *or* $G_{22}$ is bounded, the same property follows for $G_{20}$ , $G_{22}$ and $G_{20}$ , $G_{21}$ respectively.

**Remark 14:** If $G_{20}$ *is* bounded, from below, the same property holds for $G_{21}$ *and* $G_{22}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{21}$ is bounded from below. 218

**Remark 15:** If $T_{20}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t),t\right)=(b_{21}')^{(3)}$ then $T_{21}\to\infty$. 219

**Definition of** $(m)^{(3)}$ and $\varepsilon_3$ :

Indeed let $t_3$ be so that for $t > t_3$

$(b_{21})^{(3)}-(b_i'')^{(3)}\left((G_{23})(t),t\right)<\varepsilon_3, T_{20}(t)>(m)^{(3)}$

Then $\frac{dT_{21}}{dt}\geq(a_{21})^{(3)}(m)^{(3)}-\varepsilon_3 T_{21}$ which leads to 220

$T_{21}\geq\left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1-e^{-\varepsilon_3 t})+T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t}=\frac{1}{2}$ it results

$T_{21}\geq\left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right),\quad t=log\frac{2}{\varepsilon_3}$ By taking now $\varepsilon_3$ sufficiently small one sees that $T_{21}$ is unbounded.
The same property holds for $T_{22}$ if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t),t\right)=(b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}$ , $\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}}<1$ and to choose 221

$(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\left[(\hat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \quad 222$$

$$\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \quad 223$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself 224

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric 225

$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{24})^{(4)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{27}}),(\widetilde{T_{27}})$ : $\left((\widetilde{G_{27}}),(\widetilde{T_{27}})\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{24})^{(4)} \left|G_{25}^{(1)}-G_{25}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)}s_{(24)}} e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\, ds_{(24)} +$$

$$\int_0^t \{(a'_{24})^{(4)}\left|G_{24}^{(1)}-G_{24}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)}s_{(24)}} e^{-(\widehat{M}_{24})^{(4)}s_{(24)}} +$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)}, s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)}s_{(24)}} e^{(\widehat{M}_{24})^{(4)}s_{(24)}} +$$

$$G_{24}^{(2)}\left|(a''_{24})^{(4)}\left(T_{25}^{(1)}, s_{(24)}\right)-(a''_{24})^{(4)}\left(T_{25}^{(2)}, s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)}s_{(24)}} e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\} ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)}t} \leq \qquad (226)$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)}+\ (a'_{24})^{(4)}+(\widehat{A}_{24})^{(4)}+(\widehat{P}_{24})^{(4)}(\widehat{k}_{24})^{(4)}\right) d\left(\left((G_{27})^{(1)}, (T_{27})^{(1)};\ (G_{27})^{(2)}, (T_{27})^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

**Remark 16:** The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}\ and\ (\widehat{Q}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_+$. (227)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i = 24,25,26$ depend only on $T_{25}$ and respectively on $(G_{27})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 17:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ (228)

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(4)}-(a_i'')^{(4)}(T_{25}(s_{(24)}),s_{(24)})\}ds_{(24)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(4)}t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2\ and\ \left((\widehat{M}_{24})^{(4)}\right)_3$ : (229)

**Remark 18:** if $G_{24}$ is bounded, the same property have also $G_{25}\ and\ G_{26}$ . indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a'_{25})^{(4)}G_{25}$ and by integrating

$$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1/(a'_{25})^{(4)}$$

In the same way , one can obtain

$$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2/(a'_{26})^{(4)}$$

If $G_{25}$ *or* $G_{26}$ is bounded, the same property follows for $G_{24}$ , $G_{26}$ and $G_{24}$ , $G_{25}$ respectively.

**Remark 19:** If $G_{24}$ *is* bounded, from below, the same property holds for $G_{25}$ *and* $G_{26}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{25}$ is bounded from below. 230

**Remark 20:** If $T_{24}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}((G_{27})(t),t)) = (b_{25}')^{(4)}$ then $T_{25} \to \infty$. 231

**Definition of** $(m)^{(4)}$ and $\varepsilon_4$ :

Indeed let $t_4$ be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t),t) < \varepsilon_4, T_{24}(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right)$, $t = log\frac{2}{\varepsilon_4}$ By taking now $\varepsilon_4$ sufficiently small one sees that $T_{25}$ is unbounded.
The same property holds for $T_{26}$ if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t),t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for $G_{29}, G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}$ , $\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)} + \left((\hat{P}_{28})^{(5)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{28})^{(5)}\right] \leq (\hat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$$d\left(((G_{31})^{(1)},(T_{31})^{(1)}),((G_{31})^{(2)},(T_{31})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

**Definition of** $\widetilde{(G_{31})}, \widetilde{(T_{31})}$ : $\left(\widetilde{(G_{31})}, \widetilde{(T_{31})}\right) = \mathcal{A}^{(5)}\left((G_{31}), (T_{31})\right)$

It results

$$\left|\tilde{G}_{28}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{28})^{(5)} \left|G_{29}^{(1)} - G_{29}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}}\, ds_{(28)} +$$

$$\int_0^t \{(a'_{28})^{(5)} \left|G_{28}^{(1)} - G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} +$$

$$(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right)\left|G_{28}^{(1)} - G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}} +$$

$$G_{28}^{(2)} \left|(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right) - (a''_{28})^{(5)}\left(T_{29}^{(2)}, s_{(28)}\right)\right| \; e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}} \} ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$$\left|(G_{31})^{(1)} - (G_{31})^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} t} \leq \qquad (237)$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)} + (a'_{28})^{(5)} + (\widehat{A}_{28})^{(5)} + (\widehat{P}_{28})^{(5)} (\widehat{k}_{28})^{(5)}\right) d\left(\left((G_{31})^{(1)}, (T_{31})^{(1)}; (G_{31})^{(2)}, (T_{31})^{(2)}\right)\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 21:** The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$ *and* $(\widehat{Q}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$ respectively of $\mathbb{R}_+$. (238)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on $T_{29}$ and respectively on $(G_{31})$(*and not on* $t$) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 22:** There does not exist any $t$ where $G_i(t) = 0$ *and* $T_i(t) = 0$ (239)

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(5)} - (a_i'')^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(5)} t)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2$ *and* $\left((\widehat{M}_{28})^{(5)}\right)_3$ : (240)

**Remark 23:** if $G_{28}$ is bounded, the same property have also $G_{29}$ *and* $G_{30}$ . indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a'_{29})^{(5)} G_{29}$ and by integrating

$$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1 / (a'_{29})^{(5)}$$

In the same way , one can obtain

$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2/(a'_{30})^{(5)}$

If $G_{29}$ *or* $G_{30}$ is bounded, the same property follows for $G_{28}$, $G_{30}$ and $G_{28}$, $G_{29}$ respectively.

**Remark 24:** If $G_{28}$ *is* bounded, from below, the same property holds for $G_{29}$ *and* $G_{30}$. The proof is analogous with the preceding one. An analogous property is true if $G_{29}$ is bounded from below. 241

**Remark 25:** If $T_{28}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}((G_{31})(t),t)) = (b'_{29})^{(5)}$ then $T_{29} \to \infty$. 242

**Definition of** $(m)^{(5)}$ and $\varepsilon_5$ :

Indeed let $t_5$ be so that for $t > t_5$

$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t),t) < \varepsilon_5, T_{28}(t) > (m)^{(5)}$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\frac{2}{\varepsilon_5}$ By taking now $\varepsilon_5$ sufficiently small one sees that $T_{29}$ is unbounded.
The same property holds for $T_{30}$ if $\lim_{t\to\infty}(b_{30}'')^{(5)}\left((G_{31})(t),t\right) = (b'_{30})^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{33}, G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}$, $\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$ and to choose 244

$(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[(\hat{P}_{32})^{(6)} + \left((\hat{P}_{32})^{(6)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{32})^{(6)} \quad 245$$

$$\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[\left((\hat{Q}_{32})^{(6)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{32})^{(6)}\right] \leq (\hat{Q}_{32})^{(6)} \quad 246$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric 247

$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right) =$

$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{32})^{(6)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{32})^{(6)}t}\}$

Indeed if we denote

**Definition of** $\widetilde{(G_{35})}, \widetilde{(T_{35})}$ : $\left(\widetilde{(G_{35})}, \widetilde{(T_{35})}\right) = \mathcal{A}^{(6)}\left((G_{35}), (T_{35})\right)$

It results

$$\left|\tilde{G}_{32}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{32})^{(6)} \left|G_{33}^{(1)} - G_{33}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}}\, ds_{(32)} +$$

$$\int_0^t \{(a'_{32})^{(6)} \left|G_{32}^{(1)} - G_{32}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} +$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)}, s_{(32)}\right)\left|G_{32}^{(1)} - G_{32}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}} +$$

$$G_{32}^{(2)} \left|(a''_{32})^{(6)}\left(T_{33}^{(1)}, s_{(32)}\right) - (a''_{32})^{(6)}\left(T_{33}^{(2)}, s_{(32)}\right)\right| \; e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}} \} ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|(G_{35})^{(1)} - (G_{35})^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} t} \leq \qquad (248)$$
$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)} + (a'_{32})^{(6)} + (\widehat{A}_{32})^{(6)} + (\widehat{P}_{32})^{(6)} (\widehat{k}_{32})^{(6)}\right) d\left(\left((G_{35})^{(1)}, (T_{35})^{(1)}; \; (G_{35})^{(2)}, (T_{35})^{(2)}\right)\right)$$

And analogous inequalities for $G_i \; and \; T_i$. Taking into account the hypothesis the result follows

**Remark 26:** The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)} t} \; and \; (\widehat{Q}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)} t}$ respectively of $\mathbb{R}_+$. (249)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i = 32{,}33{,}34$ depend only on $T_{33}$ and respectively on $(G_{35})(and \; not \; on \; t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 27:** There does not exist any $t$ where $G_i\,(t) = 0 \; and \; T_i\,(t) = 0$ (250)

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(6)} - (a_i'')^{(6)}(T_{33}(s_{(32)}), s_{(32)})\} ds_{(32)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(6)} t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{32})^{(6)}\right)_1, \left((\widehat{M}_{32})^{(6)}\right)_2 \; and \; \left((\widehat{M}_{32})^{(6)}\right)_3$ : (251)

**Remark 28:** if $G_{32}$ is bounded, the same property have also $G_{33} \; and \; G_{34}$ . indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a'_{33})^{(6)} G_{33}$ and by integrating

$$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_1 / (a'_{33})^{(6)}$$

In the same way , one can obtain

$$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_2/(a'_{34})^{(6)}$$

If $G_{33}$ *or* $G_{34}$ is bounded, the same property follows for $G_{32}$ , $G_{34}$ and $G_{32}$ , $G_{33}$ respectively.

**Remark 29:** If $G_{32}$ *is* bounded, from below, the same property holds for $G_{33}$ *and* $G_{34}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{33}$ is bounded from below. 252

**Remark 30:** If $T_{32}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}((G_{35})(t),t)) = (b'_{33})^{(6)}$ then $T_{33} \to \infty$. 253

**Definition of** $(m)^{(6)}$ and $\varepsilon_6$ :

Indeed let $t_6$ be so that for $t > t_6$

$$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t),t\right) < \varepsilon_6, T_{32}(t) > (m)^{(6)}$$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right)$, $t = log\frac{2}{\varepsilon_6}$ By taking now $\varepsilon_6$ sufficiently small one sees that $T_{33}$ is unbounded.
The same property holds for $T_{34}$ if $\lim_{t\to\infty}(b''_{34})^{(6)}\left((G_{35})(t),t(t),t\right) = (b'_{34})^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{37}$ , $G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}$ , $\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}} < 1$ and to choose
$(\widehat{P}_{36})^{(7)}$ and $(\widehat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[(\widehat{P}_{36})^{(7)} + \left((\widehat{P}_{36})^{(7)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{36})^{(7)} \quad 256$$

$$\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[\left((\widehat{Q}_{36})^{(7)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{36})^{(7)}\right] \leq (\widehat{Q}_{36})^{(7)} \quad 257$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself
The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric 258

$$d\left(\left((G_{39})^{(1)},(T_{39})^{(1)}\right),\left((G_{39})^{(2)},(T_{39})^{(2)}\right)\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\tilde{M}_{36})^{(7)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\tilde{M}_{36})^{(7)}t}\}$$

Indeed if we denote

**Definition of** $\widetilde{(G_{39})},\widetilde{(T_{39})}:\left(\widetilde{(G_{39})},\widetilde{(T_{39})}\right)=\mathcal{A}^{(7)}((G_{39}),(T_{39}))$
It results

$$\left|\tilde{G}_{36}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{36})^{(7)}\left|G_{37}^{(1)}-G_{37}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\,ds_{(36)}+$$
$$\int_0^t\{(a'_{36})^{(7)}\left|G_{36}^{(1)}-G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$(a''_{36})^{(7)}\left(T_{37}^{(1)},s_{(36)}\right)\left|G_{36}^{(1)}-G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$G_{36}^{(2)}\left|(a''_{36})^{(7)}\left(T_{37}^{(1)},s_{(36)}\right)-(a''_{36})^{(7)}\left(T_{37}^{(2)},s_{(36)}\right)\right|\ e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\}ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses on it follows

$$\left|(G_{39})^{(1)}-(G_{39})^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}t}\le \qquad 259$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}}\left((a_{36})^{(7)}+(a'_{36})^{(7)}+(\widehat{A}_{36})^{(7)}+(\widehat{P}_{36})^{(7)}(\widehat{k}_{36})^{(7)}\right)d\left(((G_{39})^{(1)},(T_{39})^{(1)};\ (G_{39})^{(2)},(T_{39})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

**Remark 31:** The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}\ and\ (\widehat{Q}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ respectively of $\mathbb{R}_+$. 260

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(7)}$ and $(b_i'')^{(7)}, i=36{,}37{,}38$ depend only on $\mathrm{T}_{37}$ and respectively on $(G_{39})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 32:** There does not exist any $t$ where $G_i\,(t)=0\ and\ T_i\,(t)=0$ 261

it results

$$G_i\,(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(7)}-(a_i'')^{(7)}(T_{37}(s_{(36)}),s_{(36)})\}ds_{(36)}\right]}\ge 0$$

$$T_i\,(t)\ge T_i^0e^{\left(-(b_i')^{(7)}t\right)}>0\quad \text{for t}>0$$

**Definition of** $\left((\widehat{M}_{36})^{(7)}\right)_1,\left((\widehat{M}_{36})^{(7)}\right)_2\ and\ \left((\widehat{M}_{36})^{(7)}\right)_3:$ 262

**Remark 33:** if $G_{36}$ is bounded, the same property have also $G_{37}\ and\ G_{38}$ . indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a'_{37})^{(7)} G_{37}$ and by integrating

$$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1/(a'_{37})^{(7)}$$

In the same way , one can obtain

$$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2/(a'_{38})^{(7)}$$

If $G_{37}$ *or* $G_{38}$ is bounded, the same property follows for $G_{36}$ , $G_{38}$ and $G_{36}$ , $G_{37}$ respectively.

**Remark 34:** If $G_{36}$ *is* bounded, from below, the same property holds for $G_{37}$ *and* $G_{38}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{37}$ is bounded from below. 263

**Remark 35:** If $T_{36}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(7)}((G_{39})(t),t)) = (b'_{37})^{(7)}$ then $T_{37} \to \infty$. 264

**Definition of** $(m)^{(7)}$ and $\varepsilon_7$ :

Indeed let $t_7$ be so that for $t > t_7$

$$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t),t) < \varepsilon_7, T_{36}(t) > (m)^{(7)}$$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right)$, $t = log\frac{2}{\varepsilon_7}$ By taking now $\varepsilon_7$ sufficiently small one sees that $T_{37}$ is unbounded.
The same property holds for $T_{38}$ if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t),t\right) = (b'_{38})^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}$ , $\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}} < 1$ and to choose 266
$(\widehat{P}_{40})^{(8)}$ and $(\widehat{Q}_{40})^{(8)}$ large to have

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[(\widehat{P}_{40})^{(8)} + \left((\widehat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{40})^{(8)} \quad 267$$

$$\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[\left((\widehat{Q}_{40})^{(8)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{40})^{(8)}\right] \leq (\widehat{Q}_{40})^{(8)} \quad 268$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)},(T_{43})^{(1)}),((G_{43})^{(2)},(T_{43})^{(2)})\right)= \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

**Definition of** $\widetilde{(G_{43})},\widetilde{(T_{43})}$ : $\left(\widetilde{(G_{43})},\widetilde{(T_{43})}\right)=\mathcal{A}^{(8)}((G_{43}),(T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{40})^{(8)}\left|G_{41}^{(1)}-G_{41}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\,ds_{(40)}+$$
$$\int_0^t\{(a'_{40})^{(8)}\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$G_{40}^{(2)}\left|(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)-(a''_{40})^{(8)}\left(T_{41}^{(2)},s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\}ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0,t]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)}-(G_{43})^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}t}\le \quad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)}+(a'_{40})^{(8)}+(\widehat{A}_{40})^{(8)}+(\widehat{P}_{40})^{(8)}(\widehat{k}_{40})^{(8)}\right)d\left(((G_{43})^{(1)},(T_{43})^{(1)};\ (G_{43})^{(2)},(T_{43})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

**Remark 36:** The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}\ and\ (\widehat{Q}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ respectively of $\mathbb{R}_+$. 274

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i=40,41,42$ depend only on $T_{41}$ and respectively on $(G_{43})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 37** There does not exist any $t$ where $G_i\,(t)=0\ and\ T_i\,(t)=0$ 275

it results

$$G_i\,(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(8)}-(a_i'')^{(8)}(T_{41}(s_{(40)}),s_{(40)})\}ds_{(40)}\right]}\ge 0$$

$$T_i\,(t)\ge T_i^0e^{\left(-(b_i')^{(8)}t\right)}>0\quad \text{for t}>0$$

**Definition of** $\left((\widehat{M}_{40})^{(8)}\right)_1, \left((\widehat{M}_{40})^{(8)}\right)_2$ *and* $\left((\widehat{M}_{40})^{(8)}\right)_3$ : 276

**Remark 38:** if $G_{40}$ is bounded, the same property have also $G_{41}$ *and* $G_{42}$ . indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a'_{41})^{(8)} G_{41}$ and by integrating

$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1/(a'_{41})^{(8)}$

In the same way , one can obtain

$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2/(a'_{42})^{(8)}$

If $G_{41}$ *or* $G_{42}$ is bounded, the same property follows for $G_{40}$ , $G_{42}$ and $G_{40}$ , $G_{41}$ respectively.

**Remark 39:** If $G_{40}$ *is* bounded, from below, the same property holds for $G_{41}$ *and* $G_{42}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{41}$ is bounded from below. 277

**Remark 40:** If $T_{40}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}((G_{43})(t),t)) = (b'_{41})^{(8)}$ then $T_{41} \to \infty$. 278

**Definition of** $(m)^{(8)}$ and $\varepsilon_8$ :

Indeed let $t_8$ be so that for $t > t_8$

$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t),t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right), \quad t = log\frac{2}{\varepsilon_8}$ By taking now $\varepsilon_8$ sufficiently small one sees that $T_{41}$ is unbounded.
The same property holds for $T_{42}$ if $\lim_{t\to\infty}(b''_{42})^{(8)}\left((G_{43})(t),t(t),t\right) = (b'_{42})^{(8)}$

It is now sufficient to take $\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}$ , $\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}} < 1$ and to choose $(\widehat{P}_{44})^{(9)}$ and $(\widehat{Q}_{44})^{(9)}$ large to have 279 A

$$\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[(\widehat{P}_{44})^{(9)} + \left((\widehat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[\left((\widehat{Q}_{44})^{(9)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{44})^{(9)}\right] \leq (\widehat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)},(T_{47})^{(1)}),((G_{47})^{(2)},(T_{47})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{47}}),(\widetilde{T_{47}}):\left((\widetilde{G_{47}}),(\widetilde{T_{47}})\right)=\mathcal{A}^{(9)}\left((G_{47}),(T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{44})^{(9)}\left|G_{45}^{(1)}-G_{45}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\,ds_{(44)}+$$
$$\int_0^t\{(a'_{44})^{(9)}\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$G_{44}^{(2)}\left|(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)-(a''_{44})^{(9)}\left(T_{45}^{(2)},s_{(44)}\right)\right|\;e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\}ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}t}\le$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)}+(a'_{44})^{(9)}+(\widehat{A}_{44})^{(9)}+(\widehat{P}_{44})^{(9)}(\widehat{k}_{44})^{(9)}\right)d\left(((G_{47})^{(1)},(T_{47})^{(1)};\;(G_{47})^{(2)},(T_{47})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis (39,35,36) the result follows

**Remark 41:** The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}\ and\ (\widehat{Q}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i=44,45,46$ depend only on $T_{45}$ and respectively on $(G_{47})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 42:** There does not exist any $t$ where $G_i\,(t)=0\ and\ T_i\,(t)=0$

From 99 to 44 it results

$$G_i\,(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(9)}-(a_i'')^{(9)}(T_{45}(s_{(44)}),s_{(44)})\}ds_{(44)}\right]}\ge 0$$

$$T_i\,(t)\ge T_i^0e^{(-(b_i')^{(9)}t)}>0\quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{44})^{(9)}\right)_1,\left((\widehat{M}_{44})^{(9)}\right)_2\ and\ \left((\widehat{M}_{44})^{(9)}\right)_3$ :

**Remark 43:** if $G_{44}$ is bounded, the same property have also $G_{45}\ and\ G_{46}$ . indeed if

$G_{44} < (\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt} \leq \left((\widehat{M}_{44})^{(9)}\right)_1 - (a'_{45})^{(9)} G_{45}$ and by integrating

$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_1 / (a'_{45})^{(9)}$

In the same way , one can obtain

$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_2 / (a'_{46})^{(9)}$

If $G_{45}$ *or* $G_{46}$ is bounded, the same property follows for $G_{44}$ , $G_{46}$ and $G_{44}$ , $G_{45}$ respectively.

**Remark 44:** If $G_{44}$ *is* bounded, from below, the same property holds for $G_{45}$ *and* $G_{46}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{45}$ is bounded from below.

**Remark 45:** If $T_{44}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t),t\right) = (b'_{45})^{(9)}$ then $T_{45} \to \infty$.

**Definition of** $(m)^{(9)}$ and $\varepsilon_9$ :

Indeed let $t_9$ be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t),t\right) < \varepsilon_9, T_{44}(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right)$, $t = log\frac{2}{\varepsilon_9}$ By taking now $\varepsilon_9$ sufficiently small one sees that $T_{45}$ is unbounded.
The same property holds for $T_{46}$ if $\lim_{t\to\infty}(b''_{46})^{(9)}\left((G_{47})(t),t\right) = (b'_{46})^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

**Behavior of the solutions of equation** 280

**Theorem** If we denote and define

**Definition of** $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ :

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)} \leq -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14},t) + (a''_{14})^{(1)}(T_{14},t) \leq -(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)} \leq -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G,t) - (b''_{14})^{(1)}(G,t) \leq -(\tau_1)^{(1)}$

**Definition of** $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$ : 281

By $(\nu_1)^{(1)} > 0$ , $(\nu_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$ , $(u_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

**Definition of** $(\bar{\nu}_1)^{(1)}, , (\bar{\nu}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$ : 282

By $(\bar{\nu}_1)^{(1)} > 0$ , $(\bar{\nu}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$ , $(\bar{u}_2)^{(1)} < 0$ the roots of the equations

$(a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

**Definition of** $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (v_0)^{(1)}$ :- 283

If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by

$(m_2)^{(1)} = (v_0)^{(1)}, (m_1)^{(1)} = (v_1)^{(1)},\ if\ (v_0)^{(1)} < (v_1)^{(1)}$

$(m_2)^{(1)} = (v_1)^{(1)}, (m_1)^{(1)} = (\bar{v}_1)^{(1)}, if\ (v_1)^{(1)} < (v_0)^{(1)} < (\bar{v}_1)^{(1)},$

and $\boxed{(v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$

$(m_2)^{(1)} = (v_1)^{(1)}, (m_1)^{(1)} = (v_0)^{(1)},\ if\ (\bar{v}_1)^{(1)} < (v_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)},\ if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)},$

and $\boxed{(u_0)^{(1)} = \frac{T_{13}^0}{T_{14}^0}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$

are defined

Then the solution of global equations satisfies the inequalities 285

$$G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{13}(t) \le G_{13}^0 e^{(S_1)^{(1)}t}$$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)}t}$$

$$\left(\frac{(a_{15})^{(1)} G_{13}^0}{(m_1)^{(1)}((S_1)^{(1)} - (p_{13})^{(1)} - (S_2)^{(1)})}\left[e^{((S_1)^{(1)} - (p_{13})^{(1)})t} - e^{-(S_2)^{(1)}t}\right] + G_{15}^0 e^{-(S_2)^{(1)}t} \le G_{15}(t) \le \right. \quad 286$$

$$\left. \frac{(a_{15})^{(1)} G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)} - (a'_{15})^{(1)})}\left[e^{(S_1)^{(1)}t} - e^{-(a'_{15})^{(1)}t}\right] + G_{15}^0 e^{-(a'_{15})^{(1)}t}\right)$$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)}t} \le T_{13}(t) \le T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t}} \quad 287$$

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)}t} \le T_{13}(t) \le \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t} \quad 288$$

$$\frac{(b_{15})^{(1)} T_{13}^0}{(\mu_1)^{(1)}((R_1)^{(1)} - (b'_{15})^{(1)})}\left[e^{(R_1)^{(1)}t} - e^{-(b'_{15})^{(1)}t}\right] + T_{15}^0 e^{-(b'_{15})^{(1)}t} \le T_{15}(t) \le \quad 289$$

$$\frac{(a_{15})^{(1)} T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)} + (r_{13})^{(1)} + (R_2)^{(1)})}\left[e^{((R_1)^{(1)} + (r_{13})^{(1)})t} - e^{-(R_2)^{(1)}t}\right] + T_{15}^0 e^{-(R_2)^{(1)}t}$$

**Definition of** $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:- 290

Where $(S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$

$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$

$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$

$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$

**Behavior of the solutions of equation** 291

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ : 292

$(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ four constants satisfying
$-(\sigma_2)^{(2)} \le -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(T_{17}, t) + (a''_{17})^{(2)}(T_{17}, t) \le -(\sigma_1)^{(2)}$ 293

$-(\tau_2)^{(2)} \le -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}\big((G_{19}), t\big) - (b''_{17})^{(2)}\big((G_{19}), t\big) \le -(\tau_1)^{(2)}$ 294

**Definition of** $(\nu_1)^{(2)}, (\nu_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$ : 295

By $(\nu_1)^{(2)} > 0$ , $(\nu_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0$ , $(u_2)^{(2)} < 0$ the roots 296

of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 297

and $(b_{14})^{(2)}\big(u^{(2)}\big)^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and 298

**Definition of** $(\bar{\nu}_1)^{(2)},, (\bar{\nu}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$ : 299

By $(\bar{\nu}_1)^{(2)} > 0$ , $(\bar{\nu}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0$ , $(\bar{u}_2)^{(2)} < 0$ the 300

roots of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 301

and $(b_{17})^{(2)}\big(u^{(2)}\big)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 302

**Definition of** $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ :- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{G^0_{16}}{G^0_{17}}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)},\ \boldsymbol{if}\ (u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}, \boldsymbol{if}\ (u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)},$

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)},\ \boldsymbol{if}\ (\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)} t}$

$(p_i)^{(2)}$ is defined by equation

$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)} t}$ 311

$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)} t}\right] + G_{18}^0 e^{-(S_2)^{(2)} t} \le G_{18}(t) \le\right.$ 312

$\left.\frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)} t} - e^{-(a'_{18})^{(2)} t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)} t}\right)$

$\boxed{T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}}$ 313

$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}$ 314

$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)} t} - e^{-(b'_{18})^{(2)} t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)} t} \le T_{18}(t) \le$ 315

$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})}\left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)} t}\right] + T_{18}^0 e^{-(R_2)^{(2)} t}$

**Definition of** $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$

$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)}$ 318

$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$

**Behavior of the solutions** 319

**Theorem 3:** If we denote and define

**Definition of** $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ :

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}\big((G_{23}), t\big) \le -(\tau_1)^{(3)}$$

**Definition of** $(\nu_1)^{(3)}, (\nu_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$ : 320

By $(\nu_1)^{(3)} > 0$ , $(\nu_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$ , $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\big(\nu^{(3)}\big)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{\nu}_1)^{(3)} > 0$ , $(\bar{\nu}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$ , $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\big(\nu^{(3)}\big)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

**Definition of** $(m_1)^{(3)}$ , $(m_2)^{(3)}$ , $(\mu_1)^{(3)}, (\mu_2)^{(3)}$ :- 321

If we define $(m_1)^{(3)}$ , $(m_2)^{(3)}$ , $(\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (\nu_0)^{(3)}, (m_1)^{(3)} = (\nu_1)^{(3)}$, ***if*** $(\nu_0)^{(3)} < (\nu_1)^{(3)}$

$(m_2)^{(3)} = (\nu_1)^{(3)}, (m_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ , ***if*** $(\nu_1)^{(3)} < (\nu_0)^{(3)} < (\bar{\nu}_1)^{(3)}$,

and $\boxed{(\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}}$

$(m_2)^{(3)} = (\nu_1)^{(3)}, (m_1)^{(3)} = (\nu_0)^{(3)}$, ***if*** $(\bar{\nu}_1)^{(3)} < (\nu_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}$, ***if*** $(u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$ , ***if*** $(u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}$, and $\boxed{(u_0)^{(3)} = \frac{T^0_{20}}{T^0_{21}}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}$, ***if*** $(\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$$G^0_{20} e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G^0_{20} e^{(S_1)^{(3)} t}$$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}} G^0_{20} e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}} G^0_{20} e^{(S_1)^{(3)} t} \quad 323$$

$$\Big(\frac{(a_{22})^{(3)} G^0_{20}}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\Big[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)} t}\Big] + G^0_{22} e^{-(S_2)^{(3)} t} \le G_{22}(t) \le \quad 324$$

$$\frac{(a_{22})^{(3)} G^0_{20}}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}[e^{(S_1)^{(3)} t} - e^{-(a'_{22})^{(3)} t}] + G^0_{22} e^{-(a'_{22})^{(3)} t}\Big)$$

$$\boxed{T_{20}^{0}e^{(R_1)^{(3)}t} \le T_{20}(t) \le T_{20}^{0}e^{((R_1)^{(3)}+(r_{20})^{(3)})t}} \quad 325$$

$$\frac{1}{(\mu_1)^{(3)}}T_{20}^{0}e^{(R_1)^{(3)}t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}}T_{20}^{0}e^{((R_1)^{(3)}+(r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)}T_{20}^{0}}{(\mu_1)^{(3)}((R_1)^{(3)}-(b'_{22})^{(3)})}\left[e^{(R_1)^{(3)}t} - e^{-(b'_{22})^{(3)}t}\right] + T_{22}^{0}e^{-(b'_{22})^{(3)}t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)}T_{20}^{0}}{(\mu_2)^{(3)}((R_1)^{(3)}+(r_{20})^{(3)}+(R_2)^{(3)})}\left[e^{((R_1)^{(3)}+(r_{20})^{(3)})t} - e^{-(R_2)^{(3)}t}\right] + T_{22}^{0}e^{-(R_2)^{(3)}t}$$

**Definition of** $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

**Behavior of the solutions of equation**

**Theorem:** If we denote and define

**Definition of** $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ :

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$

$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}\big((G_{27}), t\big) - (b''_{25})^{(4)}\big((G_{27}), t\big) \le -(\tau_1)^{(4)}$

**Definition of** $(\nu_1)^{(4)}, (\nu_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, \nu^{(4)}, u^{(4)}$ : 329

By $(\nu_1)^{(4)} > 0$ , $(\nu_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$ , $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\big(\nu^{(4)}\big)^2 + (\sigma_1)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\big(u^{(4)}\big)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(4)},, (\bar{\nu}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$ : 330

By $(\bar{\nu}_1)^{(4)} > 0$ , $(\bar{\nu}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$ , $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\big(\nu^{(4)}\big)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\big(u^{(4)}\big)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

**Definition of** $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}, (\nu_0)^{(4)}$ :-

If we define $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (\nu_0)^{(4)}, (m_1)^{(4)} = (\nu_1)^{(4)}$, ***if*** $(\nu_0)^{(4)} < (\nu_1)^{(4)}$

$(m_2)^{(4)} = (\nu_1)^{(4)}, (m_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ , ***if*** $(\nu_4)^{(4)} < (\nu_0)^{(4)} < (\bar{\nu}_1)^{(4)}$,

and $\boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (\nu_4)^{(4)}, (m_1)^{(4)} = (\nu_0)^{(4)},\ \boldsymbol{if}\ (\bar{\nu}_4)^{(4)} < (\nu_0)^{(4)}$

and analogously

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$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)},\ \boldsymbol{if}\ (u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}, \boldsymbol{if}\ (u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)},$

and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}, \boldsymbol{if}\ (\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

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$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)} t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)} t} \quad (333)$$

$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)} t}\right] + G_{26}^0 e^{-(S_2)^{(4)} t} \le G_{26}(t) \le \right. \quad (334)$$

*(a26)4G240(m2)4(S1)4−(a26′)4e(S1)4t−e−(a26′)4t+ G260e−(a26′)4t*

$$\boxed{T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}} T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}} T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t} \quad (335)$$

$$\frac{(b_{26})^{(4)} T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)} - (b'_{26})^{(4)})}\left[e^{(R_1)^{(4)} t} - e^{-(b'_{26})^{(4)} t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)} t} \le T_{26}(t) \le \quad (336)$$

$$\frac{(a_{26})^{(4)} T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)} + (r_{24})^{(4)} + (R_2)^{(4)})}\left[e^{((R_1)^{(4)} + (r_{24})^{(4)})t} - e^{-(R_2)^{(4)} t}\right] + T_{26}^0 e^{-(R_2)^{(4)} t}$$

**Definition of** $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

**Behavior of the solutions of equation** 338

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ :

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

**Definition of** $(\nu_1)^{(5)}, (\nu_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, \nu^{(5)}, u^{(5)}$ : 339

By $(\nu_1)^{(5)} > 0$ , $(\nu_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$ , $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(5)}, , (\bar{\nu}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$ : 340

By $(\bar{\nu}_1)^{(5)} > 0$ , $(\bar{\nu}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$ , $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

**Definition of** $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (\nu_0)^{(5)}$ :-

If we define $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$$(m_2)^{(5)} = (\nu_0)^{(5)}, (m_1)^{(5)} = (\nu_1)^{(5)}, \ \boldsymbol{if}\ (\nu_0)^{(5)} < (\nu_1)^{(5)}$$

$$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\bar{\nu}_1)^{(5)}, \boldsymbol{if}\ (\nu_1)^{(5)} < (\nu_0)^{(5)} < (\bar{\nu}_1)^{(5)},$$

and $\boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}}$

$$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$$

and analogously 341

$$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)}, \ \boldsymbol{if}\ (u_0)^{(5)} < (u_1)^{(5)}$$

$$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}, \boldsymbol{if}\ (u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)},$$

and $\boxed{(u_0)^{(5)} = \frac{T^0_{28}}{T^0_{29}}}$

$$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}, \boldsymbol{if}\ (\bar{u}_1)^{(5)} < (u_0)^{(5)}$$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$$G^0_{28} e^{\left((S_1)^{(5)} - (p_{28})^{(5)}\right)t} \le G_{28}(t) \le G^0_{28} e^{(S_1)^{(5)}t}$$

where $(p_i)^{(5)}$ is defined by equation

$$\frac{1}{(m_5)^{(5)}} G^0_{28} e^{\left((S_1)^{(5)} - (p_{28})^{(5)}\right)t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}} G^0_{28} e^{(S_1)^{(5)}t}$$ 343

$$\left(\frac{(a_{30})^{(5)}G_{28}^{0}}{(m_1)^{(5)}((S_1)^{(5)}-(p_{28})^{(5)}-(S_2)^{(5)})}\left[e^{((S_1)^{(5)}-(p_{28})^{(5)})t}-e^{-(S_2)^{(5)}t}\right]+G_{30}^{0}e^{-(S_2)^{(5)}t}\leq G_{30}(t)\leq\right. \quad 344$$

*(a30)5G280(m2)5(S1)5−(a30′)5e(S1)5t−e−(a30′)5t+ G300e−(a30′)5t*

$$\boxed{T_{28}^{0}e^{(R_1)^{(5)}t}\leq T_{28}(t)\leq T_{28}^{0}e^{((R_1)^{(5)}+(r_{28})^{(5)})t}} \quad 345$$

$$\frac{1}{(\mu_1)^{(5)}}T_{28}^{0}e^{(R_1)^{(5)}t}\leq T_{28}(t)\leq\frac{1}{(\mu_2)^{(5)}}T_{28}^{0}e^{((R_1)^{(5)}+(r_{28})^{(5)})t} \quad 346$$

$$\frac{(b_{30})^{(5)}T_{28}^{0}}{(\mu_1)^{(5)}((R_1)^{(5)}-(b'_{30})^{(5)})}\left[e^{(R_1)^{(5)}t}-e^{-(b'_{30})^{(5)}t}\right]+T_{30}^{0}e^{-(b'_{30})^{(5)}t}\leq T_{30}(t)\leq \quad 347$$

$$\frac{(a_{30})^{(5)}T_{28}^{0}}{(\mu_2)^{(5)}((R_1)^{(5)}+(r_{28})^{(5)}+(R_2)^{(5)})}\left[e^{((R_1)^{(5)}+(r_{28})^{(5)})t}-e^{-(R_2)^{(5)}t}\right]+T_{30}^{0}e^{-(R_2)^{(5)}t}$$

**Definition of** $(S_1)^{(5)},(S_2)^{(5)},(R_1)^{(5)},(R_2)^{(5)}$:- 348

Where $(S_1)^{(5)}=(a_{28})^{(5)}(m_2)^{(5)}-(a'_{28})^{(5)}$

$(S_2)^{(5)}=(a_{30})^{(5)}-(p_{30})^{(5)}$

$(R_1)^{(5)}=(b_{28})^{(5)}(\mu_2)^{(5)}-(b'_{28})^{(5)}$

$(R_2)^{(5)}=(b'_{30})^{(5)}-(r_{30})^{(5)}$

**Behavior of the solutions of equation** 349

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(6)},(\sigma_2)^{(6)},(\tau_1)^{(6)},(\tau_2)^{(6)}$ :

$(\sigma_1)^{(6)},(\sigma_2)^{(6)},(\tau_1)^{(6)},(\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)}\leq-(a'_{32})^{(6)}+(a'_{33})^{(6)}-(a''_{32})^{(6)}(T_{33},t)+(a''_{33})^{(6)}(T_{33},t)\leq-(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)}\leq-(b'_{32})^{(6)}+(b'_{33})^{(6)}-(b''_{32})^{(6)}\big((G_{35}),t\big)-(b''_{33})^{(6)}\big((G_{35}),t\big)\leq-(\tau_1)^{(6)}$$

**Definition of** $(v_1)^{(6)},(v_2)^{(6)},(u_1)^{(6)},(u_2)^{(6)},v^{(6)},u^{(6)}$ : 350

By $(v_1)^{(6)}>0$ , $(v_2)^{(6)}<0$ and respectively $(u_1)^{(6)}>0$ , $(u_2)^{(6)}<0$ the roots of the equations $(a_{33})^{(6)}\big(v^{(6)}\big)^2+(\sigma_1)^{(6)}v^{(6)}-(a_{32})^{(6)}=0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2+(\tau_1)^{(6)}u^{(6)}-(b_{32})^{(6)}=0$ and

**Definition of** $(\bar{v}_1)^{(6)},,(\bar{v}_2)^{(6)},(\bar{u}_1)^{(6)},(\bar{u}_2)^{(6)}$ : 351

By $(\bar{v}_1)^{(6)}>0$ , $(\bar{v}_2)^{(6)}<0$ and respectively $(\bar{u}_1)^{(6)}>0$ , $(\bar{u}_2)^{(6)}<0$ the roots of the equations $(a_{33})^{(6)}\big(v^{(6)}\big)^2+(\sigma_2)^{(6)}v^{(6)}-(a_{32})^{(6)}=0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2+(\tau_2)^{(6)}u^{(6)}-(b_{32})^{(6)}=0$

**Definition of** $(m_1)^{(6)},(m_2)^{(6)},(\mu_1)^{(6)},(\mu_2)^{(6)},(v_0)^{(6)}$ :-

If we define $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}$ by

$(m_2)^{(6)} = (v_0)^{(6)}, (m_1)^{(6)} = (v_1)^{(6)}$, ***if*** $(v_0)^{(6)} < (v_1)^{(6)}$

$(m_2)^{(6)} = (v_1)^{(6)}, (m_1)^{(6)} = (\bar{v}_6)^{(6)}$, ***if*** $(v_1)^{(6)} < (v_0)^{(6)} < (\bar{v}_1)^{(6)}$,

and $\boxed{(v_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$

$(m_2)^{(6)} = (v_1)^{(6)}, (m_1)^{(6)} = (v_0)^{(6)}$, ***if*** $(\bar{v}_1)^{(6)} < (v_0)^{(6)}$

and analogously 352

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}$, ***if*** $(u_0)^{(6)} < (u_1)^{(6)}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}$, ***if*** $(u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)}$,

and $\boxed{(u_0)^{(6)} = \frac{T_{32}^0}{T_{33}^0}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}$, ***if*** $(\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$

Then the solution of global equations satisfies the inequalities 353

$$G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \leq G_{32}(t) \leq G_{32}^0 e^{(S_1)^{(6)} t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}} G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \leq G_{33}(t) \leq \frac{1}{(m_2)^{(6)}} G_{32}^0 e^{(S_1)^{(6)} t} \quad 354$$

$$\left( \frac{(a_{34})^{(6)} G_{32}^0}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})} \left[ e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)} t} \right] + G_{34}^0 e^{-(S_2)^{(6)} t} \leq G_{34}(t) \leq \right. \quad 355$$

*(a34)6G320(m2)6(S1)6−(a34′)6e(S1)6t−e−(a34′)6t+ G340e−(a34′)6t*

$$\boxed{T_{32}^0 e^{(R_1)^{(6)} t} \leq T_{32}(t) \leq T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}} T_{32}^0 e^{(R_1)^{(6)} t} \leq T_{32}(t) \leq \frac{1}{(\mu_2)^{(6)}} T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)} T_{32}^0}{(\mu_1)^{(6)}((R_1)^{(6)} - (b'_{34})^{(6)})} \left[ e^{(R_1)^{(6)} t} - e^{-(b'_{34})^{(6)} t} \right] + T_{34}^0 e^{-(b'_{34})^{(6)} t} \leq T_{34}(t) \leq \quad 358$$

$$\frac{(a_{34})^{(6)} T_{32}^0}{(\mu_2)^{(6)}((R_1)^{(6)} + (r_{32})^{(6)} + (R_2)^{(6)})} \left[ e^{((R_1)^{(6)} + (r_{32})^{(6)})t} - e^{-(R_2)^{(6)} t} \right] + T_{34}^0 e^{-(R_2)^{(6)} t}$$

**Definition of** $(S_1)^{(6)}, (S_2)^{(6)}, (R_1)^{(6)}, (R_2)^{(6)}$:- 359

Where $(S_1)^{(6)} = (a_{32})^{(6)} (m_2)^{(6)} - (a'_{32})^{(6)}$

$(S_2)^{(6)} = (a_{34})^{(6)} - (p_{34})^{(6)}$

$(R_1)^{(6)} = (b_{32})^{(6)} (\mu_2)^{(6)} - (b'_{32})^{(6)}$

$$(R_2)^{(6)} = (b'_{34})^{(6)} - (r_{34})^{(6)}$$

**Behavior of the solutions of equation**

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$ :

$(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)} \le -(a'_{36})^{(7)} + (a'_{37})^{(7)} - (a''_{36})^{(7)}(T_{37}, t) + (a''_{37})^{(7)}(T_{37}, t) \le -(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)} \le -(b'_{36})^{(7)} + (b'_{37})^{(7)} - (b''_{36})^{(7)}\big((G_{39}), t\big) - (b''_{37})^{(7)}\big((G_{39}), t\big) \le -(\tau_1)^{(7)}$$

**Definition of** $(\nu_1)^{(7)}, (\nu_2)^{(7)}, (u_1)^{(7)}, (u_2)^{(7)}, \nu^{(7)}, u^{(7)}$ : 361

By $(\nu_1)^{(7)} > 0, (\nu_2)^{(7)} < 0$ and respectively $(u_1)^{(7)} > 0, (u_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_1)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(7)},, (\bar{\nu}_2)^{(7)}, (\bar{u}_1)^{(7)}, (\bar{u}_2)^{(7)}$ : 362

By $(\bar{\nu}_1)^{(7)} > 0, (\bar{\nu}_2)^{(7)} < 0$ and respectively $(\bar{u}_1)^{(7)} > 0, (\bar{u}_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

**Definition of** $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (\nu_0)^{(7)}$ :-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$$(m_2)^{(7)} = (\nu_0)^{(7)}, (m_1)^{(7)} = (\nu_1)^{(7)}, \ \boldsymbol{if}\ (\nu_0)^{(7)} < (\nu_1)^{(7)}$$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\bar{\nu}_1)^{(7)}, \boldsymbol{if}\ (\nu_1)^{(7)} < (\nu_0)^{(7)} < (\bar{\nu}_1)^{(7)},$$

and $\boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\nu_0)^{(7)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(7)} < (\nu_0)^{(7)}$$

and analogously 363

$$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}, \ \boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$$

and $\boxed{(u_0)^{(7)} = \frac{T^0_{36}}{T^0_{37}}}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)}$ where $(u_1)^{(7)}, (\bar{u}_1)^{(7)}$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^0 e^{\left((S_1)^{(7)} - (p_{36})^{(7)}\right)t} \leq G_{36}(t) \leq G_{36}^0 e^{(S_1)^{(7)}t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{\left((S_1)^{(7)} - (p_{36})^{(7)}\right)t} \leq G_{37}(t) \leq \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)}t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)}\left((S_1)^{(7)} - (p_{36})^{(7)} - (S_2)^{(7)}\right)}\left[e^{\left((S_1)^{(7)} - (p_{36})^{(7)}\right)t} - e^{-(S_2)^{(7)}t}\right] + G_{38}^0 e^{-(S_2)^{(7)}t} \leq G_{38}(t) \leq \right. \quad 366$$

$$\left. \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)}\left((S_1)^{(7)} - (a'_{38})^{(7)}\right)}\left[e^{(S_1)^{(7)}t} - e^{-(a'_{38})^{(7)}t}\right] + G_{38}^0 e^{-(a'_{38})^{(7)}t}\right)$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)}t} \leq T_{36}(t) \leq T_{36}^0 e^{\left((R_1)^{(7)} + (r_{36})^{(7)}\right)t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)}t} \leq T_{36}(t) \leq \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{\left((R_1)^{(7)} + (r_{36})^{(7)}\right)t} \quad 368$$

$$\frac{(b_{38})^{(7)} T_{36}^0}{(\mu_1)^{(7)}\left((R_1)^{(7)} - (b'_{38})^{(7)}\right)}\left[e^{(R_1)^{(7)}t} - e^{-(b'_{38})^{(7)}t}\right] + T_{38}^0 e^{-(b'_{38})^{(7)}t} \leq T_{38}(t) \leq \quad 369$$

$$\frac{(a_{38})^{(7)} T_{36}^0}{(\mu_2)^{(7)}\left((R_1)^{(7)} + (r_{36})^{(7)} + (R_2)^{(7)}\right)}\left[e^{\left((R_1)^{(7)} + (r_{36})^{(7)}\right)t} - e^{-(R_2)^{(7)}t}\right] + T_{38}^0 e^{-(R_2)^{(7)}t}$$

**Definition of** $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$$

$$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$$

$$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$$

**Behavior of the solutions of equation** 371

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ :

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$$-(\sigma_2)^{(8)} \leq -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \leq -(\sigma_1)^{(8)}$$

$$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\big((G_{43}),t\big) - (b''_{41})^{(8)}\big((G_{43}),t\big) \le -(\tau_1)^{(8)}$$

**Definition of** $(\nu_1)^{(8)}, (\nu_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, \nu^{(8)}, u^{(8)}$ : 372

By $(\nu_1)^{(8)} > 0$ , $(\nu_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0$ , $(u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(8)},, (\bar{\nu}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$ :

By $(\bar{\nu}_1)^{(8)} > 0$ , $(\bar{\nu}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0$ , $(\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

**Definition of** $(m_1)^{(8)}$ , $(m_2)^{(8)}$ , $(\mu_1)^{(8)}, (\mu_2)^{(8)}, (\nu_0)^{(8)}$ :-

If we define $(m_1)^{(8)}$ , $(m_2)^{(8)}$ , $(\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$(m_2)^{(8)} = (\nu_0)^{(8)}, (m_1)^{(8)} = (\nu_1)^{(8)}$, ***if*** $(\nu_0)^{(8)} < (\nu_1)^{(8)}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ , ***if*** $(\nu_1)^{(8)} < (\nu_0)^{(8)} < (\bar{\nu}_1)^{(8)}$,
and $\boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\nu_0)^{(8)}$, ***if*** $(\bar{\nu}_1)^{(8)} < (\nu_0)^{(8)}$

and analogously 374

$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)}$, ***if*** $(u_0)^{(8)} < (u_1)^{(8)}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}$ , ***if*** $(u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)}$,
and $\boxed{(u_0)^{(8)} = \frac{T^0_{40}}{T^0_{41}}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}$, ***if*** $(\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G^0_{40}e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{40}(t) \le G^0_{40}e^{(S_1)^{(8)}t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}} G_{40}^0 e^{((S_1)^{(8)}-(p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}} G_{40}^0 e^{(S_1)^{(8)}t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)}G_{40}^0}{(m_1)^{(8)}((S_1)^{(8)}-(p_{40})^{(8)}-(S_2)^{(8)})}\left[e^{((S_1)^{(8)}-(p_{40})^{(8)})t} - e^{-(S_2)^{(8)}t}\right] + G_{42}^0 e^{-(S_2)^{(8)}t} \le G_{42}(t) \le \right. \quad 377$$

$$\left. \frac{(a_{42})^{(8)}G_{40}^0}{(m_2)^{(8)}((S_1)^{(8)}-(a'_{42})^{(8)})}[e^{(S_1)^{(8)}t} - e^{-(a'_{42})^{(8)}t}] + G_{42}^0 e^{-(a'_{42})^{(8)}t}\right)$$

$$\boxed{T_{40}^0 e^{(R_1)^{(8)}t} \le T_{40}(t) \le T_{40}^0 e^{((R_1)^{(8)}+(r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}} T_{40}^0 e^{(R_1)^{(8)}t} \le T_{40}(t) \le \frac{1}{(\mu_2)^{(8)}} T_{40}^0 e^{((R_1)^{(8)}+(r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)}T_{40}^0}{(\mu_1)^{(8)}((R_1)^{(8)}-(b'_{42})^{(8)})}\left[e^{(R_1)^{(8)}t} - e^{-(b'_{42})^{(8)}t}\right] + T_{42}^0 e^{-(b'_{42})^{(8)}t} \le T_{42}(t) \le \quad 380$$

$$\frac{(a_{42})^{(8)}T_{40}^0}{(\mu_2)^{(8)}((R_1)^{(8)}+(r_{40})^{(8)}+(R_2)^{(8)})}\left[e^{((R_1)^{(8)}+(r_{40})^{(8)})t} - e^{-(R_2)^{(8)}t}\right] + T_{42}^0 e^{-(R_2)^{(8)}t}$$

**Definition of** $(S_1)^{(8)}, (S_2)^{(8)}, (R_1)^{(8)}, (R_2)^{(8)}$:- 381

Where $(S_1)^{(8)} = (a_{40})^{(8)}(m_2)^{(8)} - (a'_{40})^{(8)}$

$(S_2)^{(8)} = (a_{42})^{(8)} - (p_{42})^{(8)}$

$(R_1)^{(8)} = (b_{40})^{(8)}(\mu_2)^{(8)} - (b'_{40})^{(8)}$

$(R_2)^{(8)} = (b'_{42})^{(8)} - (r_{42})^{(8)}$

**Behavior of the solutions of equation 37 to 92** 382

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ :

$(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ four constants satisfying

$$-(\sigma_2)^{(9)} \le -(a'_{44})^{(9)} + (a'_{45})^{(9)} - (a''_{44})^{(9)}(T_{45}, t) + (a''_{45})^{(9)}(T_{45}, t) \le -(\sigma_1)^{(9)}$$

$$-(\tau_2)^{(9)} \le -(b'_{44})^{(9)} + (b'_{45})^{(9)} - (b''_{44})^{(9)}\big((G_{47}), t\big) - (b''_{45})^{(9)}\big((G_{47}), t\big) \le -(\tau_1)^{(9)}$$

**Definition of** $(\nu_1)^{(9)}, (\nu_2)^{(9)}, (u_1)^{(9)}, (u_2)^{(9)}, \nu^{(9)}, u^{(9)}$ :

By $(\nu_1)^{(9)} > 0$ , $(\nu_2)^{(9)} < 0$ and respectively $(u_1)^{(9)} > 0$ , $(u_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_1)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(9)}, , (\bar{\nu}_2)^{(9)}, (\bar{u}_1)^{(9)}, (\bar{u}_2)^{(9)}$ :

By $(\bar{\nu}_1)^{(9)} > 0$ , $(\bar{\nu}_2)^{(9)} < 0$ and respectively $(\bar{u}_1)^{(9)} > 0$ , $(\bar{u}_2)^{(9)} < 0$ the

roots of the equations $(a_{45})^{(9)}\left(v^{(9)}\right)^2+(\sigma_2)^{(9)}v^{(9)}-(a_{44})^{(9)}=0$
and $(b_{45})^{(9)}\left(u^{(9)}\right)^2+(\tau_2)^{(9)}u^{(9)}-(b_{44})^{(9)}=0$

**Definition of** $(m_1)^{(9)},(m_2)^{(9)},(\mu_1)^{(9)},(\mu_2)^{(9)},(v_0)^{(9)}$ :-

If we define $(m_1)^{(9)},(m_2)^{(9)},(\mu_1)^{(9)},(\mu_2)^{(9)}$ by

$(m_2)^{(9)}=(v_0)^{(9)},(m_1)^{(9)}=(v_1)^{(9)},\ \boldsymbol{if}\ (v_0)^{(9)}<(v_1)^{(9)}$

$(m_2)^{(9)}=(v_1)^{(9)},(m_1)^{(9)}=(\bar{v}_1)^{(9)},\boldsymbol{if}\ (v_1)^{(9)}<(v_0)^{(9)}<(\bar{v}_1)^{(9)}$,
and $\boxed{(v_0)^{(9)}=\frac{G_{44}^0}{G_{45}^0}}$

$(m_2)^{(9)}=(v_1)^{(9)},(m_1)^{(9)}=(v_0)^{(9)},\ \boldsymbol{if}\ (\bar{v}_1)^{(9)}<(v_0)^{(9)}$

and analogously

$(\mu_2)^{(9)}=(u_0)^{(9)},(\mu_1)^{(9)}=(u_1)^{(9)},\ \boldsymbol{if}\ (u_0)^{(9)}<(u_1)^{(9)}$

$(\mu_2)^{(9)}=(u_1)^{(9)},(\mu_1)^{(9)}=(\bar{u}_1)^{(9)},\boldsymbol{if}\ (u_1)^{(9)}<(u_0)^{(9)}<(\bar{u}_1)^{(9)}$,
and $\boxed{(u_0)^{(9)}=\frac{T_{44}^0}{T_{45}^0}}$

$(\mu_2)^{(9)}=(u_1)^{(9)},(\mu_1)^{(9)}=(u_0)^{(9)},\boldsymbol{if}\ (\bar{u}_1)^{(9)}<(u_0)^{(9)}$ where $(u_1)^{(9)},(\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G_{44}^0e^{\left((S_1)^{(9)}-(p_{44})^{(9)}\right)t}\le G_{44}(t)\le G_{44}^0e^{(S_1)^{(9)}t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}}G_{44}^0e^{\left((S_1)^{(9)}-(p_{44})^{(9)}\right)t}\le G_{45}(t)\le\frac{1}{(m_2)^{(9)}}G_{44}^0e^{(S_1)^{(9)}t}$$

$$\left(\frac{(a_{46})^{(9)}G_{44}^0}{(m_1)^{(9)}\left((S_1)^{(9)}-(p_{44})^{(9)}-(S_2)^{(9)}\right)}\left[e^{\left((S_1)^{(9)}-(p_{44})^{(9)}\right)t}-e^{-(S_2)^{(9)}t}\right]+G_{46}^0e^{-(S_2)^{(9)}t}\le G_{46}(t)\le\right.$$
$$\left.\frac{(a_{46})^{(9)}G_{44}^0}{(m_2)^{(9)}\left((S_1)^{(9)}-(a'_{46})^{(9)}\right)}\left[e^{(S_1)^{(9)}t}-e^{-(a'_{46})^{(9)}t}\right]+\ G_{46}^0e^{-(a'_{46})^{(9)}t}\right)$$

$$\boxed{T_{44}^0e^{(R_1)^{(9)}t}\le T_{44}(t)\le T_{44}^0e^{\left((R_1)^{(9)}+(r_{44})^{(9)}\right)t}}$$

$$\frac{1}{(\mu_1)^{(9)}}T_{44}^0e^{(R_1)^{(9)}t}\le T_{44}(t)\le\frac{1}{(\mu_2)^{(9)}}T_{44}^0e^{\left((R_1)^{(9)}+(r_{44})^{(9)}\right)t}$$

$$\frac{(b_{46})^{(9)}T_{44}^0}{(\mu_1)^{(9)}\left((R_1)^{(9)}-(b'_{46})^{(9)}\right)}\left[e^{(R_1)^{(9)}t}-e^{-(b'_{46})^{(9)}t}\right]+T_{46}^0e^{-(b'_{46})^{(9)}t}\le T_{46}(t)\le$$

$$\frac{(a_{46})^{(9)}T_{44}^0}{(\mu_2)^{(9)}\left((R_1)^{(9)}+(r_{44})^{(9)}+(R_2)^{(9)}\right)}\left[e^{\left((R_1)^{(9)}+(r_{44})^{(9)}\right)t}-e^{-(R_2)^{(9)}t}\right]+T_{46}^0e^{-(R_2)^{(9)}t}$$

**Definition of** $(S_1)^{(9)},(S_2)^{(9)},(R_1)^{(9)},(R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$

**Proof :** From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right) - (a''_{14})^{(1)}(T_{14}, t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

**Definition of** $v^{(1)}$ :- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)}\right) \le \frac{dv^{(1)}}{dt} \le -\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)}\right)$$

From which one obtains

**Definition of** $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$ :-

For $0 < \boxed{(v_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}} < (v_1)^{(1)} < (\bar{v}_1)^{(1)}$

$$v^{(1)}(t) \ge \frac{(v_1)^{(1)} + (C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_0)^{(1)}\right)t\right]}}{1 + (C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(v_1)^{(1)} - (v_0)^{(1)}}{(v_0)^{(1)} - (v_2)^{(1)}}}$$

it follows $(v_0)^{(1)} \le v^{(1)}(t) \le (v_1)^{(1)}$

In the same manner , we get 384

$$v^{(1)}(t) \le \frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{v}_1)^{(1)} - (v_0)^{(1)}}{(v_0)^{(1)} - (\bar{v}_2)^{(1)}}}$$

From which we deduce $(v_0)^{(1)} \le v^{(1)}(t) \le (\bar{v}_1)^{(1)}$

If $0 < (v_1)^{(1)} < (v_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}} < (\bar{v}_1)^{(1)}$ we find like in the previous case, 385

$$(v_1)^{(1)} \le \frac{(v_1)^{(1)} + (C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_2)^{(1)}\right)t\right]}}{1 + (C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_2)^{(1)}\right)t\right]}} \le v^{(1)}(t) \le$$

$$\frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}} \le (\bar{v}_1)^{(1)}$$

If $0 < (\nu_1)^{(1)} \leq (\bar{\nu}_1)^{(1)} \leq \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$ , we obtain 386

$$(\nu_1)^{(1)} \leq \nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)} + (\bar{C})^{(1)}(\bar{\nu}_2)^{(1)} e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)} - (\bar{\nu}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)} e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)} - (\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\nu_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(1)}(t)$ :-

$$(m_2)^{(1)} \leq \nu^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{\nu^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(1)}(t)$ :-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{13}'')^{(1)} = (a_{14}'')^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)} G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b_{13}'')^{(1)} = (b_{14}'')^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)} T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

**Proof** : From global equations we obtain 387

$$\frac{d\nu^{(2)}}{dt} = (a_{16})^{(2)} - \left((a_{16}')^{(2)} - (a_{17}')^{(2)} + (a_{16}'')^{(2)}(T_{17}, t)\right) - (a_{17}'')^{(2)}(T_{17}, t)\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

**Definition of** $\nu^{(2)}$ :- $\boxed{\nu^{(2)} = \frac{G_{16}}{G_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \leq \frac{d\nu^{(2)}}{dt} \leq -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

**Definition of** $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$ :-

For $0 < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \geq \frac{(\nu_1)^{(2)}+(\mathrm{C})^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_0)^{(2)}\right)t\right]}}{1+(\mathrm{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\mathrm{C})^{(2)}=\frac{(\nu_1)^{(2)}-(\nu_0)^{(2)}}{(\nu_0)^{(2)}-(\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)}+(\bar{\mathrm{C}})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{\mathrm{C}})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{\mathrm{C}})^{(2)}=\frac{(\bar{\nu}_1)^{(2)}-(\nu_0)^{(2)}}{(\nu_0)^{(2)}-(\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{\mathrm{G}_{16}^0}{\mathrm{G}_{17}^0} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \leq \frac{(\nu_1)^{(2)}+(\mathrm{C})^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}}{1+(\mathrm{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}} \leq \nu^{(2)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(2)}+(\bar{\mathrm{C}})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{\mathrm{C}})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \leq (\bar{\nu}_1)^{(2)} \leq (\nu_0)^{(2)} = \frac{\mathrm{G}_{16}^0}{\mathrm{G}_{17}^0}$ , we obtain 394

$$(\nu_1)^{(2)} \leq \nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)}+(\bar{\mathrm{C}})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{\mathrm{C}})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(2)}(t)$ :- 395

$$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}, \quad \boxed{\nu^{(2)}(t)=\frac{G_{16}(t)}{G_{17}(t)}}$$

In a completely analogous way, we obtain 396

**Definition of** $u^{(2)}(t)$ :-

$$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}, \quad \boxed{u^{(2)}(t)=\frac{T_{16}(t)}{T_{17}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :** 397

If $(a_{16}'')^{(2)} = (a_{17}'')^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)}G_{17}(t)$

Analogously if $(b''_{16})^{(2)} = (b''_{17})^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)}T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

**Proof** : From global equations we obtain 398

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right) - (a''_{21})^{(3)}(T_{21}, t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

**Definition of** $\nu^{(3)}$ :- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right) \leq \frac{d\nu^{(3)}}{dt} \leq -\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right)$$

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From which one obtains

For $0 < (\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\nu_1)^{(3)} < (\bar{\nu}_1)^{(3)}$

$$\nu^{(3)}(t) \geq \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(\nu_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\nu_2)^{(3)}}}$$

it follows $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\nu_1)^{(3)}$

In the same manner , we get 401

$$\nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)} + (\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{\nu}_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\bar{\nu}_2)^{(3)}}}$$

**Definition of** $(\bar{\nu}_1)^{(3)}$ :-

From which we deduce $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\bar{\nu}_1)^{(3)}$

If $0 < (\nu_1)^{(3)} < (\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\bar{\nu}_1)^{(3)}$ we find like in the previous case, 402

$$(\nu_1)^{(3)} \leq \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_2)^{(3)}\right)t\right]}}{1 + (C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_2)^{(3)}\right)t\right]}} \leq \nu^{(3)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(3)} + (\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\bar{\nu}_1)^{(3)}$$

If $0 < (\nu_1)^{(3)} \leq (\bar{\nu}_1)^{(3)} \leq (\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}$ , we obtain 403

$$(\nu_1)^{(3)} \leq \nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\nu_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(3)}(t)$ :-

$$(m_2)^{(3)} \leq \nu^{(3)}(t) \leq (m_1)^{(3)}, \quad \boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(3)}(t)$ :-

$$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}, \quad \boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{20})^{(3)} = (a''_{21})^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)}G_{21}(t)$

Analogously if $(b''_{20})^{(3)} = (b''_{21})^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)}T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

**Proof** : From global equations we obtain 404

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right) - (a''_{25})^{(4)}(T_{25},t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

**Definition of** $\nu^{(4)}$ :- $\quad \boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right) \leq \frac{d\nu^{(4)}}{dt} \leq -\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_4)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$ :-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \geq \frac{(\nu_1)^{(4)}+(C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_0)^{(4)}\right)t\right]}}{4+(C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)}-(\nu_0)^{(4)}}{(\nu_0)^{(4)}-(\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{4+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)}-(\nu_0)^{(4)}}{(\nu_0)^{(4)}-(\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \leq \frac{(\nu_1)^{(4)}+(C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_2)^{(4)}\right)t\right]}}{1+(C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_2)^{(4)}\right)t\right]}} \leq \nu^{(4)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \leq (\bar{\nu}_1)^{(4)} \leq \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$ , we obtain 407

$$(\nu_1)^{(4)} \leq \nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(4)}(t)$ :-

$$(m_2)^{(4)} \leq \nu^{(4)}(t) \leq (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(4)}(t)$ :-

$$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a_{24}'')^{(4)} = (a_{25}'')^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)}G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b_{24}'')^{(4)} = (b_{25}'')^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)}T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

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**Proof** : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a_{28}')^{(5)} - (a_{29}')^{(5)} + (a_{28}'')^{(5)}(T_{29}, t)\right) - (a_{29}'')^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

**Definition of** $\nu^{(5)}$ :- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$ :-

For $0 < \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \leq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}}{1 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}} \leq \nu^{(5)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \leq (\bar{\nu}_1)^{(5)} \leq \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$ , we obtain 411

$$(\nu_1)^{(5)} \leq \nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(5)}(t)$ :-

$$(m_2)^{(5)} \leq \nu^{(5)}(t) \leq (m_1)^{(5)}, \quad \boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(5)}(t)$ :-

$$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}, \quad \boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{28})^{(5)} = (a''_{29})^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)} G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b''_{28})^{(5)} = (b''_{29})^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$ if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)} T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

**Proof** : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a'_{32})^{(6)} - (a'_{33})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right) - (a''_{33})^{(6)}(T_{33}, t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

**Definition of** $\nu^{(6)}$ :- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \leq \frac{d\nu^{(6)}}{dt} \leq -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$ :-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \geq \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\nu_1)^{(6)}$

In the same manner , we get 413

$$\nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{\nu}_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\bar{\nu}_1)^{(6)}$

If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 414

$$(\nu_1)^{(6)} \leq \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}} \leq \nu^{(6)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\bar{\nu}_1)^{(6)}$$

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If $0 < (\nu_1)^{(6)} \leq (\bar{\nu}_1)^{(6)} \leq \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$, we obtain

$$(\nu_1)^{(6)} \leq \nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(6)}(t)$ :-

$$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}, \quad \boxed{\nu^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(6)}(t)$ :-

$$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}, \quad \boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)}G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)}T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

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**Proof** : From global equations we obtain

$$\frac{d\nu^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right) - (a''_{37})^{(7)}(T_{37}, t)\nu^{(7)} - (a_{37})^{(7)}\nu^{(7)}$$

**Definition of** $\nu^{(7)}$ :- $\boxed{\nu^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right) \leq \frac{d\nu^{(7)}}{dt} \leq -\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(7)}, (\nu_0)^{(7)}$ :-

For $0 < \boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}} < (\nu_1)^{(7)} < (\bar{\nu}_1)^{(7)}$

$$\nu^{(7)}(t) \geq \frac{(\nu_1)^{(7)}+(C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_0)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)}=\frac{(\nu_1)^{(7)}-(\nu_0)^{(7)}}{(\nu_0)^{(7)}-(\nu_2)^{(7)}}}$$

it follows $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\nu_1)^{(7)}$

In the same manner , we get 417

$$\nu^{(7)}(t) \leq \frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)}=\frac{(\bar{\nu}_1)^{(7)}-(\nu_0)^{(7)}}{(\nu_0)^{(7)}-(\bar{\nu}_2)^{(7)}}}$$

From which we deduce $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\bar{\nu}_1)^{(7)}$

If $0 < (\nu_1)^{(7)} < (\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0} < (\bar{\nu}_1)^{(7)}$ we find like in the previous case, 418

$$(\nu_1)^{(7)} \leq \frac{(\nu_1)^{(7)}+(C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}} \leq \nu^{(7)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \leq (\bar{\nu}_1)^{(7)}$$

If $0 < (\nu_1)^{(7)} \leq (\bar{\nu}_1)^{(7)} \leq \boxed{(\nu_0)^{(7)}=\frac{G_{36}^0}{G_{37}^0}}$ , we obtain 419

$$(\nu_1)^{(7)} \leq \nu^{(7)}(t) \leq \frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \leq (\nu_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(7)}(t)$ :-

$$(m_2)^{(7)} \leq \nu^{(7)}(t) \leq (m_1)^{(7)}, \quad \boxed{\nu^{(7)}(t)=\frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(7)}(t)$ :- 420

$$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}, \quad \boxed{u^{(7)}(t)=\frac{T_{36}(t)}{T_{37}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $\nu^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)}G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for**

**the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)} T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

**Proof** : From global equations we obtain 421

$$\frac{d\nu^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right) - (a''_{41})^{(8)}(T_{41}, t)\nu^{(8)} - (a_{41})^{(8)}\nu^{(8)}$$

**Definition of** $\nu^{(8)}$ :- $\boxed{\nu^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right) \le \frac{d\nu^{(8)}}{dt} \le -\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$ :-

For $0 < \boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$

$$\nu^{(8)}(t) \ge \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \le \nu^{(8)}(t) \le (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \le \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)} e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \le \nu^{(8)}(t) \le (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \le \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}}{1 + (C)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}} \le \nu^{(8)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)} e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \le (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \le (\bar{\nu}_1)^{(8)} \le \boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}}$ , we obtain 424

$$(v_1)^{(8)} \leq v^{(8)}(t) \leq \frac{(\bar{v}_1)^{(8)}+(\bar{C})^{(8)}(\bar{v}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{v}_1)^{(8)}-(\bar{v}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{v}_1)^{(8)}-(\bar{v}_2)^{(8)}\right)t\right]}} \leq (v_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $v^{(8)}(t)$ :-

$$(m_2)^{(8)} \leq v^{(8)}(t) \leq (m_1)^{(8)}, \quad \boxed{v^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(8)}(t)$ :-

$$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}, \quad \boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(v_1)^{(8)} = (\bar{v}_1)^{(8)}$ if in addition $(v_0)^{(8)} = (v_1)^{(8)}$ then $v^{(8)}(t) = (v_0)^{(8)}$ and as a consequence $G_{40}(t) = (v_0)^{(8)}G_{41}(t)$ **this also defines** $(v_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then $(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)}T_{41}(t)$ This is an important consequence of the relation between $(v_1)^{(8)}$ and $(\bar{v}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

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**Proof** : From 99,20,44,22,23,44 we obtain

$$\frac{dv^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right) - (a''_{45})^{(9)}(T_{45},t)v^{(9)} - (a_{45})^{(9)}v^{(9)}$$

**Definition of** $v^{(9)}$ :- $\boxed{v^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)}\right) \leq \frac{dv^{(9)}}{dt} \leq -\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_1)^{(9)}v^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

**Definition of** $(\bar{v}_1)^{(9)}, (v_0)^{(9)}$ :-

For $0 < \boxed{(v_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (v_1)^{(9)} < (\bar{v}_1)^{(9)}$

$$v^{(9)}(t) \geq \frac{(v_1)^{(9)}+(C)^{(9)}(v_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((v_1)^{(9)}-(v_0)^{(9)}\right)t\right]}}{1+(C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((v_1)^{(9)}-(v_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(v_1)^{(9)}-(v_0)^{(9)}}{(v_0)^{(9)}-(v_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\nu_9)^{(9)}$

In the same manner , we get

$$\nu^{(9)}(t) \leq \frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)}-(\nu_0)^{(9)}}{(\nu_0)^{(9)}-(\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \leq \frac{(\nu_1)^{(9)}+(C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}}{1+(C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}} \leq \nu^{(9)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \leq (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \leq (\bar{\nu}_1)^{(9)} \leq \boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$ , we obtain

$$(\nu_1)^{(9)} \leq \nu^{(9)}(t) \leq \frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \leq (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(9)}(t)$ :-

$$(m_2)^{(9)} \leq \nu^{(9)}(t) \leq (m_1)^{(9)}, \quad \boxed{\nu^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(9)}(t)$ :-

$$(\mu_2)^{(9)} \leq u^{(9)}(t) \leq (\mu_1)^{(9)}, \quad \boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)}G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then $(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)}T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following

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**Theorem :** If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on $t$ , and the conditions with the notations

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0$ ,

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$

$with\ \ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(2)} and\ (b_i'')^{(2)}$ are independent on t , and the conditions with the notations 426

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$ 427

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 428

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$ , 429

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 430

$with\ \ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(3)} and\ (b_i'')^{(3)}$ are independent on $t$ , and the conditions with the notations 431

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$ ,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

**Theorem :** If $(a_i'')^{(4)} and\ (b_i'')^{(4)}$ are independent on $t$ , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$ ,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(5)} and\ (b_i'')^{(5)}$ are independent on $t$ , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$ ,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

**Theorem** If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on $t$ , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$ ,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(7)} and\ (b_i'')^{(7)}$ are independent on $t$ , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$ ,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(8)} and\ (b_i'')^{(8)}$ are independent on $t$ , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$ ,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(9)} and\ (b_i'')^{(9)}$ are independent on $t$ , and the conditions (with the notations 436 A
45,46,27,28)

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$ ,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0 \quad 437$$

$$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0 \quad 438$$

$$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0 \quad 439$$

$$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0 \quad 440$$

$$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ]T_{14} = 0 \quad 441$$

$$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0 \quad 442$$

has a unique positive solution , which is an equilibrium solution for the system

$$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0 \quad 443$$

$$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0 \quad 444$$

$$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0 \quad 445$$

$$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\ ]T_{16} = 0 \quad 446$$

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\ ]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\ ]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\ ]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\ ]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\ ]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27})\right)]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27})\right)]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27})\right)]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0 \quad 479$$

$$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0 \quad 480$$

$$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0 \quad 481$$

$$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0 \quad 482$$

$$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0 \quad 483$$

$$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0 \quad 484$$

$$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0 \quad 484\text{A}$$

$$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$$

$$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$$

$$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$$

$$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$$

$$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$$

**Proof:** 485

(a) Indeed the first two equations have a nontrivial solution $G_{13}, G_{14}$ if

$$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$$

**Proof:** 486

(t) Indeed the first two equations have a nontrivial solution $G_{16}, G_{17}$ if

$$\mathrm{F}(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$$

**Proof:** 487

(a) Indeed the first two equations have a nontrivial solution $G_{20}, G_{21}$ if

$$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$$

**Proof:** 488

(a) Indeed the first two equations have a nontrivial solution $G_{24}, G_{25}$ if

$$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$$

**Proof:** 489

(a) Indeed the first two equations have a nontrivial solution $G_{28}, G_{29}$ if

$$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$$

**Proof:** 490

(a) Indeed the first two equations have a nontrivial solution $G_{32}, G_{33}$ if

$$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$$

**Proof:** 491

(a) Indeed the first two equations have a nontrivial solution $G_{36}, G_{37}$ if

$$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$$

**Proof:** 492

(a) Indeed the first two equations have a nontrivial solution $G_{40}, G_{41}$ if

$$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$$

**Proof:** 492 A

(a) Indeed the first two equations have a nontrivial solution $G_{44}, G_{45}$ if

$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$

**Definition and uniqueness of** $T^*_{14}$ :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique $T^*_{14}$ for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

**Definition and uniqueness of** $T^*_{17}$ :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique $T^*_{17}$ for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]} \qquad 495$$

**Definition and uniqueness of** $T^*_{21}$ :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique $T^*_{21}$ for which $f(T^*_{21}) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

**Definition and uniqueness of** $T^*_{25}$ :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that there exists a unique $T^*_{25}$ for which $f(T^*_{25}) = 0$. With this value , we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T^*_{25})]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T^*_{25})]}$$

**Definition and uniqueness of** $T^*_{29}$ :- 498

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(5)}(T_{29})$ being increasing, it follows that there exists a unique $T^*_{29}$ for which $f(T^*_{29}) = 0$. With this value , we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T^*_{29})]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T^*_{29})]}$$

**Definition and uniqueness of** $T^*_{33}$ :- 499

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(6)}(T_{33})$ being increasing, it follows that

there exists a unique $T_{33}^{*}$ for which $f(T_{33}^{*})=0$. With this value , we obtain from the three first equations

$$G_{32}=\frac{(a_{32})^{(6)}G_{33}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^{*})]} \quad , \quad G_{34}=\frac{(a_{34})^{(6)}G_{33}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^{*})]}$$

**Definition and uniqueness of** $T_{37}^{*}$ :- 500

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(7)}(T_{37})$ being increasing, it follows that there exists a unique $T_{37}^{*}$ for which $f(T_{37}^{*})=0$. With this value , we obtain from the three first equations

$$G_{36}=\frac{(a_{36})^{(7)}G_{37}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^{*})]} \quad , \quad G_{38}=\frac{(a_{38})^{(7)}G_{37}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^{*})]}$$

**Definition and uniqueness of** $T_{41}^{*}$ :- 501

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique $T_{41}^{*}$ for which $f(T_{41}^{*})=0$. With this value , we obtain from the three first equations

$$G_{40}=\frac{(a_{40})^{(8)}G_{41}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T_{41}^{*})]} \quad , \quad G_{42}=\frac{(a_{42})^{(8)}G_{41}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T_{41}^{*})]}$$

**Definition and uniqueness of** $T_{45}^{*}$ :- 501 A

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(9)}(T_{45})$ being increasing, it follows that there exists a unique $T_{45}^{*}$ for which $f(T_{45}^{*})=0$. With this value , we obtain from the three first equations

$$G_{44}=\frac{(a_{44})^{(9)}G_{45}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T_{45}^{*})]} \quad , \quad G_{46}=\frac{(a_{46})^{(9)}G_{45}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T_{45}^{*})]}$$

By the same argument, the equations admit solutions $G_{13}, G_{14}$ if 502

$$\varphi(G)=(b'_{13})^{(1)}(b'_{14})^{(1)}-(b_{13})^{(1)}(b_{14})^{(1)}-$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G)+(b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right]+(b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G)=0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{14}$ taking into account the hypothesis $\varphi(0)>0$ , $\varphi(\infty)<0$ it follows that there exists a unique $G_{14}^{*}$ such that $\varphi(G^{*})=0$

By the same argument, the equations admit solutions $G_{16}, G_{17}$ if 503

$$\varphi(G_{19})=(b'_{16})^{(2)}(b'_{17})^{(2)}-(b_{16})^{(2)}(b_{17})^{(2)}-$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19})+(b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right]+(b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19})=0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{17}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{14}^{*}$ such that $\varphi((G_{19})^{*}) = 0$ 504

By the same argument, the equations admit solutions $G_{20}, G_{21}$ if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right] + (b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{21}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{21}^{*}$ such that $\varphi((G_{23})^{*}) = 0$

By the same argument, the equations admit solutions $G_{24}, G_{25}$ if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right] + (b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{25}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{25}^{*}$ such that $\varphi((G_{27})^{*}) = 0$

By the same argument, the equations admit solutions $G_{28}, G_{29}$ if 507

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right] + (b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{29}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{29}^{*}$ such that $\varphi((G_{31})^{*}) = 0$

By the same argument, the equations admit solutions $G_{32}, G_{33}$ if 508

$$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$$

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right] + (b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{33}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{33}^{*}$ such that $\varphi({G_{35}}^{*}) = 0$

By the same argument, the equations admit solutions $G_{36}, G_{37}$ if 509

$$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$$

$$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right] + (b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{37}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{37}^{*}$ such that $\varphi({G_{39}}^{*}) = 0$

By the same argument, the equations admit solutions $G_{40}, G_{41}$ if 510

$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$
$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right] + (b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{41}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{41}$ such that $\varphi(G_{43}{}^*) = 0$

By the same argument, the equations 92,93 admit solutions $G_{44}, G_{45}$ if
$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$

$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right] + (b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{45}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{45}$ such that $\varphi((G_{47})^*) = 0$

Finally we obtain the unique solution 511

$G^*_{14}$ given by $\varphi(G^*) = 0$ , $T^*_{14}$ given by $f(T^*_{14}) = 0$ and

$$G^*_{13} = \frac{(a_{13})^{(1)}G^*_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G^*_{15} = \frac{(a_{15})^{(1)}G^*_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

$$T^*_{13} = \frac{(b_{13})^{(1)}T^*_{14}}{[(b'_{13})^{(1)}-(b''_{13})^{(1)}(G^*)]} \quad , \quad T^*_{15} = \frac{(b_{15})^{(1)}T^*_{14}}{[(b'_{15})^{(1)}-(b''_{15})^{(1)}(G^*)]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

$G^*_{17}$ given by $\varphi((G_{19})^*) = 0$ , $T^*_{17}$ given by $f(T^*_{17}) = 0$ and 512

$$G^*_{16} = \frac{(a_{16})^{(2)}G^*_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G^*_{18} = \frac{(a_{18})^{(2)}G^*_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]} \quad 513$$

$$T^*_{16} = \frac{(b_{16})^{(2)}T^*_{17}}{[(b'_{16})^{(2)}-(b''_{16})^{(2)}((G_{19})^*)]} \quad , \quad T^*_{18} = \frac{(b_{18})^{(2)}T^*_{17}}{[(b'_{18})^{(2)}-(b''_{18})^{(2)}((G_{19})^*)]} \quad 514$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

$G^*_{21}$ given by $\varphi((G_{23})^*) = 0$ , $T^*_{21}$ given by $f(T^*_{21}) = 0$ and

$$G^*_{20} = \frac{(a_{20})^{(3)}G^*_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G^*_{22} = \frac{(a_{22})^{(3)}G^*_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

$$T^*_{20} = \frac{(b_{20})^{(3)}T^*_{21}}{[(b'_{20})^{(3)}-(b''_{20})^{(3)}(G_{23}{}^*)]} \quad , \quad T^*_{22} = \frac{(b_{22})^{(3)}T^*_{21}}{[(b'_{22})^{(3)}-(b''_{22})^{(3)}(G_{23}{}^*)]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

$G_{25}^{*}$ given by $\varphi(G_{27}) = 0$ , $T_{25}^{*}$ given by $f(T_{25}^{*}) = 0$ and

$$G_{24}^{*} = \frac{(a_{24})^{(4)} G_{25}^{*}}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}^{*} = \frac{(a_{26})^{(4)} G_{25}^{*}}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^{*})]}$$

$$T_{24}^{*} = \frac{(b_{24})^{(4)} T_{25}^{*}}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^{*})]} \quad , \quad T_{26}^{*} = \frac{(b_{26})^{(4)} T_{25}^{*}}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^{*})]} \qquad 517$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

$G_{29}^{*}$ given by $\varphi((G_{31})^{*}) = 0$ , $T_{29}^{*}$ given by $f(T_{29}^{*}) = 0$ and

$$G_{28}^{*} = \frac{(a_{28})^{(5)} G_{29}^{*}}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}^{*} = \frac{(a_{30})^{(5)} G_{29}^{*}}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^{*})]}$$

$$T_{28}^{*} = \frac{(b_{28})^{(5)} T_{29}^{*}}{[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31})^{*})]} \quad , \quad T_{30}^{*} = \frac{(b_{30})^{(5)} T_{29}^{*}}{[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31})^{*})]} \qquad 519$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

$G_{33}^{*}$ given by $\varphi((G_{35})^{*}) = 0$ , $T_{33}^{*}$ given by $f(T_{33}^{*}) = 0$ and

$$G_{32}^{*} = \frac{(a_{32})^{(6)} G_{33}^{*}}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^{*})]} \quad , \quad G_{34}^{*} = \frac{(a_{34})^{(6)} G_{33}^{*}}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^{*})]}$$

$$T_{32}^{*} = \frac{(b_{32})^{(6)} T_{33}^{*}}{[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35})^{*})]} \quad , \quad T_{34}^{*} = \frac{(b_{34})^{(6)} T_{33}^{*}}{[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35})^{*})]} \qquad 521$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

$G_{37}^{*}$ given by $\varphi((G_{39})^{*}) = 0$ , $T_{37}^{*}$ given by $f(T_{37}^{*}) = 0$ and

$$G_{36}^{*} = \frac{(a_{36})^{(7)} G_{37}^{*}}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^{*})]} \quad , \quad G_{38}^{*} = \frac{(a_{38})^{(7)} G_{37}^{*}}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^{*})]}$$

$$T_{36}^{*} = \frac{(b_{36})^{(7)} T_{37}^{*}}{[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39})^{*})]} \quad , \quad T_{38}^{*} = \frac{(b_{38})^{(7)} T_{37}^{*}}{[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39})^{*})]}$$

Finally we obtain the unique solution 523

$G_{41}^{*}$ given by $\varphi((G_{43})^{*}) = 0$ , $T_{41}^{*}$ given by $f(T_{41}^{*}) = 0$ and

$$G_{40}^{*} = \frac{(a_{40})^{(8)} G_{41}^{*}}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^{*})]} \quad , \quad G_{42}^{*} = \frac{(a_{42})^{(8)} G_{41}^{*}}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^{*})]}$$

$$T_{40}^{*} = \frac{(b_{40})^{(8)} T_{41}^{*}}{[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43})^{*})]} \quad , \quad T_{42}^{*} = \frac{(b_{42})^{(8)} T_{41}^{*}}{[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43})^{*})]}$$

Finally we obtain the unique solution of 89 to 99 523

A

$G_{45}^{*}$ given by $\varphi((G_{47})^{*}) = 0$ , $T_{45}^{*}$ given by $f(T_{45}^{*}) = 0$ and

$$G_{44}^{*} = \frac{(a_{44})^{(9)} G_{45}^{*}}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^{*})]} \quad , \quad G_{46}^{*} = \frac{(a_{46})^{(9)} G_{45}^{*}}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^{*})]}$$

$$T_{44}^{*} = \frac{(b_{44})^{(9)} T_{45}^{*}}{[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47})^{*})]} \quad , \quad T_{46}^{*} = \frac{(b_{46})^{(9)} T_{45}^{*}}{[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47})^{*})]}$$

**ASYMPTOTIC STABILITY ANALYSIS** 524

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

**Proof:** Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^{*} + \mathbb{G}_i \qquad , T_i = T_i^{*} + \mathbb{T}_i$$

$$\frac{\partial (a''_{14})^{(1)}}{\partial T_{14}}(T_{14}^{*}) = (q_{14})^{(1)} \quad , \frac{\partial (b_i'')^{(1)}}{\partial G_j}(G^{*}) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a'_{13})^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)} G_{13}^{*}\mathbb{T}_{14} \quad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a'_{14})^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)} G_{14}^{*}\mathbb{T}_{14} \quad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a'_{15})^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)} G_{15}^{*}\mathbb{T}_{14} \quad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b'_{13})^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)} T_{13}^{*}\mathbb{G}_j\right) \quad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b'_{14})^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)} T_{14}^{*}\mathbb{G}_j\right) \quad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b'_{15})^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)} T_{15}^{*}\mathbb{G}_j\right) \quad 530$$

**ASYMPTOTIC STABILITY ANALYSIS** 531

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(2)}$ and $(b_i'')^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i \tag{532}$$

$$\frac{\partial(a_{17}'')^{(2)}}{\partial T_{17}}(T_{17}^*) = (q_{17})^{(2)} \quad , \frac{\partial(b_i'')^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \tag{533}$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a_{16}')^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G_{16}^*\mathbb{T}_{17} \tag{534}$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a_{17}')^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G_{17}^*\mathbb{T}_{17} \tag{535}$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a_{18}')^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G_{18}^*\mathbb{T}_{17} \tag{536}$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b_{16}')^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T_{16}^*\mathbb{G}_j\right) \tag{537}$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b_{17}')^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}T_{17}^*\mathbb{G}_j\right) \tag{538}$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b_{18}')^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}T_{18}^*\mathbb{G}_j\right) \tag{539}$$

**ASYMPTOTIC STABILITY ANALYSIS** 540

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(3)}$ *and* $(b_i'')^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{21}'')^{(3)}}{\partial T_{21}}(T_{21}^*) = (q_{21})^{(3)} \quad , \frac{\partial(b_i'')^{(3)}}{\partial G_j}((G_{23})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a_{20}')^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G_{20}^*\mathbb{T}_{21} \tag{541}$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a_{21}')^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G_{21}^*\mathbb{T}_{21} \tag{542}$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a_{22}')^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G_{22}^*\mathbb{T}_{21} \tag{543}$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b_{20}')^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T_{20}^*\mathbb{G}_j\right) \tag{544}$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b_{21}')^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T_{21}^*\mathbb{G}_j\right) \tag{545}$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b'_{22})^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T^*_{22}\mathbb{G}_j\right) \quad 546$$

**ASYMPTOTIC STABILITY ANALYSIS** 547

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(4)}\ and\ (b''_i)^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{25})^{(4)}}{\partial T_{25}}(T^*_{25}) = (q_{25})^{(4)} \ , \ \frac{\partial(b''_i)^{(4)}}{\partial G_j}((G_{27})^*\ ) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\left((a'_{24})^{(4)} + (p_{24})^{(4)}\right)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G^*_{24}\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\left((a'_{25})^{(4)} + (p_{25})^{(4)}\right)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G^*_{25}\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\left((a'_{26})^{(4)} + (p_{26})^{(4)}\right)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G^*_{26}\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\left((b'_{24})^{(4)} - (r_{24})^{(4)}\right)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(24)(j)}T^*_{24}\mathbb{G}_j\right) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\left((b'_{25})^{(4)} - (r_{25})^{(4)}\right)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\left(s_{(25)(j)}T^*_{25}\mathbb{G}_j\right) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T^*_{26}\mathbb{G}_j\right) \quad 554$$

**ASYMPTOTIC STABILITY ANALYSIS** 555

**Theorem 5:** If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(5)}\ and\ (b''_i)^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 556

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{29})^{(5)}}{\partial T_{29}}(T^*_{29}) = (q_{29})^{(5)} \ , \ \frac{\partial(b''_i)^{(5)}}{\partial G_j}(\ (G_{31})^*\ ) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a'_{28})^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G^*_{28}\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a'_{29})^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G^*_{29}\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a'_{30})^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G^*_{30}\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b'_{28})^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T^*_{28}\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b'_{29})^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T^*_{29}\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b'_{30})^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T^*_{30}\mathbb{G}_j\right) \quad 562$$

**ASYMPTOTIC STABILITY ANALYSIS** 563

**Theorem 6:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}$ *and* $(b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{33})^{(6)}}{\partial T_{33}}(T^*_{33}) = (q_{33})^{(6)} \quad , \frac{\partial(b_i'')^{(6)}}{\partial G_j}\left((G_{35})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a'_{32})^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G^*_{32}\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a'_{33})^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G^*_{33}\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a'_{34})^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G^*_{34}\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b'_{32})^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T^*_{32}\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b'_{33})^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T^*_{33}\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b'_{34})^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T^*_{34}\mathbb{G}_j\right) \quad 570$$

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**Theorem 7:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 572

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{37}'')^{(7)}}{\partial T_{37}}(T_{37}^{*}) = (q_{37})^{(7)} \quad , \frac{\partial(b_i'')^{(7)}}{\partial G_j}\left( (G_{39})^{**} \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a_{36}')^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G_{36}^{*}\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a_{37}')^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G_{37}^{*}\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a_{38}')^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G_{38}^{*}\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b_{36}')^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T_{36}^{*}\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b_{37}')^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T_{37}^{*}\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b_{38}')^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T_{38}^{*}\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

## ASYMPTOTIC STABILITY ANALYSIS

**Theorem 8:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 580

$$G_i = G_i^{*} + \mathbb{G}_i \qquad , T_i = T_i^{*} + \mathbb{T}_i$$

$$\frac{\partial(a_{41}'')^{(8)}}{\partial T_{41}}(T_{41}^{*}) = (q_{41})^{(8)} \quad , \frac{\partial(b_i'')^{(8)}}{\partial G_j}\left( (G_{43})^{*} \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a_{40}')^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G_{40}^{*}\mathbb{T}_{41} \quad 581$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a_{41}')^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G_{41}^{*}\mathbb{T}_{41} \quad 582$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a_{42}')^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G_{42}^{*}\mathbb{T}_{41} \quad 583$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b_{40}')^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T_{40}^{*}\mathbb{G}_j\right) \quad 584$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b_{41}')^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T_{41}^{*}\mathbb{G}_j\right) \quad 585$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b'_{42})^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T^*_{42}\mathbb{G}_j\right) \quad 586$$

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**ASYMPTOTIC STABILITY ANALYSIS** A

**Theorem 9:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}\ and\ (b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

**Proof: Denote**

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{45})^{(9)}}{\partial T_{45}}(T^*_{45}) = (q_{45})^{(9)} \ , \ \frac{\partial(b_i'')^{(9)}}{\partial G_j}(\ (G_{47})^*\ ) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a'_{44})^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)}G^*_{44}\mathbb{T}_{45} \quad 586\ B$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a'_{45})^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)}G^*_{45}\mathbb{T}_{45} \quad 586\ C$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a'_{46})^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)}G^*_{46}\mathbb{T}_{45} \quad 586\ D$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b'_{44})^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)}T^*_{44}\mathbb{G}_j\right) \quad 586\ E$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b'_{45})^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)}T^*_{45}\mathbb{G}_j\right) \quad 586\ F$$

$$\frac{d\mathbb{T}_{46}}{dt} = -\left((b'_{46})^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T^*_{46}\mathbb{G}_j\right) \quad 586\ G$$

**The characteristic equation of this system is** 587

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(q_{13})^{(1)}G^*_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)}T^*_{14} + (b_{14})^{(1)}s_{(13),(14)}T^*_{14}\right)$$

$$+\left(\left((\lambda)^{(1)} + (a'_{14})^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)}G^*_{13} + (a_{13})^{(1)}(q_{14})^{(1)}G^*_{14}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)}T^*_{14} + (b_{14})^{(1)}s_{(13),(13)}T^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left(\ (a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left(\ (b'_{13})^{(1)} + (b'_{14})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^2+\left((a'_{13})^{(1)}+(a'_{14})^{(1)}+(p_{13})^{(1)}+(p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)}+(a'_{13})^{(1)}+(p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^*_{14}+(a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}+(b'_{13})^{(1)}-(r_{13})^{(1)}\right)s_{(14),(15)}T^*_{14}+(b_{14})^{(1)}s_{(13),(15)}T^*_{13}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(2)}+(b'_{18})^{(2)}-(r_{18})^{(2)}\right)\{\left((\lambda)^{(2)}+(a'_{18})^{(2)}+(p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^*_{17}+(a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^*_{17}\right)$$

$$+\left(\left((\lambda)^{(2)}+(a'_{17})^{(2)}+(p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^*_{16}+(a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2+\left((b'_{16})^{(2)}+(b'_{17})^{(2)}-(r_{16})^{(2)}+(r_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$+\left(\left((\lambda)^{(2)}\right)^2+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}+(a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^*_{16}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\right)\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left(\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^*_{20}\right)\right]$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^*_{21}+(b_{21})^{(3)}s_{(20),(21)}T^*_{21}\right)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^*_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^*_{21}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^*_{21}+(b_{21})^{(3)}s_{(20),(20)}T^*_{20}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^2+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$+\left(\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^*_{20}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^*_{21}+(b_{21})^{(3)}s_{(20),(22)}T^*_{20}\right)\}=0$$

$$+$$

$$(\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left(\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^*_{24}\right)\right]$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^*_{25}+(b_{25})^{(4)}s_{(24),(25)}T^*_{25}\right)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^*_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^*_{25}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^*_{25}+(b_{25})^{(4)}s_{(24),(24)}T^*_{24}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^2+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$+\left(\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^*_{24}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^*_{25}+(b_{25})^{(4)}s_{(24),(26)}T^*_{24}\right)\}=0$$

$$+$$

$$(\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^*_{28}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^*_{29}+(b_{29})^{(5)}s_{(28),(29)}T^*_{29}\right)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^*_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^*_{29}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^*_{29}+(b_{29})^{(5)}s_{(28),(28)}T^*_{28}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^2+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$+\left(\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^*_{28}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^*_{29}+(b_{29})^{(5)}s_{(28),(30)}T^*_{28}\right)\}=0$$

$$+$$

$$(\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left(\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^*_{32}\right)\right]$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^*_{33}+(b_{33})^{(6)}s_{(32),(33)}T^*_{33}\right)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^*_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^*_{33}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^*_{33}+(b_{33})^{(6)}s_{(32),(32)}T^*_{32}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^2+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$+\left(\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^*_{32}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^*_{33}+(b_{33})^{(6)}s_{(32),(34)}T^*_{32}\right)\}=0$$

$$+$$

$$(\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^*_{36}\right)\right]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^*_{37}+(b_{37})^{(7)}s_{(36),(37)}T^*_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^*_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^*_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^*_{37}+(b_{37})^{(7)}s_{(36),(36)}T^*_{36}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^2+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$+\left(\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^*_{36}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^*_{37}+(b_{37})^{(7)}s_{(36),(38)}T^*_{36}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\right)\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^*_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^*_{41}+(b_{41})^{(8)}s_{(40),(41)}T^*_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^*_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^*_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^*_{41}+(b_{41})^{(8)}s_{(40),(40)}T^*_{40}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^2+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$+\left(\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^*_{40}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^*_{41}+(b_{41})^{(8)}s_{(40),(42)}T^*_{40}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\right)\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^*_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^*_{45}+(b_{45})^{(9)}s_{(44),(45)}T^*_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^*_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^*_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^*_{45}+(b_{45})^{(9)}s_{(44),(44)}T^*_{44}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^2+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$+\left(\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^*_{44}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^*_{45}+(b_{45})^{(9)}s_{(44),(46)}T^*_{44}\right)\}=0$$

**And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.**

## SECTION TWENTY THREE

## Regularity And Chaos

### INTRODUCTION—VARIABLES USED

**Regularity and chaos in 0+ states of the interacting boson model using quantum measures S. Karampagia, Dennis Bonatsos, and R. F. Casten Phys. Rev. C 91, 054325 – Published 26 May 2015**

(1) Background: Statistical measures of chaos have long been used (e) in the study of chaotic dynamics in (eb) the framework of (e) the interacting boson model.
(2) The use of large numbers of bosons renders (eb) possible additional studies of chaos that can provide (eb) a direct comparison with similar classical studies of chaos.
(3) Purpose: Authors intend to provide complete quantum chaotic dynamics at (eb) zero angular momentum in (eb) the vicinity of the arc of regularity and link the results of the study of chaos using statistical measures with (e&eb) those of the study of chaos using classical measures.
(4) Method: Statistical measures of chaos are applied on (e&eb) the spectrum and the transition intensities of (e) 0+ states in the framework of (e) the interacting boson model.
(5) Results: The energy dependence of chaos is provided for the first time using (e) statistical measures of chaos. The position of the arc of regularity was also found to be (=) stable in the limit of large boson numbers
**(6)** Conclusions: The results of the study of chaos using statistical measures are consistent with (e&eb) previous studies using classical measures of chaos, as well as with (e&eb) studies using statistical measures of chaos, but for small number of bosons and states with angular momentum greater than 2. DOI: **http://dx.doi.org/10.1103/PhysRevC.91.054325 ©2015 American Physical Society**

### NOTATION

**Module One**

Background:

Statistical measures of chaos have long been used (e) in the study of **chaotic dynamics** in (eb) the framework of the interacting boson model

$G_{13}$ : Category one of **chaotic dynamics;** framework of the interacting boson model

$G_{14}$ : Category two of SAS

$G_{15}$ : Category three of SAS

$T_{13}$ : Category one of framework of the interacting boson model **; chaotic dynamics**

$T_{14}$ : Category two of SAS

$T_{15}$ : Category three of SAS

**Module Two**

The use of large numbers of bosons renders (eb) possible additional studies of chaos that can provide (eb) a direct comparison with similar classical studies of chaos.

$G_{16}$ : Category one of use of large numbers of bosons

$G_{17}$ : Category two of SAS

$G_{18}$ : Category three of SAS

$T_{16}$ : Category one of direct comparison with similar classical studies of chaos.

$T_{17}$ : Category two of SAS

$T_{18}$ : Category three of SAS

**Module three**

Purpose:

Authors intend to provide complete quantum chaotic dynamics at (eb) zero angular momentum in (eb) the vicinity of the arc of regularity and link the results of the study of chaos using statistical measures with (e&eb) those of the study of chaos using classical measures

$G_{20}$ : Category one of complete quantum chaotic dynamics

$G_{21}$ : Category two of SAS

$G_{22}$ : Category three of SAS

$T_{20}$ : Category one of zero angular momentum in (eb) the vicinity of the arc of regularity and link the results of the study of chaos using statistical measures with (e&eb) those of the study of chaos using classical measures

$T_{21}$ : Category two of SAS

$T_{22}$ : Category three of SAS

**Module four**

Authors intend to provide complete quantum chaotic dynamics at (eb) zero angular momentum in (eb) the vicinity of the arc of regularity and link the results of the study of chaos using statistical measures with (e&eb) those of the study of chaos using classical measures

$G_{24}$ : Category one of complete quantum chaotic dynamics

$G_{25}$ : Category two of SAS

$G_{26}$ : Category three of SAS

$T_{24}$ : Category one of zero angular momentum in (eb) the vicinity of the arc of regularity and link the results of the study of chaos using statistical measures with (e&eb) those of the study of chaos using classical measures

$T_{25}$ : Category two of SAS

$T_{26}$ : Category three of SAS

**Module five**

Authors intend to provide complete quantum chaotic dynamics at zero angular momentum in the vicinity of the arc of regularity and link the results of the study of chaos using statistical measures with (e&eb) those of the study of **chaos using classical measures**

$G_{28}$ : Category one of complete quantum chaotic dynamics at zero angular momentum in the vicinity of the arc of regularity and link the results of the study of chaos using statistical measures; **chaos using classical measures**

$G_{29}$ : Category two of SAS

$G_{30}$ : Category three of SAS

$T_{28}$ : Category one of **chaos using classical measures** ;complete quantum chaotic dynamics at zero angular momentum in the vicinity of the arc of regularity and link the results of the study of chaos using statistical measures

$T_{29}$ : Category two of SAS

$T_{30}$ : Category three of SAS

**Module six**

Statistical measures of chaos are applied on (e&eb) the spectrum and the transition intensities of (e) 0+ states in the framework of (e) the interacting boson model

$G_{32}$ : Category one of **Statistical measures of chaos;** spectrum and the transition intensities of (e) 0+ states in the framework of (e) the interacting boson model

$G_{33}$ : Category two of SAS

$G_{34}$ : Category three of SAS

$T_{32}$ : Category one of spectrum and the transition intensities of (e) 0+ states in the framework of (e) the interacting boson model **;Statistical measures of chaos**

$T_{33}$ : Category two of SAS

$T_{34}$ : Category three of SAS

**Module seven**

Statistical measures of chaos are applied on the spectrum and the transition intensities of 0+ states in the framework of the **interacting boson model**

$G_{36}$ : Category one of Statistical measures of chaos are applied on the spectrum and the transition intensities of 0+ states; **interacting boson model**

$G_{37}$ : Category two of SAS

$G_{38}$ : Category three of SAS

$T_{36}$ : Category one of **interacting boson model;** Statistical measures of chaos are applied on the spectrum and the transition intensities of 0+ states

$T_{37}$ : Category two of SAS

$T_{38}$ : Category three of SAS

**Module eight**

Results: The energy dependence of chaos is provided for the first time using (e) statistical measures of chaos.

The position of the arc of regularity was also found to be (=) stable in the limit of large boson numbers

$G_{40}$ : Category one of position of the arc of regularity

$G_{41}$ : Category two of SAS

$G_{42}$ : Category three of SAS

$T_{40}$ : Category one of stable in the limit of large boson numbers

$T_{41}$ : Category two of SAS

$T_{42}$ : Category three of SAS

**Module Nine**

Conclusions:

The results of the study of chaos using statistical measures are consistent with previous studies using classical measures of chaos, as well as with (e&eb) **studies using statistical measures of chaos, but for small number of bosons and states with angular momentum greater than 2.**

DOI: **http://dx.doi.org/10.1103/PhysRevC.91.054325 ©2015 American Physical Society**

$G_{44}$ : Category one of results of the study of chaos using statistical measures are consistent with previous studies using classical measures of chaos; **studies using statistical measures of chaos, but for small number of bosons and states with angular momentum greater than 2.**

$G_{45}$ : Category two of SAS

$G_{46}$ : Category three of SAS

$T_{44}$ : Category one of **studies using statistical measures of chaos, but for small number of bosons and states with angular momentum greater than 2;**results of the study of chaos using statistical measures are consistent with previous studies using classical measures of chaos

$T_{45}$ : Category two of SAS

$T_{46}$ : Category three of SAS

**The Coefficients:**

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}$ , $(b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}$,
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}$, $(b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}\ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$ ,
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)}$,
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)}$,
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)}$,
are Dissipation coefficients

**Module Numbered One**

**The differential system of this model is now (Module Numbered one)**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \quad 1$$
$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad 2$$
$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15} \quad 3$$
$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13} \quad 4$$
$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14} \quad 5$$
$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)\right]T_{15} \quad 6$$
$+(a''_{13})^{(1)}(T_{14}, t) =$ First augmentation factor
$-(b''_{13})^{(1)}(G, t) =$ First detritions factor

**Module Numbered Two**

**The differential system of this model is now ( Module numbered two)**

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right]G_{16} \quad 7$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t)\right]G_{17} \quad 8$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t)\right]G_{18} \quad 9$$

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\left((G_{19}), t\right)\right]T_{16} \quad 10$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\left((G_{19}), t\right)\right]T_{17} \quad 11$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\left((G_{19}), t\right)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17}, t) =$ First augmentation factor

$-(b''_{16})^{(2)}\left((G_{19}), t\right) =$ First detritions factor

**Module Numbered Three**

**The differential system of this model is now (Module numbered three)**

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right]G_{20} \quad 13$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}, t)\right]G_{21} \quad 14$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}, t)\right]G_{22} \quad 15$$

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}, t)\right]T_{20} \quad 16$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}, t)\right]T_{21} \quad 17$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}, t)\right]T_{22} \quad 18$$

$+(a''_{20})^{(3)}(T_{21}, t) =$ First augmentation factor

$-(b''_{20})^{(3)}(G_{23}, t) =$ First detritions factor

**Module Numbered Four**

**The differential system of this model is now (Module numbered Four)**

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)\right]G_{24} \quad 19$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}, t)\right]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}, t)\right]G_{26} \quad 21$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27}), t\right)\right]T_{24} \quad 22$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27}), t\right)\right]T_{25} \quad 23$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27}), t\right)\right]T_{26} \quad 24$$

$+(a''_{24})^{(4)}(T_{25}, t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}\left((G_{27}), t\right) =$ **First detritions factor**

**Module Numbered Five**:

**The differential system of this model is now (Module number five)**

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right]G_{28} \quad 25$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}, t)\right]G_{29} \quad 26$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}, t)\right]G_{30} \quad 27$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}\left((G_{31}), t\right)\right]T_{28} \quad 28$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}\left((G_{31}), t\right)\right]T_{29} \quad 29$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}\left((G_{31}), t\right)\right]T_{30} \quad 30$$

$+(a''_{28})^{(5)}(T_{29},t) =$ **First augmentation factor**
$-(b''_{28})^{(5)}\big((G_{31}),t\big) =$ **First detritions factor**
**Module Numbered Six**

**The differential system of this model is now (Module numbered Six)**

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \big[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33},t)\big]G_{32} \quad 31$$
$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \big[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33},t)\big]G_{33} \quad 32$$
$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \big[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33},t)\big]G_{34} \quad 33$$
$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \big[(b'_{32})^{(6)} - (b''_{32})^{(6)}\big((G_{35}),t\big)\big]T_{32} \quad 34$$
$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \big[(b'_{33})^{(6)} - (b''_{33})^{(6)}\big((G_{35}),t\big)\big]T_{33} \quad 35$$
$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \big[(b'_{34})^{(6)} - (b''_{34})^{(6)}\big((G_{35}),t\big)\big]T_{34} \quad 36$$

$+(a''_{32})^{(6)}(T_{33},t) =$ **First augmentation factor**
**Module Numbered Seven:**

**The differential system of this model is now (Seventh Module)**

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \big[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\big]G_{36} \quad 37$$
$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \big[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37},t)\big]G_{37} \quad 38$$
$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \big[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37},t)\big]G_{38} \quad 39$$
$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \big[(b'_{36})^{(7)} - (b''_{36})^{(7)}\big((G_{39}),t\big)\big]T_{36} \quad 40$$
$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \big[(b'_{37})^{(7)} - (b''_{37})^{(7)}\big((G_{39}),t\big)\big]T_{37} \quad 41$$
$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \big[(b'_{38})^{(7)} - (b''_{38})^{(7)}\big((G_{39}),t\big)\big]T_{38} \quad 42$$

$+(a''_{36})^{(7)}(T_{37},t) =$ **First augmentation factor**
**Module Numbered Eight**

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \big[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\big]G_{40} \quad 43$$
$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \big[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41},t)\big]G_{41} \quad 44$$
$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \big[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41},t)\big]G_{42} \quad 45$$
$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \big[(b'_{40})^{(8)} - (b''_{40})^{(8)}\big((G_{43}),t\big)\big]T_{40} \quad 46$$
$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \big[(b'_{41})^{(8)} - (b''_{41})^{(8)}\big((G_{43}),t\big)\big]T_{41} \quad 47$$
$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \big[(b'_{42})^{(8)} - (b''_{42})^{(8)}\big((G_{43}),t\big)\big]T_{42} \quad 48$$

**Module Numbered Nine**

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \big[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\big]G_{44} \quad 49$$
$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \big[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\big]G_{45} \quad 50$$
$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \big[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\big]G_{46} \quad 51$$
$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \big[(b'_{44})^{(9)} - (b''_{44})^{(9)}\big((G_{47}),t\big)\big]T_{44} \quad 52$$

$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}((G_{47}),t)\right]T_{45}$ 53

$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47}),t)\right]T_{46}$ 54

$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}((G_{47}),t) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)} & +(a''_{13})^{(1)}(T_{14},t) & +(a''_{16})^{(2,2,)}(T_{17},t) & +(a''_{20})^{(3,3,)}(T_{21},t) \\ +(a''_{24})^{(4,4,4,4,)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,)}(T_{33},t) & \\ +(a''_{36})^{(7,7)}(T_{37},t) & +(a''_{40})^{(8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) & \end{bmatrix} G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)} & +(a''_{14})^{(1)}(T_{14},t) & +(a''_{17})^{(2,2,)}(T_{17},t) & +(a''_{21})^{(3,3,)}(T_{21},t) \\ +(a''_{25})^{(4,4,4,4,)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,)}(T_{33},t) & \\ +(a''_{37})^{(7,7)}(T_{37},t) & +(a''_{41})^{(8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) & \end{bmatrix} G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)} & +(a''_{15})^{(1)}(T_{14},t) & +(a''_{18})^{(2,2,)}(T_{17},t) & +(a''_{22})^{(3,3,)}(T_{21},t) \\ +(a''_{26})^{(4,4,4,4,)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,)}(T_{33},t) & \\ +(a''_{38})^{(7,7)}(T_{37},t) & +(a''_{42})^{(8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) & \end{bmatrix} G_{15} \quad 57$$

Where $(a''_{13})^{(1)}(T_{14},t)$, $(a''_{14})^{(1)}(T_{14},t)$, $(a''_{15})^{(1)}(T_{14},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,)}(T_{17},t)$, $+(a''_{17})^{(2,2,)}(T_{17},t)$, $+(a''_{18})^{(2,2,)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,)}(T_{21},t)$, $+(a''_{21})^{(3,3,)}(T_{21},t)$, $+(a''_{22})^{(3,3,)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{38})^{(7,7)}(T_{37},t)$ $+(a''_{37})^{(7,7)}(T_{37},t)$ $+(a''_{36})^{(7,7)}(T_{37},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{40})^{(8,8)}(T_{41},t)$ $+(a''_{41})^{(8,8)}(T_{41},t)$ $+(a''_{42})^{(8,8)}(T_{41},t)$ are eight augmentation coefficient for 1,2,3

$+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)} & -(b''_{13})^{(1)}(G,t) & -(b''_{16})^{(2,2,)}(G_{19},t) & -(b''_{20})^{(3,3,)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,)}(G_{35},t) & \\ -(b''_{36})^{(7,7,)}(G_{39},t) & -(b''_{40})^{(8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) & \end{bmatrix} T_{13} \quad 58$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)} & -(b''_{14})^{(1)}(G,t) & -(b''_{17})^{(2,2,)}(G_{19},t) & -(b''_{21})^{(3,3,)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,)}(G_{35},t) & \\ -(b''_{37})^{(7,7,)}(G_{39},t) & -(b''_{41})^{(8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) & \end{bmatrix} T_{14} \quad 59$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)}\boxed{-(b''_{15})^{(1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15} \quad (60)$$

Where $\boxed{-(b''_{13})^{(1)}(G,t)}$, $\boxed{-(b''_{14})^{(1)}(G,t)}$, $\boxed{-(b''_{15})^{(1)}(G,t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8)}(G_{43},t)}$ $\boxed{-(b''_{41})^{(8,8)}(G_{43},t)}$ $\boxed{-(b''_{42})^{(8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \begin{bmatrix} (a'_{16})^{(2)}\boxed{+(a''_{16})^{(2)}(T_{17},t)} & \boxed{+(a''_{13})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{16} \quad (61)$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \begin{bmatrix} (a'_{17})^{(2)}\boxed{+(a''_{17})^{(2)}(T_{17},t)} & \boxed{+(a''_{14})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{17} \quad (62)$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \begin{bmatrix} (a'_{18})^{(2)}\boxed{+(a''_{18})^{(2)}(T_{17},t)} & \boxed{+(a''_{15})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{18} \quad (63)$$

Where $\boxed{+(a''_{16})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,)}(T_{14},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{36})^{(7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7)}(T_{37},t)$ are seventh augmentation coefficient for category 1, 2 and 3

$+(a''_{40})^{(8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8)}(T_{41},t)$ are eight augmentation coefficient for category 1, 2 and 3

$+(a''_{44})^{(9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9)}(T_{45},t)$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \begin{bmatrix} (b'_{16})^{(2)} & -(b''_{16})^{(2)}(G_{19},t) & -(b''_{13})^{(1,1,)}(G,t) & -(b''_{20})^{(3,3,3,)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6)}(G_{35},t) \\ -(b''_{36})^{(7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9)}(G_{47},t) \end{bmatrix} T_{16} \quad 64$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)} & -(b''_{17})^{(2)}(G_{19},t) & -(b''_{14})^{(1,1,)}(G,t) & -(b''_{21})^{(3,3,3,)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6)}(G_{35},t) \\ -(b''_{37})^{(7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9)}(G_{47},t) \end{bmatrix} T_{17} \quad 65$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)} & -(b''_{18})^{(2)}(G_{19},t) & -(b''_{15})^{(1,1,)}(G,t) & -(b''_{22})^{(3,3,3,)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6)}(G_{35},t) \\ -(b''_{38})^{(7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9)}(G_{47},t) \end{bmatrix} T_{18} \quad 66$$

where $-(b''_{16})^{(2)}(G_{19},t)$, $-(b''_{17})^{(2)}(G_{19},t)$, $-(b''_{18})^{(2)}(G_{19},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{13})^{(1,1,)}(G,t)$, $-(b''_{14})^{(1,1,)}(G,t)$, $-(b''_{15})^{(1,1,)}(G,t)$ are second detrition coefficients for category 1,2 and 3

$-(b''_{20})^{(3,3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,)}(G_{23},t)$ are third detrition coefficients for category 1,2 and 3

$-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1,2 and 3

$-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1,2 and 3

$-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients for category 1,2 and 3

$-(b''_{36})^{(7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2 and 3

$-(b''_{40})^{(8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1,2 and 3

$-(b''_{44})^{(9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \begin{bmatrix} (a'_{20})^{(3)} & +(a''_{20})^{(3)}(T_{21},t) & +(a''_{16})^{(2,2,2)}(T_{17},t) & +(a''_{13})^{(1,1,1,)}(T_{14},t) \\ +(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t) & \\ +(a''_{36})^{(7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9)}(T_{45},t) & \end{bmatrix} G_{20} \quad (67)$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \begin{bmatrix} (a'_{21})^{(3)} & +(a''_{21})^{(3)}(T_{21},t) & +(a''_{17})^{(2,2,2)}(T_{17},t) & +(a''_{14})^{(1,1,1,)}(T_{14},t) \\ +(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t) & \\ +(a''_{37})^{(7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9)}(T_{45},t) & \end{bmatrix} G_{21} \quad (68)$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \begin{bmatrix} (a'_{22})^{(3)} & +(a''_{22})^{(3)}(T_{21},t) & +(a''_{18})^{(2,2,2)}(T_{17},t) & +(a''_{15})^{(1,1,1,)}(T_{14},t) \\ +(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t) & \\ +(a''_{38})^{(7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9)}(T_{45},t) & \end{bmatrix} G_{22} \quad (69)$$

$+(a''_{20})^{(3)}(T_{21},t)$, $+(a''_{21})^{(3)}(T_{21},t)$, $+(a''_{22})^{(3)}(T_{21},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2)}(T_{17},t)$ are second augmentation coefficients for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,)}(T_{14},t)$ are third augmentation coefficients for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficients for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficients for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficients for category 1, 2 and 3

$+(a''_{36})^{(7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1, 2 and 3

$+(a''_{40})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8)}(T_{41},t)$ are eight augmentation coefficients for category 1, 2 and 3

$+(a''_{44})^{(9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b'_{20})^{(3)} & -(b''_{20})^{(3)}(G_{23},t) & -(b''_{16})^{(2,2,2)}(G_{19},t) & -(b''_{13})^{(1,1,1,)}(G,t) \\ -(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t) & \\ -(b''_{36})^{(7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9)}(G_{47},t) & \end{bmatrix} T_{20} \quad (70)$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b'_{21})^{(3)} & -(b''_{21})^{(3)}(G_{23},t) & -(b''_{17})^{(2,2,2)}(G_{19},t) & -(b''_{14})^{(1,1,1,)}(G,t) \\ -(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t) & \\ -(b''_{37})^{(7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9)}(G_{47},t) & \end{bmatrix} T_{21} \quad (71)$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)}\boxed{-(b''_{22})^{(3)}(G_{23},t)} & \boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{22} \quad 72$$

$\boxed{-(b''_{20})^{(3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3)}(G_{23},t)}$ *are first detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}$ *are second detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{13})^{(1,1,1,)}(G,t)}, \boxed{-(b''_{14})^{(1,1,1,)}(G,t)}, \boxed{-(b''_{15})^{(1,1,1,)}(G,t)}$ *are third detrition coefficients for category 1,2 and 3*

$\boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)}, \boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)}, \boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)}$ *are fourth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)}, \boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)}, \boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)}$ *are fifth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)}, \boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)}, \boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)}$ *are sixth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{36})^{(7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{37})^{(7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)}, \boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)}, \boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \begin{bmatrix} (a'_{24})^{(4)}\boxed{+(a''_{24})^{(4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{24} \quad 73$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \begin{bmatrix} (a'_{25})^{(4)}\boxed{+(a''_{25})^{(4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{25} \quad 74$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \begin{bmatrix} (a'_{26})^{(4)}\boxed{+(a''_{26})^{(4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{26} \quad 75$$

$\boxed{(a''_{24})^{(4)}(T_{25},t)}, \boxed{(a''_{25})^{(4)}(T_{25},t)}, \boxed{(a''_{26})^{(4)}(T_{25},t)}$ *are first augmentation coefficients category* 1, 2 3

$\boxed{+(a''_{28})^{(5,5,)}(T_{29},t)}, \boxed{+(a''_{29})^{(5,5,)}(T_{29},t)}, \boxed{+(a''_{30})^{(5,5,)}(T_{29},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}, \boxed{+(a''_{33})^{(6,6,)}(T_{33},t)}, \boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)}$

$are\ fourth\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$
$+(a_{16}'')^{(2,2,2,2)}(T_{17},t)$, $+(a_{17}'')^{(2,2,2,2)}(T_{17},t)$, $+(a_{18}'')^{(2,2,2,2)}(T_{17},t)$
$are\ fifth\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$
$+(a_{20}'')^{(3,3,3,3)}(T_{21},t)$, $+(a_{21}'')^{(3,3,3,3)}(T_{21},t)$, $+(a_{22}'')^{(3,3,3,3)}(T_{21},t)$
$are\ sixth\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$
$+(a_{36}'')^{(7,7,7,7,7)}(T_{37},t)$, $+(a_{37}'')^{(7,7,7,7,7)}(T_{37},t)$, $+(a_{38}'')^{(7,7,7,7,7)}(T_{37},t)$
$are\ seventh\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$
$+(a_{40}'')^{(8,8,8,8,8)}(T_{41},t)$, $+(a_{41}'')^{(8,8,8,8,8)}(T_{41},t)$, $+(a_{42}'')^{(8,8,8,8,8)}(T_{41},t)$
$are\ eighth\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$
$+(a_{46}'')^{(9,9,9,9)}(T_{45},t)$, $+(a_{45}'')^{(9,9,9,9)}(T_{45},t)$, $+(a_{44}'')^{(9,9,9,9)}(T_{45},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \begin{bmatrix} (b_{24}')^{(4)}\left[-(b_{24}'')^{(4)}(G_{27},t)\right] & \left[-(b_{28}'')^{(5,5,)}(G_{31},t)\right] & \left[-(b_{32}'')^{(6,6,)}(G_{35},t)\right] \\ \left[-(b_{13}'')^{(1,1,1,1)}(G,t)\right] & \left[-(b_{16}'')^{(2,2,2,2)}(G_{19},t)\right] & \left[-(b_{20}'')^{(3,3,3,3)}(G_{23},t)\right] \\ \left[-(b_{36}'')^{(7,7,7,7,7)}(G_{39},t)\right] & \left[-(b_{40}'')^{(8,8,8,8,8)}(G_{43},t)\right] & \left[-(b_{44}'')^{(9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{24} \quad (76)$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \begin{bmatrix} (b_{25}')^{(4)}\left[-(b_{25}'')^{(4)}(G_{27},t)\right] & \left[-(b_{29}'')^{(5,5,)}(G_{31},t)\right] & \left[-(b_{33}'')^{(6,6,)}(G_{35},t)\right] \\ \left[-(b_{14}'')^{(1,1,1,1)}(G,t)\right] & \left[-(b_{17}'')^{(2,2,2,2)}(G_{19},t)\right] & \left[-(b_{21}'')^{(3,3,3,3)}(G_{23},t)\right] \\ \left[-(b_{37}'')^{(7,7,7,7,7)}(G_{39},t)\right] & \left[-(b_{41}'')^{(8,8,8,8,8)}(G_{43},t)\right] & \left[-(b_{45}'')^{(9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{25} \quad (77)$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b_{26}')^{(4)}\left[-(b_{26}'')^{(4)}(G_{27},t)\right] & \left[-(b_{30}'')^{(5,5,)}(G_{31},t)\right] & \left[-(b_{34}'')^{(6,6,)}(G_{35},t)\right] \\ \left[-(b_{15}'')^{(1,1,1,1)}(G,t)\right] & \left[-(b_{18}'')^{(2,2,2,2)}(G_{19},t)\right] & \left[-(b_{22}'')^{(3,3,3,3)}(G_{23},t)\right] \\ \left[-(b_{38}'')^{(7,7,7,7,7)}(G_{39},t)\right] & \left[-(b_{42}'')^{(8,8,8,8,8)}(G_{43},t)\right] & \left[-(b_{46}'')^{(9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{26} \quad (78)$$

$Where\ -(b_{24}'')^{(4)}(G_{27},t)$, $-(b_{25}'')^{(4)}(G_{27},t)$, $-(b_{26}'')^{(4)}(G_{27},t)$ $are\ first\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b_{28}'')^{(5,5,)}(G_{31},t)$, $-(b_{29}'')^{(5,5,)}(G_{31},t)$, $-(b_{30}'')^{(5,5,)}(G_{31},t)$ $are\ second\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b_{32}'')^{(6,6,)}(G_{35},t)$, $-(b_{33}'')^{(6,6,)}(G_{35},t)$, $-(b_{34}'')^{(6,6,)}(G_{35},t)$ $are\ third\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b_{13}'')^{(1,1,1,1)}(G,t)$, $-(b_{14}'')^{(1,1,1,1)}(G,t)$, $-(b_{15}'')^{(1,1,1,1)}(G,t)$
$are\ fourth\ \ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b_{16}'')^{(2,2,2,2)}(G_{19},t)$, $-(b_{17}'')^{(2,2,2,2)}(G_{19},t)$, $-(b_{18}'')^{(2,2,2,2)}(G_{19},t)$
$are\ fifth\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b_{20}'')^{(3,3,3,3)}(G_{23},t)$, $-(b_{21}'')^{(3,3,3,3)}(G_{23},t)$, $-(b_{22}'')^{(3,3,3,3)}(G_{23},t)$
$are\ sixth\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b_{36}'')^{(7,7,7,7,7)}(G_{39},t)$, $-(b_{37}'')^{(7,7,7,7,7)}(G_{39},t)$, $-(b_{38}'')^{(7,7,7,7,7)}(G_{39},t)$
$are\ seventh\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b_{40}'')^{(8,8,8,8,8)}(G_{43},t)$, $-(b_{41}'')^{(8,8,8,8,8)}(G_{43},t)$, $-(b_{42}'')^{(8,8,8,8,8)}(G_{43},t)$
$are\ eighth\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b_{46}'')^{(9,9,9,9)}(G_{47},t)$, $-(b_{45}'')^{(9,9,9,9)}(G_{47},t)$, $-(b_{44}'')^{(9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \begin{bmatrix} (a'_{28})^{(5)}\boxed{+(a''_{28})^{(5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{28} \quad 79$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \begin{bmatrix} (a'_{29})^{(5)}\boxed{+(a''_{29})^{(5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{29} \quad 80$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \begin{bmatrix} (a'_{30})^{(5)}\boxed{+(a''_{30})^{(5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{30} \quad 81$$

*Where* $\boxed{+(a''_{28})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5)}(T_{29},t)}$ *are first augmentation coefficients for category* 1, 2 *and* 3

*And* $\boxed{+(a''_{24})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,)}(T_{25},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)}$ are fourth augmentation coefficients for category 1,2, and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)}$ are fifth augmentation coefficients for category 1,2,and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)}$ are sixth augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)}$ are seventh augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)}$ are eighth augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \begin{bmatrix} (b'_{28})^{(5)}\boxed{-(b''_{28})^{(5)}(G_{31},t)} & \boxed{-(b''_{24})^{(4,4,)}(G_{27},t)} & \boxed{-(b''_{32})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{28} \quad 82$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b'_{29})^{(5)}\boxed{-(b''_{29})^{(5)}(G_{31},t)} & \boxed{-(b''_{25})^{(4,4,)}(G_{27},t)} & \boxed{-(b''_{33})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{29} \quad 83$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b'_{30})^{(5)}\boxed{-(b''_{30})^{(5)}(G_{31},t)} & \boxed{-(b''_{26})^{(4,4,)}(G_{27},t)} & \boxed{-(b''_{34})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{30} \quad 84$$

$where\ \boxed{-(b''_{28})^{(5)}(G_{31},t)}\ ,\boxed{-(b''_{29})^{(5)}(G_{31},t)}\ ,\boxed{-(b''_{30})^{(5)}(G_{31},t)}\ \ are\ first\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b''_{24})^{(4,4,)}(G_{27},t)}\ ,\boxed{-(b''_{25})^{(4,4,)}(G_{27},t)}\ ,\boxed{-(b''_{26})^{(4,4,)}(G_{27},t)}\ are\ second\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b''_{32})^{(6,6,6)}(G_{35},t)}\ ,\boxed{-(b''_{33})^{(6,6,6)}(G_{35},t)}\ ,\boxed{-(b''_{34})^{(6,6,6)}(G_{35},t)}\ are\ third\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b''_{13})^{(1,1,1,1,1)}(G,t)}\ ,\boxed{-(b''_{14})^{(1,1,1,1,1)}(G,t)}\ ,\boxed{-(b''_{15})^{(1,1,1,1,1,)}(G,t)}$ are fourth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)}\ ,\boxed{-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)}\ ,\boxed{-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)}\ ,\boxed{-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)}\ ,\boxed{-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)}\ ,\boxed{-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)}\ ,\boxed{-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)}\ ,\boxed{-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)}\ ,\boxed{-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)}\ ,\boxed{-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)}\ ,\boxed{-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[\begin{array}{c} (a'_{32})^{(6)}\boxed{+(a''_{32})^{(6)}(T_{33},t)}\ \boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}\ \boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}\ \boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}\ \boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{32} \quad 85$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[\begin{array}{c} (a'_{33})^{(6)}\boxed{+(a''_{33})^{(6)}(T_{33},t)}\ \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}\ \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}\ \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}\ \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{33} \quad 86$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[\begin{array}{c} (a'_{34})^{(6)}\boxed{+(a''_{34})^{(6)}(T_{33},t)}\ \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}\ \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}\ \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}\ \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{34} \quad 87$$

$\boxed{+(a''_{32})^{(6)}(T_{33},t)}\ ,\boxed{+(a''_{33})^{(6)}(T_{33},t)}\ ,\boxed{+(a''_{34})^{(6)}(T_{33},t)}\ \ are\ \ first\ \ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}\ ,\boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}\ ,\boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}\ are\ second\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}\ ,\boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}\ ,\boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}\ \ are\ third\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}\ ,\boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}\ ,\boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}$ - are fourth augmentation coefficients

$\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}\ ,\boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}\ ,\boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}$ - fifth augmentation

coefficients
$+(a_{20}'')^{(3,3,3,3,3,3)}(T_{21},t)$, $+(a_{21}'')^{(3,3,3,3,3,3)}(T_{21},t)$, $+(a_{22}'')^{(3,3,3,3,3,3)}(T_{21},t)$ sixth augmentation
coefficients
$+(a_{36}'')^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a_{37}'')^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a_{38}'')^{(7,7,7,7,7,7,7)}(T_{37},t)$
seventh augmentation coefficients
$+(a_{40}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a_{41}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a_{42}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t)$
Eighth augmentation coefficients
$+(a_{44}'')^{(9,9,9,9,9,9,9)}(T_{45},t)$, $+(a_{45}'')^{(9,9,9,9,9,9,9)}(T_{45},t)$, $+(a_{46}'')^{(9,9,9,9,9,9,9)}(T_{45},t)$ ninth augmentation
coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \begin{bmatrix} (b_{32}')^{(6)} & -(b_{32}'')^{(6)}(G_{35},t) & -(b_{28}'')^{(5,5,5)}(G_{31},t) & -(b_{24}'')^{(4,4,4,)}(G_{27},t) \\ -(b_{13}'')^{(1,1,1,1,1,1)}(G,t) & -(b_{16}'')^{(2,2,2,2,2,2)}(G_{19},t) & -(b_{20}'')^{(3,3,3,3,3,3)}(G_{23},t) & \\ -(b_{36}'')^{(7,7,7,7,7,7,7)}(G_{39},t) & -(b_{40}'')^{(8,8,8,8,8,8,8)}(G_{43},t) & -(b_{44}'')^{(9,9,9,9,9,9)}(G_{47},t) & \end{bmatrix} T_{32} \quad 88$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \begin{bmatrix} (b_{33}')^{(6)} & -(b_{33}'')^{(6)}(G_{35},t) & -(b_{29}'')^{(5,5,5)}(G_{31},t) & -(b_{25}'')^{(4,4,4,)}(G_{27},t) \\ -(b_{14}'')^{(1,1,1,1,1,1)}(G,t) & -(b_{17}'')^{(2,2,2,2,2,2)}(G_{19},t) & -(b_{21}'')^{(3,3,3,3,3,3)}(G_{23},t) & \\ -(b_{37}'')^{(7,7,7,7,7,7,7)}(G_{39},t) & -(b_{41}'')^{(8,8,8,8,8,8,8)}(G_{43},t) & -(b_{45}'')^{(9,9,9,9,9,9)}(G_{47},t) & \end{bmatrix} T_{33} \quad 89$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b_{34}')^{(6)} & -(b_{34}'')^{(6)}(G_{35},t) & -(b_{30}'')^{(5,5,5)}(G_{31},t) & -(b_{26}'')^{(4,4,4,)}(G_{27},t) \\ -(b_{15}'')^{(1,1,1,1,1,1)}(G,t) & -(b_{18}'')^{(2,2,2,2,2,2)}(G_{19},t) & -(b_{22}'')^{(3,3,3,3,3,3)}(G_{23},t) & \\ -(b_{38}'')^{(7,7,7,7,7,7,7)}(G_{39},t) & -(b_{42}'')^{(8,8,8,8,8,8,8)}(G_{43},t) & -(b_{46}'')^{(9,9,9,9,9,9)}(G_{47},t) & \end{bmatrix} T_{34} \quad 90$$

$-(b_{32}'')^{(6)}(G_{35},t)$, $-(b_{33}'')^{(6)}(G_{35},t)$, $-(b_{34}'')^{(6)}(G_{35},t)$ *are first detrition coefficients for category* 1, 2 *and* 3
$-(b_{28}'')^{(5,5,5)}(G_{31},t)$, $-(b_{29}'')^{(5,5,5)}(G_{31},t)$, $-(b_{30}'')^{(5,5,5)}(G_{31},t)$ *are second detrition coefficients for category* 1, 2 *and* 3
$-(b_{24}'')^{(4,4,4,)}(G_{27},t)$, $-(b_{25}'')^{(4,4,4,)}(G_{27},t)$, $-(b_{26}'')^{(4,4,4,)}(G_{27},t)$ *are third detrition coefficients for category* 1,2 *and* 3
$-(b_{13}'')^{(1,1,1,1,1,1)}(G,t)$, $-(b_{14}'')^{(1,1,1,1,1,1)}(G,t)$, $-(b_{15}'')^{(1,1,1,1,1,1)}(G,t)$ are fourth detrition coefficients for category 1, 2, and 3
$-(b_{16}'')^{(2,2,2,2,2,2)}(G_{19},t)$, $-(b_{17}'')^{(2,2,2,2,2,2)}(G_{19},t)$, $-(b_{18}'')^{(2,2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1, 2, and 3
$-(b_{20}'')^{(3,3,3,3,3,3)}(G_{23},t)$, $-(b_{21}'')^{(3,3,3,3,3,3)}(G_{23},t)$, $-(b_{22}'')^{(3,3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1, 2, and 3
$-(b_{36}'')^{(7,7,7,7,7,7,7)}(G_{39},t)$, $-(b_{37}'')^{(7,7,7,7,7,7,7)}(G_{39},t)$, $-(b_{38}'')^{(7,7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2, and 3
$-(b_{40}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)$, $-(b_{41}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)$, $-(b_{42}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)$
are eighth detrition coefficients for category 1, 2, and 3
$-(b_{46}'')^{(9,9,9,9,9,9)}(G_{47},t)$, $-(b_{45}'')^{(9,9,9,9,9,9)}(G_{47},t)$, $-(b_{44}'')^{(9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \qquad 91$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a'_{36})^{(7)}\boxed{+(a''_{36})^{(7)}(T_{37},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \qquad 92$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a'_{37})^{(7)}\boxed{+(a''_{37})^{(7)}(T_{37},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{38}}{dt} \qquad 93$$

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a'_{38})^{(7)}\boxed{+(a''_{38})^{(7)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{(a''_{36})^{(7)}(T_{37},t)}$, $\boxed{(a''_{37})^{(7)}(T_{37},t)}$, $\boxed{(a''_{38})^{(7)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{36}}{dt} = \qquad 94$$

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b'_{36})^{(7)}\boxed{-(b''_{36})^{(7)}(G_{39},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{37}}{dt} =$$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b'_{37})^{(7)}\boxed{-(b''_{37})^{(7)}(G_{39},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{38}}{dt} =$$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b'_{38})^{(7)}\boxed{-(b''_{38})^{(7)}(G_{39},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{40}}{dt}$$



$$= (a_{40})^{(8)}G_{41} - \begin{bmatrix} (a'_{40})^{(8)}\boxed{+(a''_{40})^{(8)}(T_{41},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{41}}{dt}$$

$$= (a_{41})^{(8)}G_{40} - \begin{bmatrix} (a'_{41})^{(8)}\boxed{+(a''_{41})^{(8)}(T_{41},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \begin{bmatrix} (a'_{42})^{(8)}\boxed{+(a''_{42})^{(8)}(T_{41},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a''_{40})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)}\boxed{-(b''_{40})^{(8)}(G_{43},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)}\boxed{-(b''_{41})^{(8)}(G_{43},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)}\boxed{-(b''_{42})^{(8)}(G_{43},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{44}}{dt} \qquad 96$$

$$= (a_{44})^{(9)}G_{45} - \left[\begin{array}{ccc} (a'_{44})^{(9)}\boxed{+(a''_{44})^{(9)}(T_{45},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{array}\right] G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[\begin{array}{ccc} (a'_{45})^{(9)}\boxed{+(a''_{45})^{(9)}(T_{45},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{array}\right] G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[\begin{array}{ccc} (a'_{46})^{(9)}\boxed{+(a''_{46})^{(9)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{array}\right] G_{15}$$

Where $\boxed{+(a''_{44})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$+(a_{16}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)$ , $+(a_{17}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)$ , $+(a_{18}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a_{20}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)$ , $+(a_{21}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)$ , $+(a_{22}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a_{24}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)$ , $+(a_{25}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)$ , $+(a_{26}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a_{28}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)$, $+(a_{29}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)$ , $+(a_{30}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a_{32}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a_{33}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$ , $+(a_{34}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a_{13}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$ , $+(a_{14}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$ , $+(a_{15}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$ are Seventh augmentation coefficient for category 1, 2 and 3

$+(a_{38}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ $+(a_{37}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ $+(a_{36}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ are eighth augmentation coefficient for 1,2,3

$+(a_{40}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)$ ,$+(a_{42}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a_{41}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} =$$

$$(b_{44})^{(9)}T_{45} -$$

$$\begin{bmatrix} (b_{44}')^{(9)}\boxed{-(b_{44}'')^{(9)}(G_{47},t)} & \boxed{-(b_{16}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{20}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{24}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{28}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{32}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{13}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{36}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b_{45}')^{(9)}\boxed{-(b_{45}'')^{(9)}(G_{47},t)} & \boxed{-(b_{17}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{21}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{25}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{29}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{33}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{14}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{37}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b_{46}')^{(9)}\boxed{-(b_{46}'')^{(9)}(G_{47},t)} & \boxed{-(b_{18}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{22}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{26}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{30}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{34}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{15}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{38}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{15}$$

Where $-(b_{44}'')^{(9)}(G_{47},t)$ , $-(b_{45}'')^{(9)}(G_{47},t)$ , $-(b_{46}'')^{(9)}(G_{47},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b_{16}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)$ , $-(b_{17}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)$ , $-(b_{18}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b_{20}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)$ , $-(b_{21}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)$ , $-(b_{22}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{24}'')^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{25}'')^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{26}'')^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{28}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{32}'')^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{34}'')^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{37}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{36}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a_i')^{(1)}, (a_i'')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (b_i'')^{(1)} > 0, \quad i,j = 13,14,15 \qquad 97$$

The functions $(a_i'')^{(1)}, (b_i'')^{(1)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a_i'')^{(1)}(T_{14},t) \leq (p_i)^{(1)} \leq (\hat{A}_{13})^{(1)}$$

$$(b_i'')^{(1)}(G,t) \leq (r_i)^{(1)} \leq (b_i')^{(1)} \leq (\hat{B}_{13})^{(1)}$$

$$lim_{T_2\to\infty}(a_i'')^{(1)}(T_{14},t) = (p_i)^{(1)} \qquad 98$$

$$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = (r_i)^{(1)}$$

**Definition of** $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$ :

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$$|(a_i'')^{(1)}(T_{14}',t) - (a_i'')^{(1)}(T_{14},t)| \leq (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G',t) - (b_i'')^{(1)}(G,t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}',t)$ and $(a_i'')^{(1)}(T_{14},t)$ . $(T_{14}',t)$ and $(T_{14},t)$ are points belonging to the interval $[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}]$. It is to be noted that $(a_i'')^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14},t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$ : 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$

**Definition of** $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$ : 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}$, $(r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)} (\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)} (\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i, j = 16,17,18$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17}, t) \leq (p_i)^{(2)} \leq (\hat{A}_{16})^{(2)} \quad 102$$

$$(b_i'')^{(2)}(G_{19}, t) \leq (r_i)^{(2)} \leq (b_i')^{(2)} \leq (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2 \to \infty} (a_i'')^{(2)}(T_{17}, t) = (p_i)^{(2)} \quad 104$$

$$\lim_{G \to \infty} (b_i'')^{(2)}((G_{19}), t) = (r_i)^{(2)} \quad 105$$

**Definition of** $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$ : 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}', t) - (a_i'')^{(2)}(T_{17}, t)| \leq (\hat{k}_{16})^{(2)} |T_{17} - T_{17}'| e^{-(\hat{M}_{16})^{(2)} t} \quad 107$$

$$|(b_i'')^{(2)}((G_{19})', t) - (b_i'')^{(2)}((G_{19}), t)| < (\hat{k}_{16})^{(2)} ||(G_{19}) - (G_{19})'|| e^{-(\hat{M}_{16})^{(2)} t} \quad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}', t)$ and$(a_i'')^{(2)}(T_{17}, t)$ . $(T_{17}', t)$ and $(T_{17}, t)$ are points belonging to the interval $[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}]$ . It is to be noted that $(a_i'')^{(2)}(T_{17}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17}, t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$ :

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)},$ are positive constants 109

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$$

**Definition of** $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$ :

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and\ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1 \quad 110$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(b_i)^{(2)} + (b_i')^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1 \quad 111$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i, j = 20,21,22 \quad 112$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(3)}$, $(r_i)^{(3)}$:

$$(a_i'')^{(3)}(T_{21}, t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23}, t) \leq (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(3)}(T_{21}, t) = (p_i)^{(3)} \quad 113$$

$$\lim_{G \to \infty}(b_i'')^{(3)}(G_{23}, t) = (r_i)^{(3)}$$

**Definition of** $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$ :

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: 114

$$|(a_i'')^{(3)}(T_{21}', t) - (a_i'')^{(3)}(T_{21}, t)| \leq (\hat{k}_{20})^{(3)} |T_{21} - T_{21}'| e^{-(\hat{M}_{20})^{(3)} t}$$

$$|(b_i'')^{(3)}(G_{23}', t) - (b_i'')^{(3)}(G_{23}, t)| < (\hat{k}_{20})^{(3)} ||G_{23} - G_{23}'|| e^{-(\hat{M}_{20})^{(3)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}', t)$ and $(a_i'')^{(3)}(T_{21}, t)$ . $(T_{21}', t)$ And $(T_{21}, t)$ are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21}, t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$ : 115

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}$ , $\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with 116
$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$ and the constants
$(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}, i = 20,21,22,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(a_i)^{(3)} + (a_i')^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)} (\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(b_i)^{(3)} + (b_i')^{(3)} + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)} (\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i, j = 24,25,26$$ 117

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(4)}$, $(r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25}, t) \leq (p_i)^{(4)} \leq (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}((G_{27}), t) \leq (r_i)^{(4)} \leq (b_i')^{(4)} \leq (\hat{B}_{24})^{(4)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(4)}(T_{25}, t) = (p_i)^{(4)}$$ 118

$$\lim_{G \to \infty}(b_i'')^{(4)}((G_{27}), t) = (r_i)^{(4)}$$

**Definition of** $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$ :

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24,25,26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}', t) - (a_i'')^{(4)}(T_{25}, t)| \leq (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})', t) - (b_i'')^{(4)}((G_{27}), t)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}', t)$ and$(a_i'')^{(4)}(T_{25}, t)$ . $(T_{25}', t)$ and $(T_{25}, t)$ are points belonging to the interval $[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}]$. It is to be noted that $(a_i'')^{(4)}(T_{25}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25}, t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$ : 120

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)},$ are positive constants

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}, \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$$

**Definition of** $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$ : 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and\ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}, i = 24,25,26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(a_i)^{(4)} + (a_i')^{(4)} + (\hat{A}_{24})^{(4)} + (\hat{P}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(b_i)^{(4)} + (b_i')^{(4)} + (\hat{B}_{24})^{(4)} + (\hat{Q}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i, j = 28,29,30$$ 122

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(5)}$, $(r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29}, t) \le (p_i)^{(5)} \le (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}((G_{31}), t) \le (r_i)^{(5)} \le (b_i')^{(5)} \le (\hat{B}_{28})^{(5)}$$

$$lim_{T_2 \to \infty} (a_i'')^{(5)}(T_{29}, t) = (p_i)^{(5)}$$ 123

$$\lim_{G \to \infty} (b_i'')^{(5)}(G_{31}, t) = (r_i)^{(5)}$$

**Definition of** $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$ :

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}', t) - (a_i'')^{(5)}(T_{29}, t)| \le (\hat{k}_{28})^{(5)} |T_{29} - T_{29}'| e^{-(\hat{M}_{28})^{(5)} t}$$

$$|(b_i'')^{(5)}((G_{31})', t) - (b_i'')^{(5)}((G_{31}), t)| < (\hat{k}_{28})^{(5)} ||(G_{31}) - (G_{31})'|| e^{-(\hat{M}_{28})^{(5)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}', t)$ and $(a_i'')^{(5)}(T_{29}, t)$ . $(T_{29}', t)$ and $(T_{29}, t)$ are points belonging to the interval $\left[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\right]$. It is to be noted that $(a_i'')^{(5)}(T_{29}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$ : 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)},$ are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}, \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

**Definition of** $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$ : 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and\ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(a_i)^{(5)} + (a_i')^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(b_i)^{(5)} + (b_i')^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i, j = 32,33,34$$ 127

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(6)}$, $(r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33}, t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}), t) \leq (r_i)^{(6)} \leq (b_i')^{(6)} \leq (\hat{B}_{32})^{(6)}$$

$$lim_{T_2 \to \infty} (a_i'')^{(6)} (T_{33}, t) = (p_i)^{(6)}$$ 128

$$\lim_{G \to \infty} (b_i'')^{(6)} ((G_{35}), t) = (r_i)^{(6)}$$

**Definition of** $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$ :

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}', t) - (a_i'')^{(6)}(T_{33}, t)| \leq (\hat{k}_{32})^{(6)} |T_{33} - T_{33}'| e^{-(\hat{M}_{32})^{(6)} t}$$

$$|(b_i'')^{(6)}((G_{35})', t) - (b_i'')^{(6)}((G_{35}), t)| < (\hat{k}_{32})^{(6)} ||(G_{35}) - (G_{35})'|| e^{-(\hat{M}_{32})^{(6)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}', t)$ and $(a_i'')^{(6)}(T_{33}, t)$ . $(T_{33}', t)$ and $(T_{33}, t)$ are points belonging to the interval $[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}]$. It is to be noted that $(a_i'')^{(6)}(T_{33}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$ : 129

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)},$ are positive constants

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$$

**Definition of** $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$ : 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and\ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32,33,34,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(a_i)^{(6)} + (a_i')^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)} (\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(b_i)^{(6)} + (b_i')^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)} (\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

(QQQQQ) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i, j = 36,37,38$ 131

(RRRRR) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(7)}$, $(r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37}, t) \leq (p_i)^{(7)} \leq (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39}, t) \leq (r_i)^{(7)} \leq (b_i')^{(7)} \leq (\hat{B}_{36})^{(7)}$$

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*(SSSSS)* $\lim_{T_2 \to \infty} (a_i'')^{(7)}(T_{37}, t) = (p_i)^{(7)}$

*(TTTTT)*

$\lim_{G \to \infty} (b_i'')^{(7)}((G_{39}), t) = (r_i)^{(7)}$

**Definition of** $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$ :

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$$|(a_i'')^{(7)}(T_{37}', t) - (a_i'')^{(7)}(T_{37}, t)| \leq (\hat{k}_{36})^{(7)} |T_{37} - T_{37}'| e^{-(\hat{M}_{36})^{(7)} t}$$

$$|(b_i'')^{(7)}((G_{39})', t) - (b_i'')^{(7)}((G_{39}), t)| < (\hat{k}_{36})^{(7)} ||(G_{39}) - (G_{39})'|| e^{-(\hat{M}_{36})^{(7)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}', t)$ and $(a_i'')^{(7)}(T_{37}, t)$ . $(T_{37}', t)$ and $(T_{37}, t)$ are points belonging to the interval $[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}]$ . It is to be noted that $(a_i'')^{(7)}(T_{37}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37}, t)$ , the **first augmentation coefficient** attributable to the

system, would be absolutely continuous.

**Definition of** $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$ : 134

(UUUUU) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)},$ are positive constants

$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$

**Definition of** $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$ : 135

(VVVVV) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(b_i)^{(7)} + (b_i')^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40,41,42$ 136

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

**Definition of** $(p_i)^{(8)}, (r_i)^{(8)}$: 137

$(a_i'')^{(8)}(T_{41}, t) \le (p_i)^{(8)} \le (\hat{A}_{40})^{(8)}$ 138

$(b_i'')^{(8)}((G_{43}), t) \le (r_i)^{(8)} \le (b_i')^{(8)} \le (\hat{B}_{40})^{(8)}$ 139

$\lim_{T_2 \to \infty} (a_i'')^{(8)}(T_{41}, t) = (p_i)^{(8)}$ 140

$\lim_{G \to \infty} (b_i'')^{(8)}((G_{43}), t) = (r_i)^{(8)}$ 141

**Definition of** $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$ :

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \le (\hat{k}_{40})^{(8)} |T_{41} - T_{41}'| e^{-(\hat{M}_{40})^{(8)} t}$ 142

$|(b_i'')^{(8)}((G_{43})', t) - (b_i'')^{(8)}((G_{43}), t)| < (\hat{k}_{40})^{(8)} ||(G_{43}) - (G_{43})'|| e^{-(\hat{M}_{40})^{(8)} t}$ 143

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}', t)$ and $(a_i'')^{(8)}(T_{41}, t)$ . $(T_{41}', t)$ and $(T_{41}, t)$ are points belonging to the interval $\left[(\hat{k}_{40})^{(8)}, (\widehat{M}_{40})^{(8)}\right]$. It is to be noted that $(a_i'')^{(8)}(T_{41}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{40})^{(8)} = 1$ then the function $(a_i'')^{(8)}(T_{41}, t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\widehat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$ :

$(\widehat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)},$ are positive constants

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}, \frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}} < 1 \qquad 144$$

**Definition of** $(\hat{P}_{40})^{(8)}, (\hat{Q}_{40})^{(8)}$ :

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\widehat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}, (\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)}, (a_i')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}, i = 40,41,42,$
Satisfy the inequalities

$$\frac{1}{(\widehat{M}_{40})^{(8)}}[(a_i)^{(8)} + (a_i')^{(8)} + (\hat{A}_{40})^{(8)} + (\hat{P}_{40})^{(8)} (\hat{k}_{40})^{(8)}] < 1 \qquad 145$$

$$\frac{1}{(\widehat{M}_{40})^{(8)}}[(b_i)^{(8)} + (b_i')^{(8)} + (\hat{B}_{40})^{(8)} + (\hat{Q}_{40})^{(8)} (\hat{k}_{40})^{(8)}] < 1 \qquad 146$$

Where we suppose

$$(a_i)^{(9)}, (a_i')^{(9)}, (a_i'')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (b_i'')^{(9)} > 0, \quad i, j = 44,45,46 \qquad 146\text{A}$$

The functions $(a_i'')^{(9)}, (b_i'')^{(9)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(9)}$, $(r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45}, t) \leq (p_i)^{(9)} \leq (\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47}, t) \leq (r_i)^{(9)} \leq (b_i')^{(9)} \leq (\hat{B}_{44})^{(9)}$$

$$\lim_{T_2 \to \infty} (a_i'')^{(9)}(T_{45}, t) = (p_i)^{(9)}$$

$$\lim_{G \to \infty} (b_i'')^{(9)}(G_{47}, t) = (r_i)^{(9)}$$

**Definition of** $(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}$ :

Where $\boxed{(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}}$ are positive constants and $\boxed{i = 44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}', t) - (a_i'')^{(9)}(T_{45}, t)| \leq (\hat{k}_{44})^{(9)} |T_{45} - T_{45}'| e^{-(\widehat{M}_{44})^{(9)} t}$$

$$|(b_i'')^{(9)}((G_{47})', t) - (b_i'')^{(9)}\big((G_{47}), t\big)| < (\hat{k}_{44})^{(9)}||(G_{47}) - (G_{47})'||e^{-(\hat{M}_{44})^{(9)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}', t)$ and$(a_i'')^{(9)}(T_{45}, t)$ . $(T_{45}', t)$ and $(T_{45}, t)$ are points belonging to the interval $\big[(\hat{k}_{44})^{(9)}, (\hat{M}_{44})^{(9)}\big]$. It is to be noted that $(a_i'')^{(9)}(T_{45}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{44})^{(9)} = 1$ then the function $(a_i'')^{(9)}(T_{45}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$ :

$(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$, are positive constants

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}} \ , \frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$$

**Definition of** $(\hat{P}_{44})^{(9)}, (\hat{Q}_{44})^{(9)}$ :

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}, (\hat{A}_{44})^{(9)} and\ (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)}, (a_i')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (p_i)^{(9)}, \ (r_i)^{(9)}, i = 44,45,46,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{44})^{(9)}}[\,(a_i)^{(9)} + (a_i')^{(9)} + \ (\hat{A}_{44})^{(9)} + \ (\hat{P}_{44})^{(9)}\,(\hat{k}_{44})^{(9)}] < 1$$

$$\frac{1}{(\hat{M}_{44})^{(9)}}[\ (b_i)^{(9)} + (b_i')^{(9)} + \ (\hat{B}_{44})^{(9)} + \ (\hat{Q}_{44})^{(9)}\ (\hat{k}_{44})^{(9)}] < 1$$

**Theorem 1:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \big(\hat{P}_{13}\big)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \ , \ \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 2 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

**Definition of** $G_i(0)\,, T_i(0)$

$$G_i(t) \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} \quad , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} \quad , \quad T_i(0) = T_i^0 > 0$$

**Theorem 3 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t) \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t} \quad , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t} \quad , \quad T_i(0) = T_i^0 > 0$$

**Theorem 4 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

**Definition of** $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \left(\hat{P}_{24}\right)^{(4)} e^{(\hat{M}_{24})^{(4)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{24}\right)^{(4)} e^{(\hat{M}_{24})^{(4)}t} \;,\quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 5 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

**Definition of** $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \left(\hat{P}_{28}\right)^{(5)} e^{(\hat{M}_{28})^{(5)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{28}\right)^{(5)} e^{(\hat{M}_{28})^{(5)}t} \;,\quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 6 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

**Definition of** $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \left(\hat{P}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t} \;,\quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 7:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

**Definition of** $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \left(\hat{P}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t} \;,\quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 8:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

**Definition of** $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \left(\hat{P}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t} \;,\quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 9:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

**Definition of** $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \left(\hat{P}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

**Proof:** Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \leq (\hat{P}_{13})^{(1)} \text{ , } T_i^0 \leq (\hat{Q}_{13})^{(1)}, \quad 155$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t} \quad 156$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t} \quad 157$$

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[ (a_{13})^{(1)} G_{14}(s_{(13)}) - \left( (a'_{13})^{(1)} + a''_{13})^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \right) G_{13}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[ (a_{14})^{(1)} G_{13}(s_{(13)}) - \left( (a'_{14})^{(1)} + (a''_{14})^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \right) G_{14}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[ (a_{15})^{(1)} G_{14}(s_{(13)}) - \left( (a'_{15})^{(1)} + (a''_{15})^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \right) G_{15}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[ (b_{13})^{(1)} T_{14}(s_{(13)}) - \left( (b'_{13})^{(1)} - (b''_{13})^{(1)} (G(s_{(13)}), s_{(13)}) \right) T_{13}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[ (b_{14})^{(1)} T_{13}(s_{(13)}) - \left( (b'_{14})^{(1)} - (b''_{14})^{(1)} (G(s_{(13)}), s_{(13)}) \right) T_{14}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{T}_{15}(t) = T_{15}^0 + \int_0^t \left[ (b_{15})^{(1)} T_{14}(s_{(13)}) - \left( (b'_{15})^{(1)} - (b''_{15})^{(1)} (G(s_{(13)}), s_{(13)}) \right) T_{15}(s_{(13)}) \right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \leq (\hat{P}_{16})^{(2)} \text{ , } T_i^0 \leq (\hat{Q}_{16})^{(2)},$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$$

By 160

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[ (a_{16})^{(2)} G_{17}(s_{(16)}) - \left( (a'_{16})^{(2)} + a''_{16})^{(2)} (T_{17}(s_{(16)}), s_{(16)}) \right) G_{16}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[ (a_{17})^{(2)} G_{16}(s_{(16)}) - \left( (a'_{17})^{(2)} + (a''_{17})^{(2)} (T_{17}(s_{(16)}), s_{(17)}) \right) G_{17}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[ (a_{18})^{(2)} G_{17}(s_{(16)}) - \left( (a'_{18})^{(2)} + (a''_{18})^{(2)} (T_{17}(s_{(16)}), s_{(16)}) \right) G_{18}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^{0} + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^{0} + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^{0} + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{18}(s_{(16)})\right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{20})^{(3)}$, $T_i^0 \le (\hat{Q}_{20})^{(3)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\bar{M}_{20})^{(3)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\bar{M}_{20})^{(3)} t}$

By 161

$$\bar{G}_{20}(t) = G_{20}^{0} + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''_{20})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^{0} + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^{0} + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{22}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^{0} + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^{0} + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{\mathrm{T}}_{22}(\mathrm{t}) = \mathrm{T}_{22}^{0} + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{22}(s_{(20)})\right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{24})^{(4)}$, $T_i^0 \le (\hat{Q}_{24})^{(4)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\bar{M}_{24})^{(4)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\bar{M}_{24})^{(4)} t}$

By 162

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{26}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{26}(t) = T_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{26}(s_{(24)})\right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{28})^{(5)},\ T_i^0 \le (\hat{Q}_{28})^{(5)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

By



$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{30}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{30}(t) = T_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{30}(s_{(28)})\right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{32})^{(6)},\ T_i^0 \le (\hat{Q}_{32})^{(6)},$$

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\tilde{M}_{32})^{(6)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\tilde{M}_{32})^{(6)} t}$

By 164

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[ (a_{32})^{(6)} G_{33}(s_{(32)}) - \left( (a'_{32})^{(6)} + a''_{32})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[ (a_{33})^{(6)} G_{32}(s_{(32)}) - \left( (a'_{33})^{(6)} + (a''_{33})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[ (a_{34})^{(6)} G_{33}(s_{(32)}) - \left( (a'_{34})^{(6)} + (a''_{34})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{34}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[ (b_{32})^{(6)} T_{33}(s_{(32)}) - \left( (b'_{32})^{(6)} - (b''_{32})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[ (b_{33})^{(6)} T_{32}(s_{(32)}) - \left( (b'_{33})^{(6)} - (b''_{33})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{34}(t) = T_{34}^0 + \int_0^t \left[ (b_{34})^{(6)} T_{33}(s_{(32)}) - \left( (b'_{34})^{(6)} - (b''_{34})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{34}(s_{(32)}) \right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\tilde{M}_{36})^{(7)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\tilde{M}_{36})^{(7)} t}$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[ (a_{36})^{(7)} G_{37}(s_{(36)}) - \left( (a'_{36})^{(7)} + a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[ (a_{37})^{(7)} G_{36}(s_{(36)}) - \left( (a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[ (a_{38})^{(7)} G_{37}(s_{(36)}) - \left( (a'_{38})^{(7)} + (a''_{38})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{38}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[ (b_{36})^{(7)} T_{37}(s_{(36)}) - \left( (b'_{36})^{(7)} - (b''_{36})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[ (b_{37})^{(7)} T_{36}(s_{(36)}) - \left( (b'_{37})^{(7)} - (b''_{37})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\overline{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^{0} + \int_0^t \left[ (b_{38})^{(7)} T_{37}(s_{(36)}) - \left( (b'_{38})^{(7)} - (b''_{38})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{38}(s_{(36)}) \right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{40})^{(8)}, T_i^0 \le (\hat{Q}_{40})^{(8)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[ (a_{40})^{(8)} G_{41}(s_{(40)}) - \left( (a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[ (a_{41})^{(8)} G_{40}(s_{(40)}) - \left( (a'_{41})^{(8)} + (a''_{41})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[ (a_{42})^{(8)} G_{41}(s_{(40)}) - \left( (a'_{42})^{(8)} + (a''_{42})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{42}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^0 + \int_0^t \left[ (b_{40})^{(8)} T_{41}(s_{(40)}) - \left( (b'_{40})^{(8)} - (b''_{40})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^0 + \int_0^t \left[ (b_{41})^{(8)} T_{40}(s_{(40)}) - \left( (b'_{41})^{(8)} - (b''_{41})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\overline{\mathrm{T}}_{42}(\mathrm{t}) = \mathrm{T}_{42}^{0} + \int_0^t \left[ (b_{42})^{(8)} T_{41}(s_{(40)}) - \left( (b'_{42})^{(8)} - (b''_{42})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{42}(s_{(40)}) \right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** 166
Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy A

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{44})^{(9)}, T_i^0 \le (\hat{Q}_{44})^{(9)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[ (a_{44})^{(9)} G_{45}(s_{(44)}) - \left( (a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45}(s_{(44)}), s_{(44)})\right) G_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}(s_{(44)}), s_{(44)})\right) G_{46}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{44}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{\mathrm{T}}_{46}(\mathrm{t}) = \mathrm{T}_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{46}(s_{(44)})\right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}}\right)\right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left(e^{(\hat{M}_{13})^{(1)} t} - 1\right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\hat{M}_{13})^{(1)} t} \le \frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{P}_{13})^{(1)} + G_{14}^0\right) e^{\left(-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0}\right)} + (\hat{P}_{13})^{(1)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \le G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}}\right)\right] ds_{(16)} = \quad 169$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1\right)$$

From which it follows that 170

$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \le \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left[\left((\hat{P}_{16})^{(2)} + G_{17}^0\right) e^{\left(-\frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0}\right)} + (\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t) \le G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} \left(G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} s_{(20)}}\right)\right] ds_{(20)} =$$

$$\left(1 + (a_{20})^{(3)} t\right) G_{21}^0 + \frac{(a_{20})^{(3)} (\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left(e^{(\hat{M}_{20})^{(3)} t} - 1\right)$$

From which it follows that 172

$$(G_{20}(t)-G_{20}^{0})e^{-(\hat{M}_{20})^{(3)}t}\leq\frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left[((\hat{P}_{20})^{(3)}+G_{21}^{0})e^{\left(-\frac{(\hat{P}_{20})^{(3)}+G_{21}^{0}}{G_{21}^{0}}\right)}+(\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 173

$$G_{24}(t)\leq G_{24}^{0}+\int_{0}^{t}\left[(a_{24})^{(4)}\left(G_{25}^{0}+(\hat{P}_{24})^{(4)}e^{(\hat{M}_{24})^{(4)}s_{(24)}}\right)\right]ds_{(24)}=$$

$$\left(1+(a_{24})^{(4)}t\right)G_{25}^{0}+\frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left(e^{(\hat{M}_{24})^{(4)}t}-1\right)$$

From which it follows that 174

$$(G_{24}(t)-G_{24}^{0})e^{-(\hat{M}_{24})^{(4)}t}\leq\frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left[((\hat{P}_{24})^{(4)}+G_{25}^{0})e^{\left(-\frac{(\hat{P}_{24})^{(4)}+G_{25}^{0}}{G_{25}^{0}}\right)}+(\hat{P}_{24})^{(4)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t)\leq G_{28}^{0}+\int_{0}^{t}\left[(a_{28})^{(5)}\left(G_{29}^{0}+(\hat{P}_{28})^{(5)}e^{(\hat{M}_{28})^{(5)}s_{(28)}}\right)\right]ds_{(28)}=$$

$$\left(1+(a_{28})^{(5)}t\right)G_{29}^{0}+\frac{(a_{28})^{(5)}(\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left(e^{(\hat{M}_{28})^{(5)}t}-1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t}\leq\frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t)\leq G_{32}^{0}+\int_{0}^{t}\left[(a_{32})^{(6)}\left(G_{33}^{0}+(\hat{P}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\right)\right]ds_{(32)}=$$

$$\left(1+(a_{32})^{(6)}t\right)G_{33}^{0}+\frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left(e^{(\hat{M}_{32})^{(6)}t}-1\right)$$

From which it follows that 177

$$(G_{32}(t)-G_{32}^{0})e^{-(\hat{M}_{32})^{(6)}t}\leq\frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left[((\hat{P}_{32})^{(6)}+G_{33}^{0})e^{\left(-\frac{(\hat{P}_{32})^{(6)}+G_{33}^{0}}{G_{33}^{0}}\right)}+(\hat{P}_{32})^{(6)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}$ , $G_{26}, T_{24}, T_{25}, T_{26}$

(u) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t) \le G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} \left(G_{37}^0 + (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} s_{(36)}}\right)\right] ds_{(36)} =$$
$$\left(1 + (a_{36})^{(7)} t\right) G_{37}^0 + \frac{(a_{36})^{(7)} (\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left(e^{(\hat{M}_{36})^{(7)} t} - 1\right)$$

From which it follows that

$$(G_{36}(t) - G_{36}^0) e^{-(\hat{M}_{36})^{(7)} t} \le \frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left[((\hat{P}_{36})^{(7)} + G_{37}^0) e^{\left(-\frac{(\hat{P}_{36})^{(7)} + G_{37}^0}{G_{37}^0}\right)} + (\hat{P}_{36})^{(7)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t) \le G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} \left(G_{41}^0 + (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} s_{(40)}}\right)\right] ds_{(40)} = \quad 180$$
$$\left(1 + (a_{40})^{(8)} t\right) G_{41}^0 + \frac{(a_{40})^{(8)} (\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}} \left(e^{(\hat{M}_{40})^{(8)} t} - 1\right)$$

From which it follows that 181

$$(G_{40}(t) - G_{40}^0) e^{-(\hat{M}_{40})^{(8)} t} \le \frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}} \left[((\hat{P}_{40})^{(8)} + G_{41}^0) e^{\left(-\frac{(\hat{P}_{40})^{(8)} + G_{41}^0}{G_{41}^0}\right)} + (\hat{P}_{40})^{(8)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 8

Analogous inequalities hold also for $G_{41}$ , $G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t) \le G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} \left(G_{45}^0 + (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} s_{(44)}}\right)\right] ds_{(44)} =$$
$$\left(1 + (a_{44})^{(9)} t\right) G_{45}^0 + \frac{(a_{44})^{(9)} (\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left(e^{(\hat{M}_{44})^{(9)} t} - 1\right)$$

From which it follows that

$$(G_{44}(t) - G_{44}^0) e^{-(\hat{M}_{44})^{(9)} t} \le \frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left[((\hat{P}_{44})^{(9)} + G_{45}^0) e^{\left(-\frac{(\hat{P}_{44})^{(9)} + G_{45}^0}{G_{45}^0}\right)} + (\hat{P}_{44})^{(9)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 9

Analogous inequalities hold also for $G_{45}$ , $G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}$ , $\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$ and to choose 182

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[(\hat{P}_{13})^{(1)}+\left((\hat{P}_{13})^{(1)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{13})^{(1)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{13})^{(1)} \quad 183$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[\left((\hat{Q}_{13})^{(1)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{13})^{(1)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{13})^{(1)}\right]\leq(\hat{Q}_{13})^{(1)} \quad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)},T^{(1)}\right),\left(G^{(2)},T^{(2)}\right)\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{13})^{(1)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{13})^{(1)}t}\}$$

Indeed if we denote

**Definition of** $\tilde{G},\tilde{T}:\left(\tilde{G},\tilde{T}\right)=\mathcal{A}^{(1)}(G,T)$

It results

$$\left|\tilde{G}_{13}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{13})^{(1)}\left|G_{14}^{(1)}-G_{14}^{(2)}\right|e^{-(\hat{M}_{13})^{(1)}s_{(13)}}e^{(\hat{M}_{13})^{(1)}s_{(13)}}ds_{(13)}+$$

$$\int_0^t\{(a'_{13})^{(1)}\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\hat{M}_{13})^{(1)}s_{(13)}}e^{-(\hat{M}_{13})^{(1)}s_{(13)}}+$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\hat{M}_{13})^{(1)}s_{(13)}}e^{(\hat{M}_{13})^{(1)}s_{(13)}}+$$

$$G_{13}^{(2)}\left|(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)-(a''_{13})^{(1)}\left(T_{14}^{(2)},s_{(13)}\right)\right|\ e^{-(\hat{M}_{13})^{(1)}s_{(13)}}e^{(\hat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|G^{(1)}-G^{(2)}\right|e^{-(\hat{M}_{13})^{(1)}t}\leq \quad 186$$
$$\frac{1}{(\hat{M}_{13})^{(1)}}\left((a_{13})^{(1)}+(a'_{13})^{(1)}+(\hat{A}_{13})^{(1)}+(\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}\right)d\left(\left(G^{(1)},T^{(1)};\ G^{(2)},T^{(2)}\right)\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 1:** The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\hat{P}_{13})^{(1)}e^{(\hat{M}_{13})^{(1)}t}$ *and* $(\hat{Q}_{13})^{(1)}e^{(\hat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i = 13,14,15$ depend only on $T_{14}$ and respectively on $G(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any $t$ where $G_i(t) = 0\ and\ T_i(t) = 0$

From 19 to 24 it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(1)} - (a_i'')^{(1)}(T_{14}(s_{(13)}), s_{(13)})\} ds_{(13)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(1)} t)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2\ and\ \left((\widehat{M}_{13})^{(1)}\right)_3$ : 187

**Remark 3**: if $G_{13}$ is bounded, the same property have also $G_{14}\ and\ G_{15}$ . indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a_{14}')^{(1)} G_{14}$ and by integrating

$$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)} \left((\widehat{M}_{13})^{(1)}\right)_1 / (a_{14}')^{(1)}$$

In the same way , one can obtain

$$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)} \left((\widehat{M}_{13})^{(1)}\right)_2 / (a_{15}')^{(1)}$$

If $G_{14}\ or\ G_{15}$ is bounded, the same property follows for $G_{13}$ , $G_{15}$ and $G_{13}$ , $G_{14}$ respectively.

**Remark 4:** If $G_{13}$ *is* bounded, from below, the same property holds for $G_{14}\ and\ G_{15}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{14}$ is bounded from below. 188

**Remark 5:** If $T_{13}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}(G(t), t)) = (b_{14}')^{(1)}$ then $T_{14} \to \infty$. 189

**Definition of** $(m)^{(1)}$ and $\varepsilon_1$ :

Indeed let $t_1$ be so that for $t > t_1$

$$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t), t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right)$, $t = log\frac{2}{\varepsilon_1}$ By taking now $\varepsilon_1$ sufficiently small one sees that $T_{14}$ is unbounded.
The same property holds for $T_{15}$ if $\lim_{t\to\infty}(b_{15}'')^{(1)}(G(t), t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}$ , $\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}\ and\ (\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[(\widehat{P}_{16})^{(2)}+\left((\hat{P}_{16})^{(2)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{16})^{(2)}\right]\leq(\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

**Definition of** $\widetilde{G_{19}},\widetilde{T_{19}}$ : $\left(\widetilde{G_{19}},\widetilde{T_{19}}\right)=\mathcal{A}^{(2)}(G_{19},T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{16})^{(2)}\left|G_{17}^{(1)}-G_{17}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)}+$$

$$\int_0^t\{(a'_{16})^{(2)}\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$G_{16}^{(2)}\left|(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)-(a''_{16})^{(2)}\left(T_{17}^{(2)},s_{(16)}\right)\right|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0,t]$ 197

From the hypotheses it follows

$$\left|(G_{19})^{(1)}-(G_{19})^{(2)}\right|\mathrm{e}^{-(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}\leq$$
$$\frac{1}{(\widehat{\mathrm{M}}_{16})^{(2)}}\left((a_{16})^{(2)}+(a'_{16})^{(2)}+(\widehat{\mathrm{A}}_{16})^{(2)}+(\widehat{\mathrm{P}}_{16})^{(2)}(\hat{k}_{16})^{(2)}\right)\mathrm{d}\left(((G_{19})^{(1)},(T_{19})^{(1)};\ (G_{19})^{(2)},(T_{19})^{(2)})\right)$$

And analogous inequalities for $G_i$ and $T_i$. Taking into account the hypothesis the result follows 198

**Remark 6:** The fact that we supposed $(a''_{16})^{(2)}$ and $(b''_{16})^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{\mathrm{P}}_{16})^{(2)}\mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}$ and $(\widehat{\mathrm{Q}}_{16})^{(2)}\mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i=16,17,18$ depend only on $\mathrm{T}_{17}$ and respectively on

$(G_{19})$(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 7:** There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$ 200

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(2)} - (a_i'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})\} ds_{(16)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(2)}t)} > 0 \quad \text{for } t > 0$$

**Definition of** $\left((\widehat{M}_{16})^{(2)}\right)_1$, $\left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$: 201

**Remark 8:** if $G_{16}$ is bounded, the same property have also $G_{17}$ and $G_{18}$. indeed if

$G_{16} < (\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt} \leq \left((\widehat{M}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)} G_{17}$ and by integrating

$$G_{17} \leq \left((\widehat{M}_{16})^{(2)}\right)_2 = G_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1 / (a_{17}')^{(2)}$$

In the same way , one can obtain

$$G_{18} \leq \left((\widehat{M}_{16})^{(2)}\right)_3 = G_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2 / (a_{18}')^{(2)}$$

If $G_{17}$ or $G_{18}$ is bounded, the same property follows for $G_{16}$, $G_{18}$ and $G_{16}$, $G_{17}$ respectively.

**Remark 9:** If $G_{16}$ is bounded, from below, the same property holds for $G_{17}$ and $G_{18}$. The proof is analogous with the preceding one. An analogous property is true if $G_{17}$ is bounded from below. 202

**Remark 10:** If $T_{16}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t), t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$. 203

**Definition of** $(m)^{(2)}$ and $\varepsilon_2$ :

Indeed let $t_2$ be so that for $t > t_2$

$$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right)$, $t = \log\frac{2}{\varepsilon_2}$ By taking now $\varepsilon_2$ sufficiently small one sees that $T_{17}$ is unbounded. 205
The same property holds for $T_{18}$ if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t), t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[(\widehat{P}_{20})^{(3)}+\left((\hat{P}_{20})^{(3)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{20})^{(3)}\right]\leq(\hat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself 210

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)},(T_{23})^{(1)}),((G_{23})^{(2)},(T_{23})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

**Definition of** $\widetilde{G_{23}},\widetilde{T_{23}}:(\widetilde{(G_{23})},\widetilde{(T_{23})})=\mathcal{A}^{(3)}((G_{23}),(T_{23}))$

It results 213

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{20})^{(3)}\left|G_{21}^{(1)}-G_{21}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}ds_{(20)}+$$

$$\int_0^t\{(a'_{20})^{(3)}\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$G_{20}^{(2)}\left|(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)},s_{(20)}\right)\right|\ e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\}ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}t}\leq \quad 214$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+(a'_{20})^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\widehat{k}_{20})^{(3)}\right)d\left(((G_{23})^{(1)},(T_{23})^{(1)};\ (G_{23})^{(2)},(T_{23})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 11:** The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ *and* $(\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i=20,21,22$ depend only on $T_{21}$ and respectively on

$(G_{23})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 12:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 216

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(3)} - (a_i'')^{(3)}(T_{21}(s_{(20)}), s_{(20)})\} ds_{(20)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(3)} t)} > 0$ for t > 0

**Definition of** $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2\ and\ \left((\widehat{M}_{20})^{(3)}\right)_3$ : 217

**Remark 13:** if $G_{20}$ is bounded, the same property have also $G_{21}\ and\ G_{22}$ . indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a_{21}')^{(3)} G_{21}$ and by integrating

$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1 / (a_{21}')^{(3)}$

In the same way , one can obtain

$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2 / (a_{22}')^{(3)}$

If $G_{21}\ or\ G_{22}$ is bounded, the same property follows for $G_{20}$ , $G_{22}$ and $G_{20}$ , $G_{21}$ respectively.

**Remark 14:** If $G_{20}$ *is* bounded, from below, the same property holds for $G_{21}\ and\ G_{22}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{21}$ is bounded from below. 218

**Remark 15:** If $T_{20}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t), t\right)) = (b_{21}')^{(3)}$ then $T_{21} \to \infty$. 219

**Definition of** $(m)^{(3)}$ and $\varepsilon_3$ :

Indeed let $t_3$ be so that for $t > t_3$

$(b_{21})^{(3)} - (b_i'')^{(3)}\left((G_{23})(t), t\right) < \varepsilon_3, T_{20}\,(t) > (m)^{(3)}$

Then $\frac{dT_{21}}{dt} \geq (a_{21})^{(3)}(m)^{(3)} - \varepsilon_3 T_{21}$ which leads to 220

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t} = \frac{1}{2}$ it results

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_3}$ By taking now $\varepsilon_3$ sufficiently small one sees that $T_{21}$ is unbounded.
The same property holds for $T_{22}$ if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t), t\right) = (b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}$ , $\frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}} < 1$ and to choose 221

$(\widehat{P}_{24})^{(4)}$ and $(\widehat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[(\widehat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \tag{222}$$

$$\frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \tag{223}$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself (224)

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric (225)

$$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{24})^{(4)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{27}}),(\widetilde{T_{27}})$ : $\left((\widetilde{G_{27}}),(\widetilde{T_{27}})\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{24})^{(4)}\left|G_{25}^{(1)}-G_{25}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\,ds_{(24)}+$$

$$\int_0^t\{(a'_{24})^{(4)}\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$G_{24}^{(2)}\left|(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)-(a''_{24})^{(4)}\left(T_{25}^{(2)},s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\}ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}t}\leq \tag{226}$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)}+(a'_{24})^{(4)}+(\widehat{A}_{24})^{(4)}+(\widehat{P}_{24})^{(4)}(\hat{k}_{24})^{(4)}\right)d\left(((G_{27})^{(1)},(T_{27})^{(1)};\ (G_{27})^{(2)},(T_{27})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 16:** The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ *and* $(\widehat{Q}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_+$. (227)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i=24,25,26$ depend only on $T_{25}$ and respectively on

$(G_{27})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 17:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 228

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(4)} - (a_i'')^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(4)} t)} > 0$ for t > 0

**Definition of** $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2\ and\ \left((\widehat{M}_{24})^{(4)}\right)_3$ : 229

**Remark 18:** if $G_{24}$ is bounded, the same property have also $G_{25}\ and\ G_{26}$ . indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a_{25}')^{(4)} G_{25}$ and by integrating

$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1 / (a_{25}')^{(4)}$

In the same way , one can obtain

$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2 / (a_{26}')^{(4)}$

If $G_{25}\ or\ G_{26}$ is bounded, the same property follows for $G_{24}$ , $G_{26}$ and $G_{24}$ , $G_{25}$ respectively.

**Remark 19:** If $G_{24}$ *is* bounded, from below, the same property holds for $G_{25}\ and\ G_{26}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{25}$ is bounded from below. 230

**Remark 20:** If $T_{24}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}\ ((G_{27})(t), t)) = (b_{25}')^{(4)}$ then $T_{25} \to \infty$. 231

**Definition of** $(m)^{(4)}$ and $\varepsilon_4$ :

Indeed let $t_4$ be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t), t) < \varepsilon_4, T_{24}\,(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_4}$ By taking now $\varepsilon_4$ sufficiently small one sees that $T_{25}$ is unbounded.
The same property holds for $T_{26}$ if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t), t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for $G_{29}$ , $G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}$ , $\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)}+\left((\hat{P}_{28})^{(5)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{28})^{(5)}\right]\leq(\hat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$$d\left(((G_{31})^{(1)},(T_{31})^{(1)}),((G_{31})^{(2)},(T_{31})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\hat{M}_{28})^{(5)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\hat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{31}}),(\widetilde{T_{31}})$ : $\left((\widetilde{G_{31}}),(\widetilde{T_{31}})\right)=\mathcal{A}^{(5)}\left((G_{31}),(T_{31})\right)$

It results

$$|\tilde{G}_{28}^{(1)}-\tilde{G}_i^{(2)}|\leq\int_0^t(a_{28})^{(5)}|G_{29}^{(1)}-G_{29}^{(2)}|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\,ds_{(28)}+$$

$$\int_0^t\{(a'_{28})^{(5)}|G_{28}^{(1)}-G_{28}^{(2)}|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$(a''_{28})^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)|G_{28}^{(1)}-G_{28}^{(2)}|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$G_{28}^{(2)}|(a''_{28})^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)-(a''_{28})^{(5)}\left(T_{29}^{(2)},s_{(28)}\right)|\;e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\}ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on it follows

$$|(G_{31})^{(1)}-(G_{31})^{(2)}|e^{-(\widehat{M}_{28})^{(5)}t}\leq \quad 237$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)}+(a'_{28})^{(5)}+(\widehat{A}_{28})^{(5)}+(\widehat{P}_{28})^{(5)}(\widehat{k}_{28})^{(5)}\right)d\left(((G_{31})^{(1)},(T_{31})^{(1)};\;(G_{31})^{(2)},(T_{31})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 21:** The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\hat{P}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ *and* $(\hat{Q}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ respectively of $\mathbb{R}_+$. 238

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on $T_{29}$ and respectively on $(G_{31})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 22:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 239

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(5)} - (a_i'')^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(5)} t\right)} > 0$ for t > 0

**Definition of** $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2\ and\ \left((\widehat{M}_{28})^{(5)}\right)_3$ : 240

**Remark 23:** if $G_{28}$ is bounded, the same property have also $G_{29}\ and\ G_{30}$ . indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a_{29}')^{(5)} G_{29}$ and by integrating

$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1 / (a_{29}')^{(5)}$

In the same way , one can obtain

$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2 / (a_{30}')^{(5)}$

If $G_{29}\ or\ G_{30}$ is bounded, the same property follows for $G_{28}$ , $G_{30}$ and $G_{28}$ , $G_{29}$ respectively.

**Remark 24:** If $G_{28}$ *is* bounded, from below, the same property holds for $G_{29}\ and\ G_{30}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{29}$ is bounded from below. 241

**Remark 25:** If $T_{28}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}\,((G_{31})(t), t)) = (b_{29}')^{(5)}$ then $T_{29} \to \infty$. 242

**Definition of** $(m)^{(5)}$ and $\varepsilon_5$ :

Indeed let $t_5$ be so that for $t > t_5$

$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t), t) < \varepsilon_5, T_{28}\,(t) > (m)^{(5)}$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_5}$ By taking now $\varepsilon_5$ sufficiently small one sees that $T_{29}$ is unbounded.
The same property holds for $T_{30}$ if $\lim_{t\to\infty}(b_{30}'')^{(5)}\left((G_{31})(t), t\right) = (b_{30}')^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{33}\,, G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\ , \frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}} < 1$ and to choose 244

$(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[(\hat{P}_{32})^{(6)}+\left((\hat{P}_{32})^{(6)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{32})^{(6)} \tag{245}$$

$$\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[\left((\hat{Q}_{32})^{(6)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{32})^{(6)}\right]\leq(\hat{Q}_{32})^{(6)} \tag{246}$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric (247)

$$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\hat{M}_{32})^{(6)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\hat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{35}}),(\widetilde{T_{35}})$ : $\left((\widetilde{G_{35}}),(\widetilde{T_{35}})\right)=\mathcal{A}^{(6)}\left((G_{35}),(T_{35})\right)$

It results

$$|\tilde{G}_{32}^{(1)}-\tilde{G}_i^{(2)}|\leq\int_0^t(a_{32})^{(6)}|G_{33}^{(1)}-G_{33}^{(2)}|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\,ds_{(32)}+$$

$$\int_0^t\{(a'_{32})^{(6)}|G_{32}^{(1)}-G_{32}^{(2)}|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)|G_{32}^{(1)}-G_{32}^{(2)}|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$G_{32}^{(2)}|(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)-(a''_{32})^{(6)}\left(T_{33}^{(2)},s_{(32)}\right)|\;e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\}ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$|(G_{35})^{(1)}-(G_{35})^{(2)}|e^{-(\widehat{M}_{32})^{(6)}t}\leq \tag{248}$$
$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)}+(a'_{32})^{(6)}+(\widehat{A}_{32})^{(6)}+(\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}\right)d\left(((G_{35})^{(1)},(T_{35})^{(1)};\;(G_{35})^{(2)},(T_{35})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 26:** The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\hat{P}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ *and* $(\hat{Q}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ respectively of $\mathbb{R}_+$. (249)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i = 32,33,34$ depend only on $T_{33}$ and respectively on $(G_{35})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 27:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 250

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(6)} - (a_i'')^{(6)}(T_{33}(s_{(32)}), s_{(32)})\} ds_{(32)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(6)} t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{32})^{(6)}\right)_1, \left((\widehat{M}_{32})^{(6)}\right)_2\ and\ \left((\widehat{M}_{32})^{(6)}\right)_3$ : 251

**Remark 28:** if $G_{32}$ is bounded, the same property have also $G_{33}\ and\ G_{34}$ . indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a_{33}')^{(6)} G_{33}$ and by integrating

$$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_1 / (a_{33}')^{(6)}$$

In the same way , one can obtain

$$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_2 / (a_{34}')^{(6)}$$

If $G_{33}\ or\ G_{34}$ is bounded, the same property follows for $G_{32}$ , $G_{34}$ and $G_{32}$ , $G_{33}$ respectively.

**Remark 29:** If $G_{32}$ *is* bounded, from below, the same property holds for $G_{33}\ and\ G_{34}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{33}$ is bounded from below. 252

**Remark 30:** If $T_{32}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}\,((G_{35})(t), t)) = (b_{33}')^{(6)}$ then $T_{33} \to \infty$. 253

**Definition of** $(m)^{(6)}$ and $\varepsilon_6$ :

Indeed let $t_6$ be so that for $t > t_6$

$$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t), t\right) < \varepsilon_6, T_{32}\,(t) > (m)^{(6)}$$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right)$, $t = log\,\frac{2}{\varepsilon_6}$ By taking now $\varepsilon_6$ sufficiently small one sees that $T_{33}$ is unbounded.
The same property holds for $T_{34}$ if $\lim_{t\to\infty}(b_{34}'')^{(6)}\left((G_{35})(t), t(t), t\right) = (b_{34}')^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{37}, G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}$ , $\frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$ and to choose

$(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[(\hat{P}_{36})^{(7)}+\left((\hat{P}_{36})^{(7)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{36})^{(7)} \tag{256}$$

$$\frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[\left((\hat{Q}_{36})^{(7)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{36})^{(7)}\right]\leq(\hat{Q}_{36})^{(7)} \tag{257}$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself
The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric (258)

$$d\left(((G_{39})^{(1)},(T_{39})^{(1)}),((G_{39})^{(2)},(T_{39})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\hat{M}_{36})^{(7)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\hat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

**Definition of** $\widetilde{(G_{39})},\widetilde{(T_{39})}:(\widetilde{(G_{39})},\widetilde{(T_{39})})=\mathcal{A}^{(7)}((G_{39}),(T_{39}))$
It results

$$|\tilde{G}_{36}^{(1)}-\tilde{G}_i^{(2)}|\leq\int_0^t(a_{36})^{(7)}|G_{37}^{(1)}-G_{37}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\,ds_{(36)}+$$
$$\int_0^t\{(a'_{36})^{(7)}|G_{36}^{(1)}-G_{36}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$(a''_{36})^{(7)}(T_{37}^{(1)},s_{(36)})|G_{36}^{(1)}-G_{36}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$G_{36}^{(2)}|(a''_{36})^{(7)}(T_{37}^{(1)},s_{(36)})-(a''_{36})^{(7)}(T_{37}^{(2)},s_{(36)})|\;e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\}ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on it follows

$$|(G_{39})^{(1)}-(G_{39})^{(2)}|e^{-(\widehat{M}_{36})^{(7)}t}\leq \tag{259}$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}}\left((a_{36})^{(7)}+(a'_{36})^{(7)}+(\widehat{A}_{36})^{(7)}+(\widehat{P}_{36})^{(7)}(\widehat{k}_{36})^{(7)}\right)d\left(((G_{39})^{(1)},(T_{39})^{(1)};\;(G_{39})^{(2)},(T_{39})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 31:** The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ *and* $(\widehat{Q}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ respectively of $\mathbb{R}_+$. (260)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(7)}$ and $(b_i'')^{(7)}, i = 36,37,38$ depend only on $T_{37}$ and respectively on $(G_{39})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 32:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 261

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(7)} - (a_i'')^{(7)}(T_{37}(s_{(36)}), s_{(36)})\} ds_{(36)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(7)} t\right)} > 0$ for t > 0

**Definition of** $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2\ and\ \left((\widehat{M}_{36})^{(7)}\right)_3$ : 262

**Remark 33:** if $G_{36}$ is bounded, the same property have also $G_{37}\ and\ G_{38}$ . indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a_{37}')^{(7)} G_{37}$ and by integrating

$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1 / (a_{37}')^{(7)}$

In the same way , one can obtain

$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2 / (a_{38}')^{(7)}$

If $G_{37}\ or\ G_{38}$ is bounded, the same property follows for $G_{36}$ , $G_{38}$ and $G_{36}$ , $G_{37}$ respectively.

**Remark 34:** If $G_{36}$ *is* bounded, from below, the same property holds for $G_{37}\ and\ G_{38}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{37}$ is bounded from below. 263

**Remark 35:** If $T_{36}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(7)}\ ((G_{39})(t), t)) = (b_{37}')^{(7)}$ then $T_{37} \to \infty$. 264

**Definition of** $(m)^{(7)}$ and $\varepsilon_7$ :

Indeed let $t_7$ be so that for $t > t_7$

$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t), t) < \varepsilon_7, T_{36}\,(t) > (m)^{(7)}$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_7}$ By taking now $\varepsilon_7$ sufficiently small one sees that $T_{37}$ is unbounded.
The same property holds for $T_{38}$ if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t), t\right) = (b_{38}')^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}$ , $\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}} < 1$ and to choose $(\widehat{P}_{40})^{(8)}$ and $(\widehat{Q}_{40})^{(8)}$ large to have 266

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[(\widehat{P}_{40})^{(8)} + \left((\hat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{40})^{(8)} \quad 267$$

$$\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{40})^{(8)}\right] \leq (\hat{Q}_{40})^{(8)} \quad 268$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)}, (T_{43})^{(1)}), ((G_{43})^{(2)}, (T_{43})^{(2)})\right) = \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

**Definition of** $\widetilde{(G_{43})}, \widetilde{(T_{43})}$ : $\left(\widetilde{(G_{43})}, \widetilde{(T_{43})}\right) = \mathcal{A}^{(8)}((G_{43}), (T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{40})^{(8)}\left|G_{41}^{(1)} - G_{41}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\,ds_{(40)} +$$
$$\int_0^t\{(a'_{40})^{(8)}\left|G_{40}^{(1)} - G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{-(\widehat{M}_{40})^{(8)}s_{(40)}} +$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right)\left|G_{40}^{(1)} - G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}} +$$
$$G_{40}^{(2)}\left|(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right) - (a''_{40})^{(8)}\left(T_{41}^{(2)}, s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\}ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0, t]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)} - (G_{43})^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}t} \leq \quad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)} + (a'_{40})^{(8)} + (\widehat{A}_{40})^{(8)} + (\widehat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}\right)d\left(((G_{43})^{(1)}, (T_{43})^{(1)};\ (G_{43})^{(2)}, (T_{43})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 36:** The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate 274

condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ *and* $(\widehat{Q}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i = 40{,}41{,}42$ depend only on $T_{41}$ and respectively on $(G_{43})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 37** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 275

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t\{(a_i')^{(8)}-(a_i'')^{(8)}(T_{41}(s_{(40)}),s_{(40)})\}ds_{(40)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(8)}t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{40})^{(8)}\right)_1, \left((\widehat{M}_{40})^{(8)}\right)_2\ and\ \left((\widehat{M}_{40})^{(8)}\right)_3$ : 276

**Remark 38:** if $G_{40}$ is bounded, the same property have also $G_{41}\ and\ G_{42}$ . indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a_{41}')^{(8)}G_{41}$ and by integrating

$$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1/(a_{41}')^{(8)}$$

In the same way , one can obtain

$$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2/(a_{42}')^{(8)}$$

If $G_{41}\ or\ G_{42}$ is bounded, the same property follows for $G_{40}$ , $G_{42}$ and $G_{40}$ , $G_{41}$ respectively.

**Remark 39:** If $G_{40}$ *is* bounded, from below, the same property holds for $G_{41}\ and\ G_{42}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{41}$ is bounded from below. 277

**Remark 40:** If $T_{40}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}\ ((G_{43})(t),t)) = (b_{41}')^{(8)}$ then $T_{41} \to \infty$. 278

**Definition of** $(m)^{(8)}$ and $\varepsilon_8$ :

Indeed let $t_8$ be so that for $t\ > t_8$

$$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t),t\right) < \varepsilon_8, T_{40}\,(t) > (m)^{(8)}$$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right)$, $t = log\frac{2}{\varepsilon_8}$ By taking now $\varepsilon_8$ sufficiently small one sees that $T_{41}$ is unbounded.
The same property holds for $T_{42}$ if $\lim_{t\to\infty}(b''_{42})^{(8)}\left((G_{43})(t), t(t), t\right) = (b'_{42})^{(8)}$



It is now sufficient to take $\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}$ , $\frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$ and to choose $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ large to have

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}\left[(\hat{P}_{44})^{(9)} + \left((\hat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}}\left[\left((\hat{Q}_{44})^{(9)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{44})^{(9)}\right] \leq (\hat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)}, (T_{47})^{(1)}), ((G_{47})^{(2)}, (T_{47})^{(2)})\right) =$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{44})^{(9)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{47}}), (\widetilde{T_{47}}) : \left((\widetilde{G_{47}}), (\widetilde{T_{47}})\right) = \mathcal{A}^{(9)}\left((G_{47}), (T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{44})^{(9)}\left|G_{45}^{(1)} - G_{45}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\,ds_{(44)} +$$
$$\int_0^t\{(a'_{44})^{(9)}\left|G_{44}^{(1)} - G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{-(\widehat{M}_{44})^{(9)}s_{(44)}} +$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right)\left|G_{44}^{(1)} - G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}} +$$
$$G_{44}^{(2)}\left|(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right) - (a''_{44})^{(9)}\left(T_{45}^{(2)}, s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\}ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)} - G^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}t} \leq$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)} + (a'_{44})^{(9)} + (\widehat{A}_{44})^{(9)} + (\widehat{P}_{44})^{(9)}(\hat{k}_{44})^{(9)}\right)d\left(((G_{47})^{(1)}, (T_{47})^{(1)};\ (G_{47})^{(2)}, (T_{47})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis (39,35,36) the result follows

**Remark 41:** The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by

$(\widehat{P}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ *and* $(\widehat{Q}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i = 44{,}45{,}46$ depend only on $T_{45}$ and respectively on $(G_{47})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 42:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$

From 99 to 44 it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t\{(a_i')^{(9)}-(a_i'')^{(9)}(T_{45}(s_{(44)}),s_{(44)})\}ds_{(44)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(9)}t)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{44})^{(9)}\right)_1, \left((\widehat{M}_{44})^{(9)}\right)_2$ *and* $\left((\widehat{M}_{44})^{(9)}\right)_3$ :

**Remark 43:** if $G_{44}$ is bounded, the same property have also $G_{45}\ and\ G_{46}$ . indeed if

$G_{44} < (\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt} \leq \left((\widehat{M}_{44})^{(9)}\right)_1 - (a_{45}')^{(9)}G_{45}$ and by integrating

$$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_1/(a_{45}')^{(9)}$$

In the same way , one can obtain

$$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_2/(a_{46}')^{(9)}$$

If $G_{45}\ or\ G_{46}$ is bounded, the same property follows for $G_{44}$ , $G_{46}$ and $G_{44}$ , $G_{45}$ respectively.

**Remark 44:** If $G_{44}$ *is* bounded, from below, the same property holds for $G_{45}\ and\ G_{46}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{45}$ is bounded from below.

**Remark 45:** If $T_{44}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t),t\right) = (b_{45}')^{(9)}$ then $T_{45} \to \infty$.

**Definition of** $(m)^{(9)}$ and $\varepsilon_9$ :

Indeed let $t_9$ be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t),t\right) < \varepsilon_9, T_{44}\,(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right), \quad t = log\frac{2}{\varepsilon_9}$ By taking now $\varepsilon_9$ sufficiently small one sees that $T_{45}$ is unbounded.
The same property holds for $T_{46}$ if $\lim_{t\to\infty}(b_{46}'')^{(9)}\left((G_{47})(t),t\right) = (b_{46}')^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

**Behavior of the solutions of equation** 280

**Theorem** If we denote and define

**Definition of** $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ :

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)} \le -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14}, t) + (a''_{14})^{(1)}(T_{14}, t) \le -(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)} \le -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G, t) - (b''_{14})^{(1)}(G, t) \le -(\tau_1)^{(1)}$

**Definition of** $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$ : 281

By $(\nu_1)^{(1)} > 0$, $(\nu_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$, $(u_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

**Definition of** $(\bar{\nu}_1)^{(1)},, (\bar{\nu}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$ : 282

By $(\bar{\nu}_1)^{(1)} > 0$, $(\bar{\nu}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$, $(\bar{u}_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

**Definition of** $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (\nu_0)^{(1)}$ :- 283

If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by
$(m_2)^{(1)} = (\nu_0)^{(1)}, (m_1)^{(1)} = (\nu_1)^{(1)}$, $if\ (\nu_0)^{(1)} < (\nu_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}, if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)}$,

and $\boxed{(\nu_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)}$, $if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)}$, $if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)}$,

and $\boxed{(u_0)^{(1)} = \frac{T^0_{13}}{T^0_{14}}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$

are defined

Then the solution of global equations satisfies the inequalities 285

$G^0_{13}e^{\left((S_1)^{(1)} - (p_{13})^{(1)}\right)t} \le G_{13}(t) \le G^0_{13}e^{(S_1)^{(1)}t}$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)}-(p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)}t}$$

$$\left(\frac{(a_{15})^{(1)}G_{13}^0}{(m_1)^{(1)}((S_1)^{(1)}-(p_{13})^{(1)}-(S_2)^{(1)})}\left[e^{((S_1)^{(1)}-(p_{13})^{(1)})t} - e^{-(S_2)^{(1)}t}\right] + G_{15}^0 e^{-(S_2)^{(1)}t} \le G_{15}(t) \le \right. \quad (286)$$

$$\left. \frac{(a_{15})^{(1)}G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)}-(a'_{15})^{(1)})}[e^{(S_1)^{(1)}t} - e^{-(a'_{15})^{(1)}t}] + G_{15}^0 e^{-(a'_{15})^{(1)}t}\right)$$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)}t} \le T_{13}(t) \le T_{13}^0 e^{((R_1)^{(1)}+(r_{13})^{(1)})t}} \quad (287)$$

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)}t} \le T_{13}(t) \le \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)}+(r_{13})^{(1)})t} \quad (288)$$

$$\frac{(b_{15})^{(1)}T_{13}^0}{(\mu_1)^{(1)}((R_1)^{(1)}-(b'_{15})^{(1)})}\left[e^{(R_1)^{(1)}t} - e^{-(b'_{15})^{(1)}t}\right] + T_{15}^0 e^{-(b'_{15})^{(1)}t} \le T_{15}(t) \le \quad (289)$$

$$\frac{(a_{15})^{(1)}T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)}+(r_{13})^{(1)}+(R_2)^{(1)})}\left[e^{((R_1)^{(1)}+(r_{13})^{(1)})t} - e^{-(R_2)^{(1)}t}\right] + T_{15}^0 e^{-(R_2)^{(1)}t}$$

**Definition of** $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:- (290)

Where $(S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$

$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$

$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$

$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$

**Behavior of the solutions of equation** (291)

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ : (292)

$(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ four constants satisfying

$$-(\sigma_2)^{(2)} \le -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(T_{17}, t) + (a''_{17})^{(2)}(T_{17}, t) \le -(\sigma_1)^{(2)} \quad (293)$$

$$-(\tau_2)^{(2)} \le -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}((G_{19}), t) - (b''_{17})^{(2)}((G_{19}), t) \le -(\tau_1)^{(2)} \quad (294)$$

**Definition of** $(\nu_1)^{(2)}, (\nu_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$ : (295)

By $(\nu_1)^{(2)} > 0$, $(\nu_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0$, $(u_2)^{(2)} < 0$ the roots (296)

of the equations $(a_{17})^{(2)}(\nu^{(2)})^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ (297)

and $(b_{14})^{(2)}(u^{(2)})^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and (298)

**Definition of** $(\bar{\nu}_1)^{(2)}, , (\bar{\nu}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$ : (299)

By $(\bar{\nu}_1)^{(2)} > 0$, $(\bar{\nu}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0$, $(\bar{u}_2)^{(2)} < 0$ the (300)

roots of the equations $(a_{17})^{(2)}\left(v^{(2)}\right)^2 + (\sigma_2)^{(2)}v^{(2)} - (a_{16})^{(2)} = 0$ 301

and $(b_{17})^{(2)}\left(u^{(2)}\right)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 302

**Definition of** $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ :- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (v_0)^{(2)}, (m_1)^{(2)} = (v_1)^{(2)}$, ***if*** $(v_0)^{(2)} < (v_1)^{(2)}$ 305

$(m_2)^{(2)} = (v_1)^{(2)}, (m_1)^{(2)} = (\bar{v}_1)^{(2)}$, ***if*** $(v_1)^{(2)} < (v_0)^{(2)} < (\bar{v}_1)^{(2)}$, 306

and $\boxed{(v_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}}$

$(m_2)^{(2)} = (v_1)^{(2)}, (m_1)^{(2)} = (v_0)^{(2)}$, ***if*** $(\bar{v}_1)^{(2)} < (v_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, ***if*** $(u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$, ***if*** $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)}$, ***if*** $(\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)}t}$$

$(p_i)^{(2)}$ is defined by equation

$$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)}t} \quad 311$$

$$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)}t}\right] + G_{18}^0 e^{-(S_2)^{(2)}t} \le G_{18}(t) \le \right. \quad 312$$

$$\left. \frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)}t} - e^{-(a'_{18})^{(2)}t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)}t}\right)$$

$$\boxed{T_{16}^0 e^{(R_1)^{(2)}t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}} \quad 313$$

$$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)}t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t} \quad 314$$

$$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)}t} - e^{-(b'_{18})^{(2)}t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)}t} \le T_{18}(t) \le \quad 315$$

$$\frac{(a_{18})^{(2)}T_{16}^{0}}{(\mu_2)^{(2)}((R_1)^{(2)}+(r_{16})^{(2)}+(R_2)^{(2)})}\left[e^{((R_1)^{(2)}+(r_{16})^{(2)})t}-e^{-(R_2)^{(2)}t}\right]+T_{18}^{0}e^{-(R_2)^{(2)}t}$$

**Definition of** $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$$

$$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)}$$ 318

$$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$$

**Behavior of the solutions** 319

**Theorem 3:** If we denote and define

**Definition of** $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ :

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}((G_{23}), t) \le -(\tau_1)^{(3)}$$

**Definition of** $(v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$ : 320

By $(v_1)^{(3)} > 0$, $(v_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations
$(a_{21})^{(3)}(v^{(3)})^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$
and $(b_{21})^{(3)}(u^{(3)})^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{v}_1)^{(3)} > 0$, $(\bar{v}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}(v^{(3)})^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}(u^{(3)})^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

**Definition of** $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ :- 321

If we define $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ by
$(m_2)^{(3)} = (v_0)^{(3)}, (m_1)^{(3)} = (v_1)^{(3)}$, ***if*** $(v_0)^{(3)} < (v_1)^{(3)}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (\bar{v}_1)^{(3)}$, ***if*** $(v_1)^{(3)} < (v_0)^{(3)} < (\bar{v}_1)^{(3)}$,

and $\boxed{(v_0)^{(3)} = \frac{G_{20}^{0}}{G_{21}^{0}}}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_0)^{(3)}$, ***if*** $(\bar{v}_1)^{(3)} < (v_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)},\ \boldsymbol{if}\ (u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}, \boldsymbol{if}\ (u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)},$ and $\boxed{(u_0)^{(3)} = \frac{T_{20}^0}{T_{21}^0}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)},\ \boldsymbol{if}\ (\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G_{20}^0 e^{(S_1)^{(3)} t}$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)} t} \quad 323$$

$$\left(\frac{(a_{22})^{(3)} G_{20}^0}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\left[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)} t}\right] + G_{22}^0 e^{-(S_2)^{(3)} t} \le G_{22}(t) \le \right. \quad 324$$

$$\left. \frac{(a_{22})^{(3)} G_{20}^0}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}\left[e^{(S_1)^{(3)} t} - e^{-(a'_{22})^{(3)} t}\right] + G_{22}^0 e^{-(a'_{22})^{(3)} t}\right)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t}} \quad 325$$

$$\frac{1}{(\mu_1)^{(3)}} T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}} T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)} T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)} - (b'_{22})^{(3)})}\left[e^{(R_1)^{(3)} t} - e^{-(b'_{22})^{(3)} t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)} t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)} T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)} + (r_{20})^{(3)} + (R_2)^{(3)})}\left[e^{((R_1)^{(3)} + (r_{20})^{(3)})t} - e^{-(R_2)^{(3)} t}\right] + T_{22}^0 e^{-(R_2)^{(3)} t}$$

**Definition of** $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

**Behavior of the solutions of equation**

**Theorem:** If we denote and define

**Definition of** $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ :

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$

$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}((G_{27}), t) - (b''_{25})^{(4)}((G_{27}), t) \le -(\tau_1)^{(4)}$

**Definition of** $(v_1)^{(4)}, (v_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, v^{(4)}, u^{(4)}$ : 329

By $(v_1)^{(4)} > 0$ , $(v_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$ , $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_1)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

**Definition of** $(\bar{v}_1)^{(4)},, (\bar{v}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$ : 330

By $(\bar{v}_1)^{(4)} > 0$ , $(\bar{v}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$ , $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

**Definition of** $(m_1)^{(4)}$ , $(m_2)^{(4)}$ , $(\mu_1)^{(4)}, (\mu_2)^{(4)}, (\nu_0)^{(4)}$ :-

If we define $(m_1)^{(4)}$ , $(m_2)^{(4)}$ , $(\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (v_0)^{(4)}, (m_1)^{(4)} = (v_1)^{(4)}$, $\boldsymbol{if}$ $(v_0)^{(4)} < (v_1)^{(4)}$

$(m_2)^{(4)} = (v_1)^{(4)}, (m_1)^{(4)} = (\bar{v}_1)^{(4)}$ , $\boldsymbol{if}$ $(v_4)^{(4)} < (v_0)^{(4)} < (\bar{v}_1)^{(4)}$,
and $\boxed{(v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (v_4)^{(4)}, (m_1)^{(4)} = (v_0)^{(4)}$, $\boldsymbol{if}$ $(\bar{v}_4)^{(4)} < (v_0)^{(4)}$

and analogously

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$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)}$, $\boldsymbol{if}$ $(u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}$ , $\boldsymbol{if}$ $(u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)}$,
and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}, \boldsymbol{if}$ $(\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

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$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)}t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)}t}$$

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$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)}t}\right] + G_{26}^0 e^{-(S_2)^{(4)}t} \le G_{26}(t) \le\right.$$ 334

*(a26)4G240(m2)4(S1)4−(a26′)4e(S1)4t−e−(a26′)4t+ G260e−(a26′)4t*

$$\boxed{T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}}T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}}T_{24}^0 e^{((R_1)^{(4)}+(r_{24})^{(4)})t} \quad 335$$

$$\frac{(b_{26})^{(4)}T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)}-(b'_{26})^{(4)})}\left[e^{(R_1)^{(4)}t} - e^{-(b'_{26})^{(4)}t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)}t} \le T_{26}(t) \le \quad 336$$

$$\frac{(a_{26})^{(4)}T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)}+(r_{24})^{(4)}+(R_2)^{(4)})}\left[e^{((R_1)^{(4)}+(r_{24})^{(4)})t} - e^{-(R_2)^{(4)}t}\right] + T_{26}^0 e^{-(R_2)^{(4)}t}$$

**Definition of** $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

**Behavior of the solutions of equation** 338

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ :

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

**Definition of** $(v_1)^{(5)}, (v_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, v^{(5)}, u^{(5)}$ : 339

By $(v_1)^{(5)} > 0$, $(v_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$, $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(v^{(5)}\big)^2 + (\sigma_1)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

**Definition of** $(\bar{v}_1)^{(5)}, , (\bar{v}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$ : 340

By $(\bar{v}_1)^{(5)} > 0$, $(\bar{v}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$, $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(v^{(5)}\big)^2 + (\sigma_2)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

**Definition of** $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (v_0)^{(5)}$ :-

If we define $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (v_0)^{(5)}, (m_1)^{(5)} = (v_1)^{(5)}$, ***if*** $(v_0)^{(5)} < (v_1)^{(5)}$

$(m_2)^{(5)} = (v_1)^{(5)}, (m_1)^{(5)} = (\bar{v}_1)^{(5)}$, ***if*** $(v_1)^{(5)} < (v_0)^{(5)} < (\bar{v}_1)^{(5)}$,
and $\boxed{(v_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)},\ \boldsymbol{if}\ (u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}, \boldsymbol{if}\ (u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)},$

and $\boxed{(u_0)^{(5)} = \frac{T_{28}^0}{T_{29}^0}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}, \boldsymbol{if}\ (\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{28}(t) \le G_{28}^0 e^{(S_1)^{(5)} t}$

where $(p_i)^{(5)}$ is defined by equation

$\frac{1}{(m_5)^{(5)}} G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}} G_{28}^0 e^{(S_1)^{(5)} t}$ 343

$\left(\frac{(a_{30})^{(5)} G_{28}^0}{(m_1)^{(5)}((S_1)^{(5)} - (p_{28})^{(5)} - (S_2)^{(5)})}\left[e^{((S_1)^{(5)} - (p_{28})^{(5)})t} - e^{-(S_2)^{(5)} t}\right] + G_{30}^0 e^{-(S_2)^{(5)} t} \le G_{30}(t) \le\right.$ 344

*(a30)5G280(m2)5(S1)5−(a30′)5e(S1)5t−e−(a30′)5t+ G300e−(a30′)5t*

$\boxed{T_{28}^0 e^{(R_1)^{(5)} t} \le T_{28}(t) \le T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}}$ 345

$\frac{1}{(\mu_1)^{(5)}} T_{28}^0 e^{(R_1)^{(5)} t} \le T_{28}(t) \le \frac{1}{(\mu_2)^{(5)}} T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}$ 346

$\frac{(b_{30})^{(5)} T_{28}^0}{(\mu_1)^{(5)}((R_1)^{(5)} - (b'_{30})^{(5)})}\left[e^{(R_1)^{(5)} t} - e^{-(b'_{30})^{(5)} t}\right] + T_{30}^0 e^{-(b'_{30})^{(5)} t} \le T_{30}(t) \le$ 347

$\frac{(a_{30})^{(5)} T_{28}^0}{(\mu_2)^{(5)}((R_1)^{(5)} + (r_{28})^{(5)} + (R_2)^{(5)})}\left[e^{((R_1)^{(5)} + (r_{28})^{(5)})t} - e^{-(R_2)^{(5)} t}\right] + T_{30}^0 e^{-(R_2)^{(5)} t}$

**Definition of** $(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$:- 348

Where $(S_1)^{(5)} = (a_{28})^{(5)}(m_2)^{(5)} - (a'_{28})^{(5)}$

$(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}$

$(R_1)^{(5)} = (b_{28})^{(5)}(\mu_2)^{(5)} - (b'_{28})^{(5)}$

$(R_2)^{(5)} = (b'_{30})^{(5)} - (r_{30})^{(5)}$

**Behavior of the solutions of equation** 349

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ :

$(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)} \le -(a'_{32})^{(6)} + (a'_{33})^{(6)} - (a''_{32})^{(6)}(T_{33}, t) + (a''_{33})^{(6)}(T_{33}, t) \le -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)} \le -(b'_{32})^{(6)} + (b'_{33})^{(6)} - (b''_{32})^{(6)}\big((G_{35}), t\big) - (b''_{33})^{(6)}\big((G_{35}), t\big) \le -(\tau_1)^{(6)}$$

**Definition of** $(\nu_1)^{(6)}, (\nu_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, \nu^{(6)}, u^{(6)}$ : 350

By $(\nu_1)^{(6)} > 0$ , $(\nu_2)^{(6)} < 0$ and respectively $(u_1)^{(6)} > 0$ , $(u_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(6)}, , (\bar{\nu}_2)^{(6)}, (\bar{u}_1)^{(6)}, (\bar{u}_2)^{(6)}$ : 351

By $(\bar{\nu}_1)^{(6)} > 0$ , $(\bar{\nu}_2)^{(6)} < 0$ and respectively $(\bar{u}_1)^{(6)} > 0$ , $(\bar{u}_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$

**Definition of** $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}, (\nu_0)^{(6)}$ :-

If we define $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}$ by

$$(m_2)^{(6)} = (\nu_0)^{(6)}, (m_1)^{(6)} = (\nu_1)^{(6)}, \ \boldsymbol{if}\ (\nu_0)^{(6)} < (\nu_1)^{(6)}$$

$$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}, \boldsymbol{if}\ (\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)},$$

and $\boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}}$

$$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$$

and analogously 352

$$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}, \ \boldsymbol{if}\ (u_0)^{(6)} < (u_1)^{(6)}$$

$$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}, \boldsymbol{if}\ (u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)},$$

and $\boxed{(u_0)^{(6)} = \frac{T^0_{32}}{T^0_{33}}}$

$$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}, \boldsymbol{if}\ (\bar{u}_1)^{(6)} < (u_0)^{(6)} \text{ where } (u_1)^{(6)}, (\bar{u}_1)^{(6)}$$

Then the solution of global equations satisfies the inequalities 353

$$G^0_{32}e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{32}(t) \le G^0_{32}e^{(S_1)^{(6)}t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}}G^0_{32}e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{33}(t) \le \frac{1}{(m_2)^{(6)}}G^0_{32}e^{(S_1)^{(6)}t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)}G^0_{32}}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})}\left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)}t}\right] + G^0_{34}e^{-(S_2)^{(6)}t} \le G_{34}(t) \le \right. \quad 355$$

$$\frac{(a_{34})^{(6)}G_{32}^{0}}{(m_2)^{(6)}((S_1)^{(6)}-(a'_{34})^{(6)})}\left[e^{(S_1)^{(6)}t}-e^{-(a'_{34})^{(6)}t}\right]+\ G_{34}^{0}e^{-(a'_{34})^{(6)}t}\Big)$$

$$\boxed{T_{32}^{0}e^{(R_1)^{(6)}t}\le T_{32}(t)\le T_{32}^{0}e^{((R_1)^{(6)}+(r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}}T_{32}^{0}e^{(R_1)^{(6)}t}\le T_{32}(t)\le\frac{1}{(\mu_2)^{(6)}}T_{32}^{0}e^{((R_1)^{(6)}+(r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)}T_{32}^{0}}{(\mu_1)^{(6)}((R_1)^{(6)}-(b'_{34})^{(6)})}\left[e^{(R_1)^{(6)}t}-e^{-(b'_{34})^{(6)}t}\right]+T_{34}^{0}e^{-(b'_{34})^{(6)}t}\le T_{34}(t)\le \quad 358$$

$$\frac{(a_{34})^{(6)}T_{32}^{0}}{(\mu_2)^{(6)}((R_1)^{(6)}+(r_{32})^{(6)}+(R_2)^{(6)})}\left[e^{((R_1)^{(6)}+(r_{32})^{(6)})t}-e^{-(R_2)^{(6)}t}\right]+T_{34}^{0}e^{-(R_2)^{(6)}t}$$

**Definition of** $(S_1)^{(6)},(S_2)^{(6)},(R_1)^{(6)},(R_2)^{(6)}$:- 359

Where $(S_1)^{(6)}=(a_{32})^{(6)}(m_2)^{(6)}-(a'_{32})^{(6)}$

$(S_2)^{(6)}=(a_{34})^{(6)}-(p_{34})^{(6)}$

$(R_1)^{(6)}=(b_{32})^{(6)}(\mu_2)^{(6)}-(b'_{32})^{(6)}$

$(R_2)^{(6)}=(b'_{34})^{(6)}-(r_{34})^{(6)}$

**Behavior of the solutions of equation**

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(7)},(\sigma_2)^{(7)},(\tau_1)^{(7)},(\tau_2)^{(7)}$ :

$(\sigma_1)^{(7)},(\sigma_2)^{(7)},(\tau_1)^{(7)},(\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)}\le-(a'_{36})^{(7)}+(a'_{37})^{(7)}-(a''_{36})^{(7)}(T_{37},t)+(a''_{37})^{(7)}(T_{37},t)\le-(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)}\le-(b'_{36})^{(7)}+(b'_{37})^{(7)}-(b''_{36})^{(7)}\big((G_{39}),t\big)-(b''_{37})^{(7)}\big((G_{39}),t\big)\le-(\tau_1)^{(7)}$$

**Definition of** $(v_1)^{(7)},(v_2)^{(7)},(u_1)^{(7)},(u_2)^{(7)},v^{(7)},u^{(7)}$ : 361

By $(v_1)^{(7)}>0,(v_2)^{(7)}<0$ and respectively $(u_1)^{(7)}>0,(u_2)^{(7)}<0$ the roots of the equations $(a_{37})^{(7)}\big(v^{(7)}\big)^2+(\sigma_1)^{(7)}v^{(7)}-(a_{36})^{(7)}=0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2+(\tau_1)^{(7)}u^{(7)}-(b_{36})^{(7)}=0$ and

**Definition of** $(\bar{v}_1)^{(7)},,(\bar{v}_2)^{(7)},(\bar{u}_1)^{(7)},(\bar{u}_2)^{(7)}$ : 362

By $(\bar{v}_1)^{(7)}>0,(\bar{v}_2)^{(7)}<0$ and respectively $(\bar{u}_1)^{(7)}>0,(\bar{u}_2)^{(7)}<0$ the roots of the equations $(a_{37})^{(7)}\big(v^{(7)}\big)^2+(\sigma_2)^{(7)}v^{(7)}-(a_{36})^{(7)}=0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2+(\tau_2)^{(7)}u^{(7)}-(b_{36})^{(7)}=0$

**Definition of** $(m_1)^{(7)},(m_2)^{(7)},(\mu_1)^{(7)},(\mu_2)^{(7)},(v_0)^{(7)}$ :-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$$(m_2)^{(7)} = (\nu_0)^{(7)}, (m_1)^{(7)} = (\nu_1)^{(7)}, \ \boldsymbol{if}\ (\nu_0)^{(7)} < (\nu_1)^{(7)}$$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\bar{\nu}_1)^{(7)}, \boldsymbol{if}\ (\nu_1)^{(7)} < (\nu_0)^{(7)} < (\bar{\nu}_1)^{(7)},$$

and $\boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\nu_0)^{(7)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(7)} < (\nu_0)^{(7)}$$

and analogously 363

$$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}, \ \boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$$

and $\boxed{(u_0)^{(7)} = \frac{T_{36}^0}{T_{37}^0}}$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)} \text{ where } (u_1)^{(7)}, (\bar{u}_1)^{(7)}$$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^0 e^{(S_1)^{(7)} t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)} t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)}((S_1)^{(7)} - (p_{36})^{(7)} - (S_2)^{(7)})}\left[e^{((S_1)^{(7)} - (p_{36})^{(7)})t} - e^{-(S_2)^{(7)} t}\right] + G_{38}^0 e^{-(S_2)^{(7)} t} \le G_{38}(t) \le \right. \quad 366$$

$$\left. \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)}((S_1)^{(7)} - (a'_{38})^{(7)})}\left[e^{(S_1)^{(7)} t} - e^{-(a'_{38})^{(7)} t}\right] + G_{38}^0 e^{-(a'_{38})^{(7)} t}\right)$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t} \quad 368$$

$$\frac{(b_{38})^{(7)} T_{36}^0}{(\mu_1)^{(7)}((R_1)^{(7)} - (b'_{38})^{(7)})}\left[e^{(R_1)^{(7)} t} - e^{-(b'_{38})^{(7)} t}\right] + T_{38}^0 e^{-(b'_{38})^{(7)} t} \le T_{38}(t) \le \quad 369$$

$$\frac{(a_{38})^{(7)} T_{36}^0}{(\mu_2)^{(7)}((R_1)^{(7)} + (r_{36})^{(7)} + (R_2)^{(7)})}\left[e^{((R_1)^{(7)} + (r_{36})^{(7)})t} - e^{-(R_2)^{(7)} t}\right] + T_{38}^0 e^{-(R_2)^{(7)} t}$$

**Definition of** $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$

$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$

$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$

**Behavior of the solutions of equation** 371

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ :

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$-(\sigma_2)^{(8)} \le -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \le -(\sigma_1)^{(8)}$

$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\big((G_{43}), t\big) - (b''_{41})^{(8)}\big((G_{43}), t\big) \le -(\tau_1)^{(8)}$

**Definition of** $(\nu_1)^{(8)}, (\nu_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, \nu^{(8)}, u^{(8)}$ : 372

By $(\nu_1)^{(8)} > 0, (\nu_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0, (u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(8)}, , (\bar{\nu}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$ :

By $(\bar{\nu}_1)^{(8)} > 0, (\bar{\nu}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0, (\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

**Definition of** $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}, (\nu_0)^{(8)}$ :-

If we define $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$(m_2)^{(8)} = (\nu_0)^{(8)}, (m_1)^{(8)} = (\nu_1)^{(8)}$, ***if*** $(\nu_0)^{(8)} < (\nu_1)^{(8)}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\bar{\nu}_1)^{(8)}$, ***if*** $(\nu_1)^{(8)} < (\nu_0)^{(8)} < (\bar{\nu}_1)^{(8)}$,
and $\boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\nu_0)^{(8)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(8)} < (\nu_0)^{(8)}$

and analogously 374

$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)},\ \boldsymbol{if}\ (u_0)^{(8)} < (u_1)^{(8)}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}, \boldsymbol{if}\ (u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)},$

and $\boxed{(u_0)^{(8)} = \frac{T_{40}^0}{T_{41}^0}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}, \boldsymbol{if}\ (\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^0 e^{(S_1)^{(8)} t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}} G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}} G_{40}^0 e^{(S_1)^{(8)} t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)} G_{40}^0}{(m_1)^{(8)}((S_1)^{(8)} - (p_{40})^{(8)} - (S_2)^{(8)})}\left[e^{((S_1)^{(8)} - (p_{40})^{(8)})t} - e^{-(S_2)^{(8)} t}\right] + G_{42}^0 e^{-(S_2)^{(8)} t} \le G_{42}(t) \le \right. \quad 377$$

$$\left. \frac{(a_{42})^{(8)} G_{40}^0}{(m_2)^{(8)}((S_1)^{(8)} - (a'_{42})^{(8)})}\left[e^{(S_1)^{(8)} t} - e^{-(a'_{42})^{(8)} t}\right] + G_{42}^0 e^{-(a'_{42})^{(8)} t}\right)$$

$$\boxed{T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}} T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le \frac{1}{(\mu_2)^{(8)}} T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)} T_{40}^0}{(\mu_1)^{(8)}((R_1)^{(8)} - (b'_{42})^{(8)})}\left[e^{(R_1)^{(8)} t} - e^{-(b'_{42})^{(8)} t}\right] + T_{42}^0 e^{-(b'_{42})^{(8)} t} \le T_{42}(t) \le \quad 380$$

$$\frac{(a_{42})^{(8)} T_{40}^0}{(\mu_2)^{(8)}((R_1)^{(8)} + (r_{40})^{(8)} + (R_2)^{(8)})}\left[e^{((R_1)^{(8)} + (r_{40})^{(8)})t} - e^{-(R_2)^{(8)} t}\right] + T_{42}^0 e^{-(R_2)^{(8)} t}$$

**Definition of** $(S_1)^{(8)}, (S_2)^{(8)}, (R_1)^{(8)}, (R_2)^{(8)}$:- 381

Where $(S_1)^{(8)} = (a_{40})^{(8)}(m_2)^{(8)} - (a'_{40})^{(8)}$

$(S_2)^{(8)} = (a_{42})^{(8)} - (p_{42})^{(8)}$

$(R_1)^{(8)} = (b_{40})^{(8)}(\mu_2)^{(8)} - (b'_{40})^{(8)}$

$(R_2)^{(8)} = (b'_{42})^{(8)} - (r_{42})^{(8)}$

**Behavior of the solutions of equation 37 to 92** 382

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ :

$(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ four constants satisfying

$$-(\sigma_2)^{(9)} \le -(a'_{44})^{(9)} + (a'_{45})^{(9)} - (a''_{44})^{(9)}(T_{45}, t) + (a''_{45})^{(9)}(T_{45}, t) \le -(\sigma_1)^{(9)}$$

$$-(\tau_2)^{(9)} \le -(b'_{44})^{(9)} + (b'_{45})^{(9)} - (b''_{44})^{(9)}\big((G_{47}), t\big) - (b''_{45})^{(9)}\big((G_{47}), t\big) \le -(\tau_1)^{(9)}$$

**Definition of** $(\nu_1)^{(9)}, (\nu_2)^{(9)}, (u_1)^{(9)}, (u_2)^{(9)}, \nu^{(9)}, u^{(9)}$ :

By $(\nu_1)^{(9)} > 0$ , $(\nu_2)^{(9)} < 0$ and respectively $(u_1)^{(9)} > 0$ , $(u_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_1)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(9)}, , (\bar{\nu}_2)^{(9)}, (\bar{u}_1)^{(9)}, (\bar{u}_2)^{(9)}$ :

By $(\bar{\nu}_1)^{(9)} > 0$ , $(\bar{\nu}_2)^{(9)} < 0$ and respectively $(\bar{u}_1)^{(9)} > 0$ , $(\bar{u}_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_2)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$

**Definition of** $(m_1)^{(9)}, (m_2)^{(9)}, (\mu_1)^{(9)}, (\mu_2)^{(9)}, (\nu_0)^{(9)}$ :-

If we define $(m_1)^{(9)}, (m_2)^{(9)}, (\mu_1)^{(9)}, (\mu_2)^{(9)}$ by

$$(m_2)^{(9)} = (\nu_0)^{(9)}, (m_1)^{(9)} = (\nu_1)^{(9)}, \ \boldsymbol{if}\ (\nu_0)^{(9)} < (\nu_1)^{(9)}$$

$$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\bar{\nu}_1)^{(9)}, \boldsymbol{if}\ (\nu_1)^{(9)} < (\nu_0)^{(9)} < (\bar{\nu}_1)^{(9)},$$

and $\boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$

$$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\nu_0)^{(9)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(9)} < (\nu_0)^{(9)}$$

and analogously

$$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)}, \ \boldsymbol{if}\ (u_0)^{(9)} < (u_1)^{(9)}$$

$$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}, \boldsymbol{if}\ (u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)},$$

and $\boxed{(u_0)^{(9)} = \frac{T^0_{44}}{T^0_{45}}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}, \boldsymbol{if}\ (\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G^0_{44} e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{44}(t) \le G^0_{44} e^{(S_1)^{(9)} t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}} G_{44}^0 e^{((S_1)^{(9)}-(p_{44})^{(9)})t} \le G_{45}(t) \le \frac{1}{(m_2)^{(9)}} G_{44}^0 e^{(S_1)^{(9)}t}$$

$$\left( \frac{(a_{46})^{(9)} G_{44}^0}{(m_1)^{(9)}((S_1)^{(9)}-(p_{44})^{(9)}-(S_2)^{(9)})} \left[ e^{((S_1)^{(9)}-(p_{44})^{(9)})t} - e^{-(S_2)^{(9)}t} \right] + G_{46}^0 e^{-(S_2)^{(9)}t} \le G_{46}(t) \le \right.$$

$$\left. \frac{(a_{46})^{(9)} G_{44}^0}{(m_2)^{(9)}((S_1)^{(9)}-(a'_{46})^{(9)})} [e^{(S_1)^{(9)}t} - e^{-(a'_{46})^{(9)}t}] + G_{46}^0 e^{-(a'_{46})^{(9)}t} \right)$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le T_{44}^0 e^{((R_1)^{(9)}+(r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}} T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le \frac{1}{(\mu_2)^{(9)}} T_{44}^0 e^{((R_1)^{(9)}+(r_{44})^{(9)})t}$$

$$\frac{(b_{46})^{(9)} T_{44}^0}{(\mu_1)^{(9)}((R_1)^{(9)}-(b'_{46})^{(9)})} \left[ e^{(R_1)^{(9)}t} - e^{-(b'_{46})^{(9)}t} \right] + T_{46}^0 e^{-(b'_{46})^{(9)}t} \le T_{46}(t) \le$$

$$\frac{(a_{46})^{(9)} T_{44}^0}{(\mu_2)^{(9)}((R_1)^{(9)}+(r_{44})^{(9)}+(R_2)^{(9)})} \left[ e^{((R_1)^{(9)}+(r_{44})^{(9)})t} - e^{-(R_2)^{(9)}t} \right] + T_{46}^0 e^{-(R_2)^{(9)}t}$$

**Definition of** $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$

**Proof :** From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left( (a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14}, t) \right) - (a''_{14})^{(1)}(T_{14}, t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

**Definition of** $v^{(1)}$ :- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left( (a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)} \right) \le \frac{dv^{(1)}}{dt} \le -\left( (a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)} \right)$$

From which one obtains

**Definition of** $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$ :-

For $0 < \boxed{(v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}} < (v_1)^{(1)} < (\bar{v}_1)^{(1)}$

$$v^{(1)}(t) \ge \frac{(v_1)^{(1)} + (C)^{(1)}(v_2)^{(1)} e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}}{1+(C)^{(1)} e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(v_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(v_2)^{(1)}}}$$

it follows $(\nu_0)^{(1)} \leq \nu^{(1)}(t) \leq (\nu_1)^{(1)}$

In the same manner , we get 384

$$\nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{\nu}_1)^{(1)}-(\nu_0)^{(1)}}{(\nu_0)^{(1)}-(\bar{\nu}_2)^{(1)}}}$$

From which we deduce $(\nu_0)^{(1)} \leq \nu^{(1)}(t) \leq (\bar{\nu}_1)^{(1)}$

If $0 < (\nu_1)^{(1)} < (\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0} < (\bar{\nu}_1)^{(1)}$ we find like in the previous case, 385

$$(\nu_1)^{(1)} \leq \frac{(\nu_1)^{(1)}+(C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}} \leq \nu^{(1)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\bar{\nu}_1)^{(1)}$$

If $0 < (\nu_1)^{(1)} \leq (\bar{\nu}_1)^{(1)} \leq \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$ , we obtain 386

$$(\nu_1)^{(1)} \leq \nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\nu_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(1)}(t)$ :-

$$(m_2)^{(1)} \leq \nu^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{\nu^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(1)}(t)$ :-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{13})^{(1)} = (a''_{14})^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)}G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)}T_{14}(t)$ This is an important

consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

**Proof** : From global equations we obtain 387

$$\frac{d\nu^{(2)}}{dt} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right) - (a''_{17})^{(2)}(T_{17}, t)\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

**Definition of** $\nu^{(2)}$ :- $\boxed{\nu^{(2)} = \frac{G_{16}}{G_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \le \frac{d\nu^{(2)}}{dt} \le -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

**Definition of** $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$ :-

For $0 < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \ge \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}}{1 + (C)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(C)^{(2)} = \frac{(\nu_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \le \nu^{(2)}(t) \le (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \le \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \le \nu^{(2)}(t) \le (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \le \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}}{1 + (C)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}} \le \nu^{(2)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \le (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \le (\bar{\nu}_1)^{(2)} \le (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$ , we obtain 394

$$(\nu_1)^{(2)} \le \nu^{(2)}(t) \le \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \le (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(2)}(t)$ :- 395

$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}$, $\boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$

In a completely analogous way, we obtain 396

**Definition of** $u^{(2)}(t)$ :-

$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}$, $\boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :** 397

If $(a''_{16})^{(2)} = (a''_{17})^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)} G_{17}(t)$

Analogously if $(b''_{16})^{(2)} = (b''_{17})^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)} T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

**Proof** : From global equations we obtain 398

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right) - (a''_{21})^{(3)}(T_{21}, t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

**Definition of** $\nu^{(3)}$ :- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right) \leq \frac{d\nu^{(3)}}{dt} \leq -\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right)$$

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From which one obtains

For $0 < (\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\nu_1)^{(3)} < (\bar{\nu}_1)^{(3)}$

$$\nu^{(3)}(t) \geq \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(\nu_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\nu_2)^{(3)}}}$$

it follows $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\nu_1)^{(3)}$

In the same manner , we get 401

$$\nu^{(3)}(t) \le \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{\nu}_1)^{(3)}-(\nu_0)^{(3)}}{(\nu_0)^{(3)}-(\bar{\nu}_2)^{(3)}}}$$

**Definition of** $(\bar{\nu}_1)^{(3)}$ :-

From which we deduce $(\nu_0)^{(3)} \le \nu^{(3)}(t) \le (\bar{\nu}_1)^{(3)}$

If $0 < (\nu_1)^{(3)} < (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (\bar{\nu}_1)^{(3)}$ we find like in the previous case, 402

$$(\nu_1)^{(3)} \le \frac{(\nu_1)^{(3)}+(C)^{(3)}(\nu_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}} \le \nu^{(3)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \le (\bar{\nu}_1)^{(3)}$$

If $0 < (\nu_1)^{(3)} \le (\bar{\nu}_1)^{(3)} \le (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}$ , we obtain 403

$$(\nu_1)^{(3)} \le \nu^{(3)}(t) \le \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \le (\nu_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(3)}(t)$ :-

$$(m_2)^{(3)} \le \nu^{(3)}(t) \le (m_1)^{(3)}, \quad \boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(3)}(t)$ :-

$$(\mu_2)^{(3)} \le u^{(3)}(t) \le (\mu_1)^{(3)}, \quad \boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{20}'')^{(3)} = (a_{21}'')^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)}G_{21}(t)$

Analogously if $(b_{20}'')^{(3)} = (b_{21}'')^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)}T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

**Proof** : From global equations we obtain 404

$$\frac{dv^{(4)}}{dt} = (a_{24})^{(4)} - \left((a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right) - (a''_{25})^{(4)}(T_{25},t)v^{(4)} - (a_{25})^{(4)}v^{(4)}$$

**Definition of** $v^{(4)}$ :- $\boxed{v^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)}\right) \le \frac{dv^{(4)}}{dt} \le -\left((a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_4)^{(4)}v^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$ :-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \ge \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \le \nu^{(4)}(t) \le (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \le \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \le \nu^{(4)}(t) \le (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \le \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}}{1 + (C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}} \le \nu^{(4)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \le (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \le (\bar{\nu}_1)^{(4)} \le \boxed{(\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}}}$ , we obtain 407

$$(\nu_1)^{(4)} \le \nu^{(4)}(t) \le \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \le (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(4)}(t)$ :-

$$(m_2)^{(4)} \le \nu^{(4)}(t) \le (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(4)}(t)$ :-

$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}$, $\boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{24})^{(4)} = (a''_{25})^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)} G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b''_{24})^{(4)} = (b''_{25})^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)} T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

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**Proof** : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a'_{28})^{(5)} - (a'_{29})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right) - (a''_{29})^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

**Definition of** $\nu^{(5)}$ :- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$ :-

For $0 < \boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \le \frac{(\nu_1)^{(5)}+(C)^{(5)}(\nu_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)}-(\nu_2)^{(5)}\right)t\right]}}{1+(C)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)}-(\nu_2)^{(5)}\right)t\right]}} \le \nu^{(5)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \le (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \le (\bar{\nu}_1)^{(5)} \le \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain 411

$$(\nu_1)^{(5)} \le \nu^{(5)}(t) \le \frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \le (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(5)}(t)$ :-

$(m_2)^{(5)} \le \nu^{(5)}(t) \le (m_1)^{(5)}$, $\boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$

In a completely analogous way, we obtain

**Definition of** $u^{(5)}(t)$ :-

$(\mu_2)^{(5)} \le u^{(5)}(t) \le (\mu_1)^{(5)}$, $\boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a_{28}'')^{(5)} = (a_{29}'')^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)}G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b_{28}'')^{(5)} = (b_{29}'')^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)}T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

**Proof** : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a_{32}')^{(6)} - (a_{33}')^{(6)} + (a_{32}'')^{(6)}(T_{33},t)\right) - (a_{33}'')^{(6)}(T_{33},t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

**Definition of** $\nu^{(6)}$ :- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \le \frac{d\nu^{(6)}}{dt} \le -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$ :-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \geq \frac{(\nu_1)^{(6)}+(C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_0)^{(6)}\right)t\right]}}{1+(C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)}-(\nu_0)^{(6)}}{(\nu_0)^{(6)}-(\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\nu_1)^{(6)}$

In the same manner , we get 413

$$\nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{\nu}_1)^{(6)}-(\nu_0)^{(6)}}{(\nu_0)^{(6)}-(\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\bar{\nu}_1)^{(6)}$

If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 414

$$(\nu_1)^{(6)} \leq \frac{(\nu_1)^{(6)}+(C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_2)^{(6)}\right)t\right]}}{1+(C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_2)^{(6)}\right)t\right]}} \leq \nu^{(6)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\bar{\nu}_1)^{(6)}$$

If $0 < (\nu_1)^{(6)} \leq (\bar{\nu}_1)^{(6)} \leq \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$ , we obtain 415

$$(\nu_1)^{(6)} \leq \nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(6)}(t)$ :-

$$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}, \quad \boxed{\nu^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(6)}(t)$ :-

$$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}, \quad \boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)}G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)}T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

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**Proof** : From global equations we obtain

$$\frac{dv^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right) - (a''_{37})^{(7)}(T_{37},t)v^{(7)} - (a_{37})^{(7)}v^{(7)}$$

**Definition of** $v^{(7)}$ :- $\boxed{v^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_2)^{(7)}v^{(7)} - (a_{36})^{(7)}\right) \le \frac{dv^{(7)}}{dt} \le -\left((a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_1)^{(7)}v^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

**Definition of** $(\bar{v}_1)^{(7)}, (v_0)^{(7)}$ :-

For $0 < \boxed{(v_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}} < (v_1)^{(7)} < (\bar{v}_1)^{(7)}$

$$v^{(7)}(t) \ge \frac{(v_1)^{(7)} + (C)^{(7)}(v_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_0)^{(7)}\right)t\right]}}{1 + (C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(v_1)^{(7)} - (v_0)^{(7)}}{(v_0)^{(7)} - (v_2)^{(7)}}}$$

it follows $(v_0)^{(7)} \le v^{(7)}(t) \le (v_1)^{(7)}$

In the same manner , we get 417

$$v^{(7)}(t) \le \frac{(\bar{v}_1)^{(7)} + (\bar{C})^{(7)}(\bar{v}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{v}_1)^{(7)} - (v_0)^{(7)}}{(v_0)^{(7)} - (\bar{v}_2)^{(7)}}}$$

From which we deduce $(v_0)^{(7)} \le v^{(7)}(t) \le (\bar{v}_1)^{(7)}$

If $0 < (v_1)^{(7)} < (v_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}} < (\bar{v}_1)^{(7)}$ we find like in the previous case, 418

$$(v_1)^{(7)} \le \frac{(v_1)^{(7)} + (C)^{(7)}(v_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_2)^{(7)}\right)t\right]}}{1 + (C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_2)^{(7)}\right)t\right]}} \le v^{(7)}(t) \le$$

$$\frac{(\bar{v}_1)^{(7)} + (\bar{C})^{(7)}(\bar{v}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}} \le (\bar{v}_1)^{(7)}$$

If $0 < (v_1)^{(7)} \le (\bar{v}_1)^{(7)} \le \boxed{(v_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$ , we obtain 419

$$(v_1)^{(7)} \le v^{(7)}(t) \le \frac{(\bar{v}_1)^{(7)} + (\bar{C})^{(7)}(\bar{v}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}} \le (v_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $v^{(7)}(t)$ :-

$$(m_2)^{(7)} \leq v^{(7)}(t) \leq (m_1)^{(7)}, \quad \boxed{v^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(7)}(t)$ :- 420

$$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}, \quad \boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $v^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)} G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)} T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

**Proof** : From global equations we obtain 421

$$\frac{dv^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right) - (a''_{41})^{(8)}(T_{41}, t) v^{(8)} - (a_{41})^{(8)} v^{(8)}$$

**Definition of** $v^{(8)}$ :- $\boxed{v^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_2)^{(8)} v^{(8)} - (a_{40})^{(8)}\right) \leq \frac{dv^{(8)}}{dt} \leq -\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_1)^{(8)} v^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$ :-

$$\text{For } 0 < \boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$$

$$v^{(8)}(t) \geq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)}-(\nu_0)^{(8)}}{(\nu_0)^{(8)}-(\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \leq \frac{(\nu_1)^{(8)}+(C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}}{1+(C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}} \leq \nu^{(8)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \leq (\bar{\nu}_1)^{(8)} \leq \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$ , we obtain 424

$$(\nu_1)^{(8)} \leq \nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(8)}(t)$ :-

$$(m_2)^{(8)} \leq \nu^{(8)}(t) \leq (m_1)^{(8)}, \quad \boxed{\nu^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(8)}(t)$ :-

$$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}, \quad \boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $\nu^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)}G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then $(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)}T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

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**Proof** : From 99,20,44,22,23,44 we obtain A

$$\frac{dv^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right) - (a''_{45})^{(9)}(T_{45},t)v^{(9)} - (a_{45})^{(9)}v^{(9)}$$

**Definition of** $v^{(9)}$ :- $\boxed{v^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)}\right) \le \frac{dv^{(9)}}{dt} \le -\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_1)^{(9)}v^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$ :-

For $0 < \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$

$$v^{(9)}(t) \ge \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \le v^{(9)}(t) \le (\nu_9)^{(9)}$

In the same manner , we get

$$v^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \le v^{(9)}(t) \le (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \le \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}}{1 + (C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}} \le v^{(9)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \le (\bar{\nu}_1)^{(9)} \le \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$ , we obtain

$$(\nu_1)^{(9)} \le v^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $v^{(9)}(t)$ :-

$$(m_2)^{(9)} \le v^{(9)}(t) \le (m_1)^{(9)}, \quad \boxed{v^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(9)}(t)$ :-

$(\mu_2)^{(9)} \leq u^{(9)}(t) \leq (\mu_1)^{(9)}$, $\boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)} G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then
$(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)} T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following 425

**Theorem :** If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on $t$ , and the conditions with the notations

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0$ ,

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$

$with\ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a''_i)^{(2)} and\ (b''_i)^{(2)}$ are independent on t , and the conditions with the notations 426

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$ 427

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 428

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$ , 429

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 430

$with\ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a''_i)^{(3)} and\ (b''_i)^{(3)}$ are independent on $t$ , and the conditions with the notations 431

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$ ,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

**Theorem :** If $(a_i'')^{(4)} and\ (b_i'')^{(4)}$ are independent on $t$ , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$ ,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(5)} and\ (b_i'')^{(5)}$ are independent on $t$ , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$ ,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

**Theorem** If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on $t$ , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$ ,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(7)} and\ (b_i'')^{(7)}$ are independent on $t$ , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$ ,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(8)} and\ (b_i'')^{(8)}$ are independent on $t$ , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$ ,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(9)} and\ (b_i'')^{(9)}$ are independent on $t$ , and the conditions (with the notations 436 A
45,46,27,28)

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$ ,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0$ 437

$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0$ 438

$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0$ 439

$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0$ 440

$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ]T_{14} = 0$ 441

$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0$ 442

has a unique positive solution , which is an equilibrium solution for the system

$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0$ 443

$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0$ 444

$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0$ 445

$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\ ]T_{16} = 0$ 446

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\,]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\,]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\,]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\,]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\,]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\big((G_{27})\big)\,]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\big((G_{27})\big)\,]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\big((G_{27})\big)\,]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})\,]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})\,]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})\,]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})\,]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})\,]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})\,]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0 \quad 479$$

$$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0 \quad 480$$

$$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0 \quad 481$$

$$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0 \quad 482$$

$$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0 \quad 483$$

$$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0 \quad 484$$

$$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0 \quad 484\text{A}$$

$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$

$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$

$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$

$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$

$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$

**Proof:** 485

(a) Indeed the first two equations have a nontrivial solution $G_{13}, G_{14}$ if

$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$

**Proof:** 486

(u) Indeed the first two equations have a nontrivial solution $G_{16}, G_{17}$ if

$\mathrm{F}(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$

**Proof:** 487

(a) Indeed the first two equations have a nontrivial solution $G_{20}, G_{21}$ if

$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$

**Proof:** 488

(a) Indeed the first two equations have a nontrivial solution $G_{24}, G_{25}$ if

$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$

**Proof:** 489

(a) Indeed the first two equations have a nontrivial solution $G_{28}, G_{29}$ if

$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$

**Proof:** 490

(a) Indeed the first two equations have a nontrivial solution $G_{32}, G_{33}$ if

$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) +$

$(a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$

**Proof:** 491

(a) Indeed the first two equations have a nontrivial solution $G_{36}, G_{37}$ if

$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$

**Proof:** 492

(a) Indeed the first two equations have a nontrivial solution $G_{40}, G_{41}$ if

$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$

**Proof:** 492 A

(a) Indeed the first two equations have a nontrivial solution $G_{44}, G_{45}$ if

$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$

**Definition and uniqueness of** $T^*_{14}$ :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique $T^*_{14}$ for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

**Definition and uniqueness of** $T^*_{17}$ :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique $T^*_{17}$ for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]}$$ 495

**Definition and uniqueness of** $T^*_{21}$ :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique $T^*_{21}$ for which $f(T^*_{21}) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

**Definition and uniqueness of** $T^*_{25}$ :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that

there exists a unique $T_{25}^{*}$ for which $f(T_{25}^{*})=0$. With this value , we obtain from the three first equations

$$G_{24}=\frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}=\frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T_{25}^{*})]}$$

**Definition and uniqueness of** $T_{29}^{*}$ :- 498

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(5)}(T_{29})$ being increasing, it follows that there exists a unique $T_{29}^{*}$ for which $f(T_{29}^{*})=0$. With this value , we obtain from the three first equations

$$G_{28}=\frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}=\frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T_{29}^{*})]}$$

**Definition and uniqueness of** $T_{33}^{*}$ :- 499

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(6)}(T_{33})$ being increasing, it follows that there exists a unique $T_{33}^{*}$ for which $f(T_{33}^{*})=0$. With this value , we obtain from the three first equations

$$G_{32}=\frac{(a_{32})^{(6)}G_{33}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^{*})]} \quad , \quad G_{34}=\frac{(a_{34})^{(6)}G_{33}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^{*})]}$$

**Definition and uniqueness of** $T_{37}^{*}$ :- 500

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(7)}(T_{37})$ being increasing, it follows that there exists a unique $T_{37}^{*}$ for which $f(T_{37}^{*})=0$. With this value , we obtain from the three first equations

$$G_{36}=\frac{(a_{36})^{(7)}G_{37}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^{*})]} \quad , \quad G_{38}=\frac{(a_{38})^{(7)}G_{37}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^{*})]}$$

**Definition and uniqueness of** $T_{41}^{*}$ :- 501

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique $T_{41}^{*}$ for which $f(T_{41}^{*})=0$. With this value , we obtain from the three first equations

$$G_{40}=\frac{(a_{40})^{(8)}G_{41}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T_{41}^{*})]} \quad , \quad G_{42}=\frac{(a_{42})^{(8)}G_{41}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T_{41}^{*})]}$$

**Definition and uniqueness of** $T_{45}^{*}$ :- 501 A

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(9)}(T_{45})$ being increasing, it follows that there exists a unique $T_{45}^{*}$ for which $f(T_{45}^{*})=0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)}G_{45}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T^*_{45})]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)}G_{45}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T^*_{45})]}$$

By the same argument, the equations admit solutions $G_{13}, G_{14}$ if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right]+(b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{14}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{14}$ such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions $G_{16}, G_{17}$ if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right]+(b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ 504
is a decreasing function in $G_{17}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{14}$ such that $\varphi((G_{19})^*) = 0$

By the same argument, the equations admit solutions $G_{20}, G_{21}$ if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right]+(b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{21}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{21}$ such that $\varphi((G_{23})^*) = 0$

By the same argument, the equations admit solutions $G_{24}, G_{25}$ if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right]+(b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{25}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{25}$ such that $\varphi((G_{27})^*) = 0$

By the same argument, the equations admit solutions $G_{28}, G_{29}$ if 507

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right]+(b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{29}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that

there exists a unique $G_{29}^{*}$ such that $\varphi((G_{31})^{*}) = 0$

By the same argument, the equations admit solutions $G_{32}, G_{33}$ if 508
$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right] + (b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{33}$ taking into account the hypothesis $\varphi(0) > 0\,, \varphi(\infty) < 0$ it follows that there exists a unique $G_{33}^{*}$ such that $\varphi(G_{35}{}^{*}) = 0$

By the same argument, the equations admit solutions $G_{36}, G_{37}$ if 509
$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$
$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right] + (b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{37}$ taking into account the hypothesis $\varphi(0) > 0\,, \varphi(\infty) < 0$ it follows that there exists a unique $G_{37}^{*}$ such that $\varphi(G_{39}{}^{*}) = 0$

By the same argument, the equations admit solutions $G_{40}, G_{41}$ if 510
$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$
$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right] + (b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{41}$ taking into account the hypothesis $\varphi(0) > 0\,, \varphi(\infty) < 0$ it follows that there exists a unique $G_{41}^{*}$ such that $\varphi(G_{43}{}^{*}) = 0$

By the same argument, the equations 92,93 admit solutions $G_{44}, G_{45}$ if
$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$

$$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right] + (b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{45}$ taking into account the hypothesis $\varphi(0) > 0\,, \varphi(\infty) < 0$ it follows that there exists a unique $G_{45}^{*}$ such that $\varphi((G_{47})^{*}) = 0$

Finally we obtain the unique solution 511

$G_{14}^{*}$ given by $\varphi(G^{*}) = 0\,, T_{14}^{*}$ given by $f(T_{14}^{*}) = 0$ and

$$G_{13}^{*} = \frac{(a_{13})^{(1)}G_{14}^{*}}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^{*})]} \quad , \quad G_{15}^{*} = \frac{(a_{15})^{(1)}G_{14}^{*}}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^{*})]}$$

$$T_{13}^{*} = \frac{(b_{13})^{(1)}T_{14}^{*}}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^{*})]} \quad , \quad T_{15}^{*} = \frac{(b_{15})^{(1)}T_{14}^{*}}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^{*})]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

$G_{17}^{*}$ given by $\varphi((G_{19})^{*}) = 0$ , $T_{17}^{*}$ given by $f(T_{17}^{*}) = 0$ and 512

$$G_{16}^{*} = \frac{(a_{16})^{(2)} G_{17}^{*}}{[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}^{*})]} \quad , \quad G_{18}^{*} = \frac{(a_{18})^{(2)} G_{17}^{*}}{[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}^{*})]} \tag{513}$$

$$T_{16}^{*} = \frac{(b_{16})^{(2)} T_{17}^{*}}{[(b'_{16})^{(2)} - (b''_{16})^{(2)}((G_{19})^{*})]} \quad , \quad T_{18}^{*} = \frac{(b_{18})^{(2)} T_{17}^{*}}{[(b'_{18})^{(2)} - (b''_{18})^{(2)}((G_{19})^{*})]} \tag{514}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

$G_{21}^{*}$ given by $\varphi((G_{23})^{*}) = 0$ , $T_{21}^{*}$ given by $f(T_{21}^{*}) = 0$ and

$$G_{20}^{*} = \frac{(a_{20})^{(3)} G_{21}^{*}}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^{*})]} \quad , \quad G_{22}^{*} = \frac{(a_{22})^{(3)} G_{21}^{*}}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^{*})]}$$

$$T_{20}^{*} = \frac{(b_{20})^{(3)} T_{21}^{*}}{[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}{}^{*})]} \quad , \quad T_{22}^{*} = \frac{(b_{22})^{(3)} T_{21}^{*}}{[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}{}^{*})]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

$G_{25}^{*}$ given by $\varphi(G_{27}) = 0$ , $T_{25}^{*}$ given by $f(T_{25}^{*}) = 0$ and

$$G_{24}^{*} = \frac{(a_{24})^{(4)} G_{25}^{*}}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}^{*} = \frac{(a_{26})^{(4)} G_{25}^{*}}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^{*})]}$$

$$T_{24}^{*} = \frac{(b_{24})^{(4)} T_{25}^{*}}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^{*})]} \quad , \quad T_{26}^{*} = \frac{(b_{26})^{(4)} T_{25}^{*}}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^{*})]} \tag{517}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

$G_{29}^{*}$ given by $\varphi((G_{31})^{*}) = 0$ , $T_{29}^{*}$ given by $f(T_{29}^{*}) = 0$ and

$$G_{28}^{*} = \frac{(a_{28})^{(5)} G_{29}^{*}}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}^{*} = \frac{(a_{30})^{(5)} G_{29}^{*}}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^{*})]}$$

$$T_{28}^{*} = \frac{(b_{28})^{(5)} T_{29}^{*}}{[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31})^{*})]} \quad , \quad T_{30}^{*} = \frac{(b_{30})^{(5)} T_{29}^{*}}{[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31})^{*})]} \tag{519}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

$G_{33}^{*}$ given by $\varphi((G_{35})^{*}) = 0$ , $T_{33}^{*}$ given by $f(T_{33}^{*}) = 0$ and

$$G_{32}^{*} = \frac{(a_{32})^{(6)} G_{33}^{*}}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^{*})]} \quad , \quad G_{34}^{*} = \frac{(a_{34})^{(6)} G_{33}^{*}}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^{*})]}$$

$$T_{32}^{*} = \frac{(b_{32})^{(6)} T_{33}^{*}}{[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35})^{*})]} \quad , \quad T_{34}^{*} = \frac{(b_{34})^{(6)} T_{33}^{*}}{[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35})^{*})]} \tag{521}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

$G_{37}^*$ given by $\varphi((G_{39})^*) = 0$ , $T_{37}^*$ given by $f(T_{37}^*) = 0$ and

$$G_{36}^* = \frac{(a_{36})^{(7)} G_{37}^*}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^*)]} \quad , \quad G_{38}^* = \frac{(a_{38})^{(7)} G_{37}^*}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^*)]}$$

$$T_{36}^* = \frac{(b_{36})^{(7)} T_{37}^*}{[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39})^*)]} \quad , \quad T_{38}^* = \frac{(b_{38})^{(7)} T_{37}^*}{[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39})^*)]}$$

Finally we obtain the unique solution 523

$G_{41}^*$ given by $\varphi((G_{43})^*) = 0$ , $T_{41}^*$ given by $f(T_{41}^*) = 0$ and

$$G_{40}^* = \frac{(a_{40})^{(8)} G_{41}^*}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42}^* = \frac{(a_{42})^{(8)} G_{41}^*}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

$$T_{40}^* = \frac{(b_{40})^{(8)} T_{41}^*}{[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43})^*)]} \quad , \quad T_{42}^* = \frac{(b_{42})^{(8)} T_{41}^*}{[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43})^*)]}$$

Finally we obtain the unique solution of 89 to 99 523 A

$G_{45}^*$ given by $\varphi((G_{47})^*) = 0$ , $T_{45}^*$ given by $f(T_{45}^*) = 0$ and

$$G_{44}^* = \frac{(a_{44})^{(9)} G_{45}^*}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46}^* = \frac{(a_{46})^{(9)} G_{45}^*}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

$$T_{44}^* = \frac{(b_{44})^{(9)} T_{45}^*}{[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47})^*)]} \quad , \quad T_{46}^* = \frac{(b_{46})^{(9)} T_{45}^*}{[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47})^*)]}$$

## ASYMPTOTIC STABILITY ANALYSIS 524

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

**Proof:** Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a''_{14})^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \frac{\partial (b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a'_{13})^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)}G^*_{13}\mathbb{T}_{14} \quad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a'_{14})^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)}G^*_{14}\mathbb{T}_{14} \quad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a'_{15})^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)}G^*_{15}\mathbb{T}_{14} \quad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b'_{13})^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)}T^*_{13}\mathbb{G}_j\right) \quad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b'_{14})^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)}T^*_{14}\mathbb{G}_j\right) \quad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b'_{15})^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T^*_{15}\mathbb{G}_j\right) \quad 530$$

**ASYMPTOTIC STABILITY ANALYSIS** 531

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(2)}$ and $(b''_i)^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G^*_i + \mathbb{G}_i \quad , T_i = T^*_i + \mathbb{T}_i \quad 532$$

$$\frac{\partial(a''_{17})^{(2)}}{\partial T_{17}}(T^*_{17}) = (q_{17})^{(2)} \quad , \frac{\partial(b''_i)^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \quad 533$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a'_{16})^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G^*_{16}\mathbb{T}_{17} \quad 534$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a'_{17})^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G^*_{17}\mathbb{T}_{17} \quad 535$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a'_{18})^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G^*_{18}\mathbb{T}_{17} \quad 536$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b'_{16})^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T^*_{16}\mathbb{G}_j\right) \quad 537$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b'_{17})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}T^*_{17}\mathbb{G}_j\right) \quad 538$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b'_{18})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}T^*_{18}\mathbb{G}_j\right) \quad 539$$

**ASYMPTOTIC STABILITY ANALYSIS** 540

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(3)}$ *and* $(b''_i)^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{21}'')^{(3)}}{\partial T_{21}}(T_{21}^*) = (q_{21})^{(3)} \quad , \frac{\partial (b_i'')^{(3)}}{\partial G_j}(\,(G_{23})^*\,) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\big((a_{20}')^{(3)} + (p_{20})^{(3)}\big)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G_{20}^*\mathbb{T}_{21} \quad 541$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\big((a_{21}')^{(3)} + (p_{21})^{(3)}\big)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G_{21}^*\mathbb{T}_{21} \quad 542$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\big((a_{22}')^{(3)} + (p_{22})^{(3)}\big)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G_{22}^*\mathbb{T}_{21} \quad 543$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\big((b_{20}')^{(3)} - (r_{20})^{(3)}\big)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\big(s_{(20)(j)}T_{20}^*\mathbb{G}_j\big) \quad 544$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\big((b_{21}')^{(3)} - (r_{21})^{(3)}\big)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\big(s_{(21)(j)}T_{21}^*\mathbb{G}_j\big) \quad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\big((b_{22}')^{(3)} - (r_{22})^{(3)}\big)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\big(s_{(22)(j)}T_{22}^*\mathbb{G}_j\big) \quad 546$$

**ASYMPTOTIC STABILITY ANALYSIS** 547

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}\ and\ (b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{25}'')^{(4)}}{\partial T_{25}}(T_{25}^*) = (q_{25})^{(4)} \quad , \frac{\partial (b_i'')^{(4)}}{\partial G_j}((G_{27})^*\,) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\big((a_{24}')^{(4)} + (p_{24})^{(4)}\big)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G_{24}^*\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\big((a_{25}')^{(4)} + (p_{25})^{(4)}\big)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G_{25}^*\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\big((a_{26}')^{(4)} + (p_{26})^{(4)}\big)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G_{26}^*\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\big((b_{24}')^{(4)} - (r_{24})^{(4)}\big)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\big(s_{(24)(j)}T_{24}^*\mathbb{G}_j\big) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\big((b_{25}')^{(4)} - (r_{25})^{(4)}\big)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\big(s_{(25)(j)}T_{25}^*\mathbb{G}_j\big) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T^*_{26}\mathbb{G}_j\right) \quad 554$$

**ASYMPTOTIC STABILITY ANALYSIS** 555

**Theorem 5:** If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(5)}\ and\ (b''_i)^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 556

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{29})^{(5)}}{\partial T_{29}}(T^*_{29}) = (q_{29})^{(5)} \ , \ \frac{\partial(b''_i)^{(5)}}{\partial G_j}\left((G_{31})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a'_{28})^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G^*_{28}\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a'_{29})^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G^*_{29}\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a'_{30})^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G^*_{30}\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b'_{28})^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T^*_{28}\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b'_{29})^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T^*_{29}\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b'_{30})^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T^*_{30}\mathbb{G}_j\right) \quad 562$$

**ASYMPTOTIC STABILITY ANALYSIS** 563

**Theorem 6:** If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(6)}\ and\ (b''_i)^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{33})^{(6)}}{\partial T_{33}}(T^*_{33}) = (q_{33})^{(6)} \ , \ \frac{\partial(b''_i)^{(6)}}{\partial G_j}\left((G_{35})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a'_{32})^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G^*_{32}\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a'_{33})^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G^*_{33}\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a'_{34})^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G^*_{34}\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b'_{32})^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T^*_{32}\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b'_{33})^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T^*_{33}\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b'_{34})^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T^*_{34}\mathbb{G}_j\right) \quad 570$$

**ASYMPTOTIC STABILITY ANALYSIS** 571

**Theorem 7:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 572

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{37})^{(7)}}{\partial T_{37}}(T^*_{37}) = (q_{37})^{(7)} \quad , \frac{\partial(b_i'')^{(7)}}{\partial G_j}\left((G_{39})^{**}\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a'_{36})^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G^*_{36}\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a'_{37})^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G^*_{37}\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a'_{38})^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G^*_{38}\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b'_{36})^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T^*_{36}\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b'_{37})^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T^*_{37}\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b'_{38})^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T^*_{38}\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

**ASYMPTOTIC STABILITY ANALYSIS**

**Theorem 8:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{41}'')^{(8)}}{\partial T_{41}}(T_{41}^*) = (q_{41})^{(8)} \quad , \frac{\partial (b_i'')^{(8)}}{\partial G_j}(\,(G_{43})^*\,) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\big((a_{40}')^{(8)} + (p_{40})^{(8)}\big)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G_{40}^*\mathbb{T}_{41} \quad (581)$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\big((a_{41}')^{(8)} + (p_{41})^{(8)}\big)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G_{41}^*\mathbb{T}_{41} \quad (582)$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\big((a_{42}')^{(8)} + (p_{42})^{(8)}\big)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G_{42}^*\mathbb{T}_{41} \quad (583)$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\big((b_{40}')^{(8)} - (r_{40})^{(8)}\big)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\big(s_{(40)(j)}T_{40}^*\mathbb{G}_j\big) \quad (584)$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\big((b_{41}')^{(8)} - (r_{41})^{(8)}\big)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\big(s_{(41)(j)}T_{41}^*\mathbb{G}_j\big) \quad (585)$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\big((b_{42}')^{(8)} - (r_{42})^{(8)}\big)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\big(s_{(42)(j)}T_{42}^*\mathbb{G}_j\big) \quad (586)$$

(586 A)

**ASYMPTOTIC STABILITY ANALYSIS**

**Theorem 9:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}\ and\ (b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.
**Proof:** Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{45}'')^{(9)}}{\partial T_{45}}(T_{45}^*) = (q_{45})^{(9)} \quad , \frac{\partial (b_i'')^{(9)}}{\partial G_j}(\,(G_{47})^*\,) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\big((a_{44}')^{(9)} + (p_{44})^{(9)}\big)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)}G_{44}^*\mathbb{T}_{45} \quad (586\,B)$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\big((a_{45}')^{(9)} + (p_{45})^{(9)}\big)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)}G_{45}^*\mathbb{T}_{45} \quad (586\,C)$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\big((a_{46}')^{(9)} + (p_{46})^{(9)}\big)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)}G_{46}^*\mathbb{T}_{45} \quad (586\,D)$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\big((b_{44}')^{(9)} - (r_{44})^{(9)}\big)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\big(s_{(44)(j)}T_{44}^*\mathbb{G}_j\big) \quad (586\,E)$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\big((b_{45}')^{(9)} - (r_{45})^{(9)}\big)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\big(s_{(45)(j)}T_{45}^*\mathbb{G}_j\big) \quad (586\,F)$$

$\frac{d\mathbb{T}_{46}}{dt} = -\left((b'_{46})^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T^*_{46}\mathbb{G}_j\right)$ 586

G

587

**The characteristic equation of this system is**

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(q_{13})^{(1)}G^*_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)}T^*_{14} + (b_{14})^{(1)}s_{(13),(14)}T^*_{14}\right)$$

$$+\left(\left((\lambda)^{(1)} + (a'_{14})^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)}G^*_{13} + (a_{13})^{(1)}(q_{14})^{(1)}G^*_{14}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)}T^*_{14} + (b_{14})^{(1)}s_{(13),(13)}T^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((b'_{13})^{(1)} + (b'_{14})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(15)}T^*_{14} + (b_{14})^{(1)}s_{(13),(15)}T^*_{13}\right)\} = 0$$

$+$

$$(\lambda)^{(2)} + (b'_{18})^{(2)} - (r_{18})^{(2)}\{\left((\lambda)^{(2)} + (a'_{18})^{(2)} + (p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^*_{17} + (a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^*_{17}\right)$$

$$+\left(\left((\lambda)^{(2)} + (a'_{17})^{(2)} + (p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^*_{16} + (a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}\right)$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2 + \left((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2 + \left((b'_{16})^{(2)} + (b'_{17})^{(2)} - (r_{16})^{(2)} + (r_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$+\left(\left((\lambda)^{(2)}\right)^2 + \left((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17} + (a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^{*}_{16}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\right)\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)\Big]$$

$$\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(21)}T^{*}_{21}\Big)$$

$$+\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^{*}_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^{*}_{21}\Big)$$

$$\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(20)}T^{*}_{20}\Big)$$

$$\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\Big)$$

$$\left((\lambda)^{(3)}\right)^{2}+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\Big)$$

$$+\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)$$

$$\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(22)}T^{*}_{20}\Big)\}=0$$

$$+$$

$$\left((\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\right)\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)\Big]$$

$$\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(25)}T^{*}_{25}\Big)$$

$$+\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^{*}_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^{*}_{25}\Big)$$

$$\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(24)}T^{*}_{24}\Big)$$

$$\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\Big)$$

$$\left((\lambda)^{(4)}\right)^{2}+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\Big)$$

$$+\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(26)}T^{*}_{24}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\right)\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(29)}T^{*}_{29}\right)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^{*}_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^{*}_{29}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(28)}T^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$+\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(30)}T^{*}_{28}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\right)\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left(\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)\right]$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(33)}T^{*}_{33}\right)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^{*}_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^{*}_{33}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(32)}T^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$+\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(34)}T^{*}_{32}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\right)\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)\right]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(37)}T^{*}_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^{*}_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^{*}_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(36)}T^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$+\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(38)}T^{*}_{36}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\right)\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(41)}T^{*}_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^{*}_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^{*}_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(40)}T^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$+\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(42)}T^{*}_{40}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\right)\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(45)}T^{*}_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^{*}_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^{*}_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(44)}T^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$+\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(46)}T^{*}_{44}\right)\}=0$$

**And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.**

## SECTION TWENTY FOUR

## Mind Influences Brain Through Proactive Spin

### INTRODUCTION—VARIABLES USED

**DOI: 10.14704/nq.2007.5.2.128 On Dark Chemistry: What's Dark Matter and How Mind Influences Brain Through Proactive Spin Huping Hu, Maoxin Wu NeuroQuantology**

(1) "Dark chemistry" is a hypothetical chemistry based on (e) the hypothetical axion dark matter. Benjamin is commendable for boldly going where no one has gone before but may find himself still

in the "bright" territory instead of the "dark" side, if he is willing to use Occam's razor to (e) cut out "dark" things and replace them with non-local effects.

(2) Based on their recent experimental findings, contentions are two-fold: (1) dark matter is (=) likely the cosmological manifestation of (e) quantum entanglement;

(3) And (2) the hypothetical axion dark matter is, therefore, replaceable by (e&eb) non-local effects mediated by (e&eb) the primordial spin processes.

(4) They also discuss the cause of (e) apparent dark energy. In particular, they explore the issue how **mind influences the (e&eb) brain through said spin processes.**

**(5)** Our thoughts are that the manifestation of (e) free will is intrinsically associated with (e&eb) the nuclear and/or electron spin processes inside (eb) the varying high electric voltage environment of (e) the neural membranes and proteins which likely enable (eb) the said spin processes to be "proactive," that is, being able to utilize non-local energy (potential) and quantum information to influence (e&eb) brain activities through (e&eb) spin chemistry and possibly other chemical/physical processes in defiance of (e) the second law of thermodynamics.

## NOTATION

### Module One

"Dark chemistry" is a hypothetical chemistry based on (e) the **hypothetical axion dark matter**.

Benjamin is commendable for boldly going where no one has gone before but may find himself still in the "bright" territory instead of the "dark" side, if he is willing to use Occam's razor to (e) cut out "dark" things and replace them with non-local effects

$G_{13}$ : Category one of "Dark chemistry" ; **hypothetical axion dark matter**

$G_{14}$ : Category two of SAS

$G_{15}$ : Category three of SAS

$T_{13}$ : Category one of **hypothetical axion dark matter** ;"Dark chemistry"

$T_{14}$ : Category two of SAS

$T_{15}$ : Category three of SAS

### Module Two

Benjamin is commendable for boldly going where no one has gone before but may find himself still in the "bright" territory instead of the "dark" side, if he is willing to use **Occam's razor** to (e) cut out "dark" things and replace them with non-local effects

$G_{16}$ : Category one of **Occam's razor;** replace them with non-local effects

$G_{17}$ : Category two of SAS

$G_{18}$ : Category three of SAS

$T_{16}$ : Category one of replace them with non-local effects **;Occam's razor**

$T_{17}$ : Category two of SAS

$T_{18}$ : Category three of SAS

### Module three

Benjamin is commendable for boldly going where no one has gone before but may find himself still in the

"bright" territory instead of the "dark" side, if he is willing to use **Occam's razor to cut out "dark" things** and replace them with non-local effects

$G_{20}$ : Category one of **Occam's razor to cut out "dark" things;** non-local effects

$G_{21}$ : Category two of SAS

$G_{22}$ : Category three of SAS

$T_{20}$ : Category one of non-local effects **;Occam's razor to cut out "dark" things**

$T_{21}$ : Category two of SAS

$T_{22}$ : Category three of SAS

**Module four**

Based on their recent experimental findings, contentions are two-fold: (1) dark matter is (=) likely the cosmological manifestation of (e) quantum entanglement

$G_{24}$ : Category one of dark matter

$G_{25}$ : Category two of SAS

$G_{26}$ : Category three of SAS

$T_{24}$ : Category one of cosmological manifestation of (e) quantum entanglement

$T_{25}$ : Category two of SAS

$T_{26}$ : Category three of SAS

**Module five**

Based on their recent experimental findings, contentions are two-fold:

(1) **dark matter is likely the cosmological manifestation** of (e) quantum entanglement

$G_{28}$ : Category one of **dark matter is likely the cosmological manifestation;** quantum entanglement

$G_{29}$ : Category two of SAS

$G_{30}$ : Category three of SAS

$T_{28}$ : Category one of quantum entanglement **;dark matter is likely the cosmological manifestation**

$T_{29}$ : Category two of SAS

$T_{30}$ : Category three of SAS

**Module six**

And (2) the hypothetical axion dark matter is, therefore, replaceable by (e&eb) non-local effects mediated by (e&eb) the primordial spin processes

$G_{32}$ : Category one of **hypothetical axion dark matter;** non-local effects mediated by (e&eb) the

primordial spin processes

$G_{33}$ : Category two of SAS

$G_{34}$ : Category three of SAS

$T_{32}$ : Category one of non-local effects mediated by (e&eb) the primordial spin processes; **hypothetical axion dark matter**

$T_{33}$ : Category two of SAS

$T_{34}$ : Category three of SAS

**Module seven**

hypothetical axion dark matter is, therefore, replaceable by non-local effects mediated by (e&eb) the **primordial spin processes**

$G_{36}$ : Category one of hypothetical axion dark matter is, therefore, replaceable by non-local effects ; **primordial spin processes**

$G_{37}$ : Category two of SAS

$G_{38}$ : Category three of SAS

$T_{36}$ : Category one of **primordial spin processes;** hypothetical axion dark matter is, therefore, replaceable by non-local effects

$T_{37}$ : Category two of SAS

$T_{38}$ : Category three of SAS

**Module eight**

They also discuss the cause of (e) apparent dark energy.
Model throws light on the mechanism of how mind influences brain through spin processes.

In particular, they explore the issue how **mind influences the (e&eb)** brain through said spin processes
$G_{40}$ : Category one of **mind;** brain through said spin processes

$G_{41}$ : Category two of SAS

$G_{42}$ : Category three of SAS

$T_{40}$ : Category one of brain through said spin processes; **mind**

$T_{41}$ : Category two of SAS

$T_{42}$ : Category three of SAS

**Module Nine**

Our thoughts are that the manifestation of (e) free will is intrinsically associated with (e&eb) the nuclear

and/or electron spin processes inside (eb) the varying high electric voltage environment of (e) the neural membranes and proteins which likely enable (eb) the said spin processes to be "proactive," that is, being able to utilize non-local energy (potential) and quantum information to influence (e&eb) brain activities through (e&eb) spin chemistry and possibly other chemical/physical processes in defiance of (e) the second law of thermodynamics

$G_{44}$ : Category one of thoughts; **manifestation of free will**

$G_{45}$ : Category two of SAS

$G_{46}$ : Category three of SAS

$T_{44}$ : Category one of **manifestation of free will;** thoughts

$T_{45}$ : Category two of SAS

$T_{46}$ : Category three of SAS

**The Coefficients:**

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}$ , $(b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}$,
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}$, $(b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}\ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$ ,
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)}$,
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)}$,
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)}$,
are Dissipation coefficients

**Module Numbered One**

**The differential system of this model is now (Module Numbered one)**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \quad 1$$
$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad 2$$
$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15} \quad 3$$
$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13} \quad 4$$
$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14} \quad 5$$

$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G,t)\right]T_{15}$ 6

$+(a''_{13})^{(1)}(T_{14},t) =$ First augmentation factor

$-(b''_{13})^{(1)}(G,t) =$ First detritions factor

**Module Numbered Two**

**The differential system of this model is now ( Module numbered two)**

$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17},t)\right]G_{16}$ 7

$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17},t)\right]G_{17}$ 8

$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17},t)\right]G_{18}$ 9

$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\left((G_{19}),t\right)\right]T_{16}$ 10

$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\left((G_{19}),t\right)\right]T_{17}$ 11

$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\left((G_{19}),t\right)\right]T_{18}$ 12

$+(a''_{16})^{(2)}(T_{17},t) =$ First augmentation factor

$-(b''_{16})^{(2)}\left((G_{19}),t\right) =$ First detritions factor

**Module Numbered Three**

**The differential system of this model is now (Module numbered three)**

$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21},t)\right]G_{20}$ 13

$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21},t)\right]G_{21}$ 14

$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21},t)\right]G_{22}$ 15

$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23},t)\right]T_{20}$ 16

$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23},t)\right]T_{21}$ 17

$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23},t)\right]T_{22}$ 18

$+(a''_{20})^{(3)}(T_{21},t) =$ First augmentation factor

$-(b''_{20})^{(3)}(G_{23},t) =$ First detritions factor

**Module Numbered Four**

**The differential system of this model is now (Module numbered Four)**

$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right]G_{24}$ 19

$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25},t)\right]G_{25}$ 20

$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25},t)\right]G_{26}$ 21

$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27}),t\right)\right]T_{24}$ 22

$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27}),t\right)\right]T_{25}$ 23

$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27}),t\right)\right]T_{26}$ 24

$+(a''_{24})^{(4)}(T_{25},t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}\left((G_{27}),t\right) =$ **First detritions factor**

**Module Numbered Five**:

**The differential system of this model is now (Module number five)**

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right]G_{28} \quad 25$$
$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}, t)\right]G_{29} \quad 26$$
$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}, t)\right]G_{30} \quad 27$$
$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31}), t)\right]T_{28} \quad 28$$
$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}((G_{31}), t)\right]T_{29} \quad 29$$
$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31}), t)\right]T_{30} \quad 30$$

$+(a''_{28})^{(5)}(T_{29}, t) =$ **First augmentation factor**

$-(b''_{28})^{(5)}((G_{31}), t) =$ **First detritions factor**

**Module Numbered Six**

**The differential system of this model is now (Module numbered Six)**

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right]G_{32} \quad 31$$
$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}, t)\right]G_{33} \quad 32$$
$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}, t)\right]G_{34} \quad 33$$
$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35}), t)\right]T_{32} \quad 34$$
$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}((G_{35}), t)\right]T_{33} \quad 35$$
$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35}), t)\right]T_{34} \quad 36$$

$+(a''_{32})^{(6)}(T_{33}, t) =$ **First augmentation factor**

**Module Numbered Seven:**

**The differential system of this model is now (Seventh Module)**

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right]G_{36} \quad 37$$
$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37}, t)\right]G_{37} \quad 38$$
$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}, t)\right]G_{38} \quad 39$$
$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39}), t)\right]T_{36} \quad 40$$
$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}((G_{39}), t)\right]T_{37} \quad 41$$
$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39}), t)\right]T_{38} \quad 42$$

$+(a''_{36})^{(7)}(T_{37}, t) =$ **First augmentation factor**

**Module Numbered Eight**

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right]G_{40} \quad 43$$
$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41}, t)\right]G_{41} \quad 44$$
$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}, t)\right]G_{42} \quad 45$$
$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43}), t)\right]T_{40} \quad 46$$
$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}((G_{43}), t)\right]T_{41} \quad 47$$
$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43}), t)\right]T_{42} \quad 48$$

**Module Numbered Nine**

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}\left((G_{47}),t\right)\right]T_{44} \quad 52$$

$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}\left((G_{47}),t\right)\right]T_{45} \quad 53$$

$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}\left((G_{47}),t\right)\right]T_{46} \quad 54$$

$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}\left((G_{47}),t\right) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)}\boxed{+(a''_{13})^{(1)}(T_{14},t)}\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7)}(T_{37},t)}\boxed{+(a''_{40})^{(8,8)}(T_{41},t)}\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)}\boxed{+(a''_{14})^{(1)}(T_{14},t)}\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7)}(T_{37},t)}\boxed{+(a''_{41})^{(8,8)}(T_{41},t)}\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)}\boxed{+(a''_{15})^{(1)}(T_{14},t)}\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7)}(T_{37},t)}\boxed{+(a''_{42})^{(8,8)}(T_{41},t)}\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15} \quad 57$$

Where $\boxed{(a''_{13})^{(1)}(T_{14},t)}$, $\boxed{(a''_{14})^{(1)}(T_{14},t)}$, $\boxed{(a''_{15})^{(1)}(T_{14},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7)}(T_{37},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8)}(T_{41},t)}$ $\boxed{+(a''_{41})^{(8,8)}(T_{41},t)}$ $\boxed{+(a''_{42})^{(8,8)}(T_{41},t)}$ are eight augmentation coefficient for 1,2,3

$\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)}\boxed{-(b''_{13})^{(1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13} \quad (58)$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)}\boxed{-(b''_{14})^{(1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14} \quad (59)$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)}\boxed{-(b''_{15})^{(1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15} \quad (60)$$

Where $\boxed{-(b''_{13})^{(1)}(G,t)}$, $\boxed{-(b''_{14})^{(1)}(G,t)}$, $\boxed{-(b''_{15})^{(1)}(G,t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8)}(G_{43},t)}$ $\boxed{-(b''_{41})^{(8,8)}(G_{43},t)}$ $\boxed{-(b''_{42})^{(8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \begin{bmatrix} (a'_{16})^{(2)}\boxed{+(a''_{16})^{(2)}(T_{17},t)} & \boxed{+(a''_{13})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{16} \quad (61)$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \begin{bmatrix} (a'_{17})^{(2)}\boxed{+(a''_{17})^{(2)}(T_{17},t)} & \boxed{+(a''_{14})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{17} \quad (62)$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \begin{bmatrix} (a'_{18})^{(2)}\boxed{+(a''_{18})^{(2)}(T_{17},t)} & \boxed{+(a''_{15})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{18} \quad (63)$$

Where $\boxed{+(a''_{16})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,)}(T_{14},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)}$ are eight augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{44})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9)}(T_{45},t)}$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \begin{bmatrix} (b'_{16})^{(2)}\boxed{-(b''_{16})^{(2)}(G_{19},t)} & \boxed{-(b''_{13})^{(1,1,)}(G,t)} & \boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{16} \quad (64)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)}\boxed{-(b''_{17})^{(2)}(G_{19},t)} & \boxed{-(b''_{14})^{(1,1,)}(G,t)} & \boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{17} \quad (65)$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)}\boxed{-(b''_{18})^{(2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,)}(G,t)} & \boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{18} \quad (66)$$

where $\boxed{-(b''_{16})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2)}(G_{19},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,)}(G,t)}$ are second detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)}$ are fourth detrition

coefficients for category 1,2 and 3

$-(b_{28}'')^{(5,5,5,5,5)}(G_{31},t)$, $-(b_{29}'')^{(5,5,5,5,5)}(G_{31},t)$, $-(b_{30}'')^{(5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1,2 and 3

$-(b_{32}'')^{(6,6,6,6,6)}(G_{35},t)$, $-(b_{33}'')^{(6,6,6,6,6)}(G_{35},t)$, $-(b_{34}'')^{(6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients for category 1,2 and 3

$-(b_{36}'')^{(7,7,7)}(G_{39},t)$, $-(b_{37}'')^{(7,7,7)}(G_{39},t)$, $-(b_{38}'')^{(7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2 and 3

$-(b_{40}'')^{(8,8,8)}(G_{43},t)$, $-(b_{41}'')^{(8,8,8)}(G_{43},t)$, $-(b_{42}'')^{(8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1,2 and 3

$-(b_{44}'')^{(9,9)}(G_{47},t)$, $-(b_{46}'')^{(9,9)}(G_{47},t)$, $-(b_{45}'')^{(9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[\begin{array}{c}(a_{20}')^{(3)}+(a_{20}'')^{(3)}(T_{21},t)\ +(a_{16}'')^{(2,2,2)}(T_{17},t)\ +(a_{13}'')^{(1,1,1,)}(T_{14},t)\\ +(a_{24}'')^{(4,4,4,4,4,4)}(T_{25},t)\ +(a_{28}'')^{(5,5,5,5,5,5)}(T_{29},t)\ +(a_{32}'')^{(6,6,6,6,6,6)}(T_{33},t)\\ +(a_{36}'')^{(7,7,7,7)}(T_{37},t)\ +(a_{40}'')^{(8,8,8,8)}(T_{41},t)\ +(a_{44}'')^{(9,9,9)}(T_{45},t)\end{array}\right]G_{20} \quad 67$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[\begin{array}{c}(a_{21}')^{(3)}+(a_{21}'')^{(3)}(T_{21},t)\ +(a_{17}'')^{(2,2,2)}(T_{17},t)\ +(a_{14}'')^{(1,1,1,)}(T_{14},t)\\ +(a_{25}'')^{(4,4,4,4,4,4)}(T_{25},t)\ +(a_{29}'')^{(5,5,5,5,5,5)}(T_{29},t)\ +(a_{33}'')^{(6,6,6,6,6,6)}(T_{33},t)\\ +(a_{37}'')^{(7,7,7,7)}(T_{37},t)\ +(a_{41}'')^{(8,8,8,8)}(T_{41},t)\ +(a_{45}'')^{(9,9,9)}(T_{45},t)\end{array}\right]G_{21} \quad 68$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[\begin{array}{c}(a_{22}')^{(3)}+(a_{22}'')^{(3)}(T_{21},t)\ +(a_{18}'')^{(2,2,2)}(T_{17},t)\ +(a_{15}'')^{(1,1,1,)}(T_{14},t)\\ +(a_{26}'')^{(4,4,4,4,4,4)}(T_{25},t)\ +(a_{30}'')^{(5,5,5,5,5,5)}(T_{29},t)\ +(a_{34}'')^{(6,6,6,6,6,6)}(T_{33},t)\\ +(a_{38}'')^{(7,7,7,7)}(T_{37},t)\ +(a_{42}'')^{(8,8,8,8)}(T_{41},t)\ +(a_{46}'')^{(9,9,9)}(T_{45},t)\end{array}\right]G_{22} \quad 69$$

$+(a_{20}'')^{(3)}(T_{21},t)$, $+(a_{21}'')^{(3)}(T_{21},t)$, $+(a_{22}'')^{(3)}(T_{21},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a_{16}'')^{(2,2,2)}(T_{17},t)$, $+(a_{17}'')^{(2,2,2)}(T_{17},t)$, $+(a_{18}'')^{(2,2,2)}(T_{17},t)$ are second augmentation coefficients for category 1, 2 and 3

$+(a_{13}'')^{(1,1,1,)}(T_{14},t)$, $+(a_{14}'')^{(1,1,1,)}(T_{14},t)$, $+(a_{15}'')^{(1,1,1,)}(T_{14},t)$ are third augmentation coefficients for category 1, 2 and 3

$+(a_{24}'')^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a_{25}'')^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a_{26}'')^{(4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficients for category 1, 2 and 3

$+(a_{28}'')^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a_{29}'')^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a_{30}'')^{(5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficients for category 1, 2 and 3

$+(a_{32}'')^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a_{33}'')^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a_{34}'')^{(6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficients for category 1, 2 and 3

$+(a_{36}'')^{(7,7,7,7)}(T_{37},t)$, $+(a_{37}'')^{(7,7,7,7)}(T_{37},t)$, $+(a_{38}'')^{(7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1, 2 and 3

$+(a_{40}'')^{(8,8,8,8)}(T_{41},t)$, $+(a_{41}'')^{(8,8,8,8)}(T_{41},t)$, $+(a_{42}'')^{(8,8,8,8)}(T_{41},t)$ are eight augmentation coefficients for category 1, 2 and 3

$+(a_{44}'')^{(9,9,9)}(T_{45},t)$, $+(a_{45}'')^{(9,9,9)}(T_{45},t)$, $+(a_{46}'')^{(9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for

category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b'_{20})^{(3)}\boxed{-(b''_{20})^{(3)}(G_{23},t)}\boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}\boxed{-(b''_{13})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)}\boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)}\boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7)}(G_{39},t)}\boxed{-(b''_{40})^{(8,8,8,8)}(G_{43},t)}\boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{20} \quad (70)$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b'_{21})^{(3)}\boxed{-(b''_{21})^{(3)}(G_{23},t)}\boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}\boxed{-(b''_{14})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)}\boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)}\boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7)}(G_{39},t)}\boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)}\boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{21} \quad (71)$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)}\boxed{-(b''_{22})^{(3)}(G_{23},t)}\boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}\boxed{-(b''_{15})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)}\boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)}\boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)}\boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)}\boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{22} \quad (72)$$

$\boxed{-(b''_{20})^{(3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3)}(G_{23},t)}$ *are first detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}$ *are second detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{13})^{(1,1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,)}(G,t)}$ *are third detrition coefficients for category 1,2 and 3*

$\boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)}$ *are fourth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)}$ *are fifth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)}$ *are sixth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{36})^{(7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7)}(G_{39},t)}$ $\boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \begin{bmatrix} (a'_{24})^{(4)}\boxed{+(a''_{24})^{(4)}(T_{25},t)}\boxed{+(a''_{28})^{(5,5,)}(T_{29},t)}\boxed{+(a''_{32})^{(6,6,)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)}\boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)}\boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)}\boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)}\boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{24} \quad (73)$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \begin{bmatrix} (a'_{25})^{(4)}\boxed{+(a''_{25})^{(4)}(T_{25},t)}\boxed{+(a''_{29})^{(5,5,)}(T_{29},t)}\boxed{+(a''_{33})^{(6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)}\boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)}\boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)}\boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)}\boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{25} \quad (74)$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \begin{bmatrix} (a'_{26})^{(4)} & +(a''_{26})^{(4)}(T_{25},t) & +(a''_{30})^{(5,5,)}(T_{29},t) & +(a''_{34})^{(6,6,)}(T_{33},t) \\ +(a''_{15})^{(1,1,1,1)}(T_{14},t) & +(a''_{18})^{(2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3)}(T_{21},t) & \\ +(a''_{38})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9)}(T_{45},t) & \end{bmatrix} G_{26} \quad 75$$

$(a''_{24})^{(4)}(T_{25},t)$, $(a''_{25})^{(4)}(T_{25},t)$, $(a''_{26})^{(4)}(T_{25},t)$ *are first augmentation coefficients category* 1, 2 3

$+(a''_{28})^{(5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,)}(T_{29},t)$ *are second augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{32})^{(6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,)}(T_{33},t)$ *are third augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{13})^{(1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1)}(T_{14},t)$ *are fourth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{16})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2)}(T_{17},t)$ *are fifth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{20})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3)}(T_{21},t)$ *are sixth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)$ *are seventh augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)$ *are eighth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{46})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9)}(T_{45},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \begin{bmatrix} (b'_{24})^{(4)} & -(b''_{24})^{(4)}(G_{27},t) & -(b''_{28})^{(5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,)}(G_{35},t) \\ -(b''_{13})^{(1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3)}(G_{23},t) & \\ -(b''_{36})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9)}(G_{47},t) & \end{bmatrix} T_{24} \quad 76$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \begin{bmatrix} (b'_{25})^{(4)} & -(b''_{25})^{(4)}(G_{27},t) & -(b''_{29})^{(5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,)}(G_{35},t) \\ -(b''_{14})^{(1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3)}(G_{23},t) & \\ -(b''_{37})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9)}(G_{47},t) & \end{bmatrix} T_{25} \quad 77$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)} & -(b''_{26})^{(4)}(G_{27},t) & -(b''_{30})^{(5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,)}(G_{35},t) \\ -(b''_{15})^{(1,1,1,1)}(G,t) & -(b''_{18})^{(2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3)}(G_{23},t) & \\ -(b''_{38})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9)}(G_{47},t) & \end{bmatrix} T_{26} \quad 78$$

*Where* $-(b''_{24})^{(4)}(G_{27},t)$, $-(b''_{25})^{(4)}(G_{27},t)$, $-(b''_{26})^{(4)}(G_{27},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{28})^{(5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,)}(G_{31},t)$ *are second detrition coefficients for category* 1, 2 *and* 3

$-(b''_{32})^{(6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,)}(G_{35},t)$ *are third detrition coefficients for category* 1, 2 *and* 3

$-(b''_{13})^{(1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1)}(G,t)$

*are fourth detrition coefficients for category* 1, 2 *and* 3

$-(b''_{16})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2)}(G_{19},t)$

*are fifth detrition coefficients for category* 1, 2 *and* 3

$-(b''_{20})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3)}(G_{23},t)$

*are sixth detrition coefficients for category* 1, 2 *and* 3

$-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)$

*are seventh detrition coefficients for category* 1, 2 *and* 3

$-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)$

*are eighth detrition coefficients for category* 1, 2 *and* 3

$-(b''_{46})^{(9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \begin{bmatrix} (a'_{28})^{(5)} & +(a''_{28})^{(5)}(T_{29},t) & +(a''_{24})^{(4,4,)}(T_{25},t) & +(a''_{32})^{(6,6,6)}(T_{33},t) \\ +(a''_{13})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{16})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3,3)}(T_{21},t) \\ +(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{28} \quad (79)$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \begin{bmatrix} (a'_{29})^{(5)} & +(a''_{29})^{(5)}(T_{29},t) & +(a''_{25})^{(4,4,)}(T_{25},t) & +(a''_{33})^{(6,6,6)}(T_{33},t) \\ +(a''_{14})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{17})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3,3)}(T_{21},t) \\ +(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{29} \quad (80)$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \begin{bmatrix} (a'_{30})^{(5)} & +(a''_{30})^{(5)}(T_{29},t) & +(a''_{26})^{(4,4,)}(T_{25},t) & +(a''_{34})^{(6,6,6)}(T_{33},t) \\ +(a''_{15})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{18})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3,3)}(T_{21},t) \\ +(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{30} \quad (81)$$

*Where* $+(a''_{28})^{(5)}(T_{29},t)$, $+(a''_{29})^{(5)}(T_{29},t)$, $+(a''_{30})^{(5)}(T_{29},t)$ *are first augmentation coefficients for category* 1, 2 *and* 3

*And* $+(a''_{24})^{(4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,)}(T_{25},t)$ *are second augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{32})^{(6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6)}(T_{33},t)$ *are third augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)$ are fourth augmentation coefficients for category 1,2, and 3

$+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)$ are fifth augmentation coefficients for category 1,2,and 3

$+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)$ are sixth augmentation coefficients for category 1,2, 3

$+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1,2, 3

$+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)$ are eighth augmentation coefficients for category 1,2, 3

$+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)$ are ninth augmentation

coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \begin{bmatrix} (b'_{28})^{(5)}\boxed{-(b''_{28})^{(5)}(G_{31},t)} & \boxed{-(b''_{24})^{(4,4,)}(G_{27},t)} & \boxed{-(b''_{32})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{28} \quad (82)$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b'_{29})^{(5)}\boxed{-(b''_{29})^{(5)}(G_{31},t)} & \boxed{-(b''_{25})^{(4,4,)}(G_{27},t)} & \boxed{-(b''_{33})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{29} \quad (83)$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b'_{30})^{(5)}\boxed{-(b''_{30})^{(5)}(G_{31},t)} & \boxed{-(b''_{26})^{(4,4,)}(G_{27},t)} & \boxed{-(b''_{34})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{30} \quad (84)$$

*where* $\boxed{-(b''_{28})^{(5)}(G_{31},t)}$ , $\boxed{-(b''_{29})^{(5)}(G_{31},t)}$ , $\boxed{-(b''_{30})^{(5)}(G_{31},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{24})^{(4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,)}(G_{27},t)}$ *are second detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{32})^{(6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6)}(G_{35},t)}$ *are third detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1)}(G,t)}$ , $\boxed{-(b''_{15})^{(1,1,1,1,1,)}(G,t)}$ are fourth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)}$ , $\boxed{-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)}$ , $\boxed{-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)}$ , $\boxed{-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)}$ , $\boxed{-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)}$ , $\boxed{-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)}$ , $\boxed{-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \begin{bmatrix} (a'_{32})^{(6)}\boxed{+(a''_{32})^{(6)}(T_{33},t)} & \boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{32} \quad (85)$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \begin{bmatrix} (a'_{33})^{(6)}\boxed{+(a''_{33})^{(6)}(T_{33},t)} & \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{33} \quad (86)$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \begin{bmatrix} (a'_{34})^{(6)}\boxed{+(a''_{34})^{(6)}(T_{33},t)} & \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{34} \quad 87$$

$\boxed{+(a''_{32})^{(6)}(T_{33},t)}, \boxed{+(a''_{33})^{(6)}(T_{33},t)}, \boxed{+(a''_{34})^{(6)}(T_{33},t)}$ *are first augmentation coefficients for category* $1, 2$ *and* $3$

$\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}$ *are second augmentation coefficients for category* $1, 2$ *and* $3$

$\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}$ *are third augmentation coefficients for category* $1, 2$ *and* $3$

$\boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}$ - are fourth augmentation coefficients

$\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}$ - fifth augmentation coefficients

$\boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)}$ sixth augmentation coefficients

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)}$ seventh augmentation coefficients

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}$ Eighth augmentation coefficients

$\boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)}$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \begin{bmatrix} (b'_{32})^{(6)}\boxed{-(b''_{32})^{(6)}(G_{35},t)} & \boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{32} \quad 88$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \begin{bmatrix} (b'_{33})^{(6)}\boxed{-(b''_{33})^{(6)}(G_{35},t)} & \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{33} \quad 89$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)}\boxed{-(b''_{34})^{(6)}(G_{35},t)} & \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{34} \quad 90$$

$\boxed{-(b''_{32})^{(6)}(G_{35},t)}, \boxed{-(b''_{33})^{(6)}(G_{35},t)}, \boxed{-(b''_{34})^{(6)}(G_{35},t)}$ *are first detrition coefficients for category* $1, 2$ *and* $3$

$\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}$ *are second detrition coefficients for category* $1, 2$ *and* $3$

$\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)}$ *are third detrition coefficients*

$for\ category\ 1,2\ and\ 3$

$-(b_{13}'')^{(1,1,1,1,1,1,1)}(G,t)$, $-(b_{14}'')^{(1,1,1,1,1,1,1)}(G,t)$, $-(b_{15}'')^{(1,1,1,1,1,1,1)}(G,t)$ are fourth detrition coefficients for category 1, 2, and 3

$-(b_{16}'')^{(2,2,2,2,2,2,2)}(G_{19},t)$, $-(b_{17}'')^{(2,2,2,2,2,2,2)}(G_{19},t)$, $-(b_{18}'')^{(2,2,2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1, 2, and 3

$-(b_{20}'')^{(3,3,3,3,3,3,3)}(G_{23},t)$, $-(b_{21}'')^{(3,3,3,3,3,3,3)}(G_{23},t)$, $-(b_{22}'')^{(3,3,3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1, 2, and 3

$-(b_{36}'')^{(7,7,7,7,7,7,7)}(G_{39},t)$, $-(b_{37}'')^{(7,7,7,7,7,7,7)}(G_{39},t)$, $-(b_{38}'')^{(7,7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2, and 3

$-(b_{40}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)$, $-(b_{41}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)$, $-(b_{42}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1, 2, and 3

$-(b_{46}'')^{(9,9,9,9,9,9,9)}(G_{47},t)$, $-(b_{45}'')^{(9,9,9,9,9,9,9)}(G_{47},t)$, $-(b_{44}'')^{(9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \qquad 91$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a_{36}')^{(7)} & +(a_{36}'')^{(7)}(T_{37},t) & +(a_{16}'')^{(2,2,2,2,2,2,2)}(T_{17},t) & +(a_{20}'')^{(3,3,3,3,3,3,3)}(T_{21},t) \\ & +(a_{24}'')^{(4,4,4,4,4,4,4)}(T_{25},t) & +(a_{28}'')^{(5,5,5,5,5,5,5)}(T_{29},t) & +(a_{32}'')^{(6,6,6,6,6,6,6)}(T_{33},t) \\ & +(a_{13}'')^{(1,1,1,1,1,1,1)}(T_{14},t) & +(a_{40}'')^{(8,8,8,8,8,8,8,8,)}(T_{41},t) & +(a_{44}'')^{(9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \qquad 92$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a_{37}')^{(7)} & +(a_{37}'')^{(7)}(T_{37},t) & +(a_{17}'')^{(2,2,2,2,2,2,2)}(T_{17},t) & +(a_{21}'')^{(3,3,3,3,3,3,3)}(T_{21},t) \\ & +(a_{25}'')^{(4,4,4,4,4,4,4)}(T_{25},t) & +(a_{29}'')^{(5,5,5,5,5,5,5)}(T_{29},t) & +(a_{33}'')^{(6,6,6,6,6,6,6)}(T_{33},t) \\ & +(a_{13}'')^{(1,1,1,1,1,1,1)}(T_{14},t) & +(a_{41}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t) & +(a_{45}'')^{(9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{14}$$

$$\frac{dG_{38}}{dt} \qquad 93$$

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a_{38}')^{(7)} & +(a_{38}'')^{(7)}(T_{37},t) & +(a_{18}'')^{(2,2,2,2,2,2,2)}(T_{17},t) & +(a_{22}'')^{(3,3,3,3,3,3,3)}(T_{21},t) \\ & +(a_{26}'')^{(4,4,4,4,4,4,4)}(T_{25},t) & +(a_{30}'')^{(5,5,5,5,5,5,5)}(T_{29},t) & +(a_{34}'')^{(6,6,6,6,6,6,6)}(T_{33},t) \\ & +(a_{15}'')^{(1,1,1,1,1,1,1)}(T_{14},t) & +(a_{42}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t) & +(a_{46}'')^{(9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{15}$$

Where $(a_{36}'')^{(7)}(T_{37},t)$, $(a_{37}'')^{(7)}(T_{37},t)$, $(a_{38}'')^{(7)}(T_{37},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a_{16}'')^{(2,2,2,2,2,2,2)}(T_{17},t)$, $+(a_{17}'')^{(2,2,2,2,2,2,2)}(T_{17},t)$, $+(a_{18}'')^{(2,2,2,2,2,2,2)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a_{20}'')^{(3,3,3,3,3,3,3)}(T_{21},t)$, $+(a_{21}'')^{(3,3,3,3,3,3,3)}(T_{21},t)$, $+(a_{22}'')^{(3,3,3,3,3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a_{24}'')^{(4,4,4,4,4,4,4)}(T_{25},t)$, $+(a_{25}'')^{(4,4,4,4,4,4,4)}(T_{25},t)$, $+(a_{26}'')^{(4,4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a_{28}'')^{(5,5,5,5,5,5,5)}(T_{29},t)$, $+(a_{29}'')^{(5,5,5,5,5,5,5)}(T_{29},t)$, $+(a_{30}'')^{(5,5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a_{32}'')^{(6,6,6,6,6,6,6)}(T_{33},t)$, $+(a_{33}'')^{(6,6,6,6,6,6,6)}(T_{33},t)$, $+(a_{34}'')^{(6,6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation

coefficient for category 1, 2 and 3
$+(a_{13}'')^{(1,1,1,1,1,1,1)}(T_{14},t)$, $+(a_{13}'')^{(1,1,1,1,1,1,1)}(T_{14},t)$, $+(a_{15}'')^{(1,1,1,1,1,1,1)}(T_{14},t)$ are seventh augmentation coefficient for category 1, 2 and 3
$+(a_{42}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a_{41}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a_{40}'')^{(8,8,8,8,8,8,8,8,)}(T_{41},t)$ are eighth augmentation coefficient for 1,2,3
$+(a_{46}'')^{(9,9,9,9,9,9,9)}(T_{45},t)$, $+(a_{45}'')^{(9,9,9,9,9,9,9)}(T_{45},t)$, $+(a_{44}'')^{(9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3



$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \begin{bmatrix} (b_{36}')^{(7)}\boxed{-(b_{36}'')^{(7)}(G_{39},t)} & \boxed{-(b_{16}'')^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{20}'')^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{24}'')^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{28}'')^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{32}'')^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{13}'')^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b_{44}'')^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \begin{bmatrix} (b_{37}')^{(7)}\boxed{-(b_{37}'')^{(7)}(G_{39},t)} & \boxed{-(b_{17}'')^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{21}'')^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{25}'')^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{29}'')^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{33}'')^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{14}'')^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b_{45}'')^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \begin{bmatrix} (b_{38}')^{(7)}\boxed{-(b_{38}'')^{(7)}(G_{39},t)} & \boxed{-(b_{18}'')^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{22}'')^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{26}'')^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{30}'')^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{34}'')^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{15}'')^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b_{46}'')^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b_{36}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{37}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3
$\boxed{-(b_{16}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{17}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{18}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3
$\boxed{-(b_{20}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{21}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{22}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3
$\boxed{-(b_{24}'')^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{25}'')^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{26}'')^{(4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3
$\boxed{-(b_{28}'')^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3
$\boxed{-(b_{32}'')^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{34}'')^{(6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3
$\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3
$\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3



$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \begin{bmatrix} (a'_{40})^{(8)}\boxed{+(a''_{40})^{(8)}(T_{41},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \begin{bmatrix} (a'_{41})^{(8)}\boxed{+(a''_{41})^{(8)}(T_{41},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \begin{bmatrix} (a'_{42})^{(8)}\boxed{+(a''_{42})^{(8)}(T_{41},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a''_{40})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} =$$

$$(b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)}\boxed{-(b''_{40})^{(8)}(G_{43},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} =$$

$$(b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)}\boxed{-(b''_{41})^{(8)}(G_{43},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} =$$

$$(b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)}\boxed{-(b''_{42})^{(8)}(G_{43},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3



$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \begin{bmatrix} (a'_{44})^{(9)}\boxed{+(a''_{44})^{(9)}(T_{45},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \begin{bmatrix} (a'_{45})^{(9)}\boxed{+(a''_{45})^{(9)}(T_{45},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \begin{bmatrix} (a'_{46})^{(9)}\boxed{+(a''_{46})^{(9)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a''_{44})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ are Seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} -$$

$$\begin{bmatrix} (b'_{44})^{(9)}\boxed{-(b''_{44})^{(9)}(G_{47},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b'_{45})^{(9)}\boxed{-(b''_{45})^{(9)}(G_{47},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b'_{46})^{(9)}\boxed{-(b''_{46})^{(9)}(G_{47},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{44})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9)}(G_{47},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a'_i)^{(1)}, (a''_i)^{(1)}, (b_i)^{(1)}, (b'_i)^{(1)}, (b''_i)^{(1)} > 0, \quad i,j = 13,14,15 \qquad 97$$

The functions $(a''_i)^{(1)}, (b''_i)^{(1)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$(a_i'')^{(1)}(T_{14},t) \leq (p_i)^{(1)} \leq (\hat{A}_{13})^{(1)}$

$(b_i'')^{(1)}(G,t) \leq (r_i)^{(1)} \leq (b_i')^{(1)} \leq (\hat{B}_{13})^{(1)}$

$lim_{T_2\to\infty}(a_i'')^{(1)}(T_{14},t) = (p_i)^{(1)}$ 98

$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = (r_i)^{(1)}$

**Definition of** $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$ :

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$|(a_i'')^{(1)}(T_{14}',t) - (a_i'')^{(1)}(T_{14},t)| \leq (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$

$|(b_i'')^{(1)}(G',t) - (b_i'')^{(1)}(G,t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}',t)$ and $(a_i'')^{(1)}(T_{14},t)$ . $(T_{14}',t)$ and $(T_{14},t)$ are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14},t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$ : 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}$ , $\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$

**Definition of** $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$ : 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$

$\frac{1}{(\hat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$

Where we suppose

$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i,j = 16,17,18$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17},t) \le (p_i)^{(2)} \le \left(\hat{A}_{16}\right)^{(2)} \quad 102$$

$$(b_i'')^{(2)}(G_{19},t) \le (r_i)^{(2)} \le (b_i')^{(2)} \le (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2\to\infty}(a_i'')^{(2)}(T_{17},t) = (p_i)^{(2)} \quad 104$$

$$\lim_{G\to\infty}(b_i'')^{(2)}\left((G_{19}),t\right) = (r_i)^{(2)} \quad 105$$

**Definition of** $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$ : 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}',t) - (a_i'')^{(2)}(T_{17},t)| \le (\hat{k}_{16})^{(2)}|T_{17} - T_{17}'|e^{-(\hat{M}_{16})^{(2)}t} \quad 107$$

$$|(b_i'')^{(2)}((G_{19})',t) - (b_i'')^{(2)}\left((G_{19}),t\right)| < (\hat{k}_{16})^{(2)}||(G_{19}) - (G_{19})'||e^{-(\hat{M}_{16})^{(2)}t} \quad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}',t)$ and$(a_i'')^{(2)}(T_{17},t)$ . $(T_{17}',t)$ and $(T_{17},t)$ are points belonging to the interval $\left[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}\right]$ . It is to be noted that $(a_i'')^{(2)}(T_{17},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17},t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$ :

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants 109

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$$

**Definition of** $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$ :

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1 \quad 110$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(b_i)^{(2)} + (b_i')^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1 \quad 111$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i,j = 20,21,22 \quad 112$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(3)}$, $(\mathrm{r_i})^{(3)}$:

$$(a_i'')^{(3)}(T_{21},t) \le (p_i)^{(3)} \le (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23},t) \le (r_i)^{(3)} \le (b_i')^{(3)} \le (\hat{B}_{20})^{(3)}$$

$$lim_{T_2\to\infty}(a_i'')^{(3)}(T_{21},t) = (p_i)^{(3)} \qquad 113$$

$$\lim_{G\to\infty}(b_i'')^{(3)}(G_{23},t) = (r_i)^{(3)}$$

**Definition of** $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$ :

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: 114

$$|(a_i'')^{(3)}(T_{21}',t) - (a_i'')^{(3)}(T_{21},t)| \le (\hat{k}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\hat{M}_{20})^{(3)}t}$$

$$|(b_i'')^{(3)}(G_{23}',t) - (b_i'')^{(3)}(G_{23},t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}'||e^{-(\hat{M}_{20})^{(3)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}',t)$ and$(a_i'')^{(3)}(T_{21},t)$ . $(T_{21}',t)$ And $(T_{21},t)$ are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$ : 115

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$ and the constants $(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}, i = 20,21,22,$ satisfy the inequalities 116

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(a_i)^{(3)} + (a_i')^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(b_i)^{(3)} + (b_i')^{(3)} + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i,j = 24,25,26 \qquad 117$$

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(4)}$, $(r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25},t) \le (p_i)^{(4)} \le (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}\big((G_{27}),t\big) \le (r_i)^{(4)} \le (b_i')^{(4)} \le (\hat{B}_{24})^{(4)}$$

$$lim_{T_2\to\infty}(a_i'')^{(4)}(T_{25},t) = (p_i)^{(4)} \quad (118)$$

$$\lim_{G\to\infty}(b_i'')^{(4)}\big((G_{27}),t\big) = (r_i)^{(4)}$$

**Definition of** $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$ :

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24,25,26}$

They satisfy Lipschitz condition: (119)

$$|(a_i'')^{(4)}(T_{25}',t) - (a_i'')^{(4)}(T_{25},t)| \le (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})',t) - (b_i'')^{(4)}\big((G_{27}),t\big)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}',t)$ and$(a_i'')^{(4)}(T_{25},t)$ . $(T_{25}',t)$ and $(T_{25},t)$ are points belonging to the interval $\big[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}\big]$. It is to be noted that $(a_i'')^{(4)}(T_{25},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25},t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$ : (120)

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$, are positive constants

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}, \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$$

**Definition of** $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$ : (121)

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and\ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}, i = 24,25,26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(a_i)^{(4)} + (a_i')^{(4)} + (\hat{A}_{24})^{(4)} + (\hat{P}_{24})^{(4)}(\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(b_i)^{(4)} + (b_i')^{(4)} + (\hat{B}_{24})^{(4)} + (\hat{Q}_{24})^{(4)}(\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i,j = 28,29,30 \quad (122)$$

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(5)}$, $(r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29},t) \le (p_i)^{(5)} \le (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}\big((G_{31}),t\big) \le (r_i)^{(5)} \le (b_i')^{(5)} \le (\hat{B}_{28})^{(5)}$$

$$lim_{T_2\to\infty}(a_i'')^{(5)}(T_{29},t) = (p_i)^{(5)} \qquad 123$$

$$\lim_{G\to\infty}(b_i'')^{(5)}(G_{31},t) = (r_i)^{(5)}$$

**Definition of** $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$ :

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}',t) - (a_i'')^{(5)}(T_{29},t)| \le (\hat{k}_{28})^{(5)}|T_{29} - T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})',t) - (b_i'')^{(5)}\big((G_{31}),t\big)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}',t)$ and$(a_i'')^{(5)}(T_{29},t)$ . $(T_{29}',t)$ and $(T_{29},t)$ are points belonging to the interval $\big[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\big]$. It is to be noted that $(a_i'')^{(5)}(T_{29},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29},t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$ : 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}, \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

**Definition of** $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$ : 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and\ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(a_i)^{(5)} + (a_i')^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)}(\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(b_i)^{(5)} + (b_i')^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)}(\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i,j = 32,33,34 \qquad 127$$

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(6)}$, $(r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33},t) \le (p_i)^{(6)} \le (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}),t) \le (r_i)^{(6)} \le (b_i')^{(6)} \le (\hat{B}_{32})^{(6)}$$

$$lim_{T_2\to\infty}(a_i'')^{(6)}(T_{33},t) = (p_i)^{(6)} \qquad 128$$

$$\lim_{G\to\infty}(b_i'')^{(6)}\left((G_{35}),t\right) = (r_i)^{(6)}$$

**Definition of** $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$ :

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}',t) - (a_i'')^{(6)}(T_{33},t)| \le (\hat{k}_{32})^{(6)}|T_{33} - T_{33}'|e^{-(\hat{M}_{32})^{(6)}t}$$

$$|(b_i'')^{(6)}((G_{35})',t) - (b_i'')^{(6)}\left((G_{35}),t\right)| < (\hat{k}_{32})^{(6)}||(G_{35}) - (G_{35})'||e^{-(\hat{M}_{32})^{(6)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}',t)$ and$(a_i'')^{(6)}(T_{33},t)$ . $(T_{33}',t)$ and $(T_{33},t)$ are points belonging to the interval $\left[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}\right]$. It is to be noted that $(a_i'')^{(6)}(T_{33},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33},t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$ : 129

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$, are positive constants

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}} \ , \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$$

**Definition of** $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$ : 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and\ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32,33,34,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(a_i)^{(6)} + (a_i')^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(b_i)^{(6)} + (b_i')^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

(WWWWW) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i,j = 36,37,38$ 131

(XXXXX) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(7)}$, $(r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37},t) \le (p_i)^{(7)} \le (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39},t) \le (r_i)^{(7)} \le (b_i')^{(7)} \le (\hat{B}_{36})^{(7)}$$

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*(YYYYY)* $\lim_{T_2\to\infty} (a_i'')^{(7)}(T_{37},t) = (p_i)^{(7)}$

*(ZZZZZ)*

$\lim_{G\to\infty}(b_i'')^{(7)}\big((G_{39}),t\big) = (r_i)^{(7)}$

**Definition of** $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$ :

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$$|(a_i'')^{(7)}(T_{37}',t) - (a_i'')^{(7)}(T_{37},t)| \le (\hat{k}_{36})^{(7)}|T_{37} - T_{37}'|e^{-(\hat{M}_{36})^{(7)}t}$$

$$|(b_i'')^{(7)}((G_{39})',t) - (b_i'')^{(7)}\big((G_{39}),t\big)| < (\hat{k}_{36})^{(7)}||(G_{39}) - (G_{39})'||e^{-(\hat{M}_{36})^{(7)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}',t)$ and$(a_i'')^{(7)}(T_{37},t)$ . $(T_{37}',t)$ and $(T_{37},t)$ are points belonging to the interval $\big[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}\big]$. It is to be noted that $(a_i'')^{(7)}(T_{37},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37},t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$ : 134

(AAAAAA) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$, are positive constants

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$$

**Definition of** $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$ : 135

(BBBBBB) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)}(\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(b_i)^{(7)} + (b_i')^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)}(\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40,41,42 \tag{136}$$

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

**Definition of** $(p_i)^{(8)}$, $(r_i)^{(8)}$: (137)

$$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)} \tag{138}$$

$$(b_i'')^{(8)}((G_{43}), t) \leq (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)} \tag{139}$$

$$\lim_{T_2 \to \infty} (a_i'')^{(8)}(T_{41}, t) = (p_i)^{(8)} \tag{140}$$

$$\lim_{G \to \infty} (b_i'')^{(8)}\left((G_{43}), t\right) = (r_i)^{(8)} \tag{141}$$

**Definition of** $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$ :

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)} |T_{41} - T_{41}'| e^{-(\hat{M}_{40})^{(8)} t} \tag{142}$$

$$|(b_i'')^{(8)}((G_{43})', t) - (b_i'')^{(8)}\left((G_{43}), t\right)| < (\hat{k}_{40})^{(8)} ||(G_{43}) - (G_{43})'|| e^{-(\hat{M}_{40})^{(8)} t} \tag{143}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}', t)$ and $(a_i'')^{(8)}(T_{41}, t)$ . $(T_{41}', t)$ and $(T_{41}, t)$ are points belonging to the interval $\left[(\hat{k}_{40})^{(8)}, (\hat{M}_{40})^{(8)}\right]$. It is to be noted that $(a_i'')^{(8)}(T_{41}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{40})^{(8)} = 1$ then the function $(a_i'')^{(8)}(T_{41}, t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$ :

$(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$, are positive constants

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}, \frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1 \tag{144}$$

**Definition of** $(\hat{P}_{40})^{(8)}, (\hat{Q}_{40})^{(8)}$ :

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}, (\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)}, (a_i')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (p_i)^{(8)}$, $(r_i)^{(8)}, i = 40,41,42,$
Satisfy the inequalities

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(a_i)^{(8)} + (a_i')^{(8)} + (\hat{A}_{40})^{(8)} + (\hat{P}_{40})^{(8)} (\hat{k}_{40})^{(8)}] < 1 \tag{145}$$

$$\frac{1}{(\widehat{M}_{40})^{(8)}}[\ (b_i)^{(8)} + (b_i')^{(8)} + \ (\hat{B}_{40})^{(8)} + \ (\hat{Q}_{40})^{(8)}\ (\hat{k}_{40})^{(8)}] < 1 \qquad 146$$

Where we suppose

$$(a_i)^{(9)}, (a_i')^{(9)}, (a_i'')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (b_i'')^{(9)} > 0, \quad i,j = 44,45,46 \qquad 146\ A$$

The functions $(a_i'')^{(9)}, (b_i'')^{(9)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(9)}$, $(r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45}, t) \leq (p_i)^{(9)} \leq (\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47}, t) \leq \ (r_i)^{(9)} \leq (b_i')^{(9)} \leq (\hat{B}_{44})^{(9)}$$

$$\lim_{T_2 \to \infty} (a_i'')^{(9)}(T_{45}, t) = (p_i)^{(9)}$$

$$\lim_{G \to \infty} (b_i'')^{(9)}(G_{47}, t) = \ (r_i)^{(9)}$$

**Definition of** $(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}$ :

Where $\boxed{(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}, (p_i)^{(9)}, \ (r_i)^{(9)}}$ are positive constants and $\boxed{i = 44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}', t) - (a_i'')^{(9)}(T_{45}, t)| \leq (\hat{k}_{44})^{(9)} |T_{45} - T_{45}'| e^{-(\widehat{M}_{44})^{(9)} t}$$

$$|(b_i'')^{(9)}((G_{47})', t) - (b_i'')^{(9)}\big((G_{47}), t\big)| < (\hat{k}_{44})^{(9)} ||(G_{47}) - (G_{47})'|| e^{-(\widehat{M}_{44})^{(9)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}', t)$ and$(a_i'')^{(9)}(T_{45}, t)$ . $(T_{45}', t)$ and $(T_{45}, t)$ are points belonging to the interval $\left[(\hat{k}_{44})^{(9)}, (\widehat{M}_{44})^{(9)}\right]$. It is to be noted that $(a_i'')^{(9)}(T_{45}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{44})^{(9)} = 1$ then the function $(a_i'')^{(9)}(T_{45}, t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\widehat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$ :

$(\widehat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$, are positive constants

$$\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}} \ , \frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}} < 1$$

**Definition of** $(\hat{P}_{44})^{(9)}, (\hat{Q}_{44})^{(9)}$ :

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\widehat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}, (\hat{A}_{44})^{(9)} and\ (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)}, (a_i')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (p_i)^{(9)}, \ (r_i)^{(9)}, i = 44,45,46,$
satisfy the inequalities

$$\frac{1}{(\widehat{M}_{44})^{(9)}}[(a_i)^{(9)} + (a_i')^{(9)} + (\hat{A}_{44})^{(9)} + (\hat{P}_{44})^{(9)}(\hat{k}_{44})^{(9)}] < 1$$

$$\frac{1}{(\widehat{M}_{44})^{(9)}}[(b_i)^{(9)} + (b_i')^{(9)} + (\hat{B}_{44})^{(9)} + (\hat{Q}_{44})^{(9)}(\hat{k}_{44})^{(9)}] < 1$$

**Theorem 1:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0), T_i(0)$ :

$$G_i(t) \le (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 2 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

**Definition of** $G_i(0), T_i(0)$

$$G_i(t) \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} \quad , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} \quad , \quad T_i(0) = T_i^0 > 0$$

**Theorem 3 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t) \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t} \quad , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t} \quad , \quad T_i(0) = T_i^0 > 0$$

**Theorem 4 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

**Definition of** $G_i(0), T_i(0)$ :

$$G_i(t) \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 5 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

**Definition of** $G_i(0), T_i(0)$ :

$$G_i(t) \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 6 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

**Definition of** $G_i(0), T_i(0)$ :

$$G_i(t) \le (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 7:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

**Definition of** $G_i(0), T_i(0)$ :

$$G_i(t) \le (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$
$$T_i(t) \le (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 8:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

**Definition of** $G_i(0), T_i(0)$ :

$$G_i(t) \le (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$
$$T_i(t) \le (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t} \quad , \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 9:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

**Definition of** $G_i(0), T_i(0)$ :

$$G_i(t) \le (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

**Proof:** Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$$G_i(0) = G_i^0 \ , \ T_i(0) = T_i^0 \ , \ G_i^0 \le (\hat{P}_{13})^{(1)} , T_i^0 \le (\hat{Q}_{13})^{(1)}, \quad 155$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t} \quad 156$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t} \quad 157$$

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[ (a_{13})^{(1)} G_{14}(s_{(13)}) - \left( (a'_{13})^{(1)} + a''_{13})^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \right) G_{13}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[ (a_{14})^{(1)} G_{13}(s_{(13)}) - \left( (a'_{14})^{(1)} + (a''_{14})^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \right) G_{14}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[ (a_{15})^{(1)} G_{14}(s_{(13)}) - \left( (a'_{15})^{(1)} + (a''_{15})^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \right) G_{15}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[ (b_{13})^{(1)} T_{14}(s_{(13)}) - \left( (b'_{13})^{(1)} - (b''_{13})^{(1)} (G(s_{(13)}), s_{(13)}) \right) T_{13}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^{0} + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)}\left(G(s_{(13)}), s_{(13)}\right)\right) T_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{\mathrm{T}}_{15}(\mathrm{t}) = \mathrm{T}_{15}^{0} + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)}\left(G(s_{(13)}), s_{(13)}\right)\right) T_{15}(s_{(13)})\right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{16})^{(2)}, T_i^0 \le (\hat{Q}_{16})^{(2)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$$

By 160

$$\bar{G}_{16}(t) = G_{16}^{0} + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''_{16})^{(2)}\left(T_{17}(s_{(16)}), s_{(16)}\right)\right) G_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^{0} + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)}\left(T_{17}(s_{(16)}), s_{(17)}\right)\right) G_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^{0} + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)}\left(T_{17}(s_{(16)}), s_{(16)}\right)\right) G_{18}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^{0} + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)}\left(G_{19}(s_{(16)}), s_{(16)}\right)\right) T_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^{0} + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)}\left(G_{19}(s_{(16)}), s_{(16)}\right)\right) T_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^{0} + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)}\left(G_{19}(s_{(16)}), s_{(16)}\right)\right) T_{18}(s_{(16)})\right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{20})^{(3)}, T_i^0 \le (\hat{Q}_{20})^{(3)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$$

By 161

$$\bar{G}_{20}(t) = G_{20}^{0} + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''_{20})^{(3)}\left(T_{21}(s_{(20)}), s_{(20)}\right)\right) G_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{22}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^0 + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^0 + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{\mathrm{T}}_{22}(\mathrm{t}) = \mathrm{T}_{22}^0 + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{22}(s_{(20)})\right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{24})^{(4)}, T_i^0 \le (\hat{Q}_{24})^{(4)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

By 162

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{26}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{\mathrm{T}}_{26}(\mathrm{t}) = \mathrm{T}_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{26}(s_{(24)})\right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{28})^{(5)}, T_i^0 \le (\hat{Q}_{28})^{(5)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

By 163

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{30}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{\mathrm{T}}_{30}(\mathrm{t}) = \mathrm{T}_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{30}(s_{(28)})\right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{32})^{(6)}, T_i^0 \le (\hat{Q}_{32})^{(6)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$$

By 164

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a'_{32})^{(6)} + a''_{32})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{34}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{\mathrm{T}}_{34}(\mathrm{t}) = \mathrm{T}_{34}^0 + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{34}(s_{(32)})\right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$ , $T_i(0) = T_i^0$ , $G_i^0 \leq (\hat{P}_{36})^{(7)}$ , $T_i^0 \leq (\hat{Q}_{36})^{(7)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[ (a_{36})^{(7)} G_{37}(s_{(36)}) - \left( (a'_{36})^{(7)} + a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[ (a_{37})^{(7)} G_{36}(s_{(36)}) - \left( (a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[ (a_{38})^{(7)} G_{37}(s_{(36)}) - \left( (a'_{38})^{(7)} + (a''_{38})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{38}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[ (b_{36})^{(7)} T_{37}(s_{(36)}) - \left( (b'_{36})^{(7)} - (b''_{36})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[ (b_{37})^{(7)} T_{36}(s_{(36)}) - \left( (b'_{37})^{(7)} - (b''_{37})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{38}(t) = T_{38}^0 + \int_0^t \left[ (b_{38})^{(7)} T_{37}(s_{(36)}) - \left( (b'_{38})^{(7)} - (b''_{38})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{38}(s_{(36)}) \right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i$: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$ , $T_i(0) = T_i^0$ , $G_i^0 \leq (\hat{P}_{40})^{(8)}$ , $T_i^0 \leq (\hat{Q}_{40})^{(8)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{40})^{(8)} e^{(\bar{M}_{40})^{(8)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{40})^{(8)} e^{(\bar{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[ (a_{40})^{(8)} G_{41}(s_{(40)}) - \left( (a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[ (a_{41})^{(8)} G_{40}(s_{(40)}) - \left( (a'_{41})^{(8)} + (a''_{41})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)}\left(T_{41}(s_{(40)}), s_{(40)}\right)\right) G_{42}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^0 + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)}\left(G_{43}(s_{(40)}), s_{(40)}\right)\right) T_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^0 + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)}\left(G_{43}(s_{(40)}), s_{(40)}\right)\right) T_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{\mathrm{T}}_{42}(\mathrm{t}) = \mathrm{T}_{42}^0 + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)}\left(G_{43}(s_{(40)}), s_{(40)}\right)\right) T_{42}(s_{(40)})\right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** 166

Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy A

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{44})^{(9)},\ T_i^0 \le (\hat{Q}_{44})^{(9)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''_{44})^{(9)}\left(T_{45}(s_{(44)}), s_{(44)}\right)\right) G_{44}(s_{(44)})\right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)}\left(T_{45}(s_{(44)}), s_{(44)}\right)\right) G_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)}\left(T_{45}(s_{(44)}), s_{(44)}\right)\right) G_{46}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)}\left(G_{47}(s_{(44)}), s_{(44)}\right)\right) T_{44}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)}\left(G_{47}(s_{(44)}), s_{(44)}\right)\right) T_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{\mathrm{T}}_{46}(\mathrm{t}) = \mathrm{T}_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)}\left(G_{47}(s_{(44)}), s_{(44)}\right)\right) T_{46}(s_{(44)})\right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}}\right)\right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left(e^{(\hat{M}_{13})^{(1)} t} - 1\right)$$

From which it follows that 168

$$(G_{13}(t)-G_{13}^0)e^{-(\hat{M}_{13})^{(1)}t}\le\frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}}\left[((\hat{P}_{13})^{(1)}+G_{14}^0)e^{\left(-\frac{(\hat{P}_{13})^{(1)}+G_{14}^0}{G_{14}^0}\right)}+(\hat{P}_{13})^{(1)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t)\le G_{16}^0+\int_0^t\left[(a_{16})^{(2)}\left(G_{17}^0+(\hat{P}_{16})^{(6)}e^{(\hat{M}_{16})^{(2)}s_{(16)}}\right)\right]ds_{(16)}= \quad (169)$$

$$\left(1+(a_{16})^{(2)}t\right)G_{17}^0+\frac{(a_{16})^{(2)}(\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}}\left(e^{(\hat{M}_{16})^{(2)}t}-1\right)$$

From which it follows that (170)

$$(G_{16}(t)-G_{16}^0)e^{-(\hat{M}_{16})^{(2)}t}\le\frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}}\left[((\hat{P}_{16})^{(2)}+G_{17}^0)e^{\left(-\frac{(\hat{P}_{16})^{(2)}+G_{17}^0}{G_{17}^0}\right)}+(\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that (171)

$$G_{20}(t)\le G_{20}^0+\int_0^t\left[(a_{20})^{(3)}\left(G_{21}^0+(\hat{P}_{20})^{(3)}e^{(\hat{M}_{20})^{(3)}s_{(20)}}\right)\right]ds_{(20)}=$$

$$\left(1+(a_{20})^{(3)}t\right)G_{21}^0+\frac{(a_{20})^{(3)}(\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left(e^{(\hat{M}_{20})^{(3)}t}-1\right)$$

From which it follows that (172)

$$(G_{20}(t)-G_{20}^0)e^{-(\hat{M}_{20})^{(3)}t}\le\frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left[((\hat{P}_{20})^{(3)}+G_{21}^0)e^{\left(-\frac{(\hat{P}_{20})^{(3)}+G_{21}^0}{G_{21}^0}\right)}+(\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that (173)

$$G_{24}(t)\le G_{24}^0+\int_0^t\left[(a_{24})^{(4)}\left(G_{25}^0+(\hat{P}_{24})^{(4)}e^{(\hat{M}_{24})^{(4)}s_{(24)}}\right)\right]ds_{(24)}=$$

$$\left(1+(a_{24})^{(4)}t\right)G_{25}^0+\frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left(e^{(\hat{M}_{24})^{(4)}t}-1\right)$$

From which it follows that (174)

$$(G_{24}(t)-G_{24}^0)e^{-(\hat{M}_{24})^{(4)}t}\le\frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left[((\hat{P}_{24})^{(4)}+G_{25}^0)e^{\left(-\frac{(\hat{P}_{24})^{(4)}+G_{25}^0}{G_{25}^0}\right)}+(\hat{P}_{24})^{(4)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t) \le G_{28}^0 + \int_0^t \left[ (a_{28})^{(5)} \left( G_{29}^0 + (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} s_{(28)}} \right) \right] ds_{(28)} =$$

$$\left(1 + (a_{28})^{(5)} t\right) G_{29}^0 + \frac{(a_{28})^{(5)} (\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}} \left( e^{(\hat{M}_{28})^{(5)} t} - 1 \right)$$

From which it follows that 175

$$(G_{28}(t) - G_{28}^0) e^{-(\hat{M}_{28})^{(5)} t} \le \frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}} \left[ \left( (\hat{P}_{28})^{(5)} + G_{29}^0 \right) e^{\left( - \frac{(\hat{P}_{28})^{(5)} + G_{29}^0}{G_{29}^0} \right)} + (\hat{P}_{28})^{(5)} \right]$$

$(G_i^0)$ is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t) \le G_{32}^0 + \int_0^t \left[ (a_{32})^{(6)} \left( G_{33}^0 + (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} s_{(32)}} \right) \right] ds_{(32)} =$$

$$\left(1 + (a_{32})^{(6)} t\right) G_{33}^0 + \frac{(a_{32})^{(6)} (\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}} \left( e^{(\hat{M}_{32})^{(6)} t} - 1 \right)$$

From which it follows that 177

$$(G_{32}(t) - G_{32}^0) e^{-(\hat{M}_{32})^{(6)} t} \le \frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}} \left[ \left( (\hat{P}_{32})^{(6)} + G_{33}^0 \right) e^{\left( - \frac{(\hat{P}_{32})^{(6)} + G_{33}^0}{G_{33}^0} \right)} + (\hat{P}_{32})^{(6)} \right]$$

$(G_i^0)$ is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

(v) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t) \le G_{36}^0 + \int_0^t \left[ (a_{36})^{(7)} \left( G_{37}^0 + (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} s_{(36)}} \right) \right] ds_{(36)} =$$

$$\left(1 + (a_{36})^{(7)} t\right) G_{37}^0 + \frac{(a_{36})^{(7)} (\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left( e^{(\hat{M}_{36})^{(7)} t} - 1 \right)$$

From which it follows that

$$(G_{36}(t) - G_{36}^0) e^{-(\hat{M}_{36})^{(7)} t} \le \frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left[ \left( (\hat{P}_{36})^{(7)} + G_{37}^0 \right) e^{\left( - \frac{(\hat{P}_{36})^{(7)} + G_{37}^0}{G_{37}^0} \right)} + (\hat{P}_{36})^{(7)} \right]$$

$(G_i^0)$ is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t) \le G_{40}^0 + \int_0^t \left[ (a_{40})^{(8)} \left( G_{41}^0 + (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} s_{(40)}} \right) \right] ds_{(40)} = \quad 180$$

$$\left(1 + (a_{40})^{(8)} t\right) G_{41}^0 + \frac{(a_{40})^{(8)} (\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}} \left( e^{(\hat{M}_{40})^{(8)} t} - 1 \right)$$

From which it follows that 181

$$(G_{40}(t)-G_{40}^{0})e^{-(\hat{M}_{40})^{(8)}t}\leq\frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left[((\hat{P}_{40})^{(8)}+G_{41}^{0})e^{\left(-\frac{(\hat{P}_{40})^{(8)}+G_{41}^{0}}{G_{41}^{0}}\right)}+(\hat{P}_{40})^{(8)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 8
Analogous inequalities hold also for $G_{41}, G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t)\leq G_{44}^{0}+\int_{0}^{t}\left[(a_{44})^{(9)}\left(G_{45}^{0}+(\hat{P}_{44})^{(9)}e^{(\hat{M}_{44})^{(9)}s_{(44)}}\right)\right]ds_{(44)}=$$

$$\left(1+(a_{44})^{(9)}t\right)G_{45}^{0}+\frac{(a_{44})^{(9)}(\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}}\left(e^{(\hat{M}_{44})^{(9)}t}-1\right)$$

From which it follows that

$$(G_{44}(t)-G_{44}^{0})e^{-(\hat{M}_{44})^{(9)}t}\leq\frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}}\left[((\hat{P}_{44})^{(9)}+G_{45}^{0})e^{\left(-\frac{(\hat{P}_{44})^{(9)}+G_{45}^{0}}{G_{45}^{0}}\right)}+(\hat{P}_{44})^{(9)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 9

Analogous inequalities hold also for $G_{45}, G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}<1$ and to choose 182

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[(\hat{P}_{13})^{(1)}+((\hat{P}_{13})^{(1)}+G_j^0)e^{-\left(\frac{(\hat{P}_{13})^{(1)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{13})^{(1)} \quad 183$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[((\hat{Q}_{13})^{(1)}+T_j^0)e^{-\left(\frac{(\hat{Q}_{13})^{(1)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{13})^{(1)}\right]\leq(\hat{Q}_{13})^{(1)} \quad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left((G^{(1)},T^{(1)}),(G^{(2)},T^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\hat{M}_{13})^{(1)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\hat{M}_{13})^{(1)}t}\}$$

Indeed if we denote

**Definition of** $\tilde{G},\tilde{T}:(\tilde{G},\tilde{T})=\mathcal{A}^{(1)}(G,T)$

It results

$$\left|\tilde{G}_{13}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{13})^{(1)} \left|G_{14}^{(1)}-G_{14}^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\, ds_{(13)} +$$

$$\int_0^t \{(a'_{13})^{(1)} \left|G_{13}^{(1)}-G_{13}^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} +$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)}, s_{(13)}\right)\left|G_{13}^{(1)}-G_{13}^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} e^{(\widehat{M}_{13})^{(1)}s_{(13)}} +$$

$$G_{13}^{(2)} \left|(a''_{13})^{(1)}\left(T_{14}^{(1)}, s_{(13)}\right) - (a''_{13})^{(1)}\left(T_{14}^{(2)}, s_{(13)}\right)\right|\ e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\} ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\left|G^{(1)}-G^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)}t} \leq \qquad (186)$$
$$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)} + (a'_{13})^{(1)} + (\widehat{A}_{13})^{(1)} + (\widehat{P}_{13})^{(1)}(\widehat{k}_{13})^{(1)}\right) d\left(\left(G^{(1)}, T^{(1)};\ G^{(2)}, T^{(2)}\right)\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 1:** The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t}$ *and* $(\widehat{Q}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i = 13,14,15$ depend only on $T_{14}$ and respectively on $G(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any $t$ where $G_i(t) = 0$ *and* $T_i(t) = 0$

From 19 to 24 it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(1)} - (a_i'')^{(1)}(T_{14}(s_{(13)}), s_{(13)})\} ds_{(13)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(1)}t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2$ *and* $\left((\widehat{M}_{13})^{(1)}\right)_3$ : (187)

**Remark 3**: if $G_{13}$ is bounded, the same property have also $G_{14}$ *and* $G_{15}$ . indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a'_{14})^{(1)} G_{14}$ and by integrating

$$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1 / (a'_{14})^{(1)}$$

In the same way , one can obtain

$$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2 / (a'_{15})^{(1)}$$

If $G_{14}$ *or* $G_{15}$ is bounded, the same property follows for $G_{13}$ , $G_{15}$ and $G_{13}$ , $G_{14}$ respectively.

**Remark 4:** If $G_{13}$ *is* bounded, from below, the same property holds for $G_{14}$ *and* $G_{15}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{14}$ is bounded from below. 188

**Remark 5:** If $T_{13}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}\,(G(t),t)) = (b_{14}')^{(1)}$ then $T_{14} \to \infty$. 189

**Definition of** $(m)^{(1)}$ and $\varepsilon_1$ :

Indeed let $t_1$ be so that for $t > t_1$

$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t),t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_1}$ By taking now $\varepsilon_1$ sufficiently small one sees that $T_{14}$ is unbounded.
The same property holds for $T_{15}$ if $\lim_{t\to\infty}(b_{15}'')^{(1)}\,(G(t),t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}$ , $\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}$ *and* $(\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[(\hat{P}_{16})^{(2)} + \left((\hat{P}_{16})^{(2)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{16})^{(2)}\right] \leq (\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

**Definition of** $\widetilde{G_{19}}, \widetilde{T_{19}}$ : $\left(\widetilde{G_{19}}, \widetilde{T_{19}}\right) = \mathcal{A}^{(2)}(G_{19}, T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{16})^{(2)} \left|G_{17}^{(1)}-G_{17}^{(2)}\right| e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\, ds_{(16)} +$$

$$\int_0^t \{(a'_{16})^{(2)} \left|G_{16}^{(1)}-G_{16}^{(2)}\right| e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} +$$

$$(a''_{16})^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right)\left|G_{16}^{(1)}-G_{16}^{(2)}\right| e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} e^{(\widehat{M}_{16})^{(2)}s_{(16)}} +$$

$$G_{16}^{(2)} \left|(a''_{16})^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right)-(a''_{16})^{(2)}\left(T_{17}^{(2)}, s_{(16)}\right)\right|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\} ds_{(16)}$$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0, t]$ 197

From the hypotheses it follows

$$\left|(G_{19})^{(1)}-(G_{19})^{(2)}\right| e^{-(\widehat{M}_{16})^{(2)}t} \leq$$
$$\frac{1}{(\widehat{M}_{16})^{(2)}}\left((a_{16})^{(2)}+\ (a'_{16})^{(2)}+(\widehat{A}_{16})^{(2)}+(\widehat{P}_{16})^{(2)}(\widehat{k}_{16})^{(2)}\right) d\left(((G_{19})^{(1)}, (T_{19})^{(1)};\ (G_{19})^{(2)}, (T_{19})^{(2)})\right)$$

And analogous inequalities for $G_i$ and $T_i$. Taking into account the hypothesis the result follows 198

**Remark 6:** The fact that we supposed $(a''_{16})^{(2)}$ and $(b''_{16})^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{16})^{(2)} e^{(\widehat{M}_{16})^{(2)}t}$ and $(\widehat{Q}_{16})^{(2)} e^{(\widehat{M}_{16})^{(2)}t}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i = 16,17,18$ depend only on $T_{17}$ and respectively on $(G_{19})$(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 7:** There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$ 200

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(2)}-(a_i'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})\} ds_{(16)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(2)}t)} > 0 \quad \text{for } t > 0$$

**Definition of** $\left((\widehat{M}_{16})^{(2)}\right)_1$, $\left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$: 201

**Remark 8:** if $G_{16}$ is bounded, the same property have also $G_{17}$ and $G_{18}$ . indeed if

$G_{16} < (\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt} \leq \left((\widehat{M}_{16})^{(2)}\right)_1 - (a'_{17})^{(2)} G_{17}$ and by integrating

$$G_{17} \leq \left((\widehat{M}_{16})^{(2)}\right)_2 = G_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1 / (a'_{17})^{(2)}$$

In the same way , one can obtain

$$G_{18} \leq \left((\widehat{M}_{16})^{(2)}\right)_3 = G_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2 / (a'_{18})^{(2)}$$

If $G_{17}$ or $G_{18}$ is bounded, the same property follows for $G_{16}$, $G_{18}$ and $G_{16}$, $G_{17}$ respectively.

**Remark 9**: If $G_{16}$ is bounded, from below, the same property holds for $G_{17}$ and $G_{18}$. The proof is analogous with the preceding one. An analogous property is true if $G_{17}$ is bounded from below. 202

**Remark 10:** If $T_{16}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t),t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$. 203

**Definition of** $(m)^{(2)}$ and $\varepsilon_2$ :

Indeed let $t_2$ be so that for $t > t_2$

$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t),t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right), \quad t = \log\frac{2}{\varepsilon_2}$ By taking now $\varepsilon_2$ sufficiently small one sees that $T_{17}$ is unbounded. 205
The same property holds for $T_{18}$ if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t),t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\bar{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\bar{M}_{20})^{(3)}} < 1$ and to choose 207

$(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\bar{M}_{20})^{(3)}}\left[(\hat{P}_{20})^{(3)} + \left((\hat{P}_{20})^{(3)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\bar{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{20})^{(3)}\right] \leq (\hat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself 210

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)},(T_{23})^{(1)}),((G_{23})^{(2)},(T_{23})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\bar{M}_{20})^{(3)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\bar{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

**Definition of** $\widetilde{G_{23}}, \widetilde{T_{23}}$ :$\left(\widetilde{(G_{23})},\widetilde{(T_{23})}\right) = \mathcal{A}^{(3)}\left((G_{23}),(T_{23})\right)$

It results 213

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_{0}^{t}(a_{20})^{(3)}\left|G_{21}^{(1)}-G_{21}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\, ds_{(20)} +$$

$$\int_{0}^{t}\{(a'_{20})^{(3)}\left|G_{20}^{(1)}-G_{20}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} +$$

$$(a''_{20})^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{(\widehat{M}_{20})^{(3)}s_{(20)}} +$$

$$G_{20}^{(2)}\left|(a''_{20})^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)}, s_{(20)}\right)\right|\ e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\} ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}t} \leq \qquad 214$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+\ (a'_{20})^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\widehat{k}_{20})^{(3)}\right) d\left(\left((G_{23})^{(1)},(T_{23})^{(1)};\ (G_{23})^{(2)},(T_{23})^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

**Remark 11:** The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}\ and\ (\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_{+}$. 215

If instead of proving the existence of the solution on $\mathbb{R}_{+}$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i = 20,21,22$ depend only on $T_{21}$ and respectively on $(G_{23})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 12:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 216

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t\{(a_i')^{(3)}-(a_i'')^{(3)}(T_{21}(s_{(20)}),s_{(20)})\}ds_{(20)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(3)}t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2\ and\ \left((\widehat{M}_{20})^{(3)}\right)_3$ : 217

**Remark 13:** if $G_{20}$ is bounded, the same property have also $G_{21}\ and\ G_{22}$ . indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a'_{21})^{(3)}G_{21}$ and by integrating

$$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1/(a'_{21})^{(3)}$$

In the same way , one can obtain

$$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2/(a'_{22})^{(3)}$$

If $G_{21}$ *or* $G_{22}$ is bounded, the same property follows for $G_{20}$ , $G_{22}$ and $G_{20}$ , $G_{21}$ respectively.

**Remark 14:** If $G_{20}$ *is* bounded, from below, the same property holds for $G_{21}$ *and* $G_{22}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{21}$ is bounded from below. 218

**Remark 15:** If $T_{20}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t),t\right)=(b_{21}')^{(3)}$ then $T_{21}\to\infty$. 219

**Definition of** $(m)^{(3)}$ and $\varepsilon_3$ :

Indeed let $t_3$ be so that for $t > t_3$

$(b_{21})^{(3)}-(b_i'')^{(3)}\left((G_{23})(t),t\right)<\varepsilon_3, T_{20}(t)>(m)^{(3)}$

Then $\frac{dT_{21}}{dt}\geq(a_{21})^{(3)}(m)^{(3)}-\varepsilon_3 T_{21}$ which leads to 220

$T_{21}\geq\left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1-e^{-\varepsilon_3 t})+T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t}=\frac{1}{2}$ it results

$T_{21}\geq\left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right),\quad t=log\frac{2}{\varepsilon_3}$ By taking now $\varepsilon_3$ sufficiently small one sees that $T_{21}$ is unbounded.
The same property holds for $T_{22}$ if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t),t\right)=(b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}$ , $\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}}<1$ and to choose 221

$(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\left[(\hat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \quad 222$$

$$\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \quad 223$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself 224

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric 225

$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{24})^{(4)}t}\}$$

Indeed if we denote

**Definition of** $\widetilde{(G_{27})},\widetilde{(T_{27})}$ : $\left(\widetilde{(G_{27})},\widetilde{(T_{27})}\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{24})^{(4)} \left|G_{25}^{(1)}-G_{25}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)}s_{(24)}} e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\, ds_{(24)} +$$

$$\int_0^t \{(a'_{24})^{(4)}\left|G_{24}^{(1)}-G_{24}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)}s_{(24)}} e^{-(\widehat{M}_{24})^{(4)}s_{(24)}} +$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)}, s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)}s_{(24)}} e^{(\widehat{M}_{24})^{(4)}s_{(24)}} +$$

$$G_{24}^{(2)}\left|(a''_{24})^{(4)}\left(T_{25}^{(1)}, s_{(24)}\right)-(a''_{24})^{(4)}\left(T_{25}^{(2)}, s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)}s_{(24)}} e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\} ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)}t} \leq \qquad (226)$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)}+\ (a'_{24})^{(4)}+(\widehat{A}_{24})^{(4)}+(\widehat{P}_{24})^{(4)}(\widehat{k}_{24})^{(4)}\right) d\left(\left((G_{27})^{(1)}, (T_{27})^{(1)};\ (G_{27})^{(2)}, (T_{27})^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

**Remark 16:** The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}\ and\ (\widehat{Q}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_+$. (227)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i = 24,25,26$ depend only on $T_{25}$ and respectively on $(G_{27})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 17:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ (228)

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(4)}-(a_i'')^{(4)}(T_{25}(s_{(24)}),s_{(24)})\}ds_{(24)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(4)}t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2\ and\ \left((\widehat{M}_{24})^{(4)}\right)_3$ : (229)

**Remark 18:** if $G_{24}$ is bounded, the same property have also $G_{25}\ and\ G_{26}$ . indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a'_{25})^{(4)}G_{25}$ and by integrating

$$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1/(a'_{25})^{(4)}$$

In the same way , one can obtain

$$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2/(a'_{26})^{(4)}$$

If $G_{25}$ *or* $G_{26}$ is bounded, the same property follows for $G_{24}$ , $G_{26}$ and $G_{24}$ , $G_{25}$ respectively.

**Remark 19:** If $G_{24}$ *is* bounded, from below, the same property holds for $G_{25}$ *and* $G_{26}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{25}$ is bounded from below. 230

**Remark 20:** If $T_{24}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}((G_{27})(t),t)) = (b_{25}')^{(4)}$ then $T_{25} \to \infty$. 231

**Definition of** $(m)^{(4)}$ and $\varepsilon_4$ :

Indeed let $t_4$ be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t),t) < \varepsilon_4, T_{24}(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right)$, $t = log\frac{2}{\varepsilon_4}$ By taking now $\varepsilon_4$ sufficiently small one sees that $T_{25}$ is unbounded.
The same property holds for $T_{26}$ if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t),t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for $G_{29}, G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}$ , $\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)} + \left((\hat{P}_{28})^{(5)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{28})^{(5)}\right] \leq (\hat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$$d\left(((G_{31})^{(1)},(T_{31})^{(1)}),((G_{31})^{(2)},(T_{31})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

**Definition of** $\widetilde{(G_{31})}, \widetilde{(T_{31})}$ : $\left(\widetilde{(G_{31})}, \widetilde{(T_{31})}\right) = \mathcal{A}^{(5)}\left((G_{31}), (T_{31})\right)$

It results

$\left|\tilde{G}_{28}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{28})^{(5)} \left|G_{29}^{(1)} - G_{29}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}}\, ds_{(28)} +$

$\int_0^t \{(a'_{28})^{(5)} \left|G_{28}^{(1)} - G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} +$

$(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right)\left|G_{28}^{(1)} - G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}} +$

$G_{28}^{(2)} \left|(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right) - (a''_{28})^{(5)}\left(T_{29}^{(2)}, s_{(28)}\right)\right|\ e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}} \} ds_{(28)}$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$\left|(G_{31})^{(1)} - (G_{31})^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} t} \leq$ 237

$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)} + (a'_{28})^{(5)} + (\widehat{A}_{28})^{(5)} + (\widehat{P}_{28})^{(5)} (\widehat{k}_{28})^{(5)}\right) d\left(\left((G_{31})^{(1)}, (T_{31})^{(1)};\ (G_{31})^{(2)}, (T_{31})^{(2)}\right)\right)$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 21:** The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$ *and* $(\widehat{Q}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$ respectively of $\mathbb{R}_+$. 238

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on $T_{29}$ and respectively on $(G_{31})$(*and not on* $t$) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 22:** There does not exist any $t$ where $G_i(t) = 0$ *and* $T_i(t) = 0$ 239

it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(5)} - (a_i'')^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(5)} t\right)} > 0$ for t > 0

**Definition of** $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2$ *and* $\left((\widehat{M}_{28})^{(5)}\right)_3$ : 240

**Remark 23:** if $G_{28}$ is bounded, the same property have also $G_{29}$ *and* $G_{30}$ . indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a'_{29})^{(5)} G_{29}$ and by integrating

$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1 / (a'_{29})^{(5)}$

In the same way , one can obtain

$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2/(a'_{30})^{(5)}$

If $G_{29}$ *or* $G_{30}$ is bounded, the same property follows for $G_{28}$, $G_{30}$ and $G_{28}$, $G_{29}$ respectively.

**Remark 24:** If $G_{28}$ *is* bounded, from below, the same property holds for $G_{29}$ *and* $G_{30}$. The proof is analogous with the preceding one. An analogous property is true if $G_{29}$ is bounded from below. 241

**Remark 25:** If $T_{28}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}((G_{31})(t),t)) = (b'_{29})^{(5)}$ then $T_{29} \to \infty$. 242

**Definition of** $(m)^{(5)}$ and $\varepsilon_5$ :

Indeed let $t_5$ be so that for $t > t_5$

$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t),t) < \varepsilon_5, T_{28}(t) > (m)^{(5)}$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\frac{2}{\varepsilon_5}$ By taking now $\varepsilon_5$ sufficiently small one sees that $T_{29}$ is unbounded.
The same property holds for $T_{30}$ if $\lim_{t\to\infty}(b_{30}'')^{(5)}\left((G_{31})(t),t\right) = (b'_{30})^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{33}, G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}$, $\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$ and to choose 244

$(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[(\hat{P}_{32})^{(6)} + \left((\hat{P}_{32})^{(6)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{32})^{(6)} \quad 245$$

$$\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[\left((\hat{Q}_{32})^{(6)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{32})^{(6)}\right] \leq (\hat{Q}_{32})^{(6)} \quad 246$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric 247

$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right) =$

$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{32})^{(6)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{32})^{(6)}t}\}$

Indeed if we denote

**Definition of** $\widetilde{(G_{35})}, \widetilde{(T_{35})}$ : $\left(\widetilde{(G_{35})}, \widetilde{(T_{35})}\right) = \mathcal{A}^{(6)}\left((G_{35}), (T_{35})\right)$

It results

$$\left|\tilde{G}_{32}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{32})^{(6)} \left|G_{33}^{(1)} - G_{33}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}}\, ds_{(32)} +$$

$$\int_0^t \{(a'_{32})^{(6)} \left|G_{32}^{(1)} - G_{32}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} +$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)}, s_{(32)}\right)\left|G_{32}^{(1)} - G_{32}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}} +$$

$$G_{32}^{(2)} \left|(a''_{32})^{(6)}\left(T_{33}^{(1)}, s_{(32)}\right) - (a''_{32})^{(6)}\left(T_{33}^{(2)}, s_{(32)}\right)\right| \; e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}} \} ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|(G_{35})^{(1)} - (G_{35})^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} t} \leq \qquad (248)$$
$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)} + (a'_{32})^{(6)} + (\widehat{A}_{32})^{(6)} + (\widehat{P}_{32})^{(6)} (\widehat{k}_{32})^{(6)}\right) d\left(\left((G_{35})^{(1)}, (T_{35})^{(1)}; \; (G_{35})^{(2)}, (T_{35})^{(2)}\right)\right)$$

And analogous inequalities for $G_i \; and \; T_i$. Taking into account the hypothesis the result follows

**Remark 26:** The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)} t} \; and \; (\widehat{Q}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)} t}$ respectively of $\mathbb{R}_+$. (249)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i = 32{,}33{,}34$ depend only on $T_{33}$ and respectively on $(G_{35})(and \; not \; on \; t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 27:** There does not exist any $t$ where $G_i\,(t) = 0 \; and \; T_i\,(t) = 0$ (250)

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(6)} - (a_i'')^{(6)}(T_{33}(s_{(32)}), s_{(32)})\} ds_{(32)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(6)} t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{32})^{(6)}\right)_1, \left((\widehat{M}_{32})^{(6)}\right)_2 \; and \; \left((\widehat{M}_{32})^{(6)}\right)_3$ : (251)

**Remark 28:** if $G_{32}$ is bounded, the same property have also $G_{33} \; and \; G_{34}$ . indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a'_{33})^{(6)} G_{33}$ and by integrating

$$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_1 / (a'_{33})^{(6)}$$

In the same way , one can obtain

$$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_2/(a'_{34})^{(6)}$$

If $G_{33}$ *or* $G_{34}$ is bounded, the same property follows for $G_{32}$ , $G_{34}$ and $G_{32}$ , $G_{33}$ respectively.

**Remark 29:** If $G_{32}$ *is* bounded, from below, the same property holds for $G_{33}$ *and* $G_{34}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{33}$ is bounded from below. 252

**Remark 30:** If $T_{32}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}((G_{35})(t),t)) = (b'_{33})^{(6)}$ then $T_{33} \to \infty$. 253

**Definition of** $(m)^{(6)}$ and $\varepsilon_6$ :

Indeed let $t_6$ be so that for $t > t_6$

$$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t),t\right) < \varepsilon_6, T_{32}(t) > (m)^{(6)}$$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right)$, $t = log\frac{2}{\varepsilon_6}$ By taking now $\varepsilon_6$ sufficiently small one sees that $T_{33}$ is unbounded.
The same property holds for $T_{34}$ if $\lim_{t\to\infty}(b''_{34})^{(6)}\left((G_{35})(t),t(t),t\right) = (b'_{34})^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{37}$ , $G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}$ , $\frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$ and to choose
$(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[(\hat{P}_{36})^{(7)} + \left((\hat{P}_{36})^{(7)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{36})^{(7)} \quad 256$$

$$\frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[\left((\hat{Q}_{36})^{(7)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{36})^{(7)}\right] \leq (\hat{Q}_{36})^{(7)} \quad 257$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself
The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric 258

$$d\left(\left((G_{39})^{(1)},(T_{39})^{(1)}\right),\left((G_{39})^{(2)},(T_{39})^{(2)}\right)\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\tilde{M}_{36})^{(7)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\tilde{M}_{36})^{(7)}t}\}$$

Indeed if we denote

**Definition of** $\widetilde{(G_{39})},\widetilde{(T_{39})}:\left(\widetilde{(G_{39})},\widetilde{(T_{39})}\right)=\mathcal{A}^{(7)}((G_{39}),(T_{39}))$
It results

$$\left|\tilde{G}_{36}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{36})^{(7)}\left|G_{37}^{(1)}-G_{37}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\,ds_{(36)}+$$
$$\int_0^t\{(a'_{36})^{(7)}\left|G_{36}^{(1)}-G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$(a''_{36})^{(7)}\left(T_{37}^{(1)},s_{(36)}\right)\left|G_{36}^{(1)}-G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$G_{36}^{(2)}\left|(a''_{36})^{(7)}\left(T_{37}^{(1)},s_{(36)}\right)-(a''_{36})^{(7)}\left(T_{37}^{(2)},s_{(36)}\right)\right|\ e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\}ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses on it follows

$$\left|(G_{39})^{(1)}-(G_{39})^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}t}\le \qquad 259$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}}\left((a_{36})^{(7)}+(a'_{36})^{(7)}+(\widehat{A}_{36})^{(7)}+(\widehat{P}_{36})^{(7)}(\widehat{k}_{36})^{(7)}\right)d\left(((G_{39})^{(1)},(T_{39})^{(1)};\ (G_{39})^{(2)},(T_{39})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

**Remark 31:** The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}\ and\ (\widehat{Q}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ respectively of $\mathbb{R}_+$. 260

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(7)}$ and $(b_i'')^{(7)},i=36,37,38$ depend only on $\mathrm{T}_{37}$ and respectively on $(G_{39})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 32:** There does not exist any $t$ where $G_i\,(t)=0\ and\ T_i\,(t)=0$ 261

it results

$$G_i\,(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(7)}-(a_i'')^{(7)}(T_{37}(s_{(36)}),s_{(36)})\}ds_{(36)}\right]}\ge 0$$

$$T_i\,(t)\ge T_i^0e^{\left(-(b_i')^{(7)}t\right)}>0\quad \text{for t}>0$$

**Definition of** $\left((\widehat{M}_{36})^{(7)}\right)_1,\left((\widehat{M}_{36})^{(7)}\right)_2\ and\ \left((\widehat{M}_{36})^{(7)}\right)_3:$ 262

**Remark 33:** if $G_{36}$ is bounded, the same property have also $G_{37}\ and\ G_{38}$ . indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a'_{37})^{(7)} G_{37}$ and by integrating

$$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1/(a'_{37})^{(7)}$$

In the same way , one can obtain

$$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2/(a'_{38})^{(7)}$$

If $G_{37}$ *or* $G_{38}$ is bounded, the same property follows for $G_{36}$ , $G_{38}$ and $G_{36}$ , $G_{37}$ respectively.

**Remark 34:** If $G_{36}$ *is* bounded, from below, the same property holds for $G_{37}$ *and* $G_{38}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{37}$ is bounded from below. 263

**Remark 35:** If $T_{36}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(7)}((G_{39})(t),t)) = (b'_{37})^{(7)}$ then $T_{37} \to \infty$. 264

**Definition of** $(m)^{(7)}$ and $\varepsilon_7$ :

Indeed let $t_7$ be so that for $t > t_7$

$$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t),t) < \varepsilon_7, T_{36}(t) > (m)^{(7)}$$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right), \quad t = log\frac{2}{\varepsilon_7}$$ By taking now $\varepsilon_7$ sufficiently small one sees that $T_{37}$ is unbounded.
The same property holds for $T_{38}$ if $\lim_{t\to\infty}(b''_{38})^{(7)}\left((G_{39})(t),t\right) = (b'_{38})^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}$ , $\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1$ and to choose 266
$(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ large to have

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[(\hat{P}_{40})^{(8)} + \left((\hat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{40})^{(8)}$$ 267

$$\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{40})^{(8)}\right] \leq (\hat{Q}_{40})^{(8)}$$ 268

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)},(T_{43})^{(1)}),((G_{43})^{(2)},(T_{43})^{(2)})\right)= \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

**Definition of** $\widetilde{(G_{43})},\widetilde{(T_{43})}$ : $\left(\widetilde{(G_{43})},\widetilde{(T_{43})}\right)=\mathcal{A}^{(8)}((G_{43}),(T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{40})^{(8)}\left|G_{41}^{(1)}-G_{41}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\,ds_{(40)}+$$
$$\int_0^t\{(a'_{40})^{(8)}\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$G_{40}^{(2)}\left|(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)-(a''_{40})^{(8)}\left(T_{41}^{(2)},s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\}ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0,t]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)}-(G_{43})^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}t}\le \quad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)}+(a'_{40})^{(8)}+(\widehat{A}_{40})^{(8)}+(\widehat{P}_{40})^{(8)}(\widehat{k}_{40})^{(8)}\right)d\left(((G_{43})^{(1)},(T_{43})^{(1)};\ (G_{43})^{(2)},(T_{43})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

**Remark 36:** The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}\ and\ (\widehat{Q}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ respectively of $\mathbb{R}_+$. 274

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i=40,41,42$ depend only on $T_{41}$ and respectively on $(G_{43})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 37** There does not exist any $t$ where $G_i\,(t)=0\ and\ T_i\,(t)=0$ 275

it results

$$G_i\,(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(8)}-(a_i'')^{(8)}(T_{41}(s_{(40)}),s_{(40)})\}ds_{(40)}\right]}\ge 0$$

$$T_i\,(t)\ge T_i^0e^{\left(-(b_i')^{(8)}t\right)}>0\quad \text{for t}>0$$

**Definition of** $\left((\widehat{M}_{40})^{(8)}\right)_1, \left((\widehat{M}_{40})^{(8)}\right)_2$ *and* $\left((\widehat{M}_{40})^{(8)}\right)_3$ : 276

**Remark 38:** if $G_{40}$ is bounded, the same property have also $G_{41}$ *and* $G_{42}$ . indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a'_{41})^{(8)} G_{41}$ and by integrating

$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1/(a'_{41})^{(8)}$

In the same way , one can obtain

$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2/(a'_{42})^{(8)}$

If $G_{41}$ *or* $G_{42}$ is bounded, the same property follows for $G_{40}$ , $G_{42}$ and $G_{40}$ , $G_{41}$ respectively.

**Remark 39:** If $G_{40}$ *is* bounded, from below, the same property holds for $G_{41}$ *and* $G_{42}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{41}$ is bounded from below. 277

**Remark 40:** If $T_{40}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}((G_{43})(t),t)) = (b'_{41})^{(8)}$ then $T_{41} \to \infty$. 278

**Definition of** $(m)^{(8)}$ and $\varepsilon_8$ :

Indeed let $t_8$ be so that for $t > t_8$

$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t),t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right), \quad t = log\frac{2}{\varepsilon_8}$ By taking now $\varepsilon_8$ sufficiently small one sees that $T_{41}$ is unbounded.
The same property holds for $T_{42}$ if $\lim_{t\to\infty}(b''_{42})^{(8)}\left((G_{43})(t),t(t),t\right) = (b'_{42})^{(8)}$

It is now sufficient to take $\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}$ , $\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}} < 1$ and to choose $(\widehat{P}_{44})^{(9)}$ and $(\widehat{Q}_{44})^{(9)}$ large to have 279 A

$$\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[(\widehat{P}_{44})^{(9)} + \left((\widehat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[\left((\widehat{Q}_{44})^{(9)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{44})^{(9)}\right] \leq (\widehat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)},(T_{47})^{(1)}),((G_{47})^{(2)},(T_{47})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

**Definition of** $\widetilde{(G_{47})},\widetilde{(T_{47})}:\left(\widetilde{(G_{47})},\widetilde{(T_{47})}\right)=\mathcal{A}^{(9)}\left((G_{47}),(T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{44})^{(9)}\left|G_{45}^{(1)}-G_{45}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\,ds_{(44)}+$$
$$\int_0^t\{(a'_{44})^{(9)}\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$G_{44}^{(2)}\left|(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)-(a''_{44})^{(9)}\left(T_{45}^{(2)},s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\}ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}t}\le$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)}+(a'_{44})^{(9)}+(\widehat{A}_{44})^{(9)}+(\widehat{P}_{44})^{(9)}(\widehat{k}_{44})^{(9)}\right)d\left(((G_{47})^{(1)},(T_{47})^{(1)};\ (G_{47})^{(2)},(T_{47})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis (39,35,36) the result follows

**Remark 41:** The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}\ and\ (\widehat{Q}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i=44{,}45{,}46$ depend only on $T_{45}$ and respectively on $(G_{47})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 42:** There does not exist any $t$ where $G_i\,(t)=0\ and\ T_i\,(t)=0$

From 99 to 44 it results

$$G_i\,(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(9)}-(a_i'')^{(9)}(T_{45}(s_{(44)}),s_{(44)})\}ds_{(44)}\right]}\ge 0$$

$$T_i\,(t)\ge T_i^0e^{\left(-(b_i')^{(9)}t\right)}>0\quad \text{for t}>0$$

**Definition of** $\left((\widehat{M}_{44})^{(9)}\right)_1,\left((\widehat{M}_{44})^{(9)}\right)_2\ and\ \left((\widehat{M}_{44})^{(9)}\right)_3$ :

**Remark 43:** if $G_{44}$ is bounded, the same property have also $G_{45}\ and\ G_{46}$ . indeed if

$G_{44} < (\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt} \leq \left((\widehat{M}_{44})^{(9)}\right)_1 - (a'_{45})^{(9)} G_{45}$ and by integrating

$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_1 / (a'_{45})^{(9)}$

In the same way , one can obtain

$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_2 / (a'_{46})^{(9)}$

If $G_{45}$ *or* $G_{46}$ is bounded, the same property follows for $G_{44}$ , $G_{46}$ and $G_{44}$ , $G_{45}$ respectively.

**Remark 44:** If $G_{44}$ *is* bounded, from below, the same property holds for $G_{45}$ *and* $G_{46}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{45}$ is bounded from below.

**Remark 45:** If $T_{44}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t),t\right) = (b'_{45})^{(9)}$ then $T_{45} \to \infty$.

**Definition of** $(m)^{(9)}$ and $\varepsilon_9$ :

Indeed let $t_9$ be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t),t\right) < \varepsilon_9, T_{44}(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right)$, $t = log\frac{2}{\varepsilon_9}$ By taking now $\varepsilon_9$ sufficiently small one sees that $T_{45}$ is unbounded.
The same property holds for $T_{46}$ if $\lim_{t\to\infty}(b''_{46})^{(9)}\left((G_{47})(t),t\right) = (b'_{46})^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

**Behavior of the solutions of equation** 280

**Theorem** If we denote and define

**Definition of** $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ :

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)} \leq -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14},t) + (a''_{14})^{(1)}(T_{14},t) \leq -(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)} \leq -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G,t) - (b''_{14})^{(1)}(G,t) \leq -(\tau_1)^{(1)}$

**Definition of** $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$ : 281

By $(\nu_1)^{(1)} > 0$ , $(\nu_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$ , $(u_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

**Definition of** $(\bar{\nu}_1)^{(1)}, , (\bar{\nu}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$ : 282

By $(\bar{\nu}_1)^{(1)} > 0$ , $(\bar{\nu}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$ , $(\bar{u}_2)^{(1)} < 0$ the roots of the equations

$(a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

**Definition of** $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (v_0)^{(1)}$ :- 283

If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by

$(m_2)^{(1)} = (v_0)^{(1)}, (m_1)^{(1)} = (v_1)^{(1)},\ if\ (v_0)^{(1)} < (v_1)^{(1)}$

$(m_2)^{(1)} = (v_1)^{(1)}, (m_1)^{(1)} = (\bar{v}_1)^{(1)}, if\ (v_1)^{(1)} < (v_0)^{(1)} < (\bar{v}_1)^{(1)},$

and $\boxed{(v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$

$(m_2)^{(1)} = (v_1)^{(1)}, (m_1)^{(1)} = (v_0)^{(1)},\ if\ (\bar{v}_1)^{(1)} < (v_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)},\ if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)},$

and $\boxed{(u_0)^{(1)} = \frac{T_{13}^0}{T_{14}^0}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$

are defined

Then the solution of global equations satisfies the inequalities 285

$$G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{13}(t) \le G_{13}^0 e^{(S_1)^{(1)}t}$$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)}t}$$

$$\left(\frac{(a_{15})^{(1)} G_{13}^0}{(m_1)^{(1)}((S_1)^{(1)} - (p_{13})^{(1)} - (S_2)^{(1)})}\left[e^{((S_1)^{(1)} - (p_{13})^{(1)})t} - e^{-(S_2)^{(1)}t}\right] + G_{15}^0 e^{-(S_2)^{(1)}t} \le G_{15}(t) \le \right. \quad 286$$

$$\left. \frac{(a_{15})^{(1)} G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)} - (a'_{15})^{(1)})}\left[e^{(S_1)^{(1)}t} - e^{-(a'_{15})^{(1)}t}\right] + G_{15}^0 e^{-(a'_{15})^{(1)}t}\right)$$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)}t} \le T_{13}(t) \le T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t}} \quad 287$$

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)}t} \le T_{13}(t) \le \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t} \quad 288$$

$$\frac{(b_{15})^{(1)} T_{13}^0}{(\mu_1)^{(1)}((R_1)^{(1)} - (b'_{15})^{(1)})}\left[e^{(R_1)^{(1)}t} - e^{-(b'_{15})^{(1)}t}\right] + T_{15}^0 e^{-(b'_{15})^{(1)}t} \le T_{15}(t) \le \quad 289$$

$$\frac{(a_{15})^{(1)} T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)} + (r_{13})^{(1)} + (R_2)^{(1)})}\left[e^{((R_1)^{(1)} + (r_{13})^{(1)})t} - e^{-(R_2)^{(1)}t}\right] + T_{15}^0 e^{-(R_2)^{(1)}t}$$

**Definition of** $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:- 290

Where $(S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$

$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$

$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$

$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$

**Behavior of the solutions of equation** 291

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ : 292

$(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ four constants satisfying
$-(\sigma_2)^{(2)} \le -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(T_{17}, t) + (a''_{17})^{(2)}(T_{17}, t) \le -(\sigma_1)^{(2)}$ 293

$-(\tau_2)^{(2)} \le -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}\big((G_{19}), t\big) - (b''_{17})^{(2)}\big((G_{19}), t\big) \le -(\tau_1)^{(2)}$ 294

**Definition of** $(\nu_1)^{(2)}, (\nu_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$ : 295

By $(\nu_1)^{(2)} > 0$ , $(\nu_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0$ , $(u_2)^{(2)} < 0$ the roots 296

of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 297

and $(b_{14})^{(2)}\big(u^{(2)}\big)^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and 298

**Definition of** $(\bar{\nu}_1)^{(2)},, (\bar{\nu}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$ : 299

By $(\bar{\nu}_1)^{(2)} > 0$ , $(\bar{\nu}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0$ , $(\bar{u}_2)^{(2)} < 0$ the 300

roots of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 301

and $(b_{17})^{(2)}\big(u^{(2)}\big)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 302

**Definition of** $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ :- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)},\ \boldsymbol{if}\ (u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}, \boldsymbol{if}\ (u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)},$

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)},\ \boldsymbol{if}\ (\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)} t}$

$(p_i)^{(2)}$ is defined by equation

$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)} t}$ 311

$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)} t}\right] + G_{18}^0 e^{-(S_2)^{(2)} t} \le G_{18}(t) \le\right.$ 312

$\left.\frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)} t} - e^{-(a'_{18})^{(2)} t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)} t}\right)$

$\boxed{T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}}$ 313

$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}$ 314

$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)} t} - e^{-(b'_{18})^{(2)} t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)} t} \le T_{18}(t) \le$ 315

$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})}\left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)} t}\right] + T_{18}^0 e^{-(R_2)^{(2)} t}$

**Definition of** $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$

$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)}$ 318

$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$

**Behavior of the solutions** 319

**Theorem 3:** If we denote and define

**Definition of** $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ :

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}\big((G_{23}), t\big) \le -(\tau_1)^{(3)}$$

**Definition of** $(\nu_1)^{(3)}, (\nu_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$ : 320

By $(\nu_1)^{(3)} > 0$ , $(\nu_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$ , $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\big(\nu^{(3)}\big)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{\nu}_1)^{(3)} > 0$ , $(\bar{\nu}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$ , $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\big(\nu^{(3)}\big)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

**Definition of** $(m_1)^{(3)}$ , $(m_2)^{(3)}$ , $(\mu_1)^{(3)}, (\mu_2)^{(3)}$ :- 321

If we define $(m_1)^{(3)}$ , $(m_2)^{(3)}$ , $(\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (\nu_0)^{(3)}, (m_1)^{(3)} = (\nu_1)^{(3)}$, ***if*** $(\nu_0)^{(3)} < (\nu_1)^{(3)}$

$(m_2)^{(3)} = (\nu_1)^{(3)}, (m_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ , ***if*** $(\nu_1)^{(3)} < (\nu_0)^{(3)} < (\bar{\nu}_1)^{(3)}$,

and $\boxed{(\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}}$

$(m_2)^{(3)} = (\nu_1)^{(3)}, (m_1)^{(3)} = (\nu_0)^{(3)}$, ***if*** $(\bar{\nu}_1)^{(3)} < (\nu_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}$, ***if*** $(u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$ , ***if*** $(u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}$, and $\boxed{(u_0)^{(3)} = \frac{T^0_{20}}{T^0_{21}}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}$, ***if*** $(\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$$G^0_{20} e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G^0_{20} e^{(S_1)^{(3)} t}$$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}} G^0_{20} e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}} G^0_{20} e^{(S_1)^{(3)} t} \quad 323$$

$$\Big(\frac{(a_{22})^{(3)} G^0_{20}}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\Big[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)} t}\Big] + G^0_{22} e^{-(S_2)^{(3)} t} \le G_{22}(t) \le \quad 324$$

$$\frac{(a_{22})^{(3)} G^0_{20}}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}[e^{(S_1)^{(3)} t} - e^{-(a'_{22})^{(3)} t}] + G^0_{22} e^{-(a'_{22})^{(3)} t}\Big)$$

$$\boxed{T_{20}^{0}e^{(R_1)^{(3)}t} \le T_{20}(t) \le T_{20}^{0}e^{((R_1)^{(3)}+(r_{20})^{(3)})t}} \quad 325$$

$$\frac{1}{(\mu_1)^{(3)}}T_{20}^{0}e^{(R_1)^{(3)}t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}}T_{20}^{0}e^{((R_1)^{(3)}+(r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)}T_{20}^{0}}{(\mu_1)^{(3)}((R_1)^{(3)}-(b'_{22})^{(3)})}\left[e^{(R_1)^{(3)}t} - e^{-(b'_{22})^{(3)}t}\right] + T_{22}^{0}e^{-(b'_{22})^{(3)}t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)}T_{20}^{0}}{(\mu_2)^{(3)}((R_1)^{(3)}+(r_{20})^{(3)}+(R_2)^{(3)})}\left[e^{((R_1)^{(3)}+(r_{20})^{(3)})t} - e^{-(R_2)^{(3)}t}\right] + T_{22}^{0}e^{-(R_2)^{(3)}t}$$

**Definition of** $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

**Behavior of the solutions of equation**

**Theorem:** If we denote and define

**Definition of** $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ :

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$$

$$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}\left((G_{27}), t\right) - (b''_{25})^{(4)}\left((G_{27}), t\right) \le -(\tau_1)^{(4)}$$

**Definition of** $(\nu_1)^{(4)}, (\nu_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, \nu^{(4)}, u^{(4)}$ : 329

By $(\nu_1)^{(4)} > 0$ , $(\nu_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$ , $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_1)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(4)},, (\bar{\nu}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$ : 330

By $(\bar{\nu}_1)^{(4)} > 0$ , $(\bar{\nu}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$ , $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

**Definition of** $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}, (\nu_0)^{(4)}$ :-

If we define $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (\nu_0)^{(4)}, (m_1)^{(4)} = (\nu_1)^{(4)}$, ***if*** $(\nu_0)^{(4)} < (\nu_1)^{(4)}$

$(m_2)^{(4)} = (\nu_1)^{(4)}, (m_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ , ***if*** $(\nu_4)^{(4)} < (\nu_0)^{(4)} < (\bar{\nu}_1)^{(4)}$,

and $\boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (\nu_4)^{(4)}, (m_1)^{(4)} = (\nu_0)^{(4)},\ \boldsymbol{if}\ (\bar{\nu}_4)^{(4)} < (\nu_0)^{(4)}$

and analogously

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$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)},\ \boldsymbol{if}\ (u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}, \boldsymbol{if}\ (u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)},$

and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}, \boldsymbol{if}\ (\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

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$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)} t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)} t} \quad 333$$

$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)} t}\right] + G_{26}^0 e^{-(S_2)^{(4)} t} \le G_{26}(t) \le \right. \quad 334$$

*(a26)4G240(m2)4(S1)4−(a26′)4e(S1)4t−e−(a26′)4t+ G260e−(a26′)4t*

$$\boxed{T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}} T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}} T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t} \quad 335$$

$$\frac{(b_{26})^{(4)} T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)} - (b'_{26})^{(4)})}\left[e^{(R_1)^{(4)} t} - e^{-(b'_{26})^{(4)} t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)} t} \le T_{26}(t) \le \quad 336$$

$$\frac{(a_{26})^{(4)} T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)} + (r_{24})^{(4)} + (R_2)^{(4)})}\left[e^{((R_1)^{(4)} + (r_{24})^{(4)})t} - e^{-(R_2)^{(4)} t}\right] + T_{26}^0 e^{-(R_2)^{(4)} t}$$

**Definition of** $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

**Behavior of the solutions of equation** 338

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ :

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

**Definition of** $(\nu_1)^{(5)}, (\nu_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, \nu^{(5)}, u^{(5)}$ : 339

By $(\nu_1)^{(5)} > 0$ , $(\nu_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$ , $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(5)}, , (\bar{\nu}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$ : 340

By $(\bar{\nu}_1)^{(5)} > 0$ , $(\bar{\nu}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$ , $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

**Definition of** $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (\nu_0)^{(5)}$ :-

If we define $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (\nu_0)^{(5)}, (m_1)^{(5)} = (\nu_1)^{(5)}$, ***if*** $(\nu_0)^{(5)} < (\nu_1)^{(5)}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ , ***if*** $(\nu_1)^{(5)} < (\nu_0)^{(5)} < (\bar{\nu}_1)^{(5)}$,
and $\boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)}$, ***if*** $(\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)}$, ***if*** $(u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}$ , ***if*** $(u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)}$,
and $\boxed{(u_0)^{(5)} = \frac{T^0_{28}}{T^0_{29}}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}$, ***if*** $(\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$$G^0_{28} e^{\left((S_1)^{(5)} - (p_{28})^{(5)}\right)t} \le G_{28}(t) \le G^0_{28} e^{(S_1)^{(5)}t}$$

where $(p_i)^{(5)}$ is defined by equation

$$\frac{1}{(m_5)^{(5)}} G^0_{28} e^{\left((S_1)^{(5)} - (p_{28})^{(5)}\right)t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}} G^0_{28} e^{(S_1)^{(5)}t} \quad 343$$

$$\left(\frac{(a_{30})^{(5)}G_{28}^{0}}{(m_1)^{(5)}((S_1)^{(5)}-(p_{28})^{(5)}-(S_2)^{(5)})}\left[e^{((S_1)^{(5)}-(p_{28})^{(5)})t}-e^{-(S_2)^{(5)}t}\right]+G_{30}^{0}e^{-(S_2)^{(5)}t}\leq G_{30}(t)\leq\right. \quad 344$$

*(a30)5G280(m2)5(S1)5−(a30′)5e(S1)5t−e−(a30′)5t+ G300e−(a30′)5t*

$$\boxed{T_{28}^{0}e^{(R_1)^{(5)}t}\leq T_{28}(t)\leq T_{28}^{0}e^{((R_1)^{(5)}+(r_{28})^{(5)})t}} \quad 345$$

$$\frac{1}{(\mu_1)^{(5)}}T_{28}^{0}e^{(R_1)^{(5)}t}\leq T_{28}(t)\leq\frac{1}{(\mu_2)^{(5)}}T_{28}^{0}e^{((R_1)^{(5)}+(r_{28})^{(5)})t} \quad 346$$

$$\frac{(b_{30})^{(5)}T_{28}^{0}}{(\mu_1)^{(5)}((R_1)^{(5)}-(b'_{30})^{(5)})}\left[e^{(R_1)^{(5)}t}-e^{-(b'_{30})^{(5)}t}\right]+T_{30}^{0}e^{-(b'_{30})^{(5)}t}\leq T_{30}(t)\leq \quad 347$$

$$\frac{(a_{30})^{(5)}T_{28}^{0}}{(\mu_2)^{(5)}((R_1)^{(5)}+(r_{28})^{(5)}+(R_2)^{(5)})}\left[e^{((R_1)^{(5)}+(r_{28})^{(5)})t}-e^{-(R_2)^{(5)}t}\right]+T_{30}^{0}e^{-(R_2)^{(5)}t}$$

**Definition of** $(S_1)^{(5)},(S_2)^{(5)},(R_1)^{(5)},(R_2)^{(5)}$:- 348

Where $(S_1)^{(5)}=(a_{28})^{(5)}(m_2)^{(5)}-(a'_{28})^{(5)}$

$(S_2)^{(5)}=(a_{30})^{(5)}-(p_{30})^{(5)}$

$(R_1)^{(5)}=(b_{28})^{(5)}(\mu_2)^{(5)}-(b'_{28})^{(5)}$

$(R_2)^{(5)}=(b'_{30})^{(5)}-(r_{30})^{(5)}$

**Behavior of the solutions of equation** 349

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(6)},(\sigma_2)^{(6)},(\tau_1)^{(6)},(\tau_2)^{(6)}$ :

$(\sigma_1)^{(6)},(\sigma_2)^{(6)},(\tau_1)^{(6)},(\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)}\leq-(a'_{32})^{(6)}+(a'_{33})^{(6)}-(a''_{32})^{(6)}(T_{33},t)+(a''_{33})^{(6)}(T_{33},t)\leq-(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)}\leq-(b'_{32})^{(6)}+(b'_{33})^{(6)}-(b''_{32})^{(6)}\big((G_{35}),t\big)-(b''_{33})^{(6)}\big((G_{35}),t\big)\leq-(\tau_1)^{(6)}$$

**Definition of** $(v_1)^{(6)},(v_2)^{(6)},(u_1)^{(6)},(u_2)^{(6)},v^{(6)},u^{(6)}$ : 350

By $(v_1)^{(6)}>0$ , $(v_2)^{(6)}<0$ and respectively $(u_1)^{(6)}>0$ , $(u_2)^{(6)}<0$ the roots of the equations $(a_{33})^{(6)}\big(v^{(6)}\big)^2+(\sigma_1)^{(6)}v^{(6)}-(a_{32})^{(6)}=0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2+(\tau_1)^{(6)}u^{(6)}-(b_{32})^{(6)}=0$ and

**Definition of** $(\bar{v}_1)^{(6)},,(\bar{v}_2)^{(6)},(\bar{u}_1)^{(6)},(\bar{u}_2)^{(6)}$ : 351

By $(\bar{v}_1)^{(6)}>0$ , $(\bar{v}_2)^{(6)}<0$ and respectively $(\bar{u}_1)^{(6)}>0$ , $(\bar{u}_2)^{(6)}<0$ the roots of the equations $(a_{33})^{(6)}\big(v^{(6)}\big)^2+(\sigma_2)^{(6)}v^{(6)}-(a_{32})^{(6)}=0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2+(\tau_2)^{(6)}u^{(6)}-(b_{32})^{(6)}=0$

**Definition of** $(m_1)^{(6)},(m_2)^{(6)},(\mu_1)^{(6)},(\mu_2)^{(6)},(v_0)^{(6)}$ :-

If we define $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}$ by

$(m_2)^{(6)} = (v_0)^{(6)}, (m_1)^{(6)} = (v_1)^{(6)}$, ***if*** $(v_0)^{(6)} < (v_1)^{(6)}$

$(m_2)^{(6)} = (v_1)^{(6)}, (m_1)^{(6)} = (\bar{v}_6)^{(6)}$, ***if*** $(v_1)^{(6)} < (v_0)^{(6)} < (\bar{v}_1)^{(6)}$,

and $\boxed{(v_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$

$(m_2)^{(6)} = (v_1)^{(6)}, (m_1)^{(6)} = (v_0)^{(6)}$, ***if*** $(\bar{v}_1)^{(6)} < (v_0)^{(6)}$

and analogously 352

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}$, ***if*** $(u_0)^{(6)} < (u_1)^{(6)}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}$, ***if*** $(u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)}$,

and $\boxed{(u_0)^{(6)} = \frac{T_{32}^0}{T_{33}^0}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}$, ***if*** $(\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$

Then the solution of global equations satisfies the inequalities 353

$$G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{32}(t) \le G_{32}^0 e^{(S_1)^{(6)} t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}} G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{33}(t) \le \frac{1}{(m_2)^{(6)}} G_{32}^0 e^{(S_1)^{(6)} t} \quad 354$$

$$\left( \frac{(a_{34})^{(6)} G_{32}^0}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})} \left[ e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)} t} \right] + G_{34}^0 e^{-(S_2)^{(6)} t} \le G_{34}(t) \le \right. \quad 355$$

*(a34)6G320(m2)6(S1)6−(a34′)6e(S1)6t−e−(a34′)6t+ G340e−(a34′)6t*

$$\boxed{T_{32}^0 e^{(R_1)^{(6)} t} \le T_{32}(t) \le T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}} T_{32}^0 e^{(R_1)^{(6)} t} \le T_{32}(t) \le \frac{1}{(\mu_2)^{(6)}} T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)} T_{32}^0}{(\mu_1)^{(6)}((R_1)^{(6)} - (b'_{34})^{(6)})} \left[ e^{(R_1)^{(6)} t} - e^{-(b'_{34})^{(6)} t} \right] + T_{34}^0 e^{-(b'_{34})^{(6)} t} \le T_{34}(t) \le \quad 358$$

$$\frac{(a_{34})^{(6)} T_{32}^0}{(\mu_2)^{(6)}((R_1)^{(6)} + (r_{32})^{(6)} + (R_2)^{(6)})} \left[ e^{((R_1)^{(6)} + (r_{32})^{(6)})t} - e^{-(R_2)^{(6)} t} \right] + T_{34}^0 e^{-(R_2)^{(6)} t}$$

**Definition of** $(S_1)^{(6)}, (S_2)^{(6)}, (R_1)^{(6)}, (R_2)^{(6)}$:- 359

Where $(S_1)^{(6)} = (a_{32})^{(6)} (m_2)^{(6)} - (a'_{32})^{(6)}$

$(S_2)^{(6)} = (a_{34})^{(6)} - (p_{34})^{(6)}$

$(R_1)^{(6)} = (b_{32})^{(6)} (\mu_2)^{(6)} - (b'_{32})^{(6)}$

$$(R_2)^{(6)} = (b'_{34})^{(6)} - (r_{34})^{(6)}$$

**Behavior of the solutions of equation**

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$ :

$(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)} \le -(a'_{36})^{(7)} + (a'_{37})^{(7)} - (a''_{36})^{(7)}(T_{37}, t) + (a''_{37})^{(7)}(T_{37}, t) \le -(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)} \le -(b'_{36})^{(7)} + (b'_{37})^{(7)} - (b''_{36})^{(7)}\big((G_{39}), t\big) - (b''_{37})^{(7)}\big((G_{39}), t\big) \le -(\tau_1)^{(7)}$$

**Definition of** $(\nu_1)^{(7)}, (\nu_2)^{(7)}, (u_1)^{(7)}, (u_2)^{(7)}, \nu^{(7)}, u^{(7)}$ : 361

By $(\nu_1)^{(7)} > 0, (\nu_2)^{(7)} < 0$ and respectively $(u_1)^{(7)} > 0, (u_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_1)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(7)},, (\bar{\nu}_2)^{(7)}, (\bar{u}_1)^{(7)}, (\bar{u}_2)^{(7)}$ : 362

By $(\bar{\nu}_1)^{(7)} > 0, (\bar{\nu}_2)^{(7)} < 0$ and respectively $(\bar{u}_1)^{(7)} > 0, (\bar{u}_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

**Definition of** $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (\nu_0)^{(7)}$ :-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$$(m_2)^{(7)} = (\nu_0)^{(7)}, (m_1)^{(7)} = (\nu_1)^{(7)}, \ \boldsymbol{if}\ (\nu_0)^{(7)} < (\nu_1)^{(7)}$$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\bar{\nu}_1)^{(7)}, \boldsymbol{if}\ (\nu_1)^{(7)} < (\nu_0)^{(7)} < (\bar{\nu}_1)^{(7)},$$

and $\boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\nu_0)^{(7)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(7)} < (\nu_0)^{(7)}$$

and analogously 363

$$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}, \ \boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$$

and $\boxed{(u_0)^{(7)} = \frac{T^0_{36}}{T^0_{37}}}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)}$ where $(u_1)^{(7)}, (\bar{u}_1)^{(7)}$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^0 e^{\left((S_1)^{(7)} - (p_{36})^{(7)}\right)t} \leq G_{36}(t) \leq G_{36}^0 e^{(S_1)^{(7)} t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{\left((S_1)^{(7)} - (p_{36})^{(7)}\right)t} \leq G_{37}(t) \leq \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)} t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)}\left((S_1)^{(7)} - (p_{36})^{(7)} - (S_2)^{(7)}\right)}\left[e^{\left((S_1)^{(7)} - (p_{36})^{(7)}\right)t} - e^{-(S_2)^{(7)} t}\right] + G_{38}^0 e^{-(S_2)^{(7)} t} \leq G_{38}(t) \leq \right. \quad 366$$

$$\left. \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)}\left((S_1)^{(7)} - (a'_{38})^{(7)}\right)}\left[e^{(S_1)^{(7)} t} - e^{-(a'_{38})^{(7)} t}\right] + G_{38}^0 e^{-(a'_{38})^{(7)} t}\right)$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)} t} \leq T_{36}(t) \leq T_{36}^0 e^{\left((R_1)^{(7)} + (r_{36})^{(7)}\right)t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)} t} \leq T_{36}(t) \leq \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{\left((R_1)^{(7)} + (r_{36})^{(7)}\right)t} \quad 368$$

$$\frac{(b_{38})^{(7)} T_{36}^0}{(\mu_1)^{(7)}\left((R_1)^{(7)} - (b'_{38})^{(7)}\right)}\left[e^{(R_1)^{(7)} t} - e^{-(b'_{38})^{(7)} t}\right] + T_{38}^0 e^{-(b'_{38})^{(7)} t} \leq T_{38}(t) \leq \quad 369$$

$$\frac{(a_{38})^{(7)} T_{36}^0}{(\mu_2)^{(7)}\left((R_1)^{(7)} + (r_{36})^{(7)} + (R_2)^{(7)}\right)}\left[e^{\left((R_1)^{(7)} + (r_{36})^{(7)}\right)t} - e^{-(R_2)^{(7)} t}\right] + T_{38}^0 e^{-(R_2)^{(7)} t}$$

**Definition of** $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$$

$$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$$

$$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$$

**Behavior of the solutions of equation** 371

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ :

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$$-(\sigma_2)^{(8)} \leq -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \leq -(\sigma_1)^{(8)}$$

$$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\big((G_{43}),t\big) - (b''_{41})^{(8)}\big((G_{43}),t\big) \le -(\tau_1)^{(8)}$$

**Definition of** $(\nu_1)^{(8)}, (\nu_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, \nu^{(8)}, u^{(8)}$ : 372

By $(\nu_1)^{(8)} > 0$ , $(\nu_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0$ , $(u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(8)},, (\bar{\nu}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$ :

By $(\bar{\nu}_1)^{(8)} > 0$ , $(\bar{\nu}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0$ , $(\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

**Definition of** $(m_1)^{(8)}$ , $(m_2)^{(8)}$ , $(\mu_1)^{(8)}, (\mu_2)^{(8)}, (\nu_0)^{(8)}$ :-

If we define $(m_1)^{(8)}$ , $(m_2)^{(8)}$ , $(\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$(m_2)^{(8)} = (\nu_0)^{(8)}, (m_1)^{(8)} = (\nu_1)^{(8)}$, ***if*** $(\nu_0)^{(8)} < (\nu_1)^{(8)}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ , ***if*** $(\nu_1)^{(8)} < (\nu_0)^{(8)} < (\bar{\nu}_1)^{(8)}$,
and $\boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\nu_0)^{(8)}$, ***if*** $(\bar{\nu}_1)^{(8)} < (\nu_0)^{(8)}$

and analogously 374

$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)}$, ***if*** $(u_0)^{(8)} < (u_1)^{(8)}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}$ , ***if*** $(u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)}$,
and $\boxed{(u_0)^{(8)} = \frac{T^0_{40}}{T^0_{41}}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}$, ***if*** $(\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G^0_{40}e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{40}(t) \le G^0_{40}e^{(S_1)^{(8)}t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}} G_{40}^0 e^{((S_1)^{(8)}-(p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}} G_{40}^0 e^{(S_1)^{(8)}t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)}G_{40}^0}{(m_1)^{(8)}((S_1)^{(8)}-(p_{40})^{(8)}-(S_2)^{(8)})}\left[e^{((S_1)^{(8)}-(p_{40})^{(8)})t} - e^{-(S_2)^{(8)}t}\right] + G_{42}^0 e^{-(S_2)^{(8)}t} \le G_{42}(t) \le \right. \quad 377$$

$$\left. \frac{(a_{42})^{(8)}G_{40}^0}{(m_2)^{(8)}((S_1)^{(8)}-(a'_{42})^{(8)})}[e^{(S_1)^{(8)}t} - e^{-(a'_{42})^{(8)}t}] + G_{42}^0 e^{-(a'_{42})^{(8)}t}\right)$$

$$\boxed{T_{40}^0 e^{(R_1)^{(8)}t} \le T_{40}(t) \le T_{40}^0 e^{((R_1)^{(8)}+(r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}} T_{40}^0 e^{(R_1)^{(8)}t} \le T_{40}(t) \le \frac{1}{(\mu_2)^{(8)}} T_{40}^0 e^{((R_1)^{(8)}+(r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)}T_{40}^0}{(\mu_1)^{(8)}((R_1)^{(8)}-(b'_{42})^{(8)})}\left[e^{(R_1)^{(8)}t} - e^{-(b'_{42})^{(8)}t}\right] + T_{42}^0 e^{-(b'_{42})^{(8)}t} \le T_{42}(t) \le \quad 380$$

$$\frac{(a_{42})^{(8)}T_{40}^0}{(\mu_2)^{(8)}((R_1)^{(8)}+(r_{40})^{(8)}+(R_2)^{(8)})}\left[e^{((R_1)^{(8)}+(r_{40})^{(8)})t} - e^{-(R_2)^{(8)}t}\right] + T_{42}^0 e^{-(R_2)^{(8)}t}$$

**Definition of** $(S_1)^{(8)}, (S_2)^{(8)}, (R_1)^{(8)}, (R_2)^{(8)}$:- 381

Where $(S_1)^{(8)} = (a_{40})^{(8)}(m_2)^{(8)} - (a'_{40})^{(8)}$

$$(S_2)^{(8)} = (a_{42})^{(8)} - (p_{42})^{(8)}$$

$$(R_1)^{(8)} = (b_{40})^{(8)}(\mu_2)^{(8)} - (b'_{40})^{(8)}$$

$$(R_2)^{(8)} = (b'_{42})^{(8)} - (r_{42})^{(8)}$$

**Behavior of the solutions of equation 37 to 92** 382

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ :

$(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ four constants satisfying

$$-(\sigma_2)^{(9)} \le -(a'_{44})^{(9)} + (a'_{45})^{(9)} - (a''_{44})^{(9)}(T_{45}, t) + (a''_{45})^{(9)}(T_{45}, t) \le -(\sigma_1)^{(9)}$$

$$-(\tau_2)^{(9)} \le -(b'_{44})^{(9)} + (b'_{45})^{(9)} - (b''_{44})^{(9)}\big((G_{47}), t\big) - (b''_{45})^{(9)}\big((G_{47}), t\big) \le -(\tau_1)^{(9)}$$

**Definition of** $(\nu_1)^{(9)}, (\nu_2)^{(9)}, (u_1)^{(9)}, (u_2)^{(9)}, \nu^{(9)}, u^{(9)}$ :

By $(\nu_1)^{(9)} > 0$, $(\nu_2)^{(9)} < 0$ and respectively $(u_1)^{(9)} > 0$, $(u_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_1)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(9)}, , (\bar{\nu}_2)^{(9)}, (\bar{u}_1)^{(9)}, (\bar{u}_2)^{(9)}$ :

By $(\bar{\nu}_1)^{(9)} > 0$, $(\bar{\nu}_2)^{(9)} < 0$ and respectively $(\bar{u}_1)^{(9)} > 0$, $(\bar{u}_2)^{(9)} < 0$ the

roots of the equations $(a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\left(u^{(9)}\right)^2 + (\tau_2)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$

**Definition of** $(m_1)^{(9)}, (m_2)^{(9)}, (\mu_1)^{(9)}, (\mu_2)^{(9)}, (v_0)^{(9)}$ :-

If we define $(m_1)^{(9)}, (m_2)^{(9)}, (\mu_1)^{(9)}, (\mu_2)^{(9)}$ by

$(m_2)^{(9)} = (v_0)^{(9)}, (m_1)^{(9)} = (v_1)^{(9)}$, ***if*** $(v_0)^{(9)} < (v_1)^{(9)}$

$(m_2)^{(9)} = (v_1)^{(9)}, (m_1)^{(9)} = (\bar{v}_1)^{(9)}$, ***if*** $(v_1)^{(9)} < (v_0)^{(9)} < (\bar{v}_1)^{(9)}$,
and $\boxed{(v_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$

$(m_2)^{(9)} = (v_1)^{(9)}, (m_1)^{(9)} = (v_0)^{(9)}$, ***if*** $(\bar{v}_1)^{(9)} < (v_0)^{(9)}$

and analogously

$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)}$, ***if*** $(u_0)^{(9)} < (u_1)^{(9)}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}$, ***if*** $(u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)}$,
and $\boxed{(u_0)^{(9)} = \frac{T_{44}^0}{T_{45}^0}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}$, ***if*** $(\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G_{44}^0 e^{\left((S_1)^{(9)} - (p_{44})^{(9)}\right)t} \le G_{44}(t) \le G_{44}^0 e^{(S_1)^{(9)}t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}} G_{44}^0 e^{\left((S_1)^{(9)} - (p_{44})^{(9)}\right)t} \le G_{45}(t) \le \frac{1}{(m_2)^{(9)}} G_{44}^0 e^{(S_1)^{(9)}t}$$

$$\left(\frac{(a_{46})^{(9)} G_{44}^0}{(m_1)^{(9)}\left((S_1)^{(9)} - (p_{44})^{(9)} - (S_2)^{(9)}\right)}\left[e^{\left((S_1)^{(9)} - (p_{44})^{(9)}\right)t} - e^{-(S_2)^{(9)}t}\right] + G_{46}^0 e^{-(S_2)^{(9)}t} \le G_{46}(t) \le \right.$$
$$\left.\frac{(a_{46})^{(9)} G_{44}^0}{(m_2)^{(9)}\left((S_1)^{(9)} - (a'_{46})^{(9)}\right)}\left[e^{(S_1)^{(9)}t} - e^{-(a'_{46})^{(9)}t}\right] + G_{46}^0 e^{-(a'_{46})^{(9)}t}\right)$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le T_{44}^0 e^{\left((R_1)^{(9)} + (r_{44})^{(9)}\right)t}}$$

$$\frac{1}{(\mu_1)^{(9)}} T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le \frac{1}{(\mu_2)^{(9)}} T_{44}^0 e^{\left((R_1)^{(9)} + (r_{44})^{(9)}\right)t}$$

$$\frac{(b_{46})^{(9)} T_{44}^0}{(\mu_1)^{(9)}\left((R_1)^{(9)} - (b'_{46})^{(9)}\right)}\left[e^{(R_1)^{(9)}t} - e^{-(b'_{46})^{(9)}t}\right] + T_{46}^0 e^{-(b'_{46})^{(9)}t} \le T_{46}(t) \le$$

$$\frac{(a_{46})^{(9)} T_{44}^0}{(\mu_2)^{(9)}\left((R_1)^{(9)} + (r_{44})^{(9)} + (R_2)^{(9)}\right)}\left[e^{\left((R_1)^{(9)} + (r_{44})^{(9)}\right)t} - e^{-(R_2)^{(9)}t}\right] + T_{46}^0 e^{-(R_2)^{(9)}t}$$

**Definition of** $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$

**Proof :** From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right) - (a''_{14})^{(1)}(T_{14}, t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

**Definition of** $v^{(1)}$ :- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)}\right) \le \frac{dv^{(1)}}{dt} \le -\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)}\right)$$

From which one obtains

**Definition of** $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$ :-

For $0 < \boxed{(v_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}} < (v_1)^{(1)} < (\bar{v}_1)^{(1)}$

$$v^{(1)}(t) \ge \frac{(v_1)^{(1)} + (C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_0)^{(1)}\right)t\right]}}{1 + (C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(v_1)^{(1)} - (v_0)^{(1)}}{(v_0)^{(1)} - (v_2)^{(1)}}}$$

it follows $(v_0)^{(1)} \le v^{(1)}(t) \le (v_1)^{(1)}$

In the same manner , we get 384

$$v^{(1)}(t) \le \frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{v}_1)^{(1)} - (v_0)^{(1)}}{(v_0)^{(1)} - (\bar{v}_2)^{(1)}}}$$

From which we deduce $(v_0)^{(1)} \le v^{(1)}(t) \le (\bar{v}_1)^{(1)}$

If $0 < (v_1)^{(1)} < (v_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}} < (\bar{v}_1)^{(1)}$ we find like in the previous case, 385

$$(v_1)^{(1)} \le \frac{(v_1)^{(1)} + (C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_2)^{(1)}\right)t\right]}}{1 + (C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_2)^{(1)}\right)t\right]}} \le v^{(1)}(t) \le$$

$$\frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}} \le (\bar{v}_1)^{(1)}$$

If $0 < (\nu_1)^{(1)} \leq (\bar{\nu}_1)^{(1)} \leq \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$ , we obtain 386

$$(\nu_1)^{(1)} \leq \nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)} + (\bar{C})^{(1)}(\bar{\nu}_2)^{(1)} e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)} - (\bar{\nu}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)} e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)} - (\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\nu_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(1)}(t)$ :-

$$(m_2)^{(1)} \leq \nu^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{\nu^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(1)}(t)$ :-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{13}'')^{(1)} = (a_{14}'')^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)} G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b_{13}'')^{(1)} = (b_{14}'')^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)} T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

**Proof** : From global equations we obtain 387

$$\frac{d\nu^{(2)}}{dt} = (a_{16})^{(2)} - \left((a_{16}')^{(2)} - (a_{17}')^{(2)} + (a_{16}'')^{(2)}(T_{17}, t)\right) - (a_{17}'')^{(2)}(T_{17}, t)\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

**Definition of** $\nu^{(2)}$ :- $\boxed{\nu^{(2)} = \frac{G_{16}}{G_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \leq \frac{d\nu^{(2)}}{dt} \leq -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

**Definition of** $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$ :-

For $0 < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \geq \frac{(\nu_1)^{(2)}+(\mathrm{C})^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_0)^{(2)}\right)t\right]}}{1+(\mathrm{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\mathrm{C})^{(2)}=\frac{(\nu_1)^{(2)}-(\nu_0)^{(2)}}{(\nu_0)^{(2)}-(\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)}+(\bar{\mathrm{C}})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{\mathrm{C}})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{\mathrm{C}})^{(2)}=\frac{(\bar{\nu}_1)^{(2)}-(\nu_0)^{(2)}}{(\nu_0)^{(2)}-(\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{\mathrm{G}_{16}^0}{\mathrm{G}_{17}^0} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \leq \frac{(\nu_1)^{(2)}+(\mathrm{C})^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}}{1+(\mathrm{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}} \leq \nu^{(2)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(2)}+(\bar{\mathrm{C}})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{\mathrm{C}})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \leq (\bar{\nu}_1)^{(2)} \leq (\nu_0)^{(2)} = \frac{\mathrm{G}_{16}^0}{\mathrm{G}_{17}^0}$ , we obtain 394

$$(\nu_1)^{(2)} \leq \nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)}+(\bar{\mathrm{C}})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{\mathrm{C}})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(2)}(t)$ :- 395

$$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}, \quad \boxed{\nu^{(2)}(t)=\frac{G_{16}(t)}{G_{17}(t)}}$$

In a completely analogous way, we obtain 396

**Definition of** $u^{(2)}(t)$ :-

$$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}, \quad \boxed{u^{(2)}(t)=\frac{T_{16}(t)}{T_{17}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :** 397

If $(a_{16}'')^{(2)} = (a_{17}'')^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)}G_{17}(t)$

Analogously if $(b''_{16})^{(2)} = (b''_{17})^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)}T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

**Proof** : From global equations we obtain 398

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right) - (a''_{21})^{(3)}(T_{21}, t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

**Definition of** $\nu^{(3)}$ :- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right) \leq \frac{d\nu^{(3)}}{dt} \leq -\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right)$$

400

From which one obtains

For $0 < (\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\nu_1)^{(3)} < (\bar{\nu}_1)^{(3)}$

$$\nu^{(3)}(t) \geq \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(\nu_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\nu_2)^{(3)}}}$$

it follows $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\nu_1)^{(3)}$

In the same manner , we get 401

$$\nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)} + (\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{\nu}_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\bar{\nu}_2)^{(3)}}}$$

**Definition of** $(\bar{\nu}_1)^{(3)}$ :-

From which we deduce $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\bar{\nu}_1)^{(3)}$

If $0 < (\nu_1)^{(3)} < (\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\bar{\nu}_1)^{(3)}$ we find like in the previous case, 402

$$(\nu_1)^{(3)} \leq \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_2)^{(3)}\right)t\right]}}{1 + (C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_2)^{(3)}\right)t\right]}} \leq \nu^{(3)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(3)} + (\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\bar{\nu}_1)^{(3)}$$

If $0 < (\nu_1)^{(3)} \leq (\bar{\nu}_1)^{(3)} \leq (\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}$ , we obtain 403

$$(\nu_1)^{(3)} \leq \nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\nu_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(3)}(t)$ :-

$$(m_2)^{(3)} \leq \nu^{(3)}(t) \leq (m_1)^{(3)}, \quad \boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(3)}(t)$ :-

$$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}, \quad \boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{20})^{(3)} = (a''_{21})^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)}G_{21}(t)$

Analogously if $(b''_{20})^{(3)} = (b''_{21})^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)}T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

**Proof** : From global equations we obtain 404

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right) - (a''_{25})^{(4)}(T_{25},t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

**Definition of** $\nu^{(4)}$ :- $\quad \boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right) \leq \frac{d\nu^{(4)}}{dt} \leq -\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_4)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$ :-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \geq \frac{(\nu_1)^{(4)}+(C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_0)^{(4)}\right)t\right]}}{4+(C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)}-(\nu_0)^{(4)}}{(\nu_0)^{(4)}-(\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \le \frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{4+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)}-(\nu_0)^{(4)}}{(\nu_0)^{(4)}-(\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \le \nu^{(4)}(t) \le (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \le \frac{(\nu_1)^{(4)}+(C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_2)^{(4)}\right)t\right]}}{1+(C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_2)^{(4)}\right)t\right]}} \le \nu^{(4)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \le (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \le (\bar{\nu}_1)^{(4)} \le \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$ , we obtain 407

$$(\nu_1)^{(4)} \le \nu^{(4)}(t) \le \frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \le (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(4)}(t)$ :-

$$(m_2)^{(4)} \le \nu^{(4)}(t) \le (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(4)}(t)$ :-

$$(\mu_2)^{(4)} \le u^{(4)}(t) \le (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a_{24}'')^{(4)} = (a_{25}'')^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)}G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b_{24}'')^{(4)} = (b_{25}'')^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)}T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

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**Proof** : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a_{28}')^{(5)} - (a_{29}')^{(5)} + (a_{28}'')^{(5)}(T_{29},t)\right) - (a_{29}'')^{(5)}(T_{29},t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

**Definition of** $\nu^{(5)}$ :- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$ :-

For $0 < \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \leq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}}{1 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}} \leq \nu^{(5)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \leq (\bar{\nu}_1)^{(5)} \leq \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$ , we obtain 411

$$(\nu_1)^{(5)} \leq \nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(5)}(t)$ :-

$(m_2)^{(5)} \leq \nu^{(5)}(t) \leq (m_1)^{(5)}$, $\boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$

In a completely analogous way, we obtain

**Definition of** $u^{(5)}(t)$ :-

$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}$, $\boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{28})^{(5)} = (a''_{29})^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)} G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b''_{28})^{(5)} = (b''_{29})^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$ if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)} T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

**Proof** : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a'_{32})^{(6)} - (a'_{33})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right) - (a''_{33})^{(6)}(T_{33}, t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

**Definition of** $\nu^{(6)}$ :- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \leq \frac{d\nu^{(6)}}{dt} \leq -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$ :-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \geq \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\nu_1)^{(6)}$

In the same manner , we get 413

$$\nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{\nu}_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\bar{\nu}_1)^{(6)}$

If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 414

$$(\nu_1)^{(6)} \leq \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}} \leq \nu^{(6)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\bar{\nu}_1)^{(6)}$$

If $0 < (\nu_1)^{(6)} \leq (\bar{\nu}_1)^{(6)} \leq \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$, we obtain

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$$(\nu_1)^{(6)} \leq \nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(6)}(t)$ :-

$$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}, \quad \boxed{\nu^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(6)}(t)$ :-

$$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}, \quad \boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)}G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)}T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

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**Proof** : From global equations we obtain

$$\frac{d\nu^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right) - (a''_{37})^{(7)}(T_{37}, t)\nu^{(7)} - (a_{37})^{(7)}\nu^{(7)}$$

**Definition of** $\nu^{(7)}$ :- $\boxed{\nu^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right) \leq \frac{d\nu^{(7)}}{dt} \leq -\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(7)}, (\nu_0)^{(7)}$ :-

For $0 < \boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}} < (\nu_1)^{(7)} < (\bar{\nu}_1)^{(7)}$

$$\nu^{(7)}(t) \geq \frac{(\nu_1)^{(7)}+(C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_0)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)}=\frac{(\nu_1)^{(7)}-(\nu_0)^{(7)}}{(\nu_0)^{(7)}-(\nu_2)^{(7)}}}$$

it follows $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\nu_1)^{(7)}$

In the same manner , we get 417

$$\nu^{(7)}(t) \leq \frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)}=\frac{(\bar{\nu}_1)^{(7)}-(\nu_0)^{(7)}}{(\nu_0)^{(7)}-(\bar{\nu}_2)^{(7)}}}$$

From which we deduce $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\bar{\nu}_1)^{(7)}$

If $0 < (\nu_1)^{(7)} < (\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0} < (\bar{\nu}_1)^{(7)}$ we find like in the previous case, 418

$$(\nu_1)^{(7)} \leq \frac{(\nu_1)^{(7)}+(C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}} \leq \nu^{(7)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \leq (\bar{\nu}_1)^{(7)}$$

If $0 < (\nu_1)^{(7)} \leq (\bar{\nu}_1)^{(7)} \leq \boxed{(\nu_0)^{(7)}=\frac{G_{36}^0}{G_{37}^0}}$ , we obtain 419

$$(\nu_1)^{(7)} \leq \nu^{(7)}(t) \leq \frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \leq (\nu_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(7)}(t)$ :-

$$(m_2)^{(7)} \leq \nu^{(7)}(t) \leq (m_1)^{(7)}, \quad \boxed{\nu^{(7)}(t)=\frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(7)}(t)$ :- 420

$$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}, \quad \boxed{u^{(7)}(t)=\frac{T_{36}(t)}{T_{37}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $\nu^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)}G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for**

**the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)}T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

**Proof** : From global equations we obtain 421

$$\frac{d\nu^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right) - (a''_{41})^{(8)}(T_{41}, t)\nu^{(8)} - (a_{41})^{(8)}\nu^{(8)}$$

**Definition of** $\nu^{(8)}$ :- $\boxed{\nu^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right) \le \frac{d\nu^{(8)}}{dt} \le -\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$ :-

For $0 < \boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$

$$\nu^{(8)}(t) \ge \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \le \nu^{(8)}(t) \le (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \le \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \le \nu^{(8)}(t) \le (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \le \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}} \le \nu^{(8)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \le (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \le (\bar{\nu}_1)^{(8)} \le \boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}}$ , we obtain 424

$$(v_1)^{(8)} \leq v^{(8)}(t) \leq \frac{(\bar{v}_1)^{(8)}+(\bar{C})^{(8)}(\bar{v}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{v}_1)^{(8)}-(\bar{v}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{v}_1)^{(8)}-(\bar{v}_2)^{(8)}\right)t\right]}} \leq (v_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $v^{(8)}(t)$ :-

$$(m_2)^{(8)} \leq v^{(8)}(t) \leq (m_1)^{(8)}, \quad \boxed{v^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(8)}(t)$ :-

$$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}, \quad \boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(v_1)^{(8)} = (\bar{v}_1)^{(8)}$ if in addition $(v_0)^{(8)} = (v_1)^{(8)}$ then $v^{(8)}(t) = (v_0)^{(8)}$ and as a consequence $G_{40}(t) = (v_0)^{(8)}G_{41}(t)$ **this also defines** $(v_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then $(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)}T_{41}(t)$ This is an important consequence of the relation between $(v_1)^{(8)}$ and $(\bar{v}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

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**Proof** : From 99,20,44,22,23,44 we obtain

$$\frac{dv^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45}, t)\right) - (a''_{45})^{(9)}(T_{45}, t)v^{(9)} - (a_{45})^{(9)}v^{(9)}$$

**Definition of** $v^{(9)}$ :- $\boxed{v^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)}\right) \leq \frac{dv^{(9)}}{dt} \leq -\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_1)^{(9)}v^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

**Definition of** $(\bar{v}_1)^{(9)}, (v_0)^{(9)}$ :-

For $0 < \boxed{(v_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (v_1)^{(9)} < (\bar{v}_1)^{(9)}$

$$v^{(9)}(t) \geq \frac{(v_1)^{(9)}+(C)^{(9)}(v_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((v_1)^{(9)}-(v_0)^{(9)}\right)t\right]}}{1+(C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((v_1)^{(9)}-(v_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(v_1)^{(9)}-(v_0)^{(9)}}{(v_0)^{(9)}-(v_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\nu_9)^{(9)}$

In the same manner , we get

$$\nu^{(9)}(t) \leq \frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)}-(\nu_0)^{(9)}}{(\nu_0)^{(9)}-(\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \leq \frac{(\nu_1)^{(9)}+(C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}}{1+(C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}} \leq \nu^{(9)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \leq (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \leq (\bar{\nu}_1)^{(9)} \leq \boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$ , we obtain

$$(\nu_1)^{(9)} \leq \nu^{(9)}(t) \leq \frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \leq (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(9)}(t)$ :-

$$(m_2)^{(9)} \leq \nu^{(9)}(t) \leq (m_1)^{(9)}, \quad \boxed{\nu^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(9)}(t)$ :-

$$(\mu_2)^{(9)} \leq u^{(9)}(t) \leq (\mu_1)^{(9)}, \quad \boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)}G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then $(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)}T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following

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**Theorem :** If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on $t$ , and the conditions with the notations

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0$ ,

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$

$with\ \ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(2)} and\ (b_i'')^{(2)}$ are independent on t , and the conditions with the notations 426

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$ 427

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 428

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$ , 429

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 430

$with\ \ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(3)} and\ (b_i'')^{(3)}$ are independent on $t$ , and the conditions with the notations 431

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$ ,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

**Theorem :** If $(a_i'')^{(4)} and\ (b_i'')^{(4)}$ are independent on $t$ , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$ ,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(5)} and\ (b_i'')^{(5)}$ are independent on $t$ , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$ ,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

**Theorem** If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on $t$ , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$ ,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(7)} and\ (b_i'')^{(7)}$ are independent on $t$ , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$ ,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(8)} and\ (b_i'')^{(8)}$ are independent on $t$ , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$ ,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(9)} and\ (b_i'')^{(9)}$ are independent on $t$ , and the conditions (with the notations 436 A
45,46,27,28)

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$ ,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0 \quad 437$$

$$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0 \quad 438$$

$$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0 \quad 439$$

$$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0 \quad 440$$

$$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ]T_{14} = 0 \quad 441$$

$$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0 \quad 442$$

has a unique positive solution , which is an equilibrium solution for the system

$$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0 \quad 443$$

$$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0 \quad 444$$

$$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0 \quad 445$$

$$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\ ]T_{16} = 0 \quad 446$$

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\ ]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\ ]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\ ]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\ ]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\ ]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27})\right)]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27})\right)]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27})\right)]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0 \quad 479$$

$$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0 \quad 480$$

$$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0 \quad 481$$

$$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0 \quad 482$$

$$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0 \quad 483$$

$$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0 \quad 484$$

$$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0 \quad 484\text{A}$$

$$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$$

$$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$$

$$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$$

$$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$$

$$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$$

**Proof:** 485

(a) Indeed the first two equations have a nontrivial solution $G_{13}, G_{14}$ if

$$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$$

**Proof:** 486

(v) Indeed the first two equations have a nontrivial solution $G_{16}, G_{17}$ if

$$\mathrm{F}(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$$

**Proof:** 487

(a) Indeed the first two equations have a nontrivial solution $G_{20}, G_{21}$ if

$$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$$

**Proof:** 488

(a) Indeed the first two equations have a nontrivial solution $G_{24}, G_{25}$ if

$$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$$

**Proof:** 489

(a) Indeed the first two equations have a nontrivial solution $G_{28}, G_{29}$ if

$$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$$

**Proof:** 490

(a) Indeed the first two equations have a nontrivial solution $G_{32}, G_{33}$ if

$$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$$

**Proof:** 491

(a) Indeed the first two equations have a nontrivial solution $G_{36}, G_{37}$ if

$$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$$

**Proof:** 492

(a) Indeed the first two equations have a nontrivial solution $G_{40}, G_{41}$ if

$$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$$

**Proof:** 492 A

(a) Indeed the first two equations have a nontrivial solution $G_{44}, G_{45}$ if

$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$

**Definition and uniqueness of** $T^*_{14}$ :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique $T^*_{14}$ for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

**Definition and uniqueness of** $T^*_{17}$ :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique $T^*_{17}$ for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]} \qquad 495$$

**Definition and uniqueness of** $T^*_{21}$ :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique $T^*_{21}$ for which $f(T^*_{21}) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

**Definition and uniqueness of** $T^*_{25}$ :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that there exists a unique $T^*_{25}$ for which $f(T^*_{25}) = 0$. With this value , we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T^*_{25})]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T^*_{25})]}$$

**Definition and uniqueness of** $T^*_{29}$ :- 498

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(5)}(T_{29})$ being increasing, it follows that there exists a unique $T^*_{29}$ for which $f(T^*_{29}) = 0$. With this value , we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T^*_{29})]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T^*_{29})]}$$

**Definition and uniqueness of** $T^*_{33}$ :- 499

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(6)}(T_{33})$ being increasing, it follows that

there exists a unique $T_{33}^{*}$ for which $f(T_{33}^{*})=0$. With this value , we obtain from the three first equations

$$G_{32}=\frac{(a_{32})^{(6)}G_{33}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^{*})]} \quad , \quad G_{34}=\frac{(a_{34})^{(6)}G_{33}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^{*})]}$$

**Definition and uniqueness of** $T_{37}^{*}$ :- 500

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(7)}(T_{37})$ being increasing, it follows that there exists a unique $T_{37}^{*}$ for which $f(T_{37}^{*})=0$. With this value , we obtain from the three first equations

$$G_{36}=\frac{(a_{36})^{(7)}G_{37}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^{*})]} \quad , \quad G_{38}=\frac{(a_{38})^{(7)}G_{37}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^{*})]}$$

**Definition and uniqueness of** $T_{41}^{*}$ :- 501

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique $T_{41}^{*}$ for which $f(T_{41}^{*})=0$. With this value , we obtain from the three first equations

$$G_{40}=\frac{(a_{40})^{(8)}G_{41}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T_{41}^{*})]} \quad , \quad G_{42}=\frac{(a_{42})^{(8)}G_{41}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T_{41}^{*})]}$$

**Definition and uniqueness of** $T_{45}^{*}$ :- 501 A

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(9)}(T_{45})$ being increasing, it follows that there exists a unique $T_{45}^{*}$ for which $f(T_{45}^{*})=0$. With this value , we obtain from the three first equations

$$G_{44}=\frac{(a_{44})^{(9)}G_{45}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T_{45}^{*})]} \quad , \quad G_{46}=\frac{(a_{46})^{(9)}G_{45}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T_{45}^{*})]}$$

By the same argument, the equations admit solutions $G_{13}, G_{14}$ if 502

$$\varphi(G)=(b'_{13})^{(1)}(b'_{14})^{(1)}-(b_{13})^{(1)}(b_{14})^{(1)}-$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G)+(b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right]+(b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G)=0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{14}$ taking into account the hypothesis $\varphi(0)>0$ , $\varphi(\infty)<0$ it follows that there exists a unique $G_{14}^{*}$ such that $\varphi(G^{*})=0$

By the same argument, the equations admit solutions $G_{16}, G_{17}$ if 503

$$\varphi(G_{19})=(b'_{16})^{(2)}(b'_{17})^{(2)}-(b_{16})^{(2)}(b_{17})^{(2)}-$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19})+(b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right]+(b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19})=0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{17}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{14}^{*}$ such that $\varphi((G_{19})^{*}) = 0$ 504

By the same argument, the equations admit solutions $G_{20}, G_{21}$ if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right] + (b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{21}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{21}^{*}$ such that $\varphi((G_{23})^{*}) = 0$

By the same argument, the equations admit solutions $G_{24}, G_{25}$ if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right] + (b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{25}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{25}^{*}$ such that $\varphi((G_{27})^{*}) = 0$

By the same argument, the equations admit solutions $G_{28}, G_{29}$ if 507

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right] + (b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{29}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{29}^{*}$ such that $\varphi((G_{31})^{*}) = 0$

By the same argument, the equations admit solutions $G_{32}, G_{33}$ if 508

$$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$$

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right] + (b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{33}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{33}^{*}$ such that $\varphi({G_{35}}^{*}) = 0$

By the same argument, the equations admit solutions $G_{36}, G_{37}$ if 509

$$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$$
$$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right] + (b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{37}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{37}^{*}$ such that $\varphi({G_{39}}^{*}) = 0$

By the same argument, the equations admit solutions $G_{40}, G_{41}$ if 510

$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$
$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right] + (b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{41}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{41}$ such that $\varphi(G_{43}{}^*) = 0$

By the same argument, the equations 92,93 admit solutions $G_{44}, G_{45}$ if
$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$

$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right] + (b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{45}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{45}$ such that $\varphi((G_{47})^*) = 0$

Finally we obtain the unique solution 511

$G^*_{14}$ given by $\varphi(G^*) = 0$ , $T^*_{14}$ given by $f(T^*_{14}) = 0$ and

$$G^*_{13} = \frac{(a_{13})^{(1)}G^*_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G^*_{15} = \frac{(a_{15})^{(1)}G^*_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

$$T^*_{13} = \frac{(b_{13})^{(1)}T^*_{14}}{[(b'_{13})^{(1)}-(b''_{13})^{(1)}(G^*)]} \quad , \quad T^*_{15} = \frac{(b_{15})^{(1)}T^*_{14}}{[(b'_{15})^{(1)}-(b''_{15})^{(1)}(G^*)]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

$G^*_{17}$ given by $\varphi((G_{19})^*) = 0$ , $T^*_{17}$ given by $f(T^*_{17}) = 0$ and 512

$$G^*_{16} = \frac{(a_{16})^{(2)}G^*_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G^*_{18} = \frac{(a_{18})^{(2)}G^*_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]} \quad 513$$

$$T^*_{16} = \frac{(b_{16})^{(2)}T^*_{17}}{[(b'_{16})^{(2)}-(b''_{16})^{(2)}((G_{19})^*)]} \quad , \quad T^*_{18} = \frac{(b_{18})^{(2)}T^*_{17}}{[(b'_{18})^{(2)}-(b''_{18})^{(2)}((G_{19})^*)]} \quad 514$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

$G^*_{21}$ given by $\varphi((G_{23})^*) = 0$ , $T^*_{21}$ given by $f(T^*_{21}) = 0$ and

$$G^*_{20} = \frac{(a_{20})^{(3)}G^*_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G^*_{22} = \frac{(a_{22})^{(3)}G^*_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

$$T^*_{20} = \frac{(b_{20})^{(3)}T^*_{21}}{[(b'_{20})^{(3)}-(b''_{20})^{(3)}(G_{23}{}^*)]} \quad , \quad T^*_{22} = \frac{(b_{22})^{(3)}T^*_{21}}{[(b'_{22})^{(3)}-(b''_{22})^{(3)}(G_{23}{}^*)]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

$G_{25}^{*}$ given by $\varphi(G_{27}) = 0$ , $T_{25}^{*}$ given by $f(T_{25}^{*}) = 0$ and

$$G_{24}^{*} = \frac{(a_{24})^{(4)} G_{25}^{*}}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}^{*} = \frac{(a_{26})^{(4)} G_{25}^{*}}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^{*})]}$$

$$T_{24}^{*} = \frac{(b_{24})^{(4)} T_{25}^{*}}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^{*})]} \quad , \quad T_{26}^{*} = \frac{(b_{26})^{(4)} T_{25}^{*}}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^{*})]} \qquad 517$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

$G_{29}^{*}$ given by $\varphi((G_{31})^{*}) = 0$ , $T_{29}^{*}$ given by $f(T_{29}^{*}) = 0$ and

$$G_{28}^{*} = \frac{(a_{28})^{(5)} G_{29}^{*}}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}^{*} = \frac{(a_{30})^{(5)} G_{29}^{*}}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^{*})]}$$

$$T_{28}^{*} = \frac{(b_{28})^{(5)} T_{29}^{*}}{[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31})^{*})]} \quad , \quad T_{30}^{*} = \frac{(b_{30})^{(5)} T_{29}^{*}}{[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31})^{*})]} \qquad 519$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

$G_{33}^{*}$ given by $\varphi((G_{35})^{*}) = 0$ , $T_{33}^{*}$ given by $f(T_{33}^{*}) = 0$ and

$$G_{32}^{*} = \frac{(a_{32})^{(6)} G_{33}^{*}}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^{*})]} \quad , \quad G_{34}^{*} = \frac{(a_{34})^{(6)} G_{33}^{*}}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^{*})]}$$

$$T_{32}^{*} = \frac{(b_{32})^{(6)} T_{33}^{*}}{[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35})^{*})]} \quad , \quad T_{34}^{*} = \frac{(b_{34})^{(6)} T_{33}^{*}}{[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35})^{*})]} \qquad 521$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

$G_{37}^{*}$ given by $\varphi((G_{39})^{*}) = 0$ , $T_{37}^{*}$ given by $f(T_{37}^{*}) = 0$ and

$$G_{36}^{*} = \frac{(a_{36})^{(7)} G_{37}^{*}}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^{*})]} \quad , \quad G_{38}^{*} = \frac{(a_{38})^{(7)} G_{37}^{*}}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^{*})]}$$

$$T_{36}^{*} = \frac{(b_{36})^{(7)} T_{37}^{*}}{[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39})^{*})]} \quad , \quad T_{38}^{*} = \frac{(b_{38})^{(7)} T_{37}^{*}}{[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39})^{*})]}$$

Finally we obtain the unique solution 523

$G_{41}^{*}$ given by $\varphi((G_{43})^{*}) = 0$ , $T_{41}^{*}$ given by $f(T_{41}^{*}) = 0$ and

$$G_{40}^{*} = \frac{(a_{40})^{(8)} G_{41}^{*}}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^{*})]} \quad , \quad G_{42}^{*} = \frac{(a_{42})^{(8)} G_{41}^{*}}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^{*})]}$$

$$T_{40}^{*} = \frac{(b_{40})^{(8)} T_{41}^{*}}{[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43})^{*})]} \quad , \quad T_{42}^{*} = \frac{(b_{42})^{(8)} T_{41}^{*}}{[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43})^{*})]}$$

Finally we obtain the unique solution of 89 to 99 523

A

$G_{45}^{*}$ given by $\varphi((G_{47})^{*}) = 0$ , $T_{45}^{*}$ given by $f(T_{45}^{*}) = 0$ and

$$G_{44}^{*} = \frac{(a_{44})^{(9)} G_{45}^{*}}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^{*})]} \quad , \quad G_{46}^{*} = \frac{(a_{46})^{(9)} G_{45}^{*}}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^{*})]}$$

$$T_{44}^{*} = \frac{(b_{44})^{(9)} T_{45}^{*}}{[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47})^{*})]} \quad , \quad T_{46}^{*} = \frac{(b_{46})^{(9)} T_{45}^{*}}{[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47})^{*})]}$$

**ASYMPTOTIC STABILITY ANALYSIS** 524

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

**Proof:** Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^{*} + \mathbb{G}_i \qquad , T_i = T_i^{*} + \mathbb{T}_i$$

$$\frac{\partial (a''_{14})^{(1)}}{\partial T_{14}}(T_{14}^{*}) = (q_{14})^{(1)} \quad , \frac{\partial (b_i'')^{(1)}}{\partial G_j}(G^{*}) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a'_{13})^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)} G_{13}^{*}\mathbb{T}_{14} \quad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a'_{14})^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)} G_{14}^{*}\mathbb{T}_{14} \quad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a'_{15})^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)} G_{15}^{*}\mathbb{T}_{14} \quad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b'_{13})^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)} T_{13}^{*}\mathbb{G}_j\right) \quad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b'_{14})^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)} T_{14}^{*}\mathbb{G}_j\right) \quad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b'_{15})^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)} T_{15}^{*}\mathbb{G}_j\right) \quad 530$$

**ASYMPTOTIC STABILITY ANALYSIS** 531

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(2)}$ and $(b_i'')^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i \tag{532}$$

$$\frac{\partial(a_{17}'')^{(2)}}{\partial T_{17}}(T_{17}^*) = (q_{17})^{(2)} \quad , \frac{\partial(b_i'')^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \tag{533}$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a_{16}')^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G_{16}^*\mathbb{T}_{17} \tag{534}$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a_{17}')^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G_{17}^*\mathbb{T}_{17} \tag{535}$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a_{18}')^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G_{18}^*\mathbb{T}_{17} \tag{536}$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b_{16}')^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T_{16}^*\mathbb{G}_j\right) \tag{537}$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b_{17}')^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}T_{17}^*\mathbb{G}_j\right) \tag{538}$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b_{18}')^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}T_{18}^*\mathbb{G}_j\right) \tag{539}$$

**ASYMPTOTIC STABILITY ANALYSIS** 540

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(3)}$ *and* $(b_i'')^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{21}'')^{(3)}}{\partial T_{21}}(T_{21}^*) = (q_{21})^{(3)} \quad , \frac{\partial(b_i'')^{(3)}}{\partial G_j}((G_{23})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a_{20}')^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G_{20}^*\mathbb{T}_{21} \tag{541}$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a_{21}')^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G_{21}^*\mathbb{T}_{21} \tag{542}$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a_{22}')^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G_{22}^*\mathbb{T}_{21} \tag{543}$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b_{20}')^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T_{20}^*\mathbb{G}_j\right) \tag{544}$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b_{21}')^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T_{21}^*\mathbb{G}_j\right) \tag{545}$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b'_{22})^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T^*_{22}\mathbb{G}_j\right) \quad 546$$

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**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}\ and\ (b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{25})^{(4)}}{\partial T_{25}}(T^*_{25}) = (q_{25})^{(4)} \ , \ \frac{\partial(b_i'')^{(4)}}{\partial G_j}((G_{27})^*\ ) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\left((a'_{24})^{(4)} + (p_{24})^{(4)}\right)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G^*_{24}\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\left((a'_{25})^{(4)} + (p_{25})^{(4)}\right)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G^*_{25}\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\left((a'_{26})^{(4)} + (p_{26})^{(4)}\right)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G^*_{26}\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\left((b'_{24})^{(4)} - (r_{24})^{(4)}\right)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(24)(j)}T^*_{24}\mathbb{G}_j\right) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\left((b'_{25})^{(4)} - (r_{25})^{(4)}\right)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\left(s_{(25)(j)}T^*_{25}\mathbb{G}_j\right) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T^*_{26}\mathbb{G}_j\right) \quad 554$$

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**Theorem 5:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}\ and\ (b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 556

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{29})^{(5)}}{\partial T_{29}}(T^*_{29}) = (q_{29})^{(5)} \ , \ \frac{\partial(b_i'')^{(5)}}{\partial G_j}(\ (G_{31})^*\ ) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a'_{28})^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G^*_{28}\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a'_{29})^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G^*_{29}\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a'_{30})^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G^*_{30}\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b'_{28})^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T^*_{28}\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b'_{29})^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T^*_{29}\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b'_{30})^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T^*_{30}\mathbb{G}_j\right) \quad 562$$

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**Theorem 6:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}$ *and* $(b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{33})^{(6)}}{\partial T_{33}}(T^*_{33}) = (q_{33})^{(6)} \quad , \frac{\partial(b_i'')^{(6)}}{\partial G_j}\left((G_{35})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a'_{32})^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G^*_{32}\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a'_{33})^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G^*_{33}\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a'_{34})^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G^*_{34}\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b'_{32})^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T^*_{32}\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b'_{33})^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T^*_{33}\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b'_{34})^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T^*_{34}\mathbb{G}_j\right) \quad 570$$

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**Theorem 7:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 572

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{37}'')^{(7)}}{\partial T_{37}}(T_{37}^{*}) = (q_{37})^{(7)} \quad , \frac{\partial(b_i'')^{(7)}}{\partial G_j}\left( (G_{39})^{**} \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a_{36}')^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G_{36}^{*}\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a_{37}')^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G_{37}^{*}\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a_{38}')^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G_{38}^{*}\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b_{36}')^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T_{36}^{*}\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b_{37}')^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T_{37}^{*}\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b_{38}')^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T_{38}^{*}\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

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**Theorem 8:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 580

$$G_i = G_i^{*} + \mathbb{G}_i \qquad , T_i = T_i^{*} + \mathbb{T}_i$$

$$\frac{\partial(a_{41}'')^{(8)}}{\partial T_{41}}(T_{41}^{*}) = (q_{41})^{(8)} \quad , \frac{\partial(b_i'')^{(8)}}{\partial G_j}\left( (G_{43})^{*} \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a_{40}')^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G_{40}^{*}\mathbb{T}_{41} \quad 581$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a_{41}')^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G_{41}^{*}\mathbb{T}_{41} \quad 582$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a_{42}')^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G_{42}^{*}\mathbb{T}_{41} \quad 583$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b_{40}')^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T_{40}^{*}\mathbb{G}_j\right) \quad 584$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b_{41}')^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T_{41}^{*}\mathbb{G}_j\right) \quad 585$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b'_{42})^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T^*_{42}\mathbb{G}_j\right) \quad 586$$

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**ASYMPTOTIC STABILITY ANALYSIS** A

**Theorem 9:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}\ and\ (b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

**Proof: Denote**

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{45})^{(9)}}{\partial T_{45}}(T^*_{45}) = (q_{45})^{(9)} \ , \ \frac{\partial(b_i'')^{(9)}}{\partial G_j}(\ (G_{47})^*\ ) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a'_{44})^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)}G^*_{44}\mathbb{T}_{45} \quad 586\ B$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a'_{45})^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)}G^*_{45}\mathbb{T}_{45} \quad 586\ C$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a'_{46})^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)}G^*_{46}\mathbb{T}_{45} \quad 586\ D$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b'_{44})^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)}T^*_{44}\mathbb{G}_j\right) \quad 586\ E$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b'_{45})^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)}T^*_{45}\mathbb{G}_j\right) \quad 586\ F$$

$$\frac{d\mathbb{T}_{46}}{dt} = -\left((b'_{46})^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T^*_{46}\mathbb{G}_j\right) \quad 586\ G$$

**The characteristic equation of this system is** 587

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(q_{13})^{(1)}G^*_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)}T^*_{14} + (b_{14})^{(1)}s_{(13),(14)}T^*_{14}\right)$$

$$+\left(\left((\lambda)^{(1)} + (a'_{14})^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)}G^*_{13} + (a_{13})^{(1)}(q_{14})^{(1)}G^*_{14}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)}T^*_{14} + (b_{14})^{(1)}s_{(13),(13)}T^*_{13}\right)$$

$$\left((\lambda)^{(1)}\right)^2 + \left(\ (a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\Big)$$

$$\left((\lambda)^{(1)}\right)^2 + \left(\ (b'_{13})^{(1)} + (b'_{14})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)(\lambda)^{(1)}\Big)$$

$$+\left(\left((\lambda)^{(1)}\right)^2+\left((a'_{13})^{(1)}+(a'_{14})^{(1)}+(p_{13})^{(1)}+(p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)}+(a'_{13})^{(1)}+(p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^*_{14}+(a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}+(b'_{13})^{(1)}-(r_{13})^{(1)}\right)s_{(14),(15)}T^*_{14}+(b_{14})^{(1)}s_{(13),(15)}T^*_{13}\right)\}=0$$

$$+$$

$$(\lambda)^{(2)}+(b'_{18})^{(2)}-(r_{18})^{(2)}\}\{\left((\lambda)^{(2)}+(a'_{18})^{(2)}+(p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^*_{17}+(a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^*_{17}\right)$$

$$+\left(\left((\lambda)^{(2)}+(a'_{17})^{(2)}+(p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^*_{16}+(a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2+\left((b'_{16})^{(2)}+(b'_{17})^{(2)}-(r_{16})^{(2)}+(r_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$+\left(\left((\lambda)^{(2)}\right)^2+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}+(a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^*_{16}\right)\}=0$$

$$+$$

$$(\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\}\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left(\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^*_{20}\right)\right]$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^*_{21}+(b_{21})^{(3)}s_{(20),(21)}T^*_{21}\right)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^*_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^*_{21}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^*_{21}+(b_{21})^{(3)}s_{(20),(20)}T^*_{20}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^2+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$+\left(\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^*_{20}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^*_{21}+(b_{21})^{(3)}s_{(20),(22)}T^*_{20}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\right)\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left(\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^*_{24}\right)\right]$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^*_{25}+(b_{25})^{(4)}s_{(24),(25)}T^*_{25}\right)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^*_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^*_{25}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^*_{25}+(b_{25})^{(4)}s_{(24),(24)}T^*_{24}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^2+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$+\left(\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^*_{24}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^*_{25}+(b_{25})^{(4)}s_{(24),(26)}T^*_{24}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\right)\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^*_{28}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^*_{29}+(b_{29})^{(5)}s_{(28),(29)}T^*_{29}\right)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^*_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^*_{29}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^*_{29}+(b_{29})^{(5)}s_{(28),(28)}T^*_{28}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^2+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$+\left(\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^*_{28}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^*_{29}+(b_{29})^{(5)}s_{(28),(30)}T^*_{28}\right)\}=0$$

$$+$$

$$(\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left(\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^*_{32}\right)\right]$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^*_{33}+(b_{33})^{(6)}s_{(32),(33)}T^*_{33}\right)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^*_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^*_{33}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^*_{33}+(b_{33})^{(6)}s_{(32),(32)}T^*_{32}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^2+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$+\left(\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^*_{32}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^*_{33}+(b_{33})^{(6)}s_{(32),(34)}T^*_{32}\right)\}=0$$

$$+$$

$$(\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^*_{36}\right)\right]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^*_{37}+(b_{37})^{(7)}s_{(36),(37)}T^*_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^*_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^*_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^*_{37}+(b_{37})^{(7)}s_{(36),(36)}T^*_{36}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^2+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$+\left(\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^*_{36}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^*_{37}+(b_{37})^{(7)}s_{(36),(38)}T^*_{36}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\right)\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^*_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^*_{41}+(b_{41})^{(8)}s_{(40),(41)}T^*_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^*_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^*_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^*_{41}+(b_{41})^{(8)}s_{(40),(40)}T^*_{40}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^2+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$+\left(\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^*_{40}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^*_{41}+(b_{41})^{(8)}s_{(40),(42)}T^*_{40}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\right)\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^*_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^*_{45}+(b_{45})^{(9)}s_{(44),(45)}T^*_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^*_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^*_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^*_{45}+(b_{45})^{(9)}s_{(44),(44)}T^*_{44}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^2+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$+\left(\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^*_{44}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^*_{45}+(b_{45})^{(9)}s_{(44),(46)}T^*_{44}\right)\}=0$$

**And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.**

## SECTION TWENTY FIVE

## Thoughts Are The Manifestation Of Free Will

### INTRODUCTION—VARIABLES USED

Our thoughts are that the manifestation of (e) free will is intrinsically associated with (e&eb) the nuclear and/or electron spin processes inside (eb) the varying high electric voltage environment of (e) the neural membranes and proteins which likely enable (eb) the said spin processes to be "proactive," that is, being able to utilize non-local energy (potential) and quantum information to influence (e&eb) brain activities through (e&eb) spin chemistry and possibly other chemical/physical processes in defiance of (e) the second law of thermodynamics

### NOTATION

#### Module One

$G_{13}$ : Category one of thoughts are that the manifestation of free will; nuclear and/or electron spin processes inside (eb) the varying high electric voltage environment of (e) the neural membranes and proteins which likely enable (eb) the said spin processes to be "proactive," that is, being able to utilize non-local energy (potential) and quantum information to influence (e&eb) brain activities through (e&eb) spin chemistry and possibly other chemical/physical processes in defiance of (e) the second law of thermodynamics

$G_{14}$ : Category two of SAS

$G_{15}$ : Category three of SAS

$T_{13}$ : Category one of nuclear and/or electron spin processes inside (eb) the varying high electric voltage environment of (e) the neural membranes and proteins which likely enable (eb) the said spin processes to be "proactive," that is, being able to utilize non-local energy (potential) and quantum information to influence (e&eb) brain activities through (e&eb) spin chemistry and possibly other chemical/physical processes in

defiance of (e) the second law of thermodynamics; thoughts are that the manifestation of free will

$T_{14}$ : Category two of SAS

$T_{15}$ : Category three of SAS

**Module Two**

Our thoughts are that the manifestation of free will is intrinsically associated with the nuclear and/or electron spin processes inside (eb) the varying high electric voltage environment of (e) the neural membranes and proteins which likely enable (eb) the said spin processes to be "proactive," that is, being able to utilize non-local energy (potential) and quantum information to influence (e&eb) brain activities through (e&eb) spin chemistry and possibly other chemical/physical processes in defiance of (e) the second law of thermodynamics

$G_{16}$ : Category one of thoughts are that the manifestation of free will is intrinsically associated with the nuclear and/or electron spin processes

$G_{17}$ : Category two of SAS

$G_{18}$ : Category three of SAS

$T_{16}$ : Category one ofvarying high electric voltage environment of (e) the neural membranes and proteins which likely enable (eb) the said spin processes to be "proactive," that is, being able to utilize non-local energy (potential) and quantum information to influence (e&eb) brain activities through (e&eb) spin chemistry and possibly other chemical/physical processes in defiance of (e) the second law of thermodynamics

$T_{17}$ : Category two of SAS

$T_{18}$ : Category three of SAS

**Module three**

Our thoughts are that the manifestation of free will is intrinsically associated with the nuclear and/or electron spin processes inside the varying high electric voltage environment of (e) the neural membranes and proteins which likely enable (eb) the said spin processes to be "proactive," that is, being able to utilize non-local energy (potential) and quantum information to influence (e&eb) brain activities through (e&eb) spin chemistry and possibly other chemical/physical processes in defiance of (e) the second law of thermodynamics

$G_{20}$ : Category one of thoughts are that the manifestation of free will is intrinsically associated with the nuclear and/or electron spin processes inside the varying high electric voltage environment; **neural membranes and proteins which likely enable (eb) the said spin processes to be "proactive," that is, being able to utilize non-local energy (potential) and quantum information to influence (e&eb) brain activities through (e&eb) spin chemistry and possibly other chemical/physical processes in defiance of (e) the second law of thermodynamics**

$G_{21}$ : Category two of SAS

$G_{22}$ : Category three of SAS

$T_{20}$ : Category one of**neural membranes and proteins which likely enable (eb) the said spin processes to be "proactive," that is, being able to utilize non-local energy (potential) and quantum information to influence (e&eb) brain activities through (e&eb) spin chemistry and possibly other chemical/physical processes in defiance of (e) the second law of thermodynamics;** thoughts are that the manifestation of free will is intrinsically associated with the nuclear and/or electron spin processes inside

the varying high electric voltage environment

$T_{21}$ : Category two of SAS

$T_{22}$ : Category three of SAS

**Module four**

Our thoughts are that the manifestation of free will is intrinsically associated with the nuclear and/or electron spin processes inside the varying high electric voltage environment of the neural membranes and proteins which likely enable (eb) the said spin processes to be "proactive," that is, being able to utilize non-local energy (potential) and quantum information to influence (e&eb) brain activities through (e&eb) spin chemistry and possibly other chemical/physical processes in defiance of (e) the second law of thermodynamics

$G_{24}$ : Category one of thoughts are that the manifestation of free will is intrinsically associated with the nuclear and/or electron spin processes inside the varying high electric voltage environment of the neural membranes and proteins

$G_{25}$ : Category two of SAS

$G_{26}$ : Category three of SAS

$T_{24}$ : Category one ofsaid spin processes to be "proactive," that is, being able to utilize non-local energy (potential) and quantum information to influence (e&eb) brain activities through (e&eb) spin chemistry and possibly other chemical/physical processes in defiance of (e) the second law of thermodynamics

$T_{25}$ : Category two of SAS

$T_{26}$ : Category three of SAS

**Module five**

Our thoughts are that the manifestation of free will is intrinsically associated with the nuclear and/or electron spin processes inside the varying high electric voltage environment of the neural membranes and proteins which likely enable the said spin processes to be "proactive," that is, being able to utilize non-local energy (potential) and quantum information to influence (e&eb) brain activities through (e&eb) spin chemistry and possibly other chemical/physical processes in defiance of (e) the second law of thermodynamics

$G_{28}$ : Category one of thoughts are that the manifestation of free will is intrinsically associated with the nuclear and/or electron spin processes inside the varying high electric voltage environment of the neural membranes and proteins which likely enable the said spin processes to be "proactive

$G_{29}$ : Category two of SAS

$G_{30}$ : Category three of SAS

$T_{28}$ : Category one of ability to utilize non-local energy (potential) and quantum information to influence (e&eb) brain activities through (e&eb) spin chemistry and possibly other chemical/physical processes in defiance of (e) the second law of thermodynamics

$T_{29}$ : Category two of SAS

$T_{30}$ : Category three of SAS

**Module six**

Our thoughts are that the manifestation of free will is intrinsically associated with the nuclear and/or electron spin processes inside the varying high electric voltage environment of the neural membranes and proteins which likely enable the said spin processes to be "proactive," that is, being able to utilize non-local energy (potential) and quantum information to influence (e&eb) brain activities through (e&eb) spin chemistry and possibly other chemical/physical processes in defiance of (e) the second law of thermodynamics

$G_{32}$ : Category one of thoughts are that the manifestation of free will is intrinsically associated with the nuclear and/or electron spin processes inside the varying high electric voltage environment of the neural membranes and proteins which likely enable the said spin processes to be "proactive," that is, being able to utilize; **brain activities through (e&eb) spin chemistry and possibly other chemical/physical processes in defiance of (e) the second law of thermodynamics**

$G_{33}$ : Category two of SAS

$G_{34}$ : Category three of SAS

$T_{32}$ : Category one of **brain activities through (e&eb) spin chemistry and possibly other chemical/physical processes in defiance of (e) the second law of thermodynamics;** thoughts are that the manifestation of free will is intrinsically associated with the nuclear and/or electron spin processes inside the varying high electric voltage environment of the neural membranes and proteins which likely enable the said spin processes to be "proactive," that is, being able to utilize

$T_{33}$ : Category two of SAS

$T_{34}$ : Category three of SAS

**Module seven**

Our thoughts are that the manifestation of free will is intrinsically associated with the nuclear and/or electron spin processes inside the varying high electric voltage environment of the neural membranes and proteins which likely enable the said spin processes to be "proactive," that is, being able to utilize non-local energy (potential) and quantum information to influence brain activities through (e&eb) spin chemistry and possibly other chemical/physical processes in defiance of (e) the second law of thermodynamics

$G_{36}$ : Category one of thoughts are that the manifestation of free will is intrinsically associated with the nuclear and/or electron spin processes inside the varying high electric voltage environment of the neural membranes and proteins which likely enable the said spin processes to be "proactive," that is, being able to utilize non-local energy (potential) and quantum information to influence brain activities; **spin chemistry and possibly other chemical/physical processes in defiance of (e) the second law of thermodynamics**

$G_{37}$ : Category two of SAS

$G_{38}$ : Category three of SAS

$T_{36}$ : Category one of **spin chemistry and possibly other chemical/physical processes in defiance of (e) the second law of thermodynamics**; thoughts are that the manifestation of free will is intrinsically associated with the nuclear and/or electron spin processes inside the varying high electric voltage environment of the neural membranes and proteins which likely enable the said spin processes to be "proactive," that is, being able to utilize non-local energy (potential) and quantum information to influence brain activities

$T_{37}$ : Category two of SAS

$T_{38}$ : Category three of SAS

**Module eight**

Our thoughts are that the manifestation of free will is intrinsically associated with the nuclear and/or electron spin processes inside the **varying high electric voltage environment of the neural membranes** and proteins which likely enable the said spin processes to be "proactive," that is, being able to utilize non-local energy (potential) and quantum information to influence brain activities through spin chemistry and possibly other chemical/physical processes in defiance of (e) the second law of thermodynamics

$G_{40}$ : Category one of second law of thermodynamics

$G_{41}$ : Category two of SAS

$G_{42}$ : Category three of SAS

$T_{40}$ : Category one of thoughts are that the manifestation of free will is intrinsically associated with the nuclear and/or electron spin processes inside the varying high electric voltage environment of the neural membranes and proteins which likely enable the said spin processes to be "proactive," that is, being able to utilize non-local energy (potential) and quantum information to influence brain activities through spin chemistry and possibly other chemical/physical processes

$T_{41}$ : Category two of SAS

$T_{42}$ : Category three of SAS

**Module Nine**

$G_{44}$ : Category one of **varying high electric voltage environment of the neural membranes;** spin dynamics

$G_{45}$ : Category two of SAS

$G_{46}$ : Category three of SAS

$T_{44}$ : Category one of spin dynamics; **varying high electric voltage environment of the neural membranes**

$T_{45}$ : Category two of SAS

$T_{46}$ : Category three of SAS

**The Coefficients:**

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}\ ,\ (b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}$,
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)},\ \ (b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}\ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$ ,
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)}$,
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)}$,
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)}$,
are Dissipation coefficients

**Module Numbered One**

**The differential system of this model is now (Module Numbered one)**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \quad 1$$
$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad 2$$
$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15} \quad 3$$
$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13} \quad 4$$
$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14} \quad 5$$
$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)\right]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14}, t) =$ First augmentation factor
$-(b''_{13})^{(1)}(G, t) =$ First detritions factor

**Module Numbered Two**

**The differential system of this model is now ( Module numbered two)**

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right]G_{16} \quad 7$$
$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t)\right]G_{17} \quad 8$$
$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t)\right]G_{18} \quad 9$$
$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\left((G_{19}), t\right)\right]T_{16} \quad 10$$
$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\left((G_{19}), t\right)\right]T_{17} \quad 11$$
$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\left((G_{19}), t\right)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17}, t) =$ First augmentation factor
$-(b''_{16})^{(2)}\left((G_{19}), t\right) =$ First detritions factor

**Module Numbered Three**

**The differential system of this model is now (Module numbered three)**

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21},t)\right]G_{20} \quad 13$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21},t)\right]G_{21} \quad 14$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21},t)\right]G_{22} \quad 15$$

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23},t)\right]T_{20} \quad 16$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23},t)\right]T_{21} \quad 17$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23},t)\right]T_{22} \quad 18$$

$+(a''_{20})^{(3)}(T_{21},t) =$ First augmentation factor

$-(b''_{20})^{(3)}(G_{23},t) =$ First detritions factor

**Module Numbered Four**

**The differential system of this model is now (Module numbered Four)**

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right]G_{24} \quad 19$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25},t)\right]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25},t)\right]G_{26} \quad 21$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}\big((G_{27}),t\big)\right]T_{24} \quad 22$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}\big((G_{27}),t\big)\right]T_{25} \quad 23$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}\big((G_{27}),t\big)\right]T_{26} \quad 24$$

$+(a''_{24})^{(4)}(T_{25},t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}\big((G_{27}),t\big) =$ **First detritions factor**

**Module Numbered Five**:

**The differential system of this model is now (Module number five)**

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29},t)\right]G_{28} \quad 25$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29},t)\right]G_{29} \quad 26$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29},t)\right]G_{30} \quad 27$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}\big((G_{31}),t\big)\right]T_{28} \quad 28$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}\big((G_{31}),t\big)\right]T_{29} \quad 29$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}\big((G_{31}),t\big)\right]T_{30} \quad 30$$

$+(a''_{28})^{(5)}(T_{29},t) =$ **First augmentation factor**

$-(b''_{28})^{(5)}\big((G_{31}),t\big) =$ **First detritions factor**

**Module Numbered Six**

**The differential system of this model is now (Module numbered Six)**

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33},t)\right]G_{32} \quad 31$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33},t)\right]G_{33} \quad 32$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33},t)\right]G_{34} \quad 33$$

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35}),t)\right]T_{32} \quad 34$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}((G_{35}),t)\right]T_{33} \quad 35$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35}),t)\right]T_{34} \quad 36$$

$+(a''_{32})^{(6)}(T_{33},t) =$ **First augmentation factor**

**Module Numbered Seven:**

**The differential system of this model is now (Seventh Module)**

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right]G_{36} \quad 37$$

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37},t)\right]G_{37} \quad 38$$

$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37},t)\right]G_{38} \quad 39$$

$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39}),t)\right]T_{36} \quad 40$$

$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}((G_{39}),t)\right]T_{37} \quad 41$$

$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39}),t)\right]T_{38} \quad 42$$

$+(a''_{36})^{(7)}(T_{37},t) =$ **First augmentation factor**

**Module Numbered Eight**

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right]G_{40} \quad 43$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41},t)\right]G_{41} \quad 44$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41},t)\right]G_{42} \quad 45$$

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43}),t)\right]T_{40} \quad 46$$

$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}((G_{43}),t)\right]T_{41} \quad 47$$

$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43}),t)\right]T_{42} \quad 48$$

**Module Numbered Nine**

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47}),t)\right]T_{44} \quad 52$$

$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}((G_{47}),t)\right]T_{45} \quad 53$$

$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47}),t)\right]T_{46} \quad 54$$

$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}((G_{47}),t) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[\begin{array}{c} (a'_{13})^{(1)}\boxed{+(a''_{13})^{(1)}(T_{14},t)}\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7)}(T_{37},t)}\boxed{+(a''_{40})^{(8,8)}(T_{41},t)}\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right]G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)}\boxed{+(a''_{14})^{(1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)}\boxed{+(a''_{15})^{(1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15} \quad 57$$

Where $\boxed{(a''_{13})^{(1)}(T_{14},t)}$, $\boxed{(a''_{14})^{(1)}(T_{14},t)}$, $\boxed{(a''_{15})^{(1)}(T_{14},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7)}(T_{37},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8)}(T_{41},t)}$ $\boxed{+(a''_{41})^{(8,8)}(T_{41},t)}$ $\boxed{+(a''_{42})^{(8,8)}(T_{41},t)}$ are eight augmentation coefficient for 1,2,3

$\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)}\boxed{-(b''_{13})^{(1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13} \quad 58$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)}\boxed{-(b''_{14})^{(1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14} \quad 59$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)}\boxed{-(b''_{15})^{(1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15} \quad 60$$

Where $\boxed{-(b''_{13})^{(1)}(G,t)}$, $\boxed{-(b''_{14})^{(1)}(G,t)}$, $\boxed{-(b''_{15})^{(1)}(G,t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{20}'')^{(3,3,)}(G_{23},t)}$, $\boxed{-(b_{21}'')^{(3,3,)}(G_{23},t)}$, $\boxed{-(b_{22}'')^{(3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{24}'')^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b_{25}'')^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b_{26}'')^{(4,4,4,4,)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{28}'')^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{32}'')^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b_{34}'')^{(6,6,6,6,)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{37}'')^{(7,7,)}(G_{39},t)}$, $\boxed{-(b_{36}'')^{(7,7,)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{40}'')^{(8,8)}(G_{43},t)}$ $\boxed{-(b_{41}'')^{(8,8)}(G_{43},t)}$ $\boxed{-(b_{42}'')^{(8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{44}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{45}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{46}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[\begin{array}{ccc} (a_{16}')^{(2)}\boxed{+(a_{16}'')^{(2)}(T_{17},t)} & \boxed{+(a_{13}'')^{(1,1,)}(T_{14},t)} & \boxed{+(a_{20}'')^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a_{24}'')^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{28}'')^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{32}'')^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{36}'')^{(7,7,7)}(T_{37},t)} & \boxed{+(a_{40}'')^{(8,8,8)}(T_{41},t)} & \boxed{+(a_{44}'')^{(9,9)}(T_{45},t)} \end{array}\right] G_{16} \quad (61)$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[\begin{array}{ccc} (a_{17}')^{(2)}\boxed{+(a_{17}'')^{(2)}(T_{17},t)} & \boxed{+(a_{14}'')^{(1,1,)}(T_{14},t)} & \boxed{+(a_{21}'')^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a_{25}'')^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{29}'')^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{33}'')^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{37}'')^{(7,7,7)}(T_{37},t)} & \boxed{+(a_{41}'')^{(8,8,8)}(T_{41},t)} & \boxed{+(a_{45}'')^{(9,9)}(T_{45},t)} \end{array}\right] G_{17} \quad (62)$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[\begin{array}{ccc} (a_{18}')^{(2)}\boxed{+(a_{18}'')^{(2)}(T_{17},t)} & \boxed{+(a_{15}'')^{(1,1,)}(T_{14},t)} & \boxed{+(a_{22}'')^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a_{26}'')^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{30}'')^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{34}'')^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{38}'')^{(7,7,7)}(T_{37},t)} & \boxed{+(a_{42}'')^{(8,8,8)}(T_{41},t)} & \boxed{+(a_{46}'')^{(9,9)}(T_{45},t)} \end{array}\right] G_{18} \quad (63)$$

Where $\boxed{+(a_{16}'')^{(2)}(T_{17},t)}$, $\boxed{+(a_{17}'')^{(2)}(T_{17},t)}$, $\boxed{+(a_{18}'')^{(2)}(T_{17},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a_{13}'')^{(1,1,)}(T_{14},t)}$, $\boxed{+(a_{14}'')^{(1,1,)}(T_{14},t)}$, $\boxed{+(a_{15}'')^{(1,1,)}(T_{14},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{20}'')^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a_{21}'')^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a_{22}'')^{(3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{24}'')^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{25}'')^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{26}'')^{(4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{28}'')^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{29}'')^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{30}'')^{(5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{32}'')^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{33}'')^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{34}'')^{(6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{36}'')^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a_{37}'')^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a_{38}'')^{(7,7,7)}(T_{37},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$+(a''_{40})^{(8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8)}(T_{41},t)$ are eight augmentation coefficient for category 1, 2 and 3

$+(a''_{44})^{(9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9)}(T_{45},t)$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \begin{bmatrix} (b'_{16})^{(2)} & -(b''_{16})^{(2)}(G_{19},t) & -(b''_{13})^{(1,1,)}(G,t) & -(b''_{20})^{(3,3,3,)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6)}(G_{35},t) & \\ -(b''_{36})^{(7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9)}(G_{47},t) & \end{bmatrix} T_{16} \quad (64)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)} & -(b''_{17})^{(2)}(G_{19},t) & -(b''_{14})^{(1,1,)}(G,t) & -(b''_{21})^{(3,3,3,)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6)}(G_{35},t) & \\ -(b''_{37})^{(7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9)}(G_{47},t) & \end{bmatrix} T_{17} \quad (65)$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)} & -(b''_{18})^{(2)}(G_{19},t) & -(b''_{15})^{(1,1,)}(G,t) & -(b''_{22})^{(3,3,3,)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6)}(G_{35},t) & \\ -(b''_{38})^{(7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9)}(G_{47},t) & \end{bmatrix} T_{18} \quad (66)$$

where $-(b''_{16})^{(2)}(G_{19},t)$, $-(b''_{17})^{(2)}(G_{19},t)$, $-(b''_{18})^{(2)}(G_{19},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{13})^{(1,1,)}(G,t)$, $-(b''_{14})^{(1,1,)}(G,t)$, $-(b''_{15})^{(1,1,)}(G,t)$ are second detrition coefficients for category 1,2 and 3

$-(b''_{20})^{(3,3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,)}(G_{23},t)$ are third detrition coefficients for category 1,2 and 3

$-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1,2 and 3

$-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1,2 and 3

$-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients for category 1,2 and 3

$-(b''_{36})^{(7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2 and 3

$-(b''_{40})^{(8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1,2 and 3

$-(b''_{44})^{(9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \begin{bmatrix} (a'_{20})^{(3)} & +(a''_{20})^{(3)}(T_{21},t) & +(a''_{16})^{(2,2,2)}(T_{17},t) & +(a''_{13})^{(1,1,1,)}(T_{14},t) \\ +(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t) & \\ +(a''_{36})^{(7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9)}(T_{45},t) & \end{bmatrix} G_{20} \quad (67)$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \begin{bmatrix} (a'_{21})^{(3)}\boxed{+(a''_{21})^{(3)}(T_{21},t)}\ \boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}\ \boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{21} \quad (68)$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \begin{bmatrix} (a'_{22})^{(3)}\boxed{+(a''_{22})^{(3)}(T_{21},t)}\ \boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}\ \boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{22} \quad (69)$$

$\boxed{+(a''_{20})^{(3)}(T_{21},t)}, \boxed{+(a''_{21})^{(3)}(T_{21},t)}, \boxed{+(a''_{22})^{(3)}(T_{21},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}, \boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}, \boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}$ are second augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)}$ are third augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)}, \boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)}, \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)}, \boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)}, \boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)}, \boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)}, \boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)}$ are seventh augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)}$ are eight augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{44})^{(9,9,9)}(T_{45},t)}, \boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)}, \boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)}$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b'_{20})^{(3)}\boxed{-(b''_{20})^{(3)}(G_{23},t)}\ \boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}\ \boxed{-(b''_{13})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)}\ \boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)}\ \boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7)}(G_{39},t)}\ \boxed{-(b''_{40})^{(8,8,8,8)}(G_{43},t)}\ \boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{20} \quad (70)$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b'_{21})^{(3)}\boxed{-(b''_{21})^{(3)}(G_{23},t)}\ \boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}\ \boxed{-(b''_{14})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)}\ \boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)}\ \boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7)}(G_{39},t)}\ \boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)}\ \boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{21} \quad (71)$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)}\boxed{-(b''_{22})^{(3)}(G_{23},t)}\ \boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}\ \boxed{-(b''_{15})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)}\ \boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)}\ \boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)}\ \boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)}\ \boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{22} \quad (72)$$

$\boxed{-(b''_{20})^{(3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3)}(G_{23},t)}$ *are first detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}$ *are second detrition coefficients for*

*category 1, 2 and 3*

$\boxed{-(b_{13}'')^{(1,1,1,)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,)}(G,t)}$, $\boxed{-(b_{15}'')^{(1,1,1,)}(G,t)}$ *are third detrition coefficients for category 1,2 and 3*

$\boxed{-(b_{24}'')^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{25}'')^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{26}'')^{(4,4,4,4,4,4)}(G_{27},t)}$ *are fourth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b_{28}'')^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5)}(G_{31},t)}$ *are fifth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b_{32}'')^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{34}'')^{(6,6,6,6,6,6)}(G_{35},t)}$ *are sixth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b_{36}'')^{(7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{37}'')^{(7,7,7,7)}(G_{39},t)}$ $\boxed{-(b_{38}'')^{(7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{40}'')^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{42}'')^{(8,8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{46}'')^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b_{45}'')^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b_{44}'')^{(9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[\begin{array}{ccc} (a_{24}')^{(4)} & \boxed{+(a_{24}'')^{(4)}(T_{25},t)} & \boxed{+(a_{28}'')^{(5,5,)}(T_{29},t)} \quad \boxed{+(a_{32}'')^{(6,6,)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a_{16}'')^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a_{20}'')^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{36}'')^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{40}'')^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a_{44}'')^{(9,9,9,9)}(T_{45},t)} \end{array}\right] G_{24} \quad (73)$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[\begin{array}{ccc} (a_{25}')^{(4)} & \boxed{+(a_{25}'')^{(4)}(T_{25},t)} & \boxed{+(a_{29}'')^{(5,5,)}(T_{29},t)} \quad \boxed{+(a_{33}'')^{(6,6)}(T_{33},t)} \\ \boxed{+(a_{14}'')^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a_{17}'')^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a_{21}'')^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{37}'')^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{41}'')^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a_{45}'')^{(9,9,9,9)}(T_{45},t)} \end{array}\right] G_{25} \quad (74)$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[\begin{array}{ccc} (a_{26}')^{(4)} & \boxed{+(a_{26}'')^{(4)}(T_{25},t)} & \boxed{+(a_{30}'')^{(5,5,)}(T_{29},t)} \quad \boxed{+(a_{34}'')^{(6,6,)}(T_{33},t)} \\ \boxed{+(a_{15}'')^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a_{18}'')^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a_{22}'')^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{38}'')^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{42}'')^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a_{46}'')^{(9,9,9,9)}(T_{45},t)} \end{array}\right] G_{26} \quad (75)$$

$\boxed{(a_{24}'')^{(4)}(T_{25},t)}$, $\boxed{(a_{25}'')^{(4)}(T_{25},t)}$, $\boxed{(a_{26}'')^{(4)}(T_{25},t)}$ *are first augmentation coefficients category* $1, 2$ $3$

$\boxed{+(a_{28}'')^{(5,5,)}(T_{29},t)}$, $\boxed{+(a_{29}'')^{(5,5,)}(T_{29},t)}$, $\boxed{+(a_{30}'')^{(5,5,)}(T_{29},t)}$ *are second augmentation coefficient for category* $1, 2$ *and* $3$

$\boxed{+(a_{32}'')^{(6,6,)}(T_{33},t)}$, $\boxed{+(a_{33}'')^{(6,6,)}(T_{33},t)}$, $\boxed{+(a_{34}'')^{(6,6,)}(T_{33},t)}$ *are third augmentation coefficient for category* $1, 2$ *and* $3$

$\boxed{+(a_{13}'')^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a_{14}'')^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a_{15}'')^{(1,1,1,1)}(T_{14},t)}$ *are fourth augmentation coefficients for category* $1, 2$ *and* $3$

$\boxed{+(a_{16}'')^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{17}'')^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{18}'')^{(2,2,2,2)}(T_{17},t)}$ *are fifth augmentation coefficients for category* $1, 2$ *and* $3$

$\boxed{+(a_{20}'')^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{21}'')^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{22}'')^{(3,3,3,3)}(T_{21},t)}$ *are sixth augmentation coefficients for category* $1, 2$ *and* $3$

$\boxed{+(a_{36}'')^{(7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a_{37}'')^{(7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a_{38}'')^{(7,7,7,7,7)}(T_{37},t)}$ *are seventh augmentation coefficients for category* $1, 2$ *and* $3$

$+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)$

*are eighth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{46})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9)}(T_{45},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \begin{bmatrix} (b'_{24})^{(4)} & -(b''_{24})^{(4)}(G_{27},t) & -(b''_{28})^{(5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,)}(G_{35},t) \\ & -(b''_{13})^{(1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{24} \quad (76)$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \begin{bmatrix} (b'_{25})^{(4)} & -(b''_{25})^{(4)}(G_{27},t) & -(b''_{29})^{(5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{25} \quad (77)$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)} & -(b''_{26})^{(4)}(G_{27},t) & -(b''_{30})^{(5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1)}(G,t) & -(b''_{18})^{(2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{26} \quad (78)$$

*Where* $-(b''_{24})^{(4)}(G_{27},t)$, $-(b''_{25})^{(4)}(G_{27},t)$, $-(b''_{26})^{(4)}(G_{27},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{28})^{(5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,)}(G_{31},t)$ *are second detrition coefficients for category* 1, 2 *and* 3

$-(b''_{32})^{(6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,)}(G_{35},t)$ *are third detrition coefficients for category* 1, 2 *and* 3

$-(b''_{13})^{(1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1)}(G,t)$

*are fourth detrition coefficients for category* 1, 2 *and* 3

$-(b''_{16})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2)}(G_{19},t)$

*are fifth detrition coefficients for category* 1, 2 *and* 3

$-(b''_{20})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3)}(G_{23},t)$

*are sixth detrition coefficients for category* 1, 2 *and* 3

$-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)$

*are seventh detrition coefficients for category* 1, 2 *and* 3

$-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)$

*are eighth detrition coefficients for category* 1, 2 *and* 3

$-(b''_{46})^{(9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \begin{bmatrix} (a'_{28})^{(5)} & +(a''_{28})^{(5)}(T_{29},t) & +(a''_{24})^{(4,4,)}(T_{25},t) & +(a''_{32})^{(6,6,6)}(T_{33},t) \\ & +(a''_{13})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{16})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3,3)}(T_{21},t) \\ & +(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{28} \quad (79)$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \begin{bmatrix} (a'_{29})^{(5)} & +(a''_{29})^{(5)}(T_{29},t) & +(a''_{25})^{(4,4,)}(T_{25},t) & +(a''_{33})^{(6,6,6)}(T_{33},t) \\ +(a''_{14})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{17})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3,3)}(T_{21},t) \\ +(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{29} \quad (80)$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \begin{bmatrix} (a'_{30})^{(5)} & +(a''_{30})^{(5)}(T_{29},t) & +(a''_{26})^{(4,4,)}(T_{25},t) & +(a''_{34})^{(6,6,6)}(T_{33},t) \\ +(a''_{15})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{18})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3,3)}(T_{21},t) \\ +(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{30} \quad (81)$$

*Where* $+(a''_{28})^{(5)}(T_{29},t)$, $+(a''_{29})^{(5)}(T_{29},t)$, $+(a''_{30})^{(5)}(T_{29},t)$ *are first augmentation coefficients for category* 1, 2 *and* 3

*And* $+(a''_{24})^{(4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,)}(T_{25},t)$ *are second augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{32})^{(6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6)}(T_{33},t)$ *are third augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)$ are fourth augmentation coefficients for category 1,2, and 3

$+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)$ are fifth augmentation coefficients for category 1,2,and 3

$+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)$ are sixth augmentation coefficients for category 1,2, 3

$+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)$ , $+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1,2, 3

$+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)$ are eighth augmentation coefficients for category 1,2, 3

$+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \begin{bmatrix} (b'_{28})^{(5)} & -(b''_{28})^{(5)}(G_{31},t) & -(b''_{24})^{(4,4,)}(G_{27},t) & -(b''_{32})^{(6,6,6)}(G_{35},t) \\ -(b''_{13})^{(1,1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3)}(G_{23},t) \\ -(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{28} \quad (82)$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b'_{29})^{(5)} & -(b''_{29})^{(5)}(G_{31},t) & -(b''_{25})^{(4,4,)}(G_{27},t) & -(b''_{33})^{(6,6,6)}(G_{35},t) \\ -(b''_{14})^{(1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3)}(G_{23},t) \\ -(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{29} \quad (83)$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b'_{30})^{(5)} & -(b''_{30})^{(5)}(G_{31},t) & -(b''_{26})^{(4,4,)}(G_{27},t) & -(b''_{34})^{(6,6,6)}(G_{35},t) \\ -(b''_{15})^{(1,1,1,1,1,)}(G,t) & -(b''_{18})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3)}(G_{23},t) \\ -(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{30} \quad (84)$$

*where* $-(b''_{28})^{(5)}(G_{31},t)$ , $-(b''_{29})^{(5)}(G_{31},t)$ , $-(b''_{30})^{(5)}(G_{31},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{24})^{(4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,)}(G_{27},t)$ *are second detrition coefficients*

$for\ category\ 1,2\ and\ 3$

$\boxed{-(b''_{32})^{(6,6,6)}(G_{35},t)}, \boxed{-(b''_{33})^{(6,6,6)}(G_{35},t)}, \boxed{-(b''_{34})^{(6,6,6)}(G_{35},t)}$ $are\ third\ detrition\ coefficients$

$for\ category\ 1,2\ and\ 3$

$\boxed{-(b''_{13})^{(1,1,1,1,1)}(G,t)}, \boxed{-(b''_{14})^{(1,1,1,1,1)}(G,t)}, \boxed{-(b''_{15})^{(1,1,1,1,1,)}(G,t)}$ are fourth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[\begin{array}{c} (a'_{32})^{(6)}\boxed{+(a''_{32})^{(6)}(T_{33},t)}\ \boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}\ \boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}\ \boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}\ \boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{32} \quad (85)$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[\begin{array}{c} (a'_{33})^{(6)}\boxed{+(a''_{33})^{(6)}(T_{33},t)}\ \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}\ \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}\ \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}\ \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{33} \quad (86)$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[\begin{array}{c} (a'_{34})^{(6)}\boxed{+(a''_{34})^{(6)}(T_{33},t)}\ \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}\ \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}\ \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}\ \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{34} \quad (87)$$

$\boxed{+(a''_{32})^{(6)}(T_{33},t)}, \boxed{+(a''_{33})^{(6)}(T_{33},t)}, \boxed{+(a''_{34})^{(6)}(T_{33},t)}$ $are\ first\ augmentation\ coefficients$

$for\ category\ 1,2\ and\ 3$

$\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}$ $are\ second\ augmentation$

$coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}$ $are\ third\ augmentation$

$coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}$ - are fourth augmentation coefficients

$\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}$ - fifth augmentation coefficients

$\boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)}$ sixth augmentation coefficients

$+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)$
seventh augmentation coefficients
$+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)$
Eighth augmentation coefficients
$+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \begin{bmatrix} (b'_{32})^{(6)} & -(b''_{32})^{(6)}(G_{35},t) & -(b''_{28})^{(5,5,5)}(G_{31},t) & -(b''_{24})^{(4,4,4,)}(G_{27},t) \\ & -(b''_{13})^{(1,1,1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{32} \quad (88)$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \begin{bmatrix} (b'_{33})^{(6)} & -(b''_{33})^{(6)}(G_{35},t) & -(b''_{29})^{(5,5,5)}(G_{31},t) & -(b''_{25})^{(4,4,4,)}(G_{27},t) \\ & -(b''_{14})^{(1,1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{33} \quad (89)$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)} & -(b''_{34})^{(6)}(G_{35},t) & -(b''_{30})^{(5,5,5)}(G_{31},t) & -(b''_{26})^{(4,4,4,)}(G_{27},t) \\ & -(b''_{15})^{(1,1,1,1,1,1)}(G,t) & -(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{34} \quad (90)$$

$-(b''_{32})^{(6)}(G_{35},t)$, $-(b''_{33})^{(6)}(G_{35},t)$, $-(b''_{34})^{(6)}(G_{35},t)$ *are first detrition coefficients for category* 1, 2 *and* 3
$-(b''_{28})^{(5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5)}(G_{31},t)$ *are second detrition coefficients for category* 1, 2 *and* 3
$-(b''_{24})^{(4,4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,)}(G_{27},t)$ *are third detrition coefficients for category* 1,2 *and* 3
$-(b''_{13})^{(1,1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1,1,1)}(G,t)$ are fourth detrition coefficients for category 1, 2, and 3
$-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1, 2, and 3
$-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1, 2, and 3
$-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2, and 3
$-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)$
are eighth detrition coefficients for category 1, 2, and 3
$-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \quad 91$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a'_{36})^{(7)}\boxed{+(a''_{36})^{(7)}(T_{37},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \quad 92$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a'_{37})^{(7)}\boxed{+(a''_{37})^{(7)}(T_{37},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{38}}{dt} \quad 93$$

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a'_{38})^{(7)}\boxed{+(a''_{38})^{(7)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{(a''_{36})^{(7)}(T_{37},t)}$, $\boxed{(a''_{37})^{(7)}(T_{37},t)}$, $\boxed{(a''_{38})^{(7)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{36}}{dt} = \quad 94$$

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b'_{36})^{(7)}\boxed{-(b''_{36})^{(7)}(G_{39},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{37}}{dt} =$$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b'_{37})^{(7)}\boxed{-(b''_{37})^{(7)}(G_{39},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{38}}{dt} =$$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b'_{38})^{(7)}\boxed{-(b''_{38})^{(7)}(G_{39},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{40}}{dt}$$



$$= (a_{40})^{(8)}G_{41} - \begin{bmatrix} (a'_{40})^{(8)}\boxed{+(a''_{40})^{(8)}(T_{41},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{41}}{dt}$$

$$= (a_{41})^{(8)}G_{40} - \begin{bmatrix} (a'_{41})^{(8)}\boxed{+(a''_{41})^{(8)}(T_{41},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \begin{bmatrix} (a'_{42})^{(8)}\boxed{+(a''_{42})^{(8)}(T_{41},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a''_{40})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)}\boxed{-(b''_{40})^{(8)}(G_{43},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)}\boxed{-(b''_{41})^{(8)}(G_{43},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)}\boxed{-(b''_{42})^{(8)}(G_{43},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{44}}{dt} \qquad 96$$

$$= (a_{44})^{(9)}G_{45} - \left[\begin{array}{ccc} (a'_{44})^{(9)}\boxed{+(a''_{44})^{(9)}(T_{45},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} \end{array}\right] G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[\begin{array}{ccc} (a'_{45})^{(9)}\boxed{+(a''_{45})^{(9)}(T_{45},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} \end{array}\right] G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[\begin{array}{ccc} (a'_{46})^{(9)}\boxed{+(a''_{46})^{(9)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} \end{array}\right] G_{15}$$

Where $\boxed{+(a''_{44})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$+(a_{16}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)$ , $+(a_{17}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)$ , $+(a_{18}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a_{20}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)$ , $+(a_{21}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)$ , $+(a_{22}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a_{24}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)$ , $+(a_{25}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)$ , $+(a_{26}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a_{28}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)$, $+(a_{29}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)$ , $+(a_{30}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a_{32}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a_{33}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$ , $+(a_{34}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a_{13}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$ , $+(a_{14}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$ , $+(a_{15}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$ are Seventh augmentation coefficient for category 1, 2 and 3

$+(a_{38}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ $+(a_{37}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ $+(a_{36}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ are eighth augmentation coefficient for 1,2,3

$+(a_{40}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)$ ,$+(a_{42}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a_{41}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} =$$

$$(b_{44})^{(9)}T_{45} -$$

$$\begin{bmatrix} (b_{44}')^{(9)}\boxed{-(b_{44}'')^{(9)}(G_{47},t)} & \boxed{-(b_{16}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{20}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{24}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{28}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{32}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{13}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{36}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b_{45}')^{(9)}\boxed{-(b_{45}'')^{(9)}(G_{47},t)} & \boxed{-(b_{17}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{21}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{25}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{29}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{33}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{14}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{37}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b_{46}')^{(9)}\boxed{-(b_{46}'')^{(9)}(G_{47},t)} & \boxed{-(b_{18}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{22}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{26}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{30}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{34}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{15}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{38}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b_{44}'')^{(9)}(G_{47},t)}$ , $\boxed{-(b_{45}'')^{(9)}(G_{47},t)}$ , $\boxed{-(b_{46}'')^{(9)}(G_{47},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{16}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$ , $\boxed{-(b_{17}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$ , $\boxed{-(b_{18}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{20}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$ , $\boxed{-(b_{21}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$ , $\boxed{-(b_{22}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{24}'')^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{25}'')^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{26}'')^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{28}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{32}'')^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{34}'')^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{37}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{36}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a_i')^{(1)}, (a_i'')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (b_i'')^{(1)} > 0, \quad i,j = 13,14,15 \qquad 97$$

The functions $(a_i'')^{(1)}, (b_i'')^{(1)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a_i'')^{(1)}(T_{14},t) \leq (p_i)^{(1)} \leq (\hat{A}_{13})^{(1)}$$

$$(b_i'')^{(1)}(G,t) \leq (r_i)^{(1)} \leq (b_i')^{(1)} \leq (\hat{B}_{13})^{(1)}$$

$$lim_{T_2\to\infty}(a_i'')^{(1)}(T_{14},t) = (p_i)^{(1)} \qquad 98$$

$$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = (r_i)^{(1)}$$

**Definition of** $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$ :

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$$|(a_i'')^{(1)}(T_{14}',t) - (a_i'')^{(1)}(T_{14},t)| \leq (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G',t) - (b_i'')^{(1)}(G,t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}',t)$ and $(a_i'')^{(1)}(T_{14},t)$ . $(T_{14}',t)$ and $(T_{14},t)$ are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14},t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$ : 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$

**Definition of** $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$ : 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}$, $(r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)} (\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)} (\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i, j = 16,17,18$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17}, t) \leq (p_i)^{(2)} \leq (\hat{A}_{16})^{(2)} \quad 102$$

$$(b_i'')^{(2)}(G_{19}, t) \leq (r_i)^{(2)} \leq (b_i')^{(2)} \leq (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2 \to \infty} (a_i'')^{(2)}(T_{17}, t) = (p_i)^{(2)} \quad 104$$

$$\lim_{G \to \infty} (b_i'')^{(2)}((G_{19}), t) = (r_i)^{(2)} \quad 105$$

**Definition of** $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$ : 106

Where $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}$ are positive constants and $i = 16,17,18$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}', t) - (a_i'')^{(2)}(T_{17}, t)| \leq (\hat{k}_{16})^{(2)} |T_{17} - T_{17}'| e^{-(\hat{M}_{16})^{(2)} t} \quad 107$$

$$|(b_i'')^{(2)}((G_{19})', t) - (b_i'')^{(2)}((G_{19}), t)| < (\hat{k}_{16})^{(2)} ||(G_{19}) - (G_{19})'|| e^{-(\hat{M}_{16})^{(2)} t} \quad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}', t)$ and$(a_i'')^{(2)}(T_{17}, t)$ . $(T_{17}', t)$ and $(T_{17}, t)$ are points belonging to the interval $[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}]$ . It is to be noted that $(a_i'')^{(2)}(T_{17}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17}, t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$ :

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)},$ are positive constants 109

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$$

**Definition of** $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$ :

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and\ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1 \quad 110$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(b_i)^{(2)} + (b_i')^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1 \quad 111$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i, j = 20,21,22 \quad 112$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(3)}$, $(r_i)^{(3)}$:

$$(a_i'')^{(3)}(T_{21}, t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23}, t) \leq (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(3)}(T_{21}, t) = (p_i)^{(3)} \quad 113$$

$$\lim_{G \to \infty}(b_i'')^{(3)}(G_{23}, t) = (r_i)^{(3)}$$

**Definition of** $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$ :

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: 114

$$|(a_i'')^{(3)}(T_{21}', t) - (a_i'')^{(3)}(T_{21}, t)| \leq (\hat{k}_{20})^{(3)} |T_{21} - T_{21}'| e^{-(\hat{M}_{20})^{(3)} t}$$

$$|(b_i'')^{(3)}(G_{23}', t) - (b_i'')^{(3)}(G_{23}, t)| < (\hat{k}_{20})^{(3)} ||G_{23} - G_{23}'|| e^{-(\hat{M}_{20})^{(3)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}', t)$ and $(a_i'')^{(3)}(T_{21}, t)$ . $(T_{21}', t)$ And $(T_{21}, t)$ are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21}, t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$ : 115

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}} \ , \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with 116
$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$ and the constants
$(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, \ (r_i)^{(3)}, i = 20,21,22,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{20})^{(3)}}[\,(a_i)^{(3)} + (a_i')^{(3)} + \ (\hat{A}_{20})^{(3)} + \ (\hat{P}_{20})^{(3)}\,(\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[\ (b_i)^{(3)} + (b_i')^{(3)} + \ (\hat{B}_{20})^{(3)} + \ (\hat{Q}_{20})^{(3)}\ \ (\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i,j = 24,25,26 \qquad 117$$

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(4)}, \ (r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25}, t) \le (p_i)^{(4)} \le (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}\big((G_{27}), t\big) \le \ (r_i)^{(4)} \le (b_i')^{(4)} \le (\hat{B}_{24})^{(4)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(4)}\,(T_{25}, t) = (p_i)^{(4)} \qquad 118$$

$$\lim_{G \to \infty}(b_i'')^{(4)}\,\big((G_{27}), t\big) = \ (r_i)^{(4)}$$

**Definition of** $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$ :

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24,25,26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}', t) - (a_i'')^{(4)}(T_{25}, t)| \le (\hat{k}_{24})^{(4)}|T_{25} - \ T_{25}'|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})', t) - (b_i'')^{(4)}\big((G_{27}), t\big)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}', t)$ and$(a_i'')^{(4)}(T_{25}, t)$ . $(T_{25}', t)$ and $(T_{25}, t)$ are points belonging to the interval $\big[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}\big]$. It is to be noted that $(a_i'')^{(4)}(T_{25}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25}, t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$ : 120

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)},$ are positive constants

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}, \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$$

**Definition of** $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$ : 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and\ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}, i = 24,25,26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(a_i)^{(4)} + (a_i')^{(4)} + (\hat{A}_{24})^{(4)} + (\hat{P}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(b_i)^{(4)} + (b_i')^{(4)} + (\hat{B}_{24})^{(4)} + (\hat{Q}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i, j = 28,29,30$$ 122

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(5)}$, $(r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29}, t) \le (p_i)^{(5)} \le (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}((G_{31}), t) \le (r_i)^{(5)} \le (b_i')^{(5)} \le (\hat{B}_{28})^{(5)}$$

$$lim_{T_2 \to \infty} (a_i'')^{(5)}(T_{29}, t) = (p_i)^{(5)}$$ 123

$$\lim_{G \to \infty} (b_i'')^{(5)}(G_{31}, t) = (r_i)^{(5)}$$

**Definition of** $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$ :

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}', t) - (a_i'')^{(5)}(T_{29}, t)| \le (\hat{k}_{28})^{(5)} |T_{29} - T_{29}'| e^{-(\hat{M}_{28})^{(5)} t}$$

$$|(b_i'')^{(5)}((G_{31})', t) - (b_i'')^{(5)}((G_{31}), t)| < (\hat{k}_{28})^{(5)} ||(G_{31}) - (G_{31})'|| e^{-(\hat{M}_{28})^{(5)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}', t)$ and $(a_i'')^{(5)}(T_{29}, t)$ . $(T_{29}', t)$ and $(T_{29}, t)$ are points belonging to the interval $\left[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\right]$. It is to be noted that $(a_i'')^{(5)}(T_{29}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$ : 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)},$ are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}, \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

**Definition of** $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$ : 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and\ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(a_i)^{(5)} + (a_i')^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(b_i)^{(5)} + (b_i')^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i, j = 32,33,34$$ 127

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(6)}$, $(r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33}, t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}), t) \leq (r_i)^{(6)} \leq (b_i')^{(6)} \leq (\hat{B}_{32})^{(6)}$$

$$lim_{T_2 \to \infty} (a_i'')^{(6)} (T_{33}, t) = (p_i)^{(6)}$$ 128

$$\lim_{G \to \infty} (b_i'')^{(6)} \left((G_{35}), t\right) = (r_i)^{(6)}$$

**Definition of** $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$ :

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}', t) - (a_i'')^{(6)}(T_{33}, t)| \leq (\hat{k}_{32})^{(6)} |T_{33} - T_{33}'| e^{-(\hat{M}_{32})^{(6)} t}$$

$$|(b_i'')^{(6)}((G_{35})', t) - (b_i'')^{(6)}\left((G_{35}), t\right)| < (\hat{k}_{32})^{(6)} ||(G_{35}) - (G_{35})'|| e^{-(\hat{M}_{32})^{(6)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}', t)$ and $(a_i'')^{(6)}(T_{33}, t)$ . $(T_{33}', t)$ and $(T_{33}, t)$ are points belonging to the interval $\left[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}\right]$. It is to be noted that $(a_i'')^{(6)}(T_{33}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$ : 129

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)},$ are positive constants

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$$

**Definition of** $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$ : 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and\ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32,33,34,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(a_i)^{(6)} + (a_i')^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)} (\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(b_i)^{(6)} + (b_i')^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)} (\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

(CCCCCC) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i, j = 36,37,38$ 131

(DDDDDD) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(7)}$, $(r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37}, t) \leq (p_i)^{(7)} \leq (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39}, t) \leq (r_i)^{(7)} \leq (b_i')^{(7)} \leq (\hat{B}_{36})^{(7)}$$

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*(EEEEEE)* $\lim_{T_2 \to \infty} (a_i'')^{(7)}(T_{37}, t) = (p_i)^{(7)}$

*(FFFFFF)*

$\lim_{G \to \infty} (b_i'')^{(7)}((G_{39}), t) = (r_i)^{(7)}$

**Definition of** $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$ :

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$$|(a_i'')^{(7)}(T_{37}', t) - (a_i'')^{(7)}(T_{37}, t)| \leq (\hat{k}_{36})^{(7)} |T_{37} - T_{37}'| e^{-(\hat{M}_{36})^{(7)} t}$$

$$|(b_i'')^{(7)}((G_{39})', t) - (b_i'')^{(7)}((G_{39}), t)| < (\hat{k}_{36})^{(7)} ||(G_{39}) - (G_{39})'|| e^{-(\hat{M}_{36})^{(7)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}', t)$ and $(a_i'')^{(7)}(T_{37}, t)$ . $(T_{37}', t)$ and $(T_{37}, t)$ are points belonging to the interval $[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}]$ . It is to be noted that $(a_i'')^{(7)}(T_{37}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37}, t)$ , the **first augmentation coefficient** attributable to the

system, would be absolutely continuous.

**Definition of** $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$ : 134

(GGGGGG) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$, are positive constants

$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$

**Definition of** $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$ : 135

(HHHHHH) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(b_i)^{(7)} + (b_i')^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40,41,42$ 136

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

**Definition of** $(p_i)^{(8)}$, $(r_i)^{(8)}$: 137

$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)}$ 138

$(b_i'')^{(8)}((G_{43}), t) \leq (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)}$ 139

$\lim_{T_2 \to \infty} (a_i'')^{(8)}(T_{41}, t) = (p_i)^{(8)}$ 140

$\lim_{G \to \infty} (b_i'')^{(8)}((G_{43}), t) = (r_i)^{(8)}$ 141

**Definition of** $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$ :

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)} |T_{41} - T_{41}'| e^{-(\hat{M}_{40})^{(8)} t}$ 142

$|(b_i'')^{(8)}((G_{43})', t) - (b_i'')^{(8)}((G_{43}), t)| < (\hat{k}_{40})^{(8)} ||(G_{43}) - (G_{43})'|| e^{-(\hat{M}_{40})^{(8)} t}$ 143

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}', t)$ and $(a_i'')^{(8)}(T_{41}, t)$ . $(T_{41}', t)$ and $(T_{41}, t)$ are points belonging to the interval $\left[(\hat{k}_{40})^{(8)}, (\widehat{M}_{40})^{(8)}\right]$. It is to be noted that $(a_i'')^{(8)}(T_{41}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{40})^{(8)} = 1$ then the function $(a_i'')^{(8)}(T_{41}, t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\widehat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$ :

$(\widehat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)},$ are positive constants

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}, \frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}} < 1 \qquad 144$$

**Definition of** $(\hat{P}_{40})^{(8)}, (\hat{Q}_{40})^{(8)}$ :

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\widehat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}, (\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)}, (a_i')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}, i = 40,41,42,$
Satisfy the inequalities

$$\frac{1}{(\widehat{M}_{40})^{(8)}}[(a_i)^{(8)} + (a_i')^{(8)} + (\hat{A}_{40})^{(8)} + (\hat{P}_{40})^{(8)} (\hat{k}_{40})^{(8)}] < 1 \qquad 145$$

$$\frac{1}{(\widehat{M}_{40})^{(8)}}[(b_i)^{(8)} + (b_i')^{(8)} + (\hat{B}_{40})^{(8)} + (\hat{Q}_{40})^{(8)} (\hat{k}_{40})^{(8)}] < 1 \qquad 146$$

Where we suppose

$$(a_i)^{(9)}, (a_i')^{(9)}, (a_i'')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (b_i'')^{(9)} > 0, \quad i, j = 44,45,46 \qquad 146\text{A}$$

The functions $(a_i'')^{(9)}, (b_i'')^{(9)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(9)}$, $(r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45}, t) \leq (p_i)^{(9)} \leq (\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47}, t) \leq (r_i)^{(9)} \leq (b_i')^{(9)} \leq (\hat{B}_{44})^{(9)}$$

$$\lim_{T_2 \to \infty} (a_i'')^{(9)}(T_{45}, t) = (p_i)^{(9)}$$

$$\lim_{G \to \infty} (b_i'')^{(9)}(G_{47}, t) = (r_i)^{(9)}$$

**Definition of** $(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}$ :

Where $\boxed{(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}}$ are positive constants and $\boxed{i = 44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}', t) - (a_i'')^{(9)}(T_{45}, t)| \leq (\hat{k}_{44})^{(9)} |T_{45} - T_{45}'| e^{-(\widehat{M}_{44})^{(9)} t}$$

$$|(b_i'')^{(9)}((G_{47})', t) - (b_i'')^{(9)}\big((G_{47}), t\big)| < (\hat{k}_{44})^{(9)}||(G_{47}) - (G_{47})'||e^{-(\hat{M}_{44})^{(9)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}', t)$ and$(a_i'')^{(9)}(T_{45}, t)$ . $(T_{45}', t)$ and $(T_{45}, t)$ are points belonging to the interval $\big[(\hat{k}_{44})^{(9)}, (\hat{M}_{44})^{(9)}\big]$. It is to be noted that $(a_i'')^{(9)}(T_{45}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{44})^{(9)} = 1$ then the function $(a_i'')^{(9)}(T_{45}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$ :

$(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$, are positive constants

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}} \ , \frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$$

**Definition of** $(\hat{P}_{44})^{(9)}, (\hat{Q}_{44})^{(9)}$ :

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}, (\hat{A}_{44})^{(9)} and\ (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)}, (a_i')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (p_i)^{(9)}, \ (r_i)^{(9)}, i = 44,45,46,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{44})^{(9)}}[\,(a_i)^{(9)} + (a_i')^{(9)} + \ (\hat{A}_{44})^{(9)} + \ (\hat{P}_{44})^{(9)}\,(\hat{k}_{44})^{(9)}] < 1$$

$$\frac{1}{(\hat{M}_{44})^{(9)}}[\ (b_i)^{(9)} + (b_i')^{(9)} + \ (\hat{B}_{44})^{(9)} + \ (\hat{Q}_{44})^{(9)}\ (\hat{k}_{44})^{(9)}] < 1$$

**Theorem 1:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \big(\hat{P}_{13}\big)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \ , \ \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 2 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

**Definition of** $G_i(0)\,, T_i(0)$

$$G_i(t) \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} \quad , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} \quad , \quad T_i(0) = T_i^0 > 0$$

**Theorem 3 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t) \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t} \quad , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t} \quad , \quad T_i(0) = T_i^0 > 0$$

**Theorem 4 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

**Definition of** $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \left(\hat{P}_{24}\right)^{(4)} e^{(\hat{M}_{24})^{(4)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{24}\right)^{(4)} e^{(\hat{M}_{24})^{(4)}t} \;,\quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 5 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

**Definition of** $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \left(\hat{P}_{28}\right)^{(5)} e^{(\hat{M}_{28})^{(5)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{28}\right)^{(5)} e^{(\hat{M}_{28})^{(5)}t} \;,\quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 6 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

**Definition of** $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \left(\hat{P}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t} \;,\quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 7:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

**Definition of** $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \left(\hat{P}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t} \;,\quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 8:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

**Definition of** $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \left(\hat{P}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t} \;,\quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 9:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

**Definition of** $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \left(\hat{P}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

**Proof:** Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \leq (\hat{P}_{13})^{(1)} \text{ , } T_i^0 \leq (\hat{Q}_{13})^{(1)}, \quad 155$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t} \quad 156$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t} \quad 157$$

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[ (a_{13})^{(1)} G_{14}(s_{(13)}) - \left( (a'_{13})^{(1)} + a''_{13})^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \right) G_{13}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[ (a_{14})^{(1)} G_{13}(s_{(13)}) - \left( (a'_{14})^{(1)} + (a''_{14})^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \right) G_{14}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[ (a_{15})^{(1)} G_{14}(s_{(13)}) - \left( (a'_{15})^{(1)} + (a''_{15})^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \right) G_{15}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[ (b_{13})^{(1)} T_{14}(s_{(13)}) - \left( (b'_{13})^{(1)} - (b''_{13})^{(1)} (G(s_{(13)}), s_{(13)}) \right) T_{13}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[ (b_{14})^{(1)} T_{13}(s_{(13)}) - \left( (b'_{14})^{(1)} - (b''_{14})^{(1)} (G(s_{(13)}), s_{(13)}) \right) T_{14}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{T}_{15}(t) = T_{15}^0 + \int_0^t \left[ (b_{15})^{(1)} T_{14}(s_{(13)}) - \left( (b'_{15})^{(1)} - (b''_{15})^{(1)} (G(s_{(13)}), s_{(13)}) \right) T_{15}(s_{(13)}) \right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \leq (\hat{P}_{16})^{(2)} \text{ , } T_i^0 \leq (\hat{Q}_{16})^{(2)},$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$$

By 160

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[ (a_{16})^{(2)} G_{17}(s_{(16)}) - \left( (a'_{16})^{(2)} + a''_{16})^{(2)} (T_{17}(s_{(16)}), s_{(16)}) \right) G_{16}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[ (a_{17})^{(2)} G_{16}(s_{(16)}) - \left( (a'_{17})^{(2)} + (a''_{17})^{(2)} (T_{17}(s_{(16)}), s_{(17)}) \right) G_{17}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[ (a_{18})^{(2)} G_{17}(s_{(16)}) - \left( (a'_{18})^{(2)} + (a''_{18})^{(2)} (T_{17}(s_{(16)}), s_{(16)}) \right) G_{18}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^{0} + \int_0^t \left[ (b_{16})^{(2)} T_{17}(s_{(16)}) - \left( (b'_{16})^{(2)} - (b''_{16})^{(2)} (G_{19}(s_{(16)}), s_{(16)}) \right) T_{16}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^{0} + \int_0^t \left[ (b_{17})^{(2)} T_{16}(s_{(16)}) - \left( (b'_{17})^{(2)} - (b''_{17})^{(2)} (G_{19}(s_{(16)}), s_{(16)}) \right) T_{17}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^{0} + \int_0^t \left[ (b_{18})^{(2)} T_{17}(s_{(16)}) - \left( (b'_{18})^{(2)} - (b''_{18})^{(2)} (G_{19}(s_{(16)}), s_{(16)}) \right) T_{18}(s_{(16)}) \right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i : \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \le (\hat{P}_{20})^{(3)} \text{ , } T_i^0 \le (\hat{Q}_{20})^{(3)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\bar{M}_{20})^{(3)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\bar{M}_{20})^{(3)} t}$$

By 161

$$\bar{G}_{20}(t) = G_{20}^{0} + \int_0^t \left[ (a_{20})^{(3)} G_{21}(s_{(20)}) - \left( (a'_{20})^{(3)} + a''_{20})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^{0} + \int_0^t \left[ (a_{21})^{(3)} G_{20}(s_{(20)}) - \left( (a'_{21})^{(3)} + (a''_{21})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{21}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^{0} + \int_0^t \left[ (a_{22})^{(3)} G_{21}(s_{(20)}) - \left( (a'_{22})^{(3)} + (a''_{22})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{22}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^{0} + \int_0^t \left[ (b_{20})^{(3)} T_{21}(s_{(20)}) - \left( (b'_{20})^{(3)} - (b''_{20})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^{0} + \int_0^t \left[ (b_{21})^{(3)} T_{20}(s_{(20)}) - \left( (b'_{21})^{(3)} - (b''_{21})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{21}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{22}(t) = T_{22}^{0} + \int_0^t \left[ (b_{22})^{(3)} T_{21}(s_{(20)}) - \left( (b'_{22})^{(3)} - (b''_{22})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{22}(s_{(20)}) \right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i : \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \le (\hat{P}_{24})^{(4)} \text{ , } T_i^0 \le (\hat{Q}_{24})^{(4)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\bar{M}_{24})^{(4)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\bar{M}_{24})^{(4)} t}$$

By 162

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{26}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{\mathrm{T}}_{26}(\mathrm{t}) = \mathrm{T}_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{26}(s_{(24)})\right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{28})^{(5)}, T_i^0 \le (\hat{Q}_{28})^{(5)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

By 163

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{30}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{\mathrm{T}}_{30}(\mathrm{t}) = \mathrm{T}_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{30}(s_{(28)})\right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{32})^{(6)}, T_i^0 \le (\hat{Q}_{32})^{(6)},$$

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\tilde{M}_{32})^{(6)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\tilde{M}_{32})^{(6)} t}$

By 164

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[ (a_{32})^{(6)} G_{33}(s_{(32)}) - \left( (a'_{32})^{(6)} + a''_{32})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[ (a_{33})^{(6)} G_{32}(s_{(32)}) - \left( (a'_{33})^{(6)} + (a''_{33})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[ (a_{34})^{(6)} G_{33}(s_{(32)}) - \left( (a'_{34})^{(6)} + (a''_{34})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{34}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[ (b_{32})^{(6)} T_{33}(s_{(32)}) - \left( (b'_{32})^{(6)} - (b''_{32})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[ (b_{33})^{(6)} T_{32}(s_{(32)}) - \left( (b'_{33})^{(6)} - (b''_{33})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{\mathrm{T}}_{34}(\mathrm{t}) = \mathrm{T}_{34}^0 + \int_0^t \left[ (b_{34})^{(6)} T_{33}(s_{(32)}) - \left( (b'_{34})^{(6)} - (b''_{34})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{34}(s_{(32)}) \right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\tilde{M}_{36})^{(7)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\tilde{M}_{36})^{(7)} t}$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[ (a_{36})^{(7)} G_{37}(s_{(36)}) - \left( (a'_{36})^{(7)} + a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[ (a_{37})^{(7)} G_{36}(s_{(36)}) - \left( (a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[ (a_{38})^{(7)} G_{37}(s_{(36)}) - \left( (a'_{38})^{(7)} + (a''_{38})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{38}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[ (b_{36})^{(7)} T_{37}(s_{(36)}) - \left( (b'_{36})^{(7)} - (b''_{36})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[ (b_{37})^{(7)} T_{36}(s_{(36)}) - \left( (b'_{37})^{(7)} - (b''_{37})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\overline{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^{0} + \int_0^t \left[ (b_{38})^{(7)} T_{37}(s_{(36)}) - \left( (b'_{38})^{(7)} - (b''_{38})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{38}(s_{(36)}) \right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{40})^{(8)}, T_i^0 \le (\hat{Q}_{40})^{(8)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[ (a_{40})^{(8)} G_{41}(s_{(40)}) - \left( (a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[ (a_{41})^{(8)} G_{40}(s_{(40)}) - \left( (a'_{41})^{(8)} + (a''_{41})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[ (a_{42})^{(8)} G_{41}(s_{(40)}) - \left( (a'_{42})^{(8)} + (a''_{42})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{42}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^0 + \int_0^t \left[ (b_{40})^{(8)} T_{41}(s_{(40)}) - \left( (b'_{40})^{(8)} - (b''_{40})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^0 + \int_0^t \left[ (b_{41})^{(8)} T_{40}(s_{(40)}) - \left( (b'_{41})^{(8)} - (b''_{41})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\overline{\mathrm{T}}_{42}(\mathrm{t}) = \mathrm{T}_{42}^{0} + \int_0^t \left[ (b_{42})^{(8)} T_{41}(s_{(40)}) - \left( (b'_{42})^{(8)} - (b''_{42})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{42}(s_{(40)}) \right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** 166
Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy A

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{44})^{(9)}, T_i^0 \le (\hat{Q}_{44})^{(9)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[ (a_{44})^{(9)} G_{45}(s_{(44)}) - \left( (a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45}(s_{(44)}), s_{(44)})\right) G_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}(s_{(44)}), s_{(44)})\right) G_{46}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{44}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{46}(t) = T_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{46}(s_{(44)})\right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}}\right)\right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left(e^{(\hat{M}_{13})^{(1)} t} - 1\right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\hat{M}_{13})^{(1)} t} \le \frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{P}_{13})^{(1)} + G_{14}^0\right) e^{\left(-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0}\right)} + (\hat{P}_{13})^{(1)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \le G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}}\right)\right] ds_{(16)} = \quad 169$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1\right)$$

From which it follows that 170

$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \le \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left[\left((\hat{P}_{16})^{(2)} + G_{17}^0\right) e^{\left(-\frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0}\right)} + (\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t) \le G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} \left(G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} s_{(20)}}\right)\right] ds_{(20)} =$$

$$\left(1 + (a_{20})^{(3)} t\right) G_{21}^0 + \frac{(a_{20})^{(3)} (\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left(e^{(\hat{M}_{20})^{(3)} t} - 1\right)$$

From which it follows that 172

$$(G_{20}(t)-G_{20}^{0})e^{-(\hat{M}_{20})^{(3)}t}\leq\frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left[((\hat{P}_{20})^{(3)}+G_{21}^{0})e^{\left(-\frac{(\hat{P}_{20})^{(3)}+G_{21}^{0}}{G_{21}^{0}}\right)}+(\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 173

$$G_{24}(t)\leq G_{24}^{0}+\int_{0}^{t}\left[(a_{24})^{(4)}\left(G_{25}^{0}+(\hat{P}_{24})^{(4)}e^{(\hat{M}_{24})^{(4)}s_{(24)}}\right)\right]ds_{(24)}=$$

$$\left(1+(a_{24})^{(4)}t\right)G_{25}^{0}+\frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left(e^{(\hat{M}_{24})^{(4)}t}-1\right)$$

From which it follows that 174

$$(G_{24}(t)-G_{24}^{0})e^{-(\hat{M}_{24})^{(4)}t}\leq\frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left[((\hat{P}_{24})^{(4)}+G_{25}^{0})e^{\left(-\frac{(\hat{P}_{24})^{(4)}+G_{25}^{0}}{G_{25}^{0}}\right)}+(\hat{P}_{24})^{(4)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t)\leq G_{28}^{0}+\int_{0}^{t}\left[(a_{28})^{(5)}\left(G_{29}^{0}+(\hat{P}_{28})^{(5)}e^{(\hat{M}_{28})^{(5)}s_{(28)}}\right)\right]ds_{(28)}=$$

$$\left(1+(a_{28})^{(5)}t\right)G_{29}^{0}+\frac{(a_{28})^{(5)}(\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left(e^{(\hat{M}_{28})^{(5)}t}-1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t}\leq\frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t)\leq G_{32}^{0}+\int_{0}^{t}\left[(a_{32})^{(6)}\left(G_{33}^{0}+(\hat{P}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\right)\right]ds_{(32)}=$$

$$\left(1+(a_{32})^{(6)}t\right)G_{33}^{0}+\frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left(e^{(\hat{M}_{32})^{(6)}t}-1\right)$$

From which it follows that 177

$$(G_{32}(t)-G_{32}^{0})e^{-(\hat{M}_{32})^{(6)}t}\leq\frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left[((\hat{P}_{32})^{(6)}+G_{33}^{0})e^{\left(-\frac{(\hat{P}_{32})^{(6)}+G_{33}^{0}}{G_{33}^{0}}\right)}+(\hat{P}_{32})^{(6)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}$ , $G_{26}, T_{24}, T_{25}, T_{26}$

(w) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t) \le G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} \left(G_{37}^0 + (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} s_{(36)}}\right)\right] ds_{(36)} =$$
$$\left(1 + (a_{36})^{(7)} t\right) G_{37}^0 + \frac{(a_{36})^{(7)} (\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left(e^{(\hat{M}_{36})^{(7)} t} - 1\right)$$

From which it follows that

$$(G_{36}(t) - G_{36}^0) e^{-(\hat{M}_{36})^{(7)} t} \le \frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left[((\hat{P}_{36})^{(7)} + G_{37}^0) e^{\left(-\frac{(\hat{P}_{36})^{(7)} + G_{37}^0}{G_{37}^0}\right)} + (\hat{P}_{36})^{(7)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t) \le G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} \left(G_{41}^0 + (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} s_{(40)}}\right)\right] ds_{(40)} = \quad 180$$
$$\left(1 + (a_{40})^{(8)} t\right) G_{41}^0 + \frac{(a_{40})^{(8)} (\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}} \left(e^{(\hat{M}_{40})^{(8)} t} - 1\right)$$

From which it follows that 181

$$(G_{40}(t) - G_{40}^0) e^{-(\hat{M}_{40})^{(8)} t} \le \frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}} \left[((\hat{P}_{40})^{(8)} + G_{41}^0) e^{\left(-\frac{(\hat{P}_{40})^{(8)} + G_{41}^0}{G_{41}^0}\right)} + (\hat{P}_{40})^{(8)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 8

Analogous inequalities hold also for $G_{41}$ , $G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t) \le G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} \left(G_{45}^0 + (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} s_{(44)}}\right)\right] ds_{(44)} =$$
$$\left(1 + (a_{44})^{(9)} t\right) G_{45}^0 + \frac{(a_{44})^{(9)} (\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left(e^{(\hat{M}_{44})^{(9)} t} - 1\right)$$

From which it follows that

$$(G_{44}(t) - G_{44}^0) e^{-(\hat{M}_{44})^{(9)} t} \le \frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left[((\hat{P}_{44})^{(9)} + G_{45}^0) e^{\left(-\frac{(\hat{P}_{44})^{(9)} + G_{45}^0}{G_{45}^0}\right)} + (\hat{P}_{44})^{(9)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 9

Analogous inequalities hold also for $G_{45}$ , $G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}$ , $\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$ and to choose 182

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[(\hat{P}_{13})^{(1)}+\left((\hat{P}_{13})^{(1)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{13})^{(1)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{13})^{(1)} \quad 183$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[\left((\hat{Q}_{13})^{(1)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{13})^{(1)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{13})^{(1)}\right]\leq(\hat{Q}_{13})^{(1)} \quad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)},T^{(1)}\right),\left(G^{(2)},T^{(2)}\right)\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{13})^{(1)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{13})^{(1)}t}\}$$

Indeed if we denote

**Definition of** $\tilde{G},\tilde{T}:\left(\tilde{G},\tilde{T}\right)=\mathcal{A}^{(1)}(G,T)$

It results

$$\left|\tilde{G}_{13}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{13})^{(1)}\left|G_{14}^{(1)}-G_{14}^{(2)}\right|e^{-(\hat{M}_{13})^{(1)}s_{(13)}}e^{(\hat{M}_{13})^{(1)}s_{(13)}}\,ds_{(13)}+$$

$$\int_0^t\{(a'_{13})^{(1)}\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\hat{M}_{13})^{(1)}s_{(13)}}e^{-(\hat{M}_{13})^{(1)}s_{(13)}}+$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\hat{M}_{13})^{(1)}s_{(13)}}e^{(\hat{M}_{13})^{(1)}s_{(13)}}+$$

$$G_{13}^{(2)}\left|(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)-(a''_{13})^{(1)}\left(T_{14}^{(2)},s_{(13)}\right)\right|\ e^{-(\hat{M}_{13})^{(1)}s_{(13)}}e^{(\hat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|G^{(1)}-G^{(2)}\right|e^{-(\hat{M}_{13})^{(1)}t}\leq \quad 186$$
$$\frac{1}{(\hat{M}_{13})^{(1)}}\left((a_{13})^{(1)}+(a'_{13})^{(1)}+(\hat{A}_{13})^{(1)}+(\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}\right)d\left(\left(G^{(1)},T^{(1)};\,G^{(2)},T^{(2)}\right)\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 1:** The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\hat{P}_{13})^{(1)}e^{(\hat{M}_{13})^{(1)}t}$ *and* $(\hat{Q}_{13})^{(1)}e^{(\hat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i = 13,14,15$ depend only on $T_{14}$ and respectively on $G(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any $t$ where $G_i(t) = 0\ and\ T_i(t) = 0$

From 19 to 24 it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(1)} - (a_i'')^{(1)}(T_{14}(s_{(13)}), s_{(13)})\} ds_{(13)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(1)} t)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2\ and\ \left((\widehat{M}_{13})^{(1)}\right)_3$ : 187

**Remark 3**: if $G_{13}$ is bounded, the same property have also $G_{14}\ and\ G_{15}$ . indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a_{14}')^{(1)} G_{14}$ and by integrating

$$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)} \left((\widehat{M}_{13})^{(1)}\right)_1 / (a_{14}')^{(1)}$$

In the same way , one can obtain

$$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)} \left((\widehat{M}_{13})^{(1)}\right)_2 / (a_{15}')^{(1)}$$

If $G_{14}\ or\ G_{15}$ is bounded, the same property follows for $G_{13}$ , $G_{15}$ and $G_{13}$ , $G_{14}$ respectively.

**Remark 4:** If $G_{13}$ *is* bounded, from below, the same property holds for $G_{14}\ and\ G_{15}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{14}$ is bounded from below. 188

**Remark 5:** If $T_{13}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}(G(t), t)) = (b_{14}')^{(1)}$ then $T_{14} \to \infty$. 189

**Definition of** $(m)^{(1)}$ and $\varepsilon_1$ :

Indeed let $t_1$ be so that for $t > t_1$

$$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t), t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right)$, $t = log\frac{2}{\varepsilon_1}$ By taking now $\varepsilon_1$ sufficiently small one sees that $T_{14}$ is unbounded.
The same property holds for $T_{15}$ if $\lim_{t\to\infty}(b_{15}'')^{(1)}(G(t), t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}$ , $\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}\ and\ (\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[(\widehat{P}_{16})^{(2)}+\left((\hat{P}_{16})^{(2)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{16})^{(2)}\right]\leq(\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

**Definition of** $\widetilde{G_{19}},\widetilde{T_{19}}$ : $\left(\widetilde{G_{19}},\widetilde{T_{19}}\right)=\mathcal{A}^{(2)}(G_{19},T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{16})^{(2)}\left|G_{17}^{(1)}-G_{17}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)}+$$

$$\int_0^t\{(a'_{16})^{(2)}\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$G_{16}^{(2)}\left|(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)-(a''_{16})^{(2)}\left(T_{17}^{(2)},s_{(16)}\right)\right|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0,t]$ 197

From the hypotheses it follows

$$\left|(G_{19})^{(1)}-(G_{19})^{(2)}\right|\mathrm{e}^{-(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}\leq$$
$$\frac{1}{(\widehat{\mathrm{M}}_{16})^{(2)}}\left((a_{16})^{(2)}+(a'_{16})^{(2)}+(\widehat{\mathrm{A}}_{16})^{(2)}+(\widehat{\mathrm{P}}_{16})^{(2)}(\hat{k}_{16})^{(2)}\right)\mathrm{d}\left(((G_{19})^{(1)},(T_{19})^{(1)};\ (G_{19})^{(2)},(T_{19})^{(2)})\right)$$

And analogous inequalities for $G_i$ and $T_i$. Taking into account the hypothesis the result follows 198

**Remark 6:** The fact that we supposed $(a''_{16})^{(2)}$ and $(b''_{16})^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{\mathrm{P}}_{16})^{(2)}\mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}$ and $(\widehat{\mathrm{Q}}_{16})^{(2)}\mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i=16,17,18$ depend only on $\mathrm{T}_{17}$ and respectively on

$(G_{19})$(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 7:** There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$ 200

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(2)} - (a_i'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})\} ds_{(16)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(2)}t)} > 0 \quad \text{for } t > 0$$

**Definition of** $\left((\widehat{M}_{16})^{(2)}\right)_1$, $\left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$: 201

**Remark 8:** if $G_{16}$ is bounded, the same property have also $G_{17}$ and $G_{18}$. indeed if

$G_{16} < (\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt} \leq \left((\widehat{M}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)} G_{17}$ and by integrating

$$G_{17} \leq \left((\widehat{M}_{16})^{(2)}\right)_2 = G_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1 / (a_{17}')^{(2)}$$

In the same way , one can obtain

$$G_{18} \leq \left((\widehat{M}_{16})^{(2)}\right)_3 = G_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2 / (a_{18}')^{(2)}$$

If $G_{17}$ or $G_{18}$ is bounded, the same property follows for $G_{16}$, $G_{18}$ and $G_{16}$, $G_{17}$ respectively.

**Remark 9:** If $G_{16}$ is bounded, from below, the same property holds for $G_{17}$ and $G_{18}$. The proof is analogous with the preceding one. An analogous property is true if $G_{17}$ is bounded from below. 202

**Remark 10:** If $T_{16}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t), t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$. 203

**Definition of** $(m)^{(2)}$ and $\varepsilon_2$ :

Indeed let $t_2$ be so that for $t > t_2$

$$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right)$, $t = \log\frac{2}{\varepsilon_2}$ By taking now $\varepsilon_2$ sufficiently small one sees that $T_{17}$ is unbounded. 205
The same property holds for $T_{18}$ if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t), t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[(\widehat{P}_{20})^{(3)}+\left((\hat{P}_{20})^{(3)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{20})^{(3)}\right]\leq(\hat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself 210

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)},(T_{23})^{(1)}),((G_{23})^{(2)},(T_{23})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

**Definition of** $\widetilde{G_{23}},\widetilde{T_{23}}:(\widetilde{(G_{23})},\widetilde{(T_{23})})=\mathcal{A}^{(3)}((G_{23}),(T_{23}))$

It results 213

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{20})^{(3)}\left|G_{21}^{(1)}-G_{21}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}ds_{(20)}+$$

$$\int_0^t\{(a'_{20})^{(3)}\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$G_{20}^{(2)}\left|(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)},s_{(20)}\right)\right|\ e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\}ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses it follows

$$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}t}\leq \quad 214$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+(a'_{20})^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\widehat{k}_{20})^{(3)}\right)d\left(((G_{23})^{(1)},(T_{23})^{(1)};\ (G_{23})^{(2)},(T_{23})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 11:** The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ *and* $(\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i=20,21,22$ depend only on $T_{21}$ and respectively on

$(G_{23})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 12:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 216

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(3)} - (a_i'')^{(3)}(T_{21}(s_{(20)}), s_{(20)})\} ds_{(20)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(3)} t)} > 0$ for t > 0

**Definition of** $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2\ and\ \left((\widehat{M}_{20})^{(3)}\right)_3$ : 217

**Remark 13:** if $G_{20}$ is bounded, the same property have also $G_{21}\ and\ G_{22}$ . indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a_{21}')^{(3)} G_{21}$ and by integrating

$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1 / (a_{21}')^{(3)}$

In the same way , one can obtain

$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2 / (a_{22}')^{(3)}$

If $G_{21}\ or\ G_{22}$ is bounded, the same property follows for $G_{20}$ , $G_{22}$ and $G_{20}$ , $G_{21}$ respectively.

**Remark 14:** If $G_{20}$ *is* bounded, from below, the same property holds for $G_{21}\ and\ G_{22}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{21}$ is bounded from below. 218

**Remark 15:** If $T_{20}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t), t\right)) = (b_{21}')^{(3)}$ then $T_{21} \to \infty$. 219

**Definition of** $(m)^{(3)}$ and $\varepsilon_3$ :

Indeed let $t_3$ be so that for $t > t_3$

$(b_{21})^{(3)} - (b_i'')^{(3)}\left((G_{23})(t), t\right) < \varepsilon_3, T_{20}\,(t) > (m)^{(3)}$

Then $\frac{dT_{21}}{dt} \geq (a_{21})^{(3)}(m)^{(3)} - \varepsilon_3 T_{21}$ which leads to 220

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t} = \frac{1}{2}$ it results

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right),\ \ t = log\,\frac{2}{\varepsilon_3}$ By taking now $\varepsilon_3$ sufficiently small one sees that $T_{21}$ is unbounded.
The same property holds for $T_{22}$ if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t), t\right) = (b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}$ , $\frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}} < 1$ and to choose 221

$(\widehat{P}_{24})^{(4)}$ and $(\widehat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[(\widehat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \tag{222}$$

$$\frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \tag{223}$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself (224)

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric (225)

$$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{24})^{(4)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{27}}),(\widetilde{T_{27}})$ : $\left((\widetilde{G_{27}}),(\widetilde{T_{27}})\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{24})^{(4)}\left|G_{25}^{(1)}-G_{25}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\,ds_{(24)}+$$

$$\int_0^t\{(a'_{24})^{(4)}\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$G_{24}^{(2)}\left|(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)-(a''_{24})^{(4)}\left(T_{25}^{(2)},s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\}ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}t}\leq \tag{226}$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)}+(a'_{24})^{(4)}+(\widehat{A}_{24})^{(4)}+(\widehat{P}_{24})^{(4)}(\hat{k}_{24})^{(4)}\right)d\left(((G_{27})^{(1)},(T_{27})^{(1)};\ (G_{27})^{(2)},(T_{27})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 16:** The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ *and* $(\widehat{Q}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_+$. (227)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i=24,25,26$ depend only on $T_{25}$ and respectively on

$(G_{27})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 17:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 228

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(4)} - (a_i'')^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(4)} t)} > 0$ for t > 0

**Definition of** $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2\ and\ \left((\widehat{M}_{24})^{(4)}\right)_3$ : 229

**Remark 18:** if $G_{24}$ is bounded, the same property have also $G_{25}\ and\ G_{26}$ . indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a_{25}')^{(4)} G_{25}$ and by integrating

$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1 / (a_{25}')^{(4)}$

In the same way , one can obtain

$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2 / (a_{26}')^{(4)}$

If $G_{25}\ or\ G_{26}$ is bounded, the same property follows for $G_{24}$ , $G_{26}$ and $G_{24}$ , $G_{25}$ respectively.

**Remark 19:** If $G_{24}$ *is* bounded, from below, the same property holds for $G_{25}\ and\ G_{26}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{25}$ is bounded from below. 230

**Remark 20:** If $T_{24}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}\ ((G_{27})(t), t)) = (b_{25}')^{(4)}$ then $T_{25}\ \to \infty$. 231

**Definition of** $(m)^{(4)}$ and $\varepsilon_4$ :

Indeed let $t_4$ be so that for $t\ > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t), t) < \varepsilon_4, T_{24}\,(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25}\ \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25}\ \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_4}$ By taking now $\varepsilon_4$ sufficiently small one sees that $T_{25}$ is unbounded.
The same property holds for $T_{26}$ if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t), t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for $G_{29}$ , $G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}$ , $\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)}+\left((\hat{P}_{28})^{(5)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{28})^{(5)} \tag{234}$$

$$\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{28})^{(5)}\right]\leq(\hat{Q}_{28})^{(5)} \tag{235}$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric (236)

$$d\left(((G_{31})^{(1)},(T_{31})^{(1)}),((G_{31})^{(2)},(T_{31})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\hat{M}_{28})^{(5)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\hat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{31}}),(\widetilde{T_{31}})$ : $\left((\widetilde{G_{31}}),(\widetilde{T_{31}})\right)=\mathcal{A}^{(5)}\left((G_{31}),(T_{31})\right)$

It results

$$|\tilde{G}_{28}^{(1)}-\tilde{G}_i^{(2)}|\leq\int_0^t(a_{28})^{(5)}|G_{29}^{(1)}-G_{29}^{(2)}|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\,ds_{(28)}+$$

$$\int_0^t\{(a'_{28})^{(5)}|G_{28}^{(1)}-G_{28}^{(2)}|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$(a''_{28})^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)|G_{28}^{(1)}-G_{28}^{(2)}|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$G_{28}^{(2)}|(a''_{28})^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)-(a''_{28})^{(5)}\left(T_{29}^{(2)},s_{(28)}\right)|\;e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\}ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on it follows

$$|(G_{31})^{(1)}-(G_{31})^{(2)}|e^{-(\widehat{M}_{28})^{(5)}t}\leq \tag{237}$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)}+(a'_{28})^{(5)}+(\widehat{A}_{28})^{(5)}+(\widehat{P}_{28})^{(5)}(\widehat{k}_{28})^{(5)}\right)d\left(((G_{31})^{(1)},(T_{31})^{(1)};\;(G_{31})^{(2)},(T_{31})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 21:** The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\hat{P}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ *and* $(\hat{Q}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ respectively of $\mathbb{R}_+$. (238)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on $T_{29}$ and respectively on $(G_{31})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 22:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 239

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(5)} - (a_i'')^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(5)} t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2\ and\ \left((\widehat{M}_{28})^{(5)}\right)_3$ : 240

**Remark 23:** if $G_{28}$ is bounded, the same property have also $G_{29}\ and\ G_{30}$ . indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a_{29}')^{(5)} G_{29}$ and by integrating

$$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1 / (a_{29}')^{(5)}$$

In the same way , one can obtain

$$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2 / (a_{30}')^{(5)}$$

If $G_{29}\ or\ G_{30}$ is bounded, the same property follows for $G_{28}$ , $G_{30}$ and $G_{28}$ , $G_{29}$ respectively.

**Remark 24:** If $G_{28}$ *is* bounded, from below, the same property holds for $G_{29}\ and\ G_{30}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{29}$ is bounded from below. 241

**Remark 25:** If $T_{28}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}\,((G_{31})(t), t)) = (b_{29}')^{(5)}$ then $T_{29} \to \infty$. 242

**Definition of** $(m)^{(5)}$ and $\varepsilon_5$ :

Indeed let $t_5$ be so that for $t > t_5$

$$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t), t) < \varepsilon_5, T_{28}\,(t) > (m)^{(5)}$$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_5}$ By taking now $\varepsilon_5$ sufficiently small one sees that $T_{29}$ is unbounded.
The same property holds for $T_{30}$ if $\lim_{t\to\infty}(b_{30}'')^{(5)}\left((G_{31})(t), t\right) = (b_{30}')^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{33}$ , $G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}$ , $\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}} < 1$ and to choose 244

$(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[(\hat{P}_{32})^{(6)}+\left((\hat{P}_{32})^{(6)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{32})^{(6)} \tag{245}$$

$$\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[\left((\hat{Q}_{32})^{(6)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{32})^{(6)}\right]\leq(\hat{Q}_{32})^{(6)} \tag{246}$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric (247)

$$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\hat{M}_{32})^{(6)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\hat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{35}}),(\widetilde{T_{35}})$ : $\left((\widetilde{G_{35}}),(\widetilde{T_{35}})\right)=\mathcal{A}^{(6)}\left((G_{35}),(T_{35})\right)$

It results

$$|\tilde{G}_{32}^{(1)}-\tilde{G}_i^{(2)}|\leq\int_0^t(a_{32})^{(6)}|G_{33}^{(1)}-G_{33}^{(2)}|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\,ds_{(32)}+$$

$$\int_0^t\{(a'_{32})^{(6)}|G_{32}^{(1)}-G_{32}^{(2)}|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)|G_{32}^{(1)}-G_{32}^{(2)}|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$G_{32}^{(2)}|(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)-(a''_{32})^{(6)}\left(T_{33}^{(2)},s_{(32)}\right)|\;e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\}ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$|(G_{35})^{(1)}-(G_{35})^{(2)}|e^{-(\widehat{M}_{32})^{(6)}t}\leq \tag{248}$$
$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)}+(a'_{32})^{(6)}+(\widehat{A}_{32})^{(6)}+(\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}\right)d\left(((G_{35})^{(1)},(T_{35})^{(1)};\;(G_{35})^{(2)},(T_{35})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 26:** The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\hat{P}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ *and* $(\hat{Q}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ respectively of $\mathbb{R}_+$. (249)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i = 32,33,34$ depend only on $T_{33}$ and respectively on $(G_{35})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 27:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 250

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(6)} - (a_i'')^{(6)}(T_{33}(s_{(32)}), s_{(32)})\} ds_{(32)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(6)} t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{32})^{(6)}\right)_1, \left((\widehat{M}_{32})^{(6)}\right)_2\ and\ \left((\widehat{M}_{32})^{(6)}\right)_3$ : 251

**Remark 28:** if $G_{32}$ is bounded, the same property have also $G_{33}\ and\ G_{34}$ . indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a_{33}')^{(6)} G_{33}$ and by integrating

$$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)} \left((\widehat{M}_{32})^{(6)}\right)_1 / (a_{33}')^{(6)}$$

In the same way , one can obtain

$$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)} \left((\widehat{M}_{32})^{(6)}\right)_2 / (a_{34}')^{(6)}$$

If $G_{33}\ or\ G_{34}$ is bounded, the same property follows for $G_{32}$ , $G_{34}$ and $G_{32}$ , $G_{33}$ respectively.

**Remark 29:** If $G_{32}$ *is* bounded, from below, the same property holds for $G_{33}\ and\ G_{34}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{33}$ is bounded from below. 252

**Remark 30:** If $T_{32}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}\ ((G_{35})(t), t)) = (b_{33}')^{(6)}$ then $T_{33} \to \infty$. 253

**Definition of** $(m)^{(6)}$ and $\varepsilon_6$ :

Indeed let $t_6$ be so that for $t > t_6$

$$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t), t\right) < \varepsilon_6, T_{32}\,(t) > (m)^{(6)}$$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right)$, $t = log\,\frac{2}{\varepsilon_6}$ By taking now $\varepsilon_6$ sufficiently small one sees that $T_{33}$ is unbounded.
The same property holds for $T_{34}$ if $\lim_{t\to\infty}(b_{34}'')^{(6)}\left((G_{35})(t), t(t), t\right) = (b_{34}')^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{37}, G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}$ , $\frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$ and to choose

$(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[(\hat{P}_{36})^{(7)}+\left((\hat{P}_{36})^{(7)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{36})^{(7)} \tag{256}$$

$$\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[\left((\hat{Q}_{36})^{(7)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{36})^{(7)}\right]\leq(\hat{Q}_{36})^{(7)} \tag{257}$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself
The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric (258)

$$d\left(((G_{39})^{(1)},(T_{39})^{(1)}),((G_{39})^{(2)},(T_{39})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\tilde{M}_{36})^{(7)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\tilde{M}_{36})^{(7)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{39}}),(\widetilde{T_{39}}):\left((\widetilde{G_{39}}),(\widetilde{T_{39}})\right)=\mathcal{A}^{(7)}((G_{39}),(T_{39}))$
It results

$$\left|\tilde{G}_{36}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{36})^{(7)}\left|G_{37}^{(1)}-G_{37}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\,ds_{(36)}+$$
$$\int_0^t\{(a'_{36})^{(7)}\left|G_{36}^{(1)}-G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$(a''_{36})^{(7)}\left(T_{37}^{(1)},s_{(36)}\right)\left|G_{36}^{(1)}-G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$G_{36}^{(2)}\left|(a''_{36})^{(7)}\left(T_{37}^{(1)},s_{(36)}\right)-(a''_{36})^{(7)}\left(T_{37}^{(2)},s_{(36)}\right)\right|\;e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\}ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on it follows

$$\left|(G_{39})^{(1)}-(G_{39})^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}t}\leq \tag{259}$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}}\left((a_{36})^{(7)}+(a'_{36})^{(7)}+(\hat{A}_{36})^{(7)}+(\hat{P}_{36})^{(7)}(\hat{k}_{36})^{(7)}\right)d\left(((G_{39})^{(1)},(T_{39})^{(1)};\;(G_{39})^{(2)},(T_{39})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 31:** The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\hat{P}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ *and* $(\hat{Q}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ respectively of $\mathbb{R}_+$. (260)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(7)}$ and $(b_i'')^{(7)}, i = 36,37,38$ depend only on $T_{37}$ and respectively on $(G_{39})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 32:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 261

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(7)} - (a_i'')^{(7)}(T_{37}(s_{(36)}), s_{(36)})\} ds_{(36)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(7)} t\right)} > 0$ for t > 0

**Definition of** $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2\ and\ \left((\widehat{M}_{36})^{(7)}\right)_3$ : 262

**Remark 33:** if $G_{36}$ is bounded, the same property have also $G_{37}\ and\ G_{38}$ . indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a_{37}')^{(7)} G_{37}$ and by integrating

$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1 / (a_{37}')^{(7)}$

In the same way , one can obtain

$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2 / (a_{38}')^{(7)}$

If $G_{37}\ or\ G_{38}$ is bounded, the same property follows for $G_{36}$ , $G_{38}$ and $G_{36}$ , $G_{37}$ respectively.

**Remark 34:** If $G_{36}$ *is* bounded, from below, the same property holds for $G_{37}\ and\ G_{38}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{37}$ is bounded from below. 263

**Remark 35:** If $T_{36}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(7)}\ ((G_{39})(t), t)) = (b_{37}')^{(7)}$ then $T_{37} \to \infty$. 264

**Definition of** $(m)^{(7)}$ and $\varepsilon_7$ :

Indeed let $t_7$ be so that for $t > t_7$

$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t), t) < \varepsilon_7, T_{36}\,(t) > (m)^{(7)}$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_7}$ By taking now $\varepsilon_7$ sufficiently small one sees that $T_{37}$ is unbounded.
The same property holds for $T_{38}$ if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t), t\right) = (b_{38}')^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}$ , $\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}} < 1$ and to choose $(\widehat{P}_{40})^{(8)}$ and $(\widehat{Q}_{40})^{(8)}$ large to have 266

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[(\widehat{P}_{40})^{(8)} + \left((\hat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{40})^{(8)} \quad 267$$

$$\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{40})^{(8)}\right] \leq (\hat{Q}_{40})^{(8)} \quad 268$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)}, (T_{43})^{(1)}), ((G_{43})^{(2)}, (T_{43})^{(2)})\right) = \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

**Definition of** $\widetilde{(G_{43})}, \widetilde{(T_{43})}$ : $\left(\widetilde{(G_{43})}, \widetilde{(T_{43})}\right) = \mathcal{A}^{(8)}((G_{43}), (T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{40})^{(8)}\left|G_{41}^{(1)} - G_{41}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\,ds_{(40)} +$$
$$\int_0^t\{(a'_{40})^{(8)}\left|G_{40}^{(1)} - G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{-(\widehat{M}_{40})^{(8)}s_{(40)}} +$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right)\left|G_{40}^{(1)} - G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}} +$$
$$G_{40}^{(2)}\left|(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right) - (a''_{40})^{(8)}\left(T_{41}^{(2)}, s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\}ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0, t]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)} - (G_{43})^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}t} \leq \quad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)} + (a'_{40})^{(8)} + (\widehat{A}_{40})^{(8)} + (\widehat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}\right)d\left(((G_{43})^{(1)}, (T_{43})^{(1)};\ (G_{43})^{(2)}, (T_{43})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

**Remark 36:** The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate 274

condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ $and$ $(\widehat{Q}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i = 40,41,42$ depend only on $T_{41}$ and respectively on $(G_{43})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 37** There does not exist any $t$ where $G_i(t) = 0\ and\ T_i(t) = 0$ 275

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(8)} - (a_i'')^{(8)}(T_{41}(s_{(40)}), s_{(40)})\} ds_{(40)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(8)}t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{40})^{(8)}\right)_1, \left((\widehat{M}_{40})^{(8)}\right)_2$ $and$ $\left((\widehat{M}_{40})^{(8)}\right)_3$ : 276

**Remark 38:** if $G_{40}$ is bounded, the same property have also $G_{41}$ $and$ $G_{42}$ . indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a_{41}')^{(8)}G_{41}$ and by integrating

$$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1/(a_{41}')^{(8)}$$

In the same way , one can obtain

$$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2/(a_{42}')^{(8)}$$

If $G_{41}$ $or$ $G_{42}$ is bounded, the same property follows for $G_{40}$ , $G_{42}$ and $G_{40}$ , $G_{41}$ respectively.

**Remark 39:** If $G_{40}$ $is$ bounded, from below, the same property holds for $G_{41}$ $and$ $G_{42}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{41}$ is bounded from below. 277

**Remark 40:** If $T_{40}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}((G_{43})(t), t)) = (b_{41}')^{(8)}$ then $T_{41} \to \infty$. 278

**Definition of** $(m)^{(8)}$ and $\varepsilon_8$ :

Indeed let $t_8$ be so that for $t > t_8$

$$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t), t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right)$, $t = log\frac{2}{\varepsilon_8}$ By taking now $\varepsilon_8$ sufficiently small one sees that $T_{41}$ is unbounded.
The same property holds for $T_{42}$ if $\lim_{t\to\infty}(b''_{42})^{(8)}\left((G_{43})(t), t(t), t\right) = (b'_{42})^{(8)}$



It is now sufficient to take $\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}$ , $\frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$ and to choose $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ large to have

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}\left[(\hat{P}_{44})^{(9)} + \left((\hat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}}\left[\left((\hat{Q}_{44})^{(9)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{44})^{(9)}\right] \leq (\hat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)}, (T_{47})^{(1)}), ((G_{47})^{(2)}, (T_{47})^{(2)})\right) =$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{44})^{(9)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{47}}), (\widetilde{T_{47}}) : \left((\widetilde{G_{47}}), (\widetilde{T_{47}})\right) = \mathcal{A}^{(9)}\left((G_{47}), (T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{44})^{(9)}\left|G_{45}^{(1)} - G_{45}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\,ds_{(44)} +$$
$$\int_0^t\{(a'_{44})^{(9)}\left|G_{44}^{(1)} - G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{-(\widehat{M}_{44})^{(9)}s_{(44)}} +$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right)\left|G_{44}^{(1)} - G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}} +$$
$$G_{44}^{(2)}\left|(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right) - (a''_{44})^{(9)}\left(T_{45}^{(2)}, s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\}ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)} - G^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}t} \leq$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)} + (a'_{44})^{(9)} + (\widehat{A}_{44})^{(9)} + (\widehat{P}_{44})^{(9)}(\hat{k}_{44})^{(9)}\right)d\left(((G_{47})^{(1)}, (T_{47})^{(1)};\ (G_{47})^{(2)}, (T_{47})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis (39,35,36) the result follows

**Remark 41:** The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by

$(\widehat{P}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ *and* $(\widehat{Q}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i = 44{,}45{,}46$ depend only on $T_{45}$ and respectively on $(G_{47})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 42:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$

From 99 to 44 it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t\{(a_i')^{(9)}-(a_i'')^{(9)}(T_{45}(s_{(44)}),s_{(44)})\}ds_{(44)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(9)}t)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{44})^{(9)}\right)_1, \left((\widehat{M}_{44})^{(9)}\right)_2$ *and* $\left((\widehat{M}_{44})^{(9)}\right)_3$ :

**Remark 43:** if $G_{44}$ is bounded, the same property have also $G_{45}\ and\ G_{46}$ . indeed if

$G_{44} < (\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt} \leq \left((\widehat{M}_{44})^{(9)}\right)_1 - (a_{45}')^{(9)}G_{45}$ and by integrating

$$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_1/(a_{45}')^{(9)}$$

In the same way , one can obtain

$$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_2/(a_{46}')^{(9)}$$

If $G_{45}\ or\ G_{46}$ is bounded, the same property follows for $G_{44}$ , $G_{46}$ and $G_{44}$ , $G_{45}$ respectively.

**Remark 44:** If $G_{44}$ *is* bounded, from below, the same property holds for $G_{45}\ and\ G_{46}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{45}$ is bounded from below.

**Remark 45:** If $T_{44}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t),t\right) = (b_{45}')^{(9)}$ then $T_{45} \to \infty$.

**Definition of** $(m)^{(9)}$ and $\varepsilon_9$ :

Indeed let $t_9$ be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t),t\right) < \varepsilon_9, T_{44}\,(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right), \quad t = log\frac{2}{\varepsilon_9}$ By taking now $\varepsilon_9$ sufficiently small one sees that $T_{45}$ is unbounded.
The same property holds for $T_{46}$ if $\lim_{t\to\infty}(b_{46}'')^{(9)}\left((G_{47})(t),t\right) = (b_{46}')^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

**Behavior of the solutions of equation** 280

**Theorem** If we denote and define

**Definition of** $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ :

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)} \le -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14}, t) + (a''_{14})^{(1)}(T_{14}, t) \le -(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)} \le -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G, t) - (b''_{14})^{(1)}(G, t) \le -(\tau_1)^{(1)}$

**Definition of** $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$ : 281

By $(\nu_1)^{(1)} > 0$, $(\nu_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$, $(u_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

**Definition of** $(\bar{\nu}_1)^{(1)},, (\bar{\nu}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$ : 282

By $(\bar{\nu}_1)^{(1)} > 0$, $(\bar{\nu}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$, $(\bar{u}_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

**Definition of** $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (\nu_0)^{(1)}$ :- 283

If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by
$(m_2)^{(1)} = (\nu_0)^{(1)}, (m_1)^{(1)} = (\nu_1)^{(1)}$, $if\ (\nu_0)^{(1)} < (\nu_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}, if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)}$,

and $\boxed{(\nu_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)}$, $if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)}$, $if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)}$,

and $\boxed{(u_0)^{(1)} = \frac{T^0_{13}}{T^0_{14}}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$

are defined

Then the solution of global equations satisfies the inequalities 285

$G^0_{13}e^{\left((S_1)^{(1)} - (p_{13})^{(1)}\right)t} \le G_{13}(t) \le G^0_{13}e^{(S_1)^{(1)}t}$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)}-(p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)}t}$$

$$\left(\frac{(a_{15})^{(1)}G_{13}^0}{(m_1)^{(1)}((S_1)^{(1)}-(p_{13})^{(1)}-(S_2)^{(1)})}\left[e^{((S_1)^{(1)}-(p_{13})^{(1)})t} - e^{-(S_2)^{(1)}t}\right] + G_{15}^0 e^{-(S_2)^{(1)}t} \le G_{15}(t) \le \right. \quad 286$$

$$\left. \frac{(a_{15})^{(1)}G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)}-(a'_{15})^{(1)})}[e^{(S_1)^{(1)}t} - e^{-(a'_{15})^{(1)}t}] + G_{15}^0 e^{-(a'_{15})^{(1)}t}\right)$$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)}t} \le T_{13}(t) \le T_{13}^0 e^{((R_1)^{(1)}+(r_{13})^{(1)})t}} \quad 287$$

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)}t} \le T_{13}(t) \le \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)}+(r_{13})^{(1)})t} \quad 288$$

$$\frac{(b_{15})^{(1)}T_{13}^0}{(\mu_1)^{(1)}((R_1)^{(1)}-(b'_{15})^{(1)})}\left[e^{(R_1)^{(1)}t} - e^{-(b'_{15})^{(1)}t}\right] + T_{15}^0 e^{-(b'_{15})^{(1)}t} \le T_{15}(t) \le \quad 289$$

$$\frac{(a_{15})^{(1)}T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)}+(r_{13})^{(1)}+(R_2)^{(1)})}\left[e^{((R_1)^{(1)}+(r_{13})^{(1)})t} - e^{-(R_2)^{(1)}t}\right] + T_{15}^0 e^{-(R_2)^{(1)}t}$$

**Definition of** $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:- 290

Where $(S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$

$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$

$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$

$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$

**Behavior of the solutions of equation** 291

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ : 292

$(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ four constants satisfying

$-(\sigma_2)^{(2)} \le -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(T_{17}, t) + (a''_{17})^{(2)}(T_{17}, t) \le -(\sigma_1)^{(2)}$ 293

$-(\tau_2)^{(2)} \le -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}((G_{19}), t) - (b''_{17})^{(2)}((G_{19}), t) \le -(\tau_1)^{(2)}$ 294

**Definition of** $(\nu_1)^{(2)}, (\nu_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$ : 295

By $(\nu_1)^{(2)} > 0$, $(\nu_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0$, $(u_2)^{(2)} < 0$ the roots 296

of the equations $(a_{17})^{(2)}(\nu^{(2)})^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 297

and $(b_{14})^{(2)}(u^{(2)})^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and 298

**Definition of** $(\bar{\nu}_1)^{(2)}, , (\bar{\nu}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$ : 299

By $(\bar{\nu}_1)^{(2)} > 0$, $(\bar{\nu}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0$, $(\bar{u}_2)^{(2)} < 0$ the 300

roots of the equations $(a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 301

and $(b_{17})^{(2)}\left(u^{(2)}\right)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 302

**Definition of** $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ :- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, ***if*** $(u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$, ***if*** $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)}$, ***if*** $(\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)}t}$

$(p_i)^{(2)}$ is defined by equation

$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)}t}$ 311

$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)}t}\right] + G_{18}^0 e^{-(S_2)^{(2)}t} \le G_{18}(t) \le\right.$ 312

$\left.\frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)}t} - e^{-(a'_{18})^{(2)}t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)}t}\right)$

$\boxed{T_{16}^0 e^{(R_1)^{(2)}t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}}$ 313

$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)}t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}$ 314

$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)}t} - e^{-(b'_{18})^{(2)}t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)}t} \le T_{18}(t) \le$ 315

$$\frac{(a_{18})^{(2)}T_{16}^{0}}{(\mu_2)^{(2)}((R_1)^{(2)}+(r_{16})^{(2)}+(R_2)^{(2)})}\left[e^{((R_1)^{(2)}+(r_{16})^{(2)})t}-e^{-(R_2)^{(2)}t}\right]+T_{18}^{0}e^{-(R_2)^{(2)}t}$$

**Definition of** $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$$

$$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)}$$ 318

$$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$$

**Behavior of the solutions** 319

**Theorem 3:** If we denote and define

**Definition of** $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ :

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}\big((G_{23}), t\big) \le -(\tau_1)^{(3)}$$

**Definition of** $(v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$ : 320

By $(v_1)^{(3)} > 0$, $(v_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{v}_1)^{(3)} > 0$, $(\bar{v}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

**Definition of** $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ :- 321

If we define $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (v_0)^{(3)}, (m_1)^{(3)} = (v_1)^{(3)}$, ***if*** $(v_0)^{(3)} < (v_1)^{(3)}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (\bar{v}_1)^{(3)}$, ***if*** $(v_1)^{(3)} < (v_0)^{(3)} < (\bar{v}_1)^{(3)}$,

and $\boxed{(v_0)^{(3)} = \frac{G_{20}^{0}}{G_{21}^{0}}}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_0)^{(3)}$, ***if*** $(\bar{v}_1)^{(3)} < (v_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)},\ \boldsymbol{if}\ (u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}, \boldsymbol{if}\ (u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)},$ and $\boxed{(u_0)^{(3)} = \frac{T_{20}^0}{T_{21}^0}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)},\ \boldsymbol{if}\ (\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G_{20}^0 e^{(S_1)^{(3)} t}$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)} t} \quad 323$$

$$\left(\frac{(a_{22})^{(3)} G_{20}^0}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\left[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)} t}\right] + G_{22}^0 e^{-(S_2)^{(3)} t} \le G_{22}(t) \le \right. \quad 324$$

$$\left. \frac{(a_{22})^{(3)} G_{20}^0}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}\left[e^{(S_1)^{(3)} t} - e^{-(a'_{22})^{(3)} t}\right] + G_{22}^0 e^{-(a'_{22})^{(3)} t}\right)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t}} \quad 325$$

$$\frac{1}{(\mu_1)^{(3)}} T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}} T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)} T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)} - (b'_{22})^{(3)})}\left[e^{(R_1)^{(3)} t} - e^{-(b'_{22})^{(3)} t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)} t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)} T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)} + (r_{20})^{(3)} + (R_2)^{(3)})}\left[e^{((R_1)^{(3)} + (r_{20})^{(3)})t} - e^{-(R_2)^{(3)} t}\right] + T_{22}^0 e^{-(R_2)^{(3)} t}$$

**Definition of** $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

**Behavior of the solutions of equation**

**Theorem:** If we denote and define

**Definition of** $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ :

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$

$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}((G_{27}), t) - (b''_{25})^{(4)}((G_{27}), t) \le -(\tau_1)^{(4)}$

**Definition of** $(v_1)^{(4)}, (v_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, v^{(4)}, u^{(4)}$ : 329

By $(v_1)^{(4)} > 0$ , $(v_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$ , $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_1)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

**Definition of** $(\bar{v}_1)^{(4)},, (\bar{v}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$ : 330

By $(\bar{v}_1)^{(4)} > 0$ , $(\bar{v}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$ , $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

**Definition of** $(m_1)^{(4)}$ , $(m_2)^{(4)}$ , $(\mu_1)^{(4)}, (\mu_2)^{(4)}, (\nu_0)^{(4)}$ :-

If we define $(m_1)^{(4)}$ , $(m_2)^{(4)}$ , $(\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (v_0)^{(4)}, (m_1)^{(4)} = (v_1)^{(4)}$, ***if*** $(v_0)^{(4)} < (v_1)^{(4)}$

$(m_2)^{(4)} = (v_1)^{(4)}, (m_1)^{(4)} = (\bar{v}_1)^{(4)}$ , ***if*** $(v_4)^{(4)} < (v_0)^{(4)} < (\bar{v}_1)^{(4)}$,
and $\boxed{(v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (v_4)^{(4)}, (m_1)^{(4)} = (v_0)^{(4)}$, ***if*** $(\bar{v}_4)^{(4)} < (v_0)^{(4)}$

and analogously

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$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)}$, ***if*** $(u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}$ , ***if*** $(u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)}$,
and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}$, ***if*** $(\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

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$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)}t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)}t}$$

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$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)}t}\right] + G_{26}^0 e^{-(S_2)^{(4)}t} \le G_{26}(t) \le\right.$$ 334

*(a26)4G240(m2)4(S1)4−(a26′)4e(S1)4t−e−(a26′)4t+ G260e−(a26′)4t*

$$\boxed{T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}}T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}}T_{24}^0 e^{((R_1)^{(4)}+(r_{24})^{(4)})t} \quad 335$$

$$\frac{(b_{26})^{(4)}T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)}-(b'_{26})^{(4)})}\left[e^{(R_1)^{(4)}t} - e^{-(b'_{26})^{(4)}t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)}t} \le T_{26}(t) \le \quad 336$$

$$\frac{(a_{26})^{(4)}T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)}+(r_{24})^{(4)}+(R_2)^{(4)})}\left[e^{((R_1)^{(4)}+(r_{24})^{(4)})t} - e^{-(R_2)^{(4)}t}\right] + T_{26}^0 e^{-(R_2)^{(4)}t}$$

**Definition of** $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

**Behavior of the solutions of equation** 338

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ :

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

**Definition of** $(v_1)^{(5)}, (v_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, v^{(5)}, u^{(5)}$ : 339

By $(v_1)^{(5)} > 0$, $(v_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$, $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(v^{(5)}\big)^2 + (\sigma_1)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

**Definition of** $(\bar{v}_1)^{(5)}, , (\bar{v}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$ : 340

By $(\bar{v}_1)^{(5)} > 0$, $(\bar{v}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$, $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(v^{(5)}\big)^2 + (\sigma_2)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

**Definition of** $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (v_0)^{(5)}$ :-

If we define $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (v_0)^{(5)}, (m_1)^{(5)} = (v_1)^{(5)}$, ***if*** $(v_0)^{(5)} < (v_1)^{(5)}$

$(m_2)^{(5)} = (v_1)^{(5)}, (m_1)^{(5)} = (\bar{v}_1)^{(5)}$, ***if*** $(v_1)^{(5)} < (v_0)^{(5)} < (\bar{v}_1)^{(5)}$,
and $\boxed{(v_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)},\ \boldsymbol{if}\ (u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}, \boldsymbol{if}\ (u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)},$

and $\boxed{(u_0)^{(5)} = \frac{T_{28}^0}{T_{29}^0}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}, \boldsymbol{if}\ (\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{28}(t) \le G_{28}^0 e^{(S_1)^{(5)} t}$

where $(p_i)^{(5)}$ is defined by equation

$\frac{1}{(m_5)^{(5)}} G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}} G_{28}^0 e^{(S_1)^{(5)} t}$ 343

$\left(\frac{(a_{30})^{(5)} G_{28}^0}{(m_1)^{(5)}((S_1)^{(5)} - (p_{28})^{(5)} - (S_2)^{(5)})}\left[e^{((S_1)^{(5)} - (p_{28})^{(5)})t} - e^{-(S_2)^{(5)} t}\right] + G_{30}^0 e^{-(S_2)^{(5)} t} \le G_{30}(t) \le\right.$ 344

*(a30)5G280(m2)5(S1)5−(a30′)5e(S1)5t−e−(a30′)5t+ G300e−(a30′)5t*

$\boxed{T_{28}^0 e^{(R_1)^{(5)} t} \le T_{28}(t) \le T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}}$ 345

$\frac{1}{(\mu_1)^{(5)}} T_{28}^0 e^{(R_1)^{(5)} t} \le T_{28}(t) \le \frac{1}{(\mu_2)^{(5)}} T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}$ 346

$\frac{(b_{30})^{(5)} T_{28}^0}{(\mu_1)^{(5)}((R_1)^{(5)} - (b'_{30})^{(5)})}\left[e^{(R_1)^{(5)} t} - e^{-(b'_{30})^{(5)} t}\right] + T_{30}^0 e^{-(b'_{30})^{(5)} t} \le T_{30}(t) \le$ 347

$\frac{(a_{30})^{(5)} T_{28}^0}{(\mu_2)^{(5)}((R_1)^{(5)} + (r_{28})^{(5)} + (R_2)^{(5)})}\left[e^{((R_1)^{(5)} + (r_{28})^{(5)})t} - e^{-(R_2)^{(5)} t}\right] + T_{30}^0 e^{-(R_2)^{(5)} t}$

**Definition of** $(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$:- 348

Where $(S_1)^{(5)} = (a_{28})^{(5)}(m_2)^{(5)} - (a'_{28})^{(5)}$

$(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}$

$(R_1)^{(5)} = (b_{28})^{(5)}(\mu_2)^{(5)} - (b'_{28})^{(5)}$

$(R_2)^{(5)} = (b'_{30})^{(5)} - (r_{30})^{(5)}$

**Behavior of the solutions of equation** 349

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ :

$(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)} \le -(a'_{32})^{(6)} + (a'_{33})^{(6)} - (a''_{32})^{(6)}(T_{33}, t) + (a''_{33})^{(6)}(T_{33}, t) \le -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)} \le -(b'_{32})^{(6)} + (b'_{33})^{(6)} - (b''_{32})^{(6)}\big((G_{35}), t\big) - (b''_{33})^{(6)}\big((G_{35}), t\big) \le -(\tau_1)^{(6)}$$

**Definition of** $(\nu_1)^{(6)}, (\nu_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, \nu^{(6)}, u^{(6)}$ : 350

By $(\nu_1)^{(6)} > 0$, $(\nu_2)^{(6)} < 0$ and respectively $(u_1)^{(6)} > 0$, $(u_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(6)},, (\bar{\nu}_2)^{(6)}, (\bar{u}_1)^{(6)}, (\bar{u}_2)^{(6)}$ : 351

By $(\bar{\nu}_1)^{(6)} > 0$, $(\bar{\nu}_2)^{(6)} < 0$ and respectively $(\bar{u}_1)^{(6)} > 0$, $(\bar{u}_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$

**Definition of** $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}, (\nu_0)^{(6)}$ :-

If we define $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}$ by

$$(m_2)^{(6)} = (\nu_0)^{(6)}, (m_1)^{(6)} = (\nu_1)^{(6)}, \ \boldsymbol{if}\ (\nu_0)^{(6)} < (\nu_1)^{(6)}$$

$$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}, \boldsymbol{if}\ (\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)},$$

and $\boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}}$

$$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$$

and analogously 352

$$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}, \ \boldsymbol{if}\ (u_0)^{(6)} < (u_1)^{(6)}$$

$$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}, \boldsymbol{if}\ (u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)},$$

and $\boxed{(u_0)^{(6)} = \frac{T^0_{32}}{T^0_{33}}}$

$$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}, \boldsymbol{if}\ (\bar{u}_1)^{(6)} < (u_0)^{(6)} \text{ where } (u_1)^{(6)}, (\bar{u}_1)^{(6)}$$

Then the solution of global equations satisfies the inequalities 353

$$G^0_{32}e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{32}(t) \le G^0_{32}e^{(S_1)^{(6)}t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}}G^0_{32}e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{33}(t) \le \frac{1}{(m_2)^{(6)}}G^0_{32}e^{(S_1)^{(6)}t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)}G^0_{32}}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})}\left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)}t}\right] + G^0_{34}e^{-(S_2)^{(6)}t} \le G_{34}(t) \le \right. \quad 355$$

$$\frac{(a_{34})^{(6)}G_{32}^{0}}{(m_2)^{(6)}((S_1)^{(6)}-(a'_{34})^{(6)})}\left[e^{(S_1)^{(6)}t}-e^{-(a'_{34})^{(6)}t}\right]+\ G_{34}^{0}e^{-(a'_{34})^{(6)}t}\Big)$$

$$\boxed{T_{32}^{0}e^{(R_1)^{(6)}t}\le T_{32}(t)\le T_{32}^{0}e^{((R_1)^{(6)}+(r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}}T_{32}^{0}e^{(R_1)^{(6)}t}\le T_{32}(t)\le\frac{1}{(\mu_2)^{(6)}}T_{32}^{0}e^{((R_1)^{(6)}+(r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)}T_{32}^{0}}{(\mu_1)^{(6)}((R_1)^{(6)}-(b'_{34})^{(6)})}\left[e^{(R_1)^{(6)}t}-e^{-(b'_{34})^{(6)}t}\right]+T_{34}^{0}e^{-(b'_{34})^{(6)}t}\le T_{34}(t)\le \quad 358$$

$$\frac{(a_{34})^{(6)}T_{32}^{0}}{(\mu_2)^{(6)}((R_1)^{(6)}+(r_{32})^{(6)}+(R_2)^{(6)})}\left[e^{((R_1)^{(6)}+(r_{32})^{(6)})t}-e^{-(R_2)^{(6)}t}\right]+T_{34}^{0}e^{-(R_2)^{(6)}t}$$

**Definition of** $(S_1)^{(6)},(S_2)^{(6)},(R_1)^{(6)},(R_2)^{(6)}$:- 359

Where $(S_1)^{(6)}=(a_{32})^{(6)}(m_2)^{(6)}-(a'_{32})^{(6)}$

$(S_2)^{(6)}=(a_{34})^{(6)}-(p_{34})^{(6)}$

$(R_1)^{(6)}=(b_{32})^{(6)}(\mu_2)^{(6)}-(b'_{32})^{(6)}$

$(R_2)^{(6)}=(b'_{34})^{(6)}-(r_{34})^{(6)}$

**Behavior of the solutions of equation**

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(7)},(\sigma_2)^{(7)},(\tau_1)^{(7)},(\tau_2)^{(7)}$ :

$(\sigma_1)^{(7)},(\sigma_2)^{(7)},(\tau_1)^{(7)},(\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)}\le-(a'_{36})^{(7)}+(a'_{37})^{(7)}-(a''_{36})^{(7)}(T_{37},t)+(a''_{37})^{(7)}(T_{37},t)\le-(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)}\le-(b'_{36})^{(7)}+(b'_{37})^{(7)}-(b''_{36})^{(7)}\big((G_{39}),t\big)-(b''_{37})^{(7)}\big((G_{39}),t\big)\le-(\tau_1)^{(7)}$$

**Definition of** $(v_1)^{(7)},(v_2)^{(7)},(u_1)^{(7)},(u_2)^{(7)},v^{(7)},u^{(7)}$ : 361

By $(v_1)^{(7)}>0,(v_2)^{(7)}<0$ and respectively $(u_1)^{(7)}>0,(u_2)^{(7)}<0$ the roots of the equations $(a_{37})^{(7)}\big(v^{(7)}\big)^2+(\sigma_1)^{(7)}v^{(7)}-(a_{36})^{(7)}=0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2+(\tau_1)^{(7)}u^{(7)}-(b_{36})^{(7)}=0$ and

**Definition of** $(\bar{v}_1)^{(7)},,(\bar{v}_2)^{(7)},(\bar{u}_1)^{(7)},(\bar{u}_2)^{(7)}$ : 362

By $(\bar{v}_1)^{(7)}>0,(\bar{v}_2)^{(7)}<0$ and respectively $(\bar{u}_1)^{(7)}>0,(\bar{u}_2)^{(7)}<0$ the roots of the equations $(a_{37})^{(7)}\big(v^{(7)}\big)^2+(\sigma_2)^{(7)}v^{(7)}-(a_{36})^{(7)}=0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2+(\tau_2)^{(7)}u^{(7)}-(b_{36})^{(7)}=0$

**Definition of** $(m_1)^{(7)},(m_2)^{(7)},(\mu_1)^{(7)},(\mu_2)^{(7)},(v_0)^{(7)}$ :-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$$(m_2)^{(7)} = (\nu_0)^{(7)}, (m_1)^{(7)} = (\nu_1)^{(7)}, \ \boldsymbol{if}\ (\nu_0)^{(7)} < (\nu_1)^{(7)}$$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\bar{\nu}_1)^{(7)}, \boldsymbol{if}\ (\nu_1)^{(7)} < (\nu_0)^{(7)} < (\bar{\nu}_1)^{(7)},$$

and $\boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\nu_0)^{(7)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(7)} < (\nu_0)^{(7)}$$

and analogously 363

$$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}, \ \boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$$

and $\boxed{(u_0)^{(7)} = \frac{T_{36}^0}{T_{37}^0}}$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)} \text{ where } (u_1)^{(7)}, (\bar{u}_1)^{(7)}$$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^0 e^{(S_1)^{(7)} t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)} t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)}((S_1)^{(7)} - (p_{36})^{(7)} - (S_2)^{(7)})}\left[e^{((S_1)^{(7)} - (p_{36})^{(7)})t} - e^{-(S_2)^{(7)} t}\right] + G_{38}^0 e^{-(S_2)^{(7)} t} \le G_{38}(t) \le \right. \quad 366$$

$$\left. \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)}((S_1)^{(7)} - (a'_{38})^{(7)})}\left[e^{(S_1)^{(7)} t} - e^{-(a'_{38})^{(7)} t}\right] + G_{38}^0 e^{-(a'_{38})^{(7)} t}\right)$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t} \quad 368$$

$$\frac{(b_{38})^{(7)} T_{36}^0}{(\mu_1)^{(7)}((R_1)^{(7)} - (b'_{38})^{(7)})}\left[e^{(R_1)^{(7)} t} - e^{-(b'_{38})^{(7)} t}\right] + T_{38}^0 e^{-(b'_{38})^{(7)} t} \le T_{38}(t) \le \quad 369$$

$$\frac{(a_{38})^{(7)} T_{36}^0}{(\mu_2)^{(7)}((R_1)^{(7)} + (r_{36})^{(7)} + (R_2)^{(7)})}\left[e^{((R_1)^{(7)} + (r_{36})^{(7)})t} - e^{-(R_2)^{(7)} t}\right] + T_{38}^0 e^{-(R_2)^{(7)} t}$$

**Definition of** $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$

$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$

$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$

**Behavior of the solutions of equation** 371

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ :

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$-(\sigma_2)^{(8)} \le -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \le -(\sigma_1)^{(8)}$

$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\big((G_{43}), t\big) - (b''_{41})^{(8)}\big((G_{43}), t\big) \le -(\tau_1)^{(8)}$

**Definition of** $(\nu_1)^{(8)}, (\nu_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, \nu^{(8)}, u^{(8)}$ : 372

By $(\nu_1)^{(8)} > 0, (\nu_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0, (u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(8)}, , (\bar{\nu}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$ :

By $(\bar{\nu}_1)^{(8)} > 0, (\bar{\nu}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0, (\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

**Definition of** $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}, (\nu_0)^{(8)}$ :-

If we define $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$(m_2)^{(8)} = (\nu_0)^{(8)}, (m_1)^{(8)} = (\nu_1)^{(8)}$, ***if*** $(\nu_0)^{(8)} < (\nu_1)^{(8)}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\bar{\nu}_1)^{(8)}$, ***if*** $(\nu_1)^{(8)} < (\nu_0)^{(8)} < (\bar{\nu}_1)^{(8)}$,
and $\boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}}$

$$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\nu_0)^{(8)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(8)} < (\nu_0)^{(8)}$$

and analogously 374

$$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)},\ \boldsymbol{if}\ (u_0)^{(8)} < (u_1)^{(8)}$$

$$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}, \boldsymbol{if}\ (u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)},$$

and $\boxed{(u_0)^{(8)} = \frac{T_{40}^0}{T_{41}^0}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}, \boldsymbol{if}\ (\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^0 e^{(S_1)^{(8)} t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}} G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}} G_{40}^0 e^{(S_1)^{(8)} t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)} G_{40}^0}{(m_1)^{(8)}((S_1)^{(8)} - (p_{40})^{(8)} - (S_2)^{(8)})}\left[e^{((S_1)^{(8)} - (p_{40})^{(8)})t} - e^{-(S_2)^{(8)} t}\right] + G_{42}^0 e^{-(S_2)^{(8)} t} \le G_{42}(t) \le \right. \quad 377$$

$$\left. \frac{(a_{42})^{(8)} G_{40}^0}{(m_2)^{(8)}((S_1)^{(8)} - (a'_{42})^{(8)})}\left[e^{(S_1)^{(8)} t} - e^{-(a'_{42})^{(8)} t}\right] + G_{42}^0 e^{-(a'_{42})^{(8)} t}\right)$$

$$\boxed{T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}} T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le \frac{1}{(\mu_2)^{(8)}} T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)} T_{40}^0}{(\mu_1)^{(8)}((R_1)^{(8)} - (b'_{42})^{(8)})}\left[e^{(R_1)^{(8)} t} - e^{-(b'_{42})^{(8)} t}\right] + T_{42}^0 e^{-(b'_{42})^{(8)} t} \le T_{42}(t) \le \quad 380$$

$$\frac{(a_{42})^{(8)} T_{40}^0}{(\mu_2)^{(8)}((R_1)^{(8)} + (r_{40})^{(8)} + (R_2)^{(8)})}\left[e^{((R_1)^{(8)} + (r_{40})^{(8)})t} - e^{-(R_2)^{(8)} t}\right] + T_{42}^0 e^{-(R_2)^{(8)} t}$$

**Definition of** $(S_1)^{(8)}, (S_2)^{(8)}, (R_1)^{(8)}, (R_2)^{(8)}$:- 381

Where $(S_1)^{(8)} = (a_{40})^{(8)}(m_2)^{(8)} - (a'_{40})^{(8)}$

$$(S_2)^{(8)} = (a_{42})^{(8)} - (p_{42})^{(8)}$$

$$(R_1)^{(8)} = (b_{40})^{(8)}(\mu_2)^{(8)} - (b'_{40})^{(8)}$$

$$(R_2)^{(8)} = (b'_{42})^{(8)} - (r_{42})^{(8)}$$

**Behavior of the solutions of equation 37 to 92** 382

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ :

$(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ four constants satisfying

$$-(\sigma_2)^{(9)} \le -(a'_{44})^{(9)} + (a'_{45})^{(9)} - (a''_{44})^{(9)}(T_{45}, t) + (a''_{45})^{(9)}(T_{45}, t) \le -(\sigma_1)^{(9)}$$

$$-(\tau_2)^{(9)} \le -(b'_{44})^{(9)} + (b'_{45})^{(9)} - (b''_{44})^{(9)}\big((G_{47}), t\big) - (b''_{45})^{(9)}\big((G_{47}), t\big) \le -(\tau_1)^{(9)}$$

**Definition of** $(\nu_1)^{(9)}, (\nu_2)^{(9)}, (u_1)^{(9)}, (u_2)^{(9)}, \nu^{(9)}, u^{(9)}$ :

By $(\nu_1)^{(9)} > 0$ , $(\nu_2)^{(9)} < 0$ and respectively $(u_1)^{(9)} > 0$ , $(u_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_1)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(9)}, , (\bar{\nu}_2)^{(9)}, (\bar{u}_1)^{(9)}, (\bar{u}_2)^{(9)}$ :

By $(\bar{\nu}_1)^{(9)} > 0$ , $(\bar{\nu}_2)^{(9)} < 0$ and respectively $(\bar{u}_1)^{(9)} > 0$ , $(\bar{u}_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_2)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$

**Definition of** $(m_1)^{(9)}, (m_2)^{(9)}, (\mu_1)^{(9)}, (\mu_2)^{(9)}, (\nu_0)^{(9)}$ :-

If we define $(m_1)^{(9)}, (m_2)^{(9)}, (\mu_1)^{(9)}, (\mu_2)^{(9)}$ by

$$(m_2)^{(9)} = (\nu_0)^{(9)}, (m_1)^{(9)} = (\nu_1)^{(9)}, \ \boldsymbol{if}\ (\nu_0)^{(9)} < (\nu_1)^{(9)}$$

$$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\bar{\nu}_1)^{(9)}, \boldsymbol{if}\ (\nu_1)^{(9)} < (\nu_0)^{(9)} < (\bar{\nu}_1)^{(9)},$$

and $\boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$

$$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\nu_0)^{(9)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(9)} < (\nu_0)^{(9)}$$

and analogously

$$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)}, \ \boldsymbol{if}\ (u_0)^{(9)} < (u_1)^{(9)}$$

$$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}, \boldsymbol{if}\ (u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)},$$

and $\boxed{(u_0)^{(9)} = \frac{T^0_{44}}{T^0_{45}}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}, \boldsymbol{if}\ (\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G^0_{44} e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{44}(t) \le G^0_{44} e^{(S_1)^{(9)} t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}}G_{44}^0 e^{\left((S_1)^{(9)}-(p_{44})^{(9)}\right)t} \le G_{45}(t) \le \frac{1}{(m_2)^{(9)}}G_{44}^0 e^{(S_1)^{(9)}t}$$

$$\left(\frac{(a_{46})^{(9)}G_{44}^0}{(m_1)^{(9)}\left((S_1)^{(9)}-(p_{44})^{(9)}-(S_2)^{(9)}\right)}\left[e^{\left((S_1)^{(9)}-(p_{44})^{(9)}\right)t}-e^{-(S_2)^{(9)}t}\right]+G_{46}^0 e^{-(S_2)^{(9)}t} \le G_{46}(t) \le \right.$$

$$\left.\frac{(a_{46})^{(9)}G_{44}^0}{(m_2)^{(9)}\left((S_1)^{(9)}-(a'_{46})^{(9)}\right)}\left[e^{(S_1)^{(9)}t}-e^{-(a'_{46})^{(9)}t}\right]+\ G_{46}^0 e^{-(a'_{46})^{(9)}t}\right)$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le T_{44}^0 e^{\left((R_1)^{(9)}+(r_{44})^{(9)}\right)t}}$$

$$\frac{1}{(\mu_1)^{(9)}}T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le \frac{1}{(\mu_2)^{(9)}}T_{44}^0 e^{\left((R_1)^{(9)}+(r_{44})^{(9)}\right)t}$$

$$\frac{(b_{46})^{(9)}T_{44}^0}{(\mu_1)^{(9)}\left((R_1)^{(9)}-(b'_{46})^{(9)}\right)}\left[e^{(R_1)^{(9)}t}-e^{-(b'_{46})^{(9)}t}\right]+T_{46}^0 e^{-(b'_{46})^{(9)}t} \le T_{46}(t) \le$$

$$\frac{(a_{46})^{(9)}T_{44}^0}{(\mu_2)^{(9)}\left((R_1)^{(9)}+(r_{44})^{(9)}+(R_2)^{(9)}\right)}\left[e^{\left((R_1)^{(9)}+(r_{44})^{(9)}\right)t}-e^{-(R_2)^{(9)}t}\right]+T_{46}^0 e^{-(R_2)^{(9)}t}$$

**Definition of** $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$

**Proof :** From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14},t)\right) - (a''_{14})^{(1)}(T_{14},t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

**Definition of** $v^{(1)}$ :- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)}\right) \le \frac{dv^{(1)}}{dt} \le -\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)}\right)$$

From which one obtains

**Definition of** $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$ :-

For $0 < \boxed{(v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}} < (v_1)^{(1)} < (\bar{v}_1)^{(1)}$

$$v^{(1)}(t) \ge \frac{(v_1)^{(1)}+(C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(v_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(v_2)^{(1)}}}$$

it follows $(\nu_0)^{(1)} \leq \nu^{(1)}(t) \leq (\nu_1)^{(1)}$

In the same manner , we get 384

$$\nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{\nu}_1)^{(1)}-(\nu_0)^{(1)}}{(\nu_0)^{(1)}-(\bar{\nu}_2)^{(1)}}}$$

From which we deduce $(\nu_0)^{(1)} \leq \nu^{(1)}(t) \leq (\bar{\nu}_1)^{(1)}$

If $0 < (\nu_1)^{(1)} < (\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0} < (\bar{\nu}_1)^{(1)}$ we find like in the previous case, 385

$$(\nu_1)^{(1)} \leq \frac{(\nu_1)^{(1)}+(C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}} \leq \nu^{(1)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\bar{\nu}_1)^{(1)}$$

If $0 < (\nu_1)^{(1)} \leq (\bar{\nu}_1)^{(1)} \leq \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$ , we obtain 386

$$(\nu_1)^{(1)} \leq \nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\nu_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(1)}(t)$ :-

$$(m_2)^{(1)} \leq \nu^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{\nu^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(1)}(t)$ :-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{13})^{(1)} = (a''_{14})^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)}G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)}T_{14}(t)$ This is an important

consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

**Proof** : From global equations we obtain 387

$$\frac{d\nu^{(2)}}{dt} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right) - (a''_{17})^{(2)}(T_{17}, t)\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

**Definition of** $\nu^{(2)}$ :- $\boxed{\nu^{(2)} = \frac{G_{16}}{G_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \leq \frac{d\nu^{(2)}}{dt} \leq -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

**Definition of** $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$ :-

For $0 < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \geq \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}}{1 + (C)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(C)^{(2)} = \frac{(\nu_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \leq \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}}{1 + (C)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}} \leq \nu^{(2)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \leq (\bar{\nu}_1)^{(2)} \leq (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$ , we obtain 394

$$(\nu_1)^{(2)} \leq \nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(2)}(t)$ :- 395

$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}$, $\boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$

In a completely analogous way, we obtain 396

**Definition of** $u^{(2)}(t)$ :-

$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}$, $\boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :** 397

If $(a''_{16})^{(2)} = (a''_{17})^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)} G_{17}(t)$

Analogously if $(b''_{16})^{(2)} = (b''_{17})^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)} T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

**Proof** : From global equations we obtain 398

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right) - (a''_{21})^{(3)}(T_{21}, t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

**Definition of** $\nu^{(3)}$ :- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right) \leq \frac{d\nu^{(3)}}{dt} \leq -\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right)$$

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From which one obtains

For $0 < (\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\nu_1)^{(3)} < (\bar{\nu}_1)^{(3)}$

$$\nu^{(3)}(t) \geq \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(\nu_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\nu_2)^{(3)}}}$$

it follows $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\nu_1)^{(3)}$

In the same manner , we get 401

$$\nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{\nu}_1)^{(3)}-(\nu_0)^{(3)}}{(\nu_0)^{(3)}-(\bar{\nu}_2)^{(3)}}}$$

**Definition of** $(\bar{\nu}_1)^{(3)}$ :-

From which we deduce $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\bar{\nu}_1)^{(3)}$

If $0 < (\nu_1)^{(3)} < (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (\bar{\nu}_1)^{(3)}$ we find like in the previous case, 402

$$(\nu_1)^{(3)} \leq \frac{(\nu_1)^{(3)}+(C)^{(3)}(\nu_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}} \leq \nu^{(3)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\bar{\nu}_1)^{(3)}$$

If $0 < (\nu_1)^{(3)} \leq (\bar{\nu}_1)^{(3)} \leq (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}$ , we obtain 403

$$(\nu_1)^{(3)} \leq \nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\nu_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(3)}(t)$ :-

$$(m_2)^{(3)} \leq \nu^{(3)}(t) \leq (m_1)^{(3)}, \quad \boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(3)}(t)$ :-

$$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}, \quad \boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{20}'')^{(3)} = (a_{21}'')^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)}G_{21}(t)$

Analogously if $(b_{20}'')^{(3)} = (b_{21}'')^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)}T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

**Proof** : From global equations we obtain 404

$$\frac{dv^{(4)}}{dt} = (a_{24})^{(4)} - \left((a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right) - (a''_{25})^{(4)}(T_{25},t)v^{(4)} - (a_{25})^{(4)}v^{(4)}$$

**Definition of** $v^{(4)}$ :- $\boxed{v^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)}\right) \le \frac{dv^{(4)}}{dt} \le -\left((a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_4)^{(4)}v^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$ :-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \ge \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \le \nu^{(4)}(t) \le (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \le \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \le \nu^{(4)}(t) \le (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \le \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}}{1 + (C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}} \le \nu^{(4)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \le (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \le (\bar{\nu}_1)^{(4)} \le \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$ , we obtain 407

$$(\nu_1)^{(4)} \le \nu^{(4)}(t) \le \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \le (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(4)}(t)$ :-

$$(m_2)^{(4)} \le \nu^{(4)}(t) \le (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(4)}(t)$ :-

$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}$, $\boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{24})^{(4)} = (a''_{25})^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)} G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b''_{24})^{(4)} = (b''_{25})^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)} T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

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**Proof** : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a'_{28})^{(5)} - (a'_{29})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right) - (a''_{29})^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

**Definition of** $\nu^{(5)}$ :- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$ :-

For $0 < \boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \le \frac{(\nu_1)^{(5)}+(C)^{(5)}(\nu_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)}-(\nu_2)^{(5)}\right)t\right]}}{1+(C)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)}-(\nu_2)^{(5)}\right)t\right]}} \le \nu^{(5)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \le (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \le (\bar{\nu}_1)^{(5)} \le \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain 411

$$(\nu_1)^{(5)} \le \nu^{(5)}(t) \le \frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \le (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(5)}(t)$ :-

$(m_2)^{(5)} \le \nu^{(5)}(t) \le (m_1)^{(5)}$, $\boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$

In a completely analogous way, we obtain

**Definition of** $u^{(5)}(t)$ :-

$(\mu_2)^{(5)} \le u^{(5)}(t) \le (\mu_1)^{(5)}$, $\boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a_{28}'')^{(5)} = (a_{29}'')^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)}G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b_{28}'')^{(5)} = (b_{29}'')^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)}T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

**Proof** : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a_{32}')^{(6)} - (a_{33}')^{(6)} + (a_{32}'')^{(6)}(T_{33},t)\right) - (a_{33}'')^{(6)}(T_{33},t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

**Definition of** $\nu^{(6)}$ :- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \le \frac{d\nu^{(6)}}{dt} \le -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$ :-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \geq \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\nu_1)^{(6)}$

In the same manner , we get 413

$$\nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{\nu}_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\bar{\nu}_1)^{(6)}$

If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 414

$$(\nu_1)^{(6)} \leq \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}} \leq \nu^{(6)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\bar{\nu}_1)^{(6)}$$

If $0 < (\nu_1)^{(6)} \leq (\bar{\nu}_1)^{(6)} \leq \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$ , we obtain 415

$$(\nu_1)^{(6)} \leq \nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(6)}(t)$ :-

$$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}, \quad \boxed{\nu^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(6)}(t)$ :-

$$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}, \quad \boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)} G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)} T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

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**Proof** : From global equations we obtain

$$\frac{dv^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right) - (a''_{37})^{(7)}(T_{37},t)v^{(7)} - (a_{37})^{(7)}v^{(7)}$$

**Definition of** $v^{(7)}$ :- $\boxed{v^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_2)^{(7)}v^{(7)} - (a_{36})^{(7)}\right) \le \frac{dv^{(7)}}{dt} \le -\left((a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_1)^{(7)}v^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

**Definition of** $(\bar{v}_1)^{(7)}, (v_0)^{(7)}$ :-

For $0 < \boxed{(v_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}} < (v_1)^{(7)} < (\bar{v}_1)^{(7)}$

$$v^{(7)}(t) \ge \frac{(v_1)^{(7)} + (C)^{(7)}(v_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_0)^{(7)}\right)t\right]}}{1 + (C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(v_1)^{(7)} - (v_0)^{(7)}}{(v_0)^{(7)} - (v_2)^{(7)}}}$$

it follows $(v_0)^{(7)} \le v^{(7)}(t) \le (v_1)^{(7)}$

In the same manner , we get 417

$$v^{(7)}(t) \le \frac{(\bar{v}_1)^{(7)} + (\bar{C})^{(7)}(\bar{v}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{v}_1)^{(7)} - (v_0)^{(7)}}{(v_0)^{(7)} - (\bar{v}_2)^{(7)}}}$$

From which we deduce $(v_0)^{(7)} \le v^{(7)}(t) \le (\bar{v}_1)^{(7)}$

If $0 < (v_1)^{(7)} < (v_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}} < (\bar{v}_1)^{(7)}$ we find like in the previous case, 418

$$(v_1)^{(7)} \le \frac{(v_1)^{(7)} + (C)^{(7)}(v_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_2)^{(7)}\right)t\right]}}{1 + (C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_2)^{(7)}\right)t\right]}} \le v^{(7)}(t) \le$$

$$\frac{(\bar{v}_1)^{(7)} + (\bar{C})^{(7)}(\bar{v}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}} \le (\bar{v}_1)^{(7)}$$

If $0 < (v_1)^{(7)} \le (\bar{v}_1)^{(7)} \le \boxed{(v_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$ , we obtain 419

$$(v_1)^{(7)} \le v^{(7)}(t) \le \frac{(\bar{v}_1)^{(7)} + (\bar{C})^{(7)}(\bar{v}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}} \le (v_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $v^{(7)}(t)$ :-

$$(m_2)^{(7)} \leq v^{(7)}(t) \leq (m_1)^{(7)}, \quad \boxed{v^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(7)}(t)$ :- 420

$$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}, \quad \boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $v^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)} G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)} T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

**Proof** : From global equations we obtain 421

$$\frac{dv^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right) - (a''_{41})^{(8)}(T_{41}, t) v^{(8)} - (a_{41})^{(8)} v^{(8)}$$

**Definition of** $v^{(8)}$ :- $\boxed{v^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_2)^{(8)} v^{(8)} - (a_{40})^{(8)}\right) \leq \frac{dv^{(8)}}{dt} \leq -\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_1)^{(8)} v^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$ :-

$$\text{For } 0 < \boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$$

$$v^{(8)}(t) \geq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)}-(\nu_0)^{(8)}}{(\nu_0)^{(8)}-(\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \leq \frac{(\nu_1)^{(8)}+(C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}}{1+(C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}} \leq \nu^{(8)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \leq (\bar{\nu}_1)^{(8)} \leq \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$ , we obtain 424

$$(\nu_1)^{(8)} \leq \nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(8)}(t)$ :-

$$(m_2)^{(8)} \leq \nu^{(8)}(t) \leq (m_1)^{(8)}, \quad \boxed{\nu^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(8)}(t)$ :-

$$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}, \quad \boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $\nu^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)}G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then $(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)}T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

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**Proof** : From 99,20,44,22,23,44 we obtain A

$$\frac{dv^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right) - (a''_{45})^{(9)}(T_{45},t)v^{(9)} - (a_{45})^{(9)}v^{(9)}$$

**Definition of** $v^{(9)}$ :- $\boxed{v^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)}\right) \le \frac{dv^{(9)}}{dt} \le -\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_1)^{(9)}v^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$ :-

For $0 < \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$

$$v^{(9)}(t) \ge \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \le v^{(9)}(t) \le (\nu_9)^{(9)}$

In the same manner , we get

$$v^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \le v^{(9)}(t) \le (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \le \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}}{1 + (C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}} \le v^{(9)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \le (\bar{\nu}_1)^{(9)} \le \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$ , we obtain

$$(\nu_1)^{(9)} \le v^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $v^{(9)}(t)$ :-

$$(m_2)^{(9)} \le v^{(9)}(t) \le (m_1)^{(9)}, \quad \boxed{v^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(9)}(t)$ :-

$(\mu_2)^{(9)} \leq u^{(9)}(t) \leq (\mu_1)^{(9)}$, $\boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)} G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then
$(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)} T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following 425

**Theorem :** If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on $t$ , and the conditions with the notations

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0$ ,

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$

$with\ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a''_i)^{(2)} and\ (b''_i)^{(2)}$ are independent on t , and the conditions with the notations 426

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$ 427

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 428

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$ , 429

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 430

$with\ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a''_i)^{(3)} and\ (b''_i)^{(3)}$ are independent on $t$ , and the conditions with the notations 431

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$ ,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

**Theorem :** If $(a_i'')^{(4)} and\ (b_i'')^{(4)}$ are independent on $t$ , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$ ,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(5)} and\ (b_i'')^{(5)}$ are independent on $t$ , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$ ,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

**Theorem** If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on $t$ , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$ ,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(7)} and\ (b_i'')^{(7)}$ are independent on $t$ , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$ ,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(8)} and\ (b_i'')^{(8)}$ are independent on $t$ , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$ ,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(9)} and\ (b_i'')^{(9)}$ are independent on $t$ , and the conditions (with the notations 436 A
45,46,27,28)

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$ ,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0$ 437

$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0$ 438

$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0$ 439

$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0$ 440

$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ]T_{14} = 0$ 441

$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0$ 442

has a unique positive solution , which is an equilibrium solution for the system

$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0$ 443

$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0$ 444

$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0$ 445

$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\ ]T_{16} = 0$ 446

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\,]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\,]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\,]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\,]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\,]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\big((G_{27})\big)\,]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\big((G_{27})\big)\,]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\big((G_{27})\big)\,]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})\,]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})\,]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})\,]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})\,]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})\,]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})\,]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0 \quad 479$$

$$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0 \quad 480$$

$$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0 \quad 481$$

$$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0 \quad 482$$

$$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0 \quad 483$$

$$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0 \quad 484$$

$$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0 \quad 484\text{A}$$

$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$

$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$

$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$

$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$

$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$

**Proof:** 485

(a) Indeed the first two equations have a nontrivial solution $G_{13}, G_{14}$ if

$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$

**Proof:** 486

(w) Indeed the first two equations have a nontrivial solution $G_{16}, G_{17}$ if

$\mathrm{F}(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$

**Proof:** 487

(a) Indeed the first two equations have a nontrivial solution $G_{20}, G_{21}$ if

$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$

**Proof:** 488

(a) Indeed the first two equations have a nontrivial solution $G_{24}, G_{25}$ if

$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$

**Proof:** 489

(a) Indeed the first two equations have a nontrivial solution $G_{28}, G_{29}$ if

$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$

**Proof:** 490

(a) Indeed the first two equations have a nontrivial solution $G_{32}, G_{33}$ if

$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) +$

$(a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$

**Proof:** 491

(a) Indeed the first two equations have a nontrivial solution $G_{36}, G_{37}$ if

$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$

**Proof:** 492

(a) Indeed the first two equations have a nontrivial solution $G_{40}, G_{41}$ if

$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$

**Proof:** 492 A

(a) Indeed the first two equations have a nontrivial solution $G_{44}, G_{45}$ if

$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$

**Definition and uniqueness of** $T^*_{14}$ :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique $T^*_{14}$ for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

**Definition and uniqueness of** $T^*_{17}$ :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique $T^*_{17}$ for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]}$$ 495

**Definition and uniqueness of** $T^*_{21}$ :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique $T^*_{21}$ for which $f(T^*_{21}) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

**Definition and uniqueness of** $T^*_{25}$ :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that

there exists a unique $T_{25}^{*}$ for which $f(T_{25}^{*})=0$. With this value , we obtain from the three first equations

$$G_{24}=\frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}=\frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T_{25}^{*})]}$$

**Definition and uniqueness of** $T_{29}^{*}$ :- 498

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(5)}(T_{29})$ being increasing, it follows that there exists a unique $T_{29}^{*}$ for which $f(T_{29}^{*})=0$. With this value , we obtain from the three first equations

$$G_{28}=\frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}=\frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T_{29}^{*})]}$$

**Definition and uniqueness of** $T_{33}^{*}$ :- 499

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(6)}(T_{33})$ being increasing, it follows that there exists a unique $T_{33}^{*}$ for which $f(T_{33}^{*})=0$. With this value , we obtain from the three first equations

$$G_{32}=\frac{(a_{32})^{(6)}G_{33}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^{*})]} \quad , \quad G_{34}=\frac{(a_{34})^{(6)}G_{33}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^{*})]}$$

**Definition and uniqueness of** $T_{37}^{*}$ :- 500

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(7)}(T_{37})$ being increasing, it follows that there exists a unique $T_{37}^{*}$ for which $f(T_{37}^{*})=0$. With this value , we obtain from the three first equations

$$G_{36}=\frac{(a_{36})^{(7)}G_{37}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^{*})]} \quad , \quad G_{38}=\frac{(a_{38})^{(7)}G_{37}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^{*})]}$$

**Definition and uniqueness of** $T_{41}^{*}$ :- 501

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique $T_{41}^{*}$ for which $f(T_{41}^{*})=0$. With this value , we obtain from the three first equations

$$G_{40}=\frac{(a_{40})^{(8)}G_{41}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T_{41}^{*})]} \quad , \quad G_{42}=\frac{(a_{42})^{(8)}G_{41}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T_{41}^{*})]}$$

**Definition and uniqueness of** $T_{45}^{*}$ :- 501 A

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(9)}(T_{45})$ being increasing, it follows that there exists a unique $T_{45}^{*}$ for which $f(T_{45}^{*})=0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)}G_{45}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T^*_{45})]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)}G_{45}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T^*_{45})]}$$

By the same argument, the equations admit solutions $G_{13}, G_{14}$ if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right]+(b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{14}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{14}$ such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions $G_{16}, G_{17}$ if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right]+(b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ 504
is a decreasing function in $G_{17}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{14}$ such that $\varphi((G_{19})^*) = 0$

By the same argument, the equations admit solutions $G_{20}, G_{21}$ if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right]+(b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{21}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{21}$ such that $\varphi((G_{23})^*) = 0$

By the same argument, the equations admit solutions $G_{24}, G_{25}$ if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right]+(b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{25}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{25}$ such that $\varphi((G_{27})^*) = 0$

By the same argument, the equations admit solutions $G_{28}, G_{29}$ if 507

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right]+(b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{29}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that

there exists a unique $G_{29}^{*}$ such that $\varphi((G_{31})^{*}) = 0$

By the same argument, the equations admit solutions $G_{32}, G_{33}$ if 508
$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$

$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right] + (b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{33}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{33}^{*}$ such that $\varphi(G_{35}{}^{*}) = 0$

By the same argument, the equations admit solutions $G_{36}, G_{37}$ if 509
$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$
$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right] + (b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{37}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{37}^{*}$ such that $\varphi(G_{39}{}^{*}) = 0$

By the same argument, the equations admit solutions $G_{40}, G_{41}$ if 510
$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$
$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right] + (b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{41}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{41}^{*}$ such that $\varphi(G_{43}{}^{*}) = 0$

By the same argument, the equations 92,93 admit solutions $G_{44}, G_{45}$ if
$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$

$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right] + (b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{45}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{45}^{*}$ such that $\varphi((G_{47})^{*}) = 0$

Finally we obtain the unique solution 511

$G_{14}^{*}$ given by $\varphi(G^{*}) = 0$ , $T_{14}^{*}$ given by $f(T_{14}^{*}) = 0$ and

$$G_{13}^{*} = \frac{(a_{13})^{(1)}G_{14}^{*}}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^{*})]} \quad , \quad G_{15}^{*} = \frac{(a_{15})^{(1)}G_{14}^{*}}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^{*})]}$$

$$T_{13}^{*} = \frac{(b_{13})^{(1)}T_{14}^{*}}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^{*})]} \quad , \quad T_{15}^{*} = \frac{(b_{15})^{(1)}T_{14}^{*}}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^{*})]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

$G_{17}^{*}$ given by $\varphi((G_{19})^{*}) = 0$ , $T_{17}^{*}$ given by $f(T_{17}^{*}) = 0$ and 512

$$G_{16}^{*} = \frac{(a_{16})^{(2)} G_{17}^{*}}{[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}^{*})]} \quad , \quad G_{18}^{*} = \frac{(a_{18})^{(2)} G_{17}^{*}}{[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}^{*})]} \qquad 513$$

$$T_{16}^{*} = \frac{(b_{16})^{(2)} T_{17}^{*}}{[(b'_{16})^{(2)} - (b''_{16})^{(2)}((G_{19})^{*})]} \quad , \quad T_{18}^{*} = \frac{(b_{18})^{(2)} T_{17}^{*}}{[(b'_{18})^{(2)} - (b''_{18})^{(2)}((G_{19})^{*})]} \qquad 514$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

$G_{21}^{*}$ given by $\varphi((G_{23})^{*}) = 0$ , $T_{21}^{*}$ given by $f(T_{21}^{*}) = 0$ and

$$G_{20}^{*} = \frac{(a_{20})^{(3)} G_{21}^{*}}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^{*})]} \quad , \quad G_{22}^{*} = \frac{(a_{22})^{(3)} G_{21}^{*}}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^{*})]}$$

$$T_{20}^{*} = \frac{(b_{20})^{(3)} T_{21}^{*}}{[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}{}^{*})]} \quad , \quad T_{22}^{*} = \frac{(b_{22})^{(3)} T_{21}^{*}}{[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}{}^{*})]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

$G_{25}^{*}$ given by $\varphi(G_{27}) = 0$ , $T_{25}^{*}$ given by $f(T_{25}^{*}) = 0$ and

$$G_{24}^{*} = \frac{(a_{24})^{(4)} G_{25}^{*}}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}^{*} = \frac{(a_{26})^{(4)} G_{25}^{*}}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^{*})]}$$

$$T_{24}^{*} = \frac{(b_{24})^{(4)} T_{25}^{*}}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^{*})]} \quad , \quad T_{26}^{*} = \frac{(b_{26})^{(4)} T_{25}^{*}}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^{*})]} \qquad 517$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

$G_{29}^{*}$ given by $\varphi((G_{31})^{*}) = 0$ , $T_{29}^{*}$ given by $f(T_{29}^{*}) = 0$ and

$$G_{28}^{*} = \frac{(a_{28})^{(5)} G_{29}^{*}}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}^{*} = \frac{(a_{30})^{(5)} G_{29}^{*}}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^{*})]}$$

$$T_{28}^{*} = \frac{(b_{28})^{(5)} T_{29}^{*}}{[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31})^{*})]} \quad , \quad T_{30}^{*} = \frac{(b_{30})^{(5)} T_{29}^{*}}{[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31})^{*})]} \qquad 519$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

$G_{33}^{*}$ given by $\varphi((G_{35})^{*}) = 0$ , $T_{33}^{*}$ given by $f(T_{33}^{*}) = 0$ and

$$G_{32}^{*} = \frac{(a_{32})^{(6)} G_{33}^{*}}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^{*})]} \quad , \quad G_{34}^{*} = \frac{(a_{34})^{(6)} G_{33}^{*}}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^{*})]}$$

$$T_{32}^{*} = \frac{(b_{32})^{(6)} T_{33}^{*}}{[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35})^{*})]} \quad , \quad T_{34}^{*} = \frac{(b_{34})^{(6)} T_{33}^{*}}{[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35})^{*})]} \qquad 521$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

$G_{37}^*$ given by $\varphi((G_{39})^*) = 0$ , $T_{37}^*$ given by $f(T_{37}^*) = 0$ and

$$G_{36}^* = \frac{(a_{36})^{(7)} G_{37}^*}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^*)]} \quad , \quad G_{38}^* = \frac{(a_{38})^{(7)} G_{37}^*}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^*)]}$$

$$T_{36}^* = \frac{(b_{36})^{(7)} T_{37}^*}{[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39})^*)]} \quad , \quad T_{38}^* = \frac{(b_{38})^{(7)} T_{37}^*}{[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39})^*)]}$$

Finally we obtain the unique solution 523

$G_{41}^*$ given by $\varphi((G_{43})^*) = 0$ , $T_{41}^*$ given by $f(T_{41}^*) = 0$ and

$$G_{40}^* = \frac{(a_{40})^{(8)} G_{41}^*}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42}^* = \frac{(a_{42})^{(8)} G_{41}^*}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

$$T_{40}^* = \frac{(b_{40})^{(8)} T_{41}^*}{[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43})^*)]} \quad , \quad T_{42}^* = \frac{(b_{42})^{(8)} T_{41}^*}{[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43})^*)]}$$

Finally we obtain the unique solution of 89 to 99 523 A

$G_{45}^*$ given by $\varphi((G_{47})^*) = 0$ , $T_{45}^*$ given by $f(T_{45}^*) = 0$ and

$$G_{44}^* = \frac{(a_{44})^{(9)} G_{45}^*}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46}^* = \frac{(a_{46})^{(9)} G_{45}^*}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

$$T_{44}^* = \frac{(b_{44})^{(9)} T_{45}^*}{[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47})^*)]} \quad , \quad T_{46}^* = \frac{(b_{46})^{(9)} T_{45}^*}{[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47})^*)]}$$

## ASYMPTOTIC STABILITY ANALYSIS 524

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

**Proof:** Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a''_{14})^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \frac{\partial (b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a'_{13})^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)}G^*_{13}\mathbb{T}_{14} \quad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a'_{14})^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)}G^*_{14}\mathbb{T}_{14} \quad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a'_{15})^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)}G^*_{15}\mathbb{T}_{14} \quad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b'_{13})^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)}T^*_{13}\mathbb{G}_j\right) \quad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b'_{14})^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)}T^*_{14}\mathbb{G}_j\right) \quad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b'_{15})^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T^*_{15}\mathbb{G}_j\right) \quad 530$$

**ASYMPTOTIC STABILITY ANALYSIS** 531

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(2)}$ and $(b''_i)^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G^*_i + \mathbb{G}_i \quad , T_i = T^*_i + \mathbb{T}_i \quad 532$$

$$\frac{\partial(a''_{17})^{(2)}}{\partial T_{17}}(T^*_{17}) = (q_{17})^{(2)} \quad , \frac{\partial(b''_i)^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \quad 533$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a'_{16})^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G^*_{16}\mathbb{T}_{17} \quad 534$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a'_{17})^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G^*_{17}\mathbb{T}_{17} \quad 535$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a'_{18})^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G^*_{18}\mathbb{T}_{17} \quad 536$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b'_{16})^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T^*_{16}\mathbb{G}_j\right) \quad 537$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b'_{17})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}T^*_{17}\mathbb{G}_j\right) \quad 538$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b'_{18})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}T^*_{18}\mathbb{G}_j\right) \quad 539$$

**ASYMPTOTIC STABILITY ANALYSIS** 540

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(3)}$ *and* $(b''_i)^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{21}'')^{(3)}}{\partial T_{21}}(T_{21}^*) = (q_{21})^{(3)} \quad , \frac{\partial (b_i'')^{(3)}}{\partial G_j}(\,(G_{23})^*\,) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\big((a_{20}')^{(3)} + (p_{20})^{(3)}\big)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G_{20}^*\mathbb{T}_{21} \quad 541$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\big((a_{21}')^{(3)} + (p_{21})^{(3)}\big)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G_{21}^*\mathbb{T}_{21} \quad 542$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\big((a_{22}')^{(3)} + (p_{22})^{(3)}\big)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G_{22}^*\mathbb{T}_{21} \quad 543$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\big((b_{20}')^{(3)} - (r_{20})^{(3)}\big)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\big(s_{(20)(j)}T_{20}^*\mathbb{G}_j\big) \quad 544$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\big((b_{21}')^{(3)} - (r_{21})^{(3)}\big)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\big(s_{(21)(j)}T_{21}^*\mathbb{G}_j\big) \quad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\big((b_{22}')^{(3)} - (r_{22})^{(3)}\big)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\big(s_{(22)(j)}T_{22}^*\mathbb{G}_j\big) \quad 546$$

**ASYMPTOTIC STABILITY ANALYSIS** 547

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}\ and\ (b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{25}'')^{(4)}}{\partial T_{25}}(T_{25}^*) = (q_{25})^{(4)} \quad , \frac{\partial (b_i'')^{(4)}}{\partial G_j}((G_{27})^*\,) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\big((a_{24}')^{(4)} + (p_{24})^{(4)}\big)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G_{24}^*\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\big((a_{25}')^{(4)} + (p_{25})^{(4)}\big)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G_{25}^*\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\big((a_{26}')^{(4)} + (p_{26})^{(4)}\big)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G_{26}^*\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\big((b_{24}')^{(4)} - (r_{24})^{(4)}\big)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\big(s_{(24)(j)}T_{24}^*\mathbb{G}_j\big) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\big((b_{25}')^{(4)} - (r_{25})^{(4)}\big)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\big(s_{(25)(j)}T_{25}^*\mathbb{G}_j\big) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T^*_{26}\mathbb{G}_j\right) \quad 554$$

**ASYMPTOTIC STABILITY ANALYSIS** 555

**Theorem 5:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}\ and\ (b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 556

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{29})^{(5)}}{\partial T_{29}}(T^*_{29}) = (q_{29})^{(5)} \ , \ \frac{\partial(b_i'')^{(5)}}{\partial G_j}\left((G_{31})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a'_{28})^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G^*_{28}\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a'_{29})^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G^*_{29}\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a'_{30})^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G^*_{30}\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b'_{28})^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T^*_{28}\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b'_{29})^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T^*_{29}\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b'_{30})^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T^*_{30}\mathbb{G}_j\right) \quad 562$$

**ASYMPTOTIC STABILITY ANALYSIS** 563

**Theorem 6:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}\ and\ (b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{33})^{(6)}}{\partial T_{33}}(T^*_{33}) = (q_{33})^{(6)} \ , \ \frac{\partial(b_i'')^{(6)}}{\partial G_j}\left((G_{35})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a'_{32})^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G^*_{32}\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a'_{33})^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G^*_{33}\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a'_{34})^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G^*_{34}\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b'_{32})^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T^*_{32}\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b'_{33})^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T^*_{33}\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b'_{34})^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T^*_{34}\mathbb{G}_j\right) \quad 570$$

**ASYMPTOTIC STABILITY ANALYSIS** 571

**Theorem 7:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 572

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{37})^{(7)}}{\partial T_{37}}(T^*_{37}) = (q_{37})^{(7)} \quad , \frac{\partial(b_i'')^{(7)}}{\partial G_j}\left((G_{39})^{**}\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a'_{36})^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G^*_{36}\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a'_{37})^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G^*_{37}\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a'_{38})^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G^*_{38}\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b'_{36})^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T^*_{36}\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b'_{37})^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T^*_{37}\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b'_{38})^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T^*_{38}\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

**ASYMPTOTIC STABILITY ANALYSIS**

**Theorem 8:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{41}'')^{(8)}}{\partial T_{41}}(T_{41}^*) = (q_{41})^{(8)} \quad , \frac{\partial (b_i'')^{(8)}}{\partial G_j}\left( (G_{43})^* \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a_{40}')^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G_{40}^*\mathbb{T}_{41} \quad (581)$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a_{41}')^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G_{41}^*\mathbb{T}_{41} \quad (582)$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a_{42}')^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G_{42}^*\mathbb{T}_{41} \quad (583)$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b_{40}')^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T_{40}^*\mathbb{G}_j\right) \quad (584)$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b_{41}')^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T_{41}^*\mathbb{G}_j\right) \quad (585)$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b_{42}')^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T_{42}^*\mathbb{G}_j\right) \quad (586)$$

(586 A)

**ASYMPTOTIC STABILITY ANALYSIS**

**Theorem 9:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}$ *and* $(b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.
**Proof:** Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{45}'')^{(9)}}{\partial T_{45}}(T_{45}^*) = (q_{45})^{(9)} \quad , \frac{\partial (b_i'')^{(9)}}{\partial G_j}\left( (G_{47})^* \right) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a_{44}')^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)}G_{44}^*\mathbb{T}_{45} \quad (586\,B)$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a_{45}')^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)}G_{45}^*\mathbb{T}_{45} \quad (586\,C)$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a_{46}')^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)}G_{46}^*\mathbb{T}_{45} \quad (586\,D)$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b_{44}')^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)}T_{44}^*\mathbb{G}_j\right) \quad (586\,E)$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b_{45}')^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)}T_{45}^*\mathbb{G}_j\right) \quad (586\,F)$$

$\frac{d\mathbb{T}_{46}}{dt} = -\left((b'_{46})^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T^*_{46}\mathbb{G}_j\right)$ 586

G

587

**The characteristic equation of this system is**

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(q_{13})^{(1)}G^*_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)}T^*_{14} + (b_{14})^{(1)}s_{(13),(14)}T^*_{14}\right)$$

$$+\left(\left((\lambda)^{(1)} + (a'_{14})^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)}G^*_{13} + (a_{13})^{(1)}(q_{14})^{(1)}G^*_{14}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)}T^*_{14} + (b_{14})^{(1)}s_{(13),(13)}T^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((b'_{13})^{(1)} + (b'_{14})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(15)}T^*_{14} + (b_{14})^{(1)}s_{(13),(15)}T^*_{13}\right)\} = 0$$

$+$

$$(\lambda)^{(2)} + (b'_{18})^{(2)} - (r_{18})^{(2)}\{\left((\lambda)^{(2)} + (a'_{18})^{(2)} + (p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^*_{17} + (a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^*_{17}\right)$$

$$+\left(\left((\lambda)^{(2)} + (a'_{17})^{(2)} + (p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^*_{16} + (a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}\right)$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2 + \left((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2 + \left((b'_{16})^{(2)} + (b'_{17})^{(2)} - (r_{16})^{(2)} + (r_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$+\left(\left((\lambda)^{(2)}\right)^2 + \left((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17} + (a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^{*}_{16}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\right)\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left(\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)\right]$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(21)}T^{*}_{21}\right)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^{*}_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^{*}_{21}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(20)}T^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$+\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(22)}T^{*}_{20}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\right)\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left(\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)\right]$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(25)}T^{*}_{25}\right)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^{*}_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^{*}_{25}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(24)}T^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$+\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(26)}T^{*}_{24}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\right)\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(29)}T^{*}_{29}\right)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^{*}_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^{*}_{29}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(28)}T^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$+\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(30)}T^{*}_{28}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\right)\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left(\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)\right]$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(33)}T^{*}_{33}\right)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^{*}_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^{*}_{33}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(32)}T^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$+\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(34)}T^{*}_{32}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\right)\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)\right]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(37)}T^{*}_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^{*}_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^{*}_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(36)}T^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$+\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(38)}T^{*}_{36}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\right)\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(41)}T^{*}_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^{*}_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^{*}_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(40)}T^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$+\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(42)}T^{*}_{40}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\right)\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(45)}T^{*}_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^{*}_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^{*}_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(44)}T^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$+\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(46)}T^{*}_{44}\right)\}=0$$

**And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.**

## SECTION TWENTY SIX

## Dynamics In Large-Scale Functional Brain Networks

### INTRODUCTION—VARIABLES USED

**Ising-like dynamics in large-scale functional brain networks Daniel Fraiman, Pablo Balenzuela, Jennifer Foss, and Dante R. Chialvo Phys. Rev. E 79, 061922 – Published 23 June 2009**

(1) Brain "rest" is defined—more or less unsuccessfully—as (=) the state in which there is no explicit brain input or output.
(2) This work focuses on the question of whether such state can be comparable to (e&eb) any known dynamical state.
(3) For that purpose, correlation networks from human brain functional magnetic resonance imaging are contrasted with (e&eb) correlation networks extracted from numerical simulations of the Ising model in two dimensions at (eb) different temperatures.
(4) For the critical temperature Tc, striking similarities appear in (eb) the most relevant statistical properties, making the two networks indistinguishable from (e) each other.
**(5)** These results are interpreted here as lending support to (e) the conjecture that the dynamics of the functioning brain is (=) near a critical point. **http://dx.doi.org/10.1103/PhysRevE.79.061922**

**Temporal Networks Part of the series Understanding Complex Systems pp 175-190 Date: 15 April 2013 Time Allocation in Social Networks: Correlation between Social Structure and Human Communication Dynamics Giovanna Miritello, Rubén Lara, Esteban Moro**

**(6)** Recent research has shown the deep impact of the dynamics of human interactions (or temporal social networks) on (e&eb) the spreading of information, opinion formation, etc.
**(7)** In general, the bursty nature of human interactions lowers (e) the interaction between people to the extent that both the speed and reach of information diffusion are (=) diminished.
**(8)** Using a large database of 20 million users of mobile phone calls authors show evidence this effect is not homogeneous in the social network but in fact, there is a large correlation between this effect and (e&eb) the social topological structure around a given individual.
**(9)** In particular, authors show that social relations of hubs in a network are (=) relatively weaker from the dynamical point than (e) those that are poorer connected in (eb) the information diffusion process.
**(10)** Results show the importance of the temporal patterns of communication when (e) analyzing and modeling dynamical process on social networks.

## NOTATION

### Module One

Brain "rest" is defined—more or less unsuccessfully—as (=) the state in which there is no explicit brain input or output

$G_{13}$ : Category one of Brain "rest"

$G_{14}$ : Category two of SAS

$G_{15}$ : Category three of SAS

$T_{13}$ : Category one of **state in which there is no explicit brain input or output**

$T_{14}$ : Category two of SAS

$T_{15}$ : Category three of SAS

### Module Two

This work focuses on the question of whether such state can be comparable to (e&eb) any known dynamical state

We assume the proposition to be affirmative and give a model.

$G_{16}$ : Category one of **state in which there is no explicit brain input or output;** known dynamical state

$G_{17}$ : Category two of SAS

$G_{18}$ : Category three of SAS

$T_{16}$ : Category one of known dynamical state; **state in which there is no explicit brain input or output**
$T_{17}$ : Category two of SAS

$T_{18}$ : Category three of SAS

**Module three**

For that purpose, correlation networks from human brain functional magnetic resonance imaging are contrasted with (e&eb) correlation networks extracted from numerical simulations of the Ising model in two dimensions at (eb) different temperatures

$G_{20}$ : Category one of **correlation networks from human brain functional magnetic resonance imaging;** correlation networks extracted from numerical simulations of the Ising model in two dimensions at (eb) different temperatures

$G_{21}$ : Category two of SAS

$G_{22}$ : Category three of SAS

$T_{20}$ : Category one of correlation networks extracted from numerical simulations of the Ising model in two dimensions at (eb) different temperatures; **correlation networks from human brain functional magnetic resonance imaging**

$T_{21}$ : Category two of SAS

$T_{22}$ : Category three of SAS

**Module four**

For that purpose, correlation networks from human brain functional magnetic resonance imaging are contrasted with correlation networks extracted from numerical simulations of the Ising model in two dimensions at (eb) different temperatures

$G_{24}$ : Category one of correlation networks from human brain functional magnetic resonance imaging are contrasted with correlation networks extracted from numerical simulations of the Ising model in two dimensions

$G_{25}$ : Category two of SAS

$G_{26}$ : Category three of SAS

$T_{24}$ : Category one of different temperatures

$T_{25}$ : Category two of SAS

$T_{26}$ : Category three of SAS

**Module five**

For the critical temperature Tc, striking similarities appear in (eb) the most relevant statistical properties,

making the two networks indistinguishable from (e) each other

$G_{28}$ : Category one of striking similarities appear for the critical temperature Tc

$G_{29}$ : Category two of SAS

$G_{30}$ : Category three of SAS

$T_{28}$ : Category one of most relevant statistical properties, making the two networks indistinguishable from each other

$T_{29}$ : Category two of SAS

$T_{30}$ : Category three of SAS

**Module six**

These **results are interpreted here as lending support** to (e) the conjecture that the dynamics of the functioning brain is (=) near a critical point.

**http://dx.doi.org/10.1103/PhysRevE.79.061922**

$G_{32}$ : Category one of **results are interpreted here as lending support;** conjecture that the dynamics of the functioning brain is (=) near a critical point

$G_{33}$ : Category two of SAS

$G_{34}$ : Category three of SAS

$T_{32}$ : Category one of conjecture that the dynamics of the functioning brain is (=) near a critical point; **results are interpreted here as lending support**

$T_{33}$ : Category two of SAS

$T_{34}$ : Category three of SAS

**Module seven**

**Electroencephalography and Clinical Neurophysiology Volume 79, Issue 2, August 1991, Pages 81-93 Neural mechanisms underlying brain waves: from neural membranes to networks:. Fernando Lopes da Silva**

(1) In this review, a number of experimental findings and theoretical concepts that have led to (eb) new insights into the mechanisms underlying brain waves are presented.
(2) At the cellular level, the new evidence that certain types of neuron have (e) intrinsic oscillatory properties that may underlie (e&eb) rhythmic EEG activities is discussed.
(3) In particular, the question of whether spindle oscillations are (=) autonomous or ∝ input-dependent is addressed.
(4) At the neural network level, the main circuits of the thalamus and cortex that are responsible for the occurrence and modulation of (e&eb) spindles and alpha activity are described.
(5) In addition, the properties of rhythmic activities outside the alpha band are considered, particularly in relation to (e&eb) the prominent beta activity of the visual cortex.
(6) At the theoretical level, the possibility that neural networks may behave as (e&eb) complex dynamic systems with (e&eb) the properties of deterministic chaos is discussed.
(7) Finally, the fact that brain rhythms may have (e) functional implications for the working of neural

networks is examined in relation to 2 cases: the possibility that oscillations may subserve (e&eb) a gating function and those oscillations may play a role (e&eb) in the formation of assemblies of neurons that represent given stimulus patterns.

At the cellular level, the new evidence that certain types of neuron have (e) intrinsic oscillatory properties that may underlie (e&eb) rhythmic EEG activities is discussed

$G_{36}$ : Category one of **certain types of neuron;** intrinsic oscillatory properties that may underlie (e&eb) rhythmic EEG activities

$G_{37}$ : Category two of SAS

$G_{38}$ : Category three of SAS

$T_{36}$ : Category one of intrinsic oscillatory properties that may underlie (e&eb) rhythmic EEG activities ;**certain types of neuron**

$T_{37}$ : Category two of SAS

$T_{38}$ : Category three of SAS

**Module eight**

At the cellular level, the new evidence that **certain types of neuron have intrinsic oscillatory properties** that may underlie (e&eb) rhythmic EEG activities is discussed

$G_{40}$ : Category one of **certain types of neuron have intrinsic oscillatory properties;** rhythmic EEG activities

$G_{41}$ : Category two of SAS

$G_{42}$ : Category three of SAS

$T_{40}$ : Category one of rhythmic EEG activities; **certain types of neuron have intrinsic oscillatory properties**

$T_{41}$ : Category two of SAS

$T_{42}$ : Category three of SAS

**Module Nine**

Finally, the fact that brain rhythms may have (e) functional implications for the working of neural networks is examined in relation to 2 cases:

the possibility that **oscillations may subserve** (e&eb) a gating function and those oscillations may play a role (e&eb) in the formation of assemblies of neurons that represent given stimulus patterns

$G_{44}$ : Category one of **oscillations may subserve;** gating function and those oscillations may play a role (e&eb) in the formation of assemblies of neurons that represent given stimulus patterns

$G_{45}$ : Category two of SAS

$G_{46}$ : Category three of SAS

$T_{44}$ : Category one of gating function and those oscillations may play a role (e&eb) in the formation of assemblies of neurons that represent given stimulus patterns **;oscillations may subserve**

$T_{45}$ : Category two of SAS

$T_{46}$ : Category three of SAS

**The Coefficients:**

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}$ , $(b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}$,
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}$, $(b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}\ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$ ,
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)}$,
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)}$,
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)}$,
are Dissipation coefficients

**Module Numbered One**

**The differential system of this model is now (Module Numbered one)**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \quad 1$$
$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad 2$$
$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15} \quad 3$$
$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13} \quad 4$$
$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14} \quad 5$$
$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)\right]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14}, t) =$ First augmentation factor
$-(b''_{13})^{(1)}(G, t) =$ First detritions factor

**Module Numbered Two**

**The differential system of this model is now ( Module numbered two)**

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right]G_{16} \quad 7$$
$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t)\right]G_{17} \quad 8$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t)\right]G_{18} \quad 9$$

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\left((G_{19}), t\right)\right]T_{16} \quad 10$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\left((G_{19}), t\right)\right]T_{17} \quad 11$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\left((G_{19}), t\right)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17}, t) =$ First augmentation factor

$-(b''_{16})^{(2)}\left((G_{19}), t\right) =$ First detritions factor

**Module Numbered Three**

**The differential system of this model is now (Module numbered three)**

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right]G_{20} \quad 13$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}, t)\right]G_{21} \quad 14$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}, t)\right]G_{22} \quad 15$$

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}, t)\right]T_{20} \quad 16$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}, t)\right]T_{21} \quad 17$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}, t)\right]T_{22} \quad 18$$

$+(a''_{20})^{(3)}(T_{21}, t) =$ First augmentation factor

$-(b''_{20})^{(3)}(G_{23}, t) =$ First detritions factor

**Module Numbered Four**

**The differential system of this model is now (Module numbered Four)**

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)\right]G_{24} \quad 19$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}, t)\right]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}, t)\right]G_{26} \quad 21$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27}), t\right)\right]T_{24} \quad 22$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27}), t\right)\right]T_{25} \quad 23$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27}), t\right)\right]T_{26} \quad 24$$

$+(a''_{24})^{(4)}(T_{25}, t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}\left((G_{27}), t\right) =$ **First detritions factor**

**Module Numbered Five**:

**The differential system of this model is now (Module number five)**

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right]G_{28} \quad 25$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}, t)\right]G_{29} \quad 26$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}, t)\right]G_{30} \quad 27$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}\left((G_{31}), t\right)\right]T_{28} \quad 28$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}\left((G_{31}), t\right)\right]T_{29} \quad 29$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}\left((G_{31}), t\right)\right]T_{30} \quad 30$$

$+(a''_{28})^{(5)}(T_{29}, t) =$ **First augmentation factor**

$-(b''_{28})^{(5)}((G_{31}),t) =$ **First detritions factor**

**Module Numbered Six**

**The differential system of this model is now (Module numbered Six)**

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33},t)\right]G_{32} \quad 31$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33},t)\right]G_{33} \quad 32$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33},t)\right]G_{34} \quad 33$$

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35}),t)\right]T_{32} \quad 34$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}((G_{35}),t)\right]T_{33} \quad 35$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35}),t)\right]T_{34} \quad 36$$

$+(a''_{32})^{(6)}(T_{33},t) =$ **First augmentation factor**

**Module Numbered Seven:**

**The differential system of this model is now (Seventh Module)**

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right]G_{36} \quad 37$$

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37},t)\right]G_{37} \quad 38$$

$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37},t)\right]G_{38} \quad 39$$

$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39}),t)\right]T_{36} \quad 40$$

$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}((G_{39}),t)\right]T_{37} \quad 41$$

$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39}),t)\right]T_{38} \quad 42$$

$+(a''_{36})^{(7)}(T_{37},t) =$ **First augmentation factor**

**Module Numbered Eight**

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right]G_{40} \quad 43$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41},t)\right]G_{41} \quad 44$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41},t)\right]G_{42} \quad 45$$

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43}),t)\right]T_{40} \quad 46$$

$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}((G_{43}),t)\right]T_{41} \quad 47$$

$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43}),t)\right]T_{42} \quad 48$$

**Module Numbered Nine**

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47}),t)\right]T_{44} \quad 52$$

$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}((G_{47}),t)\right]T_{45} \quad 53$$

$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47}),t)\right]T_{46} \quad 54$$

$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}((G_{47}),t) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)}\boxed{+(a''_{13})^{(1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)}\boxed{+(a''_{14})^{(1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)}\boxed{+(a''_{15})^{(1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15} \quad 57$$

Where $\boxed{(a''_{13})^{(1)}(T_{14},t)}$, $\boxed{(a''_{14})^{(1)}(T_{14},t)}$, $\boxed{(a''_{15})^{(1)}(T_{14},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7)}(T_{37},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8)}(T_{41},t)}$ $\boxed{+(a''_{41})^{(8,8)}(T_{41},t)}$ $\boxed{+(a''_{42})^{(8,8)}(T_{41},t)}$ are eight augmentation coefficient for 1,2,3

$\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)}\boxed{-(b''_{13})^{(1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13} \quad 58$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)}\boxed{-(b''_{14})^{(1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14} \quad 59$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)}\boxed{-(b''_{15})^{(1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15} \quad (60)$$

Where $\boxed{-(b''_{13})^{(1)}(G,t)}$, $\boxed{-(b''_{14})^{(1)}(G,t)}$, $\boxed{-(b''_{15})^{(1)}(G,t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8)}(G_{43},t)}$ $\boxed{-(b''_{41})^{(8,8)}(G_{43},t)}$ $\boxed{-(b''_{42})^{(8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \begin{bmatrix} (a'_{16})^{(2)}\boxed{+(a''_{16})^{(2)}(T_{17},t)} & \boxed{+(a''_{13})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{16} \quad (61)$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \begin{bmatrix} (a'_{17})^{(2)}\boxed{+(a''_{17})^{(2)}(T_{17},t)} & \boxed{+(a''_{14})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{17} \quad (62)$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \begin{bmatrix} (a'_{18})^{(2)}\boxed{+(a''_{18})^{(2)}(T_{17},t)} & \boxed{+(a''_{15})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{18} \quad (63)$$

Where $\boxed{+(a''_{16})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,)}(T_{14},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{36})^{(7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7)}(T_{37},t)$ are seventh augmentation coefficient for category 1, 2 and 3

$+(a''_{40})^{(8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8)}(T_{41},t)$ are eight augmentation coefficient for category 1, 2 and 3

$+(a''_{44})^{(9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9)}(T_{45},t)$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \begin{bmatrix} (b'_{16})^{(2)} & -(b''_{16})^{(2)}(G_{19},t) & -(b''_{13})^{(1,1,)}(G,t) & -(b''_{20})^{(3,3,3,)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6)}(G_{35},t) \\ -(b''_{36})^{(7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9)}(G_{47},t) \end{bmatrix} T_{16} \quad 64$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)} & -(b''_{17})^{(2)}(G_{19},t) & -(b''_{14})^{(1,1,)}(G,t) & -(b''_{21})^{(3,3,3,)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6)}(G_{35},t) \\ -(b''_{37})^{(7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9)}(G_{47},t) \end{bmatrix} T_{17} \quad 65$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)} & -(b''_{18})^{(2)}(G_{19},t) & -(b''_{15})^{(1,1,)}(G,t) & -(b''_{22})^{(3,3,3,)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6)}(G_{35},t) \\ -(b''_{38})^{(7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9)}(G_{47},t) \end{bmatrix} T_{18} \quad 66$$

where $-(b''_{16})^{(2)}(G_{19},t)$, $-(b''_{17})^{(2)}(G_{19},t)$, $-(b''_{18})^{(2)}(G_{19},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{13})^{(1,1,)}(G,t)$, $-(b''_{14})^{(1,1,)}(G,t)$, $-(b''_{15})^{(1,1,)}(G,t)$ are second detrition coefficients for category 1,2 and 3

$-(b''_{20})^{(3,3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,)}(G_{23},t)$ are third detrition coefficients for category 1,2 and 3

$-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1,2 and 3

$-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1,2 and 3

$-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients for category 1,2 and 3

$-(b''_{36})^{(7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2 and 3

$-(b''_{40})^{(8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1,2 and 3

$-(b''_{44})^{(9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \begin{bmatrix} (a'_{20})^{(3)} & +(a''_{20})^{(3)}(T_{21},t) & +(a''_{16})^{(2,2,2)}(T_{17},t) & +(a''_{13})^{(1,1,1,)}(T_{14},t) \\ +(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t) & \\ +(a''_{36})^{(7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9)}(T_{45},t) & \end{bmatrix} G_{20} \quad (67)$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \begin{bmatrix} (a'_{21})^{(3)} & +(a''_{21})^{(3)}(T_{21},t) & +(a''_{17})^{(2,2,2)}(T_{17},t) & +(a''_{14})^{(1,1,1,)}(T_{14},t) \\ +(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t) & \\ +(a''_{37})^{(7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9)}(T_{45},t) & \end{bmatrix} G_{21} \quad (68)$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \begin{bmatrix} (a'_{22})^{(3)} & +(a''_{22})^{(3)}(T_{21},t) & +(a''_{18})^{(2,2,2)}(T_{17},t) & +(a''_{15})^{(1,1,1,)}(T_{14},t) \\ +(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t) & \\ +(a''_{38})^{(7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9)}(T_{45},t) & \end{bmatrix} G_{22} \quad (69)$$

$+(a''_{20})^{(3)}(T_{21},t)$, $+(a''_{21})^{(3)}(T_{21},t)$, $+(a''_{22})^{(3)}(T_{21},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2)}(T_{17},t)$ are second augmentation coefficients for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,)}(T_{14},t)$ are third augmentation coefficients for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficients for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficients for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficients for category 1, 2 and 3

$+(a''_{36})^{(7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1, 2 and 3

$+(a''_{40})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8)}(T_{41},t)$ are eight augmentation coefficients for category 1, 2 and 3

$+(a''_{44})^{(9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b'_{20})^{(3)} & -(b''_{20})^{(3)}(G_{23},t) & -(b''_{16})^{(2,2,2)}(G_{19},t) & -(b''_{13})^{(1,1,1,)}(G,t) \\ -(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t) & \\ -(b''_{36})^{(7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9)}(G_{47},t) & \end{bmatrix} T_{20} \quad (70)$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b'_{21})^{(3)} & -(b''_{21})^{(3)}(G_{23},t) & -(b''_{17})^{(2,2,2)}(G_{19},t) & -(b''_{14})^{(1,1,1,)}(G,t) \\ -(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t) & \\ -(b''_{37})^{(7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9)}(G_{47},t) & \end{bmatrix} T_{21} \quad (71)$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)}\boxed{-(b''_{22})^{(3)}(G_{23},t)} & \boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{22} \quad 72$$

$\boxed{-(b''_{20})^{(3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3)}(G_{23},t)}$ *are first detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}$ *are second detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{13})^{(1,1,1,)}(G,t)}, \boxed{-(b''_{14})^{(1,1,1,)}(G,t)}, \boxed{-(b''_{15})^{(1,1,1,)}(G,t)}$ *are third detrition coefficients for category 1,2 and 3*

$\boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)}, \boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)}, \boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)}$ *are fourth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)}, \boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)}, \boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)}$ *are fifth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)}, \boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)}, \boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)}$ *are sixth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{36})^{(7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{37})^{(7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)}, \boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)}, \boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \begin{bmatrix} (a'_{24})^{(4)}\boxed{+(a''_{24})^{(4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{24} \quad 73$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \begin{bmatrix} (a'_{25})^{(4)}\boxed{+(a''_{25})^{(4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{25} \quad 74$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \begin{bmatrix} (a'_{26})^{(4)}\boxed{+(a''_{26})^{(4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{26} \quad 75$$

$\boxed{(a''_{24})^{(4)}(T_{25},t)}, \boxed{(a''_{25})^{(4)}(T_{25},t)}, \boxed{(a''_{26})^{(4)}(T_{25},t)}$ *are first augmentation coefficients category* $1, 2\ 3$

$\boxed{+(a''_{28})^{(5,5,)}(T_{29},t)}, \boxed{+(a''_{29})^{(5,5,)}(T_{29},t)}, \boxed{+(a''_{30})^{(5,5,)}(T_{29},t)}$ *are second augmentation coefficient for category* $1, 2$ *and* $3$

$\boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}, \boxed{+(a''_{33})^{(6,6,)}(T_{33},t)}, \boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}$ *are third augmentation coefficient for category* $1, 2$ *and* $3$

$\boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)}$

$are\ fourth\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$
$+(a''_{16})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2)}(T_{17},t)$
$are\ fifth\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$
$+(a''_{20})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3)}(T_{21},t)$
$are\ sixth\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$
$+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)$
$are\ seventh\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$
$+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)$
$are\ eighth\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$
$+(a''_{46})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9)}(T_{45},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \begin{bmatrix} (b'_{24})^{(4)} & -(b''_{24})^{(4)}(G_{27},t) & -(b''_{28})^{(5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,)}(G_{35},t) \\ & -(b''_{13})^{(1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{24} \quad 76$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \begin{bmatrix} (b'_{25})^{(4)} & -(b''_{25})^{(4)}(G_{27},t) & -(b''_{29})^{(5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{25} \quad 77$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)} & -(b''_{26})^{(4)}(G_{27},t) & -(b''_{30})^{(5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1)}(G,t) & -(b''_{18})^{(2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{26} \quad 78$$

$Where$ $-(b''_{24})^{(4)}(G_{27},t)$, $-(b''_{25})^{(4)}(G_{27},t)$, $-(b''_{26})^{(4)}(G_{27},t)$ $are\ first\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b''_{28})^{(5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,)}(G_{31},t)$ $are\ second\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b''_{32})^{(6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,)}(G_{35},t)$ $are\ third\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b''_{13})^{(1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1)}(G,t)$
$are\ fourth\ \ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b''_{16})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2)}(G_{19},t)$
$are\ fifth\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b''_{20})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3)}(G_{23},t)$
$are\ sixth\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)$
$are\ seventh\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)$
$are\ eighth\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b''_{46})^{(9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \begin{bmatrix} (a'_{28})^{(5)}\boxed{+(a''_{28})^{(5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{28} \quad 79$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \begin{bmatrix} (a'_{29})^{(5)}\boxed{+(a''_{29})^{(5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{29} \quad 80$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \begin{bmatrix} (a'_{30})^{(5)}\boxed{+(a''_{30})^{(5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{30} \quad 81$$

*Where* $\boxed{+(a''_{28})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5)}(T_{29},t)}$ *are first augmentation coefficients for category* 1, 2 *and* 3

*And* $\boxed{+(a''_{24})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,)}(T_{25},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)}$ are fourth augmentation coefficients for category 1,2, and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)}$ are fifth augmentation coefficients for category 1,2,and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)}$ are sixth augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)}$ are seventh augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)}$ are eighth augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \begin{bmatrix} (b'_{28})^{(5)}\boxed{-(b''_{28})^{(5)}(G_{31},t)} & \boxed{-(b''_{24})^{(4,4,)}(G_{27},t)} & \boxed{-(b''_{32})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{28} \quad 82$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b'_{29})^{(5)}\boxed{-(b''_{29})^{(5)}(G_{31},t)} & \boxed{-(b''_{25})^{(4,4,)}(G_{27},t)} & \boxed{-(b''_{33})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{29} \quad 83$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b'_{30})^{(5)}\boxed{-(b''_{30})^{(5)}(G_{31},t)} & \boxed{-(b''_{26})^{(4,4,)}(G_{27},t)} & \boxed{-(b''_{34})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{30} \quad 84$$

where $-(b''_{28})^{(5)}(G_{31},t)$ , $-(b''_{29})^{(5)}(G_{31},t)$ , $-(b''_{30})^{(5)}(G_{31},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,)}(G_{27},t)$ are second detrition coefficients for category 1,2 and 3

$-(b''_{32})^{(6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6)}(G_{35},t)$ are third detrition coefficients for category 1,2 and 3

$-(b''_{13})^{(1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1)}(G,t)$ , $-(b''_{15})^{(1,1,1,1,1,)}(G,t)$ are fourth detrition coefficients for category 1,2, and 3

$-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1,2, and 3

$-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1,2, and 3

$-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2, and 3

$-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1,2, and 3

$-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[\begin{array}{c} (a'_{32})^{(6)}\left[+(a''_{32})^{(6)}(T_{33},t)\right]\left[+(a''_{28})^{(5,5,5)}(T_{29},t)\right]\left[+(a''_{24})^{(4,4,4,)}(T_{25},t)\right] \\ \left[+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)\right]\left[+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)\right]\left[+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)\right] \\ \left[+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)\right]\left[+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)\right]\left[+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)\right] \end{array}\right] G_{32} \quad (85)$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[\begin{array}{c} (a'_{33})^{(6)}\left[+(a''_{33})^{(6)}(T_{33},t)\right]\left[+(a''_{29})^{(5,5,5)}(T_{29},t)\right]\left[+(a''_{25})^{(4,4,4,)}(T_{25},t)\right] \\ \left[+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)\right]\left[+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)\right]\left[+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)\right] \\ \left[+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)\right]\left[+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)\right]\left[+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)\right] \end{array}\right] G_{33} \quad (86)$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[\begin{array}{c} (a'_{34})^{(6)}\left[+(a''_{34})^{(6)}(T_{33},t)\right]\left[+(a''_{30})^{(5,5,5)}(T_{29},t)\right]\left[+(a''_{26})^{(4,4,4,)}(T_{25},t)\right] \\ \left[+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)\right]\left[+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)\right]\left[+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)\right] \\ \left[+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)\right]\left[+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)\right]\left[+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)\right] \end{array}\right] G_{34} \quad (87)$$

$+(a''_{32})^{(6)}(T_{33},t)$, $+(a''_{33})^{(6)}(T_{33},t)$, $+(a''_{34})^{(6)}(T_{33},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{28})^{(5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5)}(T_{29},t)$ are second augmentation coefficients for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,)}(T_{25},t)$ are third augmentation coefficients for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)$ - are fourth augmentation coefficients

$+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)$ - fifth augmentation

coefficients
$+(a_{20}'')^{(3,3,3,3,3,3)}(T_{21},t)$, $+(a_{21}'')^{(3,3,3,3,3,3)}(T_{21},t)$, $+(a_{22}'')^{(3,3,3,3,3,3)}(T_{21},t)$ sixth augmentation coefficients
$+(a_{36}'')^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a_{37}'')^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a_{38}'')^{(7,7,7,7,7,7,7)}(T_{37},t)$
seventh augmentation coefficients
$+(a_{40}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a_{41}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a_{42}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t)$
Eighth augmentation coefficients
$+(a_{44}'')^{(9,9,9,9,9,9,9)}(T_{45},t)$, $+(a_{45}'')^{(9,9,9,9,9,9,9)}(T_{45},t)$, $+(a_{46}'')^{(9,9,9,9,9,9,9)}(T_{45},t)$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \begin{bmatrix} (b_{32}')^{(6)} & -(b_{32}'')^{(6)}(G_{35},t) & -(b_{28}'')^{(5,5,5)}(G_{31},t) & -(b_{24}'')^{(4,4,4,)}(G_{27},t) \\ -(b_{13}'')^{(1,1,1,1,1,1)}(G,t) & -(b_{16}'')^{(2,2,2,2,2,2)}(G_{19},t) & -(b_{20}'')^{(3,3,3,3,3,3)}(G_{23},t) & \\ -(b_{36}'')^{(7,7,7,7,7,7,7)}(G_{39},t) & -(b_{40}'')^{(8,8,8,8,8,8,8)}(G_{43},t) & -(b_{44}'')^{(9,9,9,9,9,9)}(G_{47},t) & \end{bmatrix} T_{32} \quad 88$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \begin{bmatrix} (b_{33}')^{(6)} & -(b_{33}'')^{(6)}(G_{35},t) & -(b_{29}'')^{(5,5,5)}(G_{31},t) & -(b_{25}'')^{(4,4,4,)}(G_{27},t) \\ -(b_{14}'')^{(1,1,1,1,1,1)}(G,t) & -(b_{17}'')^{(2,2,2,2,2,2)}(G_{19},t) & -(b_{21}'')^{(3,3,3,3,3,3)}(G_{23},t) & \\ -(b_{37}'')^{(7,7,7,7,7,7,7)}(G_{39},t) & -(b_{41}'')^{(8,8,8,8,8,8,8)}(G_{43},t) & -(b_{45}'')^{(9,9,9,9,9,9)}(G_{47},t) & \end{bmatrix} T_{33} \quad 89$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b_{34}')^{(6)} & -(b_{34}'')^{(6)}(G_{35},t) & -(b_{30}'')^{(5,5,5)}(G_{31},t) & -(b_{26}'')^{(4,4,4,)}(G_{27},t) \\ -(b_{15}'')^{(1,1,1,1,1,1)}(G,t) & -(b_{18}'')^{(2,2,2,2,2,2)}(G_{19},t) & -(b_{22}'')^{(3,3,3,3,3,3)}(G_{23},t) & \\ -(b_{38}'')^{(7,7,7,7,7,7,7)}(G_{39},t) & -(b_{42}'')^{(8,8,8,8,8,8,8)}(G_{43},t) & -(b_{46}'')^{(9,9,9,9,9,9)}(G_{47},t) & \end{bmatrix} T_{34} \quad 90$$

$-(b_{32}'')^{(6)}(G_{35},t)$, $-(b_{33}'')^{(6)}(G_{35},t)$, $-(b_{34}'')^{(6)}(G_{35},t)$ *are first detrition coefficients for category* 1, 2 *and* 3
$-(b_{28}'')^{(5,5,5)}(G_{31},t)$, $-(b_{29}'')^{(5,5,5)}(G_{31},t)$, $-(b_{30}'')^{(5,5,5)}(G_{31},t)$ *are second detrition coefficients for category* 1, 2 *and* 3
$-(b_{24}'')^{(4,4,4,)}(G_{27},t)$, $-(b_{25}'')^{(4,4,4,)}(G_{27},t)$, $-(b_{26}'')^{(4,4,4,)}(G_{27},t)$ *are third detrition coefficients for category* 1,2 *and* 3
$-(b_{13}'')^{(1,1,1,1,1,1)}(G,t)$, $-(b_{14}'')^{(1,1,1,1,1,1)}(G,t)$, $-(b_{15}'')^{(1,1,1,1,1,1)}(G,t)$ are fourth detrition coefficients for category 1, 2, and 3
$-(b_{16}'')^{(2,2,2,2,2,2)}(G_{19},t)$, $-(b_{17}'')^{(2,2,2,2,2,2)}(G_{19},t)$, $-(b_{18}'')^{(2,2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1, 2, and 3
$-(b_{20}'')^{(3,3,3,3,3,3)}(G_{23},t)$, $-(b_{21}'')^{(3,3,3,3,3,3)}(G_{23},t)$, $-(b_{22}'')^{(3,3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1, 2, and 3
$-(b_{36}'')^{(7,7,7,7,7,7,7)}(G_{39},t)$, $-(b_{37}'')^{(7,7,7,7,7,7,7)}(G_{39},t)$, $-(b_{38}'')^{(7,7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2, and 3
$-(b_{40}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)$, $-(b_{41}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)$, $-(b_{42}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)$
are eighth detrition coefficients for category 1, 2, and 3
$-(b_{46}'')^{(9,9,9,9,9,9)}(G_{47},t)$, $-(b_{45}'')^{(9,9,9,9,9,9)}(G_{47},t)$, $-(b_{44}'')^{(9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \quad 91$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a'_{36})^{(7)}\boxed{+(a''_{36})^{(7)}(T_{37},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \quad 92$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a'_{37})^{(7)}\boxed{+(a''_{37})^{(7)}(T_{37},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{38}}{dt} \quad 93$$

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a'_{38})^{(7)}\boxed{+(a''_{38})^{(7)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{(a''_{36})^{(7)}(T_{37},t)}$, $\boxed{(a''_{37})^{(7)}(T_{37},t)}$, $\boxed{(a''_{38})^{(7)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{36}}{dt} = \quad 94$$

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b'_{36})^{(7)}\boxed{-(b''_{36})^{(7)}(G_{39},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{37}}{dt} =$$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b'_{37})^{(7)}\boxed{-(b''_{37})^{(7)}(G_{39},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{38}}{dt} =$$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b'_{38})^{(7)}\boxed{-(b''_{38})^{(7)}(G_{39},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{40}}{dt}$$



$$= (a_{40})^{(8)}G_{41} - \begin{bmatrix} (a'_{40})^{(8)}\boxed{+(a''_{40})^{(8)}(T_{41},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{41}}{dt}$$

$$= (a_{41})^{(8)}G_{40} - \begin{bmatrix} (a'_{41})^{(8)}\boxed{+(a''_{41})^{(8)}(T_{41},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \begin{bmatrix} (a'_{42})^{(8)}\boxed{+(a''_{42})^{(8)}(T_{41},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a''_{40})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)}\boxed{-(b''_{40})^{(8)}(G_{43},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)}\boxed{-(b''_{41})^{(8)}(G_{43},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)}\boxed{-(b''_{42})^{(8)}(G_{43},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{44}}{dt} \qquad 96$$

$$= (a_{44})^{(9)}G_{45} - \left[\begin{array}{ccc} (a'_{44})^{(9)}\boxed{+(a''_{44})^{(9)}(T_{45},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{array}\right] G_{13}$$

$$\frac{dG_{45}}{dt}$$

$$= (a_{45})^{(9)}G_{44} - \left[\begin{array}{ccc} (a'_{45})^{(9)}\boxed{+(a''_{45})^{(9)}(T_{45},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{array}\right] G_{14}$$

$$\frac{dG_{46}}{dt}$$

$$= (a_{46})^{(9)}G_{45} - \left[\begin{array}{ccc} (a'_{46})^{(9)}\boxed{+(a''_{46})^{(9)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{array}\right] G_{15}$$

Where $\boxed{+(a''_{44})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$+(a_{16}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)$ , $+(a_{17}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)$ , $+(a_{18}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a_{20}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)$ , $+(a_{21}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)$ , $+(a_{22}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a_{24}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)$ , $+(a_{25}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)$ , $+(a_{26}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a_{28}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)$, $+(a_{29}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)$ , $+(a_{30}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a_{32}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a_{33}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$ , $+(a_{34}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a_{13}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$ , $+(a_{14}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$ , $+(a_{15}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$ are Seventh augmentation coefficient for category 1, 2 and 3

$+(a_{38}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ $+(a_{37}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ $+(a_{36}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ are eighth augmentation coefficient for 1,2,3

$+(a_{40}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)$ ,$+(a_{42}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a_{41}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} =$$

$$(b_{44})^{(9)}T_{45} -$$

$$\begin{bmatrix} (b_{44}')^{(9)}\boxed{-(b_{44}'')^{(9)}(G_{47},t)} & \boxed{-(b_{16}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{20}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{24}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{28}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{32}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{13}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{36}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b_{45}')^{(9)}\boxed{-(b_{45}'')^{(9)}(G_{47},t)} & \boxed{-(b_{17}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{21}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{25}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{29}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{33}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{14}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{37}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b_{46}')^{(9)}\boxed{-(b_{46}'')^{(9)}(G_{47},t)} & \boxed{-(b_{18}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{22}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{26}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{30}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{34}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{15}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{38}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{15}$$

Where $-(b_{44}'')^{(9)}(G_{47},t)$ , $-(b_{45}'')^{(9)}(G_{47},t)$ , $-(b_{46}'')^{(9)}(G_{47},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b_{16}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)$ , $-(b_{17}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)$ , $-(b_{18}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b_{20}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)$ , $-(b_{21}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)$ , $-(b_{22}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{24}'')^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{25}'')^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{26}'')^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{28}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{32}'')^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{34}'')^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{37}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{36}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a_i')^{(1)}, (a_i'')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (b_i'')^{(1)} > 0, \quad i,j = 13,14,15 \qquad 97$$

The functions $(a_i'')^{(1)}, (b_i'')^{(1)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a_i'')^{(1)}(T_{14},t) \le (p_i)^{(1)} \le (\hat{A}_{13})^{(1)}$$

$$(b_i'')^{(1)}(G,t) \le (r_i)^{(1)} \le (b_i')^{(1)} \le (\hat{B}_{13})^{(1)}$$

$$lim_{T_2\to\infty}(a_i'')^{(1)}(T_{14},t) = (p_i)^{(1)} \qquad 98$$

$$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = (r_i)^{(1)}$$

**Definition of** $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$ :

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$$|(a_i'')^{(1)}(T_{14}',t) - (a_i'')^{(1)}(T_{14},t)| \le (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G',t) - (b_i'')^{(1)}(G,t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}',t)$ and $(a_i'')^{(1)}(T_{14},t)$ . $(T_{14}',t)$ and $(T_{14},t)$ are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14},t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$ : 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$

**Definition of** $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$ : 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}$, $(r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)} (\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)} (\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i, j = 16,17,18$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17}, t) \le (p_i)^{(2)} \le (\hat{A}_{16})^{(2)} \quad 102$$

$$(b_i'')^{(2)}(G_{19}, t) \le (r_i)^{(2)} \le (b_i')^{(2)} \le (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2 \to \infty} (a_i'')^{(2)}(T_{17}, t) = (p_i)^{(2)} \quad 104$$

$$\lim_{G \to \infty} (b_i'')^{(2)}((G_{19}), t) = (r_i)^{(2)} \quad 105$$

**Definition of** $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$ : 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}', t) - (a_i'')^{(2)}(T_{17}, t)| \le (\hat{k}_{16})^{(2)} |T_{17} - T_{17}'| e^{-(\hat{M}_{16})^{(2)} t} \quad 107$$

$$|(b_i'')^{(2)}((G_{19})', t) - (b_i'')^{(2)}((G_{19}), t)| < (\hat{k}_{16})^{(2)} ||(G_{19}) - (G_{19})'|| e^{-(\hat{M}_{16})^{(2)} t} \quad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}', t)$ and$(a_i'')^{(2)}(T_{17}, t)$ . $(T_{17}', t)$ and $(T_{17}, t)$ are points belonging to the interval $[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}]$ . It is to be noted that $(a_i'')^{(2)}(T_{17}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17}, t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$ :

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)},$ are positive constants 109

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$$

**Definition of** $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$ :

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and\ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1 \quad 110$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(b_i)^{(2)} + (b_i')^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1 \quad 111$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i, j = 20,21,22 \quad 112$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(3)}$, $(r_i)^{(3)}$:

$$(a_i'')^{(3)}(T_{21}, t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23}, t) \leq (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(3)}(T_{21}, t) = (p_i)^{(3)} \quad 113$$

$$\lim_{G \to \infty}(b_i'')^{(3)}(G_{23}, t) = (r_i)^{(3)}$$

**Definition of** $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$ :

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: 114

$$|(a_i'')^{(3)}(T_{21}', t) - (a_i'')^{(3)}(T_{21}, t)| \leq (\hat{k}_{20})^{(3)} |T_{21} - T_{21}'| e^{-(\hat{M}_{20})^{(3)} t}$$

$$|(b_i'')^{(3)}(G_{23}', t) - (b_i'')^{(3)}(G_{23}, t)| < (\hat{k}_{20})^{(3)} ||G_{23} - G_{23}'|| e^{-(\hat{M}_{20})^{(3)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}', t)$ and$(a_i'')^{(3)}(T_{21}, t)$ . $(T_{21}', t)$ And $(T_{21}, t)$ are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21}, t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$ : 115

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}$ , $\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with 116
$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$ and the constants
$(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}, i = 20,21,22,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(a_i)^{(3)} + (a_i')^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)} (\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(b_i)^{(3)} + (b_i')^{(3)} + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)} (\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i, j = 24,25,26$$ 117

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(4)}$, $(r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25}, t) \le (p_i)^{(4)} \le (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}((G_{27}), t) \le (r_i)^{(4)} \le (b_i')^{(4)} \le (\hat{B}_{24})^{(4)}$$

$$lim_{T_2 \to \infty} (a_i'')^{(4)}(T_{25}, t) = (p_i)^{(4)}$$ 118

$$\lim_{G \to \infty} (b_i'')^{(4)}((G_{27}), t) = (r_i)^{(4)}$$

**Definition of** $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$ :

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24,25,26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}', t) - (a_i'')^{(4)}(T_{25}, t)| \le (\hat{k}_{24})^{(4)} |T_{25} - T_{25}'| e^{-(\hat{M}_{24})^{(4)} t}$$

$$|(b_i'')^{(4)}((G_{27})', t) - (b_i'')^{(4)}((G_{27}), t)| < (\hat{k}_{24})^{(4)} ||(G_{27}) - (G_{27})'|| e^{-(\hat{M}_{24})^{(4)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}', t)$ and $(a_i'')^{(4)}(T_{25}, t)$ . $(T_{25}', t)$ and $(T_{25}, t)$ are points belonging to the interval $\left[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}\right]$. It is to be noted that $(a_i'')^{(4)}(T_{25}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25}, t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$ : 120

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)},$ are positive constants

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}, \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$$

**Definition of** $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$ : 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and\ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}, i = 24,25,26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(a_i)^{(4)} + (a_i')^{(4)} + (\hat{A}_{24})^{(4)} + (\hat{P}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(b_i)^{(4)} + (b_i')^{(4)} + (\hat{B}_{24})^{(4)} + (\hat{Q}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i, j = 28,29,30$$ 122

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(5)}$, $(r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29}, t) \le (p_i)^{(5)} \le (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}((G_{31}), t) \le (r_i)^{(5)} \le (b_i')^{(5)} \le (\hat{B}_{28})^{(5)}$$

$$lim_{T_2 \to \infty} (a_i'')^{(5)}(T_{29}, t) = (p_i)^{(5)}$$ 123

$$\lim_{G \to \infty} (b_i'')^{(5)}(G_{31}, t) = (r_i)^{(5)}$$

**Definition of** $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$ :

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}', t) - (a_i'')^{(5)}(T_{29}, t)| \le (\hat{k}_{28})^{(5)} |T_{29} - T_{29}'| e^{-(\hat{M}_{28})^{(5)} t}$$

$$|(b_i'')^{(5)}((G_{31})', t) - (b_i'')^{(5)}((G_{31}), t)| < (\hat{k}_{28})^{(5)} ||(G_{31}) - (G_{31})'|| e^{-(\hat{M}_{28})^{(5)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}', t)$ and $(a_i'')^{(5)}(T_{29}, t)$ . $(T_{29}', t)$ and $(T_{29}, t)$ are points belonging to the interval $\left[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\right]$. It is to be noted that $(a_i'')^{(5)}(T_{29}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$ : 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)},$ are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}, \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

**Definition of** $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$ : 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and\ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(a_i)^{(5)} + (a_i')^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(b_i)^{(5)} + (b_i')^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i, j = 32,33,34$$ 127

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(6)}$, $(r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33}, t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}), t) \leq (r_i)^{(6)} \leq (b_i')^{(6)} \leq (\hat{B}_{32})^{(6)}$$

$$lim_{T_2 \to \infty} (a_i'')^{(6)} (T_{33}, t) = (p_i)^{(6)}$$ 128

$$\lim_{G \to \infty} (b_i'')^{(6)} \left((G_{35}), t\right) = (r_i)^{(6)}$$

**Definition of** $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$ :

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}', t) - (a_i'')^{(6)}(T_{33}, t)| \leq (\hat{k}_{32})^{(6)} |T_{33} - T_{33}'| e^{-(\hat{M}_{32})^{(6)} t}$$

$$|(b_i'')^{(6)}((G_{35})', t) - (b_i'')^{(6)}\left((G_{35}), t\right)| < (\hat{k}_{32})^{(6)} ||(G_{35}) - (G_{35})'|| e^{-(\hat{M}_{32})^{(6)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}', t)$ and $(a_i'')^{(6)}(T_{33}, t)$ . $(T_{33}', t)$ and $(T_{33}, t)$ are points belonging to the interval $\left[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}\right]$. It is to be noted that $(a_i'')^{(6)}(T_{33}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$ : 129

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)},$ are positive constants

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$$

**Definition of** $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$ : 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and\ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32,33,34,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(a_i)^{(6)} + (a_i')^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)} (\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(b_i)^{(6)} + (b_i')^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)} (\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

(IIIIII) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i, j = 36,37,38$ 131

(JJJJJJ) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(7)}$, $(r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37}, t) \leq (p_i)^{(7)} \leq (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39}, t) \leq (r_i)^{(7)} \leq (b_i')^{(7)} \leq (\hat{B}_{36})^{(7)}$$

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*(KKKKKK)* $\lim_{T_2 \to \infty} (a_i'')^{(7)}(T_{37}, t) = (p_i)^{(7)}$

*(LLLLLL)*

$\lim_{G \to \infty} (b_i'')^{(7)}((G_{39}), t) = (r_i)^{(7)}$

**Definition of** $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$ :

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$$|(a_i'')^{(7)}(T_{37}', t) - (a_i'')^{(7)}(T_{37}, t)| \leq (\hat{k}_{36})^{(7)} |T_{37} - T_{37}'| e^{-(\hat{M}_{36})^{(7)} t}$$

$$|(b_i'')^{(7)}((G_{39})', t) - (b_i'')^{(7)}((G_{39}), t)| < (\hat{k}_{36})^{(7)} ||(G_{39}) - (G_{39})'|| e^{-(\hat{M}_{36})^{(7)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}', t)$ and $(a_i'')^{(7)}(T_{37}, t)$ . $(T_{37}', t)$ and $(T_{37}, t)$ are points belonging to the interval $[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}]$ . It is to be noted that $(a_i'')^{(7)}(T_{37}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37}, t)$ , the **first augmentation coefficient** attributable to the

system, would be absolutely continuous.

**Definition of** $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$ : 134

(MMMMMM) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)},$ are positive constants

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$$

**Definition of** $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$ : 135

(NNNNNN) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(b_i)^{(7)} + (b_i')^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40,41,42 \qquad 136$$

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

**Definition of** $(p_i)^{(8)}, (r_i)^{(8)}$: 137

$$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)} \qquad 138$$

$$(b_i'')^{(8)}((G_{43}), t) \leq (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)} \qquad 139$$

$$\lim_{T_2 \to \infty} (a_i'')^{(8)}(T_{41}, t) = (p_i)^{(8)} \qquad 140$$

$$\lim_{G \to \infty} (b_i'')^{(8)}((G_{43}), t) = (r_i)^{(8)} \qquad 141$$

**Definition of** $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$ :

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)} |T_{41} - T_{41}'| e^{-(\hat{M}_{40})^{(8)} t} \qquad 142$$

$$|(b_i'')^{(8)}((G_{43})', t) - (b_i'')^{(8)}((G_{43}), t)| < (\hat{k}_{40})^{(8)} ||(G_{43}) - (G_{43})'|| e^{-(\hat{M}_{40})^{(8)} t} \qquad 143$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}', t)$ and $(a_i'')^{(8)}(T_{41}, t)$ . $(T_{41}', t)$ and $(T_{41}, t)$ are points belonging to the interval $\left[(\hat{k}_{40})^{(8)}, (\widehat{M}_{40})^{(8)}\right]$. It is to be noted that $(a_i'')^{(8)}(T_{41}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{40})^{(8)} = 1$ then the function $(a_i'')^{(8)}(T_{41}, t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\widehat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$ :

$(\widehat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)},$ are positive constants

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}, \frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}} < 1 \qquad 144$$

**Definition of** $(\hat{P}_{40})^{(8)}, (\hat{Q}_{40})^{(8)}$ :

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\widehat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}, (\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)}, (a_i')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}, i = 40,41,42,$
Satisfy the inequalities

$$\frac{1}{(\widehat{M}_{40})^{(8)}}[(a_i)^{(8)} + (a_i')^{(8)} + (\hat{A}_{40})^{(8)} + (\hat{P}_{40})^{(8)} (\hat{k}_{40})^{(8)}] < 1 \qquad 145$$

$$\frac{1}{(\widehat{M}_{40})^{(8)}}[(b_i)^{(8)} + (b_i')^{(8)} + (\hat{B}_{40})^{(8)} + (\hat{Q}_{40})^{(8)} (\hat{k}_{40})^{(8)}] < 1 \qquad 146$$

Where we suppose

$$(a_i)^{(9)}, (a_i')^{(9)}, (a_i'')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (b_i'')^{(9)} > 0, \quad i, j = 44,45,46 \qquad 146\text{A}$$

The functions $(a_i'')^{(9)}, (b_i'')^{(9)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(9)}$, $(r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45}, t) \leq (p_i)^{(9)} \leq (\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47}, t) \leq (r_i)^{(9)} \leq (b_i')^{(9)} \leq (\hat{B}_{44})^{(9)}$$

$$\lim_{T_2 \to \infty} (a_i'')^{(9)}(T_{45}, t) = (p_i)^{(9)}$$

$$\lim_{G \to \infty} (b_i'')^{(9)}(G_{47}, t) = (r_i)^{(9)}$$

**Definition of** $(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}$ :

Where $\boxed{(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}}$ are positive constants and $\boxed{i = 44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}', t) - (a_i'')^{(9)}(T_{45}, t)| \leq (\hat{k}_{44})^{(9)} |T_{45} - T_{45}'| e^{-(\widehat{M}_{44})^{(9)} t}$$

$$|(b_i'')^{(9)}((G_{47})', t) - (b_i'')^{(9)}\big((G_{47}), t\big)| < (\hat{k}_{44})^{(9)}||(G_{47}) - (G_{47})'||e^{-(\hat{M}_{44})^{(9)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}', t)$ and$(a_i'')^{(9)}(T_{45}, t)$ . $(T_{45}', t)$ and $(T_{45}, t)$ are points belonging to the interval $\big[(\hat{k}_{44})^{(9)}, (\hat{M}_{44})^{(9)}\big]$. It is to be noted that $(a_i'')^{(9)}(T_{45}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{44})^{(9)} = 1$ then the function $(a_i'')^{(9)}(T_{45}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$ :

$(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$, are positive constants

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}} \ , \frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$$

**Definition of** $(\hat{P}_{44})^{(9)}, (\hat{Q}_{44})^{(9)}$ :

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}, (\hat{A}_{44})^{(9)} and\ (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)}, (a_i')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (p_i)^{(9)}, \ (r_i)^{(9)}, i = 44,45,46,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{44})^{(9)}}[\,(a_i)^{(9)} + (a_i')^{(9)} + \ (\hat{A}_{44})^{(9)} + \ (\hat{P}_{44})^{(9)}\,(\hat{k}_{44})^{(9)}] < 1$$

$$\frac{1}{(\hat{M}_{44})^{(9)}}[\ (b_i)^{(9)} + (b_i')^{(9)} + \ (\hat{B}_{44})^{(9)} + \ (\hat{Q}_{44})^{(9)}\ (\hat{k}_{44})^{(9)}] < 1$$

**Theorem 1:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \big(\hat{P}_{13}\big)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \ , \ \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 2 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

**Definition of** $G_i(0)\,, T_i(0)$

$$G_i(t) \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} \quad , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} \quad , \quad T_i(0) = T_i^0 > 0$$

**Theorem 3 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t) \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t} \quad , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t} \quad , \quad T_i(0) = T_i^0 > 0$$

**Theorem 4 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

**Definition of** $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \left(\hat{P}_{24}\right)^{(4)} e^{(\hat{M}_{24})^{(4)}t} \;,\; \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{24}\right)^{(4)} e^{(\hat{M}_{24})^{(4)}t} \;,\; \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 5 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

**Definition of** $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \left(\hat{P}_{28}\right)^{(5)} e^{(\hat{M}_{28})^{(5)}t} \;,\; \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{28}\right)^{(5)} e^{(\hat{M}_{28})^{(5)}t} \;,\; \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 6 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

**Definition of** $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \left(\hat{P}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t} \;,\; \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t} \;,\; \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 7:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

**Definition of** $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \left(\hat{P}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t} \;,\; \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t} \;,\; \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 8:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

**Definition of** $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \left(\hat{P}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t} \;,\; \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t} \;,\; \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 9:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

**Definition of** $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \left(\hat{P}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t} \;,\; \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

**Proof:** Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \leq (\hat{P}_{13})^{(1)} \text{ , } T_i^0 \leq (\hat{Q}_{13})^{(1)}, \quad 155$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t} \quad 156$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t} \quad 157$$

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[ (a_{13})^{(1)} G_{14}(s_{(13)}) - \left( (a'_{13})^{(1)} + a''_{13})^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \right) G_{13}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[ (a_{14})^{(1)} G_{13}(s_{(13)}) - \left( (a'_{14})^{(1)} + (a''_{14})^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \right) G_{14}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[ (a_{15})^{(1)} G_{14}(s_{(13)}) - \left( (a'_{15})^{(1)} + (a''_{15})^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \right) G_{15}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[ (b_{13})^{(1)} T_{14}(s_{(13)}) - \left( (b'_{13})^{(1)} - (b''_{13})^{(1)} (G(s_{(13)}), s_{(13)}) \right) T_{13}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[ (b_{14})^{(1)} T_{13}(s_{(13)}) - \left( (b'_{14})^{(1)} - (b''_{14})^{(1)} (G(s_{(13)}), s_{(13)}) \right) T_{14}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{T}_{15}(t) = T_{15}^0 + \int_0^t \left[ (b_{15})^{(1)} T_{14}(s_{(13)}) - \left( (b'_{15})^{(1)} - (b''_{15})^{(1)} (G(s_{(13)}), s_{(13)}) \right) T_{15}(s_{(13)}) \right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \leq (\hat{P}_{16})^{(2)} \text{ , } T_i^0 \leq (\hat{Q}_{16})^{(2)},$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$$

By 160

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[ (a_{16})^{(2)} G_{17}(s_{(16)}) - \left( (a'_{16})^{(2)} + a''_{16})^{(2)} (T_{17}(s_{(16)}), s_{(16)}) \right) G_{16}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[ (a_{17})^{(2)} G_{16}(s_{(16)}) - \left( (a'_{17})^{(2)} + (a''_{17})^{(2)} (T_{17}(s_{(16)}), s_{(17)}) \right) G_{17}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[ (a_{18})^{(2)} G_{17}(s_{(16)}) - \left( (a'_{18})^{(2)} + (a''_{18})^{(2)} (T_{17}(s_{(16)}), s_{(16)}) \right) G_{18}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^{0} + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^{0} + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^{0} + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{18}(s_{(16)})\right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{20})^{(3)}$, $T_i^0 \le (\hat{Q}_{20})^{(3)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\tilde{M}_{20})^{(3)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\tilde{M}_{20})^{(3)} t}$

By 161

$$\bar{G}_{20}(t) = G_{20}^{0} + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''_{20})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^{0} + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^{0} + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{22}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^{0} + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^{0} + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{22}(t) = T_{22}^{0} + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{22}(s_{(20)})\right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{24})^{(4)}$, $T_i^0 \le (\hat{Q}_{24})^{(4)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\tilde{M}_{24})^{(4)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\tilde{M}_{24})^{(4)} t}$

By 162

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{26}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{26}(t) = T_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{26}(s_{(24)})\right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{28})^{(5)}, T_i^0 \le (\hat{Q}_{28})^{(5)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

By 163

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{30}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{30}(t) = T_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{30}(s_{(28)})\right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{32})^{(6)}, T_i^0 \le (\hat{Q}_{32})^{(6)},$$

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\tilde{M}_{32})^{(6)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\tilde{M}_{32})^{(6)} t}$

By 164

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[ (a_{32})^{(6)} G_{33}(s_{(32)}) - \left( (a'_{32})^{(6)} + a''_{32})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[ (a_{33})^{(6)} G_{32}(s_{(32)}) - \left( (a'_{33})^{(6)} + (a''_{33})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[ (a_{34})^{(6)} G_{33}(s_{(32)}) - \left( (a'_{34})^{(6)} + (a''_{34})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{34}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[ (b_{32})^{(6)} T_{33}(s_{(32)}) - \left( (b'_{32})^{(6)} - (b''_{32})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[ (b_{33})^{(6)} T_{32}(s_{(32)}) - \left( (b'_{33})^{(6)} - (b''_{33})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{\mathrm{T}}_{34}(\mathrm{t}) = \mathrm{T}_{34}^0 + \int_0^t \left[ (b_{34})^{(6)} T_{33}(s_{(32)}) - \left( (b'_{34})^{(6)} - (b''_{34})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{34}(s_{(32)}) \right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\tilde{M}_{36})^{(7)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\tilde{M}_{36})^{(7)} t}$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[ (a_{36})^{(7)} G_{37}(s_{(36)}) - \left( (a'_{36})^{(7)} + a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[ (a_{37})^{(7)} G_{36}(s_{(36)}) - \left( (a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[ (a_{38})^{(7)} G_{37}(s_{(36)}) - \left( (a'_{38})^{(7)} + (a''_{38})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{38}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[ (b_{36})^{(7)} T_{37}(s_{(36)}) - \left( (b'_{36})^{(7)} - (b''_{36})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[ (b_{37})^{(7)} T_{36}(s_{(36)}) - \left( (b'_{37})^{(7)} - (b''_{37})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\overline{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^{0} + \int_0^t \left[ (b_{38})^{(7)} T_{37}(s_{(36)}) - \left( (b'_{38})^{(7)} - (b''_{38})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{38}(s_{(36)}) \right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$ , $T_i(0) = T_i^0$ , $G_i^0 \le (\hat{P}_{40})^{(8)}$ , $T_i^0 \le (\hat{Q}_{40})^{(8)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[ (a_{40})^{(8)} G_{41}(s_{(40)}) - \left( (a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[ (a_{41})^{(8)} G_{40}(s_{(40)}) - \left( (a'_{41})^{(8)} + (a''_{41})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[ (a_{42})^{(8)} G_{41}(s_{(40)}) - \left( (a'_{42})^{(8)} + (a''_{42})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{42}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^0 + \int_0^t \left[ (b_{40})^{(8)} T_{41}(s_{(40)}) - \left( (b'_{40})^{(8)} - (b''_{40})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^0 + \int_0^t \left[ (b_{41})^{(8)} T_{40}(s_{(40)}) - \left( (b'_{41})^{(8)} - (b''_{41})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\overline{\mathrm{T}}_{42}(\mathrm{t}) = \mathrm{T}_{42}^{0} + \int_0^t \left[ (b_{42})^{(8)} T_{41}(s_{(40)}) - \left( (b'_{42})^{(8)} - (b''_{42})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{42}(s_{(40)}) \right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** 166
Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy A

$G_i(0) = G_i^0$ , $T_i(0) = T_i^0$ , $G_i^0 \le (\hat{P}_{44})^{(9)}$ , $T_i^0 \le (\hat{Q}_{44})^{(9)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[ (a_{44})^{(9)} G_{45}(s_{(44)}) - \left( (a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45}(s_{(44)}), s_{(44)})\right) G_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}(s_{(44)}), s_{(44)})\right) G_{46}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{44}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{\mathrm{T}}_{46}(\mathrm{t}) = \mathrm{T}_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{46}(s_{(44)})\right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}}\right)\right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left(e^{(\hat{M}_{13})^{(1)} t} - 1\right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\hat{M}_{13})^{(1)} t} \le \frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{P}_{13})^{(1)} + G_{14}^0\right) e^{\left(-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0}\right)} + (\hat{P}_{13})^{(1)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \le G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}}\right)\right] ds_{(16)} = \quad 169$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1\right)$$

From which it follows that 170

$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \le \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left[\left((\hat{P}_{16})^{(2)} + G_{17}^0\right) e^{\left(-\frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0}\right)} + (\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t) \le G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} \left(G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} s_{(20)}}\right)\right] ds_{(20)} =$$

$$\left(1 + (a_{20})^{(3)} t\right) G_{21}^0 + \frac{(a_{20})^{(3)} (\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left(e^{(\hat{M}_{20})^{(3)} t} - 1\right)$$

From which it follows that 172

$$(G_{20}(t)-G_{20}^{0})e^{-(\hat{M}_{20})^{(3)}t}\leq\frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left[((\hat{P}_{20})^{(3)}+G_{21}^{0})e^{\left(-\frac{(\hat{P}_{20})^{(3)}+G_{21}^{0}}{G_{21}^{0}}\right)}+(\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 173

$$G_{24}(t)\leq G_{24}^{0}+\int_{0}^{t}\left[(a_{24})^{(4)}\left(G_{25}^{0}+(\hat{P}_{24})^{(4)}e^{(\hat{M}_{24})^{(4)}s_{(24)}}\right)\right]ds_{(24)}=$$

$$\left(1+(a_{24})^{(4)}t\right)G_{25}^{0}+\frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left(e^{(\hat{M}_{24})^{(4)}t}-1\right)$$

From which it follows that 174

$$(G_{24}(t)-G_{24}^{0})e^{-(\hat{M}_{24})^{(4)}t}\leq\frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left[((\hat{P}_{24})^{(4)}+G_{25}^{0})e^{\left(-\frac{(\hat{P}_{24})^{(4)}+G_{25}^{0}}{G_{25}^{0}}\right)}+(\hat{P}_{24})^{(4)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t)\leq G_{28}^{0}+\int_{0}^{t}\left[(a_{28})^{(5)}\left(G_{29}^{0}+(\hat{P}_{28})^{(5)}e^{(\hat{M}_{28})^{(5)}s_{(28)}}\right)\right]ds_{(28)}=$$

$$\left(1+(a_{28})^{(5)}t\right)G_{29}^{0}+\frac{(a_{28})^{(5)}(\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left(e^{(\hat{M}_{28})^{(5)}t}-1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t}\leq\frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t)\leq G_{32}^{0}+\int_{0}^{t}\left[(a_{32})^{(6)}\left(G_{33}^{0}+(\hat{P}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\right)\right]ds_{(32)}=$$

$$\left(1+(a_{32})^{(6)}t\right)G_{33}^{0}+\frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left(e^{(\hat{M}_{32})^{(6)}t}-1\right)$$

From which it follows that 177

$$(G_{32}(t)-G_{32}^{0})e^{-(\hat{M}_{32})^{(6)}t}\leq\frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left[((\hat{P}_{32})^{(6)}+G_{33}^{0})e^{\left(-\frac{(\hat{P}_{32})^{(6)}+G_{33}^{0}}{G_{33}^{0}}\right)}+(\hat{P}_{32})^{(6)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}$ , $G_{26}, T_{24}, T_{25}, T_{26}$

(x) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t) \le G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} \left(G_{37}^0 + (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} s_{(36)}}\right)\right] ds_{(36)} =$$
$$\left(1 + (a_{36})^{(7)} t\right) G_{37}^0 + \frac{(a_{36})^{(7)} (\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left(e^{(\hat{M}_{36})^{(7)} t} - 1\right)$$

From which it follows that

$$(G_{36}(t) - G_{36}^0) e^{-(\hat{M}_{36})^{(7)} t} \le \frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left[((\hat{P}_{36})^{(7)} + G_{37}^0) e^{\left(-\frac{(\hat{P}_{36})^{(7)} + G_{37}^0}{G_{37}^0}\right)} + (\hat{P}_{36})^{(7)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t) \le G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} \left(G_{41}^0 + (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} s_{(40)}}\right)\right] ds_{(40)} = \quad 180$$
$$\left(1 + (a_{40})^{(8)} t\right) G_{41}^0 + \frac{(a_{40})^{(8)} (\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}} \left(e^{(\hat{M}_{40})^{(8)} t} - 1\right)$$

From which it follows that 181

$$(G_{40}(t) - G_{40}^0) e^{-(\hat{M}_{40})^{(8)} t} \le \frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}} \left[((\hat{P}_{40})^{(8)} + G_{41}^0) e^{\left(-\frac{(\hat{P}_{40})^{(8)} + G_{41}^0}{G_{41}^0}\right)} + (\hat{P}_{40})^{(8)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 8

Analogous inequalities hold also for $G_{41}$ , $G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t) \le G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} \left(G_{45}^0 + (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} s_{(44)}}\right)\right] ds_{(44)} =$$
$$\left(1 + (a_{44})^{(9)} t\right) G_{45}^0 + \frac{(a_{44})^{(9)} (\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left(e^{(\hat{M}_{44})^{(9)} t} - 1\right)$$

From which it follows that

$$(G_{44}(t) - G_{44}^0) e^{-(\hat{M}_{44})^{(9)} t} \le \frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left[((\hat{P}_{44})^{(9)} + G_{45}^0) e^{\left(-\frac{(\hat{P}_{44})^{(9)} + G_{45}^0}{G_{45}^0}\right)} + (\hat{P}_{44})^{(9)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 9

Analogous inequalities hold also for $G_{45}$ , $G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}$ , $\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$ and to choose 182

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[(\hat{P}_{13})^{(1)}+\left((\hat{P}_{13})^{(1)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{13})^{(1)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{13})^{(1)} \quad 183$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[\left((\hat{Q}_{13})^{(1)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{13})^{(1)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{13})^{(1)}\right]\leq(\hat{Q}_{13})^{(1)} \quad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)},T^{(1)}\right),\left(G^{(2)},T^{(2)}\right)\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{13})^{(1)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{13})^{(1)}t}\}$$

Indeed if we denote

**Definition of** $\tilde{G},\tilde{T}:\left(\tilde{G},\tilde{T}\right)=\mathcal{A}^{(1)}(G,T)$

It results

$$\left|\tilde{G}_{13}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{13})^{(1)}\left|G_{14}^{(1)}-G_{14}^{(2)}\right|e^{-(\hat{M}_{13})^{(1)}s_{(13)}}e^{(\hat{M}_{13})^{(1)}s_{(13)}}\,ds_{(13)}+$$

$$\int_0^t\{(a'_{13})^{(1)}\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\hat{M}_{13})^{(1)}s_{(13)}}e^{-(\hat{M}_{13})^{(1)}s_{(13)}}+$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\hat{M}_{13})^{(1)}s_{(13)}}e^{(\hat{M}_{13})^{(1)}s_{(13)}}+$$

$$G_{13}^{(2)}\left|(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)-(a''_{13})^{(1)}\left(T_{14}^{(2)},s_{(13)}\right)\right|\ e^{-(\hat{M}_{13})^{(1)}s_{(13)}}e^{(\hat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|G^{(1)}-G^{(2)}\right|e^{-(\hat{M}_{13})^{(1)}t}\leq \quad 186$$
$$\frac{1}{(\hat{M}_{13})^{(1)}}\left((a_{13})^{(1)}+(a'_{13})^{(1)}+(\hat{A}_{13})^{(1)}+(\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}\right)d\left(\left(G^{(1)},T^{(1)};\,G^{(2)},T^{(2)}\right)\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 1:** The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\hat{P}_{13})^{(1)}e^{(\hat{M}_{13})^{(1)}t}$ *and* $(\hat{Q}_{13})^{(1)}e^{(\hat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i = 13,14,15$ depend only on $T_{14}$ and respectively on $G(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any $t$ where $G_i(t) = 0\ and\ T_i(t) = 0$

From 19 to 24 it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(1)} - (a_i'')^{(1)}(T_{14}(s_{(13)}), s_{(13)})\} ds_{(13)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(1)} t)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2\ and\ \left((\widehat{M}_{13})^{(1)}\right)_3$ : 187

**Remark 3**: if $G_{13}$ is bounded, the same property have also $G_{14}\ and\ G_{15}$ . indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a_{14}')^{(1)} G_{14}$ and by integrating

$$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)} \left((\widehat{M}_{13})^{(1)}\right)_1 / (a_{14}')^{(1)}$$

In the same way , one can obtain

$$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)} \left((\widehat{M}_{13})^{(1)}\right)_2 / (a_{15}')^{(1)}$$

If $G_{14}\ or\ G_{15}$ is bounded, the same property follows for $G_{13}$ , $G_{15}$ and $G_{13}$ , $G_{14}$ respectively.

**Remark 4:** If $G_{13}$ *is* bounded, from below, the same property holds for $G_{14}\ and\ G_{15}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{14}$ is bounded from below. 188

**Remark 5:** If $T_{13}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}(G(t), t)) = (b_{14}')^{(1)}$ then $T_{14} \to \infty$. 189

**Definition of** $(m)^{(1)}$ and $\varepsilon_1$ :

Indeed let $t_1$ be so that for $t > t_1$

$$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t), t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right)$, $t = log\frac{2}{\varepsilon_1}$ By taking now $\varepsilon_1$ sufficiently small one sees that $T_{14}$ is unbounded.
The same property holds for $T_{15}$ if $\lim_{t\to\infty}(b_{15}'')^{(1)}(G(t), t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}$ , $\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}\ and\ (\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[(\widehat{P}_{16})^{(2)}+\left((\hat{P}_{16})^{(2)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{16})^{(2)}\right]\leq(\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

**Definition of** $\widetilde{G_{19}},\widetilde{T_{19}}$ : $\left(\widetilde{G_{19}},\widetilde{T_{19}}\right)=\mathcal{A}^{(2)}(G_{19},T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{16})^{(2)}\left|G_{17}^{(1)}-G_{17}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)}+$$

$$\int_0^t\{(a'_{16})^{(2)}\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$G_{16}^{(2)}\left|(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)-(a''_{16})^{(2)}\left(T_{17}^{(2)},s_{(16)}\right)\right|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0,t]$ 197

From the hypotheses it follows

$$\left|(G_{19})^{(1)}-(G_{19})^{(2)}\right|\mathrm{e}^{-(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}\leq$$
$$\frac{1}{(\widehat{\mathrm{M}}_{16})^{(2)}}\left((a_{16})^{(2)}+(a'_{16})^{(2)}+(\widehat{\mathrm{A}}_{16})^{(2)}+(\widehat{\mathrm{P}}_{16})^{(2)}(\hat{k}_{16})^{(2)}\right)\mathrm{d}\left(((G_{19})^{(1)},(T_{19})^{(1)};\ (G_{19})^{(2)},(T_{19})^{(2)})\right)$$

And analogous inequalities for $G_i$ and $T_i$. Taking into account the hypothesis the result follows 198

**Remark 6:** The fact that we supposed $(a''_{16})^{(2)}$ and $(b''_{16})^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{\mathrm{P}}_{16})^{(2)}\mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}$ and $(\widehat{\mathrm{Q}}_{16})^{(2)}\mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i=16,17,18$ depend only on $\mathrm{T}_{17}$ and respectively on

$(G_{19})$(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 7:** There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$ 200

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(2)} - (a_i'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})\} ds_{(16)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(2)}t)} > 0 \quad \text{for } t > 0$$

**Definition of** $\left((\widehat{M}_{16})^{(2)}\right)_1$, $\left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$: 201

**Remark 8:** if $G_{16}$ is bounded, the same property have also $G_{17}$ and $G_{18}$. indeed if

$G_{16} < (\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt} \leq \left((\widehat{M}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)} G_{17}$ and by integrating

$$G_{17} \leq \left((\widehat{M}_{16})^{(2)}\right)_2 = G_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1 / (a_{17}')^{(2)}$$

In the same way , one can obtain

$$G_{18} \leq \left((\widehat{M}_{16})^{(2)}\right)_3 = G_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2 / (a_{18}')^{(2)}$$

If $G_{17}$ or $G_{18}$ is bounded, the same property follows for $G_{16}$, $G_{18}$ and $G_{16}$, $G_{17}$ respectively.

**Remark 9:** If $G_{16}$ is bounded, from below, the same property holds for $G_{17}$ and $G_{18}$. The proof is analogous with the preceding one. An analogous property is true if $G_{17}$ is bounded from below. 202

**Remark 10:** If $T_{16}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t), t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$. 203

**Definition of** $(m)^{(2)}$ and $\varepsilon_2$ :

Indeed let $t_2$ be so that for $t > t_2$

$$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right)$, $t = \log\frac{2}{\varepsilon_2}$ By taking now $\varepsilon_2$ sufficiently small one sees that $T_{17}$ is unbounded. 205
The same property holds for $T_{18}$ if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t), t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[(\widehat{P}_{20})^{(3)}+\left((\hat{P}_{20})^{(3)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{20})^{(3)}\right]\leq(\hat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself 210

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)},(T_{23})^{(1)}),((G_{23})^{(2)},(T_{23})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

**Definition of** $\widetilde{G_{23}},\widetilde{T_{23}}:(\widetilde{(G_{23})},\widetilde{(T_{23})})=\mathcal{A}^{(3)}((G_{23}),(T_{23}))$

It results 213

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{20})^{(3)}\left|G_{21}^{(1)}-G_{21}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}ds_{(20)}+$$

$$\int_0^t\{(a'_{20})^{(3)}\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$G_{20}^{(2)}\left|(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)},s_{(20)}\right)\right|\ e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\}ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses it follows

$$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}t}\leq \quad 214$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+(a'_{20})^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\widehat{k}_{20})^{(3)}\right)d\left(((G_{23})^{(1)},(T_{23})^{(1)};\ (G_{23})^{(2)},(T_{23})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 11:** The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ *and* $(\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i=20,21,22$ depend only on $T_{21}$ and respectively on

$(G_{23})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 12:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 216

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(3)} - (a_i'')^{(3)}(T_{21}(s_{(20)}), s_{(20)})\} ds_{(20)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(3)} t)} > 0$ for t > 0

**Definition of** $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2\ and\ \left((\widehat{M}_{20})^{(3)}\right)_3$ : 217

**Remark 13:** if $G_{20}$ is bounded, the same property have also $G_{21}\ and\ G_{22}$ . indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a_{21}')^{(3)} G_{21}$ and by integrating

$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1 / (a_{21}')^{(3)}$

In the same way , one can obtain

$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2 / (a_{22}')^{(3)}$

If $G_{21}\ or\ G_{22}$ is bounded, the same property follows for $G_{20}$ , $G_{22}$ and $G_{20}$ , $G_{21}$ respectively.

**Remark 14:** If $G_{20}$ *is* bounded, from below, the same property holds for $G_{21}\ and\ G_{22}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{21}$ is bounded from below. 218

**Remark 15:** If $T_{20}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t), t\right)) = (b_{21}')^{(3)}$ then $T_{21} \to \infty$. 219

**Definition of** $(m)^{(3)}$ and $\varepsilon_3$ :

Indeed let $t_3$ be so that for $t > t_3$

$(b_{21})^{(3)} - (b_i'')^{(3)}\left((G_{23})(t), t\right) < \varepsilon_3, T_{20}\,(t) > (m)^{(3)}$

Then $\frac{dT_{21}}{dt} \geq (a_{21})^{(3)}(m)^{(3)} - \varepsilon_3 T_{21}$ which leads to 220

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t} = \frac{1}{2}$ it results

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_3}$ By taking now $\varepsilon_3$ sufficiently small one sees that $T_{21}$ is unbounded.
The same property holds for $T_{22}$ if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t), t\right) = (b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}$ , $\frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}} < 1$ and to choose 221

$(\widehat{P}_{24})^{(4)}$ and $(\widehat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[(\widehat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \tag{222}$$

$$\frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \tag{223}$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself (224)

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric (225)

$$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{24})^{(4)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{27}}),(\widetilde{T_{27}})$ : $\left((\widetilde{G_{27}}),(\widetilde{T_{27}})\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{24})^{(4)}\left|G_{25}^{(1)}-G_{25}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\,ds_{(24)}+$$

$$\int_0^t\{(a'_{24})^{(4)}\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$G_{24}^{(2)}\left|(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)-(a''_{24})^{(4)}\left(T_{25}^{(2)},s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\}ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}t}\leq \tag{226}$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)}+(a'_{24})^{(4)}+(\widehat{A}_{24})^{(4)}+(\widehat{P}_{24})^{(4)}(\hat{k}_{24})^{(4)}\right)d\left(((G_{27})^{(1)},(T_{27})^{(1)};\ (G_{27})^{(2)},(T_{27})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 16:** The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ *and* $(\widehat{Q}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_+$. (227)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i=24,25,26$ depend only on $T_{25}$ and respectively on

$(G_{27})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 17:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 228

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(4)} - (a_i'')^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(4)} t)} > 0$ for t > 0

**Definition of** $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2\ and\ \left((\widehat{M}_{24})^{(4)}\right)_3$ : 229

**Remark 18:** if $G_{24}$ is bounded, the same property have also $G_{25}\ and\ G_{26}$ . indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a_{25}')^{(4)} G_{25}$ and by integrating

$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1 / (a_{25}')^{(4)}$

In the same way , one can obtain

$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2 / (a_{26}')^{(4)}$

If $G_{25}\ or\ G_{26}$ is bounded, the same property follows for $G_{24}$ , $G_{26}$ and $G_{24}$ , $G_{25}$ respectively.

**Remark 19:** If $G_{24}$ *is* bounded, from below, the same property holds for $G_{25}\ and\ G_{26}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{25}$ is bounded from below. 230

**Remark 20:** If $T_{24}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}\ ((G_{27})(t), t)) = (b_{25}')^{(4)}$ then $T_{25} \to \infty$. 231

**Definition of** $(m)^{(4)}$ and $\varepsilon_4$ :

Indeed let $t_4$ be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t), t) < \varepsilon_4, T_{24}\,(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_4}$ By taking now $\varepsilon_4$ sufficiently small one sees that $T_{25}$ is unbounded.
The same property holds for $T_{26}$ if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t), t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for $G_{29}$ , $G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}$ , $\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)}+\left((\hat{P}_{28})^{(5)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{28})^{(5)}\right]\leq(\hat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$$d\left(((G_{31})^{(1)},(T_{31})^{(1)}),((G_{31})^{(2)},(T_{31})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\widehat{M}_{28})^{(5)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\widehat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{31}}),(\widetilde{T_{31}})$ : $\left((\widetilde{G_{31}}),(\widetilde{T_{31}})\right)=\mathcal{A}^{(5)}\left((G_{31}),(T_{31})\right)$

It results

$$|\tilde{G}_{28}^{(1)}-\tilde{G}_i^{(2)}|\leq\int_0^t(a_{28})^{(5)}|G_{29}^{(1)}-G_{29}^{(2)}|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\,ds_{(28)}+$$

$$\int_0^t\{(a'_{28})^{(5)}|G_{28}^{(1)}-G_{28}^{(2)}|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$(a''_{28})^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)|G_{28}^{(1)}-G_{28}^{(2)}|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$G_{28}^{(2)}|(a''_{28})^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)-(a''_{28})^{(5)}\left(T_{29}^{(2)},s_{(28)}\right)|\;e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\}ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on it follows

$$|(G_{31})^{(1)}-(G_{31})^{(2)}|e^{-(\widehat{M}_{28})^{(5)}t}\leq \quad 237$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)}+(a'_{28})^{(5)}+(\widehat{A}_{28})^{(5)}+(\hat{P}_{28})^{(5)}(\hat{k}_{28})^{(5)}\right)d\left(((G_{31})^{(1)},(T_{31})^{(1)};\;(G_{31})^{(2)},(T_{31})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 21:** The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\hat{P}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ *and* $(\hat{Q}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ respectively of $\mathbb{R}_+$. 238

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on $T_{29}$ and respectively on $(G_{31})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 22:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 239

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(5)} - (a_i'')^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(5)} t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2\ and\ \left((\widehat{M}_{28})^{(5)}\right)_3$ : 240

**Remark 23:** if $G_{28}$ is bounded, the same property have also $G_{29}\ and\ G_{30}$ . indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a_{29}')^{(5)} G_{29}$ and by integrating

$$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1 / (a_{29}')^{(5)}$$

In the same way , one can obtain

$$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2 / (a_{30}')^{(5)}$$

If $G_{29}\ or\ G_{30}$ is bounded, the same property follows for $G_{28}$ , $G_{30}$ and $G_{28}$ , $G_{29}$ respectively.

**Remark 24:** If $G_{28}$ *is* bounded, from below, the same property holds for $G_{29}\ and\ G_{30}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{29}$ is bounded from below. 241

**Remark 25:** If $T_{28}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}\,((G_{31})(t), t)) = (b_{29}')^{(5)}$ then $T_{29} \to \infty$. 242

**Definition of** $(m)^{(5)}$ and $\varepsilon_5$ :

Indeed let $t_5$ be so that for $t > t_5$

$$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t), t) < \varepsilon_5, T_{28}\,(t) > (m)^{(5)}$$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_5}$ By taking now $\varepsilon_5$ sufficiently small one sees that $T_{29}$ is unbounded.
The same property holds for $T_{30}$ if $\lim_{t\to\infty}(b_{30}'')^{(5)}\left((G_{31})(t), t\right) = (b_{30}')^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{33}$ , $G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}$ , $\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}} < 1$ and to choose 244

$(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[(\hat{P}_{32})^{(6)}+\left((\hat{P}_{32})^{(6)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{32})^{(6)} \tag{245}$$

$$\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[\left((\hat{Q}_{32})^{(6)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{32})^{(6)}\right]\leq(\hat{Q}_{32})^{(6)} \tag{246}$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric (247)

$$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\hat{M}_{32})^{(6)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\hat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{35}}),(\widetilde{T_{35}})$ : $\left((\widetilde{G_{35}}),(\widetilde{T_{35}})\right)=\mathcal{A}^{(6)}\left((G_{35}),(T_{35})\right)$

It results

$$|\tilde{G}_{32}^{(1)}-\tilde{G}_i^{(2)}|\leq\int_0^t(a_{32})^{(6)}|G_{33}^{(1)}-G_{33}^{(2)}|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}ds_{(32)}+$$

$$\int_0^t\{(a'_{32})^{(6)}|G_{32}^{(1)}-G_{32}^{(2)}|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)|G_{32}^{(1)}-G_{32}^{(2)}|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$G_{32}^{(2)}|(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)-(a''_{32})^{(6)}\left(T_{33}^{(2)},s_{(32)}\right)|\ e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\}ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$|(G_{35})^{(1)}-(G_{35})^{(2)}|e^{-(\widehat{M}_{32})^{(6)}t}\leq \tag{248}$$
$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)}+(a'_{32})^{(6)}+(\widehat{A}_{32})^{(6)}+(\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}\right)d\left(((G_{35})^{(1)},(T_{35})^{(1)};\ (G_{35})^{(2)},(T_{35})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 26:** The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\hat{P}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ *and* $(\hat{Q}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ respectively of $\mathbb{R}_+$. (249)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i = 32,33,34$ depend only on $T_{33}$ and respectively on $(G_{35})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 27:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 250

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(6)} - (a_i'')^{(6)}(T_{33}(s_{(32)}), s_{(32)})\} ds_{(32)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(6)} t\right)} > 0$ for t > 0

**Definition of** $\left((\widehat{M}_{32})^{(6)}\right)_1, \left((\widehat{M}_{32})^{(6)}\right)_2\ and\ \left((\widehat{M}_{32})^{(6)}\right)_3$ : 251

**Remark 28:** if $G_{32}$ is bounded, the same property have also $G_{33}\ and\ G_{34}$ . indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a_{33}')^{(6)} G_{33}$ and by integrating

$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_1 / (a_{33}')^{(6)}$

In the same way , one can obtain

$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_2 / (a_{34}')^{(6)}$

If $G_{33}\ or\ G_{34}$ is bounded, the same property follows for $G_{32}$ , $G_{34}$ and $G_{32}$ , $G_{33}$ respectively.

**Remark 29:** If $G_{32}$ *is* bounded, from below, the same property holds for $G_{33}\ and\ G_{34}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{33}$ is bounded from below. 252

**Remark 30:** If $T_{32}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}\ ((G_{35})(t), t)) = (b_{33}')^{(6)}$ then $T_{33}\ \to \infty$. 253

**Definition of** $(m)^{(6)}$ and $\varepsilon_6$ :

Indeed let $t_6$ be so that for $t\ > t_6$

$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t), t\right) < \varepsilon_6, T_{32}\,(t) > (m)^{(6)}$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33}\ \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33}\ \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_6}$ By taking now $\varepsilon_6$ sufficiently small one sees that $T_{33}$ is unbounded.
The same property holds for $T_{34}$ if $\lim_{t\to\infty}(b_{34}'')^{(6)}\left((G_{35})(t), t(t), t\right) = (b_{34}')^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{37}\ , G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\ , \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$ and to choose

$(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[(\hat{P}_{36})^{(7)}+\left((\hat{P}_{36})^{(7)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{36})^{(7)} \tag{256}$$

$$\frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[\left((\hat{Q}_{36})^{(7)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{36})^{(7)}\right]\leq(\hat{Q}_{36})^{(7)} \tag{257}$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself
The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric (258)

$$d\left(((G_{39})^{(1)},(T_{39})^{(1)}),((G_{39})^{(2)},(T_{39})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\hat{M}_{36})^{(7)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\hat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{39}}),(\widetilde{T_{39}}):\left((\widetilde{G_{39}}),(\widetilde{T_{39}})\right)=\mathcal{A}^{(7)}((G_{39}),(T_{39}))$
It results

$$|\tilde{G}_{36}^{(1)}-\tilde{G}_i^{(2)}|\leq\int_0^t(a_{36})^{(7)}|G_{37}^{(1)}-G_{37}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\,ds_{(36)}+$$
$$\int_0^t\{(a'_{36})^{(7)}|G_{36}^{(1)}-G_{36}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$(a''_{36})^{(7)}(T_{37}^{(1)},s_{(36)})|G_{36}^{(1)}-G_{36}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$G_{36}^{(2)}|(a''_{36})^{(7)}(T_{37}^{(1)},s_{(36)})-(a''_{36})^{(7)}(T_{37}^{(2)},s_{(36)})|\;e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\}ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on it follows

$$|(G_{39})^{(1)}-(G_{39})^{(2)}|e^{-(\widehat{M}_{36})^{(7)}t}\leq \tag{259}$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}}\left((a_{36})^{(7)}+(a'_{36})^{(7)}+(\widehat{A}_{36})^{(7)}+(\widehat{P}_{36})^{(7)}(\widehat{k}_{36})^{(7)}\right)d\left(((G_{39})^{(1)},(T_{39})^{(1)};\;(G_{39})^{(2)},(T_{39})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 31:** The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ *and* $(\widehat{Q}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ respectively of $\mathbb{R}_+$. (260)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(7)}$ and $(b_i'')^{(7)}, i = 36,37,38$ depend only on $T_{37}$ and respectively on $(G_{39})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 32:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 261

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(7)} - (a_i'')^{(7)}(T_{37}(s_{(36)}), s_{(36)})\} ds_{(36)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(7)} t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2\ and\ \left((\widehat{M}_{36})^{(7)}\right)_3$ : 262

**Remark 33:** if $G_{36}$ is bounded, the same property have also $G_{37}\ and\ G_{38}$ . indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a_{37}')^{(7)} G_{37}$ and by integrating

$$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1 / (a_{37}')^{(7)}$$

In the same way , one can obtain

$$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2 / (a_{38}')^{(7)}$$

If $G_{37}\ or\ G_{38}$ is bounded, the same property follows for $G_{36}$ , $G_{38}$ and $G_{36}$ , $G_{37}$ respectively.

**Remark 34:** If $G_{36}$ *is* bounded, from below, the same property holds for $G_{37}\ and\ G_{38}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{37}$ is bounded from below. 263

**Remark 35:** If $T_{36}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(7)}\ ((G_{39})(t), t)) = (b_{37}')^{(7)}$ then $T_{37} \to \infty$. 264

**Definition of** $(m)^{(7)}$ and $\varepsilon_7$ :

Indeed let $t_7$ be so that for $t > t_7$

$$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t), t) < \varepsilon_7, T_{36}\,(t) > (m)^{(7)}$$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_7}$ By taking now $\varepsilon_7$ sufficiently small one sees that $T_{37}$ is unbounded.
The same property holds for $T_{38}$ if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t), t\right) = (b_{38}')^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}$ , $\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}} < 1$ and to choose $(\widehat{P}_{40})^{(8)}$ and $(\widehat{Q}_{40})^{(8)}$ large to have 266

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[(\widehat{P}_{40})^{(8)} + \left((\hat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{40})^{(8)} \quad 267$$

$$\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{40})^{(8)}\right] \leq (\hat{Q}_{40})^{(8)} \quad 268$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)}, (T_{43})^{(1)}), ((G_{43})^{(2)}, (T_{43})^{(2)})\right) = \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

**Definition of** $\widetilde{(G_{43})}, \widetilde{(T_{43})}$ : $\left(\widetilde{(G_{43})}, \widetilde{(T_{43})}\right) = \mathcal{A}^{(8)}((G_{43}), (T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{40})^{(8)}\left|G_{41}^{(1)} - G_{41}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\,ds_{(40)} +$$
$$\int_0^t\{(a'_{40})^{(8)}\left|G_{40}^{(1)} - G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{-(\widehat{M}_{40})^{(8)}s_{(40)}} +$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right)\left|G_{40}^{(1)} - G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}} +$$
$$G_{40}^{(2)}\left|(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right) - (a''_{40})^{(8)}\left(T_{41}^{(2)}, s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\}ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0, t]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)} - (G_{43})^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}t} \leq \quad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)} + (a'_{40})^{(8)} + (\widehat{A}_{40})^{(8)} + (\widehat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}\right)d\left(((G_{43})^{(1)}, (T_{43})^{(1)};\ (G_{43})^{(2)}, (T_{43})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

**Remark 36:** The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate 274

condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ *and* $(\widehat{Q}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i = 40,41,42$ depend only on $T_{41}$ and respectively on $(G_{43})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 37** There does not exist any $t$ where $G_i(t) = 0\ and\ T_i(t) = 0$ 275

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(8)} - (a_i'')^{(8)}(T_{41}(s_{(40)}), s_{(40)})\} ds_{(40)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(8)}t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{40})^{(8)}\right)_1, \left((\widehat{M}_{40})^{(8)}\right)_2\ and\ \left((\widehat{M}_{40})^{(8)}\right)_3$ : 276

**Remark 38:** if $G_{40}$ is bounded, the same property have also $G_{41}\ and\ G_{42}$ . indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a_{41}')^{(8)}G_{41}$ and by integrating

$$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1/(a_{41}')^{(8)}$$

In the same way , one can obtain

$$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2/(a_{42}')^{(8)}$$

If $G_{41}\ or\ G_{42}$ is bounded, the same property follows for $G_{40}$ , $G_{42}$ and $G_{40}$ , $G_{41}$ respectively.

**Remark 39:** If $G_{40}$ *is* bounded, from below, the same property holds for $G_{41}\ and\ G_{42}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{41}$ is bounded from below. 277

**Remark 40:** If $T_{40}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}((G_{43})(t), t)) = (b_{41}')^{(8)}$ then $T_{41} \to \infty$. 278

**Definition of** $(m)^{(8)}$ and $\varepsilon_8$ :

Indeed let $t_8$ be so that for $t > t_8$

$$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t), t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right)$, $t = log\frac{2}{\varepsilon_8}$ By taking now $\varepsilon_8$ sufficiently small one sees that $T_{41}$ is unbounded.
The same property holds for $T_{42}$ if $\lim_{t\to\infty}(b''_{42})^{(8)}\left((G_{43})(t), t(t), t\right) = (b'_{42})^{(8)}$



It is now sufficient to take $\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}$ , $\frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$ and to choose $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ large to have

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}\left[(\hat{P}_{44})^{(9)} + \left((\hat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}}\left[\left((\hat{Q}_{44})^{(9)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{44})^{(9)}\right] \leq (\hat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)}, (T_{47})^{(1)}), ((G_{47})^{(2)}, (T_{47})^{(2)})\right) =$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{44})^{(9)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{47}}), (\widetilde{T_{47}}) : \left((\widetilde{G_{47}}), (\widetilde{T_{47}})\right) = \mathcal{A}^{(9)}\left((G_{47}), (T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{44})^{(9)}\left|G_{45}^{(1)} - G_{45}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\,ds_{(44)} +$$
$$\int_0^t\{(a'_{44})^{(9)}\left|G_{44}^{(1)} - G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{-(\widehat{M}_{44})^{(9)}s_{(44)}} +$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right)\left|G_{44}^{(1)} - G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}} +$$
$$G_{44}^{(2)}\left|(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right) - (a''_{44})^{(9)}\left(T_{45}^{(2)}, s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\}ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)} - G^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}t} \leq$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)} + (a'_{44})^{(9)} + (\widehat{A}_{44})^{(9)} + (\widehat{P}_{44})^{(9)}(\hat{k}_{44})^{(9)}\right)d\left(((G_{47})^{(1)}, (T_{47})^{(1)};\ (G_{47})^{(2)}, (T_{47})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis (39,35,36) the result follows

**Remark 41:** The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by

$(\widehat{P}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ *and* $(\widehat{Q}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i = 44{,}45{,}46$ depend only on $T_{45}$ and respectively on $(G_{47})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 42:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$

From 99 to 44 it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t\{(a_i')^{(9)}-(a_i'')^{(9)}(T_{45}(s_{(44)}),s_{(44)})\}ds_{(44)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(9)}t)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{44})^{(9)}\right)_1, \left((\widehat{M}_{44})^{(9)}\right)_2$ *and* $\left((\widehat{M}_{44})^{(9)}\right)_3$ :

**Remark 43:** if $G_{44}$ is bounded, the same property have also $G_{45}\ and\ G_{46}$ . indeed if

$G_{44} < (\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt} \leq \left((\widehat{M}_{44})^{(9)}\right)_1 - (a_{45}')^{(9)}G_{45}$ and by integrating

$$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_1/(a_{45}')^{(9)}$$

In the same way , one can obtain

$$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_2/(a_{46}')^{(9)}$$

If $G_{45}\ or\ G_{46}$ is bounded, the same property follows for $G_{44}$ , $G_{46}$ and $G_{44}$ , $G_{45}$ respectively.

**Remark 44:** If $G_{44}$ *is* bounded, from below, the same property holds for $G_{45}\ and\ G_{46}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{45}$ is bounded from below.

**Remark 45:** If $T_{44}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t),t\right) = (b_{45}')^{(9)}$ then $T_{45} \to \infty$.

**Definition of** $(m)^{(9)}$ and $\varepsilon_9$ :

Indeed let $t_9$ be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t),t\right) < \varepsilon_9, T_{44}\,(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right), \quad t = log\frac{2}{\varepsilon_9}$ By taking now $\varepsilon_9$ sufficiently small one sees that $T_{45}$ is unbounded.
The same property holds for $T_{46}$ if $\lim_{t\to\infty}(b_{46}'')^{(9)}\left((G_{47})(t),t\right) = (b_{46}')^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

**Behavior of the solutions of equation** 280

**Theorem** If we denote and define

**Definition of** $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ :

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)} \le -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14}, t) + (a''_{14})^{(1)}(T_{14}, t) \le -(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)} \le -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G, t) - (b''_{14})^{(1)}(G, t) \le -(\tau_1)^{(1)}$

**Definition of** $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$ : 281

By $(\nu_1)^{(1)} > 0$, $(\nu_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$, $(u_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

**Definition of** $(\bar{\nu}_1)^{(1)},, (\bar{\nu}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$ : 282

By $(\bar{\nu}_1)^{(1)} > 0$, $(\bar{\nu}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$, $(\bar{u}_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

**Definition of** $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (\nu_0)^{(1)}$ :- 283

If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by
$(m_2)^{(1)} = (\nu_0)^{(1)}, (m_1)^{(1)} = (\nu_1)^{(1)}$, $if\ (\nu_0)^{(1)} < (\nu_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}, if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)}$,

and $\boxed{(\nu_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)}$, $if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)}$, $if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)}$,

and $\boxed{(u_0)^{(1)} = \frac{T^0_{13}}{T^0_{14}}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$

are defined

Then the solution of global equations satisfies the inequalities 285

$G^0_{13}e^{\left((S_1)^{(1)} - (p_{13})^{(1)}\right)t} \le G_{13}(t) \le G^0_{13}e^{(S_1)^{(1)}t}$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)}-(p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)}t}$$

$$\left(\frac{(a_{15})^{(1)}G_{13}^0}{(m_1)^{(1)}((S_1)^{(1)}-(p_{13})^{(1)}-(S_2)^{(1)})}\left[e^{((S_1)^{(1)}-(p_{13})^{(1)})t} - e^{-(S_2)^{(1)}t}\right] + G_{15}^0 e^{-(S_2)^{(1)}t} \le G_{15}(t) \le \right. \quad (286)$$

$$\left. \frac{(a_{15})^{(1)}G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)}-(a'_{15})^{(1)})}[e^{(S_1)^{(1)}t} - e^{-(a'_{15})^{(1)}t}] + G_{15}^0 e^{-(a'_{15})^{(1)}t}\right)$$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)}t} \le T_{13}(t) \le T_{13}^0 e^{((R_1)^{(1)}+(r_{13})^{(1)})t}} \quad (287)$$

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)}t} \le T_{13}(t) \le \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)}+(r_{13})^{(1)})t} \quad (288)$$

$$\frac{(b_{15})^{(1)}T_{13}^0}{(\mu_1)^{(1)}((R_1)^{(1)}-(b'_{15})^{(1)})}\left[e^{(R_1)^{(1)}t} - e^{-(b'_{15})^{(1)}t}\right] + T_{15}^0 e^{-(b'_{15})^{(1)}t} \le T_{15}(t) \le \quad (289)$$

$$\frac{(a_{15})^{(1)}T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)}+(r_{13})^{(1)}+(R_2)^{(1)})}\left[e^{((R_1)^{(1)}+(r_{13})^{(1)})t} - e^{-(R_2)^{(1)}t}\right] + T_{15}^0 e^{-(R_2)^{(1)}t}$$

**Definition of** $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:- (290)

Where $(S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$

$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$

$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$

$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$

**Behavior of the solutions of equation** (291)

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$: (292)

$(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ four constants satisfying

$$-(\sigma_2)^{(2)} \le -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(T_{17}, t) + (a''_{17})^{(2)}(T_{17}, t) \le -(\sigma_1)^{(2)} \quad (293)$$

$$-(\tau_2)^{(2)} \le -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}((G_{19}), t) - (b''_{17})^{(2)}((G_{19}), t) \le -(\tau_1)^{(2)} \quad (294)$$

**Definition of** $(\nu_1)^{(2)}, (\nu_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$: (295)

By $(\nu_1)^{(2)} > 0$, $(\nu_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0$, $(u_2)^{(2)} < 0$ the roots (296)

of the equations $(a_{17})^{(2)}(\nu^{(2)})^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ (297)

and $(b_{14})^{(2)}(u^{(2)})^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and (298)

**Definition of** $(\bar{\nu}_1)^{(2)}, , (\bar{\nu}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$: (299)

By $(\bar{\nu}_1)^{(2)} > 0$, $(\bar{\nu}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0$, $(\bar{u}_2)^{(2)} < 0$ the (300)

roots of the equations $(a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 301

and $(b_{17})^{(2)}\left(u^{(2)}\right)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 302

**Definition of** $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ :- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, ***if*** $(u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$, ***if*** $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)}$, ***if*** $(\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)}t}$

$(p_i)^{(2)}$ is defined by equation

$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)}t}$ 311

$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)}t}\right] + G_{18}^0 e^{-(S_2)^{(2)}t} \le G_{18}(t) \le\right.$ 312

$\left.\frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)}t} - e^{-(a'_{18})^{(2)}t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)}t}\right)$

$\boxed{T_{16}^0 e^{(R_1)^{(2)}t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}}$ 313

$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)}t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}$ 314

$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)}t} - e^{-(b'_{18})^{(2)}t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)}t} \le T_{18}(t) \le$ 315

$$\frac{(a_{18})^{(2)}T_{16}^{0}}{(\mu_2)^{(2)}((R_1)^{(2)}+(r_{16})^{(2)}+(R_2)^{(2)})}\left[e^{((R_1)^{(2)}+(r_{16})^{(2)})t}-e^{-(R_2)^{(2)}t}\right]+T_{18}^{0}e^{-(R_2)^{(2)}t}$$

**Definition of** $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$$

$$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)}$$ 318

$$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$$

**Behavior of the solutions** 319

**Theorem 3:** If we denote and define

**Definition of** $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ :

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}\big((G_{23}), t\big) \le -(\tau_1)^{(3)}$$

**Definition of** $(v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$ : 320

By $(v_1)^{(3)} > 0$, $(v_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{v}_1)^{(3)} > 0$, $(\bar{v}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

**Definition of** $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ :- 321

If we define $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (v_0)^{(3)}, (m_1)^{(3)} = (v_1)^{(3)}$, ***if*** $(v_0)^{(3)} < (v_1)^{(3)}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (\bar{v}_1)^{(3)}$, ***if*** $(v_1)^{(3)} < (v_0)^{(3)} < (\bar{v}_1)^{(3)}$,

and $\boxed{(v_0)^{(3)} = \frac{G_{20}^{0}}{G_{21}^{0}}}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_0)^{(3)}$, ***if*** $(\bar{v}_1)^{(3)} < (v_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)},\ \boldsymbol{if}\ (u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}, \boldsymbol{if}\ (u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)},$ and $\boxed{(u_0)^{(3)} = \frac{T_{20}^0}{T_{21}^0}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)},\ \boldsymbol{if}\ (\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G_{20}^0 e^{(S_1)^{(3)} t}$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)} t} \quad 323$$

$$\left(\frac{(a_{22})^{(3)} G_{20}^0}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\left[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)} t}\right] + G_{22}^0 e^{-(S_2)^{(3)} t} \le G_{22}(t) \le \right. \quad 324$$

$$\left. \frac{(a_{22})^{(3)} G_{20}^0}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}\left[e^{(S_1)^{(3)} t} - e^{-(a'_{22})^{(3)} t}\right] + G_{22}^0 e^{-(a'_{22})^{(3)} t}\right)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t}} \quad 325$$

$$\frac{1}{(\mu_1)^{(3)}} T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}} T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)} T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)} - (b'_{22})^{(3)})}\left[e^{(R_1)^{(3)} t} - e^{-(b'_{22})^{(3)} t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)} t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)} T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)} + (r_{20})^{(3)} + (R_2)^{(3)})}\left[e^{((R_1)^{(3)} + (r_{20})^{(3)})t} - e^{-(R_2)^{(3)} t}\right] + T_{22}^0 e^{-(R_2)^{(3)} t}$$

**Definition of** $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

**Behavior of the solutions of equation**

**Theorem:** If we denote and define

**Definition of** $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ :

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$

$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}((G_{27}), t) - (b''_{25})^{(4)}((G_{27}), t) \le -(\tau_1)^{(4)}$

**Definition of** $(v_1)^{(4)}, (v_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, v^{(4)}, u^{(4)}$ : 329

By $(v_1)^{(4)} > 0$ , $(v_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$ , $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_1)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

**Definition of** $(\bar{v}_1)^{(4)},, (\bar{v}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$ : 330

By $(\bar{v}_1)^{(4)} > 0$ , $(\bar{v}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$ , $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

**Definition of** $(m_1)^{(4)}$ , $(m_2)^{(4)}$ , $(\mu_1)^{(4)}, (\mu_2)^{(4)}, (\nu_0)^{(4)}$ :-

If we define $(m_1)^{(4)}$ , $(m_2)^{(4)}$ , $(\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (v_0)^{(4)}, (m_1)^{(4)} = (v_1)^{(4)}$, ***if*** $(v_0)^{(4)} < (v_1)^{(4)}$

$(m_2)^{(4)} = (v_1)^{(4)}, (m_1)^{(4)} = (\bar{v}_1)^{(4)}$ , ***if*** $(v_4)^{(4)} < (v_0)^{(4)} < (\bar{v}_1)^{(4)}$,
and $\boxed{(v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (v_4)^{(4)}, (m_1)^{(4)} = (v_0)^{(4)}$, ***if*** $(\bar{v}_4)^{(4)} < (v_0)^{(4)}$

and analogously

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$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)}$, ***if*** $(u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}$ , ***if*** $(u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)}$,
and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}$, ***if*** $(\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

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$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)}t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)}t}$$

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$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)}t}\right] + G_{26}^0 e^{-(S_2)^{(4)}t} \le G_{26}(t) \le\right.$$ 334

*(a26)4G240(m2)4(S1)4−(a26′)4e(S1)4t−e−(a26′)4t+ G260e−(a26′)4t*

$$\boxed{T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}}T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}}T_{24}^0 e^{((R_1)^{(4)}+(r_{24})^{(4)})t} \quad 335$$

$$\frac{(b_{26})^{(4)}T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)}-(b'_{26})^{(4)})}\left[e^{(R_1)^{(4)}t} - e^{-(b'_{26})^{(4)}t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)}t} \le T_{26}(t) \le \quad 336$$

$$\frac{(a_{26})^{(4)}T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)}+(r_{24})^{(4)}+(R_2)^{(4)})}\left[e^{((R_1)^{(4)}+(r_{24})^{(4)})t} - e^{-(R_2)^{(4)}t}\right] + T_{26}^0 e^{-(R_2)^{(4)}t}$$

**Definition of** $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

**Behavior of the solutions of equation** 338

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ :

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

**Definition of** $(v_1)^{(5)}, (v_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, v^{(5)}, u^{(5)}$ : 339

By $(v_1)^{(5)} > 0$ , $(v_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$ , $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(v^{(5)}\big)^2 + (\sigma_1)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

**Definition of** $(\bar{v}_1)^{(5)}, , (\bar{v}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$ : 340

By $(\bar{v}_1)^{(5)} > 0$ , $(\bar{v}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$ , $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(v^{(5)}\big)^2 + (\sigma_2)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

**Definition of** $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (v_0)^{(5)}$ :-

If we define $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (v_0)^{(5)}, (m_1)^{(5)} = (v_1)^{(5)}$, ***if*** $(v_0)^{(5)} < (v_1)^{(5)}$

$(m_2)^{(5)} = (v_1)^{(5)}, (m_1)^{(5)} = (\bar{v}_1)^{(5)}$ , ***if*** $(v_1)^{(5)} < (v_0)^{(5)} < (\bar{v}_1)^{(5)}$,
and $\boxed{(v_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)},\ \boldsymbol{if}\ (u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}, \boldsymbol{if}\ (u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)},$

and $\boxed{(u_0)^{(5)} = \frac{T_{28}^0}{T_{29}^0}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}, \boldsymbol{if}\ (\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{28}(t) \le G_{28}^0 e^{(S_1)^{(5)} t}$

where $(p_i)^{(5)}$ is defined by equation

$\frac{1}{(m_5)^{(5)}} G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}} G_{28}^0 e^{(S_1)^{(5)} t}$ 343

$\left(\frac{(a_{30})^{(5)} G_{28}^0}{(m_1)^{(5)}((S_1)^{(5)} - (p_{28})^{(5)} - (S_2)^{(5)})}\left[e^{((S_1)^{(5)} - (p_{28})^{(5)})t} - e^{-(S_2)^{(5)} t}\right] + G_{30}^0 e^{-(S_2)^{(5)} t} \le G_{30}(t) \le\right.$ 344

*(a30)5G280(m2)5(S1)5−(a30′)5e(S1)5t−e−(a30′)5t+ G300e−(a30′)5t*

$\boxed{T_{28}^0 e^{(R_1)^{(5)} t} \le T_{28}(t) \le T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}}$ 345

$\frac{1}{(\mu_1)^{(5)}} T_{28}^0 e^{(R_1)^{(5)} t} \le T_{28}(t) \le \frac{1}{(\mu_2)^{(5)}} T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}$ 346

$\frac{(b_{30})^{(5)} T_{28}^0}{(\mu_1)^{(5)}((R_1)^{(5)} - (b'_{30})^{(5)})}\left[e^{(R_1)^{(5)} t} - e^{-(b'_{30})^{(5)} t}\right] + T_{30}^0 e^{-(b'_{30})^{(5)} t} \le T_{30}(t) \le$ 347

$\frac{(a_{30})^{(5)} T_{28}^0}{(\mu_2)^{(5)}((R_1)^{(5)} + (r_{28})^{(5)} + (R_2)^{(5)})}\left[e^{((R_1)^{(5)} + (r_{28})^{(5)})t} - e^{-(R_2)^{(5)} t}\right] + T_{30}^0 e^{-(R_2)^{(5)} t}$

**Definition of** $(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$:- 348

Where $(S_1)^{(5)} = (a_{28})^{(5)}(m_2)^{(5)} - (a'_{28})^{(5)}$

$(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}$

$(R_1)^{(5)} = (b_{28})^{(5)}(\mu_2)^{(5)} - (b'_{28})^{(5)}$

$(R_2)^{(5)} = (b'_{30})^{(5)} - (r_{30})^{(5)}$

**Behavior of the solutions of equation** 349

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ :

$(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)} \le -(a'_{32})^{(6)} + (a'_{33})^{(6)} - (a''_{32})^{(6)}(T_{33}, t) + (a''_{33})^{(6)}(T_{33}, t) \le -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)} \le -(b'_{32})^{(6)} + (b'_{33})^{(6)} - (b''_{32})^{(6)}\big((G_{35}), t\big) - (b''_{33})^{(6)}\big((G_{35}), t\big) \le -(\tau_1)^{(6)}$$

**Definition of** $(\nu_1)^{(6)}, (\nu_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, \nu^{(6)}, u^{(6)}$ : 350

By $(\nu_1)^{(6)} > 0$ , $(\nu_2)^{(6)} < 0$ and respectively $(u_1)^{(6)} > 0$ , $(u_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(6)},, (\bar{\nu}_2)^{(6)}, (\bar{u}_1)^{(6)}, (\bar{u}_2)^{(6)}$ : 351

By $(\bar{\nu}_1)^{(6)} > 0$ , $(\bar{\nu}_2)^{(6)} < 0$ and respectively $(\bar{u}_1)^{(6)} > 0$ , $(\bar{u}_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$

**Definition of** $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}, (\nu_0)^{(6)}$ :-

If we define $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}$ by

$$(m_2)^{(6)} = (\nu_0)^{(6)}, (m_1)^{(6)} = (\nu_1)^{(6)}, \ \boldsymbol{if}\ (\nu_0)^{(6)} < (\nu_1)^{(6)}$$

$$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}, \boldsymbol{if}\ (\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)},$$

and $\boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}}$

$$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$$

and analogously 352

$$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}, \ \boldsymbol{if}\ (u_0)^{(6)} < (u_1)^{(6)}$$

$$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}, \boldsymbol{if}\ (u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)},$$

and $\boxed{(u_0)^{(6)} = \frac{T^0_{32}}{T^0_{33}}}$

$$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}, \boldsymbol{if}\ (\bar{u}_1)^{(6)} < (u_0)^{(6)} \text{ where } (u_1)^{(6)}, (\bar{u}_1)^{(6)}$$

Then the solution of global equations satisfies the inequalities 353

$$G^0_{32}e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{32}(t) \le G^0_{32}e^{(S_1)^{(6)}t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}}G^0_{32}e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{33}(t) \le \frac{1}{(m_2)^{(6)}}G^0_{32}e^{(S_1)^{(6)}t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)}G^0_{32}}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})}\left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)}t}\right] + G^0_{34}e^{-(S_2)^{(6)}t} \le G_{34}(t) \le \right. \quad 355$$

$$\frac{(a_{34})^{(6)}G_{32}^{0}}{(m_2)^{(6)}((S_1)^{(6)}-(a'_{34})^{(6)})}\left[e^{(S_1)^{(6)}t}-e^{-(a'_{34})^{(6)}t}\right]+\ G_{34}^{0}e^{-(a'_{34})^{(6)}t}\Big)$$

$$\boxed{T_{32}^{0}e^{(R_1)^{(6)}t}\le T_{32}(t)\le T_{32}^{0}e^{((R_1)^{(6)}+(r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}}T_{32}^{0}e^{(R_1)^{(6)}t}\le T_{32}(t)\le\frac{1}{(\mu_2)^{(6)}}T_{32}^{0}e^{((R_1)^{(6)}+(r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)}T_{32}^{0}}{(\mu_1)^{(6)}((R_1)^{(6)}-(b'_{34})^{(6)})}\left[e^{(R_1)^{(6)}t}-e^{-(b'_{34})^{(6)}t}\right]+T_{34}^{0}e^{-(b'_{34})^{(6)}t}\le T_{34}(t)\le \quad 358$$

$$\frac{(a_{34})^{(6)}T_{32}^{0}}{(\mu_2)^{(6)}((R_1)^{(6)}+(r_{32})^{(6)}+(R_2)^{(6)})}\left[e^{((R_1)^{(6)}+(r_{32})^{(6)})t}-e^{-(R_2)^{(6)}t}\right]+T_{34}^{0}e^{-(R_2)^{(6)}t}$$

**Definition of** $(S_1)^{(6)},(S_2)^{(6)},(R_1)^{(6)},(R_2)^{(6)}$:- 359

Where $(S_1)^{(6)}=(a_{32})^{(6)}(m_2)^{(6)}-(a'_{32})^{(6)}$

$(S_2)^{(6)}=(a_{34})^{(6)}-(p_{34})^{(6)}$

$(R_1)^{(6)}=(b_{32})^{(6)}(\mu_2)^{(6)}-(b'_{32})^{(6)}$

$(R_2)^{(6)}=(b'_{34})^{(6)}-(r_{34})^{(6)}$

**Behavior of the solutions of equation**

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(7)},(\sigma_2)^{(7)},(\tau_1)^{(7)},(\tau_2)^{(7)}$ :

$(\sigma_1)^{(7)},(\sigma_2)^{(7)},(\tau_1)^{(7)},(\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)}\le-(a'_{36})^{(7)}+(a'_{37})^{(7)}-(a''_{36})^{(7)}(T_{37},t)+(a''_{37})^{(7)}(T_{37},t)\le-(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)}\le-(b'_{36})^{(7)}+(b'_{37})^{(7)}-(b''_{36})^{(7)}\big((G_{39}),t\big)-(b''_{37})^{(7)}\big((G_{39}),t\big)\le-(\tau_1)^{(7)}$$

**Definition of** $(v_1)^{(7)},(v_2)^{(7)},(u_1)^{(7)},(u_2)^{(7)},v^{(7)},u^{(7)}$ : 361

By $(v_1)^{(7)}>0,(v_2)^{(7)}<0$ and respectively $(u_1)^{(7)}>0,(u_2)^{(7)}<0$ the roots of the equations $(a_{37})^{(7)}\big(v^{(7)}\big)^2+(\sigma_1)^{(7)}v^{(7)}-(a_{36})^{(7)}=0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2+(\tau_1)^{(7)}u^{(7)}-(b_{36})^{(7)}=0$ and

**Definition of** $(\bar{v}_1)^{(7)},,(\bar{v}_2)^{(7)},(\bar{u}_1)^{(7)},(\bar{u}_2)^{(7)}$ : 362

By $(\bar{v}_1)^{(7)}>0,(\bar{v}_2)^{(7)}<0$ and respectively $(\bar{u}_1)^{(7)}>0,(\bar{u}_2)^{(7)}<0$ the roots of the equations $(a_{37})^{(7)}\big(v^{(7)}\big)^2+(\sigma_2)^{(7)}v^{(7)}-(a_{36})^{(7)}=0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2+(\tau_2)^{(7)}u^{(7)}-(b_{36})^{(7)}=0$

**Definition of** $(m_1)^{(7)},(m_2)^{(7)},(\mu_1)^{(7)},(\mu_2)^{(7)},(v_0)^{(7)}$ :-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$$(m_2)^{(7)} = (\nu_0)^{(7)}, (m_1)^{(7)} = (\nu_1)^{(7)}, \ \boldsymbol{if}\ (\nu_0)^{(7)} < (\nu_1)^{(7)}$$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\bar{\nu}_1)^{(7)}, \boldsymbol{if}\ (\nu_1)^{(7)} < (\nu_0)^{(7)} < (\bar{\nu}_1)^{(7)},$$

and $\boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\nu_0)^{(7)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(7)} < (\nu_0)^{(7)}$$

and analogously 363

$$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}, \ \boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$$

and $\boxed{(u_0)^{(7)} = \frac{T_{36}^0}{T_{37}^0}}$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)} \text{ where } (u_1)^{(7)}, (\bar{u}_1)^{(7)}$$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^0 e^{(S_1)^{(7)} t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)} t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)}((S_1)^{(7)} - (p_{36})^{(7)} - (S_2)^{(7)})}\left[e^{((S_1)^{(7)} - (p_{36})^{(7)})t} - e^{-(S_2)^{(7)} t}\right] + G_{38}^0 e^{-(S_2)^{(7)} t} \le G_{38}(t) \le \right. \quad 366$$

$$\left. \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)}((S_1)^{(7)} - (a'_{38})^{(7)})}\left[e^{(S_1)^{(7)} t} - e^{-(a'_{38})^{(7)} t}\right] + G_{38}^0 e^{-(a'_{38})^{(7)} t}\right)$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t} \quad 368$$

$$\frac{(b_{38})^{(7)} T_{36}^0}{(\mu_1)^{(7)}((R_1)^{(7)} - (b'_{38})^{(7)})}\left[e^{(R_1)^{(7)} t} - e^{-(b'_{38})^{(7)} t}\right] + T_{38}^0 e^{-(b'_{38})^{(7)} t} \le T_{38}(t) \le \quad 369$$

$$\frac{(a_{38})^{(7)} T_{36}^0}{(\mu_2)^{(7)}((R_1)^{(7)} + (r_{36})^{(7)} + (R_2)^{(7)})}\left[e^{((R_1)^{(7)} + (r_{36})^{(7)})t} - e^{-(R_2)^{(7)} t}\right] + T_{38}^0 e^{-(R_2)^{(7)} t}$$

**Definition of** $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$

$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$

$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$

**Behavior of the solutions of equation** 371

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ :

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$-(\sigma_2)^{(8)} \le -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \le -(\sigma_1)^{(8)}$

$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\big((G_{43}), t\big) - (b''_{41})^{(8)}\big((G_{43}), t\big) \le -(\tau_1)^{(8)}$

**Definition of** $(\nu_1)^{(8)}, (\nu_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, \nu^{(8)}, u^{(8)}$ : 372

By $(\nu_1)^{(8)} > 0, (\nu_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0, (u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(8)}, , (\bar{\nu}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$ :

By $(\bar{\nu}_1)^{(8)} > 0, (\bar{\nu}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0, (\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

**Definition of** $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}, (\nu_0)^{(8)}$ :-

If we define $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$(m_2)^{(8)} = (\nu_0)^{(8)}, (m_1)^{(8)} = (\nu_1)^{(8)}$, ***if*** $(\nu_0)^{(8)} < (\nu_1)^{(8)}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\bar{\nu}_1)^{(8)}$, ***if*** $(\nu_1)^{(8)} < (\nu_0)^{(8)} < (\bar{\nu}_1)^{(8)}$,
and $\boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\nu_0)^{(8)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(8)} < (\nu_0)^{(8)}$

and analogously 374

$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)},\ \boldsymbol{if}\ (u_0)^{(8)} < (u_1)^{(8)}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}, \boldsymbol{if}\ (u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)},$

and $\boxed{(u_0)^{(8)} = \frac{T_{40}^0}{T_{41}^0}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}, \boldsymbol{if}\ (\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^0 e^{(S_1)^{(8)} t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}} G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}} G_{40}^0 e^{(S_1)^{(8)} t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)} G_{40}^0}{(m_1)^{(8)}((S_1)^{(8)} - (p_{40})^{(8)} - (S_2)^{(8)})}\left[e^{((S_1)^{(8)} - (p_{40})^{(8)})t} - e^{-(S_2)^{(8)} t}\right] + G_{42}^0 e^{-(S_2)^{(8)} t} \le G_{42}(t) \le \right. \quad 377$$

$$\left. \frac{(a_{42})^{(8)} G_{40}^0}{(m_2)^{(8)}((S_1)^{(8)} - (a'_{42})^{(8)})}\left[e^{(S_1)^{(8)} t} - e^{-(a'_{42})^{(8)} t}\right] + G_{42}^0 e^{-(a'_{42})^{(8)} t}\right)$$

$$\boxed{T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}} T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le \frac{1}{(\mu_2)^{(8)}} T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)} T_{40}^0}{(\mu_1)^{(8)}((R_1)^{(8)} - (b'_{42})^{(8)})}\left[e^{(R_1)^{(8)} t} - e^{-(b'_{42})^{(8)} t}\right] + T_{42}^0 e^{-(b'_{42})^{(8)} t} \le T_{42}(t) \le \quad 380$$

$$\frac{(a_{42})^{(8)} T_{40}^0}{(\mu_2)^{(8)}((R_1)^{(8)} + (r_{40})^{(8)} + (R_2)^{(8)})}\left[e^{((R_1)^{(8)} + (r_{40})^{(8)})t} - e^{-(R_2)^{(8)} t}\right] + T_{42}^0 e^{-(R_2)^{(8)} t}$$

**Definition of** $(S_1)^{(8)}, (S_2)^{(8)}, (R_1)^{(8)}, (R_2)^{(8)}$:- 381

Where $(S_1)^{(8)} = (a_{40})^{(8)}(m_2)^{(8)} - (a'_{40})^{(8)}$

$(S_2)^{(8)} = (a_{42})^{(8)} - (p_{42})^{(8)}$

$(R_1)^{(8)} = (b_{40})^{(8)}(\mu_2)^{(8)} - (b'_{40})^{(8)}$

$(R_2)^{(8)} = (b'_{42})^{(8)} - (r_{42})^{(8)}$

**Behavior of the solutions of equation 37 to 92** 382

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ :

$(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ four constants satisfying

$$-(\sigma_2)^{(9)} \le -(a'_{44})^{(9)} + (a'_{45})^{(9)} - (a''_{44})^{(9)}(T_{45}, t) + (a''_{45})^{(9)}(T_{45}, t) \le -(\sigma_1)^{(9)}$$

$$-(\tau_2)^{(9)} \le -(b'_{44})^{(9)} + (b'_{45})^{(9)} - (b''_{44})^{(9)}\big((G_{47}), t\big) - (b''_{45})^{(9)}\big((G_{47}), t\big) \le -(\tau_1)^{(9)}$$

**Definition of** $(\nu_1)^{(9)}, (\nu_2)^{(9)}, (u_1)^{(9)}, (u_2)^{(9)}, \nu^{(9)}, u^{(9)}$ :

By $(\nu_1)^{(9)} > 0$ , $(\nu_2)^{(9)} < 0$ and respectively $(u_1)^{(9)} > 0$ , $(u_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_1)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(9)}, , (\bar{\nu}_2)^{(9)}, (\bar{u}_1)^{(9)}, (\bar{u}_2)^{(9)}$ :

By $(\bar{\nu}_1)^{(9)} > 0$ , $(\bar{\nu}_2)^{(9)} < 0$ and respectively $(\bar{u}_1)^{(9)} > 0$ , $(\bar{u}_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_2)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$

**Definition of** $(m_1)^{(9)}, (m_2)^{(9)}, (\mu_1)^{(9)}, (\mu_2)^{(9)}, (\nu_0)^{(9)}$ :-

If we define $(m_1)^{(9)}, (m_2)^{(9)}, (\mu_1)^{(9)}, (\mu_2)^{(9)}$ by

$$(m_2)^{(9)} = (\nu_0)^{(9)}, (m_1)^{(9)} = (\nu_1)^{(9)}, \ \boldsymbol{if}\ (\nu_0)^{(9)} < (\nu_1)^{(9)}$$

$$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\bar{\nu}_1)^{(9)}, \boldsymbol{if}\ (\nu_1)^{(9)} < (\nu_0)^{(9)} < (\bar{\nu}_1)^{(9)},$$

and $\boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$

$$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\nu_0)^{(9)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(9)} < (\nu_0)^{(9)}$$

and analogously

$$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)}, \ \boldsymbol{if}\ (u_0)^{(9)} < (u_1)^{(9)}$$

$$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}, \boldsymbol{if}\ (u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)},$$

and $\boxed{(u_0)^{(9)} = \frac{T^0_{44}}{T^0_{45}}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}, \boldsymbol{if}\ (\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G^0_{44} e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{44}(t) \le G^0_{44} e^{(S_1)^{(9)} t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}}G_{44}^0 e^{\left((S_1)^{(9)}-(p_{44})^{(9)}\right)t} \le G_{45}(t) \le \frac{1}{(m_2)^{(9)}}G_{44}^0 e^{(S_1)^{(9)}t}$$

$$\left(\frac{(a_{46})^{(9)}G_{44}^0}{(m_1)^{(9)}\left((S_1)^{(9)}-(p_{44})^{(9)}-(S_2)^{(9)}\right)}\left[e^{\left((S_1)^{(9)}-(p_{44})^{(9)}\right)t}-e^{-(S_2)^{(9)}t}\right]+G_{46}^0 e^{-(S_2)^{(9)}t} \le G_{46}(t) \le \right.$$

$$\left.\frac{(a_{46})^{(9)}G_{44}^0}{(m_2)^{(9)}\left((S_1)^{(9)}-(a'_{46})^{(9)}\right)}\left[e^{(S_1)^{(9)}t}-e^{-(a'_{46})^{(9)}t}\right]+\ G_{46}^0 e^{-(a'_{46})^{(9)}t}\right)$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le T_{44}^0 e^{\left((R_1)^{(9)}+(r_{44})^{(9)}\right)t}}$$

$$\frac{1}{(\mu_1)^{(9)}}T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le \frac{1}{(\mu_2)^{(9)}}T_{44}^0 e^{\left((R_1)^{(9)}+(r_{44})^{(9)}\right)t}$$

$$\frac{(b_{46})^{(9)}T_{44}^0}{(\mu_1)^{(9)}\left((R_1)^{(9)}-(b'_{46})^{(9)}\right)}\left[e^{(R_1)^{(9)}t}-e^{-(b'_{46})^{(9)}t}\right]+T_{46}^0 e^{-(b'_{46})^{(9)}t} \le T_{46}(t) \le$$

$$\frac{(a_{46})^{(9)}T_{44}^0}{(\mu_2)^{(9)}\left((R_1)^{(9)}+(r_{44})^{(9)}+(R_2)^{(9)}\right)}\left[e^{\left((R_1)^{(9)}+(r_{44})^{(9)}\right)t}-e^{-(R_2)^{(9)}t}\right]+T_{46}^0 e^{-(R_2)^{(9)}t}$$

**Definition of** $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$

**Proof :** From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14},t)\right) - (a''_{14})^{(1)}(T_{14},t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

**Definition of** $v^{(1)}$ :- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)}\right) \le \frac{dv^{(1)}}{dt} \le -\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)}\right)$$

From which one obtains

**Definition of** $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$ :-

For $0 < \boxed{(v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}} < (v_1)^{(1)} < (\bar{v}_1)^{(1)}$

$$v^{(1)}(t) \ge \frac{(v_1)^{(1)}+(C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(v_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(v_2)^{(1)}}}$$

it follows $(\nu_0)^{(1)} \leq \nu^{(1)}(t) \leq (\nu_1)^{(1)}$

In the same manner , we get 384

$$\nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{\nu}_1)^{(1)}-(\nu_0)^{(1)}}{(\nu_0)^{(1)}-(\bar{\nu}_2)^{(1)}}}$$

From which we deduce $(\nu_0)^{(1)} \leq \nu^{(1)}(t) \leq (\bar{\nu}_1)^{(1)}$

If $0 < (\nu_1)^{(1)} < (\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0} < (\bar{\nu}_1)^{(1)}$ we find like in the previous case, 385

$$(\nu_1)^{(1)} \leq \frac{(\nu_1)^{(1)}+(C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}} \leq \nu^{(1)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\bar{\nu}_1)^{(1)}$$

If $0 < (\nu_1)^{(1)} \leq (\bar{\nu}_1)^{(1)} \leq \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$ , we obtain 386

$$(\nu_1)^{(1)} \leq \nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\nu_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(1)}(t)$ :-

$$(m_2)^{(1)} \leq \nu^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{\nu^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(1)}(t)$ :-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{13})^{(1)} = (a''_{14})^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)}G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)}T_{14}(t)$ This is an important

consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

**Proof** : From global equations we obtain 387

$$\frac{d\nu^{(2)}}{dt} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right) - (a''_{17})^{(2)}(T_{17}, t)\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

**Definition of** $\nu^{(2)}$ :- $\boxed{\nu^{(2)} = \frac{G_{16}}{G_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \le \frac{d\nu^{(2)}}{dt} \le -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

**Definition of** $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$ :-

For $0 < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \ge \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}}{1 + (C)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(C)^{(2)} = \frac{(\nu_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \le \nu^{(2)}(t) \le (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \le \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \le \nu^{(2)}(t) \le (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \le \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}}{1 + (C)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}} \le \nu^{(2)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \le (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \le (\bar{\nu}_1)^{(2)} \le (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$ , we obtain 394

$$(\nu_1)^{(2)} \le \nu^{(2)}(t) \le \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \le (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(2)}(t)$ :- 395

$$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}, \quad \boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$$

In a completely analogous way, we obtain 396

**Definition of** $u^{(2)}(t)$ :-

$$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}, \quad \boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :** 397

If $(a_{16}'')^{(2)} = (a_{17}'')^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)} G_{17}(t)$

Analogously if $(b_{16}'')^{(2)} = (b_{17}'')^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)} T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

**Proof** : From global equations we obtain 398

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a_{20}')^{(3)} - (a_{21}')^{(3)} + (a_{20}'')^{(3)}(T_{21}, t)\right) - (a_{21}'')^{(3)}(T_{21}, t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

**Definition of** $\nu^{(3)}$ :- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right) \leq \frac{d\nu^{(3)}}{dt} \leq -\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right)$$

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From which one obtains

For $0 < (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (\nu_1)^{(3)} < (\bar{\nu}_1)^{(3)}$

$$\nu^{(3)}(t) \geq \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(\nu_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\nu_2)^{(3)}}}$$

it follows $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\nu_1)^{(3)}$

In the same manner , we get 401

$$\nu^{(3)}(t) \le \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{\nu}_1)^{(3)}-(\nu_0)^{(3)}}{(\nu_0)^{(3)}-(\bar{\nu}_2)^{(3)}}}$$

**Definition of** $(\bar{\nu}_1)^{(3)}$ :-

From which we deduce $(\nu_0)^{(3)} \le \nu^{(3)}(t) \le (\bar{\nu}_1)^{(3)}$

If $0 < (\nu_1)^{(3)} < (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (\bar{\nu}_1)^{(3)}$ we find like in the previous case, 402

$$(\nu_1)^{(3)} \le \frac{(\nu_1)^{(3)}+(C)^{(3)}(\nu_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}} \le \nu^{(3)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \le (\bar{\nu}_1)^{(3)}$$

If $0 < (\nu_1)^{(3)} \le (\bar{\nu}_1)^{(3)} \le (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}$ , we obtain 403

$$(\nu_1)^{(3)} \le \nu^{(3)}(t) \le \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \le (\nu_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(3)}(t)$ :-

$$(m_2)^{(3)} \le \nu^{(3)}(t) \le (m_1)^{(3)}, \quad \boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(3)}(t)$ :-

$$(\mu_2)^{(3)} \le u^{(3)}(t) \le (\mu_1)^{(3)}, \quad \boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{20}'')^{(3)} = (a_{21}'')^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)}G_{21}(t)$

Analogously if $(b_{20}'')^{(3)} = (b_{21}'')^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)}T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

**Proof** : From global equations we obtain 404

$$\frac{dv^{(4)}}{dt} = (a_{24})^{(4)} - \left((a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right) - (a''_{25})^{(4)}(T_{25},t)v^{(4)} - (a_{25})^{(4)}v^{(4)}$$

**Definition of** $v^{(4)}$ :- $\boxed{v^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)}\right) \le \frac{dv^{(4)}}{dt} \le -\left((a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_4)^{(4)}v^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$ :-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \ge \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \le \nu^{(4)}(t) \le (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \le \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \le \nu^{(4)}(t) \le (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \le \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}}{1 + (C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}} \le \nu^{(4)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \le (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \le (\bar{\nu}_1)^{(4)} \le \boxed{(\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}}}$ , we obtain 407

$$(\nu_1)^{(4)} \le \nu^{(4)}(t) \le \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \le (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(4)}(t)$ :-

$$(m_2)^{(4)} \le \nu^{(4)}(t) \le (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(4)}(t)$ :-

$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}$, $\boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{24})^{(4)} = (a''_{25})^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)} G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b''_{24})^{(4)} = (b''_{25})^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)} T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

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**Proof** : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a'_{28})^{(5)} - (a'_{29})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right) - (a''_{29})^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

**Definition of** $\nu^{(5)}$ :- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$ :-

For $0 < \boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \le \frac{(\nu_1)^{(5)}+(C)^{(5)}(\nu_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)}-(\nu_2)^{(5)}\right)t\right]}}{1+(C)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)}-(\nu_2)^{(5)}\right)t\right]}} \le \nu^{(5)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \le (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \le (\bar{\nu}_1)^{(5)} \le \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain 411

$$(\nu_1)^{(5)} \le \nu^{(5)}(t) \le \frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \le (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(5)}(t)$ :-

$(m_2)^{(5)} \le \nu^{(5)}(t) \le (m_1)^{(5)}$, $\boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$

In a completely analogous way, we obtain

**Definition of** $u^{(5)}(t)$ :-

$(\mu_2)^{(5)} \le u^{(5)}(t) \le (\mu_1)^{(5)}$, $\boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a_{28}'')^{(5)} = (a_{29}'')^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)}G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b_{28}'')^{(5)} = (b_{29}'')^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)}T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

**Proof** : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a_{32}')^{(6)} - (a_{33}')^{(6)} + (a_{32}'')^{(6)}(T_{33}, t)\right) - (a_{33}'')^{(6)}(T_{33}, t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

**Definition of** $\nu^{(6)}$ :- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \le \frac{d\nu^{(6)}}{dt} \le -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$ :-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \geq \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\nu_1)^{(6)}$

In the same manner , we get 413

$$\nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{\nu}_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\bar{\nu}_1)^{(6)}$

If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 414

$$(\nu_1)^{(6)} \leq \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}} \leq \nu^{(6)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\bar{\nu}_1)^{(6)}$$

If $0 < (\nu_1)^{(6)} \leq (\bar{\nu}_1)^{(6)} \leq \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$ , we obtain 415

$$(\nu_1)^{(6)} \leq \nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(6)}(t)$ :-

$$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}, \quad \boxed{\nu^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(6)}(t)$ :-

$$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}, \quad \boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)} G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)} T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

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**Proof** : From global equations we obtain

$$\frac{dv^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right) - (a''_{37})^{(7)}(T_{37},t)v^{(7)} - (a_{37})^{(7)}v^{(7)}$$

**Definition of** $v^{(7)}$ :- $\boxed{v^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_2)^{(7)}v^{(7)} - (a_{36})^{(7)}\right) \le \frac{dv^{(7)}}{dt} \le -\left((a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_1)^{(7)}v^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

**Definition of** $(\bar{v}_1)^{(7)}, (v_0)^{(7)}$ :-

For $0 < \boxed{(v_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}} < (v_1)^{(7)} < (\bar{v}_1)^{(7)}$

$$v^{(7)}(t) \ge \frac{(v_1)^{(7)} + (C)^{(7)}(v_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_0)^{(7)}\right)t\right]}}{1 + (C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(v_1)^{(7)} - (v_0)^{(7)}}{(v_0)^{(7)} - (v_2)^{(7)}}}$$

it follows $(v_0)^{(7)} \le v^{(7)}(t) \le (v_1)^{(7)}$

In the same manner , we get 417

$$v^{(7)}(t) \le \frac{(\bar{v}_1)^{(7)} + (\bar{C})^{(7)}(\bar{v}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{v}_1)^{(7)} - (v_0)^{(7)}}{(v_0)^{(7)} - (\bar{v}_2)^{(7)}}}$$

From which we deduce $(v_0)^{(7)} \le v^{(7)}(t) \le (\bar{v}_1)^{(7)}$

If $0 < (v_1)^{(7)} < (v_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}} < (\bar{v}_1)^{(7)}$ we find like in the previous case, 418

$$(v_1)^{(7)} \le \frac{(v_1)^{(7)} + (C)^{(7)}(v_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_2)^{(7)}\right)t\right]}}{1 + (C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_2)^{(7)}\right)t\right]}} \le v^{(7)}(t) \le$$

$$\frac{(\bar{v}_1)^{(7)} + (\bar{C})^{(7)}(\bar{v}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}} \le (\bar{v}_1)^{(7)}$$

If $0 < (v_1)^{(7)} \le (\bar{v}_1)^{(7)} \le \boxed{(v_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$ , we obtain 419

$$(v_1)^{(7)} \le v^{(7)}(t) \le \frac{(\bar{v}_1)^{(7)} + (\bar{C})^{(7)}(\bar{v}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}} \le (v_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $v^{(7)}(t)$ :-

$$(m_2)^{(7)} \leq v^{(7)}(t) \leq (m_1)^{(7)}, \quad \boxed{v^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(7)}(t)$ :- 420

$$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}, \quad \boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $v^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)} G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)} T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

**Proof** : From global equations we obtain 421

$$\frac{dv^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right) - (a''_{41})^{(8)}(T_{41}, t) v^{(8)} - (a_{41})^{(8)} v^{(8)}$$

**Definition of** $v^{(8)}$ :- $\boxed{v^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_2)^{(8)} v^{(8)} - (a_{40})^{(8)}\right) \leq \frac{dv^{(8)}}{dt} \leq -\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_1)^{(8)} v^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$ :-

$$\text{For } 0 < \boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$$

$$v^{(8)}(t) \geq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)}-(\nu_0)^{(8)}}{(\nu_0)^{(8)}-(\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \leq \frac{(\nu_1)^{(8)}+(C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}}{1+(C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}} \leq \nu^{(8)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \leq (\bar{\nu}_1)^{(8)} \leq \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$ , we obtain 424

$$(\nu_1)^{(8)} \leq \nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(8)}(t)$ :-

$$(m_2)^{(8)} \leq \nu^{(8)}(t) \leq (m_1)^{(8)}, \quad \boxed{\nu^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(8)}(t)$ :-

$$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}, \quad \boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $\nu^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)}G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then $(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)}T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

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**Proof** : From 99,20,44,22,23,44 we obtain A

$$\frac{dv^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right) - (a''_{45})^{(9)}(T_{45},t)v^{(9)} - (a_{45})^{(9)}v^{(9)}$$

**Definition of** $v^{(9)}$ :- $\boxed{v^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)}\right) \le \frac{dv^{(9)}}{dt} \le -\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_1)^{(9)}v^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$ :-

For $0 < \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$

$$v^{(9)}(t) \ge \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \le v^{(9)}(t) \le (\nu_9)^{(9)}$

In the same manner , we get

$$v^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \le v^{(9)}(t) \le (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \le \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}}{1 + (C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}} \le v^{(9)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \le (\bar{\nu}_1)^{(9)} \le \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$ , we obtain

$$(\nu_1)^{(9)} \le v^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $v^{(9)}(t)$ :-

$$(m_2)^{(9)} \le v^{(9)}(t) \le (m_1)^{(9)}, \quad \boxed{v^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(9)}(t)$ :-

$(\mu_2)^{(9)} \leq u^{(9)}(t) \leq (\mu_1)^{(9)}$, $\boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)} G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then
$(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)} T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following 425

**Theorem :** If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on $t$ , and the conditions with the notations

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0$ ,

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$

$with\ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a''_i)^{(2)} and\ (b''_i)^{(2)}$ are independent on t , and the conditions with the notations 426

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$ 427

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 428

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$ , 429

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 430

$with\ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a''_i)^{(3)} and\ (b''_i)^{(3)}$ are independent on $t$ , and the conditions with the notations 431

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$ ,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

**Theorem :** If $(a_i'')^{(4)} and\ (b_i'')^{(4)}$ are independent on $t$ , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$ ,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(5)} and\ (b_i'')^{(5)}$ are independent on $t$ , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$ ,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

**Theorem** If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on $t$ , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$ ,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(7)} and\ (b_i'')^{(7)}$ are independent on $t$ , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$ ,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(8)} and\ (b_i'')^{(8)}$ are independent on $t$ , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$ ,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(9)} and\ (b_i'')^{(9)}$ are independent on $t$ , and the conditions (with the notations 436 A
45,46,27,28)

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$ ,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0$ 437

$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0$ 438

$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0$ 439

$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0$ 440

$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ]T_{14} = 0$ 441

$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0$ 442

has a unique positive solution , which is an equilibrium solution for the system

$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0$ 443

$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0$ 444

$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0$ 445

$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\ ]T_{16} = 0$ 446

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\,]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\,]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\,]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\,]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\,]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\big((G_{27})\big)\,]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\big((G_{27})\big)\,]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\big((G_{27})\big)\,]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})\,]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})\,]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})\,]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})\,]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})\,]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})\,]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0 \quad 479$$

$$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0 \quad 480$$

$$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0 \quad 481$$

$$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0 \quad 482$$

$$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0 \quad 483$$

$$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0 \quad 484$$

$$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0 \quad 484\text{A}$$

$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$

$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$

$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$

$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$

$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$

**Proof:** 485

(a) Indeed the first two equations have a nontrivial solution $G_{13}, G_{14}$ if

$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$

**Proof:** 486

(x) Indeed the first two equations have a nontrivial solution $G_{16}, G_{17}$ if

$\mathrm{F}(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$

**Proof:** 487

(a) Indeed the first two equations have a nontrivial solution $G_{20}, G_{21}$ if

$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$

**Proof:** 488

(a) Indeed the first two equations have a nontrivial solution $G_{24}, G_{25}$ if

$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$

**Proof:** 489

(a) Indeed the first two equations have a nontrivial solution $G_{28}, G_{29}$ if

$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$

**Proof:** 490

(a) Indeed the first two equations have a nontrivial solution $G_{32}, G_{33}$ if

$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) +$

$(a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$

**Proof:** 491

(a) Indeed the first two equations have a nontrivial solution $G_{36}, G_{37}$ if

$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$

**Proof:** 492

(a) Indeed the first two equations have a nontrivial solution $G_{40}, G_{41}$ if

$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$

**Proof:** 492 A

(a) Indeed the first two equations have a nontrivial solution $G_{44}, G_{45}$ if

$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$

**Definition and uniqueness of** $T^*_{14}$ :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique $T^*_{14}$ for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

**Definition and uniqueness of** $T^*_{17}$ :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique $T^*_{17}$ for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]}$$ 495

**Definition and uniqueness of** $T^*_{21}$ :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique $T^*_{21}$ for which $f(T^*_{21}) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

**Definition and uniqueness of** $T^*_{25}$ :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that

there exists a unique $T_{25}^{*}$ for which $f(T_{25}^{*})=0$. With this value , we obtain from the three first equations

$$G_{24}=\frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}=\frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T_{25}^{*})]}$$

**Definition and uniqueness of** $T_{29}^{*}$ :- 498

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(5)}(T_{29})$ being increasing, it follows that there exists a unique $T_{29}^{*}$ for which $f(T_{29}^{*})=0$. With this value , we obtain from the three first equations

$$G_{28}=\frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}=\frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T_{29}^{*})]}$$

**Definition and uniqueness of** $T_{33}^{*}$ :- 499

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(6)}(T_{33})$ being increasing, it follows that there exists a unique $T_{33}^{*}$ for which $f(T_{33}^{*})=0$. With this value , we obtain from the three first equations

$$G_{32}=\frac{(a_{32})^{(6)}G_{33}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^{*})]} \quad , \quad G_{34}=\frac{(a_{34})^{(6)}G_{33}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^{*})]}$$

**Definition and uniqueness of** $T_{37}^{*}$ :- 500

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(7)}(T_{37})$ being increasing, it follows that there exists a unique $T_{37}^{*}$ for which $f(T_{37}^{*})=0$. With this value , we obtain from the three first equations

$$G_{36}=\frac{(a_{36})^{(7)}G_{37}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^{*})]} \quad , \quad G_{38}=\frac{(a_{38})^{(7)}G_{37}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^{*})]}$$

**Definition and uniqueness of** $T_{41}^{*}$ :- 501

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique $T_{41}^{*}$ for which $f(T_{41}^{*})=0$. With this value , we obtain from the three first equations

$$G_{40}=\frac{(a_{40})^{(8)}G_{41}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T_{41}^{*})]} \quad , \quad G_{42}=\frac{(a_{42})^{(8)}G_{41}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T_{41}^{*})]}$$

**Definition and uniqueness of** $T_{45}^{*}$ :- 501 A

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(9)}(T_{45})$ being increasing, it follows that there exists a unique $T_{45}^{*}$ for which $f(T_{45}^{*})=0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)}G_{45}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T^*_{45})]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)}G_{45}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T^*_{45})]}$$

By the same argument, the equations admit solutions $G_{13}, G_{14}$ if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right]+(b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{14}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{14}$ such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions $G_{16}, G_{17}$ if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right]+(b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ 504
is a decreasing function in $G_{17}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{14}$ such that $\varphi((G_{19})^*) = 0$

By the same argument, the equations admit solutions $G_{20}, G_{21}$ if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right]+(b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{21}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{21}$ such that $\varphi((G_{23})^*) = 0$

By the same argument, the equations admit solutions $G_{24}, G_{25}$ if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right]+(b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{25}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{25}$ such that $\varphi((G_{27})^*) = 0$

By the same argument, the equations admit solutions $G_{28}, G_{29}$ if 507

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right]+(b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{29}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that

there exists a unique $G_{29}^{*}$ such that $\varphi((G_{31})^{*}) = 0$

By the same argument, the equations admit solutions $G_{32}, G_{33}$ if 508
$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$

$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right] + (b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{33}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{33}^{*}$ such that $\varphi(G_{35}{}^{*}) = 0$

By the same argument, the equations admit solutions $G_{36}, G_{37}$ if 509
$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$
$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right] + (b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{37}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{37}^{*}$ such that $\varphi(G_{39}{}^{*}) = 0$

By the same argument, the equations admit solutions $G_{40}, G_{41}$ if 510
$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$
$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right] + (b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{41}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{41}^{*}$ such that $\varphi(G_{43}{}^{*}) = 0$

By the same argument, the equations 92,93 admit solutions $G_{44}, G_{45}$ if
$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$

$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right] + (b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{45}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{45}^{*}$ such that $\varphi((G_{47})^{*}) = 0$

Finally we obtain the unique solution 511

$G_{14}^{*}$ given by $\varphi(G^{*}) = 0$ , $T_{14}^{*}$ given by $f(T_{14}^{*}) = 0$ and

$$G_{13}^{*} = \frac{(a_{13})^{(1)}G_{14}^{*}}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^{*})]} \quad , \quad G_{15}^{*} = \frac{(a_{15})^{(1)}G_{14}^{*}}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^{*})]}$$

$$T_{13}^{*} = \frac{(b_{13})^{(1)}T_{14}^{*}}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^{*})]} \quad , \quad T_{15}^{*} = \frac{(b_{15})^{(1)}T_{14}^{*}}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^{*})]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

$G_{17}^{*}$ given by $\varphi((G_{19})^{*}) = 0$ , $T_{17}^{*}$ given by $f(T_{17}^{*}) = 0$ and 512

$$G_{16}^{*} = \frac{(a_{16})^{(2)} G_{17}^{*}}{[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}^{*})]} \quad , \quad G_{18}^{*} = \frac{(a_{18})^{(2)} G_{17}^{*}}{[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}^{*})]} \qquad 513$$

$$T_{16}^{*} = \frac{(b_{16})^{(2)} T_{17}^{*}}{[(b'_{16})^{(2)} - (b''_{16})^{(2)}((G_{19})^{*})]} \quad , \quad T_{18}^{*} = \frac{(b_{18})^{(2)} T_{17}^{*}}{[(b'_{18})^{(2)} - (b''_{18})^{(2)}((G_{19})^{*})]} \qquad 514$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

$G_{21}^{*}$ given by $\varphi((G_{23})^{*}) = 0$ , $T_{21}^{*}$ given by $f(T_{21}^{*}) = 0$ and

$$G_{20}^{*} = \frac{(a_{20})^{(3)} G_{21}^{*}}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^{*})]} \quad , \quad G_{22}^{*} = \frac{(a_{22})^{(3)} G_{21}^{*}}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^{*})]}$$

$$T_{20}^{*} = \frac{(b_{20})^{(3)} T_{21}^{*}}{[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}{}^{*})]} \quad , \quad T_{22}^{*} = \frac{(b_{22})^{(3)} T_{21}^{*}}{[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}{}^{*})]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

$G_{25}^{*}$ given by $\varphi(G_{27}) = 0$ , $T_{25}^{*}$ given by $f(T_{25}^{*}) = 0$ and

$$G_{24}^{*} = \frac{(a_{24})^{(4)} G_{25}^{*}}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}^{*} = \frac{(a_{26})^{(4)} G_{25}^{*}}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^{*})]}$$

$$T_{24}^{*} = \frac{(b_{24})^{(4)} T_{25}^{*}}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^{*})]} \quad , \quad T_{26}^{*} = \frac{(b_{26})^{(4)} T_{25}^{*}}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^{*})]} \qquad 517$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

$G_{29}^{*}$ given by $\varphi((G_{31})^{*}) = 0$ , $T_{29}^{*}$ given by $f(T_{29}^{*}) = 0$ and

$$G_{28}^{*} = \frac{(a_{28})^{(5)} G_{29}^{*}}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}^{*} = \frac{(a_{30})^{(5)} G_{29}^{*}}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^{*})]}$$

$$T_{28}^{*} = \frac{(b_{28})^{(5)} T_{29}^{*}}{[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31})^{*})]} \quad , \quad T_{30}^{*} = \frac{(b_{30})^{(5)} T_{29}^{*}}{[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31})^{*})]} \qquad 519$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

$G_{33}^{*}$ given by $\varphi((G_{35})^{*}) = 0$ , $T_{33}^{*}$ given by $f(T_{33}^{*}) = 0$ and

$$G_{32}^{*} = \frac{(a_{32})^{(6)} G_{33}^{*}}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^{*})]} \quad , \quad G_{34}^{*} = \frac{(a_{34})^{(6)} G_{33}^{*}}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^{*})]}$$

$$T_{32}^{*} = \frac{(b_{32})^{(6)} T_{33}^{*}}{[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35})^{*})]} \quad , \quad T_{34}^{*} = \frac{(b_{34})^{(6)} T_{33}^{*}}{[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35})^{*})]} \qquad 521$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

$G_{37}^*$ given by $\varphi((G_{39})^*) = 0$ , $T_{37}^*$ given by $f(T_{37}^*) = 0$ and

$$G_{36}^* = \frac{(a_{36})^{(7)} G_{37}^*}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^*)]} \quad , \quad G_{38}^* = \frac{(a_{38})^{(7)} G_{37}^*}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^*)]}$$

$$T_{36}^* = \frac{(b_{36})^{(7)} T_{37}^*}{[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39})^*)]} \quad , \quad T_{38}^* = \frac{(b_{38})^{(7)} T_{37}^*}{[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39})^*)]}$$

Finally we obtain the unique solution 523

$G_{41}^*$ given by $\varphi((G_{43})^*) = 0$ , $T_{41}^*$ given by $f(T_{41}^*) = 0$ and

$$G_{40}^* = \frac{(a_{40})^{(8)} G_{41}^*}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42}^* = \frac{(a_{42})^{(8)} G_{41}^*}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

$$T_{40}^* = \frac{(b_{40})^{(8)} T_{41}^*}{[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43})^*)]} \quad , \quad T_{42}^* = \frac{(b_{42})^{(8)} T_{41}^*}{[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43})^*)]}$$

Finally we obtain the unique solution of 89 to 99 523 A

$G_{45}^*$ given by $\varphi((G_{47})^*) = 0$ , $T_{45}^*$ given by $f(T_{45}^*) = 0$ and

$$G_{44}^* = \frac{(a_{44})^{(9)} G_{45}^*}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46}^* = \frac{(a_{46})^{(9)} G_{45}^*}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

$$T_{44}^* = \frac{(b_{44})^{(9)} T_{45}^*}{[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47})^*)]} \quad , \quad T_{46}^* = \frac{(b_{46})^{(9)} T_{45}^*}{[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47})^*)]}$$

## ASYMPTOTIC STABILITY ANALYSIS 524

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

**Proof:** Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a''_{14})^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \frac{\partial (b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a'_{13})^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)}G^*_{13}\mathbb{T}_{14} \quad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a'_{14})^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)}G^*_{14}\mathbb{T}_{14} \quad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a'_{15})^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)}G^*_{15}\mathbb{T}_{14} \quad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b'_{13})^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)}T^*_{13}\mathbb{G}_j\right) \quad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b'_{14})^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)}T^*_{14}\mathbb{G}_j\right) \quad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b'_{15})^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T^*_{15}\mathbb{G}_j\right) \quad 530$$

**ASYMPTOTIC STABILITY ANALYSIS** 531

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(2)}$ and $(b''_i)^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G^*_i + \mathbb{G}_i \quad , T_i = T^*_i + \mathbb{T}_i \quad 532$$

$$\frac{\partial(a''_{17})^{(2)}}{\partial T_{17}}(T^*_{17}) = (q_{17})^{(2)} \quad , \frac{\partial(b''_i)^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \quad 533$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a'_{16})^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G^*_{16}\mathbb{T}_{17} \quad 534$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a'_{17})^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G^*_{17}\mathbb{T}_{17} \quad 535$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a'_{18})^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G^*_{18}\mathbb{T}_{17} \quad 536$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b'_{16})^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T^*_{16}\mathbb{G}_j\right) \quad 537$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b'_{17})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}T^*_{17}\mathbb{G}_j\right) \quad 538$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b'_{18})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}T^*_{18}\mathbb{G}_j\right) \quad 539$$

**ASYMPTOTIC STABILITY ANALYSIS** 540

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(3)}$ *and* $(b''_i)^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{21}'')^{(3)}}{\partial T_{21}}(T_{21}^*) = (q_{21})^{(3)} \quad , \frac{\partial (b_i'')^{(3)}}{\partial G_j}(\,(G_{23})^*\,) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\big((a_{20}')^{(3)} + (p_{20})^{(3)}\big)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G_{20}^*\mathbb{T}_{21} \quad 541$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\big((a_{21}')^{(3)} + (p_{21})^{(3)}\big)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G_{21}^*\mathbb{T}_{21} \quad 542$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\big((a_{22}')^{(3)} + (p_{22})^{(3)}\big)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G_{22}^*\mathbb{T}_{21} \quad 543$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\big((b_{20}')^{(3)} - (r_{20})^{(3)}\big)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\big(s_{(20)(j)}T_{20}^*\mathbb{G}_j\big) \quad 544$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\big((b_{21}')^{(3)} - (r_{21})^{(3)}\big)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\big(s_{(21)(j)}T_{21}^*\mathbb{G}_j\big) \quad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\big((b_{22}')^{(3)} - (r_{22})^{(3)}\big)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\big(s_{(22)(j)}T_{22}^*\mathbb{G}_j\big) \quad 546$$

**ASYMPTOTIC STABILITY ANALYSIS** 547

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}\ and\ (b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{25}'')^{(4)}}{\partial T_{25}}(T_{25}^*) = (q_{25})^{(4)} \quad , \frac{\partial (b_i'')^{(4)}}{\partial G_j}((G_{27})^*\,) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\big((a_{24}')^{(4)} + (p_{24})^{(4)}\big)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G_{24}^*\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\big((a_{25}')^{(4)} + (p_{25})^{(4)}\big)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G_{25}^*\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\big((a_{26}')^{(4)} + (p_{26})^{(4)}\big)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G_{26}^*\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\big((b_{24}')^{(4)} - (r_{24})^{(4)}\big)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\big(s_{(24)(j)}T_{24}^*\mathbb{G}_j\big) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\big((b_{25}')^{(4)} - (r_{25})^{(4)}\big)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\big(s_{(25)(j)}T_{25}^*\mathbb{G}_j\big) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T^*_{26}\mathbb{G}_j\right) \quad 554$$

**ASYMPTOTIC STABILITY ANALYSIS** 555

**Theorem 5:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}\ and\ (b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 556

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{29})^{(5)}}{\partial T_{29}}(T^*_{29}) = (q_{29})^{(5)} \ , \ \frac{\partial(b_i'')^{(5)}}{\partial G_j}\left((G_{31})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a'_{28})^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G^*_{28}\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a'_{29})^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G^*_{29}\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a'_{30})^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G^*_{30}\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b'_{28})^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T^*_{28}\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b'_{29})^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T^*_{29}\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b'_{30})^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T^*_{30}\mathbb{G}_j\right) \quad 562$$

**ASYMPTOTIC STABILITY ANALYSIS** 563

**Theorem 6:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}\ and\ (b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{33})^{(6)}}{\partial T_{33}}(T^*_{33}) = (q_{33})^{(6)} \ , \ \frac{\partial(b_i'')^{(6)}}{\partial G_j}\left((G_{35})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a'_{32})^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G^*_{32}\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a'_{33})^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G^*_{33}\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a'_{34})^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G^*_{34}\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b'_{32})^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T^*_{32}\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b'_{33})^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T^*_{33}\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b'_{34})^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T^*_{34}\mathbb{G}_j\right) \quad 570$$

**ASYMPTOTIC STABILITY ANALYSIS** 571

**Theorem 7:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 572

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{37})^{(7)}}{\partial T_{37}}(T^*_{37}) = (q_{37})^{(7)} \quad , \frac{\partial(b_i'')^{(7)}}{\partial G_j}\left((G_{39})^{**}\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a'_{36})^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G^*_{36}\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a'_{37})^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G^*_{37}\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a'_{38})^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G^*_{38}\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b'_{36})^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T^*_{36}\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b'_{37})^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T^*_{37}\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b'_{38})^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T^*_{38}\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

**ASYMPTOTIC STABILITY ANALYSIS**

**Theorem 8:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{41}'')^{(8)}}{\partial T_{41}} (T_{41}^*) = (q_{41})^{(8)} \quad , \frac{\partial (b_i'')^{(8)}}{\partial G_j} ( (G_{43})^* ) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\big((a_{40}')^{(8)} + (p_{40})^{(8)}\big)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)} G_{40}^* \mathbb{T}_{41} \quad (581)$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\big((a_{41}')^{(8)} + (p_{41})^{(8)}\big)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)} G_{41}^* \mathbb{T}_{41} \quad (582)$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\big((a_{42}')^{(8)} + (p_{42})^{(8)}\big)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)} G_{42}^* \mathbb{T}_{41} \quad (583)$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\big((b_{40}')^{(8)} - (r_{40})^{(8)}\big)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\big(s_{(40)(j)} T_{40}^* \mathbb{G}_j\big) \quad (584)$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\big((b_{41}')^{(8)} - (r_{41})^{(8)}\big)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\big(s_{(41)(j)} T_{41}^* \mathbb{G}_j\big) \quad (585)$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\big((b_{42}')^{(8)} - (r_{42})^{(8)}\big)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\big(s_{(42)(j)} T_{42}^* \mathbb{G}_j\big) \quad (586)$$

(586 A)

**ASYMPTOTIC STABILITY ANALYSIS**

**Theorem 9:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}$ *and* $(b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

**Proof:** Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{45}'')^{(9)}}{\partial T_{45}} (T_{45}^*) = (q_{45})^{(9)} \quad , \frac{\partial (b_i'')^{(9)}}{\partial G_j} ( (G_{47})^* ) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\big((a_{44}')^{(9)} + (p_{44})^{(9)}\big)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)} G_{44}^* \mathbb{T}_{45} \quad (586\,B)$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\big((a_{45}')^{(9)} + (p_{45})^{(9)}\big)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)} G_{45}^* \mathbb{T}_{45} \quad (586\,C)$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\big((a_{46}')^{(9)} + (p_{46})^{(9)}\big)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)} G_{46}^* \mathbb{T}_{45} \quad (586\,D)$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\big((b_{44}')^{(9)} - (r_{44})^{(9)}\big)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\big(s_{(44)(j)} T_{44}^* \mathbb{G}_j\big) \quad (586\,E)$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\big((b_{45}')^{(9)} - (r_{45})^{(9)}\big)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\big(s_{(45)(j)} T_{45}^* \mathbb{G}_j\big) \quad (586\,F)$$

$\frac{d\mathbb{T}_{46}}{dt} = -\left((b'_{46})^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T^*_{46}\mathbb{G}_j\right)$ 586

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**The characteristic equation of this system is** 587

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(q_{13})^{(1)}G^*_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)}T^*_{14} + (b_{14})^{(1)}s_{(13),(14)}T^*_{14}\right)$$

$$+\left(\left((\lambda)^{(1)} + (a'_{14})^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)}G^*_{13} + (a_{13})^{(1)}(q_{14})^{(1)}G^*_{14}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)}T^*_{14} + (b_{14})^{(1)}s_{(13),(13)}T^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((b'_{13})^{(1)} + (b'_{14})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(15)}T^*_{14} + (b_{14})^{(1)}s_{(13),(15)}T^*_{13}\right)\} = 0$$

$$+$$

$$(\lambda)^{(2)} + (b'_{18})^{(2)} - (r_{18})^{(2)}\{\left((\lambda)^{(2)} + (a'_{18})^{(2)} + (p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^*_{17} + (a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^*_{17}\right)$$

$$+\left(\left((\lambda)^{(2)} + (a'_{17})^{(2)} + (p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^*_{16} + (a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}\right)$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2 + \left((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2 + \left((b'_{16})^{(2)} + (b'_{17})^{(2)} - (r_{16})^{(2)} + (r_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$+\left(\left((\lambda)^{(2)}\right)^2 + \left((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17} + (a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^{*}_{16}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\right)\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left(\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)\right]$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(21)}T^{*}_{21}\right)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^{*}_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^{*}_{21}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(20)}T^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$+\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(22)}T^{*}_{20}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\right)\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left(\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)\right]$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(25)}T^{*}_{25}\right)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^{*}_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^{*}_{25}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(24)}T^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$+\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(26)}T^{*}_{24}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\right)\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(29)}T^{*}_{29}\right)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^{*}_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^{*}_{29}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(28)}T^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$+\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(30)}T^{*}_{28}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\right)\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left(\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)\right]$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(33)}T^{*}_{33}\right)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^{*}_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^{*}_{33}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(32)}T^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$+\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(34)}T^{*}_{32}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\right)\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)\right]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(37)}T^{*}_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^{*}_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^{*}_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(36)}T^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$+\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(38)}T^{*}_{36}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\right)\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(41)}T^{*}_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^{*}_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^{*}_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(40)}T^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$+\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(42)}T^{*}_{40}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\right)\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(45)}T^{*}_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^{*}_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^{*}_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(44)}T^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$+\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(46)}T^{*}_{44}\right)\}=0$$

**And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.**

## SECTION TWENTY SEVEN

## Planck's Constant And The Objective And Subjective Consciousness

### INTRODUCTION—VARIABLES USED

**Antonella Vannini Quantum Biosystems 2008, 2, 165-184 165 Quantum Models of Consciousness email: antonella.vannini@uniroma1.it**

In this paper a short review of quantum models of consciousness is presented. In the conclusions two criteria are offered in order to classify these models, and a choice is operated towards those models which integrate

quantum mechanics with special relativity.

1. CHRONOLOGICAL ORDER OF QUANTUM MODELS OF CONSCIOUSNESS

1.1. **Alfred Lotka: Planck's constant and the objective and subjective consciousness -**

1925.

(1) Lotka (1925) suggested his model before Heisenberg's uncertainty principle and the formulation of the interpretation of Copenhagen.

(2) Lotka's model can be considered the first quantum model of consciousness.

(3) Lotka suggested the existence of two types of consciousness. The first one, which he called deterministic, corresponds to (e&eb) the objectivity of the external world; the second one, which he called subjective, corresponds to (e&eb) the inner world (qualia).

According to Lotka, consciousness of the first type (deterministic) is relative to (e&eb) all those phenomena which take place above (e) Planck's constant.

(82) At this level it was considered that the deterministic laws of classical physics took place. Consciousness of the second type (subjective) would, on the contrary, take place below (e) Planck's constant, where (e) the deterministic laws of classical physics do not operate.

**Antonella Vannini Quantum Biosystems 2008, 2, 165-184 166**

**1.2. Niels Bohr model:**

Consciousness creates (eb) reality through (e&eb) the collapse of the wave function - 1930. The interpretation of Copenhagen proposed by Niels Bohr and Werner Heisenberg, suggests a direct link between consciousness and (e&eb) Quantum Mechanics. This interpretation attributes to (e) consciousness an explicit role, through (e&eb) the act of measurement and observation, forcing (eb) the wave function to collapse into (e&eb) a particle and determining (eb) in this way reality itself. According to Bohr and Heisenberg consciousness would be (=) an immanent property of reality which exists before (e) the creation of reality.

**NOTATION**

**Module One**

the possibility that **oscillations may subserve a gating function and those oscillations** may play a role (e&eb) in the formation of assemblies of neurons that represent given stimulus patterns

$G_{13}$ : Category one of **oscillations may subserve a gating function and those oscillations;** formation of assemblies of neurons that represent given stimulus patterns

$G_{14}$ : Category two of SAS

$G_{15}$ : Category three of SAS

$T_{13}$ : Category one of formation of assemblies of neurons that represent given stimulus patterns **;oscillations may subserve a gating function and those oscillations**

$T_{14}$ : Category two of SAS

$T_{15}$ : Category three of SAS

**Module Two**

**Electroencephalography and Clinical Neurophysiology Volume 79, Issue 2, August 1991, Pages 81-93 Neural mechanisms underlying brain waves: from neural membranes to networks:. Fernando Lopes da Silva**

(1) In particular, the question of whether spindle oscillations are (=) autonomous or ∝ input-dependent is addressed.
(2) At the neural network level, the main circuits of the thalamus and cortex that are responsible for the occurrence and modulation of (e&eb) spindles and alpha activity are described.
(3) In addition, the properties of rhythmic activities outside the alpha band are considered, particularly in relation to (e&eb) the prominent beta activity of the visual cortex.
(4) At the theoretical level, the possibility that neural networks may behave as (e&eb) complex dynamic systems with (e&eb) the properties of deterministic chaos is discussed.
(5) Finally, the fact that brain rhythms may have (e) functional implications for the working of neural networks is examined in relation to 2 cases: the possibility that oscillations may subserve (e&eb) a gating function and those oscillations may play a role (e&eb) in the formation of assemblies of neurons that represent given stimulus patterns.

In particular, the question of whether **spindle oscillations** are (=) autonomous or ∝ input-dependent is addressed

$G_{16}$ : Category one of **spindle oscillations**

$G_{17}$ : Category two of SAS

$G_{18}$ : Category three of SAS

$T_{16}$ : Category one of autonomous

$T_{17}$ : Category two of SAS

$T_{18}$ : Category three of SAS

**Module three**

$G_{20}$ : Category one of **spindle oscillations**

$G_{21}$ : Category two of SAS

$G_{22}$ : Category three of SAS

$T_{20}$ : Category one of input-dependent

$T_{21}$ : Category two of SAS

$T_{22}$ : Category three of SAS

**Module four**

At the neural network level, the main circuits of the thalamus and cortex that are responsible for the occurrence and modulation of (e&eb) **spindles and alpha activity** are described

$G_{24}$ : Category one of main circuits of the thalamus and cortex that are responsible for the occurrence and modulation; **spindles and alpha activity**

$G_{25}$ : Category two of SAS

$G_{26}$ : Category three of SAS

$T_{24}$ : Category one of **spindles and alpha activity** ; main circuits of the thalamus and cortex that are responsible for the occurrence and modulation

$T_{25}$ : Category two of SAS

$T_{26}$ : Category three of SAS

**Module five**

In addition, the properties of rhythmic activities outside the alpha band are considered, particularly in relation to (e&eb) the prominent beta activity of the visual cortex

$G_{28}$ : Category one of **properties of rhythmic activities outside the alpha band;** prominent beta activity of the visual cortex

$G_{29}$ : Category two of SAS

$G_{30}$ : Category three of SAS

$T_{28}$ : Category one of prominent beta activity of the visual cortex; **properties of rhythmic activities outside the alpha band**

$T_{29}$ : Category two of SAS

$T_{30}$ : Category three of SAS

**Module six**

At the theoretical level, the possibility that neural networks may behave as (e&eb) complex dynamic systems with (e&eb) the properties of deterministic chaos is discussed

$G_{32}$ : Category one of **possibility that neural networks;** complex dynamic systems with (e&eb) the properties of deterministic chaos

$G_{33}$ : Category two of SAS

$G_{34}$ : Category three of SAS

$T_{32}$ : Category one of complex dynamic systems with (e&eb) the properties of deterministic chaos; **possibility that neural networks**

$T_{33}$ : Category two of SAS

$T_{34}$ : Category three of SAS

**Module seven**

At the theoretical level, the **possibility that neural networks may behave as complex dynamic systems** with (e&eb) the properties of deterministic chaos is discussed

$G_{36}$ : Category one of **possibility that neural networks may behave as complex dynamic systems;** properties of deterministic chaos

$G_{37}$ : Category two of SAS

$G_{38}$ : Category three of SAS

$T_{36}$ : Category one of properties of deterministic chaos **;possibility that neural networks may behave as complex dynamic systems**

$T_{37}$ : Category two of SAS

$T_{38}$ : Category three of SAS

**Module eight**

Lotka suggested the existence of two types of consciousness

The first one, which he called deterministic, corresponds to (e&eb) the **objectivity of the external world**; the second one, which he called subjective, corresponds to (e&eb) the inner world (qualia).

According to Lotka, consciousness of the first type (deterministic) is relative to (e&eb) all those phenomena which take place above (e) Planck's constant

$G_{40}$ : Category one of deterministic consciousness; **objectivity of the external world**

$G_{41}$ : Category two of SAS

$G_{42}$ : Category three of SAS

$T_{40}$ : Category one of **objectivity of the external world;** deterministic consciousness

$T_{41}$ : Category two of SAS

$T_{42}$ : Category three of SAS

**Module Nine**

$G_{44}$ : Category one of **subjective consciousness;** inner world (qualia).

$G_{45}$ : Category two of SAS

$G_{46}$ : Category three of SAS

$T_{44}$ : Category one of inner world (qualia). ; **subjective consciousness**

$T_{45}$ : Category two of SAS

$T_{46}$ : Category three of SAS

**The Coefficients:**

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}\ ,\ (b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)},$
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)},\ \ (b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}\ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}\ ,$
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)},$
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)},$
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)},$
are Dissipation coefficients

**Module Numbered One**

**The differential system of this model is now (Module Numbered one)**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \quad 1$$
$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad 2$$
$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15} \quad 3$$
$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13} \quad 4$$
$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14} \quad 5$$
$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)\right]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14}, t) =$ First augmentation factor
$-(b''_{13})^{(1)}(G, t) =$ First detritions factor

**Module Numbered Two**

**The differential system of this model is now ( Module numbered two)**

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right]G_{16} \quad 7$$
$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t)\right]G_{17} \quad 8$$
$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t)\right]G_{18} \quad 9$$
$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\big((G_{19}), t\big)\right]T_{16} \quad 10$$
$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\big((G_{19}), t\big)\right]T_{17} \quad 11$$
$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\big((G_{19}), t\big)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17}, t) =$ First augmentation factor
$-(b''_{16})^{(2)}\big((G_{19}), t\big) =$ First detritions factor

**Module Numbered Three**

**The differential system of this model is now (Module numbered three)**

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21},t)\right]G_{20} \quad 13$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21},t)\right]G_{21} \quad 14$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21},t)\right]G_{22} \quad 15$$

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23},t)\right]T_{20} \quad 16$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23},t)\right]T_{21} \quad 17$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23},t)\right]T_{22} \quad 18$$

$+(a''_{20})^{(3)}(T_{21},t) =$ First augmentation factor

$-(b''_{20})^{(3)}(G_{23},t) =$ First detritions factor

**Module Numbered Four**

**The differential system of this model is now (Module numbered Four)**

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right]G_{24} \quad 19$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25},t)\right]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25},t)\right]G_{26} \quad 21$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27}),t\right)\right]T_{24} \quad 22$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27}),t\right)\right]T_{25} \quad 23$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27}),t\right)\right]T_{26} \quad 24$$

$+(a''_{24})^{(4)}(T_{25},t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}\left((G_{27}),t\right) =$ **First detritions factor**

**Module Numbered Five**:

**The differential system of this model is now (Module number five)**

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29},t)\right]G_{28} \quad 25$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29},t)\right]G_{29} \quad 26$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29},t)\right]G_{30} \quad 27$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}\left((G_{31}),t\right)\right]T_{28} \quad 28$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}\left((G_{31}),t\right)\right]T_{29} \quad 29$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}\left((G_{31}),t\right)\right]T_{30} \quad 30$$

$+(a''_{28})^{(5)}(T_{29},t) =$ **First augmentation factor**

$-(b''_{28})^{(5)}\left((G_{31}),t\right) =$ **First detritions factor**

**Module Numbered Six**

**The differential system of this model is now (Module numbered Six)**

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33},t)\right]G_{32} \quad 31$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33},t)\right]G_{33} \quad 32$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}, t)\right]G_{34} \quad 33$$
$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35}), t)\right]T_{32} \quad 34$$
$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}((G_{35}), t)\right]T_{33} \quad 35$$
$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35}), t)\right]T_{34} \quad 36$$
$+(a''_{32})^{(6)}(T_{33}, t) =$ **First augmentation factor**

**Module Numbered Seven:**

**The differential system of this model is now (Seventh Module)**

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right]G_{36} \quad 37$$
$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37}, t)\right]G_{37} \quad 38$$
$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}, t)\right]G_{38} \quad 39$$
$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39}), t)\right]T_{36} \quad 40$$
$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}((G_{39}), t)\right]T_{37} \quad 41$$
$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39}), t)\right]T_{38} \quad 42$$
$+(a''_{36})^{(7)}(T_{37}, t) =$ **First augmentation factor**

**Module Numbered Eight**

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right]G_{40} \quad 43$$
$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41}, t)\right]G_{41} \quad 44$$
$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}, t)\right]G_{42} \quad 45$$
$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43}), t)\right]T_{40} \quad 46$$
$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}((G_{43}), t)\right]T_{41} \quad 47$$
$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43}), t)\right]T_{42} \quad 48$$

**Module Numbered Nine**

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}, t)\right]G_{44} \quad 49$$
$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45}, t)\right]G_{45} \quad 50$$
$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}, t)\right]G_{46} \quad 51$$
$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47}), t)\right]T_{44} \quad 52$$
$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}((G_{47}), t)\right]T_{45} \quad 53$$
$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47}), t)\right]T_{46} \quad 54$$
$+(a''_{44})^{(9)}(T_{45}, t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}((G_{47}), t) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[\begin{array}{ccc} (a'_{13})^{(1)}\boxed{+(a''_{13})^{(1)}(T_{14}, t)} & \boxed{+(a''_{16})^{(2,2,)}(T_{17}, t)} & \boxed{+(a''_{20})^{(3,3,)}(T_{21}, t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25}, t)} & \boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29}, t)} & \boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33}, t)} \\ \boxed{+(a''_{36})^{(7,7)}(T_{37}, t)} & \boxed{+(a''_{40})^{(8,8)}(T_{41}, t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45}, t)} \end{array}\right]G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)}\boxed{+(a''_{14})^{(1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14} \quad (56)$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)}\boxed{+(a''_{15})^{(1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15} \quad (57)$$

Where $\boxed{(a''_{13})^{(1)}(T_{14},t)}$, $\boxed{(a''_{14})^{(1)}(T_{14},t)}$, $\boxed{(a''_{15})^{(1)}(T_{14},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7)}(T_{37},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8)}(T_{41},t)}$ $\boxed{+(a''_{41})^{(8,8)}(T_{41},t)}$ $\boxed{+(a''_{42})^{(8,8)}(T_{41},t)}$ are eight augmentation coefficient for 1,2,3

$\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)}\boxed{-(b''_{13})^{(1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13} \quad (58)$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)}\boxed{-(b''_{14})^{(1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14} \quad (59)$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)}\boxed{-(b''_{15})^{(1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15} \quad (60)$$

Where $\boxed{-(b''_{13})^{(1)}(G,t)}$, $\boxed{-(b''_{14})^{(1)}(G,t)}$, $\boxed{-(b''_{15})^{(1)}(G,t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8)}(G_{43},t)}$ $\boxed{-(b''_{41})^{(8,8)}(G_{43},t)}$ $\boxed{-(b''_{42})^{(8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[\begin{array}{ccc} (a'_{16})^{(2)}\boxed{+(a''_{16})^{(2)}(T_{17},t)} & \boxed{+(a''_{13})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9)}(T_{45},t)} \end{array}\right] G_{16} \quad (61)$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[\begin{array}{ccc} (a'_{17})^{(2)}\boxed{+(a''_{17})^{(2)}(T_{17},t)} & \boxed{+(a''_{14})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9)}(T_{45},t)} \end{array}\right] G_{17} \quad (62)$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[\begin{array}{ccc} (a'_{18})^{(2)}\boxed{+(a''_{18})^{(2)}(T_{17},t)} & \boxed{+(a''_{15})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9)}(T_{45},t)} \end{array}\right] G_{18} \quad (63)$$

Where $\boxed{+(a''_{16})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,)}(T_{14},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$+(a''_{40})^{(8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8)}(T_{41},t)$ are eight augmentation coefficient for category 1, 2 and 3

$+(a''_{44})^{(9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9)}(T_{45},t)$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \begin{bmatrix} (b'_{16})^{(2)} & -(b''_{16})^{(2)}(G_{19},t) & -(b''_{13})^{(1,1,)}(G,t) & -(b''_{20})^{(3,3,3,)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6)}(G_{35},t) & \\ -(b''_{36})^{(7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9)}(G_{47},t) & \end{bmatrix} T_{16} \quad (64)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)} & -(b''_{17})^{(2)}(G_{19},t) & -(b''_{14})^{(1,1,)}(G,t) & -(b''_{21})^{(3,3,3,)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6)}(G_{35},t) & \\ -(b''_{37})^{(7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9)}(G_{47},t) & \end{bmatrix} T_{17} \quad (65)$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)} & -(b''_{18})^{(2)}(G_{19},t) & -(b''_{15})^{(1,1,)}(G,t) & -(b''_{22})^{(3,3,3,)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6)}(G_{35},t) & \\ -(b''_{38})^{(7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9)}(G_{47},t) & \end{bmatrix} T_{18} \quad (66)$$

where $-(b''_{16})^{(2)}(G_{19},t)$, $-(b''_{17})^{(2)}(G_{19},t)$, $-(b''_{18})^{(2)}(G_{19},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{13})^{(1,1,)}(G,t)$, $-(b''_{14})^{(1,1,)}(G,t)$, $-(b''_{15})^{(1,1,)}(G,t)$ are second detrition coefficients for category 1,2 and 3

$-(b''_{20})^{(3,3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,)}(G_{23},t)$ are third detrition coefficients for category 1,2 and 3

$-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1,2 and 3

$-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1,2 and 3

$-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients for category 1,2 and 3

$-(b''_{36})^{(7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2 and 3

$-(b''_{40})^{(8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1,2 and 3

$-(b''_{44})^{(9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \begin{bmatrix} (a'_{20})^{(3)} & +(a''_{20})^{(3)}(T_{21},t) & +(a''_{16})^{(2,2,2)}(T_{17},t) & +(a''_{13})^{(1,1,1,)}(T_{14},t) \\ +(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t) & \\ +(a''_{36})^{(7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9)}(T_{45},t) & \end{bmatrix} G_{20} \quad (67)$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \begin{bmatrix} (a'_{21})^{(3)}\boxed{+(a''_{21})^{(3)}(T_{21},t)}\ \boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}\ \boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{21} \quad 68$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \begin{bmatrix} (a'_{22})^{(3)}\boxed{+(a''_{22})^{(3)}(T_{21},t)}\ \boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}\ \boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{22} \quad 69$$

$\boxed{+(a''_{20})^{(3)}(T_{21},t)}, \boxed{+(a''_{21})^{(3)}(T_{21},t)}, \boxed{+(a''_{22})^{(3)}(T_{21},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}, \boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}, \boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}$ are second augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)}$ are third augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)}, \boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)}, \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)}, \boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)}, \boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)}, \boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)}, \boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)}$ are seventh augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)}$ are eight augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{44})^{(9,9,9)}(T_{45},t)}, \boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)}, \boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)}$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b'_{20})^{(3)}\boxed{-(b''_{20})^{(3)}(G_{23},t)}\ \boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}\ \boxed{-(b''_{13})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)}\ \boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)}\ \boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7)}(G_{39},t)}\ \boxed{-(b''_{40})^{(8,8,8,8)}(G_{43},t)}\ \boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{20} \quad 70$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b'_{21})^{(3)}\boxed{-(b''_{21})^{(3)}(G_{23},t)}\ \boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}\ \boxed{-(b''_{14})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)}\ \boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)}\ \boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7)}(G_{39},t)}\ \boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)}\ \boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{21} \quad 71$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)}\boxed{-(b''_{22})^{(3)}(G_{23},t)}\ \boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}\ \boxed{-(b''_{15})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)}\ \boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)}\ \boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)}\ \boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)}\ \boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{22} \quad 72$$

$\boxed{-(b''_{20})^{(3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3)}(G_{23},t)}$ *are first detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}$ *are second detrition coefficients for*

*category 1, 2 and 3*

$\boxed{-(b_{13}'')^{(1,1,1,)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,)}(G,t)}$, $\boxed{-(b_{15}'')^{(1,1,1,)}(G,t)}$ *are third detrition coefficients for category 1,2 and 3*

$\boxed{-(b_{24}'')^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{25}'')^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{26}'')^{(4,4,4,4,4,4)}(G_{27},t)}$ *are fourth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b_{28}'')^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5)}(G_{31},t)}$ *are fifth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b_{32}'')^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{34}'')^{(6,6,6,6,6,6)}(G_{35},t)}$ *are sixth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b_{36}'')^{(7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{37}'')^{(7,7,7,7)}(G_{39},t)}$ $\boxed{-(b_{38}'')^{(7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{40}'')^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{42}'')^{(8,8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{46}'')^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b_{45}'')^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b_{44}'')^{(9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[\begin{array}{ccc} (a_{24}')^{(4)} & \boxed{+(a_{24}'')^{(4)}(T_{25},t)} & \boxed{+(a_{28}'')^{(5,5,)}(T_{29},t)} \quad \boxed{+(a_{32}'')^{(6,6,)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a_{16}'')^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a_{20}'')^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{36}'')^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{40}'')^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a_{44}'')^{(9,9,9,9)}(T_{45},t)} \end{array}\right] G_{24} \quad (73)$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[\begin{array}{ccc} (a_{25}')^{(4)} & \boxed{+(a_{25}'')^{(4)}(T_{25},t)} & \boxed{+(a_{29}'')^{(5,5,)}(T_{29},t)} \quad \boxed{+(a_{33}'')^{(6,6)}(T_{33},t)} \\ \boxed{+(a_{14}'')^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a_{17}'')^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a_{21}'')^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{37}'')^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{41}'')^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a_{45}'')^{(9,9,9,9)}(T_{45},t)} \end{array}\right] G_{25} \quad (74)$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[\begin{array}{ccc} (a_{26}')^{(4)} & \boxed{+(a_{26}'')^{(4)}(T_{25},t)} & \boxed{+(a_{30}'')^{(5,5,)}(T_{29},t)} \quad \boxed{+(a_{34}'')^{(6,6,)}(T_{33},t)} \\ \boxed{+(a_{15}'')^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a_{18}'')^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a_{22}'')^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{38}'')^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{42}'')^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a_{46}'')^{(9,9,9,9)}(T_{45},t)} \end{array}\right] G_{26} \quad (75)$$

$\boxed{(a_{24}'')^{(4)}(T_{25},t)}$, $\boxed{(a_{25}'')^{(4)}(T_{25},t)}$, $\boxed{(a_{26}'')^{(4)}(T_{25},t)}$ *are first augmentation coefficients category* $1, 2$ $3$

$\boxed{+(a_{28}'')^{(5,5,)}(T_{29},t)}$, $\boxed{+(a_{29}'')^{(5,5,)}(T_{29},t)}$, $\boxed{+(a_{30}'')^{(5,5,)}(T_{29},t)}$ *are second augmentation coefficient for category* $1, 2$ *and* $3$

$\boxed{+(a_{32}'')^{(6,6,)}(T_{33},t)}$, $\boxed{+(a_{33}'')^{(6,6,)}(T_{33},t)}$, $\boxed{+(a_{34}'')^{(6,6,)}(T_{33},t)}$ *are third augmentation coefficient for category* $1, 2$ *and* $3$

$\boxed{+(a_{13}'')^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a_{14}'')^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a_{15}'')^{(1,1,1,1)}(T_{14},t)}$ *are fourth augmentation coefficients for category* $1, 2$ *and* $3$

$\boxed{+(a_{16}'')^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{17}'')^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{18}'')^{(2,2,2,2)}(T_{17},t)}$ *are fifth augmentation coefficients for category* $1, 2$ *and* $3$

$\boxed{+(a_{20}'')^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{21}'')^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{22}'')^{(3,3,3,3)}(T_{21},t)}$ *are sixth augmentation coefficients for category* $1, 2$ *and* $3$

$\boxed{+(a_{36}'')^{(7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a_{37}'')^{(7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a_{38}'')^{(7,7,7,7,7)}(T_{37},t)}$ *are seventh augmentation coefficients for category* $1, 2$ *and* $3$

$+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)$

*are eighth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{46})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9)}(T_{45},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt}=(b_{24})^{(4)}T_{25}-\begin{bmatrix}(b'_{24})^{(4)} & -(b''_{24})^{(4)}(G_{27},t) & -(b''_{28})^{(5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,)}(G_{35},t)\\ & -(b''_{13})^{(1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3)}(G_{23},t)\\ & -(b''_{36})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9)}(G_{47},t)\end{bmatrix}T_{24} \quad (76)$$

$$\frac{dT_{25}}{dt}=(b_{25})^{(4)}T_{24}-\begin{bmatrix}(b'_{25})^{(4)} & -(b''_{25})^{(4)}(G_{27},t) & -(b''_{29})^{(5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,)}(G_{35},t)\\ & -(b''_{14})^{(1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3)}(G_{23},t)\\ & -(b''_{37})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9)}(G_{47},t)\end{bmatrix}T_{25} \quad (77)$$

$$\frac{dT_{26}}{dt}=(b_{26})^{(4)}T_{25}-\begin{bmatrix}(b'_{26})^{(4)} & -(b''_{26})^{(4)}(G_{27},t) & -(b''_{30})^{(5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,)}(G_{35},t)\\ & -(b''_{15})^{(1,1,1,1)}(G,t) & -(b''_{18})^{(2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3)}(G_{23},t)\\ & -(b''_{38})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9)}(G_{47},t)\end{bmatrix}T_{26} \quad (78)$$

*Where* $-(b''_{24})^{(4)}(G_{27},t)$, $-(b''_{25})^{(4)}(G_{27},t)$, $-(b''_{26})^{(4)}(G_{27},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{28})^{(5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,)}(G_{31},t)$ *are second detrition coefficients for category* 1, 2 *and* 3

$-(b''_{32})^{(6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,)}(G_{35},t)$ *are third detrition coefficients for category* 1, 2 *and* 3

$-(b''_{13})^{(1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1)}(G,t)$

*are fourth detrition coefficients for category* 1, 2 *and* 3

$-(b''_{16})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2)}(G_{19},t)$

*are fifth detrition coefficients for category* 1, 2 *and* 3

$-(b''_{20})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3)}(G_{23},t)$

*are sixth detrition coefficients for category* 1, 2 *and* 3

$-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)$

*are seventh detrition coefficients for category* 1, 2 *and* 3

$-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)$

*are eighth detrition coefficients for category* 1, 2 *and* 3

$-(b''_{46})^{(9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt}=(a_{28})^{(5)}G_{29}-\begin{bmatrix}(a'_{28})^{(5)} & +(a''_{28})^{(5)}(T_{29},t) & +(a''_{24})^{(4,4,)}(T_{25},t) & +(a''_{32})^{(6,6,6)}(T_{33},t)\\ & +(a''_{13})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{16})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3,3)}(T_{21},t)\\ & +(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9)}(T_{45},t)\end{bmatrix}G_{28} \quad (79)$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \begin{bmatrix} (a'_{29})^{(5)} & +(a''_{29})^{(5)}(T_{29},t) & +(a''_{25})^{(4,4,)}(T_{25},t) & +(a''_{33})^{(6,6,6)}(T_{33},t) \\ +(a''_{14})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{17})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3,3)}(T_{21},t) \\ +(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{29} \quad (80)$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \begin{bmatrix} (a'_{30})^{(5)} & +(a''_{30})^{(5)}(T_{29},t) & +(a''_{26})^{(4,4,)}(T_{25},t) & +(a''_{34})^{(6,6,6)}(T_{33},t) \\ +(a''_{15})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{18})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3,3)}(T_{21},t) \\ +(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{30} \quad (81)$$

*Where* $+(a''_{28})^{(5)}(T_{29},t)$, $+(a''_{29})^{(5)}(T_{29},t)$, $+(a''_{30})^{(5)}(T_{29},t)$ *are first augmentation coefficients for category* 1, 2 *and* 3

*And* $+(a''_{24})^{(4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,)}(T_{25},t)$ *are second augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{32})^{(6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6)}(T_{33},t)$ *are third augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)$ are fourth augmentation coefficients for category 1,2, and 3

$+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)$ are fifth augmentation coefficients for category 1,2,and 3

$+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)$ are sixth augmentation coefficients for category 1,2, 3

$+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)$ , $+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1,2, 3

$+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)$ are eighth augmentation coefficients for category 1,2, 3

$+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \begin{bmatrix} (b'_{28})^{(5)} & -(b''_{28})^{(5)}(G_{31},t) & -(b''_{24})^{(4,4,)}(G_{27},t) & -(b''_{32})^{(6,6,6)}(G_{35},t) \\ -(b''_{13})^{(1,1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3)}(G_{23},t) \\ -(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{28} \quad (82)$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b'_{29})^{(5)} & -(b''_{29})^{(5)}(G_{31},t) & -(b''_{25})^{(4,4,)}(G_{27},t) & -(b''_{33})^{(6,6,6)}(G_{35},t) \\ -(b''_{14})^{(1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3)}(G_{23},t) \\ -(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{29} \quad (83)$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b'_{30})^{(5)} & -(b''_{30})^{(5)}(G_{31},t) & -(b''_{26})^{(4,4,)}(G_{27},t) & -(b''_{34})^{(6,6,6)}(G_{35},t) \\ -(b''_{15})^{(1,1,1,1,1,)}(G,t) & -(b''_{18})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3)}(G_{23},t) \\ -(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{30} \quad (84)$$

*where* $-(b''_{28})^{(5)}(G_{31},t)$ , $-(b''_{29})^{(5)}(G_{31},t)$ , $-(b''_{30})^{(5)}(G_{31},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{24})^{(4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,)}(G_{27},t)$ *are second detrition coefficients*

$for\ category\ 1,2\ and\ 3$

$\boxed{-(b''_{32})^{(6,6,6)}(G_{35},t)}, \boxed{-(b''_{33})^{(6,6,6)}(G_{35},t)}, \boxed{-(b''_{34})^{(6,6,6)}(G_{35},t)}$ $are\ third\ detrition\ coefficients$

$for\ category\ 1,2\ and\ 3$

$\boxed{-(b''_{13})^{(1,1,1,1,1)}(G,t)}, \boxed{-(b''_{14})^{(1,1,1,1,1)}(G,t)}, \boxed{-(b''_{15})^{(1,1,1,1,1,)}(G,t)}$ are fourth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \begin{bmatrix} (a'_{32})^{(6)}\boxed{+(a''_{32})^{(6)}(T_{33},t)}\ \boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}\ \boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}\ \boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}\ \boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{32} \quad 85$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \begin{bmatrix} (a'_{33})^{(6)}\boxed{+(a''_{33})^{(6)}(T_{33},t)}\ \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}\ \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}\ \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}\ \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{33} \quad 86$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \begin{bmatrix} (a'_{34})^{(6)}\boxed{+(a''_{34})^{(6)}(T_{33},t)}\ \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}\ \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}\ \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}\ \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{34} \quad 87$$

$\boxed{+(a''_{32})^{(6)}(T_{33},t)}, \boxed{+(a''_{33})^{(6)}(T_{33},t)}, \boxed{+(a''_{34})^{(6)}(T_{33},t)}$ $are\ first\ augmentation\ coefficients$

$for\ category\ 1,2\ and\ 3$

$\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}$ $are\ second\ augmentation$

$coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}$ $are\ third\ augmentation$

$coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}$ - are fourth augmentation coefficients

$\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}$ - fifth augmentation coefficients

$\boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)}$ sixth augmentation coefficients

$+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)$
seventh augmentation coefficients
$+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)$
Eighth augmentation coefficients
$+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \begin{bmatrix} (b'_{32})^{(6)} & -(b''_{32})^{(6)}(G_{35},t) & -(b''_{28})^{(5,5,5)}(G_{31},t) & -(b''_{24})^{(4,4,4,)}(G_{27},t) \\ & -(b''_{13})^{(1,1,1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{32} \quad (88)$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \begin{bmatrix} (b'_{33})^{(6)} & -(b''_{33})^{(6)}(G_{35},t) & -(b''_{29})^{(5,5,5)}(G_{31},t) & -(b''_{25})^{(4,4,4,)}(G_{27},t) \\ & -(b''_{14})^{(1,1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{33} \quad (89)$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)} & -(b''_{34})^{(6)}(G_{35},t) & -(b''_{30})^{(5,5,5)}(G_{31},t) & -(b''_{26})^{(4,4,4,)}(G_{27},t) \\ & -(b''_{15})^{(1,1,1,1,1,1)}(G,t) & -(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{34} \quad (90)$$

$-(b''_{32})^{(6)}(G_{35},t)$, $-(b''_{33})^{(6)}(G_{35},t)$, $-(b''_{34})^{(6)}(G_{35},t)$ *are first detrition coefficients for category* 1, 2 *and* 3
$-(b''_{28})^{(5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5)}(G_{31},t)$ *are second detrition coefficients for category* 1, 2 *and* 3
$-(b''_{24})^{(4,4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,)}(G_{27},t)$ *are third detrition coefficients for category* 1,2 *and* 3
$-(b''_{13})^{(1,1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1,1,1)}(G,t)$ are fourth detrition coefficients for category 1, 2, and 3
$-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1, 2, and 3
$-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1, 2, and 3
$-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2, and 3
$-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)$
are eighth detrition coefficients for category 1, 2, and 3
$-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \qquad 91$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a'_{36})^{(7)}\boxed{+(a''_{36})^{(7)}(T_{37},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \qquad 92$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a'_{37})^{(7)}\boxed{+(a''_{37})^{(7)}(T_{37},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{38}}{dt} \qquad 93$$

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a'_{38})^{(7)}\boxed{+(a''_{38})^{(7)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{(a''_{36})^{(7)}(T_{37},t)}$, $\boxed{(a''_{37})^{(7)}(T_{37},t)}$, $\boxed{(a''_{38})^{(7)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{36}}{dt} = \qquad 94$$

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b'_{36})^{(7)}\boxed{-(b''_{36})^{(7)}(G_{39},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{37}}{dt} =$$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b'_{37})^{(7)}\boxed{-(b''_{37})^{(7)}(G_{39},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{38}}{dt} =$$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b'_{38})^{(7)}\boxed{-(b''_{38})^{(7)}(G_{39},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{40}}{dt}$$



$$= (a_{40})^{(8)}G_{41} - \begin{bmatrix} (a'_{40})^{(8)}\boxed{+(a''_{40})^{(8)}(T_{41},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{41}}{dt}$$

$$= (a_{41})^{(8)}G_{40} - \begin{bmatrix} (a'_{41})^{(8)}\boxed{+(a''_{41})^{(8)}(T_{41},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \begin{bmatrix} (a'_{42})^{(8)}\boxed{+(a''_{42})^{(8)}(T_{41},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a''_{40})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)}\boxed{-(b''_{40})^{(8)}(G_{43},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)}\boxed{-(b''_{41})^{(8)}(G_{43},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)}\boxed{-(b''_{42})^{(8)}(G_{43},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{44}}{dt} \qquad 96$$

$$= (a_{44})^{(9)}G_{45} - \left[\begin{array}{ccc} (a'_{44})^{(9)}\boxed{+(a''_{44})^{(9)}(T_{45},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} \end{array}\right] G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[\begin{array}{ccc} (a'_{45})^{(9)}\boxed{+(a''_{45})^{(9)}(T_{45},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} \end{array}\right] G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[\begin{array}{ccc} (a'_{46})^{(9)}\boxed{+(a''_{46})^{(9)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} \end{array}\right] G_{15}$$

Where $\boxed{+(a''_{44})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$+(a_{16}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)$ , $+(a_{17}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)$ , $+(a_{18}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a_{20}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)$ , $+(a_{21}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)$ , $+(a_{22}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a_{24}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)$ , $+(a_{25}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)$ , $+(a_{26}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a_{28}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)$, $+(a_{29}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)$ , $+(a_{30}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a_{32}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a_{33}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$ , $+(a_{34}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a_{13}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$ , $+(a_{14}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$ , $+(a_{15}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$ are Seventh augmentation coefficient for category 1, 2 and 3

$+(a_{38}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ $+(a_{37}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ $+(a_{36}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ are eighth augmentation coefficient for 1,2,3

$+(a_{40}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)$ ,$+(a_{42}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a_{41}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} =$$

$$(b_{44})^{(9)}T_{45} -$$

$$\begin{bmatrix} (b_{44}')^{(9)}\boxed{-(b_{44}'')^{(9)}(G_{47},t)} & \boxed{-(b_{16}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{20}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{24}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{28}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{32}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{13}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{36}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b_{45}')^{(9)}\boxed{-(b_{45}'')^{(9)}(G_{47},t)} & \boxed{-(b_{17}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{21}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{25}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{29}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{33}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{14}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{37}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b_{46}')^{(9)}\boxed{-(b_{46}'')^{(9)}(G_{47},t)} & \boxed{-(b_{18}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{22}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{26}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{30}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{34}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{15}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{38}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{15}$$

Where $-(b_{44}'')^{(9)}(G_{47},t)$ , $-(b_{45}'')^{(9)}(G_{47},t)$ , $-(b_{46}'')^{(9)}(G_{47},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b_{16}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)$ , $-(b_{17}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)$ , $-(b_{18}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b_{20}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)$ , $-(b_{21}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)$ , $-(b_{22}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{24}'')^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{25}'')^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{26}'')^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{28}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{32}'')^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{34}'')^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{37}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{36}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a_i')^{(1)}, (a_i'')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (b_i'')^{(1)} > 0, \quad i,j = 13,14,15 \qquad 97$$

The functions $(a_i'')^{(1)}, (b_i'')^{(1)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a_i'')^{(1)}(T_{14},t) \le (p_i)^{(1)} \le (\hat{A}_{13})^{(1)}$$

$$(b_i'')^{(1)}(G,t) \le (r_i)^{(1)} \le (b_i')^{(1)} \le (\hat{B}_{13})^{(1)}$$

$$lim_{T_2\to\infty}(a_i'')^{(1)}(T_{14},t) = (p_i)^{(1)} \qquad 98$$

$$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = (r_i)^{(1)}$$

**Definition of** $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$ :

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$$|(a_i'')^{(1)}(T_{14}',t) - (a_i'')^{(1)}(T_{14},t)| \le (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G',t) - (b_i'')^{(1)}(G,t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}',t)$ and $(a_i'')^{(1)}(T_{14},t)$ . $(T_{14}',t)$ and $(T_{14},t)$ are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14},t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$ : 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$

**Definition of** $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$ : 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}$, $(r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)} (\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)} (\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i, j = 16,17,18$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17}, t) \leq (p_i)^{(2)} \leq (\hat{A}_{16})^{(2)} \quad 102$$

$$(b_i'')^{(2)}(G_{19}, t) \leq (r_i)^{(2)} \leq (b_i')^{(2)} \leq (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2 \to \infty} (a_i'')^{(2)}(T_{17}, t) = (p_i)^{(2)} \quad 104$$

$$\lim_{G \to \infty} (b_i'')^{(2)}((G_{19}), t) = (r_i)^{(2)} \quad 105$$

**Definition of** $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$ : 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}', t) - (a_i'')^{(2)}(T_{17}, t)| \leq (\hat{k}_{16})^{(2)} |T_{17} - T_{17}'| e^{-(\hat{M}_{16})^{(2)} t} \quad 107$$

$$|(b_i'')^{(2)}((G_{19})', t) - (b_i'')^{(2)}((G_{19}), t)| < (\hat{k}_{16})^{(2)} ||(G_{19}) - (G_{19})'|| e^{-(\hat{M}_{16})^{(2)} t} \quad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}', t)$ and$(a_i'')^{(2)}(T_{17}, t)$ . $(T_{17}', t)$ and $(T_{17}, t)$ are points belonging to the interval $[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}]$ . It is to be noted that $(a_i'')^{(2)}(T_{17}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17}, t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$ :

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)},$ are positive constants 109

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$$

**Definition of** $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$ :

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and\ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1 \quad 110$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(b_i)^{(2)} + (b_i')^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1 \quad 111$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i, j = 20,21,22 \quad 112$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(3)}$, $(r_i)^{(3)}$:

$$(a_i'')^{(3)}(T_{21}, t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23}, t) \leq (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(3)}(T_{21}, t) = (p_i)^{(3)} \quad 113$$

$$\lim_{G \to \infty}(b_i'')^{(3)}(G_{23}, t) = (r_i)^{(3)}$$

**Definition of** $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$ :

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: 114

$$|(a_i'')^{(3)}(T_{21}', t) - (a_i'')^{(3)}(T_{21}, t)| \leq (\hat{k}_{20})^{(3)} |T_{21} - T_{21}'| e^{-(\hat{M}_{20})^{(3)} t}$$

$$|(b_i'')^{(3)}(G_{23}', t) - (b_i'')^{(3)}(G_{23}, t)| < (\hat{k}_{20})^{(3)} ||G_{23} - G_{23}'|| e^{-(\hat{M}_{20})^{(3)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}', t)$ and$(a_i'')^{(3)}(T_{21}, t)$ . $(T_{21}', t)$ And $(T_{21}, t)$ are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21}, t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$ : 115

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}$ , $\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with 116
$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$ and the constants
$(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}, i = 20,21,22,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(a_i)^{(3)} + (a_i')^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)} (\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(b_i)^{(3)} + (b_i')^{(3)} + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)} (\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i, j = 24,25,26$$ 117

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(4)}$, $(r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25}, t) \leq (p_i)^{(4)} \leq (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}((G_{27}), t) \leq (r_i)^{(4)} \leq (b_i')^{(4)} \leq (\hat{B}_{24})^{(4)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(4)}(T_{25}, t) = (p_i)^{(4)}$$ 118

$$\lim_{G \to \infty}(b_i'')^{(4)}((G_{27}), t) = (r_i)^{(4)}$$

**Definition of** $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$ :

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24,25,26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}', t) - (a_i'')^{(4)}(T_{25}, t)| \leq (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})', t) - (b_i'')^{(4)}((G_{27}), t)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}', t)$ and$(a_i'')^{(4)}(T_{25}, t)$ . $(T_{25}', t)$ and $(T_{25}, t)$ are points belonging to the interval $[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}]$. It is to be noted that $(a_i'')^{(4)}(T_{25}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25}, t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$ : 120

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)},$ are positive constants

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}, \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$$

**Definition of** $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$ : 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and\ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}, i = 24,25,26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(a_i)^{(4)} + (a_i')^{(4)} + (\hat{A}_{24})^{(4)} + (\hat{P}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(b_i)^{(4)} + (b_i')^{(4)} + (\hat{B}_{24})^{(4)} + (\hat{Q}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i, j = 28,29,30$$ 122

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(5)}$, $(r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29}, t) \le (p_i)^{(5)} \le (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}((G_{31}), t) \le (r_i)^{(5)} \le (b_i')^{(5)} \le (\hat{B}_{28})^{(5)}$$

$$lim_{T_2 \to \infty} (a_i'')^{(5)}(T_{29}, t) = (p_i)^{(5)}$$ 123

$$\lim_{G \to \infty} (b_i'')^{(5)}(G_{31}, t) = (r_i)^{(5)}$$

**Definition of** $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$ :

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}', t) - (a_i'')^{(5)}(T_{29}, t)| \le (\hat{k}_{28})^{(5)} |T_{29} - T_{29}'| e^{-(\hat{M}_{28})^{(5)} t}$$

$$|(b_i'')^{(5)}((G_{31})', t) - (b_i'')^{(5)}((G_{31}), t)| < (\hat{k}_{28})^{(5)} ||(G_{31}) - (G_{31})'|| e^{-(\hat{M}_{28})^{(5)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}', t)$ and $(a_i'')^{(5)}(T_{29}, t)$ . $(T_{29}', t)$ and $(T_{29}, t)$ are points belonging to the interval $\left[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\right]$. It is to be noted that $(a_i'')^{(5)}(T_{29}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$ : 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)},$ are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}, \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

**Definition of** $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$ : 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and\ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(a_i)^{(5)} + (a_i')^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(b_i)^{(5)} + (b_i')^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i, j = 32,33,34$$ 127

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(6)}$, $(r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33}, t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}), t) \leq (r_i)^{(6)} \leq (b_i')^{(6)} \leq (\hat{B}_{32})^{(6)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(6)}(T_{33}, t) = (p_i)^{(6)}$$ 128

$$\lim_{G \to \infty}(b_i'')^{(6)}((G_{35}), t) = (r_i)^{(6)}$$

**Definition of** $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$ :

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}', t) - (a_i'')^{(6)}(T_{33}, t)| \leq (\hat{k}_{32})^{(6)} |T_{33} - T_{33}'| e^{-(\hat{M}_{32})^{(6)} t}$$

$$|(b_i'')^{(6)}((G_{35})', t) - (b_i'')^{(6)}((G_{35}), t)| < (\hat{k}_{32})^{(6)} ||(G_{35}) - (G_{35})'|| e^{-(\hat{M}_{32})^{(6)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}', t)$ and $(a_i'')^{(6)}(T_{33}, t)$ . $(T_{33}', t)$ and $(T_{33}, t)$ are points belonging to the interval $[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}]$. It is to be noted that $(a_i'')^{(6)}(T_{33}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$ : 129

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)},$ are positive constants

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$$

**Definition of** $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$ : 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and\ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32,33,34,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(a_i)^{(6)} + (a_i')^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)} (\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(b_i)^{(6)} + (b_i')^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)} (\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

(OOOOOO) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i, j = 36,37,38$ 131

(PPPPPP) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(7)}$, $(r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37}, t) \leq (p_i)^{(7)} \leq (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39}, t) \leq (r_i)^{(7)} \leq (b_i')^{(7)} \leq (\hat{B}_{36})^{(7)}$$

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*(QQQQQQ)* $\lim_{T_2 \to \infty} (a_i'')^{(7)}(T_{37}, t) = (p_i)^{(7)}$

*(RRRRRR)*

$\lim_{G \to \infty} (b_i'')^{(7)}((G_{39}), t) = (r_i)^{(7)}$

**Definition of** $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$ :

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$$|(a_i'')^{(7)}(T_{37}', t) - (a_i'')^{(7)}(T_{37}, t)| \leq (\hat{k}_{36})^{(7)} |T_{37} - T_{37}'| e^{-(\hat{M}_{36})^{(7)} t}$$

$$|(b_i'')^{(7)}((G_{39})', t) - (b_i'')^{(7)}((G_{39}), t)| < (\hat{k}_{36})^{(7)} ||(G_{39}) - (G_{39})'|| e^{-(\hat{M}_{36})^{(7)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}', t)$ and $(a_i'')^{(7)}(T_{37}, t)$ . $(T_{37}', t)$ and $(T_{37}, t)$ are points belonging to the interval $[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}]$ . It is to be noted that $(a_i'')^{(7)}(T_{37}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37}, t)$ , the **first augmentation coefficient** attributable to the

system, would be absolutely continuous.

**Definition of** $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$ : 134

(SSSSSS) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)},$ are positive constants

$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$

**Definition of** $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$ : 135

(TTTTTT) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(b_i)^{(7)} + (b_i')^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40,41,42$ 136

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

**Definition of** $(p_i)^{(8)}, (r_i)^{(8)}$: 137

$(a_i'')^{(8)}(T_{41}, t) \le (p_i)^{(8)} \le (\hat{A}_{40})^{(8)}$ 138

$(b_i'')^{(8)}((G_{43}), t) \le (r_i)^{(8)} \le (b_i')^{(8)} \le (\hat{B}_{40})^{(8)}$ 139

$\lim_{T_2 \to \infty} (a_i'')^{(8)}(T_{41}, t) = (p_i)^{(8)}$ 140

$\lim_{G \to \infty} (b_i'')^{(8)}((G_{43}), t) = (r_i)^{(8)}$ 141

**Definition of** $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$ :

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \le (\hat{k}_{40})^{(8)} |T_{41} - T_{41}'| e^{-(\hat{M}_{40})^{(8)} t}$ 142

$|(b_i'')^{(8)}((G_{43})', t) - (b_i'')^{(8)}((G_{43}), t)| < (\hat{k}_{40})^{(8)} ||(G_{43}) - (G_{43})'|| e^{-(\hat{M}_{40})^{(8)} t}$ 143

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}', t)$ and $(a_i'')^{(8)}(T_{41}, t)$ . $(T_{41}', t)$ and $(T_{41}, t)$ are points belonging to the interval $\left[(\hat{k}_{40})^{(8)}, (\widehat{M}_{40})^{(8)}\right]$. It is to be noted that $(a_i'')^{(8)}(T_{41}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{40})^{(8)} = 1$ then the function $(a_i'')^{(8)}(T_{41}, t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\widehat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$ :

$(\widehat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)},$ are positive constants

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}, \frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}} < 1 \qquad 144$$

**Definition of** $(\hat{P}_{40})^{(8)}, (\hat{Q}_{40})^{(8)}$ :

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\widehat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}, (\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)}, (a_i')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}, i = 40,41,42,$
Satisfy the inequalities

$$\frac{1}{(\widehat{M}_{40})^{(8)}}[(a_i)^{(8)} + (a_i')^{(8)} + (\hat{A}_{40})^{(8)} + (\hat{P}_{40})^{(8)} (\hat{k}_{40})^{(8)}] < 1 \qquad 145$$

$$\frac{1}{(\widehat{M}_{40})^{(8)}}[(b_i)^{(8)} + (b_i')^{(8)} + (\hat{B}_{40})^{(8)} + (\hat{Q}_{40})^{(8)} (\hat{k}_{40})^{(8)}] < 1 \qquad 146$$

Where we suppose

$$(a_i)^{(9)}, (a_i')^{(9)}, (a_i'')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (b_i'')^{(9)} > 0, \quad i, j = 44,45,46 \qquad 146\text{A}$$

The functions $(a_i'')^{(9)}, (b_i'')^{(9)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(9)}$, $(r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45}, t) \leq (p_i)^{(9)} \leq (\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47}, t) \leq (r_i)^{(9)} \leq (b_i')^{(9)} \leq (\hat{B}_{44})^{(9)}$$

$$\lim_{T_2 \to \infty} (a_i'')^{(9)}(T_{45}, t) = (p_i)^{(9)}$$

$$\lim_{G \to \infty} (b_i'')^{(9)}(G_{47}, t) = (r_i)^{(9)}$$

**Definition of** $(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}$ :

Where $\boxed{(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}}$ are positive constants and $\boxed{i = 44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}', t) - (a_i'')^{(9)}(T_{45}, t)| \leq (\hat{k}_{44})^{(9)} |T_{45} - T_{45}'| e^{-(\widehat{M}_{44})^{(9)} t}$$

$$|(b_i'')^{(9)}((G_{47})', t) - (b_i'')^{(9)}\big((G_{47}), t\big)| < (\hat{k}_{44})^{(9)}||(G_{47}) - (G_{47})'||e^{-(\hat{M}_{44})^{(9)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}', t)$ and$(a_i'')^{(9)}(T_{45}, t)$ . $(T_{45}', t)$ and $(T_{45}, t)$ are points belonging to the interval $\big[(\hat{k}_{44})^{(9)}, (\hat{M}_{44})^{(9)}\big]$. It is to be noted that $(a_i'')^{(9)}(T_{45}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{44})^{(9)} = 1$ then the function $(a_i'')^{(9)}(T_{45}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$ :

$(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$, are positive constants

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}} \ , \frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$$

**Definition of** $(\hat{P}_{44})^{(9)}, (\hat{Q}_{44})^{(9)}$ :

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}, (\hat{A}_{44})^{(9)} and\ (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)}, (a_i')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (p_i)^{(9)}, \ (r_i)^{(9)}, i = 44,45,46,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{44})^{(9)}}[\,(a_i)^{(9)} + (a_i')^{(9)} + \ (\hat{A}_{44})^{(9)} + \ (\hat{P}_{44})^{(9)}\,(\hat{k}_{44})^{(9)}] < 1$$

$$\frac{1}{(\hat{M}_{44})^{(9)}}[\ (b_i)^{(9)} + (b_i')^{(9)} + \ (\hat{B}_{44})^{(9)} + \ (\hat{Q}_{44})^{(9)}\ (\hat{k}_{44})^{(9)}] < 1$$

**Theorem 1:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \big(\hat{P}_{13}\big)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \ , \ \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 2 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

**Definition of** $G_i(0)\,, T_i(0)$

$$G_i(t) \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} \quad , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} \quad , \quad T_i(0) = T_i^0 > 0$$

**Theorem 3 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t) \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t} \quad , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t} \quad , \quad T_i(0) = T_i^0 > 0$$

**Theorem 4 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

**Definition of** $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \left(\hat{P}_{24}\right)^{(4)} e^{(\hat{M}_{24})^{(4)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{24}\right)^{(4)} e^{(\hat{M}_{24})^{(4)}t} \;,\quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 5 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

**Definition of** $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \left(\hat{P}_{28}\right)^{(5)} e^{(\hat{M}_{28})^{(5)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{28}\right)^{(5)} e^{(\hat{M}_{28})^{(5)}t} \;,\quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 6 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

**Definition of** $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \left(\hat{P}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t} \;,\quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 7:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

**Definition of** $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \left(\hat{P}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t} \;,\quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 8:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

**Definition of** $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \left(\hat{P}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t} \;,\quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 9:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

**Definition of** $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \left(\hat{P}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

**Proof:** Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \leq (\hat{P}_{13})^{(1)} \text{ , } T_i^0 \leq (\hat{Q}_{13})^{(1)}, \quad 155$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t} \quad 156$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t} \quad 157$$

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[ (a_{13})^{(1)} G_{14}(s_{(13)}) - \left( (a'_{13})^{(1)} + a''_{13})^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \right) G_{13}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[ (a_{14})^{(1)} G_{13}(s_{(13)}) - \left( (a'_{14})^{(1)} + (a''_{14})^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \right) G_{14}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[ (a_{15})^{(1)} G_{14}(s_{(13)}) - \left( (a'_{15})^{(1)} + (a''_{15})^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \right) G_{15}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[ (b_{13})^{(1)} T_{14}(s_{(13)}) - \left( (b'_{13})^{(1)} - (b''_{13})^{(1)} (G(s_{(13)}), s_{(13)}) \right) T_{13}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[ (b_{14})^{(1)} T_{13}(s_{(13)}) - \left( (b'_{14})^{(1)} - (b''_{14})^{(1)} (G(s_{(13)}), s_{(13)}) \right) T_{14}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{T}_{15}(t) = T_{15}^0 + \int_0^t \left[ (b_{15})^{(1)} T_{14}(s_{(13)}) - \left( (b'_{15})^{(1)} - (b''_{15})^{(1)} (G(s_{(13)}), s_{(13)}) \right) T_{15}(s_{(13)}) \right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \leq (\hat{P}_{16})^{(2)} \text{ , } T_i^0 \leq (\hat{Q}_{16})^{(2)},$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$$

By 160

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[ (a_{16})^{(2)} G_{17}(s_{(16)}) - \left( (a'_{16})^{(2)} + a''_{16})^{(2)} (T_{17}(s_{(16)}), s_{(16)}) \right) G_{16}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[ (a_{17})^{(2)} G_{16}(s_{(16)}) - \left( (a'_{17})^{(2)} + (a''_{17})^{(2)} (T_{17}(s_{(16)}), s_{(17)}) \right) G_{17}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[ (a_{18})^{(2)} G_{17}(s_{(16)}) - \left( (a'_{18})^{(2)} + (a''_{18})^{(2)} (T_{17}(s_{(16)}), s_{(16)}) \right) G_{18}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^{0} + \int_0^t \left[ (b_{16})^{(2)} T_{17}(s_{(16)}) - \left( (b'_{16})^{(2)} - (b''_{16})^{(2)} (G_{19}(s_{(16)}), s_{(16)}) \right) T_{16}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^{0} + \int_0^t \left[ (b_{17})^{(2)} T_{16}(s_{(16)}) - \left( (b'_{17})^{(2)} - (b''_{17})^{(2)} (G_{19}(s_{(16)}), s_{(16)}) \right) T_{17}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^{0} + \int_0^t \left[ (b_{18})^{(2)} T_{17}(s_{(16)}) - \left( (b'_{18})^{(2)} - (b''_{18})^{(2)} (G_{19}(s_{(16)}), s_{(16)}) \right) T_{18}(s_{(16)}) \right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i : \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \le (\hat{P}_{20})^{(3)} \text{ , } T_i^0 \le (\hat{Q}_{20})^{(3)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\bar{M}_{20})^{(3)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\bar{M}_{20})^{(3)} t}$$

By 161

$$\bar{G}_{20}(t) = G_{20}^{0} + \int_0^t \left[ (a_{20})^{(3)} G_{21}(s_{(20)}) - \left( (a'_{20})^{(3)} + a''_{20})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^{0} + \int_0^t \left[ (a_{21})^{(3)} G_{20}(s_{(20)}) - \left( (a'_{21})^{(3)} + (a''_{21})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{21}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^{0} + \int_0^t \left[ (a_{22})^{(3)} G_{21}(s_{(20)}) - \left( (a'_{22})^{(3)} + (a''_{22})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{22}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^{0} + \int_0^t \left[ (b_{20})^{(3)} T_{21}(s_{(20)}) - \left( (b'_{20})^{(3)} - (b''_{20})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^{0} + \int_0^t \left[ (b_{21})^{(3)} T_{20}(s_{(20)}) - \left( (b'_{21})^{(3)} - (b''_{21})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{21}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{22}(t) = T_{22}^{0} + \int_0^t \left[ (b_{22})^{(3)} T_{21}(s_{(20)}) - \left( (b'_{22})^{(3)} - (b''_{22})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{22}(s_{(20)}) \right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i : \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \le (\hat{P}_{24})^{(4)} \text{ , } T_i^0 \le (\hat{Q}_{24})^{(4)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\bar{M}_{24})^{(4)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\bar{M}_{24})^{(4)} t}$$

By 162

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{26}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{\mathrm{T}}_{26}(\mathrm{t}) = \mathrm{T}_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{26}(s_{(24)})\right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{28})^{(5)}, T_i^0 \le (\hat{Q}_{28})^{(5)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

By 163

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{30}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{\mathrm{T}}_{30}(\mathrm{t}) = \mathrm{T}_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{30}(s_{(28)})\right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{32})^{(6)}, T_i^0 \le (\hat{Q}_{32})^{(6)},$$

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\bar{M}_{32})^{(6)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\bar{M}_{32})^{(6)} t}$

By 164

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[ (a_{32})^{(6)} G_{33}(s_{(32)}) - \left( (a'_{32})^{(6)} + a''_{32})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[ (a_{33})^{(6)} G_{32}(s_{(32)}) - \left( (a'_{33})^{(6)} + (a''_{33})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[ (a_{34})^{(6)} G_{33}(s_{(32)}) - \left( (a'_{34})^{(6)} + (a''_{34})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{34}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[ (b_{32})^{(6)} T_{33}(s_{(32)}) - \left( (b'_{32})^{(6)} - (b''_{32})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[ (b_{33})^{(6)} T_{32}(s_{(32)}) - \left( (b'_{33})^{(6)} - (b''_{33})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{\mathrm{T}}_{34}(\mathrm{t}) = \mathrm{T}_{34}^0 + \int_0^t \left[ (b_{34})^{(6)} T_{33}(s_{(32)}) - \left( (b'_{34})^{(6)} - (b''_{34})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{34}(s_{(32)}) \right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[ (a_{36})^{(7)} G_{37}(s_{(36)}) - \left( (a'_{36})^{(7)} + a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[ (a_{37})^{(7)} G_{36}(s_{(36)}) - \left( (a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[ (a_{38})^{(7)} G_{37}(s_{(36)}) - \left( (a'_{38})^{(7)} + (a''_{38})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{38}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[ (b_{36})^{(7)} T_{37}(s_{(36)}) - \left( (b'_{36})^{(7)} - (b''_{36})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[ (b_{37})^{(7)} T_{36}(s_{(36)}) - \left( (b'_{37})^{(7)} - (b''_{37})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\overline{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^{0} + \int_0^t \left[ (b_{38})^{(7)} T_{37}(s_{(36)}) - \left( (b'_{38})^{(7)} - (b''_{38})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{38}(s_{(36)}) \right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$ , $T_i(0) = T_i^0$ , $G_i^0 \leq (\hat{P}_{40})^{(8)}$ , $T_i^0 \leq (\hat{Q}_{40})^{(8)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[ (a_{40})^{(8)} G_{41}(s_{(40)}) - \left( (a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[ (a_{41})^{(8)} G_{40}(s_{(40)}) - \left( (a'_{41})^{(8)} + (a''_{41})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[ (a_{42})^{(8)} G_{41}(s_{(40)}) - \left( (a'_{42})^{(8)} + (a''_{42})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{42}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^0 + \int_0^t \left[ (b_{40})^{(8)} T_{41}(s_{(40)}) - \left( (b'_{40})^{(8)} - (b''_{40})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^0 + \int_0^t \left[ (b_{41})^{(8)} T_{40}(s_{(40)}) - \left( (b'_{41})^{(8)} - (b''_{41})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\overline{\mathrm{T}}_{42}(\mathrm{t}) = \mathrm{T}_{42}^{0} + \int_0^t \left[ (b_{42})^{(8)} T_{41}(s_{(40)}) - \left( (b'_{42})^{(8)} - (b''_{42})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{42}(s_{(40)}) \right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** 166
Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy A

$G_i(0) = G_i^0$ , $T_i(0) = T_i^0$ , $G_i^0 \leq (\hat{P}_{44})^{(9)}$ , $T_i^0 \leq (\hat{Q}_{44})^{(9)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[ (a_{44})^{(9)} G_{45}(s_{(44)}) - \left( (a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45}(s_{(44)}), s_{(44)})\right) G_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}(s_{(44)}), s_{(44)})\right) G_{46}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{44}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{46}(t) = T_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{46}(s_{(44)})\right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}}\right)\right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left(e^{(\hat{M}_{13})^{(1)} t} - 1\right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\hat{M}_{13})^{(1)} t} \le \frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{P}_{13})^{(1)} + G_{14}^0\right) e^{\left(-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0}\right)} + (\hat{P}_{13})^{(1)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \le G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}}\right)\right] ds_{(16)} = \quad 169$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1\right)$$

From which it follows that 170

$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \le \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left[\left((\hat{P}_{16})^{(2)} + G_{17}^0\right) e^{\left(-\frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0}\right)} + (\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t) \le G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} \left(G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} s_{(20)}}\right)\right] ds_{(20)} =$$

$$\left(1 + (a_{20})^{(3)} t\right) G_{21}^0 + \frac{(a_{20})^{(3)} (\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left(e^{(\hat{M}_{20})^{(3)} t} - 1\right)$$

From which it follows that 172

$$(G_{20}(t)-G_{20}^{0})e^{-(\hat{M}_{20})^{(3)}t}\leq\frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left[((\hat{P}_{20})^{(3)}+G_{21}^{0})e^{\left(-\frac{(\hat{P}_{20})^{(3)}+G_{21}^{0}}{G_{21}^{0}}\right)}+(\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 173

$$G_{24}(t)\leq G_{24}^{0}+\int_{0}^{t}\left[(a_{24})^{(4)}\left(G_{25}^{0}+(\hat{P}_{24})^{(4)}e^{(\hat{M}_{24})^{(4)}s_{(24)}}\right)\right]ds_{(24)}=$$

$$\left(1+(a_{24})^{(4)}t\right)G_{25}^{0}+\frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left(e^{(\hat{M}_{24})^{(4)}t}-1\right)$$

From which it follows that 174

$$(G_{24}(t)-G_{24}^{0})e^{-(\hat{M}_{24})^{(4)}t}\leq\frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left[((\hat{P}_{24})^{(4)}+G_{25}^{0})e^{\left(-\frac{(\hat{P}_{24})^{(4)}+G_{25}^{0}}{G_{25}^{0}}\right)}+(\hat{P}_{24})^{(4)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t)\leq G_{28}^{0}+\int_{0}^{t}\left[(a_{28})^{(5)}\left(G_{29}^{0}+(\hat{P}_{28})^{(5)}e^{(\hat{M}_{28})^{(5)}s_{(28)}}\right)\right]ds_{(28)}=$$

$$\left(1+(a_{28})^{(5)}t\right)G_{29}^{0}+\frac{(a_{28})^{(5)}(\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left(e^{(\hat{M}_{28})^{(5)}t}-1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t}\leq\frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t)\leq G_{32}^{0}+\int_{0}^{t}\left[(a_{32})^{(6)}\left(G_{33}^{0}+(\hat{P}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\right)\right]ds_{(32)}=$$

$$\left(1+(a_{32})^{(6)}t\right)G_{33}^{0}+\frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left(e^{(\hat{M}_{32})^{(6)}t}-1\right)$$

From which it follows that 177

$$(G_{32}(t)-G_{32}^{0})e^{-(\hat{M}_{32})^{(6)}t}\leq\frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left[((\hat{P}_{32})^{(6)}+G_{33}^{0})e^{\left(-\frac{(\hat{P}_{32})^{(6)}+G_{33}^{0}}{G_{33}^{0}}\right)}+(\hat{P}_{32})^{(6)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}$ , $G_{26}, T_{24}, T_{25}, T_{26}$

(y) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t) \le G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} \left(G_{37}^0 + (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} s_{(36)}}\right)\right] ds_{(36)} =$$
$$\left(1 + (a_{36})^{(7)} t\right) G_{37}^0 + \frac{(a_{36})^{(7)} (\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left(e^{(\hat{M}_{36})^{(7)} t} - 1\right)$$

From which it follows that

$$(G_{36}(t) - G_{36}^0) e^{-(\hat{M}_{36})^{(7)} t} \le \frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left[((\hat{P}_{36})^{(7)} + G_{37}^0) e^{\left(-\frac{(\hat{P}_{36})^{(7)} + G_{37}^0}{G_{37}^0}\right)} + (\hat{P}_{36})^{(7)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t) \le G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} \left(G_{41}^0 + (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} s_{(40)}}\right)\right] ds_{(40)} = \quad 180$$
$$\left(1 + (a_{40})^{(8)} t\right) G_{41}^0 + \frac{(a_{40})^{(8)} (\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}} \left(e^{(\hat{M}_{40})^{(8)} t} - 1\right)$$

From which it follows that 181

$$(G_{40}(t) - G_{40}^0) e^{-(\hat{M}_{40})^{(8)} t} \le \frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}} \left[((\hat{P}_{40})^{(8)} + G_{41}^0) e^{\left(-\frac{(\hat{P}_{40})^{(8)} + G_{41}^0}{G_{41}^0}\right)} + (\hat{P}_{40})^{(8)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 8

Analogous inequalities hold also for $G_{41}$ , $G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t) \le G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} \left(G_{45}^0 + (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} s_{(44)}}\right)\right] ds_{(44)} =$$
$$\left(1 + (a_{44})^{(9)} t\right) G_{45}^0 + \frac{(a_{44})^{(9)} (\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left(e^{(\hat{M}_{44})^{(9)} t} - 1\right)$$

From which it follows that

$$(G_{44}(t) - G_{44}^0) e^{-(\hat{M}_{44})^{(9)} t} \le \frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left[((\hat{P}_{44})^{(9)} + G_{45}^0) e^{\left(-\frac{(\hat{P}_{44})^{(9)} + G_{45}^0}{G_{45}^0}\right)} + (\hat{P}_{44})^{(9)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 9

Analogous inequalities hold also for $G_{45}$ , $G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}$ , $\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$ and to choose 182

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[(\hat{P}_{13})^{(1)}+\left((\hat{P}_{13})^{(1)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{13})^{(1)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{13})^{(1)} \quad 183$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[\left((\hat{Q}_{13})^{(1)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{13})^{(1)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{13})^{(1)}\right]\leq(\hat{Q}_{13})^{(1)} \quad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)},T^{(1)}\right),\left(G^{(2)},T^{(2)}\right)\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{13})^{(1)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{13})^{(1)}t}\}$$

Indeed if we denote

**Definition of** $\tilde{G},\tilde{T}:\left(\tilde{G},\tilde{T}\right)=\mathcal{A}^{(1)}(G,T)$

It results

$$\left|\tilde{G}_{13}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{13})^{(1)}\left|G_{14}^{(1)}-G_{14}^{(2)}\right|e^{-(\hat{M}_{13})^{(1)}s_{(13)}}e^{(\hat{M}_{13})^{(1)}s_{(13)}}ds_{(13)}+$$

$$\int_0^t\{(a'_{13})^{(1)}\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\hat{M}_{13})^{(1)}s_{(13)}}e^{-(\hat{M}_{13})^{(1)}s_{(13)}}+$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\hat{M}_{13})^{(1)}s_{(13)}}e^{(\hat{M}_{13})^{(1)}s_{(13)}}+$$

$$G_{13}^{(2)}\left|(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)-(a''_{13})^{(1)}\left(T_{14}^{(2)},s_{(13)}\right)\right|\ e^{-(\hat{M}_{13})^{(1)}s_{(13)}}e^{(\hat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|G^{(1)}-G^{(2)}\right|e^{-(\hat{M}_{13})^{(1)}t}\leq \quad 186$$
$$\frac{1}{(\hat{M}_{13})^{(1)}}\left((a_{13})^{(1)}+(a'_{13})^{(1)}+(\hat{A}_{13})^{(1)}+(\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}\right)d\left(\left(G^{(1)},T^{(1)};\ G^{(2)},T^{(2)}\right)\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 1:** The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\hat{P}_{13})^{(1)}e^{(\hat{M}_{13})^{(1)}t}$ *and* $(\hat{Q}_{13})^{(1)}e^{(\hat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i = 13,14,15$ depend only on $T_{14}$ and respectively on $G(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any $t$ where $G_i(t) = 0\ and\ T_i(t) = 0$

From 19 to 24 it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(1)} - (a_i'')^{(1)}(T_{14}(s_{(13)}), s_{(13)})\} ds_{(13)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(1)} t)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2\ and\ \left((\widehat{M}_{13})^{(1)}\right)_3$ : 187

**Remark 3**: if $G_{13}$ is bounded, the same property have also $G_{14}\ and\ G_{15}$ . indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a_{14}')^{(1)} G_{14}$ and by integrating

$$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)} \left((\widehat{M}_{13})^{(1)}\right)_1 / (a_{14}')^{(1)}$$

In the same way , one can obtain

$$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)} \left((\widehat{M}_{13})^{(1)}\right)_2 / (a_{15}')^{(1)}$$

If $G_{14}\ or\ G_{15}$ is bounded, the same property follows for $G_{13}$ , $G_{15}$ and $G_{13}$ , $G_{14}$ respectively.

**Remark 4:** If $G_{13}$ *is* bounded, from below, the same property holds for $G_{14}\ and\ G_{15}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{14}$ is bounded from below. 188

**Remark 5:** If $T_{13}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}(G(t), t)) = (b_{14}')^{(1)}$ then $T_{14} \to \infty$. 189

**Definition of** $(m)^{(1)}$ and $\varepsilon_1$ :

Indeed let $t_1$ be so that for $t > t_1$

$$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t), t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right)$, $t = log\frac{2}{\varepsilon_1}$ By taking now $\varepsilon_1$ sufficiently small one sees that $T_{14}$ is unbounded.
The same property holds for $T_{15}$ if $\lim_{t\to\infty}(b_{15}'')^{(1)}(G(t), t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}$ , $\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}\ and\ (\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[(\widehat{P}_{16})^{(2)}+\left((\hat{P}_{16})^{(2)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{16})^{(2)}\right]\leq(\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

**Definition of** $\widetilde{G_{19}},\widetilde{T_{19}}$ : $\left(\widetilde{G_{19}},\widetilde{T_{19}}\right)=\mathcal{A}^{(2)}(G_{19},T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{16})^{(2)}\left|G_{17}^{(1)}-G_{17}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)}+$$

$$\int_0^t\{(a'_{16})^{(2)}\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$G_{16}^{(2)}\left|(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)-(a''_{16})^{(2)}\left(T_{17}^{(2)},s_{(16)}\right)\right|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0,t]$ 197

From the hypotheses it follows

$$\left|(G_{19})^{(1)}-(G_{19})^{(2)}\right|\mathrm{e}^{-(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}\leq$$
$$\frac{1}{(\widehat{\mathrm{M}}_{16})^{(2)}}\left((a_{16})^{(2)}+(a'_{16})^{(2)}+(\widehat{\mathrm{A}}_{16})^{(2)}+(\widehat{\mathrm{P}}_{16})^{(2)}(\hat{k}_{16})^{(2)}\right)\mathrm{d}\left(((G_{19})^{(1)},(T_{19})^{(1)};\ (G_{19})^{(2)},(T_{19})^{(2)})\right)$$

And analogous inequalities for $G_i$ and $T_i$. Taking into account the hypothesis the result follows 198

**Remark 6:** The fact that we supposed $(a''_{16})^{(2)}$ and $(b''_{16})^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{\mathrm{P}}_{16})^{(2)}\mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}$ and $(\widehat{\mathrm{Q}}_{16})^{(2)}\mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i=16,17,18$ depend only on $\mathrm{T}_{17}$ and respectively on

$(G_{19})$(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 7:** There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$ 200

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(2)} - (a_i'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})\} ds_{(16)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(2)}t)} > 0 \quad \text{for } t > 0$$

**Definition of** $\left((\widehat{M}_{16})^{(2)}\right)_1$, $\left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$: 201

**Remark 8:** if $G_{16}$ is bounded, the same property have also $G_{17}$ and $G_{18}$. indeed if

$G_{16} < (\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt} \leq \left((\widehat{M}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)} G_{17}$ and by integrating

$$G_{17} \leq \left((\widehat{M}_{16})^{(2)}\right)_2 = G_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1 / (a_{17}')^{(2)}$$

In the same way , one can obtain

$$G_{18} \leq \left((\widehat{M}_{16})^{(2)}\right)_3 = G_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2 / (a_{18}')^{(2)}$$

If $G_{17}$ or $G_{18}$ is bounded, the same property follows for $G_{16}$, $G_{18}$ and $G_{16}$, $G_{17}$ respectively.

**Remark 9:** If $G_{16}$ is bounded, from below, the same property holds for $G_{17}$ and $G_{18}$. The proof is analogous with the preceding one. An analogous property is true if $G_{17}$ is bounded from below. 202

**Remark 10:** If $T_{16}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t), t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$. 203

**Definition of** $(m)^{(2)}$ and $\varepsilon_2$ :

Indeed let $t_2$ be so that for $t > t_2$

$$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right)$, $t = \log\frac{2}{\varepsilon_2}$ By taking now $\varepsilon_2$ sufficiently small one sees that $T_{17}$ is unbounded. 205
The same property holds for $T_{18}$ if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t), t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[(\widehat{P}_{20})^{(3)}+\left((\hat{P}_{20})^{(3)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{20})^{(3)}\right]\leq(\hat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself 210

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)},(T_{23})^{(1)}),((G_{23})^{(2)},(T_{23})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

**Definition of** $\widetilde{G_{23}},\widetilde{T_{23}}:(\widetilde{(G_{23})},\widetilde{(T_{23})})=\mathcal{A}^{(3)}((G_{23}),(T_{23}))$

It results 213

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{20})^{(3)}\left|G_{21}^{(1)}-G_{21}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\,ds_{(20)}+$$

$$\int_0^t\{(a'_{20})^{(3)}\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$G_{20}^{(2)}\left|(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)},s_{(20)}\right)\right|\ e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\}ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses it follows

$$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}t}\leq \quad 214$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+(a'_{20})^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\widehat{k}_{20})^{(3)}\right)d\left(((G_{23})^{(1)},(T_{23})^{(1)};\ (G_{23})^{(2)},(T_{23})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

**Remark 11:** The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}\ and\ (\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i=20,21,22$ depend only on $\mathrm{T}_{21}$ and respectively on

$(G_{23})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 12:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 216

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(3)} - (a_i'')^{(3)}(T_{21}(s_{(20)}), s_{(20)})\} ds_{(20)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(3)} t)} > 0$ for t > 0

**Definition of** $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2\ and\ \left((\widehat{M}_{20})^{(3)}\right)_3$ : 217

**Remark 13:** if $G_{20}$ is bounded, the same property have also $G_{21}\ and\ G_{22}$ . indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a_{21}')^{(3)} G_{21}$ and by integrating

$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1 / (a_{21}')^{(3)}$

In the same way , one can obtain

$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2 / (a_{22}')^{(3)}$

If $G_{21}\ or\ G_{22}$ is bounded, the same property follows for $G_{20}$ , $G_{22}$ and $G_{20}$ , $G_{21}$ respectively.

**Remark 14:** If $G_{20}$ *is* bounded, from below, the same property holds for $G_{21}\ and\ G_{22}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{21}$ is bounded from below. 218

**Remark 15:** If $T_{20}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t), t\right)) = (b_{21}')^{(3)}$ then $T_{21} \to \infty$. 219

**Definition of** $(m)^{(3)}$ and $\varepsilon_3$ :

Indeed let $t_3$ be so that for $t > t_3$

$(b_{21})^{(3)} - (b_i'')^{(3)}\left((G_{23})(t), t\right) < \varepsilon_3, T_{20}\,(t) > (m)^{(3)}$

Then $\frac{dT_{21}}{dt} \geq (a_{21})^{(3)}(m)^{(3)} - \varepsilon_3 T_{21}$ which leads to 220

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t} = \frac{1}{2}$ it results

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right),\ \ t = log\,\frac{2}{\varepsilon_3}$ By taking now $\varepsilon_3$ sufficiently small one sees that $T_{21}$ is unbounded.
The same property holds for $T_{22}$ if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t), t\right) = (b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}$ , $\frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}} < 1$ and to choose 221

$(\widehat{P}_{24})^{(4)}$ and $(\widehat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[(\widehat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \tag{222}$$

$$\frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \tag{223}$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself (224)

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric (225)

$$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{24})^{(4)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{27}}),(\widetilde{T_{27}})$ : $\left((\widetilde{G_{27}}),(\widetilde{T_{27}})\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{24})^{(4)}\left|G_{25}^{(1)}-G_{25}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\,ds_{(24)}+$$

$$\int_0^t\{(a'_{24})^{(4)}\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$G_{24}^{(2)}\left|(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)-(a''_{24})^{(4)}\left(T_{25}^{(2)},s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\}ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}t}\leq \tag{226}$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)}+(a'_{24})^{(4)}+(\widehat{A}_{24})^{(4)}+(\widehat{P}_{24})^{(4)}(\hat{k}_{24})^{(4)}\right)d\left(((G_{27})^{(1)},(T_{27})^{(1)};\ (G_{27})^{(2)},(T_{27})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 16:** The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ *and* $(\widehat{Q}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_+$. (227)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i=24,25,26$ depend only on $T_{25}$ and respectively on

$(G_{27})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 17:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 228

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(4)} - (a_i'')^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(4)} t)} > 0$ for t > 0

**Definition of** $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2\ and\ \left((\widehat{M}_{24})^{(4)}\right)_3$ : 229

**Remark 18:** if $G_{24}$ is bounded, the same property have also $G_{25}\ and\ G_{26}$ . indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a_{25}')^{(4)} G_{25}$ and by integrating

$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1 / (a_{25}')^{(4)}$

In the same way , one can obtain

$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2 / (a_{26}')^{(4)}$

If $G_{25}\ or\ G_{26}$ is bounded, the same property follows for $G_{24}$ , $G_{26}$ and $G_{24}$ , $G_{25}$ respectively.

**Remark 19:** If $G_{24}$ *is* bounded, from below, the same property holds for $G_{25}\ and\ G_{26}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{25}$ is bounded from below. 230

**Remark 20:** If $T_{24}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}\ ((G_{27})(t), t)) = (b_{25}')^{(4)}$ then $T_{25} \to \infty$. 231

**Definition of** $(m)^{(4)}$ and $\varepsilon_4$ :

Indeed let $t_4$ be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t), t) < \varepsilon_4, T_{24}\,(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_4}$ By taking now $\varepsilon_4$ sufficiently small one sees that $T_{25}$ is unbounded.
The same property holds for $T_{26}$ if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t), t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for $G_{29}$ , $G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}$ , $\frac{(b_i)^{(5)}}{(\widehat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}\left[(\widehat{P}_{28})^{(5)}+\left((\hat{P}_{28})^{(5)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{28})^{(5)} \tag{234}$$

$$\frac{(b_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{28})^{(5)}\right]\leq(\hat{Q}_{28})^{(5)} \tag{235}$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric (236)

$$d\left(((G_{31})^{(1)},(T_{31})^{(1)}),((G_{31})^{(2)},(T_{31})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\widehat{M}_{28})^{(5)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\widehat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{31}}),(\widetilde{T_{31}})$ : $\left((\widetilde{G_{31}}),(\widetilde{T_{31}})\right)=\mathcal{A}^{(5)}\left((G_{31}),(T_{31})\right)$

It results

$$|\tilde{G}_{28}^{(1)}-\tilde{G}_i^{(2)}|\leq\int_0^t(a_{28})^{(5)}|G_{29}^{(1)}-G_{29}^{(2)}|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\,ds_{(28)}+$$

$$\int_0^t\{(a'_{28})^{(5)}|G_{28}^{(1)}-G_{28}^{(2)}|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$(a''_{28})^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)|G_{28}^{(1)}-G_{28}^{(2)}|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$G_{28}^{(2)}|(a''_{28})^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)-(a''_{28})^{(5)}\left(T_{29}^{(2)},s_{(28)}\right)|\;e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\}ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on it follows

$$|(G_{31})^{(1)}-(G_{31})^{(2)}|e^{-(\widehat{M}_{28})^{(5)}t}\leq \tag{237}$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)}+(a'_{28})^{(5)}+(\widehat{A}_{28})^{(5)}+(\widehat{P}_{28})^{(5)}(\hat{k}_{28})^{(5)}\right)d\left(((G_{31})^{(1)},(T_{31})^{(1)};\;(G_{31})^{(2)},(T_{31})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 21:** The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ *and* $(\widehat{Q}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ respectively of $\mathbb{R}_+$. (238)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on $T_{29}$ and respectively on $(G_{31})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 22:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 239

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(5)} - (a_i'')^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(5)} t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2\ and\ \left((\widehat{M}_{28})^{(5)}\right)_3$ : 240

**Remark 23:** if $G_{28}$ is bounded, the same property have also $G_{29}\ and\ G_{30}$ . indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a_{29}')^{(5)} G_{29}$ and by integrating

$$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1 / (a_{29}')^{(5)}$$

In the same way , one can obtain

$$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2 / (a_{30}')^{(5)}$$

If $G_{29}\ or\ G_{30}$ is bounded, the same property follows for $G_{28}$ , $G_{30}$ and $G_{28}$ , $G_{29}$ respectively.

**Remark 24:** If $G_{28}$ *is* bounded, from below, the same property holds for $G_{29}\ and\ G_{30}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{29}$ is bounded from below. 241

**Remark 25:** If $T_{28}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}\,((G_{31})(t), t)) = (b_{29}')^{(5)}$ then $T_{29} \to \infty$. 242

**Definition of** $(m)^{(5)}$ and $\varepsilon_5$ :

Indeed let $t_5$ be so that for $t > t_5$

$$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t), t) < \varepsilon_5, T_{28}\,(t) > (m)^{(5)}$$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_5}$ By taking now $\varepsilon_5$ sufficiently small one sees that $T_{29}$ is unbounded.
The same property holds for $T_{30}$ if $\lim_{t\to\infty}(b_{30}'')^{(5)}\left((G_{31})(t), t\right) = (b_{30}')^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{33}\,, G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\ , \frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}} < 1$ and to choose 244

$(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[(\hat{P}_{32})^{(6)}+\left((\hat{P}_{32})^{(6)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{32})^{(6)} \tag{245}$$

$$\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[\left((\hat{Q}_{32})^{(6)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{32})^{(6)}\right]\leq(\hat{Q}_{32})^{(6)} \tag{246}$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric (247)

$$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\hat{M}_{32})^{(6)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\hat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{35}}),(\widetilde{T_{35}})$ : $\left((\widetilde{G_{35}}),(\widetilde{T_{35}})\right)=\mathcal{A}^{(6)}\left((G_{35}),(T_{35})\right)$

It results

$$|\tilde{G}_{32}^{(1)}-\tilde{G}_i^{(2)}|\leq\int_0^t(a_{32})^{(6)}|G_{33}^{(1)}-G_{33}^{(2)}|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\,ds_{(32)}+$$

$$\int_0^t\{(a'_{32})^{(6)}|G_{32}^{(1)}-G_{32}^{(2)}|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)|G_{32}^{(1)}-G_{32}^{(2)}|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$G_{32}^{(2)}|(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)-(a''_{32})^{(6)}\left(T_{33}^{(2)},s_{(32)}\right)|\;e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\}ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$|(G_{35})^{(1)}-(G_{35})^{(2)}|e^{-(\widehat{M}_{32})^{(6)}t}\leq \tag{248}$$
$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)}+(a'_{32})^{(6)}+(\widehat{A}_{32})^{(6)}+(\hat{P}_{32})^{(6)}(\hat{k}_{32})^{(6)}\right)d\left(((G_{35})^{(1)},(T_{35})^{(1)};\;(G_{35})^{(2)},(T_{35})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 26:** The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\hat{P}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ *and* $(\hat{Q}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ respectively of $\mathbb{R}_+$. (249)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i = 32,33,34$ depend only on $T_{33}$ and respectively on $(G_{35})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 27:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 250

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(6)} - (a_i'')^{(6)}(T_{33}(s_{(32)}), s_{(32)})\} ds_{(32)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(6)} t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{32})^{(6)}\right)_1, \left((\widehat{M}_{32})^{(6)}\right)_2\ and\ \left((\widehat{M}_{32})^{(6)}\right)_3$ : 251

**Remark 28:** if $G_{32}$ is bounded, the same property have also $G_{33}\ and\ G_{34}$ . indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a_{33}')^{(6)} G_{33}$ and by integrating

$$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_1 / (a_{33}')^{(6)}$$

In the same way , one can obtain

$$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_2 / (a_{34}')^{(6)}$$

If $G_{33}\ or\ G_{34}$ is bounded, the same property follows for $G_{32}$ , $G_{34}$ and $G_{32}$ , $G_{33}$ respectively.

**Remark 29:** If $G_{32}$ *is* bounded, from below, the same property holds for $G_{33}\ and\ G_{34}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{33}$ is bounded from below. 252

**Remark 30:** If $T_{32}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}\ ((G_{35})(t), t)) = (b_{33}')^{(6)}$ then $T_{33} \to \infty$. 253

**Definition of** $(m)^{(6)}$ and $\varepsilon_6$ :

Indeed let $t_6$ be so that for $t > t_6$

$$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t), t\right) < \varepsilon_6, T_{32}\,(t) > (m)^{(6)}$$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right)$, $t = log\,\frac{2}{\varepsilon_6}$ By taking now $\varepsilon_6$ sufficiently small one sees that $T_{33}$ is unbounded.
The same property holds for $T_{34}$ if $\lim_{t\to\infty}(b_{34}'')^{(6)}\left((G_{35})(t), t(t), t\right) = (b_{34}')^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{37}, G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}$ , $\frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$ and to choose

$(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[(\hat{P}_{36})^{(7)}+\left((\hat{P}_{36})^{(7)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{36})^{(7)} \tag{256}$$

$$\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[\left((\hat{Q}_{36})^{(7)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{36})^{(7)}\right]\leq(\hat{Q}_{36})^{(7)} \tag{257}$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself
The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric (258)

$$d\left(((G_{39})^{(1)},(T_{39})^{(1)}),((G_{39})^{(2)},(T_{39})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\widehat{M}_{36})^{(7)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\widehat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{39}}),(\widetilde{T_{39}}):\left((\widetilde{G_{39}}),(\widetilde{T_{39}})\right)=\mathcal{A}^{(7)}((G_{39}),(T_{39}))$
It results

$$|\tilde{G}_{36}^{(1)}-\tilde{G}_i^{(2)}|\leq\int_0^t(a_{36})^{(7)}|G_{37}^{(1)}-G_{37}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\,ds_{(36)}+$$
$$\int_0^t\{(a'_{36})^{(7)}|G_{36}^{(1)}-G_{36}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$(a''_{36})^{(7)}(T_{37}^{(1)},s_{(36)})|G_{36}^{(1)}-G_{36}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$G_{36}^{(2)}|(a''_{36})^{(7)}(T_{37}^{(1)},s_{(36)})-(a''_{36})^{(7)}(T_{37}^{(2)},s_{(36)})|\;e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\}ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on it follows

$$|(G_{39})^{(1)}-(G_{39})^{(2)}|e^{-(\widehat{M}_{36})^{(7)}t}\leq \tag{259}$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}}\left((a_{36})^{(7)}+(a'_{36})^{(7)}+(\hat{A}_{36})^{(7)}+(\hat{P}_{36})^{(7)}(\hat{k}_{36})^{(7)}\right)d\left(((G_{39})^{(1)},(T_{39})^{(1)};\;(G_{39})^{(2)},(T_{39})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 31:** The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\hat{P}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ *and* $(\hat{Q}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ respectively of $\mathbb{R}_+$. (260)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(7)}$ and $(b_i'')^{(7)}, i = 36,37,38$ depend only on $T_{37}$ and respectively on $(G_{39})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 32:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 261

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(7)} - (a_i'')^{(7)}(T_{37}(s_{(36)}), s_{(36)})\} ds_{(36)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(7)} t\right)} > 0$ for t > 0

**Definition of** $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2\ and\ \left((\widehat{M}_{36})^{(7)}\right)_3$ : 262

**Remark 33:** if $G_{36}$ is bounded, the same property have also $G_{37}\ and\ G_{38}$ . indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a_{37}')^{(7)} G_{37}$ and by integrating

$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1 / (a_{37}')^{(7)}$

In the same way , one can obtain

$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2 / (a_{38}')^{(7)}$

If $G_{37}\ or\ G_{38}$ is bounded, the same property follows for $G_{36}$ , $G_{38}$ and $G_{36}$ , $G_{37}$ respectively.

**Remark 34:** If $G_{36}$ *is* bounded, from below, the same property holds for $G_{37}\ and\ G_{38}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{37}$ is bounded from below. 263

**Remark 35:** If $T_{36}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(7)}\ ((G_{39})(t), t)) = (b_{37}')^{(7)}$ then $T_{37} \to \infty$. 264

**Definition of** $(m)^{(7)}$ and $\varepsilon_7$ :

Indeed let $t_7$ be so that for $t > t_7$

$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t), t) < \varepsilon_7, T_{36}\,(t) > (m)^{(7)}$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_7}$ By taking now $\varepsilon_7$ sufficiently small one sees that $T_{37}$ is unbounded.
The same property holds for $T_{38}$ if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t), t\right) = (b_{38}')^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}$ , $\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}} < 1$ and to choose $(\widehat{P}_{40})^{(8)}$ and $(\widehat{Q}_{40})^{(8)}$ large to have 266

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[(\widehat{P}_{40})^{(8)} + \left((\hat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{40})^{(8)} \quad 267$$

$$\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{40})^{(8)}\right] \leq (\hat{Q}_{40})^{(8)} \quad 268$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)}, (T_{43})^{(1)}), ((G_{43})^{(2)}, (T_{43})^{(2)})\right) = \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

**Definition of** $\widetilde{(G_{43})}, \widetilde{(T_{43})}$ : $\left(\widetilde{(G_{43})}, \widetilde{(T_{43})}\right) = \mathcal{A}^{(8)}((G_{43}), (T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{40})^{(8)}\left|G_{41}^{(1)} - G_{41}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\,ds_{(40)} +$$
$$\int_0^t\{(a'_{40})^{(8)}\left|G_{40}^{(1)} - G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{-(\widehat{M}_{40})^{(8)}s_{(40)}} +$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right)\left|G_{40}^{(1)} - G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}} +$$
$$G_{40}^{(2)}\left|(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right) - (a''_{40})^{(8)}\left(T_{41}^{(2)}, s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\}ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0, t]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)} - (G_{43})^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}t} \leq \quad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)} + (a'_{40})^{(8)} + (\widehat{A}_{40})^{(8)} + (\widehat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}\right)d\left(((G_{43})^{(1)}, (T_{43})^{(1)};\ (G_{43})^{(2)}, (T_{43})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

**Remark 36:** The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate 274

condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ $and$ $(\widehat{Q}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i = 40,41,42$ depend only on $T_{41}$ and respectively on $(G_{43})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 37** There does not exist any $t$ where $G_i(t) = 0\ and\ T_i(t) = 0$ 275

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(8)} - (a_i'')^{(8)}(T_{41}(s_{(40)}), s_{(40)})\} ds_{(40)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(8)}t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{40})^{(8)}\right)_1, \left((\widehat{M}_{40})^{(8)}\right)_2$ $and$ $\left((\widehat{M}_{40})^{(8)}\right)_3$ : 276

**Remark 38:** if $G_{40}$ is bounded, the same property have also $G_{41}$ $and$ $G_{42}$ . indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a_{41}')^{(8)}G_{41}$ and by integrating

$$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1/(a_{41}')^{(8)}$$

In the same way , one can obtain

$$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2/(a_{42}')^{(8)}$$

If $G_{41}$ $or$ $G_{42}$ is bounded, the same property follows for $G_{40}$ , $G_{42}$ and $G_{40}$ , $G_{41}$ respectively.

**Remark 39:** If $G_{40}$ $is$ bounded, from below, the same property holds for $G_{41}$ $and$ $G_{42}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{41}$ is bounded from below. 277

**Remark 40:** If $T_{40}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}((G_{43})(t), t)) = (b_{41}')^{(8)}$ then $T_{41} \to \infty$. 278

**Definition of** $(m)^{(8)}$ and $\varepsilon_8$ :

Indeed let $t_8$ be so that for $t > t_8$

$$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t), t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right)$, $t = log\frac{2}{\varepsilon_8}$ By taking now $\varepsilon_8$ sufficiently small one sees that $T_{41}$ is unbounded.
The same property holds for $T_{42}$ if $\lim_{t\to\infty}(b''_{42})^{(8)}\left((G_{43})(t), t(t), t\right) = (b'_{42})^{(8)}$



It is now sufficient to take $\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}$ , $\frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$ and to choose $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ large to have

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}\left[(\hat{P}_{44})^{(9)} + \left((\hat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}}\left[\left((\hat{Q}_{44})^{(9)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{44})^{(9)}\right] \leq (\hat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)}, (T_{47})^{(1)}), ((G_{47})^{(2)}, (T_{47})^{(2)})\right) =$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{44})^{(9)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{47}}), (\widetilde{T_{47}}) : \left((\widetilde{G_{47}}), (\widetilde{T_{47}})\right) = \mathcal{A}^{(9)}\left((G_{47}), (T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{44})^{(9)}\left|G_{45}^{(1)} - G_{45}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\,ds_{(44)} +$$
$$\int_0^t\{(a'_{44})^{(9)}\left|G_{44}^{(1)} - G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{-(\widehat{M}_{44})^{(9)}s_{(44)}} +$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right)\left|G_{44}^{(1)} - G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}} +$$
$$G_{44}^{(2)}\left|(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right) - (a''_{44})^{(9)}\left(T_{45}^{(2)}, s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\}ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)} - G^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}t} \leq$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)} + (a'_{44})^{(9)} + (\widehat{A}_{44})^{(9)} + (\widehat{P}_{44})^{(9)}(\hat{k}_{44})^{(9)}\right)d\left(((G_{47})^{(1)}, (T_{47})^{(1)};\ (G_{47})^{(2)}, (T_{47})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis (39,35,36) the result follows

**Remark 41:** The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by

$(\widehat{P}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ *and* $(\widehat{Q}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i = 44{,}45{,}46$ depend only on $T_{45}$ and respectively on $(G_{47})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 42:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$

From 99 to 44 it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t\{(a_i')^{(9)}-(a_i'')^{(9)}(T_{45}(s_{(44)}),s_{(44)})\}ds_{(44)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(9)}t)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{44})^{(9)}\right)_1, \left((\widehat{M}_{44})^{(9)}\right)_2$ *and* $\left((\widehat{M}_{44})^{(9)}\right)_3$ :

**Remark 43:** if $G_{44}$ is bounded, the same property have also $G_{45}\ and\ G_{46}$ . indeed if

$G_{44} < (\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt} \leq \left((\widehat{M}_{44})^{(9)}\right)_1 - (a_{45}')^{(9)}G_{45}$ and by integrating

$$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_1/(a_{45}')^{(9)}$$

In the same way , one can obtain

$$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_2/(a_{46}')^{(9)}$$

If $G_{45}\ or\ G_{46}$ is bounded, the same property follows for $G_{44}$ , $G_{46}$ and $G_{44}$ , $G_{45}$ respectively.

**Remark 44:** If $G_{44}$ *is* bounded, from below, the same property holds for $G_{45}\ and\ G_{46}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{45}$ is bounded from below.

**Remark 45:** If $T_{44}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t),t\right) = (b_{45}')^{(9)}$ then $T_{45} \to \infty$.

**Definition of** $(m)^{(9)}$ and $\varepsilon_9$ :

Indeed let $t_9$ be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t),t\right) < \varepsilon_9, T_{44}\,(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right), \quad t = log\frac{2}{\varepsilon_9}$ By taking now $\varepsilon_9$ sufficiently small one sees that $T_{45}$ is unbounded.
The same property holds for $T_{46}$ if $\lim_{t\to\infty}(b_{46}'')^{(9)}\left((G_{47})(t),t\right) = (b_{46}')^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

**Behavior of the solutions of equation** 280

**Theorem** If we denote and define

**Definition of** $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ :

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)} \le -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14}, t) + (a''_{14})^{(1)}(T_{14}, t) \le -(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)} \le -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G, t) - (b''_{14})^{(1)}(G, t) \le -(\tau_1)^{(1)}$

**Definition of** $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$ : 281

By $(\nu_1)^{(1)} > 0$, $(\nu_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$, $(u_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

**Definition of** $(\bar{\nu}_1)^{(1)},, (\bar{\nu}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$ : 282

By $(\bar{\nu}_1)^{(1)} > 0$, $(\bar{\nu}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$, $(\bar{u}_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

**Definition of** $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (\nu_0)^{(1)}$ :- 283

If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by
$(m_2)^{(1)} = (\nu_0)^{(1)}, (m_1)^{(1)} = (\nu_1)^{(1)}$, $if\ (\nu_0)^{(1)} < (\nu_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}, if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)}$,

and $\boxed{(\nu_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)}$, $if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)}$, $if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)}$,

and $\boxed{(u_0)^{(1)} = \frac{T^0_{13}}{T^0_{14}}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$

are defined

Then the solution of global equations satisfies the inequalities 285

$G^0_{13}e^{\left((S_1)^{(1)} - (p_{13})^{(1)}\right)t} \le G_{13}(t) \le G^0_{13}e^{(S_1)^{(1)}t}$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)}-(p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)}t}$$

$$\left(\frac{(a_{15})^{(1)}G_{13}^0}{(m_1)^{(1)}((S_1)^{(1)}-(p_{13})^{(1)}-(S_2)^{(1)})}\left[e^{((S_1)^{(1)}-(p_{13})^{(1)})t} - e^{-(S_2)^{(1)}t}\right] + G_{15}^0 e^{-(S_2)^{(1)}t} \le G_{15}(t) \le \right. \quad (286)$$

$$\left. \frac{(a_{15})^{(1)}G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)}-(a'_{15})^{(1)})}[e^{(S_1)^{(1)}t} - e^{-(a'_{15})^{(1)}t}] + G_{15}^0 e^{-(a'_{15})^{(1)}t}\right)$$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)}t} \le T_{13}(t) \le T_{13}^0 e^{((R_1)^{(1)}+(r_{13})^{(1)})t}} \quad (287)$$

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)}t} \le T_{13}(t) \le \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)}+(r_{13})^{(1)})t} \quad (288)$$

$$\frac{(b_{15})^{(1)}T_{13}^0}{(\mu_1)^{(1)}((R_1)^{(1)}-(b'_{15})^{(1)})}\left[e^{(R_1)^{(1)}t} - e^{-(b'_{15})^{(1)}t}\right] + T_{15}^0 e^{-(b'_{15})^{(1)}t} \le T_{15}(t) \le \quad (289)$$

$$\frac{(a_{15})^{(1)}T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)}+(r_{13})^{(1)}+(R_2)^{(1)})}\left[e^{((R_1)^{(1)}+(r_{13})^{(1)})t} - e^{-(R_2)^{(1)}t}\right] + T_{15}^0 e^{-(R_2)^{(1)}t}$$

**Definition of** $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:- (290)

Where $(S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$

$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$

$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$

$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$

**Behavior of the solutions of equation** (291)

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ : (292)

$(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ four constants satisfying

$-(\sigma_2)^{(2)} \le -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(T_{17}, t) + (a''_{17})^{(2)}(T_{17}, t) \le -(\sigma_1)^{(2)}$ (293)

$-(\tau_2)^{(2)} \le -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}((G_{19}), t) - (b''_{17})^{(2)}((G_{19}), t) \le -(\tau_1)^{(2)}$ (294)

**Definition of** $(\nu_1)^{(2)}, (\nu_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$ : (295)

By $(\nu_1)^{(2)} > 0$, $(\nu_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0$, $(u_2)^{(2)} < 0$ the roots (296)

of the equations $(a_{17})^{(2)}(\nu^{(2)})^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ (297)

and $(b_{14})^{(2)}(u^{(2)})^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and (298)

**Definition of** $(\bar{\nu}_1)^{(2)}, , (\bar{\nu}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$ : (299)

By $(\bar{\nu}_1)^{(2)} > 0$, $(\bar{\nu}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0$, $(\bar{u}_2)^{(2)} < 0$ the (300)

roots of the equations $(a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 301

and $(b_{17})^{(2)}\left(u^{(2)}\right)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 302

**Definition of** $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ :- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, ***if*** $(u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$, ***if*** $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)}$, ***if*** $(\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)}t}$$

$(p_i)^{(2)}$ is defined by equation

$$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)}t} \quad 311$$

$$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)}t}\right] + G_{18}^0 e^{-(S_2)^{(2)}t} \le G_{18}(t) \le \right. \quad 312$$

$$\left. \frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)}t} - e^{-(a'_{18})^{(2)}t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)}t}\right)$$

$$\boxed{T_{16}^0 e^{(R_1)^{(2)}t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}} \quad 313$$

$$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)}t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t} \quad 314$$

$$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)}t} - e^{-(b'_{18})^{(2)}t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)}t} \le T_{18}(t) \le \quad 315$$

$$\frac{(a_{18})^{(2)}T_{16}^{0}}{(\mu_2)^{(2)}((R_1)^{(2)}+(r_{16})^{(2)}+(R_2)^{(2)})}\left[e^{((R_1)^{(2)}+(r_{16})^{(2)})t}-e^{-(R_2)^{(2)}t}\right]+T_{18}^{0}e^{-(R_2)^{(2)}t}$$

**Definition of** $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$$

$$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)}$$ 318

$$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$$

**Behavior of the solutions** 319

**Theorem 3:** If we denote and define

**Definition of** $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ :

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}\big((G_{23}), t\big) \le -(\tau_1)^{(3)}$$

**Definition of** $(v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$ : 320

By $(v_1)^{(3)} > 0$, $(v_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{v}_1)^{(3)} > 0$, $(\bar{v}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

**Definition of** $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ :- 321

If we define $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (v_0)^{(3)}, (m_1)^{(3)} = (v_1)^{(3)}$, ***if*** $(v_0)^{(3)} < (v_1)^{(3)}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (\bar{v}_1)^{(3)}$, ***if*** $(v_1)^{(3)} < (v_0)^{(3)} < (\bar{v}_1)^{(3)}$,

and $\boxed{(v_0)^{(3)} = \frac{G_{20}^{0}}{G_{21}^{0}}}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_0)^{(3)}$, ***if*** $(\bar{v}_1)^{(3)} < (v_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)},$ ***if*** $(u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$ , ***if*** $(u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)},$ and $\boxed{(u_0)^{(3)} = \frac{T_{20}^0}{T_{21}^0}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)},$ ***if*** $(\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G_{20}^0 e^{(S_1)^{(3)} t}$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)} t} \quad 323$$

$$\left(\frac{(a_{22})^{(3)} G_{20}^0}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\left[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)} t}\right] + G_{22}^0 e^{-(S_2)^{(3)} t} \le G_{22}(t) \le \right. \quad 324$$

$$\left. \frac{(a_{22})^{(3)} G_{20}^0}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}\left[e^{(S_1)^{(3)} t} - e^{-(a'_{22})^{(3)} t}\right] + G_{22}^0 e^{-(a'_{22})^{(3)} t}\right)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t}} \quad 325$$

$$\frac{1}{(\mu_1)^{(3)}} T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}} T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)} T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)} - (b'_{22})^{(3)})}\left[e^{(R_1)^{(3)} t} - e^{-(b'_{22})^{(3)} t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)} t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)} T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)} + (r_{20})^{(3)} + (R_2)^{(3)})}\left[e^{((R_1)^{(3)} + (r_{20})^{(3)})t} - e^{-(R_2)^{(3)} t}\right] + T_{22}^0 e^{-(R_2)^{(3)} t}$$

**Definition of** $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

**Behavior of the solutions of equation**

**Theorem:** If we denote and define

**Definition of** $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ :

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$

$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}((G_{27}), t) - (b''_{25})^{(4)}((G_{27}), t) \le -(\tau_1)^{(4)}$

**Definition of** $(v_1)^{(4)}, (v_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, v^{(4)}, u^{(4)}$ : 329

By $(v_1)^{(4)} > 0$ , $(v_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$ , $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_1)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

**Definition of** $(\bar{v}_1)^{(4)},, (\bar{v}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$ : 330

By $(\bar{v}_1)^{(4)} > 0$ , $(\bar{v}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$ , $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

**Definition of** $(m_1)^{(4)}$ , $(m_2)^{(4)}$ , $(\mu_1)^{(4)}, (\mu_2)^{(4)}, (\nu_0)^{(4)}$ :-

If we define $(m_1)^{(4)}$ , $(m_2)^{(4)}$ , $(\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (v_0)^{(4)}, (m_1)^{(4)} = (v_1)^{(4)}$, ***if*** $(v_0)^{(4)} < (v_1)^{(4)}$

$(m_2)^{(4)} = (v_1)^{(4)}, (m_1)^{(4)} = (\bar{v}_1)^{(4)}$ , ***if*** $(v_4)^{(4)} < (v_0)^{(4)} < (\bar{v}_1)^{(4)}$,
and $\boxed{(v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (v_4)^{(4)}, (m_1)^{(4)} = (v_0)^{(4)}$, ***if*** $(\bar{v}_4)^{(4)} < (v_0)^{(4)}$

and analogously

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$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)}$, ***if*** $(u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}$ , ***if*** $(u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)}$,
and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}$, ***if*** $(\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

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$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)}t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)}t}$$

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$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)}t}\right] + G_{26}^0 e^{-(S_2)^{(4)}t} \le G_{26}(t) \le\right.$$ 334

*(a26)4G240(m2)4(S1)4−(a26′)4e(S1)4t−e−(a26′)4t+ G260e−(a26′)4t*

$$\boxed{T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}}T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}}T_{24}^0 e^{((R_1)^{(4)}+(r_{24})^{(4)})t} \quad 335$$

$$\frac{(b_{26})^{(4)}T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)}-(b'_{26})^{(4)})}\left[e^{(R_1)^{(4)}t} - e^{-(b'_{26})^{(4)}t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)}t} \le T_{26}(t) \le \quad 336$$

$$\frac{(a_{26})^{(4)}T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)}+(r_{24})^{(4)}+(R_2)^{(4)})}\left[e^{((R_1)^{(4)}+(r_{24})^{(4)})t} - e^{-(R_2)^{(4)}t}\right] + T_{26}^0 e^{-(R_2)^{(4)}t}$$

**Definition of** $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

**Behavior of the solutions of equation** 338

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ :

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

**Definition of** $(v_1)^{(5)}, (v_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, v^{(5)}, u^{(5)}$ : 339

By $(v_1)^{(5)} > 0$ , $(v_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$ , $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(v^{(5)}\big)^2 + (\sigma_1)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

**Definition of** $(\bar{v}_1)^{(5)}, , (\bar{v}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$ : 340

By $(\bar{v}_1)^{(5)} > 0$ , $(\bar{v}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$ , $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(v^{(5)}\big)^2 + (\sigma_2)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

**Definition of** $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (v_0)^{(5)}$ :-

If we define $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (v_0)^{(5)}, (m_1)^{(5)} = (v_1)^{(5)}$, ***if*** $(v_0)^{(5)} < (v_1)^{(5)}$

$(m_2)^{(5)} = (v_1)^{(5)}, (m_1)^{(5)} = (\bar{v}_1)^{(5)}$ , ***if*** $(v_1)^{(5)} < (v_0)^{(5)} < (\bar{v}_1)^{(5)}$,
and $\boxed{(v_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)},\ \boldsymbol{if}\ (u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}, \boldsymbol{if}\ (u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)},$

and $\boxed{(u_0)^{(5)} = \frac{T_{28}^0}{T_{29}^0}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}, \boldsymbol{if}\ (\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{28}(t) \le G_{28}^0 e^{(S_1)^{(5)} t}$

where $(p_i)^{(5)}$ is defined by equation

$\frac{1}{(m_5)^{(5)}} G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}} G_{28}^0 e^{(S_1)^{(5)} t}$ 343

$\left(\frac{(a_{30})^{(5)} G_{28}^0}{(m_1)^{(5)}((S_1)^{(5)} - (p_{28})^{(5)} - (S_2)^{(5)})}\left[e^{((S_1)^{(5)} - (p_{28})^{(5)})t} - e^{-(S_2)^{(5)} t}\right] + G_{30}^0 e^{-(S_2)^{(5)} t} \le G_{30}(t) \le\right.$ 344

*(a30)5G280(m2)5(S1)5−(a30′)5e(S1)5t−e−(a30′)5t+ G300e−(a30′)5t*

$\boxed{T_{28}^0 e^{(R_1)^{(5)} t} \le T_{28}(t) \le T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}}$ 345

$\frac{1}{(\mu_1)^{(5)}} T_{28}^0 e^{(R_1)^{(5)} t} \le T_{28}(t) \le \frac{1}{(\mu_2)^{(5)}} T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}$ 346

$\frac{(b_{30})^{(5)} T_{28}^0}{(\mu_1)^{(5)}((R_1)^{(5)} - (b'_{30})^{(5)})}\left[e^{(R_1)^{(5)} t} - e^{-(b'_{30})^{(5)} t}\right] + T_{30}^0 e^{-(b'_{30})^{(5)} t} \le T_{30}(t) \le$ 347

$\frac{(a_{30})^{(5)} T_{28}^0}{(\mu_2)^{(5)}((R_1)^{(5)} + (r_{28})^{(5)} + (R_2)^{(5)})}\left[e^{((R_1)^{(5)} + (r_{28})^{(5)})t} - e^{-(R_2)^{(5)} t}\right] + T_{30}^0 e^{-(R_2)^{(5)} t}$

**Definition of** $(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$:- 348

Where $(S_1)^{(5)} = (a_{28})^{(5)}(m_2)^{(5)} - (a'_{28})^{(5)}$

$(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}$

$(R_1)^{(5)} = (b_{28})^{(5)}(\mu_2)^{(5)} - (b'_{28})^{(5)}$

$(R_2)^{(5)} = (b'_{30})^{(5)} - (r_{30})^{(5)}$

**Behavior of the solutions of equation** 349

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ :

$(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)} \le -(a'_{32})^{(6)} + (a'_{33})^{(6)} - (a''_{32})^{(6)}(T_{33}, t) + (a''_{33})^{(6)}(T_{33}, t) \le -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)} \le -(b'_{32})^{(6)} + (b'_{33})^{(6)} - (b''_{32})^{(6)}\big((G_{35}), t\big) - (b''_{33})^{(6)}\big((G_{35}), t\big) \le -(\tau_1)^{(6)}$$

**Definition of** $(\nu_1)^{(6)}, (\nu_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, \nu^{(6)}, u^{(6)}$ : 350

By $(\nu_1)^{(6)} > 0$, $(\nu_2)^{(6)} < 0$ and respectively $(u_1)^{(6)} > 0$, $(u_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(6)},, (\bar{\nu}_2)^{(6)}, (\bar{u}_1)^{(6)}, (\bar{u}_2)^{(6)}$ : 351

By $(\bar{\nu}_1)^{(6)} > 0$, $(\bar{\nu}_2)^{(6)} < 0$ and respectively $(\bar{u}_1)^{(6)} > 0$, $(\bar{u}_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$

**Definition of** $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}, (\nu_0)^{(6)}$ :-

If we define $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}$ by

$$(m_2)^{(6)} = (\nu_0)^{(6)}, (m_1)^{(6)} = (\nu_1)^{(6)}, \ \boldsymbol{if}\ (\nu_0)^{(6)} < (\nu_1)^{(6)}$$

$$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}, \boldsymbol{if}\ (\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)},$$

and $\boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}}$

$$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$$

and analogously 352

$$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}, \ \boldsymbol{if}\ (u_0)^{(6)} < (u_1)^{(6)}$$

$$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}, \boldsymbol{if}\ (u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)},$$

and $\boxed{(u_0)^{(6)} = \frac{T^0_{32}}{T^0_{33}}}$

$$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}, \boldsymbol{if}\ (\bar{u}_1)^{(6)} < (u_0)^{(6)} \text{ where } (u_1)^{(6)}, (\bar{u}_1)^{(6)}$$

Then the solution of global equations satisfies the inequalities 353

$$G^0_{32}e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{32}(t) \le G^0_{32}e^{(S_1)^{(6)}t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}}G^0_{32}e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{33}(t) \le \frac{1}{(m_2)^{(6)}}G^0_{32}e^{(S_1)^{(6)}t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)}G^0_{32}}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})}\left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)}t}\right] + G^0_{34}e^{-(S_2)^{(6)}t} \le G_{34}(t) \le \right. \quad 355$$

$$\frac{(a_{34})^{(6)}G_{32}^{0}}{(m_2)^{(6)}((S_1)^{(6)}-(a'_{34})^{(6)})}\left[e^{(S_1)^{(6)}t}-e^{-(a'_{34})^{(6)}t}\right]+\ G_{34}^{0}e^{-(a'_{34})^{(6)}t}\Big)$$

$$\boxed{T_{32}^{0}e^{(R_1)^{(6)}t}\le T_{32}(t)\le T_{32}^{0}e^{((R_1)^{(6)}+(r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}}T_{32}^{0}e^{(R_1)^{(6)}t}\le T_{32}(t)\le\frac{1}{(\mu_2)^{(6)}}T_{32}^{0}e^{((R_1)^{(6)}+(r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)}T_{32}^{0}}{(\mu_1)^{(6)}((R_1)^{(6)}-(b'_{34})^{(6)})}\left[e^{(R_1)^{(6)}t}-e^{-(b'_{34})^{(6)}t}\right]+T_{34}^{0}e^{-(b'_{34})^{(6)}t}\le T_{34}(t)\le \quad 358$$

$$\frac{(a_{34})^{(6)}T_{32}^{0}}{(\mu_2)^{(6)}((R_1)^{(6)}+(r_{32})^{(6)}+(R_2)^{(6)})}\left[e^{((R_1)^{(6)}+(r_{32})^{(6)})t}-e^{-(R_2)^{(6)}t}\right]+T_{34}^{0}e^{-(R_2)^{(6)}t}$$

**Definition of** $(S_1)^{(6)},(S_2)^{(6)},(R_1)^{(6)},(R_2)^{(6)}$:- 359

Where $(S_1)^{(6)}=(a_{32})^{(6)}(m_2)^{(6)}-(a'_{32})^{(6)}$

$(S_2)^{(6)}=(a_{34})^{(6)}-(p_{34})^{(6)}$

$(R_1)^{(6)}=(b_{32})^{(6)}(\mu_2)^{(6)}-(b'_{32})^{(6)}$

$(R_2)^{(6)}=(b'_{34})^{(6)}-(r_{34})^{(6)}$

**Behavior of the solutions of equation**

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(7)},(\sigma_2)^{(7)},(\tau_1)^{(7)},(\tau_2)^{(7)}$ :

$(\sigma_1)^{(7)},(\sigma_2)^{(7)},(\tau_1)^{(7)},(\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)}\le-(a'_{36})^{(7)}+(a'_{37})^{(7)}-(a''_{36})^{(7)}(T_{37},t)+(a''_{37})^{(7)}(T_{37},t)\le-(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)}\le-(b'_{36})^{(7)}+(b'_{37})^{(7)}-(b''_{36})^{(7)}\big((G_{39}),t\big)-(b''_{37})^{(7)}\big((G_{39}),t\big)\le-(\tau_1)^{(7)}$$

**Definition of** $(v_1)^{(7)},(v_2)^{(7)},(u_1)^{(7)},(u_2)^{(7)},v^{(7)},u^{(7)}$ : 361

By $(v_1)^{(7)}>0,(v_2)^{(7)}<0$ and respectively $(u_1)^{(7)}>0,(u_2)^{(7)}<0$ the roots of the equations $(a_{37})^{(7)}\big(v^{(7)}\big)^2+(\sigma_1)^{(7)}v^{(7)}-(a_{36})^{(7)}=0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2+(\tau_1)^{(7)}u^{(7)}-(b_{36})^{(7)}=0$ and

**Definition of** $(\bar{v}_1)^{(7)},,(\bar{v}_2)^{(7)},(\bar{u}_1)^{(7)},(\bar{u}_2)^{(7)}$ : 362

By $(\bar{v}_1)^{(7)}>0,(\bar{v}_2)^{(7)}<0$ and respectively $(\bar{u}_1)^{(7)}>0,(\bar{u}_2)^{(7)}<0$ the roots of the equations $(a_{37})^{(7)}\big(v^{(7)}\big)^2+(\sigma_2)^{(7)}v^{(7)}-(a_{36})^{(7)}=0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2+(\tau_2)^{(7)}u^{(7)}-(b_{36})^{(7)}=0$

**Definition of** $(m_1)^{(7)},(m_2)^{(7)},(\mu_1)^{(7)},(\mu_2)^{(7)},(v_0)^{(7)}$ :-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$$(m_2)^{(7)} = (v_0)^{(7)}, (m_1)^{(7)} = (v_1)^{(7)}, \ \boldsymbol{if}\ (v_0)^{(7)} < (v_1)^{(7)}$$

$$(m_2)^{(7)} = (v_1)^{(7)}, (m_1)^{(7)} = (\bar{v}_1)^{(7)}, \boldsymbol{if}\ (v_1)^{(7)} < (v_0)^{(7)} < (\bar{v}_1)^{(7)},$$

and $\boxed{(v_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$

$$(m_2)^{(7)} = (v_1)^{(7)}, (m_1)^{(7)} = (v_0)^{(7)}, \ \boldsymbol{if}\ (\bar{v}_1)^{(7)} < (v_0)^{(7)}$$

and analogously 363

$$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}, \ \boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$$

and $\boxed{(u_0)^{(7)} = \frac{T_{36}^0}{T_{37}^0}}$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)} \text{ where } (u_1)^{(7)}, (\bar{u}_1)^{(7)}$$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^0 e^{(S_1)^{(7)} t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)} t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)}((S_1)^{(7)} - (p_{36})^{(7)} - (S_2)^{(7)})}\left[e^{((S_1)^{(7)} - (p_{36})^{(7)})t} - e^{-(S_2)^{(7)} t}\right] + G_{38}^0 e^{-(S_2)^{(7)} t} \le G_{38}(t) \le \right. \quad 366$$

$$\left. \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)}((S_1)^{(7)} - (a'_{38})^{(7)})}\left[e^{(S_1)^{(7)} t} - e^{-(a'_{38})^{(7)} t}\right] + G_{38}^0 e^{-(a'_{38})^{(7)} t}\right)$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t} \quad 368$$

$$\frac{(b_{38})^{(7)} T_{36}^0}{(\mu_1)^{(7)}((R_1)^{(7)} - (b'_{38})^{(7)})}\left[e^{(R_1)^{(7)} t} - e^{-(b'_{38})^{(7)} t}\right] + T_{38}^0 e^{-(b'_{38})^{(7)} t} \le T_{38}(t) \le \quad 369$$

$$\frac{(a_{38})^{(7)} T_{36}^0}{(\mu_2)^{(7)}((R_1)^{(7)} + (r_{36})^{(7)} + (R_2)^{(7)})}\left[e^{((R_1)^{(7)} + (r_{36})^{(7)})t} - e^{-(R_2)^{(7)} t}\right] + T_{38}^0 e^{-(R_2)^{(7)} t}$$

**Definition of** $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$

$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$

$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$

**Behavior of the solutions of equation** 371

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ :

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$-(\sigma_2)^{(8)} \le -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \le -(\sigma_1)^{(8)}$

$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\big((G_{43}), t\big) - (b''_{41})^{(8)}\big((G_{43}), t\big) \le -(\tau_1)^{(8)}$

**Definition of** $(\nu_1)^{(8)}, (\nu_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, \nu^{(8)}, u^{(8)}$ : 372

By $(\nu_1)^{(8)} > 0, (\nu_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0, (u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(8)}, , (\bar{\nu}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$ :

By $(\bar{\nu}_1)^{(8)} > 0, (\bar{\nu}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0, (\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

**Definition of** $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}, (\nu_0)^{(8)}$ :-

If we define $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$(m_2)^{(8)} = (\nu_0)^{(8)}, (m_1)^{(8)} = (\nu_1)^{(8)}$, ***if*** $(\nu_0)^{(8)} < (\nu_1)^{(8)}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\bar{\nu}_1)^{(8)}$, ***if*** $(\nu_1)^{(8)} < (\nu_0)^{(8)} < (\bar{\nu}_1)^{(8)}$,
and $\boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\nu_0)^{(8)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(8)} < (\nu_0)^{(8)}$

and analogously 374

$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)},\ \boldsymbol{if}\ (u_0)^{(8)} < (u_1)^{(8)}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}, \boldsymbol{if}\ (u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)},$

and $\boxed{(u_0)^{(8)} = \frac{T_{40}^0}{T_{41}^0}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}, \boldsymbol{if}\ (\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^0 e^{(S_1)^{(8)} t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}} G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}} G_{40}^0 e^{(S_1)^{(8)} t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)} G_{40}^0}{(m_1)^{(8)}((S_1)^{(8)} - (p_{40})^{(8)} - (S_2)^{(8)})}\left[e^{((S_1)^{(8)} - (p_{40})^{(8)})t} - e^{-(S_2)^{(8)} t}\right] + G_{42}^0 e^{-(S_2)^{(8)} t} \le G_{42}(t) \le \right. \quad 377$$

$$\left. \frac{(a_{42})^{(8)} G_{40}^0}{(m_2)^{(8)}((S_1)^{(8)} - (a'_{42})^{(8)})}\left[e^{(S_1)^{(8)} t} - e^{-(a'_{42})^{(8)} t}\right] + G_{42}^0 e^{-(a'_{42})^{(8)} t}\right)$$

$$\boxed{T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}} T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le \frac{1}{(\mu_2)^{(8)}} T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)} T_{40}^0}{(\mu_1)^{(8)}((R_1)^{(8)} - (b'_{42})^{(8)})}\left[e^{(R_1)^{(8)} t} - e^{-(b'_{42})^{(8)} t}\right] + T_{42}^0 e^{-(b'_{42})^{(8)} t} \le T_{42}(t) \le \quad 380$$

$$\frac{(a_{42})^{(8)} T_{40}^0}{(\mu_2)^{(8)}((R_1)^{(8)} + (r_{40})^{(8)} + (R_2)^{(8)})}\left[e^{((R_1)^{(8)} + (r_{40})^{(8)})t} - e^{-(R_2)^{(8)} t}\right] + T_{42}^0 e^{-(R_2)^{(8)} t}$$

**Definition of** $(S_1)^{(8)}, (S_2)^{(8)}, (R_1)^{(8)}, (R_2)^{(8)}$:- 381

Where $(S_1)^{(8)} = (a_{40})^{(8)}(m_2)^{(8)} - (a'_{40})^{(8)}$

$(S_2)^{(8)} = (a_{42})^{(8)} - (p_{42})^{(8)}$

$(R_1)^{(8)} = (b_{40})^{(8)}(\mu_2)^{(8)} - (b'_{40})^{(8)}$

$(R_2)^{(8)} = (b'_{42})^{(8)} - (r_{42})^{(8)}$

**Behavior of the solutions of equation 37 to 92** 382

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ :

$(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ four constants satisfying

$$-(\sigma_2)^{(9)} \le -(a'_{44})^{(9)} + (a'_{45})^{(9)} - (a''_{44})^{(9)}(T_{45}, t) + (a''_{45})^{(9)}(T_{45}, t) \le -(\sigma_1)^{(9)}$$

$$-(\tau_2)^{(9)} \le -(b'_{44})^{(9)} + (b'_{45})^{(9)} - (b''_{44})^{(9)}\big((G_{47}), t\big) - (b''_{45})^{(9)}\big((G_{47}), t\big) \le -(\tau_1)^{(9)}$$

**Definition of** $(\nu_1)^{(9)}, (\nu_2)^{(9)}, (u_1)^{(9)}, (u_2)^{(9)}, \nu^{(9)}, u^{(9)}$ :

By $(\nu_1)^{(9)} > 0$ , $(\nu_2)^{(9)} < 0$ and respectively $(u_1)^{(9)} > 0$ , $(u_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_1)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(9)}, , (\bar{\nu}_2)^{(9)}, (\bar{u}_1)^{(9)}, (\bar{u}_2)^{(9)}$ :

By $(\bar{\nu}_1)^{(9)} > 0$ , $(\bar{\nu}_2)^{(9)} < 0$ and respectively $(\bar{u}_1)^{(9)} > 0$ , $(\bar{u}_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_2)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$

**Definition of** $(m_1)^{(9)}, (m_2)^{(9)}, (\mu_1)^{(9)}, (\mu_2)^{(9)}, (\nu_0)^{(9)}$ :-

If we define $(m_1)^{(9)}, (m_2)^{(9)}, (\mu_1)^{(9)}, (\mu_2)^{(9)}$ by

$$(m_2)^{(9)} = (\nu_0)^{(9)}, (m_1)^{(9)} = (\nu_1)^{(9)}, \ \boldsymbol{if}\ (\nu_0)^{(9)} < (\nu_1)^{(9)}$$

$$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\bar{\nu}_1)^{(9)}, \boldsymbol{if}\ (\nu_1)^{(9)} < (\nu_0)^{(9)} < (\bar{\nu}_1)^{(9)},$$

and $\boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$

$$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\nu_0)^{(9)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(9)} < (\nu_0)^{(9)}$$

and analogously

$$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)}, \ \boldsymbol{if}\ (u_0)^{(9)} < (u_1)^{(9)}$$

$$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}, \boldsymbol{if}\ (u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)},$$

and $\boxed{(u_0)^{(9)} = \frac{T^0_{44}}{T^0_{45}}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}, \boldsymbol{if}\ (\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G^0_{44} e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{44}(t) \le G^0_{44} e^{(S_1)^{(9)} t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}}G_{44}^0 e^{((S_1)^{(9)}-(p_{44})^{(9)})t} \le G_{45}(t) \le \frac{1}{(m_2)^{(9)}}G_{44}^0 e^{(S_1)^{(9)}t}$$

$$\left(\frac{(a_{46})^{(9)}G_{44}^0}{(m_1)^{(9)}((S_1)^{(9)}-(p_{44})^{(9)}-(S_2)^{(9)})}\left[e^{((S_1)^{(9)}-(p_{44})^{(9)})t}-e^{-(S_2)^{(9)}t}\right]+G_{46}^0 e^{-(S_2)^{(9)}t} \le G_{46}(t) \le \right.$$

$$\left.\frac{(a_{46})^{(9)}G_{44}^0}{(m_2)^{(9)}((S_1)^{(9)}-(a'_{46})^{(9)})}\left[e^{(S_1)^{(9)}t}-e^{-(a'_{46})^{(9)}t}\right]+\ G_{46}^0 e^{-(a'_{46})^{(9)}t}\right)$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le T_{44}^0 e^{((R_1)^{(9)}+(r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}}T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le \frac{1}{(\mu_2)^{(9)}}T_{44}^0 e^{((R_1)^{(9)}+(r_{44})^{(9)})t}$$

$$\frac{(b_{46})^{(9)}T_{44}^0}{(\mu_1)^{(9)}((R_1)^{(9)}-(b'_{46})^{(9)})}\left[e^{(R_1)^{(9)}t}-e^{-(b'_{46})^{(9)}t}\right]+T_{46}^0 e^{-(b'_{46})^{(9)}t} \le T_{46}(t) \le$$

$$\frac{(a_{46})^{(9)}T_{44}^0}{(\mu_2)^{(9)}((R_1)^{(9)}+(r_{44})^{(9)}+(R_2)^{(9)})}\left[e^{((R_1)^{(9)}+(r_{44})^{(9)})t}-e^{-(R_2)^{(9)}t}\right]+T_{46}^0 e^{-(R_2)^{(9)}t}$$

**Definition of** $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$

**Proof :** From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14},t)\right) - (a''_{14})^{(1)}(T_{14},t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

**Definition of** $v^{(1)}$ :- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)}\right) \le \frac{dv^{(1)}}{dt} \le -\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)}\right)$$

From which one obtains

**Definition of** $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$ :-

For $0 < \boxed{(v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}} < (v_1)^{(1)} < (\bar{v}_1)^{(1)}$

$$v^{(1)}(t) \ge \frac{(v_1)^{(1)}+(C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(v_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(v_2)^{(1)}}}$$

it follows $(\nu_0)^{(1)} \leq \nu^{(1)}(t) \leq (\nu_1)^{(1)}$

In the same manner , we get 384

$$\nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{\nu}_1)^{(1)}-(\nu_0)^{(1)}}{(\nu_0)^{(1)}-(\bar{\nu}_2)^{(1)}}}$$

From which we deduce $(\nu_0)^{(1)} \leq \nu^{(1)}(t) \leq (\bar{\nu}_1)^{(1)}$

If $0 < (\nu_1)^{(1)} < (\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0} < (\bar{\nu}_1)^{(1)}$ we find like in the previous case, 385

$$(\nu_1)^{(1)} \leq \frac{(\nu_1)^{(1)}+(C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}} \leq \nu^{(1)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\bar{\nu}_1)^{(1)}$$

If $0 < (\nu_1)^{(1)} \leq (\bar{\nu}_1)^{(1)} \leq \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$ , we obtain 386

$$(\nu_1)^{(1)} \leq \nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\nu_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(1)}(t)$ :-

$$(m_2)^{(1)} \leq \nu^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{\nu^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(1)}(t)$ :-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{13})^{(1)} = (a''_{14})^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)}G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)}T_{14}(t)$ This is an important

consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

**Proof** : From global equations we obtain 387

$$\frac{d\nu^{(2)}}{dt} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right) - (a''_{17})^{(2)}(T_{17}, t)\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

**Definition of** $\nu^{(2)}$ :- $\boxed{\nu^{(2)} = \frac{G_{16}}{G_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \le \frac{d\nu^{(2)}}{dt} \le -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

**Definition of** $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$ :-

For $0 < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \ge \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}}{1 + (C)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(C)^{(2)} = \frac{(\nu_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \le \nu^{(2)}(t) \le (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \le \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \le \nu^{(2)}(t) \le (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \le \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}}{1 + (C)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}} \le \nu^{(2)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \le (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \le (\bar{\nu}_1)^{(2)} \le (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$ , we obtain 394

$$(\nu_1)^{(2)} \le \nu^{(2)}(t) \le \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \le (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(2)}(t)$ :- 395

$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}$, $\boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$

In a completely analogous way, we obtain 396

**Definition of** $u^{(2)}(t)$ :-

$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}$, $\boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :** 397

If $(a''_{16})^{(2)} = (a''_{17})^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)} G_{17}(t)$

Analogously if $(b''_{16})^{(2)} = (b''_{17})^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)} T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

**Proof** : From global equations we obtain 398

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right) - (a''_{21})^{(3)}(T_{21}, t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

**Definition of** $\nu^{(3)}$ :- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right) \leq \frac{d\nu^{(3)}}{dt} \leq -\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right)$$

400

From which one obtains

For $0 < (\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\nu_1)^{(3)} < (\bar{\nu}_1)^{(3)}$

$$\nu^{(3)}(t) \geq \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(\nu_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\nu_2)^{(3)}}}$$

it follows $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\nu_1)^{(3)}$

In the same manner , we get 401

$$\nu^{(3)}(t) \le \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{\nu}_1)^{(3)}-(\nu_0)^{(3)}}{(\nu_0)^{(3)}-(\bar{\nu}_2)^{(3)}}}$$

**Definition of** $(\bar{\nu}_1)^{(3)}$ :-

From which we deduce $(\nu_0)^{(3)} \le \nu^{(3)}(t) \le (\bar{\nu}_1)^{(3)}$

If $0 < (\nu_1)^{(3)} < (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (\bar{\nu}_1)^{(3)}$ we find like in the previous case, 402

$$(\nu_1)^{(3)} \le \frac{(\nu_1)^{(3)}+(C)^{(3)}(\nu_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}} \le \nu^{(3)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \le (\bar{\nu}_1)^{(3)}$$

If $0 < (\nu_1)^{(3)} \le (\bar{\nu}_1)^{(3)} \le (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}$ , we obtain 403

$$(\nu_1)^{(3)} \le \nu^{(3)}(t) \le \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \le (\nu_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(3)}(t)$ :-

$$(m_2)^{(3)} \le \nu^{(3)}(t) \le (m_1)^{(3)}, \quad \boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(3)}(t)$ :-

$$(\mu_2)^{(3)} \le u^{(3)}(t) \le (\mu_1)^{(3)}, \quad \boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{20}'')^{(3)} = (a_{21}'')^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)}G_{21}(t)$

Analogously if $(b_{20}'')^{(3)} = (b_{21}'')^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)}T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

**Proof** : From global equations we obtain 404

$$\frac{dv^{(4)}}{dt} = (a_{24})^{(4)} - \left((a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right) - (a''_{25})^{(4)}(T_{25},t)v^{(4)} - (a_{25})^{(4)}v^{(4)}$$

**Definition of** $v^{(4)}$ :- $\boxed{v^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)}\right) \le \frac{dv^{(4)}}{dt} \le -\left((a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_4)^{(4)}v^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$ :-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \ge \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \le \nu^{(4)}(t) \le (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \le \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \le \nu^{(4)}(t) \le (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \le \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}}{1 + (C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}} \le \nu^{(4)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \le (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \le (\bar{\nu}_1)^{(4)} \le \boxed{(\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}}}$ , we obtain 407

$$(\nu_1)^{(4)} \le \nu^{(4)}(t) \le \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \le (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(4)}(t)$ :-

$$(m_2)^{(4)} \le \nu^{(4)}(t) \le (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(4)}(t)$ :-

$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}$, $\boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{24})^{(4)} = (a''_{25})^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)} G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b''_{24})^{(4)} = (b''_{25})^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)} T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

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**Proof** : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a'_{28})^{(5)} - (a'_{29})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right) - (a''_{29})^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

**Definition of** $\nu^{(5)}$ :- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$ :-

For $0 < \boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \le \frac{(\nu_1)^{(5)}+(C)^{(5)}(\nu_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)}-(\nu_2)^{(5)}\right)t\right]}}{1+(C)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)}-(\nu_2)^{(5)}\right)t\right]}} \le \nu^{(5)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \le (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \le (\bar{\nu}_1)^{(5)} \le \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain 411

$$(\nu_1)^{(5)} \le \nu^{(5)}(t) \le \frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \le (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(5)}(t)$ :-

$(m_2)^{(5)} \le \nu^{(5)}(t) \le (m_1)^{(5)}$, $\boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$

In a completely analogous way, we obtain

**Definition of** $u^{(5)}(t)$ :-

$(\mu_2)^{(5)} \le u^{(5)}(t) \le (\mu_1)^{(5)}$, $\boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a_{28}'')^{(5)} = (a_{29}'')^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)}G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b_{28}'')^{(5)} = (b_{29}'')^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)}T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

**Proof** : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a_{32}')^{(6)} - (a_{33}')^{(6)} + (a_{32}'')^{(6)}(T_{33},t)\right) - (a_{33}'')^{(6)}(T_{33},t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

**Definition of** $\nu^{(6)}$ :- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \le \frac{d\nu^{(6)}}{dt} \le -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$ :-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \geq \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\nu_1)^{(6)}$

In the same manner , we get 413

$$\nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{\nu}_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\bar{\nu}_1)^{(6)}$

If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 414

$$(\nu_1)^{(6)} \leq \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}} \leq \nu^{(6)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\bar{\nu}_1)^{(6)}$$

If $0 < (\nu_1)^{(6)} \leq (\bar{\nu}_1)^{(6)} \leq \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$ , we obtain 415

$$(\nu_1)^{(6)} \leq \nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(6)}(t)$ :-

$$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}, \quad \boxed{\nu^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(6)}(t)$ :-

$$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}, \quad \boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)} G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)} T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

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**Proof** : From global equations we obtain

$$\frac{dv^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right) - (a''_{37})^{(7)}(T_{37},t)v^{(7)} - (a_{37})^{(7)}v^{(7)}$$

**Definition of** $v^{(7)}$ :- $\boxed{v^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_2)^{(7)}v^{(7)} - (a_{36})^{(7)}\right) \le \frac{dv^{(7)}}{dt} \le -\left((a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_1)^{(7)}v^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

**Definition of** $(\bar{v}_1)^{(7)}, (v_0)^{(7)}$ :-

For $0 < \boxed{(v_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}} < (v_1)^{(7)} < (\bar{v}_1)^{(7)}$

$$v^{(7)}(t) \ge \frac{(v_1)^{(7)} + (C)^{(7)}(v_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_0)^{(7)}\right)t\right]}}{1 + (C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(v_1)^{(7)} - (v_0)^{(7)}}{(v_0)^{(7)} - (v_2)^{(7)}}}$$

it follows $(v_0)^{(7)} \le v^{(7)}(t) \le (v_1)^{(7)}$

In the same manner , we get 417

$$v^{(7)}(t) \le \frac{(\bar{v}_1)^{(7)} + (\bar{C})^{(7)}(\bar{v}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{v}_1)^{(7)} - (v_0)^{(7)}}{(v_0)^{(7)} - (\bar{v}_2)^{(7)}}}$$

From which we deduce $(v_0)^{(7)} \le v^{(7)}(t) \le (\bar{v}_1)^{(7)}$

If $0 < (v_1)^{(7)} < (v_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}} < (\bar{v}_1)^{(7)}$ we find like in the previous case, 418

$$(v_1)^{(7)} \le \frac{(v_1)^{(7)} + (C)^{(7)}(v_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_2)^{(7)}\right)t\right]}}{1 + (C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_2)^{(7)}\right)t\right]}} \le v^{(7)}(t) \le$$

$$\frac{(\bar{v}_1)^{(7)} + (\bar{C})^{(7)}(\bar{v}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}} \le (\bar{v}_1)^{(7)}$$

If $0 < (v_1)^{(7)} \le (\bar{v}_1)^{(7)} \le \boxed{(v_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$ , we obtain 419

$$(v_1)^{(7)} \le v^{(7)}(t) \le \frac{(\bar{v}_1)^{(7)} + (\bar{C})^{(7)}(\bar{v}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}} \le (v_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $v^{(7)}(t)$ :-

$$(m_2)^{(7)} \leq v^{(7)}(t) \leq (m_1)^{(7)}, \quad \boxed{v^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(7)}(t)$ :- 420

$$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}, \quad \boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $v^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)} G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)} T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

**Proof** : From global equations we obtain 421

$$\frac{dv^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right) - (a''_{41})^{(8)}(T_{41}, t) v^{(8)} - (a_{41})^{(8)} v^{(8)}$$

**Definition of** $v^{(8)}$ :- $\boxed{v^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_2)^{(8)} v^{(8)} - (a_{40})^{(8)}\right) \leq \frac{dv^{(8)}}{dt} \leq -\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_1)^{(8)} v^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$ :-

$$\text{For } 0 < \boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$$

$$v^{(8)}(t) \geq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)}-(\nu_0)^{(8)}}{(\nu_0)^{(8)}-(\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \leq \frac{(\nu_1)^{(8)}+(C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}}{1+(C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}} \leq \nu^{(8)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \leq (\bar{\nu}_1)^{(8)} \leq \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$ , we obtain 424

$$(\nu_1)^{(8)} \leq \nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(8)}(t)$ :-

$$(m_2)^{(8)} \leq \nu^{(8)}(t) \leq (m_1)^{(8)}, \quad \boxed{\nu^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(8)}(t)$ :-

$$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}, \quad \boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $\nu^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)}G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then $(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)}T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

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**Proof** : From 99,20,44,22,23,44 we obtain A

$$\frac{dv^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right) - (a''_{45})^{(9)}(T_{45},t)v^{(9)} - (a_{45})^{(9)}v^{(9)}$$

**Definition of** $v^{(9)}$ :- $\boxed{v^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)}\right) \le \frac{dv^{(9)}}{dt} \le -\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_1)^{(9)}v^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

**Definition of** $(\bar{v}_1)^{(9)}, (v_0)^{(9)}$ :-

For $0 < \boxed{(v_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (v_1)^{(9)} < (\bar{v}_1)^{(9)}$

$$v^{(9)}(t) \ge \frac{(v_1)^{(9)} + (C)^{(9)}(v_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((v_1)^{(9)} - (v_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((v_1)^{(9)} - (v_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(v_1)^{(9)} - (v_0)^{(9)}}{(v_0)^{(9)} - (v_2)^{(9)}}}$$

it follows $(v_0)^{(9)} \le v^{(9)}(t) \le (v_9)^{(9)}$

In the same manner , we get

$$v^{(9)}(t) \le \frac{(\bar{v}_1)^{(9)} + (\bar{C})^{(9)}(\bar{v}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{v}_1)^{(9)} - (\bar{v}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{v}_1)^{(9)} - (\bar{v}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{v}_1)^{(9)} - (v_0)^{(9)}}{(v_0)^{(9)} - (\bar{v}_2)^{(9)}}}$$

From which we deduce $(v_0)^{(9)} \le v^{(9)}(t) \le (\bar{v}_1)^{(9)}$

If $0 < (v_1)^{(9)} < (v_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}} < (\bar{v}_1)^{(9)}$ we find like in the previous case,

$$(v_1)^{(9)} \le \frac{(v_1)^{(9)} + (C)^{(9)}(v_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((v_1)^{(9)} - (v_2)^{(9)}\right)t\right]}}{1 + (C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((v_1)^{(9)} - (v_2)^{(9)}\right)t\right]}} \le v^{(9)}(t) \le$$

$$\frac{(\bar{v}_1)^{(9)} + (\bar{C})^{(9)}(\bar{v}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{v}_1)^{(9)} - (\bar{v}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{v}_1)^{(9)} - (\bar{v}_2)^{(9)}\right)t\right]}} \le (\bar{v}_1)^{(9)}$$

If $0 < (v_1)^{(9)} \le (\bar{v}_1)^{(9)} \le \boxed{(v_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$ , we obtain

$$(v_1)^{(9)} \le v^{(9)}(t) \le \frac{(\bar{v}_1)^{(9)} + (\bar{C})^{(9)}(\bar{v}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{v}_1)^{(9)} - (\bar{v}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{v}_1)^{(9)} - (\bar{v}_2)^{(9)}\right)t\right]}} \le (v_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $v^{(9)}(t)$ :-

$$(m_2)^{(9)} \le v^{(9)}(t) \le (m_1)^{(9)}, \quad \boxed{v^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(9)}(t)$ :-

$(\mu_2)^{(9)} \leq u^{(9)}(t) \leq (\mu_1)^{(9)}$, $\boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)} G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then
$(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)} T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following 425

**Theorem :** If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on $t$ , and the conditions with the notations

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0$ ,

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$

$with\ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a''_i)^{(2)} and\ (b''_i)^{(2)}$ are independent on t , and the conditions with the notations 426

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$ 427

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 428

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$ , 429

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 430

$with\ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a''_i)^{(3)} and\ (b''_i)^{(3)}$ are independent on $t$ , and the conditions with the notations 431

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$ ,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

**Theorem :** If $(a_i'')^{(4)} and\ (b_i'')^{(4)}$ are independent on $t$ , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$ ,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(5)} and\ (b_i'')^{(5)}$ are independent on $t$ , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$ ,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

**Theorem** If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on $t$ , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$ ,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(7)} and\ (b_i'')^{(7)}$ are independent on $t$ , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$ ,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(8)} and\ (b_i'')^{(8)}$ are independent on $t$ , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$ ,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(9)} and\ (b_i'')^{(9)}$ are independent on $t$ , and the conditions (with the notations 436 A
45,46,27,28)

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$ ,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0$ 437

$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0$ 438

$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0$ 439

$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0$ 440

$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ]T_{14} = 0$ 441

$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0$ 442

has a unique positive solution , which is an equilibrium solution for the system

$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0$ 443

$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0$ 444

$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0$ 445

$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\ ]T_{16} = 0$ 446

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\,]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\,]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\,]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\,]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\,]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\big((G_{27})\big)\,]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\big((G_{27})\big)\,]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\big((G_{27})\big)\,]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})\,]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})\,]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})\,]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})\,]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})\,]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})\,]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0 \quad 479$$

$$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0 \quad 480$$

$$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0 \quad 481$$

$$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0 \quad 482$$

$$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0 \quad 483$$

$$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0 \quad 484$$

$$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0 \quad 484\text{A}$$

$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$

$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$

$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$

$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$

$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$

**Proof:** 485

(a) Indeed the first two equations have a nontrivial solution $G_{13}, G_{14}$ if

$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$

**Proof:** 486

(y) Indeed the first two equations have a nontrivial solution $G_{16}, G_{17}$ if

$\mathrm{F}(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$

**Proof:** 487

(a) Indeed the first two equations have a nontrivial solution $G_{20}, G_{21}$ if

$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$

**Proof:** 488

(a) Indeed the first two equations have a nontrivial solution $G_{24}, G_{25}$ if

$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$

**Proof:** 489

(a) Indeed the first two equations have a nontrivial solution $G_{28}, G_{29}$ if

$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$

**Proof:** 490

(a) Indeed the first two equations have a nontrivial solution $G_{32}, G_{33}$ if

$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) +$

$(a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$

**Proof:** 491

(a) Indeed the first two equations have a nontrivial solution $G_{36}, G_{37}$ if

$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$

**Proof:** 492

(a) Indeed the first two equations have a nontrivial solution $G_{40}, G_{41}$ if

$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$

**Proof:** 492 A

(a) Indeed the first two equations have a nontrivial solution $G_{44}, G_{45}$ if

$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$

**Definition and uniqueness of** $T^*_{14}$ :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique $T^*_{14}$ for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

**Definition and uniqueness of** $T^*_{17}$ :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique $T^*_{17}$ for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]}$$ 495

**Definition and uniqueness of** $T^*_{21}$ :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique $T^*_{21}$ for which $f(T^*_{21}) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

**Definition and uniqueness of** $T^*_{25}$ :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that

there exists a unique $T_{25}^{*}$ for which $f(T_{25}^{*})=0$. With this value , we obtain from the three first equations

$$G_{24}=\frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}=\frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T_{25}^{*})]}$$

**Definition and uniqueness of** $T_{29}^{*}$ :- 498

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(5)}(T_{29})$ being increasing, it follows that there exists a unique $T_{29}^{*}$ for which $f(T_{29}^{*})=0$. With this value , we obtain from the three first equations

$$G_{28}=\frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}=\frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T_{29}^{*})]}$$

**Definition and uniqueness of** $T_{33}^{*}$ :- 499

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(6)}(T_{33})$ being increasing, it follows that there exists a unique $T_{33}^{*}$ for which $f(T_{33}^{*})=0$. With this value , we obtain from the three first equations

$$G_{32}=\frac{(a_{32})^{(6)}G_{33}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^{*})]} \quad , \quad G_{34}=\frac{(a_{34})^{(6)}G_{33}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^{*})]}$$

**Definition and uniqueness of** $T_{37}^{*}$ :- 500

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(7)}(T_{37})$ being increasing, it follows that there exists a unique $T_{37}^{*}$ for which $f(T_{37}^{*})=0$. With this value , we obtain from the three first equations

$$G_{36}=\frac{(a_{36})^{(7)}G_{37}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^{*})]} \quad , \quad G_{38}=\frac{(a_{38})^{(7)}G_{37}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^{*})]}$$

**Definition and uniqueness of** $T_{41}^{*}$ :- 501

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique $T_{41}^{*}$ for which $f(T_{41}^{*})=0$. With this value , we obtain from the three first equations

$$G_{40}=\frac{(a_{40})^{(8)}G_{41}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T_{41}^{*})]} \quad , \quad G_{42}=\frac{(a_{42})^{(8)}G_{41}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T_{41}^{*})]}$$

**Definition and uniqueness of** $T_{45}^{*}$ :- 501 A

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(9)}(T_{45})$ being increasing, it follows that there exists a unique $T_{45}^{*}$ for which $f(T_{45}^{*})=0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)}G_{45}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T^*_{45})]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)}G_{45}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T^*_{45})]}$$

By the same argument, the equations admit solutions $G_{13}, G_{14}$ if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right]+(b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{14}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{14}$ such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions $G_{16}, G_{17}$ if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right]+(b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ 504
is a decreasing function in $G_{17}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{14}$ such that $\varphi((G_{19})^*) = 0$

By the same argument, the equations admit solutions $G_{20}, G_{21}$ if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right]+(b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{21}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{21}$ such that $\varphi((G_{23})^*) = 0$

By the same argument, the equations admit solutions $G_{24}, G_{25}$ if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right]+(b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{25}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{25}$ such that $\varphi((G_{27})^*) = 0$

By the same argument, the equations admit solutions $G_{28}, G_{29}$ if 507

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right]+(b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{29}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that

there exists a unique $G_{29}^{*}$ such that $\varphi((G_{31})^{*}) = 0$

By the same argument, the equations admit solutions $G_{32}, G_{33}$ if 508
$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$

$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right] + (b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{33}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{33}^{*}$ such that $\varphi(G_{35}{}^{*}) = 0$

By the same argument, the equations admit solutions $G_{36}, G_{37}$ if 509
$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$
$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right] + (b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{37}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{37}^{*}$ such that $\varphi(G_{39}{}^{*}) = 0$

By the same argument, the equations admit solutions $G_{40}, G_{41}$ if 510
$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$
$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right] + (b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{41}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{41}^{*}$ such that $\varphi(G_{43}{}^{*}) = 0$

By the same argument, the equations 92,93 admit solutions $G_{44}, G_{45}$ if
$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$

$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right] + (b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{45}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{45}^{*}$ such that $\varphi((G_{47})^{*}) = 0$

Finally we obtain the unique solution 511

$G_{14}^{*}$ given by $\varphi(G^{*}) = 0$ , $T_{14}^{*}$ given by $f(T_{14}^{*}) = 0$ and

$$G_{13}^{*} = \frac{(a_{13})^{(1)}G_{14}^{*}}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^{*})]} \quad , \quad G_{15}^{*} = \frac{(a_{15})^{(1)}G_{14}^{*}}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^{*})]}$$

$$T_{13}^{*} = \frac{(b_{13})^{(1)}T_{14}^{*}}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^{*})]} \quad , \quad T_{15}^{*} = \frac{(b_{15})^{(1)}T_{14}^{*}}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^{*})]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

$G_{17}^{*}$ given by $\varphi((G_{19})^{*}) = 0$ , $T_{17}^{*}$ given by $f(T_{17}^{*}) = 0$ and 512

$$G_{16}^{*} = \frac{(a_{16})^{(2)} G_{17}^{*}}{[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}^{*})]} \quad , \quad G_{18}^{*} = \frac{(a_{18})^{(2)} G_{17}^{*}}{[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}^{*})]} \tag{513}$$

$$T_{16}^{*} = \frac{(b_{16})^{(2)} T_{17}^{*}}{[(b'_{16})^{(2)} - (b''_{16})^{(2)}((G_{19})^{*})]} \quad , \quad T_{18}^{*} = \frac{(b_{18})^{(2)} T_{17}^{*}}{[(b'_{18})^{(2)} - (b''_{18})^{(2)}((G_{19})^{*})]} \tag{514}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

$G_{21}^{*}$ given by $\varphi((G_{23})^{*}) = 0$ , $T_{21}^{*}$ given by $f(T_{21}^{*}) = 0$ and

$$G_{20}^{*} = \frac{(a_{20})^{(3)} G_{21}^{*}}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^{*})]} \quad , \quad G_{22}^{*} = \frac{(a_{22})^{(3)} G_{21}^{*}}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^{*})]}$$

$$T_{20}^{*} = \frac{(b_{20})^{(3)} T_{21}^{*}}{[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}{}^{*})]} \quad , \quad T_{22}^{*} = \frac{(b_{22})^{(3)} T_{21}^{*}}{[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}{}^{*})]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

$G_{25}^{*}$ given by $\varphi(G_{27}) = 0$ , $T_{25}^{*}$ given by $f(T_{25}^{*}) = 0$ and

$$G_{24}^{*} = \frac{(a_{24})^{(4)} G_{25}^{*}}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}^{*} = \frac{(a_{26})^{(4)} G_{25}^{*}}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^{*})]}$$

$$T_{24}^{*} = \frac{(b_{24})^{(4)} T_{25}^{*}}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^{*})]} \quad , \quad T_{26}^{*} = \frac{(b_{26})^{(4)} T_{25}^{*}}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^{*})]} \tag{517}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

$G_{29}^{*}$ given by $\varphi((G_{31})^{*}) = 0$ , $T_{29}^{*}$ given by $f(T_{29}^{*}) = 0$ and

$$G_{28}^{*} = \frac{(a_{28})^{(5)} G_{29}^{*}}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}^{*} = \frac{(a_{30})^{(5)} G_{29}^{*}}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^{*})]}$$

$$T_{28}^{*} = \frac{(b_{28})^{(5)} T_{29}^{*}}{[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31})^{*})]} \quad , \quad T_{30}^{*} = \frac{(b_{30})^{(5)} T_{29}^{*}}{[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31})^{*})]} \tag{519}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

$G_{33}^{*}$ given by $\varphi((G_{35})^{*}) = 0$ , $T_{33}^{*}$ given by $f(T_{33}^{*}) = 0$ and

$$G_{32}^{*} = \frac{(a_{32})^{(6)} G_{33}^{*}}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^{*})]} \quad , \quad G_{34}^{*} = \frac{(a_{34})^{(6)} G_{33}^{*}}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^{*})]}$$

$$T_{32}^{*} = \frac{(b_{32})^{(6)} T_{33}^{*}}{[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35})^{*})]} \quad , \quad T_{34}^{*} = \frac{(b_{34})^{(6)} T_{33}^{*}}{[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35})^{*})]} \tag{521}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

$G_{37}^*$ given by $\varphi((G_{39})^*) = 0$ , $T_{37}^*$ given by $f(T_{37}^*) = 0$ and

$$G_{36}^* = \frac{(a_{36})^{(7)} G_{37}^*}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^*)]} \quad , \quad G_{38}^* = \frac{(a_{38})^{(7)} G_{37}^*}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^*)]}$$

$$T_{36}^* = \frac{(b_{36})^{(7)} T_{37}^*}{[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39})^*)]} \quad , \quad T_{38}^* = \frac{(b_{38})^{(7)} T_{37}^*}{[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39})^*)]}$$

Finally we obtain the unique solution 523

$G_{41}^*$ given by $\varphi((G_{43})^*) = 0$ , $T_{41}^*$ given by $f(T_{41}^*) = 0$ and

$$G_{40}^* = \frac{(a_{40})^{(8)} G_{41}^*}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42}^* = \frac{(a_{42})^{(8)} G_{41}^*}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

$$T_{40}^* = \frac{(b_{40})^{(8)} T_{41}^*}{[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43})^*)]} \quad , \quad T_{42}^* = \frac{(b_{42})^{(8)} T_{41}^*}{[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43})^*)]}$$

Finally we obtain the unique solution of 89 to 99 523 A

$G_{45}^*$ given by $\varphi((G_{47})^*) = 0$ , $T_{45}^*$ given by $f(T_{45}^*) = 0$ and

$$G_{44}^* = \frac{(a_{44})^{(9)} G_{45}^*}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46}^* = \frac{(a_{46})^{(9)} G_{45}^*}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

$$T_{44}^* = \frac{(b_{44})^{(9)} T_{45}^*}{[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47})^*)]} \quad , \quad T_{46}^* = \frac{(b_{46})^{(9)} T_{45}^*}{[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47})^*)]}$$

## ASYMPTOTIC STABILITY ANALYSIS 524

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

**Proof:** Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a''_{14})^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \frac{\partial (b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a'_{13})^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)}G^*_{13}\mathbb{T}_{14} \quad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a'_{14})^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)}G^*_{14}\mathbb{T}_{14} \quad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a'_{15})^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)}G^*_{15}\mathbb{T}_{14} \quad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b'_{13})^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)}T^*_{13}\mathbb{G}_j\right) \quad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b'_{14})^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)}T^*_{14}\mathbb{G}_j\right) \quad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b'_{15})^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T^*_{15}\mathbb{G}_j\right) \quad 530$$

**ASYMPTOTIC STABILITY ANALYSIS** 531

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(2)}$ and $(b''_i)^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G^*_i + \mathbb{G}_i \quad , T_i = T^*_i + \mathbb{T}_i \quad 532$$

$$\frac{\partial(a''_{17})^{(2)}}{\partial T_{17}}(T^*_{17}) = (q_{17})^{(2)} \quad , \frac{\partial(b''_i)^{(2)}}{\partial G_j}(\,(G_{19})^*\,) = s_{ij} \quad 533$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a'_{16})^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G^*_{16}\mathbb{T}_{17} \quad 534$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a'_{17})^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G^*_{17}\mathbb{T}_{17} \quad 535$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a'_{18})^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G^*_{18}\mathbb{T}_{17} \quad 536$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b'_{16})^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T^*_{16}\mathbb{G}_j\right) \quad 537$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b'_{17})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}T^*_{17}\mathbb{G}_j\right) \quad 538$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b'_{18})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}T^*_{18}\mathbb{G}_j\right) \quad 539$$

**ASYMPTOTIC STABILITY ANALYSIS** 540

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(3)}$ *and* $(b''_i)^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{21}'')^{(3)}}{\partial T_{21}}(T_{21}^*) = (q_{21})^{(3)} \quad , \frac{\partial (b_i'')^{(3)}}{\partial G_j}(\,(G_{23})^*\,) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\big((a_{20}')^{(3)} + (p_{20})^{(3)}\big)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G_{20}^*\mathbb{T}_{21} \quad 541$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\big((a_{21}')^{(3)} + (p_{21})^{(3)}\big)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G_{21}^*\mathbb{T}_{21} \quad 542$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\big((a_{22}')^{(3)} + (p_{22})^{(3)}\big)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G_{22}^*\mathbb{T}_{21} \quad 543$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\big((b_{20}')^{(3)} - (r_{20})^{(3)}\big)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\big(s_{(20)(j)}T_{20}^*\mathbb{G}_j\big) \quad 544$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\big((b_{21}')^{(3)} - (r_{21})^{(3)}\big)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\big(s_{(21)(j)}T_{21}^*\mathbb{G}_j\big) \quad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\big((b_{22}')^{(3)} - (r_{22})^{(3)}\big)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\big(s_{(22)(j)}T_{22}^*\mathbb{G}_j\big) \quad 546$$

**ASYMPTOTIC STABILITY ANALYSIS** 547

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}\ and\ (b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{25}'')^{(4)}}{\partial T_{25}}(T_{25}^*) = (q_{25})^{(4)} \quad , \frac{\partial (b_i'')^{(4)}}{\partial G_j}((G_{27})^*\,) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\big((a_{24}')^{(4)} + (p_{24})^{(4)}\big)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G_{24}^*\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\big((a_{25}')^{(4)} + (p_{25})^{(4)}\big)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G_{25}^*\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\big((a_{26}')^{(4)} + (p_{26})^{(4)}\big)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G_{26}^*\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\big((b_{24}')^{(4)} - (r_{24})^{(4)}\big)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\big(s_{(24)(j)}T_{24}^*\mathbb{G}_j\big) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\big((b_{25}')^{(4)} - (r_{25})^{(4)}\big)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\big(s_{(25)(j)}T_{25}^*\mathbb{G}_j\big) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T^*_{26}\mathbb{G}_j\right) \quad 554$$

**ASYMPTOTIC STABILITY ANALYSIS** 555

**Theorem 5:** If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(5)}\ and\ (b''_i)^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 556

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{29})^{(5)}}{\partial T_{29}}(T^*_{29}) = (q_{29})^{(5)} \ , \ \frac{\partial(b''_i)^{(5)}}{\partial G_j}\left((G_{31})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a'_{28})^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G^*_{28}\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a'_{29})^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G^*_{29}\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a'_{30})^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G^*_{30}\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b'_{28})^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T^*_{28}\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b'_{29})^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T^*_{29}\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b'_{30})^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T^*_{30}\mathbb{G}_j\right) \quad 562$$

**ASYMPTOTIC STABILITY ANALYSIS** 563

**Theorem 6:** If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(6)}\ and\ (b''_i)^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{33})^{(6)}}{\partial T_{33}}(T^*_{33}) = (q_{33})^{(6)} \ , \ \frac{\partial(b''_i)^{(6)}}{\partial G_j}\left((G_{35})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a'_{32})^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G^*_{32}\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a'_{33})^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G^*_{33}\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a'_{34})^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G^*_{34}\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b'_{32})^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T^*_{32}\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b'_{33})^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T^*_{33}\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b'_{34})^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T^*_{34}\mathbb{G}_j\right) \quad 570$$

**ASYMPTOTIC STABILITY ANALYSIS** 571

**Theorem 7:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 572

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{37})^{(7)}}{\partial T_{37}}(T^*_{37}) = (q_{37})^{(7)} \quad , \frac{\partial(b_i'')^{(7)}}{\partial G_j}\left((G_{39})^{**}\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a'_{36})^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G^*_{36}\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a'_{37})^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G^*_{37}\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a'_{38})^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G^*_{38}\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b'_{36})^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T^*_{36}\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b'_{37})^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T^*_{37}\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b'_{38})^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T^*_{38}\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

**ASYMPTOTIC STABILITY ANALYSIS**

**Theorem 8:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{41}'')^{(8)}}{\partial T_{41}}(T_{41}^*) = (q_{41})^{(8)} \quad , \frac{\partial (b_i'')^{(8)}}{\partial G_j}\left( (G_{43})^* \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a_{40}')^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G_{40}^*\mathbb{T}_{41} \quad (581)$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a_{41}')^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G_{41}^*\mathbb{T}_{41} \quad (582)$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a_{42}')^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G_{42}^*\mathbb{T}_{41} \quad (583)$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b_{40}')^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T_{40}^*\mathbb{G}_j\right) \quad (584)$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b_{41}')^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T_{41}^*\mathbb{G}_j\right) \quad (585)$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b_{42}')^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T_{42}^*\mathbb{G}_j\right) \quad (586)$$

(586 A)

**ASYMPTOTIC STABILITY ANALYSIS**

**Theorem 9:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}$ *and* $(b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

**Proof:** Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{45}'')^{(9)}}{\partial T_{45}}(T_{45}^*) = (q_{45})^{(9)} \quad , \frac{\partial (b_i'')^{(9)}}{\partial G_j}\left( (G_{47})^* \right) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a_{44}')^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)}G_{44}^*\mathbb{T}_{45} \quad (586\,B)$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a_{45}')^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)}G_{45}^*\mathbb{T}_{45} \quad (586\,C)$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a_{46}')^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)}G_{46}^*\mathbb{T}_{45} \quad (586\,D)$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b_{44}')^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)}T_{44}^*\mathbb{G}_j\right) \quad (586\,E)$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b_{45}')^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)}T_{45}^*\mathbb{G}_j\right) \quad (586\,F)$$

$\frac{d\mathbb{T}_{46}}{dt} = -\left((b'_{46})^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T^*_{46}\mathbb{G}_j\right)$ 586

G

**The characteristic equation of this system is** 587

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(q_{13})^{(1)}G^*_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)}T^*_{14} + (b_{14})^{(1)}s_{(13),(14)}T^*_{14}\right)$$

$$+\left(\left((\lambda)^{(1)} + (a'_{14})^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)}G^*_{13} + (a_{13})^{(1)}(q_{14})^{(1)}G^*_{14}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)}T^*_{14} + (b_{14})^{(1)}s_{(13),(13)}T^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((b'_{13})^{(1)} + (b'_{14})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(15)}T^*_{14} + (b_{14})^{(1)}s_{(13),(15)}T^*_{13}\right)\} = 0$$

$+$

$$(\lambda)^{(2)} + (b'_{18})^{(2)} - (r_{18})^{(2)}\{\left((\lambda)^{(2)} + (a'_{18})^{(2)} + (p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^*_{17} + (a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^*_{17}\right)$$

$$+\left(\left((\lambda)^{(2)} + (a'_{17})^{(2)} + (p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^*_{16} + (a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}\right)$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2 + \left((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2 + \left((b'_{16})^{(2)} + (b'_{17})^{(2)} - (r_{16})^{(2)} + (r_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$+\left(\left((\lambda)^{(2)}\right)^2 + \left((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17} + (a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^{*}_{16}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\right)\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left(\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)\right]$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(21)}T^{*}_{21}\right)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^{*}_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^{*}_{21}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(20)}T^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$+\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(22)}T^{*}_{20}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\right)\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left(\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)\right]$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(25)}T^{*}_{25}\right)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^{*}_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^{*}_{25}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(24)}T^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$+\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(26)}T^{*}_{24}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\right)\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(29)}T^{*}_{29}\right)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^{*}_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^{*}_{29}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(28)}T^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$+\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(30)}T^{*}_{28}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\right)\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left(\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)\right]$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(33)}T^{*}_{33}\right)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^{*}_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^{*}_{33}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(32)}T^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$+\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(34)}T^{*}_{32}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\right)\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)\right]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(37)}T^{*}_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^{*}_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^{*}_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(36)}T^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$+\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(38)}T^{*}_{36}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\right)\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(41)}T^{*}_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^{*}_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^{*}_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(40)}T^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$+\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(42)}T^{*}_{40}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\right)\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(45)}T^{*}_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^{*}_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^{*}_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(44)}T^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$+\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(46)}T^{*}_{44}\right)\}=0$$

**And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.**

## SECTION TWENTY EIGHT

## Objective And Subjective Consciousness

### INTRODUCTION—VARIABLES USED

**Alfred Lotka: Planck's constant and the objective and subjective consciousness** -1925.

(1) According to Lotka, consciousness of the first type (deterministic) is relative to (e&eb) all those phenomena which take place above (e) Planck's constant.

(2) At this level it was considered that the deterministic laws of classical physics took place. Consciousness

of the second type (subjective) would, on the contrary, take place below (e) Planck's constant, where (e) the deterministic laws of classical physics do not operate.

**Antonella Vannini Quantum Biosystems 2008, 2, 165-184 166**

**1.2. Niels Bohr model:**

Consciousness creates (eb) reality through (e&eb) the collapse of the wave function –

The interpretation of Copenhagen proposed by Niels Bohr and Werner Heisenberg, suggests a direct link between consciousness and (e&eb) Quantum Mechanics.

This interpretation attributes to (e) consciousness an explicit role, through (e&eb) the act of measurement and observation, forcing (eb) the wave function to collapse into (e&eb) a particle and determining (eb) in this way reality itself.

According to Bohr and Heisenberg consciousness would be (=) an immanent property of reality which exists before (e) the creation of reality.

**NOTATION**

**Module One**

According to Lotka, consciousness of the first type (deterministic) is relative to (e&eb) all those phenomena which take place above (e) Planck's constant

$G_{13}$ : Category one of **consciousness of the first type (deterministic);** all those phenomena which take place above (e) Planck's constant

$G_{14}$ : Category two of SAS

$G_{15}$ : Category three of SAS

$T_{13}$ : Category one of all those phenomena which take place above (e) Planck's constant **;consciousness of the first type (deterministic)**

$T_{14}$ : Category two of SAS

$T_{15}$ : Category three of SAS

**Module Two**

According to Lotka, consciousness of the first type (deterministic) is relative to all those phenomena which take place above (e) **Planck's constant**

$G_{16}$ : Category one of consciousness of the first type (deterministic) is relative to all those phenomena which take place; **Planck's constant (take all the three categories as equivalent)**

$G_{17}$ : Category two of SAS

$G_{18}$ : Category three of SAS

$T_{16}$ : Category one of **Planck's constant (take all the three categories as equivalent);**consciousness of the first type (deterministic) is relative to all those phenomena which take place

$T_{17}$ : Category two of SAS

$T_{18}$ : Category three of SAS

**Module three**

At this level it was considered that the deterministic laws of classical physics took place.

Consciousness of the second type (subjective) would, on the contrary, take place below (e) Planck's constant, where (e) the deterministic laws of classical physics do not operate

$G_{20}$ : Category one of Consciousness of the second type (subjective); Planck's constant, where (e) the deterministic laws of classical physics do not operate

$G_{21}$ : Category two of SAS

$G_{22}$ : Category three of SAS

$T_{20}$ : Category one of Planck's constant, where (e) the deterministic laws of classical physics do not operate ;Consciousness of the second type (subjective)

$T_{21}$ : Category two of SAS

$T_{22}$ : Category three of SAS

**Module four**

Consciousness of the second type (subjective) would, on the contrary, take place below Planck's constant, where (e) the deterministic laws of classical physics do not operate

$G_{24}$ : Category one of deterministic laws of classical physics do not operate

$G_{25}$ : Category two of SAS

$G_{26}$ : Category three of SAS

$T_{24}$ : Category one of Consciousness of the second type (subjective) would, on the contrary, take place below Planck's constant

$T_{25}$ : Category two of SAS

$T_{26}$ : Category three of SAS

**Module five**

Consciousness creates (eb) reality through (e&eb) the collapse of the wave function - 1930. The interpretation of Copenhagen proposed by Niels Bohr and Werner Heisenberg, suggests a direct link between consciousness and (e&eb) Quantum Mechanics.

This interpretation attributes to (e) consciousness an explicit role, through (e&eb) the act of measurement and observation, forcing (eb) the wave function to collapse into (e&eb) a particle and determining (eb) in this way reality itself.

According to Bohr and Heisenberg consciousness would be (=) an immanent property of reality which exists before (e) the creation of reality.

$G_{28}$ : Category one of Consciousness

$G_{29}$ : Category two of SAS

$G_{30}$ : Category three of SAS

$T_{28}$ : Category one of reality through (e&eb) the collapse of the wave function - 1930. The interpretation of Copenhagen proposed by Niels Bohr and Werner Heisenberg, suggests a direct link between consciousness and (e&eb) Quantum Mechanics

$T_{29}$ : Category two of SAS

$T_{30}$ : Category three of SAS

**Module six**

**Consciousness creates reality** through (e&eb) the collapse of the wave function - 1930.

The interpretation of Copenhagen proposed by Niels Bohr and Werner Heisenberg, suggests a direct link between consciousness and (e&eb) Quantum Mechanics

$G_{32}$ : Category one of **Consciousness creates reality;** collapse of the wave function

$G_{33}$ : Category two of SAS

$G_{34}$ : Category three of SAS

$T_{32}$ : Category one of collapse of the wave function ;**Consciousness creates reality**

$T_{33}$ : Category two of SAS

$T_{34}$ : Category three of SAS

**Module seven**

The interpretation of Copenhagen proposed by Niels Bohr and Werner Heisenberg, suggests a direct link between **consciousness** and (e&eb) Quantum Mechanics

$G_{36}$ : Category one of **consciousness;** Quantum Mechanics

$G_{37}$ : Category two of SAS

$G_{38}$ : Category three of SAS

$T_{36}$ : Category one of Quantum Mechanics; **consciousness**

$T_{37}$ : Category two of SAS

$T_{38}$ : Category three of SAS

**Module eight**

This interpretation attributes to (e) consciousness an explicit role, through (e&eb) the act of measurement and observation, forcing (eb) the wave function to collapse into (e&eb) a particle and determining (eb) in this way reality itself.

$G_{40}$ : Category one of **interpretation attributes**; consciousness an explicit role, through (e&eb) the act of measurement and observation, forcing (eb) the wave function to collapse into (e&eb) a particle and determining (eb) in this way reality itself

$G_{41}$ : Category two of SAS

$G_{42}$ : Category three of SAS

$T_{40}$ : Category one of consciousness an explicit role, through (e&eb) the act of measurement and observation, forcing (eb) the wave function to collapse into (e&eb) a particle and determining (eb) in this way reality itself; **interpretation attributes**

$T_{41}$ : Category two of SAS

$T_{42}$ : Category three of SAS

**Module Nine**

This interpretation attributes to consciousness an explicit role, through (e&eb) the act of measurement and observation, forcing (eb) the wave function to collapse into (e&eb) a particle and determining (eb) in this way reality itself

$G_{44}$ : Category one of **interpretation attributes to consciousness an explicit role;** act of measurement and observation, forcing (eb) the wave function to collapse into (e&eb) a particle and determining (eb) in this way reality itself

$G_{45}$ : Category two of SAS

$G_{46}$ : Category three of SAS

$T_{44}$ : Category one of act of measurement and observation, forcing (eb) the wave function to collapse into (e&eb) a particle and determining (eb) in this way reality itself; **interpretation attributes to consciousness an explicit role**

$T_{45}$ : Category two of SAS

$T_{46}$ : Category three of SAS

**The Coefficients:**

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}$ , $(b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}$,
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients

$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}, \ (b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)} \ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$ ,
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)},$
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)},$
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)},$
are Dissipation coefficients

**Module Numbered One**

**The differential system of this model is now (Module Numbered one)**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \quad 1$$
$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad 2$$
$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15} \quad 3$$
$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13} \quad 4$$
$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14} \quad 5$$
$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)\right]T_{15} \quad 6$$
$+(a''_{13})^{(1)}(T_{14}, t) =$ First augmentation factor
$-(b''_{13})^{(1)}(G, t) =$ First detritions factor

**Module Numbered Two**

**The differential system of this model is now ( Module numbered two)**

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right]G_{16} \quad 7$$
$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t)\right]G_{17} \quad 8$$
$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t)\right]G_{18} \quad 9$$
$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\big((G_{19}), t\big)\right]T_{16} \quad 10$$
$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\big((G_{19}), t\big)\right]T_{17} \quad 11$$
$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\big((G_{19}), t\big)\right]T_{18} \quad 12$$
$+(a''_{16})^{(2)}(T_{17}, t) =$ First augmentation factor
$-(b''_{16})^{(2)}\big((G_{19}), t\big) =$ First detritions factor

**Module Numbered Three**

**The differential system of this model is now (Module numbered three)**

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right]G_{20} \quad 13$$
$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}, t)\right]G_{21} \quad 14$$
$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}, t)\right]G_{22} \quad 15$$
$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}, t)\right]T_{20} \quad 16$$
$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}, t)\right]T_{21} \quad 17$$

$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}, t)\right]T_{22}$ 18

$+(a''_{20})^{(3)}(T_{21}, t) =$ First augmentation factor

$-(b''_{20})^{(3)}(G_{23}, t) =$ First detritions factor

**Module Numbered Four**

**The differential system of this model is now (Module numbered Four)**

$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)\right]G_{24}$ 19

$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}, t)\right]G_{25}$ 20

$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}, t)\right]G_{26}$ 21

$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27}), t)\right]T_{24}$ 22

$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}((G_{27}), t)\right]T_{25}$ 23

$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27}), t)\right]T_{26}$ 24

$+(a''_{24})^{(4)}(T_{25}, t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}((G_{27}), t) =$ **First detritions factor**

**Module Numbered Five**:

**The differential system of this model is now (Module number five)**

$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right]G_{28}$ 25

$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}, t)\right]G_{29}$ 26

$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}, t)\right]G_{30}$ 27

$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31}), t)\right]T_{28}$ 28

$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}((G_{31}), t)\right]T_{29}$ 29

$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31}), t)\right]T_{30}$ 30

$+(a''_{28})^{(5)}(T_{29}, t) =$ **First augmentation factor**

$-(b''_{28})^{(5)}((G_{31}), t) =$ **First detritions factor**

**Module Numbered Six**

**The differential system of this model is now (Module numbered Six)**

$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right]G_{32}$ 31

$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}, t)\right]G_{33}$ 32

$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}, t)\right]G_{34}$ 33

$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35}), t)\right]T_{32}$ 34

$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}((G_{35}), t)\right]T_{33}$ 35

$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35}), t)\right]T_{34}$ 36

$+(a''_{32})^{(6)}(T_{33}, t) =$ **First augmentation factor**

**Module Numbered Seven:**

**The differential system of this model is now (Seventh Module)**

$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right]G_{36}$ 37

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37},t)\right]G_{37} \quad 38$$
$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37},t)\right]G_{38} \quad 39$$
$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39}),t)\right]T_{36} \quad 40$$
$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}((G_{39}),t)\right]T_{37} \quad 41$$
$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39}),t)\right]T_{38} \quad 42$$

$+(a''_{36})^{(7)}(T_{37},t) =$ **First augmentation factor**

**Module Numbered Eight**

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right]G_{40} \quad 43$$
$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41},t)\right]G_{41} \quad 44$$
$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41},t)\right]G_{42} \quad 45$$
$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43}),t)\right]T_{40} \quad 46$$
$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}((G_{43}),t)\right]T_{41} \quad 47$$
$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43}),t)\right]T_{42} \quad 48$$

**Module Numbered Nine**

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$
$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$
$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$
$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47}),t)\right]T_{44} \quad 52$$
$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}((G_{47}),t)\right]T_{45} \quad 53$$
$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47}),t)\right]T_{46} \quad 54$$

$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}((G_{47}),t) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)} & \boxed{+(a''_{13})^{(1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)} & \boxed{+(a''_{14})^{(1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)} & \boxed{+(a''_{15})^{(1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15} \quad 57$$

Where $\boxed{(a''_{13})^{(1)}(T_{14},t)}$, $\boxed{(a''_{14})^{(1)}(T_{14},t)}$, $\boxed{(a''_{15})^{(1)}(T_{14},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7)}(T_{37},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8)}(T_{41},t)}$ $\boxed{+(a''_{41})^{(8,8)}(T_{41},t)}$ $\boxed{+(a''_{42})^{(8,8)}(T_{41},t)}$ are eight augmentation coefficient for 1,2,3

$\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)}\boxed{-(b''_{13})^{(1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13} \quad (58)$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)}\boxed{-(b''_{14})^{(1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14} \quad (59)$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)}\boxed{-(b''_{15})^{(1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15} \quad (60)$$

Where $\boxed{-(b''_{13})^{(1)}(G,t)}$, $\boxed{-(b''_{14})^{(1)}(G,t)}$, $\boxed{-(b''_{15})^{(1)}(G,t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{37})^{(7,7,)}(G_{39},t)$, $-(b''_{36})^{(7,7,)}(G_{39},t)$, $-(b''_{38})^{(7,7,)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8)}(G_{43},t)$ $-(b''_{41})^{(8,8)}(G_{43},t)$ $-(b''_{42})^{(8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt}=(a_{16})^{(2)}G_{17}-\left[\begin{array}{ccc}(a'_{16})^{(2)}+(a''_{16})^{(2)}(T_{17},t) & +(a''_{13})^{(1,1,)}(T_{14},t) & +(a''_{20})^{(3,3,3)}(T_{21},t)\\ +(a''_{24})^{(4,4,4,4,4)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,5)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,6)}(T_{33},t)\\ +(a''_{36})^{(7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9)}(T_{45},t)\end{array}\right]G_{16} \quad 61$$

$$\frac{dG_{17}}{dt}=(a_{17})^{(2)}G_{16}-\left[\begin{array}{ccc}(a'_{17})^{(2)}+(a''_{17})^{(2)}(T_{17},t) & +(a''_{14})^{(1,1,)}(T_{14},t) & +(a''_{21})^{(3,3,3)}(T_{21},t)\\ +(a''_{25})^{(4,4,4,4,4)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,5)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,6)}(T_{33},t)\\ +(a''_{37})^{(7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9)}(T_{45},t)\end{array}\right]G_{17} \quad 62$$

$$\frac{dG_{18}}{dt}=(a_{18})^{(2)}G_{17}-\left[\begin{array}{ccc}(a'_{18})^{(2)}+(a''_{18})^{(2)}(T_{17},t) & +(a''_{15})^{(1,1,)}(T_{14},t) & +(a''_{22})^{(3,3,3)}(T_{21},t)\\ +(a''_{26})^{(4,4,4,4,4)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,5)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,6)}(T_{33},t)\\ +(a''_{38})^{(7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9)}(T_{45},t)\end{array}\right]G_{18} \quad 63$$

Where $+(a''_{16})^{(2)}(T_{17},t)$, $+(a''_{17})^{(2)}(T_{17},t)$, $+(a''_{18})^{(2)}(T_{17},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{13})^{(1,1,)}(T_{14},t)$, $+(a''_{14})^{(1,1,)}(T_{14},t)$, $+(a''_{15})^{(1,1,)}(T_{14},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{36})^{(7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7)}(T_{37},t)$ are seventh augmentation coefficient for category 1, 2 and 3

$+(a''_{40})^{(8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8)}(T_{41},t)$ are eight augmentation coefficient for category 1, 2 and 3

$+(a''_{44})^{(9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9)}(T_{45},t)$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt}=(b_{16})^{(2)}T_{17}-\left[\begin{array}{ccc}(b'_{16})^{(2)}-(b''_{16})^{(2)}(G_{19},t) & -(b''_{13})^{(1,1,)}(G,t) & -(b''_{20})^{(3,3,3,)}(G_{23},t)\\ -(b''_{24})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6)}(G_{35},t)\\ -(b''_{36})^{(7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9)}(G_{47},t)\end{array}\right]T_{16} \quad 64$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)}\boxed{-(b''_{17})^{(2)}(G_{19},t)} & \boxed{-(b''_{14})^{(1,1,)}(G,t)} & \boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{17} \quad (65)$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)}\boxed{-(b''_{18})^{(2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,)}(G,t)} & \boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{18} \quad (66)$$

where $\boxed{-(b''_{16})^{(2)}(G_{19},t)}$ , $\boxed{-(b''_{17})^{(2)}(G_{19},t)}$ , $\boxed{-(b''_{18})^{(2)}(G_{19},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,)}(G,t)}$ are second detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{44})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \begin{bmatrix} (a'_{20})^{(3)}\boxed{+(a''_{20})^{(3)}(T_{21},t)} & \boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{20} \quad (67)$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \begin{bmatrix} (a'_{21})^{(3)}\boxed{+(a''_{21})^{(3)}(T_{21},t)} & \boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{21} \quad (68)$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \begin{bmatrix} (a'_{22})^{(3)}\boxed{+(a''_{22})^{(3)}(T_{21},t)} & \boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{22} \quad (69)$$

$\boxed{+(a''_{20})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3)}(T_{21},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}$ are second augmentation coefficients

for category 1, 2 and 3
$+(a_{13}'')^{(1,1,1,)}(T_{14},t)$, $+(a_{14}'')^{(1,1,1,)}(T_{14},t)$, $+(a_{15}'')^{(1,1,1,)}(T_{14},t)$ are third augmentation coefficients for category 1, 2 and 3
$+(a_{24}'')^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a_{25}'')^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a_{26}'')^{(4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficients for category 1, 2 and 3
$+(a_{28}'')^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a_{29}'')^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a_{30}'')^{(5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficients for category 1, 2 and 3
$+(a_{32}'')^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a_{33}'')^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a_{34}'')^{(6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficients for category 1, 2 and 3
$+(a_{36}'')^{(7,7,7,7)}(T_{37},t)$, $+(a_{37}'')^{(7,7,7,7)}(T_{37},t)$, $+(a_{38}'')^{(7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1, 2 and 3
$+(a_{40}'')^{(8,8,8,8)}(T_{41},t)$, $+(a_{41}'')^{(8,8,8,8)}(T_{41},t)$, $+(a_{42}'')^{(8,8,8,8)}(T_{41},t)$ are eight augmentation coefficients for category 1, 2 and 3
$+(a_{44}'')^{(9,9,9)}(T_{45},t)$, $+(a_{45}'')^{(9,9,9)}(T_{45},t)$, $+(a_{46}'')^{(9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b_{20}')^{(3)} & -(b_{20}'')^{(3)}(G_{23},t) & -(b_{16}'')^{(2,2,2)}(G_{19},t) & -(b_{13}'')^{(1,1,1,)}(G,t) \\ -(b_{24}'')^{(4,4,4,4,4,4)}(G_{27},t) & -(b_{28}'')^{(5,5,5,5,5,5)}(G_{31},t) & -(b_{32}'')^{(6,6,6,6,6,6)}(G_{35},t) & \\ -(b_{36}'')^{(7,7,7,7)}(G_{39},t) & -(b_{40}'')^{(8,8,8,8)}(G_{43},t) & -(b_{44}'')^{(9,9,9)}(G_{47},t) & \end{bmatrix} T_{20} \quad (70)$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b_{21}')^{(3)} & -(b_{21}'')^{(3)}(G_{23},t) & -(b_{17}'')^{(2,2,2)}(G_{19},t) & -(b_{14}'')^{(1,1,1,)}(G,t) \\ -(b_{25}'')^{(4,4,4,4,4,4)}(G_{27},t) & -(b_{29}'')^{(5,5,5,5,5,5)}(G_{31},t) & -(b_{33}'')^{(6,6,6,6,6,6)}(G_{35},t) & \\ -(b_{37}'')^{(7,7,7,7)}(G_{39},t) & -(b_{41}'')^{(8,8,8,8)}(G_{43},t) & -(b_{45}'')^{(9,9,9)}(G_{47},t) & \end{bmatrix} T_{21} \quad (71)$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b_{22}')^{(3)} & -(b_{22}'')^{(3)}(G_{23},t) & -(b_{18}'')^{(2,2,2)}(G_{19},t) & -(b_{15}'')^{(1,1,1,)}(G,t) \\ -(b_{26}'')^{(4,4,4,4,4,4)}(G_{27},t) & -(b_{30}'')^{(5,5,5,5,5,5)}(G_{31},t) & -(b_{34}'')^{(6,6,6,6,6,6)}(G_{35},t) & \\ -(b_{38}'')^{(7,7,7,7)}(G_{39},t) & -(b_{42}'')^{(8,8,8,8)}(G_{43},t) & -(b_{46}'')^{(9,9,9)}(G_{47},t) & \end{bmatrix} T_{22} \quad (72)$$

$-(b_{20}'')^{(3)}(G_{23},t)$, $-(b_{21}'')^{(3)}(G_{23},t)$, $-(b_{22}'')^{(3)}(G_{23},t)$ *are first detrition coefficients for category 1, 2 and 3*
$-(b_{16}'')^{(2,2,2)}(G_{19},t)$, $-(b_{17}'')^{(2,2,2)}(G_{19},t)$, $-(b_{18}'')^{(2,2,2)}(G_{19},t)$ *are second detrition coefficients for category 1, 2 and 3*
$-(b_{13}'')^{(1,1,1,)}(G,t)$, $-(b_{14}'')^{(1,1,1,)}(G,t)$, $-(b_{15}'')^{(1,1,1,)}(G,t)$ *are third detrition coefficients for category 1,2 and 3*
$-(b_{24}'')^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b_{25}'')^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b_{26}'')^{(4,4,4,4,4,4)}(G_{27},t)$ *are fourth detrition coefficients for category 1, 2 and 3*
$-(b_{28}'')^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b_{29}'')^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b_{30}'')^{(5,5,5,5,5,5)}(G_{31},t)$ *are fifth detrition coefficients for category 1, 2 and 3*
$-(b_{32}'')^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b_{33}'')^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b_{34}'')^{(6,6,6,6,6,6)}(G_{35},t)$ *are sixth detrition coefficients for category 1, 2 and 3*
$-(b_{36}'')^{(7,7,7,7)}(G_{39},t)$, $-(b_{37}'')^{(7,7,7,7)}(G_{39},t)$ $-(b_{38}'')^{(7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients

for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt}=(a_{24})^{(4)}G_{25}-\begin{bmatrix}(a'_{24})^{(4)}\boxed{+(a''_{24})^{(4)}(T_{25},t)}&\boxed{+(a''_{28})^{(5,5,)}(T_{29},t)}&\boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}\\ \boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)}&\boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)}&\boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)}&\boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)}&\boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{24} \quad (73)$$

$$\frac{dG_{25}}{dt}=(a_{25})^{(4)}G_{24}-\begin{bmatrix}(a'_{25})^{(4)}\boxed{+(a''_{25})^{(4)}(T_{25},t)}&\boxed{+(a''_{29})^{(5,5,)}(T_{29},t)}&\boxed{+(a''_{33})^{(6,6)}(T_{33},t)}\\ \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)}&\boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)}&\boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)}&\boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)}&\boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{25} \quad (74)$$

$$\frac{dG_{26}}{dt}=(a_{26})^{(4)}G_{25}-\begin{bmatrix}(a'_{26})^{(4)}\boxed{+(a''_{26})^{(4)}(T_{25},t)}&\boxed{+(a''_{30})^{(5,5,)}(T_{29},t)}&\boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}\\ \boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)}&\boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)}&\boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)}&\boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)}&\boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{26} \quad (75)$$

$\boxed{(a''_{24})^{(4)}(T_{25},t)}$, $\boxed{(a''_{25})^{(4)}(T_{25},t)}$, $\boxed{(a''_{26})^{(4)}(T_{25},t)}$ *are first augmentation coefficients category* 1, 2 3

$\boxed{+(a''_{28})^{(5,5,)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,)}(T_{29},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)}$
*are fourth augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)}$
*are fifth augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)}$
*are sixth augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)}$
*are seventh augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)}$
*are eighth augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt}=(b_{24})^{(4)}T_{25}-\begin{bmatrix}(b'_{24})^{(4)}\boxed{-(b''_{24})^{(4)}(G_{27},t)}&\boxed{-(b''_{28})^{(5,5,)}(G_{31},t)}&\boxed{-(b''_{32})^{(6,6,)}(G_{35},t)}\\ \boxed{-(b''_{13})^{(1,1,1,1)}(G,t)}&\boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)}&\boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)}\\ \boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)}&\boxed{-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)}&\boxed{-(b''_{44})^{(9,9,9,9)}(G_{47},t)}\end{bmatrix}T_{24} \quad (76)$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \begin{bmatrix} (b'_{25})^{(4)} & -(b''_{25})^{(4)}(G_{27},t) & -(b''_{29})^{(5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{25} \quad (77)$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)} & -(b''_{26})^{(4)}(G_{27},t) & -(b''_{30})^{(5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1)}(G,t) & -(b''_{18})^{(2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{26} \quad (78)$$

*Where* $-(b''_{24})^{(4)}(G_{27},t)$, $-(b''_{25})^{(4)}(G_{27},t)$, $-(b''_{26})^{(4)}(G_{27},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{28})^{(5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,)}(G_{31},t)$ *are second detrition coefficients for category* 1, 2 *and* 3

$-(b''_{32})^{(6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,)}(G_{35},t)$ *are third detrition coefficients for category* 1, 2 *and* 3

$-(b''_{13})^{(1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1)}(G,t)$
*are fourth detrition coefficients for category* 1, 2 *and* 3

$-(b''_{16})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2)}(G_{19},t)$
*are fifth detrition coefficients for category* 1, 2 *and* 3

$-(b''_{20})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3)}(G_{23},t)$
*are sixth detrition coefficients for category* 1, 2 *and* 3

$-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)$
*are seventh detrition coefficients for category* 1, 2 *and* 3

$-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)$
*are eighth detrition coefficients for category* 1, 2 *and* 3

$-(b''_{46})^{(9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \begin{bmatrix} (a'_{28})^{(5)} & +(a''_{28})^{(5)}(T_{29},t) & +(a''_{24})^{(4,4,)}(T_{25},t) & +(a''_{32})^{(6,6,6)}(T_{33},t) \\ & +(a''_{13})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{16})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3,3)}(T_{21},t) \\ & +(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{28} \quad (79)$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \begin{bmatrix} (a'_{29})^{(5)} & +(a''_{29})^{(5)}(T_{29},t) & +(a''_{25})^{(4,4,)}(T_{25},t) & +(a''_{33})^{(6,6,6)}(T_{33},t) \\ & +(a''_{14})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{17})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3,3)}(T_{21},t) \\ & +(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{29} \quad (80)$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \begin{bmatrix} (a'_{30})^{(5)} & +(a''_{30})^{(5)}(T_{29},t) & +(a''_{26})^{(4,4,)}(T_{25},t) & +(a''_{34})^{(6,6,6)}(T_{33},t) \\ & +(a''_{15})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{18})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3,3)}(T_{21},t) \\ & +(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{30} \quad (81)$$

*Where* $+(a''_{28})^{(5)}(T_{29},t)$, $+(a''_{29})^{(5)}(T_{29},t)$, $+(a''_{30})^{(5)}(T_{29},t)$ *are first augmentation coefficients for category* 1, 2 *and* 3

*And* $+(a''_{24})^{(4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,)}(T_{25},t)$ *are second augmentation*

$coefficient\ for\ category\ 1,2\ and\ 3$
$+(a''_{32})^{(6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6)}(T_{33},t)$ $are\ third\ \ augmentation$
$coefficient\ for\ category\ 1,2\ and\ 3$
$+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)$ are fourth augmentation coefficients for category 1,2, and 3
$+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)$ are fifth augmentation coefficients for category 1,2,and 3
$+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)$ are sixth augmentation coefficients for category 1,2, 3
$+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)$ , $+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1,2, 3
$+(a''_{40})^{(8,8\ ,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)$ are eighth augmentation coefficients for category 1,2, 3
$+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt}=(b_{28})^{(5)}T_{29}-\begin{bmatrix}(b'_{28})^{(5)} & -(b''_{28})^{(5)}(G_{31},t) & -(b''_{24})^{(4,4,)}(G_{27},t) & -(b''_{32})^{(6,6,6)}(G_{35},t)\\ -(b''_{13})^{(1,1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3)}(G_{23},t)\\ -(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9)}(G_{47},t)\end{bmatrix}T_{28} \quad 82$$

$$\frac{dT_{29}}{dt}=(b_{29})^{(5)}T_{28}-\begin{bmatrix}(b'_{29})^{(5)} & -(b''_{29})^{(5)}(G_{31},t) & -(b''_{25})^{(4,4,)}(G_{27},t) & -(b''_{33})^{(6,6,6)}(G_{35},t)\\ -(b''_{14})^{(1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3)}(G_{23},t)\\ -(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9)}(G_{47},t)\end{bmatrix}T_{29} \quad 83$$

$$\frac{dT_{30}}{dt}=(b_{30})^{(5)}T_{29}-\begin{bmatrix}(b'_{30})^{(5)} & -(b''_{30})^{(5)}(G_{31},t) & -(b''_{26})^{(4,4,)}(G_{27},t) & -(b''_{34})^{(6,6,6)}(G_{35},t)\\ -(b''_{15})^{(1,1,1,1,1,)}(G,t) & -(b''_{18})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3)}(G_{23},t)\\ -(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9)}(G_{47},t)\end{bmatrix}T_{30} \quad 84$$

$where$ $-(b''_{28})^{(5)}(G_{31},t)$ , $-(b''_{29})^{(5)}(G_{31},t)$ , $-(b''_{30})^{(5)}(G_{31},t)$ $are\ first\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b''_{24})^{(4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,)}(G_{27},t)$ $are\ second\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b''_{32})^{(6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6)}(G_{35},t)$ $are\ \ third\ \ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b''_{13})^{(1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1)}(G,t)$ , $-(b''_{15})^{(1,1,1,1,1,)}(G,t)$ are fourth detrition coefficients for category 1,2, and 3
$-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1,2, and 3
$-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1,2, and 3
$-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} \quad 85$$
$$-\left[\begin{array}{ccc} (a'_{32})^{(6)}\boxed{+(a''_{32})^{(6)}(T_{33},t)} & \boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)} \end{array}\right]G_{32}$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[\begin{array}{ccc} (a'_{33})^{(6)}\boxed{+(a''_{33})^{(6)}(T_{33},t)} & \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)} \end{array}\right]G_{33} \quad 86$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[\begin{array}{ccc} (a'_{34})^{(6)}\boxed{+(a''_{34})^{(6)}(T_{33},t)} & \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)} \end{array}\right]G_{34} \quad 87$$

$\boxed{+(a''_{32})^{(6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6)}(T_{33},t)}$ *are first augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}$ *are second augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}$ *are third augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}$ - are fourth augmentation coefficients

$\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}$ - fifth augmentation coefficients

$\boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)}$ sixth augmentation coefficients

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)}$ seventh augmentation coefficients

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}$ Eighth augmentation coefficients

$\boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)}$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[\begin{array}{ccc} (b'_{32})^{(6)}\boxed{-(b''_{32})^{(6)}(G_{35},t)} & \boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)} \end{array}\right]T_{32} \quad 88$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \begin{bmatrix} (b'_{33})^{(6)}\boxed{-(b''_{33})^{(6)}(G_{35},t)} & \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{33} \quad (89)$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)}\boxed{-(b''_{34})^{(6)}(G_{35},t)} & \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{34} \quad (90)$$

$\boxed{-(b''_{32})^{(6)}(G_{35},t)}, \boxed{-(b''_{33})^{(6)}(G_{35},t)}, \boxed{-(b''_{34})^{(6)}(G_{35},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}$ *are second detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)}$ *are third detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}$ are fourth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \quad (91)$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a'_{36})^{(7)}\boxed{+(a''_{36})^{(7)}(T_{37},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \quad (92)$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a'_{37})^{(7)}\boxed{+(a''_{37})^{(7)}(T_{37},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$\frac{dG_{38}}{dt}$ *93*

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a'_{38})^{(7)}\boxed{+(a''_{38})^{(7)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{(a''_{36})^{(7)}(T_{37},t)}$, $\boxed{(a''_{37})^{(7)}(T_{37},t)}$, $\boxed{(a''_{38})^{(7)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,)}(T_{41},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$\frac{dT_{36}}{dt} =$ 94

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b'_{36})^{(7)}\boxed{-(b''_{36})^{(7)}(G_{39},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$\frac{dT_{37}}{dt} =$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b'_{37})^{(7)}\boxed{-(b''_{37})^{(7)}(G_{39},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$\frac{dT_{38}}{dt} =$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b'_{38})^{(7)}\boxed{-(b''_{38})^{(7)}(G_{39},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b_{36}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{37}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{16}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{17}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{18}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{20}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{21}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{22}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{24}'')^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{25}'')^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{26}'')^{(4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{28}'')^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{32}'')^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{34}'')^{(6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{46}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{45}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{44}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3



$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \begin{bmatrix} (a_{40}')^{(8)}\boxed{+(a_{40}'')^{(8)}(T_{41},t)} & \boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{20}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{24}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{28}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{32}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{36}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{44}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \begin{bmatrix} (a_{41}')^{(8)}\boxed{+(a_{41}'')^{(8)}(T_{41},t)} & \boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{21}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{25}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{29}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{33}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{37}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{45}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \begin{bmatrix} (a_{42}')^{(8)}\boxed{+(a_{42}'')^{(8)}(T_{41},t)} & \boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{22}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{26}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{30}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{34}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{15}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{38}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{46}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a_{40}'')^{(8)}(T_{41},t)}$, $\boxed{+(a_{41}'')^{(8)}(T_{41},t)}$, $\boxed{+(a_{42}'')^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second

augmentation coefficient for category 1, 2 and 3
$+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)$ , $+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)$ , $+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3
$+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)$ , $+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)$ , $+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3
$+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)$ , $+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3
$+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)$ , $+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3
$+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ $+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ $+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ are seventh augmentation coefficient for 1,2,3
$+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$ , $+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$ , $+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$ are eighth augmentation coefficient for 1,2,3
$+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$ , $+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$ , $+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} =$$

$$(b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)}\left[-(b''_{40})^{(8)}(G_{43},t)\right] & \left[-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} =$$

$$(b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)}\left[-(b''_{41})^{(8)}(G_{43},t)\right] & \left[-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} =$$

$$(b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)}\left[-(b''_{42})^{(8)}(G_{43},t)\right] & \left[-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{15}$$

Where $-(b''_{36})^{(7)}(G_{39},t)$ , $-(b''_{37})^{(7)}(G_{39},t)$ , $-(b''_{38})^{(7)}(G_{39},t)$ are first detrition coefficients for category 1, 2 and 3
$-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$ , $-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$ , $-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3
$-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$ , $-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$ , $-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3
$-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)$ , $-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)$ , $-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{28}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{32}'')^{(6,6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{38}'')^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{36}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{37}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{44}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{45}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{46}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

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$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \begin{bmatrix} (a_{44}')^{(9)}\boxed{+(a_{44}'')^{(9)}(T_{45},t)} & \boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{20}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{24}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{28}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{32}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{36}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{40}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \begin{bmatrix} (a_{45}')^{(9)}\boxed{+(a_{45}'')^{(9)}(T_{45},t)} & \boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{21}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{25}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{29}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{33}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{14}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{37}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{41}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \begin{bmatrix} (a_{46}')^{(9)}\boxed{+(a_{46}'')^{(9)}(T_{37},t)} & \boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{22}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{26}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{30}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{34}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{15}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{38}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{42}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a_{44}'')^{(9)}(T_{45},t)}$, $\boxed{+(a_{45}'')^{(9)}(T_{45},t)}$, $\boxed{+(a_{46}'')^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{20}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{21}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{22}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{24}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{25}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{26}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{28}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{29}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{30}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$ are Seventh augmentation coefficient for category 1, 2 and 3

$+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ $+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ $+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ are eighth augmentation coefficient for 1,2,3

$+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} =$$

$$(b_{44})^{(9)}T_{45} -$$

$$\begin{bmatrix} (b'_{44})^{(9)}\,-(b''_{44})^{(9)}(G_{47},t) & -(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t) & -(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t) \end{bmatrix} T_{13}$$

$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b'_{45})^{(9)}\,-(b''_{45})^{(9)}(G_{47},t) & -(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t) & -(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t) \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b'_{46})^{(9)}\,-(b''_{46})^{(9)}(G_{47},t) & -(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t) & -(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t) \end{bmatrix} T_{15}$$

Where $-(b''_{44})^{(9)}(G_{47},t)$, $-(b''_{45})^{(9)}(G_{47},t)$, $-(b''_{46})^{(9)}(G_{47},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)$, $-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{37}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{36}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a_i')^{(1)}, (a_i'')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (b_i'')^{(1)} > 0, \quad i,j = 13,14,15 \qquad 97$$

The functions $(a_i'')^{(1)}, (b_i'')^{(1)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a_i'')^{(1)}(T_{14},t) \leq (p_i)^{(1)} \leq (\hat{A}_{13})^{(1)}$$

$$(b_i'')^{(1)}(G,t) \leq (r_i)^{(1)} \leq (b_i')^{(1)} \leq (\hat{B}_{13})^{(1)}$$

$$lim_{T_2\to\infty}(a_i'')^{(1)}(T_{14},t) = (p_i)^{(1)} \qquad 98$$

$$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = (r_i)^{(1)}$$

**Definition of** $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$ :

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$$|(a_i'')^{(1)}(T_{14}',t) - (a_i'')^{(1)}(T_{14},t)| \leq (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G',t) - (b_i'')^{(1)}(G,t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}',t)$ and $(a_i'')^{(1)}(T_{14},t)$ . $(T_{14}',t)$ and $(T_{14},t)$ are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14},t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$ : 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$$

**Definition of** $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$ : 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i,j = 16,17,18$$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17}, t) \leq (p_i)^{(2)} \leq (\hat{A}_{16})^{(2)} \quad 102$$

$$(b_i'')^{(2)}(G_{19}, t) \leq (r_i)^{(2)} \leq (b_i')^{(2)} \leq (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2 \to \infty} (a_i'')^{(2)}(T_{17}, t) = (p_i)^{(2)} \quad 104$$

$$\lim_{G \to \infty} (b_i'')^{(2)}((G_{19}), t) = (r_i)^{(2)} \quad 105$$

**Definition of** $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$ : 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}', t) - (a_i'')^{(2)}(T_{17}, t)| \leq (\hat{k}_{16})^{(2)}|T_{17} - T_{17}'|e^{-(\hat{M}_{16})^{(2)}t} \quad 107$$

$$|(b_i'')^{(2)}((G_{19})', t) - (b_i'')^{(2)}((G_{19}), t)| < (\hat{k}_{16})^{(2)}||(G_{19}) - (G_{19})'||e^{-(\hat{M}_{16})^{(2)}t} \quad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}', t)$ and$(a_i'')^{(2)}(T_{17}, t)$ . $(T_{17}', t)$ and $(T_{17}, t)$ are points belonging to the interval $[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}]$ . It is to be noted that $(a_i'')^{(2)}(T_{17}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17}, t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$ :

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants 109

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$$

**Definition of** $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$ :

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1 \quad (110)$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(b_i)^{(2)} + (b_i')^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1 \quad (111)$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i,j = 20,21,22 \quad (112)$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(3)}$, $(r_i)^{(3)}$:

$$(a_i'')^{(3)}(T_{21}, t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23}, t) \leq (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(3)}(T_{21}, t) = (p_i)^{(3)} \quad (113)$$

$$\lim_{G \to \infty}(b_i'')^{(3)}(G_{23}, t) = (r_i)^{(3)}$$

**Definition of** $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$ :

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: (114)

$$|(a_i'')^{(3)}(T_{21}', t) - (a_i'')^{(3)}(T_{21}, t)| \leq (\hat{k}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\hat{M}_{20})^{(3)}t}$$

$$|(b_i'')^{(3)}(G_{23}', t) - (b_i'')^{(3)}(G_{23}, t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}'||e^{-(\hat{M}_{20})^{(3)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}', t)$ and $(a_i'')^{(3)}(T_{21}, t)$ . $(T_{21}', t)$ And $(T_{21}, t)$ are points belonging to the interval $[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}]$. It is to be noted that $(a_i'')^{(3)}(T_{21}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$ : (115)

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$ and the constants $(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}, i = 20,21,22,$ satisfy the inequalities 116

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(a_i)^{(3)} + (a_i')^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(b_i)^{(3)} + (b_i')^{(3)} + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i, j = 24,25,26 \qquad 117$$

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(4)}$, $(r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25}, t) \le (p_i)^{(4)} \le (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}((G_{27}), t) \le (r_i)^{(4)} \le (b_i')^{(4)} \le (\hat{B}_{24})^{(4)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(4)}(T_{25}, t) = (p_i)^{(4)} \qquad 118$$

$$\lim_{G \to \infty}(b_i'')^{(4)}((G_{27}), t) = (r_i)^{(4)}$$

**Definition of** $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$ :

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24,25,26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}', t) - (a_i'')^{(4)}(T_{25}, t)| \le (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})', t) - (b_i'')^{(4)}((G_{27}), t)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}', t)$ and$(a_i'')^{(4)}(T_{25}, t)$ . $(T_{25}', t)$ and $(T_{25}, t)$ are points belonging to the interval $[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}]$. It is to be noted that $(a_i'')^{(4)}(T_{25}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$ : 120

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$, are positive constants

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}, \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$$

**Definition of** $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$ : 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and\ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}, i = 24,25,26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(a_i)^{(4)} + (a_i')^{(4)} + (\hat{A}_{24})^{(4)} + (\hat{P}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(b_i)^{(4)} + (b_i')^{(4)} + (\hat{B}_{24})^{(4)} + (\hat{Q}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i, j = 28,29,30 \qquad 122$$

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(5)}$, $(r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29}, t) \le (p_i)^{(5)} \le (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}((G_{31}), t) \le (r_i)^{(5)} \le (b_i')^{(5)} \le (\hat{B}_{28})^{(5)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(5)}(T_{29}, t) = (p_i)^{(5)} \qquad 123$$

$$\lim_{G \to \infty}(b_i'')^{(5)}(G_{31}, t) = (r_i)^{(5)}$$

**Definition of** $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$ :

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}', t) - (a_i'')^{(5)}(T_{29}, t)| \le (\hat{k}_{28})^{(5)}|T_{29} - T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})', t) - (b_i'')^{(5)}((G_{31}), t)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}', t)$ and $(a_i'')^{(5)}(T_{29}, t)$ . $(T_{29}', t)$ and $(T_{29}, t)$ are points belonging to the interval $[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}]$. It is to be noted that $(a_i'')^{(5)}(T_{29}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$ : 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}, \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

**Definition of** $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$ : 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\widehat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and\ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\widehat{M}_{28})^{(5)}}[(a_i)^{(5)} + (a_i')^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\widehat{M}_{28})^{(5)}}[(b_i)^{(5)} + (b_i')^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i,j = 32,33,34$$ 127

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(6)}$, $(r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33}, t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}), t) \leq (r_i)^{(6)} \leq (b_i')^{(6)} \leq (\hat{B}_{32})^{(6)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(6)}(T_{33}, t) = (p_i)^{(6)}$$ 128

$$\lim_{G \to \infty}(b_i'')^{(6)}((G_{35}), t) = (r_i)^{(6)}$$

**Definition of** $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$ :

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}', t) - (a_i'')^{(6)}(T_{33}, t)| \leq (\hat{k}_{32})^{(6)}|T_{33} - T_{33}'|e^{-(\widehat{M}_{32})^{(6)}t}$$

$$|(b_i'')^{(6)}((G_{35})', t) - (b_i'')^{(6)}((G_{35}), t)| < (\hat{k}_{32})^{(6)}||(G_{35}) - (G_{35})'||e^{-(\widehat{M}_{32})^{(6)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}', t)$ and $(a_i'')^{(6)}(T_{33}, t)$ . $(T_{33}', t)$ and $(T_{33}, t)$ are points belonging to the interval $[(\hat{k}_{32})^{(6)}, (\widehat{M}_{32})^{(6)}]$. It is to be noted that $(a_i'')^{(6)}(T_{33}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33}, t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\widehat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$ : 129

$(\widehat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$, are positive constants

$$\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}} < 1$$

**Definition of** $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$ : 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and\ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32,33,34,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(a_i)^{(6)} + (a_i')^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)} (\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(b_i)^{(6)} + (b_i')^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)} (\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

(UUUUUU) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i, j = 36,37,38$ 131

(VVVVVV) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(7)}$, $(r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37}, t) \leq (p_i)^{(7)} \leq (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39}, t) \leq (r_i)^{(7)} \leq (b_i')^{(7)} \leq (\hat{B}_{36})^{(7)}$$

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*(WWWWWW)* $\lim_{T_2 \to \infty} (a_i'')^{(7)}(T_{37}, t) = (p_i)^{(7)}$

*(XXXXXX)*

$\lim_{G \to \infty} (b_i'')^{(7)}((G_{39}), t) = (r_i)^{(7)}$

**Definition of** $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$ :

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$$|(a_i'')^{(7)}(T_{37}', t) - (a_i'')^{(7)}(T_{37}, t)| \leq (\hat{k}_{36})^{(7)} |T_{37} - T_{37}'| e^{-(\hat{M}_{36})^{(7)} t}$$

$$|(b_i'')^{(7)}((G_{39})', t) - (b_i'')^{(7)}((G_{39}), t)| < (\hat{k}_{36})^{(7)} ||(G_{39}) - (G_{39})'|| e^{-(\hat{M}_{36})^{(7)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}', t)$ and $(a_i'')^{(7)}(T_{37}, t)$ . $(T_{37}', t)$ and $(T_{37}, t)$ are points belonging to the interval $[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}]$ . It is to be noted that $(a_i'')^{(7)}(T_{37}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37}, t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$ : 134

(YYYYYY) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)},$ are positive constants

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$$

**Definition of** $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$ : 135

(ZZZZZZ) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(b_i)^{(7)} + (b_i')^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40,41,42 \qquad 136$$

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

**Definition of** $(p_i)^{(8)}$, $(r_i)^{(8)}$: 137

$$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)} \qquad 138$$

$$(b_i'')^{(8)}((G_{43}), t) \leq (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)} \qquad 139$$

$$\lim_{T_2 \to \infty} (a_i'')^{(8)}(T_{41}, t) = (p_i)^{(8)} \qquad 140$$

$$\lim_{G \to \infty} (b_i'')^{(8)}\big((G_{43}), t\big) = (r_i)^{(8)} \qquad 141$$

**Definition of** $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$ :

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)} |T_{41} - T_{41}'| e^{-(\hat{M}_{40})^{(8)} t} \qquad 142$$

$$|(b_i'')^{(8)}((G_{43})', t) - (b_i'')^{(8)}\big((G_{43}), t\big)| < (\hat{k}_{40})^{(8)} ||(G_{43}) - (G_{43})'|| e^{-(\hat{M}_{40})^{(8)} t} \qquad 143$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}', t)$ and $(a_i'')^{(8)}(T_{41}, t)$ . $(T_{41}', t)$ and $(T_{41}, t)$ are points belonging to the interval $\big[(\hat{k}_{40})^{(8)}, (\hat{M}_{40})^{(8)}\big]$. It is to be noted that $(a_i'')^{(8)}(T_{41}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{40})^{(8)} = 1$

then the function $(a_i'')^{(8)}(T_{41},t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$ :

$(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)},$ are positive constants

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}, \frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1 \qquad 144$$

**Definition of** $(\hat{P}_{40})^{(8)}, (\hat{Q}_{40})^{(8)}$ :

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}, (\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)}, (a_i')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (p_i)^{(8)}, \ (r_i)^{(8)}, i = 40,41,42,$

Satisfy the inequalities

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(a_i)^{(8)} + (a_i')^{(8)} + (\hat{A}_{40})^{(8)} + (\hat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}] < 1 \qquad 145$$

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(b_i)^{(8)} + (b_i')^{(8)} + (\hat{B}_{40})^{(8)} + (\hat{Q}_{40})^{(8)}(\hat{k}_{40})^{(8)}] < 1 \qquad 146$$

Where we suppose

$$(a_i)^{(9)}, (a_i')^{(9)}, (a_i'')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (b_i'')^{(9)} > 0, \quad i,j = 44,45,46 \qquad 146\text{A}$$

The functions $(a_i'')^{(9)}, (b_i'')^{(9)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(9)}, \ (r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45},t) \le (p_i)^{(9)} \le (\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47},t) \le (r_i)^{(9)} \le (b_i')^{(9)} \le (\hat{B}_{44})^{(9)}$$

$$\lim_{T_2\to\infty} (a_i'')^{(9)}(T_{45},t) = (p_i)^{(9)}$$

$$\lim_{G\to\infty} (b_i'')^{(9)}(G_{47},t) = (r_i)^{(9)}$$

**Definition of** $(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}$ :

Where $\boxed{(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}, (p_i)^{(9)}, \ (r_i)^{(9)}}$ are positive constants and $\boxed{i = 44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}',t) - (a_i'')^{(9)}(T_{45},t)| \le (\hat{k}_{44})^{(9)}|T_{45} - T_{45}'|e^{-(\hat{M}_{44})^{(9)}t}$$

$$|(b_i'')^{(9)}((G_{47})',t) - (b_i'')^{(9)}((G_{47}),t)| < (\hat{k}_{44})^{(9)}||(G_{47}) - (G_{47})'||e^{-(\hat{M}_{44})^{(9)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}',t)$ and $(a_i'')^{(9)}(T_{45},t)$ . $(T_{45}',t)$ and $(T_{45},t)$ are points belonging to the interval $[(\hat{k}_{44})^{(9)}, (\hat{M}_{44})^{(9)}]$ . It is

to be noted that $(a_i'')^{(9)}(T_{45},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{44})^{(9)}=1$ then the function $(a_i'')^{(9)}(T_{45},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\widehat{M}_{44})^{(9)},(\hat{k}_{44})^{(9)}$ :

$(\widehat{M}_{44})^{(9)},(\hat{k}_{44})^{(9)}$, are positive constants

$$\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\ ,\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}<1$$

**Definition of** $(\hat{P}_{44})^{(9)},(\hat{Q}_{44})^{(9)}$ :

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\widehat{M}_{44})^{(9)},(\hat{k}_{44})^{(9)},(\hat{A}_{44})^{(9)} and\ (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)},(a_i')^{(9)},(b_i)^{(9)},(b_i')^{(9)},(p_i)^{(9)},\ (r_i)^{(9)}, i=44,45,46,$ satisfy the inequalities

$$\frac{1}{(\widehat{M}_{44})^{(9)}}[(a_i)^{(9)}+(a_i')^{(9)}+\ (\hat{A}_{44})^{(9)}+\ (\hat{P}_{44})^{(9)}\ (\hat{k}_{44})^{(9)}]<1$$

$$\frac{1}{(\widehat{M}_{44})^{(9)}}[\ (b_i)^{(9)}+(b_i')^{(9)}+\ (\hat{B}_{44})^{(9)}+\ (\hat{Q}_{44})^{(9)}\ (\hat{k}_{44})^{(9)}]<1$$

**Theorem 1:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0)\ ,T_i(0)$ :

$$G_i(t)\le\ (\hat{P}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}\ ,\ \boxed{G_i(0)=G_i^0>0}$$

$$T_i(t)\le\ (\hat{Q}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}\ ,\ \boxed{T_i(0)=T_i^0>0}$$

**Theorem 2 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

**Definition of** $G_i(0)\ ,T_i(0)$

$$G_i(t)\le\ (\hat{P}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}\ ,\ \ G_i(0)=G_i^0>0$$

$$T_i(t)\le\ (\hat{Q}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}\ ,\ \ T_i(0)=T_i^0>0$$

**Theorem 3 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t)\le\ (\hat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}\ ,\ \ G_i(0)=G_i^0>0$$

$$T_i(t)\le\ (\hat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}\ ,\ \ T_i(0)=T_i^0>0$$

**Theorem 4 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

**Definition of** $G_i(0)\ ,T_i(0)$ :

$G_i(t) \leq \left(\hat{P}_{24}\right)^{(4)} e^{(\hat{M}_{24})^{(4)}t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{24}\right)^{(4)} e^{(\hat{M}_{24})^{(4)}t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Theorem 5 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

**Definition of** $G_i(0)$ , $T_i(0)$ :

$G_i(t) \leq \left(\hat{P}_{28}\right)^{(5)} e^{(\hat{M}_{28})^{(5)}t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{28}\right)^{(5)} e^{(\hat{M}_{28})^{(5)}t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Theorem 6 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

**Definition of** $G_i(0)$ , $T_i(0)$ :

$G_i(t) \leq \left(\hat{P}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Theorem 7:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

**Definition of** $G_i(0)$ , $T_i(0)$ :

$G_i(t) \leq \left(\hat{P}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Theorem 8:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

**Definition of** $G_i(0)$ , $T_i(0)$ :

$G_i(t) \leq \left(\hat{P}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Theorem 9:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

**Definition of** $G_i(0)$ , $T_i(0)$ :

$G_i(t) \leq \left(\hat{P}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Proof:** Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{13})^{(1)}, T_i^0 \le (\hat{Q}_{13})^{(1)}, \quad 155$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t} \quad 156$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t} \quad 157$$

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}(s_{(13)}) - \left((a'_{13})^{(1)} + a''_{13})^{(1)} (T_{14}(s_{(13)}), s_{(13)})\right) G_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)} (T_{14}(s_{(13)}), s_{(13)})\right) G_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)} (T_{14}(s_{(13)}), s_{(13)})\right) G_{15}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)} (G(s_{(13)}), s_{(13)})\right) T_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)} (G(s_{(13)}), s_{(13)})\right) T_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{15}(t) = T_{15}^0 + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)} (G(s_{(13)}), s_{(13)})\right) T_{15}(s_{(13)})\right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{16})^{(2)}, T_i^0 \le (\hat{Q}_{16})^{(2)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$$

By 160

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''_{16})^{(2)} (T_{17}(s_{(16)}), s_{(16)})\right) G_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)} (T_{17}(s_{(16)}), s_{(17)})\right) G_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)} (T_{17}(s_{(16)}), s_{(16)})\right) G_{18}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)} (G_{19}(s_{(16)}), s_{(16)})\right) T_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^{0} + \int_0^t \left[ (b_{17})^{(2)} T_{16}(s_{(16)}) - \left( (b'_{17})^{(2)} - (b''_{17})^{(2)} (G_{19}(s_{(16)}), s_{(16)}) \right) T_{17}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^{0} + \int_0^t \left[ (b_{18})^{(2)} T_{17}(s_{(16)}) - \left( (b'_{18})^{(2)} - (b''_{18})^{(2)} (G_{19}(s_{(16)}), s_{(16)}) \right) T_{18}(s_{(16)}) \right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \le (\hat{P}_{20})^{(3)} \text{ , } T_i^0 \le (\hat{Q}_{20})^{(3)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$$

By 161

$$\bar{G}_{20}(t) = G_{20}^{0} + \int_0^t \left[ (a_{20})^{(3)} G_{21}(s_{(20)}) - \left( (a'_{20})^{(3)} + a''_{20})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^{0} + \int_0^t \left[ (a_{21})^{(3)} G_{20}(s_{(20)}) - \left( (a'_{21})^{(3)} + (a''_{21})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{21}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^{0} + \int_0^t \left[ (a_{22})^{(3)} G_{21}(s_{(20)}) - \left( (a'_{22})^{(3)} + (a''_{22})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{22}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^{0} + \int_0^t \left[ (b_{20})^{(3)} T_{21}(s_{(20)}) - \left( (b'_{20})^{(3)} - (b''_{20})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^{0} + \int_0^t \left[ (b_{21})^{(3)} T_{20}(s_{(20)}) - \left( (b'_{21})^{(3)} - (b''_{21})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{21}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{22}(t) = T_{22}^{0} + \int_0^t \left[ (b_{22})^{(3)} T_{21}(s_{(20)}) - \left( (b'_{22})^{(3)} - (b''_{22})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{22}(s_{(20)}) \right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \le (\hat{P}_{24})^{(4)} \text{ , } T_i^0 \le (\hat{Q}_{24})^{(4)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

By 162

$$\bar{G}_{24}(t) = G_{24}^{0} + \int_0^t \left[ (a_{24})^{(4)} G_{25}(s_{(24)}) - \left( (a'_{24})^{(4)} + a''_{24})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{26}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{26}(t) = T_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{26}(s_{(24)})\right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{28})^{(5)}, T_i^0 \le (\hat{Q}_{28})^{(5)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

By



$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{30}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{30}(t) = T_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{30}(s_{(28)})\right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{32})^{(6)}, T_i^0 \le (\hat{Q}_{32})^{(6)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\bar{M}_{32})^{(6)} t}$

By 164

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[ (a_{32})^{(6)} G_{33}(s_{(32)}) - \left( (a'_{32})^{(6)} + a''_{32})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[ (a_{33})^{(6)} G_{32}(s_{(32)}) - \left( (a'_{33})^{(6)} + (a''_{33})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[ (a_{34})^{(6)} G_{33}(s_{(32)}) - \left( (a'_{34})^{(6)} + (a''_{34})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{34}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[ (b_{32})^{(6)} T_{33}(s_{(32)}) - \left( (b'_{32})^{(6)} - (b''_{32})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[ (b_{33})^{(6)} T_{32}(s_{(32)}) - \left( (b'_{33})^{(6)} - (b''_{33})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{\mathrm{T}}_{34}(\mathrm{t}) = \mathrm{T}_{34}^0 + \int_0^t \left[ (b_{34})^{(6)} T_{33}(s_{(32)}) - \left( (b'_{34})^{(6)} - (b''_{34})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{34}(s_{(32)}) \right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[ (a_{36})^{(7)} G_{37}(s_{(36)}) - \left( (a'_{36})^{(7)} + a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[ (a_{37})^{(7)} G_{36}(s_{(36)}) - \left( (a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[ (a_{38})^{(7)} G_{37}(s_{(36)}) - \left( (a'_{38})^{(7)} + (a''_{38})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{38}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[ (b_{36})^{(7)} T_{37}(s_{(36)}) - \left( (b'_{36})^{(7)} - (b''_{36})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[ (b_{37})^{(7)} T_{36}(s_{(36)}) - \left( (b'_{37})^{(7)} - (b''_{37})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^0 + \int_0^t \left[ (b_{38})^{(7)} T_{37}(s_{(36)}) - \left( (b'_{38})^{(7)} - (b''_{38})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{38}(s_{(36)}) \right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0,t)$

**Proof:**

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{40})^{(8)}$, $T_i^0 \le (\hat{Q}_{40})^{(8)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} G_{41}(s_{(40)}) - \left((a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[(a_{41})^{(8)} G_{40}(s_{(40)}) - \left((a'_{41})^{(8)} + (a''_{41})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{42}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^0 + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)} (G_{43}(s_{(40)}), s_{(40)})\right) T_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^0 + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)} (G_{43}(s_{(40)}), s_{(40)})\right) T_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{42}(t) = T_{42}^0 + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)} (G_{43}(s_{(40)}), s_{(40)})\right) T_{42}(s_{(40)})\right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0,t)$

**Proof:** 166
Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ A
which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{44})^{(9)}$, $T_i^0 \le (\hat{Q}_{44})^{(9)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)})\right) G_{44}(s_{(44)})\right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)} (T_{45}(s_{(44)}), s_{(44)})\right) G_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}(s_{(44)}), s_{(44)})\right) G_{46}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{44}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{\mathrm{T}}_{46}(\mathrm{t}) = \mathrm{T}_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{46}(s_{(44)})\right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}}\right)\right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left(e^{(\hat{M}_{13})^{(1)} t} - 1\right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\hat{M}_{13})^{(1)} t} \le \frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{P}_{13})^{(1)} + G_{14}^0\right) e^{\left(-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0}\right)} + (\hat{P}_{13})^{(1)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \le G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}}\right)\right] ds_{(16)} = \quad 169$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1\right)$$

From which it follows that 170

$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \le \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left[\left((\hat{P}_{16})^{(2)} + G_{17}^0\right) e^{\left(-\frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0}\right)} + (\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t) \le G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} \left(G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} s_{(20)}}\right)\right] ds_{(20)} =$$

$$\left(1 + (a_{20})^{(3)} t\right) G_{21}^0 + \frac{(a_{20})^{(3)} (\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left(e^{(\hat{M}_{20})^{(3)} t} - 1\right)$$

From which it follows that 172

$$(G_{20}(t)-G_{20}^{0})e^{-(\hat{M}_{20})^{(3)}t}\leq\frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left[((\hat{P}_{20})^{(3)}+G_{21}^{0})e^{\left(-\frac{(\hat{P}_{20})^{(3)}+G_{21}^{0}}{G_{21}^{0}}\right)}+(\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 173

$$G_{24}(t)\leq G_{24}^{0}+\int_{0}^{t}\left[(a_{24})^{(4)}\left(G_{25}^{0}+(\hat{P}_{24})^{(4)}e^{(\hat{M}_{24})^{(4)}s_{(24)}}\right)\right]ds_{(24)}=$$

$$\left(1+(a_{24})^{(4)}t\right)G_{25}^{0}+\frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left(e^{(\hat{M}_{24})^{(4)}t}-1\right)$$

From which it follows that 174

$$(G_{24}(t)-G_{24}^{0})e^{-(\hat{M}_{24})^{(4)}t}\leq\frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left[((\hat{P}_{24})^{(4)}+G_{25}^{0})e^{\left(-\frac{(\hat{P}_{24})^{(4)}+G_{25}^{0}}{G_{25}^{0}}\right)}+(\hat{P}_{24})^{(4)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t)\leq G_{28}^{0}+\int_{0}^{t}\left[(a_{28})^{(5)}\left(G_{29}^{0}+(\hat{P}_{28})^{(5)}e^{(\hat{M}_{28})^{(5)}s_{(28)}}\right)\right]ds_{(28)}=$$

$$\left(1+(a_{28})^{(5)}t\right)G_{29}^{0}+\frac{(a_{28})^{(5)}(\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left(e^{(\hat{M}_{28})^{(5)}t}-1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t}\leq\frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t)\leq G_{32}^{0}+\int_{0}^{t}\left[(a_{32})^{(6)}\left(G_{33}^{0}+(\hat{P}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\right)\right]ds_{(32)}=$$

$$\left(1+(a_{32})^{(6)}t\right)G_{33}^{0}+\frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left(e^{(\hat{M}_{32})^{(6)}t}-1\right)$$

From which it follows that 177

$$(G_{32}(t)-G_{32}^{0})e^{-(\hat{M}_{32})^{(6)}t}\leq\frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left[((\hat{P}_{32})^{(6)}+G_{33}^{0})e^{\left(-\frac{(\hat{P}_{32})^{(6)}+G_{33}^{0}}{G_{33}^{0}}\right)}+(\hat{P}_{32})^{(6)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

(z) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t) \le G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} \left(G_{37}^0 + (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} s_{(36)}}\right)\right] ds_{(36)} =$$
$$\left(1 + (a_{36})^{(7)} t\right) G_{37}^0 + \frac{(a_{36})^{(7)} (\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left(e^{(\hat{M}_{36})^{(7)} t} - 1\right)$$

From which it follows that

$$(G_{36}(t) - G_{36}^0) e^{-(\hat{M}_{36})^{(7)} t} \le \frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left[((\hat{P}_{36})^{(7)} + G_{37}^0) e^{\left(-\frac{(\hat{P}_{36})^{(7)} + G_{37}^0}{G_{37}^0}\right)} + (\hat{P}_{36})^{(7)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t) \le G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} \left(G_{41}^0 + (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} s_{(40)}}\right)\right] ds_{(40)} = \quad 180$$
$$\left(1 + (a_{40})^{(8)} t\right) G_{41}^0 + \frac{(a_{40})^{(8)} (\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}} \left(e^{(\hat{M}_{40})^{(8)} t} - 1\right)$$

From which it follows that 181

$$(G_{40}(t) - G_{40}^0) e^{-(\hat{M}_{40})^{(8)} t} \le \frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}} \left[((\hat{P}_{40})^{(8)} + G_{41}^0) e^{\left(-\frac{(\hat{P}_{40})^{(8)} + G_{41}^0}{G_{41}^0}\right)} + (\hat{P}_{40})^{(8)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 8

Analogous inequalities hold also for $G_{41}, G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t) \le G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} \left(G_{45}^0 + (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} s_{(44)}}\right)\right] ds_{(44)} =$$
$$\left(1 + (a_{44})^{(9)} t\right) G_{45}^0 + \frac{(a_{44})^{(9)} (\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left(e^{(\hat{M}_{44})^{(9)} t} - 1\right)$$

From which it follows that

$$(G_{44}(t) - G_{44}^0) e^{-(\hat{M}_{44})^{(9)} t} \le \frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left[((\hat{P}_{44})^{(9)} + G_{45}^0) e^{\left(-\frac{(\hat{P}_{44})^{(9)} + G_{45}^0}{G_{45}^0}\right)} + (\hat{P}_{44})^{(9)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 9

Analogous inequalities hold also for $G_{45}, G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$ and to choose 182

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\widehat{M}_{13})^{(1)}}\left[(\widehat{P}_{13})^{(1)}+\left((\hat{P}_{13})^{(1)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{13})^{(1)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{13})^{(1)} \quad 183$$

$$\frac{(b_i)^{(1)}}{(\widehat{M}_{13})^{(1)}}\left[\left((\hat{Q}_{13})^{(1)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{13})^{(1)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{13})^{(1)}\right]\leq(\hat{Q}_{13})^{(1)} \quad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)},T^{(1)}\right),\left(G^{(2)},T^{(2)}\right)\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{13})^{(1)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{13})^{(1)}t}\}$$

Indeed if we denote

**Definition of** $\tilde{G},\tilde{T}:\left(\tilde{G},\tilde{T}\right)=\mathcal{A}^{(1)}(G,T)$

It results

$$\left|\tilde{G}_{13}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{13})^{(1)}\left|G_{14}^{(1)}-G_{14}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\,ds_{(13)}+$$

$$\int_0^t\{(a'_{13})^{(1)}\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$G_{13}^{(2)}\left|(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)-(a''_{13})^{(1)}\left(T_{14}^{(2)},s_{(13)}\right)\right|\ e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses it follows

$$\left|G^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}t}\leq \quad 186$$
$$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)}+(a'_{13})^{(1)}+(\widehat{A}_{13})^{(1)}+(\widehat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}\right)d\left(\left(G^{(1)},T^{(1)};\,G^{(2)},T^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

**Remark 1:** The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}\ and\ (\widehat{Q}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i=13,14,15$ depend only on $\mathrm{T}_{14}$ and respectively on

$G(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$

From 19 to 24 it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(1)} - (a_i'')^{(1)}(T_{14}(s_{(13)}), s_{(13)})\} ds_{(13)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(1)} t)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2\ and\ \left((\widehat{M}_{13})^{(1)}\right)_3$ : 187

**Remark 3**: if $G_{13}$ is bounded, the same property have also $G_{14}\ and\ G_{15}$ . indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a_{14}')^{(1)} G_{14}$ and by integrating

$$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1 / (a_{14}')^{(1)}$$

In the same way , one can obtain

$$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2 / (a_{15}')^{(1)}$$

If $G_{14}\ or\ G_{15}$ is bounded, the same property follows for $G_{13}$ , $G_{15}$ and $G_{13}$ , $G_{14}$ respectively.

**Remark 4:** If $G_{13}$ $is$ bounded, from below, the same property holds for $G_{14}\ and\ G_{15}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{14}$ is bounded from below. 188

**Remark 5:** If $T_{13}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}\,(G(t), t)) = (b_{14}')^{(1)}$ then $T_{14} \to \infty$. 189

**Definition of** $(m)^{(1)}$ and $\varepsilon_1$ :

Indeed let $t_1$ be so that for $t > t_1$

$$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t), t) < \varepsilon_1, T_{13}\,(t) > (m)^{(1)}$$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right)$, $t = log\,\frac{2}{\varepsilon_1}$ By taking now $\varepsilon_1$ sufficiently small one sees that $T_{14}$ is unbounded.
The same property holds for $T_{15}$ if $\lim_{t\to\infty}(b_{15}'')^{(1)}\,(G(t), t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}$ , $\frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}\ and\ (\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[(\widehat{P}_{16})^{(2)}+\left((\hat{P}_{16})^{(2)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{16})^{(2)}\right]\leq(\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

**Definition of** $\widetilde{G_{19}},\widetilde{T_{19}}$ : $\left(\widetilde{G_{19}},\widetilde{T_{19}}\right)=\mathcal{A}^{(2)}(G_{19},T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{16})^{(2)}\left|G_{17}^{(1)}-G_{17}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)}+$$

$$\int_0^t\{(a'_{16})^{(2)}\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$G_{16}^{(2)}\left|(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)-(a''_{16})^{(2)}\left(T_{17}^{(2)},s_{(16)}\right)\right|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0,t]$ 197

From the hypotheses it follows

$$\left|(G_{19})^{(1)}-(G_{19})^{(2)}\right|\mathrm{e}^{-(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}\leq$$
$$\frac{1}{(\widehat{\mathrm{M}}_{16})^{(2)}}\left((a_{16})^{(2)}+(a'_{16})^{(2)}+(\widehat{\mathrm{A}}_{16})^{(2)}+(\widehat{\mathrm{P}}_{16})^{(2)}(\hat{k}_{16})^{(2)}\right)\mathrm{d}\left(((G_{19})^{(1)},(T_{19})^{(1)};\ (G_{19})^{(2)},(T_{19})^{(2)})\right)$$

And analogous inequalities for $G_i$ and $T_i$. Taking into account the hypothesis the result follows 198

**Remark 6:** The fact that we supposed $(a''_{16})^{(2)}$ and $(b''_{16})^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{\mathrm{P}}_{16})^{(2)}\mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}$ and $(\widehat{\mathrm{Q}}_{16})^{(2)}\mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i=16,17,18$ depend only on $\mathrm{T}_{17}$ and respectively on

$(G_{19})$(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 7:** There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$ 200

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(2)} - (a_i'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})\} ds_{(16)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(2)}t)} > 0 \quad \text{for } t > 0$$

**Definition of** $\left((\widehat{M}_{16})^{(2)}\right)_1$, $\left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$: 201

**Remark 8:** if $G_{16}$ is bounded, the same property have also $G_{17}$ and $G_{18}$. indeed if

$G_{16} < (\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt} \leq \left((\widehat{M}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)} G_{17}$ and by integrating

$$G_{17} \leq \left((\widehat{M}_{16})^{(2)}\right)_2 = G_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1 / (a_{17}')^{(2)}$$

In the same way , one can obtain

$$G_{18} \leq \left((\widehat{M}_{16})^{(2)}\right)_3 = G_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2 / (a_{18}')^{(2)}$$

If $G_{17}$ or $G_{18}$ is bounded, the same property follows for $G_{16}$, $G_{18}$ and $G_{16}$, $G_{17}$ respectively.

**Remark 9:** If $G_{16}$ is bounded, from below, the same property holds for $G_{17}$ and $G_{18}$. The proof is analogous with the preceding one. An analogous property is true if $G_{17}$ is bounded from below. 202

**Remark 10:** If $T_{16}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t), t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$. 203

**Definition of** $(m)^{(2)}$ and $\varepsilon_2$ :

Indeed let $t_2$ be so that for $t > t_2$

$$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right)$, $t = \log\frac{2}{\varepsilon_2}$ By taking now $\varepsilon_2$ sufficiently small one sees that $T_{17}$ is unbounded. 205
The same property holds for $T_{18}$ if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t), t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[(\widehat{P}_{20})^{(3)}+\left((\hat{P}_{20})^{(3)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{20})^{(3)}\right]\leq(\hat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself 210

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)},(T_{23})^{(1)}),((G_{23})^{(2)},(T_{23})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

**Definition of** $\widetilde{G_{23}},\widetilde{T_{23}}:(\widetilde{(G_{23})},\widetilde{(T_{23})})=\mathcal{A}^{(3)}((G_{23}),(T_{23}))$

It results 213

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{20})^{(3)}\left|G_{21}^{(1)}-G_{21}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}ds_{(20)}+$$

$$\int_0^t\{(a'_{20})^{(3)}\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$G_{20}^{(2)}\left|(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)},s_{(20)}\right)\right|\ e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\}ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses it follows

$$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}t}\leq \quad 214$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+(a'_{20})^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\widehat{k}_{20})^{(3)}\right)d\left(((G_{23})^{(1)},(T_{23})^{(1)};\ (G_{23})^{(2)},(T_{23})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 11:** The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ *and* $(\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i=20,21,22$ depend only on $T_{21}$ and respectively on

$(G_{23})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 12:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 216

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(3)} - (a_i'')^{(3)}(T_{21}(s_{(20)}), s_{(20)})\} ds_{(20)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(3)} t)} > 0$ for t > 0

**Definition of** $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2\ and\ \left((\widehat{M}_{20})^{(3)}\right)_3$ : 217

**Remark 13:** if $G_{20}$ is bounded, the same property have also $G_{21}\ and\ G_{22}$ . indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a_{21}')^{(3)} G_{21}$ and by integrating

$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1 / (a_{21}')^{(3)}$

In the same way , one can obtain

$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2 / (a_{22}')^{(3)}$

If $G_{21}\ or\ G_{22}$ is bounded, the same property follows for $G_{20}$ , $G_{22}$ and $G_{20}$ , $G_{21}$ respectively.

**Remark 14:** If $G_{20}$ *is* bounded, from below, the same property holds for $G_{21}\ and\ G_{22}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{21}$ is bounded from below. 218

**Remark 15:** If $T_{20}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t), t\right)) = (b_{21}')^{(3)}$ then $T_{21} \to \infty$. 219

**Definition of** $(m)^{(3)}$ and $\varepsilon_3$ :

Indeed let $t_3$ be so that for $t > t_3$

$(b_{21})^{(3)} - (b_i'')^{(3)}\left((G_{23})(t), t\right) < \varepsilon_3, T_{20}\,(t) > (m)^{(3)}$

Then $\frac{dT_{21}}{dt} \geq (a_{21})^{(3)}(m)^{(3)} - \varepsilon_3 T_{21}$ which leads to 220

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t} = \frac{1}{2}$ it results

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_3}$ By taking now $\varepsilon_3$ sufficiently small one sees that $T_{21}$ is unbounded.
The same property holds for $T_{22}$ if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t), t\right) = (b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}$ , $\frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}} < 1$ and to choose 221

$(\widehat{P}_{24})^{(4)}$ and $(\widehat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[(\widehat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \tag{222}$$

$$\frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \tag{223}$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself (224)

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric (225)

$$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{24})^{(4)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{27}}),(\widetilde{T_{27}})$ : $\left((\widetilde{G_{27}}),(\widetilde{T_{27}})\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{24})^{(4)}\left|G_{25}^{(1)}-G_{25}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\,ds_{(24)}+$$

$$\int_0^t\{(a'_{24})^{(4)}\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$G_{24}^{(2)}\left|(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)-(a''_{24})^{(4)}\left(T_{25}^{(2)},s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\}ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}t}\leq \tag{226}$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)}+(a'_{24})^{(4)}+(\widehat{A}_{24})^{(4)}+(\widehat{P}_{24})^{(4)}(\widehat{k}_{24})^{(4)}\right)d\left(((G_{27})^{(1)},(T_{27})^{(1)};\ (G_{27})^{(2)},(T_{27})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 16:** The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ *and* $(\widehat{Q}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_+$. (227)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i=24,25,26$ depend only on $T_{25}$ and respectively on

$(G_{27})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 17:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 228

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(4)} - (a_i'')^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(4)} t)} > 0$ for t > 0

**Definition of** $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2\ and\ \left((\widehat{M}_{24})^{(4)}\right)_3$ : 229

**Remark 18:** if $G_{24}$ is bounded, the same property have also $G_{25}\ and\ G_{26}$ . indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a_{25}')^{(4)} G_{25}$ and by integrating

$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1 / (a_{25}')^{(4)}$

In the same way , one can obtain

$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2 / (a_{26}')^{(4)}$

If $G_{25}\ or\ G_{26}$ is bounded, the same property follows for $G_{24}$ , $G_{26}$ and $G_{24}$ , $G_{25}$ respectively.

**Remark 19:** If $G_{24}$ *is* bounded, from below, the same property holds for $G_{25}\ and\ G_{26}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{25}$ is bounded from below. 230

**Remark 20:** If $T_{24}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}\ ((G_{27})(t), t)) = (b_{25}')^{(4)}$ then $T_{25} \to \infty$. 231

**Definition of** $(m)^{(4)}$ and $\varepsilon_4$ :

Indeed let $t_4$ be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t), t) < \varepsilon_4, T_{24}\,(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_4}$ By taking now $\varepsilon_4$ sufficiently small one sees that $T_{25}$ is unbounded.
The same property holds for $T_{26}$ if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t), t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for $G_{29}$ , $G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}$ , $\frac{(b_i)^{(5)}}{(\widehat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)}+\left((\hat{P}_{28})^{(5)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{28})^{(5)}\right]\leq(\hat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$$d\left(((G_{31})^{(1)},(T_{31})^{(1)}),((G_{31})^{(2)},(T_{31})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\hat{M}_{28})^{(5)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\hat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{31}}),(\widetilde{T_{31}})$ : $\left((\widetilde{G_{31}}),(\widetilde{T_{31}})\right)=\mathcal{A}^{(5)}\left((G_{31}),(T_{31})\right)$

It results

$$|\tilde{G}_{28}^{(1)}-\tilde{G}_i^{(2)}|\leq\int_0^t(a_{28})^{(5)}|G_{29}^{(1)}-G_{29}^{(2)}|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\,ds_{(28)}+$$

$$\int_0^t\{(a'_{28})^{(5)}|G_{28}^{(1)}-G_{28}^{(2)}|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$(a''_{28})^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)|G_{28}^{(1)}-G_{28}^{(2)}|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$G_{28}^{(2)}|(a''_{28})^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)-(a''_{28})^{(5)}\left(T_{29}^{(2)},s_{(28)}\right)|\;e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\}ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on it follows

$$|(G_{31})^{(1)}-(G_{31})^{(2)}|e^{-(\widehat{M}_{28})^{(5)}t}\leq \quad 237$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)}+(a'_{28})^{(5)}+(\widehat{A}_{28})^{(5)}+(\widehat{P}_{28})^{(5)}(\widehat{k}_{28})^{(5)}\right)d\left(((G_{31})^{(1)},(T_{31})^{(1)};\;(G_{31})^{(2)},(T_{31})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 21:** The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\hat{P}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ *and* $(\hat{Q}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ respectively of $\mathbb{R}_+$. 238

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on $T_{29}$ and respectively on $(G_{31})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 22:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 239

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(5)} - (a_i'')^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(5)} t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2\ and\ \left((\widehat{M}_{28})^{(5)}\right)_3$ : 240

**Remark 23:** if $G_{28}$ is bounded, the same property have also $G_{29}\ and\ G_{30}$ . indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a_{29}')^{(5)} G_{29}$ and by integrating

$$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1 / (a_{29}')^{(5)}$$

In the same way , one can obtain

$$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2 / (a_{30}')^{(5)}$$

If $G_{29}\ or\ G_{30}$ is bounded, the same property follows for $G_{28}$ , $G_{30}$ and $G_{28}$ , $G_{29}$ respectively.

**Remark 24:** If $G_{28}$ *is* bounded, from below, the same property holds for $G_{29}\ and\ G_{30}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{29}$ is bounded from below. 241

**Remark 25:** If $T_{28}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}\,((G_{31})(t), t)) = (b_{29}')^{(5)}$ then $T_{29} \to \infty$. 242

**Definition of** $(m)^{(5)}$ and $\varepsilon_5$ :

Indeed let $t_5$ be so that for $t > t_5$

$$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t), t) < \varepsilon_5, T_{28}\,(t) > (m)^{(5)}$$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_5}$ By taking now $\varepsilon_5$ sufficiently small one sees that $T_{29}$ is unbounded.
The same property holds for $T_{30}$ if $\lim_{t\to\infty}(b_{30}'')^{(5)}\left((G_{31})(t), t\right) = (b_{30}')^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{33}\,, G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\ , \frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}} < 1$ and to choose 244

$(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[(\hat{P}_{32})^{(6)}+\left((\hat{P}_{32})^{(6)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{32})^{(6)} \tag{245}$$

$$\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[\left((\hat{Q}_{32})^{(6)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{32})^{(6)}\right]\leq(\hat{Q}_{32})^{(6)} \tag{246}$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric (247)

$$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\hat{M}_{32})^{(6)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\hat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{35}}),(\widetilde{T_{35}})$ : $\left((\widetilde{G_{35}}),(\widetilde{T_{35}})\right)=\mathcal{A}^{(6)}\left((G_{35}),(T_{35})\right)$

It results

$$|\tilde{G}_{32}^{(1)}-\tilde{G}_i^{(2)}|\leq\int_0^t(a_{32})^{(6)}|G_{33}^{(1)}-G_{33}^{(2)}|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\,ds_{(32)}+$$

$$\int_0^t\{(a'_{32})^{(6)}|G_{32}^{(1)}-G_{32}^{(2)}|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)|G_{32}^{(1)}-G_{32}^{(2)}|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$G_{32}^{(2)}|(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)-(a''_{32})^{(6)}\left(T_{33}^{(2)},s_{(32)}\right)|\;e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\}ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$|(G_{35})^{(1)}-(G_{35})^{(2)}|e^{-(\widehat{M}_{32})^{(6)}t}\leq \tag{248}$$
$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)}+(a'_{32})^{(6)}+(\widehat{A}_{32})^{(6)}+(\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}\right)d\left(((G_{35})^{(1)},(T_{35})^{(1)};\;(G_{35})^{(2)},(T_{35})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 26:** The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\hat{P}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ *and* $(\hat{Q}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ respectively of $\mathbb{R}_+$. (249)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i = 32,33,34$ depend only on $T_{33}$ and respectively on $(G_{35})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 27:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 250

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(6)} - (a_i'')^{(6)}(T_{33}(s_{(32)}), s_{(32)})\} ds_{(32)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(6)} t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{32})^{(6)}\right)_1, \left((\widehat{M}_{32})^{(6)}\right)_2\ and\ \left((\widehat{M}_{32})^{(6)}\right)_3$ : 251

**Remark 28:** if $G_{32}$ is bounded, the same property have also $G_{33}\ and\ G_{34}$ . indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a_{33}')^{(6)} G_{33}$ and by integrating

$$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)} \left((\widehat{M}_{32})^{(6)}\right)_1 / (a_{33}')^{(6)}$$

In the same way , one can obtain

$$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)} \left((\widehat{M}_{32})^{(6)}\right)_2 / (a_{34}')^{(6)}$$

If $G_{33}\ or\ G_{34}$ is bounded, the same property follows for $G_{32}$ , $G_{34}$ and $G_{32}$ , $G_{33}$ respectively.

**Remark 29:** If $G_{32}$ *is* bounded, from below, the same property holds for $G_{33}\ and\ G_{34}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{33}$ is bounded from below. 252

**Remark 30:** If $T_{32}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}\ ((G_{35})(t), t)) = (b_{33}')^{(6)}$ then $T_{33} \to \infty$. 253

**Definition of** $(m)^{(6)}$ and $\varepsilon_6$ :

Indeed let $t_6$ be so that for $t > t_6$

$$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t), t\right) < \varepsilon_6, T_{32}\,(t) > (m)^{(6)}$$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right)$, $t = log\,\frac{2}{\varepsilon_6}$ By taking now $\varepsilon_6$ sufficiently small one sees that $T_{33}$ is unbounded.
The same property holds for $T_{34}$ if $\lim_{t\to\infty}(b_{34}'')^{(6)}\left((G_{35})(t), t(t), t\right) = (b_{34}')^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{37}, G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}$ , $\frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$ and to choose

$(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[(\hat{P}_{36})^{(7)}+\left((\hat{P}_{36})^{(7)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{36})^{(7)} \tag{256}$$

$$\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[\left((\hat{Q}_{36})^{(7)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{36})^{(7)}\right]\leq(\hat{Q}_{36})^{(7)} \tag{257}$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself
The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric (258)

$$d\left(((G_{39})^{(1)},(T_{39})^{(1)}),((G_{39})^{(2)},(T_{39})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{36})^{(7)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{39}}),(\widetilde{T_{39}}):\left((\widetilde{G_{39}}),(\widetilde{T_{39}})\right)=\mathcal{A}^{(7)}((G_{39}),(T_{39}))$
It results

$$\left|\tilde{G}_{36}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{36})^{(7)}\left|G_{37}^{(1)}-G_{37}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\,ds_{(36)}+$$
$$\int_0^t\{(a'_{36})^{(7)}\left|G_{36}^{(1)}-G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$(a''_{36})^{(7)}\left(T_{37}^{(1)},s_{(36)}\right)\left|G_{36}^{(1)}-G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$G_{36}^{(2)}\left|(a''_{36})^{(7)}\left(T_{37}^{(1)},s_{(36)}\right)-(a''_{36})^{(7)}\left(T_{37}^{(2)},s_{(36)}\right)\right|\;e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\}ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on it follows

$$\left|(G_{39})^{(1)}-(G_{39})^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}t}\leq \tag{259}$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}}\left((a_{36})^{(7)}+(a'_{36})^{(7)}+(\hat{A}_{36})^{(7)}+(\hat{P}_{36})^{(7)}(\hat{k}_{36})^{(7)}\right)d\left(((G_{39})^{(1)},(T_{39})^{(1)};\;(G_{39})^{(2)},(T_{39})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 31:** The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\hat{P}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ *and* $(\hat{Q}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ respectively of $\mathbb{R}_+$. (260)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(7)}$ and $(b_i'')^{(7)}, i = 36,37,38$ depend only on $T_{37}$ and respectively on $(G_{39})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 32:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 261

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(7)} - (a_i'')^{(7)}(T_{37}(s_{(36)}), s_{(36)})\} ds_{(36)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(7)} t\right)} > 0$ for t > 0

**Definition of** $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2\ and\ \left((\widehat{M}_{36})^{(7)}\right)_3$ : 262

**Remark 33:** if $G_{36}$ is bounded, the same property have also $G_{37}\ and\ G_{38}$ . indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a_{37}')^{(7)} G_{37}$ and by integrating

$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1 / (a_{37}')^{(7)}$

In the same way , one can obtain

$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2 / (a_{38}')^{(7)}$

If $G_{37}\ or\ G_{38}$ is bounded, the same property follows for $G_{36}$ , $G_{38}$ and $G_{36}$ , $G_{37}$ respectively.

**Remark 34:** If $G_{36}$ *is* bounded, from below, the same property holds for $G_{37}\ and\ G_{38}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{37}$ is bounded from below. 263

**Remark 35:** If $T_{36}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(7)}\ ((G_{39})(t), t)) = (b_{37}')^{(7)}$ then $T_{37} \to \infty$. 264

**Definition of** $(m)^{(7)}$ and $\varepsilon_7$ :

Indeed let $t_7$ be so that for $t > t_7$

$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t), t) < \varepsilon_7, T_{36}\,(t) > (m)^{(7)}$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_7}$ By taking now $\varepsilon_7$ sufficiently small one sees that $T_{37}$ is unbounded.
The same property holds for $T_{38}$ if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t), t\right) = (b_{38}')^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}$ , $\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}} < 1$ and to choose $(\widehat{P}_{40})^{(8)}$ and $(\widehat{Q}_{40})^{(8)}$ large to have 266

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[(\widehat{P}_{40})^{(8)} + \left((\hat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{40})^{(8)} \quad 267$$

$$\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{40})^{(8)}\right] \leq (\hat{Q}_{40})^{(8)} \quad 268$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)}, (T_{43})^{(1)}), ((G_{43})^{(2)}, (T_{43})^{(2)})\right) = \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

**Definition of** $\widetilde{(G_{43})}, \widetilde{(T_{43})}$ : $\left(\widetilde{(G_{43})}, \widetilde{(T_{43})}\right) = \mathcal{A}^{(8)}((G_{43}), (T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{40})^{(8)}\left|G_{41}^{(1)} - G_{41}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\,ds_{(40)} +$$
$$\int_0^t\{(a'_{40})^{(8)}\left|G_{40}^{(1)} - G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{-(\widehat{M}_{40})^{(8)}s_{(40)}} +$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right)\left|G_{40}^{(1)} - G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}} +$$
$$G_{40}^{(2)}\left|(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right) - (a''_{40})^{(8)}\left(T_{41}^{(2)}, s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\}ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0, t]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)} - (G_{43})^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}t} \leq \quad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)} + (a'_{40})^{(8)} + (\widehat{A}_{40})^{(8)} + (\widehat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}\right)d\left(((G_{43})^{(1)}, (T_{43})^{(1)};\ (G_{43})^{(2)}, (T_{43})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

**Remark 36:** The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate 274

condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ *and* $(\widehat{Q}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i = 40{,}41{,}42$ depend only on $T_{41}$ and respectively on $(G_{43})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 37** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 275

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t\{(a_i')^{(8)}-(a_i'')^{(8)}(T_{41}(s_{(40)}),s_{(40)})\}ds_{(40)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(8)}t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{40})^{(8)}\right)_1, \left((\widehat{M}_{40})^{(8)}\right)_2\ and\ \left((\widehat{M}_{40})^{(8)}\right)_3$ : 276

**Remark 38:** if $G_{40}$ is bounded, the same property have also $G_{41}\ and\ G_{42}$ . indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a_{41}')^{(8)}G_{41}$ and by integrating

$$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1/(a_{41}')^{(8)}$$

In the same way , one can obtain

$$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2/(a_{42}')^{(8)}$$

If $G_{41}\ or\ G_{42}$ is bounded, the same property follows for $G_{40}$ , $G_{42}$ and $G_{40}$ , $G_{41}$ respectively.

**Remark 39:** If $G_{40}$ *is* bounded, from below, the same property holds for $G_{41}\ and\ G_{42}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{41}$ is bounded from below. 277

**Remark 40:** If $T_{40}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}\ ((G_{43})(t),t)) = (b_{41}')^{(8)}$ then $T_{41} \to \infty$. 278

**Definition of** $(m)^{(8)}$ and $\varepsilon_8$ :

Indeed let $t_8$ be so that for $t\ > t_8$

$$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t),t\right) < \varepsilon_8, T_{40}\,(t) > (m)^{(8)}$$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right)$, $t = log\frac{2}{\varepsilon_8}$ By taking now $\varepsilon_8$ sufficiently small one sees that $T_{41}$ is unbounded.
The same property holds for $T_{42}$ if $\lim_{t\to\infty}(b''_{42})^{(8)}\left((G_{43})(t), t(t), t\right) = (b'_{42})^{(8)}$



It is now sufficient to take $\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}$ , $\frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$ and to choose $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ large to have

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}\left[(\hat{P}_{44})^{(9)} + \left((\hat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}}\left[\left((\hat{Q}_{44})^{(9)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{44})^{(9)}\right] \leq (\hat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)}, (T_{47})^{(1)}), ((G_{47})^{(2)}, (T_{47})^{(2)})\right) =$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{44})^{(9)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{47}}), (\widetilde{T_{47}}) : \left((\widetilde{G_{47}}), (\widetilde{T_{47}})\right) = \mathcal{A}^{(9)}\left((G_{47}), (T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{44})^{(9)}\left|G_{45}^{(1)} - G_{45}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\,ds_{(44)} +$$
$$\int_0^t\{(a'_{44})^{(9)}\left|G_{44}^{(1)} - G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{-(\widehat{M}_{44})^{(9)}s_{(44)}} +$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right)\left|G_{44}^{(1)} - G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}} +$$
$$G_{44}^{(2)}\left|(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right) - (a''_{44})^{(9)}\left(T_{45}^{(2)}, s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\}ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)} - G^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}t} \leq$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)} + (a'_{44})^{(9)} + (\widehat{A}_{44})^{(9)} + (\widehat{P}_{44})^{(9)}(\hat{k}_{44})^{(9)}\right)d\left(((G_{47})^{(1)}, (T_{47})^{(1)};\ (G_{47})^{(2)}, (T_{47})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis (39,35,36) the result follows

**Remark 41:** The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by

$(\widehat{P}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ *and* $(\widehat{Q}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i = 44{,}45{,}46$ depend only on $T_{45}$ and respectively on $(G_{47})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 42:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$

From 99 to 44 it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t\{(a_i')^{(9)}-(a_i'')^{(9)}(T_{45}(s_{(44)}),s_{(44)})\}ds_{(44)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(9)}t)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{44})^{(9)}\right)_1, \left((\widehat{M}_{44})^{(9)}\right)_2$ *and* $\left((\widehat{M}_{44})^{(9)}\right)_3$ :

**Remark 43:** if $G_{44}$ is bounded, the same property have also $G_{45}\ and\ G_{46}$ . indeed if

$G_{44} < (\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt} \leq \left((\widehat{M}_{44})^{(9)}\right)_1 - (a_{45}')^{(9)}G_{45}$ and by integrating

$$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_1/(a_{45}')^{(9)}$$

In the same way , one can obtain

$$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_2/(a_{46}')^{(9)}$$

If $G_{45}\ or\ G_{46}$ is bounded, the same property follows for $G_{44}$ , $G_{46}$ and $G_{44}$ , $G_{45}$ respectively.

**Remark 44:** If $G_{44}$ *is* bounded, from below, the same property holds for $G_{45}\ and\ G_{46}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{45}$ is bounded from below.

**Remark 45:** If $T_{44}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t),t\right) = (b_{45}')^{(9)}$ then $T_{45} \to \infty$.

**Definition of** $(m)^{(9)}$ and $\varepsilon_9$ :

Indeed let $t_9$ be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t),t\right) < \varepsilon_9, T_{44}\,(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right), \quad t = log\frac{2}{\varepsilon_9}$ By taking now $\varepsilon_9$ sufficiently small one sees that $T_{45}$ is unbounded.
The same property holds for $T_{46}$ if $\lim_{t\to\infty}(b_{46}'')^{(9)}\left((G_{47})(t),t\right) = (b_{46}')^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

**Behavior of the solutions of equation** 280

**Theorem** If we denote and define

**Definition of** $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ :

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)} \le -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14}, t) + (a''_{14})^{(1)}(T_{14}, t) \le -(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)} \le -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G, t) - (b''_{14})^{(1)}(G, t) \le -(\tau_1)^{(1)}$

**Definition of** $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$ : 281

By $(\nu_1)^{(1)} > 0$ , $(\nu_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$ , $(u_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

**Definition of** $(\bar{\nu}_1)^{(1)}, , (\bar{\nu}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$ : 282

By $(\bar{\nu}_1)^{(1)} > 0$ , $(\bar{\nu}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$ , $(\bar{u}_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

**Definition of** $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (\nu_0)^{(1)}$ :- 283

If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by
$(m_2)^{(1)} = (\nu_0)^{(1)}, (m_1)^{(1)} = (\nu_1)^{(1)}$, $if\ (\nu_0)^{(1)} < (\nu_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}, if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)}$,

and $\boxed{(\nu_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)}$, $if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)}$, $if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)}$,

and $\boxed{(u_0)^{(1)} = \frac{T^0_{13}}{T^0_{14}}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$

are defined

Then the solution of global equations satisfies the inequalities 285

$G^0_{13}e^{\left((S_1)^{(1)} - (p_{13})^{(1)}\right)t} \le G_{13}(t) \le G^0_{13}e^{(S_1)^{(1)}t}$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)}-(p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)}t}$$

$$\left(\frac{(a_{15})^{(1)}G_{13}^0}{(m_1)^{(1)}((S_1)^{(1)}-(p_{13})^{(1)}-(S_2)^{(1)})}\left[e^{((S_1)^{(1)}-(p_{13})^{(1)})t} - e^{-(S_2)^{(1)}t}\right] + G_{15}^0 e^{-(S_2)^{(1)}t} \le G_{15}(t) \le \right. \quad (286)$$

$$\left. \frac{(a_{15})^{(1)}G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)}-(a'_{15})^{(1)})}[e^{(S_1)^{(1)}t} - e^{-(a'_{15})^{(1)}t}] + G_{15}^0 e^{-(a'_{15})^{(1)}t}\right)$$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)}t} \le T_{13}(t) \le T_{13}^0 e^{((R_1)^{(1)}+(r_{13})^{(1)})t}} \quad (287)$$

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)}t} \le T_{13}(t) \le \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)}+(r_{13})^{(1)})t} \quad (288)$$

$$\frac{(b_{15})^{(1)}T_{13}^0}{(\mu_1)^{(1)}((R_1)^{(1)}-(b'_{15})^{(1)})}\left[e^{(R_1)^{(1)}t} - e^{-(b'_{15})^{(1)}t}\right] + T_{15}^0 e^{-(b'_{15})^{(1)}t} \le T_{15}(t) \le \quad (289)$$

$$\frac{(a_{15})^{(1)}T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)}+(r_{13})^{(1)}+(R_2)^{(1)})}\left[e^{((R_1)^{(1)}+(r_{13})^{(1)})t} - e^{-(R_2)^{(1)}t}\right] + T_{15}^0 e^{-(R_2)^{(1)}t}$$

**Definition of** $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:- (290)

Where $(S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$

$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$

$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$

$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$

**Behavior of the solutions of equation** (291)

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ : (292)

$(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ four constants satisfying

$$-(\sigma_2)^{(2)} \le -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(T_{17}, t) + (a''_{17})^{(2)}(T_{17}, t) \le -(\sigma_1)^{(2)} \quad (293)$$

$$-(\tau_2)^{(2)} \le -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}((G_{19}), t) - (b''_{17})^{(2)}((G_{19}), t) \le -(\tau_1)^{(2)} \quad (294)$$

**Definition of** $(\nu_1)^{(2)}, (\nu_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$ : (295)

By $(\nu_1)^{(2)} > 0$, $(\nu_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0$, $(u_2)^{(2)} < 0$ the roots (296)

of the equations $(a_{17})^{(2)}(\nu^{(2)})^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ (297)

and $(b_{14})^{(2)}(u^{(2)})^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and (298)

**Definition of** $(\bar{\nu}_1)^{(2)}, , (\bar{\nu}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$ : (299)

By $(\bar{\nu}_1)^{(2)} > 0$, $(\bar{\nu}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0$, $(\bar{u}_2)^{(2)} < 0$ the (300)

roots of the equations $(a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 301

and $(b_{17})^{(2)}\left(u^{(2)}\right)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 302

**Definition of** $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ :- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, ***if*** $(u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$, ***if*** $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)}$, ***if*** $(\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)}t}$$

$(p_i)^{(2)}$ is defined by equation

$$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)}t} \quad 311$$

$$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)}t}\right] + G_{18}^0 e^{-(S_2)^{(2)}t} \le G_{18}(t) \le \right. \quad 312$$

$$\left. \frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)}t} - e^{-(a'_{18})^{(2)}t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)}t}\right)$$

$$\boxed{T_{16}^0 e^{(R_1)^{(2)}t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}} \quad 313$$

$$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)}t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t} \quad 314$$

$$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)}t} - e^{-(b'_{18})^{(2)}t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)}t} \le T_{18}(t) \le \quad 315$$

$$\frac{(a_{18})^{(2)}T_{16}^{0}}{(\mu_2)^{(2)}((R_1)^{(2)}+(r_{16})^{(2)}+(R_2)^{(2)})}\left[e^{((R_1)^{(2)}+(r_{16})^{(2)})t}-e^{-(R_2)^{(2)}t}\right]+T_{18}^{0}e^{-(R_2)^{(2)}t}$$

**Definition of** $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$$

$$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)}$$ 318

$$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$$

**Behavior of the solutions** 319

**Theorem 3:** If we denote and define

**Definition of** $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ :

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}((G_{23}), t) \le -(\tau_1)^{(3)}$$

**Definition of** $(v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$ : 320

By $(v_1)^{(3)} > 0$, $(v_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}(v^{(3)})^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}(u^{(3)})^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{v}_1)^{(3)} > 0$, $(\bar{v}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}(v^{(3)})^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}(u^{(3)})^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

**Definition of** $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ :- 321

If we define $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (v_0)^{(3)}, (m_1)^{(3)} = (v_1)^{(3)}$, ***if*** $(v_0)^{(3)} < (v_1)^{(3)}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (\bar{v}_1)^{(3)}$, ***if*** $(v_1)^{(3)} < (v_0)^{(3)} < (\bar{v}_1)^{(3)}$,

and $\boxed{(v_0)^{(3)} = \frac{G_{20}^{0}}{G_{21}^{0}}}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_0)^{(3)}$, ***if*** $(\bar{v}_1)^{(3)} < (v_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)},\ \boldsymbol{if}\ (u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}, \boldsymbol{if}\ (u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)},$ and $\boxed{(u_0)^{(3)} = \frac{T_{20}^0}{T_{21}^0}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)},\ \boldsymbol{if}\ (\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G_{20}^0 e^{(S_1)^{(3)} t}$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)} t} \quad 323$$

$$\left(\frac{(a_{22})^{(3)} G_{20}^0}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\left[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)} t}\right] + G_{22}^0 e^{-(S_2)^{(3)} t} \le G_{22}(t) \le \right. \quad 324$$

$$\left. \frac{(a_{22})^{(3)} G_{20}^0}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}\left[e^{(S_1)^{(3)} t} - e^{-(a'_{22})^{(3)} t}\right] + G_{22}^0 e^{-(a'_{22})^{(3)} t}\right)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t}} \quad 325$$

$$\frac{1}{(\mu_1)^{(3)}} T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}} T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)} T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)} - (b'_{22})^{(3)})}\left[e^{(R_1)^{(3)} t} - e^{-(b'_{22})^{(3)} t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)} t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)} T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)} + (r_{20})^{(3)} + (R_2)^{(3)})}\left[e^{((R_1)^{(3)} + (r_{20})^{(3)})t} - e^{-(R_2)^{(3)} t}\right] + T_{22}^0 e^{-(R_2)^{(3)} t}$$

**Definition of** $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

**Behavior of the solutions of equation**

**Theorem:** If we denote and define

**Definition of** $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ :

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$

$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}((G_{27}), t) - (b''_{25})^{(4)}((G_{27}), t) \le -(\tau_1)^{(4)}$

**Definition of** $(\nu_1)^{(4)}, (\nu_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, \nu^{(4)}, u^{(4)}$ : 329

By $(\nu_1)^{(4)} > 0$ , $(\nu_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$ , $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_1)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(4)},, (\bar{\nu}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$ : 330

By $(\bar{\nu}_1)^{(4)} > 0$ , $(\bar{\nu}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$ , $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

**Definition of** $(m_1)^{(4)}$ , $(m_2)^{(4)}$ , $(\mu_1)^{(4)}, (\mu_2)^{(4)}, (\nu_0)^{(4)}$ :-

If we define $(m_1)^{(4)}$ , $(m_2)^{(4)}$ , $(\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (\nu_0)^{(4)}, (m_1)^{(4)} = (\nu_1)^{(4)}$, ***if*** $(\nu_0)^{(4)} < (\nu_1)^{(4)}$

$(m_2)^{(4)} = (\nu_1)^{(4)}, (m_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ , ***if*** $(\nu_4)^{(4)} < (\nu_0)^{(4)} < (\bar{\nu}_1)^{(4)}$,
and $\boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (\nu_4)^{(4)}, (m_1)^{(4)} = (\nu_0)^{(4)}$, ***if*** $(\bar{\nu}_4)^{(4)} < (\nu_0)^{(4)}$

and analogously

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$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)}$, ***if*** $(u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}$ , ***if*** $(u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)}$,
and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}$, ***if*** $(\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

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$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)}t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)}t}$$

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$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)}t}\right] + G_{26}^0 e^{-(S_2)^{(4)}t} \le G_{26}(t) \le\right.$$ 334

*(a26)4G240(m2)4(S1)4−(a26′)4e(S1)4t−e−(a26′)4t+ G260e−(a26′)4t*

$$\boxed{T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}}T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}}T_{24}^0 e^{((R_1)^{(4)}+(r_{24})^{(4)})t} \quad 335$$

$$\frac{(b_{26})^{(4)}T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)}-(b'_{26})^{(4)})}\left[e^{(R_1)^{(4)}t} - e^{-(b'_{26})^{(4)}t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)}t} \le T_{26}(t) \le \quad 336$$

$$\frac{(a_{26})^{(4)}T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)}+(r_{24})^{(4)}+(R_2)^{(4)})}\left[e^{((R_1)^{(4)}+(r_{24})^{(4)})t} - e^{-(R_2)^{(4)}t}\right] + T_{26}^0 e^{-(R_2)^{(4)}t}$$

**Definition of** $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

**Behavior of the solutions of equation** 338

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ :

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

**Definition of** $(v_1)^{(5)}, (v_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, v^{(5)}, u^{(5)}$ : 339

By $(v_1)^{(5)} > 0$ , $(v_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$ , $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(v^{(5)}\big)^2 + (\sigma_1)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

**Definition of** $(\bar{v}_1)^{(5)}, , (\bar{v}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$ : 340

By $(\bar{v}_1)^{(5)} > 0$ , $(\bar{v}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$ , $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(v^{(5)}\big)^2 + (\sigma_2)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

**Definition of** $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (v_0)^{(5)}$ :-

If we define $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (v_0)^{(5)}, (m_1)^{(5)} = (v_1)^{(5)}$, ***if*** $(v_0)^{(5)} < (v_1)^{(5)}$

$(m_2)^{(5)} = (v_1)^{(5)}, (m_1)^{(5)} = (\bar{v}_1)^{(5)}$ , ***if*** $(v_1)^{(5)} < (v_0)^{(5)} < (\bar{v}_1)^{(5)}$,
and $\boxed{(v_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)},\ \boldsymbol{if}\ (u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}, \boldsymbol{if}\ (u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)},$

and $\boxed{(u_0)^{(5)} = \frac{T_{28}^0}{T_{29}^0}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}, \boldsymbol{if}\ (\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{28}(t) \le G_{28}^0 e^{(S_1)^{(5)} t}$

where $(p_i)^{(5)}$ is defined by equation

$\frac{1}{(m_5)^{(5)}} G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}} G_{28}^0 e^{(S_1)^{(5)} t}$ 343

$\left(\frac{(a_{30})^{(5)} G_{28}^0}{(m_1)^{(5)}((S_1)^{(5)} - (p_{28})^{(5)} - (S_2)^{(5)})}\left[e^{((S_1)^{(5)} - (p_{28})^{(5)})t} - e^{-(S_2)^{(5)} t}\right] + G_{30}^0 e^{-(S_2)^{(5)} t} \le G_{30}(t) \le\right.$ 344

*(a30)5G280(m2)5(S1)5−(a30′)5e(S1)5t−e−(a30′)5t+ G300e−(a30′)5t*

$\boxed{T_{28}^0 e^{(R_1)^{(5)} t} \le T_{28}(t) \le T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}}$ 345

$\frac{1}{(\mu_1)^{(5)}} T_{28}^0 e^{(R_1)^{(5)} t} \le T_{28}(t) \le \frac{1}{(\mu_2)^{(5)}} T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}$ 346

$\frac{(b_{30})^{(5)} T_{28}^0}{(\mu_1)^{(5)}((R_1)^{(5)} - (b'_{30})^{(5)})}\left[e^{(R_1)^{(5)} t} - e^{-(b'_{30})^{(5)} t}\right] + T_{30}^0 e^{-(b'_{30})^{(5)} t} \le T_{30}(t) \le$ 347

$\frac{(a_{30})^{(5)} T_{28}^0}{(\mu_2)^{(5)}((R_1)^{(5)} + (r_{28})^{(5)} + (R_2)^{(5)})}\left[e^{((R_1)^{(5)} + (r_{28})^{(5)})t} - e^{-(R_2)^{(5)} t}\right] + T_{30}^0 e^{-(R_2)^{(5)} t}$

**Definition of** $(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$:- 348

Where $(S_1)^{(5)} = (a_{28})^{(5)}(m_2)^{(5)} - (a'_{28})^{(5)}$

$(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}$

$(R_1)^{(5)} = (b_{28})^{(5)}(\mu_2)^{(5)} - (b'_{28})^{(5)}$

$(R_2)^{(5)} = (b'_{30})^{(5)} - (r_{30})^{(5)}$

**Behavior of the solutions of equation** 349

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ :

$(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)} \le -(a'_{32})^{(6)} + (a'_{33})^{(6)} - (a''_{32})^{(6)}(T_{33}, t) + (a''_{33})^{(6)}(T_{33}, t) \le -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)} \le -(b'_{32})^{(6)} + (b'_{33})^{(6)} - (b''_{32})^{(6)}\big((G_{35}), t\big) - (b''_{33})^{(6)}\big((G_{35}), t\big) \le -(\tau_1)^{(6)}$$

**Definition of** $(\nu_1)^{(6)}, (\nu_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, \nu^{(6)}, u^{(6)}$ : 350

By $(\nu_1)^{(6)} > 0$, $(\nu_2)^{(6)} < 0$ and respectively $(u_1)^{(6)} > 0$, $(u_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(6)},, (\bar{\nu}_2)^{(6)}, (\bar{u}_1)^{(6)}, (\bar{u}_2)^{(6)}$ : 351

By $(\bar{\nu}_1)^{(6)} > 0$, $(\bar{\nu}_2)^{(6)} < 0$ and respectively $(\bar{u}_1)^{(6)} > 0$, $(\bar{u}_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$

**Definition of** $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}, (\nu_0)^{(6)}$ :-

If we define $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}$ by

$$(m_2)^{(6)} = (\nu_0)^{(6)}, (m_1)^{(6)} = (\nu_1)^{(6)}, \ \boldsymbol{if}\ (\nu_0)^{(6)} < (\nu_1)^{(6)}$$

$$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}, \boldsymbol{if}\ (\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)},$$

and $\boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}}$

$$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$$

and analogously 352

$$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}, \ \boldsymbol{if}\ (u_0)^{(6)} < (u_1)^{(6)}$$

$$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}, \boldsymbol{if}\ (u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)},$$

and $\boxed{(u_0)^{(6)} = \frac{T^0_{32}}{T^0_{33}}}$

$$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}, \boldsymbol{if}\ (\bar{u}_1)^{(6)} < (u_0)^{(6)} \text{ where } (u_1)^{(6)}, (\bar{u}_1)^{(6)}$$

Then the solution of global equations satisfies the inequalities 353

$$G^0_{32}e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{32}(t) \le G^0_{32}e^{(S_1)^{(6)}t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}}G^0_{32}e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{33}(t) \le \frac{1}{(m_2)^{(6)}}G^0_{32}e^{(S_1)^{(6)}t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)}G^0_{32}}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})}\left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)}t}\right] + G^0_{34}e^{-(S_2)^{(6)}t} \le G_{34}(t) \le \right. \quad 355$$

$$\frac{(a_{34})^{(6)}G_{32}^{0}}{(m_2)^{(6)}((S_1)^{(6)}-(a'_{34})^{(6)})}\left[e^{(S_1)^{(6)}t}-e^{-(a'_{34})^{(6)}t}\right]+\ G_{34}^{0}e^{-(a'_{34})^{(6)}t}\Big)$$

$$\boxed{T_{32}^{0}e^{(R_1)^{(6)}t}\le T_{32}(t)\le T_{32}^{0}e^{((R_1)^{(6)}+(r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}}T_{32}^{0}e^{(R_1)^{(6)}t}\le T_{32}(t)\le\frac{1}{(\mu_2)^{(6)}}T_{32}^{0}e^{((R_1)^{(6)}+(r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)}T_{32}^{0}}{(\mu_1)^{(6)}((R_1)^{(6)}-(b'_{34})^{(6)})}\left[e^{(R_1)^{(6)}t}-e^{-(b'_{34})^{(6)}t}\right]+T_{34}^{0}e^{-(b'_{34})^{(6)}t}\le T_{34}(t)\le \quad 358$$

$$\frac{(a_{34})^{(6)}T_{32}^{0}}{(\mu_2)^{(6)}((R_1)^{(6)}+(r_{32})^{(6)}+(R_2)^{(6)})}\left[e^{((R_1)^{(6)}+(r_{32})^{(6)})t}-e^{-(R_2)^{(6)}t}\right]+T_{34}^{0}e^{-(R_2)^{(6)}t}$$

**Definition of** $(S_1)^{(6)},(S_2)^{(6)},(R_1)^{(6)},(R_2)^{(6)}$:- 359

Where $(S_1)^{(6)}=(a_{32})^{(6)}(m_2)^{(6)}-(a'_{32})^{(6)}$

$(S_2)^{(6)}=(a_{34})^{(6)}-(p_{34})^{(6)}$

$(R_1)^{(6)}=(b_{32})^{(6)}(\mu_2)^{(6)}-(b'_{32})^{(6)}$

$(R_2)^{(6)}=(b'_{34})^{(6)}-(r_{34})^{(6)}$

**Behavior of the solutions of equation**

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(7)},(\sigma_2)^{(7)},(\tau_1)^{(7)},(\tau_2)^{(7)}$ :

$(\sigma_1)^{(7)},(\sigma_2)^{(7)},(\tau_1)^{(7)},(\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)}\le-(a'_{36})^{(7)}+(a'_{37})^{(7)}-(a''_{36})^{(7)}(T_{37},t)+(a''_{37})^{(7)}(T_{37},t)\le-(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)}\le-(b'_{36})^{(7)}+(b'_{37})^{(7)}-(b''_{36})^{(7)}\big((G_{39}),t\big)-(b''_{37})^{(7)}\big((G_{39}),t\big)\le-(\tau_1)^{(7)}$$

**Definition of** $(v_1)^{(7)},(v_2)^{(7)},(u_1)^{(7)},(u_2)^{(7)},v^{(7)},u^{(7)}$ : 361

By $(v_1)^{(7)}>0,(v_2)^{(7)}<0$ and respectively $(u_1)^{(7)}>0,(u_2)^{(7)}<0$ the roots of the equations $(a_{37})^{(7)}\big(v^{(7)}\big)^2+(\sigma_1)^{(7)}v^{(7)}-(a_{36})^{(7)}=0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2+(\tau_1)^{(7)}u^{(7)}-(b_{36})^{(7)}=0$ and

**Definition of** $(\bar{v}_1)^{(7)},,(\bar{v}_2)^{(7)},(\bar{u}_1)^{(7)},(\bar{u}_2)^{(7)}$ : 362

By $(\bar{v}_1)^{(7)}>0,(\bar{v}_2)^{(7)}<0$ and respectively $(\bar{u}_1)^{(7)}>0,(\bar{u}_2)^{(7)}<0$ the roots of the equations $(a_{37})^{(7)}\big(v^{(7)}\big)^2+(\sigma_2)^{(7)}v^{(7)}-(a_{36})^{(7)}=0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2+(\tau_2)^{(7)}u^{(7)}-(b_{36})^{(7)}=0$

**Definition of** $(m_1)^{(7)},(m_2)^{(7)},(\mu_1)^{(7)},(\mu_2)^{(7)},(v_0)^{(7)}$ :-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$$(m_2)^{(7)} = (\nu_0)^{(7)}, (m_1)^{(7)} = (\nu_1)^{(7)}, \ \boldsymbol{if}\ (\nu_0)^{(7)} < (\nu_1)^{(7)}$$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\bar{\nu}_1)^{(7)}, \boldsymbol{if}\ (\nu_1)^{(7)} < (\nu_0)^{(7)} < (\bar{\nu}_1)^{(7)},$$

and $\boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\nu_0)^{(7)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(7)} < (\nu_0)^{(7)}$$

and analogously 363

$$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}, \ \boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$$

and $\boxed{(u_0)^{(7)} = \frac{T_{36}^0}{T_{37}^0}}$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)} \text{ where } (u_1)^{(7)}, (\bar{u}_1)^{(7)}$$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^0 e^{(S_1)^{(7)} t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)} t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)}((S_1)^{(7)} - (p_{36})^{(7)} - (S_2)^{(7)})}\left[e^{((S_1)^{(7)} - (p_{36})^{(7)})t} - e^{-(S_2)^{(7)} t}\right] + G_{38}^0 e^{-(S_2)^{(7)} t} \le G_{38}(t) \le \right. \quad 366$$

$$\left. \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)}((S_1)^{(7)} - (a'_{38})^{(7)})}\left[e^{(S_1)^{(7)} t} - e^{-(a'_{38})^{(7)} t}\right] + G_{38}^0 e^{-(a'_{38})^{(7)} t}\right)$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t} \quad 368$$

$$\frac{(b_{38})^{(7)} T_{36}^0}{(\mu_1)^{(7)}((R_1)^{(7)} - (b'_{38})^{(7)})}\left[e^{(R_1)^{(7)} t} - e^{-(b'_{38})^{(7)} t}\right] + T_{38}^0 e^{-(b'_{38})^{(7)} t} \le T_{38}(t) \le \quad 369$$

$$\frac{(a_{38})^{(7)} T_{36}^0}{(\mu_2)^{(7)}((R_1)^{(7)} + (r_{36})^{(7)} + (R_2)^{(7)})}\left[e^{((R_1)^{(7)} + (r_{36})^{(7)})t} - e^{-(R_2)^{(7)} t}\right] + T_{38}^0 e^{-(R_2)^{(7)} t}$$

**Definition of** $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$

$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$

$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$

**Behavior of the solutions of equation** 371

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ :

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$-(\sigma_2)^{(8)} \le -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \le -(\sigma_1)^{(8)}$

$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\big((G_{43}), t\big) - (b''_{41})^{(8)}\big((G_{43}), t\big) \le -(\tau_1)^{(8)}$

**Definition of** $(\nu_1)^{(8)}, (\nu_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, \nu^{(8)}, u^{(8)}$ : 372

By $(\nu_1)^{(8)} > 0, (\nu_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0, (u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(8)}, , (\bar{\nu}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$ :

By $(\bar{\nu}_1)^{(8)} > 0, (\bar{\nu}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0, (\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

**Definition of** $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}, (\nu_0)^{(8)}$ :-

If we define $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$(m_2)^{(8)} = (\nu_0)^{(8)}, (m_1)^{(8)} = (\nu_1)^{(8)}$, ***if*** $(\nu_0)^{(8)} < (\nu_1)^{(8)}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\bar{\nu}_1)^{(8)}$, ***if*** $(\nu_1)^{(8)} < (\nu_0)^{(8)} < (\bar{\nu}_1)^{(8)}$,
and $\boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\nu_0)^{(8)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(8)} < (\nu_0)^{(8)}$

and analogously 374

$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)},\ \boldsymbol{if}\ (u_0)^{(8)} < (u_1)^{(8)}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}, \boldsymbol{if}\ (u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)},$

and $\boxed{(u_0)^{(8)} = \frac{T_{40}^0}{T_{41}^0}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}, \boldsymbol{if}\ (\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^0 e^{(S_1)^{(8)} t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}} G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}} G_{40}^0 e^{(S_1)^{(8)} t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)} G_{40}^0}{(m_1)^{(8)}((S_1)^{(8)} - (p_{40})^{(8)} - (S_2)^{(8)})}\left[e^{((S_1)^{(8)} - (p_{40})^{(8)})t} - e^{-(S_2)^{(8)} t}\right] + G_{42}^0 e^{-(S_2)^{(8)} t} \le G_{42}(t) \le \right. \quad 377$$

$$\left. \frac{(a_{42})^{(8)} G_{40}^0}{(m_2)^{(8)}((S_1)^{(8)} - (a'_{42})^{(8)})}\left[e^{(S_1)^{(8)} t} - e^{-(a'_{42})^{(8)} t}\right] + G_{42}^0 e^{-(a'_{42})^{(8)} t}\right)$$

$$\boxed{T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}} T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le \frac{1}{(\mu_2)^{(8)}} T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)} T_{40}^0}{(\mu_1)^{(8)}((R_1)^{(8)} - (b'_{42})^{(8)})}\left[e^{(R_1)^{(8)} t} - e^{-(b'_{42})^{(8)} t}\right] + T_{42}^0 e^{-(b'_{42})^{(8)} t} \le T_{42}(t) \le \quad 380$$

$$\frac{(a_{42})^{(8)} T_{40}^0}{(\mu_2)^{(8)}((R_1)^{(8)} + (r_{40})^{(8)} + (R_2)^{(8)})}\left[e^{((R_1)^{(8)} + (r_{40})^{(8)})t} - e^{-(R_2)^{(8)} t}\right] + T_{42}^0 e^{-(R_2)^{(8)} t}$$

**Definition of** $(S_1)^{(8)}, (S_2)^{(8)}, (R_1)^{(8)}, (R_2)^{(8)}$:- 381

Where $(S_1)^{(8)} = (a_{40})^{(8)}(m_2)^{(8)} - (a'_{40})^{(8)}$

$(S_2)^{(8)} = (a_{42})^{(8)} - (p_{42})^{(8)}$

$(R_1)^{(8)} = (b_{40})^{(8)}(\mu_2)^{(8)} - (b'_{40})^{(8)}$

$(R_2)^{(8)} = (b'_{42})^{(8)} - (r_{42})^{(8)}$

**Behavior of the solutions of equation 37 to 92** 382

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ :

$(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ four constants satisfying

$$-(\sigma_2)^{(9)} \le -(a'_{44})^{(9)} + (a'_{45})^{(9)} - (a''_{44})^{(9)}(T_{45}, t) + (a''_{45})^{(9)}(T_{45}, t) \le -(\sigma_1)^{(9)}$$

$$-(\tau_2)^{(9)} \le -(b'_{44})^{(9)} + (b'_{45})^{(9)} - (b''_{44})^{(9)}\big((G_{47}), t\big) - (b''_{45})^{(9)}\big((G_{47}), t\big) \le -(\tau_1)^{(9)}$$

**Definition of** $(\nu_1)^{(9)}, (\nu_2)^{(9)}, (u_1)^{(9)}, (u_2)^{(9)}, \nu^{(9)}, u^{(9)}$ :

By $(\nu_1)^{(9)} > 0$ , $(\nu_2)^{(9)} < 0$ and respectively $(u_1)^{(9)} > 0$ , $(u_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_1)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(9)}, , (\bar{\nu}_2)^{(9)}, (\bar{u}_1)^{(9)}, (\bar{u}_2)^{(9)}$ :

By $(\bar{\nu}_1)^{(9)} > 0$ , $(\bar{\nu}_2)^{(9)} < 0$ and respectively $(\bar{u}_1)^{(9)} > 0$ , $(\bar{u}_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_2)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$

**Definition of** $(m_1)^{(9)}, (m_2)^{(9)}, (\mu_1)^{(9)}, (\mu_2)^{(9)}, (\nu_0)^{(9)}$ :-

If we define $(m_1)^{(9)}, (m_2)^{(9)}, (\mu_1)^{(9)}, (\mu_2)^{(9)}$ by

$$(m_2)^{(9)} = (\nu_0)^{(9)}, (m_1)^{(9)} = (\nu_1)^{(9)}, \ \boldsymbol{if}\ (\nu_0)^{(9)} < (\nu_1)^{(9)}$$

$$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\bar{\nu}_1)^{(9)}, \boldsymbol{if}\ (\nu_1)^{(9)} < (\nu_0)^{(9)} < (\bar{\nu}_1)^{(9)},$$

and $\boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$

$$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\nu_0)^{(9)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(9)} < (\nu_0)^{(9)}$$

and analogously

$$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)}, \ \boldsymbol{if}\ (u_0)^{(9)} < (u_1)^{(9)}$$

$$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}, \boldsymbol{if}\ (u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)},$$

and $\boxed{(u_0)^{(9)} = \frac{T^0_{44}}{T^0_{45}}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}, \boldsymbol{if}\ (\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G^0_{44} e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{44}(t) \le G^0_{44} e^{(S_1)^{(9)} t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}}G_{44}^0 e^{((S_1)^{(9)}-(p_{44})^{(9)})t} \le G_{45}(t) \le \frac{1}{(m_2)^{(9)}}G_{44}^0 e^{(S_1)^{(9)}t}$$

$$\left(\frac{(a_{46})^{(9)}G_{44}^0}{(m_1)^{(9)}((S_1)^{(9)}-(p_{44})^{(9)}-(S_2)^{(9)})}\left[e^{((S_1)^{(9)}-(p_{44})^{(9)})t} - e^{-(S_2)^{(9)}t}\right] + G_{46}^0 e^{-(S_2)^{(9)}t} \le G_{46}(t) \le \right.$$

$$\left.\frac{(a_{46})^{(9)}G_{44}^0}{(m_2)^{(9)}((S_1)^{(9)}-(a'_{46})^{(9)})}[e^{(S_1)^{(9)}t} - e^{-(a'_{46})^{(9)}t}] + G_{46}^0 e^{-(a'_{46})^{(9)}t}\right)$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le T_{44}^0 e^{((R_1)^{(9)}+(r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}}T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le \frac{1}{(\mu_2)^{(9)}}T_{44}^0 e^{((R_1)^{(9)}+(r_{44})^{(9)})t}$$

$$\frac{(b_{46})^{(9)}T_{44}^0}{(\mu_1)^{(9)}((R_1)^{(9)}-(b'_{46})^{(9)})}\left[e^{(R_1)^{(9)}t} - e^{-(b'_{46})^{(9)}t}\right] + T_{46}^0 e^{-(b'_{46})^{(9)}t} \le T_{46}(t) \le$$

$$\frac{(a_{46})^{(9)}T_{44}^0}{(\mu_2)^{(9)}((R_1)^{(9)}+(r_{44})^{(9)}+(R_2)^{(9)})}\left[e^{((R_1)^{(9)}+(r_{44})^{(9)})t} - e^{-(R_2)^{(9)}t}\right] + T_{46}^0 e^{-(R_2)^{(9)}t}$$

**Definition of** $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$

**Proof :** From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right) - (a''_{14})^{(1)}(T_{14}, t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

**Definition of** $v^{(1)}$ :- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)}\right) \le \frac{dv^{(1)}}{dt} \le -\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)}\right)$$

From which one obtains

**Definition of** $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$ :-

For $0 < \boxed{(v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}} < (v_1)^{(1)} < (\bar{v}_1)^{(1)}$

$$v^{(1)}(t) \ge \frac{(v_1)^{(1)}+(C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(v_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(v_2)^{(1)}}}$$

it follows $(\nu_0)^{(1)} \leq \nu^{(1)}(t) \leq (\nu_1)^{(1)}$

In the same manner , we get 384

$$\nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{\nu}_1)^{(1)}-(\nu_0)^{(1)}}{(\nu_0)^{(1)}-(\bar{\nu}_2)^{(1)}}}$$

From which we deduce $(\nu_0)^{(1)} \leq \nu^{(1)}(t) \leq (\bar{\nu}_1)^{(1)}$

If $0 < (\nu_1)^{(1)} < (\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0} < (\bar{\nu}_1)^{(1)}$ we find like in the previous case, 385

$$(\nu_1)^{(1)} \leq \frac{(\nu_1)^{(1)}+(C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}} \leq \nu^{(1)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\bar{\nu}_1)^{(1)}$$

If $0 < (\nu_1)^{(1)} \leq (\bar{\nu}_1)^{(1)} \leq \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$ , we obtain 386

$$(\nu_1)^{(1)} \leq \nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\nu_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(1)}(t)$ :-

$$(m_2)^{(1)} \leq \nu^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{\nu^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(1)}(t)$ :-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{13})^{(1)} = (a''_{14})^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)}G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)}T_{14}(t)$ This is an important

consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

**Proof** : From global equations we obtain 387

$$\frac{d\nu^{(2)}}{dt} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right) - (a''_{17})^{(2)}(T_{17}, t)\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

**Definition of** $\nu^{(2)}$ :- $\boxed{\nu^{(2)} = \frac{G_{16}}{G_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \le \frac{d\nu^{(2)}}{dt} \le -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

**Definition of** $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$ :-

For $0 < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \ge \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}}{1 + (C)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(C)^{(2)} = \frac{(\nu_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \le \nu^{(2)}(t) \le (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \le \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \le \nu^{(2)}(t) \le (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \le \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}}{1 + (C)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}} \le \nu^{(2)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \le (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \le (\bar{\nu}_1)^{(2)} \le (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$ , we obtain 394

$$(\nu_1)^{(2)} \le \nu^{(2)}(t) \le \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \le (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(2)}(t)$ :- 395

$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}$, $\boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$

In a completely analogous way, we obtain 396

**Definition of** $u^{(2)}(t)$ :-

$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}$, $\boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :** 397

If $(a_{16}'')^{(2)} = (a_{17}'')^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)} G_{17}(t)$

Analogously if $(b_{16}'')^{(2)} = (b_{17}'')^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)} T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

**Proof** : From global equations we obtain 398

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a_{20}')^{(3)} - (a_{21}')^{(3)} + (a_{20}'')^{(3)}(T_{21}, t)\right) - (a_{21}'')^{(3)}(T_{21}, t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

**Definition of** $\nu^{(3)}$ :- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right) \leq \frac{d\nu^{(3)}}{dt} \leq -\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right)$$

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From which one obtains

For $0 < (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (\nu_1)^{(3)} < (\bar{\nu}_1)^{(3)}$

$$\nu^{(3)}(t) \geq \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(\nu_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\nu_2)^{(3)}}}$$

it follows $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\nu_1)^{(3)}$

In the same manner , we get 401

$$\nu^{(3)}(t) \le \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{\nu}_1)^{(3)}-(\nu_0)^{(3)}}{(\nu_0)^{(3)}-(\bar{\nu}_2)^{(3)}}}$$

**Definition of** $(\bar{\nu}_1)^{(3)}$ :-

From which we deduce $(\nu_0)^{(3)} \le \nu^{(3)}(t) \le (\bar{\nu}_1)^{(3)}$

If $0 < (\nu_1)^{(3)} < (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (\bar{\nu}_1)^{(3)}$ we find like in the previous case, 402

$$(\nu_1)^{(3)} \le \frac{(\nu_1)^{(3)}+(C)^{(3)}(\nu_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}} \le \nu^{(3)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \le (\bar{\nu}_1)^{(3)}$$

If $0 < (\nu_1)^{(3)} \le (\bar{\nu}_1)^{(3)} \le (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}$ , we obtain 403

$$(\nu_1)^{(3)} \le \nu^{(3)}(t) \le \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \le (\nu_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(3)}(t)$ :-

$$(m_2)^{(3)} \le \nu^{(3)}(t) \le (m_1)^{(3)}, \quad \boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(3)}(t)$ :-

$$(\mu_2)^{(3)} \le u^{(3)}(t) \le (\mu_1)^{(3)}, \quad \boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{20}'')^{(3)} = (a_{21}'')^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)}G_{21}(t)$

Analogously if $(b_{20}'')^{(3)} = (b_{21}'')^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)}T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

**Proof** : From global equations we obtain 404

$$\frac{dv^{(4)}}{dt} = (a_{24})^{(4)} - \left((a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right) - (a''_{25})^{(4)}(T_{25},t)v^{(4)} - (a_{25})^{(4)}v^{(4)}$$

**Definition of** $v^{(4)}$ :- $\boxed{v^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)}\right) \le \frac{dv^{(4)}}{dt} \le -\left((a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_4)^{(4)}v^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$ :-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \ge \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \le \nu^{(4)}(t) \le (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \le \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \le \nu^{(4)}(t) \le (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \le \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}}{1 + (C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}} \le \nu^{(4)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \le (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \le (\bar{\nu}_1)^{(4)} \le \boxed{(\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}}}$ , we obtain 407

$$(\nu_1)^{(4)} \le \nu^{(4)}(t) \le \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \le (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(4)}(t)$ :-

$$(m_2)^{(4)} \le \nu^{(4)}(t) \le (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(4)}(t)$ :-

$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}$, $\boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{24})^{(4)} = (a''_{25})^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)} G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b''_{24})^{(4)} = (b''_{25})^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)} T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

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**Proof** : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a'_{28})^{(5)} - (a'_{29})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right) - (a''_{29})^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

**Definition of** $\nu^{(5)}$ :- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$ :-

For $0 < \boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \le \frac{(\nu_1)^{(5)}+(C)^{(5)}(\nu_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)}-(\nu_2)^{(5)}\right)t\right]}}{1+(C)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)}-(\nu_2)^{(5)}\right)t\right]}} \le \nu^{(5)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \le (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \le (\bar{\nu}_1)^{(5)} \le \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain 411

$$(\nu_1)^{(5)} \le \nu^{(5)}(t) \le \frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \le (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(5)}(t)$ :-

$(m_2)^{(5)} \le \nu^{(5)}(t) \le (m_1)^{(5)}$, $\boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$

In a completely analogous way, we obtain

**Definition of** $u^{(5)}(t)$ :-

$(\mu_2)^{(5)} \le u^{(5)}(t) \le (\mu_1)^{(5)}$, $\boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a_{28}'')^{(5)} = (a_{29}'')^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)}G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b_{28}'')^{(5)} = (b_{29}'')^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)}T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

**Proof** : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a_{32}')^{(6)} - (a_{33}')^{(6)} + (a_{32}'')^{(6)}(T_{33},t)\right) - (a_{33}'')^{(6)}(T_{33},t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

**Definition of** $\nu^{(6)}$ :- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \le \frac{d\nu^{(6)}}{dt} \le -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$ :-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \geq \frac{(\nu_1)^{(6)}+(C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_0)^{(6)}\right)t\right]}}{1+(C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)}-(\nu_0)^{(6)}}{(\nu_0)^{(6)}-(\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\nu_1)^{(6)}$

In the same manner , we get 413

$$\nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{\nu}_1)^{(6)}-(\nu_0)^{(6)}}{(\nu_0)^{(6)}-(\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\bar{\nu}_1)^{(6)}$

If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 414

$$(\nu_1)^{(6)} \leq \frac{(\nu_1)^{(6)}+(C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_2)^{(6)}\right)t\right]}}{1+(C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_2)^{(6)}\right)t\right]}} \leq \nu^{(6)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\bar{\nu}_1)^{(6)}$$

If $0 < (\nu_1)^{(6)} \leq (\bar{\nu}_1)^{(6)} \leq \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$ , we obtain 415

$$(\nu_1)^{(6)} \leq \nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(6)}(t)$ :-

$$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}, \quad \boxed{\nu^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(6)}(t)$ :-

$$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}, \quad \boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)}G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)}T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

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**Proof** : From global equations we obtain

$$\frac{dv^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right) - (a''_{37})^{(7)}(T_{37},t)v^{(7)} - (a_{37})^{(7)}v^{(7)}$$

**Definition of** $v^{(7)}$ :- $\boxed{v^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_2)^{(7)}v^{(7)} - (a_{36})^{(7)}\right) \le \frac{dv^{(7)}}{dt} \le -\left((a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_1)^{(7)}v^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

**Definition of** $(\bar{v}_1)^{(7)}, (v_0)^{(7)}$ :-

For $0 < \boxed{(v_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}} < (v_1)^{(7)} < (\bar{v}_1)^{(7)}$

$$v^{(7)}(t) \ge \frac{(v_1)^{(7)} + (C)^{(7)}(v_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_0)^{(7)}\right)t\right]}}{1 + (C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(v_1)^{(7)} - (v_0)^{(7)}}{(v_0)^{(7)} - (v_2)^{(7)}}}$$

it follows $(v_0)^{(7)} \le v^{(7)}(t) \le (v_1)^{(7)}$

In the same manner , we get 417

$$v^{(7)}(t) \le \frac{(\bar{v}_1)^{(7)} + (\bar{C})^{(7)}(\bar{v}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{v}_1)^{(7)} - (v_0)^{(7)}}{(v_0)^{(7)} - (\bar{v}_2)^{(7)}}}$$

From which we deduce $(v_0)^{(7)} \le v^{(7)}(t) \le (\bar{v}_1)^{(7)}$

If $0 < (v_1)^{(7)} < (v_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}} < (\bar{v}_1)^{(7)}$ we find like in the previous case, 418

$$(v_1)^{(7)} \le \frac{(v_1)^{(7)} + (C)^{(7)}(v_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_2)^{(7)}\right)t\right]}}{1 + (C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_2)^{(7)}\right)t\right]}} \le v^{(7)}(t) \le$$

$$\frac{(\bar{v}_1)^{(7)} + (\bar{C})^{(7)}(\bar{v}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}} \le (\bar{v}_1)^{(7)}$$

If $0 < (v_1)^{(7)} \le (\bar{v}_1)^{(7)} \le \boxed{(v_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$ , we obtain 419

$$(v_1)^{(7)} \le v^{(7)}(t) \le \frac{(\bar{v}_1)^{(7)} + (\bar{C})^{(7)}(\bar{v}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}} \le (v_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $v^{(7)}(t)$ :-

$$(m_2)^{(7)} \leq v^{(7)}(t) \leq (m_1)^{(7)}, \quad \boxed{v^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(7)}(t)$ :- 420

$$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}, \quad \boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $v^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)} G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)} T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

**Proof** : From global equations we obtain 421

$$\frac{dv^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right) - (a''_{41})^{(8)}(T_{41}, t) v^{(8)} - (a_{41})^{(8)} v^{(8)}$$

**Definition of** $v^{(8)}$ :- $\boxed{v^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_2)^{(8)} v^{(8)} - (a_{40})^{(8)}\right) \leq \frac{dv^{(8)}}{dt} \leq -\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_1)^{(8)} v^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$ :-

$$\text{For } 0 < \boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$$

$$v^{(8)}(t) \geq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)}-(\nu_0)^{(8)}}{(\nu_0)^{(8)}-(\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \leq \frac{(\nu_1)^{(8)}+(C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}}{1+(C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}} \leq \nu^{(8)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \leq (\bar{\nu}_1)^{(8)} \leq \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$ , we obtain 424

$$(\nu_1)^{(8)} \leq \nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(8)}(t)$ :-

$$(m_2)^{(8)} \leq \nu^{(8)}(t) \leq (m_1)^{(8)}, \quad \boxed{\nu^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(8)}(t)$ :-

$$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}, \quad \boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $\nu^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)}G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then $(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)}T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

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**Proof** : From 99,20,44,22,23,44 we obtain A

$$\frac{dv^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right) - (a''_{45})^{(9)}(T_{45},t)v^{(9)} - (a_{45})^{(9)}v^{(9)}$$

**Definition of** $v^{(9)}$ :- $\boxed{v^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)}\right) \le \frac{dv^{(9)}}{dt} \le -\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_1)^{(9)}v^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$ :-

For $0 < \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$

$$v^{(9)}(t) \ge \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \le v^{(9)}(t) \le (\nu_9)^{(9)}$

In the same manner , we get

$$v^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \le v^{(9)}(t) \le (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \le \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}}{1 + (C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}} \le v^{(9)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \le (\bar{\nu}_1)^{(9)} \le \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$ , we obtain

$$(\nu_1)^{(9)} \le v^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $v^{(9)}(t)$ :-

$$(m_2)^{(9)} \le v^{(9)}(t) \le (m_1)^{(9)}, \quad \boxed{v^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(9)}(t)$ :-

$(\mu_2)^{(9)} \leq u^{(9)}(t) \leq (\mu_1)^{(9)}$, $\boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)} G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then
$(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)} T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following 425

**Theorem :** If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on $t$ , and the conditions with the notations

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0$ ,

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$

$with\ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a''_i)^{(2)} and\ (b''_i)^{(2)}$ are independent on t , and the conditions with the notations 426

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$ 427

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 428

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$ , 429

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 430

$with\ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a''_i)^{(3)} and\ (b''_i)^{(3)}$ are independent on $t$ , and the conditions with the notations 431

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$ ,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

**Theorem :** If $(a_i'')^{(4)} and\ (b_i'')^{(4)}$ are independent on $t$ , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$ ,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(5)} and\ (b_i'')^{(5)}$ are independent on $t$ , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$ ,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

**Theorem** If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on $t$ , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$ ,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(7)} and\ (b_i'')^{(7)}$ are independent on $t$ , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$ ,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(8)} and\ (b_i'')^{(8)}$ are independent on $t$ , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$ ,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(9)} and\ (b_i'')^{(9)}$ are independent on $t$ , and the conditions (with the notations 436 A
45,46,27,28)

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$ ,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0$ 437

$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0$ 438

$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0$ 439

$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0$ 440

$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ]T_{14} = 0$ 441

$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0$ 442

has a unique positive solution , which is an equilibrium solution for the system

$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0$ 443

$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0$ 444

$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0$ 445

$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\ ]T_{16} = 0$ 446

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\,]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\,]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\,]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\,]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\,]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\big((G_{27})\big)\,]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\big((G_{27})\big)\,]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\big((G_{27})\big)\,]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})\,]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})\,]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})\,]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})\,]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})\,]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})\,]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0 \quad 479$$

$$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0 \quad 480$$

$$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0 \quad 481$$

$$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0 \quad 482$$

$$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0 \quad 483$$

$$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0 \quad 484$$

$$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0 \quad 484\text{A}$$

$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$

$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$

$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$

$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$

$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$

**Proof:** 485

(a) Indeed the first two equations have a nontrivial solution $G_{13}, G_{14}$ if

$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$

**Proof:** 486

(z) Indeed the first two equations have a nontrivial solution $G_{16}, G_{17}$ if

$\mathrm{F}(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$

**Proof:** 487

(a) Indeed the first two equations have a nontrivial solution $G_{20}, G_{21}$ if

$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$

**Proof:** 488

(a) Indeed the first two equations have a nontrivial solution $G_{24}, G_{25}$ if

$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$

**Proof:** 489

(a) Indeed the first two equations have a nontrivial solution $G_{28}, G_{29}$ if

$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$

**Proof:** 490

(a) Indeed the first two equations have a nontrivial solution $G_{32}, G_{33}$ if

$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) +$

$(a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$

**Proof:** 491

(a) Indeed the first two equations have a nontrivial solution $G_{36}, G_{37}$ if

$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$

**Proof:** 492

(a) Indeed the first two equations have a nontrivial solution $G_{40}, G_{41}$ if

$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$

**Proof:** 492 A

(a) Indeed the first two equations have a nontrivial solution $G_{44}, G_{45}$ if

$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$

**Definition and uniqueness of** $T^*_{14}$ :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique $T^*_{14}$ for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

**Definition and uniqueness of** $T^*_{17}$ :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique $T^*_{17}$ for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]}$$ 495

**Definition and uniqueness of** $T^*_{21}$ :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique $T^*_{21}$ for which $f(T^*_{21}) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

**Definition and uniqueness of** $T^*_{25}$ :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that

there exists a unique $T_{25}^{*}$ for which $f(T_{25}^{*})=0$. With this value , we obtain from the three first equations

$$G_{24}=\frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}=\frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T_{25}^{*})]}$$

**Definition and uniqueness of** $T_{29}^{*}$ :- 498

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(5)}(T_{29})$ being increasing, it follows that there exists a unique $T_{29}^{*}$ for which $f(T_{29}^{*})=0$. With this value , we obtain from the three first equations

$$G_{28}=\frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}=\frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T_{29}^{*})]}$$

**Definition and uniqueness of** $T_{33}^{*}$ :- 499

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(6)}(T_{33})$ being increasing, it follows that there exists a unique $T_{33}^{*}$ for which $f(T_{33}^{*})=0$. With this value , we obtain from the three first equations

$$G_{32}=\frac{(a_{32})^{(6)}G_{33}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^{*})]} \quad , \quad G_{34}=\frac{(a_{34})^{(6)}G_{33}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^{*})]}$$

**Definition and uniqueness of** $T_{37}^{*}$ :- 500

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(7)}(T_{37})$ being increasing, it follows that there exists a unique $T_{37}^{*}$ for which $f(T_{37}^{*})=0$. With this value , we obtain from the three first equations

$$G_{36}=\frac{(a_{36})^{(7)}G_{37}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^{*})]} \quad , \quad G_{38}=\frac{(a_{38})^{(7)}G_{37}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^{*})]}$$

**Definition and uniqueness of** $T_{41}^{*}$ :- 501

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique $T_{41}^{*}$ for which $f(T_{41}^{*})=0$. With this value , we obtain from the three first equations

$$G_{40}=\frac{(a_{40})^{(8)}G_{41}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T_{41}^{*})]} \quad , \quad G_{42}=\frac{(a_{42})^{(8)}G_{41}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T_{41}^{*})]}$$

**Definition and uniqueness of** $T_{45}^{*}$ :- 501 A

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(9)}(T_{45})$ being increasing, it follows that there exists a unique $T_{45}^{*}$ for which $f(T_{45}^{*})=0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)}G_{45}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T^*_{45})]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)}G_{45}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T^*_{45})]}$$

By the same argument, the equations admit solutions $G_{13}, G_{14}$ if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right]+(b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{14}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{14}$ such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions $G_{16}, G_{17}$ if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right]+(b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ 504
is a decreasing function in $G_{17}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{14}$ such that $\varphi((G_{19})^*) = 0$

By the same argument, the equations admit solutions $G_{20}, G_{21}$ if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right]+(b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{21}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{21}$ such that $\varphi((G_{23})^*) = 0$

By the same argument, the equations admit solutions $G_{24}, G_{25}$ if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right]+(b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{25}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{25}$ such that $\varphi((G_{27})^*) = 0$

By the same argument, the equations admit solutions $G_{28}, G_{29}$ if 507

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right]+(b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{29}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that

there exists a unique $G_{29}^{*}$ such that $\varphi((G_{31})^{*}) = 0$

By the same argument, the equations admit solutions $G_{32}, G_{33}$ if 508
$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$

$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right] + (b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{33}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{33}^{*}$ such that $\varphi(G_{35}{}^{*}) = 0$

By the same argument, the equations admit solutions $G_{36}, G_{37}$ if 509
$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$
$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right] + (b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{37}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{37}^{*}$ such that $\varphi(G_{39}{}^{*}) = 0$

By the same argument, the equations admit solutions $G_{40}, G_{41}$ if 510
$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$
$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right] + (b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{41}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{41}^{*}$ such that $\varphi(G_{43}{}^{*}) = 0$

By the same argument, the equations 92,93 admit solutions $G_{44}, G_{45}$ if
$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$

$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right] + (b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{45}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{45}^{*}$ such that $\varphi((G_{47})^{*}) = 0$

Finally we obtain the unique solution 511

$G_{14}^{*}$ given by $\varphi(G^{*}) = 0$ , $T_{14}^{*}$ given by $f(T_{14}^{*}) = 0$ and

$$G_{13}^{*} = \frac{(a_{13})^{(1)}G_{14}^{*}}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^{*})]} \quad , \quad G_{15}^{*} = \frac{(a_{15})^{(1)}G_{14}^{*}}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^{*})]}$$

$$T_{13}^{*} = \frac{(b_{13})^{(1)}T_{14}^{*}}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^{*})]} \quad , \quad T_{15}^{*} = \frac{(b_{15})^{(1)}T_{14}^{*}}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^{*})]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

$G_{17}^{*}$ given by $\varphi((G_{19})^{*}) = 0$ , $T_{17}^{*}$ given by $f(T_{17}^{*}) = 0$ and 512

$$G_{16}^{*} = \frac{(a_{16})^{(2)} G_{17}^{*}}{[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}^{*})]} \quad , \quad G_{18}^{*} = \frac{(a_{18})^{(2)} G_{17}^{*}}{[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}^{*})]} \tag{513}$$

$$T_{16}^{*} = \frac{(b_{16})^{(2)} T_{17}^{*}}{[(b'_{16})^{(2)} - (b''_{16})^{(2)}((G_{19})^{*})]} \quad , \quad T_{18}^{*} = \frac{(b_{18})^{(2)} T_{17}^{*}}{[(b'_{18})^{(2)} - (b''_{18})^{(2)}((G_{19})^{*})]} \tag{514}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

$G_{21}^{*}$ given by $\varphi((G_{23})^{*}) = 0$ , $T_{21}^{*}$ given by $f(T_{21}^{*}) = 0$ and

$$G_{20}^{*} = \frac{(a_{20})^{(3)} G_{21}^{*}}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^{*})]} \quad , \quad G_{22}^{*} = \frac{(a_{22})^{(3)} G_{21}^{*}}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^{*})]}$$

$$T_{20}^{*} = \frac{(b_{20})^{(3)} T_{21}^{*}}{[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}{}^{*})]} \quad , \quad T_{22}^{*} = \frac{(b_{22})^{(3)} T_{21}^{*}}{[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}{}^{*})]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

$G_{25}^{*}$ given by $\varphi(G_{27}) = 0$ , $T_{25}^{*}$ given by $f(T_{25}^{*}) = 0$ and

$$G_{24}^{*} = \frac{(a_{24})^{(4)} G_{25}^{*}}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}^{*} = \frac{(a_{26})^{(4)} G_{25}^{*}}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^{*})]}$$

$$T_{24}^{*} = \frac{(b_{24})^{(4)} T_{25}^{*}}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^{*})]} \quad , \quad T_{26}^{*} = \frac{(b_{26})^{(4)} T_{25}^{*}}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^{*})]} \tag{517}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

$G_{29}^{*}$ given by $\varphi((G_{31})^{*}) = 0$ , $T_{29}^{*}$ given by $f(T_{29}^{*}) = 0$ and

$$G_{28}^{*} = \frac{(a_{28})^{(5)} G_{29}^{*}}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}^{*} = \frac{(a_{30})^{(5)} G_{29}^{*}}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^{*})]}$$

$$T_{28}^{*} = \frac{(b_{28})^{(5)} T_{29}^{*}}{[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31})^{*})]} \quad , \quad T_{30}^{*} = \frac{(b_{30})^{(5)} T_{29}^{*}}{[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31})^{*})]} \tag{519}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

$G_{33}^{*}$ given by $\varphi((G_{35})^{*}) = 0$ , $T_{33}^{*}$ given by $f(T_{33}^{*}) = 0$ and

$$G_{32}^{*} = \frac{(a_{32})^{(6)} G_{33}^{*}}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^{*})]} \quad , \quad G_{34}^{*} = \frac{(a_{34})^{(6)} G_{33}^{*}}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^{*})]}$$

$$T_{32}^{*} = \frac{(b_{32})^{(6)} T_{33}^{*}}{[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35})^{*})]} \quad , \quad T_{34}^{*} = \frac{(b_{34})^{(6)} T_{33}^{*}}{[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35})^{*})]} \tag{521}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

$G_{37}^*$ given by $\varphi((G_{39})^*) = 0$ , $T_{37}^*$ given by $f(T_{37}^*) = 0$ and

$$G_{36}^* = \frac{(a_{36})^{(7)} G_{37}^*}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^*)]} \quad , \quad G_{38}^* = \frac{(a_{38})^{(7)} G_{37}^*}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^*)]}$$

$$T_{36}^* = \frac{(b_{36})^{(7)} T_{37}^*}{[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39})^*)]} \quad , \quad T_{38}^* = \frac{(b_{38})^{(7)} T_{37}^*}{[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39})^*)]}$$

Finally we obtain the unique solution 523

$G_{41}^*$ given by $\varphi((G_{43})^*) = 0$ , $T_{41}^*$ given by $f(T_{41}^*) = 0$ and

$$G_{40}^* = \frac{(a_{40})^{(8)} G_{41}^*}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42}^* = \frac{(a_{42})^{(8)} G_{41}^*}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

$$T_{40}^* = \frac{(b_{40})^{(8)} T_{41}^*}{[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43})^*)]} \quad , \quad T_{42}^* = \frac{(b_{42})^{(8)} T_{41}^*}{[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43})^*)]}$$

Finally we obtain the unique solution of 89 to 99 523 A

$G_{45}^*$ given by $\varphi((G_{47})^*) = 0$ , $T_{45}^*$ given by $f(T_{45}^*) = 0$ and

$$G_{44}^* = \frac{(a_{44})^{(9)} G_{45}^*}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46}^* = \frac{(a_{46})^{(9)} G_{45}^*}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

$$T_{44}^* = \frac{(b_{44})^{(9)} T_{45}^*}{[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47})^*)]} \quad , \quad T_{46}^* = \frac{(b_{46})^{(9)} T_{45}^*}{[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47})^*)]}$$

## ASYMPTOTIC STABILITY ANALYSIS 524

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

**Proof:** Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a''_{14})^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \frac{\partial (b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a'_{13})^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)}G^*_{13}\mathbb{T}_{14} \quad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a'_{14})^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)}G^*_{14}\mathbb{T}_{14} \quad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a'_{15})^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)}G^*_{15}\mathbb{T}_{14} \quad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b'_{13})^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)}T^*_{13}\mathbb{G}_j\right) \quad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b'_{14})^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)}T^*_{14}\mathbb{G}_j\right) \quad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b'_{15})^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T^*_{15}\mathbb{G}_j\right) \quad 530$$

**ASYMPTOTIC STABILITY ANALYSIS** 531

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(2)}$ and $(b''_i)^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G^*_i + \mathbb{G}_i \quad , T_i = T^*_i + \mathbb{T}_i \quad 532$$

$$\frac{\partial(a''_{17})^{(2)}}{\partial T_{17}}(T^*_{17}) = (q_{17})^{(2)} \quad , \frac{\partial(b''_i)^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \quad 533$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a'_{16})^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G^*_{16}\mathbb{T}_{17} \quad 534$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a'_{17})^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G^*_{17}\mathbb{T}_{17} \quad 535$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a'_{18})^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G^*_{18}\mathbb{T}_{17} \quad 536$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b'_{16})^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T^*_{16}\mathbb{G}_j\right) \quad 537$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b'_{17})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}T^*_{17}\mathbb{G}_j\right) \quad 538$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b'_{18})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}T^*_{18}\mathbb{G}_j\right) \quad 539$$

**ASYMPTOTIC STABILITY ANALYSIS** 540

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(3)}$ *and* $(b''_i)^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{21}'')^{(3)}}{\partial T_{21}}(T_{21}^*) = (q_{21})^{(3)} \quad , \frac{\partial (b_i'')^{(3)}}{\partial G_j}(\,(G_{23})^*\,) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\big((a_{20}')^{(3)} + (p_{20})^{(3)}\big)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G_{20}^*\mathbb{T}_{21} \quad 541$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\big((a_{21}')^{(3)} + (p_{21})^{(3)}\big)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G_{21}^*\mathbb{T}_{21} \quad 542$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\big((a_{22}')^{(3)} + (p_{22})^{(3)}\big)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G_{22}^*\mathbb{T}_{21} \quad 543$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\big((b_{20}')^{(3)} - (r_{20})^{(3)}\big)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\big(s_{(20)(j)}T_{20}^*\mathbb{G}_j\big) \quad 544$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\big((b_{21}')^{(3)} - (r_{21})^{(3)}\big)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\big(s_{(21)(j)}T_{21}^*\mathbb{G}_j\big) \quad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\big((b_{22}')^{(3)} - (r_{22})^{(3)}\big)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\big(s_{(22)(j)}T_{22}^*\mathbb{G}_j\big) \quad 546$$

**ASYMPTOTIC STABILITY ANALYSIS** 547

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}\ and\ (b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{25}'')^{(4)}}{\partial T_{25}}(T_{25}^*) = (q_{25})^{(4)} \quad , \frac{\partial (b_i'')^{(4)}}{\partial G_j}((G_{27})^*\,) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\big((a_{24}')^{(4)} + (p_{24})^{(4)}\big)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G_{24}^*\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\big((a_{25}')^{(4)} + (p_{25})^{(4)}\big)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G_{25}^*\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\big((a_{26}')^{(4)} + (p_{26})^{(4)}\big)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G_{26}^*\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\big((b_{24}')^{(4)} - (r_{24})^{(4)}\big)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\big(s_{(24)(j)}T_{24}^*\mathbb{G}_j\big) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\big((b_{25}')^{(4)} - (r_{25})^{(4)}\big)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\big(s_{(25)(j)}T_{25}^*\mathbb{G}_j\big) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T^*_{26}\mathbb{G}_j\right) \quad 554$$

**ASYMPTOTIC STABILITY ANALYSIS** 555

**Theorem 5:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}\ and\ (b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 556

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{29})^{(5)}}{\partial T_{29}}(T^*_{29}) = (q_{29})^{(5)} \ , \ \frac{\partial(b_i'')^{(5)}}{\partial G_j}\left((G_{31})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a'_{28})^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G^*_{28}\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a'_{29})^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G^*_{29}\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a'_{30})^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G^*_{30}\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b'_{28})^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T^*_{28}\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b'_{29})^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T^*_{29}\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b'_{30})^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T^*_{30}\mathbb{G}_j\right) \quad 562$$

**ASYMPTOTIC STABILITY ANALYSIS** 563

**Theorem 6:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}\ and\ (b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{33})^{(6)}}{\partial T_{33}}(T^*_{33}) = (q_{33})^{(6)} \ , \ \frac{\partial(b_i'')^{(6)}}{\partial G_j}\left((G_{35})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a'_{32})^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G^*_{32}\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a'_{33})^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G^*_{33}\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a'_{34})^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G^*_{34}\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b'_{32})^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T^*_{32}\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b'_{33})^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T^*_{33}\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b'_{34})^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T^*_{34}\mathbb{G}_j\right) \quad 570$$

**ASYMPTOTIC STABILITY ANALYSIS** 571

**Theorem 7:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 572

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{37})^{(7)}}{\partial T_{37}}(T^*_{37}) = (q_{37})^{(7)} \quad , \frac{\partial(b_i'')^{(7)}}{\partial G_j}\left((G_{39})^{**}\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a'_{36})^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G^*_{36}\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a'_{37})^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G^*_{37}\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a'_{38})^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G^*_{38}\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b'_{36})^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T^*_{36}\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b'_{37})^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T^*_{37}\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b'_{38})^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T^*_{38}\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

**ASYMPTOTIC STABILITY ANALYSIS**

**Theorem 8:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{41}'')^{(8)}}{\partial T_{41}}(T_{41}^*) = (q_{41})^{(8)} \quad , \frac{\partial (b_i'')^{(8)}}{\partial G_j}\left( (G_{43})^* \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a_{40}')^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G_{40}^*\mathbb{T}_{41} \quad (581)$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a_{41}')^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G_{41}^*\mathbb{T}_{41} \quad (582)$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a_{42}')^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G_{42}^*\mathbb{T}_{41} \quad (583)$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b_{40}')^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T_{40}^*\mathbb{G}_j\right) \quad (584)$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b_{41}')^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T_{41}^*\mathbb{G}_j\right) \quad (585)$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b_{42}')^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T_{42}^*\mathbb{G}_j\right) \quad (586)$$

(586 A)

## ASYMPTOTIC STABILITY ANALYSIS

**Theorem 9:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}$ *and* $(b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

**Proof:** Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{45}'')^{(9)}}{\partial T_{45}}(T_{45}^*) = (q_{45})^{(9)} \quad , \frac{\partial (b_i'')^{(9)}}{\partial G_j}\left( (G_{47})^* \right) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a_{44}')^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)}G_{44}^*\mathbb{T}_{45} \quad (586\,B)$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a_{45}')^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)}G_{45}^*\mathbb{T}_{45} \quad (586\,C)$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a_{46}')^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)}G_{46}^*\mathbb{T}_{45} \quad (586\,D)$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b_{44}')^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)}T_{44}^*\mathbb{G}_j\right) \quad (586\,E)$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b_{45}')^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)}T_{45}^*\mathbb{G}_j\right) \quad (586\,F)$$

$\frac{d\mathbb{T}_{46}}{dt} = -\left((b'_{46})^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T^*_{46}\mathbb{G}_j\right)$ 586

G

587

**The characteristic equation of this system is**

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(q_{13})^{(1)}G^*_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)}T^*_{14} + (b_{14})^{(1)}s_{(13),(14)}T^*_{14}\right)$$

$$+\left(\left((\lambda)^{(1)} + (a'_{14})^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)}G^*_{13} + (a_{13})^{(1)}(q_{14})^{(1)}G^*_{14}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)}T^*_{14} + (b_{14})^{(1)}s_{(13),(13)}T^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((b'_{13})^{(1)} + (b'_{14})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(15)}T^*_{14} + (b_{14})^{(1)}s_{(13),(15)}T^*_{13}\right)\} = 0$$

$+$

$$(\lambda)^{(2)} + (b'_{18})^{(2)} - (r_{18})^{(2)}\{\left((\lambda)^{(2)} + (a'_{18})^{(2)} + (p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^*_{17} + (a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^*_{17}\right)$$

$$+\left(\left((\lambda)^{(2)} + (a'_{17})^{(2)} + (p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^*_{16} + (a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}\right)$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2 + \left((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2 + \left((b'_{16})^{(2)} + (b'_{17})^{(2)} - (r_{16})^{(2)} + (r_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$+\left(\left((\lambda)^{(2)}\right)^2 + \left((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17} + (a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^{*}_{16}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\right)\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left(\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)\right]$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(21)}T^{*}_{21}\right)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^{*}_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^{*}_{21}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(20)}T^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$+\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(22)}T^{*}_{20}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\right)\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left(\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)\right]$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(25)}T^{*}_{25}\right)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^{*}_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^{*}_{25}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(24)}T^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$+\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(26)}T^{*}_{24}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\right)\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(29)}T^{*}_{29}\right)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^{*}_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^{*}_{29}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(28)}T^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$+\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(30)}T^{*}_{28}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\right)\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left(\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)\right]$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(33)}T^{*}_{33}\right)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^{*}_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^{*}_{33}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(32)}T^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$+\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(34)}T^{*}_{32}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\right)\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)\right]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(37)}T^{*}_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^{*}_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^{*}_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(36)}T^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$+\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(38)}T^{*}_{36}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\right)\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(41)}T^{*}_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^{*}_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^{*}_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(40)}T^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$+\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(42)}T^{*}_{40}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\right)\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(45)}T^{*}_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^{*}_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^{*}_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(44)}T^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$+\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(46)}T^{*}_{44}\right)\}=0$$

**And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.**

## SECTION TWENTY NINE

## Consciousness Would Be An Immanent Property Of Reality Which Exists Before The Creation Of Reality

### INTRODUCTION—VARIABLES USED

This interpretation attributes to consciousness an explicit role, through (e&eb) the act of measurement and observation, forcing (eb) the wave function to collapse into (e&eb) a particle and determining (eb) in this way reality itself

## NOTATION

This interpretation attributes to consciousness an explicit role, through the act of measurement and observation, forcing (eb) the wave function to collapse into (e&eb) a particle and determining (eb) in this way reality itself

### Module One

$G_{13}$ : Category one of interpretation attributes to consciousness an explicit role, through the act of measurement and observation

$G_{14}$ : Category two of SAS

$G_{15}$ : Category three of SAS

$T_{13}$ : Category one of wave function to collapse into (e&eb) a particle and determining (eb) in this way reality itself

$T_{14}$ : Category two of SAS

$T_{15}$ : Category three of SAS

### Module Two

This interpretation attributes to consciousness an explicit role, through the act of measurement and observation, forcing the wave function to collapse into (e&eb) a particle and determining (eb) in this way reality itself

$G_{16}$ : Category one of **interpretation attributes to consciousness an explicit role, through the act of measurement and observation, forcing the wave function to collapse;** particle and determining (eb) in this way reality itself

$G_{17}$ : Category two of SAS

$G_{18}$ : Category three of SAS

$T_{16}$ : Category one of particle and determining (eb) in this way reality itself; **interpretation attributes to consciousness an explicit role, through the act of measurement and observation, forcing the wave function to collapse**

$T_{17}$ : Category two of SAS

$T_{18}$ : Category three of SAS

### Module three

This interpretation attributes to consciousness an explicit role, through the act of measurement and observation, forcing the wave function to collapse into a particle and determining (eb) in this way reality itself

$G_{20}$ : Category one of interpretation attributes to consciousness an explicit role, through the act of measurement and observation, forcing the wave function to collapse into a particle; way reality itself

$G_{21}$ : Category two of SAS

$G_{22}$ : Category three of SAS

$T_{20}$ : Category one of i way reality itself ;interpretation attributes to consciousness an explicit role, through the act of measurement and observation, forcing the wave function to collapse into a particle

$T_{21}$ : Category two of SAS

$T_{22}$ : Category three of SAS

**Module four**

According to Bohr and Heisenberg consciousness would be (=) an immanent property of reality which exists before (e) the creation of reality.

$G_{24}$ : Category one of consciousness

$G_{25}$ : Category two of SAS

$G_{26}$ : Category three of SAS

$T_{24}$ : Category one of immanent property of reality which exists before (e) the creation of reality.

$T_{25}$ : Category two of SAS

$T_{26}$ : Category three of SAS

**Module five**

According to Bohr and Heisenberg consciousness would be an immanent property of reality which exists before (e) the **creation of reality**

$G_{28}$ : Category one of consciousness would be an immanent property of reality which exists; **creation of reality**

$G_{29}$ : Category two of SAS

$G_{30}$ : Category three of SAS

$T_{28}$ : Category one of **creation of reality;** consciousness would be an immanent property of reality which exists

$T_{29}$ : Category two of SAS

$T_{30}$ : Category three of SAS

**Module six**

**A neuronal network model linking subjective reports and objective physiological data during conscious perception Stanislas Dehaene, Claire Sergent, and Jean-Pierre Changeux**

(1) The subjective experience of perceiving visual stimuli is accompanied by (e&eb) objective neuronal activity patterns such as (=) sustained activity in primary visual area (V1), amplification of perceptual processing, correlation across distant regions, joint parietal, frontal, and cingulate activation, γ-band oscillations, and P300 waveform
(2) Authors describe a neuronal network model that aims at explaining how those physiological parameters may cohere with (e&eb) conscious reports.
(3) The model proposes that the step of conscious perception, referred to as access awareness, is related

to (e&eb) the entry of processed visual stimuli into (e&eb) a global brain state that links distant areas including the prefrontal cortex through (e&eb) reciprocal connections, and thus makes (eb) perceptual information reportable by multiple means.

(4) Authors use the model to simulate a classical psychological paradigm: the attentional blink. In addition to reproducing (eb) the main objective and subjective features of this paradigm, the model predicts (eb) an unique property of nonlinear transition from nonconscious processing to (e&eb) subjective perception.

(5) This all-or-none dynamics of conscious perception was verified behaviorally in (eb) human subjects

The **subjective experience of perceiving visual stimuli** is accompanied by (e&eb) objective neuronal activity patterns such as (=) sustained activity in primary visual area (V1), amplification of perceptual processing, correlation across distant regions, joint parietal, frontal, and cingulate activation, γ-band oscillations, and P300 waveform

$G_{32}$ : Category one of **subjective experience of perceiving visual stimuli;** objective neuronal activity patterns such as (=) sustained activity in primary visual area (V1), amplification of perceptual processing, correlation across distant regions, joint parietal, frontal, and cingulate activation, γ-band oscillations, and P300 waveform

$G_{33}$ : Category two of SAS

$G_{34}$ : Category three of SAS

$T_{32}$ : Category one of objective neuronal activity patterns such as (=) sustained activity in primary visual area (V1), amplification of perceptual processing, correlation across distant regions, joint parietal, frontal, and cingulate activation, γ-band oscillations, and P300 waveform; **subjective experience of perceiving visual stimuli**

$T_{33}$ : Category two of SAS

$T_{34}$ : Category three of SAS

**Module seven**

The subjective experience of perceiving visual stimuli is accompanied by objective neuronal activity patterns such as (=) sustained activity in primary visual area (V1), amplification of perceptual processing, correlation across distant regions, joint parietal, frontal, and cingulate activation, γ-band oscillations, and P300 waveform

$G_{36}$ : Category one of subjective experience of perceiving visual stimuli is accompanied by objective neuronal activity patterns

$G_{37}$ : Category two of SAS

$G_{38}$ : Category three of SAS

$T_{36}$ : Category one of sustained activity in primary visual area (V1), amplification of perceptual processing, correlation across distant regions, joint parietal, frontal, and cingulate activation, γ-band oscillations, and P300 waveform

$T_{37}$ : Category two of SAS

$T_{38}$ : Category three of SAS

**Module eight**

Authors describe a neuronal network model that aims at explaining how those physiological parameters may cohere with (e&eb) **conscious reports**

$G_{40}$ : Category one of neuronal network model that aims at explaining how those physiological parameters may cohere; **conscious reports**

$G_{41}$ : Category two of SAS

$G_{42}$ : Category three of SAS

$T_{40}$ : Category one of **conscious reports;** neuronal network model that aims at explaining how those physiological parameters may cohere

$T_{41}$ : Category two of SAS

$T_{42}$ : Category three of SAS

**Module Nine**

The model proposes that the step of conscious perception, referred to as access awareness, is related to (e&eb) the entry of processed visual stimuli into (e&eb) a global brain state that links distant areas including the prefrontal cortex through (e&eb) reciprocal connections, and thus makes (eb) perceptual information reportable by multiple means

$G_{44}$ : Category one of **model proposes that the step of conscious perception, referred to as access awareness;** entry of processed visual stimuli into (e&eb) a global brain state that links distant areas including the prefrontal cortex through (e&eb) reciprocal connections, and thus makes (eb) perceptual information reportable by multiple means

$G_{45}$ : Category two of SAS

$G_{46}$ : Category three of SAS

$T_{44}$ : Category one of entry of processed visual stimuli into (e&eb) a global brain state that links distant areas including the prefrontal cortex through (e&eb) reciprocal connections, and thus makes (eb) perceptual information reportable by multiple means; **model proposes that the step of conscious perception, referred to as access awareness**

$T_{45}$ : Category two of SAS

$T_{46}$ : Category three of SAS

**The Coefficients:**

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}\ ,\ (b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}$,
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)},\ \ (b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}\ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$ ,
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)}$,
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)}$,
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)}$,
are Dissipation coefficients

**Module Numbered One**

**The differential system of this model is now (Module Numbered one)**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \quad 1$$
$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad 2$$
$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15} \quad 3$$
$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13} \quad 4$$
$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14} \quad 5$$
$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)\right]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14}, t) =$ First augmentation factor
$-(b''_{13})^{(1)}(G, t) =$ First detritions factor

**Module Numbered Two**

**The differential system of this model is now ( Module numbered two)**

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right]G_{16} \quad 7$$
$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t)\right]G_{17} \quad 8$$
$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t)\right]G_{18} \quad 9$$
$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\left((G_{19}), t\right)\right]T_{16} \quad 10$$
$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\left((G_{19}), t\right)\right]T_{17} \quad 11$$
$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\left((G_{19}), t\right)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17}, t) =$ First augmentation factor
$-(b''_{16})^{(2)}\left((G_{19}), t\right) =$ First detritions factor

**Module Numbered Three**

**The differential system of this model is now (Module numbered three)**

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21},t)\right]G_{20} \quad 13$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21},t)\right]G_{21} \quad 14$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21},t)\right]G_{22} \quad 15$$

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23},t)\right]T_{20} \quad 16$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23},t)\right]T_{21} \quad 17$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23},t)\right]T_{22} \quad 18$$

$+(a''_{20})^{(3)}(T_{21},t) =$ First augmentation factor

$-(b''_{20})^{(3)}(G_{23},t) =$ First detritions factor

**Module Numbered Four**

**The differential system of this model is now (Module numbered Four)**

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right]G_{24} \quad 19$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25},t)\right]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25},t)\right]G_{26} \quad 21$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}\big((G_{27}),t\big)\right]T_{24} \quad 22$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}\big((G_{27}),t\big)\right]T_{25} \quad 23$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}\big((G_{27}),t\big)\right]T_{26} \quad 24$$

$+(a''_{24})^{(4)}(T_{25},t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}\big((G_{27}),t\big) =$ **First detritions factor**

**Module Numbered Five**:

**The differential system of this model is now (Module number five)**

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29},t)\right]G_{28} \quad 25$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29},t)\right]G_{29} \quad 26$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29},t)\right]G_{30} \quad 27$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}\big((G_{31}),t\big)\right]T_{28} \quad 28$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}\big((G_{31}),t\big)\right]T_{29} \quad 29$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}\big((G_{31}),t\big)\right]T_{30} \quad 30$$

$+(a''_{28})^{(5)}(T_{29},t) =$ **First augmentation factor**

$-(b''_{28})^{(5)}\big((G_{31}),t\big) =$ **First detritions factor**

**Module Numbered Six**

**The differential system of this model is now (Module numbered Six)**

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33},t)\right]G_{32} \quad 31$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33},t)\right]G_{33} \quad 32$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33},t)\right]G_{34} \quad 33$$

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35}),t)\right]T_{32} \quad 34$$
$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}((G_{35}),t)\right]T_{33} \quad 35$$
$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35}),t)\right]T_{34} \quad 36$$
$+(a''_{32})^{(6)}(T_{33},t) =$ **First augmentation factor**

**Module Numbered Seven:**

**The differential system of this model is now (Seventh Module)**

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right]G_{36} \quad 37$$
$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37},t)\right]G_{37} \quad 38$$
$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37},t)\right]G_{38} \quad 39$$
$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39}),t)\right]T_{36} \quad 40$$
$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}((G_{39}),t)\right]T_{37} \quad 41$$
$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39}),t)\right]T_{38} \quad 42$$
$+(a''_{36})^{(7)}(T_{37},t) =$ **First augmentation factor**

**Module Numbered Eight**

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right]G_{40} \quad 43$$
$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41},t)\right]G_{41} \quad 44$$
$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41},t)\right]G_{42} \quad 45$$
$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43}),t)\right]T_{40} \quad 46$$
$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}((G_{43}),t)\right]T_{41} \quad 47$$
$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43}),t)\right]T_{42} \quad 48$$

**Module Numbered Nine**

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$
$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$
$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$
$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47}),t)\right]T_{44} \quad 52$$
$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}((G_{47}),t)\right]T_{45} \quad 53$$
$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47}),t)\right]T_{46} \quad 54$$
$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**
$-(b''_{44})^{(9)}((G_{47}),t) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)} & \boxed{+(a''_{13})^{(1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)} & \\ \boxed{+(a''_{36})^{(7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} & \end{bmatrix} G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)}\boxed{+(a''_{14})^{(1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14} \quad (56)$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)}\boxed{+(a''_{15})^{(1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15} \quad (57)$$

Where $\boxed{(a''_{13})^{(1)}(T_{14},t)}$, $\boxed{(a''_{14})^{(1)}(T_{14},t)}$, $\boxed{(a''_{15})^{(1)}(T_{14},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7)}(T_{37},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8)}(T_{41},t)}$ $\boxed{+(a''_{41})^{(8,8)}(T_{41},t)}$ $\boxed{+(a''_{42})^{(8,8)}(T_{41},t)}$ are eight augmentation coefficient for 1,2,3

$\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)}\boxed{-(b''_{13})^{(1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13} \quad (58)$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)}\boxed{-(b''_{14})^{(1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14} \quad (59)$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)}\boxed{-(b''_{15})^{(1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15} \quad (60)$$

Where $\boxed{-(b''_{13})^{(1)}(G,t)}$, $\boxed{-(b''_{14})^{(1)}(G,t)}$, $\boxed{-(b''_{15})^{(1)}(G,t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{20}'')^{(3,3,)}(G_{23},t)}$, $\boxed{-(b_{21}'')^{(3,3,)}(G_{23},t)}$, $\boxed{-(b_{22}'')^{(3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{24}'')^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b_{25}'')^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b_{26}'')^{(4,4,4,4,)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{28}'')^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{32}'')^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b_{34}'')^{(6,6,6,6,)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{37}'')^{(7,7,)}(G_{39},t)}$, $\boxed{-(b_{36}'')^{(7,7,)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{40}'')^{(8,8)}(G_{43},t)}$ $\boxed{-(b_{41}'')^{(8,8)}(G_{43},t)}$ $\boxed{-(b_{42}'')^{(8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{44}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{45}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{46}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[\begin{array}{ccc} (a_{16}')^{(2)}\boxed{+(a_{16}'')^{(2)}(T_{17},t)} & \boxed{+(a_{13}'')^{(1,1,)}(T_{14},t)} & \boxed{+(a_{20}'')^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a_{24}'')^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{28}'')^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{32}'')^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{36}'')^{(7,7,7)}(T_{37},t)} & \boxed{+(a_{40}'')^{(8,8,8)}(T_{41},t)} & \boxed{+(a_{44}'')^{(9,9)}(T_{45},t)} \end{array}\right] G_{16} \quad (61)$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[\begin{array}{ccc} (a_{17}')^{(2)}\boxed{+(a_{17}'')^{(2)}(T_{17},t)} & \boxed{+(a_{14}'')^{(1,1,)}(T_{14},t)} & \boxed{+(a_{21}'')^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a_{25}'')^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{29}'')^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{33}'')^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{37}'')^{(7,7,7)}(T_{37},t)} & \boxed{+(a_{41}'')^{(8,8,8)}(T_{41},t)} & \boxed{+(a_{45}'')^{(9,9)}(T_{45},t)} \end{array}\right] G_{17} \quad (62)$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[\begin{array}{ccc} (a_{18}')^{(2)}\boxed{+(a_{18}'')^{(2)}(T_{17},t)} & \boxed{+(a_{15}'')^{(1,1,)}(T_{14},t)} & \boxed{+(a_{22}'')^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a_{26}'')^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{30}'')^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{34}'')^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{38}'')^{(7,7,7)}(T_{37},t)} & \boxed{+(a_{42}'')^{(8,8,8)}(T_{41},t)} & \boxed{+(a_{46}'')^{(9,9)}(T_{45},t)} \end{array}\right] G_{18} \quad (63)$$

Where $\boxed{+(a_{16}'')^{(2)}(T_{17},t)}$, $\boxed{+(a_{17}'')^{(2)}(T_{17},t)}$, $\boxed{+(a_{18}'')^{(2)}(T_{17},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a_{13}'')^{(1,1,)}(T_{14},t)}$, $\boxed{+(a_{14}'')^{(1,1,)}(T_{14},t)}$, $\boxed{+(a_{15}'')^{(1,1,)}(T_{14},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{20}'')^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a_{21}'')^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a_{22}'')^{(3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{24}'')^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{25}'')^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{26}'')^{(4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{28}'')^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{29}'')^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{30}'')^{(5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{32}'')^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{33}'')^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{34}'')^{(6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{36}'')^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a_{37}'')^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a_{38}'')^{(7,7,7)}(T_{37},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$+(a''_{40})^{(8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8)}(T_{41},t)$ are eight augmentation coefficient for category 1, 2 and 3

$+(a''_{44})^{(9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9)}(T_{45},t)$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \begin{bmatrix} (b'_{16})^{(2)} & -(b''_{16})^{(2)}(G_{19},t) & -(b''_{13})^{(1,1,)}(G,t) & -(b''_{20})^{(3,3,3,)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6)}(G_{35},t) & \\ -(b''_{36})^{(7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9)}(G_{47},t) & \end{bmatrix} T_{16} \quad (64)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)} & -(b''_{17})^{(2)}(G_{19},t) & -(b''_{14})^{(1,1,)}(G,t) & -(b''_{21})^{(3,3,3,)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6)}(G_{35},t) & \\ -(b''_{37})^{(7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9)}(G_{47},t) & \end{bmatrix} T_{17} \quad (65)$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)} & -(b''_{18})^{(2)}(G_{19},t) & -(b''_{15})^{(1,1,)}(G,t) & -(b''_{22})^{(3,3,3,)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6)}(G_{35},t) & \\ -(b''_{38})^{(7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9)}(G_{47},t) & \end{bmatrix} T_{18} \quad (66)$$

where $-(b''_{16})^{(2)}(G_{19},t)$, $-(b''_{17})^{(2)}(G_{19},t)$, $-(b''_{18})^{(2)}(G_{19},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{13})^{(1,1,)}(G,t)$, $-(b''_{14})^{(1,1,)}(G,t)$, $-(b''_{15})^{(1,1,)}(G,t)$ are second detrition coefficients for category 1,2 and 3

$-(b''_{20})^{(3,3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,)}(G_{23},t)$ are third detrition coefficients for category 1,2 and 3

$-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1,2 and 3

$-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1,2 and 3

$-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients for category 1,2 and 3

$-(b''_{36})^{(7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2 and 3

$-(b''_{40})^{(8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1,2 and 3

$-(b''_{44})^{(9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \begin{bmatrix} (a'_{20})^{(3)} & +(a''_{20})^{(3)}(T_{21},t) & +(a''_{16})^{(2,2,2)}(T_{17},t) & +(a''_{13})^{(1,1,1,)}(T_{14},t) \\ +(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t) & \\ +(a''_{36})^{(7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9)}(T_{45},t) & \end{bmatrix} G_{20} \quad (67)$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \begin{bmatrix} (a'_{21})^{(3)}\boxed{+(a''_{21})^{(3)}(T_{21},t)}\ \boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}\ \boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{21} \quad 68$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \begin{bmatrix} (a'_{22})^{(3)}\boxed{+(a''_{22})^{(3)}(T_{21},t)}\ \boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}\ \boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{22} \quad 69$$

$\boxed{+(a''_{20})^{(3)}(T_{21},t)}, \boxed{+(a''_{21})^{(3)}(T_{21},t)}, \boxed{+(a''_{22})^{(3)}(T_{21},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}, \boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}, \boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}$ are second augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)}$ are third augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)}, \boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)}, \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)}, \boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)}, \boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)}, \boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)}, \boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)}$ are seventh augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)}$ are eight augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{44})^{(9,9,9)}(T_{45},t)}, \boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)}, \boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)}$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b'_{20})^{(3)}\boxed{-(b''_{20})^{(3)}(G_{23},t)}\ \boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}\ \boxed{-(b''_{13})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)}\ \boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)}\ \boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7)}(G_{39},t)}\ \boxed{-(b''_{40})^{(8,8,8,8)}(G_{43},t)}\ \boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{20} \quad 70$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b'_{21})^{(3)}\boxed{-(b''_{21})^{(3)}(G_{23},t)}\ \boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}\ \boxed{-(b''_{14})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)}\ \boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)}\ \boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7)}(G_{39},t)}\ \boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)}\ \boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{21} \quad 71$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)}\boxed{-(b''_{22})^{(3)}(G_{23},t)}\ \boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}\ \boxed{-(b''_{15})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)}\ \boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)}\ \boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)}\ \boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)}\ \boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{22} \quad 72$$

$\boxed{-(b''_{20})^{(3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3)}(G_{23},t)}$ *are first detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}$ *are second detrition coefficients for*

*category 1, 2 and 3*

$\boxed{-(b_{13}'')^{(1,1,1,)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,)}(G,t)}$, $\boxed{-(b_{15}'')^{(1,1,1,)}(G,t)}$ *are third detrition coefficients for category 1,2 and 3*

$\boxed{-(b_{24}'')^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{25}'')^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{26}'')^{(4,4,4,4,4,4)}(G_{27},t)}$ *are fourth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b_{28}'')^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5)}(G_{31},t)}$ *are fifth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b_{32}'')^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{34}'')^{(6,6,6,6,6,6)}(G_{35},t)}$ *are sixth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b_{36}'')^{(7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{37}'')^{(7,7,7,7)}(G_{39},t)}$ $\boxed{-(b_{38}'')^{(7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{40}'')^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{42}'')^{(8,8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{46}'')^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b_{45}'')^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b_{44}'')^{(9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \begin{bmatrix} (a_{24}')^{(4)} & \boxed{+(a_{24}'')^{(4)}(T_{25},t)} & \boxed{+(a_{28}'')^{(5,5,)}(T_{29},t)} & \boxed{+(a_{32}'')^{(6,6,)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a_{16}'')^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a_{20}'')^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{36}'')^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{40}'')^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a_{44}'')^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{24} \quad 73$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \begin{bmatrix} (a_{25}')^{(4)} & \boxed{+(a_{25}'')^{(4)}(T_{25},t)} & \boxed{+(a_{29}'')^{(5,5,)}(T_{29},t)} & \boxed{+(a_{33}'')^{(6,6)}(T_{33},t)} \\ \boxed{+(a_{14}'')^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a_{17}'')^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a_{21}'')^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{37}'')^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{41}'')^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a_{45}'')^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{25} \quad 74$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \begin{bmatrix} (a_{26}')^{(4)} & \boxed{+(a_{26}'')^{(4)}(T_{25},t)} & \boxed{+(a_{30}'')^{(5,5,)}(T_{29},t)} & \boxed{+(a_{34}'')^{(6,6,)}(T_{33},t)} \\ \boxed{+(a_{15}'')^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a_{18}'')^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a_{22}'')^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{38}'')^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{42}'')^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a_{46}'')^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{26} \quad 75$$

$\boxed{(a_{24}'')^{(4)}(T_{25},t)}$, $\boxed{(a_{25}'')^{(4)}(T_{25},t)}$, $\boxed{(a_{26}'')^{(4)}(T_{25},t)}$ *are first augmentation coefficients category* 1, 2 3

$\boxed{+(a_{28}'')^{(5,5,)}(T_{29},t)}$, $\boxed{+(a_{29}'')^{(5,5,)}(T_{29},t)}$, $\boxed{+(a_{30}'')^{(5,5,)}(T_{29},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a_{32}'')^{(6,6,)}(T_{33},t)}$, $\boxed{+(a_{33}'')^{(6,6,)}(T_{33},t)}$, $\boxed{+(a_{34}'')^{(6,6,)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a_{13}'')^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a_{14}'')^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a_{15}'')^{(1,1,1,1)}(T_{14},t)}$ *are fourth augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a_{16}'')^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{17}'')^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{18}'')^{(2,2,2,2)}(T_{17},t)}$ *are fifth augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a_{20}'')^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{21}'')^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{22}'')^{(3,3,3,3)}(T_{21},t)}$ *are sixth augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a_{36}'')^{(7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a_{37}'')^{(7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a_{38}'')^{(7,7,7,7,7)}(T_{37},t)}$ *are seventh augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)$

*are eighth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{46})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9)}(T_{45},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \begin{bmatrix} (b'_{24})^{(4)} & -(b''_{24})^{(4)}(G_{27},t) & -(b''_{28})^{(5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,)}(G_{35},t) \\ & -(b''_{13})^{(1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{24} \quad (76)$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \begin{bmatrix} (b'_{25})^{(4)} & -(b''_{25})^{(4)}(G_{27},t) & -(b''_{29})^{(5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{25} \quad (77)$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)} & -(b''_{26})^{(4)}(G_{27},t) & -(b''_{30})^{(5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1)}(G,t) & -(b''_{18})^{(2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{26} \quad (78)$$

*Where* $-(b''_{24})^{(4)}(G_{27},t)$, $-(b''_{25})^{(4)}(G_{27},t)$, $-(b''_{26})^{(4)}(G_{27},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{28})^{(5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,)}(G_{31},t)$ *are second detrition coefficients for category* 1, 2 *and* 3

$-(b''_{32})^{(6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,)}(G_{35},t)$ *are third detrition coefficients for category* 1, 2 *and* 3

$-(b''_{13})^{(1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1)}(G,t)$

*are fourth detrition coefficients for category* 1, 2 *and* 3

$-(b''_{16})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2)}(G_{19},t)$

*are fifth detrition coefficients for category* 1, 2 *and* 3

$-(b''_{20})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3)}(G_{23},t)$

*are sixth detrition coefficients for category* 1, 2 *and* 3

$-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)$

*are seventh detrition coefficients for category* 1, 2 *and* 3

$-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)$

*are eighth detrition coefficients for category* 1, 2 *and* 3

$-(b''_{46})^{(9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \begin{bmatrix} (a'_{28})^{(5)} & +(a''_{28})^{(5)}(T_{29},t) & +(a''_{24})^{(4,4,)}(T_{25},t) & +(a''_{32})^{(6,6,6)}(T_{33},t) \\ & +(a''_{13})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{16})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3,3)}(T_{21},t) \\ & +(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{28} \quad (79)$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \begin{bmatrix} (a'_{29})^{(5)} & +(a''_{29})^{(5)}(T_{29},t) & +(a''_{25})^{(4,4,)}(T_{25},t) & +(a''_{33})^{(6,6,6)}(T_{33},t) \\ +(a''_{14})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{17})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3,3)}(T_{21},t) \\ +(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{29} \quad (80)$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \begin{bmatrix} (a'_{30})^{(5)} & +(a''_{30})^{(5)}(T_{29},t) & +(a''_{26})^{(4,4,)}(T_{25},t) & +(a''_{34})^{(6,6,6)}(T_{33},t) \\ +(a''_{15})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{18})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3,3)}(T_{21},t) \\ +(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{30} \quad (81)$$

*Where* $+(a''_{28})^{(5)}(T_{29},t)$, $+(a''_{29})^{(5)}(T_{29},t)$, $+(a''_{30})^{(5)}(T_{29},t)$ *are first augmentation coefficients for category* 1, 2 *and* 3

*And* $+(a''_{24})^{(4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,)}(T_{25},t)$ *are second augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{32})^{(6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6)}(T_{33},t)$ *are third augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)$ are fourth augmentation coefficients for category 1,2, and 3

$+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)$ are fifth augmentation coefficients for category 1,2,and 3

$+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)$ are sixth augmentation coefficients for category 1,2, 3

$+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)$ , $+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1,2, 3

$+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)$ are eighth augmentation coefficients for category 1,2, 3

$+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \begin{bmatrix} (b'_{28})^{(5)} & -(b''_{28})^{(5)}(G_{31},t) & -(b''_{24})^{(4,4,)}(G_{27},t) & -(b''_{32})^{(6,6,6)}(G_{35},t) \\ -(b''_{13})^{(1,1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3)}(G_{23},t) \\ -(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{28} \quad (82)$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b'_{29})^{(5)} & -(b''_{29})^{(5)}(G_{31},t) & -(b''_{25})^{(4,4,)}(G_{27},t) & -(b''_{33})^{(6,6,6)}(G_{35},t) \\ -(b''_{14})^{(1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3)}(G_{23},t) \\ -(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{29} \quad (83)$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b'_{30})^{(5)} & -(b''_{30})^{(5)}(G_{31},t) & -(b''_{26})^{(4,4,)}(G_{27},t) & -(b''_{34})^{(6,6,6)}(G_{35},t) \\ -(b''_{15})^{(1,1,1,1,1,)}(G,t) & -(b''_{18})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3)}(G_{23},t) \\ -(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{30} \quad (84)$$

*where* $-(b''_{28})^{(5)}(G_{31},t)$ , $-(b''_{29})^{(5)}(G_{31},t)$ , $-(b''_{30})^{(5)}(G_{31},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{24})^{(4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,)}(G_{27},t)$ *are second detrition coefficients*

$for\ category\ 1,2\ and\ 3$

$\boxed{-(b''_{32})^{(6,6,6)}(G_{35},t)}, \boxed{-(b''_{33})^{(6,6,6)}(G_{35},t)}, \boxed{-(b''_{34})^{(6,6,6)}(G_{35},t)}$ $are\ third\ detrition\ coefficients$

$for\ category\ 1,2\ and\ 3$

$\boxed{-(b''_{13})^{(1,1,1,1,1)}(G,t)}, \boxed{-(b''_{14})^{(1,1,1,1,1)}(G,t)}, \boxed{-(b''_{15})^{(1,1,1,1,1,)}(G,t)}$ are fourth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[\begin{array}{c} (a'_{32})^{(6)}\boxed{+(a''_{32})^{(6)}(T_{33},t)}\ \boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}\ \boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}\ \boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}\ \boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{32} \quad 85$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[\begin{array}{c} (a'_{33})^{(6)}\boxed{+(a''_{33})^{(6)}(T_{33},t)}\ \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}\ \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}\ \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}\ \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{33} \quad 86$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[\begin{array}{c} (a'_{34})^{(6)}\boxed{+(a''_{34})^{(6)}(T_{33},t)}\ \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}\ \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}\ \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}\ \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{34} \quad 87$$

$\boxed{+(a''_{32})^{(6)}(T_{33},t)}, \boxed{+(a''_{33})^{(6)}(T_{33},t)}, \boxed{+(a''_{34})^{(6)}(T_{33},t)}$ $are\ first\ augmentation\ coefficients$

$for\ category\ 1,2\ and\ 3$

$\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}$ $are\ second\ augmentation$

$coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}$ $are\ third\ augmentation$

$coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}$ - are fourth augmentation coefficients

$\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}$ - fifth augmentation coefficients

$\boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)}$ sixth augmentation coefficients

$+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)$
seventh augmentation coefficients
$+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)$
Eighth augmentation coefficients
$+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt}=(b_{32})^{(6)}T_{33}-\begin{bmatrix}(b'_{32})^{(6)} & -(b''_{32})^{(6)}(G_{35},t) & -(b''_{28})^{(5,5,5)}(G_{31},t) & -(b''_{24})^{(4,4,4,)}(G_{27},t)\\ -(b''_{13})^{(1,1,1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)\\ -(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)\end{bmatrix}T_{32} \quad (88)$$

$$\frac{dT_{33}}{dt}=(b_{33})^{(6)}T_{32}-\begin{bmatrix}(b'_{33})^{(6)} & -(b''_{33})^{(6)}(G_{35},t) & -(b''_{29})^{(5,5,5)}(G_{31},t) & -(b''_{25})^{(4,4,4,)}(G_{27},t)\\ -(b''_{14})^{(1,1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)\\ -(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)\end{bmatrix}T_{33} \quad (89)$$

$$\frac{dT_{34}}{dt}=(b_{34})^{(6)}T_{33}-\begin{bmatrix}(b'_{34})^{(6)} & -(b''_{34})^{(6)}(G_{35},t) & -(b''_{30})^{(5,5,5)}(G_{31},t) & -(b''_{26})^{(4,4,4,)}(G_{27},t)\\ -(b''_{15})^{(1,1,1,1,1,1)}(G,t) & -(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)\\ -(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)\end{bmatrix}T_{34} \quad (90)$$

$-(b''_{32})^{(6)}(G_{35},t)$, $-(b''_{33})^{(6)}(G_{35},t)$, $-(b''_{34})^{(6)}(G_{35},t)$ *are first detrition coefficients for category* 1, 2 *and* 3
$-(b''_{28})^{(5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5)}(G_{31},t)$ *are second detrition coefficients for category* 1, 2 *and* 3
$-(b''_{24})^{(4,4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,)}(G_{27},t)$ *are third detrition coefficients for category* 1,2 *and* 3
$-(b''_{13})^{(1,1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1,1,1)}(G,t)$ are fourth detrition coefficients for category 1, 2, and 3
$-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1, 2, and 3
$-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1, 2, and 3
$-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2, and 3
$-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)$
are eighth detrition coefficients for category 1, 2, and 3
$-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \qquad 91$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a'_{36})^{(7)}\boxed{+(a''_{36})^{(7)}(T_{37},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \qquad 92$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a'_{37})^{(7)}\boxed{+(a''_{37})^{(7)}(T_{37},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{38}}{dt} \qquad 93$$

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a'_{38})^{(7)}\boxed{+(a''_{38})^{(7)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{(a''_{36})^{(7)}(T_{37},t)}$, $\boxed{(a''_{37})^{(7)}(T_{37},t)}$, $\boxed{(a''_{38})^{(7)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{36}}{dt} = \qquad 94$$

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b'_{36})^{(7)}\boxed{-(b''_{36})^{(7)}(G_{39},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{37}}{dt} =$$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b'_{37})^{(7)}\boxed{-(b''_{37})^{(7)}(G_{39},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{38}}{dt} =$$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b'_{38})^{(7)}\boxed{-(b''_{38})^{(7)}(G_{39},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{40}}{dt}$$



$$= (a_{40})^{(8)}G_{41} - \begin{bmatrix} (a'_{40})^{(8)}\boxed{+(a''_{40})^{(8)}(T_{41},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{41}}{dt}$$

$$= (a_{41})^{(8)}G_{40} - \begin{bmatrix} (a'_{41})^{(8)}\boxed{+(a''_{41})^{(8)}(T_{41},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \begin{bmatrix} (a'_{42})^{(8)}\boxed{+(a''_{42})^{(8)}(T_{41},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a''_{40})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)}\boxed{-(b''_{40})^{(8)}(G_{43},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)}\boxed{-(b''_{41})^{(8)}(G_{43},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)}\boxed{-(b''_{42})^{(8)}(G_{43},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{44}}{dt} \qquad 96$$

$$= (a_{44})^{(9)}G_{45} - \left[\begin{array}{ccc} (a'_{44})^{(9)}\boxed{+(a''_{44})^{(9)}(T_{45},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} \end{array}\right] G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[\begin{array}{ccc} (a'_{45})^{(9)}\boxed{+(a''_{45})^{(9)}(T_{45},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} \end{array}\right] G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[\begin{array}{ccc} (a'_{46})^{(9)}\boxed{+(a''_{46})^{(9)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} \end{array}\right] G_{15}$$

Where $\boxed{+(a''_{44})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$+(a_{16}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)$ , $+(a_{17}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)$ , $+(a_{18}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a_{20}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)$ , $+(a_{21}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)$ , $+(a_{22}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a_{24}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)$ , $+(a_{25}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)$ , $+(a_{26}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a_{28}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)$, $+(a_{29}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)$ , $+(a_{30}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a_{32}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a_{33}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$ , $+(a_{34}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a_{13}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$ , $+(a_{14}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$ , $+(a_{15}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$ are Seventh augmentation coefficient for category 1, 2 and 3

$+(a_{38}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ $+(a_{37}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ $+(a_{36}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ are eighth augmentation coefficient for 1,2,3

$+(a_{40}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)$ ,$+(a_{42}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a_{41}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} =$$

$$(b_{44})^{(9)}T_{45} -$$

$$\begin{bmatrix} (b_{44}')^{(9)}\boxed{-(b_{44}'')^{(9)}(G_{47},t)} & \boxed{-(b_{16}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{20}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{24}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{28}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{32}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{13}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{36}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b_{45}')^{(9)}\boxed{-(b_{45}'')^{(9)}(G_{47},t)} & \boxed{-(b_{17}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{21}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{25}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{29}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{33}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{14}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{37}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b_{46}')^{(9)}\boxed{-(b_{46}'')^{(9)}(G_{47},t)} & \boxed{-(b_{18}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{22}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{26}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{30}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{34}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{15}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{38}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b_{44}'')^{(9)}(G_{47},t)}$ , $\boxed{-(b_{45}'')^{(9)}(G_{47},t)}$ , $\boxed{-(b_{46}'')^{(9)}(G_{47},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{16}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$ , $\boxed{-(b_{17}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$ , $\boxed{-(b_{18}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{20}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$ , $\boxed{-(b_{21}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$ , $\boxed{-(b_{22}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{24}'')^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{25}'')^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{26}'')^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{28}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{32}'')^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{34}'')^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{37}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{36}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a_i')^{(1)}, (a_i'')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (b_i'')^{(1)} > 0, \quad i,j = 13,14,15 \qquad 97$$

The functions $(a_i'')^{(1)}, (b_i'')^{(1)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a_i'')^{(1)}(T_{14},t) \leq (p_i)^{(1)} \leq (\hat{A}_{13})^{(1)}$$

$$(b_i'')^{(1)}(G,t) \leq (r_i)^{(1)} \leq (b_i')^{(1)} \leq (\hat{B}_{13})^{(1)}$$

$$lim_{T_2\to\infty}(a_i'')^{(1)}(T_{14},t) = (p_i)^{(1)} \qquad 98$$

$$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = (r_i)^{(1)}$$

**Definition of** $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$ :

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$$|(a_i'')^{(1)}(T_{14}',t) - (a_i'')^{(1)}(T_{14},t)| \leq (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G',t) - (b_i'')^{(1)}(G,t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}',t)$ and $(a_i'')^{(1)}(T_{14},t)$ . $(T_{14}',t)$ and $(T_{14},t)$ are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14},t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$ : 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}$ , $\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$

**Definition of** $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$ : 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}$, $(r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)} (\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)} (\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i, j = 16,17,18$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17}, t) \leq (p_i)^{(2)} \leq (\hat{A}_{16})^{(2)} \quad 102$$

$$(b_i'')^{(2)}(G_{19}, t) \leq (r_i)^{(2)} \leq (b_i')^{(2)} \leq (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2 \to \infty} (a_i'')^{(2)}(T_{17}, t) = (p_i)^{(2)} \quad 104$$

$$\lim_{G \to \infty} (b_i'')^{(2)}((G_{19}), t) = (r_i)^{(2)} \quad 105$$

**Definition of** $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$ : 106

Where $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}$ are positive constants and $i = 16,17,18$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}', t) - (a_i'')^{(2)}(T_{17}, t)| \leq (\hat{k}_{16})^{(2)} |T_{17} - T_{17}'| e^{-(\hat{M}_{16})^{(2)} t} \quad 107$$

$$|(b_i'')^{(2)}((G_{19})', t) - (b_i'')^{(2)}((G_{19}), t)| < (\hat{k}_{16})^{(2)} ||(G_{19}) - (G_{19})'|| e^{-(\hat{M}_{16})^{(2)} t} \quad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}', t)$ and$(a_i'')^{(2)}(T_{17}, t)$ . $(T_{17}', t)$ and $(T_{17}, t)$ are points belonging to the interval $[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}]$ . It is to be noted that $(a_i'')^{(2)}(T_{17}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17}, t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$ :

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)},$ are positive constants 109

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$$

**Definition of** $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$ :

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and\ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1 \quad 110$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(b_i)^{(2)} + (b_i')^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1 \quad 111$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i, j = 20,21,22 \quad 112$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(3)}$, $(r_i)^{(3)}$:

$$(a_i'')^{(3)}(T_{21}, t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23}, t) \leq (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(3)}(T_{21}, t) = (p_i)^{(3)} \quad 113$$

$$\lim_{G \to \infty}(b_i'')^{(3)}(G_{23}, t) = (r_i)^{(3)}$$

**Definition of** $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$ :

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: 114

$$|(a_i'')^{(3)}(T_{21}', t) - (a_i'')^{(3)}(T_{21}, t)| \leq (\hat{k}_{20})^{(3)} |T_{21} - T_{21}'| e^{-(\hat{M}_{20})^{(3)} t}$$

$$|(b_i'')^{(3)}(G_{23}', t) - (b_i'')^{(3)}(G_{23}, t)| < (\hat{k}_{20})^{(3)} ||G_{23} - G_{23}'|| e^{-(\hat{M}_{20})^{(3)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}', t)$ and$(a_i'')^{(3)}(T_{21}, t)$ . $(T_{21}', t)$ And $(T_{21}, t)$ are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21}, t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$ : 115

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}$ , $\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with 116
$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$ and the constants
$(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}, i = 20,21,22,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(a_i)^{(3)} + (a_i')^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)} (\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(b_i)^{(3)} + (b_i')^{(3)} + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)} (\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i, j = 24,25,26$$ 117

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(4)}$, $(r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25}, t) \leq (p_i)^{(4)} \leq (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}((G_{27}), t) \leq (r_i)^{(4)} \leq (b_i')^{(4)} \leq (\hat{B}_{24})^{(4)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(4)}(T_{25}, t) = (p_i)^{(4)}$$ 118

$$\lim_{G \to \infty}(b_i'')^{(4)}((G_{27}), t) = (r_i)^{(4)}$$

**Definition of** $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$ :

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24,25,26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}', t) - (a_i'')^{(4)}(T_{25}, t)| \leq (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})', t) - (b_i'')^{(4)}((G_{27}), t)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}', t)$ and$(a_i'')^{(4)}(T_{25}, t)$ . $(T_{25}', t)$ and $(T_{25}, t)$ are points belonging to the interval $[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}]$. It is to be noted that $(a_i'')^{(4)}(T_{25}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25}, t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$ : 120

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)},$ are positive constants

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}, \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$$

**Definition of** $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$ : 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and\ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}, i = 24,25,26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(a_i)^{(4)} + (a_i')^{(4)} + (\hat{A}_{24})^{(4)} + (\hat{P}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(b_i)^{(4)} + (b_i')^{(4)} + (\hat{B}_{24})^{(4)} + (\hat{Q}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i, j = 28,29,30$$ 122

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(5)}$, $(r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29}, t) \le (p_i)^{(5)} \le (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}((G_{31}), t) \le (r_i)^{(5)} \le (b_i')^{(5)} \le (\hat{B}_{28})^{(5)}$$

$$lim_{T_2 \to \infty} (a_i'')^{(5)} (T_{29}, t) = (p_i)^{(5)}$$ 123

$$\lim_{G \to \infty} (b_i'')^{(5)} (G_{31}, t) = (r_i)^{(5)}$$

**Definition of** $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$ :

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}', t) - (a_i'')^{(5)}(T_{29}, t)| \le (\hat{k}_{28})^{(5)} |T_{29} - T_{29}'| e^{-(\hat{M}_{28})^{(5)} t}$$

$$|(b_i'')^{(5)}((G_{31})', t) - (b_i'')^{(5)}((G_{31}), t)| < (\hat{k}_{28})^{(5)} ||(G_{31}) - (G_{31})'|| e^{-(\hat{M}_{28})^{(5)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}', t)$ and $(a_i'')^{(5)}(T_{29}, t)$ . $(T_{29}', t)$ and $(T_{29}, t)$ are points belonging to the interval $\left[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\right]$. It is to be noted that $(a_i'')^{(5)}(T_{29}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$ : 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)},$ are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}, \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

**Definition of** $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$ : 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and\ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(a_i)^{(5)} + (a_i')^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(b_i)^{(5)} + (b_i')^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i, j = 32,33,34$$ 127

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(6)}$, $(r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33}, t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}), t) \leq (r_i)^{(6)} \leq (b_i')^{(6)} \leq (\hat{B}_{32})^{(6)}$$

$$lim_{T_2 \to \infty} (a_i'')^{(6)} (T_{33}, t) = (p_i)^{(6)}$$ 128

$$\lim_{G \to \infty} (b_i'')^{(6)} ((G_{35}), t) = (r_i)^{(6)}$$

**Definition of** $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$ :

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}', t) - (a_i'')^{(6)}(T_{33}, t)| \leq (\hat{k}_{32})^{(6)} |T_{33} - T_{33}'| e^{-(\hat{M}_{32})^{(6)} t}$$

$$|(b_i'')^{(6)}((G_{35})', t) - (b_i'')^{(6)}((G_{35}), t)| < (\hat{k}_{32})^{(6)} ||(G_{35}) - (G_{35})'|| e^{-(\hat{M}_{32})^{(6)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}', t)$ and $(a_i'')^{(6)}(T_{33}, t)$ . $(T_{33}', t)$ and $(T_{33}, t)$ are points belonging to the interval $[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}]$. It is to be noted that $(a_i'')^{(6)}(T_{33}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$ : 129

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)},$ are positive constants

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$$

**Definition of** $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$ : 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and\ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32,33,34,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(a_i)^{(6)} + (a_i')^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)} (\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(b_i)^{(6)} + (b_i')^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)} (\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

(AAAAAAA) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i, j = 36,37,38$ 131

(BBBBBBB) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(7)}$, $(r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37}, t) \leq (p_i)^{(7)} \leq (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39}, t) \leq (r_i)^{(7)} \leq (b_i')^{(7)} \leq (\hat{B}_{36})^{(7)}$$

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*(CCCCCCC)* $\lim_{T_2 \to \infty} (a_i'')^{(7)}(T_{37}, t) = (p_i)^{(7)}$

*(DDDDDDD)*

$\lim_{G \to \infty} (b_i'')^{(7)}((G_{39}), t) = (r_i)^{(7)}$

**Definition of** $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$ :

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$$|(a_i'')^{(7)}(T_{37}', t) - (a_i'')^{(7)}(T_{37}, t)| \leq (\hat{k}_{36})^{(7)} |T_{37} - T_{37}'| e^{-(\hat{M}_{36})^{(7)} t}$$

$$|(b_i'')^{(7)}((G_{39})', t) - (b_i'')^{(7)}((G_{39}), t)| < (\hat{k}_{36})^{(7)} ||(G_{39}) - (G_{39})'|| e^{-(\hat{M}_{36})^{(7)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}', t)$ and $(a_i'')^{(7)}(T_{37}, t)$ . $(T_{37}', t)$ and $(T_{37}, t)$ are points belonging to the interval $[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}]$ . It is to be noted that $(a_i'')^{(7)}(T_{37}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37}, t)$ , the **first augmentation coefficient** attributable to the

system, would be absolutely continuous.

**Definition of** $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$ : 134

(EEEEEEE) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)},$ are positive constants

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$$

**Definition of** $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$ : 135

(FFFFFFF) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(b_i)^{(7)} + (b_i')^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40,41,42 \qquad 136$$

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

**Definition of** $(p_i)^{(8)}$, $(r_i)^{(8)}$: 137

$$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)} \qquad 138$$

$$(b_i'')^{(8)}((G_{43}), t) \leq (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)} \qquad 139$$

$$\lim_{T_2 \to \infty} (a_i'')^{(8)}(T_{41}, t) = (p_i)^{(8)} \qquad 140$$

$$\lim_{G \to \infty} (b_i'')^{(8)}((G_{43}), t) = (r_i)^{(8)} \qquad 141$$

**Definition of** $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$ :

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)} |T_{41} - T_{41}'| e^{-(\hat{M}_{40})^{(8)} t} \qquad 142$$

$$|(b_i'')^{(8)}((G_{43})', t) - (b_i'')^{(8)}((G_{43}), t)| < (\hat{k}_{40})^{(8)} ||(G_{43}) - (G_{43})'|| e^{-(\hat{M}_{40})^{(8)} t} \qquad 143$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}', t)$ and $(a_i'')^{(8)}(T_{41}, t)$ . $(T_{41}', t)$ and $(T_{41}, t)$ are points belonging to the interval $\left[(\hat{k}_{40})^{(8)}, (\widehat{M}_{40})^{(8)}\right]$. It is to be noted that $(a_i'')^{(8)}(T_{41}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{40})^{(8)} = 1$ then the function $(a_i'')^{(8)}(T_{41}, t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\widehat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$ :

$(\widehat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)},$ are positive constants

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}, \frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}} < 1 \qquad 144$$

**Definition of** $(\hat{P}_{40})^{(8)}, (\hat{Q}_{40})^{(8)}$ :

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\widehat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}, (\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)}, (a_i')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}, i = 40,41,42,$
Satisfy the inequalities

$$\frac{1}{(\widehat{M}_{40})^{(8)}}[(a_i)^{(8)} + (a_i')^{(8)} + (\hat{A}_{40})^{(8)} + (\hat{P}_{40})^{(8)} (\hat{k}_{40})^{(8)}] < 1 \qquad 145$$

$$\frac{1}{(\widehat{M}_{40})^{(8)}}[(b_i)^{(8)} + (b_i')^{(8)} + (\hat{B}_{40})^{(8)} + (\hat{Q}_{40})^{(8)} (\hat{k}_{40})^{(8)}] < 1 \qquad 146$$

Where we suppose

$$(a_i)^{(9)}, (a_i')^{(9)}, (a_i'')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (b_i'')^{(9)} > 0, \quad i, j = 44,45,46 \qquad 146\,A$$

The functions $(a_i'')^{(9)}, (b_i'')^{(9)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(9)}$, $(r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45}, t) \leq (p_i)^{(9)} \leq (\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47}, t) \leq (r_i)^{(9)} \leq (b_i')^{(9)} \leq (\hat{B}_{44})^{(9)}$$

$$\lim_{T_2 \to \infty} (a_i'')^{(9)}(T_{45}, t) = (p_i)^{(9)}$$

$$\lim_{G \to \infty} (b_i'')^{(9)}(G_{47}, t) = (r_i)^{(9)}$$

**Definition of** $(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}$ :

Where $\boxed{(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}}$ are positive constants and $\boxed{i = 44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}', t) - (a_i'')^{(9)}(T_{45}, t)| \leq (\hat{k}_{44})^{(9)} |T_{45} - T_{45}'| e^{-(\widehat{M}_{44})^{(9)} t}$$

$$|(b_i'')^{(9)}((G_{47})', t) - (b_i'')^{(9)}\big((G_{47}), t\big)| < (\hat{k}_{44})^{(9)}||(G_{47}) - (G_{47})'||e^{-(\hat{M}_{44})^{(9)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}', t)$ and$(a_i'')^{(9)}(T_{45}, t)$ . $(T_{45}', t)$ and $(T_{45}, t)$ are points belonging to the interval $\big[(\hat{k}_{44})^{(9)}, (\hat{M}_{44})^{(9)}\big]$. It is to be noted that $(a_i'')^{(9)}(T_{45}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{44})^{(9)} = 1$ then the function $(a_i'')^{(9)}(T_{45}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$ :

$(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$, are positive constants

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}} \ , \frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$$

**Definition of** $(\hat{P}_{44})^{(9)}, (\hat{Q}_{44})^{(9)}$ :

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}, (\hat{A}_{44})^{(9)} and\ (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)}, (a_i')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (p_i)^{(9)}, \ (r_i)^{(9)}, i = 44,45,46,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{44})^{(9)}}[\,(a_i)^{(9)} + (a_i')^{(9)} + \ (\hat{A}_{44})^{(9)} + \ (\hat{P}_{44})^{(9)}\,(\hat{k}_{44})^{(9)}] < 1$$

$$\frac{1}{(\hat{M}_{44})^{(9)}}[\ (b_i)^{(9)} + (b_i')^{(9)} + \ (\hat{B}_{44})^{(9)} + \ (\hat{Q}_{44})^{(9)}\ (\hat{k}_{44})^{(9)}] < 1$$

**Theorem 1:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \big(\hat{P}_{13}\big)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \ , \ \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 2 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

**Definition of** $G_i(0)\,, T_i(0)$

$$G_i(t) \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} \quad , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} \quad , \quad T_i(0) = T_i^0 > 0$$

**Theorem 3 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t) \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t} \quad , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t} \quad , \quad T_i(0) = T_i^0 > 0$$

**Theorem 4 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

**Definition of** $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \left(\hat{P}_{24}\right)^{(4)} e^{(\hat{M}_{24})^{(4)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{24}\right)^{(4)} e^{(\hat{M}_{24})^{(4)}t} \;,\quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 5 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

**Definition of** $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \left(\hat{P}_{28}\right)^{(5)} e^{(\hat{M}_{28})^{(5)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{28}\right)^{(5)} e^{(\hat{M}_{28})^{(5)}t} \;,\quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 6 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

**Definition of** $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \left(\hat{P}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t} \;,\quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 7:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

**Definition of** $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \left(\hat{P}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t} \;,\quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 8:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

**Definition of** $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \left(\hat{P}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t} \;,\quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 9:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

**Definition of** $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \left(\hat{P}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$T_i(t) \leq (\hat{Q}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Proof:** Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$G_i(0) = G_i^0$ , $T_i(0) = T_i^0$ , $G_i^0 \leq (\hat{P}_{13})^{(1)}$ , $T_i^0 \leq (\hat{Q}_{13})^{(1)}$, 155

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t}$ 156

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t}$ 157

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[ (a_{13})^{(1)} G_{14}(s_{(13)}) - \left( (a'_{13})^{(1)} + a''_{13})^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \right) G_{13}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[ (a_{14})^{(1)} G_{13}(s_{(13)}) - \left( (a'_{14})^{(1)} + (a''_{14})^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \right) G_{14}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[ (a_{15})^{(1)} G_{14}(s_{(13)}) - \left( (a'_{15})^{(1)} + (a''_{15})^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \right) G_{15}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[ (b_{13})^{(1)} T_{14}(s_{(13)}) - \left( (b'_{13})^{(1)} - (b''_{13})^{(1)} (G(s_{(13)}), s_{(13)}) \right) T_{13}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[ (b_{14})^{(1)} T_{13}(s_{(13)}) - \left( (b'_{14})^{(1)} - (b''_{14})^{(1)} (G(s_{(13)}), s_{(13)}) \right) T_{14}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{T}_{15}(t) = T_{15}^0 + \int_0^t \left[ (b_{15})^{(1)} T_{14}(s_{(13)}) - \left( (b'_{15})^{(1)} - (b''_{15})^{(1)} (G(s_{(13)}), s_{(13)}) \right) T_{15}(s_{(13)}) \right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$ , $T_i(0) = T_i^0$ , $G_i^0 \leq (\hat{P}_{16})^{(2)}$ , $T_i^0 \leq (\hat{Q}_{16})^{(2)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

By 160

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[ (a_{16})^{(2)} G_{17}(s_{(16)}) - \left( (a'_{16})^{(2)} + a''_{16})^{(2)} (T_{17}(s_{(16)}), s_{(16)}) \right) G_{16}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[ (a_{17})^{(2)} G_{16}(s_{(16)}) - \left( (a'_{17})^{(2)} + (a''_{17})^{(2)} (T_{17}(s_{(16)}), s_{(17)}) \right) G_{17}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[ (a_{18})^{(2)} G_{17}(s_{(16)}) - \left( (a'_{18})^{(2)} + (a''_{18})^{(2)} (T_{17}(s_{(16)}), s_{(16)}) \right) G_{18}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^{0} + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^{0} + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^{0} + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{18}(s_{(16)})\right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{20})^{(3)}$, $T_i^0 \le (\hat{Q}_{20})^{(3)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\bar{M}_{20})^{(3)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\bar{M}_{20})^{(3)} t}$

By 161

$$\bar{G}_{20}(t) = G_{20}^{0} + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''_{20})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^{0} + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^{0} + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{22}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^{0} + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^{0} + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{22}(t) = T_{22}^{0} + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{22}(s_{(20)})\right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{24})^{(4)}$, $T_i^0 \le (\hat{Q}_{24})^{(4)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\bar{M}_{24})^{(4)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\bar{M}_{24})^{(4)} t}$

By 162

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{26}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{\mathrm{T}}_{26}(\mathrm{t}) = \mathrm{T}_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{26}(s_{(24)})\right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i$: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$ , $T_i(0) = T_i^0$ , $G_i^0 \le (\hat{P}_{28})^{(5)}$ , $T_i^0 \le (\hat{Q}_{28})^{(5)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

By 163

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{30}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{\mathrm{T}}_{30}(\mathrm{t}) = \mathrm{T}_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{30}(s_{(28)})\right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i$: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$ , $T_i(0) = T_i^0$ , $G_i^0 \le (\hat{P}_{32})^{(6)}$ , $T_i^0 \le (\hat{Q}_{32})^{(6)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\bar{M}_{32})^{(6)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\bar{M}_{32})^{(6)} t}$

By 164

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[ (a_{32})^{(6)} G_{33}(s_{(32)}) - \left( (a'_{32})^{(6)} + a''_{32})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[ (a_{33})^{(6)} G_{32}(s_{(32)}) - \left( (a'_{33})^{(6)} + (a''_{33})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[ (a_{34})^{(6)} G_{33}(s_{(32)}) - \left( (a'_{34})^{(6)} + (a''_{34})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{34}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[ (b_{32})^{(6)} T_{33}(s_{(32)}) - \left( (b'_{32})^{(6)} - (b''_{32})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[ (b_{33})^{(6)} T_{32}(s_{(32)}) - \left( (b'_{33})^{(6)} - (b''_{33})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{\mathrm{T}}_{34}(\mathrm{t}) = \mathrm{T}_{34}^0 + \int_0^t \left[ (b_{34})^{(6)} T_{33}(s_{(32)}) - \left( (b'_{34})^{(6)} - (b''_{34})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{34}(s_{(32)}) \right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[ (a_{36})^{(7)} G_{37}(s_{(36)}) - \left( (a'_{36})^{(7)} + a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[ (a_{37})^{(7)} G_{36}(s_{(36)}) - \left( (a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[ (a_{38})^{(7)} G_{37}(s_{(36)}) - \left( (a'_{38})^{(7)} + (a''_{38})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{38}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[ (b_{36})^{(7)} T_{37}(s_{(36)}) - \left( (b'_{36})^{(7)} - (b''_{36})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[ (b_{37})^{(7)} T_{36}(s_{(36)}) - \left( (b'_{37})^{(7)} - (b''_{37})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\overline{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^{0} + \int_0^t \left[ (b_{38})^{(7)} T_{37}(s_{(36)}) - \left( (b'_{38})^{(7)} - (b''_{38})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{38}(s_{(36)}) \right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{40})^{(8)}, T_i^0 \le (\hat{Q}_{40})^{(8)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[ (a_{40})^{(8)} G_{41}(s_{(40)}) - \left( (a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[ (a_{41})^{(8)} G_{40}(s_{(40)}) - \left( (a'_{41})^{(8)} + (a''_{41})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[ (a_{42})^{(8)} G_{41}(s_{(40)}) - \left( (a'_{42})^{(8)} + (a''_{42})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{42}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^0 + \int_0^t \left[ (b_{40})^{(8)} T_{41}(s_{(40)}) - \left( (b'_{40})^{(8)} - (b''_{40})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^0 + \int_0^t \left[ (b_{41})^{(8)} T_{40}(s_{(40)}) - \left( (b'_{41})^{(8)} - (b''_{41})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\overline{\mathrm{T}}_{42}(\mathrm{t}) = \mathrm{T}_{42}^{0} + \int_0^t \left[ (b_{42})^{(8)} T_{41}(s_{(40)}) - \left( (b'_{42})^{(8)} - (b''_{42})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{42}(s_{(40)}) \right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** 166
Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy A

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{44})^{(9)}, T_i^0 \le (\hat{Q}_{44})^{(9)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[ (a_{44})^{(9)} G_{45}(s_{(44)}) - \left( (a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45}(s_{(44)}), s_{(44)})\right) G_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}(s_{(44)}), s_{(44)})\right) G_{46}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{44}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{46}(t) = T_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{46}(s_{(44)})\right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}}\right)\right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left(e^{(\hat{M}_{13})^{(1)} t} - 1\right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\hat{M}_{13})^{(1)} t} \le \frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{P}_{13})^{(1)} + G_{14}^0\right) e^{\left(-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0}\right)} + (\hat{P}_{13})^{(1)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \le G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}}\right)\right] ds_{(16)} = \quad 169$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1\right)$$

From which it follows that 170

$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \le \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left[\left((\hat{P}_{16})^{(2)} + G_{17}^0\right) e^{\left(-\frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0}\right)} + (\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t) \le G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} \left(G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} s_{(20)}}\right)\right] ds_{(20)} =$$

$$\left(1 + (a_{20})^{(3)} t\right) G_{21}^0 + \frac{(a_{20})^{(3)} (\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left(e^{(\hat{M}_{20})^{(3)} t} - 1\right)$$

From which it follows that 172

$$(G_{20}(t)-G_{20}^{0})e^{-(\hat{M}_{20})^{(3)}t}\leq\frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left[((\hat{P}_{20})^{(3)}+G_{21}^{0})e^{\left(-\frac{(\hat{P}_{20})^{(3)}+G_{21}^{0}}{G_{21}^{0}}\right)}+(\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 173

$$G_{24}(t)\leq G_{24}^{0}+\int_{0}^{t}\left[(a_{24})^{(4)}\left(G_{25}^{0}+(\hat{P}_{24})^{(4)}e^{(\hat{M}_{24})^{(4)}s_{(24)}}\right)\right]ds_{(24)}=$$

$$\left(1+(a_{24})^{(4)}t\right)G_{25}^{0}+\frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left(e^{(\hat{M}_{24})^{(4)}t}-1\right)$$

From which it follows that 174

$$(G_{24}(t)-G_{24}^{0})e^{-(\hat{M}_{24})^{(4)}t}\leq\frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left[((\hat{P}_{24})^{(4)}+G_{25}^{0})e^{\left(-\frac{(\hat{P}_{24})^{(4)}+G_{25}^{0}}{G_{25}^{0}}\right)}+(\hat{P}_{24})^{(4)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t)\leq G_{28}^{0}+\int_{0}^{t}\left[(a_{28})^{(5)}\left(G_{29}^{0}+(\hat{P}_{28})^{(5)}e^{(\hat{M}_{28})^{(5)}s_{(28)}}\right)\right]ds_{(28)}=$$

$$\left(1+(a_{28})^{(5)}t\right)G_{29}^{0}+\frac{(a_{28})^{(5)}(\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left(e^{(\hat{M}_{28})^{(5)}t}-1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t}\leq\frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t)\leq G_{32}^{0}+\int_{0}^{t}\left[(a_{32})^{(6)}\left(G_{33}^{0}+(\hat{P}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\right)\right]ds_{(32)}=$$

$$\left(1+(a_{32})^{(6)}t\right)G_{33}^{0}+\frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left(e^{(\hat{M}_{32})^{(6)}t}-1\right)$$

From which it follows that 177

$$(G_{32}(t)-G_{32}^{0})e^{-(\hat{M}_{32})^{(6)}t}\leq\frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left[((\hat{P}_{32})^{(6)}+G_{33}^{0})e^{\left(-\frac{(\hat{P}_{32})^{(6)}+G_{33}^{0}}{G_{33}^{0}}\right)}+(\hat{P}_{32})^{(6)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

(aa) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t) \le G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} \left(G_{37}^0 + (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} s_{(36)}}\right)\right] ds_{(36)} =$$
$$\left(1 + (a_{36})^{(7)} t\right) G_{37}^0 + \frac{(a_{36})^{(7)} (\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left(e^{(\hat{M}_{36})^{(7)} t} - 1\right)$$

From which it follows that

$$(G_{36}(t) - G_{36}^0) e^{-(\hat{M}_{36})^{(7)} t} \le \frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left[((\hat{P}_{36})^{(7)} + G_{37}^0) e^{\left(-\frac{(\hat{P}_{36})^{(7)} + G_{37}^0}{G_{37}^0}\right)} + (\hat{P}_{36})^{(7)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t) \le G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} \left(G_{41}^0 + (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} s_{(40)}}\right)\right] ds_{(40)} = \quad 180$$
$$\left(1 + (a_{40})^{(8)} t\right) G_{41}^0 + \frac{(a_{40})^{(8)} (\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}} \left(e^{(\hat{M}_{40})^{(8)} t} - 1\right)$$

From which it follows that 181

$$(G_{40}(t) - G_{40}^0) e^{-(\hat{M}_{40})^{(8)} t} \le \frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}} \left[((\hat{P}_{40})^{(8)} + G_{41}^0) e^{\left(-\frac{(\hat{P}_{40})^{(8)} + G_{41}^0}{G_{41}^0}\right)} + (\hat{P}_{40})^{(8)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 8

Analogous inequalities hold also for $G_{41}, G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t) \le G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} \left(G_{45}^0 + (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} s_{(44)}}\right)\right] ds_{(44)} =$$
$$\left(1 + (a_{44})^{(9)} t\right) G_{45}^0 + \frac{(a_{44})^{(9)} (\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left(e^{(\hat{M}_{44})^{(9)} t} - 1\right)$$

From which it follows that

$$(G_{44}(t) - G_{44}^0) e^{-(\hat{M}_{44})^{(9)} t} \le \frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left[((\hat{P}_{44})^{(9)} + G_{45}^0) e^{\left(-\frac{(\hat{P}_{44})^{(9)} + G_{45}^0}{G_{45}^0}\right)} + (\hat{P}_{44})^{(9)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 9

Analogous inequalities hold also for $G_{45}, G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$ and to choose 182

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[(\hat{P}_{13})^{(1)}+\left((\hat{P}_{13})^{(1)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{13})^{(1)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{13})^{(1)} \quad 183$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[\left((\hat{Q}_{13})^{(1)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{13})^{(1)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{13})^{(1)}\right]\leq(\hat{Q}_{13})^{(1)} \quad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)},T^{(1)}\right),\left(G^{(2)},T^{(2)}\right)\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{13})^{(1)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{13})^{(1)}t}\}$$

Indeed if we denote

**Definition of** $\tilde{G},\tilde{T}:\left(\tilde{G},\tilde{T}\right)=\mathcal{A}^{(1)}(G,T)$

It results

$$\left|\tilde{G}_{13}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{13})^{(1)}\left|G_{14}^{(1)}-G_{14}^{(2)}\right|e^{-(\hat{M}_{13})^{(1)}s_{(13)}}e^{(\hat{M}_{13})^{(1)}s_{(13)}}\,ds_{(13)}+$$

$$\int_0^t\{(a'_{13})^{(1)}\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\hat{M}_{13})^{(1)}s_{(13)}}e^{-(\hat{M}_{13})^{(1)}s_{(13)}}+$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\hat{M}_{13})^{(1)}s_{(13)}}e^{(\hat{M}_{13})^{(1)}s_{(13)}}+$$

$$G_{13}^{(2)}\left|(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)-(a''_{13})^{(1)}\left(T_{14}^{(2)},s_{(13)}\right)\right|\ e^{-(\hat{M}_{13})^{(1)}s_{(13)}}e^{(\hat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|G^{(1)}-G^{(2)}\right|e^{-(\hat{M}_{13})^{(1)}t}\leq \quad 186$$
$$\frac{1}{(\hat{M}_{13})^{(1)}}\left((a_{13})^{(1)}+(a'_{13})^{(1)}+(\hat{A}_{13})^{(1)}+(\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}\right)d\left(\left(G^{(1)},T^{(1)};\,G^{(2)},T^{(2)}\right)\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 1:** The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\hat{P}_{13})^{(1)}e^{(\hat{M}_{13})^{(1)}t}$ *and* $(\hat{Q}_{13})^{(1)}e^{(\hat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i = 13,14,15$ depend only on $T_{14}$ and respectively on $G(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any $t$ where $G_i(t) = 0\ and\ T_i(t) = 0$

From 19 to 24 it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(1)} - (a_i'')^{(1)}(T_{14}(s_{(13)}), s_{(13)})\} ds_{(13)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(1)} t)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2\ and\ \left((\widehat{M}_{13})^{(1)}\right)_3$ : 187

**Remark 3**: if $G_{13}$ is bounded, the same property have also $G_{14}\ and\ G_{15}$ . indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a_{14}')^{(1)} G_{14}$ and by integrating

$$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)} \left((\widehat{M}_{13})^{(1)}\right)_1 / (a_{14}')^{(1)}$$

In the same way , one can obtain

$$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)} \left((\widehat{M}_{13})^{(1)}\right)_2 / (a_{15}')^{(1)}$$

If $G_{14}\ or\ G_{15}$ is bounded, the same property follows for $G_{13}$ , $G_{15}$ and $G_{13}$ , $G_{14}$ respectively.

**Remark 4:** If $G_{13}$ *is* bounded, from below, the same property holds for $G_{14}\ and\ G_{15}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{14}$ is bounded from below. 188

**Remark 5:** If $T_{13}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}(G(t), t)) = (b_{14}')^{(1)}$ then $T_{14} \to \infty$. 189

**Definition of** $(m)^{(1)}$ and $\varepsilon_1$ :

Indeed let $t_1$ be so that for $t > t_1$

$$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t), t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right)$, $t = log\frac{2}{\varepsilon_1}$ By taking now $\varepsilon_1$ sufficiently small one sees that $T_{14}$ is unbounded.
The same property holds for $T_{15}$ if $\lim_{t\to\infty}(b_{15}'')^{(1)}(G(t), t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}$ , $\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}\ and\ (\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[(\widehat{P}_{16})^{(2)}+\left((\hat{P}_{16})^{(2)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{16})^{(2)}\right]\leq(\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

**Definition of** $\widetilde{G_{19}},\widetilde{T_{19}}$ : $\left(\widetilde{G_{19}},\widetilde{T_{19}}\right)=\mathcal{A}^{(2)}(G_{19},T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{16})^{(2)}\left|G_{17}^{(1)}-G_{17}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)}+$$

$$\int_0^t\{(a'_{16})^{(2)}\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$G_{16}^{(2)}\left|(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)-(a''_{16})^{(2)}\left(T_{17}^{(2)},s_{(16)}\right)\right|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0,t]$ 197

From the hypotheses it follows

$$\left|(G_{19})^{(1)}-(G_{19})^{(2)}\right|\mathrm{e}^{-(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}\leq$$
$$\frac{1}{(\widehat{\mathrm{M}}_{16})^{(2)}}\left((a_{16})^{(2)}+(a'_{16})^{(2)}+(\widehat{\mathrm{A}}_{16})^{(2)}+(\widehat{\mathrm{P}}_{16})^{(2)}(\hat{k}_{16})^{(2)}\right)\mathrm{d}\left(((G_{19})^{(1)},(T_{19})^{(1)};\ (G_{19})^{(2)},(T_{19})^{(2)})\right)$$

And analogous inequalities for $G_i$ and $T_i$. Taking into account the hypothesis the result follows 198

**Remark 6:** The fact that we supposed $(a''_{16})^{(2)}$ and $(b''_{16})^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{\mathrm{P}}_{16})^{(2)}\mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}$ and $(\widehat{\mathrm{Q}}_{16})^{(2)}\mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i=16,17,18$ depend only on $\mathrm{T}_{17}$ and respectively on

$(G_{19})$(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 7:** There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$ 200

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(2)} - (a_i'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})\} ds_{(16)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(2)}t)} > 0 \quad \text{for } t > 0$$

**Definition of** $\left((\widehat{M}_{16})^{(2)}\right)_1$, $\left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$: 201

**Remark 8:** if $G_{16}$ is bounded, the same property have also $G_{17}$ and $G_{18}$. indeed if

$G_{16} < (\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt} \leq \left((\widehat{M}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)} G_{17}$ and by integrating

$$G_{17} \leq \left((\widehat{M}_{16})^{(2)}\right)_2 = G_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1 / (a_{17}')^{(2)}$$

In the same way , one can obtain

$$G_{18} \leq \left((\widehat{M}_{16})^{(2)}\right)_3 = G_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2 / (a_{18}')^{(2)}$$

If $G_{17}$ or $G_{18}$ is bounded, the same property follows for $G_{16}$, $G_{18}$ and $G_{16}$, $G_{17}$ respectively.

**Remark 9:** If $G_{16}$ is bounded, from below, the same property holds for $G_{17}$ and $G_{18}$. The proof is analogous with the preceding one. An analogous property is true if $G_{17}$ is bounded from below. 202

**Remark 10:** If $T_{16}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t), t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$. 203

**Definition of** $(m)^{(2)}$ and $\varepsilon_2$ :

Indeed let $t_2$ be so that for $t > t_2$

$$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right)$, $t = \log\frac{2}{\varepsilon_2}$ By taking now $\varepsilon_2$ sufficiently small one sees that $T_{17}$ is unbounded. 205
The same property holds for $T_{18}$ if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t), t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[(\widehat{P}_{20})^{(3)}+\left((\hat{P}_{20})^{(3)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{20})^{(3)}\right]\leq(\hat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself 210

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)},(T_{23})^{(1)}),((G_{23})^{(2)},(T_{23})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

**Definition of** $\widetilde{G_{23}},\widetilde{T_{23}}:(\widetilde{(G_{23})},\widetilde{(T_{23})})=\mathcal{A}^{(3)}((G_{23}),(T_{23}))$

It results 213

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{20})^{(3)}\left|G_{21}^{(1)}-G_{21}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\,ds_{(20)}+$$

$$\int_0^t\{(a'_{20})^{(3)}\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$G_{20}^{(2)}\left|(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)},s_{(20)}\right)\right|\ e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\}ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses it follows

$$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}t}\leq \quad 214$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+(a'_{20})^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\widehat{k}_{20})^{(3)}\right)d\left(((G_{23})^{(1)},(T_{23})^{(1)};\ (G_{23})^{(2)},(T_{23})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

**Remark 11:** The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}\ and\ (\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i=20,21,22$ depend only on $\mathrm{T}_{21}$ and respectively on

$(G_{23})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 12:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 216

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(3)} - (a_i'')^{(3)}(T_{21}(s_{(20)}), s_{(20)})\} ds_{(20)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(3)} t)} > 0$ for t > 0

**Definition of** $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2\ and\ \left((\widehat{M}_{20})^{(3)}\right)_3$ : 217

**Remark 13:** if $G_{20}$ is bounded, the same property have also $G_{21}\ and\ G_{22}$ . indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a_{21}')^{(3)} G_{21}$ and by integrating

$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1 / (a_{21}')^{(3)}$

In the same way , one can obtain

$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2 / (a_{22}')^{(3)}$

If $G_{21}\ or\ G_{22}$ is bounded, the same property follows for $G_{20}$ , $G_{22}$ and $G_{20}$ , $G_{21}$ respectively.

**Remark 14:** If $G_{20}$ *is* bounded, from below, the same property holds for $G_{21}\ and\ G_{22}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{21}$ is bounded from below. 218

**Remark 15:** If $T_{20}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t), t\right)) = (b_{21}')^{(3)}$ then $T_{21} \to \infty$. 219

**Definition of** $(m)^{(3)}$ and $\varepsilon_3$ :

Indeed let $t_3$ be so that for $t > t_3$

$(b_{21})^{(3)} - (b_i'')^{(3)}\left((G_{23})(t), t\right) < \varepsilon_3, T_{20}\,(t) > (m)^{(3)}$

Then $\frac{dT_{21}}{dt} \geq (a_{21})^{(3)}(m)^{(3)} - \varepsilon_3 T_{21}$ which leads to 220

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t} = \frac{1}{2}$ it results

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_3}$ By taking now $\varepsilon_3$ sufficiently small one sees that $T_{21}$ is unbounded.
The same property holds for $T_{22}$ if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t), t\right) = (b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}$ , $\frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}} < 1$ and to choose 221

$(\widehat{P}_{24})^{(4)}$ and $(\widehat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[(\widehat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \tag{222}$$

$$\frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \tag{223}$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself (224)

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric (225)

$$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{24})^{(4)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{27}}),(\widetilde{T_{27}})$ : $\left((\widetilde{G_{27}}),(\widetilde{T_{27}})\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{24})^{(4)}\left|G_{25}^{(1)}-G_{25}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\,ds_{(24)}+$$

$$\int_0^t\{(a'_{24})^{(4)}\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$G_{24}^{(2)}\left|(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)-(a''_{24})^{(4)}\left(T_{25}^{(2)},s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\}ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}t}\leq \tag{226}$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)}+(a'_{24})^{(4)}+(\widehat{A}_{24})^{(4)}+(\widehat{P}_{24})^{(4)}(\hat{k}_{24})^{(4)}\right)d\left(((G_{27})^{(1)},(T_{27})^{(1)};\ (G_{27})^{(2)},(T_{27})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 16:** The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ *and* $(\widehat{Q}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_+$. (227)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i=24,25,26$ depend only on $T_{25}$ and respectively on

$(G_{27})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 17:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 228

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(4)} - (a_i'')^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(4)} t)} > 0$ for t > 0

**Definition of** $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2\ and\ \left((\widehat{M}_{24})^{(4)}\right)_3$ : 229

**Remark 18:** if $G_{24}$ is bounded, the same property have also $G_{25}\ and\ G_{26}$ . indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a_{25}')^{(4)} G_{25}$ and by integrating

$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1 / (a_{25}')^{(4)}$

In the same way , one can obtain

$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2 / (a_{26}')^{(4)}$

If $G_{25}\ or\ G_{26}$ is bounded, the same property follows for $G_{24}$ , $G_{26}$ and $G_{24}$ , $G_{25}$ respectively.

**Remark 19:** If $G_{24}$ *is* bounded, from below, the same property holds for $G_{25}\ and\ G_{26}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{25}$ is bounded from below. 230

**Remark 20:** If $T_{24}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}\ ((G_{27})(t), t)) = (b_{25}')^{(4)}$ then $T_{25} \to \infty$. 231

**Definition of** $(m)^{(4)}$ and $\varepsilon_4$ :

Indeed let $t_4$ be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t), t) < \varepsilon_4, T_{24}\,(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_4}$ By taking now $\varepsilon_4$ sufficiently small one sees that $T_{25}$ is unbounded.
The same property holds for $T_{26}$ if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t), t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for $G_{29}$ , $G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}$ , $\frac{(b_i)^{(5)}}{(\widehat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)}+\left((\hat{P}_{28})^{(5)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{28})^{(5)}\right]\leq(\hat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$$d\left(((G_{31})^{(1)},(T_{31})^{(1)}),((G_{31})^{(2)},(T_{31})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\widehat{M}_{28})^{(5)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\widehat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{31}}),(\widetilde{T_{31}})$ : $\left((\widetilde{G_{31}}),(\widetilde{T_{31}})\right)=\mathcal{A}^{(5)}\left((G_{31}),(T_{31})\right)$

It results

$$|\tilde{G}_{28}^{(1)}-\tilde{G}_i^{(2)}|\leq\int_0^t(a_{28})^{(5)}|G_{29}^{(1)}-G_{29}^{(2)}|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\,ds_{(28)}+$$

$$\int_0^t\{(a'_{28})^{(5)}|G_{28}^{(1)}-G_{28}^{(2)}|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$(a''_{28})^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)|G_{28}^{(1)}-G_{28}^{(2)}|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$G_{28}^{(2)}|(a''_{28})^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)-(a''_{28})^{(5)}\left(T_{29}^{(2)},s_{(28)}\right)|\;e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\}ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on it follows

$$|(G_{31})^{(1)}-(G_{31})^{(2)}|e^{-(\widehat{M}_{28})^{(5)}t}\leq \quad 237$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)}+(a'_{28})^{(5)}+(\widehat{A}_{28})^{(5)}+(\hat{P}_{28})^{(5)}(\hat{k}_{28})^{(5)}\right)d\left(((G_{31})^{(1)},(T_{31})^{(1)};\;(G_{31})^{(2)},(T_{31})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 21:** The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\hat{P}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ *and* $(\hat{Q}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ respectively of $\mathbb{R}_+$. 238

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on $T_{29}$ and respectively on $(G_{31})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 22:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 239

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(5)} - (a_i'')^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(5)} t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2\ and\ \left((\widehat{M}_{28})^{(5)}\right)_3$ : 240

**Remark 23:** if $G_{28}$ is bounded, the same property have also $G_{29}\ and\ G_{30}$ . indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a_{29}')^{(5)} G_{29}$ and by integrating

$$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1 / (a_{29}')^{(5)}$$

In the same way , one can obtain

$$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2 / (a_{30}')^{(5)}$$

If $G_{29}\ or\ G_{30}$ is bounded, the same property follows for $G_{28}$ , $G_{30}$ and $G_{28}$ , $G_{29}$ respectively.

**Remark 24:** If $G_{28}$ *is* bounded, from below, the same property holds for $G_{29}\ and\ G_{30}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{29}$ is bounded from below. 241

**Remark 25:** If $T_{28}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}\,((G_{31})(t), t)) = (b_{29}')^{(5)}$ then $T_{29} \to \infty$. 242

**Definition of** $(m)^{(5)}$ and $\varepsilon_5$ :

Indeed let $t_5$ be so that for $t > t_5$

$$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t), t) < \varepsilon_5, T_{28}\,(t) > (m)^{(5)}$$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_5}$ By taking now $\varepsilon_5$ sufficiently small one sees that $T_{29}$ is unbounded.
The same property holds for $T_{30}$ if $\lim_{t\to\infty}(b_{30}'')^{(5)}\left((G_{31})(t), t\right) = (b_{30}')^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{33}\,, G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\ , \frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}} < 1$ and to choose 244

$(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[(\hat{P}_{32})^{(6)}+\left((\hat{P}_{32})^{(6)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{32})^{(6)} \quad 245$$

$$\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[\left((\hat{Q}_{32})^{(6)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{32})^{(6)}\right]\leq(\hat{Q}_{32})^{(6)} \quad 246$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric 247

$$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\hat{M}_{32})^{(6)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\hat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{35}}),(\widetilde{T_{35}})$ : $\left((\widetilde{G_{35}}),(\widetilde{T_{35}})\right)=\mathcal{A}^{(6)}\left((G_{35}),(T_{35})\right)$

It results

$$|\tilde{G}_{32}^{(1)}-\tilde{G}_i^{(2)}|\leq\int_0^t(a_{32})^{(6)}|G_{33}^{(1)}-G_{33}^{(2)}|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}ds_{(32)}+$$

$$\int_0^t\{(a'_{32})^{(6)}|G_{32}^{(1)}-G_{32}^{(2)}|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)|G_{32}^{(1)}-G_{32}^{(2)}|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$G_{32}^{(2)}|(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)-(a''_{32})^{(6)}\left(T_{33}^{(2)},s_{(32)}\right)|\ e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\}ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$|(G_{35})^{(1)}-(G_{35})^{(2)}|e^{-(\widehat{M}_{32})^{(6)}t}\leq \quad 248$$
$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)}+(a'_{32})^{(6)}+(\widehat{A}_{32})^{(6)}+(\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}\right)d\left(((G_{35})^{(1)},(T_{35})^{(1)};\ (G_{35})^{(2)},(T_{35})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 26:** The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\hat{P}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ *and* $(\hat{Q}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ respectively of $\mathbb{R}_+$. 249

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i = 32,33,34$ depend only on $T_{33}$ and respectively on $(G_{35})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 27:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 250

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(6)} - (a_i'')^{(6)}(T_{33}(s_{(32)}), s_{(32)})\} ds_{(32)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(6)} t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{32})^{(6)}\right)_1, \left((\widehat{M}_{32})^{(6)}\right)_2\ and\ \left((\widehat{M}_{32})^{(6)}\right)_3$ : 251

**Remark 28:** if $G_{32}$ is bounded, the same property have also $G_{33}\ and\ G_{34}$ . indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a_{33}')^{(6)} G_{33}$ and by integrating

$$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)} \left((\widehat{M}_{32})^{(6)}\right)_1 / (a_{33}')^{(6)}$$

In the same way , one can obtain

$$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)} \left((\widehat{M}_{32})^{(6)}\right)_2 / (a_{34}')^{(6)}$$

If $G_{33}\ or\ G_{34}$ is bounded, the same property follows for $G_{32}$ , $G_{34}$ and $G_{32}$ , $G_{33}$ respectively.

**Remark 29:** If $G_{32}$ *is* bounded, from below, the same property holds for $G_{33}\ and\ G_{34}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{33}$ is bounded from below. 252

**Remark 30:** If $T_{32}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}\ ((G_{35})(t), t)) = (b_{33}')^{(6)}$ then $T_{33} \to \infty$. 253

**Definition of** $(m)^{(6)}$ and $\varepsilon_6$ :

Indeed let $t_6$ be so that for $t > t_6$

$$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t), t\right) < \varepsilon_6, T_{32}\,(t) > (m)^{(6)}$$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right)$, $t = log\,\frac{2}{\varepsilon_6}$ By taking now $\varepsilon_6$ sufficiently small one sees that $T_{33}$ is unbounded.
The same property holds for $T_{34}$ if $\lim_{t\to\infty}(b_{34}'')^{(6)}\left((G_{35})(t), t(t), t\right) = (b_{34}')^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{37}, G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}$ , $\frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$ and to choose

$(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[(\hat{P}_{36})^{(7)}+\left((\hat{P}_{36})^{(7)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{36})^{(7)} \tag{256}$$

$$\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[\left((\hat{Q}_{36})^{(7)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{36})^{(7)}\right]\leq(\hat{Q}_{36})^{(7)} \tag{257}$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself
The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric (258)

$$d\left(((G_{39})^{(1)},(T_{39})^{(1)}),((G_{39})^{(2)},(T_{39})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{36})^{(7)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{39}}),(\widetilde{T_{39}}):\left((\widetilde{G_{39}}),(\widetilde{T_{39}})\right)=\mathcal{A}^{(7)}((G_{39}),(T_{39}))$
It results

$$\left|\tilde{G}_{36}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{36})^{(7)}\left|G_{37}^{(1)}-G_{37}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\,ds_{(36)}+$$
$$\int_0^t\{(a'_{36})^{(7)}\left|G_{36}^{(1)}-G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$(a''_{36})^{(7)}\left(T_{37}^{(1)},s_{(36)}\right)\left|G_{36}^{(1)}-G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$G_{36}^{(2)}\left|(a''_{36})^{(7)}\left(T_{37}^{(1)},s_{(36)}\right)-(a''_{36})^{(7)}\left(T_{37}^{(2)},s_{(36)}\right)\right|\;e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\}ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on it follows

$$\left|(G_{39})^{(1)}-(G_{39})^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}t}\leq \tag{259}$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}}\left((a_{36})^{(7)}+(a'_{36})^{(7)}+(\hat{A}_{36})^{(7)}+(\hat{P}_{36})^{(7)}(\hat{k}_{36})^{(7)}\right)d\left(((G_{39})^{(1)},(T_{39})^{(1)};\;(G_{39})^{(2)},(T_{39})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 31:** The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\hat{P}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ *and* $(\hat{Q}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ respectively of $\mathbb{R}_+$. (260)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(7)}$ and $(b_i'')^{(7)}, i = 36,37,38$ depend only on $T_{37}$ and respectively on $(G_{39})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 32:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 261

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(7)} - (a_i'')^{(7)}(T_{37}(s_{(36)}), s_{(36)})\} ds_{(36)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(7)} t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2\ and\ \left((\widehat{M}_{36})^{(7)}\right)_3$ : 262

**Remark 33:** if $G_{36}$ is bounded, the same property have also $G_{37}\ and\ G_{38}$ . indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a_{37}')^{(7)} G_{37}$ and by integrating

$$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1 / (a_{37}')^{(7)}$$

In the same way , one can obtain

$$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2 / (a_{38}')^{(7)}$$

If $G_{37}\ or\ G_{38}$ is bounded, the same property follows for $G_{36}$ , $G_{38}$ and $G_{36}$ , $G_{37}$ respectively.

**Remark 34:** If $G_{36}$ *is* bounded, from below, the same property holds for $G_{37}\ and\ G_{38}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{37}$ is bounded from below. 263

**Remark 35:** If $T_{36}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(7)}\ ((G_{39})(t), t)) = (b_{37}')^{(7)}$ then $T_{37} \to \infty$. 264

**Definition of** $(m)^{(7)}$ and $\varepsilon_7$ :

Indeed let $t_7$ be so that for $t > t_7$

$$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t), t) < \varepsilon_7, T_{36}\,(t) > (m)^{(7)}$$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_7}$ By taking now $\varepsilon_7$ sufficiently small one sees that $T_{37}$ is unbounded.
The same property holds for $T_{38}$ if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t), t\right) = (b_{38}')^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}$ , $\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}} < 1$ and to choose $(\widehat{P}_{40})^{(8)}$ and $(\widehat{Q}_{40})^{(8)}$ large to have 266

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[(\widehat{P}_{40})^{(8)} + \left((\hat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{40})^{(8)} \quad 267$$

$$\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{40})^{(8)}\right] \leq (\hat{Q}_{40})^{(8)} \quad 268$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)}, (T_{43})^{(1)}), ((G_{43})^{(2)}, (T_{43})^{(2)})\right) = \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

**Definition of** $\widetilde{(G_{43})}, \widetilde{(T_{43})}$ : $\left(\widetilde{(G_{43})}, \widetilde{(T_{43})}\right) = \mathcal{A}^{(8)}((G_{43}), (T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{40})^{(8)}\left|G_{41}^{(1)} - G_{41}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\,ds_{(40)} +$$
$$\int_0^t\{(a'_{40})^{(8)}\left|G_{40}^{(1)} - G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{-(\widehat{M}_{40})^{(8)}s_{(40)}} +$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right)\left|G_{40}^{(1)} - G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}} +$$
$$G_{40}^{(2)}\left|(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right) - (a''_{40})^{(8)}\left(T_{41}^{(2)}, s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\}ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0, t]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)} - (G_{43})^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}t} \leq \quad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)} + (a'_{40})^{(8)} + (\widehat{A}_{40})^{(8)} + (\widehat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}\right)d\left(((G_{43})^{(1)}, (T_{43})^{(1)};\ (G_{43})^{(2)}, (T_{43})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

**Remark 36:** The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate 274

condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ $and$ $(\widehat{Q}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i = 40,41,42$ depend only on $T_{41}$ and respectively on $(G_{43})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 37** There does not exist any $t$ where $G_i(t) = 0\ and\ T_i(t) = 0$ 275

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(8)} - (a_i'')^{(8)}(T_{41}(s_{(40)}), s_{(40)})\} ds_{(40)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(8)}t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{40})^{(8)}\right)_1, \left((\widehat{M}_{40})^{(8)}\right)_2$ $and$ $\left((\widehat{M}_{40})^{(8)}\right)_3$ : 276

**Remark 38:** if $G_{40}$ is bounded, the same property have also $G_{41}\ and\ G_{42}$ . indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a_{41}')^{(8)}G_{41}$ and by integrating

$$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1/(a_{41}')^{(8)}$$

In the same way , one can obtain

$$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2/(a_{42}')^{(8)}$$

If $G_{41}\ or\ G_{42}$ is bounded, the same property follows for $G_{40}$ , $G_{42}$ and $G_{40}$ , $G_{41}$ respectively.

**Remark 39:** If $G_{40}$ $is$ bounded, from below, the same property holds for $G_{41}\ and\ G_{42}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{41}$ is bounded from below. 277

**Remark 40:** If $T_{40}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}((G_{43})(t), t)) = (b_{41}')^{(8)}$ then $T_{41} \to \infty$. 278

**Definition of** $(m)^{(8)}$ and $\varepsilon_8$ :

Indeed let $t_8$ be so that for $t > t_8$

$$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t), t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right)$, $t = log\frac{2}{\varepsilon_8}$ By taking now $\varepsilon_8$ sufficiently small one sees that $T_{41}$ is unbounded.
The same property holds for $T_{42}$ if $\lim_{t\to\infty}(b''_{42})^{(8)}\left((G_{43})(t), t(t), t\right) = (b'_{42})^{(8)}$



It is now sufficient to take $\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}$ , $\frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$ and to choose $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ large to have

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}\left[(\hat{P}_{44})^{(9)} + \left((\hat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}}\left[\left((\hat{Q}_{44})^{(9)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{44})^{(9)}\right] \leq (\hat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)}, (T_{47})^{(1)}), ((G_{47})^{(2)}, (T_{47})^{(2)})\right) =$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{44})^{(9)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{47}}), (\widetilde{T_{47}}) : \left((\widetilde{G_{47}}), (\widetilde{T_{47}})\right) = \mathcal{A}^{(9)}\left((G_{47}), (T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{44})^{(9)}\left|G_{45}^{(1)} - G_{45}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\,ds_{(44)} +$$
$$\int_0^t\{(a'_{44})^{(9)}\left|G_{44}^{(1)} - G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{-(\widehat{M}_{44})^{(9)}s_{(44)}} +$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right)\left|G_{44}^{(1)} - G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}} +$$
$$G_{44}^{(2)}\left|(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right) - (a''_{44})^{(9)}\left(T_{45}^{(2)}, s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\}ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)} - G^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}t} \leq$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)} + (a'_{44})^{(9)} + (\widehat{A}_{44})^{(9)} + (\widehat{P}_{44})^{(9)}(\hat{k}_{44})^{(9)}\right)d\left(((G_{47})^{(1)}, (T_{47})^{(1)};\ (G_{47})^{(2)}, (T_{47})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis (39,35,36) the result follows

**Remark 41:** The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by

$(\widehat{P}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ *and* $(\widehat{Q}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i = 44{,}45{,}46$ depend only on $T_{45}$ and respectively on $(G_{47})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 42:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$

From 99 to 44 it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t\{(a_i')^{(9)}-(a_i'')^{(9)}(T_{45}(s_{(44)}),s_{(44)})\}ds_{(44)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(9)}t)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{44})^{(9)}\right)_1, \left((\widehat{M}_{44})^{(9)}\right)_2$ *and* $\left((\widehat{M}_{44})^{(9)}\right)_3$ :

**Remark 43:** if $G_{44}$ is bounded, the same property have also $G_{45}\ and\ G_{46}$ . indeed if

$G_{44} < (\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt} \leq \left((\widehat{M}_{44})^{(9)}\right)_1 - (a_{45}')^{(9)}G_{45}$ and by integrating

$$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_1/(a_{45}')^{(9)}$$

In the same way , one can obtain

$$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_2/(a_{46}')^{(9)}$$

If $G_{45}\ or\ G_{46}$ is bounded, the same property follows for $G_{44}$ , $G_{46}$ and $G_{44}$ , $G_{45}$ respectively.

**Remark 44:** If $G_{44}$ *is* bounded, from below, the same property holds for $G_{45}\ and\ G_{46}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{45}$ is bounded from below.

**Remark 45:** If $T_{44}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t),t\right) = (b_{45}')^{(9)}$ then $T_{45} \to \infty$.

**Definition of** $(m)^{(9)}$ and $\varepsilon_9$ :

Indeed let $t_9$ be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t),t\right) < \varepsilon_9, T_{44}\,(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right), \quad t = log\frac{2}{\varepsilon_9}$ By taking now $\varepsilon_9$ sufficiently small one sees that $T_{45}$ is unbounded.
The same property holds for $T_{46}$ if $\lim_{t\to\infty}(b_{46}'')^{(9)}\left((G_{47})(t),t\right) = (b_{46}')^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

**Behavior of the solutions of equation** 280

**Theorem** If we denote and define

**Definition of** $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ :

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)} \le -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14}, t) + (a''_{14})^{(1)}(T_{14}, t) \le -(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)} \le -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G, t) - (b''_{14})^{(1)}(G, t) \le -(\tau_1)^{(1)}$

**Definition of** $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$ : 281

By $(\nu_1)^{(1)} > 0$, $(\nu_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$, $(u_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

**Definition of** $(\bar{\nu}_1)^{(1)},, (\bar{\nu}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$ : 282

By $(\bar{\nu}_1)^{(1)} > 0$, $(\bar{\nu}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$, $(\bar{u}_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

**Definition of** $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (\nu_0)^{(1)}$ :- 283

If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by
$(m_2)^{(1)} = (\nu_0)^{(1)}, (m_1)^{(1)} = (\nu_1)^{(1)}$, $if\ (\nu_0)^{(1)} < (\nu_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}, if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)}$,

and $\boxed{(\nu_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)}$, $if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)}$, $if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)}$,

and $\boxed{(u_0)^{(1)} = \frac{T^0_{13}}{T^0_{14}}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$

are defined

Then the solution of global equations satisfies the inequalities 285

$G^0_{13}e^{\left((S_1)^{(1)} - (p_{13})^{(1)}\right)t} \le G_{13}(t) \le G^0_{13}e^{(S_1)^{(1)}t}$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)}-(p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)}t}$$

$$\left(\frac{(a_{15})^{(1)}G_{13}^0}{(m_1)^{(1)}((S_1)^{(1)}-(p_{13})^{(1)}-(S_2)^{(1)})}\left[e^{((S_1)^{(1)}-(p_{13})^{(1)})t} - e^{-(S_2)^{(1)}t}\right] + G_{15}^0 e^{-(S_2)^{(1)}t} \le G_{15}(t) \le \right. \quad (286)$$

$$\left. \frac{(a_{15})^{(1)}G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)}-(a'_{15})^{(1)})}\left[e^{(S_1)^{(1)}t} - e^{-(a'_{15})^{(1)}t}\right] + G_{15}^0 e^{-(a'_{15})^{(1)}t}\right)$$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)}t} \le T_{13}(t) \le T_{13}^0 e^{((R_1)^{(1)}+(r_{13})^{(1)})t}} \quad (287)$$

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)}t} \le T_{13}(t) \le \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)}+(r_{13})^{(1)})t} \quad (288)$$

$$\frac{(b_{15})^{(1)}T_{13}^0}{(\mu_1)^{(1)}((R_1)^{(1)}-(b'_{15})^{(1)})}\left[e^{(R_1)^{(1)}t} - e^{-(b'_{15})^{(1)}t}\right] + T_{15}^0 e^{-(b'_{15})^{(1)}t} \le T_{15}(t) \le \quad (289)$$

$$\frac{(a_{15})^{(1)}T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)}+(r_{13})^{(1)}+(R_2)^{(1)})}\left[e^{((R_1)^{(1)}+(r_{13})^{(1)})t} - e^{-(R_2)^{(1)}t}\right] + T_{15}^0 e^{-(R_2)^{(1)}t}$$

**Definition of** $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:- (290)

Where $(S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$

$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$

$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$

$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$

**Behavior of the solutions of equation** (291)

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ : (292)

$(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ four constants satisfying

$-(\sigma_2)^{(2)} \le -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(T_{17}, t) + (a''_{17})^{(2)}(T_{17}, t) \le -(\sigma_1)^{(2)}$ (293)

$-(\tau_2)^{(2)} \le -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}((G_{19}), t) - (b''_{17})^{(2)}((G_{19}), t) \le -(\tau_1)^{(2)}$ (294)

**Definition of** $(\nu_1)^{(2)}, (\nu_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$ : (295)

By $(\nu_1)^{(2)} > 0$, $(\nu_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0$, $(u_2)^{(2)} < 0$ the roots (296)

of the equations $(a_{17})^{(2)}(\nu^{(2)})^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ (297)

and $(b_{14})^{(2)}(u^{(2)})^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and (298)

**Definition of** $(\bar{\nu}_1)^{(2)}, , (\bar{\nu}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$ : (299)

By $(\bar{\nu}_1)^{(2)} > 0$, $(\bar{\nu}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0$, $(\bar{u}_2)^{(2)} < 0$ the (300)

roots of the equations $(a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 301

and $(b_{17})^{(2)}\left(u^{(2)}\right)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 302

**Definition of** $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ :- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, ***if*** $(u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$, ***if*** $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)}$, ***if*** $(\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)}t}$

$(p_i)^{(2)}$ is defined by equation

$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)}t}$ 311

$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)}t}\right] + G_{18}^0 e^{-(S_2)^{(2)}t} \le G_{18}(t) \le\right.$ 312

$\left.\frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)}t} - e^{-(a'_{18})^{(2)}t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)}t}\right)$

$\boxed{T_{16}^0 e^{(R_1)^{(2)}t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}}$ 313

$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)}t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}$ 314

$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)}t} - e^{-(b'_{18})^{(2)}t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)}t} \le T_{18}(t) \le$ 315

$$\frac{(a_{18})^{(2)}T_{16}^{0}}{(\mu_2)^{(2)}((R_1)^{(2)}+(r_{16})^{(2)}+(R_2)^{(2)})}\left[e^{((R_1)^{(2)}+(r_{16})^{(2)})t}-e^{-(R_2)^{(2)}t}\right]+T_{18}^{0}e^{-(R_2)^{(2)}t}$$

**Definition of** $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$$

$$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)}$$ 318

$$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$$

**Behavior of the solutions** 319

**Theorem 3:** If we denote and define

**Definition of** $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ :

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}\big((G_{23}), t\big) \le -(\tau_1)^{(3)}$$

**Definition of** $(v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$ : 320

By $(v_1)^{(3)} > 0$, $(v_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{v}_1)^{(3)} > 0$, $(\bar{v}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

**Definition of** $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ :- 321

If we define $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (v_0)^{(3)}, (m_1)^{(3)} = (v_1)^{(3)}$, ***if*** $(v_0)^{(3)} < (v_1)^{(3)}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (\bar{v}_1)^{(3)}$, ***if*** $(v_1)^{(3)} < (v_0)^{(3)} < (\bar{v}_1)^{(3)}$,

and $\boxed{(v_0)^{(3)} = \frac{G_{20}^{0}}{G_{21}^{0}}}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_0)^{(3)}$, ***if*** $(\bar{v}_1)^{(3)} < (v_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)},\ \boldsymbol{if}\ (u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}, \boldsymbol{if}\ (u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)},$ and $\boxed{(u_0)^{(3)} = \frac{T_{20}^0}{T_{21}^0}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)},\ \boldsymbol{if}\ (\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G_{20}^0 e^{(S_1)^{(3)} t}$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)} t} \quad 323$$

$$\left(\frac{(a_{22})^{(3)} G_{20}^0}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\left[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)} t}\right] + G_{22}^0 e^{-(S_2)^{(3)} t} \le G_{22}(t) \le \right. \quad 324$$

$$\left. \frac{(a_{22})^{(3)} G_{20}^0}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}\left[e^{(S_1)^{(3)} t} - e^{-(a'_{22})^{(3)} t}\right] + G_{22}^0 e^{-(a'_{22})^{(3)} t}\right)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t}} \quad 325$$

$$\frac{1}{(\mu_1)^{(3)}} T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}} T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)} T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)} - (b'_{22})^{(3)})}\left[e^{(R_1)^{(3)} t} - e^{-(b'_{22})^{(3)} t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)} t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)} T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)} + (r_{20})^{(3)} + (R_2)^{(3)})}\left[e^{((R_1)^{(3)} + (r_{20})^{(3)})t} - e^{-(R_2)^{(3)} t}\right] + T_{22}^0 e^{-(R_2)^{(3)} t}$$

**Definition of** $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

**Behavior of the solutions of equation**

**Theorem:** If we denote and define

**Definition of** $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ :

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$

$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}((G_{27}), t) - (b''_{25})^{(4)}((G_{27}), t) \le -(\tau_1)^{(4)}$

**Definition of** $(v_1)^{(4)}, (v_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, v^{(4)}, u^{(4)}$ : 329

By $(v_1)^{(4)} > 0$ , $(v_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$ , $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_1)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

**Definition of** $(\bar{v}_1)^{(4)},, (\bar{v}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$ : 330

By $(\bar{v}_1)^{(4)} > 0$ , $(\bar{v}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$ , $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

**Definition of** $(m_1)^{(4)}$ , $(m_2)^{(4)}$ , $(\mu_1)^{(4)}, (\mu_2)^{(4)}, (\nu_0)^{(4)}$ :-

If we define $(m_1)^{(4)}$ , $(m_2)^{(4)}$ , $(\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (v_0)^{(4)}, (m_1)^{(4)} = (v_1)^{(4)}$, $\boldsymbol{if}$ $(v_0)^{(4)} < (v_1)^{(4)}$

$(m_2)^{(4)} = (v_1)^{(4)}, (m_1)^{(4)} = (\bar{v}_1)^{(4)}$ , $\boldsymbol{if}$ $(v_4)^{(4)} < (v_0)^{(4)} < (\bar{v}_1)^{(4)}$,
and $\boxed{(v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (v_4)^{(4)}, (m_1)^{(4)} = (v_0)^{(4)}$, $\boldsymbol{if}$ $(\bar{v}_4)^{(4)} < (v_0)^{(4)}$

and analogously 331

$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)}$, $\boldsymbol{if}$ $(u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}$ , $\boldsymbol{if}$ $(u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)}$,
and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}, \boldsymbol{if}$ $(\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities 332

$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)}t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)}t} \quad 333$$

$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)}t}\right] + G_{26}^0 e^{-(S_2)^{(4)}t} \le G_{26}(t) \le \right. \quad 334$$

*(a26)4G240(m2)4(S1)4−(a26′)4e(S1)4t−e−(a26′)4t+ G260e−(a26′)4t*

$$\boxed{T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}}T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}}T_{24}^0 e^{((R_1)^{(4)}+(r_{24})^{(4)})t} \quad 335$$

$$\frac{(b_{26})^{(4)}T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)}-(b'_{26})^{(4)})}\left[e^{(R_1)^{(4)}t} - e^{-(b'_{26})^{(4)}t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)}t} \le T_{26}(t) \le \quad 336$$

$$\frac{(a_{26})^{(4)}T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)}+(r_{24})^{(4)}+(R_2)^{(4)})}\left[e^{((R_1)^{(4)}+(r_{24})^{(4)})t} - e^{-(R_2)^{(4)}t}\right] + T_{26}^0 e^{-(R_2)^{(4)}t}$$

**Definition of** $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

**Behavior of the solutions of equation** 338

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ :

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

**Definition of** $(v_1)^{(5)}, (v_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, v^{(5)}, u^{(5)}$ : 339

By $(v_1)^{(5)} > 0$, $(v_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$, $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(v^{(5)}\big)^2 + (\sigma_1)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

**Definition of** $(\bar{v}_1)^{(5)}, , (\bar{v}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$ : 340

By $(\bar{v}_1)^{(5)} > 0$, $(\bar{v}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$, $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(v^{(5)}\big)^2 + (\sigma_2)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

**Definition of** $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (v_0)^{(5)}$ :-

If we define $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (v_0)^{(5)}, (m_1)^{(5)} = (v_1)^{(5)}$, ***if*** $(v_0)^{(5)} < (v_1)^{(5)}$

$(m_2)^{(5)} = (v_1)^{(5)}, (m_1)^{(5)} = (\bar{v}_1)^{(5)}$, ***if*** $(v_1)^{(5)} < (v_0)^{(5)} < (\bar{v}_1)^{(5)}$,
and $\boxed{(v_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)},\ \boldsymbol{if}\ (u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}, \boldsymbol{if}\ (u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)},$

and $\boxed{(u_0)^{(5)} = \frac{T_{28}^0}{T_{29}^0}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}, \boldsymbol{if}\ (\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{28}(t) \le G_{28}^0 e^{(S_1)^{(5)} t}$

where $(p_i)^{(5)}$ is defined by equation

$\frac{1}{(m_5)^{(5)}} G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}} G_{28}^0 e^{(S_1)^{(5)} t}$ 343

$\left(\frac{(a_{30})^{(5)} G_{28}^0}{(m_1)^{(5)}((S_1)^{(5)} - (p_{28})^{(5)} - (S_2)^{(5)})}\left[e^{((S_1)^{(5)} - (p_{28})^{(5)})t} - e^{-(S_2)^{(5)} t}\right] + G_{30}^0 e^{-(S_2)^{(5)} t} \le G_{30}(t) \le\right.$ 344

*(a30)5G280(m2)5(S1)5−(a30′)5e(S1)5t−e−(a30′)5t+ G300e−(a30′)5t*

$\boxed{T_{28}^0 e^{(R_1)^{(5)} t} \le T_{28}(t) \le T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}}$ 345

$\frac{1}{(\mu_1)^{(5)}} T_{28}^0 e^{(R_1)^{(5)} t} \le T_{28}(t) \le \frac{1}{(\mu_2)^{(5)}} T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}$ 346

$\frac{(b_{30})^{(5)} T_{28}^0}{(\mu_1)^{(5)}((R_1)^{(5)} - (b'_{30})^{(5)})}\left[e^{(R_1)^{(5)} t} - e^{-(b'_{30})^{(5)} t}\right] + T_{30}^0 e^{-(b'_{30})^{(5)} t} \le T_{30}(t) \le$ 347

$\frac{(a_{30})^{(5)} T_{28}^0}{(\mu_2)^{(5)}((R_1)^{(5)} + (r_{28})^{(5)} + (R_2)^{(5)})}\left[e^{((R_1)^{(5)} + (r_{28})^{(5)})t} - e^{-(R_2)^{(5)} t}\right] + T_{30}^0 e^{-(R_2)^{(5)} t}$

**Definition of** $(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$:- 348

Where $(S_1)^{(5)} = (a_{28})^{(5)}(m_2)^{(5)} - (a'_{28})^{(5)}$

$(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}$

$(R_1)^{(5)} = (b_{28})^{(5)}(\mu_2)^{(5)} - (b'_{28})^{(5)}$

$(R_2)^{(5)} = (b'_{30})^{(5)} - (r_{30})^{(5)}$

**Behavior of the solutions of equation** 349

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ :

$(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)} \le -(a'_{32})^{(6)} + (a'_{33})^{(6)} - (a''_{32})^{(6)}(T_{33}, t) + (a''_{33})^{(6)}(T_{33}, t) \le -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)} \le -(b'_{32})^{(6)} + (b'_{33})^{(6)} - (b''_{32})^{(6)}\big((G_{35}), t\big) - (b''_{33})^{(6)}\big((G_{35}), t\big) \le -(\tau_1)^{(6)}$$

**Definition of** $(\nu_1)^{(6)}, (\nu_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, \nu^{(6)}, u^{(6)}$ : 350

By $(\nu_1)^{(6)} > 0$, $(\nu_2)^{(6)} < 0$ and respectively $(u_1)^{(6)} > 0$, $(u_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(6)},, (\bar{\nu}_2)^{(6)}, (\bar{u}_1)^{(6)}, (\bar{u}_2)^{(6)}$ : 351

By $(\bar{\nu}_1)^{(6)} > 0$, $(\bar{\nu}_2)^{(6)} < 0$ and respectively $(\bar{u}_1)^{(6)} > 0$, $(\bar{u}_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$

**Definition of** $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}, (\nu_0)^{(6)}$ :-

If we define $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}$ by

$$(m_2)^{(6)} = (\nu_0)^{(6)}, (m_1)^{(6)} = (\nu_1)^{(6)}, \ \boldsymbol{if}\ (\nu_0)^{(6)} < (\nu_1)^{(6)}$$

$$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}, \boldsymbol{if}\ (\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)},$$

and $\boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}}$

$$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$$

and analogously 352

$$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}, \ \boldsymbol{if}\ (u_0)^{(6)} < (u_1)^{(6)}$$

$$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}, \boldsymbol{if}\ (u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)},$$

and $\boxed{(u_0)^{(6)} = \frac{T^0_{32}}{T^0_{33}}}$

$$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}, \boldsymbol{if}\ (\bar{u}_1)^{(6)} < (u_0)^{(6)} \text{ where } (u_1)^{(6)}, (\bar{u}_1)^{(6)}$$

Then the solution of global equations satisfies the inequalities 353

$$G^0_{32}e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{32}(t) \le G^0_{32}e^{(S_1)^{(6)}t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}}G^0_{32}e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{33}(t) \le \frac{1}{(m_2)^{(6)}}G^0_{32}e^{(S_1)^{(6)}t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)}G^0_{32}}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})}\left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)}t}\right] + G^0_{34}e^{-(S_2)^{(6)}t} \le G_{34}(t) \le \right. \quad 355$$

$$\frac{(a_{34})^{(6)}G_{32}^{0}}{(m_2)^{(6)}((S_1)^{(6)}-(a'_{34})^{(6)})}\left[e^{(S_1)^{(6)}t}-e^{-(a'_{34})^{(6)}t}\right]+\ G_{34}^{0}e^{-(a'_{34})^{(6)}t}\Big)$$

$$\boxed{T_{32}^{0}e^{(R_1)^{(6)}t}\le T_{32}(t)\le T_{32}^{0}e^{((R_1)^{(6)}+(r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}}T_{32}^{0}e^{(R_1)^{(6)}t}\le T_{32}(t)\le\frac{1}{(\mu_2)^{(6)}}T_{32}^{0}e^{((R_1)^{(6)}+(r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)}T_{32}^{0}}{(\mu_1)^{(6)}((R_1)^{(6)}-(b'_{34})^{(6)})}\left[e^{(R_1)^{(6)}t}-e^{-(b'_{34})^{(6)}t}\right]+T_{34}^{0}e^{-(b'_{34})^{(6)}t}\le T_{34}(t)\le \quad 358$$

$$\frac{(a_{34})^{(6)}T_{32}^{0}}{(\mu_2)^{(6)}((R_1)^{(6)}+(r_{32})^{(6)}+(R_2)^{(6)})}\left[e^{((R_1)^{(6)}+(r_{32})^{(6)})t}-e^{-(R_2)^{(6)}t}\right]+T_{34}^{0}e^{-(R_2)^{(6)}t}$$

**Definition of** $(S_1)^{(6)},(S_2)^{(6)},(R_1)^{(6)},(R_2)^{(6)}$:- 359

Where $(S_1)^{(6)}=(a_{32})^{(6)}(m_2)^{(6)}-(a'_{32})^{(6)}$

$(S_2)^{(6)}=(a_{34})^{(6)}-(p_{34})^{(6)}$

$(R_1)^{(6)}=(b_{32})^{(6)}(\mu_2)^{(6)}-(b'_{32})^{(6)}$

$(R_2)^{(6)}=(b'_{34})^{(6)}-(r_{34})^{(6)}$

**Behavior of the solutions of equation**

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(7)},(\sigma_2)^{(7)},(\tau_1)^{(7)},(\tau_2)^{(7)}$ :

$(\sigma_1)^{(7)},(\sigma_2)^{(7)},(\tau_1)^{(7)},(\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)}\le-(a'_{36})^{(7)}+(a'_{37})^{(7)}-(a''_{36})^{(7)}(T_{37},t)+(a''_{37})^{(7)}(T_{37},t)\le-(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)}\le-(b'_{36})^{(7)}+(b'_{37})^{(7)}-(b''_{36})^{(7)}\big((G_{39}),t\big)-(b''_{37})^{(7)}\big((G_{39}),t\big)\le-(\tau_1)^{(7)}$$

**Definition of** $(v_1)^{(7)},(v_2)^{(7)},(u_1)^{(7)},(u_2)^{(7)},v^{(7)},u^{(7)}$ : 361

By $(v_1)^{(7)}>0,(v_2)^{(7)}<0$ and respectively $(u_1)^{(7)}>0,(u_2)^{(7)}<0$ the roots of the equations $(a_{37})^{(7)}\big(v^{(7)}\big)^2+(\sigma_1)^{(7)}v^{(7)}-(a_{36})^{(7)}=0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2+(\tau_1)^{(7)}u^{(7)}-(b_{36})^{(7)}=0$ and

**Definition of** $(\bar{v}_1)^{(7)},,(\bar{v}_2)^{(7)},(\bar{u}_1)^{(7)},(\bar{u}_2)^{(7)}$ : 362

By $(\bar{v}_1)^{(7)}>0,(\bar{v}_2)^{(7)}<0$ and respectively $(\bar{u}_1)^{(7)}>0,(\bar{u}_2)^{(7)}<0$ the roots of the equations $(a_{37})^{(7)}\big(v^{(7)}\big)^2+(\sigma_2)^{(7)}v^{(7)}-(a_{36})^{(7)}=0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2+(\tau_2)^{(7)}u^{(7)}-(b_{36})^{(7)}=0$

**Definition of** $(m_1)^{(7)},(m_2)^{(7)},(\mu_1)^{(7)},(\mu_2)^{(7)},(v_0)^{(7)}$ :-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$$(m_2)^{(7)} = (\nu_0)^{(7)}, (m_1)^{(7)} = (\nu_1)^{(7)}, \ \boldsymbol{if}\ (\nu_0)^{(7)} < (\nu_1)^{(7)}$$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\bar{\nu}_1)^{(7)}, \boldsymbol{if}\ (\nu_1)^{(7)} < (\nu_0)^{(7)} < (\bar{\nu}_1)^{(7)},$$

and $\boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\nu_0)^{(7)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(7)} < (\nu_0)^{(7)}$$

and analogously 363

$$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}, \ \boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$$

and $\boxed{(u_0)^{(7)} = \frac{T_{36}^0}{T_{37}^0}}$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)} \text{ where } (u_1)^{(7)}, (\bar{u}_1)^{(7)}$$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^0 e^{(S_1)^{(7)} t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)} t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)}((S_1)^{(7)} - (p_{36})^{(7)} - (S_2)^{(7)})}\left[e^{((S_1)^{(7)} - (p_{36})^{(7)})t} - e^{-(S_2)^{(7)} t}\right] + G_{38}^0 e^{-(S_2)^{(7)} t} \le G_{38}(t) \le \right. \quad 366$$

$$\left. \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)}((S_1)^{(7)} - (a'_{38})^{(7)})}\left[e^{(S_1)^{(7)} t} - e^{-(a'_{38})^{(7)} t}\right] + G_{38}^0 e^{-(a'_{38})^{(7)} t}\right)$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t} \quad 368$$

$$\frac{(b_{38})^{(7)} T_{36}^0}{(\mu_1)^{(7)}((R_1)^{(7)} - (b'_{38})^{(7)})}\left[e^{(R_1)^{(7)} t} - e^{-(b'_{38})^{(7)} t}\right] + T_{38}^0 e^{-(b'_{38})^{(7)} t} \le T_{38}(t) \le \quad 369$$

$$\frac{(a_{38})^{(7)} T_{36}^0}{(\mu_2)^{(7)}((R_1)^{(7)} + (r_{36})^{(7)} + (R_2)^{(7)})}\left[e^{((R_1)^{(7)} + (r_{36})^{(7)})t} - e^{-(R_2)^{(7)} t}\right] + T_{38}^0 e^{-(R_2)^{(7)} t}$$

**Definition of** $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$

$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$

$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$

**Behavior of the solutions of equation** 371

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ :

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$-(\sigma_2)^{(8)} \le -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \le -(\sigma_1)^{(8)}$

$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\big((G_{43}), t\big) - (b''_{41})^{(8)}\big((G_{43}), t\big) \le -(\tau_1)^{(8)}$

**Definition of** $(\nu_1)^{(8)}, (\nu_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, \nu^{(8)}, u^{(8)}$ : 372

By $(\nu_1)^{(8)} > 0, (\nu_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0, (u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(8)}, , (\bar{\nu}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$ :

By $(\bar{\nu}_1)^{(8)} > 0, (\bar{\nu}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0, (\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

**Definition of** $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}, (\nu_0)^{(8)}$ :-

If we define $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$(m_2)^{(8)} = (\nu_0)^{(8)}, (m_1)^{(8)} = (\nu_1)^{(8)}$, ***if*** $(\nu_0)^{(8)} < (\nu_1)^{(8)}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\bar{\nu}_1)^{(8)}$, ***if*** $(\nu_1)^{(8)} < (\nu_0)^{(8)} < (\bar{\nu}_1)^{(8)}$,
and $\boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\nu_0)^{(8)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(8)} < (\nu_0)^{(8)}$

and analogously 374

$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)},\ \boldsymbol{if}\ (u_0)^{(8)} < (u_1)^{(8)}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}, \boldsymbol{if}\ (u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)},$

and $\boxed{(u_0)^{(8)} = \frac{T_{40}^0}{T_{41}^0}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}, \boldsymbol{if}\ (\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^0 e^{(S_1)^{(8)} t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}} G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}} G_{40}^0 e^{(S_1)^{(8)} t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)} G_{40}^0}{(m_1)^{(8)}((S_1)^{(8)} - (p_{40})^{(8)} - (S_2)^{(8)})}\left[e^{((S_1)^{(8)} - (p_{40})^{(8)})t} - e^{-(S_2)^{(8)} t}\right] + G_{42}^0 e^{-(S_2)^{(8)} t} \le G_{42}(t) \le \right. \quad 377$$

$$\left. \frac{(a_{42})^{(8)} G_{40}^0}{(m_2)^{(8)}((S_1)^{(8)} - (a'_{42})^{(8)})}\left[e^{(S_1)^{(8)} t} - e^{-(a'_{42})^{(8)} t}\right] + G_{42}^0 e^{-(a'_{42})^{(8)} t}\right)$$

$$\boxed{T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}} T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le \frac{1}{(\mu_2)^{(8)}} T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)} T_{40}^0}{(\mu_1)^{(8)}((R_1)^{(8)} - (b'_{42})^{(8)})}\left[e^{(R_1)^{(8)} t} - e^{-(b'_{42})^{(8)} t}\right] + T_{42}^0 e^{-(b'_{42})^{(8)} t} \le T_{42}(t) \le \quad 380$$

$$\frac{(a_{42})^{(8)} T_{40}^0}{(\mu_2)^{(8)}((R_1)^{(8)} + (r_{40})^{(8)} + (R_2)^{(8)})}\left[e^{((R_1)^{(8)} + (r_{40})^{(8)})t} - e^{-(R_2)^{(8)} t}\right] + T_{42}^0 e^{-(R_2)^{(8)} t}$$

**Definition of** $(S_1)^{(8)}, (S_2)^{(8)}, (R_1)^{(8)}, (R_2)^{(8)}$:- 381

Where $(S_1)^{(8)} = (a_{40})^{(8)}(m_2)^{(8)} - (a'_{40})^{(8)}$

$(S_2)^{(8)} = (a_{42})^{(8)} - (p_{42})^{(8)}$

$(R_1)^{(8)} = (b_{40})^{(8)}(\mu_2)^{(8)} - (b'_{40})^{(8)}$

$(R_2)^{(8)} = (b'_{42})^{(8)} - (r_{42})^{(8)}$

**Behavior of the solutions of equation 37 to 92** 382

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ :

$(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ four constants satisfying

$$-(\sigma_2)^{(9)} \le -(a'_{44})^{(9)} + (a'_{45})^{(9)} - (a''_{44})^{(9)}(T_{45}, t) + (a''_{45})^{(9)}(T_{45}, t) \le -(\sigma_1)^{(9)}$$

$$-(\tau_2)^{(9)} \le -(b'_{44})^{(9)} + (b'_{45})^{(9)} - (b''_{44})^{(9)}\big((G_{47}), t\big) - (b''_{45})^{(9)}\big((G_{47}), t\big) \le -(\tau_1)^{(9)}$$

**Definition of** $(\nu_1)^{(9)}, (\nu_2)^{(9)}, (u_1)^{(9)}, (u_2)^{(9)}, \nu^{(9)}, u^{(9)}$ :

By $(\nu_1)^{(9)} > 0$ , $(\nu_2)^{(9)} < 0$ and respectively $(u_1)^{(9)} > 0$ , $(u_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_1)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(9)}, , (\bar{\nu}_2)^{(9)}, (\bar{u}_1)^{(9)}, (\bar{u}_2)^{(9)}$ :

By $(\bar{\nu}_1)^{(9)} > 0$ , $(\bar{\nu}_2)^{(9)} < 0$ and respectively $(\bar{u}_1)^{(9)} > 0$ , $(\bar{u}_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_2)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$

**Definition of** $(m_1)^{(9)}, (m_2)^{(9)}, (\mu_1)^{(9)}, (\mu_2)^{(9)}, (\nu_0)^{(9)}$ :-

If we define $(m_1)^{(9)}, (m_2)^{(9)}, (\mu_1)^{(9)}, (\mu_2)^{(9)}$ by

$$(m_2)^{(9)} = (\nu_0)^{(9)}, (m_1)^{(9)} = (\nu_1)^{(9)}, \ \boldsymbol{if}\ (\nu_0)^{(9)} < (\nu_1)^{(9)}$$

$$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\bar{\nu}_1)^{(9)}, \boldsymbol{if}\ (\nu_1)^{(9)} < (\nu_0)^{(9)} < (\bar{\nu}_1)^{(9)},$$

and $\boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$

$$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\nu_0)^{(9)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(9)} < (\nu_0)^{(9)}$$

and analogously

$$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)}, \ \boldsymbol{if}\ (u_0)^{(9)} < (u_1)^{(9)}$$

$$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}, \boldsymbol{if}\ (u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)},$$

and $\boxed{(u_0)^{(9)} = \frac{T^0_{44}}{T^0_{45}}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}, \boldsymbol{if}\ (\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G^0_{44} e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{44}(t) \le G^0_{44} e^{(S_1)^{(9)} t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}} G_{44}^0 e^{\left((S_1)^{(9)}-(p_{44})^{(9)}\right)t} \le G_{45}(t) \le \frac{1}{(m_2)^{(9)}} G_{44}^0 e^{(S_1)^{(9)}t}$$

(

$$\frac{(a_{46})^{(9)} G_{44}^0}{(m_1)^{(9)}\left((S_1)^{(9)}-(p_{44})^{(9)}-(S_2)^{(9)}\right)}\left[e^{\left((S_1)^{(9)}-(p_{44})^{(9)}\right)t} - e^{-(S_2)^{(9)}t}\right] + G_{46}^0 e^{-(S_2)^{(9)}t} \le G_{46}(t) \le$$

$$\frac{(a_{46})^{(9)} G_{44}^0}{(m_2)^{(9)}\left((S_1)^{(9)}-(a'_{46})^{(9)}\right)}\left[e^{(S_1)^{(9)}t} - e^{-(a'_{46})^{(9)}t}\right] + G_{46}^0 e^{-(a'_{46})^{(9)}t})$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le T_{44}^0 e^{\left((R_1)^{(9)}+(r_{44})^{(9)}\right)t}}$$

$$\frac{1}{(\mu_1)^{(9)}} T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le \frac{1}{(\mu_2)^{(9)}} T_{44}^0 e^{\left((R_1)^{(9)}+(r_{44})^{(9)}\right)t}$$

$$\frac{(b_{46})^{(9)} T_{44}^0}{(\mu_1)^{(9)}\left((R_1)^{(9)}-(b'_{46})^{(9)}\right)}\left[e^{(R_1)^{(9)}t} - e^{-(b'_{46})^{(9)}t}\right] + T_{46}^0 e^{-(b'_{46})^{(9)}t} \le T_{46}(t) \le$$

$$\frac{(a_{46})^{(9)} T_{44}^0}{(\mu_2)^{(9)}\left((R_1)^{(9)}+(r_{44})^{(9)}+(R_2)^{(9)}\right)}\left[e^{\left((R_1)^{(9)}+(r_{44})^{(9)}\right)t} - e^{-(R_2)^{(9)}t}\right] + T_{46}^0 e^{-(R_2)^{(9)}t}$$

**Definition of** $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$

**Proof :** From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right) - (a''_{14})^{(1)}(T_{14}, t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

**Definition of** $v^{(1)}$ :- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)}\right) \le \frac{dv^{(1)}}{dt} \le -\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(1)}, (\nu_0)^{(1)}$ :-

For $0 < \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}} < (\nu_1)^{(1)} < (\bar{\nu}_1)^{(1)}$

$$v^{(1)}(t) \ge \frac{(\nu_1)^{(1)}+(C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_0)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(\nu_1)^{(1)}-(\nu_0)^{(1)}}{(\nu_0)^{(1)}-(\nu_2)^{(1)}}}$$

it follows $(\nu_0)^{(1)} \leq \nu^{(1)}(t) \leq (\nu_1)^{(1)}$

In the same manner , we get 384

$$\nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{\nu}_1)^{(1)}-(\nu_0)^{(1)}}{(\nu_0)^{(1)}-(\bar{\nu}_2)^{(1)}}}$$

From which we deduce $(\nu_0)^{(1)} \leq \nu^{(1)}(t) \leq (\bar{\nu}_1)^{(1)}$

If $0 < (\nu_1)^{(1)} < (\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0} < (\bar{\nu}_1)^{(1)}$ we find like in the previous case, 385

$$(\nu_1)^{(1)} \leq \frac{(\nu_1)^{(1)}+(C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}} \leq \nu^{(1)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\bar{\nu}_1)^{(1)}$$

If $0 < (\nu_1)^{(1)} \leq (\bar{\nu}_1)^{(1)} \leq \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$ , we obtain 386

$$(\nu_1)^{(1)} \leq \nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\nu_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(1)}(t)$ :-

$$(m_2)^{(1)} \leq \nu^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{\nu^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(1)}(t)$ :-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{13})^{(1)} = (a''_{14})^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)}G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)}T_{14}(t)$ This is an important

consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

**Proof** : From global equations we obtain 387

$$\frac{d\nu^{(2)}}{dt} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right) - (a''_{17})^{(2)}(T_{17}, t)\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

**Definition of** $\nu^{(2)}$ :- $\boxed{\nu^{(2)} = \frac{G_{16}}{G_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \le \frac{d\nu^{(2)}}{dt} \le -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

**Definition of** $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$ :-

For $0 < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \ge \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}}{1 + (C)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(C)^{(2)} = \frac{(\nu_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \le \nu^{(2)}(t) \le (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \le \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \le \nu^{(2)}(t) \le (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \le \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}}{1 + (C)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}} \le \nu^{(2)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \le (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \le (\bar{\nu}_1)^{(2)} \le (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$ , we obtain 394

$$(\nu_1)^{(2)} \le \nu^{(2)}(t) \le \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \le (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(2)}(t)$ :- 395

$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}$, $\boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$

In a completely analogous way, we obtain 396

**Definition of** $u^{(2)}(t)$ :-

$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}$, $\boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :** 397

If $(a''_{16})^{(2)} = (a''_{17})^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)} G_{17}(t)$

Analogously if $(b''_{16})^{(2)} = (b''_{17})^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)} T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

**Proof** : From global equations we obtain 398

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right) - (a''_{21})^{(3)}(T_{21}, t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

**Definition of** $\nu^{(3)}$ :- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right) \leq \frac{d\nu^{(3)}}{dt} \leq -\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right)$$

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From which one obtains

For $0 < (\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\nu_1)^{(3)} < (\bar{\nu}_1)^{(3)}$

$$\nu^{(3)}(t) \geq \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(\nu_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\nu_2)^{(3)}}}$$

it follows $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\nu_1)^{(3)}$

In the same manner , we get 401

$$\nu^{(3)}(t) \le \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{\nu}_1)^{(3)}-(\nu_0)^{(3)}}{(\nu_0)^{(3)}-(\bar{\nu}_2)^{(3)}}}$$

**Definition of** $(\bar{\nu}_1)^{(3)}$ :-

From which we deduce $(\nu_0)^{(3)} \le \nu^{(3)}(t) \le (\bar{\nu}_1)^{(3)}$

If $0 < (\nu_1)^{(3)} < (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (\bar{\nu}_1)^{(3)}$ we find like in the previous case, 402

$$(\nu_1)^{(3)} \le \frac{(\nu_1)^{(3)}+(C)^{(3)}(\nu_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}} \le \nu^{(3)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \le (\bar{\nu}_1)^{(3)}$$

If $0 < (\nu_1)^{(3)} \le (\bar{\nu}_1)^{(3)} \le (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}$ , we obtain 403

$$(\nu_1)^{(3)} \le \nu^{(3)}(t) \le \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \le (\nu_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(3)}(t)$ :-

$$(m_2)^{(3)} \le \nu^{(3)}(t) \le (m_1)^{(3)}, \quad \boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(3)}(t)$ :-

$$(\mu_2)^{(3)} \le u^{(3)}(t) \le (\mu_1)^{(3)}, \quad \boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{20}'')^{(3)} = (a_{21}'')^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)}G_{21}(t)$

Analogously if $(b_{20}'')^{(3)} = (b_{21}'')^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)}T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

**Proof** : From global equations we obtain 404

$$\frac{dv^{(4)}}{dt} = (a_{24})^{(4)} - \left((a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right) - (a''_{25})^{(4)}(T_{25},t)v^{(4)} - (a_{25})^{(4)}v^{(4)}$$

**Definition of** $v^{(4)}$ :- $\boxed{v^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)}\right) \le \frac{dv^{(4)}}{dt} \le -\left((a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_4)^{(4)}v^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$ :-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \ge \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \le \nu^{(4)}(t) \le (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \le \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \le \nu^{(4)}(t) \le (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \le \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}}{1 + (C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}} \le \nu^{(4)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \le (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \le (\bar{\nu}_1)^{(4)} \le \boxed{(\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}}}$ , we obtain 407

$$(\nu_1)^{(4)} \le \nu^{(4)}(t) \le \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \le (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(4)}(t)$ :-

$$(m_2)^{(4)} \le \nu^{(4)}(t) \le (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(4)}(t)$ :-

$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}$, $\boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{24})^{(4)} = (a''_{25})^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)} G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b''_{24})^{(4)} = (b''_{25})^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)} T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

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**Proof** : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a'_{28})^{(5)} - (a'_{29})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right) - (a''_{29})^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

**Definition of** $\nu^{(5)}$ :- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$ :-

For $0 < \boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \le \frac{(\nu_1)^{(5)}+(C)^{(5)}(\nu_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)}-(\nu_2)^{(5)}\right)t\right]}}{1+(C)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)}-(\nu_2)^{(5)}\right)t\right]}} \le \nu^{(5)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \le (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \le (\bar{\nu}_1)^{(5)} \le \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain 411

$$(\nu_1)^{(5)} \le \nu^{(5)}(t) \le \frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \le (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(5)}(t)$ :-

$(m_2)^{(5)} \le \nu^{(5)}(t) \le (m_1)^{(5)}$, $\boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$

In a completely analogous way, we obtain

**Definition of** $u^{(5)}(t)$ :-

$(\mu_2)^{(5)} \le u^{(5)}(t) \le (\mu_1)^{(5)}$, $\boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a_{28}'')^{(5)} = (a_{29}'')^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)}G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b_{28}'')^{(5)} = (b_{29}'')^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)}T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

**Proof** : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a_{32}')^{(6)} - (a_{33}')^{(6)} + (a_{32}'')^{(6)}(T_{33},t)\right) - (a_{33}'')^{(6)}(T_{33},t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

**Definition of** $\nu^{(6)}$ :- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \le \frac{d\nu^{(6)}}{dt} \le -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$ :-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \geq \frac{(\nu_1)^{(6)}+(C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_0)^{(6)}\right)t\right]}}{1+(C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)}-(\nu_0)^{(6)}}{(\nu_0)^{(6)}-(\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\nu_1)^{(6)}$

In the same manner , we get 413

$$\nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{\nu}_1)^{(6)}-(\nu_0)^{(6)}}{(\nu_0)^{(6)}-(\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\bar{\nu}_1)^{(6)}$

If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 414

$$(\nu_1)^{(6)} \leq \frac{(\nu_1)^{(6)}+(C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_2)^{(6)}\right)t\right]}}{1+(C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_2)^{(6)}\right)t\right]}} \leq \nu^{(6)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\bar{\nu}_1)^{(6)}$$

If $0 < (\nu_1)^{(6)} \leq (\bar{\nu}_1)^{(6)} \leq \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$ , we obtain 415

$$(\nu_1)^{(6)} \leq \nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(6)}(t)$ :-

$$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}, \quad \boxed{\nu^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(6)}(t)$ :-

$$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}, \quad \boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)}G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)}T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

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**Proof** : From global equations we obtain

$$\frac{dv^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right) - (a''_{37})^{(7)}(T_{37},t)v^{(7)} - (a_{37})^{(7)}v^{(7)}$$

**Definition of** $v^{(7)}$ :- $\boxed{v^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_2)^{(7)}v^{(7)} - (a_{36})^{(7)}\right) \le \frac{dv^{(7)}}{dt} \le -\left((a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_1)^{(7)}v^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

**Definition of** $(\bar{v}_1)^{(7)}, (v_0)^{(7)}$ :-

For $0 < \boxed{(v_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}} < (v_1)^{(7)} < (\bar{v}_1)^{(7)}$

$$v^{(7)}(t) \ge \frac{(v_1)^{(7)} + (C)^{(7)}(v_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_0)^{(7)}\right)t\right]}}{1 + (C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(v_1)^{(7)} - (v_0)^{(7)}}{(v_0)^{(7)} - (v_2)^{(7)}}}$$

it follows $(v_0)^{(7)} \le v^{(7)}(t) \le (v_1)^{(7)}$

In the same manner , we get 417

$$v^{(7)}(t) \le \frac{(\bar{v}_1)^{(7)} + (\bar{C})^{(7)}(\bar{v}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{v}_1)^{(7)} - (v_0)^{(7)}}{(v_0)^{(7)} - (\bar{v}_2)^{(7)}}}$$

From which we deduce $(v_0)^{(7)} \le v^{(7)}(t) \le (\bar{v}_1)^{(7)}$

If $0 < (v_1)^{(7)} < (v_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}} < (\bar{v}_1)^{(7)}$ we find like in the previous case, 418

$$(v_1)^{(7)} \le \frac{(v_1)^{(7)} + (C)^{(7)}(v_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_2)^{(7)}\right)t\right]}}{1 + (C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_2)^{(7)}\right)t\right]}} \le v^{(7)}(t) \le$$

$$\frac{(\bar{v}_1)^{(7)} + (\bar{C})^{(7)}(\bar{v}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}} \le (\bar{v}_1)^{(7)}$$

If $0 < (v_1)^{(7)} \le (\bar{v}_1)^{(7)} \le \boxed{(v_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$ , we obtain 419

$$(v_1)^{(7)} \le v^{(7)}(t) \le \frac{(\bar{v}_1)^{(7)} + (\bar{C})^{(7)}(\bar{v}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}} \le (v_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $v^{(7)}(t)$ :-

$$(m_2)^{(7)} \leq v^{(7)}(t) \leq (m_1)^{(7)}, \quad \boxed{v^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(7)}(t)$ :- 420

$$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}, \quad \boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $v^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)} G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)} T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

**Proof** : From global equations we obtain 421

$$\frac{dv^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right) - (a''_{41})^{(8)}(T_{41}, t) v^{(8)} - (a_{41})^{(8)} v^{(8)}$$

**Definition of** $v^{(8)}$ :- $\boxed{v^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_2)^{(8)} v^{(8)} - (a_{40})^{(8)}\right) \leq \frac{dv^{(8)}}{dt} \leq -\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_1)^{(8)} v^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$ :-

$$\text{For } 0 < \boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$$

$$v^{(8)}(t) \geq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)}-(\nu_0)^{(8)}}{(\nu_0)^{(8)}-(\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \leq \frac{(\nu_1)^{(8)}+(C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}}{1+(C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}} \leq \nu^{(8)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \leq (\bar{\nu}_1)^{(8)} \leq \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$ , we obtain 424

$$(\nu_1)^{(8)} \leq \nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(8)}(t)$ :-

$$(m_2)^{(8)} \leq \nu^{(8)}(t) \leq (m_1)^{(8)}, \quad \boxed{\nu^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(8)}(t)$ :-

$$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}, \quad \boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $\nu^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)}G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then $(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)}T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

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**Proof** : From 99,20,44,22,23,44 we obtain A

$$\frac{dv^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right) - (a''_{45})^{(9)}(T_{45},t)v^{(9)} - (a_{45})^{(9)}v^{(9)}$$

**Definition of** $v^{(9)}$ :- $\boxed{v^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)}\right) \le \frac{dv^{(9)}}{dt} \le -\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_1)^{(9)}v^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$ :-

For $0 < \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$

$$v^{(9)}(t) \ge \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \le v^{(9)}(t) \le (\nu_9)^{(9)}$

In the same manner , we get

$$v^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \le v^{(9)}(t) \le (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \le \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}}{1 + (C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}} \le v^{(9)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \le (\bar{\nu}_1)^{(9)} \le \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$ , we obtain

$$(\nu_1)^{(9)} \le v^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $v^{(9)}(t)$ :-

$$(m_2)^{(9)} \le v^{(9)}(t) \le (m_1)^{(9)}, \quad \boxed{v^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(9)}(t)$ :-

$(\mu_2)^{(9)} \leq u^{(9)}(t) \leq (\mu_1)^{(9)}$, $\boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)} G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then
$(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)} T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following 425

**Theorem :** If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on $t$ , and the conditions with the notations

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0$ ,

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$

$with\ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a''_i)^{(2)} and\ (b''_i)^{(2)}$ are independent on t , and the conditions with the notations 426

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$ 427

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 428

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$ , 429

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 430

$with\ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a''_i)^{(3)} and\ (b''_i)^{(3)}$ are independent on $t$ , and the conditions with the notations 431

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$ ,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

**Theorem :** If $(a_i'')^{(4)} and\ (b_i'')^{(4)}$ are independent on $t$ , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$ ,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(5)} and\ (b_i'')^{(5)}$ are independent on $t$ , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$ ,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

**Theorem** If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on $t$ , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$ ,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(7)} and\ (b_i'')^{(7)}$ are independent on $t$ , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$ ,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(8)} and\ (b_i'')^{(8)}$ are independent on $t$ , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$ ,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(9)} and\ (b_i'')^{(9)}$ are independent on $t$ , and the conditions (with the notations 436 A
45,46,27,28)

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$ ,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0$ 437

$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0$ 438

$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0$ 439

$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0$ 440

$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ]T_{14} = 0$ 441

$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0$ 442

has a unique positive solution , which is an equilibrium solution for the system

$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0$ 443

$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0$ 444

$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0$ 445

$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\ ]T_{16} = 0$ 446

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\,]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\,]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\,]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\,]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\,]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\big((G_{27})\big)\,]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\big((G_{27})\big)\,]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\big((G_{27})\big)\,]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})\,]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})\,]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})\,]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})\,]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})\,]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})\,]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0 \quad 479$$

$$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0 \quad 480$$

$$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0 \quad 481$$

$$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0 \quad 482$$

$$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0 \quad 483$$

$$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0 \quad 484$$

$$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0 \quad 484\text{A}$$

$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$

$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$

$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$

$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$

$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$

**Proof:** 485

(a) Indeed the first two equations have a nontrivial solution $G_{13}, G_{14}$ if

$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$

**Proof:** 486

(aa) Indeed the first two equations have a nontrivial solution $G_{16}, G_{17}$ if

$\mathrm{F}(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$

**Proof:** 487

(a) Indeed the first two equations have a nontrivial solution $G_{20}, G_{21}$ if

$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$

**Proof:** 488

(a) Indeed the first two equations have a nontrivial solution $G_{24}, G_{25}$ if

$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$

**Proof:** 489

(a) Indeed the first two equations have a nontrivial solution $G_{28}, G_{29}$ if

$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$

**Proof:** 490

(a) Indeed the first two equations have a nontrivial solution $G_{32}, G_{33}$ if

$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) +$

$(a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$

**Proof:** 491

(a) Indeed the first two equations have a nontrivial solution $G_{36}, G_{37}$ if

$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$

**Proof:** 492

(a) Indeed the first two equations have a nontrivial solution $G_{40}, G_{41}$ if

$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$

**Proof:** 492 A

(a) Indeed the first two equations have a nontrivial solution $G_{44}, G_{45}$ if

$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$

**Definition and uniqueness of** $T^*_{14}$ :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique $T^*_{14}$ for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

**Definition and uniqueness of** $T^*_{17}$ :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique $T^*_{17}$ for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]}$$ 495

**Definition and uniqueness of** $T^*_{21}$ :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique $T^*_{21}$ for which $f(T^*_{21}) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

**Definition and uniqueness of** $T^*_{25}$ :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that

there exists a unique $T_{25}^{*}$ for which $f(T_{25}^{*})=0$. With this value , we obtain from the three first equations

$$G_{24}=\frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}=\frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T_{25}^{*})]}$$

**Definition and uniqueness of** $T_{29}^{*}$ :- 498

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(5)}(T_{29})$ being increasing, it follows that there exists a unique $T_{29}^{*}$ for which $f(T_{29}^{*})=0$. With this value , we obtain from the three first equations

$$G_{28}=\frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}=\frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T_{29}^{*})]}$$

**Definition and uniqueness of** $T_{33}^{*}$ :- 499

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(6)}(T_{33})$ being increasing, it follows that there exists a unique $T_{33}^{*}$ for which $f(T_{33}^{*})=0$. With this value , we obtain from the three first equations

$$G_{32}=\frac{(a_{32})^{(6)}G_{33}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^{*})]} \quad , \quad G_{34}=\frac{(a_{34})^{(6)}G_{33}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^{*})]}$$

**Definition and uniqueness of** $T_{37}^{*}$ :- 500

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(7)}(T_{37})$ being increasing, it follows that there exists a unique $T_{37}^{*}$ for which $f(T_{37}^{*})=0$. With this value , we obtain from the three first equations

$$G_{36}=\frac{(a_{36})^{(7)}G_{37}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^{*})]} \quad , \quad G_{38}=\frac{(a_{38})^{(7)}G_{37}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^{*})]}$$

**Definition and uniqueness of** $T_{41}^{*}$ :- 501

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique $T_{41}^{*}$ for which $f(T_{41}^{*})=0$. With this value , we obtain from the three first equations

$$G_{40}=\frac{(a_{40})^{(8)}G_{41}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T_{41}^{*})]} \quad , \quad G_{42}=\frac{(a_{42})^{(8)}G_{41}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T_{41}^{*})]}$$

**Definition and uniqueness of** $T_{45}^{*}$ :- 501 A

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(9)}(T_{45})$ being increasing, it follows that there exists a unique $T_{45}^{*}$ for which $f(T_{45}^{*})=0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)} G_{45}}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T^*_{45})]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)} G_{45}}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T^*_{45})]}$$

By the same argument, the equations admit solutions $G_{13}, G_{14}$ if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right] + (b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{14}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{14}$ such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions $G_{16}, G_{17}$ if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right] + (b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ 504
is a decreasing function in $G_{17}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{14}$ such that $\varphi((G_{19})^*) = 0$

By the same argument, the equations admit solutions $G_{20}, G_{21}$ if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right] + (b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{21}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{21}$ such that $\varphi((G_{23})^*) = 0$

By the same argument, the equations admit solutions $G_{24}, G_{25}$ if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right] + (b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{25}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{25}$ such that $\varphi((G_{27})^*) = 0$

By the same argument, the equations admit solutions $G_{28}, G_{29}$ if 507

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right] + (b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{29}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that

there exists a unique $G_{29}^{*}$ such that $\varphi((G_{31})^{*}) = 0$

By the same argument, the equations admit solutions $G_{32}, G_{33}$ if 508
$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$

$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right] + (b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{33}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{33}^{*}$ such that $\varphi(G_{35}{}^{*}) = 0$

By the same argument, the equations admit solutions $G_{36}, G_{37}$ if 509
$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$
$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right] + (b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{37}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{37}^{*}$ such that $\varphi(G_{39}{}^{*}) = 0$

By the same argument, the equations admit solutions $G_{40}, G_{41}$ if 510
$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$
$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right] + (b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{41}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{41}^{*}$ such that $\varphi(G_{43}{}^{*}) = 0$

By the same argument, the equations 92,93 admit solutions $G_{44}, G_{45}$ if
$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$

$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right] + (b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{45}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{45}^{*}$ such that $\varphi((G_{47})^{*}) = 0$

Finally we obtain the unique solution 511

$G_{14}^{*}$ given by $\varphi(G^{*}) = 0$ , $T_{14}^{*}$ given by $f(T_{14}^{*}) = 0$ and

$$G_{13}^{*} = \frac{(a_{13})^{(1)}G_{14}^{*}}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^{*})]} \quad , \quad G_{15}^{*} = \frac{(a_{15})^{(1)}G_{14}^{*}}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^{*})]}$$

$$T_{13}^{*} = \frac{(b_{13})^{(1)}T_{14}^{*}}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^{*})]} \quad , \quad T_{15}^{*} = \frac{(b_{15})^{(1)}T_{14}^{*}}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^{*})]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

$G_{17}^{*}$ given by $\varphi((G_{19})^{*}) = 0$ , $T_{17}^{*}$ given by $f(T_{17}^{*}) = 0$ and 512

$$G_{16}^{*} = \frac{(a_{16})^{(2)} G_{17}^{*}}{[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}^{*})]} \quad , \quad G_{18}^{*} = \frac{(a_{18})^{(2)} G_{17}^{*}}{[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}^{*})]} \qquad 513$$

$$T_{16}^{*} = \frac{(b_{16})^{(2)} T_{17}^{*}}{[(b'_{16})^{(2)} - (b''_{16})^{(2)}((G_{19})^{*})]} \quad , \quad T_{18}^{*} = \frac{(b_{18})^{(2)} T_{17}^{*}}{[(b'_{18})^{(2)} - (b''_{18})^{(2)}((G_{19})^{*})]} \qquad 514$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

$G_{21}^{*}$ given by $\varphi((G_{23})^{*}) = 0$ , $T_{21}^{*}$ given by $f(T_{21}^{*}) = 0$ and

$$G_{20}^{*} = \frac{(a_{20})^{(3)} G_{21}^{*}}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^{*})]} \quad , \quad G_{22}^{*} = \frac{(a_{22})^{(3)} G_{21}^{*}}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^{*})]}$$

$$T_{20}^{*} = \frac{(b_{20})^{(3)} T_{21}^{*}}{[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}{}^{*})]} \quad , \quad T_{22}^{*} = \frac{(b_{22})^{(3)} T_{21}^{*}}{[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}{}^{*})]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

$G_{25}^{*}$ given by $\varphi(G_{27}) = 0$ , $T_{25}^{*}$ given by $f(T_{25}^{*}) = 0$ and

$$G_{24}^{*} = \frac{(a_{24})^{(4)} G_{25}^{*}}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}^{*} = \frac{(a_{26})^{(4)} G_{25}^{*}}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^{*})]}$$

$$T_{24}^{*} = \frac{(b_{24})^{(4)} T_{25}^{*}}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^{*})]} \quad , \quad T_{26}^{*} = \frac{(b_{26})^{(4)} T_{25}^{*}}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^{*})]} \qquad 517$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

$G_{29}^{*}$ given by $\varphi((G_{31})^{*}) = 0$ , $T_{29}^{*}$ given by $f(T_{29}^{*}) = 0$ and

$$G_{28}^{*} = \frac{(a_{28})^{(5)} G_{29}^{*}}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}^{*} = \frac{(a_{30})^{(5)} G_{29}^{*}}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^{*})]}$$

$$T_{28}^{*} = \frac{(b_{28})^{(5)} T_{29}^{*}}{[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31})^{*})]} \quad , \quad T_{30}^{*} = \frac{(b_{30})^{(5)} T_{29}^{*}}{[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31})^{*})]} \qquad 519$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

$G_{33}^{*}$ given by $\varphi((G_{35})^{*}) = 0$ , $T_{33}^{*}$ given by $f(T_{33}^{*}) = 0$ and

$$G_{32}^{*} = \frac{(a_{32})^{(6)} G_{33}^{*}}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^{*})]} \quad , \quad G_{34}^{*} = \frac{(a_{34})^{(6)} G_{33}^{*}}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^{*})]}$$

$$T_{32}^{*} = \frac{(b_{32})^{(6)} T_{33}^{*}}{[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35})^{*})]} \quad , \quad T_{34}^{*} = \frac{(b_{34})^{(6)} T_{33}^{*}}{[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35})^{*})]} \qquad 521$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

$G_{37}^*$ given by $\varphi((G_{39})^*) = 0$ , $T_{37}^*$ given by $f(T_{37}^*) = 0$ and

$$G_{36}^* = \frac{(a_{36})^{(7)} G_{37}^*}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^*)]} \quad , \quad G_{38}^* = \frac{(a_{38})^{(7)} G_{37}^*}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^*)]}$$

$$T_{36}^* = \frac{(b_{36})^{(7)} T_{37}^*}{[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39})^*)]} \quad , \quad T_{38}^* = \frac{(b_{38})^{(7)} T_{37}^*}{[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39})^*)]}$$

Finally we obtain the unique solution 523

$G_{41}^*$ given by $\varphi((G_{43})^*) = 0$ , $T_{41}^*$ given by $f(T_{41}^*) = 0$ and

$$G_{40}^* = \frac{(a_{40})^{(8)} G_{41}^*}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42}^* = \frac{(a_{42})^{(8)} G_{41}^*}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

$$T_{40}^* = \frac{(b_{40})^{(8)} T_{41}^*}{[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43})^*)]} \quad , \quad T_{42}^* = \frac{(b_{42})^{(8)} T_{41}^*}{[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43})^*)]}$$

Finally we obtain the unique solution of 89 to 99 523 A

$G_{45}^*$ given by $\varphi((G_{47})^*) = 0$ , $T_{45}^*$ given by $f(T_{45}^*) = 0$ and

$$G_{44}^* = \frac{(a_{44})^{(9)} G_{45}^*}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46}^* = \frac{(a_{46})^{(9)} G_{45}^*}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

$$T_{44}^* = \frac{(b_{44})^{(9)} T_{45}^*}{[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47})^*)]} \quad , \quad T_{46}^* = \frac{(b_{46})^{(9)} T_{45}^*}{[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47})^*)]}$$

## ASYMPTOTIC STABILITY ANALYSIS 524

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

**Proof:** Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a''_{14})^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \frac{\partial (b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a'_{13})^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)}G^*_{13}\mathbb{T}_{14} \quad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a'_{14})^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)}G^*_{14}\mathbb{T}_{14} \quad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a'_{15})^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)}G^*_{15}\mathbb{T}_{14} \quad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b'_{13})^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)}T^*_{13}\mathbb{G}_j\right) \quad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b'_{14})^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)}T^*_{14}\mathbb{G}_j\right) \quad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b'_{15})^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T^*_{15}\mathbb{G}_j\right) \quad 530$$

**ASYMPTOTIC STABILITY ANALYSIS** 531

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(2)}$ and $(b''_i)^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G^*_i + \mathbb{G}_i \quad , T_i = T^*_i + \mathbb{T}_i \quad 532$$

$$\frac{\partial(a''_{17})^{(2)}}{\partial T_{17}}(T^*_{17}) = (q_{17})^{(2)} \quad , \frac{\partial(b''_i)^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \quad 533$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a'_{16})^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G^*_{16}\mathbb{T}_{17} \quad 534$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a'_{17})^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G^*_{17}\mathbb{T}_{17} \quad 535$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a'_{18})^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G^*_{18}\mathbb{T}_{17} \quad 536$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b'_{16})^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T^*_{16}\mathbb{G}_j\right) \quad 537$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b'_{17})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}T^*_{17}\mathbb{G}_j\right) \quad 538$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b'_{18})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}T^*_{18}\mathbb{G}_j\right) \quad 539$$

**ASYMPTOTIC STABILITY ANALYSIS** 540

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(3)}$ *and* $(b''_i)^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{21}'')^{(3)}}{\partial T_{21}}(T_{21}^*) = (q_{21})^{(3)} \quad , \frac{\partial (b_i'')^{(3)}}{\partial G_j}(\,(G_{23})^*\,) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\big((a_{20}')^{(3)} + (p_{20})^{(3)}\big)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G_{20}^*\mathbb{T}_{21} \qquad 541$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\big((a_{21}')^{(3)} + (p_{21})^{(3)}\big)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G_{21}^*\mathbb{T}_{21} \qquad 542$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\big((a_{22}')^{(3)} + (p_{22})^{(3)}\big)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G_{22}^*\mathbb{T}_{21} \qquad 543$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\big((b_{20}')^{(3)} - (r_{20})^{(3)}\big)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\big(s_{(20)(j)}T_{20}^*\mathbb{G}_j\big) \qquad 544$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\big((b_{21}')^{(3)} - (r_{21})^{(3)}\big)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\big(s_{(21)(j)}T_{21}^*\mathbb{G}_j\big) \qquad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\big((b_{22}')^{(3)} - (r_{22})^{(3)}\big)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\big(s_{(22)(j)}T_{22}^*\mathbb{G}_j\big) \qquad 546$$

**ASYMPTOTIC STABILITY ANALYSIS** 547

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}\ and\ (b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{25}'')^{(4)}}{\partial T_{25}}(T_{25}^*) = (q_{25})^{(4)} \quad , \frac{\partial (b_i'')^{(4)}}{\partial G_j}((G_{27})^*\,) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\big((a_{24}')^{(4)} + (p_{24})^{(4)}\big)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G_{24}^*\mathbb{T}_{25} \qquad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\big((a_{25}')^{(4)} + (p_{25})^{(4)}\big)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G_{25}^*\mathbb{T}_{25} \qquad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\big((a_{26}')^{(4)} + (p_{26})^{(4)}\big)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G_{26}^*\mathbb{T}_{25} \qquad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\big((b_{24}')^{(4)} - (r_{24})^{(4)}\big)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\big(s_{(24)(j)}T_{24}^*\mathbb{G}_j\big) \qquad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\big((b_{25}')^{(4)} - (r_{25})^{(4)}\big)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\big(s_{(25)(j)}T_{25}^*\mathbb{G}_j\big) \qquad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T^*_{26}\mathbb{G}_j\right) \quad 554$$

**ASYMPTOTIC STABILITY ANALYSIS** 555

**Theorem 5:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}\ and\ (b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 556

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{29})^{(5)}}{\partial T_{29}}(T^*_{29}) = (q_{29})^{(5)} \ , \ \frac{\partial(b_i'')^{(5)}}{\partial G_j}((G_{31})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a'_{28})^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G^*_{28}\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a'_{29})^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G^*_{29}\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a'_{30})^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G^*_{30}\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b'_{28})^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T^*_{28}\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b'_{29})^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T^*_{29}\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b'_{30})^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T^*_{30}\mathbb{G}_j\right) \quad 562$$

**ASYMPTOTIC STABILITY ANALYSIS** 563

**Theorem 6:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}\ and\ (b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{33})^{(6)}}{\partial T_{33}}(T^*_{33}) = (q_{33})^{(6)} \ , \ \frac{\partial(b_i'')^{(6)}}{\partial G_j}((G_{35})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a'_{32})^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G^*_{32}\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a'_{33})^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G^*_{33}\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a'_{34})^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G^*_{34}\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b'_{32})^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T^*_{32}\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b'_{33})^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T^*_{33}\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b'_{34})^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T^*_{34}\mathbb{G}_j\right) \quad 570$$

**ASYMPTOTIC STABILITY ANALYSIS** 571

**Theorem 7:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 572

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{37})^{(7)}}{\partial T_{37}}(T^*_{37}) = (q_{37})^{(7)} \quad , \frac{\partial(b_i'')^{(7)}}{\partial G_j}\left((G_{39})^{**}\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a'_{36})^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G^*_{36}\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a'_{37})^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G^*_{37}\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a'_{38})^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G^*_{38}\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b'_{36})^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T^*_{36}\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b'_{37})^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T^*_{37}\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b'_{38})^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T^*_{38}\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

**ASYMPTOTIC STABILITY ANALYSIS**

**Theorem 8:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{41}'')^{(8)}}{\partial T_{41}}(T_{41}^*) = (q_{41})^{(8)} \quad , \frac{\partial (b_i'')^{(8)}}{\partial G_j}\left( (G_{43})^* \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a_{40}')^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G_{40}^*\mathbb{T}_{41} \quad (581)$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a_{41}')^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G_{41}^*\mathbb{T}_{41} \quad (582)$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a_{42}')^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G_{42}^*\mathbb{T}_{41} \quad (583)$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b_{40}')^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T_{40}^*\mathbb{G}_j\right) \quad (584)$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b_{41}')^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T_{41}^*\mathbb{G}_j\right) \quad (585)$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b_{42}')^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T_{42}^*\mathbb{G}_j\right) \quad (586)$$

(586 A)

**ASYMPTOTIC STABILITY ANALYSIS**

**Theorem 9:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}$ *and* $(b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.
**Proof:** Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{45}'')^{(9)}}{\partial T_{45}}(T_{45}^*) = (q_{45})^{(9)} \quad , \frac{\partial (b_i'')^{(9)}}{\partial G_j}\left( (G_{47})^* \right) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a_{44}')^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)}G_{44}^*\mathbb{T}_{45} \quad (586\,B)$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a_{45}')^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)}G_{45}^*\mathbb{T}_{45} \quad (586\,C)$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a_{46}')^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)}G_{46}^*\mathbb{T}_{45} \quad (586\,D)$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b_{44}')^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)}T_{44}^*\mathbb{G}_j\right) \quad (586\,E)$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b_{45}')^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)}T_{45}^*\mathbb{G}_j\right) \quad (586\,F)$$

$\frac{d\mathbb{T}_{46}}{dt} = -\left((b'_{46})^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T^*_{46}\mathbb{G}_j\right)$ 586

G

587

**The characteristic equation of this system is**

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(q_{13})^{(1)}G^*_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)}T^*_{14} + (b_{14})^{(1)}s_{(13),(14)}T^*_{14}\right)$$

$$+\left(\left((\lambda)^{(1)} + (a'_{14})^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)}G^*_{13} + (a_{13})^{(1)}(q_{14})^{(1)}G^*_{14}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)}T^*_{14} + (b_{14})^{(1)}s_{(13),(13)}T^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((b'_{13})^{(1)} + (b'_{14})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(15)}T^*_{14} + (b_{14})^{(1)}s_{(13),(15)}T^*_{13}\right)\} = 0$$

$+$

$$(\lambda)^{(2)} + (b'_{18})^{(2)} - (r_{18})^{(2)}\{\left((\lambda)^{(2)} + (a'_{18})^{(2)} + (p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^*_{17} + (a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^*_{17}\right)$$

$$+\left(\left((\lambda)^{(2)} + (a'_{17})^{(2)} + (p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^*_{16} + (a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}\right)$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2 + \left((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2 + \left((b'_{16})^{(2)} + (b'_{17})^{(2)} - (r_{16})^{(2)} + (r_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$+\left(\left((\lambda)^{(2)}\right)^2 + \left((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17} + (a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^{*}_{16}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\right)\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left(\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)\right]$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(21)}T^{*}_{21}\right)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^{*}_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^{*}_{21}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(20)}T^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$+\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(22)}T^{*}_{20}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\right)\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left(\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)\right]$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(25)}T^{*}_{25}\right)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^{*}_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^{*}_{25}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(24)}T^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$+\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(26)}T^{*}_{24}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\right)\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(29)}T^{*}_{29}\right)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^{*}_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^{*}_{29}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(28)}T^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$+\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(30)}T^{*}_{28}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\right)\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left(\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)\right]$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(33)}T^{*}_{33}\right)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^{*}_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^{*}_{33}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(32)}T^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$+\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(34)}T^{*}_{32}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\right)\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)\right]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(37)}T^{*}_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^{*}_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^{*}_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(36)}T^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$+\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(38)}T^{*}_{36}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\right)\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(41)}T^{*}_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^{*}_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^{*}_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(40)}T^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$+\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(42)}T^{*}_{40}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\right)\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(45)}T^{*}_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^{*}_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^{*}_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(44)}T^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$+\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(46)}T^{*}_{44}\right)\}=0$$

**And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.**

## SECTION THIRTY

## Subjective Reports And Objective Physiological Data During Conscious Perception

### INTRODUCTION—VARIABLES USED

**A neuronal network model linking subjective reports and objective physiological data during conscious perception Stanislas Dehaene, Claire Sergent, and Jean-Pierre Changeux**

(1) Authors use the model to simulate a classical psychological paradigm: the attentional blink
(2) In addition to reproducing (eb) the main objective and subjective features of this paradigm, the

model predicts (eb) an unique property of nonlinear transition from nonconscious processing to (e&eb) subjective perception.

(3) This all-or-none dynamics of conscious perception was verified behaviorally in (eb) human subjects

**Exploring "fringe" consciousness: The subjective experience of perceptual fluency and its objective bases Rolf Rebera, Pascal Wurtzb, Thomas D Zimmermann b Exploring "fringe" consciousness: The subjective experience of perceptual fluency and its objective bases Rolf Rebera, Pascal Wurtzb, Thomas D Zimmermann b**

(4) Perceptual fluency is (=) the subjective experience of ease with which an incoming stimulus is processed.

(5) Although perceptual fluency is assessed by speed of processing, it remains unclear how objective speed is related to (e&eb) subjective experiences of fluency.

(6) Authors present evidence that speed at different stages of the perceptual process contributes to perceptual fluency.

(7) In an experiment, figure-ground contrast influenced detection of briefly presented words, but not their identification at longer exposure durations.

(8) Conversely, font in which the word was written influenced identification, but not detection. Both contrast and font influenced subjective fluency.

(9) These findings suggest that speed of processing at different stages condensed into a unified subjective experience of perceptual fluency.

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**Context effects in recognition memory: The role of familiarity and recollection Original Research Article Pages 20-38 Wendy A. McKenzie, Guy Tiberghien**

**(11)** A variant of the process dissociation procedure was coupled with a manipulation of response signal lag to assess whether manipulations of context affect one or both of the familiarity and search processes described by the dual process model of recognition.

**(12)** Participants studied a list of word pairs (context + target) followed by a recognition test with target words presented in the same or different context, and in the same or different form as study (singular/plural).

**(13)** Participants were asked to recognize any target word regardless of changes to form (inclusion), or to only recognise words that were presented in the same form (exclusion).

**(14)** The standard context reinstatement effect was evident even at the short response lags.

**(15)** Analyses of the estimates of the contributions of familiarity and search processes suggest that the context effect demonstrated here can be attributed in part to the influence of familiarity on recognition, whereas the effect on recollection was less clear.

## NOTATION

### Module One

Authors use the model to simulate a classical psychological paradigm: the attentional blink

In addition to reproducing (eb) the main objective and subjective features of this paradigm, the model predicts (eb) an unique property of nonlinear transition from nonconscious processing to (e&eb) subjective

perception

$G_{13}$ : Category one of nonconscious processing to (e&eb) subjective perception

$G_{14}$ : Category two of SAS

$G_{15}$ : Category three of SAS

$T_{13}$ : Category one of unique property of nonlinear transition

$T_{14}$ : Category two of SAS

$T_{15}$ : Category three of SAS

**Module Two**

Authors use the model to simulate a classical psychological paradigm: the attentional blink

In addition to reproducing (eb) the main objective and subjective features of this paradigm, the model predicts (eb) an **unique property of nonlinear transition from nonconscious processing** to (e&eb) subjective perception

$G_{16}$ : Category one of **unique property of nonlinear transition from nonconscious processing;** subjective perception

$G_{17}$ : Category two of SAS

$G_{18}$ : Category three of SAS

$T_{16}$ : Category one of subjective perception **;unique property of nonlinear transition from nonconscious processing**

$T_{17}$ : Category two of SAS

$T_{18}$ : Category three of SAS

**Module three**

Perceptual fluency is (=) the subjective experience of ease with which an incoming stimulus is processed

$G_{20}$ : Category one of Perceptual fluency

$G_{21}$ : Category two of SAS

$G_{22}$ : Category three of SAS

$T_{20}$ : Category one of subjective experience of ease with which an incoming stimulus is processed

$T_{21}$ : Category two of SAS

$T_{22}$ : Category three of SAS

**Module four**

**Perceptual fluency is the subjective experience of ease** with which an incoming stimulus is processed

$G_{24}$ : Category one of **Perceptual fluency is the subjective experience of ease;** incoming stimulus is processed

$G_{25}$ : Category two of SAS

$G_{26}$ : Category three of SAS

$T_{24}$ : Category one of incoming stimulus is processed **;Perceptual fluency is the subjective experience of ease**

$T_{25}$ : Category two of SAS

$T_{26}$ : Category three of SAS

**Module five**

Although perceptual fluency is assessed by speed of processing, it remains unclear how **objective speed** is related to (e&eb) subjective experiences of fluency

Model throws some light on the relationship

$G_{28}$ : Category one of **objective speed;** subjective experiences of fluency

$G_{29}$ : Category two of SAS

$G_{30}$ : Category three of SAS

$T_{28}$ : Category one of subjective experiences of fluency **;objective speed**

$T_{29}$ : Category two of SAS

$T_{30}$ : Category three of SAS

**Module six**

Authors present evidence that **speed at different stages of the perceptual process** contributes to perceptual fluency.

In an experiment, figure-ground contrast influenced detection of briefly presented words, but not their identification at longer exposure durations

$G_{32}$ : Category one of **speed at different stages of the perceptual process;** perceptual fluency

$G_{33}$ : Category two of SAS

$G_{34}$ : Category three of SAS

$T_{32}$ : Category one of perceptual fluency; **speed at different stages of the perceptual process**

$T_{33}$ : Category two of SAS

$T_{34}$ : Category three of SAS

**Module seven**

In an experiment, **figure-ground contrast** influenced detection of briefly presented words, but not their identification at longer exposure durations

$G_{36}$ : Category one of **figure-ground contrast;** detection of briefly presented words

$G_{37}$ : Category two of SAS

$G_{38}$ : Category three of SAS

$T_{36}$ : Category one of detection of briefly presented words **;figure-ground contrast**

$T_{37}$ : Category two of SAS

$T_{38}$ : Category three of SAS

**Module eight**

**Take AC=DC=0**

$G_{40}$ : Category one of **figure-ground contrast;** their identification at longer exposure durations

$G_{41}$ : Category two of SAS

$G_{42}$ : Category three of SAS

$T_{40}$ : Category one of their identification at longer exposure durations; **figure-ground contrast**

$T_{41}$ : Category two of SAS

$T_{42}$ : Category three of SAS

**Module Nine**

Conversely, **font in which the word was written** influenced identification, but not detection. Both contrast and font influenced subjective fluency

$G_{44}$ : Category one of **font in which the word was written;** identification

$G_{45}$ : Category two of SAS

$G_{46}$ : Category three of SAS

$T_{44}$ : Category one of identification; **font in which the word was written**

$T_{45}$ : Category two of SAS

$T_{46}$ : Category three of SAS

**The Coefficients:**

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}$ , $(b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)},$
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$

$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}, \ (b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)} \ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$ ,
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)},$
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)},$
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)},$
are Dissipation coefficients

**Module Numbered One**

**The differential system of this model is now (Module Numbered one)**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \quad 1$$
$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad 2$$
$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15} \quad 3$$
$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13} \quad 4$$
$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14} \quad 5$$
$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)\right]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14}, t) =$ First augmentation factor
$-(b''_{13})^{(1)}(G, t) =$ First detritions factor

**Module Numbered Two**

**The differential system of this model is now ( Module numbered two)**

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right]G_{16} \quad 7$$
$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t)\right]G_{17} \quad 8$$
$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t)\right]G_{18} \quad 9$$
$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\left((G_{19}), t\right)\right]T_{16} \quad 10$$
$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\left((G_{19}), t\right)\right]T_{17} \quad 11$$
$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\left((G_{19}), t\right)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17}, t) =$ First augmentation factor
$-(b''_{16})^{(2)}\left((G_{19}), t\right) =$ First detritions factor

**Module Numbered Three**

**The differential system of this model is now (Module numbered three)**

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right]G_{20} \quad 13$$
$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}, t)\right]G_{21} \quad 14$$
$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}, t)\right]G_{22} \quad 15$$

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23},t)\right]T_{20} \quad 16$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23},t)\right]T_{21} \quad 17$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23},t)\right]T_{22} \quad 18$$

$+(a''_{20})^{(3)}(T_{21},t) =$ First augmentation factor

$-(b''_{20})^{(3)}(G_{23},t) =$ First detritions factor

**Module Numbered Four**

**The differential system of this model is now (Module numbered Four)**

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right]G_{24} \quad 19$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25},t)\right]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25},t)\right]G_{26} \quad 21$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27}),t\right)\right]T_{24} \quad 22$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27}),t\right)\right]T_{25} \quad 23$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27}),t\right)\right]T_{26} \quad 24$$

$+(a''_{24})^{(4)}(T_{25},t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}\left((G_{27}),t\right) =$ **First detritions factor**

**Module Numbered Five**:

**The differential system of this model is now (Module number five)**

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29},t)\right]G_{28} \quad 25$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29},t)\right]G_{29} \quad 26$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29},t)\right]G_{30} \quad 27$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}\left((G_{31}),t\right)\right]T_{28} \quad 28$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}\left((G_{31}),t\right)\right]T_{29} \quad 29$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}\left((G_{31}),t\right)\right]T_{30} \quad 30$$

$+(a''_{28})^{(5)}(T_{29},t) =$ **First augmentation factor**

$-(b''_{28})^{(5)}\left((G_{31}),t\right) =$ **First detritions factor**

**Module Numbered Six**

**The differential system of this model is now (Module numbered Six)**

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33},t)\right]G_{32} \quad 31$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33},t)\right]G_{33} \quad 32$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33},t)\right]G_{34} \quad 33$$

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}\left((G_{35}),t\right)\right]T_{32} \quad 34$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}\left((G_{35}),t\right)\right]T_{33} \quad 35$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}\left((G_{35}),t\right)\right]T_{34} \quad 36$$

$+(a''_{32})^{(6)}(T_{33},t) =$ **First augmentation factor**

**Module Numbered Seven:**

**The differential system of this model is now (Seventh Module)**

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right]G_{36} \quad 37$$

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37},t)\right]G_{37} \quad 38$$

$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37},t)\right]G_{38} \quad 39$$

$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39}),t)\right]T_{36} \quad 40$$

$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}((G_{39}),t)\right]T_{37} \quad 41$$

$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39}),t)\right]T_{38} \quad 42$$

$+(a''_{36})^{(7)}(T_{37},t) =$ **First augmentation factor**

**Module Numbered Eight**

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right]G_{40} \quad 43$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41},t)\right]G_{41} \quad 44$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41},t)\right]G_{42} \quad 45$$

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43}),t)\right]T_{40} \quad 46$$

$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}((G_{43}),t)\right]T_{41} \quad 47$$

$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43}),t)\right]T_{42} \quad 48$$

**Module Numbered Nine**

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47}),t)\right]T_{44} \quad 52$$

$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}((G_{47}),t)\right]T_{45} \quad 53$$

$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47}),t)\right]T_{46} \quad 54$$

$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}((G_{47}),t) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)} & +(a''_{13})^{(1)}(T_{14},t) & +(a''_{16})^{(2,2,)}(T_{17},t) & +(a''_{20})^{(3,3,)}(T_{21},t) \\ +(a''_{24})^{(4,4,4,4,)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,)}(T_{33},t) & \\ +(a''_{36})^{(7,7)}(T_{37},t) & +(a''_{40})^{(8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) & \end{bmatrix} G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)} & +(a''_{14})^{(1)}(T_{14},t) & +(a''_{17})^{(2,2,)}(T_{17},t) & +(a''_{21})^{(3,3,)}(T_{21},t) \\ +(a''_{25})^{(4,4,4,4,)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,)}(T_{33},t) & \\ +(a''_{37})^{(7,7)}(T_{37},t) & +(a''_{41})^{(8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) & \end{bmatrix} G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[\begin{array}{c} (a'_{15})^{(1)}\boxed{+(a''_{15})^{(1)}(T_{14},t)}\ \boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}\ \boxed{+(a''_{22})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)}\ \boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)}\ \boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7)}(T_{37},t)}\ \boxed{+(a''_{42})^{(8,8)}(T_{41},t)}\ \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{15} \quad (57)$$

Where $\boxed{(a''_{13})^{(1)}(T_{14},t)}$, $\boxed{(a''_{14})^{(1)}(T_{14},t)}$, $\boxed{(a''_{15})^{(1)}(T_{14},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7)}(T_{37},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8)}(T_{41},t)}$ $\boxed{+(a''_{41})^{(8,8)}(T_{41},t)}$ $\boxed{+(a''_{42})^{(8,8)}(T_{41},t)}$ are eight augmentation coefficient for 1,2,3

$\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[\begin{array}{c} (b'_{13})^{(1)}\boxed{-(b''_{13})^{(1)}(G,t)}\ \boxed{-(b''_{16})^{(2,2,)}(G_{19},t)}\ \boxed{-(b''_{20})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)}\ \boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)}\ \boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,)}(G_{39},t)}\ \boxed{-(b''_{40})^{(8,8)}(G_{43},t)}\ \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{array}\right] T_{13} \quad (58)$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[\begin{array}{c} (b'_{14})^{(1)}\boxed{-(b''_{14})^{(1)}(G,t)}\ \boxed{-(b''_{17})^{(2,2,)}(G_{19},t)}\ \boxed{-(b''_{21})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)}\ \boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)}\ \boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,)}(G_{39},t)}\ \boxed{-(b''_{41})^{(8,8)}(G_{43},t)}\ \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{array}\right] T_{14} \quad (59)$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[\begin{array}{c} (b'_{15})^{(1)}\boxed{-(b''_{15})^{(1)}(G,t)}\ \boxed{-(b''_{18})^{(2,2,)}(G_{19},t)}\ \boxed{-(b''_{22})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)}\ \boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)}\ \boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}\ \boxed{-(b''_{42})^{(8,8)}(G_{43},t)}\ \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{array}\right] T_{15} \quad (60)$$

Where $\boxed{-(b''_{13})^{(1)}(G,t)}$, $\boxed{-(b''_{14})^{(1)}(G,t)}$, $\boxed{-(b''_{15})^{(1)}(G,t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)}$ are fourth detrition coefficients

for category 1, 2 and 3
$-(b_{28}'')^{(5,5,5,5,)}(G_{31},t)$ , $-(b_{29}'')^{(5,5,5,5,)}(G_{31},t)$ , $-(b_{30}'')^{(5,5,5,5,)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3
$-(b_{32}'')^{(6,6,6,6,)}(G_{35},t)$ , $-(b_{33}'')^{(6,6,6,6,)}(G_{35},t)$ , $-(b_{34}'')^{(6,6,6,6,)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3
$-(b_{37}'')^{(7,7,)}(G_{39},t)$ , $-(b_{36}'')^{(7,7,)}(G_{39},t)$ , $-(b_{38}'')^{(7,7,)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3
$-(b_{40}'')^{(8,8)}(G_{43},t)$ $-(b_{41}'')^{(8,8)}(G_{43},t)$ $-(b_{42}'')^{(8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3
$-(b_{44}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$ , $-(b_{45}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$ , $-(b_{46}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \begin{bmatrix} (a_{16}')^{(2)} & +(a_{16}'')^{(2)}(T_{17},t) & +(a_{13}'')^{(1,1,)}(T_{14},t) & +(a_{20}'')^{(3,3,3)}(T_{21},t) \\ +(a_{24}'')^{(4,4,4,4,4)}(T_{25},t) & +(a_{28}'')^{(5,5,5,5,5)}(T_{29},t) & +(a_{32}'')^{(6,6,6,6,6)}(T_{33},t) & \\ +(a_{36}'')^{(7,7,7)}(T_{37},t) & +(a_{40}'')^{(8,8,8)}(T_{41},t) & +(a_{44}'')^{(9,9)}(T_{45},t) & \end{bmatrix} G_{16} \quad 61$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \begin{bmatrix} (a_{17}')^{(2)} & +(a_{17}'')^{(2)}(T_{17},t) & +(a_{14}'')^{(1,1,)}(T_{14},t) & +(a_{21}'')^{(3,3,3)}(T_{21},t) \\ +(a_{25}'')^{(4,4,4,4,4)}(T_{25},t) & +(a_{29}'')^{(5,5,5,5,5)}(T_{29},t) & +(a_{33}'')^{(6,6,6,6,6)}(T_{33},t) & \\ +(a_{37}'')^{(7,7,7)}(T_{37},t) & +(a_{41}'')^{(8,8,8)}(T_{41},t) & +(a_{45}'')^{(9,9)}(T_{45},t) & \end{bmatrix} G_{17} \quad 62$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \begin{bmatrix} (a_{18}')^{(2)} & +(a_{18}'')^{(2)}(T_{17},t) & +(a_{15}'')^{(1,1,)}(T_{14},t) & +(a_{22}'')^{(3,3,3)}(T_{21},t) \\ +(a_{26}'')^{(4,4,4,4,4)}(T_{25},t) & +(a_{30}'')^{(5,5,5,5,5)}(T_{29},t) & +(a_{34}'')^{(6,6,6,6,6)}(T_{33},t) & \\ +(a_{38}'')^{(7,7,7)}(T_{37},t) & +(a_{42}'')^{(8,8,8)}(T_{41},t) & +(a_{46}'')^{(9,9)}(T_{45},t) & \end{bmatrix} G_{18} \quad 63$$

Where $+(a_{16}'')^{(2)}(T_{17},t)$ , $+(a_{17}'')^{(2)}(T_{17},t)$ , $+(a_{18}'')^{(2)}(T_{17},t)$ are first augmentation coefficients for category 1, 2 and 3
$+(a_{13}'')^{(1,1,)}(T_{14},t)$ , $+(a_{14}'')^{(1,1,)}(T_{14},t)$ , $+(a_{15}'')^{(1,1,)}(T_{14},t)$ are second augmentation coefficient for category 1, 2 and 3
$+(a_{20}'')^{(3,3,3)}(T_{21},t)$ , $+(a_{21}'')^{(3,3,3)}(T_{21},t)$ , $+(a_{22}'')^{(3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3
$+(a_{24}'')^{(4,4,4,4,4)}(T_{25},t)$ , $+(a_{25}'')^{(4,4,4,4,4)}(T_{25},t)$ , $+(a_{26}'')^{(4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3
$+(a_{28}'')^{(5,5,5,5,5)}(T_{29},t)$ , $+(a_{29}'')^{(5,5,5,5,5)}(T_{29},t)$ , $+(a_{30}'')^{(5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3
$+(a_{32}'')^{(6,6,6,6,6)}(T_{33},t)$ , $+(a_{33}'')^{(6,6,6,6,6)}(T_{33},t)$ , $+(a_{34}'')^{(6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3
$+(a_{36}'')^{(7,7,7)}(T_{37},t)$ , $+(a_{37}'')^{(7,7,7)}(T_{37},t)$ , $+(a_{38}'')^{(7,7,7)}(T_{37},t)$ are seventh augmentation coefficient for category 1, 2 and 3
$+(a_{40}'')^{(8,8,8)}(T_{41},t)$ , $+(a_{41}'')^{(8,8,8)}(T_{41},t)$ , $+(a_{42}'')^{(8,8,8)}(T_{41},t)$ are eight augmentation coefficient for category 1, 2 and 3
$+(a_{44}'')^{(9,9)}(T_{45},t)$ , $+(a_{45}'')^{(9,9)}(T_{45},t)$ , $+(a_{46}'')^{(9,9)}(T_{45},t)$ are ninth augmentation coefficient for

category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[\begin{array}{ccc} (b'_{16})^{(2)}\boxed{-(b''_{16})^{(2)}(G_{19},t)} & \boxed{-(b''_{13})^{(1,1,)}(G,t)} & \boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9)}(G_{47},t)} \end{array}\right] T_{16} \quad (64)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[\begin{array}{ccc} (b'_{17})^{(2)}\boxed{-(b''_{17})^{(2)}(G_{19},t)} & \boxed{-(b''_{14})^{(1,1,)}(G,t)} & \boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9)}(G_{47},t)} \end{array}\right] T_{17} \quad (65)$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[\begin{array}{ccc} (b'_{18})^{(2)}\boxed{-(b''_{18})^{(2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,)}(G,t)} & \boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9)}(G_{47},t)} \end{array}\right] T_{18} \quad (66)$$

where $\boxed{-(b''_{16})^{(2)}(G_{19},t)}$ , $\boxed{-(b''_{17})^{(2)}(G_{19},t)}$ , $\boxed{-(b''_{18})^{(2)}(G_{19},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,)}(G,t)}$ are second detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{44})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[\begin{array}{ccc} (a'_{20})^{(3)}\boxed{+(a''_{20})^{(3)}(T_{21},t)} & \boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9)}(T_{45},t)} \end{array}\right] G_{20} \quad (67)$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[\begin{array}{ccc} (a'_{21})^{(3)}\boxed{+(a''_{21})^{(3)}(T_{21},t)} & \boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)} \end{array}\right] G_{21} \quad (68)$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \begin{bmatrix} (a'_{22})^{(3)}\boxed{+(a''_{22})^{(3)}(T_{21},t)} & \boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{22} \quad (69)$$

$\boxed{+(a''_{20})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3)}(T_{21},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}$ are second augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)}$ are third augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)}$ are seventh augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)}$ are eight augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{44})^{(9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)}$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b'_{20})^{(3)}\boxed{-(b''_{20})^{(3)}(G_{23},t)} & \boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)} & \boxed{-(b''_{13})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{20} \quad (70)$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b'_{21})^{(3)}\boxed{-(b''_{21})^{(3)}(G_{23},t)} & \boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)} & \boxed{-(b''_{14})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{21} \quad (71)$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)}\boxed{-(b''_{22})^{(3)}(G_{23},t)} & \boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{22} \quad (72)$$

$\boxed{-(b''_{20})^{(3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3)}(G_{23},t)}$ *are first detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}$ *are second detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{13})^{(1,1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,)}(G,t)}$ *are third detrition coefficients for category 1,2 and 3*

$\boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)}$ *are fourth detrition*

*coefficients for category 1, 2 and 3*

$-(b_{28}'')^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b_{29}'')^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b_{30}'')^{(5,5,5,5,5,5)}(G_{31},t)$ *are fifth detrition coefficients for category 1, 2 and 3*

$-(b_{32}'')^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b_{33}'')^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b_{34}'')^{(6,6,6,6,6,6)}(G_{35},t)$ *are sixth detrition coefficients for category 1, 2 and 3*

$-(b_{36}'')^{(7,7,7,7)}(G_{39},t)$, $-(b_{37}'')^{(7,7,7,7)}(G_{39},t)$ $-(b_{38}'')^{(7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b_{40}'')^{(8,8,8,8)}(G_{43},t)$, $-(b_{41}'')^{(8,8,8,8)}(G_{43},t)$, $-(b_{42}'')^{(8,8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$-(b_{46}'')^{(9,9,9)}(G_{47},t)$, $-(b_{45}'')^{(9,9,9)}(G_{47},t)$, $-(b_{44}'')^{(9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt}=(a_{24})^{(4)}G_{25}-\left[\begin{array}{ccc}(a_{24}')^{(4)}+(a_{24}'')^{(4)}(T_{25},t) & +(a_{28}'')^{(5,5,)}(T_{29},t) & +(a_{32}'')^{(6,6,)}(T_{33},t)\\ +(a_{13}'')^{(1,1,1,1)}(T_{14},t) & +(a_{16}'')^{(2,2,2,2)}(T_{17},t) & +(a_{20}'')^{(3,3,3,3)}(T_{21},t)\\ +(a_{36}'')^{(7,7,7,7,7)}(T_{37},t) & +(a_{40}'')^{(8,8,8,8,8)}(T_{41},t) & +(a_{44}'')^{(9,9,9,9)}(T_{45},t)\end{array}\right]G_{24} \quad (73)$$

$$\frac{dG_{25}}{dt}=(a_{25})^{(4)}G_{24}-\left[\begin{array}{ccc}(a_{25}')^{(4)}+(a_{25}'')^{(4)}(T_{25},t) & +(a_{29}'')^{(5,5,)}(T_{29},t) & +(a_{33}'')^{(6,6)}(T_{33},t)\\ +(a_{14}'')^{(1,1,1,1)}(T_{14},t) & +(a_{17}'')^{(2,2,2,2)}(T_{17},t) & +(a_{21}'')^{(3,3,3,3)}(T_{21},t)\\ +(a_{37}'')^{(7,7,7,7,7)}(T_{37},t) & +(a_{41}'')^{(8,8,8,8,8)}(T_{41},t) & +(a_{45}'')^{(9,9,9,9)}(T_{45},t)\end{array}\right]G_{25} \quad (74)$$

$$\frac{dG_{26}}{dt}=(a_{26})^{(4)}G_{25}-\left[\begin{array}{ccc}(a_{26}')^{(4)}+(a_{26}'')^{(4)}(T_{25},t) & +(a_{30}'')^{(5,5,)}(T_{29},t) & +(a_{34}'')^{(6,6,)}(T_{33},t)\\ +(a_{15}'')^{(1,1,1,1)}(T_{14},t) & +(a_{18}'')^{(2,2,2,2)}(T_{17},t) & +(a_{22}'')^{(3,3,3,3)}(T_{21},t)\\ +(a_{38}'')^{(7,7,7,7,7)}(T_{37},t) & +(a_{42}'')^{(8,8,8,8,8)}(T_{41},t) & +(a_{46}'')^{(9,9,9,9)}(T_{45},t)\end{array}\right]G_{26} \quad (75)$$

$(a_{24}'')^{(4)}(T_{25},t)$, $(a_{25}'')^{(4)}(T_{25},t)$, $(a_{26}'')^{(4)}(T_{25},t)$ *are first augmentation coefficients category* $1,2\ 3$

$+(a_{28}'')^{(5,5,)}(T_{29},t)$, $+(a_{29}'')^{(5,5,)}(T_{29},t)$, $+(a_{30}'')^{(5,5,)}(T_{29},t)$ *are second augmentation coefficient for category* $1,2$ *and* $3$

$+(a_{32}'')^{(6,6,)}(T_{33},t)$, $+(a_{33}'')^{(6,6,)}(T_{33},t)$, $+(a_{34}'')^{(6,6,)}(T_{33},t)$ *are third augmentation coefficient for category* $1,2$ *and* $3$

$+(a_{13}'')^{(1,1,1,1)}(T_{14},t)$, $+(a_{14}'')^{(1,1,1,1)}(T_{14},t)$, $+(a_{15}'')^{(1,1,1,1)}(T_{14},t)$ *are fourth augmentation coefficients for category* $1,2$ *and* $3$

$+(a_{16}'')^{(2,2,2,2)}(T_{17},t)$, $+(a_{17}'')^{(2,2,2,2)}(T_{17},t)$, $+(a_{18}'')^{(2,2,2,2)}(T_{17},t)$ *are fifth augmentation coefficients for category* $1,2$ *and* $3$

$+(a_{20}'')^{(3,3,3,3)}(T_{21},t)$, $+(a_{21}'')^{(3,3,3,3)}(T_{21},t)$, $+(a_{22}'')^{(3,3,3,3)}(T_{21},t)$ *are sixth augmentation coefficients for category* $1,2$ *and* $3$

$+(a_{36}'')^{(7,7,7,7,7)}(T_{37},t)$, $+(a_{37}'')^{(7,7,7,7,7)}(T_{37},t)$, $+(a_{38}'')^{(7,7,7,7,7)}(T_{37},t)$ *are seventh augmentation coefficients for category* $1,2$ *and* $3$

$+(a_{40}'')^{(8,8,8,8,8)}(T_{41},t)$, $+(a_{41}'')^{(8,8,8,8,8)}(T_{41},t)$, $+(a_{42}'')^{(8,8,8,8,8)}(T_{41},t)$ *are eighth augmentation coefficients for category* $1,2$ *and* $3$

$+(a_{46}'')^{(9,9,9,9)}(T_{45},t)$, $+(a_{45}'')^{(9,9,9,9)}(T_{45},t)$, $+(a_{44}'')^{(9,9,9,9)}(T_{45},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \begin{bmatrix} (b'_{24})^{(4)} & -(b''_{24})^{(4)}(G_{27},t) & -(b''_{28})^{(5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,)}(G_{35},t) \\ & -(b''_{13})^{(1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{24} \quad (76)$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \begin{bmatrix} (b'_{25})^{(4)} & -(b''_{25})^{(4)}(G_{27},t) & -(b''_{29})^{(5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{25} \quad (77)$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)} & -(b''_{26})^{(4)}(G_{27},t) & -(b''_{30})^{(5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1)}(G,t) & -(b''_{18})^{(2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{26} \quad (78)$$

*Where* $-(b''_{24})^{(4)}(G_{27},t)$, $-(b''_{25})^{(4)}(G_{27},t)$, $-(b''_{26})^{(4)}(G_{27},t)$ *are first detrition coefficients for category* $1, 2$ *and* $3$

$-(b''_{28})^{(5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,)}(G_{31},t)$ *are second detrition coefficients for category* $1, 2$ *and* $3$

$-(b''_{32})^{(6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,)}(G_{35},t)$ *are third detrition coefficients for category* $1, 2$ *and* $3$

$-(b''_{13})^{(1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1)}(G,t)$
*are fourth detrition coefficients for category* $1, 2$ *and* $3$

$-(b''_{16})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2)}(G_{19},t)$
*are fifth detrition coefficients for category* $1, 2$ *and* $3$

$-(b''_{20})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3)}(G_{23},t)$
*are sixth detrition coefficients for category* $1, 2$ *and* $3$

$-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)$
*are seventh detrition coefficients for category* $1, 2$ *and* $3$

$-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)$
*are eighth detrition coefficients for category* $1, 2$ *and* $3$

$-(b''_{46})^{(9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \begin{bmatrix} (a'_{28})^{(5)} & +(a''_{28})^{(5)}(T_{29},t) & +(a''_{24})^{(4,4,)}(T_{25},t) & +(a''_{32})^{(6,6,6)}(T_{33},t) \\ & +(a''_{13})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{16})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3,3)}(T_{21},t) \\ & +(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{28} \quad (79)$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \begin{bmatrix} (a'_{29})^{(5)} & +(a''_{29})^{(5)}(T_{29},t) & +(a''_{25})^{(4,4,)}(T_{25},t) & +(a''_{33})^{(6,6,6)}(T_{33},t) \\ & +(a''_{14})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{17})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3,3)}(T_{21},t) \\ & +(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{29} \quad (80)$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \begin{bmatrix} (a'_{30})^{(5)} & +(a''_{30})^{(5)}(T_{29},t) & +(a''_{26})^{(4,4,)}(T_{25},t) & +(a''_{34})^{(6,6,6)}(T_{33},t) \\ +(a''_{15})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{18})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3,3)}(T_{21},t) \\ +(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{30} \quad (81)$$

*Where* $+(a''_{28})^{(5)}(T_{29},t)$, $+(a''_{29})^{(5)}(T_{29},t)$, $+(a''_{30})^{(5)}(T_{29},t)$ *are first augmentation coefficients for category* 1, 2 *and* 3

*And* $+(a''_{24})^{(4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,)}(T_{25},t)$ *are second augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{32})^{(6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6)}(T_{33},t)$ *are third augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)$ are fourth augmentation coefficients for category 1,2, and 3

$+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)$ are fifth augmentation coefficients for category 1,2,and 3

$+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)$ are sixth augmentation coefficients for category 1,2, 3

$+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1,2, 3

$+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)$ are eighth augmentation coefficients for category 1,2, 3

$+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \begin{bmatrix} (b'_{28})^{(5)} & -(b''_{28})^{(5)}(G_{31},t) & -(b''_{24})^{(4,4,)}(G_{27},t) & -(b''_{32})^{(6,6,6)}(G_{35},t) \\ -(b''_{13})^{(1,1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3)}(G_{23},t) \\ -(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{28} \quad (82)$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b'_{29})^{(5)} & -(b''_{29})^{(5)}(G_{31},t) & -(b''_{25})^{(4,4,)}(G_{27},t) & -(b''_{33})^{(6,6,6)}(G_{35},t) \\ -(b''_{14})^{(1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3)}(G_{23},t) \\ -(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{29} \quad (83)$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b'_{30})^{(5)} & -(b''_{30})^{(5)}(G_{31},t) & -(b''_{26})^{(4,4,)}(G_{27},t) & -(b''_{34})^{(6,6,6)}(G_{35},t) \\ -(b''_{15})^{(1,1,1,1,1,)}(G,t) & -(b''_{18})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3)}(G_{23},t) \\ -(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{30} \quad (84)$$

*where* $-(b''_{28})^{(5)}(G_{31},t)$, $-(b''_{29})^{(5)}(G_{31},t)$, $-(b''_{30})^{(5)}(G_{31},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{24})^{(4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,)}(G_{27},t)$ *are second detrition coefficients for category* 1,2 *and* 3

$-(b''_{32})^{(6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6)}(G_{35},t)$ *are third detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1)}(G,t)}$ , $\boxed{-(b''_{15})^{(1,1,1,1,1,)}(G,t)}$ are fourth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[\begin{array}{ccc} (a'_{32})^{(6)}\boxed{+(a''_{32})^{(6)}(T_{33},t)} & \boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{32} \quad (85)$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[\begin{array}{ccc} (a'_{33})^{(6)}\boxed{+(a''_{33})^{(6)}(T_{33},t)} & \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{33} \quad (86)$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[\begin{array}{ccc} (a'_{34})^{(6)}\boxed{+(a''_{34})^{(6)}(T_{33},t)} & \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{34} \quad (87)$$

$\boxed{+(a''_{32})^{(6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6)}(T_{33},t)}$ *are first augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}$ *are second augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}$ *are third augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}$ - are fourth augmentation coefficients

$\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}$ - fifth augmentation coefficients

$\boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)}$ sixth augmentation coefficients

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)}$ seventh augmentation coefficients

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}$

Eighth augmentation coefficients
$+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt}=(b_{32})^{(6)}T_{33}-\begin{bmatrix}(b'_{32})^{(6)}-(b''_{32})^{(6)}(G_{35},t) & -(b''_{28})^{(5,5,5)}(G_{31},t) & -(b''_{24})^{(4,4,4,)}(G_{27},t)\\ -(b''_{13})^{(1,1,1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)\\ -(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)\end{bmatrix}T_{32} \quad (88)$$

$$\frac{dT_{33}}{dt}=(b_{33})^{(6)}T_{32}-\begin{bmatrix}(b'_{33})^{(6)}-(b''_{33})^{(6)}(G_{35},t) & -(b''_{29})^{(5,5,5)}(G_{31},t) & -(b''_{25})^{(4,4,4,)}(G_{27},t)\\ -(b''_{14})^{(1,1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)\\ -(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)\end{bmatrix}T_{33} \quad (89)$$

$$\frac{dT_{34}}{dt}=(b_{34})^{(6)}T_{33}-\begin{bmatrix}(b'_{34})^{(6)}-(b''_{34})^{(6)}(G_{35},t) & -(b''_{30})^{(5,5,5)}(G_{31},t) & -(b''_{26})^{(4,4,4,)}(G_{27},t)\\ -(b''_{15})^{(1,1,1,1,1,1)}(G,t) & -(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)\\ -(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)\end{bmatrix}T_{34} \quad (90)$$

$-(b''_{32})^{(6)}(G_{35},t)$, $-(b''_{33})^{(6)}(G_{35},t)$, $-(b''_{34})^{(6)}(G_{35},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{28})^{(5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5)}(G_{31},t)$ *are second detrition coefficients for category* 1, 2 *and* 3

$-(b''_{24})^{(4,4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,)}(G_{27},t)$ *are third detrition coefficients for category* 1,2 *and* 3

$-(b''_{13})^{(1,1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1,1,1)}(G,t)$ are fourth detrition coefficients for category 1, 2, and 3

$-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1, 2, and 3

$-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1, 2, and 3

$-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2, and 3

$-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1, 2, and 3

$-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \quad (91)$$

$$=(a_{36})^{(7)}G_{37}-\begin{bmatrix}(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37},t) & +(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)\\ +(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)\\ +(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t) & +(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)\end{bmatrix}G_{13}$$

$$\frac{dG_{37}}{dt} \qquad 92$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a'_{37})^{(7)}\boxed{+(a''_{37})^{(7)}(T_{37},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{38}}{dt} \qquad 93$$

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a'_{38})^{(7)}\boxed{+(a''_{38})^{(7)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{(a''_{36})^{(7)}(T_{37},t)}$, $\boxed{(a''_{37})^{(7)}(T_{37},t)}$, $\boxed{(a''_{38})^{(7)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,)}(T_{41},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{36}}{dt} = \qquad 94$$

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b'_{36})^{(7)}\boxed{-(b''_{36})^{(7)}(G_{39},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{37}}{dt} =$$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b'_{37})^{(7)}\boxed{-(b''_{37})^{(7)}(G_{39},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{38}}{dt} =$$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b'_{38})^{(7)}\boxed{-(b''_{38})^{(7)}(G_{39},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{40}}{dt} \qquad 95$$

$$= (a_{40})^{(8)}G_{41} - \begin{bmatrix} (a'_{40})^{(8)}\boxed{+(a''_{40})^{(8)}(T_{41},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{41}}{dt}$$

$$= (a_{41})^{(8)}G_{40} - \begin{bmatrix} (a'_{41})^{(8)}\boxed{+(a''_{41})^{(8)}(T_{41},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \begin{bmatrix} (a'_{42})^{(8)}\boxed{+(a''_{42})^{(8)}(T_{41},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a''_{40})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)}\boxed{-(b''_{40})^{(8)}(G_{43},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)}\boxed{-(b''_{41})^{(8)}(G_{43},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)}\boxed{-(b''_{42})^{(8)}(G_{43},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{44}}{dt} \qquad 96$$

$$= (a_{44})^{(9)}G_{45} - \left[\begin{array}{ccc} (a'_{44})^{(9)}\boxed{+(a''_{44})^{(9)}(T_{45},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{array}\right] G_{13}$$

$$\frac{dG_{45}}{dt}$$

$$= (a_{45})^{(9)}G_{44} - \left[\begin{array}{ccc} (a'_{45})^{(9)}\boxed{+(a''_{45})^{(9)}(T_{45},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{array}\right] G_{14}$$

$$\frac{dG_{46}}{dt}$$

$$= (a_{46})^{(9)}G_{45} - \left[\begin{array}{ccc} (a'_{46})^{(9)}\boxed{+(a''_{46})^{(9)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{array}\right] G_{15}$$

Where $\boxed{+(a''_{44})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$+(a_{16}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)$ , $+(a_{17}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)$ , $+(a_{18}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a_{20}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)$ , $+(a_{21}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)$ , $+(a_{22}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a_{24}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)$ , $+(a_{25}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)$ , $+(a_{26}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a_{28}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)$, $+(a_{29}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)$ , $+(a_{30}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a_{32}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a_{33}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$ , $+(a_{34}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a_{13}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$ , $+(a_{14}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$ , $+(a_{15}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$ are Seventh augmentation coefficient for category 1, 2 and 3

$+(a_{38}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ $+(a_{37}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ $+(a_{36}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ are eighth augmentation coefficient for 1,2,3

$+(a_{40}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)$ ,$+(a_{42}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a_{41}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} =$$

$$(b_{44})^{(9)}T_{45} -$$

$$\begin{bmatrix} (b_{44}')^{(9)}\boxed{-(b_{44}'')^{(9)}(G_{47},t)} & \boxed{-(b_{16}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{20}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{24}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{28}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{32}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{13}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{36}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b_{45}')^{(9)}\boxed{-(b_{45}'')^{(9)}(G_{47},t)} & \boxed{-(b_{17}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{21}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{25}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{29}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{33}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{14}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{37}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b_{46}')^{(9)}\boxed{-(b_{46}'')^{(9)}(G_{47},t)} & \boxed{-(b_{18}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{22}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{26}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{30}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{34}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{15}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{38}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{15}$$

Where $-(b_{44}'')^{(9)}(G_{47},t)$ , $-(b_{45}'')^{(9)}(G_{47},t)$ , $-(b_{46}'')^{(9)}(G_{47},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b_{16}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)$ , $-(b_{17}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)$ , $-(b_{18}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b_{20}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)$ , $-(b_{21}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)$ , $-(b_{22}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{24}'')^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{25}'')^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{26}'')^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{28}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{32}'')^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{34}'')^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{37}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{36}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a_i')^{(1)}, (a_i'')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (b_i'')^{(1)} > 0, \quad i,j = 13,14,15 \qquad 97$$

The functions $(a_i'')^{(1)}, (b_i'')^{(1)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a_i'')^{(1)}(T_{14},t) \leq (p_i)^{(1)} \leq (\hat{A}_{13})^{(1)}$$

$$(b_i'')^{(1)}(G,t) \leq (r_i)^{(1)} \leq (b_i')^{(1)} \leq (\hat{B}_{13})^{(1)}$$

$$lim_{T_2\to\infty}(a_i'')^{(1)}(T_{14},t) = (p_i)^{(1)} \qquad 98$$

$$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = (r_i)^{(1)}$$

**Definition of** $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$ :

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$$|(a_i'')^{(1)}(T_{14}',t) - (a_i'')^{(1)}(T_{14},t)| \leq (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G',t) - (b_i'')^{(1)}(G,t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}',t)$ and $(a_i'')^{(1)}(T_{14},t)$ . $(T_{14}',t)$ and $(T_{14},t)$ are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14},t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$ : 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$

**Definition of** $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$ : 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}$, $(r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)} (\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)} (\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i, j = 16,17,18$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17}, t) \leq (p_i)^{(2)} \leq (\hat{A}_{16})^{(2)} \quad 102$$

$$(b_i'')^{(2)}(G_{19}, t) \leq (r_i)^{(2)} \leq (b_i')^{(2)} \leq (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2 \to \infty} (a_i'')^{(2)}(T_{17}, t) = (p_i)^{(2)} \quad 104$$

$$\lim_{G \to \infty} (b_i'')^{(2)}((G_{19}), t) = (r_i)^{(2)} \quad 105$$

**Definition of** $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$ : 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}', t) - (a_i'')^{(2)}(T_{17}, t)| \leq (\hat{k}_{16})^{(2)} |T_{17} - T_{17}'| e^{-(\hat{M}_{16})^{(2)} t} \quad 107$$

$$|(b_i'')^{(2)}((G_{19})', t) - (b_i'')^{(2)}((G_{19}), t)| < (\hat{k}_{16})^{(2)} ||(G_{19}) - (G_{19})'|| e^{-(\hat{M}_{16})^{(2)} t} \quad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}', t)$ and$(a_i'')^{(2)}(T_{17}, t)$ . $(T_{17}', t)$ and $(T_{17}, t)$ are points belonging to the interval $[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}]$ . It is to be noted that $(a_i'')^{(2)}(T_{17}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17}, t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$ :

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)},$ are positive constants 109

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$$

**Definition of** $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$ :

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and\ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1 \quad 110$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(b_i)^{(2)} + (b_i')^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1 \quad 111$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i, j = 20,21,22 \quad 112$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(3)}$, $(r_i)^{(3)}$:

$$(a_i'')^{(3)}(T_{21}, t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23}, t) \leq (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(3)}(T_{21}, t) = (p_i)^{(3)} \quad 113$$

$$\lim_{G \to \infty}(b_i'')^{(3)}(G_{23}, t) = (r_i)^{(3)}$$

**Definition of** $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$ :

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: 114

$$|(a_i'')^{(3)}(T_{21}', t) - (a_i'')^{(3)}(T_{21}, t)| \leq (\hat{k}_{20})^{(3)} |T_{21} - T_{21}'| e^{-(\hat{M}_{20})^{(3)} t}$$

$$|(b_i'')^{(3)}(G_{23}', t) - (b_i'')^{(3)}(G_{23}, t)| < (\hat{k}_{20})^{(3)} ||G_{23} - G_{23}'|| e^{-(\hat{M}_{20})^{(3)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}', t)$ and$(a_i'')^{(3)}(T_{21}, t)$ . $(T_{21}', t)$ And $(T_{21}, t)$ are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21}, t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$ : 115

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}$ , $\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with 116
$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$ and the constants
$(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}, i = 20,21,22,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(a_i)^{(3)} + (a_i')^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)} (\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(b_i)^{(3)} + (b_i')^{(3)} + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)} (\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i, j = 24,25,26$$ 117

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(4)}$, $(r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25}, t) \leq (p_i)^{(4)} \leq (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}((G_{27}), t) \leq (r_i)^{(4)} \leq (b_i')^{(4)} \leq (\hat{B}_{24})^{(4)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(4)}(T_{25}, t) = (p_i)^{(4)}$$ 118

$$\lim_{G \to \infty}(b_i'')^{(4)}((G_{27}), t) = (r_i)^{(4)}$$

**Definition of** $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$ :

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24,25,26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}', t) - (a_i'')^{(4)}(T_{25}, t)| \leq (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})', t) - (b_i'')^{(4)}((G_{27}), t)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}', t)$ and$(a_i'')^{(4)}(T_{25}, t)$ . $(T_{25}', t)$ and $(T_{25}, t)$ are points belonging to the interval $[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}]$. It is to be noted that $(a_i'')^{(4)}(T_{25}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25}, t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$ : 120

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)},$ are positive constants

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}, \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$$

**Definition of** $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$ : 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and\ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}, i = 24,25,26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(a_i)^{(4)} + (a_i')^{(4)} + (\hat{A}_{24})^{(4)} + (\hat{P}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(b_i)^{(4)} + (b_i')^{(4)} + (\hat{B}_{24})^{(4)} + (\hat{Q}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i, j = 28,29,30$$ 122

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(5)}$, $(r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29}, t) \le (p_i)^{(5)} \le (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}((G_{31}), t) \le (r_i)^{(5)} \le (b_i')^{(5)} \le (\hat{B}_{28})^{(5)}$$

$$lim_{T_2 \to \infty} (a_i'')^{(5)} (T_{29}, t) = (p_i)^{(5)}$$ 123

$$\lim_{G \to \infty} (b_i'')^{(5)} (G_{31}, t) = (r_i)^{(5)}$$

**Definition of** $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$ :

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}', t) - (a_i'')^{(5)}(T_{29}, t)| \le (\hat{k}_{28})^{(5)} |T_{29} - T_{29}'| e^{-(\hat{M}_{28})^{(5)} t}$$

$$|(b_i'')^{(5)}((G_{31})', t) - (b_i'')^{(5)}((G_{31}), t)| < (\hat{k}_{28})^{(5)} ||(G_{31}) - (G_{31})'|| e^{-(\hat{M}_{28})^{(5)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}', t)$ and $(a_i'')^{(5)}(T_{29}, t)$ . $(T_{29}', t)$ and $(T_{29}, t)$ are points belonging to the interval $\left[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\right]$. It is to be noted that $(a_i'')^{(5)}(T_{29}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$ : 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)},$ are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}, \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

**Definition of** $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$ : 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and\ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(a_i)^{(5)} + (a_i')^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(b_i)^{(5)} + (b_i')^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i, j = 32,33,34$$ 127

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(6)}$, $(r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33}, t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}), t) \leq (r_i)^{(6)} \leq (b_i')^{(6)} \leq (\hat{B}_{32})^{(6)}$$

$$lim_{T_2 \to \infty} (a_i'')^{(6)} (T_{33}, t) = (p_i)^{(6)}$$ 128

$$\lim_{G \to \infty} (b_i'')^{(6)} ((G_{35}), t) = (r_i)^{(6)}$$

**Definition of** $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$ :

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}', t) - (a_i'')^{(6)}(T_{33}, t)| \leq (\hat{k}_{32})^{(6)} |T_{33} - T_{33}'| e^{-(\hat{M}_{32})^{(6)} t}$$

$$|(b_i'')^{(6)}((G_{35})', t) - (b_i'')^{(6)}((G_{35}), t)| < (\hat{k}_{32})^{(6)} ||(G_{35}) - (G_{35})'|| e^{-(\hat{M}_{32})^{(6)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}', t)$ and $(a_i'')^{(6)}(T_{33}, t)$ . $(T_{33}', t)$ and $(T_{33}, t)$ are points belonging to the interval $[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}]$. It is to be noted that $(a_i'')^{(6)}(T_{33}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$ : 129

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)},$ are positive constants

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$$

**Definition of** $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$ : 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and\ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32,33,34,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(a_i)^{(6)} + (a_i')^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)} (\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(b_i)^{(6)} + (b_i')^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)} (\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

(GGGGGGG) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i, j = 36,37,38$ 131

(HHHHHHH) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(7)}$, $(r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37}, t) \le (p_i)^{(7)} \le (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39}, t) \le (r_i)^{(7)} \le (b_i')^{(7)} \le (\hat{B}_{36})^{(7)}$$

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*(IIIIIII)* $\lim_{T_2 \to \infty} (a_i'')^{(7)}(T_{37}, t) = (p_i)^{(7)}$

*(JJJJJJJ)*

$\lim_{G \to \infty} (b_i'')^{(7)}((G_{39}), t) = (r_i)^{(7)}$

**Definition of** $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$ :

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$$|(a_i'')^{(7)}(T_{37}', t) - (a_i'')^{(7)}(T_{37}, t)| \le (\hat{k}_{36})^{(7)} |T_{37} - T_{37}'| e^{-(\hat{M}_{36})^{(7)} t}$$

$$|(b_i'')^{(7)}((G_{39})', t) - (b_i'')^{(7)}((G_{39}), t)| < (\hat{k}_{36})^{(7)} ||(G_{39}) - (G_{39})'|| e^{-(\hat{M}_{36})^{(7)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}', t)$ and $(a_i'')^{(7)}(T_{37}, t)$ . $(T_{37}', t)$ and $(T_{37}, t)$ are points belonging to the interval $[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}]$ . It is to be noted that $(a_i'')^{(7)}(T_{37}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37}, t)$ , the **first augmentation coefficient** attributable to the

system, would be absolutely continuous.

**Definition of** $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$ : 134

(KKKKKKK) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$, are positive constants

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$$

**Definition of** $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$ : 135

(LLLLLLL) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(b_i)^{(7)} + (b_i')^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40,41,42 \qquad 136$$

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

**Definition of** $(p_i)^{(8)}$, $(r_i)^{(8)}$: 137

$$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)} \qquad 138$$

$$(b_i'')^{(8)}((G_{43}), t) \leq (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)} \qquad 139$$

$$\lim_{T_2 \to \infty} (a_i'')^{(8)}(T_{41}, t) = (p_i)^{(8)} \qquad 140$$

$$\lim_{G \to \infty} (b_i'')^{(8)}((G_{43}), t) = (r_i)^{(8)} \qquad 141$$

**Definition of** $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$ :

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)} |T_{41} - T_{41}'| e^{-(\hat{M}_{40})^{(8)} t} \qquad 142$$

$$|(b_i'')^{(8)}((G_{43})', t) - (b_i'')^{(8)}((G_{43}), t)| < (\hat{k}_{40})^{(8)} ||(G_{43}) - (G_{43})'|| e^{-(\hat{M}_{40})^{(8)} t} \qquad 143$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}', t)$ and $(a_i'')^{(8)}(T_{41}, t)$ . $(T_{41}', t)$ and $(T_{41}, t)$ are points belonging to the interval $\left[(\hat{k}_{40})^{(8)}, (\widehat{M}_{40})^{(8)}\right]$. It is to be noted that $(a_i'')^{(8)}(T_{41}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{40})^{(8)} = 1$ then the function $(a_i'')^{(8)}(T_{41}, t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\widehat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$ :

$(\widehat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)},$ are positive constants

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}, \frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}} < 1 \qquad 144$$

**Definition of** $(\hat{P}_{40})^{(8)}, (\hat{Q}_{40})^{(8)}$ :

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\widehat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}, (\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)}, (a_i')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}, i = 40,41,42,$
Satisfy the inequalities

$$\frac{1}{(\widehat{M}_{40})^{(8)}}[(a_i)^{(8)} + (a_i')^{(8)} + (\hat{A}_{40})^{(8)} + (\hat{P}_{40})^{(8)} (\hat{k}_{40})^{(8)}] < 1 \qquad 145$$

$$\frac{1}{(\widehat{M}_{40})^{(8)}}[(b_i)^{(8)} + (b_i')^{(8)} + (\hat{B}_{40})^{(8)} + (\hat{Q}_{40})^{(8)} (\hat{k}_{40})^{(8)}] < 1 \qquad 146$$

Where we suppose

$$(a_i)^{(9)}, (a_i')^{(9)}, (a_i'')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (b_i'')^{(9)} > 0, \quad i, j = 44,45,46 \qquad 146\,A$$

The functions $(a_i'')^{(9)}, (b_i'')^{(9)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(9)}$, $(r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45}, t) \leq (p_i)^{(9)} \leq (\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47}, t) \leq (r_i)^{(9)} \leq (b_i')^{(9)} \leq (\hat{B}_{44})^{(9)}$$

$$\lim_{T_2 \to \infty} (a_i'')^{(9)}(T_{45}, t) = (p_i)^{(9)}$$

$$\lim_{G \to \infty} (b_i'')^{(9)}(G_{47}, t) = (r_i)^{(9)}$$

**Definition of** $(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}$ :

Where $\boxed{(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}}$ are positive constants and $\boxed{i = 44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}', t) - (a_i'')^{(9)}(T_{45}, t)| \leq (\hat{k}_{44})^{(9)} |T_{45} - T_{45}'| e^{-(\widehat{M}_{44})^{(9)} t}$$

$$|(b_i'')^{(9)}((G_{47})', t) - (b_i'')^{(9)}\big((G_{47}), t\big)| < (\hat{k}_{44})^{(9)}||(G_{47}) - (G_{47})'||e^{-(\hat{M}_{44})^{(9)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}', t)$ and$(a_i'')^{(9)}(T_{45}, t)$ . $(T_{45}', t)$ and $(T_{45}, t)$ are points belonging to the interval $\big[(\hat{k}_{44})^{(9)}, (\hat{M}_{44})^{(9)}\big]$. It is to be noted that $(a_i'')^{(9)}(T_{45}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{44})^{(9)} = 1$ then the function $(a_i'')^{(9)}(T_{45}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$ :

$(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$, are positive constants

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}} \ , \frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$$

**Definition of** $(\hat{P}_{44})^{(9)}, (\hat{Q}_{44})^{(9)}$ :

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}, (\hat{A}_{44})^{(9)} and\ (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)}, (a_i')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (p_i)^{(9)}, \ (r_i)^{(9)}, i = 44,45,46,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{44})^{(9)}}[\,(a_i)^{(9)} + (a_i')^{(9)} + \ (\hat{A}_{44})^{(9)} + \ (\hat{P}_{44})^{(9)}\,(\hat{k}_{44})^{(9)}] < 1$$

$$\frac{1}{(\hat{M}_{44})^{(9)}}[\ (b_i)^{(9)} + (b_i')^{(9)} + \ (\hat{B}_{44})^{(9)} + \ (\hat{Q}_{44})^{(9)}\ (\hat{k}_{44})^{(9)}] < 1$$

**Theorem 1:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \big(\hat{P}_{13}\big)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \ , \ \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 2 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

**Definition of** $G_i(0)\,, T_i(0)$

$$G_i(t) \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} \quad , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} \quad , \quad T_i(0) = T_i^0 > 0$$

**Theorem 3 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t) \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t} \quad , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t} \quad , \quad T_i(0) = T_i^0 > 0$$

**Theorem 4 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

**Definition of** $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \left(\hat{P}_{24}\right)^{(4)} e^{(\hat{M}_{24})^{(4)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{24}\right)^{(4)} e^{(\hat{M}_{24})^{(4)}t} \;,\quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 5 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

**Definition of** $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \left(\hat{P}_{28}\right)^{(5)} e^{(\hat{M}_{28})^{(5)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{28}\right)^{(5)} e^{(\hat{M}_{28})^{(5)}t} \;,\quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 6 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

**Definition of** $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \left(\hat{P}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t} \;,\quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 7:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

**Definition of** $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \left(\hat{P}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t} \;,\quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 8:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

**Definition of** $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \left(\hat{P}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t} \;,\quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 9:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

**Definition of** $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \left(\hat{P}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

**Proof:** Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \leq (\hat{P}_{13})^{(1)} \text{ , } T_i^0 \leq (\hat{Q}_{13})^{(1)}, \quad 155$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t} \quad 156$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t} \quad 157$$

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[ (a_{13})^{(1)} G_{14}(s_{(13)}) - \left( (a'_{13})^{(1)} + a''_{13})^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \right) G_{13}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[ (a_{14})^{(1)} G_{13}(s_{(13)}) - \left( (a'_{14})^{(1)} + (a''_{14})^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \right) G_{14}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[ (a_{15})^{(1)} G_{14}(s_{(13)}) - \left( (a'_{15})^{(1)} + (a''_{15})^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \right) G_{15}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[ (b_{13})^{(1)} T_{14}(s_{(13)}) - \left( (b'_{13})^{(1)} - (b''_{13})^{(1)} (G(s_{(13)}), s_{(13)}) \right) T_{13}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[ (b_{14})^{(1)} T_{13}(s_{(13)}) - \left( (b'_{14})^{(1)} - (b''_{14})^{(1)} (G(s_{(13)}), s_{(13)}) \right) T_{14}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{T}_{15}(t) = T_{15}^0 + \int_0^t \left[ (b_{15})^{(1)} T_{14}(s_{(13)}) - \left( (b'_{15})^{(1)} - (b''_{15})^{(1)} (G(s_{(13)}), s_{(13)}) \right) T_{15}(s_{(13)}) \right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \leq (\hat{P}_{16})^{(2)} \text{ , } T_i^0 \leq (\hat{Q}_{16})^{(2)},$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$$

By 160

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[ (a_{16})^{(2)} G_{17}(s_{(16)}) - \left( (a'_{16})^{(2)} + a''_{16})^{(2)} (T_{17}(s_{(16)}), s_{(16)}) \right) G_{16}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[ (a_{17})^{(2)} G_{16}(s_{(16)}) - \left( (a'_{17})^{(2)} + (a''_{17})^{(2)} (T_{17}(s_{(16)}), s_{(17)}) \right) G_{17}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[ (a_{18})^{(2)} G_{17}(s_{(16)}) - \left( (a'_{18})^{(2)} + (a''_{18})^{(2)} (T_{17}(s_{(16)}), s_{(16)}) \right) G_{18}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^{0} + \int_0^t \left[ (b_{16})^{(2)} T_{17}(s_{(16)}) - \left( (b'_{16})^{(2)} - (b''_{16})^{(2)} (G_{19}(s_{(16)}), s_{(16)}) \right) T_{16}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^{0} + \int_0^t \left[ (b_{17})^{(2)} T_{16}(s_{(16)}) - \left( (b'_{17})^{(2)} - (b''_{17})^{(2)} (G_{19}(s_{(16)}), s_{(16)}) \right) T_{17}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^{0} + \int_0^t \left[ (b_{18})^{(2)} T_{17}(s_{(16)}) - \left( (b'_{18})^{(2)} - (b''_{18})^{(2)} (G_{19}(s_{(16)}), s_{(16)}) \right) T_{18}(s_{(16)}) \right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i : \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \le (\hat{P}_{20})^{(3)} \text{ , } T_i^0 \le (\hat{Q}_{20})^{(3)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\tilde{M}_{20})^{(3)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\tilde{M}_{20})^{(3)} t}$$

By 161

$$\bar{G}_{20}(t) = G_{20}^{0} + \int_0^t \left[ (a_{20})^{(3)} G_{21}(s_{(20)}) - \left( (a'_{20})^{(3)} + a''_{20})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^{0} + \int_0^t \left[ (a_{21})^{(3)} G_{20}(s_{(20)}) - \left( (a'_{21})^{(3)} + (a''_{21})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{21}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^{0} + \int_0^t \left[ (a_{22})^{(3)} G_{21}(s_{(20)}) - \left( (a'_{22})^{(3)} + (a''_{22})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{22}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^{0} + \int_0^t \left[ (b_{20})^{(3)} T_{21}(s_{(20)}) - \left( (b'_{20})^{(3)} - (b''_{20})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^{0} + \int_0^t \left[ (b_{21})^{(3)} T_{20}(s_{(20)}) - \left( (b'_{21})^{(3)} - (b''_{21})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{21}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{22}(t) = T_{22}^{0} + \int_0^t \left[ (b_{22})^{(3)} T_{21}(s_{(20)}) - \left( (b'_{22})^{(3)} - (b''_{22})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{22}(s_{(20)}) \right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i : \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \le (\hat{P}_{24})^{(4)} \text{ , } T_i^0 \le (\hat{Q}_{24})^{(4)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\tilde{M}_{24})^{(4)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\tilde{M}_{24})^{(4)} t}$$

By 162

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{26}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{26}(t) = T_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{26}(s_{(24)})\right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{28})^{(5)}$, $T_i^0 \le (\hat{Q}_{28})^{(5)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

By 163

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{30}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{30}(t) = T_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{30}(s_{(28)})\right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{32})^{(6)}$, $T_i^0 \le (\hat{Q}_{32})^{(6)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\bar{M}_{32})^{(6)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\bar{M}_{32})^{(6)} t}$

By 164

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[ (a_{32})^{(6)} G_{33}(s_{(32)}) - \left( (a'_{32})^{(6)} + a''_{32})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[ (a_{33})^{(6)} G_{32}(s_{(32)}) - \left( (a'_{33})^{(6)} + (a''_{33})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[ (a_{34})^{(6)} G_{33}(s_{(32)}) - \left( (a'_{34})^{(6)} + (a''_{34})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{34}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[ (b_{32})^{(6)} T_{33}(s_{(32)}) - \left( (b'_{32})^{(6)} - (b''_{32})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[ (b_{33})^{(6)} T_{32}(s_{(32)}) - \left( (b'_{33})^{(6)} - (b''_{33})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{34}(t) = T_{34}^0 + \int_0^t \left[ (b_{34})^{(6)} T_{33}(s_{(32)}) - \left( (b'_{34})^{(6)} - (b''_{34})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{34}(s_{(32)}) \right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[ (a_{36})^{(7)} G_{37}(s_{(36)}) - \left( (a'_{36})^{(7)} + a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[ (a_{37})^{(7)} G_{36}(s_{(36)}) - \left( (a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[ (a_{38})^{(7)} G_{37}(s_{(36)}) - \left( (a'_{38})^{(7)} + (a''_{38})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{38}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[ (b_{36})^{(7)} T_{37}(s_{(36)}) - \left( (b'_{36})^{(7)} - (b''_{36})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[ (b_{37})^{(7)} T_{36}(s_{(36)}) - \left( (b'_{37})^{(7)} - (b''_{37})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\overline{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^{0} + \int_0^t \left[ (b_{38})^{(7)} T_{37}(s_{(36)}) - \left( (b'_{38})^{(7)} - (b''_{38})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{38}(s_{(36)}) \right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{40})^{(8)}, T_i^0 \le (\hat{Q}_{40})^{(8)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[ (a_{40})^{(8)} G_{41}(s_{(40)}) - \left( (a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[ (a_{41})^{(8)} G_{40}(s_{(40)}) - \left( (a'_{41})^{(8)} + (a''_{41})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[ (a_{42})^{(8)} G_{41}(s_{(40)}) - \left( (a'_{42})^{(8)} + (a''_{42})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{42}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^0 + \int_0^t \left[ (b_{40})^{(8)} T_{41}(s_{(40)}) - \left( (b'_{40})^{(8)} - (b''_{40})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^0 + \int_0^t \left[ (b_{41})^{(8)} T_{40}(s_{(40)}) - \left( (b'_{41})^{(8)} - (b''_{41})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\overline{\mathrm{T}}_{42}(\mathrm{t}) = \mathrm{T}_{42}^{0} + \int_0^t \left[ (b_{42})^{(8)} T_{41}(s_{(40)}) - \left( (b'_{42})^{(8)} - (b''_{42})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{42}(s_{(40)}) \right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** 166
Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy A

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{44})^{(9)}, T_i^0 \le (\hat{Q}_{44})^{(9)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[ (a_{44})^{(9)} G_{45}(s_{(44)}) - \left( (a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45}(s_{(44)}), s_{(44)})\right) G_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}(s_{(44)}), s_{(44)})\right) G_{46}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{44}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{46}(t) = T_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{46}(s_{(44)})\right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}}\right)\right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left(e^{(\hat{M}_{13})^{(1)} t} - 1\right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\hat{M}_{13})^{(1)} t} \le \frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{P}_{13})^{(1)} + G_{14}^0\right) e^{\left(-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0}\right)} + (\hat{P}_{13})^{(1)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \le G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}}\right)\right] ds_{(16)} = \quad 169$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1\right)$$

From which it follows that 170

$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \le \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left[\left((\hat{P}_{16})^{(2)} + G_{17}^0\right) e^{\left(-\frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0}\right)} + (\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t) \le G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} \left(G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} s_{(20)}}\right)\right] ds_{(20)} =$$

$$\left(1 + (a_{20})^{(3)} t\right) G_{21}^0 + \frac{(a_{20})^{(3)} (\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left(e^{(\hat{M}_{20})^{(3)} t} - 1\right)$$

From which it follows that 172

$$(G_{20}(t)-G_{20}^{0})e^{-(\hat{M}_{20})^{(3)}t}\leq\frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left[((\hat{P}_{20})^{(3)}+G_{21}^{0})e^{\left(-\frac{(\hat{P}_{20})^{(3)}+G_{21}^{0}}{G_{21}^{0}}\right)}+(\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 173

$$G_{24}(t)\leq G_{24}^{0}+\int_{0}^{t}\left[(a_{24})^{(4)}\left(G_{25}^{0}+(\hat{P}_{24})^{(4)}e^{(\hat{M}_{24})^{(4)}s_{(24)}}\right)\right]ds_{(24)}=$$

$$\left(1+(a_{24})^{(4)}t\right)G_{25}^{0}+\frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left(e^{(\hat{M}_{24})^{(4)}t}-1\right)$$

From which it follows that 174

$$(G_{24}(t)-G_{24}^{0})e^{-(\hat{M}_{24})^{(4)}t}\leq\frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left[((\hat{P}_{24})^{(4)}+G_{25}^{0})e^{\left(-\frac{(\hat{P}_{24})^{(4)}+G_{25}^{0}}{G_{25}^{0}}\right)}+(\hat{P}_{24})^{(4)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t)\leq G_{28}^{0}+\int_{0}^{t}\left[(a_{28})^{(5)}\left(G_{29}^{0}+(\hat{P}_{28})^{(5)}e^{(\hat{M}_{28})^{(5)}s_{(28)}}\right)\right]ds_{(28)}=$$

$$\left(1+(a_{28})^{(5)}t\right)G_{29}^{0}+\frac{(a_{28})^{(5)}(\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left(e^{(\hat{M}_{28})^{(5)}t}-1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t}\leq\frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t)\leq G_{32}^{0}+\int_{0}^{t}\left[(a_{32})^{(6)}\left(G_{33}^{0}+(\hat{P}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\right)\right]ds_{(32)}=$$

$$\left(1+(a_{32})^{(6)}t\right)G_{33}^{0}+\frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left(e^{(\hat{M}_{32})^{(6)}t}-1\right)$$

From which it follows that 177

$$(G_{32}(t)-G_{32}^{0})e^{-(\hat{M}_{32})^{(6)}t}\leq\frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left[((\hat{P}_{32})^{(6)}+G_{33}^{0})e^{\left(-\frac{(\hat{P}_{32})^{(6)}+G_{33}^{0}}{G_{33}^{0}}\right)}+(\hat{P}_{32})^{(6)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

(bb) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t) \le G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} \left(G_{37}^0 + (\hat{P}_{36})^{(7)} e^{(\tilde{M}_{36})^{(7)} s_{(36)}}\right)\right] ds_{(36)} =$$
$$\left(1 + (a_{36})^{(7)} t\right) G_{37}^0 + \frac{(a_{36})^{(7)} (\hat{P}_{36})^{(7)}}{(\tilde{M}_{36})^{(7)}} \left(e^{(\tilde{M}_{36})^{(7)} t} - 1\right)$$

From which it follows that

$$(G_{36}(t) - G_{36}^0) e^{-(\tilde{M}_{36})^{(7)} t} \le \frac{(a_{36})^{(7)}}{(\tilde{M}_{36})^{(7)}} \left[\left((\hat{P}_{36})^{(7)} + G_{37}^0\right) e^{\left(-\frac{(\hat{P}_{36})^{(7)} + G_{37}^0}{G_{37}^0}\right)} + (\hat{P}_{36})^{(7)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t) \le G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} \left(G_{41}^0 + (\hat{P}_{40})^{(8)} e^{(\tilde{M}_{40})^{(8)} s_{(40)}}\right)\right] ds_{(40)} = \quad 180$$
$$\left(1 + (a_{40})^{(8)} t\right) G_{41}^0 + \frac{(a_{40})^{(8)} (\hat{P}_{40})^{(8)}}{(\tilde{M}_{40})^{(8)}} \left(e^{(\tilde{M}_{40})^{(8)} t} - 1\right)$$

From which it follows that 181

$$(G_{40}(t) - G_{40}^0) e^{-(\tilde{M}_{40})^{(8)} t} \le \frac{(a_{40})^{(8)}}{(\tilde{M}_{40})^{(8)}} \left[\left((\hat{P}_{40})^{(8)} + G_{41}^0\right) e^{\left(-\frac{(\hat{P}_{40})^{(8)} + G_{41}^0}{G_{41}^0}\right)} + (\hat{P}_{40})^{(8)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 8

Analogous inequalities hold also for $G_{41}, G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t) \le G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} \left(G_{45}^0 + (\hat{P}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} s_{(44)}}\right)\right] ds_{(44)} =$$
$$\left(1 + (a_{44})^{(9)} t\right) G_{45}^0 + \frac{(a_{44})^{(9)} (\hat{P}_{44})^{(9)}}{(\tilde{M}_{44})^{(9)}} \left(e^{(\tilde{M}_{44})^{(9)} t} - 1\right)$$

From which it follows that

$$(G_{44}(t) - G_{44}^0) e^{-(\tilde{M}_{44})^{(9)} t} \le \frac{(a_{44})^{(9)}}{(\tilde{M}_{44})^{(9)}} \left[\left((\hat{P}_{44})^{(9)} + G_{45}^0\right) e^{\left(-\frac{(\hat{P}_{44})^{(9)} + G_{45}^0}{G_{45}^0}\right)} + (\hat{P}_{44})^{(9)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 9

Analogous inequalities hold also for $G_{45}, G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\tilde{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\tilde{M}_{13})^{(1)}} < 1$ and to choose 182

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[(\hat{P}_{13})^{(1)}+\left((\hat{P}_{13})^{(1)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{13})^{(1)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{13})^{(1)} \quad 183$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[\left((\hat{Q}_{13})^{(1)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{13})^{(1)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{13})^{(1)}\right]\leq(\hat{Q}_{13})^{(1)} \quad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)},T^{(1)}\right),\left(G^{(2)},T^{(2)}\right)\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{13})^{(1)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{13})^{(1)}t}\}$$

Indeed if we denote

**Definition of** $\tilde{G},\tilde{T}:\left(\tilde{G},\tilde{T}\right)=\mathcal{A}^{(1)}(G,T)$

It results

$$\left|\tilde{G}_{13}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{13})^{(1)}\left|G_{14}^{(1)}-G_{14}^{(2)}\right|e^{-(\hat{M}_{13})^{(1)}s_{(13)}}e^{(\hat{M}_{13})^{(1)}s_{(13)}}\,ds_{(13)}+$$

$$\int_0^t\{(a'_{13})^{(1)}\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\hat{M}_{13})^{(1)}s_{(13)}}e^{-(\hat{M}_{13})^{(1)}s_{(13)}}+$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\hat{M}_{13})^{(1)}s_{(13)}}e^{(\hat{M}_{13})^{(1)}s_{(13)}}+$$

$$G_{13}^{(2)}\left|(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)-(a''_{13})^{(1)}\left(T_{14}^{(2)},s_{(13)}\right)\right|\ e^{-(\hat{M}_{13})^{(1)}s_{(13)}}e^{(\hat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|G^{(1)}-G^{(2)}\right|e^{-(\hat{M}_{13})^{(1)}t}\leq \quad 186$$
$$\frac{1}{(\hat{M}_{13})^{(1)}}\left((a_{13})^{(1)}+(a'_{13})^{(1)}+(\hat{A}_{13})^{(1)}+(\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}\right)d\left(\left(G^{(1)},T^{(1)};\,G^{(2)},T^{(2)}\right)\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 1:** The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\hat{P}_{13})^{(1)}e^{(\hat{M}_{13})^{(1)}t}$ *and* $(\hat{Q}_{13})^{(1)}e^{(\hat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i = 13,14,15$ depend only on $T_{14}$ and respectively on $G(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any $t$ where $G_i(t) = 0\ and\ T_i(t) = 0$

From 19 to 24 it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(1)} - (a_i'')^{(1)}(T_{14}(s_{(13)}), s_{(13)})\} ds_{(13)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(1)} t)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2\ and\ \left((\widehat{M}_{13})^{(1)}\right)_3$ : 187

**Remark 3**: if $G_{13}$ is bounded, the same property have also $G_{14}\ and\ G_{15}$ . indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a_{14}')^{(1)} G_{14}$ and by integrating

$$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)} \left((\widehat{M}_{13})^{(1)}\right)_1 / (a_{14}')^{(1)}$$

In the same way , one can obtain

$$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)} \left((\widehat{M}_{13})^{(1)}\right)_2 / (a_{15}')^{(1)}$$

If $G_{14}\ or\ G_{15}$ is bounded, the same property follows for $G_{13}$ , $G_{15}$ and $G_{13}$ , $G_{14}$ respectively.

**Remark 4:** If $G_{13}$ *is* bounded, from below, the same property holds for $G_{14}\ and\ G_{15}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{14}$ is bounded from below. 188

**Remark 5:** If $T_{13}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}(G(t), t)) = (b_{14}')^{(1)}$ then $T_{14} \to \infty$. 189

**Definition of** $(m)^{(1)}$ and $\varepsilon_1$ :

Indeed let $t_1$ be so that for $t > t_1$

$$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t), t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right)$, $t = log\frac{2}{\varepsilon_1}$ By taking now $\varepsilon_1$ sufficiently small one sees that $T_{14}$ is unbounded.
The same property holds for $T_{15}$ if $\lim_{t\to\infty}(b_{15}'')^{(1)}(G(t), t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}$ , $\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}\ and\ (\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[(\widehat{P}_{16})^{(2)}+\left((\hat{P}_{16})^{(2)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{16})^{(2)}\right]\leq(\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

**Definition of** $\widetilde{G_{19}},\widetilde{T_{19}}$ : $\left(\widetilde{G_{19}},\widetilde{T_{19}}\right)=\mathcal{A}^{(2)}(G_{19},T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{16})^{(2)}\left|G_{17}^{(1)}-G_{17}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)}+$$

$$\int_0^t\{(a'_{16})^{(2)}\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$G_{16}^{(2)}\left|(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)-(a''_{16})^{(2)}\left(T_{17}^{(2)},s_{(16)}\right)\right|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0,t]$ 197

From the hypotheses it follows

$$\left|(G_{19})^{(1)}-(G_{19})^{(2)}\right|\mathrm{e}^{-(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}\leq$$
$$\frac{1}{(\widehat{\mathrm{M}}_{16})^{(2)}}\left((a_{16})^{(2)}+(a'_{16})^{(2)}+(\widehat{\mathrm{A}}_{16})^{(2)}+(\widehat{\mathrm{P}}_{16})^{(2)}(\hat{k}_{16})^{(2)}\right)\mathrm{d}\left(((G_{19})^{(1)},(T_{19})^{(1)};\ (G_{19})^{(2)},(T_{19})^{(2)})\right)$$

And analogous inequalities for $G_i$ and $T_i$. Taking into account the hypothesis the result follows 198

**Remark 6:** The fact that we supposed $(a''_{16})^{(2)}$ and $(b''_{16})^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{\mathrm{P}}_{16})^{(2)}\mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}$ and $(\widehat{\mathrm{Q}}_{16})^{(2)}\mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i=16,17,18$ depend only on $\mathrm{T}_{17}$ and respectively on

$(G_{19})$(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 7:** There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$ 200

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(2)} - (a_i'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})\} ds_{(16)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(2)}t)} > 0 \quad \text{for } t > 0$$

**Definition of** $\left((\widehat{M}_{16})^{(2)}\right)_1$, $\left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$: 201

**Remark 8:** if $G_{16}$ is bounded, the same property have also $G_{17}$ and $G_{18}$. indeed if

$G_{16} < (\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt} \leq \left((\widehat{M}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)} G_{17}$ and by integrating

$$G_{17} \leq \left((\widehat{M}_{16})^{(2)}\right)_2 = G_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1 / (a_{17}')^{(2)}$$

In the same way , one can obtain

$$G_{18} \leq \left((\widehat{M}_{16})^{(2)}\right)_3 = G_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2 / (a_{18}')^{(2)}$$

If $G_{17}$ or $G_{18}$ is bounded, the same property follows for $G_{16}$, $G_{18}$ and $G_{16}$, $G_{17}$ respectively.

**Remark 9:** If $G_{16}$ is bounded, from below, the same property holds for $G_{17}$ and $G_{18}$. The proof is analogous with the preceding one. An analogous property is true if $G_{17}$ is bounded from below. 202

**Remark 10:** If $T_{16}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t), t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$. 203

**Definition of** $(m)^{(2)}$ and $\varepsilon_2$ :

Indeed let $t_2$ be so that for $t > t_2$

$$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right)$, $t = \log\frac{2}{\varepsilon_2}$ By taking now $\varepsilon_2$ sufficiently small one sees that $T_{17}$ is unbounded. 205
The same property holds for $T_{18}$ if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t), t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[(\widehat{P}_{20})^{(3)}+\left((\hat{P}_{20})^{(3)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{20})^{(3)}\right]\leq(\hat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself 210

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)},(T_{23})^{(1)}),((G_{23})^{(2)},(T_{23})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

**Definition of** $\widetilde{G_{23}},\widetilde{T_{23}}:(\widetilde{(G_{23})},\widetilde{(T_{23})})=\mathcal{A}^{(3)}((G_{23}),(T_{23}))$

It results 213

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{20})^{(3)}\left|G_{21}^{(1)}-G_{21}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\,ds_{(20)}+$$

$$\int_0^t\{(a'_{20})^{(3)}\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$G_{20}^{(2)}\left|(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)},s_{(20)}\right)\right|\ e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\}ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses it follows

$$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}t}\leq \quad 214$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+(a'_{20})^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\widehat{k}_{20})^{(3)}\right)d\left(((G_{23})^{(1)},(T_{23})^{(1)};\ (G_{23})^{(2)},(T_{23})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

**Remark 11:** The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}\ and\ (\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i=20,21,22$ depend only on $\mathrm{T}_{21}$ and respectively on

$(G_{23})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 12:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 216

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(3)} - (a_i'')^{(3)}(T_{21}(s_{(20)}), s_{(20)})\} ds_{(20)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(3)} t)} > 0$ for t > 0

**Definition of** $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2\ and\ \left((\widehat{M}_{20})^{(3)}\right)_3$ : 217

**Remark 13:** if $G_{20}$ is bounded, the same property have also $G_{21}\ and\ G_{22}$ . indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a_{21}')^{(3)} G_{21}$ and by integrating

$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1 / (a_{21}')^{(3)}$

In the same way , one can obtain

$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2 / (a_{22}')^{(3)}$

If $G_{21}\ or\ G_{22}$ is bounded, the same property follows for $G_{20}$ , $G_{22}$ and $G_{20}$ , $G_{21}$ respectively.

**Remark 14:** If $G_{20}\ \ is$ bounded, from below, the same property holds for $G_{21}\ and\ G_{22}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{21}$ is bounded from below. 218

**Remark 15:** If $T_{20}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t), t\right)) = (b_{21}')^{(3)}$ then $T_{21} \to \infty$. 219

**Definition of** $(m)^{(3)}$ and $\varepsilon_3$ :

Indeed let $t_3$ be so that for $t > t_3$

$(b_{21})^{(3)} - (b_i'')^{(3)}\left((G_{23})(t), t\right) < \varepsilon_3, T_{20}\,(t) > (m)^{(3)}$

Then $\frac{dT_{21}}{dt} \geq (a_{21})^{(3)}(m)^{(3)} - \varepsilon_3 T_{21}$ which leads to 220

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t} = \frac{1}{2}$ it results

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right),\ \ t = log\,\frac{2}{\varepsilon_3}$ By taking now $\varepsilon_3$ sufficiently small one sees that $T_{21}$ is unbounded.
The same property holds for $T_{22}$ if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t), t\right) = (b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}$ , $\frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}} < 1$ and to choose 221

$(\widehat{P}_{24})^{(4)}$ and $(\widehat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[(\widehat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \tag{222}$$

$$\frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \tag{223}$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself (224)

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric (225)

$$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{24})^{(4)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{27}}),(\widetilde{T_{27}})$ : $\left((\widetilde{G_{27}}),(\widetilde{T_{27}})\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{24})^{(4)}\left|G_{25}^{(1)}-G_{25}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\,ds_{(24)}+$$

$$\int_0^t\{(a'_{24})^{(4)}\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$G_{24}^{(2)}\left|(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)-(a''_{24})^{(4)}\left(T_{25}^{(2)},s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\}ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}t}\leq \tag{226}$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)}+(a'_{24})^{(4)}+(\widehat{A}_{24})^{(4)}+(\widehat{P}_{24})^{(4)}(\hat{k}_{24})^{(4)}\right)d\left(((G_{27})^{(1)},(T_{27})^{(1)};\ (G_{27})^{(2)},(T_{27})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 16:** The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ *and* $(\widehat{Q}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_+$. (227)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i=24,25,26$ depend only on $T_{25}$ and respectively on

$(G_{27})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 17:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 228

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(4)} - (a_i'')^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(4)} t)} > 0$ for t > 0

**Definition of** $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2\ and\ \left((\widehat{M}_{24})^{(4)}\right)_3$ : 229

**Remark 18:** if $G_{24}$ is bounded, the same property have also $G_{25}\ and\ G_{26}$ . indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a_{25}')^{(4)} G_{25}$ and by integrating

$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1 / (a_{25}')^{(4)}$

In the same way , one can obtain

$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2 / (a_{26}')^{(4)}$

If $G_{25}\ or\ G_{26}$ is bounded, the same property follows for $G_{24}$ , $G_{26}$ and $G_{24}$ , $G_{25}$ respectively.

**Remark 19:** If $G_{24}$ *is* bounded, from below, the same property holds for $G_{25}\ and\ G_{26}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{25}$ is bounded from below. 230

**Remark 20:** If $T_{24}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}\ ((G_{27})(t), t)) = (b_{25}')^{(4)}$ then $T_{25} \to \infty$. 231

**Definition of** $(m)^{(4)}$ and $\varepsilon_4$ :

Indeed let $t_4$ be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t), t) < \varepsilon_4, T_{24}\,(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_4}$ By taking now $\varepsilon_4$ sufficiently small one sees that $T_{25}$ is unbounded.
The same property holds for $T_{26}$ if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t), t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for $G_{29}$ , $G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}$ , $\frac{(b_i)^{(5)}}{(\widehat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)}+\left((\hat{P}_{28})^{(5)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{28})^{(5)} \tag{234}$$

$$\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{28})^{(5)}\right]\leq(\hat{Q}_{28})^{(5)} \tag{235}$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric (236)

$$d\left(((G_{31})^{(1)},(T_{31})^{(1)}),((G_{31})^{(2)},(T_{31})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\hat{M}_{28})^{(5)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\hat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{31}}),(\widetilde{T_{31}})$ : $\left((\widetilde{G_{31}}),(\widetilde{T_{31}})\right)=\mathcal{A}^{(5)}\left((G_{31}),(T_{31})\right)$

It results

$$|\tilde{G}_{28}^{(1)}-\tilde{G}_i^{(2)}|\leq\int_0^t(a_{28})^{(5)}|G_{29}^{(1)}-G_{29}^{(2)}|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\,ds_{(28)}+$$

$$\int_0^t\{(a'_{28})^{(5)}|G_{28}^{(1)}-G_{28}^{(2)}|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$(a''_{28})^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)|G_{28}^{(1)}-G_{28}^{(2)}|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$G_{28}^{(2)}|(a''_{28})^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)-(a''_{28})^{(5)}\left(T_{29}^{(2)},s_{(28)}\right)|\;e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\}ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on it follows

$$|(G_{31})^{(1)}-(G_{31})^{(2)}|e^{-(\widehat{M}_{28})^{(5)}t}\leq \tag{237}$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)}+(a'_{28})^{(5)}+(\widehat{A}_{28})^{(5)}+(\widehat{P}_{28})^{(5)}(\widehat{k}_{28})^{(5)}\right)d\left(((G_{31})^{(1)},(T_{31})^{(1)};\;(G_{31})^{(2)},(T_{31})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 21:** The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\hat{P}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ *and* $(\hat{Q}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ respectively of $\mathbb{R}_+$. (238)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on $T_{29}$ and respectively on $(G_{31})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 22:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 239

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(5)} - (a_i'')^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(5)} t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2\ and\ \left((\widehat{M}_{28})^{(5)}\right)_3$ : 240

**Remark 23:** if $G_{28}$ is bounded, the same property have also $G_{29}\ and\ G_{30}$ . indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a_{29}')^{(5)} G_{29}$ and by integrating

$$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1 / (a_{29}')^{(5)}$$

In the same way , one can obtain

$$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2 / (a_{30}')^{(5)}$$

If $G_{29}\ or\ G_{30}$ is bounded, the same property follows for $G_{28}$ , $G_{30}$ and $G_{28}$ , $G_{29}$ respectively.

**Remark 24:** If $G_{28}$ *is* bounded, from below, the same property holds for $G_{29}\ and\ G_{30}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{29}$ is bounded from below. 241

**Remark 25:** If $T_{28}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}\,((G_{31})(t), t)) = (b_{29}')^{(5)}$ then $T_{29} \to \infty$. 242

**Definition of** $(m)^{(5)}$ and $\varepsilon_5$ :

Indeed let $t_5$ be so that for $t > t_5$

$$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t), t) < \varepsilon_5, T_{28}\,(t) > (m)^{(5)}$$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_5}$ By taking now $\varepsilon_5$ sufficiently small one sees that $T_{29}$ is unbounded.
The same property holds for $T_{30}$ if $\lim_{t\to\infty}(b_{30}'')^{(5)}\left((G_{31})(t), t\right) = (b_{30}')^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{33}\,, G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\ , \frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}} < 1$ and to choose 244

$(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[(\hat{P}_{32})^{(6)}+\left((\hat{P}_{32})^{(6)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{32})^{(6)} \tag{245}$$

$$\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[\left((\hat{Q}_{32})^{(6)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{32})^{(6)}\right]\leq(\hat{Q}_{32})^{(6)} \tag{246}$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric (247)

$$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\hat{M}_{32})^{(6)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\hat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{35}}),(\widetilde{T_{35}})$ : $\left((\widetilde{G_{35}}),(\widetilde{T_{35}})\right)=\mathcal{A}^{(6)}\left((G_{35}),(T_{35})\right)$

It results

$$|\tilde{G}_{32}^{(1)}-\tilde{G}_i^{(2)}|\leq\int_0^t(a_{32})^{(6)}|G_{33}^{(1)}-G_{33}^{(2)}|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\,ds_{(32)}+$$

$$\int_0^t\{(a'_{32})^{(6)}|G_{32}^{(1)}-G_{32}^{(2)}|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)|G_{32}^{(1)}-G_{32}^{(2)}|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$G_{32}^{(2)}|(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)-(a''_{32})^{(6)}\left(T_{33}^{(2)},s_{(32)}\right)|\;e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\}ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$|(G_{35})^{(1)}-(G_{35})^{(2)}|e^{-(\widehat{M}_{32})^{(6)}t}\leq \tag{248}$$
$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)}+(a'_{32})^{(6)}+(\widehat{A}_{32})^{(6)}+(\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}\right)d\left(((G_{35})^{(1)},(T_{35})^{(1)};\;(G_{35})^{(2)},(T_{35})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 26:** The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\hat{P}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ *and* $(\hat{Q}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ respectively of $\mathbb{R}_+$. (249)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i = 32,33,34$ depend only on $T_{33}$ and respectively on $(G_{35})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 27:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 250

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(6)} - (a_i'')^{(6)}(T_{33}(s_{(32)}), s_{(32)})\} ds_{(32)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(6)} t\right)} > 0$ for t > 0

**Definition of** $\left((\widehat{M}_{32})^{(6)}\right)_1, \left((\widehat{M}_{32})^{(6)}\right)_2\ and\ \left((\widehat{M}_{32})^{(6)}\right)_3$ : 251

**Remark 28:** if $G_{32}$ is bounded, the same property have also $G_{33}\ and\ G_{34}$ . indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a_{33}')^{(6)} G_{33}$ and by integrating

$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_1 / (a_{33}')^{(6)}$

In the same way , one can obtain

$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_2 / (a_{34}')^{(6)}$

If $G_{33}\ or\ G_{34}$ is bounded, the same property follows for $G_{32}$ , $G_{34}$ and $G_{32}$ , $G_{33}$ respectively.

**Remark 29:** If $G_{32}$ *is* bounded, from below, the same property holds for $G_{33}\ and\ G_{34}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{33}$ is bounded from below. 252

**Remark 30:** If $T_{32}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}\ ((G_{35})(t), t)) = (b_{33}')^{(6)}$ then $T_{33}\ \to \infty$. 253

**Definition of** $(m)^{(6)}$ and $\varepsilon_6$ :

Indeed let $t_6$ be so that for $t\ > t_6$

$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t), t\right) < \varepsilon_6, T_{32}\,(t) > (m)^{(6)}$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33}\ \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33}\ \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_6}$ By taking now $\varepsilon_6$ sufficiently small one sees that $T_{33}$ is unbounded.
The same property holds for $T_{34}$ if $\lim_{t\to\infty}(b_{34}'')^{(6)}\left((G_{35})(t), t(t), t\right) = (b_{34}')^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{37}\ , G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\ , \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$ and to choose

$(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[(\hat{P}_{36})^{(7)}+\left((\hat{P}_{36})^{(7)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{36})^{(7)} \tag{256}$$

$$\frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[\left((\hat{Q}_{36})^{(7)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{36})^{(7)}\right]\leq(\hat{Q}_{36})^{(7)} \tag{257}$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric (258)

$$d\left(((G_{39})^{(1)},(T_{39})^{(1)}),((G_{39})^{(2)},(T_{39})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\tilde{M}_{36})^{(7)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\tilde{M}_{36})^{(7)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{39}}),(\widetilde{T_{39}}):\left((\widetilde{G_{39}}),(\widetilde{T_{39}})\right)=\mathcal{A}^{(7)}((G_{39}),(T_{39}))$

It results

$$|\tilde{G}_{36}^{(1)}-\tilde{G}_i^{(2)}|\leq\int_0^t(a_{36})^{(7)}|G_{37}^{(1)}-G_{37}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\,ds_{(36)}+$$
$$\int_0^t\{(a'_{36})^{(7)}|G_{36}^{(1)}-G_{36}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$(a''_{36})^{(7)}(T_{37}^{(1)},s_{(36)})|G_{36}^{(1)}-G_{36}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$G_{36}^{(2)}|(a''_{36})^{(7)}(T_{37}^{(1)},s_{(36)})-(a''_{36})^{(7)}(T_{37}^{(2)},s_{(36)})|\;e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\}ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on it follows

$$|(G_{39})^{(1)}-(G_{39})^{(2)}|e^{-(\widehat{M}_{36})^{(7)}t}\leq \tag{259}$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}}\left((a_{36})^{(7)}+(a'_{36})^{(7)}+(\widehat{A}_{36})^{(7)}+(\widehat{P}_{36})^{(7)}(\widehat{k}_{36})^{(7)}\right)d\left(((G_{39})^{(1)},(T_{39})^{(1)};\;(G_{39})^{(2)},(T_{39})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 31:** The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ *and* $(\widehat{Q}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ respectively of $\mathbb{R}_+$. (260)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(7)}$ and $(b_i'')^{(7)}, i = 36,37,38$ depend only on $T_{37}$ and respectively on $(G_{39})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 32:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 261

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(7)} - (a_i'')^{(7)}(T_{37}(s_{(36)}), s_{(36)})\} ds_{(36)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(7)} t\right)} > 0$ for t > 0

**Definition of** $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2\ and\ \left((\widehat{M}_{36})^{(7)}\right)_3$ : 262

**Remark 33:** if $G_{36}$ is bounded, the same property have also $G_{37}\ and\ G_{38}$ . indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a_{37}')^{(7)} G_{37}$ and by integrating

$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1 / (a_{37}')^{(7)}$

In the same way , one can obtain

$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2 / (a_{38}')^{(7)}$

If $G_{37}\ or\ G_{38}$ is bounded, the same property follows for $G_{36}$ , $G_{38}$ and $G_{36}$ , $G_{37}$ respectively.

**Remark 34:** If $G_{36}$ *is* bounded, from below, the same property holds for $G_{37}\ and\ G_{38}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{37}$ is bounded from below. 263

**Remark 35:** If $T_{36}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(7)}\ ((G_{39})(t), t)) = (b_{37}')^{(7)}$ then $T_{37} \to \infty$. 264

**Definition of** $(m)^{(7)}$ and $\varepsilon_7$ :

Indeed let $t_7$ be so that for $t > t_7$

$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t), t) < \varepsilon_7, T_{36}\,(t) > (m)^{(7)}$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_7}$ By taking now $\varepsilon_7$ sufficiently small one sees that $T_{37}$ is unbounded.
The same property holds for $T_{38}$ if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t), t\right) = (b_{38}')^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}$ , $\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}} < 1$ and to choose $(\widehat{P}_{40})^{(8)}$ and $(\widehat{Q}_{40})^{(8)}$ large to have 266

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[(\widehat{P}_{40})^{(8)} + \left((\hat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{40})^{(8)} \quad 267$$

$$\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{40})^{(8)}\right] \leq (\hat{Q}_{40})^{(8)} \quad 268$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)}, (T_{43})^{(1)}), ((G_{43})^{(2)}, (T_{43})^{(2)})\right) = \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

**Definition of** $\widetilde{(G_{43})}, \widetilde{(T_{43})}$ : $\left(\widetilde{(G_{43})}, \widetilde{(T_{43})}\right) = \mathcal{A}^{(8)}((G_{43}), (T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{40})^{(8)}\left|G_{41}^{(1)} - G_{41}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\,ds_{(40)} +$$
$$\int_0^t\{(a'_{40})^{(8)}\left|G_{40}^{(1)} - G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{-(\widehat{M}_{40})^{(8)}s_{(40)}} +$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right)\left|G_{40}^{(1)} - G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}} +$$
$$G_{40}^{(2)}\left|(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right) - (a''_{40})^{(8)}\left(T_{41}^{(2)}, s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\}ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0, t]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)} - (G_{43})^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}t} \leq \quad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)} + (a'_{40})^{(8)} + (\widehat{A}_{40})^{(8)} + (\widehat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}\right)d\left(((G_{43})^{(1)}, (T_{43})^{(1)};\ (G_{43})^{(2)}, (T_{43})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

**Remark 36:** The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate 274

condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ $and$ $(\widehat{Q}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i = 40,41,42$ depend only on $T_{41}$ and respectively on $(G_{43})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 37** There does not exist any $t$ where $G_i(t) = 0\ and\ T_i(t) = 0$ 275

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(8)} - (a_i'')^{(8)}(T_{41}(s_{(40)}), s_{(40)})\} ds_{(40)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(8)}t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{40})^{(8)}\right)_1, \left((\widehat{M}_{40})^{(8)}\right)_2$ $and$ $\left((\widehat{M}_{40})^{(8)}\right)_3$ : 276

**Remark 38:** if $G_{40}$ is bounded, the same property have also $G_{41}$ $and$ $G_{42}$ . indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a_{41}')^{(8)}G_{41}$ and by integrating

$$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1/(a_{41}')^{(8)}$$

In the same way , one can obtain

$$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2/(a_{42}')^{(8)}$$

If $G_{41}$ $or$ $G_{42}$ is bounded, the same property follows for $G_{40}$ , $G_{42}$ and $G_{40}$ , $G_{41}$ respectively.

**Remark 39:** If $G_{40}$ $is$ bounded, from below, the same property holds for $G_{41}$ $and$ $G_{42}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{41}$ is bounded from below. 277

**Remark 40:** If $T_{40}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}((G_{43})(t), t)) = (b_{41}')^{(8)}$ then $T_{41} \to \infty$. 278

**Definition of** $(m)^{(8)}$ and $\varepsilon_8$ :

Indeed let $t_8$ be so that for $t > t_8$

$$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t), t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right)$, $t = log\frac{2}{\varepsilon_8}$ By taking now $\varepsilon_8$ sufficiently small one sees that $T_{41}$ is unbounded.
The same property holds for $T_{42}$ if $\lim_{t\to\infty}(b''_{42})^{(8)}\left((G_{43})(t), t(t), t\right) = (b'_{42})^{(8)}$



It is now sufficient to take $\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}$ , $\frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$ and to choose $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ large to have

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}\left[(\hat{P}_{44})^{(9)} + \left((\hat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}}\left[\left((\hat{Q}_{44})^{(9)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{44})^{(9)}\right] \leq (\hat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)}, (T_{47})^{(1)}), ((G_{47})^{(2)}, (T_{47})^{(2)})\right) =$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{44})^{(9)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{47}}), (\widetilde{T_{47}}) : \left((\widetilde{G_{47}}), (\widetilde{T_{47}})\right) = \mathcal{A}^{(9)}\left((G_{47}), (T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{44})^{(9)}\left|G_{45}^{(1)} - G_{45}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\,ds_{(44)} +$$
$$\int_0^t\{(a'_{44})^{(9)}\left|G_{44}^{(1)} - G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{-(\widehat{M}_{44})^{(9)}s_{(44)}} +$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right)\left|G_{44}^{(1)} - G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}} +$$
$$G_{44}^{(2)}\left|(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right) - (a''_{44})^{(9)}\left(T_{45}^{(2)}, s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\}ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)} - G^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}t} \leq$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)} + (a'_{44})^{(9)} + (\widehat{A}_{44})^{(9)} + (\widehat{P}_{44})^{(9)}(\hat{k}_{44})^{(9)}\right)d\left(((G_{47})^{(1)}, (T_{47})^{(1)};\ (G_{47})^{(2)}, (T_{47})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis (39,35,36) the result follows

**Remark 41:** The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by

$(\widehat{P}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ *and* $(\widehat{Q}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i = 44{,}45{,}46$ depend only on $T_{45}$ and respectively on $(G_{47})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 42:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$

From 99 to 44 it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t\{(a_i')^{(9)}-(a_i'')^{(9)}(T_{45}(s_{(44)}),s_{(44)})\}ds_{(44)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(9)}t)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{44})^{(9)}\right)_1, \left((\widehat{M}_{44})^{(9)}\right)_2\ and\ \left((\widehat{M}_{44})^{(9)}\right)_3$ :

**Remark 43:** if $G_{44}$ is bounded, the same property have also $G_{45}\ and\ G_{46}$ . indeed if

$G_{44} < (\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt} \leq \left((\widehat{M}_{44})^{(9)}\right)_1 - (a_{45}')^{(9)}G_{45}$ and by integrating

$$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_1/(a_{45}')^{(9)}$$

In the same way , one can obtain

$$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_2/(a_{46}')^{(9)}$$

If $G_{45}\ or\ G_{46}$ is bounded, the same property follows for $G_{44}$ , $G_{46}$ and $G_{44}$ , $G_{45}$ respectively.

**Remark 44:** If $G_{44}$ *is* bounded, from below, the same property holds for $G_{45}\ and\ G_{46}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{45}$ is bounded from below.

**Remark 45:** If $T_{44}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t),t\right) = (b_{45}')^{(9)}$ then $T_{45} \to \infty$.

**Definition of** $(m)^{(9)}$ and $\varepsilon_9$ :

Indeed let $t_9$ be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t),t\right) < \varepsilon_9, T_{44}\,(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right), \quad t = log\frac{2}{\varepsilon_9}$ By taking now $\varepsilon_9$ sufficiently small one sees that $T_{45}$ is unbounded.
The same property holds for $T_{46}$ if $\lim_{t\to\infty}(b_{46}'')^{(9)}\left((G_{47})(t),t\right) = (b_{46}')^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

**Behavior of the solutions of equation** 280

**Theorem** If we denote and define

**Definition of** $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ :

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)} \le -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14}, t) + (a''_{14})^{(1)}(T_{14}, t) \le -(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)} \le -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G, t) - (b''_{14})^{(1)}(G, t) \le -(\tau_1)^{(1)}$

**Definition of** $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$ : 281

By $(\nu_1)^{(1)} > 0$, $(\nu_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$, $(u_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

**Definition of** $(\bar{\nu}_1)^{(1)},, (\bar{\nu}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$ : 282

By $(\bar{\nu}_1)^{(1)} > 0$, $(\bar{\nu}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$, $(\bar{u}_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

**Definition of** $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (\nu_0)^{(1)}$ :- 283

If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by
$(m_2)^{(1)} = (\nu_0)^{(1)}, (m_1)^{(1)} = (\nu_1)^{(1)}$, $if\ (\nu_0)^{(1)} < (\nu_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}, if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)}$,

and $\boxed{(\nu_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)}$, $if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)}$, $if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)}$,

and $\boxed{(u_0)^{(1)} = \frac{T^0_{13}}{T^0_{14}}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$

are defined

Then the solution of global equations satisfies the inequalities 285

$G^0_{13}e^{\left((S_1)^{(1)} - (p_{13})^{(1)}\right)t} \le G_{13}(t) \le G^0_{13}e^{(S_1)^{(1)}t}$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)}-(p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)}t}$$

$$\left(\frac{(a_{15})^{(1)}G_{13}^0}{(m_1)^{(1)}((S_1)^{(1)}-(p_{13})^{(1)}-(S_2)^{(1)})}\left[e^{((S_1)^{(1)}-(p_{13})^{(1)})t} - e^{-(S_2)^{(1)}t}\right] + G_{15}^0 e^{-(S_2)^{(1)}t} \le G_{15}(t) \le \right. \quad (286)$$

$$\left. \frac{(a_{15})^{(1)}G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)}-(a'_{15})^{(1)})}[e^{(S_1)^{(1)}t} - e^{-(a'_{15})^{(1)}t}] + G_{15}^0 e^{-(a'_{15})^{(1)}t}\right)$$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)}t} \le T_{13}(t) \le T_{13}^0 e^{((R_1)^{(1)}+(r_{13})^{(1)})t}} \quad (287)$$

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)}t} \le T_{13}(t) \le \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)}+(r_{13})^{(1)})t} \quad (288)$$

$$\frac{(b_{15})^{(1)}T_{13}^0}{(\mu_1)^{(1)}((R_1)^{(1)}-(b'_{15})^{(1)})}\left[e^{(R_1)^{(1)}t} - e^{-(b'_{15})^{(1)}t}\right] + T_{15}^0 e^{-(b'_{15})^{(1)}t} \le T_{15}(t) \le \quad (289)$$

$$\frac{(a_{15})^{(1)}T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)}+(r_{13})^{(1)}+(R_2)^{(1)})}\left[e^{((R_1)^{(1)}+(r_{13})^{(1)})t} - e^{-(R_2)^{(1)}t}\right] + T_{15}^0 e^{-(R_2)^{(1)}t}$$

**Definition of** $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:- (290)

Where $(S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$

$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$

$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$

$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$

**Behavior of the solutions of equation** (291)

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$: (292)

$(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ four constants satisfying

$$-(\sigma_2)^{(2)} \le -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(T_{17}, t) + (a''_{17})^{(2)}(T_{17}, t) \le -(\sigma_1)^{(2)} \quad (293)$$

$$-(\tau_2)^{(2)} \le -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}((G_{19}), t) - (b''_{17})^{(2)}((G_{19}), t) \le -(\tau_1)^{(2)} \quad (294)$$

**Definition of** $(\nu_1)^{(2)}, (\nu_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$: (295)

By $(\nu_1)^{(2)} > 0$, $(\nu_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0$, $(u_2)^{(2)} < 0$ the roots (296)

of the equations $(a_{17})^{(2)}(\nu^{(2)})^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ (297)

and $(b_{14})^{(2)}(u^{(2)})^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and (298)

**Definition of** $(\bar{\nu}_1)^{(2)}, , (\bar{\nu}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$: (299)

By $(\bar{\nu}_1)^{(2)} > 0$, $(\bar{\nu}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0$, $(\bar{u}_2)^{(2)} < 0$ the (300)

roots of the equations $(a_{17})^{(2)}\left(v^{(2)}\right)^2 + (\sigma_2)^{(2)}v^{(2)} - (a_{16})^{(2)} = 0$ 301

and $(b_{17})^{(2)}\left(u^{(2)}\right)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 302

**Definition of** $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ :- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (v_0)^{(2)}, (m_1)^{(2)} = (v_1)^{(2)}$, ***if*** $(v_0)^{(2)} < (v_1)^{(2)}$ 305

$(m_2)^{(2)} = (v_1)^{(2)}, (m_1)^{(2)} = (\bar{v}_1)^{(2)}$, ***if*** $(v_1)^{(2)} < (v_0)^{(2)} < (\bar{v}_1)^{(2)}$, 306

and $\boxed{(v_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}}$

$(m_2)^{(2)} = (v_1)^{(2)}, (m_1)^{(2)} = (v_0)^{(2)}$, ***if*** $(\bar{v}_1)^{(2)} < (v_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, ***if*** $(u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$, ***if*** $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)}$, ***if*** $(\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)}t}$

$(p_i)^{(2)}$ is defined by equation

$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)}t}$ 311

$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)}t}\right] + G_{18}^0 e^{-(S_2)^{(2)}t} \le G_{18}(t) \le\right.$ 312

$\left.\frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)}t} - e^{-(a'_{18})^{(2)}t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)}t}\right)$

$\boxed{T_{16}^0 e^{(R_1)^{(2)}t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}}$ 313

$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)}t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}$ 314

$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)}t} - e^{-(b'_{18})^{(2)}t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)}t} \le T_{18}(t) \le$ 315

$$\frac{(a_{18})^{(2)}T_{16}^{0}}{(\mu_2)^{(2)}((R_1)^{(2)}+(r_{16})^{(2)}+(R_2)^{(2)})}\left[e^{((R_1)^{(2)}+(r_{16})^{(2)})t}-e^{-(R_2)^{(2)}t}\right]+T_{18}^{0}e^{-(R_2)^{(2)}t}$$

**Definition of** $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$$

$$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)}$$ 318

$$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$$

**Behavior of the solutions** 319

**Theorem 3:** If we denote and define

**Definition of** $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ :

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}((G_{23}), t) \le -(\tau_1)^{(3)}$$

**Definition of** $(v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$ : 320

By $(v_1)^{(3)} > 0$, $(v_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations
$(a_{21})^{(3)}(v^{(3)})^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$
and $(b_{21})^{(3)}(u^{(3)})^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{v}_1)^{(3)} > 0$, $(\bar{v}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}(v^{(3)})^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}(u^{(3)})^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

**Definition of** $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ :- 321

If we define $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ by
$(m_2)^{(3)} = (v_0)^{(3)}, (m_1)^{(3)} = (v_1)^{(3)}$, ***if*** $(v_0)^{(3)} < (v_1)^{(3)}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (\bar{v}_1)^{(3)}$, ***if*** $(v_1)^{(3)} < (v_0)^{(3)} < (\bar{v}_1)^{(3)}$,

and $\boxed{(v_0)^{(3)} = \frac{G_{20}^{0}}{G_{21}^{0}}}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_0)^{(3)}$, ***if*** $(\bar{v}_1)^{(3)} < (v_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)},\ \boldsymbol{if}\ (u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}, \boldsymbol{if}\ (u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)},$ and $\boxed{(u_0)^{(3)} = \frac{T_{20}^0}{T_{21}^0}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)},\ \boldsymbol{if}\ (\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G_{20}^0 e^{(S_1)^{(3)} t}$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)} t} \quad 323$$

$$\left(\frac{(a_{22})^{(3)} G_{20}^0}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\left[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)} t}\right] + G_{22}^0 e^{-(S_2)^{(3)} t} \le G_{22}(t) \le \right. \quad 324$$

$$\left. \frac{(a_{22})^{(3)} G_{20}^0}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}\left[e^{(S_1)^{(3)} t} - e^{-(a'_{22})^{(3)} t}\right] + G_{22}^0 e^{-(a'_{22})^{(3)} t}\right)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t}} \quad 325$$

$$\frac{1}{(\mu_1)^{(3)}} T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}} T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)} T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)} - (b'_{22})^{(3)})}\left[e^{(R_1)^{(3)} t} - e^{-(b'_{22})^{(3)} t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)} t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)} T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)} + (r_{20})^{(3)} + (R_2)^{(3)})}\left[e^{((R_1)^{(3)} + (r_{20})^{(3)})t} - e^{-(R_2)^{(3)} t}\right] + T_{22}^0 e^{-(R_2)^{(3)} t}$$

**Definition of** $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

**Behavior of the solutions of equation**

**Theorem:** If we denote and define

**Definition of** $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ :

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$

$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}((G_{27}), t) - (b''_{25})^{(4)}((G_{27}), t) \le -(\tau_1)^{(4)}$

**Definition of** $(v_1)^{(4)}, (v_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, v^{(4)}, u^{(4)}$ : 329

By $(v_1)^{(4)} > 0$ , $(v_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$ , $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_1)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

**Definition of** $(\bar{v}_1)^{(4)},, (\bar{v}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$ : 330

By $(\bar{v}_1)^{(4)} > 0$ , $(\bar{v}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$ , $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

**Definition of** $(m_1)^{(4)}$ , $(m_2)^{(4)}$ , $(\mu_1)^{(4)}, (\mu_2)^{(4)}, (\nu_0)^{(4)}$ :-

If we define $(m_1)^{(4)}$ , $(m_2)^{(4)}$ , $(\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (v_0)^{(4)}, (m_1)^{(4)} = (v_1)^{(4)}$, ***if*** $(v_0)^{(4)} < (v_1)^{(4)}$

$(m_2)^{(4)} = (v_1)^{(4)}, (m_1)^{(4)} = (\bar{v}_1)^{(4)}$ , ***if*** $(v_4)^{(4)} < (v_0)^{(4)} < (\bar{v}_1)^{(4)}$,
and $\boxed{(v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (v_4)^{(4)}, (m_1)^{(4)} = (v_0)^{(4)}$, ***if*** $(\bar{v}_4)^{(4)} < (v_0)^{(4)}$

and analogously

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$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)}$, ***if*** $(u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}$ , ***if*** $(u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)}$,
and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}$, ***if*** $(\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

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$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)}t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)}t}$$

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$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)}t}\right] + G_{26}^0 e^{-(S_2)^{(4)}t} \le G_{26}(t) \le\right.$$ 334

*(a26)4G240(m2)4(S1)4−(a26′)4e(S1)4t−e−(a26′)4t+ G260e−(a26′)4t*

$$\boxed{T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}}T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}}T_{24}^0 e^{((R_1)^{(4)}+(r_{24})^{(4)})t} \quad 335$$

$$\frac{(b_{26})^{(4)}T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)}-(b'_{26})^{(4)})}\left[e^{(R_1)^{(4)}t} - e^{-(b'_{26})^{(4)}t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)}t} \le T_{26}(t) \le \quad 336$$

$$\frac{(a_{26})^{(4)}T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)}+(r_{24})^{(4)}+(R_2)^{(4)})}\left[e^{((R_1)^{(4)}+(r_{24})^{(4)})t} - e^{-(R_2)^{(4)}t}\right] + T_{26}^0 e^{-(R_2)^{(4)}t}$$

**Definition of** $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

**Behavior of the solutions of equation** 338

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ :

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

**Definition of** $(v_1)^{(5)}, (v_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, v^{(5)}, u^{(5)}$ : 339

By $(v_1)^{(5)} > 0$, $(v_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$, $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(v^{(5)}\big)^2 + (\sigma_1)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

**Definition of** $(\bar{v}_1)^{(5)}, , (\bar{v}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$ : 340

By $(\bar{v}_1)^{(5)} > 0$, $(\bar{v}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$, $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(v^{(5)}\big)^2 + (\sigma_2)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

**Definition of** $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (v_0)^{(5)}$ :-

If we define $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (v_0)^{(5)}, (m_1)^{(5)} = (v_1)^{(5)}$, ***if*** $(v_0)^{(5)} < (v_1)^{(5)}$

$(m_2)^{(5)} = (v_1)^{(5)}, (m_1)^{(5)} = (\bar{v}_1)^{(5)}$, ***if*** $(v_1)^{(5)} < (v_0)^{(5)} < (\bar{v}_1)^{(5)}$,
and $\boxed{(v_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)},\ \boldsymbol{if}\ (u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}, \boldsymbol{if}\ (u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)},$

and $\boxed{(u_0)^{(5)} = \frac{T_{28}^0}{T_{29}^0}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}, \boldsymbol{if}\ (\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{28}(t) \le G_{28}^0 e^{(S_1)^{(5)} t}$

where $(p_i)^{(5)}$ is defined by equation

$\frac{1}{(m_5)^{(5)}} G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}} G_{28}^0 e^{(S_1)^{(5)} t}$ 343

$\left(\frac{(a_{30})^{(5)} G_{28}^0}{(m_1)^{(5)}((S_1)^{(5)} - (p_{28})^{(5)} - (S_2)^{(5)})}\left[e^{((S_1)^{(5)} - (p_{28})^{(5)})t} - e^{-(S_2)^{(5)} t}\right] + G_{30}^0 e^{-(S_2)^{(5)} t} \le G_{30}(t) \le\right.$ 344

*(a30)5G280(m2)5(S1)5−(a30′)5e(S1)5t−e−(a30′)5t+ G300e−(a30′)5t*

$\boxed{T_{28}^0 e^{(R_1)^{(5)} t} \le T_{28}(t) \le T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}}$ 345

$\frac{1}{(\mu_1)^{(5)}} T_{28}^0 e^{(R_1)^{(5)} t} \le T_{28}(t) \le \frac{1}{(\mu_2)^{(5)}} T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}$ 346

$\frac{(b_{30})^{(5)} T_{28}^0}{(\mu_1)^{(5)}((R_1)^{(5)} - (b'_{30})^{(5)})}\left[e^{(R_1)^{(5)} t} - e^{-(b'_{30})^{(5)} t}\right] + T_{30}^0 e^{-(b'_{30})^{(5)} t} \le T_{30}(t) \le$ 347

$\frac{(a_{30})^{(5)} T_{28}^0}{(\mu_2)^{(5)}((R_1)^{(5)} + (r_{28})^{(5)} + (R_2)^{(5)})}\left[e^{((R_1)^{(5)} + (r_{28})^{(5)})t} - e^{-(R_2)^{(5)} t}\right] + T_{30}^0 e^{-(R_2)^{(5)} t}$

**Definition of** $(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$:- 348

Where $(S_1)^{(5)} = (a_{28})^{(5)}(m_2)^{(5)} - (a'_{28})^{(5)}$

$(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}$

$(R_1)^{(5)} = (b_{28})^{(5)}(\mu_2)^{(5)} - (b'_{28})^{(5)}$

$(R_2)^{(5)} = (b'_{30})^{(5)} - (r_{30})^{(5)}$

**Behavior of the solutions of equation** 349

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ :

$(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)} \le -(a'_{32})^{(6)} + (a'_{33})^{(6)} - (a''_{32})^{(6)}(T_{33}, t) + (a''_{33})^{(6)}(T_{33}, t) \le -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)} \le -(b'_{32})^{(6)} + (b'_{33})^{(6)} - (b''_{32})^{(6)}\big((G_{35}), t\big) - (b''_{33})^{(6)}\big((G_{35}), t\big) \le -(\tau_1)^{(6)}$$

**Definition of** $(\nu_1)^{(6)}, (\nu_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, \nu^{(6)}, u^{(6)}$ : 350

By $(\nu_1)^{(6)} > 0$, $(\nu_2)^{(6)} < 0$ and respectively $(u_1)^{(6)} > 0$, $(u_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(6)},, (\bar{\nu}_2)^{(6)}, (\bar{u}_1)^{(6)}, (\bar{u}_2)^{(6)}$ : 351

By $(\bar{\nu}_1)^{(6)} > 0$, $(\bar{\nu}_2)^{(6)} < 0$ and respectively $(\bar{u}_1)^{(6)} > 0$, $(\bar{u}_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$

**Definition of** $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}, (\nu_0)^{(6)}$ :-

If we define $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}$ by

$$(m_2)^{(6)} = (\nu_0)^{(6)}, (m_1)^{(6)} = (\nu_1)^{(6)}, \ \boldsymbol{if}\ (\nu_0)^{(6)} < (\nu_1)^{(6)}$$

$$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}, \boldsymbol{if}\ (\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)},$$

and $\boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}}$

$$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$$

and analogously 352

$$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}, \ \boldsymbol{if}\ (u_0)^{(6)} < (u_1)^{(6)}$$

$$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}, \boldsymbol{if}\ (u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)},$$

and $\boxed{(u_0)^{(6)} = \frac{T^0_{32}}{T^0_{33}}}$

$$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}, \boldsymbol{if}\ (\bar{u}_1)^{(6)} < (u_0)^{(6)} \text{ where } (u_1)^{(6)}, (\bar{u}_1)^{(6)}$$

Then the solution of global equations satisfies the inequalities 353

$$G^0_{32}e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{32}(t) \le G^0_{32}e^{(S_1)^{(6)}t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}}G^0_{32}e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{33}(t) \le \frac{1}{(m_2)^{(6)}}G^0_{32}e^{(S_1)^{(6)}t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)}G^0_{32}}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})}\left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)}t}\right] + G^0_{34}e^{-(S_2)^{(6)}t} \le G_{34}(t) \le \right. \quad 355$$

$$\frac{(a_{34})^{(6)}G_{32}^{0}}{(m_2)^{(6)}((S_1)^{(6)}-(a'_{34})^{(6)})}\left[e^{(S_1)^{(6)}t}-e^{-(a'_{34})^{(6)}t}\right]+\ G_{34}^{0}e^{-(a'_{34})^{(6)}t}\Big)$$

$$\boxed{T_{32}^{0}e^{(R_1)^{(6)}t}\le T_{32}(t)\le T_{32}^{0}e^{((R_1)^{(6)}+(r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}}T_{32}^{0}e^{(R_1)^{(6)}t}\le T_{32}(t)\le\frac{1}{(\mu_2)^{(6)}}T_{32}^{0}e^{((R_1)^{(6)}+(r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)}T_{32}^{0}}{(\mu_1)^{(6)}((R_1)^{(6)}-(b'_{34})^{(6)})}\left[e^{(R_1)^{(6)}t}-e^{-(b'_{34})^{(6)}t}\right]+T_{34}^{0}e^{-(b'_{34})^{(6)}t}\le T_{34}(t)\le \quad 358$$

$$\frac{(a_{34})^{(6)}T_{32}^{0}}{(\mu_2)^{(6)}((R_1)^{(6)}+(r_{32})^{(6)}+(R_2)^{(6)})}\left[e^{((R_1)^{(6)}+(r_{32})^{(6)})t}-e^{-(R_2)^{(6)}t}\right]+T_{34}^{0}e^{-(R_2)^{(6)}t}$$

**Definition of** $(S_1)^{(6)},(S_2)^{(6)},(R_1)^{(6)},(R_2)^{(6)}$:- 359

Where $(S_1)^{(6)}=(a_{32})^{(6)}(m_2)^{(6)}-(a'_{32})^{(6)}$

$(S_2)^{(6)}=(a_{34})^{(6)}-(p_{34})^{(6)}$

$(R_1)^{(6)}=(b_{32})^{(6)}(\mu_2)^{(6)}-(b'_{32})^{(6)}$

$(R_2)^{(6)}=(b'_{34})^{(6)}-(r_{34})^{(6)}$

**Behavior of the solutions of equation**

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(7)},(\sigma_2)^{(7)},(\tau_1)^{(7)},(\tau_2)^{(7)}$ :

$(\sigma_1)^{(7)},(\sigma_2)^{(7)},(\tau_1)^{(7)},(\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)}\le-(a'_{36})^{(7)}+(a'_{37})^{(7)}-(a''_{36})^{(7)}(T_{37},t)+(a''_{37})^{(7)}(T_{37},t)\le-(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)}\le-(b'_{36})^{(7)}+(b'_{37})^{(7)}-(b''_{36})^{(7)}\big((G_{39}),t\big)-(b''_{37})^{(7)}\big((G_{39}),t\big)\le-(\tau_1)^{(7)}$$

**Definition of** $(v_1)^{(7)},(v_2)^{(7)},(u_1)^{(7)},(u_2)^{(7)},v^{(7)},u^{(7)}$ : 361

By $(v_1)^{(7)}>0,(v_2)^{(7)}<0$ and respectively $(u_1)^{(7)}>0,(u_2)^{(7)}<0$ the roots of the equations $(a_{37})^{(7)}\big(v^{(7)}\big)^2+(\sigma_1)^{(7)}v^{(7)}-(a_{36})^{(7)}=0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2+(\tau_1)^{(7)}u^{(7)}-(b_{36})^{(7)}=0$ and

**Definition of** $(\bar{v}_1)^{(7)},,(\bar{v}_2)^{(7)},(\bar{u}_1)^{(7)},(\bar{u}_2)^{(7)}$ : 362

By $(\bar{v}_1)^{(7)}>0,(\bar{v}_2)^{(7)}<0$ and respectively $(\bar{u}_1)^{(7)}>0,(\bar{u}_2)^{(7)}<0$ the roots of the equations $(a_{37})^{(7)}\big(v^{(7)}\big)^2+(\sigma_2)^{(7)}v^{(7)}-(a_{36})^{(7)}=0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2+(\tau_2)^{(7)}u^{(7)}-(b_{36})^{(7)}=0$

**Definition of** $(m_1)^{(7)},(m_2)^{(7)},(\mu_1)^{(7)},(\mu_2)^{(7)},(v_0)^{(7)}$ :-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$$(m_2)^{(7)} = (\nu_0)^{(7)}, (m_1)^{(7)} = (\nu_1)^{(7)}, \ \boldsymbol{if}\ (\nu_0)^{(7)} < (\nu_1)^{(7)}$$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\bar{\nu}_1)^{(7)}, \boldsymbol{if}\ (\nu_1)^{(7)} < (\nu_0)^{(7)} < (\bar{\nu}_1)^{(7)},$$

and $\boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\nu_0)^{(7)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(7)} < (\nu_0)^{(7)}$$

and analogously 363

$$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}, \ \boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$$

and $\boxed{(u_0)^{(7)} = \frac{T_{36}^0}{T_{37}^0}}$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)} \text{ where } (u_1)^{(7)}, (\bar{u}_1)^{(7)}$$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^0 e^{(S_1)^{(7)} t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)} t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)}((S_1)^{(7)} - (p_{36})^{(7)} - (S_2)^{(7)})}\left[e^{((S_1)^{(7)} - (p_{36})^{(7)})t} - e^{-(S_2)^{(7)} t}\right] + G_{38}^0 e^{-(S_2)^{(7)} t} \le G_{38}(t) \le \right. \quad 366$$

$$\left. \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)}((S_1)^{(7)} - (a'_{38})^{(7)})}\left[e^{(S_1)^{(7)} t} - e^{-(a'_{38})^{(7)} t}\right] + G_{38}^0 e^{-(a'_{38})^{(7)} t}\right)$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t} \quad 368$$

$$\frac{(b_{38})^{(7)} T_{36}^0}{(\mu_1)^{(7)}((R_1)^{(7)} - (b'_{38})^{(7)})}\left[e^{(R_1)^{(7)} t} - e^{-(b'_{38})^{(7)} t}\right] + T_{38}^0 e^{-(b'_{38})^{(7)} t} \le T_{38}(t) \le \quad 369$$

$$\frac{(a_{38})^{(7)} T_{36}^0}{(\mu_2)^{(7)}((R_1)^{(7)} + (r_{36})^{(7)} + (R_2)^{(7)})}\left[e^{((R_1)^{(7)} + (r_{36})^{(7)})t} - e^{-(R_2)^{(7)} t}\right] + T_{38}^0 e^{-(R_2)^{(7)} t}$$

**Definition of** $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$

$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$

$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$

**Behavior of the solutions of equation** 371

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ :

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$-(\sigma_2)^{(8)} \le -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \le -(\sigma_1)^{(8)}$

$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\big((G_{43}), t\big) - (b''_{41})^{(8)}\big((G_{43}), t\big) \le -(\tau_1)^{(8)}$

**Definition of** $(\nu_1)^{(8)}, (\nu_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, \nu^{(8)}, u^{(8)}$ : 372

By $(\nu_1)^{(8)} > 0, (\nu_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0, (u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(8)}, , (\bar{\nu}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$ :

By $(\bar{\nu}_1)^{(8)} > 0, (\bar{\nu}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0, (\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

**Definition of** $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}, (\nu_0)^{(8)}$ :-

If we define $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$(m_2)^{(8)} = (\nu_0)^{(8)}, (m_1)^{(8)} = (\nu_1)^{(8)}$, ***if*** $(\nu_0)^{(8)} < (\nu_1)^{(8)}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\bar{\nu}_1)^{(8)}$, ***if*** $(\nu_1)^{(8)} < (\nu_0)^{(8)} < (\bar{\nu}_1)^{(8)}$,
and $\boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\nu_0)^{(8)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(8)} < (\nu_0)^{(8)}$

and analogously 374

$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)},\ \boldsymbol{if}\ (u_0)^{(8)} < (u_1)^{(8)}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}, \boldsymbol{if}\ (u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)},$

and $\boxed{(u_0)^{(8)} = \frac{T_{40}^0}{T_{41}^0}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}, \boldsymbol{if}\ (\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^0 e^{(S_1)^{(8)} t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}} G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}} G_{40}^0 e^{(S_1)^{(8)} t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)} G_{40}^0}{(m_1)^{(8)}((S_1)^{(8)} - (p_{40})^{(8)} - (S_2)^{(8)})}\left[e^{((S_1)^{(8)} - (p_{40})^{(8)})t} - e^{-(S_2)^{(8)} t}\right] + G_{42}^0 e^{-(S_2)^{(8)} t} \le G_{42}(t) \le \right. \quad 377$$

$$\left. \frac{(a_{42})^{(8)} G_{40}^0}{(m_2)^{(8)}((S_1)^{(8)} - (a'_{42})^{(8)})}\left[e^{(S_1)^{(8)} t} - e^{-(a'_{42})^{(8)} t}\right] + G_{42}^0 e^{-(a'_{42})^{(8)} t}\right)$$

$$\boxed{T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}} T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le \frac{1}{(\mu_2)^{(8)}} T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)} T_{40}^0}{(\mu_1)^{(8)}((R_1)^{(8)} - (b'_{42})^{(8)})}\left[e^{(R_1)^{(8)} t} - e^{-(b'_{42})^{(8)} t}\right] + T_{42}^0 e^{-(b'_{42})^{(8)} t} \le T_{42}(t) \le \quad 380$$

$$\frac{(a_{42})^{(8)} T_{40}^0}{(\mu_2)^{(8)}((R_1)^{(8)} + (r_{40})^{(8)} + (R_2)^{(8)})}\left[e^{((R_1)^{(8)} + (r_{40})^{(8)})t} - e^{-(R_2)^{(8)} t}\right] + T_{42}^0 e^{-(R_2)^{(8)} t}$$

**Definition of** $(S_1)^{(8)}, (S_2)^{(8)}, (R_1)^{(8)}, (R_2)^{(8)}$:- 381

Where $(S_1)^{(8)} = (a_{40})^{(8)}(m_2)^{(8)} - (a'_{40})^{(8)}$

$(S_2)^{(8)} = (a_{42})^{(8)} - (p_{42})^{(8)}$

$(R_1)^{(8)} = (b_{40})^{(8)}(\mu_2)^{(8)} - (b'_{40})^{(8)}$

$(R_2)^{(8)} = (b'_{42})^{(8)} - (r_{42})^{(8)}$

**Behavior of the solutions of equation 37 to 92** 382

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ :

$(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ four constants satisfying

$$-(\sigma_2)^{(9)} \le -(a'_{44})^{(9)} + (a'_{45})^{(9)} - (a''_{44})^{(9)}(T_{45}, t) + (a''_{45})^{(9)}(T_{45}, t) \le -(\sigma_1)^{(9)}$$

$$-(\tau_2)^{(9)} \le -(b'_{44})^{(9)} + (b'_{45})^{(9)} - (b''_{44})^{(9)}\big((G_{47}), t\big) - (b''_{45})^{(9)}\big((G_{47}), t\big) \le -(\tau_1)^{(9)}$$

**Definition of** $(\nu_1)^{(9)}, (\nu_2)^{(9)}, (u_1)^{(9)}, (u_2)^{(9)}, \nu^{(9)}, u^{(9)}$ :

By $(\nu_1)^{(9)} > 0$ , $(\nu_2)^{(9)} < 0$ and respectively $(u_1)^{(9)} > 0$ , $(u_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_1)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(9)}, , (\bar{\nu}_2)^{(9)}, (\bar{u}_1)^{(9)}, (\bar{u}_2)^{(9)}$ :

By $(\bar{\nu}_1)^{(9)} > 0$ , $(\bar{\nu}_2)^{(9)} < 0$ and respectively $(\bar{u}_1)^{(9)} > 0$ , $(\bar{u}_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_2)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$

**Definition of** $(m_1)^{(9)}, (m_2)^{(9)}, (\mu_1)^{(9)}, (\mu_2)^{(9)}, (\nu_0)^{(9)}$ :-

If we define $(m_1)^{(9)}, (m_2)^{(9)}, (\mu_1)^{(9)}, (\mu_2)^{(9)}$ by

$$(m_2)^{(9)} = (\nu_0)^{(9)}, (m_1)^{(9)} = (\nu_1)^{(9)}, \ \boldsymbol{if}\ (\nu_0)^{(9)} < (\nu_1)^{(9)}$$

$$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\bar{\nu}_1)^{(9)}, \boldsymbol{if}\ (\nu_1)^{(9)} < (\nu_0)^{(9)} < (\bar{\nu}_1)^{(9)},$$

and $\boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$

$$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\nu_0)^{(9)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(9)} < (\nu_0)^{(9)}$$

and analogously

$$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)}, \ \boldsymbol{if}\ (u_0)^{(9)} < (u_1)^{(9)}$$

$$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}, \boldsymbol{if}\ (u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)},$$

and $\boxed{(u_0)^{(9)} = \frac{T^0_{44}}{T^0_{45}}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}, \boldsymbol{if}\ (\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G^0_{44} e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{44}(t) \le G^0_{44} e^{(S_1)^{(9)} t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}}G_{44}^0 e^{((S_1)^{(9)}-(p_{44})^{(9)})t} \le G_{45}(t) \le \frac{1}{(m_2)^{(9)}}G_{44}^0 e^{(S_1)^{(9)}t}$$

(

$$\frac{(a_{46})^{(9)}G_{44}^0}{(m_1)^{(9)}((S_1)^{(9)}-(p_{44})^{(9)}-(S_2)^{(9)})}\left[e^{((S_1)^{(9)}-(p_{44})^{(9)})t}-e^{-(S_2)^{(9)}t}\right]+G_{46}^0 e^{-(S_2)^{(9)}t} \le G_{46}(t) \le$$

$$\frac{(a_{46})^{(9)}G_{44}^0}{(m_2)^{(9)}((S_1)^{(9)}-(a'_{46})^{(9)})}[e^{(S_1)^{(9)}t}-e^{-(a'_{46})^{(9)}t}]+\ G_{46}^0 e^{-(a'_{46})^{(9)}t})$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le T_{44}^0 e^{((R_1)^{(9)}+(r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}}T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le \frac{1}{(\mu_2)^{(9)}}T_{44}^0 e^{((R_1)^{(9)}+(r_{44})^{(9)})t}$$

$$\frac{(b_{46})^{(9)}T_{44}^0}{(\mu_1)^{(9)}((R_1)^{(9)}-(b'_{46})^{(9)})}\left[e^{(R_1)^{(9)}t}-e^{-(b'_{46})^{(9)}t}\right]+T_{46}^0 e^{-(b'_{46})^{(9)}t} \le T_{46}(t) \le$$

$$\frac{(a_{46})^{(9)}T_{44}^0}{(\mu_2)^{(9)}((R_1)^{(9)}+(r_{44})^{(9)}+(R_2)^{(9)})}\left[e^{((R_1)^{(9)}+(r_{44})^{(9)})t}-e^{-(R_2)^{(9)}t}\right]+T_{46}^0 e^{-(R_2)^{(9)}t}$$

**Definition of** $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$

**Proof :** From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14},t)\right) - (a''_{14})^{(1)}(T_{14},t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

**Definition of** $v^{(1)}$ :- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)}\right) \le \frac{dv^{(1)}}{dt} \le -\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)}\right)$$

From which one obtains

**Definition of** $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$ :-

For $0 < \boxed{(v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}} < (v_1)^{(1)} < (\bar{v}_1)^{(1)}$

$$v^{(1)}(t) \ge \frac{(v_1)^{(1)}+(C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(v_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(v_2)^{(1)}}}$$

it follows $(\nu_0)^{(1)} \leq \nu^{(1)}(t) \leq (\nu_1)^{(1)}$

In the same manner , we get 384

$$\nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{\nu}_1)^{(1)}-(\nu_0)^{(1)}}{(\nu_0)^{(1)}-(\bar{\nu}_2)^{(1)}}}$$

From which we deduce $(\nu_0)^{(1)} \leq \nu^{(1)}(t) \leq (\bar{\nu}_1)^{(1)}$

If $0 < (\nu_1)^{(1)} < (\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0} < (\bar{\nu}_1)^{(1)}$ we find like in the previous case, 385

$$(\nu_1)^{(1)} \leq \frac{(\nu_1)^{(1)}+(C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}} \leq \nu^{(1)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\bar{\nu}_1)^{(1)}$$

If $0 < (\nu_1)^{(1)} \leq (\bar{\nu}_1)^{(1)} \leq \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$ , we obtain 386

$$(\nu_1)^{(1)} \leq \nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\nu_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(1)}(t)$ :-

$$(m_2)^{(1)} \leq \nu^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{\nu^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(1)}(t)$ :-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{13})^{(1)} = (a''_{14})^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)}G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)}T_{14}(t)$ This is an important

consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

**Proof** : From global equations we obtain 387

$$\frac{d\nu^{(2)}}{dt} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right) - (a''_{17})^{(2)}(T_{17}, t)\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

**Definition of** $\nu^{(2)}$ :- $\boxed{\nu^{(2)} = \frac{G_{16}}{G_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \leq \frac{d\nu^{(2)}}{dt} \leq -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

**Definition of** $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$ :-

For $0 < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \geq \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}}{1 + (C)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(C)^{(2)} = \frac{(\nu_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \leq \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}}{1 + (C)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}} \leq \nu^{(2)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \leq (\bar{\nu}_1)^{(2)} \leq (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$ , we obtain 394

$$(\nu_1)^{(2)} \leq \nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(2)}(t)$ :- 395

$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}$, $\boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$

In a completely analogous way, we obtain 396

**Definition of** $u^{(2)}(t)$ :-

$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}$, $\boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :** 397

If $(a''_{16})^{(2)} = (a''_{17})^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)} G_{17}(t)$

Analogously if $(b''_{16})^{(2)} = (b''_{17})^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)} T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

**Proof** : From global equations we obtain 398

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right) - (a''_{21})^{(3)}(T_{21}, t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

**Definition of** $\nu^{(3)}$ :- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right) \leq \frac{d\nu^{(3)}}{dt} \leq -\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right)$$

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From which one obtains

For $0 < (\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\nu_1)^{(3)} < (\bar{\nu}_1)^{(3)}$

$$\nu^{(3)}(t) \geq \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(\nu_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\nu_2)^{(3)}}}$$

it follows $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\nu_1)^{(3)}$

In the same manner , we get 401

$$\nu^{(3)}(t) \le \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{\nu}_1)^{(3)}-(\nu_0)^{(3)}}{(\nu_0)^{(3)}-(\bar{\nu}_2)^{(3)}}}$$

**Definition of** $(\bar{\nu}_1)^{(3)}$ :-

From which we deduce $(\nu_0)^{(3)} \le \nu^{(3)}(t) \le (\bar{\nu}_1)^{(3)}$

If $0 < (\nu_1)^{(3)} < (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (\bar{\nu}_1)^{(3)}$ we find like in the previous case, 402

$$(\nu_1)^{(3)} \le \frac{(\nu_1)^{(3)}+(C)^{(3)}(\nu_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}} \le \nu^{(3)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \le (\bar{\nu}_1)^{(3)}$$

If $0 < (\nu_1)^{(3)} \le (\bar{\nu}_1)^{(3)} \le (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}$ , we obtain 403

$$(\nu_1)^{(3)} \le \nu^{(3)}(t) \le \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \le (\nu_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(3)}(t)$ :-

$$(m_2)^{(3)} \le \nu^{(3)}(t) \le (m_1)^{(3)}, \quad \boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(3)}(t)$ :-

$$(\mu_2)^{(3)} \le u^{(3)}(t) \le (\mu_1)^{(3)}, \quad \boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{20}'')^{(3)} = (a_{21}'')^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)}G_{21}(t)$

Analogously if $(b_{20}'')^{(3)} = (b_{21}'')^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)}T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

**Proof** : From global equations we obtain 404

$$\frac{dv^{(4)}}{dt} = (a_{24})^{(4)} - \left((a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right) - (a''_{25})^{(4)}(T_{25},t)v^{(4)} - (a_{25})^{(4)}v^{(4)}$$

**Definition of** $v^{(4)}$ :- $\boxed{v^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)}\right) \le \frac{dv^{(4)}}{dt} \le -\left((a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_4)^{(4)}v^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$ :-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \ge \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \le \nu^{(4)}(t) \le (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \le \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \le \nu^{(4)}(t) \le (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \le \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}}{1 + (C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}} \le \nu^{(4)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \le (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \le (\bar{\nu}_1)^{(4)} \le \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$ , we obtain 407

$$(\nu_1)^{(4)} \le \nu^{(4)}(t) \le \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \le (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(4)}(t)$ :-

$$(m_2)^{(4)} \le \nu^{(4)}(t) \le (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(4)}(t)$ :-

$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}$, $\boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{24})^{(4)} = (a''_{25})^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)} G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b''_{24})^{(4)} = (b''_{25})^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)} T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

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**Proof** : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a'_{28})^{(5)} - (a'_{29})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right) - (a''_{29})^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

**Definition of** $\nu^{(5)}$ :- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$ :-

For $0 < \boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \le \frac{(\nu_1)^{(5)}+(C)^{(5)}(\nu_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)}-(\nu_2)^{(5)}\right)t\right]}}{1+(C)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)}-(\nu_2)^{(5)}\right)t\right]}} \le \nu^{(5)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \le (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \le (\bar{\nu}_1)^{(5)} \le \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain 411

$$(\nu_1)^{(5)} \le \nu^{(5)}(t) \le \frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \le (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(5)}(t)$ :-

$(m_2)^{(5)} \le \nu^{(5)}(t) \le (m_1)^{(5)}$, $\boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$

In a completely analogous way, we obtain

**Definition of** $u^{(5)}(t)$ :-

$(\mu_2)^{(5)} \le u^{(5)}(t) \le (\mu_1)^{(5)}$, $\boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a_{28}'')^{(5)} = (a_{29}'')^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)}G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b_{28}'')^{(5)} = (b_{29}'')^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)}T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

**Proof** : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a_{32}')^{(6)} - (a_{33}')^{(6)} + (a_{32}'')^{(6)}(T_{33},t)\right) - (a_{33}'')^{(6)}(T_{33},t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

**Definition of** $\nu^{(6)}$ :- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \le \frac{d\nu^{(6)}}{dt} \le -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$ :-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \geq \frac{(\nu_1)^{(6)}+(C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_0)^{(6)}\right)t\right]}}{1+(C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)}-(\nu_0)^{(6)}}{(\nu_0)^{(6)}-(\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\nu_1)^{(6)}$

In the same manner , we get 413

$$\nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{\nu}_1)^{(6)}-(\nu_0)^{(6)}}{(\nu_0)^{(6)}-(\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\bar{\nu}_1)^{(6)}$

If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 414

$$(\nu_1)^{(6)} \leq \frac{(\nu_1)^{(6)}+(C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_2)^{(6)}\right)t\right]}}{1+(C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_2)^{(6)}\right)t\right]}} \leq \nu^{(6)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\bar{\nu}_1)^{(6)}$$

If $0 < (\nu_1)^{(6)} \leq (\bar{\nu}_1)^{(6)} \leq \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$ , we obtain 415

$$(\nu_1)^{(6)} \leq \nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(6)}(t)$ :-

$$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}, \quad \boxed{\nu^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(6)}(t)$ :-

$$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}, \quad \boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)}G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)}T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

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**Proof** : From global equations we obtain

$$\frac{dv^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right) - (a''_{37})^{(7)}(T_{37},t)v^{(7)} - (a_{37})^{(7)}v^{(7)}$$

**Definition of** $v^{(7)}$ :- $\boxed{v^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_2)^{(7)}v^{(7)} - (a_{36})^{(7)}\right) \le \frac{dv^{(7)}}{dt} \le -\left((a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_1)^{(7)}v^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

**Definition of** $(\bar{v}_1)^{(7)}, (v_0)^{(7)}$ :-

For $0 < \boxed{(v_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}} < (v_1)^{(7)} < (\bar{v}_1)^{(7)}$

$$v^{(7)}(t) \ge \frac{(v_1)^{(7)} + (C)^{(7)}(v_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_0)^{(7)}\right)t\right]}}{1 + (C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(v_1)^{(7)} - (v_0)^{(7)}}{(v_0)^{(7)} - (v_2)^{(7)}}}$$

it follows $(v_0)^{(7)} \le v^{(7)}(t) \le (v_1)^{(7)}$

In the same manner , we get 417

$$v^{(7)}(t) \le \frac{(\bar{v}_1)^{(7)} + (\bar{C})^{(7)}(\bar{v}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{v}_1)^{(7)} - (v_0)^{(7)}}{(v_0)^{(7)} - (\bar{v}_2)^{(7)}}}$$

From which we deduce $(v_0)^{(7)} \le v^{(7)}(t) \le (\bar{v}_1)^{(7)}$

If $0 < (v_1)^{(7)} < (v_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}} < (\bar{v}_1)^{(7)}$ we find like in the previous case, 418

$$(v_1)^{(7)} \le \frac{(v_1)^{(7)} + (C)^{(7)}(v_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_2)^{(7)}\right)t\right]}}{1 + (C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_2)^{(7)}\right)t\right]}} \le v^{(7)}(t) \le$$

$$\frac{(\bar{v}_1)^{(7)} + (\bar{C})^{(7)}(\bar{v}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}} \le (\bar{v}_1)^{(7)}$$

If $0 < (v_1)^{(7)} \le (\bar{v}_1)^{(7)} \le \boxed{(v_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$ , we obtain 419

$$(v_1)^{(7)} \le v^{(7)}(t) \le \frac{(\bar{v}_1)^{(7)} + (\bar{C})^{(7)}(\bar{v}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}} \le (v_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $v^{(7)}(t)$ :-

$$(m_2)^{(7)} \leq v^{(7)}(t) \leq (m_1)^{(7)}, \quad \boxed{v^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(7)}(t)$ :- 420

$$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}, \quad \boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $v^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)} G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)} T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

**Proof** : From global equations we obtain 421

$$\frac{dv^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right) - (a''_{41})^{(8)}(T_{41}, t)v^{(8)} - (a_{41})^{(8)}v^{(8)}$$

**Definition of** $v^{(8)}$ :- $\boxed{v^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_2)^{(8)}v^{(8)} - (a_{40})^{(8)}\right) \leq \frac{dv^{(8)}}{dt} \leq -\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_1)^{(8)}v^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$ :-

$$\text{For } 0 < \boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$$

$$v^{(8)}(t) \geq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)}-(\nu_0)^{(8)}}{(\nu_0)^{(8)}-(\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \leq \frac{(\nu_1)^{(8)}+(C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}}{1+(C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}} \leq \nu^{(8)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \leq (\bar{\nu}_1)^{(8)} \leq \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$ , we obtain 424

$$(\nu_1)^{(8)} \leq \nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(8)}(t)$ :-

$$(m_2)^{(8)} \leq \nu^{(8)}(t) \leq (m_1)^{(8)}, \quad \boxed{\nu^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(8)}(t)$ :-

$$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}, \quad \boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $\nu^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)}G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then $(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)}T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

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**Proof** : From 99,20,44,22,23,44 we obtain A

$$\frac{dv^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right) - (a''_{45})^{(9)}(T_{45},t)v^{(9)} - (a_{45})^{(9)}v^{(9)}$$

**Definition of** $v^{(9)}$ :- $\boxed{v^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)}\right) \le \frac{dv^{(9)}}{dt} \le -\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_1)^{(9)}v^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$ :-

For $0 < \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$

$$v^{(9)}(t) \ge \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \le v^{(9)}(t) \le (\nu_9)^{(9)}$

In the same manner , we get

$$v^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \le v^{(9)}(t) \le (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \le \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}}{1 + (C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}} \le v^{(9)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \le (\bar{\nu}_1)^{(9)} \le \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$ , we obtain

$$(\nu_1)^{(9)} \le v^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $v^{(9)}(t)$ :-

$$(m_2)^{(9)} \le v^{(9)}(t) \le (m_1)^{(9)}, \quad \boxed{v^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(9)}(t)$ :-

$(\mu_2)^{(9)} \leq u^{(9)}(t) \leq (\mu_1)^{(9)}$, $\boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)} G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then
$(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)} T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following 425

**Theorem :** If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on $t$ , and the conditions with the notations

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0$ ,

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$

$with\ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a''_i)^{(2)} and\ (b''_i)^{(2)}$ are independent on t , and the conditions with the notations 426

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$ 427

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 428

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$ , 429

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 430

$with\ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a''_i)^{(3)} and\ (b''_i)^{(3)}$ are independent on $t$ , and the conditions with the notations 431

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$ ,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

**Theorem :** If $(a_i'')^{(4)} and\ (b_i'')^{(4)}$ are independent on $t$ , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$ ,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(5)} and\ (b_i'')^{(5)}$ are independent on $t$ , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$ ,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

**Theorem** If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on $t$ , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$ ,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(7)} and\ (b_i'')^{(7)}$ are independent on $t$ , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$ ,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(8)} and\ (b_i'')^{(8)}$ are independent on $t$ , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$ ,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(9)} and\ (b_i'')^{(9)}$ are independent on $t$ , and the conditions (with the notations 436 A
45,46,27,28)

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$ ,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0$ 437

$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0$ 438

$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0$ 439

$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0$ 440

$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ]T_{14} = 0$ 441

$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0$ 442

has a unique positive solution , which is an equilibrium solution for the system

$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0$ 443

$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0$ 444

$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0$ 445

$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\ ]T_{16} = 0$ 446

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\,]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\,]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\,]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\,]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\,]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\big((G_{27})\big)\,]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\big((G_{27})\big)\,]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\big((G_{27})\big)\,]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})\,]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})\,]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})\,]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})\,]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})\,]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})\,]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0 \quad 479$$

$$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0 \quad 480$$

$$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0 \quad 481$$

$$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0 \quad 482$$

$$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0 \quad 483$$

$$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0 \quad 484$$

$$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0 \quad 484\text{A}$$

$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$

$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$

$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$

$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$

$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$

**Proof:** 485

(a) Indeed the first two equations have a nontrivial solution $G_{13}, G_{14}$ if

$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$

**Proof:** 486

(bb) Indeed the first two equations have a nontrivial solution $G_{16}, G_{17}$ if

$\mathrm{F}(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$

**Proof:** 487

(a) Indeed the first two equations have a nontrivial solution $G_{20}, G_{21}$ if

$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$

**Proof:** 488

(a) Indeed the first two equations have a nontrivial solution $G_{24}, G_{25}$ if

$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$

**Proof:** 489

(a) Indeed the first two equations have a nontrivial solution $G_{28}, G_{29}$ if

$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$

**Proof:** 490

(a) Indeed the first two equations have a nontrivial solution $G_{32}, G_{33}$ if

$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) +$

$(a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$

**Proof:** 491

(a) Indeed the first two equations have a nontrivial solution $G_{36}, G_{37}$ if

$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$

**Proof:** 492

(a) Indeed the first two equations have a nontrivial solution $G_{40}, G_{41}$ if

$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$

**Proof:** 492 A

(a) Indeed the first two equations have a nontrivial solution $G_{44}, G_{45}$ if

$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$

**Definition and uniqueness of** $T^*_{14}$ :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique $T^*_{14}$ for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

**Definition and uniqueness of** $T^*_{17}$ :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique $T^*_{17}$ for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]}$$ 495

**Definition and uniqueness of** $T^*_{21}$ :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique $T^*_{21}$ for which $f(T^*_{21}) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

**Definition and uniqueness of** $T^*_{25}$ :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that

there exists a unique $T_{25}^{*}$ for which $f(T_{25}^{*})=0$. With this value , we obtain from the three first equations

$$G_{24}=\frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}=\frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T_{25}^{*})]}$$

**Definition and uniqueness of** $T_{29}^{*}$ :- 498

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(5)}(T_{29})$ being increasing, it follows that there exists a unique $T_{29}^{*}$ for which $f(T_{29}^{*})=0$. With this value , we obtain from the three first equations

$$G_{28}=\frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}=\frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T_{29}^{*})]}$$

**Definition and uniqueness of** $T_{33}^{*}$ :- 499

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(6)}(T_{33})$ being increasing, it follows that there exists a unique $T_{33}^{*}$ for which $f(T_{33}^{*})=0$. With this value , we obtain from the three first equations

$$G_{32}=\frac{(a_{32})^{(6)}G_{33}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^{*})]} \quad , \quad G_{34}=\frac{(a_{34})^{(6)}G_{33}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^{*})]}$$

**Definition and uniqueness of** $T_{37}^{*}$ :- 500

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(7)}(T_{37})$ being increasing, it follows that there exists a unique $T_{37}^{*}$ for which $f(T_{37}^{*})=0$. With this value , we obtain from the three first equations

$$G_{36}=\frac{(a_{36})^{(7)}G_{37}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^{*})]} \quad , \quad G_{38}=\frac{(a_{38})^{(7)}G_{37}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^{*})]}$$

**Definition and uniqueness of** $T_{41}^{*}$ :- 501

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique $T_{41}^{*}$ for which $f(T_{41}^{*})=0$. With this value , we obtain from the three first equations

$$G_{40}=\frac{(a_{40})^{(8)}G_{41}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T_{41}^{*})]} \quad , \quad G_{42}=\frac{(a_{42})^{(8)}G_{41}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T_{41}^{*})]}$$

**Definition and uniqueness of** $T_{45}^{*}$ :- 501 A

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(9)}(T_{45})$ being increasing, it follows that there exists a unique $T_{45}^{*}$ for which $f(T_{45}^{*})=0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)}G_{45}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T^*_{45})]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)}G_{45}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T^*_{45})]}$$

By the same argument, the equations admit solutions $G_{13}, G_{14}$ if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right]+(b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{14}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{14}$ such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions $G_{16}, G_{17}$ if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right]+(b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ 504
is a decreasing function in $G_{17}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{14}$ such that $\varphi((G_{19})^*) = 0$

By the same argument, the equations admit solutions $G_{20}, G_{21}$ if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right]+(b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{21}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{21}$ such that $\varphi((G_{23})^*) = 0$

By the same argument, the equations admit solutions $G_{24}, G_{25}$ if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right]+(b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{25}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{25}$ such that $\varphi((G_{27})^*) = 0$

By the same argument, the equations admit solutions $G_{28}, G_{29}$ if 507

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right]+(b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{29}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that

there exists a unique $G_{29}^{*}$ such that $\varphi((G_{31})^{*}) = 0$

By the same argument, the equations admit solutions $G_{32}, G_{33}$ if 508
$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right] + (b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{33}$ taking into account the hypothesis $\varphi(0) > 0\,, \varphi(\infty) < 0$ it follows that there exists a unique $G_{33}^{*}$ such that $\varphi(G_{35}{}^{*}) = 0$

By the same argument, the equations admit solutions $G_{36}, G_{37}$ if 509
$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$
$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right] + (b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{37}$ taking into account the hypothesis $\varphi(0) > 0\,, \varphi(\infty) < 0$ it follows that there exists a unique $G_{37}^{*}$ such that $\varphi(G_{39}{}^{*}) = 0$

By the same argument, the equations admit solutions $G_{40}, G_{41}$ if 510
$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$
$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right] + (b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{41}$ taking into account the hypothesis $\varphi(0) > 0\,, \varphi(\infty) < 0$ it follows that there exists a unique $G_{41}^{*}$ such that $\varphi(G_{43}{}^{*}) = 0$

By the same argument, the equations 92,93 admit solutions $G_{44}, G_{45}$ if
$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$

$$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right] + (b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{45}$ taking into account the hypothesis $\varphi(0) > 0\,, \varphi(\infty) < 0$ it follows that there exists a unique $G_{45}^{*}$ such that $\varphi((G_{47})^{*}) = 0$

Finally we obtain the unique solution 511

$G_{14}^{*}$ given by $\varphi(G^{*}) = 0\,, T_{14}^{*}$ given by $f(T_{14}^{*}) = 0$ and

$$G_{13}^{*} = \frac{(a_{13})^{(1)}G_{14}^{*}}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^{*})]} \quad , \quad G_{15}^{*} = \frac{(a_{15})^{(1)}G_{14}^{*}}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^{*})]}$$

$$T_{13}^{*} = \frac{(b_{13})^{(1)}T_{14}^{*}}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^{*})]} \quad , \quad T_{15}^{*} = \frac{(b_{15})^{(1)}T_{14}^{*}}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^{*})]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

$G_{17}^{*}$ given by $\varphi((G_{19})^{*}) = 0$ , $T_{17}^{*}$ given by $f(T_{17}^{*}) = 0$ and 512

$$G_{16}^{*} = \frac{(a_{16})^{(2)} G_{17}^{*}}{[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}^{*})]} \quad , \quad G_{18}^{*} = \frac{(a_{18})^{(2)} G_{17}^{*}}{[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}^{*})]} \qquad 513$$

$$T_{16}^{*} = \frac{(b_{16})^{(2)} T_{17}^{*}}{[(b'_{16})^{(2)} - (b''_{16})^{(2)}((G_{19})^{*})]} \quad , \quad T_{18}^{*} = \frac{(b_{18})^{(2)} T_{17}^{*}}{[(b'_{18})^{(2)} - (b''_{18})^{(2)}((G_{19})^{*})]} \qquad 514$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

$G_{21}^{*}$ given by $\varphi((G_{23})^{*}) = 0$ , $T_{21}^{*}$ given by $f(T_{21}^{*}) = 0$ and

$$G_{20}^{*} = \frac{(a_{20})^{(3)} G_{21}^{*}}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^{*})]} \quad , \quad G_{22}^{*} = \frac{(a_{22})^{(3)} G_{21}^{*}}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^{*})]}$$

$$T_{20}^{*} = \frac{(b_{20})^{(3)} T_{21}^{*}}{[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}{}^{*})]} \quad , \quad T_{22}^{*} = \frac{(b_{22})^{(3)} T_{21}^{*}}{[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}{}^{*})]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

$G_{25}^{*}$ given by $\varphi(G_{27}) = 0$ , $T_{25}^{*}$ given by $f(T_{25}^{*}) = 0$ and

$$G_{24}^{*} = \frac{(a_{24})^{(4)} G_{25}^{*}}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}^{*} = \frac{(a_{26})^{(4)} G_{25}^{*}}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^{*})]}$$

$$T_{24}^{*} = \frac{(b_{24})^{(4)} T_{25}^{*}}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^{*})]} \quad , \quad T_{26}^{*} = \frac{(b_{26})^{(4)} T_{25}^{*}}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^{*})]} \qquad 517$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

$G_{29}^{*}$ given by $\varphi((G_{31})^{*}) = 0$ , $T_{29}^{*}$ given by $f(T_{29}^{*}) = 0$ and

$$G_{28}^{*} = \frac{(a_{28})^{(5)} G_{29}^{*}}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}^{*} = \frac{(a_{30})^{(5)} G_{29}^{*}}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^{*})]}$$

$$T_{28}^{*} = \frac{(b_{28})^{(5)} T_{29}^{*}}{[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31})^{*})]} \quad , \quad T_{30}^{*} = \frac{(b_{30})^{(5)} T_{29}^{*}}{[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31})^{*})]} \qquad 519$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

$G_{33}^{*}$ given by $\varphi((G_{35})^{*}) = 0$ , $T_{33}^{*}$ given by $f(T_{33}^{*}) = 0$ and

$$G_{32}^{*} = \frac{(a_{32})^{(6)} G_{33}^{*}}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^{*})]} \quad , \quad G_{34}^{*} = \frac{(a_{34})^{(6)} G_{33}^{*}}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^{*})]}$$

$$T_{32}^{*} = \frac{(b_{32})^{(6)} T_{33}^{*}}{[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35})^{*})]} \quad , \quad T_{34}^{*} = \frac{(b_{34})^{(6)} T_{33}^{*}}{[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35})^{*})]} \qquad 521$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

$G_{37}^*$ given by $\varphi((G_{39})^*) = 0$ , $T_{37}^*$ given by $f(T_{37}^*) = 0$ and

$$G_{36}^* = \frac{(a_{36})^{(7)} G_{37}^*}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^*)]} \quad , \quad G_{38}^* = \frac{(a_{38})^{(7)} G_{37}^*}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^*)]}$$

$$T_{36}^* = \frac{(b_{36})^{(7)} T_{37}^*}{[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39})^*)]} \quad , \quad T_{38}^* = \frac{(b_{38})^{(7)} T_{37}^*}{[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39})^*)]}$$

Finally we obtain the unique solution 523

$G_{41}^*$ given by $\varphi((G_{43})^*) = 0$ , $T_{41}^*$ given by $f(T_{41}^*) = 0$ and

$$G_{40}^* = \frac{(a_{40})^{(8)} G_{41}^*}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42}^* = \frac{(a_{42})^{(8)} G_{41}^*}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

$$T_{40}^* = \frac{(b_{40})^{(8)} T_{41}^*}{[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43})^*)]} \quad , \quad T_{42}^* = \frac{(b_{42})^{(8)} T_{41}^*}{[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43})^*)]}$$

Finally we obtain the unique solution of 89 to 99 523 A

$G_{45}^*$ given by $\varphi((G_{47})^*) = 0$ , $T_{45}^*$ given by $f(T_{45}^*) = 0$ and

$$G_{44}^* = \frac{(a_{44})^{(9)} G_{45}^*}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46}^* = \frac{(a_{46})^{(9)} G_{45}^*}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

$$T_{44}^* = \frac{(b_{44})^{(9)} T_{45}^*}{[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47})^*)]} \quad , \quad T_{46}^* = \frac{(b_{46})^{(9)} T_{45}^*}{[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47})^*)]}$$

## ASYMPTOTIC STABILITY ANALYSIS 524

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

**Proof:** Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a''_{14})^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \frac{\partial (b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a'_{13})^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)}G^*_{13}\mathbb{T}_{14} \quad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a'_{14})^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)}G^*_{14}\mathbb{T}_{14} \quad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a'_{15})^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)}G^*_{15}\mathbb{T}_{14} \quad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b'_{13})^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)}T^*_{13}\mathbb{G}_j\right) \quad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b'_{14})^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)}T^*_{14}\mathbb{G}_j\right) \quad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b'_{15})^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T^*_{15}\mathbb{G}_j\right) \quad 530$$

**ASYMPTOTIC STABILITY ANALYSIS** 531

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(2)}$ and $(b''_i)^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G^*_i + \mathbb{G}_i \quad , T_i = T^*_i + \mathbb{T}_i \quad 532$$

$$\frac{\partial(a''_{17})^{(2)}}{\partial T_{17}}(T^*_{17}) = (q_{17})^{(2)} \quad , \frac{\partial(b''_i)^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \quad 533$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a'_{16})^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G^*_{16}\mathbb{T}_{17} \quad 534$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a'_{17})^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G^*_{17}\mathbb{T}_{17} \quad 535$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a'_{18})^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G^*_{18}\mathbb{T}_{17} \quad 536$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b'_{16})^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T^*_{16}\mathbb{G}_j\right) \quad 537$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b'_{17})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}T^*_{17}\mathbb{G}_j\right) \quad 538$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b'_{18})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}T^*_{18}\mathbb{G}_j\right) \quad 539$$

**ASYMPTOTIC STABILITY ANALYSIS** 540

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(3)}$ *and* $(b''_i)^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{21}'')^{(3)}}{\partial T_{21}}(T_{21}^*) = (q_{21})^{(3)} \quad , \frac{\partial (b_i'')^{(3)}}{\partial G_j}(\,(G_{23})^*\,) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\big((a_{20}')^{(3)} + (p_{20})^{(3)}\big)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G_{20}^*\mathbb{T}_{21} \qquad 541$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\big((a_{21}')^{(3)} + (p_{21})^{(3)}\big)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G_{21}^*\mathbb{T}_{21} \qquad 542$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\big((a_{22}')^{(3)} + (p_{22})^{(3)}\big)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G_{22}^*\mathbb{T}_{21} \qquad 543$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\big((b_{20}')^{(3)} - (r_{20})^{(3)}\big)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\big(s_{(20)(j)}T_{20}^*\mathbb{G}_j\big) \qquad 544$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\big((b_{21}')^{(3)} - (r_{21})^{(3)}\big)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\big(s_{(21)(j)}T_{21}^*\mathbb{G}_j\big) \qquad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\big((b_{22}')^{(3)} - (r_{22})^{(3)}\big)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\big(s_{(22)(j)}T_{22}^*\mathbb{G}_j\big) \qquad 546$$

**ASYMPTOTIC STABILITY ANALYSIS** 547

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}\ and\ (b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{25}'')^{(4)}}{\partial T_{25}}(T_{25}^*) = (q_{25})^{(4)} \quad , \frac{\partial (b_i'')^{(4)}}{\partial G_j}((G_{27})^*\,) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\big((a_{24}')^{(4)} + (p_{24})^{(4)}\big)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G_{24}^*\mathbb{T}_{25} \qquad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\big((a_{25}')^{(4)} + (p_{25})^{(4)}\big)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G_{25}^*\mathbb{T}_{25} \qquad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\big((a_{26}')^{(4)} + (p_{26})^{(4)}\big)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G_{26}^*\mathbb{T}_{25} \qquad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\big((b_{24}')^{(4)} - (r_{24})^{(4)}\big)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\big(s_{(24)(j)}T_{24}^*\mathbb{G}_j\big) \qquad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\big((b_{25}')^{(4)} - (r_{25})^{(4)}\big)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\big(s_{(25)(j)}T_{25}^*\mathbb{G}_j\big) \qquad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T^*_{26}\mathbb{G}_j\right) \quad 554$$

**ASYMPTOTIC STABILITY ANALYSIS** 555

**Theorem 5:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}\ and\ (b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 556

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{29})^{(5)}}{\partial T_{29}}(T^*_{29}) = (q_{29})^{(5)} \ , \ \frac{\partial(b_i'')^{(5)}}{\partial G_j}\left((G_{31})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a'_{28})^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G^*_{28}\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a'_{29})^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G^*_{29}\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a'_{30})^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G^*_{30}\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b'_{28})^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T^*_{28}\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b'_{29})^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T^*_{29}\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b'_{30})^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T^*_{30}\mathbb{G}_j\right) \quad 562$$

**ASYMPTOTIC STABILITY ANALYSIS** 563

**Theorem 6:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}\ and\ (b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{33})^{(6)}}{\partial T_{33}}(T^*_{33}) = (q_{33})^{(6)} \ , \ \frac{\partial(b_i'')^{(6)}}{\partial G_j}\left((G_{35})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a'_{32})^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G^*_{32}\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a'_{33})^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G^*_{33}\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a'_{34})^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G^*_{34}\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b'_{32})^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T^*_{32}\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b'_{33})^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T^*_{33}\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b'_{34})^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T^*_{34}\mathbb{G}_j\right) \quad 570$$

**ASYMPTOTIC STABILITY ANALYSIS** 571

**Theorem 7:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 572

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{37})^{(7)}}{\partial T_{37}}(T^*_{37}) = (q_{37})^{(7)} \quad , \frac{\partial(b_i'')^{(7)}}{\partial G_j}\left((G_{39})^{**}\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a'_{36})^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G^*_{36}\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a'_{37})^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G^*_{37}\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a'_{38})^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G^*_{38}\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b'_{36})^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T^*_{36}\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b'_{37})^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T^*_{37}\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b'_{38})^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T^*_{38}\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

**ASYMPTOTIC STABILITY ANALYSIS**

**Theorem 8:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{41}'')^{(8)}}{\partial T_{41}}(T_{41}^*) = (q_{41})^{(8)} \quad , \frac{\partial (b_i'')^{(8)}}{\partial G_j}\left( (G_{43})^* \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a_{40}')^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G_{40}^*\mathbb{T}_{41} \quad (581)$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a_{41}')^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G_{41}^*\mathbb{T}_{41} \quad (582)$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a_{42}')^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G_{42}^*\mathbb{T}_{41} \quad (583)$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b_{40}')^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T_{40}^*\mathbb{G}_j\right) \quad (584)$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b_{41}')^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T_{41}^*\mathbb{G}_j\right) \quad (585)$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b_{42}')^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T_{42}^*\mathbb{G}_j\right) \quad (586)$$

(586 A)

**ASYMPTOTIC STABILITY ANALYSIS**

**Theorem 9:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}$ *and* $(b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.
**Proof:** Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{45}'')^{(9)}}{\partial T_{45}}(T_{45}^*) = (q_{45})^{(9)} \quad , \frac{\partial (b_i'')^{(9)}}{\partial G_j}\left( (G_{47})^* \right) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a_{44}')^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)}G_{44}^*\mathbb{T}_{45} \quad (586\,B)$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a_{45}')^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)}G_{45}^*\mathbb{T}_{45} \quad (586\,C)$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a_{46}')^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)}G_{46}^*\mathbb{T}_{45} \quad (586\,D)$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b_{44}')^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)}T_{44}^*\mathbb{G}_j\right) \quad (586\,E)$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b_{45}')^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)}T_{45}^*\mathbb{G}_j\right) \quad (586\,F)$$

$\frac{d\mathbb{T}_{46}}{dt} = -\left((b'_{46})^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T^*_{46}\mathbb{G}_j\right)$ 586

G

**The characteristic equation of this system is** 587

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(q_{13})^{(1)}G^*_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)}T^*_{14} + (b_{14})^{(1)}s_{(13),(14)}T^*_{14}\right)$$

$$+\left(\left((\lambda)^{(1)} + (a'_{14})^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)}G^*_{13} + (a_{13})^{(1)}(q_{14})^{(1)}G^*_{14}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)}T^*_{14} + (b_{14})^{(1)}s_{(13),(13)}T^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((b'_{13})^{(1)} + (b'_{14})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(15)}T^*_{14} + (b_{14})^{(1)}s_{(13),(15)}T^*_{13}\right)\} = 0$$

$+$

$$(\lambda)^{(2)} + (b'_{18})^{(2)} - (r_{18})^{(2)}\{\left((\lambda)^{(2)} + (a'_{18})^{(2)} + (p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^*_{17} + (a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^*_{17}\right)$$

$$+\left(\left((\lambda)^{(2)} + (a'_{17})^{(2)} + (p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^*_{16} + (a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}\right)$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2 + \left((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2 + \left((b'_{16})^{(2)} + (b'_{17})^{(2)} - (r_{16})^{(2)} + (r_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$+\left(\left((\lambda)^{(2)}\right)^2 + \left((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17} + (a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^{*}_{16}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\right)\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left(\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)\right]$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(21)}T^{*}_{21}\right)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^{*}_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^{*}_{21}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(20)}T^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$+\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(22)}T^{*}_{20}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\right)\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left(\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)\right]$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(25)}T^{*}_{25}\right)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^{*}_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^{*}_{25}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(24)}T^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$+\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(26)}T^{*}_{24}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\right)\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(29)}T^{*}_{29}\right)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^{*}_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^{*}_{29}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(28)}T^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$+\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(30)}T^{*}_{28}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\right)\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left(\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)\right]$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(33)}T^{*}_{33}\right)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^{*}_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^{*}_{33}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(32)}T^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$+\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)$$

$$\left((\lambda)^{(6)} + (b'_{32})^{(6)} - (r_{32})^{(6)}\right)s_{(33),(34)}T^*_{33} + (b_{33})^{(6)}s_{(32),(34)}T^*_{32}\Big)\} = 0$$

$+$

$$(\lambda)^{(7)} + (b'_{38})^{(7)} - (r_{38})^{(7)})\{\left((\lambda)^{(7)} + (a'_{38})^{(7)} + (p_{38})^{(7)}\right)$$

$$\left[\left((\lambda)^{(7)} + (a'_{36})^{(7)} + (p_{36})^{(7)}\right)(q_{37})^{(7)}G^*_{37} + (a_{37})^{(7)}(q_{36})^{(7)}G^*_{36}\right)\Big]$$

$$\left((\lambda)^{(7)} + (b'_{36})^{(7)} - (r_{36})^{(7)}\right)s_{(37),(37)}T^*_{37} + (b_{37})^{(7)}s_{(36),(37)}T^*_{37}\Big)$$

$$+\left((\lambda)^{(7)} + (a'_{37})^{(7)} + (p_{37})^{(7)}\right)(q_{36})^{(7)}G^*_{36} + (a_{36})^{(7)}(q_{37})^{(7)}G^*_{37}\Big)$$

$$\left((\lambda)^{(7)} + (b'_{36})^{(7)} - (r_{36})^{(7)}\right)s_{(37),(36)}T^*_{37} + (b_{37})^{(7)}s_{(36),(36)}T^*_{36}\Big)$$

$$\left((\lambda)^{(7)}\right)^2 + \left((a'_{36})^{(7)} + (a'_{37})^{(7)} + (p_{36})^{(7)} + (p_{37})^{(7)}\right)(\lambda)^{(7)}\Big)$$

$$\left((\lambda)^{(7)}\right)^2 + \left((b'_{36})^{(7)} + (b'_{37})^{(7)} - (r_{36})^{(7)} + (r_{37})^{(7)}\right)(\lambda)^{(7)}\Big)$$

$$+\left(\left((\lambda)^{(7)}\right)^2 + \left((a'_{36})^{(7)} + (a'_{37})^{(7)} + (p_{36})^{(7)} + (p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)} + (a'_{36})^{(7)} + (p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^*_{37} + (a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^*_{36}\right)$$

$$\left((\lambda)^{(7)} + (b'_{36})^{(7)} - (r_{36})^{(7)}\right)s_{(37),(38)}T^*_{37} + (b_{37})^{(7)}s_{(36),(38)}T^*_{36}\Big)\} = 0$$

$+$

$$(\lambda)^{(8)} + (b'_{42})^{(8)} - (r_{42})^{(8)})\{\left((\lambda)^{(8)} + (a'_{42})^{(8)} + (p_{42})^{(8)}\right)$$

$$\left[\left((\lambda)^{(8)} + (a'_{40})^{(8)} + (p_{40})^{(8)}\right)(q_{41})^{(8)}G^*_{41} + (a_{41})^{(8)}(q_{40})^{(8)}G^*_{40}\right)\Big]$$

$$\left((\lambda)^{(8)} + (b'_{40})^{(8)} - (r_{40})^{(8)}\right)s_{(41),(41)}T^*_{41} + (b_{41})^{(8)}s_{(40),(41)}T^*_{41}\Big)$$

$$+\left((\lambda)^{(8)} + (a'_{41})^{(8)} + (p_{41})^{(8)}\right)(q_{40})^{(8)}G^*_{40} + (a_{40})^{(8)}(q_{41})^{(8)}G^*_{41}\Big)$$

$$\left((\lambda)^{(8)} + (b'_{40})^{(8)} - (r_{40})^{(8)}\right)s_{(41),(40)}T^*_{41} + (b_{41})^{(8)}s_{(40),(40)}T^*_{40}\Big)$$

$$\left((\lambda)^{(8)}\right)^2 + \left((a'_{40})^{(8)} + (a'_{41})^{(8)} + (p_{40})^{(8)} + (p_{41})^{(8)}\right)(\lambda)^{(8)}\Big)$$

$$\left((\lambda)^{(8)}\right)^2 + \left((b'_{40})^{(8)} + (b'_{41})^{(8)} - (r_{40})^{(8)} + (r_{41})^{(8)}\right)(\lambda)^{(8)}\Big)$$

$$+\left(\left((\lambda)^{(8)}\right)^2 + \left((a'_{40})^{(8)} + (a'_{41})^{(8)} + (p_{40})^{(8)} + (p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(42)}T^{*}_{40}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\right)\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(45)}T^{*}_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^{*}_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^{*}_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(44)}T^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$+\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(46)}T^{*}_{44}\right)\}=0$$

**And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.**

## SECTION THIRTY ONE

## "Fringe" Consciousness

### INTRODUCTION—VARIABLES USED

**Exploring "fringe" consciousness: The subjective experience of perceptual fluency and its objective bases Rolf Rebera, Pascal Wurtzb, Thomas D Zimmermann b Exploring "fringe" consciousness: The subjective experience of perceptual fluency and its objective bases Rolf Rebera, Pascal Wurtzb, Thomas D Zimmermann b**

(1) These findings suggest that speed of processing at different stages condensed into a unified subjective experience of perceptual fluency.
(2) This research was supported by the Swiss National Science Foundation (Grant No. 61-57881.99 to Rolf Reber). We thank Josephine Cock, Doris Eckstein, Thedra Fazendeiro, Dörthe Heinemann, Walter Perrig, Mark Price, Bruce Whittlesea, Piotr Winkielman, and an anonymous reviewer for comments on earlier versions of the manuscript. Corresponding author. Fax: +47-55-58-98-79 Copyright © 2003 Elsevier Inc. All rights reserved.

**Context effects in recognition memory: The role of familiarity and recollection Original Research Article Pages 20-38 Wendy A. McKenzie, Guy Tiberghien**

**(3)** A variant of the process dissociation procedure was coupled with a manipulation of response signal lag to assess whether manipulations of context affect one or both of the familiarity and search processes described by the dual process model of recognition.
**(4)** Participants studied a list of word pairs (context + target) followed by a recognition test with target words presented in the same or different context, and in the same or different form as study (singular/plural).
**(5)** Participants were asked to recognize any target word regardless of changes to form (inclusion), or to only recognise words that were presented in the same form (exclusion).
**(6)** The standard context reinstatement effect was evident even at the short response lags.
**(7)** Analyses of the estimates of the contributions of familiarity and search processes suggest that the context effect demonstrated here can be attributed in part to the influence of familiarity on recognition, whereas the effect on recollection was less clear.

## NOTATION

### Module One

Conversely, **font in which the word was written** influenced identification, but not detection.

Both contrast and font influenced subjective fluency

$G_{13}$ : Category one of **font in which the word was written;** detection (Take AC+DC=0)

$G_{14}$ : Category two of SAS

$G_{15}$ : Category three of SAS

$T_{13}$ : Category one of detection **;font in which the word was written**

$T_{14}$ : Category two of SAS

$T_{15}$ : Category three of SAS

### Module Two

Both **contrast and font** influenced subjective fluency

$G_{16}$ : Category one of **contrast and font;** subjective fluency

$G_{17}$ : Category two of SAS

$G_{18}$ : Category three of SAS

$T_{16}$ : Category one of subjective fluency **;contrast and font**

$T_{17}$ : Category two of SAS

$T_{18}$ : Category three of SAS

**Module three**

These findings suggest that speed of processing at different stages condensed into a unified subjective experience of perceptual fluency

$G_{20}$ : Category one of findings

$G_{21}$ : Category two of SAS

$G_{22}$ : Category three of SAS

$T_{20}$ : Category one of speed of processing at different stages condensed into a unified subjective experience of perceptual fluency

$T_{21}$ : Category two of SAS

$T_{22}$ : Category three of SAS

**Module four**

speed of processing at different stages condensed into a unified subjective experience of perceptual fluency

$G_{24}$ : Category one of speed of processing at different stages

$G_{25}$ : Category two of SAS

$G_{26}$ : Category three of SAS

$T_{24}$ : Category one of unified subjective experience of perceptual fluency

$T_{25}$ : Category two of SAS

$T_{26}$ : Category three of SAS

**Module five**

A **variant of the process dissociation procedure** was coupled with a manipulation of response signal lag to assess whether manipulations of context affect one or both of the familiarity and search processes described by the dual process model of recognition

$G_{28}$ : Category one of **variant of the process dissociation procedure;** manipulation of response signal lag to assess whether manipulations of context affect one or both of the familiarity and search processes described by the dual process model of recognition

$G_{29}$ : Category two of SAS

$G_{30}$ : Category three of SAS

$T_{28}$ : Category one of manipulation of response signal lag to assess whether manipulations of context affect one or both of the familiarity and search processes described by the dual process model of recognition

**;variant of the process dissociation procedure**

$T_{29}$ : Category two of SAS

$T_{30}$ : Category three of SAS

**Module six**

A variant of the process dissociation procedure was coupled with a manipulation of response signal lag to assess whether manipulations of context affect one or both of the familiarity and search processes described by the dual process model of recognition

$G_{32}$ : Category one of **variant of the process dissociation procedure was coupled with a manipulation of response signal lag;** manipulations of context affect one or both of the familiarity and search processes described by the dual process model of recognition

$G_{33}$ : Category two of SAS

$G_{34}$ : Category three of SAS

$T_{32}$ : Category one of manipulations of context affect one or both of the familiarity and search processes described by the dual process model of recognition; **variant of the process dissociation procedure was coupled with a manipulation of response signal lag**

$T_{33}$ : Category two of SAS

$T_{34}$ : Category three of SAS

**Module seven**

A **variant of the process dissociation procedure was coupled with a manipulation of response signal lag to assess whether manipulations of context** affect one or both of the familiarity and search processes described by the dual process model of recognition

$G_{36}$ : Category one of **variant of the process dissociation procedure was coupled with a manipulation of response signal lag to assess whether manipulations of context;** one or both of the familiarity and search processes described by the dual process model of recognition

$G_{37}$ : Category two of SAS

$G_{38}$ : Category three of SAS

$T_{36}$ : Category one of one or both of the familiarity and search processes described by the dual process model of recognition **;variant of the process dissociation procedure was coupled with a manipulation of response signal lag to assess whether manipulations of context**

$T_{37}$ : Category two of SAS

$T_{38}$ : Category three of SAS

**Module eight**

**standard context reinstatement effect** was evident even at the short response lags

$G_{40}$ : Category one of **standard context reinstatement effect;** short response lags

$G_{41}$ : Category two of SAS

$G_{42}$ : Category three of SAS

$T_{40}$ : Category one of short response lags; **standard context reinstatement effect**

$T_{41}$ : Category two of SAS

$T_{42}$ : Category three of SAS

**Module Nine**

Analyses of the estimates of the contributions of familiarity and search processes suggest that the context effect demonstrated here can be attributed in part to the influence of familiarity on recognition, whereas the effect on recollection was less clear.

$G_{44}$ : Category one of estimates of the contributions of familiarity and search processes

$G_{45}$ : Category two of SAS

$G_{46}$ : Category three of SAS

$T_{44}$ : Category one of context effect demonstrated here can be attributed in part to the influence of familiarity on recognition, whereas the effect on recollection was less clear

$T_{45}$ : Category two of SAS

$T_{46}$ : Category three of SAS

**The Coefficients:**

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}$ , $(b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}$,
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}, \ (b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}\ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$ ,
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)}$,
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)}$,
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)}$,

are Dissipation coefficients

**Module Numbered One**

**The differential system of this model is now (Module Numbered one)**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14},t)\right]G_{13} \quad 1$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14},t)\right]G_{14} \quad 2$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14},t)\right]G_{15} \quad 3$$

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G,t)\right]T_{13} \quad 4$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G,t)\right]T_{14} \quad 5$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G,t)\right]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14},t) =$ First augmentation factor

$-(b''_{13})^{(1)}(G,t) =$ First detritions factor

**Module Numbered Two**

**The differential system of this model is now ( Module numbered two)**

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17},t)\right]G_{16} \quad 7$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17},t)\right]G_{17} \quad 8$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17},t)\right]G_{18} \quad 9$$

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\big((G_{19}),t\big)\right]T_{16} \quad 10$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\big((G_{19}),t\big)\right]T_{17} \quad 11$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\big((G_{19}),t\big)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17},t) =$ First augmentation factor

$-(b''_{16})^{(2)}\big((G_{19}),t\big) =$ First detritions factor

**Module Numbered Three**

**The differential system of this model is now (Module numbered three)**

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21},t)\right]G_{20} \quad 13$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21},t)\right]G_{21} \quad 14$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21},t)\right]G_{22} \quad 15$$

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23},t)\right]T_{20} \quad 16$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23},t)\right]T_{21} \quad 17$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23},t)\right]T_{22} \quad 18$$

$+(a''_{20})^{(3)}(T_{21},t) =$ First augmentation factor

$-(b''_{20})^{(3)}(G_{23},t) =$ First detritions factor

**Module Numbered Four**

**The differential system of this model is now (Module numbered Four)**

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)\right]G_{24} \quad 19$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}, t)\right]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}, t)\right]G_{26} \quad 21$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27}), t\right)\right]T_{24} \quad 22$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27}), t\right)\right]T_{25} \quad 23$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27}), t\right)\right]T_{26} \quad 24$$

$+(a''_{24})^{(4)}(T_{25}, t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}\left((G_{27}), t\right) =$ **First detritions factor**

**Module Numbered Five**:

**The differential system of this model is now (Module number five)**

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right]G_{28} \quad 25$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}, t)\right]G_{29} \quad 26$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}, t)\right]G_{30} \quad 27$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}\left((G_{31}), t\right)\right]T_{28} \quad 28$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}\left((G_{31}), t\right)\right]T_{29} \quad 29$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}\left((G_{31}), t\right)\right]T_{30} \quad 30$$

$+(a''_{28})^{(5)}(T_{29}, t) =$ **First augmentation factor**

$-(b''_{28})^{(5)}\left((G_{31}), t\right) =$ **First detritions factor**

**Module Numbered Six**

**The differential system of this model is now (Module numbered Six)**

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right]G_{32} \quad 31$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}, t)\right]G_{33} \quad 32$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}, t)\right]G_{34} \quad 33$$

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}\left((G_{35}), t\right)\right]T_{32} \quad 34$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}\left((G_{35}), t\right)\right]T_{33} \quad 35$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}\left((G_{35}), t\right)\right]T_{34} \quad 36$$

$+(a''_{32})^{(6)}(T_{33}, t) =$ **First augmentation factor**

**Module Numbered Seven:**

**The differential system of this model is now (Seventh Module)**

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right]G_{36} \quad 37$$

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37}, t)\right]G_{37} \quad 38$$

$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}, t)\right]G_{38} \quad 39$$

$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}\left((G_{39}), t\right)\right]T_{36} \quad 40$$

$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}\left((G_{39}), t\right)\right]T_{37} \quad 41$$

$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39}),t)\right]T_{38} \quad 42$$
$$+(a''_{36})^{(7)}(T_{37},t) = \textbf{ First augmentation factor}$$

**Module Numbered Eight**

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right]G_{40} \quad 43$$
$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41},t)\right]G_{41} \quad 44$$
$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41},t)\right]G_{42} \quad 45$$
$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43}),t)\right]T_{40} \quad 46$$
$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}((G_{43}),t)\right]T_{41} \quad 47$$
$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43}),t)\right]T_{42} \quad 48$$

**Module Numbered Nine**

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$
$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$
$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$
$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47}),t)\right]T_{44} \quad 52$$
$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}((G_{47}),t)\right]T_{45} \quad 53$$
$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47}),t)\right]T_{46} \quad 54$$
$$+(a''_{44})^{(9)}(T_{45},t) = \textbf{ First augmentation factor}$$
$$-(b''_{44})^{(9)}((G_{47}),t) = \textbf{ First detrition factor}$$

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[\begin{array}{ccc} (a'_{13})^{(1)} & +(a''_{13})^{(1)}(T_{14},t) & +(a''_{16})^{(2,2,)}(T_{17},t) \quad +(a''_{20})^{(3,3,)}(T_{21},t) \\ +(a''_{24})^{(4,4,4,4,)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{36})^{(7,7)}(T_{37},t) & +(a''_{40})^{(8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{array}\right]G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[\begin{array}{ccc} (a'_{14})^{(1)} & +(a''_{14})^{(1)}(T_{14},t) & +(a''_{17})^{(2,2,)}(T_{17},t) \quad +(a''_{21})^{(3,3,)}(T_{21},t) \\ +(a''_{25})^{(4,4,4,4,)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{37})^{(7,7)}(T_{37},t) & +(a''_{41})^{(8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{array}\right]G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[\begin{array}{ccc} (a'_{15})^{(1)} & +(a''_{15})^{(1)}(T_{14},t) & +(a''_{18})^{(2,2,)}(T_{17},t) \quad +(a''_{22})^{(3,3,)}(T_{21},t) \\ +(a''_{26})^{(4,4,4,4,)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{38})^{(7,7)}(T_{37},t) & +(a''_{42})^{(8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{array}\right]G_{15} \quad 57$$

Where $\boxed{(a''_{13})^{(1)}(T_{14},t)}$, $\boxed{(a''_{14})^{(1)}(T_{14},t)}$, $\boxed{(a''_{15})^{(1)}(T_{14},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,)}(T_{25},t)$ , $+(a''_{25})^{(4,4,4,4,)}(T_{25},t)$ , $+(a''_{26})^{(4,4,4,4,)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,)}(T_{29},t)$ , $+(a''_{30})^{(5,5,5,5,)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,)}(T_{33},t)$ , $+(a''_{34})^{(6,6,6,6,)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{38})^{(7,7)}(T_{37},t)$ $+(a''_{37})^{(7,7)}(T_{37},t)$ $+(a''_{36})^{(7,7)}(T_{37},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{40})^{(8,8)}(T_{41},t)$ $+(a''_{41})^{(8,8)}(T_{41},t)$ $+(a''_{42})^{(8,8)}(T_{41},t)$ are eight augmentation coefficient for 1,2,3

$+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$ , $+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$ , $+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)} & -(b''_{13})^{(1)}(G,t) & -(b''_{16})^{(2,2,)}(G_{19},t) & -(b''_{20})^{(3,3,)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{36})^{(7,7,)}(G_{39},t) & -(b''_{40})^{(8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{13} \quad (58)$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)} & -(b''_{14})^{(1)}(G,t) & -(b''_{17})^{(2,2,)}(G_{19},t) & -(b''_{21})^{(3,3,)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{37})^{(7,7,)}(G_{39},t) & -(b''_{41})^{(8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{14} \quad (59)$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)} & -(b''_{15})^{(1)}(G,t) & -(b''_{18})^{(2,2,)}(G_{19},t) & -(b''_{22})^{(3,3,)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{38})^{(7,7,)}(G_{39},t) & -(b''_{42})^{(8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{15} \quad (60)$$

Where $-(b''_{13})^{(1)}(G,t)$ , $-(b''_{14})^{(1)}(G,t)$ , $-(b''_{15})^{(1)}(G,t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,)}(G_{19},t)$ , $-(b''_{17})^{(2,2,)}(G_{19},t)$ , $-(b''_{18})^{(2,2,)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,)}(G_{23},t)$ , $-(b''_{21})^{(3,3,)}(G_{23},t)$ , $-(b''_{22})^{(3,3,)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,)}(G_{27},t)$ , $-(b''_{25})^{(4,4,4,4,)}(G_{27},t)$ , $-(b''_{26})^{(4,4,4,4,)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,)}(G_{31},t)$ , $-(b''_{29})^{(5,5,5,5,)}(G_{31},t)$ , $-(b''_{30})^{(5,5,5,5,)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,)}(G_{35},t)$ , $-(b''_{33})^{(6,6,6,6,)}(G_{35},t)$ , $-(b''_{34})^{(6,6,6,6,)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{37})^{(7,7,)}(G_{39},t)$ , $-(b''_{36})^{(7,7,)}(G_{39},t)$ , $-(b''_{38})^{(7,7,)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8)}(G_{43},t)$ $-(b''_{41})^{(8,8)}(G_{43},t)$ $-(b''_{42})^{(8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \begin{bmatrix} (a'_{16})^{(2)} & \boxed{+(a''_{16})^{(2)}(T_{17},t)} & \boxed{+(a''_{13})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)} & \\ \boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9)}(T_{45},t)} & \end{bmatrix} G_{16} \quad (61)$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \begin{bmatrix} (a'_{17})^{(2)} & \boxed{+(a''_{17})^{(2)}(T_{17},t)} & \boxed{+(a''_{14})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)} & \\ \boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9)}(T_{45},t)} & \end{bmatrix} G_{17} \quad (62)$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \begin{bmatrix} (a'_{18})^{(2)} & \boxed{+(a''_{18})^{(2)}(T_{17},t)} & \boxed{+(a''_{15})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)} & \\ \boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9)}(T_{45},t)} & \end{bmatrix} G_{18} \quad (63)$$

Where $\boxed{+(a''_{16})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,)}(T_{14},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)}$ are eight augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{44})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9)}(T_{45},t)}$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \begin{bmatrix} (b'_{16})^{(2)} & \boxed{-(b''_{16})^{(2)}(G_{19},t)} & \boxed{-(b''_{13})^{(1,1,)}(G,t)} & \boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)} & \\ \boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9)}(G_{47},t)} & \end{bmatrix} T_{16} \quad (64)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)} & \boxed{-(b''_{17})^{(2)}(G_{19},t)} & \boxed{-(b''_{14})^{(1,1,)}(G,t)} & \boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)} & \\ \boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9)}(G_{47},t)} & \end{bmatrix} T_{17} \quad (65)$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)}\boxed{-(b''_{18})^{(2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,)}(G,t)} & \boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{18} \quad 66$$

where $\boxed{-(b''_{16})^{(2)}(G_{19},t)}$ , $\boxed{-(b''_{17})^{(2)}(G_{19},t)}$ , $\boxed{-(b''_{18})^{(2)}(G_{19},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,)}(G,t)}$ are second detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{44})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \begin{bmatrix} (a'_{20})^{(3)}\boxed{+(a''_{20})^{(3)}(T_{21},t)} & \boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{20} \quad 67$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \begin{bmatrix} (a'_{21})^{(3)}\boxed{+(a''_{21})^{(3)}(T_{21},t)} & \boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{21} \quad 68$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \begin{bmatrix} (a'_{22})^{(3)}\boxed{+(a''_{22})^{(3)}(T_{21},t)} & \boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{22} \quad 69$$

$\boxed{+(a''_{20})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3)}(T_{21},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}$ are second augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)}$ are third augmentation coefficients for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficients for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficients for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficients for category 1, 2 and 3

$+(a''_{36})^{(7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1, 2 and 3

$+(a''_{40})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8)}(T_{41},t)$ are eight augmentation coefficients for category 1, 2 and 3

$+(a''_{44})^{(9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b'_{20})^{(3)} & -(b''_{20})^{(3)}(G_{23},t) & -(b''_{16})^{(2,2,2)}(G_{19},t) & -(b''_{13})^{(1,1,1,)}(G,t) \\ -(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{36})^{(7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{20} \quad (70)$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b'_{21})^{(3)} & -(b''_{21})^{(3)}(G_{23},t) & -(b''_{17})^{(2,2,2)}(G_{19},t) & -(b''_{14})^{(1,1,1,)}(G,t) \\ -(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{37})^{(7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{21} \quad (71)$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)} & -(b''_{22})^{(3)}(G_{23},t) & -(b''_{18})^{(2,2,2)}(G_{19},t) & -(b''_{15})^{(1,1,1,)}(G,t) \\ -(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{38})^{(7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{22} \quad (72)$$

$-(b''_{20})^{(3)}(G_{23},t)$, $-(b''_{21})^{(3)}(G_{23},t)$, $-(b''_{22})^{(3)}(G_{23},t)$ *are first detrition coefficients for category 1, 2 and 3*

$-(b''_{16})^{(2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2)}(G_{19},t)$ *are second detrition coefficients for category 1, 2 and 3*

$-(b''_{13})^{(1,1,1,)}(G,t)$, $-(b''_{14})^{(1,1,1,)}(G,t)$, $-(b''_{15})^{(1,1,1,)}(G,t)$ *are third detrition coefficients for category 1,2 and 3*

$-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)$ *are fourth detrition coefficients for category 1, 2 and 3*

$-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)$ *are fifth detrition coefficients for category 1, 2 and 3*

$-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)$ *are sixth detrition coefficients for category 1, 2 and 3*

$-(b''_{36})^{(7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt}=(a_{24})^{(4)}G_{25}-\begin{bmatrix}(a'_{24})^{(4)}\boxed{+(a''_{24})^{(4)}(T_{25},t)}&\boxed{+(a''_{28})^{(5,5,)}(T_{29},t)}&\boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}\\ \boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)}&\boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)}&\boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)}&\boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)}&\boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{24} \quad (73)$$

$$\frac{dG_{25}}{dt}=(a_{25})^{(4)}G_{24}-\begin{bmatrix}(a'_{25})^{(4)}\boxed{+(a''_{25})^{(4)}(T_{25},t)}&\boxed{+(a''_{29})^{(5,5,)}(T_{29},t)}&\boxed{+(a''_{33})^{(6,6)}(T_{33},t)}\\ \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)}&\boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)}&\boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)}&\boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)}&\boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{25} \quad (74)$$

$$\frac{dG_{26}}{dt}=(a_{26})^{(4)}G_{25}-\begin{bmatrix}(a'_{26})^{(4)}\boxed{+(a''_{26})^{(4)}(T_{25},t)}&\boxed{+(a''_{30})^{(5,5,)}(T_{29},t)}&\boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}\\ \boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)}&\boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)}&\boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)}&\boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)}&\boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{26} \quad (75)$$

$\boxed{(a''_{24})^{(4)}(T_{25},t)}$, $\boxed{(a''_{25})^{(4)}(T_{25},t)}$, $\boxed{(a''_{26})^{(4)}(T_{25},t)}$ *are first augmentation coefficients category* 1, 2 3

$\boxed{+(a''_{28})^{(5,5,)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,)}(T_{29},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)}$
*are fourth augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)}$
*are fifth augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)}$
*are sixth augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)}$
*are seventh augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)}$
*are eighth augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt}=(b_{24})^{(4)}T_{25}-\begin{bmatrix}(b'_{24})^{(4)}\boxed{-(b''_{24})^{(4)}(G_{27},t)}&\boxed{-(b''_{28})^{(5,5,)}(G_{31},t)}&\boxed{-(b''_{32})^{(6,6,)}(G_{35},t)}\\ \boxed{-(b''_{13})^{(1,1,1,1)}(G,t)}&\boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)}&\boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)}\\ \boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)}&\boxed{-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)}&\boxed{-(b''_{44})^{(9,9,9,9)}(G_{47},t)}\end{bmatrix}T_{24} \quad (76)$$

$$\frac{dT_{25}}{dt}=(b_{25})^{(4)}T_{24}-\begin{bmatrix}(b'_{25})^{(4)}\boxed{-(b''_{25})^{(4)}(G_{27},t)}&\boxed{-(b''_{29})^{(5,5,)}(G_{31},t)}&\boxed{-(b''_{33})^{(6,6,)}(G_{35},t)}\\ \boxed{-(b''_{14})^{(1,1,1,1)}(G,t)}&\boxed{-(b''_{17})^{(2,2,2,2)}(G_{19},t)}&\boxed{-(b''_{21})^{(3,3,3,3)}(G_{23},t)}\\ \boxed{-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)}&\boxed{-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)}&\boxed{-(b''_{45})^{(9,9,9,9)}(G_{47},t)}\end{bmatrix}T_{25} \quad (77)$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)}\boxed{-(b''_{26})^{(4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{26} \quad 78$$

*Where* $\boxed{-(b''_{24})^{(4)}(G_{27},t)}, \boxed{-(b''_{25})^{(4)}(G_{27},t)}, \boxed{-(b''_{26})^{(4)}(G_{27},t)}$ *are first detrition coefficients for category* $1, 2$ *and* $3$

$\boxed{-(b''_{28})^{(5,5,)}(G_{31},t)}, \boxed{-(b''_{29})^{(5,5,)}(G_{31},t)}, \boxed{-(b''_{30})^{(5,5,)}(G_{31},t)}$ *are second detrition coefficients for category* $1, 2$ *and* $3$

$\boxed{-(b''_{32})^{(6,6,)}(G_{35},t)}, \boxed{-(b''_{33})^{(6,6,)}(G_{35},t)}, \boxed{-(b''_{34})^{(6,6,)}(G_{35},t)}$ *are third detrition coefficients for category* $1, 2$ *and* $3$

$\boxed{-(b''_{13})^{(1,1,1,1)}(G,t)}, \boxed{-(b''_{14})^{(1,1,1,1)}(G,t)}, \boxed{-(b''_{15})^{(1,1,1,1)}(G,t)}$

*are fourth detrition coefficients for category* $1, 2$ *and* $3$

$\boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2,2)}(G_{19},t)}$

*are fifth detrition coefficients for category* $1, 2$ *and* $3$

$\boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3,3,3,3)}(G_{23},t)}$

*are sixth detrition coefficients for category* $1, 2$ *and* $3$

$\boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)}$

*are seventh detrition coefficients for category* $1, 2$ *and* $3$

$\boxed{-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)}$

*are eighth detrition coefficients for category* $1, 2$ *and* $3$

$\boxed{-(b''_{46})^{(9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{45})^{(9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{44})^{(9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \begin{bmatrix} (a'_{28})^{(5)}\boxed{+(a''_{28})^{(5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{28} \quad 79$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \begin{bmatrix} (a'_{29})^{(5)}\boxed{+(a''_{29})^{(5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{29} \quad 80$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \begin{bmatrix} (a'_{30})^{(5)}\boxed{+(a''_{30})^{(5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{30} \quad 81$$

*Where* $\boxed{+(a''_{28})^{(5)}(T_{29},t)}, \boxed{+(a''_{29})^{(5)}(T_{29},t)}, \boxed{+(a''_{30})^{(5)}(T_{29},t)}$ *are first augmentation coefficients for category* $1, 2$ *and* $3$

*And* $\boxed{+(a''_{24})^{(4,4,)}(T_{25},t)}, \boxed{+(a''_{25})^{(4,4,)}(T_{25},t)}, \boxed{+(a''_{26})^{(4,4,)}(T_{25},t)}$ *are second augmentation coefficient for category* $1, 2$ *and* $3$

$\boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)}, \boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)}, \boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)}$ *are third augmentation coefficient for category* $1, 2$ *and* $3$

$+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)$ are fourth augmentation coefficients for category 1,2, and 3

$+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)$ are fifth augmentation coefficients for category 1,2,and 3

$+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)$ are sixth augmentation coefficients for category 1,2, 3

$+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)$ , $+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1,2, 3

$+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)$ are eighth augmentation coefficients for category 1,2, 3

$+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \begin{bmatrix} (b'_{28})^{(5)} & -(b''_{28})^{(5)}(G_{31},t) & -(b''_{24})^{(4,4,)}(G_{27},t) & -(b''_{32})^{(6,6,6)}(G_{35},t) \\ & -(b''_{13})^{(1,1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{28} \quad 82$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b'_{29})^{(5)} & -(b''_{29})^{(5)}(G_{31},t) & -(b''_{25})^{(4,4,)}(G_{27},t) & -(b''_{33})^{(6,6,6)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{29} \quad 83$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b'_{30})^{(5)} & -(b''_{30})^{(5)}(G_{31},t) & -(b''_{26})^{(4,4,)}(G_{27},t) & -(b''_{34})^{(6,6,6)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1,1,)}(G,t) & -(b''_{18})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{30} \quad 84$$

*where* $-(b''_{28})^{(5)}(G_{31},t)$ , $-(b''_{29})^{(5)}(G_{31},t)$ , $-(b''_{30})^{(5)}(G_{31},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{24})^{(4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,)}(G_{27},t)$ *are second detrition coefficients for category* 1,2 *and* 3

$-(b''_{32})^{(6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6)}(G_{35},t)$ *are third detrition coefficients for category* 1,2 *and* 3

$-(b''_{13})^{(1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1)}(G,t)$ , $-(b''_{15})^{(1,1,1,1,1,)}(G,t)$ are fourth detrition coefficients for category 1,2, and 3

$-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1,2, and 3

$-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1,2, and 3

$-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2, and 3

$-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} \quad 85$$

$$-\left[\begin{array}{ccc} (a'_{32})^{(6)}\boxed{+(a''_{32})^{(6)}(T_{33},t)} & \boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)} \end{array}\right]G_{32}$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[\begin{array}{ccc} (a'_{33})^{(6)}\boxed{+(a''_{33})^{(6)}(T_{33},t)} & \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)} \end{array}\right]G_{33} \quad 86$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[\begin{array}{ccc} (a'_{34})^{(6)}\boxed{+(a''_{34})^{(6)}(T_{33},t)} & \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)} \end{array}\right]G_{34} \quad 87$$

$\boxed{+(a''_{32})^{(6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6)}(T_{33},t)}$ *are first augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}$ *are second augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}$ *are third augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}$ - are fourth augmentation coefficients

$\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}$ - fifth augmentation coefficients

$\boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)}$ sixth augmentation coefficients

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)}$ seventh augmentation coefficients

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}$ Eighth augmentation coefficients

$\boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)}$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[\begin{array}{ccc} (b'_{32})^{(6)}\boxed{-(b''_{32})^{(6)}(G_{35},t)} & \boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)} \end{array}\right]T_{32} \quad 88$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \begin{bmatrix} (b'_{33})^{(6)}\boxed{-(b''_{33})^{(6)}(G_{35},t)} & \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{33} \quad (89)$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)}\boxed{-(b''_{34})^{(6)}(G_{35},t)} & \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{34} \quad (90)$$

$\boxed{-(b''_{32})^{(6)}(G_{35},t)}, \boxed{-(b''_{33})^{(6)}(G_{35},t)}, \boxed{-(b''_{34})^{(6)}(G_{35},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}$ *are second detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)}$ *are third detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}$ are fourth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \quad (91)$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a'_{36})^{(7)}\boxed{+(a''_{36})^{(7)}(T_{37},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \quad (92)$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a'_{37})^{(7)}\boxed{+(a''_{37})^{(7)}(T_{37},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$\frac{dG_{38}}{dt}$ *93*

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a'_{38})^{(7)}\boxed{+(a''_{38})^{(7)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{(a''_{36})^{(7)}(T_{37},t)}$, $\boxed{(a''_{37})^{(7)}(T_{37},t)}$, $\boxed{(a''_{38})^{(7)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,)}(T_{41},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$\frac{dT_{36}}{dt} =$ 94

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b'_{36})^{(7)}\boxed{-(b''_{36})^{(7)}(G_{39},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$\frac{dT_{37}}{dt} =$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b'_{37})^{(7)}\boxed{-(b''_{37})^{(7)}(G_{39},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$\frac{dT_{38}}{dt} =$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b'_{38})^{(7)}\boxed{-(b''_{38})^{(7)}(G_{39},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b_{36}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{37}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{16}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{17}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{18}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{20}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{21}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{22}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{24}'')^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{25}'')^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{26}'')^{(4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{28}'')^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{32}'')^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{34}'')^{(6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{46}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{45}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{44}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3



$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \begin{bmatrix} (a_{40}')^{(8)}\boxed{+(a_{40}'')^{(8)}(T_{41},t)} & \boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{20}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{24}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{28}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{32}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{36}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{44}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \begin{bmatrix} (a_{41}')^{(8)}\boxed{+(a_{41}'')^{(8)}(T_{41},t)} & \boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{21}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{25}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{29}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{33}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{37}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{45}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \begin{bmatrix} (a_{42}')^{(8)}\boxed{+(a_{42}'')^{(8)}(T_{41},t)} & \boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{22}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{26}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{30}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{34}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{15}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{38}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{46}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a_{40}'')^{(8)}(T_{41},t)}$, $\boxed{+(a_{41}'')^{(8)}(T_{41},t)}$, $\boxed{+(a_{42}'')^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second

augmentation coefficient for category 1, 2 and 3
$+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)$ , $+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)$ , $+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3
$+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)$ , $+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)$ , $+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3
$+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)$ , $+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3
$+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)$ , $+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3
$+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ $+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ $+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ are seventh augmentation coefficient for 1,2,3
$+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$ , $+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$ , $+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$ are eighth augmentation coefficient for 1,2,3
$+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$ , $+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$ , $+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} =$$

$$(b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)}\left[-(b''_{40})^{(8)}(G_{43},t)\right] & \left[-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} =$$

$$(b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)}\left[-(b''_{41})^{(8)}(G_{43},t)\right] & \left[-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} =$$

$$(b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)}\left[-(b''_{42})^{(8)}(G_{43},t)\right] & \left[-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{15}$$

Where $-(b''_{36})^{(7)}(G_{39},t)$ , $-(b''_{37})^{(7)}(G_{39},t)$ , $-(b''_{38})^{(7)}(G_{39},t)$ are first detrition coefficients for category 1, 2 and 3
$-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$ , $-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$ , $-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3
$-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$ , $-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$ , $-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3
$-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)$ , $-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)$ , $-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{28}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{32}'')^{(6,6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{38}'')^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{36}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{37}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{44}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{45}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{46}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

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$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \begin{bmatrix} (a_{44}')^{(9)}\boxed{+(a_{44}'')^{(9)}(T_{45},t)} & \boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{20}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{24}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{28}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{32}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{36}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{40}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \begin{bmatrix} (a_{45}')^{(9)}\boxed{+(a_{45}'')^{(9)}(T_{45},t)} & \boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{21}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{25}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{29}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{33}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{14}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{37}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{41}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \begin{bmatrix} (a_{46}')^{(9)}\boxed{+(a_{46}'')^{(9)}(T_{37},t)} & \boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{22}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{26}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{30}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{34}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{15}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{38}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{42}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a_{44}'')^{(9)}(T_{45},t)}$, $\boxed{+(a_{45}'')^{(9)}(T_{45},t)}$, $\boxed{+(a_{46}'')^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{20}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{21}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{22}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{24}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{25}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{26}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{28}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{29}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{30}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ are Seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} =$$

$$(b_{44})^{(9)}T_{45} -$$

$$\begin{bmatrix} (b'_{44})^{(9)}\boxed{-(b''_{44})^{(9)}(G_{47},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b'_{45})^{(9)}\boxed{-(b''_{45})^{(9)}(G_{47},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b'_{46})^{(9)}\boxed{-(b''_{46})^{(9)}(G_{47},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{44})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9)}(G_{47},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{37}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{36}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a_i')^{(1)}, (a_i'')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (b_i'')^{(1)} > 0, \quad i,j = 13,14,15 \qquad 97$$

The functions $(a_i'')^{(1)}, (b_i'')^{(1)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a_i'')^{(1)}(T_{14},t) \leq (p_i)^{(1)} \leq (\hat{A}_{13})^{(1)}$$

$$(b_i'')^{(1)}(G,t) \leq (r_i)^{(1)} \leq (b_i')^{(1)} \leq (\hat{B}_{13})^{(1)}$$

$$lim_{T_2\to\infty}(a_i'')^{(1)}(T_{14},t) = (p_i)^{(1)} \qquad 98$$

$$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = (r_i)^{(1)}$$

**Definition of** $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$ :

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$$|(a_i'')^{(1)}(T_{14}',t) - (a_i'')^{(1)}(T_{14},t)| \leq (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G',t) - (b_i'')^{(1)}(G,t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}',t)$ and $(a_i'')^{(1)}(T_{14},t)$ . $(T_{14}',t)$ and $(T_{14},t)$ are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14},t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$ : 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$$

**Definition of** $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$ : 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i,j = 16,17,18$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17}, t) \leq (p_i)^{(2)} \leq (\hat{A}_{16})^{(2)} \quad 102$$

$$(b_i'')^{(2)}(G_{19}, t) \leq (r_i)^{(2)} \leq (b_i')^{(2)} \leq (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2 \to \infty} (a_i'')^{(2)}(T_{17}, t) = (p_i)^{(2)} \quad 104$$

$$\lim_{G \to \infty} (b_i'')^{(2)}((G_{19}), t) = (r_i)^{(2)} \quad 105$$

**Definition of** $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$ : 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}', t) - (a_i'')^{(2)}(T_{17}, t)| \leq (\hat{k}_{16})^{(2)}|T_{17} - T_{17}'|e^{-(\hat{M}_{16})^{(2)}t} \quad 107$$

$$|(b_i'')^{(2)}((G_{19})', t) - (b_i'')^{(2)}((G_{19}), t)| < (\hat{k}_{16})^{(2)}||(G_{19}) - (G_{19})'||e^{-(\hat{M}_{16})^{(2)}t} \quad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}', t)$ and$(a_i'')^{(2)}(T_{17}, t)$ . $(T_{17}', t)$ and $(T_{17}, t)$ are points belonging to the interval $[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}]$ . It is to be noted that $(a_i'')^{(2)}(T_{17}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17}, t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$ :

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants 109

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$$

**Definition of** $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$ :

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1 \quad (110)$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(b_i)^{(2)} + (b_i')^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1 \quad (111)$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i,j = 20,21,22 \quad (112)$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(3)}$, $(r_i)^{(3)}$:

$$(a_i'')^{(3)}(T_{21}, t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23}, t) \leq (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(3)}(T_{21}, t) = (p_i)^{(3)} \quad (113)$$

$$\lim_{G \to \infty}(b_i'')^{(3)}(G_{23}, t) = (r_i)^{(3)}$$

**Definition of** $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$ :

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: (114)

$$|(a_i'')^{(3)}(T_{21}', t) - (a_i'')^{(3)}(T_{21}, t)| \leq (\hat{k}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\hat{M}_{20})^{(3)}t}$$

$$|(b_i'')^{(3)}(G_{23}', t) - (b_i'')^{(3)}(G_{23}, t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}'||e^{-(\hat{M}_{20})^{(3)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}', t)$ and $(a_i'')^{(3)}(T_{21}, t)$ . $(T_{21}', t)$ And $(T_{21}, t)$ are points belonging to the interval $[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}]$. It is to be noted that $(a_i'')^{(3)}(T_{21}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$ : (115)

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$ and the constants $(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}, i = 20,21,22,$ satisfy the inequalities 116

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(a_i)^{(3)} + (a_i')^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(b_i)^{(3)} + (b_i')^{(3)} + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i, j = 24,25,26 \qquad 117$$

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(4)}$, $(r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25}, t) \le (p_i)^{(4)} \le (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}((G_{27}), t) \le (r_i)^{(4)} \le (b_i')^{(4)} \le (\hat{B}_{24})^{(4)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(4)}(T_{25}, t) = (p_i)^{(4)} \qquad 118$$

$$\lim_{G \to \infty}(b_i'')^{(4)}((G_{27}), t) = (r_i)^{(4)}$$

**Definition of** $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$ :

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24,25,26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}', t) - (a_i'')^{(4)}(T_{25}, t)| \le (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})', t) - (b_i'')^{(4)}((G_{27}), t)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}', t)$ and $(a_i'')^{(4)}(T_{25}, t)$ . $(T_{25}', t)$ and $(T_{25}, t)$ are points belonging to the interval $[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}]$. It is to be noted that $(a_i'')^{(4)}(T_{25}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$ : 120

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$, are positive constants

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}, \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$$

**Definition of** $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$ : 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and\ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}, i = 24,25,26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(a_i)^{(4)} + (a_i')^{(4)} + (\hat{A}_{24})^{(4)} + (\hat{P}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(b_i)^{(4)} + (b_i')^{(4)} + (\hat{B}_{24})^{(4)} + (\hat{Q}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i, j = 28,29,30 \qquad 122$$

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(5)}$, $(r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29}, t) \le (p_i)^{(5)} \le (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}((G_{31}), t) \le (r_i)^{(5)} \le (b_i')^{(5)} \le (\hat{B}_{28})^{(5)}$$

$$lim_{T_2 \to \infty} (a_i'')^{(5)}(T_{29}, t) = (p_i)^{(5)} \qquad 123$$

$$\lim_{G \to \infty} (b_i'')^{(5)}(G_{31}, t) = (r_i)^{(5)}$$

**Definition of** $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$ :

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}', t) - (a_i'')^{(5)}(T_{29}, t)| \le (\hat{k}_{28})^{(5)} |T_{29} - T_{29}'| e^{-(\hat{M}_{28})^{(5)} t}$$

$$|(b_i'')^{(5)}((G_{31})', t) - (b_i'')^{(5)}((G_{31}), t)| < (\hat{k}_{28})^{(5)} ||(G_{31}) - (G_{31})'|| e^{-(\hat{M}_{28})^{(5)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}', t)$ and $(a_i'')^{(5)}(T_{29}, t)$ . $(T_{29}', t)$ and $(T_{29}, t)$ are points belonging to the interval $\left[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\right]$. It is to be noted that $(a_i'')^{(5)}(T_{29}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29}, t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$ : 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}, \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

**Definition of** $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$ : 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\widehat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and\ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\widehat{M}_{28})^{(5)}}[(a_i)^{(5)} + (a_i')^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\widehat{M}_{28})^{(5)}}[(b_i)^{(5)} + (b_i')^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i,j = 32,33,34$$ 127

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(6)}$, $(r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33}, t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}), t) \leq (r_i)^{(6)} \leq (b_i')^{(6)} \leq (\hat{B}_{32})^{(6)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(6)}(T_{33}, t) = (p_i)^{(6)}$$ 128

$$\lim_{G \to \infty}(b_i'')^{(6)}((G_{35}), t) = (r_i)^{(6)}$$

**Definition of** $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$ :

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}', t) - (a_i'')^{(6)}(T_{33}, t)| \leq (\hat{k}_{32})^{(6)}|T_{33} - T_{33}'|e^{-(\widehat{M}_{32})^{(6)}t}$$

$$|(b_i'')^{(6)}((G_{35})', t) - (b_i'')^{(6)}((G_{35}), t)| < (\hat{k}_{32})^{(6)}||(G_{35}) - (G_{35})'||e^{-(\widehat{M}_{32})^{(6)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}', t)$ and $(a_i'')^{(6)}(T_{33}, t)$ . $(T_{33}', t)$ and $(T_{33}, t)$ are points belonging to the interval $[(\hat{k}_{32})^{(6)}, (\widehat{M}_{32})^{(6)}]$. It is to be noted that $(a_i'')^{(6)}(T_{33}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33}, t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\widehat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$ : 129

$(\widehat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$, are positive constants

$$\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}} < 1$$

**Definition of** $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$ : 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and\ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32,33,34,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(a_i)^{(6)} + (a_i')^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)} (\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(b_i)^{(6)} + (b_i')^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)} (\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

(MMMMMMM) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i, j = 36,37,38$ 131

(NNNNNNN) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(7)}$, $(r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37}, t) \leq (p_i)^{(7)} \leq (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39}, t) \leq (r_i)^{(7)} \leq (b_i')^{(7)} \leq (\hat{B}_{36})^{(7)}$$

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*(OOOOOOO)* $\lim_{T_2 \to \infty} (a_i'')^{(7)}(T_{37}, t) = (p_i)^{(7)}$

*(PPPPPPP)*

$\lim_{G \to \infty} (b_i'')^{(7)}((G_{39}), t) = (r_i)^{(7)}$

**Definition of** $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$ :

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$$|(a_i'')^{(7)}(T_{37}', t) - (a_i'')^{(7)}(T_{37}, t)| \leq (\hat{k}_{36})^{(7)} |T_{37} - T_{37}'| e^{-(\hat{M}_{36})^{(7)} t}$$

$$|(b_i'')^{(7)}((G_{39})', t) - (b_i'')^{(7)}((G_{39}), t)| < (\hat{k}_{36})^{(7)} ||(G_{39}) - (G_{39})'|| e^{-(\hat{M}_{36})^{(7)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}', t)$ and $(a_i'')^{(7)}(T_{37}, t)$ . $(T_{37}', t)$ and $(T_{37}, t)$ are points belonging to the interval $[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}]$ . It is to be noted that $(a_i'')^{(7)}(T_{37}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37}, t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$ : 134

(QQQQQQQ) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)},$ are positive constants

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$$

**Definition of** $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$ : 135

(RRRRRRR) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(b_i)^{(7)} + (b_i')^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i, j = 40,41,42$$ 136

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

**Definition of** $(p_i)^{(8)}$, $(r_i)^{(8)}$: 137

$$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)}$$ 138

$$(b_i'')^{(8)}((G_{43}), t) \leq (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)}$$ 139

$$\lim_{T_2 \to \infty} (a_i'')^{(8)}(T_{41}, t) = (p_i)^{(8)}$$ 140

$$\lim_{G \to \infty} (b_i'')^{(8)}((G_{43}), t) = (r_i)^{(8)}$$ 141

**Definition of** $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$ :

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)} |T_{41} - T_{41}'| e^{-(\hat{M}_{40})^{(8)} t}$$ 142

$$|(b_i'')^{(8)}((G_{43})', t) - (b_i'')^{(8)}((G_{43}), t)| < (\hat{k}_{40})^{(8)} ||(G_{43}) - (G_{43})'|| e^{-(\hat{M}_{40})^{(8)} t}$$ 143

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}', t)$ and $(a_i'')^{(8)}(T_{41}, t)$ . $(T_{41}', t)$ and $(T_{41}, t)$ are points belonging to the interval $[(\hat{k}_{40})^{(8)}, (\hat{M}_{40})^{(8)}]$. It is to be noted that $(a_i'')^{(8)}(T_{41}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{40})^{(8)} = 1$

then the function $(a_i'')^{(8)}(T_{41},t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)}$ :

$(\hat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)},$ are positive constants

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\ ,\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}}<1 \qquad 144$$

**Definition of** $(\hat{P}_{40})^{(8)},(\hat{Q}_{40})^{(8)}$ :

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\hat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)},(\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)},(a_i')^{(8)},(b_i)^{(8)},(b_i')^{(8)},(p_i)^{(8)},\ (r_i)^{(8)},i=40,41,42,$

Satisfy the inequalities

$$\frac{1}{(\hat{M}_{40})^{(8)}}[\,(a_i)^{(8)}+(a_i')^{(8)}+\ (\hat{A}_{40})^{(8)}+\ (\hat{P}_{40})^{(8)}\,(\hat{k}_{40})^{(8)}]<1 \qquad 145$$

$$\frac{1}{(\hat{M}_{40})^{(8)}}[\ (b_i)^{(8)}+(b_i')^{(8)}+\ (\hat{B}_{40})^{(8)}+\ (\hat{Q}_{40})^{(8)}\ (\hat{k}_{40})^{(8)}]<1 \qquad 146$$

Where we suppose

$$(a_i)^{(9)},(a_i')^{(9)},(a_i'')^{(9)},(b_i)^{(9)},(b_i')^{(9)},(b_i'')^{(9)}>0,\quad i,j=44,45,46 \qquad 146\,A$$

The functions $(a_i'')^{(9)},(b_i'')^{(9)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(9)},\ (r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45},t)\le(p_i)^{(9)}\le(\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47},t)\le\ (r_i)^{(9)}\le(b_i')^{(9)}\le(\hat{B}_{44})^{(9)}$$

$$\lim_{T_2\to\infty}(a_i'')^{(9)}(T_{45},t)=(p_i)^{(9)}$$

$$\lim_{G\to\infty}(b_i'')^{(9)}(G_{47},t)=\ (r_i)^{(9)}$$

**Definition of** $(\hat{A}_{44})^{(9)},(\hat{B}_{44})^{(9)}$ :

Where $\boxed{(\hat{A}_{44})^{(9)},(\hat{B}_{44})^{(9)},(p_i)^{(9)},\ (r_i)^{(9)}}$ are positive constants and $\boxed{i=44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}',t)-(a_i'')^{(9)}(T_{45},t)|\le(\hat{k}_{44})^{(9)}|T_{45}-\ T_{45}'|e^{-(\hat{M}_{44})^{(9)}t}$$

$$|(b_i'')^{(9)}((G_{47})',t)-(b_i'')^{(9)}\big((G_{47}),t\big)|<(\hat{k}_{44})^{(9)}||(G_{47})-(G_{47})'||e^{-(\hat{M}_{44})^{(9)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}',t)$ and$(a_i'')^{(9)}(T_{45},t)$ . $(T_{45}',t)$ and $(T_{45},t)$ are points belonging to the interval $\big[(\hat{k}_{44})^{(9)},(\hat{M}_{44})^{(9)}\big]$. It is

to be noted that $(a_i'')^{(9)}(T_{45},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{44})^{(9)}=1$ then the function $(a_i'')^{(9)}(T_{45},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\widehat{M}_{44})^{(9)},(\hat{k}_{44})^{(9)}$ :

$(\widehat{M}_{44})^{(9)},(\hat{k}_{44})^{(9)}$, are positive constants

$$\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\ ,\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}<1$$

**Definition of** $(\hat{P}_{44})^{(9)},(\hat{Q}_{44})^{(9)}$ :

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\widehat{M}_{44})^{(9)},(\hat{k}_{44})^{(9)},(\hat{A}_{44})^{(9)} and\ (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)},(a_i')^{(9)},(b_i)^{(9)},(b_i')^{(9)},(p_i)^{(9)},\ (r_i)^{(9)}, i=44,45,46,$
satisfy the inequalities

$$\frac{1}{(\widehat{M}_{44})^{(9)}}[(a_i)^{(9)}+(a_i')^{(9)}+\ (\hat{A}_{44})^{(9)}+\ (\hat{P}_{44})^{(9)}\ (\hat{k}_{44})^{(9)}]<1$$

$$\frac{1}{(\widehat{M}_{44})^{(9)}}[\ (b_i)^{(9)}+(b_i')^{(9)}+\ (\hat{B}_{44})^{(9)}+\ (\hat{Q}_{44})^{(9)}\ (\hat{k}_{44})^{(9)}]<1$$

**Theorem 1:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0)\ ,T_i(0)$ :

$$G_i(t)\le\ (\hat{P}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}\ ,\ \boxed{G_i(0)=G_i^0>0}$$

$$T_i(t)\le\ (\hat{Q}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}\ ,\ \boxed{T_i(0)=T_i^0>0}$$

**Theorem 2 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

**Definition of** $G_i(0)\ ,T_i(0)$

$$G_i(t)\le\ (\hat{P}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}\ ,\ \ G_i(0)=G_i^0>0$$

$$T_i(t)\le\ (\hat{Q}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}\ ,\ \ T_i(0)=T_i^0>0$$

**Theorem 3 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t)\le\ (\hat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}\ ,\ \ G_i(0)=G_i^0>0$$

$$T_i(t)\le\ (\hat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}\ ,\ \ T_i(0)=T_i^0>0$$

**Theorem 4 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

**Definition of** $G_i(0)\ ,T_i(0)$ :

$G_i(t) \leq \left(\hat{P}_{24}\right)^{(4)} e^{(\hat{M}_{24})^{(4)}t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{24}\right)^{(4)} e^{(\hat{M}_{24})^{(4)}t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Theorem 5 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

**Definition of** $G_i(0)$ , $T_i(0)$ :

$G_i(t) \leq \left(\hat{P}_{28}\right)^{(5)} e^{(\hat{M}_{28})^{(5)}t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{28}\right)^{(5)} e^{(\hat{M}_{28})^{(5)}t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Theorem 6 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

**Definition of** $G_i(0)$ , $T_i(0)$ :

$G_i(t) \leq \left(\hat{P}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Theorem 7:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

**Definition of** $G_i(0)$ , $T_i(0)$ :

$G_i(t) \leq \left(\hat{P}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Theorem 8:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

**Definition of** $G_i(0)$ , $T_i(0)$ :

$G_i(t) \leq \left(\hat{P}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Theorem 9:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

**Definition of** $G_i(0)$ , $T_i(0)$ :

$G_i(t) \leq \left(\hat{P}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Proof:** Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i$: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \le (\hat{P}_{13})^{(1)} \text{ , } T_i^0 \le (\hat{Q}_{13})^{(1)}, \quad 155$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t} \quad 156$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t} \quad 157$$

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}(s_{(13)}) - \left((a'_{13})^{(1)} + a''_{13})^{(1)} (T_{14}(s_{(13)}), s_{(13)})\right) G_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)} (T_{14}(s_{(13)}), s_{(13)})\right) G_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)} (T_{14}(s_{(13)}), s_{(13)})\right) G_{15}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)} (G(s_{(13)}), s_{(13)})\right) T_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)} (G(s_{(13)}), s_{(13)})\right) T_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{15}(t) = T_{15}^0 + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)} (G(s_{(13)}), s_{(13)})\right) T_{15}(s_{(13)})\right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i$: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \le (\hat{P}_{16})^{(2)} \text{ , } T_i^0 \le (\hat{Q}_{16})^{(2)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$$

By 160

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''_{16})^{(2)} (T_{17}(s_{(16)}), s_{(16)})\right) G_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)} (T_{17}(s_{(16)}), s_{(17)})\right) G_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)} (T_{17}(s_{(16)}), s_{(16)})\right) G_{18}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)} (G_{19}(s_{(16)}), s_{(16)})\right) T_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^{0} + \int_0^t \left[ (b_{17})^{(2)} T_{16}(s_{(16)}) - \left( (b'_{17})^{(2)} - (b''_{17})^{(2)} (G_{19}(s_{(16)}), s_{(16)}) \right) T_{17}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^{0} + \int_0^t \left[ (b_{18})^{(2)} T_{17}(s_{(16)}) - \left( (b'_{18})^{(2)} - (b''_{18})^{(2)} (G_{19}(s_{(16)}), s_{(16)}) \right) T_{18}(s_{(16)}) \right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \le (\hat{P}_{20})^{(3)} \text{ , } T_i^0 \le (\hat{Q}_{20})^{(3)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$$

By 161

$$\bar{G}_{20}(t) = G_{20}^{0} + \int_0^t \left[ (a_{20})^{(3)} G_{21}(s_{(20)}) - \left( (a'_{20})^{(3)} + a''_{20})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^{0} + \int_0^t \left[ (a_{21})^{(3)} G_{20}(s_{(20)}) - \left( (a'_{21})^{(3)} + (a''_{21})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{21}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^{0} + \int_0^t \left[ (a_{22})^{(3)} G_{21}(s_{(20)}) - \left( (a'_{22})^{(3)} + (a''_{22})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{22}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^{0} + \int_0^t \left[ (b_{20})^{(3)} T_{21}(s_{(20)}) - \left( (b'_{20})^{(3)} - (b''_{20})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^{0} + \int_0^t \left[ (b_{21})^{(3)} T_{20}(s_{(20)}) - \left( (b'_{21})^{(3)} - (b''_{21})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{21}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{\mathrm{T}}_{22}(\mathrm{t}) = \mathrm{T}_{22}^{0} + \int_0^t \left[ (b_{22})^{(3)} T_{21}(s_{(20)}) - \left( (b'_{22})^{(3)} - (b''_{22})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{22}(s_{(20)}) \right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \le (\hat{P}_{24})^{(4)} \text{ , } T_i^0 \le (\hat{Q}_{24})^{(4)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

By 162

$$\bar{G}_{24}(t) = G_{24}^{0} + \int_0^t \left[ (a_{24})^{(4)} G_{25}(s_{(24)}) - \left( (a'_{24})^{(4)} + a''_{24})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{26}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{26}(t) = T_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{26}(s_{(24)})\right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{28})^{(5)}, T_i^0 \le (\hat{Q}_{28})^{(5)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

By 163

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{30}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{30}(t) = T_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{30}(s_{(28)})\right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{32})^{(6)}, T_i^0 \le (\hat{Q}_{32})^{(6)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\bar{M}_{32})^{(6)} t}$

By 164

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[ (a_{32})^{(6)} G_{33}(s_{(32)}) - \left( (a'_{32})^{(6)} + a''_{32})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[ (a_{33})^{(6)} G_{32}(s_{(32)}) - \left( (a'_{33})^{(6)} + (a''_{33})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[ (a_{34})^{(6)} G_{33}(s_{(32)}) - \left( (a'_{34})^{(6)} + (a''_{34})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{34}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[ (b_{32})^{(6)} T_{33}(s_{(32)}) - \left( (b'_{32})^{(6)} - (b''_{32})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[ (b_{33})^{(6)} T_{32}(s_{(32)}) - \left( (b'_{33})^{(6)} - (b''_{33})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{34}(t) = T_{34}^0 + \int_0^t \left[ (b_{34})^{(6)} T_{33}(s_{(32)}) - \left( (b'_{34})^{(6)} - (b''_{34})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{34}(s_{(32)}) \right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[ (a_{36})^{(7)} G_{37}(s_{(36)}) - \left( (a'_{36})^{(7)} + a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[ (a_{37})^{(7)} G_{36}(s_{(36)}) - \left( (a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[ (a_{38})^{(7)} G_{37}(s_{(36)}) - \left( (a'_{38})^{(7)} + (a''_{38})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{38}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[ (b_{36})^{(7)} T_{37}(s_{(36)}) - \left( (b'_{36})^{(7)} - (b''_{36})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[ (b_{37})^{(7)} T_{36}(s_{(36)}) - \left( (b'_{37})^{(7)} - (b''_{37})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{38}(t) = T_{38}^0 + \int_0^t \left[ (b_{38})^{(7)} T_{37}(s_{(36)}) - \left( (b'_{38})^{(7)} - (b''_{38})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{38}(s_{(36)}) \right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0,t)$

**Proof:**

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{40})^{(8)}$, $T_i^0 \le (\hat{Q}_{40})^{(8)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} G_{41}(s_{(40)}) - \left((a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[(a_{41})^{(8)} G_{40}(s_{(40)}) - \left((a'_{41})^{(8)} + (a''_{41})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{42}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^0 + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)} (G_{43}(s_{(40)}), s_{(40)})\right) T_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^0 + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)} (G_{43}(s_{(40)}), s_{(40)})\right) T_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{42}(t) = T_{42}^0 + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)} (G_{43}(s_{(40)}), s_{(40)})\right) T_{42}(s_{(40)})\right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0,t)$

**Proof:** 166
Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ A
which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{44})^{(9)}$, $T_i^0 \le (\hat{Q}_{44})^{(9)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)})\right) G_{44}(s_{(44)})\right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)} (T_{45}(s_{(44)}), s_{(44)})\right) G_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}(s_{(44)}), s_{(44)})\right) G_{46}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{44}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{\mathrm{T}}_{46}(\mathrm{t}) = \mathrm{T}_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{46}(s_{(44)})\right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}}\right)\right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left(e^{(\hat{M}_{13})^{(1)} t} - 1\right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\hat{M}_{13})^{(1)} t} \le \frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{P}_{13})^{(1)} + G_{14}^0\right) e^{\left(-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0}\right)} + (\hat{P}_{13})^{(1)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \le G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}}\right)\right] ds_{(16)} = \quad 169$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1\right)$$

From which it follows that 170

$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \le \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left[\left((\hat{P}_{16})^{(2)} + G_{17}^0\right) e^{\left(-\frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0}\right)} + (\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t) \le G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} \left(G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} s_{(20)}}\right)\right] ds_{(20)} =$$

$$\left(1 + (a_{20})^{(3)} t\right) G_{21}^0 + \frac{(a_{20})^{(3)} (\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left(e^{(\hat{M}_{20})^{(3)} t} - 1\right)$$

From which it follows that 172

$$(G_{20}(t)-G_{20}^{0})e^{-(\hat{M}_{20})^{(3)}t}\leq\frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left[((\hat{P}_{20})^{(3)}+G_{21}^{0})e^{\left(-\frac{(\hat{P}_{20})^{(3)}+G_{21}^{0}}{G_{21}^{0}}\right)}+(\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 173

$$G_{24}(t)\leq G_{24}^{0}+\int_{0}^{t}\left[(a_{24})^{(4)}\left(G_{25}^{0}+(\hat{P}_{24})^{(4)}e^{(\hat{M}_{24})^{(4)}s_{(24)}}\right)\right]ds_{(24)}=$$

$$\left(1+(a_{24})^{(4)}t\right)G_{25}^{0}+\frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left(e^{(\hat{M}_{24})^{(4)}t}-1\right)$$

From which it follows that 174

$$(G_{24}(t)-G_{24}^{0})e^{-(\hat{M}_{24})^{(4)}t}\leq\frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left[((\hat{P}_{24})^{(4)}+G_{25}^{0})e^{\left(-\frac{(\hat{P}_{24})^{(4)}+G_{25}^{0}}{G_{25}^{0}}\right)}+(\hat{P}_{24})^{(4)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t)\leq G_{28}^{0}+\int_{0}^{t}\left[(a_{28})^{(5)}\left(G_{29}^{0}+(\hat{P}_{28})^{(5)}e^{(\hat{M}_{28})^{(5)}s_{(28)}}\right)\right]ds_{(28)}=$$

$$\left(1+(a_{28})^{(5)}t\right)G_{29}^{0}+\frac{(a_{28})^{(5)}(\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left(e^{(\hat{M}_{28})^{(5)}t}-1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t}\leq\frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t)\leq G_{32}^{0}+\int_{0}^{t}\left[(a_{32})^{(6)}\left(G_{33}^{0}+(\hat{P}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\right)\right]ds_{(32)}=$$

$$\left(1+(a_{32})^{(6)}t\right)G_{33}^{0}+\frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left(e^{(\hat{M}_{32})^{(6)}t}-1\right)$$

From which it follows that 177

$$(G_{32}(t)-G_{32}^{0})e^{-(\hat{M}_{32})^{(6)}t}\leq\frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left[((\hat{P}_{32})^{(6)}+G_{33}^{0})e^{\left(-\frac{(\hat{P}_{32})^{(6)}+G_{33}^{0}}{G_{33}^{0}}\right)}+(\hat{P}_{32})^{(6)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}$ , $G_{26}, T_{24}, T_{25}, T_{26}$

(cc) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t) \le G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} \left(G_{37}^0 + (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} s_{(36)}}\right)\right] ds_{(36)} =$$
$$\left(1 + (a_{36})^{(7)} t\right) G_{37}^0 + \frac{(a_{36})^{(7)} (\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left(e^{(\hat{M}_{36})^{(7)} t} - 1\right)$$

From which it follows that

$$(G_{36}(t) - G_{36}^0) e^{-(\hat{M}_{36})^{(7)} t} \le \frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left[((\hat{P}_{36})^{(7)} + G_{37}^0) e^{\left(-\frac{(\hat{P}_{36})^{(7)} + G_{37}^0}{G_{37}^0}\right)} + (\hat{P}_{36})^{(7)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t) \le G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} \left(G_{41}^0 + (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} s_{(40)}}\right)\right] ds_{(40)} = \quad 180$$
$$\left(1 + (a_{40})^{(8)} t\right) G_{41}^0 + \frac{(a_{40})^{(8)} (\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}} \left(e^{(\hat{M}_{40})^{(8)} t} - 1\right)$$

From which it follows that 181

$$(G_{40}(t) - G_{40}^0) e^{-(\hat{M}_{40})^{(8)} t} \le \frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}} \left[((\hat{P}_{40})^{(8)} + G_{41}^0) e^{\left(-\frac{(\hat{P}_{40})^{(8)} + G_{41}^0}{G_{41}^0}\right)} + (\hat{P}_{40})^{(8)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 8

Analogous inequalities hold also for $G_{41}$ , $G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t) \le G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} \left(G_{45}^0 + (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} s_{(44)}}\right)\right] ds_{(44)} =$$
$$\left(1 + (a_{44})^{(9)} t\right) G_{45}^0 + \frac{(a_{44})^{(9)} (\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left(e^{(\hat{M}_{44})^{(9)} t} - 1\right)$$

From which it follows that

$$(G_{44}(t) - G_{44}^0) e^{-(\hat{M}_{44})^{(9)} t} \le \frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left[((\hat{P}_{44})^{(9)} + G_{45}^0) e^{\left(-\frac{(\hat{P}_{44})^{(9)} + G_{45}^0}{G_{45}^0}\right)} + (\hat{P}_{44})^{(9)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 9

Analogous inequalities hold also for $G_{45}$ , $G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}$ , $\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$ and to choose 182

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\widehat{M}_{13})^{(1)}}\left[(\widehat{P}_{13})^{(1)}+\left((\hat{P}_{13})^{(1)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{13})^{(1)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{13})^{(1)} \quad 183$$

$$\frac{(b_i)^{(1)}}{(\widehat{M}_{13})^{(1)}}\left[\left((\hat{Q}_{13})^{(1)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{13})^{(1)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{13})^{(1)}\right]\leq(\hat{Q}_{13})^{(1)} \quad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)},T^{(1)}\right),\left(G^{(2)},T^{(2)}\right)\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{13})^{(1)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{13})^{(1)}t}\}$$

Indeed if we denote

**Definition of** $\tilde{G},\tilde{T}:\left(\tilde{G},\tilde{T}\right)=\mathcal{A}^{(1)}(G,T)$

It results

$$\left|\tilde{G}_{13}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{13})^{(1)}\left|G_{14}^{(1)}-G_{14}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\,ds_{(13)}+$$

$$\int_0^t\{(a'_{13})^{(1)}\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$G_{13}^{(2)}\left|(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)-(a''_{13})^{(1)}\left(T_{14}^{(2)},s_{(13)}\right)\right|\ e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses it follows

$$\left|G^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}t}\leq \quad 186$$
$$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)}+(a'_{13})^{(1)}+(\widehat{A}_{13})^{(1)}+(\widehat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}\right)d\left(\left(G^{(1)},T^{(1)};\,G^{(2)},T^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

**Remark 1:** The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}\ and\ (\widehat{Q}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i=13,14,15$ depend only on $\mathrm{T}_{14}$ and respectively on

$G(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$

From 19 to 24 it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(1)} - (a_i'')^{(1)}(T_{14}(s_{(13)}), s_{(13)})\} ds_{(13)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(1)} t)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2\ and\ \left((\widehat{M}_{13})^{(1)}\right)_3$ : 187

**Remark 3**: if $G_{13}$ is bounded, the same property have also $G_{14}\ and\ G_{15}$ . indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a_{14}')^{(1)} G_{14}$ and by integrating

$$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1 / (a_{14}')^{(1)}$$

In the same way , one can obtain

$$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2 / (a_{15}')^{(1)}$$

If $G_{14}\ or\ G_{15}$ is bounded, the same property follows for $G_{13}$ , $G_{15}$ and $G_{13}$ , $G_{14}$ respectively.

**Remark 4:** If $G_{13}$ $is$ bounded, from below, the same property holds for $G_{14}\ and\ G_{15}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{14}$ is bounded from below. 188

**Remark 5:** If $T_{13}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}\,(G(t), t)) = (b_{14}')^{(1)}$ then $T_{14} \to \infty$. 189

**Definition of** $(m)^{(1)}$ and $\varepsilon_1$ :

Indeed let $t_1$ be so that for $t > t_1$

$$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t), t) < \varepsilon_1, T_{13}\,(t) > (m)^{(1)}$$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right)$, $t = log\,\frac{2}{\varepsilon_1}$ By taking now $\varepsilon_1$ sufficiently small one sees that $T_{14}$ is unbounded.
The same property holds for $T_{15}$ if $\lim_{t\to\infty}(b_{15}'')^{(1)}\,(G(t), t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}$ , $\frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}\ and\ (\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[(\widehat{P}_{16})^{(2)}+\left((\hat{P}_{16})^{(2)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{16})^{(2)}\right]\leq(\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

**Definition of** $\widetilde{G_{19}},\widetilde{T_{19}}$ : $\left(\widetilde{G_{19}},\widetilde{T_{19}}\right)=\mathcal{A}^{(2)}(G_{19},T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{16})^{(2)}\left|G_{17}^{(1)}-G_{17}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)}+$$

$$\int_0^t\{(a'_{16})^{(2)}\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$G_{16}^{(2)}\left|(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)-(a''_{16})^{(2)}\left(T_{17}^{(2)},s_{(16)}\right)\right|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0,t]$ 197

From the hypotheses it follows

$$\left|(G_{19})^{(1)}-(G_{19})^{(2)}\right|\mathrm{e}^{-(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}\leq$$
$$\frac{1}{(\widehat{\mathrm{M}}_{16})^{(2)}}\left((a_{16})^{(2)}+(a'_{16})^{(2)}+(\widehat{\mathrm{A}}_{16})^{(2)}+(\widehat{\mathrm{P}}_{16})^{(2)}(\hat{k}_{16})^{(2)}\right)\mathrm{d}\left(((G_{19})^{(1)},(T_{19})^{(1)};\ (G_{19})^{(2)},(T_{19})^{(2)})\right)$$

And analogous inequalities for $G_i$ and $T_i$. Taking into account the hypothesis the result follows 198

**Remark 6:** The fact that we supposed $(a''_{16})^{(2)}$ and $(b''_{16})^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{\mathrm{P}}_{16})^{(2)}\mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}$ and $(\widehat{\mathrm{Q}}_{16})^{(2)}\mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i=16,17,18$ depend only on $\mathrm{T}_{17}$ and respectively on

$(G_{19})$(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 7:** There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$ 200

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(2)} - (a_i'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})\} ds_{(16)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(2)}t)} > 0 \quad \text{for } t > 0$$

**Definition of** $\left((\widehat{M}_{16})^{(2)}\right)_1$, $\left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$: 201

**Remark 8:** if $G_{16}$ is bounded, the same property have also $G_{17}$ and $G_{18}$. indeed if

$G_{16} < (\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt} \leq \left((\widehat{M}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)} G_{17}$ and by integrating

$$G_{17} \leq \left((\widehat{M}_{16})^{(2)}\right)_2 = G_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1 / (a_{17}')^{(2)}$$

In the same way , one can obtain

$$G_{18} \leq \left((\widehat{M}_{16})^{(2)}\right)_3 = G_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2 / (a_{18}')^{(2)}$$

If $G_{17}$ or $G_{18}$ is bounded, the same property follows for $G_{16}$, $G_{18}$ and $G_{16}$, $G_{17}$ respectively.

**Remark 9:** If $G_{16}$ is bounded, from below, the same property holds for $G_{17}$ and $G_{18}$. The proof is analogous with the preceding one. An analogous property is true if $G_{17}$ is bounded from below. 202

**Remark 10:** If $T_{16}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t), t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$. 203

**Definition of** $(m)^{(2)}$ and $\varepsilon_2$ :

Indeed let $t_2$ be so that for $t > t_2$

$$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right)$, $t = \log\frac{2}{\varepsilon_2}$ By taking now $\varepsilon_2$ sufficiently small one sees that $T_{17}$ is unbounded. 205
The same property holds for $T_{18}$ if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t), t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[(\widehat{P}_{20})^{(3)}+\left((\hat{P}_{20})^{(3)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{20})^{(3)}\right]\leq(\hat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself 210

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)},(T_{23})^{(1)}),((G_{23})^{(2)},(T_{23})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

**Definition of** $\widetilde{G_{23}},\widetilde{T_{23}}:(\widetilde{(G_{23})},\widetilde{(T_{23})})=\mathcal{A}^{(3)}((G_{23}),(T_{23}))$

It results 213

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{20})^{(3)}\left|G_{21}^{(1)}-G_{21}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\,ds_{(20)}+$$

$$\int_0^t\{(a'_{20})^{(3)}\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$G_{20}^{(2)}\left|(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)},s_{(20)}\right)\right|\ e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\}ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses it follows

$$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}t}\leq \quad 214$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+(a'_{20})^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\widehat{k}_{20})^{(3)}\right)d\left(((G_{23})^{(1)},(T_{23})^{(1)};\ (G_{23})^{(2)},(T_{23})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 11:** The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ *and* $(\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i=20,21,22$ depend only on $T_{21}$ and respectively on

$(G_{23})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 12:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 216

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(3)} - (a_i'')^{(3)}(T_{21}(s_{(20)}), s_{(20)})\} ds_{(20)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(3)} t)} > 0$ for t > 0

**Definition of** $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2\ and\ \left((\widehat{M}_{20})^{(3)}\right)_3$ : 217

**Remark 13:** if $G_{20}$ is bounded, the same property have also $G_{21}\ and\ G_{22}$ . indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a_{21}')^{(3)} G_{21}$ and by integrating

$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1 / (a_{21}')^{(3)}$

In the same way , one can obtain

$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2 / (a_{22}')^{(3)}$

If $G_{21}\ or\ G_{22}$ is bounded, the same property follows for $G_{20}$ , $G_{22}$ and $G_{20}$ , $G_{21}$ respectively.

**Remark 14:** If $G_{20}$ *is* bounded, from below, the same property holds for $G_{21}\ and\ G_{22}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{21}$ is bounded from below. 218

**Remark 15:** If $T_{20}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t), t\right)) = (b_{21}')^{(3)}$ then $T_{21} \to \infty$. 219

**Definition of** $(m)^{(3)}$ and $\varepsilon_3$ :

Indeed let $t_3$ be so that for $t > t_3$

$(b_{21})^{(3)} - (b_i'')^{(3)}\left((G_{23})(t), t\right) < \varepsilon_3, T_{20}\,(t) > (m)^{(3)}$

Then $\frac{dT_{21}}{dt} \geq (a_{21})^{(3)}(m)^{(3)} - \varepsilon_3 T_{21}$ which leads to 220

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t} = \frac{1}{2}$ it results

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_3}$ By taking now $\varepsilon_3$ sufficiently small one sees that $T_{21}$ is unbounded.
The same property holds for $T_{22}$ if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t), t\right) = (b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}$ , $\frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}} < 1$ and to choose 221

$(\widehat{P}_{24})^{(4)}$ and $(\widehat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[(\widehat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \tag{222}$$

$$\frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \tag{223}$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself (224)

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric (225)

$$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{24})^{(4)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{27}}),(\widetilde{T_{27}})$ : $\left((\widetilde{G_{27}}),(\widetilde{T_{27}})\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{24})^{(4)}\left|G_{25}^{(1)}-G_{25}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\,ds_{(24)}+$$

$$\int_0^t\{(a'_{24})^{(4)}\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$G_{24}^{(2)}\left|(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)-(a''_{24})^{(4)}\left(T_{25}^{(2)},s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\}ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}t}\leq \tag{226}$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)}+(a'_{24})^{(4)}+(\widehat{A}_{24})^{(4)}+(\widehat{P}_{24})^{(4)}(\hat{k}_{24})^{(4)}\right)d\left(((G_{27})^{(1)},(T_{27})^{(1)};\ (G_{27})^{(2)},(T_{27})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 16:** The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ *and* $(\widehat{Q}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_+$. (227)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i=24,25,26$ depend only on $T_{25}$ and respectively on

$(G_{27})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 17:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 228

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(4)} - (a_i'')^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(4)} t)} > 0$ for t > 0

**Definition of** $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2\ and\ \left((\widehat{M}_{24})^{(4)}\right)_3$ : 229

**Remark 18:** if $G_{24}$ is bounded, the same property have also $G_{25}\ and\ G_{26}$ . indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a_{25}')^{(4)} G_{25}$ and by integrating

$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1 / (a_{25}')^{(4)}$

In the same way , one can obtain

$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2 / (a_{26}')^{(4)}$

If $G_{25}\ or\ G_{26}$ is bounded, the same property follows for $G_{24}$ , $G_{26}$ and $G_{24}$ , $G_{25}$ respectively.

**Remark 19:** If $G_{24}$ *is* bounded, from below, the same property holds for $G_{25}\ and\ G_{26}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{25}$ is bounded from below. 230

**Remark 20:** If $T_{24}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}\ ((G_{27})(t), t)) = (b_{25}')^{(4)}$ then $T_{25} \to \infty$. 231

**Definition of** $(m)^{(4)}$ and $\varepsilon_4$ :

Indeed let $t_4$ be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t), t) < \varepsilon_4, T_{24}\,(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_4}$ By taking now $\varepsilon_4$ sufficiently small one sees that $T_{25}$ is unbounded.
The same property holds for $T_{26}$ if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t), t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for $G_{29}$ , $G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}$ , $\frac{(b_i)^{(5)}}{(\widehat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)}+\left((\hat{P}_{28})^{(5)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{28})^{(5)}\right]\leq(\hat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$$d\left(((G_{31})^{(1)},(T_{31})^{(1)}),((G_{31})^{(2)},(T_{31})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\hat{M}_{28})^{(5)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\hat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

**Definition of** $\widetilde{(G_{31})},\widetilde{(T_{31})}$ : $\left(\widetilde{(G_{31})},\widetilde{(T_{31})}\right)=\mathcal{A}^{(5)}\left((G_{31}),(T_{31})\right)$

It results

$$|\tilde{G}_{28}^{(1)}-\tilde{G}_i^{(2)}|\leq\int_0^t(a_{28})^{(5)}|G_{29}^{(1)}-G_{29}^{(2)}|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\,ds_{(28)}+$$

$$\int_0^t\{(a'_{28})^{(5)}|G_{28}^{(1)}-G_{28}^{(2)}|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$(a''_{28})^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)|G_{28}^{(1)}-G_{28}^{(2)}|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$G_{28}^{(2)}|(a''_{28})^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)-(a''_{28})^{(5)}\left(T_{29}^{(2)},s_{(28)}\right)|\;e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\}ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on it follows

$$|(G_{31})^{(1)}-(G_{31})^{(2)}|e^{-(\widehat{M}_{28})^{(5)}t}\leq \quad 237$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)}+(a'_{28})^{(5)}+(\widehat{A}_{28})^{(5)}+(\widehat{P}_{28})^{(5)}(\widehat{k}_{28})^{(5)}\right)d\left(((G_{31})^{(1)},(T_{31})^{(1)};\;(G_{31})^{(2)},(T_{31})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 21:** The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\hat{P}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ *and* $(\hat{Q}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ respectively of $\mathbb{R}_+$. 238

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on $T_{29}$ and respectively on $(G_{31})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 22:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 239

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(5)} - (a_i'')^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(5)} t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2\ and\ \left((\widehat{M}_{28})^{(5)}\right)_3$ : 240

**Remark 23:** if $G_{28}$ is bounded, the same property have also $G_{29}\ and\ G_{30}$ . indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a_{29}')^{(5)} G_{29}$ and by integrating

$$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1 / (a_{29}')^{(5)}$$

In the same way , one can obtain

$$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2 / (a_{30}')^{(5)}$$

If $G_{29}\ or\ G_{30}$ is bounded, the same property follows for $G_{28}$ , $G_{30}$ and $G_{28}$ , $G_{29}$ respectively.

**Remark 24:** If $G_{28}$ *is* bounded, from below, the same property holds for $G_{29}\ and\ G_{30}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{29}$ is bounded from below. 241

**Remark 25:** If $T_{28}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}\,((G_{31})(t), t)) = (b_{29}')^{(5)}$ then $T_{29} \to \infty$. 242

**Definition of** $(m)^{(5)}$ and $\varepsilon_5$ :

Indeed let $t_5$ be so that for $t > t_5$

$$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t), t) < \varepsilon_5, T_{28}\,(t) > (m)^{(5)}$$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_5}$ By taking now $\varepsilon_5$ sufficiently small one sees that $T_{29}$ is unbounded.
The same property holds for $T_{30}$ if $\lim_{t\to\infty}(b_{30}'')^{(5)}\left((G_{31})(t), t\right) = (b_{30}')^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{33}\,, G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\ , \frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}} < 1$ and to choose 244

$(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[(\hat{P}_{32})^{(6)}+\left((\hat{P}_{32})^{(6)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{32})^{(6)} \tag{245}$$

$$\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[\left((\hat{Q}_{32})^{(6)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{32})^{(6)}\right]\leq(\hat{Q}_{32})^{(6)} \tag{246}$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric (247)

$$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\hat{M}_{32})^{(6)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\hat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{35}}),(\widetilde{T_{35}})$ : $\left((\widetilde{G_{35}}),(\widetilde{T_{35}})\right)=\mathcal{A}^{(6)}\left((G_{35}),(T_{35})\right)$

It results

$$|\tilde{G}_{32}^{(1)}-\tilde{G}_i^{(2)}|\leq\int_0^t(a_{32})^{(6)}|G_{33}^{(1)}-G_{33}^{(2)}|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\,ds_{(32)}+$$

$$\int_0^t\{(a'_{32})^{(6)}|G_{32}^{(1)}-G_{32}^{(2)}|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)|G_{32}^{(1)}-G_{32}^{(2)}|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$G_{32}^{(2)}|(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)-(a''_{32})^{(6)}\left(T_{33}^{(2)},s_{(32)}\right)|\;e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\}ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$|(G_{35})^{(1)}-(G_{35})^{(2)}|e^{-(\widehat{M}_{32})^{(6)}t}\leq \tag{248}$$
$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)}+(a'_{32})^{(6)}+(\widehat{A}_{32})^{(6)}+(\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}\right)d\left(((G_{35})^{(1)},(T_{35})^{(1)};\;(G_{35})^{(2)},(T_{35})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 26:** The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\hat{P}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ *and* $(\hat{Q}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ respectively of $\mathbb{R}_+$. (249)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i = 32,33,34$ depend only on $T_{33}$ and respectively on $(G_{35})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 27:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 250

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(6)} - (a_i'')^{(6)}(T_{33}(s_{(32)}), s_{(32)})\} ds_{(32)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(6)} t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{32})^{(6)}\right)_1, \left((\widehat{M}_{32})^{(6)}\right)_2\ and\ \left((\widehat{M}_{32})^{(6)}\right)_3$ : 251

**Remark 28:** if $G_{32}$ is bounded, the same property have also $G_{33}\ and\ G_{34}$ . indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a_{33}')^{(6)} G_{33}$ and by integrating

$$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)} \left((\widehat{M}_{32})^{(6)}\right)_1 / (a_{33}')^{(6)}$$

In the same way , one can obtain

$$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)} \left((\widehat{M}_{32})^{(6)}\right)_2 / (a_{34}')^{(6)}$$

If $G_{33}\ or\ G_{34}$ is bounded, the same property follows for $G_{32}$ , $G_{34}$ and $G_{32}$ , $G_{33}$ respectively.

**Remark 29:** If $G_{32}$ *is* bounded, from below, the same property holds for $G_{33}\ and\ G_{34}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{33}$ is bounded from below. 252

**Remark 30:** If $T_{32}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}\ ((G_{35})(t), t)) = (b_{33}')^{(6)}$ then $T_{33} \to \infty$. 253

**Definition of** $(m)^{(6)}$ and $\varepsilon_6$ :

Indeed let $t_6$ be so that for $t > t_6$

$$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t), t\right) < \varepsilon_6, T_{32}\,(t) > (m)^{(6)}$$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right)$, $t = log\,\frac{2}{\varepsilon_6}$ By taking now $\varepsilon_6$ sufficiently small one sees that $T_{33}$ is unbounded.
The same property holds for $T_{34}$ if $\lim_{t\to\infty}(b_{34}'')^{(6)}\left((G_{35})(t), t(t), t\right) = (b_{34}')^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{37}, G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}$ , $\frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$ and to choose

$(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[(\hat{P}_{36})^{(7)}+\left((\hat{P}_{36})^{(7)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{36})^{(7)} \tag{256}$$

$$\frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[\left((\hat{Q}_{36})^{(7)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{36})^{(7)}\right]\leq(\hat{Q}_{36})^{(7)} \tag{257}$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself
The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric (258)

$$d\left(((G_{39})^{(1)},(T_{39})^{(1)}),((G_{39})^{(2)},(T_{39})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\tilde{M}_{36})^{(7)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\tilde{M}_{36})^{(7)}t}\}$$

Indeed if we denote

**Definition of** $\widetilde{(G_{39})},\widetilde{(T_{39})}:(\widetilde{(G_{39})},\widetilde{(T_{39})})=\mathcal{A}^{(7)}((G_{39}),(T_{39}))$
It results

$$\left|\tilde{G}_{36}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{36})^{(7)}\left|G_{37}^{(1)}-G_{37}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\,ds_{(36)}+$$
$$\int_0^t\{(a'_{36})^{(7)}\left|G_{36}^{(1)}-G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$(a''_{36})^{(7)}(T_{37}^{(1)},s_{(36)})\left|G_{36}^{(1)}-G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$G_{36}^{(2)}\left|(a''_{36})^{(7)}(T_{37}^{(1)},s_{(36)})-(a''_{36})^{(7)}(T_{37}^{(2)},s_{(36)})\right|\;e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\}ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on it follows

$$\left|(G_{39})^{(1)}-(G_{39})^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}t}\leq \tag{259}$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}}\left((a_{36})^{(7)}+(a'_{36})^{(7)}+(\widehat{A}_{36})^{(7)}+(\widehat{P}_{36})^{(7)}(\widehat{k}_{36})^{(7)}\right)d\left(((G_{39})^{(1)},(T_{39})^{(1)};\;(G_{39})^{(2)},(T_{39})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 31:** The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ *and* $(\widehat{Q}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ respectively of $\mathbb{R}_+$. (260)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(7)}$ and $(b_i'')^{(7)}, i = 36,37,38$ depend only on $T_{37}$ and respectively on $(G_{39})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 32:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 261

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(7)} - (a_i'')^{(7)}(T_{37}(s_{(36)}), s_{(36)})\} ds_{(36)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(7)} t\right)} > 0$ for t > 0

**Definition of** $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2\ and\ \left((\widehat{M}_{36})^{(7)}\right)_3$ : 262

**Remark 33:** if $G_{36}$ is bounded, the same property have also $G_{37}\ and\ G_{38}$ . indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a_{37}')^{(7)} G_{37}$ and by integrating

$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1 / (a_{37}')^{(7)}$

In the same way , one can obtain

$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2 / (a_{38}')^{(7)}$

If $G_{37}\ or\ G_{38}$ is bounded, the same property follows for $G_{36}$ , $G_{38}$ and $G_{36}$ , $G_{37}$ respectively.

**Remark 34:** If $G_{36}$ *is* bounded, from below, the same property holds for $G_{37}\ and\ G_{38}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{37}$ is bounded from below. 263

**Remark 35:** If $T_{36}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(7)}\ ((G_{39})(t), t)) = (b_{37}')^{(7)}$ then $T_{37} \to \infty$. 264

**Definition of** $(m)^{(7)}$ and $\varepsilon_7$ :

Indeed let $t_7$ be so that for $t > t_7$

$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t), t) < \varepsilon_7, T_{36}\,(t) > (m)^{(7)}$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_7}$ By taking now $\varepsilon_7$ sufficiently small one sees that $T_{37}$ is unbounded.
The same property holds for $T_{38}$ if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t), t\right) = (b_{38}')^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}$ , $\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}} < 1$ and to choose $(\widehat{P}_{40})^{(8)}$ and $(\widehat{Q}_{40})^{(8)}$ large to have 266

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[(\widehat{P}_{40})^{(8)} + \left((\hat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{40})^{(8)} \quad 267$$

$$\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{40})^{(8)}\right] \leq (\hat{Q}_{40})^{(8)} \quad 268$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)}, (T_{43})^{(1)}), ((G_{43})^{(2)}, (T_{43})^{(2)})\right) = \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

**Definition of** $\widetilde{(G_{43})}, \widetilde{(T_{43})}$ : $\left(\widetilde{(G_{43})}, \widetilde{(T_{43})}\right) = \mathcal{A}^{(8)}((G_{43}), (T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{40})^{(8)}\left|G_{41}^{(1)} - G_{41}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\,ds_{(40)} +$$
$$\int_0^t\{(a'_{40})^{(8)}\left|G_{40}^{(1)} - G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{-(\widehat{M}_{40})^{(8)}s_{(40)}} +$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right)\left|G_{40}^{(1)} - G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}} +$$
$$G_{40}^{(2)}\left|(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right) - (a''_{40})^{(8)}\left(T_{41}^{(2)}, s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\}ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0, t]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)} - (G_{43})^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}t} \leq \quad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)} + (a'_{40})^{(8)} + (\widehat{A}_{40})^{(8)} + (\widehat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}\right)d\left(((G_{43})^{(1)}, (T_{43})^{(1)};\ (G_{43})^{(2)}, (T_{43})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

**Remark 36:** The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate 274

condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ *and* $(\widehat{Q}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i = 40{,}41{,}42$ depend only on $T_{41}$ and respectively on $(G_{43})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 37** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 275

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t\{(a_i')^{(8)}-(a_i'')^{(8)}(T_{41}(s_{(40)}),s_{(40)})\}ds_{(40)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(8)}t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{40})^{(8)}\right)_1, \left((\widehat{M}_{40})^{(8)}\right)_2\ and\ \left((\widehat{M}_{40})^{(8)}\right)_3$ : 276

**Remark 38:** if $G_{40}$ is bounded, the same property have also $G_{41}\ and\ G_{42}$ . indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a_{41}')^{(8)}G_{41}$ and by integrating

$$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1/(a_{41}')^{(8)}$$

In the same way , one can obtain

$$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2/(a_{42}')^{(8)}$$

If $G_{41}\ or\ G_{42}$ is bounded, the same property follows for $G_{40}$ , $G_{42}$ and $G_{40}$ , $G_{41}$ respectively.

**Remark 39:** If $G_{40}$ *is* bounded, from below, the same property holds for $G_{41}\ and\ G_{42}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{41}$ is bounded from below. 277

**Remark 40:** If $T_{40}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}\ ((G_{43})(t),t)) = (b_{41}')^{(8)}$ then $T_{41} \to \infty$. 278

**Definition of** $(m)^{(8)}$ and $\varepsilon_8$ :

Indeed let $t_8$ be so that for $t\ > t_8$

$$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t),t\right) < \varepsilon_8, T_{40}\,(t) > (m)^{(8)}$$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right)$, $t = log\frac{2}{\varepsilon_8}$ By taking now $\varepsilon_8$ sufficiently small one sees that $T_{41}$ is unbounded.
The same property holds for $T_{42}$ if $\lim_{t\to\infty}(b''_{42})^{(8)}\left((G_{43})(t), t(t), t\right) = (b'_{42})^{(8)}$



It is now sufficient to take $\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}$ , $\frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$ and to choose $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ large to have

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}\left[(\hat{P}_{44})^{(9)} + \left((\hat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}}\left[\left((\hat{Q}_{44})^{(9)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{44})^{(9)}\right] \leq (\hat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)}, (T_{47})^{(1)}), ((G_{47})^{(2)}, (T_{47})^{(2)})\right) =$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{44})^{(9)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{47}}), (\widetilde{T_{47}}) : \left((\widetilde{G_{47}}), (\widetilde{T_{47}})\right) = \mathcal{A}^{(9)}\left((G_{47}), (T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{44})^{(9)}\left|G_{45}^{(1)} - G_{45}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\,ds_{(44)} +$$
$$\int_0^t\{(a'_{44})^{(9)}\left|G_{44}^{(1)} - G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{-(\widehat{M}_{44})^{(9)}s_{(44)}} +$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right)\left|G_{44}^{(1)} - G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}} +$$
$$G_{44}^{(2)}\left|(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right) - (a''_{44})^{(9)}\left(T_{45}^{(2)}, s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\}ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)} - G^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}t} \leq$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)} + (a'_{44})^{(9)} + (\widehat{A}_{44})^{(9)} + (\widehat{P}_{44})^{(9)}(\hat{k}_{44})^{(9)}\right)d\left(((G_{47})^{(1)}, (T_{47})^{(1)};\ (G_{47})^{(2)}, (T_{47})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis (39,35,36) the result follows

**Remark 41:** The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by

$(\widehat{P}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ *and* $(\widehat{Q}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i = 44{,}45{,}46$ depend only on $T_{45}$ and respectively on $(G_{47})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 42:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$

From 99 to 44 it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t\{(a_i')^{(9)}-(a_i'')^{(9)}(T_{45}(s_{(44)}),s_{(44)})\}ds_{(44)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(9)}t)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{44})^{(9)}\right)_1, \left((\widehat{M}_{44})^{(9)}\right)_2\ and\ \left((\widehat{M}_{44})^{(9)}\right)_3$ :

**Remark 43:** if $G_{44}$ is bounded, the same property have also $G_{45}\ and\ G_{46}$ . indeed if

$G_{44} < (\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt} \leq \left((\widehat{M}_{44})^{(9)}\right)_1 - (a_{45}')^{(9)}G_{45}$ and by integrating

$$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_1/(a_{45}')^{(9)}$$

In the same way , one can obtain

$$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_2/(a_{46}')^{(9)}$$

If $G_{45}\ or\ G_{46}$ is bounded, the same property follows for $G_{44}$ , $G_{46}$ and $G_{44}$ , $G_{45}$ respectively.

**Remark 44:** If $G_{44}$ *is* bounded, from below, the same property holds for $G_{45}\ and\ G_{46}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{45}$ is bounded from below.

**Remark 45:** If $T_{44}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t),t\right) = (b_{45}')^{(9)}$ then $T_{45} \to \infty$.

**Definition of** $(m)^{(9)}$ and $\varepsilon_9$ :

Indeed let $t_9$ be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t),t\right) < \varepsilon_9, T_{44}\,(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right), \quad t = log\frac{2}{\varepsilon_9}$ By taking now $\varepsilon_9$ sufficiently small one sees that $T_{45}$ is unbounded.
The same property holds for $T_{46}$ if $\lim_{t\to\infty}(b_{46}'')^{(9)}\left((G_{47})(t),t\right) = (b_{46}')^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

**Behavior of the solutions of equation** 280

**Theorem** If we denote and define

**Definition of** $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ :

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)} \le -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14}, t) + (a''_{14})^{(1)}(T_{14}, t) \le -(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)} \le -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G, t) - (b''_{14})^{(1)}(G, t) \le -(\tau_1)^{(1)}$

**Definition of** $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$ : 281

By $(\nu_1)^{(1)} > 0$, $(\nu_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$, $(u_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

**Definition of** $(\bar{\nu}_1)^{(1)},, (\bar{\nu}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$ : 282

By $(\bar{\nu}_1)^{(1)} > 0$, $(\bar{\nu}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$, $(\bar{u}_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

**Definition of** $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (\nu_0)^{(1)}$ :- 283

If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by
$(m_2)^{(1)} = (\nu_0)^{(1)}, (m_1)^{(1)} = (\nu_1)^{(1)}$, $if\ (\nu_0)^{(1)} < (\nu_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}, if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)}$,

and $\boxed{(\nu_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)}$, $if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)}$, $if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)}$,

and $\boxed{(u_0)^{(1)} = \frac{T^0_{13}}{T^0_{14}}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$

are defined

Then the solution of global equations satisfies the inequalities 285

$G^0_{13}e^{\left((S_1)^{(1)} - (p_{13})^{(1)}\right)t} \le G_{13}(t) \le G^0_{13}e^{(S_1)^{(1)}t}$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)}-(p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)}t}$$

$$\left(\frac{(a_{15})^{(1)}G_{13}^0}{(m_1)^{(1)}((S_1)^{(1)}-(p_{13})^{(1)}-(S_2)^{(1)})}\left[e^{((S_1)^{(1)}-(p_{13})^{(1)})t} - e^{-(S_2)^{(1)}t}\right] + G_{15}^0 e^{-(S_2)^{(1)}t} \le G_{15}(t) \le \right. \quad (286)$$

$$\left. \frac{(a_{15})^{(1)}G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)}-(a'_{15})^{(1)})}\left[e^{(S_1)^{(1)}t} - e^{-(a'_{15})^{(1)}t}\right] + G_{15}^0 e^{-(a'_{15})^{(1)}t}\right)$$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)}t} \le T_{13}(t) \le T_{13}^0 e^{((R_1)^{(1)}+(r_{13})^{(1)})t}} \quad (287)$$

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)}t} \le T_{13}(t) \le \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)}+(r_{13})^{(1)})t} \quad (288)$$

$$\frac{(b_{15})^{(1)}T_{13}^0}{(\mu_1)^{(1)}((R_1)^{(1)}-(b'_{15})^{(1)})}\left[e^{(R_1)^{(1)}t} - e^{-(b'_{15})^{(1)}t}\right] + T_{15}^0 e^{-(b'_{15})^{(1)}t} \le T_{15}(t) \le \quad (289)$$

$$\frac{(a_{15})^{(1)}T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)}+(r_{13})^{(1)}+(R_2)^{(1)})}\left[e^{((R_1)^{(1)}+(r_{13})^{(1)})t} - e^{-(R_2)^{(1)}t}\right] + T_{15}^0 e^{-(R_2)^{(1)}t}$$

**Definition of** $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:- (290)

Where $(S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$

$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$

$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$

$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$

**Behavior of the solutions of equation** (291)

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ : (292)

$(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ four constants satisfying

$$-(\sigma_2)^{(2)} \le -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(T_{17}, t) + (a''_{17})^{(2)}(T_{17}, t) \le -(\sigma_1)^{(2)} \quad (293)$$

$$-(\tau_2)^{(2)} \le -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}\big((G_{19}), t\big) - (b''_{17})^{(2)}\big((G_{19}), t\big) \le -(\tau_1)^{(2)} \quad (294)$$

**Definition of** $(\nu_1)^{(2)}, (\nu_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$ : (295)

By $(\nu_1)^{(2)} > 0$, $(\nu_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0$, $(u_2)^{(2)} < 0$ the roots (296)

of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ (297)

and $(b_{14})^{(2)}\big(u^{(2)}\big)^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and (298)

**Definition of** $(\bar{\nu}_1)^{(2)}, , (\bar{\nu}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$ : (299)

By $(\bar{\nu}_1)^{(2)} > 0$, $(\bar{\nu}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0$, $(\bar{u}_2)^{(2)} < 0$ the (300)

roots of the equations $(a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 301

and $(b_{17})^{(2)}\left(u^{(2)}\right)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 302

**Definition of** $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ :- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, ***if*** $(u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$, ***if*** $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)}$, ***if*** $(\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)}t}$

$(p_i)^{(2)}$ is defined by equation

$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)}t}$ 311

$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)}t}\right] + G_{18}^0 e^{-(S_2)^{(2)}t} \le G_{18}(t) \le\right.$ 312

$\left.\frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)}t} - e^{-(a'_{18})^{(2)}t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)}t}\right)$

$\boxed{T_{16}^0 e^{(R_1)^{(2)}t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}}$ 313

$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)}t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}$ 314

$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)}t} - e^{-(b'_{18})^{(2)}t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)}t} \le T_{18}(t) \le$ 315

$$\frac{(a_{18})^{(2)}T_{16}^{0}}{(\mu_2)^{(2)}((R_1)^{(2)}+(r_{16})^{(2)}+(R_2)^{(2)})}\left[e^{((R_1)^{(2)}+(r_{16})^{(2)})t}-e^{-(R_2)^{(2)}t}\right]+T_{18}^{0}e^{-(R_2)^{(2)}t}$$

**Definition of** $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$$

$$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)}$$ 318

$$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$$

**Behavior of the solutions** 319

**Theorem 3:** If we denote and define

**Definition of** $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ :

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}\big((G_{23}), t\big) \le -(\tau_1)^{(3)}$$

**Definition of** $(v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$ : 320

By $(v_1)^{(3)} > 0$, $(v_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{v}_1)^{(3)} > 0$, $(\bar{v}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

**Definition of** $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ :- 321

If we define $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (v_0)^{(3)}, (m_1)^{(3)} = (v_1)^{(3)}$, ***if*** $(v_0)^{(3)} < (v_1)^{(3)}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (\bar{v}_1)^{(3)}$, ***if*** $(v_1)^{(3)} < (v_0)^{(3)} < (\bar{v}_1)^{(3)}$,

and $\boxed{(v_0)^{(3)} = \frac{G_{20}^{0}}{G_{21}^{0}}}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_0)^{(3)}$, ***if*** $(\bar{v}_1)^{(3)} < (v_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)},\ \boldsymbol{if}\ (u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}, \boldsymbol{if}\ (u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)},$ and $\boxed{(u_0)^{(3)} = \frac{T_{20}^0}{T_{21}^0}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)},\ \boldsymbol{if}\ (\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G_{20}^0 e^{(S_1)^{(3)} t}$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)} t} \quad 323$$

$$\left(\frac{(a_{22})^{(3)} G_{20}^0}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\left[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)} t}\right] + G_{22}^0 e^{-(S_2)^{(3)} t} \le G_{22}(t) \le \right. \quad 324$$

$$\left. \frac{(a_{22})^{(3)} G_{20}^0}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}\left[e^{(S_1)^{(3)} t} - e^{-(a'_{22})^{(3)} t}\right] + G_{22}^0 e^{-(a'_{22})^{(3)} t}\right)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t}} \quad 325$$

$$\frac{1}{(\mu_1)^{(3)}} T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}} T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)} T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)} - (b'_{22})^{(3)})}\left[e^{(R_1)^{(3)} t} - e^{-(b'_{22})^{(3)} t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)} t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)} T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)} + (r_{20})^{(3)} + (R_2)^{(3)})}\left[e^{((R_1)^{(3)} + (r_{20})^{(3)})t} - e^{-(R_2)^{(3)} t}\right] + T_{22}^0 e^{-(R_2)^{(3)} t}$$

**Definition of** $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

**Behavior of the solutions of equation**

**Theorem:** If we denote and define

**Definition of** $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ :

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$

$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}((G_{27}), t) - (b''_{25})^{(4)}((G_{27}), t) \le -(\tau_1)^{(4)}$

**Definition of** $(v_1)^{(4)}, (v_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, v^{(4)}, u^{(4)}$ : 329

By $(v_1)^{(4)} > 0$ , $(v_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$ , $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_1)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

**Definition of** $(\bar{v}_1)^{(4)},, (\bar{v}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$ : 330

By $(\bar{v}_1)^{(4)} > 0$ , $(\bar{v}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$ , $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

**Definition of** $(m_1)^{(4)}$ , $(m_2)^{(4)}$ , $(\mu_1)^{(4)}, (\mu_2)^{(4)}, (\nu_0)^{(4)}$ :-

If we define $(m_1)^{(4)}$ , $(m_2)^{(4)}$ , $(\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (v_0)^{(4)}, (m_1)^{(4)} = (v_1)^{(4)}$, ***if*** $(v_0)^{(4)} < (v_1)^{(4)}$

$(m_2)^{(4)} = (v_1)^{(4)}, (m_1)^{(4)} = (\bar{v}_1)^{(4)}$ , ***if*** $(v_4)^{(4)} < (v_0)^{(4)} < (\bar{v}_1)^{(4)}$,
and $\boxed{(v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (v_4)^{(4)}, (m_1)^{(4)} = (v_0)^{(4)}$, ***if*** $(\bar{v}_4)^{(4)} < (v_0)^{(4)}$

and analogously

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$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)}$, ***if*** $(u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}$ , ***if*** $(u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)}$,
and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}$, ***if*** $(\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

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$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)}t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)}t}$$

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$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)}t}\right] + G_{26}^0 e^{-(S_2)^{(4)}t} \le G_{26}(t) \le\right.$$ 334

*(a26)4G240(m2)4(S1)4−(a26′)4e(S1)4t−e−(a26′)4t+ G260e−(a26′)4t*

$$\boxed{T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}}T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}}T_{24}^0 e^{((R_1)^{(4)}+(r_{24})^{(4)})t} \quad 335$$

$$\frac{(b_{26})^{(4)}T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)}-(b'_{26})^{(4)})}\left[e^{(R_1)^{(4)}t} - e^{-(b'_{26})^{(4)}t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)}t} \le T_{26}(t) \le \quad 336$$

$$\frac{(a_{26})^{(4)}T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)}+(r_{24})^{(4)}+(R_2)^{(4)})}\left[e^{((R_1)^{(4)}+(r_{24})^{(4)})t} - e^{-(R_2)^{(4)}t}\right] + T_{26}^0 e^{-(R_2)^{(4)}t}$$

**Definition of** $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

**Behavior of the solutions of equation** 338

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ :

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

**Definition of** $(v_1)^{(5)}, (v_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, v^{(5)}, u^{(5)}$ : 339

By $(v_1)^{(5)} > 0$ , $(v_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$ , $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(v^{(5)}\big)^2 + (\sigma_1)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

**Definition of** $(\bar{v}_1)^{(5)}, , (\bar{v}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$ : 340

By $(\bar{v}_1)^{(5)} > 0$ , $(\bar{v}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$ , $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(v^{(5)}\big)^2 + (\sigma_2)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

**Definition of** $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (v_0)^{(5)}$ :-

If we define $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (v_0)^{(5)}, (m_1)^{(5)} = (v_1)^{(5)}$, ***if*** $(v_0)^{(5)} < (v_1)^{(5)}$

$(m_2)^{(5)} = (v_1)^{(5)}, (m_1)^{(5)} = (\bar{v}_1)^{(5)}$ , ***if*** $(v_1)^{(5)} < (v_0)^{(5)} < (\bar{v}_1)^{(5)}$,
and $\boxed{(v_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)},\ \boldsymbol{if}\ (u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}, \boldsymbol{if}\ (u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)},$

and $\boxed{(u_0)^{(5)} = \frac{T_{28}^0}{T_{29}^0}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}, \boldsymbol{if}\ (\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{28}(t) \le G_{28}^0 e^{(S_1)^{(5)} t}$

where $(p_i)^{(5)}$ is defined by equation

$\frac{1}{(m_5)^{(5)}} G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}} G_{28}^0 e^{(S_1)^{(5)} t}$ 343

$\left(\frac{(a_{30})^{(5)} G_{28}^0}{(m_1)^{(5)}((S_1)^{(5)} - (p_{28})^{(5)} - (S_2)^{(5)})}\left[e^{((S_1)^{(5)} - (p_{28})^{(5)})t} - e^{-(S_2)^{(5)} t}\right] + G_{30}^0 e^{-(S_2)^{(5)} t} \le G_{30}(t) \le\right.$ 344

*(a30)5G280(m2)5(S1)5−(a30′)5e(S1)5t−e−(a30′)5t+ G300e−(a30′)5t*

$\boxed{T_{28}^0 e^{(R_1)^{(5)} t} \le T_{28}(t) \le T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}}$ 345

$\frac{1}{(\mu_1)^{(5)}} T_{28}^0 e^{(R_1)^{(5)} t} \le T_{28}(t) \le \frac{1}{(\mu_2)^{(5)}} T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}$ 346

$\frac{(b_{30})^{(5)} T_{28}^0}{(\mu_1)^{(5)}((R_1)^{(5)} - (b'_{30})^{(5)})}\left[e^{(R_1)^{(5)} t} - e^{-(b'_{30})^{(5)} t}\right] + T_{30}^0 e^{-(b'_{30})^{(5)} t} \le T_{30}(t) \le$ 347

$\frac{(a_{30})^{(5)} T_{28}^0}{(\mu_2)^{(5)}((R_1)^{(5)} + (r_{28})^{(5)} + (R_2)^{(5)})}\left[e^{((R_1)^{(5)} + (r_{28})^{(5)})t} - e^{-(R_2)^{(5)} t}\right] + T_{30}^0 e^{-(R_2)^{(5)} t}$

**Definition of** $(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$:- 348

Where $(S_1)^{(5)} = (a_{28})^{(5)}(m_2)^{(5)} - (a'_{28})^{(5)}$

$(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}$

$(R_1)^{(5)} = (b_{28})^{(5)}(\mu_2)^{(5)} - (b'_{28})^{(5)}$

$(R_2)^{(5)} = (b'_{30})^{(5)} - (r_{30})^{(5)}$

**Behavior of the solutions of equation** 349

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ :

$(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)} \le -(a'_{32})^{(6)} + (a'_{33})^{(6)} - (a''_{32})^{(6)}(T_{33}, t) + (a''_{33})^{(6)}(T_{33}, t) \le -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)} \le -(b'_{32})^{(6)} + (b'_{33})^{(6)} - (b''_{32})^{(6)}\big((G_{35}), t\big) - (b''_{33})^{(6)}\big((G_{35}), t\big) \le -(\tau_1)^{(6)}$$

**Definition of** $(\nu_1)^{(6)}, (\nu_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, \nu^{(6)}, u^{(6)}$ : 350

By $(\nu_1)^{(6)} > 0$, $(\nu_2)^{(6)} < 0$ and respectively $(u_1)^{(6)} > 0$, $(u_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(6)},, (\bar{\nu}_2)^{(6)}, (\bar{u}_1)^{(6)}, (\bar{u}_2)^{(6)}$ : 351

By $(\bar{\nu}_1)^{(6)} > 0$, $(\bar{\nu}_2)^{(6)} < 0$ and respectively $(\bar{u}_1)^{(6)} > 0$, $(\bar{u}_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$

**Definition of** $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}, (\nu_0)^{(6)}$ :-

If we define $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}$ by

$$(m_2)^{(6)} = (\nu_0)^{(6)}, (m_1)^{(6)} = (\nu_1)^{(6)}, \ \boldsymbol{if}\ (\nu_0)^{(6)} < (\nu_1)^{(6)}$$

$$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}, \boldsymbol{if}\ (\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)},$$

and $\boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}}$

$$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$$

and analogously 352

$$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}, \ \boldsymbol{if}\ (u_0)^{(6)} < (u_1)^{(6)}$$

$$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}, \boldsymbol{if}\ (u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)},$$

and $\boxed{(u_0)^{(6)} = \frac{T^0_{32}}{T^0_{33}}}$

$$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}, \boldsymbol{if}\ (\bar{u}_1)^{(6)} < (u_0)^{(6)} \text{ where } (u_1)^{(6)}, (\bar{u}_1)^{(6)}$$

Then the solution of global equations satisfies the inequalities 353

$$G^0_{32}e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{32}(t) \le G^0_{32}e^{(S_1)^{(6)}t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}}G^0_{32}e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{33}(t) \le \frac{1}{(m_2)^{(6)}}G^0_{32}e^{(S_1)^{(6)}t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)}G^0_{32}}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})}\left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)}t}\right] + G^0_{34}e^{-(S_2)^{(6)}t} \le G_{34}(t) \le \right. \quad 355$$

$$\frac{(a_{34})^{(6)}G_{32}^{0}}{(m_2)^{(6)}((S_1)^{(6)}-(a'_{34})^{(6)})}\left[e^{(S_1)^{(6)}t}-e^{-(a'_{34})^{(6)}t}\right]+\ G_{34}^{0}e^{-(a'_{34})^{(6)}t}\Big)$$

$$\boxed{T_{32}^{0}e^{(R_1)^{(6)}t}\le T_{32}(t)\le T_{32}^{0}e^{((R_1)^{(6)}+(r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}}T_{32}^{0}e^{(R_1)^{(6)}t}\le T_{32}(t)\le\frac{1}{(\mu_2)^{(6)}}T_{32}^{0}e^{((R_1)^{(6)}+(r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)}T_{32}^{0}}{(\mu_1)^{(6)}((R_1)^{(6)}-(b'_{34})^{(6)})}\left[e^{(R_1)^{(6)}t}-e^{-(b'_{34})^{(6)}t}\right]+T_{34}^{0}e^{-(b'_{34})^{(6)}t}\le T_{34}(t)\le \quad 358$$

$$\frac{(a_{34})^{(6)}T_{32}^{0}}{(\mu_2)^{(6)}((R_1)^{(6)}+(r_{32})^{(6)}+(R_2)^{(6)})}\left[e^{((R_1)^{(6)}+(r_{32})^{(6)})t}-e^{-(R_2)^{(6)}t}\right]+T_{34}^{0}e^{-(R_2)^{(6)}t}$$

**Definition of** $(S_1)^{(6)},(S_2)^{(6)},(R_1)^{(6)},(R_2)^{(6)}$:- 359

Where $(S_1)^{(6)}=(a_{32})^{(6)}(m_2)^{(6)}-(a'_{32})^{(6)}$

$(S_2)^{(6)}=(a_{34})^{(6)}-(p_{34})^{(6)}$

$(R_1)^{(6)}=(b_{32})^{(6)}(\mu_2)^{(6)}-(b'_{32})^{(6)}$

$(R_2)^{(6)}=(b'_{34})^{(6)}-(r_{34})^{(6)}$

**Behavior of the solutions of equation**

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(7)},(\sigma_2)^{(7)},(\tau_1)^{(7)},(\tau_2)^{(7)}$:

$(\sigma_1)^{(7)},(\sigma_2)^{(7)},(\tau_1)^{(7)},(\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)}\le-(a'_{36})^{(7)}+(a'_{37})^{(7)}-(a''_{36})^{(7)}(T_{37},t)+(a''_{37})^{(7)}(T_{37},t)\le-(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)}\le-(b'_{36})^{(7)}+(b'_{37})^{(7)}-(b''_{36})^{(7)}\big((G_{39}),t\big)-(b''_{37})^{(7)}\big((G_{39}),t\big)\le-(\tau_1)^{(7)}$$

**Definition of** $(v_1)^{(7)},(v_2)^{(7)},(u_1)^{(7)},(u_2)^{(7)},v^{(7)},u^{(7)}$: 361

By $(v_1)^{(7)}>0,(v_2)^{(7)}<0$ and respectively $(u_1)^{(7)}>0,(u_2)^{(7)}<0$ the roots of the equations $(a_{37})^{(7)}\big(v^{(7)}\big)^2+(\sigma_1)^{(7)}v^{(7)}-(a_{36})^{(7)}=0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2+(\tau_1)^{(7)}u^{(7)}-(b_{36})^{(7)}=0$ and

**Definition of** $(\bar{v}_1)^{(7)},,(\bar{v}_2)^{(7)},(\bar{u}_1)^{(7)},(\bar{u}_2)^{(7)}$: 362

By $(\bar{v}_1)^{(7)}>0,(\bar{v}_2)^{(7)}<0$ and respectively $(\bar{u}_1)^{(7)}>0,(\bar{u}_2)^{(7)}<0$ the roots of the equations $(a_{37})^{(7)}\big(v^{(7)}\big)^2+(\sigma_2)^{(7)}v^{(7)}-(a_{36})^{(7)}=0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2+(\tau_2)^{(7)}u^{(7)}-(b_{36})^{(7)}=0$

**Definition of** $(m_1)^{(7)},(m_2)^{(7)},(\mu_1)^{(7)},(\mu_2)^{(7)},(v_0)^{(7)}$:-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$$(m_2)^{(7)} = (\nu_0)^{(7)}, (m_1)^{(7)} = (\nu_1)^{(7)}, \ \boldsymbol{if}\ (\nu_0)^{(7)} < (\nu_1)^{(7)}$$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\bar{\nu}_1)^{(7)}, \boldsymbol{if}\ (\nu_1)^{(7)} < (\nu_0)^{(7)} < (\bar{\nu}_1)^{(7)},$$

and $\boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\nu_0)^{(7)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(7)} < (\nu_0)^{(7)}$$

and analogously 363

$$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}, \ \boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$$

and $\boxed{(u_0)^{(7)} = \frac{T_{36}^0}{T_{37}^0}}$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)} \text{ where } (u_1)^{(7)}, (\bar{u}_1)^{(7)}$$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^0 e^{(S_1)^{(7)} t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)} t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)}((S_1)^{(7)} - (p_{36})^{(7)} - (S_2)^{(7)})}\left[e^{((S_1)^{(7)} - (p_{36})^{(7)})t} - e^{-(S_2)^{(7)} t}\right] + G_{38}^0 e^{-(S_2)^{(7)} t} \le G_{38}(t) \le \right. \quad 366$$

$$\left. \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)}((S_1)^{(7)} - (a'_{38})^{(7)})}\left[e^{(S_1)^{(7)} t} - e^{-(a'_{38})^{(7)} t}\right] + G_{38}^0 e^{-(a'_{38})^{(7)} t}\right)$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t} \quad 368$$

$$\frac{(b_{38})^{(7)} T_{36}^0}{(\mu_1)^{(7)}((R_1)^{(7)} - (b'_{38})^{(7)})}\left[e^{(R_1)^{(7)} t} - e^{-(b'_{38})^{(7)} t}\right] + T_{38}^0 e^{-(b'_{38})^{(7)} t} \le T_{38}(t) \le \quad 369$$

$$\frac{(a_{38})^{(7)} T_{36}^0}{(\mu_2)^{(7)}((R_1)^{(7)} + (r_{36})^{(7)} + (R_2)^{(7)})}\left[e^{((R_1)^{(7)} + (r_{36})^{(7)})t} - e^{-(R_2)^{(7)} t}\right] + T_{38}^0 e^{-(R_2)^{(7)} t}$$

**Definition of** $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$

$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$

$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$

**Behavior of the solutions of equation** 371

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ :

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$-(\sigma_2)^{(8)} \le -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \le -(\sigma_1)^{(8)}$

$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\big((G_{43}), t\big) - (b''_{41})^{(8)}\big((G_{43}), t\big) \le -(\tau_1)^{(8)}$

**Definition of** $(\nu_1)^{(8)}, (\nu_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, \nu^{(8)}, u^{(8)}$ : 372

By $(\nu_1)^{(8)} > 0, (\nu_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0, (u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(8)}, , (\bar{\nu}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$ :

By $(\bar{\nu}_1)^{(8)} > 0, (\bar{\nu}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0, (\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

**Definition of** $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}, (\nu_0)^{(8)}$ :-

If we define $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$(m_2)^{(8)} = (\nu_0)^{(8)}, (m_1)^{(8)} = (\nu_1)^{(8)}$, ***if*** $(\nu_0)^{(8)} < (\nu_1)^{(8)}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\bar{\nu}_1)^{(8)}$, ***if*** $(\nu_1)^{(8)} < (\nu_0)^{(8)} < (\bar{\nu}_1)^{(8)}$,
and $\boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\nu_0)^{(8)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(8)} < (\nu_0)^{(8)}$

and analogously 374

$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)},\ \boldsymbol{if}\ (u_0)^{(8)} < (u_1)^{(8)}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}, \boldsymbol{if}\ (u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)},$

and $\boxed{(u_0)^{(8)} = \frac{T_{40}^0}{T_{41}^0}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}, \boldsymbol{if}\ (\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^0 e^{(S_1)^{(8)} t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}} G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}} G_{40}^0 e^{(S_1)^{(8)} t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)} G_{40}^0}{(m_1)^{(8)}((S_1)^{(8)} - (p_{40})^{(8)} - (S_2)^{(8)})}\left[e^{((S_1)^{(8)} - (p_{40})^{(8)})t} - e^{-(S_2)^{(8)} t}\right] + G_{42}^0 e^{-(S_2)^{(8)} t} \le G_{42}(t) \le \right. \quad 377$$

$$\left. \frac{(a_{42})^{(8)} G_{40}^0}{(m_2)^{(8)}((S_1)^{(8)} - (a'_{42})^{(8)})}\left[e^{(S_1)^{(8)} t} - e^{-(a'_{42})^{(8)} t}\right] + G_{42}^0 e^{-(a'_{42})^{(8)} t}\right)$$

$$\boxed{T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}} T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le \frac{1}{(\mu_2)^{(8)}} T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)} T_{40}^0}{(\mu_1)^{(8)}((R_1)^{(8)} - (b'_{42})^{(8)})}\left[e^{(R_1)^{(8)} t} - e^{-(b'_{42})^{(8)} t}\right] + T_{42}^0 e^{-(b'_{42})^{(8)} t} \le T_{42}(t) \le \quad 380$$

$$\frac{(a_{42})^{(8)} T_{40}^0}{(\mu_2)^{(8)}((R_1)^{(8)} + (r_{40})^{(8)} + (R_2)^{(8)})}\left[e^{((R_1)^{(8)} + (r_{40})^{(8)})t} - e^{-(R_2)^{(8)} t}\right] + T_{42}^0 e^{-(R_2)^{(8)} t}$$

**Definition of** $(S_1)^{(8)}, (S_2)^{(8)}, (R_1)^{(8)}, (R_2)^{(8)}$:- 381

Where $(S_1)^{(8)} = (a_{40})^{(8)}(m_2)^{(8)} - (a'_{40})^{(8)}$

$(S_2)^{(8)} = (a_{42})^{(8)} - (p_{42})^{(8)}$

$(R_1)^{(8)} = (b_{40})^{(8)}(\mu_2)^{(8)} - (b'_{40})^{(8)}$

$(R_2)^{(8)} = (b'_{42})^{(8)} - (r_{42})^{(8)}$

**Behavior of the solutions of equation 37 to 92** 382

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ :

$(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ four constants satisfying

$$-(\sigma_2)^{(9)} \le -(a'_{44})^{(9)} + (a'_{45})^{(9)} - (a''_{44})^{(9)}(T_{45}, t) + (a''_{45})^{(9)}(T_{45}, t) \le -(\sigma_1)^{(9)}$$

$$-(\tau_2)^{(9)} \le -(b'_{44})^{(9)} + (b'_{45})^{(9)} - (b''_{44})^{(9)}\big((G_{47}), t\big) - (b''_{45})^{(9)}\big((G_{47}), t\big) \le -(\tau_1)^{(9)}$$

**Definition of** $(\nu_1)^{(9)}, (\nu_2)^{(9)}, (u_1)^{(9)}, (u_2)^{(9)}, \nu^{(9)}, u^{(9)}$ :

By $(\nu_1)^{(9)} > 0$ , $(\nu_2)^{(9)} < 0$ and respectively $(u_1)^{(9)} > 0$ , $(u_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$ and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_1)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(9)}, , (\bar{\nu}_2)^{(9)}, (\bar{u}_1)^{(9)}, (\bar{u}_2)^{(9)}$ :

By $(\bar{\nu}_1)^{(9)} > 0$ , $(\bar{\nu}_2)^{(9)} < 0$ and respectively $(\bar{u}_1)^{(9)} > 0$ , $(\bar{u}_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$ and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_2)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$

**Definition of** $(m_1)^{(9)}, (m_2)^{(9)}, (\mu_1)^{(9)}, (\mu_2)^{(9)}, (\nu_0)^{(9)}$ :-

If we define $(m_1)^{(9)}, (m_2)^{(9)}, (\mu_1)^{(9)}, (\mu_2)^{(9)}$ by

$$(m_2)^{(9)} = (\nu_0)^{(9)}, (m_1)^{(9)} = (\nu_1)^{(9)}, \ \boldsymbol{if}\ (\nu_0)^{(9)} < (\nu_1)^{(9)}$$

$$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\bar{\nu}_1)^{(9)}, \boldsymbol{if}\ (\nu_1)^{(9)} < (\nu_0)^{(9)} < (\bar{\nu}_1)^{(9)},$$

and $\boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$

$$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\nu_0)^{(9)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(9)} < (\nu_0)^{(9)}$$

and analogously

$$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)}, \ \boldsymbol{if}\ (u_0)^{(9)} < (u_1)^{(9)}$$

$$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}, \boldsymbol{if}\ (u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)},$$

and $\boxed{(u_0)^{(9)} = \frac{T^0_{44}}{T^0_{45}}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}, \boldsymbol{if}\ (\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G^0_{44} e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{44}(t) \le G^0_{44} e^{(S_1)^{(9)} t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}} G_{44}^0 e^{\left((S_1)^{(9)}-(p_{44})^{(9)}\right)t} \le G_{45}(t) \le \frac{1}{(m_2)^{(9)}} G_{44}^0 e^{(S_1)^{(9)}t}$$

(

$$\frac{(a_{46})^{(9)} G_{44}^0}{(m_1)^{(9)}\left((S_1)^{(9)}-(p_{44})^{(9)}-(S_2)^{(9)}\right)}\left[e^{\left((S_1)^{(9)}-(p_{44})^{(9)}\right)t} - e^{-(S_2)^{(9)}t}\right] + G_{46}^0 e^{-(S_2)^{(9)}t} \le G_{46}(t) \le$$

$$\frac{(a_{46})^{(9)} G_{44}^0}{(m_2)^{(9)}\left((S_1)^{(9)}-(a'_{46})^{(9)}\right)}\left[e^{(S_1)^{(9)}t} - e^{-(a'_{46})^{(9)}t}\right] + G_{46}^0 e^{-(a'_{46})^{(9)}t})$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le T_{44}^0 e^{\left((R_1)^{(9)}+(r_{44})^{(9)}\right)t}}$$

$$\frac{1}{(\mu_1)^{(9)}} T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le \frac{1}{(\mu_2)^{(9)}} T_{44}^0 e^{\left((R_1)^{(9)}+(r_{44})^{(9)}\right)t}$$

$$\frac{(b_{46})^{(9)} T_{44}^0}{(\mu_1)^{(9)}\left((R_1)^{(9)}-(b'_{46})^{(9)}\right)}\left[e^{(R_1)^{(9)}t} - e^{-(b'_{46})^{(9)}t}\right] + T_{46}^0 e^{-(b'_{46})^{(9)}t} \le T_{46}(t) \le$$

$$\frac{(a_{46})^{(9)} T_{44}^0}{(\mu_2)^{(9)}\left((R_1)^{(9)}+(r_{44})^{(9)}+(R_2)^{(9)}\right)}\left[e^{\left((R_1)^{(9)}+(r_{44})^{(9)}\right)t} - e^{-(R_2)^{(9)}t}\right] + T_{46}^0 e^{-(R_2)^{(9)}t}$$

**Definition of** $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$

**Proof :** From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right) - (a''_{14})^{(1)}(T_{14}, t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

**Definition of** $v^{(1)}$ :- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)}\right) \le \frac{dv^{(1)}}{dt} \le -\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(1)}, (\nu_0)^{(1)}$ :-

For $0 < \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}} < (\nu_1)^{(1)} < (\bar{\nu}_1)^{(1)}$

$$\nu^{(1)}(t) \ge \frac{(\nu_1)^{(1)} + (C)^{(1)}(\nu_2)^{(1)} e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_0)^{(1)}\right)t\right]}}{1+(C)^{(1)} e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(\nu_1)^{(1)}-(\nu_0)^{(1)}}{(\nu_0)^{(1)}-(\nu_2)^{(1)}}}$$

it follows $(\nu_0)^{(1)} \leq \nu^{(1)}(t) \leq (\nu_1)^{(1)}$

In the same manner , we get 384

$$\nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{\nu}_1)^{(1)}-(\nu_0)^{(1)}}{(\nu_0)^{(1)}-(\bar{\nu}_2)^{(1)}}}$$

From which we deduce $(\nu_0)^{(1)} \leq \nu^{(1)}(t) \leq (\bar{\nu}_1)^{(1)}$

If $0 < (\nu_1)^{(1)} < (\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0} < (\bar{\nu}_1)^{(1)}$ we find like in the previous case, 385

$$(\nu_1)^{(1)} \leq \frac{(\nu_1)^{(1)}+(C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}} \leq \nu^{(1)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\bar{\nu}_1)^{(1)}$$

If $0 < (\nu_1)^{(1)} \leq (\bar{\nu}_1)^{(1)} \leq \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$ , we obtain 386

$$(\nu_1)^{(1)} \leq \nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\nu_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(1)}(t)$ :-

$$(m_2)^{(1)} \leq \nu^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{\nu^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(1)}(t)$ :-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{13})^{(1)} = (a''_{14})^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)}G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)}T_{14}(t)$ This is an important

consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

**Proof** : From global equations we obtain 387

$$\frac{d\nu^{(2)}}{dt} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right) - (a''_{17})^{(2)}(T_{17}, t)\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

**Definition of** $\nu^{(2)}$ :- $\boxed{\nu^{(2)} = \frac{G_{16}}{G_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \le \frac{d\nu^{(2)}}{dt} \le -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

**Definition of** $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$ :-

For $0 < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \ge \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}}{1 + (C)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(C)^{(2)} = \frac{(\nu_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \le \nu^{(2)}(t) \le (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \le \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \le \nu^{(2)}(t) \le (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \le \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}}{1 + (C)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}} \le \nu^{(2)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \le (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \le (\bar{\nu}_1)^{(2)} \le (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$ , we obtain 394

$$(\nu_1)^{(2)} \le \nu^{(2)}(t) \le \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \le (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(2)}(t)$ :- 395

$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}$, $\boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$

In a completely analogous way, we obtain 396

**Definition of** $u^{(2)}(t)$ :-

$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}$, $\boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :** 397

If $(a_{16}'')^{(2)} = (a_{17}'')^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)} G_{17}(t)$

Analogously if $(b_{16}'')^{(2)} = (b_{17}'')^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)} T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

**Proof** : From global equations we obtain 398

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a_{20}')^{(3)} - (a_{21}')^{(3)} + (a_{20}'')^{(3)}(T_{21}, t)\right) - (a_{21}'')^{(3)}(T_{21}, t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

**Definition of** $\nu^{(3)}$ :- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right) \leq \frac{d\nu^{(3)}}{dt} \leq -\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right)$$

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From which one obtains

For $0 < (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (\nu_1)^{(3)} < (\bar{\nu}_1)^{(3)}$

$$\nu^{(3)}(t) \geq \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(\nu_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\nu_2)^{(3)}}}$$

it follows $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\nu_1)^{(3)}$

In the same manner , we get 401

$$\nu^{(3)}(t) \le \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{\nu}_1)^{(3)}-(\nu_0)^{(3)}}{(\nu_0)^{(3)}-(\bar{\nu}_2)^{(3)}}}$$

**Definition of** $(\bar{\nu}_1)^{(3)}$ :-

From which we deduce $(\nu_0)^{(3)} \le \nu^{(3)}(t) \le (\bar{\nu}_1)^{(3)}$

If $0 < (\nu_1)^{(3)} < (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (\bar{\nu}_1)^{(3)}$ we find like in the previous case, 402

$$(\nu_1)^{(3)} \le \frac{(\nu_1)^{(3)}+(C)^{(3)}(\nu_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}} \le \nu^{(3)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \le (\bar{\nu}_1)^{(3)}$$

If $0 < (\nu_1)^{(3)} \le (\bar{\nu}_1)^{(3)} \le (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}$ , we obtain 403

$$(\nu_1)^{(3)} \le \nu^{(3)}(t) \le \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \le (\nu_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(3)}(t)$ :-

$$(m_2)^{(3)} \le \nu^{(3)}(t) \le (m_1)^{(3)}, \quad \boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(3)}(t)$ :-

$$(\mu_2)^{(3)} \le u^{(3)}(t) \le (\mu_1)^{(3)}, \quad \boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{20}'')^{(3)} = (a_{21}'')^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)}G_{21}(t)$

Analogously if $(b_{20}'')^{(3)} = (b_{21}'')^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)}T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

**Proof** : From global equations we obtain 404

$$\frac{dv^{(4)}}{dt} = (a_{24})^{(4)} - \left((a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right) - (a''_{25})^{(4)}(T_{25},t)v^{(4)} - (a_{25})^{(4)}v^{(4)}$$

**Definition of** $v^{(4)}$ :- $\boxed{v^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)}\right) \le \frac{dv^{(4)}}{dt} \le -\left((a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_4)^{(4)}v^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$ :-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \ge \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \le \nu^{(4)}(t) \le (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \le \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \le \nu^{(4)}(t) \le (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \le \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}}{1 + (C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}} \le \nu^{(4)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \le (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \le (\bar{\nu}_1)^{(4)} \le \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$ , we obtain 407

$$(\nu_1)^{(4)} \le \nu^{(4)}(t) \le \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \le (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(4)}(t)$ :-

$$(m_2)^{(4)} \le \nu^{(4)}(t) \le (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(4)}(t)$ :-

$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}$, $\boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{24})^{(4)} = (a''_{25})^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)} G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b''_{24})^{(4)} = (b''_{25})^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)} T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

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**Proof** : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a'_{28})^{(5)} - (a'_{29})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right) - (a''_{29})^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

**Definition of** $\nu^{(5)}$ :- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$ :-

For $0 < \boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \le \frac{(\nu_1)^{(5)}+(C)^{(5)}(\nu_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)}-(\nu_2)^{(5)}\right)t\right]}}{1+(C)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)}-(\nu_2)^{(5)}\right)t\right]}} \le \nu^{(5)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \le (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \le (\bar{\nu}_1)^{(5)} \le \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain 411

$$(\nu_1)^{(5)} \le \nu^{(5)}(t) \le \frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \le (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(5)}(t)$ :-

$(m_2)^{(5)} \le \nu^{(5)}(t) \le (m_1)^{(5)}$, $\boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$

In a completely analogous way, we obtain

**Definition of** $u^{(5)}(t)$ :-

$(\mu_2)^{(5)} \le u^{(5)}(t) \le (\mu_1)^{(5)}$, $\boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a_{28}'')^{(5)} = (a_{29}'')^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)}G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b_{28}'')^{(5)} = (b_{29}'')^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)}T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

**Proof** : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a_{32}')^{(6)} - (a_{33}')^{(6)} + (a_{32}'')^{(6)}(T_{33},t)\right) - (a_{33}'')^{(6)}(T_{33},t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

**Definition of** $\nu^{(6)}$ :- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \le \frac{d\nu^{(6)}}{dt} \le -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$ :-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \geq \frac{(\nu_1)^{(6)}+(C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_0)^{(6)}\right)t\right]}}{1+(C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)}-(\nu_0)^{(6)}}{(\nu_0)^{(6)}-(\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\nu_1)^{(6)}$

In the same manner , we get 413

$$\nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{\nu}_1)^{(6)}-(\nu_0)^{(6)}}{(\nu_0)^{(6)}-(\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\bar{\nu}_1)^{(6)}$

If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 414

$$(\nu_1)^{(6)} \leq \frac{(\nu_1)^{(6)}+(C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_2)^{(6)}\right)t\right]}}{1+(C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_2)^{(6)}\right)t\right]}} \leq \nu^{(6)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\bar{\nu}_1)^{(6)}$$

If $0 < (\nu_1)^{(6)} \leq (\bar{\nu}_1)^{(6)} \leq \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$ , we obtain 415

$$(\nu_1)^{(6)} \leq \nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(6)}(t)$ :-

$$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}, \quad \boxed{\nu^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(6)}(t)$ :-

$$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}, \quad \boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)}G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)}T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

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**Proof** : From global equations we obtain

$$\frac{dv^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right) - (a''_{37})^{(7)}(T_{37},t)v^{(7)} - (a_{37})^{(7)}v^{(7)}$$

**Definition of** $v^{(7)}$ :- $\boxed{v^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_2)^{(7)}v^{(7)} - (a_{36})^{(7)}\right) \le \frac{dv^{(7)}}{dt} \le -\left((a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_1)^{(7)}v^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

**Definition of** $(\bar{v}_1)^{(7)}, (v_0)^{(7)}$ :-

For $0 < \boxed{(v_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}} < (v_1)^{(7)} < (\bar{v}_1)^{(7)}$

$$v^{(7)}(t) \ge \frac{(v_1)^{(7)} + (C)^{(7)}(v_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_0)^{(7)}\right)t\right]}}{1 + (C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(v_1)^{(7)} - (v_0)^{(7)}}{(v_0)^{(7)} - (v_2)^{(7)}}}$$

it follows $(v_0)^{(7)} \le v^{(7)}(t) \le (v_1)^{(7)}$

In the same manner , we get 417

$$v^{(7)}(t) \le \frac{(\bar{v}_1)^{(7)} + (\bar{C})^{(7)}(\bar{v}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{v}_1)^{(7)} - (v_0)^{(7)}}{(v_0)^{(7)} - (\bar{v}_2)^{(7)}}}$$

From which we deduce $(v_0)^{(7)} \le v^{(7)}(t) \le (\bar{v}_1)^{(7)}$

If $0 < (v_1)^{(7)} < (v_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}} < (\bar{v}_1)^{(7)}$ we find like in the previous case, 418

$$(v_1)^{(7)} \le \frac{(v_1)^{(7)} + (C)^{(7)}(v_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_2)^{(7)}\right)t\right]}}{1 + (C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_2)^{(7)}\right)t\right]}} \le v^{(7)}(t) \le$$

$$\frac{(\bar{v}_1)^{(7)} + (\bar{C})^{(7)}(\bar{v}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}} \le (\bar{v}_1)^{(7)}$$

If $0 < (v_1)^{(7)} \le (\bar{v}_1)^{(7)} \le \boxed{(v_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$ , we obtain 419

$$(v_1)^{(7)} \le v^{(7)}(t) \le \frac{(\bar{v}_1)^{(7)} + (\bar{C})^{(7)}(\bar{v}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}} \le (v_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $v^{(7)}(t)$ :-

$$(m_2)^{(7)} \leq v^{(7)}(t) \leq (m_1)^{(7)}, \quad \boxed{v^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(7)}(t)$ :- 420

$$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}, \quad \boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $v^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)} G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)} T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

**Proof** : From global equations we obtain 421

$$\frac{dv^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right) - (a''_{41})^{(8)}(T_{41}, t) v^{(8)} - (a_{41})^{(8)} v^{(8)}$$

**Definition of** $v^{(8)}$ :- $\boxed{v^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_2)^{(8)} v^{(8)} - (a_{40})^{(8)}\right) \leq \frac{dv^{(8)}}{dt} \leq -\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_1)^{(8)} v^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$ :-

$$\text{For } 0 < \boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$$

$$v^{(8)}(t) \geq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)}-(\nu_0)^{(8)}}{(\nu_0)^{(8)}-(\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \leq \frac{(\nu_1)^{(8)}+(C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}}{1+(C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}} \leq \nu^{(8)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \leq (\bar{\nu}_1)^{(8)} \leq \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$ , we obtain 424

$$(\nu_1)^{(8)} \leq \nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(8)}(t)$ :-

$$(m_2)^{(8)} \leq \nu^{(8)}(t) \leq (m_1)^{(8)}, \quad \boxed{\nu^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(8)}(t)$ :-

$$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}, \quad \boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $\nu^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)}G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then $(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)}T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

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**Proof** : From 99,20,44,22,23,44 we obtain A

$$\frac{dv^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right) - (a''_{45})^{(9)}(T_{45},t)v^{(9)} - (a_{45})^{(9)}v^{(9)}$$

**Definition of** $v^{(9)}$ :- $\boxed{v^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)}\right) \le \frac{dv^{(9)}}{dt} \le -\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_1)^{(9)}v^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$ :-

For $0 < \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$

$$v^{(9)}(t) \ge \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \le v^{(9)}(t) \le (\nu_9)^{(9)}$

In the same manner , we get

$$v^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \le v^{(9)}(t) \le (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \le \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}}{1 + (C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}} \le v^{(9)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \le (\bar{\nu}_1)^{(9)} \le \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$ , we obtain

$$(\nu_1)^{(9)} \le v^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $v^{(9)}(t)$ :-

$$(m_2)^{(9)} \le v^{(9)}(t) \le (m_1)^{(9)}, \quad \boxed{v^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(9)}(t)$ :-

$(\mu_2)^{(9)} \leq u^{(9)}(t) \leq (\mu_1)^{(9)}$, $\boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)} G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then
$(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)} T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following 425

**Theorem :** If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on $t$ , and the conditions with the notations

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0$ ,

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$

$with\ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a''_i)^{(2)} and\ (b''_i)^{(2)}$ are independent on t , and the conditions with the notations 426

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$ 427

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 428

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$ , 429

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 430

$with\ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a''_i)^{(3)} and\ (b''_i)^{(3)}$ are independent on $t$ , and the conditions with the notations 431

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$ ,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

**Theorem :** If $(a_i'')^{(4)} and\ (b_i'')^{(4)}$ are independent on $t$ , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$ ,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(5)} and\ (b_i'')^{(5)}$ are independent on $t$ , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$ ,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

**Theorem** If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on $t$ , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$ ,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(7)} and\ (b_i'')^{(7)}$ are independent on $t$ , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$ ,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(8)} and\ (b_i'')^{(8)}$ are independent on $t$ , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$ ,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(9)} and\ (b_i'')^{(9)}$ are independent on $t$ , and the conditions (with the notations 436 A
45,46,27,28)

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$ ,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0$ 437

$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0$ 438

$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0$ 439

$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0$ 440

$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ]T_{14} = 0$ 441

$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0$ 442

has a unique positive solution , which is an equilibrium solution for the system

$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0$ 443

$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0$ 444

$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0$ 445

$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\ ]T_{16} = 0$ 446

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\,]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\,]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\,]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\,]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\,]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\big((G_{27})\big)\,]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\big((G_{27})\big)\,]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\big((G_{27})\big)\,]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})\,]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})\,]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})\,]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})\,]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})\,]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})\,]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0 \quad 479$$

$$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0 \quad 480$$

$$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0 \quad 481$$

$$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0 \quad 482$$

$$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0 \quad 483$$

$$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0 \quad 484$$

$$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0 \quad 484\text{A}$$

$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$

$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$

$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$

$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$

$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$

**Proof:** 485

(a) Indeed the first two equations have a nontrivial solution $G_{13}, G_{14}$ if

$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$

**Proof:** 486

(cc) Indeed the first two equations have a nontrivial solution $G_{16}, G_{17}$ if

$\mathrm{F}(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$

**Proof:** 487

(a) Indeed the first two equations have a nontrivial solution $G_{20}, G_{21}$ if

$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$

**Proof:** 488

(a) Indeed the first two equations have a nontrivial solution $G_{24}, G_{25}$ if

$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$

**Proof:** 489

(a) Indeed the first two equations have a nontrivial solution $G_{28}, G_{29}$ if

$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$

**Proof:** 490

(a) Indeed the first two equations have a nontrivial solution $G_{32}, G_{33}$ if

$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) +$

$(a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$

**Proof:** 491

(a) Indeed the first two equations have a nontrivial solution $G_{36}, G_{37}$ if

$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$

**Proof:** 492

(a) Indeed the first two equations have a nontrivial solution $G_{40}, G_{41}$ if

$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$

**Proof:** 492 A

(a) Indeed the first two equations have a nontrivial solution $G_{44}, G_{45}$ if

$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$

**Definition and uniqueness of** $T^*_{14}$ :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique $T^*_{14}$ for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

**Definition and uniqueness of** $T^*_{17}$ :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique $T^*_{17}$ for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]}$$ 495

**Definition and uniqueness of** $T^*_{21}$ :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique $T^*_{21}$ for which $f(T^*_{21}) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

**Definition and uniqueness of** $T^*_{25}$ :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that

there exists a unique $T_{25}^{*}$ for which $f(T_{25}^{*})=0$. With this value , we obtain from the three first equations

$$G_{24}=\frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}=\frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T_{25}^{*})]}$$

**Definition and uniqueness of** $T_{29}^{*}$ :- 498

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(5)}(T_{29})$ being increasing, it follows that there exists a unique $T_{29}^{*}$ for which $f(T_{29}^{*})=0$. With this value , we obtain from the three first equations

$$G_{28}=\frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}=\frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T_{29}^{*})]}$$

**Definition and uniqueness of** $T_{33}^{*}$ :- 499

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(6)}(T_{33})$ being increasing, it follows that there exists a unique $T_{33}^{*}$ for which $f(T_{33}^{*})=0$. With this value , we obtain from the three first equations

$$G_{32}=\frac{(a_{32})^{(6)}G_{33}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^{*})]} \quad , \quad G_{34}=\frac{(a_{34})^{(6)}G_{33}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^{*})]}$$

**Definition and uniqueness of** $T_{37}^{*}$ :- 500

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(7)}(T_{37})$ being increasing, it follows that there exists a unique $T_{37}^{*}$ for which $f(T_{37}^{*})=0$. With this value , we obtain from the three first equations

$$G_{36}=\frac{(a_{36})^{(7)}G_{37}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^{*})]} \quad , \quad G_{38}=\frac{(a_{38})^{(7)}G_{37}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^{*})]}$$

**Definition and uniqueness of** $T_{41}^{*}$ :- 501

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique $T_{41}^{*}$ for which $f(T_{41}^{*})=0$. With this value , we obtain from the three first equations

$$G_{40}=\frac{(a_{40})^{(8)}G_{41}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T_{41}^{*})]} \quad , \quad G_{42}=\frac{(a_{42})^{(8)}G_{41}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T_{41}^{*})]}$$

**Definition and uniqueness of** $T_{45}^{*}$ :- 501 A

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(9)}(T_{45})$ being increasing, it follows that there exists a unique $T_{45}^{*}$ for which $f(T_{45}^{*})=0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)}G_{45}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T^*_{45})]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)}G_{45}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T^*_{45})]}$$

By the same argument, the equations admit solutions $G_{13}, G_{14}$ if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right]+(b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{14}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{14}$ such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions $G_{16}, G_{17}$ if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right]+(b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ 504
is a decreasing function in $G_{17}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{14}$ such that $\varphi((G_{19})^*) = 0$

By the same argument, the equations admit solutions $G_{20}, G_{21}$ if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right]+(b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{21}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{21}$ such that $\varphi((G_{23})^*) = 0$

By the same argument, the equations admit solutions $G_{24}, G_{25}$ if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right]+(b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{25}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{25}$ such that $\varphi((G_{27})^*) = 0$

By the same argument, the equations admit solutions $G_{28}, G_{29}$ if 507

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right]+(b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{29}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that

there exists a unique $G_{29}^{*}$ such that $\varphi((G_{31})^{*}) = 0$

By the same argument, the equations admit solutions $G_{32}, G_{33}$ if 508
$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$

$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right] + (b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{33}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{33}^{*}$ such that $\varphi(G_{35}{}^{*}) = 0$

By the same argument, the equations admit solutions $G_{36}, G_{37}$ if 509
$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$
$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right] + (b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{37}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{37}^{*}$ such that $\varphi(G_{39}{}^{*}) = 0$

By the same argument, the equations admit solutions $G_{40}, G_{41}$ if 510
$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$
$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right] + (b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{41}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{41}^{*}$ such that $\varphi(G_{43}{}^{*}) = 0$

By the same argument, the equations 92,93 admit solutions $G_{44}, G_{45}$ if
$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$

$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right] + (b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{45}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{45}^{*}$ such that $\varphi((G_{47})^{*}) = 0$

Finally we obtain the unique solution 511

$G_{14}^{*}$ given by $\varphi(G^{*}) = 0$ , $T_{14}^{*}$ given by $f(T_{14}^{*}) = 0$ and

$$G_{13}^{*} = \frac{(a_{13})^{(1)}G_{14}^{*}}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^{*})]} \quad , \quad G_{15}^{*} = \frac{(a_{15})^{(1)}G_{14}^{*}}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^{*})]}$$

$$T_{13}^{*} = \frac{(b_{13})^{(1)}T_{14}^{*}}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^{*})]} \quad , \quad T_{15}^{*} = \frac{(b_{15})^{(1)}T_{14}^{*}}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^{*})]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

$G_{17}^{*}$ given by $\varphi((G_{19})^{*}) = 0$ , $T_{17}^{*}$ given by $f(T_{17}^{*}) = 0$ and 512

$$G_{16}^{*} = \frac{(a_{16})^{(2)} G_{17}^{*}}{[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}^{*})]} \quad , \quad G_{18}^{*} = \frac{(a_{18})^{(2)} G_{17}^{*}}{[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}^{*})]} \qquad 513$$

$$T_{16}^{*} = \frac{(b_{16})^{(2)} T_{17}^{*}}{[(b'_{16})^{(2)} - (b''_{16})^{(2)}((G_{19})^{*})]} \quad , \quad T_{18}^{*} = \frac{(b_{18})^{(2)} T_{17}^{*}}{[(b'_{18})^{(2)} - (b''_{18})^{(2)}((G_{19})^{*})]} \qquad 514$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

$G_{21}^{*}$ given by $\varphi((G_{23})^{*}) = 0$ , $T_{21}^{*}$ given by $f(T_{21}^{*}) = 0$ and

$$G_{20}^{*} = \frac{(a_{20})^{(3)} G_{21}^{*}}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^{*})]} \quad , \quad G_{22}^{*} = \frac{(a_{22})^{(3)} G_{21}^{*}}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^{*})]}$$

$$T_{20}^{*} = \frac{(b_{20})^{(3)} T_{21}^{*}}{[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}{}^{*})]} \quad , \quad T_{22}^{*} = \frac{(b_{22})^{(3)} T_{21}^{*}}{[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}{}^{*})]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

$G_{25}^{*}$ given by $\varphi(G_{27}) = 0$ , $T_{25}^{*}$ given by $f(T_{25}^{*}) = 0$ and

$$G_{24}^{*} = \frac{(a_{24})^{(4)} G_{25}^{*}}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}^{*} = \frac{(a_{26})^{(4)} G_{25}^{*}}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^{*})]}$$

$$T_{24}^{*} = \frac{(b_{24})^{(4)} T_{25}^{*}}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^{*})]} \quad , \quad T_{26}^{*} = \frac{(b_{26})^{(4)} T_{25}^{*}}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^{*})]} \qquad 517$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

$G_{29}^{*}$ given by $\varphi((G_{31})^{*}) = 0$ , $T_{29}^{*}$ given by $f(T_{29}^{*}) = 0$ and

$$G_{28}^{*} = \frac{(a_{28})^{(5)} G_{29}^{*}}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}^{*} = \frac{(a_{30})^{(5)} G_{29}^{*}}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^{*})]}$$

$$T_{28}^{*} = \frac{(b_{28})^{(5)} T_{29}^{*}}{[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31})^{*})]} \quad , \quad T_{30}^{*} = \frac{(b_{30})^{(5)} T_{29}^{*}}{[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31})^{*})]} \qquad 519$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

$G_{33}^{*}$ given by $\varphi((G_{35})^{*}) = 0$ , $T_{33}^{*}$ given by $f(T_{33}^{*}) = 0$ and

$$G_{32}^{*} = \frac{(a_{32})^{(6)} G_{33}^{*}}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^{*})]} \quad , \quad G_{34}^{*} = \frac{(a_{34})^{(6)} G_{33}^{*}}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^{*})]}$$

$$T_{32}^{*} = \frac{(b_{32})^{(6)} T_{33}^{*}}{[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35})^{*})]} \quad , \quad T_{34}^{*} = \frac{(b_{34})^{(6)} T_{33}^{*}}{[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35})^{*})]} \qquad 521$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

$G_{37}^*$ given by $\varphi((G_{39})^*) = 0$ , $T_{37}^*$ given by $f(T_{37}^*) = 0$ and

$$G_{36}^* = \frac{(a_{36})^{(7)} G_{37}^*}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^*)]} \quad , \quad G_{38}^* = \frac{(a_{38})^{(7)} G_{37}^*}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^*)]}$$

$$T_{36}^* = \frac{(b_{36})^{(7)} T_{37}^*}{[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39})^*)]} \quad , \quad T_{38}^* = \frac{(b_{38})^{(7)} T_{37}^*}{[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39})^*)]}$$

Finally we obtain the unique solution 523

$G_{41}^*$ given by $\varphi((G_{43})^*) = 0$ , $T_{41}^*$ given by $f(T_{41}^*) = 0$ and

$$G_{40}^* = \frac{(a_{40})^{(8)} G_{41}^*}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42}^* = \frac{(a_{42})^{(8)} G_{41}^*}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

$$T_{40}^* = \frac{(b_{40})^{(8)} T_{41}^*}{[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43})^*)]} \quad , \quad T_{42}^* = \frac{(b_{42})^{(8)} T_{41}^*}{[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43})^*)]}$$

Finally we obtain the unique solution of 89 to 99 523 A

$G_{45}^*$ given by $\varphi((G_{47})^*) = 0$ , $T_{45}^*$ given by $f(T_{45}^*) = 0$ and

$$G_{44}^* = \frac{(a_{44})^{(9)} G_{45}^*}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46}^* = \frac{(a_{46})^{(9)} G_{45}^*}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

$$T_{44}^* = \frac{(b_{44})^{(9)} T_{45}^*}{[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47})^*)]} \quad , \quad T_{46}^* = \frac{(b_{46})^{(9)} T_{45}^*}{[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47})^*)]}$$

## ASYMPTOTIC STABILITY ANALYSIS 524

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

**Proof:** Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a''_{14})^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \frac{\partial (b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a'_{13})^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)}G^*_{13}\mathbb{T}_{14} \quad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a'_{14})^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)}G^*_{14}\mathbb{T}_{14} \quad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a'_{15})^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)}G^*_{15}\mathbb{T}_{14} \quad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b'_{13})^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)}T^*_{13}\mathbb{G}_j\right) \quad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b'_{14})^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)}T^*_{14}\mathbb{G}_j\right) \quad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b'_{15})^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T^*_{15}\mathbb{G}_j\right) \quad 530$$

**ASYMPTOTIC STABILITY ANALYSIS** 531

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(2)}$ and $(b''_i)^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G^*_i + \mathbb{G}_i \quad , T_i = T^*_i + \mathbb{T}_i \quad 532$$

$$\frac{\partial(a''_{17})^{(2)}}{\partial T_{17}}(T^*_{17}) = (q_{17})^{(2)} \quad , \frac{\partial(b''_i)^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \quad 533$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a'_{16})^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G^*_{16}\mathbb{T}_{17} \quad 534$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a'_{17})^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G^*_{17}\mathbb{T}_{17} \quad 535$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a'_{18})^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G^*_{18}\mathbb{T}_{17} \quad 536$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b'_{16})^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T^*_{16}\mathbb{G}_j\right) \quad 537$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b'_{17})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}T^*_{17}\mathbb{G}_j\right) \quad 538$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b'_{18})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}T^*_{18}\mathbb{G}_j\right) \quad 539$$

**ASYMPTOTIC STABILITY ANALYSIS** 540

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(3)}$ *and* $(b''_i)^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{21}'')^{(3)}}{\partial T_{21}}(T_{21}^*) = (q_{21})^{(3)} \quad , \frac{\partial (b_i'')^{(3)}}{\partial G_j}(\,(G_{23})^*\,) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\big((a_{20}')^{(3)} + (p_{20})^{(3)}\big)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G_{20}^*\mathbb{T}_{21} \quad 541$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\big((a_{21}')^{(3)} + (p_{21})^{(3)}\big)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G_{21}^*\mathbb{T}_{21} \quad 542$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\big((a_{22}')^{(3)} + (p_{22})^{(3)}\big)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G_{22}^*\mathbb{T}_{21} \quad 543$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\big((b_{20}')^{(3)} - (r_{20})^{(3)}\big)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\big(s_{(20)(j)}T_{20}^*\mathbb{G}_j\big) \quad 544$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\big((b_{21}')^{(3)} - (r_{21})^{(3)}\big)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\big(s_{(21)(j)}T_{21}^*\mathbb{G}_j\big) \quad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\big((b_{22}')^{(3)} - (r_{22})^{(3)}\big)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\big(s_{(22)(j)}T_{22}^*\mathbb{G}_j\big) \quad 546$$

**ASYMPTOTIC STABILITY ANALYSIS** 547

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}\ and\ (b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{25}'')^{(4)}}{\partial T_{25}}(T_{25}^*) = (q_{25})^{(4)} \quad , \frac{\partial (b_i'')^{(4)}}{\partial G_j}((G_{27})^*\,) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\big((a_{24}')^{(4)} + (p_{24})^{(4)}\big)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G_{24}^*\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\big((a_{25}')^{(4)} + (p_{25})^{(4)}\big)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G_{25}^*\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\big((a_{26}')^{(4)} + (p_{26})^{(4)}\big)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G_{26}^*\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\big((b_{24}')^{(4)} - (r_{24})^{(4)}\big)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\big(s_{(24)(j)}T_{24}^*\mathbb{G}_j\big) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\big((b_{25}')^{(4)} - (r_{25})^{(4)}\big)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\big(s_{(25)(j)}T_{25}^*\mathbb{G}_j\big) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T^*_{26}\mathbb{G}_j\right) \quad 554$$

**ASYMPTOTIC STABILITY ANALYSIS** 555

**Theorem 5:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}\ and\ (b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 556

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{29})^{(5)}}{\partial T_{29}}(T^*_{29}) = (q_{29})^{(5)} \ , \ \frac{\partial(b_i'')^{(5)}}{\partial G_j}\left((G_{31})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a'_{28})^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G^*_{28}\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a'_{29})^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G^*_{29}\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a'_{30})^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G^*_{30}\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b'_{28})^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T^*_{28}\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b'_{29})^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T^*_{29}\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b'_{30})^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T^*_{30}\mathbb{G}_j\right) \quad 562$$

**ASYMPTOTIC STABILITY ANALYSIS** 563

**Theorem 6:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}\ and\ (b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{33})^{(6)}}{\partial T_{33}}(T^*_{33}) = (q_{33})^{(6)} \ , \ \frac{\partial(b_i'')^{(6)}}{\partial G_j}\left((G_{35})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a'_{32})^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G^*_{32}\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a'_{33})^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G^*_{33}\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a'_{34})^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G^*_{34}\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b'_{32})^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T^*_{32}\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b'_{33})^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T^*_{33}\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b'_{34})^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T^*_{34}\mathbb{G}_j\right) \quad 570$$

**ASYMPTOTIC STABILITY ANALYSIS** 571

**Theorem 7:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 572

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{37})^{(7)}}{\partial T_{37}}(T^*_{37}) = (q_{37})^{(7)} \quad , \frac{\partial(b_i'')^{(7)}}{\partial G_j}\left((G_{39})^{**}\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a'_{36})^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G^*_{36}\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a'_{37})^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G^*_{37}\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a'_{38})^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G^*_{38}\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b'_{36})^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T^*_{36}\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b'_{37})^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T^*_{37}\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b'_{38})^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T^*_{38}\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

**ASYMPTOTIC STABILITY ANALYSIS**

**Theorem 8:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{41}'')^{(8)}}{\partial T_{41}}(T_{41}^*) = (q_{41})^{(8)} \quad , \frac{\partial (b_i'')^{(8)}}{\partial G_j}\left( (G_{43})^* \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a_{40}')^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G_{40}^*\mathbb{T}_{41} \quad (581)$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a_{41}')^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G_{41}^*\mathbb{T}_{41} \quad (582)$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a_{42}')^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G_{42}^*\mathbb{T}_{41} \quad (583)$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b_{40}')^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T_{40}^*\mathbb{G}_j\right) \quad (584)$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b_{41}')^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T_{41}^*\mathbb{G}_j\right) \quad (585)$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b_{42}')^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T_{42}^*\mathbb{G}_j\right) \quad (586)$$

(586 A)

**ASYMPTOTIC STABILITY ANALYSIS**

**Theorem 9:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}$ *and* $(b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.
**Proof:** Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{45}'')^{(9)}}{\partial T_{45}}(T_{45}^*) = (q_{45})^{(9)} \quad , \frac{\partial (b_i'')^{(9)}}{\partial G_j}\left( (G_{47})^* \right) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a_{44}')^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)}G_{44}^*\mathbb{T}_{45} \quad (586\,B)$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a_{45}')^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)}G_{45}^*\mathbb{T}_{45} \quad (586\,C)$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a_{46}')^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)}G_{46}^*\mathbb{T}_{45} \quad (586\,D)$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b_{44}')^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)}T_{44}^*\mathbb{G}_j\right) \quad (586\,E)$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b_{45}')^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)}T_{45}^*\mathbb{G}_j\right) \quad (586\,F)$$

$\frac{d\mathbb{T}_{46}}{dt} = -\left((b'_{46})^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T^*_{46}\mathbb{G}_j\right)$ 586

G

587

**The characteristic equation of this system is**

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(q_{13})^{(1)}G^*_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)}T^*_{14} + (b_{14})^{(1)}s_{(13),(14)}T^*_{14}\right)$$

$$+\left(\left((\lambda)^{(1)} + (a'_{14})^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)}G^*_{13} + (a_{13})^{(1)}(q_{14})^{(1)}G^*_{14}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)}T^*_{14} + (b_{14})^{(1)}s_{(13),(13)}T^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((b'_{13})^{(1)} + (b'_{14})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(15)}T^*_{14} + (b_{14})^{(1)}s_{(13),(15)}T^*_{13}\right)\} = 0$$

$$+$$

$$(\lambda)^{(2)} + (b'_{18})^{(2)} - (r_{18})^{(2)}\{\left((\lambda)^{(2)} + (a'_{18})^{(2)} + (p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^*_{17} + (a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^*_{17}\right)$$

$$+\left(\left((\lambda)^{(2)} + (a'_{17})^{(2)} + (p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^*_{16} + (a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}\right)$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2 + \left((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2 + \left((b'_{16})^{(2)} + (b'_{17})^{(2)} - (r_{16})^{(2)} + (r_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$+\left(\left((\lambda)^{(2)}\right)^2 + \left((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17} + (a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^{*}_{16}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\right)\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left(\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)\right]$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(21)}T^{*}_{21}\right)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^{*}_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^{*}_{21}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(20)}T^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$+\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(22)}T^{*}_{20}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\right)\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left(\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)\right]$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(25)}T^{*}_{25}\right)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^{*}_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^{*}_{25}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(24)}T^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$+\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(26)}T^{*}_{24}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\right)\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(29)}T^{*}_{29}\right)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^{*}_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^{*}_{29}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(28)}T^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$+\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(30)}T^{*}_{28}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\right)\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left(\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)\right]$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(33)}T^{*}_{33}\right)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^{*}_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^{*}_{33}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(32)}T^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$+\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(34)}T^{*}_{32}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\right)\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)\right]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(37)}T^{*}_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^{*}_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^{*}_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(36)}T^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$+\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(38)}T^{*}_{36}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\right)\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(41)}T^{*}_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^{*}_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^{*}_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(40)}T^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$+\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(42)}T^{*}_{40}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\right)\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(45)}T^{*}_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^{*}_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^{*}_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(44)}T^{*}_{44}\right)$$

$$\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\Big)$$

$$\left((\lambda)^{(9)}\right)^{2}+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\Big)$$

$$+\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(46)}T^{*}_{44}\right)\}=0$$

**And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.**

## SECTION THIRTY TWO

## Advanced Waves And Syntropy

### INTRODUCTION—VARIABLES USED

**Source: Wikipedia: Time line**

**Luigi Fantappiè: advanced waves and syntropy - 1941**.

(1) Fantappiè (1991) starts from the d'Alembert operator which in Klein-Gordon's relativistic generalization of Schrödinger's wave equation has (e) two wave solutions: retarded waves which diverge from (e) the past to the future and anticipated waves which diverge from the future to (e) the past, and which for us, moving forward in time, correspond to (e&eb) converging waves, attractors/absorbers.

(2) Studying the mathematical properties of these waves Fantappiè discovered that the diverging waves are governed by (e&eb) the law of entropy, whereas (e) the converging waves are governed by(e&eb) a symmetrical law, which concentrates (e&eb)energy, produces (eb) differentiation, structures and order, and which Fantappiè named syntropy.

(3) Fantappiè recognized the properties of the law of syntropy in the living systems, and concluded that living systems are a consequence of (e) anticipated waves.

**(4)** Starting from the mathematical properties of the laws of syntropy and entropy, Fantappiè arrived to the formulation of a model of consciousness based on (e) the following elements:

(5) **Free will:** this is generally considered a fundamental element of (e) consciousness, is seen as the consequence of (e) a permanent state of choice between information coming from the past and (e&eb) emotions coming from the future. **Antonella Vannini Quantum Biosystems 2008, 2, 165-184 167**

**(6)** **Feeling of life**: another basic component of consciousness is, according to Fantappiè, a direct consequence of (e) waves moving backward in time, from the future to (e&eb) the past. Fantappiè says that when physical senses vanish, as in deep meditation, (eb) people experience states of consciousness in which past, present and future coexist. The coexistence of past, present and future would be a direct consequence of (e) the coexistence of advanced and retarded waves.

**(7)** **Non local memory**: Fantappiè suggests the existence of non local correlations in the universe, as a consequence of (e) the existence of advanced waves. For example, in living systems, memory could be (=) a manifestation of non local links with (e&eb) past events which according to Klein-Gordon's equation still exist. Fantappiè's model is similar to the transactional interpretation of quantum mechanics and to the model of consciousness suggested by **Chris King (1989).**

## NOTATION

### Module One

(i) Fantappiè (1991) starts from the d'Alembert operator which in Klein-Gordon's relativistic generalization of Schrödinger's wave equation has (e) two wave solutions:

(ii) retarded waves which diverge from (e) the past to the future and anticipated waves which diverge from the future to (e) the past, and which for us, moving forward in time, correspond to (e&eb) converging waves, attractors/absorbers.

$G_{13}$ : Category one of two wave solutions

$G_{14}$ : Category two of SAS

$G_{15}$ : Category three of SAS

$T_{13}$ : Category one of d'Alembert operator which in Klein-Gordon's relativistic generalization of Schrödinger's wave equation

$T_{14}$ : Category two of SAS

$T_{15}$ : Category three of SAS

**Module Two**

**retarded waves which diverge from the past** to (e&eb) the future and

anticipated waves which diverge from the future to (e) the past, and which for us, moving forward in time, correspond to (e&eb) converging waves, attractors/absorbers

$G_{16}$ : Category one of **retarded waves which diverge from the past;** future

$G_{17}$ : Category two of SAS

$G_{18}$ : Category three of SAS

$T_{16}$ : Category one of future **;retarded waves which diverge from the past**

$T_{17}$ : Category two of SAS

$T_{18}$ : Category three of SAS

**Module three**

anticipated waves which diverge from the future to (e) the past, and which for us, moving forward in time, correspond to (e&eb) converging waves, attractors/absorbers

$G_{20}$ : Category one of **anticipated waves which diverge from the future;** past, and which for us, moving forward in time, correspond to (e&eb) converging waves, attractors/absorbers

$G_{21}$ : Category two of SAS

$G_{22}$ : Category three of SAS

$T_{20}$ : Category one of past, and which for us, moving forward in time, correspond to (e&eb) converging waves, attractors/absorbers **;anticipated waves which diverge from the future**

$T_{21}$ : Category two of SAS

$T_{22}$ : Category three of SAS

**Module four**

anticipated waves which diverge from the future to the past, and which for us, moving forward in time, correspond to (e&eb) **converging waves, attractors/absorbers**

$G_{24}$ : Category one of anticipated waves which diverge from the future to the past, and which for us, moving forward in time; **converging waves, attractors/absorbers**

$G_{25}$ : Category two of SAS

$G_{26}$ : Category three of SAS

$T_{24}$ : Category one of **converging waves, attractors/absorbers** ; anticipated waves which diverge from the future to the past, and which for us, moving forward in time,

$T_{25}$ : Category two of SAS

$T_{26}$ : Category three of SAS

**Module five**

Studying the mathematical properties of these waves Fantappiè discovered that the **diverging waves** are governed by (e&eb) the law of entropy, whereas (e) the converging waves are governed by(e&eb) a symmetrical law, which concentrates (e&eb)energy, produces (eb) differentiation, structures and order, and which Fantappiè named syntropy

$G_{28}$ : Category one of **diverging waves;** law of entropy

$G_{29}$ : Category two of SAS

$G_{30}$ : Category three of SAS

$T_{28}$ : Category one of law of entropy **;diverging waves**

$T_{29}$ : Category two of SAS

$T_{30}$ : Category three of SAS

**Module six**

**converging waves** are governed by(e&eb) a symmetrical law, which concentrates (e&eb)energy, produces (eb) differentiation, structures and order, and which Fantappiè named syntropy

$G_{32}$ : Category one of **converging waves;** symmetrical law, which concentrates (e&eb)energy, produces (eb) differentiation, structures and order, and which Fantappiè named syntropy

$G_{33}$ : Category two of SAS

$G_{34}$ : Category three of SAS

$T_{32}$ : Category one of symmetrical law, which concentrates (e&eb)energy, produces (eb) differentiation, structures and order, and which Fantappiè named syntropy **;converging waves**

$T_{33}$ : Category two of SAS

$T_{34}$ : Category three of SAS

**Module seven**

converging waves are governed by a symmetrical law, which concentrates (e&eb) **energy**, produces (eb) differentiation, structures and order, and which Fantappiè named syntropy

$G_{36}$ : Category one of **converging waves** are governed by a symmetrical law, which concentrates; **energy**

$G_{37}$ : Category two of SAS

$G_{38}$ : Category three of SAS

$T_{36}$ : Category one of **energy ;converging waves** are governed by a symmetrical law, which concentrates

$T_{37}$ : Category two of SAS

$T_{38}$ : Category three of SAS

**Module eight**

converging waves are governed by a symmetrical law, which concentrates **energy**, produces (eb) differentiation, structures and order, and which Fantappiè named syntropy

$G_{40}$ : Category one of converging waves are governed by a symmetrical law, which concentrates **energy**

$G_{41}$ : Category two of SAS

$G_{42}$ : Category three of SAS

$T_{40}$ : Category one of differentiation, structures and order, and which Fantappiè named syntropy

$T_{41}$ : Category two of SAS

$T_{42}$ : Category three of SAS

**Module Nine**

Fantappiè recognized the properties of the law of syntropy in the living systems, and concluded that living systems are a consequence of (e) anticipated waves

$G_{44}$ : Category one of anticipated waves

$G_{45}$ : Category two of SAS

$G_{46}$ : Category three of SAS

$T_{44}$ : Category one of properties of the law of syntropy in the living systems, and concluded that living systems

$T_{45}$ : Category two of SAS

$T_{46}$ : Category three of SAS

**The Coefficients:**

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}$ , $(b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)},$
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}, \ (b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$

$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}\ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}\ ,$
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)},$
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)},$
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)},$
are Dissipation coefficients

**Module Numbered One**

**The differential system of this model is now (Module Numbered one)**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \quad 1$$
$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad 2$$
$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15} \quad 3$$
$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13} \quad 4$$
$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14} \quad 5$$
$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)\right]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14}, t) =$ First augmentation factor
$-(b''_{13})^{(1)}(G, t) =$ First detritions factor

**Module Numbered Two**

**The differential system of this model is now ( Module numbered two)**

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right]G_{16} \quad 7$$
$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t)\right]G_{17} \quad 8$$
$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t)\right]G_{18} \quad 9$$
$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\big((G_{19}), t\big)\right]T_{16} \quad 10$$
$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\big((G_{19}), t\big)\right]T_{17} \quad 11$$
$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\big((G_{19}), t\big)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17}, t) =$ First augmentation factor
$-(b''_{16})^{(2)}\big((G_{19}), t\big) =$ First detritions factor

**Module Numbered Three**

**The differential system of this model is now (Module numbered three)**

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right]G_{20} \quad 13$$
$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}, t)\right]G_{21} \quad 14$$
$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}, t)\right]G_{22} \quad 15$$
$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}, t)\right]T_{20} \quad 16$$
$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}, t)\right]T_{21} \quad 17$$
$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}, t)\right]T_{22} \quad 18$$

$+(a''_{20})^{(3)}(T_{21},t) =$ First augmentation factor
$-(b''_{20})^{(3)}(G_{23},t) =$ First detritions factor

**Module Numbered Four**

**The differential system of this model is now (Module numbered Four)**

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right]G_{24} \quad 19$$
$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25},t)\right]G_{25} \quad 20$$
$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25},t)\right]G_{26} \quad 21$$
$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27}),t)\right]T_{24} \quad 22$$
$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}((G_{27}),t)\right]T_{25} \quad 23$$
$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27}),t)\right]T_{26} \quad 24$$

$+(a''_{24})^{(4)}(T_{25},t) =$ **First augmentation factor**
$-(b''_{24})^{(4)}((G_{27}),t) =$ **First detritions factor**

**Module Numbered Five**:

**The differential system of this model is now (Module number five)**

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29},t)\right]G_{28} \quad 25$$
$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29},t)\right]G_{29} \quad 26$$
$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29},t)\right]G_{30} \quad 27$$
$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31}),t)\right]T_{28} \quad 28$$
$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}((G_{31}),t)\right]T_{29} \quad 29$$
$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31}),t)\right]T_{30} \quad 30$$

$+(a''_{28})^{(5)}(T_{29},t) =$ **First augmentation factor**
$-(b''_{28})^{(5)}((G_{31}),t) =$ **First detritions factor**

**Module Numbered Six**

**The differential system of this model is now (Module numbered Six)**

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33},t)\right]G_{32} \quad 31$$
$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33},t)\right]G_{33} \quad 32$$
$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33},t)\right]G_{34} \quad 33$$
$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35}),t)\right]T_{32} \quad 34$$
$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}((G_{35}),t)\right]T_{33} \quad 35$$
$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35}),t)\right]T_{34} \quad 36$$

$+(a''_{32})^{(6)}(T_{33},t) =$ **First augmentation factor**

**Module Numbered Seven:**

**The differential system of this model is now (Seventh Module)**

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right]G_{36} \quad 37$$
$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37},t)\right]G_{37} \quad 38$$

$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37},t)\right]G_{38} \quad 39$$
$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39}),t)\right]T_{36} \quad 40$$
$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}((G_{39}),t)\right]T_{37} \quad 41$$
$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39}),t)\right]T_{38} \quad 42$$
$+(a''_{36})^{(7)}(T_{37},t) =$ **First augmentation factor**

**Module Numbered Eight**

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right]G_{40} \quad 43$$
$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41},t)\right]G_{41} \quad 44$$
$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41},t)\right]G_{42} \quad 45$$
$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43}),t)\right]T_{40} \quad 46$$
$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}((G_{43}),t)\right]T_{41} \quad 47$$
$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43}),t)\right]T_{42} \quad 48$$

**Module Numbered Nine**

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$
$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$
$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$
$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47}),t)\right]T_{44} \quad 52$$
$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}((G_{47}),t)\right]T_{45} \quad 53$$
$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47}),t)\right]T_{46} \quad 54$$
$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}((G_{47}),t) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)} & \boxed{+(a''_{13})^{(1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)} & \boxed{+(a''_{14})^{(1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)} & \boxed{+(a''_{15})^{(1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15} \quad 57$$

Where $\boxed{(a''_{13})^{(1)}(T_{14},t)}$, $\boxed{(a''_{14})^{(1)}(T_{14},t)}$, $\boxed{(a''_{15})^{(1)}(T_{14},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}$ are second augmentation coefficient for

category 1, 2 and 3
$+(a''_{20})^{(3,3,)}(T_{21},t)$, $+(a''_{21})^{(3,3,)}(T_{21},t)$, $+(a''_{22})^{(3,3,)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3
$+(a''_{24})^{(4,4,4,4,)}(T_{25},t)$ , $+(a''_{25})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3
$+(a''_{28})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3
$+(a''_{32})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3
$+(a''_{38})^{(7,7)}(T_{37},t)$ $+(a''_{37})^{(7,7)}(T_{37},t)$ $+(a''_{36})^{(7,7)}(T_{37},t)$ are seventh augmentation coefficient for 1,2,3
$+(a''_{40})^{(8,8)}(T_{41},t)$ $+(a''_{41})^{(8,8)}(T_{41},t)$ $+(a''_{42})^{(8,8)}(T_{41},t)$ are eight augmentation coefficient for 1,2,3
$+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)} & -(b''_{13})^{(1)}(G,t) & -(b''_{16})^{(2,2,)}(G_{19},t) & -(b''_{20})^{(3,3,)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{36})^{(7,7,)}(G_{39},t) & -(b''_{40})^{(8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{13} \quad 58$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)} & -(b''_{14})^{(1)}(G,t) & -(b''_{17})^{(2,2,)}(G_{19},t) & -(b''_{21})^{(3,3,)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{37})^{(7,7,)}(G_{39},t) & -(b''_{41})^{(8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{14} \quad 59$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)} & -(b''_{15})^{(1)}(G,t) & -(b''_{18})^{(2,2,)}(G_{19},t) & -(b''_{22})^{(3,3,)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{38})^{(7,7,)}(G_{39},t) & -(b''_{42})^{(8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{15} \quad 60$$

Where $-(b''_{13})^{(1)}(G,t)$, $-(b''_{14})^{(1)}(G,t)$, $-(b''_{15})^{(1)}(G,t)$ are first detrition coefficients for category 1, 2 and 3
$-(b''_{16})^{(2,2,)}(G_{19},t)$, $-(b''_{17})^{(2,2,)}(G_{19},t)$, $-(b''_{18})^{(2,2,)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3
$-(b''_{20})^{(3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3
$-(b''_{24})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3
$-(b''_{28})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3
$-(b''_{32})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3
$-(b''_{37})^{(7,7,)}(G_{39},t)$, $-(b''_{36})^{(7,7,)}(G_{39},t)$, $-(b''_{38})^{(7,7,)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8)}(G_{43},t)}$ $\boxed{-(b''_{41})^{(8,8)}(G_{43},t)}$ $\boxed{-(b''_{42})^{(8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[\begin{array}{c} (a'_{16})^{(2)}\boxed{+(a''_{16})^{(2)}(T_{17},t)}\ \boxed{+(a''_{13})^{(1,1,)}(T_{14},t)}\ \boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)}\ \boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)}\ \boxed{+(a''_{44})^{(9,9)}(T_{45},t)} \end{array}\right] G_{16} \quad (61)$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[\begin{array}{c} (a'_{17})^{(2)}\boxed{+(a''_{17})^{(2)}(T_{17},t)}\ \boxed{+(a''_{14})^{(1,1,)}(T_{14},t)}\ \boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)}\ \boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)}\ \boxed{+(a''_{45})^{(9,9)}(T_{45},t)} \end{array}\right] G_{17} \quad (62)$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[\begin{array}{c} (a'_{18})^{(2)}\boxed{+(a''_{18})^{(2)}(T_{17},t)}\ \boxed{+(a''_{15})^{(1,1,)}(T_{14},t)}\ \boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)}\ \boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)}\ \boxed{+(a''_{46})^{(9,9)}(T_{45},t)} \end{array}\right] G_{18} \quad (63)$$

Where $\boxed{+(a''_{16})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,)}(T_{14},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)}$ are eight augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{44})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9)}(T_{45},t)}$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[\begin{array}{c} (b'_{16})^{(2)}\boxed{-(b''_{16})^{(2)}(G_{19},t)}\ \boxed{-(b''_{13})^{(1,1,)}(G,t)}\ \boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)}\ \boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)}\ \boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)}\ \boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)}\ \boxed{-(b''_{44})^{(9,9)}(G_{47},t)} \end{array}\right] T_{16} \quad (64)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)}\boxed{-(b''_{17})^{(2)}(G_{19},t)} & \boxed{-(b''_{14})^{(1,1,)}(G,t)} & \boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{17} \quad 65$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)}\boxed{-(b''_{18})^{(2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,)}(G,t)} & \boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{18} \quad 66$$

where $\boxed{-(b''_{16})^{(2)}(G_{19},t)}$ , $\boxed{-(b''_{17})^{(2)}(G_{19},t)}$ , $\boxed{-(b''_{18})^{(2)}(G_{19},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,)}(G,t)}$ are second detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{44})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \begin{bmatrix} (a'_{20})^{(3)}\boxed{+(a''_{20})^{(3)}(T_{21},t)} & \boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{20} \quad 67$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \begin{bmatrix} (a'_{21})^{(3)}\boxed{+(a''_{21})^{(3)}(T_{21},t)} & \boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{21} \quad 68$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \begin{bmatrix} (a'_{22})^{(3)}\boxed{+(a''_{22})^{(3)}(T_{21},t)} & \boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{22} \quad 69$$

$\boxed{+(a''_{20})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3)}(T_{21},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}$ are second augmentation coefficients

for category 1, 2 and 3

$+(a_{13}'')^{(1,1,1,)}(T_{14},t)$, $+(a_{14}'')^{(1,1,1,)}(T_{14},t)$, $+(a_{15}'')^{(1,1,1,)}(T_{14},t)$ are third augmentation coefficients for category 1, 2 and 3

$+(a_{24}'')^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a_{25}'')^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a_{26}'')^{(4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficients for category 1, 2 and 3

$+(a_{28}'')^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a_{29}'')^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a_{30}'')^{(5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficients for category 1, 2 and 3

$+(a_{32}'')^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a_{33}'')^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a_{34}'')^{(6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficients for category 1, 2 and 3

$+(a_{36}'')^{(7,7,7,7)}(T_{37},t)$, $+(a_{37}'')^{(7,7,7,7)}(T_{37},t)$, $+(a_{38}'')^{(7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1, 2 and 3

$+(a_{40}'')^{(8,8,8,8)}(T_{41},t)$, $+(a_{41}'')^{(8,8,8,8)}(T_{41},t)$, $+(a_{42}'')^{(8,8,8,8)}(T_{41},t)$ are eight augmentation coefficients for category 1, 2 and 3

$+(a_{44}'')^{(9,9,9)}(T_{45},t)$, $+(a_{45}'')^{(9,9,9)}(T_{45},t)$, $+(a_{46}'')^{(9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b_{20}')^{(3)} & -(b_{20}'')^{(3)}(G_{23},t) & -(b_{16}'')^{(2,2,2)}(G_{19},t) & -(b_{13}'')^{(1,1,1,)}(G,t) \\ & -(b_{24}'')^{(4,4,4,4,4,4)}(G_{27},t) & -(b_{28}'')^{(5,5,5,5,5,5)}(G_{31},t) & -(b_{32}'')^{(6,6,6,6,6,6)}(G_{35},t) \\ & -(b_{36}'')^{(7,7,7,7)}(G_{39},t) & -(b_{40}'')^{(8,8,8,8)}(G_{43},t) & -(b_{44}'')^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{20} \quad (70)$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b_{21}')^{(3)} & -(b_{21}'')^{(3)}(G_{23},t) & -(b_{17}'')^{(2,2,2)}(G_{19},t) & -(b_{14}'')^{(1,1,1,)}(G,t) \\ & -(b_{25}'')^{(4,4,4,4,4,4)}(G_{27},t) & -(b_{29}'')^{(5,5,5,5,5,5)}(G_{31},t) & -(b_{33}'')^{(6,6,6,6,6,6)}(G_{35},t) \\ & -(b_{37}'')^{(7,7,7,7)}(G_{39},t) & -(b_{41}'')^{(8,8,8,8)}(G_{43},t) & -(b_{45}'')^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{21} \quad (71)$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b_{22}')^{(3)} & -(b_{22}'')^{(3)}(G_{23},t) & -(b_{18}'')^{(2,2,2)}(G_{19},t) & -(b_{15}'')^{(1,1,1,)}(G,t) \\ & -(b_{26}'')^{(4,4,4,4,4,4)}(G_{27},t) & -(b_{30}'')^{(5,5,5,5,5,5)}(G_{31},t) & -(b_{34}'')^{(6,6,6,6,6,6)}(G_{35},t) \\ & -(b_{38}'')^{(7,7,7,7)}(G_{39},t) & -(b_{42}'')^{(8,8,8,8)}(G_{43},t) & -(b_{46}'')^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{22} \quad (72)$$

$-(b_{20}'')^{(3)}(G_{23},t)$, $-(b_{21}'')^{(3)}(G_{23},t)$, $-(b_{22}'')^{(3)}(G_{23},t)$ *are first detrition coefficients for category 1, 2 and 3*

$-(b_{16}'')^{(2,2,2)}(G_{19},t)$, $-(b_{17}'')^{(2,2,2)}(G_{19},t)$, $-(b_{18}'')^{(2,2,2)}(G_{19},t)$ *are second detrition coefficients for category 1, 2 and 3*

$-(b_{13}'')^{(1,1,1,)}(G,t)$, $-(b_{14}'')^{(1,1,1,)}(G,t)$, $-(b_{15}'')^{(1,1,1,)}(G,t)$ *are third detrition coefficients for category 1,2 and 3*

$-(b_{24}'')^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b_{25}'')^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b_{26}'')^{(4,4,4,4,4,4)}(G_{27},t)$ *are fourth detrition coefficients for category 1, 2 and 3*

$-(b_{28}'')^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b_{29}'')^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b_{30}'')^{(5,5,5,5,5,5)}(G_{31},t)$ *are fifth detrition coefficients for category 1, 2 and 3*

$-(b_{32}'')^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b_{33}'')^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b_{34}'')^{(6,6,6,6,6,6)}(G_{35},t)$ *are sixth detrition coefficients for category 1, 2 and 3*

$-(b_{36}'')^{(7,7,7,7)}(G_{39},t)$, $-(b_{37}'')^{(7,7,7,7)}(G_{39},t)$, $-(b_{38}'')^{(7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients

for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt}=(a_{24})^{(4)}G_{25}-\begin{bmatrix}(a'_{24})^{(4)} & \boxed{+(a''_{24})^{(4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}\\ \boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{24} \quad (73)$$

$$\frac{dG_{25}}{dt}=(a_{25})^{(4)}G_{24}-\begin{bmatrix}(a'_{25})^{(4)} & \boxed{+(a''_{25})^{(4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6)}(T_{33},t)}\\ \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{25} \quad (74)$$

$$\frac{dG_{26}}{dt}=(a_{26})^{(4)}G_{25}-\begin{bmatrix}(a'_{26})^{(4)} & \boxed{+(a''_{26})^{(4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}\\ \boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{26} \quad (75)$$

$\boxed{(a''_{24})^{(4)}(T_{25},t)}$, $\boxed{(a''_{25})^{(4)}(T_{25},t)}$, $\boxed{(a''_{26})^{(4)}(T_{25},t)}$ *are first augmentation coefficients category* 1, 2 3

$\boxed{+(a''_{28})^{(5,5,)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,)}(T_{29},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)}$
*are fourth augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)}$
*are fifth augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)}$
*are sixth augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)}$
*are seventh augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)}$
*are eighth augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt}=(b_{24})^{(4)}T_{25}-\begin{bmatrix}(b'_{24})^{(4)} & \boxed{-(b''_{24})^{(4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,)}(G_{35},t)}\\ \boxed{-(b''_{13})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)}\\ \boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9)}(G_{47},t)}\end{bmatrix}T_{24} \quad (76)$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \begin{bmatrix} (b'_{25})^{(4)}\boxed{-(b''_{25})^{(4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{25} \quad (77)$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)}\boxed{-(b''_{26})^{(4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{26} \quad (78)$$

*Where* $\boxed{-(b''_{24})^{(4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4)}(G_{27},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{28})^{(5,5,)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,)}(G_{31},t)}$ *are second detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{32})^{(6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,)}(G_{35},t)}$ *are third detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,1)}(G,t)}$
*are fourth detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2)}(G_{19},t)}$
*are fifth detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3)}(G_{23},t)}$
*are sixth detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)}$
*are seventh detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)}$
*are eighth detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{46})^{(9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \begin{bmatrix} (a'_{28})^{(5)}\boxed{+(a''_{28})^{(5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{28} \quad (79)$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \begin{bmatrix} (a'_{29})^{(5)}\boxed{+(a''_{29})^{(5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{29} \quad (80)$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \begin{bmatrix} (a'_{30})^{(5)}\boxed{+(a''_{30})^{(5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{30} \quad (81)$$

*Where* $\boxed{+(a''_{28})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5)}(T_{29},t)}$ *are first augmentation coefficients for category* 1, 2 *and* 3

*And* $\boxed{+(a''_{24})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,)}(T_{25},t)}$ *are second augmentation*

$coefficient\ for\ category\ 1,2\ and\ 3$
$+(a''_{32})^{(6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6)}(T_{33},t)$ $are\ third\ \ augmentation$
$coefficient\ for\ category\ 1,2\ and\ 3$
$+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)$ are fourth augmentation coefficients for category 1,2, and 3
$+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)$ are fifth augmentation coefficients for category 1,2,and 3
$+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)$ are sixth augmentation coefficients for category 1,2, 3
$+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)$ , $+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1,2, 3
$+(a''_{40})^{(8,8\ ,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)$ are eighth augmentation coefficients for category 1,2, 3
$+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt}=(b_{28})^{(5)}T_{29}-\begin{bmatrix}(b'_{28})^{(5)} & -(b''_{28})^{(5)}(G_{31},t) & -(b''_{24})^{(4,4,)}(G_{27},t) & -(b''_{32})^{(6,6,6)}(G_{35},t)\\ & -(b''_{13})^{(1,1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3)}(G_{23},t)\\ & -(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9)}(G_{47},t)\end{bmatrix}T_{28} \quad 82$$

$$\frac{dT_{29}}{dt}=(b_{29})^{(5)}T_{28}-\begin{bmatrix}(b'_{29})^{(5)} & -(b''_{29})^{(5)}(G_{31},t) & -(b''_{25})^{(4,4,)}(G_{27},t) & -(b''_{33})^{(6,6,6)}(G_{35},t)\\ & -(b''_{14})^{(1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3)}(G_{23},t)\\ & -(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9)}(G_{47},t)\end{bmatrix}T_{29} \quad 83$$

$$\frac{dT_{30}}{dt}=(b_{30})^{(5)}T_{29}-\begin{bmatrix}(b'_{30})^{(5)} & -(b''_{30})^{(5)}(G_{31},t) & -(b''_{26})^{(4,4,)}(G_{27},t) & -(b''_{34})^{(6,6,6)}(G_{35},t)\\ & -(b''_{15})^{(1,1,1,1,1,)}(G,t) & -(b''_{18})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3)}(G_{23},t)\\ & -(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9)}(G_{47},t)\end{bmatrix}T_{30} \quad 84$$

$where$ $-(b''_{28})^{(5)}(G_{31},t)$ , $-(b''_{29})^{(5)}(G_{31},t)$ , $-(b''_{30})^{(5)}(G_{31},t)$ $are\ first\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b''_{24})^{(4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,)}(G_{27},t)$ $are\ second\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b''_{32})^{(6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6)}(G_{35},t)$ $are\ \ third\ \ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b''_{13})^{(1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1)}(G,t)$ , $-(b''_{15})^{(1,1,1,1,1,)}(G,t)$ are fourth detrition coefficients for category 1,2, and 3
$-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1,2, and 3
$-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1,2, and 3
$-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[\begin{array}{ccc} (a'_{32})^{(6)}\boxed{+(a''_{32})^{(6)}(T_{33},t)} & \boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{32} \quad (85)$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[\begin{array}{ccc} (a'_{33})^{(6)}\boxed{+(a''_{33})^{(6)}(T_{33},t)} & \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{33} \quad (86)$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[\begin{array}{ccc} (a'_{34})^{(6)}\boxed{+(a''_{34})^{(6)}(T_{33},t)} & \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{34} \quad (87)$$

$\boxed{+(a''_{32})^{(6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6)}(T_{33},t)}$ *are first augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}$ *are second augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}$ *are third augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}$ - are fourth augmentation coefficients

$\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}$ - fifth augmentation coefficients

$\boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)}$ sixth augmentation coefficients

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)}$ seventh augmentation coefficients

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}$ Eighth augmentation coefficients

$\boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)}$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[\begin{array}{ccc} (b'_{32})^{(6)}\boxed{-(b''_{32})^{(6)}(G_{35},t)} & \boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)} \end{array}\right] T_{32} \quad (88)$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \begin{bmatrix} (b'_{33})^{(6)}\boxed{-(b''_{33})^{(6)}(G_{35},t)}\boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}\boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}\boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)}\boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{33} \quad (89)$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)}\boxed{-(b''_{34})^{(6)}(G_{35},t)}\boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}\boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}\boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)}\boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{34} \quad (90)$$

$\boxed{-(b''_{32})^{(6)}(G_{35},t)}, \boxed{-(b''_{33})^{(6)}(G_{35},t)}, \boxed{-(b''_{34})^{(6)}(G_{35},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}$ *are second detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)}$ *are third detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}$ are fourth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \quad (91)$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a'_{36})^{(7)}\boxed{+(a''_{36})^{(7)}(T_{37},t)}\boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)}\boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)}\boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)}\boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)}\boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \quad (92)$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a'_{37})^{(7)}\boxed{+(a''_{37})^{(7)}(T_{37},t)}\boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)}\boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)}\boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)}\boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}\boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$\frac{dG_{38}}{dt}$ *93*

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a'_{38})^{(7)}\boxed{+(a''_{38})^{(7)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{(a''_{36})^{(7)}(T_{37},t)}$, $\boxed{(a''_{37})^{(7)}(T_{37},t)}$, $\boxed{(a''_{38})^{(7)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,)}(T_{41},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$\frac{dT_{36}}{dt} =$ 94

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b'_{36})^{(7)}\boxed{-(b''_{36})^{(7)}(G_{39},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$\frac{dT_{37}}{dt} =$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b'_{37})^{(7)}\boxed{-(b''_{37})^{(7)}(G_{39},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$\frac{dT_{38}}{dt} =$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b'_{38})^{(7)}\boxed{-(b''_{38})^{(7)}(G_{39},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b_{36}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{37}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{16}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{17}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{18}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{20}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{21}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{22}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{24}'')^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{25}'')^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{26}'')^{(4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{28}'')^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{32}'')^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{34}'')^{(6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{46}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{45}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{44}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3



$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[\begin{array}{ccc} (a_{40}')^{(8)}\boxed{+(a_{40}'')^{(8)}(T_{41},t)} & \boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{20}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{24}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{28}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{32}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{36}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{44}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{13}$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[\begin{array}{ccc} (a_{41}')^{(8)}\boxed{+(a_{41}'')^{(8)}(T_{41},t)} & \boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{21}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{25}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{29}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{33}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{37}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{45}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[\begin{array}{ccc} (a_{42}')^{(8)}\boxed{+(a_{42}'')^{(8)}(T_{41},t)} & \boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{22}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{26}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{30}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{34}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{15}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{38}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{46}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{15}$$

Where $\boxed{+(a_{40}'')^{(8)}(T_{41},t)}$, $\boxed{+(a_{41}'')^{(8)}(T_{41},t)}$, $\boxed{+(a_{42}'')^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second

augmentation coefficient for category 1, 2 and 3
$+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)$ , $+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)$ , $+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3
$+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)$ , $+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)$ , $+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3
$+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)$ , $+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3
$+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)$ , $+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3
$+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ $+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ $+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ are seventh augmentation coefficient for 1,2,3
$+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$ , $+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$ , $+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$ are eighth augmentation coefficient for 1,2,3
$+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$ , $+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$ , $+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} =$$

$$(b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)}\left[-(b''_{40})^{(8)}(G_{43},t)\right] & \left[-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} =$$

$$(b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)}\left[-(b''_{41})^{(8)}(G_{43},t)\right] & \left[-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} =$$

$$(b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)}\left[-(b''_{42})^{(8)}(G_{43},t)\right] & \left[-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{15}$$

Where $-(b''_{36})^{(7)}(G_{39},t)$ , $-(b''_{37})^{(7)}(G_{39},t)$ , $-(b''_{38})^{(7)}(G_{39},t)$ are first detrition coefficients for category 1, 2 and 3
$-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$ , $-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$ , $-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3
$-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$ , $-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$ , $-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3
$-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)$ , $-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)$ , $-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3



$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \begin{bmatrix} (a'_{44})^{(9)}\boxed{+(a''_{44})^{(9)}(T_{45},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \begin{bmatrix} (a'_{45})^{(9)}\boxed{+(a''_{45})^{(9)}(T_{45},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \begin{bmatrix} (a'_{46})^{(9)}\boxed{+(a''_{46})^{(9)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a''_{44})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,1,1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1,1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1,1,1,1,1,1)}(T_{14},t)$ are Seventh augmentation coefficient for category 1, 2 and 3

$+(a''_{38})^{(7,7,7,7,7,7,7,7,7,7)}(T_{37},t)$ $+(a''_{37})^{(7,7,7,7,7,7,7,7,7,7)}(T_{37},t)$ $+(a''_{36})^{(7,7,7,7,7,7,7,7,7,7)}(T_{37},t)$ are eighth augmentation coefficient for 1,2,3

$+(a''_{40})^{(8,8,8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8,8,8,8,8)}(T_{41},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} =$$

$$(b_{44})^{(9)}T_{45} -$$

$$\begin{bmatrix} (b'_{44})^{(9)} & -(b''_{44})^{(9)}(G_{47},t) & -(b''_{16})^{(2,2,2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6,6,6,6,6,6)}(G_{35},t) & \\ -(b''_{13})^{(1,1,1,1,1,1,1,1,1,1)}(G,t) & -(b''_{36})^{(7,7,7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8,8,8,8,8)}(G_{43},t) & \end{bmatrix} T_{13}$$

$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b'_{45})^{(9)} & -(b''_{45})^{(9)}(G_{47},t) & -(b''_{17})^{(2,2,2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6,6,6,6,6,6)}(G_{35},t) & \\ -(b''_{14})^{(1,1,1,1,1,1,1,1,1,1)}(G,t) & -(b''_{37})^{(7,7,7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8,8,8,8,8)}(G_{43},t) & \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b'_{46})^{(9)} & -(b''_{46})^{(9)}(G_{47},t) & -(b''_{18})^{(2,2,2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6,6,6,6,6,6)}(G_{35},t) & \\ -(b''_{15})^{(1,1,1,1,1,1,1,1,1,1)}(G,t) & -(b''_{38})^{(7,7,7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8,8,8,8,8)}(G_{43},t) & \end{bmatrix} T_{15}$$

Where $-(b''_{44})^{(9)}(G_{47},t)$, $-(b''_{45})^{(9)}(G_{47},t)$, $-(b''_{46})^{(9)}(G_{47},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6,6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{15})^{(1,1,1,1,1,1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1,1,1,1,1,1)}(G,t)$, $-(b''_{13})^{(1,1,1,1,1,1,1,1,1,1)}(G,t)$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{37}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{36}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a_i')^{(1)}, (a_i'')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (b_i'')^{(1)} > 0, \quad i,j = 13,14,15 \qquad 97$$

The functions $(a_i'')^{(1)}, (b_i'')^{(1)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a_i'')^{(1)}(T_{14},t) \leq (p_i)^{(1)} \leq (\hat{A}_{13})^{(1)}$$

$$(b_i'')^{(1)}(G,t) \leq (r_i)^{(1)} \leq (b_i')^{(1)} \leq (\hat{B}_{13})^{(1)}$$

$$lim_{T_2\to\infty}(a_i'')^{(1)}(T_{14},t) = (p_i)^{(1)} \qquad 98$$

$$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = (r_i)^{(1)}$$

**Definition of** $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$ :

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$$|(a_i'')^{(1)}(T_{14}',t) - (a_i'')^{(1)}(T_{14},t)| \leq (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G',t) - (b_i'')^{(1)}(G,t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}',t)$ and $(a_i'')^{(1)}(T_{14},t)$ . $(T_{14}',t)$ and $(T_{14},t)$ are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14},t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$ : 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$$

**Definition of** $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$ : 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i,j = 16,17,18$$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17}, t) \leq (p_i)^{(2)} \leq (\hat{A}_{16})^{(2)} \qquad 102$$

$$(b_i'')^{(2)}(G_{19}, t) \leq (r_i)^{(2)} \leq (b_i')^{(2)} \leq (\hat{B}_{16})^{(2)} \qquad 103$$

$$\lim_{T_2 \to \infty} (a_i'')^{(2)}(T_{17}, t) = (p_i)^{(2)} \qquad 104$$

$$\lim_{G \to \infty} (b_i'')^{(2)}((G_{19}), t) = (r_i)^{(2)} \qquad 105$$

**Definition of** $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$ : 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}', t) - (a_i'')^{(2)}(T_{17}, t)| \leq (\hat{k}_{16})^{(2)}|T_{17} - T_{17}'|e^{-(\hat{M}_{16})^{(2)}t} \qquad 107$$

$$|(b_i'')^{(2)}((G_{19})', t) - (b_i'')^{(2)}((G_{19}), t)| < (\hat{k}_{16})^{(2)}||(G_{19}) - (G_{19})'||e^{-(\hat{M}_{16})^{(2)}t} \qquad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}', t)$ and$(a_i'')^{(2)}(T_{17}, t)$ . $(T_{17}', t)$ and $(T_{17}, t)$ are points belonging to the interval $[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}]$ . It is to be noted that $(a_i'')^{(2)}(T_{17}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17}, t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$ :

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants 109

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$$

**Definition of** $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$ :

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1 \quad (110)$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(b_i)^{(2)} + (b_i')^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1 \quad (111)$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i,j = 20,21,22 \quad (112)$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(3)}$, $(r_i)^{(3)}$:

$$(a_i'')^{(3)}(T_{21}, t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23}, t) \leq (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(3)}(T_{21}, t) = (p_i)^{(3)} \quad (113)$$

$$\lim_{G \to \infty}(b_i'')^{(3)}(G_{23}, t) = (r_i)^{(3)}$$

**Definition of** $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$ :

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: (114)

$$|(a_i'')^{(3)}(T_{21}', t) - (a_i'')^{(3)}(T_{21}, t)| \leq (\hat{k}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\hat{M}_{20})^{(3)}t}$$

$$|(b_i'')^{(3)}(G_{23}', t) - (b_i'')^{(3)}(G_{23}, t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}'||e^{-(\hat{M}_{20})^{(3)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}', t)$ and $(a_i'')^{(3)}(T_{21}, t)$ . $(T_{21}', t)$ And $(T_{21}, t)$ are points belonging to the interval $[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}]$. It is to be noted that $(a_i'')^{(3)}(T_{21}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$ : (115)

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$ and the constants $(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}, i = 20,21,22,$ satisfy the inequalities 116

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(a_i)^{(3)} + (a_i')^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(b_i)^{(3)} + (b_i')^{(3)} + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i, j = 24,25,26 \qquad 117$$

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(4)}$, $(r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25}, t) \le (p_i)^{(4)} \le (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}((G_{27}), t) \le (r_i)^{(4)} \le (b_i')^{(4)} \le (\hat{B}_{24})^{(4)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(4)}(T_{25}, t) = (p_i)^{(4)} \qquad 118$$

$$\lim_{G \to \infty}(b_i'')^{(4)}((G_{27}), t) = (r_i)^{(4)}$$

**Definition of** $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$ :

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24,25,26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}', t) - (a_i'')^{(4)}(T_{25}, t)| \le (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})', t) - (b_i'')^{(4)}((G_{27}), t)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}', t)$ and $(a_i'')^{(4)}(T_{25}, t)$ . $(T_{25}', t)$ and $(T_{25}, t)$ are points belonging to the interval $[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}]$. It is to be noted that $(a_i'')^{(4)}(T_{25}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$ : 120

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$, are positive constants

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}, \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$$

**Definition of** $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$ : 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and\ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}, i = 24,25,26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(a_i)^{(4)} + (a_i')^{(4)} + (\hat{A}_{24})^{(4)} + (\hat{P}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(b_i)^{(4)} + (b_i')^{(4)} + (\hat{B}_{24})^{(4)} + (\hat{Q}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i,j = 28,29,30 \qquad 122$$

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(5)}$, $(r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29}, t) \le (p_i)^{(5)} \le (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}((G_{31}), t) \le (r_i)^{(5)} \le (b_i')^{(5)} \le (\hat{B}_{28})^{(5)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(5)}(T_{29}, t) = (p_i)^{(5)} \qquad 123$$

$$\lim_{G \to \infty}(b_i'')^{(5)}(G_{31}, t) = (r_i)^{(5)}$$

**Definition of** $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$ :

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}', t) - (a_i'')^{(5)}(T_{29}, t)| \le (\hat{k}_{28})^{(5)}|T_{29} - T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})', t) - (b_i'')^{(5)}((G_{31}), t)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}', t)$ and $(a_i'')^{(5)}(T_{29}, t)$ . $(T_{29}', t)$ and $(T_{29}, t)$ are points belonging to the interval $[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}]$. It is to be noted that $(a_i'')^{(5)}(T_{29}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29}, t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$ : 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}, \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

**Definition of** $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$ : 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and\ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(a_i)^{(5)} + (a_i')^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(b_i)^{(5)} + (b_i')^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i,j = 32,33,34$$ 127

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(6)}$, $(r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33}, t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}), t) \leq (r_i)^{(6)} \leq (b_i')^{(6)} \leq (\hat{B}_{32})^{(6)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(6)}(T_{33}, t) = (p_i)^{(6)}$$ 128

$$\lim_{G \to \infty}(b_i'')^{(6)}((G_{35}), t) = (r_i)^{(6)}$$

**Definition of** $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$ :

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}', t) - (a_i'')^{(6)}(T_{33}, t)| \leq (\hat{k}_{32})^{(6)}|T_{33} - T_{33}'|e^{-(\hat{M}_{32})^{(6)}t}$$

$$|(b_i'')^{(6)}((G_{35})', t) - (b_i'')^{(6)}((G_{35}), t)| < (\hat{k}_{32})^{(6)}||(G_{35}) - (G_{35})'||e^{-(\hat{M}_{32})^{(6)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}', t)$ and $(a_i'')^{(6)}(T_{33}, t)$ . $(T_{33}', t)$ and $(T_{33}, t)$ are points belonging to the interval $[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}]$. It is to be noted that $(a_i'')^{(6)}(T_{33}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33}, t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$ : 129

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$, are positive constants

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$$

**Definition of** $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$ : 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and\ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32,33,34,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(a_i)^{(6)} + (a_i')^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)} (\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(b_i)^{(6)} + (b_i')^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)} (\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

(SSSSSSS) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i, j = 36,37,38$ 131

(TTTTTTT) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(7)}, (r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37}, t) \leq (p_i)^{(7)} \leq (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39}, t) \leq (r_i)^{(7)} \leq (b_i')^{(7)} \leq (\hat{B}_{36})^{(7)}$$

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*(UUUUUUU)* $\lim_{T_2 \to \infty} (a_i'')^{(7)}(T_{37}, t) = (p_i)^{(7)}$

*(VVVVVVV)*

$\lim_{G \to \infty} (b_i'')^{(7)}((G_{39}), t) = (r_i)^{(7)}$

**Definition of** $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$ :

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$$|(a_i'')^{(7)}(T_{37}', t) - (a_i'')^{(7)}(T_{37}, t)| \leq (\hat{k}_{36})^{(7)} |T_{37} - T_{37}'| e^{-(\hat{M}_{36})^{(7)} t}$$

$$|(b_i'')^{(7)}((G_{39})', t) - (b_i'')^{(7)}((G_{39}), t)| < (\hat{k}_{36})^{(7)} ||(G_{39}) - (G_{39})'|| e^{-(\hat{M}_{36})^{(7)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}', t)$ and $(a_i'')^{(7)}(T_{37}, t)$ . $(T_{37}', t)$ and $(T_{37}, t)$ are points belonging to the interval $[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}]$ . It is to be noted that $(a_i'')^{(7)}(T_{37}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37}, t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$ : 134

(WWWWWWW) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)},$ are positive constants

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$$

**Definition of** $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$ : 135

(XXXXXXX) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(b_i)^{(7)} + (b_i')^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i, j = 40,41,42 \qquad 136$$

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

**Definition of** $(p_i)^{(8)}$, $(r_i)^{(8)}$: 137

$$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)} \qquad 138$$

$$(b_i'')^{(8)}((G_{43}), t) \leq (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)} \qquad 139$$

$$\lim_{T_2 \to \infty} (a_i'')^{(8)}(T_{41}, t) = (p_i)^{(8)} \qquad 140$$

$$\lim_{G \to \infty} (b_i'')^{(8)}\big((G_{43}), t\big) = (r_i)^{(8)} \qquad 141$$

**Definition of** $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$ :

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)} |T_{41} - T_{41}'| e^{-(\hat{M}_{40})^{(8)} t} \qquad 142$$

$$|(b_i'')^{(8)}((G_{43})', t) - (b_i'')^{(8)}\big((G_{43}), t\big)| < (\hat{k}_{40})^{(8)} ||(G_{43}) - (G_{43})'|| e^{-(\hat{M}_{40})^{(8)} t} \qquad 143$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}', t)$ and $(a_i'')^{(8)}(T_{41}, t)$ . $(T_{41}', t)$ and $(T_{41}, t)$ are points belonging to the interval $\big[(\hat{k}_{40})^{(8)}, (\hat{M}_{40})^{(8)}\big]$. It is to be noted that $(a_i'')^{(8)}(T_{41}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{40})^{(8)} = 1$

then the function $(a_i'')^{(8)}(T_{41},t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)}$ :

$(\hat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)},$ are positive constants

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\ ,\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}}<1 \qquad 144$$

**Definition of** $(\hat{P}_{40})^{(8)},(\hat{Q}_{40})^{(8)}$ :

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\hat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)},(\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)},(a_i')^{(8)},(b_i)^{(8)},(b_i')^{(8)},(p_i)^{(8)},\ (r_i)^{(8)},i=40,41,42,$

Satisfy the inequalities

$$\frac{1}{(\hat{M}_{40})^{(8)}}[\,(a_i)^{(8)}+(a_i')^{(8)}+\ (\hat{A}_{40})^{(8)}+\ (\hat{P}_{40})^{(8)}\,(\hat{k}_{40})^{(8)}]<1 \qquad 145$$

$$\frac{1}{(\hat{M}_{40})^{(8)}}[\ (b_i)^{(8)}+(b_i')^{(8)}+\ (\hat{B}_{40})^{(8)}+\ (\hat{Q}_{40})^{(8)}\ \ (\hat{k}_{40})^{(8)}]<1 \qquad 146$$

Where we suppose

$$(a_i)^{(9)},(a_i')^{(9)},(a_i'')^{(9)},(b_i)^{(9)},(b_i')^{(9)},(b_i'')^{(9)}>0,\qquad i,j=44,45,46 \qquad 146\,A$$

The functions $(a_i'')^{(9)},(b_i'')^{(9)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(9)},\ (r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45},t)\le(p_i)^{(9)}\le(\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47},t)\le\ (r_i)^{(9)}\le(b_i')^{(9)}\le(\hat{B}_{44})^{(9)}$$

$$\lim_{T_2\to\infty}(a_i'')^{(9)}(T_{45},t)=(p_i)^{(9)}$$

$$\lim_{G\to\infty}(b_i'')^{(9)}(G_{47},t)=\ (r_i)^{(9)}$$

**Definition of** $(\hat{A}_{44})^{(9)},(\hat{B}_{44})^{(9)}$ :

Where $\boxed{(\hat{A}_{44})^{(9)},(\hat{B}_{44})^{(9)},(p_i)^{(9)},\ (r_i)^{(9)}}$ are positive constants and $\boxed{i=44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}',t)-(a_i'')^{(9)}(T_{45},t)|\le(\hat{k}_{44})^{(9)}|T_{45}-\ T_{45}'|e^{-(\hat{M}_{44})^{(9)}t}$$

$$|(b_i'')^{(9)}((G_{47})',t)-(b_i'')^{(9)}\big((G_{47}),t\big)|<(\hat{k}_{44})^{(9)}||(G_{47})-(G_{47})'||e^{-(\hat{M}_{44})^{(9)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}',t)$ and$(a_i'')^{(9)}(T_{45},t)$ . $(T_{45}',t)$ and $(T_{45},t)$ are points belonging to the interval $\big[(\hat{k}_{44})^{(9)},(\hat{M}_{44})^{(9)}\big]$. It is

to be noted that $(a_i'')^{(9)}(T_{45},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{44})^{(9)}=1$ then the function $(a_i'')^{(9)}(T_{45},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\widehat{M}_{44})^{(9)},(\hat{k}_{44})^{(9)}$ :

$(\widehat{M}_{44})^{(9)},(\hat{k}_{44})^{(9)}$, are positive constants

$$\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\ ,\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}<1$$

**Definition of** $(\hat{P}_{44})^{(9)},(\hat{Q}_{44})^{(9)}$ :

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\widehat{M}_{44})^{(9)},(\hat{k}_{44})^{(9)},(\hat{A}_{44})^{(9)} and\ (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)},(a_i')^{(9)},(b_i)^{(9)},(b_i')^{(9)},(p_i)^{(9)},\ (r_i)^{(9)}, i=44,45,46,$
satisfy the inequalities

$$\frac{1}{(\widehat{M}_{44})^{(9)}}[(a_i)^{(9)}+(a_i')^{(9)}+\ (\hat{A}_{44})^{(9)}+\ (\hat{P}_{44})^{(9)}\ (\hat{k}_{44})^{(9)}]<1$$

$$\frac{1}{(\widehat{M}_{44})^{(9)}}[\ (b_i)^{(9)}+(b_i')^{(9)}+\ (\hat{B}_{44})^{(9)}+\ (\hat{Q}_{44})^{(9)}\ (\hat{k}_{44})^{(9)}]<1$$

**Theorem 1:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0)\ ,T_i(0)$ :

$$G_i(t)\le\ (\hat{P}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}\ ,\ \boxed{G_i(0)=G_i^0>0}$$

$$T_i(t)\le\ (\hat{Q}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}\ ,\ \boxed{T_i(0)=T_i^0>0}$$

**Theorem 2 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

**Definition of** $G_i(0)\ ,T_i(0)$

$$G_i(t)\le\ (\hat{P}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}\ ,\ G_i(0)=G_i^0>0$$

$$T_i(t)\le\ (\hat{Q}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}\ ,\ T_i(0)=T_i^0>0$$

**Theorem 3 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t)\le\ (\hat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}\ ,\ G_i(0)=G_i^0>0$$

$$T_i(t)\le\ (\hat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}\ ,\ T_i(0)=T_i^0>0$$

**Theorem 4 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

**Definition of** $G_i(0)\ ,T_i(0)$ :

$G_i(t) \leq \left(\hat{P}_{24}\right)^{(4)} e^{(\hat{M}_{24})^{(4)}t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{24}\right)^{(4)} e^{(\hat{M}_{24})^{(4)}t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Theorem 5 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

**Definition of** $G_i(0)$ , $T_i(0)$ :

$G_i(t) \leq \left(\hat{P}_{28}\right)^{(5)} e^{(\hat{M}_{28})^{(5)}t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{28}\right)^{(5)} e^{(\hat{M}_{28})^{(5)}t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Theorem 6 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

**Definition of** $G_i(0)$ , $T_i(0)$ :

$G_i(t) \leq \left(\hat{P}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Theorem 7:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

**Definition of** $G_i(0)$ , $T_i(0)$ :

$G_i(t) \leq \left(\hat{P}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Theorem 8:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

**Definition of** $G_i(0)$ , $T_i(0)$ :

$G_i(t) \leq \left(\hat{P}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Theorem 9:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

**Definition of** $G_i(0)$ , $T_i(0)$ :

$G_i(t) \leq \left(\hat{P}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Proof:** Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i$: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \le (\hat{P}_{13})^{(1)} \text{ , } T_i^0 \le (\hat{Q}_{13})^{(1)}, \quad 155$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t} \quad 156$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t} \quad 157$$

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}(s_{(13)}) - \left((a'_{13})^{(1)} + a''_{13})^{(1)} (T_{14}(s_{(13)}), s_{(13)})\right) G_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)} (T_{14}(s_{(13)}), s_{(13)})\right) G_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)} (T_{14}(s_{(13)}), s_{(13)})\right) G_{15}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)} (G(s_{(13)}), s_{(13)})\right) T_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)} (G(s_{(13)}), s_{(13)})\right) T_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{15}(t) = T_{15}^0 + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)} (G(s_{(13)}), s_{(13)})\right) T_{15}(s_{(13)})\right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i$: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \le (\hat{P}_{16})^{(2)} \text{ , } T_i^0 \le (\hat{Q}_{16})^{(2)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$$

By 160

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''_{16})^{(2)} (T_{17}(s_{(16)}), s_{(16)})\right) G_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)} (T_{17}(s_{(16)}), s_{(17)})\right) G_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)} (T_{17}(s_{(16)}), s_{(16)})\right) G_{18}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)} (G_{19}(s_{(16)}), s_{(16)})\right) T_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^{0} + \int_0^t \left[ (b_{17})^{(2)} T_{16}(s_{(16)}) - \left( (b'_{17})^{(2)} - (b''_{17})^{(2)} (G_{19}(s_{(16)}), s_{(16)}) \right) T_{17}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^{0} + \int_0^t \left[ (b_{18})^{(2)} T_{17}(s_{(16)}) - \left( (b'_{18})^{(2)} - (b''_{18})^{(2)} (G_{19}(s_{(16)}), s_{(16)}) \right) T_{18}(s_{(16)}) \right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{20})^{(3)}, T_i^0 \le (\hat{Q}_{20})^{(3)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$$

By 161

$$\bar{G}_{20}(t) = G_{20}^0 + \int_0^t \left[ (a_{20})^{(3)} G_{21}(s_{(20)}) - \left( (a'_{20})^{(3)} + a''_{20})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[ (a_{21})^{(3)} G_{20}(s_{(20)}) - \left( (a'_{21})^{(3)} + (a''_{21})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{21}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[ (a_{22})^{(3)} G_{21}(s_{(20)}) - \left( (a'_{22})^{(3)} + (a''_{22})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{22}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^0 + \int_0^t \left[ (b_{20})^{(3)} T_{21}(s_{(20)}) - \left( (b'_{20})^{(3)} - (b''_{20})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^0 + \int_0^t \left[ (b_{21})^{(3)} T_{20}(s_{(20)}) - \left( (b'_{21})^{(3)} - (b''_{21})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{21}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{22}(t) = T_{22}^0 + \int_0^t \left[ (b_{22})^{(3)} T_{21}(s_{(20)}) - \left( (b'_{22})^{(3)} - (b''_{22})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{22}(s_{(20)}) \right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{24})^{(4)}, T_i^0 \le (\hat{Q}_{24})^{(4)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

By 162

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[ (a_{24})^{(4)} G_{25}(s_{(24)}) - \left( (a'_{24})^{(4)} + a''_{24})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{26}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{26}(t) = T_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{26}(s_{(24)})\right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{28})^{(5)}, T_i^0 \le (\hat{Q}_{28})^{(5)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

By 163

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{30}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{30}(t) = T_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{30}(s_{(28)})\right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{32})^{(6)}, T_i^0 \le (\hat{Q}_{32})^{(6)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\bar{M}_{32})^{(6)} t}$

By 164

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[ (a_{32})^{(6)} G_{33}(s_{(32)}) - \left( (a'_{32})^{(6)} + a''_{32})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[ (a_{33})^{(6)} G_{32}(s_{(32)}) - \left( (a'_{33})^{(6)} + (a''_{33})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[ (a_{34})^{(6)} G_{33}(s_{(32)}) - \left( (a'_{34})^{(6)} + (a''_{34})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{34}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[ (b_{32})^{(6)} T_{33}(s_{(32)}) - \left( (b'_{32})^{(6)} - (b''_{32})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[ (b_{33})^{(6)} T_{32}(s_{(32)}) - \left( (b'_{33})^{(6)} - (b''_{33})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{\mathrm{T}}_{34}(\mathrm{t}) = \mathrm{T}_{34}^0 + \int_0^t \left[ (b_{34})^{(6)} T_{33}(s_{(32)}) - \left( (b'_{34})^{(6)} - (b''_{34})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{34}(s_{(32)}) \right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[ (a_{36})^{(7)} G_{37}(s_{(36)}) - \left( (a'_{36})^{(7)} + a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[ (a_{37})^{(7)} G_{36}(s_{(36)}) - \left( (a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[ (a_{38})^{(7)} G_{37}(s_{(36)}) - \left( (a'_{38})^{(7)} + (a''_{38})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{38}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[ (b_{36})^{(7)} T_{37}(s_{(36)}) - \left( (b'_{36})^{(7)} - (b''_{36})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[ (b_{37})^{(7)} T_{36}(s_{(36)}) - \left( (b'_{37})^{(7)} - (b''_{37})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^0 + \int_0^t \left[ (b_{38})^{(7)} T_{37}(s_{(36)}) - \left( (b'_{38})^{(7)} - (b''_{38})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{38}(s_{(36)}) \right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0,t)$

**Proof:**

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{40})^{(8)}$, $T_i^0 \le (\hat{Q}_{40})^{(8)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} G_{41}(s_{(40)}) - \left((a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[(a_{41})^{(8)} G_{40}(s_{(40)}) - \left((a'_{41})^{(8)} + (a''_{41})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{42}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^0 + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)} (G_{43}(s_{(40)}), s_{(40)})\right) T_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^0 + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)} (G_{43}(s_{(40)}), s_{(40)})\right) T_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{42}(t) = T_{42}^0 + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)} (G_{43}(s_{(40)}), s_{(40)})\right) T_{42}(s_{(40)})\right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0,t)$

**Proof:** 166
Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ A
which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{44})^{(9)}$, $T_i^0 \le (\hat{Q}_{44})^{(9)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)})\right) G_{44}(s_{(44)})\right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)} (T_{45}(s_{(44)}), s_{(44)})\right) G_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}(s_{(44)}), s_{(44)})\right) G_{46}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{44}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{\mathrm{T}}_{46}(\mathrm{t}) = \mathrm{T}_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{46}(s_{(44)})\right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}}\right)\right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left(e^{(\hat{M}_{13})^{(1)} t} - 1\right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\hat{M}_{13})^{(1)} t} \le \frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{P}_{13})^{(1)} + G_{14}^0\right) e^{\left(-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0}\right)} + (\hat{P}_{13})^{(1)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \le G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}}\right)\right] ds_{(16)} = \quad 169$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1\right)$$

From which it follows that 170

$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \le \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left[\left((\hat{P}_{16})^{(2)} + G_{17}^0\right) e^{\left(-\frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0}\right)} + (\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t) \le G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} \left(G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} s_{(20)}}\right)\right] ds_{(20)} =$$

$$\left(1 + (a_{20})^{(3)} t\right) G_{21}^0 + \frac{(a_{20})^{(3)} (\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left(e^{(\hat{M}_{20})^{(3)} t} - 1\right)$$

From which it follows that 172

$$(G_{20}(t)-G_{20}^{0})e^{-(\hat{M}_{20})^{(3)}t}\leq\frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left[((\hat{P}_{20})^{(3)}+G_{21}^{0})e^{\left(-\frac{(\hat{P}_{20})^{(3)}+G_{21}^{0}}{G_{21}^{0}}\right)}+(\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 173

$$G_{24}(t)\leq G_{24}^{0}+\int_{0}^{t}\left[(a_{24})^{(4)}\left(G_{25}^{0}+(\hat{P}_{24})^{(4)}e^{(\hat{M}_{24})^{(4)}s_{(24)}}\right)\right]ds_{(24)}=$$

$$\left(1+(a_{24})^{(4)}t\right)G_{25}^{0}+\frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left(e^{(\hat{M}_{24})^{(4)}t}-1\right)$$

From which it follows that 174

$$(G_{24}(t)-G_{24}^{0})e^{-(\hat{M}_{24})^{(4)}t}\leq\frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left[((\hat{P}_{24})^{(4)}+G_{25}^{0})e^{\left(-\frac{(\hat{P}_{24})^{(4)}+G_{25}^{0}}{G_{25}^{0}}\right)}+(\hat{P}_{24})^{(4)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t)\leq G_{28}^{0}+\int_{0}^{t}\left[(a_{28})^{(5)}\left(G_{29}^{0}+(\hat{P}_{28})^{(5)}e^{(\hat{M}_{28})^{(5)}s_{(28)}}\right)\right]ds_{(28)}=$$

$$\left(1+(a_{28})^{(5)}t\right)G_{29}^{0}+\frac{(a_{28})^{(5)}(\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left(e^{(\hat{M}_{28})^{(5)}t}-1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t}\leq\frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t)\leq G_{32}^{0}+\int_{0}^{t}\left[(a_{32})^{(6)}\left(G_{33}^{0}+(\hat{P}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\right)\right]ds_{(32)}=$$

$$\left(1+(a_{32})^{(6)}t\right)G_{33}^{0}+\frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left(e^{(\hat{M}_{32})^{(6)}t}-1\right)$$

From which it follows that 177

$$(G_{32}(t)-G_{32}^{0})e^{-(\hat{M}_{32})^{(6)}t}\leq\frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left[((\hat{P}_{32})^{(6)}+G_{33}^{0})e^{\left(-\frac{(\hat{P}_{32})^{(6)}+G_{33}^{0}}{G_{33}^{0}}\right)}+(\hat{P}_{32})^{(6)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}$ , $G_{26}, T_{24}, T_{25}, T_{26}$

(dd) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t) \le G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} \left(G_{37}^0 + (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} s_{(36)}}\right)\right] ds_{(36)} =$$
$$\left(1 + (a_{36})^{(7)} t\right) G_{37}^0 + \frac{(a_{36})^{(7)} (\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left(e^{(\hat{M}_{36})^{(7)} t} - 1\right)$$

From which it follows that

$$(G_{36}(t) - G_{36}^0) e^{-(\hat{M}_{36})^{(7)} t} \le \frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left[((\hat{P}_{36})^{(7)} + G_{37}^0) e^{\left(-\frac{(\hat{P}_{36})^{(7)} + G_{37}^0}{G_{37}^0}\right)} + (\hat{P}_{36})^{(7)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t) \le G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} \left(G_{41}^0 + (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} s_{(40)}}\right)\right] ds_{(40)} = \quad 180$$
$$\left(1 + (a_{40})^{(8)} t\right) G_{41}^0 + \frac{(a_{40})^{(8)} (\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}} \left(e^{(\hat{M}_{40})^{(8)} t} - 1\right)$$

From which it follows that 181

$$(G_{40}(t) - G_{40}^0) e^{-(\hat{M}_{40})^{(8)} t} \le \frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}} \left[((\hat{P}_{40})^{(8)} + G_{41}^0) e^{\left(-\frac{(\hat{P}_{40})^{(8)} + G_{41}^0}{G_{41}^0}\right)} + (\hat{P}_{40})^{(8)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 8

Analogous inequalities hold also for $G_{41}$ , $G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t) \le G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} \left(G_{45}^0 + (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} s_{(44)}}\right)\right] ds_{(44)} =$$
$$\left(1 + (a_{44})^{(9)} t\right) G_{45}^0 + \frac{(a_{44})^{(9)} (\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left(e^{(\hat{M}_{44})^{(9)} t} - 1\right)$$

From which it follows that

$$(G_{44}(t) - G_{44}^0) e^{-(\hat{M}_{44})^{(9)} t} \le \frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left[((\hat{P}_{44})^{(9)} + G_{45}^0) e^{\left(-\frac{(\hat{P}_{44})^{(9)} + G_{45}^0}{G_{45}^0}\right)} + (\hat{P}_{44})^{(9)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 9

Analogous inequalities hold also for $G_{45}$ , $G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}$ , $\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$ and to choose 182

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\widehat{M}_{13})^{(1)}}\left[(\widehat{P}_{13})^{(1)}+\left((\hat{P}_{13})^{(1)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{13})^{(1)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{13})^{(1)} \quad 183$$

$$\frac{(b_i)^{(1)}}{(\widehat{M}_{13})^{(1)}}\left[\left((\hat{Q}_{13})^{(1)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{13})^{(1)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{13})^{(1)}\right]\leq(\hat{Q}_{13})^{(1)} \quad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)},T^{(1)}\right),\left(G^{(2)},T^{(2)}\right)\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\tilde{M}_{13})^{(1)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\tilde{M}_{13})^{(1)}t}\}$$

Indeed if we denote

**Definition of** $\tilde{G},\tilde{T}:\left(\tilde{G},\tilde{T}\right)=\mathcal{A}^{(1)}(G,T)$

It results

$$\left|\tilde{G}_{13}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{13})^{(1)}\left|G_{14}^{(1)}-G_{14}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\,ds_{(13)}+$$

$$\int_0^t\{(a'_{13})^{(1)}\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$G_{13}^{(2)}\left|(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)-(a''_{13})^{(1)}\left(T_{14}^{(2)},s_{(13)}\right)\right|\ e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses it follows

$$\left|G^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}t}\leq \quad 186$$
$$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)}+(a'_{13})^{(1)}+(\widehat{A}_{13})^{(1)}+(\widehat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}\right)d\left(\left(G^{(1)},T^{(1)};\,G^{(2)},T^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

**Remark 1:** The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}\ and\ (\widehat{Q}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i=13,14,15$ depend only on $\mathrm{T}_{14}$ and respectively on

$G(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$

From 19 to 24 it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(1)} - (a_i'')^{(1)}(T_{14}(s_{(13)}), s_{(13)})\} ds_{(13)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(1)} t)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2\ and\ \left((\widehat{M}_{13})^{(1)}\right)_3$ : 187

**Remark 3**: if $G_{13}$ is bounded, the same property have also $G_{14}\ and\ G_{15}$ . indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a_{14}')^{(1)} G_{14}$ and by integrating

$$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1 / (a_{14}')^{(1)}$$

In the same way , one can obtain

$$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2 / (a_{15}')^{(1)}$$

If $G_{14}\ or\ G_{15}$ is bounded, the same property follows for $G_{13}$ , $G_{15}$ and $G_{13}$ , $G_{14}$ respectively.

**Remark 4:** If $G_{13}$ $is$ bounded, from below, the same property holds for $G_{14}\ and\ G_{15}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{14}$ is bounded from below. 188

**Remark 5:** If $T_{13}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}\,(G(t), t)) = (b_{14}')^{(1)}$ then $T_{14} \to \infty$. 189

**Definition of** $(m)^{(1)}$ and $\varepsilon_1$ :

Indeed let $t_1$ be so that for $t > t_1$

$$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t), t) < \varepsilon_1, T_{13}\,(t) > (m)^{(1)}$$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right)$, $t = log\,\frac{2}{\varepsilon_1}$ By taking now $\varepsilon_1$ sufficiently small one sees that $T_{14}$ is unbounded.
The same property holds for $T_{15}$ if $\lim_{t\to\infty}(b_{15}'')^{(1)}\,(G(t), t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}$ , $\frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}\ and\ (\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[(\widehat{P}_{16})^{(2)}+\left((\hat{P}_{16})^{(2)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{16})^{(2)}\right]\leq(\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

**Definition of** $\widetilde{G_{19}},\widetilde{T_{19}}$ : $\left(\widetilde{G_{19}},\widetilde{T_{19}}\right)=\mathcal{A}^{(2)}(G_{19},T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{16})^{(2)}\left|G_{17}^{(1)}-G_{17}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)}+$$

$$\int_0^t\{(a'_{16})^{(2)}\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$G_{16}^{(2)}\left|(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)-(a''_{16})^{(2)}\left(T_{17}^{(2)},s_{(16)}\right)\right|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0,t]$ 197

From the hypotheses it follows

$$\left|(G_{19})^{(1)}-(G_{19})^{(2)}\right|\mathrm{e}^{-(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}\leq$$
$$\frac{1}{(\widehat{\mathrm{M}}_{16})^{(2)}}\left((a_{16})^{(2)}+(a'_{16})^{(2)}+(\widehat{\mathrm{A}}_{16})^{(2)}+(\widehat{\mathrm{P}}_{16})^{(2)}(\hat{k}_{16})^{(2)}\right)\mathrm{d}\left(((G_{19})^{(1)},(T_{19})^{(1)};\ (G_{19})^{(2)},(T_{19})^{(2)})\right)$$

And analogous inequalities for $G_i$ and $T_i$. Taking into account the hypothesis the result follows 198

**Remark 6:** The fact that we supposed $(a''_{16})^{(2)}$ and $(b''_{16})^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{\mathrm{P}}_{16})^{(2)}\mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}$ and $(\widehat{\mathrm{Q}}_{16})^{(2)}\mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i=16,17,18$ depend only on $\mathrm{T}_{17}$ and respectively on

$(G_{19})$(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 7:** There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$ 200

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(2)} - (a_i'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})\} ds_{(16)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(2)}t)} > 0 \quad \text{for } t > 0$$

**Definition of** $\left((\widehat{M}_{16})^{(2)}\right)_1$, $\left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$: 201

**Remark 8:** if $G_{16}$ is bounded, the same property have also $G_{17}$ and $G_{18}$. indeed if

$G_{16} < (\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt} \leq \left((\widehat{M}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)} G_{17}$ and by integrating

$$G_{17} \leq \left((\widehat{M}_{16})^{(2)}\right)_2 = G_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1 / (a_{17}')^{(2)}$$

In the same way , one can obtain

$$G_{18} \leq \left((\widehat{M}_{16})^{(2)}\right)_3 = G_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2 / (a_{18}')^{(2)}$$

If $G_{17}$ or $G_{18}$ is bounded, the same property follows for $G_{16}$, $G_{18}$ and $G_{16}$, $G_{17}$ respectively.

**Remark 9:** If $G_{16}$ is bounded, from below, the same property holds for $G_{17}$ and $G_{18}$. The proof is analogous with the preceding one. An analogous property is true if $G_{17}$ is bounded from below. 202

**Remark 10:** If $T_{16}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t), t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$. 203

**Definition of** $(m)^{(2)}$ and $\varepsilon_2$ :

Indeed let $t_2$ be so that for $t > t_2$

$$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right)$, $t = \log\frac{2}{\varepsilon_2}$ By taking now $\varepsilon_2$ sufficiently small one sees that $T_{17}$ is unbounded. 205
The same property holds for $T_{18}$ if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t), t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[(\widehat{P}_{20})^{(3)}+\left((\hat{P}_{20})^{(3)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{20})^{(3)}\right]\leq(\hat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself 210

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)},(T_{23})^{(1)}),((G_{23})^{(2)},(T_{23})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

**Definition of** $\widetilde{G_{23}},\widetilde{T_{23}}:(\widetilde{(G_{23})},\widetilde{(T_{23})})=\mathcal{A}^{(3)}((G_{23}),(T_{23}))$

It results 213

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{20})^{(3)}\left|G_{21}^{(1)}-G_{21}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\,ds_{(20)}+$$

$$\int_0^t\{(a'_{20})^{(3)}\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$G_{20}^{(2)}\left|(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)},s_{(20)}\right)\right|\ e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\}ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses it follows

$$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}t}\leq \quad 214$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+(a'_{20})^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\widehat{k}_{20})^{(3)}\right)d\left(((G_{23})^{(1)},(T_{23})^{(1)};\ (G_{23})^{(2)},(T_{23})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 11:** The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ *and* $(\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i=20,21,22$ depend only on $T_{21}$ and respectively on

$(G_{23})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 12:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 216

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(3)} - (a_i'')^{(3)}(T_{21}(s_{(20)}), s_{(20)})\} ds_{(20)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(3)} t)} > 0$ for t > 0

**Definition of** $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2\ and\ \left((\widehat{M}_{20})^{(3)}\right)_3$ : 217

**Remark 13:** if $G_{20}$ is bounded, the same property have also $G_{21}\ and\ G_{22}$ . indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a_{21}')^{(3)} G_{21}$ and by integrating

$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1 / (a_{21}')^{(3)}$

In the same way , one can obtain

$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2 / (a_{22}')^{(3)}$

If $G_{21}\ or\ G_{22}$ is bounded, the same property follows for $G_{20}$ , $G_{22}$ and $G_{20}$ , $G_{21}$ respectively.

**Remark 14:** If $G_{20}$ *is* bounded, from below, the same property holds for $G_{21}\ and\ G_{22}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{21}$ is bounded from below. 218

**Remark 15:** If $T_{20}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t), t\right)) = (b_{21}')^{(3)}$ then $T_{21} \to \infty$. 219

**Definition of** $(m)^{(3)}$ and $\varepsilon_3$ :

Indeed let $t_3$ be so that for $t > t_3$

$(b_{21})^{(3)} - (b_i'')^{(3)}\left((G_{23})(t), t\right) < \varepsilon_3, T_{20}\,(t) > (m)^{(3)}$

Then $\frac{dT_{21}}{dt} \geq (a_{21})^{(3)}(m)^{(3)} - \varepsilon_3 T_{21}$ which leads to 220

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t} = \frac{1}{2}$ it results

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_3}$ By taking now $\varepsilon_3$ sufficiently small one sees that $T_{21}$ is unbounded.
The same property holds for $T_{22}$ if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t), t\right) = (b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}$ , $\frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}} < 1$ and to choose 221

$(\widehat{P}_{24})^{(4)}$ and $(\widehat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[(\widehat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \tag{222}$$

$$\frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \tag{223}$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself (224)

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric (225)

$$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{24})^{(4)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{27}}),(\widetilde{T_{27}})$ : $\left((\widetilde{G_{27}}),(\widetilde{T_{27}})\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{24})^{(4)}\left|G_{25}^{(1)}-G_{25}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\,ds_{(24)}+$$

$$\int_0^t\{(a'_{24})^{(4)}\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$G_{24}^{(2)}\left|(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)-(a''_{24})^{(4)}\left(T_{25}^{(2)},s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\}ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}t}\leq \tag{226}$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)}+(a'_{24})^{(4)}+(\widehat{A}_{24})^{(4)}+(\widehat{P}_{24})^{(4)}(\hat{k}_{24})^{(4)}\right)d\left(((G_{27})^{(1)},(T_{27})^{(1)};\ (G_{27})^{(2)},(T_{27})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 16:** The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ *and* $(\widehat{Q}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_+$. (227)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i=24,25,26$ depend only on $T_{25}$ and respectively on

$(G_{27})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 17:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 228

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(4)} - (a_i'')^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(4)} t)} > 0$ for t > 0

**Definition of** $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2\ and\ \left((\widehat{M}_{24})^{(4)}\right)_3$ : 229

**Remark 18:** if $G_{24}$ is bounded, the same property have also $G_{25}\ and\ G_{26}$ . indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a_{25}')^{(4)} G_{25}$ and by integrating

$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1 / (a_{25}')^{(4)}$

In the same way , one can obtain

$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2 / (a_{26}')^{(4)}$

If $G_{25}\ or\ G_{26}$ is bounded, the same property follows for $G_{24}$ , $G_{26}$ and $G_{24}$ , $G_{25}$ respectively.

**Remark 19:** If $G_{24}$ *is* bounded, from below, the same property holds for $G_{25}\ and\ G_{26}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{25}$ is bounded from below. 230

**Remark 20:** If $T_{24}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}\ ((G_{27})(t), t)) = (b_{25}')^{(4)}$ then $T_{25} \to \infty$. 231

**Definition of** $(m)^{(4)}$ and $\varepsilon_4$ :

Indeed let $t_4$ be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t), t) < \varepsilon_4, T_{24}\,(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_4}$ By taking now $\varepsilon_4$ sufficiently small one sees that $T_{25}$ is unbounded.
The same property holds for $T_{26}$ if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t), t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for $G_{29}$ , $G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}$ , $\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)}+\left((\hat{P}_{28})^{(5)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{28})^{(5)}\right]\leq(\hat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$$d\left(((G_{31})^{(1)},(T_{31})^{(1)}),((G_{31})^{(2)},(T_{31})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\hat{M}_{28})^{(5)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\hat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{31}}),(\widetilde{T_{31}})$ : $\left((\widetilde{G_{31}}),(\widetilde{T_{31}})\right)=\mathcal{A}^{(5)}\left((G_{31}),(T_{31})\right)$

It results

$$|\tilde{G}_{28}^{(1)}-\tilde{G}_i^{(2)}|\leq\int_0^t(a_{28})^{(5)}|G_{29}^{(1)}-G_{29}^{(2)}|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\,ds_{(28)}+$$

$$\int_0^t\{(a'_{28})^{(5)}|G_{28}^{(1)}-G_{28}^{(2)}|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$(a''_{28})^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)|G_{28}^{(1)}-G_{28}^{(2)}|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$G_{28}^{(2)}|(a''_{28})^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)-(a''_{28})^{(5)}\left(T_{29}^{(2)},s_{(28)}\right)|\;e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\}ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on it follows

$$|(G_{31})^{(1)}-(G_{31})^{(2)}|e^{-(\widehat{M}_{28})^{(5)}t}\leq \quad 237$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)}+(a'_{28})^{(5)}+(\widehat{A}_{28})^{(5)}+(\widehat{P}_{28})^{(5)}(\widehat{k}_{28})^{(5)}\right)d\left(((G_{31})^{(1)},(T_{31})^{(1)};\;(G_{31})^{(2)},(T_{31})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 21:** The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\hat{P}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ *and* $(\hat{Q}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ respectively of $\mathbb{R}_+$. 238

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on $T_{29}$ and respectively on $(G_{31})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 22:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 239

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(5)} - (a_i'')^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(5)} t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2\ and\ \left((\widehat{M}_{28})^{(5)}\right)_3$ : 240

**Remark 23:** if $G_{28}$ is bounded, the same property have also $G_{29}\ and\ G_{30}$ . indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a_{29}')^{(5)} G_{29}$ and by integrating

$$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1 / (a_{29}')^{(5)}$$

In the same way , one can obtain

$$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2 / (a_{30}')^{(5)}$$

If $G_{29}\ or\ G_{30}$ is bounded, the same property follows for $G_{28}$ , $G_{30}$ and $G_{28}$ , $G_{29}$ respectively.

**Remark 24:** If $G_{28}$ *is* bounded, from below, the same property holds for $G_{29}\ and\ G_{30}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{29}$ is bounded from below. 241

**Remark 25:** If $T_{28}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}\,((G_{31})(t), t)) = (b_{29}')^{(5)}$ then $T_{29} \to \infty$. 242

**Definition of** $(m)^{(5)}$ and $\varepsilon_5$ :

Indeed let $t_5$ be so that for $t > t_5$

$$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t), t) < \varepsilon_5, T_{28}\,(t) > (m)^{(5)}$$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_5}$ By taking now $\varepsilon_5$ sufficiently small one sees that $T_{29}$ is unbounded.
The same property holds for $T_{30}$ if $\lim_{t\to\infty}(b_{30}'')^{(5)}\left((G_{31})(t), t\right) = (b_{30}')^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{33}\,, G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\ , \frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}} < 1$ and to choose 244

$(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[(\hat{P}_{32})^{(6)}+\left((\hat{P}_{32})^{(6)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{32})^{(6)} \tag{245}$$

$$\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[\left((\hat{Q}_{32})^{(6)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{32})^{(6)}\right]\leq(\hat{Q}_{32})^{(6)} \tag{246}$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric (247)

$$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\hat{M}_{32})^{(6)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\hat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{35}}),(\widetilde{T_{35}})$ : $\left((\widetilde{G_{35}}),(\widetilde{T_{35}})\right)=\mathcal{A}^{(6)}\left((G_{35}),(T_{35})\right)$

It results

$$|\tilde{G}_{32}^{(1)}-\tilde{G}_i^{(2)}|\leq\int_0^t(a_{32})^{(6)}|G_{33}^{(1)}-G_{33}^{(2)}|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\,ds_{(32)}+$$

$$\int_0^t\{(a'_{32})^{(6)}|G_{32}^{(1)}-G_{32}^{(2)}|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)|G_{32}^{(1)}-G_{32}^{(2)}|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$G_{32}^{(2)}|(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)-(a''_{32})^{(6)}\left(T_{33}^{(2)},s_{(32)}\right)|\;e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\}ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$|(G_{35})^{(1)}-(G_{35})^{(2)}|e^{-(\widehat{M}_{32})^{(6)}t}\leq \tag{248}$$
$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)}+(a'_{32})^{(6)}+(\widehat{A}_{32})^{(6)}+(\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}\right)d\left(((G_{35})^{(1)},(T_{35})^{(1)};\;(G_{35})^{(2)},(T_{35})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 26:** The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\hat{P}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ *and* $(\hat{Q}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ respectively of $\mathbb{R}_+$. (249)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i = 32,33,34$ depend only on $T_{33}$ and respectively on $(G_{35})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 27:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 250

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(6)} - (a_i'')^{(6)}(T_{33}(s_{(32)}), s_{(32)})\} ds_{(32)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(6)} t\right)} > 0$ for t > 0

**Definition of** $\left((\widehat{M}_{32})^{(6)}\right)_1, \left((\widehat{M}_{32})^{(6)}\right)_2\ and\ \left((\widehat{M}_{32})^{(6)}\right)_3$ : 251

**Remark 28:** if $G_{32}$ is bounded, the same property have also $G_{33}\ and\ G_{34}$ . indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a_{33}')^{(6)} G_{33}$ and by integrating

$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_1 / (a_{33}')^{(6)}$

In the same way , one can obtain

$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_2 / (a_{34}')^{(6)}$

If $G_{33}\ or\ G_{34}$ is bounded, the same property follows for $G_{32}$ , $G_{34}$ and $G_{32}$ , $G_{33}$ respectively.

**Remark 29:** If $G_{32}$ *is* bounded, from below, the same property holds for $G_{33}\ and\ G_{34}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{33}$ is bounded from below. 252

**Remark 30:** If $T_{32}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}\,((G_{35})(t), t)) = (b_{33}')^{(6)}$ then $T_{33} \to \infty$. 253

**Definition of** $(m)^{(6)}$ and $\varepsilon_6$ :

Indeed let $t_6$ be so that for $t > t_6$

$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t), t\right) < \varepsilon_6, T_{32}\,(t) > (m)^{(6)}$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_6}$ By taking now $\varepsilon_6$ sufficiently small one sees that $T_{33}$ is unbounded.
The same property holds for $T_{34}$ if $\lim_{t\to\infty}(b_{34}'')^{(6)}\left((G_{35})(t), t(t), t\right) = (b_{34}')^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{37}, G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}$ , $\frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$ and to choose

$(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[(\hat{P}_{36})^{(7)}+\left((\hat{P}_{36})^{(7)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{36})^{(7)} \tag{256}$$

$$\frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[\left((\hat{Q}_{36})^{(7)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{36})^{(7)}\right]\leq(\hat{Q}_{36})^{(7)} \tag{257}$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric (258)

$$d\left(((G_{39})^{(1)},(T_{39})^{(1)}),((G_{39})^{(2)},(T_{39})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\tilde{M}_{36})^{(7)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\tilde{M}_{36})^{(7)}t}\}$$

Indeed if we denote

**Definition of** $\widetilde{(G_{39})},\widetilde{(T_{39})}:(\widetilde{(G_{39})},\widetilde{(T_{39})})=\mathcal{A}^{(7)}((G_{39}),(T_{39}))$

It results

$$|\tilde{G}_{36}^{(1)}-\tilde{G}_i^{(2)}|\leq\int_0^t(a_{36})^{(7)}|G_{37}^{(1)}-G_{37}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\,ds_{(36)}+$$
$$\int_0^t\{(a'_{36})^{(7)}|G_{36}^{(1)}-G_{36}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$(a''_{36})^{(7)}(T_{37}^{(1)},s_{(36)})|G_{36}^{(1)}-G_{36}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$G_{36}^{(2)}|(a''_{36})^{(7)}(T_{37}^{(1)},s_{(36)})-(a''_{36})^{(7)}(T_{37}^{(2)},s_{(36)})|\;e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\}ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on it follows

$$|(G_{39})^{(1)}-(G_{39})^{(2)}|e^{-(\widehat{M}_{36})^{(7)}t}\leq \tag{259}$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}}\left((a_{36})^{(7)}+(a'_{36})^{(7)}+(\widehat{A}_{36})^{(7)}+(\widehat{P}_{36})^{(7)}(\widehat{k}_{36})^{(7)}\right)d\left(((G_{39})^{(1)},(T_{39})^{(1)};\;(G_{39})^{(2)},(T_{39})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 31:** The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ *and* $(\widehat{Q}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ respectively of $\mathbb{R}_+$. (260)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(7)}$ and $(b_i'')^{(7)}, i = 36,37,38$ depend only on $T_{37}$ and respectively on $(G_{39})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 32:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 261

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(7)} - (a_i'')^{(7)}(T_{37}(s_{(36)}), s_{(36)})\} ds_{(36)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(7)} t\right)} > 0$ for t > 0

**Definition of** $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2\ and\ \left((\widehat{M}_{36})^{(7)}\right)_3$ : 262

**Remark 33:** if $G_{36}$ is bounded, the same property have also $G_{37}\ and\ G_{38}$ . indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a_{37}')^{(7)} G_{37}$ and by integrating

$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1 / (a_{37}')^{(7)}$

In the same way , one can obtain

$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2 / (a_{38}')^{(7)}$

If $G_{37}\ or\ G_{38}$ is bounded, the same property follows for $G_{36}$ , $G_{38}$ and $G_{36}$ , $G_{37}$ respectively.

**Remark 34:** If $G_{36}$ *is* bounded, from below, the same property holds for $G_{37}\ and\ G_{38}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{37}$ is bounded from below. 263

**Remark 35:** If $T_{36}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(7)}\ ((G_{39})(t), t)) = (b_{37}')^{(7)}$ then $T_{37} \to \infty$. 264

**Definition of** $(m)^{(7)}$ and $\varepsilon_7$ :

Indeed let $t_7$ be so that for $t > t_7$

$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t), t) < \varepsilon_7, T_{36}\,(t) > (m)^{(7)}$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_7}$ By taking now $\varepsilon_7$ sufficiently small one sees that $T_{37}$ is unbounded.
The same property holds for $T_{38}$ if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t), t\right) = (b_{38}')^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}$ , $\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}} < 1$ and to choose $(\widehat{P}_{40})^{(8)}$ and $(\widehat{Q}_{40})^{(8)}$ large to have 266

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[(\widehat{P}_{40})^{(8)} + \left((\hat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{40})^{(8)} \quad 267$$

$$\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{40})^{(8)}\right] \leq (\hat{Q}_{40})^{(8)} \quad 268$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)}, (T_{43})^{(1)}), ((G_{43})^{(2)}, (T_{43})^{(2)})\right) = \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

**Definition of** $\widetilde{(G_{43})}, \widetilde{(T_{43})}$ : $\left(\widetilde{(G_{43})}, \widetilde{(T_{43})}\right) = \mathcal{A}^{(8)}((G_{43}), (T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{40})^{(8)}\left|G_{41}^{(1)} - G_{41}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\,ds_{(40)} +$$
$$\int_0^t\{(a'_{40})^{(8)}\left|G_{40}^{(1)} - G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{-(\widehat{M}_{40})^{(8)}s_{(40)}} +$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right)\left|G_{40}^{(1)} - G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}} +$$
$$G_{40}^{(2)}\left|(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right) - (a''_{40})^{(8)}\left(T_{41}^{(2)}, s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\}ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0, t]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)} - (G_{43})^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}t} \leq \quad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)} + (a'_{40})^{(8)} + (\widehat{A}_{40})^{(8)} + (\widehat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}\right)d\left(((G_{43})^{(1)}, (T_{43})^{(1)};\ (G_{43})^{(2)}, (T_{43})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

**Remark 36:** The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate 274

condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ *and* $(\widehat{Q}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i = 40{,}41{,}42$ depend only on $T_{41}$ and respectively on $(G_{43})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 37** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 275

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(8)} - (a_i'')^{(8)}(T_{41}(s_{(40)}), s_{(40)})\} ds_{(40)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(8)}t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{40})^{(8)}\right)_1, \left((\widehat{M}_{40})^{(8)}\right)_2\ and\ \left((\widehat{M}_{40})^{(8)}\right)_3$ : 276

**Remark 38:** if $G_{40}$ is bounded, the same property have also $G_{41}\ and\ G_{42}$ . indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a_{41}')^{(8)}G_{41}$ and by integrating

$$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1/(a_{41}')^{(8)}$$

In the same way , one can obtain

$$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2/(a_{42}')^{(8)}$$

If $G_{41}\ or\ G_{42}$ is bounded, the same property follows for $G_{40}$ , $G_{42}$ and $G_{40}$ , $G_{41}$ respectively.

**Remark 39:** If $G_{40}$ *is* bounded, from below, the same property holds for $G_{41}\ and\ G_{42}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{41}$ is bounded from below. 277

**Remark 40:** If $T_{40}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}\ ((G_{43})(t), t)) = (b_{41}')^{(8)}$ then $T_{41} \to \infty$. 278

**Definition of** $(m)^{(8)}$ and $\varepsilon_8$ :

Indeed let $t_8$ be so that for $t > t_8$

$$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t), t\right) < \varepsilon_8, T_{40}\,(t) > (m)^{(8)}$$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right)$, $t = log\frac{2}{\varepsilon_8}$ By taking now $\varepsilon_8$ sufficiently small one sees that $T_{41}$ is unbounded.
The same property holds for $T_{42}$ if $\lim_{t\to\infty}(b''_{42})^{(8)}\left((G_{43})(t), t(t), t\right) = (b'_{42})^{(8)}$



It is now sufficient to take $\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}$ , $\frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$ and to choose $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ large to have

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}\left[(\hat{P}_{44})^{(9)} + \left((\hat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}}\left[\left((\hat{Q}_{44})^{(9)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{44})^{(9)}\right] \leq (\hat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)}, (T_{47})^{(1)}), ((G_{47})^{(2)}, (T_{47})^{(2)})\right) =$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{44})^{(9)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{47}}), (\widetilde{T_{47}}) : \left((\widetilde{G_{47}}), (\widetilde{T_{47}})\right) = \mathcal{A}^{(9)}\left((G_{47}), (T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{44})^{(9)}\left|G_{45}^{(1)} - G_{45}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\,ds_{(44)} +$$
$$\int_0^t\{(a'_{44})^{(9)}\left|G_{44}^{(1)} - G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{-(\widehat{M}_{44})^{(9)}s_{(44)}} +$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right)\left|G_{44}^{(1)} - G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}} +$$
$$G_{44}^{(2)}\left|(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right) - (a''_{44})^{(9)}\left(T_{45}^{(2)}, s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\}ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)} - G^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}t} \leq$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)} + (a'_{44})^{(9)} + (\widehat{A}_{44})^{(9)} + (\widehat{P}_{44})^{(9)}(\hat{k}_{44})^{(9)}\right)d\left(((G_{47})^{(1)}, (T_{47})^{(1)};\ (G_{47})^{(2)}, (T_{47})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis (39,35,36) the result follows

**Remark 41:** The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by

$(\widehat{P}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ *and* $(\widehat{Q}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i = 44{,}45{,}46$ depend only on $T_{45}$ and respectively on $(G_{47})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 42:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$

From 99 to 44 it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t\{(a_i')^{(9)}-(a_i'')^{(9)}(T_{45}(s_{(44)}),s_{(44)})\}ds_{(44)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(9)}t)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{44})^{(9)}\right)_1, \left((\widehat{M}_{44})^{(9)}\right)_2$ *and* $\left((\widehat{M}_{44})^{(9)}\right)_3$ :

**Remark 43:** if $G_{44}$ is bounded, the same property have also $G_{45}\ and\ G_{46}$ . indeed if

$G_{44} < (\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt} \leq \left((\widehat{M}_{44})^{(9)}\right)_1 - (a_{45}')^{(9)}G_{45}$ and by integrating

$$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_1/(a_{45}')^{(9)}$$

In the same way , one can obtain

$$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_2/(a_{46}')^{(9)}$$

If $G_{45}\ or\ G_{46}$ is bounded, the same property follows for $G_{44}$ , $G_{46}$ and $G_{44}$ , $G_{45}$ respectively.

**Remark 44:** If $G_{44}$ *is* bounded, from below, the same property holds for $G_{45}\ and\ G_{46}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{45}$ is bounded from below.

**Remark 45:** If $T_{44}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t),t\right) = (b_{45}')^{(9)}$ then $T_{45} \to \infty$.

**Definition of** $(m)^{(9)}$ and $\varepsilon_9$ :

Indeed let $t_9$ be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t),t\right) < \varepsilon_9, T_{44}\,(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right), \quad t = log\frac{2}{\varepsilon_9}$ By taking now $\varepsilon_9$ sufficiently small one sees that $T_{45}$ is unbounded.
The same property holds for $T_{46}$ if $\lim_{t\to\infty}(b_{46}'')^{(9)}\left((G_{47})(t),t\right) = (b_{46}')^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

**Behavior of the solutions of equation** 280

**Theorem** If we denote and define

**Definition of** $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ :

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)} \le -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14}, t) + (a''_{14})^{(1)}(T_{14}, t) \le -(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)} \le -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G, t) - (b''_{14})^{(1)}(G, t) \le -(\tau_1)^{(1)}$

**Definition of** $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$ : 281

By $(\nu_1)^{(1)} > 0$ , $(\nu_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$ , $(u_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

**Definition of** $(\bar{\nu}_1)^{(1)}, , (\bar{\nu}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$ : 282

By $(\bar{\nu}_1)^{(1)} > 0$ , $(\bar{\nu}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$ , $(\bar{u}_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

**Definition of** $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (\nu_0)^{(1)}$ :- 283

If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by
$(m_2)^{(1)} = (\nu_0)^{(1)}, (m_1)^{(1)} = (\nu_1)^{(1)}, \ if\ (\nu_0)^{(1)} < (\nu_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}, if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)},$

and $\boxed{(\nu_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)}, \ if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)}, \ if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)},$

and $\boxed{(u_0)^{(1)} = \frac{T^0_{13}}{T^0_{14}}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$

are defined

Then the solution of global equations satisfies the inequalities 285

$G^0_{13}e^{\left((S_1)^{(1)} - (p_{13})^{(1)}\right)t} \le G_{13}(t) \le G^0_{13}e^{(S_1)^{(1)}t}$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)}-(p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)}t}$$

$$\left(\frac{(a_{15})^{(1)}G_{13}^0}{(m_1)^{(1)}((S_1)^{(1)}-(p_{13})^{(1)}-(S_2)^{(1)})}\left[e^{((S_1)^{(1)}-(p_{13})^{(1)})t} - e^{-(S_2)^{(1)}t}\right] + G_{15}^0 e^{-(S_2)^{(1)}t} \le G_{15}(t) \le \right. \quad (286)$$

$$\left. \frac{(a_{15})^{(1)}G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)}-(a'_{15})^{(1)})}[e^{(S_1)^{(1)}t} - e^{-(a'_{15})^{(1)}t}] + G_{15}^0 e^{-(a'_{15})^{(1)}t}\right)$$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)}t} \le T_{13}(t) \le T_{13}^0 e^{((R_1)^{(1)}+(r_{13})^{(1)})t}} \quad (287)$$

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)}t} \le T_{13}(t) \le \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)}+(r_{13})^{(1)})t} \quad (288)$$

$$\frac{(b_{15})^{(1)}T_{13}^0}{(\mu_1)^{(1)}((R_1)^{(1)}-(b'_{15})^{(1)})}\left[e^{(R_1)^{(1)}t} - e^{-(b'_{15})^{(1)}t}\right] + T_{15}^0 e^{-(b'_{15})^{(1)}t} \le T_{15}(t) \le \quad (289)$$

$$\frac{(a_{15})^{(1)}T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)}+(r_{13})^{(1)}+(R_2)^{(1)})}\left[e^{((R_1)^{(1)}+(r_{13})^{(1)})t} - e^{-(R_2)^{(1)}t}\right] + T_{15}^0 e^{-(R_2)^{(1)}t}$$

**Definition of** $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:- (290)

Where $(S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$

$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$

$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$

$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$

**Behavior of the solutions of equation** (291)

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ : (292)

$(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ four constants satisfying

$$-(\sigma_2)^{(2)} \le -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(T_{17}, t) + (a''_{17})^{(2)}(T_{17}, t) \le -(\sigma_1)^{(2)} \quad (293)$$

$$-(\tau_2)^{(2)} \le -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}((G_{19}), t) - (b''_{17})^{(2)}((G_{19}), t) \le -(\tau_1)^{(2)} \quad (294)$$

**Definition of** $(\nu_1)^{(2)}, (\nu_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$ : (295)

By $(\nu_1)^{(2)} > 0$, $(\nu_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0$, $(u_2)^{(2)} < 0$ the roots (296)

of the equations $(a_{17})^{(2)}(\nu^{(2)})^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ (297)

and $(b_{14})^{(2)}(u^{(2)})^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and (298)

**Definition of** $(\bar{\nu}_1)^{(2)}, , (\bar{\nu}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$ : (299)

By $(\bar{\nu}_1)^{(2)} > 0$, $(\bar{\nu}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0$, $(\bar{u}_2)^{(2)} < 0$ the (300)

roots of the equations $(a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 301

and $(b_{17})^{(2)}\left(u^{(2)}\right)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 302

**Definition of** $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ :- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, ***if*** $(u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$, ***if*** $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)}$, ***if*** $(\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)}t}$

$(p_i)^{(2)}$ is defined by equation

$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)}t}$ 311

$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)}t}\right] + G_{18}^0 e^{-(S_2)^{(2)}t} \le G_{18}(t) \le\right.$ 312

$\left.\frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)}t} - e^{-(a'_{18})^{(2)}t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)}t}\right)$

$\boxed{T_{16}^0 e^{(R_1)^{(2)}t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}}$ 313

$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)}t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}$ 314

$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)}t} - e^{-(b'_{18})^{(2)}t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)}t} \le T_{18}(t) \le$ 315

$$\frac{(a_{18})^{(2)}T_{16}^{0}}{(\mu_2)^{(2)}((R_1)^{(2)}+(r_{16})^{(2)}+(R_2)^{(2)})}\left[e^{((R_1)^{(2)}+(r_{16})^{(2)})t}-e^{-(R_2)^{(2)}t}\right]+T_{18}^{0}e^{-(R_2)^{(2)}t}$$

**Definition of** $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$$

$$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)}$$ 318

$$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$$

**Behavior of the solutions** 319

**Theorem 3:** If we denote and define

**Definition of** $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ :

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}\big((G_{23}), t\big) \le -(\tau_1)^{(3)}$$

**Definition of** $(v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$ : 320

By $(v_1)^{(3)} > 0$, $(v_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{v}_1)^{(3)} > 0$, $(\bar{v}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

**Definition of** $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ :- 321

If we define $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (v_0)^{(3)}, (m_1)^{(3)} = (v_1)^{(3)}$, ***if*** $(v_0)^{(3)} < (v_1)^{(3)}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (\bar{v}_1)^{(3)}$, ***if*** $(v_1)^{(3)} < (v_0)^{(3)} < (\bar{v}_1)^{(3)}$,

and $\boxed{(v_0)^{(3)} = \frac{G_{20}^{0}}{G_{21}^{0}}}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_0)^{(3)}$, ***if*** $(\bar{v}_1)^{(3)} < (v_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)},\ \boldsymbol{if}\ (u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}, \boldsymbol{if}\ (u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)},$ and $\boxed{(u_0)^{(3)} = \frac{T_{20}^0}{T_{21}^0}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)},\ \boldsymbol{if}\ (\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G_{20}^0 e^{(S_1)^{(3)} t}$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)} t} \quad 323$$

$$\left(\frac{(a_{22})^{(3)} G_{20}^0}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\left[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)} t}\right] + G_{22}^0 e^{-(S_2)^{(3)} t} \le G_{22}(t) \le \right. \quad 324$$

$$\left. \frac{(a_{22})^{(3)} G_{20}^0}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}\left[e^{(S_1)^{(3)} t} - e^{-(a'_{22})^{(3)} t}\right] + G_{22}^0 e^{-(a'_{22})^{(3)} t}\right)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t}} \quad 325$$

$$\frac{1}{(\mu_1)^{(3)}} T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}} T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)} T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)} - (b'_{22})^{(3)})}\left[e^{(R_1)^{(3)} t} - e^{-(b'_{22})^{(3)} t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)} t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)} T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)} + (r_{20})^{(3)} + (R_2)^{(3)})}\left[e^{((R_1)^{(3)} + (r_{20})^{(3)})t} - e^{-(R_2)^{(3)} t}\right] + T_{22}^0 e^{-(R_2)^{(3)} t}$$

**Definition of** $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

**Behavior of the solutions of equation**

**Theorem:** If we denote and define

**Definition of** $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ :

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$

$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}((G_{27}), t) - (b''_{25})^{(4)}((G_{27}), t) \le -(\tau_1)^{(4)}$

**Definition of** $(v_1)^{(4)}, (v_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, v^{(4)}, u^{(4)}$ : 329

By $(v_1)^{(4)} > 0$ , $(v_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$ , $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_1)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

**Definition of** $(\bar{v}_1)^{(4)},, (\bar{v}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$ : 330

By $(\bar{v}_1)^{(4)} > 0$ , $(\bar{v}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$ , $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

**Definition of** $(m_1)^{(4)}$ , $(m_2)^{(4)}$ , $(\mu_1)^{(4)}, (\mu_2)^{(4)}, (\nu_0)^{(4)}$ :-

If we define $(m_1)^{(4)}$ , $(m_2)^{(4)}$ , $(\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (v_0)^{(4)}, (m_1)^{(4)} = (v_1)^{(4)}$, ***if*** $(v_0)^{(4)} < (v_1)^{(4)}$

$(m_2)^{(4)} = (v_1)^{(4)}, (m_1)^{(4)} = (\bar{v}_1)^{(4)}$ , ***if*** $(v_4)^{(4)} < (v_0)^{(4)} < (\bar{v}_1)^{(4)}$,
and $\boxed{(v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (v_4)^{(4)}, (m_1)^{(4)} = (v_0)^{(4)}$, ***if*** $(\bar{v}_4)^{(4)} < (v_0)^{(4)}$

and analogously

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$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)}$, ***if*** $(u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}$ , ***if*** $(u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)}$,
and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}$, ***if*** $(\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

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$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)}t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)}t}$$

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$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)}t}\right] + G_{26}^0 e^{-(S_2)^{(4)}t} \le G_{26}(t) \le\right.$$ 334

*(a26)4G240(m2)4(S1)4−(a26′)4e(S1)4t−e−(a26′)4t+ G260e−(a26′)4t*

$$\boxed{T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}}T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}}T_{24}^0 e^{((R_1)^{(4)}+(r_{24})^{(4)})t} \quad 335$$

$$\frac{(b_{26})^{(4)}T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)}-(b'_{26})^{(4)})}\left[e^{(R_1)^{(4)}t} - e^{-(b'_{26})^{(4)}t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)}t} \le T_{26}(t) \le \quad 336$$

$$\frac{(a_{26})^{(4)}T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)}+(r_{24})^{(4)}+(R_2)^{(4)})}\left[e^{((R_1)^{(4)}+(r_{24})^{(4)})t} - e^{-(R_2)^{(4)}t}\right] + T_{26}^0 e^{-(R_2)^{(4)}t}$$

**Definition of** $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

**Behavior of the solutions of equation** 338

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ :

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

**Definition of** $(v_1)^{(5)}, (v_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, v^{(5)}, u^{(5)}$ : 339

By $(v_1)^{(5)} > 0$, $(v_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$, $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(v^{(5)}\big)^2 + (\sigma_1)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

**Definition of** $(\bar{v}_1)^{(5)}, , (\bar{v}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$ : 340

By $(\bar{v}_1)^{(5)} > 0$, $(\bar{v}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$, $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(v^{(5)}\big)^2 + (\sigma_2)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

**Definition of** $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (v_0)^{(5)}$ :-

If we define $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (v_0)^{(5)}, (m_1)^{(5)} = (v_1)^{(5)}$, ***if*** $(v_0)^{(5)} < (v_1)^{(5)}$

$(m_2)^{(5)} = (v_1)^{(5)}, (m_1)^{(5)} = (\bar{v}_1)^{(5)}$, ***if*** $(v_1)^{(5)} < (v_0)^{(5)} < (\bar{v}_1)^{(5)}$,
and $\boxed{(v_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)},\ \boldsymbol{if}\ (u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}, \boldsymbol{if}\ (u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)},$

and $\boxed{(u_0)^{(5)} = \frac{T_{28}^0}{T_{29}^0}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}, \boldsymbol{if}\ (\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{28}(t) \le G_{28}^0 e^{(S_1)^{(5)} t}$

where $(p_i)^{(5)}$ is defined by equation

$\frac{1}{(m_5)^{(5)}} G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}} G_{28}^0 e^{(S_1)^{(5)} t}$ 343

$\left(\frac{(a_{30})^{(5)} G_{28}^0}{(m_1)^{(5)}((S_1)^{(5)} - (p_{28})^{(5)} - (S_2)^{(5)})}\left[e^{((S_1)^{(5)} - (p_{28})^{(5)})t} - e^{-(S_2)^{(5)} t}\right] + G_{30}^0 e^{-(S_2)^{(5)} t} \le G_{30}(t) \le\right.$ 344

*(a30)5G280(m2)5(S1)5−(a30′)5e(S1)5t−e−(a30′)5t+ G300e−(a30′)5t*

$\boxed{T_{28}^0 e^{(R_1)^{(5)} t} \le T_{28}(t) \le T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}}$ 345

$\frac{1}{(\mu_1)^{(5)}} T_{28}^0 e^{(R_1)^{(5)} t} \le T_{28}(t) \le \frac{1}{(\mu_2)^{(5)}} T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}$ 346

$\frac{(b_{30})^{(5)} T_{28}^0}{(\mu_1)^{(5)}((R_1)^{(5)} - (b'_{30})^{(5)})}\left[e^{(R_1)^{(5)} t} - e^{-(b'_{30})^{(5)} t}\right] + T_{30}^0 e^{-(b'_{30})^{(5)} t} \le T_{30}(t) \le$ 347

$\frac{(a_{30})^{(5)} T_{28}^0}{(\mu_2)^{(5)}((R_1)^{(5)} + (r_{28})^{(5)} + (R_2)^{(5)})}\left[e^{((R_1)^{(5)} + (r_{28})^{(5)})t} - e^{-(R_2)^{(5)} t}\right] + T_{30}^0 e^{-(R_2)^{(5)} t}$

**Definition of** $(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$:- 348

Where $(S_1)^{(5)} = (a_{28})^{(5)}(m_2)^{(5)} - (a'_{28})^{(5)}$

$(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}$

$(R_1)^{(5)} = (b_{28})^{(5)}(\mu_2)^{(5)} - (b'_{28})^{(5)}$

$(R_2)^{(5)} = (b'_{30})^{(5)} - (r_{30})^{(5)}$

**Behavior of the solutions of equation** 349

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ :

$(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)} \le -(a'_{32})^{(6)} + (a'_{33})^{(6)} - (a''_{32})^{(6)}(T_{33}, t) + (a''_{33})^{(6)}(T_{33}, t) \le -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)} \le -(b'_{32})^{(6)} + (b'_{33})^{(6)} - (b''_{32})^{(6)}\big((G_{35}), t\big) - (b''_{33})^{(6)}\big((G_{35}), t\big) \le -(\tau_1)^{(6)}$$

**Definition of** $(\nu_1)^{(6)}, (\nu_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, \nu^{(6)}, u^{(6)}$ : 350

By $(\nu_1)^{(6)} > 0$, $(\nu_2)^{(6)} < 0$ and respectively $(u_1)^{(6)} > 0$, $(u_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(6)},, (\bar{\nu}_2)^{(6)}, (\bar{u}_1)^{(6)}, (\bar{u}_2)^{(6)}$ : 351

By $(\bar{\nu}_1)^{(6)} > 0$, $(\bar{\nu}_2)^{(6)} < 0$ and respectively $(\bar{u}_1)^{(6)} > 0$, $(\bar{u}_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$

**Definition of** $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}, (\nu_0)^{(6)}$ :-

If we define $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}$ by

$$(m_2)^{(6)} = (\nu_0)^{(6)}, (m_1)^{(6)} = (\nu_1)^{(6)}, \ \boldsymbol{if}\ (\nu_0)^{(6)} < (\nu_1)^{(6)}$$

$$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}, \boldsymbol{if}\ (\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)},$$

and $\boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}}$

$$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$$

and analogously 352

$$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}, \ \boldsymbol{if}\ (u_0)^{(6)} < (u_1)^{(6)}$$

$$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}, \boldsymbol{if}\ (u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)},$$

and $\boxed{(u_0)^{(6)} = \frac{T^0_{32}}{T^0_{33}}}$

$$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}, \boldsymbol{if}\ (\bar{u}_1)^{(6)} < (u_0)^{(6)} \text{ where } (u_1)^{(6)}, (\bar{u}_1)^{(6)}$$

Then the solution of global equations satisfies the inequalities 353

$$G^0_{32}e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{32}(t) \le G^0_{32}e^{(S_1)^{(6)}t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}}G^0_{32}e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{33}(t) \le \frac{1}{(m_2)^{(6)}}G^0_{32}e^{(S_1)^{(6)}t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)}G^0_{32}}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})}\left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)}t}\right] + G^0_{34}e^{-(S_2)^{(6)}t} \le G_{34}(t) \le \right. \quad 355$$

$$\frac{(a_{34})^{(6)}G_{32}^{0}}{(m_2)^{(6)}((S_1)^{(6)}-(a'_{34})^{(6)})}\left[e^{(S_1)^{(6)}t}-e^{-(a'_{34})^{(6)}t}\right]+\ G_{34}^{0}e^{-(a'_{34})^{(6)}t}\Big)$$

$$\boxed{T_{32}^{0}e^{(R_1)^{(6)}t}\le T_{32}(t)\le T_{32}^{0}e^{((R_1)^{(6)}+(r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}}T_{32}^{0}e^{(R_1)^{(6)}t}\le T_{32}(t)\le\frac{1}{(\mu_2)^{(6)}}T_{32}^{0}e^{((R_1)^{(6)}+(r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)}T_{32}^{0}}{(\mu_1)^{(6)}((R_1)^{(6)}-(b'_{34})^{(6)})}\left[e^{(R_1)^{(6)}t}-e^{-(b'_{34})^{(6)}t}\right]+T_{34}^{0}e^{-(b'_{34})^{(6)}t}\le T_{34}(t)\le \quad 358$$

$$\frac{(a_{34})^{(6)}T_{32}^{0}}{(\mu_2)^{(6)}((R_1)^{(6)}+(r_{32})^{(6)}+(R_2)^{(6)})}\left[e^{((R_1)^{(6)}+(r_{32})^{(6)})t}-e^{-(R_2)^{(6)}t}\right]+T_{34}^{0}e^{-(R_2)^{(6)}t}$$

**Definition of** $(S_1)^{(6)},(S_2)^{(6)},(R_1)^{(6)},(R_2)^{(6)}$:- 359

Where $(S_1)^{(6)}=(a_{32})^{(6)}(m_2)^{(6)}-(a'_{32})^{(6)}$

$(S_2)^{(6)}=(a_{34})^{(6)}-(p_{34})^{(6)}$

$(R_1)^{(6)}=(b_{32})^{(6)}(\mu_2)^{(6)}-(b'_{32})^{(6)}$

$(R_2)^{(6)}=(b'_{34})^{(6)}-(r_{34})^{(6)}$

**Behavior of the solutions of equation**

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(7)},(\sigma_2)^{(7)},(\tau_1)^{(7)},(\tau_2)^{(7)}$:

$(\sigma_1)^{(7)},(\sigma_2)^{(7)},(\tau_1)^{(7)},(\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)}\le-(a'_{36})^{(7)}+(a'_{37})^{(7)}-(a''_{36})^{(7)}(T_{37},t)+(a''_{37})^{(7)}(T_{37},t)\le-(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)}\le-(b'_{36})^{(7)}+(b'_{37})^{(7)}-(b''_{36})^{(7)}\big((G_{39}),t\big)-(b''_{37})^{(7)}\big((G_{39}),t\big)\le-(\tau_1)^{(7)}$$

**Definition of** $(v_1)^{(7)},(v_2)^{(7)},(u_1)^{(7)},(u_2)^{(7)},v^{(7)},u^{(7)}$: 361

By $(v_1)^{(7)}>0,(v_2)^{(7)}<0$ and respectively $(u_1)^{(7)}>0,(u_2)^{(7)}<0$ the roots of the equations $(a_{37})^{(7)}\big(v^{(7)}\big)^2+(\sigma_1)^{(7)}v^{(7)}-(a_{36})^{(7)}=0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2+(\tau_1)^{(7)}u^{(7)}-(b_{36})^{(7)}=0$ and

**Definition of** $(\bar{v}_1)^{(7)},,(\bar{v}_2)^{(7)},(\bar{u}_1)^{(7)},(\bar{u}_2)^{(7)}$: 362

By $(\bar{v}_1)^{(7)}>0,(\bar{v}_2)^{(7)}<0$ and respectively $(\bar{u}_1)^{(7)}>0,(\bar{u}_2)^{(7)}<0$ the roots of the equations $(a_{37})^{(7)}\big(v^{(7)}\big)^2+(\sigma_2)^{(7)}v^{(7)}-(a_{36})^{(7)}=0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2+(\tau_2)^{(7)}u^{(7)}-(b_{36})^{(7)}=0$

**Definition of** $(m_1)^{(7)},(m_2)^{(7)},(\mu_1)^{(7)},(\mu_2)^{(7)},(v_0)^{(7)}$:-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$$(m_2)^{(7)} = (\nu_0)^{(7)}, (m_1)^{(7)} = (\nu_1)^{(7)}, \ \boldsymbol{if}\ (\nu_0)^{(7)} < (\nu_1)^{(7)}$$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\bar{\nu}_1)^{(7)}, \boldsymbol{if}\ (\nu_1)^{(7)} < (\nu_0)^{(7)} < (\bar{\nu}_1)^{(7)},$$

and $\boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\nu_0)^{(7)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(7)} < (\nu_0)^{(7)}$$

and analogously 363

$$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}, \ \boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$$

and $\boxed{(u_0)^{(7)} = \frac{T_{36}^0}{T_{37}^0}}$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)} \text{ where } (u_1)^{(7)}, (\bar{u}_1)^{(7)}$$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^0 e^{(S_1)^{(7)} t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)} t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)}((S_1)^{(7)} - (p_{36})^{(7)} - (S_2)^{(7)})}\left[e^{((S_1)^{(7)} - (p_{36})^{(7)})t} - e^{-(S_2)^{(7)} t}\right] + G_{38}^0 e^{-(S_2)^{(7)} t} \le G_{38}(t) \le \right. \quad 366$$

$$\left. \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)}((S_1)^{(7)} - (a'_{38})^{(7)})}\left[e^{(S_1)^{(7)} t} - e^{-(a'_{38})^{(7)} t}\right] + G_{38}^0 e^{-(a'_{38})^{(7)} t}\right)$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t} \quad 368$$

$$\frac{(b_{38})^{(7)} T_{36}^0}{(\mu_1)^{(7)}((R_1)^{(7)} - (b'_{38})^{(7)})}\left[e^{(R_1)^{(7)} t} - e^{-(b'_{38})^{(7)} t}\right] + T_{38}^0 e^{-(b'_{38})^{(7)} t} \le T_{38}(t) \le \quad 369$$

$$\frac{(a_{38})^{(7)} T_{36}^0}{(\mu_2)^{(7)}((R_1)^{(7)} + (r_{36})^{(7)} + (R_2)^{(7)})}\left[e^{((R_1)^{(7)} + (r_{36})^{(7)})t} - e^{-(R_2)^{(7)} t}\right] + T_{38}^0 e^{-(R_2)^{(7)} t}$$

**Definition of** $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$

$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$

$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$

**Behavior of the solutions of equation** 371

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ :

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$-(\sigma_2)^{(8)} \le -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \le -(\sigma_1)^{(8)}$

$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\big((G_{43}), t\big) - (b''_{41})^{(8)}\big((G_{43}), t\big) \le -(\tau_1)^{(8)}$

**Definition of** $(\nu_1)^{(8)}, (\nu_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, \nu^{(8)}, u^{(8)}$ : 372

By $(\nu_1)^{(8)} > 0, (\nu_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0, (u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(8)}, , (\bar{\nu}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$ :

By $(\bar{\nu}_1)^{(8)} > 0, (\bar{\nu}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0, (\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

**Definition of** $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}, (\nu_0)^{(8)}$ :-

If we define $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$(m_2)^{(8)} = (\nu_0)^{(8)}, (m_1)^{(8)} = (\nu_1)^{(8)}$, ***if*** $(\nu_0)^{(8)} < (\nu_1)^{(8)}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\bar{\nu}_1)^{(8)}$, ***if*** $(\nu_1)^{(8)} < (\nu_0)^{(8)} < (\bar{\nu}_1)^{(8)}$,
and $\boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\nu_0)^{(8)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(8)} < (\nu_0)^{(8)}$

and analogously 374

$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)},\ \boldsymbol{if}\ (u_0)^{(8)} < (u_1)^{(8)}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}, \boldsymbol{if}\ (u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)},$

and $\boxed{(u_0)^{(8)} = \frac{T_{40}^0}{T_{41}^0}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}, \boldsymbol{if}\ (\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^0 e^{(S_1)^{(8)} t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}} G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}} G_{40}^0 e^{(S_1)^{(8)} t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)} G_{40}^0}{(m_1)^{(8)}((S_1)^{(8)} - (p_{40})^{(8)} - (S_2)^{(8)})}\left[e^{((S_1)^{(8)} - (p_{40})^{(8)})t} - e^{-(S_2)^{(8)} t}\right] + G_{42}^0 e^{-(S_2)^{(8)} t} \le G_{42}(t) \le \right. \quad 377$$

$$\left. \frac{(a_{42})^{(8)} G_{40}^0}{(m_2)^{(8)}((S_1)^{(8)} - (a'_{42})^{(8)})}\left[e^{(S_1)^{(8)} t} - e^{-(a'_{42})^{(8)} t}\right] + G_{42}^0 e^{-(a'_{42})^{(8)} t}\right)$$

$$\boxed{T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}} T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le \frac{1}{(\mu_2)^{(8)}} T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)} T_{40}^0}{(\mu_1)^{(8)}((R_1)^{(8)} - (b'_{42})^{(8)})}\left[e^{(R_1)^{(8)} t} - e^{-(b'_{42})^{(8)} t}\right] + T_{42}^0 e^{-(b'_{42})^{(8)} t} \le T_{42}(t) \le \quad 380$$

$$\frac{(a_{42})^{(8)} T_{40}^0}{(\mu_2)^{(8)}((R_1)^{(8)} + (r_{40})^{(8)} + (R_2)^{(8)})}\left[e^{((R_1)^{(8)} + (r_{40})^{(8)})t} - e^{-(R_2)^{(8)} t}\right] + T_{42}^0 e^{-(R_2)^{(8)} t}$$

**Definition of** $(S_1)^{(8)}, (S_2)^{(8)}, (R_1)^{(8)}, (R_2)^{(8)}$:- 381

Where $(S_1)^{(8)} = (a_{40})^{(8)}(m_2)^{(8)} - (a'_{40})^{(8)}$

$(S_2)^{(8)} = (a_{42})^{(8)} - (p_{42})^{(8)}$

$(R_1)^{(8)} = (b_{40})^{(8)}(\mu_2)^{(8)} - (b'_{40})^{(8)}$

$(R_2)^{(8)} = (b'_{42})^{(8)} - (r_{42})^{(8)}$

**Behavior of the solutions of equation 37 to 92** 382

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ :

$(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ four constants satisfying

$$-(\sigma_2)^{(9)} \le -(a'_{44})^{(9)} + (a'_{45})^{(9)} - (a''_{44})^{(9)}(T_{45}, t) + (a''_{45})^{(9)}(T_{45}, t) \le -(\sigma_1)^{(9)}$$

$$-(\tau_2)^{(9)} \le -(b'_{44})^{(9)} + (b'_{45})^{(9)} - (b''_{44})^{(9)}\big((G_{47}), t\big) - (b''_{45})^{(9)}\big((G_{47}), t\big) \le -(\tau_1)^{(9)}$$

**Definition of** $(\nu_1)^{(9)}, (\nu_2)^{(9)}, (u_1)^{(9)}, (u_2)^{(9)}, \nu^{(9)}, u^{(9)}$ :

By $(\nu_1)^{(9)} > 0$ , $(\nu_2)^{(9)} < 0$ and respectively $(u_1)^{(9)} > 0$ , $(u_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_1)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(9)}, , (\bar{\nu}_2)^{(9)}, (\bar{u}_1)^{(9)}, (\bar{u}_2)^{(9)}$ :

By $(\bar{\nu}_1)^{(9)} > 0$ , $(\bar{\nu}_2)^{(9)} < 0$ and respectively $(\bar{u}_1)^{(9)} > 0$ , $(\bar{u}_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_2)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$

**Definition of** $(m_1)^{(9)}, (m_2)^{(9)}, (\mu_1)^{(9)}, (\mu_2)^{(9)}, (\nu_0)^{(9)}$ :-

If we define $(m_1)^{(9)}, (m_2)^{(9)}, (\mu_1)^{(9)}, (\mu_2)^{(9)}$ by

$$(m_2)^{(9)} = (\nu_0)^{(9)}, (m_1)^{(9)} = (\nu_1)^{(9)}, \ \boldsymbol{if}\ (\nu_0)^{(9)} < (\nu_1)^{(9)}$$

$$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\bar{\nu}_1)^{(9)}, \boldsymbol{if}\ (\nu_1)^{(9)} < (\nu_0)^{(9)} < (\bar{\nu}_1)^{(9)},$$

and $\boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$

$$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\nu_0)^{(9)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(9)} < (\nu_0)^{(9)}$$

and analogously

$$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)}, \ \boldsymbol{if}\ (u_0)^{(9)} < (u_1)^{(9)}$$

$$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}, \boldsymbol{if}\ (u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)},$$

and $\boxed{(u_0)^{(9)} = \frac{T^0_{44}}{T^0_{45}}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}, \boldsymbol{if}\ (\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G^0_{44} e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{44}(t) \le G^0_{44} e^{(S_1)^{(9)} t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}}G_{44}^0 e^{((S_1)^{(9)}-(p_{44})^{(9)})t} \le G_{45}(t) \le \frac{1}{(m_2)^{(9)}}G_{44}^0 e^{(S_1)^{(9)}t}$$

(

$$\frac{(a_{46})^{(9)}G_{44}^0}{(m_1)^{(9)}((S_1)^{(9)}-(p_{44})^{(9)}-(S_2)^{(9)})}\left[e^{((S_1)^{(9)}-(p_{44})^{(9)})t}-e^{-(S_2)^{(9)}t}\right]+G_{46}^0 e^{-(S_2)^{(9)}t} \le G_{46}(t) \le$$

$$\frac{(a_{46})^{(9)}G_{44}^0}{(m_2)^{(9)}((S_1)^{(9)}-(a'_{46})^{(9)})}[e^{(S_1)^{(9)}t}-e^{-(a'_{46})^{(9)}t}]+\ G_{46}^0 e^{-(a'_{46})^{(9)}t})$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le T_{44}^0 e^{((R_1)^{(9)}+(r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}}T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le \frac{1}{(\mu_2)^{(9)}}T_{44}^0 e^{((R_1)^{(9)}+(r_{44})^{(9)})t}$$

$$\frac{(b_{46})^{(9)}T_{44}^0}{(\mu_1)^{(9)}((R_1)^{(9)}-(b'_{46})^{(9)})}\left[e^{(R_1)^{(9)}t}-e^{-(b'_{46})^{(9)}t}\right]+T_{46}^0 e^{-(b'_{46})^{(9)}t} \le T_{46}(t) \le$$

$$\frac{(a_{46})^{(9)}T_{44}^0}{(\mu_2)^{(9)}((R_1)^{(9)}+(r_{44})^{(9)}+(R_2)^{(9)})}\left[e^{((R_1)^{(9)}+(r_{44})^{(9)})t}-e^{-(R_2)^{(9)}t}\right]+T_{46}^0 e^{-(R_2)^{(9)}t}$$

**Definition of** $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$

**Proof :** From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14},t)\right) - (a''_{14})^{(1)}(T_{14},t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

**Definition of** $v^{(1)}$ :- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)}\right) \le \frac{dv^{(1)}}{dt} \le -\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)}\right)$$

From which one obtains

**Definition of** $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$ :-

For $0 < \boxed{(v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}} < (v_1)^{(1)} < (\bar{v}_1)^{(1)}$

$$v^{(1)}(t) \ge \frac{(v_1)^{(1)}+(C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(v_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(v_2)^{(1)}}}$$

it follows $(\nu_0)^{(1)} \leq \nu^{(1)}(t) \leq (\nu_1)^{(1)}$

In the same manner , we get 384

$$\nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{\nu}_1)^{(1)}-(\nu_0)^{(1)}}{(\nu_0)^{(1)}-(\bar{\nu}_2)^{(1)}}}$$

From which we deduce $(\nu_0)^{(1)} \leq \nu^{(1)}(t) \leq (\bar{\nu}_1)^{(1)}$

If $0 < (\nu_1)^{(1)} < (\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0} < (\bar{\nu}_1)^{(1)}$ we find like in the previous case, 385

$$(\nu_1)^{(1)} \leq \frac{(\nu_1)^{(1)}+(C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}} \leq \nu^{(1)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\bar{\nu}_1)^{(1)}$$

If $0 < (\nu_1)^{(1)} \leq (\bar{\nu}_1)^{(1)} \leq \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$ , we obtain 386

$$(\nu_1)^{(1)} \leq \nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\nu_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(1)}(t)$ :-

$$(m_2)^{(1)} \leq \nu^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{\nu^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(1)}(t)$ :-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{13})^{(1)} = (a''_{14})^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)}G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)}T_{14}(t)$ This is an important

consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

**Proof** : From global equations we obtain 387

$$\frac{d\nu^{(2)}}{dt} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right) - (a''_{17})^{(2)}(T_{17}, t)\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

**Definition of** $\nu^{(2)}$ :- $\boxed{\nu^{(2)} = \frac{G_{16}}{G_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \leq \frac{d\nu^{(2)}}{dt} \leq -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

**Definition of** $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$ :-

For $0 < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \geq \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}}{1 + (C)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(C)^{(2)} = \frac{(\nu_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \leq \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}}{1 + (C)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}} \leq \nu^{(2)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \leq (\bar{\nu}_1)^{(2)} \leq (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$ , we obtain 394

$$(\nu_1)^{(2)} \leq \nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(2)}(t)$ :- 395

$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}$, $\boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$

In a completely analogous way, we obtain 396

**Definition of** $u^{(2)}(t)$ :-

$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}$, $\boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :** 397

If $(a''_{16})^{(2)} = (a''_{17})^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)} G_{17}(t)$

Analogously if $(b''_{16})^{(2)} = (b''_{17})^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)} T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

**Proof** : From global equations we obtain 398

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right) - (a''_{21})^{(3)}(T_{21}, t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

**Definition of** $\nu^{(3)}$ :- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right) \leq \frac{d\nu^{(3)}}{dt} \leq -\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right)$$

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From which one obtains

For $0 < (\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\nu_1)^{(3)} < (\bar{\nu}_1)^{(3)}$

$$\nu^{(3)}(t) \geq \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(\nu_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\nu_2)^{(3)}}}$$

it follows $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\nu_1)^{(3)}$

In the same manner , we get 401

$$\nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{\nu}_1)^{(3)}-(\nu_0)^{(3)}}{(\nu_0)^{(3)}-(\bar{\nu}_2)^{(3)}}}$$

**Definition of** $(\bar{\nu}_1)^{(3)}$ :-

From which we deduce $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\bar{\nu}_1)^{(3)}$

If $0 < (\nu_1)^{(3)} < (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (\bar{\nu}_1)^{(3)}$ we find like in the previous case, 402

$$(\nu_1)^{(3)} \leq \frac{(\nu_1)^{(3)}+(C)^{(3)}(\nu_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}} \leq \nu^{(3)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\bar{\nu}_1)^{(3)}$$

If $0 < (\nu_1)^{(3)} \leq (\bar{\nu}_1)^{(3)} \leq (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}$ , we obtain 403

$$(\nu_1)^{(3)} \leq \nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\nu_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(3)}(t)$ :-

$$(m_2)^{(3)} \leq \nu^{(3)}(t) \leq (m_1)^{(3)}, \quad \boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(3)}(t)$ :-

$$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}, \quad \boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{20}'')^{(3)} = (a_{21}'')^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)}G_{21}(t)$

Analogously if $(b_{20}'')^{(3)} = (b_{21}'')^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)}T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

**Proof** : From global equations we obtain 404

$$\frac{dv^{(4)}}{dt} = (a_{24})^{(4)} - \left((a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right) - (a''_{25})^{(4)}(T_{25},t)v^{(4)} - (a_{25})^{(4)}v^{(4)}$$

**Definition of** $v^{(4)}$ :- $\boxed{v^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)}\right) \le \frac{dv^{(4)}}{dt} \le -\left((a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_4)^{(4)}v^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$ :-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \ge \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \le \nu^{(4)}(t) \le (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \le \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \le \nu^{(4)}(t) \le (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \le \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}}{1 + (C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}} \le \nu^{(4)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \le (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \le (\bar{\nu}_1)^{(4)} \le \boxed{(\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}}}$ , we obtain 407

$$(\nu_1)^{(4)} \le \nu^{(4)}(t) \le \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \le (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(4)}(t)$ :-

$$(m_2)^{(4)} \le \nu^{(4)}(t) \le (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(4)}(t)$ :-

$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}$, $\boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{24})^{(4)} = (a''_{25})^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)} G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b''_{24})^{(4)} = (b''_{25})^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)} T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

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**Proof** : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a'_{28})^{(5)} - (a'_{29})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right) - (a''_{29})^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

**Definition of** $\nu^{(5)}$ :- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$ :-

For $0 < \boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \le \frac{(\nu_1)^{(5)}+(C)^{(5)}(\nu_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)}-(\nu_2)^{(5)}\right)t\right]}}{1+(C)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)}-(\nu_2)^{(5)}\right)t\right]}} \le \nu^{(5)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \le (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \le (\bar{\nu}_1)^{(5)} \le \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain 411

$$(\nu_1)^{(5)} \le \nu^{(5)}(t) \le \frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \le (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(5)}(t)$ :-

$(m_2)^{(5)} \le \nu^{(5)}(t) \le (m_1)^{(5)}$, $\boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$

In a completely analogous way, we obtain

**Definition of** $u^{(5)}(t)$ :-

$(\mu_2)^{(5)} \le u^{(5)}(t) \le (\mu_1)^{(5)}$, $\boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a_{28}'')^{(5)} = (a_{29}'')^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)}G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b_{28}'')^{(5)} = (b_{29}'')^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)}T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

**Proof** : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a_{32}')^{(6)} - (a_{33}')^{(6)} + (a_{32}'')^{(6)}(T_{33}, t)\right) - (a_{33}'')^{(6)}(T_{33}, t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

**Definition of** $\nu^{(6)}$ :- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \le \frac{d\nu^{(6)}}{dt} \le -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$ :-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \geq \frac{(\nu_1)^{(6)}+(C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_0)^{(6)}\right)t\right]}}{1+(C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)}-(\nu_0)^{(6)}}{(\nu_0)^{(6)}-(\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\nu_1)^{(6)}$

In the same manner , we get 413

$$\nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{\nu}_1)^{(6)}-(\nu_0)^{(6)}}{(\nu_0)^{(6)}-(\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\bar{\nu}_1)^{(6)}$

If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 414

$$(\nu_1)^{(6)} \leq \frac{(\nu_1)^{(6)}+(C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_2)^{(6)}\right)t\right]}}{1+(C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_2)^{(6)}\right)t\right]}} \leq \nu^{(6)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\bar{\nu}_1)^{(6)}$$

If $0 < (\nu_1)^{(6)} \leq (\bar{\nu}_1)^{(6)} \leq \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$ , we obtain 415

$$(\nu_1)^{(6)} \leq \nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(6)}(t)$ :-

$$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}, \quad \boxed{\nu^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(6)}(t)$ :-

$$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}, \quad \boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)}G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)}T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

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**Proof** : From global equations we obtain

$$\frac{dv^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right) - (a''_{37})^{(7)}(T_{37},t)v^{(7)} - (a_{37})^{(7)}v^{(7)}$$

**Definition of** $v^{(7)}$ :- $\boxed{v^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_2)^{(7)}v^{(7)} - (a_{36})^{(7)}\right) \le \frac{dv^{(7)}}{dt} \le -\left((a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_1)^{(7)}v^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

**Definition of** $(\bar{v}_1)^{(7)}, (v_0)^{(7)}$ :-

For $0 < \boxed{(v_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}} < (v_1)^{(7)} < (\bar{v}_1)^{(7)}$

$$v^{(7)}(t) \ge \frac{(v_1)^{(7)} + (C)^{(7)}(v_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_0)^{(7)}\right)t\right]}}{1 + (C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(v_1)^{(7)} - (v_0)^{(7)}}{(v_0)^{(7)} - (v_2)^{(7)}}}$$

it follows $(v_0)^{(7)} \le v^{(7)}(t) \le (v_1)^{(7)}$

In the same manner , we get 417

$$v^{(7)}(t) \le \frac{(\bar{v}_1)^{(7)} + (\bar{C})^{(7)}(\bar{v}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{v}_1)^{(7)} - (v_0)^{(7)}}{(v_0)^{(7)} - (\bar{v}_2)^{(7)}}}$$

From which we deduce $(v_0)^{(7)} \le v^{(7)}(t) \le (\bar{v}_1)^{(7)}$

If $0 < (v_1)^{(7)} < (v_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}} < (\bar{v}_1)^{(7)}$ we find like in the previous case, 418

$$(v_1)^{(7)} \le \frac{(v_1)^{(7)} + (C)^{(7)}(v_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_2)^{(7)}\right)t\right]}}{1 + (C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_2)^{(7)}\right)t\right]}} \le v^{(7)}(t) \le$$

$$\frac{(\bar{v}_1)^{(7)} + (\bar{C})^{(7)}(\bar{v}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}} \le (\bar{v}_1)^{(7)}$$

If $0 < (v_1)^{(7)} \le (\bar{v}_1)^{(7)} \le \boxed{(v_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$ , we obtain 419

$$(v_1)^{(7)} \le v^{(7)}(t) \le \frac{(\bar{v}_1)^{(7)} + (\bar{C})^{(7)}(\bar{v}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}} \le (v_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $v^{(7)}(t)$ :-

$$(m_2)^{(7)} \leq v^{(7)}(t) \leq (m_1)^{(7)}, \quad \boxed{v^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(7)}(t)$ :- 420

$$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}, \quad \boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $v^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)} G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)} T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

**Proof** : From global equations we obtain 421

$$\frac{dv^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right) - (a''_{41})^{(8)}(T_{41}, t) v^{(8)} - (a_{41})^{(8)} v^{(8)}$$

**Definition of** $v^{(8)}$ :- $\boxed{v^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_2)^{(8)} v^{(8)} - (a_{40})^{(8)}\right) \leq \frac{dv^{(8)}}{dt} \leq -\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_1)^{(8)} v^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$ :-

$$\text{For } 0 < \boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$$

$$v^{(8)}(t) \geq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)}-(\nu_0)^{(8)}}{(\nu_0)^{(8)}-(\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \leq \frac{(\nu_1)^{(8)}+(C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}}{1+(C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}} \leq \nu^{(8)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \leq (\bar{\nu}_1)^{(8)} \leq \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$ , we obtain 424

$$(\nu_1)^{(8)} \leq \nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(8)}(t)$ :-

$$(m_2)^{(8)} \leq \nu^{(8)}(t) \leq (m_1)^{(8)}, \quad \boxed{\nu^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(8)}(t)$ :-

$$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}, \quad \boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $\nu^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)}G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then $(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)}T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

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**Proof** : From 99,20,44,22,23,44 we obtain A

$$\frac{dv^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right) - (a''_{45})^{(9)}(T_{45},t)v^{(9)} - (a_{45})^{(9)}v^{(9)}$$

**Definition of** $v^{(9)}$ :- $\boxed{v^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)}\right) \le \frac{dv^{(9)}}{dt} \le -\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_1)^{(9)}v^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$ :-

For $0 < \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$

$$v^{(9)}(t) \ge \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \le v^{(9)}(t) \le (\nu_9)^{(9)}$

In the same manner , we get

$$v^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \le v^{(9)}(t) \le (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \le \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}}{1 + (C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}} \le v^{(9)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \le (\bar{\nu}_1)^{(9)} \le \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$ , we obtain

$$(\nu_1)^{(9)} \le v^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $v^{(9)}(t)$ :-

$$(m_2)^{(9)} \le v^{(9)}(t) \le (m_1)^{(9)}, \quad \boxed{v^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(9)}(t)$ :-

$(\mu_2)^{(9)} \leq u^{(9)}(t) \leq (\mu_1)^{(9)}$, $\boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)} G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then
$(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)} T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following 425

**Theorem :** If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on $t$ , and the conditions with the notations

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0$ ,

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$

$with\ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a''_i)^{(2)} and\ (b''_i)^{(2)}$ are independent on t , and the conditions with the notations 426

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$ 427

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 428

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$ , 429

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 430

$with\ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a''_i)^{(3)} and\ (b''_i)^{(3)}$ are independent on $t$ , and the conditions with the notations 431

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$ ,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

**Theorem :** If $(a_i'')^{(4)} and\ (b_i'')^{(4)}$ are independent on $t$ , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$ ,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(5)} and\ (b_i'')^{(5)}$ are independent on $t$ , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$ ,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

**Theorem** If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on $t$ , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$ ,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(7)} and\ (b_i'')^{(7)}$ are independent on $t$ , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$ ,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(8)} and\ (b_i'')^{(8)}$ are independent on $t$ , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$ ,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(9)} and\ (b_i'')^{(9)}$ are independent on $t$ , and the conditions (with the notations 436 A
45,46,27,28)

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$ ,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0$ 437

$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0$ 438

$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0$ 439

$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0$ 440

$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ]T_{14} = 0$ 441

$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0$ 442

has a unique positive solution , which is an equilibrium solution for the system

$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0$ 443

$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0$ 444

$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0$ 445

$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\ ]T_{16} = 0$ 446

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\,]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\,]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\,]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\,]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\,]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\big((G_{27})\big)\,]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\big((G_{27})\big)\,]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\big((G_{27})\big)\,]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})\,]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})\,]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})\,]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})\,]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})\,]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})\,]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0 \quad 479$$

$$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0 \quad 480$$

$$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0 \quad 481$$

$$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0 \quad 482$$

$$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0 \quad 483$$

$$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0 \quad 484$$

$$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0 \quad 484\text{A}$$

$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$

$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$

$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$

$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$

$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$

**Proof:** 485

(a) Indeed the first two equations have a nontrivial solution $G_{13}, G_{14}$ if

$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$

**Proof:** 486

(dd) Indeed the first two equations have a nontrivial solution $G_{16}, G_{17}$ if

$\mathrm{F}(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$

**Proof:** 487

(a) Indeed the first two equations have a nontrivial solution $G_{20}, G_{21}$ if

$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$

**Proof:** 488

(a) Indeed the first two equations have a nontrivial solution $G_{24}, G_{25}$ if

$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$

**Proof:** 489

(a) Indeed the first two equations have a nontrivial solution $G_{28}, G_{29}$ if

$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$

**Proof:** 490

(a) Indeed the first two equations have a nontrivial solution $G_{32}, G_{33}$ if

$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) +$

$(a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$

**Proof:** 491

(a) Indeed the first two equations have a nontrivial solution $G_{36}, G_{37}$ if

$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$

**Proof:** 492

(a) Indeed the first two equations have a nontrivial solution $G_{40}, G_{41}$ if

$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$

**Proof:** 492 A

(a) Indeed the first two equations have a nontrivial solution $G_{44}, G_{45}$ if

$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$

**Definition and uniqueness of** $T^*_{14}$ :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique $T^*_{14}$ for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

**Definition and uniqueness of** $T^*_{17}$ :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique $T^*_{17}$ for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]}$$ 495

**Definition and uniqueness of** $T^*_{21}$ :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique $T^*_{21}$ for which $f(T^*_{21}) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

**Definition and uniqueness of** $T^*_{25}$ :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that

there exists a unique $T_{25}^{*}$ for which $f(T_{25}^{*})=0$. With this value , we obtain from the three first equations

$$G_{24}=\frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}=\frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T_{25}^{*})]}$$

**Definition and uniqueness of** $T_{29}^{*}$ :- 498

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(5)}(T_{29})$ being increasing, it follows that there exists a unique $T_{29}^{*}$ for which $f(T_{29}^{*})=0$. With this value , we obtain from the three first equations

$$G_{28}=\frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}=\frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T_{29}^{*})]}$$

**Definition and uniqueness of** $T_{33}^{*}$ :- 499

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(6)}(T_{33})$ being increasing, it follows that there exists a unique $T_{33}^{*}$ for which $f(T_{33}^{*})=0$. With this value , we obtain from the three first equations

$$G_{32}=\frac{(a_{32})^{(6)}G_{33}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^{*})]} \quad , \quad G_{34}=\frac{(a_{34})^{(6)}G_{33}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^{*})]}$$

**Definition and uniqueness of** $T_{37}^{*}$ :- 500

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(7)}(T_{37})$ being increasing, it follows that there exists a unique $T_{37}^{*}$ for which $f(T_{37}^{*})=0$. With this value , we obtain from the three first equations

$$G_{36}=\frac{(a_{36})^{(7)}G_{37}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^{*})]} \quad , \quad G_{38}=\frac{(a_{38})^{(7)}G_{37}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^{*})]}$$

**Definition and uniqueness of** $T_{41}^{*}$ :- 501

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique $T_{41}^{*}$ for which $f(T_{41}^{*})=0$. With this value , we obtain from the three first equations

$$G_{40}=\frac{(a_{40})^{(8)}G_{41}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T_{41}^{*})]} \quad , \quad G_{42}=\frac{(a_{42})^{(8)}G_{41}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T_{41}^{*})]}$$

**Definition and uniqueness of** $T_{45}^{*}$ :- 501 A

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(9)}(T_{45})$ being increasing, it follows that there exists a unique $T_{45}^{*}$ for which $f(T_{45}^{*})=0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)}G_{45}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T^*_{45})]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)}G_{45}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T^*_{45})]}$$

By the same argument, the equations admit solutions $G_{13}, G_{14}$ if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right]+(b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{14}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{14}$ such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions $G_{16}, G_{17}$ if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right]+(b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ 504
is a decreasing function in $G_{17}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{14}$ such that $\varphi((G_{19})^*) = 0$

By the same argument, the equations admit solutions $G_{20}, G_{21}$ if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right]+(b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{21}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{21}$ such that $\varphi((G_{23})^*) = 0$

By the same argument, the equations admit solutions $G_{24}, G_{25}$ if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right]+(b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{25}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{25}$ such that $\varphi((G_{27})^*) = 0$

By the same argument, the equations admit solutions $G_{28}, G_{29}$ if 507

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right]+(b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{29}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that

there exists a unique $G_{29}^{*}$ such that $\varphi((G_{31})^{*}) = 0$

By the same argument, the equations admit solutions $G_{32}, G_{33}$ if 508
$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$

$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right] + (b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{33}$ taking into account the hypothesis $\varphi(0) > 0\,, \varphi(\infty) < 0$ it follows that there exists a unique $G_{33}^{*}$ such that $\varphi(G_{35}{}^{*}) = 0$

By the same argument, the equations admit solutions $G_{36}, G_{37}$ if 509
$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$
$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right] + (b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{37}$ taking into account the hypothesis $\varphi(0) > 0\,, \varphi(\infty) < 0$ it follows that there exists a unique $G_{37}^{*}$ such that $\varphi(G_{39}{}^{*}) = 0$

By the same argument, the equations admit solutions $G_{40}, G_{41}$ if 510
$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$
$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right] + (b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{41}$ taking into account the hypothesis $\varphi(0) > 0\,, \varphi(\infty) < 0$ it follows that there exists a unique $G_{41}^{*}$ such that $\varphi(G_{43}{}^{*}) = 0$

By the same argument, the equations 92,93 admit solutions $G_{44}, G_{45}$ if
$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$

$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right] + (b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{45}$ taking into account the hypothesis $\varphi(0) > 0\,, \varphi(\infty) < 0$ it follows that there exists a unique $G_{45}^{*}$ such that $\varphi((G_{47})^{*}) = 0$

Finally we obtain the unique solution 511

$G_{14}^{*}$ given by $\varphi(G^{*}) = 0\,, T_{14}^{*}$ given by $f(T_{14}^{*}) = 0$ and

$$G_{13}^{*} = \frac{(a_{13})^{(1)}G_{14}^{*}}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^{*})]} \quad , \quad G_{15}^{*} = \frac{(a_{15})^{(1)}G_{14}^{*}}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^{*})]}$$

$$T_{13}^{*} = \frac{(b_{13})^{(1)}T_{14}^{*}}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^{*})]} \quad , \quad T_{15}^{*} = \frac{(b_{15})^{(1)}T_{14}^{*}}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^{*})]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

$G_{17}^{*}$ given by $\varphi((G_{19})^{*}) = 0$ , $T_{17}^{*}$ given by $f(T_{17}^{*}) = 0$ and 512

$$G_{16}^{*} = \frac{(a_{16})^{(2)} G_{17}^{*}}{[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}^{*})]} \quad , \quad G_{18}^{*} = \frac{(a_{18})^{(2)} G_{17}^{*}}{[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}^{*})]} \qquad 513$$

$$T_{16}^{*} = \frac{(b_{16})^{(2)} T_{17}^{*}}{[(b'_{16})^{(2)} - (b''_{16})^{(2)}((G_{19})^{*})]} \quad , \quad T_{18}^{*} = \frac{(b_{18})^{(2)} T_{17}^{*}}{[(b'_{18})^{(2)} - (b''_{18})^{(2)}((G_{19})^{*})]} \qquad 514$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

$G_{21}^{*}$ given by $\varphi((G_{23})^{*}) = 0$ , $T_{21}^{*}$ given by $f(T_{21}^{*}) = 0$ and

$$G_{20}^{*} = \frac{(a_{20})^{(3)} G_{21}^{*}}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^{*})]} \quad , \quad G_{22}^{*} = \frac{(a_{22})^{(3)} G_{21}^{*}}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^{*})]}$$

$$T_{20}^{*} = \frac{(b_{20})^{(3)} T_{21}^{*}}{[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}{}^{*})]} \quad , \quad T_{22}^{*} = \frac{(b_{22})^{(3)} T_{21}^{*}}{[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}{}^{*})]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

$G_{25}^{*}$ given by $\varphi(G_{27}) = 0$ , $T_{25}^{*}$ given by $f(T_{25}^{*}) = 0$ and

$$G_{24}^{*} = \frac{(a_{24})^{(4)} G_{25}^{*}}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}^{*} = \frac{(a_{26})^{(4)} G_{25}^{*}}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^{*})]}$$

$$T_{24}^{*} = \frac{(b_{24})^{(4)} T_{25}^{*}}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^{*})]} \quad , \quad T_{26}^{*} = \frac{(b_{26})^{(4)} T_{25}^{*}}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^{*})]} \qquad 517$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

$G_{29}^{*}$ given by $\varphi((G_{31})^{*}) = 0$ , $T_{29}^{*}$ given by $f(T_{29}^{*}) = 0$ and

$$G_{28}^{*} = \frac{(a_{28})^{(5)} G_{29}^{*}}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}^{*} = \frac{(a_{30})^{(5)} G_{29}^{*}}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^{*})]}$$

$$T_{28}^{*} = \frac{(b_{28})^{(5)} T_{29}^{*}}{[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31})^{*})]} \quad , \quad T_{30}^{*} = \frac{(b_{30})^{(5)} T_{29}^{*}}{[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31})^{*})]} \qquad 519$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

$G_{33}^{*}$ given by $\varphi((G_{35})^{*}) = 0$ , $T_{33}^{*}$ given by $f(T_{33}^{*}) = 0$ and

$$G_{32}^{*} = \frac{(a_{32})^{(6)} G_{33}^{*}}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^{*})]} \quad , \quad G_{34}^{*} = \frac{(a_{34})^{(6)} G_{33}^{*}}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^{*})]}$$

$$T_{32}^{*} = \frac{(b_{32})^{(6)} T_{33}^{*}}{[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35})^{*})]} \quad , \quad T_{34}^{*} = \frac{(b_{34})^{(6)} T_{33}^{*}}{[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35})^{*})]} \qquad 521$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

$G_{37}^*$ given by $\varphi((G_{39})^*) = 0$ , $T_{37}^*$ given by $f(T_{37}^*) = 0$ and

$$G_{36}^* = \frac{(a_{36})^{(7)} G_{37}^*}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^*)]} \quad , \quad G_{38}^* = \frac{(a_{38})^{(7)} G_{37}^*}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^*)]}$$

$$T_{36}^* = \frac{(b_{36})^{(7)} T_{37}^*}{[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39})^*)]} \quad , \quad T_{38}^* = \frac{(b_{38})^{(7)} T_{37}^*}{[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39})^*)]}$$

Finally we obtain the unique solution 523

$G_{41}^*$ given by $\varphi((G_{43})^*) = 0$ , $T_{41}^*$ given by $f(T_{41}^*) = 0$ and

$$G_{40}^* = \frac{(a_{40})^{(8)} G_{41}^*}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42}^* = \frac{(a_{42})^{(8)} G_{41}^*}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

$$T_{40}^* = \frac{(b_{40})^{(8)} T_{41}^*}{[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43})^*)]} \quad , \quad T_{42}^* = \frac{(b_{42})^{(8)} T_{41}^*}{[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43})^*)]}$$

Finally we obtain the unique solution of 89 to 99 523 A

$G_{45}^*$ given by $\varphi((G_{47})^*) = 0$ , $T_{45}^*$ given by $f(T_{45}^*) = 0$ and

$$G_{44}^* = \frac{(a_{44})^{(9)} G_{45}^*}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46}^* = \frac{(a_{46})^{(9)} G_{45}^*}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

$$T_{44}^* = \frac{(b_{44})^{(9)} T_{45}^*}{[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47})^*)]} \quad , \quad T_{46}^* = \frac{(b_{46})^{(9)} T_{45}^*}{[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47})^*)]}$$

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**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

**Proof:** Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a''_{14})^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \frac{\partial (b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a'_{13})^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)}G^*_{13}\mathbb{T}_{14} \quad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a'_{14})^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)}G^*_{14}\mathbb{T}_{14} \quad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a'_{15})^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)}G^*_{15}\mathbb{T}_{14} \quad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b'_{13})^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)}T^*_{13}\mathbb{G}_j\right) \quad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b'_{14})^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)}T^*_{14}\mathbb{G}_j\right) \quad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b'_{15})^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T^*_{15}\mathbb{G}_j\right) \quad 530$$

**ASYMPTOTIC STABILITY ANALYSIS** 531

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(2)}$ and $(b''_i)^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G^*_i + \mathbb{G}_i \quad , T_i = T^*_i + \mathbb{T}_i \quad 532$$

$$\frac{\partial(a''_{17})^{(2)}}{\partial T_{17}}(T^*_{17}) = (q_{17})^{(2)} \quad , \frac{\partial(b''_i)^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \quad 533$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a'_{16})^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G^*_{16}\mathbb{T}_{17} \quad 534$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a'_{17})^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G^*_{17}\mathbb{T}_{17} \quad 535$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a'_{18})^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G^*_{18}\mathbb{T}_{17} \quad 536$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b'_{16})^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T^*_{16}\mathbb{G}_j\right) \quad 537$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b'_{17})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}T^*_{17}\mathbb{G}_j\right) \quad 538$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b'_{18})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}T^*_{18}\mathbb{G}_j\right) \quad 539$$

**ASYMPTOTIC STABILITY ANALYSIS** 540

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(3)}$ *and* $(b''_i)^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{21}'')^{(3)}}{\partial T_{21}}(T_{21}^*) = (q_{21})^{(3)} \quad , \frac{\partial (b_i'')^{(3)}}{\partial G_j}(\,(G_{23})^*\,) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\big((a_{20}')^{(3)} + (p_{20})^{(3)}\big)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G_{20}^*\mathbb{T}_{21} \quad 541$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\big((a_{21}')^{(3)} + (p_{21})^{(3)}\big)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G_{21}^*\mathbb{T}_{21} \quad 542$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\big((a_{22}')^{(3)} + (p_{22})^{(3)}\big)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G_{22}^*\mathbb{T}_{21} \quad 543$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\big((b_{20}')^{(3)} - (r_{20})^{(3)}\big)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\big(s_{(20)(j)}T_{20}^*\mathbb{G}_j\big) \quad 544$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\big((b_{21}')^{(3)} - (r_{21})^{(3)}\big)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\big(s_{(21)(j)}T_{21}^*\mathbb{G}_j\big) \quad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\big((b_{22}')^{(3)} - (r_{22})^{(3)}\big)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\big(s_{(22)(j)}T_{22}^*\mathbb{G}_j\big) \quad 546$$

**ASYMPTOTIC STABILITY ANALYSIS** 547

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}\ and\ (b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{25}'')^{(4)}}{\partial T_{25}}(T_{25}^*) = (q_{25})^{(4)} \quad , \frac{\partial (b_i'')^{(4)}}{\partial G_j}((G_{27})^*\,) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\big((a_{24}')^{(4)} + (p_{24})^{(4)}\big)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G_{24}^*\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\big((a_{25}')^{(4)} + (p_{25})^{(4)}\big)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G_{25}^*\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\big((a_{26}')^{(4)} + (p_{26})^{(4)}\big)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G_{26}^*\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\big((b_{24}')^{(4)} - (r_{24})^{(4)}\big)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\big(s_{(24)(j)}T_{24}^*\mathbb{G}_j\big) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\big((b_{25}')^{(4)} - (r_{25})^{(4)}\big)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\big(s_{(25)(j)}T_{25}^*\mathbb{G}_j\big) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T^*_{26}\mathbb{G}_j\right) \quad 554$$

**ASYMPTOTIC STABILITY ANALYSIS** 555

**Theorem 5:** If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(5)}\ and\ (b''_i)^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 556

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{29})^{(5)}}{\partial T_{29}}(T^*_{29}) = (q_{29})^{(5)} \ , \ \frac{\partial(b''_i)^{(5)}}{\partial G_j}((G_{31})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a'_{28})^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G^*_{28}\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a'_{29})^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G^*_{29}\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a'_{30})^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G^*_{30}\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b'_{28})^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T^*_{28}\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b'_{29})^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T^*_{29}\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b'_{30})^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T^*_{30}\mathbb{G}_j\right) \quad 562$$

**ASYMPTOTIC STABILITY ANALYSIS** 563

**Theorem 6:** If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(6)}\ and\ (b''_i)^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{33})^{(6)}}{\partial T_{33}}(T^*_{33}) = (q_{33})^{(6)} \ , \ \frac{\partial(b''_i)^{(6)}}{\partial G_j}((G_{35})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a'_{32})^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G^*_{32}\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a'_{33})^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G^*_{33}\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a'_{34})^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G^*_{34}\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b'_{32})^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T^*_{32}\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b'_{33})^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T^*_{33}\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b'_{34})^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T^*_{34}\mathbb{G}_j\right) \quad 570$$

**ASYMPTOTIC STABILITY ANALYSIS** 571

**Theorem 7:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 572

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{37})^{(7)}}{\partial T_{37}}(T^*_{37}) = (q_{37})^{(7)} \quad , \frac{\partial(b_i'')^{(7)}}{\partial G_j}\left((G_{39})^{**}\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a'_{36})^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G^*_{36}\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a'_{37})^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G^*_{37}\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a'_{38})^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G^*_{38}\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b'_{36})^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T^*_{36}\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b'_{37})^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T^*_{37}\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b'_{38})^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T^*_{38}\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

**ASYMPTOTIC STABILITY ANALYSIS**

**Theorem 8:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{41}'')^{(8)}}{\partial T_{41}}(T_{41}^*) = (q_{41})^{(8)} \quad , \frac{\partial (b_i'')^{(8)}}{\partial G_j}\left( (G_{43})^* \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a_{40}')^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G_{40}^*\mathbb{T}_{41} \quad (581)$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a_{41}')^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G_{41}^*\mathbb{T}_{41} \quad (582)$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a_{42}')^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G_{42}^*\mathbb{T}_{41} \quad (583)$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b_{40}')^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T_{40}^*\mathbb{G}_j\right) \quad (584)$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b_{41}')^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T_{41}^*\mathbb{G}_j\right) \quad (585)$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b_{42}')^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T_{42}^*\mathbb{G}_j\right) \quad (586)$$

(586 A)

**ASYMPTOTIC STABILITY ANALYSIS**

**Theorem 9:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}$ *and* $(b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

**Proof:** Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{45}'')^{(9)}}{\partial T_{45}}(T_{45}^*) = (q_{45})^{(9)} \quad , \frac{\partial (b_i'')^{(9)}}{\partial G_j}\left( (G_{47})^* \right) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a_{44}')^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)}G_{44}^*\mathbb{T}_{45} \quad (586\,B)$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a_{45}')^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)}G_{45}^*\mathbb{T}_{45} \quad (586\,C)$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a_{46}')^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)}G_{46}^*\mathbb{T}_{45} \quad (586\,D)$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b_{44}')^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)}T_{44}^*\mathbb{G}_j\right) \quad (586\,E)$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b_{45}')^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)}T_{45}^*\mathbb{G}_j\right) \quad (586\,F)$$

$\frac{d\mathbb{T}_{46}}{dt} = -\left((b'_{46})^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T^*_{46}\mathbb{G}_j\right)$ 586

G

587

**The characteristic equation of this system is**

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(q_{13})^{(1)}G^*_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)}T^*_{14} + (b_{14})^{(1)}s_{(13),(14)}T^*_{14}\right)$$

$$+\left(\left((\lambda)^{(1)} + (a'_{14})^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)}G^*_{13} + (a_{13})^{(1)}(q_{14})^{(1)}G^*_{14}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)}T^*_{14} + (b_{14})^{(1)}s_{(13),(13)}T^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((b'_{13})^{(1)} + (b'_{14})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(15)}T^*_{14} + (b_{14})^{(1)}s_{(13),(15)}T^*_{13}\right)\} = 0$$

$+$

$$(\lambda)^{(2)} + (b'_{18})^{(2)} - (r_{18})^{(2)}\{\left((\lambda)^{(2)} + (a'_{18})^{(2)} + (p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^*_{17} + (a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^*_{17}\right)$$

$$+\left(\left((\lambda)^{(2)} + (a'_{17})^{(2)} + (p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^*_{16} + (a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}\right)$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2 + \left((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2 + \left((b'_{16})^{(2)} + (b'_{17})^{(2)} - (r_{16})^{(2)} + (r_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$+\left(\left((\lambda)^{(2)}\right)^2 + \left((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17} + (a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^{*}_{16}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\right)\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left(\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)\right]$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(21)}T^{*}_{21}\right)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^{*}_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^{*}_{21}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(20)}T^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$+\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(22)}T^{*}_{20}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\right)\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left(\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)\right]$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(25)}T^{*}_{25}\right)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^{*}_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^{*}_{25}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(24)}T^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$+\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(26)}T^{*}_{24}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\right)\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(29)}T^{*}_{29}\right)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^{*}_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^{*}_{29}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(28)}T^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$+\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(30)}T^{*}_{28}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\right)\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left(\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)\right]$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(33)}T^{*}_{33}\right)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^{*}_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^{*}_{33}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(32)}T^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$+\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(34)}T^{*}_{32}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\right)\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)\right]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(37)}T^{*}_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^{*}_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^{*}_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(36)}T^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$+\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(38)}T^{*}_{36}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\right)\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(41)}T^{*}_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^{*}_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^{*}_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(40)}T^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$+\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(42)}T^{*}_{40}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\right)\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(45)}T^{*}_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^{*}_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^{*}_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(44)}T^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$+\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(46)}T^{*}_{44}\right)\}=0$$

**And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.**

## SECTION THIRTY THREE

## Free Will, Feeling Of Life And Non Local Memory

## INTRODUCTION—VARIABLES USED

**Source: Wikipedia: Time line**

**Luigi Fantappiè: advanced waves and syntropy - 1941**.

**(1)** Starting from the mathematical properties of the laws of syntropy and entropy, Fantappiè arrived to the formulation of a model of consciousness based on (e) the following elements:

(2) **Free will:** this is generally considered a fundamental element of (e) consciousness, is seen as the consequence of (e) a permanent state of choice between information coming from the past and (e&eb) emotions coming from the future. **Antonella Vannini Quantum Biosystems 2008, 2, 165-184 167**

**(3) Feeling of life**: another basic component of consciousness is, according to Fantappiè, a direct consequence of (e) waves moving backward in time, from the future to (e&eb) the past. Fantappiè says that when physical senses vanish, as in deep meditation, (eb) people experience states of consciousness in which past, present and future coexist. The coexistence of past, present and future would be a direct consequence of (e) the coexistence of advanced and retarded waves.

**(4)** **Non local memory**: Fantappiè suggests the existence of non local correlations in the universe, as a consequence of (e) the existence of advanced waves. For example, in living systems, memory could be (=) a manifestation of non local links with (e&eb) past events which according to Klein-Gordon's equation still exist. Fantappiè's model is similar to the transactional interpretation of quantum mechanics and to the model of consciousness suggested by **Chris King (1989).**

## DECONSTRUCTING CONSCIOUSNESS IN SEARCH OF USEFUL DEFINITIONS FOR APPLIED LINGUISTICS Richard Schmidt

The "conscious" vs. "unconscious" controversy is a core issue in both second and foreign language pedagogy and applied linguistics theory, but at the present time there is no generally agreed upon set of constructs and terminology that unifies discussion. It is proposed in this paper that it is possible to standardize somewhat the theoretical concepts that are relevant to understanding the role of conscious and unconscious processes in language learning through reference to four rather different senses of consciousness that are common in everyday usage and surface in some way in technical terminology and as the theoretical issues in the study of learning: consciousness as intentionality (the intentional/incidental learning' contrast), consciousness as attention (focal attention and "noticing" vs. peripheral attention), consciousness as awareness (the contrasts between explicit/implicit learning and knowledge), and consciousness as control (controlled vs. automatic processing, automaticity, explicit/implicit memory). Avoiding the use of terms like conscious and unconscious as umbrella terms and specifying the precise contrasts (dichotomies or continua) at issue should help to unify discussion, enhance comparability across studies, and improve our understanding of what research results in the field may actually imply about learning. This approach does not solve the "problem" of consciousness simply by specifying clear constructs, since intentionality, attention, awareness and control all have a pronounced phenomenological feel to them. This may not necessarily be a problem, however, since there is reason to suspect (though not, of course, to assume) that consciousness as a subjective phenomenon and language learning may turn out in the end to be intimately connected. The "conscious" versus "unconscious" controversy has always been a core issue in the field of second and foreign language pedagogy. Traditional methods such as grammar translation stress the importance of consciousness in learning. In the traditional view, the job of the teacher is to provide an analysis of the target language, as well as some of the differences between the target and native languages. The learner's task is to learn the rules provided and to practice their application by mapping meanings (often provided in the native language) onto the lexicon and structure of the target language. Conscious knowledge of morphological paradigms and syntactic rules is considered crucial, under the assumption is that such knowledge results eventually in the

ability to use the language for communicative purposes (the strong interface position). Errors are considered symptomatic of ignorance, forgetting, lack of attention, or laziness. While defense of the traditional view has been rare in recent years, R. Ellis (1993) has recently recommended the use of grammatical consciousness-raising exercises (based on a structural curriculum) to foster explicit knowledge, to be used as a complement to a functional or task-based syllabus intended to promote implicit knowledge. Ellis' rationale for this combination is based on a weak interface position: explicit declarative knowledge cannot directly become implicit procedural knowledge, but can foster its development through "intake facilitation," causing learners to pay attention to formal features of the input and to notice the gap between these features and those of their interlanguage (Schmidt, 1990; Schmidt & Frota, 1986). Other pedagogical approaches stress the role of unconscious processes in language learning almost exclusively. The "natural approach" of Krashen and Terrell (1983) is derived from Krashen's distinction between conscious learning and subconscious acquisition and the argument that there is no interface between the two. 1t follows that instruction should be based on motivated communicative interaction or other exposure to natural input and that rule explanation, drill, and error correction should be eschewed. "Communicative 1anguage teaching" (e.g. as promoted by the British Council around the world) is based primarily on educational principles rather than a fully articulated theory of learning, but also stresses the importance of learning through the practice of activities that approximate real communication as closely as possible, assuming that learners gain linguistic form by seeking situational meaning rather than by concentrating consciously on linguistic form. Still other pedagogical methods embody a variety of intermediate positions. The audio-lingual method de-emphasizes the role of conscious learning by eliminating direct rule explanation (in principle if not always in practice), under the influence of the behaviorist dictum that awareness of stimulus-response contingencies is irrelevant to the establishment of the desired responses. However, audio-lingual methodology also derives from the belief that proficiency in a foreign language depends on the gradual internalization of structures that are produced with a great deal of conscious effort in the beginning stages, with spontaneous, unreflecting performance emerging as the end result of extensive drill. The "silent way" also avoids externally provided explanations of rules, while stressing the importance of internal awareness achieved when learners consciously engage with the target language

## NOTATION

### Module One
### Free will:

**Freewill** is generally considered a fundamental element of consciousness, is seen as the consequence of (e) a permanent state of choice between information coming from the past and (e&eb) emotions coming from the future.

**Antonella Vannini Quantum Biosystems 2008, 2, 165-184 167**

$G_{13}$ : Category one of consciousness

$G_{14}$ : Category two of SAS

$G_{15}$ : Category three of SAS

$T_{13}$ : Category one of **Freewill** is generally considered a fundamental element

$T_{14}$ : Category two of SAS

$T_{15}$ : Category three of SAS

**Module Two**

**Freewill** is generally considered a fundamental element of consciousness, is seen as the consequence of (e) a **permanent state of choice between information coming from the past and (e&eb) emotions coming from the future**

**Antonella Vannini Quantum Biosystems 2008, 2, 165-184 167**

$G_{16}$ : Category one of **Freewill** is generally considered a fundamental element of consciousness; **permanent state of choice between information coming from the past and (e&eb) emotions coming from the future**

$G_{17}$ : Category two of SAS

$G_{18}$ : Category three of SAS

$T_{16}$ : Category one of **permanent state of choice between information coming from the past and (e&eb) emotions coming from the future ; Freewill** is generally considered a fundamental element of consciousness

$T_{17}$ : Category two of SAS

$T_{18}$ : Category three of SAS

**Module three**

**Feeling of life is** another basic component of consciousness, according to Fantappiè, a direct consequence of (e) waves moving backward in time, from the future to (e&eb) the past.

Fantappiè says that when physical senses vanish, as in deep meditation, (eb) people experience states of consciousness in which past, present and future coexist.

The coexistence of past, present and future would be a direct consequence of (e) the coexistence of advanced and retarded waves

$G_{20}$ : Category one of **Feeling of life**

$G_{21}$ : Category two of SAS

$G_{22}$ : Category three of SAS

$T_{20}$ : Category one of basic component of consciousness, according to Fantappiè, a direct consequence of (e) waves moving backward in time, from the future to (e&eb) the past

$T_{21}$ : Category two of SAS

$T_{22}$ : Category three of SAS

**Module four**

**Feeling of life is** another basic component of consciousness, according to Fantappiè, a direct consequence of (e) waves moving backward in time, from the future to (e&eb) the past

$G_{24}$ : Category one of waves moving backward in time, from the future to (e&eb) the past

$G_{25}$ : Category two of SAS

$G_{26}$ : Category three of SAS

$T_{24}$ : Category one of **Feeling of life is** another basic component of consciousness, according to Fantappiè, a direct consequence

$T_{25}$ : Category two of SAS

$T_{26}$ : Category three of SAS

**Module five**

**Feeling of life is** another basic component of consciousness, according to Fantappiè, a direct consequence of waves moving backward in time, from the future to (e&eb) the **past**

$G_{28}$ : Category one of **Feeling of life is** another basic component of consciousness, according to Fantappiè, a direct consequence of waves moving backward in time, from the future; **past**

$G_{29}$ : Category two of SAS

$G_{30}$ : Category three of SAS

$T_{28}$ : Category one of **past ; Feeling of life is** another basic component of consciousness, according to Fantappiè, a direct consequence of waves moving backward in time, from the future

$T_{29}$ : Category two of SAS

$T_{30}$ : Category three of SAS

**Module six**

Fantappiè says that when physical senses vanish, as in deep meditation, (eb) people experience states of consciousness in which past, present and future coexist

$G_{32}$ : Category one of meditation

$G_{33}$ : Category two of SAS

$G_{34}$ : Category three of SAS

$T_{32}$ : Category one of vanishment of physical senses

$T_{33}$ : Category two of SAS

$T_{34}$ : Category three of SAS

**Module seven**

Fantappiè says that when physical senses vanish, as in deep meditation, (eb) people experience states of consciousness in which past, present and future coexist

Model means the coexistence of past, future and present

$G_{36}$ : Category one of past; present (take AC=DC=0)

$G_{37}$ : Category two of SAS

$G_{38}$ : Category three of SAS

$T_{36}$ : Category one of present; past (take AC=DC=0)

$T_{37}$ : Category two of SAS

$T_{38}$ : Category three of SAS

**Module eight**

$G_{40}$ : Category one of present; future (take AC=DC=0)

$G_{41}$ : Category two of SAS

$G_{42}$ : Category three of SAS

$T_{40}$ : Category one of future; present (take AC=DC=0)

$T_{41}$ : Category two of SAS

$T_{42}$ : Category three of SAS

**Module Nine**

The coexistence of past, present and future would be a direct consequence of (e) the coexistence of advanced and retarded waves

$G_{44}$ : Category one of coexistence of advanced and retarded waves

$G_{45}$ : Category two of SAS

$G_{46}$ : Category three of SAS

$T_{44}$ : Category one of coexistence of past, present and future

$T_{45}$ : Category two of SAS

$T_{46}$ : Category three of SAS

**The Coefficients:**

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}\ ,\ (b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)},$
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients

$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}, \ (b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)} \ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$ ,
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)},$
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)},$
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)},$
are Dissipation coefficients

**Module Numbered One**

**The differential system of this model is now (Module Numbered one)**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \quad 1$$
$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad 2$$
$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15} \quad 3$$
$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13} \quad 4$$
$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14} \quad 5$$
$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)\right]T_{15} \quad 6$$
$+(a''_{13})^{(1)}(T_{14}, t) =$ First augmentation factor
$-(b''_{13})^{(1)}(G, t) =$ First detritions factor

**Module Numbered Two**

**The differential system of this model is now ( Module numbered two)**

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right]G_{16} \quad 7$$
$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t)\right]G_{17} \quad 8$$
$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t)\right]G_{18} \quad 9$$
$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\big((G_{19}), t\big)\right]T_{16} \quad 10$$
$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\big((G_{19}), t\big)\right]T_{17} \quad 11$$
$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\big((G_{19}), t\big)\right]T_{18} \quad 12$$
$+(a''_{16})^{(2)}(T_{17}, t) =$ First augmentation factor
$-(b''_{16})^{(2)}\big((G_{19}), t\big) =$ First detritions factor

**Module Numbered Three**

**The differential system of this model is now (Module numbered three)**

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right]G_{20} \quad 13$$
$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}, t)\right]G_{21} \quad 14$$
$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}, t)\right]G_{22} \quad 15$$
$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}, t)\right]T_{20} \quad 16$$
$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}, t)\right]T_{21} \quad 17$$

$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}, t)\right]T_{22}$ 18

$+(a''_{20})^{(3)}(T_{21}, t) =$ First augmentation factor

$-(b''_{20})^{(3)}(G_{23}, t) =$ First detritions factor

**Module Numbered Four**

**The differential system of this model is now (Module numbered Four)**

$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)\right]G_{24}$ 19

$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}, t)\right]G_{25}$ 20

$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}, t)\right]G_{26}$ 21

$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27}), t)\right]T_{24}$ 22

$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}((G_{27}), t)\right]T_{25}$ 23

$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27}), t)\right]T_{26}$ 24

$+(a''_{24})^{(4)}(T_{25}, t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}((G_{27}), t) =$ **First detritions factor**

**Module Numbered Five**:

**The differential system of this model is now (Module number five)**

$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right]G_{28}$ 25

$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}, t)\right]G_{29}$ 26

$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}, t)\right]G_{30}$ 27

$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31}), t)\right]T_{28}$ 28

$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}((G_{31}), t)\right]T_{29}$ 29

$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31}), t)\right]T_{30}$ 30

$+(a''_{28})^{(5)}(T_{29}, t) =$ **First augmentation factor**

$-(b''_{28})^{(5)}((G_{31}), t) =$ **First detritions factor**

**Module Numbered Six**

**The differential system of this model is now (Module numbered Six)**

$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right]G_{32}$ 31

$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}, t)\right]G_{33}$ 32

$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}, t)\right]G_{34}$ 33

$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35}), t)\right]T_{32}$ 34

$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}((G_{35}), t)\right]T_{33}$ 35

$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35}), t)\right]T_{34}$ 36

$+(a''_{32})^{(6)}(T_{33}, t) =$ **First augmentation factor**

**Module Numbered Seven:**

**The differential system of this model is now (Seventh Module)**

$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right]G_{36}$ 37

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37},t)\right]G_{37} \quad 38$$
$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37},t)\right]G_{38} \quad 39$$
$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39}),t)\right]T_{36} \quad 40$$
$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}((G_{39}),t)\right]T_{37} \quad 41$$
$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39}),t)\right]T_{38} \quad 42$$

$+(a''_{36})^{(7)}(T_{37},t) =$ **First augmentation factor**

**Module Numbered Eight**

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right]G_{40} \quad 43$$
$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41},t)\right]G_{41} \quad 44$$
$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41},t)\right]G_{42} \quad 45$$
$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43}),t)\right]T_{40} \quad 46$$
$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}((G_{43}),t)\right]T_{41} \quad 47$$
$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43}),t)\right]T_{42} \quad 48$$

**Module Numbered Nine**

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$
$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$
$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$
$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47}),t)\right]T_{44} \quad 52$$
$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}((G_{47}),t)\right]T_{45} \quad 53$$
$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47}),t)\right]T_{46} \quad 54$$

$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}((G_{47}),t) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)}\boxed{+(a''_{13})^{(1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)}\boxed{+(a''_{14})^{(1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)}\boxed{+(a''_{15})^{(1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15} \quad 57$$

Where $\boxed{(a''_{13})^{(1)}(T_{14},t)}$, $\boxed{(a''_{14})^{(1)}(T_{14},t)}$, $\boxed{(a''_{15})^{(1)}(T_{14},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7)}(T_{37},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8)}(T_{41},t)}$ $\boxed{+(a''_{41})^{(8,8)}(T_{41},t)}$ $\boxed{+(a''_{42})^{(8,8)}(T_{41},t)}$ are eight augmentation coefficient for 1,2,3

$\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)}\boxed{-(b''_{13})^{(1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13} \quad 58$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)}\boxed{-(b''_{14})^{(1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14} \quad 59$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)}\boxed{-(b''_{15})^{(1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15} \quad 60$$

Where $\boxed{-(b''_{13})^{(1)}(G,t)}$, $\boxed{-(b''_{14})^{(1)}(G,t)}$, $\boxed{-(b''_{15})^{(1)}(G,t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{37})^{(7,7,)}(G_{39},t)$, $-(b''_{36})^{(7,7,)}(G_{39},t)$, $-(b''_{38})^{(7,7,)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8)}(G_{43},t)$ $-(b''_{41})^{(8,8)}(G_{43},t)$ $-(b''_{42})^{(8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt}=(a_{16})^{(2)}G_{17}-\left[\begin{array}{ccc}(a'_{16})^{(2)}+(a''_{16})^{(2)}(T_{17},t) & +(a''_{13})^{(1,1,)}(T_{14},t) & +(a''_{20})^{(3,3,3)}(T_{21},t)\\ +(a''_{24})^{(4,4,4,4,4)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,5)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,6)}(T_{33},t)\\ +(a''_{36})^{(7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9)}(T_{45},t)\end{array}\right]G_{16} \quad (61)$$

$$\frac{dG_{17}}{dt}=(a_{17})^{(2)}G_{16}-\left[\begin{array}{ccc}(a'_{17})^{(2)}+(a''_{17})^{(2)}(T_{17},t) & +(a''_{14})^{(1,1,)}(T_{14},t) & +(a''_{21})^{(3,3,3)}(T_{21},t)\\ +(a''_{25})^{(4,4,4,4,4)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,5)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,6)}(T_{33},t)\\ +(a''_{37})^{(7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9)}(T_{45},t)\end{array}\right]G_{17} \quad (62)$$

$$\frac{dG_{18}}{dt}=(a_{18})^{(2)}G_{17}-\left[\begin{array}{ccc}(a'_{18})^{(2)}+(a''_{18})^{(2)}(T_{17},t) & +(a''_{15})^{(1,1,)}(T_{14},t) & +(a''_{22})^{(3,3,3)}(T_{21},t)\\ +(a''_{26})^{(4,4,4,4,4)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,5)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,6)}(T_{33},t)\\ +(a''_{38})^{(7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9)}(T_{45},t)\end{array}\right]G_{18} \quad (63)$$

Where $+(a''_{16})^{(2)}(T_{17},t)$, $+(a''_{17})^{(2)}(T_{17},t)$, $+(a''_{18})^{(2)}(T_{17},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{13})^{(1,1,)}(T_{14},t)$, $+(a''_{14})^{(1,1,)}(T_{14},t)$, $+(a''_{15})^{(1,1,)}(T_{14},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{36})^{(7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7)}(T_{37},t)$ are seventh augmentation coefficient for category 1, 2 and 3

$+(a''_{40})^{(8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8)}(T_{41},t)$ are eight augmentation coefficient for category 1, 2 and 3

$+(a''_{44})^{(9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9)}(T_{45},t)$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt}=(b_{16})^{(2)}T_{17}-\left[\begin{array}{ccc}(b'_{16})^{(2)}-(b''_{16})^{(2)}(G_{19},t) & -(b''_{13})^{(1,1,)}(G,t) & -(b''_{20})^{(3,3,3,)}(G_{23},t)\\ -(b''_{24})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6)}(G_{35},t)\\ -(b''_{36})^{(7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9)}(G_{47},t)\end{array}\right]T_{16} \quad (64)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)}\boxed{-(b''_{17})^{(2)}(G_{19},t)} & \boxed{-(b''_{14})^{(1,1,)}(G,t)} & \boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{17} \quad 65$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)}\boxed{-(b''_{18})^{(2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,)}(G,t)} & \boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{18} \quad 66$$

where $\boxed{-(b''_{16})^{(2)}(G_{19},t)}$ , $\boxed{-(b''_{17})^{(2)}(G_{19},t)}$ , $\boxed{-(b''_{18})^{(2)}(G_{19},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,)}(G,t)}$ are second detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{44})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \begin{bmatrix} (a'_{20})^{(3)}\boxed{+(a''_{20})^{(3)}(T_{21},t)} & \boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{20} \quad 67$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \begin{bmatrix} (a'_{21})^{(3)}\boxed{+(a''_{21})^{(3)}(T_{21},t)} & \boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{21} \quad 68$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \begin{bmatrix} (a'_{22})^{(3)}\boxed{+(a''_{22})^{(3)}(T_{21},t)} & \boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{22} \quad 69$$

$\boxed{+(a''_{20})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3)}(T_{21},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}$ are second augmentation coefficients

for category 1, 2 and 3

$+(a_{13}'')^{(1,1,1,)}(T_{14},t)$, $+(a_{14}'')^{(1,1,1,)}(T_{14},t)$, $+(a_{15}'')^{(1,1,1,)}(T_{14},t)$ are third augmentation coefficients for category 1, 2 and 3

$+(a_{24}'')^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a_{25}'')^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a_{26}'')^{(4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficients for category 1, 2 and 3

$+(a_{28}'')^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a_{29}'')^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a_{30}'')^{(5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficients for category 1, 2 and 3

$+(a_{32}'')^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a_{33}'')^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a_{34}'')^{(6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficients for category 1, 2 and 3

$+(a_{36}'')^{(7,7,7,7)}(T_{37},t)$, $+(a_{37}'')^{(7,7,7,7)}(T_{37},t)$, $+(a_{38}'')^{(7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1, 2 and 3

$+(a_{40}'')^{(8,8,8,8)}(T_{41},t)$, $+(a_{41}'')^{(8,8,8,8)}(T_{41},t)$, $+(a_{42}'')^{(8,8,8,8)}(T_{41},t)$ are eight augmentation coefficients for category 1, 2 and 3

$+(a_{44}'')^{(9,9,9)}(T_{45},t)$, $+(a_{45}'')^{(9,9,9)}(T_{45},t)$, $+(a_{46}'')^{(9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b_{20}')^{(3)} & -(b_{20}'')^{(3)}(G_{23},t) & -(b_{16}'')^{(2,2,2)}(G_{19},t) & -(b_{13}'')^{(1,1,1,)}(G,t) \\ -(b_{24}'')^{(4,4,4,4,4,4)}(G_{27},t) & -(b_{28}'')^{(5,5,5,5,5,5)}(G_{31},t) & -(b_{32}'')^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b_{36}'')^{(7,7,7,7)}(G_{39},t) & -(b_{40}'')^{(8,8,8,8)}(G_{43},t) & -(b_{44}'')^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{20} \quad (70)$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b_{21}')^{(3)} & -(b_{21}'')^{(3)}(G_{23},t) & -(b_{17}'')^{(2,2,2)}(G_{19},t) & -(b_{14}'')^{(1,1,1,)}(G,t) \\ -(b_{25}'')^{(4,4,4,4,4,4)}(G_{27},t) & -(b_{29}'')^{(5,5,5,5,5,5)}(G_{31},t) & -(b_{33}'')^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b_{37}'')^{(7,7,7,7)}(G_{39},t) & -(b_{41}'')^{(8,8,8,8)}(G_{43},t) & -(b_{45}'')^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{21} \quad (71)$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b_{22}')^{(3)} & -(b_{22}'')^{(3)}(G_{23},t) & -(b_{18}'')^{(2,2,2)}(G_{19},t) & -(b_{15}'')^{(1,1,1,)}(G,t) \\ -(b_{26}'')^{(4,4,4,4,4,4)}(G_{27},t) & -(b_{30}'')^{(5,5,5,5,5,5)}(G_{31},t) & -(b_{34}'')^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b_{38}'')^{(7,7,7,7)}(G_{39},t) & -(b_{42}'')^{(8,8,8,8)}(G_{43},t) & -(b_{46}'')^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{22} \quad (72)$$

$-(b_{20}'')^{(3)}(G_{23},t)$, $-(b_{21}'')^{(3)}(G_{23},t)$, $-(b_{22}'')^{(3)}(G_{23},t)$ *are first detrition coefficients for category 1, 2 and 3*

$-(b_{16}'')^{(2,2,2)}(G_{19},t)$, $-(b_{17}'')^{(2,2,2)}(G_{19},t)$, $-(b_{18}'')^{(2,2,2)}(G_{19},t)$ *are second detrition coefficients for category 1, 2 and 3*

$-(b_{13}'')^{(1,1,1,)}(G,t)$, $-(b_{14}'')^{(1,1,1,)}(G,t)$, $-(b_{15}'')^{(1,1,1,)}(G,t)$ *are third detrition coefficients for category 1,2 and 3*

$-(b_{24}'')^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b_{25}'')^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b_{26}'')^{(4,4,4,4,4,4)}(G_{27},t)$ *are fourth detrition coefficients for category 1, 2 and 3*

$-(b_{28}'')^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b_{29}'')^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b_{30}'')^{(5,5,5,5,5,5)}(G_{31},t)$ *are fifth detrition coefficients for category 1, 2 and 3*

$-(b_{32}'')^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b_{33}'')^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b_{34}'')^{(6,6,6,6,6,6)}(G_{35},t)$ *are sixth detrition coefficients for category 1, 2 and 3*

$-(b_{36}'')^{(7,7,7,7)}(G_{39},t)$, $-(b_{37}'')^{(7,7,7,7)}(G_{39},t)$, $-(b_{38}'')^{(7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients

for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \begin{bmatrix} (a'_{24})^{(4)} & \boxed{+(a''_{24})^{(4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,)}(T_{33},t)} \\ & \boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)} \\ & \boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{24} \quad 73$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \begin{bmatrix} (a'_{25})^{(4)} & \boxed{+(a''_{25})^{(4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6)}(T_{33},t)} \\ & \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)} \\ & \boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{25} \quad 74$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \begin{bmatrix} (a'_{26})^{(4)} & \boxed{+(a''_{26})^{(4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,)}(T_{33},t)} \\ & \boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)} \\ & \boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{26} \quad 75$$

$\boxed{(a''_{24})^{(4)}(T_{25},t)}$, $\boxed{(a''_{25})^{(4)}(T_{25},t)}$, $\boxed{(a''_{26})^{(4)}(T_{25},t)}$ *are first augmentation coefficients category* 1, 2 3

$\boxed{+(a''_{28})^{(5,5,)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,)}(T_{29},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)}$
*are fourth augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)}$
*are fifth augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)}$
*are sixth augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)}$
*are seventh augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)}$
*are eighth augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \begin{bmatrix} (b'_{24})^{(4)} & \boxed{-(b''_{24})^{(4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,)}(G_{35},t)} \\ & \boxed{-(b''_{13})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)} \\ & \boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{24} \quad 76$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \begin{bmatrix} (b'_{25})^{(4)} & -(b''_{25})^{(4)}(G_{27},t) & -(b''_{29})^{(5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{25} \quad (77)$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)} & -(b''_{26})^{(4)}(G_{27},t) & -(b''_{30})^{(5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1)}(G,t) & -(b''_{18})^{(2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{26} \quad (78)$$

*Where* $-(b''_{24})^{(4)}(G_{27},t)$, $-(b''_{25})^{(4)}(G_{27},t)$, $-(b''_{26})^{(4)}(G_{27},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{28})^{(5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,)}(G_{31},t)$ *are second detrition coefficients for category* 1, 2 *and* 3

$-(b''_{32})^{(6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,)}(G_{35},t)$ *are third detrition coefficients for category* 1, 2 *and* 3

$-(b''_{13})^{(1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1)}(G,t)$
*are fourth detrition coefficients for category* 1, 2 *and* 3

$-(b''_{16})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2)}(G_{19},t)$
*are fifth detrition coefficients for category* 1, 2 *and* 3

$-(b''_{20})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3)}(G_{23},t)$
*are sixth detrition coefficients for category* 1, 2 *and* 3

$-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)$
*are seventh detrition coefficients for category* 1, 2 *and* 3

$-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)$
*are eighth detrition coefficients for category* 1, 2 *and* 3

$-(b''_{46})^{(9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \begin{bmatrix} (a'_{28})^{(5)} & +(a''_{28})^{(5)}(T_{29},t) & +(a''_{24})^{(4,4,)}(T_{25},t) & +(a''_{32})^{(6,6,6)}(T_{33},t) \\ & +(a''_{13})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{16})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3,3)}(T_{21},t) \\ & +(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{28} \quad (79)$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \begin{bmatrix} (a'_{29})^{(5)} & +(a''_{29})^{(5)}(T_{29},t) & +(a''_{25})^{(4,4,)}(T_{25},t) & +(a''_{33})^{(6,6,6)}(T_{33},t) \\ & +(a''_{14})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{17})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3,3)}(T_{21},t) \\ & +(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{29} \quad (80)$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \begin{bmatrix} (a'_{30})^{(5)} & +(a''_{30})^{(5)}(T_{29},t) & +(a''_{26})^{(4,4,)}(T_{25},t) & +(a''_{34})^{(6,6,6)}(T_{33},t) \\ & +(a''_{15})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{18})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3,3)}(T_{21},t) \\ & +(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{30} \quad (81)$$

*Where* $+(a''_{28})^{(5)}(T_{29},t)$, $+(a''_{29})^{(5)}(T_{29},t)$, $+(a''_{30})^{(5)}(T_{29},t)$ *are first augmentation coefficients for category* 1, 2 *and* 3

*And* $+(a''_{24})^{(4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,)}(T_{25},t)$ *are second augmentation*

$coefficient\ for\ category\ 1,2\ and\ 3$
$+(a''_{32})^{(6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6)}(T_{33},t)$ $are\ third\ \ augmentation$
$coefficient\ for\ category\ 1,2\ and\ 3$
$+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)$ are fourth augmentation coefficients for category 1,2, and 3
$+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)$ are fifth augmentation coefficients for category 1,2,and 3
$+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)$ are sixth augmentation coefficients for category 1,2, 3
$+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)$ , $+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1,2, 3
$+(a''_{40})^{(8,8\ ,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)$ are eighth augmentation coefficients for category 1,2, 3
$+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \begin{bmatrix} (b'_{28})^{(5)} & -(b''_{28})^{(5)}(G_{31},t) & -(b''_{24})^{(4,4,)}(G_{27},t) & -(b''_{32})^{(6,6,6)}(G_{35},t) \\ & -(b''_{13})^{(1,1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{28} \quad 82$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b'_{29})^{(5)} & -(b''_{29})^{(5)}(G_{31},t) & -(b''_{25})^{(4,4,)}(G_{27},t) & -(b''_{33})^{(6,6,6)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{29} \quad 83$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b'_{30})^{(5)} & -(b''_{30})^{(5)}(G_{31},t) & -(b''_{26})^{(4,4,)}(G_{27},t) & -(b''_{34})^{(6,6,6)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1,1,)}(G,t) & -(b''_{18})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{30} \quad 84$$

$where$ $-(b''_{28})^{(5)}(G_{31},t)$ , $-(b''_{29})^{(5)}(G_{31},t)$ , $-(b''_{30})^{(5)}(G_{31},t)$ $are\ first\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b''_{24})^{(4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,)}(G_{27},t)$ $are\ second\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b''_{32})^{(6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6)}(G_{35},t)$ $are\ \ third\ \ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b''_{13})^{(1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1)}(G,t)$ , $-(b''_{15})^{(1,1,1,1,1,)}(G,t)$ are fourth detrition coefficients for category 1,2, and 3
$-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1,2, and 3
$-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1,2, and 3
$-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[\begin{array}{ccc} (a'_{32})^{(6)}\boxed{+(a''_{32})^{(6)}(T_{33},t)} & \boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{32} \quad (85)$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[\begin{array}{ccc} (a'_{33})^{(6)}\boxed{+(a''_{33})^{(6)}(T_{33},t)} & \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{33} \quad (86)$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[\begin{array}{ccc} (a'_{34})^{(6)}\boxed{+(a''_{34})^{(6)}(T_{33},t)} & \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{34} \quad (87)$$

$\boxed{+(a''_{32})^{(6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6)}(T_{33},t)}$ *are first augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}$ *are second augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}$ *are third augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}$ - are fourth augmentation coefficients

$\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}$ - fifth augmentation coefficients

$\boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)}$ sixth augmentation coefficients

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)}$ seventh augmentation coefficients

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}$ Eighth augmentation coefficients

$\boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)}$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[\begin{array}{ccc} (b'_{32})^{(6)}\boxed{-(b''_{32})^{(6)}(G_{35},t)} & \boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)} \end{array}\right] T_{32} \quad (88)$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \begin{bmatrix} (b'_{33})^{(6)}\boxed{-(b''_{33})^{(6)}(G_{35},t)}\boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}\boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}\boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)}\boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{33} \quad (89)$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)}\boxed{-(b''_{34})^{(6)}(G_{35},t)}\boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}\boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}\boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)}\boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{34} \quad (90)$$

$\boxed{-(b''_{32})^{(6)}(G_{35},t)}, \boxed{-(b''_{33})^{(6)}(G_{35},t)}, \boxed{-(b''_{34})^{(6)}(G_{35},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}$ *are second detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)}$ *are third detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}$ are fourth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \quad (91)$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a'_{36})^{(7)}\boxed{+(a''_{36})^{(7)}(T_{37},t)}\boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)}\boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)}\boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)}\boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)}\boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \quad (92)$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a'_{37})^{(7)}\boxed{+(a''_{37})^{(7)}(T_{37},t)}\boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)}\boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)}\boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)}\boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}\boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{38}}{dt} \qquad 93$$

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a'_{38})^{(7)}\boxed{+(a''_{38})^{(7)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{(a''_{36})^{(7)}(T_{37},t)}$, $\boxed{(a''_{37})^{(7)}(T_{37},t)}$, $\boxed{(a''_{38})^{(7)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,)}(T_{41},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{36}}{dt} = \qquad 94$$

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b'_{36})^{(7)}\boxed{-(b''_{36})^{(7)}(G_{39},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{37}}{dt} =$$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b'_{37})^{(7)}\boxed{-(b''_{37})^{(7)}(G_{39},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{38}}{dt} =$$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b'_{38})^{(7)}\boxed{-(b''_{38})^{(7)}(G_{39},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b_{36}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{37}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{16}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{17}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{18}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{20}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{21}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{22}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{24}'')^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{25}'')^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{26}'')^{(4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{28}'')^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{32}'')^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{34}'')^{(6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{46}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{45}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{44}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3



$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \begin{bmatrix} (a_{40}')^{(8)}\boxed{+(a_{40}'')^{(8)}(T_{41},t)} & \boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{20}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{24}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{28}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{32}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{36}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{44}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \begin{bmatrix} (a_{41}')^{(8)}\boxed{+(a_{41}'')^{(8)}(T_{41},t)} & \boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{21}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{25}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{29}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{33}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{37}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{45}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \begin{bmatrix} (a_{42}')^{(8)}\boxed{+(a_{42}'')^{(8)}(T_{41},t)} & \boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{22}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{26}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{30}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{34}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{15}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{38}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{46}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a_{40}'')^{(8)}(T_{41},t)}$, $\boxed{+(a_{41}'')^{(8)}(T_{41},t)}$, $\boxed{+(a_{42}'')^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second

augmentation coefficient for category 1, 2 and 3
$+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)$ , $+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)$ , $+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3
$+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)$ , $+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)$ , $+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3
$+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)$ , $+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3
$+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)$ , $+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3
$+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ $+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ $+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ are seventh augmentation coefficient for 1,2,3
$+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$ , $+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$ , $+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$ are eighth augmentation coefficient for 1,2,3
$+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$ , $+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$ , $+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} =$$

$$(b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)}\left[-(b''_{40})^{(8)}(G_{43},t)\right] & \left[-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} =$$

$$(b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)}\left[-(b''_{41})^{(8)}(G_{43},t)\right] & \left[-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} =$$

$$(b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)}\left[-(b''_{42})^{(8)}(G_{43},t)\right] & \left[-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{15}$$

Where $-(b''_{36})^{(7)}(G_{39},t)$ , $-(b''_{37})^{(7)}(G_{39},t)$ , $-(b''_{38})^{(7)}(G_{39},t)$ are first detrition coefficients for category 1, 2 and 3
$-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$ , $-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$ , $-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3
$-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$ , $-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$ , $-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3
$-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)$ , $-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)$ , $-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3



$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \begin{bmatrix} (a'_{44})^{(9)}\boxed{+(a''_{44})^{(9)}(T_{45},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \begin{bmatrix} (a'_{45})^{(9)}\boxed{+(a''_{45})^{(9)}(T_{45},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \begin{bmatrix} (a'_{46})^{(9)}\boxed{+(a''_{46})^{(9)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a''_{44})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,1,1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1,1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1,1,1,1,1,1)}(T_{14},t)$ are Seventh augmentation coefficient for category 1, 2 and 3

$+(a''_{38})^{(7,7,7,7,7,7,7,7,7,7)}(T_{37},t)$ $+(a''_{37})^{(7,7,7,7,7,7,7,7,7,7)}(T_{37},t)$ $+(a''_{36})^{(7,7,7,7,7,7,7,7,7,7)}(T_{37},t)$ are eighth augmentation coefficient for 1,2,3

$+(a''_{40})^{(8,8,8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8,8,8,8,8)}(T_{41},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} =$$

$$(b_{44})^{(9)}T_{45} -$$

$$\begin{bmatrix} (b'_{44})^{(9)} & -(b''_{44})^{(9)}(G_{47},t) & -(b''_{16})^{(2,2,2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6,6,6,6,6,6)}(G_{35},t) & \\ -(b''_{13})^{(1,1,1,1,1,1,1,1,1,1)}(G,t) & -(b''_{36})^{(7,7,7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8,8,8,8,8)}(G_{43},t) & \end{bmatrix} T_{13}$$

$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b'_{45})^{(9)} & -(b''_{45})^{(9)}(G_{47},t) & -(b''_{17})^{(2,2,2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6,6,6,6,6,6)}(G_{35},t) & \\ -(b''_{14})^{(1,1,1,1,1,1,1,1,1,1)}(G,t) & -(b''_{37})^{(7,7,7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8,8,8,8,8)}(G_{43},t) & \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b'_{46})^{(9)} & -(b''_{46})^{(9)}(G_{47},t) & -(b''_{18})^{(2,2,2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6,6,6,6,6,6)}(G_{35},t) & \\ -(b''_{15})^{(1,1,1,1,1,1,1,1,1,1)}(G,t) & -(b''_{38})^{(7,7,7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8,8,8,8,8)}(G_{43},t) & \end{bmatrix} T_{15}$$

Where $-(b''_{44})^{(9)}(G_{47},t)$, $-(b''_{45})^{(9)}(G_{47},t)$, $-(b''_{46})^{(9)}(G_{47},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6,6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{15})^{(1,1,1,1,1,1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1,1,1,1,1,1)}(G,t)$, $-(b''_{13})^{(1,1,1,1,1,1,1,1,1,1)}(G,t)$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{37}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{36}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a_i')^{(1)}, (a_i'')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (b_i'')^{(1)} > 0, \quad i,j = 13,14,15 \qquad 97$$

The functions $(a_i'')^{(1)}, (b_i'')^{(1)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a_i'')^{(1)}(T_{14},t) \leq (p_i)^{(1)} \leq (\hat{A}_{13})^{(1)}$$

$$(b_i'')^{(1)}(G,t) \leq (r_i)^{(1)} \leq (b_i')^{(1)} \leq (\hat{B}_{13})^{(1)}$$

$$lim_{T_2\to\infty}(a_i'')^{(1)}(T_{14},t) = (p_i)^{(1)} \qquad 98$$

$$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = (r_i)^{(1)}$$

**Definition of** $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$ :

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$$|(a_i'')^{(1)}(T_{14}',t) - (a_i'')^{(1)}(T_{14},t)| \leq (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G',t) - (b_i'')^{(1)}(G,t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}',t)$ and $(a_i'')^{(1)}(T_{14},t)$ . $(T_{14}',t)$ and $(T_{14},t)$ are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14},t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$ : 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$$

**Definition of** $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$ : 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i,j = 16,17,18$$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17}, t) \leq (p_i)^{(2)} \leq (\hat{A}_{16})^{(2)} \quad 102$$

$$(b_i'')^{(2)}(G_{19}, t) \leq (r_i)^{(2)} \leq (b_i')^{(2)} \leq (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2 \to \infty} (a_i'')^{(2)}(T_{17}, t) = (p_i)^{(2)} \quad 104$$

$$\lim_{G \to \infty} (b_i'')^{(2)}((G_{19}), t) = (r_i)^{(2)} \quad 105$$

**Definition of** $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$ : 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}', t) - (a_i'')^{(2)}(T_{17}, t)| \leq (\hat{k}_{16})^{(2)}|T_{17} - T_{17}'|e^{-(\hat{M}_{16})^{(2)}t} \quad 107$$

$$|(b_i'')^{(2)}((G_{19})', t) - (b_i'')^{(2)}((G_{19}), t)| < (\hat{k}_{16})^{(2)}||(G_{19}) - (G_{19})'||e^{-(\hat{M}_{16})^{(2)}t} \quad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}', t)$ and$(a_i'')^{(2)}(T_{17}, t)$ . $(T_{17}', t)$ and $(T_{17}, t)$ are points belonging to the interval $[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}]$ . It is to be noted that $(a_i'')^{(2)}(T_{17}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17}, t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$ :

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants 109

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$$

**Definition of** $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$ :

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1 \quad (110)$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(b_i)^{(2)} + (b_i')^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1 \quad (111)$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i,j = 20,21,22 \quad (112)$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(3)}$, $(r_i)^{(3)}$:

$$(a_i'')^{(3)}(T_{21}, t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23}, t) \leq (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(3)}(T_{21}, t) = (p_i)^{(3)} \quad (113)$$

$$\lim_{G \to \infty}(b_i'')^{(3)}(G_{23}, t) = (r_i)^{(3)}$$

**Definition of** $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$ :

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: (114)

$$|(a_i'')^{(3)}(T_{21}', t) - (a_i'')^{(3)}(T_{21}, t)| \leq (\hat{k}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\hat{M}_{20})^{(3)}t}$$

$$|(b_i'')^{(3)}(G_{23}', t) - (b_i'')^{(3)}(G_{23}, t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}'||e^{-(\hat{M}_{20})^{(3)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}', t)$ and $(a_i'')^{(3)}(T_{21}, t)$ . $(T_{21}', t)$ And $(T_{21}, t)$ are points belonging to the interval $[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}]$. It is to be noted that $(a_i'')^{(3)}(T_{21}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$ : (115)

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$ and the constants $(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}, i = 20,21,22,$ satisfy the inequalities 116

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(a_i)^{(3)} + (a_i')^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(b_i)^{(3)} + (b_i')^{(3)} + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i, j = 24,25,26 \qquad 117$$

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(4)}$, $(r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25}, t) \le (p_i)^{(4)} \le (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}((G_{27}), t) \le (r_i)^{(4)} \le (b_i')^{(4)} \le (\hat{B}_{24})^{(4)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(4)}(T_{25}, t) = (p_i)^{(4)} \qquad 118$$

$$\lim_{G \to \infty}(b_i'')^{(4)}((G_{27}), t) = (r_i)^{(4)}$$

**Definition of** $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$ :

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24,25,26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}', t) - (a_i'')^{(4)}(T_{25}, t)| \le (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})', t) - (b_i'')^{(4)}((G_{27}), t)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}', t)$ and$(a_i'')^{(4)}(T_{25}, t)$ . $(T_{25}', t)$ and $(T_{25}, t)$ are points belonging to the interval $[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}]$. It is to be noted that $(a_i'')^{(4)}(T_{25}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$ : 120

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$, are positive constants

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}, \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$$

**Definition of** $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$ : 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and\ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}, i = 24,25,26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(a_i)^{(4)} + (a_i')^{(4)} + (\hat{A}_{24})^{(4)} + (\hat{P}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(b_i)^{(4)} + (b_i')^{(4)} + (\hat{B}_{24})^{(4)} + (\hat{Q}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i,j = 28,29,30 \qquad 122$$

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(5)}$, $(r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29}, t) \le (p_i)^{(5)} \le (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}((G_{31}), t) \le (r_i)^{(5)} \le (b_i')^{(5)} \le (\hat{B}_{28})^{(5)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(5)}(T_{29}, t) = (p_i)^{(5)} \qquad 123$$

$$\lim_{G \to \infty}(b_i'')^{(5)}(G_{31}, t) = (r_i)^{(5)}$$

**Definition of** $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$ :

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}', t) - (a_i'')^{(5)}(T_{29}, t)| \le (\hat{k}_{28})^{(5)}|T_{29} - T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})', t) - (b_i'')^{(5)}((G_{31}), t)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}', t)$ and $(a_i'')^{(5)}(T_{29}, t)$ . $(T_{29}', t)$ and $(T_{29}, t)$ are points belonging to the interval $[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}]$. It is to be noted that $(a_i'')^{(5)}(T_{29}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29}, t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$ : 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}, \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

**Definition of** $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$ : 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and\ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(a_i)^{(5)} + (a_i')^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(b_i)^{(5)} + (b_i')^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i,j = 32,33,34$$ 127

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(6)}$, $(r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33}, t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}), t) \leq (r_i)^{(6)} \leq (b_i')^{(6)} \leq (\hat{B}_{32})^{(6)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(6)}(T_{33}, t) = (p_i)^{(6)}$$ 128

$$\lim_{G \to \infty}(b_i'')^{(6)}((G_{35}), t) = (r_i)^{(6)}$$

**Definition of** $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$ :

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}', t) - (a_i'')^{(6)}(T_{33}, t)| \leq (\hat{k}_{32})^{(6)}|T_{33} - T_{33}'|e^{-(\hat{M}_{32})^{(6)}t}$$

$$|(b_i'')^{(6)}((G_{35})', t) - (b_i'')^{(6)}((G_{35}), t)| < (\hat{k}_{32})^{(6)}||(G_{35}) - (G_{35})'||e^{-(\hat{M}_{32})^{(6)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}', t)$ and $(a_i'')^{(6)}(T_{33}, t)$ . $(T_{33}', t)$ and $(T_{33}, t)$ are points belonging to the interval $[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}]$. It is to be noted that $(a_i'')^{(6)}(T_{33}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33}, t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$ : 129

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$, are positive constants

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$$

**Definition of** $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$ : 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and\ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32,33,34,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(a_i)^{(6)} + (a_i')^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)} (\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(b_i)^{(6)} + (b_i')^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)} (\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

(YYYYYYY) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i, j = 36,37,38$ 131

(ZZZZZZZ) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(7)}, (r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37}, t) \leq (p_i)^{(7)} \leq (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39}, t) \leq (r_i)^{(7)} \leq (b_i')^{(7)} \leq (\hat{B}_{36})^{(7)}$$

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*(AAAAAAAA)* $\lim_{T_2 \to \infty} (a_i'')^{(7)}(T_{37}, t) = (p_i)^{(7)}$

*(BBBBBBBB)*

$\lim_{G \to \infty} (b_i'')^{(7)}((G_{39}), t) = (r_i)^{(7)}$

**Definition of** $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$ :

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$$|(a_i'')^{(7)}(T_{37}', t) - (a_i'')^{(7)}(T_{37}, t)| \leq (\hat{k}_{36})^{(7)} |T_{37} - T_{37}'| e^{-(\hat{M}_{36})^{(7)} t}$$

$$|(b_i'')^{(7)}((G_{39})', t) - (b_i'')^{(7)}((G_{39}), t)| < (\hat{k}_{36})^{(7)} ||(G_{39}) - (G_{39})'|| e^{-(\hat{M}_{36})^{(7)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}', t)$ and $(a_i'')^{(7)}(T_{37}, t)$ . $(T_{37}', t)$ and $(T_{37}, t)$ are points belonging to the interval $[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}]$ . It is to be noted that $(a_i'')^{(7)}(T_{37}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37}, t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$ : 134

(CCCCCCCC) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)},$ are positive constants

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$$

**Definition of** $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$ : 135

(DDDDDDDD) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(b_i)^{(7)} + (b_i')^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40,41,42 \qquad 136$$

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

**Definition of** $(p_i)^{(8)}$, $(r_i)^{(8)}$: 137

$$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)} \qquad 138$$

$$(b_i'')^{(8)}((G_{43}), t) \leq (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)} \qquad 139$$

$$\lim_{T_2 \to \infty} (a_i'')^{(8)}(T_{41}, t) = (p_i)^{(8)} \qquad 140$$

$$\lim_{G \to \infty} (b_i'')^{(8)}\big((G_{43}), t\big) = (r_i)^{(8)} \qquad 141$$

**Definition of** $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$ :

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)} |T_{41} - T_{41}'| e^{-(\hat{M}_{40})^{(8)} t} \qquad 142$$

$$|(b_i'')^{(8)}((G_{43})', t) - (b_i'')^{(8)}\big((G_{43}), t\big)| < (\hat{k}_{40})^{(8)} ||(G_{43}) - (G_{43})'|| e^{-(\hat{M}_{40})^{(8)} t} \qquad 143$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}', t)$ and $(a_i'')^{(8)}(T_{41}, t)$ . $(T_{41}', t)$ and $(T_{41}, t)$ are points belonging to the interval $\big[(\hat{k}_{40})^{(8)}, (\hat{M}_{40})^{(8)}\big]$. It is to be noted that $(a_i'')^{(8)}(T_{41}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{40})^{(8)} = 1$

then the function $(a_i'')^{(8)}(T_{41},t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$ :

$(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)},$ are positive constants

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}, \frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1 \qquad 144$$

**Definition of** $(\hat{P}_{40})^{(8)}, (\hat{Q}_{40})^{(8)}$ :

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}, (\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)}, (a_i')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (p_i)^{(8)}, \ (r_i)^{(8)}, i = 40,41,42,$

Satisfy the inequalities

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(a_i)^{(8)} + (a_i')^{(8)} + (\hat{A}_{40})^{(8)} + (\hat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}] < 1 \qquad 145$$

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(b_i)^{(8)} + (b_i')^{(8)} + (\hat{B}_{40})^{(8)} + (\hat{Q}_{40})^{(8)}(\hat{k}_{40})^{(8)}] < 1 \qquad 146$$

Where we suppose

$$(a_i)^{(9)}, (a_i')^{(9)}, (a_i'')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (b_i'')^{(9)} > 0, \quad i,j = 44,45,46 \qquad 146\text{A}$$

The functions $(a_i'')^{(9)}, (b_i'')^{(9)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(9)}, \ (r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45},t) \le (p_i)^{(9)} \le (\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47},t) \le (r_i)^{(9)} \le (b_i')^{(9)} \le (\hat{B}_{44})^{(9)}$$

$$\lim_{T_2\to\infty} (a_i'')^{(9)}(T_{45},t) = (p_i)^{(9)}$$

$$\lim_{G\to\infty} (b_i'')^{(9)}(G_{47},t) = (r_i)^{(9)}$$

**Definition of** $(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}$ :

Where $\boxed{(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}, (p_i)^{(9)}, \ (r_i)^{(9)}}$ are positive constants and $\boxed{i = 44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}',t) - (a_i'')^{(9)}(T_{45},t)| \le (\hat{k}_{44})^{(9)}|T_{45} - T_{45}'|e^{-(\hat{M}_{44})^{(9)}t}$$

$$|(b_i'')^{(9)}((G_{47})',t) - (b_i'')^{(9)}((G_{47}),t)| < (\hat{k}_{44})^{(9)}||(G_{47}) - (G_{47})'||e^{-(\hat{M}_{44})^{(9)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}',t)$ and$(a_i'')^{(9)}(T_{45},t)$ . $(T_{45}',t)$ and $(T_{45},t)$ are points belonging to the interval $\left[(\hat{k}_{44})^{(9)}, (\hat{M}_{44})^{(9)}\right]$. It is

to be noted that $(a_i'')^{(9)}(T_{45},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{44})^{(9)}=1$ then the function $(a_i'')^{(9)}(T_{45},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\widehat{M}_{44})^{(9)},(\hat{k}_{44})^{(9)}$ :

$(\widehat{M}_{44})^{(9)},(\hat{k}_{44})^{(9)}$, are positive constants

$$\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\ ,\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}<1$$

**Definition of** $(\hat{P}_{44})^{(9)},(\hat{Q}_{44})^{(9)}$ :

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\widehat{M}_{44})^{(9)},(\hat{k}_{44})^{(9)},(\hat{A}_{44})^{(9)} and\ (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)},(a_i')^{(9)},(b_i)^{(9)},(b_i')^{(9)},(p_i)^{(9)},\ (r_i)^{(9)}, i=44,45,46,$ satisfy the inequalities

$$\frac{1}{(\widehat{M}_{44})^{(9)}}[(a_i)^{(9)}+(a_i')^{(9)}+\ (\hat{A}_{44})^{(9)}+\ (\hat{P}_{44})^{(9)}\ (\hat{k}_{44})^{(9)}]<1$$

$$\frac{1}{(\widehat{M}_{44})^{(9)}}[\ (b_i)^{(9)}+(b_i')^{(9)}+\ (\hat{B}_{44})^{(9)}+\ (\hat{Q}_{44})^{(9)}\ (\hat{k}_{44})^{(9)}]<1$$

**Theorem 1:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0)\ ,T_i(0)$ :

$$G_i(t)\le\ (\hat{P}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}\ ,\ \boxed{G_i(0)=G_i^0>0}$$

$$T_i(t)\le\ (\hat{Q}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}\ ,\ \boxed{T_i(0)=T_i^0>0}$$

**Theorem 2 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

**Definition of** $G_i(0)\ ,T_i(0)$

$$G_i(t)\le\ (\hat{P}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}\ ,\ \ G_i(0)=G_i^0>0$$

$$T_i(t)\le\ (\hat{Q}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}\ ,\ \ T_i(0)=T_i^0>0$$

**Theorem 3 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t)\le\ (\hat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}\ ,\ \ G_i(0)=G_i^0>0$$

$$T_i(t)\le\ (\hat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}\ ,\ \ T_i(0)=T_i^0>0$$

**Theorem 4 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

**Definition of** $G_i(0)\ ,T_i(0)$ :

$G_i(t) \leq \left(\hat{P}_{24}\right)^{(4)} e^{(\hat{M}_{24})^{(4)}t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{24}\right)^{(4)} e^{(\hat{M}_{24})^{(4)}t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Theorem 5 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

**Definition of** $G_i(0)$ , $T_i(0)$ :

$G_i(t) \leq \left(\hat{P}_{28}\right)^{(5)} e^{(\hat{M}_{28})^{(5)}t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{28}\right)^{(5)} e^{(\hat{M}_{28})^{(5)}t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Theorem 6 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

**Definition of** $G_i(0)$ , $T_i(0)$ :

$G_i(t) \leq \left(\hat{P}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Theorem 7:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

**Definition of** $G_i(0)$ , $T_i(0)$ :

$G_i(t) \leq \left(\hat{P}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Theorem 8:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

**Definition of** $G_i(0)$ , $T_i(0)$ :

$G_i(t) \leq \left(\hat{P}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Theorem 9:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

**Definition of** $G_i(0)$ , $T_i(0)$ :

$G_i(t) \leq \left(\hat{P}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Proof:** Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i$: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \le (\hat{P}_{13})^{(1)} \text{ , } T_i^0 \le (\hat{Q}_{13})^{(1)}, \quad 155$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t} \quad 156$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t} \quad 157$$

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}(s_{(13)}) - \left((a'_{13})^{(1)} + a''_{13})^{(1)} (T_{14}(s_{(13)}), s_{(13)})\right) G_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)} (T_{14}(s_{(13)}), s_{(13)})\right) G_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)} (T_{14}(s_{(13)}), s_{(13)})\right) G_{15}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)} (G(s_{(13)}), s_{(13)})\right) T_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)} (G(s_{(13)}), s_{(13)})\right) T_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{15}(t) = T_{15}^0 + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)} (G(s_{(13)}), s_{(13)})\right) T_{15}(s_{(13)})\right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i$: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \le (\hat{P}_{16})^{(2)} \text{ , } T_i^0 \le (\hat{Q}_{16})^{(2)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$$

By 160

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''_{16})^{(2)} (T_{17}(s_{(16)}), s_{(16)})\right) G_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)} (T_{17}(s_{(16)}), s_{(17)})\right) G_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)} (T_{17}(s_{(16)}), s_{(16)})\right) G_{18}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)} (G_{19}(s_{(16)}), s_{(16)})\right) T_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^{0} + \int_0^t \left[ (b_{17})^{(2)} T_{16}(s_{(16)}) - \left( (b'_{17})^{(2)} - (b''_{17})^{(2)} (G_{19}(s_{(16)}), s_{(16)}) \right) T_{17}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^{0} + \int_0^t \left[ (b_{18})^{(2)} T_{17}(s_{(16)}) - \left( (b'_{18})^{(2)} - (b''_{18})^{(2)} (G_{19}(s_{(16)}), s_{(16)}) \right) T_{18}(s_{(16)}) \right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \leq (\hat{P}_{20})^{(3)} \text{ , } T_i^0 \leq (\hat{Q}_{20})^{(3)},$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$$

By 161

$$\bar{G}_{20}(t) = G_{20}^{0} + \int_0^t \left[ (a_{20})^{(3)} G_{21}(s_{(20)}) - \left( (a'_{20})^{(3)} + a''_{20})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^{0} + \int_0^t \left[ (a_{21})^{(3)} G_{20}(s_{(20)}) - \left( (a'_{21})^{(3)} + (a''_{21})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{21}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^{0} + \int_0^t \left[ (a_{22})^{(3)} G_{21}(s_{(20)}) - \left( (a'_{22})^{(3)} + (a''_{22})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{22}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^{0} + \int_0^t \left[ (b_{20})^{(3)} T_{21}(s_{(20)}) - \left( (b'_{20})^{(3)} - (b''_{20})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^{0} + \int_0^t \left[ (b_{21})^{(3)} T_{20}(s_{(20)}) - \left( (b'_{21})^{(3)} - (b''_{21})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{21}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{\mathrm{T}}_{22}(\mathrm{t}) = \mathrm{T}_{22}^{0} + \int_0^t \left[ (b_{22})^{(3)} T_{21}(s_{(20)}) - \left( (b'_{22})^{(3)} - (b''_{22})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{22}(s_{(20)}) \right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \leq (\hat{P}_{24})^{(4)} \text{ , } T_i^0 \leq (\hat{Q}_{24})^{(4)},$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

By 162

$$\bar{G}_{24}(t) = G_{24}^{0} + \int_0^t \left[ (a_{24})^{(4)} G_{25}(s_{(24)}) - \left( (a'_{24})^{(4)} + a''_{24})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{26}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{26}(t) = T_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{26}(s_{(24)})\right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{28})^{(5)}, T_i^0 \le (\hat{Q}_{28})^{(5)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

By 163

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{30}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{30}(t) = T_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{30}(s_{(28)})\right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{32})^{(6)}, T_i^0 \le (\hat{Q}_{32})^{(6)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\bar{M}_{32})^{(6)} t}$

By 164

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[ (a_{32})^{(6)} G_{33}(s_{(32)}) - \left( (a'_{32})^{(6)} + a''_{32})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[ (a_{33})^{(6)} G_{32}(s_{(32)}) - \left( (a'_{33})^{(6)} + (a''_{33})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[ (a_{34})^{(6)} G_{33}(s_{(32)}) - \left( (a'_{34})^{(6)} + (a''_{34})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{34}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[ (b_{32})^{(6)} T_{33}(s_{(32)}) - \left( (b'_{32})^{(6)} - (b''_{32})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[ (b_{33})^{(6)} T_{32}(s_{(32)}) - \left( (b'_{33})^{(6)} - (b''_{33})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{\mathrm{T}}_{34}(\mathrm{t}) = \mathrm{T}_{34}^0 + \int_0^t \left[ (b_{34})^{(6)} T_{33}(s_{(32)}) - \left( (b'_{34})^{(6)} - (b''_{34})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{34}(s_{(32)}) \right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[ (a_{36})^{(7)} G_{37}(s_{(36)}) - \left( (a'_{36})^{(7)} + a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[ (a_{37})^{(7)} G_{36}(s_{(36)}) - \left( (a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[ (a_{38})^{(7)} G_{37}(s_{(36)}) - \left( (a'_{38})^{(7)} + (a''_{38})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{38}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[ (b_{36})^{(7)} T_{37}(s_{(36)}) - \left( (b'_{36})^{(7)} - (b''_{36})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[ (b_{37})^{(7)} T_{36}(s_{(36)}) - \left( (b'_{37})^{(7)} - (b''_{37})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^0 + \int_0^t \left[ (b_{38})^{(7)} T_{37}(s_{(36)}) - \left( (b'_{38})^{(7)} - (b''_{38})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{38}(s_{(36)}) \right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0,t)$

**Proof:**

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{40})^{(8)}$, $T_i^0 \le (\hat{Q}_{40})^{(8)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} G_{41}(s_{(40)}) - \left((a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[(a_{41})^{(8)} G_{40}(s_{(40)}) - \left((a'_{41})^{(8)} + (a''_{41})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{42}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^0 + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)} (G_{43}(s_{(40)}), s_{(40)})\right) T_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^0 + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)} (G_{43}(s_{(40)}), s_{(40)})\right) T_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{42}(t) = T_{42}^0 + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)} (G_{43}(s_{(40)}), s_{(40)})\right) T_{42}(s_{(40)})\right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0,t)$

**Proof:** 166
Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ A which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{44})^{(9)}$, $T_i^0 \le (\hat{Q}_{44})^{(9)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)})\right) G_{44}(s_{(44)})\right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)} (T_{45}(s_{(44)}), s_{(44)})\right) G_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}(s_{(44)}), s_{(44)})\right) G_{46}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{44}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{\mathrm{T}}_{46}(\mathrm{t}) = \mathrm{T}_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{46}(s_{(44)})\right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}}\right)\right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left(e^{(\hat{M}_{13})^{(1)} t} - 1\right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\hat{M}_{13})^{(1)} t} \le \frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{P}_{13})^{(1)} + G_{14}^0\right) e^{\left(-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0}\right)} + (\hat{P}_{13})^{(1)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \le G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}}\right)\right] ds_{(16)} = \quad 169$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1\right)$$

From which it follows that 170

$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \le \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left[\left((\hat{P}_{16})^{(2)} + G_{17}^0\right) e^{\left(-\frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0}\right)} + (\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t) \le G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} \left(G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} s_{(20)}}\right)\right] ds_{(20)} =$$

$$\left(1 + (a_{20})^{(3)} t\right) G_{21}^0 + \frac{(a_{20})^{(3)} (\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left(e^{(\hat{M}_{20})^{(3)} t} - 1\right)$$

From which it follows that 172

$$(G_{20}(t)-G_{20}^{0})e^{-(\hat{M}_{20})^{(3)}t}\leq\frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left[((\hat{P}_{20})^{(3)}+G_{21}^{0})e^{\left(-\frac{(\hat{P}_{20})^{(3)}+G_{21}^{0}}{G_{21}^{0}}\right)}+(\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 173

$$G_{24}(t)\leq G_{24}^{0}+\int_{0}^{t}\left[(a_{24})^{(4)}\left(G_{25}^{0}+(\hat{P}_{24})^{(4)}e^{(\hat{M}_{24})^{(4)}s_{(24)}}\right)\right]ds_{(24)}=$$

$$\left(1+(a_{24})^{(4)}t\right)G_{25}^{0}+\frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left(e^{(\hat{M}_{24})^{(4)}t}-1\right)$$

From which it follows that 174

$$(G_{24}(t)-G_{24}^{0})e^{-(\hat{M}_{24})^{(4)}t}\leq\frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left[((\hat{P}_{24})^{(4)}+G_{25}^{0})e^{\left(-\frac{(\hat{P}_{24})^{(4)}+G_{25}^{0}}{G_{25}^{0}}\right)}+(\hat{P}_{24})^{(4)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t)\leq G_{28}^{0}+\int_{0}^{t}\left[(a_{28})^{(5)}\left(G_{29}^{0}+(\hat{P}_{28})^{(5)}e^{(\hat{M}_{28})^{(5)}s_{(28)}}\right)\right]ds_{(28)}=$$

$$\left(1+(a_{28})^{(5)}t\right)G_{29}^{0}+\frac{(a_{28})^{(5)}(\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left(e^{(\hat{M}_{28})^{(5)}t}-1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t}\leq\frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t)\leq G_{32}^{0}+\int_{0}^{t}\left[(a_{32})^{(6)}\left(G_{33}^{0}+(\hat{P}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\right)\right]ds_{(32)}=$$

$$\left(1+(a_{32})^{(6)}t\right)G_{33}^{0}+\frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left(e^{(\hat{M}_{32})^{(6)}t}-1\right)$$

From which it follows that 177

$$(G_{32}(t)-G_{32}^{0})e^{-(\hat{M}_{32})^{(6)}t}\leq\frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left[((\hat{P}_{32})^{(6)}+G_{33}^{0})e^{\left(-\frac{(\hat{P}_{32})^{(6)}+G_{33}^{0}}{G_{33}^{0}}\right)}+(\hat{P}_{32})^{(6)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}$ , $G_{26}, T_{24}, T_{25}, T_{26}$

(ee) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t) \le G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} \left(G_{37}^0 + (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} s_{(36)}}\right)\right] ds_{(36)} =$$
$$\left(1 + (a_{36})^{(7)} t\right) G_{37}^0 + \frac{(a_{36})^{(7)} (\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left(e^{(\hat{M}_{36})^{(7)} t} - 1\right)$$

From which it follows that

$$(G_{36}(t) - G_{36}^0) e^{-(\hat{M}_{36})^{(7)} t} \le \frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left[((\hat{P}_{36})^{(7)} + G_{37}^0) e^{\left(-\frac{(\hat{P}_{36})^{(7)} + G_{37}^0}{G_{37}^0}\right)} + (\hat{P}_{36})^{(7)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t) \le G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} \left(G_{41}^0 + (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} s_{(40)}}\right)\right] ds_{(40)} = \quad 180$$
$$\left(1 + (a_{40})^{(8)} t\right) G_{41}^0 + \frac{(a_{40})^{(8)} (\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}} \left(e^{(\hat{M}_{40})^{(8)} t} - 1\right)$$

From which it follows that 181

$$(G_{40}(t) - G_{40}^0) e^{-(\hat{M}_{40})^{(8)} t} \le \frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}} \left[((\hat{P}_{40})^{(8)} + G_{41}^0) e^{\left(-\frac{(\hat{P}_{40})^{(8)} + G_{41}^0}{G_{41}^0}\right)} + (\hat{P}_{40})^{(8)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 8

Analogous inequalities hold also for $G_{41}$ , $G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t) \le G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} \left(G_{45}^0 + (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} s_{(44)}}\right)\right] ds_{(44)} =$$
$$\left(1 + (a_{44})^{(9)} t\right) G_{45}^0 + \frac{(a_{44})^{(9)} (\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left(e^{(\hat{M}_{44})^{(9)} t} - 1\right)$$

From which it follows that

$$(G_{44}(t) - G_{44}^0) e^{-(\hat{M}_{44})^{(9)} t} \le \frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left[((\hat{P}_{44})^{(9)} + G_{45}^0) e^{\left(-\frac{(\hat{P}_{44})^{(9)} + G_{45}^0}{G_{45}^0}\right)} + (\hat{P}_{44})^{(9)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 9

Analogous inequalities hold also for $G_{45}$ , $G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}$ , $\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$ and to choose 182

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\widehat{M}_{13})^{(1)}}\left[(\widehat{P}_{13})^{(1)}+\left((\hat{P}_{13})^{(1)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{13})^{(1)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{13})^{(1)} \quad 183$$

$$\frac{(b_i)^{(1)}}{(\widehat{M}_{13})^{(1)}}\left[\left((\hat{Q}_{13})^{(1)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{13})^{(1)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{13})^{(1)}\right]\leq(\hat{Q}_{13})^{(1)} \quad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)},T^{(1)}\right),\left(G^{(2)},T^{(2)}\right)\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{13})^{(1)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{13})^{(1)}t}\}$$

Indeed if we denote

**Definition of** $\tilde{G},\tilde{T}:\left(\tilde{G},\tilde{T}\right)=\mathcal{A}^{(1)}(G,T)$

It results

$$\left|\tilde{G}_{13}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{13})^{(1)}\left|G_{14}^{(1)}-G_{14}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\,ds_{(13)}+$$

$$\int_0^t\{(a'_{13})^{(1)}\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$G_{13}^{(2)}\left|(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)-(a''_{13})^{(1)}\left(T_{14}^{(2)},s_{(13)}\right)\right|\ e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses it follows

$$\left|G^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}t}\leq \quad 186$$
$$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)}+(a'_{13})^{(1)}+(\widehat{A}_{13})^{(1)}+(\widehat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}\right)d\left(\left(G^{(1)},T^{(1)};\,G^{(2)},T^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

**Remark 1:** The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}\ and\ (\widehat{Q}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i=13{,}14{,}15$ depend only on $\mathrm{T}_{14}$ and respectively on

$G(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$

From 19 to 24 it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(1)} - (a_i'')^{(1)}(T_{14}(s_{(13)}), s_{(13)})\} ds_{(13)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(1)} t)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2\ and\ \left((\widehat{M}_{13})^{(1)}\right)_3$ : 187

**Remark 3**: if $G_{13}$ is bounded, the same property have also $G_{14}\ and\ G_{15}$ . indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a_{14}')^{(1)} G_{14}$ and by integrating

$$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1 / (a_{14}')^{(1)}$$

In the same way , one can obtain

$$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2 / (a_{15}')^{(1)}$$

If $G_{14}\ or\ G_{15}$ is bounded, the same property follows for $G_{13}$ , $G_{15}$ and $G_{13}$ , $G_{14}$ respectively.

**Remark 4:** If $G_{13}$ $is$ bounded, from below, the same property holds for $G_{14}\ and\ G_{15}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{14}$ is bounded from below. 188

**Remark 5:** If $T_{13}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}\,(G(t), t)) = (b_{14}')^{(1)}$ then $T_{14} \to \infty$. 189

**Definition of** $(m)^{(1)}$ and $\varepsilon_1$ :

Indeed let $t_1$ be so that for $t > t_1$

$$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t), t) < \varepsilon_1, T_{13}\,(t) > (m)^{(1)}$$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right)$, $t = log\,\frac{2}{\varepsilon_1}$ By taking now $\varepsilon_1$ sufficiently small one sees that $T_{14}$ is unbounded.
The same property holds for $T_{15}$ if $\lim_{t\to\infty}(b_{15}'')^{(1)}\,(G(t), t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}$ , $\frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}\ and\ (\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[(\widehat{P}_{16})^{(2)}+\left((\hat{P}_{16})^{(2)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{16})^{(2)}\right]\leq(\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

**Definition of** $\widetilde{G_{19}},\widetilde{T_{19}}$ : $\left(\widetilde{G_{19}},\widetilde{T_{19}}\right)=\mathcal{A}^{(2)}(G_{19},T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{16})^{(2)}\left|G_{17}^{(1)}-G_{17}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)}+$$

$$\int_0^t\{(a'_{16})^{(2)}\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$G_{16}^{(2)}\left|(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)-(a''_{16})^{(2)}\left(T_{17}^{(2)},s_{(16)}\right)\right|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0,t]$ 197

From the hypotheses it follows

$$\left|(G_{19})^{(1)}-(G_{19})^{(2)}\right|\mathrm{e}^{-(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}\leq$$
$$\frac{1}{(\widehat{\mathrm{M}}_{16})^{(2)}}\left((a_{16})^{(2)}+(a'_{16})^{(2)}+(\widehat{\mathrm{A}}_{16})^{(2)}+(\widehat{\mathrm{P}}_{16})^{(2)}(\hat{k}_{16})^{(2)}\right)\mathrm{d}\left(((G_{19})^{(1)},(T_{19})^{(1)};\ (G_{19})^{(2)},(T_{19})^{(2)})\right)$$

And analogous inequalities for $G_i$ and $T_i$. Taking into account the hypothesis the result follows 198

**Remark 6:** The fact that we supposed $(a''_{16})^{(2)}$ and $(b''_{16})^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{\mathrm{P}}_{16})^{(2)}\mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}$ and $(\widehat{\mathrm{Q}}_{16})^{(2)}\mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i=16,17,18$ depend only on $\mathrm{T}_{17}$ and respectively on

$(G_{19})$(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 7:** There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$ 200

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(2)} - (a_i'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})\} ds_{(16)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(2)}t)} > 0 \quad \text{for } t > 0$$

**Definition of** $\left((\widehat{M}_{16})^{(2)}\right)_1$, $\left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$: 201

**Remark 8:** if $G_{16}$ is bounded, the same property have also $G_{17}$ and $G_{18}$. indeed if

$G_{16} < (\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt} \leq \left((\widehat{M}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)} G_{17}$ and by integrating

$$G_{17} \leq \left((\widehat{M}_{16})^{(2)}\right)_2 = G_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1 / (a_{17}')^{(2)}$$

In the same way , one can obtain

$$G_{18} \leq \left((\widehat{M}_{16})^{(2)}\right)_3 = G_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2 / (a_{18}')^{(2)}$$

If $G_{17}$ or $G_{18}$ is bounded, the same property follows for $G_{16}$, $G_{18}$ and $G_{16}$, $G_{17}$ respectively.

**Remark 9:** If $G_{16}$ is bounded, from below, the same property holds for $G_{17}$ and $G_{18}$. The proof is analogous with the preceding one. An analogous property is true if $G_{17}$ is bounded from below. 202

**Remark 10:** If $T_{16}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t), t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$. 203

**Definition of** $(m)^{(2)}$ and $\varepsilon_2$ :

Indeed let $t_2$ be so that for $t > t_2$

$$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right)$, $t = \log\frac{2}{\varepsilon_2}$ By taking now $\varepsilon_2$ sufficiently small one sees that $T_{17}$ is unbounded. 205
The same property holds for $T_{18}$ if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t), t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[(\widehat{P}_{20})^{(3)}+\left((\hat{P}_{20})^{(3)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{20})^{(3)}\right]\leq(\hat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself 210

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)},(T_{23})^{(1)}),((G_{23})^{(2)},(T_{23})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

**Definition of** $\widetilde{G_{23}},\widetilde{T_{23}}:(\widetilde{(G_{23})},\widetilde{(T_{23})})=\mathcal{A}^{(3)}((G_{23}),(T_{23}))$

It results 213

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{20})^{(3)}\left|G_{21}^{(1)}-G_{21}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}ds_{(20)}+$$

$$\int_0^t\{(a'_{20})^{(3)}\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$G_{20}^{(2)}\left|(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)},s_{(20)}\right)\right|\ e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\}ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}t}\leq \quad 214$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+(a'_{20})^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\widehat{k}_{20})^{(3)}\right)d\left(((G_{23})^{(1)},(T_{23})^{(1)};\ (G_{23})^{(2)},(T_{23})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 11:** The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ *and* $(\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i=20,21,22$ depend only on $T_{21}$ and respectively on

$(G_{23})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 12:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 216

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(3)} - (a_i'')^{(3)}(T_{21}(s_{(20)}), s_{(20)})\} ds_{(20)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(3)} t)} > 0$ for t > 0

**Definition of** $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2\ and\ \left((\widehat{M}_{20})^{(3)}\right)_3$ : 217

**Remark 13:** if $G_{20}$ is bounded, the same property have also $G_{21}\ and\ G_{22}$ . indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a_{21}')^{(3)} G_{21}$ and by integrating

$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1 / (a_{21}')^{(3)}$

In the same way , one can obtain

$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2 / (a_{22}')^{(3)}$

If $G_{21}\ or\ G_{22}$ is bounded, the same property follows for $G_{20}$ , $G_{22}$ and $G_{20}$ , $G_{21}$ respectively.

**Remark 14:** If $G_{20}\ \ is$ bounded, from below, the same property holds for $G_{21}\ and\ G_{22}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{21}$ is bounded from below. 218

**Remark 15:** If $T_{20}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t), t\right)) = (b_{21}')^{(3)}$ then $T_{21} \to \infty$. 219

**Definition of** $(m)^{(3)}$ and $\varepsilon_3$ :

Indeed let $t_3$ be so that for $t > t_3$

$(b_{21})^{(3)} - (b_i'')^{(3)}\left((G_{23})(t), t\right) < \varepsilon_3, T_{20}\,(t) > (m)^{(3)}$

Then $\frac{dT_{21}}{dt} \geq (a_{21})^{(3)}(m)^{(3)} - \varepsilon_3 T_{21}$ which leads to 220

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t} = \frac{1}{2}$ it results

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right),\ \ t = log\,\frac{2}{\varepsilon_3}$ By taking now $\varepsilon_3$ sufficiently small one sees that $T_{21}$ is unbounded.
The same property holds for $T_{22}$ if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t), t\right) = (b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}$ , $\frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}} < 1$ and to choose 221

$(\widehat{P}_{24})^{(4)}$ and $(\widehat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[(\widehat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \tag{222}$$

$$\frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \tag{223}$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself (224)

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric (225)

$$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{24})^{(4)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{27}}),(\widetilde{T_{27}})$ : $\left((\widetilde{G_{27}}),(\widetilde{T_{27}})\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{24})^{(4)}\left|G_{25}^{(1)}-G_{25}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\,ds_{(24)}+$$

$$\int_0^t\{(a'_{24})^{(4)}\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$G_{24}^{(2)}\left|(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)-(a''_{24})^{(4)}\left(T_{25}^{(2)},s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\}ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}t}\leq \tag{226}$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)}+(a'_{24})^{(4)}+(\widehat{A}_{24})^{(4)}+(\widehat{P}_{24})^{(4)}(\hat{k}_{24})^{(4)}\right)d\left(((G_{27})^{(1)},(T_{27})^{(1)};\ (G_{27})^{(2)},(T_{27})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 16:** The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ *and* $(\widehat{Q}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_+$. (227)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i=24,25,26$ depend only on $T_{25}$ and respectively on

$(G_{27})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 17:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 228

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(4)} - (a_i'')^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(4)} t)} > 0$ for t > 0

**Definition of** $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2\ and\ \left((\widehat{M}_{24})^{(4)}\right)_3$ : 229

**Remark 18:** if $G_{24}$ is bounded, the same property have also $G_{25}\ and\ G_{26}$ . indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a_{25}')^{(4)} G_{25}$ and by integrating

$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1 / (a_{25}')^{(4)}$

In the same way , one can obtain

$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2 / (a_{26}')^{(4)}$

If $G_{25}\ or\ G_{26}$ is bounded, the same property follows for $G_{24}$ , $G_{26}$ and $G_{24}$ , $G_{25}$ respectively.

**Remark 19:** If $G_{24}$ *is* bounded, from below, the same property holds for $G_{25}\ and\ G_{26}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{25}$ is bounded from below. 230

**Remark 20:** If $T_{24}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}\ ((G_{27})(t), t)) = (b_{25}')^{(4)}$ then $T_{25} \to \infty$. 231

**Definition of** $(m)^{(4)}$ and $\varepsilon_4$ :

Indeed let $t_4$ be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t), t) < \varepsilon_4, T_{24}\,(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_4}$ By taking now $\varepsilon_4$ sufficiently small one sees that $T_{25}$ is unbounded.
The same property holds for $T_{26}$ if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t), t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for $G_{29}$ , $G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}$ , $\frac{(b_i)^{(5)}}{(\widehat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}\left[(\widehat{P}_{28})^{(5)}+\left((\hat{P}_{28})^{(5)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{28})^{(5)} \tag{234}$$

$$\frac{(b_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{28})^{(5)}\right]\leq(\hat{Q}_{28})^{(5)} \tag{235}$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric (236)

$$d\left(((G_{31})^{(1)},(T_{31})^{(1)}),((G_{31})^{(2)},(T_{31})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{28})^{(5)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{31}}),(\widetilde{T_{31}})$ : $\left((\widetilde{G_{31}}),(\widetilde{T_{31}})\right)=\mathcal{A}^{(5)}\left((G_{31}),(T_{31})\right)$

It results

$$\left|\tilde{G}_{28}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{28})^{(5)}\left|G_{29}^{(1)}-G_{29}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\,ds_{(28)}+$$

$$\int_0^t\{(a'_{28})^{(5)}\left|G_{28}^{(1)}-G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$(a''_{28})^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)\left|G_{28}^{(1)}-G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$G_{28}^{(2)}\left|(a''_{28})^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)-(a''_{28})^{(5)}\left(T_{29}^{(2)},s_{(28)}\right)\right|\ e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\}ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on it follows

$$\left|(G_{31})^{(1)}-(G_{31})^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}t}\leq \tag{237}$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)}+(a'_{28})^{(5)}+(\widehat{A}_{28})^{(5)}+(\widehat{P}_{28})^{(5)}(\hat{k}_{28})^{(5)}\right)d\left(((G_{31})^{(1)},(T_{31})^{(1)};\ (G_{31})^{(2)},(T_{31})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 21:** The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ *and* $(\widehat{Q}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ respectively of $\mathbb{R}_+$. (238)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on $T_{29}$ and respectively on $(G_{31})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 22:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 239

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(5)} - (a_i'')^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(5)} t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2\ and\ \left((\widehat{M}_{28})^{(5)}\right)_3$ : 240

**Remark 23:** if $G_{28}$ is bounded, the same property have also $G_{29}\ and\ G_{30}$ . indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a_{29}')^{(5)} G_{29}$ and by integrating

$$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1 / (a_{29}')^{(5)}$$

In the same way , one can obtain

$$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2 / (a_{30}')^{(5)}$$

If $G_{29}\ or\ G_{30}$ is bounded, the same property follows for $G_{28}$ , $G_{30}$ and $G_{28}$ , $G_{29}$ respectively.

**Remark 24:** If $G_{28}$ *is* bounded, from below, the same property holds for $G_{29}\ and\ G_{30}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{29}$ is bounded from below. 241

**Remark 25:** If $T_{28}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}\,((G_{31})(t), t)) = (b_{29}')^{(5)}$ then $T_{29} \to \infty$. 242

**Definition of** $(m)^{(5)}$ and $\varepsilon_5$ :

Indeed let $t_5$ be so that for $t > t_5$

$$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t), t) < \varepsilon_5, T_{28}\,(t) > (m)^{(5)}$$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_5}$ By taking now $\varepsilon_5$ sufficiently small one sees that $T_{29}$ is unbounded.
The same property holds for $T_{30}$ if $\lim_{t\to\infty}(b_{30}'')^{(5)}\left((G_{31})(t), t\right) = (b_{30}')^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{33}\,, G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\ , \frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}} < 1$ and to choose 244

$(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[(\hat{P}_{32})^{(6)}+\left((\hat{P}_{32})^{(6)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{32})^{(6)} \tag{245}$$

$$\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[\left((\hat{Q}_{32})^{(6)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{32})^{(6)}\right]\leq(\hat{Q}_{32})^{(6)} \tag{246}$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric (247)

$$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\hat{M}_{32})^{(6)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\hat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{35}}),(\widetilde{T_{35}})$ : $\left((\widetilde{G_{35}}),(\widetilde{T_{35}})\right)=\mathcal{A}^{(6)}\left((G_{35}),(T_{35})\right)$

It results

$$|\tilde{G}_{32}^{(1)}-\tilde{G}_i^{(2)}|\leq\int_0^t(a_{32})^{(6)}|G_{33}^{(1)}-G_{33}^{(2)}|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\,ds_{(32)}+$$

$$\int_0^t\{(a'_{32})^{(6)}|G_{32}^{(1)}-G_{32}^{(2)}|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)|G_{32}^{(1)}-G_{32}^{(2)}|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$G_{32}^{(2)}|(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)-(a''_{32})^{(6)}\left(T_{33}^{(2)},s_{(32)}\right)|\;e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\}ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$|(G_{35})^{(1)}-(G_{35})^{(2)}|e^{-(\widehat{M}_{32})^{(6)}t}\leq \tag{248}$$
$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)}+(a'_{32})^{(6)}+(\widehat{A}_{32})^{(6)}+(\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}\right)d\left(((G_{35})^{(1)},(T_{35})^{(1)};\;(G_{35})^{(2)},(T_{35})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 26:** The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\hat{P}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ *and* $(\hat{Q}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ respectively of $\mathbb{R}_+$. (249)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i = 32,33,34$ depend only on $T_{33}$ and respectively on $(G_{35})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 27:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 250

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(6)} - (a_i'')^{(6)}(T_{33}(s_{(32)}), s_{(32)})\} ds_{(32)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(6)} t\right)} > 0$ for t > 0

**Definition of** $\left((\widehat{M}_{32})^{(6)}\right)_1, \left((\widehat{M}_{32})^{(6)}\right)_2\ and\ \left((\widehat{M}_{32})^{(6)}\right)_3$ : 251

**Remark 28:** if $G_{32}$ is bounded, the same property have also $G_{33}\ and\ G_{34}$ . indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a_{33}')^{(6)} G_{33}$ and by integrating

$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_1 / (a_{33}')^{(6)}$

In the same way , one can obtain

$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_2 / (a_{34}')^{(6)}$

If $G_{33}\ or\ G_{34}$ is bounded, the same property follows for $G_{32}$ , $G_{34}$ and $G_{32}$ , $G_{33}$ respectively.

**Remark 29:** If $G_{32}$ *is* bounded, from below, the same property holds for $G_{33}\ and\ G_{34}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{33}$ is bounded from below. 252

**Remark 30:** If $T_{32}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}\ ((G_{35})(t), t)) = (b_{33}')^{(6)}$ then $T_{33}\ \to \infty$. 253

**Definition of** $(m)^{(6)}$ and $\varepsilon_6$ :

Indeed let $t_6$ be so that for $t\ > t_6$

$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t), t\right) < \varepsilon_6, T_{32}\,(t) > (m)^{(6)}$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33}\ \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33}\ \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_6}$ By taking now $\varepsilon_6$ sufficiently small one sees that $T_{33}$ is unbounded.
The same property holds for $T_{34}$ if $\lim_{t\to\infty}(b_{34}'')^{(6)}\left((G_{35})(t), t(t), t\right) = (b_{34}')^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{37}\ , G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\ , \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$ and to choose

$(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[(\hat{P}_{36})^{(7)}+\left((\hat{P}_{36})^{(7)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{36})^{(7)} \tag{256}$$

$$\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[\left((\hat{Q}_{36})^{(7)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{36})^{(7)}\right]\leq(\hat{Q}_{36})^{(7)} \tag{257}$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself
The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric (258)

$$d\left(((G_{39})^{(1)},(T_{39})^{(1)}),((G_{39})^{(2)},(T_{39})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{36})^{(7)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{39}}),(\widetilde{T_{39}}):\left((\widetilde{G_{39}}),(\widetilde{T_{39}})\right)=\mathcal{A}^{(7)}((G_{39}),(T_{39}))$
It results

$$\left|\tilde{G}_{36}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{36})^{(7)}\left|G_{37}^{(1)}-G_{37}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\,ds_{(36)}+$$
$$\int_0^t\{(a'_{36})^{(7)}\left|G_{36}^{(1)}-G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$(a''_{36})^{(7)}\left(T_{37}^{(1)},s_{(36)}\right)\left|G_{36}^{(1)}-G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$G_{36}^{(2)}\left|(a''_{36})^{(7)}\left(T_{37}^{(1)},s_{(36)}\right)-(a''_{36})^{(7)}\left(T_{37}^{(2)},s_{(36)}\right)\right|\;e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\}ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on it follows

$$\left|(G_{39})^{(1)}-(G_{39})^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}t}\leq \tag{259}$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}}\left((a_{36})^{(7)}+(a'_{36})^{(7)}+(\hat{A}_{36})^{(7)}+(\hat{P}_{36})^{(7)}(\hat{k}_{36})^{(7)}\right)d\left(((G_{39})^{(1)},(T_{39})^{(1)};\;(G_{39})^{(2)},(T_{39})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 31:** The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\hat{P}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ *and* $(\hat{Q}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ respectively of $\mathbb{R}_+$. (260)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(7)}$ and $(b_i'')^{(7)}, i = 36,37,38$ depend only on $T_{37}$ and respectively on $(G_{39})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 32:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 261

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(7)} - (a_i'')^{(7)}(T_{37}(s_{(36)}), s_{(36)})\} ds_{(36)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(7)} t)} > 0$ for t > 0

**Definition of** $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2\ and\ \left((\widehat{M}_{36})^{(7)}\right)_3$ : 262

**Remark 33:** if $G_{36}$ is bounded, the same property have also $G_{37}\ and\ G_{38}$ . indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a_{37}')^{(7)} G_{37}$ and by integrating

$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1 / (a_{37}')^{(7)}$

In the same way , one can obtain

$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2 / (a_{38}')^{(7)}$

If $G_{37}\ or\ G_{38}$ is bounded, the same property follows for $G_{36}$ , $G_{38}$ and $G_{36}$ , $G_{37}$ respectively.

**Remark 34:** If $G_{36}$ *is* bounded, from below, the same property holds for $G_{37}\ and\ G_{38}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{37}$ is bounded from below. 263

**Remark 35:** If $T_{36}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(7)}\ ((G_{39})(t), t)) = (b_{37}')^{(7)}$ then $T_{37} \to \infty$. 264

**Definition of** $(m)^{(7)}$ and $\varepsilon_7$ :

Indeed let $t_7$ be so that for $t > t_7$

$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t), t) < \varepsilon_7, T_{36}\,(t) > (m)^{(7)}$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_7}$ By taking now $\varepsilon_7$ sufficiently small one sees that $T_{37}$ is unbounded.
The same property holds for $T_{38}$ if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t), t\right) = (b_{38}')^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}$ , $\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}} < 1$ and to choose $(\widehat{P}_{40})^{(8)}$ and $(\widehat{Q}_{40})^{(8)}$ large to have 266

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[(\widehat{P}_{40})^{(8)} + \left((\hat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{40})^{(8)} \quad 267$$

$$\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{40})^{(8)}\right] \leq (\hat{Q}_{40})^{(8)} \quad 268$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)}, (T_{43})^{(1)}), ((G_{43})^{(2)}, (T_{43})^{(2)})\right) = \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

**Definition of** $\widetilde{(G_{43})}, \widetilde{(T_{43})}$ : $\left(\widetilde{(G_{43})}, \widetilde{(T_{43})}\right) = \mathcal{A}^{(8)}((G_{43}), (T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{40})^{(8)}\left|G_{41}^{(1)} - G_{41}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\,ds_{(40)} +$$
$$\int_0^t\{(a'_{40})^{(8)}\left|G_{40}^{(1)} - G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{-(\widehat{M}_{40})^{(8)}s_{(40)}} +$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right)\left|G_{40}^{(1)} - G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}} +$$
$$G_{40}^{(2)}\left|(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right) - (a''_{40})^{(8)}\left(T_{41}^{(2)}, s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\}ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0, t]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)} - (G_{43})^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}t} \leq \quad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)} + (a'_{40})^{(8)} + (\widehat{A}_{40})^{(8)} + (\widehat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}\right)d\left(((G_{43})^{(1)}, (T_{43})^{(1)};\ (G_{43})^{(2)}, (T_{43})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

**Remark 36:** The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate 274

condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ *and* $(\widehat{Q}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i = 40{,}41{,}42$ depend only on $T_{41}$ and respectively on $(G_{43})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 37** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 275

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(8)} - (a_i'')^{(8)}(T_{41}(s_{(40)}), s_{(40)})\} ds_{(40)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(8)}t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{40})^{(8)}\right)_1, \left((\widehat{M}_{40})^{(8)}\right)_2\ and\ \left((\widehat{M}_{40})^{(8)}\right)_3$ : 276

**Remark 38:** if $G_{40}$ is bounded, the same property have also $G_{41}\ and\ G_{42}$ . indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a_{41}')^{(8)}G_{41}$ and by integrating

$$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1/(a_{41}')^{(8)}$$

In the same way , one can obtain

$$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2/(a_{42}')^{(8)}$$

If $G_{41}\ or\ G_{42}$ is bounded, the same property follows for $G_{40}$ , $G_{42}$ and $G_{40}$ , $G_{41}$ respectively.

**Remark 39:** If $G_{40}$ *is* bounded, from below, the same property holds for $G_{41}\ and\ G_{42}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{41}$ is bounded from below. 277

**Remark 40:** If $T_{40}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}\ ((G_{43})(t), t)) = (b_{41}')^{(8)}$ then $T_{41} \to \infty$. 278

**Definition of** $(m)^{(8)}$ and $\varepsilon_8$ :

Indeed let $t_8$ be so that for $t > t_8$

$$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t), t\right) < \varepsilon_8, T_{40}\,(t) > (m)^{(8)}$$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right)$, $t = log\frac{2}{\varepsilon_8}$ By taking now $\varepsilon_8$ sufficiently small one sees that $T_{41}$ is unbounded.
The same property holds for $T_{42}$ if $\lim_{t\to\infty}(b''_{42})^{(8)}\left((G_{43})(t), t(t), t\right) = (b'_{42})^{(8)}$



It is now sufficient to take $\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}$ , $\frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$ and to choose $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ large to have

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}\left[(\hat{P}_{44})^{(9)} + \left((\hat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}}\left[\left((\hat{Q}_{44})^{(9)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{44})^{(9)}\right] \leq (\hat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)}, (T_{47})^{(1)}), ((G_{47})^{(2)}, (T_{47})^{(2)})\right) =$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{44})^{(9)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{47}}), (\widetilde{T_{47}}) : \left((\widetilde{G_{47}}), (\widetilde{T_{47}})\right) = \mathcal{A}^{(9)}\left((G_{47}), (T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{44})^{(9)}\left|G_{45}^{(1)} - G_{45}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\,ds_{(44)} +$$
$$\int_0^t\{(a'_{44})^{(9)}\left|G_{44}^{(1)} - G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{-(\widehat{M}_{44})^{(9)}s_{(44)}} +$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right)\left|G_{44}^{(1)} - G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}} +$$
$$G_{44}^{(2)}\left|(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right) - (a''_{44})^{(9)}\left(T_{45}^{(2)}, s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\}ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)} - G^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}t} \leq$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)} + (a'_{44})^{(9)} + (\widehat{A}_{44})^{(9)} + (\widehat{P}_{44})^{(9)}(\hat{k}_{44})^{(9)}\right)d\left(((G_{47})^{(1)}, (T_{47})^{(1)};\ (G_{47})^{(2)}, (T_{47})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis (39,35,36) the result follows

**Remark 41:** The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by

$(\widehat{P}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ *and* $(\widehat{Q}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i = 44{,}45{,}46$ depend only on $T_{45}$ and respectively on $(G_{47})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 42:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$

From 99 to 44 it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t\{(a_i')^{(9)}-(a_i'')^{(9)}(T_{45}(s_{(44)}),s_{(44)})\}ds_{(44)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(9)}t)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{44})^{(9)}\right)_1, \left((\widehat{M}_{44})^{(9)}\right)_2\ and\ \left((\widehat{M}_{44})^{(9)}\right)_3$ :

**Remark 43:** if $G_{44}$ is bounded, the same property have also $G_{45}\ and\ G_{46}$ . indeed if

$G_{44} < (\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt} \leq \left((\widehat{M}_{44})^{(9)}\right)_1 - (a_{45}')^{(9)}G_{45}$ and by integrating

$$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_1/(a_{45}')^{(9)}$$

In the same way , one can obtain

$$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_2/(a_{46}')^{(9)}$$

If $G_{45}\ or\ G_{46}$ is bounded, the same property follows for $G_{44}$ , $G_{46}$ and $G_{44}$ , $G_{45}$ respectively.

**Remark 44:** If $G_{44}$ *is* bounded, from below, the same property holds for $G_{45}\ and\ G_{46}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{45}$ is bounded from below.

**Remark 45:** If $T_{44}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t),t\right) = (b_{45}')^{(9)}$ then $T_{45} \to \infty$.

**Definition of** $(m)^{(9)}$ and $\varepsilon_9$ :

Indeed let $t_9$ be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t),t\right) < \varepsilon_9, T_{44}\,(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right), \quad t = log\frac{2}{\varepsilon_9}$ By taking now $\varepsilon_9$ sufficiently small one sees that $T_{45}$ is unbounded.
The same property holds for $T_{46}$ if $\lim_{t\to\infty}(b_{46}'')^{(9)}\left((G_{47})(t),t\right) = (b_{46}')^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

**Behavior of the solutions of equation** 280

**Theorem** If we denote and define

**Definition of** $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ :

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)} \le -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14}, t) + (a''_{14})^{(1)}(T_{14}, t) \le -(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)} \le -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G, t) - (b''_{14})^{(1)}(G, t) \le -(\tau_1)^{(1)}$

**Definition of** $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$ : 281

By $(\nu_1)^{(1)} > 0$, $(\nu_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$, $(u_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

**Definition of** $(\bar{\nu}_1)^{(1)},, (\bar{\nu}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$ : 282

By $(\bar{\nu}_1)^{(1)} > 0$, $(\bar{\nu}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$, $(\bar{u}_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

**Definition of** $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (\nu_0)^{(1)}$ :- 283

If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by
$(m_2)^{(1)} = (\nu_0)^{(1)}, (m_1)^{(1)} = (\nu_1)^{(1)}$, $if\ (\nu_0)^{(1)} < (\nu_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}, if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)}$,

and $\boxed{(\nu_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)}$, $if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)}$, $if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)}$,

and $\boxed{(u_0)^{(1)} = \frac{T^0_{13}}{T^0_{14}}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$

are defined

Then the solution of global equations satisfies the inequalities 285

$G^0_{13}e^{\left((S_1)^{(1)} - (p_{13})^{(1)}\right)t} \le G_{13}(t) \le G^0_{13}e^{(S_1)^{(1)}t}$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)}-(p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)}t}$$

$$\left(\frac{(a_{15})^{(1)}G_{13}^0}{(m_1)^{(1)}((S_1)^{(1)}-(p_{13})^{(1)}-(S_2)^{(1)})}\left[e^{((S_1)^{(1)}-(p_{13})^{(1)})t} - e^{-(S_2)^{(1)}t}\right] + G_{15}^0 e^{-(S_2)^{(1)}t} \le G_{15}(t) \le \right. \quad (286)$$

$$\left. \frac{(a_{15})^{(1)}G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)}-(a'_{15})^{(1)})}[e^{(S_1)^{(1)}t} - e^{-(a'_{15})^{(1)}t}] + G_{15}^0 e^{-(a'_{15})^{(1)}t}\right)$$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)}t} \le T_{13}(t) \le T_{13}^0 e^{((R_1)^{(1)}+(r_{13})^{(1)})t}} \quad (287)$$

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)}t} \le T_{13}(t) \le \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)}+(r_{13})^{(1)})t} \quad (288)$$

$$\frac{(b_{15})^{(1)}T_{13}^0}{(\mu_1)^{(1)}((R_1)^{(1)}-(b'_{15})^{(1)})}\left[e^{(R_1)^{(1)}t} - e^{-(b'_{15})^{(1)}t}\right] + T_{15}^0 e^{-(b'_{15})^{(1)}t} \le T_{15}(t) \le \quad (289)$$

$$\frac{(a_{15})^{(1)}T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)}+(r_{13})^{(1)}+(R_2)^{(1)})}\left[e^{((R_1)^{(1)}+(r_{13})^{(1)})t} - e^{-(R_2)^{(1)}t}\right] + T_{15}^0 e^{-(R_2)^{(1)}t}$$

**Definition of** $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:- (290)

Where $(S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$

$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$

$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$

$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$

**Behavior of the solutions of equation** (291)

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ : (292)

$(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ four constants satisfying

$$-(\sigma_2)^{(2)} \le -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(T_{17}, t) + (a''_{17})^{(2)}(T_{17}, t) \le -(\sigma_1)^{(2)} \quad (293)$$

$$-(\tau_2)^{(2)} \le -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}((G_{19}), t) - (b''_{17})^{(2)}((G_{19}), t) \le -(\tau_1)^{(2)} \quad (294)$$

**Definition of** $(\nu_1)^{(2)}, (\nu_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$ : (295)

By $(\nu_1)^{(2)} > 0$, $(\nu_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0$, $(u_2)^{(2)} < 0$ the roots (296)

of the equations $(a_{17})^{(2)}(\nu^{(2)})^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ (297)

and $(b_{14})^{(2)}(u^{(2)})^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and (298)

**Definition of** $(\bar{\nu}_1)^{(2)}, , (\bar{\nu}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$ : (299)

By $(\bar{\nu}_1)^{(2)} > 0$, $(\bar{\nu}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0$, $(\bar{u}_2)^{(2)} < 0$ the (300)

roots of the equations $(a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 301

and $(b_{17})^{(2)}\left(u^{(2)}\right)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 302

**Definition of** $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ :- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, ***if*** $(u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$, ***if*** $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)}$, ***if*** $(\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)}t}$

$(p_i)^{(2)}$ is defined by equation

$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)}t}$ 311

$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)}t}\right] + G_{18}^0 e^{-(S_2)^{(2)}t} \le G_{18}(t) \le\right.$ 312

$\left.\frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)}t} - e^{-(a'_{18})^{(2)}t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)}t}\right)$

$\boxed{T_{16}^0 e^{(R_1)^{(2)}t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}}$ 313

$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)}t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}$ 314

$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)}t} - e^{-(b'_{18})^{(2)}t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)}t} \le T_{18}(t) \le$ 315

$$\frac{(a_{18})^{(2)}T_{16}^{0}}{(\mu_2)^{(2)}((R_1)^{(2)}+(r_{16})^{(2)}+(R_2)^{(2)})}\left[e^{((R_1)^{(2)}+(r_{16})^{(2)})t}-e^{-(R_2)^{(2)}t}\right]+T_{18}^{0}e^{-(R_2)^{(2)}t}$$

**Definition of** $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$$

$$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)}$$ 318

$$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$$

**Behavior of the solutions** 319

**Theorem 3:** If we denote and define

**Definition of** $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ :

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}\big((G_{23}), t\big) \le -(\tau_1)^{(3)}$$

**Definition of** $(v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$ : 320

By $(v_1)^{(3)} > 0$, $(v_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{v}_1)^{(3)} > 0$, $(\bar{v}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

**Definition of** $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ :- 321

If we define $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (v_0)^{(3)}, (m_1)^{(3)} = (v_1)^{(3)}$, ***if*** $(v_0)^{(3)} < (v_1)^{(3)}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (\bar{v}_1)^{(3)}$, ***if*** $(v_1)^{(3)} < (v_0)^{(3)} < (\bar{v}_1)^{(3)}$,

and $\boxed{(v_0)^{(3)} = \frac{G_{20}^{0}}{G_{21}^{0}}}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_0)^{(3)}$, ***if*** $(\bar{v}_1)^{(3)} < (v_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)},\ \boldsymbol{if}\ (u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}, \boldsymbol{if}\ (u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)},$ and $\boxed{(u_0)^{(3)} = \frac{T_{20}^0}{T_{21}^0}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)},\ \boldsymbol{if}\ (\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G_{20}^0 e^{(S_1)^{(3)} t}$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)} t} \quad 323$$

$$\left(\frac{(a_{22})^{(3)} G_{20}^0}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\left[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)} t}\right] + G_{22}^0 e^{-(S_2)^{(3)} t} \le G_{22}(t) \le \right. \quad 324$$

$$\left. \frac{(a_{22})^{(3)} G_{20}^0}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}\left[e^{(S_1)^{(3)} t} - e^{-(a'_{22})^{(3)} t}\right] + G_{22}^0 e^{-(a'_{22})^{(3)} t}\right)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t}} \quad 325$$

$$\frac{1}{(\mu_1)^{(3)}} T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}} T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)} T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)} - (b'_{22})^{(3)})}\left[e^{(R_1)^{(3)} t} - e^{-(b'_{22})^{(3)} t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)} t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)} T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)} + (r_{20})^{(3)} + (R_2)^{(3)})}\left[e^{((R_1)^{(3)} + (r_{20})^{(3)})t} - e^{-(R_2)^{(3)} t}\right] + T_{22}^0 e^{-(R_2)^{(3)} t}$$

**Definition of** $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

**Behavior of the solutions of equation**

**Theorem:** If we denote and define

**Definition of** $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ :

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$

$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}((G_{27}), t) - (b''_{25})^{(4)}((G_{27}), t) \le -(\tau_1)^{(4)}$

**Definition of** $(v_1)^{(4)}, (v_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, v^{(4)}, u^{(4)}$ : 329

By $(v_1)^{(4)} > 0$ , $(v_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$ , $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_1)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

**Definition of** $(\bar{v}_1)^{(4)},, (\bar{v}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$ : 330

By $(\bar{v}_1)^{(4)} > 0$ , $(\bar{v}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$ , $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

**Definition of** $(m_1)^{(4)}$ , $(m_2)^{(4)}$ , $(\mu_1)^{(4)}, (\mu_2)^{(4)}, (\nu_0)^{(4)}$ :-

If we define $(m_1)^{(4)}$ , $(m_2)^{(4)}$ , $(\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (v_0)^{(4)}, (m_1)^{(4)} = (v_1)^{(4)}$, ***if*** $(v_0)^{(4)} < (v_1)^{(4)}$

$(m_2)^{(4)} = (v_1)^{(4)}, (m_1)^{(4)} = (\bar{v}_1)^{(4)}$ , ***if*** $(v_4)^{(4)} < (v_0)^{(4)} < (\bar{v}_1)^{(4)}$,
and $\boxed{(v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (v_4)^{(4)}, (m_1)^{(4)} = (v_0)^{(4)}$, ***if*** $(\bar{v}_4)^{(4)} < (v_0)^{(4)}$

and analogously

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$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)}$, ***if*** $(u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}$ , ***if*** $(u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)}$,
and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}$, ***if*** $(\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

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$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)}t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)}t}$$

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$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)}t}\right] + G_{26}^0 e^{-(S_2)^{(4)}t} \le G_{26}(t) \le\right.$$ 334

*(a26)4G240(m2)4(S1)4−(a26′)4e(S1)4t−e−(a26′)4t+ G260e−(a26′)4t*

$$\boxed{T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}}T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}}T_{24}^0 e^{((R_1)^{(4)}+(r_{24})^{(4)})t} \quad 335$$

$$\frac{(b_{26})^{(4)}T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)}-(b'_{26})^{(4)})}\left[e^{(R_1)^{(4)}t} - e^{-(b'_{26})^{(4)}t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)}t} \le T_{26}(t) \le \quad 336$$

$$\frac{(a_{26})^{(4)}T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)}+(r_{24})^{(4)}+(R_2)^{(4)})}\left[e^{((R_1)^{(4)}+(r_{24})^{(4)})t} - e^{-(R_2)^{(4)}t}\right] + T_{26}^0 e^{-(R_2)^{(4)}t}$$

**Definition of** $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

**Behavior of the solutions of equation** 338

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ :

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

**Definition of** $(v_1)^{(5)}, (v_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, v^{(5)}, u^{(5)}$ : 339

By $(v_1)^{(5)} > 0$, $(v_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$, $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(v^{(5)}\big)^2 + (\sigma_1)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

**Definition of** $(\bar{v}_1)^{(5)}, , (\bar{v}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$ : 340

By $(\bar{v}_1)^{(5)} > 0$, $(\bar{v}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$, $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(v^{(5)}\big)^2 + (\sigma_2)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

**Definition of** $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (v_0)^{(5)}$ :-

If we define $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (v_0)^{(5)}, (m_1)^{(5)} = (v_1)^{(5)}$, ***if*** $(v_0)^{(5)} < (v_1)^{(5)}$

$(m_2)^{(5)} = (v_1)^{(5)}, (m_1)^{(5)} = (\bar{v}_1)^{(5)}$, ***if*** $(v_1)^{(5)} < (v_0)^{(5)} < (\bar{v}_1)^{(5)}$,
and $\boxed{(v_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)},\ \boldsymbol{if}\ (u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}, \boldsymbol{if}\ (u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)},$

and $\boxed{(u_0)^{(5)} = \frac{T_{28}^0}{T_{29}^0}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}, \boldsymbol{if}\ (\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{28}(t) \le G_{28}^0 e^{(S_1)^{(5)} t}$

where $(p_i)^{(5)}$ is defined by equation

$\frac{1}{(m_5)^{(5)}} G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}} G_{28}^0 e^{(S_1)^{(5)} t}$ 343

$\left(\frac{(a_{30})^{(5)} G_{28}^0}{(m_1)^{(5)}((S_1)^{(5)} - (p_{28})^{(5)} - (S_2)^{(5)})}\left[e^{((S_1)^{(5)} - (p_{28})^{(5)})t} - e^{-(S_2)^{(5)} t}\right] + G_{30}^0 e^{-(S_2)^{(5)} t} \le G_{30}(t) \le\right.$ 344

*(a30)5G280(m2)5(S1)5−(a30′)5e(S1)5t−e−(a30′)5t+ G300e−(a30′)5t*

$\boxed{T_{28}^0 e^{(R_1)^{(5)} t} \le T_{28}(t) \le T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}}$ 345

$\frac{1}{(\mu_1)^{(5)}} T_{28}^0 e^{(R_1)^{(5)} t} \le T_{28}(t) \le \frac{1}{(\mu_2)^{(5)}} T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}$ 346

$\frac{(b_{30})^{(5)} T_{28}^0}{(\mu_1)^{(5)}((R_1)^{(5)} - (b'_{30})^{(5)})}\left[e^{(R_1)^{(5)} t} - e^{-(b'_{30})^{(5)} t}\right] + T_{30}^0 e^{-(b'_{30})^{(5)} t} \le T_{30}(t) \le$ 347

$\frac{(a_{30})^{(5)} T_{28}^0}{(\mu_2)^{(5)}((R_1)^{(5)} + (r_{28})^{(5)} + (R_2)^{(5)})}\left[e^{((R_1)^{(5)} + (r_{28})^{(5)})t} - e^{-(R_2)^{(5)} t}\right] + T_{30}^0 e^{-(R_2)^{(5)} t}$

**Definition of** $(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$:- 348

Where $(S_1)^{(5)} = (a_{28})^{(5)}(m_2)^{(5)} - (a'_{28})^{(5)}$

$(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}$

$(R_1)^{(5)} = (b_{28})^{(5)}(\mu_2)^{(5)} - (b'_{28})^{(5)}$

$(R_2)^{(5)} = (b'_{30})^{(5)} - (r_{30})^{(5)}$

**Behavior of the solutions of equation** 349

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ :

$(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)} \le -(a'_{32})^{(6)} + (a'_{33})^{(6)} - (a''_{32})^{(6)}(T_{33}, t) + (a''_{33})^{(6)}(T_{33}, t) \le -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)} \le -(b'_{32})^{(6)} + (b'_{33})^{(6)} - (b''_{32})^{(6)}\big((G_{35}), t\big) - (b''_{33})^{(6)}\big((G_{35}), t\big) \le -(\tau_1)^{(6)}$$

**Definition of** $(\nu_1)^{(6)}, (\nu_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, \nu^{(6)}, u^{(6)}$ : 350

By $(\nu_1)^{(6)} > 0$ , $(\nu_2)^{(6)} < 0$ and respectively $(u_1)^{(6)} > 0$ , $(u_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(6)},, (\bar{\nu}_2)^{(6)}, (\bar{u}_1)^{(6)}, (\bar{u}_2)^{(6)}$ : 351

By $(\bar{\nu}_1)^{(6)} > 0$ , $(\bar{\nu}_2)^{(6)} < 0$ and respectively $(\bar{u}_1)^{(6)} > 0$ , $(\bar{u}_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$

**Definition of** $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}, (\nu_0)^{(6)}$ :-

If we define $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}$ by

$$(m_2)^{(6)} = (\nu_0)^{(6)}, (m_1)^{(6)} = (\nu_1)^{(6)}, \ \boldsymbol{if}\ (\nu_0)^{(6)} < (\nu_1)^{(6)}$$

$$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}, \boldsymbol{if}\ (\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)},$$

and $\boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}}$

$$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$$

and analogously 352

$$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}, \ \boldsymbol{if}\ (u_0)^{(6)} < (u_1)^{(6)}$$

$$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}, \boldsymbol{if}\ (u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)},$$

and $\boxed{(u_0)^{(6)} = \frac{T^0_{32}}{T^0_{33}}}$

$$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}, \boldsymbol{if}\ (\bar{u}_1)^{(6)} < (u_0)^{(6)} \text{ where } (u_1)^{(6)}, (\bar{u}_1)^{(6)}$$

Then the solution of global equations satisfies the inequalities 353

$$G^0_{32}e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{32}(t) \le G^0_{32}e^{(S_1)^{(6)}t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}}G^0_{32}e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{33}(t) \le \frac{1}{(m_2)^{(6)}}G^0_{32}e^{(S_1)^{(6)}t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)}G^0_{32}}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})}\left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)}t}\right] + G^0_{34}e^{-(S_2)^{(6)}t} \le G_{34}(t) \le \right. \quad 355$$

$$\frac{(a_{34})^{(6)}G_{32}^{0}}{(m_2)^{(6)}((S_1)^{(6)}-(a'_{34})^{(6)})}\left[e^{(S_1)^{(6)}t}-e^{-(a'_{34})^{(6)}t}\right]+\ G_{34}^{0}e^{-(a'_{34})^{(6)}t}\Big)$$

$$\boxed{T_{32}^{0}e^{(R_1)^{(6)}t}\le T_{32}(t)\le T_{32}^{0}e^{((R_1)^{(6)}+(r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}}T_{32}^{0}e^{(R_1)^{(6)}t}\le T_{32}(t)\le \frac{1}{(\mu_2)^{(6)}}T_{32}^{0}e^{((R_1)^{(6)}+(r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)}T_{32}^{0}}{(\mu_1)^{(6)}((R_1)^{(6)}-(b'_{34})^{(6)})}\left[e^{(R_1)^{(6)}t}-e^{-(b'_{34})^{(6)}t}\right]+T_{34}^{0}e^{-(b'_{34})^{(6)}t}\le T_{34}(t)\le \quad 358$$

$$\frac{(a_{34})^{(6)}T_{32}^{0}}{(\mu_2)^{(6)}((R_1)^{(6)}+(r_{32})^{(6)}+(R_2)^{(6)})}\left[e^{((R_1)^{(6)}+(r_{32})^{(6)})t}-e^{-(R_2)^{(6)}t}\right]+T_{34}^{0}e^{-(R_2)^{(6)}t}$$

**Definition of** $(S_1)^{(6)},(S_2)^{(6)},(R_1)^{(6)},(R_2)^{(6)}$:- 359

Where $(S_1)^{(6)}=(a_{32})^{(6)}(m_2)^{(6)}-(a'_{32})^{(6)}$

$(S_2)^{(6)}=(a_{34})^{(6)}-(p_{34})^{(6)}$

$(R_1)^{(6)}=(b_{32})^{(6)}(\mu_2)^{(6)}-(b'_{32})^{(6)}$

$(R_2)^{(6)}=(b'_{34})^{(6)}-(r_{34})^{(6)}$

**Behavior of the solutions of equation**

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(7)},(\sigma_2)^{(7)},(\tau_1)^{(7)},(\tau_2)^{(7)}$ :

$(\sigma_1)^{(7)},(\sigma_2)^{(7)},(\tau_1)^{(7)},(\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)}\le -(a'_{36})^{(7)}+(a'_{37})^{(7)}-(a''_{36})^{(7)}(T_{37},t)+(a''_{37})^{(7)}(T_{37},t)\le -(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)}\le -(b'_{36})^{(7)}+(b'_{37})^{(7)}-(b''_{36})^{(7)}\big((G_{39}),t\big)-(b''_{37})^{(7)}\big((G_{39}),t\big)\le -(\tau_1)^{(7)}$$

**Definition of** $(v_1)^{(7)},(v_2)^{(7)},(u_1)^{(7)},(u_2)^{(7)},v^{(7)},u^{(7)}$ : 361

By $(v_1)^{(7)}>0,(v_2)^{(7)}<0$ and respectively $(u_1)^{(7)}>0,(u_2)^{(7)}<0$ the roots of the equations $(a_{37})^{(7)}\big(v^{(7)}\big)^2+(\sigma_1)^{(7)}v^{(7)}-(a_{36})^{(7)}=0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2+(\tau_1)^{(7)}u^{(7)}-(b_{36})^{(7)}=0$ and

**Definition of** $(\bar{v}_1)^{(7)},,(\bar{v}_2)^{(7)},(\bar{u}_1)^{(7)},(\bar{u}_2)^{(7)}$ : 362

By $(\bar{v}_1)^{(7)}>0,(\bar{v}_2)^{(7)}<0$ and respectively $(\bar{u}_1)^{(7)}>0,(\bar{u}_2)^{(7)}<0$ the roots of the equations $(a_{37})^{(7)}\big(v^{(7)}\big)^2+(\sigma_2)^{(7)}v^{(7)}-(a_{36})^{(7)}=0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2+(\tau_2)^{(7)}u^{(7)}-(b_{36})^{(7)}=0$

**Definition of** $(m_1)^{(7)},(m_2)^{(7)},(\mu_1)^{(7)},(\mu_2)^{(7)},(v_0)^{(7)}$ :-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$$(m_2)^{(7)} = (\nu_0)^{(7)}, (m_1)^{(7)} = (\nu_1)^{(7)}, \ \boldsymbol{if}\ (\nu_0)^{(7)} < (\nu_1)^{(7)}$$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\bar{\nu}_1)^{(7)}, \boldsymbol{if}\ (\nu_1)^{(7)} < (\nu_0)^{(7)} < (\bar{\nu}_1)^{(7)},$$

and $\boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\nu_0)^{(7)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(7)} < (\nu_0)^{(7)}$$

and analogously 363

$$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}, \ \boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$$

and $\boxed{(u_0)^{(7)} = \frac{T_{36}^0}{T_{37}^0}}$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)} \text{ where } (u_1)^{(7)}, (\bar{u}_1)^{(7)}$$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^0 e^{(S_1)^{(7)} t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)} t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)}((S_1)^{(7)} - (p_{36})^{(7)} - (S_2)^{(7)})}\left[e^{((S_1)^{(7)} - (p_{36})^{(7)})t} - e^{-(S_2)^{(7)} t}\right] + G_{38}^0 e^{-(S_2)^{(7)} t} \le G_{38}(t) \le \right. \quad 366$$

$$\left. \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)}((S_1)^{(7)} - (a'_{38})^{(7)})}\left[e^{(S_1)^{(7)} t} - e^{-(a'_{38})^{(7)} t}\right] + G_{38}^0 e^{-(a'_{38})^{(7)} t}\right)$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t} \quad 368$$

$$\frac{(b_{38})^{(7)} T_{36}^0}{(\mu_1)^{(7)}((R_1)^{(7)} - (b'_{38})^{(7)})}\left[e^{(R_1)^{(7)} t} - e^{-(b'_{38})^{(7)} t}\right] + T_{38}^0 e^{-(b'_{38})^{(7)} t} \le T_{38}(t) \le \quad 369$$

$$\frac{(a_{38})^{(7)} T_{36}^0}{(\mu_2)^{(7)}((R_1)^{(7)} + (r_{36})^{(7)} + (R_2)^{(7)})}\left[e^{((R_1)^{(7)} + (r_{36})^{(7)})t} - e^{-(R_2)^{(7)} t}\right] + T_{38}^0 e^{-(R_2)^{(7)} t}$$

**Definition of** $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$

$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$

$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$

**Behavior of the solutions of equation** 371

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ :

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$-(\sigma_2)^{(8)} \le -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \le -(\sigma_1)^{(8)}$

$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\big((G_{43}), t\big) - (b''_{41})^{(8)}\big((G_{43}), t\big) \le -(\tau_1)^{(8)}$

**Definition of** $(\nu_1)^{(8)}, (\nu_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, \nu^{(8)}, u^{(8)}$ : 372

By $(\nu_1)^{(8)} > 0, (\nu_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0, (u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(8)}, , (\bar{\nu}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$ :

By $(\bar{\nu}_1)^{(8)} > 0, (\bar{\nu}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0, (\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

**Definition of** $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}, (\nu_0)^{(8)}$ :-

If we define $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$(m_2)^{(8)} = (\nu_0)^{(8)}, (m_1)^{(8)} = (\nu_1)^{(8)}$, ***if*** $(\nu_0)^{(8)} < (\nu_1)^{(8)}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\bar{\nu}_1)^{(8)}$, ***if*** $(\nu_1)^{(8)} < (\nu_0)^{(8)} < (\bar{\nu}_1)^{(8)}$,
and $\boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\nu_0)^{(8)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(8)} < (\nu_0)^{(8)}$

and analogously 374

$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)},\ \boldsymbol{if}\ (u_0)^{(8)} < (u_1)^{(8)}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}, \boldsymbol{if}\ (u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)},$

and $\boxed{(u_0)^{(8)} = \frac{T_{40}^0}{T_{41}^0}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}, \boldsymbol{if}\ (\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^0 e^{(S_1)^{(8)} t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}} G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}} G_{40}^0 e^{(S_1)^{(8)} t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)} G_{40}^0}{(m_1)^{(8)}((S_1)^{(8)} - (p_{40})^{(8)} - (S_2)^{(8)})}\left[e^{((S_1)^{(8)} - (p_{40})^{(8)})t} - e^{-(S_2)^{(8)} t}\right] + G_{42}^0 e^{-(S_2)^{(8)} t} \le G_{42}(t) \le \right. \quad 377$$

$$\left. \frac{(a_{42})^{(8)} G_{40}^0}{(m_2)^{(8)}((S_1)^{(8)} - (a'_{42})^{(8)})}\left[e^{(S_1)^{(8)} t} - e^{-(a'_{42})^{(8)} t}\right] + G_{42}^0 e^{-(a'_{42})^{(8)} t}\right)$$

$$\boxed{T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}} T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le \frac{1}{(\mu_2)^{(8)}} T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)} T_{40}^0}{(\mu_1)^{(8)}((R_1)^{(8)} - (b'_{42})^{(8)})}\left[e^{(R_1)^{(8)} t} - e^{-(b'_{42})^{(8)} t}\right] + T_{42}^0 e^{-(b'_{42})^{(8)} t} \le T_{42}(t) \le \quad 380$$

$$\frac{(a_{42})^{(8)} T_{40}^0}{(\mu_2)^{(8)}((R_1)^{(8)} + (r_{40})^{(8)} + (R_2)^{(8)})}\left[e^{((R_1)^{(8)} + (r_{40})^{(8)})t} - e^{-(R_2)^{(8)} t}\right] + T_{42}^0 e^{-(R_2)^{(8)} t}$$

**Definition of** $(S_1)^{(8)}, (S_2)^{(8)}, (R_1)^{(8)}, (R_2)^{(8)}$:- 381

Where $(S_1)^{(8)} = (a_{40})^{(8)}(m_2)^{(8)} - (a'_{40})^{(8)}$

$(S_2)^{(8)} = (a_{42})^{(8)} - (p_{42})^{(8)}$

$(R_1)^{(8)} = (b_{40})^{(8)}(\mu_2)^{(8)} - (b'_{40})^{(8)}$

$(R_2)^{(8)} = (b'_{42})^{(8)} - (r_{42})^{(8)}$

**Behavior of the solutions of equation 37 to 92** 382

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ :

$(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ four constants satisfying

$$-(\sigma_2)^{(9)} \le -(a'_{44})^{(9)} + (a'_{45})^{(9)} - (a''_{44})^{(9)}(T_{45}, t) + (a''_{45})^{(9)}(T_{45}, t) \le -(\sigma_1)^{(9)}$$

$$-(\tau_2)^{(9)} \le -(b'_{44})^{(9)} + (b'_{45})^{(9)} - (b''_{44})^{(9)}\big((G_{47}), t\big) - (b''_{45})^{(9)}\big((G_{47}), t\big) \le -(\tau_1)^{(9)}$$

**Definition of** $(\nu_1)^{(9)}, (\nu_2)^{(9)}, (u_1)^{(9)}, (u_2)^{(9)}, \nu^{(9)}, u^{(9)}$ :

By $(\nu_1)^{(9)} > 0$ , $(\nu_2)^{(9)} < 0$ and respectively $(u_1)^{(9)} > 0$ , $(u_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$ and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_1)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(9)}, , (\bar{\nu}_2)^{(9)}, (\bar{u}_1)^{(9)}, (\bar{u}_2)^{(9)}$ :

By $(\bar{\nu}_1)^{(9)} > 0$ , $(\bar{\nu}_2)^{(9)} < 0$ and respectively $(\bar{u}_1)^{(9)} > 0$ , $(\bar{u}_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$ and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_2)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$

**Definition of** $(m_1)^{(9)}, (m_2)^{(9)}, (\mu_1)^{(9)}, (\mu_2)^{(9)}, (\nu_0)^{(9)}$ :-

If we define $(m_1)^{(9)}, (m_2)^{(9)}, (\mu_1)^{(9)}, (\mu_2)^{(9)}$ by

$$(m_2)^{(9)} = (\nu_0)^{(9)}, (m_1)^{(9)} = (\nu_1)^{(9)}, \ \boldsymbol{if}\ (\nu_0)^{(9)} < (\nu_1)^{(9)}$$

$$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\bar{\nu}_1)^{(9)}, \boldsymbol{if}\ (\nu_1)^{(9)} < (\nu_0)^{(9)} < (\bar{\nu}_1)^{(9)},$$

and $\boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$

$$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\nu_0)^{(9)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(9)} < (\nu_0)^{(9)}$$

and analogously

$$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)}, \ \boldsymbol{if}\ (u_0)^{(9)} < (u_1)^{(9)}$$

$$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}, \boldsymbol{if}\ (u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)},$$

and $\boxed{(u_0)^{(9)} = \frac{T^0_{44}}{T^0_{45}}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}, \boldsymbol{if}\ (\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G^0_{44} e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{44}(t) \le G^0_{44} e^{(S_1)^{(9)} t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}} G_{44}^0 e^{((S_1)^{(9)}-(p_{44})^{(9)})t} \le G_{45}(t) \le \frac{1}{(m_2)^{(9)}} G_{44}^0 e^{(S_1)^{(9)}t}$$

(

$$\frac{(a_{46})^{(9)} G_{44}^0}{(m_1)^{(9)}((S_1)^{(9)}-(p_{44})^{(9)}-(S_2)^{(9)})}\left[e^{((S_1)^{(9)}-(p_{44})^{(9)})t} - e^{-(S_2)^{(9)}t}\right] + G_{46}^0 e^{-(S_2)^{(9)}t} \le G_{46}(t) \le$$

$$\frac{(a_{46})^{(9)} G_{44}^0}{(m_2)^{(9)}((S_1)^{(9)}-(a'_{46})^{(9)})}[e^{(S_1)^{(9)}t} - e^{-(a'_{46})^{(9)}t}] + G_{46}^0 e^{-(a'_{46})^{(9)}t})$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le T_{44}^0 e^{((R_1)^{(9)}+(r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}} T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le \frac{1}{(\mu_2)^{(9)}} T_{44}^0 e^{((R_1)^{(9)}+(r_{44})^{(9)})t}$$

$$\frac{(b_{46})^{(9)} T_{44}^0}{(\mu_1)^{(9)}((R_1)^{(9)}-(b'_{46})^{(9)})}\left[e^{(R_1)^{(9)}t} - e^{-(b'_{46})^{(9)}t}\right] + T_{46}^0 e^{-(b'_{46})^{(9)}t} \le T_{46}(t) \le$$

$$\frac{(a_{46})^{(9)} T_{44}^0}{(\mu_2)^{(9)}((R_1)^{(9)}+(r_{44})^{(9)}+(R_2)^{(9)})}\left[e^{((R_1)^{(9)}+(r_{44})^{(9)})t} - e^{-(R_2)^{(9)}t}\right] + T_{46}^0 e^{-(R_2)^{(9)}t}$$

**Definition of** $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$

**Proof :** From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right) - (a''_{14})^{(1)}(T_{14}, t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

**Definition of** $v^{(1)}$ :- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)}\right) \le \frac{dv^{(1)}}{dt} \le -\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)}\right)$$

From which one obtains

**Definition of** $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$ :-

For $0 < \boxed{(v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}} < (v_1)^{(1)} < (\bar{v}_1)^{(1)}$

$$v^{(1)}(t) \ge \frac{(v_1)^{(1)}+(C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(v_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(v_2)^{(1)}}}$$

it follows $(\nu_0)^{(1)} \leq \nu^{(1)}(t) \leq (\nu_1)^{(1)}$

In the same manner , we get 384

$$\nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{\nu}_1)^{(1)}-(\nu_0)^{(1)}}{(\nu_0)^{(1)}-(\bar{\nu}_2)^{(1)}}}$$

From which we deduce $(\nu_0)^{(1)} \leq \nu^{(1)}(t) \leq (\bar{\nu}_1)^{(1)}$

If $0 < (\nu_1)^{(1)} < (\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0} < (\bar{\nu}_1)^{(1)}$ we find like in the previous case, 385

$$(\nu_1)^{(1)} \leq \frac{(\nu_1)^{(1)}+(C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}} \leq \nu^{(1)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\bar{\nu}_1)^{(1)}$$

If $0 < (\nu_1)^{(1)} \leq (\bar{\nu}_1)^{(1)} \leq \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$ , we obtain 386

$$(\nu_1)^{(1)} \leq \nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\nu_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(1)}(t)$ :-

$$(m_2)^{(1)} \leq \nu^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{\nu^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(1)}(t)$ :-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{13})^{(1)} = (a''_{14})^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)}G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)}T_{14}(t)$ This is an important

consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

**Proof** : From global equations we obtain 387

$$\frac{d\nu^{(2)}}{dt} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right) - (a''_{17})^{(2)}(T_{17}, t)\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

**Definition of** $\nu^{(2)}$ :- $\boxed{\nu^{(2)} = \frac{G_{16}}{G_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \le \frac{d\nu^{(2)}}{dt} \le -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

**Definition of** $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$ :-

For $0 < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \ge \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}}{1 + (C)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(C)^{(2)} = \frac{(\nu_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \le \nu^{(2)}(t) \le (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \le \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \le \nu^{(2)}(t) \le (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \le \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}}{1 + (C)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}} \le \nu^{(2)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \le (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \le (\bar{\nu}_1)^{(2)} \le (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$ , we obtain 394

$$(\nu_1)^{(2)} \le \nu^{(2)}(t) \le \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \le (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(2)}(t)$ :- 395

$$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}, \quad \boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$$

In a completely analogous way, we obtain 396

**Definition of** $u^{(2)}(t)$ :-

$$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}, \quad \boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :** 397

If $(a_{16}'')^{(2)} = (a_{17}'')^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)} G_{17}(t)$

Analogously if $(b_{16}'')^{(2)} = (b_{17}'')^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)} T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

**Proof** : From global equations we obtain 398

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a_{20}')^{(3)} - (a_{21}')^{(3)} + (a_{20}'')^{(3)}(T_{21}, t)\right) - (a_{21}'')^{(3)}(T_{21}, t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

**Definition of** $\nu^{(3)}$ :- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right) \leq \frac{d\nu^{(3)}}{dt} \leq -\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right)$$

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From which one obtains

For $0 < (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (\nu_1)^{(3)} < (\bar{\nu}_1)^{(3)}$

$$\nu^{(3)}(t) \geq \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(\nu_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\nu_2)^{(3)}}}$$

it follows $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\nu_1)^{(3)}$

In the same manner , we get 401

$$\nu^{(3)}(t) \le \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{\nu}_1)^{(3)}-(\nu_0)^{(3)}}{(\nu_0)^{(3)}-(\bar{\nu}_2)^{(3)}}}$$

**Definition of** $(\bar{\nu}_1)^{(3)}$ :-

From which we deduce $(\nu_0)^{(3)} \le \nu^{(3)}(t) \le (\bar{\nu}_1)^{(3)}$

If $0 < (\nu_1)^{(3)} < (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (\bar{\nu}_1)^{(3)}$ we find like in the previous case, 402

$$(\nu_1)^{(3)} \le \frac{(\nu_1)^{(3)}+(C)^{(3)}(\nu_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}} \le \nu^{(3)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \le (\bar{\nu}_1)^{(3)}$$

If $0 < (\nu_1)^{(3)} \le (\bar{\nu}_1)^{(3)} \le (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}$ , we obtain 403

$$(\nu_1)^{(3)} \le \nu^{(3)}(t) \le \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \le (\nu_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(3)}(t)$ :-

$$(m_2)^{(3)} \le \nu^{(3)}(t) \le (m_1)^{(3)}, \quad \boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(3)}(t)$ :-

$$(\mu_2)^{(3)} \le u^{(3)}(t) \le (\mu_1)^{(3)}, \quad \boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{20}'')^{(3)} = (a_{21}'')^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)}G_{21}(t)$

Analogously if $(b_{20}'')^{(3)} = (b_{21}'')^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)}T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

**Proof** : From global equations we obtain 404

$$\frac{dv^{(4)}}{dt} = (a_{24})^{(4)} - \left((a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right) - (a''_{25})^{(4)}(T_{25},t)v^{(4)} - (a_{25})^{(4)}v^{(4)}$$

**Definition of** $v^{(4)}$ :- $\boxed{v^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)}\right) \le \frac{dv^{(4)}}{dt} \le -\left((a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_4)^{(4)}v^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$ :-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \ge \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \le \nu^{(4)}(t) \le (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \le \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \le \nu^{(4)}(t) \le (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \le \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}}{1 + (C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}} \le \nu^{(4)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \le (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \le (\bar{\nu}_1)^{(4)} \le \boxed{(\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}}}$ , we obtain 407

$$(\nu_1)^{(4)} \le \nu^{(4)}(t) \le \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \le (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(4)}(t)$ :-

$$(m_2)^{(4)} \le \nu^{(4)}(t) \le (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(4)}(t)$ :-

$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}$, $\boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{24})^{(4)} = (a''_{25})^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)} G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b''_{24})^{(4)} = (b''_{25})^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)} T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

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**Proof** : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a'_{28})^{(5)} - (a'_{29})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right) - (a''_{29})^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

**Definition of** $\nu^{(5)}$ :- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$ :-

For $0 < \boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \le \frac{(\nu_1)^{(5)}+(C)^{(5)}(\nu_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)}-(\nu_2)^{(5)}\right)t\right]}}{1+(C)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)}-(\nu_2)^{(5)}\right)t\right]}} \le \nu^{(5)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \le (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \le (\bar{\nu}_1)^{(5)} \le \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain 411

$$(\nu_1)^{(5)} \le \nu^{(5)}(t) \le \frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \le (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(5)}(t)$ :-

$(m_2)^{(5)} \le \nu^{(5)}(t) \le (m_1)^{(5)}$, $\boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$

In a completely analogous way, we obtain

**Definition of** $u^{(5)}(t)$ :-

$(\mu_2)^{(5)} \le u^{(5)}(t) \le (\mu_1)^{(5)}$, $\boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a_{28}'')^{(5)} = (a_{29}'')^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)}G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b_{28}'')^{(5)} = (b_{29}'')^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)}T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

**Proof** : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a_{32}')^{(6)} - (a_{33}')^{(6)} + (a_{32}'')^{(6)}(T_{33},t)\right) - (a_{33}'')^{(6)}(T_{33},t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

**Definition of** $\nu^{(6)}$ :- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \le \frac{d\nu^{(6)}}{dt} \le -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$ :-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \geq \frac{(\nu_1)^{(6)}+(C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_0)^{(6)}\right)t\right]}}{1+(C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)}-(\nu_0)^{(6)}}{(\nu_0)^{(6)}-(\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\nu_1)^{(6)}$

In the same manner , we get 413

$$\nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{\nu}_1)^{(6)}-(\nu_0)^{(6)}}{(\nu_0)^{(6)}-(\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\bar{\nu}_1)^{(6)}$

If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 414

$$(\nu_1)^{(6)} \leq \frac{(\nu_1)^{(6)}+(C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_2)^{(6)}\right)t\right]}}{1+(C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_2)^{(6)}\right)t\right]}} \leq \nu^{(6)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\bar{\nu}_1)^{(6)}$$

If $0 < (\nu_1)^{(6)} \leq (\bar{\nu}_1)^{(6)} \leq \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$ , we obtain 415

$$(\nu_1)^{(6)} \leq \nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(6)}(t)$ :-

$$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}, \quad \boxed{\nu^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(6)}(t)$ :-

$$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}, \quad \boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)}G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)}T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

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**Proof** : From global equations we obtain

$$\frac{dv^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right) - (a''_{37})^{(7)}(T_{37},t)v^{(7)} - (a_{37})^{(7)}v^{(7)}$$

**Definition of** $v^{(7)}$ :- $\boxed{v^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_2)^{(7)}v^{(7)} - (a_{36})^{(7)}\right) \le \frac{dv^{(7)}}{dt} \le -\left((a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_1)^{(7)}v^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

**Definition of** $(\bar{v}_1)^{(7)}, (v_0)^{(7)}$ :-

For $0 < \boxed{(v_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}} < (v_1)^{(7)} < (\bar{v}_1)^{(7)}$

$$v^{(7)}(t) \ge \frac{(v_1)^{(7)} + (C)^{(7)}(v_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_0)^{(7)}\right)t\right]}}{1 + (C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(v_1)^{(7)} - (v_0)^{(7)}}{(v_0)^{(7)} - (v_2)^{(7)}}}$$

it follows $(v_0)^{(7)} \le v^{(7)}(t) \le (v_1)^{(7)}$

In the same manner , we get 417

$$v^{(7)}(t) \le \frac{(\bar{v}_1)^{(7)} + (\bar{C})^{(7)}(\bar{v}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{v}_1)^{(7)} - (v_0)^{(7)}}{(v_0)^{(7)} - (\bar{v}_2)^{(7)}}}$$

From which we deduce $(v_0)^{(7)} \le v^{(7)}(t) \le (\bar{v}_1)^{(7)}$

If $0 < (v_1)^{(7)} < (v_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}} < (\bar{v}_1)^{(7)}$ we find like in the previous case, 418

$$(v_1)^{(7)} \le \frac{(v_1)^{(7)} + (C)^{(7)}(v_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_2)^{(7)}\right)t\right]}}{1 + (C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_2)^{(7)}\right)t\right]}} \le v^{(7)}(t) \le$$

$$\frac{(\bar{v}_1)^{(7)} + (\bar{C})^{(7)}(\bar{v}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}} \le (\bar{v}_1)^{(7)}$$

If $0 < (v_1)^{(7)} \le (\bar{v}_1)^{(7)} \le \boxed{(v_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$ , we obtain 419

$$(v_1)^{(7)} \le v^{(7)}(t) \le \frac{(\bar{v}_1)^{(7)} + (\bar{C})^{(7)}(\bar{v}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}} \le (v_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $v^{(7)}(t)$ :-

$$(m_2)^{(7)} \leq v^{(7)}(t) \leq (m_1)^{(7)}, \quad \boxed{v^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(7)}(t)$ :- 420

$$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}, \quad \boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $v^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)} G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)} T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

**Proof** : From global equations we obtain 421

$$\frac{dv^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right) - (a''_{41})^{(8)}(T_{41}, t) v^{(8)} - (a_{41})^{(8)} v^{(8)}$$

**Definition of** $v^{(8)}$ :- $\boxed{v^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_2)^{(8)} v^{(8)} - (a_{40})^{(8)}\right) \leq \frac{dv^{(8)}}{dt} \leq -\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_1)^{(8)} v^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$ :-

$$\text{For } 0 < \boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$$

$$v^{(8)}(t) \geq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)}-(\nu_0)^{(8)}}{(\nu_0)^{(8)}-(\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \leq \frac{(\nu_1)^{(8)}+(C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}}{1+(C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}} \leq \nu^{(8)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \leq (\bar{\nu}_1)^{(8)} \leq \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$ , we obtain 424

$$(\nu_1)^{(8)} \leq \nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(8)}(t)$ :-

$$(m_2)^{(8)} \leq \nu^{(8)}(t) \leq (m_1)^{(8)}, \quad \boxed{\nu^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(8)}(t)$ :-

$$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}, \quad \boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $\nu^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)}G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then $(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)}T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

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**Proof** : From 99,20,44,22,23,44 we obtain A

$$\frac{dv^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right) - (a''_{45})^{(9)}(T_{45},t)v^{(9)} - (a_{45})^{(9)}v^{(9)}$$

**Definition of** $v^{(9)}$ :- $\boxed{v^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)}\right) \le \frac{dv^{(9)}}{dt} \le -\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_1)^{(9)}v^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$ :-

For $0 < \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$

$$v^{(9)}(t) \ge \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \le v^{(9)}(t) \le (\nu_9)^{(9)}$

In the same manner , we get

$$v^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \le v^{(9)}(t) \le (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \le \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}}{1 + (C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}} \le v^{(9)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \le (\bar{\nu}_1)^{(9)} \le \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$ , we obtain

$$(\nu_1)^{(9)} \le v^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $v^{(9)}(t)$ :-

$$(m_2)^{(9)} \le v^{(9)}(t) \le (m_1)^{(9)}, \quad \boxed{v^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(9)}(t)$ :-

$(\mu_2)^{(9)} \leq u^{(9)}(t) \leq (\mu_1)^{(9)}$, $\boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)} G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then
$(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)} T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following 425

**Theorem :** If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on $t$ , and the conditions with the notations

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0$ ,

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$

$with\ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a''_i)^{(2)} and\ (b''_i)^{(2)}$ are independent on t , and the conditions with the notations 426

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$ 427

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 428

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$ , 429

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 430

$with\ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a''_i)^{(3)} and\ (b''_i)^{(3)}$ are independent on $t$ , and the conditions with the notations 431

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$ ,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

**Theorem :** If $(a_i'')^{(4)} and\ (b_i'')^{(4)}$ are independent on $t$ , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$ ,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(5)} and\ (b_i'')^{(5)}$ are independent on $t$ , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$ ,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

**Theorem** If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on $t$ , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$ ,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(7)} and\ (b_i'')^{(7)}$ are independent on $t$ , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$ ,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(8)} and\ (b_i'')^{(8)}$ are independent on $t$ , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$ ,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(9)} and\ (b_i'')^{(9)}$ are independent on $t$ , and the conditions (with the notations 436 A
45,46,27,28)

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$ ,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0$ 437

$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0$ 438

$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0$ 439

$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0$ 440

$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ]T_{14} = 0$ 441

$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0$ 442

has a unique positive solution , which is an equilibrium solution for the system

$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0$ 443

$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0$ 444

$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0$ 445

$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\ ]T_{16} = 0$ 446

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\,]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\,]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\,]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\,]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\,]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\big((G_{27})\big)\,]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\big((G_{27})\big)\,]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\big((G_{27})\big)\,]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})\,]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})\,]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})\,]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})\,]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})\,]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})\,]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0 \quad 479$$

$$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0 \quad 480$$

$$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0 \quad 481$$

$$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0 \quad 482$$

$$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0 \quad 483$$

$$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0 \quad 484$$

$$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0 \quad 484\text{A}$$

$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$

$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$

$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$

$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$

$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$

**Proof:** 485

(a) Indeed the first two equations have a nontrivial solution $G_{13}, G_{14}$ if

$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$

**Proof:** 486

(ee) Indeed the first two equations have a nontrivial solution $G_{16}, G_{17}$ if

$\mathrm{F}(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$

**Proof:** 487

(a) Indeed the first two equations have a nontrivial solution $G_{20}, G_{21}$ if

$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$

**Proof:** 488

(a) Indeed the first two equations have a nontrivial solution $G_{24}, G_{25}$ if

$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$

**Proof:** 489

(a) Indeed the first two equations have a nontrivial solution $G_{28}, G_{29}$ if

$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$

**Proof:** 490

(a) Indeed the first two equations have a nontrivial solution $G_{32}, G_{33}$ if

$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) +$

$(a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$

**Proof:** 491

(a) Indeed the first two equations have a nontrivial solution $G_{36}, G_{37}$ if

$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$

**Proof:** 492

(a) Indeed the first two equations have a nontrivial solution $G_{40}, G_{41}$ if

$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$

**Proof:** 492 A

(a) Indeed the first two equations have a nontrivial solution $G_{44}, G_{45}$ if

$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$

**Definition and uniqueness of** $T^*_{14}$ :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique $T^*_{14}$ for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

**Definition and uniqueness of** $T^*_{17}$ :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique $T^*_{17}$ for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]}$$ 495

**Definition and uniqueness of** $T^*_{21}$ :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique $T^*_{21}$ for which $f(T^*_{21}) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

**Definition and uniqueness of** $T^*_{25}$ :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that

there exists a unique $T_{25}^{*}$ for which $f(T_{25}^{*})=0$. With this value , we obtain from the three first equations

$$G_{24}=\frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}=\frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T_{25}^{*})]}$$

**Definition and uniqueness of** $T_{29}^{*}$ :- 498

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(5)}(T_{29})$ being increasing, it follows that there exists a unique $T_{29}^{*}$ for which $f(T_{29}^{*})=0$. With this value , we obtain from the three first equations

$$G_{28}=\frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}=\frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T_{29}^{*})]}$$

**Definition and uniqueness of** $T_{33}^{*}$ :- 499

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(6)}(T_{33})$ being increasing, it follows that there exists a unique $T_{33}^{*}$ for which $f(T_{33}^{*})=0$. With this value , we obtain from the three first equations

$$G_{32}=\frac{(a_{32})^{(6)}G_{33}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^{*})]} \quad , \quad G_{34}=\frac{(a_{34})^{(6)}G_{33}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^{*})]}$$

**Definition and uniqueness of** $T_{37}^{*}$ :- 500

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(7)}(T_{37})$ being increasing, it follows that there exists a unique $T_{37}^{*}$ for which $f(T_{37}^{*})=0$. With this value , we obtain from the three first equations

$$G_{36}=\frac{(a_{36})^{(7)}G_{37}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^{*})]} \quad , \quad G_{38}=\frac{(a_{38})^{(7)}G_{37}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^{*})]}$$

**Definition and uniqueness of** $T_{41}^{*}$ :- 501

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique $T_{41}^{*}$ for which $f(T_{41}^{*})=0$. With this value , we obtain from the three first equations

$$G_{40}=\frac{(a_{40})^{(8)}G_{41}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T_{41}^{*})]} \quad , \quad G_{42}=\frac{(a_{42})^{(8)}G_{41}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T_{41}^{*})]}$$

**Definition and uniqueness of** $T_{45}^{*}$ :- 501 A

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(9)}(T_{45})$ being increasing, it follows that there exists a unique $T_{45}^{*}$ for which $f(T_{45}^{*})=0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)}G_{45}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T^*_{45})]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)}G_{45}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T^*_{45})]}$$

By the same argument, the equations admit solutions $G_{13}, G_{14}$ if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right]+(b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{14}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{14}$ such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions $G_{16}, G_{17}$ if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right]+(b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ 504
is a decreasing function in $G_{17}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{14}$ such that $\varphi((G_{19})^*) = 0$

By the same argument, the equations admit solutions $G_{20}, G_{21}$ if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right]+(b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{21}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{21}$ such that $\varphi((G_{23})^*) = 0$

By the same argument, the equations admit solutions $G_{24}, G_{25}$ if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right]+(b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{25}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{25}$ such that $\varphi((G_{27})^*) = 0$

By the same argument, the equations admit solutions $G_{28}, G_{29}$ if 507

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right]+(b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{29}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that

there exists a unique $G_{29}^{*}$ such that $\varphi((G_{31})^{*}) = 0$

By the same argument, the equations admit solutions $G_{32}, G_{33}$ if 508
$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$

$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right] + (b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{33}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{33}^{*}$ such that $\varphi(G_{35}{}^{*}) = 0$

By the same argument, the equations admit solutions $G_{36}, G_{37}$ if 509
$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$
$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right] + (b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{37}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{37}^{*}$ such that $\varphi(G_{39}{}^{*}) = 0$

By the same argument, the equations admit solutions $G_{40}, G_{41}$ if 510
$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$
$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right] + (b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{41}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{41}^{*}$ such that $\varphi(G_{43}{}^{*}) = 0$

By the same argument, the equations 92,93 admit solutions $G_{44}, G_{45}$ if
$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$

$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right] + (b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{45}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{45}^{*}$ such that $\varphi((G_{47})^{*}) = 0$

Finally we obtain the unique solution 511

$G_{14}^{*}$ given by $\varphi(G^{*}) = 0$ , $T_{14}^{*}$ given by $f(T_{14}^{*}) = 0$ and

$$G_{13}^{*} = \frac{(a_{13})^{(1)}G_{14}^{*}}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^{*})]} \quad , \quad G_{15}^{*} = \frac{(a_{15})^{(1)}G_{14}^{*}}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^{*})]}$$

$$T_{13}^{*} = \frac{(b_{13})^{(1)}T_{14}^{*}}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^{*})]} \quad , \quad T_{15}^{*} = \frac{(b_{15})^{(1)}T_{14}^{*}}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^{*})]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

$G_{17}^{*}$ given by $\varphi((G_{19})^{*}) = 0$ , $T_{17}^{*}$ given by $f(T_{17}^{*}) = 0$ and 512

$$G_{16}^{*} = \frac{(a_{16})^{(2)} G_{17}^{*}}{[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}^{*})]} \quad , \quad G_{18}^{*} = \frac{(a_{18})^{(2)} G_{17}^{*}}{[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}^{*})]} \tag{513}$$

$$T_{16}^{*} = \frac{(b_{16})^{(2)} T_{17}^{*}}{[(b'_{16})^{(2)} - (b''_{16})^{(2)}((G_{19})^{*})]} \quad , \quad T_{18}^{*} = \frac{(b_{18})^{(2)} T_{17}^{*}}{[(b'_{18})^{(2)} - (b''_{18})^{(2)}((G_{19})^{*})]} \tag{514}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

$G_{21}^{*}$ given by $\varphi((G_{23})^{*}) = 0$ , $T_{21}^{*}$ given by $f(T_{21}^{*}) = 0$ and

$$G_{20}^{*} = \frac{(a_{20})^{(3)} G_{21}^{*}}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^{*})]} \quad , \quad G_{22}^{*} = \frac{(a_{22})^{(3)} G_{21}^{*}}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^{*})]}$$

$$T_{20}^{*} = \frac{(b_{20})^{(3)} T_{21}^{*}}{[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}{}^{*})]} \quad , \quad T_{22}^{*} = \frac{(b_{22})^{(3)} T_{21}^{*}}{[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}{}^{*})]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

$G_{25}^{*}$ given by $\varphi(G_{27}) = 0$ , $T_{25}^{*}$ given by $f(T_{25}^{*}) = 0$ and

$$G_{24}^{*} = \frac{(a_{24})^{(4)} G_{25}^{*}}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}^{*} = \frac{(a_{26})^{(4)} G_{25}^{*}}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^{*})]}$$

$$T_{24}^{*} = \frac{(b_{24})^{(4)} T_{25}^{*}}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^{*})]} \quad , \quad T_{26}^{*} = \frac{(b_{26})^{(4)} T_{25}^{*}}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^{*})]} \tag{517}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

$G_{29}^{*}$ given by $\varphi((G_{31})^{*}) = 0$ , $T_{29}^{*}$ given by $f(T_{29}^{*}) = 0$ and

$$G_{28}^{*} = \frac{(a_{28})^{(5)} G_{29}^{*}}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}^{*} = \frac{(a_{30})^{(5)} G_{29}^{*}}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^{*})]}$$

$$T_{28}^{*} = \frac{(b_{28})^{(5)} T_{29}^{*}}{[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31})^{*})]} \quad , \quad T_{30}^{*} = \frac{(b_{30})^{(5)} T_{29}^{*}}{[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31})^{*})]} \tag{519}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

$G_{33}^{*}$ given by $\varphi((G_{35})^{*}) = 0$ , $T_{33}^{*}$ given by $f(T_{33}^{*}) = 0$ and

$$G_{32}^{*} = \frac{(a_{32})^{(6)} G_{33}^{*}}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^{*})]} \quad , \quad G_{34}^{*} = \frac{(a_{34})^{(6)} G_{33}^{*}}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^{*})]}$$

$$T_{32}^{*} = \frac{(b_{32})^{(6)} T_{33}^{*}}{[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35})^{*})]} \quad , \quad T_{34}^{*} = \frac{(b_{34})^{(6)} T_{33}^{*}}{[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35})^{*})]} \tag{521}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

$G_{37}^*$ given by $\varphi((G_{39})^*) = 0$ , $T_{37}^*$ given by $f(T_{37}^*) = 0$ and

$$G_{36}^* = \frac{(a_{36})^{(7)} G_{37}^*}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^*)]} \quad , \quad G_{38}^* = \frac{(a_{38})^{(7)} G_{37}^*}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^*)]}$$

$$T_{36}^* = \frac{(b_{36})^{(7)} T_{37}^*}{[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39})^*)]} \quad , \quad T_{38}^* = \frac{(b_{38})^{(7)} T_{37}^*}{[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39})^*)]}$$

Finally we obtain the unique solution 523

$G_{41}^*$ given by $\varphi((G_{43})^*) = 0$ , $T_{41}^*$ given by $f(T_{41}^*) = 0$ and

$$G_{40}^* = \frac{(a_{40})^{(8)} G_{41}^*}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42}^* = \frac{(a_{42})^{(8)} G_{41}^*}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

$$T_{40}^* = \frac{(b_{40})^{(8)} T_{41}^*}{[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43})^*)]} \quad , \quad T_{42}^* = \frac{(b_{42})^{(8)} T_{41}^*}{[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43})^*)]}$$

Finally we obtain the unique solution of 89 to 99 523 A

$G_{45}^*$ given by $\varphi((G_{47})^*) = 0$ , $T_{45}^*$ given by $f(T_{45}^*) = 0$ and

$$G_{44}^* = \frac{(a_{44})^{(9)} G_{45}^*}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46}^* = \frac{(a_{46})^{(9)} G_{45}^*}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

$$T_{44}^* = \frac{(b_{44})^{(9)} T_{45}^*}{[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47})^*)]} \quad , \quad T_{46}^* = \frac{(b_{46})^{(9)} T_{45}^*}{[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47})^*)]}$$

## ASYMPTOTIC STABILITY ANALYSIS 524

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

**Proof:** Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a''_{14})^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \frac{\partial (b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a'_{13})^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)}G^*_{13}\mathbb{T}_{14} \quad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a'_{14})^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)}G^*_{14}\mathbb{T}_{14} \quad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a'_{15})^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)}G^*_{15}\mathbb{T}_{14} \quad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b'_{13})^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)}T^*_{13}\mathbb{G}_j\right) \quad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b'_{14})^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)}T^*_{14}\mathbb{G}_j\right) \quad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b'_{15})^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T^*_{15}\mathbb{G}_j\right) \quad 530$$

**ASYMPTOTIC STABILITY ANALYSIS** 531

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(2)}$ and $(b''_i)^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G^*_i + \mathbb{G}_i \quad , T_i = T^*_i + \mathbb{T}_i \quad 532$$

$$\frac{\partial(a''_{17})^{(2)}}{\partial T_{17}}(T^*_{17}) = (q_{17})^{(2)} \quad , \frac{\partial(b''_i)^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \quad 533$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a'_{16})^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G^*_{16}\mathbb{T}_{17} \quad 534$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a'_{17})^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G^*_{17}\mathbb{T}_{17} \quad 535$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a'_{18})^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G^*_{18}\mathbb{T}_{17} \quad 536$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b'_{16})^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T^*_{16}\mathbb{G}_j\right) \quad 537$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b'_{17})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}T^*_{17}\mathbb{G}_j\right) \quad 538$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b'_{18})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}T^*_{18}\mathbb{G}_j\right) \quad 539$$

**ASYMPTOTIC STABILITY ANALYSIS** 540

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(3)}$ *and* $(b''_i)^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{21}'')^{(3)}}{\partial T_{21}}(T_{21}^*) = (q_{21})^{(3)} \quad , \frac{\partial (b_i'')^{(3)}}{\partial G_j}(\,(G_{23})^*\,) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\big((a_{20}')^{(3)} + (p_{20})^{(3)}\big)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G_{20}^*\mathbb{T}_{21} \quad 541$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\big((a_{21}')^{(3)} + (p_{21})^{(3)}\big)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G_{21}^*\mathbb{T}_{21} \quad 542$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\big((a_{22}')^{(3)} + (p_{22})^{(3)}\big)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G_{22}^*\mathbb{T}_{21} \quad 543$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\big((b_{20}')^{(3)} - (r_{20})^{(3)}\big)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\big(s_{(20)(j)}T_{20}^*\mathbb{G}_j\big) \quad 544$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\big((b_{21}')^{(3)} - (r_{21})^{(3)}\big)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\big(s_{(21)(j)}T_{21}^*\mathbb{G}_j\big) \quad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\big((b_{22}')^{(3)} - (r_{22})^{(3)}\big)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\big(s_{(22)(j)}T_{22}^*\mathbb{G}_j\big) \quad 546$$

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**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}\ and\ (b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{25}'')^{(4)}}{\partial T_{25}}(T_{25}^*) = (q_{25})^{(4)} \quad , \frac{\partial (b_i'')^{(4)}}{\partial G_j}((G_{27})^*\,) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\big((a_{24}')^{(4)} + (p_{24})^{(4)}\big)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G_{24}^*\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\big((a_{25}')^{(4)} + (p_{25})^{(4)}\big)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G_{25}^*\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\big((a_{26}')^{(4)} + (p_{26})^{(4)}\big)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G_{26}^*\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\big((b_{24}')^{(4)} - (r_{24})^{(4)}\big)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\big(s_{(24)(j)}T_{24}^*\mathbb{G}_j\big) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\big((b_{25}')^{(4)} - (r_{25})^{(4)}\big)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\big(s_{(25)(j)}T_{25}^*\mathbb{G}_j\big) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T^*_{26}\mathbb{G}_j\right) \quad 554$$

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**Theorem 5:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}\ and\ (b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 556

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{29})^{(5)}}{\partial T_{29}}(T^*_{29}) = (q_{29})^{(5)} \ , \ \frac{\partial(b_i'')^{(5)}}{\partial G_j}\left((G_{31})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a'_{28})^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G^*_{28}\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a'_{29})^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G^*_{29}\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a'_{30})^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G^*_{30}\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b'_{28})^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T^*_{28}\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b'_{29})^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T^*_{29}\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b'_{30})^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T^*_{30}\mathbb{G}_j\right) \quad 562$$

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**Theorem 6:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}\ and\ (b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{33})^{(6)}}{\partial T_{33}}(T^*_{33}) = (q_{33})^{(6)} \ , \ \frac{\partial(b_i'')^{(6)}}{\partial G_j}\left((G_{35})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a'_{32})^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G^*_{32}\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a'_{33})^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G^*_{33}\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a'_{34})^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G^*_{34}\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b'_{32})^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T^*_{32}\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b'_{33})^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T^*_{33}\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b'_{34})^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T^*_{34}\mathbb{G}_j\right) \quad 570$$

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**Theorem 7:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 572

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{37})^{(7)}}{\partial T_{37}}(T^*_{37}) = (q_{37})^{(7)} \quad , \frac{\partial(b_i'')^{(7)}}{\partial G_j}\left((G_{39})^{**}\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a'_{36})^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G^*_{36}\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a'_{37})^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G^*_{37}\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a'_{38})^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G^*_{38}\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b'_{36})^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T^*_{36}\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b'_{37})^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T^*_{37}\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b'_{38})^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T^*_{38}\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

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**Theorem 8:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{41}'')^{(8)}}{\partial T_{41}}(T_{41}^*) = (q_{41})^{(8)} \quad , \frac{\partial (b_i'')^{(8)}}{\partial G_j}\left( (G_{43})^* \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a_{40}')^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G_{40}^*\mathbb{T}_{41} \quad (581)$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a_{41}')^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G_{41}^*\mathbb{T}_{41} \quad (582)$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a_{42}')^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G_{42}^*\mathbb{T}_{41} \quad (583)$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b_{40}')^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T_{40}^*\mathbb{G}_j\right) \quad (584)$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b_{41}')^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T_{41}^*\mathbb{G}_j\right) \quad (585)$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b_{42}')^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T_{42}^*\mathbb{G}_j\right) \quad (586)$$

(586 A)

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**Theorem 9:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}$ *and* $(b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

**Proof:** Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{45}'')^{(9)}}{\partial T_{45}}(T_{45}^*) = (q_{45})^{(9)} \quad , \frac{\partial (b_i'')^{(9)}}{\partial G_j}\left( (G_{47})^* \right) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a_{44}')^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)}G_{44}^*\mathbb{T}_{45} \quad (586\,B)$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a_{45}')^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)}G_{45}^*\mathbb{T}_{45} \quad (586\,C)$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a_{46}')^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)}G_{46}^*\mathbb{T}_{45} \quad (586\,D)$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b_{44}')^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)}T_{44}^*\mathbb{G}_j\right) \quad (586\,E)$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b_{45}')^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)}T_{45}^*\mathbb{G}_j\right) \quad (586\,F)$$

$\frac{d\mathbb{T}_{46}}{dt} = -\left((b'_{46})^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T^*_{46}\mathbb{G}_j\right)$ 586

G

587

**The characteristic equation of this system is**

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(q_{13})^{(1)}G^*_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)}T^*_{14} + (b_{14})^{(1)}s_{(13),(14)}T^*_{14}\right)$$

$$+\left(\left((\lambda)^{(1)} + (a'_{14})^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)}G^*_{13} + (a_{13})^{(1)}(q_{14})^{(1)}G^*_{14}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)}T^*_{14} + (b_{14})^{(1)}s_{(13),(13)}T^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((b'_{13})^{(1)} + (b'_{14})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(15)}T^*_{14} + (b_{14})^{(1)}s_{(13),(15)}T^*_{13}\right)\} = 0$$

$+$

$$(\lambda)^{(2)} + (b'_{18})^{(2)} - (r_{18})^{(2)}\{\left((\lambda)^{(2)} + (a'_{18})^{(2)} + (p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^*_{17} + (a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^*_{17}\right)$$

$$+\left(\left((\lambda)^{(2)} + (a'_{17})^{(2)} + (p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^*_{16} + (a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}\right)$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2 + \left((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2 + \left((b'_{16})^{(2)} + (b'_{17})^{(2)} - (r_{16})^{(2)} + (r_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$+\left(\left((\lambda)^{(2)}\right)^2 + \left((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17} + (a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^{*}_{16}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\right)\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left(\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)\right]$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(21)}T^{*}_{21}\right)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^{*}_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^{*}_{21}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(20)}T^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$+\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(22)}T^{*}_{20}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\right)\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left(\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)\right]$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(25)}T^{*}_{25}\right)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^{*}_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^{*}_{25}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(24)}T^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$+\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(26)}T^{*}_{24}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\right)\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(29)}T^{*}_{29}\right)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^{*}_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^{*}_{29}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(28)}T^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$+\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(30)}T^{*}_{28}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\right)\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left(\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)\right]$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(33)}T^{*}_{33}\right)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^{*}_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^{*}_{33}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(32)}T^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$+\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(34)}T^{*}_{32}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\right)\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)\right]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(37)}T^{*}_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^{*}_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^{*}_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(36)}T^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$+\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(38)}T^{*}_{36}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\right)\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(41)}T^{*}_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^{*}_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^{*}_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(40)}T^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$+\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^*_{40}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^*_{41}+(b_{41})^{(8)}s_{(40),(42)}T^*_{40}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\right)\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^*_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^*_{45}+(b_{45})^{(9)}s_{(44),(45)}T^*_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^*_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^*_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^*_{45}+(b_{45})^{(9)}s_{(44),(44)}T^*_{44}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^2+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$+\left(\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^*_{44}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^*_{45}+(b_{45})^{(9)}s_{(44),(46)}T^*_{44}\right)\}=0$$

**And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.**

## SECTION THIRTY FOUR

## Psycho-Space

### INTRODUCTION—VARIABLES USED

**Source: Time line of models of consciousness (Wikipedia)**

(1) **James Culbertson**: **psycho-space - 1963**. According to Culberston (1963) memory, which is at the basis of (e) consciousness, is a consequence of (e) the change of state of matter in time.

(2) In other words, memory is equivalent to (eb) different states of matter in time. From this point of

view memory is not formed by (e) data stored in the brain, but by links between (e&eb) different states of space-time, and consciousness would therefore be located outside (e) the brain, in the space-time.

**(3)** In order to describe this concept Culberston coined the term psycho-space. All objects of nature change their state in (eb) time. Therefore, according to Culberston, all objects of nature could be endowed with (e&eb) consciousness.

**Horoomi Umezawa and Luigi Maria Ricciardi: Quantum Field Theory (QFT) – 1967**

(4) Ricciardi and Umezawa (1967) suggested a model of consciousness based on (e) Quantum Field Theory (QFT).
(5) This model starts from the work carried by Fröhlich on Bose-Einstein condensates. The functions of the brain are seen as consequence of (e) collective quantum order states.
(6) For example, memory is Antonella Vannini Quantum Biosystems 2008, 2, 165-184 168 associated to (e&eb) vacuum states.
(7) In QFT vacuum states are (=) the lowest level of energy in which, by definition, (eb) no particle is present.
(8) The stability of vacuum states makes them ideal as (=) a unit of memory.
(9) Umezawa and Ricciardi underline that one of the properties of vacuum states is that of (e) developing correlations and order which can extend to (eb) the macroscopic level, producing (eb) fields which can affect (e&eb) the neural system.
(10) According to Umezawa, consciousness would be the result of (e) the sum of quantum processes, while the neural systems would be limited to (e) the transmission of macroscopic signals. The model of Umezawa was further developed by Giuseppe Vitiello.

**The Quarterly Journal of Experimental Psychology Section A: Human Experimental Psychology Volume 48, Issue 1, 1995: Is there a Relationship between Task Demand and Storage Space in Tests of Working Memory Capacity? DOI: 10.1080/14640749508401379 J.N. Towsea* & G.J. Hitchb pages 108-124 Published online: 29 May 2007**

(1) This paper considers working memory capacity, critically examining the hypothesis that counting span (the ability to count arrays of objects and store count totals) reflects a trade-off in resources available for processing and (e&eb) short-term storage.
(2) Previous evidence interpreted as favouring this hypothesis has confounded task difficulty with counting time. Experiment 1 validated a manipulation of (e&eb) the attentional demands of counting in which target objects were differentiated from (e&eb) non-targets by (e&eb) either a single feature (colour) or a feature conjunction (a combination of line orientations).
(3) The results confirmed that the two presentations involved (e&eb) qualitatively different attentional loads.
(4) Experiment 2 used these displays to compare counting span for children aged 6 to 11, both with and without (e) an adjustment of target numerosity to control for differences in processing time.
(5) At all ages, span was lower when counting took longer, but there was no difference between feature and conjunction arrays once counting time was accounted for.
(6) These results argue against a resource trade-off interpretation of (e&eb) counting span.
(7) Rather, they support a hypothesis of resource-switching among children, implying (eb) that counting span acts as (e&eb) a measure of time-based forgetting.

**How many memory systems are there? Tulving, Endel American Psychologist, Vol 40(4), Apr 1985, 385-398. http://dx.doi.org/10.1037/0003-066X.40.4.385**

**(8)** Proposes a ternary classificatory scheme of memory in which procedural, semantic, and episodic memory constitute (e) a monohierarchical arrangement.

**(9)** In this scheme, episodic memory is a specialized subsystem of (e) semantic memory, which in turn is a specialized subsystem of (e) procedural memory.

**(10)** The 3 memory systems differ from one another in a number of ways, including (e) the kind of consciousness that characterizes their operations.

**(11)** The ternary scheme overlaps with (e&eb) dichotomies and trichotomies of memory proposed by others. Evidence for multiple systems is reviewed, and illustrative data are provided from experiments.

**(12)** Direct priming effects were found to be both functionally and stochastically independent of recognition memory. (100 ref) (PsycINFO Database Record (c) 2012 APA, all rights reserved)

## NOTATION

### Module One

The coexistence of past, present and future would be a direct consequence of (e) the coexistence of advanced and retarded waves

$G_{13}$ : Category one of advanced waves; retarded waves

$G_{14}$ : Category two of SAS

$G_{15}$ : Category three of SAS

$T_{13}$ : Category one of retarded waves; advanced waves

$T_{14}$ : Category two of SAS

$T_{15}$ : Category three of SAS

### Module Two

**James Culbertson**: **psycho-space - 1963**.

According to Culberston (1963) memory, which is at the basis of (e) **consciousness**, is a consequence of (e) the change of state of matter in time

$G_{16}$ : Category one of memory; **consciousness**

$G_{17}$ : Category two of SAS

$G_{18}$ : Category three of SAS

$T_{16}$ : Category one of **consciousness;** memory

$T_{17}$ : Category two of SAS

$T_{18}$ : Category three of SAS

### Module three

According to Culberston (1963) memory, which is at the basis of **consciousness**, is a consequence of (e) the change of state of matter in time

$G_{20}$ : Category one of change of state of matter in time

$G_{21}$ : Category two of SAS

$G_{22}$ : Category three of SAS

$T_{20}$ : Category one of) memory, which is at the basis of **consciousness**

$T_{21}$ : Category two of SAS

$T_{22}$ : Category three of SAS

**Module four**

This model starts from the work carried by Fröhlich on Bose-Einstein condensates.

The functions of the brain are seen as consequence of (e) collective quantum order states

$G_{24}$ : Category one of collective quantum order states

$G_{25}$ : Category two of SAS

$G_{26}$ : Category three of SAS

$T_{24}$ : Category one of functions of the brain

$T_{25}$ : Category two of SAS

$T_{26}$ : Category three of SAS

**Module five**

For example, memory is Antonella Vannini Quantum Biosystems 2008, 2, 165-184 168 associated to (e&eb) **vacuum states**

$G_{28}$ : Category one of memory; **vacuum states**

$G_{29}$ : Category two of SAS

$G_{30}$ : Category three of SAS

$T_{28}$ : Category one of **vacuum states;** memory

$T_{29}$ : Category two of SAS

$T_{30}$ : Category three of SAS

**Module six**

In QFT vacuum states are (=) the lowest level of energy in which, by definition, (eb) no particle is present.

$G_{32}$ : Category one of QFT vacuum states

$G_{33}$ : Category two of SAS

$G_{34}$ : Category three of SAS

$T_{32}$ : Category one of lowest level of energy in which, by definition, no particle is present.

$T_{33}$ : Category two of SAS

$T_{34}$ : Category three of SAS

**Module seven**

The stability of vacuum states makes them ideal as (=) a **unit of memory**

$G_{36}$ : Category one of stability of vacuum states

$G_{37}$ : Category two of SAS

$G_{38}$ : Category three of SAS

$T_{36}$ : Category one of unit of memory

$T_{37}$ : Category two of SAS

$T_{38}$ : Category three of SAS

**Module eight**

Umezawa and Ricciardi underline that one of the properties of vacuum states is that of (e) developing correlations and order which can extend to (eb) the macroscopic level, producing (eb) fields which can affect (e&eb) the neural system

$G_{40}$ : Category one of properties of vacuum states is that of (e) developing correlations and order which can extend to (eb) the macroscopic level

$G_{41}$ : Category two of SAS

$G_{42}$ : Category three of SAS

$T_{40}$ : Category one of fields which can affect (e&eb) the neural system

$T_{41}$ : Category two of SAS

$T_{42}$ : Category three of SAS

**Module Nine**

Umezawa and Ricciardi underline that one of the properties of vacuum states is that of (e) developing correlations and order which can extend to the macroscopic level, producing fields which can affect (e&eb) the **neural system**

$G_{44}$ : Category one of Umezawa and Ricciardi underline that one of the properties of vacuum states is that of (e) developing correlations and order which can extend to the macroscopic level, producing fields; **neural system**

$G_{45}$ : Category two of SAS

$G_{46}$ : Category three of SAS

$T_{44}$ : Category one of **neural system** ;Umezawa and Ricciardi underline that one of the properties of vacuum states is that of (e) developing correlations and order which can extend to the macroscopic level, producing fields

$T_{45}$ : Category two of SAS

$T_{46}$ : Category three of SAS

**The Coefficients:**

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}$ , $(b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}$,
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}$, $(b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}\ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$ ,
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)}$,
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)}$,
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)}$,
are Dissipation coefficients

**Module Numbered One**

**The differential system of this model is now (Module Numbered one)**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \quad 1$$
$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad 2$$
$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15} \quad 3$$
$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13} \quad 4$$
$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14} \quad 5$$
$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)\right]T_{15} \quad 6$$
$+(a''_{13})^{(1)}(T_{14}, t) =$ First augmentation factor
$-(b''_{13})^{(1)}(G, t) =$ First detritions factor

**Module Numbered Two**

**The differential system of this model is now ( Module numbered two)**

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right]G_{16} \quad 7$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t)\right]G_{17} \quad 8$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t)\right]G_{18} \quad 9$$

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\left((G_{19}), t\right)\right]T_{16} \quad 10$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\left((G_{19}), t\right)\right]T_{17} \quad 11$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\left((G_{19}), t\right)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17}, t) =$ First augmentation factor

$-(b''_{16})^{(2)}\left((G_{19}), t\right) =$ First detritions factor

**Module Numbered Three**

**The differential system of this model is now (Module numbered three)**

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right]G_{20} \quad 13$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}, t)\right]G_{21} \quad 14$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}, t)\right]G_{22} \quad 15$$

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}, t)\right]T_{20} \quad 16$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}, t)\right]T_{21} \quad 17$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}, t)\right]T_{22} \quad 18$$

$+(a''_{20})^{(3)}(T_{21}, t) =$ First augmentation factor

$-(b''_{20})^{(3)}(G_{23}, t) =$ First detritions factor

**Module Numbered Four**

**The differential system of this model is now (Module numbered Four)**

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)\right]G_{24} \quad 19$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}, t)\right]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}, t)\right]G_{26} \quad 21$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27}), t\right)\right]T_{24} \quad 22$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27}), t\right)\right]T_{25} \quad 23$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27}), t\right)\right]T_{26} \quad 24$$

$+(a''_{24})^{(4)}(T_{25}, t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}\left((G_{27}), t\right) =$ **First detritions factor**

**Module Numbered Five**:

**The differential system of this model is now (Module number five)**

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right]G_{28} \quad 25$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}, t)\right]G_{29} \quad 26$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}, t)\right]G_{30} \quad 27$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}\left((G_{31}), t\right)\right]T_{28} \quad 28$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}\left((G_{31}), t\right)\right]T_{29} \quad 29$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}\left((G_{31}), t\right)\right]T_{30} \quad 30$$

$+(a''_{28})^{(5)}(T_{29},t) =$ **First augmentation factor**
$-(b''_{28})^{(5)}\big((G_{31}),t\big) =$ **First detritions factor**
**Module Numbered Six**

**The differential system of this model is now (Module numbered Six)**

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \big[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33},t)\big]G_{32} \quad 31$$
$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \big[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33},t)\big]G_{33} \quad 32$$
$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \big[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33},t)\big]G_{34} \quad 33$$
$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \big[(b'_{32})^{(6)} - (b''_{32})^{(6)}\big((G_{35}),t\big)\big]T_{32} \quad 34$$
$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \big[(b'_{33})^{(6)} - (b''_{33})^{(6)}\big((G_{35}),t\big)\big]T_{33} \quad 35$$
$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \big[(b'_{34})^{(6)} - (b''_{34})^{(6)}\big((G_{35}),t\big)\big]T_{34} \quad 36$$

$+(a''_{32})^{(6)}(T_{33},t) =$ **First augmentation factor**
**Module Numbered Seven:**

**The differential system of this model is now (Seventh Module)**

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \big[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\big]G_{36} \quad 37$$
$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \big[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37},t)\big]G_{37} \quad 38$$
$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \big[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37},t)\big]G_{38} \quad 39$$
$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \big[(b'_{36})^{(7)} - (b''_{36})^{(7)}\big((G_{39}),t\big)\big]T_{36} \quad 40$$
$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \big[(b'_{37})^{(7)} - (b''_{37})^{(7)}\big((G_{39}),t\big)\big]T_{37} \quad 41$$
$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \big[(b'_{38})^{(7)} - (b''_{38})^{(7)}\big((G_{39}),t\big)\big]T_{38} \quad 42$$

$+(a''_{36})^{(7)}(T_{37},t) =$ **First augmentation factor**
**Module Numbered Eight**

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \big[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\big]G_{40} \quad 43$$
$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \big[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41},t)\big]G_{41} \quad 44$$
$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \big[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41},t)\big]G_{42} \quad 45$$
$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \big[(b'_{40})^{(8)} - (b''_{40})^{(8)}\big((G_{43}),t\big)\big]T_{40} \quad 46$$
$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \big[(b'_{41})^{(8)} - (b''_{41})^{(8)}\big((G_{43}),t\big)\big]T_{41} \quad 47$$
$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \big[(b'_{42})^{(8)} - (b''_{42})^{(8)}\big((G_{43}),t\big)\big]T_{42} \quad 48$$

**Module Numbered Nine**

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \big[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\big]G_{44} \quad 49$$
$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \big[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\big]G_{45} \quad 50$$
$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \big[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\big]G_{46} \quad 51$$
$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \big[(b'_{44})^{(9)} - (b''_{44})^{(9)}\big((G_{47}),t\big)\big]T_{44} \quad 52$$

$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}((G_{47}),t)\right]T_{45}$ 53

$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47}),t)\right]T_{46}$ 54

$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}((G_{47}),t) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)} & +(a''_{13})^{(1)}(T_{14},t) & +(a''_{16})^{(2,2,)}(T_{17},t) & +(a''_{20})^{(3,3,)}(T_{21},t) \\ +(a''_{24})^{(4,4,4,4,)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{36})^{(7,7)}(T_{37},t) & +(a''_{40})^{(8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)} & +(a''_{14})^{(1)}(T_{14},t) & +(a''_{17})^{(2,2,)}(T_{17},t) & +(a''_{21})^{(3,3,)}(T_{21},t) \\ +(a''_{25})^{(4,4,4,4,)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{37})^{(7,7)}(T_{37},t) & +(a''_{41})^{(8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)} & +(a''_{15})^{(1)}(T_{14},t) & +(a''_{18})^{(2,2,)}(T_{17},t) & +(a''_{22})^{(3,3,)}(T_{21},t) \\ +(a''_{26})^{(4,4,4,4,)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{38})^{(7,7)}(T_{37},t) & +(a''_{42})^{(8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{15} \quad 57$$

Where $(a''_{13})^{(1)}(T_{14},t)$, $(a''_{14})^{(1)}(T_{14},t)$, $(a''_{15})^{(1)}(T_{14},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,)}(T_{17},t)$, $+(a''_{17})^{(2,2,)}(T_{17},t)$, $+(a''_{18})^{(2,2,)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,)}(T_{21},t)$, $+(a''_{21})^{(3,3,)}(T_{21},t)$, $+(a''_{22})^{(3,3,)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{38})^{(7,7)}(T_{37},t)$ $+(a''_{37})^{(7,7)}(T_{37},t)$ $+(a''_{36})^{(7,7)}(T_{37},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{40})^{(8,8)}(T_{41},t)$ $+(a''_{41})^{(8,8)}(T_{41},t)$ $+(a''_{42})^{(8,8)}(T_{41},t)$ are eight augmentation coefficient for 1,2,3

$+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)} & -(b''_{13})^{(1)}(G,t) & -(b''_{16})^{(2,2,)}(G_{19},t) & -(b''_{20})^{(3,3,)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{36})^{(7,7,)}(G_{39},t) & -(b''_{40})^{(8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{13} \quad 58$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)} & -(b''_{14})^{(1)}(G,t) & -(b''_{17})^{(2,2,)}(G_{19},t) & -(b''_{21})^{(3,3,)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{37})^{(7,7,)}(G_{39},t) & -(b''_{41})^{(8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{14} \quad 59$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)}\boxed{-(b''_{15})^{(1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15} \quad (60)$$

Where $\boxed{-(b''_{13})^{(1)}(G,t)}$, $\boxed{-(b''_{14})^{(1)}(G,t)}$, $\boxed{-(b''_{15})^{(1)}(G,t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8)}(G_{43},t)}$ $\boxed{-(b''_{41})^{(8,8)}(G_{43},t)}$ $\boxed{-(b''_{42})^{(8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \begin{bmatrix} (a'_{16})^{(2)}\boxed{+(a''_{16})^{(2)}(T_{17},t)} & \boxed{+(a''_{13})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{16} \quad (61)$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \begin{bmatrix} (a'_{17})^{(2)}\boxed{+(a''_{17})^{(2)}(T_{17},t)} & \boxed{+(a''_{14})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{17} \quad (62)$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \begin{bmatrix} (a'_{18})^{(2)}\boxed{+(a''_{18})^{(2)}(T_{17},t)} & \boxed{+(a''_{15})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{18} \quad (63)$$

Where $\boxed{+(a''_{16})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,)}(T_{14},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{36})^{(7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7)}(T_{37},t)$ are seventh augmentation coefficient for category 1, 2 and 3

$+(a''_{40})^{(8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8)}(T_{41},t)$ are eight augmentation coefficient for category 1, 2 and 3

$+(a''_{44})^{(9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9)}(T_{45},t)$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \begin{bmatrix} (b'_{16})^{(2)} & -(b''_{16})^{(2)}(G_{19},t) & -(b''_{13})^{(1,1,)}(G,t) & -(b''_{20})^{(3,3,3,)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6)}(G_{35},t) \\ -(b''_{36})^{(7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9)}(G_{47},t) \end{bmatrix} T_{16} \quad 64$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)} & -(b''_{17})^{(2)}(G_{19},t) & -(b''_{14})^{(1,1,)}(G,t) & -(b''_{21})^{(3,3,3,)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6)}(G_{35},t) \\ -(b''_{37})^{(7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9)}(G_{47},t) \end{bmatrix} T_{17} \quad 65$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)} & -(b''_{18})^{(2)}(G_{19},t) & -(b''_{15})^{(1,1,)}(G,t) & -(b''_{22})^{(3,3,3,)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6)}(G_{35},t) \\ -(b''_{38})^{(7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9)}(G_{47},t) \end{bmatrix} T_{18} \quad 66$$

where $-(b''_{16})^{(2)}(G_{19},t)$, $-(b''_{17})^{(2)}(G_{19},t)$, $-(b''_{18})^{(2)}(G_{19},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{13})^{(1,1,)}(G,t)$, $-(b''_{14})^{(1,1,)}(G,t)$, $-(b''_{15})^{(1,1,)}(G,t)$ are second detrition coefficients for category 1,2 and 3

$-(b''_{20})^{(3,3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,)}(G_{23},t)$ are third detrition coefficients for category 1,2 and 3

$-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1,2 and 3

$-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1,2 and 3

$-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients for category 1,2 and 3

$-(b''_{36})^{(7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2 and 3

$-(b''_{40})^{(8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1,2 and 3

$-(b''_{44})^{(9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \begin{bmatrix} (a'_{20})^{(3)} & +(a''_{20})^{(3)}(T_{21},t) & +(a''_{16})^{(2,2,2)}(T_{17},t) & +(a''_{13})^{(1,1,1,)}(T_{14},t) \\ +(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t) & \\ +(a''_{36})^{(7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9)}(T_{45},t) & \end{bmatrix} G_{20} \quad (67)$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \begin{bmatrix} (a'_{21})^{(3)} & +(a''_{21})^{(3)}(T_{21},t) & +(a''_{17})^{(2,2,2)}(T_{17},t) & +(a''_{14})^{(1,1,1,)}(T_{14},t) \\ +(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t) & \\ +(a''_{37})^{(7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9)}(T_{45},t) & \end{bmatrix} G_{21} \quad (68)$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \begin{bmatrix} (a'_{22})^{(3)} & +(a''_{22})^{(3)}(T_{21},t) & +(a''_{18})^{(2,2,2)}(T_{17},t) & +(a''_{15})^{(1,1,1,)}(T_{14},t) \\ +(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t) & \\ +(a''_{38})^{(7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9)}(T_{45},t) & \end{bmatrix} G_{22} \quad (69)$$

$+(a''_{20})^{(3)}(T_{21},t)$, $+(a''_{21})^{(3)}(T_{21},t)$, $+(a''_{22})^{(3)}(T_{21},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2)}(T_{17},t)$ are second augmentation coefficients for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,)}(T_{14},t)$ are third augmentation coefficients for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficients for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficients for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficients for category 1, 2 and 3

$+(a''_{36})^{(7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1, 2 and 3

$+(a''_{40})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8)}(T_{41},t)$ are eight augmentation coefficients for category 1, 2 and 3

$+(a''_{44})^{(9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b'_{20})^{(3)} & -(b''_{20})^{(3)}(G_{23},t) & -(b''_{16})^{(2,2,2)}(G_{19},t) & -(b''_{13})^{(1,1,1,)}(G,t) \\ -(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t) & \\ -(b''_{36})^{(7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9)}(G_{47},t) & \end{bmatrix} T_{20} \quad (70)$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b'_{21})^{(3)} & -(b''_{21})^{(3)}(G_{23},t) & -(b''_{17})^{(2,2,2)}(G_{19},t) & -(b''_{14})^{(1,1,1,)}(G,t) \\ -(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t) & \\ -(b''_{37})^{(7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9)}(G_{47},t) & \end{bmatrix} T_{21} \quad (71)$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)}\boxed{-(b''_{22})^{(3)}(G_{23},t)} & \boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{22} \quad 72$$

$\boxed{-(b''_{20})^{(3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3)}(G_{23},t)}$ *are first detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}$ *are second detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{13})^{(1,1,1,)}(G,t)}, \boxed{-(b''_{14})^{(1,1,1,)}(G,t)}, \boxed{-(b''_{15})^{(1,1,1,)}(G,t)}$ *are third detrition coefficients for category 1,2 and 3*

$\boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)}, \boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)}, \boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)}$ *are fourth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)}, \boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)}, \boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)}$ *are fifth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)}, \boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)}, \boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)}$ *are sixth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{36})^{(7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{37})^{(7,7,7,7)}(G_{39},t)} \boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)}, \boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)}, \boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \begin{bmatrix} (a'_{24})^{(4)}\boxed{+(a''_{24})^{(4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{24} \quad 73$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \begin{bmatrix} (a'_{25})^{(4)}\boxed{+(a''_{25})^{(4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{25} \quad 74$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \begin{bmatrix} (a'_{26})^{(4)}\boxed{+(a''_{26})^{(4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{26} \quad 75$$

$\boxed{(a''_{24})^{(4)}(T_{25},t)}, \boxed{(a''_{25})^{(4)}(T_{25},t)}, \boxed{(a''_{26})^{(4)}(T_{25},t)}$ *are first augmentation coefficients category* $1, 2\ 3$

$\boxed{+(a''_{28})^{(5,5,)}(T_{29},t)}, \boxed{+(a''_{29})^{(5,5,)}(T_{29},t)}, \boxed{+(a''_{30})^{(5,5,)}(T_{29},t)}$ *are second augmentation coefficient for category* $1, 2$ *and* $3$

$\boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}, \boxed{+(a''_{33})^{(6,6,)}(T_{33},t)}, \boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}$ *are third augmentation coefficient for category* $1, 2$ *and* $3$

$\boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)}$

$are\ fourth\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$
$+(a''_{16})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2)}(T_{17},t)$
$are\ fifth\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$
$+(a''_{20})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3)}(T_{21},t)$
$are\ sixth\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$
$+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)$
$are\ seventh\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$
$+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)$
$are\ eighth\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$
$+(a''_{46})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9)}(T_{45},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \begin{bmatrix} (b'_{24})^{(4)}\left[-(b''_{24})^{(4)}(G_{27},t)\right] & \left[-(b''_{28})^{(5,5,)}(G_{31},t)\right] & \left[-(b''_{32})^{(6,6,)}(G_{35},t)\right] \\ \left[-(b''_{13})^{(1,1,1,1)}(G,t)\right] & \left[-(b''_{16})^{(2,2,2,2)}(G_{19},t)\right] & \left[-(b''_{20})^{(3,3,3,3)}(G_{23},t)\right] \\ \left[-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)\right] & \left[-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)\right] & \left[-(b''_{44})^{(9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{24} \quad (76)$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \begin{bmatrix} (b'_{25})^{(4)}\left[-(b''_{25})^{(4)}(G_{27},t)\right] & \left[-(b''_{29})^{(5,5,)}(G_{31},t)\right] & \left[-(b''_{33})^{(6,6,)}(G_{35},t)\right] \\ \left[-(b''_{14})^{(1,1,1,1)}(G,t)\right] & \left[-(b''_{17})^{(2,2,2,2)}(G_{19},t)\right] & \left[-(b''_{21})^{(3,3,3,3)}(G_{23},t)\right] \\ \left[-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)\right] & \left[-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)\right] & \left[-(b''_{45})^{(9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{25} \quad (77)$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)}\left[-(b''_{26})^{(4)}(G_{27},t)\right] & \left[-(b''_{30})^{(5,5,)}(G_{31},t)\right] & \left[-(b''_{34})^{(6,6,)}(G_{35},t)\right] \\ \left[-(b''_{15})^{(1,1,1,1)}(G,t)\right] & \left[-(b''_{18})^{(2,2,2,2)}(G_{19},t)\right] & \left[-(b''_{22})^{(3,3,3,3)}(G_{23},t)\right] \\ \left[-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)\right] & \left[-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)\right] & \left[-(b''_{46})^{(9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{26} \quad (78)$$

$Where\ \left[-(b''_{24})^{(4)}(G_{27},t)\right]$, $\left[-(b''_{25})^{(4)}(G_{27},t)\right]$, $\left[-(b''_{26})^{(4)}(G_{27},t)\right]\ are\ first\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$\left[-(b''_{28})^{(5,5,)}(G_{31},t)\right]$, $\left[-(b''_{29})^{(5,5,)}(G_{31},t)\right]$, $\left[-(b''_{30})^{(5,5,)}(G_{31},t)\right]\ are\ second\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$\left[-(b''_{32})^{(6,6,)}(G_{35},t)\right]$, $\left[-(b''_{33})^{(6,6,)}(G_{35},t)\right]$, $\left[-(b''_{34})^{(6,6,)}(G_{35},t)\right]\ are\ third\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$\left[-(b''_{13})^{(1,1,1,1)}(G,t)\right]$, $\left[-(b''_{14})^{(1,1,1,1)}(G,t)\right]$, $\left[-(b''_{15})^{(1,1,1,1)}(G,t)\right]$
$are\ fourth\ \ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$\left[-(b''_{16})^{(2,2,2,2)}(G_{19},t)\right]$, $\left[-(b''_{17})^{(2,2,2,2)}(G_{19},t)\right]$, $\left[-(b''_{18})^{(2,2,2,2)}(G_{19},t)\right]$
$are\ fifth\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$\left[-(b''_{20})^{(3,3,3,3)}(G_{23},t)\right]$, $\left[-(b''_{21})^{(3,3,3,3)}(G_{23},t)\right]$, $\left[-(b''_{22})^{(3,3,3,3)}(G_{23},t)\right]$
$are\ sixth\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$\left[-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)\right]$, $\left[-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)\right]$, $\left[-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)\right]$
$are\ seventh\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$\left[-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)\right]$, $\left[-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)\right]$, $\left[-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)\right]$
$are\ eighth\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$\left[-(b''_{46})^{(9,9,9,9)}(G_{47},t)\right]$, $\left[-(b''_{45})^{(9,9,9,9)}(G_{47},t)\right]$, $\left[-(b''_{44})^{(9,9,9,9)}(G_{47},t)\right]$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[\begin{array}{ccc} (a'_{28})^{(5)}\boxed{+(a''_{28})^{(5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{28} \quad (79)$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[\begin{array}{ccc} (a'_{29})^{(5)}\boxed{+(a''_{29})^{(5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{29} \quad (80)$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[\begin{array}{ccc} (a'_{30})^{(5)}\boxed{+(a''_{30})^{(5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{30} \quad (81)$$

*Where* $\boxed{+(a''_{28})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5)}(T_{29},t)}$ *are first augmentation coefficients for category* 1, 2 *and* 3

*And* $\boxed{+(a''_{24})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,)}(T_{25},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)}$ are fourth augmentation coefficients for category 1,2, and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)}$ are fifth augmentation coefficients for category 1,2,and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)}$ are sixth augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)}$ are seventh augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)}$ are eighth augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[\begin{array}{ccc} (b'_{28})^{(5)}\boxed{-(b''_{28})^{(5)}(G_{31},t)} & \boxed{-(b''_{24})^{(4,4,)}(G_{27},t)} & \boxed{-(b''_{32})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)} \end{array}\right] T_{28} \quad (82)$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[\begin{array}{ccc} (b'_{29})^{(5)}\boxed{-(b''_{29})^{(5)}(G_{31},t)} & \boxed{-(b''_{25})^{(4,4,)}(G_{27},t)} & \boxed{-(b''_{33})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)} \end{array}\right] T_{29} \quad (83)$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[\begin{array}{ccc} (b'_{30})^{(5)}\boxed{-(b''_{30})^{(5)}(G_{31},t)} & \boxed{-(b''_{26})^{(4,4,)}(G_{27},t)} & \boxed{-(b''_{34})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)} \end{array}\right] T_{30} \quad (84)$$

where $-(b''_{28})^{(5)}(G_{31},t)$ , $-(b''_{29})^{(5)}(G_{31},t)$ , $-(b''_{30})^{(5)}(G_{31},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,)}(G_{27},t)$ are second detrition coefficients for category 1,2 and 3

$-(b''_{32})^{(6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6)}(G_{35},t)$ are third detrition coefficients for category 1,2 and 3

$-(b''_{13})^{(1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1)}(G,t)$ , $-(b''_{15})^{(1,1,1,1,1,)}(G,t)$ are fourth detrition coefficients for category 1,2, and 3

$-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1,2, and 3

$-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1,2, and 3

$-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2, and 3

$-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1,2, and 3

$-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[\begin{array}{c} (a'_{32})^{(6)}\left[+(a''_{32})^{(6)}(T_{33},t)\right]\left[+(a''_{28})^{(5,5,5)}(T_{29},t)\right]\left[+(a''_{24})^{(4,4,4,)}(T_{25},t)\right] \\ \left[+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)\right]\left[+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)\right]\left[+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)\right] \\ \left[+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)\right]\left[+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)\right]\left[+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)\right] \end{array}\right] G_{32} \quad (85)$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[\begin{array}{c} (a'_{33})^{(6)}\left[+(a''_{33})^{(6)}(T_{33},t)\right]\left[+(a''_{29})^{(5,5,5)}(T_{29},t)\right]\left[+(a''_{25})^{(4,4,4,)}(T_{25},t)\right] \\ \left[+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)\right]\left[+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)\right]\left[+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)\right] \\ \left[+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)\right]\left[+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)\right]\left[+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)\right] \end{array}\right] G_{33} \quad (86)$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[\begin{array}{c} (a'_{34})^{(6)}\left[+(a''_{34})^{(6)}(T_{33},t)\right]\left[+(a''_{30})^{(5,5,5)}(T_{29},t)\right]\left[+(a''_{26})^{(4,4,4,)}(T_{25},t)\right] \\ \left[+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)\right]\left[+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)\right]\left[+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)\right] \\ \left[+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)\right]\left[+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)\right]\left[+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)\right] \end{array}\right] G_{34} \quad (87)$$

$+(a''_{32})^{(6)}(T_{33},t)$, $+(a''_{33})^{(6)}(T_{33},t)$, $+(a''_{34})^{(6)}(T_{33},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{28})^{(5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5)}(T_{29},t)$ are second augmentation coefficients for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,)}(T_{25},t)$ are third augmentation coefficients for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)$ - are fourth augmentation coefficients

$+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)$ - fifth augmentation

coefficients
$+(a_{20}'')^{(3,3,3,3,3,3)}(T_{21},t)$, $+(a_{21}'')^{(3,3,3,3,3,3)}(T_{21},t)$, $+(a_{22}'')^{(3,3,3,3,3,3)}(T_{21},t)$ sixth augmentation coefficients
$+(a_{36}'')^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a_{37}'')^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a_{38}'')^{(7,7,7,7,7,7,7)}(T_{37},t)$
seventh augmentation coefficients
$+(a_{40}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a_{41}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a_{42}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t)$
Eighth augmentation coefficients
$+(a_{44}'')^{(9,9,9,9,9,9,9)}(T_{45},t)$, $+(a_{45}'')^{(9,9,9,9,9,9,9)}(T_{45},t)$, $+(a_{46}'')^{(9,9,9,9,9,9,9)}(T_{45},t)$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \begin{bmatrix} (b_{32}')^{(6)} & -(b_{32}'')^{(6)}(G_{35},t) & -(b_{28}'')^{(5,5,5)}(G_{31},t) & -(b_{24}'')^{(4,4,4,)}(G_{27},t) \\ -(b_{13}'')^{(1,1,1,1,1,1)}(G,t) & -(b_{16}'')^{(2,2,2,2,2,2)}(G_{19},t) & -(b_{20}'')^{(3,3,3,3,3,3)}(G_{23},t) & \\ -(b_{36}'')^{(7,7,7,7,7,7,7)}(G_{39},t) & -(b_{40}'')^{(8,8,8,8,8,8,8)}(G_{43},t) & -(b_{44}'')^{(9,9,9,9,9,9)}(G_{47},t) & \end{bmatrix} T_{32} \quad 88$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \begin{bmatrix} (b_{33}')^{(6)} & -(b_{33}'')^{(6)}(G_{35},t) & -(b_{29}'')^{(5,5,5)}(G_{31},t) & -(b_{25}'')^{(4,4,4,)}(G_{27},t) \\ -(b_{14}'')^{(1,1,1,1,1,1)}(G,t) & -(b_{17}'')^{(2,2,2,2,2,2)}(G_{19},t) & -(b_{21}'')^{(3,3,3,3,3,3)}(G_{23},t) & \\ -(b_{37}'')^{(7,7,7,7,7,7,7)}(G_{39},t) & -(b_{41}'')^{(8,8,8,8,8,8,8)}(G_{43},t) & -(b_{45}'')^{(9,9,9,9,9,9)}(G_{47},t) & \end{bmatrix} T_{33} \quad 89$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b_{34}')^{(6)} & -(b_{34}'')^{(6)}(G_{35},t) & -(b_{30}'')^{(5,5,5)}(G_{31},t) & -(b_{26}'')^{(4,4,4,)}(G_{27},t) \\ -(b_{15}'')^{(1,1,1,1,1,1)}(G,t) & -(b_{18}'')^{(2,2,2,2,2,2)}(G_{19},t) & -(b_{22}'')^{(3,3,3,3,3,3)}(G_{23},t) & \\ -(b_{38}'')^{(7,7,7,7,7,7,7)}(G_{39},t) & -(b_{42}'')^{(8,8,8,8,8,8,8)}(G_{43},t) & -(b_{46}'')^{(9,9,9,9,9,9)}(G_{47},t) & \end{bmatrix} T_{34} \quad 90$$

$-(b_{32}'')^{(6)}(G_{35},t)$, $-(b_{33}'')^{(6)}(G_{35},t)$, $-(b_{34}'')^{(6)}(G_{35},t)$ *are first detrition coefficients for category* 1, 2 *and* 3
$-(b_{28}'')^{(5,5,5)}(G_{31},t)$, $-(b_{29}'')^{(5,5,5)}(G_{31},t)$, $-(b_{30}'')^{(5,5,5)}(G_{31},t)$ *are second detrition coefficients for category* 1, 2 *and* 3
$-(b_{24}'')^{(4,4,4,)}(G_{27},t)$, $-(b_{25}'')^{(4,4,4,)}(G_{27},t)$, $-(b_{26}'')^{(4,4,4,)}(G_{27},t)$ *are third detrition coefficients for category* 1,2 *and* 3
$-(b_{13}'')^{(1,1,1,1,1,1)}(G,t)$, $-(b_{14}'')^{(1,1,1,1,1,1)}(G,t)$, $-(b_{15}'')^{(1,1,1,1,1,1)}(G,t)$ are fourth detrition coefficients for category 1, 2, and 3
$-(b_{16}'')^{(2,2,2,2,2,2)}(G_{19},t)$, $-(b_{17}'')^{(2,2,2,2,2,2)}(G_{19},t)$, $-(b_{18}'')^{(2,2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1, 2, and 3
$-(b_{20}'')^{(3,3,3,3,3,3)}(G_{23},t)$, $-(b_{21}'')^{(3,3,3,3,3,3)}(G_{23},t)$, $-(b_{22}'')^{(3,3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1, 2, and 3
$-(b_{36}'')^{(7,7,7,7,7,7,7)}(G_{39},t)$, $-(b_{37}'')^{(7,7,7,7,7,7,7)}(G_{39},t)$, $-(b_{38}'')^{(7,7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2, and 3
$-(b_{40}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)$, $-(b_{41}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)$, $-(b_{42}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)$
are eighth detrition coefficients for category 1, 2, and 3
$-(b_{46}'')^{(9,9,9,9,9,9)}(G_{47},t)$, $-(b_{45}'')^{(9,9,9,9,9,9)}(G_{47},t)$, $-(b_{44}'')^{(9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \qquad 91$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a'_{36})^{(7)}\boxed{+(a''_{36})^{(7)}(T_{37},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \qquad 92$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a'_{37})^{(7)}\boxed{+(a''_{37})^{(7)}(T_{37},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{38}}{dt} \qquad 93$$

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a'_{38})^{(7)}\boxed{+(a''_{38})^{(7)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{(a''_{36})^{(7)}(T_{37},t)}$, $\boxed{(a''_{37})^{(7)}(T_{37},t)}$, $\boxed{(a''_{38})^{(7)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{36}}{dt} = \qquad 94$$

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b'_{36})^{(7)}\boxed{-(b''_{36})^{(7)}(G_{39},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{37}}{dt} =$$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b'_{37})^{(7)}\boxed{-(b''_{37})^{(7)}(G_{39},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{38}}{dt} =$$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b'_{38})^{(7)}\boxed{-(b''_{38})^{(7)}(G_{39},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{40}}{dt}$$



$$= (a_{40})^{(8)}G_{41} - \begin{bmatrix} (a'_{40})^{(8)}\boxed{+(a''_{40})^{(8)}(T_{41},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{41}}{dt}$$

$$= (a_{41})^{(8)}G_{40} - \begin{bmatrix} (a'_{41})^{(8)}\boxed{+(a''_{41})^{(8)}(T_{41},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \begin{bmatrix} (a'_{42})^{(8)}\boxed{+(a''_{42})^{(8)}(T_{41},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a''_{40})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)}\boxed{-(b''_{40})^{(8)}(G_{43},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)}\boxed{-(b''_{41})^{(8)}(G_{43},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)}\boxed{-(b''_{42})^{(8)}(G_{43},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{44}}{dt} \qquad 96$$

$$= (a_{44})^{(9)}G_{45} - \left[\begin{array}{ccc} (a'_{44})^{(9)}\boxed{+(a''_{44})^{(9)}(T_{45},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} \end{array}\right] G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[\begin{array}{ccc} (a'_{45})^{(9)}\boxed{+(a''_{45})^{(9)}(T_{45},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} \end{array}\right] G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[\begin{array}{ccc} (a'_{46})^{(9)}\boxed{+(a''_{46})^{(9)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} \end{array}\right] G_{15}$$

Where $\boxed{+(a''_{44})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$+(a_{16}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)$ , $+(a_{17}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)$ , $+(a_{18}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a_{20}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)$ , $+(a_{21}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)$ , $+(a_{22}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a_{24}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)$ , $+(a_{25}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)$ , $+(a_{26}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a_{28}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)$, $+(a_{29}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)$ , $+(a_{30}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a_{32}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a_{33}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$ , $+(a_{34}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a_{13}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$ , $+(a_{14}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$ , $+(a_{15}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$ are Seventh augmentation coefficient for category 1, 2 and 3

$+(a_{38}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ $+(a_{37}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ $+(a_{36}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ are eighth augmentation coefficient for 1,2,3

$+(a_{40}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)$ ,$+(a_{42}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a_{41}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} =$$

$$(b_{44})^{(9)}T_{45} -$$

$$\begin{bmatrix} (b_{44}')^{(9)}\boxed{-(b_{44}'')^{(9)}(G_{47},t)} & \boxed{-(b_{16}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{20}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{24}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{28}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{32}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{13}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{36}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b_{45}')^{(9)}\boxed{-(b_{45}'')^{(9)}(G_{47},t)} & \boxed{-(b_{17}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{21}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{25}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{29}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{33}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{14}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{37}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b_{46}')^{(9)}\boxed{-(b_{46}'')^{(9)}(G_{47},t)} & \boxed{-(b_{18}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{22}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{26}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{30}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{34}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{15}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{38}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b_{44}'')^{(9)}(G_{47},t)}$ , $\boxed{-(b_{45}'')^{(9)}(G_{47},t)}$ , $\boxed{-(b_{46}'')^{(9)}(G_{47},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{16}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$ , $\boxed{-(b_{17}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$ , $\boxed{-(b_{18}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{20}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$ , $\boxed{-(b_{21}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$ , $\boxed{-(b_{22}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{24}'')^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{25}'')^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{26}'')^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{28}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{32}'')^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{34}'')^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{37}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{36}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a_i')^{(1)}, (a_i'')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (b_i'')^{(1)} > 0, \quad i,j = 13,14,15 \qquad 97$$

The functions $(a_i'')^{(1)}, (b_i'')^{(1)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a_i'')^{(1)}(T_{14},t) \leq (p_i)^{(1)} \leq (\hat{A}_{13})^{(1)}$$

$$(b_i'')^{(1)}(G,t) \leq (r_i)^{(1)} \leq (b_i')^{(1)} \leq (\hat{B}_{13})^{(1)}$$

$$lim_{T_2\to\infty}(a_i'')^{(1)}(T_{14},t) = (p_i)^{(1)} \qquad 98$$

$$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = (r_i)^{(1)}$$

**Definition of** $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$ :

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$$|(a_i'')^{(1)}(T_{14}',t) - (a_i'')^{(1)}(T_{14},t)| \leq (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G',t) - (b_i'')^{(1)}(G,t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}',t)$ and $(a_i'')^{(1)}(T_{14},t)$ . $(T_{14}',t)$ and $(T_{14},t)$ are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14},t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$ : 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$

**Definition of** $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$ : 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}$, $(r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)} (\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)} (\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i, j = 16,17,18$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17}, t) \leq (p_i)^{(2)} \leq (\hat{A}_{16})^{(2)} \quad 102$$

$$(b_i'')^{(2)}(G_{19}, t) \leq (r_i)^{(2)} \leq (b_i')^{(2)} \leq (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2 \to \infty} (a_i'')^{(2)}(T_{17}, t) = (p_i)^{(2)} \quad 104$$

$$\lim_{G \to \infty} (b_i'')^{(2)}((G_{19}), t) = (r_i)^{(2)} \quad 105$$

**Definition of** $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$ : 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}', t) - (a_i'')^{(2)}(T_{17}, t)| \leq (\hat{k}_{16})^{(2)} |T_{17} - T_{17}'| e^{-(\hat{M}_{16})^{(2)} t} \quad 107$$

$$|(b_i'')^{(2)}((G_{19})', t) - (b_i'')^{(2)}((G_{19}), t)| < (\hat{k}_{16})^{(2)} ||(G_{19}) - (G_{19})'|| e^{-(\hat{M}_{16})^{(2)} t} \quad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}', t)$ and$(a_i'')^{(2)}(T_{17}, t)$ . $(T_{17}', t)$ and $(T_{17}, t)$ are points belonging to the interval $[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}]$ . It is to be noted that $(a_i'')^{(2)}(T_{17}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17}, t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$ :

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)},$ are positive constants 109

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$$

**Definition of** $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$ :

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and\ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1 \quad 110$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(b_i)^{(2)} + (b_i')^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1 \quad 111$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i, j = 20,21,22 \quad 112$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(3)}$, $(r_i)^{(3)}$:

$$(a_i'')^{(3)}(T_{21}, t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23}, t) \leq (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(3)}(T_{21}, t) = (p_i)^{(3)} \quad 113$$

$$\lim_{G \to \infty}(b_i'')^{(3)}(G_{23}, t) = (r_i)^{(3)}$$

**Definition of** $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$ :

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: 114

$$|(a_i'')^{(3)}(T_{21}', t) - (a_i'')^{(3)}(T_{21}, t)| \leq (\hat{k}_{20})^{(3)} |T_{21} - T_{21}'| e^{-(\hat{M}_{20})^{(3)} t}$$

$$|(b_i'')^{(3)}(G_{23}', t) - (b_i'')^{(3)}(G_{23}, t)| < (\hat{k}_{20})^{(3)} ||G_{23} - G_{23}'|| e^{-(\hat{M}_{20})^{(3)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}', t)$ and$(a_i'')^{(3)}(T_{21}, t)$ . $(T_{21}', t)$ And $(T_{21}, t)$ are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21}, t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$ : 115

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}$ , $\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with 116
$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$ and the constants
$(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}, i = 20,21,22,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(a_i)^{(3)} + (a_i')^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)} (\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(b_i)^{(3)} + (b_i')^{(3)} + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)} (\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i, j = 24,25,26$$ 117

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(4)}$, $(r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25}, t) \leq (p_i)^{(4)} \leq (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}((G_{27}), t) \leq (r_i)^{(4)} \leq (b_i')^{(4)} \leq (\hat{B}_{24})^{(4)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(4)}(T_{25}, t) = (p_i)^{(4)}$$ 118

$$\lim_{G \to \infty}(b_i'')^{(4)}((G_{27}), t) = (r_i)^{(4)}$$

**Definition of** $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$ :

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24,25,26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}', t) - (a_i'')^{(4)}(T_{25}, t)| \leq (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})', t) - (b_i'')^{(4)}((G_{27}), t)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}', t)$ and$(a_i'')^{(4)}(T_{25}, t)$ . $(T_{25}', t)$ and $(T_{25}, t)$ are points belonging to the interval $[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}]$. It is to be noted that $(a_i'')^{(4)}(T_{25}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25}, t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$ : 120

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)},$ are positive constants

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}, \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$$

**Definition of** $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$ : 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and\ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}, i = 24,25,26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(a_i)^{(4)} + (a_i')^{(4)} + (\hat{A}_{24})^{(4)} + (\hat{P}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(b_i)^{(4)} + (b_i')^{(4)} + (\hat{B}_{24})^{(4)} + (\hat{Q}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i, j = 28,29,30$$ 122

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(5)}$, $(r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29}, t) \le (p_i)^{(5)} \le (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}((G_{31}), t) \le (r_i)^{(5)} \le (b_i')^{(5)} \le (\hat{B}_{28})^{(5)}$$

$$lim_{T_2 \to \infty} (a_i'')^{(5)} (T_{29}, t) = (p_i)^{(5)}$$ 123

$$\lim_{G \to \infty} (b_i'')^{(5)} (G_{31}, t) = (r_i)^{(5)}$$

**Definition of** $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$ :

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}', t) - (a_i'')^{(5)}(T_{29}, t)| \le (\hat{k}_{28})^{(5)} |T_{29} - T_{29}'| e^{-(\hat{M}_{28})^{(5)} t}$$

$$|(b_i'')^{(5)}((G_{31})', t) - (b_i'')^{(5)}((G_{31}), t)| < (\hat{k}_{28})^{(5)} ||(G_{31}) - (G_{31})'|| e^{-(\hat{M}_{28})^{(5)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}', t)$ and $(a_i'')^{(5)}(T_{29}, t)$ . $(T_{29}', t)$ and $(T_{29}, t)$ are points belonging to the interval $\left[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\right]$. It is to be noted that $(a_i'')^{(5)}(T_{29}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$ : 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)},$ are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}, \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

**Definition of** $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$ : 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and\ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(a_i)^{(5)} + (a_i')^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(b_i)^{(5)} + (b_i')^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i, j = 32,33,34$$ 127

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(6)}$, $(r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33}, t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}), t) \leq (r_i)^{(6)} \leq (b_i')^{(6)} \leq (\hat{B}_{32})^{(6)}$$

$$lim_{T_2 \to \infty} (a_i'')^{(6)} (T_{33}, t) = (p_i)^{(6)}$$ 128

$$\lim_{G \to \infty} (b_i'')^{(6)} ((G_{35}), t) = (r_i)^{(6)}$$

**Definition of** $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$ :

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}', t) - (a_i'')^{(6)}(T_{33}, t)| \leq (\hat{k}_{32})^{(6)} |T_{33} - T_{33}'| e^{-(\hat{M}_{32})^{(6)} t}$$

$$|(b_i'')^{(6)}((G_{35})', t) - (b_i'')^{(6)}((G_{35}), t)| < (\hat{k}_{32})^{(6)} ||(G_{35}) - (G_{35})'|| e^{-(\hat{M}_{32})^{(6)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}', t)$ and $(a_i'')^{(6)}(T_{33}, t)$ . $(T_{33}', t)$ and $(T_{33}, t)$ are points belonging to the interval $[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}]$. It is to be noted that $(a_i'')^{(6)}(T_{33}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$ : 129

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)},$ are positive constants

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$$

**Definition of** $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$ : 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and\ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32,33,34,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(a_i)^{(6)} + (a_i')^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)} (\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(b_i)^{(6)} + (b_i')^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)} (\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

(EEEEEEEE) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i, j = 36,37,38$ 131

(FFFFFFFF) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(7)}$, $(r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37}, t) \le (p_i)^{(7)} \le (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39}, t) \le (r_i)^{(7)} \le (b_i')^{(7)} \le (\hat{B}_{36})^{(7)}$$

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*(GGGGGGGG)* $\lim_{T_2 \to \infty} (a_i'')^{(7)}(T_{37}, t) = (p_i)^{(7)}$

*(HHHHHHHH)*

$\lim_{G \to \infty} (b_i'')^{(7)}((G_{39}), t) = (r_i)^{(7)}$

**Definition of** $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$ :

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$$|(a_i'')^{(7)}(T_{37}', t) - (a_i'')^{(7)}(T_{37}, t)| \le (\hat{k}_{36})^{(7)} |T_{37} - T_{37}'| e^{-(\hat{M}_{36})^{(7)} t}$$

$$|(b_i'')^{(7)}((G_{39})', t) - (b_i'')^{(7)}((G_{39}), t)| < (\hat{k}_{36})^{(7)} ||(G_{39}) - (G_{39})'|| e^{-(\hat{M}_{36})^{(7)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}', t)$ and $(a_i'')^{(7)}(T_{37}, t)$ . $(T_{37}', t)$ and $(T_{37}, t)$ are points belonging to the interval $[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}]$ . It is to be noted that $(a_i'')^{(7)}(T_{37}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37}, t)$ , the **first augmentation coefficient** attributable to the

system, would be absolutely continuous.

**Definition of** $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$ : 134

(IIIIIIII) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$, are positive constants

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$$

**Definition of** $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$ : 135

(JJJJJJJJ) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(b_i)^{(7)} + (b_i')^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40,41,42$$ 136

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

**Definition of** $(p_i)^{(8)}$, $(r_i)^{(8)}$: 137

$$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)}$$ 138

$$(b_i'')^{(8)}((G_{43}), t) \leq (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)}$$ 139

$$\lim_{T_2 \to \infty} (a_i'')^{(8)}(T_{41}, t) = (p_i)^{(8)}$$ 140

$$\lim_{G \to \infty} (b_i'')^{(8)}((G_{43}), t) = (r_i)^{(8)}$$ 141

**Definition of** $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$ :

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)} |T_{41} - T_{41}'| e^{-(\hat{M}_{40})^{(8)} t}$$ 142

$$|(b_i'')^{(8)}((G_{43})', t) - (b_i'')^{(8)}((G_{43}), t)| < (\hat{k}_{40})^{(8)} ||(G_{43}) - (G_{43})'|| e^{-(\hat{M}_{40})^{(8)} t}$$ 143

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}', t)$ and $(a_i'')^{(8)}(T_{41}, t)$ . $(T_{41}', t)$ and $(T_{41}, t)$ are points belonging to the interval $\left[(\hat{k}_{40})^{(8)}, (\widehat{M}_{40})^{(8)}\right]$. It is to be noted that $(a_i'')^{(8)}(T_{41}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{40})^{(8)} = 1$ then the function $(a_i'')^{(8)}(T_{41}, t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\widehat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$ :

$(\widehat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)},$ are positive constants

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}, \frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}} < 1 \qquad 144$$

**Definition of** $(\hat{P}_{40})^{(8)}, (\hat{Q}_{40})^{(8)}$ :

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\widehat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}, (\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)}, (a_i')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}, i = 40,41,42,$
Satisfy the inequalities

$$\frac{1}{(\widehat{M}_{40})^{(8)}}[(a_i)^{(8)} + (a_i')^{(8)} + (\hat{A}_{40})^{(8)} + (\hat{P}_{40})^{(8)} (\hat{k}_{40})^{(8)}] < 1 \qquad 145$$

$$\frac{1}{(\widehat{M}_{40})^{(8)}}[(b_i)^{(8)} + (b_i')^{(8)} + (\hat{B}_{40})^{(8)} + (\hat{Q}_{40})^{(8)} (\hat{k}_{40})^{(8)}] < 1 \qquad 146$$

Where we suppose

$$(a_i)^{(9)}, (a_i')^{(9)}, (a_i'')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (b_i'')^{(9)} > 0, \quad i, j = 44,45,46 \qquad 146\,A$$

The functions $(a_i'')^{(9)}, (b_i'')^{(9)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(9)}$, $(r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45}, t) \leq (p_i)^{(9)} \leq (\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47}, t) \leq (r_i)^{(9)} \leq (b_i')^{(9)} \leq (\hat{B}_{44})^{(9)}$$

$$\lim_{T_2 \to \infty} (a_i'')^{(9)}(T_{45}, t) = (p_i)^{(9)}$$

$$\lim_{G \to \infty} (b_i'')^{(9)}(G_{47}, t) = (r_i)^{(9)}$$

**Definition of** $(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}$ :

Where $\boxed{(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}}$ are positive constants and $\boxed{i = 44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}', t) - (a_i'')^{(9)}(T_{45}, t)| \leq (\hat{k}_{44})^{(9)} |T_{45} - T_{45}'| e^{-(\widehat{M}_{44})^{(9)} t}$$

$$|(b_i'')^{(9)}((G_{47})', t) - (b_i'')^{(9)}\big((G_{47}), t\big)| < (\hat{k}_{44})^{(9)}||(G_{47}) - (G_{47})'||e^{-(\hat{M}_{44})^{(9)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}', t)$ and$(a_i'')^{(9)}(T_{45}, t)$ . $(T_{45}', t)$ and $(T_{45}, t)$ are points belonging to the interval $\big[(\hat{k}_{44})^{(9)}, (\hat{M}_{44})^{(9)}\big]$. It is to be noted that $(a_i'')^{(9)}(T_{45}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{44})^{(9)} = 1$ then the function $(a_i'')^{(9)}(T_{45}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$ :

$(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$, are positive constants

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}} \ , \frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$$

**Definition of** $(\hat{P}_{44})^{(9)}, (\hat{Q}_{44})^{(9)}$ :

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}, (\hat{A}_{44})^{(9)} and\ (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)}, (a_i')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (p_i)^{(9)}, \ (r_i)^{(9)}, i = 44,45,46,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{44})^{(9)}}[\,(a_i)^{(9)} + (a_i')^{(9)} + \ (\hat{A}_{44})^{(9)} + \ (\hat{P}_{44})^{(9)}\,(\hat{k}_{44})^{(9)}] < 1$$

$$\frac{1}{(\hat{M}_{44})^{(9)}}[\ (b_i)^{(9)} + (b_i')^{(9)} + \ (\hat{B}_{44})^{(9)} + \ (\hat{Q}_{44})^{(9)}\ (\hat{k}_{44})^{(9)}] < 1$$

**Theorem 1:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \big(\hat{P}_{13}\big)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \ , \ \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 2 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

**Definition of** $G_i(0)\,, T_i(0)$

$$G_i(t) \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} \quad , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} \quad , \quad T_i(0) = T_i^0 > 0$$

**Theorem 3 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t) \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t} \quad , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t} \quad , \quad T_i(0) = T_i^0 > 0$$

**Theorem 4 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

**Definition of** $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \left(\hat{P}_{24}\right)^{(4)} e^{(\hat{M}_{24})^{(4)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{24}\right)^{(4)} e^{(\hat{M}_{24})^{(4)}t} \;,\quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 5 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

**Definition of** $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \left(\hat{P}_{28}\right)^{(5)} e^{(\hat{M}_{28})^{(5)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{28}\right)^{(5)} e^{(\hat{M}_{28})^{(5)}t} \;,\quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 6 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

**Definition of** $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \left(\hat{P}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t} \;,\quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 7:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

**Definition of** $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \left(\hat{P}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t} \;,\quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 8:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

**Definition of** $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \left(\hat{P}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t} \;,\quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 9:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

**Definition of** $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \left(\hat{P}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

**Proof:** Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \leq (\hat{P}_{13})^{(1)} \text{ , } T_i^0 \leq (\hat{Q}_{13})^{(1)}, \quad 155$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t} \quad 156$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t} \quad 157$$

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[ (a_{13})^{(1)} G_{14}(s_{(13)}) - \left( (a'_{13})^{(1)} + a''_{13})^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \right) G_{13}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[ (a_{14})^{(1)} G_{13}(s_{(13)}) - \left( (a'_{14})^{(1)} + (a''_{14})^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \right) G_{14}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[ (a_{15})^{(1)} G_{14}(s_{(13)}) - \left( (a'_{15})^{(1)} + (a''_{15})^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \right) G_{15}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[ (b_{13})^{(1)} T_{14}(s_{(13)}) - \left( (b'_{13})^{(1)} - (b''_{13})^{(1)} (G(s_{(13)}), s_{(13)}) \right) T_{13}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[ (b_{14})^{(1)} T_{13}(s_{(13)}) - \left( (b'_{14})^{(1)} - (b''_{14})^{(1)} (G(s_{(13)}), s_{(13)}) \right) T_{14}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{T}_{15}(t) = T_{15}^0 + \int_0^t \left[ (b_{15})^{(1)} T_{14}(s_{(13)}) - \left( (b'_{15})^{(1)} - (b''_{15})^{(1)} (G(s_{(13)}), s_{(13)}) \right) T_{15}(s_{(13)}) \right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \leq (\hat{P}_{16})^{(2)} \text{ , } T_i^0 \leq (\hat{Q}_{16})^{(2)},$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$$

By 160

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[ (a_{16})^{(2)} G_{17}(s_{(16)}) - \left( (a'_{16})^{(2)} + a''_{16})^{(2)} (T_{17}(s_{(16)}), s_{(16)}) \right) G_{16}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[ (a_{17})^{(2)} G_{16}(s_{(16)}) - \left( (a'_{17})^{(2)} + (a''_{17})^{(2)} (T_{17}(s_{(16)}), s_{(17)}) \right) G_{17}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[ (a_{18})^{(2)} G_{17}(s_{(16)}) - \left( (a'_{18})^{(2)} + (a''_{18})^{(2)} (T_{17}(s_{(16)}), s_{(16)}) \right) G_{18}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^{0} + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^{0} + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^{0} + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{18}(s_{(16)})\right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{20})^{(3)}$, $T_i^0 \le (\hat{Q}_{20})^{(3)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\bar{M}_{20})^{(3)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\bar{M}_{20})^{(3)} t}$

By 161

$$\bar{G}_{20}(t) = G_{20}^{0} + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''_{20})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^{0} + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^{0} + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{22}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^{0} + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^{0} + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{22}(t) = T_{22}^{0} + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{22}(s_{(20)})\right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{24})^{(4)}$, $T_i^0 \le (\hat{Q}_{24})^{(4)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\bar{M}_{24})^{(4)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\bar{M}_{24})^{(4)} t}$

By 162

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{26}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{\mathrm{T}}_{26}(\mathrm{t}) = \mathrm{T}_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{26}(s_{(24)})\right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i$: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$ , $T_i(0) = T_i^0$ , $G_i^0 \le (\hat{P}_{28})^{(5)}$ , $T_i^0 \le (\hat{Q}_{28})^{(5)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

By 163

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{30}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{\mathrm{T}}_{30}(\mathrm{t}) = \mathrm{T}_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{30}(s_{(28)})\right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i$: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$ , $T_i(0) = T_i^0$ , $G_i^0 \le (\hat{P}_{32})^{(6)}$ , $T_i^0 \le (\hat{Q}_{32})^{(6)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\tilde{M}_{32})^{(6)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\tilde{M}_{32})^{(6)} t}$

By 164

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[ (a_{32})^{(6)} G_{33}(s_{(32)}) - \left( (a'_{32})^{(6)} + a''_{32})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[ (a_{33})^{(6)} G_{32}(s_{(32)}) - \left( (a'_{33})^{(6)} + (a''_{33})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[ (a_{34})^{(6)} G_{33}(s_{(32)}) - \left( (a'_{34})^{(6)} + (a''_{34})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{34}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[ (b_{32})^{(6)} T_{33}(s_{(32)}) - \left( (b'_{32})^{(6)} - (b''_{32})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[ (b_{33})^{(6)} T_{32}(s_{(32)}) - \left( (b'_{33})^{(6)} - (b''_{33})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{34}(t) = T_{34}^0 + \int_0^t \left[ (b_{34})^{(6)} T_{33}(s_{(32)}) - \left( (b'_{34})^{(6)} - (b''_{34})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{34}(s_{(32)}) \right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\tilde{M}_{36})^{(7)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\tilde{M}_{36})^{(7)} t}$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[ (a_{36})^{(7)} G_{37}(s_{(36)}) - \left( (a'_{36})^{(7)} + a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[ (a_{37})^{(7)} G_{36}(s_{(36)}) - \left( (a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[ (a_{38})^{(7)} G_{37}(s_{(36)}) - \left( (a'_{38})^{(7)} + (a''_{38})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{38}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[ (b_{36})^{(7)} T_{37}(s_{(36)}) - \left( (b'_{36})^{(7)} - (b''_{36})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[ (b_{37})^{(7)} T_{36}(s_{(36)}) - \left( (b'_{37})^{(7)} - (b''_{37})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\overline{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^{0} + \int_0^t \left[ (b_{38})^{(7)} T_{37}(s_{(36)}) - \left( (b'_{38})^{(7)} - (b''_{38})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{38}(s_{(36)}) \right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \le (\hat{P}_{40})^{(8)} \text{ , } T_i^0 \le (\hat{Q}_{40})^{(8)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[ (a_{40})^{(8)} G_{41}(s_{(40)}) - \left( (a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[ (a_{41})^{(8)} G_{40}(s_{(40)}) - \left( (a'_{41})^{(8)} + (a''_{41})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[ (a_{42})^{(8)} G_{41}(s_{(40)}) - \left( (a'_{42})^{(8)} + (a''_{42})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{42}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^0 + \int_0^t \left[ (b_{40})^{(8)} T_{41}(s_{(40)}) - \left( (b'_{40})^{(8)} - (b''_{40})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^0 + \int_0^t \left[ (b_{41})^{(8)} T_{40}(s_{(40)}) - \left( (b'_{41})^{(8)} - (b''_{41})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\overline{\mathrm{T}}_{42}(\mathrm{t}) = \mathrm{T}_{42}^{0} + \int_0^t \left[ (b_{42})^{(8)} T_{41}(s_{(40)}) - \left( (b'_{42})^{(8)} - (b''_{42})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{42}(s_{(40)}) \right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** 166

Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy A

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \le (\hat{P}_{44})^{(9)} \text{ , } T_i^0 \le (\hat{Q}_{44})^{(9)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[ (a_{44})^{(9)} G_{45}(s_{(44)}) - \left( (a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45}(s_{(44)}), s_{(44)})\right) G_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}(s_{(44)}), s_{(44)})\right) G_{46}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{44}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{46}(t) = T_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{46}(s_{(44)})\right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}}\right)\right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left(e^{(\hat{M}_{13})^{(1)} t} - 1\right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\hat{M}_{13})^{(1)} t} \le \frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{P}_{13})^{(1)} + G_{14}^0\right) e^{\left(-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0}\right)} + (\hat{P}_{13})^{(1)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \le G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}}\right)\right] ds_{(16)} = \quad 169$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1\right)$$

From which it follows that 170

$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \le \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left[\left((\hat{P}_{16})^{(2)} + G_{17}^0\right) e^{\left(-\frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0}\right)} + (\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t) \le G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} \left(G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} s_{(20)}}\right)\right] ds_{(20)} =$$

$$\left(1 + (a_{20})^{(3)} t\right) G_{21}^0 + \frac{(a_{20})^{(3)} (\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left(e^{(\hat{M}_{20})^{(3)} t} - 1\right)$$

From which it follows that 172

$$(G_{20}(t)-G_{20}^{0})e^{-(\hat{M}_{20})^{(3)}t}\leq\frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left[((\hat{P}_{20})^{(3)}+G_{21}^{0})e^{\left(-\frac{(\hat{P}_{20})^{(3)}+G_{21}^{0}}{G_{21}^{0}}\right)}+(\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 173

$$G_{24}(t)\leq G_{24}^{0}+\int_{0}^{t}\left[(a_{24})^{(4)}\left(G_{25}^{0}+(\hat{P}_{24})^{(4)}e^{(\hat{M}_{24})^{(4)}s_{(24)}}\right)\right]ds_{(24)}=$$

$$\left(1+(a_{24})^{(4)}t\right)G_{25}^{0}+\frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left(e^{(\hat{M}_{24})^{(4)}t}-1\right)$$

From which it follows that 174

$$(G_{24}(t)-G_{24}^{0})e^{-(\hat{M}_{24})^{(4)}t}\leq\frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left[((\hat{P}_{24})^{(4)}+G_{25}^{0})e^{\left(-\frac{(\hat{P}_{24})^{(4)}+G_{25}^{0}}{G_{25}^{0}}\right)}+(\hat{P}_{24})^{(4)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t)\leq G_{28}^{0}+\int_{0}^{t}\left[(a_{28})^{(5)}\left(G_{29}^{0}+(\hat{P}_{28})^{(5)}e^{(\hat{M}_{28})^{(5)}s_{(28)}}\right)\right]ds_{(28)}=$$

$$\left(1+(a_{28})^{(5)}t\right)G_{29}^{0}+\frac{(a_{28})^{(5)}(\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left(e^{(\hat{M}_{28})^{(5)}t}-1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t}\leq\frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t)\leq G_{32}^{0}+\int_{0}^{t}\left[(a_{32})^{(6)}\left(G_{33}^{0}+(\hat{P}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\right)\right]ds_{(32)}=$$

$$\left(1+(a_{32})^{(6)}t\right)G_{33}^{0}+\frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left(e^{(\hat{M}_{32})^{(6)}t}-1\right)$$

From which it follows that 177

$$(G_{32}(t)-G_{32}^{0})e^{-(\hat{M}_{32})^{(6)}t}\leq\frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left[((\hat{P}_{32})^{(6)}+G_{33}^{0})e^{\left(-\frac{(\hat{P}_{32})^{(6)}+G_{33}^{0}}{G_{33}^{0}}\right)}+(\hat{P}_{32})^{(6)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}$ , $G_{26}, T_{24}, T_{25}, T_{26}$

(ff) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t) \le G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} \left(G_{37}^0 + (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} s_{(36)}}\right)\right] ds_{(36)} =$$
$$\left(1 + (a_{36})^{(7)} t\right) G_{37}^0 + \frac{(a_{36})^{(7)} (\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left(e^{(\hat{M}_{36})^{(7)} t} - 1\right)$$

From which it follows that

$$(G_{36}(t) - G_{36}^0) e^{-(\hat{M}_{36})^{(7)} t} \le \frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left[((\hat{P}_{36})^{(7)} + G_{37}^0) e^{\left(-\frac{(\hat{P}_{36})^{(7)} + G_{37}^0}{G_{37}^0}\right)} + (\hat{P}_{36})^{(7)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t) \le G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} \left(G_{41}^0 + (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} s_{(40)}}\right)\right] ds_{(40)} = \quad 180$$
$$\left(1 + (a_{40})^{(8)} t\right) G_{41}^0 + \frac{(a_{40})^{(8)} (\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}} \left(e^{(\hat{M}_{40})^{(8)} t} - 1\right)$$

From which it follows that 181

$$(G_{40}(t) - G_{40}^0) e^{-(\hat{M}_{40})^{(8)} t} \le \frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}} \left[((\hat{P}_{40})^{(8)} + G_{41}^0) e^{\left(-\frac{(\hat{P}_{40})^{(8)} + G_{41}^0}{G_{41}^0}\right)} + (\hat{P}_{40})^{(8)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 8

Analogous inequalities hold also for $G_{41}$ , $G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t) \le G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} \left(G_{45}^0 + (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} s_{(44)}}\right)\right] ds_{(44)} =$$
$$\left(1 + (a_{44})^{(9)} t\right) G_{45}^0 + \frac{(a_{44})^{(9)} (\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left(e^{(\hat{M}_{44})^{(9)} t} - 1\right)$$

From which it follows that

$$(G_{44}(t) - G_{44}^0) e^{-(\hat{M}_{44})^{(9)} t} \le \frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left[((\hat{P}_{44})^{(9)} + G_{45}^0) e^{\left(-\frac{(\hat{P}_{44})^{(9)} + G_{45}^0}{G_{45}^0}\right)} + (\hat{P}_{44})^{(9)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 9

Analogous inequalities hold also for $G_{45}$ , $G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}$ , $\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$ and to choose 182

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[(\hat{P}_{13})^{(1)}+\left((\hat{P}_{13})^{(1)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{13})^{(1)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{13})^{(1)} \quad 183$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[\left((\hat{Q}_{13})^{(1)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{13})^{(1)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{13})^{(1)}\right]\leq(\hat{Q}_{13})^{(1)} \quad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)},T^{(1)}\right),\left(G^{(2)},T^{(2)}\right)\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{13})^{(1)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{13})^{(1)}t}\}$$

Indeed if we denote

**Definition of** $\tilde{G},\tilde{T}:\left(\tilde{G},\tilde{T}\right)=\mathcal{A}^{(1)}(G,T)$

It results

$$\left|\tilde{G}_{13}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{13})^{(1)}\left|G_{14}^{(1)}-G_{14}^{(2)}\right|e^{-(\hat{M}_{13})^{(1)}s_{(13)}}e^{(\hat{M}_{13})^{(1)}s_{(13)}}ds_{(13)}+$$

$$\int_0^t\{(a'_{13})^{(1)}\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\hat{M}_{13})^{(1)}s_{(13)}}e^{-(\hat{M}_{13})^{(1)}s_{(13)}}+$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\hat{M}_{13})^{(1)}s_{(13)}}e^{(\hat{M}_{13})^{(1)}s_{(13)}}+$$

$$G_{13}^{(2)}\left|(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)-(a''_{13})^{(1)}\left(T_{14}^{(2)},s_{(13)}\right)\right|\ e^{-(\hat{M}_{13})^{(1)}s_{(13)}}e^{(\hat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|G^{(1)}-G^{(2)}\right|e^{-(\hat{M}_{13})^{(1)}t}\leq \quad 186$$
$$\frac{1}{(\hat{M}_{13})^{(1)}}\left((a_{13})^{(1)}+(a'_{13})^{(1)}+(\hat{A}_{13})^{(1)}+(\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}\right)d\left(\left(G^{(1)},T^{(1)};\ G^{(2)},T^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

**Remark 1:** The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\hat{P}_{13})^{(1)}e^{(\hat{M}_{13})^{(1)}t}\ and\ (\hat{Q}_{13})^{(1)}e^{(\hat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i = 13,14,15$ depend only on $T_{14}$ and respectively on $G(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any $t$ where $G_i(t) = 0\ and\ T_i(t) = 0$

From 19 to 24 it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(1)} - (a_i'')^{(1)}(T_{14}(s_{(13)}), s_{(13)})\} ds_{(13)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(1)} t)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2\ and\ \left((\widehat{M}_{13})^{(1)}\right)_3$ : 187

**Remark 3**: if $G_{13}$ is bounded, the same property have also $G_{14}\ and\ G_{15}$ . indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a_{14}')^{(1)} G_{14}$ and by integrating

$$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)} \left((\widehat{M}_{13})^{(1)}\right)_1 / (a_{14}')^{(1)}$$

In the same way , one can obtain

$$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)} \left((\widehat{M}_{13})^{(1)}\right)_2 / (a_{15}')^{(1)}$$

If $G_{14}\ or\ G_{15}$ is bounded, the same property follows for $G_{13}$ , $G_{15}$ and $G_{13}$ , $G_{14}$ respectively.

**Remark 4:** If $G_{13}$ *is* bounded, from below, the same property holds for $G_{14}\ and\ G_{15}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{14}$ is bounded from below. 188

**Remark 5:** If $T_{13}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}(G(t), t)) = (b_{14}')^{(1)}$ then $T_{14} \to \infty$. 189

**Definition of** $(m)^{(1)}$ and $\varepsilon_1$ :

Indeed let $t_1$ be so that for $t > t_1$

$$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t), t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\frac{2}{\varepsilon_1}$ By taking now $\varepsilon_1$ sufficiently small one sees that $T_{14}$ is unbounded.
The same property holds for $T_{15}$ if $\lim_{t\to\infty}(b_{15}'')^{(1)}(G(t), t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}$ , $\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}\ and\ (\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[(\widehat{P}_{16})^{(2)}+\left((\hat{P}_{16})^{(2)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{16})^{(2)}\right]\leq(\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

**Definition of** $\widetilde{G_{19}},\widetilde{T_{19}}$ : $\left(\widetilde{G_{19}},\widetilde{T_{19}}\right)=\mathcal{A}^{(2)}(G_{19},T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{16})^{(2)}\left|G_{17}^{(1)}-G_{17}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)}+$$

$$\int_0^t\{(a'_{16})^{(2)}\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$G_{16}^{(2)}\left|(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)-(a''_{16})^{(2)}\left(T_{17}^{(2)},s_{(16)}\right)\right|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0,t]$ 197

From the hypotheses it follows

$$\left|(G_{19})^{(1)}-(G_{19})^{(2)}\right|\mathrm{e}^{-(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}\leq$$
$$\frac{1}{(\widehat{\mathrm{M}}_{16})^{(2)}}\left((a_{16})^{(2)}+(a'_{16})^{(2)}+(\widehat{\mathrm{A}}_{16})^{(2)}+(\widehat{\mathrm{P}}_{16})^{(2)}(\hat{k}_{16})^{(2)}\right)\mathrm{d}\left(((G_{19})^{(1)},(T_{19})^{(1)};\ (G_{19})^{(2)},(T_{19})^{(2)})\right)$$

And analogous inequalities for $G_i$ and $T_i$. Taking into account the hypothesis the result follows 198

**Remark 6:** The fact that we supposed $(a''_{16})^{(2)}$ and $(b''_{16})^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{\mathrm{P}}_{16})^{(2)}\mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}$ and $(\widehat{\mathrm{Q}}_{16})^{(2)}\mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i=16,17,18$ depend only on $\mathrm{T}_{17}$ and respectively on

$(G_{19})$(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 7:** There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$ 200

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(2)} - (a_i'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})\} ds_{(16)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(2)}t)} > 0 \quad \text{for } t > 0$$

**Definition of** $\left((\widehat{M}_{16})^{(2)}\right)_1$, $\left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$: 201

**Remark 8:** if $G_{16}$ is bounded, the same property have also $G_{17}$ and $G_{18}$. indeed if

$G_{16} < (\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt} \leq \left((\widehat{M}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)} G_{17}$ and by integrating

$$G_{17} \leq \left((\widehat{M}_{16})^{(2)}\right)_2 = G_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1 / (a_{17}')^{(2)}$$

In the same way , one can obtain

$$G_{18} \leq \left((\widehat{M}_{16})^{(2)}\right)_3 = G_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2 / (a_{18}')^{(2)}$$

If $G_{17}$ or $G_{18}$ is bounded, the same property follows for $G_{16}$, $G_{18}$ and $G_{16}$, $G_{17}$ respectively.

**Remark 9:** If $G_{16}$ is bounded, from below, the same property holds for $G_{17}$ and $G_{18}$. The proof is analogous with the preceding one. An analogous property is true if $G_{17}$ is bounded from below. 202

**Remark 10:** If $T_{16}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t), t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$. 203

**Definition of** $(m)^{(2)}$ and $\varepsilon_2$ :

Indeed let $t_2$ be so that for $t > t_2$

$$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right)$, $t = \log\frac{2}{\varepsilon_2}$ By taking now $\varepsilon_2$ sufficiently small one sees that $T_{17}$ is unbounded. 205
The same property holds for $T_{18}$ if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t), t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[(\widehat{P}_{20})^{(3)}+\left((\hat{P}_{20})^{(3)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{20})^{(3)}\right]\leq(\hat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself 210

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)},(T_{23})^{(1)}),((G_{23})^{(2)},(T_{23})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

**Definition of** $\widetilde{G_{23}},\widetilde{T_{23}}:(\widetilde{(G_{23})},\widetilde{(T_{23})})=\mathcal{A}^{(3)}((G_{23}),(T_{23}))$

It results 213

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{20})^{(3)}\left|G_{21}^{(1)}-G_{21}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\,ds_{(20)}+$$

$$\int_0^t\{(a'_{20})^{(3)}\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$G_{20}^{(2)}\left|(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)},s_{(20)}\right)\right|\;e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\}ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses it follows

$$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}t}\leq \quad 214$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+(a'_{20})^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\widehat{k}_{20})^{(3)}\right)d\left(((G_{23})^{(1)},(T_{23})^{(1)};\;(G_{23})^{(2)},(T_{23})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

**Remark 11:** The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}\ and\ (\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i=20,21,22$ depend only on $\mathrm{T}_{21}$ and respectively on

$(G_{23})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 12:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 216

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(3)} - (a_i'')^{(3)}(T_{21}(s_{(20)}), s_{(20)})\} ds_{(20)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(3)} t)} > 0$ for t > 0

**Definition of** $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2\ and\ \left((\widehat{M}_{20})^{(3)}\right)_3$ : 217

**Remark 13:** if $G_{20}$ is bounded, the same property have also $G_{21}\ and\ G_{22}$ . indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a_{21}')^{(3)} G_{21}$ and by integrating

$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1 / (a_{21}')^{(3)}$

In the same way , one can obtain

$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2 / (a_{22}')^{(3)}$

If $G_{21}\ or\ G_{22}$ is bounded, the same property follows for $G_{20}$ , $G_{22}$ and $G_{20}$ , $G_{21}$ respectively.

**Remark 14:** If $G_{20}$ *is* bounded, from below, the same property holds for $G_{21}\ and\ G_{22}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{21}$ is bounded from below. 218

**Remark 15:** If $T_{20}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t), t\right)) = (b_{21}')^{(3)}$ then $T_{21} \to \infty$. 219

**Definition of** $(m)^{(3)}$ and $\varepsilon_3$ :

Indeed let $t_3$ be so that for $t > t_3$

$(b_{21})^{(3)} - (b_i'')^{(3)}\left((G_{23})(t), t\right) < \varepsilon_3, T_{20}\,(t) > (m)^{(3)}$

Then $\frac{dT_{21}}{dt} \geq (a_{21})^{(3)}(m)^{(3)} - \varepsilon_3 T_{21}$ which leads to 220

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t} = \frac{1}{2}$ it results

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_3}$ By taking now $\varepsilon_3$ sufficiently small one sees that $T_{21}$ is unbounded.
The same property holds for $T_{22}$ if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t), t\right) = (b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}$ , $\frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}} < 1$ and to choose 221

$(\widehat{P}_{24})^{(4)}$ and $(\widehat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[(\widehat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \tag{222}$$

$$\frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \tag{223}$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself (224)

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric (225)

$$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{24})^{(4)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{27}}),(\widetilde{T_{27}})$ : $\left((\widetilde{G_{27}}),(\widetilde{T_{27}})\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{24})^{(4)}\left|G_{25}^{(1)}-G_{25}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\,ds_{(24)}+$$

$$\int_0^t\{(a'_{24})^{(4)}\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$G_{24}^{(2)}\left|(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)-(a''_{24})^{(4)}\left(T_{25}^{(2)},s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\}ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}t}\leq \tag{226}$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)}+(a'_{24})^{(4)}+(\widehat{A}_{24})^{(4)}+(\widehat{P}_{24})^{(4)}(\hat{k}_{24})^{(4)}\right)d\left(((G_{27})^{(1)},(T_{27})^{(1)};\ (G_{27})^{(2)},(T_{27})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 16:** The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ *and* $(\widehat{Q}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_+$. (227)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i=24,25,26$ depend only on $T_{25}$ and respectively on

$(G_{27})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 17:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 228

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(4)} - (a_i'')^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(4)} t)} > 0$ for t > 0

**Definition of** $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2\ and\ \left((\widehat{M}_{24})^{(4)}\right)_3$ : 229

**Remark 18:** if $G_{24}$ is bounded, the same property have also $G_{25}\ and\ G_{26}$ . indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a_{25}')^{(4)} G_{25}$ and by integrating

$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1 / (a_{25}')^{(4)}$

In the same way , one can obtain

$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2 / (a_{26}')^{(4)}$

If $G_{25}\ or\ G_{26}$ is bounded, the same property follows for $G_{24}$ , $G_{26}$ and $G_{24}$ , $G_{25}$ respectively.

**Remark 19:** If $G_{24}$ *is* bounded, from below, the same property holds for $G_{25}\ and\ G_{26}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{25}$ is bounded from below. 230

**Remark 20:** If $T_{24}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}\ ((G_{27})(t), t)) = (b_{25}')^{(4)}$ then $T_{25}\ \to \infty$. 231

**Definition of** $(m)^{(4)}$ and $\varepsilon_4$ :

Indeed let $t_4$ be so that for $t\ > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t), t) < \varepsilon_4, T_{24}\,(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25}\ \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25}\ \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_4}$ By taking now $\varepsilon_4$ sufficiently small one sees that $T_{25}$ is unbounded.
The same property holds for $T_{26}$ if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t), t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for $G_{29}$ , $G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}$ , $\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)}+\left((\hat{P}_{28})^{(5)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{28})^{(5)}\right]\leq(\hat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$$d\left(((G_{31})^{(1)},(T_{31})^{(1)}),((G_{31})^{(2)},(T_{31})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\widehat{M}_{28})^{(5)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\widehat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{31}}),(\widetilde{T_{31}})$ : $\left((\widetilde{G_{31}}),(\widetilde{T_{31}})\right)=\mathcal{A}^{(5)}\left((G_{31}),(T_{31})\right)$

It results

$$|\tilde{G}_{28}^{(1)}-\tilde{G}_i^{(2)}|\leq\int_0^t(a_{28})^{(5)}|G_{29}^{(1)}-G_{29}^{(2)}|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\,ds_{(28)}+$$

$$\int_0^t\{(a'_{28})^{(5)}|G_{28}^{(1)}-G_{28}^{(2)}|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$(a''_{28})^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)|G_{28}^{(1)}-G_{28}^{(2)}|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$G_{28}^{(2)}|(a''_{28})^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)-(a''_{28})^{(5)}\left(T_{29}^{(2)},s_{(28)}\right)|\;e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\}ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on it follows

$$|(G_{31})^{(1)}-(G_{31})^{(2)}|e^{-(\widehat{M}_{28})^{(5)}t}\leq \quad 237$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)}+(a'_{28})^{(5)}+(\widehat{A}_{28})^{(5)}+(\hat{P}_{28})^{(5)}(\hat{k}_{28})^{(5)}\right)d\left(((G_{31})^{(1)},(T_{31})^{(1)};\;(G_{31})^{(2)},(T_{31})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 21:** The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\hat{P}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ *and* $(\hat{Q}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ respectively of $\mathbb{R}_+$. 238

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on $T_{29}$ and respectively on $(G_{31})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 22:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 239

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(5)} - (a_i'')^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(5)} t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2\ and\ \left((\widehat{M}_{28})^{(5)}\right)_3$ : 240

**Remark 23:** if $G_{28}$ is bounded, the same property have also $G_{29}\ and\ G_{30}$ . indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a_{29}')^{(5)} G_{29}$ and by integrating

$$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1 / (a_{29}')^{(5)}$$

In the same way , one can obtain

$$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2 / (a_{30}')^{(5)}$$

If $G_{29}\ or\ G_{30}$ is bounded, the same property follows for $G_{28}$ , $G_{30}$ and $G_{28}$ , $G_{29}$ respectively.

**Remark 24:** If $G_{28}$ *is* bounded, from below, the same property holds for $G_{29}\ and\ G_{30}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{29}$ is bounded from below. 241

**Remark 25:** If $T_{28}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}\,((G_{31})(t), t)) = (b_{29}')^{(5)}$ then $T_{29} \to \infty$. 242

**Definition of** $(m)^{(5)}$ and $\varepsilon_5$ :

Indeed let $t_5$ be so that for $t > t_5$

$$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t), t) < \varepsilon_5, T_{28}\,(t) > (m)^{(5)}$$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_5}$ By taking now $\varepsilon_5$ sufficiently small one sees that $T_{29}$ is unbounded.
The same property holds for $T_{30}$ if $\lim_{t\to\infty}(b_{30}'')^{(5)}\left((G_{31})(t), t\right) = (b_{30}')^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{33}$ , $G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}$ , $\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}} < 1$ and to choose 244

$(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[(\hat{P}_{32})^{(6)}+\left((\hat{P}_{32})^{(6)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{32})^{(6)} \tag{245}$$

$$\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[\left((\hat{Q}_{32})^{(6)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{32})^{(6)}\right]\leq(\hat{Q}_{32})^{(6)} \tag{246}$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric (247)

$$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\hat{M}_{32})^{(6)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\hat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{35}}),(\widetilde{T_{35}})$ : $\left((\widetilde{G_{35}}),(\widetilde{T_{35}})\right)=\mathcal{A}^{(6)}\left((G_{35}),(T_{35})\right)$

It results

$$|\tilde{G}_{32}^{(1)}-\tilde{G}_i^{(2)}|\leq\int_0^t(a_{32})^{(6)}|G_{33}^{(1)}-G_{33}^{(2)}|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\,ds_{(32)}+$$

$$\int_0^t\{(a'_{32})^{(6)}|G_{32}^{(1)}-G_{32}^{(2)}|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)|G_{32}^{(1)}-G_{32}^{(2)}|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$G_{32}^{(2)}|(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)-(a''_{32})^{(6)}\left(T_{33}^{(2)},s_{(32)}\right)|\;e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\}ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$|(G_{35})^{(1)}-(G_{35})^{(2)}|e^{-(\widehat{M}_{32})^{(6)}t}\leq \tag{248}$$
$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)}+(a'_{32})^{(6)}+(\widehat{A}_{32})^{(6)}+(\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}\right)d\left(((G_{35})^{(1)},(T_{35})^{(1)};\;(G_{35})^{(2)},(T_{35})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 26:** The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\hat{P}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ *and* $(\hat{Q}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ respectively of $\mathbb{R}_+$. (249)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i = 32,33,34$ depend only on $T_{33}$ and respectively on $(G_{35})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 27:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 250

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(6)} - (a_i'')^{(6)}(T_{33}(s_{(32)}), s_{(32)})\} ds_{(32)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(6)} t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{32})^{(6)}\right)_1, \left((\widehat{M}_{32})^{(6)}\right)_2\ and\ \left((\widehat{M}_{32})^{(6)}\right)_3$ : 251

**Remark 28:** if $G_{32}$ is bounded, the same property have also $G_{33}\ and\ G_{34}$ . indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a_{33}')^{(6)} G_{33}$ and by integrating

$$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)} \left((\widehat{M}_{32})^{(6)}\right)_1 / (a_{33}')^{(6)}$$

In the same way , one can obtain

$$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)} \left((\widehat{M}_{32})^{(6)}\right)_2 / (a_{34}')^{(6)}$$

If $G_{33}\ or\ G_{34}$ is bounded, the same property follows for $G_{32}$ , $G_{34}$ and $G_{32}$ , $G_{33}$ respectively.

**Remark 29:** If $G_{32}$ *is* bounded, from below, the same property holds for $G_{33}\ and\ G_{34}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{33}$ is bounded from below. 252

**Remark 30:** If $T_{32}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}\ ((G_{35})(t), t)) = (b_{33}')^{(6)}$ then $T_{33} \to \infty$. 253

**Definition of** $(m)^{(6)}$ and $\varepsilon_6$ :

Indeed let $t_6$ be so that for $t > t_6$

$$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t), t\right) < \varepsilon_6, T_{32}\,(t) > (m)^{(6)}$$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right)$, $t = log\,\frac{2}{\varepsilon_6}$ By taking now $\varepsilon_6$ sufficiently small one sees that $T_{33}$ is unbounded.
The same property holds for $T_{34}$ if $\lim_{t\to\infty}(b_{34}'')^{(6)}\left((G_{35})(t), t(t), t\right) = (b_{34}')^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{37}, G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}$ , $\frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$ and to choose

$(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[(\hat{P}_{36})^{(7)}+\left((\hat{P}_{36})^{(7)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{36})^{(7)} \tag{256}$$

$$\frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[\left((\hat{Q}_{36})^{(7)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{36})^{(7)}\right]\leq(\hat{Q}_{36})^{(7)} \tag{257}$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself
The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric (258)

$$d\left(((G_{39})^{(1)},(T_{39})^{(1)}),((G_{39})^{(2)},(T_{39})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\tilde{M}_{36})^{(7)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\tilde{M}_{36})^{(7)}t}\}$$

Indeed if we denote

**Definition of** $\widetilde{(G_{39})},\widetilde{(T_{39})}:(\widetilde{(G_{39})},\widetilde{(T_{39})})=\mathcal{A}^{(7)}((G_{39}),(T_{39}))$
It results

$$|\tilde{G}_{36}^{(1)}-\tilde{G}_i^{(2)}|\leq\int_0^t(a_{36})^{(7)}|G_{37}^{(1)}-G_{37}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\,ds_{(36)}+$$
$$\int_0^t\{(a'_{36})^{(7)}|G_{36}^{(1)}-G_{36}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$(a''_{36})^{(7)}(T_{37}^{(1)},s_{(36)})|G_{36}^{(1)}-G_{36}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$G_{36}^{(2)}|(a''_{36})^{(7)}(T_{37}^{(1)},s_{(36)})-(a''_{36})^{(7)}(T_{37}^{(2)},s_{(36)})|\;e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\}ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on it follows

$$|(G_{39})^{(1)}-(G_{39})^{(2)}|e^{-(\widehat{M}_{36})^{(7)}t}\leq \tag{259}$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}}\left((a_{36})^{(7)}+(a'_{36})^{(7)}+(\hat{A}_{36})^{(7)}+(\hat{P}_{36})^{(7)}(\hat{k}_{36})^{(7)}\right)d\left(((G_{39})^{(1)},(T_{39})^{(1)};\;(G_{39})^{(2)},(T_{39})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 31:** The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\hat{P}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ *and* $(\hat{Q}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ respectively of $\mathbb{R}_+$. (260)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(7)}$ and $(b_i'')^{(7)}, i = 36,37,38$ depend only on $T_{37}$ and respectively on $(G_{39})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 32:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 261

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(7)} - (a_i'')^{(7)}(T_{37}(s_{(36)}), s_{(36)})\} ds_{(36)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(7)} t\right)} > 0$ for t > 0

**Definition of** $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2\ and\ \left((\widehat{M}_{36})^{(7)}\right)_3$ : 262

**Remark 33:** if $G_{36}$ is bounded, the same property have also $G_{37}\ and\ G_{38}$ . indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a_{37}')^{(7)} G_{37}$ and by integrating

$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1 / (a_{37}')^{(7)}$

In the same way , one can obtain

$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2 / (a_{38}')^{(7)}$

If $G_{37}\ or\ G_{38}$ is bounded, the same property follows for $G_{36}$ , $G_{38}$ and $G_{36}$ , $G_{37}$ respectively.

**Remark 34:** If $G_{36}$ *is* bounded, from below, the same property holds for $G_{37}\ and\ G_{38}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{37}$ is bounded from below. 263

**Remark 35:** If $T_{36}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(7)}\ ((G_{39})(t), t)) = (b_{37}')^{(7)}$ then $T_{37} \to \infty$. 264

**Definition of** $(m)^{(7)}$ and $\varepsilon_7$ :

Indeed let $t_7$ be so that for $t > t_7$

$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t), t) < \varepsilon_7, T_{36}\,(t) > (m)^{(7)}$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_7}$ By taking now $\varepsilon_7$ sufficiently small one sees that $T_{37}$ is unbounded.
The same property holds for $T_{38}$ if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t), t\right) = (b_{38}')^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}$ , $\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}} < 1$ and to choose $(\widehat{P}_{40})^{(8)}$ and $(\widehat{Q}_{40})^{(8)}$ large to have 266

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[(\widehat{P}_{40})^{(8)} + \left((\hat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{40})^{(8)} \quad 267$$

$$\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{40})^{(8)}\right] \leq (\hat{Q}_{40})^{(8)} \quad 268$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)}, (T_{43})^{(1)}), ((G_{43})^{(2)}, (T_{43})^{(2)})\right) = \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

**Definition of** $\widetilde{(G_{43})}, \widetilde{(T_{43})}$ : $\left(\widetilde{(G_{43})}, \widetilde{(T_{43})}\right) = \mathcal{A}^{(8)}((G_{43}), (T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{40})^{(8)}\left|G_{41}^{(1)} - G_{41}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\,ds_{(40)} +$$
$$\int_0^t\{(a'_{40})^{(8)}\left|G_{40}^{(1)} - G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{-(\widehat{M}_{40})^{(8)}s_{(40)}} +$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right)\left|G_{40}^{(1)} - G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}} +$$
$$G_{40}^{(2)}\left|(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right) - (a''_{40})^{(8)}\left(T_{41}^{(2)}, s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\}ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0, t]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)} - (G_{43})^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}t} \leq \quad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)} + (a'_{40})^{(8)} + (\widehat{A}_{40})^{(8)} + (\widehat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}\right)d\left(((G_{43})^{(1)}, (T_{43})^{(1)};\ (G_{43})^{(2)}, (T_{43})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

**Remark 36:** The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate 274

condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ *and* $(\widehat{Q}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i = 40{,}41{,}42$ depend only on $T_{41}$ and respectively on $(G_{43})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 37** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 275

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t\{(a_i')^{(8)}-(a_i'')^{(8)}(T_{41}(s_{(40)}),s_{(40)})\}ds_{(40)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(8)}t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{40})^{(8)}\right)_1, \left((\widehat{M}_{40})^{(8)}\right)_2\ and\ \left((\widehat{M}_{40})^{(8)}\right)_3$ : 276

**Remark 38:** if $G_{40}$ is bounded, the same property have also $G_{41}\ and\ G_{42}$ . indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a_{41}')^{(8)}G_{41}$ and by integrating

$$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1/(a_{41}')^{(8)}$$

In the same way , one can obtain

$$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2/(a_{42}')^{(8)}$$

If $G_{41}\ or\ G_{42}$ is bounded, the same property follows for $G_{40}$ , $G_{42}$ and $G_{40}$ , $G_{41}$ respectively.

**Remark 39:** If $G_{40}$ *is* bounded, from below, the same property holds for $G_{41}\ and\ G_{42}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{41}$ is bounded from below. 277

**Remark 40:** If $T_{40}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}\ ((G_{43})(t),t)) = (b_{41}')^{(8)}$ then $T_{41} \to \infty$. 278

**Definition of** $(m)^{(8)}$ and $\varepsilon_8$ :

Indeed let $t_8$ be so that for $t\ > t_8$

$$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t),t\right) < \varepsilon_8, T_{40}\,(t) > (m)^{(8)}$$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right)$, $t = log\frac{2}{\varepsilon_8}$ By taking now $\varepsilon_8$ sufficiently small one sees that $T_{41}$ is unbounded.
The same property holds for $T_{42}$ if $\lim_{t\to\infty}(b''_{42})^{(8)}\left((G_{43})(t), t(t), t\right) = (b'_{42})^{(8)}$



It is now sufficient to take $\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}$ , $\frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$ and to choose $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ large to have

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}\left[(\hat{P}_{44})^{(9)} + \left((\hat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}}\left[\left((\hat{Q}_{44})^{(9)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{44})^{(9)}\right] \leq (\hat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)}, (T_{47})^{(1)}), ((G_{47})^{(2)}, (T_{47})^{(2)})\right) =$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{44})^{(9)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{47}}), (\widetilde{T_{47}}) : \left((\widetilde{G_{47}}), (\widetilde{T_{47}})\right) = \mathcal{A}^{(9)}\left((G_{47}), (T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{44})^{(9)}\left|G_{45}^{(1)} - G_{45}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\,ds_{(44)} +$$
$$\int_0^t\{(a'_{44})^{(9)}\left|G_{44}^{(1)} - G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{-(\widehat{M}_{44})^{(9)}s_{(44)}} +$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right)\left|G_{44}^{(1)} - G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}} +$$
$$G_{44}^{(2)}\left|(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right) - (a''_{44})^{(9)}\left(T_{45}^{(2)}, s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\}ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)} - G^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}t} \leq$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)} + (a'_{44})^{(9)} + (\widehat{A}_{44})^{(9)} + (\widehat{P}_{44})^{(9)}(\hat{k}_{44})^{(9)}\right)d\left(((G_{47})^{(1)}, (T_{47})^{(1)};\ (G_{47})^{(2)}, (T_{47})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis (39,35,36) the result follows

**Remark 41:** The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by

$(\widehat{P}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ *and* $(\widehat{Q}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i = 44{,}45{,}46$ depend only on $T_{45}$ and respectively on $(G_{47})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 42:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$

From 99 to 44 it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t\{(a_i')^{(9)}-(a_i'')^{(9)}(T_{45}(s_{(44)}),s_{(44)})\}ds_{(44)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(9)}t)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{44})^{(9)}\right)_1, \left((\widehat{M}_{44})^{(9)}\right)_2$ *and* $\left((\widehat{M}_{44})^{(9)}\right)_3$ :

**Remark 43:** if $G_{44}$ is bounded, the same property have also $G_{45}\ and\ G_{46}$ . indeed if

$G_{44} < (\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt} \leq \left((\widehat{M}_{44})^{(9)}\right)_1 - (a_{45}')^{(9)}G_{45}$ and by integrating

$$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_1/(a_{45}')^{(9)}$$

In the same way , one can obtain

$$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_2/(a_{46}')^{(9)}$$

If $G_{45}\ or\ G_{46}$ is bounded, the same property follows for $G_{44}$ , $G_{46}$ and $G_{44}$ , $G_{45}$ respectively.

**Remark 44:** If $G_{44}$ *is* bounded, from below, the same property holds for $G_{45}\ and\ G_{46}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{45}$ is bounded from below.

**Remark 45:** If $T_{44}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t),t\right) = (b_{45}')^{(9)}$ then $T_{45} \to \infty$.

**Definition of** $(m)^{(9)}$ and $\varepsilon_9$ :

Indeed let $t_9$ be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t),t\right) < \varepsilon_9, T_{44}\,(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right), \quad t = log\frac{2}{\varepsilon_9}$ By taking now $\varepsilon_9$ sufficiently small one sees that $T_{45}$ is unbounded.
The same property holds for $T_{46}$ if $\lim_{t\to\infty}(b_{46}'')^{(9)}\left((G_{47})(t),t\right) = (b_{46}')^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

**Behavior of the solutions of equation** 280

**Theorem** If we denote and define

**Definition of** $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ :

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)} \le -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14}, t) + (a''_{14})^{(1)}(T_{14}, t) \le -(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)} \le -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G, t) - (b''_{14})^{(1)}(G, t) \le -(\tau_1)^{(1)}$

**Definition of** $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$ : 281

By $(\nu_1)^{(1)} > 0$, $(\nu_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$, $(u_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

**Definition of** $(\bar{\nu}_1)^{(1)},, (\bar{\nu}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$ : 282

By $(\bar{\nu}_1)^{(1)} > 0$, $(\bar{\nu}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$, $(\bar{u}_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

**Definition of** $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (\nu_0)^{(1)}$ :- 283

If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by
$(m_2)^{(1)} = (\nu_0)^{(1)}, (m_1)^{(1)} = (\nu_1)^{(1)}$, $if\ (\nu_0)^{(1)} < (\nu_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}, if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)}$,

and $\boxed{(\nu_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)}$, $if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)}$, $if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)}$,

and $\boxed{(u_0)^{(1)} = \frac{T^0_{13}}{T^0_{14}}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$

are defined

Then the solution of global equations satisfies the inequalities 285

$G^0_{13}e^{\left((S_1)^{(1)} - (p_{13})^{(1)}\right)t} \le G_{13}(t) \le G^0_{13}e^{(S_1)^{(1)}t}$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)}-(p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)}t}$$

$$\left(\frac{(a_{15})^{(1)}G_{13}^0}{(m_1)^{(1)}((S_1)^{(1)}-(p_{13})^{(1)}-(S_2)^{(1)})}\left[e^{((S_1)^{(1)}-(p_{13})^{(1)})t} - e^{-(S_2)^{(1)}t}\right] + G_{15}^0 e^{-(S_2)^{(1)}t} \le G_{15}(t) \le \right. \quad (286)$$

$$\left. \frac{(a_{15})^{(1)}G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)}-(a'_{15})^{(1)})}[e^{(S_1)^{(1)}t} - e^{-(a'_{15})^{(1)}t}] + G_{15}^0 e^{-(a'_{15})^{(1)}t}\right)$$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)}t} \le T_{13}(t) \le T_{13}^0 e^{((R_1)^{(1)}+(r_{13})^{(1)})t}} \quad (287)$$

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)}t} \le T_{13}(t) \le \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)}+(r_{13})^{(1)})t} \quad (288)$$

$$\frac{(b_{15})^{(1)}T_{13}^0}{(\mu_1)^{(1)}((R_1)^{(1)}-(b'_{15})^{(1)})}\left[e^{(R_1)^{(1)}t} - e^{-(b'_{15})^{(1)}t}\right] + T_{15}^0 e^{-(b'_{15})^{(1)}t} \le T_{15}(t) \le \quad (289)$$

$$\frac{(a_{15})^{(1)}T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)}+(r_{13})^{(1)}+(R_2)^{(1)})}\left[e^{((R_1)^{(1)}+(r_{13})^{(1)})t} - e^{-(R_2)^{(1)}t}\right] + T_{15}^0 e^{-(R_2)^{(1)}t}$$

**Definition of** $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:- (290)

Where $(S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$

$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$

$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$

$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$

**Behavior of the solutions of equation** (291)

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ : (292)

$(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ four constants satisfying

$$-(\sigma_2)^{(2)} \le -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(T_{17}, t) + (a''_{17})^{(2)}(T_{17}, t) \le -(\sigma_1)^{(2)} \quad (293)$$

$$-(\tau_2)^{(2)} \le -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}((G_{19}), t) - (b''_{17})^{(2)}((G_{19}), t) \le -(\tau_1)^{(2)} \quad (294)$$

**Definition of** $(\nu_1)^{(2)}, (\nu_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$ : (295)

By $(\nu_1)^{(2)} > 0$, $(\nu_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0$, $(u_2)^{(2)} < 0$ the roots (296)

of the equations $(a_{17})^{(2)}(\nu^{(2)})^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ (297)

and $(b_{14})^{(2)}(u^{(2)})^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and (298)

**Definition of** $(\bar{\nu}_1)^{(2)}, , (\bar{\nu}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$ : (299)

By $(\bar{\nu}_1)^{(2)} > 0$, $(\bar{\nu}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0$, $(\bar{u}_2)^{(2)} < 0$ the (300)

roots of the equations $(a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 301

and $(b_{17})^{(2)}\left(u^{(2)}\right)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 302

**Definition of** $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ :- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, ***if*** $(u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$, ***if*** $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)}$, ***if*** $(\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)}t}$

$(p_i)^{(2)}$ is defined by equation

$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)}t}$ 311

$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)}t}\right] + G_{18}^0 e^{-(S_2)^{(2)}t} \le G_{18}(t) \le\right.$ 312

$\left.\frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)}t} - e^{-(a'_{18})^{(2)}t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)}t}\right)$

$\boxed{T_{16}^0 e^{(R_1)^{(2)}t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}}$ 313

$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)}t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}$ 314

$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)}t} - e^{-(b'_{18})^{(2)}t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)}t} \le T_{18}(t) \le$ 315

$$\frac{(a_{18})^{(2)}T_{16}^{0}}{(\mu_2)^{(2)}((R_1)^{(2)}+(r_{16})^{(2)}+(R_2)^{(2)})}\left[e^{((R_1)^{(2)}+(r_{16})^{(2)})t}-e^{-(R_2)^{(2)}t}\right]+T_{18}^{0}e^{-(R_2)^{(2)}t}$$

**Definition of** $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$$

$$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)}$$ 318

$$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$$

**Behavior of the solutions** 319

**Theorem 3:** If we denote and define

**Definition of** $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ :

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}\big((G_{23}), t\big) \le -(\tau_1)^{(3)}$$

**Definition of** $(v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$ : 320

By $(v_1)^{(3)} > 0$, $(v_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{v}_1)^{(3)} > 0$, $(\bar{v}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

**Definition of** $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ :- 321

If we define $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (v_0)^{(3)}, (m_1)^{(3)} = (v_1)^{(3)}$, ***if*** $(v_0)^{(3)} < (v_1)^{(3)}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (\bar{v}_1)^{(3)}$, ***if*** $(v_1)^{(3)} < (v_0)^{(3)} < (\bar{v}_1)^{(3)}$,

and $\boxed{(v_0)^{(3)} = \frac{G_{20}^{0}}{G_{21}^{0}}}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_0)^{(3)}$, ***if*** $(\bar{v}_1)^{(3)} < (v_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)},\ \boldsymbol{if}\ (u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}, \boldsymbol{if}\ (u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)},$ and $\boxed{(u_0)^{(3)} = \frac{T_{20}^0}{T_{21}^0}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)},\ \boldsymbol{if}\ (\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G_{20}^0 e^{(S_1)^{(3)} t}$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)} t} \quad 323$$

$$\left(\frac{(a_{22})^{(3)} G_{20}^0}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\left[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)} t}\right] + G_{22}^0 e^{-(S_2)^{(3)} t} \le G_{22}(t) \le \right. \quad 324$$

$$\left. \frac{(a_{22})^{(3)} G_{20}^0}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}\left[e^{(S_1)^{(3)} t} - e^{-(a'_{22})^{(3)} t}\right] + G_{22}^0 e^{-(a'_{22})^{(3)} t}\right)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t}} \quad 325$$

$$\frac{1}{(\mu_1)^{(3)}} T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}} T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)} T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)} - (b'_{22})^{(3)})}\left[e^{(R_1)^{(3)} t} - e^{-(b'_{22})^{(3)} t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)} t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)} T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)} + (r_{20})^{(3)} + (R_2)^{(3)})}\left[e^{((R_1)^{(3)} + (r_{20})^{(3)})t} - e^{-(R_2)^{(3)} t}\right] + T_{22}^0 e^{-(R_2)^{(3)} t}$$

**Definition of** $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

**Behavior of the solutions of equation**

**Theorem:** If we denote and define

**Definition of** $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ :

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$

$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}((G_{27}), t) - (b''_{25})^{(4)}((G_{27}), t) \le -(\tau_1)^{(4)}$

**Definition of** $(v_1)^{(4)}, (v_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, v^{(4)}, u^{(4)}$ : 329

By $(v_1)^{(4)} > 0$ , $(v_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$ , $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_1)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

**Definition of** $(\bar{v}_1)^{(4)},, (\bar{v}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$ : 330

By $(\bar{v}_1)^{(4)} > 0$ , $(\bar{v}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$ , $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

**Definition of** $(m_1)^{(4)}$ , $(m_2)^{(4)}$ , $(\mu_1)^{(4)}, (\mu_2)^{(4)}, (\nu_0)^{(4)}$ :-

If we define $(m_1)^{(4)}$ , $(m_2)^{(4)}$ , $(\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (v_0)^{(4)}, (m_1)^{(4)} = (v_1)^{(4)}$, ***if*** $(v_0)^{(4)} < (v_1)^{(4)}$

$(m_2)^{(4)} = (v_1)^{(4)}, (m_1)^{(4)} = (\bar{v}_1)^{(4)}$ , ***if*** $(v_4)^{(4)} < (v_0)^{(4)} < (\bar{v}_1)^{(4)}$,
and $\boxed{(v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (v_4)^{(4)}, (m_1)^{(4)} = (v_0)^{(4)}$, ***if*** $(\bar{v}_4)^{(4)} < (v_0)^{(4)}$

and analogously

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$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)}$, ***if*** $(u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}$ , ***if*** $(u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)}$,
and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}$, ***if*** $(\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

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$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)}t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)}t}$$

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$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)}t}\right] + G_{26}^0 e^{-(S_2)^{(4)}t} \le G_{26}(t) \le\right.$$ 334

*(a26)4G240(m2)4(S1)4−(a26′)4e(S1)4t−e−(a26′)4t+ G260e−(a26′)4t*

$$\boxed{T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}}T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}}T_{24}^0 e^{((R_1)^{(4)}+(r_{24})^{(4)})t} \quad 335$$

$$\frac{(b_{26})^{(4)}T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)}-(b'_{26})^{(4)})}\left[e^{(R_1)^{(4)}t} - e^{-(b'_{26})^{(4)}t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)}t} \le T_{26}(t) \le \quad 336$$

$$\frac{(a_{26})^{(4)}T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)}+(r_{24})^{(4)}+(R_2)^{(4)})}\left[e^{((R_1)^{(4)}+(r_{24})^{(4)})t} - e^{-(R_2)^{(4)}t}\right] + T_{26}^0 e^{-(R_2)^{(4)}t}$$

**Definition of** $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

**Behavior of the solutions of equation** 338

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ :

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

**Definition of** $(v_1)^{(5)}, (v_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, v^{(5)}, u^{(5)}$ : 339

By $(v_1)^{(5)} > 0$ , $(v_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$ , $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(v^{(5)}\big)^2 + (\sigma_1)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

**Definition of** $(\bar{v}_1)^{(5)}, , (\bar{v}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$ : 340

By $(\bar{v}_1)^{(5)} > 0$ , $(\bar{v}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$ , $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(v^{(5)}\big)^2 + (\sigma_2)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

**Definition of** $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (v_0)^{(5)}$ :-

If we define $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (v_0)^{(5)}, (m_1)^{(5)} = (v_1)^{(5)}$, ***if*** $(v_0)^{(5)} < (v_1)^{(5)}$

$(m_2)^{(5)} = (v_1)^{(5)}, (m_1)^{(5)} = (\bar{v}_1)^{(5)}$ , ***if*** $(v_1)^{(5)} < (v_0)^{(5)} < (\bar{v}_1)^{(5)}$,
and $\boxed{(v_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)},\ \boldsymbol{if}\ (u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}, \boldsymbol{if}\ (u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)},$

and $\boxed{(u_0)^{(5)} = \frac{T_{28}^0}{T_{29}^0}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}, \boldsymbol{if}\ (\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{28}(t) \le G_{28}^0 e^{(S_1)^{(5)} t}$

where $(p_i)^{(5)}$ is defined by equation

$\frac{1}{(m_5)^{(5)}} G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}} G_{28}^0 e^{(S_1)^{(5)} t}$ 343

$\left(\frac{(a_{30})^{(5)} G_{28}^0}{(m_1)^{(5)}((S_1)^{(5)} - (p_{28})^{(5)} - (S_2)^{(5)})}\left[e^{((S_1)^{(5)} - (p_{28})^{(5)})t} - e^{-(S_2)^{(5)} t}\right] + G_{30}^0 e^{-(S_2)^{(5)} t} \le G_{30}(t) \le\right.$ 344

*(a30)5G280(m2)5(S1)5−(a30′)5e(S1)5t−e−(a30′)5t+ G300e−(a30′)5t*

$\boxed{T_{28}^0 e^{(R_1)^{(5)} t} \le T_{28}(t) \le T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}}$ 345

$\frac{1}{(\mu_1)^{(5)}} T_{28}^0 e^{(R_1)^{(5)} t} \le T_{28}(t) \le \frac{1}{(\mu_2)^{(5)}} T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}$ 346

$\frac{(b_{30})^{(5)} T_{28}^0}{(\mu_1)^{(5)}((R_1)^{(5)} - (b'_{30})^{(5)})}\left[e^{(R_1)^{(5)} t} - e^{-(b'_{30})^{(5)} t}\right] + T_{30}^0 e^{-(b'_{30})^{(5)} t} \le T_{30}(t) \le$ 347

$\frac{(a_{30})^{(5)} T_{28}^0}{(\mu_2)^{(5)}((R_1)^{(5)} + (r_{28})^{(5)} + (R_2)^{(5)})}\left[e^{((R_1)^{(5)} + (r_{28})^{(5)})t} - e^{-(R_2)^{(5)} t}\right] + T_{30}^0 e^{-(R_2)^{(5)} t}$

**Definition of** $(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$:- 348

Where $(S_1)^{(5)} = (a_{28})^{(5)}(m_2)^{(5)} - (a'_{28})^{(5)}$

$(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}$

$(R_1)^{(5)} = (b_{28})^{(5)}(\mu_2)^{(5)} - (b'_{28})^{(5)}$

$(R_2)^{(5)} = (b'_{30})^{(5)} - (r_{30})^{(5)}$

**Behavior of the solutions of equation** 349

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ :

$(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)} \le -(a'_{32})^{(6)} + (a'_{33})^{(6)} - (a''_{32})^{(6)}(T_{33}, t) + (a''_{33})^{(6)}(T_{33}, t) \le -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)} \le -(b'_{32})^{(6)} + (b'_{33})^{(6)} - (b''_{32})^{(6)}\big((G_{35}), t\big) - (b''_{33})^{(6)}\big((G_{35}), t\big) \le -(\tau_1)^{(6)}$$

**Definition of** $(\nu_1)^{(6)}, (\nu_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, \nu^{(6)}, u^{(6)}$ : 350

By $(\nu_1)^{(6)} > 0$ , $(\nu_2)^{(6)} < 0$ and respectively $(u_1)^{(6)} > 0$ , $(u_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(6)}, , (\bar{\nu}_2)^{(6)}, (\bar{u}_1)^{(6)}, (\bar{u}_2)^{(6)}$ : 351

By $(\bar{\nu}_1)^{(6)} > 0$ , $(\bar{\nu}_2)^{(6)} < 0$ and respectively $(\bar{u}_1)^{(6)} > 0$ , $(\bar{u}_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$

**Definition of** $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}, (\nu_0)^{(6)}$ :-

If we define $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}$ by

$$(m_2)^{(6)} = (\nu_0)^{(6)}, (m_1)^{(6)} = (\nu_1)^{(6)}, \ \boldsymbol{if}\ (\nu_0)^{(6)} < (\nu_1)^{(6)}$$

$$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}, \boldsymbol{if}\ (\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)},$$

and $\boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}}$

$$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$$

and analogously 352

$$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}, \ \boldsymbol{if}\ (u_0)^{(6)} < (u_1)^{(6)}$$

$$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}, \boldsymbol{if}\ (u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)},$$

and $\boxed{(u_0)^{(6)} = \frac{T^0_{32}}{T^0_{33}}}$

$$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}, \boldsymbol{if}\ (\bar{u}_1)^{(6)} < (u_0)^{(6)} \text{ where } (u_1)^{(6)}, (\bar{u}_1)^{(6)}$$

Then the solution of global equations satisfies the inequalities 353

$$G^0_{32}e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{32}(t) \le G^0_{32}e^{(S_1)^{(6)}t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}}G^0_{32}e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{33}(t) \le \frac{1}{(m_2)^{(6)}}G^0_{32}e^{(S_1)^{(6)}t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)}G^0_{32}}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})}\left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)}t}\right] + G^0_{34}e^{-(S_2)^{(6)}t} \le G_{34}(t) \le \right. \quad 355$$

$$\frac{(a_{34})^{(6)}G_{32}^{0}}{(m_2)^{(6)}((S_1)^{(6)}-(a'_{34})^{(6)})}\left[e^{(S_1)^{(6)}t}-e^{-(a'_{34})^{(6)}t}\right]+\ G_{34}^{0}e^{-(a'_{34})^{(6)}t}\Big)$$

$$\boxed{T_{32}^{0}e^{(R_1)^{(6)}t}\le T_{32}(t)\le T_{32}^{0}e^{((R_1)^{(6)}+(r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}}T_{32}^{0}e^{(R_1)^{(6)}t}\le T_{32}(t)\le\frac{1}{(\mu_2)^{(6)}}T_{32}^{0}e^{((R_1)^{(6)}+(r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)}T_{32}^{0}}{(\mu_1)^{(6)}((R_1)^{(6)}-(b'_{34})^{(6)})}\left[e^{(R_1)^{(6)}t}-e^{-(b'_{34})^{(6)}t}\right]+T_{34}^{0}e^{-(b'_{34})^{(6)}t}\le T_{34}(t)\le \quad 358$$

$$\frac{(a_{34})^{(6)}T_{32}^{0}}{(\mu_2)^{(6)}((R_1)^{(6)}+(r_{32})^{(6)}+(R_2)^{(6)})}\left[e^{((R_1)^{(6)}+(r_{32})^{(6)})t}-e^{-(R_2)^{(6)}t}\right]+T_{34}^{0}e^{-(R_2)^{(6)}t}$$

**Definition of** $(S_1)^{(6)},(S_2)^{(6)},(R_1)^{(6)},(R_2)^{(6)}$:- 359

Where $(S_1)^{(6)}=(a_{32})^{(6)}(m_2)^{(6)}-(a'_{32})^{(6)}$

$(S_2)^{(6)}=(a_{34})^{(6)}-(p_{34})^{(6)}$

$(R_1)^{(6)}=(b_{32})^{(6)}(\mu_2)^{(6)}-(b'_{32})^{(6)}$

$(R_2)^{(6)}=(b'_{34})^{(6)}-(r_{34})^{(6)}$

**Behavior of the solutions of equation**

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(7)},(\sigma_2)^{(7)},(\tau_1)^{(7)},(\tau_2)^{(7)}$ :

$(\sigma_1)^{(7)},(\sigma_2)^{(7)},(\tau_1)^{(7)},(\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)}\le-(a'_{36})^{(7)}+(a'_{37})^{(7)}-(a''_{36})^{(7)}(T_{37},t)+(a''_{37})^{(7)}(T_{37},t)\le-(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)}\le-(b'_{36})^{(7)}+(b'_{37})^{(7)}-(b''_{36})^{(7)}\big((G_{39}),t\big)-(b''_{37})^{(7)}\big((G_{39}),t\big)\le-(\tau_1)^{(7)}$$

**Definition of** $(v_1)^{(7)},(v_2)^{(7)},(u_1)^{(7)},(u_2)^{(7)},v^{(7)},u^{(7)}$ : 361

By $(v_1)^{(7)}>0,(v_2)^{(7)}<0$ and respectively $(u_1)^{(7)}>0,(u_2)^{(7)}<0$ the roots of the equations $(a_{37})^{(7)}\big(v^{(7)}\big)^2+(\sigma_1)^{(7)}v^{(7)}-(a_{36})^{(7)}=0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2+(\tau_1)^{(7)}u^{(7)}-(b_{36})^{(7)}=0$ and

**Definition of** $(\bar{v}_1)^{(7)},,(\bar{v}_2)^{(7)},(\bar{u}_1)^{(7)},(\bar{u}_2)^{(7)}$ : 362

By $(\bar{v}_1)^{(7)}>0,(\bar{v}_2)^{(7)}<0$ and respectively $(\bar{u}_1)^{(7)}>0,(\bar{u}_2)^{(7)}<0$ the roots of the equations $(a_{37})^{(7)}\big(v^{(7)}\big)^2+(\sigma_2)^{(7)}v^{(7)}-(a_{36})^{(7)}=0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2+(\tau_2)^{(7)}u^{(7)}-(b_{36})^{(7)}=0$

**Definition of** $(m_1)^{(7)},(m_2)^{(7)},(\mu_1)^{(7)},(\mu_2)^{(7)},(v_0)^{(7)}$ :-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$$(m_2)^{(7)} = (\nu_0)^{(7)}, (m_1)^{(7)} = (\nu_1)^{(7)}, \ \boldsymbol{if}\ (\nu_0)^{(7)} < (\nu_1)^{(7)}$$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\bar{\nu}_1)^{(7)}, \boldsymbol{if}\ (\nu_1)^{(7)} < (\nu_0)^{(7)} < (\bar{\nu}_1)^{(7)},$$

and $\boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\nu_0)^{(7)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(7)} < (\nu_0)^{(7)}$$

and analogously 363

$$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}, \ \boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$$

and $\boxed{(u_0)^{(7)} = \frac{T_{36}^0}{T_{37}^0}}$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)} \text{ where } (u_1)^{(7)}, (\bar{u}_1)^{(7)}$$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^0 e^{(S_1)^{(7)} t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)} t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)}((S_1)^{(7)} - (p_{36})^{(7)} - (S_2)^{(7)})}\left[e^{((S_1)^{(7)} - (p_{36})^{(7)})t} - e^{-(S_2)^{(7)} t}\right] + G_{38}^0 e^{-(S_2)^{(7)} t} \le G_{38}(t) \le \right. \quad 366$$

$$\left. \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)}((S_1)^{(7)} - (a'_{38})^{(7)})}\left[e^{(S_1)^{(7)} t} - e^{-(a'_{38})^{(7)} t}\right] + G_{38}^0 e^{-(a'_{38})^{(7)} t}\right)$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t} \quad 368$$

$$\frac{(b_{38})^{(7)} T_{36}^0}{(\mu_1)^{(7)}((R_1)^{(7)} - (b'_{38})^{(7)})}\left[e^{(R_1)^{(7)} t} - e^{-(b'_{38})^{(7)} t}\right] + T_{38}^0 e^{-(b'_{38})^{(7)} t} \le T_{38}(t) \le \quad 369$$

$$\frac{(a_{38})^{(7)} T_{36}^0}{(\mu_2)^{(7)}((R_1)^{(7)} + (r_{36})^{(7)} + (R_2)^{(7)})}\left[e^{((R_1)^{(7)} + (r_{36})^{(7)})t} - e^{-(R_2)^{(7)} t}\right] + T_{38}^0 e^{-(R_2)^{(7)} t}$$

**Definition of** $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$

$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$

$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$

**Behavior of the solutions of equation** 371

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ :

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$-(\sigma_2)^{(8)} \le -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \le -(\sigma_1)^{(8)}$

$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\big((G_{43}), t\big) - (b''_{41})^{(8)}\big((G_{43}), t\big) \le -(\tau_1)^{(8)}$

**Definition of** $(\nu_1)^{(8)}, (\nu_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, \nu^{(8)}, u^{(8)}$ : 372

By $(\nu_1)^{(8)} > 0, (\nu_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0, (u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(8)}, , (\bar{\nu}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$ :

By $(\bar{\nu}_1)^{(8)} > 0, (\bar{\nu}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0, (\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

**Definition of** $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}, (\nu_0)^{(8)}$ :-

If we define $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$(m_2)^{(8)} = (\nu_0)^{(8)}, (m_1)^{(8)} = (\nu_1)^{(8)}$, ***if*** $(\nu_0)^{(8)} < (\nu_1)^{(8)}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\bar{\nu}_1)^{(8)}$, ***if*** $(\nu_1)^{(8)} < (\nu_0)^{(8)} < (\bar{\nu}_1)^{(8)}$,
and $\boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\nu_0)^{(8)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(8)} < (\nu_0)^{(8)}$

and analogously 374

$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)},\ \boldsymbol{if}\ (u_0)^{(8)} < (u_1)^{(8)}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}, \boldsymbol{if}\ (u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)},$

and $\boxed{(u_0)^{(8)} = \frac{T_{40}^0}{T_{41}^0}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}, \boldsymbol{if}\ (\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^0 e^{(S_1)^{(8)} t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}} G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}} G_{40}^0 e^{(S_1)^{(8)} t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)} G_{40}^0}{(m_1)^{(8)}((S_1)^{(8)} - (p_{40})^{(8)} - (S_2)^{(8)})}\left[e^{((S_1)^{(8)} - (p_{40})^{(8)})t} - e^{-(S_2)^{(8)} t}\right] + G_{42}^0 e^{-(S_2)^{(8)} t} \le G_{42}(t) \le \right. \quad 377$$

$$\left. \frac{(a_{42})^{(8)} G_{40}^0}{(m_2)^{(8)}((S_1)^{(8)} - (a'_{42})^{(8)})}\left[e^{(S_1)^{(8)} t} - e^{-(a'_{42})^{(8)} t}\right] + G_{42}^0 e^{-(a'_{42})^{(8)} t}\right)$$

$$\boxed{T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}} T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le \frac{1}{(\mu_2)^{(8)}} T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)} T_{40}^0}{(\mu_1)^{(8)}((R_1)^{(8)} - (b'_{42})^{(8)})}\left[e^{(R_1)^{(8)} t} - e^{-(b'_{42})^{(8)} t}\right] + T_{42}^0 e^{-(b'_{42})^{(8)} t} \le T_{42}(t) \le \quad 380$$

$$\frac{(a_{42})^{(8)} T_{40}^0}{(\mu_2)^{(8)}((R_1)^{(8)} + (r_{40})^{(8)} + (R_2)^{(8)})}\left[e^{((R_1)^{(8)} + (r_{40})^{(8)})t} - e^{-(R_2)^{(8)} t}\right] + T_{42}^0 e^{-(R_2)^{(8)} t}$$

**Definition of** $(S_1)^{(8)}, (S_2)^{(8)}, (R_1)^{(8)}, (R_2)^{(8)}$:- 381

Where $(S_1)^{(8)} = (a_{40})^{(8)}(m_2)^{(8)} - (a'_{40})^{(8)}$

$(S_2)^{(8)} = (a_{42})^{(8)} - (p_{42})^{(8)}$

$(R_1)^{(8)} = (b_{40})^{(8)}(\mu_2)^{(8)} - (b'_{40})^{(8)}$

$(R_2)^{(8)} = (b'_{42})^{(8)} - (r_{42})^{(8)}$

**Behavior of the solutions of equation 37 to 92** 382

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ :

$(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ four constants satisfying

$$-(\sigma_2)^{(9)} \le -(a'_{44})^{(9)} + (a'_{45})^{(9)} - (a''_{44})^{(9)}(T_{45}, t) + (a''_{45})^{(9)}(T_{45}, t) \le -(\sigma_1)^{(9)}$$

$$-(\tau_2)^{(9)} \le -(b'_{44})^{(9)} + (b'_{45})^{(9)} - (b''_{44})^{(9)}\big((G_{47}), t\big) - (b''_{45})^{(9)}\big((G_{47}), t\big) \le -(\tau_1)^{(9)}$$

**Definition of** $(\nu_1)^{(9)}, (\nu_2)^{(9)}, (u_1)^{(9)}, (u_2)^{(9)}, \nu^{(9)}, u^{(9)}$ :

By $(\nu_1)^{(9)} > 0$ , $(\nu_2)^{(9)} < 0$ and respectively $(u_1)^{(9)} > 0$ , $(u_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$ and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_1)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(9)}, , (\bar{\nu}_2)^{(9)}, (\bar{u}_1)^{(9)}, (\bar{u}_2)^{(9)}$ :

By $(\bar{\nu}_1)^{(9)} > 0$ , $(\bar{\nu}_2)^{(9)} < 0$ and respectively $(\bar{u}_1)^{(9)} > 0$ , $(\bar{u}_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$ and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_2)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$

**Definition of** $(m_1)^{(9)}, (m_2)^{(9)}, (\mu_1)^{(9)}, (\mu_2)^{(9)}, (\nu_0)^{(9)}$ :-

If we define $(m_1)^{(9)}, (m_2)^{(9)}, (\mu_1)^{(9)}, (\mu_2)^{(9)}$ by

$$(m_2)^{(9)} = (\nu_0)^{(9)}, (m_1)^{(9)} = (\nu_1)^{(9)}, \ \boldsymbol{if}\ (\nu_0)^{(9)} < (\nu_1)^{(9)}$$

$$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\bar{\nu}_1)^{(9)}, \boldsymbol{if}\ (\nu_1)^{(9)} < (\nu_0)^{(9)} < (\bar{\nu}_1)^{(9)},$$

and $\boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$

$$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\nu_0)^{(9)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(9)} < (\nu_0)^{(9)}$$

and analogously

$$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)}, \ \boldsymbol{if}\ (u_0)^{(9)} < (u_1)^{(9)}$$

$$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}, \boldsymbol{if}\ (u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)},$$

and $\boxed{(u_0)^{(9)} = \frac{T^0_{44}}{T^0_{45}}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}, \boldsymbol{if}\ (\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G^0_{44} e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{44}(t) \le G^0_{44} e^{(S_1)^{(9)} t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}}G_{44}^0 e^{((S_1)^{(9)}-(p_{44})^{(9)})t} \le G_{45}(t) \le \frac{1}{(m_2)^{(9)}}G_{44}^0 e^{(S_1)^{(9)}t}$$

$$\left(\frac{(a_{46})^{(9)}G_{44}^0}{(m_1)^{(9)}((S_1)^{(9)}-(p_{44})^{(9)}-(S_2)^{(9)})}\left[e^{((S_1)^{(9)}-(p_{44})^{(9)})t} - e^{-(S_2)^{(9)}t}\right] + G_{46}^0 e^{-(S_2)^{(9)}t} \le G_{46}(t) \le \right.$$

$$\left.\frac{(a_{46})^{(9)}G_{44}^0}{(m_2)^{(9)}((S_1)^{(9)}-(a'_{46})^{(9)})}[e^{(S_1)^{(9)}t} - e^{-(a'_{46})^{(9)}t}] + G_{46}^0 e^{-(a'_{46})^{(9)}t}\right)$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le T_{44}^0 e^{((R_1)^{(9)}+(r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}}T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le \frac{1}{(\mu_2)^{(9)}}T_{44}^0 e^{((R_1)^{(9)}+(r_{44})^{(9)})t}$$

$$\frac{(b_{46})^{(9)}T_{44}^0}{(\mu_1)^{(9)}((R_1)^{(9)}-(b'_{46})^{(9)})}\left[e^{(R_1)^{(9)}t} - e^{-(b'_{46})^{(9)}t}\right] + T_{46}^0 e^{-(b'_{46})^{(9)}t} \le T_{46}(t) \le$$

$$\frac{(a_{46})^{(9)}T_{44}^0}{(\mu_2)^{(9)}((R_1)^{(9)}+(r_{44})^{(9)}+(R_2)^{(9)})}\left[e^{((R_1)^{(9)}+(r_{44})^{(9)})t} - e^{-(R_2)^{(9)}t}\right] + T_{46}^0 e^{-(R_2)^{(9)}t}$$

**Definition of** $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$

**Proof :** From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14},t)\right) - (a''_{14})^{(1)}(T_{14},t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

**Definition of** $v^{(1)}$ :- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}(v^{(1)})^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)}\right) \le \frac{dv^{(1)}}{dt} \le -\left((a_{14})^{(1)}(v^{(1)})^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)}\right)$$

From which one obtains

**Definition of** $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$ :-

For $0 < \boxed{(v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}} < (v_1)^{(1)} < (\bar{v}_1)^{(1)}$

$$v^{(1)}(t) \ge \frac{(v_1)^{(1)}+(C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}((v_1)^{(1)}-(v_0)^{(1)})t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}((v_1)^{(1)}-(v_0)^{(1)})t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(v_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(v_2)^{(1)}}}$$

it follows $(\nu_0)^{(1)} \leq \nu^{(1)}(t) \leq (\nu_1)^{(1)}$

In the same manner , we get 384

$$\nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{\nu}_1)^{(1)}-(\nu_0)^{(1)}}{(\nu_0)^{(1)}-(\bar{\nu}_2)^{(1)}}}$$

From which we deduce $(\nu_0)^{(1)} \leq \nu^{(1)}(t) \leq (\bar{\nu}_1)^{(1)}$

If $0 < (\nu_1)^{(1)} < (\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0} < (\bar{\nu}_1)^{(1)}$ we find like in the previous case, 385

$$(\nu_1)^{(1)} \leq \frac{(\nu_1)^{(1)}+(C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}} \leq \nu^{(1)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\bar{\nu}_1)^{(1)}$$

If $0 < (\nu_1)^{(1)} \leq (\bar{\nu}_1)^{(1)} \leq \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$ , we obtain 386

$$(\nu_1)^{(1)} \leq \nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\nu_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(1)}(t)$ :-

$$(m_2)^{(1)} \leq \nu^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{\nu^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(1)}(t)$ :-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{13})^{(1)} = (a''_{14})^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)}G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)}T_{14}(t)$ This is an important

consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

**Proof** : From global equations we obtain 387

$$\frac{d\nu^{(2)}}{dt} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right) - (a''_{17})^{(2)}(T_{17}, t)\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

**Definition of** $\nu^{(2)}$ :- $\boxed{\nu^{(2)} = \frac{G_{16}}{G_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \le \frac{d\nu^{(2)}}{dt} \le -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

**Definition of** $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$ :-

For $0 < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \ge \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}}{1 + (C)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(C)^{(2)} = \frac{(\nu_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \le \nu^{(2)}(t) \le (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \le \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \le \nu^{(2)}(t) \le (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \le \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}}{1 + (C)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}} \le \nu^{(2)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \le (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \le (\bar{\nu}_1)^{(2)} \le (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$ , we obtain 394

$$(\nu_1)^{(2)} \le \nu^{(2)}(t) \le \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \le (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(2)}(t)$ :- 395

$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}$, $\boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$

In a completely analogous way, we obtain 396

**Definition of** $u^{(2)}(t)$ :-

$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}$, $\boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :** 397

If $(a''_{16})^{(2)} = (a''_{17})^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)} G_{17}(t)$

Analogously if $(b''_{16})^{(2)} = (b''_{17})^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)} T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

**Proof** : From global equations we obtain 398

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right) - (a''_{21})^{(3)}(T_{21}, t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

**Definition of** $\nu^{(3)}$ :- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right) \leq \frac{d\nu^{(3)}}{dt} \leq -\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right)$$

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From which one obtains

For $0 < (\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\nu_1)^{(3)} < (\bar{\nu}_1)^{(3)}$

$$\nu^{(3)}(t) \geq \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(\nu_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\nu_2)^{(3)}}}$$

it follows $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\nu_1)^{(3)}$

In the same manner , we get 401

$$\nu^{(3)}(t) \le \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{\nu}_1)^{(3)}-(\nu_0)^{(3)}}{(\nu_0)^{(3)}-(\bar{\nu}_2)^{(3)}}}$$

**Definition of** $(\bar{\nu}_1)^{(3)}$ :-

From which we deduce $(\nu_0)^{(3)} \le \nu^{(3)}(t) \le (\bar{\nu}_1)^{(3)}$

If $0 < (\nu_1)^{(3)} < (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (\bar{\nu}_1)^{(3)}$ we find like in the previous case, 402

$$(\nu_1)^{(3)} \le \frac{(\nu_1)^{(3)}+(C)^{(3)}(\nu_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}} \le \nu^{(3)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \le (\bar{\nu}_1)^{(3)}$$

If $0 < (\nu_1)^{(3)} \le (\bar{\nu}_1)^{(3)} \le (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}$ , we obtain 403

$$(\nu_1)^{(3)} \le \nu^{(3)}(t) \le \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \le (\nu_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(3)}(t)$ :-

$$(m_2)^{(3)} \le \nu^{(3)}(t) \le (m_1)^{(3)}, \quad \boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(3)}(t)$ :-

$$(\mu_2)^{(3)} \le u^{(3)}(t) \le (\mu_1)^{(3)}, \quad \boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{20}'')^{(3)} = (a_{21}'')^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)}G_{21}(t)$

Analogously if $(b_{20}'')^{(3)} = (b_{21}'')^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)}T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

**Proof** : From global equations we obtain 404

$$\frac{dv^{(4)}}{dt} = (a_{24})^{(4)} - \left((a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right) - (a''_{25})^{(4)}(T_{25},t)v^{(4)} - (a_{25})^{(4)}v^{(4)}$$

**Definition of** $v^{(4)}$ :- $\boxed{v^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)}\right) \le \frac{dv^{(4)}}{dt} \le -\left((a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_4)^{(4)}v^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$ :-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \ge \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \le \nu^{(4)}(t) \le (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \le \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \le \nu^{(4)}(t) \le (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \le \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}}{1 + (C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}} \le \nu^{(4)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \le (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \le (\bar{\nu}_1)^{(4)} \le \boxed{(\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}}}$ , we obtain 407

$$(\nu_1)^{(4)} \le \nu^{(4)}(t) \le \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \le (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(4)}(t)$ :-

$$(m_2)^{(4)} \le \nu^{(4)}(t) \le (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(4)}(t)$ :-

$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}$, $\boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{24})^{(4)} = (a''_{25})^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)} G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b''_{24})^{(4)} = (b''_{25})^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)} T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

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**Proof** : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a'_{28})^{(5)} - (a'_{29})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right) - (a''_{29})^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

**Definition of** $\nu^{(5)}$ :- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$ :-

For $0 < \boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \le \frac{(\nu_1)^{(5)}+(C)^{(5)}(\nu_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)}-(\nu_2)^{(5)}\right)t\right]}}{1+(C)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)}-(\nu_2)^{(5)}\right)t\right]}} \le \nu^{(5)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \le (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \le (\bar{\nu}_1)^{(5)} \le \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain 411

$$(\nu_1)^{(5)} \le \nu^{(5)}(t) \le \frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \le (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(5)}(t)$ :-

$(m_2)^{(5)} \le \nu^{(5)}(t) \le (m_1)^{(5)}$, $\boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$

In a completely analogous way, we obtain

**Definition of** $u^{(5)}(t)$ :-

$(\mu_2)^{(5)} \le u^{(5)}(t) \le (\mu_1)^{(5)}$, $\boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a_{28}'')^{(5)} = (a_{29}'')^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)}G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b_{28}'')^{(5)} = (b_{29}'')^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)}T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

**Proof** : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a_{32}')^{(6)} - (a_{33}')^{(6)} + (a_{32}'')^{(6)}(T_{33},t)\right) - (a_{33}'')^{(6)}(T_{33},t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

**Definition of** $\nu^{(6)}$ :- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \le \frac{d\nu^{(6)}}{dt} \le -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$ :-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \geq \frac{(\nu_1)^{(6)}+(C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_0)^{(6)}\right)t\right]}}{1+(C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)}-(\nu_0)^{(6)}}{(\nu_0)^{(6)}-(\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\nu_1)^{(6)}$

In the same manner , we get 413

$$\nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{\nu}_1)^{(6)}-(\nu_0)^{(6)}}{(\nu_0)^{(6)}-(\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\bar{\nu}_1)^{(6)}$

If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 414

$$(\nu_1)^{(6)} \leq \frac{(\nu_1)^{(6)}+(C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_2)^{(6)}\right)t\right]}}{1+(C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_2)^{(6)}\right)t\right]}} \leq \nu^{(6)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\bar{\nu}_1)^{(6)}$$

If $0 < (\nu_1)^{(6)} \leq (\bar{\nu}_1)^{(6)} \leq \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$ , we obtain 415

$$(\nu_1)^{(6)} \leq \nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(6)}(t)$ :-

$$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}, \quad \boxed{\nu^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(6)}(t)$ :-

$$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}, \quad \boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)}G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)}T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

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**Proof** : From global equations we obtain

$$\frac{dv^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right) - (a''_{37})^{(7)}(T_{37},t)v^{(7)} - (a_{37})^{(7)}v^{(7)}$$

**Definition of** $v^{(7)}$ :- $\boxed{v^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_2)^{(7)}v^{(7)} - (a_{36})^{(7)}\right) \le \frac{dv^{(7)}}{dt} \le -\left((a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_1)^{(7)}v^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

**Definition of** $(\bar{v}_1)^{(7)}, (v_0)^{(7)}$ :-

For $0 < \boxed{(v_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}} < (v_1)^{(7)} < (\bar{v}_1)^{(7)}$

$$v^{(7)}(t) \ge \frac{(v_1)^{(7)} + (C)^{(7)}(v_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_0)^{(7)}\right)t\right]}}{1 + (C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(v_1)^{(7)} - (v_0)^{(7)}}{(v_0)^{(7)} - (v_2)^{(7)}}}$$

it follows $(v_0)^{(7)} \le v^{(7)}(t) \le (v_1)^{(7)}$

In the same manner , we get 417

$$v^{(7)}(t) \le \frac{(\bar{v}_1)^{(7)} + (\bar{C})^{(7)}(\bar{v}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{v}_1)^{(7)} - (v_0)^{(7)}}{(v_0)^{(7)} - (\bar{v}_2)^{(7)}}}$$

From which we deduce $(v_0)^{(7)} \le v^{(7)}(t) \le (\bar{v}_1)^{(7)}$

If $0 < (v_1)^{(7)} < (v_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}} < (\bar{v}_1)^{(7)}$ we find like in the previous case, 418

$$(v_1)^{(7)} \le \frac{(v_1)^{(7)} + (C)^{(7)}(v_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_2)^{(7)}\right)t\right]}}{1 + (C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_2)^{(7)}\right)t\right]}} \le v^{(7)}(t) \le$$

$$\frac{(\bar{v}_1)^{(7)} + (\bar{C})^{(7)}(\bar{v}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}} \le (\bar{v}_1)^{(7)}$$

If $0 < (v_1)^{(7)} \le (\bar{v}_1)^{(7)} \le \boxed{(v_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$ , we obtain 419

$$(v_1)^{(7)} \le v^{(7)}(t) \le \frac{(\bar{v}_1)^{(7)} + (\bar{C})^{(7)}(\bar{v}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}} \le (v_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $v^{(7)}(t)$ :-

$$(m_2)^{(7)} \leq v^{(7)}(t) \leq (m_1)^{(7)}, \quad \boxed{v^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(7)}(t)$ :- 420

$$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}, \quad \boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $v^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)} G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)} T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

**Proof** : From global equations we obtain 421

$$\frac{dv^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right) - (a''_{41})^{(8)}(T_{41}, t) v^{(8)} - (a_{41})^{(8)} v^{(8)}$$

**Definition of** $v^{(8)}$ :- $\boxed{v^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_2)^{(8)} v^{(8)} - (a_{40})^{(8)}\right) \leq \frac{dv^{(8)}}{dt} \leq -\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_1)^{(8)} v^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$ :-

$$\text{For } 0 < \boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$$

$$v^{(8)}(t) \geq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)}-(\nu_0)^{(8)}}{(\nu_0)^{(8)}-(\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \leq \frac{(\nu_1)^{(8)}+(C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}}{1+(C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}} \leq \nu^{(8)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \leq (\bar{\nu}_1)^{(8)} \leq \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$ , we obtain 424

$$(\nu_1)^{(8)} \leq \nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(8)}(t)$ :-

$$(m_2)^{(8)} \leq \nu^{(8)}(t) \leq (m_1)^{(8)}, \quad \boxed{\nu^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(8)}(t)$ :-

$$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}, \quad \boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $\nu^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)}G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then $(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)}T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

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**Proof** : From 99,20,44,22,23,44 we obtain A

$$\frac{dv^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right) - (a''_{45})^{(9)}(T_{45},t)v^{(9)} - (a_{45})^{(9)}v^{(9)}$$

**Definition of** $v^{(9)}$ :- $\boxed{v^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)}\right) \le \frac{dv^{(9)}}{dt} \le -\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_1)^{(9)}v^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$ :-

For $0 < \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$

$$v^{(9)}(t) \ge \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \le v^{(9)}(t) \le (\nu_9)^{(9)}$

In the same manner , we get

$$v^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \le v^{(9)}(t) \le (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \le \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}}{1 + (C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}} \le v^{(9)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \le (\bar{\nu}_1)^{(9)} \le \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$ , we obtain

$$(\nu_1)^{(9)} \le v^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $v^{(9)}(t)$ :-

$$(m_2)^{(9)} \le v^{(9)}(t) \le (m_1)^{(9)}, \quad \boxed{v^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(9)}(t)$ :-

$(\mu_2)^{(9)} \leq u^{(9)}(t) \leq (\mu_1)^{(9)}$, $\boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)} G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then
$(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)} T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following 425

**Theorem :** If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on $t$ , and the conditions with the notations

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0$ ,

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$

$with\ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a''_i)^{(2)} and\ (b''_i)^{(2)}$ are independent on t , and the conditions with the notations 426

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$ 427

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 428

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$ , 429

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 430

$with\ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a''_i)^{(3)} and\ (b''_i)^{(3)}$ are independent on $t$ , and the conditions with the notations 431

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$ ,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

**Theorem :** If $(a_i'')^{(4)} and\ (b_i'')^{(4)}$ are independent on $t$ , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$ ,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(5)} and\ (b_i'')^{(5)}$ are independent on $t$ , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$ ,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

**Theorem** If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on $t$ , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$ ,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(7)} and\ (b_i'')^{(7)}$ are independent on $t$ , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$ ,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(8)} and\ (b_i'')^{(8)}$ are independent on $t$ , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$ ,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(9)} and\ (b_i'')^{(9)}$ are independent on $t$ , and the conditions (with the notations 436 A
45,46,27,28)

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$ ,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0$ 437

$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0$ 438

$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0$ 439

$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0$ 440

$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ]T_{14} = 0$ 441

$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0$ 442

has a unique positive solution , which is an equilibrium solution for the system

$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0$ 443

$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0$ 444

$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0$ 445

$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\ ]T_{16} = 0$ 446

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\,]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\,]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\,]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\,]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\,]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\big((G_{27})\big)\,]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\big((G_{27})\big)\,]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\big((G_{27})\big)\,]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})\,]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})\,]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})\,]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})\,]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})\,]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})\,]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0 \quad 479$$

$$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0 \quad 480$$

$$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0 \quad 481$$

$$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0 \quad 482$$

$$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0 \quad 483$$

$$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0 \quad 484$$

$$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0 \quad 484\text{A}$$

$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$

$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$

$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$

$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$

$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$

**Proof:** 485

(a) Indeed the first two equations have a nontrivial solution $G_{13}, G_{14}$ if

$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$

**Proof:** 486

(ff) Indeed the first two equations have a nontrivial solution $G_{16}, G_{17}$ if

$\mathrm{F}(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$

**Proof:** 487

(a) Indeed the first two equations have a nontrivial solution $G_{20}, G_{21}$ if

$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$

**Proof:** 488

(a) Indeed the first two equations have a nontrivial solution $G_{24}, G_{25}$ if

$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$

**Proof:** 489

(a) Indeed the first two equations have a nontrivial solution $G_{28}, G_{29}$ if

$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$

**Proof:** 490

(a) Indeed the first two equations have a nontrivial solution $G_{32}, G_{33}$ if

$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) +$

$(a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$

**Proof:** 491

(a) Indeed the first two equations have a nontrivial solution $G_{36}, G_{37}$ if

$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$

**Proof:** 492

(a) Indeed the first two equations have a nontrivial solution $G_{40}, G_{41}$ if

$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$

**Proof:** 492 A

(a) Indeed the first two equations have a nontrivial solution $G_{44}, G_{45}$ if

$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$

**Definition and uniqueness of** $T^*_{14}$ :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique $T^*_{14}$ for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

**Definition and uniqueness of** $T^*_{17}$ :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique $T^*_{17}$ for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]}$$ 495

**Definition and uniqueness of** $T^*_{21}$ :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique $T^*_{21}$ for which $f(T^*_{21}) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

**Definition and uniqueness of** $T^*_{25}$ :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that

there exists a unique $T_{25}^{*}$ for which $f(T_{25}^{*})=0$. With this value , we obtain from the three first equations

$$G_{24}=\frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}=\frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T_{25}^{*})]}$$

**Definition and uniqueness of** $T_{29}^{*}$ :- 498

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(5)}(T_{29})$ being increasing, it follows that there exists a unique $T_{29}^{*}$ for which $f(T_{29}^{*})=0$. With this value , we obtain from the three first equations

$$G_{28}=\frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}=\frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T_{29}^{*})]}$$

**Definition and uniqueness of** $T_{33}^{*}$ :- 499

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(6)}(T_{33})$ being increasing, it follows that there exists a unique $T_{33}^{*}$ for which $f(T_{33}^{*})=0$. With this value , we obtain from the three first equations

$$G_{32}=\frac{(a_{32})^{(6)}G_{33}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^{*})]} \quad , \quad G_{34}=\frac{(a_{34})^{(6)}G_{33}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^{*})]}$$

**Definition and uniqueness of** $T_{37}^{*}$ :- 500

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(7)}(T_{37})$ being increasing, it follows that there exists a unique $T_{37}^{*}$ for which $f(T_{37}^{*})=0$. With this value , we obtain from the three first equations

$$G_{36}=\frac{(a_{36})^{(7)}G_{37}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^{*})]} \quad , \quad G_{38}=\frac{(a_{38})^{(7)}G_{37}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^{*})]}$$

**Definition and uniqueness of** $T_{41}^{*}$ :- 501

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique $T_{41}^{*}$ for which $f(T_{41}^{*})=0$. With this value , we obtain from the three first equations

$$G_{40}=\frac{(a_{40})^{(8)}G_{41}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T_{41}^{*})]} \quad , \quad G_{42}=\frac{(a_{42})^{(8)}G_{41}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T_{41}^{*})]}$$

**Definition and uniqueness of** $T_{45}^{*}$ :- 501 A

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(9)}(T_{45})$ being increasing, it follows that there exists a unique $T_{45}^{*}$ for which $f(T_{45}^{*})=0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)}G_{45}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T^*_{45})]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)}G_{45}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T^*_{45})]}$$

By the same argument, the equations admit solutions $G_{13}, G_{14}$ if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right]+(b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{14}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{14}$ such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions $G_{16}, G_{17}$ if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right]+(b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ 504
is a decreasing function in $G_{17}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{14}$ such that $\varphi((G_{19})^*) = 0$

By the same argument, the equations admit solutions $G_{20}, G_{21}$ if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right]+(b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{21}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{21}$ such that $\varphi((G_{23})^*) = 0$

By the same argument, the equations admit solutions $G_{24}, G_{25}$ if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right]+(b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{25}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{25}$ such that $\varphi((G_{27})^*) = 0$

By the same argument, the equations admit solutions $G_{28}, G_{29}$ if 507

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right]+(b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{29}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that

there exists a unique $G_{29}^{*}$ such that $\varphi((G_{31})^{*}) = 0$

By the same argument, the equations admit solutions $G_{32}, G_{33}$ if 508
$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right] + (b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{33}$ taking into account the hypothesis $\varphi(0) > 0\,, \varphi(\infty) < 0$ it follows that there exists a unique $G_{33}^{*}$ such that $\varphi(G_{35}{}^{*}) = 0$

By the same argument, the equations admit solutions $G_{36}, G_{37}$ if 509
$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$
$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right] + (b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{37}$ taking into account the hypothesis $\varphi(0) > 0\,, \varphi(\infty) < 0$ it follows that there exists a unique $G_{37}^{*}$ such that $\varphi(G_{39}{}^{*}) = 0$

By the same argument, the equations admit solutions $G_{40}, G_{41}$ if 510
$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$
$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right] + (b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{41}$ taking into account the hypothesis $\varphi(0) > 0\,, \varphi(\infty) < 0$ it follows that there exists a unique $G_{41}^{*}$ such that $\varphi(G_{43}{}^{*}) = 0$

By the same argument, the equations 92,93 admit solutions $G_{44}, G_{45}$ if
$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$

$$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right] + (b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{45}$ taking into account the hypothesis $\varphi(0) > 0\,, \varphi(\infty) < 0$ it follows that there exists a unique $G_{45}^{*}$ such that $\varphi((G_{47})^{*}) = 0$

Finally we obtain the unique solution 511

$G_{14}^{*}$ given by $\varphi(G^{*}) = 0\,, T_{14}^{*}$ given by $f(T_{14}^{*}) = 0$ and

$$G_{13}^{*} = \frac{(a_{13})^{(1)}G_{14}^{*}}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^{*})]} \quad , \quad G_{15}^{*} = \frac{(a_{15})^{(1)}G_{14}^{*}}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^{*})]}$$

$$T_{13}^{*} = \frac{(b_{13})^{(1)}T_{14}^{*}}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^{*})]} \quad , \quad T_{15}^{*} = \frac{(b_{15})^{(1)}T_{14}^{*}}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^{*})]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

$G_{17}^{*}$ given by $\varphi((G_{19})^{*}) = 0$ , $T_{17}^{*}$ given by $f(T_{17}^{*}) = 0$ and 512

$$G_{16}^{*} = \frac{(a_{16})^{(2)} G_{17}^{*}}{[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}^{*})]} \quad , \quad G_{18}^{*} = \frac{(a_{18})^{(2)} G_{17}^{*}}{[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}^{*})]} \tag{513}$$

$$T_{16}^{*} = \frac{(b_{16})^{(2)} T_{17}^{*}}{[(b'_{16})^{(2)} - (b''_{16})^{(2)}((G_{19})^{*})]} \quad , \quad T_{18}^{*} = \frac{(b_{18})^{(2)} T_{17}^{*}}{[(b'_{18})^{(2)} - (b''_{18})^{(2)}((G_{19})^{*})]} \tag{514}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

$G_{21}^{*}$ given by $\varphi((G_{23})^{*}) = 0$ , $T_{21}^{*}$ given by $f(T_{21}^{*}) = 0$ and

$$G_{20}^{*} = \frac{(a_{20})^{(3)} G_{21}^{*}}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^{*})]} \quad , \quad G_{22}^{*} = \frac{(a_{22})^{(3)} G_{21}^{*}}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^{*})]}$$

$$T_{20}^{*} = \frac{(b_{20})^{(3)} T_{21}^{*}}{[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}{}^{*})]} \quad , \quad T_{22}^{*} = \frac{(b_{22})^{(3)} T_{21}^{*}}{[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}{}^{*})]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

$G_{25}^{*}$ given by $\varphi(G_{27}) = 0$ , $T_{25}^{*}$ given by $f(T_{25}^{*}) = 0$ and

$$G_{24}^{*} = \frac{(a_{24})^{(4)} G_{25}^{*}}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}^{*} = \frac{(a_{26})^{(4)} G_{25}^{*}}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^{*})]}$$

$$T_{24}^{*} = \frac{(b_{24})^{(4)} T_{25}^{*}}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^{*})]} \quad , \quad T_{26}^{*} = \frac{(b_{26})^{(4)} T_{25}^{*}}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^{*})]} \tag{517}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

$G_{29}^{*}$ given by $\varphi((G_{31})^{*}) = 0$ , $T_{29}^{*}$ given by $f(T_{29}^{*}) = 0$ and

$$G_{28}^{*} = \frac{(a_{28})^{(5)} G_{29}^{*}}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}^{*} = \frac{(a_{30})^{(5)} G_{29}^{*}}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^{*})]}$$

$$T_{28}^{*} = \frac{(b_{28})^{(5)} T_{29}^{*}}{[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31})^{*})]} \quad , \quad T_{30}^{*} = \frac{(b_{30})^{(5)} T_{29}^{*}}{[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31})^{*})]} \tag{519}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

$G_{33}^{*}$ given by $\varphi((G_{35})^{*}) = 0$ , $T_{33}^{*}$ given by $f(T_{33}^{*}) = 0$ and

$$G_{32}^{*} = \frac{(a_{32})^{(6)} G_{33}^{*}}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^{*})]} \quad , \quad G_{34}^{*} = \frac{(a_{34})^{(6)} G_{33}^{*}}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^{*})]}$$

$$T_{32}^{*} = \frac{(b_{32})^{(6)} T_{33}^{*}}{[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35})^{*})]} \quad , \quad T_{34}^{*} = \frac{(b_{34})^{(6)} T_{33}^{*}}{[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35})^{*})]} \tag{521}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

$G_{37}^*$ given by $\varphi((G_{39})^*) = 0$ , $T_{37}^*$ given by $f(T_{37}^*) = 0$ and

$$G_{36}^* = \frac{(a_{36})^{(7)} G_{37}^*}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^*)]} \quad , \quad G_{38}^* = \frac{(a_{38})^{(7)} G_{37}^*}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^*)]}$$

$$T_{36}^* = \frac{(b_{36})^{(7)} T_{37}^*}{[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39})^*)]} \quad , \quad T_{38}^* = \frac{(b_{38})^{(7)} T_{37}^*}{[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39})^*)]}$$

Finally we obtain the unique solution 523

$G_{41}^*$ given by $\varphi((G_{43})^*) = 0$ , $T_{41}^*$ given by $f(T_{41}^*) = 0$ and

$$G_{40}^* = \frac{(a_{40})^{(8)} G_{41}^*}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42}^* = \frac{(a_{42})^{(8)} G_{41}^*}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

$$T_{40}^* = \frac{(b_{40})^{(8)} T_{41}^*}{[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43})^*)]} \quad , \quad T_{42}^* = \frac{(b_{42})^{(8)} T_{41}^*}{[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43})^*)]}$$

Finally we obtain the unique solution of 89 to 99 523 A

$G_{45}^*$ given by $\varphi((G_{47})^*) = 0$ , $T_{45}^*$ given by $f(T_{45}^*) = 0$ and

$$G_{44}^* = \frac{(a_{44})^{(9)} G_{45}^*}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46}^* = \frac{(a_{46})^{(9)} G_{45}^*}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

$$T_{44}^* = \frac{(b_{44})^{(9)} T_{45}^*}{[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47})^*)]} \quad , \quad T_{46}^* = \frac{(b_{46})^{(9)} T_{45}^*}{[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47})^*)]}$$

## ASYMPTOTIC STABILITY ANALYSIS 524

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

**Proof:** Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a''_{14})^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \frac{\partial (b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a'_{13})^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)}G^*_{13}\mathbb{T}_{14} \quad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a'_{14})^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)}G^*_{14}\mathbb{T}_{14} \quad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a'_{15})^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)}G^*_{15}\mathbb{T}_{14} \quad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b'_{13})^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)}T^*_{13}\mathbb{G}_j\right) \quad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b'_{14})^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)}T^*_{14}\mathbb{G}_j\right) \quad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b'_{15})^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T^*_{15}\mathbb{G}_j\right) \quad 530$$

**ASYMPTOTIC STABILITY ANALYSIS** 531

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(2)}$ and $(b''_i)^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G^*_i + \mathbb{G}_i \quad , T_i = T^*_i + \mathbb{T}_i \quad 532$$

$$\frac{\partial(a''_{17})^{(2)}}{\partial T_{17}}(T^*_{17}) = (q_{17})^{(2)} \quad , \frac{\partial(b''_i)^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \quad 533$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a'_{16})^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G^*_{16}\mathbb{T}_{17} \quad 534$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a'_{17})^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G^*_{17}\mathbb{T}_{17} \quad 535$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a'_{18})^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G^*_{18}\mathbb{T}_{17} \quad 536$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b'_{16})^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T^*_{16}\mathbb{G}_j\right) \quad 537$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b'_{17})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}T^*_{17}\mathbb{G}_j\right) \quad 538$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b'_{18})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}T^*_{18}\mathbb{G}_j\right) \quad 539$$

**ASYMPTOTIC STABILITY ANALYSIS** 540

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(3)}$ *and* $(b''_i)^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{21}'')^{(3)}}{\partial T_{21}}(T_{21}^*) = (q_{21})^{(3)} \quad , \frac{\partial (b_i'')^{(3)}}{\partial G_j}(\,(G_{23})^*\,) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\big((a_{20}')^{(3)} + (p_{20})^{(3)}\big)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G_{20}^*\mathbb{T}_{21} \quad 541$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\big((a_{21}')^{(3)} + (p_{21})^{(3)}\big)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G_{21}^*\mathbb{T}_{21} \quad 542$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\big((a_{22}')^{(3)} + (p_{22})^{(3)}\big)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G_{22}^*\mathbb{T}_{21} \quad 543$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\big((b_{20}')^{(3)} - (r_{20})^{(3)}\big)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\big(s_{(20)(j)}T_{20}^*\mathbb{G}_j\big) \quad 544$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\big((b_{21}')^{(3)} - (r_{21})^{(3)}\big)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\big(s_{(21)(j)}T_{21}^*\mathbb{G}_j\big) \quad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\big((b_{22}')^{(3)} - (r_{22})^{(3)}\big)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\big(s_{(22)(j)}T_{22}^*\mathbb{G}_j\big) \quad 546$$

**ASYMPTOTIC STABILITY ANALYSIS** 547

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}\ and\ (b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{25}'')^{(4)}}{\partial T_{25}}(T_{25}^*) = (q_{25})^{(4)} \quad , \frac{\partial (b_i'')^{(4)}}{\partial G_j}((G_{27})^*\,) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\big((a_{24}')^{(4)} + (p_{24})^{(4)}\big)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G_{24}^*\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\big((a_{25}')^{(4)} + (p_{25})^{(4)}\big)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G_{25}^*\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\big((a_{26}')^{(4)} + (p_{26})^{(4)}\big)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G_{26}^*\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\big((b_{24}')^{(4)} - (r_{24})^{(4)}\big)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\big(s_{(24)(j)}T_{24}^*\mathbb{G}_j\big) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\big((b_{25}')^{(4)} - (r_{25})^{(4)}\big)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\big(s_{(25)(j)}T_{25}^*\mathbb{G}_j\big) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T^*_{26}\mathbb{G}_j\right) \quad 554$$

**ASYMPTOTIC STABILITY ANALYSIS** 555

**Theorem 5:** If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(5)}\ and\ (b''_i)^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 556

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{29})^{(5)}}{\partial T_{29}}(T^*_{29}) = (q_{29})^{(5)} \ , \ \frac{\partial(b''_i)^{(5)}}{\partial G_j}((G_{31})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a'_{28})^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G^*_{28}\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a'_{29})^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G^*_{29}\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a'_{30})^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G^*_{30}\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b'_{28})^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T^*_{28}\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b'_{29})^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T^*_{29}\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b'_{30})^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T^*_{30}\mathbb{G}_j\right) \quad 562$$

**ASYMPTOTIC STABILITY ANALYSIS** 563

**Theorem 6:** If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(6)}\ and\ (b''_i)^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{33})^{(6)}}{\partial T_{33}}(T^*_{33}) = (q_{33})^{(6)} \ , \ \frac{\partial(b''_i)^{(6)}}{\partial G_j}((G_{35})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a'_{32})^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G^*_{32}\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a'_{33})^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G^*_{33}\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a'_{34})^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G^*_{34}\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b'_{32})^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T^*_{32}\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b'_{33})^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T^*_{33}\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b'_{34})^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T^*_{34}\mathbb{G}_j\right) \quad 570$$

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**Theorem 7:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 572

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{37})^{(7)}}{\partial T_{37}}(T^*_{37}) = (q_{37})^{(7)} \quad , \frac{\partial(b_i'')^{(7)}}{\partial G_j}\left((G_{39})^{**}\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a'_{36})^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G^*_{36}\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a'_{37})^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G^*_{37}\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a'_{38})^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G^*_{38}\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b'_{36})^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T^*_{36}\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b'_{37})^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T^*_{37}\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b'_{38})^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T^*_{38}\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

**ASYMPTOTIC STABILITY ANALYSIS**

**Theorem 8:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{41}'')^{(8)}}{\partial T_{41}}(T_{41}^*) = (q_{41})^{(8)} \quad , \frac{\partial (b_i'')^{(8)}}{\partial G_j}\left( (G_{43})^* \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a_{40}')^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G_{40}^*\mathbb{T}_{41} \quad (581)$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a_{41}')^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G_{41}^*\mathbb{T}_{41} \quad (582)$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a_{42}')^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G_{42}^*\mathbb{T}_{41} \quad (583)$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b_{40}')^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T_{40}^*\mathbb{G}_j\right) \quad (584)$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b_{41}')^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T_{41}^*\mathbb{G}_j\right) \quad (585)$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b_{42}')^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T_{42}^*\mathbb{G}_j\right) \quad (586)$$

(586 A)

## ASYMPTOTIC STABILITY ANALYSIS

**Theorem 9:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}$ *and* $(b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

**Proof:** Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{45}'')^{(9)}}{\partial T_{45}}(T_{45}^*) = (q_{45})^{(9)} \quad , \frac{\partial (b_i'')^{(9)}}{\partial G_j}\left( (G_{47})^* \right) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a_{44}')^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)}G_{44}^*\mathbb{T}_{45} \quad (586\,B)$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a_{45}')^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)}G_{45}^*\mathbb{T}_{45} \quad (586\,C)$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a_{46}')^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)}G_{46}^*\mathbb{T}_{45} \quad (586\,D)$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b_{44}')^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)}T_{44}^*\mathbb{G}_j\right) \quad (586\,E)$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b_{45}')^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)}T_{45}^*\mathbb{G}_j\right) \quad (586\,F)$$

$\frac{d\mathbb{T}_{46}}{dt} = -\left((b'_{46})^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T^*_{46}\mathbb{G}_j\right)$ 586

G

587

**The characteristic equation of this system is**

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(q_{13})^{(1)}G^*_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)}T^*_{14} + (b_{14})^{(1)}s_{(13),(14)}T^*_{14}\right)$$

$$+\left(\left((\lambda)^{(1)} + (a'_{14})^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)}G^*_{13} + (a_{13})^{(1)}(q_{14})^{(1)}G^*_{14}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)}T^*_{14} + (b_{14})^{(1)}s_{(13),(13)}T^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((b'_{13})^{(1)} + (b'_{14})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(15)}T^*_{14} + (b_{14})^{(1)}s_{(13),(15)}T^*_{13}\right)\} = 0$$

$+$

$$(\lambda)^{(2)} + (b'_{18})^{(2)} - (r_{18})^{(2)}\{\left((\lambda)^{(2)} + (a'_{18})^{(2)} + (p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^*_{17} + (a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^*_{17}\right)$$

$$+\left(\left((\lambda)^{(2)} + (a'_{17})^{(2)} + (p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^*_{16} + (a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}\right)$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2 + \left((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2 + \left((b'_{16})^{(2)} + (b'_{17})^{(2)} - (r_{16})^{(2)} + (r_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$+\left(\left((\lambda)^{(2)}\right)^2 + \left((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17} + (a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^{*}_{16}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\right)\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left(\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)\right]$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(21)}T^{*}_{21}\right)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^{*}_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^{*}_{21}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(20)}T^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$+\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(22)}T^{*}_{20}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\right)\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left(\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)\right]$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(25)}T^{*}_{25}\right)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^{*}_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^{*}_{25}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(24)}T^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$+\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(26)}T^{*}_{24}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\right)\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(29)}T^{*}_{29}\right)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^{*}_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^{*}_{29}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(28)}T^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$+\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(30)}T^{*}_{28}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\right)\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left(\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)\right]$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(33)}T^{*}_{33}\right)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^{*}_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^{*}_{33}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(32)}T^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$+\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)$$

$$\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(34)}T^{*}_{32}\Big)\}=0$$

$$+$$

$$(\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^{*}_{36}\Big)\right]$$

$$\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(37)}T^{*}_{37}\Big)$$

$$+\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^{*}_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^{*}_{37}\Big)$$

$$\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(36)}T^{*}_{36}\Big)$$

$$\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\Big)$$

$$\left((\lambda)^{(7)}\right)^{2}+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\Big)$$

$$+\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\Big)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)$$

$$\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(38)}T^{*}_{36}\Big)\}=0$$

$$+$$

$$(\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^{*}_{40}\Big)\right]$$

$$\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(41)}T^{*}_{41}\Big)$$

$$+\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^{*}_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^{*}_{41}\Big)$$

$$\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(40)}T^{*}_{40}\Big)$$

$$\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\Big)$$

$$\left((\lambda)^{(8)}\right)^{2}+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\Big)$$

$$+\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\Big)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(42)}T^{*}_{40}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\right)\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(45)}T^{*}_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^{*}_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^{*}_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(44)}T^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$+\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(46)}T^{*}_{44}\right)\}=0$$

**And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.**

## SECTION THIRTY FIVE

## Memory Systems

### INTRODUCTION—VARIABLES USED

**The Quarterly Journal of Experimental Psychology Section A: Human Experimental Psychology Volume 48, Issue 1, 1995: Is there a Relationship between Task Demand and Storage Space in Tests of Working Memory Capacity? DOI: 10.1080/14640749508401379 J.N. Towsea* & G.J. Hitchb pages 108-124 Published online: 29 May 2007**

(1) This paper considers working memory capacity, critically examining the hypothesis that counting span (the ability to count arrays of objects and store count totals) reflects a trade-off in resources available for processing and (e&eb) short-term storage.
(2) Previous evidence interpreted as favouring this hypothesis has confounded task difficulty with counting time. Experiment 1 validated a manipulation of (e&eb) the attentional demands of counting in which target objects were differentiated from (e&eb) non-targets by (e&eb) either a single feature (colour) or a feature conjunction (a combination of line orientations).
(3) The results confirmed that the two presentations involved (e&eb) qualitatively different attentional loads.
(4) Experiment 2 used these displays to compare counting span for children aged 6 to 11, both with and without (e) an adjustment of target numerosity to control for (e&eb) differences in processing time.
(5) At all ages, span was lower when counting took longer, but there was no difference between feature and conjunction arrays once counting time was accounted for.
(6) These results argue against a resource trade-off interpretation of (e&eb) counting span.
(7) Rather, they support a hypothesis of resource-switching among children, implying (eb) that counting span acts as (e&eb) a measure of time-based forgetting.

**How many memory systems are there? Tulving, Endel American Psychologist, Vol 40(4), Apr 1985, 385-398. http://dx.doi.org/10.1037/0003-066X.40.4.385**

**(8)** Proposes a ternary classificatory scheme of memory in which procedural, semantic, and episodic memory constitute (e) a monohierarchical arrangement.
**(9)** In this scheme, episodic memory is a specialized subsystem of (e) semantic memory, which in turn is a specialized subsystem of (e) procedural memory.
**(10)** The 3 memory systems differ from one another in a number of ways, including (e) the kind of consciousness that characterizes their operations.
**(11)** The ternary scheme overlaps with (e&eb) dichotomies and trichotomies of memory proposed by others. Evidence for multiple systems is reviewed, and illustrative data are provided from experiments.
**(12)** Direct priming effects were found to be both functionally and stochastically independent of recognition memory. (100 ref) (PsycINFO Database Record (c) 2012 APA, all rights reserved)

## NOTATION

### Module One

According to Umezawa, consciousness would be the result of (e) the sum of quantum processes, while the neural systems would be limited to (e) the transmission of macroscopic signals. The model of Umezawa was further developed by Giuseppe Vitiello

$G_{13}$ : Category one of sum of quantum processes

$G_{14}$ : Category two of SAS

$G_{15}$ : Category three of SAS

$T_{13}$ : Category one of consciousness

$T_{14}$ : Category two of SAS

$T_{15}$ : Category three of SAS

### Module Two

**neural systems** would be limited to (e) the transmission of macroscopic signals

$G_{16}$ : Category one of **neural systems;** transmission of macroscopic signals

$G_{17}$ : Category two of SAS

$G_{18}$ : Category three of SAS

$T_{16}$ : Category one of transmission of macroscopic signals; **neural systems**

$T_{17}$ : Category two of SAS

$T_{18}$ : Category three of SAS

**Module three**

This paper considers working memory capacity, critically examining the hypothesis that counting span (the ability to count arrays of objects and store count totals) reflects a trade-off in resources available for processing and (e&eb) short-term storage

$G_{20}$ : Category one of **working memory capacity, critically examining the hypothesis that counting span (the ability to count arrays of objects and store count totals);** trade-off in resources available for processing and (e&eb) short-term storage

$G_{21}$ : Category two of SAS

$G_{22}$ : Category three of SAS

$T_{20}$ : Category one of trade-off in resources available for processing and (e&eb) short-term storage **;working memory capacity, critically examining the hypothesis that counting span (the ability to count arrays of objects and store count totals)**

$T_{21}$ : Category two of SAS

$T_{22}$ : Category three of SAS

**Module four**

This paper considers working memory capacity, critically examining the hypothesis that counting span (the ability to count arrays of objects and store count totals) reflects a trade-off in resources available for processing and (e&eb) **short-term storage**

$G_{24}$ : Category one of working memory capacity, critically examining the hypothesis that counting span (the ability to count arrays of objects and store count totals) reflects a trade-off in resources available for processing; **short-term storage**

$G_{25}$ : Category two of SAS

$G_{26}$ : Category three of SAS

$T_{24}$ : Category one of **short-term storage;** working memory capacity, critically examining the hypothesis that counting span (the ability to count arrays of objects and store count totals) reflects a trade-off in resources available for processing

$T_{25}$ : Category two of SAS

$T_{26}$ : Category three of SAS

**Module five**

Previous evidence interpreted as favouring this hypothesis has confounded task difficulty with counting time.

**Experiment 1 (many such similar experiments could be conducted) validated a manipulation of the attentional demands of counting in which target objects were differentiated** from (e&eb) non-targets by (e&eb) either a single feature (colour) or a feature conjunction (a combination of line orientations)

$G_{28}$ : Category one of **Experiment 1 validated a manipulation of the attentional demands of counting in which target objects were differentiated (many such experiments could be conducted);** non-targets by (e&eb) either a single feature (colour) or a feature conjunction (a combination of line orientations)

$G_{29}$ : Category two of SAS

$G_{30}$ : Category three of SAS

$T_{28}$ : Category one of non-targets by (e&eb) either a single feature (colour) or a feature conjunction (a combination of line orientations); **Experiment 1 validated a manipulation of the attentional demands of counting in which target objects were differentiated (many such experiments could be conducted)**

$T_{29}$ : Category two of SAS

$T_{30}$ : Category three of SAS

**Module six**

**Experiment 1 validated a manipulation of the attentional demands of counting in which target objects were differentiated from non-targets** by (e&eb) either a single feature (colour) or a feature conjunction (a combination of line orientations)

$G_{32}$ : Category one of **Experiment 1 validated a manipulation of the attentional demands of counting in which target objects were differentiated from non-targets;** either a single feature (colour) or a feature conjunction (a combination of line orientations)

$G_{33}$ : Category two of SAS

$G_{34}$ : Category three of SAS

$T_{32}$ : Category one of either a single feature (colour) or a feature conjunction (a combination of line orientations);**Experiment 1 validated a manipulation of the attentional demands of counting in which target objects were differentiated from non-targets**

$T_{33}$ : Category two of SAS

$T_{34}$ : Category three of SAS

**Module seven**

The results confirmed that the **two presentations** involved (e&eb) qualitatively different attentional loads

$G_{36}$ : Category one of **two presentations;** qualitatively different attentional loads

$G_{37}$ : Category two of SAS

$G_{38}$ : Category three of SAS

$T_{36}$ : Category one of qualitatively different attentional loads; **two presentations**

$T_{37}$ : Category two of SAS

$T_{38}$ : Category three of SAS

**Module eight**

These results argue against a resource trade-off interpretation of (e&eb) **counting span**

$G_{40}$ : Category one of results argue against a resource trade-off interpretation; **counting span**

$G_{41}$ : Category two of SAS

$G_{42}$ : Category three of SAS

$T_{40}$ : Category one of **counting span;** results argue against a resource trade-off interpretation

$T_{41}$ : Category two of SAS

$T_{42}$ : Category three of SAS

**Module Nine**

Results support a hypothesis of resource-switching among children, implying (eb) that counting span acts as (e&eb) a measure of time-based forgetting

**counting span** acts as (e&eb) a measure of time-based forgetting

$G_{44}$ : Category one of **counting span;** measure of time-based forgetting

$G_{45}$ : Category two of SAS

$G_{46}$ : Category three of SAS

$T_{44}$ : Category one of measure of time-based forgetting **;counting span**

$T_{45}$ : Category two of SAS

$T_{46}$ : Category three of SAS

**The Coefficients:**

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}$ , $(b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}$,
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$

$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}, \ (b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)} \ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$ ,
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)}$,
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)}$,
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)}$,
are Dissipation coefficients

**Module Numbered One**

**The differential system of this model is now (Module Numbered one)**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \quad 1$$
$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad 2$$
$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15} \quad 3$$
$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13} \quad 4$$
$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14} \quad 5$$
$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)\right]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14}, t) =$ First augmentation factor
$-(b''_{13})^{(1)}(G, t) =$ First detritions factor

**Module Numbered Two**

**The differential system of this model is now ( Module numbered two)**

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right]G_{16} \quad 7$$
$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t)\right]G_{17} \quad 8$$
$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t)\right]G_{18} \quad 9$$
$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\big((G_{19}), t\big)\right]T_{16} \quad 10$$
$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\big((G_{19}), t\big)\right]T_{17} \quad 11$$
$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\big((G_{19}), t\big)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17}, t) =$ First augmentation factor
$-(b''_{16})^{(2)}\big((G_{19}), t\big) =$ First detritions factor

**Module Numbered Three**

**The differential system of this model is now (Module numbered three)**

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right]G_{20} \quad 13$$
$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}, t)\right]G_{21} \quad 14$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}, t)\right]G_{22} \quad 15$$
$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}, t)\right]T_{20} \quad 16$$
$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}, t)\right]T_{21} \quad 17$$
$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}, t)\right]T_{22} \quad 18$$
$+(a''_{20})^{(3)}(T_{21}, t) =$ First augmentation factor
$-(b''_{20})^{(3)}(G_{23}, t) =$ First detritions factor

**Module Numbered Four**

**The differential system of this model is now (Module numbered Four)**

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)\right]G_{24} \quad 19$$
$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}, t)\right]G_{25} \quad 20$$
$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}, t)\right]G_{26} \quad 21$$
$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27}), t\right)\right]T_{24} \quad 22$$
$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27}), t\right)\right]T_{25} \quad 23$$
$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27}), t\right)\right]T_{26} \quad 24$$
$+(a''_{24})^{(4)}(T_{25}, t) =$ **First augmentation factor**
$-(b''_{24})^{(4)}\left((G_{27}), t\right) =$ **First detritions factor**

**Module Numbered Five**:

**The differential system of this model is now (Module number five)**

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right]G_{28} \quad 25$$
$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}, t)\right]G_{29} \quad 26$$
$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}, t)\right]G_{30} \quad 27$$
$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}\left((G_{31}), t\right)\right]T_{28} \quad 28$$
$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}\left((G_{31}), t\right)\right]T_{29} \quad 29$$
$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}\left((G_{31}), t\right)\right]T_{30} \quad 30$$
$+(a''_{28})^{(5)}(T_{29}, t) =$ **First augmentation factor**
$-(b''_{28})^{(5)}\left((G_{31}), t\right) =$ **First detritions factor**

**Module Numbered Six**

**The differential system of this model is now (Module numbered Six)**

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right]G_{32} \quad 31$$
$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}, t)\right]G_{33} \quad 32$$
$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}, t)\right]G_{34} \quad 33$$
$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}\left((G_{35}), t\right)\right]T_{32} \quad 34$$
$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}\left((G_{35}), t\right)\right]T_{33} \quad 35$$
$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}\left((G_{35}), t\right)\right]T_{34} \quad 36$$
$+(a''_{32})^{(6)}(T_{33}, t) =$ **First augmentation factor**

**Module Numbered Seven:**

**The differential system of this model is now (Seventh Module)**

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right]G_{36} \quad 37$$

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37},t)\right]G_{37} \quad 38$$

$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37},t)\right]G_{38} \quad 39$$

$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}\left((G_{39}),t\right)\right]T_{36} \quad 40$$

$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}\left((G_{39}),t\right)\right]T_{37} \quad 41$$

$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}\left((G_{39}),t\right)\right]T_{38} \quad 42$$

$+(a''_{36})^{(7)}(T_{37},t) =$ **First augmentation factor**

**Module Numbered Eight**

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right]G_{40} \quad 43$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41},t)\right]G_{41} \quad 44$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41},t)\right]G_{42} \quad 45$$

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}\left((G_{43}),t\right)\right]T_{40} \quad 46$$

$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}\left((G_{43}),t\right)\right]T_{41} \quad 47$$

$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}\left((G_{43}),t\right)\right]T_{42} \quad 48$$

**Module Numbered Nine**

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}\left((G_{47}),t\right)\right]T_{44} \quad 52$$

$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}\left((G_{47}),t\right)\right]T_{45} \quad 53$$

$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}\left((G_{47}),t\right)\right]T_{46} \quad 54$$

$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}\left((G_{47}),t\right) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)} & \boxed{+(a''_{13})^{(1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)} & \boxed{+(a''_{14})^{(1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[\begin{array}{ccc} (a'_{15})^{(1)}\boxed{+(a''_{15})^{(1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{15} \quad (57)$$

Where $\boxed{(a''_{13})^{(1)}(T_{14},t)}$, $\boxed{(a''_{14})^{(1)}(T_{14},t)}$, $\boxed{(a''_{15})^{(1)}(T_{14},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7)}(T_{37},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8)}(T_{41},t)}$ $\boxed{+(a''_{41})^{(8,8)}(T_{41},t)}$ $\boxed{+(a''_{42})^{(8,8)}(T_{41},t)}$ are eight augmentation coefficient for 1,2,3

$\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[\begin{array}{ccc} (b'_{13})^{(1)}\boxed{-(b''_{13})^{(1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{array}\right] T_{13} \quad (58)$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[\begin{array}{ccc} (b'_{14})^{(1)}\boxed{-(b''_{14})^{(1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{array}\right] T_{14} \quad (59)$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[\begin{array}{ccc} (b'_{15})^{(1)}\boxed{-(b''_{15})^{(1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{array}\right] T_{15} \quad (60)$$

Where $\boxed{-(b''_{13})^{(1)}(G,t)}$, $\boxed{-(b''_{14})^{(1)}(G,t)}$, $\boxed{-(b''_{15})^{(1)}(G,t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)}$ are fourth detrition coefficients

for category 1, 2 and 3
$-(b_{28}'')^{(5,5,5,5,)}(G_{31},t)$ , $-(b_{29}'')^{(5,5,5,5,)}(G_{31},t)$ , $-(b_{30}'')^{(5,5,5,5,)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3
$-(b_{32}'')^{(6,6,6,6,)}(G_{35},t)$ , $-(b_{33}'')^{(6,6,6,6,)}(G_{35},t)$ , $-(b_{34}'')^{(6,6,6,6,)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3
$-(b_{37}'')^{(7,7,)}(G_{39},t)$ , $-(b_{36}'')^{(7,7,)}(G_{39},t)$ , $-(b_{38}'')^{(7,7,)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3
$-(b_{40}'')^{(8,8)}(G_{43},t)$ $-(b_{41}'')^{(8,8)}(G_{43},t)$ $-(b_{42}'')^{(8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3
$-(b_{44}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$ , $-(b_{45}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$ , $-(b_{46}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \begin{bmatrix} (a_{16}')^{(2)} & +(a_{16}'')^{(2)}(T_{17},t) & +(a_{13}'')^{(1,1,)}(T_{14},t) & +(a_{20}'')^{(3,3,3)}(T_{21},t) \\ +(a_{24}'')^{(4,4,4,4,4)}(T_{25},t) & +(a_{28}'')^{(5,5,5,5,5)}(T_{29},t) & +(a_{32}'')^{(6,6,6,6,6)}(T_{33},t) & \\ +(a_{36}'')^{(7,7,7)}(T_{37},t) & +(a_{40}'')^{(8,8,8)}(T_{41},t) & +(a_{44}'')^{(9,9)}(T_{45},t) & \end{bmatrix} G_{16} \quad 61$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \begin{bmatrix} (a_{17}')^{(2)} & +(a_{17}'')^{(2)}(T_{17},t) & +(a_{14}'')^{(1,1,)}(T_{14},t) & +(a_{21}'')^{(3,3,3)}(T_{21},t) \\ +(a_{25}'')^{(4,4,4,4,4)}(T_{25},t) & +(a_{29}'')^{(5,5,5,5,5)}(T_{29},t) & +(a_{33}'')^{(6,6,6,6,6)}(T_{33},t) & \\ +(a_{37}'')^{(7,7,7)}(T_{37},t) & +(a_{41}'')^{(8,8,8)}(T_{41},t) & +(a_{45}'')^{(9,9)}(T_{45},t) & \end{bmatrix} G_{17} \quad 62$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \begin{bmatrix} (a_{18}')^{(2)} & +(a_{18}'')^{(2)}(T_{17},t) & +(a_{15}'')^{(1,1,)}(T_{14},t) & +(a_{22}'')^{(3,3,3)}(T_{21},t) \\ +(a_{26}'')^{(4,4,4,4,4)}(T_{25},t) & +(a_{30}'')^{(5,5,5,5,5)}(T_{29},t) & +(a_{34}'')^{(6,6,6,6,6)}(T_{33},t) & \\ +(a_{38}'')^{(7,7,7)}(T_{37},t) & +(a_{42}'')^{(8,8,8)}(T_{41},t) & +(a_{46}'')^{(9,9)}(T_{45},t) & \end{bmatrix} G_{18} \quad 63$$

Where $+(a_{16}'')^{(2)}(T_{17},t)$ , $+(a_{17}'')^{(2)}(T_{17},t)$ , $+(a_{18}'')^{(2)}(T_{17},t)$ are first augmentation coefficients for category 1, 2 and 3
$+(a_{13}'')^{(1,1,)}(T_{14},t)$ , $+(a_{14}'')^{(1,1,)}(T_{14},t)$ , $+(a_{15}'')^{(1,1,)}(T_{14},t)$ are second augmentation coefficient for category 1, 2 and 3
$+(a_{20}'')^{(3,3,3)}(T_{21},t)$ , $+(a_{21}'')^{(3,3,3)}(T_{21},t)$ , $+(a_{22}'')^{(3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3
$+(a_{24}'')^{(4,4,4,4,4)}(T_{25},t)$ , $+(a_{25}'')^{(4,4,4,4,4)}(T_{25},t)$ , $+(a_{26}'')^{(4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3
$+(a_{28}'')^{(5,5,5,5,5)}(T_{29},t)$ , $+(a_{29}'')^{(5,5,5,5,5)}(T_{29},t)$ , $+(a_{30}'')^{(5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3
$+(a_{32}'')^{(6,6,6,6,6)}(T_{33},t)$ , $+(a_{33}'')^{(6,6,6,6,6)}(T_{33},t)$ , $+(a_{34}'')^{(6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3
$+(a_{36}'')^{(7,7,7)}(T_{37},t)$ , $+(a_{37}'')^{(7,7,7)}(T_{37},t)$ , $+(a_{38}'')^{(7,7,7)}(T_{37},t)$ are seventh augmentation coefficient for category 1, 2 and 3
$+(a_{40}'')^{(8,8,8)}(T_{41},t)$ , $+(a_{41}'')^{(8,8,8)}(T_{41},t)$ , $+(a_{42}'')^{(8,8,8)}(T_{41},t)$ are eight augmentation coefficient for category 1, 2 and 3
$+(a_{44}'')^{(9,9)}(T_{45},t)$ , $+(a_{45}'')^{(9,9)}(T_{45},t)$ , $+(a_{46}'')^{(9,9)}(T_{45},t)$ are ninth augmentation coefficient for

category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[\begin{array}{ccc} (b'_{16})^{(2)}\boxed{-(b''_{16})^{(2)}(G_{19},t)} & \boxed{-(b''_{13})^{(1,1,)}(G,t)} & \boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9)}(G_{47},t)} \end{array}\right] T_{16} \quad (64)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[\begin{array}{ccc} (b'_{17})^{(2)}\boxed{-(b''_{17})^{(2)}(G_{19},t)} & \boxed{-(b''_{14})^{(1,1,)}(G,t)} & \boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9)}(G_{47},t)} \end{array}\right] T_{17} \quad (65)$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[\begin{array}{ccc} (b'_{18})^{(2)}\boxed{-(b''_{18})^{(2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,)}(G,t)} & \boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9)}(G_{47},t)} \end{array}\right] T_{18} \quad (66)$$

where $\boxed{-(b''_{16})^{(2)}(G_{19},t)}$ , $\boxed{-(b''_{17})^{(2)}(G_{19},t)}$ , $\boxed{-(b''_{18})^{(2)}(G_{19},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,)}(G,t)}$ are second detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{44})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[\begin{array}{ccc} (a'_{20})^{(3)}\boxed{+(a''_{20})^{(3)}(T_{21},t)} & \boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9)}(T_{45},t)} \end{array}\right] G_{20} \quad (67)$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[\begin{array}{ccc} (a'_{21})^{(3)}\boxed{+(a''_{21})^{(3)}(T_{21},t)} & \boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)} \end{array}\right] G_{21} \quad (68)$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \begin{bmatrix} (a'_{22})^{(3)}\boxed{+(a''_{22})^{(3)}(T_{21},t)} & \boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{22} \quad (69)$$

$\boxed{+(a''_{20})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3)}(T_{21},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}$ are second augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)}$ are third augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)}$ are seventh augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)}$ are eight augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{44})^{(9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)}$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b'_{20})^{(3)}\boxed{-(b''_{20})^{(3)}(G_{23},t)} & \boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)} & \boxed{-(b''_{13})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{20} \quad (70)$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b'_{21})^{(3)}\boxed{-(b''_{21})^{(3)}(G_{23},t)} & \boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)} & \boxed{-(b''_{14})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{21} \quad (71)$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)}\boxed{-(b''_{22})^{(3)}(G_{23},t)} & \boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{22} \quad (72)$$

$\boxed{-(b''_{20})^{(3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3)}(G_{23},t)}$ *are first detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}$ *are second detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{13})^{(1,1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,)}(G,t)}$ *are third detrition coefficients for category 1,2 and 3*

$\boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)}$ *are fourth detrition*

*coefficients for category 1, 2 and 3*

$\boxed{-(b_{28}'')^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5)}(G_{31},t)}$ *are fifth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b_{32}'')^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{34}'')^{(6,6,6,6,6,6)}(G_{35},t)}$ *are sixth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b_{36}'')^{(7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{37}'')^{(7,7,7,7)}(G_{39},t)}$ $\boxed{-(b_{38}'')^{(7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{40}'')^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{42}'')^{(8,8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{46}'')^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b_{45}'')^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b_{44}'')^{(9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[\begin{array}{ccc} (a_{24}')^{(4)} & \boxed{+(a_{24}'')^{(4)}(T_{25},t)} & \boxed{+(a_{28}'')^{(5,5,)}(T_{29},t)} \quad \boxed{+(a_{32}'')^{(6,6,)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a_{16}'')^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a_{20}'')^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{36}'')^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{40}'')^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a_{44}'')^{(9,9,9,9)}(T_{45},t)} \end{array}\right] G_{24} \quad (73)$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[\begin{array}{ccc} (a_{25}')^{(4)} & \boxed{+(a_{25}'')^{(4)}(T_{25},t)} & \boxed{+(a_{29}'')^{(5,5,)}(T_{29},t)} \quad \boxed{+(a_{33}'')^{(6,6)}(T_{33},t)} \\ \boxed{+(a_{14}'')^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a_{17}'')^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a_{21}'')^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{37}'')^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{41}'')^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a_{45}'')^{(9,9,9,9)}(T_{45},t)} \end{array}\right] G_{25} \quad (74)$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[\begin{array}{ccc} (a_{26}')^{(4)} & \boxed{+(a_{26}'')^{(4)}(T_{25},t)} & \boxed{+(a_{30}'')^{(5,5,)}(T_{29},t)} \quad \boxed{+(a_{34}'')^{(6,6,)}(T_{33},t)} \\ \boxed{+(a_{15}'')^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a_{18}'')^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a_{22}'')^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{38}'')^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{42}'')^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a_{46}'')^{(9,9,9,9)}(T_{45},t)} \end{array}\right] G_{26} \quad (75)$$

$\boxed{(a_{24}'')^{(4)}(T_{25},t)}$, $\boxed{(a_{25}'')^{(4)}(T_{25},t)}$, $\boxed{(a_{26}'')^{(4)}(T_{25},t)}$ *are first augmentation coefficients category* 1, 2 3

$\boxed{+(a_{28}'')^{(5,5,)}(T_{29},t)}$, $\boxed{+(a_{29}'')^{(5,5,)}(T_{29},t)}$, $\boxed{+(a_{30}'')^{(5,5,)}(T_{29},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a_{32}'')^{(6,6,)}(T_{33},t)}$, $\boxed{+(a_{33}'')^{(6,6,)}(T_{33},t)}$, $\boxed{+(a_{34}'')^{(6,6,)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a_{13}'')^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a_{14}'')^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a_{15}'')^{(1,1,1,1)}(T_{14},t)}$ *are fourth augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a_{16}'')^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{17}'')^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{18}'')^{(2,2,2,2)}(T_{17},t)}$ *are fifth augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a_{20}'')^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{21}'')^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{22}'')^{(3,3,3,3)}(T_{21},t)}$ *are sixth augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a_{36}'')^{(7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a_{37}'')^{(7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a_{38}'')^{(7,7,7,7,7)}(T_{37},t)}$ *are seventh augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a_{40}'')^{(8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a_{41}'')^{(8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a_{42}'')^{(8,8,8,8,8)}(T_{41},t)}$ *are eighth augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a_{46}'')^{(9,9,9,9)}(T_{45},t)}$, $\boxed{+(a_{45}'')^{(9,9,9,9)}(T_{45},t)}$, $\boxed{+(a_{44}'')^{(9,9,9,9)}(T_{45},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \begin{bmatrix} (b'_{24})^{(4)} & -(b''_{24})^{(4)}(G_{27},t) & -(b''_{28})^{(5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,)}(G_{35},t) \\ & -(b''_{13})^{(1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{24} \quad (76)$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \begin{bmatrix} (b'_{25})^{(4)} & -(b''_{25})^{(4)}(G_{27},t) & -(b''_{29})^{(5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{25} \quad (77)$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)} & -(b''_{26})^{(4)}(G_{27},t) & -(b''_{30})^{(5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1)}(G,t) & -(b''_{18})^{(2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{26} \quad (78)$$

*Where* $-(b''_{24})^{(4)}(G_{27},t)$, $-(b''_{25})^{(4)}(G_{27},t)$, $-(b''_{26})^{(4)}(G_{27},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{28})^{(5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,)}(G_{31},t)$ *are second detrition coefficients for category* 1, 2 *and* 3

$-(b''_{32})^{(6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,)}(G_{35},t)$ *are third detrition coefficients for category* 1, 2 *and* 3

$-(b''_{13})^{(1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1)}(G,t)$
*are fourth detrition coefficients for category* 1, 2 *and* 3

$-(b''_{16})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2)}(G_{19},t)$
*are fifth detrition coefficients for category* 1, 2 *and* 3

$-(b''_{20})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3)}(G_{23},t)$
*are sixth detrition coefficients for category* 1, 2 *and* 3

$-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)$
*are seventh detrition coefficients for category* 1, 2 *and* 3

$-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)$
*are eighth detrition coefficients for category* 1, 2 *and* 3

$-(b''_{46})^{(9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \begin{bmatrix} (a'_{28})^{(5)} & +(a''_{28})^{(5)}(T_{29},t) & +(a''_{24})^{(4,4,)}(T_{25},t) & +(a''_{32})^{(6,6,6)}(T_{33},t) \\ & +(a''_{13})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{16})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3,3)}(T_{21},t) \\ & +(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{28} \quad (79)$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \begin{bmatrix} (a'_{29})^{(5)} & +(a''_{29})^{(5)}(T_{29},t) & +(a''_{25})^{(4,4,)}(T_{25},t) & +(a''_{33})^{(6,6,6)}(T_{33},t) \\ & +(a''_{14})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{17})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3,3)}(T_{21},t) \\ & +(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{29} \quad (80)$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[\begin{array}{ccc} (a'_{30})^{(5)} & +(a''_{30})^{(5)}(T_{29},t) & +(a''_{26})^{(4,4,)}(T_{25},t) \quad +(a''_{34})^{(6,6,6)}(T_{33},t) \\ +(a''_{15})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{18})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3,3)}(T_{21},t) \\ +(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9)}(T_{45},t) \end{array}\right] G_{30} \quad 81$$

*Where* $+(a''_{28})^{(5)}(T_{29},t)$, $+(a''_{29})^{(5)}(T_{29},t)$, $+(a''_{30})^{(5)}(T_{29},t)$ *are first augmentation coefficients for category* 1, 2 *and* 3

*And* $+(a''_{24})^{(4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,)}(T_{25},t)$ *are second augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{32})^{(6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6)}(T_{33},t)$ *are third augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)$ are fourth augmentation coefficients for category 1,2, and 3

$+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)$ are fifth augmentation coefficients for category 1,2,and 3

$+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)$ are sixth augmentation coefficients for category 1,2, 3

$+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)$ , $+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1,2, 3

$+(a''_{40})^{(8,8\,,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)$ are eighth augmentation coefficients for category 1,2, 3

$+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[\begin{array}{ccc} (b'_{28})^{(5)} & -(b''_{28})^{(5)}(G_{31},t) & -(b''_{24})^{(4,4,)}(G_{27},t) \quad -(b''_{32})^{(6,6,6)}(G_{35},t) \\ -(b''_{13})^{(1,1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3)}(G_{23},t) \\ -(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9)}(G_{47},t) \end{array}\right] T_{28} \quad 82$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[\begin{array}{ccc} (b'_{29})^{(5)} & -(b''_{29})^{(5)}(G_{31},t) & -(b''_{25})^{(4,4,)}(G_{27},t) \quad -(b''_{33})^{(6,6,6)}(G_{35},t) \\ -(b''_{14})^{(1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3)}(G_{23},t) \\ -(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9)}(G_{47},t) \end{array}\right] T_{29} \quad 83$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[\begin{array}{ccc} (b'_{30})^{(5)} & -(b''_{30})^{(5)}(G_{31},t) & -(b''_{26})^{(4,4,)}(G_{27},t) \quad -(b''_{34})^{(6,6,6)}(G_{35},t) \\ -(b''_{15})^{(1,1,1,1,1,)}(G,t) & -(b''_{18})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3)}(G_{23},t) \\ -(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9)}(G_{47},t) \end{array}\right] T_{30} \quad 84$$

*where* $-(b''_{28})^{(5)}(G_{31},t)$ , $-(b''_{29})^{(5)}(G_{31},t)$ , $-(b''_{30})^{(5)}(G_{31},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{24})^{(4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,)}(G_{27},t)$ *are second detrition coefficients for category* 1,2 *and* 3

$-(b''_{32})^{(6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6)}(G_{35},t)$ *are third detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1)}(G,t)}$ , $\boxed{-(b''_{15})^{(1,1,1,1,1,)}(G,t)}$ are fourth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \begin{bmatrix} (a'_{32})^{(6)}\boxed{+(a''_{32})^{(6)}(T_{33},t)} & \boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{32} \quad (85)$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \begin{bmatrix} (a'_{33})^{(6)}\boxed{+(a''_{33})^{(6)}(T_{33},t)} & \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{33} \quad (86)$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \begin{bmatrix} (a'_{34})^{(6)}\boxed{+(a''_{34})^{(6)}(T_{33},t)} & \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{34} \quad (87)$$

$\boxed{+(a''_{32})^{(6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6)}(T_{33},t)}$ *are first augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}$ *are second augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}$ *are third augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}$ - are fourth augmentation coefficients

$\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}$ - fifth augmentation coefficients

$\boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)}$ sixth augmentation coefficients

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)}$ seventh augmentation coefficients

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}$

Eighth augmentation coefficients
$+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt}=(b_{32})^{(6)}T_{33}-\begin{bmatrix}(b'_{32})^{(6)}-(b''_{32})^{(6)}(G_{35},t) & -(b''_{28})^{(5,5,5)}(G_{31},t) & -(b''_{24})^{(4,4,4,)}(G_{27},t)\\ -(b''_{13})^{(1,1,1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)\\ -(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)\end{bmatrix}T_{32} \quad (88)$$

$$\frac{dT_{33}}{dt}=(b_{33})^{(6)}T_{32}-\begin{bmatrix}(b'_{33})^{(6)}-(b''_{33})^{(6)}(G_{35},t) & -(b''_{29})^{(5,5,5)}(G_{31},t) & -(b''_{25})^{(4,4,4,)}(G_{27},t)\\ -(b''_{14})^{(1,1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)\\ -(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)\end{bmatrix}T_{33} \quad (89)$$

$$\frac{dT_{34}}{dt}=(b_{34})^{(6)}T_{33}-\begin{bmatrix}(b'_{34})^{(6)}-(b''_{34})^{(6)}(G_{35},t) & -(b''_{30})^{(5,5,5)}(G_{31},t) & -(b''_{26})^{(4,4,4,)}(G_{27},t)\\ -(b''_{15})^{(1,1,1,1,1,1)}(G,t) & -(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)\\ -(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)\end{bmatrix}T_{34} \quad (90)$$

$-(b''_{32})^{(6)}(G_{35},t)$, $-(b''_{33})^{(6)}(G_{35},t)$, $-(b''_{34})^{(6)}(G_{35},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{28})^{(5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5)}(G_{31},t)$ *are second detrition coefficients for category* 1, 2 *and* 3

$-(b''_{24})^{(4,4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,)}(G_{27},t)$ *are third detrition coefficients for category* 1,2 *and* 3

$-(b''_{13})^{(1,1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1,1,1)}(G,t)$ are fourth detrition coefficients for category 1, 2, and 3

$-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1, 2, and 3

$-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1, 2, and 3

$-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2, and 3

$-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1, 2, and 3

$-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \quad (91)$$

$$=(a_{36})^{(7)}G_{37}-\begin{bmatrix}(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37},t) & +(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)\\ +(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)\\ +(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t) & +(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)\end{bmatrix}G_{13}$$

$$\frac{dG_{37}}{dt} \quad 92$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a'_{37})^{(7)}\boxed{+(a''_{37})^{(7)}(T_{37},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{38}}{dt} \quad 93$$

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a'_{38})^{(7)}\boxed{+(a''_{38})^{(7)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{(a''_{36})^{(7)}(T_{37},t)}$, $\boxed{(a''_{37})^{(7)}(T_{37},t)}$, $\boxed{(a''_{38})^{(7)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,)}(T_{41},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{36}}{dt} = \quad 94$$

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b'_{36})^{(7)}\boxed{-(b''_{36})^{(7)}(G_{39},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{37}}{dt} =$$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b'_{37})^{(7)}\boxed{-(b''_{37})^{(7)}(G_{39},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{38}}{dt} =$$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b'_{38})^{(7)}\boxed{-(b''_{38})^{(7)}(G_{39},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{40}}{dt} \qquad 95$$

$$= (a_{40})^{(8)}G_{41} - \begin{bmatrix} (a'_{40})^{(8)}\boxed{+(a''_{40})^{(8)}(T_{41},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{41}}{dt}$$

$$= (a_{41})^{(8)}G_{40} - \begin{bmatrix} (a'_{41})^{(8)}\boxed{+(a''_{41})^{(8)}(T_{41},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \begin{bmatrix} (a'_{42})^{(8)}\boxed{+(a''_{42})^{(8)}(T_{41},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a''_{40})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)}\boxed{-(b''_{40})^{(8)}(G_{43},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)}\boxed{-(b''_{41})^{(8)}(G_{43},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)}\boxed{-(b''_{42})^{(8)}(G_{43},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{44}}{dt} \qquad 96$$

$$= (a_{44})^{(9)}G_{45} - \left[\begin{array}{ccc} (a'_{44})^{(9)}\boxed{+(a''_{44})^{(9)}(T_{45},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} \end{array}\right] G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[\begin{array}{ccc} (a'_{45})^{(9)}\boxed{+(a''_{45})^{(9)}(T_{45},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} \end{array}\right] G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[\begin{array}{ccc} (a'_{46})^{(9)}\boxed{+(a''_{46})^{(9)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} \end{array}\right] G_{15}$$

Where $\boxed{+(a''_{44})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{20}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{21}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{22}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{24}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{25}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{26}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{28}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{29}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{30}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{32}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{33}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{34}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a_{14}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a_{15}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ are Seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{38}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a_{37}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a_{36}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a_{40}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a_{42}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a_{41}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} =$$

$$(b_{44})^{(9)}T_{45} -$$

$$\begin{bmatrix} (b_{44}')^{(9)}\boxed{-(b_{44}'')^{(9)}(G_{47},t)} & \boxed{-(b_{16}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{20}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{24}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{28}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{32}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{13}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{36}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b_{45}')^{(9)}\boxed{-(b_{45}'')^{(9)}(G_{47},t)} & \boxed{-(b_{17}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{21}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{25}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{29}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{33}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{14}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{37}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b_{46}')^{(9)}\boxed{-(b_{46}'')^{(9)}(G_{47},t)} & \boxed{-(b_{18}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{22}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{26}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{30}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{34}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{15}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{38}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b_{44}'')^{(9)}(G_{47},t)}$, $\boxed{-(b_{45}'')^{(9)}(G_{47},t)}$, $\boxed{-(b_{46}'')^{(9)}(G_{47},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{16}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{17}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{18}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{20}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{21}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{22}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{24}'')^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{25}'')^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{26}'')^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{28}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{32}'')^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{34}'')^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{37}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{36}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a_i')^{(1)}, (a_i'')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (b_i'')^{(1)} > 0, \quad i,j = 13,14,15 \qquad 97$$

The functions $(a_i'')^{(1)}, (b_i'')^{(1)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a_i'')^{(1)}(T_{14},t) \leq (p_i)^{(1)} \leq (\hat{A}_{13})^{(1)}$$

$$(b_i'')^{(1)}(G,t) \leq (r_i)^{(1)} \leq (b_i')^{(1)} \leq (\hat{B}_{13})^{(1)}$$

$$lim_{T_2\to\infty}(a_i'')^{(1)}(T_{14},t) = (p_i)^{(1)} \qquad 98$$

$$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = (r_i)^{(1)}$$

**Definition of** $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$ :

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$$|(a_i'')^{(1)}(T_{14}',t) - (a_i'')^{(1)}(T_{14},t)| \leq (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G',t) - (b_i'')^{(1)}(G,t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}',t)$ and $(a_i'')^{(1)}(T_{14},t)$ . $(T_{14}',t)$ and $(T_{14},t)$ are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14},t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$ : 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$

**Definition of** $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$ : 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}$, $(r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)} (\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)} (\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i, j = 16,17,18$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17}, t) \leq (p_i)^{(2)} \leq (\hat{A}_{16})^{(2)} \quad 102$$

$$(b_i'')^{(2)}(G_{19}, t) \leq (r_i)^{(2)} \leq (b_i')^{(2)} \leq (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2 \to \infty} (a_i'')^{(2)}(T_{17}, t) = (p_i)^{(2)} \quad 104$$

$$\lim_{G \to \infty} (b_i'')^{(2)}((G_{19}), t) = (r_i)^{(2)} \quad 105$$

**Definition of** $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$ : 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}', t) - (a_i'')^{(2)}(T_{17}, t)| \leq (\hat{k}_{16})^{(2)} |T_{17} - T_{17}'| e^{-(\hat{M}_{16})^{(2)} t} \quad 107$$

$$|(b_i'')^{(2)}((G_{19})', t) - (b_i'')^{(2)}((G_{19}), t)| < (\hat{k}_{16})^{(2)} ||(G_{19}) - (G_{19})'|| e^{-(\hat{M}_{16})^{(2)} t} \quad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}', t)$ and$(a_i'')^{(2)}(T_{17}, t)$ . $(T_{17}', t)$ and $(T_{17}, t)$ are points belonging to the interval $[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}]$ . It is to be noted that $(a_i'')^{(2)}(T_{17}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17}, t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$ :

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)},$ are positive constants 109

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$$

**Definition of** $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$ :

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and\ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1 \quad 110$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(b_i)^{(2)} + (b_i')^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1 \quad 111$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i, j = 20,21,22 \quad 112$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(3)}$, $(r_i)^{(3)}$:

$$(a_i'')^{(3)}(T_{21}, t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23}, t) \leq (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(3)}(T_{21}, t) = (p_i)^{(3)} \quad 113$$

$$\lim_{G \to \infty}(b_i'')^{(3)}(G_{23}, t) = (r_i)^{(3)}$$

**Definition of** $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$ :

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: 114

$$|(a_i'')^{(3)}(T_{21}', t) - (a_i'')^{(3)}(T_{21}, t)| \leq (\hat{k}_{20})^{(3)} |T_{21} - T_{21}'| e^{-(\hat{M}_{20})^{(3)} t}$$

$$|(b_i'')^{(3)}(G_{23}', t) - (b_i'')^{(3)}(G_{23}, t)| < (\hat{k}_{20})^{(3)} ||G_{23} - G_{23}'|| e^{-(\hat{M}_{20})^{(3)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}', t)$ and$(a_i'')^{(3)}(T_{21}, t)$ . $(T_{21}', t)$ And $(T_{21}, t)$ are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21}, t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$ : 115

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}$ , $\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with 116
$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$ and the constants
$(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}, i = 20,21,22,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(a_i)^{(3)} + (a_i')^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)} (\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(b_i)^{(3)} + (b_i')^{(3)} + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)} (\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i, j = 24,25,26$$ 117

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(4)}$, $(r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25}, t) \le (p_i)^{(4)} \le (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}((G_{27}), t) \le (r_i)^{(4)} \le (b_i')^{(4)} \le (\hat{B}_{24})^{(4)}$$

$$lim_{T_2 \to \infty} (a_i'')^{(4)}(T_{25}, t) = (p_i)^{(4)}$$ 118

$$\lim_{G \to \infty} (b_i'')^{(4)}((G_{27}), t) = (r_i)^{(4)}$$

**Definition of** $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$ :

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24,25,26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}', t) - (a_i'')^{(4)}(T_{25}, t)| \le (\hat{k}_{24})^{(4)} |T_{25} - T_{25}'| e^{-(\hat{M}_{24})^{(4)} t}$$

$$|(b_i'')^{(4)}((G_{27})', t) - (b_i'')^{(4)}((G_{27}), t)| < (\hat{k}_{24})^{(4)} ||(G_{27}) - (G_{27})'|| e^{-(\hat{M}_{24})^{(4)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}', t)$ and $(a_i'')^{(4)}(T_{25}, t)$ . $(T_{25}', t)$ and $(T_{25}, t)$ are points belonging to the interval $\left[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}\right]$. It is to be noted that $(a_i'')^{(4)}(T_{25}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25}, t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$ : 120

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)},$ are positive constants

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}, \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$$

**Definition of** $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$ : 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and\ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}, i = 24,25,26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(a_i)^{(4)} + (a_i')^{(4)} + (\hat{A}_{24})^{(4)} + (\hat{P}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(b_i)^{(4)} + (b_i')^{(4)} + (\hat{B}_{24})^{(4)} + (\hat{Q}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i, j = 28,29,30$$ 122

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(5)}$, $(r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29}, t) \le (p_i)^{(5)} \le (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}((G_{31}), t) \le (r_i)^{(5)} \le (b_i')^{(5)} \le (\hat{B}_{28})^{(5)}$$

$$lim_{T_2 \to \infty} (a_i'')^{(5)} (T_{29}, t) = (p_i)^{(5)}$$ 123

$$\lim_{G \to \infty} (b_i'')^{(5)} (G_{31}, t) = (r_i)^{(5)}$$

**Definition of** $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$ :

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}', t) - (a_i'')^{(5)}(T_{29}, t)| \le (\hat{k}_{28})^{(5)} |T_{29} - T_{29}'| e^{-(\hat{M}_{28})^{(5)} t}$$

$$|(b_i'')^{(5)}((G_{31})', t) - (b_i'')^{(5)}((G_{31}), t)| < (\hat{k}_{28})^{(5)} ||(G_{31}) - (G_{31})'|| e^{-(\hat{M}_{28})^{(5)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}', t)$ and $(a_i'')^{(5)}(T_{29}, t)$ . $(T_{29}', t)$ and $(T_{29}, t)$ are points belonging to the interval $\left[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\right]$. It is to be noted that $(a_i'')^{(5)}(T_{29}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$ : 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)},$ are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}, \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

**Definition of** $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$ : 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and\ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(a_i)^{(5)} + (a_i')^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(b_i)^{(5)} + (b_i')^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i, j = 32,33,34$$ 127

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(6)}$, $(r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33}, t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}), t) \leq (r_i)^{(6)} \leq (b_i')^{(6)} \leq (\hat{B}_{32})^{(6)}$$

$$lim_{T_2 \to \infty} (a_i'')^{(6)} (T_{33}, t) = (p_i)^{(6)}$$ 128

$$\lim_{G \to \infty} (b_i'')^{(6)} \left((G_{35}), t\right) = (r_i)^{(6)}$$

**Definition of** $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$ :

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}', t) - (a_i'')^{(6)}(T_{33}, t)| \leq (\hat{k}_{32})^{(6)} |T_{33} - T_{33}'| e^{-(\hat{M}_{32})^{(6)} t}$$

$$|(b_i'')^{(6)}((G_{35})', t) - (b_i'')^{(6)}\left((G_{35}), t\right)| < (\hat{k}_{32})^{(6)} ||(G_{35}) - (G_{35})'|| e^{-(\hat{M}_{32})^{(6)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}', t)$ and $(a_i'')^{(6)}(T_{33}, t)$ . $(T_{33}', t)$ and $(T_{33}, t)$ are points belonging to the interval $\left[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}\right]$. It is to be noted that $(a_i'')^{(6)}(T_{33}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$ : 129

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)},$ are positive constants

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$$

**Definition of** $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$ : 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and\ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32,33,34,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(a_i)^{(6)} + (a_i')^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)} (\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(b_i)^{(6)} + (b_i')^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)} (\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

(KKKKKKKK) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i, j = 36,37,38$ 131

(LLLLLLLL) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(7)}$, $(r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37}, t) \le (p_i)^{(7)} \le (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39}, t) \le (r_i)^{(7)} \le (b_i')^{(7)} \le (\hat{B}_{36})^{(7)}$$

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*(MMMMMMMM)* $\lim_{T_2 \to \infty} (a_i'')^{(7)}(T_{37}, t) = (p_i)^{(7)}$

*(NNNNNNNN)*

$\lim_{G \to \infty} (b_i'')^{(7)}((G_{39}), t) = (r_i)^{(7)}$

**Definition of** $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$ :

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$$|(a_i'')^{(7)}(T_{37}', t) - (a_i'')^{(7)}(T_{37}, t)| \le (\hat{k}_{36})^{(7)} |T_{37} - T_{37}'| e^{-(\hat{M}_{36})^{(7)} t}$$

$$|(b_i'')^{(7)}((G_{39})', t) - (b_i'')^{(7)}((G_{39}), t)| < (\hat{k}_{36})^{(7)} ||(G_{39}) - (G_{39})'|| e^{-(\hat{M}_{36})^{(7)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}', t)$ and $(a_i'')^{(7)}(T_{37}, t)$ . $(T_{37}', t)$ and $(T_{37}, t)$ are points belonging to the interval $[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}]$ . It is to be noted that $(a_i'')^{(7)}(T_{37}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37}, t)$ , the **first augmentation coefficient** attributable to the

system, would be absolutely continuous.

**Definition of** $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$ : 134

(OOOOOOOO) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)},$ are positive constants

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$$

**Definition of** $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$ : 135

(PPPPPPPP) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(b_i)^{(7)} + (b_i')^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40,41,42 \qquad 136$$

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

**Definition of** $(p_i)^{(8)}$, $(r_i)^{(8)}$: 137

$$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)} \qquad 138$$

$$(b_i'')^{(8)}((G_{43}), t) \leq (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)} \qquad 139$$

$$\lim_{T_2 \to \infty} (a_i'')^{(8)}(T_{41}, t) = (p_i)^{(8)} \qquad 140$$

$$\lim_{G \to \infty} (b_i'')^{(8)}((G_{43}), t) = (r_i)^{(8)} \qquad 141$$

**Definition of** $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$ :

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)} |T_{41} - T_{41}'| e^{-(\hat{M}_{40})^{(8)} t} \qquad 142$$

$$|(b_i'')^{(8)}((G_{43})', t) - (b_i'')^{(8)}((G_{43}), t)| < (\hat{k}_{40})^{(8)} ||(G_{43}) - (G_{43})'|| e^{-(\hat{M}_{40})^{(8)} t} \qquad 143$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}', t)$ and $(a_i'')^{(8)}(T_{41}, t)$ . $(T_{41}', t)$ and $(T_{41}, t)$ are points belonging to the interval $\left[(\hat{k}_{40})^{(8)}, (\widehat{M}_{40})^{(8)}\right]$. It is to be noted that $(a_i'')^{(8)}(T_{41}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{40})^{(8)} = 1$ then the function $(a_i'')^{(8)}(T_{41}, t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\widehat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$ :

$(\widehat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)},$ are positive constants

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}, \frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}} < 1 \qquad 144$$

**Definition of** $(\hat{P}_{40})^{(8)}, (\hat{Q}_{40})^{(8)}$ :

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\widehat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}, (\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)}, (a_i')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}, i = 40,41,42,$
Satisfy the inequalities

$$\frac{1}{(\widehat{M}_{40})^{(8)}}[(a_i)^{(8)} + (a_i')^{(8)} + (\hat{A}_{40})^{(8)} + (\hat{P}_{40})^{(8)} (\hat{k}_{40})^{(8)}] < 1 \qquad 145$$

$$\frac{1}{(\widehat{M}_{40})^{(8)}}[(b_i)^{(8)} + (b_i')^{(8)} + (\hat{B}_{40})^{(8)} + (\hat{Q}_{40})^{(8)} (\hat{k}_{40})^{(8)}] < 1 \qquad 146$$

Where we suppose

$$(a_i)^{(9)}, (a_i')^{(9)}, (a_i'')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (b_i'')^{(9)} > 0, \quad i, j = 44,45,46 \qquad 146\,A$$

The functions $(a_i'')^{(9)}, (b_i'')^{(9)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(9)}$, $(r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45}, t) \leq (p_i)^{(9)} \leq (\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47}, t) \leq (r_i)^{(9)} \leq (b_i')^{(9)} \leq (\hat{B}_{44})^{(9)}$$

$$\lim_{T_2 \to \infty} (a_i'')^{(9)}(T_{45}, t) = (p_i)^{(9)}$$

$$\lim_{G \to \infty} (b_i'')^{(9)}(G_{47}, t) = (r_i)^{(9)}$$

**Definition of** $(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}$ :

Where $\boxed{(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}}$ are positive constants and $\boxed{i = 44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}', t) - (a_i'')^{(9)}(T_{45}, t)| \leq (\hat{k}_{44})^{(9)} |T_{45} - T_{45}'| e^{-(\widehat{M}_{44})^{(9)} t}$$

$$|(b_i'')^{(9)}((G_{47})', t) - (b_i'')^{(9)}\big((G_{47}), t\big)| < (\hat{k}_{44})^{(9)}||(G_{47}) - (G_{47})'||e^{-(\hat{M}_{44})^{(9)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}', t)$ and$(a_i'')^{(9)}(T_{45}, t)$ . $(T_{45}', t)$ and $(T_{45}, t)$ are points belonging to the interval $\big[(\hat{k}_{44})^{(9)}, (\hat{M}_{44})^{(9)}\big]$. It is to be noted that $(a_i'')^{(9)}(T_{45}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{44})^{(9)} = 1$ then the function $(a_i'')^{(9)}(T_{45}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$ :

$(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$, are positive constants

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}} \ , \frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$$

**Definition of** $(\hat{P}_{44})^{(9)}, (\hat{Q}_{44})^{(9)}$ :

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}, (\hat{A}_{44})^{(9)} and\ (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)}, (a_i')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (p_i)^{(9)}, \ (r_i)^{(9)}, i = 44,45,46,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{44})^{(9)}}[\,(a_i)^{(9)} + (a_i')^{(9)} + \ (\hat{A}_{44})^{(9)} + \ (\hat{P}_{44})^{(9)}\,(\hat{k}_{44})^{(9)}] < 1$$

$$\frac{1}{(\hat{M}_{44})^{(9)}}[\ (b_i)^{(9)} + (b_i')^{(9)} + \ (\hat{B}_{44})^{(9)} + \ (\hat{Q}_{44})^{(9)}\ (\hat{k}_{44})^{(9)}] < 1$$

**Theorem 1:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \big(\hat{P}_{13}\big)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \ , \ \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 2 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

**Definition of** $G_i(0)\,, T_i(0)$

$$G_i(t) \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} \quad , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} \quad , \quad T_i(0) = T_i^0 > 0$$

**Theorem 3 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t) \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t} \quad , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t} \quad , \quad T_i(0) = T_i^0 > 0$$

**Theorem 4 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

**Definition of** $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \left(\hat{P}_{24}\right)^{(4)} e^{(\hat{M}_{24})^{(4)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{24}\right)^{(4)} e^{(\hat{M}_{24})^{(4)}t} \;,\quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 5 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

**Definition of** $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \left(\hat{P}_{28}\right)^{(5)} e^{(\hat{M}_{28})^{(5)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{28}\right)^{(5)} e^{(\hat{M}_{28})^{(5)}t} \;,\quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 6 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

**Definition of** $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \left(\hat{P}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t} \;,\quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 7:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

**Definition of** $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \left(\hat{P}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t} \;,\quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 8:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

**Definition of** $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \left(\hat{P}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t} \;,\quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 9:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

**Definition of** $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \left(\hat{P}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

**Proof:** Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \leq (\hat{P}_{13})^{(1)} \text{ , } T_i^0 \leq (\hat{Q}_{13})^{(1)}, \quad 155$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t} \quad 156$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t} \quad 157$$

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[ (a_{13})^{(1)} G_{14}(s_{(13)}) - \left( (a'_{13})^{(1)} + a''_{13})^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \right) G_{13}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[ (a_{14})^{(1)} G_{13}(s_{(13)}) - \left( (a'_{14})^{(1)} + (a''_{14})^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \right) G_{14}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[ (a_{15})^{(1)} G_{14}(s_{(13)}) - \left( (a'_{15})^{(1)} + (a''_{15})^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \right) G_{15}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[ (b_{13})^{(1)} T_{14}(s_{(13)}) - \left( (b'_{13})^{(1)} - (b''_{13})^{(1)} (G(s_{(13)}), s_{(13)}) \right) T_{13}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[ (b_{14})^{(1)} T_{13}(s_{(13)}) - \left( (b'_{14})^{(1)} - (b''_{14})^{(1)} (G(s_{(13)}), s_{(13)}) \right) T_{14}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{T}_{15}(t) = T_{15}^0 + \int_0^t \left[ (b_{15})^{(1)} T_{14}(s_{(13)}) - \left( (b'_{15})^{(1)} - (b''_{15})^{(1)} (G(s_{(13)}), s_{(13)}) \right) T_{15}(s_{(13)}) \right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \leq (\hat{P}_{16})^{(2)} \text{ , } T_i^0 \leq (\hat{Q}_{16})^{(2)},$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$$

By 160

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[ (a_{16})^{(2)} G_{17}(s_{(16)}) - \left( (a'_{16})^{(2)} + a''_{16})^{(2)} (T_{17}(s_{(16)}), s_{(16)}) \right) G_{16}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[ (a_{17})^{(2)} G_{16}(s_{(16)}) - \left( (a'_{17})^{(2)} + (a''_{17})^{(2)} (T_{17}(s_{(16)}), s_{(17)}) \right) G_{17}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[ (a_{18})^{(2)} G_{17}(s_{(16)}) - \left( (a'_{18})^{(2)} + (a''_{18})^{(2)} (T_{17}(s_{(16)}), s_{(16)}) \right) G_{18}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^{0} + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^{0} + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^{0} + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{18}(s_{(16)})\right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{20})^{(3)}$, $T_i^0 \le (\hat{Q}_{20})^{(3)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\bar{M}_{20})^{(3)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\bar{M}_{20})^{(3)} t}$

By 161

$$\bar{G}_{20}(t) = G_{20}^{0} + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''_{20})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^{0} + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^{0} + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{22}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^{0} + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^{0} + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{\mathrm{T}}_{22}(\mathrm{t}) = \mathrm{T}_{22}^{0} + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{22}(s_{(20)})\right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{24})^{(4)}$, $T_i^0 \le (\hat{Q}_{24})^{(4)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\bar{M}_{24})^{(4)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\bar{M}_{24})^{(4)} t}$

By 162

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{26}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{\mathrm{T}}_{26}(\mathrm{t}) = \mathrm{T}_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{26}(s_{(24)})\right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$ , $T_i(0) = T_i^0$ , $G_i^0 \leq (\hat{P}_{28})^{(5)}$ , $T_i^0 \leq (\hat{Q}_{28})^{(5)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

By 163

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{30}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{\mathrm{T}}_{30}(\mathrm{t}) = \mathrm{T}_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{30}(s_{(28)})\right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$ , $T_i(0) = T_i^0$ , $G_i^0 \leq (\hat{P}_{32})^{(6)}$ , $T_i^0 \leq (\hat{Q}_{32})^{(6)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\tilde{M}_{32})^{(6)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\tilde{M}_{32})^{(6)} t}$

By 164

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[ (a_{32})^{(6)} G_{33}(s_{(32)}) - \left( (a'_{32})^{(6)} + a''_{32})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[ (a_{33})^{(6)} G_{32}(s_{(32)}) - \left( (a'_{33})^{(6)} + (a''_{33})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[ (a_{34})^{(6)} G_{33}(s_{(32)}) - \left( (a'_{34})^{(6)} + (a''_{34})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{34}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[ (b_{32})^{(6)} T_{33}(s_{(32)}) - \left( (b'_{32})^{(6)} - (b''_{32})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[ (b_{33})^{(6)} T_{32}(s_{(32)}) - \left( (b'_{33})^{(6)} - (b''_{33})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{34}(t) = T_{34}^0 + \int_0^t \left[ (b_{34})^{(6)} T_{33}(s_{(32)}) - \left( (b'_{34})^{(6)} - (b''_{34})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{34}(s_{(32)}) \right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\tilde{M}_{36})^{(7)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\tilde{M}_{36})^{(7)} t}$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[ (a_{36})^{(7)} G_{37}(s_{(36)}) - \left( (a'_{36})^{(7)} + a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[ (a_{37})^{(7)} G_{36}(s_{(36)}) - \left( (a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[ (a_{38})^{(7)} G_{37}(s_{(36)}) - \left( (a'_{38})^{(7)} + (a''_{38})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{38}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[ (b_{36})^{(7)} T_{37}(s_{(36)}) - \left( (b'_{36})^{(7)} - (b''_{36})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[ (b_{37})^{(7)} T_{36}(s_{(36)}) - \left( (b'_{37})^{(7)} - (b''_{37})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\overline{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^{0} + \int_0^t \left[ (b_{38})^{(7)} T_{37}(s_{(36)}) - \left( (b'_{38})^{(7)} - (b''_{38})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{38}(s_{(36)}) \right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{40})^{(8)}, T_i^0 \le (\hat{Q}_{40})^{(8)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[ (a_{40})^{(8)} G_{41}(s_{(40)}) - \left( (a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[ (a_{41})^{(8)} G_{40}(s_{(40)}) - \left( (a'_{41})^{(8)} + (a''_{41})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[ (a_{42})^{(8)} G_{41}(s_{(40)}) - \left( (a'_{42})^{(8)} + (a''_{42})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{42}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^0 + \int_0^t \left[ (b_{40})^{(8)} T_{41}(s_{(40)}) - \left( (b'_{40})^{(8)} - (b''_{40})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^0 + \int_0^t \left[ (b_{41})^{(8)} T_{40}(s_{(40)}) - \left( (b'_{41})^{(8)} - (b''_{41})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\overline{\mathrm{T}}_{42}(\mathrm{t}) = \mathrm{T}_{42}^{0} + \int_0^t \left[ (b_{42})^{(8)} T_{41}(s_{(40)}) - \left( (b'_{42})^{(8)} - (b''_{42})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{42}(s_{(40)}) \right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** 166
Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy A

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{44})^{(9)}, T_i^0 \le (\hat{Q}_{44})^{(9)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[ (a_{44})^{(9)} G_{45}(s_{(44)}) - \left( (a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45}(s_{(44)}), s_{(44)})\right) G_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}(s_{(44)}), s_{(44)})\right) G_{46}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{44}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{\mathrm{T}}_{46}(\mathrm{t}) = \mathrm{T}_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{46}(s_{(44)})\right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}}\right)\right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left(e^{(\hat{M}_{13})^{(1)} t} - 1\right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\hat{M}_{13})^{(1)} t} \le \frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{P}_{13})^{(1)} + G_{14}^0\right) e^{\left(-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0}\right)} + (\hat{P}_{13})^{(1)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \le G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}}\right)\right] ds_{(16)} = \quad 169$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1\right)$$

From which it follows that 170

$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \le \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left[\left((\hat{P}_{16})^{(2)} + G_{17}^0\right) e^{\left(-\frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0}\right)} + (\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t) \le G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} \left(G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} s_{(20)}}\right)\right] ds_{(20)} =$$

$$\left(1 + (a_{20})^{(3)} t\right) G_{21}^0 + \frac{(a_{20})^{(3)} (\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left(e^{(\hat{M}_{20})^{(3)} t} - 1\right)$$

From which it follows that 172

$$(G_{20}(t)-G_{20}^{0})e^{-(\hat{M}_{20})^{(3)}t}\leq\frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left[((\hat{P}_{20})^{(3)}+G_{21}^{0})e^{\left(-\frac{(\hat{P}_{20})^{(3)}+G_{21}^{0}}{G_{21}^{0}}\right)}+(\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 173

$$G_{24}(t)\leq G_{24}^{0}+\int_{0}^{t}\left[(a_{24})^{(4)}\left(G_{25}^{0}+(\hat{P}_{24})^{(4)}e^{(\hat{M}_{24})^{(4)}s_{(24)}}\right)\right]ds_{(24)}=$$

$$\left(1+(a_{24})^{(4)}t\right)G_{25}^{0}+\frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left(e^{(\hat{M}_{24})^{(4)}t}-1\right)$$

From which it follows that 174

$$(G_{24}(t)-G_{24}^{0})e^{-(\hat{M}_{24})^{(4)}t}\leq\frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left[((\hat{P}_{24})^{(4)}+G_{25}^{0})e^{\left(-\frac{(\hat{P}_{24})^{(4)}+G_{25}^{0}}{G_{25}^{0}}\right)}+(\hat{P}_{24})^{(4)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t)\leq G_{28}^{0}+\int_{0}^{t}\left[(a_{28})^{(5)}\left(G_{29}^{0}+(\hat{P}_{28})^{(5)}e^{(\hat{M}_{28})^{(5)}s_{(28)}}\right)\right]ds_{(28)}=$$

$$\left(1+(a_{28})^{(5)}t\right)G_{29}^{0}+\frac{(a_{28})^{(5)}(\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left(e^{(\hat{M}_{28})^{(5)}t}-1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t}\leq\frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t)\leq G_{32}^{0}+\int_{0}^{t}\left[(a_{32})^{(6)}\left(G_{33}^{0}+(\hat{P}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\right)\right]ds_{(32)}=$$

$$\left(1+(a_{32})^{(6)}t\right)G_{33}^{0}+\frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left(e^{(\hat{M}_{32})^{(6)}t}-1\right)$$

From which it follows that 177

$$(G_{32}(t)-G_{32}^{0})e^{-(\hat{M}_{32})^{(6)}t}\leq\frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left[((\hat{P}_{32})^{(6)}+G_{33}^{0})e^{\left(-\frac{(\hat{P}_{32})^{(6)}+G_{33}^{0}}{G_{33}^{0}}\right)}+(\hat{P}_{32})^{(6)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

(gg) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t) \le G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} \left(G_{37}^0 + (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} s_{(36)}}\right)\right] ds_{(36)} =$$
$$\left(1 + (a_{36})^{(7)} t\right) G_{37}^0 + \frac{(a_{36})^{(7)} (\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left(e^{(\hat{M}_{36})^{(7)} t} - 1\right)$$

From which it follows that

$$(G_{36}(t) - G_{36}^0) e^{-(\hat{M}_{36})^{(7)} t} \le \frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left[\left((\hat{P}_{36})^{(7)} + G_{37}^0\right) e^{\left(-\frac{(\hat{P}_{36})^{(7)} + G_{37}^0}{G_{37}^0}\right)} + (\hat{P}_{36})^{(7)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t) \le G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} \left(G_{41}^0 + (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} s_{(40)}}\right)\right] ds_{(40)} = \quad 180$$
$$\left(1 + (a_{40})^{(8)} t\right) G_{41}^0 + \frac{(a_{40})^{(8)} (\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}} \left(e^{(\hat{M}_{40})^{(8)} t} - 1\right)$$

From which it follows that 181

$$(G_{40}(t) - G_{40}^0) e^{-(\hat{M}_{40})^{(8)} t} \le \frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}} \left[\left((\hat{P}_{40})^{(8)} + G_{41}^0\right) e^{\left(-\frac{(\hat{P}_{40})^{(8)} + G_{41}^0}{G_{41}^0}\right)} + (\hat{P}_{40})^{(8)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 8

Analogous inequalities hold also for $G_{41}, G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t) \le G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} \left(G_{45}^0 + (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} s_{(44)}}\right)\right] ds_{(44)} =$$
$$\left(1 + (a_{44})^{(9)} t\right) G_{45}^0 + \frac{(a_{44})^{(9)} (\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left(e^{(\hat{M}_{44})^{(9)} t} - 1\right)$$

From which it follows that

$$(G_{44}(t) - G_{44}^0) e^{-(\hat{M}_{44})^{(9)} t} \le \frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left[\left((\hat{P}_{44})^{(9)} + G_{45}^0\right) e^{\left(-\frac{(\hat{P}_{44})^{(9)} + G_{45}^0}{G_{45}^0}\right)} + (\hat{P}_{44})^{(9)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 9

Analogous inequalities hold also for $G_{45}, G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$ and to choose 182

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[(\hat{P}_{13})^{(1)}+\left((\hat{P}_{13})^{(1)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{13})^{(1)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{13})^{(1)} \quad 183$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[\left((\hat{Q}_{13})^{(1)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{13})^{(1)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{13})^{(1)}\right]\leq(\hat{Q}_{13})^{(1)} \quad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)},T^{(1)}\right),\left(G^{(2)},T^{(2)}\right)\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{13})^{(1)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{13})^{(1)}t}\}$$

Indeed if we denote

**Definition of** $\tilde{G},\tilde{T}:\left(\tilde{G},\tilde{T}\right)=\mathcal{A}^{(1)}(G,T)$

It results

$$\left|\tilde{G}_{13}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{13})^{(1)}\left|G_{14}^{(1)}-G_{14}^{(2)}\right|e^{-(\hat{M}_{13})^{(1)}s_{(13)}}e^{(\hat{M}_{13})^{(1)}s_{(13)}}ds_{(13)}+$$

$$\int_0^t\{(a'_{13})^{(1)}\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\hat{M}_{13})^{(1)}s_{(13)}}e^{-(\hat{M}_{13})^{(1)}s_{(13)}}+$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\hat{M}_{13})^{(1)}s_{(13)}}e^{(\hat{M}_{13})^{(1)}s_{(13)}}+$$

$$G_{13}^{(2)}\left|(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)-(a''_{13})^{(1)}\left(T_{14}^{(2)},s_{(13)}\right)\right|\ e^{-(\hat{M}_{13})^{(1)}s_{(13)}}e^{(\hat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|G^{(1)}-G^{(2)}\right|e^{-(\hat{M}_{13})^{(1)}t}\leq \quad 186$$
$$\frac{1}{(\hat{M}_{13})^{(1)}}\left((a_{13})^{(1)}+(a'_{13})^{(1)}+(\hat{A}_{13})^{(1)}+(\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}\right)d\left(\left(G^{(1)},T^{(1)};\ G^{(2)},T^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

**Remark 1:** The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\hat{P}_{13})^{(1)}e^{(\hat{M}_{13})^{(1)}t}\ and\ (\hat{Q}_{13})^{(1)}e^{(\hat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i = 13,14,15$ depend only on $T_{14}$ and respectively on $G(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any $t$ where $G_i(t) = 0\ and\ T_i(t) = 0$

From 19 to 24 it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(1)} - (a_i'')^{(1)}(T_{14}(s_{(13)}), s_{(13)})\} ds_{(13)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(1)} t)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2\ and\ \left((\widehat{M}_{13})^{(1)}\right)_3$ : 187

**Remark 3**: if $G_{13}$ is bounded, the same property have also $G_{14}\ and\ G_{15}$ . indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a_{14}')^{(1)} G_{14}$ and by integrating

$$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)} \left((\widehat{M}_{13})^{(1)}\right)_1 / (a_{14}')^{(1)}$$

In the same way , one can obtain

$$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)} \left((\widehat{M}_{13})^{(1)}\right)_2 / (a_{15}')^{(1)}$$

If $G_{14}\ or\ G_{15}$ is bounded, the same property follows for $G_{13}$ , $G_{15}$ and $G_{13}$ , $G_{14}$ respectively.

**Remark 4:** If $G_{13}$ *is* bounded, from below, the same property holds for $G_{14}\ and\ G_{15}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{14}$ is bounded from below. 188

**Remark 5:** If $T_{13}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}(G(t), t)) = (b_{14}')^{(1)}$ then $T_{14} \to \infty$. 189

**Definition of** $(m)^{(1)}$ and $\varepsilon_1$ :

Indeed let $t_1$ be so that for $t > t_1$

$$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t), t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right)$, $t = log\frac{2}{\varepsilon_1}$ By taking now $\varepsilon_1$ sufficiently small one sees that $T_{14}$ is unbounded.
The same property holds for $T_{15}$ if $\lim_{t\to\infty}(b_{15}'')^{(1)}(G(t), t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}$ , $\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}\ and\ (\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[(\widehat{P}_{16})^{(2)}+\left((\hat{P}_{16})^{(2)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{16})^{(2)}\right]\leq(\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

**Definition of** $\widetilde{G_{19}},\widetilde{T_{19}}$ : $\left(\widetilde{G_{19}},\widetilde{T_{19}}\right)=\mathcal{A}^{(2)}(G_{19},T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{16})^{(2)}\left|G_{17}^{(1)}-G_{17}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)}+$$

$$\int_0^t\{(a'_{16})^{(2)}\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$G_{16}^{(2)}\left|(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)-(a''_{16})^{(2)}\left(T_{17}^{(2)},s_{(16)}\right)\right|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0,t]$ 197

From the hypotheses it follows

$$\left|(G_{19})^{(1)}-(G_{19})^{(2)}\right|\mathrm{e}^{-(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}\leq$$
$$\frac{1}{(\widehat{\mathrm{M}}_{16})^{(2)}}\left((a_{16})^{(2)}+(a'_{16})^{(2)}+(\widehat{\mathrm{A}}_{16})^{(2)}+(\widehat{\mathrm{P}}_{16})^{(2)}(\hat{k}_{16})^{(2)}\right)\mathrm{d}\left(((G_{19})^{(1)},(T_{19})^{(1)};\ (G_{19})^{(2)},(T_{19})^{(2)})\right)$$

And analogous inequalities for $G_i$ and $T_i$. Taking into account the hypothesis the result follows 198

**Remark 6:** The fact that we supposed $(a''_{16})^{(2)}$ and $(b''_{16})^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{\mathrm{P}}_{16})^{(2)}\mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}$ and $(\widehat{\mathrm{Q}}_{16})^{(2)}\mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i=16,17,18$ depend only on $\mathrm{T}_{17}$ and respectively on

$(G_{19})$(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 7:** There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$ 200

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(2)} - (a_i'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})\} ds_{(16)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(2)}t)} > 0 \quad \text{for } t > 0$$

**Definition of** $\left((\widehat{M}_{16})^{(2)}\right)_1$, $\left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$: 201

**Remark 8:** if $G_{16}$ is bounded, the same property have also $G_{17}$ and $G_{18}$. indeed if

$G_{16} < (\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt} \leq \left((\widehat{M}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)} G_{17}$ and by integrating

$$G_{17} \leq \left((\widehat{M}_{16})^{(2)}\right)_2 = G_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1 / (a_{17}')^{(2)}$$

In the same way , one can obtain

$$G_{18} \leq \left((\widehat{M}_{16})^{(2)}\right)_3 = G_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2 / (a_{18}')^{(2)}$$

If $G_{17}$ or $G_{18}$ is bounded, the same property follows for $G_{16}$, $G_{18}$ and $G_{16}$, $G_{17}$ respectively.

**Remark 9:** If $G_{16}$ is bounded, from below, the same property holds for $G_{17}$ and $G_{18}$. The proof is analogous with the preceding one. An analogous property is true if $G_{17}$ is bounded from below. 202

**Remark 10:** If $T_{16}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t), t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$. 203

**Definition of** $(m)^{(2)}$ and $\varepsilon_2$ :

Indeed let $t_2$ be so that for $t > t_2$

$$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right)$, $t = \log\frac{2}{\varepsilon_2}$ By taking now $\varepsilon_2$ sufficiently small one sees that $T_{17}$ is unbounded. 205
The same property holds for $T_{18}$ if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t), t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[(\widehat{P}_{20})^{(3)}+\left((\hat{P}_{20})^{(3)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{20})^{(3)}\right]\leq(\hat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself 210

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)},(T_{23})^{(1)}),((G_{23})^{(2)},(T_{23})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

**Definition of** $\widetilde{G_{23}},\widetilde{T_{23}}:(\widetilde{(G_{23})},\widetilde{(T_{23})})=\mathcal{A}^{(3)}((G_{23}),(T_{23}))$

It results 213

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{20})^{(3)}\left|G_{21}^{(1)}-G_{21}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}ds_{(20)}+$$

$$\int_0^t\{(a'_{20})^{(3)}\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$G_{20}^{(2)}\left|(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)},s_{(20)}\right)\right|\ e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\}ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses it follows

$$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}t}\leq \quad 214$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+(a'_{20})^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\widehat{k}_{20})^{(3)}\right)d\left(((G_{23})^{(1)},(T_{23})^{(1)};\ (G_{23})^{(2)},(T_{23})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 11:** The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ *and* $(\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i=20,21,22$ depend only on $T_{21}$ and respectively on

$(G_{23})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 12:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 216

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(3)} - (a_i'')^{(3)}(T_{21}(s_{(20)}), s_{(20)})\} ds_{(20)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(3)} t)} > 0$ for t > 0

**Definition of** $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2\ and\ \left((\widehat{M}_{20})^{(3)}\right)_3$ : 217

**Remark 13:** if $G_{20}$ is bounded, the same property have also $G_{21}\ and\ G_{22}$ . indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a_{21}')^{(3)} G_{21}$ and by integrating

$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1 / (a_{21}')^{(3)}$

In the same way , one can obtain

$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2 / (a_{22}')^{(3)}$

If $G_{21}\ or\ G_{22}$ is bounded, the same property follows for $G_{20}$ , $G_{22}$ and $G_{20}$ , $G_{21}$ respectively.

**Remark 14:** If $G_{20}$ *is* bounded, from below, the same property holds for $G_{21}\ and\ G_{22}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{21}$ is bounded from below. 218

**Remark 15:** If $T_{20}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t), t\right)) = (b_{21}')^{(3)}$ then $T_{21} \to \infty$. 219

**Definition of** $(m)^{(3)}$ and $\varepsilon_3$ :

Indeed let $t_3$ be so that for $t > t_3$

$(b_{21})^{(3)} - (b_i'')^{(3)}\left((G_{23})(t), t\right) < \varepsilon_3, T_{20}\,(t) > (m)^{(3)}$

Then $\frac{dT_{21}}{dt} \geq (a_{21})^{(3)}(m)^{(3)} - \varepsilon_3 T_{21}$ which leads to 220

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t} = \frac{1}{2}$ it results

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right),\ \ t = log\,\frac{2}{\varepsilon_3}$ By taking now $\varepsilon_3$ sufficiently small one sees that $T_{21}$ is unbounded.
The same property holds for $T_{22}$ if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t), t\right) = (b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}$ , $\frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}} < 1$ and to choose 221

$(\widehat{P}_{24})^{(4)}$ and $(\widehat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[(\widehat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \tag{222}$$

$$\frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \tag{223}$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself (224)

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric (225)

$$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{24})^{(4)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{27}}),(\widetilde{T_{27}})$ : $\left((\widetilde{G_{27}}),(\widetilde{T_{27}})\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{24})^{(4)}\left|G_{25}^{(1)}-G_{25}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\,ds_{(24)}+$$

$$\int_0^t\{(a'_{24})^{(4)}\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$G_{24}^{(2)}\left|(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)-(a''_{24})^{(4)}\left(T_{25}^{(2)},s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\}ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}t}\leq \tag{226}$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)}+(a'_{24})^{(4)}+(\widehat{A}_{24})^{(4)}+(\widehat{P}_{24})^{(4)}(\hat{k}_{24})^{(4)}\right)d\left(((G_{27})^{(1)},(T_{27})^{(1)};\ (G_{27})^{(2)},(T_{27})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 16:** The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ *and* $(\widehat{Q}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_+$. (227)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i=24,25,26$ depend only on $T_{25}$ and respectively on

$(G_{27})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 17:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 228

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(4)} - (a_i'')^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(4)} t)} > 0$ for t > 0

**Definition of** $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2\ and\ \left((\widehat{M}_{24})^{(4)}\right)_3$ : 229

**Remark 18:** if $G_{24}$ is bounded, the same property have also $G_{25}\ and\ G_{26}$ . indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a_{25}')^{(4)} G_{25}$ and by integrating

$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1 / (a_{25}')^{(4)}$

In the same way , one can obtain

$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2 / (a_{26}')^{(4)}$

If $G_{25}\ or\ G_{26}$ is bounded, the same property follows for $G_{24}$ , $G_{26}$ and $G_{24}$ , $G_{25}$ respectively.

**Remark 19:** If $G_{24}$ *is* bounded, from below, the same property holds for $G_{25}\ and\ G_{26}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{25}$ is bounded from below. 230

**Remark 20:** If $T_{24}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}\ ((G_{27})(t), t)) = (b_{25}')^{(4)}$ then $T_{25} \to \infty$. 231

**Definition of** $(m)^{(4)}$ and $\varepsilon_4$ :

Indeed let $t_4$ be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t), t) < \varepsilon_4, T_{24}\,(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_4}$ By taking now $\varepsilon_4$ sufficiently small one sees that $T_{25}$ is unbounded.
The same property holds for $T_{26}$ if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t), t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for $G_{29}$ , $G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}$ , $\frac{(b_i)^{(5)}}{(\widehat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)}+\left((\hat{P}_{28})^{(5)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{28})^{(5)} \tag{234}$$

$$\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{28})^{(5)}\right]\leq(\hat{Q}_{28})^{(5)} \tag{235}$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric (236)

$$d\left(((G_{31})^{(1)},(T_{31})^{(1)}),((G_{31})^{(2)},(T_{31})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\hat{M}_{28})^{(5)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\hat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

**Definition of** $\widetilde{(G_{31})},\widetilde{(T_{31})}$ : $\left(\widetilde{(G_{31})},\widetilde{(T_{31})}\right)=\mathcal{A}^{(5)}((G_{31}),(T_{31}))$

It results

$$|\tilde{G}_{28}^{(1)}-\tilde{G}_i^{(2)}|\leq\int_0^t(a_{28})^{(5)}|G_{29}^{(1)}-G_{29}^{(2)}|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\,ds_{(28)}+$$

$$\int_0^t\{(a'_{28})^{(5)}|G_{28}^{(1)}-G_{28}^{(2)}|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$(a''_{28})^{(5)}(T_{29}^{(1)},s_{(28)})|G_{28}^{(1)}-G_{28}^{(2)}|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$G_{28}^{(2)}|(a''_{28})^{(5)}(T_{29}^{(1)},s_{(28)})-(a''_{28})^{(5)}(T_{29}^{(2)},s_{(28)})|\;e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\}ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on it follows

$$|(G_{31})^{(1)}-(G_{31})^{(2)}|e^{-(\widehat{M}_{28})^{(5)}t}\leq \tag{237}$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}((a_{28})^{(5)}+(a'_{28})^{(5)}+(\widehat{A}_{28})^{(5)}+(\widehat{P}_{28})^{(5)}(\widehat{k}_{28})^{(5)})d\left(((G_{31})^{(1)},(T_{31})^{(1)};\,(G_{31})^{(2)},(T_{31})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 21:** The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\hat{P}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ *and* $(\hat{Q}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ respectively of $\mathbb{R}_+$. (238)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on $T_{29}$ and respectively on $(G_{31})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 22:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 239

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(5)} - (a_i'')^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(5)} t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2\ and\ \left((\widehat{M}_{28})^{(5)}\right)_3$ : 240

**Remark 23:** if $G_{28}$ is bounded, the same property have also $G_{29}\ and\ G_{30}$ . indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a_{29}')^{(5)} G_{29}$ and by integrating

$$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1 / (a_{29}')^{(5)}$$

In the same way , one can obtain

$$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2 / (a_{30}')^{(5)}$$

If $G_{29}\ or\ G_{30}$ is bounded, the same property follows for $G_{28}$ , $G_{30}$ and $G_{28}$ , $G_{29}$ respectively.

**Remark 24:** If $G_{28}$ *is* bounded, from below, the same property holds for $G_{29}\ and\ G_{30}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{29}$ is bounded from below. 241

**Remark 25:** If $T_{28}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}\,((G_{31})(t), t)) = (b_{29}')^{(5)}$ then $T_{29} \to \infty$. 242

**Definition of** $(m)^{(5)}$ and $\varepsilon_5$ :

Indeed let $t_5$ be so that for $t > t_5$

$$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t), t) < \varepsilon_5, T_{28}\,(t) > (m)^{(5)}$$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_5}$ By taking now $\varepsilon_5$ sufficiently small one sees that $T_{29}$ is unbounded.
The same property holds for $T_{30}$ if $\lim_{t\to\infty}(b_{30}'')^{(5)}\left((G_{31})(t), t\right) = (b_{30}')^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{33}$ , $G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}$ , $\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}} < 1$ and to choose 244

$(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[(\hat{P}_{32})^{(6)}+\left((\hat{P}_{32})^{(6)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{32})^{(6)} \tag{245}$$

$$\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[\left((\hat{Q}_{32})^{(6)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{32})^{(6)}\right]\leq(\hat{Q}_{32})^{(6)} \tag{246}$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric (247)

$$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\hat{M}_{32})^{(6)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\hat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{35}}),(\widetilde{T_{35}})$ : $\left((\widetilde{G_{35}}),(\widetilde{T_{35}})\right)=\mathcal{A}^{(6)}\left((G_{35}),(T_{35})\right)$

It results

$$|\tilde{G}_{32}^{(1)}-\tilde{G}_i^{(2)}|\leq\int_0^t(a_{32})^{(6)}|G_{33}^{(1)}-G_{33}^{(2)}|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\,ds_{(32)}+$$

$$\int_0^t\{(a'_{32})^{(6)}|G_{32}^{(1)}-G_{32}^{(2)}|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)|G_{32}^{(1)}-G_{32}^{(2)}|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$G_{32}^{(2)}|(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)-(a''_{32})^{(6)}\left(T_{33}^{(2)},s_{(32)}\right)|\;e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\}ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$|(G_{35})^{(1)}-(G_{35})^{(2)}|e^{-(\widehat{M}_{32})^{(6)}t}\leq \tag{248}$$
$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)}+(a'_{32})^{(6)}+(\widehat{A}_{32})^{(6)}+(\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}\right)d\left(((G_{35})^{(1)},(T_{35})^{(1)};\;(G_{35})^{(2)},(T_{35})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 26:** The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\hat{P}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ *and* $(\hat{Q}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ respectively of $\mathbb{R}_+$. (249)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i = 32,33,34$ depend only on $T_{33}$ and respectively on $(G_{35})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 27:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 250

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(6)} - (a_i'')^{(6)}(T_{33}(s_{(32)}), s_{(32)})\} ds_{(32)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(6)} t\right)} > 0$ for t > 0

**Definition of** $\left((\widehat{M}_{32})^{(6)}\right)_1, \left((\widehat{M}_{32})^{(6)}\right)_2\ and\ \left((\widehat{M}_{32})^{(6)}\right)_3$ : 251

**Remark 28:** if $G_{32}$ is bounded, the same property have also $G_{33}\ and\ G_{34}$ . indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a_{33}')^{(6)} G_{33}$ and by integrating

$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_1 / (a_{33}')^{(6)}$

In the same way , one can obtain

$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_2 / (a_{34}')^{(6)}$

If $G_{33}\ or\ G_{34}$ is bounded, the same property follows for $G_{32}$ , $G_{34}$ and $G_{32}$ , $G_{33}$ respectively.

**Remark 29:** If $G_{32}$ *is* bounded, from below, the same property holds for $G_{33}\ and\ G_{34}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{33}$ is bounded from below. 252

**Remark 30:** If $T_{32}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}\ ((G_{35})(t), t)) = (b_{33}')^{(6)}$ then $T_{33}\ \to \infty$. 253

**Definition of** $(m)^{(6)}$ and $\varepsilon_6$ :

Indeed let $t_6$ be so that for $t\ > t_6$

$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t), t\right) < \varepsilon_6, T_{32}\,(t) > (m)^{(6)}$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33}\ \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33}\ \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_6}$ By taking now $\varepsilon_6$ sufficiently small one sees that $T_{33}$ is unbounded.
The same property holds for $T_{34}$ if $\lim_{t\to\infty}(b_{34}'')^{(6)}\ \left((G_{35})(t), t(t), t\right) = (b_{34}')^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{37}\ , G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\ , \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$ and to choose

$(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[(\hat{P}_{36})^{(7)}+\left((\hat{P}_{36})^{(7)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{36})^{(7)} \tag{256}$$

$$\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[\left((\hat{Q}_{36})^{(7)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{36})^{(7)}\right]\leq(\hat{Q}_{36})^{(7)} \tag{257}$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself
The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric (258)

$$d\left(((G_{39})^{(1)},(T_{39})^{(1)}),((G_{39})^{(2)},(T_{39})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\widehat{M}_{36})^{(7)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\widehat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

**Definition of** $\widetilde{(G_{39})},\widetilde{(T_{39})}:(\widetilde{(G_{39})},\widetilde{(T_{39})})=\mathcal{A}^{(7)}((G_{39}),(T_{39}))$
It results

$$|\tilde{G}_{36}^{(1)}-\tilde{G}_i^{(2)}|\leq\int_0^t(a_{36})^{(7)}|G_{37}^{(1)}-G_{37}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\,ds_{(36)}+$$
$$\int_0^t\{(a'_{36})^{(7)}|G_{36}^{(1)}-G_{36}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$(a''_{36})^{(7)}(T_{37}^{(1)},s_{(36)})|G_{36}^{(1)}-G_{36}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$G_{36}^{(2)}|(a''_{36})^{(7)}(T_{37}^{(1)},s_{(36)})-(a''_{36})^{(7)}(T_{37}^{(2)},s_{(36)})|\;e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\}ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on it follows

$$|(G_{39})^{(1)}-(G_{39})^{(2)}|e^{-(\widehat{M}_{36})^{(7)}t}\leq \tag{259}$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}}\left((a_{36})^{(7)}+(a'_{36})^{(7)}+(\hat{A}_{36})^{(7)}+(\hat{P}_{36})^{(7)}(\hat{k}_{36})^{(7)}\right)d\left(((G_{39})^{(1)},(T_{39})^{(1)};\,(G_{39})^{(2)},(T_{39})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 31:** The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\hat{P}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ *and* $(\hat{Q}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ respectively of $\mathbb{R}_+$. (260)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(7)}$ and $(b_i'')^{(7)}, i = 36,37,38$ depend only on $T_{37}$ and respectively on $(G_{39})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 32:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 261

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(7)} - (a_i'')^{(7)}(T_{37}(s_{(36)}), s_{(36)})\} ds_{(36)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(7)} t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2\ and\ \left((\widehat{M}_{36})^{(7)}\right)_3$ : 262

**Remark 33:** if $G_{36}$ is bounded, the same property have also $G_{37}\ and\ G_{38}$ . indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a_{37}')^{(7)} G_{37}$ and by integrating

$$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1 / (a_{37}')^{(7)}$$

In the same way , one can obtain

$$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2 / (a_{38}')^{(7)}$$

If $G_{37}\ or\ G_{38}$ is bounded, the same property follows for $G_{36}$ , $G_{38}$ and $G_{36}$ , $G_{37}$ respectively.

**Remark 34:** If $G_{36}$ *is* bounded, from below, the same property holds for $G_{37}\ and\ G_{38}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{37}$ is bounded from below. 263

**Remark 35:** If $T_{36}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(7)}\ ((G_{39})(t), t)) = (b_{37}')^{(7)}$ then $T_{37} \to \infty$. 264

**Definition of** $(m)^{(7)}$ and $\varepsilon_7$ :

Indeed let $t_7$ be so that for $t > t_7$

$$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t), t) < \varepsilon_7, T_{36}\,(t) > (m)^{(7)}$$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_7}$ By taking now $\varepsilon_7$ sufficiently small one sees that $T_{37}$ is unbounded.
The same property holds for $T_{38}$ if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t), t\right) = (b_{38}')^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}$ , $\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}} < 1$ and to choose $(\widehat{P}_{40})^{(8)}$ and $(\widehat{Q}_{40})^{(8)}$ large to have 266

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[(\widehat{P}_{40})^{(8)} + \left((\hat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{40})^{(8)} \quad 267$$

$$\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{40})^{(8)}\right] \leq (\hat{Q}_{40})^{(8)} \quad 268$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)}, (T_{43})^{(1)}), ((G_{43})^{(2)}, (T_{43})^{(2)})\right) = \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

**Definition of** $\widetilde{(G_{43})}, \widetilde{(T_{43})}$ : $\left(\widetilde{(G_{43})}, \widetilde{(T_{43})}\right) = \mathcal{A}^{(8)}((G_{43}), (T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{40})^{(8)}\left|G_{41}^{(1)} - G_{41}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\,ds_{(40)} +$$
$$\int_0^t\{(a'_{40})^{(8)}\left|G_{40}^{(1)} - G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{-(\widehat{M}_{40})^{(8)}s_{(40)}} +$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right)\left|G_{40}^{(1)} - G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}} +$$
$$G_{40}^{(2)}\left|(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right) - (a''_{40})^{(8)}\left(T_{41}^{(2)}, s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\}ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0, t]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)} - (G_{43})^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}t} \leq \quad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)} + (a'_{40})^{(8)} + (\widehat{A}_{40})^{(8)} + (\widehat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}\right)d\left(((G_{43})^{(1)}, (T_{43})^{(1)};\ (G_{43})^{(2)}, (T_{43})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

**Remark 36:** The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate 274

condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ $and$ $(\widehat{Q}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i = 40,41,42$ depend only on $T_{41}$ and respectively on $(G_{43})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 37** There does not exist any $t$ where $G_i(t) = 0\ and\ T_i(t) = 0$ 275

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(8)} - (a_i'')^{(8)}(T_{41}(s_{(40)}), s_{(40)})\} ds_{(40)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(8)}t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{40})^{(8)}\right)_1, \left((\widehat{M}_{40})^{(8)}\right)_2$ $and$ $\left((\widehat{M}_{40})^{(8)}\right)_3$ : 276

**Remark 38:** if $G_{40}$ is bounded, the same property have also $G_{41}$ $and$ $G_{42}$ . indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a_{41}')^{(8)}G_{41}$ and by integrating

$$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1/(a_{41}')^{(8)}$$

In the same way , one can obtain

$$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2/(a_{42}')^{(8)}$$

If $G_{41}$ $or$ $G_{42}$ is bounded, the same property follows for $G_{40}$ , $G_{42}$ and $G_{40}$ , $G_{41}$ respectively.

**Remark 39:** If $G_{40}$ $is$ bounded, from below, the same property holds for $G_{41}$ $and$ $G_{42}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{41}$ is bounded from below. 277

**Remark 40:** If $T_{40}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}((G_{43})(t), t)) = (b_{41}')^{(8)}$ then $T_{41} \to \infty$. 278

**Definition of** $(m)^{(8)}$ and $\varepsilon_8$ :

Indeed let $t_8$ be so that for $t > t_8$

$$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t), t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right)$, $t = log\frac{2}{\varepsilon_8}$ By taking now $\varepsilon_8$ sufficiently small one sees that $T_{41}$ is unbounded.
The same property holds for $T_{42}$ if $\lim_{t\to\infty}(b''_{42})^{(8)}\left((G_{43})(t), t(t), t\right) = (b'_{42})^{(8)}$



It is now sufficient to take $\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}$ , $\frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$ and to choose $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ large to have

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}\left[(\hat{P}_{44})^{(9)} + \left((\hat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}}\left[\left((\hat{Q}_{44})^{(9)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{44})^{(9)}\right] \leq (\hat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)}, (T_{47})^{(1)}), ((G_{47})^{(2)}, (T_{47})^{(2)})\right) =$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{44})^{(9)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{47}}), (\widetilde{T_{47}}) : \left((\widetilde{G_{47}}), (\widetilde{T_{47}})\right) = \mathcal{A}^{(9)}\left((G_{47}), (T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{44})^{(9)}\left|G_{45}^{(1)} - G_{45}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\,ds_{(44)} +$$
$$\int_0^t\{(a'_{44})^{(9)}\left|G_{44}^{(1)} - G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{-(\widehat{M}_{44})^{(9)}s_{(44)}} +$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right)\left|G_{44}^{(1)} - G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}} +$$
$$G_{44}^{(2)}\left|(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right) - (a''_{44})^{(9)}\left(T_{45}^{(2)}, s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\}ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)} - G^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}t} \leq$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)} + (a'_{44})^{(9)} + (\widehat{A}_{44})^{(9)} + (\widehat{P}_{44})^{(9)}(\hat{k}_{44})^{(9)}\right)d\left(((G_{47})^{(1)}, (T_{47})^{(1)};\ (G_{47})^{(2)}, (T_{47})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis (39,35,36) the result follows

**Remark 41:** The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by

$(\widehat{P}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ *and* $(\widehat{Q}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i = 44{,}45{,}46$ depend only on $T_{45}$ and respectively on $(G_{47})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 42:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$

From 99 to 44 it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t\{(a_i')^{(9)}-(a_i'')^{(9)}(T_{45}(s_{(44)}),s_{(44)})\}ds_{(44)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(9)}t)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{44})^{(9)}\right)_1, \left((\widehat{M}_{44})^{(9)}\right)_2$ *and* $\left((\widehat{M}_{44})^{(9)}\right)_3$ :

**Remark 43:** if $G_{44}$ is bounded, the same property have also $G_{45}\ and\ G_{46}$ . indeed if

$G_{44} < (\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt} \leq \left((\widehat{M}_{44})^{(9)}\right)_1 - (a_{45}')^{(9)}G_{45}$ and by integrating

$$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_1/(a_{45}')^{(9)}$$

In the same way , one can obtain

$$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_2/(a_{46}')^{(9)}$$

If $G_{45}\ or\ G_{46}$ is bounded, the same property follows for $G_{44}$ , $G_{46}$ and $G_{44}$ , $G_{45}$ respectively.

**Remark 44:** If $G_{44}$ *is* bounded, from below, the same property holds for $G_{45}\ and\ G_{46}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{45}$ is bounded from below.

**Remark 45:** If $T_{44}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t),t\right) = (b_{45}')^{(9)}$ then $T_{45} \to \infty$.

**Definition of** $(m)^{(9)}$ and $\varepsilon_9$ :

Indeed let $t_9$ be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t),t\right) < \varepsilon_9, T_{44}\,(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right), \quad t = log\frac{2}{\varepsilon_9}$ By taking now $\varepsilon_9$ sufficiently small one sees that $T_{45}$ is unbounded.
The same property holds for $T_{46}$ if $\lim_{t\to\infty}(b_{46}'')^{(9)}\left((G_{47})(t),t\right) = (b_{46}')^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

**Behavior of the solutions of equation** 280

**Theorem** If we denote and define

**Definition of** $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ :

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)} \le -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14}, t) + (a''_{14})^{(1)}(T_{14}, t) \le -(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)} \le -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G, t) - (b''_{14})^{(1)}(G, t) \le -(\tau_1)^{(1)}$

**Definition of** $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$ : 281

By $(\nu_1)^{(1)} > 0$, $(\nu_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$, $(u_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

**Definition of** $(\bar{\nu}_1)^{(1)},, (\bar{\nu}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$ : 282

By $(\bar{\nu}_1)^{(1)} > 0$, $(\bar{\nu}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$, $(\bar{u}_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

**Definition of** $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (\nu_0)^{(1)}$ :- 283

If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by
$(m_2)^{(1)} = (\nu_0)^{(1)}, (m_1)^{(1)} = (\nu_1)^{(1)}$, $if\ (\nu_0)^{(1)} < (\nu_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}, if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)}$,

and $\boxed{(\nu_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)}$, $if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)}$, $if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)}$,

and $\boxed{(u_0)^{(1)} = \frac{T^0_{13}}{T^0_{14}}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$

are defined

Then the solution of global equations satisfies the inequalities 285

$G^0_{13}e^{\left((S_1)^{(1)} - (p_{13})^{(1)}\right)t} \le G_{13}(t) \le G^0_{13}e^{(S_1)^{(1)}t}$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)}-(p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)}t}$$

$$\left(\frac{(a_{15})^{(1)}G_{13}^0}{(m_1)^{(1)}((S_1)^{(1)}-(p_{13})^{(1)}-(S_2)^{(1)})}\left[e^{((S_1)^{(1)}-(p_{13})^{(1)})t} - e^{-(S_2)^{(1)}t}\right] + G_{15}^0 e^{-(S_2)^{(1)}t} \le G_{15}(t) \le \right. \quad 286$$

$$\left. \frac{(a_{15})^{(1)}G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)}-(a'_{15})^{(1)})}[e^{(S_1)^{(1)}t} - e^{-(a'_{15})^{(1)}t}] + G_{15}^0 e^{-(a'_{15})^{(1)}t}\right)$$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)}t} \le T_{13}(t) \le T_{13}^0 e^{((R_1)^{(1)}+(r_{13})^{(1)})t}} \quad 287$$

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)}t} \le T_{13}(t) \le \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)}+(r_{13})^{(1)})t} \quad 288$$

$$\frac{(b_{15})^{(1)}T_{13}^0}{(\mu_1)^{(1)}((R_1)^{(1)}-(b'_{15})^{(1)})}\left[e^{(R_1)^{(1)}t} - e^{-(b'_{15})^{(1)}t}\right] + T_{15}^0 e^{-(b'_{15})^{(1)}t} \le T_{15}(t) \le \quad 289$$

$$\frac{(a_{15})^{(1)}T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)}+(r_{13})^{(1)}+(R_2)^{(1)})}\left[e^{((R_1)^{(1)}+(r_{13})^{(1)})t} - e^{-(R_2)^{(1)}t}\right] + T_{15}^0 e^{-(R_2)^{(1)}t}$$

**Definition of** $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:- 290

Where $(S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$

$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$

$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$

$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$

**Behavior of the solutions of equation** 291

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$: 292

$(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ four constants satisfying

$-(\sigma_2)^{(2)} \le -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(T_{17}, t) + (a''_{17})^{(2)}(T_{17}, t) \le -(\sigma_1)^{(2)}$ 293

$-(\tau_2)^{(2)} \le -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}((G_{19}), t) - (b''_{17})^{(2)}((G_{19}), t) \le -(\tau_1)^{(2)}$ 294

**Definition of** $(\nu_1)^{(2)}, (\nu_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$: 295

By $(\nu_1)^{(2)} > 0$, $(\nu_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0$, $(u_2)^{(2)} < 0$ the roots 296

of the equations $(a_{17})^{(2)}(\nu^{(2)})^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 297

and $(b_{14})^{(2)}(u^{(2)})^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and 298

**Definition of** $(\bar{\nu}_1)^{(2)}, , (\bar{\nu}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$: 299

By $(\bar{\nu}_1)^{(2)} > 0$, $(\bar{\nu}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0$, $(\bar{u}_2)^{(2)} < 0$ the 300

roots of the equations $(a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 301

and $(b_{17})^{(2)}\left(u^{(2)}\right)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 302

**Definition of** $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ :- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, ***if*** $(u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$, ***if*** $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)}$, ***if*** $(\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)}t}$

$(p_i)^{(2)}$ is defined by equation

$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)}t}$ 311

$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)}t}\right] + G_{18}^0 e^{-(S_2)^{(2)}t} \le G_{18}(t) \le\right.$ 312

$\left.\frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)}t} - e^{-(a'_{18})^{(2)}t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)}t}\right)$

$\boxed{T_{16}^0 e^{(R_1)^{(2)}t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}}$ 313

$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)}t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}$ 314

$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)}t} - e^{-(b'_{18})^{(2)}t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)}t} \le T_{18}(t) \le$ 315

$$\frac{(a_{18})^{(2)}T_{16}^{0}}{(\mu_2)^{(2)}((R_1)^{(2)}+(r_{16})^{(2)}+(R_2)^{(2)})}\left[e^{((R_1)^{(2)}+(r_{16})^{(2)})t}-e^{-(R_2)^{(2)}t}\right]+T_{18}^{0}e^{-(R_2)^{(2)}t}$$

**Definition of** $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$$

$$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)}$$ 318

$$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$$

**Behavior of the solutions** 319

**Theorem 3:** If we denote and define

**Definition of** $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ :

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}\big((G_{23}), t\big) \le -(\tau_1)^{(3)}$$

**Definition of** $(v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$ : 320

By $(v_1)^{(3)} > 0$, $(v_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{v}_1)^{(3)} > 0$, $(\bar{v}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

**Definition of** $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ :- 321

If we define $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (v_0)^{(3)}, (m_1)^{(3)} = (v_1)^{(3)}$, ***if*** $(v_0)^{(3)} < (v_1)^{(3)}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (\bar{v}_1)^{(3)}$, ***if*** $(v_1)^{(3)} < (v_0)^{(3)} < (\bar{v}_1)^{(3)}$,

and $\boxed{(v_0)^{(3)} = \frac{G_{20}^{0}}{G_{21}^{0}}}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_0)^{(3)}$, ***if*** $(\bar{v}_1)^{(3)} < (v_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)},\ \boldsymbol{if}\ (u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}, \boldsymbol{if}\ (u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)},$ and $\boxed{(u_0)^{(3)} = \frac{T_{20}^0}{T_{21}^0}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)},\ \boldsymbol{if}\ (\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G_{20}^0 e^{(S_1)^{(3)} t}$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)} t} \quad 323$$

$$\left(\frac{(a_{22})^{(3)} G_{20}^0}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\left[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)} t}\right] + G_{22}^0 e^{-(S_2)^{(3)} t} \le G_{22}(t) \le \right. \quad 324$$

$$\left. \frac{(a_{22})^{(3)} G_{20}^0}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}\left[e^{(S_1)^{(3)} t} - e^{-(a'_{22})^{(3)} t}\right] + G_{22}^0 e^{-(a'_{22})^{(3)} t}\right)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t}} \quad 325$$

$$\frac{1}{(\mu_1)^{(3)}} T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}} T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)} T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)} - (b'_{22})^{(3)})}\left[e^{(R_1)^{(3)} t} - e^{-(b'_{22})^{(3)} t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)} t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)} T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)} + (r_{20})^{(3)} + (R_2)^{(3)})}\left[e^{((R_1)^{(3)} + (r_{20})^{(3)})t} - e^{-(R_2)^{(3)} t}\right] + T_{22}^0 e^{-(R_2)^{(3)} t}$$

**Definition of** $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

**Behavior of the solutions of equation**

**Theorem:** If we denote and define

**Definition of** $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ :

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$

$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}((G_{27}), t) - (b''_{25})^{(4)}((G_{27}), t) \le -(\tau_1)^{(4)}$

**Definition of** $(v_1)^{(4)}, (v_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, v^{(4)}, u^{(4)}$ : 329

By $(v_1)^{(4)} > 0$ , $(v_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$ , $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_1)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

**Definition of** $(\bar{v}_1)^{(4)},, (\bar{v}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$ : 330

By $(\bar{v}_1)^{(4)} > 0$ , $(\bar{v}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$ , $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

**Definition of** $(m_1)^{(4)}$ , $(m_2)^{(4)}$ , $(\mu_1)^{(4)}, (\mu_2)^{(4)}, (\nu_0)^{(4)}$ :-

If we define $(m_1)^{(4)}$ , $(m_2)^{(4)}$ , $(\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (v_0)^{(4)}, (m_1)^{(4)} = (v_1)^{(4)}$, ***if*** $(v_0)^{(4)} < (v_1)^{(4)}$

$(m_2)^{(4)} = (v_1)^{(4)}, (m_1)^{(4)} = (\bar{v}_1)^{(4)}$ , ***if*** $(v_4)^{(4)} < (v_0)^{(4)} < (\bar{v}_1)^{(4)}$,
and $\boxed{(v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (v_4)^{(4)}, (m_1)^{(4)} = (v_0)^{(4)}$, ***if*** $(\bar{v}_4)^{(4)} < (v_0)^{(4)}$

and analogously

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$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)}$, ***if*** $(u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}$ , ***if*** $(u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)}$,
and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}$, ***if*** $(\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

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$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)}t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)}t}$$

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$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)}t}\right] + G_{26}^0 e^{-(S_2)^{(4)}t} \le G_{26}(t) \le\right.$$ 334

*(a26)4G240(m2)4(S1)4−(a26′)4e(S1)4t−e−(a26′)4t+ G260e−(a26′)4t*

$$\boxed{T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}}T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}}T_{24}^0 e^{((R_1)^{(4)}+(r_{24})^{(4)})t} \quad 335$$

$$\frac{(b_{26})^{(4)}T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)}-(b'_{26})^{(4)})}\left[e^{(R_1)^{(4)}t} - e^{-(b'_{26})^{(4)}t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)}t} \le T_{26}(t) \le \quad 336$$

$$\frac{(a_{26})^{(4)}T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)}+(r_{24})^{(4)}+(R_2)^{(4)})}\left[e^{((R_1)^{(4)}+(r_{24})^{(4)})t} - e^{-(R_2)^{(4)}t}\right] + T_{26}^0 e^{-(R_2)^{(4)}t}$$

**Definition of** $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

**Behavior of the solutions of equation** 338

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ :

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

**Definition of** $(v_1)^{(5)}, (v_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, v^{(5)}, u^{(5)}$ : 339

By $(v_1)^{(5)} > 0$ , $(v_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$ , $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(v^{(5)}\big)^2 + (\sigma_1)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

**Definition of** $(\bar{v}_1)^{(5)}, , (\bar{v}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$ : 340

By $(\bar{v}_1)^{(5)} > 0$ , $(\bar{v}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$ , $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(v^{(5)}\big)^2 + (\sigma_2)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

**Definition of** $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (v_0)^{(5)}$ :-

If we define $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (v_0)^{(5)}, (m_1)^{(5)} = (v_1)^{(5)}$, ***if*** $(v_0)^{(5)} < (v_1)^{(5)}$

$(m_2)^{(5)} = (v_1)^{(5)}, (m_1)^{(5)} = (\bar{v}_1)^{(5)}$ , ***if*** $(v_1)^{(5)} < (v_0)^{(5)} < (\bar{v}_1)^{(5)}$,
and $\boxed{(v_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)},\ \boldsymbol{if}\ (u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}, \boldsymbol{if}\ (u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)},$

and $\boxed{(u_0)^{(5)} = \frac{T_{28}^0}{T_{29}^0}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}, \boldsymbol{if}\ (\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{28}(t) \le G_{28}^0 e^{(S_1)^{(5)} t}$

where $(p_i)^{(5)}$ is defined by equation

$\frac{1}{(m_5)^{(5)}} G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}} G_{28}^0 e^{(S_1)^{(5)} t}$ 343

$\left(\frac{(a_{30})^{(5)} G_{28}^0}{(m_1)^{(5)}((S_1)^{(5)} - (p_{28})^{(5)} - (S_2)^{(5)})}\left[e^{((S_1)^{(5)} - (p_{28})^{(5)})t} - e^{-(S_2)^{(5)} t}\right] + G_{30}^0 e^{-(S_2)^{(5)} t} \le G_{30}(t) \le\right.$ 344

*(a30)5G280(m2)5(S1)5−(a30′)5e(S1)5t−e−(a30′)5t+ G300e−(a30′)5t*

$\boxed{T_{28}^0 e^{(R_1)^{(5)} t} \le T_{28}(t) \le T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}}$ 345

$\frac{1}{(\mu_1)^{(5)}} T_{28}^0 e^{(R_1)^{(5)} t} \le T_{28}(t) \le \frac{1}{(\mu_2)^{(5)}} T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}$ 346

$\frac{(b_{30})^{(5)} T_{28}^0}{(\mu_1)^{(5)}((R_1)^{(5)} - (b'_{30})^{(5)})}\left[e^{(R_1)^{(5)} t} - e^{-(b'_{30})^{(5)} t}\right] + T_{30}^0 e^{-(b'_{30})^{(5)} t} \le T_{30}(t) \le$ 347

$\frac{(a_{30})^{(5)} T_{28}^0}{(\mu_2)^{(5)}((R_1)^{(5)} + (r_{28})^{(5)} + (R_2)^{(5)})}\left[e^{((R_1)^{(5)} + (r_{28})^{(5)})t} - e^{-(R_2)^{(5)} t}\right] + T_{30}^0 e^{-(R_2)^{(5)} t}$

**Definition of** $(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$:- 348

Where $(S_1)^{(5)} = (a_{28})^{(5)}(m_2)^{(5)} - (a'_{28})^{(5)}$

$(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}$

$(R_1)^{(5)} = (b_{28})^{(5)}(\mu_2)^{(5)} - (b'_{28})^{(5)}$

$(R_2)^{(5)} = (b'_{30})^{(5)} - (r_{30})^{(5)}$

**Behavior of the solutions of equation** 349

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ :

$(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)} \le -(a'_{32})^{(6)} + (a'_{33})^{(6)} - (a''_{32})^{(6)}(T_{33}, t) + (a''_{33})^{(6)}(T_{33}, t) \le -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)} \le -(b'_{32})^{(6)} + (b'_{33})^{(6)} - (b''_{32})^{(6)}\big((G_{35}), t\big) - (b''_{33})^{(6)}\big((G_{35}), t\big) \le -(\tau_1)^{(6)}$$

**Definition of** $(\nu_1)^{(6)}, (\nu_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, \nu^{(6)}, u^{(6)}$ : 350

By $(\nu_1)^{(6)} > 0$ , $(\nu_2)^{(6)} < 0$ and respectively $(u_1)^{(6)} > 0$ , $(u_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(6)}, , (\bar{\nu}_2)^{(6)}, (\bar{u}_1)^{(6)}, (\bar{u}_2)^{(6)}$ : 351

By $(\bar{\nu}_1)^{(6)} > 0$ , $(\bar{\nu}_2)^{(6)} < 0$ and respectively $(\bar{u}_1)^{(6)} > 0$ , $(\bar{u}_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$

**Definition of** $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}, (\nu_0)^{(6)}$ :-

If we define $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}$ by

$$(m_2)^{(6)} = (\nu_0)^{(6)}, (m_1)^{(6)} = (\nu_1)^{(6)}, \ \boldsymbol{if}\ (\nu_0)^{(6)} < (\nu_1)^{(6)}$$

$$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}, \boldsymbol{if}\ (\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)},$$

and $\boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}}$

$$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$$

and analogously 352

$$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}, \ \boldsymbol{if}\ (u_0)^{(6)} < (u_1)^{(6)}$$

$$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}, \boldsymbol{if}\ (u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)},$$

and $\boxed{(u_0)^{(6)} = \frac{T^0_{32}}{T^0_{33}}}$

$$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}, \boldsymbol{if}\ (\bar{u}_1)^{(6)} < (u_0)^{(6)} \text{ where } (u_1)^{(6)}, (\bar{u}_1)^{(6)}$$

Then the solution of global equations satisfies the inequalities 353

$$G^0_{32}e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{32}(t) \le G^0_{32}e^{(S_1)^{(6)}t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}}G^0_{32}e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{33}(t) \le \frac{1}{(m_2)^{(6)}}G^0_{32}e^{(S_1)^{(6)}t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)}G^0_{32}}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})}\left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)}t}\right] + G^0_{34}e^{-(S_2)^{(6)}t} \le G_{34}(t) \le \right. \quad 355$$

$$\frac{(a_{34})^{(6)}G_{32}^{0}}{(m_2)^{(6)}((S_1)^{(6)}-(a'_{34})^{(6)})}\left[e^{(S_1)^{(6)}t}-e^{-(a'_{34})^{(6)}t}\right]+\ G_{34}^{0}e^{-(a'_{34})^{(6)}t}\Big)$$

$$\boxed{T_{32}^{0}e^{(R_1)^{(6)}t}\le T_{32}(t)\le T_{32}^{0}e^{((R_1)^{(6)}+(r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}}T_{32}^{0}e^{(R_1)^{(6)}t}\le T_{32}(t)\le\frac{1}{(\mu_2)^{(6)}}T_{32}^{0}e^{((R_1)^{(6)}+(r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)}T_{32}^{0}}{(\mu_1)^{(6)}((R_1)^{(6)}-(b'_{34})^{(6)})}\left[e^{(R_1)^{(6)}t}-e^{-(b'_{34})^{(6)}t}\right]+T_{34}^{0}e^{-(b'_{34})^{(6)}t}\le T_{34}(t)\le \quad 358$$

$$\frac{(a_{34})^{(6)}T_{32}^{0}}{(\mu_2)^{(6)}((R_1)^{(6)}+(r_{32})^{(6)}+(R_2)^{(6)})}\left[e^{((R_1)^{(6)}+(r_{32})^{(6)})t}-e^{-(R_2)^{(6)}t}\right]+T_{34}^{0}e^{-(R_2)^{(6)}t}$$

**Definition of** $(S_1)^{(6)},(S_2)^{(6)},(R_1)^{(6)},(R_2)^{(6)}$:- 359

Where $(S_1)^{(6)}=(a_{32})^{(6)}(m_2)^{(6)}-(a'_{32})^{(6)}$

$(S_2)^{(6)}=(a_{34})^{(6)}-(p_{34})^{(6)}$

$(R_1)^{(6)}=(b_{32})^{(6)}(\mu_2)^{(6)}-(b'_{32})^{(6)}$

$(R_2)^{(6)}=(b'_{34})^{(6)}-(r_{34})^{(6)}$

**Behavior of the solutions of equation**

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(7)},(\sigma_2)^{(7)},(\tau_1)^{(7)},(\tau_2)^{(7)}$ :

$(\sigma_1)^{(7)},(\sigma_2)^{(7)},(\tau_1)^{(7)},(\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)}\le-(a'_{36})^{(7)}+(a'_{37})^{(7)}-(a''_{36})^{(7)}(T_{37},t)+(a''_{37})^{(7)}(T_{37},t)\le-(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)}\le-(b'_{36})^{(7)}+(b'_{37})^{(7)}-(b''_{36})^{(7)}\big((G_{39}),t\big)-(b''_{37})^{(7)}\big((G_{39}),t\big)\le-(\tau_1)^{(7)}$$

**Definition of** $(v_1)^{(7)},(v_2)^{(7)},(u_1)^{(7)},(u_2)^{(7)},v^{(7)},u^{(7)}$ : 361

By $(v_1)^{(7)}>0,(v_2)^{(7)}<0$ and respectively $(u_1)^{(7)}>0,(u_2)^{(7)}<0$ the roots of the equations $(a_{37})^{(7)}\big(v^{(7)}\big)^2+(\sigma_1)^{(7)}v^{(7)}-(a_{36})^{(7)}=0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2+(\tau_1)^{(7)}u^{(7)}-(b_{36})^{(7)}=0$ and

**Definition of** $(\bar{v}_1)^{(7)},,(\bar{v}_2)^{(7)},(\bar{u}_1)^{(7)},(\bar{u}_2)^{(7)}$ : 362

By $(\bar{v}_1)^{(7)}>0,(\bar{v}_2)^{(7)}<0$ and respectively $(\bar{u}_1)^{(7)}>0,(\bar{u}_2)^{(7)}<0$ the roots of the equations $(a_{37})^{(7)}\big(v^{(7)}\big)^2+(\sigma_2)^{(7)}v^{(7)}-(a_{36})^{(7)}=0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2+(\tau_2)^{(7)}u^{(7)}-(b_{36})^{(7)}=0$

**Definition of** $(m_1)^{(7)},(m_2)^{(7)},(\mu_1)^{(7)},(\mu_2)^{(7)},(v_0)^{(7)}$ :-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$$(m_2)^{(7)} = (\nu_0)^{(7)}, (m_1)^{(7)} = (\nu_1)^{(7)}, \ \boldsymbol{if}\ (\nu_0)^{(7)} < (\nu_1)^{(7)}$$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\bar{\nu}_1)^{(7)}, \boldsymbol{if}\ (\nu_1)^{(7)} < (\nu_0)^{(7)} < (\bar{\nu}_1)^{(7)},$$

and $\boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\nu_0)^{(7)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(7)} < (\nu_0)^{(7)}$$

and analogously 363

$$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}, \ \boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$$

and $\boxed{(u_0)^{(7)} = \frac{T_{36}^0}{T_{37}^0}}$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)} \text{ where } (u_1)^{(7)}, (\bar{u}_1)^{(7)}$$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^0 e^{(S_1)^{(7)} t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)} t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)}((S_1)^{(7)} - (p_{36})^{(7)} - (S_2)^{(7)})}\left[e^{((S_1)^{(7)} - (p_{36})^{(7)})t} - e^{-(S_2)^{(7)} t}\right] + G_{38}^0 e^{-(S_2)^{(7)} t} \le G_{38}(t) \le \right. \quad 366$$

$$\left. \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)}((S_1)^{(7)} - (a'_{38})^{(7)})}\left[e^{(S_1)^{(7)} t} - e^{-(a'_{38})^{(7)} t}\right] + G_{38}^0 e^{-(a'_{38})^{(7)} t}\right)$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t} \quad 368$$

$$\frac{(b_{38})^{(7)} T_{36}^0}{(\mu_1)^{(7)}((R_1)^{(7)} - (b'_{38})^{(7)})}\left[e^{(R_1)^{(7)} t} - e^{-(b'_{38})^{(7)} t}\right] + T_{38}^0 e^{-(b'_{38})^{(7)} t} \le T_{38}(t) \le \quad 369$$

$$\frac{(a_{38})^{(7)} T_{36}^0}{(\mu_2)^{(7)}((R_1)^{(7)} + (r_{36})^{(7)} + (R_2)^{(7)})}\left[e^{((R_1)^{(7)} + (r_{36})^{(7)})t} - e^{-(R_2)^{(7)} t}\right] + T_{38}^0 e^{-(R_2)^{(7)} t}$$

**Definition of** $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$

$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$

$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$

**Behavior of the solutions of equation** 371

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ :

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$-(\sigma_2)^{(8)} \le -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \le -(\sigma_1)^{(8)}$

$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\big((G_{43}), t\big) - (b''_{41})^{(8)}\big((G_{43}), t\big) \le -(\tau_1)^{(8)}$

**Definition of** $(\nu_1)^{(8)}, (\nu_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, \nu^{(8)}, u^{(8)}$ : 372

By $(\nu_1)^{(8)} > 0, (\nu_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0, (u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(8)}, , (\bar{\nu}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$ :

By $(\bar{\nu}_1)^{(8)} > 0, (\bar{\nu}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0, (\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

**Definition of** $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}, (\nu_0)^{(8)}$ :-

If we define $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$(m_2)^{(8)} = (\nu_0)^{(8)}, (m_1)^{(8)} = (\nu_1)^{(8)}$, ***if*** $(\nu_0)^{(8)} < (\nu_1)^{(8)}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\bar{\nu}_1)^{(8)}$, ***if*** $(\nu_1)^{(8)} < (\nu_0)^{(8)} < (\bar{\nu}_1)^{(8)}$,
and $\boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}}$

$(m_2)^{(8)} = (v_1)^{(8)}, (m_1)^{(8)} = (v_0)^{(8)},\ \boldsymbol{if}\ (\bar{v}_1)^{(8)} < (v_0)^{(8)}$

and analogously 374

$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)},\ \boldsymbol{if}\ (u_0)^{(8)} < (u_1)^{(8)}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}, \boldsymbol{if}\ (u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)},$

and $\boxed{(u_0)^{(8)} = \frac{T_{40}^0}{T_{41}^0}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}, \boldsymbol{if}\ (\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^0 e^{(S_1)^{(8)} t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}} G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}} G_{40}^0 e^{(S_1)^{(8)} t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)} G_{40}^0}{(m_1)^{(8)}((S_1)^{(8)} - (p_{40})^{(8)} - (S_2)^{(8)})}\left[e^{((S_1)^{(8)} - (p_{40})^{(8)})t} - e^{-(S_2)^{(8)} t}\right] + G_{42}^0 e^{-(S_2)^{(8)} t} \le G_{42}(t) \le \right. \quad 377$$

$$\left. \frac{(a_{42})^{(8)} G_{40}^0}{(m_2)^{(8)}((S_1)^{(8)} - (a'_{42})^{(8)})}\left[e^{(S_1)^{(8)} t} - e^{-(a'_{42})^{(8)} t}\right] + G_{42}^0 e^{-(a'_{42})^{(8)} t}\right)$$

$$\boxed{T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}} T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le \frac{1}{(\mu_2)^{(8)}} T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)} T_{40}^0}{(\mu_1)^{(8)}((R_1)^{(8)} - (b'_{42})^{(8)})}\left[e^{(R_1)^{(8)} t} - e^{-(b'_{42})^{(8)} t}\right] + T_{42}^0 e^{-(b'_{42})^{(8)} t} \le T_{42}(t) \le \quad 380$$

$$\frac{(a_{42})^{(8)} T_{40}^0}{(\mu_2)^{(8)}((R_1)^{(8)} + (r_{40})^{(8)} + (R_2)^{(8)})}\left[e^{((R_1)^{(8)} + (r_{40})^{(8)})t} - e^{-(R_2)^{(8)} t}\right] + T_{42}^0 e^{-(R_2)^{(8)} t}$$

**Definition of** $(S_1)^{(8)}, (S_2)^{(8)}, (R_1)^{(8)}, (R_2)^{(8)}$:- 381

Where $(S_1)^{(8)} = (a_{40})^{(8)}(m_2)^{(8)} - (a'_{40})^{(8)}$

$(S_2)^{(8)} = (a_{42})^{(8)} - (p_{42})^{(8)}$

$(R_1)^{(8)} = (b_{40})^{(8)}(\mu_2)^{(8)} - (b'_{40})^{(8)}$

$(R_2)^{(8)} = (b'_{42})^{(8)} - (r_{42})^{(8)}$

**Behavior of the solutions of equation 37 to 92** 382

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ :

$(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ four constants satisfying

$$-(\sigma_2)^{(9)} \le -(a'_{44})^{(9)} + (a'_{45})^{(9)} - (a''_{44})^{(9)}(T_{45}, t) + (a''_{45})^{(9)}(T_{45}, t) \le -(\sigma_1)^{(9)}$$

$$-(\tau_2)^{(9)} \le -(b'_{44})^{(9)} + (b'_{45})^{(9)} - (b''_{44})^{(9)}\big((G_{47}), t\big) - (b''_{45})^{(9)}\big((G_{47}), t\big) \le -(\tau_1)^{(9)}$$

**Definition of** $(\nu_1)^{(9)}, (\nu_2)^{(9)}, (u_1)^{(9)}, (u_2)^{(9)}, \nu^{(9)}, u^{(9)}$ :

By $(\nu_1)^{(9)} > 0$ , $(\nu_2)^{(9)} < 0$ and respectively $(u_1)^{(9)} > 0$ , $(u_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_1)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(9)}, , (\bar{\nu}_2)^{(9)}, (\bar{u}_1)^{(9)}, (\bar{u}_2)^{(9)}$ :

By $(\bar{\nu}_1)^{(9)} > 0$ , $(\bar{\nu}_2)^{(9)} < 0$ and respectively $(\bar{u}_1)^{(9)} > 0$ , $(\bar{u}_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_2)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$

**Definition of** $(m_1)^{(9)}, (m_2)^{(9)}, (\mu_1)^{(9)}, (\mu_2)^{(9)}, (\nu_0)^{(9)}$ :-

If we define $(m_1)^{(9)}, (m_2)^{(9)}, (\mu_1)^{(9)}, (\mu_2)^{(9)}$ by

$$(m_2)^{(9)} = (\nu_0)^{(9)}, (m_1)^{(9)} = (\nu_1)^{(9)}, \ \boldsymbol{if}\ (\nu_0)^{(9)} < (\nu_1)^{(9)}$$

$$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\bar{\nu}_1)^{(9)}, \boldsymbol{if}\ (\nu_1)^{(9)} < (\nu_0)^{(9)} < (\bar{\nu}_1)^{(9)},$$

and $\boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$

$$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\nu_0)^{(9)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(9)} < (\nu_0)^{(9)}$$

and analogously

$$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)}, \ \boldsymbol{if}\ (u_0)^{(9)} < (u_1)^{(9)}$$

$$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}, \boldsymbol{if}\ (u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)},$$

and $\boxed{(u_0)^{(9)} = \frac{T^0_{44}}{T^0_{45}}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}, \boldsymbol{if}\ (\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G^0_{44} e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{44}(t) \le G^0_{44} e^{(S_1)^{(9)} t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}}G_{44}^0 e^{((S_1)^{(9)}-(p_{44})^{(9)})t} \le G_{45}(t) \le \frac{1}{(m_2)^{(9)}}G_{44}^0 e^{(S_1)^{(9)}t}$$

$$\left(\frac{(a_{46})^{(9)}G_{44}^0}{(m_1)^{(9)}((S_1)^{(9)}-(p_{44})^{(9)}-(S_2)^{(9)})}\left[e^{((S_1)^{(9)}-(p_{44})^{(9)})t}-e^{-(S_2)^{(9)}t}\right]+G_{46}^0 e^{-(S_2)^{(9)}t} \le G_{46}(t) \le \right.$$

$$\left.\frac{(a_{46})^{(9)}G_{44}^0}{(m_2)^{(9)}((S_1)^{(9)}-(a'_{46})^{(9)})}\left[e^{(S_1)^{(9)}t}-e^{-(a'_{46})^{(9)}t}\right]+\ G_{46}^0 e^{-(a'_{46})^{(9)}t}\right)$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le T_{44}^0 e^{((R_1)^{(9)}+(r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}}T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le \frac{1}{(\mu_2)^{(9)}}T_{44}^0 e^{((R_1)^{(9)}+(r_{44})^{(9)})t}$$

$$\frac{(b_{46})^{(9)}T_{44}^0}{(\mu_1)^{(9)}((R_1)^{(9)}-(b'_{46})^{(9)})}\left[e^{(R_1)^{(9)}t}-e^{-(b'_{46})^{(9)}t}\right]+T_{46}^0 e^{-(b'_{46})^{(9)}t} \le T_{46}(t) \le$$

$$\frac{(a_{46})^{(9)}T_{44}^0}{(\mu_2)^{(9)}((R_1)^{(9)}+(r_{44})^{(9)}+(R_2)^{(9)})}\left[e^{((R_1)^{(9)}+(r_{44})^{(9)})t}-e^{-(R_2)^{(9)}t}\right]+T_{46}^0 e^{-(R_2)^{(9)}t}$$

**Definition of** $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$

**Proof :** From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14},t)\right) - (a''_{14})^{(1)}(T_{14},t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

**Definition of** $v^{(1)}$ :- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)}\right) \le \frac{dv^{(1)}}{dt} \le -\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)}\right)$$

From which one obtains

**Definition of** $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$ :-

For $0 < \boxed{(v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}} < (v_1)^{(1)} < (\bar{v}_1)^{(1)}$

$$v^{(1)}(t) \ge \frac{(v_1)^{(1)}+(C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(v_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(v_2)^{(1)}}}$$

it follows $(\nu_0)^{(1)} \leq \nu^{(1)}(t) \leq (\nu_1)^{(1)}$

In the same manner , we get 384

$$\nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{\nu}_1)^{(1)}-(\nu_0)^{(1)}}{(\nu_0)^{(1)}-(\bar{\nu}_2)^{(1)}}}$$

From which we deduce $(\nu_0)^{(1)} \leq \nu^{(1)}(t) \leq (\bar{\nu}_1)^{(1)}$

If $0 < (\nu_1)^{(1)} < (\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0} < (\bar{\nu}_1)^{(1)}$ we find like in the previous case, 385

$$(\nu_1)^{(1)} \leq \frac{(\nu_1)^{(1)}+(C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}} \leq \nu^{(1)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\bar{\nu}_1)^{(1)}$$

If $0 < (\nu_1)^{(1)} \leq (\bar{\nu}_1)^{(1)} \leq \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$ , we obtain 386

$$(\nu_1)^{(1)} \leq \nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\nu_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(1)}(t)$ :-

$$(m_2)^{(1)} \leq \nu^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{\nu^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(1)}(t)$ :-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{13})^{(1)} = (a''_{14})^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)}G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)}T_{14}(t)$ This is an important

consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

**Proof** : From global equations we obtain 387

$$\frac{d\nu^{(2)}}{dt} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right) - (a''_{17})^{(2)}(T_{17}, t)\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

**Definition of** $\nu^{(2)}$ :- $\boxed{\nu^{(2)} = \frac{G_{16}}{G_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \le \frac{d\nu^{(2)}}{dt} \le -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

**Definition of** $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$ :-

For $0 < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \ge \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}}{1 + (C)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(C)^{(2)} = \frac{(\nu_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \le \nu^{(2)}(t) \le (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \le \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \le \nu^{(2)}(t) \le (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \le \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}}{1 + (C)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}} \le \nu^{(2)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \le (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \le (\bar{\nu}_1)^{(2)} \le (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$ , we obtain 394

$$(\nu_1)^{(2)} \le \nu^{(2)}(t) \le \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \le (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(2)}(t)$ :- 395

$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}$, $\boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$

In a completely analogous way, we obtain 396

**Definition of** $u^{(2)}(t)$ :-

$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}$, $\boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :** 397

If $(a''_{16})^{(2)} = (a''_{17})^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)} G_{17}(t)$

Analogously if $(b''_{16})^{(2)} = (b''_{17})^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)} T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

**Proof** : From global equations we obtain 398

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right) - (a''_{21})^{(3)}(T_{21}, t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

**Definition of** $\nu^{(3)}$ :- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right) \leq \frac{d\nu^{(3)}}{dt} \leq -\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right)$$

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From which one obtains

For $0 < (\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\nu_1)^{(3)} < (\bar{\nu}_1)^{(3)}$

$$\nu^{(3)}(t) \geq \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(\nu_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\nu_2)^{(3)}}}$$

it follows $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\nu_1)^{(3)}$

In the same manner , we get 401

$$\nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{\nu}_1)^{(3)}-(\nu_0)^{(3)}}{(\nu_0)^{(3)}-(\bar{\nu}_2)^{(3)}}}$$

**Definition of** $(\bar{\nu}_1)^{(3)}$ :-

From which we deduce $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\bar{\nu}_1)^{(3)}$

If $0 < (\nu_1)^{(3)} < (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (\bar{\nu}_1)^{(3)}$ we find like in the previous case, 402

$$(\nu_1)^{(3)} \leq \frac{(\nu_1)^{(3)}+(C)^{(3)}(\nu_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}} \leq \nu^{(3)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\bar{\nu}_1)^{(3)}$$

If $0 < (\nu_1)^{(3)} \leq (\bar{\nu}_1)^{(3)} \leq (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}$ , we obtain 403

$$(\nu_1)^{(3)} \leq \nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\nu_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(3)}(t)$ :-

$$(m_2)^{(3)} \leq \nu^{(3)}(t) \leq (m_1)^{(3)}, \quad \boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(3)}(t)$ :-

$$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}, \quad \boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{20}'')^{(3)} = (a_{21}'')^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)}G_{21}(t)$

Analogously if $(b_{20}'')^{(3)} = (b_{21}'')^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)}T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

**Proof** : From global equations we obtain 404

$$\frac{dv^{(4)}}{dt} = (a_{24})^{(4)} - \left((a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right) - (a''_{25})^{(4)}(T_{25},t)v^{(4)} - (a_{25})^{(4)}v^{(4)}$$

**Definition of** $v^{(4)}$ :- $\boxed{v^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)}\right) \le \frac{dv^{(4)}}{dt} \le -\left((a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_4)^{(4)}v^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$ :-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \ge \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \le \nu^{(4)}(t) \le (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \le \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \le \nu^{(4)}(t) \le (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \le \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}}{1 + (C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}} \le \nu^{(4)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \le (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \le (\bar{\nu}_1)^{(4)} \le \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$ , we obtain 407

$$(\nu_1)^{(4)} \le \nu^{(4)}(t) \le \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \le (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(4)}(t)$ :-

$$(m_2)^{(4)} \le \nu^{(4)}(t) \le (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(4)}(t)$ :-

$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}$, $\boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{24})^{(4)} = (a''_{25})^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)} G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b''_{24})^{(4)} = (b''_{25})^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)} T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

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**Proof** : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a'_{28})^{(5)} - (a'_{29})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right) - (a''_{29})^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

**Definition of** $\nu^{(5)}$ :- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$ :-

For $0 < \boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \le \frac{(\nu_1)^{(5)}+(C)^{(5)}(\nu_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)}-(\nu_2)^{(5)}\right)t\right]}}{1+(C)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)}-(\nu_2)^{(5)}\right)t\right]}} \le \nu^{(5)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \le (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \le (\bar{\nu}_1)^{(5)} \le \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain 411

$$(\nu_1)^{(5)} \le \nu^{(5)}(t) \le \frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \le (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(5)}(t)$ :-

$(m_2)^{(5)} \le \nu^{(5)}(t) \le (m_1)^{(5)}$, $\boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$

In a completely analogous way, we obtain

**Definition of** $u^{(5)}(t)$ :-

$(\mu_2)^{(5)} \le u^{(5)}(t) \le (\mu_1)^{(5)}$, $\boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a_{28}'')^{(5)} = (a_{29}'')^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)}G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b_{28}'')^{(5)} = (b_{29}'')^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)}T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

**Proof** : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a_{32}')^{(6)} - (a_{33}')^{(6)} + (a_{32}'')^{(6)}(T_{33},t)\right) - (a_{33}'')^{(6)}(T_{33},t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

**Definition of** $\nu^{(6)}$ :- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \le \frac{d\nu^{(6)}}{dt} \le -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$ :-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \geq \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\nu_1)^{(6)}$

In the same manner , we get 413

$$\nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{\nu}_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\bar{\nu}_1)^{(6)}$

If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 414

$$(\nu_1)^{(6)} \leq \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}} \leq \nu^{(6)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\bar{\nu}_1)^{(6)}$$

If $0 < (\nu_1)^{(6)} \leq (\bar{\nu}_1)^{(6)} \leq \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$ , we obtain 415

$$(\nu_1)^{(6)} \leq \nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(6)}(t)$ :-

$$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}, \quad \boxed{\nu^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(6)}(t)$ :-

$$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}, \quad \boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)} G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)} T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

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**Proof** : From global equations we obtain

$$\frac{dv^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right) - (a''_{37})^{(7)}(T_{37},t)v^{(7)} - (a_{37})^{(7)}v^{(7)}$$

**Definition of** $v^{(7)}$ :- $\boxed{v^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_2)^{(7)}v^{(7)} - (a_{36})^{(7)}\right) \le \frac{dv^{(7)}}{dt} \le -\left((a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_1)^{(7)}v^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

**Definition of** $(\bar{v}_1)^{(7)}, (v_0)^{(7)}$ :-

For $0 < \boxed{(v_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}} < (v_1)^{(7)} < (\bar{v}_1)^{(7)}$

$$v^{(7)}(t) \ge \frac{(v_1)^{(7)} + (C)^{(7)}(v_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_0)^{(7)}\right)t\right]}}{1 + (C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(v_1)^{(7)} - (v_0)^{(7)}}{(v_0)^{(7)} - (v_2)^{(7)}}}$$

it follows $(v_0)^{(7)} \le v^{(7)}(t) \le (v_1)^{(7)}$

In the same manner , we get 417

$$v^{(7)}(t) \le \frac{(\bar{v}_1)^{(7)} + (\bar{C})^{(7)}(\bar{v}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{v}_1)^{(7)} - (v_0)^{(7)}}{(v_0)^{(7)} - (\bar{v}_2)^{(7)}}}$$

From which we deduce $(v_0)^{(7)} \le v^{(7)}(t) \le (\bar{v}_1)^{(7)}$

If $0 < (v_1)^{(7)} < (v_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}} < (\bar{v}_1)^{(7)}$ we find like in the previous case, 418

$$(v_1)^{(7)} \le \frac{(v_1)^{(7)} + (C)^{(7)}(v_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_2)^{(7)}\right)t\right]}}{1 + (C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_2)^{(7)}\right)t\right]}} \le v^{(7)}(t) \le$$

$$\frac{(\bar{v}_1)^{(7)} + (\bar{C})^{(7)}(\bar{v}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}} \le (\bar{v}_1)^{(7)}$$

If $0 < (v_1)^{(7)} \le (\bar{v}_1)^{(7)} \le \boxed{(v_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$ , we obtain 419

$$(v_1)^{(7)} \le v^{(7)}(t) \le \frac{(\bar{v}_1)^{(7)} + (\bar{C})^{(7)}(\bar{v}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}} \le (v_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $v^{(7)}(t)$ :-

$$(m_2)^{(7)} \leq v^{(7)}(t) \leq (m_1)^{(7)}, \quad \boxed{v^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(7)}(t)$ :- 420

$$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}, \quad \boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $v^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)} G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)} T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

**Proof** : From global equations we obtain 421

$$\frac{dv^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right) - (a''_{41})^{(8)}(T_{41}, t) v^{(8)} - (a_{41})^{(8)} v^{(8)}$$

**Definition of** $v^{(8)}$ :- $\boxed{v^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_2)^{(8)} v^{(8)} - (a_{40})^{(8)}\right) \leq \frac{dv^{(8)}}{dt} \leq -\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_1)^{(8)} v^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$ :-

$$\text{For } 0 < \boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$$

$$v^{(8)}(t) \geq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)}-(\nu_0)^{(8)}}{(\nu_0)^{(8)}-(\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \leq \frac{(\nu_1)^{(8)}+(C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}}{1+(C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}} \leq \nu^{(8)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \leq (\bar{\nu}_1)^{(8)} \leq \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$ , we obtain 424

$$(\nu_1)^{(8)} \leq \nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(8)}(t)$ :-

$$(m_2)^{(8)} \leq \nu^{(8)}(t) \leq (m_1)^{(8)}, \quad \boxed{\nu^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(8)}(t)$ :-

$$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}, \quad \boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $\nu^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)}G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then $(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)}T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

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**Proof** : From 99,20,44,22,23,44 we obtain A

$$\frac{dv^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right) - (a''_{45})^{(9)}(T_{45},t)v^{(9)} - (a_{45})^{(9)}v^{(9)}$$

**Definition of** $v^{(9)}$ :- $\boxed{v^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)}\right) \le \frac{dv^{(9)}}{dt} \le -\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_1)^{(9)}v^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

**Definition of** $(\bar{v}_1)^{(9)}, (v_0)^{(9)}$ :-

For $0 < \boxed{(v_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (v_1)^{(9)} < (\bar{v}_1)^{(9)}$

$$v^{(9)}(t) \ge \frac{(v_1)^{(9)} + (C)^{(9)}(v_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((v_1)^{(9)} - (v_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((v_1)^{(9)} - (v_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(v_1)^{(9)} - (v_0)^{(9)}}{(v_0)^{(9)} - (v_2)^{(9)}}}$$

it follows $(v_0)^{(9)} \le v^{(9)}(t) \le (v_9)^{(9)}$

In the same manner , we get

$$v^{(9)}(t) \le \frac{(\bar{v}_1)^{(9)} + (\bar{C})^{(9)}(\bar{v}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{v}_1)^{(9)} - (\bar{v}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{v}_1)^{(9)} - (\bar{v}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{v}_1)^{(9)} - (v_0)^{(9)}}{(v_0)^{(9)} - (\bar{v}_2)^{(9)}}}$$

From which we deduce $(v_0)^{(9)} \le v^{(9)}(t) \le (\bar{v}_1)^{(9)}$

If $0 < (v_1)^{(9)} < (v_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}} < (\bar{v}_1)^{(9)}$ we find like in the previous case,

$$(v_1)^{(9)} \le \frac{(v_1)^{(9)} + (C)^{(9)}(v_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((v_1)^{(9)} - (v_2)^{(9)}\right)t\right]}}{1 + (C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((v_1)^{(9)} - (v_2)^{(9)}\right)t\right]}} \le v^{(9)}(t) \le$$

$$\frac{(\bar{v}_1)^{(9)} + (\bar{C})^{(9)}(\bar{v}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{v}_1)^{(9)} - (\bar{v}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{v}_1)^{(9)} - (\bar{v}_2)^{(9)}\right)t\right]}} \le (\bar{v}_1)^{(9)}$$

If $0 < (v_1)^{(9)} \le (\bar{v}_1)^{(9)} \le \boxed{(v_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$ , we obtain

$$(v_1)^{(9)} \le v^{(9)}(t) \le \frac{(\bar{v}_1)^{(9)} + (\bar{C})^{(9)}(\bar{v}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{v}_1)^{(9)} - (\bar{v}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{v}_1)^{(9)} - (\bar{v}_2)^{(9)}\right)t\right]}} \le (v_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $v^{(9)}(t)$ :-

$$(m_2)^{(9)} \le v^{(9)}(t) \le (m_1)^{(9)}, \quad \boxed{v^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(9)}(t)$ :-

$(\mu_2)^{(9)} \leq u^{(9)}(t) \leq (\mu_1)^{(9)}$, $\boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)} G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then
$(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)} T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following 425

**Theorem :** If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on $t$ , and the conditions with the notations

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0$ ,

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$

$with\ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a''_i)^{(2)} and\ (b''_i)^{(2)}$ are independent on t , and the conditions with the notations 426

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$ 427

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 428

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$ , 429

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 430

$with\ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a''_i)^{(3)} and\ (b''_i)^{(3)}$ are independent on $t$ , and the conditions with the notations 431

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$ ,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

**Theorem :** If $(a_i'')^{(4)} and\ (b_i'')^{(4)}$ are independent on $t$ , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$ ,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(5)} and\ (b_i'')^{(5)}$ are independent on $t$ , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$ ,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

**Theorem** If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on $t$ , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$ ,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(7)} and\ (b_i'')^{(7)}$ are independent on $t$ , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$ ,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(8)} and\ (b_i'')^{(8)}$ are independent on $t$ , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$ ,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(9)} and\ (b_i'')^{(9)}$ are independent on $t$ , and the conditions (with the notations 436 A
45,46,27,28)

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$ ,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0$ 437

$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0$ 438

$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0$ 439

$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0$ 440

$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ]T_{14} = 0$ 441

$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0$ 442

has a unique positive solution , which is an equilibrium solution for the system

$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0$ 443

$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0$ 444

$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0$ 445

$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\ ]T_{16} = 0$ 446

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\,]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\,]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\,]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\,]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\,]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\big((G_{27})\big)\,]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\big((G_{27})\big)\,]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\big((G_{27})\big)\,]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})\,]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})\,]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})\,]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})\,]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})\,]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})\,]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0 \quad 479$$

$$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0 \quad 480$$

$$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0 \quad 481$$

$$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0 \quad 482$$

$$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0 \quad 483$$

$$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0 \quad 484$$

$$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0 \quad 484\text{A}$$

$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$

$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$

$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$

$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$

$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$

**Proof:** 485

(a) Indeed the first two equations have a nontrivial solution $G_{13}, G_{14}$ if

$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$

**Proof:** 486

(gg) Indeed the first two equations have a nontrivial solution $G_{16}, G_{17}$ if

$\mathrm{F}(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$

**Proof:** 487

(a) Indeed the first two equations have a nontrivial solution $G_{20}, G_{21}$ if

$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$

**Proof:** 488

(a) Indeed the first two equations have a nontrivial solution $G_{24}, G_{25}$ if

$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$

**Proof:** 489

(a) Indeed the first two equations have a nontrivial solution $G_{28}, G_{29}$ if

$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$

**Proof:** 490

(a) Indeed the first two equations have a nontrivial solution $G_{32}, G_{33}$ if

$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) +$

$(a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$

**Proof:** 491

(a) Indeed the first two equations have a nontrivial solution $G_{36}, G_{37}$ if

$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$

**Proof:** 492

(a) Indeed the first two equations have a nontrivial solution $G_{40}, G_{41}$ if

$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$

**Proof:** 492 A

(a) Indeed the first two equations have a nontrivial solution $G_{44}, G_{45}$ if

$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$

**Definition and uniqueness of** $T^*_{14}$ :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique $T^*_{14}$ for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

**Definition and uniqueness of** $T^*_{17}$ :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique $T^*_{17}$ for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]}$$ 495

**Definition and uniqueness of** $T^*_{21}$ :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique $T^*_{21}$ for which $f(T^*_{21}) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

**Definition and uniqueness of** $T^*_{25}$ :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that

there exists a unique $T_{25}^{*}$ for which $f(T_{25}^{*})=0$. With this value , we obtain from the three first equations

$$G_{24}=\frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}=\frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T_{25}^{*})]}$$

**Definition and uniqueness of** $T_{29}^{*}$ :- 498

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(5)}(T_{29})$ being increasing, it follows that there exists a unique $T_{29}^{*}$ for which $f(T_{29}^{*})=0$. With this value , we obtain from the three first equations

$$G_{28}=\frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}=\frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T_{29}^{*})]}$$

**Definition and uniqueness of** $T_{33}^{*}$ :- 499

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(6)}(T_{33})$ being increasing, it follows that there exists a unique $T_{33}^{*}$ for which $f(T_{33}^{*})=0$. With this value , we obtain from the three first equations

$$G_{32}=\frac{(a_{32})^{(6)}G_{33}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^{*})]} \quad , \quad G_{34}=\frac{(a_{34})^{(6)}G_{33}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^{*})]}$$

**Definition and uniqueness of** $T_{37}^{*}$ :- 500

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(7)}(T_{37})$ being increasing, it follows that there exists a unique $T_{37}^{*}$ for which $f(T_{37}^{*})=0$. With this value , we obtain from the three first equations

$$G_{36}=\frac{(a_{36})^{(7)}G_{37}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^{*})]} \quad , \quad G_{38}=\frac{(a_{38})^{(7)}G_{37}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^{*})]}$$

**Definition and uniqueness of** $T_{41}^{*}$ :- 501

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique $T_{41}^{*}$ for which $f(T_{41}^{*})=0$. With this value , we obtain from the three first equations

$$G_{40}=\frac{(a_{40})^{(8)}G_{41}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T_{41}^{*})]} \quad , \quad G_{42}=\frac{(a_{42})^{(8)}G_{41}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T_{41}^{*})]}$$

**Definition and uniqueness of** $T_{45}^{*}$ :- 501 A

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(9)}(T_{45})$ being increasing, it follows that there exists a unique $T_{45}^{*}$ for which $f(T_{45}^{*})=0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)}G_{45}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T^*_{45})]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)}G_{45}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T^*_{45})]}$$

By the same argument, the equations admit solutions $G_{13}, G_{14}$ if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right]+(b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{14}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{14}$ such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions $G_{16}, G_{17}$ if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right]+(b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ 504
is a decreasing function in $G_{17}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{14}$ such that $\varphi((G_{19})^*) = 0$

By the same argument, the equations admit solutions $G_{20}, G_{21}$ if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right]+(b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{21}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{21}$ such that $\varphi((G_{23})^*) = 0$

By the same argument, the equations admit solutions $G_{24}, G_{25}$ if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right]+(b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{25}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{25}$ such that $\varphi((G_{27})^*) = 0$

By the same argument, the equations admit solutions $G_{28}, G_{29}$ if 507

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right]+(b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{29}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that

there exists a unique $G_{29}^{*}$ such that $\varphi((G_{31})^{*}) = 0$

By the same argument, the equations admit solutions $G_{32}, G_{33}$ if 508
$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$

$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right] + (b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{33}$ taking into account the hypothesis $\varphi(0) > 0\,, \varphi(\infty) < 0$ it follows that there exists a unique $G_{33}^{*}$ such that $\varphi(G_{35}{}^{*}) = 0$

By the same argument, the equations admit solutions $G_{36}, G_{37}$ if 509
$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$
$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right] + (b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{37}$ taking into account the hypothesis $\varphi(0) > 0\,, \varphi(\infty) < 0$ it follows that there exists a unique $G_{37}^{*}$ such that $\varphi(G_{39}{}^{*}) = 0$

By the same argument, the equations admit solutions $G_{40}, G_{41}$ if 510
$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$
$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right] + (b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{41}$ taking into account the hypothesis $\varphi(0) > 0\,, \varphi(\infty) < 0$ it follows that there exists a unique $G_{41}^{*}$ such that $\varphi(G_{43}{}^{*}) = 0$

By the same argument, the equations 92,93 admit solutions $G_{44}, G_{45}$ if
$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$

$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right] + (b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{45}$ taking into account the hypothesis $\varphi(0) > 0\,, \varphi(\infty) < 0$ it follows that there exists a unique $G_{45}^{*}$ such that $\varphi((G_{47})^{*}) = 0$

Finally we obtain the unique solution 511

$G_{14}^{*}$ given by $\varphi(G^{*}) = 0\,, T_{14}^{*}$ given by $f(T_{14}^{*}) = 0$ and

$$G_{13}^{*} = \frac{(a_{13})^{(1)}G_{14}^{*}}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^{*})]} \quad , \quad G_{15}^{*} = \frac{(a_{15})^{(1)}G_{14}^{*}}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^{*})]}$$

$$T_{13}^{*} = \frac{(b_{13})^{(1)}T_{14}^{*}}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^{*})]} \quad , \quad T_{15}^{*} = \frac{(b_{15})^{(1)}T_{14}^{*}}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^{*})]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

$G_{17}^{*}$ given by $\varphi((G_{19})^{*}) = 0$ , $T_{17}^{*}$ given by $f(T_{17}^{*}) = 0$ and 512

$$G_{16}^{*} = \frac{(a_{16})^{(2)} G_{17}^{*}}{[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}^{*})]} \quad , \quad G_{18}^{*} = \frac{(a_{18})^{(2)} G_{17}^{*}}{[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}^{*})]} \qquad 513$$

$$T_{16}^{*} = \frac{(b_{16})^{(2)} T_{17}^{*}}{[(b'_{16})^{(2)} - (b''_{16})^{(2)}((G_{19})^{*})]} \quad , \quad T_{18}^{*} = \frac{(b_{18})^{(2)} T_{17}^{*}}{[(b'_{18})^{(2)} - (b''_{18})^{(2)}((G_{19})^{*})]} \qquad 514$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

$G_{21}^{*}$ given by $\varphi((G_{23})^{*}) = 0$ , $T_{21}^{*}$ given by $f(T_{21}^{*}) = 0$ and

$$G_{20}^{*} = \frac{(a_{20})^{(3)} G_{21}^{*}}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^{*})]} \quad , \quad G_{22}^{*} = \frac{(a_{22})^{(3)} G_{21}^{*}}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^{*})]}$$

$$T_{20}^{*} = \frac{(b_{20})^{(3)} T_{21}^{*}}{[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}{}^{*})]} \quad , \quad T_{22}^{*} = \frac{(b_{22})^{(3)} T_{21}^{*}}{[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}{}^{*})]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

$G_{25}^{*}$ given by $\varphi(G_{27}) = 0$ , $T_{25}^{*}$ given by $f(T_{25}^{*}) = 0$ and

$$G_{24}^{*} = \frac{(a_{24})^{(4)} G_{25}^{*}}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}^{*} = \frac{(a_{26})^{(4)} G_{25}^{*}}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^{*})]}$$

$$T_{24}^{*} = \frac{(b_{24})^{(4)} T_{25}^{*}}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^{*})]} \quad , \quad T_{26}^{*} = \frac{(b_{26})^{(4)} T_{25}^{*}}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^{*})]} \qquad 517$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

$G_{29}^{*}$ given by $\varphi((G_{31})^{*}) = 0$ , $T_{29}^{*}$ given by $f(T_{29}^{*}) = 0$ and

$$G_{28}^{*} = \frac{(a_{28})^{(5)} G_{29}^{*}}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}^{*} = \frac{(a_{30})^{(5)} G_{29}^{*}}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^{*})]}$$

$$T_{28}^{*} = \frac{(b_{28})^{(5)} T_{29}^{*}}{[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31})^{*})]} \quad , \quad T_{30}^{*} = \frac{(b_{30})^{(5)} T_{29}^{*}}{[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31})^{*})]} \qquad 519$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

$G_{33}^{*}$ given by $\varphi((G_{35})^{*}) = 0$ , $T_{33}^{*}$ given by $f(T_{33}^{*}) = 0$ and

$$G_{32}^{*} = \frac{(a_{32})^{(6)} G_{33}^{*}}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^{*})]} \quad , \quad G_{34}^{*} = \frac{(a_{34})^{(6)} G_{33}^{*}}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^{*})]}$$

$$T_{32}^{*} = \frac{(b_{32})^{(6)} T_{33}^{*}}{[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35})^{*})]} \quad , \quad T_{34}^{*} = \frac{(b_{34})^{(6)} T_{33}^{*}}{[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35})^{*})]} \qquad 521$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

$G_{37}^*$ given by $\varphi((G_{39})^*) = 0$ , $T_{37}^*$ given by $f(T_{37}^*) = 0$ and

$$G_{36}^* = \frac{(a_{36})^{(7)} G_{37}^*}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^*)]} \quad , \quad G_{38}^* = \frac{(a_{38})^{(7)} G_{37}^*}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^*)]}$$

$$T_{36}^* = \frac{(b_{36})^{(7)} T_{37}^*}{[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39})^*)]} \quad , \quad T_{38}^* = \frac{(b_{38})^{(7)} T_{37}^*}{[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39})^*)]}$$

Finally we obtain the unique solution 523

$G_{41}^*$ given by $\varphi((G_{43})^*) = 0$ , $T_{41}^*$ given by $f(T_{41}^*) = 0$ and

$$G_{40}^* = \frac{(a_{40})^{(8)} G_{41}^*}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42}^* = \frac{(a_{42})^{(8)} G_{41}^*}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

$$T_{40}^* = \frac{(b_{40})^{(8)} T_{41}^*}{[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43})^*)]} \quad , \quad T_{42}^* = \frac{(b_{42})^{(8)} T_{41}^*}{[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43})^*)]}$$

Finally we obtain the unique solution of 89 to 99 523 A

$G_{45}^*$ given by $\varphi((G_{47})^*) = 0$ , $T_{45}^*$ given by $f(T_{45}^*) = 0$ and

$$G_{44}^* = \frac{(a_{44})^{(9)} G_{45}^*}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46}^* = \frac{(a_{46})^{(9)} G_{45}^*}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

$$T_{44}^* = \frac{(b_{44})^{(9)} T_{45}^*}{[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47})^*)]} \quad , \quad T_{46}^* = \frac{(b_{46})^{(9)} T_{45}^*}{[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47})^*)]}$$

## ASYMPTOTIC STABILITY ANALYSIS 524

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

**Proof:** Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a''_{14})^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \frac{\partial (b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a'_{13})^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)}G^*_{13}\mathbb{T}_{14} \quad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a'_{14})^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)}G^*_{14}\mathbb{T}_{14} \quad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a'_{15})^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)}G^*_{15}\mathbb{T}_{14} \quad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b'_{13})^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)}T^*_{13}\mathbb{G}_j\right) \quad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b'_{14})^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)}T^*_{14}\mathbb{G}_j\right) \quad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b'_{15})^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T^*_{15}\mathbb{G}_j\right) \quad 530$$

**ASYMPTOTIC STABILITY ANALYSIS** 531

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(2)}$ and $(b''_i)^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G^*_i + \mathbb{G}_i \quad , T_i = T^*_i + \mathbb{T}_i \quad 532$$

$$\frac{\partial(a''_{17})^{(2)}}{\partial T_{17}}(T^*_{17}) = (q_{17})^{(2)} \quad , \frac{\partial(b''_i)^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \quad 533$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a'_{16})^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G^*_{16}\mathbb{T}_{17} \quad 534$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a'_{17})^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G^*_{17}\mathbb{T}_{17} \quad 535$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a'_{18})^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G^*_{18}\mathbb{T}_{17} \quad 536$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b'_{16})^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T^*_{16}\mathbb{G}_j\right) \quad 537$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b'_{17})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}T^*_{17}\mathbb{G}_j\right) \quad 538$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b'_{18})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}T^*_{18}\mathbb{G}_j\right) \quad 539$$

**ASYMPTOTIC STABILITY ANALYSIS** 540

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(3)}$ *and* $(b''_i)^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{21}'')^{(3)}}{\partial T_{21}}(T_{21}^*) = (q_{21})^{(3)} \quad , \frac{\partial (b_i'')^{(3)}}{\partial G_j}(\,(G_{23})^*\,) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\big((a_{20}')^{(3)} + (p_{20})^{(3)}\big)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G_{20}^*\mathbb{T}_{21} \quad 541$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\big((a_{21}')^{(3)} + (p_{21})^{(3)}\big)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G_{21}^*\mathbb{T}_{21} \quad 542$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\big((a_{22}')^{(3)} + (p_{22})^{(3)}\big)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G_{22}^*\mathbb{T}_{21} \quad 543$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\big((b_{20}')^{(3)} - (r_{20})^{(3)}\big)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\big(s_{(20)(j)}T_{20}^*\mathbb{G}_j\big) \quad 544$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\big((b_{21}')^{(3)} - (r_{21})^{(3)}\big)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\big(s_{(21)(j)}T_{21}^*\mathbb{G}_j\big) \quad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\big((b_{22}')^{(3)} - (r_{22})^{(3)}\big)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\big(s_{(22)(j)}T_{22}^*\mathbb{G}_j\big) \quad 546$$

**ASYMPTOTIC STABILITY ANALYSIS** 547

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}\ and\ (b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{25}'')^{(4)}}{\partial T_{25}}(T_{25}^*) = (q_{25})^{(4)} \quad , \frac{\partial (b_i'')^{(4)}}{\partial G_j}((G_{27})^*\,) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\big((a_{24}')^{(4)} + (p_{24})^{(4)}\big)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G_{24}^*\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\big((a_{25}')^{(4)} + (p_{25})^{(4)}\big)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G_{25}^*\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\big((a_{26}')^{(4)} + (p_{26})^{(4)}\big)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G_{26}^*\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\big((b_{24}')^{(4)} - (r_{24})^{(4)}\big)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\big(s_{(24)(j)}T_{24}^*\mathbb{G}_j\big) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\big((b_{25}')^{(4)} - (r_{25})^{(4)}\big)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\big(s_{(25)(j)}T_{25}^*\mathbb{G}_j\big) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T^*_{26}\mathbb{G}_j\right) \quad 554$$

**ASYMPTOTIC STABILITY ANALYSIS** 555

**Theorem 5:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}\ and\ (b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 556

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{29})^{(5)}}{\partial T_{29}}(T^*_{29}) = (q_{29})^{(5)} \ , \ \frac{\partial(b_i'')^{(5)}}{\partial G_j}\left((G_{31})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a'_{28})^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G^*_{28}\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a'_{29})^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G^*_{29}\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a'_{30})^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G^*_{30}\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b'_{28})^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T^*_{28}\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b'_{29})^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T^*_{29}\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b'_{30})^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T^*_{30}\mathbb{G}_j\right) \quad 562$$

**ASYMPTOTIC STABILITY ANALYSIS** 563

**Theorem 6:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}\ and\ (b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{33})^{(6)}}{\partial T_{33}}(T^*_{33}) = (q_{33})^{(6)} \ , \ \frac{\partial(b_i'')^{(6)}}{\partial G_j}\left((G_{35})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a'_{32})^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G^*_{32}\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a'_{33})^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G^*_{33}\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a'_{34})^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G^*_{34}\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b'_{32})^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T^*_{32}\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b'_{33})^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T^*_{33}\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b'_{34})^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T^*_{34}\mathbb{G}_j\right) \quad 570$$

**ASYMPTOTIC STABILITY ANALYSIS** 571

**Theorem 7:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 572

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{37})^{(7)}}{\partial T_{37}}(T^*_{37}) = (q_{37})^{(7)} \quad , \frac{\partial(b_i'')^{(7)}}{\partial G_j}\left((G_{39})^{**}\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a'_{36})^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G^*_{36}\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a'_{37})^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G^*_{37}\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a'_{38})^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G^*_{38}\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b'_{36})^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T^*_{36}\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b'_{37})^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T^*_{37}\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b'_{38})^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T^*_{38}\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

**ASYMPTOTIC STABILITY ANALYSIS**

**Theorem 8:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{41}'')^{(8)}}{\partial T_{41}}(T_{41}^*) = (q_{41})^{(8)} \quad , \frac{\partial (b_i'')^{(8)}}{\partial G_j}\left( (G_{43})^* \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a_{40}')^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G_{40}^*\mathbb{T}_{41} \quad (581)$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a_{41}')^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G_{41}^*\mathbb{T}_{41} \quad (582)$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a_{42}')^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G_{42}^*\mathbb{T}_{41} \quad (583)$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b_{40}')^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T_{40}^*\mathbb{G}_j\right) \quad (584)$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b_{41}')^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T_{41}^*\mathbb{G}_j\right) \quad (585)$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b_{42}')^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T_{42}^*\mathbb{G}_j\right) \quad (586)$$

(586 A)

**ASYMPTOTIC STABILITY ANALYSIS**

**Theorem 9:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}$ *and* $(b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.
**Proof:** Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{45}'')^{(9)}}{\partial T_{45}}(T_{45}^*) = (q_{45})^{(9)} \quad , \frac{\partial (b_i'')^{(9)}}{\partial G_j}\left( (G_{47})^* \right) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a_{44}')^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)}G_{44}^*\mathbb{T}_{45} \quad (586\,B)$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a_{45}')^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)}G_{45}^*\mathbb{T}_{45} \quad (586\,C)$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a_{46}')^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)}G_{46}^*\mathbb{T}_{45} \quad (586\,D)$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b_{44}')^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)}T_{44}^*\mathbb{G}_j\right) \quad (586\,E)$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b_{45}')^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)}T_{45}^*\mathbb{G}_j\right) \quad (586\,F)$$

$\frac{d\mathbb{T}_{46}}{dt} = -\left((b'_{46})^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T^*_{46}\mathbb{G}_j\right)$ 586

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**The characteristic equation of this system is**

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(q_{13})^{(1)}G^*_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)}T^*_{14} + (b_{14})^{(1)}s_{(13),(14)}T^*_{14}\right)$$

$$+\left(\left((\lambda)^{(1)} + (a'_{14})^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)}G^*_{13} + (a_{13})^{(1)}(q_{14})^{(1)}G^*_{14}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)}T^*_{14} + (b_{14})^{(1)}s_{(13),(13)}T^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((b'_{13})^{(1)} + (b'_{14})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(15)}T^*_{14} + (b_{14})^{(1)}s_{(13),(15)}T^*_{13}\right)\} = 0$$

$+$

$$(\lambda)^{(2)} + (b'_{18})^{(2)} - (r_{18})^{(2)}\{\left((\lambda)^{(2)} + (a'_{18})^{(2)} + (p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^*_{17} + (a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^*_{17}\right)$$

$$+\left(\left((\lambda)^{(2)} + (a'_{17})^{(2)} + (p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^*_{16} + (a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}\right)$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2 + \left((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2 + \left((b'_{16})^{(2)} + (b'_{17})^{(2)} - (r_{16})^{(2)} + (r_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$+\left(\left((\lambda)^{(2)}\right)^2 + \left((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17} + (a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^{*}_{16}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\right)\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left(\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)\right]$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(21)}T^{*}_{21}\right)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^{*}_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^{*}_{21}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(20)}T^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$+\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(22)}T^{*}_{20}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\right)\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left(\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)\right]$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(25)}T^{*}_{25}\right)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^{*}_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^{*}_{25}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(24)}T^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$+\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(26)}T^{*}_{24}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\right)\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(29)}T^{*}_{29}\right)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^{*}_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^{*}_{29}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(28)}T^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$+\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(30)}T^{*}_{28}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\right)\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left(\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)\right]$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(33)}T^{*}_{33}\right)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^{*}_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^{*}_{33}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(32)}T^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$+\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(34)}T^{*}_{32}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\right)\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)\right]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(37)}T^{*}_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^{*}_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^{*}_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(36)}T^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$+\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(38)}T^{*}_{36}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\right)\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(41)}T^{*}_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^{*}_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^{*}_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(40)}T^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$+\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(42)}T^{*}_{40}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\right)\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(45)}T^{*}_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^{*}_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^{*}_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(44)}T^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$+\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(46)}T^{*}_{44}\right)\}=0$$

**And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.**

## SECTION THIRTY SIX

## Entropy And Syntropy

### INTRODUCTION—VARIABLES USED

**The conflict between entropy and syntropy: the vital needs model Ulisse Di Corpo ulisse.dicorpo@gmail.com**

(1) In this paper the vital needs model, which describes 3 main groups of conditions which living systems need to satisfy in order to survive, is discussed. This model was developed working on the **laws of entropy, syntropy and retrocausality.** The conflict between entropy and syntropy In special relativity the energy-momentum relation relates the energy of an object (E) with its momentum (p), and mass (m), where c is the speed of light: E2 = p2 c2 + m2c4

(2) This equation has a dual energy solution: one positive +E, which moves forward in time, and one negative −E, which moves backward in time. If the momentum is zero then the equation simplifies into the famous energy-mass relation: E = mc2

(3) However, in 1926 Klein and Gordon, in order to generalize the Schrödinger wave equation into (e&eb) a relativistic invariant equation, had to insert the full energy-momentum relation, arriving at (eb) a dual wave solution which characterizes (eb) the D'Alembert operator: retarded potentials which propagate from the past to (e&eb) the future (+E) and anticipated potentials which propagate (e&eb) backward, from the future to (e&eb) the past (−E).

(4) In 1928 Paul Dirac tried to solve the unacceptable negative solution by applying the energy-momentum relation to (e&eb) the study of electrons, turning them into relativistic objects.

(5) But, also in this case, the unwanted negative solution emerged (eb) in the form of electrons (+E) and its antiparticles (−E).

(6) In 1941 Luigi Fantappiè, noted that the mathematical properties of those phenomena which are determined by (e) past causes (causality, +E), such as the physical and chemical processes, are governed by (e&eb) the law of entropy, while the mathematical properties of those phenomena which are attracted towards causes located in (eb) the future (retrocausality, −E), agree with (=) a symmetrical law which Fantappiè named syntropy.

(7) Fantappiè immediately recognized the coincidence between the mathematical properties of syntropy and (=) the properties of living systems (Fantappiè, 1942).

(8) It is important to note that, as a consequence of the fact that the universe is expanding, macrocosm is governed by (e&eb) the law of entropy which forces time to flow from the past to (e&eb) the future (mechanical causation: causeÎeffect).

**The conflict between entropy and syntropy: the vital needs model Ulisse Di Corpo ulisse.dicorpo@gmail.com**

(1) Comparing the infinite of the environment (entropy) and (e&eb) finite of the living system (syntropy) produces results which tends to zero:

(2) In this equation 1 symbolizes the living system which is finite (syntropy), while (e) Infinite symbolizes the environment (entropy).

(3) The comparison between the living system (1, finite) and the environment (infinite) tends to zero. In other words, comparing ourselves with the environment which is infinite we become aware of (e) the fact that we are equal to nothing.

(4) But to be equal to nothing is equivalent to (=) death, a fact which is incompatible with (e&eb) the feeling of life.

(5) It is therefore necessary to solve this conflict between being (1) and not being (0), a conflict which consumes (e) energy and increases entropy. This conflict is generally felt as the need to give a meaning to life, for example:

**(i) Increasing our own value (through richness, power, achievement, etc.);**

**(ii) Finding a purpose in life, finality (through ideologies, religion, etc.).**

(6) In living beings with highly complex cortical systems, this need is vital because, (e) when it is not solved, it leads to (eb) the dissipation of energy, and in the most serious cases to death.

(7) The existential crisis associated with (e&eb) this conflict is accompanied by (e&eb) feelings of being useless, purposeless, reduction of energy (dissipation of energy, entropy), usually named

depression, felt in the cortical area in (eb) the form of tension, and usually strongly correlated with (e&eb) anxiety and feelings of pain in the thorax.

**The conflict between entropy and syntropy: the vital needs model Ulisse Di Corpo ulisse.dicorpo@gmail.com**

(4) This strong correlation between depression and (e&eb) anxiety is suggested by the fact that, from a mathematical point of view, the conflict between being and (e&eb) not being is solved when: Where the operator x coincides with union, which is (=) the property of –E (converging waves).
(5) In other words, when we unite ourselves (1) to (e&eb) the environment, comparing ourselves to the environment, (eb) we find our identity (= 1).
(6) This last equation permits to state that:

• when the need of meaning is answered (eb) increasing the value of the numerator (power, richness, achievement), the identity conflict is not solved, because (e) whichever is the value at the numerator compared to (e&eb) infinite it tends to zero;

• Perfect correlation between anxiety and (e&eb) depression must be observed, because (e) when the unity (x) is weak, anxiety increases (eb) and also the identity conflict and depression.

**Chronological order**

(7) According to this model a chronological order exists in the satisfaction of these needs. Initially humans have (e) to answer material needs, which are more immediate, and only afterwards they become (eb) aware of the needs of meaning and unity.
(8) Part of humanity has now satisfied (eb) material needs and is now facing the needs of (e) meaning and unity and is undergoing rising feelings of existential crisis and anxiety.
(9) Material needs can be easily recognized, whereas the needs of meaning and unity are immaterial, invisible, and more difficult to be recognized. For example:

• In order to give a meaning to their lives, people become (=) obsessed with the way they are judged. They strive for (e) power, success. They become addicted to (e) religion, ideologies and groups;

• In order to overcome anxiety they become (eb) addicted to alcohol and drugs and avoid (e) any moment of silence.

(10) The model of vital needs affirms (eb) that all these states are (=) secondary needs which originate from (e) the unsatisfied needs of meaning and unity. According to the vital needs model all behaviours (psychological, sociological and economical) can be traced back to (e) these 3 basic needs.

**Empirical evidences**

Working as a sociologist for public institutions I am always faced with the centrality of the 3 vital needs which I have just described. For example:

• in a study, just concluded, for IOM (International Organization on Migration) results showed that whilst irregular migrants come to Italy for economical reasons (material needs), a significant number would go back home immediately, because of loneliness (need of unity) and depression (need of meaning), even though their economic situation at home has not changed. It seems that the experience in Italy and the difficulties associated to the integration process lead the migrant to rediscover a set of needs, qualitatively

different from the material needs, which were met at home, thanks to the family and friends, but could not be met in Italy, causing feelings of loneliness, and depression.

• In a study conducted in the Abruzzi area of Italy, different theoretical models were compared in order to understand the origins of psychological and emotional suffering in high school students. Results show that items relative to loneliness and worthlessness obtained the highest correlations with items relative to unhappiness and suffering (SERT-Teramo, 1993).

• In several studies conducted with drug addict populations, loneliness and worthlessness emerged as the two main reasons behind drug addiction. In all these studies the correlation among loneliness (anxiety) and worthlessness (depression) is nearly perfect, supporting the hypothesis of the existence of a mathematical relation between these two needs.

**Conclusions**

The introduction of advanced waves in science might not be (=) as important in physics as it could be in life disciplines such as medicine, psychology and sociology. These disciplines now approach pathologies, illnesses and crises in a mechanistic way which leads, (eb) in an always more alarming way, to inefficiency and to an increase in costs.

## NOTATION

### Module One

In this paper the vital needs model, which describes 3 main groups of conditions which living systems need to satisfy in order to survive, is discussed.

This model was developed working on the **laws of entropy, syntropy and retrocausality.**

The conflict between entropy and syntropy

In special relativity the **energy-momentum relation relates the energy of an object (E)** with its momentum (p), and mass (m), where c is the speed of light: E2 = p2 c2 + m2c4

$G_{13}$ : Category one of **energy-momentum relation relates the energy of an object (E);** momentum (p), and mass (m), where c is the speed of light

$G_{14}$ : Category two of SAS

$G_{15}$ : Category three of SAS

$T_{13}$ : Category one of momentum (p), and mass (m), where c is the speed of light **;energy-momentum relation relates the energy of an object (E)**

$T_{14}$ : Category two of SAS

$T_{15}$ : Category three of SAS

### Module Two

E2 = p2 c2 + m2c4

$G_{16}$ : Category one of LHS of the above equation

$G_{17}$ : Category two of SAS

$G_{18}$ : Category three of SAS

$T_{16}$ : Category one of RHS of the equation

$T_{17}$ : Category two of SAS

$T_{18}$ : Category three of SAS

**Module three**

This equation has a dual energy solution: one **positive +E**, which moves forward in time, and one negative −E, which moves backward in time. If the momentum is zero then the equation simplifies into the famous energy-mass relation: E = mc2

$G_{20}$ : Category one of **positive +E;** forward in time

$G_{21}$ : Category two of SAS

$G_{22}$ : Category three of SAS

$T_{20}$ : Category one of forward in time; **positive +E**

$T_{21}$ : Category two of SAS

$T_{22}$ : Category three of SAS

**Module four**

This equation has a dual energy solution: one **positive +E**, which moves forward in time, and one **negative −E,** which moves backward in time.

If the momentum is zero then the equation simplifies into the famous energy-mass relation: E = mc2

$G_{24}$ : Category one of **negative –E;** backward in time

$G_{25}$ : Category two of SAS

$G_{26}$ : Category three of SAS

$T_{24}$ : Category one of backward in time **;negative –E;**

$T_{25}$ : Category two of SAS

$T_{26}$ : Category three of SAS

**Module five**

**E = mc2**

$G_{28}$ : Category one of E

$G_{29}$ : Category two of SAS

$G_{30}$ : Category three of SAS

$T_{28}$ : Category one of **mc2**

$T_{29}$ : Category two of SAS

$T_{30}$ : Category three of SAS

**Module six**

However, in 1926 **Klein and Gordon, in order to generalize the Schrödinger wave equation** into (e&eb) a relativistic invariant equation, had to insert the full energy-momentum relation, arriving at (eb) a dual wave solution which characterizes (eb) the D'Alembert operator: retarded potentials which propagate from the past to (e&eb) the future (+E) and anticipated potentials which propagate (e&eb) backward, from the future to (e&eb) the past (−E)

$G_{32}$ : Category one of **generalize the Schrödinger wave equation;** relativistic invariant equation, had to insert the full energy-momentum relation, arriving at (eb) a dual wave solution which characterizes (eb) the D'Alembert operator: retarded potentials which propagate from the past to (e&eb) the future (+E) and anticipated potentials which propagate (e&eb) backward, from the future to (e&eb) the past (−E)

$G_{33}$ : Category two of SAS

$G_{34}$ : Category three of SAS

$T_{32}$ : Category one of relativistic invariant equation, had to insert the full energy-momentum relation, arriving at (eb) a dual wave solution which characterizes (eb) the D'Alembert operator: retarded potentials which propagate from the past to (e&eb) the future (+E) and anticipated potentials which propagate (e&eb) backward, from the future to (e&eb) the past (−E); **generalize the Schrödinger wave equation**

$T_{33}$ : Category two of SAS

$T_{34}$ : Category three of SAS

**Module seven**

However, in 1926 Klein and Gordon, in order to generalize the Schrödinger wave equation into a relativistic invariant equation, had to insert the full energy-momentum relation, arriving at (eb) a dual wave solution which characterizes (eb) the D'Alembert operator: retarded potentials which propagate from the past to (e&eb) the future (+E) and anticipated potentials which propagate (e&eb) backward, from the future to (e&eb) the past (−E)

$G_{36}$ : Category one of generalize the Schrödinger wave equation into a relativistic invariant equation, had to insert the full energy-momentum relation

$G_{37}$ : Category two of SAS

$G_{38}$ : Category three of SAS

$T_{36}$ : Category one of dual wave solution which characterizes (eb) the D'Alembert operator: retarded potentials which propagate from the past to (e&eb) the future (+E) and anticipated potentials which propagate (e&eb) backward, from the future to (e&eb) the past (−E)

$T_{37}$ : Category two of SAS

$T_{38}$ : Category three of SAS

**Module eight**

However, in 1926 Klein and Gordon, in order to generalize the Schrödinger wave equation into a relativistic invariant equation, had to insert the full energy-momentum relation, arriving at a dual wave solution which characterizes (eb) the D'Alembert operator: retarded potentials which propagate from the past to (e&eb) the future (+E) and anticipated potentials which propagate (e&eb) backward, from the future to (e&eb) the past (−E)

$G_{40}$ : Category one of **generalize the Schrödinger wave equation into a relativistic invariant equation, had to insert the full energy-momentum relation, arriving at a dual wave solution;** D'Alembert operator: retarded potentials which propagate from the past to (e&eb) the future (+E) and anticipated potentials which propagate (e&eb) backward, from the future to (e&eb) the past (−E)

$G_{41}$ : Category two of SAS

$G_{42}$ : Category three of SAS

$T_{40}$ : Category one of D'Alembert operator: retarded potentials which propagate from the past to (e&eb) the future (+E) and anticipated potentials which propagate (e&eb) backward, from the future to (e&eb) the past (−E); **generalize the Schrödinger wave equation into a relativistic invariant equation, had to insert the full energy-momentum relation, arriving at a dual wave solution**

$T_{41}$ : Category two of SAS

$T_{42}$ : Category three of SAS

**Module Nine**

However, in 1926 Klein and Gordon, in order to generalize the Schrödinger wave equation into a relativistic invariant equation, had to insert the full energy-momentum relation, arriving at a dual wave solution which characterizes the D'Alembert operator: **retarded potentials which propagate from the past** to (e&eb) the future (+E) and anticipated potentials which propagate (e&eb) backward, from the future to (e&eb) the past (−E)

$G_{44}$ : Category one of **retarded potentials which propagate from the past;** future (+E)

$G_{45}$ : Category two of SAS

$G_{46}$ : Category three of SAS

$T_{44}$ : Category one of future (+E) ;**retarded potentials which propagate from the past**

$T_{45}$ : Category two of SAS

$T_{46}$ : Category three of SAS

**The Coefficients:**

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}\ ,\ (b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)},$
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)},\ \ (b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}\ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}\ ,$
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)},$
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)},$
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)},$
are Dissipation coefficients

**Module Numbered One**

**The differential system of this model is now (Module Numbered one)**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \quad 1$$
$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad 2$$
$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15} \quad 3$$
$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13} \quad 4$$
$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14} \quad 5$$
$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)\right]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14}, t) =$ First augmentation factor
$-(b''_{13})^{(1)}(G, t) =$ First detritions factor

**Module Numbered Two**

**The differential system of this model is now ( Module numbered two)**

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right]G_{16} \quad 7$$
$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t)\right]G_{17} \quad 8$$
$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t)\right]G_{18} \quad 9$$
$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\left((G_{19}), t\right)\right]T_{16} \quad 10$$
$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\left((G_{19}), t\right)\right]T_{17} \quad 11$$
$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\left((G_{19}), t\right)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17}, t) =$ First augmentation factor
$-(b''_{16})^{(2)}\left((G_{19}), t\right) =$ First detritions factor

**Module Numbered Three**

**The differential system of this model is now (Module numbered three)**

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21},t)\right]G_{20} \quad 13$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21},t)\right]G_{21} \quad 14$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21},t)\right]G_{22} \quad 15$$

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23},t)\right]T_{20} \quad 16$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23},t)\right]T_{21} \quad 17$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23},t)\right]T_{22} \quad 18$$

$+(a''_{20})^{(3)}(T_{21},t) =$ First augmentation factor

$-(b''_{20})^{(3)}(G_{23},t) =$ First detritions factor

**Module Numbered Four**

**The differential system of this model is now (Module numbered Four)**

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right]G_{24} \quad 19$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25},t)\right]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25},t)\right]G_{26} \quad 21$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}\big((G_{27}),t\big)\right]T_{24} \quad 22$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}\big((G_{27}),t\big)\right]T_{25} \quad 23$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}\big((G_{27}),t\big)\right]T_{26} \quad 24$$

$+(a''_{24})^{(4)}(T_{25},t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}\big((G_{27}),t\big) =$ **First detritions factor**

**Module Numbered Five**:

**The differential system of this model is now (Module number five)**

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29},t)\right]G_{28} \quad 25$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29},t)\right]G_{29} \quad 26$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29},t)\right]G_{30} \quad 27$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}\big((G_{31}),t\big)\right]T_{28} \quad 28$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}\big((G_{31}),t\big)\right]T_{29} \quad 29$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}\big((G_{31}),t\big)\right]T_{30} \quad 30$$

$+(a''_{28})^{(5)}(T_{29},t) =$ **First augmentation factor**

$-(b''_{28})^{(5)}\big((G_{31}),t\big) =$ **First detritions factor**

**Module Numbered Six**

**The differential system of this model is now (Module numbered Six)**

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33},t)\right]G_{32} \quad 31$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33},t)\right]G_{33} \quad 32$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33},t)\right]G_{34} \quad 33$$

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35}),t)\right]T_{32} \quad 34$$
$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}((G_{35}),t)\right]T_{33} \quad 35$$
$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35}),t)\right]T_{34} \quad 36$$
$+(a''_{32})^{(6)}(T_{33},t) =$ **First augmentation factor**

**Module Numbered Seven:**

**The differential system of this model is now (Seventh Module)**

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right]G_{36} \quad 37$$
$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37},t)\right]G_{37} \quad 38$$
$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37},t)\right]G_{38} \quad 39$$
$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39}),t)\right]T_{36} \quad 40$$
$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}((G_{39}),t)\right]T_{37} \quad 41$$
$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39}),t)\right]T_{38} \quad 42$$
$+(a''_{36})^{(7)}(T_{37},t) =$ **First augmentation factor**

**Module Numbered Eight**

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right]G_{40} \quad 43$$
$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41},t)\right]G_{41} \quad 44$$
$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41},t)\right]G_{42} \quad 45$$
$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43}),t)\right]T_{40} \quad 46$$
$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}((G_{43}),t)\right]T_{41} \quad 47$$
$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43}),t)\right]T_{42} \quad 48$$

**Module Numbered Nine**

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$
$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$
$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$
$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47}),t)\right]T_{44} \quad 52$$
$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}((G_{47}),t)\right]T_{45} \quad 53$$
$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47}),t)\right]T_{46} \quad 54$$
$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**
$-(b''_{44})^{(9)}((G_{47}),t) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)} & +(a''_{13})^{(1)}(T_{14},t) & +(a''_{16})^{(2,2,)}(T_{17},t) & +(a''_{20})^{(3,3,)}(T_{21},t) \\ +(a''_{24})^{(4,4,4,4,)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,)}(T_{33},t) & \\ +(a''_{36})^{(7,7)}(T_{37},t) & +(a''_{40})^{(8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) & \end{bmatrix} G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)}\boxed{+(a''_{14})^{(1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)}\boxed{+(a''_{15})^{(1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15} \quad 57$$

Where $\boxed{(a''_{13})^{(1)}(T_{14},t)}$, $\boxed{(a''_{14})^{(1)}(T_{14},t)}$, $\boxed{(a''_{15})^{(1)}(T_{14},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7)}(T_{37},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8)}(T_{41},t)}$ $\boxed{+(a''_{41})^{(8,8)}(T_{41},t)}$ $\boxed{+(a''_{42})^{(8,8)}(T_{41},t)}$ are eight augmentation coefficient for 1,2,3

$\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)}\boxed{-(b''_{13})^{(1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13} \quad 58$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)}\boxed{-(b''_{14})^{(1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14} \quad 59$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)}\boxed{-(b''_{15})^{(1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15} \quad 60$$

Where $\boxed{-(b''_{13})^{(1)}(G,t)}$, $\boxed{-(b''_{14})^{(1)}(G,t)}$, $\boxed{-(b''_{15})^{(1)}(G,t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8)}(G_{43},t)}$ $\boxed{-(b''_{41})^{(8,8)}(G_{43},t)}$ $\boxed{-(b''_{42})^{(8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[\begin{array}{ccc} (a'_{16})^{(2)}\boxed{+(a''_{16})^{(2)}(T_{17},t)} & \boxed{+(a''_{13})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9)}(T_{45},t)} \end{array}\right] G_{16} \quad (61)$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[\begin{array}{ccc} (a'_{17})^{(2)}\boxed{+(a''_{17})^{(2)}(T_{17},t)} & \boxed{+(a''_{14})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9)}(T_{45},t)} \end{array}\right] G_{17} \quad (62)$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[\begin{array}{ccc} (a'_{18})^{(2)}\boxed{+(a''_{18})^{(2)}(T_{17},t)} & \boxed{+(a''_{15})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9)}(T_{45},t)} \end{array}\right] G_{18} \quad (63)$$

Where $\boxed{+(a''_{16})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,)}(T_{14},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$+(a''_{40})^{(8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8)}(T_{41},t)$ are eight augmentation coefficient for category 1, 2 and 3

$+(a''_{44})^{(9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9)}(T_{45},t)$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \begin{bmatrix} (b'_{16})^{(2)} & -(b''_{16})^{(2)}(G_{19},t) & -(b''_{13})^{(1,1,)}(G,t) & -(b''_{20})^{(3,3,3,)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6)}(G_{35},t) & \\ -(b''_{36})^{(7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9)}(G_{47},t) & \end{bmatrix} T_{16} \quad (64)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)} & -(b''_{17})^{(2)}(G_{19},t) & -(b''_{14})^{(1,1,)}(G,t) & -(b''_{21})^{(3,3,3,)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6)}(G_{35},t) & \\ -(b''_{37})^{(7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9)}(G_{47},t) & \end{bmatrix} T_{17} \quad (65)$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)} & -(b''_{18})^{(2)}(G_{19},t) & -(b''_{15})^{(1,1,)}(G,t) & -(b''_{22})^{(3,3,3,)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6)}(G_{35},t) & \\ -(b''_{38})^{(7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9)}(G_{47},t) & \end{bmatrix} T_{18} \quad (66)$$

where $-(b''_{16})^{(2)}(G_{19},t)$, $-(b''_{17})^{(2)}(G_{19},t)$, $-(b''_{18})^{(2)}(G_{19},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{13})^{(1,1,)}(G,t)$, $-(b''_{14})^{(1,1,)}(G,t)$, $-(b''_{15})^{(1,1,)}(G,t)$ are second detrition coefficients for category 1,2 and 3

$-(b''_{20})^{(3,3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,)}(G_{23},t)$ are third detrition coefficients for category 1,2 and 3

$-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1,2 and 3

$-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1,2 and 3

$-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients for category 1,2 and 3

$-(b''_{36})^{(7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2 and 3

$-(b''_{40})^{(8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1,2 and 3

$-(b''_{44})^{(9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \begin{bmatrix} (a'_{20})^{(3)} & +(a''_{20})^{(3)}(T_{21},t) & +(a''_{16})^{(2,2,2)}(T_{17},t) & +(a''_{13})^{(1,1,1,)}(T_{14},t) \\ +(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t) & \\ +(a''_{36})^{(7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9)}(T_{45},t) & \end{bmatrix} G_{20} \quad (67)$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \begin{bmatrix} (a'_{21})^{(3)}\boxed{+(a''_{21})^{(3)}(T_{21},t)}\ \boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}\ \boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{21} \quad 68$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \begin{bmatrix} (a'_{22})^{(3)}\boxed{+(a''_{22})^{(3)}(T_{21},t)}\ \boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}\ \boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{22} \quad 69$$

$\boxed{+(a''_{20})^{(3)}(T_{21},t)}, \boxed{+(a''_{21})^{(3)}(T_{21},t)}, \boxed{+(a''_{22})^{(3)}(T_{21},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}, \boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}, \boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}$ are second augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)}$ are third augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)}, \boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)}, \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)}, \boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)}, \boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)}, \boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)}, \boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)}$ are seventh augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)}$ are eight augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{44})^{(9,9,9)}(T_{45},t)}, \boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)}, \boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)}$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b'_{20})^{(3)}\boxed{-(b''_{20})^{(3)}(G_{23},t)}\ \boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}\ \boxed{-(b''_{13})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)}\ \boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)}\ \boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7)}(G_{39},t)}\ \boxed{-(b''_{40})^{(8,8,8,8)}(G_{43},t)}\ \boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{20} \quad 70$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b'_{21})^{(3)}\boxed{-(b''_{21})^{(3)}(G_{23},t)}\ \boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}\ \boxed{-(b''_{14})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)}\ \boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)}\ \boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7)}(G_{39},t)}\ \boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)}\ \boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{21} \quad 71$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)}\boxed{-(b''_{22})^{(3)}(G_{23},t)}\ \boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}\ \boxed{-(b''_{15})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)}\ \boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)}\ \boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)}\ \boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)}\ \boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{22} \quad 72$$

$\boxed{-(b''_{20})^{(3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3)}(G_{23},t)}$ *are first detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}$ *are second detrition coefficients for*

*category 1, 2 and 3*

$\boxed{-(b_{13}'')^{(1,1,1,)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,)}(G,t)}$, $\boxed{-(b_{15}'')^{(1,1,1,)}(G,t)}$ *are third detrition coefficients for category 1,2 and 3*

$\boxed{-(b_{24}'')^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{25}'')^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{26}'')^{(4,4,4,4,4,4)}(G_{27},t)}$ *are fourth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b_{28}'')^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5)}(G_{31},t)}$ *are fifth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b_{32}'')^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{34}'')^{(6,6,6,6,6,6)}(G_{35},t)}$ *are sixth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b_{36}'')^{(7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{37}'')^{(7,7,7,7)}(G_{39},t)}$ $\boxed{-(b_{38}'')^{(7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{40}'')^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{42}'')^{(8,8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{46}'')^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b_{45}'')^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b_{44}'')^{(9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[\begin{array}{ccc} (a_{24}')^{(4)} & \boxed{+(a_{24}'')^{(4)}(T_{25},t)} & \boxed{+(a_{28}'')^{(5,5,)}(T_{29},t)} \quad \boxed{+(a_{32}'')^{(6,6,)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a_{16}'')^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a_{20}'')^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{36}'')^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{40}'')^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a_{44}'')^{(9,9,9,9)}(T_{45},t)} \end{array}\right] G_{24} \quad (73)$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[\begin{array}{ccc} (a_{25}')^{(4)} & \boxed{+(a_{25}'')^{(4)}(T_{25},t)} & \boxed{+(a_{29}'')^{(5,5,)}(T_{29},t)} \quad \boxed{+(a_{33}'')^{(6,6)}(T_{33},t)} \\ \boxed{+(a_{14}'')^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a_{17}'')^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a_{21}'')^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{37}'')^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{41}'')^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a_{45}'')^{(9,9,9,9)}(T_{45},t)} \end{array}\right] G_{25} \quad (74)$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[\begin{array}{ccc} (a_{26}')^{(4)} & \boxed{+(a_{26}'')^{(4)}(T_{25},t)} & \boxed{+(a_{30}'')^{(5,5,)}(T_{29},t)} \quad \boxed{+(a_{34}'')^{(6,6,)}(T_{33},t)} \\ \boxed{+(a_{15}'')^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a_{18}'')^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a_{22}'')^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{38}'')^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{42}'')^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a_{46}'')^{(9,9,9,9)}(T_{45},t)} \end{array}\right] G_{26} \quad (75)$$

$\boxed{(a_{24}'')^{(4)}(T_{25},t)}$, $\boxed{(a_{25}'')^{(4)}(T_{25},t)}$, $\boxed{(a_{26}'')^{(4)}(T_{25},t)}$ *are first augmentation coefficients category* $1, 2$ $3$

$\boxed{+(a_{28}'')^{(5,5,)}(T_{29},t)}$, $\boxed{+(a_{29}'')^{(5,5,)}(T_{29},t)}$, $\boxed{+(a_{30}'')^{(5,5,)}(T_{29},t)}$ *are second augmentation coefficient for category* $1, 2$ *and* $3$

$\boxed{+(a_{32}'')^{(6,6,)}(T_{33},t)}$, $\boxed{+(a_{33}'')^{(6,6,)}(T_{33},t)}$, $\boxed{+(a_{34}'')^{(6,6,)}(T_{33},t)}$ *are third augmentation coefficient for category* $1, 2$ *and* $3$

$\boxed{+(a_{13}'')^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a_{14}'')^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a_{15}'')^{(1,1,1,1)}(T_{14},t)}$ *are fourth augmentation coefficients for category* $1, 2$ *and* $3$

$\boxed{+(a_{16}'')^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{17}'')^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{18}'')^{(2,2,2,2)}(T_{17},t)}$ *are fifth augmentation coefficients for category* $1, 2$ *and* $3$

$\boxed{+(a_{20}'')^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{21}'')^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{22}'')^{(3,3,3,3)}(T_{21},t)}$ *are sixth augmentation coefficients for category* $1, 2$ *and* $3$

$\boxed{+(a_{36}'')^{(7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a_{37}'')^{(7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a_{38}'')^{(7,7,7,7,7)}(T_{37},t)}$ *are seventh augmentation coefficients for category* $1, 2$ *and* $3$

$+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)$

*are eighth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{46})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9)}(T_{45},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \begin{bmatrix} (b'_{24})^{(4)} & -(b''_{24})^{(4)}(G_{27},t) & -(b''_{28})^{(5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,)}(G_{35},t) \\ & -(b''_{13})^{(1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{24} \quad (76)$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \begin{bmatrix} (b'_{25})^{(4)} & -(b''_{25})^{(4)}(G_{27},t) & -(b''_{29})^{(5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{25} \quad (77)$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)} & -(b''_{26})^{(4)}(G_{27},t) & -(b''_{30})^{(5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1)}(G,t) & -(b''_{18})^{(2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{26} \quad (78)$$

*Where* $-(b''_{24})^{(4)}(G_{27},t)$, $-(b''_{25})^{(4)}(G_{27},t)$, $-(b''_{26})^{(4)}(G_{27},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{28})^{(5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,)}(G_{31},t)$ *are second detrition coefficients for category* 1, 2 *and* 3

$-(b''_{32})^{(6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,)}(G_{35},t)$ *are third detrition coefficients for category* 1, 2 *and* 3

$-(b''_{13})^{(1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1)}(G,t)$

*are fourth detrition coefficients for category* 1, 2 *and* 3

$-(b''_{16})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2)}(G_{19},t)$

*are fifth detrition coefficients for category* 1, 2 *and* 3

$-(b''_{20})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3)}(G_{23},t)$

*are sixth detrition coefficients for category* 1, 2 *and* 3

$-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)$

*are seventh detrition coefficients for category* 1, 2 *and* 3

$-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)$

*are eighth detrition coefficients for category* 1, 2 *and* 3

$-(b''_{46})^{(9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \begin{bmatrix} (a'_{28})^{(5)} & +(a''_{28})^{(5)}(T_{29},t) & +(a''_{24})^{(4,4,)}(T_{25},t) & +(a''_{32})^{(6,6,6)}(T_{33},t) \\ & +(a''_{13})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{16})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3,3)}(T_{21},t) \\ & +(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{28} \quad (79)$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \begin{bmatrix} (a'_{29})^{(5)} & +(a''_{29})^{(5)}(T_{29},t) & +(a''_{25})^{(4,4,)}(T_{25},t) & +(a''_{33})^{(6,6,6)}(T_{33},t) \\ +(a''_{14})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{17})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3,3)}(T_{21},t) \\ +(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{29} \quad (80)$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \begin{bmatrix} (a'_{30})^{(5)} & +(a''_{30})^{(5)}(T_{29},t) & +(a''_{26})^{(4,4,)}(T_{25},t) & +(a''_{34})^{(6,6,6)}(T_{33},t) \\ +(a''_{15})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{18})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3,3)}(T_{21},t) \\ +(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{30} \quad (81)$$

*Where* $+(a''_{28})^{(5)}(T_{29},t)$, $+(a''_{29})^{(5)}(T_{29},t)$, $+(a''_{30})^{(5)}(T_{29},t)$ *are first augmentation coefficients for category* 1, 2 *and* 3

*And* $+(a''_{24})^{(4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,)}(T_{25},t)$ *are second augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{32})^{(6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6)}(T_{33},t)$ *are third augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)$ are fourth augmentation coefficients for category 1,2, and 3

$+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)$ are fifth augmentation coefficients for category 1,2,and 3

$+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)$ are sixth augmentation coefficients for category 1,2, 3

$+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)$ , $+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1,2, 3

$+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)$ are eighth augmentation coefficients for category 1,2, 3

$+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \begin{bmatrix} (b'_{28})^{(5)} & -(b''_{28})^{(5)}(G_{31},t) & -(b''_{24})^{(4,4,)}(G_{27},t) & -(b''_{32})^{(6,6,6)}(G_{35},t) \\ -(b''_{13})^{(1,1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3)}(G_{23},t) \\ -(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{28} \quad (82)$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b'_{29})^{(5)} & -(b''_{29})^{(5)}(G_{31},t) & -(b''_{25})^{(4,4,)}(G_{27},t) & -(b''_{33})^{(6,6,6)}(G_{35},t) \\ -(b''_{14})^{(1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3)}(G_{23},t) \\ -(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{29} \quad (83)$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b'_{30})^{(5)} & -(b''_{30})^{(5)}(G_{31},t) & -(b''_{26})^{(4,4,)}(G_{27},t) & -(b''_{34})^{(6,6,6)}(G_{35},t) \\ -(b''_{15})^{(1,1,1,1,1,)}(G,t) & -(b''_{18})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3)}(G_{23},t) \\ -(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{30} \quad (84)$$

*where* $-(b''_{28})^{(5)}(G_{31},t)$ , $-(b''_{29})^{(5)}(G_{31},t)$ , $-(b''_{30})^{(5)}(G_{31},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{24})^{(4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,)}(G_{27},t)$ *are second detrition coefficients*

$for\ category\ 1,2\ and\ 3$

$\boxed{-(b''_{32})^{(6,6,6)}(G_{35},t)}, \boxed{-(b''_{33})^{(6,6,6)}(G_{35},t)}, \boxed{-(b''_{34})^{(6,6,6)}(G_{35},t)}$ $are\ third\ detrition\ coefficients$

$for\ category\ 1,2\ and\ 3$

$\boxed{-(b''_{13})^{(1,1,1,1,1)}(G,t)}, \boxed{-(b''_{14})^{(1,1,1,1,1)}(G,t)}, \boxed{-(b''_{15})^{(1,1,1,1,1,)}(G,t)}$ are fourth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} \qquad 85$$

$$-\left[\begin{matrix}(a'_{32})^{(6)}\boxed{+(a''_{32})^{(6)}(T_{33},t)}\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}\\ \boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}\boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)}\boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)}\boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)}\end{matrix}\right]G_{32}$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[\begin{matrix}(a'_{33})^{(6)}\boxed{+(a''_{33})^{(6)}(T_{33},t)}\boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}\boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}\\ \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}\boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}\boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)}\boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}\boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)}\end{matrix}\right]G_{33} \qquad 86$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[\begin{matrix}(a'_{34})^{(6)}\boxed{+(a''_{34})^{(6)}(T_{33},t)}\boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}\boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}\\ \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}\boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}\boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)}\boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}\boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)}\end{matrix}\right]G_{34} \qquad 87$$

$\boxed{+(a''_{32})^{(6)}(T_{33},t)}, \boxed{+(a''_{33})^{(6)}(T_{33},t)}, \boxed{+(a''_{34})^{(6)}(T_{33},t)}$ $are\ first\ augmentation\ coefficients$

$for\ category\ 1,2\ and\ 3$

$\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}$ $are\ second\ augmentation$

$coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}$ $are\ third\ augmentation$

$coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}$ - are fourth augmentation coefficients

$\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}$ - fifth augmentation coefficients

$\boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)}$ sixth augmentation coefficients

$+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)$
seventh augmentation coefficients
$+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)$
Eighth augmentation coefficients
$+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \begin{bmatrix} (b'_{32})^{(6)} & -(b''_{32})^{(6)}(G_{35},t) & -(b''_{28})^{(5,5,5)}(G_{31},t) & -(b''_{24})^{(4,4,4,)}(G_{27},t) \\ & -(b''_{13})^{(1,1,1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{32} \quad (88)$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \begin{bmatrix} (b'_{33})^{(6)} & -(b''_{33})^{(6)}(G_{35},t) & -(b''_{29})^{(5,5,5)}(G_{31},t) & -(b''_{25})^{(4,4,4,)}(G_{27},t) \\ & -(b''_{14})^{(1,1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{33} \quad (89)$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)} & -(b''_{34})^{(6)}(G_{35},t) & -(b''_{30})^{(5,5,5)}(G_{31},t) & -(b''_{26})^{(4,4,4,)}(G_{27},t) \\ & -(b''_{15})^{(1,1,1,1,1,1)}(G,t) & -(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{34} \quad (90)$$

$-(b''_{32})^{(6)}(G_{35},t)$, $-(b''_{33})^{(6)}(G_{35},t)$, $-(b''_{34})^{(6)}(G_{35},t)$ *are first detrition coefficients for category* 1, 2 *and* 3
$-(b''_{28})^{(5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5)}(G_{31},t)$ *are second detrition coefficients for category* 1, 2 *and* 3
$-(b''_{24})^{(4,4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,)}(G_{27},t)$ *are third detrition coefficients for category* 1,2 *and* 3
$-(b''_{13})^{(1,1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1,1,1)}(G,t)$ are fourth detrition coefficients for category 1, 2, and 3
$-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1, 2, and 3
$-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1, 2, and 3
$-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2, and 3
$-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)$
are eighth detrition coefficients for category 1, 2, and 3
$-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \quad 91$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a'_{36})^{(7)}\boxed{+(a''_{36})^{(7)}(T_{37},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \quad 92$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a'_{37})^{(7)}\boxed{+(a''_{37})^{(7)}(T_{37},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{38}}{dt} \quad 93$$

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a'_{38})^{(7)}\boxed{+(a''_{38})^{(7)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{(a''_{36})^{(7)}(T_{37},t)}$, $\boxed{(a''_{37})^{(7)}(T_{37},t)}$, $\boxed{(a''_{38})^{(7)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{36}}{dt} = \quad 94$$

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b'_{36})^{(7)}\boxed{-(b''_{36})^{(7)}(G_{39},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{37}}{dt} =$$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b'_{37})^{(7)}\boxed{-(b''_{37})^{(7)}(G_{39},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{38}}{dt} =$$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b'_{38})^{(7)}\boxed{-(b''_{38})^{(7)}(G_{39},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{40}}{dt}$$



$$= (a_{40})^{(8)}G_{41} - \begin{bmatrix} (a'_{40})^{(8)}\boxed{+(a''_{40})^{(8)}(T_{41},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{41}}{dt}$$

$$= (a_{41})^{(8)}G_{40} - \begin{bmatrix} (a'_{41})^{(8)}\boxed{+(a''_{41})^{(8)}(T_{41},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \begin{bmatrix} (a'_{42})^{(8)}\boxed{+(a''_{42})^{(8)}(T_{41},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a''_{40})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)}\boxed{-(b''_{40})^{(8)}(G_{43},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)}\boxed{-(b''_{41})^{(8)}(G_{43},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)}\boxed{-(b''_{42})^{(8)}(G_{43},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{44}}{dt} \qquad 96$$

$$= (a_{44})^{(9)}G_{45} - \left[\begin{array}{ccc} (a'_{44})^{(9)}\boxed{+(a''_{44})^{(9)}(T_{45},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{array}\right] G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[\begin{array}{ccc} (a'_{45})^{(9)}\boxed{+(a''_{45})^{(9)}(T_{45},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{array}\right] G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[\begin{array}{ccc} (a'_{46})^{(9)}\boxed{+(a''_{46})^{(9)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{array}\right] G_{15}$$

Where $\boxed{+(a''_{44})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{20}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{21}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{22}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{24}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{25}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{26}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{28}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{29}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{30}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{32}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{33}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{34}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a_{14}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a_{15}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ are Seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{38}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a_{37}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a_{36}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a_{40}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a_{42}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a_{41}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} =$$

$$(b_{44})^{(9)}T_{45} -$$

$$\begin{bmatrix} (b_{44}')^{(9)}\boxed{-(b_{44}'')^{(9)}(G_{47},t)} & \boxed{-(b_{16}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{20}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{24}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{28}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{32}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{13}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{36}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b_{45}')^{(9)}\boxed{-(b_{45}'')^{(9)}(G_{47},t)} & \boxed{-(b_{17}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{21}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{25}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{29}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{33}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{14}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{37}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b_{46}')^{(9)}\boxed{-(b_{46}'')^{(9)}(G_{47},t)} & \boxed{-(b_{18}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{22}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{26}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{30}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{34}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{15}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{38}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b_{44}'')^{(9)}(G_{47},t)}$, $\boxed{-(b_{45}'')^{(9)}(G_{47},t)}$, $\boxed{-(b_{46}'')^{(9)}(G_{47},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{16}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{17}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{18}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{20}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{21}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{22}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{24}'')^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{25}'')^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{26}'')^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{28}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{32}'')^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{34}'')^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{37}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{36}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a_i')^{(1)}, (a_i'')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (b_i'')^{(1)} > 0, \quad i,j = 13,14,15 \qquad 97$$

The functions $(a_i'')^{(1)}, (b_i'')^{(1)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a_i'')^{(1)}(T_{14},t) \leq (p_i)^{(1)} \leq (\hat{A}_{13})^{(1)}$$

$$(b_i'')^{(1)}(G,t) \leq (r_i)^{(1)} \leq (b_i')^{(1)} \leq (\hat{B}_{13})^{(1)}$$

$$lim_{T_2\to\infty}(a_i'')^{(1)}(T_{14},t) = (p_i)^{(1)} \qquad 98$$

$$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = (r_i)^{(1)}$$

**Definition of** $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$ :

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$$|(a_i'')^{(1)}(T_{14}',t) - (a_i'')^{(1)}(T_{14},t)| \leq (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G',t) - (b_i'')^{(1)}(G,t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}',t)$ and $(a_i'')^{(1)}(T_{14},t)$ . $(T_{14}',t)$ and $(T_{14},t)$ are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14},t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$ : 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$

**Definition of** $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$ : 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}$, $(r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)} (\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)} (\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i, j = 16,17,18$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17}, t) \leq (p_i)^{(2)} \leq (\hat{A}_{16})^{(2)} \quad 102$$

$$(b_i'')^{(2)}(G_{19}, t) \leq (r_i)^{(2)} \leq (b_i')^{(2)} \leq (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2 \to \infty} (a_i'')^{(2)}(T_{17}, t) = (p_i)^{(2)} \quad 104$$

$$\lim_{G \to \infty} (b_i'')^{(2)}((G_{19}), t) = (r_i)^{(2)} \quad 105$$

**Definition of** $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$ : 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}', t) - (a_i'')^{(2)}(T_{17}, t)| \leq (\hat{k}_{16})^{(2)} |T_{17} - T_{17}'| e^{-(\hat{M}_{16})^{(2)} t} \quad 107$$

$$|(b_i'')^{(2)}((G_{19})', t) - (b_i'')^{(2)}((G_{19}), t)| < (\hat{k}_{16})^{(2)} ||(G_{19}) - (G_{19})'|| e^{-(\hat{M}_{16})^{(2)} t} \quad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}', t)$ and$(a_i'')^{(2)}(T_{17}, t)$ . $(T_{17}', t)$ and $(T_{17}, t)$ are points belonging to the interval $[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}]$ . It is to be noted that $(a_i'')^{(2)}(T_{17}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17}, t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$ :

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)},$ are positive constants 109

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$$

**Definition of** $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$ :

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and\ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1 \quad 110$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(b_i)^{(2)} + (b_i')^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1 \quad 111$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i, j = 20,21,22 \quad 112$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(3)}$, $(r_i)^{(3)}$:

$$(a_i'')^{(3)}(T_{21}, t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23}, t) \leq (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(3)}(T_{21}, t) = (p_i)^{(3)} \quad 113$$

$$\lim_{G \to \infty}(b_i'')^{(3)}(G_{23}, t) = (r_i)^{(3)}$$

**Definition of** $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$ :

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: 114

$$|(a_i'')^{(3)}(T_{21}', t) - (a_i'')^{(3)}(T_{21}, t)| \leq (\hat{k}_{20})^{(3)} |T_{21} - T_{21}'| e^{-(\hat{M}_{20})^{(3)} t}$$

$$|(b_i'')^{(3)}(G_{23}', t) - (b_i'')^{(3)}(G_{23}, t)| < (\hat{k}_{20})^{(3)} ||G_{23} - G_{23}'|| e^{-(\hat{M}_{20})^{(3)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}', t)$ and $(a_i'')^{(3)}(T_{21}, t)$ . $(T_{21}', t)$ And $(T_{21}, t)$ are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21}, t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$ : 115

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}$ , $\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with 116
$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$ and the constants
$(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}, i = 20,21,22,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(a_i)^{(3)} + (a_i')^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)} (\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(b_i)^{(3)} + (b_i')^{(3)} + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)} (\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i, j = 24,25,26$$ 117

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(4)}$, $(r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25}, t) \leq (p_i)^{(4)} \leq (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}((G_{27}), t) \leq (r_i)^{(4)} \leq (b_i')^{(4)} \leq (\hat{B}_{24})^{(4)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(4)}(T_{25}, t) = (p_i)^{(4)}$$ 118

$$\lim_{G \to \infty}(b_i'')^{(4)}((G_{27}), t) = (r_i)^{(4)}$$

**Definition of** $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$ :

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24,25,26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}', t) - (a_i'')^{(4)}(T_{25}, t)| \leq (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})', t) - (b_i'')^{(4)}((G_{27}), t)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}', t)$ and$(a_i'')^{(4)}(T_{25}, t)$ . $(T_{25}', t)$ and $(T_{25}, t)$ are points belonging to the interval $[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}]$. It is to be noted that $(a_i'')^{(4)}(T_{25}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25}, t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$ : 120

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)},$ are positive constants

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}, \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$$

**Definition of** $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$ : 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and\ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}, i = 24,25,26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(a_i)^{(4)} + (a_i')^{(4)} + (\hat{A}_{24})^{(4)} + (\hat{P}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(b_i)^{(4)} + (b_i')^{(4)} + (\hat{B}_{24})^{(4)} + (\hat{Q}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i, j = 28,29,30 \qquad 122$$

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(5)}$, $(r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29}, t) \le (p_i)^{(5)} \le (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}((G_{31}), t) \le (r_i)^{(5)} \le (b_i')^{(5)} \le (\hat{B}_{28})^{(5)}$$

$$lim_{T_2 \to \infty} (a_i'')^{(5)} (T_{29}, t) = (p_i)^{(5)} \qquad 123$$

$$\lim_{G \to \infty} (b_i'')^{(5)} (G_{31}, t) = (r_i)^{(5)}$$

**Definition of** $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$ :

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}', t) - (a_i'')^{(5)}(T_{29}, t)| \le (\hat{k}_{28})^{(5)} |T_{29} - T_{29}'| e^{-(\hat{M}_{28})^{(5)} t}$$

$$|(b_i'')^{(5)}((G_{31})', t) - (b_i'')^{(5)}((G_{31}), t)| < (\hat{k}_{28})^{(5)} ||(G_{31}) - (G_{31})'|| e^{-(\hat{M}_{28})^{(5)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}', t)$ and $(a_i'')^{(5)}(T_{29}, t)$ . $(T_{29}', t)$ and $(T_{29}, t)$ are points belonging to the interval $\left[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\right]$. It is to be noted that $(a_i'')^{(5)}(T_{29}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$ : 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)},$ are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}, \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

**Definition of** $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$ : 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and\ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(a_i)^{(5)} + (a_i')^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(b_i)^{(5)} + (b_i')^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i, j = 32,33,34$$ 127

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(6)}$, $(r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33}, t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}), t) \leq (r_i)^{(6)} \leq (b_i')^{(6)} \leq (\hat{B}_{32})^{(6)}$$

$$lim_{T_2 \to \infty} (a_i'')^{(6)}(T_{33}, t) = (p_i)^{(6)}$$ 128

$$\lim_{G \to \infty} (b_i'')^{(6)}\left((G_{35}), t\right) = (r_i)^{(6)}$$

**Definition of** $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$ :

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}', t) - (a_i'')^{(6)}(T_{33}, t)| \leq (\hat{k}_{32})^{(6)} |T_{33} - T_{33}'| e^{-(\hat{M}_{32})^{(6)} t}$$

$$|(b_i'')^{(6)}((G_{35})', t) - (b_i'')^{(6)}\left((G_{35}), t\right)| < (\hat{k}_{32})^{(6)} ||(G_{35}) - (G_{35})'|| e^{-(\hat{M}_{32})^{(6)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}', t)$ and $(a_i'')^{(6)}(T_{33}, t)$ . $(T_{33}', t)$ and $(T_{33}, t)$ are points belonging to the interval $\left[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}\right]$. It is to be noted that $(a_i'')^{(6)}(T_{33}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$ : 129

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)},$ are positive constants

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$$

**Definition of** $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$ : 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and\ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32,33,34,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(a_i)^{(6)} + (a_i')^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)} (\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(b_i)^{(6)} + (b_i')^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)} (\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

(QQQQQQQQ) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i, j = 36,37,38$ 131

(RRRRRRRR) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(7)}$, $(r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37}, t) \le (p_i)^{(7)} \le (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39}, t) \le (r_i)^{(7)} \le (b_i')^{(7)} \le (\hat{B}_{36})^{(7)}$$

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*(SSSSSSSS)* $\lim_{T_2 \to \infty} (a_i'')^{(7)}(T_{37}, t) = (p_i)^{(7)}$

*(TTTTTTTT)*

$\lim_{G \to \infty} (b_i'')^{(7)}((G_{39}), t) = (r_i)^{(7)}$

**Definition of** $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$ :

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$$|(a_i'')^{(7)}(T_{37}', t) - (a_i'')^{(7)}(T_{37}, t)| \le (\hat{k}_{36})^{(7)} |T_{37} - T_{37}'| e^{-(\hat{M}_{36})^{(7)} t}$$

$$|(b_i'')^{(7)}((G_{39})', t) - (b_i'')^{(7)}((G_{39}), t)| < (\hat{k}_{36})^{(7)} ||(G_{39}) - (G_{39})'|| e^{-(\hat{M}_{36})^{(7)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}', t)$ and $(a_i'')^{(7)}(T_{37}, t)$ . $(T_{37}', t)$ and $(T_{37}, t)$ are points belonging to the interval $[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}]$ . It is to be noted that $(a_i'')^{(7)}(T_{37}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37}, t)$ , the **first augmentation coefficient** attributable to the

system, would be absolutely continuous.

**Definition of** $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$ : 134

(UUUUUUUU) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)},$ are positive constants

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$$

**Definition of** $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$ : 135

(VVVVVVVV) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(b_i)^{(7)} + (b_i')^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40,41,42 \qquad 136$$

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

**Definition of** $(p_i)^{(8)}$, $(r_i)^{(8)}$: 137

$$(a_i'')^{(8)}(T_{41}, t) \le (p_i)^{(8)} \le (\hat{A}_{40})^{(8)} \qquad 138$$

$$(b_i'')^{(8)}((G_{43}), t) \le (r_i)^{(8)} \le (b_i')^{(8)} \le (\hat{B}_{40})^{(8)} \qquad 139$$

$$\lim_{T_2 \to \infty} (a_i'')^{(8)}(T_{41}, t) = (p_i)^{(8)} \qquad 140$$

$$\lim_{G \to \infty} (b_i'')^{(8)}((G_{43}), t) = (r_i)^{(8)} \qquad 141$$

**Definition of** $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$ :

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \le (\hat{k}_{40})^{(8)} |T_{41} - T_{41}'| e^{-(\hat{M}_{40})^{(8)} t} \qquad 142$$

$$|(b_i'')^{(8)}((G_{43})', t) - (b_i'')^{(8)}((G_{43}), t)| < (\hat{k}_{40})^{(8)} ||(G_{43}) - (G_{43})'|| e^{-(\hat{M}_{40})^{(8)} t} \qquad 143$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}', t)$ and $(a_i'')^{(8)}(T_{41}, t)$ . $(T_{41}', t)$ and $(T_{41}, t)$ are points belonging to the interval $\left[(\hat{k}_{40})^{(8)}, (\widehat{M}_{40})^{(8)}\right]$. It is to be noted that $(a_i'')^{(8)}(T_{41}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{40})^{(8)} = 1$ then the function $(a_i'')^{(8)}(T_{41}, t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\widehat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$ :

$(\widehat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)},$ are positive constants

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}, \frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}} < 1 \qquad 144$$

**Definition of** $(\hat{P}_{40})^{(8)}, (\hat{Q}_{40})^{(8)}$ :

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\widehat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}, (\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)}, (a_i')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}, i = 40,41,42,$
Satisfy the inequalities

$$\frac{1}{(\widehat{M}_{40})^{(8)}}[(a_i)^{(8)} + (a_i')^{(8)} + (\hat{A}_{40})^{(8)} + (\hat{P}_{40})^{(8)} (\hat{k}_{40})^{(8)}] < 1 \qquad 145$$

$$\frac{1}{(\widehat{M}_{40})^{(8)}}[(b_i)^{(8)} + (b_i')^{(8)} + (\hat{B}_{40})^{(8)} + (\hat{Q}_{40})^{(8)} (\hat{k}_{40})^{(8)}] < 1 \qquad 146$$

Where we suppose

$$(a_i)^{(9)}, (a_i')^{(9)}, (a_i'')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (b_i'')^{(9)} > 0, \quad i, j = 44,45,46 \qquad 146\text{A}$$

The functions $(a_i'')^{(9)}, (b_i'')^{(9)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(9)}$, $(r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45}, t) \leq (p_i)^{(9)} \leq (\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47}, t) \leq (r_i)^{(9)} \leq (b_i')^{(9)} \leq (\hat{B}_{44})^{(9)}$$

$$\lim_{T_2 \to \infty} (a_i'')^{(9)}(T_{45}, t) = (p_i)^{(9)}$$

$$\lim_{G \to \infty} (b_i'')^{(9)}(G_{47}, t) = (r_i)^{(9)}$$

**Definition of** $(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}$ :

Where $\boxed{(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}}$ are positive constants and $\boxed{i = 44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}', t) - (a_i'')^{(9)}(T_{45}, t)| \leq (\hat{k}_{44})^{(9)} |T_{45} - T_{45}'| e^{-(\widehat{M}_{44})^{(9)} t}$$

$$|(b_i'')^{(9)}((G_{47})', t) - (b_i'')^{(9)}\big((G_{47}), t\big)| < (\hat{k}_{44})^{(9)}||(G_{47}) - (G_{47})'||e^{-(\hat{M}_{44})^{(9)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}', t)$ and$(a_i'')^{(9)}(T_{45}, t)$ . $(T_{45}', t)$ and $(T_{45}, t)$ are points belonging to the interval $\big[(\hat{k}_{44})^{(9)}, (\hat{M}_{44})^{(9)}\big]$. It is to be noted that $(a_i'')^{(9)}(T_{45}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{44})^{(9)} = 1$ then the function $(a_i'')^{(9)}(T_{45}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$ :

$(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$, are positive constants

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}} \ , \frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$$

**Definition of** $(\hat{P}_{44})^{(9)}, (\hat{Q}_{44})^{(9)}$ :

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}, (\hat{A}_{44})^{(9)} and\ (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)}, (a_i')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (p_i)^{(9)}, \ (r_i)^{(9)}, i = 44,45,46,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{44})^{(9)}}[\,(a_i)^{(9)} + (a_i')^{(9)} + \ (\hat{A}_{44})^{(9)} + \ (\hat{P}_{44})^{(9)}\,(\hat{k}_{44})^{(9)}] < 1$$

$$\frac{1}{(\hat{M}_{44})^{(9)}}[\ (b_i)^{(9)} + (b_i')^{(9)} + \ (\hat{B}_{44})^{(9)} + \ (\hat{Q}_{44})^{(9)}\ (\hat{k}_{44})^{(9)}] < 1$$

**Theorem 1:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \big(\hat{P}_{13}\big)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \ , \ \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 2 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

**Definition of** $G_i(0)\,, T_i(0)$

$$G_i(t) \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} \quad , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} \quad , \quad T_i(0) = T_i^0 > 0$$

**Theorem 3 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t) \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t} \quad , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t} \quad , \quad T_i(0) = T_i^0 > 0$$

**Theorem 4 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

**Definition of** $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \left(\hat{P}_{24}\right)^{(4)} e^{(\hat{M}_{24})^{(4)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{24}\right)^{(4)} e^{(\hat{M}_{24})^{(4)}t} \;,\quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 5 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

**Definition of** $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \left(\hat{P}_{28}\right)^{(5)} e^{(\hat{M}_{28})^{(5)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{28}\right)^{(5)} e^{(\hat{M}_{28})^{(5)}t} \;,\quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 6 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

**Definition of** $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \left(\hat{P}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t} \;,\quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 7:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

**Definition of** $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \left(\hat{P}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t} \;,\quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 8:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

**Definition of** $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \left(\hat{P}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t} \;,\quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 9:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

**Definition of** $G_i(0)\,, T_i(0)$ :

$$G_i(t) \le \left(\hat{P}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

**Proof:** Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \leq (\hat{P}_{13})^{(1)} \text{ , } T_i^0 \leq (\hat{Q}_{13})^{(1)}, \quad 155$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t} \quad 156$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t} \quad 157$$

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[ (a_{13})^{(1)} G_{14}(s_{(13)}) - \left( (a'_{13})^{(1)} + a''_{13})^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \right) G_{13}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[ (a_{14})^{(1)} G_{13}(s_{(13)}) - \left( (a'_{14})^{(1)} + (a''_{14})^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \right) G_{14}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[ (a_{15})^{(1)} G_{14}(s_{(13)}) - \left( (a'_{15})^{(1)} + (a''_{15})^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \right) G_{15}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[ (b_{13})^{(1)} T_{14}(s_{(13)}) - \left( (b'_{13})^{(1)} - (b''_{13})^{(1)} (G(s_{(13)}), s_{(13)}) \right) T_{13}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[ (b_{14})^{(1)} T_{13}(s_{(13)}) - \left( (b'_{14})^{(1)} - (b''_{14})^{(1)} (G(s_{(13)}), s_{(13)}) \right) T_{14}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{T}_{15}(t) = T_{15}^0 + \int_0^t \left[ (b_{15})^{(1)} T_{14}(s_{(13)}) - \left( (b'_{15})^{(1)} - (b''_{15})^{(1)} (G(s_{(13)}), s_{(13)}) \right) T_{15}(s_{(13)}) \right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \leq (\hat{P}_{16})^{(2)} \text{ , } T_i^0 \leq (\hat{Q}_{16})^{(2)},$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$$

By 160

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[ (a_{16})^{(2)} G_{17}(s_{(16)}) - \left( (a'_{16})^{(2)} + a''_{16})^{(2)} (T_{17}(s_{(16)}), s_{(16)}) \right) G_{16}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[ (a_{17})^{(2)} G_{16}(s_{(16)}) - \left( (a'_{17})^{(2)} + (a''_{17})^{(2)} (T_{17}(s_{(16)}), s_{(17)}) \right) G_{17}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[ (a_{18})^{(2)} G_{17}(s_{(16)}) - \left( (a'_{18})^{(2)} + (a''_{18})^{(2)} (T_{17}(s_{(16)}), s_{(16)}) \right) G_{18}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^{0} + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^{0} + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^{0} + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{18}(s_{(16)})\right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{20})^{(3)}$, $T_i^0 \le (\hat{Q}_{20})^{(3)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\bar{M}_{20})^{(3)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\bar{M}_{20})^{(3)} t}$

By 161

$$\bar{G}_{20}(t) = G_{20}^{0} + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''_{20})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^{0} + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^{0} + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{22}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^{0} + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^{0} + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{22}(t) = T_{22}^{0} + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{22}(s_{(20)})\right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{24})^{(4)}$, $T_i^0 \le (\hat{Q}_{24})^{(4)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\bar{M}_{24})^{(4)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\bar{M}_{24})^{(4)} t}$

By 162

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{26}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{\mathrm{T}}_{26}(\mathrm{t}) = \mathrm{T}_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{26}(s_{(24)})\right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{28})^{(5)}$, $T_i^0 \leq (\hat{Q}_{28})^{(5)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

By 163

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{30}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{\mathrm{T}}_{30}(\mathrm{t}) = \mathrm{T}_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{30}(s_{(28)})\right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{32})^{(6)}$, $T_i^0 \leq (\hat{Q}_{32})^{(6)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\bar{M}_{32})^{(6)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\bar{M}_{32})^{(6)} t}$

By 164

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[ (a_{32})^{(6)} G_{33}(s_{(32)}) - \left( (a'_{32})^{(6)} + a''_{32})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[ (a_{33})^{(6)} G_{32}(s_{(32)}) - \left( (a'_{33})^{(6)} + (a''_{33})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[ (a_{34})^{(6)} G_{33}(s_{(32)}) - \left( (a'_{34})^{(6)} + (a''_{34})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{34}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[ (b_{32})^{(6)} T_{33}(s_{(32)}) - \left( (b'_{32})^{(6)} - (b''_{32})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[ (b_{33})^{(6)} T_{32}(s_{(32)}) - \left( (b'_{33})^{(6)} - (b''_{33})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{34}(t) = T_{34}^0 + \int_0^t \left[ (b_{34})^{(6)} T_{33}(s_{(32)}) - \left( (b'_{34})^{(6)} - (b''_{34})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{34}(s_{(32)}) \right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[ (a_{36})^{(7)} G_{37}(s_{(36)}) - \left( (a'_{36})^{(7)} + a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[ (a_{37})^{(7)} G_{36}(s_{(36)}) - \left( (a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[ (a_{38})^{(7)} G_{37}(s_{(36)}) - \left( (a'_{38})^{(7)} + (a''_{38})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{38}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[ (b_{36})^{(7)} T_{37}(s_{(36)}) - \left( (b'_{36})^{(7)} - (b''_{36})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[ (b_{37})^{(7)} T_{36}(s_{(36)}) - \left( (b'_{37})^{(7)} - (b''_{37})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\overline{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^{0} + \int_0^t \left[ (b_{38})^{(7)} T_{37}(s_{(36)}) - \left( (b'_{38})^{(7)} - (b''_{38})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{38}(s_{(36)}) \right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$ , $T_i(0) = T_i^0$ , $G_i^0 \le (\hat{P}_{40})^{(8)}$ , $T_i^0 \le (\hat{Q}_{40})^{(8)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[ (a_{40})^{(8)} G_{41}(s_{(40)}) - \left( (a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[ (a_{41})^{(8)} G_{40}(s_{(40)}) - \left( (a'_{41})^{(8)} + (a''_{41})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[ (a_{42})^{(8)} G_{41}(s_{(40)}) - \left( (a'_{42})^{(8)} + (a''_{42})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{42}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^0 + \int_0^t \left[ (b_{40})^{(8)} T_{41}(s_{(40)}) - \left( (b'_{40})^{(8)} - (b''_{40})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^0 + \int_0^t \left[ (b_{41})^{(8)} T_{40}(s_{(40)}) - \left( (b'_{41})^{(8)} - (b''_{41})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\overline{\mathrm{T}}_{42}(\mathrm{t}) = \mathrm{T}_{42}^{0} + \int_0^t \left[ (b_{42})^{(8)} T_{41}(s_{(40)}) - \left( (b'_{42})^{(8)} - (b''_{42})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{42}(s_{(40)}) \right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** 166
Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy A

$G_i(0) = G_i^0$ , $T_i(0) = T_i^0$ , $G_i^0 \le (\hat{P}_{44})^{(9)}$ , $T_i^0 \le (\hat{Q}_{44})^{(9)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[ (a_{44})^{(9)} G_{45}(s_{(44)}) - \left( (a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45}(s_{(44)}), s_{(44)})\right) G_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}(s_{(44)}), s_{(44)})\right) G_{46}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{44}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{46}(t) = T_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{46}(s_{(44)})\right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}}\right)\right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left(e^{(\hat{M}_{13})^{(1)} t} - 1\right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\hat{M}_{13})^{(1)} t} \le \frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{P}_{13})^{(1)} + G_{14}^0\right) e^{\left(-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0}\right)} + (\hat{P}_{13})^{(1)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \le G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}}\right)\right] ds_{(16)} = \quad 169$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1\right)$$

From which it follows that 170

$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \le \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left[\left((\hat{P}_{16})^{(2)} + G_{17}^0\right) e^{\left(-\frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0}\right)} + (\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t) \le G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} \left(G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} s_{(20)}}\right)\right] ds_{(20)} =$$

$$\left(1 + (a_{20})^{(3)} t\right) G_{21}^0 + \frac{(a_{20})^{(3)} (\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left(e^{(\hat{M}_{20})^{(3)} t} - 1\right)$$

From which it follows that 172

$$(G_{20}(t)-G_{20}^{0})e^{-(\hat{M}_{20})^{(3)}t}\leq\frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left[((\hat{P}_{20})^{(3)}+G_{21}^{0})e^{\left(-\frac{(\hat{P}_{20})^{(3)}+G_{21}^{0}}{G_{21}^{0}}\right)}+(\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 173

$$G_{24}(t)\leq G_{24}^{0}+\int_{0}^{t}\left[(a_{24})^{(4)}\left(G_{25}^{0}+(\hat{P}_{24})^{(4)}e^{(\hat{M}_{24})^{(4)}s_{(24)}}\right)\right]ds_{(24)}=$$

$$\left(1+(a_{24})^{(4)}t\right)G_{25}^{0}+\frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left(e^{(\hat{M}_{24})^{(4)}t}-1\right)$$

From which it follows that 174

$$(G_{24}(t)-G_{24}^{0})e^{-(\hat{M}_{24})^{(4)}t}\leq\frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left[((\hat{P}_{24})^{(4)}+G_{25}^{0})e^{\left(-\frac{(\hat{P}_{24})^{(4)}+G_{25}^{0}}{G_{25}^{0}}\right)}+(\hat{P}_{24})^{(4)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t)\leq G_{28}^{0}+\int_{0}^{t}\left[(a_{28})^{(5)}\left(G_{29}^{0}+(\hat{P}_{28})^{(5)}e^{(\hat{M}_{28})^{(5)}s_{(28)}}\right)\right]ds_{(28)}=$$

$$\left(1+(a_{28})^{(5)}t\right)G_{29}^{0}+\frac{(a_{28})^{(5)}(\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left(e^{(\hat{M}_{28})^{(5)}t}-1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t}\leq\frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t)\leq G_{32}^{0}+\int_{0}^{t}\left[(a_{32})^{(6)}\left(G_{33}^{0}+(\hat{P}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\right)\right]ds_{(32)}=$$

$$\left(1+(a_{32})^{(6)}t\right)G_{33}^{0}+\frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left(e^{(\hat{M}_{32})^{(6)}t}-1\right)$$

From which it follows that 177

$$(G_{32}(t)-G_{32}^{0})e^{-(\hat{M}_{32})^{(6)}t}\leq\frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left[((\hat{P}_{32})^{(6)}+G_{33}^{0})e^{\left(-\frac{(\hat{P}_{32})^{(6)}+G_{33}^{0}}{G_{33}^{0}}\right)}+(\hat{P}_{32})^{(6)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

(hh) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t) \le G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} \left(G_{37}^0 + (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} s_{(36)}}\right)\right] ds_{(36)} =$$
$$\left(1 + (a_{36})^{(7)} t\right) G_{37}^0 + \frac{(a_{36})^{(7)} (\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left(e^{(\hat{M}_{36})^{(7)} t} - 1\right)$$

From which it follows that

$$(G_{36}(t) - G_{36}^0) e^{-(\hat{M}_{36})^{(7)} t} \le \frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left[((\hat{P}_{36})^{(7)} + G_{37}^0) e^{\left(-\frac{(\hat{P}_{36})^{(7)} + G_{37}^0}{G_{37}^0}\right)} + (\hat{P}_{36})^{(7)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t) \le G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} \left(G_{41}^0 + (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} s_{(40)}}\right)\right] ds_{(40)} = \quad 180$$
$$\left(1 + (a_{40})^{(8)} t\right) G_{41}^0 + \frac{(a_{40})^{(8)} (\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}} \left(e^{(\hat{M}_{40})^{(8)} t} - 1\right)$$

From which it follows that 181

$$(G_{40}(t) - G_{40}^0) e^{-(\hat{M}_{40})^{(8)} t} \le \frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}} \left[((\hat{P}_{40})^{(8)} + G_{41}^0) e^{\left(-\frac{(\hat{P}_{40})^{(8)} + G_{41}^0}{G_{41}^0}\right)} + (\hat{P}_{40})^{(8)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 8

Analogous inequalities hold also for $G_{41}, G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t) \le G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} \left(G_{45}^0 + (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} s_{(44)}}\right)\right] ds_{(44)} =$$
$$\left(1 + (a_{44})^{(9)} t\right) G_{45}^0 + \frac{(a_{44})^{(9)} (\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left(e^{(\hat{M}_{44})^{(9)} t} - 1\right)$$

From which it follows that

$$(G_{44}(t) - G_{44}^0) e^{-(\hat{M}_{44})^{(9)} t} \le \frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left[((\hat{P}_{44})^{(9)} + G_{45}^0) e^{\left(-\frac{(\hat{P}_{44})^{(9)} + G_{45}^0}{G_{45}^0}\right)} + (\hat{P}_{44})^{(9)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 9

Analogous inequalities hold also for $G_{45}, G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$ and to choose 182

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[(\hat{P}_{13})^{(1)}+\left((\hat{P}_{13})^{(1)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{13})^{(1)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{13})^{(1)} \quad 183$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[\left((\hat{Q}_{13})^{(1)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{13})^{(1)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{13})^{(1)}\right]\leq(\hat{Q}_{13})^{(1)} \quad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)},T^{(1)}\right),\left(G^{(2)},T^{(2)}\right)\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{13})^{(1)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{13})^{(1)}t}\}$$

Indeed if we denote

**Definition of** $\tilde{G},\tilde{T}:\left(\tilde{G},\tilde{T}\right)=\mathcal{A}^{(1)}(G,T)$

It results

$$\left|\tilde{G}_{13}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{13})^{(1)}\left|G_{14}^{(1)}-G_{14}^{(2)}\right|e^{-(\hat{M}_{13})^{(1)}s_{(13)}}e^{(\hat{M}_{13})^{(1)}s_{(13)}}\,ds_{(13)}+$$

$$\int_0^t\{(a'_{13})^{(1)}\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\hat{M}_{13})^{(1)}s_{(13)}}e^{-(\hat{M}_{13})^{(1)}s_{(13)}}+$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\hat{M}_{13})^{(1)}s_{(13)}}e^{(\hat{M}_{13})^{(1)}s_{(13)}}+$$

$$G_{13}^{(2)}\left|(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)-(a''_{13})^{(1)}\left(T_{14}^{(2)},s_{(13)}\right)\right|\ e^{-(\hat{M}_{13})^{(1)}s_{(13)}}e^{(\hat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|G^{(1)}-G^{(2)}\right|e^{-(\hat{M}_{13})^{(1)}t}\leq \quad 186$$
$$\frac{1}{(\hat{M}_{13})^{(1)}}\left((a_{13})^{(1)}+(a'_{13})^{(1)}+(\hat{A}_{13})^{(1)}+(\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}\right)d\left(\left(G^{(1)},T^{(1)};\,G^{(2)},T^{(2)}\right)\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 1:** The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\hat{P}_{13})^{(1)}e^{(\hat{M}_{13})^{(1)}t}$ *and* $(\hat{Q}_{13})^{(1)}e^{(\hat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i = 13,14,15$ depend only on $T_{14}$ and respectively on $G(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any $t$ where $G_i(t) = 0\ and\ T_i(t) = 0$

From 19 to 24 it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(1)} - (a_i'')^{(1)}(T_{14}(s_{(13)}), s_{(13)})\} ds_{(13)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(1)} t)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2\ and\ \left((\widehat{M}_{13})^{(1)}\right)_3$ : 187

**Remark 3**: if $G_{13}$ is bounded, the same property have also $G_{14}\ and\ G_{15}$ . indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a_{14}')^{(1)} G_{14}$ and by integrating

$$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)} \left((\widehat{M}_{13})^{(1)}\right)_1 / (a_{14}')^{(1)}$$

In the same way , one can obtain

$$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)} \left((\widehat{M}_{13})^{(1)}\right)_2 / (a_{15}')^{(1)}$$

If $G_{14}\ or\ G_{15}$ is bounded, the same property follows for $G_{13}$ , $G_{15}$ and $G_{13}$ , $G_{14}$ respectively.

**Remark 4:** If $G_{13}$ *is* bounded, from below, the same property holds for $G_{14}\ and\ G_{15}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{14}$ is bounded from below. 188

**Remark 5:** If $T_{13}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}(G(t), t)) = (b_{14}')^{(1)}$ then $T_{14} \to \infty$. 189

**Definition of** $(m)^{(1)}$ and $\varepsilon_1$ :

Indeed let $t_1$ be so that for $t > t_1$

$$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t), t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right)$, $t = log\frac{2}{\varepsilon_1}$ By taking now $\varepsilon_1$ sufficiently small one sees that $T_{14}$ is unbounded.
The same property holds for $T_{15}$ if $\lim_{t\to\infty}(b_{15}'')^{(1)}(G(t), t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}$ , $\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}\ and\ (\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[(\widehat{P}_{16})^{(2)}+\left((\hat{P}_{16})^{(2)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{16})^{(2)}\right]\leq(\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

**Definition of** $\widetilde{G_{19}},\widetilde{T_{19}}$ : $\left(\widetilde{G_{19}},\widetilde{T_{19}}\right)=\mathcal{A}^{(2)}(G_{19},T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{16})^{(2)}\left|G_{17}^{(1)}-G_{17}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)}+$$

$$\int_0^t\{(a'_{16})^{(2)}\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$G_{16}^{(2)}\left|(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)-(a''_{16})^{(2)}\left(T_{17}^{(2)},s_{(16)}\right)\right|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0,t]$ 197

From the hypotheses it follows

$$\left|(G_{19})^{(1)}-(G_{19})^{(2)}\right|\mathrm{e}^{-(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}\leq$$
$$\frac{1}{(\widehat{\mathrm{M}}_{16})^{(2)}}\left((a_{16})^{(2)}+(a'_{16})^{(2)}+(\widehat{\mathrm{A}}_{16})^{(2)}+(\widehat{\mathrm{P}}_{16})^{(2)}(\hat{k}_{16})^{(2)}\right)\mathrm{d}\left(((G_{19})^{(1)},(T_{19})^{(1)};\ (G_{19})^{(2)},(T_{19})^{(2)})\right)$$

And analogous inequalities for $G_i$ and $T_i$. Taking into account the hypothesis the result follows 198

**Remark 6:** The fact that we supposed $(a''_{16})^{(2)}$ and $(b''_{16})^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{\mathrm{P}}_{16})^{(2)}\mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}$ and $(\widehat{\mathrm{Q}}_{16})^{(2)}\mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i=16,17,18$ depend only on $\mathrm{T}_{17}$ and respectively on

$(G_{19})$(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 7:** There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$ 200

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(2)} - (a_i'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})\} ds_{(16)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(2)}t)} > 0 \quad \text{for } t > 0$$

**Definition of** $\left((\widehat{M}_{16})^{(2)}\right)_1$, $\left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$: 201

**Remark 8:** if $G_{16}$ is bounded, the same property have also $G_{17}$ and $G_{18}$. indeed if

$G_{16} < (\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt} \leq \left((\widehat{M}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)} G_{17}$ and by integrating

$$G_{17} \leq \left((\widehat{M}_{16})^{(2)}\right)_2 = G_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1 / (a_{17}')^{(2)}$$

In the same way , one can obtain

$$G_{18} \leq \left((\widehat{M}_{16})^{(2)}\right)_3 = G_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2 / (a_{18}')^{(2)}$$

If $G_{17}$ or $G_{18}$ is bounded, the same property follows for $G_{16}$, $G_{18}$ and $G_{16}$, $G_{17}$ respectively.

**Remark 9:** If $G_{16}$ is bounded, from below, the same property holds for $G_{17}$ and $G_{18}$. The proof is analogous with the preceding one. An analogous property is true if $G_{17}$ is bounded from below. 202

**Remark 10:** If $T_{16}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t), t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$. 203

**Definition of** $(m)^{(2)}$ and $\varepsilon_2$ :

Indeed let $t_2$ be so that for $t > t_2$

$$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right)$, $t = \log\frac{2}{\varepsilon_2}$ By taking now $\varepsilon_2$ sufficiently small one sees that $T_{17}$ is unbounded. 205
The same property holds for $T_{18}$ if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t), t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[(\widehat{P}_{20})^{(3)}+\left((\hat{P}_{20})^{(3)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{20})^{(3)}\right]\leq(\hat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself 210

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)},(T_{23})^{(1)}),((G_{23})^{(2)},(T_{23})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

**Definition of** $\widetilde{G_{23}},\widetilde{T_{23}}:(\widetilde{(G_{23})},\widetilde{(T_{23})})=\mathcal{A}^{(3)}((G_{23}),(T_{23}))$

It results 213

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{20})^{(3)}\left|G_{21}^{(1)}-G_{21}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}ds_{(20)}+$$

$$\int_0^t\{(a'_{20})^{(3)}\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$G_{20}^{(2)}\left|(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)},s_{(20)}\right)\right|\ e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\}ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses it follows

$$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}t}\leq \quad 214$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+(a'_{20})^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\widehat{k}_{20})^{(3)}\right)d\left(((G_{23})^{(1)},(T_{23})^{(1)};\ (G_{23})^{(2)},(T_{23})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 11:** The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ *and* $(\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i=20,21,22$ depend only on $T_{21}$ and respectively on

$(G_{23})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 12:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 216

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(3)} - (a_i'')^{(3)}(T_{21}(s_{(20)}), s_{(20)})\} ds_{(20)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(3)} t)} > 0$ for t > 0

**Definition of** $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2\ and\ \left((\widehat{M}_{20})^{(3)}\right)_3$ : 217

**Remark 13:** if $G_{20}$ is bounded, the same property have also $G_{21}\ and\ G_{22}$ . indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a_{21}')^{(3)} G_{21}$ and by integrating

$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1 / (a_{21}')^{(3)}$

In the same way , one can obtain

$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2 / (a_{22}')^{(3)}$

If $G_{21}\ or\ G_{22}$ is bounded, the same property follows for $G_{20}$ , $G_{22}$ and $G_{20}$ , $G_{21}$ respectively.

**Remark 14:** If $G_{20}$ *is* bounded, from below, the same property holds for $G_{21}\ and\ G_{22}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{21}$ is bounded from below. 218

**Remark 15:** If $T_{20}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t), t\right)) = (b_{21}')^{(3)}$ then $T_{21} \to \infty$. 219

**Definition of** $(m)^{(3)}$ and $\varepsilon_3$ :

Indeed let $t_3$ be so that for $t > t_3$

$(b_{21})^{(3)} - (b_i'')^{(3)}\left((G_{23})(t), t\right) < \varepsilon_3, T_{20}\,(t) > (m)^{(3)}$

Then $\frac{dT_{21}}{dt} \geq (a_{21})^{(3)}(m)^{(3)} - \varepsilon_3 T_{21}$ which leads to 220

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t} = \frac{1}{2}$ it results

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_3}$ By taking now $\varepsilon_3$ sufficiently small one sees that $T_{21}$ is unbounded.
The same property holds for $T_{22}$ if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t), t\right) = (b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}$ , $\frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}} < 1$ and to choose 221

$(\widehat{P}_{24})^{(4)}$ and $(\widehat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[(\widehat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \tag{222}$$

$$\frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \tag{223}$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself (224)

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric (225)

$$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{24})^{(4)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{27}}),(\widetilde{T_{27}})$ : $\left((\widetilde{G_{27}}),(\widetilde{T_{27}})\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{24})^{(4)}\left|G_{25}^{(1)}-G_{25}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\,ds_{(24)}+$$

$$\int_0^t\{(a'_{24})^{(4)}\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$G_{24}^{(2)}\left|(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)-(a''_{24})^{(4)}\left(T_{25}^{(2)},s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\}ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}t}\leq \tag{226}$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)}+(a'_{24})^{(4)}+(\widehat{A}_{24})^{(4)}+(\widehat{P}_{24})^{(4)}(\hat{k}_{24})^{(4)}\right)d\left(((G_{27})^{(1)},(T_{27})^{(1)};\ (G_{27})^{(2)},(T_{27})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 16:** The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ *and* $(\widehat{Q}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_+$. (227)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i=24,25,26$ depend only on $T_{25}$ and respectively on

$(G_{27})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 17:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 228

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(4)} - (a_i'')^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(4)} t)} > 0$ for t > 0

**Definition of** $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2\ and\ \left((\widehat{M}_{24})^{(4)}\right)_3$ : 229

**Remark 18:** if $G_{24}$ is bounded, the same property have also $G_{25}\ and\ G_{26}$ . indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a_{25}')^{(4)} G_{25}$ and by integrating

$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1 / (a_{25}')^{(4)}$

In the same way , one can obtain

$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2 / (a_{26}')^{(4)}$

If $G_{25}\ or\ G_{26}$ is bounded, the same property follows for $G_{24}$ , $G_{26}$ and $G_{24}$ , $G_{25}$ respectively.

**Remark 19:** If $G_{24}$ *is* bounded, from below, the same property holds for $G_{25}\ and\ G_{26}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{25}$ is bounded from below. 230

**Remark 20:** If $T_{24}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}\ ((G_{27})(t), t)) = (b_{25}')^{(4)}$ then $T_{25} \to \infty$. 231

**Definition of** $(m)^{(4)}$ and $\varepsilon_4$ :

Indeed let $t_4$ be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t), t) < \varepsilon_4, T_{24}\,(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_4}$ By taking now $\varepsilon_4$ sufficiently small one sees that $T_{25}$ is unbounded.
The same property holds for $T_{26}$ if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t), t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for $G_{29}$ , $G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}$ , $\frac{(b_i)^{(5)}}{(\widehat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)}+\left((\hat{P}_{28})^{(5)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{28})^{(5)} \tag{234}$$

$$\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{28})^{(5)}\right]\leq(\hat{Q}_{28})^{(5)} \tag{235}$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric (236)

$$d\left(((G_{31})^{(1)},(T_{31})^{(1)}),((G_{31})^{(2)},(T_{31})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\hat{M}_{28})^{(5)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\hat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{31}}),(\widetilde{T_{31}})$ : $\left((\widetilde{G_{31}}),(\widetilde{T_{31}})\right)=\mathcal{A}^{(5)}\left((G_{31}),(T_{31})\right)$

It results

$$|\tilde{G}_{28}^{(1)}-\tilde{G}_i^{(2)}|\leq\int_0^t(a_{28})^{(5)}|G_{29}^{(1)}-G_{29}^{(2)}|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\,ds_{(28)}+$$

$$\int_0^t\{(a'_{28})^{(5)}|G_{28}^{(1)}-G_{28}^{(2)}|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$(a''_{28})^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)|G_{28}^{(1)}-G_{28}^{(2)}|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$G_{28}^{(2)}|(a''_{28})^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)-(a''_{28})^{(5)}\left(T_{29}^{(2)},s_{(28)}\right)|\;e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\}ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on it follows

$$|(G_{31})^{(1)}-(G_{31})^{(2)}|e^{-(\widehat{M}_{28})^{(5)}t}\leq \tag{237}$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)}+(a'_{28})^{(5)}+(\widehat{A}_{28})^{(5)}+(\widehat{P}_{28})^{(5)}(\widehat{k}_{28})^{(5)}\right)d\left(((G_{31})^{(1)},(T_{31})^{(1)};\;(G_{31})^{(2)},(T_{31})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 21:** The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\hat{P}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ *and* $(\hat{Q}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ respectively of $\mathbb{R}_+$. (238)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on $T_{29}$ and respectively on $(G_{31})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 22:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 239

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(5)} - (a_i'')^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(5)} t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2\ and\ \left((\widehat{M}_{28})^{(5)}\right)_3$ : 240

**Remark 23:** if $G_{28}$ is bounded, the same property have also $G_{29}\ and\ G_{30}$ . indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a_{29}')^{(5)} G_{29}$ and by integrating

$$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1 / (a_{29}')^{(5)}$$

In the same way , one can obtain

$$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2 / (a_{30}')^{(5)}$$

If $G_{29}\ or\ G_{30}$ is bounded, the same property follows for $G_{28}$ , $G_{30}$ and $G_{28}$ , $G_{29}$ respectively.

**Remark 24:** If $G_{28}$ *is* bounded, from below, the same property holds for $G_{29}\ and\ G_{30}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{29}$ is bounded from below. 241

**Remark 25:** If $T_{28}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}\,((G_{31})(t), t)) = (b_{29}')^{(5)}$ then $T_{29} \to \infty$. 242

**Definition of** $(m)^{(5)}$ and $\varepsilon_5$ :

Indeed let $t_5$ be so that for $t > t_5$

$$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t), t) < \varepsilon_5, T_{28}\,(t) > (m)^{(5)}$$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_5}$ By taking now $\varepsilon_5$ sufficiently small one sees that $T_{29}$ is unbounded.
The same property holds for $T_{30}$ if $\lim_{t\to\infty}(b_{30}'')^{(5)}\left((G_{31})(t), t\right) = (b_{30}')^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{33}$ , $G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}$ , $\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}} < 1$ and to choose 244

$(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[(\hat{P}_{32})^{(6)}+\left((\hat{P}_{32})^{(6)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{32})^{(6)} \tag{245}$$

$$\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[\left((\hat{Q}_{32})^{(6)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{32})^{(6)}\right]\leq(\hat{Q}_{32})^{(6)} \tag{246}$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric (247)

$$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\hat{M}_{32})^{(6)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\hat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{35}}),(\widetilde{T_{35}})$ : $\left((\widetilde{G_{35}}),(\widetilde{T_{35}})\right)=\mathcal{A}^{(6)}\left((G_{35}),(T_{35})\right)$

It results

$$|\tilde{G}_{32}^{(1)}-\tilde{G}_i^{(2)}|\leq\int_0^t(a_{32})^{(6)}|G_{33}^{(1)}-G_{33}^{(2)}|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\,ds_{(32)}+$$

$$\int_0^t\{(a'_{32})^{(6)}|G_{32}^{(1)}-G_{32}^{(2)}|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)|G_{32}^{(1)}-G_{32}^{(2)}|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$G_{32}^{(2)}|(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)-(a''_{32})^{(6)}\left(T_{33}^{(2)},s_{(32)}\right)|\;e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\}ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$|(G_{35})^{(1)}-(G_{35})^{(2)}|e^{-(\widehat{M}_{32})^{(6)}t}\leq \tag{248}$$
$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)}+(a'_{32})^{(6)}+(\widehat{A}_{32})^{(6)}+(\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}\right)d\left(((G_{35})^{(1)},(T_{35})^{(1)};\;(G_{35})^{(2)},(T_{35})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 26:** The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\hat{P}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ *and* $(\hat{Q}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ respectively of $\mathbb{R}_+$. (249)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i = 32,33,34$ depend only on $T_{33}$ and respectively on $(G_{35})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 27:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 250

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(6)} - (a_i'')^{(6)}(T_{33}(s_{(32)}), s_{(32)})\} ds_{(32)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(6)} t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{32})^{(6)}\right)_1, \left((\widehat{M}_{32})^{(6)}\right)_2\ and\ \left((\widehat{M}_{32})^{(6)}\right)_3$ : 251

**Remark 28:** if $G_{32}$ is bounded, the same property have also $G_{33}\ and\ G_{34}$ . indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a_{33}')^{(6)} G_{33}$ and by integrating

$$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)} \left((\widehat{M}_{32})^{(6)}\right)_1 / (a_{33}')^{(6)}$$

In the same way , one can obtain

$$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)} \left((\widehat{M}_{32})^{(6)}\right)_2 / (a_{34}')^{(6)}$$

If $G_{33}\ or\ G_{34}$ is bounded, the same property follows for $G_{32}$ , $G_{34}$ and $G_{32}$ , $G_{33}$ respectively.

**Remark 29:** If $G_{32}$ *is* bounded, from below, the same property holds for $G_{33}\ and\ G_{34}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{33}$ is bounded from below. 252

**Remark 30:** If $T_{32}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}\ ((G_{35})(t), t)) = (b_{33}')^{(6)}$ then $T_{33} \to \infty$. 253

**Definition of** $(m)^{(6)}$ and $\varepsilon_6$ :

Indeed let $t_6$ be so that for $t > t_6$

$$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t), t\right) < \varepsilon_6, T_{32}\,(t) > (m)^{(6)}$$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right)$, $t = log\,\frac{2}{\varepsilon_6}$ By taking now $\varepsilon_6$ sufficiently small one sees that $T_{33}$ is unbounded.
The same property holds for $T_{34}$ if $\lim_{t\to\infty}(b_{34}'')^{(6)}\left((G_{35})(t), t(t), t\right) = (b_{34}')^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{37}, G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}$ , $\frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$ and to choose

$(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[(\hat{P}_{36})^{(7)}+\left((\hat{P}_{36})^{(7)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{36})^{(7)} \tag{256}$$

$$\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[\left((\hat{Q}_{36})^{(7)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{36})^{(7)}\right]\leq(\hat{Q}_{36})^{(7)} \tag{257}$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself
The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric (258)

$$d\left(((G_{39})^{(1)},(T_{39})^{(1)}),((G_{39})^{(2)},(T_{39})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{36})^{(7)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{39}}),(\widetilde{T_{39}}):\left((\widetilde{G_{39}}),(\widetilde{T_{39}})\right)=\mathcal{A}^{(7)}((G_{39}),(T_{39}))$
It results

$$\left|\tilde{G}_{36}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{36})^{(7)}\left|G_{37}^{(1)}-G_{37}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\,ds_{(36)}+$$
$$\int_0^t\{(a'_{36})^{(7)}\left|G_{36}^{(1)}-G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$(a''_{36})^{(7)}\left(T_{37}^{(1)},s_{(36)}\right)\left|G_{36}^{(1)}-G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$G_{36}^{(2)}\left|(a''_{36})^{(7)}\left(T_{37}^{(1)},s_{(36)}\right)-(a''_{36})^{(7)}\left(T_{37}^{(2)},s_{(36)}\right)\right|\;e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\}ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on it follows

$$\left|(G_{39})^{(1)}-(G_{39})^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}t}\leq \tag{259}$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}}\left((a_{36})^{(7)}+(a'_{36})^{(7)}+(\hat{A}_{36})^{(7)}+(\hat{P}_{36})^{(7)}(\hat{k}_{36})^{(7)}\right)d\left(((G_{39})^{(1)},(T_{39})^{(1)};\;(G_{39})^{(2)},(T_{39})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 31:** The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\hat{P}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ *and* $(\hat{Q}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ respectively of $\mathbb{R}_+$. (260)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(7)}$ and $(b_i'')^{(7)}, i = 36,37,38$ depend only on $T_{37}$ and respectively on $(G_{39})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 32:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 261

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(7)} - (a_i'')^{(7)}(T_{37}(s_{(36)}), s_{(36)})\} ds_{(36)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(7)} t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2\ and\ \left((\widehat{M}_{36})^{(7)}\right)_3$ : 262

**Remark 33:** if $G_{36}$ is bounded, the same property have also $G_{37}\ and\ G_{38}$ . indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a_{37}')^{(7)} G_{37}$ and by integrating

$$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1 / (a_{37}')^{(7)}$$

In the same way , one can obtain

$$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2 / (a_{38}')^{(7)}$$

If $G_{37}\ or\ G_{38}$ is bounded, the same property follows for $G_{36}$ , $G_{38}$ and $G_{36}$ , $G_{37}$ respectively.

**Remark 34:** If $G_{36}$ *is* bounded, from below, the same property holds for $G_{37}\ and\ G_{38}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{37}$ is bounded from below. 263

**Remark 35:** If $T_{36}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(7)}\ ((G_{39})(t), t)) = (b_{37}')^{(7)}$ then $T_{37} \to \infty$. 264

**Definition of** $(m)^{(7)}$ and $\varepsilon_7$ :

Indeed let $t_7$ be so that for $t > t_7$

$$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t), t) < \varepsilon_7, T_{36}\,(t) > (m)^{(7)}$$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_7}$ By taking now $\varepsilon_7$ sufficiently small one sees that $T_{37}$ is unbounded.
The same property holds for $T_{38}$ if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t), t\right) = (b_{38}')^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}$ , $\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}} < 1$ and to choose $(\widehat{P}_{40})^{(8)}$ and $(\widehat{Q}_{40})^{(8)}$ large to have 266

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[(\widehat{P}_{40})^{(8)} + \left((\hat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{40})^{(8)} \quad 267$$

$$\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{40})^{(8)}\right] \leq (\hat{Q}_{40})^{(8)} \quad 268$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)}, (T_{43})^{(1)}), ((G_{43})^{(2)}, (T_{43})^{(2)})\right) = \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

**Definition of** $\widetilde{(G_{43})}, \widetilde{(T_{43})}$ : $\left(\widetilde{(G_{43})}, \widetilde{(T_{43})}\right) = \mathcal{A}^{(8)}((G_{43}), (T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{40})^{(8)}\left|G_{41}^{(1)} - G_{41}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\,ds_{(40)} +$$
$$\int_0^t\{(a'_{40})^{(8)}\left|G_{40}^{(1)} - G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{-(\widehat{M}_{40})^{(8)}s_{(40)}} +$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right)\left|G_{40}^{(1)} - G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}} +$$
$$G_{40}^{(2)}\left|(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right) - (a''_{40})^{(8)}\left(T_{41}^{(2)}, s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\}ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0, t]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)} - (G_{43})^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}t} \leq \quad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)} + (a'_{40})^{(8)} + (\widehat{A}_{40})^{(8)} + (\widehat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}\right)d\left(((G_{43})^{(1)}, (T_{43})^{(1)};\ (G_{43})^{(2)}, (T_{43})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

**Remark 36:** The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate 274

condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ $and$ $(\widehat{Q}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i = 40,41,42$ depend only on $T_{41}$ and respectively on $(G_{43})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 37** There does not exist any $t$ where $G_i(t) = 0\ and\ T_i(t) = 0$ 275

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(8)} - (a_i'')^{(8)}(T_{41}(s_{(40)}), s_{(40)})\} ds_{(40)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(8)}t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{40})^{(8)}\right)_1, \left((\widehat{M}_{40})^{(8)}\right)_2$ $and$ $\left((\widehat{M}_{40})^{(8)}\right)_3$ : 276

**Remark 38:** if $G_{40}$ is bounded, the same property have also $G_{41}$ $and$ $G_{42}$ . indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a_{41}')^{(8)}G_{41}$ and by integrating

$$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1/(a_{41}')^{(8)}$$

In the same way , one can obtain

$$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2/(a_{42}')^{(8)}$$

If $G_{41}$ $or$ $G_{42}$ is bounded, the same property follows for $G_{40}$ , $G_{42}$ and $G_{40}$ , $G_{41}$ respectively.

**Remark 39:** If $G_{40}$ $is$ bounded, from below, the same property holds for $G_{41}$ $and$ $G_{42}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{41}$ is bounded from below. 277

**Remark 40:** If $T_{40}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}((G_{43})(t), t)) = (b_{41}')^{(8)}$ then $T_{41} \to \infty$. 278

**Definition of** $(m)^{(8)}$ and $\varepsilon_8$ :

Indeed let $t_8$ be so that for $t > t_8$

$$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t), t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right)$, $t = log\frac{2}{\varepsilon_8}$ By taking now $\varepsilon_8$ sufficiently small one sees that $T_{41}$ is unbounded.
The same property holds for $T_{42}$ if $\lim_{t\to\infty}(b''_{42})^{(8)}\left((G_{43})(t), t(t), t\right) = (b'_{42})^{(8)}$



It is now sufficient to take $\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}$ , $\frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$ and to choose $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ large to have

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}\left[(\hat{P}_{44})^{(9)} + \left((\hat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}}\left[\left((\hat{Q}_{44})^{(9)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{44})^{(9)}\right] \leq (\hat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)}, (T_{47})^{(1)}), ((G_{47})^{(2)}, (T_{47})^{(2)})\right) =$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{44})^{(9)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{47}}), (\widetilde{T_{47}}) : \left((\widetilde{G_{47}}), (\widetilde{T_{47}})\right) = \mathcal{A}^{(9)}\left((G_{47}), (T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{44})^{(9)}\left|G_{45}^{(1)} - G_{45}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\,ds_{(44)} +$$
$$\int_0^t\{(a'_{44})^{(9)}\left|G_{44}^{(1)} - G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{-(\widehat{M}_{44})^{(9)}s_{(44)}} +$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right)\left|G_{44}^{(1)} - G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}} +$$
$$G_{44}^{(2)}\left|(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right) - (a''_{44})^{(9)}\left(T_{45}^{(2)}, s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\}ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)} - G^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}t} \leq$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)} + (a'_{44})^{(9)} + (\widehat{A}_{44})^{(9)} + (\widehat{P}_{44})^{(9)}(\hat{k}_{44})^{(9)}\right)d\left(((G_{47})^{(1)}, (T_{47})^{(1)};\ (G_{47})^{(2)}, (T_{47})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis (39,35,36) the result follows

**Remark 41:** The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by

$(\widehat{P}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ *and* $(\widehat{Q}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i = 44{,}45{,}46$ depend only on $T_{45}$ and respectively on $(G_{47})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 42:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$

From 99 to 44 it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t\{(a_i')^{(9)}-(a_i'')^{(9)}(T_{45}(s_{(44)}),s_{(44)})\}ds_{(44)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(9)}t)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{44})^{(9)}\right)_1, \left((\widehat{M}_{44})^{(9)}\right)_2\ and\ \left((\widehat{M}_{44})^{(9)}\right)_3$ :

**Remark 43:** if $G_{44}$ is bounded, the same property have also $G_{45}\ and\ G_{46}$ . indeed if

$G_{44} < (\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt} \leq \left((\widehat{M}_{44})^{(9)}\right)_1 - (a_{45}')^{(9)}G_{45}$ and by integrating

$$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_1/(a_{45}')^{(9)}$$

In the same way , one can obtain

$$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_2/(a_{46}')^{(9)}$$

If $G_{45}\ or\ G_{46}$ is bounded, the same property follows for $G_{44}$ , $G_{46}$ and $G_{44}$ , $G_{45}$ respectively.

**Remark 44:** If $G_{44}$ *is* bounded, from below, the same property holds for $G_{45}\ and\ G_{46}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{45}$ is bounded from below.

**Remark 45:** If $T_{44}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t),t\right) = (b_{45}')^{(9)}$ then $T_{45} \to \infty$.

**Definition of** $(m)^{(9)}$ and $\varepsilon_9$ :

Indeed let $t_9$ be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t),t\right) < \varepsilon_9, T_{44}\,(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right), \quad t = log\frac{2}{\varepsilon_9}$ By taking now $\varepsilon_9$ sufficiently small one sees that $T_{45}$ is unbounded.
The same property holds for $T_{46}$ if $\lim_{t\to\infty}(b_{46}'')^{(9)}\left((G_{47})(t),t\right) = (b_{46}')^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

**Behavior of the solutions of equation** 280

**Theorem** If we denote and define

**Definition of** $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ :

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)} \le -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14}, t) + (a''_{14})^{(1)}(T_{14}, t) \le -(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)} \le -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G, t) - (b''_{14})^{(1)}(G, t) \le -(\tau_1)^{(1)}$

**Definition of** $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$ : 281

By $(\nu_1)^{(1)} > 0$, $(\nu_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$, $(u_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

**Definition of** $(\bar{\nu}_1)^{(1)},, (\bar{\nu}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$ : 282

By $(\bar{\nu}_1)^{(1)} > 0$, $(\bar{\nu}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$, $(\bar{u}_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

**Definition of** $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (\nu_0)^{(1)}$ :- 283

If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by
$(m_2)^{(1)} = (\nu_0)^{(1)}, (m_1)^{(1)} = (\nu_1)^{(1)}$, $if\ (\nu_0)^{(1)} < (\nu_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}, if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)}$,

and $\boxed{(\nu_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)}$, $if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)}$, $if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)}$,

and $\boxed{(u_0)^{(1)} = \frac{T^0_{13}}{T^0_{14}}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$

are defined

Then the solution of global equations satisfies the inequalities 285

$G^0_{13}e^{\left((S_1)^{(1)} - (p_{13})^{(1)}\right)t} \le G_{13}(t) \le G^0_{13}e^{(S_1)^{(1)}t}$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)}-(p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)}t}$$

$$\left(\frac{(a_{15})^{(1)}G_{13}^0}{(m_1)^{(1)}((S_1)^{(1)}-(p_{13})^{(1)}-(S_2)^{(1)})}\left[e^{((S_1)^{(1)}-(p_{13})^{(1)})t} - e^{-(S_2)^{(1)}t}\right] + G_{15}^0 e^{-(S_2)^{(1)}t} \le G_{15}(t) \le \right. \quad (286)$$

$$\left.\frac{(a_{15})^{(1)}G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)}-(a'_{15})^{(1)})}[e^{(S_1)^{(1)}t} - e^{-(a'_{15})^{(1)}t}] + G_{15}^0 e^{-(a'_{15})^{(1)}t}\right)$$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)}t} \le T_{13}(t) \le T_{13}^0 e^{((R_1)^{(1)}+(r_{13})^{(1)})t}} \quad (287)$$

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)}t} \le T_{13}(t) \le \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)}+(r_{13})^{(1)})t} \quad (288)$$

$$\frac{(b_{15})^{(1)}T_{13}^0}{(\mu_1)^{(1)}((R_1)^{(1)}-(b'_{15})^{(1)})}\left[e^{(R_1)^{(1)}t} - e^{-(b'_{15})^{(1)}t}\right] + T_{15}^0 e^{-(b'_{15})^{(1)}t} \le T_{15}(t) \le \quad (289)$$

$$\frac{(a_{15})^{(1)}T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)}+(r_{13})^{(1)}+(R_2)^{(1)})}\left[e^{((R_1)^{(1)}+(r_{13})^{(1)})t} - e^{-(R_2)^{(1)}t}\right] + T_{15}^0 e^{-(R_2)^{(1)}t}$$

**Definition of** $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:- (290)

Where $(S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$

$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$

$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$

$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$

**Behavior of the solutions of equation** (291)

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ : (292)

$(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ four constants satisfying

$$-(\sigma_2)^{(2)} \le -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(T_{17}, t) + (a''_{17})^{(2)}(T_{17}, t) \le -(\sigma_1)^{(2)} \quad (293)$$

$$-(\tau_2)^{(2)} \le -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}((G_{19}), t) - (b''_{17})^{(2)}((G_{19}), t) \le -(\tau_1)^{(2)} \quad (294)$$

**Definition of** $(\nu_1)^{(2)}, (\nu_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$ : (295)

By $(\nu_1)^{(2)} > 0$, $(\nu_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0$, $(u_2)^{(2)} < 0$ the roots (296)

of the equations $(a_{17})^{(2)}(\nu^{(2)})^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ (297)

and $(b_{14})^{(2)}(u^{(2)})^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and (298)

**Definition of** $(\bar{\nu}_1)^{(2)}, , (\bar{\nu}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$ : (299)

By $(\bar{\nu}_1)^{(2)} > 0$, $(\bar{\nu}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0$, $(\bar{u}_2)^{(2)} < 0$ the (300)

roots of the equations $(a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 301

and $(b_{17})^{(2)}\left(u^{(2)}\right)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 302

**Definition of** $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ :- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, ***if*** $(u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$, ***if*** $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)}$, ***if*** $(\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)}t}$

$(p_i)^{(2)}$ is defined by equation

$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)}t}$ 311

$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)}t}\right] + G_{18}^0 e^{-(S_2)^{(2)}t} \le G_{18}(t) \le\right.$ 312

$\left.\frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)}t} - e^{-(a'_{18})^{(2)}t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)}t}\right)$

$\boxed{T_{16}^0 e^{(R_1)^{(2)}t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}}$ 313

$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)}t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}$ 314

$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)}t} - e^{-(b'_{18})^{(2)}t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)}t} \le T_{18}(t) \le$ 315

$$\frac{(a_{18})^{(2)}T_{16}^{0}}{(\mu_2)^{(2)}((R_1)^{(2)}+(r_{16})^{(2)}+(R_2)^{(2)})}\left[e^{((R_1)^{(2)}+(r_{16})^{(2)})t}-e^{-(R_2)^{(2)}t}\right]+T_{18}^{0}e^{-(R_2)^{(2)}t}$$

**Definition of** $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$$

$$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)}$$ 318

$$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$$

**Behavior of the solutions** 319

**Theorem 3:** If we denote and define

**Definition of** $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ :

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}\big((G_{23}), t\big) \le -(\tau_1)^{(3)}$$

**Definition of** $(v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$ : 320

By $(v_1)^{(3)} > 0$, $(v_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{v}_1)^{(3)} > 0$, $(\bar{v}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

**Definition of** $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ :- 321

If we define $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (v_0)^{(3)}, (m_1)^{(3)} = (v_1)^{(3)}$, ***if*** $(v_0)^{(3)} < (v_1)^{(3)}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (\bar{v}_1)^{(3)}$, ***if*** $(v_1)^{(3)} < (v_0)^{(3)} < (\bar{v}_1)^{(3)}$,

and $\boxed{(v_0)^{(3)} = \frac{G_{20}^{0}}{G_{21}^{0}}}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_0)^{(3)}$, ***if*** $(\bar{v}_1)^{(3)} < (v_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)},\ \boldsymbol{if}\ (u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}, \boldsymbol{if}\ (u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)},$ and $\boxed{(u_0)^{(3)} = \frac{T_{20}^0}{T_{21}^0}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)},\ \boldsymbol{if}\ (\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G_{20}^0 e^{(S_1)^{(3)} t}$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)} t} \quad 323$$

$$\left(\frac{(a_{22})^{(3)} G_{20}^0}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\left[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)} t}\right] + G_{22}^0 e^{-(S_2)^{(3)} t} \le G_{22}(t) \le \right. \quad 324$$

$$\left. \frac{(a_{22})^{(3)} G_{20}^0}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}\left[e^{(S_1)^{(3)} t} - e^{-(a'_{22})^{(3)} t}\right] + G_{22}^0 e^{-(a'_{22})^{(3)} t}\right)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t}} \quad 325$$

$$\frac{1}{(\mu_1)^{(3)}} T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}} T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)} T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)} - (b'_{22})^{(3)})}\left[e^{(R_1)^{(3)} t} - e^{-(b'_{22})^{(3)} t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)} t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)} T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)} + (r_{20})^{(3)} + (R_2)^{(3)})}\left[e^{((R_1)^{(3)} + (r_{20})^{(3)})t} - e^{-(R_2)^{(3)} t}\right] + T_{22}^0 e^{-(R_2)^{(3)} t}$$

**Definition of** $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

**Behavior of the solutions of equation**

**Theorem:** If we denote and define

**Definition of** $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ :

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$

$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}((G_{27}), t) - (b''_{25})^{(4)}((G_{27}), t) \le -(\tau_1)^{(4)}$

**Definition of** $(v_1)^{(4)}, (v_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, v^{(4)}, u^{(4)}$ : 329

By $(v_1)^{(4)} > 0$ , $(v_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$ , $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_1)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

**Definition of** $(\bar{v}_1)^{(4)},, (\bar{v}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$ : 330

By $(\bar{v}_1)^{(4)} > 0$ , $(\bar{v}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$ , $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

**Definition of** $(m_1)^{(4)}$ , $(m_2)^{(4)}$ , $(\mu_1)^{(4)}, (\mu_2)^{(4)}, (\nu_0)^{(4)}$ :-

If we define $(m_1)^{(4)}$ , $(m_2)^{(4)}$ , $(\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (v_0)^{(4)}, (m_1)^{(4)} = (v_1)^{(4)}$, $\boldsymbol{if}$ $(v_0)^{(4)} < (v_1)^{(4)}$

$(m_2)^{(4)} = (v_1)^{(4)}, (m_1)^{(4)} = (\bar{v}_1)^{(4)}$ , $\boldsymbol{if}$ $(v_4)^{(4)} < (v_0)^{(4)} < (\bar{v}_1)^{(4)}$,
and $\boxed{(v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (v_4)^{(4)}, (m_1)^{(4)} = (v_0)^{(4)}$, $\boldsymbol{if}$ $(\bar{v}_4)^{(4)} < (v_0)^{(4)}$

and analogously

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$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)}$, $\boldsymbol{if}$ $(u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}$ , $\boldsymbol{if}$ $(u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)}$,
and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}, \boldsymbol{if}$ $(\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

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$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)}t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)}t}$$

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$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)}t}\right] + G_{26}^0 e^{-(S_2)^{(4)}t} \le G_{26}(t) \le\right.$$ 334

*(a26)4G240(m2)4(S1)4−(a26′)4e(S1)4t−e−(a26′)4t+ G260e−(a26′)4t*

$$\boxed{T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}}T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}}T_{24}^0 e^{((R_1)^{(4)}+(r_{24})^{(4)})t} \quad 335$$

$$\frac{(b_{26})^{(4)}T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)}-(b'_{26})^{(4)})}\left[e^{(R_1)^{(4)}t} - e^{-(b'_{26})^{(4)}t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)}t} \le T_{26}(t) \le \quad 336$$

$$\frac{(a_{26})^{(4)}T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)}+(r_{24})^{(4)}+(R_2)^{(4)})}\left[e^{((R_1)^{(4)}+(r_{24})^{(4)})t} - e^{-(R_2)^{(4)}t}\right] + T_{26}^0 e^{-(R_2)^{(4)}t}$$

**Definition of** $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

**Behavior of the solutions of equation** 338

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ :

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

**Definition of** $(v_1)^{(5)}, (v_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, v^{(5)}, u^{(5)}$ : 339

By $(v_1)^{(5)} > 0$ , $(v_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$ , $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(v^{(5)}\big)^2 + (\sigma_1)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

**Definition of** $(\bar{v}_1)^{(5)}, , (\bar{v}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$ : 340

By $(\bar{v}_1)^{(5)} > 0$ , $(\bar{v}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$ , $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(v^{(5)}\big)^2 + (\sigma_2)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

**Definition of** $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (v_0)^{(5)}$ :-

If we define $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (v_0)^{(5)}, (m_1)^{(5)} = (v_1)^{(5)}$, ***if*** $(v_0)^{(5)} < (v_1)^{(5)}$

$(m_2)^{(5)} = (v_1)^{(5)}, (m_1)^{(5)} = (\bar{v}_1)^{(5)}$ , ***if*** $(v_1)^{(5)} < (v_0)^{(5)} < (\bar{v}_1)^{(5)}$,
and $\boxed{(v_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)},\ \boldsymbol{if}\ (u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}, \boldsymbol{if}\ (u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)},$

and $\boxed{(u_0)^{(5)} = \frac{T_{28}^0}{T_{29}^0}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}, \boldsymbol{if}\ (\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{28}(t) \le G_{28}^0 e^{(S_1)^{(5)} t}$

where $(p_i)^{(5)}$ is defined by equation

$\frac{1}{(m_5)^{(5)}} G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}} G_{28}^0 e^{(S_1)^{(5)} t}$ 343

$\left(\frac{(a_{30})^{(5)} G_{28}^0}{(m_1)^{(5)}((S_1)^{(5)} - (p_{28})^{(5)} - (S_2)^{(5)})}\left[e^{((S_1)^{(5)} - (p_{28})^{(5)})t} - e^{-(S_2)^{(5)} t}\right] + G_{30}^0 e^{-(S_2)^{(5)} t} \le G_{30}(t) \le\right.$ 344

*(a30)5G280(m2)5(S1)5−(a30′)5e(S1)5t−e−(a30′)5t+ G300e−(a30′)5t*

$\boxed{T_{28}^0 e^{(R_1)^{(5)} t} \le T_{28}(t) \le T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}}$ 345

$\frac{1}{(\mu_1)^{(5)}} T_{28}^0 e^{(R_1)^{(5)} t} \le T_{28}(t) \le \frac{1}{(\mu_2)^{(5)}} T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}$ 346

$\frac{(b_{30})^{(5)} T_{28}^0}{(\mu_1)^{(5)}((R_1)^{(5)} - (b'_{30})^{(5)})}\left[e^{(R_1)^{(5)} t} - e^{-(b'_{30})^{(5)} t}\right] + T_{30}^0 e^{-(b'_{30})^{(5)} t} \le T_{30}(t) \le$ 347

$\frac{(a_{30})^{(5)} T_{28}^0}{(\mu_2)^{(5)}((R_1)^{(5)} + (r_{28})^{(5)} + (R_2)^{(5)})}\left[e^{((R_1)^{(5)} + (r_{28})^{(5)})t} - e^{-(R_2)^{(5)} t}\right] + T_{30}^0 e^{-(R_2)^{(5)} t}$

**Definition of** $(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$:- 348

Where $(S_1)^{(5)} = (a_{28})^{(5)}(m_2)^{(5)} - (a'_{28})^{(5)}$

$(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}$

$(R_1)^{(5)} = (b_{28})^{(5)}(\mu_2)^{(5)} - (b'_{28})^{(5)}$

$(R_2)^{(5)} = (b'_{30})^{(5)} - (r_{30})^{(5)}$

**Behavior of the solutions of equation** 349

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ :

$(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)} \le -(a'_{32})^{(6)} + (a'_{33})^{(6)} - (a''_{32})^{(6)}(T_{33}, t) + (a''_{33})^{(6)}(T_{33}, t) \le -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)} \le -(b'_{32})^{(6)} + (b'_{33})^{(6)} - (b''_{32})^{(6)}\big((G_{35}), t\big) - (b''_{33})^{(6)}\big((G_{35}), t\big) \le -(\tau_1)^{(6)}$$

**Definition of** $(\nu_1)^{(6)}, (\nu_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, \nu^{(6)}, u^{(6)}$ : 350

By $(\nu_1)^{(6)} > 0$, $(\nu_2)^{(6)} < 0$ and respectively $(u_1)^{(6)} > 0$, $(u_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(6)},, (\bar{\nu}_2)^{(6)}, (\bar{u}_1)^{(6)}, (\bar{u}_2)^{(6)}$ : 351

By $(\bar{\nu}_1)^{(6)} > 0$, $(\bar{\nu}_2)^{(6)} < 0$ and respectively $(\bar{u}_1)^{(6)} > 0$, $(\bar{u}_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$

**Definition of** $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}, (\nu_0)^{(6)}$ :-

If we define $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}$ by

$$(m_2)^{(6)} = (\nu_0)^{(6)}, (m_1)^{(6)} = (\nu_1)^{(6)}, \ \boldsymbol{if}\ (\nu_0)^{(6)} < (\nu_1)^{(6)}$$

$$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}, \boldsymbol{if}\ (\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)},$$

and $\boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}}$

$$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$$

and analogously 352

$$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}, \ \boldsymbol{if}\ (u_0)^{(6)} < (u_1)^{(6)}$$

$$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}, \boldsymbol{if}\ (u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)},$$

and $\boxed{(u_0)^{(6)} = \frac{T^0_{32}}{T^0_{33}}}$

$$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}, \boldsymbol{if}\ (\bar{u}_1)^{(6)} < (u_0)^{(6)} \text{ where } (u_1)^{(6)}, (\bar{u}_1)^{(6)}$$

Then the solution of global equations satisfies the inequalities 353

$$G^0_{32}e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{32}(t) \le G^0_{32}e^{(S_1)^{(6)}t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}}G^0_{32}e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{33}(t) \le \frac{1}{(m_2)^{(6)}}G^0_{32}e^{(S_1)^{(6)}t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)}G^0_{32}}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})}\left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)}t}\right] + G^0_{34}e^{-(S_2)^{(6)}t} \le G_{34}(t) \le \right. \quad 355$$

$$\frac{(a_{34})^{(6)}G_{32}^{0}}{(m_2)^{(6)}((S_1)^{(6)}-(a'_{34})^{(6)})}\left[e^{(S_1)^{(6)}t}-e^{-(a'_{34})^{(6)}t}\right]+\ G_{34}^{0}e^{-(a'_{34})^{(6)}t}\Big)$$

$$\boxed{T_{32}^{0}e^{(R_1)^{(6)}t}\le T_{32}(t)\le T_{32}^{0}e^{((R_1)^{(6)}+(r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}}T_{32}^{0}e^{(R_1)^{(6)}t}\le T_{32}(t)\le\frac{1}{(\mu_2)^{(6)}}T_{32}^{0}e^{((R_1)^{(6)}+(r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)}T_{32}^{0}}{(\mu_1)^{(6)}((R_1)^{(6)}-(b'_{34})^{(6)})}\left[e^{(R_1)^{(6)}t}-e^{-(b'_{34})^{(6)}t}\right]+T_{34}^{0}e^{-(b'_{34})^{(6)}t}\le T_{34}(t)\le \quad 358$$

$$\frac{(a_{34})^{(6)}T_{32}^{0}}{(\mu_2)^{(6)}((R_1)^{(6)}+(r_{32})^{(6)}+(R_2)^{(6)})}\left[e^{((R_1)^{(6)}+(r_{32})^{(6)})t}-e^{-(R_2)^{(6)}t}\right]+T_{34}^{0}e^{-(R_2)^{(6)}t}$$

**Definition of** $(S_1)^{(6)},(S_2)^{(6)},(R_1)^{(6)},(R_2)^{(6)}$:- 359

Where $(S_1)^{(6)}=(a_{32})^{(6)}(m_2)^{(6)}-(a'_{32})^{(6)}$

$(S_2)^{(6)}=(a_{34})^{(6)}-(p_{34})^{(6)}$

$(R_1)^{(6)}=(b_{32})^{(6)}(\mu_2)^{(6)}-(b'_{32})^{(6)}$

$(R_2)^{(6)}=(b'_{34})^{(6)}-(r_{34})^{(6)}$

**Behavior of the solutions of equation**

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(7)},(\sigma_2)^{(7)},(\tau_1)^{(7)},(\tau_2)^{(7)}$ :

$(\sigma_1)^{(7)},(\sigma_2)^{(7)},(\tau_1)^{(7)},(\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)}\le-(a'_{36})^{(7)}+(a'_{37})^{(7)}-(a''_{36})^{(7)}(T_{37},t)+(a''_{37})^{(7)}(T_{37},t)\le-(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)}\le-(b'_{36})^{(7)}+(b'_{37})^{(7)}-(b''_{36})^{(7)}\big((G_{39}),t\big)-(b''_{37})^{(7)}\big((G_{39}),t\big)\le-(\tau_1)^{(7)}$$

**Definition of** $(v_1)^{(7)},(v_2)^{(7)},(u_1)^{(7)},(u_2)^{(7)},v^{(7)},u^{(7)}$ : 361

By $(v_1)^{(7)}>0,(v_2)^{(7)}<0$ and respectively $(u_1)^{(7)}>0,(u_2)^{(7)}<0$ the roots of the equations $(a_{37})^{(7)}\big(v^{(7)}\big)^2+(\sigma_1)^{(7)}v^{(7)}-(a_{36})^{(7)}=0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2+(\tau_1)^{(7)}u^{(7)}-(b_{36})^{(7)}=0$ and

**Definition of** $(\bar{v}_1)^{(7)},,(\bar{v}_2)^{(7)},(\bar{u}_1)^{(7)},(\bar{u}_2)^{(7)}$ : 362

By $(\bar{v}_1)^{(7)}>0,(\bar{v}_2)^{(7)}<0$ and respectively $(\bar{u}_1)^{(7)}>0,(\bar{u}_2)^{(7)}<0$ the roots of the equations $(a_{37})^{(7)}\big(v^{(7)}\big)^2+(\sigma_2)^{(7)}v^{(7)}-(a_{36})^{(7)}=0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2+(\tau_2)^{(7)}u^{(7)}-(b_{36})^{(7)}=0$

**Definition of** $(m_1)^{(7)},(m_2)^{(7)},(\mu_1)^{(7)},(\mu_2)^{(7)},(v_0)^{(7)}$ :-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$$(m_2)^{(7)} = (\nu_0)^{(7)}, (m_1)^{(7)} = (\nu_1)^{(7)}, \ \boldsymbol{if}\ (\nu_0)^{(7)} < (\nu_1)^{(7)}$$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\bar{\nu}_1)^{(7)}, \boldsymbol{if}\ (\nu_1)^{(7)} < (\nu_0)^{(7)} < (\bar{\nu}_1)^{(7)},$$

and $\boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\nu_0)^{(7)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(7)} < (\nu_0)^{(7)}$$

and analogously 363

$$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}, \ \boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$$

and $\boxed{(u_0)^{(7)} = \frac{T_{36}^0}{T_{37}^0}}$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)} \text{ where } (u_1)^{(7)}, (\bar{u}_1)^{(7)}$$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^0 e^{(S_1)^{(7)} t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)} t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)}((S_1)^{(7)} - (p_{36})^{(7)} - (S_2)^{(7)})}\left[e^{((S_1)^{(7)} - (p_{36})^{(7)})t} - e^{-(S_2)^{(7)} t}\right] + G_{38}^0 e^{-(S_2)^{(7)} t} \le G_{38}(t) \le \right. \quad 366$$

$$\left. \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)}((S_1)^{(7)} - (a'_{38})^{(7)})}\left[e^{(S_1)^{(7)} t} - e^{-(a'_{38})^{(7)} t}\right] + G_{38}^0 e^{-(a'_{38})^{(7)} t}\right)$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t} \quad 368$$

$$\frac{(b_{38})^{(7)} T_{36}^0}{(\mu_1)^{(7)}((R_1)^{(7)} - (b'_{38})^{(7)})}\left[e^{(R_1)^{(7)} t} - e^{-(b'_{38})^{(7)} t}\right] + T_{38}^0 e^{-(b'_{38})^{(7)} t} \le T_{38}(t) \le \quad 369$$

$$\frac{(a_{38})^{(7)} T_{36}^0}{(\mu_2)^{(7)}((R_1)^{(7)} + (r_{36})^{(7)} + (R_2)^{(7)})}\left[e^{((R_1)^{(7)} + (r_{36})^{(7)})t} - e^{-(R_2)^{(7)} t}\right] + T_{38}^0 e^{-(R_2)^{(7)} t}$$

**Definition of** $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$

$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$

$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$

**Behavior of the solutions of equation** 371

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ :

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$-(\sigma_2)^{(8)} \le -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \le -(\sigma_1)^{(8)}$

$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\big((G_{43}), t\big) - (b''_{41})^{(8)}\big((G_{43}), t\big) \le -(\tau_1)^{(8)}$

**Definition of** $(\nu_1)^{(8)}, (\nu_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, \nu^{(8)}, u^{(8)}$ : 372

By $(\nu_1)^{(8)} > 0, (\nu_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0, (u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(8)}, , (\bar{\nu}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$ :

By $(\bar{\nu}_1)^{(8)} > 0, (\bar{\nu}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0, (\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

**Definition of** $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}, (\nu_0)^{(8)}$ :-

If we define $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$(m_2)^{(8)} = (\nu_0)^{(8)}, (m_1)^{(8)} = (\nu_1)^{(8)}$, ***if*** $(\nu_0)^{(8)} < (\nu_1)^{(8)}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\bar{\nu}_1)^{(8)}$, ***if*** $(\nu_1)^{(8)} < (\nu_0)^{(8)} < (\bar{\nu}_1)^{(8)}$,
and $\boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\nu_0)^{(8)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(8)} < (\nu_0)^{(8)}$

and analogously 374

$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)},\ \boldsymbol{if}\ (u_0)^{(8)} < (u_1)^{(8)}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}, \boldsymbol{if}\ (u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)},$

and $\boxed{(u_0)^{(8)} = \frac{T_{40}^0}{T_{41}^0}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}, \boldsymbol{if}\ (\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^0 e^{(S_1)^{(8)} t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}} G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}} G_{40}^0 e^{(S_1)^{(8)} t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)} G_{40}^0}{(m_1)^{(8)}((S_1)^{(8)} - (p_{40})^{(8)} - (S_2)^{(8)})}\left[e^{((S_1)^{(8)} - (p_{40})^{(8)})t} - e^{-(S_2)^{(8)} t}\right] + G_{42}^0 e^{-(S_2)^{(8)} t} \le G_{42}(t) \le \right. \quad 377$$

$$\left. \frac{(a_{42})^{(8)} G_{40}^0}{(m_2)^{(8)}((S_1)^{(8)} - (a'_{42})^{(8)})}\left[e^{(S_1)^{(8)} t} - e^{-(a'_{42})^{(8)} t}\right] + G_{42}^0 e^{-(a'_{42})^{(8)} t}\right)$$

$$\boxed{T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}} T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le \frac{1}{(\mu_2)^{(8)}} T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)} T_{40}^0}{(\mu_1)^{(8)}((R_1)^{(8)} - (b'_{42})^{(8)})}\left[e^{(R_1)^{(8)} t} - e^{-(b'_{42})^{(8)} t}\right] + T_{42}^0 e^{-(b'_{42})^{(8)} t} \le T_{42}(t) \le \quad 380$$

$$\frac{(a_{42})^{(8)} T_{40}^0}{(\mu_2)^{(8)}((R_1)^{(8)} + (r_{40})^{(8)} + (R_2)^{(8)})}\left[e^{((R_1)^{(8)} + (r_{40})^{(8)})t} - e^{-(R_2)^{(8)} t}\right] + T_{42}^0 e^{-(R_2)^{(8)} t}$$

**Definition of** $(S_1)^{(8)}, (S_2)^{(8)}, (R_1)^{(8)}, (R_2)^{(8)}$:- 381

Where $(S_1)^{(8)} = (a_{40})^{(8)}(m_2)^{(8)} - (a'_{40})^{(8)}$

$(S_2)^{(8)} = (a_{42})^{(8)} - (p_{42})^{(8)}$

$(R_1)^{(8)} = (b_{40})^{(8)}(\mu_2)^{(8)} - (b'_{40})^{(8)}$

$(R_2)^{(8)} = (b'_{42})^{(8)} - (r_{42})^{(8)}$

**Behavior of the solutions of equation 37 to 92** 382

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ :

$(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ four constants satisfying

$$-(\sigma_2)^{(9)} \le -(a'_{44})^{(9)} + (a'_{45})^{(9)} - (a''_{44})^{(9)}(T_{45}, t) + (a''_{45})^{(9)}(T_{45}, t) \le -(\sigma_1)^{(9)}$$

$$-(\tau_2)^{(9)} \le -(b'_{44})^{(9)} + (b'_{45})^{(9)} - (b''_{44})^{(9)}\big((G_{47}), t\big) - (b''_{45})^{(9)}\big((G_{47}), t\big) \le -(\tau_1)^{(9)}$$

**Definition of** $(\nu_1)^{(9)}, (\nu_2)^{(9)}, (u_1)^{(9)}, (u_2)^{(9)}, \nu^{(9)}, u^{(9)}$ :

By $(\nu_1)^{(9)} > 0$ , $(\nu_2)^{(9)} < 0$ and respectively $(u_1)^{(9)} > 0$ , $(u_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$ and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_1)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(9)}, , (\bar{\nu}_2)^{(9)}, (\bar{u}_1)^{(9)}, (\bar{u}_2)^{(9)}$ :

By $(\bar{\nu}_1)^{(9)} > 0$ , $(\bar{\nu}_2)^{(9)} < 0$ and respectively $(\bar{u}_1)^{(9)} > 0$ , $(\bar{u}_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$ and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_2)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$

**Definition of** $(m_1)^{(9)}, (m_2)^{(9)}, (\mu_1)^{(9)}, (\mu_2)^{(9)}, (\nu_0)^{(9)}$ :-

If we define $(m_1)^{(9)}, (m_2)^{(9)}, (\mu_1)^{(9)}, (\mu_2)^{(9)}$ by

$$(m_2)^{(9)} = (\nu_0)^{(9)}, (m_1)^{(9)} = (\nu_1)^{(9)}, \ \boldsymbol{if}\ (\nu_0)^{(9)} < (\nu_1)^{(9)}$$

$$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\bar{\nu}_1)^{(9)}, \boldsymbol{if}\ (\nu_1)^{(9)} < (\nu_0)^{(9)} < (\bar{\nu}_1)^{(9)},$$

and $\boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$

$$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\nu_0)^{(9)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(9)} < (\nu_0)^{(9)}$$

and analogously

$$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)}, \ \boldsymbol{if}\ (u_0)^{(9)} < (u_1)^{(9)}$$

$$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}, \boldsymbol{if}\ (u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)},$$

and $\boxed{(u_0)^{(9)} = \frac{T^0_{44}}{T^0_{45}}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}, \boldsymbol{if}\ (\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G^0_{44} e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{44}(t) \le G^0_{44} e^{(S_1)^{(9)} t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}} G_{44}^0 e^{\left((S_1)^{(9)}-(p_{44})^{(9)}\right)t} \le G_{45}(t) \le \frac{1}{(m_2)^{(9)}} G_{44}^0 e^{(S_1)^{(9)}t}$$

(

$$\frac{(a_{46})^{(9)} G_{44}^0}{(m_1)^{(9)}\left((S_1)^{(9)}-(p_{44})^{(9)}-(S_2)^{(9)}\right)}\left[e^{\left((S_1)^{(9)}-(p_{44})^{(9)}\right)t} - e^{-(S_2)^{(9)}t}\right] + G_{46}^0 e^{-(S_2)^{(9)}t} \le G_{46}(t) \le$$

$$\frac{(a_{46})^{(9)} G_{44}^0}{(m_2)^{(9)}\left((S_1)^{(9)}-(a'_{46})^{(9)}\right)}\left[e^{(S_1)^{(9)}t} - e^{-(a'_{46})^{(9)}t}\right] + G_{46}^0 e^{-(a'_{46})^{(9)}t})$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le T_{44}^0 e^{\left((R_1)^{(9)}+(r_{44})^{(9)}\right)t}}$$

$$\frac{1}{(\mu_1)^{(9)}} T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le \frac{1}{(\mu_2)^{(9)}} T_{44}^0 e^{\left((R_1)^{(9)}+(r_{44})^{(9)}\right)t}$$

$$\frac{(b_{46})^{(9)} T_{44}^0}{(\mu_1)^{(9)}\left((R_1)^{(9)}-(b'_{46})^{(9)}\right)}\left[e^{(R_1)^{(9)}t} - e^{-(b'_{46})^{(9)}t}\right] + T_{46}^0 e^{-(b'_{46})^{(9)}t} \le T_{46}(t) \le$$

$$\frac{(a_{46})^{(9)} T_{44}^0}{(\mu_2)^{(9)}\left((R_1)^{(9)}+(r_{44})^{(9)}+(R_2)^{(9)}\right)}\left[e^{\left((R_1)^{(9)}+(r_{44})^{(9)}\right)t} - e^{-(R_2)^{(9)}t}\right] + T_{46}^0 e^{-(R_2)^{(9)}t}$$

**Definition of** $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$

**Proof :** From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right) - (a''_{14})^{(1)}(T_{14}, t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

**Definition of** $v^{(1)}$ :- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)}\right) \le \frac{dv^{(1)}}{dt} \le -\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(1)}, (\nu_0)^{(1)}$ :-

For $0 < \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}} < (\nu_1)^{(1)} < (\bar{\nu}_1)^{(1)}$

$$v^{(1)}(t) \ge \frac{(\nu_1)^{(1)} + (C)^{(1)}(\nu_2)^{(1)} e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_0)^{(1)}\right)t\right]}}{1+(C)^{(1)} e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(\nu_1)^{(1)}-(\nu_0)^{(1)}}{(\nu_0)^{(1)}-(\nu_2)^{(1)}}}$$

it follows $(\nu_0)^{(1)} \leq \nu^{(1)}(t) \leq (\nu_1)^{(1)}$

In the same manner , we get 384

$$\nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{\nu}_1)^{(1)}-(\nu_0)^{(1)}}{(\nu_0)^{(1)}-(\bar{\nu}_2)^{(1)}}}$$

From which we deduce $(\nu_0)^{(1)} \leq \nu^{(1)}(t) \leq (\bar{\nu}_1)^{(1)}$

If $0 < (\nu_1)^{(1)} < (\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0} < (\bar{\nu}_1)^{(1)}$ we find like in the previous case, 385

$$(\nu_1)^{(1)} \leq \frac{(\nu_1)^{(1)}+(C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}} \leq \nu^{(1)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\bar{\nu}_1)^{(1)}$$

If $0 < (\nu_1)^{(1)} \leq (\bar{\nu}_1)^{(1)} \leq \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$ , we obtain 386

$$(\nu_1)^{(1)} \leq \nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\nu_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(1)}(t)$ :-

$$(m_2)^{(1)} \leq \nu^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{\nu^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(1)}(t)$ :-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{13})^{(1)} = (a''_{14})^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)}G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)}T_{14}(t)$ This is an important

consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

**Proof** : From global equations we obtain 387

$$\frac{d\nu^{(2)}}{dt} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right) - (a''_{17})^{(2)}(T_{17}, t)\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

**Definition of** $\nu^{(2)}$ :- $\boxed{\nu^{(2)} = \frac{G_{16}}{G_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \le \frac{d\nu^{(2)}}{dt} \le -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

**Definition of** $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$ :-

For $0 < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \ge \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}}{1 + (C)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(C)^{(2)} = \frac{(\nu_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \le \nu^{(2)}(t) \le (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \le \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \le \nu^{(2)}(t) \le (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \le \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}}{1 + (C)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}} \le \nu^{(2)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \le (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \le (\bar{\nu}_1)^{(2)} \le (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$ , we obtain 394

$$(\nu_1)^{(2)} \le \nu^{(2)}(t) \le \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \le (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(2)}(t)$ :- 395

$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}$, $\boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$

In a completely analogous way, we obtain 396

**Definition of** $u^{(2)}(t)$ :-

$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}$, $\boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :** 397

If $(a''_{16})^{(2)} = (a''_{17})^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)} G_{17}(t)$

Analogously if $(b''_{16})^{(2)} = (b''_{17})^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)} T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

**Proof** : From global equations we obtain 398

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right) - (a''_{21})^{(3)}(T_{21}, t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

**Definition of** $\nu^{(3)}$ :- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right) \leq \frac{d\nu^{(3)}}{dt} \leq -\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right)$$

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From which one obtains

For $0 < (\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\nu_1)^{(3)} < (\bar{\nu}_1)^{(3)}$

$$\nu^{(3)}(t) \geq \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(\nu_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\nu_2)^{(3)}}}$$

it follows $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\nu_1)^{(3)}$

In the same manner , we get 401

$$\nu^{(3)}(t) \le \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{\nu}_1)^{(3)}-(\nu_0)^{(3)}}{(\nu_0)^{(3)}-(\bar{\nu}_2)^{(3)}}}$$

**Definition of** $(\bar{\nu}_1)^{(3)}$ :-

From which we deduce $(\nu_0)^{(3)} \le \nu^{(3)}(t) \le (\bar{\nu}_1)^{(3)}$

If $0 < (\nu_1)^{(3)} < (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (\bar{\nu}_1)^{(3)}$ we find like in the previous case, 402

$$(\nu_1)^{(3)} \le \frac{(\nu_1)^{(3)}+(C)^{(3)}(\nu_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}} \le \nu^{(3)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \le (\bar{\nu}_1)^{(3)}$$

If $0 < (\nu_1)^{(3)} \le (\bar{\nu}_1)^{(3)} \le (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}$ , we obtain 403

$$(\nu_1)^{(3)} \le \nu^{(3)}(t) \le \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \le (\nu_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(3)}(t)$ :-

$$(m_2)^{(3)} \le \nu^{(3)}(t) \le (m_1)^{(3)}, \quad \boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(3)}(t)$ :-

$$(\mu_2)^{(3)} \le u^{(3)}(t) \le (\mu_1)^{(3)}, \quad \boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{20}'')^{(3)} = (a_{21}'')^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)}G_{21}(t)$

Analogously if $(b_{20}'')^{(3)} = (b_{21}'')^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)}T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

**Proof** : From global equations we obtain 404

$$\frac{dv^{(4)}}{dt} = (a_{24})^{(4)} - \left((a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right) - (a''_{25})^{(4)}(T_{25},t)v^{(4)} - (a_{25})^{(4)}v^{(4)}$$

**Definition of** $v^{(4)}$ :- $\boxed{v^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)}\right) \le \frac{dv^{(4)}}{dt} \le -\left((a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_4)^{(4)}v^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$ :-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \ge \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \le \nu^{(4)}(t) \le (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \le \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \le \nu^{(4)}(t) \le (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \le \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}}{1 + (C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}} \le \nu^{(4)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \le (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \le (\bar{\nu}_1)^{(4)} \le \boxed{(\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}}}$ , we obtain 407

$$(\nu_1)^{(4)} \le \nu^{(4)}(t) \le \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \le (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(4)}(t)$ :-

$$(m_2)^{(4)} \le \nu^{(4)}(t) \le (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(4)}(t)$ :-

$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}$, $\boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{24})^{(4)} = (a''_{25})^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)} G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b''_{24})^{(4)} = (b''_{25})^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)} T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

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**Proof** : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a'_{28})^{(5)} - (a'_{29})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right) - (a''_{29})^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

**Definition of** $\nu^{(5)}$ :- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$ :-

For $0 < \boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \le \frac{(\nu_1)^{(5)}+(C)^{(5)}(\nu_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)}-(\nu_2)^{(5)}\right)t\right]}}{1+(C)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)}-(\nu_2)^{(5)}\right)t\right]}} \le \nu^{(5)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \le (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \le (\bar{\nu}_1)^{(5)} \le \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain 411

$$(\nu_1)^{(5)} \le \nu^{(5)}(t) \le \frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \le (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(5)}(t)$ :-

$(m_2)^{(5)} \le \nu^{(5)}(t) \le (m_1)^{(5)}$, $\boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$

In a completely analogous way, we obtain

**Definition of** $u^{(5)}(t)$ :-

$(\mu_2)^{(5)} \le u^{(5)}(t) \le (\mu_1)^{(5)}$, $\boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a_{28}'')^{(5)} = (a_{29}'')^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)}G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b_{28}'')^{(5)} = (b_{29}'')^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)}T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

**Proof** : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a_{32}')^{(6)} - (a_{33}')^{(6)} + (a_{32}'')^{(6)}(T_{33},t)\right) - (a_{33}'')^{(6)}(T_{33},t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

**Definition of** $\nu^{(6)}$ :- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \le \frac{d\nu^{(6)}}{dt} \le -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$ :-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \geq \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\nu_1)^{(6)}$

In the same manner , we get 413

$$\nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{\nu}_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\bar{\nu}_1)^{(6)}$

If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 414

$$(\nu_1)^{(6)} \leq \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}} \leq \nu^{(6)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\bar{\nu}_1)^{(6)}$$

If $0 < (\nu_1)^{(6)} \leq (\bar{\nu}_1)^{(6)} \leq \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$ , we obtain 415

$$(\nu_1)^{(6)} \leq \nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(6)}(t)$ :-

$$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}, \quad \boxed{\nu^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(6)}(t)$ :-

$$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}, \quad \boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)} G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)} T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

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**Proof** : From global equations we obtain

$$\frac{dv^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right) - (a''_{37})^{(7)}(T_{37},t)v^{(7)} - (a_{37})^{(7)}v^{(7)}$$

**Definition of** $v^{(7)}$ :- $\boxed{v^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_2)^{(7)}v^{(7)} - (a_{36})^{(7)}\right) \le \frac{dv^{(7)}}{dt} \le -\left((a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_1)^{(7)}v^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

**Definition of** $(\bar{v}_1)^{(7)}, (v_0)^{(7)}$ :-

For $0 < \boxed{(v_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}} < (v_1)^{(7)} < (\bar{v}_1)^{(7)}$

$$v^{(7)}(t) \ge \frac{(v_1)^{(7)} + (C)^{(7)}(v_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_0)^{(7)}\right)t\right]}}{1 + (C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(v_1)^{(7)} - (v_0)^{(7)}}{(v_0)^{(7)} - (v_2)^{(7)}}}$$

it follows $(v_0)^{(7)} \le v^{(7)}(t) \le (v_1)^{(7)}$

In the same manner , we get 417

$$v^{(7)}(t) \le \frac{(\bar{v}_1)^{(7)} + (\bar{C})^{(7)}(\bar{v}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{v}_1)^{(7)} - (v_0)^{(7)}}{(v_0)^{(7)} - (\bar{v}_2)^{(7)}}}$$

From which we deduce $(v_0)^{(7)} \le v^{(7)}(t) \le (\bar{v}_1)^{(7)}$

If $0 < (v_1)^{(7)} < (v_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}} < (\bar{v}_1)^{(7)}$ we find like in the previous case, 418

$$(v_1)^{(7)} \le \frac{(v_1)^{(7)} + (C)^{(7)}(v_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_2)^{(7)}\right)t\right]}}{1 + (C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_2)^{(7)}\right)t\right]}} \le v^{(7)}(t) \le$$

$$\frac{(\bar{v}_1)^{(7)} + (\bar{C})^{(7)}(\bar{v}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}} \le (\bar{v}_1)^{(7)}$$

If $0 < (v_1)^{(7)} \le (\bar{v}_1)^{(7)} \le \boxed{(v_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$ , we obtain 419

$$(v_1)^{(7)} \le v^{(7)}(t) \le \frac{(\bar{v}_1)^{(7)} + (\bar{C})^{(7)}(\bar{v}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}} \le (v_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $v^{(7)}(t)$ :-

$$(m_2)^{(7)} \leq v^{(7)}(t) \leq (m_1)^{(7)}, \quad \boxed{v^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(7)}(t)$ :- 420

$$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}, \quad \boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $v^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)} G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)} T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

**Proof** : From global equations we obtain 421

$$\frac{dv^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right) - (a''_{41})^{(8)}(T_{41}, t)v^{(8)} - (a_{41})^{(8)}v^{(8)}$$

**Definition of** $v^{(8)}$ :- $\boxed{v^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_2)^{(8)}v^{(8)} - (a_{40})^{(8)}\right) \leq \frac{dv^{(8)}}{dt} \leq -\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_1)^{(8)}v^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$ :-

$$\text{For } 0 < \boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$$

$$v^{(8)}(t) \geq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)}-(\nu_0)^{(8)}}{(\nu_0)^{(8)}-(\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \leq \frac{(\nu_1)^{(8)}+(C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}}{1+(C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}} \leq \nu^{(8)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \leq (\bar{\nu}_1)^{(8)} \leq \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$ , we obtain 424

$$(\nu_1)^{(8)} \leq \nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(8)}(t)$ :-

$$(m_2)^{(8)} \leq \nu^{(8)}(t) \leq (m_1)^{(8)}, \quad \boxed{\nu^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(8)}(t)$ :-

$$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}, \quad \boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $\nu^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)}G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then $(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)}T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

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**Proof** : From 99,20,44,22,23,44 we obtain A

$$\frac{dv^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right) - (a''_{45})^{(9)}(T_{45},t)v^{(9)} - (a_{45})^{(9)}v^{(9)}$$

**Definition of** $v^{(9)}$ :- $\boxed{v^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)}\right) \le \frac{dv^{(9)}}{dt} \le -\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_1)^{(9)}v^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$ :-

For $0 < \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$

$$v^{(9)}(t) \ge \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \le v^{(9)}(t) \le (\nu_9)^{(9)}$

In the same manner , we get

$$v^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \le v^{(9)}(t) \le (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \le \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}}{1 + (C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}} \le v^{(9)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \le (\bar{\nu}_1)^{(9)} \le \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$ , we obtain

$$(\nu_1)^{(9)} \le v^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $v^{(9)}(t)$ :-

$$(m_2)^{(9)} \le v^{(9)}(t) \le (m_1)^{(9)}, \quad \boxed{v^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(9)}(t)$ :-

$(\mu_2)^{(9)} \leq u^{(9)}(t) \leq (\mu_1)^{(9)}$, $\boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)} G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then
$(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)} T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following 425

**Theorem :** If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on $t$ , and the conditions with the notations

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0$ ,

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$

$with\ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a''_i)^{(2)} and\ (b''_i)^{(2)}$ are independent on t , and the conditions with the notations 426

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$ 427

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 428

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$ , 429

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 430

$with\ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a''_i)^{(3)} and\ (b''_i)^{(3)}$ are independent on $t$ , and the conditions with the notations 431

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$ ,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

**Theorem :** If $(a_i'')^{(4)} and\ (b_i'')^{(4)}$ are independent on $t$ , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$ ,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(5)} and\ (b_i'')^{(5)}$ are independent on $t$ , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$ ,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

**Theorem** If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on $t$ , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$ ,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(7)} and\ (b_i'')^{(7)}$ are independent on $t$ , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$ ,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(8)} and\ (b_i'')^{(8)}$ are independent on $t$ , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$ ,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(9)} and\ (b_i'')^{(9)}$ are independent on $t$ , and the conditions (with the notations 436 A
45,46,27,28)

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$ ,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0$ 437

$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0$ 438

$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0$ 439

$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0$ 440

$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ]T_{14} = 0$ 441

$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0$ 442

has a unique positive solution , which is an equilibrium solution for the system

$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0$ 443

$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0$ 444

$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0$ 445

$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\ ]T_{16} = 0$ 446

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\,]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\,]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\,]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\,]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\,]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\big((G_{27})\big)\,]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\big((G_{27})\big)\,]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\big((G_{27})\big)\,]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})\,]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})\,]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})\,]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})\,]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})\,]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})\,]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0 \quad 479$$

$$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0 \quad 480$$

$$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0 \quad 481$$

$$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0 \quad 482$$

$$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0 \quad 483$$

$$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0 \quad 484$$

$$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0 \quad 484\text{A}$$

$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$

$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$

$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$

$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$

$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$

**Proof:** 485

(a) Indeed the first two equations have a nontrivial solution $G_{13}, G_{14}$ if

$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$

**Proof:** 486

(hh) Indeed the first two equations have a nontrivial solution $G_{16}, G_{17}$ if

$\mathrm{F}(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$

**Proof:** 487

(a) Indeed the first two equations have a nontrivial solution $G_{20}, G_{21}$ if

$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$

**Proof:** 488

(a) Indeed the first two equations have a nontrivial solution $G_{24}, G_{25}$ if

$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$

**Proof:** 489

(a) Indeed the first two equations have a nontrivial solution $G_{28}, G_{29}$ if

$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$

**Proof:** 490

(a) Indeed the first two equations have a nontrivial solution $G_{32}, G_{33}$ if

$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) +$

$(a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$

**Proof:** 491

(a) Indeed the first two equations have a nontrivial solution $G_{36}, G_{37}$ if

$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$

**Proof:** 492

(a) Indeed the first two equations have a nontrivial solution $G_{40}, G_{41}$ if

$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$

**Proof:** 492 A

(a) Indeed the first two equations have a nontrivial solution $G_{44}, G_{45}$ if

$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$

**Definition and uniqueness of** $T^*_{14}$ :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique $T^*_{14}$ for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

**Definition and uniqueness of** $T^*_{17}$ :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique $T^*_{17}$ for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]}$$ 495

**Definition and uniqueness of** $T^*_{21}$ :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique $T^*_{21}$ for which $f(T^*_{21}) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

**Definition and uniqueness of** $T^*_{25}$ :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that

there exists a unique $T_{25}^{*}$ for which $f(T_{25}^{*})=0$. With this value , we obtain from the three first equations

$$G_{24}=\frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}=\frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T_{25}^{*})]}$$

**Definition and uniqueness of** $T_{29}^{*}$ :- 498

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(5)}(T_{29})$ being increasing, it follows that there exists a unique $T_{29}^{*}$ for which $f(T_{29}^{*})=0$. With this value , we obtain from the three first equations

$$G_{28}=\frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}=\frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T_{29}^{*})]}$$

**Definition and uniqueness of** $T_{33}^{*}$ :- 499

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(6)}(T_{33})$ being increasing, it follows that there exists a unique $T_{33}^{*}$ for which $f(T_{33}^{*})=0$. With this value , we obtain from the three first equations

$$G_{32}=\frac{(a_{32})^{(6)}G_{33}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^{*})]} \quad , \quad G_{34}=\frac{(a_{34})^{(6)}G_{33}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^{*})]}$$

**Definition and uniqueness of** $T_{37}^{*}$ :- 500

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(7)}(T_{37})$ being increasing, it follows that there exists a unique $T_{37}^{*}$ for which $f(T_{37}^{*})=0$. With this value , we obtain from the three first equations

$$G_{36}=\frac{(a_{36})^{(7)}G_{37}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^{*})]} \quad , \quad G_{38}=\frac{(a_{38})^{(7)}G_{37}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^{*})]}$$

**Definition and uniqueness of** $T_{41}^{*}$ :- 501

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique $T_{41}^{*}$ for which $f(T_{41}^{*})=0$. With this value , we obtain from the three first equations

$$G_{40}=\frac{(a_{40})^{(8)}G_{41}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T_{41}^{*})]} \quad , \quad G_{42}=\frac{(a_{42})^{(8)}G_{41}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T_{41}^{*})]}$$

**Definition and uniqueness of** $T_{45}^{*}$ :- 501 A

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(9)}(T_{45})$ being increasing, it follows that there exists a unique $T_{45}^{*}$ for which $f(T_{45}^{*})=0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)} G_{45}}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T^*_{45})]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)} G_{45}}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T^*_{45})]}$$

By the same argument, the equations admit solutions $G_{13}, G_{14}$ if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right] + (b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{14}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{14}$ such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions $G_{16}, G_{17}$ if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right] + (b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ 504
is a decreasing function in $G_{17}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{14}$ such that $\varphi((G_{19})^*) = 0$

By the same argument, the equations admit solutions $G_{20}, G_{21}$ if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right] + (b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{21}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{21}$ such that $\varphi((G_{23})^*) = 0$

By the same argument, the equations admit solutions $G_{24}, G_{25}$ if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right] + (b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{25}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{25}$ such that $\varphi((G_{27})^*) = 0$

By the same argument, the equations admit solutions $G_{28}, G_{29}$ if 507

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right] + (b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{29}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that

there exists a unique $G_{29}^{*}$ such that $\varphi((G_{31})^{*}) = 0$

By the same argument, the equations admit solutions $G_{32}, G_{33}$ if 508
$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$

$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right] + (b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{33}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{33}^{*}$ such that $\varphi(G_{35}{}^{*}) = 0$

By the same argument, the equations admit solutions $G_{36}, G_{37}$ if 509
$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$
$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right] + (b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{37}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{37}^{*}$ such that $\varphi(G_{39}{}^{*}) = 0$

By the same argument, the equations admit solutions $G_{40}, G_{41}$ if 510
$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$
$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right] + (b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{41}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{41}^{*}$ such that $\varphi(G_{43}{}^{*}) = 0$

By the same argument, the equations 92,93 admit solutions $G_{44}, G_{45}$ if
$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$

$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right] + (b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{45}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{45}^{*}$ such that $\varphi((G_{47})^{*}) = 0$

Finally we obtain the unique solution 511

$G_{14}^{*}$ given by $\varphi(G^{*}) = 0$ , $T_{14}^{*}$ given by $f(T_{14}^{*}) = 0$ and

$$G_{13}^{*} = \frac{(a_{13})^{(1)}G_{14}^{*}}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^{*})]} \quad , \quad G_{15}^{*} = \frac{(a_{15})^{(1)}G_{14}^{*}}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^{*})]}$$

$$T_{13}^{*} = \frac{(b_{13})^{(1)}T_{14}^{*}}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^{*})]} \quad , \quad T_{15}^{*} = \frac{(b_{15})^{(1)}T_{14}^{*}}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^{*})]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

$G_{17}^{*}$ given by $\varphi((G_{19})^{*}) = 0$ , $T_{17}^{*}$ given by $f(T_{17}^{*}) = 0$ and 512

$$G_{16}^{*} = \frac{(a_{16})^{(2)} G_{17}^{*}}{[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}^{*})]} \quad , \quad G_{18}^{*} = \frac{(a_{18})^{(2)} G_{17}^{*}}{[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}^{*})]} \tag{513}$$

$$T_{16}^{*} = \frac{(b_{16})^{(2)} T_{17}^{*}}{[(b'_{16})^{(2)} - (b''_{16})^{(2)}((G_{19})^{*})]} \quad , \quad T_{18}^{*} = \frac{(b_{18})^{(2)} T_{17}^{*}}{[(b'_{18})^{(2)} - (b''_{18})^{(2)}((G_{19})^{*})]} \tag{514}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

$G_{21}^{*}$ given by $\varphi((G_{23})^{*}) = 0$ , $T_{21}^{*}$ given by $f(T_{21}^{*}) = 0$ and

$$G_{20}^{*} = \frac{(a_{20})^{(3)} G_{21}^{*}}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^{*})]} \quad , \quad G_{22}^{*} = \frac{(a_{22})^{(3)} G_{21}^{*}}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^{*})]}$$

$$T_{20}^{*} = \frac{(b_{20})^{(3)} T_{21}^{*}}{[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}{}^{*})]} \quad , \quad T_{22}^{*} = \frac{(b_{22})^{(3)} T_{21}^{*}}{[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}{}^{*})]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

$G_{25}^{*}$ given by $\varphi(G_{27}) = 0$ , $T_{25}^{*}$ given by $f(T_{25}^{*}) = 0$ and

$$G_{24}^{*} = \frac{(a_{24})^{(4)} G_{25}^{*}}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}^{*} = \frac{(a_{26})^{(4)} G_{25}^{*}}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^{*})]}$$

$$T_{24}^{*} = \frac{(b_{24})^{(4)} T_{25}^{*}}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^{*})]} \quad , \quad T_{26}^{*} = \frac{(b_{26})^{(4)} T_{25}^{*}}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^{*})]} \tag{517}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

$G_{29}^{*}$ given by $\varphi((G_{31})^{*}) = 0$ , $T_{29}^{*}$ given by $f(T_{29}^{*}) = 0$ and

$$G_{28}^{*} = \frac{(a_{28})^{(5)} G_{29}^{*}}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}^{*} = \frac{(a_{30})^{(5)} G_{29}^{*}}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^{*})]}$$

$$T_{28}^{*} = \frac{(b_{28})^{(5)} T_{29}^{*}}{[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31})^{*})]} \quad , \quad T_{30}^{*} = \frac{(b_{30})^{(5)} T_{29}^{*}}{[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31})^{*})]} \tag{519}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

$G_{33}^{*}$ given by $\varphi((G_{35})^{*}) = 0$ , $T_{33}^{*}$ given by $f(T_{33}^{*}) = 0$ and

$$G_{32}^{*} = \frac{(a_{32})^{(6)} G_{33}^{*}}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^{*})]} \quad , \quad G_{34}^{*} = \frac{(a_{34})^{(6)} G_{33}^{*}}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^{*})]}$$

$$T_{32}^{*} = \frac{(b_{32})^{(6)} T_{33}^{*}}{[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35})^{*})]} \quad , \quad T_{34}^{*} = \frac{(b_{34})^{(6)} T_{33}^{*}}{[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35})^{*})]} \tag{521}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

$G_{37}^*$ given by $\varphi((G_{39})^*) = 0$ , $T_{37}^*$ given by $f(T_{37}^*) = 0$ and

$$G_{36}^* = \frac{(a_{36})^{(7)} G_{37}^*}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^*)]} \quad , \quad G_{38}^* = \frac{(a_{38})^{(7)} G_{37}^*}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^*)]}$$

$$T_{36}^* = \frac{(b_{36})^{(7)} T_{37}^*}{[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39})^*)]} \quad , \quad T_{38}^* = \frac{(b_{38})^{(7)} T_{37}^*}{[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39})^*)]}$$

Finally we obtain the unique solution 523

$G_{41}^*$ given by $\varphi((G_{43})^*) = 0$ , $T_{41}^*$ given by $f(T_{41}^*) = 0$ and

$$G_{40}^* = \frac{(a_{40})^{(8)} G_{41}^*}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42}^* = \frac{(a_{42})^{(8)} G_{41}^*}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

$$T_{40}^* = \frac{(b_{40})^{(8)} T_{41}^*}{[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43})^*)]} \quad , \quad T_{42}^* = \frac{(b_{42})^{(8)} T_{41}^*}{[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43})^*)]}$$

Finally we obtain the unique solution of 89 to 99 523 A

$G_{45}^*$ given by $\varphi((G_{47})^*) = 0$ , $T_{45}^*$ given by $f(T_{45}^*) = 0$ and

$$G_{44}^* = \frac{(a_{44})^{(9)} G_{45}^*}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46}^* = \frac{(a_{46})^{(9)} G_{45}^*}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

$$T_{44}^* = \frac{(b_{44})^{(9)} T_{45}^*}{[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47})^*)]} \quad , \quad T_{46}^* = \frac{(b_{46})^{(9)} T_{45}^*}{[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47})^*)]}$$

## ASYMPTOTIC STABILITY ANALYSIS 524

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

**Proof:** Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a''_{14})^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \frac{\partial (b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a'_{13})^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)}G^*_{13}\mathbb{T}_{14} \quad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a'_{14})^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)}G^*_{14}\mathbb{T}_{14} \quad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a'_{15})^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)}G^*_{15}\mathbb{T}_{14} \quad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b'_{13})^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)}T^*_{13}\mathbb{G}_j\right) \quad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b'_{14})^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)}T^*_{14}\mathbb{G}_j\right) \quad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b'_{15})^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T^*_{15}\mathbb{G}_j\right) \quad 530$$

**ASYMPTOTIC STABILITY ANALYSIS** 531

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(2)}$ and $(b''_i)^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G^*_i + \mathbb{G}_i \quad , T_i = T^*_i + \mathbb{T}_i \quad 532$$

$$\frac{\partial(a''_{17})^{(2)}}{\partial T_{17}}(T^*_{17}) = (q_{17})^{(2)} \quad , \frac{\partial(b''_i)^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \quad 533$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a'_{16})^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G^*_{16}\mathbb{T}_{17} \quad 534$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a'_{17})^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G^*_{17}\mathbb{T}_{17} \quad 535$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a'_{18})^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G^*_{18}\mathbb{T}_{17} \quad 536$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b'_{16})^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T^*_{16}\mathbb{G}_j\right) \quad 537$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b'_{17})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}T^*_{17}\mathbb{G}_j\right) \quad 538$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b'_{18})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}T^*_{18}\mathbb{G}_j\right) \quad 539$$

**ASYMPTOTIC STABILITY ANALYSIS** 540

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(3)}$ *and* $(b''_i)^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{21}'')^{(3)}}{\partial T_{21}}(T_{21}^*) = (q_{21})^{(3)} \quad , \frac{\partial (b_i'')^{(3)}}{\partial G_j}(\,(G_{23})^*\,) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\big((a_{20}')^{(3)} + (p_{20})^{(3)}\big)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G_{20}^*\mathbb{T}_{21} \qquad 541$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\big((a_{21}')^{(3)} + (p_{21})^{(3)}\big)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G_{21}^*\mathbb{T}_{21} \qquad 542$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\big((a_{22}')^{(3)} + (p_{22})^{(3)}\big)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G_{22}^*\mathbb{T}_{21} \qquad 543$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\big((b_{20}')^{(3)} - (r_{20})^{(3)}\big)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\big(s_{(20)(j)}T_{20}^*\mathbb{G}_j\big) \qquad 544$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\big((b_{21}')^{(3)} - (r_{21})^{(3)}\big)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\big(s_{(21)(j)}T_{21}^*\mathbb{G}_j\big) \qquad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\big((b_{22}')^{(3)} - (r_{22})^{(3)}\big)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\big(s_{(22)(j)}T_{22}^*\mathbb{G}_j\big) \qquad 546$$

**ASYMPTOTIC STABILITY ANALYSIS** 547

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}\ and\ (b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{25}'')^{(4)}}{\partial T_{25}}(T_{25}^*) = (q_{25})^{(4)} \quad , \frac{\partial (b_i'')^{(4)}}{\partial G_j}((G_{27})^*\,) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\big((a_{24}')^{(4)} + (p_{24})^{(4)}\big)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G_{24}^*\mathbb{T}_{25} \qquad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\big((a_{25}')^{(4)} + (p_{25})^{(4)}\big)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G_{25}^*\mathbb{T}_{25} \qquad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\big((a_{26}')^{(4)} + (p_{26})^{(4)}\big)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G_{26}^*\mathbb{T}_{25} \qquad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\big((b_{24}')^{(4)} - (r_{24})^{(4)}\big)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\big(s_{(24)(j)}T_{24}^*\mathbb{G}_j\big) \qquad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\big((b_{25}')^{(4)} - (r_{25})^{(4)}\big)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\big(s_{(25)(j)}T_{25}^*\mathbb{G}_j\big) \qquad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T^*_{26}\mathbb{G}_j\right) \quad 554$$

**ASYMPTOTIC STABILITY ANALYSIS** 555

**Theorem 5:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}\ and\ (b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 556

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{29})^{(5)}}{\partial T_{29}}(T^*_{29}) = (q_{29})^{(5)} \ , \ \frac{\partial(b_i'')^{(5)}}{\partial G_j}\left((G_{31})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a'_{28})^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G^*_{28}\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a'_{29})^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G^*_{29}\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a'_{30})^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G^*_{30}\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b'_{28})^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T^*_{28}\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b'_{29})^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T^*_{29}\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b'_{30})^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T^*_{30}\mathbb{G}_j\right) \quad 562$$

**ASYMPTOTIC STABILITY ANALYSIS** 563

**Theorem 6:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}\ and\ (b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{33})^{(6)}}{\partial T_{33}}(T^*_{33}) = (q_{33})^{(6)} \ , \ \frac{\partial(b_i'')^{(6)}}{\partial G_j}\left((G_{35})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a'_{32})^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G^*_{32}\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a'_{33})^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G^*_{33}\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a'_{34})^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G^*_{34}\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b'_{32})^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T^*_{32}\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b'_{33})^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T^*_{33}\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b'_{34})^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T^*_{34}\mathbb{G}_j\right) \quad 570$$

**ASYMPTOTIC STABILITY ANALYSIS** 571

**Theorem 7:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 572

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{37})^{(7)}}{\partial T_{37}}(T^*_{37}) = (q_{37})^{(7)} \quad , \frac{\partial(b_i'')^{(7)}}{\partial G_j}\left((G_{39})^{**}\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a'_{36})^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G^*_{36}\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a'_{37})^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G^*_{37}\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a'_{38})^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G^*_{38}\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b'_{36})^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T^*_{36}\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b'_{37})^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T^*_{37}\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b'_{38})^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T^*_{38}\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

**ASYMPTOTIC STABILITY ANALYSIS**

**Theorem 8:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{41}'')^{(8)}}{\partial T_{41}}(T_{41}^*) = (q_{41})^{(8)} \quad , \frac{\partial (b_i'')^{(8)}}{\partial G_j}\left( (G_{43})^* \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a_{40}')^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G_{40}^*\mathbb{T}_{41} \quad (581)$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a_{41}')^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G_{41}^*\mathbb{T}_{41} \quad (582)$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a_{42}')^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G_{42}^*\mathbb{T}_{41} \quad (583)$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b_{40}')^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T_{40}^*\mathbb{G}_j\right) \quad (584)$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b_{41}')^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T_{41}^*\mathbb{G}_j\right) \quad (585)$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b_{42}')^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T_{42}^*\mathbb{G}_j\right) \quad (586)$$

(586 A)

## ASYMPTOTIC STABILITY ANALYSIS

**Theorem 9:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}$ *and* $(b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

**Proof:** Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{45}'')^{(9)}}{\partial T_{45}}(T_{45}^*) = (q_{45})^{(9)} \quad , \frac{\partial (b_i'')^{(9)}}{\partial G_j}\left( (G_{47})^* \right) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a_{44}')^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)}G_{44}^*\mathbb{T}_{45} \quad (586\,B)$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a_{45}')^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)}G_{45}^*\mathbb{T}_{45} \quad (586\,C)$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a_{46}')^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)}G_{46}^*\mathbb{T}_{45} \quad (586\,D)$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b_{44}')^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)}T_{44}^*\mathbb{G}_j\right) \quad (586\,E)$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b_{45}')^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)}T_{45}^*\mathbb{G}_j\right) \quad (586\,F)$$

$\frac{d\mathbb{T}_{46}}{dt} = -\left((b'_{46})^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T^*_{46}\mathbb{G}_j\right)$ 586

G

587

**The characteristic equation of this system is**

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(q_{13})^{(1)}G^*_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)}T^*_{14} + (b_{14})^{(1)}s_{(13),(14)}T^*_{14}\right)$$

$$+\left(\left((\lambda)^{(1)} + (a'_{14})^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)}G^*_{13} + (a_{13})^{(1)}(q_{14})^{(1)}G^*_{14}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)}T^*_{14} + (b_{14})^{(1)}s_{(13),(13)}T^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((b'_{13})^{(1)} + (b'_{14})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(15)}T^*_{14} + (b_{14})^{(1)}s_{(13),(15)}T^*_{13}\right)\} = 0$$

$$+$$

$$(\lambda)^{(2)} + (b'_{18})^{(2)} - (r_{18})^{(2)}\{\left((\lambda)^{(2)} + (a'_{18})^{(2)} + (p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^*_{17} + (a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^*_{17}\right)$$

$$+\left(\left((\lambda)^{(2)} + (a'_{17})^{(2)} + (p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^*_{16} + (a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}\right)$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2 + \left((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2 + \left((b'_{16})^{(2)} + (b'_{17})^{(2)} - (r_{16})^{(2)} + (r_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$+\left(\left((\lambda)^{(2)}\right)^2 + \left((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17} + (a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^{*}_{16}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\right)\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left(\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)\right]$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(21)}T^{*}_{21}\right)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^{*}_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^{*}_{21}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(20)}T^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$+\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(22)}T^{*}_{20}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\right)\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left(\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)\right]$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(25)}T^{*}_{25}\right)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^{*}_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^{*}_{25}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(24)}T^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$+\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(26)}T^{*}_{24}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\right)\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(29)}T^{*}_{29}\right)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^{*}_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^{*}_{29}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(28)}T^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$+\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(30)}T^{*}_{28}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\right)\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left(\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)\right]$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(33)}T^{*}_{33}\right)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^{*}_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^{*}_{33}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(32)}T^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$+\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)$$

$$\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(34)}T^{*}_{32}\Big)\}=0$$

$$+$$

$$(\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^{*}_{36}\Big)\right]$$

$$\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(37)}T^{*}_{37}\Big)$$

$$+\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^{*}_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^{*}_{37}\Big)$$

$$\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(36)}T^{*}_{36}\Big)$$

$$\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\Big)$$

$$\left((\lambda)^{(7)}\right)^{2}+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\Big)$$

$$+\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\Big)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)$$

$$\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(38)}T^{*}_{36}\Big)\}=0$$

$$+$$

$$(\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^{*}_{40}\Big)\right]$$

$$\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(41)}T^{*}_{41}\Big)$$

$$+\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^{*}_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^{*}_{41}\Big)$$

$$\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(40)}T^{*}_{40}\Big)$$

$$\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\Big)$$

$$\left((\lambda)^{(8)}\right)^{2}+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\Big)$$

$$+\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\Big)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(42)}T^{*}_{40}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\right)\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(45)}T^{*}_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^{*}_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^{*}_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(44)}T^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$+\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(46)}T^{*}_{44}\right)\}=0$$

**And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.**

## SECTION THIRTY SIX A

## Unacceptable Negative Solution By Applied The Energy-Momentum Relation To The Study Of Electrons, Turning Them Into Relativistic Objects

### INTRODUCTION—VARIABLES USED

**The conflict between entropy and syntropy: the vital needs model Ulisse Di Corpo**

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(1) In 1928 Paul Dirac tried to solve the unacceptable negative solution by applying the energy-momentum relation to (e&eb) the study of electrons, turning them into relativistic objects.
(2) But, also in this case, the unwanted negative solution emerged (eb) in the form of electrons (+E) and its antiparticles (−E).
(3) In 1941 Luigi Fantappiè, noted that the mathematical properties of those phenomena which are determined by (e) past causes (causality, +E), such as the physical and chemical processes, are governed by (e&eb) the law of entropy, while the mathematical properties of those phenomena which are attracted towards causes located in (eb) the future (retrocausality, −E), agree with (=) a symmetrical law which Fantappiè named syntropy.
(4) Fantappiè immediately recognized the coincidence between the mathematical properties of syntropy and (=) the properties of living systems (Fantappiè, 1942).
(5) It is important to note that, as a consequence of the fact that the universe is expanding, macrocosm is governed by (e&eb) the law of entropy which forces time to flow from the past to (e&eb) the future (mechanical causation: causeÎeffect).

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(8) Comparing the infinite of the environment (entropy) and (e&eb) finite of the living system (syntropy) produces results which tends to zero:
(9) In this equation 1 symbolizes the living system which is finite (syntropy), while (e) Infinite symbolizes the environment (entropy).
(10) The comparison between the living system (1, finite) and the environment (infinite) tends to zero. In other words, comparing ourselves with the environment which is infinite we become aware of (e) the fact that we are equal to nothing.
(11) But to be equal to nothing is equivalent to (=) death, a fact which is incompatible with (e&eb) the feeling of life.
(12) It is therefore necessary to solve this conflict between being (1) and not being (0), a conflict which consumes (e) energy and increases entropy. This conflict is generally felt as the need to give a meaning to life, for example:

**(iii) Increasing our own value (through richness, power, achievement, etc.);**
**(iv) Finding a purpose in life, finality (through ideologies, religion, etc.).**

(13) In living beings with highly complex cortical systems, this need is vital because, (e) when it is not solved, it leads to (eb) the dissipation of energy, and in the most serious cases to death.
(14) The existential crisis associated with (e&eb) this conflict is accompanied by (e&eb) feelings of being useless, purposeless, reduction of energy (dissipation of energy, entropy), usually named depression, felt in the cortical area in (eb) the form of tension, and usually strongly correlated with (e&eb) anxiety and feelings of pain in the thorax.

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(6) This strong correlation between depression and (e&eb) anxiety is suggested by the fact that, from a mathematical point of view, the conflict between being and (e&eb) not being is solved when: Where the operator x coincides with union, which is (=) the property of –E (converging waves).
(7) In other words, when we unite ourselves (1) to (e&eb) the environment, comparing ourselves to the environment, (eb) we find our identity (= 1).

(8) This last equation permits to state that:

• when the need of meaning is answered (eb) increasing the value of the numerator (power, richness, achievement), the identity conflict is not solved, because (e) whichever is the value at the numerator compared to (e&eb) infinite it tends to zero;

• Perfect correlation between anxiety and (e&eb) depression must be observed, because (e) when the unity (x) is weak, anxiety increases (eb) and also the identity conflict and depression.

**Chronological order**

(9) According to this model a chronological order exists in the satisfaction of these needs. Initially humans have (e) to answer material needs, which are more immediate, and only afterwards they become (eb) aware of the needs of meaning and unity.
(10) Part of humanity has now satisfied (eb) material needs and is now facing the needs of (e) meaning and unity and is undergoing rising feelings of existential crisis and anxiety.
(11) Material needs can be easily recognized, whereas the needs of meaning and unity are immaterial, invisible, and more difficult to be recognized. For example:

• In order to give a meaning to their lives, people become (=) obsessed with the way they are judged. They strive for (e) power, success. They become addicted to (e) religion, ideologies and groups;

• In order to overcome anxiety they become (eb) addicted to alcohol and drugs and avoid (e) any moment of silence.

(12) The model of vital needs affirms (eb) that all these states are (=) secondary needs which originate from (e) the unsatisfied needs of meaning and unity. According to the vital needs model all behaviours (psychological, sociological and economical) can be traced back to (e) these 3 basic needs.

**Empirical evidences**

Working as a sociologist for public institutions I am always faced with the centrality of the 3 vital needs which I have just described. For example:

• in a study, just concluded, for IOM (International Organization on Migration) results showed that whilst irregular migrants come to Italy for economical reasons (material needs), a significant number would go back home immediately, because of loneliness (need of unity) and depression (need of meaning), even though their economic situation at home has not changed. It seems that the experience in Italy and the difficulties associated to the integration process lead the migrant to rediscover a set of needs, qualitatively different from the material needs, which were met at home, thanks to the family and friends, but could not be met in Italy, causing feelings of loneliness, and depression.

• In a study conducted in the Abruzzi area of Italy, different theoretical models were compared in order to understand the origins of psychological and emotional suffering in high school students. Results show that items relative to loneliness and worthlessness obtained the highest correlations with items relative to unhappiness and suffering (SERT-Teramo, 1993).

• In several studies conducted with drug addict populations, loneliness and worthlessness emerged as the two main reasons behind drug addiction. In all these studies the correlation among loneliness (anxiety) and worthlessness (depression) is nearly perfect, supporting the hypothesis of the existence of a mathematical

relation between these two needs.

**Conclusions**

The introduction of advanced waves in science might not be (=) as important in physics as it could be in life disciplines such as medicine, psychology and sociology. These disciplines now approach pathologies, illnesses and crises in a mechanistic way which leads, (eb) in an always more alarming way, to inefficiency and to an increase in costs.

## NOTATION

### Module One

In 1928 Paul Dirac tried to solve the u**nacceptable negative solution by applying the energy-momentum relation** to (e&eb) the study of electrons, turning them into relativistic objects

$G_{13}$ : Category one of u**nacceptable negative solution by applying the energy-momentum relation;** study of electrons, turning them into relativistic objects

$G_{14}$ : Category two of SAS

$G_{15}$ : Category three of SAS

$T_{13}$ : Category one of study of electrons, turning them into relativistic objects; u**nacceptable negative solution by applying the energy-momentum relation**

$T_{14}$ : Category two of SAS

$T_{15}$ : Category three of SAS

### Module Two

But, also in this case, the unwanted negative solution emerged (eb) in the form of electrons (+E) and its antiparticles (−E)

$G_{16}$ : Category one of unwanted negative solution

$G_{17}$ : Category two of SAS

$G_{18}$ : Category three of SAS

$T_{16}$ : Category one of electrons (+E) and its antiparticles (−E)

$T_{17}$ : Category two of SAS

$T_{18}$ : Category three of SAS

### Module three

In 1941 Luigi Fantappiè, noted that the mathematical properties of those phenomena which are determined by (e) past causes (causality, +E), such as the physical and chemical processes, are governed by (e&eb) the law of entropy, while the mathematical properties of those phenomena which are attracted towards causes located in (eb) the future (retrocausality, −E), agree with (=) a symmetrical law which Fantappiè named syntropy

$G_{20}$ : Category one of past causes (causality, +E), such as the physical and chemical processes, are governed by (e&eb) the law of entropy, while the mathematical properties of those phenomena which are attracted towards causes located in (eb) the future (retrocausality, −E), agree with (=) a symmetrical law which

Fantappiè named syntropy

$G_{21}$ : Category two of SAS

$G_{22}$ : Category three of SAS

$T_{20}$ : Category one of mathematical properties of those phenomena which are determined

$T_{21}$ : Category two of SAS

$T_{22}$ : Category three of SAS

**Module four**

In 1941 Luigi Fantappiè, noted that the **mathematical properties of those phenomena which are determined by past causes (causality, +E), such as the physical and chemical processes**, are governed by (e&eb) the law of entropy, while the mathematical properties of those phenomena which are attracted towards causes located in (eb) the future (retrocausality, −E), agree with (=) a symmetrical law which Fantappiè named syntropy

$G_{24}$ : Category one of **mathematical properties of those phenomena which are determined by past causes (causality, +E), such as the physical and chemical processes;** law of entropy, while the mathematical properties of those phenomena which are attracted towards causes located in (eb) the future (retrocausality, −E), agree with (=) a symmetrical law which Fantappiè named syntropy

$G_{25}$ : Category two of SAS

$G_{26}$ : Category three of SAS

$T_{24}$ : Category one of law of entropy, while the mathematical properties of those phenomena which are attracted towards causes located in (eb) the future (retrocausality, −E), agree with (=) a symmetrical law which Fantappiè named syntropy; **mathematical properties of those phenomena which are determined by past causes (causality, +E), such as the physical and chemical processes**

$T_{25}$ : Category two of SAS

$T_{26}$ : Category three of SAS

**Module five**

mathematical properties of those phenomena which are attracted towards causes located in (eb) the future (retrocausality, −E), agree with (=) a symmetrical law which Fantappiè named syntropy

$G_{28}$ : Category one of mathematical properties of those phenomena which are attracted towards causes located(retrocausality, −E)

$G_{29}$ : Category two of SAS

$G_{30}$ : Category three of SAS

$T_{28}$ : Category one of symmetrical law which Fantappiè named syntropy

$T_{29}$ : Category two of SAS

$T_{30}$ : Category three of SAS

**Module six**

Fantappiè immediately recognized the coincidence between the mathematical properties of syntropy and (=) the properties of living systems (Fantappiè, 1942)

$G_{32}$ : Category one of mathematical properties of syntropy

$G_{33}$ : Category two of SAS

$G_{34}$ : Category three of SAS

$T_{32}$ : Category one of properties of living systems (Fantappiè, 1942)

$T_{33}$ : Category two of SAS

$T_{34}$ : Category three of SAS

**Module seven**

It is important to note that, as a consequence of the fact that the universe is expanding, macrocosm is governed by (e&eb) the law of entropy which forces time to flow from the past to (e&eb) the future (mechanical causation: causeÎeffect)

$G_{36}$ : Category one of **consequence of the fact that the universe is expanding, macrocosm;** law of entropy which forces time to flow from the past to (e&eb) the future (mechanical causation: causeÎeffect)

$G_{37}$ : Category two of SAS

$G_{38}$ : Category three of SAS

$T_{36}$ : Category one of law of entropy which forces time to flow from the past to (e&eb) the future (mechanical causation: causeÎeffect);**consequence of the fact that the universe is expanding, macrocosm**

$T_{37}$ : Category two of SAS

$T_{38}$ : Category three of SAS

**Module eight**

It is important to note that, as a consequence of the fact that the universe is expanding, macrocosm is governed by the **law of entropy which forces time to flow from the past** to (e&eb) the future (mechanical causation: causeÎeffect)

$G_{40}$ : Category one of **law of entropy which forces time to flow from the past;** future (mechanical causation: causeÎeffect)

$G_{41}$ : Category two of SAS

$G_{42}$ : Category three of SAS

$T_{40}$ : Category one of future (mechanical causation: causeÎeffect); **law of entropy which forces time to flow from the past**

$T_{41}$ : Category two of SAS

$T_{42}$ : Category three of SAS

**Module Nine**

Comparing the **infinite of the environment (entropy)** and (e&eb) finite of the living system (syntropy) produces results which tends to zero

$G_{44}$ : Category one of **infinite of the environment (entropy);** finite of the living system (syntropy) produces results which tends to zero

$G_{45}$ : Category two of SAS

$G_{46}$ : Category three of SAS

$T_{44}$ : Category one of finite of the living system (syntropy) produces results which tends to zero; **infinite of the environment (entropy)**

$T_{45}$ : Category two of SAS

$T_{46}$ : Category three of SAS

**The Coefficients:**

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}$ , $(b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}$,
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}$, $(b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}\ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$ ,
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)}$,
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)}$,
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)}$,
are Dissipation coefficients

**Module Numbered One**

**The differential system of this model is now (Module Numbered one)**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14},t)\right]G_{13} \quad 1$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14},t)\right]G_{14} \quad 2$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14},t)\right]G_{15} \quad 3$$

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G,t)\right]T_{13} \quad 4$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G,t)\right]T_{14} \quad 5$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G,t)\right]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14},t) =$ First augmentation factor

$-(b''_{13})^{(1)}(G,t) =$ First detritions factor

**Module Numbered Two**

**The differential system of this model is now ( Module numbered two)**

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17},t)\right]G_{16} \quad 7$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17},t)\right]G_{17} \quad 8$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17},t)\right]G_{18} \quad 9$$

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\left((G_{19}),t\right)\right]T_{16} \quad 10$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\left((G_{19}),t\right)\right]T_{17} \quad 11$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\left((G_{19}),t\right)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17},t) =$ First augmentation factor

$-(b''_{16})^{(2)}\left((G_{19}),t\right) =$ First detritions factor

**Module Numbered Three**

**The differential system of this model is now (Module numbered three)**

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21},t)\right]G_{20} \quad 13$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21},t)\right]G_{21} \quad 14$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21},t)\right]G_{22} \quad 15$$

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23},t)\right]T_{20} \quad 16$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23},t)\right]T_{21} \quad 17$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23},t)\right]T_{22} \quad 18$$

$+(a''_{20})^{(3)}(T_{21},t) =$ First augmentation factor

$-(b''_{20})^{(3)}(G_{23},t) =$ First detritions factor

**Module Numbered Four**

**The differential system of this model is now (Module numbered Four)**

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right]G_{24} \quad 19$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25},t)\right]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25},t)\right]G_{26} \quad 21$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27}),t\right)\right]T_{24} \quad 22$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27}),t\right)\right]T_{25} \quad 23$$

$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27}),t)\right]T_{26}$ 24

$+(a''_{24})^{(4)}(T_{25},t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}((G_{27}),t) =$ **First detritions factor**

**Module Numbered Five**:

**The differential system of this model is now (Module number five)**

$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29},t)\right]G_{28}$ 25

$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29},t)\right]G_{29}$ 26

$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29},t)\right]G_{30}$ 27

$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31}),t)\right]T_{28}$ 28

$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}((G_{31}),t)\right]T_{29}$ 29

$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31}),t)\right]T_{30}$ 30

$+(a''_{28})^{(5)}(T_{29},t) =$ **First augmentation factor**

$-(b''_{28})^{(5)}((G_{31}),t) =$ **First detritions factor**

**Module Numbered Six**

**The differential system of this model is now (Module numbered Six)**

$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33},t)\right]G_{32}$ 31

$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33},t)\right]G_{33}$ 32

$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33},t)\right]G_{34}$ 33

$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35}),t)\right]T_{32}$ 34

$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}((G_{35}),t)\right]T_{33}$ 35

$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35}),t)\right]T_{34}$ 36

$+(a''_{32})^{(6)}(T_{33},t) =$ **First augmentation factor**

**Module Numbered Seven:**

**The differential system of this model is now (Seventh Module)**

$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right]G_{36}$ 37

$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37},t)\right]G_{37}$ *38*

$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37},t)\right]G_{38}$ *39*

$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39}),t)\right]T_{36}$ *40*

$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}((G_{39}),t)\right]T_{37}$ *41*

$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39}),t)\right]T_{38}$ *42*

$+(a''_{36})^{(7)}(T_{37},t) =$ **First augmentation factor**

**Module Numbered Eight**

The differential system of this model is now

$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right]G_{40}$ *43*

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41},t)\right]G_{41} \quad 44$$
$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41},t)\right]G_{42} \quad 45$$
$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}\left((G_{43}),t\right)\right]T_{40} \quad 46$$
$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}\left((G_{43}),t\right)\right]T_{41} \quad 47$$
$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}\left((G_{43}),t\right)\right]T_{42} \quad 48$$

**Module Numbered Nine**

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$
$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$
$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$
$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}\left((G_{47}),t\right)\right]T_{44} \quad 52$$
$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}\left((G_{47}),t\right)\right]T_{45} \quad 53$$
$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}\left((G_{47}),t\right)\right]T_{46} \quad 54$$

$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}\left((G_{47}),t\right) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)} & \boxed{+(a''_{13})^{(1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)} & \boxed{+(a''_{14})^{(1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)} & \boxed{+(a''_{15})^{(1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15} \quad 57$$

Where $\boxed{(a''_{13})^{(1)}(T_{14},t)}$, $\boxed{(a''_{14})^{(1)}(T_{14},t)}$, $\boxed{(a''_{15})^{(1)}(T_{14},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{38})^{(7,7)}(T_{37},t)$ $+(a''_{37})^{(7,7)}(T_{37},t)$ $+(a''_{36})^{(7,7)}(T_{37},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{40})^{(8,8)}(T_{41},t)$ $+(a''_{41})^{(8,8)}(T_{41},t)$ $+(a''_{42})^{(8,8)}(T_{41},t)$ are eight augmentation coefficient for 1,2,3

$+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt}=(b_{13})^{(1)}T_{14}-\begin{bmatrix}(b'_{13})^{(1)}\,-(b''_{13})^{(1)}(G,t) & -(b''_{16})^{(2,2,)}(G_{19},t) & -(b''_{20})^{(3,3,)}(G_{23},t)\\ -(b''_{24})^{(4,4,4,4,)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,)}(G_{35},t)\\ -(b''_{36})^{(7,7,)}(G_{39},t) & -(b''_{40})^{(8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)\end{bmatrix}T_{13} \quad (58)$$

$$\frac{dT_{14}}{dt}=(b_{14})^{(1)}T_{13}-\begin{bmatrix}(b'_{14})^{(1)}\,-(b''_{14})^{(1)}(G,t) & -(b''_{17})^{(2,2,)}(G_{19},t) & -(b''_{21})^{(3,3,)}(G_{23},t)\\ -(b''_{25})^{(4,4,4,4,)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,)}(G_{35},t)\\ -(b''_{37})^{(7,7,)}(G_{39},t) & -(b''_{41})^{(8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)\end{bmatrix}T_{14} \quad (59)$$

$$\frac{dT_{15}}{dt}=(b_{15})^{(1)}T_{14}-\begin{bmatrix}(b'_{15})^{(1)}\,-(b''_{15})^{(1)}(G,t) & -(b''_{18})^{(2,2,)}(G_{19},t) & -(b''_{22})^{(3,3,)}(G_{23},t)\\ -(b''_{26})^{(4,4,4,4,)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,)}(G_{35},t)\\ -(b''_{38})^{(7,7,)}(G_{39},t) & -(b''_{42})^{(8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)\end{bmatrix}T_{15} \quad (60)$$

Where $-(b''_{13})^{(1)}(G,t)$, $-(b''_{14})^{(1)}(G,t)$, $-(b''_{15})^{(1)}(G,t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,)}(G_{19},t)$, $-(b''_{17})^{(2,2,)}(G_{19},t)$, $-(b''_{18})^{(2,2,)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{37})^{(7,7,)}(G_{39},t)$, $-(b''_{36})^{(7,7,)}(G_{39},t)$, $-(b''_{38})^{(7,7,)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8)}(G_{43},t)$ $-(b''_{41})^{(8,8)}(G_{43},t)$ $-(b''_{42})^{(8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt}=(a_{16})^{(2)}G_{17}-\begin{bmatrix}(a'_{16})^{(2)}\,+(a''_{16})^{(2)}(T_{17},t) & +(a''_{13})^{(1,1,)}(T_{14},t) & +(a''_{20})^{(3,3,3)}(T_{21},t)\\ +(a''_{24})^{(4,4,4,4,4)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,5)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,6)}(T_{33},t)\\ +(a''_{36})^{(7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9)}(T_{45},t)\end{bmatrix}G_{16} \quad (61)$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \begin{bmatrix} (a'_{17})^{(2)}\boxed{+(a''_{17})^{(2)}(T_{17},t)}\ \boxed{+(a''_{14})^{(1,1,)}(T_{14},t)}\ \boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)}\ \boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)}\ \boxed{+(a''_{45})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{17} \quad 62$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \begin{bmatrix} (a'_{18})^{(2)}\boxed{+(a''_{18})^{(2)}(T_{17},t)}\ \boxed{+(a''_{15})^{(1,1,)}(T_{14},t)}\ \boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)}\ \boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)}\ \boxed{+(a''_{46})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{18} \quad 63$$

Where $\boxed{+(a''_{16})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,)}(T_{14},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)}$ are eight augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{44})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9)}(T_{45},t)}$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \begin{bmatrix} (b'_{16})^{(2)}\boxed{-(b''_{16})^{(2)}(G_{19},t)}\ \boxed{-(b''_{13})^{(1,1,)}(G,t)}\ \boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)}\ \boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)}\ \boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)}\ \boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)}\ \boxed{-(b''_{44})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{16} \quad 64$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)}\boxed{-(b''_{17})^{(2)}(G_{19},t)}\ \boxed{-(b''_{14})^{(1,1,)}(G,t)}\ \boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)}\ \boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)}\ \boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)}\ \boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)}\ \boxed{-(b''_{45})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{17} \quad 65$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)}\boxed{-(b''_{18})^{(2)}(G_{19},t)}\ \boxed{-(b''_{15})^{(1,1,)}(G,t)}\ \boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)}\ \boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)}\ \boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)}\ \boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)}\ \boxed{-(b''_{46})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{18} \quad 66$$

where $\boxed{-(b''_{16})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2)}(G_{19},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,)}(G,t)}$ are second detrition coefficients for category

1,2 and 3

$\boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{44})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[\begin{array}{c} (a'_{20})^{(3)}\boxed{+(a''_{20})^{(3)}(T_{21},t)}\ \boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}\ \boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{44})^{(9,9,9)}(T_{45},t)} \end{array}\right] G_{20} \quad (67)$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[\begin{array}{c} (a'_{21})^{(3)}\boxed{+(a''_{21})^{(3)}(T_{21},t)}\ \boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}\ \boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)} \end{array}\right] G_{21} \quad (68)$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[\begin{array}{c} (a'_{22})^{(3)}\boxed{+(a''_{22})^{(3)}(T_{21},t)}\ \boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}\ \boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)} \end{array}\right] G_{22} \quad (69)$$

$\boxed{+(a''_{20})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3)}(T_{21},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}$ are second augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)}$ are third augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)}$ are seventh augmentation

coefficients for category 1, 2 and 3

$+(a''_{40})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8)}(T_{41},t)$ are eight augmentation coefficients for category 1, 2 and 3

$+(a''_{44})^{(9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b'_{20})^{(3)} & -(b''_{20})^{(3)}(G_{23},t) & -(b''_{16})^{(2,2,2)}(G_{19},t) & -(b''_{13})^{(1,1,1,)}(G,t) \\ -(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t) & \\ -(b''_{36})^{(7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9)}(G_{47},t) & \end{bmatrix} T_{20} \quad (70)$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b'_{21})^{(3)} & -(b''_{21})^{(3)}(G_{23},t) & -(b''_{17})^{(2,2,2)}(G_{19},t) & -(b''_{14})^{(1,1,1,)}(G,t) \\ -(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t) & \\ -(b''_{37})^{(7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9)}(G_{47},t) & \end{bmatrix} T_{21} \quad (71)$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)} & -(b''_{22})^{(3)}(G_{23},t) & -(b''_{18})^{(2,2,2)}(G_{19},t) & -(b''_{15})^{(1,1,1,)}(G,t) \\ -(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t) & \\ -(b''_{38})^{(7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9)}(G_{47},t) & \end{bmatrix} T_{22} \quad (72)$$

$-(b''_{20})^{(3)}(G_{23},t)$, $-(b''_{21})^{(3)}(G_{23},t)$, $-(b''_{22})^{(3)}(G_{23},t)$ *are first detrition coefficients for category 1, 2 and 3*

$-(b''_{16})^{(2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2)}(G_{19},t)$ *are second detrition coefficients for category 1, 2 and 3*

$-(b''_{13})^{(1,1,1,)}(G,t)$, $-(b''_{14})^{(1,1,1,)}(G,t)$, $-(b''_{15})^{(1,1,1,)}(G,t)$ *are third detrition coefficients for category 1,2 and 3*

$-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)$ *are fourth detrition coefficients for category 1, 2 and 3*

$-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)$ *are fifth detrition coefficients for category 1, 2 and 3*

$-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)$ *are sixth detrition coefficients for category 1, 2 and 3*

$-(b''_{36})^{(7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$-(b''_{46})^{(9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \begin{bmatrix} (a'_{24})^{(4)} & +(a''_{24})^{(4)}(T_{25},t) & +(a''_{28})^{(5,5,)}(T_{29},t) & +(a''_{32})^{(6,6,)}(T_{33},t) \\ +(a''_{13})^{(1,1,1,1)}(T_{14},t) & +(a''_{16})^{(2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3)}(T_{21},t) & \\ +(a''_{36})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9)}(T_{45},t) & \end{bmatrix} G_{24} \quad (73)$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \begin{bmatrix} (a'_{25})^{(4)} & +(a''_{25})^{(4)}(T_{25},t) & +(a''_{29})^{(5,5,)}(T_{29},t) & +(a''_{33})^{(6,6)}(T_{33},t) \\ & +(a''_{14})^{(1,1,1,1)}(T_{14},t) & +(a''_{17})^{(2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3)}(T_{21},t) \\ & +(a''_{37})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9)}(T_{45},t) \end{bmatrix} G_{25} \quad 74$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \begin{bmatrix} (a'_{26})^{(4)} & +(a''_{26})^{(4)}(T_{25},t) & +(a''_{30})^{(5,5,)}(T_{29},t) & +(a''_{34})^{(6,6,)}(T_{33},t) \\ & +(a''_{15})^{(1,1,1,1)}(T_{14},t) & +(a''_{18})^{(2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3)}(T_{21},t) \\ & +(a''_{38})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9)}(T_{45},t) \end{bmatrix} G_{26} \quad 75$$

$(a''_{24})^{(4)}(T_{25},t)$, $(a''_{25})^{(4)}(T_{25},t)$, $(a''_{26})^{(4)}(T_{25},t)$ *are first augmentation coefficients category* 1, 2 3

$+(a''_{28})^{(5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,)}(T_{29},t)$ *are second augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{32})^{(6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,)}(T_{33},t)$ *are third augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{13})^{(1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1)}(T_{14},t)$
*are fourth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{16})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2)}(T_{17},t)$
*are fifth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{20})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3)}(T_{21},t)$
*are sixth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)$
*are seventh augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)$
*are eighth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{46})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9)}(T_{45},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \begin{bmatrix} (b'_{24})^{(4)} & -(b''_{24})^{(4)}(G_{27},t) & -(b''_{28})^{(5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,)}(G_{35},t) \\ & -(b''_{13})^{(1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{24} \quad 76$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \begin{bmatrix} (b'_{25})^{(4)} & -(b''_{25})^{(4)}(G_{27},t) & -(b''_{29})^{(5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{25} \quad 77$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)} & -(b''_{26})^{(4)}(G_{27},t) & -(b''_{30})^{(5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1)}(G,t) & -(b''_{18})^{(2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{26} \quad 78$$

*Where* $-(b''_{24})^{(4)}(G_{27},t)$, $-(b''_{25})^{(4)}(G_{27},t)$, $-(b''_{26})^{(4)}(G_{27},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{28})^{(5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,)}(G_{31},t)$ *are second detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{32})^{(6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,)}(G_{35},t)}$ *are third detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,1)}(G,t)}$
*are fourth detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2)}(G_{19},t)}$
*are fifth detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3)}(G_{23},t)}$
*are sixth detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)}$
*are seventh detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)}$
*are eighth detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{46})^{(9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \begin{bmatrix} (a'_{28})^{(5)} & \boxed{+(a''_{28})^{(5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{28} \quad 79$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \begin{bmatrix} (a'_{29})^{(5)} & \boxed{+(a''_{29})^{(5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{29} \quad 80$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \begin{bmatrix} (a'_{30})^{(5)} & \boxed{+(a''_{30})^{(5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{30} \quad 81$$

*Where* $\boxed{+(a''_{28})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5)}(T_{29},t)}$ *are first augmentation coefficients for category* 1, 2 *and* 3

*And* $\boxed{+(a''_{24})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,)}(T_{25},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)}$ are fourth augmentation coefficients for category 1,2, and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)}$ are fifth augmentation coefficients for category 1,2,and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)}$ are sixth augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)}$ are seventh augmentation coefficients for category 1,2, 3

$+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)$ , $+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)$ , $+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)$ are eighth augmentation coefficients for category 1,2, 3

$+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)$ , $+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)$ , $+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \begin{bmatrix} (b'_{28})^{(5)} & -(b''_{28})^{(5)}(G_{31},t) & -(b''_{24})^{(4,4,)}(G_{27},t) & -(b''_{32})^{(6,6,6)}(G_{35},t) \\ & -(b''_{13})^{(1,1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{28} \quad 82$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b'_{29})^{(5)} & -(b''_{29})^{(5)}(G_{31},t) & -(b''_{25})^{(4,4,)}(G_{27},t) & -(b''_{33})^{(6,6,6)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{29} \quad 83$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b'_{30})^{(5)} & -(b''_{30})^{(5)}(G_{31},t) & -(b''_{26})^{(4,4,)}(G_{27},t) & -(b''_{34})^{(6,6,6)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1,1,)}(G,t) & -(b''_{18})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{30} \quad 84$$

*where* $-(b''_{28})^{(5)}(G_{31},t)$ , $-(b''_{29})^{(5)}(G_{31},t)$ , $-(b''_{30})^{(5)}(G_{31},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{24})^{(4,4,)}(G_{27},t)$ , $-(b''_{25})^{(4,4,)}(G_{27},t)$ , $-(b''_{26})^{(4,4,)}(G_{27},t)$ *are second detrition coefficients for category* 1,2 *and* 3

$-(b''_{32})^{(6,6,6)}(G_{35},t)$ , $-(b''_{33})^{(6,6,6)}(G_{35},t)$ , $-(b''_{34})^{(6,6,6)}(G_{35},t)$ *are third detrition coefficients for category* 1,2 *and* 3

$-(b''_{13})^{(1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1)}(G,t)$ , $-(b''_{15})^{(1,1,1,1,1,)}(G,t)$ are fourth detrition coefficients for category 1,2, and 3

$-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1,2, and 3

$-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1,2, and 3

$-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)$ , $-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)$ , $-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2, and 3

$-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)$ , $-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)$ , $-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1,2, and 3

$-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)$ , $-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)$ , $-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} \quad 85$$

$$- \begin{bmatrix} (a'_{32})^{(6)} & +(a''_{32})^{(6)}(T_{33},t) & +(a''_{28})^{(5,5,5)}(T_{29},t) & +(a''_{24})^{(4,4,4,)}(T_{25},t) \\ & +(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t) & +(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t) \\ & +(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{32}$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \begin{bmatrix} (a'_{33})^{(6)} & +(a''_{33})^{(6)}(T_{33},t) & +(a''_{29})^{(5,5,5)}(T_{29},t) & +(a''_{25})^{(4,4,4,)}(T_{25},t) \\ +(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t) & & +(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t) \\ +(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t) & & +(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{33} \quad 86$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \begin{bmatrix} (a'_{34})^{(6)} & +(a''_{34})^{(6)}(T_{33},t) & +(a''_{30})^{(5,5,5)}(T_{29},t) & +(a''_{26})^{(4,4,4,)}(T_{25},t) \\ +(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t) & & +(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t) \\ +(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t) & & +(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{34} \quad 87$$

$+(a''_{32})^{(6)}(T_{33},t)$, $+(a''_{33})^{(6)}(T_{33},t)$, $+(a''_{34})^{(6)}(T_{33},t)$ *are first augmentation coefficients for category* $1, 2$ *and* $3$

$+(a''_{28})^{(5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5)}(T_{29},t)$ *are second augmentation coefficients for category* $1, 2$ *and* $3$

$+(a''_{24})^{(4,4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,)}(T_{25},t)$ *are third augmentation coefficients for category* $1, 2$ *and* $3$

$+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)$ - are fourth augmentation coefficients

$+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)$ - fifth augmentation coefficients

$+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)$ sixth augmentation coefficients

$+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)$ seventh augmentation coefficients

$+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)$ Eighth augmentation coefficients

$+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \begin{bmatrix} (b'_{32})^{(6)} & -(b''_{32})^{(6)}(G_{35},t) & -(b''_{28})^{(5,5,5)}(G_{31},t) & -(b''_{24})^{(4,4,4,)}(G_{27},t) \\ -(b''_{13})^{(1,1,1,1,1,1)}(G,t) & & -(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t) & & -(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{32} \quad 88$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \begin{bmatrix} (b'_{33})^{(6)} & -(b''_{33})^{(6)}(G_{35},t) & -(b''_{29})^{(5,5,5)}(G_{31},t) & -(b''_{25})^{(4,4,4,)}(G_{27},t) \\ -(b''_{14})^{(1,1,1,1,1,1)}(G,t) & & -(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t) & & -(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{33} \quad 89$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)} & -(b''_{34})^{(6)}(G_{35},t) & -(b''_{30})^{(5,5,5)}(G_{31},t) & -(b''_{26})^{(4,4,4,)}(G_{27},t) \\ -(b''_{15})^{(1,1,1,1,1,1)}(G,t) & & -(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t) & & -(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{34} \quad 90$$

$-(b''_{32})^{(6)}(G_{35},t)$, $-(b''_{33})^{(6)}(G_{35},t)$, $-(b''_{34})^{(6)}(G_{35},t)$ *are first detrition coefficients for category* $1, 2$ *and* $3$

$\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}$ *are second detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)}$ *are third detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}$ are fourth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \qquad 91$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a'_{36})^{(7)}\boxed{+(a''_{36})^{(7)}(T_{37},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \qquad 92$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a'_{37})^{(7)}\boxed{+(a''_{37})^{(7)}(T_{37},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{38}}{dt} \qquad 93$$

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a'_{38})^{(7)}\boxed{+(a''_{38})^{(7)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{(a''_{36})^{(7)}(T_{37},t)}$, $\boxed{(a''_{37})^{(7)}(T_{37},t)}$, $\boxed{(a''_{38})^{(7)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)$ are seventh augmentation coefficient for category 1, 2 and 3

$+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)$ are eighth augmentation coefficient for 1,2,3

$+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{36}}{dt} =$$



$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b'_{36})^{(7)}\left[-(b''_{36})^{(7)}(G_{39},t)\right] & \left[-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)\right] & \left[-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{13}$$

$$\frac{dT_{37}}{dt} =$$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b'_{37})^{(7)}\left[-(b''_{37})^{(7)}(G_{39},t)\right] & \left[-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)\right] & \left[-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{14}$$

$$\frac{dT_{38}}{dt} =$$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b'_{38})^{(7)}\left[-(b''_{38})^{(7)}(G_{39},t)\right] & \left[-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)\right] & \left[-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{15}$$

Where $-(b''_{36})^{(7)}(G_{39},t)$, $-(b''_{37})^{(7)}(G_{39},t)$, $-(b''_{38})^{(7)}(G_{39},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)$, $-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)$

are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3



$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \begin{bmatrix} (a'_{40})^{(8)}\boxed{+(a''_{40})^{(8)}(T_{41},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \begin{bmatrix} (a'_{41})^{(8)}\boxed{+(a''_{41})^{(8)}(T_{41},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \begin{bmatrix} (a'_{42})^{(8)}\boxed{+(a''_{42})^{(8)}(T_{41},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a''_{40})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} =$$

$$(b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)}\left[-(b''_{40})^{(8)}(G_{43},t)\right] & \left[-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} =$$

$$(b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)}\left[-(b''_{41})^{(8)}(G_{43},t)\right] & \left[-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} =$$

$$(b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)}\left[-(b''_{42})^{(8)}(G_{43},t)\right] & \left[-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{15}$$

Where $-(b''_{36})^{(7)}(G_{39},t)$, $-(b''_{37})^{(7)}(G_{39},t)$, $-(b''_{38})^{(7)}(G_{39},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)$, $-(b''_{38})^{(7,7,)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)$ are eighth detrition coefficients for category 1, 2 and 3

$-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3



$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \begin{bmatrix} (a'_{44})^{(9)}\boxed{+(a''_{44})^{(9)}(T_{45},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \begin{bmatrix} (a'_{45})^{(9)}\boxed{+(a''_{45})^{(9)}(T_{45},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \begin{bmatrix} (a'_{46})^{(9)}\boxed{+(a''_{46})^{(9)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a''_{44})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ are Seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} -$$

$$\begin{bmatrix} (b'_{44})^{(9)}\boxed{-(b''_{44})^{(9)}(G_{47},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b'_{45})^{(9)}\boxed{-(b''_{45})^{(9)}(G_{47},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b'_{46})^{(9)}\boxed{-(b''_{46})^{(9)}(G_{47},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{44})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9)}(G_{47},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a'_i)^{(1)}, (a''_i)^{(1)}, (b_i)^{(1)}, (b'_i)^{(1)}, (b''_i)^{(1)} > 0, \quad i,j = 13,14,15$$ *97*

The functions $(a''_i)^{(1)}, (b''_i)^{(1)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$(a_i'')^{(1)}(T_{14},t) \leq (p_i)^{(1)} \leq (\hat{A}_{13})^{(1)}$

$(b_i'')^{(1)}(G,t) \leq (r_i)^{(1)} \leq (b_i')^{(1)} \leq (\hat{B}_{13})^{(1)}$

$lim_{T_2\to\infty}(a_i'')^{(1)}(T_{14},t) = (p_i)^{(1)}$ 98

$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = (r_i)^{(1)}$

**Definition of** $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$ :

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$|(a_i'')^{(1)}(T_{14}',t) - (a_i'')^{(1)}(T_{14},t)| \leq (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$

$|(b_i'')^{(1)}(G',t) - (b_i'')^{(1)}(G,t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}',t)$ and $(a_i'')^{(1)}(T_{14},t)$ . $(T_{14}',t)$ and $(T_{14},t)$ are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14},t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$ : 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}$ , $\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$

**Definition of** $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$ : 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$

$\frac{1}{(\hat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$

Where we suppose

$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i,j = 16,17,18$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17},t) \le (p_i)^{(2)} \le \left(\hat{A}_{16}\right)^{(2)} \quad 102$$

$$(b_i'')^{(2)}(G_{19},t) \le (r_i)^{(2)} \le (b_i')^{(2)} \le (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2\to\infty}(a_i'')^{(2)}(T_{17},t) = (p_i)^{(2)} \quad 104$$

$$\lim_{G\to\infty}(b_i'')^{(2)}\left((G_{19}),t\right) = (r_i)^{(2)} \quad 105$$

**Definition of** $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$ : 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}',t) - (a_i'')^{(2)}(T_{17},t)| \le (\hat{k}_{16})^{(2)}|T_{17} - T_{17}'|e^{-(\hat{M}_{16})^{(2)}t} \quad 107$$

$$|(b_i'')^{(2)}((G_{19})',t) - (b_i'')^{(2)}\left((G_{19}),t\right)| < (\hat{k}_{16})^{(2)}||(G_{19}) - (G_{19})'||e^{-(\hat{M}_{16})^{(2)}t} \quad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}',t)$ and $(a_i'')^{(2)}(T_{17},t)$ . $(T_{17}',t)$ and $(T_{17},t)$ are points belonging to the interval $\left[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}\right]$ . It is to be noted that $(a_i'')^{(2)}(T_{17},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17},t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$ :

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants 109

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$$

**Definition of** $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$ :

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1 \quad 110$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(b_i)^{(2)} + (b_i')^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1 \quad 111$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i,j = 20,21,22 \quad 112$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(3)}$, $(\mathrm{r_i})^{(3)}$:

$$(a_i'')^{(3)}(T_{21},t) \le (p_i)^{(3)} \le (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23},t) \le (r_i)^{(3)} \le (b_i')^{(3)} \le (\hat{B}_{20})^{(3)}$$

$$lim_{T_2\to\infty}(a_i'')^{(3)}(T_{21},t) = (p_i)^{(3)} \qquad 113$$

$$\lim_{G\to\infty}(b_i'')^{(3)}(G_{23},t) = (r_i)^{(3)}$$

**Definition of** $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$ :

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: 114

$$|(a_i'')^{(3)}(T_{21}',t) - (a_i'')^{(3)}(T_{21},t)| \le (\hat{k}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\hat{M}_{20})^{(3)}t}$$

$$|(b_i'')^{(3)}(G_{23}',t) - (b_i'')^{(3)}(G_{23},t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}'||e^{-(\hat{M}_{20})^{(3)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}',t)$ and$(a_i'')^{(3)}(T_{21},t)$ . $(T_{21}',t)$ And $(T_{21},t)$ are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$ : 115

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$ and the constants $(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}, i = 20,21,22,$ satisfy the inequalities 116

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(a_i)^{(3)} + (a_i')^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(b_i)^{(3)} + (b_i')^{(3)} + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i,j = 24,25,26 \qquad 117$$

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(4)}$, $(r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25},t) \le (p_i)^{(4)} \le (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}\big((G_{27}),t\big) \le (r_i)^{(4)} \le (b_i')^{(4)} \le (\hat{B}_{24})^{(4)}$$

$$lim_{T_2\to\infty}(a_i'')^{(4)}(T_{25},t) = (p_i)^{(4)} \quad (118)$$

$$\lim_{G\to\infty}(b_i'')^{(4)}\big((G_{27}),t\big) = (r_i)^{(4)}$$

**Definition of** $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$ :

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24,25,26}$

They satisfy Lipschitz condition: (119)

$$|(a_i'')^{(4)}(T_{25}',t) - (a_i'')^{(4)}(T_{25},t)| \le (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})',t) - (b_i'')^{(4)}\big((G_{27}),t\big)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}',t)$ and$(a_i'')^{(4)}(T_{25},t)$ . $(T_{25}',t)$ and $(T_{25},t)$ are points belonging to the interval $\big[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}\big]$. It is to be noted that $(a_i'')^{(4)}(T_{25},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25},t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$ : (120)

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$, are positive constants

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}, \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$$

**Definition of** $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$ : (121)

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and\ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}, i = 24,25,26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(a_i)^{(4)} + (a_i')^{(4)} + (\hat{A}_{24})^{(4)} + (\hat{P}_{24})^{(4)}(\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(b_i)^{(4)} + (b_i')^{(4)} + (\hat{B}_{24})^{(4)} + (\hat{Q}_{24})^{(4)}(\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i,j = 28,29,30 \quad (122)$$

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(5)}$, $(r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29},t) \le (p_i)^{(5)} \le (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}\big((G_{31}),t\big) \le (r_i)^{(5)} \le (b_i')^{(5)} \le (\hat{B}_{28})^{(5)}$$

$$lim_{T_2\to\infty}(a_i'')^{(5)}(T_{29},t) = (p_i)^{(5)} \qquad 123$$

$$\lim_{G\to\infty}(b_i'')^{(5)}(G_{31},t) = (r_i)^{(5)}$$

**Definition of** $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$ :

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}',t) - (a_i'')^{(5)}(T_{29},t)| \le (\hat{k}_{28})^{(5)}|T_{29} - T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})',t) - (b_i'')^{(5)}\big((G_{31}),t\big)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}',t)$ and$(a_i'')^{(5)}(T_{29},t)$ . $(T_{29}',t)$ and $(T_{29},t)$ are points belonging to the interval $\big[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\big]$. It is to be noted that $(a_i'')^{(5)}(T_{29},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29},t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$ : 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}, \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

**Definition of** $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$ : 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and\ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(a_i)^{(5)} + (a_i')^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)}(\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(b_i)^{(5)} + (b_i')^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)}(\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i,j = 32,33,34 \qquad 127$$

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(6)}$, $(r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33},t) \le (p_i)^{(6)} \le (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}),t) \le (r_i)^{(6)} \le (b_i')^{(6)} \le (\hat{B}_{32})^{(6)}$$

$$lim_{T_2\to\infty}(a_i'')^{(6)}(T_{33},t) = (p_i)^{(6)} \quad (128)$$

$$\lim_{G\to\infty}(b_i'')^{(6)}\left((G_{35}),t\right) = (r_i)^{(6)}$$

**Definition of** $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$ :

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}',t) - (a_i'')^{(6)}(T_{33},t)| \le (\hat{k}_{32})^{(6)}|T_{33} - T_{33}'|e^{-(\hat{M}_{32})^{(6)}t}$$

$$|(b_i'')^{(6)}((G_{35})',t) - (b_i'')^{(6)}\left((G_{35}),t\right)| < (\hat{k}_{32})^{(6)}||(G_{35}) - (G_{35})'||e^{-(\hat{M}_{32})^{(6)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}',t)$ and$(a_i'')^{(6)}(T_{33},t)$ . $(T_{33}',t)$ and $(T_{33},t)$ are points belonging to the interval $\left[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}\right]$. It is to be noted that $(a_i'')^{(6)}(T_{33},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33},t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$ : (129)

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$, are positive constants

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}} \ , \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$$

**Definition of** $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$ : (130)

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and\ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32,33,34,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(a_i)^{(6)} + (a_i')^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(b_i)^{(6)} + (b_i')^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

(WWWWWWWW) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i,j = 36,37,38$ (131)

(XXXXXXXX) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(7)}$, $(r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37},t) \le (p_i)^{(7)} \le (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39},t) \le (r_i)^{(7)} \le (b_i')^{(7)} \le (\hat{B}_{36})^{(7)} \quad (132)$$

*(YYYYYYYY)* $\lim_{T_2\to\infty} (a_i'')^{(7)}(T_{37},t) = (p_i)^{(7)}$

*(ZZZZZZZZ)*

$\lim_{G\to\infty}(b_i'')^{(7)}\big((G_{39}),t\big) = (r_i)^{(7)}$

**Definition of** $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$ :

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: (133)

$$|(a_i'')^{(7)}(T_{37}',t) - (a_i'')^{(7)}(T_{37},t)| \le (\hat{k}_{36})^{(7)}|T_{37} - T_{37}'|e^{-(\hat{M}_{36})^{(7)}t}$$

$$|(b_i'')^{(7)}((G_{39})',t) - (b_i'')^{(7)}\big((G_{39}),t\big)| < (\hat{k}_{36})^{(7)}||(G_{39}) - (G_{39})'||e^{-(\hat{M}_{36})^{(7)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}',t)$ and$(a_i'')^{(7)}(T_{37},t)$ . $(T_{37}',t)$ and $(T_{37},t)$ are points belonging to the interval $\big[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}\big]$. It is to be noted that $(a_i'')^{(7)}(T_{37},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37},t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$ : (134)

(AAAAAAAAA) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$, are positive constants

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$$

**Definition of** $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$ : (135)

(BBBBBBBBB) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)}(\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(b_i)^{(7)} + (b_i')^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)}(\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40,41,42 \tag{136}$$

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

**Definition of** $(p_i)^{(8)}$, $(r_i)^{(8)}$: (137)

$$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)} \tag{138}$$

$$(b_i'')^{(8)}((G_{43}), t) \leq (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)} \tag{139}$$

$$\lim_{T_2 \to \infty} (a_i'')^{(8)}(T_{41}, t) = (p_i)^{(8)} \tag{140}$$

$$\lim_{G \to \infty} (b_i'')^{(8)}\left((G_{43}), t\right) = (r_i)^{(8)} \tag{141}$$

**Definition of** $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$ :

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)} |T_{41} - T_{41}'| e^{-(\hat{M}_{40})^{(8)} t} \tag{142}$$

$$|(b_i'')^{(8)}((G_{43})', t) - (b_i'')^{(8)}\left((G_{43}), t\right)| < (\hat{k}_{40})^{(8)} ||(G_{43}) - (G_{43})'|| e^{-(\hat{M}_{40})^{(8)} t} \tag{143}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}', t)$ and $(a_i'')^{(8)}(T_{41}, t)$ . $(T_{41}', t)$ and $(T_{41}, t)$ are points belonging to the interval $\left[(\hat{k}_{40})^{(8)}, (\hat{M}_{40})^{(8)}\right]$. It is to be noted that $(a_i'')^{(8)}(T_{41}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{40})^{(8)} = 1$ then the function $(a_i'')^{(8)}(T_{41}, t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$ :

$(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$, are positive constants

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}, \frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1 \tag{144}$$

**Definition of** $(\hat{P}_{40})^{(8)}, (\hat{Q}_{40})^{(8)}$ :

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}, (\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)}, (a_i')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (p_i)^{(8)}$, $(r_i)^{(8)}, i = 40,41,42,$
Satisfy the inequalities

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(a_i)^{(8)} + (a_i')^{(8)} + (\hat{A}_{40})^{(8)} + (\hat{P}_{40})^{(8)} (\hat{k}_{40})^{(8)}] < 1 \tag{145}$$

$$\frac{1}{(\widehat{M}_{40})^{(8)}}[\ (b_i)^{(8)} + (b_i')^{(8)} + \ (\hat{B}_{40})^{(8)} + \ (\hat{Q}_{40})^{(8)}\ (\hat{k}_{40})^{(8)}] < 1 \qquad 146$$

Where we suppose

$$(a_i)^{(9)}, (a_i')^{(9)}, (a_i'')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (b_i'')^{(9)} > 0, \quad i,j = 44,45,46 \qquad 146\,A$$

The functions $(a_i'')^{(9)}, (b_i'')^{(9)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(9)}$, $(r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45}, t) \leq (p_i)^{(9)} \leq (\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47}, t) \leq \ (r_i)^{(9)} \leq (b_i')^{(9)} \leq (\hat{B}_{44})^{(9)}$$

$$\lim_{T_2 \to \infty} (a_i'')^{(9)}\,(T_{45}, t) = (p_i)^{(9)}$$

$$\lim_{G \to \infty} (b_i'')^{(9)}\,(G_{47}, t) = \ (r_i)^{(9)}$$

**Definition of** $(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}$ :

Where $\boxed{(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}, (p_i)^{(9)}, \ (r_i)^{(9)}}$ are positive constants and $\boxed{i = 44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}', t) - (a_i'')^{(9)}(T_{45}, t)| \leq (\hat{k}_{44})^{(9)} |T_{45} - \ T_{45}'| e^{-(\widehat{M}_{44})^{(9)} t}$$

$$|(b_i'')^{(9)}((G_{47})', t) - (b_i'')^{(9)}\big((G_{47}), t\big)| < (\hat{k}_{44})^{(9)} ||(G_{47}) - (G_{47})'|| e^{-(\widehat{M}_{44})^{(9)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}', t)$ and$(a_i'')^{(9)}(T_{45}, t)$ . $(T_{45}', t)$ and $(T_{45}, t)$ are points belonging to the interval $\left[(\hat{k}_{44})^{(9)}, (\widehat{M}_{44})^{(9)}\right]$. It is to be noted that $(a_i'')^{(9)}(T_{45}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{44})^{(9)} = 1$ then the function $(a_i'')^{(9)}(T_{45}, t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\widehat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$ :

$(\widehat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$, are positive constants

$$\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}} \ , \frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}} < 1$$

**Definition of** $(\hat{P}_{44})^{(9)}, (\hat{Q}_{44})^{(9)}$ :

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\widehat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}, (\hat{A}_{44})^{(9)} and\ (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)}, (a_i')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (p_i)^{(9)}, \ (r_i)^{(9)}, i = 44,45,46,$
satisfy the inequalities

$$\frac{1}{(\widehat{M}_{44})^{(9)}}[(a_i)^{(9)} + (a_i')^{(9)} + (\hat{A}_{44})^{(9)} + (\hat{P}_{44})^{(9)}(\hat{k}_{44})^{(9)}] < 1$$

$$\frac{1}{(\widehat{M}_{44})^{(9)}}[(b_i)^{(9)} + (b_i')^{(9)} + (\hat{B}_{44})^{(9)} + (\hat{Q}_{44})^{(9)}(\hat{k}_{44})^{(9)}] < 1$$

**Theorem 1:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0), T_i(0)$ :

$$G_i(t) \le (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 2 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

**Definition of** $G_i(0), T_i(0)$

$$G_i(t) \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} \quad , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} \quad , \quad T_i(0) = T_i^0 > 0$$

**Theorem 3 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t) \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t} \quad , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t} \quad , \quad T_i(0) = T_i^0 > 0$$

**Theorem 4 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

**Definition of** $G_i(0), T_i(0)$ :

$$G_i(t) \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 5 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

**Definition of** $G_i(0), T_i(0)$ :

$$G_i(t) \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

**Theorem 6 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

**Definition of** $G_i(0), T_i(0)$ :

$$G_i(t) \le (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$T_i(t) \leq (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Theorem 7:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

**Definition of** $G_i(0), T_i(0)$ :

$G_i(t) \leq (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Theorem 8:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

**Definition of** $G_i(0), T_i(0)$ :

$G_i(t) \leq (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Theorem 9:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

**Definition of** $G_i(0), T_i(0)$ :

$G_i(t) \leq (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Proof:** Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$G_i(0) = G_i^0$ , $T_i(0) = T_i^0$ , $G_i^0 \leq (\hat{P}_{13})^{(1)}$ , $T_i^0 \leq (\hat{Q}_{13})^{(1)}$, 155

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t}$ 156

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t}$ 157

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[ (a_{13})^{(1)} G_{14}(s_{(13)}) - \left( (a'_{13})^{(1)} + a''_{13})^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \right) G_{13}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[ (a_{14})^{(1)} G_{13}(s_{(13)}) - \left( (a'_{14})^{(1)} + (a''_{14})^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \right) G_{14}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[ (a_{15})^{(1)} G_{14}(s_{(13)}) - \left( (a'_{15})^{(1)} + (a''_{15})^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \right) G_{15}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[ (b_{13})^{(1)} T_{14}(s_{(13)}) - \left( (b'_{13})^{(1)} - (b''_{13})^{(1)} (G(s_{(13)}), s_{(13)}) \right) T_{13}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^{0} + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)}\left(G(s_{(13)}), s_{(13)}\right)\right) T_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{\mathrm{T}}_{15}(\mathrm{t}) = \mathrm{T}_{15}^{0} + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)}\left(G(s_{(13)}), s_{(13)}\right)\right) T_{15}(s_{(13)})\right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{16})^{(2)}, T_i^0 \le (\hat{Q}_{16})^{(2)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$$

By 160

$$\bar{G}_{16}(t) = G_{16}^{0} + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''_{16})^{(2)}\left(T_{17}(s_{(16)}), s_{(16)}\right)\right) G_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^{0} + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)}\left(T_{17}(s_{(16)}), s_{(17)}\right)\right) G_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^{0} + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)}\left(T_{17}(s_{(16)}), s_{(16)}\right)\right) G_{18}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^{0} + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)}\left(G_{19}(s_{(16)}), s_{(16)}\right)\right) T_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^{0} + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)}\left(G_{19}(s_{(16)}), s_{(16)}\right)\right) T_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^{0} + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)}\left(G_{19}(s_{(16)}), s_{(16)}\right)\right) T_{18}(s_{(16)})\right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{20})^{(3)}, T_i^0 \le (\hat{Q}_{20})^{(3)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$$

By 161

$$\bar{G}_{20}(t) = G_{20}^{0} + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''_{20})^{(3)}\left(T_{21}(s_{(20)}), s_{(20)}\right)\right) G_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{22}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^0 + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^0 + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{\mathrm{T}}_{22}(\mathrm{t}) = \mathrm{T}_{22}^0 + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{22}(s_{(20)})\right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{24})^{(4)}, T_i^0 \le (\hat{Q}_{24})^{(4)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

By



$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{26}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{\mathrm{T}}_{26}(\mathrm{t}) = \mathrm{T}_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{26}(s_{(24)})\right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{28})^{(5)}, T_i^0 \le (\hat{Q}_{28})^{(5)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

By 163

$$\bar{G}_{28}(t) = G_{28}^{0} + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^{0} + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^{0} + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{30}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^{0} + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^{0} + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{\mathrm{T}}_{30}(\mathrm{t}) = \mathrm{T}_{30}^{0} + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{30}(s_{(28)})\right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{32})^{(6)}, T_i^0 \le (\hat{Q}_{32})^{(6)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$$

By 164

$$\bar{G}_{32}(t) = G_{32}^{0} + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a'_{32})^{(6)} + a''_{32})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^{0} + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^{0} + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{34}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^{0} + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^{0} + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{\mathrm{T}}_{34}(\mathrm{t}) = \mathrm{T}_{34}^{0} + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{34}(s_{(32)})\right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$ , $T_i(0) = T_i^0$ , $G_i^0 \leq (\hat{P}_{36})^{(7)}$ , $T_i^0 \leq (\hat{Q}_{36})^{(7)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[ (a_{36})^{(7)} G_{37}(s_{(36)}) - \left( (a'_{36})^{(7)} + a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[ (a_{37})^{(7)} G_{36}(s_{(36)}) - \left( (a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[ (a_{38})^{(7)} G_{37}(s_{(36)}) - \left( (a'_{38})^{(7)} + (a''_{38})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{38}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[ (b_{36})^{(7)} T_{37}(s_{(36)}) - \left( (b'_{36})^{(7)} - (b''_{36})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[ (b_{37})^{(7)} T_{36}(s_{(36)}) - \left( (b'_{37})^{(7)} - (b''_{37})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{38}(t) = T_{38}^0 + \int_0^t \left[ (b_{38})^{(7)} T_{37}(s_{(36)}) - \left( (b'_{38})^{(7)} - (b''_{38})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{38}(s_{(36)}) \right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i$: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$ , $T_i(0) = T_i^0$ , $G_i^0 \leq (\hat{P}_{40})^{(8)}$ , $T_i^0 \leq (\hat{Q}_{40})^{(8)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[ (a_{40})^{(8)} G_{41}(s_{(40)}) - \left( (a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[ (a_{41})^{(8)} G_{40}(s_{(40)}) - \left( (a'_{41})^{(8)} + (a''_{41})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)}\left(T_{41}(s_{(40)}), s_{(40)}\right)\right) G_{42}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^0 + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)}\left(G_{43}(s_{(40)}), s_{(40)}\right)\right) T_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^0 + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)}\left(G_{43}(s_{(40)}), s_{(40)}\right)\right) T_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{\mathrm{T}}_{42}(\mathrm{t}) = \mathrm{T}_{42}^0 + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)}\left(G_{43}(s_{(40)}), s_{(40)}\right)\right) T_{42}(s_{(40)})\right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** 166

Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy A

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{44})^{(9)},\ T_i^0 \le (\hat{Q}_{44})^{(9)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''_{44})^{(9)}\left(T_{45}(s_{(44)}), s_{(44)}\right)\right) G_{44}(s_{(44)})\right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)}\left(T_{45}(s_{(44)}), s_{(44)}\right)\right) G_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)}\left(T_{45}(s_{(44)}), s_{(44)}\right)\right) G_{46}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)}\left(G_{47}(s_{(44)}), s_{(44)}\right)\right) T_{44}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)}\left(G_{47}(s_{(44)}), s_{(44)}\right)\right) T_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{\mathrm{T}}_{46}(\mathrm{t}) = \mathrm{T}_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)}\left(G_{47}(s_{(44)}), s_{(44)}\right)\right) T_{46}(s_{(44)})\right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}}\right)\right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left(e^{(\hat{M}_{13})^{(1)} t} - 1\right)$$

From which it follows that 168

$$(G_{13}(t)-G_{13}^{0})e^{-(\hat{M}_{13})^{(1)}t}\leq\frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}}\left[((\hat{P}_{13})^{(1)}+G_{14}^{0})e^{\left(-\frac{(\hat{P}_{13})^{(1)}+G_{14}^{0}}{G_{14}^{0}}\right)}+(\hat{P}_{13})^{(1)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t)\leq G_{16}^{0}+\int_{0}^{t}\left[(a_{16})^{(2)}\left(G_{17}^{0}+(\hat{P}_{16})^{(6)}e^{(\hat{M}_{16})^{(2)}s_{(16)}}\right)\right]ds_{(16)}= \quad 169$$

$$\left(1+(a_{16})^{(2)}t\right)G_{17}^{0}+\frac{(a_{16})^{(2)}(\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}}\left(e^{(\hat{M}_{16})^{(2)}t}-1\right)$$

From which it follows that 170

$$(G_{16}(t)-G_{16}^{0})e^{-(\hat{M}_{16})^{(2)}t}\leq\frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}}\left[((\hat{P}_{16})^{(2)}+G_{17}^{0})e^{\left(-\frac{(\hat{P}_{16})^{(2)}+G_{17}^{0}}{G_{17}^{0}}\right)}+(\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t)\leq G_{20}^{0}+\int_{0}^{t}\left[(a_{20})^{(3)}\left(G_{21}^{0}+(\hat{P}_{20})^{(3)}e^{(\hat{M}_{20})^{(3)}s_{(20)}}\right)\right]ds_{(20)}=$$

$$\left(1+(a_{20})^{(3)}t\right)G_{21}^{0}+\frac{(a_{20})^{(3)}(\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left(e^{(\hat{M}_{20})^{(3)}t}-1\right)$$

From which it follows that 172

$$(G_{20}(t)-G_{20}^{0})e^{-(\hat{M}_{20})^{(3)}t}\leq\frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left[((\hat{P}_{20})^{(3)}+G_{21}^{0})e^{\left(-\frac{(\hat{P}_{20})^{(3)}+G_{21}^{0}}{G_{21}^{0}}\right)}+(\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 173

$$G_{24}(t)\leq G_{24}^{0}+\int_{0}^{t}\left[(a_{24})^{(4)}\left(G_{25}^{0}+(\hat{P}_{24})^{(4)}e^{(\hat{M}_{24})^{(4)}s_{(24)}}\right)\right]ds_{(24)}=$$

$$\left(1+(a_{24})^{(4)}t\right)G_{25}^{0}+\frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left(e^{(\hat{M}_{24})^{(4)}t}-1\right)$$

From which it follows that 174

$$(G_{24}(t)-G_{24}^{0})e^{-(\hat{M}_{24})^{(4)}t}\leq\frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left[((\hat{P}_{24})^{(4)}+G_{25}^{0})e^{\left(-\frac{(\hat{P}_{24})^{(4)}+G_{25}^{0}}{G_{25}^{0}}\right)}+(\hat{P}_{24})^{(4)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t) \le G_{28}^0 + \int_0^t \left[ (a_{28})^{(5)} \left( G_{29}^0 + (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} s_{(28)}} \right) \right] ds_{(28)} =$$

$$\left(1 + (a_{28})^{(5)} t\right) G_{29}^0 + \frac{(a_{28})^{(5)} (\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}} \left( e^{(\hat{M}_{28})^{(5)} t} - 1 \right)$$

From which it follows that 175

$$(G_{28}(t) - G_{28}^0) e^{-(\hat{M}_{28})^{(5)} t} \le \frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}} \left[ \left( (\hat{P}_{28})^{(5)} + G_{29}^0 \right) e^{\left( - \frac{(\hat{P}_{28})^{(5)} + G_{29}^0}{G_{29}^0} \right)} + (\hat{P}_{28})^{(5)} \right]$$

$(G_i^0)$ is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t) \le G_{32}^0 + \int_0^t \left[ (a_{32})^{(6)} \left( G_{33}^0 + (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} s_{(32)}} \right) \right] ds_{(32)} =$$

$$\left(1 + (a_{32})^{(6)} t\right) G_{33}^0 + \frac{(a_{32})^{(6)} (\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}} \left( e^{(\hat{M}_{32})^{(6)} t} - 1 \right)$$

From which it follows that 177

$$(G_{32}(t) - G_{32}^0) e^{-(\hat{M}_{32})^{(6)} t} \le \frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}} \left[ \left( (\hat{P}_{32})^{(6)} + G_{33}^0 \right) e^{\left( - \frac{(\hat{P}_{32})^{(6)} + G_{33}^0}{G_{33}^0} \right)} + (\hat{P}_{32})^{(6)} \right]$$

$(G_i^0)$ is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

(ii) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t) \le G_{36}^0 + \int_0^t \left[ (a_{36})^{(7)} \left( G_{37}^0 + (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} s_{(36)}} \right) \right] ds_{(36)} =$$

$$\left(1 + (a_{36})^{(7)} t\right) G_{37}^0 + \frac{(a_{36})^{(7)} (\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left( e^{(\hat{M}_{36})^{(7)} t} - 1 \right)$$

From which it follows that

$$(G_{36}(t) - G_{36}^0) e^{-(\hat{M}_{36})^{(7)} t} \le \frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left[ \left( (\hat{P}_{36})^{(7)} + G_{37}^0 \right) e^{\left( - \frac{(\hat{P}_{36})^{(7)} + G_{37}^0}{G_{37}^0} \right)} + (\hat{P}_{36})^{(7)} \right]$$

$(G_i^0)$ is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t) \le G_{40}^0 + \int_0^t \left[ (a_{40})^{(8)} \left( G_{41}^0 + (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} s_{(40)}} \right) \right] ds_{(40)} = \quad 180$$

$$\left(1 + (a_{40})^{(8)} t\right) G_{41}^0 + \frac{(a_{40})^{(8)} (\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}} \left( e^{(\hat{M}_{40})^{(8)} t} - 1 \right)$$

From which it follows that 181

$$(G_{40}(t)-G_{40}^{0})e^{-(\hat{M}_{40})^{(8)}t}\leq\frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left[((\hat{P}_{40})^{(8)}+G_{41}^{0})e^{\left(-\frac{(\hat{P}_{40})^{(8)}+G_{41}^{0}}{G_{41}^{0}}\right)}+(\hat{P}_{40})^{(8)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 8

Analogous inequalities hold also for $G_{41}, G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t)\leq G_{44}^{0}+\int_{0}^{t}\left[(a_{44})^{(9)}\left(G_{45}^{0}+(\hat{P}_{44})^{(9)}e^{(\hat{M}_{44})^{(9)}s_{(44)}}\right)\right]ds_{(44)}=$$

$$\left(1+(a_{44})^{(9)}t\right)G_{45}^{0}+\frac{(a_{44})^{(9)}(\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}}\left(e^{(\hat{M}_{44})^{(9)}t}-1\right)$$

From which it follows that

$$(G_{44}(t)-G_{44}^{0})e^{-(\hat{M}_{44})^{(9)}t}\leq\frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}}\left[((\hat{P}_{44})^{(9)}+G_{45}^{0})e^{\left(-\frac{(\hat{P}_{44})^{(9)}+G_{45}^{0}}{G_{45}^{0}}\right)}+(\hat{P}_{44})^{(9)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 9

Analogous inequalities hold also for $G_{45}, G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}<1$ and to choose 182

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[(\hat{P}_{13})^{(1)}+((\hat{P}_{13})^{(1)}+G_j^0)e^{-\left(\frac{(\hat{P}_{13})^{(1)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{13})^{(1)} \quad 183$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[((\hat{Q}_{13})^{(1)}+T_j^0)e^{-\left(\frac{(\hat{Q}_{13})^{(1)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{13})^{(1)}\right]\leq(\hat{Q}_{13})^{(1)} \quad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)},T^{(1)}\right),\left(G^{(2)},T^{(2)}\right)\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{13})^{(1)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{13})^{(1)}t}\}$$

Indeed if we denote

**Definition of** $\tilde{G},\tilde{T}:\left(\tilde{G},\tilde{T}\right)=\mathcal{A}^{(1)}(G,T)$

It results

$$\left|\tilde{G}_{13}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{13})^{(1)} \left|G_{14}^{(1)}-G_{14}^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)} s_{(13)}} e^{(\widehat{M}_{13})^{(1)} s_{(13)}}\, ds_{(13)} +$$

$$\int_0^t \{(a'_{13})^{(1)} \left|G_{13}^{(1)}-G_{13}^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)} s_{(13)}} e^{-(\widehat{M}_{13})^{(1)} s_{(13)}} +$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)}, s_{(13)}\right)\left|G_{13}^{(1)}-G_{13}^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)} s_{(13)}} e^{(\widehat{M}_{13})^{(1)} s_{(13)}} +$$

$$G_{13}^{(2)} \left|(a''_{13})^{(1)}\left(T_{14}^{(1)}, s_{(13)}\right)-(a''_{13})^{(1)}\left(T_{14}^{(2)}, s_{(13)}\right)\right|\ e^{-(\widehat{M}_{13})^{(1)} s_{(13)}} e^{(\widehat{M}_{13})^{(1)} s_{(13)}}\} ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\left|G^{(1)}-G^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)} t} \leq \qquad (186)$$
$$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)} + (a'_{13})^{(1)} + (\widehat{A}_{13})^{(1)} + (\widehat{P}_{13})^{(1)}(\widehat{k}_{13})^{(1)}\right) d\left(\left(G^{(1)}, T^{(1)};\ G^{(2)}, T^{(2)}\right)\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 1:** The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)} t}$ *and* $(\widehat{Q}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)} t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i = 13,14,15$ depend only on $T_{14}$ and respectively on $G(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any $t$ where $G_i(t) = 0$ *and* $T_i(t) = 0$

From 19 to 24 it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(1)} - (a_i'')^{(1)}(T_{14}(s_{(13)}), s_{(13)})\} ds_{(13)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(1)} t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2$ *and* $\left((\widehat{M}_{13})^{(1)}\right)_3$ : (187)

**Remark 3**: if $G_{13}$ is bounded, the same property have also $G_{14}$ *and* $G_{15}$ . indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a'_{14})^{(1)} G_{14}$ and by integrating

$$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1 / (a'_{14})^{(1)}$$

In the same way , one can obtain

$$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2 / (a'_{15})^{(1)}$$

If $G_{14}$ *or* $G_{15}$ is bounded, the same property follows for $G_{13}$ , $G_{15}$ and $G_{13}$ , $G_{14}$ respectively.

**Remark 4:** If $G_{13}$ *is* bounded, from below, the same property holds for $G_{14}$ *and* $G_{15}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{14}$ is bounded from below. 188

**Remark 5:** If $T_{13}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}\,(G(t),t)) = (b_{14}')^{(1)}$ then $T_{14} \to \infty$. 189

**Definition of** $(m)^{(1)}$ and $\varepsilon_1$ :

Indeed let $t_1$ be so that for $t > t_1$

$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t),t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_1}$ By taking now $\varepsilon_1$ sufficiently small one sees that $T_{14}$ is unbounded.
The same property holds for $T_{15}$ if $\lim_{t\to\infty}(b_{15}'')^{(1)}\,(G(t),t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}$ , $\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}$ *and* $(\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[(\hat{P}_{16})^{(2)} + \left((\hat{P}_{16})^{(2)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{16})^{(2)}\right] \leq (\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

**Definition of** $\widetilde{G_{19}}, \widetilde{T_{19}}$ : $\left(\widetilde{G_{19}}, \widetilde{T_{19}}\right) = \mathcal{A}^{(2)}(G_{19}, T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{16})^{(2)} \left|G_{17}^{(1)}-G_{17}^{(2)}\right| e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\, ds_{(16)} +$$

$$\int_0^t \{(a'_{16})^{(2)} \left|G_{16}^{(1)}-G_{16}^{(2)}\right| e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} +$$

$$(a''_{16})^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right)\left|G_{16}^{(1)}-G_{16}^{(2)}\right| e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} e^{(\widehat{M}_{16})^{(2)}s_{(16)}} +$$

$$G_{16}^{(2)} \left|(a''_{16})^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right)-(a''_{16})^{(2)}\left(T_{17}^{(2)}, s_{(16)}\right)\right|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\} ds_{(16)}$$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0, t]$ 197

From the hypotheses it follows

$$\left|(G_{19})^{(1)}-(G_{19})^{(2)}\right| e^{-(\widehat{M}_{16})^{(2)}t} \leq$$
$$\frac{1}{(\widehat{M}_{16})^{(2)}}\left((a_{16})^{(2)}+\ (a'_{16})^{(2)}+(\widehat{A}_{16})^{(2)}+(\widehat{P}_{16})^{(2)}(\widehat{k}_{16})^{(2)}\right) d\left(((G_{19})^{(1)}, (T_{19})^{(1)};\ (G_{19})^{(2)}, (T_{19})^{(2)})\right)$$

And analogous inequalities for $G_i$ and $T_i$. Taking into account the hypothesis the result follows 198

**Remark 6:** The fact that we supposed $(a''_{16})^{(2)}$ and $(b''_{16})^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{16})^{(2)} e^{(\widehat{M}_{16})^{(2)}t}$ and $(\widehat{Q}_{16})^{(2)} e^{(\widehat{M}_{16})^{(2)}t}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i = 16,17,18$ depend only on $T_{17}$ and respectively on $(G_{19})$(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 7:** There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$ 200

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(2)}-(a_i'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})\} ds_{(16)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(2)}t)} > 0 \quad \text{for } t > 0$$

**Definition of** $\left((\widehat{M}_{16})^{(2)}\right)_1$, $\left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$: 201

**Remark 8:** if $G_{16}$ is bounded, the same property have also $G_{17}$ and $G_{18}$ . indeed if

$G_{16} < (\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt} \leq \left((\widehat{M}_{16})^{(2)}\right)_1 - (a'_{17})^{(2)} G_{17}$ and by integrating

$$G_{17} \leq \left((\widehat{M}_{16})^{(2)}\right)_2 = G_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1 / (a'_{17})^{(2)}$$

In the same way , one can obtain

$$G_{18} \leq \left((\widehat{M}_{16})^{(2)}\right)_3 = G_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2 / (a'_{18})^{(2)}$$

If $G_{17}$ or $G_{18}$ is bounded, the same property follows for $G_{16}$, $G_{18}$ and $G_{16}$, $G_{17}$ respectively.

**Remark 9**: If $G_{16}$ is bounded, from below, the same property holds for $G_{17}$ and $G_{18}$. The proof is analogous with the preceding one. An analogous property is true if $G_{17}$ is bounded from below. 202

**Remark 10:** If $T_{16}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t),t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$. 203

**Definition of** $(m)^{(2)}$ and $\varepsilon_2$ :

Indeed let $t_2$ be so that for $t > t_2$

$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t),t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right), \quad t = \log\frac{2}{\varepsilon_2}$ By taking now $\varepsilon_2$ sufficiently small one sees that $T_{17}$ is unbounded. 205
The same property holds for $T_{18}$ if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t),t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[(\hat{P}_{20})^{(3)} + \left((\hat{P}_{20})^{(3)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{20})^{(3)}\right] \leq (\hat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself 210

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)},(T_{23})^{(1)}),((G_{23})^{(2)},(T_{23})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{20})^{(3)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

**Definition of** $\widetilde{G_{23}}, \widetilde{T_{23}}$ :$\left(\widetilde{(G_{23})},\widetilde{(T_{23})}\right) = \mathcal{A}^{(3)}\left((G_{23}),(T_{23})\right)$

It results 213

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_{0}^{t}(a_{20})^{(3)}\left|G_{21}^{(1)}-G_{21}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\, ds_{(20)} +$$

$$\int_{0}^{t}\{(a'_{20})^{(3)}\left|G_{20}^{(1)}-G_{20}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} +$$

$$(a''_{20})^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{(\widehat{M}_{20})^{(3)}s_{(20)}} +$$

$$G_{20}^{(2)}\left|(a''_{20})^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)}, s_{(20)}\right)\right|\ e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\} ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}t} \leq \qquad 214$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+\ (a'_{20})^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\widehat{k}_{20})^{(3)}\right) d\left(\left((G_{23})^{(1)},(T_{23})^{(1)};\ (G_{23})^{(2)},(T_{23})^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

**Remark 11:** The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}\ and\ (\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_{+}$. 215

If instead of proving the existence of the solution on $\mathbb{R}_{+}$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i = 20,21,22$ depend only on $T_{21}$ and respectively on $(G_{23})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 12:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 216

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t\{(a_i')^{(3)}-(a_i'')^{(3)}(T_{21}(s_{(20)}),s_{(20)})\}ds_{(20)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(3)}t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2\ and\ \left((\widehat{M}_{20})^{(3)}\right)_3$ : 217

**Remark 13:** if $G_{20}$ is bounded, the same property have also $G_{21}\ and\ G_{22}$ . indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a'_{21})^{(3)}G_{21}$ and by integrating

$$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1/(a'_{21})^{(3)}$$

In the same way , one can obtain

$$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2/(a'_{22})^{(3)}$$

If $G_{21}$ *or* $G_{22}$ is bounded, the same property follows for $G_{20}$ , $G_{22}$ and $G_{20}$ , $G_{21}$ respectively.

**Remark 14:** If $G_{20}$ *is* bounded, from below, the same property holds for $G_{21}$ *and* $G_{22}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{21}$ is bounded from below. 218

**Remark 15:** If $T_{20}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t),t\right)=(b_{21}')^{(3)}$ then $T_{21}\to\infty$. 219

**Definition of** $(m)^{(3)}$ and $\varepsilon_3$ :

Indeed let $t_3$ be so that for $t > t_3$

$(b_{21})^{(3)}-(b_i'')^{(3)}\left((G_{23})(t),t\right)<\varepsilon_3, T_{20}(t)>(m)^{(3)}$

Then $\frac{dT_{21}}{dt}\geq(a_{21})^{(3)}(m)^{(3)}-\varepsilon_3 T_{21}$ which leads to 220

$T_{21}\geq\left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1-e^{-\varepsilon_3 t})+T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t}=\frac{1}{2}$ it results

$T_{21}\geq\left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right),\quad t=log\frac{2}{\varepsilon_3}$ By taking now $\varepsilon_3$ sufficiently small one sees that $T_{21}$ is unbounded.
The same property holds for $T_{22}$ if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t),t\right)=(b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}$ , $\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}}<1$ and to choose 221

$(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\left[(\hat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \quad 222$$

$$\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \quad 223$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself 224

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric 225

$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{24})^{(4)}t}\}$$

Indeed if we denote

**Definition of** $\widetilde{(G_{27})},\widetilde{(T_{27})}$ : $\left(\widetilde{(G_{27})},\widetilde{(T_{27})}\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_{i}^{(2)}\right|\leq\int_{0}^{t}(a_{24})^{(4)}\left|G_{25}^{(1)}-G_{25}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\,ds_{(24)}+$$

$$\int_{0}^{t}\{(a'_{24})^{(4)}\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$G_{24}^{(2)}\left|(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)-(a''_{24})^{(4)}\left(T_{25}^{(2)},s_{(24)}\right)\right|\;e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\}ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}t}\leq \qquad 226$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)}+\;(a'_{24})^{(4)}+(\widehat{A}_{24})^{(4)}+(\widehat{P}_{24})^{(4)}(\widehat{k}_{24})^{(4)}\right)d\left(\left((G_{27})^{(1)},(T_{27})^{(1)};\;(G_{27})^{(2)},(T_{27})^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

**Remark 16:** The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}\ and\ (\widehat{Q}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_{+}$. 227

If instead of proving the existence of the solution on $\mathbb{R}_{+}$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i=24,25,26$ depend only on $\mathrm{T}_{25}$ and respectively on $(G_{27})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 17:** There does not exist any $t$ where $G_i\,(t)=0\ and\ T_i\,(t)=0$ 228

it results

$$G_i\,(t)\geq G_i^0e^{\left[-\int_0^t\{(a_i')^{(4)}-(a_i'')^{(4)}(T_{25}(s_{(24)}),s_{(24)})\}ds_{(24)}\right]}\geq 0$$

$$T_i\,(t)\geq T_i^0e^{\left(-(b_i')^{(4)}t\right)}>0\quad \text{for t}>0$$

**Definition of** $\left((\widehat{M}_{24})^{(4)}\right)_1,\left((\widehat{M}_{24})^{(4)}\right)_2\ and\ \left((\widehat{M}_{24})^{(4)}\right)_3$ : 229

**Remark 18:** if $G_{24}$ is bounded, the same property have also $G_{25}\ and\ G_{26}$ . indeed if

$G_{24}<(\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt}\leq\left((\widehat{M}_{24})^{(4)}\right)_1-(a'_{25})^{(4)}G_{25}$ and by integrating

$$G_{25}\leq\left((\widehat{M}_{24})^{(4)}\right)_2=G_{25}^0+2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1/(a'_{25})^{(4)}$$

In the same way , one can obtain

$$G_{26}\leq\left((\widehat{M}_{24})^{(4)}\right)_3=G_{26}^0+2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2/(a'_{26})^{(4)}$$

If $G_{25}$ *or* $G_{26}$ is bounded, the same property follows for $G_{24}$ , $G_{26}$ and $G_{24}$ , $G_{25}$ respectively.

**Remark 19:** If $G_{24}$ *is* bounded, from below, the same property holds for $G_{25}$ *and* $G_{26}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{25}$ is bounded from below. 230

**Remark 20:** If $T_{24}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}((G_{27})(t),t)) = (b_{25}')^{(4)}$ then $T_{25} \to \infty$. 231

**Definition of** $(m)^{(4)}$ and $\varepsilon_4$ :

Indeed let $t_4$ be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t),t) < \varepsilon_4, T_{24}(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right)$, $t = log\frac{2}{\varepsilon_4}$ By taking now $\varepsilon_4$ sufficiently small one sees that $T_{25}$ is unbounded.
The same property holds for $T_{26}$ if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t),t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for $G_{29}, G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}$ , $\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)} + \left((\hat{P}_{28})^{(5)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{28})^{(5)}\right] \leq (\hat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$$d\left(((G_{31})^{(1)},(T_{31})^{(1)}),((G_{31})^{(2)},(T_{31})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

**Definition of** $\widetilde{(G_{31})}, \widetilde{(T_{31})}$ : $\left(\widetilde{(G_{31})}, \widetilde{(T_{31})}\right) = \mathcal{A}^{(5)}\left((G_{31}), (T_{31})\right)$

It results

$$\left|\tilde{G}_{28}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{28})^{(5)} \left|G_{29}^{(1)} - G_{29}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}}\, ds_{(28)} +$$

$$\int_0^t \{(a'_{28})^{(5)} \left|G_{28}^{(1)} - G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} +$$

$$(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right)\left|G_{28}^{(1)} - G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}} +$$

$$G_{28}^{(2)} \left|(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right) - (a''_{28})^{(5)}\left(T_{29}^{(2)}, s_{(28)}\right)\right| \; e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}} \} ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$$\left|(G_{31})^{(1)} - (G_{31})^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} t} \leq \qquad (237)$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)} + (a'_{28})^{(5)} + (\widehat{A}_{28})^{(5)} + (\widehat{P}_{28})^{(5)} (\widehat{k}_{28})^{(5)}\right) d\left(\left((G_{31})^{(1)}, (T_{31})^{(1)}; (G_{31})^{(2)}, (T_{31})^{(2)}\right)\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 21:** The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$ *and* $(\widehat{Q}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$ respectively of $\mathbb{R}_+$. (238)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on $T_{29}$ and respectively on $(G_{31})$(*and not on* $t$) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 22:** There does not exist any $t$ where $G_i(t) = 0$ *and* $T_i(t) = 0$ (239)

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(5)} - (a_i'')^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(5)} t)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2$ *and* $\left((\widehat{M}_{28})^{(5)}\right)_3$ : (240)

**Remark 23:** if $G_{28}$ is bounded, the same property have also $G_{29}$ *and* $G_{30}$ . indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a'_{29})^{(5)} G_{29}$ and by integrating

$$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1 / (a'_{29})^{(5)}$$

In the same way , one can obtain

$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2/(a'_{30})^{(5)}$

If $G_{29}$ *or* $G_{30}$ is bounded, the same property follows for $G_{28}$, $G_{30}$ and $G_{28}$, $G_{29}$ respectively.

**Remark 24:** If $G_{28}$ *is* bounded, from below, the same property holds for $G_{29}$ *and* $G_{30}$. The proof is analogous with the preceding one. An analogous property is true if $G_{29}$ is bounded from below. 241

**Remark 25:** If $T_{28}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}((G_{31})(t),t)) = (b'_{29})^{(5)}$ then $T_{29} \to \infty$. 242

**Definition of** $(m)^{(5)}$ and $\varepsilon_5$ :

Indeed let $t_5$ be so that for $t > t_5$

$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t),t) < \varepsilon_5, T_{28}(t) > (m)^{(5)}$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\frac{2}{\varepsilon_5}$ By taking now $\varepsilon_5$ sufficiently small one sees that $T_{29}$ is unbounded.
The same property holds for $T_{30}$ if $\lim_{t\to\infty}(b_{30}'')^{(5)}\left((G_{31})(t),t\right) = (b'_{30})^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{33}, G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}$, $\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$ and to choose 244

$(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[(\hat{P}_{32})^{(6)} + \left((\hat{P}_{32})^{(6)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{32})^{(6)} \quad 245$$

$$\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[\left((\hat{Q}_{32})^{(6)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{32})^{(6)}\right] \leq (\hat{Q}_{32})^{(6)} \quad 246$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric 247

$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right) =$

$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{32})^{(6)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{32})^{(6)}t}\}$

Indeed if we denote

**Definition of** $\widetilde{(G_{35})}, \widetilde{(T_{35})}$ : $\left(\widetilde{(G_{35})}, \widetilde{(T_{35})}\right) = \mathcal{A}^{(6)}\left((G_{35}), (T_{35})\right)$

It results

$$\left|\tilde{G}_{32}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{32})^{(6)} \left|G_{33}^{(1)} - G_{33}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}}\, ds_{(32)} +$$

$$\int_0^t \{(a'_{32})^{(6)} \left|G_{32}^{(1)} - G_{32}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} +$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)}, s_{(32)}\right)\left|G_{32}^{(1)} - G_{32}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}} +$$

$$G_{32}^{(2)} \left|(a''_{32})^{(6)}\left(T_{33}^{(1)}, s_{(32)}\right) - (a''_{32})^{(6)}\left(T_{33}^{(2)}, s_{(32)}\right)\right| \; e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}} \} ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|(G_{35})^{(1)} - (G_{35})^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} t} \leq \qquad (248)$$
$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)} + (a'_{32})^{(6)} + (\widehat{A}_{32})^{(6)} + (\widehat{P}_{32})^{(6)} (\widehat{k}_{32})^{(6)}\right) d\left(\left((G_{35})^{(1)}, (T_{35})^{(1)}; \; (G_{35})^{(2)}, (T_{35})^{(2)}\right)\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 26:** The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)} t}$ *and* $(\widehat{Q}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)} t}$ respectively of $\mathbb{R}_+$. (249)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(6)}$ and $(b''_i)^{(6)}, i = 32{,}33{,}34$ depend only on $T_{33}$ and respectively on $(G_{35})$(*and not on* $t$) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 27:** There does not exist any $t$ where $G_i(t) = 0$ *and* $T_i(t) = 0$ (250)

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a'_i)^{(6)} - (a''_i)^{(6)}(T_{33}(s_{(32)}), s_{(32)})\} ds_{(32)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b'_i)^{(6)} t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{32})^{(6)}\right)_1, \left((\widehat{M}_{32})^{(6)}\right)_2$ *and* $\left((\widehat{M}_{32})^{(6)}\right)_3$ : (251)

**Remark 28:** if $G_{32}$ is bounded, the same property have also $G_{33}$ *and* $G_{34}$ . indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a'_{33})^{(6)} G_{33}$ and by integrating

$$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_1 / (a'_{33})^{(6)}$$

In the same way , one can obtain

$$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_2/(a'_{34})^{(6)}$$

If $G_{33}$ *or* $G_{34}$ is bounded, the same property follows for $G_{32}$ , $G_{34}$ and $G_{32}$ , $G_{33}$ respectively.

**Remark 29:** If $G_{32}$ *is* bounded, from below, the same property holds for $G_{33}$ *and* $G_{34}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{33}$ is bounded from below. 252

**Remark 30:** If $T_{32}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}((G_{35})(t),t)) = (b'_{33})^{(6)}$ then $T_{33} \to \infty$. 253

**Definition of** $(m)^{(6)}$ and $\varepsilon_6$ :

Indeed let $t_6$ be so that for $t > t_6$

$$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t),t\right) < \varepsilon_6, T_{32}(t) > (m)^{(6)}$$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right)$, $t = log\frac{2}{\varepsilon_6}$ By taking now $\varepsilon_6$ sufficiently small one sees that $T_{33}$ is unbounded.
The same property holds for $T_{34}$ if $\lim_{t\to\infty}(b''_{34})^{(6)}\left((G_{35})(t),t(t),t\right) = (b'_{34})^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{37}$ , $G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}$ , $\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}} < 1$ and to choose
$(\widehat{P}_{36})^{(7)}$ and $(\widehat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[(\widehat{P}_{36})^{(7)} + \left((\widehat{P}_{36})^{(7)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{36})^{(7)} \quad 256$$

$$\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[\left((\widehat{Q}_{36})^{(7)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{36})^{(7)}\right] \leq (\widehat{Q}_{36})^{(7)} \quad 257$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself
The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric 258

$$d\left(\left((G_{39})^{(1)},(T_{39})^{(1)}\right),\left((G_{39})^{(2)},(T_{39})^{(2)}\right)\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\tilde{M}_{36})^{(7)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\tilde{M}_{36})^{(7)}t}\}$$

Indeed if we denote

**Definition of** $\widetilde{(G_{39})},\widetilde{(T_{39})}:\left(\widetilde{(G_{39})},\widetilde{(T_{39})}\right)=\mathcal{A}^{(7)}((G_{39}),(T_{39}))$
It results

$$\left|\tilde{G}_{36}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{36})^{(7)}\left|G_{37}^{(1)}-G_{37}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\,ds_{(36)}+$$
$$\int_0^t\{(a'_{36})^{(7)}\left|G_{36}^{(1)}-G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$(a''_{36})^{(7)}\left(T_{37}^{(1)},s_{(36)}\right)\left|G_{36}^{(1)}-G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$G_{36}^{(2)}\left|(a''_{36})^{(7)}\left(T_{37}^{(1)},s_{(36)}\right)-(a''_{36})^{(7)}\left(T_{37}^{(2)},s_{(36)}\right)\right|\ e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\}ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on it follows

$$\left|(G_{39})^{(1)}-(G_{39})^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}t}\le \qquad 259$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}}\left((a_{36})^{(7)}+(a'_{36})^{(7)}+(\widehat{A}_{36})^{(7)}+(\widehat{P}_{36})^{(7)}(\widehat{k}_{36})^{(7)}\right)d\left(((G_{39})^{(1)},(T_{39})^{(1)};\ (G_{39})^{(2)},(T_{39})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

**Remark 31:** The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}\ and\ (\widehat{Q}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ respectively of $\mathbb{R}_+$. 260

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(7)}$ and $(b_i'')^{(7)}, i=36,37,38$ depend only on $T_{37}$ and respectively on $(G_{39})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 32:** There does not exist any $t$ where $G_i\,(t)=0\ and\ T_i\,(t)=0$ 261

it results

$$G_i\,(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(7)}-(a_i'')^{(7)}(T_{37}(s_{(36)}),s_{(36)})\}ds_{(36)}\right]}\ge 0$$

$$T_i\,(t)\ge T_i^0e^{\left(-(b_i')^{(7)}t\right)}>0\quad \text{for t}>0$$

**Definition of** $\left((\widehat{M}_{36})^{(7)}\right)_1,\left((\widehat{M}_{36})^{(7)}\right)_2\ and\ \left((\widehat{M}_{36})^{(7)}\right)_3:$ 262

**Remark 33:** if $G_{36}$ is bounded, the same property have also $G_{37}\ and\ G_{38}$ . indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a'_{37})^{(7)} G_{37}$ and by integrating

$$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1/(a'_{37})^{(7)}$$

In the same way , one can obtain

$$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2/(a'_{38})^{(7)}$$

If $G_{37}$ *or* $G_{38}$ is bounded, the same property follows for $G_{36}$ , $G_{38}$ and $G_{36}$ , $G_{37}$ respectively.

**Remark 34:** If $G_{36}$ *is* bounded, from below, the same property holds for $G_{37}$ *and* $G_{38}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{37}$ is bounded from below. 263

**Remark 35:** If $T_{36}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(7)}((G_{39})(t),t)) = (b'_{37})^{(7)}$ then $T_{37} \to \infty$. 264

**Definition of** $(m)^{(7)}$ and $\varepsilon_7$ :

Indeed let $t_7$ be so that for $t > t_7$

$$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t),t) < \varepsilon_7, T_{36}(t) > (m)^{(7)}$$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right)$, $t = log\frac{2}{\varepsilon_7}$ By taking now $\varepsilon_7$ sufficiently small one sees that $T_{37}$ is unbounded.
The same property holds for $T_{38}$ if $\lim_{t\to\infty}(b''_{38})^{(7)}\left((G_{39})(t),t\right) = (b'_{38})^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}$ , $\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1$ and to choose 266
$(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ large to have

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[(\hat{P}_{40})^{(8)} + \left((\hat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{40})^{(8)} \quad 267$$

$$\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{40})^{(8)}\right] \leq (\hat{Q}_{40})^{(8)} \quad 268$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)},(T_{43})^{(1)}),((G_{43})^{(2)},(T_{43})^{(2)})\right)= \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

**Definition of** $\widetilde{(G_{43})},\widetilde{(T_{43})}$ : $\left(\widetilde{(G_{43})},\widetilde{(T_{43})}\right)=\mathcal{A}^{(8)}((G_{43}),(T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{40})^{(8)}\left|G_{41}^{(1)}-G_{41}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\,ds_{(40)}+$$
$$\int_0^t\{(a'_{40})^{(8)}\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$G_{40}^{(2)}\left|(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)-(a''_{40})^{(8)}\left(T_{41}^{(2)},s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\}ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0,t]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)}-(G_{43})^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}t}\le \quad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)}+(a'_{40})^{(8)}+(\widehat{A}_{40})^{(8)}+(\widehat{P}_{40})^{(8)}(\widehat{k}_{40})^{(8)}\right)d\left(((G_{43})^{(1)},(T_{43})^{(1)};\ (G_{43})^{(2)},(T_{43})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

**Remark 36:** The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}\ and\ (\widehat{Q}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ respectively of $\mathbb{R}_+$. 274

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i=40,41,42$ depend only on $T_{41}$ and respectively on $(G_{43})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 37** There does not exist any $t$ where $G_i\,(t)=0\ and\ T_i\,(t)=0$ 275

it results

$$G_i\,(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(8)}-(a_i'')^{(8)}(T_{41}(s_{(40)}),s_{(40)})\}ds_{(40)}\right]}\ge 0$$

$$T_i\,(t)\ge T_i^0e^{\left(-(b_i')^{(8)}t\right)}>0\quad \text{for t}>0$$

**Definition of** $\left((\widehat{M}_{40})^{(8)}\right)_1, \left((\widehat{M}_{40})^{(8)}\right)_2$ *and* $\left((\widehat{M}_{40})^{(8)}\right)_3$ : 276

**Remark 38:** if $G_{40}$ is bounded, the same property have also $G_{41}$ *and* $G_{42}$ . indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a'_{41})^{(8)} G_{41}$ and by integrating

$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1/(a'_{41})^{(8)}$

In the same way , one can obtain

$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2/(a'_{42})^{(8)}$

If $G_{41}$ *or* $G_{42}$ is bounded, the same property follows for $G_{40}$ , $G_{42}$ and $G_{40}$ , $G_{41}$ respectively.

**Remark 39:** If $G_{40}$ *is* bounded, from below, the same property holds for $G_{41}$ *and* $G_{42}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{41}$ is bounded from below. 277

**Remark 40:** If $T_{40}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}((G_{43})(t),t)) = (b'_{41})^{(8)}$ then $T_{41} \to \infty$. 278

**Definition of** $(m)^{(8)}$ and $\varepsilon_8$ :

Indeed let $t_8$ be so that for $t > t_8$

$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t),t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right), \quad t = log\frac{2}{\varepsilon_8}$ By taking now $\varepsilon_8$ sufficiently small one sees that $T_{41}$ is unbounded.
The same property holds for $T_{42}$ if $\lim_{t\to\infty}(b''_{42})^{(8)}\left((G_{43})(t),t(t),t\right) = (b'_{42})^{(8)}$

It is now sufficient to take $\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}$ , $\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}} < 1$ and to choose $(\widehat{P}_{44})^{(9)}$ and $(\widehat{Q}_{44})^{(9)}$ large to have 279 A

$$\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[(\widehat{P}_{44})^{(9)} + \left((\widehat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[\left((\widehat{Q}_{44})^{(9)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{44})^{(9)}\right] \leq (\widehat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)},(T_{47})^{(1)}),((G_{47})^{(2)},(T_{47})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{47}}),(\widetilde{T_{47}}):\left((\widetilde{G_{47}}),(\widetilde{T_{47}})\right)=\mathcal{A}^{(9)}\left((G_{47}),(T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{44})^{(9)}\left|G_{45}^{(1)}-G_{45}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\,ds_{(44)}+$$
$$\int_0^t\{(a'_{44})^{(9)}\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$G_{44}^{(2)}\left|(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)-(a''_{44})^{(9)}\left(T_{45}^{(2)},s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\}ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}t}\le$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)}+(a'_{44})^{(9)}+(\widehat{A}_{44})^{(9)}+(\widehat{P}_{44})^{(9)}(\widehat{k}_{44})^{(9)}\right)d\left(((G_{47})^{(1)},(T_{47})^{(1)};\ (G_{47})^{(2)},(T_{47})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis (39,35,36) the result follows

**Remark 41:** The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}\ and\ (\widehat{Q}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i=44,45,46$ depend only on $T_{45}$ and respectively on $(G_{47})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 42:** There does not exist any $t$ where $G_i\,(t)=0\ and\ T_i\,(t)=0$

From 99 to 44 it results

$$G_i\,(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(9)}-(a_i'')^{(9)}(T_{45}(s_{(44)}),s_{(44)})\}ds_{(44)}\right]}\ge 0$$

$$T_i\,(t)\ge T_i^0e^{(-(b_i')^{(9)}t)}>0\quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{44})^{(9)}\right)_1,\left((\widehat{M}_{44})^{(9)}\right)_2\ and\ \left((\widehat{M}_{44})^{(9)}\right)_3$ :

**Remark 43:** if $G_{44}$ is bounded, the same property have also $G_{45}\ and\ G_{46}$ . indeed if

$G_{44} < (\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt} \leq \left((\widehat{M}_{44})^{(9)}\right)_1 - (a'_{45})^{(9)} G_{45}$ and by integrating

$$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_1 / (a'_{45})^{(9)}$$

In the same way , one can obtain

$$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_2 / (a'_{46})^{(9)}$$

If $G_{45}$ *or* $G_{46}$ is bounded, the same property follows for $G_{44}$ , $G_{46}$ and $G_{44}$ , $G_{45}$ respectively.

**Remark 44:** If $G_{44}$ *is* bounded, from below, the same property holds for $G_{45}$ *and* $G_{46}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{45}$ is bounded from below.

**Remark 45:** If $T_{44}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t),t\right) = (b'_{45})^{(9)}$ then $T_{45} \to \infty$.

**Definition of** $(m)^{(9)}$ and $\varepsilon_9$ :

Indeed let $t_9$ be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t),t\right) < \varepsilon_9, T_{44}(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right)$, $t = log\frac{2}{\varepsilon_9}$ By taking now $\varepsilon_9$ sufficiently small one sees that $T_{45}$ is unbounded.
The same property holds for $T_{46}$ if $\lim_{t\to\infty}(b''_{46})^{(9)}\left((G_{47})(t),t\right) = (b'_{46})^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

**Behavior of the solutions of equation** 280

**Theorem** If we denote and define

**Definition of** $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ :

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)} \leq -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14},t) + (a''_{14})^{(1)}(T_{14},t) \leq -(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)} \leq -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G,t) - (b''_{14})^{(1)}(G,t) \leq -(\tau_1)^{(1)}$

**Definition of** $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$ : 281

By $(\nu_1)^{(1)} > 0$ , $(\nu_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$ , $(u_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

**Definition of** $(\bar{\nu}_1)^{(1)}, , (\bar{\nu}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$ : 282

By $(\bar{\nu}_1)^{(1)} > 0$ , $(\bar{\nu}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$ , $(\bar{u}_2)^{(1)} < 0$ the roots of the equations

$(a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

**Definition of** $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (v_0)^{(1)}$ :- 283

If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by

$(m_2)^{(1)} = (v_0)^{(1)}, (m_1)^{(1)} = (v_1)^{(1)}, \ if\ (v_0)^{(1)} < (v_1)^{(1)}$

$(m_2)^{(1)} = (v_1)^{(1)}, (m_1)^{(1)} = (\bar{v}_1)^{(1)}, if\ (v_1)^{(1)} < (v_0)^{(1)} < (\bar{v}_1)^{(1)},$

and $\boxed{(v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$

$(m_2)^{(1)} = (v_1)^{(1)}, (m_1)^{(1)} = (v_0)^{(1)}, \ if\ (\bar{v}_1)^{(1)} < (v_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)}, \ if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)},$

and $\boxed{(u_0)^{(1)} = \frac{T_{13}^0}{T_{14}^0}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$

are defined

Then the solution of global equations satisfies the inequalities 285

$$G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{13}(t) \le G_{13}^0 e^{(S_1)^{(1)}t}$$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)}t}$$

$$\left(\frac{(a_{15})^{(1)} G_{13}^0}{(m_1)^{(1)}((S_1)^{(1)} - (p_{13})^{(1)} - (S_2)^{(1)})}\left[e^{((S_1)^{(1)} - (p_{13})^{(1)})t} - e^{-(S_2)^{(1)}t}\right] + G_{15}^0 e^{-(S_2)^{(1)}t} \le G_{15}(t) \le \right. \quad 286$$

$$\left. \frac{(a_{15})^{(1)} G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)} - (a'_{15})^{(1)})}\left[e^{(S_1)^{(1)}t} - e^{-(a'_{15})^{(1)}t}\right] + G_{15}^0 e^{-(a'_{15})^{(1)}t}\right)$$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)}t} \le T_{13}(t) \le T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t}} \quad 287$$

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)}t} \le T_{13}(t) \le \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t} \quad 288$$

$$\frac{(b_{15})^{(1)} T_{13}^0}{(\mu_1)^{(1)}((R_1)^{(1)} - (b'_{15})^{(1)})}\left[e^{(R_1)^{(1)}t} - e^{-(b'_{15})^{(1)}t}\right] + T_{15}^0 e^{-(b'_{15})^{(1)}t} \le T_{15}(t) \le \quad 289$$

$$\frac{(a_{15})^{(1)} T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)} + (r_{13})^{(1)} + (R_2)^{(1)})}\left[e^{((R_1)^{(1)} + (r_{13})^{(1)})t} - e^{-(R_2)^{(1)}t}\right] + T_{15}^0 e^{-(R_2)^{(1)}t}$$

**Definition of** $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:- 290

Where $(S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$

$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$

$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$

$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$

**Behavior of the solutions of equation** 291

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ : 292

$(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ four constants satisfying
$-(\sigma_2)^{(2)} \le -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(T_{17}, t) + (a''_{17})^{(2)}(T_{17}, t) \le -(\sigma_1)^{(2)}$ 293

$-(\tau_2)^{(2)} \le -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}((G_{19}), t) - (b''_{17})^{(2)}((G_{19}), t) \le -(\tau_1)^{(2)}$ 294

**Definition of** $(\nu_1)^{(2)}, (\nu_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$ : 295

By $(\nu_1)^{(2)} > 0$ , $(\nu_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0$ , $(u_2)^{(2)} < 0$ the roots 296

of the equations $(a_{17})^{(2)}(\nu^{(2)})^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 297

and $(b_{14})^{(2)}(u^{(2)})^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and 298

**Definition of** $(\bar{\nu}_1)^{(2)},, (\bar{\nu}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$ : 299

By $(\bar{\nu}_1)^{(2)} > 0$ , $(\bar{\nu}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0$ , $(\bar{u}_2)^{(2)} < 0$ the 300

roots of the equations $(a_{17})^{(2)}(\nu^{(2)})^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 301

and $(b_{17})^{(2)}(u^{(2)})^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 302

**Definition of** $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ :- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{G^0_{16}}{G^0_{17}}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)},\ \boldsymbol{if}\ (u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}, \boldsymbol{if}\ (u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)},$

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)},\ \boldsymbol{if}\ (\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)} t}$

$(p_i)^{(2)}$ is defined by equation

$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)} t}$ 311

$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)} t}\right] + G_{18}^0 e^{-(S_2)^{(2)} t} \le G_{18}(t) \le\right.$ 312

$\left.\frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)} t} - e^{-(a'_{18})^{(2)} t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)} t}\right)$

$\boxed{T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}}$ 313

$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}$ 314

$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)} t} - e^{-(b'_{18})^{(2)} t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)} t} \le T_{18}(t) \le$ 315

$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})}\left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)} t}\right] + T_{18}^0 e^{-(R_2)^{(2)} t}$

**Definition of** $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$

$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)}$ 318

$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$

**Behavior of the solutions** 319

**Theorem 3:** If we denote and define

**Definition of** $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ :

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}\big((G_{23}), t\big) \le -(\tau_1)^{(3)}$$

**Definition of** $(v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$ : 320

By $(v_1)^{(3)} > 0$ , $(v_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$ , $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\big(v^{(3)}\big)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{v}_1)^{(3)} > 0$ , $(\bar{v}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$ , $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\big(v^{(3)}\big)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

**Definition of** $(m_1)^{(3)}$ , $(m_2)^{(3)}$ , $(\mu_1)^{(3)}, (\mu_2)^{(3)}$ :- 321

If we define $(m_1)^{(3)}$ , $(m_2)^{(3)}$ , $(\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (v_0)^{(3)}, (m_1)^{(3)} = (v_1)^{(3)}$, ***if*** $(v_0)^{(3)} < (v_1)^{(3)}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (\bar{v}_1)^{(3)}$ , ***if*** $(v_1)^{(3)} < (v_0)^{(3)} < (\bar{v}_1)^{(3)}$,

and $\boxed{(v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_0)^{(3)}$, ***if*** $(\bar{v}_1)^{(3)} < (v_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}$, ***if*** $(u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$ , ***if*** $(u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}$, and $\boxed{(u_0)^{(3)} = \frac{T^0_{20}}{T^0_{21}}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}$, ***if*** $(\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$$G^0_{20}e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G^0_{20}e^{(S_1)^{(3)}t}$$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}}G^0_{20}e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}}G^0_{20}e^{(S_1)^{(3)}t} \quad 323$$

$$\Big(\frac{(a_{22})^{(3)}G^0_{20}}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\Big[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)}t}\Big] + G^0_{22}e^{-(S_2)^{(3)}t} \le G_{22}(t) \le \quad 324$$

$$\frac{(a_{22})^{(3)}G^0_{20}}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}[e^{(S_1)^{(3)}t} - e^{-(a'_{22})^{(3)}t}] + G^0_{22}e^{-(a'_{22})^{(3)}t}\Big)$$

$$\boxed{T_{20}^{0}e^{(R_1)^{(3)}t} \le T_{20}(t) \le T_{20}^{0}e^{((R_1)^{(3)}+(r_{20})^{(3)})t}} \quad 325$$

$$\frac{1}{(\mu_1)^{(3)}}T_{20}^{0}e^{(R_1)^{(3)}t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}}T_{20}^{0}e^{((R_1)^{(3)}+(r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)}T_{20}^{0}}{(\mu_1)^{(3)}((R_1)^{(3)}-(b'_{22})^{(3)})}\left[e^{(R_1)^{(3)}t} - e^{-(b'_{22})^{(3)}t}\right] + T_{22}^{0}e^{-(b'_{22})^{(3)}t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)}T_{20}^{0}}{(\mu_2)^{(3)}((R_1)^{(3)}+(r_{20})^{(3)}+(R_2)^{(3)})}\left[e^{((R_1)^{(3)}+(r_{20})^{(3)})t} - e^{-(R_2)^{(3)}t}\right] + T_{22}^{0}e^{-(R_2)^{(3)}t}$$

**Definition of** $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$$

$$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$$

$$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$$

**Behavior of the solutions of equation**

**Theorem:** If we denote and define

**Definition of** $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ :

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$$

$$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}\left((G_{27}), t\right) - (b''_{25})^{(4)}\left((G_{27}), t\right) \le -(\tau_1)^{(4)}$$

**Definition of** $(\nu_1)^{(4)}, (\nu_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, \nu^{(4)}, u^{(4)}$ : 329

By $(\nu_1)^{(4)} > 0$ , $(\nu_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$ , $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_1)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(4)},, (\bar{\nu}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$ : 330

By $(\bar{\nu}_1)^{(4)} > 0$ , $(\bar{\nu}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$ , $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

**Definition of** $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}, (\nu_0)^{(4)}$ :-

If we define $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$$(m_2)^{(4)} = (\nu_0)^{(4)}, (m_1)^{(4)} = (\nu_1)^{(4)}, \ \boldsymbol{if}\ (\nu_0)^{(4)} < (\nu_1)^{(4)}$$

$$(m_2)^{(4)} = (\nu_1)^{(4)}, (m_1)^{(4)} = (\bar{\nu}_1)^{(4)}, \boldsymbol{if}\ (\nu_4)^{(4)} < (\nu_0)^{(4)} < (\bar{\nu}_1)^{(4)},$$

and $\boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (\nu_4)^{(4)}, (m_1)^{(4)} = (\nu_0)^{(4)},\ \boldsymbol{if}\ (\bar{\nu}_4)^{(4)} < (\nu_0)^{(4)}$

and analogously

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$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)},\ \boldsymbol{if}\ (u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}, \boldsymbol{if}\ (u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)},$

and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}, \boldsymbol{if}\ (\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

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$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)} t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)} t} \quad 333$$

$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)} t}\right] + G_{26}^0 e^{-(S_2)^{(4)} t} \le G_{26}(t) \le \right. \quad 334$$

*(a26)4G240(m2)4(S1)4−(a26′)4e(S1)4t−e−(a26′)4t+ G260e−(a26′)4t*

$$\boxed{T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}} T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}} T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t} \quad 335$$

$$\frac{(b_{26})^{(4)} T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)} - (b'_{26})^{(4)})}\left[e^{(R_1)^{(4)} t} - e^{-(b'_{26})^{(4)} t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)} t} \le T_{26}(t) \le \quad 336$$

$$\frac{(a_{26})^{(4)} T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)} + (r_{24})^{(4)} + (R_2)^{(4)})}\left[e^{((R_1)^{(4)} + (r_{24})^{(4)})t} - e^{-(R_2)^{(4)} t}\right] + T_{26}^0 e^{-(R_2)^{(4)} t}$$

**Definition of** $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

**Behavior of the solutions of equation** 338

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ :

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

**Definition of** $(\nu_1)^{(5)}, (\nu_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, \nu^{(5)}, u^{(5)}$ : 339

By $(\nu_1)^{(5)} > 0$, $(\nu_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$, $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(5)},, (\bar{\nu}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$ : 340

By $(\bar{\nu}_1)^{(5)} > 0$, $(\bar{\nu}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$, $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

**Definition of** $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (\nu_0)^{(5)}$ :-

If we define $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (\nu_0)^{(5)}, (m_1)^{(5)} = (\nu_1)^{(5)}$, ***if*** $(\nu_0)^{(5)} < (\nu_1)^{(5)}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\bar{\nu}_1)^{(5)}$, ***if*** $(\nu_1)^{(5)} < (\nu_0)^{(5)} < (\bar{\nu}_1)^{(5)}$,
and $\boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)}$, ***if*** $(\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)}$, ***if*** $(u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}$, ***if*** $(u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)}$,
and $\boxed{(u_0)^{(5)} = \frac{T^0_{28}}{T^0_{29}}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}$, ***if*** $(\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$$G^0_{28} e^{\left((S_1)^{(5)} - (p_{28})^{(5)}\right)t} \le G_{28}(t) \le G^0_{28} e^{(S_1)^{(5)}t}$$

where $(p_i)^{(5)}$ is defined by equation

$$\frac{1}{(m_5)^{(5)}} G^0_{28} e^{\left((S_1)^{(5)} - (p_{28})^{(5)}\right)t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}} G^0_{28} e^{(S_1)^{(5)}t} \quad 343$$

$$\left(\frac{(a_{30})^{(5)}G_{28}^{0}}{(m_1)^{(5)}((S_1)^{(5)}-(p_{28})^{(5)}-(S_2)^{(5)})}\left[e^{((S_1)^{(5)}-(p_{28})^{(5)})t}-e^{-(S_2)^{(5)}t}\right]+G_{30}^{0}e^{-(S_2)^{(5)}t}\leq G_{30}(t)\leq\right. \quad 344$$

*(a30)5G280(m2)5(S1)5−(a30′)5e(S1)5t−e−(a30′)5t+ G300e−(a30′)5t*

$$\boxed{T_{28}^{0}e^{(R_1)^{(5)}t}\leq T_{28}(t)\leq T_{28}^{0}e^{((R_1)^{(5)}+(r_{28})^{(5)})t}} \quad 345$$

$$\frac{1}{(\mu_1)^{(5)}}T_{28}^{0}e^{(R_1)^{(5)}t}\leq T_{28}(t)\leq\frac{1}{(\mu_2)^{(5)}}T_{28}^{0}e^{((R_1)^{(5)}+(r_{28})^{(5)})t} \quad 346$$

$$\frac{(b_{30})^{(5)}T_{28}^{0}}{(\mu_1)^{(5)}((R_1)^{(5)}-(b'_{30})^{(5)})}\left[e^{(R_1)^{(5)}t}-e^{-(b'_{30})^{(5)}t}\right]+T_{30}^{0}e^{-(b'_{30})^{(5)}t}\leq T_{30}(t)\leq \quad 347$$

$$\frac{(a_{30})^{(5)}T_{28}^{0}}{(\mu_2)^{(5)}((R_1)^{(5)}+(r_{28})^{(5)}+(R_2)^{(5)})}\left[e^{((R_1)^{(5)}+(r_{28})^{(5)})t}-e^{-(R_2)^{(5)}t}\right]+T_{30}^{0}e^{-(R_2)^{(5)}t}$$

**Definition of** $(S_1)^{(5)},(S_2)^{(5)},(R_1)^{(5)},(R_2)^{(5)}$:- 348

Where $(S_1)^{(5)}=(a_{28})^{(5)}(m_2)^{(5)}-(a'_{28})^{(5)}$

$(S_2)^{(5)}=(a_{30})^{(5)}-(p_{30})^{(5)}$

$(R_1)^{(5)}=(b_{28})^{(5)}(\mu_2)^{(5)}-(b'_{28})^{(5)}$

$(R_2)^{(5)}=(b'_{30})^{(5)}-(r_{30})^{(5)}$

**Behavior of the solutions of equation** 349

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(6)},(\sigma_2)^{(6)},(\tau_1)^{(6)},(\tau_2)^{(6)}$ :

$(\sigma_1)^{(6)},(\sigma_2)^{(6)},(\tau_1)^{(6)},(\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)}\leq-(a'_{32})^{(6)}+(a'_{33})^{(6)}-(a''_{32})^{(6)}(T_{33},t)+(a''_{33})^{(6)}(T_{33},t)\leq-(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)}\leq-(b'_{32})^{(6)}+(b'_{33})^{(6)}-(b''_{32})^{(6)}\big((G_{35}),t\big)-(b''_{33})^{(6)}\big((G_{35}),t\big)\leq-(\tau_1)^{(6)}$$

**Definition of** $(v_1)^{(6)},(v_2)^{(6)},(u_1)^{(6)},(u_2)^{(6)},v^{(6)},u^{(6)}$ : 350

By $(v_1)^{(6)}>0$ , $(v_2)^{(6)}<0$ and respectively $(u_1)^{(6)}>0$ , $(u_2)^{(6)}<0$ the roots of the equations $(a_{33})^{(6)}\big(v^{(6)}\big)^2+(\sigma_1)^{(6)}v^{(6)}-(a_{32})^{(6)}=0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2+(\tau_1)^{(6)}u^{(6)}-(b_{32})^{(6)}=0$ and

**Definition of** $(\bar{v}_1)^{(6)},,(\bar{v}_2)^{(6)},(\bar{u}_1)^{(6)},(\bar{u}_2)^{(6)}$ : 351

By $(\bar{v}_1)^{(6)}>0$ , $(\bar{v}_2)^{(6)}<0$ and respectively $(\bar{u}_1)^{(6)}>0$ , $(\bar{u}_2)^{(6)}<0$ the roots of the equations $(a_{33})^{(6)}\big(v^{(6)}\big)^2+(\sigma_2)^{(6)}v^{(6)}-(a_{32})^{(6)}=0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2+(\tau_2)^{(6)}u^{(6)}-(b_{32})^{(6)}=0$

**Definition of** $(m_1)^{(6)},(m_2)^{(6)},(\mu_1)^{(6)},(\mu_2)^{(6)},(v_0)^{(6)}$ :-

If we define $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}$ by

$(m_2)^{(6)} = (\nu_0)^{(6)}, (m_1)^{(6)} = (\nu_1)^{(6)}$, ***if*** $(\nu_0)^{(6)} < (\nu_1)^{(6)}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}$, ***if*** $(\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)}$,

and $\boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}$, ***if*** $(\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$

and analogously 352

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}$, ***if*** $(u_0)^{(6)} < (u_1)^{(6)}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}$, ***if*** $(u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)}$,

and $\boxed{(u_0)^{(6)} = \frac{T_{32}^0}{T_{33}^0}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}$, ***if*** $(\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$

Then the solution of global equations satisfies the inequalities 353

$$G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \leq G_{32}(t) \leq G_{32}^0 e^{(S_1)^{(6)} t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}} G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \leq G_{33}(t) \leq \frac{1}{(m_2)^{(6)}} G_{32}^0 e^{(S_1)^{(6)} t} \quad 354$$

$$\left( \frac{(a_{34})^{(6)} G_{32}^0}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})} \left[ e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)} t} \right] + G_{34}^0 e^{-(S_2)^{(6)} t} \leq G_{34}(t) \leq \right. \quad 355$$

*(a34)6G320(m2)6(S1)6−(a34′)6e(S1)6t−e−(a34′)6t+ G340e−(a34′)6t*

$$\boxed{T_{32}^0 e^{(R_1)^{(6)} t} \leq T_{32}(t) \leq T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}} T_{32}^0 e^{(R_1)^{(6)} t} \leq T_{32}(t) \leq \frac{1}{(\mu_2)^{(6)}} T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)} T_{32}^0}{(\mu_1)^{(6)}((R_1)^{(6)} - (b'_{34})^{(6)})} \left[ e^{(R_1)^{(6)} t} - e^{-(b'_{34})^{(6)} t} \right] + T_{34}^0 e^{-(b'_{34})^{(6)} t} \leq T_{34}(t) \leq \quad 358$$

$$\frac{(a_{34})^{(6)} T_{32}^0}{(\mu_2)^{(6)}((R_1)^{(6)} + (r_{32})^{(6)} + (R_2)^{(6)})} \left[ e^{((R_1)^{(6)} + (r_{32})^{(6)})t} - e^{-(R_2)^{(6)} t} \right] + T_{34}^0 e^{-(R_2)^{(6)} t}$$

**Definition of** $(S_1)^{(6)}, (S_2)^{(6)}, (R_1)^{(6)}, (R_2)^{(6)}$:- 359

Where $(S_1)^{(6)} = (a_{32})^{(6)} (m_2)^{(6)} - (a'_{32})^{(6)}$

$(S_2)^{(6)} = (a_{34})^{(6)} - (p_{34})^{(6)}$

$(R_1)^{(6)} = (b_{32})^{(6)} (\mu_2)^{(6)} - (b'_{32})^{(6)}$

$$(R_2)^{(6)} = (b'_{34})^{(6)} - (r_{34})^{(6)}$$

**Behavior of the solutions of equation**

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$ :

$(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)} \le -(a'_{36})^{(7)} + (a'_{37})^{(7)} - (a''_{36})^{(7)}(T_{37}, t) + (a''_{37})^{(7)}(T_{37}, t) \le -(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)} \le -(b'_{36})^{(7)} + (b'_{37})^{(7)} - (b''_{36})^{(7)}\big((G_{39}), t\big) - (b''_{37})^{(7)}\big((G_{39}), t\big) \le -(\tau_1)^{(7)}$$

**Definition of** $(\nu_1)^{(7)}, (\nu_2)^{(7)}, (u_1)^{(7)}, (u_2)^{(7)}, \nu^{(7)}, u^{(7)}$ : 361

By $(\nu_1)^{(7)} > 0, (\nu_2)^{(7)} < 0$ and respectively $(u_1)^{(7)} > 0, (u_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_1)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(7)},, (\bar{\nu}_2)^{(7)}, (\bar{u}_1)^{(7)}, (\bar{u}_2)^{(7)}$ : 362

By $(\bar{\nu}_1)^{(7)} > 0, (\bar{\nu}_2)^{(7)} < 0$ and respectively $(\bar{u}_1)^{(7)} > 0, (\bar{u}_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

**Definition of** $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (\nu_0)^{(7)}$ :-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$$(m_2)^{(7)} = (\nu_0)^{(7)}, (m_1)^{(7)} = (\nu_1)^{(7)}, \ \boldsymbol{if}\ (\nu_0)^{(7)} < (\nu_1)^{(7)}$$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\bar{\nu}_1)^{(7)}, \boldsymbol{if}\ (\nu_1)^{(7)} < (\nu_0)^{(7)} < (\bar{\nu}_1)^{(7)},$$

and $\boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\nu_0)^{(7)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(7)} < (\nu_0)^{(7)}$$

and analogously 363

$$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}, \ \boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$$

and $\boxed{(u_0)^{(7)} = \frac{T^0_{36}}{T^0_{37}}}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)}$ where $(u_1)^{(7)}, (\bar{u}_1)^{(7)}$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^0 e^{\left((S_1)^{(7)} - (p_{36})^{(7)}\right)t} \leq G_{36}(t) \leq G_{36}^0 e^{(S_1)^{(7)} t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{\left((S_1)^{(7)} - (p_{36})^{(7)}\right)t} \leq G_{37}(t) \leq \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)} t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)}\left((S_1)^{(7)} - (p_{36})^{(7)} - (S_2)^{(7)}\right)}\left[e^{\left((S_1)^{(7)} - (p_{36})^{(7)}\right)t} - e^{-(S_2)^{(7)} t}\right] + G_{38}^0 e^{-(S_2)^{(7)} t} \leq G_{38}(t) \leq \right. \quad 366$$

$$\left. \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)}\left((S_1)^{(7)} - (a'_{38})^{(7)}\right)}\left[e^{(S_1)^{(7)} t} - e^{-(a'_{38})^{(7)} t}\right] + G_{38}^0 e^{-(a'_{38})^{(7)} t}\right)$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)} t} \leq T_{36}(t) \leq T_{36}^0 e^{\left((R_1)^{(7)} + (r_{36})^{(7)}\right)t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)} t} \leq T_{36}(t) \leq \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{\left((R_1)^{(7)} + (r_{36})^{(7)}\right)t} \quad 368$$

$$\frac{(b_{38})^{(7)} T_{36}^0}{(\mu_1)^{(7)}\left((R_1)^{(7)} - (b'_{38})^{(7)}\right)}\left[e^{(R_1)^{(7)} t} - e^{-(b'_{38})^{(7)} t}\right] + T_{38}^0 e^{-(b'_{38})^{(7)} t} \leq T_{38}(t) \leq \quad 369$$

$$\frac{(a_{38})^{(7)} T_{36}^0}{(\mu_2)^{(7)}\left((R_1)^{(7)} + (r_{36})^{(7)} + (R_2)^{(7)}\right)}\left[e^{\left((R_1)^{(7)} + (r_{36})^{(7)}\right)t} - e^{-(R_2)^{(7)} t}\right] + T_{38}^0 e^{-(R_2)^{(7)} t}$$

**Definition of** $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$$

$$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$$

$$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$$

**Behavior of the solutions of equation** 371

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ :

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$$-(\sigma_2)^{(8)} \leq -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \leq -(\sigma_1)^{(8)}$$

$$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\big((G_{43}),t\big) - (b''_{41})^{(8)}\big((G_{43}),t\big) \le -(\tau_1)^{(8)}$$

**Definition of** $(\nu_1)^{(8)}, (\nu_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, \nu^{(8)}, u^{(8)}$ : 372

By $(\nu_1)^{(8)} > 0$ , $(\nu_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0$ , $(u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(8)},, (\bar{\nu}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$ :

By $(\bar{\nu}_1)^{(8)} > 0$ , $(\bar{\nu}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0$ , $(\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

**Definition of** $(m_1)^{(8)}$ , $(m_2)^{(8)}$ , $(\mu_1)^{(8)}, (\mu_2)^{(8)}, (\nu_0)^{(8)}$ :-

If we define $(m_1)^{(8)}$ , $(m_2)^{(8)}$ , $(\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$(m_2)^{(8)} = (\nu_0)^{(8)}, (m_1)^{(8)} = (\nu_1)^{(8)}$, ***if*** $(\nu_0)^{(8)} < (\nu_1)^{(8)}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ , ***if*** $(\nu_1)^{(8)} < (\nu_0)^{(8)} < (\bar{\nu}_1)^{(8)}$,
and $\boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\nu_0)^{(8)}$, ***if*** $(\bar{\nu}_1)^{(8)} < (\nu_0)^{(8)}$

and analogously 374

$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)}$, ***if*** $(u_0)^{(8)} < (u_1)^{(8)}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}$ , ***if*** $(u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)}$,
and $\boxed{(u_0)^{(8)} = \frac{T^0_{40}}{T^0_{41}}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}$, ***if*** $(\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G^0_{40}e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{40}(t) \le G^0_{40}e^{(S_1)^{(8)}t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}} G_{40}^0 e^{((S_1)^{(8)}-(p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}} G_{40}^0 e^{(S_1)^{(8)}t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)}G_{40}^0}{(m_1)^{(8)}((S_1)^{(8)}-(p_{40})^{(8)}-(S_2)^{(8)})}\left[e^{((S_1)^{(8)}-(p_{40})^{(8)})t} - e^{-(S_2)^{(8)}t}\right] + G_{42}^0 e^{-(S_2)^{(8)}t} \le G_{42}(t) \le \right. \quad 377$$

$$\left. \frac{(a_{42})^{(8)}G_{40}^0}{(m_2)^{(8)}((S_1)^{(8)}-(a'_{42})^{(8)})}[e^{(S_1)^{(8)}t} - e^{-(a'_{42})^{(8)}t}] + G_{42}^0 e^{-(a'_{42})^{(8)}t}\right)$$

$$\boxed{T_{40}^0 e^{(R_1)^{(8)}t} \le T_{40}(t) \le T_{40}^0 e^{((R_1)^{(8)}+(r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}} T_{40}^0 e^{(R_1)^{(8)}t} \le T_{40}(t) \le \frac{1}{(\mu_2)^{(8)}} T_{40}^0 e^{((R_1)^{(8)}+(r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)}T_{40}^0}{(\mu_1)^{(8)}((R_1)^{(8)}-(b'_{42})^{(8)})}\left[e^{(R_1)^{(8)}t} - e^{-(b'_{42})^{(8)}t}\right] + T_{42}^0 e^{-(b'_{42})^{(8)}t} \le T_{42}(t) \le \quad 380$$

$$\frac{(a_{42})^{(8)}T_{40}^0}{(\mu_2)^{(8)}((R_1)^{(8)}+(r_{40})^{(8)}+(R_2)^{(8)})}\left[e^{((R_1)^{(8)}+(r_{40})^{(8)})t} - e^{-(R_2)^{(8)}t}\right] + T_{42}^0 e^{-(R_2)^{(8)}t}$$

**Definition of** $(S_1)^{(8)}, (S_2)^{(8)}, (R_1)^{(8)}, (R_2)^{(8)}$:- 381

Where $(S_1)^{(8)} = (a_{40})^{(8)}(m_2)^{(8)} - (a'_{40})^{(8)}$

$(S_2)^{(8)} = (a_{42})^{(8)} - (p_{42})^{(8)}$

$(R_1)^{(8)} = (b_{40})^{(8)}(\mu_2)^{(8)} - (b'_{40})^{(8)}$

$(R_2)^{(8)} = (b'_{42})^{(8)} - (r_{42})^{(8)}$

**Behavior of the solutions of equation 37 to 92** 382

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ :

$(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ four constants satisfying

$$-(\sigma_2)^{(9)} \le -(a'_{44})^{(9)} + (a'_{45})^{(9)} - (a''_{44})^{(9)}(T_{45}, t) + (a''_{45})^{(9)}(T_{45}, t) \le -(\sigma_1)^{(9)}$$

$$-(\tau_2)^{(9)} \le -(b'_{44})^{(9)} + (b'_{45})^{(9)} - (b''_{44})^{(9)}\big((G_{47}), t\big) - (b''_{45})^{(9)}\big((G_{47}), t\big) \le -(\tau_1)^{(9)}$$

**Definition of** $(\nu_1)^{(9)}, (\nu_2)^{(9)}, (u_1)^{(9)}, (u_2)^{(9)}, \nu^{(9)}, u^{(9)}$ :

By $(\nu_1)^{(9)} > 0$ , $(\nu_2)^{(9)} < 0$ and respectively $(u_1)^{(9)} > 0$ , $(u_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_1)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(9)}, , (\bar{\nu}_2)^{(9)}, (\bar{u}_1)^{(9)}, (\bar{u}_2)^{(9)}$ :

By $(\bar{\nu}_1)^{(9)} > 0$ , $(\bar{\nu}_2)^{(9)} < 0$ and respectively $(\bar{u}_1)^{(9)} > 0$ , $(\bar{u}_2)^{(9)} < 0$ the

roots of the equations $(a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\left(u^{(9)}\right)^2 + (\tau_2)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$

**Definition of** $(m_1)^{(9)}, (m_2)^{(9)}, (\mu_1)^{(9)}, (\mu_2)^{(9)}, (v_0)^{(9)}$ :-

If we define $(m_1)^{(9)}, (m_2)^{(9)}, (\mu_1)^{(9)}, (\mu_2)^{(9)}$ by

$(m_2)^{(9)} = (v_0)^{(9)}, (m_1)^{(9)} = (v_1)^{(9)}$, ***if*** $(v_0)^{(9)} < (v_1)^{(9)}$

$(m_2)^{(9)} = (v_1)^{(9)}, (m_1)^{(9)} = (\bar{v}_1)^{(9)}$, ***if*** $(v_1)^{(9)} < (v_0)^{(9)} < (\bar{v}_1)^{(9)}$,
and $\boxed{(v_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$

$(m_2)^{(9)} = (v_1)^{(9)}, (m_1)^{(9)} = (v_0)^{(9)}$, ***if*** $(\bar{v}_1)^{(9)} < (v_0)^{(9)}$

and analogously

$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)}$, ***if*** $(u_0)^{(9)} < (u_1)^{(9)}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}$, ***if*** $(u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)}$,
and $\boxed{(u_0)^{(9)} = \frac{T_{44}^0}{T_{45}^0}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}$, ***if*** $(\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{44}(t) \le G_{44}^0 e^{(S_1)^{(9)}t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}} G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{45}(t) \le \frac{1}{(m_2)^{(9)}} G_{44}^0 e^{(S_1)^{(9)}t}$$

$$\left(\frac{(a_{46})^{(9)} G_{44}^0}{(m_1)^{(9)}((S_1)^{(9)} - (p_{44})^{(9)} - (S_2)^{(9)})}\left[e^{((S_1)^{(9)} - (p_{44})^{(9)})t} - e^{-(S_2)^{(9)}t}\right] + G_{46}^0 e^{-(S_2)^{(9)}t} \le G_{46}(t) \le \right.$$
$$\left.\frac{(a_{46})^{(9)} G_{44}^0}{(m_2)^{(9)}((S_1)^{(9)} - (a'_{46})^{(9)})}\left[e^{(S_1)^{(9)}t} - e^{-(a'_{46})^{(9)}t}\right] + G_{46}^0 e^{-(a'_{46})^{(9)}t}\right)$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}} T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le \frac{1}{(\mu_2)^{(9)}} T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}$$

$$\frac{(b_{46})^{(9)} T_{44}^0}{(\mu_1)^{(9)}((R_1)^{(9)} - (b'_{46})^{(9)})}\left[e^{(R_1)^{(9)}t} - e^{-(b'_{46})^{(9)}t}\right] + T_{46}^0 e^{-(b'_{46})^{(9)}t} \le T_{46}(t) \le$$

$$\frac{(a_{46})^{(9)} T_{44}^0}{(\mu_2)^{(9)}((R_1)^{(9)} + (r_{44})^{(9)} + (R_2)^{(9)})}\left[e^{((R_1)^{(9)} + (r_{44})^{(9)})t} - e^{-(R_2)^{(9)}t}\right] + T_{46}^0 e^{-(R_2)^{(9)}t}$$

**Definition of** $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$

**Proof :** From global equations we obtain 383

$$\frac{d\nu^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right) - (a''_{14})^{(1)}(T_{14}, t)\nu^{(1)} - (a_{14})^{(1)}\nu^{(1)}$$

**Definition of** $\nu^{(1)}$ :- $\boxed{\nu^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)}\right) \leq \frac{d\nu^{(1)}}{dt} \leq -\left((a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(1)}, (\nu_0)^{(1)}$ :-

For $0 < \boxed{(\nu_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}} < (\nu_1)^{(1)} < (\bar{\nu}_1)^{(1)}$

$$\nu^{(1)}(t) \geq \frac{(\nu_1)^{(1)} + (C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)} - (\nu_0)^{(1)}\right)t\right]}}{1 + (C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)} - (\nu_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(\nu_1)^{(1)} - (\nu_0)^{(1)}}{(\nu_0)^{(1)} - (\nu_2)^{(1)}}}$$

it follows $(\nu_0)^{(1)} \leq \nu^{(1)}(t) \leq (\nu_1)^{(1)}$

In the same manner , we get 384

$$\nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)} + (\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)} - (\bar{\nu}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)} - (\bar{\nu}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{\nu}_1)^{(1)} - (\nu_0)^{(1)}}{(\nu_0)^{(1)} - (\bar{\nu}_2)^{(1)}}}$$

From which we deduce $(\nu_0)^{(1)} \leq \nu^{(1)}(t) \leq (\bar{\nu}_1)^{(1)}$

If $0 < (\nu_1)^{(1)} < (\nu_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}} < (\bar{\nu}_1)^{(1)}$ we find like in the previous case, 385

$$(\nu_1)^{(1)} \leq \frac{(\nu_1)^{(1)} + (C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)} - (\nu_2)^{(1)}\right)t\right]}}{1 + (C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)} - (\nu_2)^{(1)}\right)t\right]}} \leq \nu^{(1)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(1)} + (\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)} - (\bar{\nu}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)} - (\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\bar{\nu}_1)^{(1)}$$

If $0 < (\nu_1)^{(1)} \leq (\bar{\nu}_1)^{(1)} \leq \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$ , we obtain 386

$$(\nu_1)^{(1)} \leq \nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)} + (\bar{C})^{(1)}(\bar{\nu}_2)^{(1)} e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)} - (\bar{\nu}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)} e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)} - (\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\nu_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(1)}(t)$ :-

$$(m_2)^{(1)} \leq \nu^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{\nu^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(1)}(t)$ :-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{13}'')^{(1)} = (a_{14}'')^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)} G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b_{13}'')^{(1)} = (b_{14}'')^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)} T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

**Proof** : From global equations we obtain 387

$$\frac{d\nu^{(2)}}{dt} = (a_{16})^{(2)} - \left((a_{16}')^{(2)} - (a_{17}')^{(2)} + (a_{16}'')^{(2)}(T_{17}, t)\right) - (a_{17}'')^{(2)}(T_{17}, t)\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

**Definition of** $\nu^{(2)}$ :- $\boxed{\nu^{(2)} = \frac{G_{16}}{G_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \leq \frac{d\nu^{(2)}}{dt} \leq -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

**Definition of** $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$ :-

For $0 < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \geq \frac{(\nu_1)^{(2)}+(\mathrm{C})^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_0)^{(2)}\right)t\right]}}{1+(\mathrm{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\mathrm{C})^{(2)}=\frac{(\nu_1)^{(2)}-(\nu_0)^{(2)}}{(\nu_0)^{(2)}-(\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)}+(\bar{\mathrm{C}})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{\mathrm{C}})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{\mathrm{C}})^{(2)}=\frac{(\bar{\nu}_1)^{(2)}-(\nu_0)^{(2)}}{(\nu_0)^{(2)}-(\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{\mathrm{G}_{16}^0}{\mathrm{G}_{17}^0} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \leq \frac{(\nu_1)^{(2)}+(\mathrm{C})^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}}{1+(\mathrm{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}} \leq \nu^{(2)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(2)}+(\bar{\mathrm{C}})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{\mathrm{C}})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \leq (\bar{\nu}_1)^{(2)} \leq (\nu_0)^{(2)} = \frac{\mathrm{G}_{16}^0}{\mathrm{G}_{17}^0}$ , we obtain 394

$$(\nu_1)^{(2)} \leq \nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)}+(\bar{\mathrm{C}})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{\mathrm{C}})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(2)}(t)$ :- 395

$$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}, \quad \boxed{\nu^{(2)}(t)=\frac{G_{16}(t)}{G_{17}(t)}}$$

In a completely analogous way, we obtain 396

**Definition of** $u^{(2)}(t)$ :-

$$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}, \quad \boxed{u^{(2)}(t)=\frac{T_{16}(t)}{T_{17}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :** 397

If $(a_{16}'')^{(2)} = (a_{17}'')^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)}G_{17}(t)$

Analogously if $(b''_{16})^{(2)} = (b''_{17})^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)}T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

**Proof** : From global equations we obtain 398

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right) - (a''_{21})^{(3)}(T_{21}, t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

**Definition of** $\nu^{(3)}$ :- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right) \leq \frac{d\nu^{(3)}}{dt} \leq -\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right)$$

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From which one obtains

For $0 < (\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\nu_1)^{(3)} < (\bar{\nu}_1)^{(3)}$

$$\nu^{(3)}(t) \geq \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(\nu_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\nu_2)^{(3)}}}$$

it follows $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\nu_1)^{(3)}$

In the same manner , we get 401

$$\nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)} + (\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{\nu}_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\bar{\nu}_2)^{(3)}}}$$

**Definition of** $(\bar{\nu}_1)^{(3)}$ :-

From which we deduce $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\bar{\nu}_1)^{(3)}$

If $0 < (\nu_1)^{(3)} < (\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\bar{\nu}_1)^{(3)}$ we find like in the previous case, 402

$$(\nu_1)^{(3)} \leq \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_2)^{(3)}\right)t\right]}}{1 + (C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_2)^{(3)}\right)t\right]}} \leq \nu^{(3)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(3)} + (\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\bar{\nu}_1)^{(3)}$$

If $0 < (\nu_1)^{(3)} \leq (\bar{\nu}_1)^{(3)} \leq (\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}$ , we obtain 403

$$(\nu_1)^{(3)} \leq \nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\nu_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(3)}(t)$ :-

$$(m_2)^{(3)} \leq \nu^{(3)}(t) \leq (m_1)^{(3)}, \quad \boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(3)}(t)$ :-

$$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}, \quad \boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{20})^{(3)} = (a''_{21})^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)}G_{21}(t)$

Analogously if $(b''_{20})^{(3)} = (b''_{21})^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)}T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

**Proof** : From global equations we obtain 404

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)\right) - (a''_{25})^{(4)}(T_{25}, t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

**Definition of** $\nu^{(4)}$ :- $\quad \boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right) \leq \frac{d\nu^{(4)}}{dt} \leq -\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_4)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$ :-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \geq \frac{(\nu_1)^{(4)}+(C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_0)^{(4)}\right)t\right]}}{4+(C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)}-(\nu_0)^{(4)}}{(\nu_0)^{(4)}-(\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{4+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)}-(\nu_0)^{(4)}}{(\nu_0)^{(4)}-(\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \leq \frac{(\nu_1)^{(4)}+(C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_2)^{(4)}\right)t\right]}}{1+(C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_2)^{(4)}\right)t\right]}} \leq \nu^{(4)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \leq (\bar{\nu}_1)^{(4)} \leq \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$ , we obtain 407

$$(\nu_1)^{(4)} \leq \nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(4)}(t)$ :-

$$(m_2)^{(4)} \leq \nu^{(4)}(t) \leq (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(4)}(t)$ :-

$$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a_{24}'')^{(4)} = (a_{25}'')^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)}G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b_{24}'')^{(4)} = (b_{25}'')^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)}T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

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**Proof** : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a_{28}')^{(5)} - (a_{29}')^{(5)} + (a_{28}'')^{(5)}(T_{29}, t)\right) - (a_{29}'')^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

**Definition of** $\nu^{(5)}$ :- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$ :-

For $0 < \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \leq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}}{1 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}} \leq \nu^{(5)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \leq (\bar{\nu}_1)^{(5)} \leq \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$ , we obtain 411

$$(\nu_1)^{(5)} \leq \nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(5)}(t)$ :-

$(m_2)^{(5)} \leq \nu^{(5)}(t) \leq (m_1)^{(5)}$, $\boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$

In a completely analogous way, we obtain

**Definition of** $u^{(5)}(t)$ :-

$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}$, $\boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{28})^{(5)} = (a''_{29})^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)} G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b''_{28})^{(5)} = (b''_{29})^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$ if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)} T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

**Proof** : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a'_{32})^{(6)} - (a'_{33})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right) - (a''_{33})^{(6)}(T_{33}, t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

**Definition of** $\nu^{(6)}$ :- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \leq \frac{d\nu^{(6)}}{dt} \leq -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$ :-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \geq \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\nu_1)^{(6)}$

In the same manner , we get 413

$$\nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{\nu}_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\bar{\nu}_1)^{(6)}$

If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 414

$$(\nu_1)^{(6)} \leq \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}} \leq \nu^{(6)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\bar{\nu}_1)^{(6)}$$

If $0 < (\nu_1)^{(6)} \leq (\bar{\nu}_1)^{(6)} \leq \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$, we obtain

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$$(\nu_1)^{(6)} \leq \nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(6)}(t)$ :-

$$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}, \quad \boxed{\nu^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(6)}(t)$ :-

$$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}, \quad \boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)}G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)}T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

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**Proof** : From global equations we obtain

$$\frac{d\nu^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right) - (a''_{37})^{(7)}(T_{37}, t)\nu^{(7)} - (a_{37})^{(7)}\nu^{(7)}$$

**Definition of** $\nu^{(7)}$ :- $\boxed{\nu^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right) \leq \frac{d\nu^{(7)}}{dt} \leq -\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(7)}, (\nu_0)^{(7)}$ :-

For $0 < \boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}} < (\nu_1)^{(7)} < (\bar{\nu}_1)^{(7)}$

$$\nu^{(7)}(t) \geq \frac{(\nu_1)^{(7)}+(C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_0)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)}=\frac{(\nu_1)^{(7)}-(\nu_0)^{(7)}}{(\nu_0)^{(7)}-(\nu_2)^{(7)}}}$$

it follows $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\nu_1)^{(7)}$

In the same manner , we get 417

$$\nu^{(7)}(t) \leq \frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)}=\frac{(\bar{\nu}_1)^{(7)}-(\nu_0)^{(7)}}{(\nu_0)^{(7)}-(\bar{\nu}_2)^{(7)}}}$$

From which we deduce $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\bar{\nu}_1)^{(7)}$

If $0 < (\nu_1)^{(7)} < (\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0} < (\bar{\nu}_1)^{(7)}$ we find like in the previous case, 418

$$(\nu_1)^{(7)} \leq \frac{(\nu_1)^{(7)}+(C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}} \leq \nu^{(7)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \leq (\bar{\nu}_1)^{(7)}$$

If $0 < (\nu_1)^{(7)} \leq (\bar{\nu}_1)^{(7)} \leq \boxed{(\nu_0)^{(7)}=\frac{G_{36}^0}{G_{37}^0}}$ , we obtain 419

$$(\nu_1)^{(7)} \leq \nu^{(7)}(t) \leq \frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \leq (\nu_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(7)}(t)$ :-

$$(m_2)^{(7)} \leq \nu^{(7)}(t) \leq (m_1)^{(7)}, \quad \boxed{\nu^{(7)}(t)=\frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(7)}(t)$ :- 420

$$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}, \quad \boxed{u^{(7)}(t)=\frac{T_{36}(t)}{T_{37}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $\nu^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)}G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for**

**the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)}T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

**Proof** : From global equations we obtain 421

$$\frac{d\nu^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right) - (a''_{41})^{(8)}(T_{41}, t)\nu^{(8)} - (a_{41})^{(8)}\nu^{(8)}$$

**Definition of** $\nu^{(8)}$ :- $\boxed{\nu^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right) \le \frac{d\nu^{(8)}}{dt} \le -\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$ :-

For $0 < \boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$

$$\nu^{(8)}(t) \ge \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \le \nu^{(8)}(t) \le (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \le \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \le \nu^{(8)}(t) \le (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \le \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}} \le \nu^{(8)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \le (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \le (\bar{\nu}_1)^{(8)} \le \boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}}$ , we obtain 424

$$(v_1)^{(8)} \leq v^{(8)}(t) \leq \frac{(\bar{v}_1)^{(8)}+(\bar{C})^{(8)}(\bar{v}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{v}_1)^{(8)}-(\bar{v}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{v}_1)^{(8)}-(\bar{v}_2)^{(8)}\right)t\right]}} \leq (v_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $v^{(8)}(t)$ :-

$$(m_2)^{(8)} \leq v^{(8)}(t) \leq (m_1)^{(8)}, \quad \boxed{v^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(8)}(t)$ :-

$$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}, \quad \boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(v_1)^{(8)} = (\bar{v}_1)^{(8)}$ if in addition $(v_0)^{(8)} = (v_1)^{(8)}$ then $v^{(8)}(t) = (v_0)^{(8)}$ and as a consequence $G_{40}(t) = (v_0)^{(8)}G_{41}(t)$ **this also defines** $(v_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then $(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)}T_{41}(t)$ This is an important consequence of the relation between $(v_1)^{(8)}$ and $(\bar{v}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

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**Proof** : From 99,20,44,22,23,44 we obtain

$$\frac{dv^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right) - (a''_{45})^{(9)}(T_{45},t)v^{(9)} - (a_{45})^{(9)}v^{(9)}$$

**Definition of** $v^{(9)}$ :- $\boxed{v^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)}\right) \leq \frac{dv^{(9)}}{dt} \leq -\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_1)^{(9)}v^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

**Definition of** $(\bar{v}_1)^{(9)}, (v_0)^{(9)}$ :-

For $0 < \boxed{(v_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (v_1)^{(9)} < (\bar{v}_1)^{(9)}$

$$v^{(9)}(t) \geq \frac{(v_1)^{(9)}+(C)^{(9)}(v_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((v_1)^{(9)}-(v_0)^{(9)}\right)t\right]}}{1+(C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((v_1)^{(9)}-(v_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(v_1)^{(9)}-(v_0)^{(9)}}{(v_0)^{(9)}-(v_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\nu_9)^{(9)}$

In the same manner , we get

$$\nu^{(9)}(t) \leq \frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)}-(\nu_0)^{(9)}}{(\nu_0)^{(9)}-(\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \leq \frac{(\nu_1)^{(9)}+(C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}}{1+(C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}} \leq \nu^{(9)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \leq (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \leq (\bar{\nu}_1)^{(9)} \leq \boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$ , we obtain

$$(\nu_1)^{(9)} \leq \nu^{(9)}(t) \leq \frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \leq (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(9)}(t)$ :-

$$(m_2)^{(9)} \leq \nu^{(9)}(t) \leq (m_1)^{(9)}, \quad \boxed{\nu^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(9)}(t)$ :-

$$(\mu_2)^{(9)} \leq u^{(9)}(t) \leq (\mu_1)^{(9)}, \quad \boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)}G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then $(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)}T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following

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**Theorem :** If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on $t$ , and the conditions with the notations

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0$ ,

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$

$with\ \ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(2)} and\ (b_i'')^{(2)}$ are independent on t , and the conditions with the notations 426

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$ 427

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 428

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$ , 429

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 430

$with\ \ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(3)} and\ (b_i'')^{(3)}$ are independent on $t$ , and the conditions with the notations 431

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$ ,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

**Theorem :** If $(a_i'')^{(4)} and\ (b_i'')^{(4)}$ are independent on $t$ , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$ ,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(5)} and\ (b_i'')^{(5)}$ are independent on $t$ , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$ ,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

**Theorem** If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on $t$ , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$ ,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(7)} and\ (b_i'')^{(7)}$ are independent on $t$ , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$ ,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(8)} and\ (b_i'')^{(8)}$ are independent on $t$ , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$ ,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(9)} and\ (b_i'')^{(9)}$ are independent on $t$ , and the conditions (with the notations 436 A
45,46,27,28)

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$ ,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0 \quad 437$$

$$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0 \quad 438$$

$$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0 \quad 439$$

$$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0 \quad 440$$

$$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ]T_{14} = 0 \quad 441$$

$$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0 \quad 442$$

has a unique positive solution , which is an equilibrium solution for the system

$$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0 \quad 443$$

$$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0 \quad 444$$

$$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0 \quad 445$$

$$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\ ]T_{16} = 0 \quad 446$$

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\ ]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\ ]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\ ]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\ ]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\ ]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27})\right)]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27})\right)]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27})\right)]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0 \quad 479$$

$$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0 \quad 480$$

$$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0 \quad 481$$

$$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0 \quad 482$$

$$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0 \quad 483$$

$$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0 \quad 484$$

$$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0 \quad 484\text{A}$$

$$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$$

$$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$$

$$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$$

$$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$$

$$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$$

**Proof:** 485

(a) Indeed the first two equations have a nontrivial solution $G_{13}, G_{14}$ if

$$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$$

**Proof:** 486

(ii) Indeed the first two equations have a nontrivial solution $G_{16}, G_{17}$ if

$$\mathrm{F}(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$$

**Proof:** 487

(a) Indeed the first two equations have a nontrivial solution $G_{20}, G_{21}$ if

$$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$$

**Proof:** 488

(a) Indeed the first two equations have a nontrivial solution $G_{24}, G_{25}$ if

$$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$$

**Proof:** 489

(a) Indeed the first two equations have a nontrivial solution $G_{28}, G_{29}$ if

$$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$$

**Proof:** 490

(a) Indeed the first two equations have a nontrivial solution $G_{32}, G_{33}$ if

$$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$$

**Proof:** 491

(a) Indeed the first two equations have a nontrivial solution $G_{36}, G_{37}$ if

$$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$$

**Proof:** 492

(a) Indeed the first two equations have a nontrivial solution $G_{40}, G_{41}$ if

$$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$$

**Proof:** 492 A

(a) Indeed the first two equations have a nontrivial solution $G_{44}, G_{45}$ if

$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$

**Definition and uniqueness of** $T^*_{14}$ :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique $T^*_{14}$ for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

**Definition and uniqueness of** $T^*_{17}$ :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique $T^*_{17}$ for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]} \qquad 495$$

**Definition and uniqueness of** $T^*_{21}$ :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique $T^*_{21}$ for which $f(T^*_{21}) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

**Definition and uniqueness of** $T^*_{25}$ :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that there exists a unique $T^*_{25}$ for which $f(T^*_{25}) = 0$. With this value , we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T^*_{25})]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T^*_{25})]}$$

**Definition and uniqueness of** $T^*_{29}$ :- 498

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(5)}(T_{29})$ being increasing, it follows that there exists a unique $T^*_{29}$ for which $f(T^*_{29}) = 0$. With this value , we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T^*_{29})]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T^*_{29})]}$$

**Definition and uniqueness of** $T^*_{33}$ :- 499

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(6)}(T_{33})$ being increasing, it follows that

there exists a unique $T_{33}^{*}$ for which $f(T_{33}^{*})=0$. With this value , we obtain from the three first equations

$$G_{32}=\frac{(a_{32})^{(6)}G_{33}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^{*})]} \quad , \quad G_{34}=\frac{(a_{34})^{(6)}G_{33}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^{*})]}$$

**Definition and uniqueness of** $T_{37}^{*}$ :- 500

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(7)}(T_{37})$ being increasing, it follows that there exists a unique $T_{37}^{*}$ for which $f(T_{37}^{*})=0$. With this value , we obtain from the three first equations

$$G_{36}=\frac{(a_{36})^{(7)}G_{37}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^{*})]} \quad , \quad G_{38}=\frac{(a_{38})^{(7)}G_{37}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^{*})]}$$

**Definition and uniqueness of** $T_{41}^{*}$ :- 501

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique $T_{41}^{*}$ for which $f(T_{41}^{*})=0$. With this value , we obtain from the three first equations

$$G_{40}=\frac{(a_{40})^{(8)}G_{41}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T_{41}^{*})]} \quad , \quad G_{42}=\frac{(a_{42})^{(8)}G_{41}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T_{41}^{*})]}$$

**Definition and uniqueness of** $T_{45}^{*}$ :- 501 A

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(9)}(T_{45})$ being increasing, it follows that there exists a unique $T_{45}^{*}$ for which $f(T_{45}^{*})=0$. With this value , we obtain from the three first equations

$$G_{44}=\frac{(a_{44})^{(9)}G_{45}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T_{45}^{*})]} \quad , \quad G_{46}=\frac{(a_{46})^{(9)}G_{45}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T_{45}^{*})]}$$

By the same argument, the equations admit solutions $G_{13}, G_{14}$ if 502

$$\varphi(G)=(b'_{13})^{(1)}(b'_{14})^{(1)}-(b_{13})^{(1)}(b_{14})^{(1)}-$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G)+(b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right]+(b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G)=0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{14}$ taking into account the hypothesis $\varphi(0)>0$ , $\varphi(\infty)<0$ it follows that there exists a unique $G_{14}^{*}$ such that $\varphi(G^{*})=0$

By the same argument, the equations admit solutions $G_{16}, G_{17}$ if 503

$$\varphi(G_{19})=(b'_{16})^{(2)}(b'_{17})^{(2)}-(b_{16})^{(2)}(b_{17})^{(2)}-$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19})+(b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right]+(b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19})=0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{17}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{14}^{*}$ such that $\varphi((G_{19})^{*}) = 0$ 504

By the same argument, the equations admit solutions $G_{20}, G_{21}$ if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right] + (b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{21}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{21}^{*}$ such that $\varphi((G_{23})^{*}) = 0$

By the same argument, the equations admit solutions $G_{24}, G_{25}$ if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right] + (b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{25}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{25}^{*}$ such that $\varphi((G_{27})^{*}) = 0$

By the same argument, the equations admit solutions $G_{28}, G_{29}$ if 507

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right] + (b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{29}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{29}^{*}$ such that $\varphi((G_{31})^{*}) = 0$

By the same argument, the equations admit solutions $G_{32}, G_{33}$ if 508

$$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$$

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right] + (b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{33}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{33}^{*}$ such that $\varphi({G_{35}}^{*}) = 0$

By the same argument, the equations admit solutions $G_{36}, G_{37}$ if 509

$$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$$

$$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right] + (b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{37}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{37}^{*}$ such that $\varphi({G_{39}}^{*}) = 0$

By the same argument, the equations admit solutions $G_{40}, G_{41}$ if 510

$$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$$
$$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right] + (b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{41}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{41}$ such that $\varphi(G_{43}{}^*) = 0$

By the same argument, the equations 92,93 admit solutions $G_{44}, G_{45}$ if

$$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$$

$$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right] + (b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{45}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{45}$ such that $\varphi((G_{47})^*) = 0$

Finally we obtain the unique solution 511

$G^*_{14}$ given by $\varphi(G^*) = 0$ , $T^*_{14}$ given by $f(T^*_{14}) = 0$ and

$$G^*_{13} = \frac{(a_{13})^{(1)}G^*_{14}}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G^*_{15} = \frac{(a_{15})^{(1)}G^*_{14}}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T^*_{14})]}$$

$$T^*_{13} = \frac{(b_{13})^{(1)}T^*_{14}}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^*)]} \quad , \quad T^*_{15} = \frac{(b_{15})^{(1)}T^*_{14}}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^*)]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

$G^*_{17}$ given by $\varphi((G_{19})^*) = 0$ , $T^*_{17}$ given by $f(T^*_{17}) = 0$ and 512

$$G^*_{16} = \frac{(a_{16})^{(2)}G^*_{17}}{[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G^*_{18} = \frac{(a_{18})^{(2)}G^*_{17}}{[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T^*_{17})]} \quad 513$$

$$T^*_{16} = \frac{(b_{16})^{(2)}T^*_{17}}{[(b'_{16})^{(2)} - (b''_{16})^{(2)}((G_{19})^*)]} \quad , \quad T^*_{18} = \frac{(b_{18})^{(2)}T^*_{17}}{[(b'_{18})^{(2)} - (b''_{18})^{(2)}((G_{19})^*)]} \quad 514$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

$G^*_{21}$ given by $\varphi((G_{23})^*) = 0$ , $T^*_{21}$ given by $f(T^*_{21}) = 0$ and

$$G^*_{20} = \frac{(a_{20})^{(3)}G^*_{21}}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G^*_{22} = \frac{(a_{22})^{(3)}G^*_{21}}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T^*_{21})]}$$

$$T^*_{20} = \frac{(b_{20})^{(3)}T^*_{21}}{[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}{}^*)]} \quad , \quad T^*_{22} = \frac{(b_{22})^{(3)}T^*_{21}}{[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}{}^*)]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

$G_{25}^{*}$ given by $\varphi(G_{27}) = 0$ , $T_{25}^{*}$ given by $f(T_{25}^{*}) = 0$ and

$$G_{24}^{*} = \frac{(a_{24})^{(4)} G_{25}^{*}}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}^{*} = \frac{(a_{26})^{(4)} G_{25}^{*}}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^{*})]}$$

$$T_{24}^{*} = \frac{(b_{24})^{(4)} T_{25}^{*}}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^{*})]} \quad , \quad T_{26}^{*} = \frac{(b_{26})^{(4)} T_{25}^{*}}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^{*})]} \qquad 517$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

$G_{29}^{*}$ given by $\varphi((G_{31})^{*}) = 0$ , $T_{29}^{*}$ given by $f(T_{29}^{*}) = 0$ and

$$G_{28}^{*} = \frac{(a_{28})^{(5)} G_{29}^{*}}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}^{*} = \frac{(a_{30})^{(5)} G_{29}^{*}}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^{*})]}$$

$$T_{28}^{*} = \frac{(b_{28})^{(5)} T_{29}^{*}}{[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31})^{*})]} \quad , \quad T_{30}^{*} = \frac{(b_{30})^{(5)} T_{29}^{*}}{[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31})^{*})]} \qquad 519$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

$G_{33}^{*}$ given by $\varphi((G_{35})^{*}) = 0$ , $T_{33}^{*}$ given by $f(T_{33}^{*}) = 0$ and

$$G_{32}^{*} = \frac{(a_{32})^{(6)} G_{33}^{*}}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^{*})]} \quad , \quad G_{34}^{*} = \frac{(a_{34})^{(6)} G_{33}^{*}}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^{*})]}$$

$$T_{32}^{*} = \frac{(b_{32})^{(6)} T_{33}^{*}}{[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35})^{*})]} \quad , \quad T_{34}^{*} = \frac{(b_{34})^{(6)} T_{33}^{*}}{[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35})^{*})]} \qquad 521$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

$G_{37}^{*}$ given by $\varphi((G_{39})^{*}) = 0$ , $T_{37}^{*}$ given by $f(T_{37}^{*}) = 0$ and

$$G_{36}^{*} = \frac{(a_{36})^{(7)} G_{37}^{*}}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^{*})]} \quad , \quad G_{38}^{*} = \frac{(a_{38})^{(7)} G_{37}^{*}}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^{*})]}$$

$$T_{36}^{*} = \frac{(b_{36})^{(7)} T_{37}^{*}}{[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39})^{*})]} \quad , \quad T_{38}^{*} = \frac{(b_{38})^{(7)} T_{37}^{*}}{[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39})^{*})]}$$

Finally we obtain the unique solution 523

$G_{41}^{*}$ given by $\varphi((G_{43})^{*}) = 0$ , $T_{41}^{*}$ given by $f(T_{41}^{*}) = 0$ and

$$G_{40}^{*} = \frac{(a_{40})^{(8)} G_{41}^{*}}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^{*})]} \quad , \quad G_{42}^{*} = \frac{(a_{42})^{(8)} G_{41}^{*}}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^{*})]}$$

$$T_{40}^{*} = \frac{(b_{40})^{(8)} T_{41}^{*}}{[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43})^{*})]} \quad , \quad T_{42}^{*} = \frac{(b_{42})^{(8)} T_{41}^{*}}{[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43})^{*})]}$$

Finally we obtain the unique solution of 89 to 99 523

A

$G_{45}^{*}$ given by $\varphi((G_{47})^{*}) = 0$ , $T_{45}^{*}$ given by $f(T_{45}^{*}) = 0$ and

$$G_{44}^{*} = \frac{(a_{44})^{(9)} G_{45}^{*}}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^{*})]} \quad , \quad G_{46}^{*} = \frac{(a_{46})^{(9)} G_{45}^{*}}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^{*})]}$$

$$T_{44}^{*} = \frac{(b_{44})^{(9)} T_{45}^{*}}{[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47})^{*})]} \quad , \quad T_{46}^{*} = \frac{(b_{46})^{(9)} T_{45}^{*}}{[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47})^{*})]}$$

## ASYMPTOTIC STABILITY ANALYSIS 524

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

**Proof:** Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^{*} + \mathbb{G}_i \qquad , T_i = T_i^{*} + \mathbb{T}_i$$

$$\frac{\partial (a''_{14})^{(1)}}{\partial T_{14}}(T_{14}^{*}) = (q_{14})^{(1)} \quad , \frac{\partial (b_i'')^{(1)}}{\partial G_j}(G^{*}) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a'_{13})^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)} G_{13}^{*}\mathbb{T}_{14} \quad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a'_{14})^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)} G_{14}^{*}\mathbb{T}_{14} \quad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a'_{15})^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)} G_{15}^{*}\mathbb{T}_{14} \quad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b'_{13})^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)} T_{13}^{*}\mathbb{G}_j\right) \quad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b'_{14})^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)} T_{14}^{*}\mathbb{G}_j\right) \quad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b'_{15})^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)} T_{15}^{*}\mathbb{G}_j\right) \quad 530$$

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**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(2)}$ and $(b_i'')^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i \tag{532}$$

$$\frac{\partial(a_{17}'')^{(2)}}{\partial T_{17}}(T_{17}^*) = (q_{17})^{(2)} \quad , \frac{\partial(b_i'')^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \tag{533}$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a_{16}')^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G_{16}^*\mathbb{T}_{17} \tag{534}$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a_{17}')^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G_{17}^*\mathbb{T}_{17} \tag{535}$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a_{18}')^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G_{18}^*\mathbb{T}_{17} \tag{536}$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b_{16}')^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T_{16}^*\mathbb{G}_j\right) \tag{537}$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b_{17}')^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}T_{17}^*\mathbb{G}_j\right) \tag{538}$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b_{18}')^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}T_{18}^*\mathbb{G}_j\right) \tag{539}$$

**ASYMPTOTIC STABILITY ANALYSIS** 540

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(3)}$ *and* $(b_i'')^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{21}'')^{(3)}}{\partial T_{21}}(T_{21}^*) = (q_{21})^{(3)} \quad , \frac{\partial(b_i'')^{(3)}}{\partial G_j}((G_{23})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a_{20}')^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G_{20}^*\mathbb{T}_{21} \tag{541}$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a_{21}')^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G_{21}^*\mathbb{T}_{21} \tag{542}$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a_{22}')^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G_{22}^*\mathbb{T}_{21} \tag{543}$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b_{20}')^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T_{20}^*\mathbb{G}_j\right) \tag{544}$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b_{21}')^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T_{21}^*\mathbb{G}_j\right) \tag{545}$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b'_{22})^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T^*_{22}\mathbb{G}_j\right) \quad 546$$

**ASYMPTOTIC STABILITY ANALYSIS** 547

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}\ and\ (b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{25})^{(4)}}{\partial T_{25}}(T^*_{25}) = (q_{25})^{(4)} \ , \ \frac{\partial(b_i'')^{(4)}}{\partial G_j}((G_{27})^*\ ) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\left((a'_{24})^{(4)} + (p_{24})^{(4)}\right)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G^*_{24}\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\left((a'_{25})^{(4)} + (p_{25})^{(4)}\right)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G^*_{25}\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\left((a'_{26})^{(4)} + (p_{26})^{(4)}\right)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G^*_{26}\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\left((b'_{24})^{(4)} - (r_{24})^{(4)}\right)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(24)(j)}T^*_{24}\mathbb{G}_j\right) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\left((b'_{25})^{(4)} - (r_{25})^{(4)}\right)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\left(s_{(25)(j)}T^*_{25}\mathbb{G}_j\right) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T^*_{26}\mathbb{G}_j\right) \quad 554$$

**ASYMPTOTIC STABILITY ANALYSIS** 555

**Theorem 5:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}\ and\ (b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 556

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{29})^{(5)}}{\partial T_{29}}(T^*_{29}) = (q_{29})^{(5)} \ , \ \frac{\partial(b_i'')^{(5)}}{\partial G_j}(\ (G_{31})^*\ ) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a'_{28})^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G^*_{28}\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a'_{29})^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G^*_{29}\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a'_{30})^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G^*_{30}\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b'_{28})^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T^*_{28}\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b'_{29})^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T^*_{29}\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b'_{30})^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T^*_{30}\mathbb{G}_j\right) \quad 562$$

**ASYMPTOTIC STABILITY ANALYSIS** 563

**Theorem 6:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}$ *and* $(b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{33}'')^{(6)}}{\partial T_{33}}(T^*_{33}) = (q_{33})^{(6)} \quad , \frac{\partial(b_i'')^{(6)}}{\partial G_j}\left((G_{35})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a'_{32})^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G^*_{32}\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a'_{33})^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G^*_{33}\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a'_{34})^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G^*_{34}\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b'_{32})^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T^*_{32}\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b'_{33})^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T^*_{33}\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b'_{34})^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T^*_{34}\mathbb{G}_j\right) \quad 570$$

**ASYMPTOTIC STABILITY ANALYSIS** 571

**Theorem 7:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 572

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{37}'')^{(7)}}{\partial T_{37}}(T_{37}^{*}) = (q_{37})^{(7)} \quad , \frac{\partial(b_i'')^{(7)}}{\partial G_j}\left( (G_{39})^{**} \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a_{36}')^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G_{36}^{*}\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a_{37}')^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G_{37}^{*}\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a_{38}')^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G_{38}^{*}\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b_{36}')^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T_{36}^{*}\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b_{37}')^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T_{37}^{*}\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b_{38}')^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T_{38}^{*}\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

## ASYMPTOTIC STABILITY ANALYSIS

**Theorem 8:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 580

$$G_i = G_i^{*} + \mathbb{G}_i \qquad , T_i = T_i^{*} + \mathbb{T}_i$$

$$\frac{\partial(a_{41}'')^{(8)}}{\partial T_{41}}(T_{41}^{*}) = (q_{41})^{(8)} \quad , \frac{\partial(b_i'')^{(8)}}{\partial G_j}\left( (G_{43})^{*} \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a_{40}')^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G_{40}^{*}\mathbb{T}_{41} \quad 581$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a_{41}')^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G_{41}^{*}\mathbb{T}_{41} \quad 582$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a_{42}')^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G_{42}^{*}\mathbb{T}_{41} \quad 583$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b_{40}')^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T_{40}^{*}\mathbb{G}_j\right) \quad 584$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b_{41}')^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T_{41}^{*}\mathbb{G}_j\right) \quad 585$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b'_{42})^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T^*_{42}\mathbb{G}_j\right)$$ 586

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**ASYMPTOTIC STABILITY ANALYSIS** A

**Theorem 9:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}\ and\ (b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

**Proof:** Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{45})^{(9)}}{\partial T_{45}}(T^*_{45}) = (q_{45})^{(9)} \ , \ \frac{\partial(b_i'')^{(9)}}{\partial G_j}((G_{47})^*) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a'_{44})^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)}G^*_{44}\mathbb{T}_{45}$$ 586 B

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a'_{45})^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)}G^*_{45}\mathbb{T}_{45}$$ 586 C

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a'_{46})^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)}G^*_{46}\mathbb{T}_{45}$$ 586 D

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b'_{44})^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)}T^*_{44}\mathbb{G}_j\right)$$ 586 E

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b'_{45})^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)}T^*_{45}\mathbb{G}_j\right)$$ 586 F

$$\frac{d\mathbb{T}_{46}}{dt} = -\left((b'_{46})^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T^*_{46}\mathbb{G}_j\right)$$ 586 G

**The characteristic equation of this system is** 587

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(q_{13})^{(1)}G^*_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)}T^*_{14} + (b_{14})^{(1)}s_{(13),(14)}T^*_{14}\right)$$

$$+\left(\left((\lambda)^{(1)} + (a'_{14})^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)}G^*_{13} + (a_{13})^{(1)}(q_{14})^{(1)}G^*_{14}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)}T^*_{14} + (b_{14})^{(1)}s_{(13),(13)}T^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((b'_{13})^{(1)} + (b'_{14})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^2+\left((a'_{13})^{(1)}+(a'_{14})^{(1)}+(p_{13})^{(1)}+(p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)}+(a'_{13})^{(1)}+(p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^*_{14}+(a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}+(b'_{13})^{(1)}-(r_{13})^{(1)}\right)s_{(14),(15)}T^*_{14}+(b_{14})^{(1)}s_{(13),(15)}T^*_{13}\right)\}=0$$

$$+$$

$$(\lambda)^{(2)}+(b'_{18})^{(2)}-(r_{18})^{(2)}\{\left((\lambda)^{(2)}+(a'_{18})^{(2)}+(p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^*_{17}+(a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^*_{17}\right)$$

$$+\left(\left((\lambda)^{(2)}+(a'_{17})^{(2)}+(p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^*_{16}+(a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2+\left((b'_{16})^{(2)}+(b'_{17})^{(2)}-(r_{16})^{(2)}+(r_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$+\left(\left((\lambda)^{(2)}\right)^2+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}+(a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^*_{16}\right)\}=0$$

$$+$$

$$(\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left(\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^*_{20}\right)\right]$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^*_{21}+(b_{21})^{(3)}s_{(20),(21)}T^*_{21}\right)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^*_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^*_{21}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^*_{21}+(b_{21})^{(3)}s_{(20),(20)}T^*_{20}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^2+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$+\left(\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^*_{20}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^*_{21}+(b_{21})^{(3)}s_{(20),(22)}T^*_{20}\right)\}=0$$

$$+$$

$$(\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left(\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^*_{24}\right)\right]$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^*_{25}+(b_{25})^{(4)}s_{(24),(25)}T^*_{25}\right)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^*_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^*_{25}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^*_{25}+(b_{25})^{(4)}s_{(24),(24)}T^*_{24}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^2+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$+\left(\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^*_{24}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^*_{25}+(b_{25})^{(4)}s_{(24),(26)}T^*_{24}\right)\}=0$$

$$+$$

$$(\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^*_{28}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^*_{29}+(b_{29})^{(5)}s_{(28),(29)}T^*_{29}\right)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^*_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^*_{29}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^*_{29}+(b_{29})^{(5)}s_{(28),(28)}T^*_{28}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^2+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$+\left(\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^*_{28}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^*_{29}+(b_{29})^{(5)}s_{(28),(30)}T^*_{28}\right)\}=0$$

$$+$$

$$(\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left(\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^*_{32}\right)\right]$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^*_{33}+(b_{33})^{(6)}s_{(32),(33)}T^*_{33}\right)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^*_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^*_{33}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^*_{33}+(b_{33})^{(6)}s_{(32),(32)}T^*_{32}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^2+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$+\left(\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^*_{32}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^*_{33}+(b_{33})^{(6)}s_{(32),(34)}T^*_{32}\right)\}=0$$

$$+$$

$$(\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^*_{36}\right)\right]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^*_{37}+(b_{37})^{(7)}s_{(36),(37)}T^*_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^*_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^*_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^*_{37}+(b_{37})^{(7)}s_{(36),(36)}T^*_{36}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^2+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$+\left(\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^*_{36}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^*_{37}+(b_{37})^{(7)}s_{(36),(38)}T^*_{36}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\right)\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^*_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^*_{41}+(b_{41})^{(8)}s_{(40),(41)}T^*_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^*_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^*_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^*_{41}+(b_{41})^{(8)}s_{(40),(40)}T^*_{40}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^2+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$+\left(\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^*_{40}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^*_{41}+(b_{41})^{(8)}s_{(40),(42)}T^*_{40}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\right)\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^*_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^*_{45}+(b_{45})^{(9)}s_{(44),(45)}T^*_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^*_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^*_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^*_{45}+(b_{45})^{(9)}s_{(44),(44)}T^*_{44}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^2+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$+\left(\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^*_{44}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^*_{45}+(b_{45})^{(9)}s_{(44),(46)}T^*_{44}\right)\}=0$$

**And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.**

## SECTION THIRTY SIX A

## Comparing Ourselves With The Environment Which Is Infinite We Become Aware Of The Fact That We Are Equal To Nothing

### INTRODUCTION—VARIABLES USED

**The conflict between entropy and syntropy: the vital needs model Ulisse Di Corpo ulisse.dicorpo@gmail.com**

(1) In this equation 1 symbolizes the living system which is finite (syntropy), while (e) Infinite symbolizes the environment (entropy).
(2) The comparison between the living system (1, finite) and the environment (infinite) tends to zero. In other words, comparing ourselves with the environment which is infinite we become aware of (e) the fact that we are equal to nothing.
(3) But to be equal to nothing is equivalent to (=) death, a fact which is incompatible with (e&eb) the feeling of life.
(4) It is therefore necessary to solve this conflict between being (1) and not being (0), a conflict which consumes (e) energy and increases entropy. This conflict is generally felt as the need to give a meaning to life, for example:
**(v) Increasing our own value (through richness, power, achievement, etc.);**
**(vi) Finding a purpose in life, finality (through ideologies, religion, etc.).**
(5) In living beings with highly complex cortical systems, this need is vital because, (e) when it is not solved, it leads to (eb) the dissipation of energy, and in the most serious cases to death.
(6) The existential crisis associated with (e&eb) this conflict is accompanied by (e&eb) feelings of being useless, purposeless, reduction of energy (dissipation of energy, entropy), usually named depression, felt in the cortical area in (eb) the form of tension, and usually strongly correlated with (e&eb) anxiety and feelings of pain in the thorax.

**The conflict between entropy and syntropy: the vital needs model Ulisse Di Corpo**

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(7) This strong correlation between depression and (e&eb) anxiety is suggested by the fact that, from a mathematical point of view, the conflict between being and (e&eb) not being is solved when: Where the operator x coincides with union, which is (=) the property of –E (converging waves).
(8) In other words, when we unite ourselves (1) to (e&eb) the environment, comparing ourselves to the environment, (eb) we find our identity (= 1).
(9) This last equation permits to state that:

• when the need of meaning is answered (eb) increasing the value of the numerator (power, richness, achievement), the identity conflict is not solved, because (e) whichever is the value at the numerator compared to (e&eb) infinite it tends to zero;

• Perfect correlation between anxiety and (e&eb) depression must be observed, because (e) when the unity (x) is weak, anxiety increases (eb) and also the identity conflict and depression.

**Chronological order**

(10) According to this model a chronological order exists in the satisfaction of these needs. Initially humans have (e) to answer material needs, which are more immediate, and only afterwards they become (eb) aware of the needs of meaning and unity.
(11) Part of humanity has now satisfied (eb) material needs and is now facing the needs of (e) meaning and unity and is undergoing rising feelings of existential crisis and anxiety.
(12) Material needs can be easily recognized, whereas the needs of meaning and unity are immaterial, invisible, and more difficult to be recognized. For example:

• In order to give a meaning to their lives, people become (=) obsessed with the way they are judged. They strive for (e) power, success. They become addicted to (e) religion, ideologies and groups;

• In order to overcome anxiety they become (eb) addicted to alcohol and drugs and avoid (e) any moment of silence.

(13) The model of vital needs affirms (eb) that all these states are (=) secondary needs which originate from (e) the unsatisfied needs of meaning and unity. According to the vital needs model all behaviours (psychological, sociological and economical) can be traced back to (e) these 3 basic needs.

**Empirical evidences**

Working as a sociologist for public institutions I am always faced with the centrality of the 3 vital needs which I have just described. For example:

• in a study, just concluded, for IOM (International Organization on Migration) results showed that whilst irregular migrants come to Italy for economical reasons (material needs), a significant number would go back home immediately, because of loneliness (need of unity) and depression (need of meaning), even though their economic situation at home has not changed. It seems that the experience in Italy and the difficulties associated to the integration process lead the migrant to rediscover a set of needs, qualitatively different from the material needs, which were met at home, thanks to the family and friends, but could not be met in Italy, causing feelings of loneliness, and depression.

• In a study conducted in the Abruzzi area of Italy, different theoretical models were compared in order to

understand the origins of psychological and emotional suffering in high school students. Results show that items relative to loneliness and worthlessness obtained the highest correlations with items relative to unhappiness and suffering (SERT-Teramo, 1993).

• In several studies conducted with drug addict populations, loneliness and worthlessness emerged as the two main reasons behind drug addiction. In all these studies the correlation among loneliness (anxiety) and worthlessness (depression) is nearly perfect, supporting the hypothesis of the existence of a mathematical relation between these two needs.

**Conclusions**

The introduction of advanced waves in science might not be (=) as important in physics as it could be in life disciplines such as medicine, psychology and sociology. These disciplines now approach pathologies, illnesses and crises in a mechanistic way which leads, (eb) in an always more alarming way, to inefficiency and to an increase in costs.

**NOTATION**

**Module One**

In this equation 1 symbolizes the living system which is finite (syntropy), while (e) Infinite symbolizes the environment (entropy)

$G_{13}$ : Category one of Infinite symbolizes the environment (entropy)

$G_{14}$ : Category two of SAS

$G_{15}$ : Category three of SAS

$T_{13}$ : Category one of equation 1 symbolizes the living system which is finite (syntropy

$T_{14}$ : Category two of SAS

$T_{15}$ : Category three of SAS

**Module Two**

The comparison between the living system (1, finite) and the environment (infinite) tends to zero.

In other words, comparing ourselves with the environment which is infinite we become aware of (e) the fact that we are equal to nothing

$G_{16}$ : Category one of comparing ourselves with the environment which is infinite

$G_{17}$ : Category two of SAS

$G_{18}$ : Category three of SAS

$T_{16}$ : Category one of we are equal to nothing (We interpret this as zero)

$T_{17}$ : Category two of SAS

$T_{18}$ : Category three of SAS

**Module three**

But to be equal to nothing is equivalent to (=) death, a fact which is incompatible with (e&eb) the feeling of

life

$G_{20}$ : Category one of equal to nothing

$G_{21}$ : Category two of SAS

$G_{22}$ : Category three of SAS

$T_{20}$ : Category one of death, a fact which is incompatible with the feeling of life

$T_{21}$ : Category two of SAS

$T_{22}$ : Category three of SAS

**Module four**

This conflict is generally felt as the need to give a meaning to life, for example:

**(vii) Increasing our own value (through richness, power, achievement, etc.)**
**(viii)**
(ix) **Finding a purpose in life, finality (through ideologies, religion, etc.)**

$G_{24}$ : Category one of conflict is generally felt

$G_{25}$ : Category two of SAS

$G_{26}$ : Category three of SAS

$T_{24}$ : Category one of meaning to life **Increasing our own value (through richness, power, achievement, etc.) Finding a purpose in life, finality (through ideologies, religion, etc.)**

$T_{25}$ : Category two of SAS

$T_{26}$ : Category three of SAS

**Module five**

In living beings with highly complex cortical systems, this need is vital because, when it is not solved, it leads to (eb) the dissipation of energy, and in the most serious cases to death

$G_{28}$ : Category one of living beings with highly complex cortical systems, this need is vital because, when it is not solved

$G_{29}$ : Category two of SAS

$G_{30}$ : Category three of SAS

$T_{28}$ : Category one of dissipation of energy, and in the most serious cases to death

$T_{29}$ : Category two of SAS

$T_{30}$ : Category three of SAS

**Module six**

The **existential crisis associated with this conflict** is accompanied by (e&eb) feelings of being useless, purposeless, reduction of energy (dissipation of energy, entropy), usually named depression, felt in the cortical area in (eb) the form of tension, and usually strongly correlated with (e&eb) anxiety and feelings of pain in the thorax

$G_{32}$ : Category one of **existential crisis associated with this conflict;** feelings of being useless, purposeless, reduction of energy (dissipation of energy, entropy), usually named depression, felt in the cortical area in (eb) the form of tension, and usually strongly correlated with (e&eb) anxiety and feelings of pain in the thorax

$G_{33}$ : Category two of SAS

$G_{34}$ : Category three of SAS

$T_{32}$ : Category one of feelings of being useless, purposeless, reduction of energy (dissipation of energy, entropy), usually named depression, felt in the cortical area in (eb) the form of tension, and usually strongly correlated with (e&eb) anxiety and feelings of pain in the thorax; **existential crisis associated with this conflict**

$T_{33}$ : Category two of SAS

$T_{34}$ : Category three of SAS

**Module seven**

The existential crisis associated with this conflict is accompanied by feelings of being useless, purposeless, reduction of energy (dissipation of energy, entropy), usually named depression, felt in the cortical area in (eb) the form of tension, and usually strongly correlated with (e&eb) anxiety and feelings of pain in the thorax

$G_{36}$ : Category one of existential crisis associated with this conflict is accompanied by feelings of being useless, purposeless, reduction of energy (dissipation of energy, entropy), usually named depression, felt in the cortical area

$G_{37}$ : Category two of SAS

$G_{38}$ : Category three of SAS

$T_{36}$ : Category one of form of tension, and usually strongly correlated with (e&eb) anxiety and feelings of pain in the thorax

$T_{37}$ : Category two of SAS

$T_{38}$ : Category three of SAS

**Module eight**

The existential crisis associated with this conflict is accompanied by feelings of being useless, purposeless, reduction of energy (dissipation of energy, entropy), usually named depression, felt in the cortical area in the form of tension, and usually strongly correlated with (e&eb) **anxiety and feelings of pain in the thorax**

$G_{40}$ : Category one of existential crisis associated with this conflict is accompanied by feelings of being useless, purposeless, reduction of energy (dissipation of energy, entropy), usually named depression, felt in the cortical area in the form of tension; **anxiety and feelings of pain in the thorax**

$G_{41}$ : Category two of SAS

$G_{42}$ : Category three of SAS

$T_{40}$ : Category one of **anxiety and feelings of pain in the thorax;** existential crisis associated with this conflict is accompanied by feelings of being useless, purposeless, reduction of energy (dissipation of energy, entropy), usually named depression, felt in the cortical area in the form of tension

$T_{41}$ : Category two of SAS

$T_{42}$ : Category three of SAS

**Module Nine**

This strong correlation between depression and (e&eb) anxiety is suggested by the fact that, from a mathematical point of view, the conflict between being and (e&eb) not being is solved when:

Where the operator x coincides with union, which is (=) the property of –E (converging waves)

$G_{44}$ : Category one of operator x coincides with union

$G_{45}$ : Category two of SAS

$G_{46}$ : Category three of SAS

$T_{44}$ : Category one of property of –E (converging waves

$T_{45}$ : Category two of SAS

$T_{46}$ : Category three of SAS

**The Coefficients:**

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}$ , $(b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)},$
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}, \ (b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}\ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$ ,

$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)},$
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)},$
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)},$
are Dissipation coefficients

**Module Numbered One**

**The differential system of this model is now (Module Numbered one)**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \quad 1$$
$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad 2$$
$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15} \quad 3$$
$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13} \quad 4$$
$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14} \quad 5$$
$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)\right]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14}, t) =$ First augmentation factor
$-(b''_{13})^{(1)}(G, t) =$ First detritions factor

**Module Numbered Two**

**The differential system of this model is now ( Module numbered two)**

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right]G_{16} \quad 7$$
$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t)\right]G_{17} \quad 8$$
$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t)\right]G_{18} \quad 9$$
$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\left((G_{19}), t\right)\right]T_{16} \quad 10$$
$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\left((G_{19}), t\right)\right]T_{17} \quad 11$$
$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\left((G_{19}), t\right)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17}, t) =$ First augmentation factor
$-(b''_{16})^{(2)}\left((G_{19}), t\right) =$ First detritions factor

**Module Numbered Three**

**The differential system of this model is now (Module numbered three)**

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right]G_{20} \quad 13$$
$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}, t)\right]G_{21} \quad 14$$
$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}, t)\right]G_{22} \quad 15$$
$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}, t)\right]T_{20} \quad 16$$
$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}, t)\right]T_{21} \quad 17$$
$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}, t)\right]T_{22} \quad 18$$

$+(a''_{20})^{(3)}(T_{21}, t) =$ First augmentation factor
$-(b''_{20})^{(3)}(G_{23}, t) =$ First detritions factor

**Module Numbered Four**

**The differential system of this model is now (Module numbered Four)**

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right]G_{24} \quad 19$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25},t)\right]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25},t)\right]G_{26} \quad 21$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27}),t)\right]T_{24} \quad 22$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}((G_{27}),t)\right]T_{25} \quad 23$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27}),t)\right]T_{26} \quad 24$$

$+(a''_{24})^{(4)}(T_{25},t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}((G_{27}),t) =$ **First detritions factor**

**Module Numbered Five**:

**The differential system of this model is now (Module number five)**

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29},t)\right]G_{28} \quad 25$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29},t)\right]G_{29} \quad 26$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29},t)\right]G_{30} \quad 27$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31}),t)\right]T_{28} \quad 28$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}((G_{31}),t)\right]T_{29} \quad 29$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31}),t)\right]T_{30} \quad 30$$

$+(a''_{28})^{(5)}(T_{29},t) =$ **First augmentation factor**

$-(b''_{28})^{(5)}((G_{31}),t) =$ **First detritions factor**

**Module Numbered Six**

**The differential system of this model is now (Module numbered Six)**

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33},t)\right]G_{32} \quad 31$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33},t)\right]G_{33} \quad 32$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33},t)\right]G_{34} \quad 33$$

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35}),t)\right]T_{32} \quad 34$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}((G_{35}),t)\right]T_{33} \quad 35$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35}),t)\right]T_{34} \quad 36$$

$+(a''_{32})^{(6)}(T_{33},t) =$ **First augmentation factor**

**Module Numbered Seven:**

**The differential system of this model is now (Seventh Module)**

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right]G_{36} \quad 37$$

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37},t)\right]G_{37} \quad 38$$

$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37},t)\right]G_{38} \quad 39$$

$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39}),t)\right]T_{36} \quad 40$$

$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}((G_{39}),t)\right]T_{37} \quad 41$$

$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39}),t)\right]T_{38} \quad 42$$

$+(a''_{36})^{(7)}(T_{37},t) =$ **First augmentation factor**

**Module Numbered Eight**

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right]G_{40} \quad 43$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41},t)\right]G_{41} \quad 44$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41},t)\right]G_{42} \quad 45$$

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43}),t)\right]T_{40} \quad 46$$

$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}((G_{43}),t)\right]T_{41} \quad 47$$

$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43}),t)\right]T_{42} \quad 48$$

**Module Numbered Nine**

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47}),t)\right]T_{44} \quad 52$$

$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}((G_{47}),t)\right]T_{45} \quad 53$$

$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47}),t)\right]T_{46} \quad 54$$

$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}((G_{47}),t) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)} & \boxed{+(a''_{13})^{(1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)} & \boxed{+(a''_{14})^{(1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)} & \boxed{+(a''_{15})^{(1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15} \quad 57$$

Where $\boxed{(a''_{13})^{(1)}(T_{14},t)}$, $\boxed{(a''_{14})^{(1)}(T_{14},t)}$, $\boxed{(a''_{15})^{(1)}(T_{14},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)}$ are third augmentation coefficient for

category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,)}(T_{25},t)$ , $+(a''_{25})^{(4,4,4,4,)}(T_{25},t)$ , $+(a''_{26})^{(4,4,4,4,)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,)}(T_{29},t)$ , $+(a''_{30})^{(5,5,5,5,)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,)}(T_{33},t)$ , $+(a''_{34})^{(6,6,6,6,)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{38})^{(7,7)}(T_{37},t)$ $+(a''_{37})^{(7,7)}(T_{37},t)$ $+(a''_{36})^{(7,7)}(T_{37},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{40})^{(8,8)}(T_{41},t)$ $+(a''_{41})^{(8,8)}(T_{41},t)$ $+(a''_{42})^{(8,8)}(T_{41},t)$ are eight augmentation coefficient for 1,2,3

$+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)} & -(b''_{13})^{(1)}(G,t) & -(b''_{16})^{(2,2,)}(G_{19},t) & -(b''_{20})^{(3,3,)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{36})^{(7,7,)}(G_{39},t) & -(b''_{40})^{(8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{13} \quad 58$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)} & -(b''_{14})^{(1)}(G,t) & -(b''_{17})^{(2,2,)}(G_{19},t) & -(b''_{21})^{(3,3,)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{37})^{(7,7,)}(G_{39},t) & -(b''_{41})^{(8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{14} \quad 59$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)} & -(b''_{15})^{(1)}(G,t) & -(b''_{18})^{(2,2,)}(G_{19},t) & -(b''_{22})^{(3,3,)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{38})^{(7,7,)}(G_{39},t) & -(b''_{42})^{(8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{15} \quad 60$$

Where $-(b''_{13})^{(1)}(G,t)$, $-(b''_{14})^{(1)}(G,t)$, $-(b''_{15})^{(1)}(G,t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,)}(G_{19},t)$, $-(b''_{17})^{(2,2,)}(G_{19},t)$, $-(b''_{18})^{(2,2,)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{37})^{(7,7,)}(G_{39},t)$, $-(b''_{36})^{(7,7,)}(G_{39},t)$, $-(b''_{38})^{(7,7,)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8)}(G_{43},t)$ $-(b''_{41})^{(8,8)}(G_{43},t)$ $-(b''_{42})^{(8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[\begin{array}{ccc} (a'_{16})^{(2)} & \boxed{+(a''_{16})^{(2)}(T_{17},t)} & \boxed{+(a''_{13})^{(1,1,)}(T_{14},t)} \quad \boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9)}(T_{45},t)} \end{array}\right] G_{16} \quad (61)$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[\begin{array}{ccc} (a'_{17})^{(2)} & \boxed{+(a''_{17})^{(2)}(T_{17},t)} & \boxed{+(a''_{14})^{(1,1,)}(T_{14},t)} \quad \boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9)}(T_{45},t)} \end{array}\right] G_{17} \quad (62)$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[\begin{array}{ccc} (a'_{18})^{(2)} & \boxed{+(a''_{18})^{(2)}(T_{17},t)} & \boxed{+(a''_{15})^{(1,1,)}(T_{14},t)} \quad \boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9)}(T_{45},t)} \end{array}\right] G_{18} \quad (63)$$

Where $\boxed{+(a''_{16})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,)}(T_{14},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)}$ are eight augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{44})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9)}(T_{45},t)}$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[\begin{array}{ccc} (b'_{16})^{(2)} & \boxed{-(b''_{16})^{(2)}(G_{19},t)} & \boxed{-(b''_{13})^{(1,1,)}(G,t)} \quad \boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9)}(G_{47},t)} \end{array}\right] T_{16} \quad (64)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[\begin{array}{ccc} (b'_{17})^{(2)} & \boxed{-(b''_{17})^{(2)}(G_{19},t)} & \boxed{-(b''_{14})^{(1,1,)}(G,t)} \quad \boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9)}(G_{47},t)} \end{array}\right] T_{17} \quad (65)$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[\begin{array}{ccc} (b'_{18})^{(2)}\boxed{-(b''_{18})^{(2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,)}(G,t)} & \boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9)}(G_{47},t)} \end{array}\right] T_{18} \quad (66)$$

where $\boxed{-(b''_{16})^{(2)}(G_{19},t)}$ , $\boxed{-(b''_{17})^{(2)}(G_{19},t)}$ , $\boxed{-(b''_{18})^{(2)}(G_{19},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,)}(G,t)}$ are second detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{44})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[\begin{array}{ccc} (a'_{20})^{(3)}\boxed{+(a''_{20})^{(3)}(T_{21},t)} & \boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9)}(T_{45},t)} \end{array}\right] G_{20} \quad (67)$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[\begin{array}{ccc} (a'_{21})^{(3)}\boxed{+(a''_{21})^{(3)}(T_{21},t)} & \boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)} \end{array}\right] G_{21} \quad (68)$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[\begin{array}{ccc} (a'_{22})^{(3)}\boxed{+(a''_{22})^{(3)}(T_{21},t)} & \boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)} \end{array}\right] G_{22} \quad (69)$$

$\boxed{+(a''_{20})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3)}(T_{21},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}$ are second augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)}$ are third augmentation coefficients for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficients for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficients for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficients for category 1, 2 and 3

$+(a''_{36})^{(7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1, 2 and 3

$+(a''_{40})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8)}(T_{41},t)$ are eight augmentation coefficients for category 1, 2 and 3

$+(a''_{44})^{(9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b'_{20})^{(3)} & -(b''_{20})^{(3)}(G_{23},t) & -(b''_{16})^{(2,2,2)}(G_{19},t) & -(b''_{13})^{(1,1,1,)}(G,t) \\ -(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{36})^{(7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{20} \quad (70)$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b'_{21})^{(3)} & -(b''_{21})^{(3)}(G_{23},t) & -(b''_{17})^{(2,2,2)}(G_{19},t) & -(b''_{14})^{(1,1,1,)}(G,t) \\ -(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{37})^{(7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{21} \quad (71)$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)} & -(b''_{22})^{(3)}(G_{23},t) & -(b''_{18})^{(2,2,2)}(G_{19},t) & -(b''_{15})^{(1,1,1,)}(G,t) \\ -(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{38})^{(7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{22} \quad (72)$$

$-(b''_{20})^{(3)}(G_{23},t)$, $-(b''_{21})^{(3)}(G_{23},t)$, $-(b''_{22})^{(3)}(G_{23},t)$ *are first detrition coefficients for category 1, 2 and 3*

$-(b''_{16})^{(2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2)}(G_{19},t)$ *are second detrition coefficients for category 1, 2 and 3*

$-(b''_{13})^{(1,1,1,)}(G,t)$, $-(b''_{14})^{(1,1,1,)}(G,t)$, $-(b''_{15})^{(1,1,1,)}(G,t)$ *are third detrition coefficients for category 1,2 and 3*

$-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)$ *are fourth detrition coefficients for category 1, 2 and 3*

$-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)$ *are fifth detrition coefficients for category 1, 2 and 3*

$-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)$ *are sixth detrition coefficients for category 1, 2 and 3*

$-(b''_{36})^{(7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt}=(a_{24})^{(4)}G_{25}-\begin{bmatrix}(a'_{24})^{(4)}\boxed{+(a''_{24})^{(4)}(T_{25},t)}&\boxed{+(a''_{28})^{(5,5,)}(T_{29},t)}&\boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}\\ \boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)}&\boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)}&\boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)}&\boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)}&\boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{24} \quad (73)$$

$$\frac{dG_{25}}{dt}=(a_{25})^{(4)}G_{24}-\begin{bmatrix}(a'_{25})^{(4)}\boxed{+(a''_{25})^{(4)}(T_{25},t)}&\boxed{+(a''_{29})^{(5,5,)}(T_{29},t)}&\boxed{+(a''_{33})^{(6,6)}(T_{33},t)}\\ \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)}&\boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)}&\boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)}&\boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)}&\boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{25} \quad (74)$$

$$\frac{dG_{26}}{dt}=(a_{26})^{(4)}G_{25}-\begin{bmatrix}(a'_{26})^{(4)}\boxed{+(a''_{26})^{(4)}(T_{25},t)}&\boxed{+(a''_{30})^{(5,5,)}(T_{29},t)}&\boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}\\ \boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)}&\boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)}&\boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)}&\boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)}&\boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{26} \quad (75)$$

$\boxed{(a''_{24})^{(4)}(T_{25},t)}$, $\boxed{(a''_{25})^{(4)}(T_{25},t)}$, $\boxed{(a''_{26})^{(4)}(T_{25},t)}$ *are first augmentation coefficients category* 1, 2 3

$\boxed{+(a''_{28})^{(5,5,)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,)}(T_{29},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)}$
*are fourth augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)}$
*are fifth augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)}$
*are sixth augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)}$
*are seventh augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)}$
*are eighth augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt}=(b_{24})^{(4)}T_{25}-\begin{bmatrix}(b'_{24})^{(4)}\boxed{-(b''_{24})^{(4)}(G_{27},t)}&\boxed{-(b''_{28})^{(5,5,)}(G_{31},t)}&\boxed{-(b''_{32})^{(6,6,)}(G_{35},t)}\\ \boxed{-(b''_{13})^{(1,1,1,1)}(G,t)}&\boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)}&\boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)}\\ \boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)}&\boxed{-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)}&\boxed{-(b''_{44})^{(9,9,9,9)}(G_{47},t)}\end{bmatrix}T_{24} \quad (76)$$

$$\frac{dT_{25}}{dt}=(b_{25})^{(4)}T_{24}-\begin{bmatrix}(b'_{25})^{(4)}\boxed{-(b''_{25})^{(4)}(G_{27},t)}&\boxed{-(b''_{29})^{(5,5,)}(G_{31},t)}&\boxed{-(b''_{33})^{(6,6,)}(G_{35},t)}\\ \boxed{-(b''_{14})^{(1,1,1,1)}(G,t)}&\boxed{-(b''_{17})^{(2,2,2,2)}(G_{19},t)}&\boxed{-(b''_{21})^{(3,3,3,3)}(G_{23},t)}\\ \boxed{-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)}&\boxed{-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)}&\boxed{-(b''_{45})^{(9,9,9,9)}(G_{47},t)}\end{bmatrix}T_{25} \quad (77)$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)} & -(b''_{26})^{(4)}(G_{27},t) & -(b''_{30})^{(5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1)}(G,t) & -(b''_{18})^{(2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{26} \quad 78$$

*Where* $-(b''_{24})^{(4)}(G_{27},t)$, $-(b''_{25})^{(4)}(G_{27},t)$, $-(b''_{26})^{(4)}(G_{27},t)$ *are first detrition coefficients for category* $1, 2$ *and* $3$

$-(b''_{28})^{(5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,)}(G_{31},t)$ *are second detrition coefficients for category* $1, 2$ *and* $3$

$-(b''_{32})^{(6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,)}(G_{35},t)$ *are third detrition coefficients for category* $1, 2$ *and* $3$

$-(b''_{13})^{(1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1)}(G,t)$

*are fourth detrition coefficients for category* $1, 2$ *and* $3$

$-(b''_{16})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2)}(G_{19},t)$

*are fifth detrition coefficients for category* $1, 2$ *and* $3$

$-(b''_{20})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3)}(G_{23},t)$

*are sixth detrition coefficients for category* $1, 2$ *and* $3$

$-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)$

*are seventh detrition coefficients for category* $1, 2$ *and* $3$

$-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)$

*are eighth detrition coefficients for category* $1, 2$ *and* $3$

$-(b''_{46})^{(9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \begin{bmatrix} (a'_{28})^{(5)} & +(a''_{28})^{(5)}(T_{29},t) & +(a''_{24})^{(4,4,)}(T_{25},t) & +(a''_{32})^{(6,6,6)}(T_{33},t) \\ & +(a''_{13})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{16})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3,3)}(T_{21},t) \\ & +(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{28} \quad 79$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \begin{bmatrix} (a'_{29})^{(5)} & +(a''_{29})^{(5)}(T_{29},t) & +(a''_{25})^{(4,4,)}(T_{25},t) & +(a''_{33})^{(6,6,6)}(T_{33},t) \\ & +(a''_{14})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{17})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3,3)}(T_{21},t) \\ & +(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{29} \quad 80$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \begin{bmatrix} (a'_{30})^{(5)} & +(a''_{30})^{(5)}(T_{29},t) & +(a''_{26})^{(4,4,)}(T_{25},t) & +(a''_{34})^{(6,6,6)}(T_{33},t) \\ & +(a''_{15})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{18})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3,3)}(T_{21},t) \\ & +(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{30} \quad 81$$

*Where* $+(a''_{28})^{(5)}(T_{29},t)$, $+(a''_{29})^{(5)}(T_{29},t)$, $+(a''_{30})^{(5)}(T_{29},t)$ *are first augmentation coefficients for category* $1, 2$ *and* $3$

*And* $+(a''_{24})^{(4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,)}(T_{25},t)$ *are second augmentation coefficient for category* $1, 2$ *and* $3$

$+(a''_{32})^{(6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6)}(T_{33},t)$ *are third augmentation coefficient for category* $1, 2$ *and* $3$

$+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)$ are fourth augmentation coefficients for category 1,2, and 3

$+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)$ are fifth augmentation coefficients for category 1,2,and 3

$+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)$ are sixth augmentation coefficients for category 1,2, 3

$+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)$ , $+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1,2, 3

$+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)$ are eighth augmentation coefficients for category 1,2, 3

$+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \begin{bmatrix} (b'_{28})^{(5)} & -(b''_{28})^{(5)}(G_{31},t) & -(b''_{24})^{(4,4,)}(G_{27},t) & -(b''_{32})^{(6,6,6)}(G_{35},t) \\ & -(b''_{13})^{(1,1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{28} \quad 82$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b'_{29})^{(5)} & -(b''_{29})^{(5)}(G_{31},t) & -(b''_{25})^{(4,4,)}(G_{27},t) & -(b''_{33})^{(6,6,6)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{29} \quad 83$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b'_{30})^{(5)} & -(b''_{30})^{(5)}(G_{31},t) & -(b''_{26})^{(4,4,)}(G_{27},t) & -(b''_{34})^{(6,6,6)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1,1,)}(G,t) & -(b''_{18})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{30} \quad 84$$

*where* $-(b''_{28})^{(5)}(G_{31},t)$ , $-(b''_{29})^{(5)}(G_{31},t)$ , $-(b''_{30})^{(5)}(G_{31},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{24})^{(4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,)}(G_{27},t)$ *are second detrition coefficients for category* 1,2 *and* 3

$-(b''_{32})^{(6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6)}(G_{35},t)$ *are third detrition coefficients for category* 1,2 *and* 3

$-(b''_{13})^{(1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1)}(G,t)$ , $-(b''_{15})^{(1,1,1,1,1,)}(G,t)$ are fourth detrition coefficients for category 1,2, and 3

$-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1,2, and 3

$-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1,2, and 3

$-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2, and 3

$-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} \quad 85$$

$$-\left[\begin{array}{ccc} (a'_{32})^{(6)}\boxed{+(a''_{32})^{(6)}(T_{33},t)} & \boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)} \end{array}\right]G_{32}$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[\begin{array}{ccc} (a'_{33})^{(6)}\boxed{+(a''_{33})^{(6)}(T_{33},t)} & \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)} \end{array}\right]G_{33} \quad 86$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[\begin{array}{ccc} (a'_{34})^{(6)}\boxed{+(a''_{34})^{(6)}(T_{33},t)} & \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)} \end{array}\right]G_{34} \quad 87$$

$\boxed{+(a''_{32})^{(6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6)}(T_{33},t)}$ *are first augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}$ *are second augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}$ *are third augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}$ - are fourth augmentation coefficients

$\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}$ - fifth augmentation coefficients

$\boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)}$ sixth augmentation coefficients

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)}$ seventh augmentation coefficients

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}$ Eighth augmentation coefficients

$\boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)}$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[\begin{array}{ccc} (b'_{32})^{(6)}\boxed{-(b''_{32})^{(6)}(G_{35},t)} & \boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)} \end{array}\right]T_{32} \quad 88$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \begin{bmatrix} (b'_{33})^{(6)}\boxed{-(b''_{33})^{(6)}(G_{35},t)} & \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{33} \quad (89)$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)}\boxed{-(b''_{34})^{(6)}(G_{35},t)} & \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{34} \quad (90)$$

$\boxed{-(b''_{32})^{(6)}(G_{35},t)}, \boxed{-(b''_{33})^{(6)}(G_{35},t)}, \boxed{-(b''_{34})^{(6)}(G_{35},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}$ *are second detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)}$ *are third detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}$ are fourth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \quad (91)$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a'_{36})^{(7)}\boxed{+(a''_{36})^{(7)}(T_{37},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \quad (92)$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a'_{37})^{(7)}\boxed{+(a''_{37})^{(7)}(T_{37},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$\frac{dG_{38}}{dt}$ *93*

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a'_{38})^{(7)}\boxed{+(a''_{38})^{(7)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{(a''_{36})^{(7)}(T_{37},t)}$, $\boxed{(a''_{37})^{(7)}(T_{37},t)}$, $\boxed{(a''_{38})^{(7)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,)}(T_{41},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$\frac{dT_{36}}{dt} =$ 94

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b'_{36})^{(7)}\boxed{-(b''_{36})^{(7)}(G_{39},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$\frac{dT_{37}}{dt} =$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b'_{37})^{(7)}\boxed{-(b''_{37})^{(7)}(G_{39},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$\frac{dT_{38}}{dt} =$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b'_{38})^{(7)}\boxed{-(b''_{38})^{(7)}(G_{39},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b_{36}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{37}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{16}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{17}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{18}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{20}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{21}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{22}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{24}'')^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{25}'')^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{26}'')^{(4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{28}'')^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{32}'')^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{34}'')^{(6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{46}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{45}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{44}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3



$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \begin{bmatrix} (a_{40}')^{(8)}\boxed{+(a_{40}'')^{(8)}(T_{41},t)} & \boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{20}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{24}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{28}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{32}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{36}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{44}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \begin{bmatrix} (a_{41}')^{(8)}\boxed{+(a_{41}'')^{(8)}(T_{41},t)} & \boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{21}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{25}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{29}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{33}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{37}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{45}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \begin{bmatrix} (a_{42}')^{(8)}\boxed{+(a_{42}'')^{(8)}(T_{41},t)} & \boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{22}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{26}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{30}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{34}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{15}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{38}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{46}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a_{40}'')^{(8)}(T_{41},t)}$, $\boxed{+(a_{41}'')^{(8)}(T_{41},t)}$, $\boxed{+(a_{42}'')^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second

augmentation coefficient for category 1, 2 and 3
$+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)$ , $+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)$ , $+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3
$+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)$ , $+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)$ , $+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3
$+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)$ , $+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3
$+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)$ , $+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3
$+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ $+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ $+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ are seventh augmentation coefficient for 1,2,3
$+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$ , $+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$ , $+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$ are eighth augmentation coefficient for 1,2,3
$+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$ , $+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$ , $+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} =$$

$$(b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)}\left[-(b''_{40})^{(8)}(G_{43},t)\right] & \left[-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} =$$

$$(b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)}\left[-(b''_{41})^{(8)}(G_{43},t)\right] & \left[-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} =$$

$$(b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)}\left[-(b''_{42})^{(8)}(G_{43},t)\right] & \left[-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{15}$$

Where $-(b''_{36})^{(7)}(G_{39},t)$ , $-(b''_{37})^{(7)}(G_{39},t)$ , $-(b''_{38})^{(7)}(G_{39},t)$ are first detrition coefficients for category 1, 2 and 3
$-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$ , $-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$ , $-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3
$-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$ , $-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$ , $-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3
$-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)$ , $-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)$ , $-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{28}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{32}'')^{(6,6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{38}'')^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{36}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{37}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{44}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{45}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{46}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

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$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \begin{bmatrix} (a_{44}')^{(9)}\boxed{+(a_{44}'')^{(9)}(T_{45},t)} & \boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{20}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{24}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{28}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{32}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{36}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{40}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \begin{bmatrix} (a_{45}')^{(9)}\boxed{+(a_{45}'')^{(9)}(T_{45},t)} & \boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{21}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{25}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{29}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{33}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{14}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{37}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{41}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \begin{bmatrix} (a_{46}')^{(9)}\boxed{+(a_{46}'')^{(9)}(T_{37},t)} & \boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{22}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{26}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{30}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{34}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{15}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{38}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{42}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a_{44}'')^{(9)}(T_{45},t)}$, $\boxed{+(a_{45}'')^{(9)}(T_{45},t)}$, $\boxed{+(a_{46}'')^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{20}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{21}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{22}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{24}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{25}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{26}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{28}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{29}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{30}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$ are Seventh augmentation coefficient for category 1, 2 and 3

$+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ $+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ $+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ are eighth augmentation coefficient for 1,2,3

$+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} =$$

$$(b_{44})^{(9)}T_{45} -$$

$$\begin{bmatrix} (b'_{44})^{(9)}\boxed{-(b''_{44})^{(9)}(G_{47},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b'_{45})^{(9)}\boxed{-(b''_{45})^{(9)}(G_{47},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b'_{46})^{(9)}\boxed{-(b''_{46})^{(9)}(G_{47},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{44})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9)}(G_{47},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{37}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{36}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a_i')^{(1)}, (a_i'')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (b_i'')^{(1)} > 0, \quad i,j = 13,14,15 \qquad 97$$

The functions $(a_i'')^{(1)}, (b_i'')^{(1)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a_i'')^{(1)}(T_{14},t) \leq (p_i)^{(1)} \leq (\hat{A}_{13})^{(1)}$$

$$(b_i'')^{(1)}(G,t) \leq (r_i)^{(1)} \leq (b_i')^{(1)} \leq (\hat{B}_{13})^{(1)}$$

$$lim_{T_2\to\infty}(a_i'')^{(1)}(T_{14},t) = (p_i)^{(1)} \qquad 98$$

$$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = (r_i)^{(1)}$$

**Definition of** $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$ :

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$$|(a_i'')^{(1)}(T_{14}',t) - (a_i'')^{(1)}(T_{14},t)| \leq (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G',t) - (b_i'')^{(1)}(G,t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}',t)$ and $(a_i'')^{(1)}(T_{14},t)$ . $(T_{14}',t)$ and $(T_{14},t)$ are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14},t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$ : 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$$

**Definition of** $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$ : 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i,j = 16,17,18$$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17}, t) \leq (p_i)^{(2)} \leq (\hat{A}_{16})^{(2)} \quad 102$$

$$(b_i'')^{(2)}(G_{19}, t) \leq (r_i)^{(2)} \leq (b_i')^{(2)} \leq (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2 \to \infty} (a_i'')^{(2)}(T_{17}, t) = (p_i)^{(2)} \quad 104$$

$$\lim_{G \to \infty} (b_i'')^{(2)}((G_{19}), t) = (r_i)^{(2)} \quad 105$$

**Definition of** $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$ : 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}', t) - (a_i'')^{(2)}(T_{17}, t)| \leq (\hat{k}_{16})^{(2)}|T_{17} - T_{17}'|e^{-(\hat{M}_{16})^{(2)}t} \quad 107$$

$$|(b_i'')^{(2)}((G_{19})', t) - (b_i'')^{(2)}((G_{19}), t)| < (\hat{k}_{16})^{(2)}||(G_{19}) - (G_{19})'||e^{-(\hat{M}_{16})^{(2)}t} \quad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}', t)$ and$(a_i'')^{(2)}(T_{17}, t)$ . $(T_{17}', t)$ and $(T_{17}, t)$ are points belonging to the interval $[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}]$ . It is to be noted that $(a_i'')^{(2)}(T_{17}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17}, t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$ :

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants 109

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$$

**Definition of** $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$ :

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1 \quad (110)$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(b_i)^{(2)} + (b_i')^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1 \quad (111)$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i,j = 20,21,22 \quad (112)$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(3)}$, $(r_i)^{(3)}$:

$$(a_i'')^{(3)}(T_{21}, t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23}, t) \leq (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(3)}(T_{21}, t) = (p_i)^{(3)} \quad (113)$$

$$\lim_{G \to \infty}(b_i'')^{(3)}(G_{23}, t) = (r_i)^{(3)}$$

**Definition of** $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$ :

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: (114)

$$|(a_i'')^{(3)}(T_{21}', t) - (a_i'')^{(3)}(T_{21}, t)| \leq (\hat{k}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\hat{M}_{20})^{(3)}t}$$

$$|(b_i'')^{(3)}(G_{23}', t) - (b_i'')^{(3)}(G_{23}, t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}'||e^{-(\hat{M}_{20})^{(3)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}', t)$ and $(a_i'')^{(3)}(T_{21}, t)$ . $(T_{21}', t)$ And $(T_{21}, t)$ are points belonging to the interval $[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}]$. It is to be noted that $(a_i'')^{(3)}(T_{21}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$ : (115)

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$ and the constants $(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}, i = 20,21,22,$ satisfy the inequalities 116

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(a_i)^{(3)} + (a_i')^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(b_i)^{(3)} + (b_i')^{(3)} + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i, j = 24,25,26 \qquad 117$$

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(4)}$, $(r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25}, t) \le (p_i)^{(4)} \le (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}((G_{27}), t) \le (r_i)^{(4)} \le (b_i')^{(4)} \le (\hat{B}_{24})^{(4)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(4)}(T_{25}, t) = (p_i)^{(4)} \qquad 118$$

$$\lim_{G \to \infty}(b_i'')^{(4)}((G_{27}), t) = (r_i)^{(4)}$$

**Definition of** $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$ :

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24,25,26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}', t) - (a_i'')^{(4)}(T_{25}, t)| \le (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})', t) - (b_i'')^{(4)}((G_{27}), t)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}', t)$ and $(a_i'')^{(4)}(T_{25}, t)$ . $(T_{25}', t)$ and $(T_{25}, t)$ are points belonging to the interval $[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}]$. It is to be noted that $(a_i'')^{(4)}(T_{25}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$ : 120

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$, are positive constants

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}, \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$$

**Definition of** $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$ : 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and\ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}, i = 24,25,26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(a_i)^{(4)} + (a_i')^{(4)} + (\hat{A}_{24})^{(4)} + (\hat{P}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(b_i)^{(4)} + (b_i')^{(4)} + (\hat{B}_{24})^{(4)} + (\hat{Q}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i,j = 28,29,30 \qquad 122$$

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(5)}$, $(r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29}, t) \le (p_i)^{(5)} \le (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}((G_{31}), t) \le (r_i)^{(5)} \le (b_i')^{(5)} \le (\hat{B}_{28})^{(5)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(5)}(T_{29}, t) = (p_i)^{(5)} \qquad 123$$

$$\lim_{G \to \infty}(b_i'')^{(5)}(G_{31}, t) = (r_i)^{(5)}$$

**Definition of** $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$ :

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}', t) - (a_i'')^{(5)}(T_{29}, t)| \le (\hat{k}_{28})^{(5)}|T_{29} - T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})', t) - (b_i'')^{(5)}((G_{31}), t)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}', t)$ and $(a_i'')^{(5)}(T_{29}, t)$ . $(T_{29}', t)$ and $(T_{29}, t)$ are points belonging to the interval $[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}]$. It is to be noted that $(a_i'')^{(5)}(T_{29}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29}, t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$ : 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}, \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

**Definition of** $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$ : 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\widehat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and\ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\widehat{M}_{28})^{(5)}}[(a_i)^{(5)} + (a_i')^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\widehat{M}_{28})^{(5)}}[(b_i)^{(5)} + (b_i')^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i,j = 32,33,34 \qquad 127$$

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(6)}$, $(r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33}, t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}), t) \leq (r_i)^{(6)} \leq (b_i')^{(6)} \leq (\hat{B}_{32})^{(6)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(6)}(T_{33}, t) = (p_i)^{(6)} \qquad 128$$

$$\lim_{G \to \infty}(b_i'')^{(6)}((G_{35}), t) = (r_i)^{(6)}$$

**Definition of** $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$ :

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}', t) - (a_i'')^{(6)}(T_{33}, t)| \leq (\hat{k}_{32})^{(6)}|T_{33} - T_{33}'|e^{-(\widehat{M}_{32})^{(6)}t}$$

$$|(b_i'')^{(6)}((G_{35})', t) - (b_i'')^{(6)}((G_{35}), t)| < (\hat{k}_{32})^{(6)}||(G_{35}) - (G_{35})'||e^{-(\widehat{M}_{32})^{(6)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}', t)$ and $(a_i'')^{(6)}(T_{33}, t)$ . $(T_{33}', t)$ and $(T_{33}, t)$ are points belonging to the interval $[(\hat{k}_{32})^{(6)}, (\widehat{M}_{32})^{(6)}]$. It is to be noted that $(a_i'')^{(6)}(T_{33}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33}, t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\widehat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$ : 129

$(\widehat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$, are positive constants

$$\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}} < 1$$

**Definition of** $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$ : 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and\ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32,33,34,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(a_i)^{(6)} + (a_i')^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)} (\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(b_i)^{(6)} + (b_i')^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)} (\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

(CCCCCCCCC) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i, j = 36,37,38$ 131

(DDDDDDDDD) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(7)}, (r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37}, t) \leq (p_i)^{(7)} \leq (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39}, t) \leq (r_i)^{(7)} \leq (b_i')^{(7)} \leq (\hat{B}_{36})^{(7)}$$

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*(EEEEEEEEE)* $\lim_{T_2 \to \infty} (a_i'')^{(7)}(T_{37}, t) = (p_i)^{(7)}$

*(FFFFFFFFF)*

$\lim_{G \to \infty} (b_i'')^{(7)}((G_{39}), t) = (r_i)^{(7)}$

**Definition of** $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$ :

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$$|(a_i'')^{(7)}(T_{37}', t) - (a_i'')^{(7)}(T_{37}, t)| \leq (\hat{k}_{36})^{(7)} |T_{37} - T_{37}'| e^{-(\hat{M}_{36})^{(7)} t}$$

$$|(b_i'')^{(7)}((G_{39})', t) - (b_i'')^{(7)}((G_{39}), t)| < (\hat{k}_{36})^{(7)} ||(G_{39}) - (G_{39})'|| e^{-(\hat{M}_{36})^{(7)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}', t)$ and $(a_i'')^{(7)}(T_{37}, t)$ . $(T_{37}', t)$ and $(T_{37}, t)$ are points belonging to the interval $[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}]$ . It is to be noted that $(a_i'')^{(7)}(T_{37}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37}, t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$ : 134

(GGGGGGGGG) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)},$ are positive constants

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$$

**Definition of** $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$ : 135

(HHHHHHHHH) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(b_i)^{(7)} + (b_i')^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i, j = 40,41,42 \qquad 136$$

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

**Definition of** $(p_i)^{(8)}$, $(r_i)^{(8)}$: 137

$$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)} \qquad 138$$

$$(b_i'')^{(8)}((G_{43}), t) \leq (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)} \qquad 139$$

$$\lim_{T_2 \to \infty} (a_i'')^{(8)}(T_{41}, t) = (p_i)^{(8)} \qquad 140$$

$$\lim_{G \to \infty} (b_i'')^{(8)}\big((G_{43}), t\big) = (r_i)^{(8)} \qquad 141$$

**Definition of** $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$ :

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)} |T_{41} - T_{41}'| e^{-(\hat{M}_{40})^{(8)} t} \qquad 142$$

$$|(b_i'')^{(8)}((G_{43})', t) - (b_i'')^{(8)}\big((G_{43}), t\big)| < (\hat{k}_{40})^{(8)} ||(G_{43}) - (G_{43})'|| e^{-(\hat{M}_{40})^{(8)} t} \qquad 143$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}', t)$ and $(a_i'')^{(8)}(T_{41}, t)$ . $(T_{41}', t)$ and $(T_{41}, t)$ are points belonging to the interval $\big[(\hat{k}_{40})^{(8)}, (\hat{M}_{40})^{(8)}\big]$. It is to be noted that $(a_i'')^{(8)}(T_{41}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{40})^{(8)} = 1$

then the function $(a_i'')^{(8)}(T_{41},t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)}$ :

$(\hat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)},$ are positive constants

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\ ,\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}}<1 \qquad 144$$

**Definition of** $(\hat{P}_{40})^{(8)},(\hat{Q}_{40})^{(8)}$ :

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\hat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)},(\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)},(a_i')^{(8)},(b_i)^{(8)},(b_i')^{(8)},(p_i)^{(8)},\ (r_i)^{(8)},i=40,41,42,$

Satisfy the inequalities

$$\frac{1}{(\hat{M}_{40})^{(8)}}[\,(a_i)^{(8)}+(a_i')^{(8)}+\ (\hat{A}_{40})^{(8)}+\ (\hat{P}_{40})^{(8)}\,(\hat{k}_{40})^{(8)}]<1 \qquad 145$$

$$\frac{1}{(\hat{M}_{40})^{(8)}}[\ (b_i)^{(8)}+(b_i')^{(8)}+\ (\hat{B}_{40})^{(8)}+\ (\hat{Q}_{40})^{(8)}\ (\hat{k}_{40})^{(8)}]<1 \qquad 146$$

Where we suppose

$$(a_i)^{(9)},(a_i')^{(9)},(a_i'')^{(9)},(b_i)^{(9)},(b_i')^{(9)},(b_i'')^{(9)}>0,\quad i,j=44,45,46 \qquad 146\,A$$

The functions $(a_i'')^{(9)},(b_i'')^{(9)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(9)},\ (r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45},t)\le(p_i)^{(9)}\le(\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47},t)\le\ (r_i)^{(9)}\le(b_i')^{(9)}\le(\hat{B}_{44})^{(9)}$$

$$\lim_{T_2\to\infty}(a_i'')^{(9)}(T_{45},t)=(p_i)^{(9)}$$

$$\lim_{G\to\infty}(b_i'')^{(9)}(G_{47},t)=\ (r_i)^{(9)}$$

**Definition of** $(\hat{A}_{44})^{(9)},(\hat{B}_{44})^{(9)}$ :

Where $\boxed{(\hat{A}_{44})^{(9)},(\hat{B}_{44})^{(9)},(p_i)^{(9)},\ (r_i)^{(9)}}$ are positive constants and $\boxed{i=44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}',t)-(a_i'')^{(9)}(T_{45},t)|\le(\hat{k}_{44})^{(9)}|T_{45}-\ T_{45}'|e^{-(\hat{M}_{44})^{(9)}t}$$

$$|(b_i'')^{(9)}((G_{47})',t)-(b_i'')^{(9)}\big((G_{47}),t\big)|<(\hat{k}_{44})^{(9)}||(G_{47})-(G_{47})'||e^{-(\hat{M}_{44})^{(9)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}',t)$ and$(a_i'')^{(9)}(T_{45},t)$ . $(T_{45}',t)$ and $(T_{45},t)$ are points belonging to the interval $\big[(\hat{k}_{44})^{(9)},(\hat{M}_{44})^{(9)}\big]$. It is

to be noted that $(a_i'')^{(9)}(T_{45},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{44})^{(9)}=1$ then the function $(a_i'')^{(9)}(T_{45},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\widehat{M}_{44})^{(9)},(\hat{k}_{44})^{(9)}$ :

$(\widehat{M}_{44})^{(9)},(\hat{k}_{44})^{(9)}$, are positive constants

$$\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\ ,\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}<1$$

**Definition of** $(\hat{P}_{44})^{(9)},(\hat{Q}_{44})^{(9)}$ :

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\widehat{M}_{44})^{(9)},(\hat{k}_{44})^{(9)},(\hat{A}_{44})^{(9)} and\ (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)},(a_i')^{(9)},(b_i)^{(9)},(b_i')^{(9)},(p_i)^{(9)},\ (r_i)^{(9)}, i=44,45,46,$ satisfy the inequalities

$$\frac{1}{(\widehat{M}_{44})^{(9)}}[(a_i)^{(9)}+(a_i')^{(9)}+\ (\hat{A}_{44})^{(9)}+\ (\hat{P}_{44})^{(9)}\ (\hat{k}_{44})^{(9)}]<1$$

$$\frac{1}{(\widehat{M}_{44})^{(9)}}[\ (b_i)^{(9)}+(b_i')^{(9)}+\ (\hat{B}_{44})^{(9)}+\ (\hat{Q}_{44})^{(9)}\ (\hat{k}_{44})^{(9)}]<1$$

**Theorem 1:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0)\ ,T_i(0)$ :

$$G_i(t)\le\ (\hat{P}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}\ ,\ \boxed{G_i(0)=G_i^0>0}$$

$$T_i(t)\le\ (\hat{Q}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}\ ,\ \boxed{T_i(0)=T_i^0>0}$$

**Theorem 2 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

**Definition of** $G_i(0)\ ,T_i(0)$

$$G_i(t)\le\ (\hat{P}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}\ ,\ \ G_i(0)=G_i^0>0$$

$$T_i(t)\le\ (\hat{Q}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}\ ,\ \ T_i(0)=T_i^0>0$$

**Theorem 3 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t)\le\ (\hat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}\ ,\ \ G_i(0)=G_i^0>0$$

$$T_i(t)\le\ (\hat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}\ ,\ \ T_i(0)=T_i^0>0$$

**Theorem 4 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

**Definition of** $G_i(0)\ ,T_i(0)$ :

$G_i(t) \leq \left(\hat{P}_{24}\right)^{(4)} e^{(\hat{M}_{24})^{(4)}t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{24}\right)^{(4)} e^{(\hat{M}_{24})^{(4)}t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Theorem 5 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

**Definition of** $G_i(0)$ , $T_i(0)$ :

$G_i(t) \leq \left(\hat{P}_{28}\right)^{(5)} e^{(\hat{M}_{28})^{(5)}t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{28}\right)^{(5)} e^{(\hat{M}_{28})^{(5)}t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Theorem 6 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

**Definition of** $G_i(0)$ , $T_i(0)$ :

$G_i(t) \leq \left(\hat{P}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Theorem 7:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

**Definition of** $G_i(0)$ , $T_i(0)$ :

$G_i(t) \leq \left(\hat{P}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Theorem 8:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

**Definition of** $G_i(0)$ , $T_i(0)$ :

$G_i(t) \leq \left(\hat{P}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Theorem 9:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

**Definition of** $G_i(0)$ , $T_i(0)$ :

$G_i(t) \leq \left(\hat{P}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Proof:** Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i$: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \le (\hat{P}_{13})^{(1)} \text{ , } T_i^0 \le (\hat{Q}_{13})^{(1)}, \quad 155$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t} \quad 156$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t} \quad 157$$

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}(s_{(13)}) - \left((a'_{13})^{(1)} + a''_{13})^{(1)} (T_{14}(s_{(13)}), s_{(13)})\right) G_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)} (T_{14}(s_{(13)}), s_{(13)})\right) G_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)} (T_{14}(s_{(13)}), s_{(13)})\right) G_{15}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)} (G(s_{(13)}), s_{(13)})\right) T_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)} (G(s_{(13)}), s_{(13)})\right) T_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{15}(t) = T_{15}^0 + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)} (G(s_{(13)}), s_{(13)})\right) T_{15}(s_{(13)})\right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i$: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \le (\hat{P}_{16})^{(2)} \text{ , } T_i^0 \le (\hat{Q}_{16})^{(2)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$$

By 160

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''_{16})^{(2)} (T_{17}(s_{(16)}), s_{(16)})\right) G_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)} (T_{17}(s_{(16)}), s_{(17)})\right) G_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)} (T_{17}(s_{(16)}), s_{(16)})\right) G_{18}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)} (G_{19}(s_{(16)}), s_{(16)})\right) T_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^{0} + \int_0^t \left[ (b_{17})^{(2)} T_{16}(s_{(16)}) - \left( (b'_{17})^{(2)} - (b''_{17})^{(2)} (G_{19}(s_{(16)}), s_{(16)}) \right) T_{17}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^{0} + \int_0^t \left[ (b_{18})^{(2)} T_{17}(s_{(16)}) - \left( (b'_{18})^{(2)} - (b''_{18})^{(2)} (G_{19}(s_{(16)}), s_{(16)}) \right) T_{18}(s_{(16)}) \right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \le (\hat{P}_{20})^{(3)} \text{ , } T_i^0 \le (\hat{Q}_{20})^{(3)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$$

By 161

$$\bar{G}_{20}(t) = G_{20}^{0} + \int_0^t \left[ (a_{20})^{(3)} G_{21}(s_{(20)}) - \left( (a'_{20})^{(3)} + a''_{20})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^{0} + \int_0^t \left[ (a_{21})^{(3)} G_{20}(s_{(20)}) - \left( (a'_{21})^{(3)} + (a''_{21})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{21}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^{0} + \int_0^t \left[ (a_{22})^{(3)} G_{21}(s_{(20)}) - \left( (a'_{22})^{(3)} + (a''_{22})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{22}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^{0} + \int_0^t \left[ (b_{20})^{(3)} T_{21}(s_{(20)}) - \left( (b'_{20})^{(3)} - (b''_{20})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^{0} + \int_0^t \left[ (b_{21})^{(3)} T_{20}(s_{(20)}) - \left( (b'_{21})^{(3)} - (b''_{21})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{21}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{\mathrm{T}}_{22}(\mathrm{t}) = \mathrm{T}_{22}^{0} + \int_0^t \left[ (b_{22})^{(3)} T_{21}(s_{(20)}) - \left( (b'_{22})^{(3)} - (b''_{22})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{22}(s_{(20)}) \right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \le (\hat{P}_{24})^{(4)} \text{ , } T_i^0 \le (\hat{Q}_{24})^{(4)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

By 162

$$\bar{G}_{24}(t) = G_{24}^{0} + \int_0^t \left[ (a_{24})^{(4)} G_{25}(s_{(24)}) - \left( (a'_{24})^{(4)} + a''_{24})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{26}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{\mathrm{T}}_{26}(\mathrm{t}) = \mathrm{T}_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{26}(s_{(24)})\right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{28})^{(5)}, T_i^0 \le (\hat{Q}_{28})^{(5)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

By 163

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{30}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{\mathrm{T}}_{30}(\mathrm{t}) = \mathrm{T}_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{30}(s_{(28)})\right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{32})^{(6)}, T_i^0 \le (\hat{Q}_{32})^{(6)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\bar{M}_{32})^{(6)} t}$

By 164

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[ (a_{32})^{(6)} G_{33}(s_{(32)}) - \left( (a'_{32})^{(6)} + a''_{32})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[ (a_{33})^{(6)} G_{32}(s_{(32)}) - \left( (a'_{33})^{(6)} + (a''_{33})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[ (a_{34})^{(6)} G_{33}(s_{(32)}) - \left( (a'_{34})^{(6)} + (a''_{34})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{34}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[ (b_{32})^{(6)} T_{33}(s_{(32)}) - \left( (b'_{32})^{(6)} - (b''_{32})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[ (b_{33})^{(6)} T_{32}(s_{(32)}) - \left( (b'_{33})^{(6)} - (b''_{33})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{34}(t) = T_{34}^0 + \int_0^t \left[ (b_{34})^{(6)} T_{33}(s_{(32)}) - \left( (b'_{34})^{(6)} - (b''_{34})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{34}(s_{(32)}) \right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[ (a_{36})^{(7)} G_{37}(s_{(36)}) - \left( (a'_{36})^{(7)} + a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[ (a_{37})^{(7)} G_{36}(s_{(36)}) - \left( (a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[ (a_{38})^{(7)} G_{37}(s_{(36)}) - \left( (a'_{38})^{(7)} + (a''_{38})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{38}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[ (b_{36})^{(7)} T_{37}(s_{(36)}) - \left( (b'_{36})^{(7)} - (b''_{36})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[ (b_{37})^{(7)} T_{36}(s_{(36)}) - \left( (b'_{37})^{(7)} - (b''_{37})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{38}(t) = T_{38}^0 + \int_0^t \left[ (b_{38})^{(7)} T_{37}(s_{(36)}) - \left( (b'_{38})^{(7)} - (b''_{38})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{38}(s_{(36)}) \right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0,t)$

**Proof:**

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{40})^{(8)}$, $T_i^0 \le (\hat{Q}_{40})^{(8)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} G_{41}(s_{(40)}) - \left((a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[(a_{41})^{(8)} G_{40}(s_{(40)}) - \left((a'_{41})^{(8)} + (a''_{41})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{42}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^0 + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)} (G_{43}(s_{(40)}), s_{(40)})\right) T_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^0 + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)} (G_{43}(s_{(40)}), s_{(40)})\right) T_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{42}(t) = T_{42}^0 + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)} (G_{43}(s_{(40)}), s_{(40)})\right) T_{42}(s_{(40)})\right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0,t)$

**Proof:** 166
Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ A which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{44})^{(9)}$, $T_i^0 \le (\hat{Q}_{44})^{(9)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)})\right) G_{44}(s_{(44)})\right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)} (T_{45}(s_{(44)}), s_{(44)})\right) G_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}(s_{(44)}), s_{(44)})\right) G_{46}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{44}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{\mathrm{T}}_{46}(\mathrm{t}) = \mathrm{T}_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{46}(s_{(44)})\right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}}\right)\right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left(e^{(\hat{M}_{13})^{(1)} t} - 1\right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\hat{M}_{13})^{(1)} t} \le \frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{P}_{13})^{(1)} + G_{14}^0\right) e^{\left(-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0}\right)} + (\hat{P}_{13})^{(1)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \le G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}}\right)\right] ds_{(16)} = \quad 169$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1\right)$$

From which it follows that 170

$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \le \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left[\left((\hat{P}_{16})^{(2)} + G_{17}^0\right) e^{\left(-\frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0}\right)} + (\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t) \le G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} \left(G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} s_{(20)}}\right)\right] ds_{(20)} =$$

$$\left(1 + (a_{20})^{(3)} t\right) G_{21}^0 + \frac{(a_{20})^{(3)} (\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left(e^{(\hat{M}_{20})^{(3)} t} - 1\right)$$

From which it follows that 172

$$(G_{20}(t)-G_{20}^{0})e^{-(\hat{M}_{20})^{(3)}t}\leq\frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left[((\hat{P}_{20})^{(3)}+G_{21}^{0})e^{\left(-\frac{(\hat{P}_{20})^{(3)}+G_{21}^{0}}{G_{21}^{0}}\right)}+(\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 173

$$G_{24}(t)\leq G_{24}^{0}+\int_{0}^{t}\left[(a_{24})^{(4)}\left(G_{25}^{0}+(\hat{P}_{24})^{(4)}e^{(\hat{M}_{24})^{(4)}s_{(24)}}\right)\right]ds_{(24)}=$$

$$\left(1+(a_{24})^{(4)}t\right)G_{25}^{0}+\frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left(e^{(\hat{M}_{24})^{(4)}t}-1\right)$$

From which it follows that 174

$$(G_{24}(t)-G_{24}^{0})e^{-(\hat{M}_{24})^{(4)}t}\leq\frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left[((\hat{P}_{24})^{(4)}+G_{25}^{0})e^{\left(-\frac{(\hat{P}_{24})^{(4)}+G_{25}^{0}}{G_{25}^{0}}\right)}+(\hat{P}_{24})^{(4)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t)\leq G_{28}^{0}+\int_{0}^{t}\left[(a_{28})^{(5)}\left(G_{29}^{0}+(\hat{P}_{28})^{(5)}e^{(\hat{M}_{28})^{(5)}s_{(28)}}\right)\right]ds_{(28)}=$$

$$\left(1+(a_{28})^{(5)}t\right)G_{29}^{0}+\frac{(a_{28})^{(5)}(\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left(e^{(\hat{M}_{28})^{(5)}t}-1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t}\leq\frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t)\leq G_{32}^{0}+\int_{0}^{t}\left[(a_{32})^{(6)}\left(G_{33}^{0}+(\hat{P}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\right)\right]ds_{(32)}=$$

$$\left(1+(a_{32})^{(6)}t\right)G_{33}^{0}+\frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left(e^{(\hat{M}_{32})^{(6)}t}-1\right)$$

From which it follows that 177

$$(G_{32}(t)-G_{32}^{0})e^{-(\hat{M}_{32})^{(6)}t}\leq\frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left[((\hat{P}_{32})^{(6)}+G_{33}^{0})e^{\left(-\frac{(\hat{P}_{32})^{(6)}+G_{33}^{0}}{G_{33}^{0}}\right)}+(\hat{P}_{32})^{(6)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}$ , $G_{26}, T_{24}, T_{25}, T_{26}$

(jj) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t) \le G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} \left(G_{37}^0 + (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} s_{(36)}}\right)\right] ds_{(36)} =$$
$$\left(1 + (a_{36})^{(7)} t\right) G_{37}^0 + \frac{(a_{36})^{(7)} (\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left(e^{(\hat{M}_{36})^{(7)} t} - 1\right)$$

From which it follows that

$$(G_{36}(t) - G_{36}^0) e^{-(\hat{M}_{36})^{(7)} t} \le \frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left[((\hat{P}_{36})^{(7)} + G_{37}^0) e^{\left(-\frac{(\hat{P}_{36})^{(7)} + G_{37}^0}{G_{37}^0}\right)} + (\hat{P}_{36})^{(7)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t) \le G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} \left(G_{41}^0 + (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} s_{(40)}}\right)\right] ds_{(40)} = \quad 180$$
$$\left(1 + (a_{40})^{(8)} t\right) G_{41}^0 + \frac{(a_{40})^{(8)} (\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}} \left(e^{(\hat{M}_{40})^{(8)} t} - 1\right)$$

From which it follows that 181

$$(G_{40}(t) - G_{40}^0) e^{-(\hat{M}_{40})^{(8)} t} \le \frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}} \left[((\hat{P}_{40})^{(8)} + G_{41}^0) e^{\left(-\frac{(\hat{P}_{40})^{(8)} + G_{41}^0}{G_{41}^0}\right)} + (\hat{P}_{40})^{(8)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 8

Analogous inequalities hold also for $G_{41}$ , $G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t) \le G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} \left(G_{45}^0 + (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} s_{(44)}}\right)\right] ds_{(44)} =$$
$$\left(1 + (a_{44})^{(9)} t\right) G_{45}^0 + \frac{(a_{44})^{(9)} (\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left(e^{(\hat{M}_{44})^{(9)} t} - 1\right)$$

From which it follows that

$$(G_{44}(t) - G_{44}^0) e^{-(\hat{M}_{44})^{(9)} t} \le \frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left[((\hat{P}_{44})^{(9)} + G_{45}^0) e^{\left(-\frac{(\hat{P}_{44})^{(9)} + G_{45}^0}{G_{45}^0}\right)} + (\hat{P}_{44})^{(9)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 9

Analogous inequalities hold also for $G_{45}$ , $G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}$ , $\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$ and to choose 182

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\widehat{M}_{13})^{(1)}}\left[(\widehat{P}_{13})^{(1)}+\left((\hat{P}_{13})^{(1)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{13})^{(1)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{13})^{(1)} \quad 183$$

$$\frac{(b_i)^{(1)}}{(\widehat{M}_{13})^{(1)}}\left[\left((\hat{Q}_{13})^{(1)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{13})^{(1)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{13})^{(1)}\right]\leq(\hat{Q}_{13})^{(1)} \quad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)},T^{(1)}\right),\left(G^{(2)},T^{(2)}\right)\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{13})^{(1)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{13})^{(1)}t}\}$$

Indeed if we denote

**Definition of** $\tilde{G},\tilde{T}:\left(\tilde{G},\tilde{T}\right)=\mathcal{A}^{(1)}(G,T)$

It results

$$\left|\tilde{G}_{13}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{13})^{(1)}\left|G_{14}^{(1)}-G_{14}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\,ds_{(13)}+$$

$$\int_0^t\{(a'_{13})^{(1)}\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$G_{13}^{(2)}\left|(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)-(a''_{13})^{(1)}\left(T_{14}^{(2)},s_{(13)}\right)\right|\ e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses it follows

$$\left|G^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}t}\leq \quad 186$$
$$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)}+(a'_{13})^{(1)}+(\widehat{A}_{13})^{(1)}+(\widehat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}\right)d\left(\left(G^{(1)},T^{(1)};\,G^{(2)},T^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

**Remark 1:** The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}\ and\ (\widehat{Q}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i=13,14,15$ depend only on $\mathrm{T}_{14}$ and respectively on

$G(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$

From 19 to 24 it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(1)} - (a_i'')^{(1)}(T_{14}(s_{(13)}), s_{(13)})\} ds_{(13)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(1)} t)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2\ and\ \left((\widehat{M}_{13})^{(1)}\right)_3$ : 187

**Remark 3**: if $G_{13}$ is bounded, the same property have also $G_{14}\ and\ G_{15}$ . indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a_{14}')^{(1)} G_{14}$ and by integrating

$$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1 / (a_{14}')^{(1)}$$

In the same way , one can obtain

$$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2 / (a_{15}')^{(1)}$$

If $G_{14}\ or\ G_{15}$ is bounded, the same property follows for $G_{13}$ , $G_{15}$ and $G_{13}$ , $G_{14}$ respectively.

**Remark 4:** If $G_{13}$ $is$ bounded, from below, the same property holds for $G_{14}\ and\ G_{15}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{14}$ is bounded from below. 188

**Remark 5:** If $T_{13}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}\,(G(t), t)) = (b_{14}')^{(1)}$ then $T_{14} \to \infty$. 189

**Definition of** $(m)^{(1)}$ and $\varepsilon_1$ :

Indeed let $t_1$ be so that for $t > t_1$

$$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t), t) < \varepsilon_1, T_{13}\,(t) > (m)^{(1)}$$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right)$, $t = log\,\frac{2}{\varepsilon_1}$ By taking now $\varepsilon_1$ sufficiently small one sees that $T_{14}$ is unbounded.
The same property holds for $T_{15}$ if $\lim_{t\to\infty}(b_{15}'')^{(1)}\,(G(t), t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}$ , $\frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}\ and\ (\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[(\widehat{P}_{16})^{(2)}+\left((\hat{P}_{16})^{(2)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{16})^{(2)}\right]\leq(\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

**Definition of** $\widetilde{G_{19}},\widetilde{T_{19}}$ : $\left(\widetilde{G_{19}},\widetilde{T_{19}}\right)=\mathcal{A}^{(2)}(G_{19},T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{16})^{(2)}\left|G_{17}^{(1)}-G_{17}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)}+$$

$$\int_0^t\{(a'_{16})^{(2)}\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$G_{16}^{(2)}\left|(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)-(a''_{16})^{(2)}\left(T_{17}^{(2)},s_{(16)}\right)\right|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0,t]$ 197

From the hypotheses it follows

$$\left|(G_{19})^{(1)}-(G_{19})^{(2)}\right|\mathrm{e}^{-(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}\leq$$
$$\frac{1}{(\widehat{\mathrm{M}}_{16})^{(2)}}\left((a_{16})^{(2)}+(a'_{16})^{(2)}+(\widehat{\mathrm{A}}_{16})^{(2)}+(\widehat{\mathrm{P}}_{16})^{(2)}(\hat{k}_{16})^{(2)}\right)\mathrm{d}\left(((G_{19})^{(1)},(T_{19})^{(1)};\ (G_{19})^{(2)},(T_{19})^{(2)})\right)$$

And analogous inequalities for $G_i$ and $T_i$. Taking into account the hypothesis the result follows 198

**Remark 6:** The fact that we supposed $(a''_{16})^{(2)}$ and $(b''_{16})^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{\mathrm{P}}_{16})^{(2)}\mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}$ and $(\widehat{\mathrm{Q}}_{16})^{(2)}\mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i=16,17,18$ depend only on $\mathrm{T}_{17}$ and respectively on

$(G_{19})$(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 7:** There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$ 200

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(2)} - (a_i'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})\} ds_{(16)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(2)}t)} > 0 \quad \text{for } t > 0$$

**Definition of** $\left((\widehat{M}_{16})^{(2)}\right)_1$, $\left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$: 201

**Remark 8:** if $G_{16}$ is bounded, the same property have also $G_{17}$ and $G_{18}$. indeed if

$G_{16} < (\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt} \leq \left((\widehat{M}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)} G_{17}$ and by integrating

$$G_{17} \leq \left((\widehat{M}_{16})^{(2)}\right)_2 = G_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1 / (a_{17}')^{(2)}$$

In the same way , one can obtain

$$G_{18} \leq \left((\widehat{M}_{16})^{(2)}\right)_3 = G_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2 / (a_{18}')^{(2)}$$

If $G_{17}$ or $G_{18}$ is bounded, the same property follows for $G_{16}$, $G_{18}$ and $G_{16}$, $G_{17}$ respectively.

**Remark 9:** If $G_{16}$ is bounded, from below, the same property holds for $G_{17}$ and $G_{18}$. The proof is analogous with the preceding one. An analogous property is true if $G_{17}$ is bounded from below. 202

**Remark 10:** If $T_{16}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t), t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$. 203

**Definition of** $(m)^{(2)}$ and $\varepsilon_2$ :

Indeed let $t_2$ be so that for $t > t_2$

$$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right)$, $t = \log\frac{2}{\varepsilon_2}$ By taking now $\varepsilon_2$ sufficiently small one sees that $T_{17}$ is unbounded. 205
The same property holds for $T_{18}$ if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t), t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[(\widehat{P}_{20})^{(3)}+\left((\hat{P}_{20})^{(3)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{20})^{(3)}\right]\leq(\hat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself 210

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)},(T_{23})^{(1)}),((G_{23})^{(2)},(T_{23})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

**Definition of** $\widetilde{G_{23}},\widetilde{T_{23}}:(\widetilde{(G_{23})},\widetilde{(T_{23})})=\mathcal{A}^{(3)}((G_{23}),(T_{23}))$

It results 213

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{20})^{(3)}\left|G_{21}^{(1)}-G_{21}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}ds_{(20)}+$$

$$\int_0^t\{(a'_{20})^{(3)}\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$G_{20}^{(2)}\left|(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)},s_{(20)}\right)\right|\ e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\}ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses it follows

$$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}t}\leq \quad 214$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+(a'_{20})^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\widehat{k}_{20})^{(3)}\right)d\left(((G_{23})^{(1)},(T_{23})^{(1)};\ (G_{23})^{(2)},(T_{23})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 11:** The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ *and* $(\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i=20,21,22$ depend only on $T_{21}$ and respectively on

$(G_{23})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 12:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 216

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(3)} - (a_i'')^{(3)}(T_{21}(s_{(20)}), s_{(20)})\} ds_{(20)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(3)} t)} > 0$ for t > 0

**Definition of** $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2\ and\ \left((\widehat{M}_{20})^{(3)}\right)_3$ : 217

**Remark 13:** if $G_{20}$ is bounded, the same property have also $G_{21}\ and\ G_{22}$ . indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a_{21}')^{(3)} G_{21}$ and by integrating

$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1 / (a_{21}')^{(3)}$

In the same way , one can obtain

$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2 / (a_{22}')^{(3)}$

If $G_{21}\ or\ G_{22}$ is bounded, the same property follows for $G_{20}$ , $G_{22}$ and $G_{20}$ , $G_{21}$ respectively.

**Remark 14:** If $G_{20}$ *is* bounded, from below, the same property holds for $G_{21}\ and\ G_{22}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{21}$ is bounded from below. 218

**Remark 15:** If $T_{20}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t), t\right)) = (b_{21}')^{(3)}$ then $T_{21} \to \infty$. 219

**Definition of** $(m)^{(3)}$ and $\varepsilon_3$ :

Indeed let $t_3$ be so that for $t > t_3$

$(b_{21})^{(3)} - (b_i'')^{(3)}\left((G_{23})(t), t\right) < \varepsilon_3, T_{20}\,(t) > (m)^{(3)}$

Then $\frac{dT_{21}}{dt} \geq (a_{21})^{(3)}(m)^{(3)} - \varepsilon_3 T_{21}$ which leads to 220

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t} = \frac{1}{2}$ it results

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_3}$ By taking now $\varepsilon_3$ sufficiently small one sees that $T_{21}$ is unbounded.
The same property holds for $T_{22}$ if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t), t\right) = (b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}$ , $\frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}} < 1$ and to choose 221

$(\widehat{P}_{24})^{(4)}$ and $(\widehat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[(\widehat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \quad 222$$

$$\frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \quad 223$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself 224

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric 225

$$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{24})^{(4)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{27}}),(\widetilde{T_{27}})$ : $\left((\widetilde{G_{27}}),(\widetilde{T_{27}})\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{24})^{(4)}\left|G_{25}^{(1)}-G_{25}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\,ds_{(24)}+$$

$$\int_0^t\{(a'_{24})^{(4)}\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$G_{24}^{(2)}\left|(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)-(a''_{24})^{(4)}\left(T_{25}^{(2)},s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\}ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}t}\leq \quad 226$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)}+(a'_{24})^{(4)}+(\widehat{A}_{24})^{(4)}+(\widehat{P}_{24})^{(4)}(\hat{k}_{24})^{(4)}\right)d\left(((G_{27})^{(1)},(T_{27})^{(1)};\ (G_{27})^{(2)},(T_{27})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 16:** The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ *and* $(\widehat{Q}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_+$. 227

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i=24,25,26$ depend only on $T_{25}$ and respectively on

$(G_{27})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 17:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 228

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(4)} - (a_i'')^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(4)} t)} > 0$ for t > 0

**Definition of** $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2\ and\ \left((\widehat{M}_{24})^{(4)}\right)_3$ : 229

**Remark 18:** if $G_{24}$ is bounded, the same property have also $G_{25}\ and\ G_{26}$ . indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a_{25}')^{(4)} G_{25}$ and by integrating

$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1 / (a_{25}')^{(4)}$

In the same way , one can obtain

$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2 / (a_{26}')^{(4)}$

If $G_{25}\ or\ G_{26}$ is bounded, the same property follows for $G_{24}$ , $G_{26}$ and $G_{24}$ , $G_{25}$ respectively.

**Remark 19:** If $G_{24}$ *is* bounded, from below, the same property holds for $G_{25}\ and\ G_{26}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{25}$ is bounded from below. 230

**Remark 20:** If $T_{24}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}\ ((G_{27})(t), t)) = (b_{25}')^{(4)}$ then $T_{25} \to \infty$. 231

**Definition of** $(m)^{(4)}$ and $\varepsilon_4$ :

Indeed let $t_4$ be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t), t) < \varepsilon_4, T_{24}\,(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_4}$ By taking now $\varepsilon_4$ sufficiently small one sees that $T_{25}$ is unbounded.
The same property holds for $T_{26}$ if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t), t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for $G_{29}$ , $G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}$ , $\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)}+\left((\hat{P}_{28})^{(5)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{28})^{(5)} \tag{234}$$

$$\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{28})^{(5)}\right]\leq(\hat{Q}_{28})^{(5)} \tag{235}$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric (236)

$$d\left(((G_{31})^{(1)},(T_{31})^{(1)}),((G_{31})^{(2)},(T_{31})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\hat{M}_{28})^{(5)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\hat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{31}}),(\widetilde{T_{31}})$ : $\left((\widetilde{G_{31}}),(\widetilde{T_{31}})\right)=\mathcal{A}^{(5)}\left((G_{31}),(T_{31})\right)$

It results

$$|\tilde{G}_{28}^{(1)}-\tilde{G}_i^{(2)}|\leq\int_0^t(a_{28})^{(5)}|G_{29}^{(1)}-G_{29}^{(2)}|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\,ds_{(28)}+$$

$$\int_0^t\{(a'_{28})^{(5)}|G_{28}^{(1)}-G_{28}^{(2)}|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$(a''_{28})^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)|G_{28}^{(1)}-G_{28}^{(2)}|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$G_{28}^{(2)}|(a''_{28})^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)-(a''_{28})^{(5)}\left(T_{29}^{(2)},s_{(28)}\right)|\;e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\}ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on it follows

$$|(G_{31})^{(1)}-(G_{31})^{(2)}|e^{-(\widehat{M}_{28})^{(5)}t}\leq \tag{237}$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)}+(a'_{28})^{(5)}+(\widehat{A}_{28})^{(5)}+(\widehat{P}_{28})^{(5)}(\widehat{k}_{28})^{(5)}\right)d\left(((G_{31})^{(1)},(T_{31})^{(1)};\;(G_{31})^{(2)},(T_{31})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 21:** The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\hat{P}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ *and* $(\hat{Q}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ respectively of $\mathbb{R}_+$. (238)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on $T_{29}$ and respectively on $(G_{31})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 22:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 239

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(5)} - (a_i'')^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(5)} t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2\ and\ \left((\widehat{M}_{28})^{(5)}\right)_3$ : 240

**Remark 23:** if $G_{28}$ is bounded, the same property have also $G_{29}\ and\ G_{30}$ . indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a_{29}')^{(5)} G_{29}$ and by integrating

$$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1 / (a_{29}')^{(5)}$$

In the same way , one can obtain

$$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2 / (a_{30}')^{(5)}$$

If $G_{29}\ or\ G_{30}$ is bounded, the same property follows for $G_{28}$ , $G_{30}$ and $G_{28}$ , $G_{29}$ respectively.

**Remark 24:** If $G_{28}$ *is* bounded, from below, the same property holds for $G_{29}\ and\ G_{30}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{29}$ is bounded from below. 241

**Remark 25:** If $T_{28}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}\,((G_{31})(t), t)) = (b_{29}')^{(5)}$ then $T_{29} \to \infty$. 242

**Definition of** $(m)^{(5)}$ and $\varepsilon_5$ :

Indeed let $t_5$ be so that for $t > t_5$

$$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t), t) < \varepsilon_5, T_{28}\,(t) > (m)^{(5)}$$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_5}$ By taking now $\varepsilon_5$ sufficiently small one sees that $T_{29}$ is unbounded.
The same property holds for $T_{30}$ if $\lim_{t\to\infty}(b_{30}'')^{(5)}\left((G_{31})(t), t\right) = (b_{30}')^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{33}$ , $G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}$ , $\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}} < 1$ and to choose 244

$(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[(\hat{P}_{32})^{(6)}+\left((\hat{P}_{32})^{(6)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{32})^{(6)} \tag{245}$$

$$\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[\left((\hat{Q}_{32})^{(6)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{32})^{(6)}\right]\leq(\hat{Q}_{32})^{(6)} \tag{246}$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric (247)

$$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\hat{M}_{32})^{(6)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\hat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{35}}),(\widetilde{T_{35}})$ : $\left((\widetilde{G_{35}}),(\widetilde{T_{35}})\right)=\mathcal{A}^{(6)}\left((G_{35}),(T_{35})\right)$

It results

$$|\tilde{G}_{32}^{(1)}-\tilde{G}_i^{(2)}|\leq\int_0^t(a_{32})^{(6)}|G_{33}^{(1)}-G_{33}^{(2)}|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\,ds_{(32)}+$$

$$\int_0^t\{(a'_{32})^{(6)}|G_{32}^{(1)}-G_{32}^{(2)}|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)|G_{32}^{(1)}-G_{32}^{(2)}|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$G_{32}^{(2)}|(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)-(a''_{32})^{(6)}\left(T_{33}^{(2)},s_{(32)}\right)|\;e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\}ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$|(G_{35})^{(1)}-(G_{35})^{(2)}|e^{-(\widehat{M}_{32})^{(6)}t}\leq \tag{248}$$
$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)}+(a'_{32})^{(6)}+(\widehat{A}_{32})^{(6)}+(\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}\right)d\left(((G_{35})^{(1)},(T_{35})^{(1)};\;(G_{35})^{(2)},(T_{35})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 26:** The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\hat{P}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ *and* $(\hat{Q}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ respectively of $\mathbb{R}_+$. (249)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i = 32,33,34$ depend only on $T_{33}$ and respectively on $(G_{35})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 27:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 250

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(6)} - (a_i'')^{(6)}(T_{33}(s_{(32)}), s_{(32)})\} ds_{(32)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(6)} t\right)} > 0$ for t > 0

**Definition of** $\left((\widehat{M}_{32})^{(6)}\right)_1, \left((\widehat{M}_{32})^{(6)}\right)_2\ and\ \left((\widehat{M}_{32})^{(6)}\right)_3$ : 251

**Remark 28:** if $G_{32}$ is bounded, the same property have also $G_{33}\ and\ G_{34}$ . indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a_{33}')^{(6)} G_{33}$ and by integrating

$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_1 / (a_{33}')^{(6)}$

In the same way , one can obtain

$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_2 / (a_{34}')^{(6)}$

If $G_{33}\ or\ G_{34}$ is bounded, the same property follows for $G_{32}$ , $G_{34}$ and $G_{32}$ , $G_{33}$ respectively.

**Remark 29:** If $G_{32}$ *is* bounded, from below, the same property holds for $G_{33}\ and\ G_{34}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{33}$ is bounded from below. 252

**Remark 30:** If $T_{32}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}\,((G_{35})(t), t)) = (b_{33}')^{(6)}$ then $T_{33} \to \infty$. 253

**Definition of** $(m)^{(6)}$ and $\varepsilon_6$ :

Indeed let $t_6$ be so that for $t > t_6$

$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t), t\right) < \varepsilon_6, T_{32}\,(t) > (m)^{(6)}$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_6}$ By taking now $\varepsilon_6$ sufficiently small one sees that $T_{33}$ is unbounded.
The same property holds for $T_{34}$ if $\lim_{t\to\infty}(b_{34}'')^{(6)}\left((G_{35})(t), t(t), t\right) = (b_{34}')^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{37}, G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}$ , $\frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$ and to choose

$(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[(\hat{P}_{36})^{(7)}+\left((\hat{P}_{36})^{(7)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{36})^{(7)} \tag{256}$$

$$\frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[\left((\hat{Q}_{36})^{(7)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{36})^{(7)}\right]\leq(\hat{Q}_{36})^{(7)} \tag{257}$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric (258)

$$d\left(((G_{39})^{(1)},(T_{39})^{(1)}),((G_{39})^{(2)},(T_{39})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\tilde{M}_{36})^{(7)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\tilde{M}_{36})^{(7)}t}\}$$

Indeed if we denote

**Definition of** $\widetilde{(G_{39})},\widetilde{(T_{39})}:(\widetilde{(G_{39})},\widetilde{(T_{39})})=\mathcal{A}^{(7)}((G_{39}),(T_{39}))$

It results

$$|\tilde{G}_{36}^{(1)}-\tilde{G}_i^{(2)}|\leq\int_0^t(a_{36})^{(7)}|G_{37}^{(1)}-G_{37}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\,ds_{(36)}+$$
$$\int_0^t\{(a'_{36})^{(7)}|G_{36}^{(1)}-G_{36}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$(a''_{36})^{(7)}(T_{37}^{(1)},s_{(36)})|G_{36}^{(1)}-G_{36}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$G_{36}^{(2)}|(a''_{36})^{(7)}(T_{37}^{(1)},s_{(36)})-(a''_{36})^{(7)}(T_{37}^{(2)},s_{(36)})|\;e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\}ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on it follows

$$|(G_{39})^{(1)}-(G_{39})^{(2)}|e^{-(\widehat{M}_{36})^{(7)}t}\leq \tag{259}$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}}\left((a_{36})^{(7)}+(a'_{36})^{(7)}+(\widehat{A}_{36})^{(7)}+(\widehat{P}_{36})^{(7)}(\widehat{k}_{36})^{(7)}\right)d\left(((G_{39})^{(1)},(T_{39})^{(1)};\;(G_{39})^{(2)},(T_{39})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 31:** The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ *and* $(\widehat{Q}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ respectively of $\mathbb{R}_+$. (260)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(7)}$ and $(b_i'')^{(7)}, i = 36,37,38$ depend only on $T_{37}$ and respectively on $(G_{39})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 32:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 261

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(7)} - (a_i'')^{(7)}(T_{37}(s_{(36)}), s_{(36)})\} ds_{(36)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(7)} t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2\ and\ \left((\widehat{M}_{36})^{(7)}\right)_3$ : 262

**Remark 33:** if $G_{36}$ is bounded, the same property have also $G_{37}\ and\ G_{38}$ . indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a_{37}')^{(7)} G_{37}$ and by integrating

$$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1 / (a_{37}')^{(7)}$$

In the same way , one can obtain

$$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2 / (a_{38}')^{(7)}$$

If $G_{37}\ or\ G_{38}$ is bounded, the same property follows for $G_{36}$ , $G_{38}$ and $G_{36}$ , $G_{37}$ respectively.

**Remark 34:** If $G_{36}$ *is* bounded, from below, the same property holds for $G_{37}\ and\ G_{38}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{37}$ is bounded from below. 263

**Remark 35:** If $T_{36}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(7)}\ ((G_{39})(t), t)) = (b_{37}')^{(7)}$ then $T_{37} \to \infty$. 264

**Definition of** $(m)^{(7)}$ and $\varepsilon_7$ :

Indeed let $t_7$ be so that for $t > t_7$

$$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t), t) < \varepsilon_7, T_{36}\,(t) > (m)^{(7)}$$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_7}$ By taking now $\varepsilon_7$ sufficiently small one sees that $T_{37}$ is unbounded.
The same property holds for $T_{38}$ if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t), t\right) = (b_{38}')^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}$ , $\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}} < 1$ and to choose $(\widehat{P}_{40})^{(8)}$ and $(\widehat{Q}_{40})^{(8)}$ large to have 266

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[(\widehat{P}_{40})^{(8)} + \left((\hat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{40})^{(8)} \quad 267$$

$$\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{40})^{(8)}\right] \leq (\hat{Q}_{40})^{(8)} \quad 268$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)}, (T_{43})^{(1)}), ((G_{43})^{(2)}, (T_{43})^{(2)})\right) = \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

**Definition of** $\widetilde{(G_{43})}, \widetilde{(T_{43})}$ : $\left(\widetilde{(G_{43})}, \widetilde{(T_{43})}\right) = \mathcal{A}^{(8)}((G_{43}), (T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{40})^{(8)}\left|G_{41}^{(1)} - G_{41}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\,ds_{(40)} +$$
$$\int_0^t\{(a'_{40})^{(8)}\left|G_{40}^{(1)} - G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{-(\widehat{M}_{40})^{(8)}s_{(40)}} +$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right)\left|G_{40}^{(1)} - G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}} +$$
$$G_{40}^{(2)}\left|(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right) - (a''_{40})^{(8)}\left(T_{41}^{(2)}, s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\}ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0, t]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)} - (G_{43})^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}t} \leq \quad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)} + (a'_{40})^{(8)} + (\widehat{A}_{40})^{(8)} + (\widehat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}\right)d\left(((G_{43})^{(1)}, (T_{43})^{(1)};\ (G_{43})^{(2)}, (T_{43})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

**Remark 36:** The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate 274

condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ *and* $(\widehat{Q}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i = 40,41,42$ depend only on $T_{41}$ and respectively on $(G_{43})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 37** There does not exist any $t$ where $G_i(t) = 0\ and\ T_i(t) = 0$ 275

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(8)} - (a_i'')^{(8)}(T_{41}(s_{(40)}), s_{(40)})\} ds_{(40)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(8)}t)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{40})^{(8)}\right)_1, \left((\widehat{M}_{40})^{(8)}\right)_2\ and\ \left((\widehat{M}_{40})^{(8)}\right)_3$ : 276

**Remark 38:** if $G_{40}$ is bounded, the same property have also $G_{41}\ and\ G_{42}$ . indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a_{41}')^{(8)}G_{41}$ and by integrating

$$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1/(a_{41}')^{(8)}$$

In the same way , one can obtain

$$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2/(a_{42}')^{(8)}$$

If $G_{41}\ or\ G_{42}$ is bounded, the same property follows for $G_{40}$ , $G_{42}$ and $G_{40}$ , $G_{41}$ respectively.

**Remark 39:** If $G_{40}$ *is* bounded, from below, the same property holds for $G_{41}\ and\ G_{42}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{41}$ is bounded from below. 277

**Remark 40:** If $T_{40}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}((G_{43})(t), t)) = (b_{41}')^{(8)}$ then $T_{41} \to \infty$. 278

**Definition of** $(m)^{(8)}$ and $\varepsilon_8$ :

Indeed let $t_8$ be so that for $t > t_8$

$$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t), t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right)$, $t = log\frac{2}{\varepsilon_8}$ By taking now $\varepsilon_8$ sufficiently small one sees that $T_{41}$ is unbounded.
The same property holds for $T_{42}$ if $\lim_{t\to\infty}(b''_{42})^{(8)}\left((G_{43})(t), t(t), t\right) = (b'_{42})^{(8)}$



It is now sufficient to take $\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}$ , $\frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$ and to choose $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ large to have

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}\left[(\hat{P}_{44})^{(9)} + \left((\hat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}}\left[\left((\hat{Q}_{44})^{(9)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{44})^{(9)}\right] \leq (\hat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)}, (T_{47})^{(1)}), ((G_{47})^{(2)}, (T_{47})^{(2)})\right) =$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{44})^{(9)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{47}}), (\widetilde{T_{47}}) : \left((\widetilde{G_{47}}), (\widetilde{T_{47}})\right) = \mathcal{A}^{(9)}\left((G_{47}), (T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{44})^{(9)}\left|G_{45}^{(1)} - G_{45}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\,ds_{(44)} +$$
$$\int_0^t\{(a'_{44})^{(9)}\left|G_{44}^{(1)} - G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{-(\widehat{M}_{44})^{(9)}s_{(44)}} +$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right)\left|G_{44}^{(1)} - G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}} +$$
$$G_{44}^{(2)}\left|(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right) - (a''_{44})^{(9)}\left(T_{45}^{(2)}, s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\}ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)} - G^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}t} \leq$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)} + (a'_{44})^{(9)} + (\widehat{A}_{44})^{(9)} + (\widehat{P}_{44})^{(9)}(\hat{k}_{44})^{(9)}\right)d\left(((G_{47})^{(1)}, (T_{47})^{(1)};\ (G_{47})^{(2)}, (T_{47})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis (39,35,36) the result follows

**Remark 41:** The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by

$(\widehat{P}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ *and* $(\widehat{Q}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i = 44{,}45{,}46$ depend only on $T_{45}$ and respectively on $(G_{47})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 42:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$

From 99 to 44 it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t\{(a_i')^{(9)}-(a_i'')^{(9)}(T_{45}(s_{(44)}),s_{(44)})\}ds_{(44)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(9)}t)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{44})^{(9)}\right)_1, \left((\widehat{M}_{44})^{(9)}\right)_2\ and\ \left((\widehat{M}_{44})^{(9)}\right)_3$ :

**Remark 43:** if $G_{44}$ is bounded, the same property have also $G_{45}\ and\ G_{46}$ . indeed if

$G_{44} < (\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt} \leq \left((\widehat{M}_{44})^{(9)}\right)_1 - (a_{45}')^{(9)}G_{45}$ and by integrating

$$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_1/(a_{45}')^{(9)}$$

In the same way , one can obtain

$$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_2/(a_{46}')^{(9)}$$

If $G_{45}\ or\ G_{46}$ is bounded, the same property follows for $G_{44}$ , $G_{46}$ and $G_{44}$ , $G_{45}$ respectively.

**Remark 44:** If $G_{44}$ *is* bounded, from below, the same property holds for $G_{45}\ and\ G_{46}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{45}$ is bounded from below.

**Remark 45:** If $T_{44}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t),t\right) = (b_{45}')^{(9)}$ then $T_{45} \to \infty$.

**Definition of** $(m)^{(9)}$ and $\varepsilon_9$ :

Indeed let $t_9$ be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t),t\right) < \varepsilon_9, T_{44}\,(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right), \quad t = log\frac{2}{\varepsilon_9}$ By taking now $\varepsilon_9$ sufficiently small one sees that $T_{45}$ is unbounded.
The same property holds for $T_{46}$ if $\lim_{t\to\infty}(b_{46}'')^{(9)}\left((G_{47})(t),t\right) = (b_{46}')^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

**Behavior of the solutions of equation** 280

**Theorem** If we denote and define

**Definition of** $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ :

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)} \le -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14}, t) + (a''_{14})^{(1)}(T_{14}, t) \le -(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)} \le -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G, t) - (b''_{14})^{(1)}(G, t) \le -(\tau_1)^{(1)}$

**Definition of** $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$ : 281

By $(\nu_1)^{(1)} > 0$, $(\nu_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$, $(u_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

**Definition of** $(\bar{\nu}_1)^{(1)},, (\bar{\nu}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$ : 282

By $(\bar{\nu}_1)^{(1)} > 0$, $(\bar{\nu}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$, $(\bar{u}_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

**Definition of** $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (\nu_0)^{(1)}$ :- 283

If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by
$(m_2)^{(1)} = (\nu_0)^{(1)}, (m_1)^{(1)} = (\nu_1)^{(1)}$, $if\ (\nu_0)^{(1)} < (\nu_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}, if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)}$,

and $\boxed{(\nu_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)}$, $if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)}$, $if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)}$,

and $\boxed{(u_0)^{(1)} = \frac{T^0_{13}}{T^0_{14}}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$

are defined

Then the solution of global equations satisfies the inequalities 285

$G^0_{13}e^{\left((S_1)^{(1)} - (p_{13})^{(1)}\right)t} \le G_{13}(t) \le G^0_{13}e^{(S_1)^{(1)}t}$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)}-(p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)}t}$$

$$\left(\frac{(a_{15})^{(1)}G_{13}^0}{(m_1)^{(1)}((S_1)^{(1)}-(p_{13})^{(1)}-(S_2)^{(1)})}\left[e^{((S_1)^{(1)}-(p_{13})^{(1)})t} - e^{-(S_2)^{(1)}t}\right] + G_{15}^0 e^{-(S_2)^{(1)}t} \le G_{15}(t) \le \right. \quad (286)$$

$$\left. \frac{(a_{15})^{(1)}G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)}-(a'_{15})^{(1)})}[e^{(S_1)^{(1)}t} - e^{-(a'_{15})^{(1)}t}] + G_{15}^0 e^{-(a'_{15})^{(1)}t}\right)$$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)}t} \le T_{13}(t) \le T_{13}^0 e^{((R_1)^{(1)}+(r_{13})^{(1)})t}} \quad (287)$$

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)}t} \le T_{13}(t) \le \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)}+(r_{13})^{(1)})t} \quad (288)$$

$$\frac{(b_{15})^{(1)}T_{13}^0}{(\mu_1)^{(1)}((R_1)^{(1)}-(b'_{15})^{(1)})}\left[e^{(R_1)^{(1)}t} - e^{-(b'_{15})^{(1)}t}\right] + T_{15}^0 e^{-(b'_{15})^{(1)}t} \le T_{15}(t) \le \quad (289)$$

$$\frac{(a_{15})^{(1)}T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)}+(r_{13})^{(1)}+(R_2)^{(1)})}\left[e^{((R_1)^{(1)}+(r_{13})^{(1)})t} - e^{-(R_2)^{(1)}t}\right] + T_{15}^0 e^{-(R_2)^{(1)}t}$$

**Definition of** $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:- (290)

Where $(S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$

$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$

$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$

$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$

**Behavior of the solutions of equation** (291)

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ : (292)

$(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ four constants satisfying

$$-(\sigma_2)^{(2)} \le -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(T_{17}, t) + (a''_{17})^{(2)}(T_{17}, t) \le -(\sigma_1)^{(2)} \quad (293)$$

$$-(\tau_2)^{(2)} \le -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}((G_{19}), t) - (b''_{17})^{(2)}((G_{19}), t) \le -(\tau_1)^{(2)} \quad (294)$$

**Definition of** $(\nu_1)^{(2)}, (\nu_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$ : (295)

By $(\nu_1)^{(2)} > 0$, $(\nu_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0$, $(u_2)^{(2)} < 0$ the roots (296)

of the equations $(a_{17})^{(2)}(\nu^{(2)})^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ (297)

and $(b_{14})^{(2)}(u^{(2)})^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and (298)

**Definition of** $(\bar{\nu}_1)^{(2)}, , (\bar{\nu}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$ : (299)

By $(\bar{\nu}_1)^{(2)} > 0$, $(\bar{\nu}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0$, $(\bar{u}_2)^{(2)} < 0$ the (300)

roots of the equations $(a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 301

and $(b_{17})^{(2)}\left(u^{(2)}\right)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 302

**Definition of** $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ :- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, ***if*** $(u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$, ***if*** $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)}$, ***if*** $(\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)}t}$

$(p_i)^{(2)}$ is defined by equation

$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)}t}$ 311

$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)}t}\right] + G_{18}^0 e^{-(S_2)^{(2)}t} \le G_{18}(t) \le\right.$ 312

$\left.\frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)}t} - e^{-(a'_{18})^{(2)}t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)}t}\right)$

$\boxed{T_{16}^0 e^{(R_1)^{(2)}t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}}$ 313

$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)}t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}$ 314

$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)}t} - e^{-(b'_{18})^{(2)}t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)}t} \le T_{18}(t) \le$ 315

$$\frac{(a_{18})^{(2)}T_{16}^{0}}{(\mu_2)^{(2)}((R_1)^{(2)}+(r_{16})^{(2)}+(R_2)^{(2)})}\left[e^{((R_1)^{(2)}+(r_{16})^{(2)})t}-e^{-(R_2)^{(2)}t}\right]+T_{18}^{0}e^{-(R_2)^{(2)}t}$$

**Definition of** $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$$

$$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)}$$ 318

$$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$$

**Behavior of the solutions** 319

**Theorem 3:** If we denote and define

**Definition of** $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ :

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}\big((G_{23}), t\big) \le -(\tau_1)^{(3)}$$

**Definition of** $(v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$ : 320

By $(v_1)^{(3)} > 0$, $(v_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{v}_1)^{(3)} > 0$, $(\bar{v}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

**Definition of** $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ :- 321

If we define $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (v_0)^{(3)}, (m_1)^{(3)} = (v_1)^{(3)}$, ***if*** $(v_0)^{(3)} < (v_1)^{(3)}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (\bar{v}_1)^{(3)}$, ***if*** $(v_1)^{(3)} < (v_0)^{(3)} < (\bar{v}_1)^{(3)}$,

and $\boxed{(v_0)^{(3)} = \frac{G_{20}^{0}}{G_{21}^{0}}}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_0)^{(3)}$, ***if*** $(\bar{v}_1)^{(3)} < (v_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)},\ \boldsymbol{if}\ (u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}, \boldsymbol{if}\ (u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)},$ and $\boxed{(u_0)^{(3)} = \frac{T_{20}^0}{T_{21}^0}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)},\ \boldsymbol{if}\ (\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G_{20}^0 e^{(S_1)^{(3)} t}$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)} t} \quad 323$$

$$\left(\frac{(a_{22})^{(3)} G_{20}^0}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\left[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)} t}\right] + G_{22}^0 e^{-(S_2)^{(3)} t} \le G_{22}(t) \le \right. \quad 324$$

$$\left. \frac{(a_{22})^{(3)} G_{20}^0}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}\left[e^{(S_1)^{(3)} t} - e^{-(a'_{22})^{(3)} t}\right] + G_{22}^0 e^{-(a'_{22})^{(3)} t}\right)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t}} \quad 325$$

$$\frac{1}{(\mu_1)^{(3)}} T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}} T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)} T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)} - (b'_{22})^{(3)})}\left[e^{(R_1)^{(3)} t} - e^{-(b'_{22})^{(3)} t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)} t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)} T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)} + (r_{20})^{(3)} + (R_2)^{(3)})}\left[e^{((R_1)^{(3)} + (r_{20})^{(3)})t} - e^{-(R_2)^{(3)} t}\right] + T_{22}^0 e^{-(R_2)^{(3)} t}$$

**Definition of** $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

**Behavior of the solutions of equation**

**Theorem:** If we denote and define

**Definition of** $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ :

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$

$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}((G_{27}), t) - (b''_{25})^{(4)}((G_{27}), t) \le -(\tau_1)^{(4)}$

**Definition of** $(v_1)^{(4)}, (v_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, v^{(4)}, u^{(4)}$ : 329

By $(v_1)^{(4)} > 0$ , $(v_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$ , $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_1)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

**Definition of** $(\bar{v}_1)^{(4)},, (\bar{v}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$ : 330

By $(\bar{v}_1)^{(4)} > 0$ , $(\bar{v}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$ , $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

**Definition of** $(m_1)^{(4)}$ , $(m_2)^{(4)}$ , $(\mu_1)^{(4)}, (\mu_2)^{(4)}, (\nu_0)^{(4)}$ :-

If we define $(m_1)^{(4)}$ , $(m_2)^{(4)}$ , $(\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (v_0)^{(4)}, (m_1)^{(4)} = (v_1)^{(4)}$, ***if*** $(v_0)^{(4)} < (v_1)^{(4)}$

$(m_2)^{(4)} = (v_1)^{(4)}, (m_1)^{(4)} = (\bar{v}_1)^{(4)}$ , ***if*** $(v_4)^{(4)} < (v_0)^{(4)} < (\bar{v}_1)^{(4)}$,
and $\boxed{(v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (v_4)^{(4)}, (m_1)^{(4)} = (v_0)^{(4)}$, ***if*** $(\bar{v}_4)^{(4)} < (v_0)^{(4)}$

and analogously

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$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)}$, ***if*** $(u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}$ , ***if*** $(u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)}$,
and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}$, ***if*** $(\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

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$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)}t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)}t}$$

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$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)}t}\right] + G_{26}^0 e^{-(S_2)^{(4)}t} \le G_{26}(t) \le\right.$$ 334

*(a26)4G240(m2)4(S1)4−(a26′)4e(S1)4t−e−(a26′)4t+ G260e−(a26′)4t*

$$\boxed{T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}}T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}}T_{24}^0 e^{((R_1)^{(4)}+(r_{24})^{(4)})t} \quad 335$$

$$\frac{(b_{26})^{(4)}T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)}-(b'_{26})^{(4)})}\left[e^{(R_1)^{(4)}t} - e^{-(b'_{26})^{(4)}t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)}t} \le T_{26}(t) \le \quad 336$$

$$\frac{(a_{26})^{(4)}T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)}+(r_{24})^{(4)}+(R_2)^{(4)})}\left[e^{((R_1)^{(4)}+(r_{24})^{(4)})t} - e^{-(R_2)^{(4)}t}\right] + T_{26}^0 e^{-(R_2)^{(4)}t}$$

**Definition of** $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

**Behavior of the solutions of equation** 338

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ :

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

**Definition of** $(v_1)^{(5)}, (v_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, v^{(5)}, u^{(5)}$ : 339

By $(v_1)^{(5)} > 0$ , $(v_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$ , $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(v^{(5)}\big)^2 + (\sigma_1)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

**Definition of** $(\bar{v}_1)^{(5)}, , (\bar{v}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$ : 340

By $(\bar{v}_1)^{(5)} > 0$ , $(\bar{v}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$ , $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(v^{(5)}\big)^2 + (\sigma_2)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

**Definition of** $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (v_0)^{(5)}$ :-

If we define $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (v_0)^{(5)}, (m_1)^{(5)} = (v_1)^{(5)}$, ***if*** $(v_0)^{(5)} < (v_1)^{(5)}$

$(m_2)^{(5)} = (v_1)^{(5)}, (m_1)^{(5)} = (\bar{v}_1)^{(5)}$ , ***if*** $(v_1)^{(5)} < (v_0)^{(5)} < (\bar{v}_1)^{(5)}$,
and $\boxed{(v_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)},\ \boldsymbol{if}\ (u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}, \boldsymbol{if}\ (u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)},$

and $\boxed{(u_0)^{(5)} = \frac{T_{28}^0}{T_{29}^0}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}, \boldsymbol{if}\ (\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{28}(t) \le G_{28}^0 e^{(S_1)^{(5)} t}$

where $(p_i)^{(5)}$ is defined by equation

$\frac{1}{(m_5)^{(5)}} G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}} G_{28}^0 e^{(S_1)^{(5)} t}$ 343

$\left(\frac{(a_{30})^{(5)} G_{28}^0}{(m_1)^{(5)}((S_1)^{(5)} - (p_{28})^{(5)} - (S_2)^{(5)})}\left[e^{((S_1)^{(5)} - (p_{28})^{(5)})t} - e^{-(S_2)^{(5)} t}\right] + G_{30}^0 e^{-(S_2)^{(5)} t} \le G_{30}(t) \le\right.$ 344

*(a30)5G280(m2)5(S1)5−(a30′)5e(S1)5t−e−(a30′)5t+ G300e−(a30′)5t*

$\boxed{T_{28}^0 e^{(R_1)^{(5)} t} \le T_{28}(t) \le T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}}$ 345

$\frac{1}{(\mu_1)^{(5)}} T_{28}^0 e^{(R_1)^{(5)} t} \le T_{28}(t) \le \frac{1}{(\mu_2)^{(5)}} T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}$ 346

$\frac{(b_{30})^{(5)} T_{28}^0}{(\mu_1)^{(5)}((R_1)^{(5)} - (b'_{30})^{(5)})}\left[e^{(R_1)^{(5)} t} - e^{-(b'_{30})^{(5)} t}\right] + T_{30}^0 e^{-(b'_{30})^{(5)} t} \le T_{30}(t) \le$ 347

$\frac{(a_{30})^{(5)} T_{28}^0}{(\mu_2)^{(5)}((R_1)^{(5)} + (r_{28})^{(5)} + (R_2)^{(5)})}\left[e^{((R_1)^{(5)} + (r_{28})^{(5)})t} - e^{-(R_2)^{(5)} t}\right] + T_{30}^0 e^{-(R_2)^{(5)} t}$

**Definition of** $(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$:- 348

Where $(S_1)^{(5)} = (a_{28})^{(5)}(m_2)^{(5)} - (a'_{28})^{(5)}$

$(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}$

$(R_1)^{(5)} = (b_{28})^{(5)}(\mu_2)^{(5)} - (b'_{28})^{(5)}$

$(R_2)^{(5)} = (b'_{30})^{(5)} - (r_{30})^{(5)}$

**Behavior of the solutions of equation** 349

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ :

$(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)} \le -(a'_{32})^{(6)} + (a'_{33})^{(6)} - (a''_{32})^{(6)}(T_{33}, t) + (a''_{33})^{(6)}(T_{33}, t) \le -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)} \le -(b'_{32})^{(6)} + (b'_{33})^{(6)} - (b''_{32})^{(6)}\big((G_{35}), t\big) - (b''_{33})^{(6)}\big((G_{35}), t\big) \le -(\tau_1)^{(6)}$$

**Definition of** $(\nu_1)^{(6)}, (\nu_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, \nu^{(6)}, u^{(6)}$ : 350

By $(\nu_1)^{(6)} > 0$ , $(\nu_2)^{(6)} < 0$ and respectively $(u_1)^{(6)} > 0$ , $(u_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(6)}, , (\bar{\nu}_2)^{(6)}, (\bar{u}_1)^{(6)}, (\bar{u}_2)^{(6)}$ : 351

By $(\bar{\nu}_1)^{(6)} > 0$ , $(\bar{\nu}_2)^{(6)} < 0$ and respectively $(\bar{u}_1)^{(6)} > 0$ , $(\bar{u}_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$

**Definition of** $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}, (\nu_0)^{(6)}$ :-

If we define $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}$ by

$$(m_2)^{(6)} = (\nu_0)^{(6)}, (m_1)^{(6)} = (\nu_1)^{(6)}, \ \boldsymbol{if}\ (\nu_0)^{(6)} < (\nu_1)^{(6)}$$

$$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}, \boldsymbol{if}\ (\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)},$$

and $\boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}}$

$$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$$

and analogously 352

$$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}, \ \boldsymbol{if}\ (u_0)^{(6)} < (u_1)^{(6)}$$

$$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}, \boldsymbol{if}\ (u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)},$$

and $\boxed{(u_0)^{(6)} = \frac{T^0_{32}}{T^0_{33}}}$

$$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}, \boldsymbol{if}\ (\bar{u}_1)^{(6)} < (u_0)^{(6)} \text{ where } (u_1)^{(6)}, (\bar{u}_1)^{(6)}$$

Then the solution of global equations satisfies the inequalities 353

$$G^0_{32}e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{32}(t) \le G^0_{32}e^{(S_1)^{(6)}t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}}G^0_{32}e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{33}(t) \le \frac{1}{(m_2)^{(6)}}G^0_{32}e^{(S_1)^{(6)}t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)}G^0_{32}}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})}\left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)}t}\right] + G^0_{34}e^{-(S_2)^{(6)}t} \le G_{34}(t) \le \right. \quad 355$$

$$\frac{(a_{34})^{(6)}G_{32}^{0}}{(m_2)^{(6)}((S_1)^{(6)}-(a'_{34})^{(6)})}\left[e^{(S_1)^{(6)}t}-e^{-(a'_{34})^{(6)}t}\right]+\ G_{34}^{0}e^{-(a'_{34})^{(6)}t}\Big)$$

$$\boxed{T_{32}^{0}e^{(R_1)^{(6)}t}\le T_{32}(t)\le T_{32}^{0}e^{((R_1)^{(6)}+(r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}}T_{32}^{0}e^{(R_1)^{(6)}t}\le T_{32}(t)\le\frac{1}{(\mu_2)^{(6)}}T_{32}^{0}e^{((R_1)^{(6)}+(r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)}T_{32}^{0}}{(\mu_1)^{(6)}((R_1)^{(6)}-(b'_{34})^{(6)})}\left[e^{(R_1)^{(6)}t}-e^{-(b'_{34})^{(6)}t}\right]+T_{34}^{0}e^{-(b'_{34})^{(6)}t}\le T_{34}(t)\le \quad 358$$

$$\frac{(a_{34})^{(6)}T_{32}^{0}}{(\mu_2)^{(6)}((R_1)^{(6)}+(r_{32})^{(6)}+(R_2)^{(6)})}\left[e^{((R_1)^{(6)}+(r_{32})^{(6)})t}-e^{-(R_2)^{(6)}t}\right]+T_{34}^{0}e^{-(R_2)^{(6)}t}$$

**Definition of** $(S_1)^{(6)},(S_2)^{(6)},(R_1)^{(6)},(R_2)^{(6)}$:- 359

Where $(S_1)^{(6)}=(a_{32})^{(6)}(m_2)^{(6)}-(a'_{32})^{(6)}$

$(S_2)^{(6)}=(a_{34})^{(6)}-(p_{34})^{(6)}$

$(R_1)^{(6)}=(b_{32})^{(6)}(\mu_2)^{(6)}-(b'_{32})^{(6)}$

$(R_2)^{(6)}=(b'_{34})^{(6)}-(r_{34})^{(6)}$

**Behavior of the solutions of equation**

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(7)},(\sigma_2)^{(7)},(\tau_1)^{(7)},(\tau_2)^{(7)}$ :

$(\sigma_1)^{(7)},(\sigma_2)^{(7)},(\tau_1)^{(7)},(\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)}\le-(a'_{36})^{(7)}+(a'_{37})^{(7)}-(a''_{36})^{(7)}(T_{37},t)+(a''_{37})^{(7)}(T_{37},t)\le-(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)}\le-(b'_{36})^{(7)}+(b'_{37})^{(7)}-(b''_{36})^{(7)}\big((G_{39}),t\big)-(b''_{37})^{(7)}\big((G_{39}),t\big)\le-(\tau_1)^{(7)}$$

**Definition of** $(v_1)^{(7)},(v_2)^{(7)},(u_1)^{(7)},(u_2)^{(7)},v^{(7)},u^{(7)}$ : 361

By $(v_1)^{(7)}>0,(v_2)^{(7)}<0$ and respectively $(u_1)^{(7)}>0,(u_2)^{(7)}<0$ the roots of the equations $(a_{37})^{(7)}\big(v^{(7)}\big)^2+(\sigma_1)^{(7)}v^{(7)}-(a_{36})^{(7)}=0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2+(\tau_1)^{(7)}u^{(7)}-(b_{36})^{(7)}=0$ and

**Definition of** $(\bar{v}_1)^{(7)},,(\bar{v}_2)^{(7)},(\bar{u}_1)^{(7)},(\bar{u}_2)^{(7)}$ : 362

By $(\bar{v}_1)^{(7)}>0,(\bar{v}_2)^{(7)}<0$ and respectively $(\bar{u}_1)^{(7)}>0,(\bar{u}_2)^{(7)}<0$ the roots of the equations $(a_{37})^{(7)}\big(v^{(7)}\big)^2+(\sigma_2)^{(7)}v^{(7)}-(a_{36})^{(7)}=0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2+(\tau_2)^{(7)}u^{(7)}-(b_{36})^{(7)}=0$

**Definition of** $(m_1)^{(7)},(m_2)^{(7)},(\mu_1)^{(7)},(\mu_2)^{(7)},(v_0)^{(7)}$ :-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$$(m_2)^{(7)} = (\nu_0)^{(7)}, (m_1)^{(7)} = (\nu_1)^{(7)}, \ \boldsymbol{if}\ (\nu_0)^{(7)} < (\nu_1)^{(7)}$$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\bar{\nu}_1)^{(7)}, \boldsymbol{if}\ (\nu_1)^{(7)} < (\nu_0)^{(7)} < (\bar{\nu}_1)^{(7)},$$

and $\boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\nu_0)^{(7)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(7)} < (\nu_0)^{(7)}$$

and analogously 363

$$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}, \ \boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$$

and $\boxed{(u_0)^{(7)} = \frac{T_{36}^0}{T_{37}^0}}$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)} \text{ where } (u_1)^{(7)}, (\bar{u}_1)^{(7)}$$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^0 e^{(S_1)^{(7)} t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)} t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)}((S_1)^{(7)} - (p_{36})^{(7)} - (S_2)^{(7)})}\left[e^{((S_1)^{(7)} - (p_{36})^{(7)})t} - e^{-(S_2)^{(7)} t}\right] + G_{38}^0 e^{-(S_2)^{(7)} t} \le G_{38}(t) \le \right. \quad 366$$

$$\left. \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)}((S_1)^{(7)} - (a'_{38})^{(7)})}\left[e^{(S_1)^{(7)} t} - e^{-(a'_{38})^{(7)} t}\right] + G_{38}^0 e^{-(a'_{38})^{(7)} t}\right)$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t} \quad 368$$

$$\frac{(b_{38})^{(7)} T_{36}^0}{(\mu_1)^{(7)}((R_1)^{(7)} - (b'_{38})^{(7)})}\left[e^{(R_1)^{(7)} t} - e^{-(b'_{38})^{(7)} t}\right] + T_{38}^0 e^{-(b'_{38})^{(7)} t} \le T_{38}(t) \le \quad 369$$

$$\frac{(a_{38})^{(7)} T_{36}^0}{(\mu_2)^{(7)}((R_1)^{(7)} + (r_{36})^{(7)} + (R_2)^{(7)})}\left[e^{((R_1)^{(7)} + (r_{36})^{(7)})t} - e^{-(R_2)^{(7)} t}\right] + T_{38}^0 e^{-(R_2)^{(7)} t}$$

**Definition of** $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$

$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$

$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$

**Behavior of the solutions of equation** 371

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ :

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$-(\sigma_2)^{(8)} \le -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \le -(\sigma_1)^{(8)}$

$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\big((G_{43}), t\big) - (b''_{41})^{(8)}\big((G_{43}), t\big) \le -(\tau_1)^{(8)}$

**Definition of** $(\nu_1)^{(8)}, (\nu_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, \nu^{(8)}, u^{(8)}$ : 372

By $(\nu_1)^{(8)} > 0, (\nu_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0, (u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(8)}, , (\bar{\nu}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$ :

By $(\bar{\nu}_1)^{(8)} > 0, (\bar{\nu}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0, (\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

**Definition of** $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}, (\nu_0)^{(8)}$ :-

If we define $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$(m_2)^{(8)} = (\nu_0)^{(8)}, (m_1)^{(8)} = (\nu_1)^{(8)}$, ***if*** $(\nu_0)^{(8)} < (\nu_1)^{(8)}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\bar{\nu}_1)^{(8)}$, ***if*** $(\nu_1)^{(8)} < (\nu_0)^{(8)} < (\bar{\nu}_1)^{(8)}$,
and $\boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\nu_0)^{(8)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(8)} < (\nu_0)^{(8)}$

and analogously 374

$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)},\ \boldsymbol{if}\ (u_0)^{(8)} < (u_1)^{(8)}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}, \boldsymbol{if}\ (u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)},$

and $\boxed{(u_0)^{(8)} = \frac{T_{40}^0}{T_{41}^0}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}, \boldsymbol{if}\ (\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^0 e^{(S_1)^{(8)} t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}} G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}} G_{40}^0 e^{(S_1)^{(8)} t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)} G_{40}^0}{(m_1)^{(8)}((S_1)^{(8)} - (p_{40})^{(8)} - (S_2)^{(8)})}\left[e^{((S_1)^{(8)} - (p_{40})^{(8)})t} - e^{-(S_2)^{(8)} t}\right] + G_{42}^0 e^{-(S_2)^{(8)} t} \le G_{42}(t) \le \right. \quad 377$$

$$\left. \frac{(a_{42})^{(8)} G_{40}^0}{(m_2)^{(8)}((S_1)^{(8)} - (a'_{42})^{(8)})}\left[e^{(S_1)^{(8)} t} - e^{-(a'_{42})^{(8)} t}\right] + G_{42}^0 e^{-(a'_{42})^{(8)} t}\right)$$

$$\boxed{T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}} T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le \frac{1}{(\mu_2)^{(8)}} T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)} T_{40}^0}{(\mu_1)^{(8)}((R_1)^{(8)} - (b'_{42})^{(8)})}\left[e^{(R_1)^{(8)} t} - e^{-(b'_{42})^{(8)} t}\right] + T_{42}^0 e^{-(b'_{42})^{(8)} t} \le T_{42}(t) \le \quad 380$$

$$\frac{(a_{42})^{(8)} T_{40}^0}{(\mu_2)^{(8)}((R_1)^{(8)} + (r_{40})^{(8)} + (R_2)^{(8)})}\left[e^{((R_1)^{(8)} + (r_{40})^{(8)})t} - e^{-(R_2)^{(8)} t}\right] + T_{42}^0 e^{-(R_2)^{(8)} t}$$

**Definition of** $(S_1)^{(8)}, (S_2)^{(8)}, (R_1)^{(8)}, (R_2)^{(8)}$:- 381

Where $(S_1)^{(8)} = (a_{40})^{(8)}(m_2)^{(8)} - (a'_{40})^{(8)}$

$(S_2)^{(8)} = (a_{42})^{(8)} - (p_{42})^{(8)}$

$(R_1)^{(8)} = (b_{40})^{(8)}(\mu_2)^{(8)} - (b'_{40})^{(8)}$

$(R_2)^{(8)} = (b'_{42})^{(8)} - (r_{42})^{(8)}$

**Behavior of the solutions of equation 37 to 92** 382

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ :

$(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ four constants satisfying

$$-(\sigma_2)^{(9)} \le -(a'_{44})^{(9)} + (a'_{45})^{(9)} - (a''_{44})^{(9)}(T_{45}, t) + (a''_{45})^{(9)}(T_{45}, t) \le -(\sigma_1)^{(9)}$$

$$-(\tau_2)^{(9)} \le -(b'_{44})^{(9)} + (b'_{45})^{(9)} - (b''_{44})^{(9)}\big((G_{47}), t\big) - (b''_{45})^{(9)}\big((G_{47}), t\big) \le -(\tau_1)^{(9)}$$

**Definition of** $(\nu_1)^{(9)}, (\nu_2)^{(9)}, (u_1)^{(9)}, (u_2)^{(9)}, \nu^{(9)}, u^{(9)}$ :

By $(\nu_1)^{(9)} > 0$ , $(\nu_2)^{(9)} < 0$ and respectively $(u_1)^{(9)} > 0$ , $(u_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_1)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(9)}, , (\bar{\nu}_2)^{(9)}, (\bar{u}_1)^{(9)}, (\bar{u}_2)^{(9)}$ :

By $(\bar{\nu}_1)^{(9)} > 0$ , $(\bar{\nu}_2)^{(9)} < 0$ and respectively $(\bar{u}_1)^{(9)} > 0$ , $(\bar{u}_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_2)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$

**Definition of** $(m_1)^{(9)}, (m_2)^{(9)}, (\mu_1)^{(9)}, (\mu_2)^{(9)}, (\nu_0)^{(9)}$ :-

If we define $(m_1)^{(9)}, (m_2)^{(9)}, (\mu_1)^{(9)}, (\mu_2)^{(9)}$ by

$$(m_2)^{(9)} = (\nu_0)^{(9)}, (m_1)^{(9)} = (\nu_1)^{(9)}, \ \boldsymbol{if}\ (\nu_0)^{(9)} < (\nu_1)^{(9)}$$

$$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\bar{\nu}_1)^{(9)}, \boldsymbol{if}\ (\nu_1)^{(9)} < (\nu_0)^{(9)} < (\bar{\nu}_1)^{(9)},$$

and $\boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$

$$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\nu_0)^{(9)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(9)} < (\nu_0)^{(9)}$$

and analogously

$$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)}, \ \boldsymbol{if}\ (u_0)^{(9)} < (u_1)^{(9)}$$

$$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}, \boldsymbol{if}\ (u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)},$$

and $\boxed{(u_0)^{(9)} = \frac{T^0_{44}}{T^0_{45}}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}, \boldsymbol{if}\ (\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G^0_{44} e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{44}(t) \le G^0_{44} e^{(S_1)^{(9)} t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}}G_{44}^0 e^{\left((S_1)^{(9)}-(p_{44})^{(9)}\right)t} \le G_{45}(t) \le \frac{1}{(m_2)^{(9)}}G_{44}^0 e^{(S_1)^{(9)}t}$$

$$\left(\frac{(a_{46})^{(9)}G_{44}^0}{(m_1)^{(9)}\left((S_1)^{(9)}-(p_{44})^{(9)}-(S_2)^{(9)}\right)}\left[e^{\left((S_1)^{(9)}-(p_{44})^{(9)}\right)t}-e^{-(S_2)^{(9)}t}\right]+G_{46}^0 e^{-(S_2)^{(9)}t} \le G_{46}(t) \le \right.$$

$$\left.\frac{(a_{46})^{(9)}G_{44}^0}{(m_2)^{(9)}\left((S_1)^{(9)}-(a'_{46})^{(9)}\right)}\left[e^{(S_1)^{(9)}t}-e^{-(a'_{46})^{(9)}t}\right]+\ G_{46}^0 e^{-(a'_{46})^{(9)}t}\right)$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le T_{44}^0 e^{\left((R_1)^{(9)}+(r_{44})^{(9)}\right)t}}$$

$$\frac{1}{(\mu_1)^{(9)}}T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le \frac{1}{(\mu_2)^{(9)}}T_{44}^0 e^{\left((R_1)^{(9)}+(r_{44})^{(9)}\right)t}$$

$$\frac{(b_{46})^{(9)}T_{44}^0}{(\mu_1)^{(9)}\left((R_1)^{(9)}-(b'_{46})^{(9)}\right)}\left[e^{(R_1)^{(9)}t}-e^{-(b'_{46})^{(9)}t}\right]+T_{46}^0 e^{-(b'_{46})^{(9)}t} \le T_{46}(t) \le$$

$$\frac{(a_{46})^{(9)}T_{44}^0}{(\mu_2)^{(9)}\left((R_1)^{(9)}+(r_{44})^{(9)}+(R_2)^{(9)}\right)}\left[e^{\left((R_1)^{(9)}+(r_{44})^{(9)}\right)t}-e^{-(R_2)^{(9)}t}\right]+T_{46}^0 e^{-(R_2)^{(9)}t}$$

**Definition of** $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$

**Proof :** From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14},t)\right) - (a''_{14})^{(1)}(T_{14},t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

**Definition of** $v^{(1)}$ :- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)}\right) \le \frac{dv^{(1)}}{dt} \le -\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)}\right)$$

From which one obtains

**Definition of** $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$ :-

For $0 < \boxed{(v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}} < (v_1)^{(1)} < (\bar{v}_1)^{(1)}$

$$v^{(1)}(t) \ge \frac{(v_1)^{(1)}+(C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(v_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(v_2)^{(1)}}}$$

it follows $(\nu_0)^{(1)} \leq \nu^{(1)}(t) \leq (\nu_1)^{(1)}$

In the same manner , we get 384

$$\nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{\nu}_1)^{(1)}-(\nu_0)^{(1)}}{(\nu_0)^{(1)}-(\bar{\nu}_2)^{(1)}}}$$

From which we deduce $(\nu_0)^{(1)} \leq \nu^{(1)}(t) \leq (\bar{\nu}_1)^{(1)}$

If $0 < (\nu_1)^{(1)} < (\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0} < (\bar{\nu}_1)^{(1)}$ we find like in the previous case, 385

$$(\nu_1)^{(1)} \leq \frac{(\nu_1)^{(1)}+(C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}} \leq \nu^{(1)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\bar{\nu}_1)^{(1)}$$

If $0 < (\nu_1)^{(1)} \leq (\bar{\nu}_1)^{(1)} \leq \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$ , we obtain 386

$$(\nu_1)^{(1)} \leq \nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\nu_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(1)}(t)$ :-

$$(m_2)^{(1)} \leq \nu^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{\nu^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(1)}(t)$ :-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{13})^{(1)} = (a''_{14})^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)}G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)}T_{14}(t)$ This is an important

consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

**Proof** : From global equations we obtain 387

$$\frac{d\nu^{(2)}}{dt} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right) - (a''_{17})^{(2)}(T_{17}, t)\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

**Definition of** $\nu^{(2)}$ :- $\boxed{\nu^{(2)} = \frac{G_{16}}{G_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \le \frac{d\nu^{(2)}}{dt} \le -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

**Definition of** $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$ :-

For $0 < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \ge \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}}{1 + (C)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(C)^{(2)} = \frac{(\nu_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \le \nu^{(2)}(t) \le (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \le \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \le \nu^{(2)}(t) \le (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \le \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}}{1 + (C)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}} \le \nu^{(2)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \le (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \le (\bar{\nu}_1)^{(2)} \le (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$ , we obtain 394

$$(\nu_1)^{(2)} \le \nu^{(2)}(t) \le \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \le (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(2)}(t)$ :- 395

$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}$, $\boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$

In a completely analogous way, we obtain 396

**Definition of** $u^{(2)}(t)$ :-

$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}$, $\boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :** 397

If $(a''_{16})^{(2)} = (a''_{17})^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)} G_{17}(t)$

Analogously if $(b''_{16})^{(2)} = (b''_{17})^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)} T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

**Proof** : From global equations we obtain 398

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right) - (a''_{21})^{(3)}(T_{21}, t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

**Definition of** $\nu^{(3)}$ :- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right) \leq \frac{d\nu^{(3)}}{dt} \leq -\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right)$$

400

From which one obtains

For $0 < (\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\nu_1)^{(3)} < (\bar{\nu}_1)^{(3)}$

$$\nu^{(3)}(t) \geq \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(\nu_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\nu_2)^{(3)}}}$$

it follows $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\nu_1)^{(3)}$

In the same manner , we get 401

$$\nu^{(3)}(t) \le \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{\nu}_1)^{(3)}-(\nu_0)^{(3)}}{(\nu_0)^{(3)}-(\bar{\nu}_2)^{(3)}}}$$

**Definition of** $(\bar{\nu}_1)^{(3)}$ :-

From which we deduce $(\nu_0)^{(3)} \le \nu^{(3)}(t) \le (\bar{\nu}_1)^{(3)}$

If $0 < (\nu_1)^{(3)} < (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (\bar{\nu}_1)^{(3)}$ we find like in the previous case, 402

$$(\nu_1)^{(3)} \le \frac{(\nu_1)^{(3)}+(C)^{(3)}(\nu_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}} \le \nu^{(3)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \le (\bar{\nu}_1)^{(3)}$$

If $0 < (\nu_1)^{(3)} \le (\bar{\nu}_1)^{(3)} \le (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}$ , we obtain 403

$$(\nu_1)^{(3)} \le \nu^{(3)}(t) \le \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \le (\nu_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(3)}(t)$ :-

$$(m_2)^{(3)} \le \nu^{(3)}(t) \le (m_1)^{(3)}, \quad \boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(3)}(t)$ :-

$$(\mu_2)^{(3)} \le u^{(3)}(t) \le (\mu_1)^{(3)}, \quad \boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{20}'')^{(3)} = (a_{21}'')^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)}G_{21}(t)$

Analogously if $(b_{20}'')^{(3)} = (b_{21}'')^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)}T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

**Proof** : From global equations we obtain 404

$$\frac{dv^{(4)}}{dt} = (a_{24})^{(4)} - \left((a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right) - (a''_{25})^{(4)}(T_{25},t)v^{(4)} - (a_{25})^{(4)}v^{(4)}$$

**Definition of** $v^{(4)}$ :- $\boxed{v^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)}\right) \le \frac{dv^{(4)}}{dt} \le -\left((a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_4)^{(4)}v^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$ :-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \ge \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \le \nu^{(4)}(t) \le (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \le \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \le \nu^{(4)}(t) \le (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \le \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}}{1 + (C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}} \le \nu^{(4)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \le (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \le (\bar{\nu}_1)^{(4)} \le \boxed{(\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}}}$ , we obtain 407

$$(\nu_1)^{(4)} \le \nu^{(4)}(t) \le \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \le (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(4)}(t)$ :-

$$(m_2)^{(4)} \le \nu^{(4)}(t) \le (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(4)}(t)$ :-

$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}$, $\boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{24})^{(4)} = (a''_{25})^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)} G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b''_{24})^{(4)} = (b''_{25})^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)} T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

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**Proof** : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a'_{28})^{(5)} - (a'_{29})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right) - (a''_{29})^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

**Definition of** $\nu^{(5)}$ :- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$ :-

For $0 < \boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \le \frac{(\nu_1)^{(5)}+(C)^{(5)}(\nu_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)}-(\nu_2)^{(5)}\right)t\right]}}{1+(C)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)}-(\nu_2)^{(5)}\right)t\right]}} \le \nu^{(5)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \le (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \le (\bar{\nu}_1)^{(5)} \le \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain 411

$$(\nu_1)^{(5)} \le \nu^{(5)}(t) \le \frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \le (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(5)}(t)$ :-

$(m_2)^{(5)} \le \nu^{(5)}(t) \le (m_1)^{(5)}$, $\boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$

In a completely analogous way, we obtain

**Definition of** $u^{(5)}(t)$ :-

$(\mu_2)^{(5)} \le u^{(5)}(t) \le (\mu_1)^{(5)}$, $\boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a_{28}'')^{(5)} = (a_{29}'')^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)}G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b_{28}'')^{(5)} = (b_{29}'')^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)}T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

**Proof** : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a_{32}')^{(6)} - (a_{33}')^{(6)} + (a_{32}'')^{(6)}(T_{33},t)\right) - (a_{33}'')^{(6)}(T_{33},t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

**Definition of** $\nu^{(6)}$ :- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \le \frac{d\nu^{(6)}}{dt} \le -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$ :-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \geq \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\nu_1)^{(6)}$

In the same manner , we get 413

$$\nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{\nu}_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\bar{\nu}_1)^{(6)}$

If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 414

$$(\nu_1)^{(6)} \leq \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}} \leq \nu^{(6)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\bar{\nu}_1)^{(6)}$$

If $0 < (\nu_1)^{(6)} \leq (\bar{\nu}_1)^{(6)} \leq \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$ , we obtain 415

$$(\nu_1)^{(6)} \leq \nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(6)}(t)$ :-

$$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}, \quad \boxed{\nu^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(6)}(t)$ :-

$$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}, \quad \boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)} G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)} T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

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**Proof** : From global equations we obtain

$$\frac{dv^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right) - (a''_{37})^{(7)}(T_{37},t)v^{(7)} - (a_{37})^{(7)}v^{(7)}$$

**Definition of** $v^{(7)}$ :- $\boxed{v^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_2)^{(7)}v^{(7)} - (a_{36})^{(7)}\right) \le \frac{dv^{(7)}}{dt} \le -\left((a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_1)^{(7)}v^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

**Definition of** $(\bar{v}_1)^{(7)}, (v_0)^{(7)}$ :-

For $0 < \boxed{(v_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}} < (v_1)^{(7)} < (\bar{v}_1)^{(7)}$

$$v^{(7)}(t) \ge \frac{(v_1)^{(7)} + (C)^{(7)}(v_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_0)^{(7)}\right)t\right]}}{1 + (C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(v_1)^{(7)} - (v_0)^{(7)}}{(v_0)^{(7)} - (v_2)^{(7)}}}$$

it follows $(v_0)^{(7)} \le v^{(7)}(t) \le (v_1)^{(7)}$

In the same manner , we get 417

$$v^{(7)}(t) \le \frac{(\bar{v}_1)^{(7)} + (\bar{C})^{(7)}(\bar{v}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{v}_1)^{(7)} - (v_0)^{(7)}}{(v_0)^{(7)} - (\bar{v}_2)^{(7)}}}$$

From which we deduce $(v_0)^{(7)} \le v^{(7)}(t) \le (\bar{v}_1)^{(7)}$

If $0 < (v_1)^{(7)} < (v_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}} < (\bar{v}_1)^{(7)}$ we find like in the previous case, 418

$$(v_1)^{(7)} \le \frac{(v_1)^{(7)} + (C)^{(7)}(v_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_2)^{(7)}\right)t\right]}}{1 + (C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_2)^{(7)}\right)t\right]}} \le v^{(7)}(t) \le$$

$$\frac{(\bar{v}_1)^{(7)} + (\bar{C})^{(7)}(\bar{v}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}} \le (\bar{v}_1)^{(7)}$$

If $0 < (v_1)^{(7)} \le (\bar{v}_1)^{(7)} \le \boxed{(v_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$ , we obtain 419

$$(v_1)^{(7)} \le v^{(7)}(t) \le \frac{(\bar{v}_1)^{(7)} + (\bar{C})^{(7)}(\bar{v}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}} \le (v_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $v^{(7)}(t)$ :-

$$(m_2)^{(7)} \leq v^{(7)}(t) \leq (m_1)^{(7)}, \quad \boxed{v^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(7)}(t)$ :- 420

$$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}, \quad \boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $v^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)} G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)} T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

**Proof** : From global equations we obtain 421

$$\frac{dv^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right) - (a''_{41})^{(8)}(T_{41}, t) v^{(8)} - (a_{41})^{(8)} v^{(8)}$$

**Definition of** $v^{(8)}$ :- $\boxed{v^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_2)^{(8)} v^{(8)} - (a_{40})^{(8)}\right) \leq \frac{dv^{(8)}}{dt} \leq -\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_1)^{(8)} v^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$ :-

$$\text{For } 0 < \boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$$

$$v^{(8)}(t) \geq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)}-(\nu_0)^{(8)}}{(\nu_0)^{(8)}-(\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \leq \frac{(\nu_1)^{(8)}+(C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}}{1+(C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}} \leq \nu^{(8)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \leq (\bar{\nu}_1)^{(8)} \leq \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$ , we obtain 424

$$(\nu_1)^{(8)} \leq \nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(8)}(t)$ :-

$$(m_2)^{(8)} \leq \nu^{(8)}(t) \leq (m_1)^{(8)}, \quad \boxed{\nu^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(8)}(t)$ :-

$$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}, \quad \boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $\nu^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)}G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then $(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)}T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

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**Proof** : From 99,20,44,22,23,44 we obtain A

$$\frac{dv^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right) - (a''_{45})^{(9)}(T_{45},t)v^{(9)} - (a_{45})^{(9)}v^{(9)}$$

**Definition of** $v^{(9)}$ :- $\boxed{v^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)}\right) \le \frac{dv^{(9)}}{dt} \le -\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_1)^{(9)}v^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

**Definition of** $(\bar{v}_1)^{(9)}, (v_0)^{(9)}$ :-

For $0 < \boxed{(v_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (v_1)^{(9)} < (\bar{v}_1)^{(9)}$

$$v^{(9)}(t) \ge \frac{(v_1)^{(9)} + (C)^{(9)}(v_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((v_1)^{(9)} - (v_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((v_1)^{(9)} - (v_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(v_1)^{(9)} - (v_0)^{(9)}}{(v_0)^{(9)} - (v_2)^{(9)}}}$$

it follows $(v_0)^{(9)} \le v^{(9)}(t) \le (v_9)^{(9)}$

In the same manner , we get

$$v^{(9)}(t) \le \frac{(\bar{v}_1)^{(9)} + (\bar{C})^{(9)}(\bar{v}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{v}_1)^{(9)} - (\bar{v}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{v}_1)^{(9)} - (\bar{v}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{v}_1)^{(9)} - (v_0)^{(9)}}{(v_0)^{(9)} - (\bar{v}_2)^{(9)}}}$$

From which we deduce $(v_0)^{(9)} \le v^{(9)}(t) \le (\bar{v}_1)^{(9)}$

If $0 < (v_1)^{(9)} < (v_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}} < (\bar{v}_1)^{(9)}$ we find like in the previous case,

$$(v_1)^{(9)} \le \frac{(v_1)^{(9)} + (C)^{(9)}(v_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((v_1)^{(9)} - (v_2)^{(9)}\right)t\right]}}{1 + (C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((v_1)^{(9)} - (v_2)^{(9)}\right)t\right]}} \le v^{(9)}(t) \le$$

$$\frac{(\bar{v}_1)^{(9)} + (\bar{C})^{(9)}(\bar{v}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{v}_1)^{(9)} - (\bar{v}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{v}_1)^{(9)} - (\bar{v}_2)^{(9)}\right)t\right]}} \le (\bar{v}_1)^{(9)}$$

If $0 < (v_1)^{(9)} \le (\bar{v}_1)^{(9)} \le \boxed{(v_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$ , we obtain

$$(v_1)^{(9)} \le v^{(9)}(t) \le \frac{(\bar{v}_1)^{(9)} + (\bar{C})^{(9)}(\bar{v}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{v}_1)^{(9)} - (\bar{v}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{v}_1)^{(9)} - (\bar{v}_2)^{(9)}\right)t\right]}} \le (v_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $v^{(9)}(t)$ :-

$$(m_2)^{(9)} \le v^{(9)}(t) \le (m_1)^{(9)}, \quad \boxed{v^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(9)}(t)$ :-

$(\mu_2)^{(9)} \leq u^{(9)}(t) \leq (\mu_1)^{(9)}$, $\boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)} G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then
$(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)} T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following 425

**Theorem :** If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on $t$ , and the conditions with the notations

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0$ ,

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$

$with\ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a''_i)^{(2)} and\ (b''_i)^{(2)}$ are independent on t , and the conditions with the notations 426

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$ 427

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 428

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$ , 429

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 430

$with\ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a''_i)^{(3)} and\ (b''_i)^{(3)}$ are independent on $t$ , and the conditions with the notations 431

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$ ,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

**Theorem :** If $(a_i'')^{(4)} and\ (b_i'')^{(4)}$ are independent on $t$ , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$ ,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(5)} and\ (b_i'')^{(5)}$ are independent on $t$ , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$ ,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

**Theorem** If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on $t$ , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$ ,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(7)} and\ (b_i'')^{(7)}$ are independent on $t$ , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$ ,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(8)} and\ (b_i'')^{(8)}$ are independent on $t$ , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$ ,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(9)} and\ (b_i'')^{(9)}$ are independent on $t$ , and the conditions (with the notations 436 A
45,46,27,28)

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$ ,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0$ 437

$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0$ 438

$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0$ 439

$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0$ 440

$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ]T_{14} = 0$ 441

$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0$ 442

has a unique positive solution , which is an equilibrium solution for the system

$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0$ 443

$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0$ 444

$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0$ 445

$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\ ]T_{16} = 0$ 446

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\,]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\,]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\,]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\,]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\,]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\big((G_{27})\big)\,]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\big((G_{27})\big)\,]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\big((G_{27})\big)\,]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})\,]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})\,]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})\,]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})\,]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})\,]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})\,]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0 \quad 479$$

$$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0 \quad 480$$

$$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0 \quad 481$$

$$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0 \quad 482$$

$$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0 \quad 483$$

$$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0 \quad 484$$

$$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0 \quad 484\text{A}$$

$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$

$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$

$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$

$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$

$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$

**Proof:** 485

(a) Indeed the first two equations have a nontrivial solution $G_{13}, G_{14}$ if

$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$

**Proof:** 486

(jj) Indeed the first two equations have a nontrivial solution $G_{16}, G_{17}$ if

$\mathrm{F}(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$

**Proof:** 487

(a) Indeed the first two equations have a nontrivial solution $G_{20}, G_{21}$ if

$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$

**Proof:** 488

(a) Indeed the first two equations have a nontrivial solution $G_{24}, G_{25}$ if

$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$

**Proof:** 489

(a) Indeed the first two equations have a nontrivial solution $G_{28}, G_{29}$ if

$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$

**Proof:** 490

(a) Indeed the first two equations have a nontrivial solution $G_{32}, G_{33}$ if

$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) +$

$(a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$

**Proof:** 491

(a) Indeed the first two equations have a nontrivial solution $G_{36}, G_{37}$ if

$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$

**Proof:** 492

(a) Indeed the first two equations have a nontrivial solution $G_{40}, G_{41}$ if

$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$

**Proof:** 492 A

(a) Indeed the first two equations have a nontrivial solution $G_{44}, G_{45}$ if

$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$

**Definition and uniqueness of** $T^*_{14}$ :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique $T^*_{14}$ for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

**Definition and uniqueness of** $T^*_{17}$ :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique $T^*_{17}$ for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]}$$ 495

**Definition and uniqueness of** $T^*_{21}$ :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique $T^*_{21}$ for which $f(T^*_{21}) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

**Definition and uniqueness of** $T^*_{25}$ :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that

there exists a unique $T_{25}^{*}$ for which $f(T_{25}^{*})=0$. With this value , we obtain from the three first equations

$$G_{24}=\frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}=\frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T_{25}^{*})]}$$

**Definition and uniqueness of** $T_{29}^{*}$ :- 498

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(5)}(T_{29})$ being increasing, it follows that there exists a unique $T_{29}^{*}$ for which $f(T_{29}^{*})=0$. With this value , we obtain from the three first equations

$$G_{28}=\frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}=\frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T_{29}^{*})]}$$

**Definition and uniqueness of** $T_{33}^{*}$ :- 499

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(6)}(T_{33})$ being increasing, it follows that there exists a unique $T_{33}^{*}$ for which $f(T_{33}^{*})=0$. With this value , we obtain from the three first equations

$$G_{32}=\frac{(a_{32})^{(6)}G_{33}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^{*})]} \quad , \quad G_{34}=\frac{(a_{34})^{(6)}G_{33}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^{*})]}$$

**Definition and uniqueness of** $T_{37}^{*}$ :- 500

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(7)}(T_{37})$ being increasing, it follows that there exists a unique $T_{37}^{*}$ for which $f(T_{37}^{*})=0$. With this value , we obtain from the three first equations

$$G_{36}=\frac{(a_{36})^{(7)}G_{37}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^{*})]} \quad , \quad G_{38}=\frac{(a_{38})^{(7)}G_{37}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^{*})]}$$

**Definition and uniqueness of** $T_{41}^{*}$ :- 501

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique $T_{41}^{*}$ for which $f(T_{41}^{*})=0$. With this value , we obtain from the three first equations

$$G_{40}=\frac{(a_{40})^{(8)}G_{41}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T_{41}^{*})]} \quad , \quad G_{42}=\frac{(a_{42})^{(8)}G_{41}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T_{41}^{*})]}$$

**Definition and uniqueness of** $T_{45}^{*}$ :- 501 A

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(9)}(T_{45})$ being increasing, it follows that there exists a unique $T_{45}^{*}$ for which $f(T_{45}^{*})=0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)}G_{45}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T^*_{45})]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)}G_{45}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T^*_{45})]}$$

By the same argument, the equations admit solutions $G_{13}, G_{14}$ if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right]+(b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{14}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{14}$ such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions $G_{16}, G_{17}$ if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right]+(b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ 504
is a decreasing function in $G_{17}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{14}$ such that $\varphi((G_{19})^*) = 0$

By the same argument, the equations admit solutions $G_{20}, G_{21}$ if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right]+(b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{21}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{21}$ such that $\varphi((G_{23})^*) = 0$

By the same argument, the equations admit solutions $G_{24}, G_{25}$ if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right]+(b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{25}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{25}$ such that $\varphi((G_{27})^*) = 0$

By the same argument, the equations admit solutions $G_{28}, G_{29}$ if 507

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right]+(b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{29}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that

there exists a unique $G_{29}^{*}$ such that $\varphi((G_{31})^{*}) = 0$

By the same argument, the equations admit solutions $G_{32}, G_{33}$ if 508
$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right] + (b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{33}$ taking into account the hypothesis $\varphi(0) > 0\,, \varphi(\infty) < 0$ it follows that there exists a unique $G_{33}^{*}$ such that $\varphi(G_{35}{}^{*}) = 0$

By the same argument, the equations admit solutions $G_{36}, G_{37}$ if 509
$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$
$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right] + (b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{37}$ taking into account the hypothesis $\varphi(0) > 0\,, \varphi(\infty) < 0$ it follows that there exists a unique $G_{37}^{*}$ such that $\varphi(G_{39}{}^{*}) = 0$

By the same argument, the equations admit solutions $G_{40}, G_{41}$ if 510
$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$
$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right] + (b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{41}$ taking into account the hypothesis $\varphi(0) > 0\,, \varphi(\infty) < 0$ it follows that there exists a unique $G_{41}^{*}$ such that $\varphi(G_{43}{}^{*}) = 0$

By the same argument, the equations 92,93 admit solutions $G_{44}, G_{45}$ if
$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$

$$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right] + (b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{45}$ taking into account the hypothesis $\varphi(0) > 0\,, \varphi(\infty) < 0$ it follows that there exists a unique $G_{45}^{*}$ such that $\varphi((G_{47})^{*}) = 0$

Finally we obtain the unique solution 511

$G_{14}^{*}$ given by $\varphi(G^{*}) = 0\,, T_{14}^{*}$ given by $f(T_{14}^{*}) = 0$ and

$$G_{13}^{*} = \frac{(a_{13})^{(1)}G_{14}^{*}}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^{*})]} \quad , \quad G_{15}^{*} = \frac{(a_{15})^{(1)}G_{14}^{*}}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^{*})]}$$

$$T_{13}^{*} = \frac{(b_{13})^{(1)}T_{14}^{*}}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^{*})]} \quad , \quad T_{15}^{*} = \frac{(b_{15})^{(1)}T_{14}^{*}}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^{*})]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

$G_{17}^{*}$ given by $\varphi((G_{19})^{*}) = 0$ , $T_{17}^{*}$ given by $f(T_{17}^{*}) = 0$ and 512

$$G_{16}^{*} = \frac{(a_{16})^{(2)} G_{17}^{*}}{[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}^{*})]} \quad , \quad G_{18}^{*} = \frac{(a_{18})^{(2)} G_{17}^{*}}{[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}^{*})]} \qquad 513$$

$$T_{16}^{*} = \frac{(b_{16})^{(2)} T_{17}^{*}}{[(b'_{16})^{(2)} - (b''_{16})^{(2)}((G_{19})^{*})]} \quad , \quad T_{18}^{*} = \frac{(b_{18})^{(2)} T_{17}^{*}}{[(b'_{18})^{(2)} - (b''_{18})^{(2)}((G_{19})^{*})]} \qquad 514$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

$G_{21}^{*}$ given by $\varphi((G_{23})^{*}) = 0$ , $T_{21}^{*}$ given by $f(T_{21}^{*}) = 0$ and

$$G_{20}^{*} = \frac{(a_{20})^{(3)} G_{21}^{*}}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^{*})]} \quad , \quad G_{22}^{*} = \frac{(a_{22})^{(3)} G_{21}^{*}}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^{*})]}$$

$$T_{20}^{*} = \frac{(b_{20})^{(3)} T_{21}^{*}}{[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}{}^{*})]} \quad , \quad T_{22}^{*} = \frac{(b_{22})^{(3)} T_{21}^{*}}{[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}{}^{*})]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

$G_{25}^{*}$ given by $\varphi(G_{27}) = 0$ , $T_{25}^{*}$ given by $f(T_{25}^{*}) = 0$ and

$$G_{24}^{*} = \frac{(a_{24})^{(4)} G_{25}^{*}}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}^{*} = \frac{(a_{26})^{(4)} G_{25}^{*}}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^{*})]}$$

$$T_{24}^{*} = \frac{(b_{24})^{(4)} T_{25}^{*}}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^{*})]} \quad , \quad T_{26}^{*} = \frac{(b_{26})^{(4)} T_{25}^{*}}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^{*})]} \qquad 517$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

$G_{29}^{*}$ given by $\varphi((G_{31})^{*}) = 0$ , $T_{29}^{*}$ given by $f(T_{29}^{*}) = 0$ and

$$G_{28}^{*} = \frac{(a_{28})^{(5)} G_{29}^{*}}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}^{*} = \frac{(a_{30})^{(5)} G_{29}^{*}}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^{*})]}$$

$$T_{28}^{*} = \frac{(b_{28})^{(5)} T_{29}^{*}}{[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31})^{*})]} \quad , \quad T_{30}^{*} = \frac{(b_{30})^{(5)} T_{29}^{*}}{[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31})^{*})]} \qquad 519$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

$G_{33}^{*}$ given by $\varphi((G_{35})^{*}) = 0$ , $T_{33}^{*}$ given by $f(T_{33}^{*}) = 0$ and

$$G_{32}^{*} = \frac{(a_{32})^{(6)} G_{33}^{*}}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^{*})]} \quad , \quad G_{34}^{*} = \frac{(a_{34})^{(6)} G_{33}^{*}}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^{*})]}$$

$$T_{32}^{*} = \frac{(b_{32})^{(6)} T_{33}^{*}}{[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35})^{*})]} \quad , \quad T_{34}^{*} = \frac{(b_{34})^{(6)} T_{33}^{*}}{[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35})^{*})]} \qquad 521$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

$G_{37}^*$ given by $\varphi((G_{39})^*) = 0$ , $T_{37}^*$ given by $f(T_{37}^*) = 0$ and

$$G_{36}^* = \frac{(a_{36})^{(7)} G_{37}^*}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^*)]} \quad , \quad G_{38}^* = \frac{(a_{38})^{(7)} G_{37}^*}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^*)]}$$

$$T_{36}^* = \frac{(b_{36})^{(7)} T_{37}^*}{[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39})^*)]} \quad , \quad T_{38}^* = \frac{(b_{38})^{(7)} T_{37}^*}{[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39})^*)]}$$

Finally we obtain the unique solution 523

$G_{41}^*$ given by $\varphi((G_{43})^*) = 0$ , $T_{41}^*$ given by $f(T_{41}^*) = 0$ and

$$G_{40}^* = \frac{(a_{40})^{(8)} G_{41}^*}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42}^* = \frac{(a_{42})^{(8)} G_{41}^*}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

$$T_{40}^* = \frac{(b_{40})^{(8)} T_{41}^*}{[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43})^*)]} \quad , \quad T_{42}^* = \frac{(b_{42})^{(8)} T_{41}^*}{[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43})^*)]}$$

Finally we obtain the unique solution of 89 to 99 523 A

$G_{45}^*$ given by $\varphi((G_{47})^*) = 0$ , $T_{45}^*$ given by $f(T_{45}^*) = 0$ and

$$G_{44}^* = \frac{(a_{44})^{(9)} G_{45}^*}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46}^* = \frac{(a_{46})^{(9)} G_{45}^*}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

$$T_{44}^* = \frac{(b_{44})^{(9)} T_{45}^*}{[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47})^*)]} \quad , \quad T_{46}^* = \frac{(b_{46})^{(9)} T_{45}^*}{[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47})^*)]}$$

## ASYMPTOTIC STABILITY ANALYSIS 524

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

**Proof:** Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a''_{14})^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \frac{\partial (b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a'_{13})^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)}G^*_{13}\mathbb{T}_{14} \quad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a'_{14})^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)}G^*_{14}\mathbb{T}_{14} \quad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a'_{15})^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)}G^*_{15}\mathbb{T}_{14} \quad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b'_{13})^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)}T^*_{13}\mathbb{G}_j\right) \quad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b'_{14})^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)}T^*_{14}\mathbb{G}_j\right) \quad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b'_{15})^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T^*_{15}\mathbb{G}_j\right) \quad 530$$

**ASYMPTOTIC STABILITY ANALYSIS** 531

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(2)}$ and $(b''_i)^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G^*_i + \mathbb{G}_i \quad , T_i = T^*_i + \mathbb{T}_i \quad 532$$

$$\frac{\partial(a''_{17})^{(2)}}{\partial T_{17}}(T^*_{17}) = (q_{17})^{(2)} \quad , \frac{\partial(b''_i)^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \quad 533$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a'_{16})^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G^*_{16}\mathbb{T}_{17} \quad 534$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a'_{17})^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G^*_{17}\mathbb{T}_{17} \quad 535$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a'_{18})^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G^*_{18}\mathbb{T}_{17} \quad 536$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b'_{16})^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T^*_{16}\mathbb{G}_j\right) \quad 537$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b'_{17})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}T^*_{17}\mathbb{G}_j\right) \quad 538$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b'_{18})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}T^*_{18}\mathbb{G}_j\right) \quad 539$$

**ASYMPTOTIC STABILITY ANALYSIS** 540

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(3)}$ *and* $(b''_i)^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{21}'')^{(3)}}{\partial T_{21}}(T_{21}^*) = (q_{21})^{(3)} \quad , \frac{\partial (b_i'')^{(3)}}{\partial G_j}(\,(G_{23})^*\,) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\big((a_{20}')^{(3)} + (p_{20})^{(3)}\big)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G_{20}^*\mathbb{T}_{21} \quad 541$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\big((a_{21}')^{(3)} + (p_{21})^{(3)}\big)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G_{21}^*\mathbb{T}_{21} \quad 542$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\big((a_{22}')^{(3)} + (p_{22})^{(3)}\big)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G_{22}^*\mathbb{T}_{21} \quad 543$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\big((b_{20}')^{(3)} - (r_{20})^{(3)}\big)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\big(s_{(20)(j)}T_{20}^*\mathbb{G}_j\big) \quad 544$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\big((b_{21}')^{(3)} - (r_{21})^{(3)}\big)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\big(s_{(21)(j)}T_{21}^*\mathbb{G}_j\big) \quad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\big((b_{22}')^{(3)} - (r_{22})^{(3)}\big)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\big(s_{(22)(j)}T_{22}^*\mathbb{G}_j\big) \quad 546$$

**ASYMPTOTIC STABILITY ANALYSIS** 547

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}\ and\ (b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{25}'')^{(4)}}{\partial T_{25}}(T_{25}^*) = (q_{25})^{(4)} \quad , \frac{\partial (b_i'')^{(4)}}{\partial G_j}((G_{27})^*\,) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\big((a_{24}')^{(4)} + (p_{24})^{(4)}\big)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G_{24}^*\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\big((a_{25}')^{(4)} + (p_{25})^{(4)}\big)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G_{25}^*\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\big((a_{26}')^{(4)} + (p_{26})^{(4)}\big)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G_{26}^*\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\big((b_{24}')^{(4)} - (r_{24})^{(4)}\big)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\big(s_{(24)(j)}T_{24}^*\mathbb{G}_j\big) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\big((b_{25}')^{(4)} - (r_{25})^{(4)}\big)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\big(s_{(25)(j)}T_{25}^*\mathbb{G}_j\big) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T^*_{26}\mathbb{G}_j\right) \quad 554$$

**ASYMPTOTIC STABILITY ANALYSIS** 555

**Theorem 5:** If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(5)}\ and\ (b''_i)^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 556

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{29})^{(5)}}{\partial T_{29}}(T^*_{29}) = (q_{29})^{(5)} \ , \ \frac{\partial(b''_i)^{(5)}}{\partial G_j}((G_{31})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a'_{28})^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G^*_{28}\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a'_{29})^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G^*_{29}\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a'_{30})^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G^*_{30}\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b'_{28})^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T^*_{28}\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b'_{29})^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T^*_{29}\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b'_{30})^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T^*_{30}\mathbb{G}_j\right) \quad 562$$

**ASYMPTOTIC STABILITY ANALYSIS** 563

**Theorem 6:** If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(6)}\ and\ (b''_i)^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{33})^{(6)}}{\partial T_{33}}(T^*_{33}) = (q_{33})^{(6)} \ , \ \frac{\partial(b''_i)^{(6)}}{\partial G_j}((G_{35})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a'_{32})^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G^*_{32}\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a'_{33})^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G^*_{33}\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a'_{34})^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G^*_{34}\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b'_{32})^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T^*_{32}\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b'_{33})^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T^*_{33}\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b'_{34})^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T^*_{34}\mathbb{G}_j\right) \quad 570$$

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**Theorem 7:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 572

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{37})^{(7)}}{\partial T_{37}}(T^*_{37}) = (q_{37})^{(7)} \quad , \frac{\partial(b_i'')^{(7)}}{\partial G_j}\left((G_{39})^{**}\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a'_{36})^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G^*_{36}\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a'_{37})^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G^*_{37}\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a'_{38})^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G^*_{38}\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b'_{36})^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T^*_{36}\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b'_{37})^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T^*_{37}\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b'_{38})^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T^*_{38}\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

**ASYMPTOTIC STABILITY ANALYSIS**

**Theorem 8:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{41}'')^{(8)}}{\partial T_{41}}(T_{41}^*) = (q_{41})^{(8)} \quad , \frac{\partial (b_i'')^{(8)}}{\partial G_j}\left( (G_{43})^* \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a_{40}')^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G_{40}^*\mathbb{T}_{41} \quad (581)$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a_{41}')^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G_{41}^*\mathbb{T}_{41} \quad (582)$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a_{42}')^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G_{42}^*\mathbb{T}_{41} \quad (583)$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b_{40}')^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T_{40}^*\mathbb{G}_j\right) \quad (584)$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b_{41}')^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T_{41}^*\mathbb{G}_j\right) \quad (585)$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b_{42}')^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T_{42}^*\mathbb{G}_j\right) \quad (586)$$

(586 A)

## ASYMPTOTIC STABILITY ANALYSIS

**Theorem 9:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}$ *and* $(b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

**Proof:** Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{45}'')^{(9)}}{\partial T_{45}}(T_{45}^*) = (q_{45})^{(9)} \quad , \frac{\partial (b_i'')^{(9)}}{\partial G_j}\left( (G_{47})^* \right) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a_{44}')^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)}G_{44}^*\mathbb{T}_{45} \quad (586\,B)$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a_{45}')^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)}G_{45}^*\mathbb{T}_{45} \quad (586\,C)$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a_{46}')^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)}G_{46}^*\mathbb{T}_{45} \quad (586\,D)$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b_{44}')^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)}T_{44}^*\mathbb{G}_j\right) \quad (586\,E)$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b_{45}')^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)}T_{45}^*\mathbb{G}_j\right) \quad (586\,F)$$

$\frac{d\mathbb{T}_{46}}{dt} = -\left((b'_{46})^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T^*_{46}\mathbb{G}_j\right)$ 586

G

**The characteristic equation of this system is** 587

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(q_{13})^{(1)}G^*_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)}T^*_{14} + (b_{14})^{(1)}s_{(13),(14)}T^*_{14}\right)$$

$$+\left(\left((\lambda)^{(1)} + (a'_{14})^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)}G^*_{13} + (a_{13})^{(1)}(q_{14})^{(1)}G^*_{14}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)}T^*_{14} + (b_{14})^{(1)}s_{(13),(13)}T^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((b'_{13})^{(1)} + (b'_{14})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(15)}T^*_{14} + (b_{14})^{(1)}s_{(13),(15)}T^*_{13}\right)\} = 0$$

$$+$$

$$(\lambda)^{(2)} + (b'_{18})^{(2)} - (r_{18})^{(2)}\{\left((\lambda)^{(2)} + (a'_{18})^{(2)} + (p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^*_{17} + (a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^*_{17}\right)$$

$$+\left(\left((\lambda)^{(2)} + (a'_{17})^{(2)} + (p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^*_{16} + (a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}\right)$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2 + \left((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2 + \left((b'_{16})^{(2)} + (b'_{17})^{(2)} - (r_{16})^{(2)} + (r_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$+\left(\left((\lambda)^{(2)}\right)^2 + \left((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17} + (a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^{*}_{16}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\right)\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left(\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)\right]$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(21)}T^{*}_{21}\right)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^{*}_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^{*}_{21}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(20)}T^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$+\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(22)}T^{*}_{20}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\right)\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left(\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)\right]$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(25)}T^{*}_{25}\right)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^{*}_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^{*}_{25}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(24)}T^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$+\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(26)}T^{*}_{24}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\right)\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(29)}T^{*}_{29}\right)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^{*}_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^{*}_{29}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(28)}T^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$+\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(30)}T^{*}_{28}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\right)\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left(\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)\right]$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(33)}T^{*}_{33}\right)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^{*}_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^{*}_{33}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(32)}T^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$+\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(34)}T^{*}_{32}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\right)\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)\right]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(37)}T^{*}_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^{*}_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^{*}_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(36)}T^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$+\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(38)}T^{*}_{36}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\right)\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(41)}T^{*}_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^{*}_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^{*}_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(40)}T^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$+\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(42)}T^{*}_{40}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\right)\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(45)}T^{*}_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^{*}_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^{*}_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(44)}T^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$+\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(46)}T^{*}_{44}\right)\}=0$$

**And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.**

## SECTION THIRTY SEVEN

## Vital Needs Model

### INTRODUCTION—VARIABLES USED

**The conflict between entropy and syntropy: the vital needs model Ulisse Di Corpo ulisse.dicorpo@gmail.com**

(1) On the contrary, in the microcosm entropy and (e&eb) syntropy are balanced and time flows (e&eb) in both directions (causality: causeÎeffect and (e&eb) retrocausality effectÍcause).

(2) For this reason, the origin of life seems possible in (eb) microcosm, while (e) it seems impossible in the macrocosm, which is governed by (e&eb), the law of entropy.
(3) The vital needs model starts from this consideration, that life originates in the microcosm and that when it grows beyond (e) the dimension of the microcosm it starts conflicting with the law of entropy which governs (e&eb) the macrocosm.
(4) The conflict between life and entropy is well known and has been discussed continuously by biologists and physicists. Schrödinger, answering the question about what permits life to contrast entropy, concluded that life feeds on (e) negative entropy (Schrödinger, 1988).
(5) The same conclusion was reached by Albert Szent-Györgyi when he used the term syntropy in order to describe (eb) the qualities of negative entropy as (=) the main property of living systems (Szent-Györgyi, 1977).
(6) This hypothesis, of a basic conflict between life (syntropy) and (e&eb) the environment (entropy), leads to (eb) the conclusion that living systems need to satisfy 3 vital conditions:

(i) acquire syntropy from (e) the microcosm;
(ii) combat the dissipative effects of (e) entropy;
**(iii)** Solve the conflict between entropy and (e&eb) syntropy.

## NOTATION

### Module One

However, in 1926 Klein and Gordon, in order to generalize the Schrödinger wave equation into a relativistic invariant equation, had to insert the full energy-momentum relation, arriving at a dual wave solution which characterizes the D'Alembert operator: **retarded potentials which propagate from the past** to (e&eb) the future (+E) and **anticipated potentials which propagate (e&eb) backward, from the future** to (e&eb) the past (−E)

$G_{13}$ : Category one of **anticipated potentials which propagate backward, from the future;** past (−E)

$G_{14}$ : Category two of SAS

$G_{15}$ : Category three of SAS

$T_{13}$ : Category one of past (−E) **anticipated potentials which propagate backward, from the future**

$T_{14}$ : Category two of SAS

$T_{15}$ : Category three of SAS

### Module Two

In other words, when **we unite ourselves** (1) to (e&eb) the environment, comparing ourselves to the environment, (eb) we find our identity (= 1).

$G_{16}$ : Category one of **we unite ourselves;** environment, comparing ourselves to the environment, (eb) we find our identity (= 1).

$G_{17}$ : Category two of SAS

$G_{18}$ : Category three of SAS

$T_{16}$ : Category one of environment, comparing ourselves to the environment, (eb) we find our identity (= 1). **we unite ourselves**

$T_{17}$ : Category two of SAS

$T_{18}$ : Category three of SAS

**Module three**

$G_{20}$ : Category one of environment; human beings

$G_{21}$ : Category two of SAS

$G_{22}$ : Category three of SAS

$T_{20}$ : Category one of human beings; environment

$T_{21}$ : Category two of SAS

$T_{22}$ : Category three of SAS

**Module four**

$G_{24}$ : Category one of result of module 3 in various cases

$G_{25}$ : Category two of SAS

$G_{26}$ : Category three of SAS

$T_{24}$ : Category one of we are 1

$T_{25}$ : Category two of SAS

$T_{26}$ : Category three of SAS

**Module five**

Perfect correlation between anxiety and (e&eb) depression must be observed, because (e) when the unity (x) is weak, anxiety increases (eb) and also the identity conflict and depression

$G_{28}$ : Category one of the unity (x) is weak

$G_{29}$ : Category two of SAS

$G_{30}$ : Category three of SAS

$T_{28}$ : Category one of anxiety increases and also the identity conflict and depression

$T_{29}$ : Category two of SAS

$T_{30}$ : Category three of SAS

**Module six**

On the contrary, in the microcosm entropy and (e&eb) syntropy are balanced and time flows (e&eb) in both directions (causality: causeÎeffect and (e&eb) retrocausality effectÍcause

$G_{32}$ : Category one of **microcosm entropy;** syntropy are balanced and time flows (e&eb) in both directions

(causality: causeÎeffect and (e&eb) retrocausality effectÍcause

$G_{33}$ : Category two of SAS

$G_{34}$ : Category three of SAS

$T_{32}$ : Category one of syntropy are balanced and time flows (e&eb) in both directions (causality: causeÎeffect and (e&eb) retrocausality effectÍcause; **microcosm entropy**

$T_{33}$ : Category two of SAS

$T_{34}$ : Category three of SAS

**Module seven**

On the contrary, in the microcosm entropy and syntropy are balanced and **time flows** (e&eb) in both directions (causality: causeÎeffect and (e&eb) retrocausality effectÍcause

$G_{36}$ : Category one of **time flows;** both directions (causality: causeÎeffect and (e&eb) retrocausality effectÍcause

$G_{37}$ : Category two of SAS

$G_{38}$ : Category three of SAS

$T_{36}$ : Category one of both directions (causality: causeÎeffect and (e&eb) retrocausality effectÍcause **;time flows**

$T_{37}$ : Category two of SAS

$T_{38}$ : Category three of SAS

**Module eight**

(causality: **causeÎeffect** and (e&eb) retrocausality effectÍcause

$G_{40}$ : Category one of **causeÎeffect;** retrocausality effectÍcause

$G_{41}$ : Category two of SAS

$G_{42}$ : Category three of SAS

$T_{40}$ : Category one of retrocausality effectÍcause **;causeÎeffect**

$T_{41}$ : Category two of SAS

$T_{42}$ : Category three of SAS

**Module Nine**

The vital needs model starts from this consideration, that life originates in the microcosm and that when it grows beyond (e) the dimension of the microcosm it starts conflicting with the law of entropy which governs (e&eb) the macrocosm

$G_{44}$ : Category one of dimension of the microcosm it starts conflicting with the law of entropy which governs (e&eb) the macrocosm

$G_{45}$ : Category two of SAS

$G_{46}$ : Category three of SAS

$T_{44}$ : Category one of life originates in the microcosm and that when it grows

$T_{45}$ : Category two of SAS

$T_{46}$ : Category three of SAS

## SECTION THIRTY SEVEN A

## Life Feeds On Negative Entropy

### INTRODUCTION—VARIABLES USED

**The conflict between entropy and syntropy: the vital needs model Ulisse Di Corpo ulisse.dicorpo@gmail.com**

(1) The conflict between life and entropy is well known and has been discussed continuously by biologists and physicists. Schrödinger, answering the question about what permits life to contrast entropy, concluded that life feeds on (e) negative entropy (Schrödinger, 1988).
(2) The same conclusion was reached by Albert Szent-Györgyi when he used the term syntropy in order to describe (eb) the qualities of negative entropy as (=) the main property of living systems (Szent-Györgyi, 1977).
(3) This hypothesis, of a basic conflict between life (syntropy) and (e&eb) the environment (entropy), leads to (eb) the conclusion that living systems need to satisfy 3 vital conditions:

(iv) acquire syntropy from (e) the microcosm;
**(v)** combat the dissipative effects of (e) entropy;
**(vi)** Solve the conflict between entropy and (e&eb) syntropy.

**Chronological order**

(1) According to this model a chronological order exists in the satisfaction of these needs. Initially humans have (e) to answer material needs, which are more immediate, and only afterwards they become (eb) aware of the needs of meaning and unity.
(2) Part of humanity has now satisfied (eb) material needs and is now facing the needs of (e) meaning and unity and is undergoing rising feelings of existential crisis and anxiety.
(3) Material needs can be easily recognized, whereas the needs of meaning and unity are immaterial, invisible, and more difficult to be recognized. For example:

• In order to give a meaning to their lives, people become (=) obsessed with the way they are judged. They strive for (e) power, success. They become addicted to (e) religion, ideologies and groups;

• In order to overcome anxiety they become (eb) addicted to alcohol and drugs and avoid (e) any moment of

silence.

(4) The model of vital needs affirms (eb) that all these states are (=) secondary needs which originate from (e) the unsatisfied needs of meaning and unity. According to the vital needs model all behaviours (psychological, sociological and economical) can be traced back to (e) these 3 basic needs.

**Empirical evidences**

Working as a sociologist for public institutions I am always faced with the centrality of the 3 vital needs which I have just described. For example:

• in a study, just concluded, for IOM (International Organization on Migration) results showed that whilst irregular migrants come to Italy for economical reasons (material needs), a significant number would go back home immediately, because of loneliness (need of unity) and depression (need of meaning), even though their economic situation at home has not changed. It seems that the experience in Italy and the difficulties associated to the integration process lead the migrant to rediscover a set of needs, qualitatively different from the material needs, which were met at home, thanks to the family and friends, but could not be met in Italy, causing feelings of loneliness, and depression.

• In a study conducted in the Abruzzi area of Italy, different theoretical models were compared in order to understand the origins of psychological and emotional suffering in high school students. Results show that items relative to loneliness and worthlessness obtained the highest correlations with items relative to unhappiness and suffering (SERT-Teramo, 1993).

• In several studies conducted with drug addict populations, loneliness and worthlessness emerged as the two main reasons behind drug addiction. In all these studies the correlation among loneliness (anxiety) and worthlessness (depression) is nearly perfect, supporting the hypothesis of the existence of a mathematical relation between these two needs.

**Conclusions**

The introduction of advanced waves in science might not be (=) as important in physics as it could be in life disciplines such as medicine, psychology and sociology.

These disciplines now approach pathologies, illnesses and crises in a mechanistic way which leads, (eb) in an always more alarming way, to inefficiency and to an increase in costs.

**NOTATION**

**Module One**

life feeds on (e) negative entropy (Schrödinger, 1988)

$G_{13}$ : Category one of negative entropy (Schrödinger, 1988)

$G_{14}$ : Category two of SAS

$G_{15}$ : Category three of SAS

$T_{13}$ : Category one of life feeds on

$T_{14}$ : Category two of SAS

$T_{15}$ : Category three of SAS

**Module Two**

The same conclusion was reached by Albert Szent-Györgyi when he used the term syntropy in order to describe (eb) the qualities of negative entropy as (=) the main property of living systems (Szent-Györgyi, 1977).

$G_{16}$ : Category one of syntropy

$G_{17}$ : Category two of SAS

$G_{18}$ : Category three of SAS

$T_{16}$ : Category one of qualities of negative entropy

$T_{17}$ : Category two of SAS

$T_{18}$ : Category three of SAS

**Module three**

$G_{20}$ : Category one of qualities of negative entropy

$G_{21}$ : Category two of SAS

$G_{22}$ : Category three of SAS

$T_{20}$ : Category one of main property of living systems (Szent-Györgyi, 1977).

$T_{21}$ : Category two of SAS

$T_{22}$ : Category three of SAS

**Module four**

This hypothesis, of a basic conflict between **life (syntropy)** and (e&eb) the environment (entropy), leads to (eb) the conclusion that living systems need to satisfy 3 vital conditions:

(vii) acquire syntropy from (e) the microcosm combat the dissipative effects of (e) entropy

$G_{24}$ : Category one of **life (syntropy);** environment (entropy

$G_{25}$ : Category two of SAS

$G_{26}$ : Category three of SAS

$T_{24}$ : Category one of environment (entropy; **life (syntropy)**

$T_{25}$ : Category two of SAS

$T_{26}$ : Category three of SAS

**Module five**

$G_{28}$ : Category one of basic conflict between **life (syntropy)** and (e&eb) the environment (entropy

$G_{29}$ : Category two of SAS

$G_{30}$ : Category three of SAS

$T_{28}$ : Category one of acquire syntropy from (e) the microcosm combat the dissipative effects of (e) entropy

$T_{29}$ : Category two of SAS

$T_{30}$ : Category three of SAS

**Module six**

Life acquires syntropy from (e) the microcosm combat the dissipative effects of (e) entropy

$G_{32}$ : Category one of syntropy from (e) the microcosm combat the dissipative effects of (e) entropy

$G_{33}$ : Category two of SAS

$G_{34}$ : Category three of SAS

$T_{32}$ : Category one of life

$T_{33}$ : Category two of SAS

$T_{34}$ : Category three of SAS

**Module seven**

combat the dissipative effects of (e) entropy

$G_{36}$ : Category one of entropy

$G_{37}$ : Category two of SAS

$G_{38}$ : Category three of SAS

$T_{36}$ : Category one of dissipative effects

$T_{37}$ : Category two of SAS

$T_{38}$ : Category three of SAS

**Module eight**

In order to give a meaning to their lives, people become (=) obsessed with the way they are judged.

They strive for (e) power, success.

They become addicted to (e) religion, ideologies and groups

$G_{40}$ : Category one of give a meaning to their lives

$G_{41}$ : Category two of SAS

$G_{42}$ : Category three of SAS

$T_{40}$ : Category one of people become obsessed with the way they are judged

$T_{41}$ : Category two of SAS

$T_{42}$ : Category three of SAS

**Module Nine**

These disciplines now approach pathologies, illnesses and crises in a mechanistic way which leads, (eb) in an always more alarming way, to inefficiency and to an increase in costs.

$G_{44}$ : Category one of mechanical ways to treat compulsive obsessive needs

$G_{45}$ : Category two of SAS

$G_{46}$ : Category three of SAS

$T_{44}$ : Category one of increase in costs, inefficiency

$T_{45}$ : Category two of SAS

$T_{46}$ : Category three of SAS

## SECTION THIRTY EIGHT

## Dissipative Effects Of Entropy

### INTRODUCTION—VARIABLES USED

**Combat the dissipative effects of entropy: material needs.**

(1) In order to combat the dissipative effects of entropy, living systems need to acquire (e) energy from outside and protect themselves from (e) the dissipative effects of entropy.
(2) In order to combat the dissipative effect of entropy: the need to acquire energy from (e) outside, for example with food; the need to reduce (e) the dissipation of energy, for example with a shelter (housing) and clothes
(3) In order to combat (e) the continuous production of waste, this is the consequence of (e) the destruction of structures under (e&eb) the effect of entropy: the need for hygienic and sanitary standards and waste disposal.
(4) When these needs are partially unsatisfied, pain is experimented in the forms of (eb) hunger, thirst, sickness, and, when (e) they are totally unsatisfied, death is the consequence.
(5) The total satisfaction of material needs leads to (eb) a state of well being which is characterized by (e) the absence of pain linked to material needs.
**(6)** Acquire syntropy from (e) the microcosm: the need of unity.

## NOTATION

### Module One

In order to combat the dissipative effects of entropy, living systems need to acquire (e) energy from outside and protect themselves from (e) the dissipative effects of entropy

$G_{13}$ : Category one of energy from outside and protect themselves from the dissipative effects of entropy

$G_{14}$ : Category two of SAS

$G_{15}$ : Category three of SAS

$T_{13}$ : Category one of living systems need to acquire In order to combat the dissipative effects of entropy

$T_{14}$ : Category two of SAS

$T_{15}$ : Category three of SAS

### Module Two

In order to combat the dissipative effect of entropy: the need to acquire energy from (e) outside, for example with food; the need to reduce (e) the dissipation of energy, for example with a shelter (housing) and clothes

$G_{16}$ : Category one of outside, for example with food

$G_{17}$ : Category two of SAS

$G_{18}$ : Category three of SAS

$T_{16}$ : Category one of acquire energy

$T_{17}$ : Category two of SAS

$T_{18}$ : Category three of SAS

### Module three

**need to reduce the dissipation of energy**, for example with a shelter (housing) and clothes

$G_{20}$ : Category one of **need to reduce the dissipation of energy;** shelter (housing) and clothes

$G_{21}$ : Category two of SAS

$G_{22}$ : Category three of SAS

$T_{20}$ : Category one of shelter (housing) and clothes **; need to reduce the dissipation of energy**

$T_{21}$ : Category two of SAS

$T_{22}$ : Category three of SAS

### Module four

In order to combat the continuous production of waste, this is the consequence of (e) the destruction of structures under (e&eb) the effect of entropy: the need for hygienic and sanitary standards and waste disposal

$G_{24}$ : Category one of destruction of structures under the effect of entropy: the need for hygienic and sanitary standards and waste disposal

$G_{25}$ : Category two of SAS

$G_{26}$ : Category three of SAS

$T_{24}$ : Category one of combat the continuous production of waste

$T_{25}$ : Category two of SAS

$T_{26}$ : Category three of SAS

**Module five**

When these needs are partially unsatisfied, pain is experienced in the forms of (eb) hunger, thirst, sickness, and, when (e) they are totally unsatisfied, death is the consequence.

$G_{28}$ : Category one of these needs are partially unsatisfied

$G_{29}$ : Category two of SAS

$G_{30}$ : Category three of SAS

$T_{28}$ : Category one of pain is experienced in the forms of hunger, thirst, sickness

$T_{29}$ : Category two of SAS

$T_{30}$ : Category three of SAS

**Module six**

The total satisfaction of material needs leads to (eb) a state of well being which is characterized by (e) the absence of pain linked to material needs.

$G_{32}$ : Category one of total satisfaction of material needs

$G_{33}$ : Category two of SAS

$G_{34}$ : Category three of SAS

$T_{32}$ : Category one of state of well being which is characterized by the absence of pain linked to material needs.

$T_{33}$ : Category two of SAS

$T_{34}$ : Category three of SAS

**Module seven**

Satisfying material needs does not stop entropy from destroying (e) the structures of the living systems:

cells die, and structures are destroyed;

the living system is therefore continuously called to repair (e&eb) the damages caused by (e) entropy

$G_{36}$ : Category one of structures of the living systems cells die, and structures are destroyed

$G_{37}$ : Category two of SAS

$G_{38}$ : Category three of SAS

$T_{36}$ : Category one of Satisfying material needs does not stop entropy from destroying

$T_{37}$ : Category two of SAS

$T_{38}$ : Category three of SAS

**Module eight**

**living system is therefore continuously called to repair** (e&eb) the damages caused by (e) entropy

$G_{40}$ : Category one of **living system is therefore continuously called to repair;** damages caused by entropy

$G_{41}$ : Category two of SAS

$G_{42}$ : Category three of SAS

$T_{40}$ : Category one of damages caused by entropy; **living system is therefore continuously called to repair**

$T_{41}$ : Category two of SAS

$T_{42}$ : Category three of SAS

**Module Nine**

Negative energy coincides with (e&eb, =) converging waves which concentrate energy

$G_{44}$ : Category one of Negative energy

$G_{45}$ : Category two of SAS

$G_{46}$ : Category three of SAS

$T_{44}$ : Category one of converging waves which concentrate energy

$T_{45}$ : Category two of SAS

$T_{46}$ : Category three of SAS

## SECTION THIRTY NINE

# Negative Energy Coincides With Converging Waves Which Concentrate Energy

## INTRODUCTION—VARIABLES USED

**The conflict between entropy and syntropy: the vital needs model Ulisse Di Corpo ulisse.dicorpo@gmail.com**

(1) In order to mend these damages the living system needs to feed on (e) syntropy, which is the only property which allows to (eb) create order and organization and to counterbalance (e&eb) (differential stages) the destructive effects of entropy.

(2) Experiments on retrocausality show that the autonomic nervous system, which supports (eb) the vital functions of the living system, should be (=) the neurophysiologic structure which acquires (e) negative energy (syntropy) from (e) the microcosm (**Radin, 2006**), feeding in this way the vital functions and the regenerative processes of the living system.

(3) For this reason, when a good connection with -E is established, feelings of warmth associated with (e&eb) well being are experienced in the areas which are governed by (e&eb) the autonomic nervous system (thorax).

**(4)** On the contrary, when (e) the connection is insufficient, a lack of energy is experienced in the form of feelings of chill and emptiness associated with suffering.

## NOTATION

### Module One

For this reason, when a good connection with -E is established, feelings of warmth associated with (e&eb) well being are experienced in the areas which are governed by (e&eb) the autonomic nervous system (thorax)

$G_{13}$ : Category one of good connection with -E is established

$G_{14}$ : Category two of SAS

$G_{15}$ : Category three of SAS

$T_{13}$ : Category one of feelings of warmth associated with (e&eb) well being are experienced in the areas which are governed by (e&eb) the autonomic nervous system (thorax)

$T_{14}$ : Category two of SAS

$T_{15}$ : Category three of SAS

### Module Two

For this reason, when a good connection with -E is established, feelings of warmth associated with (e&eb) well being are experienced in the areas which are governed by (e&eb) the autonomic nervous system (thorax)

$G_{16}$ : Category one of **good connection with -E is established, feelings of warmth;** well being are experienced in the areas which are governed by (e&eb) the autonomic nervous system (thorax)

$G_{17}$ : Category two of SAS

$G_{18}$ : Category three of SAS

$T_{16}$ : Category one of well being are experienced in the areas which are governed by (e&eb) the autonomic

nervous system (thorax);**good connection with -E is established, feelings of warmth**

$T_{17}$ : Category two of SAS

$T_{18}$ : Category three of SAS

**Module three**

For this reason, when a **good connection with -E is established, feelings of warmth associated with well being are experienced in the areas** which are governed by (e&eb) the autonomic nervous system (thorax)

$G_{20}$ : Category one of **good connection with -E is established, feelings of warmth associated with well being are experienced in the areas;** autonomic nervous system (thorax)

$G_{21}$ : Category two of SAS

$G_{22}$ : Category three of SAS

$T_{20}$ : Category one of autonomic nervous system (thorax); **good connection with -E is established, feelings of warmth associated with well being are experienced in the areas**

$T_{21}$ : Category two of SAS

$T_{22}$ : Category three of SAS

**Module four**

In order to mend these damages the living system needs to feed on (e) syntropy, which is the only property which allows to (eb) create order and organization and to counterbalance (e&eb) (differential stages) the destructive effects of entropy

$G_{24}$ : Category one of syntropy, which is the only property which allows to (eb) create order and organization and to counterbalance (e&eb) (differential stages) the destructive effects of entropy

$G_{25}$ : Category two of SAS

$G_{26}$ : Category three of SAS

$T_{24}$ : Category one of mend these damages the living system needs to feed

$T_{25}$ : Category two of SAS

$T_{26}$ : Category three of SAS

**Module five**

In order to mend these damages the living system needs to feed on syntropy, which is the only property which allows to (eb) create order and organization and to counterbalance (e&eb) (differential stages) the destructive effects of entropy

$G_{28}$ : Category one of mend these damages the living system needs to feed on syntropy, which is the only property

$G_{29}$ : Category two of SAS

$G_{30}$ : Category three of SAS

$T_{28}$ : Category one of create order and organization and to counterbalance (e&eb) (differential stages) the destructive effects of entropy

$T_{29}$ : Category two of SAS

$T_{30}$ : Category three of SAS

**Module six**

In order to mend these damages the living system needs to feed on syntropy, which is the only property which allows to create order and organization and to counterbalance (e&eb) (differential stages) the destructive effects of entropy

$G_{32}$ : Category one of **mend these damages the living system needs to feed on syntropy, which is the only property which allows to create order and organization and to counterbalance;** (differential stages) the destructive effects of entropy

$G_{33}$ : Category two of SAS

$G_{34}$ : Category three of SAS

$T_{32}$ : Category one of(differential stages) the destructive effects of entropy; **mend these damages the living system needs to feed on syntropy, which is the only property which allows to create order and organization and to counterbalance**

$T_{33}$ : Category two of SAS

$T_{34}$ : Category three of SAS

**Module seven**

On the contrary, when (e) the connection is insufficient, a lack of energy is experienced in the form of feelings of chill and emptiness associated with suffering

$G_{36}$ : Category one of connection is insufficient

$G_{37}$ : Category two of SAS

$G_{38}$ : Category three of SAS

$T_{36}$ : Category one of lack of energy is experienced in the form of feelings of chill and emptiness associated with suffering

$T_{37}$ : Category two of SAS

$T_{38}$ : Category three of SAS

**Module eight**

These feelings of suffering experienced in the area of the thorax are usually named anxiety and can take the form of fear, panic, and imminent death.

Anxiety is generally associated with (e&eb) neurovegetative symptoms such as nausea, vertigo, feelings of

suffocation. (See also What is life: Schrõdinger)

$G_{40}$ : Category one of feelings of suffering experienced in the area of the thorax

$G_{41}$ : Category two of SAS

$G_{42}$ : Category three of SAS

$T_{40}$ : Category one of anxiety and can take the form of fear, panic, and imminent death

$T_{41}$ : Category two of SAS

$T_{42}$ : Category three of SAS

**Module Nine**

**Anxiety** is generally associated with (e&eb) neurovegetative symptoms such as nausea, vertigo, feelings of suffocation. (See also What is life: Schrõdinger)

$G_{44}$ : Category one of **Anxiety;** neurovegetative symptoms such as nausea, vertigo, feelings of suffocation. (See also What is life: Schrõdinger)

$G_{45}$ : Category two of SAS

$G_{46}$ : Category three of SAS

$T_{44}$ : Category one of neurovegetative symptoms such as nausea, vertigo, feelings of suffocation. (See also What is life: Schrõdinger);**Anxiety**

$T_{45}$ : Category two of SAS

$T_{46}$ : Category three of SAS

## SECTION FORTY

## Entropy Has Inflated The Universe Towards Infinite (Diverging Waves), Syntropy (Converging Waves) Forces Living Systems To Be Finite And Localized

### INTRODUCTION—VARIABLES USED

**The conflict between entropy and syntropy: the vital needs model Ulisse Di Corpo ulisse.dicorpo@gmail.com**

(1) These feelings of suffering experienced in the area of the thorax are usually named anxiety and can take the form of fear, panic, and imminent death. Anxiety is generally associated with (e&eb) neurovegetative symptoms such as nausea, vertigo, feelings of suffocation. (See also What is life: Schrõdinger)

(2) The need for a good connection with –E is felt as (e) need of connection, of unity, and attraction, it will therefore be referred to as need of unity.

(3) When this need is not satisfied (eb) feelings of pain are experienced in the form of anxiety and pain in the **thorax area**. When this need is totally unsatisfied the system is unable to feed (e&eb) the regenerative processes and repair (e&eb) the damages produced by **entropy and death occur**. Solve the conflict between entropy and (e&eb) syntropy: the need of meaning.

(4) In order to satisfy the material needs, living systems have developed cortical systems which show their highest complexity in human beings.

(5) These cortical systems produce (eb) representations of the environment which permit (eb) the comparison of the living system with (e&eb) the environment.

**(6)** This process initiates (eb) the conflict between entropy and syntropy: while (e) entropy has inflated the universe towards infinite (diverging waves), syntropy (converging waves) forces living systems to be (=) finite and localized.

## NOTATION

### Module One

Experiments on retrocausality show that the autonomic nervous system, which supports (eb) the vital functions of the living system, should be (=) the neurophysiologic structure which acquires (e) negative energy (syntropy) from (e) the microcosm (**Radin, 2006**), feeding in this way the vital functions and the regenerative processes of the living system

$G_{13}$ : Category one of Experiments on retrocausality

$G_{14}$ : Category two of SAS

$G_{15}$ : Category three of SAS

$T_{13}$ : Category one of autonomic nervous system, which supports (eb) the vital functions of the living system, should be (=) the neurophysiologic structure which acquires (e) negative energy (syntropy) from (e) the microcosm (**Radin, 2006**), feeding in this way the vital functions and the regenerative processes of the living system

$T_{14}$ : Category two of SAS

$T_{15}$ : Category three of SAS

### Module Two

Experiments on retrocausality show that the autonomic nervous system, which supports the vital functions of the living system, should be (=) the neurophysiologic structure which acquires (e) negative energy (syntropy) from (e) the microcosm (**Radin, 2006**), feeding in this way the vital functions and the regenerative processes of the living system

$G_{16}$ : Category one of Experiments on retrocausality show that the autonomic nervous system

$G_{17}$ : Category two of SAS

$G_{18}$ : Category three of SAS

$T_{16}$ : Category one of vital functions of the living system, should be (=) the neurophysiologic structure which acquires (e) negative energy (syntropy) from (e) the microcosm (**Radin, 2006**), feeding in this way the vital functions and the regenerative processes of the living system

$T_{17}$ : Category two of SAS

$T_{18}$ : Category three of SAS

**Module three**

Experiments on retrocausality show that the autonomic nervous system, which supports the vital functions of the living system, should be the neurophysiologic structure which acquires (e) negative energy (syntropy) from (e) the microcosm (**Radin, 2006**), feeding in this way the vital functions and the regenerative processes of the living system

$G_{20}$ : Category one of Experiments on retrocausality show that the autonomic nervous system, which supports the vital functions of the living system

$G_{21}$ : Category two of SAS

$G_{22}$ : Category three of SAS

$T_{20}$ : Category one of neurophysiologic structure which acquires (e) negative energy (syntropy) from (e) the microcosm (**Radin, 2006**), feeding in this way the vital functions and the regenerative processes of the living system

$T_{21}$ : Category two of SAS

$T_{22}$ : Category three of SAS

**Module four**

Experiments on retrocausality show that the autonomic nervous system, which supports the vital functions of the living system, should be the neurophysiologic structure which acquires (e) negative energy (syntropy) from (e) the microcosm (**Radin, 2006**), feeding in this way the vital functions and the regenerative processes of the living system

$G_{24}$ : Category one of negative energy (syntropy) from (e) the microcosm (**Radin, 2006**), feeding in this way the vital functions and the regenerative processes of the living system

$G_{25}$ : Category two of SAS

$G_{26}$ : Category three of SAS

$T_{24}$ : Category one of Experiments on retrocausality show that the autonomic nervous system, which supports the vital functions of the living system, should be the neurophysiologic structure

$T_{25}$ : Category two of SAS

$T_{26}$ : Category three of SAS

**Module five**

Experiments on retrocausality show that the autonomic nervous system, which supports the vital functions of the living system, should be the neurophysiologic structure which acquires negative energy (syntropy) from (e) the microcosm (**Radin, 2006**), feeding in this way the vital functions and the regenerative processes of the living system

$G_{28}$ : Category one of microcosm (**Radin, 2006**), feeding in this way the vital functions and the regenerative processes of the living system

$G_{29}$ : Category two of SAS

$G_{30}$ : Category three of SAS

$T_{28}$ : Category one of Experiments on retrocausality show that the autonomic nervous system, which supports the vital functions of the living system, should be the neurophysiologic structure which acquires negative energy (syntropy)

$T_{29}$ : Category two of SAS

$T_{30}$ : Category three of SAS

**Module six**

The need for a good connection with –E is felt as (e) need of connection, of unity, and attraction, it will therefore be referred to as need of unity.

$G_{32}$ : Category one of need for a good connection with –E is felt

$G_{33}$ : Category two of SAS

$G_{34}$ : Category three of SAS

$T_{32}$ : Category one of need of connection, of unity, and attraction, it will therefore be referred to as need of unity

$T_{33}$ : Category two of SAS

$T_{34}$ : Category three of SAS

**Module seven**

When this need is not satisfied feelings of pain are experienced in the form of anxiety and pain in the **thorax area**.

When this need is totally unsatisfied the system is unable to feed (e&eb) the regenerative processes and repair (e&eb) the damages produced by **entropy and death occur**.

Solve the conflict between entropy and (e&eb) syntropy: the need of meaning.

$G_{36}$ : Category one of need is not satisfied

$G_{37}$ : Category two of SAS

$G_{38}$ : Category three of SAS

$T_{36}$ : Category one of feelings of pain is experienced in the form of anxiety and pain in the **thorax area**

$T_{37}$ : Category two of SAS

$T_{38}$ : Category three of SAS

**Module eight**

When this **need is totally unsatisfied the system is unable to feed** (e&eb) the regenerative processes and repair (e&eb) the damages produced by **entropy and death occur**

$G_{40}$ : Category one of **need is totally unsatisfied the system is unable to feed;** regenerative processes and repair the damages produced by **entropy and death occur**

$G_{41}$ : Category two of SAS

$G_{42}$ : Category three of SAS

$T_{40}$ : Category one of regenerative processes and repair (e&eb) the damages produced by **entropy and death occur; need is totally unsatisfied the system is unable to feed**

$T_{41}$ : Category two of SAS

$T_{42}$ : Category three of SAS

**Module Nine**

These cortical systems produce (eb) representations of the environment which permit (eb) the comparison of the living system with (e&eb) the environment.

This process initiates (eb) the conflict between entropy and syntropy:

while (e) entropy has inflated the universe towards infinite (diverging waves), syntropy (converging waves) forces living systems to be (=) finite and localized

$G_{44}$ : Category one of entropy

$G_{45}$ : Category two of SAS

$G_{46}$ : Category three of SAS

$T_{44}$ : Category one of inflated the universe towards infinite (diverging waves)

$T_{45}$ : Category two of SAS

$T_{46}$ : Category three of SAS

**The Coefficients:**

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}$ , $(b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)},$
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}, \ (b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$

$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}\ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$ ,
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)}$,
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)}$,
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)}$,
are Dissipation coefficients

**Module Numbered One**

**The differential system of this model is now (Module Numbered one)**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \quad 1$$
$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad 2$$
$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15} \quad 3$$
$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13} \quad 4$$
$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14} \quad 5$$
$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)\right]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14}, t) =$ First augmentation factor
$-(b''_{13})^{(1)}(G, t) =$ First detritions factor

**Module Numbered Two**

**The differential system of this model is now ( Module numbered two)**

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right]G_{16} \quad 7$$
$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t)\right]G_{17} \quad 8$$
$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t)\right]G_{18} \quad 9$$
$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\left((G_{19}), t\right)\right]T_{16} \quad 10$$
$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\left((G_{19}), t\right)\right]T_{17} \quad 11$$
$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\left((G_{19}), t\right)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17}, t) =$ First augmentation factor
$-(b''_{16})^{(2)}\left((G_{19}), t\right) =$ First detritions factor

**Module Numbered Three**

**The differential system of this model is now (Module numbered three)**

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right]G_{20} \quad 13$$
$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}, t)\right]G_{21} \quad 14$$
$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}, t)\right]G_{22} \quad 15$$
$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}, t)\right]T_{20} \quad 16$$
$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}, t)\right]T_{21} \quad 17$$
$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}, t)\right]T_{22} \quad 18$$

$+(a''_{20})^{(3)}(T_{21}, t) =$ First augmentation factor

$-(b''_{20})^{(3)}(G_{23},t) =$ First detritions factor

**Module Numbered Four**

**The differential system of this model is now (Module numbered Four)**

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right]G_{24} \quad 19$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25},t)\right]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25},t)\right]G_{26} \quad 21$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27}),t\right)\right]T_{24} \quad 22$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27}),t\right)\right]T_{25} \quad 23$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27}),t\right)\right]T_{26} \quad 24$$

$+(a''_{24})^{(4)}(T_{25},t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}\left((G_{27}),t\right) =$ **First detritions factor**

**Module Numbered Five**:

**The differential system of this model is now (Module number five)**

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29},t)\right]G_{28} \quad 25$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29},t)\right]G_{29} \quad 26$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29},t)\right]G_{30} \quad 27$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}\left((G_{31}),t\right)\right]T_{28} \quad 28$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}\left((G_{31}),t\right)\right]T_{29} \quad 29$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}\left((G_{31}),t\right)\right]T_{30} \quad 30$$

$+(a''_{28})^{(5)}(T_{29},t) =$ **First augmentation factor**

$-(b''_{28})^{(5)}\left((G_{31}),t\right) =$ **First detritions factor**

**Module Numbered Six**

**The differential system of this model is now (Module numbered Six)**

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33},t)\right]G_{32} \quad 31$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33},t)\right]G_{33} \quad 32$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33},t)\right]G_{34} \quad 33$$

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}\left((G_{35}),t\right)\right]T_{32} \quad 34$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}\left((G_{35}),t\right)\right]T_{33} \quad 35$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}\left((G_{35}),t\right)\right]T_{34} \quad 36$$

$+(a''_{32})^{(6)}(T_{33},t) =$ **First augmentation factor**

**Module Numbered Seven:**

**The differential system of this model is now (Seventh Module)**

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right]G_{36} \quad 37$$

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37},t)\right]G_{37} \quad 38$$

$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37},t)\right]G_{38} \quad 39$$
$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39}),t)\right]T_{36} \quad 40$$
$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}((G_{39}),t)\right]T_{37} \quad 41$$
$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39}),t)\right]T_{38} \quad 42$$
$+(a''_{36})^{(7)}(T_{37},t) =$ **First augmentation factor**

**Module Numbered Eight**

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right]G_{40} \quad 43$$
$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41},t)\right]G_{41} \quad 44$$
$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41},t)\right]G_{42} \quad 45$$
$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43}),t)\right]T_{40} \quad 46$$
$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}((G_{43}),t)\right]T_{41} \quad 47$$
$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43}),t)\right]T_{42} \quad 48$$

**Module Numbered Nine**

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$
$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$
$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$
$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47}),t)\right]T_{44} \quad 52$$
$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}((G_{47}),t)\right]T_{45} \quad 53$$
$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47}),t)\right]T_{46} \quad 54$$
$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}((G_{47}),t) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)}\boxed{+(a''_{13})^{(1)}(T_{14},t)}\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7)}(T_{37},t)}\boxed{+(a''_{40})^{(8,8)}(T_{41},t)}\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)}\boxed{+(a''_{14})^{(1)}(T_{14},t)}\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7)}(T_{37},t)}\boxed{+(a''_{41})^{(8,8)}(T_{41},t)}\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)}\boxed{+(a''_{15})^{(1)}(T_{14},t)}\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7)}(T_{37},t)}\boxed{+(a''_{42})^{(8,8)}(T_{41},t)}\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15} \quad 57$$

Where $\boxed{(a''_{13})^{(1)}(T_{14},t)}$, $\boxed{(a''_{14})^{(1)}(T_{14},t)}$, $\boxed{(a''_{15})^{(1)}(T_{14},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}$ are second augmentation coefficient for

category 1, 2 and 3
$+(a''_{20})^{(3,3,)}(T_{21},t)$, $+(a''_{21})^{(3,3,)}(T_{21},t)$, $+(a''_{22})^{(3,3,)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3
$+(a''_{24})^{(4,4,4,4,)}(T_{25},t)$ , $+(a''_{25})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3
$+(a''_{28})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3
$+(a''_{32})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3
$+(a''_{38})^{(7,7)}(T_{37},t)$ $+(a''_{37})^{(7,7)}(T_{37},t)$ $+(a''_{36})^{(7,7)}(T_{37},t)$ are seventh augmentation coefficient for 1,2,3
$+(a''_{40})^{(8,8)}(T_{41},t)$ $+(a''_{41})^{(8,8)}(T_{41},t)$ $+(a''_{42})^{(8,8)}(T_{41},t)$ are eight augmentation coefficient for 1,2,3
$+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)} & -(b''_{13})^{(1)}(G,t) & -(b''_{16})^{(2,2,)}(G_{19},t) & -(b''_{20})^{(3,3,)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{36})^{(7,7,)}(G_{39},t) & -(b''_{40})^{(8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{13} \quad 58$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)} & -(b''_{14})^{(1)}(G,t) & -(b''_{17})^{(2,2,)}(G_{19},t) & -(b''_{21})^{(3,3,)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{37})^{(7,7,)}(G_{39},t) & -(b''_{41})^{(8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{14} \quad 59$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)} & -(b''_{15})^{(1)}(G,t) & -(b''_{18})^{(2,2,)}(G_{19},t) & -(b''_{22})^{(3,3,)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{38})^{(7,7,)}(G_{39},t) & -(b''_{42})^{(8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{15} \quad 60$$

Where $-(b''_{13})^{(1)}(G,t)$, $-(b''_{14})^{(1)}(G,t)$, $-(b''_{15})^{(1)}(G,t)$ are first detrition coefficients for category 1, 2 and 3
$-(b''_{16})^{(2,2,)}(G_{19},t)$, $-(b''_{17})^{(2,2,)}(G_{19},t)$, $-(b''_{18})^{(2,2,)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3
$-(b''_{20})^{(3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3
$-(b''_{24})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3
$-(b''_{28})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3
$-(b''_{32})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3
$-(b''_{37})^{(7,7,)}(G_{39},t)$, $-(b''_{36})^{(7,7,)}(G_{39},t)$, $-(b''_{38})^{(7,7,)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8)}(G_{43},t)}$ $\boxed{-(b''_{41})^{(8,8)}(G_{43},t)}$ $\boxed{-(b''_{42})^{(8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[\begin{array}{c} (a'_{16})^{(2)}\boxed{+(a''_{16})^{(2)}(T_{17},t)}\ \boxed{+(a''_{13})^{(1,1,)}(T_{14},t)}\ \boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)}\ \boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)}\ \boxed{+(a''_{44})^{(9,9)}(T_{45},t)} \end{array}\right] G_{16} \quad (61)$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[\begin{array}{c} (a'_{17})^{(2)}\boxed{+(a''_{17})^{(2)}(T_{17},t)}\ \boxed{+(a''_{14})^{(1,1,)}(T_{14},t)}\ \boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)}\ \boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)}\ \boxed{+(a''_{45})^{(9,9)}(T_{45},t)} \end{array}\right] G_{17} \quad (62)$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[\begin{array}{c} (a'_{18})^{(2)}\boxed{+(a''_{18})^{(2)}(T_{17},t)}\ \boxed{+(a''_{15})^{(1,1,)}(T_{14},t)}\ \boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)}\ \boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)}\ \boxed{+(a''_{46})^{(9,9)}(T_{45},t)} \end{array}\right] G_{18} \quad (63)$$

Where $\boxed{+(a''_{16})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,)}(T_{14},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)}$ are eight augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{44})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9)}(T_{45},t)}$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[\begin{array}{c} (b'_{16})^{(2)}\boxed{-(b''_{16})^{(2)}(G_{19},t)}\ \boxed{-(b''_{13})^{(1,1,)}(G,t)}\ \boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)}\ \boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)}\ \boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)}\ \boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)}\ \boxed{-(b''_{44})^{(9,9)}(G_{47},t)} \end{array}\right] T_{16} \quad (64)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)}\boxed{-(b''_{17})^{(2)}(G_{19},t)} & \boxed{-(b''_{14})^{(1,1,)}(G,t)} & \boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{17} \quad (65)$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)}\boxed{-(b''_{18})^{(2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,)}(G,t)} & \boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{18} \quad (66)$$

where $\boxed{-(b''_{16})^{(2)}(G_{19},t)}$ , $\boxed{-(b''_{17})^{(2)}(G_{19},t)}$ , $\boxed{-(b''_{18})^{(2)}(G_{19},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,)}(G,t)}$ are second detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{44})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \begin{bmatrix} (a'_{20})^{(3)}\boxed{+(a''_{20})^{(3)}(T_{21},t)} & \boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{20} \quad (67)$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \begin{bmatrix} (a'_{21})^{(3)}\boxed{+(a''_{21})^{(3)}(T_{21},t)} & \boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{21} \quad (68)$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \begin{bmatrix} (a'_{22})^{(3)}\boxed{+(a''_{22})^{(3)}(T_{21},t)} & \boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{22} \quad (69)$$

$\boxed{+(a''_{20})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3)}(T_{21},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}$ are second augmentation coefficients

for category 1, 2 and 3

$+(a_{13}'')^{(1,1,1,)}(T_{14},t)$, $+(a_{14}'')^{(1,1,1,)}(T_{14},t)$, $+(a_{15}'')^{(1,1,1,)}(T_{14},t)$ are third augmentation coefficients for category 1, 2 and 3

$+(a_{24}'')^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a_{25}'')^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a_{26}'')^{(4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficients for category 1, 2 and 3

$+(a_{28}'')^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a_{29}'')^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a_{30}'')^{(5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficients for category 1, 2 and 3

$+(a_{32}'')^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a_{33}'')^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a_{34}'')^{(6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficients for category 1, 2 and 3

$+(a_{36}'')^{(7,7,7,7)}(T_{37},t)$, $+(a_{37}'')^{(7,7,7,7)}(T_{37},t)$, $+(a_{38}'')^{(7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1, 2 and 3

$+(a_{40}'')^{(8,8,8,8)}(T_{41},t)$, $+(a_{41}'')^{(8,8,8,8)}(T_{41},t)$, $+(a_{42}'')^{(8,8,8,8)}(T_{41},t)$ are eight augmentation coefficients for category 1, 2 and 3

$+(a_{44}'')^{(9,9,9)}(T_{45},t)$, $+(a_{45}'')^{(9,9,9)}(T_{45},t)$, $+(a_{46}'')^{(9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b_{20}')^{(3)} & -(b_{20}'')^{(3)}(G_{23},t) & -(b_{16}'')^{(2,2,2)}(G_{19},t) & -(b_{13}'')^{(1,1,1,)}(G,t) \\ -(b_{24}'')^{(4,4,4,4,4,4)}(G_{27},t) & -(b_{28}'')^{(5,5,5,5,5,5)}(G_{31},t) & -(b_{32}'')^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b_{36}'')^{(7,7,7,7)}(G_{39},t) & -(b_{40}'')^{(8,8,8,8)}(G_{43},t) & -(b_{44}'')^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{20} \quad (70)$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b_{21}')^{(3)} & -(b_{21}'')^{(3)}(G_{23},t) & -(b_{17}'')^{(2,2,2)}(G_{19},t) & -(b_{14}'')^{(1,1,1,)}(G,t) \\ -(b_{25}'')^{(4,4,4,4,4,4)}(G_{27},t) & -(b_{29}'')^{(5,5,5,5,5,5)}(G_{31},t) & -(b_{33}'')^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b_{37}'')^{(7,7,7,7)}(G_{39},t) & -(b_{41}'')^{(8,8,8,8)}(G_{43},t) & -(b_{45}'')^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{21} \quad (71)$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b_{22}')^{(3)} & -(b_{22}'')^{(3)}(G_{23},t) & -(b_{18}'')^{(2,2,2)}(G_{19},t) & -(b_{15}'')^{(1,1,1,)}(G,t) \\ -(b_{26}'')^{(4,4,4,4,4,4)}(G_{27},t) & -(b_{30}'')^{(5,5,5,5,5,5)}(G_{31},t) & -(b_{34}'')^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b_{38}'')^{(7,7,7,7)}(G_{39},t) & -(b_{42}'')^{(8,8,8,8)}(G_{43},t) & -(b_{46}'')^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{22} \quad (72)$$

$-(b_{20}'')^{(3)}(G_{23},t)$, $-(b_{21}'')^{(3)}(G_{23},t)$, $-(b_{22}'')^{(3)}(G_{23},t)$ *are first detrition coefficients for category 1, 2 and 3*

$-(b_{16}'')^{(2,2,2)}(G_{19},t)$, $-(b_{17}'')^{(2,2,2)}(G_{19},t)$, $-(b_{18}'')^{(2,2,2)}(G_{19},t)$ *are second detrition coefficients for category 1, 2 and 3*

$-(b_{13}'')^{(1,1,1,)}(G,t)$, $-(b_{14}'')^{(1,1,1,)}(G,t)$, $-(b_{15}'')^{(1,1,1,)}(G,t)$ *are third detrition coefficients for category 1,2 and 3*

$-(b_{24}'')^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b_{25}'')^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b_{26}'')^{(4,4,4,4,4,4)}(G_{27},t)$ *are fourth detrition coefficients for category 1, 2 and 3*

$-(b_{28}'')^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b_{29}'')^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b_{30}'')^{(5,5,5,5,5,5)}(G_{31},t)$ *are fifth detrition coefficients for category 1, 2 and 3*

$-(b_{32}'')^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b_{33}'')^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b_{34}'')^{(6,6,6,6,6,6)}(G_{35},t)$ *are sixth detrition coefficients for category 1, 2 and 3*

$-(b_{36}'')^{(7,7,7,7)}(G_{39},t)$, $-(b_{37}'')^{(7,7,7,7)}(G_{39},t)$, $-(b_{38}'')^{(7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients

for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt}=(a_{24})^{(4)}G_{25}-\begin{bmatrix}(a'_{24})^{(4)} & \boxed{+(a''_{24})^{(4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}\\ \boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{24} \quad (73)$$

$$\frac{dG_{25}}{dt}=(a_{25})^{(4)}G_{24}-\begin{bmatrix}(a'_{25})^{(4)} & \boxed{+(a''_{25})^{(4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6)}(T_{33},t)}\\ \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{25} \quad (74)$$

$$\frac{dG_{26}}{dt}=(a_{26})^{(4)}G_{25}-\begin{bmatrix}(a'_{26})^{(4)} & \boxed{+(a''_{26})^{(4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}\\ \boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{26} \quad (75)$$

$\boxed{(a''_{24})^{(4)}(T_{25},t)}$, $\boxed{(a''_{25})^{(4)}(T_{25},t)}$, $\boxed{(a''_{26})^{(4)}(T_{25},t)}$ *are first augmentation coefficients category* 1, 2 3

$\boxed{+(a''_{28})^{(5,5,)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,)}(T_{29},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)}$
*are fourth augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)}$
*are fifth augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)}$
*are sixth augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)}$
*are seventh augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)}$
*are eighth augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt}=(b_{24})^{(4)}T_{25}-\begin{bmatrix}(b'_{24})^{(4)} & \boxed{-(b''_{24})^{(4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,)}(G_{35},t)}\\ \boxed{-(b''_{13})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)}\\ \boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9)}(G_{47},t)}\end{bmatrix}T_{24} \quad (76)$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \begin{bmatrix} (b'_{25})^{(4)} & -(b''_{25})^{(4)}(G_{27},t) & -(b''_{29})^{(5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{25} \quad 77$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)} & -(b''_{26})^{(4)}(G_{27},t) & -(b''_{30})^{(5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1)}(G,t) & -(b''_{18})^{(2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{26} \quad 78$$

*Where* $-(b''_{24})^{(4)}(G_{27},t)$, $-(b''_{25})^{(4)}(G_{27},t)$, $-(b''_{26})^{(4)}(G_{27},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{28})^{(5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,)}(G_{31},t)$ *are second detrition coefficients for category* 1, 2 *and* 3

$-(b''_{32})^{(6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,)}(G_{35},t)$ *are third detrition coefficients for category* 1, 2 *and* 3

$-(b''_{13})^{(1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1)}(G,t)$
*are fourth detrition coefficients for category* 1, 2 *and* 3

$-(b''_{16})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2)}(G_{19},t)$
*are fifth detrition coefficients for category* 1, 2 *and* 3

$-(b''_{20})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3)}(G_{23},t)$
*are sixth detrition coefficients for category* 1, 2 *and* 3

$-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)$
*are seventh detrition coefficients for category* 1, 2 *and* 3

$-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)$
*are eighth detrition coefficients for category* 1, 2 *and* 3

$-(b''_{46})^{(9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \begin{bmatrix} (a'_{28})^{(5)} & +(a''_{28})^{(5)}(T_{29},t) & +(a''_{24})^{(4,4,)}(T_{25},t) & +(a''_{32})^{(6,6,6)}(T_{33},t) \\ & +(a''_{13})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{16})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3,3)}(T_{21},t) \\ & +(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{28} \quad 79$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \begin{bmatrix} (a'_{29})^{(5)} & +(a''_{29})^{(5)}(T_{29},t) & +(a''_{25})^{(4,4,)}(T_{25},t) & +(a''_{33})^{(6,6,6)}(T_{33},t) \\ & +(a''_{14})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{17})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3,3)}(T_{21},t) \\ & +(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{29} \quad 80$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \begin{bmatrix} (a'_{30})^{(5)} & +(a''_{30})^{(5)}(T_{29},t) & +(a''_{26})^{(4,4,)}(T_{25},t) & +(a''_{34})^{(6,6,6)}(T_{33},t) \\ & +(a''_{15})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{18})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3,3)}(T_{21},t) \\ & +(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{30} \quad 81$$

*Where* $+(a''_{28})^{(5)}(T_{29},t)$, $+(a''_{29})^{(5)}(T_{29},t)$, $+(a''_{30})^{(5)}(T_{29},t)$ *are first augmentation coefficients for category* 1, 2 *and* 3

*And* $+(a''_{24})^{(4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,)}(T_{25},t)$ *are second augmentation*

$coefficient\ for\ category\ 1,2\ and\ 3$
$+(a''_{32})^{(6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6)}(T_{33},t)$ $are\ third\ \ augmentation$
$coefficient\ for\ category\ 1,2\ and\ 3$
$+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)$ are fourth augmentation coefficients for category 1,2, and 3
$+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)$ are fifth augmentation coefficients for category 1,2,and 3
$+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)$ are sixth augmentation coefficients for category 1,2, 3
$+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)$ , $+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1,2, 3
$+(a''_{40})^{(8,8\ ,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)$ are eighth augmentation coefficients for category 1,2, 3
$+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt}=(b_{28})^{(5)}T_{29}-\begin{bmatrix}(b'_{28})^{(5)} & -(b''_{28})^{(5)}(G_{31},t) & -(b''_{24})^{(4,4,)}(G_{27},t) & -(b''_{32})^{(6,6,6)}(G_{35},t)\\ & -(b''_{13})^{(1,1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3)}(G_{23},t)\\ & -(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9)}(G_{47},t)\end{bmatrix}T_{28} \quad 82$$

$$\frac{dT_{29}}{dt}=(b_{29})^{(5)}T_{28}-\begin{bmatrix}(b'_{29})^{(5)} & -(b''_{29})^{(5)}(G_{31},t) & -(b''_{25})^{(4,4,)}(G_{27},t) & -(b''_{33})^{(6,6,6)}(G_{35},t)\\ & -(b''_{14})^{(1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3)}(G_{23},t)\\ & -(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9)}(G_{47},t)\end{bmatrix}T_{29} \quad 83$$

$$\frac{dT_{30}}{dt}=(b_{30})^{(5)}T_{29}-\begin{bmatrix}(b'_{30})^{(5)} & -(b''_{30})^{(5)}(G_{31},t) & -(b''_{26})^{(4,4,)}(G_{27},t) & -(b''_{34})^{(6,6,6)}(G_{35},t)\\ & -(b''_{15})^{(1,1,1,1,1,)}(G,t) & -(b''_{18})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3)}(G_{23},t)\\ & -(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9)}(G_{47},t)\end{bmatrix}T_{30} \quad 84$$

$where$ $-(b''_{28})^{(5)}(G_{31},t)$ , $-(b''_{29})^{(5)}(G_{31},t)$ , $-(b''_{30})^{(5)}(G_{31},t)$ $are\ first\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b''_{24})^{(4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,)}(G_{27},t)$ $are\ second\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b''_{32})^{(6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6)}(G_{35},t)$ $are\ \ third\ \ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b''_{13})^{(1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1)}(G,t)$ , $-(b''_{15})^{(1,1,1,1,1,)}(G,t)$ are fourth detrition coefficients for category 1,2, and 3
$-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1,2, and 3
$-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1,2, and 3
$-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[\begin{array}{ccc} (a'_{32})^{(6)}\boxed{+(a''_{32})^{(6)}(T_{33},t)} & \boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{32} \quad (85)$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[\begin{array}{ccc} (a'_{33})^{(6)}\boxed{+(a''_{33})^{(6)}(T_{33},t)} & \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{33} \quad (86)$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[\begin{array}{ccc} (a'_{34})^{(6)}\boxed{+(a''_{34})^{(6)}(T_{33},t)} & \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{34} \quad (87)$$

$\boxed{+(a''_{32})^{(6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6)}(T_{33},t)}$ *are first augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}$ *are second augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}$ *are third augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}$ - are fourth augmentation coefficients

$\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}$ - fifth augmentation coefficients

$\boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)}$ sixth augmentation coefficients

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)}$ seventh augmentation coefficients

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}$ Eighth augmentation coefficients

$\boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)}$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[\begin{array}{ccc} (b'_{32})^{(6)}\boxed{-(b''_{32})^{(6)}(G_{35},t)} & \boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)} \end{array}\right] T_{32} \quad (88)$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \begin{bmatrix} (b'_{33})^{(6)}\boxed{-(b''_{33})^{(6)}(G_{35},t)} & \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{33} \quad (89)$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)}\boxed{-(b''_{34})^{(6)}(G_{35},t)} & \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{34} \quad (90)$$

$\boxed{-(b''_{32})^{(6)}(G_{35},t)}, \boxed{-(b''_{33})^{(6)}(G_{35},t)}, \boxed{-(b''_{34})^{(6)}(G_{35},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}$ *are second detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)}$ *are third detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}$ are fourth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \quad (91)$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a'_{36})^{(7)}\boxed{+(a''_{36})^{(7)}(T_{37},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \quad (92)$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a'_{37})^{(7)}\boxed{+(a''_{37})^{(7)}(T_{37},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$\frac{dG_{38}}{dt}$ *93*

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a'_{38})^{(7)}\boxed{+(a''_{38})^{(7)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{(a''_{36})^{(7)}(T_{37},t)}$, $\boxed{(a''_{37})^{(7)}(T_{37},t)}$, $\boxed{(a''_{38})^{(7)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,)}(T_{41},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$\frac{dT_{36}}{dt} =$ 94

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b'_{36})^{(7)}\boxed{-(b''_{36})^{(7)}(G_{39},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$\frac{dT_{37}}{dt} =$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b'_{37})^{(7)}\boxed{-(b''_{37})^{(7)}(G_{39},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$\frac{dT_{38}}{dt} =$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b'_{38})^{(7)}\boxed{-(b''_{38})^{(7)}(G_{39},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b_{36}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{37}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{16}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{17}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{18}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{20}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{21}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{22}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{24}'')^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{25}'')^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{26}'')^{(4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{28}'')^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{32}'')^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{34}'')^{(6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{46}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{45}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{44}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3



$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \begin{bmatrix} (a_{40}')^{(8)}\boxed{+(a_{40}'')^{(8)}(T_{41},t)} & \boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{20}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{24}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{28}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{32}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{36}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{44}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \begin{bmatrix} (a_{41}')^{(8)}\boxed{+(a_{41}'')^{(8)}(T_{41},t)} & \boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{21}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{25}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{29}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{33}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{37}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{45}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \begin{bmatrix} (a_{42}')^{(8)}\boxed{+(a_{42}'')^{(8)}(T_{41},t)} & \boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{22}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{26}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{30}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{34}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{15}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{38}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{46}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a_{40}'')^{(8)}(T_{41},t)}$, $\boxed{+(a_{41}'')^{(8)}(T_{41},t)}$, $\boxed{+(a_{42}'')^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second

augmentation coefficient for category 1, 2 and 3
$+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)$ , $+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)$ , $+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3
$+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)$ , $+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)$ , $+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3
$+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)$ , $+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3
$+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)$ , $+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3
$+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ $+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ $+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ are seventh augmentation coefficient for 1,2,3
$+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$ , $+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$ , $+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$ are eighth augmentation coefficient for 1,2,3
$+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$ , $+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$ , $+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} =$$

$$(b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)}\left[-(b''_{40})^{(8)}(G_{43},t)\right] & \left[-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} =$$

$$(b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)}\left[-(b''_{41})^{(8)}(G_{43},t)\right] & \left[-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} =$$

$$(b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)}\left[-(b''_{42})^{(8)}(G_{43},t)\right] & \left[-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{15}$$

Where $-(b''_{36})^{(7)}(G_{39},t)$ , $-(b''_{37})^{(7)}(G_{39},t)$ , $-(b''_{38})^{(7)}(G_{39},t)$ are first detrition coefficients for category 1, 2 and 3
$-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$ , $-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$ , $-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3
$-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$ , $-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$ , $-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3
$-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)$ , $-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)$ , $-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{28}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{32}'')^{(6,6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{38}'')^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{36}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{37}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{44}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{45}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{46}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

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$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \begin{bmatrix} (a_{44}')^{(9)}\boxed{+(a_{44}'')^{(9)}(T_{45},t)} & \boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{20}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{24}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{28}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{32}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{36}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{40}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \begin{bmatrix} (a_{45}')^{(9)}\boxed{+(a_{45}'')^{(9)}(T_{45},t)} & \boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{21}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{25}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{29}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{33}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{14}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{37}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{41}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \begin{bmatrix} (a_{46}')^{(9)}\boxed{+(a_{46}'')^{(9)}(T_{37},t)} & \boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{22}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{26}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{30}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{34}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{15}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{38}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{42}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a_{44}'')^{(9)}(T_{45},t)}$, $\boxed{+(a_{45}'')^{(9)}(T_{45},t)}$, $\boxed{+(a_{46}'')^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{20}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{21}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{22}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{24}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{25}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{26}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{28}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{29}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{30}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$ are Seventh augmentation coefficient for category 1, 2 and 3

$+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ $+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ $+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ are eighth augmentation coefficient for 1,2,3

$+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} =$$

$$(b_{44})^{(9)}T_{45} -$$

$$\begin{bmatrix} (b'_{44})^{(9)}\boxed{-(b''_{44})^{(9)}(G_{47},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b'_{45})^{(9)}\boxed{-(b''_{45})^{(9)}(G_{47},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b'_{46})^{(9)}\boxed{-(b''_{46})^{(9)}(G_{47},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{44})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9)}(G_{47},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{37}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{36}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a_i')^{(1)}, (a_i'')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (b_i'')^{(1)} > 0, \quad i,j = 13,14,15 \qquad 97$$

The functions $(a_i'')^{(1)}, (b_i'')^{(1)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a_i'')^{(1)}(T_{14},t) \leq (p_i)^{(1)} \leq (\hat{A}_{13})^{(1)}$$

$$(b_i'')^{(1)}(G,t) \leq (r_i)^{(1)} \leq (b_i')^{(1)} \leq (\hat{B}_{13})^{(1)}$$

$$lim_{T_2\to\infty}(a_i'')^{(1)}(T_{14},t) = (p_i)^{(1)} \qquad 98$$

$$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = (r_i)^{(1)}$$

**Definition of** $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$ :

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$$|(a_i'')^{(1)}(T_{14}',t) - (a_i'')^{(1)}(T_{14},t)| \leq (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G',t) - (b_i'')^{(1)}(G,t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}',t)$ and $(a_i'')^{(1)}(T_{14},t)$ . $(T_{14}',t)$ and $(T_{14},t)$ are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14},t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$ : 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$$

**Definition of** $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$ : 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i,j = 16,17,18$$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17}, t) \leq (p_i)^{(2)} \leq (\hat{A}_{16})^{(2)} \quad 102$$

$$(b_i'')^{(2)}(G_{19}, t) \leq (r_i)^{(2)} \leq (b_i')^{(2)} \leq (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2 \to \infty} (a_i'')^{(2)}(T_{17}, t) = (p_i)^{(2)} \quad 104$$

$$\lim_{G \to \infty} (b_i'')^{(2)}((G_{19}), t) = (r_i)^{(2)} \quad 105$$

**Definition of** $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$ : 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}', t) - (a_i'')^{(2)}(T_{17}, t)| \leq (\hat{k}_{16})^{(2)}|T_{17} - T_{17}'|e^{-(\hat{M}_{16})^{(2)}t} \quad 107$$

$$|(b_i'')^{(2)}((G_{19})', t) - (b_i'')^{(2)}((G_{19}), t)| < (\hat{k}_{16})^{(2)}||(G_{19}) - (G_{19})'||e^{-(\hat{M}_{16})^{(2)}t} \quad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}', t)$ and$(a_i'')^{(2)}(T_{17}, t)$ . $(T_{17}', t)$ and $(T_{17}, t)$ are points belonging to the interval $[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}]$ . It is to be noted that $(a_i'')^{(2)}(T_{17}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17}, t)$ , the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$ :

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants 109

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$$

**Definition of** $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$ :

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1 \quad (110)$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(b_i)^{(2)} + (b_i')^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1 \quad (111)$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i,j = 20,21,22 \quad (112)$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(3)}$, $(r_i)^{(3)}$:

$$(a_i'')^{(3)}(T_{21}, t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23}, t) \leq (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(3)}(T_{21}, t) = (p_i)^{(3)} \quad (113)$$

$$\lim_{G \to \infty}(b_i'')^{(3)}(G_{23}, t) = (r_i)^{(3)}$$

**Definition of** $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$ :

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: (114)

$$|(a_i'')^{(3)}(T_{21}', t) - (a_i'')^{(3)}(T_{21}, t)| \leq (\hat{k}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\hat{M}_{20})^{(3)}t}$$

$$|(b_i'')^{(3)}(G_{23}', t) - (b_i'')^{(3)}(G_{23}, t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}'||e^{-(\hat{M}_{20})^{(3)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}', t)$ and $(a_i'')^{(3)}(T_{21}, t)$ . $(T_{21}', t)$ And $(T_{21}, t)$ are points belonging to the interval $[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}]$. It is to be noted that $(a_i'')^{(3)}(T_{21}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$ : (115)

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$ and the constants $(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}, i = 20,21,22,$ satisfy the inequalities 116

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(a_i)^{(3)} + (a_i')^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(b_i)^{(3)} + (b_i')^{(3)} + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i, j = 24,25,26 \qquad 117$$

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(4)}$, $(r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25}, t) \le (p_i)^{(4)} \le (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}((G_{27}), t) \le (r_i)^{(4)} \le (b_i')^{(4)} \le (\hat{B}_{24})^{(4)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(4)}(T_{25}, t) = (p_i)^{(4)} \qquad 118$$

$$\lim_{G \to \infty}(b_i'')^{(4)}((G_{27}), t) = (r_i)^{(4)}$$

**Definition of** $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$ :

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24,25,26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}', t) - (a_i'')^{(4)}(T_{25}, t)| \le (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})', t) - (b_i'')^{(4)}((G_{27}), t)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}', t)$ and$(a_i'')^{(4)}(T_{25}, t)$ . $(T_{25}', t)$ and $(T_{25}, t)$ are points belonging to the interval $[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}]$. It is to be noted that $(a_i'')^{(4)}(T_{25}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$ : 120

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$, are positive constants

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}, \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$$

**Definition of** $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$ : 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and\ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}, i = 24,25,26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(a_i)^{(4)} + (a_i')^{(4)} + (\hat{A}_{24})^{(4)} + (\hat{P}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(b_i)^{(4)} + (b_i')^{(4)} + (\hat{B}_{24})^{(4)} + (\hat{Q}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i, j = 28,29,30 \qquad 122$$

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(5)}$, $(r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29}, t) \le (p_i)^{(5)} \le (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}((G_{31}), t) \le (r_i)^{(5)} \le (b_i')^{(5)} \le (\hat{B}_{28})^{(5)}$$

$$lim_{T_2 \to \infty} (a_i'')^{(5)} (T_{29}, t) = (p_i)^{(5)} \qquad 123$$

$$\lim_{G \to \infty} (b_i'')^{(5)} (G_{31}, t) = (r_i)^{(5)}$$

**Definition of** $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$ :

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}', t) - (a_i'')^{(5)}(T_{29}, t)| \le (\hat{k}_{28})^{(5)} |T_{29} - T_{29}'| e^{-(\hat{M}_{28})^{(5)} t}$$

$$|(b_i'')^{(5)}((G_{31})', t) - (b_i'')^{(5)}((G_{31}), t)| < (\hat{k}_{28})^{(5)} ||(G_{31}) - (G_{31})'|| e^{-(\hat{M}_{28})^{(5)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}', t)$ and $(a_i'')^{(5)}(T_{29}, t)$ . $(T_{29}', t)$ and $(T_{29}, t)$ are points belonging to the interval $\left[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\right]$. It is to be noted that $(a_i'')^{(5)}(T_{29}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29}, t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$ : 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}, \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

**Definition of** $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$ : 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and\ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(a_i)^{(5)} + (a_i')^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(b_i)^{(5)} + (b_i')^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i,j = 32,33,34$$ 127

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(6)}$, $(r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33}, t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}), t) \leq (r_i)^{(6)} \leq (b_i')^{(6)} \leq (\hat{B}_{32})^{(6)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(6)}(T_{33}, t) = (p_i)^{(6)}$$ 128

$$\lim_{G \to \infty}(b_i'')^{(6)}((G_{35}), t) = (r_i)^{(6)}$$

**Definition of** $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$ :

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}', t) - (a_i'')^{(6)}(T_{33}, t)| \leq (\hat{k}_{32})^{(6)}|T_{33} - T_{33}'|e^{-(\hat{M}_{32})^{(6)}t}$$

$$|(b_i'')^{(6)}((G_{35})', t) - (b_i'')^{(6)}((G_{35}), t)| < (\hat{k}_{32})^{(6)}||(G_{35}) - (G_{35})'||e^{-(\hat{M}_{32})^{(6)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}', t)$ and $(a_i'')^{(6)}(T_{33}, t)$ . $(T_{33}', t)$ and $(T_{33}, t)$ are points belonging to the interval $[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}]$. It is to be noted that $(a_i'')^{(6)}(T_{33}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33}, t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$ : 129

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$, are positive constants

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$$

**Definition of** $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$ : 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and\ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32,33,34,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(a_i)^{(6)} + (a_i')^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)} (\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(b_i)^{(6)} + (b_i')^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)} (\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

(IIIIIIIII) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i, j = 36,37,38$ 131

(JJJJJJJJJ)The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(7)}, (r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37}, t) \leq (p_i)^{(7)} \leq (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39}, t) \leq (r_i)^{(7)} \leq (b_i')^{(7)} \leq (\hat{B}_{36})^{(7)}$$

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*(KKKKKKKKK)* $\lim_{T_2 \to \infty} (a_i'')^{(7)}(T_{37}, t) = (p_i)^{(7)}$

*(LLLLLLLLL)*

$\lim_{G \to \infty} (b_i'')^{(7)}((G_{39}), t) = (r_i)^{(7)}$

**Definition of** $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$ :

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$$|(a_i'')^{(7)}(T_{37}', t) - (a_i'')^{(7)}(T_{37}, t)| \leq (\hat{k}_{36})^{(7)} |T_{37} - T_{37}'| e^{-(\hat{M}_{36})^{(7)} t}$$

$$|(b_i'')^{(7)}((G_{39})', t) - (b_i'')^{(7)}((G_{39}), t)| < (\hat{k}_{36})^{(7)} ||(G_{39}) - (G_{39})'|| e^{-(\hat{M}_{36})^{(7)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}', t)$ and $(a_i'')^{(7)}(T_{37}, t)$ . $(T_{37}', t)$ and $(T_{37}, t)$ are points belonging to the interval $[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}]$ . It is to be noted that $(a_i'')^{(7)}(T_{37}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37}, t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$ : 134

(MMMMMMMMM) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)},$ are positive constants

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$$

**Definition of** $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$ : 135

(NNNNNNNNN) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(b_i)^{(7)} + (b_i')^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i, j = 40,41,42 \qquad 136$$

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

**Definition of** $(p_i)^{(8)}$, $(r_i)^{(8)}$: 137

$$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)} \qquad 138$$

$$(b_i'')^{(8)}((G_{43}), t) \leq (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)} \qquad 139$$

$$\lim_{T_2 \to \infty} (a_i'')^{(8)}(T_{41}, t) = (p_i)^{(8)} \qquad 140$$

$$\lim_{G \to \infty} (b_i'')^{(8)}\big((G_{43}), t\big) = (r_i)^{(8)} \qquad 141$$

**Definition of** $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$ :

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)} |T_{41} - T_{41}'| e^{-(\hat{M}_{40})^{(8)} t} \qquad 142$$

$$|(b_i'')^{(8)}((G_{43})', t) - (b_i'')^{(8)}\big((G_{43}), t\big)| < (\hat{k}_{40})^{(8)} ||(G_{43}) - (G_{43})'|| e^{-(\hat{M}_{40})^{(8)} t} \qquad 143$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}', t)$ and $(a_i'')^{(8)}(T_{41}, t)$ . $(T_{41}', t)$ and $(T_{41}, t)$ are points belonging to the interval $\big[(\hat{k}_{40})^{(8)}, (\hat{M}_{40})^{(8)}\big]$. It is to be noted that $(a_i'')^{(8)}(T_{41}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{40})^{(8)} = 1$

then the function $(a_i'')^{(8)}(T_{41},t)$ , the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\hat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)}$ :

$(\hat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)},$ are positive constants

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\ ,\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}}<1 \qquad 144$$

**Definition of** $(\hat{P}_{40})^{(8)},(\hat{Q}_{40})^{(8)}$ :

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\hat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)},(\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)},(a_i')^{(8)},(b_i)^{(8)},(b_i')^{(8)},(p_i)^{(8)},\ (r_i)^{(8)},i=40,41,42,$

Satisfy the inequalities

$$\frac{1}{(\hat{M}_{40})^{(8)}}[\,(a_i)^{(8)}+(a_i')^{(8)}+\ (\hat{A}_{40})^{(8)}+\ (\hat{P}_{40})^{(8)}\,(\hat{k}_{40})^{(8)}]<1 \qquad 145$$

$$\frac{1}{(\hat{M}_{40})^{(8)}}[\ (b_i)^{(8)}+(b_i')^{(8)}+\ (\hat{B}_{40})^{(8)}+\ (\hat{Q}_{40})^{(8)}\ (\hat{k}_{40})^{(8)}]<1 \qquad 146$$

Where we suppose

$$(a_i)^{(9)},(a_i')^{(9)},(a_i'')^{(9)},(b_i)^{(9)},(b_i')^{(9)},(b_i'')^{(9)}>0,\quad i,j=44,45,46 \qquad 146\,A$$

The functions $(a_i'')^{(9)},(b_i'')^{(9)}$ are positive continuous increasing and bounded.

**Definition of** $(p_i)^{(9)},\ (r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45},t)\le(p_i)^{(9)}\le(\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47},t)\le\ (r_i)^{(9)}\le(b_i')^{(9)}\le(\hat{B}_{44})^{(9)}$$

$$\lim_{T_2\to\infty}(a_i'')^{(9)}(T_{45},t)=(p_i)^{(9)}$$

$$\lim_{G\to\infty}(b_i'')^{(9)}(G_{47},t)=\ (r_i)^{(9)}$$

**Definition of** $(\hat{A}_{44})^{(9)},(\hat{B}_{44})^{(9)}$ :

Where $\boxed{(\hat{A}_{44})^{(9)},(\hat{B}_{44})^{(9)},(p_i)^{(9)},\ (r_i)^{(9)}}$ are positive constants and $\boxed{i=44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}',t)-(a_i'')^{(9)}(T_{45},t)|\le(\hat{k}_{44})^{(9)}|T_{45}-\ T_{45}'|e^{-(\hat{M}_{44})^{(9)}t}$$

$$|(b_i'')^{(9)}((G_{47})',t)-(b_i'')^{(9)}\big((G_{47}),t\big)|<(\hat{k}_{44})^{(9)}||(G_{47})-(G_{47})'||e^{-(\hat{M}_{44})^{(9)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}',t)$ and$(a_i'')^{(9)}(T_{45},t)$ . $(T_{45}',t)$ and $(T_{45},t)$ are points belonging to the interval $\big[(\hat{k}_{44})^{(9)},(\hat{M}_{44})^{(9)}\big]$. It is

to be noted that $(a_i'')^{(9)}(T_{45},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{44})^{(9)}=1$ then the function $(a_i'')^{(9)}(T_{45},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

**Definition of** $(\widehat{M}_{44})^{(9)},(\hat{k}_{44})^{(9)}$ :

$(\widehat{M}_{44})^{(9)},(\hat{k}_{44})^{(9)}$, are positive constants

$$\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\ ,\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}<1$$

**Definition of** $(\hat{P}_{44})^{(9)},(\hat{Q}_{44})^{(9)}$ :

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\widehat{M}_{44})^{(9)},(\hat{k}_{44})^{(9)},(\hat{A}_{44})^{(9)} and\ (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)},(a_i')^{(9)},(b_i)^{(9)},(b_i')^{(9)},(p_i)^{(9)},\ (r_i)^{(9)}, i=44,45,46,$ satisfy the inequalities

$$\frac{1}{(\widehat{M}_{44})^{(9)}}[(a_i)^{(9)}+(a_i')^{(9)}+\ (\hat{A}_{44})^{(9)}+\ (\hat{P}_{44})^{(9)}\ (\hat{k}_{44})^{(9)}]<1$$

$$\frac{1}{(\widehat{M}_{44})^{(9)}}[\ (b_i)^{(9)}+(b_i')^{(9)}+\ (\hat{B}_{44})^{(9)}+\ (\hat{Q}_{44})^{(9)}\ (\hat{k}_{44})^{(9)}]<1$$

**Theorem 1:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0)\ ,T_i(0)$ :

$$G_i(t)\le\ (\hat{P}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}\ ,\ \boxed{G_i(0)=G_i^0>0}$$

$$T_i(t)\le\ (\hat{Q}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}\ ,\ \boxed{T_i(0)=T_i^0>0}$$

**Theorem 2 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

**Definition of** $G_i(0)\ ,T_i(0)$

$$G_i(t)\le\ (\hat{P}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}\ ,\ \ G_i(0)=G_i^0>0$$

$$T_i(t)\le\ (\hat{Q}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}\ ,\ \ T_i(0)=T_i^0>0$$

**Theorem 3 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t)\le\ (\hat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}\ ,\ \ G_i(0)=G_i^0>0$$

$$T_i(t)\le\ (\hat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}\ ,\ \ T_i(0)=T_i^0>0$$

**Theorem 4 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

**Definition of** $G_i(0)\ ,T_i(0)$ :

$G_i(t) \leq \left(\hat{P}_{24}\right)^{(4)} e^{(\hat{M}_{24})^{(4)}t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{24}\right)^{(4)} e^{(\hat{M}_{24})^{(4)}t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Theorem 5 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

**Definition of** $G_i(0)$ , $T_i(0)$ :

$G_i(t) \leq \left(\hat{P}_{28}\right)^{(5)} e^{(\hat{M}_{28})^{(5)}t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{28}\right)^{(5)} e^{(\hat{M}_{28})^{(5)}t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Theorem 6 :** if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

**Definition of** $G_i(0)$ , $T_i(0)$ :

$G_i(t) \leq \left(\hat{P}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Theorem 7:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

**Definition of** $G_i(0)$ , $T_i(0)$ :

$G_i(t) \leq \left(\hat{P}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Theorem 8:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

**Definition of** $G_i(0)$ , $T_i(0)$ :

$G_i(t) \leq \left(\hat{P}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Theorem 9:** if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

**Definition of** $G_i(0)$ , $T_i(0)$ :

$G_i(t) \leq \left(\hat{P}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t}$ , $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t}$ , $\boxed{T_i(0) = T_i^0 > 0}$

**Proof:** Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i$: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \le (\hat{P}_{13})^{(1)} \text{ , } T_i^0 \le (\hat{Q}_{13})^{(1)}, \quad 155$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t} \quad 156$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t} \quad 157$$

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}(s_{(13)}) - \left((a'_{13})^{(1)} + a''_{13})^{(1)} (T_{14}(s_{(13)}), s_{(13)})\right) G_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)} (T_{14}(s_{(13)}), s_{(13)})\right) G_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)} (T_{14}(s_{(13)}), s_{(13)})\right) G_{15}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)} (G(s_{(13)}), s_{(13)})\right) T_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)} (G(s_{(13)}), s_{(13)})\right) T_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{15}(t) = T_{15}^0 + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)} (G(s_{(13)}), s_{(13)})\right) T_{15}(s_{(13)})\right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i$: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \le (\hat{P}_{16})^{(2)} \text{ , } T_i^0 \le (\hat{Q}_{16})^{(2)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$$

By 160

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''_{16})^{(2)} (T_{17}(s_{(16)}), s_{(16)})\right) G_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)} (T_{17}(s_{(16)}), s_{(17)})\right) G_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)} (T_{17}(s_{(16)}), s_{(16)})\right) G_{18}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)} (G_{19}(s_{(16)}), s_{(16)})\right) T_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^{0} + \int_0^t \left[ (b_{17})^{(2)} T_{16}(s_{(16)}) - \left( (b'_{17})^{(2)} - (b''_{17})^{(2)} (G_{19}(s_{(16)}), s_{(16)}) \right) T_{17}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^{0} + \int_0^t \left[ (b_{18})^{(2)} T_{17}(s_{(16)}) - \left( (b'_{18})^{(2)} - (b''_{18})^{(2)} (G_{19}(s_{(16)}), s_{(16)}) \right) T_{18}(s_{(16)}) \right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \le (\hat{P}_{20})^{(3)} \text{ , } T_i^0 \le (\hat{Q}_{20})^{(3)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$$

By 161

$$\bar{G}_{20}(t) = G_{20}^{0} + \int_0^t \left[ (a_{20})^{(3)} G_{21}(s_{(20)}) - \left( (a'_{20})^{(3)} + a''_{20})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^{0} + \int_0^t \left[ (a_{21})^{(3)} G_{20}(s_{(20)}) - \left( (a'_{21})^{(3)} + (a''_{21})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{21}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^{0} + \int_0^t \left[ (a_{22})^{(3)} G_{21}(s_{(20)}) - \left( (a'_{22})^{(3)} + (a''_{22})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{22}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^{0} + \int_0^t \left[ (b_{20})^{(3)} T_{21}(s_{(20)}) - \left( (b'_{20})^{(3)} - (b''_{20})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^{0} + \int_0^t \left[ (b_{21})^{(3)} T_{20}(s_{(20)}) - \left( (b'_{21})^{(3)} - (b''_{21})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{21}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{22}(t) = T_{22}^{0} + \int_0^t \left[ (b_{22})^{(3)} T_{21}(s_{(20)}) - \left( (b'_{22})^{(3)} - (b''_{22})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{22}(s_{(20)}) \right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions $G_i$ , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \le (\hat{P}_{24})^{(4)} \text{ , } T_i^0 \le (\hat{Q}_{24})^{(4)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

By 162

$$\bar{G}_{24}(t) = G_{24}^{0} + \int_0^t \left[ (a_{24})^{(4)} G_{25}(s_{(24)}) - \left( (a'_{24})^{(4)} + a''_{24})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{26}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{26}(t) = T_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{26}(s_{(24)})\right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:** Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{28})^{(5)}, T_i^0 \le (\hat{Q}_{28})^{(5)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

By



$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{30}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{30}(t) = T_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{30}(s_{(28)})\right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{32})^{(6)}, T_i^0 \le (\hat{Q}_{32})^{(6)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\bar{M}_{32})^{(6)} t}$

By 164

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[ (a_{32})^{(6)} G_{33}(s_{(32)}) - \left( (a'_{32})^{(6)} + a''_{32})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[ (a_{33})^{(6)} G_{32}(s_{(32)}) - \left( (a'_{33})^{(6)} + (a''_{33})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[ (a_{34})^{(6)} G_{33}(s_{(32)}) - \left( (a'_{34})^{(6)} + (a''_{34})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{34}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[ (b_{32})^{(6)} T_{33}(s_{(32)}) - \left( (b'_{32})^{(6)} - (b''_{32})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[ (b_{33})^{(6)} T_{32}(s_{(32)}) - \left( (b'_{33})^{(6)} - (b''_{33})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{\mathrm{T}}_{34}(\mathrm{t}) = \mathrm{T}_{34}^0 + \int_0^t \left[ (b_{34})^{(6)} T_{33}(s_{(32)}) - \left( (b'_{34})^{(6)} - (b''_{34})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{34}(s_{(32)}) \right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

**Proof:**

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[ (a_{36})^{(7)} G_{37}(s_{(36)}) - \left( (a'_{36})^{(7)} + a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[ (a_{37})^{(7)} G_{36}(s_{(36)}) - \left( (a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[ (a_{38})^{(7)} G_{37}(s_{(36)}) - \left( (a'_{38})^{(7)} + (a''_{38})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{38}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[ (b_{36})^{(7)} T_{37}(s_{(36)}) - \left( (b'_{36})^{(7)} - (b''_{36})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[ (b_{37})^{(7)} T_{36}(s_{(36)}) - \left( (b'_{37})^{(7)} - (b''_{37})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^0 + \int_0^t \left[ (b_{38})^{(7)} T_{37}(s_{(36)}) - \left( (b'_{38})^{(7)} - (b''_{38})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{38}(s_{(36)}) \right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0,t)$

**Proof:**

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{40})^{(8)}$, $T_i^0 \le (\hat{Q}_{40})^{(8)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} G_{41}(s_{(40)}) - \left((a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[(a_{41})^{(8)} G_{40}(s_{(40)}) - \left((a'_{41})^{(8)} + (a''_{41})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{42}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^0 + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)} (G_{43}(s_{(40)}), s_{(40)})\right) T_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^0 + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)} (G_{43}(s_{(40)}), s_{(40)})\right) T_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{42}(t) = T_{42}^0 + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)} (G_{43}(s_{(40)}), s_{(40)})\right) T_{42}(s_{(40)})\right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0,t)$

**Proof:** 166
Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions $G_i$, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ A
which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{44})^{(9)}$, $T_i^0 \le (\hat{Q}_{44})^{(9)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)})\right) G_{44}(s_{(44)})\right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)} (T_{45}(s_{(44)}), s_{(44)})\right) G_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}(s_{(44)}), s_{(44)})\right) G_{46}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{44}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{\mathrm{T}}_{46}(\mathrm{t}) = \mathrm{T}_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{46}(s_{(44)})\right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}}\right)\right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left(e^{(\hat{M}_{13})^{(1)} t} - 1\right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\hat{M}_{13})^{(1)} t} \le \frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{P}_{13})^{(1)} + G_{14}^0\right) e^{\left(-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0}\right)} + (\hat{P}_{13})^{(1)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \le G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}}\right)\right] ds_{(16)} = \quad 169$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1\right)$$

From which it follows that 170

$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \le \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left[\left((\hat{P}_{16})^{(2)} + G_{17}^0\right) e^{\left(-\frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0}\right)} + (\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t) \le G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} \left(G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} s_{(20)}}\right)\right] ds_{(20)} =$$

$$\left(1 + (a_{20})^{(3)} t\right) G_{21}^0 + \frac{(a_{20})^{(3)} (\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left(e^{(\hat{M}_{20})^{(3)} t} - 1\right)$$

From which it follows that 172

$$(G_{20}(t)-G_{20}^{0})e^{-(\hat{M}_{20})^{(3)}t}\leq\frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left[((\hat{P}_{20})^{(3)}+G_{21}^{0})e^{\left(-\frac{(\hat{P}_{20})^{(3)}+G_{21}^{0}}{G_{21}^{0}}\right)}+(\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 173

$$G_{24}(t)\leq G_{24}^{0}+\int_{0}^{t}\left[(a_{24})^{(4)}\left(G_{25}^{0}+(\hat{P}_{24})^{(4)}e^{(\hat{M}_{24})^{(4)}s_{(24)}}\right)\right]ds_{(24)}=$$

$$\left(1+(a_{24})^{(4)}t\right)G_{25}^{0}+\frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left(e^{(\hat{M}_{24})^{(4)}t}-1\right)$$

From which it follows that 174

$$(G_{24}(t)-G_{24}^{0})e^{-(\hat{M}_{24})^{(4)}t}\leq\frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left[((\hat{P}_{24})^{(4)}+G_{25}^{0})e^{\left(-\frac{(\hat{P}_{24})^{(4)}+G_{25}^{0}}{G_{25}^{0}}\right)}+(\hat{P}_{24})^{(4)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t)\leq G_{28}^{0}+\int_{0}^{t}\left[(a_{28})^{(5)}\left(G_{29}^{0}+(\hat{P}_{28})^{(5)}e^{(\hat{M}_{28})^{(5)}s_{(28)}}\right)\right]ds_{(28)}=$$

$$\left(1+(a_{28})^{(5)}t\right)G_{29}^{0}+\frac{(a_{28})^{(5)}(\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left(e^{(\hat{M}_{28})^{(5)}t}-1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t}\leq\frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t)\leq G_{32}^{0}+\int_{0}^{t}\left[(a_{32})^{(6)}\left(G_{33}^{0}+(\hat{P}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\right)\right]ds_{(32)}=$$

$$\left(1+(a_{32})^{(6)}t\right)G_{33}^{0}+\frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left(e^{(\hat{M}_{32})^{(6)}t}-1\right)$$

From which it follows that 177

$$(G_{32}(t)-G_{32}^{0})e^{-(\hat{M}_{32})^{(6)}t}\leq\frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left[((\hat{P}_{32})^{(6)}+G_{33}^{0})e^{\left(-\frac{(\hat{P}_{32})^{(6)}+G_{33}^{0}}{G_{33}^{0}}\right)}+(\hat{P}_{32})^{(6)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}$ , $G_{26}, T_{24}, T_{25}, T_{26}$

(kk) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t) \le G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} \left(G_{37}^0 + (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} s_{(36)}}\right)\right] ds_{(36)} =$$
$$\left(1 + (a_{36})^{(7)} t\right) G_{37}^0 + \frac{(a_{36})^{(7)} (\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left(e^{(\hat{M}_{36})^{(7)} t} - 1\right)$$

From which it follows that

$$(G_{36}(t) - G_{36}^0) e^{-(\hat{M}_{36})^{(7)} t} \le \frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left[((\hat{P}_{36})^{(7)} + G_{37}^0) e^{\left(-\frac{(\hat{P}_{36})^{(7)} + G_{37}^0}{G_{37}^0}\right)} + (\hat{P}_{36})^{(7)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t) \le G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} \left(G_{41}^0 + (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} s_{(40)}}\right)\right] ds_{(40)} = \quad 180$$
$$\left(1 + (a_{40})^{(8)} t\right) G_{41}^0 + \frac{(a_{40})^{(8)} (\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}} \left(e^{(\hat{M}_{40})^{(8)} t} - 1\right)$$

From which it follows that 181

$$(G_{40}(t) - G_{40}^0) e^{-(\hat{M}_{40})^{(8)} t} \le \frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}} \left[((\hat{P}_{40})^{(8)} + G_{41}^0) e^{\left(-\frac{(\hat{P}_{40})^{(8)} + G_{41}^0}{G_{41}^0}\right)} + (\hat{P}_{40})^{(8)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 8

Analogous inequalities hold also for $G_{41}$ , $G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t) \le G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} \left(G_{45}^0 + (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} s_{(44)}}\right)\right] ds_{(44)} =$$
$$\left(1 + (a_{44})^{(9)} t\right) G_{45}^0 + \frac{(a_{44})^{(9)} (\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left(e^{(\hat{M}_{44})^{(9)} t} - 1\right)$$

From which it follows that

$$(G_{44}(t) - G_{44}^0) e^{-(\hat{M}_{44})^{(9)} t} \le \frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left[((\hat{P}_{44})^{(9)} + G_{45}^0) e^{\left(-\frac{(\hat{P}_{44})^{(9)} + G_{45}^0}{G_{45}^0}\right)} + (\hat{P}_{44})^{(9)}\right]$$

$(G_i^0)$ is as defined in the statement of theorem 9

Analogous inequalities hold also for $G_{45}$ , $G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}$ , $\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$ and to choose 182

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\widehat{M}_{13})^{(1)}}\left[(\widehat{P}_{13})^{(1)}+\left((\hat{P}_{13})^{(1)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{13})^{(1)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{13})^{(1)} \quad 183$$

$$\frac{(b_i)^{(1)}}{(\widehat{M}_{13})^{(1)}}\left[\left((\hat{Q}_{13})^{(1)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{13})^{(1)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{13})^{(1)}\right]\leq(\hat{Q}_{13})^{(1)} \quad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)},T^{(1)}\right),\left(G^{(2)},T^{(2)}\right)\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\tilde{M}_{13})^{(1)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\tilde{M}_{13})^{(1)}t}\}$$

Indeed if we denote

**Definition of** $\tilde{G},\tilde{T}:\left(\tilde{G},\tilde{T}\right)=\mathcal{A}^{(1)}(G,T)$

It results

$$\left|\tilde{G}_{13}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{13})^{(1)}\left|G_{14}^{(1)}-G_{14}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\,ds_{(13)}+$$

$$\int_0^t\{(a'_{13})^{(1)}\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$G_{13}^{(2)}\left|(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)-(a''_{13})^{(1)}\left(T_{14}^{(2)},s_{(13)}\right)\right|\ e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses it follows

$$\left|G^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}t}\leq \quad 186$$
$$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)}+(a'_{13})^{(1)}+(\widehat{A}_{13})^{(1)}+(\widehat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}\right)d\left(\left(G^{(1)},T^{(1)};\,G^{(2)},T^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

**Remark 1:** The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}\ and\ (\widehat{Q}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i=13{,}14{,}15$ depend only on $\mathrm{T}_{14}$ and respectively on

$G(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$

From 19 to 24 it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(1)} - (a_i'')^{(1)}(T_{14}(s_{(13)}), s_{(13)})\} ds_{(13)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(1)} t)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2\ and\ \left((\widehat{M}_{13})^{(1)}\right)_3$ : 187

**Remark 3**: if $G_{13}$ is bounded, the same property have also $G_{14}\ and\ G_{15}$ . indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a_{14}')^{(1)} G_{14}$ and by integrating

$$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1 / (a_{14}')^{(1)}$$

In the same way , one can obtain

$$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2 / (a_{15}')^{(1)}$$

If $G_{14}\ or\ G_{15}$ is bounded, the same property follows for $G_{13}$ , $G_{15}$ and $G_{13}$ , $G_{14}$ respectively.

**Remark 4:** If $G_{13}$ $is$ bounded, from below, the same property holds for $G_{14}\ and\ G_{15}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{14}$ is bounded from below. 188

**Remark 5:** If $T_{13}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}\,(G(t), t)) = (b_{14}')^{(1)}$ then $T_{14} \to \infty$. 189

**Definition of** $(m)^{(1)}$ and $\varepsilon_1$ :

Indeed let $t_1$ be so that for $t > t_1$

$$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t), t) < \varepsilon_1, T_{13}\,(t) > (m)^{(1)}$$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right)$, $t = log\,\frac{2}{\varepsilon_1}$ By taking now $\varepsilon_1$ sufficiently small one sees that $T_{14}$ is unbounded.
The same property holds for $T_{15}$ if $\lim_{t\to\infty}(b_{15}'')^{(1)}\,(G(t), t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}$ , $\frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}\ and\ (\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[(\widehat{P}_{16})^{(2)}+\left((\hat{P}_{16})^{(2)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{16})^{(2)}\right]\leq(\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

**Definition of** $\widetilde{G_{19}},\widetilde{T_{19}}$ : $\left(\widetilde{G_{19}},\widetilde{T_{19}}\right)=\mathcal{A}^{(2)}(G_{19},T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{16})^{(2)}\left|G_{17}^{(1)}-G_{17}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)}+$$

$$\int_0^t\{(a'_{16})^{(2)}\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$G_{16}^{(2)}\left|(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)-(a''_{16})^{(2)}\left(T_{17}^{(2)},s_{(16)}\right)\right|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0,t]$ 197

From the hypotheses it follows

$$\left|(G_{19})^{(1)}-(G_{19})^{(2)}\right|\mathrm{e}^{-(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}\leq$$
$$\frac{1}{(\widehat{\mathrm{M}}_{16})^{(2)}}\left((a_{16})^{(2)}+(a'_{16})^{(2)}+(\widehat{\mathrm{A}}_{16})^{(2)}+(\widehat{\mathrm{P}}_{16})^{(2)}(\hat{k}_{16})^{(2)}\right)\mathrm{d}\left(((G_{19})^{(1)},(T_{19})^{(1)};\ (G_{19})^{(2)},(T_{19})^{(2)})\right)$$

And analogous inequalities for $G_i$ and $T_i$. Taking into account the hypothesis the result follows 198

**Remark 6:** The fact that we supposed $(a''_{16})^{(2)}$ and $(b''_{16})^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{\mathrm{P}}_{16})^{(2)}\mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}$ and $(\widehat{\mathrm{Q}}_{16})^{(2)}\mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i=16,17,18$ depend only on $\mathrm{T}_{17}$ and respectively on

$(G_{19})$(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 7:** There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$ 200

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(2)} - (a_i'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})\} ds_{(16)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(2)}t)} > 0 \quad \text{for } t > 0$$

**Definition of** $\left((\widehat{M}_{16})^{(2)}\right)_1$, $\left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$: 201

**Remark 8:** if $G_{16}$ is bounded, the same property have also $G_{17}$ and $G_{18}$. indeed if

$G_{16} < (\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt} \leq \left((\widehat{M}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)} G_{17}$ and by integrating

$$G_{17} \leq \left((\widehat{M}_{16})^{(2)}\right)_2 = G_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1 / (a_{17}')^{(2)}$$

In the same way , one can obtain

$$G_{18} \leq \left((\widehat{M}_{16})^{(2)}\right)_3 = G_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2 / (a_{18}')^{(2)}$$

If $G_{17}$ or $G_{18}$ is bounded, the same property follows for $G_{16}$, $G_{18}$ and $G_{16}$, $G_{17}$ respectively.

**Remark 9:** If $G_{16}$ is bounded, from below, the same property holds for $G_{17}$ and $G_{18}$. The proof is analogous with the preceding one. An analogous property is true if $G_{17}$ is bounded from below. 202

**Remark 10:** If $T_{16}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t), t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$. 203

**Definition of** $(m)^{(2)}$ and $\varepsilon_2$ :

Indeed let $t_2$ be so that for $t > t_2$

$$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right)$, $t = \log\frac{2}{\varepsilon_2}$ By taking now $\varepsilon_2$ sufficiently small one sees that $T_{17}$ is unbounded. 205
The same property holds for $T_{18}$ if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t), t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[(\widehat{P}_{20})^{(3)}+\left((\hat{P}_{20})^{(3)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{20})^{(3)}\right]\leq(\hat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying Equations into itself 210

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)},(T_{23})^{(1)}),((G_{23})^{(2)},(T_{23})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

**Definition of** $\widetilde{G_{23}},\widetilde{T_{23}}:(\widetilde{(G_{23})},\widetilde{(T_{23})})=\mathcal{A}^{(3)}((G_{23}),(T_{23}))$

It results 213

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{20})^{(3)}\left|G_{21}^{(1)}-G_{21}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\,ds_{(20)}+$$

$$\int_0^t\{(a'_{20})^{(3)}\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$G_{20}^{(2)}\left|(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)},s_{(20)}\right)\right|\ e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\}ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses it follows

$$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}t}\leq \quad 214$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+(a'_{20})^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\widehat{k}_{20})^{(3)}\right)d\left(((G_{23})^{(1)},(T_{23})^{(1)};\ (G_{23})^{(2)},(T_{23})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 11:** The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ *and* $(\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i=20,21,22$ depend only on $T_{21}$ and respectively on

$(G_{23})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 12:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 216

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(3)} - (a_i'')^{(3)}(T_{21}(s_{(20)}), s_{(20)})\} ds_{(20)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(3)} t)} > 0$ for t > 0

**Definition of** $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2\ and\ \left((\widehat{M}_{20})^{(3)}\right)_3$ : 217

**Remark 13:** if $G_{20}$ is bounded, the same property have also $G_{21}\ and\ G_{22}$ . indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a_{21}')^{(3)} G_{21}$ and by integrating

$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1 / (a_{21}')^{(3)}$

In the same way , one can obtain

$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2 / (a_{22}')^{(3)}$

If $G_{21}\ or\ G_{22}$ is bounded, the same property follows for $G_{20}$ , $G_{22}$ and $G_{20}$ , $G_{21}$ respectively.

**Remark 14:** If $G_{20}$ *is* bounded, from below, the same property holds for $G_{21}\ and\ G_{22}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{21}$ is bounded from below. 218

**Remark 15:** If $T_{20}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t), t\right)) = (b_{21}')^{(3)}$ then $T_{21} \to \infty$. 219

**Definition of** $(m)^{(3)}$ and $\varepsilon_3$ :

Indeed let $t_3$ be so that for $t > t_3$

$(b_{21})^{(3)} - (b_i'')^{(3)}\left((G_{23})(t), t\right) < \varepsilon_3, T_{20}\,(t) > (m)^{(3)}$

Then $\frac{dT_{21}}{dt} \geq (a_{21})^{(3)}(m)^{(3)} - \varepsilon_3 T_{21}$ which leads to 220

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t} = \frac{1}{2}$ it results

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_3}$ By taking now $\varepsilon_3$ sufficiently small one sees that $T_{21}$ is unbounded.
The same property holds for $T_{22}$ if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t), t\right) = (b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}$ , $\frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}} < 1$ and to choose 221

$(\widehat{P}_{24})^{(4)}$ and $(\widehat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[(\widehat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \tag{222}$$

$$\frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \tag{223}$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself (224)

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric (225)

$$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{24})^{(4)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{27}}),(\widetilde{T_{27}})$ : $\left((\widetilde{G_{27}}),(\widetilde{T_{27}})\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{24})^{(4)}\left|G_{25}^{(1)}-G_{25}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\,ds_{(24)}+$$

$$\int_0^t\{(a'_{24})^{(4)}\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$G_{24}^{(2)}\left|(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)-(a''_{24})^{(4)}\left(T_{25}^{(2)},s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\}ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}t}\leq \tag{226}$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)}+(a'_{24})^{(4)}+(\widehat{A}_{24})^{(4)}+(\widehat{P}_{24})^{(4)}(\hat{k}_{24})^{(4)}\right)d\left(((G_{27})^{(1)},(T_{27})^{(1)};\ (G_{27})^{(2)},(T_{27})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 16:** The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ *and* $(\widehat{Q}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_+$. (227)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i=24,25,26$ depend only on $T_{25}$ and respectively on

$(G_{27})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 17:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 228

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(4)} - (a_i'')^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(4)} t)} > 0$ for t > 0

**Definition of** $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2\ and\ \left((\widehat{M}_{24})^{(4)}\right)_3$ : 229

**Remark 18:** if $G_{24}$ is bounded, the same property have also $G_{25}\ and\ G_{26}$ . indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a_{25}')^{(4)} G_{25}$ and by integrating

$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1 / (a_{25}')^{(4)}$

In the same way , one can obtain

$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2 / (a_{26}')^{(4)}$

If $G_{25}\ or\ G_{26}$ is bounded, the same property follows for $G_{24}$ , $G_{26}$ and $G_{24}$ , $G_{25}$ respectively.

**Remark 19:** If $G_{24}$ *is* bounded, from below, the same property holds for $G_{25}\ and\ G_{26}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{25}$ is bounded from below. 230

**Remark 20:** If $T_{24}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}\ ((G_{27})(t), t)) = (b_{25}')^{(4)}$ then $T_{25} \to \infty$. 231

**Definition of** $(m)^{(4)}$ and $\varepsilon_4$ :

Indeed let $t_4$ be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t), t) < \varepsilon_4, T_{24}\,(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_4}$ By taking now $\varepsilon_4$ sufficiently small one sees that $T_{25}$ is unbounded.
The same property holds for $T_{26}$ if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t), t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for $G_{29}$ , $G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}$ , $\frac{(b_i)^{(5)}}{(\widehat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)}+\left((\hat{P}_{28})^{(5)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{28})^{(5)}\right]\leq(\hat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$$d\left(((G_{31})^{(1)},(T_{31})^{(1)}),((G_{31})^{(2)},(T_{31})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\hat{M}_{28})^{(5)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\hat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{31}}),(\widetilde{T_{31}})$ : $\left((\widetilde{G_{31}}),(\widetilde{T_{31}})\right)=\mathcal{A}^{(5)}\left((G_{31}),(T_{31})\right)$

It results

$$|\tilde{G}_{28}^{(1)}-\tilde{G}_i^{(2)}|\leq\int_0^t(a_{28})^{(5)}|G_{29}^{(1)}-G_{29}^{(2)}|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\,ds_{(28)}+$$

$$\int_0^t\{(a'_{28})^{(5)}|G_{28}^{(1)}-G_{28}^{(2)}|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$(a''_{28})^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)|G_{28}^{(1)}-G_{28}^{(2)}|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$G_{28}^{(2)}|(a''_{28})^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)-(a''_{28})^{(5)}\left(T_{29}^{(2)},s_{(28)}\right)|\;e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\}ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on it follows

$$|(G_{31})^{(1)}-(G_{31})^{(2)}|e^{-(\widehat{M}_{28})^{(5)}t}\leq \quad 237$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)}+(a'_{28})^{(5)}+(\widehat{A}_{28})^{(5)}+(\widehat{P}_{28})^{(5)}(\widehat{k}_{28})^{(5)}\right)d\left(((G_{31})^{(1)},(T_{31})^{(1)};\;(G_{31})^{(2)},(T_{31})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 21:** The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\hat{P}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ *and* $(\hat{Q}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ respectively of $\mathbb{R}_+$. 238

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on $T_{29}$ and respectively on $(G_{31})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 22:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 239

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(5)} - (a_i'')^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(5)} t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2\ and\ \left((\widehat{M}_{28})^{(5)}\right)_3$ : 240

**Remark 23:** if $G_{28}$ is bounded, the same property have also $G_{29}\ and\ G_{30}$ . indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a_{29}')^{(5)} G_{29}$ and by integrating

$$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1 / (a_{29}')^{(5)}$$

In the same way , one can obtain

$$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2 / (a_{30}')^{(5)}$$

If $G_{29}\ or\ G_{30}$ is bounded, the same property follows for $G_{28}$ , $G_{30}$ and $G_{28}$ , $G_{29}$ respectively.

**Remark 24:** If $G_{28}$ *is* bounded, from below, the same property holds for $G_{29}\ and\ G_{30}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{29}$ is bounded from below. 241

**Remark 25:** If $T_{28}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}\,((G_{31})(t), t)) = (b_{29}')^{(5)}$ then $T_{29} \to \infty$. 242

**Definition of** $(m)^{(5)}$ and $\varepsilon_5$ :

Indeed let $t_5$ be so that for $t > t_5$

$$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t), t) < \varepsilon_5, T_{28}\,(t) > (m)^{(5)}$$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_5}$ By taking now $\varepsilon_5$ sufficiently small one sees that $T_{29}$ is unbounded.
The same property holds for $T_{30}$ if $\lim_{t\to\infty}(b_{30}'')^{(5)}\left((G_{31})(t), t\right) = (b_{30}')^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{33}$ , $G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}$ , $\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}} < 1$ and to choose 244

$(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[(\hat{P}_{32})^{(6)}+\left((\hat{P}_{32})^{(6)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{32})^{(6)} \tag{245}$$

$$\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[\left((\hat{Q}_{32})^{(6)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{32})^{(6)}\right]\leq(\hat{Q}_{32})^{(6)} \tag{246}$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric (247)

$$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\hat{M}_{32})^{(6)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\hat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{35}}),(\widetilde{T_{35}})$ : $\left((\widetilde{G_{35}}),(\widetilde{T_{35}})\right)=\mathcal{A}^{(6)}\left((G_{35}),(T_{35})\right)$

It results

$$|\tilde{G}_{32}^{(1)}-\tilde{G}_i^{(2)}|\leq\int_0^t(a_{32})^{(6)}|G_{33}^{(1)}-G_{33}^{(2)}|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\,ds_{(32)}+$$

$$\int_0^t\{(a'_{32})^{(6)}|G_{32}^{(1)}-G_{32}^{(2)}|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)|G_{32}^{(1)}-G_{32}^{(2)}|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$G_{32}^{(2)}|(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)-(a''_{32})^{(6)}\left(T_{33}^{(2)},s_{(32)}\right)|\;e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\}ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$|(G_{35})^{(1)}-(G_{35})^{(2)}|e^{-(\widehat{M}_{32})^{(6)}t}\leq \tag{248}$$
$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)}+(a'_{32})^{(6)}+(\widehat{A}_{32})^{(6)}+(\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}\right)d\left(((G_{35})^{(1)},(T_{35})^{(1)};\;(G_{35})^{(2)},(T_{35})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 26:** The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\hat{P}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ *and* $(\hat{Q}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ respectively of $\mathbb{R}_+$. (249)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i = 32,33,34$ depend only on $T_{33}$ and respectively on $(G_{35})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 27:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 250

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(6)} - (a_i'')^{(6)}(T_{33}(s_{(32)}), s_{(32)})\} ds_{(32)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(6)} t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{32})^{(6)}\right)_1, \left((\widehat{M}_{32})^{(6)}\right)_2\ and\ \left((\widehat{M}_{32})^{(6)}\right)_3$ : 251

**Remark 28:** if $G_{32}$ is bounded, the same property have also $G_{33}\ and\ G_{34}$ . indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a_{33}')^{(6)} G_{33}$ and by integrating

$$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)} \left((\widehat{M}_{32})^{(6)}\right)_1 / (a_{33}')^{(6)}$$

In the same way , one can obtain

$$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)} \left((\widehat{M}_{32})^{(6)}\right)_2 / (a_{34}')^{(6)}$$

If $G_{33}\ or\ G_{34}$ is bounded, the same property follows for $G_{32}$ , $G_{34}$ and $G_{32}$ , $G_{33}$ respectively.

**Remark 29:** If $G_{32}$ *is* bounded, from below, the same property holds for $G_{33}\ and\ G_{34}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{33}$ is bounded from below. 252

**Remark 30:** If $T_{32}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}\ ((G_{35})(t), t)) = (b_{33}')^{(6)}$ then $T_{33} \to \infty$. 253

**Definition of** $(m)^{(6)}$ and $\varepsilon_6$ :

Indeed let $t_6$ be so that for $t > t_6$

$$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t), t\right) < \varepsilon_6, T_{32}\,(t) > (m)^{(6)}$$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right)$, $t = log\,\frac{2}{\varepsilon_6}$ By taking now $\varepsilon_6$ sufficiently small one sees that $T_{33}$ is unbounded.
The same property holds for $T_{34}$ if $\lim_{t\to\infty}(b_{34}'')^{(6)}\left((G_{35})(t), t(t), t\right) = (b_{34}')^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{37}, G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}$ , $\frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$ and to choose

$(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[(\hat{P}_{36})^{(7)}+\left((\hat{P}_{36})^{(7)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{36})^{(7)} \tag{256}$$

$$\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[\left((\hat{Q}_{36})^{(7)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{36})^{(7)}\right]\leq(\hat{Q}_{36})^{(7)} \tag{257}$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself
The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric (258)

$$d\left(((G_{39})^{(1)},(T_{39})^{(1)}),((G_{39})^{(2)},(T_{39})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{36})^{(7)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{39}}),(\widetilde{T_{39}}):\left((\widetilde{G_{39}}),(\widetilde{T_{39}})\right)=\mathcal{A}^{(7)}((G_{39}),(T_{39}))$
It results

$$\left|\tilde{G}_{36}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{36})^{(7)}\left|G_{37}^{(1)}-G_{37}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\,ds_{(36)}+$$
$$\int_0^t\{(a'_{36})^{(7)}\left|G_{36}^{(1)}-G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$(a''_{36})^{(7)}\left(T_{37}^{(1)},s_{(36)}\right)\left|G_{36}^{(1)}-G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$G_{36}^{(2)}\left|(a''_{36})^{(7)}\left(T_{37}^{(1)},s_{(36)}\right)-(a''_{36})^{(7)}\left(T_{37}^{(2)},s_{(36)}\right)\right|\;e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\}ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on it follows

$$\left|(G_{39})^{(1)}-(G_{39})^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}t}\leq \tag{259}$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}}\left((a_{36})^{(7)}+(a'_{36})^{(7)}+(\hat{A}_{36})^{(7)}+(\hat{P}_{36})^{(7)}(\hat{k}_{36})^{(7)}\right)d\left(((G_{39})^{(1)},(T_{39})^{(1)};\;(G_{39})^{(2)},(T_{39})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis the result follows

**Remark 31:** The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\hat{P}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ *and* $(\hat{Q}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ respectively of $\mathbb{R}_+$. (260)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(7)}$ and $(b_i'')^{(7)}, i = 36,37,38$ depend only on $T_{37}$ and respectively on $(G_{39})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 32:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 261

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(7)} - (a_i'')^{(7)}(T_{37}(s_{(36)}), s_{(36)})\} ds_{(36)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(7)} t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2\ and\ \left((\widehat{M}_{36})^{(7)}\right)_3$ : 262

**Remark 33:** if $G_{36}$ is bounded, the same property have also $G_{37}\ and\ G_{38}$ . indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a_{37}')^{(7)} G_{37}$ and by integrating

$$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1 / (a_{37}')^{(7)}$$

In the same way , one can obtain

$$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2 / (a_{38}')^{(7)}$$

If $G_{37}\ or\ G_{38}$ is bounded, the same property follows for $G_{36}$ , $G_{38}$ and $G_{36}$ , $G_{37}$ respectively.

**Remark 34:** If $G_{36}$ *is* bounded, from below, the same property holds for $G_{37}\ and\ G_{38}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{37}$ is bounded from below. 263

**Remark 35:** If $T_{36}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(7)}\ ((G_{39})(t), t)) = (b_{37}')^{(7)}$ then $T_{37} \to \infty$. 264

**Definition of** $(m)^{(7)}$ and $\varepsilon_7$ :

Indeed let $t_7$ be so that for $t > t_7$

$$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t), t) < \varepsilon_7, T_{36}\,(t) > (m)^{(7)}$$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_7}$ By taking now $\varepsilon_7$ sufficiently small one sees that $T_{37}$ is unbounded.
The same property holds for $T_{38}$ if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t), t\right) = (b_{38}')^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}$ , $\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}} < 1$ and to choose $(\widehat{P}_{40})^{(8)}$ and $(\widehat{Q}_{40})^{(8)}$ large to have 266

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[(\widehat{P}_{40})^{(8)} + \left((\hat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{40})^{(8)} \quad 267$$

$$\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{40})^{(8)}\right] \leq (\hat{Q}_{40})^{(8)} \quad 268$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions $G_i, T_i$ satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)}, (T_{43})^{(1)}), ((G_{43})^{(2)}, (T_{43})^{(2)})\right) = \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

**Definition of** $\widetilde{(G_{43})}, \widetilde{(T_{43})}$ : $\left(\widetilde{(G_{43})}, \widetilde{(T_{43})}\right) = \mathcal{A}^{(8)}((G_{43}), (T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{40})^{(8)}\left|G_{41}^{(1)} - G_{41}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\,ds_{(40)} +$$
$$\int_0^t\{(a'_{40})^{(8)}\left|G_{40}^{(1)} - G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{-(\widehat{M}_{40})^{(8)}s_{(40)}} +$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right)\left|G_{40}^{(1)} - G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}} +$$
$$G_{40}^{(2)}\left|(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right) - (a''_{40})^{(8)}\left(T_{41}^{(2)}, s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\}ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0, t]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)} - (G_{43})^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}t} \leq \quad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)} + (a'_{40})^{(8)} + (\widehat{A}_{40})^{(8)} + (\widehat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}\right)d\left(((G_{43})^{(1)}, (T_{43})^{(1)};\ (G_{43})^{(2)}, (T_{43})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

**Remark 36:** The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate 274

condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ *and* $(\widehat{Q}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i = 40{,}41{,}42$ depend only on $T_{41}$ and respectively on $(G_{43})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 37** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 275

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t\{(a_i')^{(8)}-(a_i'')^{(8)}(T_{41}(s_{(40)}),s_{(40)})\}ds_{(40)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(8)}t\right)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{40})^{(8)}\right)_1, \left((\widehat{M}_{40})^{(8)}\right)_2\ and\ \left((\widehat{M}_{40})^{(8)}\right)_3$ : 276

**Remark 38:** if $G_{40}$ is bounded, the same property have also $G_{41}\ and\ G_{42}$ . indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a_{41}')^{(8)}G_{41}$ and by integrating

$$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1/(a_{41}')^{(8)}$$

In the same way , one can obtain

$$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2/(a_{42}')^{(8)}$$

If $G_{41}\ or\ G_{42}$ is bounded, the same property follows for $G_{40}$ , $G_{42}$ and $G_{40}$ , $G_{41}$ respectively.

**Remark 39:** If $G_{40}$ *is* bounded, from below, the same property holds for $G_{41}\ and\ G_{42}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{41}$ is bounded from below. 277

**Remark 40:** If $T_{40}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}\ ((G_{43})(t),t)) = (b_{41}')^{(8)}$ then $T_{41} \to \infty$. 278

**Definition of** $(m)^{(8)}$ and $\varepsilon_8$ :

Indeed let $t_8$ be so that for $t\ > t_8$

$$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t),t\right) < \varepsilon_8, T_{40}\,(t) > (m)^{(8)}$$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right)$, $t = log\frac{2}{\varepsilon_8}$ By taking now $\varepsilon_8$ sufficiently small one sees that $T_{41}$ is unbounded.
The same property holds for $T_{42}$ if $\lim_{t\to\infty}(b''_{42})^{(8)}\left((G_{43})(t), t(t), t\right) = (b'_{42})^{(8)}$



It is now sufficient to take $\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}$ , $\frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$ and to choose $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ large to have

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}\left[(\hat{P}_{44})^{(9)} + \left((\hat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}}\left[\left((\hat{Q}_{44})^{(9)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{44})^{(9)}\right] \leq (\hat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions $G_i$ , $T_i$ satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)}, (T_{47})^{(1)}), ((G_{47})^{(2)}, (T_{47})^{(2)})\right) =$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{44})^{(9)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

**Definition of** $(\widetilde{G_{47}}), (\widetilde{T_{47}}) : \left((\widetilde{G_{47}}), (\widetilde{T_{47}})\right) = \mathcal{A}^{(9)}\left((G_{47}), (T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{44})^{(9)}\left|G_{45}^{(1)} - G_{45}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\,ds_{(44)} +$$
$$\int_0^t\{(a'_{44})^{(9)}\left|G_{44}^{(1)} - G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{-(\widehat{M}_{44})^{(9)}s_{(44)}} +$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right)\left|G_{44}^{(1)} - G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}} +$$
$$G_{44}^{(2)}\left|(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right) - (a''_{44})^{(9)}\left(T_{45}^{(2)}, s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\}ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)} - G^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}t} \leq$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)} + (a'_{44})^{(9)} + (\widehat{A}_{44})^{(9)} + (\widehat{P}_{44})^{(9)}(\hat{k}_{44})^{(9)}\right)d\left(((G_{47})^{(1)}, (T_{47})^{(1)};\ (G_{47})^{(2)}, (T_{47})^{(2)})\right)$$

And analogous inequalities for $G_i$ *and* $T_i$. Taking into account the hypothesis (39,35,36) the result follows

**Remark 41:** The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by

$(\widehat{P}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ $and$ $(\widehat{Q}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i = 44{,}45{,}46$ depend only on $T_{45}$ and respectively on $(G_{47})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

**Remark 42:** There does not exist any $t$ where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$

From 99 to 44 it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t\{(a_i')^{(9)}-(a_i'')^{(9)}(T_{45}(s_{(44)}),s_{(44)})\}ds_{(44)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(9)}t)} > 0 \quad \text{for t} > 0$$

**Definition of** $\left((\widehat{M}_{44})^{(9)}\right)_1, \left((\widehat{M}_{44})^{(9)}\right)_2\ and\ \left((\widehat{M}_{44})^{(9)}\right)_3$ :

**Remark 43:** if $G_{44}$ is bounded, the same property have also $G_{45}\ and\ G_{46}$ . indeed if

$G_{44} < (\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt} \leq \left((\widehat{M}_{44})^{(9)}\right)_1 - (a_{45}')^{(9)}G_{45}$ and by integrating

$$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_1/(a_{45}')^{(9)}$$

In the same way , one can obtain

$$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_2/(a_{46}')^{(9)}$$

If $G_{45}\ or\ G_{46}$ is bounded, the same property follows for $G_{44}$ , $G_{46}$ and $G_{44}$ , $G_{45}$ respectively.

**Remark 44:** If $G_{44}$ $is$ bounded, from below, the same property holds for $G_{45}\ and\ G_{46}$ . The proof is analogous with the preceding one. An analogous property is true if $G_{45}$ is bounded from below.

**Remark 45:** If $T_{44}$ is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t),t\right) = (b_{45}')^{(9)}$ then $T_{45} \to \infty$.

**Definition of** $(m)^{(9)}$ and $\varepsilon_9$ :

Indeed let $t_9$ be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t),t\right) < \varepsilon_9, T_{44}\,(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right), \quad t = log\frac{2}{\varepsilon_9}$ By taking now $\varepsilon_9$ sufficiently small one sees that $T_{45}$ is unbounded.
The same property holds for $T_{46}$ if $\lim_{t\to\infty}(b_{46}'')^{(9)}\left((G_{47})(t),t\right) = (b_{46}')^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

**Behavior of the solutions of equation** 280

**Theorem** If we denote and define

**Definition of** $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ :

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)} \le -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14}, t) + (a''_{14})^{(1)}(T_{14}, t) \le -(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)} \le -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G, t) - (b''_{14})^{(1)}(G, t) \le -(\tau_1)^{(1)}$

**Definition of** $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$ : 281

By $(\nu_1)^{(1)} > 0$, $(\nu_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$, $(u_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

**Definition of** $(\bar{\nu}_1)^{(1)},, (\bar{\nu}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$ : 282

By $(\bar{\nu}_1)^{(1)} > 0$, $(\bar{\nu}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$, $(\bar{u}_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

**Definition of** $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (\nu_0)^{(1)}$ :- 283

If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by
$(m_2)^{(1)} = (\nu_0)^{(1)}, (m_1)^{(1)} = (\nu_1)^{(1)}$, $if\ (\nu_0)^{(1)} < (\nu_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}, if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)}$,

and $\boxed{(\nu_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)}$, $if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)}$, $if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)}$,

and $\boxed{(u_0)^{(1)} = \frac{T^0_{13}}{T^0_{14}}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$

are defined

Then the solution of global equations satisfies the inequalities 285

$G^0_{13}e^{\left((S_1)^{(1)} - (p_{13})^{(1)}\right)t} \le G_{13}(t) \le G^0_{13}e^{(S_1)^{(1)}t}$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)}-(p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)}t}$$

$$\left(\frac{(a_{15})^{(1)}G_{13}^0}{(m_1)^{(1)}((S_1)^{(1)}-(p_{13})^{(1)}-(S_2)^{(1)})}\left[e^{((S_1)^{(1)}-(p_{13})^{(1)})t} - e^{-(S_2)^{(1)}t}\right] + G_{15}^0 e^{-(S_2)^{(1)}t} \le G_{15}(t) \le \right. \quad (286)$$

$$\left.\frac{(a_{15})^{(1)}G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)}-(a'_{15})^{(1)})}[e^{(S_1)^{(1)}t} - e^{-(a'_{15})^{(1)}t}] + G_{15}^0 e^{-(a'_{15})^{(1)}t}\right)$$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)}t} \le T_{13}(t) \le T_{13}^0 e^{((R_1)^{(1)}+(r_{13})^{(1)})t}} \quad (287)$$

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)}t} \le T_{13}(t) \le \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)}+(r_{13})^{(1)})t} \quad (288)$$

$$\frac{(b_{15})^{(1)}T_{13}^0}{(\mu_1)^{(1)}((R_1)^{(1)}-(b'_{15})^{(1)})}\left[e^{(R_1)^{(1)}t} - e^{-(b'_{15})^{(1)}t}\right] + T_{15}^0 e^{-(b'_{15})^{(1)}t} \le T_{15}(t) \le \quad (289)$$

$$\frac{(a_{15})^{(1)}T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)}+(r_{13})^{(1)}+(R_2)^{(1)})}\left[e^{((R_1)^{(1)}+(r_{13})^{(1)})t} - e^{-(R_2)^{(1)}t}\right] + T_{15}^0 e^{-(R_2)^{(1)}t}$$

**Definition of** $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:- (290)

Where $(S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$

$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$

$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$

$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$

**Behavior of the solutions of equation** (291)

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ : (292)

$(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ four constants satisfying

$$-(\sigma_2)^{(2)} \le -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(T_{17}, t) + (a''_{17})^{(2)}(T_{17}, t) \le -(\sigma_1)^{(2)} \quad (293)$$

$$-(\tau_2)^{(2)} \le -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}((G_{19}), t) - (b''_{17})^{(2)}((G_{19}), t) \le -(\tau_1)^{(2)} \quad (294)$$

**Definition of** $(\nu_1)^{(2)}, (\nu_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$ : (295)

By $(\nu_1)^{(2)} > 0$, $(\nu_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0$, $(u_2)^{(2)} < 0$ the roots (296)

of the equations $(a_{17})^{(2)}(\nu^{(2)})^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ (297)

and $(b_{14})^{(2)}(u^{(2)})^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and (298)

**Definition of** $(\bar{\nu}_1)^{(2)}, , (\bar{\nu}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$ : (299)

By $(\bar{\nu}_1)^{(2)} > 0$, $(\bar{\nu}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0$, $(\bar{u}_2)^{(2)} < 0$ the (300)

roots of the equations $(a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 301

and $(b_{17})^{(2)}\left(u^{(2)}\right)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 302

**Definition of** $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ :- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, ***if*** $(u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$, ***if*** $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)}$, ***if*** $(\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)}t}$

$(p_i)^{(2)}$ is defined by equation

$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)}t}$ 311

$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)}t}\right] + G_{18}^0 e^{-(S_2)^{(2)}t} \le G_{18}(t) \le\right.$ 312

$\left.\frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)}t} - e^{-(a'_{18})^{(2)}t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)}t}\right)$

$\boxed{T_{16}^0 e^{(R_1)^{(2)}t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}}$ 313

$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)}t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}$ 314

$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)}t} - e^{-(b'_{18})^{(2)}t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)}t} \le T_{18}(t) \le$ 315

$$\frac{(a_{18})^{(2)}T_{16}^{0}}{(\mu_2)^{(2)}((R_1)^{(2)}+(r_{16})^{(2)}+(R_2)^{(2)})}\left[e^{((R_1)^{(2)}+(r_{16})^{(2)})t}-e^{-(R_2)^{(2)}t}\right]+T_{18}^{0}e^{-(R_2)^{(2)}t}$$

**Definition of** $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$$

$$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)}$$ 318

$$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$$

**Behavior of the solutions** 319

**Theorem 3:** If we denote and define

**Definition of** $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ :

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}\big((G_{23}), t\big) \le -(\tau_1)^{(3)}$$

**Definition of** $(v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$ : 320

By $(v_1)^{(3)} > 0$, $(v_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{v}_1)^{(3)} > 0$, $(\bar{v}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

**Definition of** $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ :- 321

If we define $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (v_0)^{(3)}, (m_1)^{(3)} = (v_1)^{(3)}$, ***if*** $(v_0)^{(3)} < (v_1)^{(3)}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (\bar{v}_1)^{(3)}$, ***if*** $(v_1)^{(3)} < (v_0)^{(3)} < (\bar{v}_1)^{(3)}$,

and $\boxed{(v_0)^{(3)} = \frac{G_{20}^{0}}{G_{21}^{0}}}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_0)^{(3)}$, ***if*** $(\bar{v}_1)^{(3)} < (v_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)},\ \boldsymbol{if}\ (u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}, \boldsymbol{if}\ (u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)},$ and $\boxed{(u_0)^{(3)} = \frac{T_{20}^0}{T_{21}^0}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)},\ \boldsymbol{if}\ (\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G_{20}^0 e^{(S_1)^{(3)} t}$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)} t} \quad 323$$

$$\left(\frac{(a_{22})^{(3)} G_{20}^0}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\left[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)} t}\right] + G_{22}^0 e^{-(S_2)^{(3)} t} \le G_{22}(t) \le \right. \quad 324$$

$$\left. \frac{(a_{22})^{(3)} G_{20}^0}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}\left[e^{(S_1)^{(3)} t} - e^{-(a'_{22})^{(3)} t}\right] + G_{22}^0 e^{-(a'_{22})^{(3)} t}\right)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t}} \quad 325$$

$$\frac{1}{(\mu_1)^{(3)}} T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}} T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)} T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)} - (b'_{22})^{(3)})}\left[e^{(R_1)^{(3)} t} - e^{-(b'_{22})^{(3)} t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)} t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)} T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)} + (r_{20})^{(3)} + (R_2)^{(3)})}\left[e^{((R_1)^{(3)} + (r_{20})^{(3)})t} - e^{-(R_2)^{(3)} t}\right] + T_{22}^0 e^{-(R_2)^{(3)} t}$$

**Definition of** $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

**Behavior of the solutions of equation**

**Theorem:** If we denote and define

**Definition of** $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ :

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$

$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}((G_{27}), t) - (b''_{25})^{(4)}((G_{27}), t) \le -(\tau_1)^{(4)}$

**Definition of** $(\nu_1)^{(4)}, (\nu_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, \nu^{(4)}, u^{(4)}$ : 329

By $(\nu_1)^{(4)} > 0$ , $(\nu_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$ , $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_1)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(4)},, (\bar{\nu}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$ : 330

By $(\bar{\nu}_1)^{(4)} > 0$ , $(\bar{\nu}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$ , $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

**Definition of** $(m_1)^{(4)}$ , $(m_2)^{(4)}$ , $(\mu_1)^{(4)}, (\mu_2)^{(4)}, (\nu_0)^{(4)}$ :-

If we define $(m_1)^{(4)}$ , $(m_2)^{(4)}$ , $(\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (\nu_0)^{(4)}, (m_1)^{(4)} = (\nu_1)^{(4)}$, $\boldsymbol{if}\ (\nu_0)^{(4)} < (\nu_1)^{(4)}$

$(m_2)^{(4)} = (\nu_1)^{(4)}, (m_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ , $\boldsymbol{if}\ (\nu_4)^{(4)} < (\nu_0)^{(4)} < (\bar{\nu}_1)^{(4)}$,
and $\boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (\nu_4)^{(4)}, (m_1)^{(4)} = (\nu_0)^{(4)}$, $\boldsymbol{if}\ (\bar{\nu}_4)^{(4)} < (\nu_0)^{(4)}$

and analogously

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$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)}$, $\boldsymbol{if}\ (u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}$ , $\boldsymbol{if}\ (u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)}$,
and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}, \boldsymbol{if}\ (\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

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$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)}t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)}t}$$

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$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)}t}\right] + G_{26}^0 e^{-(S_2)^{(4)}t} \le G_{26}(t) \le\right.$$ 334

*(a26)4G240(m2)4(S1)4−(a26′)4e(S1)4t−e−(a26′)4t+ G260e−(a26′)4t*

$$\boxed{T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}}T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}}T_{24}^0 e^{((R_1)^{(4)}+(r_{24})^{(4)})t} \quad 335$$

$$\frac{(b_{26})^{(4)}T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)}-(b'_{26})^{(4)})}\left[e^{(R_1)^{(4)}t} - e^{-(b'_{26})^{(4)}t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)}t} \le T_{26}(t) \le \quad 336$$

$$\frac{(a_{26})^{(4)}T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)}+(r_{24})^{(4)}+(R_2)^{(4)})}\left[e^{((R_1)^{(4)}+(r_{24})^{(4)})t} - e^{-(R_2)^{(4)}t}\right] + T_{26}^0 e^{-(R_2)^{(4)}t}$$

**Definition of** $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

**Behavior of the solutions of equation** 338

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ :

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

**Definition of** $(v_1)^{(5)}, (v_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, v^{(5)}, u^{(5)}$ : 339

By $(v_1)^{(5)} > 0$ , $(v_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$ , $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(v^{(5)}\big)^2 + (\sigma_1)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

**Definition of** $(\bar{v}_1)^{(5)}, , (\bar{v}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$ : 340

By $(\bar{v}_1)^{(5)} > 0$ , $(\bar{v}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$ , $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(v^{(5)}\big)^2 + (\sigma_2)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

**Definition of** $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (v_0)^{(5)}$ :-

If we define $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (v_0)^{(5)}, (m_1)^{(5)} = (v_1)^{(5)}$, ***if*** $(v_0)^{(5)} < (v_1)^{(5)}$

$(m_2)^{(5)} = (v_1)^{(5)}, (m_1)^{(5)} = (\bar{v}_1)^{(5)}$ , ***if*** $(v_1)^{(5)} < (v_0)^{(5)} < (\bar{v}_1)^{(5)}$,
and $\boxed{(v_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)},\ \boldsymbol{if}\ (u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}, \boldsymbol{if}\ (u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)},$

and $\boxed{(u_0)^{(5)} = \frac{T_{28}^0}{T_{29}^0}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}, \boldsymbol{if}\ (\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{28}(t) \le G_{28}^0 e^{(S_1)^{(5)} t}$

where $(p_i)^{(5)}$ is defined by equation

$\frac{1}{(m_5)^{(5)}} G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}} G_{28}^0 e^{(S_1)^{(5)} t}$ 343

$\left(\frac{(a_{30})^{(5)} G_{28}^0}{(m_1)^{(5)}((S_1)^{(5)} - (p_{28})^{(5)} - (S_2)^{(5)})}\left[e^{((S_1)^{(5)} - (p_{28})^{(5)})t} - e^{-(S_2)^{(5)} t}\right] + G_{30}^0 e^{-(S_2)^{(5)} t} \le G_{30}(t) \le\right.$ 344

*(a30)5G280(m2)5(S1)5−(a30′)5e(S1)5t−e−(a30′)5t+ G300e−(a30′)5t*

$\boxed{T_{28}^0 e^{(R_1)^{(5)} t} \le T_{28}(t) \le T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}}$ 345

$\frac{1}{(\mu_1)^{(5)}} T_{28}^0 e^{(R_1)^{(5)} t} \le T_{28}(t) \le \frac{1}{(\mu_2)^{(5)}} T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}$ 346

$\frac{(b_{30})^{(5)} T_{28}^0}{(\mu_1)^{(5)}((R_1)^{(5)} - (b'_{30})^{(5)})}\left[e^{(R_1)^{(5)} t} - e^{-(b'_{30})^{(5)} t}\right] + T_{30}^0 e^{-(b'_{30})^{(5)} t} \le T_{30}(t) \le$ 347

$\frac{(a_{30})^{(5)} T_{28}^0}{(\mu_2)^{(5)}((R_1)^{(5)} + (r_{28})^{(5)} + (R_2)^{(5)})}\left[e^{((R_1)^{(5)} + (r_{28})^{(5)})t} - e^{-(R_2)^{(5)} t}\right] + T_{30}^0 e^{-(R_2)^{(5)} t}$

**Definition of** $(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$:- 348

Where $(S_1)^{(5)} = (a_{28})^{(5)}(m_2)^{(5)} - (a'_{28})^{(5)}$

$(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}$

$(R_1)^{(5)} = (b_{28})^{(5)}(\mu_2)^{(5)} - (b'_{28})^{(5)}$

$(R_2)^{(5)} = (b'_{30})^{(5)} - (r_{30})^{(5)}$

**Behavior of the solutions of equation** 349

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ :

$(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)} \le -(a'_{32})^{(6)} + (a'_{33})^{(6)} - (a''_{32})^{(6)}(T_{33}, t) + (a''_{33})^{(6)}(T_{33}, t) \le -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)} \le -(b'_{32})^{(6)} + (b'_{33})^{(6)} - (b''_{32})^{(6)}\big((G_{35}), t\big) - (b''_{33})^{(6)}\big((G_{35}), t\big) \le -(\tau_1)^{(6)}$$

**Definition of** $(\nu_1)^{(6)}, (\nu_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, \nu^{(6)}, u^{(6)}$ : 350

By $(\nu_1)^{(6)} > 0$, $(\nu_2)^{(6)} < 0$ and respectively $(u_1)^{(6)} > 0$, $(u_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(6)},, (\bar{\nu}_2)^{(6)}, (\bar{u}_1)^{(6)}, (\bar{u}_2)^{(6)}$ : 351

By $(\bar{\nu}_1)^{(6)} > 0$, $(\bar{\nu}_2)^{(6)} < 0$ and respectively $(\bar{u}_1)^{(6)} > 0$, $(\bar{u}_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(\nu^{(6)}\big)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$

**Definition of** $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}, (\nu_0)^{(6)}$ :-

If we define $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}$ by

$$(m_2)^{(6)} = (\nu_0)^{(6)}, (m_1)^{(6)} = (\nu_1)^{(6)}, \ \boldsymbol{if}\ (\nu_0)^{(6)} < (\nu_1)^{(6)}$$

$$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}, \boldsymbol{if}\ (\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)},$$

and $\boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}}$

$$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$$

and analogously 352

$$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}, \ \boldsymbol{if}\ (u_0)^{(6)} < (u_1)^{(6)}$$

$$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}, \boldsymbol{if}\ (u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)},$$

and $\boxed{(u_0)^{(6)} = \frac{T^0_{32}}{T^0_{33}}}$

$$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}, \boldsymbol{if}\ (\bar{u}_1)^{(6)} < (u_0)^{(6)} \text{ where } (u_1)^{(6)}, (\bar{u}_1)^{(6)}$$

Then the solution of global equations satisfies the inequalities 353

$$G^0_{32}e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{32}(t) \le G^0_{32}e^{(S_1)^{(6)}t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}}G^0_{32}e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{33}(t) \le \frac{1}{(m_2)^{(6)}}G^0_{32}e^{(S_1)^{(6)}t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)}G^0_{32}}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})}\left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)}t}\right] + G^0_{34}e^{-(S_2)^{(6)}t} \le G_{34}(t) \le \right. \quad 355$$

$$\frac{(a_{34})^{(6)}G_{32}^{0}}{(m_2)^{(6)}((S_1)^{(6)}-(a'_{34})^{(6)})}\left[e^{(S_1)^{(6)}t}-e^{-(a'_{34})^{(6)}t}\right]+\ G_{34}^{0}e^{-(a'_{34})^{(6)}t}\Big)$$

$$\boxed{T_{32}^{0}e^{(R_1)^{(6)}t}\le T_{32}(t)\le T_{32}^{0}e^{((R_1)^{(6)}+(r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}}T_{32}^{0}e^{(R_1)^{(6)}t}\le T_{32}(t)\le\frac{1}{(\mu_2)^{(6)}}T_{32}^{0}e^{((R_1)^{(6)}+(r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)}T_{32}^{0}}{(\mu_1)^{(6)}((R_1)^{(6)}-(b'_{34})^{(6)})}\left[e^{(R_1)^{(6)}t}-e^{-(b'_{34})^{(6)}t}\right]+T_{34}^{0}e^{-(b'_{34})^{(6)}t}\le T_{34}(t)\le \quad 358$$

$$\frac{(a_{34})^{(6)}T_{32}^{0}}{(\mu_2)^{(6)}((R_1)^{(6)}+(r_{32})^{(6)}+(R_2)^{(6)})}\left[e^{((R_1)^{(6)}+(r_{32})^{(6)})t}-e^{-(R_2)^{(6)}t}\right]+T_{34}^{0}e^{-(R_2)^{(6)}t}$$

**Definition of** $(S_1)^{(6)},(S_2)^{(6)},(R_1)^{(6)},(R_2)^{(6)}$:- 359

Where $(S_1)^{(6)}=(a_{32})^{(6)}(m_2)^{(6)}-(a'_{32})^{(6)}$

$(S_2)^{(6)}=(a_{34})^{(6)}-(p_{34})^{(6)}$

$(R_1)^{(6)}=(b_{32})^{(6)}(\mu_2)^{(6)}-(b'_{32})^{(6)}$

$(R_2)^{(6)}=(b'_{34})^{(6)}-(r_{34})^{(6)}$

**Behavior of the solutions of equation**

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(7)},(\sigma_2)^{(7)},(\tau_1)^{(7)},(\tau_2)^{(7)}$:

$(\sigma_1)^{(7)},(\sigma_2)^{(7)},(\tau_1)^{(7)},(\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)}\le-(a'_{36})^{(7)}+(a'_{37})^{(7)}-(a''_{36})^{(7)}(T_{37},t)+(a''_{37})^{(7)}(T_{37},t)\le-(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)}\le-(b'_{36})^{(7)}+(b'_{37})^{(7)}-(b''_{36})^{(7)}\big((G_{39}),t\big)-(b''_{37})^{(7)}\big((G_{39}),t\big)\le-(\tau_1)^{(7)}$$

**Definition of** $(v_1)^{(7)},(v_2)^{(7)},(u_1)^{(7)},(u_2)^{(7)},v^{(7)},u^{(7)}$: 361

By $(v_1)^{(7)}>0,(v_2)^{(7)}<0$ and respectively $(u_1)^{(7)}>0,(u_2)^{(7)}<0$ the roots of the equations $(a_{37})^{(7)}\big(v^{(7)}\big)^2+(\sigma_1)^{(7)}v^{(7)}-(a_{36})^{(7)}=0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2+(\tau_1)^{(7)}u^{(7)}-(b_{36})^{(7)}=0$ and

**Definition of** $(\bar{v}_1)^{(7)},,(\bar{v}_2)^{(7)},(\bar{u}_1)^{(7)},(\bar{u}_2)^{(7)}$: 362

By $(\bar{v}_1)^{(7)}>0,(\bar{v}_2)^{(7)}<0$ and respectively $(\bar{u}_1)^{(7)}>0,(\bar{u}_2)^{(7)}<0$ the roots of the equations $(a_{37})^{(7)}\big(v^{(7)}\big)^2+(\sigma_2)^{(7)}v^{(7)}-(a_{36})^{(7)}=0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2+(\tau_2)^{(7)}u^{(7)}-(b_{36})^{(7)}=0$

**Definition of** $(m_1)^{(7)},(m_2)^{(7)},(\mu_1)^{(7)},(\mu_2)^{(7)},(v_0)^{(7)}$:-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$$(m_2)^{(7)} = (\nu_0)^{(7)}, (m_1)^{(7)} = (\nu_1)^{(7)}, \ \boldsymbol{if}\ (\nu_0)^{(7)} < (\nu_1)^{(7)}$$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\bar{\nu}_1)^{(7)}, \boldsymbol{if}\ (\nu_1)^{(7)} < (\nu_0)^{(7)} < (\bar{\nu}_1)^{(7)},$$

and $\boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\nu_0)^{(7)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(7)} < (\nu_0)^{(7)}$$

and analogously 363

$$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}, \ \boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$$

and $\boxed{(u_0)^{(7)} = \frac{T_{36}^0}{T_{37}^0}}$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)} \text{ where } (u_1)^{(7)}, (\bar{u}_1)^{(7)}$$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^0 e^{(S_1)^{(7)} t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)} t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)}((S_1)^{(7)} - (p_{36})^{(7)} - (S_2)^{(7)})}\left[e^{((S_1)^{(7)} - (p_{36})^{(7)})t} - e^{-(S_2)^{(7)} t}\right] + G_{38}^0 e^{-(S_2)^{(7)} t} \le G_{38}(t) \le \right. \quad 366$$

$$\left. \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)}((S_1)^{(7)} - (a'_{38})^{(7)})}\left[e^{(S_1)^{(7)} t} - e^{-(a'_{38})^{(7)} t}\right] + G_{38}^0 e^{-(a'_{38})^{(7)} t}\right)$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t} \quad 368$$

$$\frac{(b_{38})^{(7)} T_{36}^0}{(\mu_1)^{(7)}((R_1)^{(7)} - (b'_{38})^{(7)})}\left[e^{(R_1)^{(7)} t} - e^{-(b'_{38})^{(7)} t}\right] + T_{38}^0 e^{-(b'_{38})^{(7)} t} \le T_{38}(t) \le \quad 369$$

$$\frac{(a_{38})^{(7)} T_{36}^0}{(\mu_2)^{(7)}((R_1)^{(7)} + (r_{36})^{(7)} + (R_2)^{(7)})}\left[e^{((R_1)^{(7)} + (r_{36})^{(7)})t} - e^{-(R_2)^{(7)} t}\right] + T_{38}^0 e^{-(R_2)^{(7)} t}$$

**Definition of** $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$

$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$

$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$

**Behavior of the solutions of equation** 371

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ :

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$-(\sigma_2)^{(8)} \le -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \le -(\sigma_1)^{(8)}$

$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\big((G_{43}), t\big) - (b''_{41})^{(8)}\big((G_{43}), t\big) \le -(\tau_1)^{(8)}$

**Definition of** $(\nu_1)^{(8)}, (\nu_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, \nu^{(8)}, u^{(8)}$ : 372

By $(\nu_1)^{(8)} > 0, (\nu_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0, (u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(8)}, , (\bar{\nu}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$ :

By $(\bar{\nu}_1)^{(8)} > 0, (\bar{\nu}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0, (\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

**Definition of** $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}, (\nu_0)^{(8)}$ :-

If we define $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$(m_2)^{(8)} = (\nu_0)^{(8)}, (m_1)^{(8)} = (\nu_1)^{(8)}$, ***if*** $(\nu_0)^{(8)} < (\nu_1)^{(8)}$

$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\bar{\nu}_1)^{(8)}$, ***if*** $(\nu_1)^{(8)} < (\nu_0)^{(8)} < (\bar{\nu}_1)^{(8)}$,
and $\boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}}$

$$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\nu_0)^{(8)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(8)} < (\nu_0)^{(8)}$$

and analogously 374

$$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)},\ \boldsymbol{if}\ (u_0)^{(8)} < (u_1)^{(8)}$$

$$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}, \boldsymbol{if}\ (u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)},$$

and $\boxed{(u_0)^{(8)} = \frac{T_{40}^0}{T_{41}^0}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}, \boldsymbol{if}\ (\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^0 e^{(S_1)^{(8)} t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}} G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}} G_{40}^0 e^{(S_1)^{(8)} t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)} G_{40}^0}{(m_1)^{(8)}((S_1)^{(8)} - (p_{40})^{(8)} - (S_2)^{(8)})}\left[e^{((S_1)^{(8)} - (p_{40})^{(8)})t} - e^{-(S_2)^{(8)} t}\right] + G_{42}^0 e^{-(S_2)^{(8)} t} \le G_{42}(t) \le \right. \quad 377$$

$$\left. \frac{(a_{42})^{(8)} G_{40}^0}{(m_2)^{(8)}((S_1)^{(8)} - (a'_{42})^{(8)})}\left[e^{(S_1)^{(8)} t} - e^{-(a'_{42})^{(8)} t}\right] + G_{42}^0 e^{-(a'_{42})^{(8)} t}\right)$$

$$\boxed{T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}} T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le \frac{1}{(\mu_2)^{(8)}} T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)} T_{40}^0}{(\mu_1)^{(8)}((R_1)^{(8)} - (b'_{42})^{(8)})}\left[e^{(R_1)^{(8)} t} - e^{-(b'_{42})^{(8)} t}\right] + T_{42}^0 e^{-(b'_{42})^{(8)} t} \le T_{42}(t) \le \quad 380$$

$$\frac{(a_{42})^{(8)} T_{40}^0}{(\mu_2)^{(8)}((R_1)^{(8)} + (r_{40})^{(8)} + (R_2)^{(8)})}\left[e^{((R_1)^{(8)} + (r_{40})^{(8)})t} - e^{-(R_2)^{(8)} t}\right] + T_{42}^0 e^{-(R_2)^{(8)} t}$$

**Definition of** $(S_1)^{(8)}, (S_2)^{(8)}, (R_1)^{(8)}, (R_2)^{(8)}$:- 381

Where $(S_1)^{(8)} = (a_{40})^{(8)}(m_2)^{(8)} - (a'_{40})^{(8)}$

$$(S_2)^{(8)} = (a_{42})^{(8)} - (p_{42})^{(8)}$$

$$(R_1)^{(8)} = (b_{40})^{(8)}(\mu_2)^{(8)} - (b'_{40})^{(8)}$$

$$(R_2)^{(8)} = (b'_{42})^{(8)} - (r_{42})^{(8)}$$

**Behavior of the solutions of equation 37 to 92** 382

**Theorem 2:** If we denote and define

**Definition of** $(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ :

$(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ four constants satisfying

$$-(\sigma_2)^{(9)} \le -(a'_{44})^{(9)} + (a'_{45})^{(9)} - (a''_{44})^{(9)}(T_{45}, t) + (a''_{45})^{(9)}(T_{45}, t) \le -(\sigma_1)^{(9)}$$

$$-(\tau_2)^{(9)} \le -(b'_{44})^{(9)} + (b'_{45})^{(9)} - (b''_{44})^{(9)}\big((G_{47}), t\big) - (b''_{45})^{(9)}\big((G_{47}), t\big) \le -(\tau_1)^{(9)}$$

**Definition of** $(\nu_1)^{(9)}, (\nu_2)^{(9)}, (u_1)^{(9)}, (u_2)^{(9)}, \nu^{(9)}, u^{(9)}$ :

By $(\nu_1)^{(9)} > 0$ , $(\nu_2)^{(9)} < 0$ and respectively $(u_1)^{(9)} > 0$ , $(u_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$ and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_1)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$ and

**Definition of** $(\bar{\nu}_1)^{(9)}, , (\bar{\nu}_2)^{(9)}, (\bar{u}_1)^{(9)}, (\bar{u}_2)^{(9)}$ :

By $(\bar{\nu}_1)^{(9)} > 0$ , $(\bar{\nu}_2)^{(9)} < 0$ and respectively $(\bar{u}_1)^{(9)} > 0$ , $(\bar{u}_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$ and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_2)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$

**Definition of** $(m_1)^{(9)}, (m_2)^{(9)}, (\mu_1)^{(9)}, (\mu_2)^{(9)}, (\nu_0)^{(9)}$ :-

If we define $(m_1)^{(9)}, (m_2)^{(9)}, (\mu_1)^{(9)}, (\mu_2)^{(9)}$ by

$$(m_2)^{(9)} = (\nu_0)^{(9)}, (m_1)^{(9)} = (\nu_1)^{(9)}, \ \boldsymbol{if}\ (\nu_0)^{(9)} < (\nu_1)^{(9)}$$

$$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\bar{\nu}_1)^{(9)}, \boldsymbol{if}\ (\nu_1)^{(9)} < (\nu_0)^{(9)} < (\bar{\nu}_1)^{(9)},$$

and $\boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$

$$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\nu_0)^{(9)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(9)} < (\nu_0)^{(9)}$$

and analogously

$$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)}, \ \boldsymbol{if}\ (u_0)^{(9)} < (u_1)^{(9)}$$

$$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}, \boldsymbol{if}\ (u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)},$$

and $\boxed{(u_0)^{(9)} = \frac{T^0_{44}}{T^0_{45}}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}, \boldsymbol{if}\ (\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G^0_{44} e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{44}(t) \le G^0_{44} e^{(S_1)^{(9)} t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}}G_{44}^0 e^{((S_1)^{(9)}-(p_{44})^{(9)})t} \le G_{45}(t) \le \frac{1}{(m_2)^{(9)}}G_{44}^0 e^{(S_1)^{(9)}t}$$

$$\left(\frac{(a_{46})^{(9)}G_{44}^0}{(m_1)^{(9)}((S_1)^{(9)}-(p_{44})^{(9)}-(S_2)^{(9)})}\left[e^{((S_1)^{(9)}-(p_{44})^{(9)})t} - e^{-(S_2)^{(9)}t}\right] + G_{46}^0 e^{-(S_2)^{(9)}t} \le G_{46}(t) \le \right.$$

$$\left.\frac{(a_{46})^{(9)}G_{44}^0}{(m_2)^{(9)}((S_1)^{(9)}-(a'_{46})^{(9)})}\left[e^{(S_1)^{(9)}t} - e^{-(a'_{46})^{(9)}t}\right] + G_{46}^0 e^{-(a'_{46})^{(9)}t}\right)$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le T_{44}^0 e^{((R_1)^{(9)}+(r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}}T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le \frac{1}{(\mu_2)^{(9)}}T_{44}^0 e^{((R_1)^{(9)}+(r_{44})^{(9)})t}$$

$$\frac{(b_{46})^{(9)}T_{44}^0}{(\mu_1)^{(9)}((R_1)^{(9)}-(b'_{46})^{(9)})}\left[e^{(R_1)^{(9)}t} - e^{-(b'_{46})^{(9)}t}\right] + T_{46}^0 e^{-(b'_{46})^{(9)}t} \le T_{46}(t) \le$$

$$\frac{(a_{46})^{(9)}T_{44}^0}{(\mu_2)^{(9)}((R_1)^{(9)}+(r_{44})^{(9)}+(R_2)^{(9)})}\left[e^{((R_1)^{(9)}+(r_{44})^{(9)})t} - e^{-(R_2)^{(9)}t}\right] + T_{46}^0 e^{-(R_2)^{(9)}t}$$

**Definition of** $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$

**Proof :** From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14},t)\right) - (a''_{14})^{(1)}(T_{14},t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

**Definition of** $v^{(1)}$ :- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)}\right) \le \frac{dv^{(1)}}{dt} \le -\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)}\right)$$

From which one obtains

**Definition of** $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$ :-

For $0 < \boxed{(v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}} < (v_1)^{(1)} < (\bar{v}_1)^{(1)}$

$$v^{(1)}(t) \ge \frac{(v_1)^{(1)}+(C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(v_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(v_2)^{(1)}}}$$

it follows $(\nu_0)^{(1)} \leq \nu^{(1)}(t) \leq (\nu_1)^{(1)}$

In the same manner , we get 384

$$\nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{\nu}_1)^{(1)}-(\nu_0)^{(1)}}{(\nu_0)^{(1)}-(\bar{\nu}_2)^{(1)}}}$$

From which we deduce $(\nu_0)^{(1)} \leq \nu^{(1)}(t) \leq (\bar{\nu}_1)^{(1)}$

If $0 < (\nu_1)^{(1)} < (\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0} < (\bar{\nu}_1)^{(1)}$ we find like in the previous case, 385

$$(\nu_1)^{(1)} \leq \frac{(\nu_1)^{(1)}+(C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}} \leq \nu^{(1)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\bar{\nu}_1)^{(1)}$$

If $0 < (\nu_1)^{(1)} \leq (\bar{\nu}_1)^{(1)} \leq \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$ , we obtain 386

$$(\nu_1)^{(1)} \leq \nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\nu_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(1)}(t)$ :-

$$(m_2)^{(1)} \leq \nu^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{\nu^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(1)}(t)$ :-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{13})^{(1)} = (a''_{14})^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)}G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)}T_{14}(t)$ This is an important

consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

**Proof** : From global equations we obtain 387

$$\frac{d\nu^{(2)}}{dt} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right) - (a''_{17})^{(2)}(T_{17}, t)\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

**Definition of** $\nu^{(2)}$ :- $\boxed{\nu^{(2)} = \frac{G_{16}}{G_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \le \frac{d\nu^{(2)}}{dt} \le -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

**Definition of** $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$ :-

For $0 < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \ge \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}}{1 + (C)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(C)^{(2)} = \frac{(\nu_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \le \nu^{(2)}(t) \le (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \le \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \le \nu^{(2)}(t) \le (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \le \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}}{1 + (C)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_2)^{(2)}\right)t\right]}} \le \nu^{(2)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \le (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \le (\bar{\nu}_1)^{(2)} \le (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$ , we obtain 394

$$(\nu_1)^{(2)} \le \nu^{(2)}(t) \le \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)}\right)t\right]}} \le (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(2)}(t)$ :- 395

$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}$, $\boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$

In a completely analogous way, we obtain 396

**Definition of** $u^{(2)}(t)$ :-

$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}$, $\boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :** 397

If $(a_{16}'')^{(2)} = (a_{17}'')^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)} G_{17}(t)$

Analogously if $(b_{16}'')^{(2)} = (b_{17}'')^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)} T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

**Proof** : From global equations we obtain 398

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a_{20}')^{(3)} - (a_{21}')^{(3)} + (a_{20}'')^{(3)}(T_{21}, t)\right) - (a_{21}'')^{(3)}(T_{21}, t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

**Definition of** $\nu^{(3)}$ :- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right) \leq \frac{d\nu^{(3)}}{dt} \leq -\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right)$$

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From which one obtains

For $0 < (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (\nu_1)^{(3)} < (\bar{\nu}_1)^{(3)}$

$$\nu^{(3)}(t) \geq \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(\nu_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\nu_2)^{(3)}}}$$

it follows $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\nu_1)^{(3)}$

In the same manner , we get 401

$$\nu^{(3)}(t) \le \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{\nu}_1)^{(3)}-(\nu_0)^{(3)}}{(\nu_0)^{(3)}-(\bar{\nu}_2)^{(3)}}}$$

**Definition of** $(\bar{\nu}_1)^{(3)}$ :-

From which we deduce $(\nu_0)^{(3)} \le \nu^{(3)}(t) \le (\bar{\nu}_1)^{(3)}$

If $0 < (\nu_1)^{(3)} < (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (\bar{\nu}_1)^{(3)}$ we find like in the previous case, 402

$$(\nu_1)^{(3)} \le \frac{(\nu_1)^{(3)}+(C)^{(3)}(\nu_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}} \le \nu^{(3)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \le (\bar{\nu}_1)^{(3)}$$

If $0 < (\nu_1)^{(3)} \le (\bar{\nu}_1)^{(3)} \le (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}$ , we obtain 403

$$(\nu_1)^{(3)} \le \nu^{(3)}(t) \le \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \le (\nu_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(3)}(t)$ :-

$$(m_2)^{(3)} \le \nu^{(3)}(t) \le (m_1)^{(3)}, \quad \boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(3)}(t)$ :-

$$(\mu_2)^{(3)} \le u^{(3)}(t) \le (\mu_1)^{(3)}, \quad \boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{20}'')^{(3)} = (a_{21}'')^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)}G_{21}(t)$

Analogously if $(b_{20}'')^{(3)} = (b_{21}'')^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)}T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

**Proof** : From global equations we obtain 404

$$\frac{dv^{(4)}}{dt} = (a_{24})^{(4)} - \left((a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right) - (a''_{25})^{(4)}(T_{25},t)v^{(4)} - (a_{25})^{(4)}v^{(4)}$$

**Definition of** $v^{(4)}$ :- $\boxed{v^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)}\right) \le \frac{dv^{(4)}}{dt} \le -\left((a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_4)^{(4)}v^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$ :-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \ge \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \le \nu^{(4)}(t) \le (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \le \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \le \nu^{(4)}(t) \le (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \le \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}}{1 + (C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}} \le \nu^{(4)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \le (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \le (\bar{\nu}_1)^{(4)} \le \boxed{(\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}}}$ , we obtain 407

$$(\nu_1)^{(4)} \le \nu^{(4)}(t) \le \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \le (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(4)}(t)$ :-

$$(m_2)^{(4)} \le \nu^{(4)}(t) \le (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(4)}(t)$ :-

$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}$, $\boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{24})^{(4)} = (a''_{25})^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)} G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b''_{24})^{(4)} = (b''_{25})^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)} T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

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**Proof** : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a'_{28})^{(5)} - (a'_{29})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right) - (a''_{29})^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

**Definition of** $\nu^{(5)}$ :- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$ :-

For $0 < \boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \le \frac{(\nu_1)^{(5)}+(C)^{(5)}(\nu_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)}-(\nu_2)^{(5)}\right)t\right]}}{1+(C)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)}-(\nu_2)^{(5)}\right)t\right]}} \le \nu^{(5)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \le (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \le (\bar{\nu}_1)^{(5)} \le \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain 411

$$(\nu_1)^{(5)} \le \nu^{(5)}(t) \le \frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \le (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(5)}(t)$ :-

$(m_2)^{(5)} \le \nu^{(5)}(t) \le (m_1)^{(5)}$, $\boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$

In a completely analogous way, we obtain

**Definition of** $u^{(5)}(t)$ :-

$(\mu_2)^{(5)} \le u^{(5)}(t) \le (\mu_1)^{(5)}$, $\boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a_{28}'')^{(5)} = (a_{29}'')^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)}G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b_{28}'')^{(5)} = (b_{29}'')^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)}T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

**Proof** : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a_{32}')^{(6)} - (a_{33}')^{(6)} + (a_{32}'')^{(6)}(T_{33},t)\right) - (a_{33}'')^{(6)}(T_{33},t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

**Definition of** $\nu^{(6)}$ :- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \le \frac{d\nu^{(6)}}{dt} \le -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$ :-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \geq \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\nu_1)^{(6)}$

In the same manner , we get 413

$$\nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{\nu}_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\bar{\nu}_1)^{(6)}$

If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 414

$$(\nu_1)^{(6)} \leq \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}} \leq \nu^{(6)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\bar{\nu}_1)^{(6)}$$

If $0 < (\nu_1)^{(6)} \leq (\bar{\nu}_1)^{(6)} \leq \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$ , we obtain 415

$$(\nu_1)^{(6)} \leq \nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(6)}(t)$ :-

$$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}, \quad \boxed{\nu^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(6)}(t)$ :-

$$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}, \quad \boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)} G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)} T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

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**Proof** : From global equations we obtain

$$\frac{dv^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right) - (a''_{37})^{(7)}(T_{37},t)v^{(7)} - (a_{37})^{(7)}v^{(7)}$$

**Definition of** $v^{(7)}$ :- $\boxed{v^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_2)^{(7)}v^{(7)} - (a_{36})^{(7)}\right) \le \frac{dv^{(7)}}{dt} \le -\left((a_{37})^{(7)}\left(v^{(7)}\right)^2 + (\sigma_1)^{(7)}v^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

**Definition of** $(\bar{v}_1)^{(7)}, (v_0)^{(7)}$ :-

For $0 < \boxed{(v_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}} < (v_1)^{(7)} < (\bar{v}_1)^{(7)}$

$$v^{(7)}(t) \ge \frac{(v_1)^{(7)} + (C)^{(7)}(v_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_0)^{(7)}\right)t\right]}}{1 + (C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(v_1)^{(7)} - (v_0)^{(7)}}{(v_0)^{(7)} - (v_2)^{(7)}}}$$

it follows $(v_0)^{(7)} \le v^{(7)}(t) \le (v_1)^{(7)}$

In the same manner , we get 417

$$v^{(7)}(t) \le \frac{(\bar{v}_1)^{(7)} + (\bar{C})^{(7)}(\bar{v}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{v}_1)^{(7)} - (v_0)^{(7)}}{(v_0)^{(7)} - (\bar{v}_2)^{(7)}}}$$

From which we deduce $(v_0)^{(7)} \le v^{(7)}(t) \le (\bar{v}_1)^{(7)}$

If $0 < (v_1)^{(7)} < (v_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}} < (\bar{v}_1)^{(7)}$ we find like in the previous case, 418

$$(v_1)^{(7)} \le \frac{(v_1)^{(7)} + (C)^{(7)}(v_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_2)^{(7)}\right)t\right]}}{1 + (C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)} - (v_2)^{(7)}\right)t\right]}} \le v^{(7)}(t) \le$$

$$\frac{(\bar{v}_1)^{(7)} + (\bar{C})^{(7)}(\bar{v}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}} \le (\bar{v}_1)^{(7)}$$

If $0 < (v_1)^{(7)} \le (\bar{v}_1)^{(7)} \le \boxed{(v_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$ , we obtain 419

$$(v_1)^{(7)} \le v^{(7)}(t) \le \frac{(\bar{v}_1)^{(7)} + (\bar{C})^{(7)}(\bar{v}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}\right)t\right]}} \le (v_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $v^{(7)}(t)$ :-

$$(m_2)^{(7)} \leq v^{(7)}(t) \leq (m_1)^{(7)}, \quad \boxed{v^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(7)}(t)$ :- 420

$$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}, \quad \boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $v^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)} G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)} T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

**Proof** : From global equations we obtain 421

$$\frac{dv^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right) - (a''_{41})^{(8)}(T_{41}, t) v^{(8)} - (a_{41})^{(8)} v^{(8)}$$

**Definition of** $v^{(8)}$ :- $\boxed{v^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_2)^{(8)} v^{(8)} - (a_{40})^{(8)}\right) \leq \frac{dv^{(8)}}{dt} \leq -\left((a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_1)^{(8)} v^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$ :-

$$\text{For } 0 < \boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$$

$$v^{(8)}(t) \geq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)}-(\nu_0)^{(8)}}{(\nu_0)^{(8)}-(\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \leq \frac{(\nu_1)^{(8)}+(C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}}{1+(C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}} \leq \nu^{(8)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \leq (\bar{\nu}_1)^{(8)} \leq \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$ , we obtain 424

$$(\nu_1)^{(8)} \leq \nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $\nu^{(8)}(t)$ :-

$$(m_2)^{(8)} \leq \nu^{(8)}(t) \leq (m_1)^{(8)}, \quad \boxed{\nu^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(8)}(t)$ :-

$$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}, \quad \boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $\nu^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)}G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then $(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)}T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

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**Proof** : From 99,20,44,22,23,44 we obtain A

$$\frac{dv^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right) - (a''_{45})^{(9)}(T_{45},t)v^{(9)} - (a_{45})^{(9)}v^{(9)}$$

**Definition of** $v^{(9)}$ :- $\boxed{v^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)}\right) \le \frac{dv^{(9)}}{dt} \le -\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_1)^{(9)}v^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

**Definition of** $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$ :-

For $0 < \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$

$$v^{(9)}(t) \ge \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \le v^{(9)}(t) \le (\nu_9)^{(9)}$

In the same manner , we get

$$v^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \le v^{(9)}(t) \le (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \le \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}}{1 + (C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}} \le v^{(9)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \le (\bar{\nu}_1)^{(9)} \le \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$ , we obtain

$$(\nu_1)^{(9)} \le v^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of** $v^{(9)}(t)$ :-

$$(m_2)^{(9)} \le v^{(9)}(t) \le (m_1)^{(9)}, \quad \boxed{v^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain

**Definition of** $u^{(9)}(t)$ :-

$(\mu_2)^{(9)} \leq u^{(9)}(t) \leq (\mu_1)^{(9)}$, $\boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

**Particular case :**

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)} G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then
$(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)} T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following 425

**Theorem :** If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on $t$ , and the conditions with the notations

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0$ ,

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$

$with\ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a''_i)^{(2)} and\ (b''_i)^{(2)}$ are independent on t , and the conditions with the notations 426

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$ 427

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 428

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$ , 429

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 430

$with\ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a''_i)^{(3)} and\ (b''_i)^{(3)}$ are independent on $t$ , and the conditions with the notations 431

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$ ,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

**Theorem :** If $(a_i'')^{(4)} and\ (b_i'')^{(4)}$ are independent on $t$ , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$ ,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(5)} and\ (b_i'')^{(5)}$ are independent on $t$ , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$ ,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

**Theorem** If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on $t$ , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$ ,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(7)} and\ (b_i'')^{(7)}$ are independent on $t$ , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$ ,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(8)} and\ (b_i'')^{(8)}$ are independent on $t$ , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$ ,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

**Theorem :** If $(a_i'')^{(9)} and\ (b_i'')^{(9)}$ are independent on $t$ , and the conditions (with the notations 436 A
45,46,27,28)

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$ ,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0$ 437

$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0$ 438

$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0$ 439

$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0$ 440

$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ]T_{14} = 0$ 441

$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0$ 442

has a unique positive solution , which is an equilibrium solution for the system

$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0$ 443

$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0$ 444

$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0$ 445

$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\ ]T_{16} = 0$ 446

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\,]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\,]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\,]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\,]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\,]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\big((G_{27})\big)\,]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\big((G_{27})\big)\,]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\big((G_{27})\big)\,]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})\,]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})\,]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})\,]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})\,]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})\,]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})\,]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0 \quad 479$$

$$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0 \quad 480$$

$$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0 \quad 481$$

$$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0 \quad 482$$

$$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0 \quad 483$$

$$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0 \quad 484$$

$$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0 \quad 484\text{A}$$

$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$

$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$

$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$

$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$

$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$

**Proof:** 485

(a) Indeed the first two equations have a nontrivial solution $G_{13}, G_{14}$ if

$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$

**Proof:** 486

(kk) Indeed the first two equations have a nontrivial solution $G_{16}, G_{17}$ if

$\mathrm{F}(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$

**Proof:** 487

(a) Indeed the first two equations have a nontrivial solution $G_{20}, G_{21}$ if

$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$

**Proof:** 488

(a) Indeed the first two equations have a nontrivial solution $G_{24}, G_{25}$ if

$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$

**Proof:** 489

(a) Indeed the first two equations have a nontrivial solution $G_{28}, G_{29}$ if

$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$

**Proof:** 490

(a) Indeed the first two equations have a nontrivial solution $G_{32}, G_{33}$ if

$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) +$

$(a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$

**Proof:** 491

(a) Indeed the first two equations have a nontrivial solution $G_{36}, G_{37}$ if

$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$

**Proof:** 492

(a) Indeed the first two equations have a nontrivial solution $G_{40}, G_{41}$ if

$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$

**Proof:** 492 A

(a) Indeed the first two equations have a nontrivial solution $G_{44}, G_{45}$ if

$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$

**Definition and uniqueness of** $T^*_{14}$ :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique $T^*_{14}$ for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

**Definition and uniqueness of** $T^*_{17}$ :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique $T^*_{17}$ for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]}$$ 495

**Definition and uniqueness of** $T^*_{21}$ :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique $T^*_{21}$ for which $f(T^*_{21}) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

**Definition and uniqueness of** $T^*_{25}$ :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that

there exists a unique $T_{25}^{*}$ for which $f(T_{25}^{*})=0$. With this value , we obtain from the three first equations

$$G_{24}=\frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}=\frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T_{25}^{*})]}$$

**Definition and uniqueness of** $T_{29}^{*}$ :- 498

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(5)}(T_{29})$ being increasing, it follows that there exists a unique $T_{29}^{*}$ for which $f(T_{29}^{*})=0$. With this value , we obtain from the three first equations

$$G_{28}=\frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}=\frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T_{29}^{*})]}$$

**Definition and uniqueness of** $T_{33}^{*}$ :- 499

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(6)}(T_{33})$ being increasing, it follows that there exists a unique $T_{33}^{*}$ for which $f(T_{33}^{*})=0$. With this value , we obtain from the three first equations

$$G_{32}=\frac{(a_{32})^{(6)}G_{33}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^{*})]} \quad , \quad G_{34}=\frac{(a_{34})^{(6)}G_{33}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^{*})]}$$

**Definition and uniqueness of** $T_{37}^{*}$ :- 500

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(7)}(T_{37})$ being increasing, it follows that there exists a unique $T_{37}^{*}$ for which $f(T_{37}^{*})=0$. With this value , we obtain from the three first equations

$$G_{36}=\frac{(a_{36})^{(7)}G_{37}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^{*})]} \quad , \quad G_{38}=\frac{(a_{38})^{(7)}G_{37}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^{*})]}$$

**Definition and uniqueness of** $T_{41}^{*}$ :- 501

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique $T_{41}^{*}$ for which $f(T_{41}^{*})=0$. With this value , we obtain from the three first equations

$$G_{40}=\frac{(a_{40})^{(8)}G_{41}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T_{41}^{*})]} \quad , \quad G_{42}=\frac{(a_{42})^{(8)}G_{41}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T_{41}^{*})]}$$

**Definition and uniqueness of** $T_{45}^{*}$ :- 501 A

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(9)}(T_{45})$ being increasing, it follows that there exists a unique $T_{45}^{*}$ for which $f(T_{45}^{*})=0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)}G_{45}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T^*_{45})]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)}G_{45}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T^*_{45})]}$$

By the same argument, the equations admit solutions $G_{13}, G_{14}$ if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right]+(b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{14}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{14}$ such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions $G_{16}, G_{17}$ if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right]+(b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ 504
is a decreasing function in $G_{17}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{14}$ such that $\varphi((G_{19})^*) = 0$

By the same argument, the equations admit solutions $G_{20}, G_{21}$ if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right]+(b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{21}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{21}$ such that $\varphi((G_{23})^*) = 0$

By the same argument, the equations admit solutions $G_{24}, G_{25}$ if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right]+(b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{25}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G^*_{25}$ such that $\varphi((G_{27})^*) = 0$

By the same argument, the equations admit solutions $G_{28}, G_{29}$ if 507

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right]+(b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{29}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that

there exists a unique $G_{29}^{*}$ such that $\varphi((G_{31})^{*}) = 0$

By the same argument, the equations admit solutions $G_{32}, G_{33}$ if 508
$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$

$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right] + (b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{33}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{33}^{*}$ such that $\varphi(G_{35}{}^{*}) = 0$

By the same argument, the equations admit solutions $G_{36}, G_{37}$ if 509
$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$
$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right] + (b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{37}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{37}^{*}$ such that $\varphi(G_{39}{}^{*}) = 0$

By the same argument, the equations admit solutions $G_{40}, G_{41}$ if 510
$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$
$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right] + (b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{41}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{41}^{*}$ such that $\varphi(G_{43}{}^{*}) = 0$

By the same argument, the equations 92,93 admit solutions $G_{44}, G_{45}$ if
$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$

$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right] + (b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in $G_{45}$ taking into account the hypothesis $\varphi(0) > 0$ , $\varphi(\infty) < 0$ it follows that there exists a unique $G_{45}^{*}$ such that $\varphi((G_{47})^{*}) = 0$

Finally we obtain the unique solution 511

$G_{14}^{*}$ given by $\varphi(G^{*}) = 0$ , $T_{14}^{*}$ given by $f(T_{14}^{*}) = 0$ and

$$G_{13}^{*} = \frac{(a_{13})^{(1)}G_{14}^{*}}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^{*})]} \quad , \quad G_{15}^{*} = \frac{(a_{15})^{(1)}G_{14}^{*}}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^{*})]}$$

$$T_{13}^{*} = \frac{(b_{13})^{(1)}T_{14}^{*}}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^{*})]} \quad , \quad T_{15}^{*} = \frac{(b_{15})^{(1)}T_{14}^{*}}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^{*})]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

$G_{17}^{*}$ given by $\varphi((G_{19})^{*}) = 0$ , $T_{17}^{*}$ given by $f(T_{17}^{*}) = 0$ and 512

$$G_{16}^{*} = \frac{(a_{16})^{(2)} G_{17}^{*}}{[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}^{*})]} \quad , \quad G_{18}^{*} = \frac{(a_{18})^{(2)} G_{17}^{*}}{[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}^{*})]} \qquad 513$$

$$T_{16}^{*} = \frac{(b_{16})^{(2)} T_{17}^{*}}{[(b'_{16})^{(2)} - (b''_{16})^{(2)}((G_{19})^{*})]} \quad , \quad T_{18}^{*} = \frac{(b_{18})^{(2)} T_{17}^{*}}{[(b'_{18})^{(2)} - (b''_{18})^{(2)}((G_{19})^{*})]} \qquad 514$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

$G_{21}^{*}$ given by $\varphi((G_{23})^{*}) = 0$ , $T_{21}^{*}$ given by $f(T_{21}^{*}) = 0$ and

$$G_{20}^{*} = \frac{(a_{20})^{(3)} G_{21}^{*}}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^{*})]} \quad , \quad G_{22}^{*} = \frac{(a_{22})^{(3)} G_{21}^{*}}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^{*})]}$$

$$T_{20}^{*} = \frac{(b_{20})^{(3)} T_{21}^{*}}{[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}{}^{*})]} \quad , \quad T_{22}^{*} = \frac{(b_{22})^{(3)} T_{21}^{*}}{[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}{}^{*})]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

$G_{25}^{*}$ given by $\varphi(G_{27}) = 0$ , $T_{25}^{*}$ given by $f(T_{25}^{*}) = 0$ and

$$G_{24}^{*} = \frac{(a_{24})^{(4)} G_{25}^{*}}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}^{*} = \frac{(a_{26})^{(4)} G_{25}^{*}}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^{*})]}$$

$$T_{24}^{*} = \frac{(b_{24})^{(4)} T_{25}^{*}}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^{*})]} \quad , \quad T_{26}^{*} = \frac{(b_{26})^{(4)} T_{25}^{*}}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^{*})]} \qquad 517$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

$G_{29}^{*}$ given by $\varphi((G_{31})^{*}) = 0$ , $T_{29}^{*}$ given by $f(T_{29}^{*}) = 0$ and

$$G_{28}^{*} = \frac{(a_{28})^{(5)} G_{29}^{*}}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}^{*} = \frac{(a_{30})^{(5)} G_{29}^{*}}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^{*})]}$$

$$T_{28}^{*} = \frac{(b_{28})^{(5)} T_{29}^{*}}{[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31})^{*})]} \quad , \quad T_{30}^{*} = \frac{(b_{30})^{(5)} T_{29}^{*}}{[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31})^{*})]} \qquad 519$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

$G_{33}^{*}$ given by $\varphi((G_{35})^{*}) = 0$ , $T_{33}^{*}$ given by $f(T_{33}^{*}) = 0$ and

$$G_{32}^{*} = \frac{(a_{32})^{(6)} G_{33}^{*}}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^{*})]} \quad , \quad G_{34}^{*} = \frac{(a_{34})^{(6)} G_{33}^{*}}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^{*})]}$$

$$T_{32}^{*} = \frac{(b_{32})^{(6)} T_{33}^{*}}{[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35})^{*})]} \quad , \quad T_{34}^{*} = \frac{(b_{34})^{(6)} T_{33}^{*}}{[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35})^{*})]} \qquad 521$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

$G_{37}^*$ given by $\varphi((G_{39})^*) = 0$ , $T_{37}^*$ given by $f(T_{37}^*) = 0$ and

$$G_{36}^* = \frac{(a_{36})^{(7)} G_{37}^*}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^*)]} \quad , \quad G_{38}^* = \frac{(a_{38})^{(7)} G_{37}^*}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^*)]}$$

$$T_{36}^* = \frac{(b_{36})^{(7)} T_{37}^*}{[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39})^*)]} \quad , \quad T_{38}^* = \frac{(b_{38})^{(7)} T_{37}^*}{[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39})^*)]}$$

Finally we obtain the unique solution 523

$G_{41}^*$ given by $\varphi((G_{43})^*) = 0$ , $T_{41}^*$ given by $f(T_{41}^*) = 0$ and

$$G_{40}^* = \frac{(a_{40})^{(8)} G_{41}^*}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42}^* = \frac{(a_{42})^{(8)} G_{41}^*}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

$$T_{40}^* = \frac{(b_{40})^{(8)} T_{41}^*}{[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43})^*)]} \quad , \quad T_{42}^* = \frac{(b_{42})^{(8)} T_{41}^*}{[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43})^*)]}$$

Finally we obtain the unique solution of 89 to 99 523 A

$G_{45}^*$ given by $\varphi((G_{47})^*) = 0$ , $T_{45}^*$ given by $f(T_{45}^*) = 0$ and

$$G_{44}^* = \frac{(a_{44})^{(9)} G_{45}^*}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46}^* = \frac{(a_{46})^{(9)} G_{45}^*}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

$$T_{44}^* = \frac{(b_{44})^{(9)} T_{45}^*}{[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47})^*)]} \quad , \quad T_{46}^* = \frac{(b_{46})^{(9)} T_{45}^*}{[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47})^*)]}$$

## ASYMPTOTIC STABILITY ANALYSIS 524

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

**Proof:** Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a''_{14})^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \frac{\partial (b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a'_{13})^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)}G^*_{13}\mathbb{T}_{14} \quad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a'_{14})^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)}G^*_{14}\mathbb{T}_{14} \quad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a'_{15})^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)}G^*_{15}\mathbb{T}_{14} \quad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b'_{13})^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)}T^*_{13}\mathbb{G}_j\right) \quad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b'_{14})^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)}T^*_{14}\mathbb{G}_j\right) \quad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b'_{15})^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T^*_{15}\mathbb{G}_j\right) \quad 530$$

**ASYMPTOTIC STABILITY ANALYSIS** 531

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(2)}$ and $(b''_i)^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G^*_i + \mathbb{G}_i \quad , T_i = T^*_i + \mathbb{T}_i \quad 532$$

$$\frac{\partial(a''_{17})^{(2)}}{\partial T_{17}}(T^*_{17}) = (q_{17})^{(2)} \quad , \frac{\partial(b''_i)^{(2)}}{\partial G_j}(\,(G_{19})^*\,) = s_{ij} \quad 533$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a'_{16})^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G^*_{16}\mathbb{T}_{17} \quad 534$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a'_{17})^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G^*_{17}\mathbb{T}_{17} \quad 535$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a'_{18})^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G^*_{18}\mathbb{T}_{17} \quad 536$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b'_{16})^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T^*_{16}\mathbb{G}_j\right) \quad 537$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b'_{17})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}T^*_{17}\mathbb{G}_j\right) \quad 538$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b'_{18})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}T^*_{18}\mathbb{G}_j\right) \quad 539$$

**ASYMPTOTIC STABILITY ANALYSIS** 540

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(3)}$ *and* $(b''_i)^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{21}'')^{(3)}}{\partial T_{21}}(T_{21}^*) = (q_{21})^{(3)} \quad , \frac{\partial (b_i'')^{(3)}}{\partial G_j}(\,(G_{23})^*\,) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\big((a_{20}')^{(3)} + (p_{20})^{(3)}\big)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G_{20}^*\mathbb{T}_{21} \qquad 541$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\big((a_{21}')^{(3)} + (p_{21})^{(3)}\big)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G_{21}^*\mathbb{T}_{21} \qquad 542$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\big((a_{22}')^{(3)} + (p_{22})^{(3)}\big)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G_{22}^*\mathbb{T}_{21} \qquad 543$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\big((b_{20}')^{(3)} - (r_{20})^{(3)}\big)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\big(s_{(20)(j)}T_{20}^*\mathbb{G}_j\big) \qquad 544$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\big((b_{21}')^{(3)} - (r_{21})^{(3)}\big)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\big(s_{(21)(j)}T_{21}^*\mathbb{G}_j\big) \qquad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\big((b_{22}')^{(3)} - (r_{22})^{(3)}\big)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\big(s_{(22)(j)}T_{22}^*\mathbb{G}_j\big) \qquad 546$$

**ASYMPTOTIC STABILITY ANALYSIS** 547

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}\ and\ (b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{25}'')^{(4)}}{\partial T_{25}}(T_{25}^*) = (q_{25})^{(4)} \quad , \frac{\partial (b_i'')^{(4)}}{\partial G_j}((G_{27})^*\,) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\big((a_{24}')^{(4)} + (p_{24})^{(4)}\big)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G_{24}^*\mathbb{T}_{25} \qquad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\big((a_{25}')^{(4)} + (p_{25})^{(4)}\big)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G_{25}^*\mathbb{T}_{25} \qquad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\big((a_{26}')^{(4)} + (p_{26})^{(4)}\big)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G_{26}^*\mathbb{T}_{25} \qquad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\big((b_{24}')^{(4)} - (r_{24})^{(4)}\big)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\big(s_{(24)(j)}T_{24}^*\mathbb{G}_j\big) \qquad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\big((b_{25}')^{(4)} - (r_{25})^{(4)}\big)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\big(s_{(25)(j)}T_{25}^*\mathbb{G}_j\big) \qquad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T^*_{26}\mathbb{G}_j\right) \quad 554$$

**ASYMPTOTIC STABILITY ANALYSIS** 555

**Theorem 5:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}\ and\ (b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 556

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{29})^{(5)}}{\partial T_{29}}(T^*_{29}) = (q_{29})^{(5)} \ , \ \frac{\partial(b_i'')^{(5)}}{\partial G_j}\left((G_{31})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a'_{28})^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G^*_{28}\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a'_{29})^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G^*_{29}\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a'_{30})^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G^*_{30}\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b'_{28})^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T^*_{28}\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b'_{29})^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T^*_{29}\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b'_{30})^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T^*_{30}\mathbb{G}_j\right) \quad 562$$

**ASYMPTOTIC STABILITY ANALYSIS** 563

**Theorem 6:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}\ and\ (b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{33})^{(6)}}{\partial T_{33}}(T^*_{33}) = (q_{33})^{(6)} \ , \ \frac{\partial(b_i'')^{(6)}}{\partial G_j}\left((G_{35})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a'_{32})^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G^*_{32}\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a'_{33})^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G^*_{33}\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a'_{34})^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G^*_{34}\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b'_{32})^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T^*_{32}\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b'_{33})^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T^*_{33}\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b'_{34})^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T^*_{34}\mathbb{G}_j\right) \quad 570$$

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**Theorem 7:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :- 572

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{37})^{(7)}}{\partial T_{37}}(T^*_{37}) = (q_{37})^{(7)} \quad , \frac{\partial(b_i'')^{(7)}}{\partial G_j}\left((G_{39})^{**}\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a'_{36})^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G^*_{36}\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a'_{37})^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G^*_{37}\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a'_{38})^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G^*_{38}\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b'_{36})^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T^*_{36}\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b'_{37})^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T^*_{37}\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b'_{38})^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T^*_{38}\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

**ASYMPTOTIC STABILITY ANALYSIS**

**Theorem 8:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{41}'')^{(8)}}{\partial T_{41}} (T_{41}^*) = (q_{41})^{(8)} \quad , \frac{\partial (b_i'')^{(8)}}{\partial G_j} ( (G_{43})^* ) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\big((a_{40}')^{(8)} + (p_{40})^{(8)}\big)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)} G_{40}^* \mathbb{T}_{41} \quad (581)$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\big((a_{41}')^{(8)} + (p_{41})^{(8)}\big)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)} G_{41}^* \mathbb{T}_{41} \quad (582)$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\big((a_{42}')^{(8)} + (p_{42})^{(8)}\big)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)} G_{42}^* \mathbb{T}_{41} \quad (583)$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\big((b_{40}')^{(8)} - (r_{40})^{(8)}\big)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\big(s_{(40)(j)} T_{40}^* \mathbb{G}_j\big) \quad (584)$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\big((b_{41}')^{(8)} - (r_{41})^{(8)}\big)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\big(s_{(41)(j)} T_{41}^* \mathbb{G}_j\big) \quad (585)$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\big((b_{42}')^{(8)} - (r_{42})^{(8)}\big)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\big(s_{(42)(j)} T_{42}^* \mathbb{G}_j\big) \quad (586)$$

(586 A)

## ASYMPTOTIC STABILITY ANALYSIS

**Theorem 9:** If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}$ *and* $(b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

**Proof:** Denote

**Definition of** $\mathbb{G}_i, \mathbb{T}_i$ :-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{45}'')^{(9)}}{\partial T_{45}} (T_{45}^*) = (q_{45})^{(9)} \quad , \frac{\partial (b_i'')^{(9)}}{\partial G_j} ( (G_{47})^* ) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\big((a_{44}')^{(9)} + (p_{44})^{(9)}\big)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)} G_{44}^* \mathbb{T}_{45} \quad (586\,B)$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\big((a_{45}')^{(9)} + (p_{45})^{(9)}\big)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)} G_{45}^* \mathbb{T}_{45} \quad (586\,C)$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\big((a_{46}')^{(9)} + (p_{46})^{(9)}\big)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)} G_{46}^* \mathbb{T}_{45} \quad (586\,D)$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\big((b_{44}')^{(9)} - (r_{44})^{(9)}\big)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\big(s_{(44)(j)} T_{44}^* \mathbb{G}_j\big) \quad (586\,E)$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\big((b_{45}')^{(9)} - (r_{45})^{(9)}\big)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\big(s_{(45)(j)} T_{45}^* \mathbb{G}_j\big) \quad (586\,F)$$

$\frac{d\mathbb{T}_{46}}{dt} = -\left((b'_{46})^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T^*_{46}\mathbb{G}_j\right)$ 586

G

587

**The characteristic equation of this system is**

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(q_{13})^{(1)}G^*_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)}T^*_{14} + (b_{14})^{(1)}s_{(13),(14)}T^*_{14}\right)$$

$$+\left(\left((\lambda)^{(1)} + (a'_{14})^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)}G^*_{13} + (a_{13})^{(1)}(q_{14})^{(1)}G^*_{14}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)}T^*_{14} + (b_{14})^{(1)}s_{(13),(13)}T^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((b'_{13})^{(1)} + (b'_{14})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(15)}T^*_{14} + (b_{14})^{(1)}s_{(13),(15)}T^*_{13}\right)\} = 0$$

$+$

$$(\lambda)^{(2)} + (b'_{18})^{(2)} - (r_{18})^{(2)}\{\left((\lambda)^{(2)} + (a'_{18})^{(2)} + (p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^*_{17} + (a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^*_{17}\right)$$

$$+\left(\left((\lambda)^{(2)} + (a'_{17})^{(2)} + (p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^*_{16} + (a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}\right)$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2 + \left((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2 + \left((b'_{16})^{(2)} + (b'_{17})^{(2)} - (r_{16})^{(2)} + (r_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$+\left(\left((\lambda)^{(2)}\right)^2 + \left((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17} + (a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^{*}_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^{*}_{16}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\right)\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left(\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)\right]$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(21)}T^{*}_{21}\right)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^{*}_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^{*}_{21}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(20)}T^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$+\left(\left((\lambda)^{(3)}\right)^{2}+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^{*}_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^{*}_{21}+(b_{21})^{(3)}s_{(20),(22)}T^{*}_{20}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\right)\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left(\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)\right]$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(25)}T^{*}_{25}\right)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^{*}_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^{*}_{25}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(24)}T^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$+\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(26)}T^{*}_{24}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\right)\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(29)}T^{*}_{29}\right)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^{*}_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^{*}_{29}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(28)}T^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$+\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(30)}T^{*}_{28}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\right)\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left(\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)\right]$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(33)}T^{*}_{33}\right)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^{*}_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^{*}_{33}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(32)}T^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$+\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(34)}T^{*}_{32}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\right)\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)\right]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(37)}T^{*}_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^{*}_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^{*}_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(36)}T^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$+\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(38)}T^{*}_{36}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\right)\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(41)}T^{*}_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^{*}_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^{*}_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(40)}T^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^{2}+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$+\left(\left((\lambda)^{(8)}\right)^{2}+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^{*}_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^{*}_{40}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^{*}_{41}+(b_{41})^{(8)}s_{(40),(42)}T^{*}_{40}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\right)\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(45)}T^{*}_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^{*}_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^{*}_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(44)}T^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^{2}+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$+\left(\left((\lambda)^{(9)}\right)^{2}+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^{*}_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^{*}_{44}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^{*}_{45}+(b_{45})^{(9)}s_{(44),(46)}T^{*}_{44}\right)\}=0$$

**And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.**

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